Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.5-Nested-general-binomial/78-1.1.5.3

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**104**]. This is test number [78].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (104)	0.00 (0)
Mathematica	99.04 (103)	0.96 (1)
Maple	98.08 (102)	1.92 (2)
Fricas	96.15 (100)	3.85 (4)
Reduce	66.35 (69)	33.65 (35)
Giac	58.65 (61)	41.35 (43)
Mupad	33.65 (35)	66.35 (69)
Maxima	29.81 (31)	70.19 (73)
Sympy	11.54 (12)	88.46 (92)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

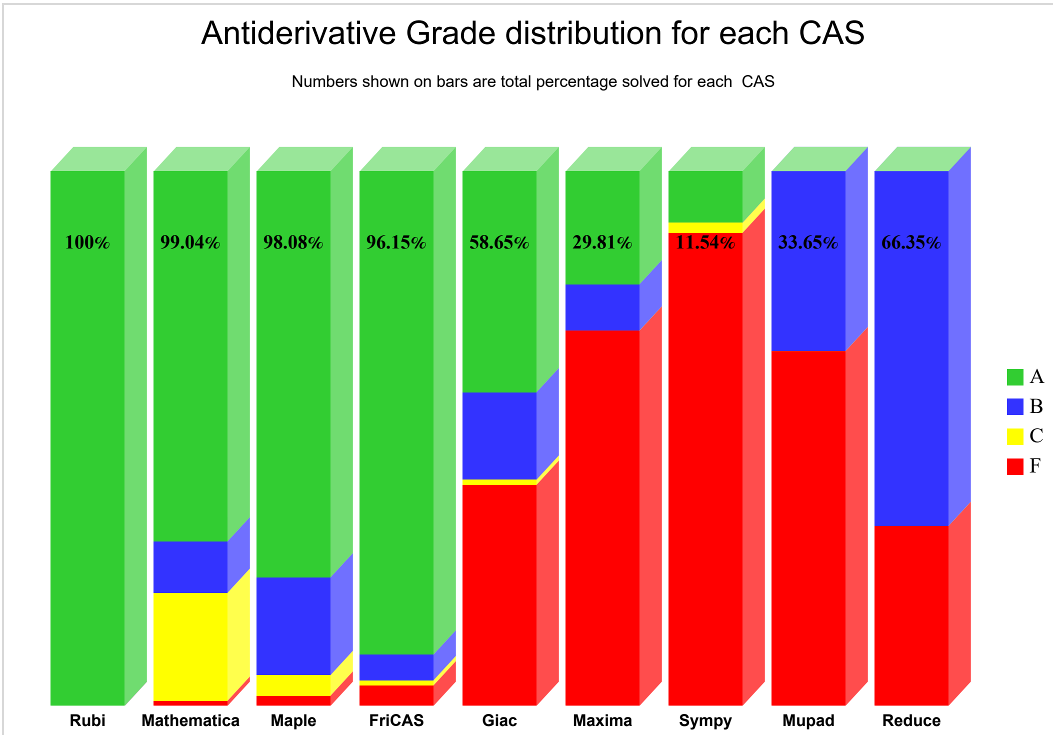
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

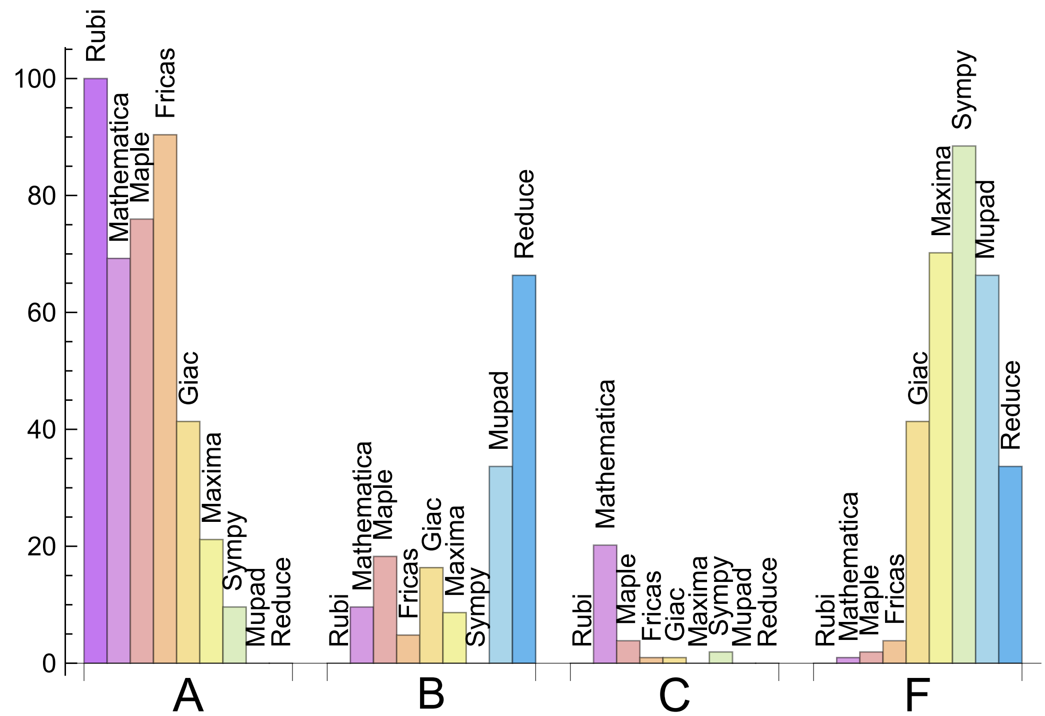
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Fricas	90.385	4.808	0.962	3.846
Maple	75.962	18.269	3.846	1.923
Mathematica	69.231	9.615	20.192	0.962
Giac	41.346	16.346	0.962	41.346
Maxima	21.154	8.654	0.000	70.192
Sympy	9.615	0.000	1.923	88.462
Mupad	0.000	33.654	0.000	66.346
Reduce	0.000	66.346	0.000	33.654

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	100.00	0.00	0.00
Maple	2	100.00	0.00	0.00
Fricas	4	50.00	25.00	25.00
Reduce	35	100.00	0.00	0.00
Giac	43	62.79	0.00	37.21
Mupad	69	0.00	100.00	0.00
Maxima	73	64.38	0.00	35.62
Sympy	92	45.65	54.35	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.07
Reduce	0.30
Fricas	0.31
Rubi	0.64
Giac	0.87
Sympy	1.02
Mathematica	1.69
Maple	1.86
Mupad	5.50

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maxima	58.84	1.35	51.00	1.15
Mupad	78.34	1.35	46.00	1.18
Sympy	90.67	1.20	64.00	0.92
Mathematica	127.19	1.06	106.00	0.85
Reduce	148.45	1.08	61.00	1.00
Rubi	169.82	1.00	115.00	1.00
Maple	242.40	1.40	153.00	1.20
Giac	259.34	2.04	51.00	1.09
Fricas	3453.48	3.88	145.50	1.10

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

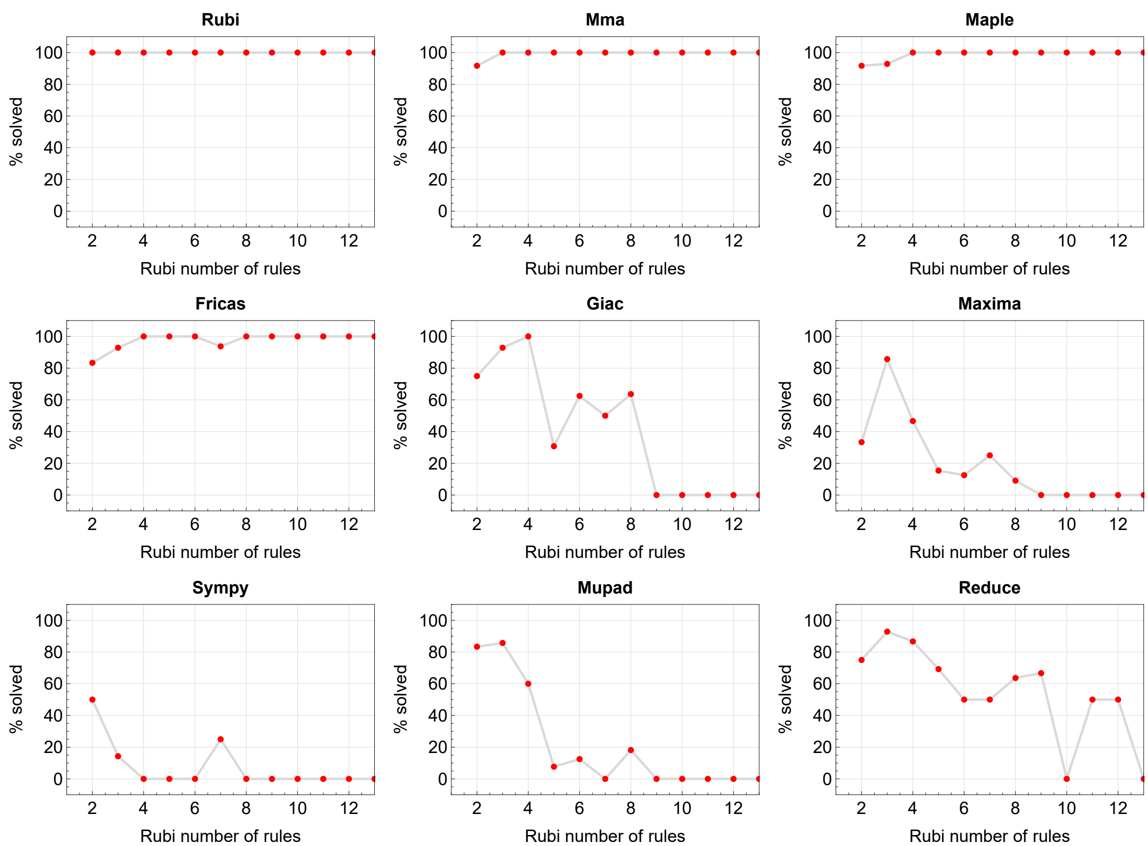


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

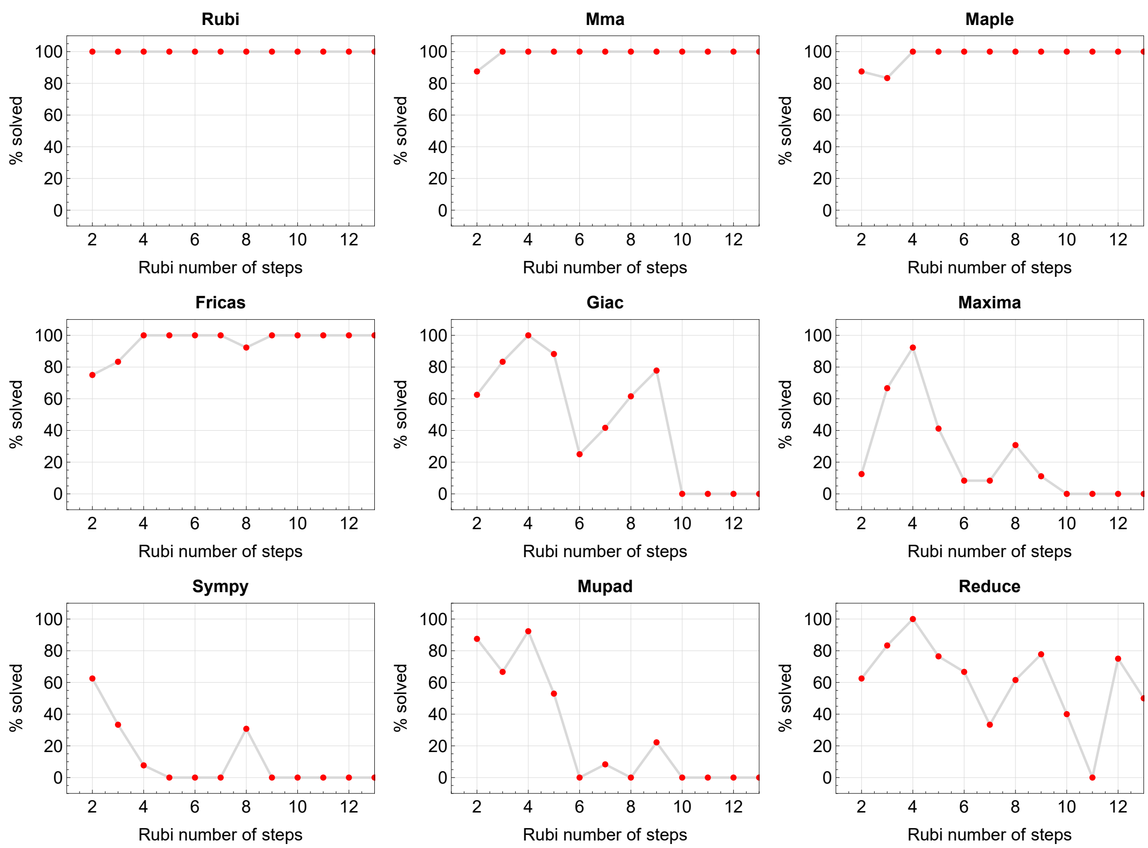


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

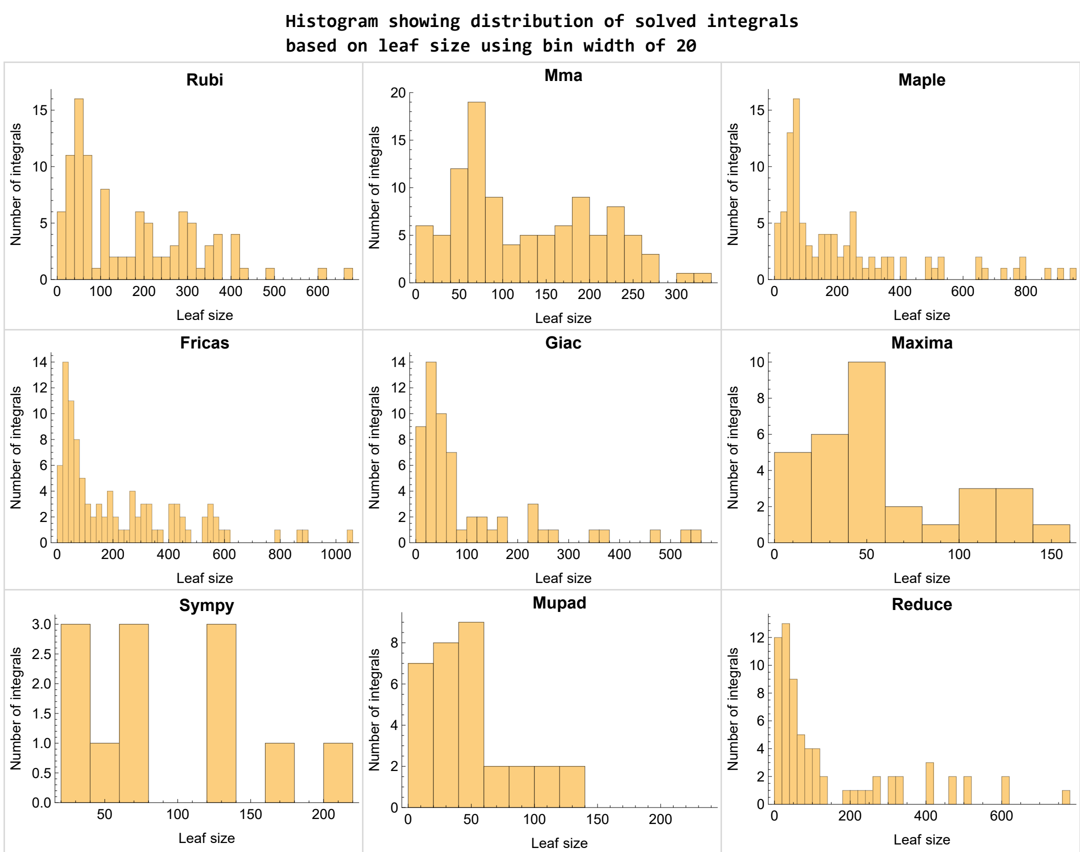


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

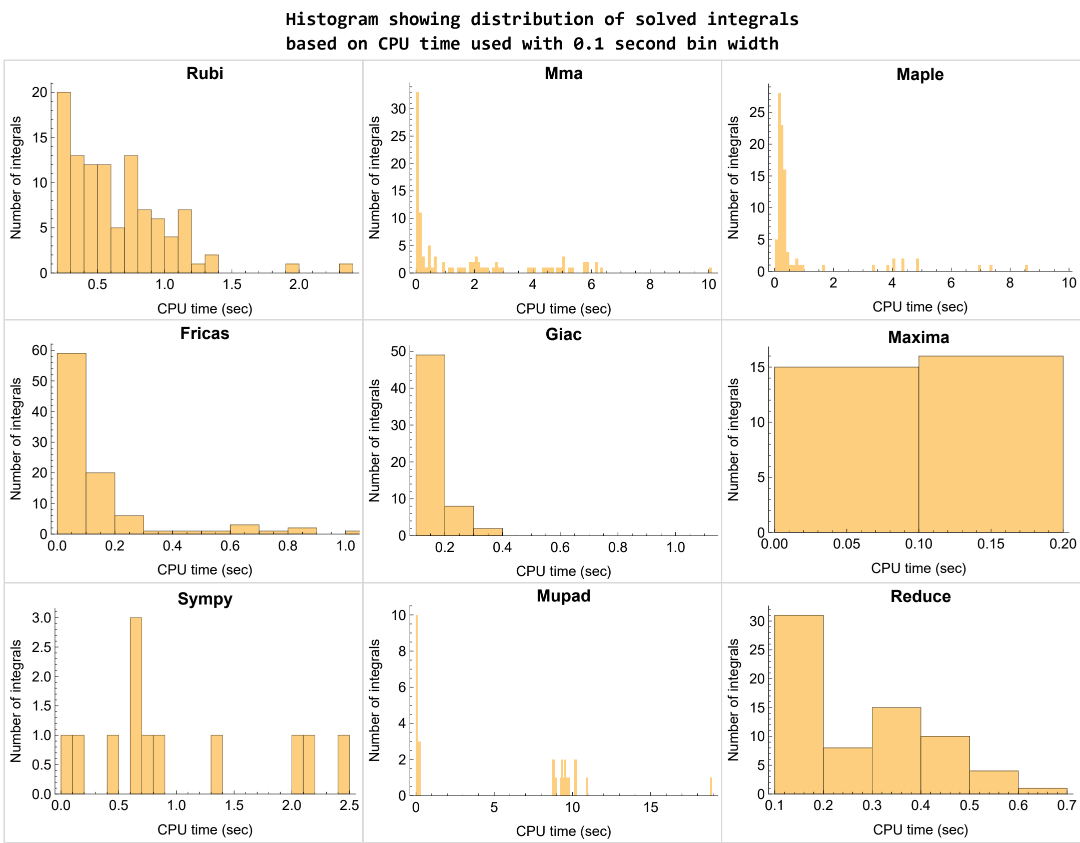


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

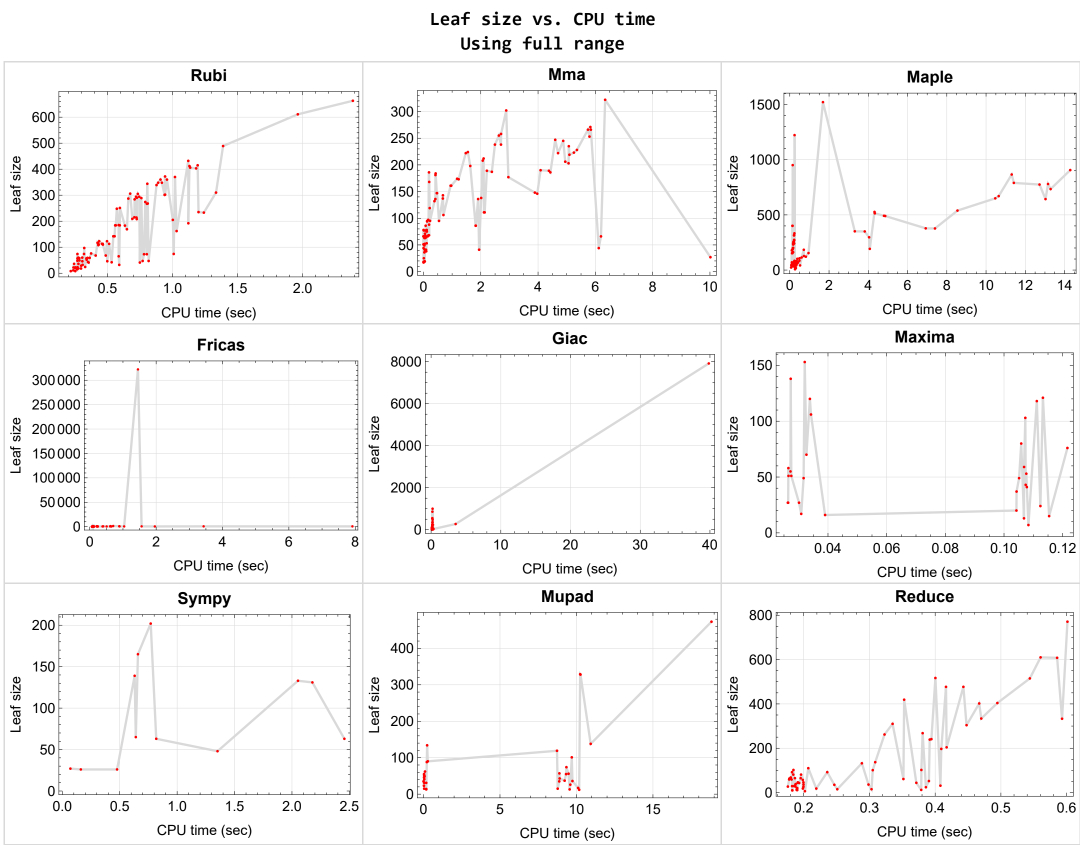


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {42, 43, 44, 46, 47, 48, 55, 56, 57, 59, 60, 61, 68, 69, 70, 71, 72, 75, 76, 77, 79, 80, 86, 87, 88, 89, 90, 91, 101}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

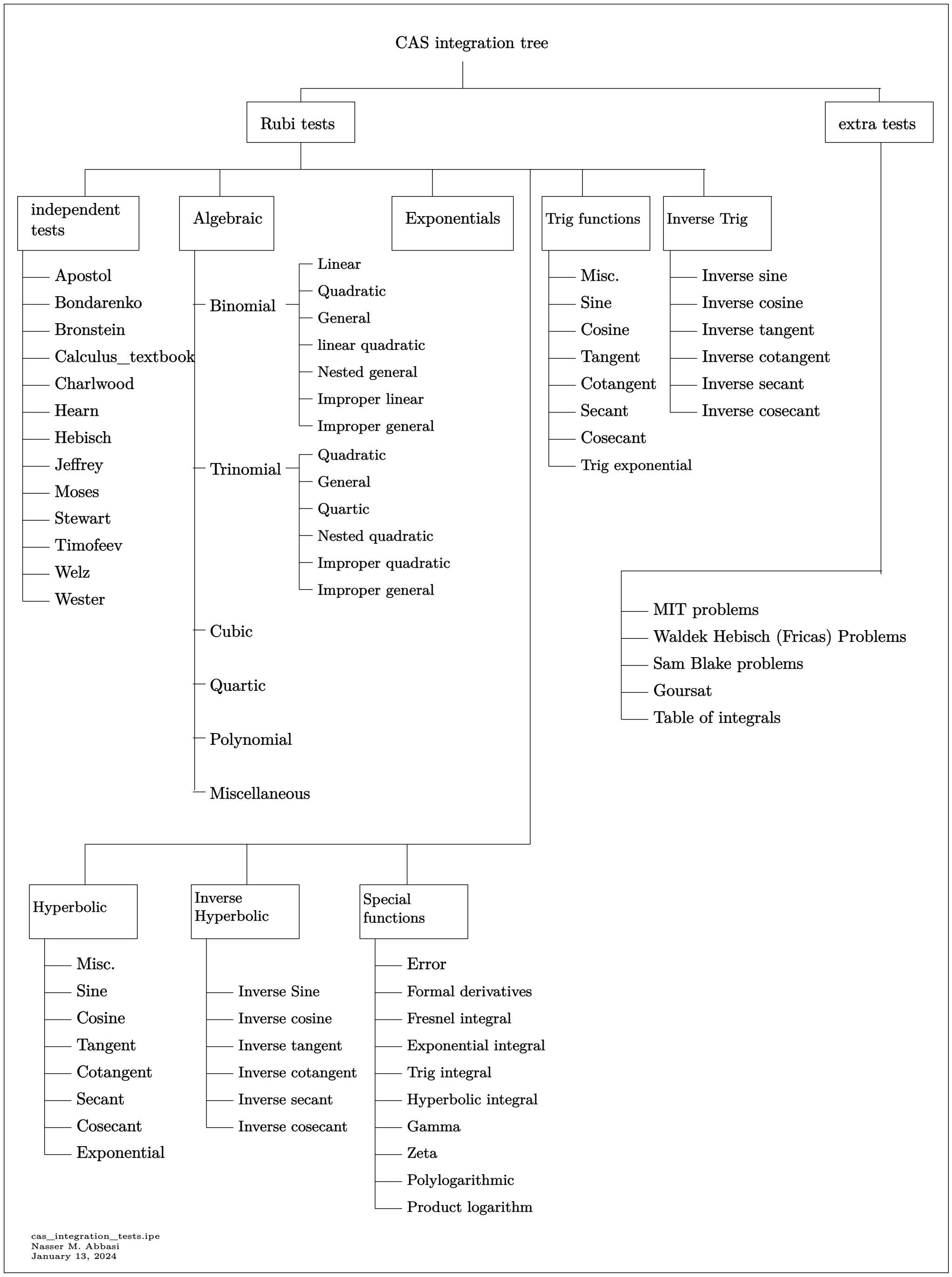
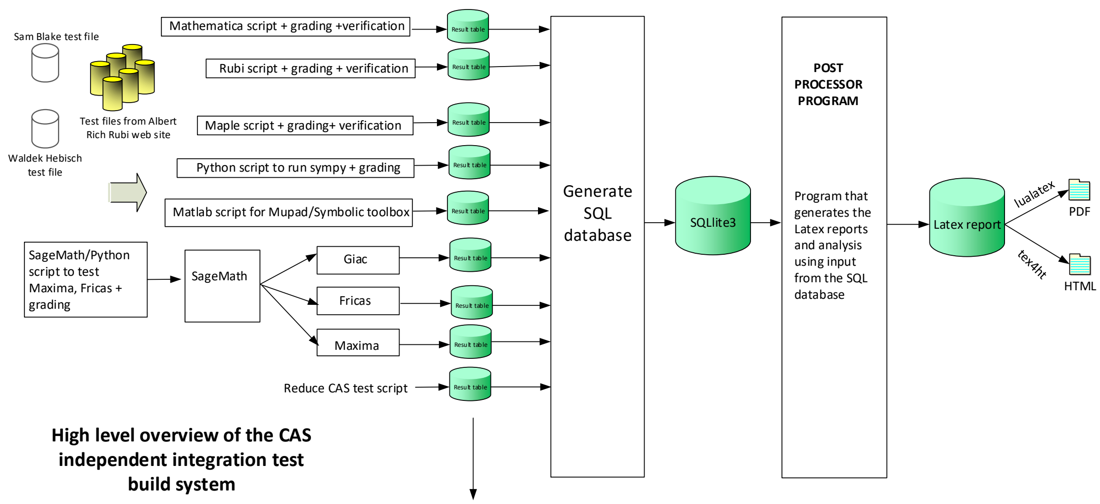


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
January 13, 2024
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 31, 32, 33, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 51, 52, 55, 56, 57, 58, 59, 60, 61, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 83, 84, 86, 87, 88, 89, 90, 91, 98, 100, 101, 102, 103, 104 }

B grade { 27, 29, 30, 34, 35, 36, 37, 68, 70, 97 }

C grade { 11, 12, 49, 50, 53, 54, 62, 63, 64, 65, 66, 67, 81, 82, 85, 92, 93, 94, 95, 96, 99 }

F normal fail { 10 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 6, 7, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 32, 33, 34, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 59, 60, 61, 62, 63, 67, 68, 69, 70, 71, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 95, 98, 101, 102, 103, 104 }

B grade { 5, 8, 9, 25, 27, 28, 29, 30, 31, 35, 37, 58, 64, 65, 66, 89, 94, 96, 97 }

C grade { 11, 12, 72, 99 }

F normal fail { 10, 100 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 36, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 101, 102, 103, 104 }

B grade { 33, 34, 35, 37, 97 }

C grade { 12 }

F normal fail { 10, 99 }

F(-1) timedout fail { 11 }

F(-2) exception fail { 100 }

Maxima

A grade { 4, 17, 21, 23, 24, 26, 27, 28, 29, 30, 31, 32, 38, 39, 40, 69, 72, 98, 101, 102, 103, 104 }

B grade { 22, 25, 33, 34, 35, 36, 37, 41, 97 }

C grade { }

F normal fail { 1, 2, 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 49, 50, 51, 52, 53, 54, 62, 63, 64, 65, 66, 67, 68, 70, 71, 73, 74, 81, 82, 83, 84, 85, 92, 93, 94, 95, 96, 99, 100 }

F(-1) timedout fail { }

F(-2) exception fail { 42, 43, 44, 45, 46, 47, 48, 55, 56, 57, 58, 59, 60, 61, 75, 76, 77, 78, 79, 80, 86, 87, 88, 89, 90, 91 }

Giac

A grade { 1, 4, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 27, 29, 30, 31, 32, 36, 38, 39, 40, 41, 42, 43, 44, 68, 69, 70, 71, 72, 74, 75, 76, 77, 98, 104 }

B grade { 2, 5, 25, 28, 33, 34, 35, 37, 46, 47, 48, 79, 80, 97, 101, 102, 103 }

C grade { 73 }

F normal fail { 3, 11, 12, 49, 50, 51, 52, 53, 54, 62, 63, 64, 65, 66, 67, 81, 82, 83, 84, 85, 92, 93, 94, 95, 96, 99, 100 }

F(-1) timedout fail { }

F(-2) exception fail { 10, 45, 55, 56, 57, 58, 59, 60, 61, 78, 86, 87, 88, 89, 90, 91 }

Mupad

A grade { }

B grade { 2, 11, 12, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 68, 69, 70, 71, 72, 97, 98 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 99, 100, 101, 102, 103, 104 }

F(-2) exception fail { }

Sympy

A grade { 11, 12, 17, 19, 20, 98, 101, 102, 103, 104 }

B grade { }

C grade { 34, 36 }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 13, 14, 15, 16, 18, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 35, 37, 38, 39, 40, 41, 68, 69, 70, 73, 74, 97, 99, 100 }

F(-1) timedout fail { 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 71, 72, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 46, 47, 48, 55, 56, 57, 59, 60, 61, 69, 73, 74, 75, 76, 77, 79, 80, 86, 87, 88, 90, 91, 97, 98, 101, 102, 103, 104 }

C grade { }

F normal fail { 10, 11, 12, 45, 49, 50, 51, 52, 53, 54, 58, 62, 63, 64, 65, 66, 67, 68, 70, 71, 72, 78, 81, 82, 83, 84, 85, 89, 92, 93, 94, 95, 96, 99, 100 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	43	36	42	0	80	0	28	24	0
N.S.	1	0.83	0.69	0.81	0.00	1.54	0.00	0.54	0.46	0.00
time (sec)	N/A	0.350	0.031	0.525	0.000	0.087	0.000	0.119	0.386	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	19	16	0	17	0	23	15	15
N.S.	1	1.00	1.73	1.45	0.00	1.55	0.00	2.09	1.36	1.36
time (sec)	N/A	0.238	0.028	0.283	0.000	0.077	0.000	0.221	0.251	8.772

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	41	40	0	34	0	0	15	0
N.S.	1	1.00	1.41	1.38	0.00	1.17	0.00	0.00	0.52	0.00
time (sec)	N/A	0.298	1.942	0.284	0.000	0.101	0.000	0.000	0.303	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	19	16	16	17	0	17	12	0
N.S.	1	1.00	1.06	0.89	0.89	0.94	0.00	0.94	0.67	0.00
time (sec)	N/A	0.297	0.041	0.293	0.039	0.075	0.000	0.138	0.379	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	38	102	0	104	0	74	36	0
N.S.	1	1.00	1.19	3.19	0.00	3.25	0.00	2.31	1.12	0.00
time (sec)	N/A	0.591	0.097	0.382	0.000	0.121	0.000	0.154	0.299	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	41	77	80	0	105	0	61	44	0
N.S.	1	0.76	1.43	1.48	0.00	1.94	0.00	1.13	0.81	0.00
time (sec)	N/A	0.749	0.071	0.362	0.000	0.098	0.000	0.138	0.371	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	74	71	62	0	141	0	46	31	0
N.S.	1	0.89	0.86	0.75	0.00	1.70	0.00	0.55	0.37	0.00
time (sec)	N/A	1.009	0.053	0.342	0.000	0.098	0.000	0.117	0.408	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	233	181	951	0	532	0	363	477	0
N.S.	1	1.07	0.83	4.36	0.00	2.44	0.00	1.67	2.19	0.00
time (sec)	N/A	1.241	0.418	0.148	0.000	0.113	0.000	0.207	0.416	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	205	184	1223	0	535	0	341	477	0
N.S.	1	0.90	0.81	5.36	0.00	2.35	0.00	1.50	2.09	0.00
time (sec)	N/A	1.002	0.425	0.243	0.000	0.100	0.000	0.190	0.443	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F	F	F(-2)	F	F(-1)
verified	N/A	Yes	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	188	310	0	0	0	0	0	0	80	0
N.S.	1	1.65	0.00	0.00	0.00	0.00	0.00	0.00	0.43	0.00
time (sec)	N/A	1.334	0.000	0.000	0.000	0.000	0.000	0.000	0.393	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F(-1)	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	490	611	66	71	0	0	131	0	26	329
N.S.	1	1.25	0.13	0.14	0.00	0.00	0.27	0.00	0.05	0.67
time (sec)	N/A	1.963	0.035	0.279	0.000	0.000	2.178	0.000	200.028	10.243

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	A	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1351	663	63	69	0	322185	133	0	67	328
N.S.	1	0.49	0.05	0.05	0.00	238.48	0.10	0.00	0.05	0.24
time (sec)	N/A	2.385	0.031	0.280	0.000	1.452	2.053	0.000	0.231	10.263

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	63	0	36	0	18	27	0
N.S.	1	1.00	0.94	1.34	0.00	0.77	0.00	0.38	0.57	0.00
time (sec)	N/A	0.816	6.119	0.340	0.000	0.078	0.000	0.135	0.176	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	63	0	36	0	18	27	0
N.S.	1	1.00	0.94	1.34	0.00	0.77	0.00	0.38	0.57	0.00
time (sec)	N/A	0.770	0.002	0.283	0.000	0.078	0.000	0.129	0.187	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	73	66	75	0	74	0	39	59	0
N.S.	1	0.70	0.63	0.71	0.00	0.70	0.00	0.37	0.56	0.00
time (sec)	N/A	0.802	6.196	0.335	0.000	0.076	0.000	0.327	0.178	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	73	66	75	0	74	0	39	59	0
N.S.	1	0.70	0.63	0.71	0.00	0.70	0.00	0.37	0.56	0.00
time (sec)	N/A	0.781	0.002	0.296	0.000	0.079	0.000	0.108	0.181	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	25	18	17	22	27	18	29	23
N.S.	1	1.00	1.09	0.78	0.74	0.96	1.17	0.78	1.26	1.00
time (sec)	N/A	0.270	0.012	0.310	0.031	0.068	0.070	0.111	0.185	0.030

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	53	45	0	44	0	57	43	31
N.S.	1	1.00	1.26	1.07	0.00	1.05	0.00	1.36	1.02	0.74
time (sec)	N/A	0.532	0.112	0.255	0.000	0.081	0.000	0.128	0.193	0.050

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	41	43	0	64	48	79	68	46
N.S.	1	1.00	0.89	0.93	0.00	1.39	1.04	1.72	1.48	1.00
time (sec)	N/A	0.499	0.089	0.089	0.000	0.101	1.352	0.122	0.180	0.038

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	68	56	0	84	63	89	63	55
N.S.	1	1.00	1.05	0.86	0.00	1.29	0.97	1.37	0.97	0.85
time (sec)	N/A	0.586	0.195	0.135	0.000	0.103	2.456	0.125	0.178	0.037

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	58	67	61	49	38	0	36	32	49
N.S.	1	1.32	1.52	1.39	1.11	0.86	0.00	0.82	0.73	1.11
time (sec)	N/A	0.284	0.071	0.162	0.105	0.100	0.000	0.135	0.187	0.072

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	60	67	62	70	58	0	40	36	51
N.S.	1	1.62	1.81	1.68	1.89	1.57	0.00	1.08	0.97	1.38
time (sec)	N/A	0.276	0.069	0.112	0.033	0.102	0.000	0.208	0.200	0.049

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	97	97	152	118	180	0	109	102	90
N.S.	1	0.95	0.95	1.49	1.16	1.76	0.00	1.07	1.00	0.88
time (sec)	N/A	0.322	0.165	0.115	0.111	0.112	0.000	0.151	0.184	0.282

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	57	64	39	43	32	0	29	26	43
N.S.	1	1.68	1.88	1.15	1.26	0.94	0.00	0.85	0.76	1.26
time (sec)	N/A	0.270	0.082	0.185	0.107	0.078	0.000	0.109	0.200	8.901

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	75	76	80	54	0	74	38	57
N.S.	1	1.00	1.53	1.55	1.63	1.10	0.00	1.51	0.78	1.16
time (sec)	N/A	0.285	0.117	0.218	0.106	0.078	0.000	0.132	0.188	8.885

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	48	52	45	37	28	0	36	20	37
N.S.	1	1.60	1.73	1.50	1.23	0.93	0.00	1.20	0.67	1.23
time (sec)	N/A	0.269	0.045	0.199	0.104	0.083	0.000	0.142	0.190	9.226

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	15	42	33	13	13	0	20	11	13
N.S.	1	1.07	3.00	2.36	0.93	0.93	0.00	1.43	0.79	0.93
time (sec)	N/A	0.267	0.030	0.194	0.107	0.071	0.000	0.159	0.190	0.180

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	88	84	53	46	0	114	81	74
N.S.	1	1.00	1.91	1.83	1.15	1.00	0.00	2.48	1.76	1.61
time (sec)	N/A	0.307	0.196	0.239	0.108	0.076	0.000	0.137	0.184	9.343

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	18	51	30	15	15	0	16	14	15
N.S.	1	1.12	3.19	1.88	0.94	0.94	0.00	1.00	0.88	0.94
time (sec)	N/A	0.276	0.030	0.279	0.115	0.086	0.000	0.133	0.189	0.045

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	63	85	24	24	0	41	29	36
N.S.	1	1.00	2.62	3.54	1.00	1.00	0.00	1.71	1.21	1.50
time (sec)	N/A	0.329	0.061	0.491	0.112	0.074	0.000	0.186	0.182	9.737

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	77	80	59	105	0	61	44	31
N.S.	1	1.00	1.88	1.95	1.44	2.56	0.00	1.49	1.07	0.76
time (sec)	N/A	0.355	0.015	0.332	0.107	0.081	0.000	0.157	0.191	0.182

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	32	19	17	27	17	0	17	12	17
N.S.	1	1.78	1.06	0.94	1.50	0.94	0.00	0.94	0.67	0.94
time (sec)	N/A	0.309	0.046	0.296	0.026	0.070	0.000	0.119	0.189	10.110

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	68	79	103	83	0	129	92	62
N.S.	1	1.09	1.26	1.46	1.91	1.54	0.00	2.39	1.70	1.15
time (sec)	N/A	0.363	0.173	0.320	0.107	0.080	0.000	0.154	0.182	0.100

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	C	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	18	7	27	18	26	14	10	14
N.S.	1	1.00	2.25	0.88	3.38	2.25	3.25	1.75	1.25	1.75
time (sec)	N/A	0.218	0.003	0.268	0.030	0.074	0.477	0.184	0.183	0.183

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	18	30	27	27	0	22	10	12
N.S.	1	1.00	2.25	3.75	3.38	3.38	0.00	2.75	1.25	1.50
time (sec)	N/A	0.253	0.004	0.095	0.026	0.071	0.000	0.134	0.182	10.178

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	51	27	49	28	63	22	18	26
N.S.	1	1.00	2.32	1.23	2.23	1.27	2.86	1.00	0.82	1.18
time (sec)	N/A	0.239	0.010	0.273	0.032	0.069	0.818	0.105	0.190	9.598

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	51	45	51	42	0	35	18	35
N.S.	1	1.00	2.32	2.05	2.32	1.91	0.00	1.59	0.82	1.59
time (sec)	N/A	0.254	0.001	0.086	0.028	0.071	0.000	0.121	0.199	8.875

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	66	43	20	39	0	42	46	138
N.S.	1	1.00	1.83	1.19	0.56	1.08	0.00	1.17	1.28	3.83
time (sec)	N/A	0.244	0.147	0.335	0.104	0.071	0.000	0.121	0.191	10.930

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	66	56	41	36	0	51	46	41
N.S.	1	1.00	1.83	1.56	1.14	1.00	0.00	1.42	1.28	1.14
time (sec)	N/A	0.277	0.002	0.167	0.108	0.085	0.000	0.122	0.200	0.054

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	74	78	60	55	46	0	47	65	473
N.S.	1	0.90	0.95	0.73	0.67	0.56	0.00	0.57	0.79	5.77
time (sec)	N/A	0.269	0.114	0.327	0.027	0.081	0.000	0.132	0.195	18.827

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	74	78	70	138	64	0	62	65	119
N.S.	1	0.90	0.95	0.85	1.68	0.78	0.00	0.76	0.79	1.45
time (sec)	N/A	0.320	0.001	0.107	0.027	0.088	0.000	0.281	0.187	8.728

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	268	198	241	0	541	0	226	334	0
N.S.	1	0.93	0.69	0.84	0.00	1.89	0.00	0.79	1.16	0.00
time (sec)	N/A	0.809	1.634	0.220	0.000	0.116	0.000	0.178	0.470	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	185	161	189	0	407	0	169	197	0
N.S.	1	0.94	0.82	0.96	0.00	2.07	0.00	0.86	1.00	0.00
time (sec)	N/A	0.586	0.948	0.145	0.000	0.104	0.000	0.161	0.409	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	110	132	154	0	313	0	138	101	0
N.S.	1	0.87	1.04	1.21	0.00	2.46	0.00	1.09	0.80	0.00
time (sec)	N/A	0.433	0.384	0.131	0.000	0.095	0.000	0.192	0.305	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	142	147	179	0	865	0	0	26	0
N.S.	1	1.04	1.08	1.32	0.00	6.36	0.00	0.00	0.19	0.00
time (sec)	N/A	0.555	0.465	0.108	0.000	0.217	0.000	0.000	200.034	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	110	137	162	0	333	0	242	132	0
N.S.	1	0.85	1.05	1.25	0.00	2.56	0.00	1.86	1.02	0.00
time (sec)	N/A	0.465	0.675	0.167	0.000	0.242	0.000	0.172	0.288	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	184	174	201	0	427	0	549	262	0
N.S.	1	0.91	0.86	1.00	0.00	2.11	0.00	2.72	1.30	0.00
time (sec)	N/A	0.595	1.176	0.172	0.000	0.509	0.000	0.180	0.323	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	267	222	257	0	561	0	1001	419	0
N.S.	1	0.91	0.76	0.87	0.00	1.91	0.00	3.40	1.43	0.00
time (sec)	N/A	0.803	1.476	0.188	0.000	1.562	0.000	0.210	0.353	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	465	372	255	490	0	273	0	0	335	0
N.S.	1	0.80	0.55	1.05	0.00	0.59	0.00	0.00	0.72	0.00
time (sec)	N/A	0.943	2.632	4.862	0.000	0.086	0.000	0.000	0.499	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	362	294	208	350	0	182	0	0	172	0
N.S.	1	0.81	0.57	0.97	0.00	0.50	0.00	0.00	0.48	0.00
time (sec)	N/A	0.720	2.068	3.819	0.000	0.098	0.000	0.000	0.426	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	248	86	184	0	150	0	0	31	0
N.S.	1	1.08	0.37	0.80	0.00	0.65	0.00	0.00	0.13	0.00
time (sec)	N/A	0.572	1.826	0.714	0.000	0.085	0.000	0.000	0.252	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	287	111	192	0	132	0	0	35	0
N.S.	1	1.21	0.47	0.81	0.00	0.56	0.00	0.00	0.15	0.00
time (sec)	N/A	0.660	2.136	4.077	0.000	0.095	0.000	0.000	0.313	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	348	238	378	0	199	0	0	136	0
N.S.	1	1.09	0.74	1.18	0.00	0.62	0.00	0.00	0.42	0.00
time (sec)	N/A	0.890	2.705	6.926	0.000	0.090	0.000	0.000	0.853	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	432	302	539	0	289	0	0	35	0
N.S.	1	1.03	0.72	1.28	0.00	0.69	0.00	0.00	0.08	0.00
time (sec)	N/A	1.121	2.889	8.543	0.000	0.096	0.000	0.000	1.736	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	302	222	313	0	553	0	0	608	0
N.S.	1	0.89	0.66	0.93	0.00	1.64	0.00	0.00	1.80	0.00
time (sec)	N/A	0.943	4.698	0.216	0.000	0.897	0.000	0.000	0.586	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	215	177	252	0	417	0	0	402	0
N.S.	1	0.87	0.72	1.02	0.00	1.69	0.00	0.00	1.63	0.00
time (sec)	N/A	0.707	2.964	0.198	0.000	0.384	0.000	0.000	0.467	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	123	136	231	0	328	0	0	241	0
N.S.	1	0.70	0.77	1.31	0.00	1.86	0.00	0.00	1.37	0.00
time (sec)	N/A	0.437	1.908	0.171	0.000	0.216	0.000	0.000	0.394	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	183	187	401	0	1049	0	0	26	0
N.S.	1	0.98	1.00	2.14	0.00	5.61	0.00	0.00	0.14	0.00
time (sec)	N/A	0.634	2.387	0.136	0.000	0.622	0.000	0.000	200.052	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	123	146	239	0	350	0	0	304	0
N.S.	1	0.69	0.82	1.34	0.00	1.96	0.00	0.00	1.70	0.00
time (sec)	N/A	0.497	3.972	0.204	0.000	0.650	0.000	0.000	0.448	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	214	186	270	0	435	0	0	515	0
N.S.	1	0.85	0.74	1.07	0.00	1.73	0.00	0.00	2.04	0.00
time (sec)	N/A	0.723	4.430	0.227	0.000	1.960	0.000	0.000	0.544	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	301	245	333	0	573	0	0	771	0
N.S.	1	0.87	0.71	0.96	0.00	1.66	0.00	0.00	2.23	0.00
time (sec)	N/A	0.939	4.878	0.236	0.000	7.915	0.000	0.000	0.601	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	513	403	266	775	0	275	0	0	935	0
N.S.	1	0.79	0.52	1.51	0.00	0.54	0.00	0.00	1.82	0.00
time (sec)	N/A	1.181	5.742	12.724	0.000	0.091	0.000	0.000	1.133	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	339	235	734	0	229	0	0	890	0
N.S.	1	0.79	0.55	1.71	0.00	0.54	0.00	0.00	2.08	0.00
time (sec)	N/A	0.877	5.074	13.289	0.000	0.090	0.000	0.000	0.989	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	305	206	527	0	173	0	0	348	0
N.S.	1	1.23	0.83	2.13	0.00	0.70	0.00	0.00	1.41	0.00
time (sec)	N/A	0.733	4.949	4.317	0.000	0.091	0.000	0.000	0.816	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	311	360	228	670	0	170	0	0	714	0
N.S.	1	1.16	0.73	2.15	0.00	0.55	0.00	0.00	2.30	0.00
time (sec)	N/A	0.957	5.357	10.644	0.000	0.107	0.000	0.000	0.904	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	395	415	253	790	0	214	0	0	0	0
N.S.	1	1.05	0.64	2.00	0.00	0.54	0.00	0.00	0.00	0.00
time (sec)	N/A	1.193	5.794	11.415	0.000	0.084	0.000	0.000	1.863	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	488	489	322	906	0	293	0	0	0	0
N.S.	1	1.00	0.66	1.86	0.00	0.60	0.00	0.00	0.00	0.00
time (sec)	N/A	1.388	6.342	14.290	0.000	0.086	0.000	0.000	2.739	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	57	95	52	0	55	0	18	44	55
N.S.	1	1.33	2.21	1.21	0.00	1.28	0.00	0.42	1.02	1.28
time (sec)	N/A	0.338	0.129	0.328	0.000	0.075	0.000	0.125	0.314	9.320

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	76	119	78	76	76	0	30	52	88
N.S.	1	1.13	1.78	1.16	1.13	1.13	0.00	0.45	0.78	1.31
time (sec)	N/A	0.374	0.217	0.352	0.122	0.082	0.000	0.134	0.390	0.216

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	F	A	F	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	59	95	68	0	55	0	22	27	56
N.S.	1	1.31	2.11	1.51	0.00	1.22	0.00	0.49	0.60	1.24
time (sec)	N/A	0.346	0.229	0.421	0.000	0.070	0.000	0.164	0.285	9.483

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	108	95	80	0	65	0	57	84	101
N.S.	1	1.11	0.98	0.82	0.00	0.67	0.00	0.59	0.87	1.04
time (sec)	N/A	0.430	0.542	0.315	0.000	0.074	0.000	0.126	0.333	9.695

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-1)	A	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	117	106	114	121	82	0	47	78	134
N.S.	1	1.15	1.04	1.12	1.19	0.80	0.00	0.46	0.76	1.31
time (sec)	N/A	0.424	0.697	0.604	0.113	0.076	0.000	0.137	0.285	0.238

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	C	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	52	49	42	0	32	0	40	35	0
N.S.	1	1.11	1.04	0.89	0.00	0.68	0.00	0.85	0.74	0.00
time (sec)	N/A	0.340	0.032	0.309	0.000	0.072	0.000	0.143	0.247	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	68	65	60	0	42	0	61	61	0
N.S.	1	0.84	0.80	0.74	0.00	0.52	0.00	0.75	0.75	0.00
time (sec)	N/A	0.409	0.044	0.304	0.000	0.079	0.000	0.121	0.352	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	274	224	243	0	545	0	235	333	0
N.S.	1	0.92	0.75	0.82	0.00	1.83	0.00	0.79	1.12	0.00
time (sec)	N/A	0.791	1.550	0.168	0.000	0.118	0.000	0.264	0.593	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	184	161	191	0	413	0	172	204	0
N.S.	1	0.90	0.79	0.93	0.00	2.01	0.00	0.84	1.00	0.00
time (sec)	N/A	0.559	0.969	0.149	0.000	0.104	0.000	0.170	0.417	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	113	136	154	0	313	0	142	102	0
N.S.	1	0.87	1.05	1.18	0.00	2.41	0.00	1.09	0.78	0.00
time (sec)	N/A	0.429	0.415	0.136	0.000	0.094	0.000	0.161	0.379	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	142	147	179	0	881	0	0	26	0
N.S.	1	1.04	1.08	1.32	0.00	6.48	0.00	0.00	0.19	0.00
time (sec)	N/A	0.545	0.474	0.108	0.000	0.211	0.000	0.000	200.038	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	113	143	162	0	333	0	238	137	0
N.S.	1	0.85	1.08	1.22	0.00	2.50	0.00	1.79	1.03	0.00
time (sec)	N/A	0.458	0.679	0.164	0.000	0.216	0.000	0.176	0.309	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	184	173	200	0	443	0	536	268	0
N.S.	1	0.87	0.82	0.95	0.00	2.10	0.00	2.54	1.27	0.00
time (sec)	N/A	0.564	1.227	0.160	0.000	0.614	0.000	0.178	0.381	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	415	370	258	492	0	276	0	0	338	0
N.S.	1	0.89	0.62	1.19	0.00	0.67	0.00	0.00	0.81	0.00
time (sec)	N/A	1.019	2.708	4.800	0.000	0.096	0.000	0.000	0.710	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	294	212	351	0	185	0	0	175	0
N.S.	1	0.94	0.68	1.12	0.00	0.59	0.00	0.00	0.56	0.00
time (sec)	N/A	0.744	2.100	3.312	0.000	0.089	0.000	0.000	0.585	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	251	86	127	0	130	0	0	34	0
N.S.	1	1.09	0.37	0.55	0.00	0.57	0.00	0.00	0.15	0.00
time (sec)	N/A	0.595	1.821	0.720	0.000	0.092	0.000	0.000	0.258	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	284	111	297	0	156	0	0	38	0
N.S.	1	1.20	0.47	1.25	0.00	0.66	0.00	0.00	0.16	0.00
time (sec)	N/A	0.709	2.111	4.041	0.000	0.085	0.000	0.000	0.306	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	348	238	377	0	206	0	0	139	0
N.S.	1	0.94	0.64	1.02	0.00	0.56	0.00	0.00	0.38	0.00
time (sec)	N/A	0.918	2.494	7.398	0.000	0.088	0.000	0.000	0.981	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	289	247	318	0	781	0	0	610	0
N.S.	1	0.83	0.71	0.91	0.00	2.23	0.00	0.00	1.74	0.00
time (sec)	N/A	0.760	4.599	0.220	0.000	0.889	0.000	0.000	0.560	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	208	190	257	0	585	0	0	404	0
N.S.	1	0.83	0.76	1.03	0.00	2.34	0.00	0.00	1.62	0.00
time (sec)	N/A	0.725	4.088	0.197	0.000	0.406	0.000	0.000	0.495	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	113	138	234	0	443	0	0	239	0
N.S.	1	0.62	0.76	1.29	0.00	2.45	0.00	0.00	1.32	0.00
time (sec)	N/A	0.463	2.015	0.184	0.000	0.242	0.000	0.000	0.392	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	169	189	401	0	1293	0	0	26	0
N.S.	1	0.90	1.01	2.13	0.00	6.88	0.00	0.00	0.14	0.00
time (sec)	N/A	0.650	2.213	0.139	0.000	0.706	0.000	0.000	200.032	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	113	148	242	0	469	0	0	310	0
N.S.	1	0.61	0.80	1.32	0.00	2.55	0.00	0.00	1.68	0.00
time (sec)	N/A	0.513	3.888	0.211	0.000	1.036	0.000	0.000	0.335	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	F(-2)	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	209	189	274	0	613	0	0	517	0
N.S.	1	0.81	0.74	1.07	0.00	2.39	0.00	0.00	2.01	0.00
time (sec)	N/A	0.691	4.378	0.221	0.000	3.431	0.000	0.000	0.400	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	477	406	271	780	0	425	0	0	937	0
N.S.	1	0.85	0.57	1.64	0.00	0.89	0.00	0.00	1.96	0.00
time (sec)	N/A	1.135	5.825	13.158	0.000	0.092	0.000	0.000	1.033	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	344	219	643	0	305	0	0	892	0
N.S.	1	0.91	0.58	1.69	0.00	0.80	0.00	0.00	2.35	0.00
time (sec)	N/A	0.806	5.095	13.030	0.000	0.120	0.000	0.000	0.902	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	306	203	514	0	251	0	0	350	0
N.S.	1	1.22	0.81	2.05	0.00	1.00	0.00	0.00	1.39	0.00
time (sec)	N/A	0.675	5.064	4.332	0.000	0.118	0.000	0.000	0.735	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	379	360	223	650	0	269	0	0	716	0
N.S.	1	0.95	0.59	1.72	0.00	0.71	0.00	0.00	1.89	0.00
time (sec)	N/A	0.907	5.250	10.470	0.000	0.127	0.000	0.000	0.852	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	412	266	866	0	374	0	0	0	0
N.S.	1	0.91	0.58	1.90	0.00	0.82	0.00	0.00	0.00	0.00
time (sec)	N/A	1.130	5.843	11.307	0.000	0.112	0.000	0.000	1.662	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	51	45	51	42	0	35	18	35
N.S.	1	1.00	2.32	2.05	2.32	1.91	0.00	1.59	0.82	1.59
time (sec)	N/A	0.251	0.012	0.082	0.027	0.098	0.000	0.332	0.219	0.002

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	17	23	7	22	26	15	6	13
N.S.	1	1.00	0.85	1.15	0.35	1.10	1.30	0.75	0.30	0.65
time (sec)	N/A	0.252	0.005	0.069	0.108	0.091	0.161	0.286	0.202	9.553

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	290	27	1522	0	0	0	0	19	0
N.S.	1	1.01	0.09	5.32	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	0.726	10.017	1.690	0.000	0.000	0.000	0.000	0.233	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	60	38	0	0	0	0	0	21	0
N.S.	1	1.30	0.83	0.00	0.00	0.00	0.00	0.00	0.46	0.00
time (sec)	N/A	0.306	0.050	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	68	86	59	58	57	65	474	57	0
N.S.	1	0.83	1.05	0.72	0.71	0.70	0.79	5.78	0.70	0.00
time (sec)	N/A	0.487	0.076	0.376	0.027	0.106	0.640	0.218	0.199	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	162	94	107	106	62	139	859	81	0
N.S.	1	1.01	0.59	0.67	0.66	0.39	0.87	5.37	0.51	0.00
time (sec)	N/A	1.031	0.128	0.496	0.034	0.078	0.630	0.184	0.196	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	192	168	121	120	76	165	7916	92	0
N.S.	1	1.01	0.88	0.64	0.63	0.40	0.87	41.66	0.48	0.00
time (sec)	N/A	1.122	0.200	0.819	0.034	0.108	0.659	39.833	0.236	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	235	186	154	153	85	202	271	110	0
N.S.	1	1.01	0.80	0.66	0.66	0.36	0.87	1.16	0.47	0.00
time (sec)	N/A	1.200	0.193	0.954	0.032	0.125	0.770	3.519	0.207	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [67] had the largest ratio of [.5000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	0.83	23	0.130
2	A	2	2	1.00	15	0.133
3	A	6	5	1.00	21	0.238
4	A	6	5	1.00	20	0.250
5	A	3	2	1.00	27	0.074
6	A	5	4	0.76	38	0.105
7	A	5	4	0.89	23	0.174
8	A	9	8	1.07	26	0.308
9	A	8	7	0.90	26	0.269
10	A	2	2	1.65	24	0.083
11	A	2	2	1.25	24	0.083
12	A	2	2	0.49	23	0.087
13	A	7	6	1.00	27	0.222
14	A	8	7	1.00	25	0.280
15	A	8	7	0.70	25	0.280
16	A	9	8	0.70	23	0.348
17	A	2	2	1.00	15	0.133
18	A	2	2	1.00	27	0.074
19	A	2	2	1.00	17	0.118
20	A	2	2	1.00	20	0.100
21	A	4	3	1.32	15	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	4	3	1.62	15	0.200
23	A	4	3	0.95	17	0.176
24	A	4	3	1.68	15	0.200
25	A	5	4	1.00	15	0.267
26	A	4	3	1.60	12	0.250
27	A	3	2	1.07	16	0.125
28	A	5	4	1.00	21	0.190
29	A	3	2	1.12	21	0.095
30	A	4	3	1.00	26	0.115
31	A	4	3	1.00	25	0.120
32	A	4	3	1.78	22	0.136
33	A	9	8	1.09	20	0.400
34	A	3	2	1.00	13	0.154
35	A	4	3	1.00	15	0.200
36	A	4	3	1.00	13	0.231
37	A	5	4	1.00	11	0.364
38	A	4	3	1.00	18	0.167
39	A	5	4	1.00	17	0.235
40	A	4	4	0.90	18	0.222
41	A	5	5	0.90	17	0.294
42	A	9	8	0.93	26	0.308
43	A	8	7	0.94	26	0.269
44	A	5	4	0.87	24	0.167
45	A	6	5	1.04	26	0.192
46	A	5	4	0.85	26	0.154
47	A	7	6	0.91	26	0.231
48	A	9	8	0.91	26	0.308
49	A	7	7	0.80	26	0.269
50	A	6	6	0.81	26	0.231
51	A	5	5	1.08	22	0.227
52	A	7	7	1.21	26	0.269
53	A	8	8	1.09	26	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	10	10	1.03	26	0.385
55	A	13	12	0.89	26	0.462
56	A	9	8	0.87	26	0.308
57	A	6	5	0.70	24	0.208
58	A	8	7	0.98	26	0.269
59	A	6	5	0.69	26	0.192
60	A	10	9	0.85	26	0.346
61	A	12	11	0.87	26	0.423
62	A	11	11	0.79	26	0.423
63	A	8	8	0.79	26	0.308
64	A	7	7	1.23	22	0.318
65	A	9	9	1.16	26	0.346
66	A	11	11	1.05	26	0.423
67	A	13	13	1.00	26	0.500
68	A	5	4	1.33	21	0.190
69	A	5	4	1.13	23	0.174
70	A	5	4	1.31	23	0.174
71	A	9	8	1.11	23	0.348
72	A	7	6	1.15	25	0.240
73	A	6	5	1.11	23	0.217
74	A	6	5	0.84	28	0.179
75	A	9	8	0.92	26	0.308
76	A	7	6	0.90	26	0.231
77	A	5	4	0.87	24	0.167
78	A	6	5	1.04	26	0.192
79	A	5	4	0.85	26	0.154
80	A	7	6	0.87	26	0.231
81	A	8	8	0.89	26	0.308
82	A	6	6	0.94	26	0.231
83	A	5	5	1.09	22	0.227
84	A	7	7	1.20	26	0.269
85	A	10	10	0.94	26	0.385

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	10	9	0.83	26	0.346
87	A	12	11	0.83	26	0.423
88	A	6	5	0.62	24	0.208
89	A	7	6	0.90	26	0.231
90	A	6	5	0.61	26	0.192
91	A	12	11	0.81	26	0.423
92	A	10	10	0.85	26	0.385
93	A	7	7	0.91	26	0.269
94	A	7	7	1.22	22	0.318
95	A	11	11	0.95	26	0.423
96	A	12	12	0.91	26	0.462
97	A	5	4	1.00	11	0.364
98	A	3	3	1.00	15	0.200
99	A	8	7	1.01	15	0.467
100	A	3	3	1.30	15	0.200
101	A	8	7	0.83	23	0.304
102	A	8	7	1.01	21	0.333
103	A	8	7	1.01	23	0.304
104	A	8	7	1.01	25	0.280

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \frac{\sqrt{(-1+x)^3}}{x(2-3x+x^2)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \frac{\sqrt{(-1+x)^3}}{x(2-3x+x^2)}\,dx = \frac{\sqrt{(-1+x)^3}\arctan\left(\sqrt{-1+x}\right)}{(-1+x)^{3/2}} - \frac{\sqrt{(-1+x)^3}\operatorname{arctanh}\left(\sqrt{-1+x}\right)}{(-1+x)^{3/2}}$$

output
```
((-1+x)^3)^(1/2)*arctan((-1+x)^(1/2))/(-1+x)^(3/2)-((-1+x)^3)^(1/2)*arctan
h((-1+x)^(1/2))/(-1+x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{(-1+x)^3}}{x(2-3x+x^2)}\,dx = \frac{\sqrt{(-1+x)^3}\left(\arctan\left(\sqrt{-1+x}\right) - \operatorname{arctanh}\left(\sqrt{-1+x}\right)\right)}{(-1+x)^{3/2}}$$

input
```
Integrate[Sqrt[(-1 + x)^3]/(x*(2 - 3*x + x^2)),x]
```

output
```
(Sqrt[(-1 + x)^3]*(ArcTan[Sqrt[-1 + x]] - ArcTanh[Sqrt[-1 + x]]))/(-1 + x)
^(3/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {2008, 1199, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{(x-1)^3}}{x\left(x^2-3x+2\right)}dx$$

$$\Big\downarrow \text{2008}$$

$$\frac{\sqrt{(x-1)^3}\int \frac{(x-1)^{3/2}}{x(x^2-3x+2)}dx}{(x-1)^{3/2}}$$

$$\Big\downarrow \text{1199}$$

$$\frac{2\sqrt{(x-1)^3}\int \left(\frac{1}{2x}-\frac{1}{2(2-x)}\right)d\sqrt{x-1}}{(x-1)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{(x-1)^3}\left(\frac{1}{2}\arctan\left(\sqrt{x-1}\right)-\frac{1}{2}\text{arctanh}\left(\sqrt{x-1}\right)\right)}{(x-1)^{3/2}}$$

input
```
Int[Sqrt[(-1 + x)^3]/(x*(2 - 3*x + x^2)),x]
```

output
```
(2*Sqrt[(-1 + x)^3]*(ArcTan[Sqrt[-1 + x]]/2 - ArcTanh[Sqrt[-1 + x]]/2))/(-
1 + x)^(3/2)
```

Defintions of rubi rules used

rule 1199
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_))/((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2), x_Symbol] :> With[{q = Denominator[m]}, Simp[q/e   Subs
t[Int[ExpandIntegrand[x^(q*(m + 1) - 1)*(((e*f - d*g)/e + g*(x^q/e))^n/((c*
d^2 - b*d*e + a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))), x],
 x], x, (d + e*x)^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && Integer
Q[n] && FractionQ[m]
```

rule 2008
```
Int[(u_.)*(Px_)^(p_), x_Symbol] :> With[{a = Rt[Coeff[Px, x, 0], Expon[Px,
x]], b = Rt[Coeff[Px, x, Expon[Px, x]], Expon[Px, x]]}, Simp[((a + b*x)^Exp
on[Px, x])^p/(a + b*x)^(Expon[Px, x]*p)   Int[u*(a + b*x)^(Expon[Px, x]*p),
 x], x] /; EqQ[Px, (a + b*x)^Expon[Px, x]]] /;  !IntegerQ[p] && PolyQ[Px, x
] && GtQ[Expon[Px, x], 1] && NeQ[Coeff[Px, x, 0], 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

method	result
default	$\frac{\sqrt{(x-1)^{3}}\,\left(\ln\left(\sqrt{x-1}-1\right)+2\arctan\left(\sqrt{x-1}\right)-\ln\left(1+\sqrt{x-1}\right)\right)}{2(x-1)^{\frac{3}{2}}}$
trager	$\frac{\ln\left(\frac{-x^{2}+2\sqrt{x^{3}-3x^{2}+3x-1}+x}{(x-2)(x-1)}\right)}{2}+\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\frac{-\operatorname{RootOf}\left(\_Z^{2}+1\right)x^{2}+3\operatorname{RootOf}\left(\_Z^{2}+1\right)x+2\sqrt{x^{3}-3x^{2}+3x-1}-2\,\text{RootC}}{x(x-1)}\right.}{2}$

input
```
int(((x-1)^3)^(1/2)/x/(x^2-3*x+2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((x-1)^3)^(1/2)*(ln((x-1)^(1/2)-1)+2*arctan((x-1)^(1/2))-ln(1+(x-1)^(1
/2)))/(x-1)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.54

$$\int \frac{\sqrt{(-1+x)^3}}{x(2-3x+x^2)}\,dx = \arctan\left(\frac{\sqrt{x^3-3x^2+3x-1}}{x-1}\right) - \frac{1}{2}\log\left(\frac{x+\sqrt{x^3-3x^2+3x-1}-1}{x-1}\right) + \frac{1}{2}\log\left(-\frac{x-\sqrt{x^3-3x^2+3x-1}-1}{x-1}\right)$$

input
```
integrate(((x-1)^3)^(1/2)/x/(x^2-3*x+2),x, algorithm="fricas")
```

output
```
arctan(sqrt(x^3 - 3*x^2 + 3*x - 1)/(x - 1)) - 1/2*log((x + sqrt(x^3 - 3*x^
2 + 3*x - 1) - 1)/(x - 1)) + 1/2*log(-(x - sqrt(x^3 - 3*x^2 + 3*x - 1) - 1
)/(x - 1))
```

Sympy [F]

$$\int \frac{\sqrt{(-1+x)^3}}{x(2-3x+x^2)}\,dx = \int \frac{\sqrt{(x-1)^3}}{x(x-2)(x-1)}\,dx$$

input
```
integrate(((x-1)**3)**(1/2)/x/(x**2-3*x+2),x)
```

output
```
Integral(sqrt((x - 1)**3)/(x*(x - 2)*(x - 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{(-1+x)^3}}{x(2-3x+x^2)}\,dx = \int \frac{\sqrt{(x-1)^3}}{(x^2-3x+2)x}\,dx$$

input
```
integrate(((x-1)^3)^(1/2)/x/(x^2-3*x+2),x, algorithm="maxima")
```

output
```
integrate(sqrt((x - 1)^3)/((x^2 - 3*x + 2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.54

$$\int \frac{\sqrt{(-1+x)^3}}{x(2-3x+x^2)}\,dx = \arctan\left(\sqrt{x-1}\right) - \frac{1}{2}\log\left(\sqrt{x-1}+1\right) + \frac{1}{2}\log\left(\left|\sqrt{x-1}-1\right|\right)$$

input
```
integrate(((x-1)^3)^(1/2)/x/(x^2-3*x+2),x, algorithm="giac")
```

output
```
arctan(sqrt(x - 1)) - 1/2*log(sqrt(x - 1) + 1) + 1/2*log(abs(sqrt(x - 1) -
 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{(-1+x)^3}}{x(2-3x+x^2)}\,dx = \int \frac{\sqrt{(x-1)^3}}{x\,(x^2-3x+2)}\,dx$$

input
```
int(((x - 1)^3)^(1/2)/(x*(x^2 - 3*x + 2)),x)
```

output
```
int(((x - 1)^3)^(1/2)/(x*(x^2 - 3*x + 2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.46

$$\int \frac{\sqrt{(-1+x)^3}}{x(2-3x+x^2)}\,dx = atan\left(\sqrt{x-1}\right) + \frac{\log\left(\sqrt{x-1}-1\right)}{2} - \frac{\log\left(\sqrt{x-1}+1\right)}{2}$$

input
```
int(((x-1)^3)^(1/2)/x/(x^2-3*x+2),x)
```

output
```
(2*atan(sqrt(x - 1)) + log(sqrt(x - 1) - 1) - log(sqrt(x - 1) + 1))/2
```

3.2 $\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} \, dx = \frac{2}{\sqrt{1+\frac{1}{x}}}$$

output
```
2/(1+1/x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2} \, dx = \frac{2x\sqrt{\frac{1+x}{x}}}{1+x}$$

input
```
Integrate[Sqrt[1 + x^(-1)]/(1 + x)^2,x]
```

output
```
(2*x*Sqrt[(1 + x)/x])/(1 + x)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {941, 793}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x}+1}}{(x+1)^2}\,dx$$
$$\Big\downarrow \; 941$$
$$\int \frac{1}{\left(\frac{1}{x}+1\right)^{3/2} x^2}dx$$
$$\Big\downarrow \; 793$$
$$\frac{2}{\sqrt{\frac{1}{x}+1}}$$

input
```
Int[Sqrt[1 + x^(-1)]/(1 + x)^2,x]
```

output
```
2/Sqrt[1 + x^(-1)]
```

Defintions of rubi rules used

rule 793
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

rule 941
```
Int[((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(p_.), x_Sym
bol] :> Int[(a + b*x^n)^p*((d + c*x^n)^q/x^(n*q)), x] /; FreeQ[{a, b, c, d,
 n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !IntegerQ[p])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.45

method	result	size
orering	$\frac{2x\sqrt{1+\frac{1}{x}}}{x+1}$	16
gosper	$\frac{2x\sqrt{\frac{x+1}{x}}}{x+1}$	18
risch	$\frac{2x\sqrt{\frac{x+1}{x}}}{x+1}$	18
trager	$\frac{2x\sqrt{-\frac{-x-1}{x}}}{x+1}$	21
default	$\frac{2\sqrt{x^2+x}\,x\sqrt{\frac{x+1}{x}}}{(x+1)\sqrt{(x+1)x}}$	32

input
```
int((1+1/x)^(1/2)/(x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
2/(x+1)*x*(1+1/x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.55

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \frac{2\,x\sqrt{\frac{x+1}{x}}}{x+1}$$

input
```
integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="fricas")
```

output
```
2*x*sqrt((x + 1)/x)/(x + 1)
```

Sympy [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \int \frac{\sqrt{1+\frac{1}{x}}}{(x+1)^2}\,dx$$

input
```
integrate((1+1/x)**(1/2)/(1+x)**2,x)
```

output
```
Integral(sqrt(1 + 1/x)/(x + 1)**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \int \frac{\sqrt{\frac{1}{x}+1}}{(x+1)^2}\,dx$$

input
```
integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(1/x + 1)/(x + 1)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(9) = 18$.

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.09

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \frac{2\,\mathrm{sgn}(x)}{x-\sqrt{x^2+x}+1} - 2\,\mathrm{sgn}(x)$$

input
```
integrate((1+1/x)^(1/2)/(1+x)^2,x, algorithm="giac")
```

output
```
2*sgn(x)/(x - sqrt(x^2 + x) + 1) - 2*sgn(x)
```

Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \frac{2\,x\,\sqrt{\frac{1}{x}+1}}{x+1}$$

input
```
int((1/x + 1)^(1/2)/(x + 1)^2,x)
```

output
```
(2*x*(1/x + 1)^(1/2))/(x + 1)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{1+\frac{1}{x}}}{(1+x)^2}\,dx = \frac{2\sqrt{x+1}+2\sqrt{x}}{\sqrt{x+1}}$$

input
```
int((1+1/x)^(1/2)/(1+x)^2,x)
```

output
```
(2*(sqrt(x + 1) + sqrt(x)))/sqrt(x + 1)
```

3.3 $\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}} \, dx = -\frac{\sqrt{1+\frac{1}{x}}\sqrt{x}\arcsin(1-2x)}{\sqrt{1+x}}$$

output
```
(1+1/x)^(1/2)*x^(1/2)*arcsin(-1+2*x)/(1+x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}} \, dx = -\arctan\left(\frac{\sqrt{\frac{1+x}{x}}(-1+2x)\sqrt{1-x^2}}{2\,(-1+x^2)}\right)$$

input
```
Integrate[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]
```

output
```
-ArcTan[(Sqrt[(1 + x)/x]*(-1 + 2*x)*Sqrt[1 - x^2])/(2*(-1 + x^2))]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1778, 516, 62, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{1-x^2}}dx$$

$$\Big\downarrow \text{1778}$$

$$\frac{\sqrt{\frac{1}{x}+1}\sqrt{x}\int \frac{\sqrt{x+1}}{\sqrt{x}\sqrt{1-x^2}}dx}{\sqrt{x+1}}$$

$$\Big\downarrow \text{516}$$

$$\frac{\sqrt{\frac{1}{x}+1}\sqrt{x}\int \frac{1}{\sqrt{1-x}\sqrt{x}}dx}{\sqrt{x+1}}$$

$$\Big\downarrow \text{62}$$

$$\frac{\sqrt{\frac{1}{x}+1}\sqrt{x}\int \frac{1}{\sqrt{x-x^2}}dx}{\sqrt{x+1}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{\sqrt{\frac{1}{x}+1}\sqrt{x}\int \frac{1}{\sqrt{1-(1-2x)^2}}d(1-2x)}{\sqrt{x+1}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\sqrt{\frac{1}{x}+1}\sqrt{x}\arcsin(1-2x)}{\sqrt{x+1}}$$

input
```
Int[Sqrt[1 + x^(-1)]/Sqrt[1 - x^2],x]
```

output
```
-((Sqrt[1 + x^(-1)]*Sqrt[x]*ArcSin[1 - 2*x])/Sqrt[1 + x])
```

Defintions of rubi rules used

rule 62
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Int[
1/Sqrt[a*c - b*(a - c)*x - b^2*x^2], x] /; FreeQ[{a, b, c, d}, x] && EqQ[b
+ d, 0] && GtQ[a + c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 516
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[(e*x)^m*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; Free
Q[{a, b, c, d, e, m, n, p}, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] ||
(GtQ[a, 0] && GtQ[c, 0] &&  !IntegerQ[n]))
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1778
```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sym
bol] :> Simp[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(x^mn*e)))^F
racPart[q]))/x^(mn*FracPart[q])   Int[x^(mn*q)*(1 + d*(1/(x^mn*e)))^q*(a +
c*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, mn, p, q}, x] && EqQ[n2, -2*mn] &&
 !IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.38

method	result	size
default	$\frac{\sqrt{\frac{x+1}{x}}\, x\sqrt{-x^{2}+1}\, \arcsin\left(2x-1\right)}{\left(x+1\right)\sqrt{-x\left(x-1\right)}}$	40

input
```
int((1+1/x)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((x+1)/x)^(1/2)*x*(-x^2+1)^(1/2)/(x+1)/(-x*(x-1))^(1/2)*arcsin(2*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = -\arctan\left(\frac{2\sqrt{-x^2+1}x\sqrt{\frac{x+1}{x}}}{2\,x^2+x-1}\right)$$

input
```
integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(2*sqrt(-x^2 + 1)*x*sqrt((x + 1)/x)/(2*x^2 + x - 1))
```

Sympy [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{-(x-1)(x+1)}}\,dx$$

input
```
integrate((1+1/x)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
Integral(sqrt(1 + 1/x)/sqrt(-(x - 1)*(x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)
```

Giac [F]

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{-x^2+1}}\,dx$$

input
```
integrate((1+1/x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(1/x + 1)/sqrt(-x^2 + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = \int \frac{\sqrt{\frac{1}{x}+1}}{\sqrt{1-x^2}}\,dx$$

input
```
int((1/x + 1)^(1/2)/(1 - x^2)^(1/2),x)
```

output
```
int((1/x + 1)^(1/2)/(1 - x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{1+\frac{1}{x}}}{\sqrt{1-x^2}}\,dx = -2\log\left(\sqrt{1-x}+\sqrt{x}\,i\right)i$$

input
```
int((1+1/x)^(1/2)/(-x^2+1)^(1/2),x)
```

output
```
- 2*log(sqrt( - x + 1) + sqrt(x)*i)*i
```

3.4 $\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 18

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = -\left((1+x)\sqrt{-1+\frac{2}{1+x}}\right)$$

output
```
-(1+x)*(-1+2/(1+x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = \frac{-1+x}{\sqrt{\frac{1-x}{1+x}}}$$

input
```
Integrate[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]
```

output
```
(-1 + x)/Sqrt[(1 - x)/(1 + x)]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1015, 25, 1016, 899, 83}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)\sqrt{\frac{2}{x+1}-1}}dx$$

$$\Big\downarrow \text{1015}$$

$$\int \frac{x}{(x+1)\sqrt{\frac{2}{x+1}-1}}d(x+1)$$

$$\Big\downarrow \text{25}$$

$$-\int -\frac{x}{(x+1)\sqrt{\frac{2}{x+1}-1}}d(x+1)$$

$$\Big\downarrow \text{1016}$$

$$-\int \frac{\frac{1}{x+1}-1}{\sqrt{\frac{2}{x+1}-1}}d(x+1)$$

$$\Big\downarrow \text{899}$$

$$\int \frac{x(x+1)^2}{\sqrt{\frac{2}{x+1}-1}}d\frac{1}{x+1}$$

$$\Big\downarrow \text{83}$$

$$-\left((x+1)\sqrt{\frac{2}{x+1}-1}\right)$$

input
```
Int[x/((1 + x)*Sqrt[-1 + 2/(1 + x)]),x]
```

output
```
-((1 + x)*Sqrt[-1 + 2/(1 + x)])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 83
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n + p + 2, 0] && EqQ[a*d*f
*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)), 0]
```

rule 899
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> -Subst[Int[(a + b/x^n)^p*((c + d/x^n)^q/x^2), x], x, 1/x] /; FreeQ[{a,
 b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && ILtQ[n, 0]
```

rule 1015
```
Int[((a_.) + (b_.)*(v_)^(n_))^(p_.)*((c_.) + (d_.)*(v_)^(n_))^(q_.)*(x_)^(m
_.), x_Symbol] :> Simp[1/Coefficient[v, x, 1]^(m + 1)   Subst[Int[SimplifyI
ntegrand[(x - Coefficient[v, x, 0])^m*(a + b*x^n)^p*(c + d*x^n)^q, x], x],
x, v], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && LinearQ[v, x] && IntegerQ[m
] && NeQ[v, x]
```

rule 1016
```
Int[(x_)^(m_.)*((c_) + (d_.)*(x_)^(mn_.))^(q_.)*((a_) + (b_.)*(x_)^(n_.))^(
p_.), x_Symbol] :> Int[x^(m - n*q)*(a + b*x^n)^p*(d + c*x^n)^q, x] /; FreeQ
[{a, b, c, d, m, n, p}, x] && EqQ[mn, -n] && IntegerQ[q] && (PosQ[n] ||  !I
ntegerQ[p])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
orering	$\frac{x-1}{\sqrt{-1+\frac{2}{x+1}}}$	16
gosper	$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$	17
risch	$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$	17
trager	$(-x-1)\sqrt{-\frac{x-1}{x+1}}$	19
default	$-\frac{\sqrt{-\frac{x-1}{x+1}}\,(x+1)\sqrt{-x^2+1}}{\sqrt{-(x-1)(x+1)}}$	37

input
```
int(x/(x+1)/(-1+2/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x-1)/(-1+2/(x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = -(x+1)\sqrt{-\frac{x-1}{x+1}}$$

input
```
integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
-(x + 1)*sqrt(-(x - 1)/(x + 1))
```

Sympy [F]

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = \int \frac{x}{\sqrt{-\frac{x-1}{x+1}}\,(x+1)}\,dx$$

input
```
integrate(x/(1+x)/(-1+2/(1+x))**(1/2),x)
```

output
```
Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = \frac{\sqrt{x+1}(x-1)}{\sqrt{-x+1}}$$

input
```
integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x + 1)*(x - 1)/sqrt(-x + 1)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = -\frac{\sqrt{-x^2+1}}{\operatorname{sgn}(x+1)}$$

input
```
integrate(x/(1+x)/(-1+2/(1+x))^(1/2),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 1)/sgn(x + 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = \int \frac{x}{(x+1)\,\sqrt{\frac{2}{x+1}-1}}\,dx$$

input
```
int(x/((x + 1)*(2/(x + 1) - 1)^(1/2)),x)
```

output
```
int(x/((x + 1)*(2/(x + 1) - 1)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x}{(1+x)\sqrt{-1+\frac{2}{1+x}}}\,dx = -\sqrt{x+1}\,\sqrt{1-x}$$

input
```
int(x/(1+x)/(-1+2/(1+x))^(1/2),x)
```

output
```
- sqrt(x + 1)*sqrt( - x + 1)
```

3.5 $\int \frac{\sqrt{a+\frac{b}{c+dx}}}{b+ac+adx} \, dx$

Optimal result

Integrand size = 27, antiderivative size = 32

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{b+ac+adx} \, dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

output
```
2*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))/a^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{b+ac+adx} \, dx = \frac{2\text{arctanh}\left(\frac{\sqrt{\frac{b+ac+adx}{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Integrate[Sqrt[a + b/(c + d*x)]/(b + a*c + a*d*x),x]
```

output
```
(2*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/(Sqrt[a]*d)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.074, Rules used = {7268, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{ac+adx+b}\,dx$$

$$\Big\downarrow \; 7268$$

$$\frac{2\int -\frac{c+dx}{b}d\sqrt{a+\frac{b}{c+dx}}}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{\sqrt{a}d}$$

input
```
Int[Sqrt[a + b/(c + d*x)]/(b + a*c + a*d*x),x]
```

output
```
(2*ArcTanh[Sqrt[a + b/(c + d*x)]/Sqrt[a]])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(26) = 52.

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.19

method	result	size
default	$\frac{\sqrt{\frac{adx+ac+b}{dx+c}}\,(dx+c)\ln\left(\frac{2a\,d^{2}x+2acd+2\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^{2}}+bd}{2\sqrt{a\,d^{2}}}\right)}{\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^{2}}}$	102

input
```
int((a+b/(d*x+c))^(1/2)/(a*d*x+a*c+b),x,method=_RETURNVERBOSE)
```

output
```
((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+
a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))/((a*d*x+a*c+b)*(d*
x+c))^(1/2)/(a*d^2)^(1/2)
```

Fricas [A] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.25

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{b+ac+adx}\,dx=\left[\frac{\log\left(2\,adx+2\,ac+2\,(dx+c)\sqrt{a}\sqrt{\frac{adx+ac+b}{dx+c}}+b\right)}{\sqrt{a}d},\ -\frac{2\,\sqrt{-a}\arctan\left(\frac{(dx+c)\sqrt{-a}\sqrt{\frac{adx+ac+b}{dx+c}}}{adx+ac+b}\right)}{ad}\right]$$

input
```
integrate((a+b/(d*x+c))^(1/2)/(a*d*x+a*c+b),x, algorithm="fricas")
```

output
```
[log(2*a*d*x + 2*a*c + 2*(d*x + c)*sqrt(a)*sqrt((a*d*x + a*c + b)/(d*x + c
)) + b)/(sqrt(a)*d), -2*sqrt(-a)*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a
*c + b)/(d*x + c))/(a*d*x + a*c + b))/(a*d)]
```

Sympy [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{b+ac+adx}\,dx = \int \frac{\sqrt{\frac{ac+adx+b}{c+dx}}}{ac+adx+b}\,dx$$

input
```
integrate((a+b/(d*x+c))**(1/2)/(a*d*x+a*c+b),x)
```

output
```
Integral(sqrt((a*c + a*d*x + b)/(c + d*x))/(a*c + a*d*x + b), x)
```

Maxima [F]

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{b+ac+adx}\,dx = \int \frac{\sqrt{a+\frac{b}{dx+c}}}{adx+ac+b}\,dx$$

input
```
integrate((a+b/(d*x+c))^(1/2)/(a*d*x+a*c+b),x, algorithm="maxima")
```

output
```
integrate(sqrt(a + b/(d*x + c))/(a*d*x + a*c + b), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(26) = 52$.

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.31

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{b+ac+adx}\,dx = -\frac{\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\right)\sqrt{a}|d|+bd\right|\right)\mathrm{sgn}(dx+c)}{\sqrt{a}|d|}$$

input
```
integrate((a+b/(d*x+c))^(1/2)/(a*d*x+a*c+b),x, algorithm="giac")
```

output
```
-log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 +
 b*d*x + b*c))*sqrt(a)*abs(d) + b*d))*sgn(d*x + c)/(sqrt(a)*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{b+ac+adx}\,dx=\int \frac{\sqrt{a+\frac{b}{c+d\,x}}}{b+a\,c+a\,d\,x}\,dx$$

input
```
int((a + b/(c + d*x))^(1/2)/(b + a*c + a*d*x),x)
```

output
```
int((a + b/(c + d*x))^(1/2)/(b + a*c + a*d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{\sqrt{a+\frac{b}{c+dx}}}{b+ac+adx}\,dx=\frac{2\sqrt{a}\,\log\left(\frac{\sqrt{adx+ac+b}+\sqrt{a}\,\sqrt{dx+c}}{\sqrt{b}}\right)}{ad}$$

input
```
int((a+b/(d*x+c))^(1/2)/(a*d*x+a*c+b),x)
```

output
```
(2*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b)))/(
a*d)
```

3.6 $\int \frac{\sqrt{\frac{b}{d}-\frac{bc-ad}{d(c+dx)}}}{a+bx}\,dx$

Optimal result

Integrand size = 38, antiderivative size = 54

$$\int \frac{\sqrt{\frac{b}{d}-\frac{bc-ad}{d(c+dx)}}}{a+bx}\,dx=\frac{2\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{b}{d}-\frac{bc-ad}{d(c+dx)}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

output
```
2*arctanh(d^(1/2)*(b/d-(-a*d+b*c)/d/(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d^(1/2
)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{\frac{b}{d}-\frac{bc-ad}{d(c+dx)}}}{a+bx}\,dx=\frac{2\sqrt{\frac{a+bx}{c+dx}}\sqrt{c+dx}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{d}\sqrt{a+bx}}$$

input
```
Integrate[Sqrt[b/d - (b*c - a*d)/(d*(c + d*x))]/(a + b*x),x]
```

output
```
(2*Sqrt[(a + b*x)/(c + d*x)]*Sqrt[c + d*x]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[a + b*x])])/(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {7239, 2055, 27, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{b}{d}-\frac{bc-ad}{d(c+dx)}}}{a+bx}dx$$

$$\Big\downarrow \ 7239$$

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}dx$$

$$\Big\downarrow \ 2055$$

$$2(bc-ad)\int \frac{1}{(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\sqrt{\frac{a+bx}{c+dx}}$$

$$\Big\downarrow \ 27$$

$$2\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\sqrt{\frac{a+bx}{c+dx}}$$

$$\Big\downarrow \ 221$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

input
```
Int[Sqrt[b/d - (b*c - a*d)/(d*(c + d*x))]/(a + b*x),x]
```

output
```
(2*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2055
```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)
 Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/
(b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1
/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Int
egerQ[r]
```

rule 7239
```
Int[u_, x_Symbol] :> With[{v = SimplifyIntegrand[u, x]}, Int[v, x] /; Simpl
erIntegrandQ[v, u, x]]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.48

method	result	size
default	$\frac{\ln\left(\frac{2bdx+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}}$	80

input
```
int((b/d-(-a*d+b*c)/d/(d*x+c))^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2)
)*(d*x+c)*((b*x+a)/(d*x+c))^(1/2)/((b*x+a)*(d*x+c))^(1/2)/(b*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.94

$$\int \frac{\sqrt{\frac{b}{d}-\frac{bc-ad}{d(c+dx)}}}{a+bx}\,dx = \left[\frac{\sqrt{bd}\log\left(2\,bdx+bc+ad+2\,\sqrt{bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}\right)}{bd}, -\frac{2\,\sqrt{-bd}\arctan\left(\frac{\sqrt{-bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{bdx+ad}\right)}{bd}\right]$$

input
```
integrate((b/d-(-a*d+b*c)/d/(d*x+c))^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x + c)*sqrt((b*x + a)/
(d*x + c)))/(b*d), -2*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*x + a
)/(d*x + c))/(b*d*x + a*d))/(b*d)]
```

Sympy [F]

$$\int \frac{\sqrt{\frac{b}{d}-\frac{bc-ad}{d(c+dx)}}}{a+bx}\,dx = \int \frac{\sqrt{\frac{d(a+bx)}{cd+d^2x}}}{a+bx}\,dx$$

input
```
integrate((b/d-(-a*d+b*c)/d/(d*x+c))**(1/2)/(b*x+a),x)
```

output
```
Integral(sqrt(d*(a + b*x)/(c*d + d**2*x))/(a + b*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{b}{d} - \frac{bc-ad}{d(c+dx)}}}{a+bx}\, dx = \int \frac{\sqrt{\frac{b}{d} - \frac{bc-ad}{(dx+c)d}}}{bx+a}\, dx$$

input
```
integrate((b/d-(-a*d+b*c)/d/(d*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(b/d - (b*c - a*d)/((d*x + c)*d))/(b*x + a), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.13

$$\int \frac{\sqrt{\frac{b}{d} - \frac{bc-ad}{d(c+dx)}}}{a+bx}\, dx = -\frac{\log\left(\left|-bc - ad - 2\sqrt{bd}\left(\sqrt{bd}x - \sqrt{bdx^2 + bcx + adx + ac}\right)\right|\right)\operatorname{sgn}(dx+c)}{\sqrt{bd}}$$

input
```
integrate((b/d-(-a*d+b*c)/d/(d*x+c))^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
-log(abs(-b*c - a*d - 2*sqrt(b*d)*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*
d*x + a*c))))*sgn(d*x + c)/sqrt(b*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{b}{d} - \frac{bc-ad}{d(c+dx)}}}{a+bx}\, dx = \int \frac{\sqrt{\frac{b}{d} + \frac{a\,d-b\,c}{d\,(c+d\,x)}}}{a+b\,x}\, dx$$

input
```
int((b/d + (a*d - b*c)/(d*(c + d*x)))^(1/2)/(a + b*x),x)
```

output
```
int((b/d + (a*d - b*c)/(d*(c + d*x)))^(1/2)/(a + b*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{\frac{b}{d}-\frac{bc-ad}{d(c+dx)}}}{a+bx}\,dx=\frac{2\sqrt{d}\,\sqrt{b}\log\left(\frac{\sqrt{d}\,\sqrt{bx+a}+\sqrt{b}\,\sqrt{dx+c}}{\sqrt{ad-bc}}\right)}{bd}$$

input
```
int((b/d-(-a*d+b*c)/d/(d*x+c))^(1/2)/(b*x+a),x)
```

output
```
(2*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqr
t(a*d - b*c)))/(b*d)
```

3.7 $\int \frac{x}{(a+x)^{3/2}\sqrt{1-\frac{2a}{a+x}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{x}{(a+x)^{3/2}\sqrt{1-\frac{2a}{a+x}}}\,dx = 2\sqrt{a+x}\sqrt{1-\frac{2a}{a+x}}+\frac{\sqrt{2}a\operatorname{arctanh}\left(\frac{\sqrt{1-\frac{2a}{a+x}}}{\sqrt{2}\sqrt{-\frac{a}{a+x}}}\right)}{\sqrt{-\frac{a}{a+x}}\sqrt{a+x}}$$

output
```
2*(a+x)^(1/2)*(1-2*a/(a+x))^(1/2)+2^(1/2)*a*arctanh(1/2*(1-2*a/(a+x))^(1/2
)*2^(1/2)/(-a/(a+x))^(1/2))/(-a/(a+x))^(1/2)/(a+x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{x}{(a+x)^{3/2}\sqrt{1-\frac{2a}{a+x}}}\,dx = \frac{-2a+2x+\sqrt{2}\sqrt{a}\sqrt{a-x}\operatorname{arctanh}\left(\frac{\sqrt{a-x}}{\sqrt{2}\sqrt{a}}\right)}{\sqrt{\frac{-a+x}{a+x}}\sqrt{a+x}}$$

input
```
Integrate[x/((a + x)^(3/2)*Sqrt[1 - (2*a)/(a + x)]),x]
```

output
```
(-2*a + 2*x + Sqrt[2]*Sqrt[a]*Sqrt[a - x]*ArcTanh[Sqrt[a - x]/(Sqrt[2]*Sqr
t[a])])/(Sqrt[(-a + x)/(a + x)]*Sqrt[a + x])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {7268, 2044, 298, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a+x)^{3/2}\sqrt{1-\frac{2a}{a+x}}}dx$$

$$\Big\downarrow \text{7268}$$

$$\sqrt{2}a\int \frac{(a+x)^{3/2}\left(2-\frac{2a}{a+x}\right)}{2\sqrt{2}a^2}d\sqrt{1-\frac{2a}{a+x}}$$

$$\Big\downarrow \text{2044}$$

$$\frac{\sqrt{2}a\int \frac{2-\frac{2a}{a+x}}{2\sqrt{2}\left(\frac{a}{a+x}\right)^{3/2}}d\sqrt{1-\frac{2a}{a+x}}}{\sqrt{\frac{a}{a+x}}\sqrt{a+x}}$$

$$\Big\downarrow \text{298}$$

$$\frac{\sqrt{2}a\left(\frac{\sqrt{2}\sqrt{1-\frac{2a}{a+x}}}{\sqrt{\frac{a}{a+x}}}-\int \frac{1}{\sqrt{2}\sqrt{\frac{a}{a+x}}}d\sqrt{1-\frac{2a}{a+x}}\right)}{\sqrt{\frac{a}{a+x}}\sqrt{a+x}}$$

$$\Big\downarrow \text{223}$$

$$\frac{\sqrt{2}a\left(\frac{\sqrt{2}\sqrt{1-\frac{2a}{a+x}}}{\sqrt{\frac{a}{a+x}}}-\arcsin\left(\sqrt{1-\frac{2a}{a+x}}\right)\right)}{\sqrt{\frac{a}{a+x}}\sqrt{a+x}}$$

input
```
Int[x/((a + x)^(3/2)*Sqrt[1 - (2*a)/(a + x)]),x]
```

output
```
(Sqrt[2]*a*((Sqrt[2]*Sqrt[1 - (2*a)/(a + x)])/Sqrt[a/(a + x)] - ArcSin[Sqr
t[1 - (2*a)/(a + x)]]))/(Sqrt[a/(a + x)]*Sqrt[a + x])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2044
```
Int[(u_.)*((c_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_))^(p_), x_Symbol] :> Simp[S
imp[(c*(a + b*x^n)^q)^p/(a + b*x^n)^(p*q)]   Int[u*(a + b*x^n)^(p*q), x], x
] /; FreeQ[{a, b, c, n, p, q}, x] && GeQ[a, 0]
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

method	result	size
default	$-\frac{\sqrt{-\frac{a-x}{a+x}}\,\sqrt{a+x}\,\left(-2\sqrt{-a+x}+\sqrt{a}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-a+x}\,\sqrt{2}}{2\sqrt{a}}\right)\right)}{\sqrt{-a+x}}$	62
risch	$-\frac{2(a-x)\sqrt{-(a+x)(a-x)}}{\sqrt{-\frac{a-x}{a+x}}\,\sqrt{a+x}\,\sqrt{(-a+x)(a+x)}}-\frac{\sqrt{a}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{-a+x}\,\sqrt{2}}{2\sqrt{a}}\right)\sqrt{-(a+x)(a-x)}\,\sqrt{-a+x}}{\sqrt{-\frac{a-x}{a+x}}\,\sqrt{a+x}\,\sqrt{(-a+x)(a+x)}}$	124

input
```
int(x/(a+x)^(3/2)/(1-2*a/(a+x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(a-x)/(a+x))^(1/2)*(a+x)^(1/2)/(-a+x)^(1/2)*(-2*(-a+x)^(1/2)+a^(1/2)*2^
(1/2)*arctan(1/2*(-a+x)^(1/2)*2^(1/2)/a^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.70

$$\int \frac{x}{(a+x)^{3/2}\sqrt{1-\frac{2a}{a+x}}}\,dx = \left[\frac{1}{2}\sqrt{2}\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{a+x}\sqrt{-\frac{a-x}{a+x}}+3a-x}{a+x}\right) + 2\sqrt{a+x}\sqrt{-\frac{a-x}{a+x}},\ \sqrt{2}\sqrt{a}\arctan\left(-\frac{\sqrt{2}\sqrt{a+x}\sqrt{a}\sqrt{-\frac{a-x}{a+x}}}{a-x}\right) + 2\sqrt{a+x}\sqrt{-\frac{a-x}{a+x}}\right]$$

input
```
integrate(x/(a+x)^(3/2)/(1-2*a/(a+x))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*sqrt(2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt(a + x)*sqrt(-(a - x)/(
a + x)) + 3*a - x)/(a + x)) + 2*sqrt(a + x)*sqrt(-(a - x)/(a + x)), sqrt(2
)*sqrt(a)*arctan(-sqrt(2)*sqrt(a + x)*sqrt(a)*sqrt(-(a - x)/(a + x))/(a -
x)) + 2*sqrt(a + x)*sqrt(-(a - x)/(a + x))]
```

Sympy [F]

$$\int \frac{x}{(a+x)^{3/2}\sqrt{1-\frac{2a}{a+x}}}\,dx = \int \frac{x}{\sqrt{\frac{-a+x}{a+x}}\,(a+x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+x)**(3/2)/(1-2*a/(a+x))**(1/2),x)
```

output
```
Integral(x/(sqrt((-a + x)/(a + x))*(a + x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{x}{(a+x)^{3/2}\sqrt{1-\frac{2a}{a+x}}}\,dx = \int \frac{x}{(a+x)^{\frac{3}{2}}\sqrt{-\frac{2\,a}{a+x}+1}}\,dx$$

input
```
integrate(x/(a+x)^(3/2)/(1-2*a/(a+x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x/((a + x)^(3/2)*sqrt(-2*a/(a + x) + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.55

$$\int \frac{x}{(a+x)^{3/2}\sqrt{1-\frac{2a}{a+x}}}\,dx = -\frac{\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{-a+x}}{2\sqrt{a}}\right)}{\mathrm{sgn}\left(a+x\right)} + \frac{2\sqrt{-a+x}}{\mathrm{sgn}\left(a+x\right)}$$

input
```
integrate(x/(a+x)^(3/2)/(1-2*a/(a+x))^(1/2),x, algorithm="giac")
```

output
```
-sqrt(2)*sqrt(a)*arctan(1/2*sqrt(2)*sqrt(-a + x)/sqrt(a))/sgn(a + x) + 2*s
qrt(-a + x)/sgn(a + x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+x)^{3/2}\sqrt{1-\frac{2a}{a+x}}}\,dx = \int \frac{x}{{(a+x)}^{3/2}\,\sqrt{1-\frac{2\,a}{a+x}}}\,dx$$

input
```
int(x/((a + x)^(3/2)*(1 - (2*a)/(a + x))^(1/2)),x)
```

output
```
int(x/((a + x)^(3/2)*(1 - (2*a)/(a + x))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.37

$$\int \frac{x}{(a+x)^{3/2}\sqrt{1-\frac{2a}{a+x}}}\,dx = -\sqrt{a}\,\sqrt{2}\,atan\left(\frac{\sqrt{-a+x}}{\sqrt{a}\,\sqrt{2}}\right)+2\sqrt{-a+x}$$

input
```
int(x/(a+x)^(3/2)/(1-2*a/(a+x))^(1/2),x)
```

output
```
 - sqrt(a)*sqrt(2)*atan(sqrt( - a + x)/(sqrt(a)*sqrt(2))) + 2*sqrt( - a +
x)
```

3.8 $\int (A+Bx+Cx^2)\sqrt{a+\frac{b}{c+dx}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 218

$$\int (A+Bx+Cx^2)\sqrt{a+\frac{b}{c+dx}}\,dx = -\frac{(b(bC+4acC-2aBd)-8a^2(c^2C-Bcd+Ad^2))\,(c+dx)\sqrt{a+\frac{b}{c+dx}}}{8a^2d^3} + \frac{(bC-12acC+6aBd)(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{12ad^3} + \frac{C(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3d^3} + \frac{b(b(bC+4acC-2aBd)+8a^2(c^2C-Bcd+Ad^2))\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{8a^{5/2}d^3}$$

output
```
-1/8*(b*(-2*B*a*d+4*C*a*c+C*b)-8*a^2*(A*d^2-B*c*d+C*c^2))*(d*x+c)*(a+b/(d*
x+c))^(1/2)/a^2/d^3+1/12*(6*B*a*d-12*C*a*c+C*b)*(d*x+c)^2*(a+b/(d*x+c))^(1
/2)/a/d^3+1/3*C*(d*x+c)^3*(a+b/(d*x+c))^(1/2)/d^3+1/8*b*(b*(-2*B*a*d+4*C*a
*c+C*b)+8*a^2*(A*d^2-B*c*d+C*c^2))*arctanh((a+b/(d*x+c))^(1/2)/a^(1/2))/a^
(5/2)/d^3
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.83

$$\int \left(A+Bx+Cx^2\right)\sqrt{a+\frac{b}{c+dx}}\,dx$$
$$=\frac{\sqrt{a}(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}\left(-3b^2C+2ab(-5cC+3Bd+Cdx)+4a^2\left(2c^2C-cd(3B+2Cx)+d^2(6A+3B\right.\right.}{24a^{5/2}d^3}$$

input
```
Integrate[(A + B*x + C*x^2)*Sqrt[a + b/(c + d*x)],x]
```

output
```
(Sqrt[a]*(c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(-3*b^2*C + 2*a*b*(-5
*c*C + 3*B*d + C*d*x) + 4*a^2*(2*c^2*C - c*d*(3*B + 2*C*x) + d^2*(6*A + 3*
B*x + 2*C*x^2))) + 3*b*(b^2*C + a*b*(4*c*C - 2*B*d) + 8*a^2*(c^2*C - B*c*d
 + A*d^2))*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/(24*a^(5/2)
*d^3)
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.07, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {7268, 2089, 1580, 25, 1471, 27, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(A+Bx+Cx^2\right)\sqrt{a+\frac{b}{c+dx}}\,dx$$

$$\Big\downarrow \text{7268}$$

$$-\frac{2b\int \frac{(c+dx)^4\left(a+\frac{b}{c+dx}\right)\left(Cb^2-\frac{(2cC-Bd)b^2}{c+dx}+\frac{\left(Cc^2-Bdc+Ad^2\right)b^2}{(c+dx)^2}\right)}{b^4}d\sqrt{a+\frac{b}{c+dx}}}{d^3}$$

$$\Big\downarrow \text{2089}$$

$$-\frac{2b\int \frac{(c+dx)^4\left(a+\frac{b}{c+dx}\right)\left(\left(Cc^2-Bdc+Ad^2\right)a^2+b(2cC-Bd)a+\left(Cc^2-Bdc+Ad^2\right)\left(a+\frac{b}{c+dx}\right)^2+b^2C-\left(b(2cC-Bd)+2a\left(Cc^2-Bdc+Ad^2\right)\right)\left(a\right.\right.}{b^4}}{d^3}$$

$\big\downarrow$ 1580

$$-\frac{2b\left(\frac{1}{6}\int \frac{(c+dx)^3\left(Cb^2+6\left(Cc^2-Bdc+Ad^2\right)\left(a+\frac{b}{c+dx}\right)^2-6\left(aCc^2+2bCc-aBdc+aAd^2-bBd\right)\left(a+\frac{b}{c+dx}\right)\right)}{b^3}d\sqrt{a+\frac{b}{c+dx}}-\frac{C(c+dx)^3\sqrt{}}{6b}\right.}{d^3}$$

$\big\downarrow$ 25

$$-\frac{2b\left(-\frac{1}{6}\int -\frac{(c+dx)^3\left(Cb^2+6\left(Cc^2-Bdc+Ad^2\right)\left(a+\frac{b}{c+dx}\right)^2-6\left(aCc^2+2bCc-aBdc+aAd^2-bBd\right)\left(a+\frac{b}{c+dx}\right)\right)}{b^3}d\sqrt{a+\frac{b}{c+dx}}-\frac{C(c+dx}{}\right.}{d^3}$$

$\big\downarrow$ 1471

$$-\frac{2b\left(\frac{1}{6}\left(\frac{\int -\frac{3(c+dx)^2\left(b(bC+4acC-2aBd)-8a\left(Cc^2-Bdc+Ad^2\right)\left(a+\frac{b}{c+dx}\right)\right)}{b^2}d\sqrt{a+\frac{b}{c+dx}}}{4a}-\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}(6aBd-12acC+bC)}{4ab}\right)-\frac{C(c+}{}\right.}{d^3}$$

$\big\downarrow$ 27

$$-\frac{2b\left(\frac{1}{6}\left(-\frac{3\int \frac{(c+dx)^2\left(b(bC+4acC-2aBd)-8a\left(Cc^2-Bdc+Ad^2\right)\left(a+\frac{b}{c+dx}\right)\right)}{b^2}d\sqrt{a+\frac{b}{c+dx}}}{4a}-\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}(6aBd-12acC+bC)}{4ab}\right)-\frac{C(c}{}\right.}{d^3}$$

$\big\downarrow$ 298

$$-\frac{2b\left(\frac{1}{6}\left(-\frac{3\left(\frac{\left(8a^2\left(Ad^2-Bcd+c^2C\right)+b(-2aBd+4acC+bC)\right)\int -\frac{c+dx}{b}d\sqrt{a+\frac{b}{c+dx}}}{2a}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}\left(b(-2aBd+4acC+bC)-8a^2\left(Ad^2-Bcd+c^2C\right)\right)}{2ab}\right)}{4a}\right.\right.}{d^3}$$

$\big\downarrow$ 219

$$-\frac{2b\left(\frac{1}{6}\left(-\frac{3\left(\frac{\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)\left(8a^2\left(Ad^2-Bcd+c^2C\right)+b(-2aBd+4acC+bC)\right)}{2a^{3/2}}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}\left(b(-2aBd+4acC+bC)-8a^2\left(Ad^2-Bcd+c^2C\right.\right.}{2ab}\right.}{4a}\right.\right.}{d^3}$$

input
```
Int[(A + B*x + C*x^2)*Sqrt[a + b/(c + d*x)],x]
```

output
```
(-2*b*(-1/6*(C*(c + d*x)^3*Sqrt[a + b/(c + d*x)])/b + (-1/4*((b*C - 12*a*c
*C + 6*a*B*d)*(c + d*x)^2*Sqrt[a + b/(c + d*x)])/(a*b) - (3*(-1/2*((b*(b*C
 + 4*a*c*C - 2*a*B*d) - 8*a^2*(c^2*C - B*c*d + A*d^2))*(c + d*x)*Sqrt[a +
b/(c + d*x)])/(a*b) + ((b*(b*C + 4*a*c*C - 2*a*B*d) + 8*a^2*(c^2*C - B*c*d
 + A*d^2))*ArcTanh[Sqrt[a + b/(c + d*x)]/Sqrt[a]])/(2*a^(3/2))))/(4*a))/6)
)/d^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 1580
```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[1/(2*e^(2*p + m/2)*
(q + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*
e^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b
*d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e
}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

rule 2089
```
Int[(u_)^(p_.)*((f_.)*(x_))^(m_.)*(z_)^(q_.), x_Symbol] :> Int[(f*x)^m*Expa
ndToSum[z, x]^q*ExpandToSum[u, x]^p, x] /; FreeQ[{f, m, p, q}, x] && Binomi
alQ[z, x] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ
[u, x])
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 950 vs. $2(198) = 396$.

Time = 0.15 (sec) , antiderivative size = 951, normalized size of antiderivative = 4.36

method	result
default	$\frac{\sqrt{\frac{adx+ac+b}{dx+c}}\,(dx+c)\left(24A\ln\left(\frac{2a\,d^2x+2acd+2\sqrt{(adx+ac+b)(dx+c)}\,\sqrt{a\,d^2}+bd}{2\sqrt{a\,d^2}}\right)a^2b\,d^3+24B\sqrt{a\,d^2}\,\sqrt{a\,d^2x^2+2adxc+a\,c^2+bdx+bc}\,a^2d^2}{}$

input
```
int((C*x^2+B*x+A)*(a+b/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)/d^3*(24*A*ln(1/2*(2*a*d^2*x+2*a
*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2
*b*d^3+24*B*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^2*
d^2*x-24*B*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^
2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b*c*d^2-48*C*(a*d^2)^(1/2)*(a*d^2*x^2+2*a
*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^2*c*d*x+24*C*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(
(a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b*c^2*d
+48*A*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^2*d^2-48*B*((a*d*x+a*c
+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^2*c*d+24*B*(a*d^2)^(1/2)*(a*d^2*x^2+2*a
*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^2*c*d-6*B*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d
^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*
a*b^2*d^2+48*C*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^2*c^2-48*C*(a
*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^2*c^2-12*C*(a*d^
2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a*b*d*x+12*C*ln(1/2*(
2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1
/2)+b*d)/(a*d^2)^(1/2))*a*b^2*c*d+12*B*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+
a*c^2+b*d*x+b*c)^(1/2)*a*b*d+16*C*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2
+b*d*x+b*c)^(3/2)*a-36*C*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*
c)^(1/2)*a*b*c+3*C*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c*d*x+a*c^2+
b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*b^3*d-6*C*(a*d^2)^(1...
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.44

$$\int \left(A+Bx+Cx^2\right)\sqrt{a+\frac{b}{c+dx}}\,dx$$

$$= \left[\frac{3\,(8\,Ca^2bc^2+8\,Aa^2bd^2+4\,Cab^2c+Cb^3-2\,(4\,Ba^2bc+Bab^2)d)\sqrt{a}\log\Big(2\,adx+2\,ac+2\,(dx+c)\sqrt{a}\sqrt{}\right.$$

$$-\frac{3\,(8\,Ca^2bc^2+8\,Aa^2bd^2+4\,Cab^2c+Cb^3-2\,(4\,Ba^2bc+Bab^2)d)\sqrt{-a}\arctan\left(\frac{(dx+c)\sqrt{-a}\sqrt{\frac{adx+ac+b}{dx+c}}}{adx+ac+b}\right)-}{}$$

input
```
integrate((C*x^2+B*x+A)*(a+b/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(8*C*a^2*b*c^2 + 8*A*a^2*b*d^2 + 4*C*a*b^2*c + C*b^3 - 2*(4*B*a^2
*b*c + B*a*b^2)*d)*sqrt(a)*log(2*a*d*x + 2*a*c + 2*(d*x + c)*sqrt(a)*sqrt(
(a*d*x + a*c + b)/(d*x + c)) + b) + 2*(8*C*a^3*d^3*x^3 + 8*C*a^3*c^3 + 24*
A*a^3*c*d^2 - 10*C*a^2*b*c^2 - 3*C*a*b^2*c + 2*(6*B*a^3*d^3 + C*a^2*b*d^2)
*x^2 - 6*(2*B*a^3*c^2 - B*a^2*b*c)*d + (24*A*a^3*d^3 + 6*B*a^2*b*d^2 - (8*
C*a^2*b*c + 3*C*a*b^2)*d)*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a^3*d^3),
 -1/24*(3*(8*C*a^2*b*c^2 + 8*A*a^2*b*d^2 + 4*C*a*b^2*c + C*b^3 - 2*(4*B*a^
2*b*c + B*a*b^2)*d)*sqrt(-a)*arctan((d*x + c)*sqrt(-a)*sqrt((a*d*x + a*c +
 b)/(d*x + c))/(a*d*x + a*c + b)) - (8*C*a^3*d^3*x^3 + 8*C*a^3*c^3 + 24*A*
a^3*c*d^2 - 10*C*a^2*b*c^2 - 3*C*a*b^2*c + 2*(6*B*a^3*d^3 + C*a^2*b*d^2)*x
^2 - 6*(2*B*a^3*c^2 - B*a^2*b*c)*d + (24*A*a^3*d^3 + 6*B*a^2*b*d^2 - (8*C*
a^2*b*c + 3*C*a*b^2)*d)*x)*sqrt((a*d*x + a*c + b)/(d*x + c)))/(a^3*d^3)]
```

Sympy [F]

$$\int \left(A+Bx+Cx^2\right)\sqrt{a+\frac{b}{c+dx}}\,dx=\int \sqrt{\frac{ac+adx+b}{c+dx}}\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(a+b/(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt((a*c + a*d*x + b)/(c + d*x))*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int \left(A+Bx+Cx^2\right)\sqrt{a+\frac{b}{c+dx}}\,dx=\int \left(Cx^2+Bx+A\right)\sqrt{a+\frac{b}{dx+c}}\,dx$$

input
```
integrate((C*x^2+B*x+A)*(a+b/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*sqrt(a + b/(d*x + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.67

$$\int \left(A+Bx+Cx^2\right)\sqrt{a+\frac{b}{c+dx}}\,dx$$
$$=\frac{1}{24}\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\Bigg(2\left(\frac{4\,Cx\mathrm{sgn}(dx+c)}{d}-\frac{4\,Ca^2cd^3\mathrm{sgn}(dx+c)-6\,Ba^2d^4\mathrm{sgn}(dx+}{a^2d^5}\right.$$
$$-\frac{(8\,Ca^2bc^2\mathrm{sgn}(dx+c)-8\,Ba^2bcd\mathrm{sgn}(dx+c)+8\,Aa^2bd^2\mathrm{sgn}(dx+c)+4\,Cab^2c\mathrm{sgn}(dx+c)-2\,Bab^2d}{16}$$

input
```
integrate((C*x^2+B*x+A)*(a+b/(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*(2*(4*C*x*sgn(d*x +
 c)/d - (4*C*a^2*c*d^3*sgn(d*x + c) - 6*B*a^2*d^4*sgn(d*x + c) - C*a*b*d^3
*sgn(d*x + c))/(a^2*d^5))*x + (8*C*a^2*c^2*d^2*sgn(d*x + c) - 12*B*a^2*c*d
^3*sgn(d*x + c) + 24*A*a^2*d^4*sgn(d*x + c) - 10*C*a*b*c*d^2*sgn(d*x + c)
+ 6*B*a*b*d^3*sgn(d*x + c) - 3*C*b^2*d^2*sgn(d*x + c))/(a^2*d^5)) - 1/16*(
8*C*a^2*b*c^2*sgn(d*x + c) - 8*B*a^2*b*c*d*sgn(d*x + c) + 8*A*a^2*b*d^2*sg
n(d*x + c) + 4*C*a*b^2*c*sgn(d*x + c) - 2*B*a*b^2*d*sgn(d*x + c) + C*b^3*s
gn(d*x + c))*log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 + 2*a*c*d
*x + a*c^2 + b*d*x + b*c))*sqrt(a)*abs(d) + b*d))/(a^(5/2)*d^2*abs(d))
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx+Cx^2)\sqrt{a+\frac{b}{c+dx}}\,dx=\int \sqrt{a+\frac{b}{c+d\,x}}\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int((a + b/(c + d*x))^(1/2)*(A + B*x + C*x^2),x)
```

output
```
int((a + b/(c + d*x))^(1/2)*(A + B*x + C*x^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.19

$$\int (A+Bx+Cx^2)\sqrt{a+\frac{b}{c+dx}}\,dx$$
$$=\frac{24\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^4d^2-12\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^3bcd+12\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^3b\,d^2x}{}$$

input
```
int((C*x^2+B*x+A)*(a+b/(d*x+c))^(1/2),x)
```

output

```
(24*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**4*d**2 - 12*sqrt(c + d*x)*sqrt(
a*c + a*d*x + b)*a**3*b*c*d + 12*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*
b*d**2*x + 8*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*c**3 - 8*sqrt(c + d*
x)*sqrt(a*c + a*d*x + b)*a**3*c**2*d*x + 8*sqrt(c + d*x)*sqrt(a*c + a*d*x
+ b)*a**3*c*d**2*x**2 + 6*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b**2*d
- 10*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b*c**2 + 2*sqrt(c + d*x)*sqr
t(a*c + a*d*x + b)*a**2*b*c*d*x - 3*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a*
b**2*c + 24*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sq
rt(b))*a**3*b*d**2 - 24*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(
c + d*x))/sqrt(b))*a**2*b**2*c*d + 24*sqrt(a)*log((sqrt(a*c + a*d*x + b) +
 sqrt(a)*sqrt(c + d*x))/sqrt(b))*a**2*b*c**3 - 6*sqrt(a)*log((sqrt(a*c + a
*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a*b**3*d + 12*sqrt(a)*log((sqr
t(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a*b**2*c**2 + 3*sqrt(
a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b**3*c)/(2
4*a**3*d**3)
```

3.9 $\int \frac{A+Bx+Cx^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 228

$$\begin{aligned}\int \frac{A+Bx+Cx^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx &= \frac{(5b^2C+6ab(2cC-Bd)+8a^2(c^2C-Bcd+Ad^2))\,(c+dx)\sqrt{a+\frac{b}{c+dx}}}{8a^3d^3} \\ &\quad - \frac{(5bC+12acC-6aBd)(c+dx)^2\sqrt{a+\frac{b}{c+dx}}}{12a^2d^3} + \frac{C(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{3ad^3} \\ &\quad - \frac{b(5b^2C+6ab(2cC-Bd)+8a^2(c^2C-Bcd+Ad^2))\,\text{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{8a^{7/2}d^3}\end{aligned}$$

output
```
1/8*(5*b^2*C+6*a*b*(-B*d+2*C*c)+8*a^2*(A*d^2-B*c*d+C*c^2))*(d*x+c)*(a+b/(d
*x+c))^(1/2)/a^3/d^3-1/12*(-6*B*a*d+12*C*a*c+5*C*b)*(d*x+c)^2*(a+b/(d*x+c)
)^(1/2)/a^2/d^3+1/3*C*(d*x+c)^3*(a+b/(d*x+c))^(1/2)/a/d^3-1/8*b*(5*b^2*C+6
*a*b*(-B*d+2*C*c)+8*a^2*(A*d^2-B*c*d+C*c^2))*arctanh((a+b/(d*x+c))^(1/2)/a
^(1/2))/a^(7/2)/d^3
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$
$$= \frac{\sqrt{a}(c+dx)\sqrt{\frac{b+ac+adx}{c+dx}}(15b^2C+2ab(13cC-9Bd-5Cdx)+4a^2(2c^2C-cd(3B+2Cx)+d^2(6A+3B}{24a^{7/2}d^3}$$

input
```
Integrate[(A + B*x + C*x^2)/Sqrt[a + b/(c + d*x)],x]
```

output
```
(Sqrt[a]*(c + d*x)*Sqrt[(b + a*c + a*d*x)/(c + d*x)]*(15*b^2*C + 2*a*b*(13
*c*C - 9*B*d - 5*C*d*x) + 4*a^2*(2*c^2*C - c*d*(3*B + 2*C*x) + d^2*(6*A +
3*B*x + 2*C*x^2))) - 3*b*(5*b^2*C + 6*a*b*(2*c*C - B*d) + 8*a^2*(c^2*C - B
*c*d + A*d^2))*ArcTanh[Sqrt[(b + a*c + a*d*x)/(c + d*x)]/Sqrt[a]])/(24*a^(
7/2)*d^3)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {7268, 2087, 1471, 25, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$

$$\Big\downarrow \text{7268}$$

$$-\frac{2b\int \frac{(c+dx)^4\left(Cb^2-\frac{(2cC-Bd)b^2}{c+dx}+\frac{\left(Cc^2-Bdc+Ad^2\right)b^2}{(c+dx)^2}\right)}{b^4}d\sqrt{a+\frac{b}{c+dx}}}{d^3}$$

$$\Big\downarrow \text{2087}$$

$$-\frac{2b\int \frac{(c+dx)^4\left(\left(Cc^2-Bdc+Ad^2\right)a^2+b(2cC-Bd)a+\left(Cc^2-Bdc+Ad^2\right)\left(a+\frac{b}{c+dx}\right)^2+b^2C-\left(b(2cC-Bd)+2a\left(Cc^2-Bdc+Ad^2\right)\right)\left(a+\frac{b}{c+dx}\right)\right)}{b^4}d}{d^3}$$

$$\Big\downarrow \; 1471$$

$$-\frac{2b\left(-\frac{\int \frac{(c+dx)^3\left(6\left(Cc^2-Bdc+Ad^2\right)a^2+6b(2cC-Bd)a-6\left(Cc^2-Bdc+Ad^2\right)\left(a+\frac{b}{c+dx}\right)a+5b^2C\right)}{b^3}d\sqrt{a+\frac{b}{c+dx}}}{6a}-\frac{C(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{6ab}\right)}{d^3}$$

$$\Big\downarrow \; 25$$

$$-\frac{2b\left(\frac{\int -\frac{(c+dx)^3\left(6\left(Cc^2-Bdc+Ad^2\right)a^2+6b(2cC-Bd)a-6\left(Cc^2-Bdc+Ad^2\right)\left(a+\frac{b}{c+dx}\right)a+5b^2C\right)}{b^3}d\sqrt{a+\frac{b}{c+dx}}}{6a}-\frac{C(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{6ab}\right)}{d^3}$$

$$\Big\downarrow \; 298$$

$$-\frac{2b\left(\frac{\frac{3\left(8a^2\left(Ad^2-Bcd+c^2C\right)+6ab(2cC-Bd)+5b^2C\right)\int \frac{(c+dx)^2}{b^2}d\sqrt{a+\frac{b}{c+dx}}}{4a}+\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}(-6aBd+12acC+5bC)}{4ab}}{6a}-\frac{C(c+dx)^3\sqrt{a+\frac{b}{c+dx}}}{6ab}\right)}{d^3}$$

$$\Big\downarrow \; 215$$

$$-\frac{2b\left(\frac{\frac{3\left(8a^2\left(Ad^2-Bcd+c^2C\right)+6ab(2cC-Bd)+5b^2C\right)\left(\frac{\int -\frac{c+dx}{b}d\sqrt{a+\frac{b}{c+dx}}}{2a}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{2ab}\right)}{4a}+\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}(-6aBd+12acC+5bC)}{4ab}}{6a}-\frac{C(c+}{}\right)}{d^3}$$

$$\Big\downarrow \; 219$$

$$-\frac{2b\left(\frac{\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+\frac{b}{c+dx}}}{\sqrt{a}}\right)}{2a^{3/2}}-\frac{(c+dx)\sqrt{a+\frac{b}{c+dx}}}{2ab}\right)\left(8a^2\left(Ad^2-Bcd+c^2C\right)+6ab(2cC-Bd)+5b^2C\right)}{4a}+\frac{(c+dx)^2\sqrt{a+\frac{b}{c+dx}}(-6aBd+12acC+5bC)}{4ab}}{6a}-\cdots\right)}{d^3}$$

input
```
Int[(A + B*x + C*x^2)/Sqrt[a + b/(c + d*x)],x]
```

output
```
(-2*b*(-1/6*(C*(c + d*x)^3*Sqrt[a + b/(c + d*x)])/(a*b) + (((5*b*C + 12*a*
c*C - 6*a*B*d)*(c + d*x)^2*Sqrt[a + b/(c + d*x)])/(4*a*b) + (3*(5*b^2*C +
6*a*b*(2*c*C - B*d) + 8*a^2*(c^2*C - B*c*d + A*d^2))*(-1/2*((c + d*x)*Sqrt
[a + b/(c + d*x)])/(a*b) + ArcTanh[Sqrt[a + b/(c + d*x)]/Sqrt[a]]/(2*a^(3/
2))))/(4*a))/(6*a)))/d^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2087
```
Int[(u_)^(q_.)*(v_)^(p_.), x_Symbol] :> Int[ExpandToSum[u, x]^q*ExpandToSum
[v, x]^p, x] /; FreeQ[{p, q}, x] && BinomialQ[u, x] && TrinomialQ[v, x] &&
 !(BinomialMatchQ[u, x] && TrinomialMatchQ[v, x])
```

rule 7268
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfQuotientOfLinears
[u, x]}, Simp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/ls
t[[2]])], x] /;  !FalseQ[lst]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1222 vs. $2(208) = 416$.

Time = 0.24 (sec) , antiderivative size = 1223, normalized size of antiderivative = 5.36

method	result	size
default	Expression too large to display	1223

input
```
int((C*x^2+B*x+A)/(a+b/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*((a*d*x+a*c+b)/(d*x+c))^(1/2)*(d*x+c)/a^3/d^3*(-24*A*ln(1/2*(2*a*d^2*
x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2)
)*a^2*b*d^3+24*B*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)
*a^2*d^2*x+24*B*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*
(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b*c*d^2-48*C*(a*d^2)^(1/2)*(a*d^2*x^
2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^2*c*d*x-24*C*ln(1/2*(2*a*d^2*x+2*a*c*
d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a^2*b*
c^2*d+48*A*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^2*d^2-48*B*((a*d*
x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)*a^2*c*d+24*B*(a*d^2)^(1/2)*(a*d^2*x^
2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*a^2*c*d-6*B*ln(1/2*(2*a*d^2*x+2*a*c*d+2
*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1
/2))*a*b^2*d^2+24*B*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1
/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*d^2+48*C*((a*d*x+a*c+b)*(d*x+c
))^(1/2)*(a*d^2)^(1/2)*a^2*c^2-48*C*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c
^2+b*d*x+b*c)^(1/2)*a^2*c^2-36*C*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+
b*d*x+b*c)^(1/2)*a*b*d*x+12*C*ln(1/2*(2*a*d^2*x+2*a*c*d+2*(a*d^2*x^2+2*a*c
*d*x+a*c^2+b*d*x+b*c)^(1/2)*(a*d^2)^(1/2)+b*d)/(a*d^2)^(1/2))*a*b^2*c*d-48
*C*ln(1/2*(2*a*d^2*x+2*a*c*d+2*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^(1/2)
+b*d)/(a*d^2)^(1/2))*a*b^2*c*d-48*B*((a*d*x+a*c+b)*(d*x+c))^(1/2)*(a*d^2)^
(1/2)*a*b*d+12*B*(a*d^2)^(1/2)*(a*d^2*x^2+2*a*c*d*x+a*c^2+b*d*x+b*c)^(1...
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.35

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$

$$= \left[\frac{3\left(8\,Ca^2bc^2+8\,Aa^2bd^2+12\,Cab^2c+5\,Cb^3-2\left(4\,Ba^2bc+3\,Bab^2\right)d\right)\sqrt{a}\log\left(2\,adx+2\,ac-2\,(dx+c)\right.}{} \right.$$

input
```
integrate((C*x^2+B*x+A)/(a+b/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/48*(3*(8*C*a^2*b*c^2 + 8*A*a^2*b*d^2 + 12*C*a*b^2*c + 5*C*b^3 - 2*(4*B*
a^2*b*c + 3*B*a*b^2)*d)*sqrt(a)*log(2*a*d*x + 2*a*c - 2*(d*x + c)*sqrt(a)*
sqrt((a*d*x + a*c + b)/(d*x + c)) + b) + 2*(8*C*a^3*d^3*x^3 + 8*C*a^3*c^3
+ 24*A*a^3*c*d^2 + 26*C*a^2*b*c^2 + 15*C*a*b^2*c + 2*(6*B*a^3*d^3 - 5*C*a^
2*b*d^2)*x^2 - 6*(2*B*a^3*c^2 + 3*B*a^2*b*c)*d + (24*A*a^3*d^3 - 18*B*a^2*
b*d^2 + (16*C*a^2*b*c + 15*C*a*b^2)*d)*x)*sqrt((a*d*x + a*c + b)/(d*x + c)
))/(a^4*d^3), 1/24*(3*(8*C*a^2*b*c^2 + 8*A*a^2*b*d^2 + 12*C*a*b^2*c + 5*C*
b^3 - 2*(4*B*a^2*b*c + 3*B*a*b^2)*d)*sqrt(-a)*arctan((d*x + c)*sqrt(-a)*sq
rt((a*d*x + a*c + b)/(d*x + c))/(a*d*x + a*c + b)) + (8*C*a^3*d^3*x^3 + 8*
C*a^3*c^3 + 24*A*a^3*c*d^2 + 26*C*a^2*b*c^2 + 15*C*a*b^2*c + 2*(6*B*a^3*d^
3 - 5*C*a^2*b*d^2)*x^2 - 6*(2*B*a^3*c^2 + 3*B*a^2*b*c)*d + (24*A*a^3*d^3 -
 18*B*a^2*b*d^2 + (16*C*a^2*b*c + 15*C*a*b^2)*d)*x)*sqrt((a*d*x + a*c + b)
/(d*x + c)))/(a^4*d^3)]
```

Sympy [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx=\int \frac{A+Bx+Cx^2}{\sqrt{\frac{ac+adx+b}{c+dx}}}\,dx$$

input
```
integrate((C*x**2+B*x+A)/(a+b/(d*x+c))**(1/2),x)
```

output
```
Integral((A + B*x + C*x**2)/sqrt((a*c + a*d*x + b)/(c + d*x)), x)
```

Maxima [F]

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx=\int \frac{Cx^2+Bx+A}{\sqrt{a+\frac{b}{dx+c}}}\,dx$$

input
```
integrate((C*x^2+B*x+A)/(a+b/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)/sqrt(a + b/(d*x + c)), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.50

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx$$
$$= \frac{1}{24}\sqrt{ad^2x^2+2\,acdx+ac^2+bdx+bc}\left(2\,x\left(\frac{4\,Cx}{ad\mathrm{sgn}\,(dx+c)}-\frac{4\,Ca^2cd^3\mathrm{sgn}(dx+c)-6\,Ba^2d^4\mathrm{sgn}(dx+\right.\right.}{a^3d^5}$$
$$+\frac{(8\,Ca^2bc^2-8\,Ba^2bcd+8\,Aa^2bd^2+12\,Cab^2c-6\,Bab^2d+5\,Cb^3)\log\left(\left|2\,acd+2\left(\sqrt{ad^2}x-\sqrt{ad^2x^2+\right.\right.\right.}{16\,a^{\frac{7}{2}}d^2|d|\mathrm{sgn}\,(dx+c)}$$

input
```
integrate((C*x^2+B*x+A)/(a+b/(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + b*d*x + b*c)*(2*x*(4*C*x/(a*d*sg
n(d*x + c)) - (4*C*a^2*c*d^3*sgn(d*x + c) - 6*B*a^2*d^4*sgn(d*x + c) + 5*C
*a*b*d^3*sgn(d*x + c))/(a^3*d^5)) + (8*C*a^2*c^2*d^2*sgn(d*x + c) - 12*B*a
^2*c*d^3*sgn(d*x + c) + 24*A*a^2*d^4*sgn(d*x + c) + 26*C*a*b*c*d^2*sgn(d*x
 + c) - 18*B*a*b*d^3*sgn(d*x + c) + 15*C*b^2*d^2*sgn(d*x + c))/(a^3*d^5))
+ 1/16*(8*C*a^2*b*c^2 - 8*B*a^2*b*c*d + 8*A*a^2*b*d^2 + 12*C*a*b^2*c - 6*B
*a*b^2*d + 5*C*b^3)*log(abs(2*a*c*d + 2*(sqrt(a*d^2)*x - sqrt(a*d^2*x^2 +
2*a*c*d*x + a*c^2 + b*d*x + b*c))*sqrt(a)*abs(d) + b*d))/(a^(7/2)*d^2*abs(
d)*sgn(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx+Cx^2}{\sqrt{a+\frac{b}{c+dx}}}\,dx = \int \frac{C\,x^2+B\,x+A}{\sqrt{a+\frac{b}{c+d\,x}}}\,dx$$

input
```
int((A + B*x + C*x^2)/(a + b/(c + d*x))^(1/2),x)
```

output
```
int((A + B*x + C*x^2)/(a + b/(c + d*x))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.09

$$\int \frac{A + Bx + Cx^2}{\sqrt{a + \frac{b}{c+dx}}}\,dx$$
$$= \frac{24\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^4d^2 - 12\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^3bcd + 12\sqrt{dx+c}\,\sqrt{adx+ac+b}\,a^3b\,d^2x -}{}$$

input
```
int((C*x^2+B*x+A)/(a+b/(d*x+c))^(1/2),x)
```

output
```
(24*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**4*d**2 - 12*sqrt(c + d*x)*sqrt(
a*c + a*d*x + b)*a**3*b*c*d + 12*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*
b*d**2*x + 8*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**3*c**3 - 8*sqrt(c + d*
x)*sqrt(a*c + a*d*x + b)*a**3*c**2*d*x + 8*sqrt(c + d*x)*sqrt(a*c + a*d*x
+ b)*a**3*c*d**2*x**2 - 18*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b**2*d
 + 26*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)*a**2*b*c**2 - 10*sqrt(c + d*x)*s
qrt(a*c + a*d*x + b)*a**2*b*c*d*x + 15*sqrt(c + d*x)*sqrt(a*c + a*d*x + b)
*a*b**2*c - 24*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))
/sqrt(b))*a**3*b*d**2 + 24*sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sq
rt(c + d*x))/sqrt(b))*a**2*b**2*c*d - 24*sqrt(a)*log((sqrt(a*c + a*d*x + b
) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a**2*b*c**3 + 18*sqrt(a)*log((sqrt(a*c
 + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a*b**3*d - 36*sqrt(a)*log(
(sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*a*b**2*c**2 - 15*
sqrt(a)*log((sqrt(a*c + a*d*x + b) + sqrt(a)*sqrt(c + d*x))/sqrt(b))*b**3*
c)/(24*a**4*d**3)
```

3.10 $\int (A+Bx+Cx^2)\left(a+\frac{b}{c+dx}\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 188

$$\int (A+Bx+Cx^2)\left(a+\frac{b}{c+dx}\right)^p\,dx$$
$$= -\frac{(a(6cC-3Bd)+bC(2-p))(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{1+p}}{6a^2d^3}+\frac{C(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{1+p}}{3ad^3}$$
$$-\frac{b(6a^2(c^2C-Bcd+Ad^2)+b(a(6cC-3Bd)+bC(2-p))(1-p))\left(a+\frac{b}{c+dx}\right)^{1+p}\text{Hypergeometric2F1}}{6a^4d^3(1+p)}$$

output
```
-1/6*(a*(-3*B*d+6*C*c)+b*C*(2-p))*(d*x+c)^2*(a+b/(d*x+c))^(p+1)/a^2/d^3+1/
3*C*(d*x+c)^3*(a+b/(d*x+c))^(p+1)/a/d^3-1/6*b*(6*a^2*(A*d^2-B*c*d+C*c^2)+b
*(a*(-3*B*d+6*C*c)+b*C*(2-p))*(1-p))*(a+b/(d*x+c))^(p+1)*hypergeom([2, p+1
],[2+p],1+b/a/(d*x+c))/a^4/d^3/(p+1)
```

Mathematica [F]

$$\int (A + Bx + Cx^2)\left(a + \frac{b}{c + dx}\right)^p dx = \int (A + Bx + Cx^2)\left(a + \frac{b}{c + dx}\right)^p dx$$

input
```
Integrate[(A + B*x + C*x^2)*(a + b/(c + d*x))^p,x]
```

output
```
Integrate[(A + B*x + C*x^2)*(a + b/(c + d*x))^p, x]
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.65, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A + Bx + Cx^2)\left(a + \frac{b}{c + dx}\right)^p dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(A\left(a + \frac{b}{c + dx}\right)^p + Bx\left(a + \frac{b}{c + dx}\right)^p + Cx^2\left(a + \frac{b}{c + dx}\right)^p\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{bB(2ac+b(-p)+b)\left(a+\frac{b}{c+dx}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{b}{a(c+dx)}+1\right)}{2a^3d^2(p+1)}-\frac{Ab\left(a+\frac{b}{c+dx}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{b}{a(c+dx)}+1\right)}{a^2d(p+1)}-\frac{C(c+dx)^2(6ac+b(2-p))\left(a+\frac{b}{c+dx}\right)^{p+1}}{6a^2d^3}-\frac{bC\left(6a^2c^2+b(1-p)(6ac+b(2-p))\right)\left(a+\frac{b}{c+dx}\right)^{p+1}\text{Hypergeometric2F1}\left(2,p+1,p+2,\frac{b}{a(c+dx)}+1\right)}{6a^4d^3(p+1)}+\frac{B(c+dx)^2\left(a+\frac{b}{c+dx}\right)^{p+1}}{2ad^2}+\frac{C(c+dx)^3\left(a+\frac{b}{c+dx}\right)^{p+1}}{3ad^3}$$

input
```
Int[(A + B*x + C*x^2)*(a + b/(c + d*x))^p,x]
```

output
```
(B*(c + d*x)^2*(a + b/(c + d*x))^(1 + p))/(2*a*d^2) - (C*(6*a*c + b*(2 - p
))*(c + d*x)^2*(a + b/(c + d*x))^(1 + p))/(6*a^2*d^3) + (C*(c + d*x)^3*(a
+ b/(c + d*x))^(1 + p))/(3*a*d^3) - (A*b*(a + b/(c + d*x))^(1 + p)*Hyperge
ometric2F1[2, 1 + p, 2 + p, 1 + b/(a*(c + d*x))])/(a^2*d*(1 + p)) - (b*C*(
6*a^2*c^2 + b*(6*a*c + b*(2 - p))*(1 - p))*(a + b/(c + d*x))^(1 + p)*Hyper
geometric2F1[2, 1 + p, 2 + p, 1 + b/(a*(c + d*x))])/(6*a^4*d^3*(1 + p)) +
(b*B*(b + 2*a*c - b*p)*(a + b/(c + d*x))^(1 + p)*Hypergeometric2F1[2, 1 +
p, 2 + p, 1 + b/(a*(c + d*x))])/(2*a^3*d^2*(1 + p))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [F]

$$\int \left(C\,x^{2}+Bx+A\right)\left(a+\frac{b}{dx+c}\right)^{p}dx$$

input
```
int((C*x^2+B*x+A)*(a+b/(d*x+c))^p,x)
```

output
```
int((C*x^2+B*x+A)*(a+b/(d*x+c))^p,x)
```

Fricas [F]

$$\int \left(A+Bx+Cx^2\right)\left(a+\frac{b}{c+dx}\right)^p\,dx=\int \left(Cx^2+Bx+A\right)\left(a+\frac{b}{dx+c}\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(a+b/(d*x+c))^p,x, algorithm="fricas")
```

output
```
integral((C*x^2 + B*x + A)*((a*d*x + a*c + b)/(d*x + c))^p, x)
```

Sympy [F]

$$\int \left(A+Bx+Cx^2\right)\left(a+\frac{b}{c+dx}\right)^p\,dx=\int \left(\frac{ac+adx+b}{c+dx}\right)^p\left(A+Bx+Cx^2\right)\,dx$$

input
```
integrate((C*x**2+B*x+A)*(a+b/(d*x+c))**p,x)
```

output
```
Integral(((a*c + a*d*x + b)/(c + d*x))**p*(A + B*x + C*x**2), x)
```

Maxima [F]

$$\int \left(A+Bx+Cx^2\right)\left(a+\frac{b}{c+dx}\right)^p\,dx=\int \left(Cx^2+Bx+A\right)\left(a+\frac{b}{dx+c}\right)^p\,dx$$

input
```
integrate((C*x^2+B*x+A)*(a+b/(d*x+c))^p,x, algorithm="maxima")
```

output
```
integrate((C*x^2 + B*x + A)*(a + b/(d*x + c))^p, x)
```

Giac [F(-2)]

Exception generated.

$$\int \left(A+Bx+Cx^2\right)\left(a+\frac{b}{c+dx}\right)^p\,dx=\text{Exception raised: RuntimeError}$$

input
```
integrate((C*x^2+B*x+A)*(a+b/(d*x+c))^p,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{-1,[
0,1,1,0]%%%} / %%%{1,[0,0,0,1]%%%} Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \left(A+Bx+Cx^2\right)\left(a+\frac{b}{c+dx}\right)^p\,dx=\int {\left(a+\frac{b}{c+d\,x}\right)}^p\,\left(C\,x^2+B\,x+A\right)\,dx$$

input
```
int((a + b/(c + d*x))^p*(A + B*x + C*x^2),x)
```

output
```
int((a + b/(c + d*x))^p*(A + B*x + C*x^2), x)
```

Reduce [F]

$$\int (A+Bx+Cx^2)\left(a+\frac{b}{c+dx}\right)^p dx = \left(\int \frac{(adx+ac+b)^p}{(dx+c)^p}dx\right)a + \left(\int \frac{(adx+ac+b)^p x^2}{(dx+c)^p}dx\right)c + \left(\int \frac{(adx+ac+b)^p x}{(dx+c)^p}dx\right)b$$

input
```
int((C*x^2+B*x+A)*(a+b/(d*x+c))^p,x)
```

output
```
int((a*c + a*d*x + b)**p/(c + d*x)**p,x)*a + int(((a*c + a*d*x + b)**p*x**
2)/(c + d*x)**p,x)*c + int(((a*c + a*d*x + b)**p*x)/(c + d*x)**p,x)*b
```

3.11 $\int \frac{1-x^2}{a-b(1-x^2)^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 490

$$\int \frac{1-x^2}{a-b(1-x^2)^4}\,dx = \frac{\arctan\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}b^{3/8}} + \frac{\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{a}\sqrt{\sqrt{a}+\sqrt{b}}b^{3/8}} - \frac{\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{a}\sqrt{\sqrt{a}+\sqrt{b}}b^{3/8}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}\right)}{4\sqrt{a}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}b^{3/8}} + \frac{\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}+\sqrt[4]{b}}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}+\sqrt[4]{b}}\sqrt[8]{b}x}{\sqrt{\sqrt{a}+\sqrt{b}}+\sqrt[4]{b}x^2}\right)}{4\sqrt{2}\sqrt{a}\sqrt{\sqrt{a}+\sqrt{b}}b^{3/8}}$$

output
```
1/4*arctan(b^(1/8)*x/(a^(1/4)-b^(1/4))^(1/2))/a^(1/2)/(a^(1/4)-b^(1/4))^(1
/2)/b^(3/8)+1/8*((a^(1/2)+b^(1/2))^(1/2)-b^(1/4))^(1/2)*arctan((((a^(1/2)+
b^(1/2))^(1/2)+b^(1/4))^(1/2)-2^(1/2)*b^(1/8)*x)/((a^(1/2)+b^(1/2))^(1/2)-
b^(1/4))^(1/2))*2^(1/2)/a^(1/2)/(a^(1/2)+b^(1/2))^(1/2)/b^(3/8)-1/8*((a^(1
/2)+b^(1/2))^(1/2)-b^(1/4))^(1/2)*arctan((((a^(1/2)+b^(1/2))^(1/2)+b^(1/4)
)^(1/2)+2^(1/2)*b^(1/8)*x)/((a^(1/2)+b^(1/2))^(1/2)-b^(1/4))^(1/2))*2^(1/2
)/a^(1/2)/(a^(1/2)+b^(1/2))^(1/2)/b^(3/8)-1/4*arctanh(b^(1/8)*x/(a^(1/4)+b
^(1/4))^(1/2))/a^(1/2)/(a^(1/4)+b^(1/4))^(1/2)/b^(3/8)+1/8*((a^(1/2)+b^(1/
2))^(1/2)+b^(1/4))^(1/2)*arctanh(2^(1/2)*((a^(1/2)+b^(1/2))^(1/2)+b^(1/4))
^(1/2)*b^(1/8)*x/((a^(1/2)+b^(1/2))^(1/2)+b^(1/4)*x^2))*2^(1/2)/a^(1/2)/(a
^(1/2)+b^(1/2))^(1/2)/b^(3/8)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.13

$$\int \frac{1-x^2}{a-b(1-x^2)^4}\,dx = \frac{\text{RootSum}\left[a-b+4b\#1^2-6b\#1^4+4b\#1^6-b\#1^8\&,\frac{\log(x-\#1)}{\#1-2\#1^3+\#1^5}\&\right]}{8b}$$

input
```
Integrate[(1 - x^2)/(a - b*(1 - x^2)^4),x]
```

output
```
RootSum[a - b + 4*b*#1^2 - 6*b*#1^4 + 4*b*#1^6 - b*#1^8 & , Log[x - #1]/(#
1 - 2*#1^3 + #1^5) & ]/(8*b)
```

Rubi [A] (verified)

Time = 1.96 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {7291, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{a-b(1-x^2)^4}\,dx$$

$$\Big\downarrow \text{7291}$$

$$\int \left(\frac{\sqrt{b}(1-x^2)}{2\sqrt{a}\left(\sqrt{a}\sqrt{b}-b(1-x^2)^2\right)}+\frac{\sqrt{b}(1-x^2)}{2\sqrt{a}\left(\sqrt{a}\sqrt{b}+b(1-x^2)^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\arctan\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}\right)}{4\sqrt{a}b^{3/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}+\frac{\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{a}b^{3/8}\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{a}b^{3/8}\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}\right)}{4\sqrt{a}b^{3/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}-\frac{\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{a}b^{3/8}\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{a}b^{3/8}\sqrt{\sqrt{a}+\sqrt{b}}}$$

input
```
Int[(1 - x^2)/(a - b*(1 - x^2)^4),x]
```

output
```
ArcTan[(b^(1/8)*x)/Sqrt[a^(1/4) - b^(1/4)]]/(4*Sqrt[a]*Sqrt[a^(1/4) - b^(1
/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt
[Sqrt[a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[a] + Sq
rt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/8)) -
(Sqrt[Sqrt[Sqrt[a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt[Sqrt[a] + Sqrt[
b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[a] + Sqrt[b]] - b^(1/4)
]])/(4*Sqrt[2]*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/8)) - ArcTanh[(b^(1/8)
*x)/Sqrt[a^(1/4) + b^(1/4)]]/(4*Sqrt[a]*Sqrt[a^(1/4) + b^(1/4)]*b^(3/8)) -
 (Sqrt[Sqrt[Sqrt[a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[a] + Sqrt[b]] - Sq
rt[2]*Sqrt[Sqrt[Sqrt[a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8
*Sqrt[2]*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[a] + S
qrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[a] + Sqrt[b]] + Sqrt[2]*Sqrt[Sqrt[Sqrt[a]
 + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[a]*Sqrt[S
qrt[a] + Sqrt[b]]*b^(3/8))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7291
```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.28 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.14

method	result	size
default	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-4\_Z^6b+6b\_Z^4-4b\_Z^2-a+b\right)}\frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{-\_R^7+3\_R^5-3\_R^3+\_R}}{8b}$	71
risch	$\frac{\sum_{\_R=\mathrm{RootOf}\left(b\_Z^8-4\_Z^6b+6b\_Z^4-4b\_Z^2-a+b\right)}\frac{\left(-\_R^2+1\right)\ln\left(x-\_R\right)}{-\_R^7+3\_R^5-3\_R^3+\_R}}{8b}$	71

input
```
int((-x^2+1)/(a-b*(-x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8/b*sum((-_R^2+1)/(-_R^7+3*_R^5-3*_R^3+_R)*ln(x-_R),_R=RootOf(_Z^8*b-4*_
Z^6*b+6*_Z^4*b-4*_Z^2*b-a+b))
```

Fricas [F(-1)]

Timed out.

$$\int \frac{1-x^2}{a-b\left(1-x^2\right)^4}\,dx=\text{Timed out}$$

input
```
integrate((-x^2+1)/(a-b*(-x^2+1)^4),x, algorithm="fricas")
```

output
```
Timed out
```

Sympy [A] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.27

$$\int \frac{1-x^2}{a-b(1-x^2)^4}\,dx$$
$$= \operatorname{RootSum}\left(t^8\cdot\left(16777216a^5b^3-16777216a^4b^4\right)+1048576t^6a^3b^3-24576t^4a^2b^2+256t^2ab-1,\ \left(t\mapsto t\log\right.\right.$$

input
```
integrate((-x**2+1)/(a-b*(-x**2+1)**4),x)
```

output
```
RootSum(_t**8*(16777216*a**5*b**3 - 16777216*a**4*b**4) + 1048576*_t**6*a*
*3*b**3 - 24576*_t**4*a**2*b**2 + 256*_t**2*a*b - 1, Lambda(_t, _t*log(-62
91456*_t**7*a**4*b**3 + 6291456*_t**7*a**3*b**4 - 65536*_t**5*a**3*b**2 -
327680*_t**5*a**2*b**3 - 512*_t**3*a**2*b + 5632*_t**3*a*b**2 - 32*_t*b +
x)))
```

Maxima [F]

$$\int \frac{1-x^2}{a-b(1-x^2)^4}\,dx = \int \frac{x^2-1}{(x^2-1)^4b-a}\,dx$$

input
```
integrate((-x^2+1)/(a-b*(-x^2+1)^4),x, algorithm="maxima")
```

output
```
integrate((x^2 - 1)/((x^2 - 1)^4*b - a), x)
```

Giac [F]

$$\int \frac{1-x^2}{a-b\left(1-x^2\right)^4}\,dx = \int \frac{x^2-1}{\left(x^2-1\right)^4 b - a}\,dx$$

input
```
integrate((-x^2+1)/(a-b*(-x^2+1)^4),x, algorithm="giac")
```

output
```
integrate((x^2 - 1)/((x^2 - 1)^4*b - a), x)
```

Mupad [B] (verification not implemented)

Time = 10.24 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.67

$$\int \frac{1-x^2}{a-b\left(1-x^2\right)^4}\,dx$$

$$= \sum_{k=1}^{8} \ln\Big(-a\,b^5\,\Big(\mathrm{root}\big(16777216\,a^5\,b^3\,z^8 - 16777216\,a^4\,b^4\,z^8 + 1048576\,a^3\,b^3\,z^6 - 24576\,a^2\,b^2\,z^4 + 256\,a\,b\,z^2$$

$$-1\Big)\,\Big(\mathrm{root}\big(16777216\,a^5\,b^3\,z^8 - 16777216\,a^4\,b^4\,z^8 + 1048576\,a^3\,b^3\,z^6 - 24576\,a^2\,b^2\,z^4 + 256\,a\,b\,z^2 - 1, z,$$

$$-\mathrm{root}\big(16777216\,a^5\,b^3\,z^8 - 16777216\,a^4\,b^4\,z^8 + 1048576\,a^3\,b^3\,z^6 - 24576\,a^2\,b^2\,z^4 + 256\,a\,b\,z^2 - 1, z, k\big)^2$$

$$-\mathrm{root}\big(16777216\,a^5\,b^3\,z^8 - 16777216\,a^4\,b^4\,z^8 + 1048576\,a^3\,b^3\,z^6 - 24576\,a^2\,b^2\,z^4 + 256\,a\,b\,z^2 - 1, z, k\big)^5$$

$$+1\Big)\Big)\,\mathrm{root}\big(16777216\,a^5\,b^3\,z^8 - 16777216\,a^4\,b^4\,z^8 + 1048576\,a^3\,b^3\,z^6 - 24576\,a^2\,b^2\,z^4$$

$$+ 256\,a\,b\,z^2 - 1, z, k\big)$$

input
```
int(-(x^2 - 1)/(a - b*(x^2 - 1)^4),x)
```

output
```
symsum(log(-a*b^5*(64*root(16777216*a^5*b^3*z^8 - 16777216*a^4*b^4*z^8 + 1
048576*a^3*b^3*z^6 - 24576*a^2*b^2*z^4 + 256*a*b*z^2 - 1, z, k)^2*a*b - 1)
*(4096*root(16777216*a^5*b^3*z^8 - 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*
z^6 - 24576*a^2*b^2*z^4 + 256*a*b*z^2 - 1, z, k)^4*a^2*b^2 - 128*root(1677
7216*a^5*b^3*z^8 - 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 - 24576*a^2*
b^2*z^4 + 256*a*b*z^2 - 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 -
 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 - 24576*a^2*b^2*z^4 + 256*a*b*
z^2 - 1, z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 - 16777216*a^4*
b^4*z^8 + 1048576*a^3*b^3*z^6 - 24576*a^2*b^2*z^4 + 256*a*b*z^2 - 1, z, k)
, k, 1, 8)
```

Reduce [F]

$$\int \frac{1-x^2}{a-b\left(1-x^2\right)^4}\,dx=\int \frac{-x^2+1}{a-b\left(-x^2+1\right)^4}dx$$

input
```
int((-x^2+1)/(a-b*(-x^2+1)^4),x)
```

output
```
int((-x^2+1)/(a-b*(-x^2+1)^4),x)
```

3.12 $\int \frac{1-x^2}{a+b\left(1-x^2\right)^4} \, dx$

Optimal result

Integrand size = 23, antiderivative size = 1351

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4} \, dx = \text{Too large to display}$$

output
```
-1/8*arctan(((-2^(1/2)*a^(1/4)+2*(a^(1/2)-2^(1/2)*a^(1/4)*b^(1/4)+b^(1/2))
^(1/2)+2*b^(1/4))^(1/2)-2*b^(1/8)*x)/(2^(1/2)*a^(1/4)+2*(a^(1/2)-2^(1/2)*a
^(1/4)*b^(1/4)+b^(1/2))^(1/2)-2*b^(1/4))^(1/2))*2^(1/2)/a^(1/4)/(2^(1/2)*a
^(1/4)+2*(a^(1/2)-2^(1/2)*a^(1/4)*b^(1/4)+b^(1/2))^(1/2)-2*b^(1/4))^(1/2)/
(a^(1/2)-2^(1/2)*a^(1/4)*b^(1/4)+b^(1/2))^(1/2)/b^(3/8)+1/8*arctan(((-2^(1
/2)*a^(1/4)+2*(a^(1/2)-2^(1/2)*a^(1/4)*b^(1/4)+b^(1/2))^(1/2)+2*b^(1/4))^(
1/2)+2*b^(1/8)*x)/(2^(1/2)*a^(1/4)+2*(a^(1/2)-2^(1/2)*a^(1/4)*b^(1/4)+b^(1
/2))^(1/2)-2*b^(1/4))^(1/2))*2^(1/2)/a^(1/4)/(2^(1/2)*a^(1/4)+2*(a^(1/2)-2
^(1/2)*a^(1/4)*b^(1/4)+b^(1/2))^(1/2)-2*b^(1/4))^(1/2)/(a^(1/2)-2^(1/2)*a^
(1/4)*b^(1/4)+b^(1/2))^(1/2)/b^(3/8)+1/8*arctan((2^(1/2)*a^(1/4)-2*(a^(1/2
)-2^(1/2)*a^(1/4)*b^(1/4)+b^(1/2))^(1/2)-2*b^(1/4))^(1/2)*b^(1/8)*x/((a^(1
/2)-2^(1/2)*a^(1/4)*b^(1/4)+b^(1/2))^(1/2)+b^(1/4)*x^2))*2^(1/2)/a^(1/4)/(
2^(1/2)*a^(1/4)-2*(a^(1/2)-2^(1/2)*a^(1/4)*b^(1/4)+b^(1/2))^(1/2)-2*b^(1/4
))^(1/2)/(a^(1/2)-2^(1/2)*a^(1/4)*b^(1/4)+b^(1/2))^(1/2)/b^(3/8)-1/8*arcta
nh(((2^(1/2)*a^(1/4)+2*(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)+b^(1/2))^(1/2)+2*b
^(1/4))^(1/2)-2*b^(1/8)*x)/(2^(1/2)*a^(1/4)-2*(a^(1/2)+2^(1/2)*a^(1/4)*b^(
1/4)+b^(1/2))^(1/2)+2*b^(1/4))^(1/2))*2^(1/2)/a^(1/4)/(2^(1/2)*a^(1/4)-2*(
a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)+b^(1/2))^(1/2)+2*b^(1/4))^(1/2)/(a^(1/2)+2
^(1/2)*a^(1/4)*b^(1/4)+b^(1/2))^(1/2)/b^(3/8)+1/8*arctanh(((2^(1/2)*a^(1/4
)+2*(a^(1/2)+2^(1/2)*a^(1/4)*b^(1/4)+b^(1/2))^(1/2)+2*b^(1/4))^(1/2)+2*...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.05

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx = -\frac{\text{RootSum}\left[a+b-4b\#1^2+6b\#1^4-4b\#1^6+b\#1^8\&,\frac{\log(x-\#1)}{\#1-2\#1^3+\#1^5}\&\right]}{8b}$$

input
```
Integrate[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

output
```
-1/8*RootSum[a + b - 4*b*#1^2 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , Log[x - #
1]/(#1 - 2*#1^3 + #1^5) & ]/b
```

Rubi [A] (verified)

Time = 2.39 (sec) , antiderivative size = 663, normalized size of antiderivative = 0.49, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {7291, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x^2}{a+b(1-x^2)^4} \, dx$$

$$\Big\downarrow \text{7291}$$

$$\int \left(-\frac{\sqrt{b}(1-x^2)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}-b(1-x^2)^2\right)}-\frac{\sqrt{b}(1-x^2)}{2\sqrt{-a}\left(\sqrt{-a}\sqrt{b}+b(1-x^2)^2\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\arctan\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}-\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}\arctan\left(\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{2}\sqrt[8]{b}x}{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}-\sqrt[4]{b}}}\right)}{4\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}+\frac{\text{arctanh}\left(\frac{\sqrt[8]{b}x}{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}\right)}{4\sqrt{-a}b^{3/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}+\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(-\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}-\frac{\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}\log\left(\sqrt{2}\sqrt[8]{b}x\sqrt{\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}}+\sqrt{\sqrt{-a}+\sqrt{b}}+\sqrt[4]{b}x^2\right)}{8\sqrt{2}\sqrt{-a}b^{3/8}\sqrt{\sqrt{-a}+\sqrt{b}}}$$

input
```
Int[(1 - x^2)/(a + b*(1 - x^2)^4),x]
```

output
```
-1/4*ArcTan[(b^(1/8)*x)/Sqrt[(-a)^(1/4) - b^(1/4)]]/(Sqrt[-a]*Sqrt[(-a)^(1
/4) - b^(1/4)]*b^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan
[(Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)] - Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[
Sqrt[-a] + Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[
b]]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] - b^(1/4)]*ArcTan[(Sqrt[Sqrt
[Sqrt[-a] + Sqrt[b]] + b^(1/4)] + Sqrt[2]*b^(1/8)*x)/Sqrt[Sqrt[Sqrt[-a] +
Sqrt[b]] - b^(1/4)]])/(4*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8)
) + ArcTanh[(b^(1/8)*x)/Sqrt[(-a)^(1/4) + b^(1/4)]]/(4*Sqrt[-a]*Sqrt[(-a)^
(1/4) + b^(1/4)]*b^(3/8)) + (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[
Sqrt[Sqrt[-a] + Sqrt[b]] - Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)
]*b^(1/8)*x + b^(1/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b
^(3/8)) - (Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*Log[Sqrt[Sqrt[-a] + Sq
rt[b]] + Sqrt[2]*Sqrt[Sqrt[Sqrt[-a] + Sqrt[b]] + b^(1/4)]*b^(1/8)*x + b^(1
/4)*x^2])/(8*Sqrt[2]*Sqrt[-a]*Sqrt[Sqrt[-a] + Sqrt[b]]*b^(3/8))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7291
```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.05

method	result	size
default	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{8}-4\_Z^{6}b+6b\_Z^{4}-4b\_Z^{2}+a+b\right)}\frac{\left(-\_R^{2}+1\right)\ln\left(x-\_R\right)}{\_R^{7}-3\_R^{5}+3\_R^{3}-\_R}}{8b}$	69
risch	$\frac{\sum_{\_R=\operatorname{RootOf}\left(b\_Z^{8}-4\_Z^{6}b+6b\_Z^{4}-4b\_Z^{2}+a+b\right)}\frac{\left(-\_R^{2}+1\right)\ln\left(x-\_R\right)}{\_R^{7}-3\_R^{5}+3\_R^{3}-\_R}}{8b}$	69

input
```
int((-x^2+1)/(a+b*(-x^2+1)^4),x,method=_RETURNVERBOSE)
```

output
```
1/8/b*sum((-_R^2+1)/(_R^7-3*_R^5+3*_R^3-_R)*ln(x-_R),_R=RootOf(_Z^8*b-4*_Z
^6*b+6*_Z^4*b-4*_Z^2*b+a+b))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.45 (sec) , antiderivative size = 322185, normalized size of antiderivative = 238.48

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.10

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx =$$
$$-\operatorname{RootSum}\left(t^8\cdot\left(16777216a^5b^3+16777216a^4b^4\right)+1048576t^6a^3b^3+24576t^4a^2b^2+256t^2ab+1,\left(t\mapsto t\right.\right.$$

input
```
integrate((-x**2+1)/(a+b*(-x**2+1)**4),x)
```

output
```
-RootSum(_t**8*(16777216*a**5*b**3 + 16777216*a**4*b**4) + 1048576*_t**6*a
**3*b**3 + 24576*_t**4*a**2*b**2 + 256*_t**2*a*b + 1, Lambda(_t, _t*log(-6
291456*_t**7*a**4*b**3 - 6291456*_t**7*a**3*b**4 + 65536*_t**5*a**3*b**2 -
 327680*_t**5*a**2*b**3 - 512*_t**3*a**2*b - 5632*_t**3*a*b**2 - 32*_t*b +
 x)))
```

Maxima [F]

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx = \int -\frac{x^2-1}{\left(x^2-1\right)^4 b + a}\,dx$$

input
```
integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="maxima")
```

output
```
-integrate((x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

Giac [F]

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx = \int -\frac{x^2-1}{\left(x^2-1\right)^4 b + a}\,dx$$

input
```
integrate((-x^2+1)/(a+b*(-x^2+1)^4),x, algorithm="giac")
```

output
```
integrate(-(x^2 - 1)/((x^2 - 1)^4*b + a), x)
```

Mupad [B] (verification not implemented)

Time = 10.26 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.24

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx$$

$$= \sum_{k=1}^{8} \ln\Bigg(a\,b^5\,\Bigg(\mathrm{root}\big(16777216\,a^5\,b^3\,z^8 + 16777216\,a^4\,b^4\,z^8 + 1048576\,a^3\,b^3\,z^6 + 24576\,a^2\,b^2\,z^4 + 256\,a\,b\,z^2 -$$

$$+1\Bigg)\,\Bigg(\mathrm{root}\big(16777216\,a^5\,b^3\,z^8 + 16777216\,a^4\,b^4\,z^8 + 1048576\,a^3\,b^3\,z^6 + 24576\,a^2\,b^2\,z^4 + 256\,a\,b\,z^2 + 1, z,$$

$$+\mathrm{root}\big(16777216\,a^5\,b^3\,z^8 + 16777216\,a^4\,b^4\,z^8 + 1048576\,a^3\,b^3\,z^6 + 24576\,a^2\,b^2\,z^4 + 256\,a\,b\,z^2 + 1, z, k\big)^2\,a$$

$$-\mathrm{root}\big(16777216\,a^5\,b^3\,z^8 + 16777216\,a^4\,b^4\,z^8 + 1048576\,a^3\,b^3\,z^6 + 24576\,a^2\,b^2\,z^4 + 256\,a\,b\,z^2 + 1, z, k\big)^5\,a$$

$$+1\Bigg)\Bigg)\,\mathrm{root}\big(16777216\,a^5\,b^3\,z^8 + 16777216\,a^4\,b^4\,z^8 + 1048576\,a^3\,b^3\,z^6 + 24576\,a^2\,b^2\,z^4$$

$$+256\,a\,b\,z^2 + 1, z, k\big)$$

input
```
int(-(x^2 - 1)/(a + b*(x^2 - 1)^4),x)
```

output
```
symsum(log(a*b^5*(64*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 10
48576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b + 1)*
(4096*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z
^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k)^4*a^2*b^2 + 128*root(16777
216*a^5*b^3*z^8 + 16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b
^2*z^4 + 256*a*b*z^2 + 1, z, k)^2*a*b - 32768*root(16777216*a^5*b^3*z^8 +
16777216*a^4*b^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z
^2 + 1, z, k)^5*a^3*b^2*x + 1))*root(16777216*a^5*b^3*z^8 + 16777216*a^4*b
^4*z^8 + 1048576*a^3*b^3*z^6 + 24576*a^2*b^2*z^4 + 256*a*b*z^2 + 1, z, k),
 k, 1, 8)
```

Reduce [F]

$$\int \frac{1-x^2}{a+b\left(1-x^2\right)^4}\,dx = -\left(\int \frac{x^2}{b\,x^8-4b\,x^6+6b\,x^4-4b\,x^2+a+b}dx\right) + \int \frac{1}{b\,x^8-4b\,x^6+6b\,x^4-4b\,x^2+a+b}dx$$

input
```
int((-x^2+1)/(a+b*(-x^2+1)^4),x)
```

output
```
 - int(x**2/(a + b*x**8 - 4*b*x**6 + 6*b*x**4 - 4*b*x**2 + b),x) + int(1/(
a + b*x**8 - 4*b*x**6 + 6*b*x**4 - 4*b*x**2 + b),x)
```

3.13 $\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2-x^2} dx$

Optimal result

Integrand size = 27, antiderivative size = 47

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2-x^2} dx = \frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\arctan\left(\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}}$$

output
```
(-x^2+1)*(1-1/(-x^2+1)^2)^(1/2)*arctan((x^2-2)^(1/2))/x/(x^2-2)^(1/2)
```

Mathematica [A] (verified)

Time = 6.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2-x^2} dx = -\frac{(-1+x^2)\sqrt{1-\frac{1}{(-1+x^2)^2}}\arctan\left(\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}}$$

input
```
Integrate[Sqrt[1 - (1 - x^2)^(-2)]/(2 - x^2),x]
```

output
```
-(((-1 + x^2)*Sqrt[1 - (-1 + x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2]])/(x*Sqrt[-2
 + x^2]))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {7273, 2467, 281, 353, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2-x^2} \, dx$$

$$\Big\downarrow \text{ 7273}$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}} \int \frac{\sqrt{(1-x^2)^2-1}}{(1-x^2)(2-x^2)}dx}{\sqrt{(1-x^2)^2-1}}$$

$$\Big\downarrow \text{ 2467}$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}} \int \frac{x\sqrt{x^2-2}}{(1-x^2)(2-x^2)}dx}{x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{ 281}$$

$$-\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}} \int \frac{x}{(1-x^2)\sqrt{x^2-2}}dx}{x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{ 353}$$

$$-\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}} \int \frac{1}{(1-x^2)\sqrt{x^2-2}}dx^2}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}} \int \frac{1}{-x^4-1}d\sqrt{x^2-2}}{x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{ 217}$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}} \arctan\left(\sqrt{x^2-2}\right)}{x\sqrt{x^2-2}}$$

input
```
Int[Sqrt[1 - (1 - x^2)^(-2)]/(2 - x^2),x]
```

output
```
((1 - x^2)*Sqrt[1 - (1 - x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2]])/(x*Sqrt[-2 + x
^2])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

rule 7273
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p])   Int[u*v^(n*p)*(b + a
/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bi
nomialQ[v, x] &&  !LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{\sqrt{\frac{x^{2}\left(x^{2}-2\right)}{\left(x^{2}-1\right)^{2}}}\,\left(x^{2}-1\right)\left(\arctan\left(\frac{x-2}{\sqrt{x^{2}-2}}\right)-\arctan\left(\frac{2+x}{\sqrt{x^{2}-2}}\right)\right)}{2x\sqrt{x^{2}-2}}$	63
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)x^{3}-2x^{2}\sqrt{-\frac{-x^{4}+2x^{2}}{x^{4}-2x^{2}+1}}-3\operatorname{RootOf}\left(\_Z^{2}+1\right)x+2\sqrt{-\frac{-x^{4}+2x^{2}}{x^{4}-2x^{2}+1}}}{x(x-1)(x+1)}\right)}{2}$	106

input
```
int((1-1/(-x^2+1)^2)^(1/2)/(-x^2+2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x^2*(x^2-2)/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((x-2)/(x^2-2)^(1/2))-ar
ctan((2+x)/(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2-x^2}\,dx=-\arctan\left(\frac{\left(x^{2}-1\right)\sqrt{\frac{x^{4}-2\,x^{2}}{x^{4}-2\,x^{2}+1}}}{x}\right)$$

input
```
integrate((1-1/(-x^2+1)^2)^(1/2)/(-x^2+2),x, algorithm="fricas")
```

output
```
-arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)
```

Sympy [F]

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2-x^2}\,dx = -\int \frac{\sqrt{\frac{x^4}{x^4-2x^2+1}-\frac{2x^2}{x^4-2x^2+1}}}{x^2-2}\,dx$$

input
```
integrate((1-1/(-x**2+1)**2)**(1/2)/(-x**2+2),x)
```

output
```
-Integral(sqrt(x**4/(x**4 - 2*x**2 + 1) - 2*x**2/(x**4 - 2*x**2 + 1))/(x**
2 - 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2-x^2}\,dx = \int -\frac{\sqrt{-\frac{1}{(x^2-1)^2}+1}}{x^2-2}\,dx$$

input
```
integrate((1-1/(-x^2+1)^2)^(1/2)/(-x^2+2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-1/(x^2 - 1)^2 + 1)/(x^2 - 2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2-x^2}\,dx = -\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)$$

input
```
integrate((1-1/(-x^2+1)^2)^(1/2)/(-x^2+2),x, algorithm="giac")
```

output
```
-arctan(sqrt(x^2 - 2))*sgn(x^3 - x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2-x^2}\,dx = \int -\frac{\sqrt{1-\frac{1}{(x^2-1)^2}}}{x^2-2}\,dx$$

input
```
int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2),x)
```

output
```
int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2-x^2}\,dx = -atan\left(\frac{\sqrt{x^2-2}\,x+x^2-2}{\sqrt{x^2-2}+x}\right)$$

input
```
int((1-1/(-x^2+1)^2)^(1/2)/(-x^2+2),x)
```

output
```
- atan((sqrt(x**2 - 2)*x + x**2 - 2)/(sqrt(x**2 - 2) + x))
```

3.14 $\int \frac{\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}}{2-x^2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 47

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx=\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\arctan\left(\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}}$$

output
```
(-x^2+1)*(1-1/(-x^2+1)^2)^(1/2)*arctan((x^2-2)^(1/2))/x/(x^2-2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx=-\frac{(-1+x^2)\sqrt{1-\frac{1}{(-1+x^2)^2}}\arctan\left(\sqrt{-2+x^2}\right)}{x\sqrt{-2+x^2}}$$

input
```
Integrate[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
```

output
```
-(((-1 + x^2)*Sqrt[1 - (-1 + x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2]])/(x*Sqrt[-2
 + x^2]))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {7273, 25, 2467, 281, 353, 73, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\frac{1}{(x^2-1)^2}}}{2-x^2} \, dx$$

$$\Big\downarrow \text{ 7273}$$

$$-\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int -\frac{\sqrt{(x^2-1)^2-1}}{(1-x^2)(2-x^2)}dx}{\sqrt{(x^2-1)^2-1}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{\sqrt{(x^2-1)^2-1}}{(1-x^2)(2-x^2)}dx}{\sqrt{(x^2-1)^2-1}}$$

$$\Big\downarrow \text{ 2467}$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{x\sqrt{x^2-2}}{(1-x^2)(2-x^2)}dx}{x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{ 281}$$

$$-\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{x}{(1-x^2)\sqrt{x^2-2}}dx}{x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{ 353}$$

$$-\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{1}{(1-x^2)\sqrt{x^2-2}}dx^2}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{1}{-x^4-1}d\sqrt{x^2-2}}{x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\arctan\left(\sqrt{x^2-2}\right)}{x\sqrt{x^2-2}}$$

input
```
Int[Sqrt[1 - (-1 + x^2)^(-2)]/(2 - x^2),x]
```

output
```
((1 - x^2)*Sqrt[1 - (1 - x^2)^(-2)]*ArcTan[Sqrt[-2 + x^2]])/(x*Sqrt[-2 + x
^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 281
```
Int[(u_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_
Symbol] :> Simp[(b/d)^p   Int[u*(c + d*x^n)^(p + q), x], x] /; FreeQ[{a, b,
 c, d, n, p, q}, x] && EqQ[b*c - a*d, 0] && IntegerQ[p] &&  !(IntegerQ[q] &
& SimplerQ[a + b*x^n, c + d*x^n])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

rule 7273
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p])   Int[u*v^(n*p)*(b + a
/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bi
nomialQ[v, x] &&  !LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.34

method	result	size
default	$-\frac{\sqrt{\frac{x^{2}\left(x^{2}-2\right)}{\left(x^{2}-1\right)^{2}}}\,\left(x^{2}-1\right)\left(\arctan\left(\frac{x-2}{\sqrt{x^{2}-2}}\right)-\arctan\left(\frac{2+x}{\sqrt{x^{2}-2}}\right)\right)}{2x\sqrt{x^{2}-2}}$	63
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)x^{3}-2x^{2}\sqrt{-\frac{-x^{4}+2x^{2}}{x^{4}-2x^{2}+1}}-3\operatorname{RootOf}\left(\_Z^{2}+1\right)x+2\sqrt{-\frac{-x^{4}+2x^{2}}{x^{4}-2x^{2}+1}}}{x(x-1)(x+1)}\right)}{2}$	106

input
```
int((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(x^2*(x^2-2)/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((x-2)/(x^2-2)^(1/2))-ar
ctan((2+x)/(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx=-\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}}{x}\right)$$

input
```
integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="fricas")
```

output
```
-arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)
```

Sympy [F]

$$\int \frac{\sqrt{1 - \frac{1}{(-1+x^2)^2}}}{2 - x^2}\,dx = -\int \frac{\sqrt{\frac{x^4}{x^4-2x^2+1} - \frac{2x^2}{x^4-2x^2+1}}}{x^2 - 2}\,dx$$

input
```
integrate((1-1/(x**2-1)**2)**(1/2)/(-x**2+2),x)
```

output
```
-Integral(sqrt(x**4/(x**4 - 2*x**2 + 1) - 2*x**2/(x**4 - 2*x**2 + 1))/(x**
2 - 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{1 - \frac{1}{(-1+x^2)^2}}}{2 - x^2}\,dx = \int -\frac{\sqrt{-\frac{1}{(x^2-1)^2} + 1}}{x^2 - 2}\,dx$$

input
```
integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="maxima")
```

output
```
-integrate(sqrt(-1/(x^2 - 1)^2 + 1)/(x^2 - 2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.38

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx = -\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)$$

input
```
integrate((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x, algorithm="giac")
```

output
```
-arctan(sqrt(x^2 - 2))*sgn(x^3 - x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx = \int -\frac{\sqrt{1-\frac{1}{(x^2-1)^2}}}{x^2-2}\,dx$$

input
```
int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2),x)
```

output
```
int(-(1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 - 2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2-x^2}\,dx = -atan\left(\frac{\sqrt{x^2-2}\,x+x^2-2}{\sqrt{x^2-2}+x}\right)$$

input
```
int((1-1/(x^2-1)^2)^(1/2)/(-x^2+2),x)
```

output
```
- atan((sqrt(x**2 - 2)*x + x**2 - 2)/(sqrt(x**2 - 2) + x))
```

3.15 $\int \frac{\sqrt{1-\frac{1}{\left(1-x^2\right)^2}}}{2+x^2} dx$

Optimal result

Integrand size = 25, antiderivative size = 105

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2+x^2} dx = -\frac{2(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\arctan\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\arctan\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

output
```
-2/3*(-x^2+1)*(1-1/(-x^2+1)^2)^(1/2)*arctan(1/2*(x^2-2)^(1/2))/x/(x^2-2)^(
1/2)+1/3*(-x^2+1)*(1-1/(-x^2+1)^2)^(1/2)*arctan((x^2-2)^(1/2))/x/(x^2-2)^(
1/2)
```

Mathematica [A] (verified)

Time = 6.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2+x^2}\,dx = \frac{(-1+x^2)\sqrt{1-\frac{1}{(-1+x^2)^2}}\left(\frac{2}{3}\arctan\left(\frac{1}{2}\sqrt{-2+x^2}\right)-\frac{1}{3}\arctan\left(\sqrt{-2+x^2}\right)\right)}{x\sqrt{-2+x^2}}$$

input
```
Integrate[Sqrt[1 - (1 - x^2)^(-2)]/(2 + x^2),x]
```

output
```
((-1 + x^2)*Sqrt[1 - (-1 + x^2)^(-2)]*((2*ArcTan[Sqrt[-2 + x^2]/2])/3 - Ar
cTan[Sqrt[-2 + x^2]]/3))/(x*Sqrt[-2 + x^2])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {7273, 2467, 435, 94, 73, 216, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{x^2+2}\,dx$$

$$\Big\downarrow \text{7273}$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{\sqrt{(1-x^2)^2-1}}{(1-x^2)(x^2+2)}dx}{\sqrt{(1-x^2)^2-1}}$$

$$\Big\downarrow \text{2467}$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{x\sqrt{x^2-2}}{(1-x^2)(x^2+2)}dx}{x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{435}$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{\sqrt{x^2-2}}{(1-x^2)(x^2+2)}dx^2}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{94}$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\left(-\frac{1}{3}\int \frac{1}{(1-x^2)\sqrt{x^2-2}}dx^2-\frac{4}{3}\int \frac{1}{\sqrt{x^2-2}(x^2+2)}dx^2\right)}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{73}$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\left(-\frac{2}{3}\int \frac{1}{-x^4-1}d\sqrt{x^2-2}-\frac{8}{3}\int \frac{1}{x^4+4}d\sqrt{x^2-2}\right)}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{216}$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\left(-\frac{2}{3}\int \frac{1}{-x^4-1}d\sqrt{x^2-2}-\frac{4}{3}\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \text{217}$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\left(\frac{2}{3}\arctan\left(\sqrt{x^2-2}\right)-\frac{4}{3}\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{2x\sqrt{x^2-2}}$$

input
```
Int[Sqrt[1 - (1 - x^2)^(-2)]/(2 + x^2),x]
```

output
```
((1 - x^2)*Sqrt[1 - (1 - x^2)^(-2)]*((-4*ArcTan[Sqrt[-2 + x^2]/2])/3 + (2*
ArcTan[Sqrt[-2 + x^2]])/3))/(2*x*Sqrt[-2 + x^2])
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

rule 7273
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p])   Int[u*v^(n*p)*(b + a
/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bi
nomialQ[v, x] &&  !LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

method	result
default	$-\frac{\sqrt{\frac{x^{2}\left(x^{2}-2\right)}{\left(x^{2}-1\right)^{2}}}\,\left(x^{2}-1\right)\left(\arctan\left(\frac{x-2}{\sqrt{x^{2}-2}}\right)-\arctan\left(\frac{2+x}{\sqrt{x^{2}-2}}\right)-4\arctan\left(\frac{\sqrt{x^{2}-2}}{2}\right)\right)}{6x\sqrt{x^{2}-2}}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)x^{7}-6\sqrt{-\frac{-x^{4}+2x^{2}}{x^{4}-2x^{2}+1}}\,x^{6}-15\operatorname{RootOf}\left(\_Z^{2}+1\right)x^{5}+22x^{4}\sqrt{-\frac{-x^{4}+2x^{2}}{x^{4}-2x^{2}+1}}+24\operatorname{RootOf}\left(\_Z^{2}+1\right.}{x(x-1)(x+1)\left(x^{2}+2\right)^{2}}\right.}{6}$

input
```
int((1-1/(-x^2+1)^2)^(1/2)/(x^2+2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(x^2*(x^2-2)/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((x-2)/(x^2-2)^(1/2))-ar
ctan((2+x)/(x^2-2)^(1/2))-4*arctan(1/2*(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2+x^2}\,dx=-\frac{1}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}}{x}\right)+\frac{2}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}}{2\,x}\right)$$

input
```
integrate((1-1/(-x^2+1)^2)^(1/2)/(x^2+2),x, algorithm="fricas")
```

output
```
-1/3*arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x) + 2/3*arcta
n(1/2*(x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)
```

Sympy [F]

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2+x^2}\,dx = \int \frac{\sqrt{\frac{x^2(x^2-2)}{x^4-2x^2+1}}}{x^2+2}\,dx$$

input
```
integrate((1-1/(-x**2+1)**2)**(1/2)/(x**2+2),x)
```

output
```
Integral(sqrt(x**2*(x**2 - 2)/(x**4 - 2*x**2 + 1))/(x**2 + 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2+x^2}\,dx = \int \frac{\sqrt{-\frac{1}{(x^2-1)^2}+1}}{x^2+2}\,dx$$

input
```
integrate((1-1/(-x^2+1)^2)^(1/2)/(x^2+2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-1/(x^2 - 1)^2 + 1)/(x^2 + 2), x)
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2+x^2}\,dx = \frac{2}{3}\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right) - \frac{1}{3}\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}\left(x^3-x\right)$$

input
```
integrate((1-1/(-x^2+1)^2)^(1/2)/(x^2+2),x, algorithm="giac")
```

output
```
2/3*arctan(1/2*sqrt(x^2 - 2))*sgn(x^3 - x) - 1/3*arctan(sqrt(x^2 - 2))*sgn
(x^3 - x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2+x^2}\,dx = \int \frac{\sqrt{1-\frac{1}{(x^2-1)^2}}}{x^2+2}\,dx$$

input
```
int((1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 + 2),x)
```

output
```
int((1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 + 2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{1-\frac{1}{(1-x^2)^2}}}{2+x^2}\,dx = -\frac{atan\left(\frac{\sqrt{x^2-2}\,x+x^2-2}{\sqrt{x^2-2}+x}\right)}{3} + \frac{2atan\left(\frac{\sqrt{x^2-2}\,x+x^2-2}{2\sqrt{x^2-2}+2x}\right)}{3}$$

input
```
int((1-1/(-x^2+1)^2)^(1/2)/(x^2+2),x)
```

output
```
( - atan((sqrt(x**2 - 2)*x + x**2 - 2)/(sqrt(x**2 - 2) + x)) + 2*atan((sqr
t(x**2 - 2)*x + x**2 - 2)/(2*sqrt(x**2 - 2) + 2*x)))/3
```

3.16 $\int \frac{\sqrt{1-\frac{1}{\left(-1+x^2\right)^2}}}{2+x^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2+x^2}\,dx = -\frac{2(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\arctan\left(\frac{1}{2}\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}} + \frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\arctan\left(\sqrt{-2+x^2}\right)}{3x\sqrt{-2+x^2}}$$

output
```
-2/3*(-x^2+1)*(1-1/(-x^2+1)^2)^(1/2)*arctan(1/2*(x^2-2)^(1/2))/x/(x^2-2)^(
1/2)+1/3*(-x^2+1)*(1-1/(-x^2+1)^2)^(1/2)*arctan((x^2-2)^(1/2))/x/(x^2-2)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.63

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2+x^2}\,dx = \frac{(-1+x^2)\sqrt{1-\frac{1}{(-1+x^2)^2}}\left(\frac{2}{3}\arctan\left(\frac{1}{2}\sqrt{-2+x^2}\right)-\frac{1}{3}\arctan\left(\sqrt{-2+x^2}\right)\right)}{x\sqrt{-2+x^2}}$$

input
```
Integrate[Sqrt[1 - (-1 + x^2)^(-2)]/(2 + x^2),x]
```

output
```
((-1 + x^2)*Sqrt[1 - (-1 + x^2)^(-2)]*((2*ArcTan[Sqrt[-2 + x^2]/2])/3 - Ar
cTan[Sqrt[-2 + x^2]]/3))/(x*Sqrt[-2 + x^2])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {7273, 25, 2467, 435, 94, 73, 216, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-\frac{1}{(x^2-1)^2}}}{x^2+2}\,dx$$

$$\Big\downarrow \text{7273}$$

$$-\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int -\frac{\sqrt{(x^2-1)^2-1}}{(1-x^2)(x^2+2)}dx}{\sqrt{(x^2-1)^2-1}}$$

$$\Big\downarrow \text{25}$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{\sqrt{(x^2-1)^2-1}}{(1-x^2)(x^2+2)}dx}{\sqrt{(x^2-1)^2-1}}$$

$$\Big\downarrow \; 2467$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{x\sqrt{x^2-2}}{(1-x^2)(x^2+2)}dx}{x\sqrt{x^2-2}}$$

$$\Big\downarrow \; 435$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\int \frac{\sqrt{x^2-2}}{(1-x^2)(x^2+2)}dx^2}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \; 94$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\left(-\frac{1}{3}\int \frac{1}{(1-x^2)\sqrt{x^2-2}}dx^2-\frac{4}{3}\int \frac{1}{\sqrt{x^2-2}(x^2+2)}dx^2\right)}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \; 73$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\left(-\frac{2}{3}\int \frac{1}{-x^4-1}d\sqrt{x^2-2}-\frac{8}{3}\int \frac{1}{x^4+4}d\sqrt{x^2-2}\right)}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \; 216$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\left(-\frac{2}{3}\int \frac{1}{-x^4-1}d\sqrt{x^2-2}-\frac{4}{3}\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{2x\sqrt{x^2-2}}$$

$$\Big\downarrow \; 217$$

$$\frac{(1-x^2)\sqrt{1-\frac{1}{(1-x^2)^2}}\left(\frac{2}{3}\arctan\left(\sqrt{x^2-2}\right)-\frac{4}{3}\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{2x\sqrt{x^2-2}}$$

input
```
Int[Sqrt[1 - (-1 + x^2)^(-2)]/(2 + x^2),x]
```

output
```
((1 - x^2)*Sqrt[1 - (1 - x^2)^(-2)]*((-4*ArcTan[Sqrt[-2 + x^2]/2])/3 + (2*
ArcTan[Sqrt[-2 + x^2]])/3))/(2*x*Sqrt[-2 + x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 435
```
Int[(x_)^(m_.)*((a_.) + (b_.)*(x_)^2)^(p_.)*((c_.) + (d_.)*(x_)^2)^(q_.)*((
e_.) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)
*(a + b*x)^p*(c + d*x)^q*(e + f*x)^r, x], x, x^2], x] /; FreeQ[{a, b, c, d,
 e, f, p, q, r}, x] && IntegerQ[(m - 1)/2]
```

rule 2467
```
Int[(Fx_.)*(Px_)^(p_), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[Px^F
racPart[p]/(x^(r*FracPart[p])*ExpandToSum[Px/x^r, x]^FracPart[p])   Int[x^(
p*r)*ExpandToSum[Px/x^r, x]^p*Fx, x], x] /; IGtQ[r, 0]] /; FreeQ[p, x] && P
olyQ[Px, x] &&  !IntegerQ[p] &&  !MonomialQ[Px, x] &&  !PolyQ[Fx, x]
```

rule 7273
```
Int[(u_.)*((a_.) + (b_.)*(v_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*v^n)^Fra
cPart[p]/(v^(n*FracPart[p])*(b + a/v^n)^FracPart[p])   Int[u*v^(n*p)*(b + a
/v^n)^p, x], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[p] && ILtQ[n, 0] && Bi
nomialQ[v, x] &&  !LinearQ[v, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.71

method	result
default	$-\frac{\sqrt{\frac{x^2(x^2-2)}{(x^2-1)^2}}\,(x^2-1)\left(\arctan\left(\frac{x-2}{\sqrt{x^2-2}}\right)-\arctan\left(\frac{2+x}{\sqrt{x^2-2}}\right)-4\arctan\left(\frac{\sqrt{x^2-2}}{2}\right)\right)}{6x\sqrt{x^2-2}}$
trager	$\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)x^7-6\sqrt{-\frac{-x^4+2x^2}{x^4-2x^2+1}}\,x^6-15\operatorname{RootOf}\left(\_Z^2+1\right)x^5+22x^4\sqrt{-\frac{-x^4+2x^2}{x^4-2x^2+1}}+24\operatorname{RootOf}\left(\_Z^2+1\right.}{x(x-1)(x+1)\left(x^2+2\right)^2}\right.}{6}$

input
```
int((1-1/(x^2-1)^2)^(1/2)/(x^2+2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(x^2*(x^2-2)/(x^2-1)^2)^(1/2)*(x^2-1)*(arctan((x-2)/(x^2-2)^(1/2))-ar
ctan((2+x)/(x^2-2)^(1/2))-4*arctan(1/2*(x^2-2)^(1/2)))/x/(x^2-2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.70

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2+x^2}\,dx = -\frac{1}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}}{x}\right) + \frac{2}{3}\arctan\left(\frac{(x^2-1)\sqrt{\frac{x^4-2\,x^2}{x^4-2\,x^2+1}}}{2\,x}\right)$$

input
```
integrate((1-1/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="fricas")
```

output
```
-1/3*arctan((x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x) + 2/3*arcta
n(1/2*(x^2 - 1)*sqrt((x^4 - 2*x^2)/(x^4 - 2*x^2 + 1))/x)
```

Sympy [F]

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2+x^2}\,dx = \int \frac{\sqrt{\frac{x^2(x^2-2)}{x^4-2x^2+1}}}{x^2+2}\,dx$$

input
```
integrate((1-1/(x**2-1)**2)**(1/2)/(x**2+2),x)
```

output
```
Integral(sqrt(x**2*(x**2 - 2)/(x**4 - 2*x**2 + 1))/(x**2 + 2), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2+x^2}\,dx = \int \frac{\sqrt{-\frac{1}{(x^2-1)^2}+1}}{x^2+2}\,dx$$

input
```
integrate((1-1/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-1/(x^2 - 1)^2 + 1)/(x^2 + 2), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.37

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2+x^2}\,dx = \frac{2}{3}\arctan\left(\frac{1}{2}\sqrt{x^2-2}\right)\operatorname{sgn}(x^3-x) - \frac{1}{3}\arctan\left(\sqrt{x^2-2}\right)\operatorname{sgn}(x^3-x)$$

input
```
integrate((1-1/(x^2-1)^2)^(1/2)/(x^2+2),x, algorithm="giac")
```

output
```
2/3*arctan(1/2*sqrt(x^2 - 2))*sgn(x^3 - x) - 1/3*arctan(sqrt(x^2 - 2))*sgn
(x^3 - x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2+x^2}\,dx = \int \frac{\sqrt{1-\frac{1}{(x^2-1)^2}}}{x^2+2}\,dx$$

input
```
int((1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 + 2),x)
```

output
```
int((1 - 1/(x^2 - 1)^2)^(1/2)/(x^2 + 2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{1-\frac{1}{(-1+x^2)^2}}}{2+x^2}\,dx = -\frac{atan\left(\frac{\sqrt{x^2-2}\,x+x^2-2}{\sqrt{x^2-2}+x}\right)}{3} + \frac{2atan\left(\frac{\sqrt{x^2-2}\,x+x^2-2}{2\sqrt{x^2-2}+2x}\right)}{3}$$

input
```
int((1-1/(x^2-1)^2)^(1/2)/(x^2+2),x)
```

output
```
( - atan((sqrt(x**2 - 2)*x + x**2 - 2)/(sqrt(x**2 - 2) + x)) + 2*atan((sqr
t(x**2 - 2)*x + x**2 - 2)/(2*sqrt(x**2 - 2) + 2*x)))/3
```

3.17 $\int x\left(1+\sqrt{1-x^2}\right)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

output
```
1/2*x^2-1/3*(-x^2+1)^(3/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.09

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=-\frac{1}{3}\left(1-x^2\right)^{3/2}+\frac{1}{2}\left(-1+x^2\right)$$

input
```
Integrate[x*(1 + Sqrt[1 - x^2]),x]
```

output
```
-1/3*(1 - x^2)^(3/2) + (-1 + x^2)/2
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(\sqrt{1-x^2}+1\right) \, dx$$

$$\Big\downarrow \text{ 2010}$$

$$\int \left(\sqrt{1-x^2}x+x\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{x^2}{2}-\frac{1}{3}\left(1-x^2\right)^{3/2}$$

input
```
Int[x*(1 + Sqrt[1 - x^2]),x]
```

output
```
x^2/2 - (1 - x^2)^(3/2)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{x^2}{2}-\frac{(-x^2+1)^{\frac{3}{2}}}{3}$	18
derivativedivides	$\frac{x^2}{2}-\frac{1}{2}-\frac{(-x^2+1)^{\frac{3}{2}}}{3}$	19
trager	$\frac{x^2}{2}+\left(\frac{x^2}{3}-\frac{1}{3}\right)\sqrt{-x^2+1}$	24
orering	$\frac{(4x^2-3)\left(\sqrt{-x^2+1}+1\right)}{6}-\frac{(x-1)(x+1)\left(\sqrt{-x^2+1}+1-\frac{x^2}{\sqrt{-x^2+1}}\right)}{6}$	55

input
```
int(x*((-x^2+1)^(1/2)+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*x^2-1/3*(-x^2+1)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{1}{2}\,x^2+\frac{1}{3}\,(x^2-1)\sqrt{-x^2+1}$$

input
```
integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*x^2 + 1/3*(x^2 - 1)*sqrt(-x^2 + 1)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.17

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{x^2\sqrt{1-x^2}}{3}+\frac{x^2}{2}-\frac{\sqrt{1-x^2}}{3}$$

input
```
integrate(x*(1+(-x**2+1)**(1/2)),x)
```

output
```
x**2*sqrt(1 - x**2)/3 + x**2/2 - sqrt(1 - x**2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.74

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{1}{2}\,x^2-\frac{1}{3}\,\left(-x^2+1\right)^{\frac{3}{2}}$$

input
```
integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
1/2*x^2 - 1/3*(-x^2 + 1)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{1}{2}\,x^2-\frac{1}{3}\,\left(-x^2+1\right)^{\frac{3}{2}}-\frac{1}{2}$$

input
```
integrate(x*(1+(-x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
1/2*x^2 - 1/3*(-x^2 + 1)^(3/2) - 1/2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{x^2}{2}+\sqrt{1-x^2}\left(\frac{x^2}{3}-\frac{1}{3}\right)$$

input
```
int(x*((1 - x^2)^(1/2) + 1),x)
```

output
```
x^2/2 + (1 - x^2)^(1/2)*(x^2/3 - 1/3)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26

$$\int x\left(1+\sqrt{1-x^2}\right)\,dx=\frac{\sqrt{-x^2+1}\,x^2}{3}-\frac{\sqrt{-x^2+1}}{3}+\frac{x^2}{2}$$

input
```
int(x*(1+(-x^2+1)^(1/2)),x)
```

output
```
(2*sqrt( - x**2 + 1)*x**2 - 2*sqrt( - x**2 + 1) + 3*x**2)/6
```

3.18 $\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx$

Optimal result

Integrand size = 27, antiderivative size = 42

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx = -\frac{1}{2x}+x+\frac{\sqrt{1+2x^2}}{2x}-\frac{\operatorname{arcsinh}\left(\sqrt{2}x\right)}{\sqrt{2}}$$

output
```
-1/2/x+x+1/2*(2*x^2+1)^(1/2)/x-1/2*arcsinh(x*2^(1/2))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx = \frac{-1+2x^2+\sqrt{1+2x^2}+\sqrt{2}x\log\left(-\sqrt{2}x+\sqrt{1+2x^2}\right)}{2x}$$

input
```
Integrate[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
```

output
```
(-1 + 2*x^2 + Sqrt[1 + 2*x^2] + Sqrt[2]*x*Log[-(Sqrt[2]*x) + Sqrt[1 + 2*x^
2]])/(2*x)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {7291, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x^2+1}}{\sqrt{2x^2+1}+1} dx$$

$$\Big\downarrow \text{7291}$$

$$\int \left(\frac{1}{-\sqrt{2x^2+1}-1}+1\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\operatorname{arcsinh}\left(\sqrt{2}x\right)}{\sqrt{2}}+\frac{\sqrt{2x^2+1}}{2x}+x-\frac{1}{2x}$$

input
```
Int[Sqrt[1 + 2*x^2]/(1 + Sqrt[1 + 2*x^2]),x]
```

output
```
-1/2*1/x + x + Sqrt[1 + 2*x^2]/(2*x) - ArcSinh[Sqrt[2]*x]/Sqrt[2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7291
```
Int[(v_)/((a_) + (b_.)*(u_)^(n_.)), x_Symbol] :> Int[ExpandIntegrand[Polyno
mialInSubst[v, u, x]/(a + b*x^n), x] /. x -> u, x] /; FreeQ[{a, b}, x] && I
GtQ[n, 0] && PolynomialInQ[v, u, x]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

method	result	size
default	$x-\frac{1}{2x}+\frac{\left(2x^{2}+1\right)^{\frac{3}{2}}}{2x}-x\sqrt{2x^{2}+1}-\frac{\operatorname{arcsinh}\left(x\sqrt{2}\right)\sqrt{2}}{2}$	45
trager	$\frac{(x-1)(1+2x)}{2x}+\frac{\sqrt{2x^{2}+1}}{2x}+\frac{\operatorname{RootOf}\left(\_Z^{2}-2\right)\ln\left(-\operatorname{RootOf}\left(\_Z^{2}-2\right)x+\sqrt{2x^{2}+1}\right)}{2}$	57

input
```
int((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
x-1/2/x+1/2/x*(2*x^2+1)^(3/2)-x*(2*x^2+1)^(1/2)-1/2*arcsinh(x*2^(1/2))*2^(
1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx=\frac{\sqrt{2}x\log\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)+2\,x^2+\sqrt{2\,x^2+1}-1}{2\,x}$$

input
```
integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*x*log(sqrt(2)*x - sqrt(2*x^2 + 1)) + 2*x^2 + sqrt(2*x^2 + 1)
- 1)/x
```

Sympy [F]

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx=\int \frac{\sqrt{2x^2+1}}{\sqrt{2x^2+1}+1}\,dx$$

input
```
integrate((2*x**2+1)**(1/2)/(1+(2*x**2+1)**(1/2)),x)
```

output
```
Integral(sqrt(2*x**2 + 1)/(sqrt(2*x**2 + 1) + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx=\int \frac{\sqrt{2\,x^2+1}}{\sqrt{2\,x^2+1}+1}\,dx$$

input
```
integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
x - integrate(1/(sqrt(2*x^2 + 1) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.36

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx=\frac{1}{2}\,\sqrt{2}\log\left(-\sqrt{2}x+\sqrt{2\,x^2+1}\right)+x-\frac{\sqrt{2}}{\left(\sqrt{2}x-\sqrt{2\,x^2+1}\right)^2-1}-\frac{1}{2\,x}$$

input
```
integrate((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*log(-sqrt(2)*x + sqrt(2*x^2 + 1)) + x - sqrt(2)/((sqrt(2)*x -
sqrt(2*x^2 + 1))^2 - 1) - 1/2/x
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx=x-\frac{\sqrt{2}\,\mathrm{asinh}\left(\sqrt{2}\,x\right)}{2}+\frac{\frac{\sqrt{2}\,\sqrt{x^2+\frac{1}{2}}}{2}-\frac{1}{2}}{x}$$

input
```
int((2*x^2 + 1)^(1/2)/((2*x^2 + 1)^(1/2) + 1),x)
```

output
```
x - (2^(1/2)*asinh(2^(1/2)*x))/2 + ((2^(1/2)*(x^2 + 1/2)^(1/2))/2 - 1/2)/x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{1+2x^2}}{1+\sqrt{1+2x^2}}\,dx = \frac{\sqrt{2x^2+1}-\sqrt{2}\log\left(\sqrt{2x^2+1}+\sqrt{2}\,x\right)x+\sqrt{2}\,x+2x^2-1}{2x}$$

input
```
int((2*x^2+1)^(1/2)/(1+(2*x^2+1)^(1/2)),x)
```

output
```
(sqrt(2*x**2 + 1) - sqrt(2)*log(sqrt(2*x**2 + 1) + sqrt(2)*x)*x + sqrt(2)*
x + 2*x**2 - 1)/(2*x)
```

3.19 $\int \frac{-1+x}{1+\sqrt{1+x^2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 46

$$\int \frac{-1+x}{1+\sqrt{1+x^2}} \, dx = -\frac{1}{x} + \sqrt{1+x^2} + \frac{\sqrt{1+x^2}}{x} - \operatorname{arcsinh}(x) - \log\left(1+\sqrt{1+x^2}\right)$$

output
```
-1/x+(x^2+1)^(1/2)+(x^2+1)^(1/2)/x-arcsinh(x)-ln(1+(x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89

$$\int \frac{-1+x}{1+\sqrt{1+x^2}} \, dx = -\frac{1}{x} + \frac{(1+x)\sqrt{1+x^2}}{x} - 4\operatorname{arctanh}\left(1-2x+2\sqrt{1+x^2}\right)$$

input
```
Integrate[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
```

output
```
-x^(-1) + ((1 + x)*Sqrt[1 + x^2])/x - 4*ArcTanh[1 - 2*x + 2*Sqrt[1 + x^2]]
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x-1}{\sqrt{x^2+1}+1} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{x}{\sqrt{x^2+1}+1}-\frac{1}{\sqrt{x^2+1}+1}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\text{arcsinh}(x)+\frac{\sqrt{x^2+1}}{x}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\frac{1}{x}$$

input
```
Int[(-1 + x)/(1 + Sqrt[1 + x^2]),x]
```

output
```
-x^(-1) + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x - ArcSinh[x] - Log[1 + Sqrt[1 +
x^2]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

method	result	size
trager	$\frac{x-1}{x}+\frac{(x+1)\sqrt{x^2+1}}{x}+2\ln\left(-\frac{\sqrt{x^2+1}-1-x}{x}\right)$	43
default	$-\frac{1}{x}+\sqrt{x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\ln(x)+\frac{(x^2+1)^{\frac{3}{2}}}{x}-x\sqrt{x^2+1}-\operatorname{arcsinh}(x)$	53
meijerg	$-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2},1\right],\left[\frac{3}{2},2\right],-x^2\right)}{2}+\frac{-4\sqrt{\pi}+4\sqrt{\pi}\,\sqrt{x^2+1}-4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^2+1}}{2}\right)}{4\sqrt{\pi}}$	58

input
```
int((x-1)/(1+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
(x-1)/x+(x+1)/x*(x^2+1)^(1/2)+2*ln(-((x^2+1)^(1/2)-1-x)/x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx = \frac{x\log\left(2\,x^2-\sqrt{x^2+1}(2\,x+1)+x+1\right)-x\log(x)-x\log\left(-x+\sqrt{x^2+1}+1\right)+\sqrt{x^2+1}(x+1)+x}{x}$$

input
```
integrate((x-1)/(1+(x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
(x*log(2*x^2 - sqrt(x^2 + 1)*(2*x + 1) + x + 1) - x*log(x) - x*log(-x + sq
rt(x^2 + 1) + 1) + sqrt(x^2 + 1)*(x + 1) + x - 1)/x
```

Sympy [A] (verification not implemented)

Time = 1.35 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx=\frac{x}{\sqrt{x^2+1}}+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+1\right)-\operatorname{asinh}(x)-\frac{1}{x}+\frac{1}{x\sqrt{x^2+1}}$$

input
```
integrate((x-1)/(1+(x**2+1)**(1/2)),x)
```

output
```
x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(sqrt(x**2 + 1) + 1) - asinh(x) - 1
/x + 1/(x*sqrt(x**2 + 1))
```

Maxima [F]

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx=\int \frac{x-1}{\sqrt{x^2+1}+1}\,dx$$

input
```
integrate((x-1)/(1+(x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
1/4*x^2 - 1/2*x - integrate(1/2*(x^3 - x^2)/(x^2 + 2*sqrt(x^2 + 1) + 2), x
)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.72

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx=\sqrt{x^2+1}-\frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1}-\frac{1}{x}+\log\left(-x+\sqrt{x^2+1}\right)-\log\left(|x|\right)-\log\left(\left|-x+\sqrt{x^2+1}+1\right|\right)+\log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

input
```
integrate((x-1)/(1+(x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
sqrt(x^2 + 1) - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + log(-x + sqrt(x^2 +
1)) - log(abs(x)) - log(abs(-x + sqrt(x^2 + 1) + 1)) + log(abs(-x + sqrt(x
^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx=\sqrt{x^2+1}-\ln\left(x\right)-\operatorname{asinh}(x)+\frac{\sqrt{x^2+1}}{x}-\frac{1}{x}+\operatorname{atan}\left(\sqrt{x^2+1}\,1\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int((x - 1)/((x^2 + 1)^(1/2) + 1),x)
```

output
```
atan((x^2 + 1)^(1/2)*1i)*1i - asinh(x) - log(x) + (x^2 + 1)^(1/2) + (x^2 +
 1)^(1/2)/x - 1/x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

$$\int \frac{-1+x}{1+\sqrt{1+x^2}}\,dx =\frac{\sqrt{x^2+1}\,x+\sqrt{x^2+1}-\log\left(\sqrt{x^2+1}+x\right)x-\log\left(\frac{\sqrt{x^2+1}\,x+\sqrt{x^2+1}+x^2+x+1}{\sqrt{x^2+1}+x}\right)x+x-1}{x}$$

input
```
int((x-1)/(1+(x^2+1)^(1/2)),x)
```

output
```
(sqrt(x**2 + 1)*x + sqrt(x**2 + 1) - log(sqrt(x**2 + 1) + x)*x - log((sqrt
(x**2 + 1)*x + sqrt(x**2 + 1) + x**2 + x + 1)/(sqrt(x**2 + 1) + x))*x + x
- 1)/x
```

3.20 $\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}} \, dx = -\frac{1}{x} - x + \sqrt{1+x^2} + \frac{\sqrt{1+x^2}}{x} + \frac{1}{2} x\sqrt{1+x^2} - \frac{\operatorname{arcsinh}(x)}{2} - \log\left(1+\sqrt{1+x^2}\right)$$

output
```
-1/x-x+(x^2+1)^(1/2)+(x^2+1)^(1/2)/x+1/2*x*(x^2+1)^(1/2)-1/2*arcsinh(x)-ln
(1+(x^2+1)^(1/2))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}} \, dx = \frac{-2(1+x^2)+\sqrt{1+x^2}(2+2x+x^2)+3x\log\left(-x+\sqrt{1+x^2}\right)-4x\log\left(1-x+\sqrt{1+x^2}\right)}{2x}$$

input
```
Integrate[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

output
```
(-2*(1 + x^2) + Sqrt[1 + x^2]*(2 + 2*x + x^2) + 3*x*Log[-x + Sqrt[1 + x^2]
] - 4*x*Log[1 - x + Sqrt[1 + x^2]])/(2*x)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+x-1}{\sqrt{x^2+1}+1} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{x^2}{\sqrt{x^2+1}+1}+\frac{x}{\sqrt{x^2+1}+1}-\frac{1}{\sqrt{x^2+1}+1}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\text{arcsinh}(x)}{2}+\frac{1}{2}\sqrt{x^2+1}x+\sqrt{x^2+1}+\frac{\sqrt{x^2+1}}{x}-\log\left(\sqrt{x^2+1}+1\right)-x-\frac{1}{x}$$

input
```
Int[(-1 + x + x^2)/(1 + Sqrt[1 + x^2]),x]
```

output
```
-x^(-1) - x + Sqrt[1 + x^2] + Sqrt[1 + x^2]/x + (x*Sqrt[1 + x^2])/2 - ArcS
inh[x]/2 - Log[1 + Sqrt[1 + x^2]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

method	result	s
default	$-x-\frac{1}{x}-\frac{x\sqrt{x^2+1}}{2}-\frac{\operatorname{arcsinh}(x)}{2}+\sqrt{x^2+1}-\operatorname{arctanh}\left(\frac{1}{\sqrt{x^2+1}}\right)-\ln(x)+\frac{(x^2+1)^{\frac{3}{2}}}{x}$	5
meijerg	$-\frac{x\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2},1\right],\left[\frac{3}{2},2\right],-x^2\right)}{2}+\frac{x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{2},1,\frac{3}{2}\right],\left[2,\frac{5}{2}\right],-x^2\right)}{6}+\frac{-4\sqrt{\pi}+4\sqrt{\pi}\,\sqrt{x^2+1}-4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{x^2+1}}{2}\right)}{4\sqrt{\pi}}$	7
trager	$-\frac{(x-1)^2}{x}+\frac{(x^2+2x+2)\sqrt{x^2+1}}{2x}+\frac{\ln\left(\frac{\sqrt{x^2+1}\,x^2-x^3+2x\sqrt{x^2+1}-2x^2+2\sqrt{x^2+1}-2x-2}{x^4}\right)}{2}$	8

input
```
int((x^2+x-1)/(1+(x^2+1)^(1/2)),x,method=_RETURNVERBOSE)
```

output
```
-x-1/x-1/2*x*(x^2+1)^(1/2)-1/2*arcsinh(x)+(x^2+1)^(1/2)-arctanh(1/(x^2+1)^
(1/2))-ln(x)+1/x*(x^2+1)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.29

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx = -\frac{2\,x^2+2\,x\log(x)+2\,x\log\left(-x+\sqrt{x^2+1}+1\right)-x\log\left(-x+\sqrt{x^2+1}\right)-2\,x\log\left(-x+\sqrt{x^2+1}-1\right)\cdots}{2\,x}$$

input
```
integrate((x^2+x-1)/(1+(x^2+1)^(1/2)),x, algorithm="fricas")
```

output
```
-1/2*(2*x^2 + 2*x*log(x) + 2*x*log(-x + sqrt(x^2 + 1) + 1) - x*log(-x + sq
rt(x^2 + 1)) - 2*x*log(-x + sqrt(x^2 + 1) - 1) - (x^2 + 2*x + 2)*sqrt(x^2
+ 1) - 2*x + 2)/x
```

Sympy [A] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx = \frac{x\sqrt{x^2+1}}{2} - x + \frac{x}{\sqrt{x^2+1}} + \sqrt{x^2+1} - \log\left(\sqrt{x^2+1}+1\right) - \frac{\operatorname{asinh}(x)}{2} - \frac{1}{x} + \frac{1}{x\sqrt{x^2+1}}$$

input
```
integrate((x**2+x-1)/(1+(x**2+1)**(1/2)),x)
```

output
```
x*sqrt(x**2 + 1)/2 - x + x/sqrt(x**2 + 1) + sqrt(x**2 + 1) - log(sqrt(x**2
 + 1) + 1) - asinh(x)/2 - 1/x + 1/(x*sqrt(x**2 + 1))
```

Maxima [F]

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx = \int \frac{x^2+x-1}{\sqrt{x^2+1}+1}\,dx$$

input
```
integrate((x^2+x-1)/(1+(x^2+1)^(1/2)),x, algorithm="maxima")
```

output
```
2*x - 5*arctan(1/2*x) + integrate((x^6 + x^5 - x^4)/(3*x^4 + 16*x^2 + (x^4
 + 8*x^2 + 16)*sqrt(x^2 + 1) + 16), x) + log(x^2 + 4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.37

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx = \frac{1}{2}\sqrt{x^2+1}(x+2) - x - \frac{2}{\left(x-\sqrt{x^2+1}\right)^2-1} - \frac{1}{x} + \frac{1}{2}\log\left(-x+\sqrt{x^2+1}\right) - \log\left(|x|\right) - \log\left(\left|-x+\sqrt{x^2+1}+1\right|\right) + \log\left(\left|-x+\sqrt{x^2+1}-1\right|\right)$$

input
```
integrate((x^2+x-1)/(1+(x^2+1)^(1/2)),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 1)*(x + 2) - x - 2/((x - sqrt(x^2 + 1))^2 - 1) - 1/x + 1/2*
log(-x + sqrt(x^2 + 1)) - log(abs(x)) - log(abs(-x + sqrt(x^2 + 1) + 1)) +
 log(abs(-x + sqrt(x^2 + 1) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.85

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx = \left(\frac{x}{2}+1\right)\sqrt{x^2+1}-\frac{\operatorname{asinh}(x)}{2}-\ln(x)-x+\frac{\sqrt{x^2+1}}{x}-\frac{1}{x}+\operatorname{atan}\left(\sqrt{x^2+1}\,1\mathrm{i}\right)1\mathrm{i}$$

input
```
int((x + x^2 - 1)/((x^2 + 1)^(1/2) + 1),x)
```

output
```
atan((x^2 + 1)^(1/2)*1i)*1i - x - asinh(x)/2 - log(x) + (x/2 + 1)*(x^2 + 1
)^(1/2) + (x^2 + 1)^(1/2)/x - 1/x
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

$$\int \frac{-1+x+x^2}{1+\sqrt{1+x^2}}\,dx = \frac{\sqrt{x^2+1}\,x^2+2\sqrt{x^2+1}\,x+2\sqrt{x^2+1}-4\log\left(\sqrt{x^2+1}+x+1\right)x+\log\left(\sqrt{x^2+1}+x\right)x-2x^2-2}{2x}$$

input
```
int((x^2+x-1)/(1+(x^2+1)^(1/2)),x)
```

output
```
(sqrt(x**2 + 1)*x**2 + 2*sqrt(x**2 + 1)*x + 2*sqrt(x**2 + 1) - 4*log(sqrt(
x**2 + 1) + x + 1)*x + log(sqrt(x**2 + 1) + x)*x - 2*x**2 - 2)/(2*x)
```

3.21 $\int \sqrt{\frac{a+x}{a-x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = -\sqrt{-1+\frac{2a}{a-x}}(a-x)+2a\arctan\left(\sqrt{-1+\frac{2a}{a-x}}\right)$$

output
```
-(-1+2*a/(a-x))^(1/2)*(a-x)+2*a*arctan((-1+2*a/(a-x))^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.52

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = \frac{\sqrt{\frac{a+x}{a-x}}\left((-a+x)\sqrt{a+x}+2a\sqrt{a-x}\arctan\left(\frac{\sqrt{a+x}}{\sqrt{a-x}}\right)\right)}{\sqrt{a+x}}$$

input
```
Integrate[Sqrt[(a + x)/(a - x)],x]
```

output
```
(Sqrt[(a + x)/(a - x)]*((-a + x)*Sqrt[a + x] + 2*a*Sqrt[a - x]*ArcTan[Sqrt
[a + x]/Sqrt[a - x]]))/Sqrt[a + x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.32, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{a+x}{a-x}}dx$$

$$\Big\downarrow \text{2051}$$

$$4a\int \frac{a+x}{(a-x)\left(\frac{a+x}{a-x}+1\right)^2}d\sqrt{\frac{a+x}{a-x}}$$

$$\Big\downarrow \text{252}$$

$$4a\left(\frac{1}{2}\int \frac{1}{\frac{a+x}{a-x}+1}d\sqrt{\frac{a+x}{a-x}}-\frac{\sqrt{\frac{a+x}{a-x}}}{2\left(\frac{a+x}{a-x}+1\right)}\right)$$

$$\Big\downarrow \text{216}$$

$$4a\left(\frac{1}{2}\arctan\left(\sqrt{\frac{a+x}{a-x}}\right)-\frac{\sqrt{\frac{a+x}{a-x}}}{2\left(\frac{a+x}{a-x}+1\right)}\right)$$

input
```
Int[Sqrt[(a + x)/(a - x)],x]
```

output
```
4*a*(-1/2*Sqrt[(a + x)/(a - x)]/(1 + (a + x)/(a - x)) + ArcTan[Sqrt[(a + x
)/(a - x)]]/2)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

method	result	size
default	$-\frac{\sqrt{\frac{a+x}{a-x}}\,(a-x)\left(\sqrt{a^{2}-x^{2}}-a\arctan\left(\frac{x}{\sqrt{a^{2}-x^{2}}}\right)\right)}{\sqrt{(a+x)(a-x)}}$	61
risch	$-\frac{(a-x)\sqrt{\frac{a+x}{a-x}}\,\sqrt{(a+x)(a-x)}}{\sqrt{-(-a+x)(a+x)}}+\frac{a\arctan\left(\frac{x}{\sqrt{a^{2}-x^{2}}}\right)\sqrt{\frac{a+x}{a-x}}\,\sqrt{(a+x)(a-x)}}{a+x}$	90

input
```
int(((a+x)/(a-x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-((a+x)/(a-x))^(1/2)*(a-x)*((a^2-x^2)^(1/2)-a*arctan(x/(a^2-x^2)^(1/2)))/(
(a+x)*(a-x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.86

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = 2\,a\arctan\left(\sqrt{\frac{a+x}{a-x}}\right) - (a-x)\sqrt{\frac{a+x}{a-x}}$$

input
```
integrate(((a+x)/(a-x))^(1/2),x, algorithm="fricas")
```

output
```
2*a*arctan(sqrt((a + x)/(a - x))) - (a - x)*sqrt((a + x)/(a - x))
```

Sympy [F]

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = \int \sqrt{\frac{a+x}{a-x}}\,dx$$

input
```
integrate(((a+x)/(a-x))**(1/2),x)
```

output
```
Integral(sqrt((a + x)/(a - x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = -2\,a\left(\frac{\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1} - \arctan\left(\sqrt{\frac{a+x}{a-x}}\right)\right)$$

input
```
integrate(((a+x)/(a-x))^(1/2),x, algorithm="maxima")
```

output
```
-2*a*(sqrt((a + x)/(a - x))/((a + x)/(a - x) + 1) - arctan(sqrt((a + x)/(a
 - x))))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.82

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = a\arcsin\left(\frac{x}{a}\right)\operatorname{sgn}(a-x)\operatorname{sgn}(a) - \sqrt{a^2-x^2}\operatorname{sgn}(a-x)$$

input
```
integrate(((a+x)/(a-x))^(1/2),x, algorithm="giac")
```

output
```
a*arcsin(x/a)*sgn(a - x)*sgn(a) - sqrt(a^2 - x^2)*sgn(a - x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = 2\,a\,\mathrm{atan}\left(\sqrt{\frac{a+x}{a-x}}\right) - \frac{2\,a\,\sqrt{\frac{a+x}{a-x}}}{\frac{a+x}{a-x}+1}$$

input
```
int(((a + x)/(a - x))^(1/2),x)
```

output
```
2*a*atan(((a + x)/(a - x))^(1/2)) - (2*a*((a + x)/(a - x))^(1/2))/((a + x)
/(a - x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \sqrt{\frac{a+x}{a-x}}\,dx = -2asin\left(\frac{\sqrt{a-x}}{\sqrt{a}\,\sqrt{2}}\right)a - \sqrt{a+x}\,\sqrt{a-x}$$

input
```
int(((a+x)/(a-x))^(1/2),x)
```

output
```
- 2*asin(sqrt(a - x)/(sqrt(a)*sqrt(2)))*a - sqrt(a + x)*sqrt(a - x)
```

3.22 $\int \sqrt{\frac{-a+x}{a+x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx=(a+x)\sqrt{1-\frac{2a}{a+x}}-2a\operatorname{arctanh}\left(\sqrt{1-\frac{2a}{a+x}}\right)$$

output
```
(a+x)*(1-2*a/(a+x))^(1/2)-2*a*arctanh((1-2*a/(a+x))^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.81

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx=\frac{\sqrt{\frac{-a+x}{a+x}}\left(\sqrt{-a+x}(a+x)-2a\sqrt{a+x}\operatorname{arctanh}\left(\frac{\sqrt{a+x}}{\sqrt{-a+x}}\right)\right)}{\sqrt{-a+x}}$$

input
```
Integrate[Sqrt[(-a + x)/(a + x)],x]
```

output
```
(Sqrt[(-a + x)/(a + x)]*(Sqrt[-a + x]*(a + x) - 2*a*Sqrt[a + x]*ArcTanh[Sq
rt[a + x]/Sqrt[-a + x]]))/Sqrt[-a + x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.62, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2051, 252, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x-a}{a+x}}dx$$

$$\Big\downarrow \text{2051}$$

$$4a\int -\frac{a-x}{(a+x)\left(\frac{a-x}{a+x}+1\right)^2}d\sqrt{-\frac{a-x}{a+x}}$$

$$\Big\downarrow \text{252}$$

$$4a\left(\frac{\sqrt{-\frac{a-x}{a+x}}}{2\left(\frac{a-x}{a+x}+1\right)}-\frac{1}{2}\int \frac{1}{\frac{a-x}{a+x}+1}d\sqrt{-\frac{a-x}{a+x}}\right)$$

$$\Big\downarrow \text{219}$$

$$4a\left(\frac{\sqrt{-\frac{a-x}{a+x}}}{2\left(\frac{a-x}{a+x}+1\right)}-\frac{1}{2}\text{arctanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)\right)$$

input
```
Int[Sqrt[(-a + x)/(a + x)],x]
```

output
```
4*a*(Sqrt[-((a - x)/(a + x))]/(2*(1 + (a - x)/(a + x))) - ArcTanh[Sqrt[-((
a - x)/(a + x))]]/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.68

method	result	size
default	$-\frac{\sqrt{-\frac{a-x}{a+x}}\,(a+x)\left(a\ln\left(x+\sqrt{-a^{2}+x^{2}}\right)-\sqrt{-a^{2}+x^{2}}\right)}{\sqrt{-(a+x)(a-x)}}$	62
risch	$\frac{(a+x)\sqrt{-\frac{a-x}{a+x}}\,\sqrt{-(a+x)(a-x)}}{\sqrt{(-a+x)(a+x)}}+\frac{a\ln\left(x+\sqrt{-a^{2}+x^{2}}\right)\sqrt{-\frac{a-x}{a+x}}\,\sqrt{-(a+x)(a-x)}}{a-x}$	92

input
```
int(((-a+x)/(a+x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(-(a-x)/(a+x))^(1/2)*(a+x)*(a*ln(x+(-a^2+x^2)^(1/2))-(-a^2+x^2)^(1/2))/(-
(a+x)*(a-x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.57

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = -a\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+a\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)+(a+x)\sqrt{-\frac{a-x}{a+x}}$$

input
```
integrate(((-a+x)/(a+x))^(1/2),x, algorithm="fricas")
```

output
```
-a*log(sqrt(-(a - x)/(a + x)) + 1) + a*log(sqrt(-(a - x)/(a + x)) - 1) + (
a + x)*sqrt(-(a - x)/(a + x))
```

Sympy [F]

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = \int \sqrt{\frac{-a+x}{a+x}}\,dx$$

input
```
integrate(((-a+x)/(a+x))**(1/2),x)
```

output
```
Integral(sqrt((-a + x)/(a + x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(33) = 66$.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.89

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = a\left(\frac{2\sqrt{-\frac{a-x}{a+x}}}{\frac{a-x}{a+x}+1}-\log\left(\sqrt{-\frac{a-x}{a+x}}+1\right)+\log\left(\sqrt{-\frac{a-x}{a+x}}-1\right)\right)$$

input
```
integrate(((-a+x)/(a+x))^(1/2),x, algorithm="maxima")
```

output
```
a*(2*sqrt(-(a - x)/(a + x))/((a - x)/(a + x) + 1) - log(sqrt(-(a - x)/(a +
 x)) + 1) + log(sqrt(-(a - x)/(a + x)) - 1))
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.08

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = a\log\left(\left|-x+\sqrt{-a^2+x^2}\right|\right)\mathrm{sgn}(a+x)+\sqrt{-a^2+x^2}\mathrm{sgn}(a+x)$$

input
```
integrate(((-a+x)/(a+x))^(1/2),x, algorithm="giac")
```

output
```
a*log(abs(-x + sqrt(-a^2 + x^2)))*sgn(a + x) + sqrt(-a^2 + x^2)*sgn(a + x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.38

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = \frac{2\,a\,\sqrt{-\frac{a-x}{a+x}}}{\frac{a-x}{a+x}+1}-2\,a\,\mathrm{atanh}\left(\sqrt{-\frac{a-x}{a+x}}\right)$$

input
```
int((-(a - x)/(a + x))^(1/2),x)
```

output
```
(2*a*(-(a - x)/(a + x))^(1/2))/((a - x)/(a + x) + 1) - 2*a*atanh((-(a - x)
/(a + x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \sqrt{\frac{-a+x}{a+x}}\,dx = \sqrt{a+x}\,\sqrt{-a+x} - 2\log\left(\frac{\sqrt{-a+x}+\sqrt{a+x}}{\sqrt{a}\,\sqrt{2}}\right)a$$

input
```
int(((-a+x)/(a+x))^(1/2),x)
```

output
```
sqrt(a + x)*sqrt( - a + x) - 2*log((sqrt( - a + x) + sqrt(a + x))/(sqrt(a)
*sqrt(2)))*a
```

3.23 $\int \sqrt{\frac{a+bx}{c+dx}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 102

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx=\frac{(c+dx)\sqrt{\frac{b}{d}-\frac{bc-ad}{d(c+dx)}}}{d}-\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{b}{d}-\frac{bc-ad}{d(c+dx)}}}{\sqrt{b}}\right)}{\sqrt{b}d^{3/2}}$$

output
```
(d*x+c)*(b/d-(-a*d+b*c)/d/(d*x+c))^(1/2)/d-(-a*d+b*c)*arctanh(d^(1/2)*(b/d
-(-a*d+b*c)/d/(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.95

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx=\frac{\sqrt{\frac{a+bx}{c+dx}}\left(\sqrt{d}(c+dx)+\frac{(-bc+ad)\sqrt{c+dx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{a+bx}}\right)}{d^{3/2}}$$

input
```
Integrate[Sqrt[(a + b*x)/(c + d*x)],x]
```

output
```
(Sqrt[(a + b*x)/(c + d*x)]*(Sqrt[d]*(c + d*x) + ((-(b*c) + a*d)*Sqrt[c + d
*x]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])/(Sqrt[d]*Sqrt[a + b*x])])/(Sqrt[b]*Sqr
t[a + b*x])))/d^(3/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {2051, 252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{a+bx}{c+dx}}dx$$

$$\Big\downarrow \text{2051}$$

$$2(bc-ad)\int \frac{a+bx}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\sqrt{\frac{a+bx}{c+dx}}$$

$$\Big\downarrow \text{252}$$

$$2(bc-ad)\left(\frac{\sqrt{\frac{a+bx}{c+dx}}}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\sqrt{\frac{a+bx}{c+dx}}}{2d}\right)$$

$$\Big\downarrow \text{221}$$

$$2(bc-ad)\left(\frac{\sqrt{\frac{a+bx}{c+dx}}}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{2\sqrt{b}d^{3/2}}\right)$$

input
```
Int[Sqrt[(a + b*x)/(c + d*x)],x]
```

output
```
2*(b*c - a*d)*(Sqrt[(a + b*x)/(c + d*x)]/(2*d*(b - (d*(a + b*x))/(c + d*x)
)) - ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]]/(2*Sqrt[b]*d^(3/
2)))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.49

method	result
default	$\frac{\sqrt{\frac{bx+a}{dx+c}}\,(dx+c)\left(\ln\left(\frac{2bdx+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)ad-\ln\left(\frac{2bdx+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)bc+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}\right)}{2\sqrt{(bx+a)(dx+c)}\,d\sqrt{bd}}$

input
```
int(((b*x+a)/(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((b*x+a)/(d*x+c))^(1/2)*(d*x+c)*(ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(
1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*a*d-ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x
+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*b*c+2*((b*x+a)*(d*x+c))^(1/2)
*(b*d)^(1/2))/((b*x+a)*(d*x+c))^(1/2)/d/(b*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.76

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx$$
$$= \left[-\frac{(bc-ad)\sqrt{bd}\log\left(2\,bdx+bc+ad+2\,\sqrt{bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}\right)-2\,(bd^2x+bcd)\sqrt{\frac{bx+a}{dx+c}}}{2\,bd^2}, \frac{(bc-ad)\sqrt{-ba}\cdots}{\cdots}\right.$$

input
```
integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((b*c - a*d)*sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x +
c)*sqrt((b*x + a)/(d*x + c))) - 2*(b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x +
c)))/(b*d^2), ((b*c - a*d)*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*
x + a)/(d*x + c))/(b*d*x + a*d)) + (b*d^2*x + b*c*d)*sqrt((b*x + a)/(d*x +
 c)))/(b*d^2)]
```

Sympy [F]

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = \int \sqrt{\frac{a+bx}{c+dx}}\,dx$$

input
```
integrate(((b*x+a)/(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt((a + b*x)/(c + d*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.16

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = \frac{(bc-ad)\sqrt{\frac{bx+a}{dx+c}}}{bd-\frac{(bx+a)d^2}{dx+c}} + \frac{(bc-ad)\log\left(\frac{d\sqrt{\frac{bx+a}{dx+c}}-\sqrt{bd}}{d\sqrt{\frac{bx+a}{dx+c}}+\sqrt{bd}}\right)}{2\sqrt{bd}d}$$

input
```
integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
(b*c - a*d)*sqrt((b*x + a)/(d*x + c))/(b*d - (b*x + a)*d^2/(d*x + c)) + 1/
2*(b*c - a*d)*log((d*sqrt((b*x + a)/(d*x + c)) - sqrt(b*d))/(d*sqrt((b*x +
 a)/(d*x + c)) + sqrt(b*d)))/(sqrt(b*d)*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = \frac{(bc\,\mathrm{sgn}(dx+c)-ad\,\mathrm{sgn}(dx+c))\log\left(\left|-bc-ad-2\sqrt{bd}\left(\sqrt{bd}x-\sqrt{bdx^2+bcx+adx+ac}\right)\right|\right)}{2\sqrt{bd}d} + \frac{\sqrt{bdx^2+bcx+adx+ac}\,\mathrm{sgn}(dx+c)}{d}$$

input
```
integrate(((b*x+a)/(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
1/2*(b*c*sgn(d*x + c) - a*d*sgn(d*x + c))*log(abs(-b*c - a*d - 2*sqrt(b*d)
*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*d*x + a*c))))/(sqrt(b*d)*d) + sqr
t(b*d*x^2 + b*c*x + a*d*x + a*c)*sgn(d*x + c)/d
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.88

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+b\,x}{c+d\,x}}}{\sqrt{b}}\right)\,(a\,d-b\,c)}{\sqrt{b}\,d^{3/2}} + \frac{(a\,d-b\,c)\,\sqrt{\frac{a+b\,x}{c+d\,x}}}{b\,d\,\left(\frac{d\,(a+b\,x)}{b\,(c+d\,x)}-1\right)}$$

input
```
int(((a + b*x)/(c + d*x))^(1/2),x)
```

output
```
(atanh((d^(1/2)*((a + b*x)/(c + d*x))^(1/2))/b^(1/2))*(a*d - b*c))/(b^(1/2
)*d^(3/2)) + ((a*d - b*c)*((a + b*x)/(c + d*x))^(1/2))/(b*d*((d*(a + b*x))
/(b*(c + d*x)) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int \sqrt{\frac{a+bx}{c+dx}}\,dx$$
$$= \frac{\sqrt{dx+c}\,\sqrt{bx+a}\,bd+\sqrt{d}\,\sqrt{b}\log\left(\frac{\sqrt{d}\,\sqrt{bx+a}+\sqrt{b}\,\sqrt{dx+c}}{\sqrt{ad-bc}}\right)ad-\sqrt{d}\,\sqrt{b}\log\left(\frac{\sqrt{d}\,\sqrt{bx+a}+\sqrt{b}\,\sqrt{dx+c}}{\sqrt{ad-bc}}\right)bc}{b\,d^2}$$

input
```
int(((b*x+a)/(d*x+c))^(1/2),x)
```

output
```
(sqrt(c + d*x)*sqrt(a + b*x)*b*d + sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b
*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*a*d - sqrt(d)*sqrt(b)*log((s
qrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqrt(a*d - b*c))*b*c)/(b*d**
2)
```

3.24 $\int \sqrt{\frac{1-x}{1+x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 34

$$\int \sqrt{\frac{1-x}{1+x}}\,dx=(1+x)\sqrt{-1+\frac{2}{1+x}}-2\arctan\left(\sqrt{-1+\frac{2}{1+x}}\right)$$

output
```
(1+x)*(-1+2/(1+x))^(1/2)-2*arctan((-1+2/(1+x))^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.88

$$\int \sqrt{\frac{1-x}{1+x}}\,dx=\frac{\sqrt{\frac{1-x}{1+x}}\sqrt{1+x}\left(\sqrt{1-x^2}-2\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)\right)}{\sqrt{1-x}}$$

input
```
Integrate[Sqrt[(1 - x)/(1 + x)],x]
```

output
```
(Sqrt[(1 - x)/(1 + x)]*Sqrt[1 + x]*(Sqrt[1 - x^2] - 2*ArcTan[Sqrt[1 - x^2]
/(-1 + x)]))/Sqrt[1 - x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.68, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{1-x}{x+1}}dx$$

$$\Big\downarrow \text{2051}$$

$$-4\int \frac{1-x}{(x+1)\left(\frac{1-x}{x+1}+1\right)^2}d\sqrt{\frac{1-x}{x+1}}$$

$$\Big\downarrow \text{252}$$

$$-4\left(\frac{1}{2}\int \frac{1}{\frac{1-x}{x+1}+1}d\sqrt{\frac{1-x}{x+1}}-\frac{\sqrt{\frac{1-x}{x+1}}}{2\left(\frac{1-x}{x+1}+1\right)}\right)$$

$$\Big\downarrow \text{216}$$

$$-4\left(\frac{1}{2}\arctan\left(\sqrt{\frac{1-x}{x+1}}\right)-\frac{\sqrt{\frac{1-x}{x+1}}}{2\left(\frac{1-x}{x+1}+1\right)}\right)$$

input
```
Int[Sqrt[(1 - x)/(1 + x)],x]
```

output
```
-4*(-1/2*Sqrt[(1 - x)/(1 + x)]/(1 + (1 - x)/(1 + x)) + ArcTan[Sqrt[(1 - x)
/(1 + x)]]/2)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.15

method	result
default	$\frac{\sqrt{-\frac{x-1}{x+1}}\,(x+1)\left(\sqrt{-x^2+1}+\arcsin(x)\right)}{\sqrt{-(x-1)(x+1)}}$
risch	$(x+1)\sqrt{-\frac{x-1}{x+1}}-\frac{\arcsin(x)\sqrt{-\frac{x-1}{x+1}}\,\sqrt{-(x-1)(x+1)}}{x-1}$
trager	$(x+1)\sqrt{-\frac{x-1}{x+1}}+\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{x-1}{x+1}}\,x+\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{\right.$

input
```
int(((1-x)/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-(x-1)/(x+1))^(1/2)*(x+1)/(-(x-1)*(x+1))^(1/2)*((-x^2+1)^(1/2)+arcsin(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = (x+1)\sqrt{-\frac{x-1}{x+1}} - 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
integrate(((1-x)/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
(x + 1)*sqrt(-(x - 1)/(x + 1)) - 2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Sympy [F]

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \int \sqrt{\frac{1-x}{x+1}}\,dx$$

input
```
integrate(((1-x)/(1+x))**(1/2),x)
```

output
```
Integral(sqrt((1 - x)/(x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -\frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1} - 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
integrate(((1-x)/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1) - 2*arctan(sqrt(-(x - 1)/(
x + 1)))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = \frac{1}{2}\,\pi\text{sgn}(x+1) + \arcsin{(x)}\,\text{sgn}(x+1) + \sqrt{-x^2+1}\text{sgn}(x+1)$$

input
```
integrate(((1-x)/(1+x))^(1/2),x, algorithm="giac")
```

output
```
1/2*pi*sgn(x + 1) + arcsin(x)*sgn(x + 1) + sqrt(-x^2 + 1)*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.26

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -2\,\text{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right) - \frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

input
```
int((-(x - 1)/(x + 1))^(1/2),x)
```

output
```
- 2*atan((-(x - 1)/(x + 1))^(1/2)) - (2*(-(x - 1)/(x + 1))^(1/2))/((x - 1)
/(x + 1) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.76

$$\int \sqrt{\frac{1-x}{1+x}}\,dx = -2asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right) + \sqrt{x+1}\,\sqrt{1-x}$$

input
```
int(((1-x)/(1+x))^(1/2),x)
```

output
```
- 2*asin(sqrt( - x + 1)/sqrt(2)) + sqrt(x + 1)*sqrt( - x + 1)
```

3.25 $\int \sqrt{\frac{-1+x}{5+3x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = \frac{1}{3}\sqrt{-1+x}\sqrt{5+3x} - \frac{8\operatorname{arcsinh}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{-1+x}\right)}{3\sqrt{3}}$$

output
```
1/3*(-1+x)^(1/2)*(5+3*x)^(1/2)-8/9*arcsinh(1/4*6^(1/2)*(-1+x)^(1/2))*3^(1/
2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.53

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = \frac{\sqrt{\frac{-1+x}{5+3x}}\left(3\sqrt{-1+x}(5+3x) - 8\sqrt{15+9x}\operatorname{arctanh}\left(\frac{\sqrt{5+3x}}{\sqrt{3}\sqrt{-1+x}}\right)\right)}{9\sqrt{-1+x}}$$

input
```
Integrate[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

output
```
(Sqrt[(-1 + x)/(5 + 3*x)]*(3*Sqrt[-1 + x]*(5 + 3*x) - 8*Sqrt[15 + 9*x]*Arc
Tanh[Sqrt[5 + 3*x]/(Sqrt[3]*Sqrt[-1 + x])]))/(9*Sqrt[-1 + x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {2050, 60, 64, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x-1}{3x+5}}dx$$

$$\Big\downarrow \text{ 2050}$$

$$\int \frac{\sqrt{x-1}}{\sqrt{3x+5}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\frac{1}{3}\sqrt{x-1}\sqrt{3x+5}-\frac{4}{3}\int \frac{1}{\sqrt{x-1}\sqrt{3x+5}}dx$$

$$\Big\downarrow \text{ 64}$$

$$\frac{1}{3}\sqrt{x-1}\sqrt{3x+5}-\frac{8}{3}\int \frac{1}{\sqrt{3(x-1)+8}}d\sqrt{x-1}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{3}\sqrt{x-1}\sqrt{3x+5}-\frac{8\text{arcsinh}\left(\frac{1}{2}\sqrt{\frac{3}{2}}\sqrt{x-1}\right)}{3\sqrt{3}}$$

input
```
Int[Sqrt[(-1 + x)/(5 + 3*x)],x]
```

output
```
(Sqrt[-1 + x]*Sqrt[5 + 3*x])/3 - (8*ArcSinh[(Sqrt[3/2]*Sqrt[-1 + x])/2])/(
3*Sqrt[3])
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 64
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[2/b   Subst[Int[1/Sqrt[c - a*(d/b) + d*(x^2/b)], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[c - a*(d/b), 0] && ( !GtQ[a - c*(b/d), 0]
 || PosQ[b])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(31) = 62$.

Time = 0.22 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.55

method	result
default	$-\frac{\sqrt{\frac{x-1}{3x+5}}\,(3x+5)\left(4\ln\left(x\sqrt{3}+\frac{\sqrt{3}}{3}+\sqrt{3x^2+2x-5}\right)\sqrt{3}-3\sqrt{3x^2+2x-5}\right)}{9\sqrt{(3x+5)(x-1)}}$
risch	$\frac{(3x+5)\sqrt{\frac{x-1}{3x+5}}}{3}-\frac{4\ln\left(\frac{(3x+1)\sqrt{3}}{3}+\sqrt{3x^2+2x-5}\right)\sqrt{3}\,\sqrt{\frac{x-1}{3x+5}}\,\sqrt{(3x+5)(x-1)}}{9(x-1)}$
trager	$5\left(\frac{x}{5}+\frac{1}{3}\right)\sqrt{-\frac{1-x}{3x+5}}-\frac{4\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(3\,\mathrm{RootOf}\left(\_Z^2-3\right)x+9\sqrt{-\frac{1-x}{3x+5}}\,x+\mathrm{RootOf}\left(\_Z^2-3\right)+15\sqrt{-\frac{1-x}{3x+5}}\right)}{9}$

input
```
int(((x-1)/(3*x+5))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/9*((x-1)/(3*x+5))^(1/2)*(3*x+5)*(4*ln(x*3^(1/2)+1/3*3^(1/2)+(3*x^2+2*x-
5)^(1/2))*3^(1/2)-3*(3*x^2+2*x-5)^(1/2))/((3*x+5)*(x-1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = \frac{1}{3}\,(3\,x+5)\sqrt{\frac{x-1}{3\,x+5}}+\frac{4}{9}\,\sqrt{3}\log\left(\sqrt{3}(3\,x+5)\sqrt{\frac{x-1}{3\,x+5}}-3\,x-1\right)$$

input
```
integrate(((x-1)/(5+3*x))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(3*x + 5)*sqrt((x - 1)/(3*x + 5)) + 4/9*sqrt(3)*log(sqrt(3)*(3*x + 5)*
sqrt((x - 1)/(3*x + 5)) - 3*x - 1)
```

Sympy [F]

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = \int \sqrt{\frac{x-1}{3x+5}}\,dx$$

input
```
integrate(((x-1)/(5+3*x))**(1/2),x)
```

output
```
Integral(sqrt((x - 1)/(3*x + 5)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(31) = 62$.

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.63

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = \frac{4}{9}\sqrt{3}\log\left(-\frac{\sqrt{3}-3\sqrt{\frac{x-1}{3x+5}}}{\sqrt{3}+3\sqrt{\frac{x-1}{3x+5}}}\right) - \frac{8\sqrt{\frac{x-1}{3x+5}}}{3\left(\frac{3(x-1)}{3x+5}-1\right)}$$

input
```
integrate(((x-1)/(5+3*x))^(1/2),x, algorithm="maxima")
```

output
```
4/9*sqrt(3)*log(-(sqrt(3) - 3*sqrt((x - 1)/(3*x + 5)))/(sqrt(3) + 3*sqrt((
x - 1)/(3*x + 5)))) - 8/3*sqrt((x - 1)/(3*x + 5))/(3*(x - 1)/(3*x + 5) - 1
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 74 vs. $2(31) = 62$.

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\begin{aligned}\int \sqrt{\frac{-1+x}{5+3x}}\,dx = & -\frac{8}{9}\sqrt{3}\log(2)\,\mathrm{sgn}(3x+5) \\ & + \frac{4}{9}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2x-5}\right)-1\right|\right)\mathrm{sgn}(3x+5) \\ & + \frac{1}{3}\sqrt{3x^2+2x-5}\,\mathrm{sgn}(3x+5)\end{aligned}$$

input
```
integrate(((x-1)/(5+3*x))^(1/2),x, algorithm="giac")
```

output
```
-8/9*sqrt(3)*log(2)*sgn(3*x + 5) + 4/9*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)*x
 - sqrt(3*x^2 + 2*x - 5)) - 1))*sgn(3*x + 5) + 1/3*sqrt(3*x^2 + 2*x - 5)*s
gn(3*x + 5)
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = -\frac{8\sqrt{3}\,\mathrm{atanh}\left(\sqrt{3}\,\sqrt{\frac{x-1}{3\,x+5}}\right)}{9} - \frac{8\,\sqrt{\frac{x-1}{3\,x+5}}}{3\,\left(\frac{3\,x-3}{3\,x+5}-1\right)}$$

input
```
int(((x - 1)/(3*x + 5))^(1/2),x)
```

output
```
- (8*3^(1/2)*atanh(3^(1/2)*((x - 1)/(3*x + 5))^(1/2)))/9 - (8*((x - 1)/(3*
x + 5))^(1/2))/(3*((3*x - 3)/(3*x + 5) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int \sqrt{\frac{-1+x}{5+3x}}\,dx = \frac{\sqrt{x-1}\,\sqrt{3x+5}}{3} - \frac{8\sqrt{3}\log\left(\frac{\sqrt{3x+5}+\sqrt{x-1}\,\sqrt{3}}{2\sqrt{2}}\right)}{9}$$

input
```
int(((x-1)/(5+3*x))^(1/2),x)
```

output
```
(3*sqrt(x - 1)*sqrt(3*x + 5) - 8*sqrt(3)*log((sqrt(3*x + 5) + sqrt(x - 1)*
sqrt(3))/(2*sqrt(2))))/9
```

3.26 $\int \sqrt{-\frac{x}{1+x}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 30

$$\int \sqrt{-\frac{x}{1+x}}\,dx = (1+x)\sqrt{-1+\frac{1}{1+x}}-\arctan\left(\sqrt{-1+\frac{1}{1+x}}\right)$$

output
```
(1+x)*(-1+1/(1+x))^(1/2)-arctan((-1+1/(1+x))^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.73

$$\int \sqrt{-\frac{x}{1+x}}\,dx = \frac{\sqrt{-\frac{x}{1+x}}\left(\sqrt{x}(1+x)+\sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)\right)}{\sqrt{x}}$$

input
```
Integrate[Sqrt[-(x/(1 + x))],x]
```

output
```
(Sqrt[-(x/(1 + x))]*(Sqrt[x]*(1 + x) + Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 +
 x]]))/Sqrt[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-\frac{x}{x+1}}dx$$

$$\Big\downarrow \text{2051}$$

$$-2\int -\frac{x}{(x+1)\left(1-\frac{x}{x+1}\right)^2}d\sqrt{-\frac{x}{x+1}}$$

$$\Big\downarrow \text{252}$$

$$-2\left(\frac{1}{2}\int \frac{1}{1-\frac{x}{x+1}}d\sqrt{-\frac{x}{x+1}}-\frac{\sqrt{-\frac{x}{x+1}}}{2\left(1-\frac{x}{x+1}\right)}\right)$$

$$\Big\downarrow \text{216}$$

$$-2\left(\frac{1}{2}\arctan\left(\sqrt{-\frac{x}{x+1}}\right)-\frac{\sqrt{-\frac{x}{x+1}}}{2\left(1-\frac{x}{x+1}\right)}\right)$$

input
```
Int[Sqrt[-(x/(1 + x))],x]
```

output
```
-2*(-1/2*Sqrt[-(x/(1 + x))]/(1 - x/(1 + x)) + ArcTan[Sqrt[-(x/(1 + x))]]/2
)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

method	result	size
risch	$(x+1)\sqrt{-\frac{x}{x+1}}-\frac{\arcsin(1+2x)\sqrt{-\frac{x}{x+1}}\,\sqrt{-(x+1)x}}{2x}$	45
default	$\frac{\sqrt{-\frac{x}{x+1}}\,(x+1)\left(2\sqrt{x^2+x}-\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\right)}{2\sqrt{(x+1)x}}$	46
trager	$2\left(\frac{1}{2}+\frac{x}{2}\right)\sqrt{-\frac{x}{x+1}}-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(2\sqrt{-\frac{x}{x+1}}\,x+2\operatorname{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-\frac{x}{x+1}}+\operatorname{RootOf}\left(\_Z^2+1\right)\right)}{2}$	69

input
```
int((-x/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x+1)*(-x/(x+1))^(1/2)-1/2*arcsin(1+2*x)*(-x/(x+1))^(1/2)*(-(x+1)*x)^(1/2)
/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \sqrt{-\frac{x}{1+x}}\, dx = (x+1)\sqrt{-\frac{x}{x+1}} - \arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

input
```
integrate((-x/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
(x + 1)*sqrt(-x/(x + 1)) - arctan(sqrt(-x/(x + 1)))
```

Sympy [F]

$$\int \sqrt{-\frac{x}{1+x}}\, dx = \int \sqrt{-\frac{x}{x+1}}\, dx$$

input
```
integrate((-x/(1+x))**(1/2),x)
```

output
```
Integral(sqrt(-x/(x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \sqrt{-\frac{x}{1+x}}\, dx = -\frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1} - 1} - \arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

input
```
integrate((-x/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(-x/(x + 1))/(x/(x + 1) - 1) - arctan(sqrt(-x/(x + 1)))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int \sqrt{-\frac{x}{1+x}}\,dx = \frac{1}{4}\,\pi \mathrm{sgn}(x+1) + \frac{1}{2}\,\arcsin\left(2\,x+1\right)\mathrm{sgn}(x+1) + \sqrt{-x^2-x}\mathrm{sgn}(x+1)$$

input
```
integrate((-x/(1+x))^(1/2),x, algorithm="giac")
```

output
```
1/4*pi*sgn(x + 1) + 1/2*arcsin(2*x + 1)*sgn(x + 1) + sqrt(-x^2 - x)*sgn(x
+ 1)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

$$\int \sqrt{-\frac{x}{1+x}}\,dx = -\mathrm{atan}\left(\sqrt{-\frac{x}{x+1}}\right) - \frac{\sqrt{-\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

input
```
int((-x/(x + 1))^(1/2),x)
```

output
```
- atan((-x/(x + 1))^(1/2)) - (-x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.67

$$\int \sqrt{-\frac{x}{1+x}}\,dx = i\left(\sqrt{x}\,\sqrt{x+1} - \log\left(\sqrt{x+1}+\sqrt{x}\right)\right)$$

input
```
int((-x/(1+x))^(1/2),x)
```

output
```
i*(sqrt(x)*sqrt(x + 1) - log(sqrt(x + 1) + sqrt(x)))
```

3.27 $\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 14

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = 2\arctan\left(\sqrt{-1+\frac{1}{1+x}}\right)$$

output
```
2*arctan((-1+1/(1+x))^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(14) = 28.

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = -\frac{2\sqrt{-\frac{x}{1+x}}\sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)}{\sqrt{x}}$$

input
```
Integrate[Sqrt[-(x/(1 + x))]/x,x]
```

output
```
(-2*Sqrt[-(x/(1 + x))]*Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]])/Sqrt[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2052, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{-\frac{x}{x+1}}}{x} \, dx$$

$$\Big\downarrow \text{2052}$$

$$-2 \int \frac{1}{\frac{x}{x+1} - 1} d\sqrt{-\frac{x}{x+1}}$$

$$\Big\downarrow \text{217}$$

$$2 \arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

input
```
Int[Sqrt[-(x/(1 + x))]/x,x]
```

output
```
2*ArcTan[Sqrt[-(x/(1 + x))]]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(12) = 24.

Time = 0.19 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

method	result
default	$\frac{\sqrt{-\frac{x}{x+1}}\,(x+1)\ln\left(\frac{1}{2}+x+\sqrt{x^{2}+x}\right)}{\sqrt{(x+1)x}}$
trager	$-\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(2\sqrt{-\frac{x}{x+1}}\,x-2\operatorname{RootOf}\left(\_Z^{2}+1\right)x+2\sqrt{-\frac{x}{x+1}}-\operatorname{RootOf}\left(\_Z^{2}+1\right)\right.$

input
```
int((-x/(x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
(-x/(x+1))^(1/2)*(x+1)/((x+1)*x)^(1/2)*ln(1/2+x+(x^2+x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = 2\,\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

input
```
integrate((-x/(1+x))^(1/2)/x,x, algorithm="fricas")
```

output
```
2*arctan(sqrt(-x/(x + 1)))
```

Sympy [F]

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = \int \frac{\sqrt{-\frac{x}{x+1}}}{x}\,dx$$

input
```
integrate((-x/(1+x))**(1/2)/x,x)
```

output
```
Integral(sqrt(-x/(x + 1))/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = 2\,\arctan\left(\sqrt{-\frac{x}{x+1}}\right)$$

input
```
integrate((-x/(1+x))^(1/2)/x,x, algorithm="maxima")
```

output
```
2*arctan(sqrt(-x/(x + 1)))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = -\frac{1}{2}\,\pi\mathrm{sgn}(x+1) - \arcsin\left(2\,x+1\right)\mathrm{sgn}(x+1)$$

input
```
integrate((-x/(1+x))^(1/2)/x,x, algorithm="giac")
```

output
```
-1/2*pi*sgn(x + 1) - arcsin(2*x + 1)*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = 2\,\mathrm{atan}\left(\sqrt{-\frac{x}{x+1}}\right)$$

input
```
int((-x/(x + 1))^(1/2)/x,x)
```

output
```
2*atan((-x/(x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{-\frac{x}{1+x}}}{x}\,dx = 2\log\left(\sqrt{x+1}+\sqrt{x}\right)i$$

input
```
int((-x/(1+x))^(1/2)/x,x)
```

output
```
2*log(sqrt(x + 1) + sqrt(x))*i
```

3.28 $\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 46

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = -\frac{\sqrt{-1+5x}\sqrt{1+7x}}{x} - 12\arctan\left(\frac{\sqrt{1+7x}}{\sqrt{-1+5x}}\right)$$

output
```
-(-1+5*x)^(1/2)*(1+7*x)^(1/2)/x-12*arctan((1+7*x)^(1/2)/(-1+5*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = -\frac{\sqrt{\frac{-1+5x}{1+7x}}\left(\sqrt{-1+5x}(1+7x)+12x\sqrt{1+7x}\arctan\left(\sqrt{35}x-\sqrt{-1+5x}\sqrt{1+7x}\right)\right)}{x\sqrt{-1+5x}}$$

input
```
Integrate[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

output
```
-((Sqrt[(-1 + 5*x)/(1 + 7*x)]*(Sqrt[-1 + 5*x]*(1 + 7*x) + 12*x*Sqrt[1 + 7*
x]*ArcTan[Sqrt[35]*x - Sqrt[-1 + 5*x]*Sqrt[1 + 7*x]]))/(x*Sqrt[-1 + 5*x]))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {2050, 105, 104, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{5x-1}{7x+1}}}{x^2} \, dx$$

$$\Big\downarrow \; 2050$$

$$\int \frac{\sqrt{5x-1}}{x^2\sqrt{7x+1}}dx$$

$$\Big\downarrow \; 105$$

$$6\int \frac{1}{x\sqrt{5x-1}\sqrt{7x+1}}dx-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x}$$

$$\Big\downarrow \; 104$$

$$12\int \frac{1}{-\frac{7x+1}{5x-1}-1}d\frac{\sqrt{7x+1}}{\sqrt{5x-1}}-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x}$$

$$\Big\downarrow \; 217$$

$$-12\arctan\left(\frac{\sqrt{7x+1}}{\sqrt{5x-1}}\right)-\frac{\sqrt{5x-1}\sqrt{7x+1}}{x}$$

input
```
Int[Sqrt[(-1 + 5*x)/(1 + 7*x)]/x^2,x]
```

output
```
-((Sqrt[-1 + 5*x]*Sqrt[1 + 7*x])/x) - 12*ArcTan[Sqrt[1 + 7*x]/Sqrt[-1 + 5*
x]]
```

Defintions of rubi rules used

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(38) = 76$.

Time = 0.24 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

method	result
risch	$-\frac{(1+7x)\sqrt{\frac{-1+5x}{1+7x}}}{x}+\frac{6\arctan\left(\frac{-2x-2}{2\sqrt{35x^2-2x-1}}\right)\sqrt{\frac{-1+5x}{1+7x}}\,\sqrt{(-1+5x)(1+7x)}}{-1+5x}$
trager	$-\frac{(1+7x)\sqrt{-\frac{1-5x}{1+7x}}}{x}-6\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\frac{7\sqrt{-\frac{1-5x}{1+7x}}\,x-\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-\frac{1-5x}{1+7x}}-\mathrm{RootOf}\left(\_Z^2+1\right)}{x}\right)$
default	$-\frac{\sqrt{\frac{-1+5x}{1+7x}}\,(1+7x)\left(-(35x^2-2x-1)^{\frac{3}{2}}+35\sqrt{35x^2-2x-1}\,x^2+6\arctan\left(\frac{x+1}{\sqrt{35x^2-2x-1}}\right)x-2\sqrt{35x^2-2x-1}\,x\right)}{\sqrt{(-1+5x)(1+7x)}\,x}$

input
```
int(((-1+5*x)/(1+7*x))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(1+7*x)/x*((-1+5*x)/(1+7*x))^(1/2)+6*arctan(1/2*(-2*x-2)/(35*x^2-2*x-1)^(
1/2))*((-1+5*x)/(1+7*x))^(1/2)*((-1+5*x)*(1+7*x))^(1/2)/(-1+5*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = \frac{12\,x\arctan\left(\sqrt{\frac{5\,x-1}{7\,x+1}}\right) - (7\,x+1)\sqrt{\frac{5\,x-1}{7\,x+1}}}{x}$$

input
```
integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(12*x*arctan(sqrt((5*x - 1)/(7*x + 1))) - (7*x + 1)*sqrt((5*x - 1)/(7*x +
1)))/x
```

Sympy [F]

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = \int \frac{\sqrt{\frac{5x-1}{7x+1}}}{x^2}\,dx$$

input
```
integrate(((-1+5*x)/(1+7*x))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((5*x - 1)/(7*x + 1))/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = -\frac{12\sqrt{\frac{5x-1}{7x+1}}}{\frac{5x-1}{7x+1}+1} + 12\arctan\left(\sqrt{\frac{5x-1}{7x+1}}\right)$$

input
```
integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-12*sqrt((5*x - 1)/(7*x + 1))/((5*x - 1)/(7*x + 1) + 1) + 12*arctan(sqrt((
5*x - 1)/(7*x + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(38) = 76.

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.48

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = \left(\sqrt{35} - 12\arctan\left(\frac{1}{7}\sqrt{35}\right)\right)\operatorname{sgn}(7x+1) + 12\arctan\left(-\sqrt{35}x + \sqrt{35x^2-2x-1}\right)\operatorname{sgn}(7x+1) - \frac{2\left(\left(\sqrt{35}x - \sqrt{35x^2-2x-1}\right)\operatorname{sgn}(7x+1) + \sqrt{35}\operatorname{sgn}(7x+1)\right)}{\left(\sqrt{35}x - \sqrt{35x^2-2x-1}\right)^2 + 1}$$

input
```
integrate(((-1+5*x)/(1+7*x))^(1/2)/x^2,x, algorithm="giac")
```

output
```
(sqrt(35) - 12*arctan(1/7*sqrt(35)))*sgn(7*x + 1) + 12*arctan(-sqrt(35)*x
+ sqrt(35*x^2 - 2*x - 1))*sgn(7*x + 1) - 2*((sqrt(35)*x - sqrt(35*x^2 - 2*
x - 1))*sgn(7*x + 1) + sqrt(35)*sgn(7*x + 1))/((sqrt(35)*x - sqrt(35*x^2 -
 2*x - 1))^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.61

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = 12\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,\sqrt{\frac{5\,x-1}{7\,x+1}}}{35}\right) - \frac{12\,\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,\sqrt{\frac{5\,x-1}{7\,x+1}}}{25\,\left(\frac{7\,x-\frac{7}{5}}{7\,x+1}+\frac{7}{5}\right)}$$

input
```
int(((5*x - 1)/(7*x + 1))^(1/2)/x^2,x)
```

output
```
12*atan((5^(1/2)*7^(1/2)*35^(1/2)*((5*x - 1)/(7*x + 1))^(1/2))/35) - (12*5
^(1/2)*7^(1/2)*35^(1/2)*((5*x - 1)/(7*x + 1))^(1/2))/(25*((7*x - 7/5)/(7*x
 + 1) + 7/5))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int \frac{\sqrt{\frac{-1+5x}{1+7x}}}{x^2}\,dx = \frac{12atan\left(\frac{\sqrt{7x+1}\,\sqrt{5}+\sqrt{5x-1}\,\sqrt{7}-\sqrt{5}}{\sqrt{7}}\right)x-12atan\left(\frac{\sqrt{7x+1}\,\sqrt{5}+\sqrt{5x-1}\,\sqrt{7}+\sqrt{5}}{\sqrt{7}}\right)x-\sqrt{5x-1}\,\sqrt{7x+1}}{x}$$

input
```
int(((-1+5*x)/(1+7*x))^(1/2)/x^2,x)
```

output
```
(12*atan((sqrt(7*x + 1)*sqrt(5) + sqrt(5*x - 1)*sqrt(7) - sqrt(5))/sqrt(7)
)*x - 12*atan((sqrt(7*x + 1)*sqrt(5) + sqrt(5*x - 1)*sqrt(7) + sqrt(5))/sq
rt(7))*x - sqrt(5*x - 1)*sqrt(7*x + 1))/x
```

3.29 $\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 16

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = 2\arctan\left(\sqrt{-1+\frac{2}{1+x}}\right)$$

output
```
2*arctan((-1+2/(1+x))^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(16) = 32.

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 3.19

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = -\frac{2\sqrt{\frac{1-x}{1+x}}\sqrt{1-x^2}\arctan\left(\frac{\sqrt{1-x^2}}{-1+x}\right)}{-1+x}$$

input
```
Integrate[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]
```

output
```
(-2*Sqrt[(1 - x)/(1 + x)]*Sqrt[1 - x^2]*ArcTan[Sqrt[1 - x^2]/(-1 + x)])/(-
1 + x)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2055, 217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{1-x}{x+1}}}{x-1} \, dx$$

$$\Big\downarrow \ 2055$$

$$-4 \int \frac{1}{-\frac{2(1-x)}{x+1}-2} d\sqrt{\frac{1-x}{x+1}}$$

$$\Big\downarrow \ 217$$

$$2 \arctan \left(\sqrt{\frac{1-x}{x+1}}\right)$$

input
```
Int[Sqrt[(1 - x)/(1 + x)]/(-1 + x),x]
```

output
```
2*ArcTan[Sqrt[(1 - x)/(1 + x)]]
```

Defintions of rubi rules used

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 2055
```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)
 Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/
(b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1
/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Int
egerQ[r]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(14) = 28$.

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

method	result	siz
default	$-\frac{\sqrt{-\frac{x-1}{x+1}}\,(x+1)\arcsin(x)}{\sqrt{-(x-1)(x+1)}}$	30
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{x-1}{x+1}}\,x-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{x-1}{x+1}}+x\right)$	52

input
```
int(((1-x)/(x+1))^(1/2)/(x-1),x,method=_RETURNVERBOSE)
```

output
```
-(-(x-1)/(x+1))^(1/2)*(x+1)/(-(x-1)*(x+1))^(1/2)*arcsin(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
integrate(((1-x)/(1+x))^(1/2)/(x-1),x, algorithm="fricas")
```

output
```
2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Sympy [F]

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = \int \frac{\sqrt{-\frac{x-1}{x+1}}}{x-1}\,dx$$

input
```
integrate(((1-x)/(1+x))**(1/2)/(x-1),x)
```

output
```
Integral(sqrt(-(x - 1)/(x + 1))/(x - 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = 2\arctan\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
integrate(((1-x)/(1+x))^(1/2)/(x-1),x, algorithm="maxima")
```

output
```
2*arctan(sqrt(-(x - 1)/(x + 1)))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = -\frac{1}{2}\,\pi \mathrm{sgn}(x+1) - \arcsin{(x)}\,\mathrm{sgn}(x+1)$$

input
```
integrate(((1-x)/(1+x))^(1/2)/(x-1),x, algorithm="giac")
```

output
```
-1/2*pi*sgn(x + 1) - arcsin(x)*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = 2\,\mathrm{atan}\left(\sqrt{-\frac{x-1}{x+1}}\right)$$

input
```
int((-(x - 1)/(x + 1))^(1/2)/(x - 1),x)
```

output
```
2*atan((-(x - 1)/(x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{\frac{1-x}{1+x}}}{-1+x}\,dx = 2asin\left(\frac{\sqrt{1-x}}{\sqrt{2}}\right)$$

input
```
int(((1-x)/(1+x))^(1/2)/(x-1),x)
```

output
```
2*asin(sqrt( - x + 1)/sqrt(2))
```

3.30 $\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 24

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx=\frac{2\arctan\left(\sqrt{-1+\frac{a+c}{c-bx}}\right)}{b}$$

output
```
2*arctan((-1+(a+c)/(-b*x+c))^(1/2))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 63 vs. 2(24) = 48.

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.62

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx=\frac{2\sqrt{c-bx}\sqrt{\frac{a+bx}{c-bx}}\arctan\left(\frac{\sqrt{a+bx}}{\sqrt{c-bx}}\right)}{b\sqrt{a+bx}}$$

input
```
Integrate[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]
```

output
```
(2*Sqrt[c - b*x]*Sqrt[(a + b*x)/(c - b*x)]*ArcTan[Sqrt[a + b*x]/Sqrt[c - b
*x]])/(b*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {2055, 27, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx} \, dx$$

$$\Big\downarrow \text{2055}$$

$$2b(a+c)\int \frac{1}{b^2(a+c)\left(\frac{a+bx}{c-bx}+1\right)}d\sqrt{\frac{a+bx}{c-bx}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\int \frac{1}{\frac{a+bx}{c-bx}+1}d\sqrt{\frac{a+bx}{c-bx}}}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{2\arctan\left(\sqrt{\frac{a+bx}{c-bx}}\right)}{b}$$

input
```
Int[Sqrt[(a + b*x)/(c - b*x)]/(a + b*x),x]
```

output
```
(2*ArcTan[Sqrt[(a + b*x)/(c - b*x)]])/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2055
```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)
 Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/
(b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1
/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Int
egerQ[r]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(22) = 44$.

Time = 0.49 (sec) , antiderivative size = 85, normalized size of antiderivative = 3.54

method	result	size
default	$-\frac{\arctan\left(\frac{\sqrt{b^2}\,(2bx+a-c)}{2b\sqrt{-(bx+a)(bx-c)}}\right)(bx-c)\sqrt{-\frac{bx+a}{bx-c}}}{\sqrt{b^2}\,\sqrt{-(bx+a)(bx-c)}}$	85

input
```
int(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-arctan(1/2*(b^2)^(1/2)/b*(2*b*x+a-c)/(-(b*x+a)*(b*x-c))^(1/2))*(b*x-c)*(-
(b*x+a)/(b*x-c))^(1/2)/(b^2)^(1/2)/(-(b*x+a)*(b*x-c))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = \frac{2\arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

input
```
integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = \int \frac{\sqrt{\frac{a+bx}{-bx+c}}}{a+bx}\,dx$$

input
```
integrate(((b*x+a)/(-b*x+c))**(1/2)/(b*x+a),x)
```

output
```
Integral(sqrt((a + b*x)/(-b*x + c))/(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = \frac{2\arctan\left(\sqrt{-\frac{bx+a}{bx-c}}\right)}{b}$$

input
```
integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
2*arctan(sqrt(-(b*x + a)/(b*x - c)))/b
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = -\frac{\arcsin\left(-\frac{2\,bx+a-c}{a+c}\right)\operatorname{sgn}(-ab-bc)\operatorname{sgn}(bx-c)}{|b|}$$

input
```
integrate(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
-arcsin(-(2*b*x + a - c)/(a + c))*sgn(-a*b - b*c)*sgn(b*x - c)/abs(b)
```

Mupad [B] (verification not implemented)

Time = 9.74 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = -\frac{2\sqrt{-b}\,\mathrm{atanh}\left(\frac{\sqrt{-b}\sqrt{\frac{a+b\,x}{c-b\,x}}}{\sqrt{b}}\right)}{b^{3/2}}$$

input
```
int(((a + b*x)/(c - b*x))^(1/2)/(a + b*x),x)
```

output
```
-(2*(-b)^(1/2)*atanh(((-b)^(1/2)*((a + b*x)/(c - b*x))^(1/2))/b^(1/2)))/b^
(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{\sqrt{\frac{a+bx}{c-bx}}}{a+bx}\,dx = \frac{2\log\left(\frac{\sqrt{bx-c}+\sqrt{bx+a}}{\sqrt{a+c}}\right)i}{b}$$

input
```
int(((b*x+a)/(-b*x+c))^(1/2)/(b*x+a),x)
```

output
```
(2*log((sqrt(b*x - c) + sqrt(a + b*x))/sqrt(a + c))*i)/b
```

3.31 $\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 41

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} \, dx = \frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

output
```
2*arctanh(d^(1/2)*((b*x+a)/(d*x+c))^(1/2)/b^(1/2))/b^(1/2)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx} \, dx = \frac{2\sqrt{\frac{a+bx}{c+dx}}\sqrt{c+dx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}\sqrt{a+bx}}\right)}{\sqrt{b}\sqrt{d}\sqrt{a+bx}}$$

input
```
Integrate[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
```

output
```
(2*Sqrt[(a + b*x)/(c + d*x)]*Sqrt[c + d*x]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x])
/(Sqrt[d]*Sqrt[a + b*x])])/(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {2055, 27, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx$$

$$\Big\downarrow \text{2055}$$

$$2(bc-ad)\int \frac{1}{(bc-ad)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\sqrt{\frac{a+bx}{c+dx}}$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\sqrt{\frac{a+bx}{c+dx}}$$

$$\Big\downarrow \text{221}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{a+bx}{c+dx}}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{d}}$$

input
```
Int[Sqrt[(a + b*x)/(c + d*x)]/(a + b*x),x]
```

output
```
(2*ArcTanh[(Sqrt[d]*Sqrt[(a + b*x)/(c + d*x)])/Sqrt[b]])/(Sqrt[b]*Sqrt[d])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2055
```
Int[(u_)^(r_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)
 Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/
(b*e - d*x^q)^(1/n + 1))*(u /. x -> ((-a)*e + c*x^q)^(1/n)/(b*e - d*x^q)^(1
/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b
, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p] && IntegerQ[1/n] && Int
egerQ[r]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(31) = 62$.

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{\ln\left(\frac{2bdx+2\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{\sqrt{(bx+a)(dx+c)}\,\sqrt{bd}}$	80

input
```
int(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
ln(1/2*(2*b*d*x+2*((b*x+a)*(d*x+c))^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2)
)*(d*x+c)*((b*x+a)/(d*x+c))^(1/2)/((b*x+a)*(d*x+c))^(1/2)/(b*d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = \left[\frac{\sqrt{bd}\log\left(2\,bdx+bc+ad+2\,\sqrt{bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}\right)}{bd}, -\frac{2\,\sqrt{-bd}\arctan\left(\frac{\sqrt{-bd}(dx+c)\sqrt{\frac{bx+a}{dx+c}}}{bdx+ad}\right)}{bd}\right]$$

input
```
integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="fricas")
```

output
```
[sqrt(b*d)*log(2*b*d*x + b*c + a*d + 2*sqrt(b*d)*(d*x + c)*sqrt((b*x + a)/
(d*x + c)))/(b*d), -2*sqrt(-b*d)*arctan(sqrt(-b*d)*(d*x + c)*sqrt((b*x + a
)/(d*x + c))/(b*d*x + a*d))/(b*d)]
```

Sympy [F]

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = \int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx$$

input
```
integrate(((b*x+a)/(d*x+c))**(1/2)/(b*x+a),x)
```

output
```
Integral(sqrt((a + b*x)/(c + d*x))/(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = -\frac{\log\left(\frac{d\sqrt{\frac{bx+a}{dx+c}}-\sqrt{bd}}{d\sqrt{\frac{bx+a}{dx+c}}+\sqrt{bd}}\right)}{\sqrt{bd}}$$

input
```
integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="maxima")
```

output
```
-log((d*sqrt((b*x + a)/(d*x + c)) - sqrt(b*d))/(d*sqrt((b*x + a)/(d*x + c)
) + sqrt(b*d)))/sqrt(b*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = -\frac{\log\left(\left|-bc-ad-2\sqrt{bd}\left(\sqrt{bd}x-\sqrt{bdx^2+bcx+adx+ac}\right)\right|\right)\operatorname{sgn}(dx+c)}{\sqrt{bd}}$$

input
```
integrate(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x, algorithm="giac")
```

output
```
-log(abs(-b*c - a*d - 2*sqrt(b*d)*(sqrt(b*d)*x - sqrt(b*d*x^2 + b*c*x + a*
d*x + a*c))))*sgn(d*x + c)/sqrt(b*d)
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = \frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d}\,\sqrt{\frac{a+b\,x}{c+d\,x}}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{d}}$$

input
```
int(((a + b*x)/(c + d*x))^(1/2)/(a + b*x),x)
```

output
```
(2*atanh((d^(1/2)*((a + b*x)/(c + d*x))^(1/2))/b^(1/2)))/(b^(1/2)*d^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.07

$$\int \frac{\sqrt{\frac{a+bx}{c+dx}}}{a+bx}\,dx = \frac{2\sqrt{d}\,\sqrt{b}\log\left(\frac{\sqrt{d}\,\sqrt{bx+a}+\sqrt{b}\,\sqrt{dx+c}}{\sqrt{ad-bc}}\right)}{bd}$$

input
```
int(((b*x+a)/(d*x+c))^(1/2)/(b*x+a),x)
```

output
```
(2*sqrt(d)*sqrt(b)*log((sqrt(d)*sqrt(a + b*x) + sqrt(b)*sqrt(c + d*x))/sqr
t(a*d - b*c)))/(b*d)
```

3.32 $\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 18

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = -\left((1+x)\sqrt{-1+\frac{2}{1+x}}\right)$$

output
```
-(1+x)*(-1+2/(1+x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = \frac{-1+x}{\sqrt{\frac{1-x}{1+x}}}$$

input
```
Integrate[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]
```

output
```
(-1 + x)/Sqrt[(1 - x)/(1 + x)]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {2056, 27, 297}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\frac{1-x}{x+1}}(x+1)}dx$$

$$\Big\downarrow \text{2056}$$

$$-4\int \frac{1-\frac{1-x}{x+1}}{2\left(\frac{1-x}{x+1}+1\right)^2}d\sqrt{\frac{1-x}{x+1}}$$

$$\Big\downarrow \text{27}$$

$$-2\int \frac{1-\frac{1-x}{x+1}}{\left(\frac{1-x}{x+1}+1\right)^2}d\sqrt{\frac{1-x}{x+1}}$$

$$\Big\downarrow \text{297}$$

$$-\frac{2\sqrt{\frac{1-x}{x+1}}}{\frac{1-x}{x+1}+1}$$

input
```
Int[x/(Sqrt[(1 - x)/(1 + x)]*(1 + x)),x]
```

output
```
(-2*Sqrt[(1 - x)/(1 + x)])/(1 + (1 - x)/(1 + x))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 297
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[c*
x*((a + b*x^2)^(p + 1)/a), x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d
, 0] && EqQ[a*d - b*c*(2*p + 3), 0]
```

rule 2056
```
Int[(u_)^(r_.)*(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)
*(x_)^(n_.)))^(p_), x_Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c
- a*d)/n)   Subst[Int[SimplifyIntegrand[x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)
^((m + 1)/n - 1)/(b*e - d*x^q)^((m + 1)/n + 1))*(u /. x -> ((-a)*e + c*x^q)
^(1/n)/(b*e - d*x^q)^(1/n))^r, x], x], x, (e*((a + b*x^n)/(c + d*x^n)))^(1/
q)], x]] /; FreeQ[{a, b, c, d, e}, x] && PolynomialQ[u, x] && FractionQ[p]
&& IntegerQ[1/n] && IntegersQ[m, r]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

method	result	size
gosper	$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$	17
risch	$\frac{x-1}{\sqrt{-\frac{x-1}{x+1}}}$	17
orering	$\frac{x-1}{\sqrt{\frac{1-x}{x+1}}}$	18
trager	$(-x-1)\sqrt{-\frac{x-1}{x+1}}$	19
default	$\frac{(x-1)\sqrt{-x^2+1}}{\sqrt{-\frac{x-1}{x+1}}\,\sqrt{-(x-1)(x+1)}}$	36

input
```
int(x/((1-x)/(x+1))^(1/2)/(x+1),x,method=_RETURNVERBOSE)
```

output
```
(x-1)/(-(x-1)/(x+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = -(x+1)\sqrt{-\frac{x-1}{x+1}}$$

input
```
integrate(x/((1-x)/(1+x))^(1/2)/(1+x),x, algorithm="fricas")
```

output
```
-(x + 1)*sqrt(-(x - 1)/(x + 1))
```

Sympy [F]

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = \int \frac{x}{\sqrt{-\frac{x-1}{x+1}}\,(x+1)}\,dx$$

input
```
integrate(x/((1-x)/(1+x))**(1/2)/(1+x),x)
```

output
```
Integral(x/(sqrt(-(x - 1)/(x + 1))*(x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.50

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = \frac{2\sqrt{-\frac{x-1}{x+1}}}{\frac{x-1}{x+1}-1}$$

input
```
integrate(x/((1-x)/(1+x))^(1/2)/(1+x),x, algorithm="maxima")
```

output
```
2*sqrt(-(x - 1)/(x + 1))/((x - 1)/(x + 1) - 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = -\frac{\sqrt{-x^2+1}}{\operatorname{sgn}(x+1)}$$

input
```
integrate(x/((1-x)/(1+x))^(1/2)/(1+x),x, algorithm="giac")
```

output
```
-sqrt(-x^2 + 1)/sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 10.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = -\sqrt{-\frac{x-1}{x+1}}\,(x+1)$$

input
```
int(x/((-(x - 1)/(x + 1))^(1/2)*(x + 1)),x)
```

output
```
-(-(x - 1)/(x + 1))^(1/2)*(x + 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{x}{\sqrt{\frac{1-x}{1+x}}(1+x)}\,dx = -\sqrt{x+1}\,\sqrt{1-x}$$

input
```
int(x/((1-x)/(1+x))^(1/2)/(1+x),x)
```

output
```
 - sqrt(x + 1)*sqrt( - x + 1)
```

3.33 $\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = \sqrt{2+x}\sqrt{3+x} - \operatorname{arcsinh}\left(\sqrt{2+x}\right) + 2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\sqrt{2+x}}{\sqrt{3+x}}\right)$$

output
```
(2+x)^(1/2)*(3+x)^(1/2)-arcsinh((2+x)^(1/2))+2*2^(1/2)*arctanh(2^(1/2)*(2+
x)^(1/2)/(3+x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = \sqrt{2+x}\sqrt{3+x} + 2\sqrt{2}\operatorname{arctanh}\left(\frac{-1-x+\sqrt{2+x}\sqrt{3+x}}{\sqrt{2}}\right) + \log\left(\sqrt{2+x}-\sqrt{3+x}\right)$$

input
```
Integrate[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

output
```
Sqrt[2 + x]*Sqrt[3 + x] + 2*Sqrt[2]*ArcTanh[(-1 - x + Sqrt[2 + x]*Sqrt[3 +
 x])/Sqrt[2]] + Log[Sqrt[2 + x] - Sqrt[3 + x]]
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2050, 171, 27, 175, 64, 104, 220, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(x+1)\sqrt{\frac{x+2}{x+3}}}dx$$

$$\Big\downarrow \text{2050}$$

$$\int \frac{x\sqrt{x+3}}{(x+1)\sqrt{x+2}}dx$$

$$\Big\downarrow \text{171}$$

$$\int -\frac{x+5}{2(x+1)\sqrt{x+2}\sqrt{x+3}}dx+\sqrt{x+2}\sqrt{x+3}$$

$$\Big\downarrow \text{27}$$

$$\sqrt{x+2}\sqrt{x+3}-\frac{1}{2}\int \frac{x+5}{(x+1)\sqrt{x+2}\sqrt{x+3}}dx$$

$$\Big\downarrow \text{175}$$

$$\frac{1}{2}\left(-\int \frac{1}{\sqrt{x+2}\sqrt{x+3}}dx-4\int \frac{1}{(x+1)\sqrt{x+2}\sqrt{x+3}}dx\right)+\sqrt{x+2}\sqrt{x+3}$$

$$\Big\downarrow \text{64}$$

$$\frac{1}{2}\left(-2\int \frac{1}{\sqrt{x+3}}d\sqrt{x+2}-4\int \frac{1}{(x+1)\sqrt{x+2}\sqrt{x+3}}dx\right)+\sqrt{x+2}\sqrt{x+3}$$

$$\Big\downarrow \text{104}$$

$$\frac{1}{2}\left(-2\int \frac{1}{\sqrt{x+3}}d\sqrt{x+2}-8\int \frac{1}{\frac{2(x+2)}{x+3}-1}d\frac{\sqrt{x+2}}{\sqrt{x+3}}\right)+\sqrt{x+2}\sqrt{x+3}$$

$$\Big\downarrow \; 220$$

$$\frac{1}{2}\left(4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x+2}}{\sqrt{x+3}}\right)-2\int\frac{1}{\sqrt{x+3}}d\sqrt{x+2}\right)+\sqrt{x+2}\sqrt{x+3}$$

$$\Big\downarrow \; 222$$

$$\frac{1}{2}\left(4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\sqrt{x+2}}{\sqrt{x+3}}\right)-2\text{arcsinh}\left(\sqrt{x+2}\right)\right)+\sqrt{x+2}\sqrt{x+3}$$

input
```
Int[x/((1 + x)*Sqrt[(2 + x)/(3 + x)]),x]
```

output
```
Sqrt[2 + x]*Sqrt[3 + x] + (-2*ArcSinh[Sqrt[2 + x]] + 4*Sqrt[2]*ArcTanh[(Sq
rt[2]*Sqrt[2 + x])/Sqrt[3 + x]])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 64
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]), x_Symbol] :> Simp
[2/b   Subst[Int[1/Sqrt[c - a*(d/b) + d*(x^2/b)], x], x, Sqrt[a + b*x]], x]
 /; FreeQ[{a, b, c, d}, x] && GtQ[c - a*(d/b), 0] && ( !GtQ[a - c*(b/d), 0]
 || PosQ[b])
```

rule 104
```
Int[(((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_))/((e_.) + (f_.)*(x
_)), x_] :> With[{q = Denominator[m]}, Simp[q   Subst[Int[x^(q*(m + 1) - 1)
/(b*e - a*f - (d*e - c*f)*x^q), x], x, (a + b*x)^(1/q)/(c + d*x)^(1/q)], x]
] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[m + n + 1, 0] && RationalQ[n] && L
tQ[-1, m, 0] && SimplerQ[a + b*x, c + d*x]
```

rule 171

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[h*(a + b*x)^m*(c + d*x)^(n + 1)*((
e + f*x)^(p + 1)/(d*f*(m + n + p + 2))), x] + Simp[1/(d*f*(m + n + p + 2))
  Int[(a + b*x)^(m - 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*g*(m + n + p + 2
) - h*(b*c*e*m + a*(d*e*(n + 1) + c*f*(p + 1))) + (b*d*f*g*(m + n + p + 2)
+ h*(a*d*f*m - b*(d*e*(m + n + 1) + c*f*(m + p + 1))))*x, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, n, p}, x] && GtQ[m, 0] && NeQ[m + n + p + 2, 0]
 && IntegersQ[2*m, 2*n, 2*p]
```

rule 175

```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_
)))/((a_.) + (b_.)*(x_)), x_] :> Simp[h/b   Int[(c + d*x)^n*(e + f*x)^p, x]
, x] + Simp[(b*g - a*h)/b   Int[(c + d*x)^n*((e + f*x)^p/(a + b*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x]
```

rule 220

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2050

```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.46

method	result
default	$-\frac{(2+x)\left(-2\sqrt{2}\,\operatorname{arctanh}\left(\frac{(7+3x)\sqrt{2}}{4\sqrt{x^2+5x+6}}\right)+\ln\left(\frac{5}{2}+x+\sqrt{x^2+5x+6}\right)-2\sqrt{x^2+5x+6}\right)}{2\sqrt{\frac{2+x}{3+x}}\,\sqrt{(3+x)(2+x)}}$
risch	$\frac{2+x}{\sqrt{\frac{2+x}{3+x}}}+\frac{\left(-\frac{\ln\left(\frac{5}{2}+x+\sqrt{x^2+5x+6}\right)}{2}+\sqrt{2}\,\operatorname{arctanh}\left(\frac{(7+3x)\sqrt{2}}{4\sqrt{(x+1)^2+3x+5}}\right)\right)\sqrt{(3+x)(2+x)}}{\sqrt{\frac{2+x}{3+x}}\,(3+x)}$
trager	$3\left(1+\frac{x}{3}\right)\sqrt{-\frac{-2-x}{3+x}}+\frac{\ln\left(2\sqrt{-\frac{-2-x}{3+x}}\,x+6\sqrt{-\frac{-2-x}{3+x}}-2x-5\right)}{2}+\operatorname{RootOf}\left(\_Z^2-2\right)\ln\left(\frac{3\operatorname{RootOf}\left(\_Z^2-2\right)x+4\right.\right.$

input
```
int(x/(x+1)/((2+x)/(3+x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2+x)*(-2*2^(1/2)*arctanh(1/4*(7+3*x)*2^(1/2)/(x^2+5*x+6)^(1/2))+ln(5
/2+x+(x^2+5*x+6)^(1/2))-2*(x^2+5*x+6)^(1/2))/((2+x)/(3+x))^(1/2)/((3+x)*(2
+x))^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. 2(40) = 80.

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.54

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = (x+3)\sqrt{\frac{x+2}{x+3}}+\sqrt{2}\log\left(\frac{2\sqrt{2}(x+3)\sqrt{\frac{x+2}{x+3}}+3x+7}{x+1}\right) - \frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}+1\right)+\frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}-1\right)$$

input
```
integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="fricas")
```

output
```
(x + 3)*sqrt((x + 2)/(x + 3)) + sqrt(2)*log((2*sqrt(2)*(x + 3)*sqrt((x + 2
)/(x + 3)) + 3*x + 7)/(x + 1)) - 1/2*log(sqrt((x + 2)/(x + 3)) + 1) + 1/2*
log(sqrt((x + 2)/(x + 3)) - 1)
```

Sympy [F]

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = \int \frac{x}{\sqrt{\frac{x+2}{x+3}}\,(x+1)}\,dx$$

input
```
integrate(x/(1+x)/((2+x)/(3+x))**(1/2),x)
```

output
```
Integral(x/(sqrt((x + 2)/(x + 3))*(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(40) = 80$.

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.91

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = -\sqrt{2}\log\left(-\frac{\sqrt{2}-2\sqrt{\frac{x+2}{x+3}}}{\sqrt{2}+2\sqrt{\frac{x+2}{x+3}}}\right) - \frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1} - \frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x+2}{x+3}}-1\right)$$

input
```
integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(2)*log(-(sqrt(2) - 2*sqrt((x + 2)/(x + 3)))/(sqrt(2) + 2*sqrt((x + 2
)/(x + 3)))) - sqrt((x + 2)/(x + 3))/((x + 2)/(x + 3) - 1) - 1/2*log(sqrt(
(x + 2)/(x + 3)) + 1) + 1/2*log(sqrt((x + 2)/(x + 3)) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(40) = 80$.

Time = 0.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.39

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = \sqrt{2}\log\left(-\frac{\sqrt{2}-2}{\sqrt{2}+2}\right)\operatorname{sgn}(x+3) - \frac{\sqrt{2}\log\left(\frac{\left|-2\,x-2\,\sqrt{2}+2\,\sqrt{x^2+5\,x+6}-2\right|}{\left|-2\,x+2\,\sqrt{2}+2\,\sqrt{x^2+5\,x+6}-2\right|}\right)}{\operatorname{sgn}(x+3)} + \frac{\log\left(\left|-2\,x+2\,\sqrt{x^2+5\,x+6}-5\right|\right)}{2\operatorname{sgn}(x+3)} + \frac{\sqrt{x^2+5\,x+6}}{\operatorname{sgn}(x+3)}$$

input
```
integrate(x/(1+x)/((2+x)/(3+x))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*log(-(sqrt(2) - 2)/(sqrt(2) + 2))*sgn(x + 3) - sqrt(2)*log(abs(-2*
x - 2*sqrt(2) + 2*sqrt(x^2 + 5*x + 6) - 2)/abs(-2*x + 2*sqrt(2) + 2*sqrt(x
^2 + 5*x + 6) - 2))/sgn(x + 3) + 1/2*log(abs(-2*x + 2*sqrt(x^2 + 5*x + 6)
- 5))/sgn(x + 3) + sqrt(x^2 + 5*x + 6)/sgn(x + 3)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = 2\sqrt{2}\,\operatorname{atanh}\left(\sqrt{2}\sqrt{\frac{x+2}{x+3}}\right) - \frac{\sqrt{\frac{x+2}{x+3}}}{\frac{x+2}{x+3}-1} - \operatorname{atanh}\left(\sqrt{\frac{x+2}{x+3}}\right)$$

input
```
int(x/(((x + 2)/(x + 3))^(1/2)*(x + 1)),x)
```

output
```
2*2^(1/2)*atanh(2^(1/2)*((x + 2)/(x + 3))^(1/2)) - ((x + 2)/(x + 3))^(1/2)
/((x + 2)/(x + 3) - 1) - atanh(((x + 2)/(x + 3))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.70

$$\int \frac{x}{(1+x)\sqrt{\frac{2+x}{3+x}}}\,dx = \sqrt{x+2}\,\sqrt{x+3} - \sqrt{2}\log\left(\sqrt{x+3}+\sqrt{x+2}-\sqrt{2}-1\right) + \sqrt{2}\log\left(\sqrt{x+3}+\sqrt{x+2}-\sqrt{2}+1\right) + \sqrt{2}\log\left(\sqrt{x+3}+\sqrt{x+2}+\sqrt{2}-1\right) - \sqrt{2}\log\left(\sqrt{x+3}+\sqrt{x+2}+\sqrt{2}+1\right) - \log\left(\sqrt{x+3}+\sqrt{x+2}\right)$$

input
```
int(x/(1+x)/((2+x)/(3+x))^(1/2),x)
```

output
```
sqrt(x + 2)*sqrt(x + 3) - sqrt(2)*log(sqrt(x + 3) + sqrt(x + 2) - sqrt(2)
- 1) + sqrt(2)*log(sqrt(x + 3) + sqrt(x + 2) - sqrt(2) + 1) + sqrt(2)*log(
sqrt(x + 3) + sqrt(x + 2) + sqrt(2) - 1) - sqrt(2)*log(sqrt(x + 3) + sqrt(
x + 2) + sqrt(2) + 1) - log(sqrt(x + 3) + sqrt(x + 2))
```

3.34 $\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 8

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = 2\text{arcsinh}\left(\sqrt{x}\right)$$

output
```
2*arcsinh(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 18 vs. $2(8) = 16$.

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = -2\log\left(-\sqrt{x}+\sqrt{1+x}\right)$$

input
```
Integrate[1/(Sqrt[x]*Sqrt[1 + x]),x]
```

output
```
-2*Log[-Sqrt[x] + Sqrt[1 + x]]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x}\sqrt{x+1}}\,dx$$

$$\Big\downarrow \; 63$$

$$2\int \frac{1}{\sqrt{x+1}}d\sqrt{x}$$

$$\Big\downarrow \; 222$$

$$2\text{arcsinh}\left(\sqrt{x}\right)$$

input
```
Int[1/(Sqrt[x]*Sqrt[1 + x]),x]
```

output
```
2*ArcSinh[Sqrt[x]]
```

Defintions of rubi rules used

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
meijerg	$2 \operatorname{arcsinh}\left(\sqrt{x}\right)$	7
default	$\frac{\sqrt{(x+1)x}\,\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)}{\sqrt{x}\,\sqrt{x+1}}$	28

input
```
int(1/x^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsinh(x^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18 vs. 2(6) = 12.

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = -\log\left(2\sqrt{x+1}\sqrt{x} - 2\,x - 1\right)$$

input
```
integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
-log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 3.25

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = \begin{cases} 2\operatorname{acosh}\left(\sqrt{x+1}\right) & \text{for } |x+1| > 1 \\ -2i\operatorname{asin}\left(\sqrt{x+1}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/x**(1/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((2*acosh(sqrt(x + 1)), Abs(x + 1) > 1), (-2*I*asin(sqrt(x + 1)),
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(6) = 12.

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = \log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right) - \log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

input
```
integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
log(sqrt(x + 1)/sqrt(x) + 1) - log(sqrt(x + 1)/sqrt(x) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 14 vs. 2(6) = 12.

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = -2\,\log\left(\sqrt{x+1}-\sqrt{x}\right)$$

input
```
integrate(1/x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-2*log(sqrt(x + 1) - sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = 4\,\mathrm{atanh}\left(\frac{\sqrt{x+1}-1}{\sqrt{x}}\right)$$

input
```
int(1/(x^(1/2)*(x + 1)^(1/2)),x)
```

output
```
4*atanh(((x + 1)^(1/2) - 1)/x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{1}{\sqrt{x}\sqrt{1+x}}\,dx = 2\log\left(\sqrt{x+1}+\sqrt{x}\right)$$

input
```
int(1/x^(1/2)/(1+x)^(1/2),x)
```

output
```
2*log(sqrt(x + 1) + sqrt(x))
```

3.35 $\int \frac{\sqrt{\frac{x}{1+x}}}{x} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 8

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x} \, dx = 2\operatorname{arcsinh}\left(\sqrt{x}\right)$$

output
```
2*arcsinh(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 18 vs. 2(8) = 16.

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 2.25

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x} \, dx = -2 \log \left(-\sqrt{x}+\sqrt{1+x}\right)$$

input
```
Integrate[Sqrt[x/(1 + x)]/x,x]
```

output
```
-2*Log[-Sqrt[x] + Sqrt[1 + x]]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2050, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{x}{x+1}}}{x} \, dx$$

$$\Big\downarrow \text{2050}$$

$$\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{63}$$

$$2\int \frac{1}{\sqrt{x+1}}d\sqrt{x}$$

$$\Big\downarrow \text{222}$$

$$2\text{arcsinh}\left(\sqrt{x}\right)$$

input
```
Int[Sqrt[x/(1 + x)]/x,x]
```

output
```
2*ArcSinh[Sqrt[x]]
```

Defintions of rubi rules used

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. $2(6) = 12$.

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 3.75

method	result	size
trager	$\ln\left(2\sqrt{\frac{x}{x+1}}\,x+2\sqrt{\frac{x}{x+1}}+2x+1\right)$	30
default	$\frac{\sqrt{\frac{x}{x+1}}\,(x+1)\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)}{\sqrt{(x+1)x}}$	32

input
```
int((x/(x+1))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
ln(2*(x/(x+1))^(1/2)*x+2*(x/(x+1))^(1/2)+2*x+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = \log\left(\sqrt{\frac{x}{x+1}}+1\right)-\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

input
```
integrate((x/(1+x))^(1/2)/x,x, algorithm="fricas")
```

output
```
log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = \int \frac{\sqrt{\frac{x}{x+1}}}{x}\,dx$$

input
```
integrate((x/(1+x))**(1/2)/x,x)
```

output
```
Integral(sqrt(x/(x + 1))/x, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(6) = 12$.

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 3.38

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = \log\left(\sqrt{\frac{x}{x+1}}+1\right) - \log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

input
```
integrate((x/(1+x))^(1/2)/x,x, algorithm="maxima")
```

output
```
log(sqrt(x/(x + 1)) + 1) - log(sqrt(x/(x + 1)) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. $2(6) = 12$.

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 2.75

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = -\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1)$$

input
```
integrate((x/(1+x))^(1/2)/x,x, algorithm="giac")
```

output
```
-log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1)
```

Mupad [B] (verification not implemented)

Time = 10.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = 2\,\mathrm{atanh}\left(\sqrt{\frac{x}{x+1}}\right)$$

input
```
int((x/(x + 1))^(1/2)/x,x)
```

output
```
2*atanh((x/(x + 1))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{\sqrt{\frac{x}{1+x}}}{x}\,dx = 2\log\left(\sqrt{x+1}+\sqrt{x}\right)$$

input
```
int((x/(1+x))^(1/2)/x,x)
```

output
```
2*log(sqrt(x + 1) + sqrt(x))
```

3.36 $\int \frac{\sqrt{x}}{\sqrt{1+x}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 22

$$\int \frac{\sqrt{x}}{\sqrt{1+x}} \, dx = \sqrt{x}\sqrt{1+x} - \operatorname{arcsinh}\left(\sqrt{x}\right)$$

output
```
x^(1/2)*(1+x)^(1/2)-arcsinh(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(22) = 44.

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \frac{\sqrt{x}}{\sqrt{1+x}} \, dx = \frac{\sqrt{\frac{x}{1+x}}\left(\sqrt{x}(1+x) + \sqrt{1+x}\log\left(-\sqrt{x} + \sqrt{1+x}\right)\right)}{\sqrt{x}}$$

input
```
Integrate[Sqrt[x]/Sqrt[1 + x],x]
```

output
```
(Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) + Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]
]))/Sqrt[x]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}}{\sqrt{x+1}}dx$$
$$\Big\downarrow \text{ 60}$$
$$\sqrt{x}\sqrt{x+1}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx$$
$$\Big\downarrow \text{ 63}$$
$$\sqrt{x}\sqrt{x+1}-\int \frac{1}{\sqrt{x+1}}d\sqrt{x}$$
$$\Big\downarrow \text{ 222}$$
$$\sqrt{x}\sqrt{x+1}-\operatorname{arcsinh}\left(\sqrt{x}\right)$$

input
```
Int[Sqrt[x]/Sqrt[1 + x],x]
```

output
```
Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.23

method	result	size
meijerg	$\frac{\sqrt{\pi}\,\sqrt{x}\,\sqrt{x+1}-\sqrt{\pi}\,\operatorname{arcsinh}\left(\sqrt{x}\right)}{\sqrt{\pi}}$	27
default	$\sqrt{x}\,\sqrt{x+1}-\frac{\sqrt{(x+1)x}\,\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)}{2\sqrt{x}\,\sqrt{x+1}}$	39
risch	$\sqrt{x}\,\sqrt{x+1}-\frac{\sqrt{(x+1)x}\,\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)}{2\sqrt{x}\,\sqrt{x+1}}$	39

input
```
int(x^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/Pi^(1/2)*(Pi^(1/2)*x^(1/2)*(x+1)^(1/2)-Pi^(1/2)*arcsinh(x^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{x}+\frac{1}{2}\,\log\left(2\,\sqrt{x+1}\sqrt{x}-2\,x-1\right)$$

input
```
integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
sqrt(x + 1)*sqrt(x) + 1/2*log(2*sqrt(x + 1)*sqrt(x) - 2*x - 1)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.82 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.86

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \begin{cases} \sqrt{x}\sqrt{x+1} - \operatorname{acosh}\left(\sqrt{x+1}\right) & \text{for } |x+1| > 1 \\ i\operatorname{asin}\left(\sqrt{x+1}\right) - \frac{i(x+1)^{\frac{3}{2}}}{\sqrt{-x}} + \frac{i\sqrt{x+1}}{\sqrt{-x}} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)/(1+x)**(1/2),x)
```

output
```
Piecewise((sqrt(x)*sqrt(x + 1) - acosh(sqrt(x + 1)), Abs(x + 1) > 1), (I*a
sin(sqrt(x + 1)) - I*(x + 1)**(3/2)/sqrt(-x) + I*sqrt(x + 1)/sqrt(-x), Tru
e))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(16) = 32.

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \frac{\sqrt{x+1}}{\sqrt{x}\left(\frac{x+1}{x}-1\right)} - \frac{1}{2}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}+1\right) + \frac{1}{2}\log\left(\frac{\sqrt{x+1}}{\sqrt{x}}-1\right)$$

input
```
integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x + 1)/(sqrt(x)*((x + 1)/x - 1)) - 1/2*log(sqrt(x + 1)/sqrt(x) + 1) +
 1/2*log(sqrt(x + 1)/sqrt(x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \sqrt{x+1}\sqrt{x} + \log\left(\sqrt{x+1} - \sqrt{x}\right)$$

input
```
integrate(x^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
sqrt(x + 1)*sqrt(x) + log(sqrt(x + 1) - sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \sqrt{x}\,\sqrt{x+1} - 2\,\mathrm{atanh}\left(\frac{\sqrt{x}}{\sqrt{x+1}-1}\right)$$

input
```
int(x^(1/2)/(x + 1)^(1/2),x)
```

output
```
x^(1/2)*(x + 1)^(1/2) - 2*atanh(x^(1/2)/((x + 1)^(1/2) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \frac{\sqrt{x}}{\sqrt{1+x}}\,dx = \sqrt{x}\,\sqrt{x+1} - \log\left(\sqrt{x+1} + \sqrt{x}\right)$$

input
```
int(x^(1/2)/(1+x)^(1/2),x)
```

output
```
sqrt(x)*sqrt(x + 1) - log(sqrt(x + 1) + sqrt(x))
```

3.37 $\int \sqrt{\frac{x}{1+x}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \sqrt{\frac{x}{1+x}}\,dx=\sqrt{x}\sqrt{1+x}-\operatorname{arcsinh}\left(\sqrt{x}\right)$$

output
```
x^(1/2)*(1+x)^(1/2)-arcsinh(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(22) = 44.

Time = 0.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \sqrt{\frac{x}{1+x}}\,dx=\frac{\sqrt{\frac{x}{1+x}}\left(\sqrt{x}(1+x)+\sqrt{1+x}\log\left(-\sqrt{x}+\sqrt{1+x}\right)\right)}{\sqrt{x}}$$

input
```
Integrate[Sqrt[x/(1 + x)],x]
```

output
```
(Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) + Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]
]))/Sqrt[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {2050, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x}{x+1}}dx$$
$$\Big\downarrow \text{2050}$$
$$\int \frac{\sqrt{x}}{\sqrt{x+1}}dx$$
$$\Big\downarrow \text{60}$$
$$\sqrt{x}\sqrt{x+1}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx$$
$$\Big\downarrow \text{63}$$
$$\sqrt{x}\sqrt{x+1}-\int \frac{1}{\sqrt{x+1}}d\sqrt{x}$$
$$\Big\downarrow \text{222}$$
$$\sqrt{x}\sqrt{x+1}-\text{arcsinh}\left(\sqrt{x}\right)$$

input
```
Int[Sqrt[x/(1 + x)],x]
```

output
```
Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(16) = 32$.

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

method	result	size
default	$\frac{\sqrt{\frac{x}{x+1}}\,(x+1)\left(2\sqrt{x^2+x}-\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\right)}{2\sqrt{(x+1)x}}$	45
risch	$(x+1)\sqrt{\frac{x}{x+1}}-\frac{\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\sqrt{\frac{x}{x+1}}\,\sqrt{(x+1)x}}{2x}$	47
trager	$2\left(\frac{1}{2}+\frac{x}{2}\right)\sqrt{\frac{x}{x+1}}-\frac{\ln\left(2\sqrt{\frac{x}{x+1}}\,x+2\sqrt{\frac{x}{x+1}}+2x+1\right)}{2}$	49

input
```
int((x/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x/(x+1))^(1/2)*(x+1)*(2*(x^2+x)^(1/2)-ln(1/2+x+(x^2+x)^(1/2)))/((x+1)
*x)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(16) = 32$.

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \sqrt{\frac{x}{1+x}}\,dx = (x+1)\sqrt{\frac{x}{x+1}} - \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

input
```
integrate((x/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
(x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x
 + 1)) - 1)
```

Sympy [F]

$$\int \sqrt{\frac{x}{1+x}}\,dx = \int \sqrt{\frac{x}{x+1}}\,dx$$

input
```
integrate((x/(1+x))**(1/2),x)
```

output
```
Integral(sqrt(x/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(16) = 32$.

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \sqrt{\frac{x}{1+x}}\,dx = -\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1} - \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

input
```
integrate((x/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(
sqrt(x/(x + 1)) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{x}{1+x}}\,dx = \frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1) + \sqrt{x^2+x}\operatorname{sgn}(x+1)$$

input
```
integrate((x/(1+x))^(1/2),x, algorithm="giac")
```

output
```
1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x
+ 1)
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{x}{1+x}}\,dx = -\operatorname{atanh}\left(\sqrt{\frac{x}{x+1}}\right) - \frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

input
```
int((x/(x + 1))^(1/2),x)
```

output
```
- atanh((x/(x + 1))^(1/2)) - (x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sqrt{\frac{x}{1+x}}\,dx = \sqrt{x}\,\sqrt{x+1} - \log\left(\sqrt{x+1}+\sqrt{x}\right)$$

input
```
int((x/(1+x))^(1/2),x)
```

output
```
sqrt(x)*sqrt(x + 1) - log(sqrt(x + 1) + sqrt(x))
```

3.38 $\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 36

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \arctan\left(\sqrt{-1+x}\sqrt{1+x}\right)$$

output
```
-(-1+x)^(1/2)*(1+x)^(1/2)/x+arctan((-1+x)^(1/2)*(1+x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}(1+x)+2x\sqrt{1+x}\arctan\left(x-\sqrt{-1+x}\sqrt{1+x}\right)\right)}{\sqrt{-1+x}x}$$

input
```
Integrate[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
```

output
```
-((Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(1 + x) + 2*x*Sqrt[1 + x]*ArcTan[x
 - Sqrt[-1 + x]*Sqrt[1 + x]]))/(Sqrt[-1 + x]*x))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {105, 103, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x-1}}{x^2\sqrt{x+1}}dx$$

$$\Big\downarrow \text{105}$$

$$\int \frac{1}{\sqrt{x-1}x\sqrt{x+1}}dx-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

$$\Big\downarrow \text{103}$$

$$\int \frac{1}{(x-1)(x+1)+1}d\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

$$\Big\downarrow \text{216}$$

$$\arctan\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

input
```
Int[Sqrt[-1 + x]/(x^2*Sqrt[1 + x]),x]
```

output
```
-((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.19

method	result	size
default	$\frac{\left(-\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)x-\sqrt{x^2-1}\right)\sqrt{x-1}\,\sqrt{x+1}}{x\sqrt{x^2-1}}$	43
risch	$-\frac{\sqrt{x-1}\,\sqrt{x+1}}{x}-\frac{\arctan\left(\frac{1}{\sqrt{x^2-1}}\right)\sqrt{(x-1)(x+1)}}{\sqrt{x-1}\,\sqrt{x+1}}$	46

input
```
int((x-1)^(1/2)/x^2/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(-arctan(1/(x^2-1)^(1/2))*x-(x^2-1)^(1/2))*(x-1)^(1/2)*(x+1)^(1/2)/x/(x^2-
1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = \frac{2\,x\arctan\left(\sqrt{x+1}\sqrt{x-1}-x\right)-\sqrt{x+1}\sqrt{x-1}-x}{x}$$

input
```
integrate((x-1)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
(2*x*arctan(sqrt(x + 1)*sqrt(x - 1) - x) - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

Sympy [F]

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = \int \frac{\sqrt{x-1}}{x^2\sqrt{x+1}}\,dx$$

input
```
integrate((x-1)**(1/2)/x**2/(1+x)**(1/2),x)
```

output
```
Integral(sqrt(x - 1)/(x**2*sqrt(x + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\frac{\sqrt{x^2-1}}{x} - \arcsin\left(\frac{1}{|x|}\right)$$

input
```
integrate((x-1)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(x^2 - 1)/x - arcsin(1/abs(x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.17

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\frac{8}{\left(\sqrt{x+1}-\sqrt{x-1}\right)^4+4} - 2\arctan\left(\frac{1}{2}\left(\sqrt{x+1}-\sqrt{x-1}\right)^2\right)$$

input
```
integrate((x-1)^(1/2)/x^2/(1+x)^(1/2),x, algorithm="giac")
```

output
```
-8/((sqrt(x + 1) - sqrt(x - 1))^4 + 4) - 2*arctan(1/2*(sqrt(x + 1) - sqrt(
x - 1))^2)
```

Mupad [B] (verification not implemented)

Time = 10.93 (sec) , antiderivative size = 138, normalized size of antiderivative = 3.83

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = -\ln\left(\frac{\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1\right)\,1\mathrm{i}+\ln\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)\,1\mathrm{i} - \frac{\sqrt{x-1}-\mathrm{i}}{4\left(\sqrt{x+1}-1\right)} - \frac{\frac{5\left(\sqrt{x-1}-\mathrm{i}\right)^2}{\left(\sqrt{x+1}-1\right)^2}+1}{\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)^3}{\left(\sqrt{x+1}-1\right)^3}+\frac{4\left(\sqrt{x-1}-\mathrm{i}\right)}{\sqrt{x+1}-1}}$$

input
```
int((x - 1)^(1/2)/(x^2*(x + 1)^(1/2)),x)
```

output
```
log(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1))*1i - log(((x - 1)^(1/2) - 1i
)^2/((x + 1)^(1/2) - 1)^2 + 1)*1i - ((x - 1)^(1/2) - 1i)/(4*((x + 1)^(1/2)
 - 1)) - ((5*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) - 1)^2 + 1)/((4*((x -
1)^(1/2) - 1i)^3)/((x + 1)^(1/2) - 1)^3 + (4*((x - 1)^(1/2) - 1i))/((x + 1
)^(1/2) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{-1+x}}{x^2\sqrt{1+x}}\,dx = \frac{2atan\left(\sqrt{x-1}+\sqrt{x+1}-1\right)x-2atan\left(\sqrt{x-1}+\sqrt{x+1}+1\right)x-\sqrt{x+1}\,\sqrt{x-1}-x}{x}$$

input
```
int((x-1)^(1/2)/x^2/(1+x)^(1/2),x)
```

output
```
(2*atan(sqrt(x - 1) + sqrt(x + 1) - 1)*x - 2*atan(sqrt(x - 1) + sqrt(x + 1
) + 1)*x - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

3.39 $\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 36

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} dx = -\frac{\sqrt{-1+x}\sqrt{1+x}}{x} + \arctan\left(\sqrt{-1+x}\sqrt{1+x}\right)$$

output
```
-(-1+x)^(1/2)*(1+x)^(1/2)/x+arctan((-1+x)^(1/2)*(1+x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.83

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} dx = -\frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}(1+x) + 2x\sqrt{1+x}\arctan\left(x - \sqrt{-1+x}\sqrt{1+x}\right)\right)}{\sqrt{-1+x}x}$$

input
```
Integrate[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
```

output
```
-((Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(1 + x) + 2*x*Sqrt[1 + x]*ArcTan[x
 - Sqrt[-1 + x]*Sqrt[1 + x]]))/(Sqrt[-1 + x]*x))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {2050, 105, 103, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{x-1}{x+1}}}{x^2} \, dx$$

$$\Big\downarrow \text{2050}$$

$$\int \frac{\sqrt{x-1}}{x^2\sqrt{x+1}}dx$$

$$\Big\downarrow \text{105}$$

$$\int \frac{1}{\sqrt{x-1}x\sqrt{x+1}}dx-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

$$\Big\downarrow \text{103}$$

$$\int \frac{1}{(x-1)(x+1)+1}d\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

$$\Big\downarrow \text{216}$$

$$\arctan\left(\sqrt{x-1}\sqrt{x+1}\right)-\frac{\sqrt{x-1}\sqrt{x+1}}{x}$$

input
```
Int[Sqrt[(-1 + x)/(1 + x)]/x^2,x]
```

output
```
-((Sqrt[-1 + x]*Sqrt[1 + x])/x) + ArcTan[Sqrt[-1 + x]*Sqrt[1 + x]]
```

Defintions of rubi rules used

rule 103
```
Int[1/(Sqrt[(a_.) + (b_.)*(x_)]*Sqrt[(c_.) + (d_.)*(x_)]*((e_.) + (f_.)*(x_
))), x_] :> Simp[b*f   Subst[Int[1/(d*(b*e - a*f)^2 + b*f^2*x^2), x], x, Sq
rt[a + b*x]*Sqrt[c + d*x]], x] /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[2*b*d
*e - f*(b*c + a*d), 0]
```

rule 105
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[n*((d*e - c*f)/((m + 1)*(b*e - a*f)))   Int[(a
+ b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d,
e, f, m, p}, x] && EqQ[m + n + p + 2, 0] && GtQ[n, 0] && (SumSimplerQ[m, 1]
 ||  !SumSimplerQ[p, 1]) && NeQ[m, -1]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.56

method	result	size
risch	$-\frac{(x+1)\sqrt{\frac{x-1}{x+1}}}{x}-\frac{\arctan\left(\frac{1}{\sqrt{x^{2}-1}}\right)\sqrt{\frac{x-1}{x+1}}\,\sqrt{(x-1)(x+1)}}{x-1}$	56
default	$\frac{\sqrt{\frac{x-1}{x+1}}\,(x+1)\left((x^{2}-1)^{\frac{3}{2}}-x^{2}\sqrt{x^{2}-1}-\arctan\left(\frac{1}{\sqrt{x^{2}-1}}\right)x\right)}{\sqrt{(x-1)(x+1)}\,x}$	59
trager	$-\frac{(x+1)\sqrt{-\frac{1-x}{x+1}}}{x}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{1-x}{x+1}}\,x+\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{1-x}{x+1}}-1}{x}\right)$	82

input
```
int(((x-1)/(x+1))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(x+1)/x*((x-1)/(x+1))^(1/2)-arctan(1/(x^2-1)^(1/2))*((x-1)/(x+1))^(1/2)*(
(x-1)*(x+1))^(1/2)/(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = \frac{2\,x\arctan\left(\sqrt{\frac{x-1}{x+1}}\right) - (x+1)\sqrt{\frac{x-1}{x+1}}}{x}$$

input
```
integrate(((x-1)/(1+x))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(2*x*arctan(sqrt((x - 1)/(x + 1))) - (x + 1)*sqrt((x - 1)/(x + 1)))/x
```

Sympy [F]

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = \int \frac{\sqrt{\frac{x-1}{x+1}}}{x^2}\,dx$$

input
```
integrate(((x-1)/(1+x))**(1/2)/x**2,x)
```

output
```
Integral(sqrt((x - 1)/(x + 1))/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = -\frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}+1} + 2\arctan\left(\sqrt{\frac{x-1}{x+1}}\right)$$

input
```
integrate(((x-1)/(1+x))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
-2*sqrt((x - 1)/(x + 1))/((x - 1)/(x + 1) + 1) + 2*arctan(sqrt((x - 1)/(x
+ 1)))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.42

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2}\,dx = -\frac{1}{2}(\pi-2)\mathrm{sgn}(x+1) + 2\arctan\left(-x+\sqrt{x^2-1}\right)\mathrm{sgn}(x+1) - \frac{2\,\mathrm{sgn}(x+1)}{\left(x-\sqrt{x^2-1}\right)^2+1}$$

input
```
integrate(((x-1)/(1+x))^(1/2)/x^2,x, algorithm="giac")
```

output
```
-1/2*(pi - 2)*sgn(x + 1) + 2*arctan(-x + sqrt(x^2 - 1))*sgn(x + 1) - 2*sgn
(x + 1)/((x - sqrt(x^2 - 1))^2 + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.14

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} \, dx = 2 \operatorname{atan}\left(\sqrt{\frac{x-1}{x+1}}\right) - \frac{2\sqrt{\frac{x-1}{x+1}}}{\frac{x-1}{x+1}+1}$$

input
```
int(((x - 1)/(x + 1))^(1/2)/x^2,x)
```

output
```
2*atan(((x - 1)/(x + 1))^(1/2)) - (2*((x - 1)/(x + 1))^(1/2))/((x - 1)/(x
+ 1) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.28

$$\int \frac{\sqrt{\frac{-1+x}{1+x}}}{x^2} \, dx = \frac{2atan\left(\sqrt{x-1}+\sqrt{x+1}-1\right) x - 2atan\left(\sqrt{x-1}+\sqrt{x+1}+1\right) x - \sqrt{x+1}\,\sqrt{x-1} - x}{x}$$

input
```
int(((x-1)/(1+x))^(1/2)/x^2,x)
```

output
```
(2*atan(sqrt(x - 1) + sqrt(x + 1) - 1)*x - 2*atan(sqrt(x - 1) + sqrt(x + 1
) + 1)*x - sqrt(x + 1)*sqrt(x - 1) - x)/x
```

3.40 $\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 82

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x}+\frac{5}{24}(-1+x)^{3/2}\sqrt{1+x} -\frac{1}{12}(-1+x)^{5/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3\operatorname{arccosh}(x)}{8}$$

output
```
-3/8*(-1+x)^(1/2)*(1+x)^(1/2)+5/24*(-1+x)^(3/2)*(1+x)^(1/2)-1/12*(-1+x)^(5
/2)*(1+x)^(1/2)+1/4*(-1+x)^(3/2)*x^2*(1+x)^(1/2)+3/8*arccosh(x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}(-16-7x+x^2-2x^3+6x^4)-18\sqrt{1+x}\log\left(\sqrt{-1+x}-\sqrt{1+x}\right)\right)}{24\sqrt{-1+x}}$$

input
```
Integrate[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

output
```
(Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(-16 - 7*x + x^2 - 2*x^3 + 6*x^4) -
18*Sqrt[1 + x]*Log[Sqrt[-1 + x] - Sqrt[1 + x]]))/(24*Sqrt[-1 + x])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {111, 164, 60, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x-1}x^3}{\sqrt{x+1}}\,dx$$

$$\Big\downarrow \text{111}$$

$$\frac{1}{4}\int \frac{(2-x)\sqrt{x-1}x}{\sqrt{x+1}}dx+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

$$\Big\downarrow \text{164}$$

$$\frac{1}{4}\left(\frac{1}{6}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{2}\int \frac{\sqrt{x-1}}{\sqrt{x+1}}dx\right)+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{1}{6}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{2}\left(\sqrt{x-1}\sqrt{x+1}-\int \frac{1}{\sqrt{x-1}\sqrt{x+1}}dx\right)\right)+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

$$\Big\downarrow \text{43}$$

$$\frac{1}{4}\left(\frac{1}{6}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{2}\left(\sqrt{x-1}\sqrt{x+1}-\text{arccosh}(x)\right)\right)+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

input
```
Int[(Sqrt[-1 + x]*x^3)/Sqrt[1 + x],x]
```

output
```
((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x
])/6 - (3*(Sqrt[-1 + x]*Sqrt[1 + x] - ArcCosh[x]))/2)/4
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.73

method	result	size
risch	$\frac{(6x^3-8x^2+9x-16)\sqrt{x-1}\,\sqrt{x+1}}{24}+\frac{3\ln\left(x+\sqrt{x^2-1}\right)\sqrt{(x-1)(x+1)}}{8\sqrt{x-1}\,\sqrt{x+1}}$	60
default	$\frac{\sqrt{x-1}\,\sqrt{x+1}\left(6x^3\sqrt{x^2-1}-8x^2\sqrt{x^2-1}+9x\sqrt{x^2-1}+9\ln\left(x+\sqrt{x^2-1}\right)-16\sqrt{x^2-1}\right)}{24\sqrt{x^2-1}}$	76

input
```
int((x-1)^(1/2)*x^3/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*x^3-8*x^2+9*x-16)*(x-1)^(1/2)*(x+1)^(1/2)+3/8*ln(x+(x^2-1)^(1/2))*
((x-1)*(x+1))^(1/2)/(x-1)^(1/2)/(x+1)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{1}{24}\left(6\,x^3-8\,x^2+9\,x-16\right)\sqrt{x+1}\sqrt{x-1} - \frac{3}{8}\,\log\left(\sqrt{x+1}\sqrt{x-1}-x\right)$$

input
```
integrate((x-1)^(1/2)*x^3/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(6*x^3 - 8*x^2 + 9*x - 16)*sqrt(x + 1)*sqrt(x - 1) - 3/8*log(sqrt(x +
 1)*sqrt(x - 1) - x)
```

Sympy [F]

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \int \frac{x^3\sqrt{x-1}}{\sqrt{x+1}}\,dx$$

input
```
integrate((x-1)**(1/2)*x**3/(1+x)**(1/2),x)
```

output
```
Integral(x**3*sqrt(x - 1)/sqrt(x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{1}{4}\left(x^2-1\right)^{\frac{3}{2}}x - \frac{1}{3}\left(x^2-1\right)^{\frac{3}{2}} + \frac{5}{8}\sqrt{x^2-1}x - \sqrt{x^2-1} + \frac{3}{8}\log\left(2\,x+2\,\sqrt{x^2-1}\right)$$

input
```
integrate((x-1)^(1/2)*x^3/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
1/4*(x^2 - 1)^(3/2)*x - 1/3*(x^2 - 1)^(3/2) + 5/8*sqrt(x^2 - 1)*x - sqrt(x
^2 - 1) + 3/8*log(2*x + 2*sqrt(x^2 - 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.57

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{1}{24}\left((2\,(3\,x-10)(x+1)+43)(x+1)-39\right)\sqrt{x+1}\sqrt{x-1} - \frac{3}{4}\log\left(\sqrt{x+1}-\sqrt{x-1}\right)$$

input
```
integrate((x-1)^(1/2)*x^3/(1+x)^(1/2),x, algorithm="giac")
```

output
```
1/24*((2*(3*x - 10)*(x + 1) + 43)*(x + 1) - 39)*sqrt(x + 1)*sqrt(x - 1) -
3/4*log(sqrt(x + 1) - sqrt(x - 1))
```

Mupad [B] (verification not implemented)

Time = 18.83 (sec) , antiderivative size = 473, normalized size of antiderivative = 5.77

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{3\,\mathrm{atanh}\left(\frac{\sqrt{x-1}-\mathrm{i}}{\sqrt{x+1}-1}\right)}{2} + \frac{\frac{23\,(\sqrt{x-1}-\mathrm{i})^3}{2\,(\sqrt{x+1}-1)^3} - \frac{(\sqrt{x-1}-\mathrm{i})^4\,64\mathrm{i}}{(\sqrt{x+1}-1)^4} + \frac{333\,(\sqrt{x-1}-\mathrm{i})^5}{2\,(\sqrt{x+1}-1)^5} + \frac{(\sqrt{x-1}-\mathrm{i})^6\,256\mathrm{i}}{3\,(\sqrt{x+1}-1)^6} + \frac{671\,(\sqrt{x-1}-\mathrm{i})^7}{2\,(\sqrt{x+1}-1)^7} - \frac{(\sqrt{x-1}-\mathrm{i})^8\,128\mathrm{i}}{3\,(\sqrt{x+1}-1)^8} + \frac{671\,(\sqrt{x-1}-\mathrm{i})^9}{2\,(\sqrt{x+1}-1)^9}}{1 + \frac{28\,(\sqrt{x-1}-\mathrm{i})^4}{(\sqrt{x+1}-1)^4} - \frac{56\,(\sqrt{x-1}-\mathrm{i})^6}{(\sqrt{x+1}-1)^6} + \frac{70\,(\sqrt{x-1}-\mathrm{i})^8}{(\sqrt{x+1}-1)^8} - \frac{56\,(\sqrt{x-1}-\mathrm{i})^{10}}{(\sqrt{x+1}-1)^{10}} + \cdots}$$

input
```
int((x^3*(x - 1)^(1/2))/(x + 1)^(1/2),x)
```

output
```
(3*atanh(((x - 1)^(1/2) - 1i)/((x + 1)^(1/2) - 1)))/2 + ((23*((x - 1)^(1/2
) - 1i)^3)/(2*((x + 1)^(1/2) - 1)^3) - (((x - 1)^(1/2) - 1i)^4*64i)/((x +
1)^(1/2) - 1)^4 + (333*((x - 1)^(1/2) - 1i)^5)/(2*((x + 1)^(1/2) - 1)^5) +
 (((x - 1)^(1/2) - 1i)^6*256i)/(3*((x + 1)^(1/2) - 1)^6) + (671*((x - 1)^(
1/2) - 1i)^7)/(2*((x + 1)^(1/2) - 1)^7) - (((x - 1)^(1/2) - 1i)^8*128i)/(3
*((x + 1)^(1/2) - 1)^8) + (671*((x - 1)^(1/2) - 1i)^9)/(2*((x + 1)^(1/2) -
 1)^9) + (((x - 1)^(1/2) - 1i)^10*256i)/(3*((x + 1)^(1/2) - 1)^10) + (333*
((x - 1)^(1/2) - 1i)^11)/(2*((x + 1)^(1/2) - 1)^11) - (((x - 1)^(1/2) - 1i
)^12*64i)/((x + 1)^(1/2) - 1)^12 + (23*((x - 1)^(1/2) - 1i)^13)/(2*((x + 1
)^(1/2) - 1)^13) - (3*((x - 1)^(1/2) - 1i)^15)/(2*((x + 1)^(1/2) - 1)^15)
 - (3*((x - 1)^(1/2) - 1i))/(2*((x + 1)^(1/2) - 1)))/((28*((x - 1)^(1/2) -
1i)^4)/((x + 1)^(1/2) - 1)^4 - (8*((x - 1)^(1/2) - 1i)^2)/((x + 1)^(1/2) -
 1)^2 - (56*((x - 1)^(1/2) - 1i)^6)/((x + 1)^(1/2) - 1)^6 + (70*((x - 1)^(
1/2) - 1i)^8)/((x + 1)^(1/2) - 1)^8 - (56*((x - 1)^(1/2) - 1i)^10)/((x + 1
)^(1/2) - 1)^10 + (28*((x - 1)^(1/2) - 1i)^12)/((x + 1)^(1/2) - 1)^12 - (8
*((x - 1)^(1/2) - 1i)^14)/((x + 1)^(1/2) - 1)^14 + ((x - 1)^(1/2) - 1i)^16
/((x + 1)^(1/2) - 1)^16 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{-1+x}x^3}{\sqrt{1+x}}\,dx = \frac{\sqrt{x+1}\,\sqrt{x-1}\,x^3}{4} - \frac{\sqrt{x+1}\,\sqrt{x-1}\,x^2}{3} + \frac{3\sqrt{x+1}\,\sqrt{x-1}\,x}{8} - \frac{2\sqrt{x+1}\,\sqrt{x-1}}{3} + \frac{3\log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right)}{4}$$

input
```
int((x-1)^(1/2)*x^3/(1+x)^(1/2),x)
```

output
```
(6*sqrt(x + 1)*sqrt(x - 1)*x**3 - 8*sqrt(x + 1)*sqrt(x - 1)*x**2 + 9*sqrt(
x + 1)*sqrt(x - 1)*x - 16*sqrt(x + 1)*sqrt(x - 1) + 18*log((sqrt(x - 1) +
sqrt(x + 1))/sqrt(2)))/24
```

3.41 $\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 82

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx = -\frac{3}{8}\sqrt{-1+x}\sqrt{1+x}+\frac{5}{24}(-1+x)^{3/2}\sqrt{1+x} -\frac{1}{12}(-1+x)^{5/2}\sqrt{1+x}+\frac{1}{4}(-1+x)^{3/2}x^2\sqrt{1+x}+\frac{3\text{arccosh}(x)}{8}$$

output
```
-3/8*(-1+x)^(1/2)*(1+x)^(1/2)+5/24*(-1+x)^(3/2)*(1+x)^(1/2)-1/12*(-1+x)^(5
/2)*(1+x)^(1/2)+1/4*(-1+x)^(3/2)*x^2*(1+x)^(1/2)+3/8*arccosh(x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx = \frac{\sqrt{\frac{-1+x}{1+x}}\left(\sqrt{-1+x}(-16-7x+x^2-2x^3+6x^4)-18\sqrt{1+x}\log\left(\sqrt{-1+x}-\sqrt{1+x}\right)\right)}{24\sqrt{-1+x}}$$

input
```
Integrate[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

output
```
(Sqrt[(-1 + x)/(1 + x)]*(Sqrt[-1 + x]*(-16 - 7*x + x^2 - 2*x^3 + 6*x^4) -
18*Sqrt[1 + x]*Log[Sqrt[-1 + x] - Sqrt[1 + x]]))/(24*Sqrt[-1 + x])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {2050, 111, 164, 60, 43}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{\frac{x-1}{x+1}}dx$$

$$\Big\downarrow \text{2050}$$

$$\int \frac{\sqrt{x-1}x^3}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{111}$$

$$\frac{1}{4}\int \frac{(2-x)\sqrt{x-1}x}{\sqrt{x+1}}dx+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

$$\Big\downarrow \text{164}$$

$$\frac{1}{4}\left(\frac{1}{6}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{2}\int \frac{\sqrt{x-1}}{\sqrt{x+1}}dx\right)+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{4}\left(\frac{1}{6}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{2}\left(\sqrt{x-1}\sqrt{x+1}-\int \frac{1}{\sqrt{x-1}\sqrt{x+1}}dx\right)\right)+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

$$\Big\downarrow \text{43}$$

$$\frac{1}{4}\left(\frac{1}{6}(7-2x)(x-1)^{3/2}\sqrt{x+1}-\frac{3}{2}\left(\sqrt{x-1}\sqrt{x+1}-\text{arccosh}(x)\right)\right)+\frac{1}{4}(x-1)^{3/2}\sqrt{x+1}x^2$$

input
```
Int[x^3*Sqrt[(-1 + x)/(1 + x)],x]
```

output
```
((-1 + x)^(3/2)*x^2*Sqrt[1 + x])/4 + (((7 - 2*x)*(-1 + x)^(3/2)*Sqrt[1 + x
])/6 - (3*(Sqrt[-1 + x]*Sqrt[1 + x] - ArcCosh[x]))/2)/4
```

Defintions of rubi rules used

rule 43
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[
ArcCosh[b*(x/a)]/(b*Sqrt[d/b]), x] /; FreeQ[{a, b, c, d}, x] && EqQ[b*c + a
*d, 0] && GtQ[a, 0] && GtQ[d/b, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 111
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/(d*f*(m + n + p + 1))), x] + Simp[1/(d*f*(m + n + p + 1))   Int[(a + b*x)
^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b*c*e*(m
 - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*(d*e*(m
 + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &
& GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

rule 164
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_
))*((g_.) + (h_.)*(x_)), x_] :> Simp[(-(a*d*f*h*(n + 2) + b*c*f*h*(m + 2) -
 b*d*(f*g + e*h)*(m + n + 3) - b*d*f*h*(m + n + 2)*x))*(a + b*x)^(m + 1)*((
c + d*x)^(n + 1)/(b^2*d^2*(m + n + 2)*(m + n + 3))), x] + Simp[(a^2*d^2*f*h
*(n + 1)*(n + 2) + a*b*d*(n + 1)*(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n +
3)) + b^2*(c^2*f*h*(m + 1)*(m + 2) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) +
d^2*e*g*(m + n + 2)*(m + n + 3)))/(b^2*d^2*(m + n + 2)*(m + n + 3))   Int[(
a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x]
&& NeQ[m + n + 2, 0] && NeQ[m + n + 3, 0]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{(6x^3-8x^2+9x-16)(x+1)\sqrt{\frac{x-1}{x+1}}}{24}+\frac{3\ln\left(x+\sqrt{x^2-1}\right)\sqrt{\frac{x-1}{x+1}}\,\sqrt{(x-1)(x+1)}}{8(x-1)}$	70
trager	$\frac{(x+1)(6x^3-8x^2+9x-16)\sqrt{-\frac{1-x}{x+1}}}{24}+\frac{3\ln\left(\sqrt{-\frac{1-x}{x+1}}\,x+\sqrt{-\frac{1-x}{x+1}}+x\right)}{8}$	71
default	$-\frac{\sqrt{\frac{x-1}{x+1}}\,(x+1)\left(-6x(x^2-1)^{\frac{3}{2}}+8((x-1)(x+1))^{\frac{3}{2}}-15x\sqrt{x^2-1}+24\sqrt{x^2-1}-9\ln\left(x+\sqrt{x^2-1}\right)\right)}{24\sqrt{(x-1)(x+1)}}$	79

input
```
int(x^3*((x-1)/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(6*x^3-8*x^2+9*x-16)*(x+1)*((x-1)/(x+1))^(1/2)+3/8*ln(x+(x^2-1)^(1/2)
)*((x-1)/(x+1))^(1/2)*((x-1)*(x+1))^(1/2)/(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.78

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx=\frac{1}{24}\left(6\,x^4-2\,x^3+x^2-7\,x-16\right)\sqrt{\frac{x-1}{x+1}}+\frac{3}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}+1\right)-\frac{3}{8}\,\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(x^3*((x-1)/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
1/24*(6*x^4 - 2*x^3 + x^2 - 7*x - 16)*sqrt((x - 1)/(x + 1)) + 3/8*log(sqrt
((x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Sympy [F]

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx = \int x^3\sqrt{\frac{x-1}{x+1}}\,dx$$

input
```
integrate(x**3*((x-1)/(1+x))**(1/2),x)
```

output
```
Integral(x**3*sqrt((x - 1)/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. 2(56) = 112.

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.68

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx = -\frac{39\left(\frac{x-1}{x+1}\right)^{\frac{7}{2}} - 31\left(\frac{x-1}{x+1}\right)^{\frac{5}{2}} + 49\left(\frac{x-1}{x+1}\right)^{\frac{3}{2}} - 9\sqrt{\frac{x-1}{x+1}}}{12\left(\frac{4(x-1)}{x+1} - \frac{6(x-1)^2}{(x+1)^2} + \frac{4(x-1)^3}{(x+1)^3} - \frac{(x-1)^4}{(x+1)^4} - 1\right)} + \frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}+1\right) - \frac{3}{8}\log\left(\sqrt{\frac{x-1}{x+1}}-1\right)$$

input
```
integrate(x^3*((x-1)/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
-1/12*(39*((x - 1)/(x + 1))^(7/2) - 31*((x - 1)/(x + 1))^(5/2) + 49*((x -
1)/(x + 1))^(3/2) - 9*sqrt((x - 1)/(x + 1)))/(4*(x - 1)/(x + 1) - 6*(x - 1
)^2/(x + 1)^2 + 4*(x - 1)^3/(x + 1)^3 - (x - 1)^4/(x + 1)^4 - 1) + 3/8*log
(sqrt((x - 1)/(x + 1)) + 1) - 3/8*log(sqrt((x - 1)/(x + 1)) - 1)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.76

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx$$
$$=-\frac{3}{8}\log\left(\left|-x+\sqrt{x^2-1}\right|\right)\operatorname{sgn}(x+1)$$
$$+\frac{1}{24}\left((2\,(3\,x\operatorname{sgn}(x+1)-4\operatorname{sgn}(x+1))x+9\operatorname{sgn}(x+1))x-16\operatorname{sgn}(x+1)\right)\sqrt{x^2-1}$$

input
```
integrate(x^3*((x-1)/(1+x))^(1/2),x, algorithm="giac")
```

output
```
-3/8*log(abs(-x + sqrt(x^2 - 1)))*sgn(x + 1) + 1/24*((2*(3*x*sgn(x + 1) -
4*sgn(x + 1))*x + 9*sgn(x + 1))*x - 16*sgn(x + 1))*sqrt(x^2 - 1)
```

Mupad [B] (verification not implemented)

Time = 8.73 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.45

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx=\frac{3\operatorname{atanh}\left(\sqrt{\frac{x-1}{x+1}}\right)}{4}-\frac{\frac{3\sqrt{\frac{x-1}{x+1}}}{4}-\frac{49\left(\frac{x-1}{x+1}\right)^{3/2}}{12}+\frac{31\left(\frac{x-1}{x+1}\right)^{5/2}}{12}-\frac{13\left(\frac{x-1}{x+1}\right)^{7/2}}{4}}{\frac{6(x-1)^2}{(x+1)^2}-\frac{4(x-1)}{x+1}-\frac{4(x-1)^3}{(x+1)^3}+\frac{(x-1)^4}{(x+1)^4}+1}$$

input
```
int(x^3*((x - 1)/(x + 1))^(1/2),x)
```

output
```
(3*atanh(((x - 1)/(x + 1))^(1/2)))/4 - ((3*((x - 1)/(x + 1))^(1/2))/4 - (4
9*((x - 1)/(x + 1))^(3/2))/12 + (31*((x - 1)/(x + 1))^(5/2))/12 - (13*((x
- 1)/(x + 1))^(7/2))/4)/((6*(x - 1)^2)/(x + 1)^2 - (4*(x - 1))/(x + 1) - (
4*(x - 1)^3)/(x + 1)^3 + (x - 1)^4/(x + 1)^4 + 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int x^3\sqrt{\frac{-1+x}{1+x}}\,dx = \frac{\sqrt{x+1}\,\sqrt{x-1}\,x^3}{4}-\frac{\sqrt{x+1}\,\sqrt{x-1}\,x^2}{3}+\frac{3\sqrt{x+1}\,\sqrt{x-1}\,x}{8}-\frac{2\sqrt{x+1}\,\sqrt{x-1}}{3}+\frac{3\log\left(\frac{\sqrt{x-1}+\sqrt{x+1}}{\sqrt{2}}\right)}{4}$$

input
```
int(x^3*((x-1)/(1+x))^(1/2),x)
```

output
```
(6*sqrt(x + 1)*sqrt(x - 1)*x**3 - 8*sqrt(x + 1)*sqrt(x - 1)*x**2 + 9*sqrt(
x + 1)*sqrt(x - 1)*x - 16*sqrt(x + 1)*sqrt(x - 1) + 18*log((sqrt(x - 1) +
sqrt(x + 1))/sqrt(2)))/24
```

3.42 $\int x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 287

$$\int x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \frac{(11b^2c^2-2abcd-a^2d^2)(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{16b^2d^3} - \frac{(13bc-ad)(c+dx^2)^2\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{24bd^3} + \frac{(c+dx^2)^3\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{6d^3} - \frac{(bc-ad)(5b^2c^2+2abcd+a^2d^2)\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{5/2}d^{7/2}}$$

output
```
1/16*(-a^2*d^2-2*a*b*c*d+11*b^2*c^2)*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^
2+c))^(1/2)/b^2/d^3-1/24*(-a*d+13*b*c)*(d*x^2+c)^2*(b*e/d-(-a*d+b*c)*e/d/(
d*x^2+c))^(1/2)/b/d^3+1/6*(d*x^2+c)^3*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/
2)/d^3-1/16*(-a*d+b*c)*(a^2*d^2+2*a*b*c*d+5*b^2*c^2)*e^(1/2)*arctanh(d^(1/
2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/b^(5/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 1.63 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.69

$$\int x^5\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-b\sqrt{d}(c+dx^2)\,(3a^2d^2-2abd(-2c+dx^2)+b^2(-15c^2+10cdx^2-8d^2x^4))-\frac{3(bc-ad)^{3/2}(5b^2c^2\ldots}{\ldots}\right.}{48b^3d^{7/2}}$$

input
```
Integrate[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(b*Sqrt[d]*(c + d*x^2)*(3*a^2*d^2 - 2
*a*b*d*(-2*c + d*x^2) + b^2*(-15*c^2 + 10*c*d*x^2 - 8*d^2*x^4))) - (3*(b*c
 - a*d)^(3/2)*(5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c
- a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/Sqrt[a + b*x^2
]))/(48*b^3*d^(7/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.81 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2053, 2052, 366, 27, 360, 27, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$

$$\Big\downarrow \; 2053$$

$$\frac{1}{2}\int x^4\sqrt{\frac{e\,(bx^2+a)}{dx^2+c}}dx^2$$

$$\Big\downarrow \; 2052$$

$$e(bc-ad)\int \frac{x^4\left(ae-cx^4\right)^2}{\left(be-dx^4\right)^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\downarrow \text{366}$$

$$e(bc-ad)\left(\frac{ex^6(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\int \frac{3ex^4\left(2bc^2dx^4+\left(b^2c^2-2abdc-a^2d^2\right)e\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{6bd^2e}\right)$$

$$\downarrow \text{27}$$

$$e(bc-ad)\left(\frac{ex^6(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\int \frac{x^4\left(2bc^2dx^4+\left(b^2c^2-2abdc-a^2d^2\right)e\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2bd^2}\right)$$

$$\downarrow \text{360}$$

$$e(bc-ad)\left(\frac{ex^6(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{e(bc-ad)(ad+3bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d(be-dx^4)^2}-\frac{\int \frac{d\left(8bc^2dx^4+(bc-ad)(3bc+ad)e\right)}{\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4d^2}}{2bd^2}\right)$$

$$\downarrow \text{27}$$

$$e(bc-ad)\left(\frac{ex^6(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{e(bc-ad)(ad+3bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d(be-dx^4)^2}-\frac{\int \frac{8bc^2dx^4+(bc-ad)(3bc+ad)e}{\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4d}}{2bd^2}\right)$$

$$\downarrow \text{298}$$

$$e(bc-ad)\left(\frac{ex^6(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{e(bc-ad)(ad+3bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d(be-dx^4)^2}-\frac{\frac{\left(-a^2d^2-2abcd+11b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2b\left(be-dx^4\right)}-\frac{\left(a^2d^2+2abcd+5b^2c^2\right)\int \frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2b}}{4d}}{2bd^2}\right)$$

$$\downarrow \text{221}$$

$$e(bc-ad)\left(\frac{ex^6(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{e(bc-ad)(ad+3bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d\left(be-dx^4\right)^2}-\frac{\frac{\left(-a^2d^2-2abcd+11b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2b\left(be-dx^4\right)}-\frac{\left(a^2d^2+2abcd+5b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\ldots}{\ldots}}}{\sqrt{\ldots}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}}{4d}}{2bd^2}\right.$$

input
```
Int[x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(b*c - a*d)*e*(((b*c - a*d)^2*e*x^6)/(6*b*d^2*(b*e - d*x^4)^3) - (((b*c -
a*d)*(3*b*c + a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*d*(b*e - d*x^4)
^2) - (((11*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2
)])/(2*b*(b*e - d*x^4)) - ((5*b^2*c^2 + 2*a*b*c*d + a^2*d^2)*ArcTanh[(Sqrt
[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(3/2)*Sqrt
[d]*Sqrt[e]))/(4*d))/(2*b*d^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\left(-8b^2d^2x^4-2ab\,d^2x^2+10b^2c\,x^2d+3a^2d^2+4abcd-15b^2c^2\right)\left(d\,x^2+c\right)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{48b^2d^3}+\frac{\left(a^3d^3+a^2bc\,d^2+3a\,b^2c^2d-5b^3c^3\right)\ln\left(\frac{\frac{1}{2}ade\cdots}{\cdots}\right)}{\cdots}$
default	$\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\left(d\,x^2+c\right)\left(-12\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,ab\,d^2x^2-36\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,b^2cd\,x^2+3\ln\left(\frac{2bd\,x^2+2\sqrt{db\,x^4\cdots}}{\cdots}\right)\cdots\right)}{\cdots}$

input
```
int(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/48*(-8*b^2*d^2*x^4-2*a*b*d^2*x^2+10*b^2*c*d*x^2+3*a^2*d^2+4*a*b*c*d-15*
b^2*c^2)*(d*x^2+c)/b^2/d^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/32*(a^3*d^3+a^2
*b*c*d^2+3*a*b^2*c^2*d-5*b^3*c^3)/b^2/d^3*ln((1/2*a*d*e+1/2*b*c*e+b*d*x^2*
e)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)*
(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.89

$$\int x^5\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$
$$= \left[-\frac{3\,(5\,b^3c^3 - 3\,ab^2c^2d - a^2bcd^2 - a^3d^3)\sqrt{\frac{e}{bd}}\log\Big(8\,b^2d^2ex^4 + 8\,(b^2cd + abd^2)ex^2 + (b^2c^2 + 6\,abcd + a^2d^2\right.$$

input
```
integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2 - a^3*d^3)*sqrt(e/(b*d
))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*
d + a^2*d^2)*e + 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 +
 a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) - 4*(8*b^2
*d^3*x^6 + 15*b^2*c^3 - 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*(b^2*c*d^2 - a*b*d^3
)*x^4 + (5*b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c)))/(b^2*d^3), 1/96*(3*(5*b^3*c^3 - 3*a*b^2*c^2*d - a^2*b*c*d^2
- a^3*d^3)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2
 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(8*b^2*d^3*x^6 +
15*b^2*c^3 - 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*(b^2*c*d^2 - a*b*d^3)*x^4 + (5*
b^2*c^2*d - 2*a*b*c*d^2 - 3*a^2*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
))/(b^2*d^3)]
```

Sympy [F(-1)]

Timed out.

$$\int x^5 \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^5 \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.79

$$\int x^5 \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$
$$= \frac{1}{96}\left(2\sqrt{bdex^4+bcex^2+adex^2+ace}\left(2\,x^2\left(\frac{4\,x^2}{d}-\frac{5\,b^2cd-abd^2}{b^2d^3}\right)+\frac{15\,b^2c^2-4\,abcd-3\,a^2d^2}{b^2d^3}\right)+\frac{3\,(}{}\right.$$
$$\left.+\,c\right)$$

input
```
integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/96*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)*(2*x^2*(4*x^2/d -
(5*b^2*c*d - a*b*d^2)/(b^2*d^3)) + (15*b^2*c^2 - 4*a*b*c*d - 3*a^2*d^2)/(b
^2*d^3)) + 3*(5*b^3*c^3*e - 3*a*b^2*c^2*d*e - a^2*b*c*d^2*e - a^3*d^3*e)*l
og(abs(-b*c*e - a*d*e - 2*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 +
b*c*e*x^2 + a*d*e*x^2 + a*c*e))))/(sqrt(b*d*e)*b^2*d^3))*sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int x^5\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx=\int x^5\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

input
```
int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.16

$$\int x^5\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \frac{\sqrt{e}\left(-3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2b\,d^3-4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a\,b^2c\,d^2+2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a\,b^2d^3x^2+15\right.}{}$$

input
```
int(x^5*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(e)*( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3 - 4*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**
2)*a*b**2*d**3*x**2 + 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**2*d - 1
0*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*x**2 + 8*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b**3*d**3*x**4 + 3*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a
+ b*x**2)*d - sqrt(d)*sqrt(c + d*x**2)*b)*a**3*d**3 + 3*sqrt(d)*sqrt(b)*lo
g( - sqrt(b)*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c + d*x**2)*b)*a**2*b*c*d**
2 + 9*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c +
 d*x**2)*b)*a*b**2*c**2*d - 15*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x
**2)*d - sqrt(d)*sqrt(c + d*x**2)*b)*b**3*c**3))/(48*b**3*d**4)
```

3.43 $\int x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 197

$$\int x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx=-\frac{(5bc-ad)(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{8bd^2}+\frac{(c+dx^2)^2\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{4d^2}+\frac{(bc-ad)(3bc+ad)\sqrt{e}\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{3/2}d^{5/2}}$$

output
```
-1/8*(-a*d+5*b*c)*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b/d^2+1
/4*(d*x^2+c)^2*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d^2+1/8*(-a*d+b*c)*(
a*d+3*b*c)*e^(1/2)*arctanh(d^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/
b^(1/2)/e^(1/2))/b^(3/2)/d^(5/2)
```

Mathematica [A] (verified)

Time = 0.95 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.82

$$\int x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{b}\sqrt{d}\sqrt{a+bx^2}(c+dx^2)\left(-3bc+ad+2bdx^2\right)+\left(3b^2c^2-2abcd-a^2d^2\right)\sqrt{c+dx^2}\operatorname{arctanh}\left(\right.\right.}{8b^{3/2}d^{5/2}\sqrt{a+bx^2}}$$

input
```
Integrate[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*(c + d
*x^2)*(-3*b*c + a*d + 2*b*d*x^2) + (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*Sqrt[
c + d*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]))/
(8*b^(3/2)*d^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2053, 2052, 25, 360, 25, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx$$

$$\Big\downarrow \; 2053$$

$$\frac{1}{2}\int x^2\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}dx^2$$

$$\Big\downarrow \; 2052$$

$$e(bc-ad)\int -\frac{x^4\left(ae-cx^4\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \; 25$$

$$-\left(e(bc-ad)\int \frac{x^4\left(ae-cx^4\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$\big\downarrow$ 360

$$e(bc-ad)\left(\frac{\int -\frac{4cdx^4+(bc-ad)e}{\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4d^2}+\frac{e(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2\left(be-dx^4\right)^2}\right)$$

$\big\downarrow$ 25

$$e(bc-ad)\left(\frac{e(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2\left(be-dx^4\right)^2}-\frac{\int \frac{4cdx^4+(bc-ad)e}{\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4d^2}\right)$$

$\big\downarrow$ 298

$$e(bc-ad)\left(\frac{e(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2\left(be-dx^4\right)^2}-\frac{\frac{(5bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2b\left(be-dx^4\right)}-\frac{(ad+3bc)\int \frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2b}}{4d^2}\right)$$

$\big\downarrow$ 221

$$e(bc-ad)\left(\frac{e(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2\left(be-dx^4\right)^2}-\frac{\frac{(5bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2b\left(be-dx^4\right)}-\frac{(ad+3bc)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}}{4d^2}\right)$$

input
```
Int[x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(b*c - a*d)*e*(((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*d^2*(b
*e - d*x^4)^2) - (((5*b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*b*(
b*e - d*x^4)) - ((3*b*c + a*d)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)])/(Sqrt[b]*Sqrt[e])])/(2*b^(3/2)*Sqrt[d]*Sqrt[e]))/(4*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.96

method	result
risch	$\frac{(2bd\,x^2+ad-3bc)\,(d\,x^2+c)\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{8b\,d^2}-\frac{(a^2d^2+2abcd-3b^2c^2)\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+bd\,x^2e}{\sqrt{bde}}+\sqrt{bde\,x^4+(ade+bce)x^2+ace}\right)\sqrt{\frac{e(b\,x^2+}{d\,x^2+}}}{16b\,d^2\sqrt{bde}\,(b\,x^2+a)}$
default	$\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)\left(4\sqrt{bd}\,\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,bd\,x^2-\ln\left(\frac{2bd\,x^2+2\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^2d^2-2\ln\left(\frac{2bd\,x^2+2}{}\right.\right.$

input
```
int(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*b*d*x^2+a*d-3*b*c)*(d*x^2+c)/b/d^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/
16*(a^2*d^2+2*a*b*c*d-3*b^2*c^2)/b/d^2*ln((1/2*a*d*e+1/2*b*c*e+b*d*x^2*e)/
(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)*(e*
(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 407, normalized size of antiderivative = 2.07

$$\int x^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$

$$=\left[-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)e-4\,(2\,b^2d\right.}{}\right.$$

$$-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\sqrt{-\frac{e}{bd}}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{bd}}}{2\,(bex^2+ae)}\right)-2\,(2\,bd^2x^4-3\,bc^2+acd-(bcd}{16\,bd^2}$$

input
```
integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^
4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2
*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((
b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) - 4*(2*b*d^2*x^4 - 3*b*c^2 + a*
c*d - (b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^2), -1/
16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2
 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 +
a*e)) - 2*(2*b*d^2*x^4 - 3*b*c^2 + a*c*d - (b*c*d - a*d^2)*x^2)*sqrt((b*e*
x^2 + a*e)/(d*x^2 + c)))/(b*d^2)]
```

Sympy [F(-1)]

Timed out.

$$\int x^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.86

$$\int x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$$
$$=\frac{1}{16}\left(2\sqrt{bdex^4+bcex^2+adex^2+ace}\left(\frac{2x^2}{d}-\frac{3bc-ad}{bd^2}\right)-\frac{(3b^2c^2e-2abcde-a^2d^2e)\log\left(\left|-bce-a\right.\right.}{}\right.$$
$$+c)$$

input
```
integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/16*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)*(2*x^2/d - (3*b*c
- a*d)/(b*d^2)) - (3*b^2*c^2*e - 2*a*b*c*d*e - a^2*d^2*e)*log(abs(-b*c*e -
 a*d*e - 2*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d
*e*x^2 + a*c*e))))/(sqrt(b*d*e)*b*d^2))*sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx=\int x^3\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

input
```
int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00

$$\int x^3\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{e}\left(\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ab\,d^2-3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,b^2cd+2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,b^2d^2x^2+\sqrt{d}\,\sqrt{b}\log\right.}{}$$

input
```
int(x^3*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(e)*(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2 - 3*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b**2*c*d + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**2*
x**2 + sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d)*sqrt(c
+ d*x**2)*b)*a**2*d**2 + 2*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)
*d + sqrt(d)*sqrt(c + d*x**2)*b)*a*b*c*d - 3*sqrt(d)*sqrt(b)*log( - sqrt(b
)*sqrt(a + b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b)*b**2*c**2))/(8*b**2*d**
3)
```

3.44 $\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 127

$$\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \frac{(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{2d} - \frac{(bc-ad)\sqrt{e}\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}d^{3/2}}$$

output
```
1/2*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d-1/2*(-a*d+b*c)*e^(1
/2)*arctanh(d^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2)
)/b^(1/2)/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.04

$$\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{b}\sqrt{d}\sqrt{a+bx^2}(c+dx^2)-(bc-ad)\sqrt{c+dx^2}\,\mathrm{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{2\sqrt{b}d^{3/2}\sqrt{a+bx^2}}$$

input
```
Integrate[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*(c + d
*x^2) - (b*c - a*d)*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqr
t[b]*Sqrt[c + d*x^2])]))/(2*Sqrt[b]*d^(3/2)*Sqrt[a + b*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2053, 2051, 252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \sqrt{\frac{e(bx^2+a)}{dx^2+c}}dx^2$$

$$\Big\downarrow \text{2051}$$

$$e(bc-ad)\int \frac{x^4}{(be-dx^4)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{252}$$

$$e(bc-ad)\left(\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d(be-dx^4)}-\frac{\int \frac{1}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2d}\right)$$

$$\Big\downarrow \text{221}$$

$$e(bc-ad)\left(\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2d(be-dx^4)}-\frac{\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2\sqrt{b}d^{3/2}\sqrt{e}}\right)$$

input
```
Int[x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(b*c - a*d)*e*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*d*(b*e - d*x^4)) - Arc
Tanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])]/(2*Sqr
t[b]*d^(3/2)*Sqrt[e]))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.21

method	result
risch	$\frac{(d\,x^2+c)\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{2d}+\frac{(ad-bc)\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+bd\,x^2e}{\sqrt{bde}}+\sqrt{bde\,x^4+(ade+bce)x^2+ace}\right)\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,\sqrt{(d\,x^2+c)(b\,x^2+a)e}}{4d\sqrt{bde}\,(b\,x^2+a)}$
default	$\frac{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)\left(a\ln\left(\frac{2bd\,x^2+2\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)d-b\ln\left(\frac{2bd\,x^2+2\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)c+2}{4\sqrt{(d\,x^2+c)(b\,x^2+a)}\,d\sqrt{bd}}$

input
```
int(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/4*(a*d-b*c)/d*ln((1/2*a*d*
e+1/2*b*c*e+b*d*x^2*e)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(
1/2))/(b*d*e)^(1/2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^
(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.46

$$\int x\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$

$$=\left[-\frac{(bc-ad)\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)e+4\,(2\,b^2d^3x^4+b^2c^2d+\right.}{8\,d}\right.$$

input
```
integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((b*c - a*d)*sqrt(e/(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^
2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b^2*d^3*x^4 + b^2*c^2*
d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
 c))*sqrt(e/(b*d))) - 4*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d,
1/4*((b*c - a*d)*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b
*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*e)) + 2*(d*x^2 + c)
*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d]
```

Sympy [F(-1)]

Timed out.

$$\int x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.09

$$\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$$
$$=\frac{1}{4}\left(\frac{2\sqrt{bdex^4+bcex^2+adex^2+ace}}{d}+\frac{(bce-ade)\sqrt{bde}\log\left(\left|-2\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2}\right.\right.\right.}{bd^2e}\right.$$
$$+c)$$

input
```
integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)/d + (b*c*e - a*d*e)
*sqrt(b*d*e)*log(abs(-2*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*
d*e*x^2 + a*c*e))*b*d - sqrt(b*d*e)*b*c - sqrt(b*d*e)*a*d))/(b*d^2*e))*sgn
(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx=\int x\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}\,dx$$

input
```
int(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.80

$$\int x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx$$
$$= \frac{\sqrt{e}\left(\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bd+\sqrt{d}\,\sqrt{b}\log\left(-\sqrt{b}\,\sqrt{b\,x^2+a}\,d-\sqrt{d}\,\sqrt{d\,x^2+c}\,b\right)ad-\sqrt{d}\,\sqrt{b}\log\left(-\sqrt{b}\,\sqrt{b}\right.\right.}{2b\,d^2}$$

input
```
int(x*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(e)*(sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d + sqrt(d)*sqrt(b)*log( - s
qrt(b)*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c + d*x**2)*b)*a*d - sqrt(d)*sqrt
(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c + d*x**2)*b)*b*c))/
(2*b*d**2)
```

3.45 $\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 136

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = -\frac{\sqrt{a}\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}}+\frac{\sqrt{b}\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}}$$

output
```
-a^(1/2)*e^(1/2)*arctanh(c^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a^
(1/2)/e^(1/2))/c^(1/2)+b^(1/2)*e^(1/2)*arctanh(d^(1/2)*(b*e/d-(-a*d+b*c)*e
/d/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x} dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\left(-\sqrt{a}\sqrt{d}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{b}\sqrt{c}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{\sqrt{c}\sqrt{d}\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-(Sqrt[a]*Sqrt[d]*ArcT
anh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]) + Sqrt[b]*Sqrt[c
]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]))/(Sqrt[c]*
Sqrt[d]*Sqrt[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2053, 2052, 25, 383, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x} dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{x^2} dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int -\frac{x^4}{(ae-cx^4)(be-dx^4)} d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \textbf{25}$$

$$-\left(e(bc-ad)\int \frac{x^4}{(ae-cx^4)(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\right)$$

$$\Big\downarrow \textbf{383}$$

$$e(bc-ad)\left(\frac{b\int \frac{1}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{bc-ad}-\frac{a\int \frac{1}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{bc-ad}\right)$$

$$\Big\downarrow \textbf{221}$$

$$e(bc-ad)\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}\sqrt{e}(bc-ad)}-\frac{\sqrt{a}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}\sqrt{e}(bc-ad)}\right)$$

input
```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x,x]
```

output
```
(b*c - a*d)*e*(-((Sqrt[a]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2
)])/(Sqrt[a]*Sqrt[e])])/(Sqrt[c]*(b*c - a*d)*Sqrt[e])) + (Sqrt[b]*ArcTanh[
(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[d]*(
b*c - a*d)*Sqrt[e]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.32

method	result
default	$\frac{\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(\ln\left(\frac{2bd\,x^{2}+2\sqrt{db\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\sqrt{ac}\,b-a\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2\sqrt{ac}\,\sqrt{db\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}+2ac}{x^{2}}\right.}{2\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{bd}\,\sqrt{ac}}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*(ln(1/2*(2*b*d*x^2+2*(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*b
-a*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2
*a*c)/x^2)*(b*d)^(1/2))/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 865, normalized size of antiderivative = 6.36

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="fricas")
```

output
```
[1/4*sqrt(b*e/d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*
c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 + (3*b*c
*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 1/4*sq
rt(a*e/c)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*
b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3
*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4), -1/2*s
qrt(-b*e/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2
+ a*e)/(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) + 1/4*sqrt(a*e/c)*log(((b^2*c^2 +
 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 -
4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c
)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4), 1/2*sqrt(-a*e/c)*arctan(1/2*((b
*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b
*e*x^2 + a^2*e)) + 1/4*sqrt(b*e/d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*
d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d
+ a*c*d^2 + (3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x
^2 + c))), 1/2*sqrt(-a*e/c)*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e
/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2*e)) - 1/2*sqrt(-b*e
/d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c))/(b^2*e*x^2 + a*b*e))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x}\,dx = \int \frac{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{x}dx$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x)
```

output
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x,x)
```

3.46 $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 130

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = -\frac{(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{2cx^2} - \frac{(bc-ad)\sqrt{e}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d\left(c+dx^2\right)}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}c^{3/2}}$$

output
```
-1/2*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c/x^2-1/2*(-a*d+b*c)
*e^(1/2)*arctanh(c^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a^(1/2)/e^
(1/2))/a^(1/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.05

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = -\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}(c+dx^2)+(bc-ad)x^2\sqrt{c+dx^2}\,\mathrm{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)\right)}{2\sqrt{a}c^{3/2}x^2\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]
```

output
```
-1/2*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*(
c + d*x^2) + (b*c - a*d)*x^2*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x
^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(Sqrt[a]*c^(3/2)*x^2*Sqrt[a + b*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2053, 2052, 252, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3} \, dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{x^4}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int \frac{x^4}{(ae-cx^4)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{252}$$

$$e(bc-ad)\left(\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(ae-cx^4\right)}-\frac{\int \frac{1}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2c}\right)$$

$$\Big\downarrow \text{221}$$

$$e(bc-ad)\left(\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2c\left(ae-cx^4\right)}-\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2\sqrt{a}c^{3/2}\sqrt{e}}\right)$$

input
```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^3,x]
```

output
```
(b*c - a*d)*e*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*c*(a*e - c*x^4)) - Arc
Tanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])]/(2*Sqr
t[a]*c^(3/2)*Sqrt[e]))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.25

method	result
risch	$-\frac{(d\,x^{2}+c)\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{2c\,x^{2}}+\frac{(ad-bc)\ln\left(\frac{2ace+(ade+bce)x^{2}+2\sqrt{ace}\,\sqrt{bde\,x^{4}+(ade+bce)x^{2}+ace}}{x^{2}}\right)\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,\sqrt{(d\,x^{2}+c)(b\,x^{2}+a)e}}{4c\sqrt{ace}\,(b\,x^{2}+a)}$
default	$-\frac{\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,(d\,x^{2}+c)\left(-2db\sqrt{db\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,x^{4}\sqrt{ac}-a^{2}\ln\left(\frac{ad\,x^{2}+bc\,x^{2}+2\sqrt{ac}\,\sqrt{db\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}+2ac}{x^{2}}\right)dc\,x^{2}+c^{2}\ln\right.}{}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/c*(d*x^2+c)/x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/4*(a*d-b*c)/c/(a*c*e)
^(1/2)*ln((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c
*e)*x^2+a*c*e)^(1/2))/x^2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2
+a)*e)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.56

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx$$
$$= \left[-\frac{(bc-ad)x^2\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2+4\,(2\,a^2c^3+(abc^2d+a^2cd^2)x^4+(abc^3+3\,a^2c^2d)x^2)\sqrt{}}{x^4}\right.}{8\,cx^2}\right.$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[-1/8*((b*c - a*d)*x^2*sqrt(e/(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*
e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^
2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(
d*x^2 + c))*sqrt(e/(a*c)))/x^4) + 4*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^
2 + c)))/(c*x^2), 1/4*((b*c - a*d)*x^2*sqrt(-e/(a*c))*arctan(1/2*((b*c + a
*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))/(b*e*x^2
 + a*e)) - 2*(d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 242 vs. $2(110) = 220$.

Time = 0.17 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.86

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx$$

$$= \frac{1}{2}\left(\frac{(bce-ade)\arctan\left(-\frac{\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}c} - \frac{\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right)}{\left(ace-\left(\sqrt{bde}x^2\right.\right.}\right.$$

$$+c)$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/2*((b*c*e - a*d*e)*arctan(-(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2
 + a*d*e*x^2 + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e)*c) - ((sqrt(b*d*e)*x^2
- sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*b*c*e + (sqrt(b*d*e)*x^
2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a*d*e + 2*sqrt(b*d*e)
*a*c*e)/((a*c*e - (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^
2 + a*c*e))^2)*c))*sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^3}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^3,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^3}\,dx$$
$$= \frac{\sqrt{e}\left(-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ac+\sqrt{c}\,\sqrt{a}\log\left(\sqrt{a}\,\sqrt{b\,x^2+a}\,c+\sqrt{c}\,\sqrt{d\,x^2+c}\,a\right)ad\,x^2-\sqrt{c}\,\sqrt{a}\log\left(\sqrt{a}\,\sqrt{b\right.\right.}{2a\,c^2x^2}$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^3,x)
```

output
```
(sqrt(e)*( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c + sqrt(c)*sqrt(a)*log(s
qrt(a)*sqrt(a + b*x**2)*c + sqrt(c)*sqrt(c + d*x**2)*a)*a*d*x**2 - sqrt(c)
*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c + sqrt(c)*sqrt(c + d*x**2)*a)*b*c*
x**2 - sqrt(c)*sqrt(a)*log(x)*a*d*x**2 + sqrt(c)*sqrt(a)*log(x)*b*c*x**2))
/(2*a*c**2*x**2)
```

3.47 $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 202

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = -\frac{(bc-5ad)\left(c+dx^2\right)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{8ac^2x^2}-\frac{\left(c+dx^2\right)^2\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{4c^2x^4}+\frac{(bc-ad)(bc+3ad)\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d\left(c+dx^2\right)}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{3/2}c^{5/2}}$$

output
```
-1/8*(-5*a*d+b*c)*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a/c^2/x
^2-1/4*(d*x^2+c)^2*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c^2/x^4+1/8*(-a*
d+b*c)*(3*a*d+b*c)*e^(1/2)*arctanh(c^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c)
)^(1/2)/a^(1/2)/e^(1/2))/a^(3/2)/c^(5/2)
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx$$
$$= \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\sqrt{c+dx^2}(-2ac-bcx^2+3adx^2)+(b^2c^2+2abcd-3a^2d^2)\,x^4\text{arctan}\right.}{8a^{3/2}c^{5/2}x^4\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5,x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2]*(-2*a*c - b*c*x^2 + 3*a*d*x^2) + (b^2*c^2 + 2*a*
b*c*d - 3*a^2*d^2)*x^4*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c +
 d*x^2])]))/(8*a^(3/2)*c^(5/2)*x^4*Sqrt[a + b*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.59 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2053, 2052, 25, 360, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx$$
$$\Big\downarrow \text{ 2053}$$
$$\frac{1}{2}\int \frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{x^6}dx^2$$
$$\Big\downarrow \text{ 2052}$$
$$e(bc-ad)\int -\frac{x^4\left(be-dx^4\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow 25$$

$$-\left(e(bc-ad)\int \frac{x^4\left(be-dx^4\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$$\Big\downarrow 360$$

$$e(bc-ad)\left(\frac{\int \frac{(bc-ad)e-4cdx^4}{\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4c^2}-\frac{e(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow 298$$

$$e(bc-ad)\left(\frac{\frac{(3ad+bc)\int \frac{1}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2a}+\frac{(bc-5ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2a\left(ae-cx^4\right)}}{4c^2}-\frac{e(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow 221$$

$$e(bc-ad)\left(\frac{\frac{(3ad+bc)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}+\frac{(bc-5ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2a\left(ae-cx^4\right)}}{4c^2}-\frac{e(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(ae-cx^4\right)^2}\right)$$

input
```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^5,x]
```

output
```
(b*c - a*d)*e*(-1/4*((b*c - a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(c^2
*(a*e - c*x^4)^2) + (((b*c - 5*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(2*
a*(a*e - c*x^4)) + ((b*c + 3*a*d)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c
 + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(3/2)*Sqrt[c]*Sqrt[e]))/(4*c^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{(d\,x^2+c)(-3ad\,x^2+bc\,x^2+2ac)\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{8c^2x^4a}-\frac{(3a^2d^2-2abcd-b^2c^2)\ln\left(\frac{2ace+(ade+bce)x^2+2\sqrt{ace}\,\sqrt{bde\,x^4+(ade+bce)x^2+ace}}{x^2}\right)}{16a\,c^2\sqrt{ace}\,(b\,x^2+a)}$
default	$-\frac{\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)\left(10d^2b\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,x^6a\sqrt{ac}+2d\,b^2\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,x^6c\sqrt{ac}+3a^3\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{}}{}\right.\right.}{}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(d*x^2+c)*(-3*a*d*x^2+b*c*x^2+2*a*c)/c^2/x^4/a*(e*(b*x^2+a)/(d*x^2+c)
)^(1/2)-1/16*(3*a^2*d^2-2*a*b*c*d-b^2*c^2)/a/c^2/(a*c*e)^(1/2)*ln((2*a*c*e
+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/
2))/x^2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)/(b*x^
2+a)
```

Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.11

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx$$

$$= \left[-\frac{(b^2c^2+2\,abcd-3\,a^2d^2)x^4\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2-4\,(2\,a^2c^3+(abc^2d+a^2cd^2)x^4+(\cdots}{x^4}\right.}{32\,ac^2x^4}\right.$$

$$-\frac{(b^2c^2+2\,abcd-3\,a^2d^2)x^4\sqrt{-\frac{e}{ac}}\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{ac}}}{2\,(bex^2+ae)}\right)+2\,((bcd-3\,ad^2)x^4+2\,ac^2+\cdots}{16\,ac^2x^4}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*sqrt(e/(a*c))*log(((b^2*c^2
+ 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 -
 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x^2)
*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) + 4*((b*c*d - 3*a*d
^2)*x^4 + 2*a*c^2 + (b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))
)/(a*c^2*x^4), -1/16*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*x^4*sqrt(-e/(a*c))
*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sq
rt(-e/(a*c))/(b*e*x^2 + a*e)) + 2*((b*c*d - 3*a*d^2)*x^4 + 2*a*c^2 + (b*c^
2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^2*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**5,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(178) = 356$.

Time = 0.18 (sec) , antiderivative size = 549, normalized size of antiderivative = 2.72

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx =$$

$$-\frac{1}{8}\left(\frac{(b^2c^2e + 2\,abcde - 3\,a^2d^2e)\arctan\left(-\frac{\sqrt{bde}x^2 - \sqrt{bdex^4 + bcex^2 + adex^2 + ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}ac^2} - \frac{\left(\sqrt{bde}x^2 - \sqrt{bdex^4 + bcex\right.}{}\right.$$

$$+ c)$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x, algorithm="giac")
```

output
```
-1/8*((b^2*c^2*e + 2*a*b*c*d*e - 3*a^2*d^2*e)*arctan(-(sqrt(b*d*e)*x^2 - s
qrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e
)*a*c^2) - ((sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*
c*e))*a*b^2*c^3*e^2 + 10*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a
*d*e*x^2 + a*c*e))*a^2*b*c^2*d*e^2 + 5*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 +
 b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a^3*c*d^2*e^2 + 8*sqrt(b*d*e)*a^3*c^2*d*e
^2 + (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3
*b^2*c^2*e + 2*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 +
 a*c*e))^3*a*b*c*d*e - 3*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a
*d*e*x^2 + a*c*e))^3*a^2*d^2*e + 8*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d
*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^2*a*b*c^2*e)/((a*c*e - (sqrt(b*d*
e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^2)^2*a*c^2))*sgn
(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^5}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^5,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^5}\,dx = \frac{\sqrt{e}\left(-2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2c^2+3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2cd\,x^2-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ab\,c^2x^2+3\sqrt{c}\,\sqrt{}\right.}{}$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^5,x)
```

output
```
(sqrt(e)*( - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c**2 + 3*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*a**2*c*d*x**2 - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a
*b*c**2*x**2 + 3*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c - sqrt(c)*
sqrt(c + d*x**2)*a)*a**2*d**2*x**4 - 2*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a
+ b*x**2)*c - sqrt(c)*sqrt(c + d*x**2)*a)*a*b*c*d*x**4 - sqrt(c)*sqrt(a)*l
og(sqrt(a)*sqrt(a + b*x**2)*c - sqrt(c)*sqrt(c + d*x**2)*a)*b**2*c**2*x**4
 - 3*sqrt(c)*sqrt(a)*log(x)*a**2*d**2*x**4 + 2*sqrt(c)*sqrt(a)*log(x)*a*b*
c*d*x**4 + sqrt(c)*sqrt(a)*log(x)*b**2*c**2*x**4))/(8*a**2*c**3*x**4)
```

3.48 $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^7}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 294

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \frac{(b^2c^2+2abcd-11a^2d^2)\,(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{16a^2c^3x^2} - \frac{(bc-13ad)\,(c+dx^2)^2\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{24ac^3x^4} - \frac{(c+dx^2)^3\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{6c^3x^6} - \frac{(bc-ad)\,(b^2c^2+2abcd+5a^2d^2)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d\left(c+dx^2\right)}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{5/2}c^{7/2}}$$

output
```
1/16*(-11*a^2*d^2+2*a*b*c*d+b^2*c^2)*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^
2+c))^(1/2)/a^2/c^3/x^2-1/24*(-13*a*d+b*c)*(d*x^2+c)^2*(b*e/d-(-a*d+b*c)*e
/d/(d*x^2+c))^(1/2)/a/c^3/x^4-1/6*(d*x^2+c)^3*(b*e/d-(-a*d+b*c)*e/d/(d*x^2
+c))^(1/2)/c^3/x^6-1/16*(-a*d+b*c)*(5*a^2*d^2+2*a*b*c*d+b^2*c^2)*e^(1/2)*a
rctanh(c^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))/a^(
5/2)/c^(7/2)
```

Mathematica [A] (verified)

Time = 1.48 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}\sqrt{c+dx^2}(3b^2c^2x^4-2abcx^2(c-2dx^2)+a^2(-8c^2+10cdx^2-15d^2x\right.}{48a^{5/2}c^{7/2}x^6\sqrt{a+bx^2}}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(Sqrt[a]*Sqrt[c]*Sqrt[a
 + b*x^2]*Sqrt[c + d*x^2]*(3*b^2*c^2*x^4 - 2*a*b*c*x^2*(c - 2*d*x^2) + a^2
*(-8*c^2 + 10*c*d*x^2 - 15*d^2*x^4)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*
c*d^2 - 5*a^3*d^3)*x^6*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c +
 d*x^2])]))/(48*a^(5/2)*c^(7/2)*x^6*Sqrt[a + b*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.80 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2053, 2052, 366, 27, 360, 27, 298, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx$$

$$\Big\downarrow \ 2053$$

$$\frac{1}{2}\int \frac{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{x^8}dx^2$$

$$\Big\downarrow \ 2052$$

$$e(bc-ad)\int \frac{x^4\left(be-dx^4\right)^2}{\left(ae-cx^4\right)^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$\Big\downarrow$ 366

$$e(bc-ad)\left(\frac{ex^6(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}-\frac{\int -\frac{3ex^4\left(\left(b^2c^2+2abdc-a^2d^2\right)e-2acd^2x^4\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{6ac^2e}\right)$$

$\Big\downarrow$ 27

$$e(bc-ad)\left(\frac{\int \frac{x^4\left(\left(b^2c^2+2abdc-a^2d^2\right)e-2acd^2x^4\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2ac^2}+\frac{ex^6(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}\right)$$

$\Big\downarrow$ 360

$$e(bc-ad)\left(\frac{\frac{e(bc-ad)(3ad+bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c\left(ae-cx^4\right)^2}-\frac{\int \frac{c\left((bc-ad)(bc+3ad)e-8acd^2x^4\right)}{\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4c^2}}{2ac^2}+\frac{ex^6(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}\right)$$

$\Big\downarrow$ 27

$$e(bc-ad)\left(\frac{\frac{e(bc-ad)(3ad+bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c\left(ae-cx^4\right)^2}-\frac{\int \frac{(bc-ad)(bc+3ad)e-8acd^2x^4}{\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4c}}{2ac^2}+\frac{ex^6(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}\right)$$

$\Big\downarrow$ 298

$$e(bc-ad)\left(\frac{\frac{e(bc-ad)(3ad+bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c\left(ae-cx^4\right)^2}-\frac{\frac{\left(5a^2d^2+2abcd+b^2c^2\right)\int \frac{1}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2a}+\frac{\left(-11a^2d^2+2abcd+b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2a\left(ae-cx^4\right)}}{4c}}{2ac^2}+\frac{ex^6(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}\right)$$

$\Big\downarrow$ 221

$$e(bc-ad)\left(\frac{\frac{e(bc-ad)(3ad+bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c(ae-cx^4)^2}-\frac{\frac{\left(-11a^2d^2+2abcd+b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2a\left(ae-cx^4\right)}+\frac{\left(5a^2d^2+2abcd+b^2c^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}}{4c}}{2ac^2}+\frac{ex^6(bc}{6ac^2\,(ae}\right.$$

input
```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^7,x]
```

output
```
(b*c - a*d)*e*(((b*c - a*d)^2*e*x^6)/(6*a*c^2*(a*e - c*x^4)^3) + (((b*c -
a*d)*(b*c + 3*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*c*(a*e - c*x^4)
^2) - (((b^2*c^2 + 2*a*b*c*d - 11*a^2*d^2)*Sqrt[(e*(a + b*x^2))/(c + d*x^2
)])/(2*a*(a*e - c*x^4)) + ((b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcTanh[(Sqrt
[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(2*a^(3/2)*Sqrt
[c]*Sqrt[e]))/(4*c))/(2*a*c^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\left(d\,x^{2}+c\right)\left(15a^{2}d^{2}x^{4}-4abcd\,x^{4}-3b^{2}c^{2}x^{4}-10a^{2}cd\,x^{2}+2ab\,c^{2}x^{2}+8a^{2}c^{2}\right)\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{48c^{3}x^{6}a^{2}}+\frac{\left(5a^{3}d^{3}-3a^{2}bc\,d^{2}-a\,b^{2}c^{2}d-b^{3}c^{3}\right)\ln\left(\right.}{}$
default	$-\frac{\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(-66\sqrt{db\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{ac}\,a^{2}b\,d^{3}x^{8}-24\sqrt{db\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{ac}\,a\,b^{2}c\,d^{2}x^{8}-6\sqrt{db\,x^{4}+ad\,x^{2}\right.}{}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(d*x^2+c)*(15*a^2*d^2*x^4-4*a*b*c*d*x^4-3*b^2*c^2*x^4-10*a^2*c*d*x^2
+2*a*b*c^2*x^2+8*a^2*c^2)/c^3/x^6/a^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/32*(
5*a^3*d^3-3*a^2*b*c*d^2-a*b^2*c^2*d-b^3*c^3)/a^2/c^3/(a*c*e)^(1/2)*ln((2*a
*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)
^(1/2))/x^2)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)/(
b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.91

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \left[-\frac{3\,(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3)x^6\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2+4\,(2\,a^2c^3+(ab\cdots}{x^4}\right.}{}\right.$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="fricas")
```

output
```
[-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*x^6*sqrt(e/
(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c
^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^
3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4
) - 4*((3*b^2*c^2*d + 4*a*b*c*d^2 - 15*a^2*d^3)*x^6 - 8*a^2*c^3 + (3*b^2*c
^3 + 2*a*b*c^2*d - 5*a^2*c*d^2)*x^4 - 2*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b
*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c^3*x^6), 1/96*(3*(b^3*c^3 + a*b^2*c^2*d
+ 3*a^2*b*c*d^2 - 5*a^3*d^3)*x^6*sqrt(-e/(a*c))*arctan(1/2*((b*c + a*d)*x^
2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e
)) + 2*((3*b^2*c^2*d + 4*a*b*c*d^2 - 15*a^2*d^3)*x^6 - 8*a^2*c^3 + (3*b^2*
c^3 + 2*a*b*c^2*d - 5*a^2*c*d^2)*x^4 - 2*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((
b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c^3*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**7,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(266) = 532$.

Time = 0.21 (sec) , antiderivative size = 1001, normalized size of antiderivative = 3.40

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \text{Too large to display}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x, algorithm="giac")
```

output
```
1/48*(3*(b^3*c^3*e + a*b^2*c^2*d*e + 3*a^2*b*c*d^2*e - 5*a^3*d^3*e)*arctan
(-(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))/sqrt
(-a*c*e))/(sqrt(-a*c*e)*a^2*c^3) - (3*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 +
b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a^2*b^3*c^5*e^3 + 51*(sqrt(b*d*e)*x^2 - sq
rt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a^3*b^2*c^4*d*e^3 + 105*(sq
rt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a^4*b*c^3
*d^2*e^3 + 33*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 +
a*c*e))*a^5*c^2*d^3*e^3 + 16*sqrt(b*d*e)*a^4*b*c^4*d*e^3 + 48*sqrt(b*d*e)*
a^5*c^3*d^2*e^3 + 8*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*
x^2 + a*c*e))^3*a*b^3*c^4*e^2 + 72*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c
*e*x^2 + a*d*e*x^2 + a*c*e))^3*a^2*b^2*c^3*d*e^2 + 24*(sqrt(b*d*e)*x^2 - s
qrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3*a^3*b*c^2*d^2*e^2 - 40*(
sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3*a^4*c
*d^3*e^2 + 48*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 +
a*d*e*x^2 + a*c*e))^2*a^2*b^2*c^4*e^2 + 144*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 -
 sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^2*a^3*b*c^3*d*e^2 - 3*(s
qrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^5*b^3*c^
3*e - 3*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)
)^5*a*b^2*c^2*d*e - 9*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*
e*x^2 + a*c*e))^5*a^2*b*c*d^2*e + 15*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^7}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^7,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 419, normalized size of antiderivative = 1.43

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^7}\,dx$$
$$= \frac{\sqrt{e}\left(-8\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^3c^3+10\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^3c^2d\,x^2-15\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^3c\,d^2x^4-2\right.}{}$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^7,x)
```

output
```
(sqrt(e)*( - 8*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*c**3 + 10*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*a**3*c**2*d*x**2 - 15*sqrt(c + d*x**2)*sqrt(a + b*
x**2)*a**3*c*d**2*x**4 - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c**3*x
**2 + 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c**2*d*x**4 + 3*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*a*b**2*c**3*x**4 + 15*sqrt(c)*sqrt(a)*log(sqrt(a)
*sqrt(a + b*x**2)*c + sqrt(c)*sqrt(c + d*x**2)*a)*a**3*d**3*x**6 - 9*sqrt(
c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c + sqrt(c)*sqrt(c + d*x**2)*a)*a*
*2*b*c*d**2*x**6 - 3*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c + sqrt
(c)*sqrt(c + d*x**2)*a)*a*b**2*c**2*d*x**6 - 3*sqrt(c)*sqrt(a)*log(sqrt(a)
*sqrt(a + b*x**2)*c + sqrt(c)*sqrt(c + d*x**2)*a)*b**3*c**3*x**6 - 15*sqrt
(c)*sqrt(a)*log(x)*a**3*d**3*x**6 + 9*sqrt(c)*sqrt(a)*log(x)*a**2*b*c*d**2
*x**6 + 3*sqrt(c)*sqrt(a)*log(x)*a*b**2*c**2*d*x**6 + 3*sqrt(c)*sqrt(a)*lo
g(x)*b**3*c**3*x**6))/(48*a**3*c**4*x**6)
```

3.49 $\int x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 465

$$\begin{aligned}
&\int x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx \\
&= -\frac{(4bc-ad)x(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{15bd^2}+\frac{x^3(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{5d} \\
&\quad+\frac{(8b^2c^2-3abcd-2a^2d^2)\,x(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{15bd^3\,(a+bx^2)} \\
&\quad-\frac{\sqrt{a}(8b^2c^2-3abcd-2a^2d^2)\,(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{3/2}d^3\,(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\
&\quad+\frac{a^{3/2}(4bc-ad)\,(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{3/2}d^2\,(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}
\end{aligned}$$

output
```
-1/15*(-a*d+4*b*c)*x*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b/d^
2+1/5*x^3*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d+1/15*(-2*a^2*
d^2-3*a*b*c*d+8*b^2*c^2)*x*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2
)/b/d^3/(b*x^2+a)-1/15*a^(1/2)*(-2*a^2*d^2-3*a*b*c*d+8*b^2*c^2)*(d*x^2+c)*
(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^
2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d^3/(b*x^2+a)/(a*(d*x^2+c)/c/(b*x^2+
a))^(1/2)+1/15*a^(3/2)*(-a*d+4*b*c)*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2
+c))^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^
(3/2)/d^2/(b*x^2+a)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.63 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.55

$$\int x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2)(-4bc+ad+3bdx^2)+ic(-8b^2c^2+3abcd+2a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1-\cdots}\right)}{15b\sqrt{\cdots}}$$

input
```
Integrate[x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*
(-4*b*c + a*d + 3*b*d*x^2) + I*c*(-8*b^2*c^2 + 3*a*b*c*d + 2*a^2*d^2)*Sqrt
[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d
)/(b*c)] - I*c*(-8*b^2*c^2 + 7*a*b*c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt
[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(15*b*Sqr
t[b/a]*d^3*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2058, 380, 444, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{x^4\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 380}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\int \frac{x^2\left((4bc-ad)x^2+3ac\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 444}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{3bd}-\frac{\int \frac{\left(8b^2c^2-3abdc-2a^2d^2\right)x^2+ac(4bc-ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{5d}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{3bd}-\frac{\left(-2a^2d^2-3abcd+8b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(4bc-ad)\int \frac{}{\sqrt{bx^2+\cdots}}}{3bd}}{5d}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{3bd}-\frac{\left(-2a^2d^2-3abcd+8b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(4bc-ad}{\sqrt{d}\sqrt{c}}}{3bd}}{5d}\right.}{\sqrt{a+bx^2}}$$

$\Big\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{3bd}-\frac{\left(-2a^2d^2-3abcd+8b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a}}{}}{3bd}}{5d}\right.}{\sqrt{a+bx^2}}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-ad)}{3bd}-\frac{\left(-2a^2d^2-3abcd+8b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right.}{3bd}}{5d}\right.}{\sqrt{a+bx^2}}$$

input

```
Int[x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*((x^3*Sqrt[a + b*x^2]*S
qrt[c + d*x^2])/(5*d) - (((4*b*c - a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
/(3*b*d) - ((8*b^2*c^2 - 3*a*b*c*d - 2*a^2*d^2)*((x*Sqrt[a + b*x^2])/(b*Sq
rt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqr
t[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*
Sqrt[c + d*x^2])) + (c^(3/2)*(4*b*c - a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a
*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b*d))/(5*d)))/Sqrt[a + b*x^2]
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 4.86 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.05

method	result
risch	$\frac{x\left(3bd\,x^{2}+ad-4bc\right)\left(d\,x^{2}+c\right)\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{15b\,d^{2}}-\frac{\left(-\frac{2\left(2a^{2}d^{2}+3abcd-8b^{2}c^{2}\right)ace\sqrt{1+\frac{x^{2}b}{a}}\,\sqrt{1+\frac{x^{2}d}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)-\text{Ell}\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}\,(ade+bce+e(ad-bc)}\right.}{}$
default	$\frac{\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(3\sqrt{-\frac{b}{a}}\,b^{2}d^{3}x^{7}+4\sqrt{-\frac{b}{a}}\,ab\,d^{3}x^{5}-\sqrt{-\frac{b}{a}}\,b^{2}c\,d^{2}x^{5}+\sqrt{-\frac{b}{a}}\,a^{2}d^{3}x^{3}-4\sqrt{-\frac{b}{a}}\,b^{2}c^{2}d\,x^{3}+\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\text{Elli}\right.}{}$

input
```
int(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*x*(3*b*d*x^2+a*d-4*b*c)*(d*x^2+c)/b/d^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)
-1/15/b/d^2*(-2*(2*a^2*d^2+3*a*b*c*d-8*b^2*c^2)*a*c*e/(-b/a)^(1/2)*(1+1/a*
x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/
2)/(a*d*e+b*c*e+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c
/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))+a^2
*c*d/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e
*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b
/e)^(1/2))-4*a*b*c^2/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/
(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(
a*d*e+b*c*e)/c/b/e)^(1/2)))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^
2+a)*e)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.59

$$\int x^4\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx =$$

$$-\frac{(8\,b^2c^3-3\,abc^2d-2\,a^2cd^2)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(8\,b^2c^3-3\,abc^2d-a^2d^3-2\,(a^2-2\,a}{}$$

input
```
integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/15*((8*b^2*c^3 - 3*a*b*c^2*d - 2*a^2*c*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*el
liptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (8*b^2*c^3 - 3*a*b*c^2*d - a^2
*d^3 - 2*(a^2 - 2*a*b)*c*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_f(arcsin(s
qrt(-c/d)/x), a*d/(b*c)) - (3*b^2*d^3*x^6 + 8*b^2*c^3 - 3*a*b*c^2*d - 2*a^
2*c*d^2 - (b^2*c*d^2 - a*b*d^3)*x^4 + 2*(2*b^2*c^2*d - a*b*c*d^2 - a^2*d^3
)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int x^4\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^4\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = \int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^4\,dx$$

input
```
integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4, x)
```

Giac [F]

$$\int x^4\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = \int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^4\,dx$$

input
```
integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx=\int x^4\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

input
```
int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \frac{\sqrt{e}\left(\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,adx-4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bcx+3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bd\,x^3-2\left(\int\frac{\sqrt{d\,x^2+c}\,\sqrt{}}{bd\,x^4+ad\,x^2}\right.\right.}{}$$

input
```
int(x^4*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(e)*(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*x - 4*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*b*c*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*x**3 - 2*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d
*x**4),x)*a**2*d**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c
+ a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*c*d + 8*int((sqrt(c + d*x**2)*sqr
t(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*c**2 -
int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 + b*d*x
**4),x)*a**2*c*d + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x*
*2 + b*c*x**2 + b*d*x**4),x)*a*b*c**2))/(15*b*d**2)
```

3.50 $\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 362

$$\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \frac{x(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{3d}-\frac{(2bc-ad)x(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{3d^2(a+bx^2)}+\frac{\sqrt{a}(2bc-ad)(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3\sqrt{b}d^2(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}-\frac{a^{3/2}(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}d(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
1/3*x*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d-1/3*(-a*d+2*b*c)*
x*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d^2/(b*x^2+a)+1/3*a^(1/
2)*(-a*d+2*b*c)*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)*EllipticE
(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/d^2/(b*x^2
+a)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*a^(3/2)*(d*x^2+c)*(b*e/d-(-a*d+b*c
)*e/d/(d*x^2+c))^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/
c)^(1/2))/b^(1/2)/d/(b*x^2+a)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.07 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.57

$$\int x^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx$$
$$=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)-ic(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+2ic\right.}{3\sqrt{\frac{b}{a}}d^2\left(a+bx^2\right)}$$

input
```
Integrate[x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)
- I*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/
(3*Sqrt[b/a]*d^2*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.81, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2058, 380, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx$$
$$\Big\downarrow \; 2058$$
$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int\frac{x^2\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{\sqrt{a+bx^2}}$$
$$\Big\downarrow \; 380$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\int \frac{(2bc-ad)x^2+ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3d}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 313}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx}{a(c+dx}}}\right.}{3d}\right.}{\sqrt{a+bx^2}}$$

input
```
Int[x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*((x*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])/(3*d) - ((2*b*c - a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^
2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*
x^2])) + (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2]))/(3*d)))/Sqrt[a + b*x^2]
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 3.82 (sec) , antiderivative size = 350, normalized size of antiderivative = 0.97

method	result
risch	$\frac{x(d\,x^2+c)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{3d} - \frac{\left(\frac{2(ad-2bc)ace\sqrt{1+\frac{x^2b}{a}}\,\sqrt{1+\frac{x^2d}{c}}\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}\,(ade+bce+e(ad-bc))}+ \right.}{3d(b\,x^2+a)}$
default	$\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)\left(\sqrt{-\frac{b}{a}}\,b\,d^2x^5+\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,x^3-2ac\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d+2\sqrt{\frac{b\,x^2+a}{a}} \right.}{3\sqrt{(d\,x^2+c)(b}$

input
```
int(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/d*x*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3/d*(2*(a*d-2*b*c)*a*c*e
/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2
+b*c*e*x^2+a*c*e)^(1/2)/(a*d*e+b*c*e+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2
),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*
e)/c/b/e)^(1/2)))+a*c/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)
/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+
(a*d*e+b*c*e)/c/b/e)^(1/2)))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x
^2+a)*e)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.50

$$\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$$
$$=\frac{(2\,bc^2-acd)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(2\,bc^2-acd+ad^2)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})}{3\,bd^2x}$$

input
```
integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
1/3*((2*b*c^2 - a*c*d)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/
d)/x), a*d/(b*c)) - (2*b*c^2 - a*c*d + a*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*ell
iptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (b*d^2*x^4 - 2*b*c^2 + a*c*d -
(b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx=\text{Timed out}$$

input
```
integrate(x**2*(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=\int\sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^2\,dx$$

input
```
integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2, x)
```

Giac [F]

$$\int x^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=\int\sqrt{\frac{(bx^2+a)e}{dx^2+c}}x^2\,dx$$

input
```
integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx=\int x^2\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}\,dx$$

input
```
int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int x^2\sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx$$

$$= \frac{\sqrt{e}\left(\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)ad-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)bc-\left(\int \frac{\sqrt{d\,x^2+c}}{bd\,x^4+ad\,x^2}\right.\right.}{3d}$$

input
```
int(x^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(e)*(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x + int((sqrt(c + d*x**2)*sqrt
(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*d - 2*int((
sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x
**4),x)*b*c - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*
c*x**2 + b*d*x**4),x)*a*c))/(3*d)
```

3.51 $\int \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 230

$$\int \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = x\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}} - \frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
x*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-c^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*
x^2+c))^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1
/2))/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+c^(1/2)*(b*e/d-(-a*d+b*c)*e/d
/(d*x^2+c))^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1
/2))/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.83 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.37

$$\int \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{\frac{c+dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{\frac{a+bx^2}{a}}}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(c + d*x^2)/c]*EllipticE[ArcSin[Sq
rt[-(d/c)]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[(a + b*x^2)/a])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2058, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,dx$$

$$\Big\downarrow \ 2058$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 324$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(a\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \ 320$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{388}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{313}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{\sqrt{a+bx^2}}$$

input
```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(b*((x*Sqrt[a + b*x^2])
/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*
x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*
x^2))]*Sqrt[c + d*x^2])))/Sqrt[a + b*x^2]
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.80

method	result
default	$\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\left(a\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-bc\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+bc\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right.\right.}{\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,d}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^
(1/2)*(a*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*d-b*c*EllipticF(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))+b*c*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)))/((
d*x^2+c)*(b*x^2+a))^(1/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.65

$$\int \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = -\frac{bc^2\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - (bc^2+ad^2)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - (bcdx^2+bc^2)}{bcdx}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-(b*c^2*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c
)) - (b*c^2 + a*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)
/x), a*d/(b*c)) - (b*c*d*x^2 + b*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(
b*c*d*x)
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Giac [F]

$$\int \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \int \sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \int \sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\frac{e\,(a+bx^2)}{c+dx^2}}\,dx = \sqrt{e}\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d\,x^2+c}dx\right)$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
sqrt(e)*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c + d*x**2),x)
```

3.52 $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 237

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = -\frac{\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{x} - \frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{b\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output

```
-(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/x-d^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d
*x^2+c))^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(
1/2))/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+b*c^(1/2)*(b*e/d-(-a*d+b*c)*
e/d/(d*x^2+c))^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)
^(1/2))/a/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.47

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx=\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)\left(-\frac{1}{x}+\frac{b\sqrt{1+\frac{bx^2}{a}}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}(a+bx^2)\sqrt{1+\frac{dx^2}{c}}}\right)}{c}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^2,x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2)*(-x^(-1) + (b*Sqrt[1 + (b*x
^2)/a]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt[-(b/a)]*(a +
b*x^2)*Sqrt[1 + (d*x^2)/c])))/c
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2058, 377, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{\sqrt{bx^2+a}}{x^2\sqrt{dx^2+c}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{377}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\int \frac{b\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\int\frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{\sqrt{a+bx^2}}$$

$$\downarrow 324$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left(c\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{\sqrt{a+bx^2}}$$

$$\downarrow 320$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left(d\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{\sqrt{a+bx^2}}$$

$$\downarrow 388$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{cx}\right)}{\sqrt{a+bx^2}}$$

$$\downarrow 313$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{\ldots}}{cx}\right.}{\sqrt{a+bx^2}}$$

input
```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^2,x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-((Sqrt[a + b*x^2]*Sqr
t[c + d*x^2])/(c*x)) + (b*(d*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (S
qrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a
*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) +
 (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)
/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
))/c))/Sqrt[a + b*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 4.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.81

method	result
default	$-\frac{\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)\left(\sqrt{-\frac{b}{a}}\,bd\,x^{4}-bc\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,x\,\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)+\sqrt{-\frac{b}{a}}\,ad\,x^{2}+\sqrt{-\frac{b}{a}}\,bc\,x^{2}+\sqrt{-\frac{b}{a}}\,ac\right)}{\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,cx\sqrt{-\frac{b}{a}}\,\sqrt{db\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}}$
risch	$-\frac{\left(d\,x^{2}+c\right)\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{cx}+\frac{b\left(\frac{c\sqrt{1+\frac{x^{2}b}{a}}\,\sqrt{1+\frac{x^{2}d}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}}-\frac{2dace\sqrt{1+\frac{x^{2}b}{a}}\,\sqrt{1+\frac{x^{2}d}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1\ldots\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\ldots}}\right.}{c\left(b\,x^{2}+a\right)\ldots}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(e*(b*x^2+a)/(d*x^2+c))^(1/2)*(d*x^2+c)*((-b/a)^(1/2)*b*d*x^4-b*c*((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2
))+(-b/a)^(1/2)*a*d*x^2+(-b/a)^(1/2)*b*c*x^2+(-b/a)^(1/2)*a*c)/((d*x^2+c)*
(b*x^2+a))^(1/2)/c/x/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.56

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \frac{bd\sqrt{\frac{ace}{d^2}}x\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - (a+b)d\sqrt{\frac{ace}{d^2}}x\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - (adx^2+ac)\sqrt{}}{acx}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
(b*d*sqrt(a*c*e/d^2)*x*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*
c)) - (a + b)*d*sqrt(a*c*e/d^2)*x*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a
)), a*d/(b*c)) - (a*d*x^2 + a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c*x
)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^2}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^2}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^2}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^2,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^2}\,dx = \sqrt{e}\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d\,x^4+c\,x^2}dx\right)$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^2,x)
```

output
```
sqrt(e)*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*x**2 + d*x**4),x)
```

3.53 $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 320

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = -\frac{(bc-2ad)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{3acx} - \frac{(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{3cx^3} - \frac{\sqrt{d}(bc-2ad)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} - \frac{b\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
-1/3*(-2*a*d+b*c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a/c/x-1/3*(d*x^2+
c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c/x^3-1/3*d^(1/2)*(-2*a*d+b*c)*(
b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2
/c)^(1/2),(1-b*c/a/d)^(1/2))/a/c^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/3
*b*d^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)*InverseJacobiAM(arctan(d
^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(
1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.71 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = -\frac{\sqrt{\frac{b}{a}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}(a+bx^2)(c+dx^2)(bcx^2+a(c-2dx^2))-ibc(-bc+2ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\right.}{3bc^2x^3(a+}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^4,x]
```

output
```
-1/3*(Sqrt[b/a]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*(
c + d*x^2)*(b*c*x^2 + a*(c - 2*d*x^2)) - I*b*c*(-(b*c) + 2*a*d)*x^3*Sqrt[1
 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/
(b*c)] + I*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*
EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(b*c^2*x^3*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2058, 377, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{\sqrt{bx^2+a}}{x^4\sqrt{dx^2+c}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 377}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\int \frac{-bdx^2+bc-2ad}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{-\frac{\int \frac{bd\left(ac-(bc-2ad)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{-\frac{bd\int \frac{ac-(bc-2ad)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{-\frac{bd\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3cx^3}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 320}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}}{3c}-\sqrt{a+\ldots}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 388}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2}{acx}}{3c}\right.}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 313$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}-\sqrt{a+}}{3c}\right.}{\sqrt{a+bx^2}}$$

input

```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^4,x]
```

output

```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-1/3*(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])/(c*x^3) + (-(((b*c - 2*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^
2])/(a*c*x)) - (b*d*(-((b*c - 2*a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^
2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 -
(b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*
x^2]))) + (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]],
1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c +
d*x^2])))/(a*c))/(3*c)))/Sqrt[a + b*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 6.93 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.18

method	result
risch	$-\frac{\left(d\,x^2+c\right)\left(-2ad\,x^2+bc\,x^2+ac\right)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{3c^2x^3a}-\frac{db\left(\frac{ac\sqrt{1+\frac{x^2b}{a}}\,\sqrt{1+\frac{x^2d}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}-\frac{2(2ad-bc)ace\sqrt{1+\frac{x^2b}{a}}}{}\right.}{}$
default	$-\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,\left(d\,x^2+c\right)\left(-2\sqrt{-\frac{b}{a}}\,ab\,d^2x^6+\sqrt{-\frac{b}{a}}\,b^2cd\,x^6-bd\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)x^3ac+\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2}{}}\right.}{}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(d*x^2+c)*(-2*a*d*x^2+b*c*x^2+a*c)/c^2/x^3/a*(e*(b*x^2+a)/(d*x^2+c))^
(1/2)-1/3/a*d*b/c^2*(a*c/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1
/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(
-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-2*(2*a*d-b*c)*a*c*e/(-b/a)^(1/2)*(1+1/a*x^2
*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/
(a*d*e+b*c*e+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/
e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))))*(e*(b
*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4} dx$$
$$= \frac{(b^2cd - 2\,abd^2)\sqrt{\frac{ace}{d^2}}x^3\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - (b^2cd - (a^2 + 2\,ab)d^2)\sqrt{\frac{ace}{d^2}}x^3\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{\right.}\right.}{3\,a^2c^2x^3}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="fricas")
```

output
```
1/3*((b^2*c*d - 2*a*b*d^2)*sqrt(a*c*e/d^2)*x^3*sqrt(-b/a)*elliptic_e(arcsi
n(x*sqrt(-b/a)), a*d/(b*c)) - (b^2*c*d - (a^2 + 2*a*b)*d^2)*sqrt(a*c*e/d^2
)*x^3*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((a*b*c*d -
 2*a^2*d^2)*x^4 + a^2*c^2 + (a*b*c^2 - a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/
(d*x^2 + c)))/(a^2*c^2*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4} dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^4}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^4, x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^4}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^4}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^4,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^4}\,dx$$
$$= \frac{\sqrt{e}\left(-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,b+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)b^2dx+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^8+ad\,x^6+bc\,x^6+ac\,x^4}dx\right)a^2cx\right)}{acx}$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^4,x)
```

output
```
(sqrt(e)*( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*b + int((sqrt(c + d*x**2)*s
qrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*d*x +
 int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**4 + a*d*x**6 + b*c*x**6 +
 b*d*x**8),x)*a**2*c*x))/(a*c*x)
```

3.54 $\int \frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 420

$$\begin{aligned}
\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx
&= \frac{(2b^2c^2+3abcd-8a^2d^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{15a^2c^2x} \\
&\quad - \frac{(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{5cx^5} - \frac{(bc-4ad)(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{15ac^2x^3} \\
&\quad + \frac{\sqrt{d}(2b^2c^2+3abcd-8a^2d^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{15a^2c^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} \\
&\quad - \frac{b\sqrt{d}(bc-4ad)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{15a^2c^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}
\end{aligned}$$

output
```
1/15*(-8*a^2*d^2+3*a*b*c*d+2*b^2*c^2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/
2)/a^2/c^2/x-1/5*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c/x^5-1/
15*(-4*a*d+b*c)*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a/c^2/x^3
+1/15*d^(1/2)*(-8*a^2*d^2+3*a*b*c*d+2*b^2*c^2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^
2+c))^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2
))/a^2/c^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/15*b*d^(1/2)*(-4*a*d+b*c)
*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c
^(1/2)),(1-b*c/a/d)^(1/2))/a^2/c^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.89 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx =$$
$$-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}(a+bx^2)\,(c+dx^2)\,(-2b^2c^2x^4+abcx^2(c-3dx^2)+a^2(3c^2-4cdx^2+8d^2x^4))+ibc(-2\right.}{}$$

input
```
Integrate[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^6,x]
```

output
```
-1/15*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2
)*(-2*b^2*c^2*x^4 + a*b*c*x^2*(c - 3*d*x^2) + a^2*(3*c^2 - 4*c*d*x^2 + 8*d
^2*x^4)) + I*b*c*(-2*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)
/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (
2*I)*b*c*(-(b^2*c^2) - a*b*c*d + 2*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1
 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(a^2*Sqrt[b
/a]*c^3*x^5*(a + b*x^2))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.03, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2058, 377, 445, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{\sqrt{bx^2+a}}{x^6\sqrt{dx^2+c}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{377}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\int \frac{-3bdx^2+bc-4ad}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{5cx^5}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{-\frac{\int \frac{2b^2c^2+3abdc-8a^2d^2+bd(bc-4ad)x^2}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-4ad)}{3acx^3}}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{5cx^5}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{-\frac{-\frac{\int -\frac{bd\left(\left(2b^2c^2+3abdc-8a^2d^2\right)x^2+ac(bc-4ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2c}{a}-\frac{8ad^2}{c}+3bd\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-4ad)}{3acx^3}}{5c}-\right.}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\Bigg(-\frac{-\frac{\int \frac{bd\left(\left(2b^2c^2+3abdc-8a^2d^2\right)x^2+ac(bc-4ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2c}{a}-\frac{8ad^2}{c}+3bd\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-4ad)}{3acx^3}}{5c} - \sqrt{\cdots}}{\sqrt{a+bx^2}}$$

$\downarrow$ 27

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\Bigg(-\frac{-\frac{bd\int \frac{\left(2b^2c^2+3abdc-8a^2d^2\right)x^2+ac(bc-4ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2c}{a}-\frac{8ad^2}{c}+3bd\right)}{x}}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-4ad)}{3acx^3}}{5c} - \sqrt{a\cdots}}{\sqrt{a+bx^2}}$$

$\downarrow$ 406

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\Bigg(-\frac{-\frac{bd\left(\left(-8a^2d^2+3abcd+2b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(bc-4ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{2b^2c}{a}-\frac{8ad^2}{c}+3\cdots\right.}{x}}{3ac}\cdots}{5c}}{\sqrt{a+bx^2}}$$

$\downarrow$ 320

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\Bigg(-\frac{-\frac{bd\left(\left(-8a^2d^2+3abcd+2b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(bc-4ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c\cdots}}{\cdots}}{3ac}\cdots}{5c}}{\sqrt{a+bx^2}}$$

$\downarrow$ 388

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{\frac{-\frac{bd\left(\left(-8a^2d^2+3abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}}{3ac}}{5c}\right.}{\sqrt{a+bx^2}}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{\frac{-\frac{bd\left(\left(-8a^2d^2+3abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-4ad)\,\text{EllipticF}\left(\arctan\left(\ldots\right.\right.}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\ldots}{ac}}{3ac}}{5c}\right.}{\sqrt{a+bx^2}}$$

input
```
Int[Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/x^6,x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-1/5*(Sqrt[a + b*x^2]*
Sqrt[c + d*x^2])/(c*x^5) + (-1/3*((b*c - 4*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d
*x^2])/(a*c*x^3) - (-((((2*b^2*c)/a + 3*b*d - (8*a*d^2)/c)*Sqrt[a + b*x^2]
*Sqrt[c + d*x^2])/x) + (b*d*((2*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*((x*Sqrt[
a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b*c - 4*a*d)*Sqrt[a + b*x^2
]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(
c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/(3*a*c))/(5*c)))
/Sqrt[a + b*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 8.54 (sec) , antiderivative size = 539, normalized size of antiderivative = 1.28

method	result
risch	$-\frac{(d\,x^2+c)\left(8a^2d^2x^4-3abcd\,x^4-2b^2c^2x^4-4a^2cd\,x^2+ab\,c^2x^2+3a^2c^2\right)\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{15c^3x^5a^2}+\frac{db\left(-\frac{2\left(8a^2d^2-3abcd-2b^2c^2\right)ace\sqrt{1+\frac{x^2b}{a}}\,\sqrt{1 \cdots}}{\sqrt{-\frac{b}{a}}\,\cdots}\cdots\right.}{\cdots}$
default	$-\frac{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)\left(8\sqrt{-\frac{b}{a}}\,a^2b\,d^3x^8-3\sqrt{-\frac{b}{a}}\,a\,b^2c\,d^2x^8-2\sqrt{-\frac{b}{a}}\,b^3c^2d\,x^8+4\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)a^{\cdots}\cdots\right.}{\cdots}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(d*x^2+c)*(8*a^2*d^2*x^4-3*a*b*c*d*x^4-2*b^2*c^2*x^4-4*a^2*c*d*x^2+a
*b*c^2*x^2+3*a^2*c^2)/c^3/x^5/a^2*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/15/a^2*d
*b/c^3*(-2*(8*a^2*d^2-3*a*b*c*d-2*b^2*c^2)*a*c*e/(-b/a)^(1/2)*(1+1/a*x^2*b
)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(a
*d*e+b*c*e+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)
^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))-a*b*c^2/
(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+
b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(
1/2))+4*a^2*c*d/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*
e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e
+b*c*e)/c/b/e)^(1/2)))*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*
e)^(1/2)/(b*x^2+a)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = -\frac{(2\,b^3c^2d+3\,ab^2cd^2-8\,a^2bd^3)\sqrt{\frac{ace}{d^2}}x^5\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(2\,b^3c^2d+(a^2b+3\,ab^2)cd^2-4}{}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-1/15*((2*b^3*c^2*d + 3*a*b^2*c*d^2 - 8*a^2*b*d^3)*sqrt(a*c*e/d^2)*x^5*sqr
t(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (2*b^3*c^2*d + (a^2*
b + 3*a*b^2)*c*d^2 - 4*(a^3 + 2*a^2*b)*d^3)*sqrt(a*c*e/d^2)*x^5*sqrt(-b/a)
*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((2*a*b^2*c^2*d + 3*a^2*b*c
*d^2 - 8*a^3*d^3)*x^6 - 3*a^3*c^3 + 2*(a*b^2*c^3 + a^2*b*c^2*d - 2*a^3*c*d
^2)*x^4 - (a^2*b*c^3 - a^3*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/
(a^3*c^3*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(1/2)/x**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^6}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^6, x)
```

Giac [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \int \frac{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}{x^6}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x, algorithm="giac")
```

output
```
integrate(sqrt((b*x^2 + a)*e/(d*x^2 + c))/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \int \frac{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}{x^6}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^6,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(1/2)/x^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{x^6}\,dx = \sqrt{e}\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{d\,x^8+c\,x^6}dx\right)$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(1/2)/x^6,x)
```

output
```
sqrt(e)*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(c*x**6 + d*x**8),x)
```

3.55 $\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 338

$$\int x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \frac{c^2(bc-ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{d^4} + \frac{(29b^2c^2-22abcd+a^2d^2)\,e(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{16bd^4} - \frac{(19bc-7ad)e(c+dx^2)^2\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{24d^4} + \frac{be(c+dx^2)^3\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{6d^4} - \frac{(bc-ad)\,(35b^2c^2-10abcd-a^2d^2)\,e^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{3/2}d^{9/2}}$$

output
```
c^2*(-a*d+b*c)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d^4+1/16*(a^2*d^2-
22*a*b*c*d+29*b^2*c^2)*e*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/
b/d^4-1/24*(-7*a*d+19*b*c)*e*(d*x^2+c)^2*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^
(1/2)/d^4+1/6*b*e*(d*x^2+c)^3*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d^4-1
/16*(-a*d+b*c)*(-a^2*d^2-10*a*b*c*d+35*b^2*c^2)*e^(3/2)*arctanh(d^(1/2)*(b
*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/b^(3/2)/d^(9/2)
```

Mathematica [A] (verified)

Time = 4.70 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.66

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(b\sqrt{d}(3a^2d^2(c+dx^2)+2abd(-50c^2-19cdx^2+7d^2x^4)+b^2(105c\right.}{}$$

input
```
Integrate[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(b*Sqrt[d]*(3*a^2*d^2*(c + d*x^2) + 2
*a*b*d*(-50*c^2 - 19*c*d*x^2 + 7*d^2*x^4) + b^2*(105*c^3 + 35*c^2*d*x^2 -
14*c*d^2*x^4 + 8*d^3*x^6)) - (3*(b*c - a*d)^(3/2)*(35*b^2*c^2 - 10*a*b*c*d
 - a^2*d^2)*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*ArcSinh[(Sqrt[d]*Sqrt[a + b*
x^2])/Sqrt[b*c - a*d]])/Sqrt[a + b*x^2]))/(48*b^2*d^(9/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.89, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2053, 2052, 366, 25, 25, 27, 360, 25, 1471, 27, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int x^4\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int \frac{x^8(ae-cx^4)^2}{(be-dx^4)^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$\Big\downarrow$ 366

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\int -\frac{x^8\left(-6bc^2dex^4+6a^2d^2e^2-5(bce-ade)^2\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{6bd^2e}\right)$$

$\Big\downarrow$ 25

$$e(bc-ad)\left(\frac{\int -\frac{ex^8\left(6bc^2dx^4+\left(5b^2c^2-10abdc-a^2d^2\right)e\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{6bd^2e}+\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}\right)$$

$\Big\downarrow$ 25

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\int \frac{ex^8\left(6bc^2dx^4+\left(5b^2c^2-10abdc-a^2d^2\right)e\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{6bd^2e}\right)$$

$\Big\downarrow$ 27

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\int \frac{x^8\left(6bc^2dx^4+\left(5b^2c^2-10abdc-a^2d^2\right)e\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{6bd^2}\right)$$

$\Big\downarrow$ 360

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{\int -\frac{24bc^2d^3x^8+4d^2(bc-ad)(11bc+ad)ex^4+bd(bc-ad)(11bc+ad)e^2}{\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4d^3}+\frac{be^2(bc-ad)(ad+11bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2\left(be-dx^4\right)^2}}{6bd^2}\right.$$

$\Big\downarrow$ 25

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{be^2(bc-ad)(ad+11bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2\left(be-dx^4\right)^2}-\frac{\int \frac{24bc^2d^3x^8+4d^2(bc-ad)(11bc+ad)ex^4+bd(bc-ad)(11bc+ad)e^2}{\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4d^3}}{6bd^2}\right.$$

$\Big\downarrow$ 1471

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{be^2(bc-ad)(ad+11bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2(be-dx^4)^2}-\frac{\frac{de\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2\left(be-dx^4\right)}-\frac{\int\frac{3bde\left(16bc^2dx^4+\left(19b^2c^2-10abd\right.\right.}{be-dx^4}}{2be}}{4d^3}}{6bd^2}\right.$$

$$\Big\downarrow \; 27$$

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{be^2(bc-ad)(ad+11bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2(be-dx^4)^2}-\frac{\frac{de\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2\left(be-dx^4\right)}-\frac{3}{2}d\int\frac{16bc^2dx^4+\left(19b^2c^2-10abdc-\right.}{be-dx^4}}{4d^3}}{6bd^2}\right.$$

$$\Big\downarrow \; 299$$

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{be^2(bc-ad)(ad+11bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2(be-dx^4)^2}-\frac{\frac{de\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2\left(be-dx^4\right)}-\frac{3}{2}d\left(e\left(-a^2d^2-10abcd+35b^2c^2\right)\right.}{4d^3}}{6bd^2}\right.$$

$$\Big\downarrow \; 221$$

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6bd^2\left(be-dx^4\right)^3}-\frac{\frac{be^2(bc-ad)(ad+11bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4d^2(be-dx^4)^2}-\frac{\frac{de\left(-5a^2d^2-50abcd+79b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2\left(be-dx^4\right)}-\frac{3}{2}d\left(\frac{\sqrt{e}\left(-a^2d^2-10abcd+35b^2c^2\right)\cdots}{\sqrt{\cdots}}\right.}{4d^3}}{6bd^2}\right.$$

input
```
Int[x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(b*c - a*d)*e*(((b*c - a*d)^2*e*x^10)/(6*b*d^2*(b*e - d*x^4)^3) - ((b*(b*c
 - a*d)*(11*b*c + a*d)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*d^2*(b*e
- d*x^4)^2) - ((d*(79*b^2*c^2 - 50*a*b*c*d - 5*a^2*d^2)*e*Sqrt[(e*(a + b*x
^2))/(c + d*x^2)])/(2*(b*e - d*x^4)) - (3*d*(-16*b*c^2*Sqrt[(e*(a + b*x^2)
)/(c + d*x^2)] + ((35*b^2*c^2 - 10*a*b*c*d - a^2*d^2)*Sqrt[e]*ArcTanh[(Sqr
t[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[b]*Sqrt[
d])))/2)/(4*d^3))/(6*b*d^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.93

method	result
risch	$\frac{(8b^2d^2x^4+14ab\,d^2x^2-22b^2c\,x^2d+3a^2d^2-52abcd+57b^2c^2)(d\,x^2+c)e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{48b\,d^4} - \left(\frac{\left(a^2d^2+10abcd-35b^2c^2\right)(ad-bc)\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}}{\sqrt{}}\right.}{2\sqrt{bd}}\right.$
default	Expression too large to display

input
```
int(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/b*(8*b^2*d^2*x^4+14*a*b*d^2*x^2-22*b^2*c*d*x^2+3*a^2*d^2-52*a*b*c*d+5
7*b^2*c^2)*(d*x^2+c)/d^4*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/16/b/d^4*(1/2*(
a^2*d^2+10*a*b*c*d-35*b^2*c^2)*(a*d-b*c)*ln((1/2*a*d*e+1/2*b*c*e+b*d*x^2*e
)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)+1
6*c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*b*(b*x^2+a)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x
^2+b*c*e*x^2+a*c*e)^(1/2))*e/(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x
^2+c)*(b*x^2+a)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 553, normalized size of antiderivative = 1.64

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx = \left[\frac{3\,(35\,b^3c^3-45\,ab^2c^2d+9\,a^2bcd^2+a^3d^3)e\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd\right.}{}\right.$$

input
```
integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(35*b^3*c^3 - 45*a*b^2*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*sqrt(e
/(b*d))*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a
*b*c*d + a^2*d^2)*e - 4*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*
d^2 + a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) + 4*(
8*b^2*d^3*e*x^6 - 14*(b^2*c*d^2 - a*b*d^3)*e*x^4 + (35*b^2*c^2*d - 38*a*b*
c*d^2 + 3*a^2*d^3)*e*x^2 + (105*b^2*c^3 - 100*a*b*c^2*d + 3*a^2*c*d^2)*e)*
sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4), 1/96*(3*(35*b^3*c^3 - 45*a*b^2
*c^2*d + 9*a^2*b*c*d^2 + a^3*d^3)*e*sqrt(-e/(b*d))*arctan(1/2*(2*b*d*x^2 +
 b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(b*d))/(b*e*x^2 + a*
e)) + 2*(8*b^2*d^3*e*x^6 - 14*(b^2*c*d^2 - a*b*d^3)*e*x^4 + (35*b^2*c^2*d
- 38*a*b*c*d^2 + 3*a^2*d^3)*e*x^2 + (105*b^2*c^3 - 100*a*b*c^2*d + 3*a^2*c
*d^2)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4)]
```

Sympy [F(-1)]

Timed out.

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x**5*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[0,5,0]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-4,[0,4,0]%%%}
,0]:[1,0,
```

Mupad [F(-1)]

Timed out.

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \int x^5\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}dx$$

input
```
int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 608, normalized size of antiderivative = 1.80

$$\int x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\frac{\sqrt{e}\,e\Big(3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2bc\,d^3+3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2b\,d^4x^2-100\sqrt{d\,}}{}$$

input
```
int(x^5*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(e)*e*(3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c*d**3 + 3*sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*a**2*b*d**4*x**2 - 100*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*a*b**2*c**2*d**2 - 38*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c*
d**3*x**2 + 14*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*d**4*x**4 + 105*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**3*d + 35*sqrt(c + d*x**2)*sqrt(a +
 b*x**2)*b**3*c**2*d**2*x**2 - 14*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c
*d**3*x**4 + 8*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*d**4*x**6 + 3*sqrt(d
)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b)*
a**3*c*d**3 + 3*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d
)*sqrt(c + d*x**2)*b)*a**3*d**4*x**2 + 27*sqrt(d)*sqrt(b)*log( - sqrt(b)*s
qrt(a + b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b)*a**2*b*c**2*d**2 + 27*sqrt
(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b
)*a**2*b*c*d**3*x**2 - 135*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)
*d + sqrt(d)*sqrt(c + d*x**2)*b)*a*b**2*c**3*d - 135*sqrt(d)*sqrt(b)*log(
- sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b)*a*b**2*c**2*d**
2*x**2 + 105*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d)*s
qrt(c + d*x**2)*b)*b**3*c**4 + 105*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a +
 b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b)*b**3*c**3*d*x**2))/(48*b**2*d**5*
(c + d*x**2))
```

3.56 $\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 247

$$\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = -\frac{c(bc-ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{d^3} - \frac{(9bc-5ad)e(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{8d^3} + \frac{be(c+dx^2)^2\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{4d^3} + \frac{3(bc-ad)(5bc-ad)e^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{b}\sqrt{e}}\right)}{8\sqrt{b}d^{7/2}}$$

output
```
-c*(-a*d+b*c)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d^3-1/8*(-5*a*d+9*b
*c)*e*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d^3+1/4*b*e*(d*x^2+
c)^2*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d^3+3/8*(-a*d+b*c)*(-a*d+5*b*c
)*e^(3/2)*arctanh(d^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^(1/2)/e
^(1/2))/b^(1/2)/d^(7/2)
```

Mathematica [A] (verified)

Time = 2.96 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.72

$$\int x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx=\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{b}\sqrt{d}\sqrt{a+bx^2}(ad(13c+5dx^2)+b(-15c^2-5cdx^2+2d^2x^4))+3\right.}{8\sqrt{b}d^{7/2}\sqrt{a+bx^2}}$$

input
```
Integrate[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b]*Sqrt[d]*Sqrt[a + b*x^2]*(a*d
*(13*c + 5*d*x^2) + b*(-15*c^2 - 5*c*d*x^2 + 2*d^2*x^4)) + 3*(5*b^2*c^2 -
6*a*b*c*d + a^2*d^2)*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[b]*Sqrt[c + d*x^2])/(Sq
rt[d]*Sqrt[a + b*x^2])]))/(8*Sqrt[b]*d^(7/2)*Sqrt[a + b*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2053, 2052, 25, 360, 1471, 27, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int x^2\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int -\frac{x^8(ae-cx^4)}{(be-dx^4)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$-\left(e(bc-ad)\int \frac{x^8\left(ae-cx^4\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$$\Big\downarrow \; 360$$

$$e(bc-ad)\left(\frac{be^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3\left(be-dx^4\right)^2}-\frac{\int \frac{4cd^2x^8+4d(bc-ad)ex^4+b(bc-ad)e^2}{(be-dx^4)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4d^3}\right)$$

$$\Big\downarrow \; 1471$$

$$e(bc-ad)\left(\frac{be^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3\left(be-dx^4\right)^2}-\frac{\frac{e(9bc-5ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2(be-dx^4)}-\frac{\int \frac{be\left(8cdx^4+(7bc-3ad)e\right)}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2be}}{4d^3}\right)$$

$$\Big\downarrow \; 27$$

$$e(bc-ad)\left(\frac{be^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3\left(be-dx^4\right)^2}-\frac{\frac{e(9bc-5ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2(be-dx^4)}-\frac{1}{2}\int \frac{8cdx^4+(7bc-3ad)e}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4d^3}\right)$$

$$\Big\downarrow \; 299$$

$$e(bc-ad)\left(\frac{be^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3\left(be-dx^4\right)^2}-\frac{\frac{1}{2}\left(8c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}-3e(5bc-ad)\int \frac{1}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\right)+\frac{e(9bc-5ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2(be-dx^4)}}{4d^3}\right)$$

$$\Big\downarrow \; 221$$

$$e(bc-ad)\left(\frac{be^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4d^3\left(be-dx^4\right)^2}-\frac{\frac{1}{2}\left(8c\sqrt{\frac{e(a+bx^2)}{c+dx^2}}-\frac{3\sqrt{e}(5bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{d}}\right)+\frac{e(9bc-5ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2(be-dx^4)}}{4d^3}\right)$$

input
```
Int[x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(b*c - a*d)*e*((b*(b*c - a*d)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*d^
3*(b*e - d*x^4)^2) - (((9*b*c - 5*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]
)/(2*(b*e - d*x^4)) + (8*c*Sqrt[(e*(a + b*x^2))/(c + d*x^2)] - (3*(5*b*c -
 a*d)*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]
*Sqrt[e])])/(Sqrt[b]*Sqrt[d]))/2)/(4*d^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.02

method	result
risch	$\frac{(2bd\,x^2+5ad-7bc)\,(d\,x^2+c)\,e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{8d^3}+\frac{\left(\frac{3(ad-5bc)(ad-bc)\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+bd\,x^2e}{\sqrt{bde}}+\sqrt{bde\,x^4+(ade+bce)x^2+ace}\right)}{2\sqrt{bde}}+\frac{8c\left(a^2d^2-\right.}{(ad-bc)\sqrt{bde}}\right.}{8d^3(b\,x^2+a)}$
default	$\frac{\left(4\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,b\,d^2x^4+3\ln\left(\frac{2bd\,x^2+2\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^2d^3x^2-18\ln\left(\frac{2bd\,x^2+2\sqrt{db\,x^4+ad\,x^2+b}}{2\sqrt{bd}}\right.\right.}{}$

input
```
int(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*b*d*x^2+5*a*d-7*b*c)*(d*x^2+c)/d^3*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+
1/8/d^3*(3/2*(a*d-5*b*c)*(a*d-b*c)*ln((1/2*a*d*e+1/2*b*c*e+b*d*x^2*e)/(b*d
*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)+8*c*(a^
2*d^2-2*a*b*c*d+b^2*c^2)*(b*x^2+a)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^
2+a*c*e)^(1/2))*e/(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^
2+a)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.69

$$\int x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\left[\frac{3\,(5\,b^2c^2-6\,abcd+a^2d^2)e\sqrt{\frac{e}{bd}}\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+\right.}{}\right.$$

$$-\frac{3\,(5\,b^2c^2-6\,abcd+a^2d^2)e\sqrt{-\frac{e}{bd}}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{bd}}}{2\,(bex^2+ae)}\right)-2\,(2\,bd^2ex^4-5\,(bcd-ad^2)ex^2-}{16\,d^3}$$

input
```
integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*sqrt(e/(b*d))*log(8*b^2*d^2*e
*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4
*(2*b^2*d^3*x^4 + b^2*c^2*d + a*b*c*d^2 + (3*b^2*c*d^2 + a*b*d^3)*x^2)*sqr
t((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(b*d))) + 4*(2*b*d^2*e*x^4 - 5*(b*c*
d - a*d^2)*e*x^2 - (15*b*c^2 - 13*a*c*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c)))/d^3, -1/16*(3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*e*sqrt(-e/(b*d))*arct
an(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-e/(
b*d))/(b*e*x^2 + a*e)) - 2*(2*b*d^2*e*x^4 - 5*(b*c*d - a*d^2)*e*x^2 - (15*
b*c^2 - 13*a*c*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d^3]
```

Sympy [F(-1)]

Timed out.

$$\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(x**3*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[0,4,0]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-4,[0,3,0]%%%}
,0]:[1,0,
```

Mupad [F(-1)]

Timed out.

$$\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int x^3 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2} dx$$

input
```
int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.63

$$\int x^3 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \frac{\sqrt{e}\, e\Big(13\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,abc\,d^2 + 5\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ab\,d^3x^2 - 15\sqrt{d\,x^2}}{}$$

input
```
int(x^3*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(e)*e*(13*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*d**2 + 5*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*a*b*d**3*x**2 - 15*sqrt(c + d*x**2)*sqrt(a + b*x**
2)*b**2*c**2*d - 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*d**2*x**2 + 2*
sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d**3*x**4 + 3*sqrt(d)*sqrt(b)*log(
- sqrt(b)*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c + d*x**2)*b)*a**2*c*d**2 + 3
*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c + d*x*
*2)*b)*a**2*d**3*x**2 - 18*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)
*d - sqrt(d)*sqrt(c + d*x**2)*b)*a*b*c**2*d - 18*sqrt(d)*sqrt(b)*log( - sq
rt(b)*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c + d*x**2)*b)*a*b*c*d**2*x**2 + 1
5*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c + d*x
**2)*b)*b**2*c**3 + 15*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d -
 sqrt(d)*sqrt(c + d*x**2)*b)*b**2*c**2*d*x**2))/(8*b*d**4*(c + d*x**2))
```

3.57 $\int x \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 24, antiderivative size = 176

$$\int x \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \frac{(bc-ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{d^2} + \frac{be(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{2d^2} - \frac{3\sqrt{b}(bc-ad)e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{b}\sqrt{e}}\right)}{2d^{5/2}}$$

output

```
(-a*d+b*c)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d^2+1/2*b*e*(d*x^2+c)*
(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d^2-3/2*b^(1/2)*(-a*d+b*c)*e^(3/2)*
arctanh(d^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/d^
(5/2)
```

Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.77

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{d}\sqrt{a+bx^2}(3bc-2ad+bdx^2)-3\sqrt{b}(bc-ad)\sqrt{c+dx^2}\text{arctanh}\left(\right.\right.}{2d^{5/2}\sqrt{a+bx^2}}$$

input
```
Integrate[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[d]*Sqrt[a + b*x^2]*(3*b*c - 2*a
*d + b*d*x^2) - 3*Sqrt[b]*(b*c - a*d)*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[d]*Sqr
t[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])]))/(2*d^(5/2)*Sqrt[a + b*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2053, 2051, 252, 262, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}dx^2$$

$$\Big\downarrow \text{ 2051}$$

$$e(bc-ad)\int\frac{x^8}{(be-dx^4)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{ 252}$$

$$e(bc-ad)\left(\frac{x^6}{2d\left(be-dx^4\right)}-\frac{3\int \frac{x^4}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2d}\right)$$

$\Big\downarrow$ 262

$$e(bc-ad)\left(\frac{x^6}{2d\left(be-dx^4\right)}-\frac{3\left(\frac{be\int \frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{d}-\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{d}\right)}{2d}\right)$$

$\Big\downarrow$ 221

$$e(bc-ad)\left(\frac{x^6}{2d\left(be-dx^4\right)}-\frac{3\left(\frac{\sqrt{b}\sqrt{e}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}-\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{d}\right)}{2d}\right)$$

input

```
Int[x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output

```
(b*c - a*d)*e*(x^6/(2*d*(b*e - d*x^4)) - (3*(-(Sqrt[(e*(a + b*x^2))/(c + d
*x^2)]/d) + (Sqrt[b]*Sqrt[e]*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)])/(Sqrt[b]*Sqrt[e])])/d^(3/2)))/(2*d))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.31

method	result
risch	$\frac{(d\,x^2+c)\,b e \sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{2 d^2}+\frac{\left(\frac{3(ad-bc)b\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+bd\,x^2e}{\sqrt{bde}}+\sqrt{bde\,x^4+(ade+bce)x^2+ace}\right)}{2\sqrt{bde}}-\frac{\left(2a^2d^2-4abcd+2b^2c^2\right)\left(b\,x^2+a\right)}{(ad-bc)\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}\right)e}{2d^2(b\,x^2+a)}$
default	$-\frac{\left(-3\ln\left(\frac{2bd\,x^2+2\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)ab\,d^2x^2+3\ln\left(\frac{2bd\,x^2+2\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)b^2cd\,x^2-2\sqrt{bd}\right.}{}$

input
```
int(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/d^2*(d*x^2+c)*b*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/2/d^2*(3/2*(a*d-b*c)
*b*ln((1/2*a*d*e+1/2*b*c*e+b*d*x^2*e)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*
e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)-(2*a^2*d^2-4*a*b*c*d+2*b^2*c^2)*(b*x^2+
a)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))*e/(b*x^2+a)*(e*(
b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.86

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\left[-\frac{3\,(bc-ad)\sqrt{\frac{be}{d}}e\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d\right.}{}\right.$$

input
```
integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*(b*c - a*d)*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*
d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d
+ a*c*d^2 + (3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x
^2 + c))) - 4*(b*d*e*x^2 + (3*b*c - 2*a*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2
+ c)))/d^2, 1/4*(3*(b*c - a*d)*sqrt(-b*e/d)*e*arctan(1/2*(2*b*d*x^2 + b*c
+ a*d)*sqrt(-b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*e*x^2 + a*b*e))
 + 2*(b*d*e*x^2 + (3*b*c - 2*a*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/d^
2]
```

Sympy [F(-1)]

Timed out.

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\text{Timed out}$$

input
```
integrate(x*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[0,3,0]%%%},[2,0,0,0]%%%}+%%%{%%{[%%%{-4,[0,2,0]%%%}
,0]:[1,0,
```

Mupad [F(-1)]

Timed out.

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \int x\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}dx$$

input
```
int(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.37

$$\int x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \frac{\sqrt{e}\,e\left(-2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a\,d^2+3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bcd+\sqrt{d\,x^2+c}\,\sqrt{b\,x\right.}{}$$

input
```
int(x*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(e)*e*( - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d**2 + 3*sqrt(c + d*x
**2)*sqrt(a + b*x**2)*b*c*d + sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d**2*x**
2 + 3*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c +
 d*x**2)*b)*a*c*d + 3*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d -
sqrt(d)*sqrt(c + d*x**2)*b)*a*d**2*x**2 - 3*sqrt(d)*sqrt(b)*log( - sqrt(b)
*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c + d*x**2)*b)*b*c**2 - 3*sqrt(d)*sqrt(
b)*log( - sqrt(b)*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c + d*x**2)*b)*b*c*d*x
**2))/(2*d**3*(c + d*x**2))
```

3.58 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 187

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = -\frac{(bc-ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{cd} - \frac{a^{3/2}e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}} + \frac{b^{3/2}e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{b}\sqrt{e}}\right)}{d^{3/2}}$$

output
```
-(-a*d+b*c)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c/d-a^(3/2)*e^(3/2)*a
rctanh(c^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))/c^(
3/2)+b^(3/2)*e^(3/2)*arctanh(d^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2
)/b^(1/2)/e^(1/2))/d^(3/2)
```

Mathematica [A] (verified)

Time = 2.39 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.00

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-a^{3/2}d^{3/2}\sqrt{c+dx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{c}\left(\sqrt{d}(-bc+ad)\sqrt{a+bx^2}\right.\right.}{c^{3/2}d^{3/2}\sqrt{a+bx^2}}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(a^(3/2)*d^(3/2)*Sqrt[c + d*x^2]*Ar
cTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]) + Sqrt[c]*(Sqr
t[d]*(-(b*c) + a*d)*Sqrt[a + b*x^2] + b^(3/2)*c*Sqrt[c + d*x^2]*ArcTanh[(S
qrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])))/(c^(3/2)*d^(3/2)*Sqr
t[a + b*x^2])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2053, 2052, 25, 381, 27, 397, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}{x^2}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$e(bc-ad)\int -\frac{x^8}{(ae-cx^4)(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{ 25}$$

$$-\left(e(bc-ad)\int \frac{x^8}{(ae-cx^4)\,(be-dx^4)}d\sqrt{\frac{e\,(bx^2+a)}{dx^2+c}}\right)$$

$\Big\downarrow$ 381

$$e(bc-ad)\left(\frac{\int \frac{e\left(abe-(bc+ad)x^4\right)}{(ae-cx^4)(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{cd}-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}\right)$$

$\Big\downarrow$ 27

$$e(bc-ad)\left(\frac{e\int \frac{abe-(bc+ad)x^4}{(ae-cx^4)(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{cd}-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}\right)$$

$\Big\downarrow$ 397

$$e(bc-ad)\left(\frac{e\left(\frac{b^2c\int \frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{bc-ad}-\frac{a^2d\int \frac{1}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{bc-ad}\right)}{cd}-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}\right)$$

$\Big\downarrow$ 221

$$e(bc-ad)\left(\frac{e\left(\frac{b^{3/2}c\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{d}\sqrt{e}(bc-ad)}-\frac{a^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{c}\sqrt{e}(bc-ad)}\right)}{cd}-\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{cd}\right)$$

input
```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x]
```

output
```
(b*c - a*d)*e*(-(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(c*d)) + (e*(-((a^(3/2)
*d*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])
/(Sqrt[c]*(b*c - a*d)*Sqrt[e])) + (b^(3/2)*c*ArcTanh[(Sqrt[d]*Sqrt[(e*(a +
 b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[d]*(b*c - a*d)*Sqrt[e])))
/(c*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(157) = 314$.

Time = 0.14 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.14

method	result
default	$\frac{\left(\ln\left(\frac{2bd\,x^2+2\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\sqrt{ac}\,b^2cd\,x^2-\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}+2ac}{x^2}\right)\sqrt{bd}\,a^2d^2x^2+\ln}{}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2*(ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a
*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*b^2*c*d*x^2-ln((a*d*x^2+b*c*x^2+2*(a*c)^(
1/2)*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*(b*d)^(1/2)*a^2*d^2*x
^2+ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d
+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*b^2*c^2-ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*(b*d)^(1/2)*a^2*c*d+2*(a*c)
^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*a*d-2*(a*c)^(1/2)*((d*x^2+c
)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*b*c)/c/d*(d*x^2+c)*(e*(b*x^2+a)/(d*x^2+c))^
(3/2)/(a*c)^(1/2)/(b*d)^(1/2)/(b*x^2+a)/((d*x^2+c)*(b*x^2+a))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 1049, normalized size of antiderivative = 5.61

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="fricas")
```

output
```
[1/4*(b*c*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2
+ (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d + a*c*d^2 +
 (3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) +
 a*d*sqrt(a*e/c)*e*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*
e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (
b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4
) - 4*(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d), -1/4*(2*b*c*
sqrt(-b*e/d)*e*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*e/d)*sqrt((b*e*x
^2 + a*e)/(d*x^2 + c))/(b^2*e*x^2 + a*b*e)) - a*d*sqrt(a*e/c)*e*log(((b^2*
c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x
^2 - 4*((b*c^2*d + a*c*d^2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(
a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) + 4*(b*c - a*d)*e*sqrt((b*e
*x^2 + a*e)/(d*x^2 + c)))/(c*d), 1/4*(2*a*d*sqrt(-a*e/c)*e*arctan(1/2*((b*
c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*
e*x^2 + a^2*e)) + b*c*sqrt(b*e/d)*e*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b
*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^3*x^4 + b*c^2*d
 + a*c*d^2 + (3*b*c*d^2 + a*d^3)*x^2)*sqrt(b*e/d)*sqrt((b*e*x^2 + a*e)/(d*
x^2 + c))) - 4*(b*c - a*d)*e*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d), 1/2
*(a*d*sqrt(-a*e/c)*e*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqr
t((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*e*x^2 + a^2*e)) - b*c*sqrt(-b*e/d)*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x}\,dx = \int \frac{\left(\frac{e(b\,x^2+a)}{d\,x^2+c}\right)^{\frac{3}{2}}}{x}dx$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x)
```

output
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x,x)
```

3.59 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 179

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \frac{(bc-ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{c^2} - \frac{ae(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{2c^2x^2} - \frac{3\sqrt{a}(bc-ad)e^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{a}\sqrt{e}}\right)}{2c^{5/2}}$$

output

```
(-a*d+b*c)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c^2-1/2*a*e*(d*x^2+c)*
(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c^2/x^2-3/2*a^(1/2)*(-a*d+b*c)*e^(3
/2)*arctanh(c^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2)
)/c^(5/2)
```

Mathematica [A] (verified)

Time = 3.97 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.82

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{c}\sqrt{a+bx^2}(2bcx^2-a(c+3dx^2))-3\sqrt{a}(bc-ad)x^2\sqrt{c+dx^2}\text{arctanh}\left(\right.\right.}{2c^{5/2}x^2\sqrt{a+bx^2}}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[c]*Sqrt[a + b*x^2]*(2*b*c*x^2 -
 a*(c + 3*d*x^2)) - 3*Sqrt[a]*(b*c - a*d)*x^2*Sqrt[c + d*x^2]*ArcTanh[(Sqr
t[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(2*c^(5/2)*x^2*Sqrt[a +
 b*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.69, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {2053, 2052, 252, 262, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}{x^4}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int \frac{x^8}{(ae-cx^4)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{252}$$

$$e(bc-ad)\left(\frac{x^6}{2c\left(ae-cx^4\right)}-\frac{3\int \frac{x^4}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2c}\right)$$

$\Big\downarrow$ 262

$$e(bc-ad)\left(\frac{x^6}{2c\left(ae-cx^4\right)}-\frac{3\left(\frac{ae\int \frac{1}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{c}-\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{c}\right)}{2c}\right)$$

$\Big\downarrow$ 221

$$e(bc-ad)\left(\frac{x^6}{2c\left(ae-cx^4\right)}-\frac{3\left(\frac{\sqrt{a}\sqrt{e}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{c^{3/2}}-\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{c}\right)}{2c}\right)$$

input
```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x]
```

output
```
(b*c - a*d)*e*(x^6/(2*c*(a*e - c*x^4)) - (3*(-(Sqrt[(e*(a + b*x^2))/(c + d
*x^2)]/c) + (Sqrt[a]*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*
x^2)])/(Sqrt[a]*Sqrt[e])])/c^(3/2)))/(2*c))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.34

method	result
risch	$-\frac{a(d x^2+c)e\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}{2c^2x^2}-\frac{\left(-\frac{(-2a^2d^2+4abcd-2b^2c^2)(b x^2+a)}{(ad-bc)\sqrt{bde x^4+ade x^2+bce x^2+ace}}-\frac{3a(ad-bc)\ln\left(\frac{2ace+(ade+bce)x^2+2\sqrt{ace}\sqrt{bde x^4+(ade+bce)x^2}}{x^2}\right.}{2\sqrt{ace}}\right.}{2c^2(b x^2+a)}$
default	$-\frac{\left(-2\sqrt{db x^4+ad x^2+bc x^2+ac}\sqrt{ac}\,b d^2x^6-3\ln\left(\frac{ad x^2+bc x^2+2\sqrt{ac}\sqrt{db x^4+ad x^2+bc x^2+ac}+2ac}{x^2}\right)a^2c d^2x^4+3\ln\left(\frac{ad x^2+bc x^2+2\sqrt{a}}{}\right.\right.}{}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/c^2*(d*x^2+c)/x^2*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/2/c^2*(-(-2*a^2
*d^2+4*a*b*c*d-2*b^2*c^2)*(b*x^2+a)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x
^2+a*c*e)^(1/2)-3/2*a*(a*d-b*c)/(a*c*e)^(1/2)*ln((2*a*c*e+(a*d*e+b*c*e)*x^
2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/x^2))*e/(b*x^
2+a)*(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.65 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.96

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \left[-\frac{3\,(bc-ad)\sqrt{\frac{ae}{c}}ex^2\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2+4\,((bc^2d+acd^2)x^4+}{x^4}\right.}{8\,c^2x^2}\right.$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[-1/8*(3*(b*c - a*d)*sqrt(a*e/c)*e*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2
)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4*((b*c^2*d + a*c*d^
2)*x^4 + 2*a*c^3 + (b*c^3 + 3*a*c^2*d)*x^2)*sqrt(a*e/c)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c)))/x^4) - 4*((2*b*c - 3*a*d)*e*x^2 - a*c*e)*sqrt((b*e*x^2 +
a*e)/(d*x^2 + c)))/(c^2*x^2), 1/4*(3*(b*c - a*d)*sqrt(-a*e/c)*e*x^2*arctan
(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt(-a*e/c)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c))/(a*b*e*x^2 + a^2*e)) + 2*((2*b*c - 3*a*d)*e*x^2 - a*c*e)*sqrt((b*e*x^2
 + a*e)/(d*x^2 + c)))/(c^2*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[0,1,0]%%%},[6,0,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1
,1,1]%%%}
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^3}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.70

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^3}\,dx = \frac{\sqrt{e}\,e\left(-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a\,c^2-3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,acd\,x^2+2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2}\right.}{}$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^3,x)
```

output
```
(sqrt(e)*e*( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c**2 - 3*sqrt(c + d*x**
2)*sqrt(a + b*x**2)*a*c*d*x**2 + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c**
2*x**2 + 3*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c + sqrt(c)*sqrt(c
 + d*x**2)*a)*a*c*d*x**2 + 3*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*
c + sqrt(c)*sqrt(c + d*x**2)*a)*a*d**2*x**4 - 3*sqrt(c)*sqrt(a)*log(sqrt(a
)*sqrt(a + b*x**2)*c + sqrt(c)*sqrt(c + d*x**2)*a)*b*c**2*x**2 - 3*sqrt(c)
*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c + sqrt(c)*sqrt(c + d*x**2)*a)*b*c*
d*x**4 - 3*sqrt(c)*sqrt(a)*log(x)*a*c*d*x**2 - 3*sqrt(c)*sqrt(a)*log(x)*a*
d**2*x**4 + 3*sqrt(c)*sqrt(a)*log(x)*b*c**2*x**2 + 3*sqrt(c)*sqrt(a)*log(x
)*b*c*d*x**4))/(2*c**3*x**2*(c + d*x**2))
```

3.60 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 252

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = -\frac{d(bc-ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{c^3} - \frac{(5bc-9ad)e(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{8c^3x^2} - \frac{ae(c+dx^2)^2\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{4c^3x^4} - \frac{3(bc-5ad)(bc-ad)e^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{a}\sqrt{e}}\right)}{8\sqrt{a}c^{7/2}}$$

output
```
-d*(-a*d+b*c)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c^3-1/8*(-9*a*d+5*b
*c)*e*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c^3/x^2-1/4*a*e*(d*
x^2+c)^2*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c^3/x^4-3/8*(-5*a*d+b*c)*(
-a*d+b*c)*e^(3/2)*arctanh(c^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a
^(1/2)/e^(1/2))/a^(1/2)/c^(7/2)
```

Mathematica [A] (verified)

Time = 4.43 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.74

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx =$$
$$-\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{a}\sqrt{c}\sqrt{a+bx^2}(bcx^2(5c+13dx^2)+a(2c^2-5cdx^2-15d^2x^4))+3(b^2c^2-6abcd+5a^2d^2)\,x^4\right.}{8\sqrt{a}c^{7/2}x^4\sqrt{a+bx^2}}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]
```

output
```
-1/8*(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]
*(b*c*x^2*(5*c + 13*d*x^2) + a*(2*c^2 - 5*c*d*x^2 - 15*d^2*x^4)) + 3*(b^2*
c^2 - 6*a*b*c*d + 5*a^2*d^2)*x^4*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a +
 b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(Sqrt[a]*c^(7/2)*x^4*Sqrt[a + b*x^2]
)
```

Rubi [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2053, 2052, 25, 360, 25, 1471, 27, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}{x^6}\,dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int -\frac{x^8\left(be-dx^4\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$\Big\downarrow$ 25

$$-\left(e(bc-ad)\int \frac{x^8\left(be-dx^4\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$\Big\downarrow$ 360

$$e(bc-ad)\left(-\frac{\int -\frac{-4c^2dx^8+4c(bc-ad)ex^4+a(bc-ad)e^2}{\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4c^3}-\frac{ae^2(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^3\left(ae-cx^4\right)^2}\right)$$

$\Big\downarrow$ 25

$$e(bc-ad)\left(\frac{\int \frac{-4c^2dx^8+4c(bc-ad)ex^4+a(bc-ad)e^2}{\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4c^3}-\frac{ae^2(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^3\left(ae-cx^4\right)^2}\right)$$

$\Big\downarrow$ 1471

$$e(bc-ad)\left(\frac{\frac{e(5bc-9ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2\left(ae-cx^4\right)}-\frac{\int \frac{ae\left((3bc-7ad)e-8cdx^4\right)}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2ae}}{4c^3}-\frac{ae^2(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^3\left(ae-cx^4\right)^2}\right)$$

$\Big\downarrow$ 27

$$e(bc-ad)\left(\frac{\frac{e(5bc-9ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2\left(ae-cx^4\right)}-\frac{1}{2}\int \frac{(3bc-7ad)e-8cdx^4}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4c^3}-\frac{ae^2(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^3\left(ae-cx^4\right)^2}\right)$$

$\Big\downarrow$ 299

$$e(bc-ad)\left(\frac{\frac{1}{2}\left(-3e(bc-5ad)\int \frac{1}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}-8d\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}\right)+\frac{e(5bc-9ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2\left(ae-cx^4\right)}}{4c^3}-\frac{ae^2(bc-ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^3\left(ae-cx^4\right)^2}\right)$$

$\Big\downarrow$ 221

$$e(bc-ad)\left(\frac{\frac{1}{2}\left(-\frac{3\sqrt{e}(bc-5ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{c}}-8d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\right)+\frac{e(5bc-9ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2(ae-cx^4)}}{4c^3}-\frac{ae^2(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^3\left(ae-cx^4\right)^2}\right.$$

input
```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x]
```

output
```
(b*c - a*d)*e*(-1/4*(a*(b*c - a*d)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/
(c^3*(a*e - c*x^4)^2) + (((5*b*c - 9*a*d)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^
2)])/(2*(a*e - c*x^4)) + (-8*d*Sqrt[(e*(a + b*x^2))/(c + d*x^2)] - (3*(b*c
 - 5*a*d)*Sqrt[e]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqr
t[a]*Sqrt[e])])/(Sqrt[a]*Sqrt[c]))/2)/(4*c^3))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{(d\,x^2+c)(-7ad\,x^2+5bc\,x^2+2ac)e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{8c^3x^4}+\frac{\left(-\frac{(15a^2d^2-18abcd+3b^2c^2)\ln\left(\frac{2ace+(ade+bce)x^2+2\sqrt{ace}\,\sqrt{bde\,x^4+(ade+bce)x^2}}{x^2}\right.}{2\sqrt{ace}}\right.}{8c^3(}$
default	Expression too large to display

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(d*x^2+c)*(-7*a*d*x^2+5*b*c*x^2+2*a*c)/c^3/x^4*e*(e*(b*x^2+a)/(d*x^2+
c))^(1/2)+1/8/c^3*(-1/2*(15*a^2*d^2-18*a*b*c*d+3*b^2*c^2)/(a*c*e)^(1/2)*ln
((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a
*c*e)^(1/2))/x^2)+8*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(b*x^2+a)/(a*d-b*c)/(b*d
*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))*e/(b*x^2+a)*(e*(b*x^2+a)/(d*x^2+c
))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.73

$$\int\frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx=\left[\frac{3\,(b^2c^2-6\,abcd+5\,a^2d^2)ex^4\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2-4\,(2}{}\right.}{}\right.$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/32*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*x^4*sqrt(e/(a*c))*log(((b^2*c
^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^
2 - 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a*b*c^3 + 3*a^2*c^2*d)*x
^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c)))/x^4) - 4*((13*b*c*d -
 15*a*d^2)*e*x^4 + 2*a*c^2*e + 5*(b*c^2 - a*c*d)*e*x^2)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c)))/(c^3*x^4), 1/16*(3*(b^2*c^2 - 6*a*b*c*d + 5*a^2*d^2)*e*x^
4*sqrt(-e/(a*c))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)
/(d*x^2 + c))*sqrt(-e/(a*c))/(b*e*x^2 + a*e)) - 2*((13*b*c*d - 15*a*d^2)*e
*x^4 + 2*a*c^2*e + 5*(b*c^2 - a*c*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c)))/(c^3*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**5,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[4,1,4]%%%},[2,7,0]%%%}+%%%{%%%{-8,[3,2,4]%%%},[2,6,
1]%%%}+%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^5}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 515, normalized size of antiderivative = 2.04

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^5}\,dx = \frac{\sqrt{e}\,e\big(-2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2c^3+5\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2c^2d\,x^2+15\sqrt{d\,x^2+c}\,\sqrt{}$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^5,x)
```

output
```
(sqrt(e)*e*( - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c**3 + 5*sqrt(c +
d*x**2)*sqrt(a + b*x**2)*a**2*c**2*d*x**2 + 15*sqrt(c + d*x**2)*sqrt(a + b
*x**2)*a**2*c*d**2*x**4 - 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c**3*x**
2 - 13*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c**2*d*x**4 + 15*sqrt(c)*sqrt
(a)*log(sqrt(a)*sqrt(a + b*x**2)*c - sqrt(c)*sqrt(c + d*x**2)*a)*a**2*c*d*
*2*x**4 + 15*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c - sqrt(c)*sqrt
(c + d*x**2)*a)*a**2*d**3*x**6 - 18*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b
*x**2)*c - sqrt(c)*sqrt(c + d*x**2)*a)*a*b*c**2*d*x**4 - 18*sqrt(c)*sqrt(a
)*log(sqrt(a)*sqrt(a + b*x**2)*c - sqrt(c)*sqrt(c + d*x**2)*a)*a*b*c*d**2*
x**6 + 3*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c - sqrt(c)*sqrt(c +
 d*x**2)*a)*b**2*c**3*x**4 + 3*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2
)*c - sqrt(c)*sqrt(c + d*x**2)*a)*b**2*c**2*d*x**6 - 15*sqrt(c)*sqrt(a)*lo
g(x)*a**2*c*d**2*x**4 - 15*sqrt(c)*sqrt(a)*log(x)*a**2*d**3*x**6 + 18*sqrt
(c)*sqrt(a)*log(x)*a*b*c**2*d*x**4 + 18*sqrt(c)*sqrt(a)*log(x)*a*b*c*d**2*
x**6 - 3*sqrt(c)*sqrt(a)*log(x)*b**2*c**3*x**4 - 3*sqrt(c)*sqrt(a)*log(x)*
b**2*c**2*d*x**6))/(8*a*c**4*x**4*(c + d*x**2))
```

3.61 $\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 346

$$\int \frac{\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \frac{d^2(bc-ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{c^4} - \frac{\left(b^2c^2-22abcd+29a^2d^2\right)e(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{16ac^4x^2} - \frac{(7bc-19ad)e(c+dx^2)^2\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{24c^4x^4} - \frac{ae(c+dx^2)^3\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{6c^4x^6} + \frac{(bc-ad)\left(b^2c^2+10abcd-35a^2d^2\right)e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d\left(c+dx^2\right)}}}{\sqrt{a}\sqrt{e}}\right)}{16a^{3/2}c^{9/2}}$$

output
```
d^2*(-a*d+b*c)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c^4-1/16*(29*a^2*d
^2-22*a*b*c*d+b^2*c^2)*e*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/
a/c^4/x^2-1/24*(-19*a*d+7*b*c)*e*(d*x^2+c)^2*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+
c))^(1/2)/c^4/x^4-1/6*a*e*(d*x^2+c)^3*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/
2)/c^4/x^6+1/16*(-a*d+b*c)*(-35*a^2*d^2+10*a*b*c*d+b^2*c^2)*e^(3/2)*arctan
h(c^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))/a^(3/2)/
c^(9/2)
```

Mathematica [A] (verified)

Time = 4.88 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.71

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{a}\sqrt{c}\sqrt{a+bx^2}(3b^2c^2x^4(c+dx^2)+2abcx^2(7c^2-19cdx^2-50d^2x^4)+\right.}{}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[a]*Sqrt[c]*Sqrt[a + b*x^2]*(3
*b^2*c^2*x^4*(c + d*x^2) + 2*a*b*c*x^2*(7*c^2 - 19*c*d*x^2 - 50*d^2*x^4) +
 a^2*(8*c^3 - 14*c^2*d*x^2 + 35*c*d^2*x^4 + 105*d^3*x^6))) + 3*(b^3*c^3 +
9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*x^6*Sqrt[c + d*x^2]*ArcTanh[(
Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])]))/(48*a^(3/2)*c^(9/2)*
x^6*Sqrt[a + b*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.94 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.87, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {2053, 2052, 366, 25, 27, 360, 25, 1471, 27, 299, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}dx$$

$\big\downarrow$ 2053

$$\frac{1}{2}\int \frac{\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}{x^8}dx^2$$

$\big\downarrow$ 2052

$$e(bc-ad)\int \frac{x^8\left(be-dx^4\right)^2}{\left(ae-cx^4\right)^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$\big\downarrow$ 366

$$e(bc-ad)\left(\frac{ex^{10}(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}-\frac{\int -\frac{x^8\left(-6acd^2ex^4+6b^2c^2e^2-5(bce-ade)^2\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{6ac^2e}\right)$$

$\big\downarrow$ 25

$$e(bc-ad)\left(\frac{\int \frac{ex^8\left(\left(b^2c^2+10abdc-5a^2d^2\right)e-6acd^2x^4\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{6ac^2e}+\frac{ex^{10}(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}\right)$$

$\big\downarrow$ 27

$$e(bc-ad)\left(\frac{\int \frac{x^8\left(\left(b^2c^2+10abdc-5a^2d^2\right)e-6acd^2x^4\right)}{\left(ae-cx^4\right)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{6ac^2}+\frac{ex^{10}(bc-ad)^2}{6ac^2\left(ae-cx^4\right)^3}\right)$$

$\big\downarrow$ 360

$$e(bc-ad)\left(\frac{\frac{\int -\frac{-24ac^3d^2x^8+4c^2(bc-ad)(bc+11ad)ex^4+ac(bc-ad)(bc+11ad)e^2}{\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4c^3}+\frac{ae^2(bc-ad)(11ad+bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2\left(ae-cx^4\right)^2}}{6ac^2}+\frac{ex^{10}(bc-ad}{6ac^2\left(ae-cx\right.}\right.$$

$\big\downarrow$ 25

$$e(bc-ad)\left(\frac{\frac{ae^2(bc-ad)(11ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2(ae-cx^4)^2}-\frac{\int \frac{-24ac^3d^2x^8+4c^2(bc-ad)(bc+11ad)ex^4+ac(bc-ad)(bc+11ad)e^2}{(ae-cx^4)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4c^3}}{6ac^2}+\frac{ex^{10}(bc-ad)^2}{6ac^2(ae-cx^4)}\right.$$

$\downarrow$ 1471

$$e(bc-ad)\left(\frac{\frac{ae^2(bc-ad)(11ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2(ae-cx^4)^2}-\frac{\frac{ce(-79a^2d^2+50abcd+5b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2(ae-cx^4)}-\frac{\int \frac{3ace((b^2c^2+10abdc-19a^2d^2)e-16acd^2x^4)}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2ae}}{4c^3}}{6ac^2}\right.$$

$\downarrow$ 27

$$e(bc-ad)\left(\frac{\frac{ae^2(bc-ad)(11ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2(ae-cx^4)^2}-\frac{\frac{ce(-79a^2d^2+50abcd+5b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2(ae-cx^4)}-\frac{3}{2}c\int \frac{(b^2c^2+10abdc-19a^2d^2)e-16acd^2x^4}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4c^3}}{6ac^2}+\right.$$

$\downarrow$ 299

$$e(bc-ad)\left(\frac{\frac{ae^2(bc-ad)(11ad+bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4c^2(ae-cx^4)^2}-\frac{\frac{ce(-79a^2d^2+50abcd+5b^2c^2)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2(ae-cx^4)}-\frac{3}{2}c\left(e(-35a^2d^2+10abcd+b^2c^2)\int \frac{1}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}+\right.}{4c^3}}{6ac^2}\right.$$

$\downarrow$ 221

$$e(bc-ad)\left(\frac{\frac{ae^2(bc-ad)(11ad+bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4c^2(ae-cx^4)^2}-\frac{\frac{ce\left(-79a^2d^2+50abcd+5b^2c^2\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2\left(ae-cx^4\right)}-\frac{3}{2}c\left(\frac{\sqrt{e}\left(-35a^2d^2+10abcd+b^2c^2\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right.}{\sqrt{a}\sqrt{c}}\right.}{4c^3}}{6ac^2}\right.$$

input
```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x]
```

output
```
(b*c - a*d)*e*(((b*c - a*d)^2*e*x^10)/(6*a*c^2*(a*e - c*x^4)^3) + ((a*(b*c
 - a*d)*(b*c + 11*a*d)*e^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*c^2*(a*e
- c*x^4)^2) - ((c*(5*b^2*c^2 + 50*a*b*c*d - 79*a^2*d^2)*e*Sqrt[(e*(a + b*x
^2))/(c + d*x^2)])/(2*(a*e - c*x^4)) - (3*c*(16*a*d^2*Sqrt[(e*(a + b*x^2))
/(c + d*x^2)] + ((b^2*c^2 + 10*a*b*c*d - 35*a^2*d^2)*Sqrt[e]*ArcTanh[(Sqrt
[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*Sqrt[c
])))/2)/(4*c^3))/(6*a*c^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 333, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{(d x^2+c)(57a^2d^2x^4-52abcd x^4+3b^2c^2x^4-22a^2cd x^2+14ab c^2x^2+8a^2c^2)e\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}{48c^4x^6a} - \frac{\left(-\frac{(35a^3d^3-45a^2bc d^2+9a b^2c^2d+b^3c}{}\right.}{}$
default	Expression too large to display

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(d*x^2+c)*(57*a^2*d^2*x^4-52*a*b*c*d*x^4+3*b^2*c^2*x^4-22*a^2*c*d*x^
2+14*a*b*c^2*x^2+8*a^2*c^2)/c^4/x^6/a*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/16
/a/c^4*(-1/2*(35*a^3*d^3-45*a^2*b*c*d^2+9*a*b^2*c^2*d+b^3*c^3)/(a*c*e)^(1/
2)*ln((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*
x^2+a*c*e)^(1/2))/x^2)+16*a*d^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(b*x^2+a)/(a*d
-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))*e/(b*x^2+a)*(e*(b*x^2+a
)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 7.92 (sec) , antiderivative size = 573, normalized size of antiderivative = 1.66

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \left[\frac{3\,(b^3c^3+9\,ab^2c^2d-45\,a^2bcd^2+35\,a^3d^3)ex^6\sqrt{\frac{e}{ac}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(\ldots}{\ldots}\right)\ldots}{\ldots}\right.$$

$$-\frac{3\,(b^3c^3+9\,ab^2c^2d-45\,a^2bcd^2+35\,a^3d^3)ex^6\sqrt{-\frac{e}{ac}}\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{e}{ac}}}{2\,(bex^2+ae)}\right)+2\,((3\,b^2c^2d-\ldots}{96\,a\ldots}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[1/192*(3*(b^3*c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*x^6*sq
rt(e/(a*c))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(
a*b*c^2 + a^2*c*d)*e*x^2 + 4*(2*a^2*c^3 + (a*b*c^2*d + a^2*c*d^2)*x^4 + (a
*b*c^3 + 3*a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(e/(a*c))
)/x^4) - 4*((3*b^2*c^2*d - 100*a*b*c*d^2 + 105*a^2*d^3)*e*x^6 + 8*a^2*c^3*
e + (3*b^2*c^3 - 38*a*b*c^2*d + 35*a^2*c*d^2)*e*x^4 + 14*(a*b*c^3 - a^2*c^
2*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^4*x^6), -1/96*(3*(b^3*
c^3 + 9*a*b^2*c^2*d - 45*a^2*b*c*d^2 + 35*a^3*d^3)*e*x^6*sqrt(-e/(a*c))*ar
ctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(
-e/(a*c))/(b*e*x^2 + a*e)) + 2*((3*b^2*c^2*d - 100*a*b*c*d^2 + 105*a^2*d^3
)*e*x^6 + 8*a^2*c^3*e + (3*b^2*c^3 - 38*a*b*c^2*d + 35*a^2*c*d^2)*e*x^4 +
14*(a*b*c^3 - a^2*c^2*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^4*
x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**7,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[5,1,5]%%%},[2,9,0]%%%}+%%%{%%%{-10,[4,2,5]%%%},[2,8
,1]%%%}+%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^7}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 771, normalized size of antiderivative = 2.23

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^7}\,dx = \text{Too large to display}$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^7,x)
```

output
```
(sqrt(e)*e*( - 8*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*c**4 + 14*sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*a**3*c**3*d*x**2 - 35*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*a**3*c**2*d**2*x**4 - 105*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*c
*d**3*x**6 - 14*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c**4*x**2 + 38*sq
rt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c**3*d*x**4 + 100*sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a**2*b*c**2*d**2*x**6 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**
2)*a*b**2*c**4*x**4 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**3*d*x*
*6 + 105*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c + sqrt(c)*sqrt(c +
 d*x**2)*a)*a**3*c*d**3*x**6 + 105*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*
x**2)*c + sqrt(c)*sqrt(c + d*x**2)*a)*a**3*d**4*x**8 - 135*sqrt(c)*sqrt(a)
*log(sqrt(a)*sqrt(a + b*x**2)*c + sqrt(c)*sqrt(c + d*x**2)*a)*a**2*b*c**2*
d**2*x**6 - 135*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c + sqrt(c)*s
qrt(c + d*x**2)*a)*a**2*b*c*d**3*x**8 + 27*sqrt(c)*sqrt(a)*log(sqrt(a)*sqr
t(a + b*x**2)*c + sqrt(c)*sqrt(c + d*x**2)*a)*a*b**2*c**3*d*x**6 + 27*sqrt
(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c + sqrt(c)*sqrt(c + d*x**2)*a)*a
*b**2*c**2*d**2*x**8 + 3*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c +
sqrt(c)*sqrt(c + d*x**2)*a)*b**3*c**4*x**6 + 3*sqrt(c)*sqrt(a)*log(sqrt(a)
*sqrt(a + b*x**2)*c + sqrt(c)*sqrt(c + d*x**2)*a)*b**3*c**3*d*x**8 - 105*s
qrt(c)*sqrt(a)*log(x)*a**3*c*d**3*x**6 - 105*sqrt(c)*sqrt(a)*log(x)*a**3*d
**4*x**8 + 135*sqrt(c)*sqrt(a)*log(x)*a**2*b*c**2*d**2*x**6 + 135*sqrt(...
```

3.62 $\int x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 513

$$\begin{aligned}\int x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx =& -\frac{ex^3(a+bx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{d} \\ & -\frac{(8bc-7ad)ex(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{5d^3}+\frac{6bex^3(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{5d^2} \\ & +\frac{(16b^2c^2-16abcd+a^2d^2)\,ex(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{5d^4\,(a+bx^2)} \\ & -\frac{\sqrt{a}(16b^2c^2-16abcd+a^2d^2)\,e(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{5\sqrt{b}d^4\,(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} \\ & +\frac{a^{3/2}(8bc-7ad)e(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{5\sqrt{b}d^3\,(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}\end{aligned}$$

output
```
-e*x^3*(b*x^2+a)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d-1/5*(-7*a*d+8*b*
c)*e*x*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d^3+6/5*b*e*x^3*(d
*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d^2+1/5*(a^2*d^2-16*a*b*c*d
+16*b^2*c^2)*e*x*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d^4/(b*x
^2+a)-1/5*a^(1/2)*(a^2*d^2-16*a*b*c*d+16*b^2*c^2)*e*(d*x^2+c)*(b*e/d-(-a*d
+b*c)*e/d/(d*x^2+c))^(1/2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(
1-a*d/b/c)^(1/2))/b^(1/2)/d^4/(b*x^2+a)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)+1/
5*a^(3/2)*(-7*a*d+8*b*c)*e*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2
)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/d^3
/(b*x^2+a)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.74 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.52

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(a+bx^2)\left(ad(7c+2dx^2)+b(-8c^2-2cdx^2+d^2x^4)\right)-ic(\right.}{}$$

input
```
Integrate[x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(a*d*(7*c
+ 2*d*x^2) + b*(-8*c^2 - 2*c*d*x^2 + d^2*x^4)) - I*c*(16*b^2*c^2 - 16*a*b*
c*d + a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh
[Sqrt[b/a]*x], (a*d)/(b*c)] + (8*I)*c*(2*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*Sq
rt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a
*d)/(b*c)]))/(5*Sqrt[b/a]*d^4*(a + b*x^2))
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.79, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {2058, 369, 27, 443, 25, 444, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{x^4(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{369}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\int \frac{3x^2\sqrt{bx^2+a}(2bx^2+a)}{\sqrt{dx^2+c}}dx}{d}-\frac{x^3(a+bx^2)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\int \frac{x^2\sqrt{bx^2+a}(2bx^2+a)}{\sqrt{dx^2+c}}dx}{d}-\frac{x^3(a+bx^2)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{443}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{\int -\frac{x^2(b(8bc-7ad)x^2+a(6bc-5ad))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}+\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}\right)}{d}-\frac{x^3(a+bx^2)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\int\frac{x^2\left(b(8bc-7ad)x^2+a(6bc-5ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5d}\right)}{d}-\frac{x^3\left(a+bx^2\right)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 444

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{3d}-\frac{\int\frac{b\left(\left(16b^2c^2-16abdc+a^2d^2\right)x^2+ac(8bc-7ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{5d}\right)}{d}-\frac{x^3(a+b}{d\sqrt{c+}}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 27

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{3d}-\frac{\int\frac{\left(16b^2c^2-16abdc+a^2d^2\right)x^2+ac(8bc-7ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{5d}\right)}{d}-\frac{x^3(a+bx^2)}{d\sqrt{c+dx}}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 406

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{3d}-\frac{\left(a^2d^2-16abcd+16b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(8bc-7ad)\int\frac{}{\sqrt{b}}}{3d}}{5d}}{d}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 320

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{3d}-\frac{\left(a^2d^2-16abcd+16b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(8bc}{\sqrt{}}}{3d}}{5d}\right.}{d}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 388

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{3d}-\frac{\left(a^2d^2-16abcd+16b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}}{}}{3d}}{5d}\right.}{d}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 313

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{3\left(\frac{2bx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5d}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{3d}-\frac{\left(a^2d^2-16abcd+16b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right.\right.}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+b}{a(c+d}}}\right.}{5d}\right.}{d}\right.}{\sqrt{a+bx^2}}$$

input
```
Int[x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-((x^3*(a + b*x^2)^(
3/2))/(d*Sqrt[c + d*x^2])) + (3*((2*b*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])
/(5*d) - (((8*b*c - 7*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d) - ((16
*b^2*c^2 - 16*a*b*c*d + a^2*d^2)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2])
- (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c
)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]
)) + (c^(3/2)*(8*b*c - 7*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))
]*Sqrt[c + d*x^2]))/(3*d))/(5*d)))/d))/Sqrt[a + b*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 12.72 (sec) , antiderivative size = 775, normalized size of antiderivative = 1.51

method	result
risch	$\frac{x\left(bd\,x^{2}+2ad-3bc\right)\left(d\,x^{2}+c\right)e\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{5d^{3}}+\frac{\left(-\frac{2\left(a^{2}d^{2}-11abcd+11b^{2}c^{2}\right)ace\sqrt{1+\frac{x^{2}b}{a}}\,\sqrt{1+\frac{x^{2}d}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)-\cdots\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}\,\left(ade+bce+e(ad-b\cdots\right.}\right.}{\cdots}$
default	$\frac{\left(\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}\right)^{\frac{3}{2}}\left(d\,x^{2}+c\right)\left(\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{-\frac{b}{a}}\,b^{2}d^{3}x^{7}+3\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{-\frac{b}{a}}\,ab\,d^{3}x^{5}-2\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{-\frac{b}{a}}\,b^{2}c\cdots\right.}{\cdots}$

input
```
int(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*x*(b*d*x^2+2*a*d-3*b*c)*(d*x^2+c)/d^3*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+
1/5/d^3*(-2*(a^2*d^2-11*a*b*c*d+11*b^2*c^2)*a*c*e/(-b/a)^(1/2)*(1+1/a*x^2*
b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(
a*d*e+b*c*e+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e
)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))-c*(7*a^
2*d^2-13*a*b*c*d+5*b^2*c^2)/d/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*
d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))+5*c^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d*((
b*d*e*x^2+a*d*e)/c/(a*d-b*c)*x/e/((x^2+c/d)*(b*d*e*x^2+a*d*e))^(1/2)+(1/c-
a*d/c/(a*d-b*c))/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d
*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*
e+b*c*e)/c/b/e)^(1/2))+2/(a*d-b*c)*b*d*a*e/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2
)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(a*d*e+b
*c*e+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)
)-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))))*e/(b*x^2+a)*
(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.54

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx =$$

$$-\frac{(16\,b^2c^3-16\,abc^2d+a^2cd^2)\sqrt{\frac{be}{d}}ex\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc}\right)-(16\,b^2c^3-16\,abc^2d-7\,a^2d^3+(a^2+8}{}$$

input
```
integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/5*((16*b^2*c^3 - 16*a*b*c^2*d + a^2*c*d^2)*sqrt(b*e/d)*e*x*sqrt(-c/d)*e
lliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (16*b^2*c^3 - 16*a*b*c^2*d -
7*a^2*d^3 + (a^2 + 8*a*b)*c*d^2)*sqrt(b*e/d)*e*x*sqrt(-c/d)*elliptic_f(arc
sin(sqrt(-c/d)/x), a*d/(b*c)) - (b^2*d^3*e*x^6 - 2*(b^2*c*d^2 - a*b*d^3)*e
*x^4 + (8*b^2*c^2*d - 9*a*b*c*d^2 + a^2*d^3)*e*x^2 + (16*b^2*c^3 - 16*a*b*
c^2*d + a^2*c*d^2)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*d^4*x)
```

Sympy [F(-1)]

Timed out.

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(x**4*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^4\, dx$$

input
```
integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4, x)
```

Giac [F]

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^4\, dx$$

input
```
integrate(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\int x^4\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}\,dx$$

input
```
int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

Reduce [F]

$$\int x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int(x^4*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output

```
(sqrt(e)*e*( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d*x + 3*sqrt(c + d
*x**2)*sqrt(a + b*x**2)*a*b*c*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*
d*x**3 - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*x**3 + sqrt(c + d*x**2
)*sqrt(a + b*x**2)*b**2*d*x**5 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*
x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 +
b*d**2*x**6),x)*a**2*b*c*d**2 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x
**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b
*d**2*x**6),x)*a**2*b*d**3*x**2 - 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2
)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4
+ b*d**2*x**6),x)*a*b**2*c**2*d - 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2
)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4
+ b*d**2*x**6),x)*a*b**2*c*d**2*x**2 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*
x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x
**4 + b*d**2*x**6),x)*b**3*c**3 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)
*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 +
 b*d**2*x**6),x)*b**3*c**2*d*x**2 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**
2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*
d**2*x**6),x)*a**3*c**2*d + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c
**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**
6),x)*a**3*c*d**2*x**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c...
```

3.63 $\int x^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 26, antiderivative size = 428

$$\int x^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = -\frac{ex(a+bx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{d} + \frac{4bex(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{3d^2} - \frac{b(8bc-7ad)ex(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{3d^3(a+bx^2)} + \frac{\sqrt{a}\sqrt{b}(8bc-7ad)e(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3d^3(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}} - \frac{a^{3/2}(4bc-3ad)e(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3\sqrt{b}cd^2(a+bx^2)\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}}$$

output
```
-e*x*(b*x^2+a)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d+4/3*b*e*x*(d*x^2+c
)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d^2-1/3*b*(-7*a*d+8*b*c)*e*x*(d*x
^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d^3/(b*x^2+a)+1/3*a^(1/2)*b^(
1/2)*(-7*a*d+8*b*c)*e*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)*Ell
ipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/d^3/(b*x^2+a
)/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)-1/3*a^(3/2)*(-3*a*d+4*b*c)*e*(d*x^2+c)*(
b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(
1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/d^2/(b*x^2+a)/(a*(d*x^2+c)/c/(b*x^2+a))
^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.07 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.55

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx =$$

$$-\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(a+bx^2)\left(3ad-b(4c+dx^2)\right)+ibc(-8bc+7ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}\right.\right.\right.}{3\sqrt{\frac{b}{a}}d^3\left(a+bx^2\right.}$$

input
```
Integrate[x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
-1/3*(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(a + b*x^2)*(3*a*
d - b*(4*c + d*x^2)) + I*b*c*(-8*b*c + 7*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
 (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + I*(8*b^2*c^2
- 11*a*b*c*d + 3*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*Elliptic
F[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*d^3*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 339, normalized size of antiderivative = 0.79, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2058, 369, 403, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{x^2(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 369}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\int \frac{\sqrt{bx^2+a}(4bx^2+a)}{\sqrt{dx^2+c}}dx}{d}-\frac{x(a+bx^2)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 403}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\frac{\int -\frac{b(8bc-7ad)x^2+a(4bc-3ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}+\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}}{d}-\frac{x(a+bx^2)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\int \frac{b(8bc-7ad)x^2+a(4bc-3ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{d}-\frac{x(a+bx^2)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 406}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{a(4bc-3ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b(8bc-7ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3d}}{d}-\frac{x(a+bx^2)^{3/2}}{d\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$\Big\downarrow$ 320

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{b(8bc-7ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(4bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3d}}{d}-\frac{x(a+\cdots}{d\sqrt{c\cdots}}\cdots\right.}{\sqrt{a+bx^2}}$$

$\Big\downarrow$ 388

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{b(8bc-7ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(4bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3d}}{d}\cdots\right.}{\sqrt{a+bx^2}}$$

$\Big\downarrow$ 313

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\frac{4bx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3d}-\frac{\frac{\sqrt{c}\sqrt{a+bx^2}(4bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b(8bc-7ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\text{arcta}\cdots\right.}{b\sqrt{d}\sqrt{c+dx^2}\cdots}\cdots\right.}{3d}}{d}\cdots\right.}{\sqrt{a+bx^2}}$$

input
```
Int[x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-((x*(a + b*x^2)^(3/
2))/(d*Sqrt[c + d*x^2])) + ((4*b*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*d)
- (b*(8*b*c - 7*a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*S
qrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b
*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c
]*(4*b*c - 3*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1
 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d
*x^2]))/(3*d))/d))/Sqrt[a + b*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 13.29 (sec) , antiderivative size = 734, normalized size of antiderivative = 1.71

method	result
default	$\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{\frac{3}{2}}(dx^2+c)\left(\sqrt{(dx^2+c)(bx^2+a)}\sqrt{-\frac{b}{a}}\,b^2d^2x^5+\sqrt{(dx^2+c)(bx^2+a)}\sqrt{-\frac{b}{a}}\,abd^2x^3+\sqrt{(dx^2+c)(bx^2+a)}\sqrt{-\frac{b}{a}}\,b^2cd\right.$
risch	$\frac{bx(dx^2+c)e\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{3d^2}+\left(-\frac{2bd(4ad-5bc)ace\sqrt{1+\frac{x^2b}{a}}\sqrt{1+\frac{x^2d}{c}}\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)-\text{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+b}{cbe}}\right)\right)}{\sqrt{-\frac{b}{a}}\sqrt{bdex^4+adex^2+bcex^2+ace}\,(ade+bce+e(ad-bc))}\right.$

input
```
int(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(
-b/a)^(1/2)*b^2*d^2*x^5+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*b*d^2*x
^3+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^3-3*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^3+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*
c)^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^3+3*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)
/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^
2*d^2-11*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d+8*((d*x^2+c)*(b*x^2+a
))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),
(a*d/b/c)^(1/2))*b^2*c^2+7*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)
*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d-8*(
(d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellipti
cE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b
/a)^(1/2)*a*b*c*d*x-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a^2
*d^2*x+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*c*d*x)/(b*x^
2+a)^2/d^3/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.54

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\frac{(8\,b^2c^3-7\,abc^2d)\sqrt{\frac{be}{d}}ex\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(8\,b^2c^3-7\,abc^2d+4\,a}{}$$

input
```
integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*((8*b^2*c^3 - 7*a*b*c^2*d)*sqrt(b*e/d)*e*x*sqrt(-c/d)*elliptic_e(arcsi
n(sqrt(-c/d)/x), a*d/(b*c)) - (8*b^2*c^3 - 7*a*b*c^2*d + 4*a*b*c*d^2 - 3*a
^2*d^3)*sqrt(b*e/d)*e*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b
*c)) + (b^2*c*d^2*e*x^4 - 4*(b^2*c^2*d - a*b*c*d^2)*e*x^2 - (8*b^2*c^3 - 7
*a*b*c^2*d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*c*d^3*x)
```

Sympy [F(-1)]

Timed out.

$$\int x^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate(x**2*(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2, x)
```

Giac [F]

$$\int x^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} x^2 \, dx$$

input
```
integrate(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\int x^2\,\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{3/2}\,dx$$

input
```
int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

Reduce [F]

$$\int x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int(x^2*(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(e)*e*(3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*d*x - 3*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a*b*c*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*c*
x**3 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x*
*2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b*c*d
**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**
2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b*d**3
*x**2 + 11*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*
x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b**2*c
**2*d + 11*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*
x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b**2*c
*d**2*x**2 - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*
c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**3
*c**3 - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x
**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**3*c**2
*d*x**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2
 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**3*c**2*d
- 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**
2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**3*c*d**2*x**2 + 3
*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x
**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*b*c**3 + 3*int(...
```

3.64 $\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$

Optimal result

Integrand size = 22, antiderivative size = 247

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \frac{bex\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{d} - \frac{(2bc-ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{\sqrt{c}d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{b\sqrt{c}e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{d^{3/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
b*e*x*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/d-(-a*d+2*b*c)*e*(b*e/d-(-a*d
+b*c)*e/d/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(
1-b*c/a/d)^(1/2))/c^(1/2)/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+b*c^(1/2
)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)*InverseJacobiAM(arctan(d^(1/2)*
x/c^(1/2)),(1-b*c/a/d)^(1/2))/d^(3/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.95 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.83

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(ibc(-2bc+ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(-bc\right.}{\sqrt{\frac{b}{a}}cd}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(I*b*c*(-2*b*c + a*d)*Sqrt[1 + (b*x^2
)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] +
(-(b*c) + a*d)*(Sqrt[b/a]*d*x*(a + b*x^2) - (2*I)*b*c*Sqrt[1 + (b*x^2)/a]*
Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(Sqr
t[b/a]*c*d^2*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2058, 315, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{(bx^2+a)^{3/2}}{(dx^2+c)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{315}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{\int \frac{b\left((2bc-ad)x^2+ac\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\int \frac{(2bc-ad)x^2+ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 406$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 320$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left((2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 388$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left((2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{cd}-\frac{x\sqrt{a+bx^2}(bc-ad)}{cd\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \; 313$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{cd}-x\right.}{\sqrt{a+bx^2}}$$

input
```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-(((b*c - a*d)*x*Sqr
t[a + b*x^2])/(c*d*Sqrt[c + d*x^2])) + (b*((2*b*c - a*d)*((x*Sqrt[a + b*x^
2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[
d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sq
rt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c +
 d*x^2))]*Sqrt[c + d*x^2])))/(c*d)))/Sqrt[a + b*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. $2(236) = 472$.

Time = 4.32 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.13

method	result
default	$\frac{\left(\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}\right)^{\frac{3}{2}}\left(d\,x^{2}+c\right)\left(\sqrt{db\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{-\frac{b}{a}}\,ab\,d^{2}x^{3}-\sqrt{db\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{-\frac{b}{a}}\,b^{2}cd\,x^{3}+2\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\right.}{}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*((b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/
2)*(-b/a)^(1/2)*a*b*d^2*x^3-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/
2)*b^2*c*d*x^3+2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c
)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d-2*((d*x^2+c)*
(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2-((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)
^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c
*d+2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2+(b*d*x^4+a*d*x^2+b*c*x^2+
a*c)^(1/2)*(-b/a)^(1/2)*a^2*d^2*x-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/
a)^(1/2)*a*b*c*d*x)/(b*x^2+a)^2/d^2/c/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^
2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.70

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx =$$

$$-\frac{(2\,bc^2-acd)\sqrt{\frac{be}{d}}ex\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - (2\,bc^2-acd+ad^2)\sqrt{\frac{be}{d}}ex\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}}{cd^2x}$$

input

```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output

```
-((2*b*c^2 - a*c*d)*sqrt(b*e/d)*e*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d
)/x), a*d/(b*c)) - (2*b*c^2 - a*c*d + a*d^2)*sqrt(b*e/d)*e*x*sqrt(-c/d)*el
liptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (b*c*d*e*x^2 + (2*b*c^2 - a*c*
d)*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(c*d^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

Giac [F]

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int \left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}} dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \int \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2} dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

Reduce [F]

$$\int \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2} dx = \frac{\sqrt{e}\,e\left(\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ax-\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{b\,d^2x^6+a\,d^2x^4+2bcd\,x^4+2acd\,x^2+b\,c^2x^2+a\,c^2}dx\right)abcd\right.}{}$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(e)*e*(sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*x - int((sqrt(c + d*x**2)*
sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2
+ 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*c*d - int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d
*x**4 + b*d**2*x**6),x)*a*b*d**2*x**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x
**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x*
*4 + b*d**2*x**6),x)*b**2*c**2 + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x*
*4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*
d**2*x**6),x)*b**2*c*d*x**2))/(c*(c + d*x**2))
```

3.65 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 311

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \frac{(bc-2ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{cdx} - \frac{(bc-ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{cdx} + \frac{(bc-2ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{c^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} + \frac{be\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{c}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}$$

output
```
(-2*a*d+b*c)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c/d/x-(-a*d+b*c)*e*(
b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c/d/x+(-2*a*d+b*c)*e*(b*e/d-(-a*d+b*
c)*e/d/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b
*c/a/d)^(1/2))/c^(3/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)+b*e*(b*e/d-
(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),
(1-b*c/a/d)^(1/2))/c^(1/2)/d^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.36 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.73

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = -\frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}d(a+bx^2)\left(ac-bcx^2+2adx^2\right)+ibc(-bc+2ad)x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right.\right.\right.}{\sqrt{\frac{b}{a}}c^2dx\left(a+bx^2\right)}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x]
```

output
```
-((e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*(a + b*x^2)*(a*c - b*c
*x^2 + 2*a*d*x^2) + I*b*c*(-(b*c) + 2*a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 +
(d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(-(b*c)
+ a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[
b/a]*x], (a*d)/(b*c)]))/(Sqrt[b/a]*c^2*d*x*(a + b*x^2)))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2058, 370, 27, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \; 2058$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{(bx^2+a)^{3/2}}{x^2(dx^2+c)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$\downarrow$ 370

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{\int \frac{a\left(-bdx^2+bc-2ad\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 27

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{a\int \frac{-bdx^2+bc-2ad}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 445

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{a\left(-\frac{\int \frac{bd\left(ac-(bc-2ad)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}\right)}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 27

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{a\left(-\frac{bd\int \frac{ac-(bc-2ad)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}\right)}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 406

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{a\left(-\frac{bd\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}\right)}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 320

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{a\left(-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-2ad)}{acx}\right)}{cd}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 388

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{a\left(-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx}}{acx}\right)}{cd}\right)}{\sqrt{a+bx^2}}$$

$\downarrow$ 313

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{a\left(-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{ac}}{cd}\right)}{\sqrt{a+bx^2}}$$

input
```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-(((b*c - a*d)*Sqrt[
a + b*x^2])/(c*d*x*Sqrt[c + d*x^2])) - (a*(-(((b*c - 2*a*d)*Sqrt[a + b*x^2
]*Sqrt[c + d*x^2])/(a*c*x)) - (b*d*(-((b*c - 2*a*d)*((x*Sqrt[a + b*x^2])/(
b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)
/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2
))]*Sqrt[c + d*x^2]))) + (c^(3/2)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d
]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x
^2))]*Sqrt[c + d*x^2])))/(a*c)))/(c*d)))/Sqrt[a + b*x^2]
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] && !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(298) = 596$.

Time = 10.64 (sec) , antiderivative size = 670, normalized size of antiderivative = 2.15

method	result
default	$-\frac{\left(\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}\right)^{\frac{3}{2}}\left(d\,x^2+c\right)\left(\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^4+\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^4-\sqrt{db\,x^4+ad\,x^2+bc\,x^2+a \ldots}\right.}{\ldots}$
risch	$-\frac{a\left(d\,x^2+c\right)e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}{c^2x}+\frac{\left(\frac{b^2c^2\sqrt{1+\frac{x^2b}{a}}\,\sqrt{1+\frac{x^2d}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)}{d\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}-\frac{2d\,a^2bce\sqrt{1+\frac{x^2b}{a}}\,\sqrt{1+\frac{x^2d}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{- \ldots}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x \ldots}}\right.}{\ldots}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/
a)^(1/2)*a*b*d^2*x^4+(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*
d^2*x^4-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^4+((d*x
^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x
*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x-((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x
^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2
))*b^2*c^2*x-2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x+((d*x^2+c)*(b
*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^
(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*
a^2*d^2*x^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*b*c*d*x^2+(b*d*x^4+
a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a^2*d^2*x^2-(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*c*d*x^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^
(1/2)*a^2*c*d)/(b*x^2+a)^2/c^2/x/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c
)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.55

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx =$$

$$-\frac{(b^2c-2\,abd)\sqrt{\frac{ace}{d^2}}ex\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(b^2c-(a^2+2\,ab)d)\sqrt{\frac{ace}{d^2}}ex\sqrt{-\frac{b}{a}}F(\arcsin\left(x\sqrt{-\frac{b}{a}}\right.}{ac^2x}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
-((b^2*c - 2*a*b*d)*sqrt(a*c*e/d^2)*e*x*sqrt(-b/a)*elliptic_e(arcsin(x*sqr
t(-b/a)), a*d/(b*c)) - (b^2*c - (a^2 + 2*a*b)*d)*sqrt(a*c*e/d^2)*e*x*sqrt(
-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (a^2*c*e - (a*b*c - 2*
a^2*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^2, x)
```

Giac [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^2}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x, algorithm="giac")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^2}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{e}\,e\left(-2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ac-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ad\,x^2+\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\right.}{}$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^2,x)
```

output
```
(sqrt(e)*e*( - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c - sqrt(c + d*x**2)*
sqrt(a + b*x**2)*a*d*x**2 + sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*x**2 + i
nt((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**
2*x**4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*c*d**2*x + int((
sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x*
*4 + b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*d**3*x**3 - int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4
+ b*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c**2*d*x - int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b
*c**2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*b**2*c*d**2*x**3 - 3*int((sqrt
(c + d*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c*
*2*x**2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*c**2*d*x - 3*int((sqrt(c + d
*x**2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**
2 + 2*b*c*d*x**4 + b*d**2*x**6),x)*a**2*c*d**2*x**3 + 3*int((sqrt(c + d*x*
*2)*sqrt(a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 +
 2*b*c*d*x**4 + b*d**2*x**6),x)*a*b*c**3*x + 3*int((sqrt(c + d*x**2)*sqrt(
a + b*x**2))/(a*c**2 + 2*a*c*d*x**2 + a*d**2*x**4 + b*c**2*x**2 + 2*b*c*d*
x**4 + b*d**2*x**6),x)*a*b*c**2*d*x**3))/(2*c**2*x*(c + d*x**2))
```

3.66 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 395

$$\begin{aligned}\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx &= -\frac{(bc-ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{cdx^3} \\ &\quad -\frac{(7bc-8ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{3c^2x}+\frac{(3bc-4ad)e(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{3c^2dx^3} \\ &\quad -\frac{\sqrt{d}(7bc-8ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3c^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} \\ &\quad +\frac{b(3bc-4ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3ac^{3/2}\sqrt{d}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\end{aligned}$$

output
```
-(-a*d+b*c)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c/d/x^3-1/3*(-8*a*d+7
*b*c)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c^2/x+1/3*(-4*a*d+3*b*c)*e*
(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c^2/d/x^3-1/3*d^(1/2)*(-8
*a*d+7*b*c)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*x/c
^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/c^(5/2)/(c*(b*x^2+a)/a/(d*x^2+
c))^(1/2)+1/3*b*(-4*a*d+3*b*c)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)*In
verseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(3/2)/d^(1/
2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.79 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.64

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}(a+bx^2)\left(-bcx^2(4c+7dx^2)+a(-c^2+4cdx^2+8d^2x^4)\right)+ibc(-7bc\right.}{}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)*(-(b*c*x^2*(4*
c + 7*d*x^2)) + a*(-c^2 + 4*c*d*x^2 + 8*d^2*x^4)) + I*b*c*(-7*b*c + 8*a*d)
*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]
*x], (a*d)/(b*c)] - (4*I)*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[
1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]))/(3*Sqrt[b/
a]*c^3*x^3*(a + b*x^2))
```

Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {2058, 370, 445, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{(bx^2+a)^{3/2}}{x^4(dx^2+c)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{370}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{\int \frac{b(2bc-3ad)x^2+a(3bc-4ad)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx^3\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{445}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{\int \frac{ad\left(b(3bc-4ad)x^2+a(7bc-8ad)\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{3cx^3}}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx^3\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\int \frac{b(3bc-4ad)x^2+a(7bc-8ad)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{3cx^3}}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx^3\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{445}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\left(-\frac{\int -\frac{ab\left(d(7bc-8ad)x^2+c(3bc-4ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{cx}\right)}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{3cx^3}}{cd}-\frac{\sqrt{a+bx^2}(\ldots}{cdx^3\sqrt{c\ldots}}\right.}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\left(\frac{\int \frac{ab\left(d(7bc-8ad)x^2+c(3bc-4ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{cx}\right)}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{3cx^3}}{cd}-\frac{\sqrt{a+bx^2}(bc-}{cdx^3\sqrt{c+d}}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 27

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\left(\frac{b\int \frac{d(7bc-8ad)x^2+c(3bc-4ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{cx}\right)}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(3bc-4ad)}{3cx^3}}{cd}-\frac{\sqrt{a+bx^2}(bc-ad}{cdx^3\sqrt{c+dx^2}}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 406

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\left(\frac{b\left(c(3bc-4ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d(7bc-8ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{cx}\right)}{3c}-\frac{\sqrt{a+bx^2}}{}}{cd}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 320

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\left(\frac{b\left(d(7bc-8ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-4ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(}{cx}}{3c}}{cd}}{\sqrt{a+bx^2}}$$

$\downarrow$ 388

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\left(\frac{b\left(d(7bc-8ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{(dx^2+c)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-4ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{c}-\sqrt{a+}}{3c}}{cd}}{\sqrt{a+bx^2}}$$

$\downarrow$ 313

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{d\left(\frac{b\left(\frac{c^{3/2}\sqrt{a+bx^2}(3bc-4ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d(7bc-8ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)\right|}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right.}}}\right.\right.}{c}}{3c}}{cd}\right.}{\sqrt{a+bx^2}}$$

input
```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-(((b*c - a*d)*Sqrt[
a + b*x^2])/(c*d*x^3*Sqrt[c + d*x^2])) - (-1/3*((3*b*c - 4*a*d)*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(c*x^3) - (d*(-(((7*b*c - 8*a*d)*Sqrt[a + b*x^2]*Sq
rt[c + d*x^2])/(c*x)) + (b*(d*(7*b*c - 8*a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt
[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[
c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sq
rt[c + d*x^2])) + (c^(3/2)*(3*b*c - 4*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTa
n[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/
(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/c))/(3*c))/(c*d)))/Sqrt[a + b*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 789 vs. $2(372) = 744$.

Time = 11.42 (sec) , antiderivative size = 790, normalized size of antiderivative = 2.00

method	result
default	$-\frac{\left(\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}\right)^{\frac{3}{2}}\left(d\,x^{2}+c\right)\left(-5\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{-\frac{b}{a}}\,ab\,d^{2}x^{6}+4\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{-\frac{b}{a}}\,b^{2}cd\,x^{6}-3\sqrt{db\,x^{4}+ad\,x^{2}+bc\,x^{2}+a}\right.}{}$
risch	$-\frac{\left(d\,x^{2}+c\right)\left(-5ad\,x^{2}+4bc\,x^{2}+ac\right)e\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}{3c^{3}x^{3}}-\frac{\left(\frac{abcd\sqrt{1+\frac{x^{2}b}{a}}\,\sqrt{1+\frac{x^{2}d}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}}-3\left(a^{2}d^{2}-2abcd+b^{2}\right.\right.}{}$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(e*(b*x^2+a)/(d*x^2+c))^(3/2)*(d*x^2+c)*(-5*((d*x^2+c)*(b*x^2+a))^(1/
2)*(-b/a)^(1/2)*a*b*d^2*x^6+4*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b^2
*c*d*x^6-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^6+3*
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^6-4*((d*x^2+c)*
(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a
)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x^3+4*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2
+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
*b^2*c^2*x^3+8*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/
c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x^3-7*((d*x^2+c
)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b
/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x^3-5*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a
)^(1/2)*a^2*d^2*x^4+4*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b^2*c^2*x^4
-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a^2*d^2*x^4+3*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*c*d*x^4-4*((d*x^2+c)*(b*x^2+a
))^(1/2)*(-b/a)^(1/2)*a^2*c*d*x^2+5*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/
2)*a*b*c^2*x^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a^2*c^2)/(b*x^2+a)
^2/c^3/x^3/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.54

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \frac{(7\,b^2cd-8\,abd^2)\sqrt{\frac{ace}{d^2}}ex^3\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-((3\,ab+7\,b^2)cd-4\,(a^2+$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="fricas")
```

output
```
1/3*((7*b^2*c*d - 8*a*b*d^2)*sqrt(a*c*e/d^2)*e*x^3*sqrt(-b/a)*elliptic_e(a
rcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((3*a*b + 7*b^2)*c*d - 4*(a^2 + 2*a*b)*d
^2)*sqrt(a*c*e/d^2)*e*x^3*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/
(b*c)) - ((7*a*b*c*d - 8*a^2*d^2)*e*x^4 + a^2*c^2*e + 4*(a*b*c^2 - a^2*c*d
)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*c^3*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**4,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^4, x)
```

Giac [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^4}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x, algorithm="giac")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^4}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^4, x)
```

Reduce [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^4}\,dx = \text{too large to display}$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^4,x)
```

output
```
(sqrt(e)*e*( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a - 8*int((sqrt(c + d*x**
2)*sqrt(a + b*x**2))/(2*a**2*c**2*d*x**2 + 4*a**2*c*d**2*x**4 + 2*a**2*d**
3*x**6 + a*b*c**3*x**2 + 4*a*b*c**2*d*x**4 + 5*a*b*c*d**2*x**6 + 2*a*b*d**
3*x**8 + b**2*c**3*x**4 + 2*b**2*c**2*d*x**6 + b**2*c*d**2*x**8),x)*a**3*c
*d**2*x**3 - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(2*a**2*c**2*d*x**2
 + 4*a**2*c*d**2*x**4 + 2*a**2*d**3*x**6 + a*b*c**3*x**2 + 4*a*b*c**2*d*x*
*4 + 5*a*b*c*d**2*x**6 + 2*a*b*d**3*x**8 + b**2*c**3*x**4 + 2*b**2*c**2*d*
x**6 + b**2*c*d**2*x**8),x)*a**3*d**3*x**5 + 4*int((sqrt(c + d*x**2)*sqrt(
a + b*x**2))/(2*a**2*c**2*d*x**2 + 4*a**2*c*d**2*x**4 + 2*a**2*d**3*x**6 +
 a*b*c**3*x**2 + 4*a*b*c**2*d*x**4 + 5*a*b*c*d**2*x**6 + 2*a*b*d**3*x**8 +
 b**2*c**3*x**4 + 2*b**2*c**2*d*x**6 + b**2*c*d**2*x**8),x)*a**2*b*c**2*d*
x**3 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(2*a**2*c**2*d*x**2 + 4*a
**2*c*d**2*x**4 + 2*a**2*d**3*x**6 + a*b*c**3*x**2 + 4*a*b*c**2*d*x**4 + 5
*a*b*c*d**2*x**6 + 2*a*b*d**3*x**8 + b**2*c**3*x**4 + 2*b**2*c**2*d*x**6 +
 b**2*c*d**2*x**8),x)*a**2*b*c*d**2*x**5 + 4*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2))/(2*a**2*c**2*d*x**2 + 4*a**2*c*d**2*x**4 + 2*a**2*d**3*x**6 + a
*b*c**3*x**2 + 4*a*b*c**2*d*x**4 + 5*a*b*c*d**2*x**6 + 2*a*b*d**3*x**8 + b
**2*c**3*x**4 + 2*b**2*c**2*d*x**6 + b**2*c*d**2*x**8),x)*a*b**2*c**3*x**3
 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(2*a**2*c**2*d*x**2 + 4*a**2*
c*d**2*x**4 + 2*a**2*d**3*x**6 + a*b*c**3*x**2 + 4*a*b*c**2*d*x**4 + 5*...
```

3.67 $\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 488

$$\begin{aligned}
\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx &= -\frac{(bc-ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{cdx^5} \\
&\quad -\frac{(b^2c^2-16abcd+16a^2d^2)\,e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{5ac^3x} \\
&\quad +\frac{(5bc-6ad)e(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{5c^2dx^5}-\frac{(7bc-8ad)e(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{5c^3x^3} \\
&\quad -\frac{\sqrt{d}(b^2c^2-16abcd+16a^2d^2)\,e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\,E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{5ac^{7/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}} \\
&\quad -\frac{b\sqrt{d}(7bc-8ad)e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}\,\mathrm{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{5ac^{5/2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}
\end{aligned}$$

output
```
-(-a*d+b*c)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c/d/x^5-1/5*(16*a^2*d
^2-16*a*b*c*d+b^2*c^2)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a/c^3/x+1/
5*(-6*a*d+5*b*c)*e*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/c^2/d/
x^5-1/5*(-8*a*d+7*b*c)*e*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/
c^3/x^3-1/5*d^(1/2)*(16*a^2*d^2-16*a*b*c*d+b^2*c^2)*e*(b*e/d-(-a*d+b*c)*e/
d/(d*x^2+c))^(1/2)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/
d)^(1/2))/a/c^(7/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)-1/5*b*d^(1/2)*(-8*a*d+
7*b*c)*e*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)*InverseJacobiAM(arctan(d^(
1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a/c^(5/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/
2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.34 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.66

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx =$$

$$-\frac{\sqrt{\frac{b}{a}}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}(a+bx^2)\left(b^2c^2x^4(c+dx^2)+abcx^2(2c^2-9cdx^2-16d^2x^4)+a^2(c^3-2c^2dx^2+8cd^2x\right.\right.}{}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x]
```

output
```
-1/5*(Sqrt[b/a]*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*(a + b*x^2)
*(b^2*c^2*x^4*(c + d*x^2) + a*b*c*x^2*(2*c^2 - 9*c*d*x^2 - 16*d^2*x^4) + a
^2*(c^3 - 2*c^2*d*x^2 + 8*c*d^2*x^4 + 16*d^3*x^6)) + I*b*c*(b^2*c^2 - 16*a
*b*c*d + 16*a^2*d^2)*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE
[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - I*b*c*(b^2*c^2 - 9*a*b*c*d + 8*a^2
*d^2)*x^5*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt
[b/a]*x], (a*d)/(b*c)]))/(b*c^4*x^5*(a + b*x^2))
```

Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2058, 370, 445, 27, 445, 25, 27, 445, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\int \frac{(bx^2+a)^{3/2}}{x^6(dx^2+c)^{3/2}}dx}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 370}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{\int \frac{b(4bc-5ad)x^2+a(5bc-6ad)}{x^6\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx^5\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{\int \frac{3ad\left(b(5bc-6ad)x^2+a(7bc-8ad)\right)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(5bc-6ad)}{5cx^5}}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx^5\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\int \frac{b(5bc-6ad)x^2+a(7bc-8ad)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(5bc-6ad)}{5cx^5}}{cd}-\frac{\sqrt{a+bx^2}(bc-ad)}{cdx^5\sqrt{c+dx^2}}\right)}{\sqrt{a+bx^2}}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(-\frac{\int -\frac{a\left(b^2c^2-16abdc+16a^2d^2-bd(7bc-8ad)x^2\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3cx^3}\right)}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(5bc-6ad)}{5cx^5}}{cd}-\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 25

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(\frac{\int \frac{a\left(b^2c^2-16abdc+16a^2d^2-bd(7bc-8ad)x^2\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3cx^3}\right)}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(5bc-6ad)}{5cx^5}}{cd}-\sqrt{\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 27

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(\frac{\int \frac{b^2c^2-16abdc+16a^2d^2-bd(7bc-8ad)x^2}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3cx^3}\right)}{5c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(5bc-6ad)}{5cx^5}}{cd}-\frac{\sqrt{a+}}{cdx}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 445

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(\frac{-\frac{\int \frac{bd\left(ac(7bc-8ad)-\left(b^2c^2-16abdc+16a^2d^2\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{b^2c}{a}+\frac{16ad^2}{c}-16bd\right)}{x}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7}{3cx^3}\right.}{5c}}{cd}\right.}{\sqrt{a+bx^2}}$$

$\downarrow$ 27

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(-\frac{-\frac{bd\int\frac{ac(7bc-8ad)-\left(b^2c^2-16abdc+16a^2d^2\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{b^2c}{a}+\frac{16ad^2}{c}-16bd\right)}{x}}{3c}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc\cdots}{3cx^3}\cdots\right.}{5c}\cdots}{cd}\cdots\right.}{\sqrt{a+bx^2}\cdots}$$

$\big\downarrow$ 406

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(-\frac{-\frac{bd\left(ac(7bc-8ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(16a^2d^2-16abcd+b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}\left(\frac{b^2c}{a}\cdots\right.}{x}}{3c}\cdots\right.}{5c}\cdots}{cd}\cdots\right.}{\sqrt{a+bx^2}\cdots}$$

$\big\downarrow$ 320

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(-\frac{-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}(7bc-8ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(16a^2d^2-16abcd+b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\cdots}{3c}\cdots\right.}{5c}\cdots}{cd}\cdots\right.}{\sqrt{a+bx^2}\cdots}$$

$\big\downarrow$ 388

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}(7bc-8ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(16a^2d^2-16abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+\cdots}}{(dx^2+c)\cdots}}{b}\cdots}{ac}\cdots}{3c}\cdots}{5c}\cdots}{cd}\cdots\right)}{\sqrt{a+bx^2}\cdots}$$

$$\Big\downarrow \; 313$$

$$\frac{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\frac{-\frac{3d\left(-\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}(7bc-8ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}-\left(16a^2d^2-16abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}\,E\cdots}{b\sqrt{d}\sqrt{c}\cdots}\cdots}{ac}\cdots}{3c}\cdots}{5c}\cdots\right)}{\sqrt{a+\cdots}}$$

input

```
Int[((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x]
```

output
```
(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2]*(-(((b*c - a*d)*Sqrt[
a + b*x^2])/(c*d*x^5*Sqrt[c + d*x^2])) - (-1/5*((5*b*c - 6*a*d)*Sqrt[a + b
*x^2]*Sqrt[c + d*x^2])/(c*x^5) - (3*d*(-1/3*((7*b*c - 8*a*d)*Sqrt[a + b*x^
2]*Sqrt[c + d*x^2])/(c*x^3) + (-((((b^2*c)/a - 16*b*d + (16*a*d^2)/c)*Sqrt
[a + b*x^2]*Sqrt[c + d*x^2])/x) - (b*d*(-((b^2*c^2 - 16*a*b*c*d + 16*a^2*d
^2)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*El
lipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*(7*b*c - 8*a*d
)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/
(3*c)))/(5*c))/(c*d)))/Sqrt[a + b*x^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 14.29 (sec) , antiderivative size = 906, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{(d\,x^2+c)(11a^2d^2x^4-11abcd\,x^4+b^2c^2x^4-3a^2cd\,x^2+2ab\,c^2x^2+a^2c^2)e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}{5c^4x^5a}+\frac{d\left(-\frac{2b(11a^2d^2-11abcd+b^2c^2)ace\sqrt{1+\frac{x^2b}{a}}}{\sqrt{-\frac{b}{a}}}\right.}{}$
default	Expression too large to display

input

```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output

```
-1/5*(d*x^2+c)*(11*a^2*d^2*x^4-11*a*b*c*d*x^4+b^2*c^2*x^4-3*a^2*c*d*x^2+2*
a*b*c^2*x^2+a^2*c^2)/c^4/x^5/a*e*(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/5/c^4/a*d
*(-2*b*(11*a^2*d^2-11*a*b*c*d+b^2*c^2)*a*c*e/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1
/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(a*d*e
+b*c*e+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/
2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))-2*a*c^2*b^2/
(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+
b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(
1/2))+3*a^2*b*c*d/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*
d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d
*e+b*c*e)/c/b/e)^(1/2))-5*(a^2*d^2-2*a*b*c*d+b^2*c^2)*a*c*((b*d*e*x^2+a*d*
e)/c/(a*d-b*c)*x/e/((x^2+c/d)*(b*d*e*x^2+a*d*e))^(1/2)+(1/c-a*d/c/(a*d-b*c
))/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x
^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e
)^(1/2))+2/(a*d-b*c)*b*d*a*e/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d
)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(a*d*e+b*c*e+e*(a*d-b*
c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*
(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))))*e/(b*x^2+a)*(e*(b*x^2+a)/(
d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.60

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \frac{(b^3c^2d-16\,ab^2cd^2+16\,a^2bd^3)\sqrt{\frac{ace}{d^2}}ex^5\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(b^3c^2d-(7\,c}{}$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="fricas")
```

output
```
1/5*((b^3*c^2*d - 16*a*b^2*c*d^2 + 16*a^2*b*d^3)*sqrt(a*c*e/d^2)*e*x^5*sqr
t(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (b^3*c^2*d - (7*a^2*
b + 16*a*b^2)*c*d^2 + 8*(a^3 + 2*a^2*b)*d^3)*sqrt(a*c*e/d^2)*e*x^5*sqrt(-b
/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((a*b^2*c^2*d - 16*a^2*b
*c*d^2 + 16*a^3*d^3)*e*x^6 + a^3*c^3*e + (a*b^2*c^3 - 9*a^2*b*c^2*d + 8*a^
3*c*d^2)*e*x^4 + 2*(a^2*b*c^3 - a^3*c^2*d)*e*x^2)*sqrt((b*e*x^2 + a*e)/(d*
x^2 + c)))/(a^2*c^4*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \text{Timed out}$$

input
```
integrate((e*(b*x**2+a)/(d*x**2+c))**(3/2)/x**6,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^6, x)
```

Giac [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \int \frac{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}{x^6}\,dx$$

input
```
integrate((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x, algorithm="giac")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)/x^6, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \int \frac{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}{x^6}\,dx$$

input
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6,x)
```

output
```
int(((e*(a + b*x^2))/(c + d*x^2))^(3/2)/x^6, x)
```

Reduce [F]

$$\int \frac{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}{x^6}\,dx = \text{too large to display}$$

input
```
int((e*(b*x^2+a)/(d*x^2+c))^(3/2)/x^6,x)
```

output
```
(sqrt(e)*e*( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*d - 2*sqrt(c + d*x
**2)*sqrt(a + b*x**2)*a**2*b*c + 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*
*2*d*x**4 - 10*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*x**4 + 30*int((sqr
t(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**2 +
 3*a**2*d**3*x**4 + 2*a*b*c**3 + 7*a*b*c**2*d*x**2 + 8*a*b*c*d**2*x**4 + 3
*a*b*d**3*x**6 + 2*b**2*c**3*x**2 + 4*b**2*c**2*d*x**4 + 2*b**2*c*d**2*x**
6),x)*a**2*b**3*c*d**3*x**5 + 30*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x*
*2)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**2 + 3*a**2*d**3*x**4 + 2*a*b*c**3 +
7*a*b*c**2*d*x**2 + 8*a*b*c*d**2*x**4 + 3*a*b*d**3*x**6 + 2*b**2*c**3*x**2
 + 4*b**2*c**2*d*x**4 + 2*b**2*c*d**2*x**6),x)*a**2*b**3*d**4*x**7 - 10*in
t((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(3*a**2*c**2*d + 6*a**2*c*d**2*
x**2 + 3*a**2*d**3*x**4 + 2*a*b*c**3 + 7*a*b*c**2*d*x**2 + 8*a*b*c*d**2*x*
*4 + 3*a*b*d**3*x**6 + 2*b**2*c**3*x**2 + 4*b**2*c**2*d*x**4 + 2*b**2*c*d*
*2*x**6),x)*a*b**4*c**2*d**2*x**5 - 10*int((sqrt(c + d*x**2)*sqrt(a + b*x*
*2)*x**2)/(3*a**2*c**2*d + 6*a**2*c*d**2*x**2 + 3*a**2*d**3*x**4 + 2*a*b*c
**3 + 7*a*b*c**2*d*x**2 + 8*a*b*c*d**2*x**4 + 3*a*b*d**3*x**6 + 2*b**2*c**
3*x**2 + 4*b**2*c**2*d*x**4 + 2*b**2*c*d**2*x**6),x)*a*b**4*c*d**3*x**7 -
20*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(3*a**2*c**2*d + 6*a**2*c*
d**2*x**2 + 3*a**2*d**3*x**4 + 2*a*b*c**3 + 7*a*b*c**2*d*x**2 + 8*a*b*c*d*
*2*x**4 + 3*a*b*d**3*x**6 + 2*b**2*c**3*x**2 + 4*b**2*c**2*d*x**4 + 2*b...
```

3.68 $\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 43

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx=\frac{1}{2}(1+x^2)\sqrt{-1+\frac{2}{1+x^2}}-\arctan\left(\sqrt{-1+\frac{2}{1+x^2}}\right)$$

output
```
1/2*(x^2+1)*(-1+2/(x^2+1))^(1/2)-arctan((-1+2/(x^2+1))^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 95 vs. 2(43) = 86.

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.21

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx=\frac{\sqrt{\frac{1-x^2}{1+x^2}}\left(\sqrt{1-x^2}(1+x^2)+4\sqrt{1+x^2}\arctan\left(\frac{\sqrt{1-x^2}}{\sqrt{2}-\sqrt{1+x^2}}\right)\right)}{2\sqrt{1-x^2}}$$

input
```
Integrate[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
```

output
```
(Sqrt[(1 - x^2)/(1 + x^2)]*(Sqrt[1 - x^2]*(1 + x^2) + 4*Sqrt[1 + x^2]*ArcT
an[Sqrt[1 - x^2]/(Sqrt[2] - Sqrt[1 + x^2])]))/(2*Sqrt[1 - x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.33, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {2053, 2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\frac{1-x^2}{x^2+1}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \sqrt{\frac{1-x^2}{x^2+1}}dx^2$$

$$\Big\downarrow \text{2051}$$

$$-2\int \frac{x^4}{(x^4+1)^2}d\sqrt{\frac{1-x^2}{x^2+1}}$$

$$\Big\downarrow \text{252}$$

$$-2\left(\frac{1}{2}\int \frac{1}{x^4+1}d\sqrt{\frac{1-x^2}{x^2+1}}-\frac{\sqrt{\frac{1-x^2}{x^2+1}}}{2(x^4+1)}\right)$$

$$\Big\downarrow \text{216}$$

$$-2\left(\frac{1}{2}\arctan\left(\sqrt{\frac{1-x^2}{x^2+1}}\right)-\frac{\sqrt{\frac{1-x^2}{x^2+1}}}{2(x^4+1)}\right)$$

input
```
Int[x*Sqrt[(1 - x^2)/(1 + x^2)],x]
```

output
```
-2*(-1/2*Sqrt[(1 - x^2)/(1 + x^2)]/(1 + x^4) + ArcTan[Sqrt[(1 - x^2)/(1 +
x^2)]]/2)
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

method	result	size
default	$\frac{\sqrt{-\frac{x^2-1}{x^2+1}}\,(x^2+1)\left(\sqrt{-x^4+1}+\arcsin(x^2)\right)}{2\sqrt{-(x^2-1)(x^2+1)}}$	52
risch	$\frac{(x^2+1)\sqrt{-\frac{x^2-1}{x^2+1}}}{2}-\frac{\arcsin(x^2)\sqrt{-\frac{x^2-1}{x^2+1}}\,\sqrt{-(x^2-1)(x^2+1)}}{2(x^2-1)}$	68
trager	$\left(\frac{x^2}{2}+\frac{1}{2}\right)\sqrt{-\frac{x^2-1}{x^2+1}}+\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}}\,x^2+\mathrm{RootOf}\left(\_Z^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}}+x^2\right)}{2}$	88

input
```
int(x*((-x^2+1)/(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-(x^2-1)/(x^2+1))^(1/2)*(x^2+1)*((-x^4+1)^(1/2)+arcsin(x^2))/(-(x^2-1
)*(x^2+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx = \frac{1}{2}\left(x^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}} - \arctan\left(\frac{\left(x^2+1\right)\sqrt{-\frac{x^2-1}{x^2+1}}-1}{x^2}\right)$$

input
```
integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="fricas")
```

output
```
1/2*(x^2 + 1)*sqrt(-(x^2 - 1)/(x^2 + 1)) - arctan(((x^2 + 1)*sqrt(-(x^2 -
1)/(x^2 + 1)) - 1)/x^2)
```

Sympy [F]

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx = \int x\sqrt{-\frac{(x-1)\,(x+1)}{x^2+1}}\,dx$$

input
```
integrate(x*((-x**2+1)/(x**2+1))**(1/2),x)
```

output
```
Integral(x*sqrt(-(x - 1)*(x + 1)/(x**2 + 1)), x)
```

Maxima [F]

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx=\int x\sqrt{-\frac{x^2-1}{x^2+1}}\,dx$$

input
```
integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(-(x^2 - 1)/(x^2 + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.42

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx=\frac{1}{2}\sqrt{-x^4+1}+\frac{1}{2}\arcsin\left(x^2\right)$$

input
```
integrate(x*((-x^2+1)/(x^2+1))^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^4 + 1) + 1/2*arcsin(x^2)
```

Mupad [B] (verification not implemented)

Time = 9.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.28

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx=-\mathrm{atan}\left(\sqrt{-\frac{x^2-1}{x^2+1}}\right)-\frac{\sqrt{-\frac{x^2-1}{x^2+1}}}{\frac{x^2-1}{x^2+1}-1}$$

input
```
int(x*(-(x^2 - 1)/(x^2 + 1))^(1/2),x)
```

output
```
- atan((-(x^2 - 1)/(x^2 + 1))^(1/2)) - (-(x^2 - 1)/(x^2 + 1))^(1/2)/((x^2
- 1)/(x^2 + 1) - 1)
```

Reduce [F]

$$\int x\sqrt{\frac{1-x^2}{1+x^2}}\,dx = \frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2} - \left(\int \frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,x}{x^4-1}dx\right)$$

input
```
int(x*((-x^2+1)/(x^2+1))^(1/2),x)
```

output
```
(sqrt( - x**2 + 1)*sqrt(x**2 + 1) - 2*int((sqrt( - x**2 + 1)*sqrt(x**2 + 1
)*x)/(x**4 - 1),x))/2
```

3.69 $\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx=\frac{(7+5x^2)\sqrt{-7+\frac{74}{7+5x^2}}}{10\sqrt{5}}-\frac{37\arctan\left(\frac{\sqrt{-7+\frac{74}{7+5x^2}}}{\sqrt{7}}\right)}{5\sqrt{35}}$$

output
```
1/50*(5*x^2+7)*(-7+74/(5*x^2+7))^(1/2)*5^(1/2)-37/175*arctan(1/7*(-7+74/(5
*x^2+7))^(1/2)*7^(1/2))*35^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.78

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx$$
$$=\frac{\sqrt{\frac{5-7x^2}{7+5x^2}}\left(35\sqrt{5-7x^2}(7+5x^2)+148\sqrt{35}\sqrt{7+5x^2}\arctan\left(\frac{\sqrt{5}\sqrt{5-7x^2}}{\sqrt{74}-\sqrt{7}\sqrt{7+5x^2}}\right)\right)}{350\sqrt{5-7x^2}}$$

input
```
Integrate[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

output
```
(Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]*(35*Sqrt[5 - 7*x^2]*(7 + 5*x^2) + 148*Sqrt[
35]*Sqrt[7 + 5*x^2]*ArcTan[(Sqrt[5]*Sqrt[5 - 7*x^2])/(Sqrt[74] - Sqrt[7]*S
qrt[7 + 5*x^2])]))/(350*Sqrt[5 - 7*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2053, 2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{\frac{5-7x^2}{5x^2+7}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \sqrt{\frac{5-7x^2}{5x^2+7}}dx^2$$

$$\Big\downarrow \text{2051}$$

$$-74\int \frac{x^4}{(5x^4+7)^2}d\sqrt{\frac{5-7x^2}{5x^2+7}}$$

$$\Big\downarrow \text{252}$$

$$-74\left(\frac{1}{10}\int \frac{1}{5x^4+7}d\sqrt{\frac{5-7x^2}{5x^2+7}}-\frac{\sqrt{\frac{5-7x^2}{5x^2+7}}}{10(5x^4+7)}\right)$$

$$\Big\downarrow \text{216}$$

$$-74\left(\frac{\arctan\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^2}{5x^2+7}}\right)}{10\sqrt{35}}-\frac{\sqrt{\frac{5-7x^2}{5x^2+7}}}{10(5x^4+7)}\right)$$

input
```
Int[x*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)],x]
```

output
```
-74*(-1/10*Sqrt[(5 - 7*x^2)/(7 + 5*x^2)]/(7 + 5*x^4) + ArcTan[Sqrt[5/7]*Sq
rt[(5 - 7*x^2)/(7 + 5*x^2)]]/(10*Sqrt[35]))
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x))/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

method	result
default	$\frac{\sqrt{-\frac{7x^2-5}{5x^2+7}}\,(5x^2+7)\left(37\sqrt{35}\,\arcsin\left(\frac{35x^2}{37}+\frac{12}{37}\right)+35\sqrt{-35x^4-24x^2+35}\right)}{350\sqrt{-(7x^2-5)(5x^2+7)}}$
risch	$\frac{(5x^2+7)\sqrt{-\frac{7x^2-5}{5x^2+7}}}{10}-\frac{37\sqrt{35}\,\arcsin\left(\frac{35x^2}{37}+\frac{12}{37}\right)\sqrt{-\frac{7x^2-5}{5x^2+7}}\,\sqrt{-(7x^2-5)(5x^2+7)}}{350(7x^2-5)}$
trager	$7\left(\frac{x^2}{14}+\frac{1}{10}\right)\sqrt{-\frac{7x^2-5}{5x^2+7}}+\frac{37\,\mathrm{RootOf}\left(\_Z^2+35\right)\ln\left(-35\,\mathrm{RootOf}\left(\_Z^2+35\right)x^2+175\sqrt{-\frac{7x^2-5}{5x^2+7}}\,x^2-12\,\mathrm{RootOf}\left(\_Z^2+3\right.\right.}{350}$

input
```
int(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/350*(-(7*x^2-5)/(5*x^2+7))^(1/2)*(5*x^2+7)*(37*35^(1/2)*arcsin(35/37*x^2
+12/37)+35*(-35*x^4-24*x^2+35)^(1/2))/(-(7*x^2-5)*(5*x^2+7))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx=\frac{1}{10}\left(5\,x^2+7\right)\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}-\frac{37}{350}\sqrt{35}\arctan\left(\frac{\sqrt{35}(5\,x^2+7)\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{35\,x^2+12}\right)$$

input
```
integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="fricas")
```

output
```
1/10*(5*x^2 + 7)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7)) - 37/350*sqrt(35)*arctan(s
qrt(35)*(5*x^2 + 7)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(35*x^2 + 12))
```

Sympy [F]

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx = \int x\sqrt{-\frac{7x^2-5}{5x^2+7}}\,dx$$

input
```
integrate(x*((-7*x**2+5)/(5*x**2+7))**(1/2),x)
```

output
```
Integral(x*sqrt(-(7*x**2 - 5)/(5*x**2 + 7)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx = -\frac{37}{175}\sqrt{35}\arctan\left(\frac{1}{7}\sqrt{35}\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}\right) - \frac{37\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{5\left(\frac{5\,(7\,x^2-5)}{5\,x^2+7}-7\right)}$$

input
```
integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="maxima")
```

output
```
-37/175*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))) - 37/
5*sqrt(-(7*x^2 - 5)/(5*x^2 + 7))/(5*(7*x^2 - 5)/(5*x^2 + 7) - 7)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.45

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx = \frac{37}{350}\sqrt{35}\arcsin\left(\frac{35}{37}\,x^2+\frac{12}{37}\right) + \frac{1}{10}\sqrt{-35\,x^4-24\,x^2+35}$$

input
```
integrate(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x, algorithm="giac")
```

output
```
37/350*sqrt(35)*arcsin(35/37*x^2 + 12/37) + 1/10*sqrt(-35*x^4 - 24*x^2 + 3
5)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx = -\frac{37\sqrt{35}\,\mathrm{atan}\left(\frac{\sqrt{5}\,\sqrt{7}\,\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{7}\right)}{175} - \frac{37\,\sqrt{5}\,\sqrt{7}\,\sqrt{35}\,\sqrt{-\frac{7\,x^2-5}{5\,x^2+7}}}{1225\,\left(\frac{5\,x^2-\frac{25}{7}}{5\,x^2+7}-1\right)}$$

input
```
int(x*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2),x)
```

output
```
- (37*35^(1/2)*atan((5^(1/2)*7^(1/2)*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2))/7))
/175 - (37*5^(1/2)*7^(1/2)*35^(1/2)*(-(7*x^2 - 5)/(5*x^2 + 7))^(1/2))/(122
5*((5*x^2 - 25/7)/(5*x^2 + 7) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int x\sqrt{\frac{5-7x^2}{7+5x^2}}\,dx = -\frac{37\sqrt{35}\,atan\left(\frac{\sqrt{5x^2+7}\,\sqrt{-7x^2+5}\,\sqrt{35}}{35x^2-25}\right)}{175} + \frac{\sqrt{5x^2+7}\,\sqrt{-7x^2+5}}{10}$$

input
```
int(x*((-7*x^2+5)/(5*x^2+7))^(1/2),x)
```

output
```
( - 74*sqrt(35)*atan((sqrt(5*x**2 + 7)*sqrt( - 7*x**2 + 5)*sqrt(35))/(35*x
**2 - 25)) + 35*sqrt(5*x**2 + 7)*sqrt( - 7*x**2 + 5))/350
```

3.70 $\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 45

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\frac{1}{3}(1+x^3)\sqrt{-1+\frac{2}{1+x^3}}-\frac{2}{3}\arctan\left(\sqrt{-1+\frac{2}{1+x^3}}\right)$$

output
```
1/3*(x^3+1)*(-1+2/(x^3+1))^(1/2)-2/3*arctan((-1+2/(x^3+1))^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 95 vs. 2(45) = 90.

Time = 0.23 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.11

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\frac{\sqrt{\frac{1-x^3}{1+x^3}}\left(\sqrt{1-x^3}(1+x^3)+4\sqrt{1+x^3}\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{2}-\sqrt{1+x^3}}\right)\right)}{3\sqrt{1-x^3}}$$

input
```
Integrate[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

output
```
(Sqrt[(1 - x^3)/(1 + x^3)]*(Sqrt[1 - x^3]*(1 + x^3) + 4*Sqrt[1 + x^3]*ArcT
an[Sqrt[1 - x^3]/(Sqrt[2] - Sqrt[1 + x^3])]))/(3*Sqrt[1 - x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.31, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {2053, 2051, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{\frac{1-x^3}{x^3+1}}dx$$
$$\Big\downarrow \text{2053}$$
$$\frac{1}{3}\int \sqrt{\frac{1-x^3}{x^3+1}}dx^3$$
$$\Big\downarrow \text{2051}$$
$$-\frac{4}{3}\int \frac{x^6}{(x^6+1)^2}d\sqrt{\frac{1-x^3}{x^3+1}}$$
$$\Big\downarrow \text{252}$$
$$-\frac{4}{3}\left(\frac{1}{2}\int \frac{1}{x^6+1}d\sqrt{\frac{1-x^3}{x^3+1}}-\frac{\sqrt{\frac{1-x^3}{x^3+1}}}{2(x^6+1)}\right)$$
$$\Big\downarrow \text{216}$$
$$-\frac{4}{3}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)-\frac{\sqrt{\frac{1-x^3}{x^3+1}}}{2(x^6+1)}\right)$$

input
```
Int[x^2*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

output
```
(-4*(-1/2*Sqrt[(1 - x^3)/(1 + x^3)]/(1 + x^6) + ArcTan[Sqrt[(1 - x^3)/(1 +
 x^3)]]/2))/3
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

method	result	size
risch	$\frac{(x^3+1)\sqrt{-\frac{x^3-1}{x^3+1}}}{3}-\frac{\arcsin(x^3)\sqrt{-\frac{x^3-1}{x^3+1}}\,\sqrt{-(x^3+1)(x^3-1)}}{3(x^3-1)}$	68
trager	$\left(\frac{x^3}{3}+\frac{1}{3}\right)\sqrt{-\frac{x^3-1}{x^3+1}}+\frac{\operatorname{RootOf}(\_Z^2+1)\ln\left(\operatorname{RootOf}(\_Z^2+1)\sqrt{-\frac{x^3-1}{x^3+1}}\,x^3+x^3+\operatorname{RootOf}(\_Z^2+1)\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3}$	88

input
```
int(x^2*((-x^3+1)/(x^3+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(x^3+1)*(-(x^3-1)/(x^3+1))^(1/2)-1/3*arcsin(x^3)*(-(x^3-1)/(x^3+1))^(1
/2)*(-(x^3+1)*(x^3-1))^(1/2)/(x^3-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.22

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \frac{1}{3}\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}} - \frac{2}{3}\arctan\left(\frac{\left(x^3+1\right)\sqrt{-\frac{x^3-1}{x^3+1}}-1}{x^3}\right)$$

input
```
integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="fricas")
```

output
```
1/3*(x^3 + 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 2/3*arctan(((x^3 + 1)*sqrt(-(x^
3 - 1)/(x^3 + 1)) - 1)/x^3)
```

Sympy [F]

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \int x^2\sqrt{-\frac{(x-1)\left(x^2+x+1\right)}{x^3+1}}\,dx$$

input
```
integrate(x**2*((-x**3+1)/(x**3+1))**(1/2),x)
```

output
```
Integral(x**2*sqrt(-(x - 1)*(x**2 + x + 1)/(x**3 + 1)), x)
```

Maxima [F]

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \int x^2\sqrt{-\frac{x^3-1}{x^3+1}}\,dx$$

input
```
integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.49

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \frac{1}{3}\left(\sqrt{-x^6+1}+\arcsin\left(x^3\right)\right)\operatorname{sgn}\left(x^3+1\right)$$

input
```
integrate(x^2*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")
```

output
```
1/3*(sqrt(-x^6 + 1) + arcsin(x^3))*sgn(x^3 + 1)
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx = -\frac{2\,\mathrm{atan}\left(\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3} - \frac{2\sqrt{-\frac{x^3-1}{x^3+1}}}{\frac{3\,(x^3-1)}{x^3+1}-3}$$

input
```
int(x^2*(-(x^3 - 1)/(x^3 + 1))^(1/2),x)
```

output
```
- (2*atan((-(x^3 - 1)/(x^3 + 1))^(1/2)))/3 - (2*(-(x^3 - 1)/(x^3 + 1))^(1/
2))/((3*(x^3 - 1))/(x^3 + 1) - 3)
```

Reduce [F]

$$\int x^2\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\int\frac{\sqrt{-x^3+1}\,\sqrt{x^3+1}\,x^2}{x^3+1}dx$$

input
```
int(x^2*((-x^3+1)/(x^3+1))^(1/2),x)
```

output
```
int((sqrt( - x**3 + 1)*sqrt(x**3 + 1)*x**2)/(x**3 + 1),x)
```

3.71 $\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \frac{1}{2}(1+x^3)\sqrt{-1+\frac{2}{1+x^3}} - \frac{7}{18}(1+x^3)^2\sqrt{-1+\frac{2}{1+x^3}} + \frac{1}{9}(1+x^3)^3\sqrt{-1+\frac{2}{1+x^3}} - \frac{1}{3}\arctan\left(\sqrt{-1+\frac{2}{1+x^3}}\right)$$

output
```
1/2*(x^3+1)*(-1+2/(x^3+1))^(1/2)-7/18*(x^3+1)^2*(-1+2/(x^3+1))^(1/2)+1/9*(
x^3+1)^3*(-1+2/(x^3+1))^(1/2)-1/3*arctan((-1+2/(x^3+1))^(1/2))
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.98

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \frac{\sqrt{\frac{1-x^3}{1+x^3}}\left(\sqrt{1-x^3}(4+x^3-x^6+2x^9)-6\sqrt{1+x^3}\arctan\left(\frac{\sqrt{1-x^3}}{\sqrt{1+x^3}}\right)\right)}{18\sqrt{1-x^3}}$$

input
```
Integrate[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

output
```
(Sqrt[(1 - x^3)/(1 + x^3)]*(Sqrt[1 - x^3]*(4 + x^3 - x^6 + 2*x^9) - 6*Sqrt
[1 + x^3]*ArcTan[Sqrt[1 - x^3]/Sqrt[1 + x^3]]))/(18*Sqrt[1 - x^3])
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {2053, 2052, 366, 27, 360, 27, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^8\sqrt{\frac{1-x^3}{x^3+1}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{3}\int x^6\sqrt{\frac{1-x^3}{x^3+1}}dx^3$$

$$\Big\downarrow \text{2052}$$

$$-\frac{4}{3}\int \frac{x^6\left(1-x^6\right)^2}{\left(x^6+1\right)^4}d\sqrt{\frac{1-x^3}{x^3+1}}$$

$$\Big\downarrow \text{366}$$

$$-\frac{4}{3}\left(\frac{2x^9}{3\left(x^6+1\right)^3}-\frac{1}{6}\int \frac{6x^6\left(1-x^6\right)}{\left(x^6+1\right)^3}d\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{4}{3}\left(\frac{2x^9}{3\left(x^6+1\right)^3}-\int \frac{x^6\left(1-x^6\right)}{\left(x^6+1\right)^3}d\sqrt{\frac{1-x^3}{x^3+1}}\right)$$

$$\Big\downarrow \text{360}$$

$$-\frac{4}{3}\left(\frac{1}{4}\int -\frac{2\left(1-2x^6\right)}{\left(x^6+1\right)^2}d\sqrt{\frac{1-x^3}{x^3+1}}+\frac{2x^9}{3\left(x^6+1\right)^3}+\frac{\sqrt{\frac{1-x^3}{x^3+1}}}{2\left(x^6+1\right)^2}\right)$$

$$\Big\downarrow \text{27}$$

$$-\frac{4}{3}\left(-\frac{1}{2}\int \frac{1-2x^6}{(x^6+1)^2}d\sqrt{\frac{1-x^3}{x^3+1}}+\frac{2x^9}{3\,(x^6+1)^3}+\frac{\sqrt{\frac{1-x^3}{x^3+1}}}{2\,(x^6+1)^2}\right)$$

$$\Big\downarrow \text{298}$$

$$-\frac{4}{3}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{x^6+1}d\sqrt{\frac{1-x^3}{x^3+1}}-\frac{3\sqrt{\frac{1-x^3}{x^3+1}}}{2\,(x^6+1)}\right)+\frac{2x^9}{3\,(x^6+1)^3}+\frac{\sqrt{\frac{1-x^3}{x^3+1}}}{2\,(x^6+1)^2}\right)$$

$$\Big\downarrow \text{216}$$

$$-\frac{4}{3}\left(\frac{1}{2}\left(\frac{1}{2}\arctan\left(\sqrt{\frac{1-x^3}{x^3+1}}\right)-\frac{3\sqrt{\frac{1-x^3}{x^3+1}}}{2\,(x^6+1)}\right)+\frac{2x^9}{3\,(x^6+1)^3}+\frac{\sqrt{\frac{1-x^3}{x^3+1}}}{2\,(x^6+1)^2}\right)$$

input
```
Int[x^8*Sqrt[(1 - x^3)/(1 + x^3)],x]
```

output
```
(-4*((2*x^9)/(3*(1 + x^6)^3) + Sqrt[(1 - x^3)/(1 + x^3)]/(2*(1 + x^6)^2) +
 ((-3*Sqrt[(1 - x^3)/(1 + x^3)])/(2*(1 + x^6)) + ArcTan[Sqrt[(1 - x^3)/(1
+ x^3)]]/2)/2))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

method	result
risch	$\frac{\left(x^{3}+1\right)\left(2x^{6}-3x^{3}+4\right)\sqrt{-\frac{x^{3}-1}{x^{3}+1}}}{18}-\frac{\arcsin\left(x^{3}\right)\sqrt{-\frac{x^{3}-1}{x^{3}+1}}\,\sqrt{-\left(x^{3}+1\right)\left(x^{3}-1\right)}}{6\left(x^{3}-1\right)}$
trager	$\frac{\left(x^{3}+1\right)\left(2x^{6}-3x^{3}+4\right)\sqrt{-\frac{x^{3}-1}{x^{3}+1}}}{18}+\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{x^{3}-1}{x^{3}+1}}\,x^{3}+x^{3}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\sqrt{-\frac{x^{3}-1}{x^{3}+1}}\right)}{6}$

input
```
int(x^8*((-x^3+1)/(x^3+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/18*(x^3+1)*(2*x^6-3*x^3+4)*(-(x^3-1)/(x^3+1))^(1/2)-1/6*arcsin(x^3)*(-(x
^3-1)/(x^3+1))^(1/2)*(-(x^3+1)*(x^3-1))^(1/2)/(x^3-1)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.67

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \frac{1}{18}\left(2\,x^9-x^6+x^3+4\right)\sqrt{-\frac{x^3-1}{x^3+1}} - \frac{1}{3}\arctan\left(\frac{(x^3+1)\sqrt{-\frac{x^3-1}{x^3+1}}-1}{x^3}\right)$$

input
```
integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="fricas")
```

output
```
1/18*(2*x^9 - x^6 + x^3 + 4)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1/3*arctan(((x^3
 + 1)*sqrt(-(x^3 - 1)/(x^3 + 1)) - 1)/x^3)
```

Sympy [F(-1)]

Timed out.

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \text{Timed out}$$

input
```
integrate(x**8*((-x**3+1)/(x**3+1))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx=\int x^8\sqrt{-\frac{x^3-1}{x^3+1}}\,dx$$

input
```
integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^8*sqrt(-(x^3 - 1)/(x^3 + 1)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.59

$$\begin{aligned}&\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx\\&=\frac{1}{6}\arcsin\left(x^3\right)\operatorname{sgn}\left(x^3+1\right)\\&\quad+\frac{1}{18}\sqrt{-x^6+1}\left(\left(2\,x^3\operatorname{sgn}\left(x^3+1\right)-3\operatorname{sgn}\left(x^3+1\right)\right)x^3+4\operatorname{sgn}\left(x^3+1\right)\right)\end{aligned}$$

input
```
integrate(x^8*((-x^3+1)/(x^3+1))^(1/2),x, algorithm="giac")
```

output
```
1/6*arcsin(x^3)*sgn(x^3 + 1) + 1/18*sqrt(-x^6 + 1)*((2*x^3*sgn(x^3 + 1) -
3*sgn(x^3 + 1))*x^3 + 4*sgn(x^3 + 1))
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

$$\begin{aligned}\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx&=\frac{2\sqrt{-\frac{x^3-1}{x^3+1}}}{9}-\frac{\operatorname{atan}\left(\sqrt{-\frac{x^3-1}{x^3+1}}\right)}{3}\\&\quad+\frac{x^3\sqrt{-\frac{x^3-1}{x^3+1}}}{18}-\frac{x^6\sqrt{-\frac{x^3-1}{x^3+1}}}{18}+\frac{x^9\sqrt{-\frac{x^3-1}{x^3+1}}}{9}\end{aligned}$$

input
```
int(x^8*(-(x^3 - 1)/(x^3 + 1))^(1/2),x)
```

output
```
(2*(-(x^3 - 1)/(x^3 + 1))^(1/2))/9 - atan((-(x^3 - 1)/(x^3 + 1))^(1/2))/3
+ (x^3*(-(x^3 - 1)/(x^3 + 1))^(1/2))/18 - (x^6*(-(x^3 - 1)/(x^3 + 1))^(1/2
))/18 + (x^9*(-(x^3 - 1)/(x^3 + 1))^(1/2))/9
```

Reduce [F]

$$\int x^8\sqrt{\frac{1-x^3}{1+x^3}}\,dx = \frac{\sqrt{-x^3+1}\,\sqrt{x^3+1}\,x^6}{9} - \frac{\sqrt{-x^3+1}\,\sqrt{x^3+1}\,x^3}{6} + \frac{2\sqrt{-x^3+1}\,\sqrt{x^3+1}}{9} - \frac{\left(\int \frac{\sqrt{-x^3+1}\,\sqrt{x^3+1}\,x^2}{x^6-1}dx\right)}{2}$$

input
```
int(x^8*((-x^3+1)/(x^3+1))^(1/2),x)
```

output
```
(2*sqrt( - x**3 + 1)*sqrt(x**3 + 1)*x**6 - 3*sqrt( - x**3 + 1)*sqrt(x**3 +
 1)*x**3 + 4*sqrt( - x**3 + 1)*sqrt(x**3 + 1) - 9*int((sqrt( - x**3 + 1)*s
qrt(x**3 + 1)*x**2)/(x**6 - 1),x))/18
```

3.72 $\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 102

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx=-\frac{27(7+5x^5)\sqrt{-7+\frac{74}{7+5x^5}}}{350\sqrt{5}}+\frac{(7+5x^5)^2\sqrt{-7+\frac{74}{7+5x^5}}}{250\sqrt{5}}+\frac{2257\arctan\left(\frac{\sqrt{-7+\frac{74}{7+5x^5}}}{\sqrt{7}}\right)}{875\sqrt{35}}$$

output
```
-27/1750*(5*x^5+7)*(-7+74/(5*x^5+7))^(1/2)*5^(1/2)+1/1250*(5*x^5+7)^2*(-7+
74/(5*x^5+7))^(1/2)*5^(1/2)+2257/30625*arctan(1/7*(-7+74/(5*x^5+7))^(1/2)*
7^(1/2))*35^(1/2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.04

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx$$
$$=\frac{\sqrt{\frac{5-7x^5}{7+5x^5}}\left(35\sqrt{5-7x^5}(-602-185x^5+175x^{10})+4514\sqrt{35}\sqrt{7+5x^5}\arctan\left(\frac{\sqrt{\frac{25}{7}-5x^5}}{\sqrt{7+5x^5}}\right)\right)}{61250\sqrt{5-7x^5}}$$

input
```
Integrate[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

output
```
(Sqrt[(5 - 7*x^5)/(7 + 5*x^5)]*(35*Sqrt[5 - 7*x^5]*(-602 - 185*x^5 + 175*x
^10) + 4514*Sqrt[35]*Sqrt[7 + 5*x^5]*ArcTan[Sqrt[25/7 - 5*x^5]/Sqrt[7 + 5*
x^5]]))/(61250*Sqrt[5 - 7*x^5])
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {2053, 2052, 360, 27, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^9\sqrt{\frac{5-7x^5}{5x^5+7}}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{5}\int x^5\sqrt{\frac{5-7x^5}{5x^5+7}}dx^5$$

$$\Big\downarrow \text{ 2052}$$

$$-\frac{148}{5}\int \frac{x^{10}(5-7x^{10})}{(5x^{10}+7)^3}d\sqrt{\frac{5-7x^5}{5x^5+7}}$$

$$\Big\downarrow \text{ 360}$$

$$-\frac{148}{5}\left(-\frac{1}{100}\int -\frac{2(37-70x^{10})}{(5x^{10}+7)^2}d\sqrt{\frac{5-7x^5}{5x^5+7}}-\frac{37\sqrt{\frac{5-7x^5}{5x^5+7}}}{50(5x^{10}+7)^2}\right)$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{148}{5}\left(\frac{1}{50}\int \frac{37-70x^{10}}{(5x^{10}+7)^2}d\sqrt{\frac{5-7x^5}{5x^5+7}}-\frac{37\sqrt{\frac{5-7x^5}{5x^5+7}}}{50(5x^{10}+7)^2}\right)$$

$$\Big\downarrow \text{ 298}$$

$$-\frac{148}{5}\left(\frac{1}{50}\left(\frac{135\sqrt{\frac{5-7x^5}{5x^5+7}}}{14\left(5x^{10}+7\right)}-\frac{61}{14}\int\frac{1}{5x^{10}+7}d\sqrt{\frac{5-7x^5}{5x^5+7}}\right)-\frac{37\sqrt{\frac{5-7x^5}{5x^5+7}}}{50\left(5x^{10}+7\right)^2}\right)$$

$$\Big\downarrow \; 216$$

$$-\frac{148}{5}\left(\frac{1}{50}\left(\frac{135\sqrt{\frac{5-7x^5}{5x^5+7}}}{14\left(5x^{10}+7\right)}-\frac{61\arctan\left(\sqrt{\frac{5}{7}}\sqrt{\frac{5-7x^5}{5x^5+7}}\right)}{14\sqrt{35}}\right)-\frac{37\sqrt{\frac{5-7x^5}{5x^5+7}}}{50\left(5x^{10}+7\right)^2}\right)$$

input
```
Int[x^9*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)],x]
```

output
```
(-148*((-37*Sqrt[(5 - 7*x^5)/(7 + 5*x^5)])/(50*(7 + 5*x^10)^2) + ((135*Sqr
t[(5 - 7*x^5)/(7 + 5*x^5)])/(14*(7 + 5*x^10)) - (61*ArcTan[Sqrt[5/7]*Sqrt[
(5 - 7*x^5)/(7 + 5*x^5)]])/(14*Sqrt[35]))/50))/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.60 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

method	result
trager	$\frac{(5x^5+7)(35x^5-86)\sqrt{-\frac{7x^5-5}{5x^5+7}}}{1750} - \frac{2257\,\mathrm{RootOf}\left(\_Z^2+35\right)\ln\left(-35\,\mathrm{RootOf}\left(\_Z^2+35\right)x^5+175\sqrt{-\frac{7x^5-5}{5x^5+7}}\,x^5-12\,\mathrm{RootOf}\left(\_Z\right.\right.}{61250}$
risch	$\frac{(5x^5+7)(35x^5-86)\sqrt{-\frac{7x^5-5}{5x^5+7}}}{1750} + \frac{2257\,\mathrm{RootOf}\left(\_Z^2+35\right)\ln\left(-35\,\mathrm{RootOf}\left(\_Z^2+35\right)x^5+35\sqrt{-35x^{10}-24x^5+35}-12\,\mathrm{RootOf}\left(\right.\right.}{61250(7x^5-5)}$

input
```
int(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/1750*(5*x^5+7)*(35*x^5-86)*(-(7*x^5-5)/(5*x^5+7))^(1/2)-2257/61250*RootO
f(_Z^2+35)*ln(-35*RootOf(_Z^2+35)*x^5+175*(-(7*x^5-5)/(5*x^5+7))^(1/2)*x^5
-12*RootOf(_Z^2+35)+245*(-(7*x^5-5)/(5*x^5+7))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.80

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \frac{1}{1750}\left(175\,x^{10}-185\,x^5-602\right)\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}} + \frac{2257}{61250}\sqrt{35}\arctan\left(\frac{\sqrt{35}(35\,x^5+12)\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}}{35\,(7\,x^5-5)}\right)$$

input
```
integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="fricas")
```

output
```
1/1750*(175*x^10 - 185*x^5 - 602)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7)) + 2257/61
250*sqrt(35)*arctan(1/35*sqrt(35)*(35*x^5 + 12)*sqrt(-(7*x^5 - 5)/(5*x^5 +
 7))/(7*x^5 - 5))
```

Sympy [F(-1)]

Timed out.

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \text{Timed out}$$

input
```
integrate(x**9*((-7*x**5+5)/(5*x**5+7))**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.19

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \frac{2257}{30625}\sqrt{35}\arctan\left(\frac{1}{7}\sqrt{35}\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}\right) - \frac{37\left(675\left(-\frac{7\,x^5-5}{5\,x^5+7}\right)^{\frac{3}{2}}+427\sqrt{-\frac{7\,x^5-5}{5\,x^5+7}}\right)}{875\left(\frac{25\,(7\,x^5-5)^2}{(5\,x^5+7)^2}-\frac{70\,(7\,x^5-5)}{5\,x^5+7}+49\right)}$$

input
```
integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="maxima")
```

output
```
2257/30625*sqrt(35)*arctan(1/7*sqrt(35)*sqrt(-(7*x^5 - 5)/(5*x^5 + 7))) -
37/875*(675*(-(7*x^5 - 5)/(5*x^5 + 7))^(3/2) + 427*sqrt(-(7*x^5 - 5)/(5*x^
5 + 7)))/(25*(7*x^5 - 5)^2/(5*x^5 + 7)^2 - 70*(7*x^5 - 5)/(5*x^5 + 7) + 49
)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.46

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx = \frac{1}{61250}\left(35\sqrt{-35\,x^{10}-24\,x^5+35}(35\,x^5-86)-2257\sqrt{35}\arcsin\left(\frac{35}{37}\,x^5+\frac{12}{37}\right)\right)\mathrm{sgn}(5\,x^5+7)$$

input
```
integrate(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x, algorithm="giac")
```

output
```
1/61250*(35*sqrt(-35*x^10 - 24*x^5 + 35)*(35*x^5 - 86) - 2257*sqrt(35)*arc
sin(35/37*x^5 + 12/37))*sgn(5*x^5 + 7)
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.31

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx=\frac{2257\sqrt{35}\,\mathrm{atan}\left(\frac{\sqrt{5}\sqrt{7}\sqrt{-\frac{7x^5-5}{5x^5+7}}}{7}\right)}{30625}-\frac{43\sqrt{5}\sqrt{7}\sqrt{35}\sqrt{-\frac{7x^5-5}{5x^5+7}}}{4375}-\frac{37\sqrt{5}\sqrt{7}\sqrt{35}\,x^5\sqrt{-\frac{7x^5-5}{5x^5+7}}}{12250}+\frac{\sqrt{5}\sqrt{7}\sqrt{35}\,x^{10}\sqrt{-\frac{7x^5-5}{5x^5+7}}}{350}$$

input
```
int(x^9*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2),x)
```

output
```
(2257*35^(1/2)*atan((5^(1/2)*7^(1/2)*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/7))
/30625 - (43*5^(1/2)*7^(1/2)*35^(1/2)*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/43
75 - (37*5^(1/2)*7^(1/2)*35^(1/2)*x^5*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/12
250 + (5^(1/2)*7^(1/2)*35^(1/2)*x^10*(-(7*x^5 - 5)/(5*x^5 + 7))^(1/2))/350
```

Reduce [F]

$$\int x^9\sqrt{\frac{5-7x^5}{7+5x^5}}\,dx=\frac{\sqrt{5x^5+7}\,\sqrt{-7x^5+5}\,x^5}{50}+\frac{7\sqrt{5x^5+7}\,\sqrt{-7x^5+5}}{120}-\frac{2257\left(\int\frac{\sqrt{5x^5+7}\,\sqrt{-7x^5+5}\,x^9}{35x^{10}+24x^5-35}dx\right)}{120}$$

input
```
int(x^9*((-7*x^5+5)/(5*x^5+7))^(1/2),x)
```

output
```
(12*sqrt(5*x**5 + 7)*sqrt( - 7*x**5 + 5)*x**5 + 35*sqrt(5*x**5 + 7)*sqrt(
- 7*x**5 + 5) - 11285*int((sqrt(5*x**5 + 7)*sqrt( - 7*x**5 + 5)*x**9)/(35*
x**10 + 24*x**5 - 35),x))/600
```

3.73 $\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2} dx = \frac{\sqrt{-1+x^2}\sqrt{1+\frac{1}{-1+x^2}}\arctan\left(\frac{\sqrt{-1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

output
```
1/2*(x^2-1)^(1/2)*(1+1/(x^2-1))^(1/2)*arctan(1/2*(x^2-1)^(1/2)*2^(1/2))*2^
(1/2)/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2} dx = \frac{\sqrt{\frac{x^2}{-1+x^2}}\sqrt{-1+x^2}\arctan\left(\frac{\sqrt{-1+x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

input
```
Integrate[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
```

output
```
(Sqrt[x^2/(-1 + x^2)]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sqrt
[2]*x)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {2058, 34, 353, 73, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1} \, dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\int \frac{\sqrt{x^2}}{\sqrt{x^2-1}(x^2+1)}dx}{\sqrt{x^2}}$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\int \frac{x}{\sqrt{x^2-1}(x^2+1)}dx}{x}$$

$$\Big\downarrow \text{353}$$

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\int \frac{1}{\sqrt{x^2-1}(x^2+1)}dx^2}{2x}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\int \frac{1}{x^4+2}d\sqrt{x^2-1}}{x}$$

$$\Big\downarrow \text{216}$$

$$\frac{\sqrt{-\frac{x^2}{1-x^2}}\sqrt{x^2-1}\arctan\left(\frac{\sqrt{x^2-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

input
```
Int[Sqrt[x^2/(-1 + x^2)]/(1 + x^2),x]
```

output
```
(Sqrt[-(x^2/(1 - x^2))]*Sqrt[-1 + x^2]*ArcTan[Sqrt[-1 + x^2]/Sqrt[2]])/(Sq
rt[2]*x)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

method	result	size
default	$\frac{\sqrt{\frac{x^{2}}{x^{2}-1}}\,\sqrt{x^{2}-1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{x^{2}-1}\,\sqrt{2}}{2}\right)}{2x}$	42
trager	$-\frac{\operatorname{RootOf}\left(\_Z^{2}+2\right)\ln\left(\frac{x^{3}\operatorname{RootOf}\left(\_Z^{2}+2\right)+4x^{2}\sqrt{\frac{x^{2}}{x^{2}-1}}-3x\operatorname{RootOf}\left(\_Z^{2}+2\right)-4\sqrt{\frac{x^{2}}{x^{2}-1}}}{x\left(x^{2}+1\right)}\right)}{4}$	74

input
```
int((x^2/(x^2-1))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2/(x^2-1))^(1/2)/x*(x^2-1)^(1/2)*2^(1/2)*arctan(1/2*(x^2-1)^(1/2)*2
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{\sqrt{2}(x^2-1)\sqrt{\frac{x^2}{x^2-1}}}{2\,x}\right)$$

input
```
integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 - 1)*sqrt(x^2/(x^2 - 1))/x)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

input
```
integrate((x**2/(x**2-1))**(1/2)/(x**2+1),x)
```

output
```
Integral(sqrt(x**2/(x**2 - 1))/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

input
```
integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2/(x^2 - 1))/(x^2 + 1), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{x^2-1}\right)\operatorname{sgn}\left(x^2-1\right)\operatorname{sgn}(x) + \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}i\sqrt{2}\right)\operatorname{sgn}(x)$$

input
```
integrate((x^2/(x^2-1))^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(x^2 - 1))*sgn(x^2 - 1)*sgn(x) + 1/2*sq
rt(2)*arctan(1/2*I*sqrt(2))*sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{x^2}{x^2-1}}}{x^2+1}\,dx$$

input
```
int((x^2/(x^2 - 1))^(1/2)/(x^2 + 1),x)
```

output
```
int((x^2/(x^2 - 1))^(1/2)/(x^2 + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.74

$$\int \frac{\sqrt{\frac{x^2}{-1+x^2}}}{1+x^2}\,dx = \frac{\sqrt{2}\,atan\left(\frac{\sqrt{x^2-1}\,x+x^2-1}{\sqrt{x^2-1}\,\sqrt{2}+\sqrt{2}\,x}\right)}{2}$$

input
```
int((x^2/(x^2-1))^(1/2)/(x^2+1),x)
```

output
```
(sqrt(2)*atan((sqrt(x**2 - 1)*x + x**2 - 1)/(sqrt(x**2 - 1)*sqrt(2) + sqrt
(2)*x)))/2
```

3.74 $\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 81

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \frac{\sqrt{-1+a+(1+a)x^2}\sqrt{\frac{1}{1+a}-\frac{1-a}{(1+a)(1-a-(1+a)x^2)}}\arctan\left(\frac{\sqrt{-1+a+(1+a)x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

output
```
1/2*(-1+a+(1+a)*x^2)^(1/2)*(1/(1+a)-(1-a)/(1+a)/(1-a-(1+a)*x^2))^(1/2)*arc
tan(1/2*(-1+a+(1+a)*x^2)^(1/2)*2^(1/2))*2^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \frac{\sqrt{-1+a+x^2+ax^2}\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}\arctan\left(\frac{\sqrt{-1+a+(1+a)x^2}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

input
```
Integrate[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
```

output
```
(Sqrt[-1 + a + x^2 + a*x^2]*Sqrt[x^2/(-1 + a + (1 + a)*x^2)]*ArcTan[Sqrt[-
1 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {2058, 34, 353, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{x^2+1}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{-\frac{x^2}{-((a+1)x^2)-a+1}}\sqrt{(a+1)x^2+a-1}\int \frac{\sqrt{x^2}}{(x^2+1)\sqrt{(a+1)x^2+a-1}}dx}{\sqrt{x^2}}$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt{-\frac{x^2}{-((a+1)x^2)-a+1}}\sqrt{(a+1)x^2+a-1}\int \frac{x}{(x^2+1)\sqrt{(a+1)x^2+a-1}}dx}{x}$$

$$\Big\downarrow \text{353}$$

$$\frac{\sqrt{-\frac{x^2}{-((a+1)x^2)-a+1}}\sqrt{(a+1)x^2+a-1}\int \frac{1}{(x^2+1)\sqrt{(a+1)x^2+a-1}}dx^2}{2x}$$

$$\Big\downarrow \text{73}$$

$$\frac{\sqrt{-\frac{x^2}{-((a+1)x^2)-a+1}}\sqrt{(a+1)x^2+a-1}\int \frac{1}{\frac{x^4}{a+1}+\frac{2}{a+1}}d\sqrt{(a+1)x^2+a-1}}{(a+1)x}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{-\frac{x^2}{-((a+1)x^2)-a+1}}\sqrt{(a+1)x^2+a-1}\arctan\left(\frac{\sqrt{(a+1)x^2+a-1}}{\sqrt{2}}\right)}{\sqrt{2}x}$$

input
```
Int[Sqrt[x^2/(-1 + a + (1 + a)*x^2)]/(1 + x^2),x]
```

output
```
(Sqrt[-(x^2/(1 - a - (1 + a)*x^2))]*Sqrt[-1 + a + (1 + a)*x^2]*ArcTan[Sqrt
[-1 + a + (1 + a)*x^2]/Sqrt[2]])/(Sqrt[2]*x)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

method	result	size
default	$\frac{\sqrt{\frac{x^{2}}{a\,x^{2}+x^{2}+a-1}}\,\sqrt{a\,x^{2}+x^{2}+a-1}\,\sqrt{2}\,\arctan\left(\frac{\sqrt{a\,x^{2}+x^{2}+a-1}\,\sqrt{2}}{2}\right)}{2x}$	60

input
```
int((x^2/(-1+a+(a+1)*x^2))^(1/2)/(x^2+1),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x^2/(a*x^2+x^2+a-1))^(1/2)/x*(a*x^2+x^2+a-1)^(1/2)*2^(1/2)*arctan(1/2
*(a*x^2+x^2+a-1)^(1/2)*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.52

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \frac{1}{4}\,\sqrt{2}\arctan\left(\frac{\sqrt{2}((a+1)x^2+a-3)\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{4\,x}\right)$$

input
```
integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*arctan(1/4*sqrt(2)*((a + 1)*x^2 + a - 3)*sqrt(x^2/((a + 1)*x^2
 + a - 1))/x)
```

Sympy [F]

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{x^2}{ax^2+a+x^2-1}}}{x^2+1}\,dx$$

input
```
integrate((x**2/(-1+a+(1+a)*x**2))**(1/2)/(x**2+1),x)
```

output
```
Integral(sqrt(x**2/(a*x**2 + a + x**2 - 1))/(x**2 + 1), x)
```

Maxima [F]

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \int \frac{\sqrt{\frac{x^2}{(a+1)x^2+a-1}}}{x^2+1}\,dx$$

input
```
integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^2/((a + 1)*x^2 + a - 1))/(x^2 + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{ax^2+x^2+a-1}\right)\operatorname{sgn}\left(ax^2+x^2+a-1\right)\operatorname{sgn}(x) - \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\sqrt{a-1}\right)\operatorname{sgn}(a-1)\operatorname{sgn}(x)$$

input
```
integrate((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a*x^2 + x^2 + a - 1))*sgn(a*x^2 + x^2
+ a - 1)*sgn(x) - 1/2*sqrt(2)*arctan(1/2*sqrt(2)*sqrt(a - 1))*sgn(a - 1)*s
gn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx=\int \frac{\sqrt{\frac{x^2}{(a+1)\,x^2+a-1}}}{x^2+1}\,dx$$

input
```
int((x^2/(a + x^2*(a + 1) - 1))^(1/2)/(x^2 + 1),x)
```

output
```
int((x^2/(a + x^2*(a + 1) - 1))^(1/2)/(x^2 + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.75

$$\int \frac{\sqrt{\frac{x^2}{-1+a+(1+a)x^2}}}{1+x^2}\,dx=\frac{\sqrt{2}\,atan\left(\frac{\sqrt{a+1}\,\sqrt{a\,x^2+x^2+a-1}\,x+a\,x^2+a+x^2-1}{\sqrt{a\,x^2+x^2+a-1}\,\sqrt{2}+\sqrt{a+1}\,\sqrt{2}\,x}\right)}{2}$$

input
```
int((x^2/(-1+a+(1+a)*x^2))^(1/2)/(x^2+1),x)
```

output
```
(sqrt(2)*atan((sqrt(a + 1)*sqrt(a*x**2 + a + x**2 - 1)*x + a*x**2 + a + x*
*2 - 1)/(sqrt(a*x**2 + a + x**2 - 1)*sqrt(2) + sqrt(a + 1)*sqrt(2)*x)))/2
```

3.75 $\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 298

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{(b^2c^2+2abcd+5a^2d^2)(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{16b^3d^2e} - \frac{(7bc+5ad)(c+dx^2)^2\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{24b^2d^2e} + \frac{(c+dx^2)^3\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{6bd^2e} + \frac{(bc-ad)(b^2c^2+2abcd+5a^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{7/2}d^{5/2}\sqrt{e}}$$

output
```
1/16*(5*a^2*d^2+2*a*b*c*d+b^2*c^2)*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+
c))^(1/2)/b^3/d^2/e-1/24*(5*a*d+7*b*c)*(d*x^2+c)^2*(b*e/d-(-a*d+b*c)*e/d/(
d*x^2+c))^(1/2)/b^2/d^2/e+1/6*(d*x^2+c)^3*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))
^(1/2)/b/d^2/e+1/16*(-a*d+b*c)*(5*a^2*d^2+2*a*b*c*d+b^2*c^2)*arctanh(d^(1/
2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/b^(7/2)/d^(5/2)
/e^(1/2)
```

Mathematica [A] (verified)

Time = 1.55 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.75

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{\sqrt{a+bx^2}\left(\sqrt{d}\sqrt{a+bx^2}\sqrt{\frac{b(c+dx^2)}{bc-ad}}(15a^2d^2-2abd(2c+5dx^2)+b^2(-3c^2+2cdx^2+8d^2x^4))+3\sqrt{bc-ad}\right.}{48b^3d^{5/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{\frac{b(c+dx^2)}{bc-ad}}}$$

input
```
Integrate[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[a + b*x^2]*(Sqrt[d]*Sqrt[a + b*x^2]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)
]*(15*a^2*d^2 - 2*a*b*d*(2*c + 5*d*x^2) + b^2*(-3*c^2 + 2*c*d*x^2 + 8*d^2*
x^4)) + 3*Sqrt[b*c - a*d]*(b^2*c^2 + 2*a*b*c*d + 5*a^2*d^2)*ArcSinh[(Sqrt[
d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]]))/(48*b^3*d^(5/2)*Sqrt[(e*(a + b*x^2)
)/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

Rubi [A] (warning: unable to verify)

Time = 0.79 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {2053, 2052, 315, 25, 27, 298, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$\Big\downarrow \ 2053$$

$$\frac{1}{2}\int \frac{x^4}{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}\,dx^2$$

$$\Big\downarrow \ 2052$$

$$e(bc-ad)\int \frac{\left(ae-cx^4\right)^2}{\left(be-dx^4\right)^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \; 315$$

$$e(bc-ad)\left(-\frac{\int -\frac{e\left(a(bc+5ad)e-3c(bc+ad)x^4\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{6bde}-\frac{(bc-ad)\left(ae-cx^4\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6bd\left(be-dx^4\right)^3}\right)$$

$$\Big\downarrow \; 25$$

$$e(bc-ad)\left(\frac{\int \frac{e\left(a(bc+5ad)e-3c(bc+ad)x^4\right)}{\left(be-dx^4\right)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{6bde}-\frac{(bc-ad)\left(ae-cx^4\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6bd\left(be-dx^4\right)^3}\right)$$

$$\Big\downarrow \; 27$$

$$e(bc-ad)\left(\frac{\int \frac{a(bc+5ad)e-3c(bc+ad)x^4}{\left(be-dx^4\right)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{6bd}-\frac{(bc-ad)\left(ae-cx^4\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6bd\left(be-dx^4\right)^3}\right)$$

$$\Big\downarrow \; 298$$

$$e(bc-ad)\left(\frac{\frac{3}{4}\left(\frac{5a^2d}{b}+2ac+\frac{bc^2}{d}\right)\int \frac{1}{\left(be-dx^4\right)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}-\frac{(bc-ad)(5ad+3bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4bd\left(be-dx^4\right)^2}}{6bd}-\frac{(bc-ad)\left(ae-cx^4\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6bd\left(be-dx^4\right)^3}\right)$$

$$\Big\downarrow \; 215$$

$$e(bc-ad)\left(\frac{\frac{3}{4}\left(\frac{5a^2d}{b}+2ac+\frac{bc^2}{d}\right)\left(\frac{\int \frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2be}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be\left(be-dx^4\right)}\right)-\frac{(bc-ad)(5ad+3bc)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4bd\left(be-dx^4\right)^2}}{6bd}-\frac{(bc-ad)\left(ae-cx^4\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6bd\left(be-d\right.}\right.$$

$$\Big\downarrow \; 221$$

$$e(bc-$$

$$ad)\left(\frac{\frac{3}{4}\left(\frac{5a^2d}{b}+2ac+\frac{bc^2}{d}\right)\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}e^{3/2}}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be(be-dx^4)}\right)-\frac{(bc-ad)(5ad+3bc)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bd(be-dx^4)^2}}{6bd}-\frac{(bc-ad)\,(ae}{6bd\,(b}\right.$$

input
```
Int[x^5/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(b*c - a*d)*e*(-1/6*((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(a*e -
c*x^4))/(b*d*(b*e - d*x^4)^3) + (-1/4*((b*c - a*d)*(3*b*c + 5*a*d)*Sqrt[(e
*(a + b*x^2))/(c + d*x^2)])/(b*d*(b*e - d*x^4)^2) + (3*(2*a*c + (b*c^2)/d
+ (5*a^2*d)/b)*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*b*e*(b*e - d*x^4)) +
ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])]/(2*
b^(3/2)*Sqrt[d]*e^(3/2))))/4)/(6*b*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.82

method	result
risch	$\frac{\left(8b^{2}d^{2}x^{4}-10ab\,d^{2}x^{2}+2b^{2}c\,x^{2}d+15a^{2}d^{2}-4abcd-3b^{2}c^{2}\right)\left(b\,x^{2}+a\right)}{48b^{3}d^{2}\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}-\frac{\left(5a^{3}d^{3}-3a^{2}bc\,d^{2}-a\,b^{2}c^{2}d-b^{3}c^{3}\right)\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+bd\,x^{2}e}{\sqrt{bde}}+\sqrt{\right.}{32b^{3}d^{2}\sqrt{bde}\,\sqrt{\frac{e\left(b\,x^{2}+\right.}{d\,x^{2}+}}}$
default	$\frac{\left(b\,x^{2}+a\right)\left(-36\sqrt{db\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}\,ab\,d^{2}x^{2}-12\sqrt{db\,x^{4}+ad\,x^{2}+bc\,x^{2}+ac}\,\sqrt{bd}\,b^{2}cd\,x^{2}-15\ln\left(\frac{2bd\,x^{2}+2\sqrt{db\,x^{4}+ad\,x^{2}+bc\,x^{2}}}{2\sqrt{bd}}\right.\right.}{}$

input
```
int(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/48*(8*b^2*d^2*x^4-10*a*b*d^2*x^2+2*b^2*c*d*x^2+15*a^2*d^2-4*a*b*c*d-3*b^
2*c^2)*(b*x^2+a)/b^3/d^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/32*(5*a^3*d^3-3*a
^2*b*c*d^2-a*b^2*c^2*d-b^3*c^3)/b^3/d^2*ln((1/2*a*d*e+1/2*b*c*e+b*d*x^2*e)
/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)/(e
*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.83

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$= \left[-\frac{3\left(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3\right)\sqrt{bde}\log\left(8\,b^2d^2ex^4+8\left(b^2cd+abd^2\right)ex^2+\left(b^2c^2+6\,abcd+a^2d\right.\right.}{}\right.$$

$$-\frac{3\left(b^3c^3+ab^2c^2d+3\,a^2bcd^2-5\,a^3d^3\right)\sqrt{-bde}\arctan\left(\frac{\left(2\,bdx^2+bc+ad\right)\sqrt{-bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\left(b^2dex^2+abde\right)}\right)-2\left(8\,b^3d^4x^6-3\,b^3\right.}{96\,b^4d}$$

input
```
integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/192*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 - 5*a^3*d^3)*sqrt(b*d*e)
*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d
+ a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqr
t(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) - 4*(8*b^3*d^4*x^6 - 3*b^3*c^3
*d - 4*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2*d^4)*x^4 - (
b^3*c^2*d^2 + 14*a*b^2*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*
x^2 + c)))/(b^4*d^3*e), -1/96*(3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2 -
5*a^3*d^3)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sq
rt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(8*b^3*d^4*x^
6 - 3*b^3*c^3*d - 4*a*b^2*c^2*d^2 + 15*a^2*b*c*d^3 + 10*(b^3*c*d^3 - a*b^2
*d^4)*x^4 - (b^3*c^2*d^2 + 14*a*b^2*c*d^3 - 15*a^2*b*d^4)*x^2)*sqrt((b*e*x
^2 + a*e)/(d*x^2 + c)))/(b^4*d^3*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.79

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{2\sqrt{bdex^4+bcex^2+adex^2+ace}\left(2\,x^2\left(\frac{4\,x^2}{be}+\frac{b^2cde-5\,abd^2e}{b^3d^2e^2}\right)-\frac{3\,b^2c^2e+4\,abcde-15\,a^2d^2e}{b^3d^2e^2}\right)-\frac{3\,(b^3c^3+ab^2c^2d+3\,a^2bcd^{2}}{}}{96\,\mathrm{sgn}\left(dx^2+c\right)}$$

input
```
integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/96*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)*(2*x^2*(4*x^2/(b*e
) + (b^2*c*d*e - 5*a*b*d^2*e)/(b^3*d^2*e^2)) - (3*b^2*c^2*e + 4*a*b*c*d*e
- 15*a^2*d^2*e)/(b^3*d^2*e^2)) - 3*(b^3*c^3 + a*b^2*c^2*d + 3*a^2*b*c*d^2
- 5*a^3*d^3)*log(abs(-b*c*e - a*d*e - 2*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqr
t(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))))/(sqrt(b*d*e)*b^3*d^2))/sgn
(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^5}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.12

$$\int \frac{x^5}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{\sqrt{e}\left(15\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2b\,d^3-4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a\,b^2c\,d^2-10\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a\,b^2d^3x^2-3\right.}{}$$

input
```
int(x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(e)*(15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**3 - 4*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*a*b**2*c*d**2 - 10*sqrt(c + d*x**2)*sqrt(a + b*x**2
)*a*b**2*d**3*x**2 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c**2*d + 2*s
qrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d**2*x**2 + 8*sqrt(c + d*x**2)*sqr
t(a + b*x**2)*b**3*d**3*x**4 + 15*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a +
b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b)*a**3*d**3 - 9*sqrt(d)*sqrt(b)*log(
 - sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b)*a**2*b*c*d**2
- 3*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d)*sqrt(c + d
*x**2)*b)*a*b**2*c**2*d - 3*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2
)*d + sqrt(d)*sqrt(c + d*x**2)*b)*b**3*c**3))/(48*b**4*d**3*e)
```

3.76 $\int \frac{x^3}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}} \, dx$

Optimal result

Integrand size = 26, antiderivative size = 205

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} \, dx = -\frac{(bc+3ad)\,(c+dx^2)\,\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{8b^2de} + \frac{(c+dx^2)^2\,\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{4bde} - \frac{(bc-ad)(bc+3ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d\left(c+dx^2\right)}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{5/2}d^{3/2}\sqrt{e}}$$

output
```
-1/8*(3*a*d+b*c)*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^2/d/e+
1/4*(d*x^2+c)^2*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b/d/e-1/8*(-a*d+b*c
)*(3*a*d+b*c)*arctanh(d^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^(1/
2)/e^(1/2))/b^(5/2)/d^(3/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.79

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{b}\sqrt{d}(a+bx^2)\sqrt{c+dx^2}(-3ad+b(c+2dx^2))-(b^2c^2+2abcd-3a^2d^2)\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{8b^{5/2}d^{3/2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

input
```
Integrate[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[b]*Sqrt[d]*(a + b*x^2)*Sqrt[c + d*x^2]*(-3*a*d + b*(c + 2*d*x^2)) -
(b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a
+ b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])/(8*b^(5/2)*d^(3/2)*Sqrt[(e*(a + b*x^
2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2053, 2052, 25, 298, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{x^2}{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int -\frac{ae-cx^4}{(be-dx^4)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$-\left(e(bc-ad)\int \frac{ae-cx^4}{(be-dx^4)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$$\Big\downarrow \text{298}$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bd\left(be-dx^4\right)^2}-\frac{(3ad+bc)\int \frac{1}{(be-dx^4)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4bd}\right)$$

$$\Big\downarrow \text{215}$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bd\left(be-dx^4\right)^2}-\frac{(3ad+bc)\left(\frac{\int \frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2be}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be(be-dx^4)}\right)}{4bd}\right)$$

$$\Big\downarrow \text{221}$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4bd\left(be-dx^4\right)^2}-\frac{(3ad+bc)\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}e^{3/2}}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be(be-dx^4)}\right)}{4bd}\right)$$

input

```
Int[x^3/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output

```
(b*c - a*d)*e*(((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*b*d*(b*e
 - d*x^4)^2) - ((b*c + 3*a*d)*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*b*e*(b
*e - d*x^4)) + ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b
]*Sqrt[e])]/(2*b^(3/2)*Sqrt[d]*e^(3/2))))/(4*b*d))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.93

method	result
risch	$-\frac{(-2bd\,x^2+3ad-bc)(b\,x^2+a)}{8b^2d\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}+\frac{(3a^2d^2-2abcd-b^2c^2)\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+bd\,x^2e}{\sqrt{bde}}+\sqrt{bde\,x^4+(ade+bce)x^2+ace}\right)\sqrt{(d\,x^2+c)(b\,x^2+}}{16b^2d\sqrt{bde}\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$-\frac{(b\,x^2+a)\left(-4\sqrt{bd}\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,bd\,x^2-3\ln\left(\frac{2bd\,x^2+2\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^2d^2+2\ln\left(\frac{2bd\,x^2+2\sqrt{db\,x^4}}{}\right.\right.}{16\sqrt{}}$

input
```
int(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-2*b*d*x^2+3*a*d-b*c)*(b*x^2+a)/b^2/d/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+
1/16*(3*a^2*d^2-2*a*b*c*d-b^2*c^2)/b^2/d*ln((1/2*a*d*e+1/2*b*c*e+b*d*x^2*e
)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)/(
e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 413, normalized size of antiderivative = 2.01

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$= \left[-\frac{(b^2c^2+2\,abcd-3\,a^2d^2)\sqrt{bde}\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)e+4\,(2\,bd\right.}{}\right.$$

input
```
integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4
+ 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b
*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))) - 4*(2*b^2*d^3*x^4 + b^2*c^2*d - 3*a*b*c*d^2 + 3*(b
^2*c*d^2 - a*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e), 1
/16*((b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*sqrt(-b*d*e)*arctan(1/2*(2*b*d*x^2
+ b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 +
 a*b*d*e)) + 2*(2*b^2*d^3*x^4 + b^2*c^2*d - 3*a*b*c*d^2 + 3*(b^2*c*d^2 - a
*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} \, dx = \text{Timed out}$$

input
```
integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.84

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{2\sqrt{bdex^4+bcex^2+adex^2+ace}\left(\frac{2x^2}{be}+\frac{bce-3ade}{b^2de^2}\right)+\frac{(b^2c^2+2abcd-3a^2d^2)\log\left(\left|-bce-ade-2\sqrt{bde}\left(\sqrt{bde}x^2-\sqrt{bdex^4+bc\cdots}\right)\right|\right)}{\sqrt{bde}b^2d}}{16\,\mathrm{sgn}\left(dx^2+c\right)}$$

input
```
integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/16*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)*(2*x^2/(b*e) + (b*
c*e - 3*a*d*e)/(b^2*d*e^2)) + (b^2*c^2 + 2*a*b*c*d - 3*a^2*d^2)*log(abs(-b
*c*e - a*d*e - 2*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2
 + a*d*e*x^2 + a*c*e))))/(sqrt(b*d*e)*b^2*d))/sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^3}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.00

$$\int \frac{x^3}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{\sqrt{e}\left(-3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ab\,d^2+\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,b^2cd+2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,b^2d^2x^2+3\sqrt{d}\,\sqrt{b}\,\mathrm{l}\right.}{}$$

input
```
int(x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(e)*( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d**2 + sqrt(c + d*x**
2)*sqrt(a + b*x**2)*b**2*c*d + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d*
*2*x**2 + 3*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d - sqrt(d)*sq
rt(c + d*x**2)*b)*a**2*d**2 - 2*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*
x**2)*d - sqrt(d)*sqrt(c + d*x**2)*b)*a*b*c*d - sqrt(d)*sqrt(b)*log( - sqr
t(b)*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c + d*x**2)*b)*b**2*c**2))/(8*b**3*
d**2*e)
```

3.77 $\int \frac{x}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=\frac{(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{2be}+\frac{(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d\left(c+dx^2\right)}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}\sqrt{e}}$$

output
```
1/2*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b/e+1/2*(-a*d+b*c)*ar
ctanh(d^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/b^(3
/2)/d^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=\frac{\sqrt{b}\sqrt{d}(a+bx^2)\left(c+dx^2\right)+(bc-ad)\sqrt{a+bx^2}\sqrt{c+dx^2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{3/2}\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(c+dx^2\right)}$$

input
```
Integrate[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[b]*Sqrt[d]*(a + b*x^2)*(c + d*x^2) + (b*c - a*d)*Sqrt[a + b*x^2]*Sqr
t[c + d*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^2])])
/(2*b^(3/2)*Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rubi [A] (warning: unable to verify)

Time = 0.43 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2053, 2051, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{2051}$$

$$e(bc-ad)\int \frac{1}{(be-dx^4)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{215}$$

$$e(bc-ad)\left(\frac{\int \frac{1}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2be}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be(be-dx^4)}\right)$$

$$\Big\downarrow \text{221}$$

$$e(bc-ad)\left(\frac{\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}e^{3/2}}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2be(be-dx^4)}\right)$$

input
```
Int[x/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(b*c - a*d)*e*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*b*e*(b*e - d*x^4)) + A
rcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])]/(2*b
^(3/2)*Sqrt[d]*e^(3/2)))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.18

method	result
risch	$\frac{b\,x^{2}+a}{2 b \sqrt{\frac{e \left(b\,x^{2}+a \right)}{d\,x^{2}+c}}}-\frac{\left(a d -b c \right) \ln \left(\frac{\frac{1}{2} a d e +\frac{1}{2} b c e +b d\,x^{2} e}{\sqrt{b d e}}+\sqrt{b d e\,x^{4}+\left(a d e +b c e \right) x^{2}+a c e}\right) \sqrt{\left(d\,x^{2}+c \right) \left(b\,x^{2}+a \right) e}}{4 b \sqrt{b d e}\, \sqrt{\frac{e \left(b\,x^{2}+a \right)}{d\,x^{2}+c}}\, \left(d\,x^{2}+c \right)}$
default	$-\frac{\left(b\,x^{2}+a \right) \left(a \ln \left(\frac{2 b d\,x^{2}+2 \sqrt{d b\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}\, \sqrt{b d}+a d +b c}{2 \sqrt{b d}}\right) d -b \ln \left(\frac{2 b d\,x^{2}+2 \sqrt{d b\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}\, \sqrt{b d}+a d +b c}{2 \sqrt{b d}}\right) c -2 \sqrt{d b\,x^{4}+a}}{4 \sqrt{\frac{e \left(b\,x^{2}+a \right)}{d\,x^{2}+c}}\, \sqrt{\left(d\,x^{2}+c \right) \left(b\,x^{2}+a \right)}\, b \sqrt{b d}}$

input
```
int(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b*(b*x^2+a)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/4*(a*d-b*c)/b*ln((1/2*a*d*
e+1/2*b*c*e+b*d*x^2*e)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(
1/2))/(b*d*e)^(1/2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^
(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.41

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$= \left[-\frac{\sqrt{bde}(bc-ad)\log\left(8\,b^2d^2ex^4+8\,(b^2cd+abd^2)ex^2+(b^2c^2+6\,abcd+a^2d^2)e-4\,(2\,bd^2x^4+bc^2+ac\right.}{8\,b^2de}\right.$$

$$\left.-\frac{\sqrt{-bde}(bc-ad)\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(b^2dex^2+abde)}\right)-2\,(bd^2x^2+bcd)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{4\,b^2de}\right]$$

input
```
integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(b*d*e)*(b*c - a*d)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)
*e*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*
d + (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))
- 4*(b*d^2*x^2 + b*c*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e), -1/4
*(sqrt(-b*d*e)*(b*c - a*d)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)
*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(b*d^2*x^2
 + b*c*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.09

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\frac{2\sqrt{bdex^4+bcex^2+adex^2+ace}}{be} - \frac{\sqrt{bde}(bc-ad)\log\left(\left|-2\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right)bd-\sqrt{bde}bc-\sqrt{bde}ad\right|\right)}{b^2de}}{4\,\mathrm{sgn}\left(dx^2+c\right)}$$

input
```
integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e)/(b*e) - sqrt(b*d*e)
*(b*c - a*d)*log(abs(-2*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*
d*e*x^2 + a*c*e))*b*d - sqrt(b*d*e)*b*c - sqrt(b*d*e)*a*d))/(b^2*d*e))/sgn
(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(x/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.78

$$\int \frac{x}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{\sqrt{e}\left(\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bd+\sqrt{d}\,\sqrt{b}\log\left(-\sqrt{b}\,\sqrt{b\,x^2+a}\,d+\sqrt{d}\,\sqrt{d\,x^2+c}\,b\right)ad-\sqrt{d}\,\sqrt{b}\log\left(-\sqrt{b}\,\sqrt{b}\right.\right.}{2b^2de}$$

input
```
int(x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(e)*(sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d + sqrt(d)*sqrt(b)*log( - s
qrt(b)*sqrt(a + b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b)*a*d - sqrt(d)*sqrt
(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b)*b*c))/
(2*b**2*d*e)
```

3.78 $\int \frac{1}{x\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 136

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=-\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d\left(c+dx^2\right)}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}}+\frac{\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d\left(c+dx^2\right)}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}}$$

output
```
-c^(1/2)*arctanh(c^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a^(1/2)/e^
(1/2))/a^(1/2)/e^(1/2)+d^(1/2)*arctanh(d^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^
2+c))^(1/2)/b^(1/2)/e^(1/2))/b^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.08

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=\frac{\sqrt{a+bx^2}\left(-\sqrt{b}\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{a}\sqrt{d}\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)\right)}{\sqrt{a}\sqrt{b}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(Sqrt[a + b*x^2]*(-(Sqrt[b]*Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqr
t[a]*Sqrt[c + d*x^2])]) + Sqrt[a]*Sqrt[d]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2]
)/(Sqrt[b]*Sqrt[c + d*x^2])]))/(Sqrt[a]*Sqrt[b]*Sqrt[(e*(a + b*x^2))/(c +
d*x^2)]*Sqrt[c + d*x^2])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2053, 2052, 25, 303, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{1}{x^2\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$e(bc-ad)\int -\frac{1}{(ae-cx^4)(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \text{ 25}$$

$$-\left(e(bc-ad)\int \frac{1}{(ae-cx^4)(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\right)$$

$$\Big\downarrow \text{ 303}$$

$$e(bc-ad)\left(\frac{d\int \frac{1}{be-dx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{e(bc-ad)}-\frac{c\int \frac{1}{ae-cx^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{e(bc-ad)}\right)$$

$$\Big\downarrow \text{ 221}$$

$$e(bc-ad)\left(\frac{\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}e^{3/2}(bc-ad)}-\frac{\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}e^{3/2}(bc-ad)}\right)$$

input
```
Int[1/(x*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(b*c - a*d)*e*(-((Sqrt[c]*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2
)])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*(b*c - a*d)*e^(3/2))) + (Sqrt[d]*ArcTanh[
(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(Sqrt[b]*(
b*c - a*d)*e^(3/2)))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.32

method	result
default	$-\frac{\left(b\,x^{2}+a\right)\left(c\ln\left(\frac{a d\,x^{2}+b c\,x^{2}+2\sqrt{a c}\,\sqrt{d b\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}+2 a c}{x^{2}}\right)\sqrt{b d}-\ln\left(\frac{2 b d\,x^{2}+2\sqrt{d b\,x^{4}+a d\,x^{2}+b c\,x^{2}+a c}\,\sqrt{b d}+a d+b c}{2\sqrt{b d}}\right)\sqrt{a c}\,d\right)}{2\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,\sqrt{\left(d\,x^{2}+c\right)\left(b\,x^{2}+a\right)}\,\sqrt{b d}\,\sqrt{a c}}$

input
```
int(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)*(c*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a*d*x^2+b*c*x
^2+a*c)^(1/2)+2*a*c)/x^2)*(b*d)^(1/2)-ln(1/2*(2*b*d*x^2+2*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*d)/(e*(b
*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/(b*d)^(1/2)/(a*c)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 881, normalized size of antiderivative = 6.48

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[1/4*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(
b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d
 + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) + 1/4*sq
rt(c/(a*e))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*
c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3
*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4), -1/2
*sqrt(-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d
*x^2 + c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) + 1/4*sqrt(c/(a*e))*log(((b^2*c
^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*
((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*
e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4), 1/2*sqrt(-c/(a*e))*arctan(1
/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*
e))/(b*c*x^2 + a*c)) + 1/4*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a
*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2
+ a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*s
qrt(d/(b*e))), 1/2*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqr
t((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) - 1/2*sqrt(
-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
 c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{x\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{x\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}dx$$

input
```
int(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
int(1/x/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

3.79 $\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 133

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{2aex^2}+\frac{(bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}\sqrt{e}}$$

output
```
-1/2*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a/e/x^2+1/2*(-a*d+b*
c)*arctanh(c^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))
/a^(3/2)/c^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{-\sqrt{a}\sqrt{c}(a+bx^2)(c+dx^2)+(bc-ad)x^2\sqrt{a+bx^2}\sqrt{c+dx^2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{2a^{3/2}\sqrt{c}x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}$$

input
```
Integrate[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(-(Sqrt[a]*Sqrt[c]*(a + b*x^2)*(c + d*x^2)) + (b*c - a*d)*x^2*Sqrt[a + b*x
^2]*Sqrt[c + d*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*
x^2])])/(2*a^(3/2)*Sqrt[c]*x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^
2))
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2053, 2052, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{1}{x^4\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int \frac{1}{(cx^4-ae)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{215}$$

$$e(bc-ad)\left(\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2ae\left(ae-cx^4\right)}-\frac{\int \frac{1}{cx^4-ae}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2ae}\right)$$

$$\Big\downarrow \text{221}$$

$$e(bc-ad)\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}e^{3/2}}+\frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{2ae\left(ae-cx^4\right)}\right)$$

input
```
Int[1/(x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(b*c - a*d)*e*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*a*e*(a*e - c*x^4)) + A
rcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])]/(2*a
^(3/2)*Sqrt[c]*e^(3/2)))
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.22

method	result
risch	$-\frac{b\,x^2+a}{2a\,x^2\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}-\frac{(ad-bc)\ln\left(\frac{2ace+(ade+bce)x^2+2\sqrt{ace}\,\sqrt{bde\,x^4+(ade+bce)x^2+ace}}{x^2}\right)\sqrt{(d\,x^2+c)(b\,x^2+a)e}}{4a\sqrt{ace}\,\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$-\frac{(b\,x^2+a)\left(-2db\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,x^4\sqrt{ac}+a^2\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}+2ac}{x^2}\right)dc\,x^2-c^2\ln\left(\frac{ad\,x^2+bc\,x}{}\right.\right.}{4\sqrt{\frac{e\left(b\,x^2\right.}{d\,x^2}}}$

input
```
int(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a*(b*x^2+a)/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/4*(a*d-b*c)/a/(a*c*e)
^(1/2)*ln((2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c
*e)*x^2+a*c*e)^(1/2))/x^2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2
+a)*e)^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.50

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$=\left[-\frac{\sqrt{ace}(bc-ad)x^2\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2-4\,((bcd+ad^2)x^4+2\,ac^2+(bc^2+3\,acd)x^2)\sqrt{ace}\sqrt{\frac{be}{d}}}{x^4}\right.}{8\,a^2cex^2}\right.$$

$$\left.-\frac{\sqrt{-ace}(bc-ad)x^2\arctan\left(\frac{\sqrt{-ace}\left((bc+ad)x^2+2\,ac\right)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(abcex^2+a^2ce)}\right)+2\,(acdx^2+ac^2)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{4\,a^2cex^2}\right]$$

input
```
integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*(sqrt(a*c*e)*(b*c - a*d)*x^2*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*
x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c*d + a*d^2)*x^4 +
 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2
+ c)))/x^4) + 4*(a*c*d*x^2 + a*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^
2*c*e*x^2), -1/4*(sqrt(-a*c*e)*(b*c - a*d)*x^2*arctan(1/2*sqrt(-a*c*e)*((b
*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*c*e*x^2 + a^
2*c*e)) + 2*(a*c*d*x^2 + a*c^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^2*c*
e*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(113) = 226$.

Time = 0.18 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.79

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{\frac{(bc-ad)\arctan\left(-\frac{\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}a}+\frac{\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right)bc+\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2-}\right.}{\left(ace-\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right)^2\right)a}}{2\,\mathrm{sgn}\left(dx^2+c\right)}$$

input
```
integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
-1/2*((b*c - a*d)*arctan(-(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 +
a*d*e*x^2 + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e)*a) + ((sqrt(b*d*e)*x^2 - s
qrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*b*c + (sqrt(b*d*e)*x^2 - s
qrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))*a*d + 2*sqrt(b*d*e)*a*c)/(
(a*c*e - (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e
))^2)*a))/sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{x^3\,\sqrt{\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03

$$\int \frac{1}{x^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{\sqrt{e}\left(-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ac+\sqrt{c}\,\sqrt{a}\log\left(\sqrt{a}\,\sqrt{b\,x^2+a}\,c-\sqrt{c}\,\sqrt{d\,x^2+c}\,a\right)ad\,x^2-\sqrt{c}\,\sqrt{a}\log\left(\sqrt{a}\,\sqrt{b}\right.}{2a^2ce\,x^2}$$

input
```
int(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(e)*( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c + sqrt(c)*sqrt(a)*log(s
qrt(a)*sqrt(a + b*x**2)*c - sqrt(c)*sqrt(c + d*x**2)*a)*a*d*x**2 - sqrt(c)
*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c - sqrt(c)*sqrt(c + d*x**2)*a)*b*c*
x**2 - sqrt(c)*sqrt(a)*log(x)*a*d*x**2 + sqrt(c)*sqrt(a)*log(x)*b*c*x**2))
/(2*a**2*c*e*x**2)
```

3.80 $\int \frac{1}{x^5\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 211

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{(3bc+ad)\left(c+dx^2\right)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{8a^2cex^2} - \frac{\left(c+dx^2\right)^2\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{4acex^4} - \frac{(bc-ad)(3bc+ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d\left(c+dx^2\right)}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{5/2}c^{3/2}\sqrt{e}}$$

output

```
1/8*(a*d+3*b*c)*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a^2/c/e/x
^2-1/4*(d*x^2+c)^2*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a/c/e/x^4-1/8*(-
a*d+b*c)*(a*d+3*b*c)*arctanh(c^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2
)/a^(1/2)/e^(1/2))/a^(5/2)/c^(3/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{a}\sqrt{c}(a+bx^2)\sqrt{c+dx^2}(3bcx^2-a(2c+dx^2))-(3b^2c^2-2abcd-a^2d^2)\,x^4\sqrt{a+bx^2}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)}{8a^{5/2}c^{3/2}x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(Sqrt[a]*Sqrt[c]*(a + b*x^2)*Sqrt[c + d*x^2]*(3*b*c*x^2 - a*(2*c + d*x^2))
 - (3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*x^4*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*
Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^(5/2)*c^(3/2)*x^4*Sqrt[(
e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.56 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.87, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2053, 2052, 25, 298, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$\Big\downarrow \; 2053$$

$$\frac{1}{2}\int \frac{1}{x^6\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}dx^2$$

$$\Big\downarrow \; 2052$$

$$e(bc-ad)\int -\frac{be-dx^4}{(ae-cx^4)^3}d\sqrt{\frac{e\,(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \ 25$$

$$-\left(e(bc-ad)\int \frac{be-dx^4}{(ae-cx^4)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\right)$$

$$\Big\downarrow \ 298$$

$$e(bc-ad)\left(-\frac{(ad+3bc)\int \frac{1}{(ae-cx^4)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4ac}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\,(ae-cx^4)^2}\right)$$

$$\Big\downarrow \ 215$$

$$e(bc-ad)\left(-\frac{(ad+3bc)\left(\frac{\int \frac{1}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2ae}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2ae(ae-cx^4)}\right)}{4ac}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\,(ae-cx^4)^2}\right)$$

$$\Big\downarrow \ 221$$

$$e(bc-ad)\left(-\frac{(ad+3bc)\left(\frac{\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{3/2}\sqrt{c}e^{3/2}}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2ae(ae-cx^4)}\right)}{4ac}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4ac\,(ae-cx^4)^2}\right)$$

input
```
Int[1/(x^5*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(b*c - a*d)*e*(-1/4*((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(a*c*(
a*e - c*x^4)^2) - ((3*b*c + a*d)*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*a*e
*(a*e - c*x^4)) + ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqr
t[a]*Sqrt[e])]/(2*a^(3/2)*Sqrt[c]*e^(3/2))))/(4*a*c))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{(b\,x^2+a)\,(ad\,x^2-3bc\,x^2+2ac)}{8a^2x^4c\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}+\frac{(a^2d^2+2abcd-3b^2c^2)\ln\left(\frac{2ace+(ade+bce)x^2+2\sqrt{ace}\,\sqrt{bde\,x^4+(ade+bce)x^2+ace}}{x^2}\right)\sqrt{(d\,x^2+c)(b\,x}{16a^2c\sqrt{ace}\,\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$-\frac{(b\,x^2+a)\left(2d^2b\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,x^6a\sqrt{ac}+10d\,b^2\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,x^6c\sqrt{ac}-a^3\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{db\,x^4+ad\,x}}{x^2}\right.\right.}{}$

input
```
int(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^2+a)*(a*d*x^2-3*b*c*x^2+2*a*c)/a^2/x^4/c/(e*(b*x^2+a)/(d*x^2+c))
^(1/2)+1/16*(a^2*d^2+2*a*b*c*d-3*b^2*c^2)/a^2/c/(a*c*e)^(1/2)*ln((2*a*c*e+
(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2
))/x^2)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)/(d*x^2
+c)
```

Fricas [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.10

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$= \left[-\frac{(3\,b^2c^2-2\,abcd-a^2d^2)\sqrt{ace}x^4\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)ex^4+8\,a^2c^2e+8\,(abc^2+a^2cd)ex^2+4\,((bcd+ad^2)x^4+2\,ac^2+(bc^2+}{x^4}\right.}{32\,a^3c^2ex^4}\right.$$

input
```
integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
[-1/32*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(a*c*e)*x^4*log(((b^2*c^2 +
6*a*b*c*d + a^2*d^2)*e*x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 + 4
*((b*c*d + a*d^2)*x^4 + 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt(
(b*e*x^2 + a*e)/(d*x^2 + c)))/x^4) + 4*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d
^2)*x^4 - 3*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/
(a^3*c^2*e*x^4), 1/16*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*sqrt(-a*c*e)*x^4*
arctan(1/2*sqrt(-a*c*e)*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*
x^2 + c))/(a*b*c*e*x^2 + a^2*c*e)) - 2*(2*a^2*c^3 - (3*a*b*c^2*d - a^2*c*d
^2)*x^4 - 3*(a*b*c^3 - a^2*c^2*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/
(a^3*c^2*e*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(187) = 374$.

Time = 0.18 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.54

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{\frac{(3b^2c^2-2abcd-a^2d^2)\arctan\left(-\frac{\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}}{\sqrt{-ace}}\right)}{\sqrt{-ace}a^2c} + \frac{5\left(\sqrt{bde}x^2-\sqrt{bdex^4+bcex^2+adex^2+ace}\right)ab^2c^3e+10\left(\sqrt{bde}x^2-\sqrt{bd}\right.}{}}{}$$

input
```
integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
1/8*((3*b^2*c^2 - 2*a*b*c*d - a^2*d^2)*arctan(-(sqrt(b*d*e)*x^2 - sqrt(b*d
*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))/sqrt(-a*c*e))/(sqrt(-a*c*e)*a^2*c
) + (5*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))
*a*b^2*c^3*e + 10*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^
2 + a*c*e))*a^2*b*c^2*d*e + (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2
+ a*d*e*x^2 + a*c*e))*a^3*c*d^2*e + 8*sqrt(b*d*e)*a^2*b*c^3*e - 3*(sqrt(b*
d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3*b^2*c^2 + 2*
(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^3*a*b*
c*d + (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^
3*a^2*d^2 + 8*sqrt(b*d*e)*(sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4 + b*c*e*x^2 +
a*d*e*x^2 + a*c*e))^2*a^2*c*d)/((a*c*e - (sqrt(b*d*e)*x^2 - sqrt(b*d*e*x^4
 + b*c*e*x^2 + a*d*e*x^2 + a*c*e))^2)^2*a^2*c))/sgn(d*x^2 + c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{x^5\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.27

$$\int \frac{1}{x^5\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{\sqrt{e}\left(-2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2c^2-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2cd\,x^2+3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ab\,c^2x^2+\sqrt{c}\,\sqrt{c}\right.}{}$$

input
```
int(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(e)*( - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c**2 - sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a**2*c*d*x**2 + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a
*b*c**2*x**2 + sqrt(c)*sqrt(a)*log( - sqrt(a)*sqrt(a + b*x**2)*c - sqrt(c)
*sqrt(c + d*x**2)*a)*a**2*d**2*x**4 + 2*sqrt(c)*sqrt(a)*log( - sqrt(a)*sqr
t(a + b*x**2)*c - sqrt(c)*sqrt(c + d*x**2)*a)*a*b*c*d*x**4 - 3*sqrt(c)*sqr
t(a)*log( - sqrt(a)*sqrt(a + b*x**2)*c - sqrt(c)*sqrt(c + d*x**2)*a)*b**2*
c**2*x**4 - sqrt(c)*sqrt(a)*log(x)*a**2*d**2*x**4 - 2*sqrt(c)*sqrt(a)*log(
x)*a*b*c*d*x**4 + 3*sqrt(c)*sqrt(a)*log(x)*b**2*c**2*x**4))/(8*a**3*c**2*e
*x**4)
```

3.81 $\int \frac{x^4}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 415

$$\begin{aligned}\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = &-\frac{(2b^2c^2+3abcd-8a^2d^2)\,x}{15b^2d^2\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} \\ &+\frac{(bc-4ad)x(a+bx^2)}{15b^2d\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}+\frac{x^3(a+bx^2)}{5b\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} \\ &+\frac{\sqrt{a}(2b^2c^2+3abcd-8a^2d^2)\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{15b^{5/2}d^2\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} \\ &-\frac{a^{3/2}(bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{15b^{5/2}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}\end{aligned}$$

output
```
-1/15*(-8*a^2*d^2+3*a*b*c*d+2*b^2*c^2)*x/b^2/d^2/(b*e/d-(-a*d+b*c)*e/d/(d*
x^2+c))^(1/2)+1/15*(-4*a*d+b*c)*x*(b*x^2+a)/b^2/d/(b*e/d-(-a*d+b*c)*e/d/(d
*x^2+c))^(1/2)+1/5*x^3*(b*x^2+a)/b/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)+
1/15*a^(1/2)*(-8*a^2*d^2+3*a*b*c*d+2*b^2*c^2)*EllipticE(b^(1/2)*x/a^(1/2)/
(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2)/d^2/(a*(d*x^2+c)/c/(b*x^2+a))
^(1/2)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-1/15*a^(3/2)*(-4*a*d+b*c)*In
verseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(5/2)/d/(a*(d
*x^2+c)/c/(b*x^2+a))^(1/2)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.71 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.62

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}dx(a+bx^2)\left(c+dx^2\right)\left(4ad-b(c+3dx^2)\right)-ic(-2b^2c^2-3abcd+8a^2d^2)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(ia\cdots}{15a^2\left(\frac{b}{a}\right)^{5/2}d^2\sqrt{\frac{e(\cdots}{c\cdots}}}$$

input
```
Integrate[x^4/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(-(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2)*(4*a*d - b*(c + 3*d*x^2))) - I*c*
(-2*b^2*c^2 - 3*a*b*c*d + 8*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/
c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*c*(-(b^2*c^2) -
a*b*c*d + 2*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*A
rcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(15*a^2*(b/a)^(5/2)*d^2*Sqrt[(e*(a + b*
x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rubi [A] (verified)

Time = 1.02 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {2058, 380, 444, 25, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{a+bx^2}\int \frac{x^4\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{380}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}-\frac{\int \frac{x^2(3ac-(bc-4ad)x^2)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{444}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}-\frac{-\frac{\int -\frac{(2b^2c^2+3abdc-8a^2d^2)x^2+ac(bc-4ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-4ad)}{3bd}}{5b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}-\frac{\frac{\int \frac{(2b^2c^2+3abdc-8a^2d^2)x^2+ac(bc-4ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-4ad)}{3bd}}{5b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}-\frac{\frac{\left(-8a^2d^2+3abcd+2b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+ac(bc-4ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}-\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(bc-4ad)}{3bd}}{5b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 320

$$\frac{\sqrt{a+bx^2}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}-\frac{\frac{\left(-8a^2d^2+3abcd+2b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}}{3bd}-\frac{x\sqrt{a+b\cdots}}{\cdots}}{5b}\cdots\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 388

$$\frac{\sqrt{a+bx^2}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}-\frac{\frac{\left(-8a^2d^2+3abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\cdots\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\cdots}{3bd}\cdots}{5b}\cdots\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 313

$$\frac{\sqrt{a+bx^2}\left(\frac{x^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}-\frac{\frac{\left(-8a^2d^2+3abcd+2b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(bc-4ad)\,\text{EllipticF}\left(\text{ar}\cdots\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+\cdots)}{a(c+\cdots)}}}\cdots}{3bd}\cdots}{5b}\cdots\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Int[x^4/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[a + b*x^2]*((x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b) - (-1/3*((b*
c - 4*a*d)*x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(b*d) + ((2*b^2*c^2 + 3*a*b*
c*d - 8*a^2*d^2)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[
a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqr
t[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(b
*c - 4*a*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*
c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])
)/(3*b*d))/(5*b)))/(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 4.80 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{x\left(-3bd\,x^{2}+4ad-bc\right)\left(b\,x^{2}+a\right)}{15b^{2}d\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}+\frac{\left(-\frac{2\left(8a^{2}d^{2}-3abcd-2b^{2}c^{2}\right)ace\sqrt{1+\frac{x^{2}b}{a}}\,\sqrt{1+\frac{x^{2}d}{c}}\,\left(\text{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)-\text{EllipticE}\left(x \cdots\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}\,\left(ade+bce+e(ad-bc)\right)}\cdots\right.}{\cdots}$
default	$-\frac{\left(b\,x^{2}+a\right)\left(-3\sqrt{-\frac{b}{a}}\,b^{2}d^{3}x^{7}+\sqrt{-\frac{b}{a}}\,ab\,d^{3}x^{5}-4\sqrt{-\frac{b}{a}}\,b^{2}c\,d^{2}x^{5}+4\sqrt{-\frac{b}{a}}\,a^{2}d^{3}x^{3}-\sqrt{-\frac{b}{a}}\,b^{2}c^{2}d\,x^{3}+4\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\text{EllipticF}\left(\cdots\right.\right.}{\cdots}$

input
```
int(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*x*(-3*b*d*x^2+4*a*d-b*c)*(b*x^2+a)/b^2/d/(e*(b*x^2+a)/(d*x^2+c))^(1/
2)+1/15/b^2/d*(-2*(8*a^2*d^2-3*a*b*c*d-2*b^2*c^2)*a*c*e/(-b/a)^(1/2)*(1+1/
a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(
1/2)/(a*d*e+b*c*e+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)
/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))-a
*b*c^2/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d
*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c
/b/e)^(1/2))+4*a^2*c*d/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2
)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1
+(a*d*e+b*c*e)/c/b/e)^(1/2)))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*
x^2+a)*e)^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.67

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{(2\,b^2c^3+3\,abc^2d-8\,a^2cd^2)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-(2\,b^2c^3+3\,abc^2d-4\,a^2d^3-(8\,a^2-ab)c}{}$$

input
```
integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
1/15*((2*b^2*c^3 + 3*a*b*c^2*d - 8*a^2*c*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*ell
iptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (2*b^2*c^3 + 3*a*b*c^2*d - 4*a^
2*d^3 - (8*a^2 - a*b)*c*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_f(arcsin(sq
rt(-c/d)/x), a*d/(b*c)) + (3*b^2*d^3*x^6 - 2*b^2*c^3 - 3*a*b*c^2*d + 8*a^2
*c*d^2 + 4*(b^2*c*d^2 - a*b*d^3)*x^4 - (b^2*c^2*d + 7*a*b*c*d^2 - 8*a^2*d^
3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*d^2*e*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

input
```
integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^4/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

input
```
integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^4/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^4}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{e}\left(-4\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,adx+\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bcx+3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,bd\,x^3+8\left(\int \frac{\sqrt{d\,x^2+c}}{bd\,x^4+ad}\right)\right)}{}$$

input
```
int(x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(e)*( - 4*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*d*x + sqrt(c + d*x**2)*
sqrt(a + b*x**2)*b*c*x + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*d*x**3 + 8*
int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 +
b*d*x**4),x)*a**2*d**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a
*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*c*d - 2*int((sqrt(c + d*x**2)*
sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b**2*c**2
 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*c*x**2 +
b*d*x**4),x)*a**2*c*d - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d
*x**2 + b*c*x**2 + b*d*x**4),x)*a*b*c**2))/(15*b**2*d*e)
```

3.82 $\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 312

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{(bc-2ad)x}{3bd\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}+\frac{x(a+bx^2)}{3b\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} - \frac{\sqrt{a}(bc-2ad)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3b^{3/2}d\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} - \frac{a^{3/2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3b^{3/2}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}$$

output
```
1/3*(-2*a*d+b*c)*x/b/d/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)+1/3*x*(b*x^2
+a)/b/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-1/3*a^(1/2)*(-2*a*d+b*c)*Elli
pticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(3/2)/d/(a*
(d*x^2+c)/c/(b*x^2+a))^(1/2)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-1/3*a^
(3/2)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(3/2)
/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.10 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.68

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{\sqrt{\frac{b}{a}}dx(a+bx^2)(c+dx^2) + ic(-bc+2ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\text{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right) - ic(-bc+ad)\sqrt{}}{3b\sqrt{\frac{b}{a}}d\sqrt{\frac{e(a+bx^2)}{c+dx^2}}(c+dx^2)}$$

input
```
Integrate[x^2/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[b/a]*d*x*(a + b*x^2)*(c + d*x^2) + I*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
 - I*c*(-(b*c) + a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*
ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])/(3*b*Sqrt[b/a]*d*Sqrt[(e*(a + b*x^2))/
(c + d*x^2)]*(c + d*x^2))
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2058, 380, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$\Big\downarrow \ 2058$$
$$\frac{\sqrt{a+bx^2}\int \frac{x^2\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 380$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}-\frac{\int \frac{ac-(bc-2ad)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 406$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}-\frac{ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 388$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)}{3b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 313$$

$$\frac{\sqrt{a+bx^2}\left(\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3b}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Int[x^2/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[a + b*x^2]*((x*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b) - (-((b*c - 2*
a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*E
llipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c
*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*Sqrt[a + b*x^
2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b)))/(Sqrt[(e*(a + b
*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 3.31 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.12

method	result
risch	$\frac{x(bx^2+a)}{3b\sqrt{\frac{e(bx^2+a)}{dx^2+c}}} - \frac{\left(\frac{ac\sqrt{1+\frac{x^2b}{a}}\sqrt{1+\frac{x^2d}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}} - \frac{2(2ad-bc)ace\sqrt{1+\frac{x^2b}{a}}\sqrt{1+\frac{x^2d}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+a\cdots}}\right.}{3b\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\,(dx^2+c)}$
default	$\frac{(bx^2+a)\left(\sqrt{-\frac{b}{a}}\,bd^2x^5+\sqrt{-\frac{b}{a}}\,ad^2x^3+\sqrt{-\frac{b}{a}}\,bcd\,x^3+ac\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)d-\sqrt{\frac{bx^2+a}{a}}\sqrt{\frac{dx^2+c}{c}}\,\mathrm{Ellipt}\cdots\right.}{3\sqrt{\frac{e(bx^2+a)}{dx^2+c}}\sqrt{(dx^2+c)(b\cdots}}$

input
```
int(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/b*x*(b*x^2+a)/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/3/b*(a*c/(-b/a)^(1/2)*(1
+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e
)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-2*(2*a*d-
b*c)*a*c*e/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4
+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(a*d*e+b*c*e+e*(a*d-b*c))*(EllipticF(x*(
-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(
a*d*e+b*c*e)/c/b/e)^(1/2))))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x
^2+a)*e)^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.59

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx =$$

$$-\frac{(bc^2-2\,acd)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) - (bc^2-2\,acd-ad^2)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}F(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{a}{b}}{3\,b^2dex}$$

input
```
integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((b*c^2 - 2*a*c*d)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c
/d)/x), a*d/(b*c)) - (b*c^2 - 2*a*c*d - a*d^2)*sqrt(b*e/d)*x*sqrt(-c/d)*el
liptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - (b*d^2*x^4 + b*c^2 - 2*a*c*d +
 2*(b*c*d - a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^2*d*e*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^2}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

input
```
integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^2}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

input
```
integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{x^2}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{\sqrt{e}\left(\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x-2\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)ad+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)bc-\left(\int \frac{\sqrt{d\,x^2+c}}{bd\,x^4+ad\,x^2}\right.}{3be}$$

input
```
int(x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(e)*(sqrt(c + d*x**2)*sqrt(a + b*x**2)*x - 2*int((sqrt(c + d*x**2)*sq
rt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*a*d + int((
sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x
**4),x)*b*c - int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c + a*d*x**2 + b*
c*x**2 + b*d*x**4),x)*a*c))/(3*b*e)
```

3.83 $\int \frac{1}{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 230

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{x}{\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} - \frac{\sqrt{a}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} + \frac{\sqrt{a}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}$$

output

```
x/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-a^(1/2)*EllipticE(b^(1/2)*x/a^(1/
2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^
(1/2)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)+a^(1/2)*InverseJacobiAM(arcta
n(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a))^
(1/2)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)
```

Mathematica [A] (verified)

Time = 1.82 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.37

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{\frac{a+bx^2}{a}}E\left(\arcsin\left(\sqrt{-\frac{b}{a}}x\right)|\frac{ad}{bc}\right)}{\sqrt{-\frac{b}{a}}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{\frac{c+dx^2}{c}}}$$

input
```
Integrate[1/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output
```
(Sqrt[(a + b*x^2)/a]*EllipticE[ArcSin[Sqrt[-(b/a)]*x], (a*d)/(b*c)])/(Sqrt
[-(b/a)]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(c + d*x^2)/c])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {2058, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{a+bx^2}\int \frac{\sqrt{dx^2+c}}{\sqrt{bx^2+a}}dx}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{324}$$

$$\frac{\sqrt{a+bx^2}\left(c\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{a+bx^2}\left(d\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\Big\downarrow$ 388

$$\frac{\sqrt{a+bx^2}\left(d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{a+bx^2}\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input

```
Int[1/Sqrt[(e*(a + b*x^2))/(c + d*x^2)],x]
```

output

```
(Sqrt[a + b*x^2]*(d*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sq
rt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*
Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)
*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/
(a*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(Sqrt[
(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{(b\,x^2+a)c\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)}{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}}$	127

input
```
int(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(b*x^2+a)*c*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))/(e*(b*x^2+a)/(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1
/2)/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{c\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}E\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}\right) - (c+d)\sqrt{\frac{be}{d}}x\sqrt{-\frac{c}{d}}F\left(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}\right) - (dx^2+c)\sqrt{\frac{bex^2+ae}{dx^2+c}}}{bex}$$

input
```
integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-(c*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) -
 (c + d)*sqrt(b*e/d)*x*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*
c)) - (d*x^2 + c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b*e*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

input
```
integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}}\,dx$$

input
```
integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt((b*x^2 + a)*e/(d*x^2 + c)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(1/((e*(a + b*x^2))/(c + d*x^2))^(1/2),x)
```

output
```
int(1/((e*(a + b*x^2))/(c + d*x^2))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b\,x^2+a}dx\right)}{e}$$

input
```
int(1/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(e)*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a + b*x**2),x))/e
```

3.84 $\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 237

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = -\frac{1}{x\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} - \frac{\sqrt{b}E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} + \frac{\sqrt{a}d\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{\sqrt{b}c\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}$$

output
```
-1/x/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-b^(1/2)*EllipticE(b^(1/2)*x/a^
(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/(a*(d*x^2+c)/c/(b*x^2+a
))^(1/2)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)+a^(1/2)*d*InverseJacobiAM(
arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(1/2)/c/(a*(d*x^2+c)/c/(b*x
^2+a))^(1/2)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)
```

Mathematica [A] (verified)

Time = 2.11 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.47

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx=\frac{(a+bx^2)\left(-\frac{1}{x}+\frac{d\sqrt{1+\frac{dx^2}{c}}E\left(\arcsin\left(\sqrt{-\frac{d}{c}}x\right)|\frac{bc}{ad}\right)}{\sqrt{-\frac{d}{c}}\sqrt{1+\frac{bx^2}{a}}(c+dx^2)}\right)}{a\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Integrate[1/(x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
((a + b*x^2)*(-x^(-1) + (d*Sqrt[1 + (d*x^2)/c]*EllipticE[ArcSin[Sqrt[-(d/c
)]*x], (b*c)/(a*d)])/(Sqrt[-(d/c)]*Sqrt[1 + (b*x^2)/a]*(c + d*x^2))))/(a*S
qrt[(e*(a + b*x^2))/(c + d*x^2)])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.20, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.269, Rules used = {2058, 377, 27, 324, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{a+bx^2}\int \frac{\sqrt{dx^2+c}}{x^2\sqrt{bx^2+a}}dx}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{377}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\int \frac{d\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ax}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\int \frac{\sqrt{bx^2+a}}{\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ax}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \ 324$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(a\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ax}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \ 320$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(b\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ax}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \ 388$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ax}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \ 313$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}+b\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)\right)}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{ax}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Int[1/(x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(Sqrt[a + b*x^2]*(-((Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*x)) + (d*(b*((x*S
qrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[A
rcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^
2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*Sqrt[a + b*x^2]*Elliptic
F[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a + b*x
^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/a))/(Sqrt[(e*(a + b*x^2))/(c + d*
x^2)]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 324
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
a   Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Simp[b   Int[x^2/(Sqr
t[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c
] && PosQ[b/a]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 4.04 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.25

method	result
default	$-\frac{\left(b\,x^2+a\right)\left(\sqrt{-\frac{b}{a}}\,bd\,x^4-\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)adx+bc\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2+c}{c}}\,x\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{ad}{bc}}\right)-bc\cdots\right.}{\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,\sqrt{(d\,x^2+c)(b\,x^2+a)}\,ax\sqrt{-\frac{b}{a}}\,\sqrt{db\,x^4+ad\cdots}}$
risch	$-\frac{b\,x^2+a}{ax\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}+\frac{d\left(\frac{a\sqrt{1+\frac{x^2b}{a}}\,\sqrt{1+\frac{x^2d}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}-\frac{2bace\sqrt{1+\frac{x^2b}{a}}\,\sqrt{1+\frac{x^2d}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade\cdots}{cb\cdots}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ac\cdots}}\right.}{a\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}\,(d\,x^2+c)}$

input
```
int(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^2+a)*((-b/a)^(1/2)*b*d*x^4-((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*E
llipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*d*x+b*c*((b*x^2+a)/a)^(1/2)*((d
*x^2+c)/c)^(1/2)*x*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))-b*c*((b*x^2+a
)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*x*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))
+(-b/a)^(1/2)*a*d*x^2+(-b/a)^(1/2)*b*c*x^2+(-b/a)^(1/2)*a*c)/(e*(b*x^2+a)/
(d*x^2+c))^(1/2)/((d*x^2+c)*(b*x^2+a))^(1/2)/a/x/(-b/a)^(1/2)/(b*d*x^4+a*d
*x^2+b*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.66

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{b^2cd\sqrt{\frac{ace}{d^2}}x\sqrt{-\frac{b}{a}}E\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}\right) - (b^2cd + a^2d^2)\sqrt{\frac{ace}{d^2}}x\sqrt{-\frac{b}{a}}F\left(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}\right) - (abcdx^2}{a^2bcex}$$

input
```
integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
(b^2*c*d*sqrt(a*c*e/d^2)*x*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d
/(b*c)) - (b^2*c*d + a^2*d^2)*sqrt(a*c*e/d^2)*x*sqrt(-b/a)*elliptic_f(arcs
in(x*sqrt(-b/a)), a*d/(b*c)) - (a*b*c*d*x^2 + a*b*c^2)*sqrt((b*e*x^2 + a*e
)/(d*x^2 + c)))/(a^2*b*c*e*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x^2}\,dx$$

input
```
integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x^2}\,dx$$

input
```
integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{x^2\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{e}\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{b\,x^4+a\,x^2}dx\right)}{e}$$

input
```
int(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(e)*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*x**2 + b*x**4),x))/e
```

3.85 $\int \frac{1}{x^4\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 370

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{-a-bx^2}{3ax^3\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}+\frac{(2bc-ad)\left(a+bx^2\right)}{3a^2x\left(c+dx^2\right)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}+\frac{\sqrt{d}(2bc-ad)\left(a+bx^2\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^2\sqrt{c}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\left(c+dx^2\right)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}-\frac{b\sqrt{c}\sqrt{d}\left(a+bx^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^2\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\left(c+dx^2\right)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}$$

output
```
1/3*(-b*x^2-a)/a/x^3/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)+1/3*(-a*d+2*b*
c)*(b*x^2+a)/a^2/x/(d*x^2+c)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)+1/3*d^
(1/2)*(-a*d+2*b*c)*(b*x^2+a)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(1/2)
,(1-b*c/a/d)^(1/2))/a^2/c^(1/2)/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)/
(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-1/3*b*c^(1/2)*d^(1/2)*(b*x^2+a)*Inv
erseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a^2/(c*(b*x^2+a)
/a/(d*x^2+c))^(1/2)/(d*x^2+c)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.49 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.64

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$
$$= \frac{-\sqrt{\frac{b}{a}}(a+bx^2)\,(c+dx^2)\,(-2bcx^2+a(c+dx^2))-ibc(-2bc+ad)x^3\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\Big(i\text{arcsinh}\Big(\sqrt{\frac{b}{a}}}{3a^2\sqrt{\frac{b}{a}}cx^3\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\,(c+d}$$

input
```
Integrate[1/(x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(-(Sqrt[b/a]*(a + b*x^2)*(c + d*x^2)*(-2*b*c*x^2 + a*(c + d*x^2))) - I*b*c
*(-2*b*c + a*d)*x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*Ar
cSinh[Sqrt[b/a]*x], (a*d)/(b*c)] + (2*I)*b*c*(-(b*c) + a*d)*x^3*Sqrt[1 + (
b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c
)])/(3*a^2*Sqrt[b/a]*c*x^3*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(c + d*x^2))
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 348, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2058, 377, 25, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx$$

$$\Big\downarrow \; 2058$$

$$\frac{\sqrt{a+bx^2}\int \frac{\sqrt{dx^2+c}}{x^4\sqrt{bx^2+a}}dx}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{377}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\int -\frac{bdx^2+2bc-ad}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ax^3}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+bx^2}\left(-\frac{\int \frac{bdx^2+2bc-ad}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ax^3}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{a+bx^2}\left(-\frac{-\frac{\int -\frac{bd\left((2bc-ad)x^2+ac\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ax^3}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+bx^2}\left(-\frac{\frac{\int \frac{bd\left((2bc-ad)x^2+ac\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ax^3}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+bx^2}\left(-\frac{\frac{bd\int \frac{(2bc-ad)x^2+ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ax^3}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{406}$$

$$\frac{\sqrt{a+bx^2}\left(-\frac{\frac{bd\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ax^3}\right)}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{320}$$

$$\frac{\sqrt{a+bx^2}\left(-\frac{\frac{bd\left((2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}}{3ax^3}\right.}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 388

$$\frac{\sqrt{a+bx^2}\left(-\frac{\frac{bd\left((2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}}{3a}-\sqrt{a+\right.}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 313

$$\frac{\sqrt{a+bx^2}\left(-\frac{\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2}{acx}\right.}{\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input

```
Int[1/(x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]),x]
```

output
```
(Sqrt[a + b*x^2]*(-1/3*(Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*x^3) - (-(((2*
b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*((2*b*c - a*d)
*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellip
ticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a
+ b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a + b*x^2]*El
lipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*Sqrt[(c*(a
 + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c))/(3*a)))/(Sqrt[(e*(a
+ b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 7.40 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.02

method	result
risch	$-\frac{\left(b\,x^{2}+a\right)\left(a d\,x^{2}-2 b c\,x^{2}+a c\right)}{3 a^{2} x^{3} c\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}-\frac{d b\left(\frac{2\left(a d-2 b c\right) a c e\sqrt{1+\frac{x^{2} b}{a}}\,\sqrt{1+\frac{x^{2} d}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{a d e+b c e}{c b e}}\right)-\operatorname{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ad}{}}\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{b d e\,x^{4}+a d e\,x^{2}+b c e\,x^{2}+a c e}\,\left(a d e+b c e+e\left(a d-b c\right)\right)}\right.}{3 a^{2} c\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}$
default	$-\frac{\left(b\,x^{2}+a\right)\left(\sqrt{-\frac{b}{a}}\,a b\,d^{2} x^{6}-2\sqrt{-\frac{b}{a}}\,b^{2} c d\,x^{6}+2 b d\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{\frac{a d}{b c}}\right)x^{3} a c-2\sqrt{\frac{b\,x^{2}+a}{a}}\,\sqrt{\frac{d\,x^{2}+c}{c}}\,\text{Ellipticl}\right.}{}$

input
```
int(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)*(a*d*x^2-2*b*c*x^2+a*c)/a^2/x^3/c/(e*(b*x^2+a)/(d*x^2+c))^(
1/2)-1/3/a^2*d*b/c*(2*(a*d-2*b*c)*a*c*e/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(
1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(a*d*e+b*c*
e+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))-E
llipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))+a*c/(-b/a)^(1/2)*
(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c
*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))/(e*(b*
x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.56

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx =$$
$$-\frac{(2\,b^2cd-abd^2)\sqrt{\frac{ace}{d^2}}x^3\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-(2\,b^2cd+(a^2-ab)d^2)\sqrt{\frac{ace}{d^2}}x^3\sqrt{-\frac{b}{a}}F(\arcsin\left(\right.}{3\,a^3cex^3}$$

input
```
integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="fricas")
```

output
```
-1/3*((2*b^2*c*d - a*b*d^2)*sqrt(a*c*e/d^2)*x^3*sqrt(-b/a)*elliptic_e(arcs
in(x*sqrt(-b/a)), a*d/(b*c)) - (2*b^2*c*d + (a^2 - a*b)*d^2)*sqrt(a*c*e/d^
2)*x^3*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((2*a*b*c*
d - a^2*d^2)*x^4 - a^2*c^2 + 2*(a*b*c^2 - a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c)))/(a^3*c*e*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**4/(e*(b*x**2+a)/(d*x**2+c))**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x^4}\,dx$$

input
```
integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{\sqrt{\frac{(bx^2+a)e}{dx^2+c}}\,x^4}\,dx$$

input
```
integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt((b*x^2 + a)*e/(d*x^2 + c))*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \int \frac{1}{x^4\,\sqrt{\frac{e\,(b\,x^2+a)}{d\,x^2+c}}}\,dx$$

input
```
int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2)),x)
```

output
```
int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}\,dx = \frac{\sqrt{e}\left(-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,d+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^2}{bd\,x^4+ad\,x^2+bc\,x^2+ac}dx\right)b\,d^2x+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}}{bd\,x^8+ad\,x^6+bc\,x^6+ac\,x^4}dx\right)a\,c^2x\right)}{acex}$$

input
```
int(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(1/2),x)
```

output
```
(sqrt(e)*( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*d + int((sqrt(c + d*x**2)*s
qrt(a + b*x**2)*x**2)/(a*c + a*d*x**2 + b*c*x**2 + b*d*x**4),x)*b*d**2*x +
 int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a*c*x**4 + a*d*x**6 + b*c*x**6 +
 b*d*x**8),x)*a*c**2*x))/(a*c*e*x)
```

3.86 $\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 350

$$\begin{aligned}\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx &= -\frac{a^2(bc-ad)}{b^4e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} \\ &\quad -\frac{(b^2c^2+10abcd-19a^2d^2)\,(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{16b^4de^2} \\ &\quad -\frac{(bc+11ad)\,(c+dx^2)^2\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{24b^3de^2}+\frac{(c+dx^2)^3\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{6b^2de^2} \\ &\quad -\frac{(bc-ad)\,(b^2c^2+10abcd-35a^2d^2)\,\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{b}\sqrt{e}}\right)}{16b^{9/2}d^{3/2}e^{3/2}}\end{aligned}$$

output
```
-a^2*(-a*d+b*c)/b^4/e/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-1/16*(-19*a^2
*d^2+10*a*b*c*d+b^2*c^2)*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/
b^4/d/e^2-1/24*(11*a*d+b*c)*(d*x^2+c)^2*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(
1/2)/b^3/d/e^2+1/6*(d*x^2+c)^3*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^2/
d/e^2-1/16*(-a*d+b*c)*(-35*a^2*d^2+10*a*b*c*d+b^2*c^2)*arctanh(d^(1/2)*(b*
e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/b^(9/2)/d^(3/2)/e^(3/
2)
```

Mathematica [A] (verified)

Time = 4.60 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.71

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{d}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\left(105a^3d^2+5a^2bd(-20c+7dx^2)+ab^2(3c^2-38cdx^2-14d^2x^4)+b^3x^2(\cdots\right.}{48b^4d^{3/2}e\sqrt{\cdots}}$$

input
```
Integrate[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(105*a^3*d^2 + 5*a^2*b*d*(-20*c
 + 7*d*x^2) + a*b^2*(3*c^2 - 38*c*d*x^2 - 14*d^2*x^4) + b^3*x^2*(3*c^2 + 1
4*c*d*x^2 + 8*d^2*x^4)) - 3*Sqrt[b*c - a*d]*(b^2*c^2 + 10*a*b*c*d - 35*a^2
*d^2)*Sqrt[a + b*x^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/
(48*b^4*d^(3/2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(
b*c - a*d)])
```

Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 289, normalized size of antiderivative = 0.83, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.346$, Rules used = {2053, 2052, 365, 25, 27, 298, 215, 215, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \textbf{2053}$$

$$\frac{1}{2}\int \frac{x^4}{\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \textbf{2052}$$

$$e(bc-ad)\int \frac{(ae-cx^4)^2}{x^4(be-dx^4)^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}$$

$$\Big\downarrow \textbf{365}$$

$$e(bc-ad)\left(\frac{\int -\frac{e(a(2bc-7ad)e-bc^2x^4)}{(be-dx^4)^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{be}-\frac{a^2e}{bx^2(be-dx^4)^3}\right)$$

$$\Big\downarrow \textbf{25}$$

$$e(bc-ad)\left(-\frac{\int \frac{e(a(2bc-7ad)e-bc^2x^4)}{(be-dx^4)^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{be}-\frac{a^2e}{bx^2(be-dx^4)^3}\right)$$

$$\Big\downarrow \textbf{27}$$

$$e(bc-ad)\left(-\frac{\int \frac{a(2bc-7ad)e-bc^2x^4}{(be-dx^4)^4}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{b}-\frac{a^2e}{bx^2(be-dx^4)^3}\right)$$

$$\Big\downarrow \textbf{298}$$

$$e(bc-ad)\left(-\frac{\frac{1}{6}\left(\frac{5a(2bc-7ad)}{b}+\frac{bc^2}{d}\right)\int \frac{1}{(be-dx^4)^3}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}-\frac{\left(\frac{bc^2}{d}-\frac{a(2bc-7ad)}{b}\right)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{6(be-dx^4)^3}}{b}-\frac{a^2e}{bx^2(be-dx^4)^3}\right)$$

$$\Big\downarrow \textbf{215}$$

$$ad)\left(-\frac{\frac{1}{6}\left(\frac{5a(2bc-7ad)}{b}+\frac{bc^2}{d}\right)\left(\frac{3\int\frac{1}{\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{4be}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4be(be-dx^4)^2}\right)-\frac{\left(\frac{bc^2}{d}-\frac{a(2bc-7ad)}{b}\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{6(be-dx^4)^3}}{b}-\frac{a^2e}{bx^2\left(be-dx\right.}\right.$$

$e(bc-$

$\downarrow$ 215

$$ad)\left(-\frac{\frac{1}{6}\left(\frac{5a(2bc-7ad)}{b}+\frac{bc^2}{d}\right)\left(\frac{3\left(\frac{\int\frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2be}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be\left(be-dx^4\right)}\right)}{4be}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4be(be-dx^4)^2}\right)-\frac{\left(\frac{bc^2}{d}-\frac{a(2bc-7ad)}{b}\right)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{6(be-dx^4)^3}}{b}-\right.$$

$e(bc-$

$\downarrow$ 221

$$e(bc-ad)\left(-\frac{a^2e}{bx^2\left(be-dx^4\right)^3}-\frac{\frac{1}{6}\left(\frac{5a(2bc-7ad)}{b}+\frac{bc^2}{d}\right)\left(\frac{3\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{3/2}\sqrt{d}e^{3/2}}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be\left(be-dx^4\right)}\right)}{4be}+\frac{\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{4be(be-dx^4)^2}\right)-\frac{\left(\frac{bc^2}{d}\cdots\right.}{}}{b}\right.$$

input
```
Int[x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(b*c - a*d)*e*(-((a^2*e)/(b*x^2*(b*e - d*x^4)^3)) - (-1/6*(((b*c^2)/d - (a
*(2*b*c - 7*a*d))/b)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(b*e - d*x^4)^3 +
(((b*c^2)/d + (5*a*(2*b*c - 7*a*d))/b)*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/
(4*b*e*(b*e - d*x^4)^2) + (3*(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]/(2*b*e*(b*
e - d*x^4)) + ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]
*Sqrt[e])]/(2*b^(3/2)*Sqrt[d]*e^(3/2))))/(4*b*e)))/6)/b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 365
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2, x
_Symbol] :> Simp[c^2*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x]
- Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*Simp[2*b*c^2*(p
+ 1) + c*(b*c - 2*a*d)*(m + 1) - a*d^2*(m + 1)*x^2, x], x], x] /; FreeQ[{a,
 b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.91

method	result
risch	$\frac{\left(8b^2d^2x^4-22ab\,d^2x^2+14b^2c\,x^2d+57a^2d^2-52abcd+3b^2c^2\right)\left(b\,x^2+a\right)}{48d\,b^4e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}-\frac{\left(\frac{\left(35a^2d^2-10abcd-b^2c^2\right)(ad-bc)\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+bd\,x^2e}{\sqrt{bde}}+\sqrt{\right.}{2\sqrt{bde}}\right.}{1}$
default	Expression too large to display

input
```
int(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/48/d*(8*b^2*d^2*x^4-22*a*b*d^2*x^2+14*b^2*c*d*x^2+57*a^2*d^2-52*a*b*c*d+
3*b^2*c^2)*(b*x^2+a)/b^4/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/16/b^4/d*(1/2*(
35*a^2*d^2-10*a*b*c*d-b^2*c^2)*(a*d-b*c)*ln((1/2*a*d*e+1/2*b*c*e+b*d*x^2*e
)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)-1
6*a^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)*d*(d*x^2+c)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x
^2+b*c*e*x^2+a*c*e)^(1/2))/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x
^2+a)*e)^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 781, normalized size of antiderivative = 2.23

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/192*(3*(a*b^3*c^3 + 9*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^
4*c^3 + 9*a*b^3*c^2*d - 45*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(b*d*e)*
log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e*x^2 + (b^2*c^2 + 6*a*b*c*d +
 a^2*d^2)*e - 4*(2*b*d^2*x^4 + b*c^2 + a*c*d + (3*b*c*d + a*d^2)*x^2)*sqrt
(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) + 4*(8*b^4*d^4*x^8 + 3*a*b^3*c^
3*d - 100*a^2*b^2*c^2*d^2 + 105*a^3*b*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^
4)*x^6 + (17*b^4*c^2*d^2 - 52*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^4 + (3*b^4*c
^3*d - 35*a*b^3*c^2*d^2 - 65*a^2*b^2*c*d^3 + 105*a^3*b*d^4)*x^2)*sqrt((b*e
*x^2 + a*e)/(d*x^2 + c)))/(b^6*d^2*e^2*x^2 + a*b^5*d^2*e^2), 1/96*(3*(a*b^
3*c^3 + 9*a^2*b^2*c^2*d - 45*a^3*b*c*d^2 + 35*a^4*d^3 + (b^4*c^3 + 9*a*b^3
*c^2*d - 45*a^2*b^2*c*d^2 + 35*a^3*b*d^3)*x^2)*sqrt(-b*d*e)*arctan(1/2*(2*
b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(b^2*d
*e*x^2 + a*b*d*e)) + 2*(8*b^4*d^4*x^8 + 3*a*b^3*c^3*d - 100*a^2*b^2*c^2*d^
2 + 105*a^3*b*c*d^3 + 2*(11*b^4*c*d^3 - 7*a*b^3*d^4)*x^6 + (17*b^4*c^2*d^2
 - 52*a*b^3*c*d^3 + 35*a^2*b^2*d^4)*x^4 + (3*b^4*c^3*d - 35*a*b^3*c^2*d^2
- 65*a^2*b^2*c*d^3 + 105*a^3*b*d^4)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))
)/(b^6*d^2*e^2*x^2 + a*b^5*d^2*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[1,0,0]%%%},[2,1,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1
,1,1]%%%}
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^5}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^5/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.74

$$\int \frac{x^5}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{e}\left(105\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^3b\,d^3-100\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2b^2c\,d^2+35\sqrt{d\,x^2+c}\right.}{}$$

input
```
int(x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(e)*(105*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*b*d**3 - 100*sqrt(c +
 d*x**2)*sqrt(a + b*x**2)*a**2*b**2*c*d**2 + 35*sqrt(c + d*x**2)*sqrt(a +
b*x**2)*a**2*b**2*d**3*x**2 + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c
**2*d - 38*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**3*c*d**2*x**2 - 14*sqrt(
c + d*x**2)*sqrt(a + b*x**2)*a*b**3*d**3*x**4 + 3*sqrt(c + d*x**2)*sqrt(a
+ b*x**2)*b**4*c**2*d*x**2 + 14*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*c*d
**2*x**4 + 8*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**4*d**3*x**6 + 105*sqrt(d
)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b)*
a**4*d**3 - 135*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d
)*sqrt(c + d*x**2)*b)*a**3*b*c*d**2 + 105*sqrt(d)*sqrt(b)*log( - sqrt(b)*s
qrt(a + b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b)*a**3*b*d**3*x**2 + 27*sqrt
(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b
)*a**2*b**2*c**2*d - 135*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d
 + sqrt(d)*sqrt(c + d*x**2)*b)*a**2*b**2*c*d**2*x**2 + 3*sqrt(d)*sqrt(b)*l
og( - sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b)*a*b**3*c**3
 + 27*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d)*sqrt(c +
 d*x**2)*b)*a*b**3*c**2*d*x**2 + 3*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a +
 b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b)*b**4*c**3*x**2))/(48*b**5*d**2*e*
*2*(a + b*x**2))
```

3.87 $\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 250

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{a(bc-ad)}{b^3e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} + \frac{(3bc-7ad)\left(c+dx^2\right)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{8b^3e^2} + \frac{\left(c+dx^2\right)^2\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{4b^2e^2} + \frac{3(bc-5ad)(bc-ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{b}\sqrt{e}}\right)}{8b^{7/2}\sqrt{d}e^{3/2}}$$

output
```
a*(-a*d+b*c)/b^3/e/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)+1/8*(-7*a*d+3*b*
c)*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^3/e^2+1/4*(d*x^2+c)^
2*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^2/e^2+3/8*(-5*a*d+b*c)*(-a*d+b*
c)*arctanh(d^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))
/b^(7/2)/d^(1/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 4.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.76

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{d}\sqrt{\frac{b(c+dx^2)}{bc-ad}}\left(-15a^2d+ab(13c-5dx^2)+b^2x^2(5c+2dx^2)\right)+3(bc-5ad)\sqrt{bc-ad}\sqrt{}}{8b^3\sqrt{d}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{\frac{b(c+dx^2)}{bc-ad}}}$$

input
```
Integrate[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[d]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)]*(-15*a^2*d + a*b*(13*c - 5*d*x^
2) + b^2*x^2*(5*c + 2*d*x^2)) + 3*(b*c - 5*a*d)*Sqrt[b*c - a*d]*Sqrt[a + b
*x^2]*ArcSinh[(Sqrt[d]*Sqrt[a + b*x^2])/Sqrt[b*c - a*d]])/(8*b^3*Sqrt[d]*e
*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[(b*(c + d*x^2))/(b*c - a*d)])
```

Rubi [A] (warning: unable to verify)

Time = 0.73 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.83, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {2053, 2052, 25, 361, 25, 27, 361, 25, 27, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{x^2}{\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 2052}$$

$$e(bc-ad)\int -\frac{ae-cx^4}{x^4\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{ 25}$$

$$-\left(e(bc-ad)\int \frac{ae-cx^4}{x^4\left(be-dx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$$\Big\downarrow \ 361$$

$$e(bc-ad)\left(\frac{1}{4}\int -\frac{4abe-3(bc-ad)x^4}{b^2ex^4\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}+\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}\right)$$

$$\Big\downarrow \ 25$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}-\frac{1}{4}\int \frac{4abe-3(bc-ad)x^4}{b^2ex^4\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$$\Big\downarrow \ 27$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}-\frac{\int \frac{4abe-3(bc-ad)x^4}{x^4(be-dx^4)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4b^2e}\right)$$

$$\Big\downarrow \ 361$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}-\frac{-\frac{1}{2}\int -\frac{8ae-\left(3c-\frac{7ad}{b}\right)x^4}{ex^4(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}-\frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be(be-dx^4)}}{4b^2e}\right)$$

$$\Big\downarrow \ 25$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}-\frac{\frac{1}{2}\int \frac{8ae-\left(3c-\frac{7ad}{b}\right)x^4}{ex^4(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}-\frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be(be-dx^4)}}{4b^2e}\right)$$

$$\Big\downarrow \ 27$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}-\frac{\frac{\int \frac{8ae-\left(3c-\frac{7ad}{b}\right)x^4}{x^4\left(be-dx^4\right)}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2e}-\frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be(be-dx^4)}}{4b^2e}\right)$$

$$\Big\downarrow \ 359$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}-\frac{\frac{-\frac{3(bc-5ad)\int \frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{b}-\frac{8a}{bx^2}}{2e}-\frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be(be-dx^4)}}{4b^2e}\right)$$

$$\Big\downarrow \ 221$$

$$e(bc-ad)\left(\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4b^2e\left(be-dx^4\right)^2}-\frac{\frac{-\frac{3(bc-5ad)\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}\sqrt{d}\sqrt{e}}-\frac{8a}{bx^2}}{2e}-\frac{(3bc-7ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2be(be-dx^4)}}{4b^2e}\right)$$

input
```
Int[x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(b*c - a*d)*e*(((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(4*b^2*e*(b
*e - d*x^4)^2) - (-1/2*((3*b*c - 7*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])
/(b*e*(b*e - d*x^4)) + ((-8*a)/(b*x^2) - (3*(b*c - 5*a*d)*ArcTanh[(Sqrt[d]
*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(b^(3/2)*Sqrt[d]*S
qrt[e]))/(2*e))/(4*b^2*e))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{(-2bd\,x^2+7ad-5bc)\,(b\,x^2+a)}{8b^3e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}+\frac{\left(\frac{3(5ad-bc)(ad-bc)\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+bd\,x^2e}{\sqrt{bde}}+\sqrt{bde\,x^4+(ade+bce)x^2+ace}\right)}{2\sqrt{bde}}-\frac{8a\left(a^2d^2-2abcd+b^2c\right.}{(ad-bc)\sqrt{bde\,x^4+ade\,x}}\right.}{8b^3e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$-\frac{(b\,x^2+a)\left(-4\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}\,b^2d\,x^4-15\ln\left(\frac{2bd\,x^2+2\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)a^2b\,d^2x^2+18\ln\left(\frac{2bd\,x^2+}{}\right.\right.}{}$

input
```
int(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-2*b*d*x^2+7*a*d-5*b*c)*(b*x^2+a)/b^3/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2
)+1/8/b^3*(3/2*(5*a*d-b*c)*(a*d-b*c)*ln((1/2*a*d*e+1/2*b*c*e+b*d*x^2*e)/(b
*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)-8*a*(
a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x^2+c)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*
x^2+a*c*e)^(1/2))/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^
(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.34

$$\int\frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=\left[\frac{3\left(ab^2c^2-6\,a^2bcd+5\,a^3d^2+(b^3c^2-6\,ab^2cd+5\,a^2bd^2)x^2\right)\sqrt{bde}\log\left(8\,b^2d^2ex^4+8\right.}{}\right.$$

$$-\frac{3\left(ab^2c^2-6\,a^2bcd+5\,a^3d^2+(b^3c^2-6\,ab^2cd+5\,a^2bd^2)x^2\right)\sqrt{-bde}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{-bde}\sqrt{\frac{bex^2+ae}{dx^2+c}}}{2\,(b^2dex^2+abde)}\right)-}{16\,(b^5de^2x}$$

input
```
integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*(a*b^2*c^2 - 6*a^2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5
*a^2*b*d^2)*x^2)*sqrt(b*d*e)*log(8*b^2*d^2*e*x^4 + 8*(b^2*c*d + a*b*d^2)*e
*x^2 + (b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e + 4*(2*b*d^2*x^4 + b*c^2 + a*c*d
+ (3*b*c*d + a*d^2)*x^2)*sqrt(b*d*e)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))) +
4*(2*b^3*d^3*x^6 + 13*a*b^2*c^2*d - 15*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5*a*b^
2*d^3)*x^4 + (5*b^3*c^2*d + 8*a*b^2*c*d^2 - 15*a^2*b*d^3)*x^2)*sqrt((b*e*x
^2 + a*e)/(d*x^2 + c)))/(b^5*d*e^2*x^2 + a*b^4*d*e^2), -1/16*(3*(a*b^2*c^2
 - 6*a^2*b*c*d + 5*a^3*d^2 + (b^3*c^2 - 6*a*b^2*c*d + 5*a^2*b*d^2)*x^2)*sq
rt(-b*d*e)*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt(-b*d*e)*sqrt((b*e*x^2 +
 a*e)/(d*x^2 + c))/(b^2*d*e*x^2 + a*b*d*e)) - 2*(2*b^3*d^3*x^6 + 13*a*b^2*
c^2*d - 15*a^2*b*c*d^2 + (7*b^3*c*d^2 - 5*a*b^2*d^3)*x^4 + (5*b^3*c^2*d +
8*a*b^2*c*d^2 - 15*a^2*b*d^3)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^5
*d*e^2*x^2 + a*b^4*d*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[1,0,0]%%%},[2,1,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1
,1,1]%%%}
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^3}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^3/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 404, normalized size of antiderivative = 1.62

$$\int \frac{x^3}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{e}\left(-15\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2b\,d^2+13\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a\,b^2cd-5\sqrt{d\,x^2+c}\,\sqrt{b}\right.}{}$$

input
```
int(x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(e)*( - 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*d**2 + 13*sqrt(c
+ d*x**2)*sqrt(a + b*x**2)*a*b**2*c*d - 5*sqrt(c + d*x**2)*sqrt(a + b*x**2
)*a*b**2*d**2*x**2 + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*c*d*x**2 + 2
*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**3*d**2*x**4 + 15*sqrt(d)*sqrt(b)*log
( - sqrt(b)*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c + d*x**2)*b)*a**3*d**2 - 1
8*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c + d*x
**2)*b)*a**2*b*c*d + 15*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d
- sqrt(d)*sqrt(c + d*x**2)*b)*a**2*b*d**2*x**2 + 3*sqrt(d)*sqrt(b)*log( -
sqrt(b)*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c + d*x**2)*b)*a*b**2*c**2 - 18*
sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d - sqrt(d)*sqrt(c + d*x**
2)*b)*a*b**2*c*d*x**2 + 3*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*
d - sqrt(d)*sqrt(c + d*x**2)*b)*b**3*c**2*x**2))/(8*b**4*d*e**2*(a + b*x**
2))
```

3.88 $\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 181

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = -\frac{bc-ad}{b^2e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}+\frac{d(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{2b^2e^2} + \frac{3\sqrt{d}(bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{b}\sqrt{e}}\right)}{2b^{5/2}e^{3/2}}$$

output

```
-(-a*d+b*c)/b^2/e/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)+1/2*d*(d*x^2+c)*(
b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^2/e^2+3/2*d^(1/2)*(-a*d+b*c)*arcta
nh(d^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^(1/2)/e^(1/2))/b^(5/2)
/e^(3/2)
```

Mathematica [A] (verified)

Time = 2.01 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.76

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{b}\sqrt{c+dx^2}(-2bc+3ad+bdx^2)+3\sqrt{d}(bc-ad)\sqrt{a+bx^2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{a+bx^2}}{\sqrt{b}\sqrt{c+dx^2}}\right)}{2b^{5/2}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

input
```
Integrate[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[b]*Sqrt[c + d*x^2]*(-2*b*c + 3*a*d + b*d*x^2) + 3*Sqrt[d]*(b*c - a*d
)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c + d*x^
2])])/(2*b^(5/2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.62, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {2053, 2051, 253, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2053}$$

$$\frac{1}{2}\int \frac{1}{\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{ 2051}$$

$$e(bc-ad)\int \frac{1}{x^4\left(be-dx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{ 253}$$

$$e(bc-ad)\left(\frac{3\int \frac{1}{x^4(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2be}+\frac{1}{2bex^2(be-dx^4)}\right)$$

$$\Big\downarrow \ 264$$

$$e(bc-ad)\left(\frac{3\left(\frac{d\int \frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{be}-\frac{1}{bex^2}\right)}{2be}+\frac{1}{2bex^2(be-dx^4)}\right)$$

$$\Big\downarrow \ 221$$

$$e(bc-ad)\left(\frac{3\left(\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}-\frac{1}{bex^2}\right)}{2be}+\frac{1}{2bex^2(be-dx^4)}\right)$$

input
```
Int[x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(b*c - a*d)*e*(1/(2*b*e*x^2*(b*e - d*x^4)) + (3*(-(1/(b*e*x^2)) + (Sqrt[d]
*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*Sqrt[e])])/(
b^(3/2)*e^(3/2))))/(2*b*e))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2051
```
Int[(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p_), x_
Symbol] :> With[{q = Denominator[p]}, Simp[q*e*((b*c - a*d)/n)   Subst[Int[
x^(q*(p + 1) - 1)*(((-a)*e + c*x^q)^(1/n - 1)/(b*e - d*x^q)^(1/n + 1)), x],
 x, (e*((a + b*x^n)/(c + d*x^n)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e}, x]
&& FractionQ[p] && IntegerQ[1/n]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.29

method	result
risch	$\frac{(b x^2+a) d}{2b^2 e\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}-\frac{\left(\frac{3d(ad-bc)\ln\left(\frac{\frac{1}{2}ade+\frac{1}{2}bce+bd x^2 e}{\sqrt{bde}}+\sqrt{bde\, x^4+(ade+bce)x^2+ace}\right)}{2\sqrt{bde}}+\frac{\left(-2a^2d^2+4abcd-2b^2c^2\right)\left(d x^2+c\right)}{(ad-bc)\sqrt{bde\, x^4+ade\, x^2+bce\, x^2+ace}}\right)\sqrt{(d x^2}{2b^2 e\sqrt{\frac{e(b x^2+a)}{d x^2+c}}\,(d x^2+c)}$
default	$\frac{(b x^2+a)\left(-3\ln\left(\frac{2bd x^2+2\sqrt{db\, x^4+ad\, x^2+bc\, x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)ab\, d^2x^2+3\ln\left(\frac{2bd x^2+2\sqrt{db\, x^4+ad\, x^2+bc\, x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)b^2cd\, x^2+}{}$

input
```
int(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*(b*x^2+a)*d/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)-1/2/b^2*(3/2*d*(a*d-b*
c)*ln((1/2*a*d*e+1/2*b*c*e+b*d*x^2*e)/(b*d*e)^(1/2)+(b*d*e*x^4+(a*d*e+b*c*
e)*x^2+a*c*e)^(1/2))/(b*d*e)^(1/2)+(-2*a^2*d^2+4*a*b*c*d-2*b^2*c^2)*(d*x^2
+c)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))/e/(e*(b*x^2+a)/
(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 443, normalized size of antiderivative = 2.45

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \left[-\frac{3\left((b^2c-abd)ex^2+(abc-a^2d)e\right)\sqrt{\frac{d}{be}}\log\left(8\,b^2d^2x^4+b^2c^2+6\,abcd+a^2d^2+8\,(b\right.}{}\right.$$

$$-\frac{3\left((b^2c-abd)ex^2+(abc-a^2d)e\right)\sqrt{-\frac{d}{be}}\arctan\left(\frac{(2\,bdx^2+bc+ad)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{d}{be}}}{2\,(bdx^2+ad)}\right)-2\,(bd^2x^4-2\,bc^2+3\,acd}{4\,(b^3e^2x^2+ab^2e^2)}$$

input
```
integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*((b^2*c - a*b*d)*e*x^2 + (a*b*c - a^2*d)*e)*sqrt(d/(b*e))*log(8*b
^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 - 4
*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e
*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) - 4*(b*d^2*x^4 - 2*b*c^2 + 3*a*c*d
 - (b*c*d - 3*a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*e^2*x^2
+ a*b^2*e^2), -1/4*(3*((b^2*c - a*b*d)*e*x^2 + (a*b*c - a^2*d)*e)*sqrt(-d/
(b*e))*arctan(1/2*(2*b*d*x^2 + b*c + a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
)*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) - 2*(b*d^2*x^4 - 2*b*c^2 + 3*a*c*d - (b*
c*d - 3*a*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^3*e^2*x^2 + a*b^
2*e^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[1,0,0]%%%},[2,1,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1
,1,1]%%%}
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.32

$$\int \frac{x}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{e}\left(3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,abd - 2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,b^2c + \sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,b^2d$$

input
```
int(x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(e)*(3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*d - 2*sqrt(c + d*x**2)*s
qrt(a + b*x**2)*b**2*c + sqrt(c + d*x**2)*sqrt(a + b*x**2)*b**2*d*x**2 + 3
*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d)*sqrt(c + d*x*
*2)*b)*a**2*d - 3*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a + b*x**2)*d + sqrt
(d)*sqrt(c + d*x**2)*b)*a*b*c + 3*sqrt(d)*sqrt(b)*log( - sqrt(b)*sqrt(a +
b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b)*a*b*d*x**2 - 3*sqrt(d)*sqrt(b)*log
( - sqrt(b)*sqrt(a + b*x**2)*d + sqrt(d)*sqrt(c + d*x**2)*b)*b**2*c*x**2))
/(2*b**3*e**2*(a + b*x**2))
```

3.89 $\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 188

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{bc-ad}{abe\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} - \frac{c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}} + \frac{d^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{\sqrt{b}\sqrt{e}}\right)}{b^{3/2}e^{3/2}}$$

output
```
(-a*d+b*c)/a/b/e/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-c^(3/2)*arctanh(c^
(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))/a^(3/2)/e^(3
/2)+d^(3/2)*arctanh(d^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/b^(1/2)
/e^(1/2))/b^(3/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 2.21 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.01

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{-b^{3/2}c^{3/2}\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right)+\sqrt{a}\left(\sqrt{b}(bc-ad)\sqrt{c+dx^2}+ad^{3/2}\sqrt{a+b}\right.}{a^{3/2}b^{3/2}e\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

input

```
Integrate[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output

```
(-(b^(3/2)*c^(3/2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt
[a]*Sqrt[c + d*x^2])]) + Sqrt[a]*(Sqrt[b]*(b*c - a*d)*Sqrt[c + d*x^2] + a*
d^(3/2)*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[d]*Sqrt[a + b*x^2])/(Sqrt[b]*Sqrt[c
+ d*x^2])]))/(a^(3/2)*b^(3/2)*e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c +
 d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2053, 2052, 25, 382, 397, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{1}{x^2\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int -\frac{1}{x^4\left(ae-cx^4\right)\left(be-dx^4\right)}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{25}$$

$$-\left(e(bc-ad)\int \frac{1}{x^4\left(ae-cx^4\right)\left(be-dx^4\right)}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$\Big\downarrow$ 382

$$e(bc-ad)\left(\frac{1}{abe^2x^2}-\frac{\int \frac{(bc+ad)e-cdx^4}{(ae-cx^4)(be-dx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{abe^2}\right)$$

$\Big\downarrow$ 397

$$e(bc-ad)\left(\frac{1}{abe^2x^2}-\frac{\frac{bc^2\int \frac{1}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{bc-ad}-\frac{ad^2\int \frac{1}{be-dx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{bc-ad}}{abe^2}\right)$$

$\Big\downarrow$ 221

$$e(bc-ad)\left(\frac{1}{abe^2x^2}-\frac{\frac{bc^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{\sqrt{a}\sqrt{e}(bc-ad)}-\frac{ad^{3/2}\text{arctanh}\left(\frac{\sqrt{d}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{b}\sqrt{e}}\right)}{\sqrt{b}\sqrt{e}(bc-ad)}}{abe^2}\right)$$

input

```
Int[1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output

```
(b*c - a*d)*e*(1/(a*b*e^2*x^2) - ((b*c^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[(e*(a +
 b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(Sqrt[a]*(b*c - a*d)*Sqrt[e]) -
 (a*d^(3/2)*ArcTanh[(Sqrt[d]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[b]*S
qrt[e])])/(Sqrt[b]*(b*c - a*d)*Sqrt[e]))/(a*b*e^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(158) = 316$.

Time = 0.14 (sec) , antiderivative size = 401, normalized size of antiderivative = 2.13

method	result
default	$-\frac{(b x^2+a)\left(\sqrt{bd}\,\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}+2ac}{x^2}\right)b^2c^2x^2-\ln\left(\frac{2bd\,x^2+2\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{bd}+ad+bc}{2\sqrt{bd}}\right)\sqrt{ac}}{}$

input
```
int(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*x^2+a)/b/a*((b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*b^2*c^2*x^2-ln(1/2*(2*b*d*x^2+2*(b
*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^
(1/2)*a*b*d^2*x^2+(b*d)^(1/2)*ln((a*d*x^2+b*c*x^2+2*(a*c)^(1/2)*(b*d*x^4+a
*d*x^2+b*c*x^2+a*c)^(1/2)+2*a*c)/x^2)*a*b*c^2-ln(1/2*(2*b*d*x^2+2*(b*d*x^4
+a*d*x^2+b*c*x^2+a*c)^(1/2)*(b*d)^(1/2)+a*d+b*c)/(b*d)^(1/2))*(a*c)^(1/2)*
a^2*d^2+2*(a*c)^(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*a*d-2*(a*c)^
(1/2)*((d*x^2+c)*(b*x^2+a))^(1/2)*(b*d)^(1/2)*b*c)/(a*c)^(1/2)/(b*d)^(1/2)
/((d*x^2+c)*(b*x^2+a))^(1/2)/(d*x^2+c)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 1293, normalized size of antiderivative = 6.88

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/4*((a*b*d*e*x^2 + a^2*d*e)*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 +
6*a*b*c*d + a^2*d^2 + 8*(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c
^2 + a*b*c*d + (3*b^2*c*d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
)*sqrt(d/(b*e))) + (b^2*c*e*x^2 + a*b*c*e)*sqrt(c/(a*e))*log(((b^2*c^2 + 6
*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*
c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2
+ a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4) + 4*(b*c^2 - a*c*d + (b*c*d - a*d^
2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^2), -1
/4*(2*(a*b*d*e*x^2 + a^2*d*e)*sqrt(-d/(b*e))*arctan(1/2*(2*b*d*x^2 + b*c +
 a*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-d/(b*e))/(b*d*x^2 + a*d)) -
(b^2*c*e*x^2 + a*b*c*e)*sqrt(c/(a*e))*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)
*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*x^2 - 4*((a*b*c*d + a^2*d^2)*x^4
+ 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))
*sqrt(c/(a*e)))/x^4) - 4*(b*c^2 - a*c*d + (b*c*d - a*d^2)*x^2)*sqrt((b*e*x
^2 + a*e)/(d*x^2 + c)))/(a*b^2*e^2*x^2 + a^2*b*e^2), 1/4*(2*(b^2*c*e*x^2 +
 a*b*c*e)*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((b*e*x^
2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) + (a*b*d*e*x^2 + a^2
*d*e)*sqrt(d/(b*e))*log(8*b^2*d^2*x^4 + b^2*c^2 + 6*a*b*c*d + a^2*d^2 + 8*
(b^2*c*d + a*b*d^2)*x^2 + 4*(2*b^2*d^2*x^4 + b^2*c^2 + a*b*c*d + (3*b^2*c*
d + a*b*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(d/(b*e))) + 4*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[1,2,2]%%%},[2,1,3,0]%%%}+%%%{%%%{-4,[2,1,2]%%%},[2,
1,2,1]%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x\left(\frac{e(b\,x^2+a)}{d\,x^2+c}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output
```
int(1/x/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

3.90 $\int \frac{1}{x^3\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 184

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = -\frac{bc-ad}{a^2e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} - \frac{c(c+dx^2)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{2a^2e^2x^2} + \frac{3\sqrt{c}(bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d\left(c+dx^2\right)}}}{\sqrt{a}\sqrt{e}}\right)}{2a^{5/2}e^{3/2}}$$

output
```
-(-a*d+b*c)/a^2/e/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-1/2*c*(d*x^2+c)*(
b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a^2/e^2/x^2+3/2*c^(1/2)*(-a*d+b*c)*a
rctanh(c^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a^(1/2)/e^(1/2))/a^(
5/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 3.89 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.80

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{-\sqrt{a}\sqrt{c+dx^2}(3bcx^2+a(c-2dx^2))+3\sqrt{c}(bc-ad)x^2\sqrt{a+bx^2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{a+bx^2}}{\sqrt{a}\sqrt{c+dx^2}}\right.}{2a^{5/2}ex^2\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(-(Sqrt[a]*Sqrt[c + d*x^2]*(3*b*c*x^2 + a*(c - 2*d*x^2))) + 3*Sqrt[c]*(b*c
 - a*d)*x^2*Sqrt[a + b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqr
t[c + d*x^2])])/(2*a^(5/2)*e*x^2*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c
+ d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.61, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {2053, 2052, 253, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2053}$$

$$\frac{1}{2}\int \frac{1}{x^4\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \text{2052}$$

$$e(bc-ad)\int \frac{1}{x^4\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \text{253}$$

$$e(bc-ad)\left(\frac{3\int \frac{1}{x^4(ae-cx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{2ae}+\frac{1}{2aex^2\left(ae-cx^4\right)}\right)$$

$\Big\downarrow$ 264

$$e(bc-ad)\left(\frac{3\left(\frac{c\int \frac{1}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{ae}-\frac{1}{aex^2}\right)}{2ae}+\frac{1}{2aex^2\left(ae-cx^4\right)}\right)$$

$\Big\downarrow$ 221

$$e(bc-ad)\left(\frac{3\left(\frac{\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}e^{3/2}}-\frac{1}{aex^2}\right)}{2ae}+\frac{1}{2aex^2\left(ae-cx^4\right)}\right)$$

input
```
Int[1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(b*c - a*d)*e*(1/(2*a*e*x^2*(a*e - c*x^4)) + (3*(-(1/(a*e*x^2)) + (Sqrt[c]
*ArcTanh[(Sqrt[c]*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(
a^(3/2)*e^(3/2))))/(2*a*e))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{c(b\,x^2+a)}{2a^2x^2e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}}+\frac{\left(\frac{\left(2a^2d^2-4abcd+2b^2c^2\right)\left(d\,x^2+c\right)}{(ad-bc)\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}}-\frac{3c(ad-bc)\ln\left(\frac{2ace+(ade+bce)x^2+2\sqrt{ace}\,\sqrt{bde\,x^4+(ade+bce)x^2+ace}}{x^2}\right)}{2\sqrt{ace}}\right.}{2a^2e\sqrt{\frac{e(b\,x^2+a)}{d\,x^2+c}}\,(d\,x^2+c)}$
default	$\frac{(b\,x^2+a)\left(2\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{ac}\,b^2d\,x^6-3\ln\left(\frac{ad\,x^2+bc\,x^2+2\sqrt{ac}\,\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}+2ac}{x^2}\right)a^2bcd\,x^4+3\ln\left(\frac{ad\,x^2+bc\,x^2}{}\right.\right.}{}$

input
```
int(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a^2*c*(b*x^2+a)/x^2/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)+1/2/a^2*((2*a^2*d
^2-4*a*b*c*d+2*b^2*c^2)*(d*x^2+c)/(a*d-b*c)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2
+a*c*e)^(1/2)-3/2*c*(a*d-b*c)/(a*c*e)^(1/2)*ln((2*a*c*e+(a*d*e+b*c*e)*x^2+
2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*c*e)^(1/2))/x^2))/e/(e*(b*x
^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 469, normalized size of antiderivative = 2.55

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=\left[-\frac{3\left((b^2c-abd)ex^4+(abc-a^2d)ex^2\right)\sqrt{\frac{c}{ae}}\log\left(\frac{(b^2c^2+6\,abcd+a^2d^2)x^4+8\,a^2c^2+8\,(abc^2+a}{}\right.}{}\right.$$

$$-\frac{3\left((b^2c-abd)ex^4+(abc-a^2d)ex^2\right)\sqrt{-\frac{c}{ae}}\arctan\left(\frac{((bc+ad)x^2+2\,ac)\sqrt{\frac{bex^2+ae}{dx^2+c}}\sqrt{-\frac{c}{ae}}}{2\,(bcx^2+ac)}\right)+2\left((3\,bcd-2\,ad^2)x^4\right.}{4\left(a^2be^2x^4+a^3e^2x^2\right)}$$

input
```
integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*((b^2*c - a*b*d)*e*x^4 + (a*b*c - a^2*d)*e*x^2)*sqrt(c/(a*e))*log
(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*x^4 + 8*a^2*c^2 + 8*(a*b*c^2 + a^2*c*d)*
x^2 - 4*((a*b*c*d + a^2*d^2)*x^4 + 2*a^2*c^2 + (a*b*c^2 + 3*a^2*c*d)*x^2)*
sqrt((b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(c/(a*e)))/x^4) + 4*((3*b*c*d - 2*a*
d^2)*x^4 + a*c^2 + (3*b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)
))/(a^2*b*e^2*x^4 + a^3*e^2*x^2), -1/4*(3*((b^2*c - a*b*d)*e*x^4 + (a*b*c
- a^2*d)*e*x^2)*sqrt(-c/(a*e))*arctan(1/2*((b*c + a*d)*x^2 + 2*a*c)*sqrt((
b*e*x^2 + a*e)/(d*x^2 + c))*sqrt(-c/(a*e))/(b*c*x^2 + a*c)) + 2*((3*b*c*d
- 2*a*d^2)*x^4 + a*c^2 + (3*b*c^2 - a*c*d)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^
2 + c)))/(a^2*b*e^2*x^4 + a^3*e^2*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**3/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[1,0,0]%%%},[6,1,0,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,
[1,1,1]%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^3\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^3*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.68

$$\int \frac{1}{x^3\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{e}\left(-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2c+2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^2d\,x^2-3\sqrt{d\,x^2+c}\,\sqrt{b\,x^2}\right.}{}$$

input
```
int(1/x^3/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(e)*( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*c + 2*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*a**2*d*x**2 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b*c*
x**2 + 3*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c - sqrt(c)*sqrt(c +
 d*x**2)*a)*a**2*d*x**2 - 3*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c
 - sqrt(c)*sqrt(c + d*x**2)*a)*a*b*c*x**2 + 3*sqrt(c)*sqrt(a)*log(sqrt(a)*
sqrt(a + b*x**2)*c - sqrt(c)*sqrt(c + d*x**2)*a)*a*b*d*x**4 - 3*sqrt(c)*sq
rt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c - sqrt(c)*sqrt(c + d*x**2)*a)*b**2*c*
x**4 - 3*sqrt(c)*sqrt(a)*log(x)*a**2*d*x**2 + 3*sqrt(c)*sqrt(a)*log(x)*a*b
*c*x**2 - 3*sqrt(c)*sqrt(a)*log(x)*a*b*d*x**4 + 3*sqrt(c)*sqrt(a)*log(x)*b
**2*c*x**4))/(2*a**3*e**2*x**2*(a + b*x**2))
```

3.91 $\int \frac{1}{x^5\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 257

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{b(bc-ad)}{a^3e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}+\frac{(7bc-3ad)\left(c+dx^2\right)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{8a^3e^2x^2} - \frac{\left(c+dx^2\right)^2\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}{4a^2e^2x^4} - \frac{3(bc-ad)(5bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d\left(c+dx^2\right)}}}{\sqrt{a}\sqrt{e}}\right)}{8a^{7/2}\sqrt{c}e^{3/2}}$$

output
```
b*(-a*d+b*c)/a^3/e/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)+1/8*(-3*a*d+7*b*
c)*(d*x^2+c)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a^3/e^2/x^2-1/4*(d*x^2
+c)^2*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a^2/e^2/x^4-3/8*(-a*d+b*c)*(-
a*d+5*b*c)*arctanh(c^(1/2)*(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)/a^(1/2)/
e^(1/2))/a^(7/2)/c^(1/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 4.38 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.74

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{a}\sqrt{c}\sqrt{c+dx^2}(15b^2cx^4+abx^2(5c-13dx^2)-a^2(2c+5dx^2))-3(5b^2c^2-6abcd}{8a^{7/2}\sqrt{c}ex^4\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\sqrt{c+dx^2}}$$

input
```
Integrate[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(Sqrt[a]*Sqrt[c]*Sqrt[c + d*x^2]*(15*b^2*c*x^4 + a*b*x^2*(5*c - 13*d*x^2)
- a^2*(2*c + 5*d*x^2)) - 3*(5*b^2*c^2 - 6*a*b*c*d + a^2*d^2)*x^4*Sqrt[a +
b*x^2]*ArcTanh[(Sqrt[c]*Sqrt[a + b*x^2])/(Sqrt[a]*Sqrt[c + d*x^2])])/(8*a^
(7/2)*Sqrt[c]*e*x^4*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Rubi [A] (warning: unable to verify)

Time = 0.69 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.81, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.423, Rules used = {2053, 2052, 25, 361, 25, 27, 361, 25, 27, 359, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 2053$$

$$\frac{1}{2}\int \frac{1}{x^6\left(\frac{e(bx^2+a)}{dx^2+c}\right)^{3/2}}dx^2$$

$$\Big\downarrow \; 2052$$

$$e(bc-ad)\int -\frac{be-dx^4}{x^4\left(ae-cx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}$$

$$\Big\downarrow \; 25$$

$$-\left(e(bc-ad)\int \frac{be-dx^4}{x^4\left(ae-cx^4\right)^3}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}\right)$$

$$\Big\downarrow \ 361$$

$$e(bc-ad)\left(\frac{1}{4}\int -\frac{3(bc-ad)x^4+4abe}{a^2ex^4\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow \ 25$$

$$e(bc-ad)\left(-\frac{1}{4}\int \frac{3(bc-ad)x^4+4abe}{a^2ex^4\left(ae-cx^4\right)^2}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow \ 27$$

$$e(bc-ad)\left(-\frac{\int \frac{3(bc-ad)x^4+4abe}{x^4\left(ae-cx^4\right)^2}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4a^2e}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow \ 361$$

$$e(bc-ad)\left(-\frac{\frac{(7bc-3ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2ae(ae-cx^4)}-\frac{1}{2}\int -\frac{\left(\frac{7bc}{a}-3d\right)x^4+8be}{ex^4(ae-cx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}}{4a^2e}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow \ 25$$

$$e(bc-ad)\left(-\frac{\frac{1}{2}\int \frac{\left(\frac{7bc}{a}-3d\right)x^4+8be}{ex^4(ae-cx^4)}d\sqrt{\frac{e(bx^2+a)}{dx^2+c}}+\frac{(7bc-3ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2ae(ae-cx^4)}}{4a^2e}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow \ 27$$

$$e(bc-ad)\left(-\frac{\frac{\int \frac{\left(\frac{7bc}{a}-3d\right)x^4+8be}{x^4\left(ae-cx^4\right)}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{2e}+\frac{(7bc-3ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2ae(ae-cx^4)}}{4a^2e}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}\right)$$

$$\Big\downarrow \ 359$$

$$e(bc-ad)\left(-\frac{\frac{\frac{3(5bc-ad)\int \frac{1}{ae-cx^4}d\sqrt{\frac{e\left(bx^2+a\right)}{dx^2+c}}}{a}-\frac{8b}{ax^2}}{2e}+\frac{(7bc-3ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2ae(ae-cx^4)}}{4a^2e}-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}\right)$$

$\downarrow$ 221

$$e(bc-ad)\left(-\frac{(bc-ad)\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}{4a^2e\left(ae-cx^4\right)^2}-\frac{\frac{\frac{3(5bc-ad)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{\sqrt{a}\sqrt{e}}\right)}{a^{3/2}\sqrt{c}\sqrt{e}}-\frac{8b}{ax^2}}{2e}+\frac{(7bc-3ad)\sqrt{\frac{e\left(a+bx^2\right)}{c+dx^2}}}{2ae(ae-cx^4)}}{4a^2e}\right)$$

input
```
Int[1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(b*c - a*d)*e*(-1/4*((b*c - a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(a^2*e
*(a*e - c*x^4)^2) - (((7*b*c - 3*a*d)*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(
2*a*e*(a*e - c*x^4)) + ((-8*b)/(a*x^2) + (3*(5*b*c - a*d)*ArcTanh[(Sqrt[c]
*Sqrt[(e*(a + b*x^2))/(c + d*x^2)])/(Sqrt[a]*Sqrt[e])])/(a^(3/2)*Sqrt[c]*S
qrt[e]))/(2*e))/(4*a^2*e))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 2052
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)))/((c_) + (d_.)*(x_)))^(p_), x_S
ymbol] :> With[{q = Denominator[p]}, Simp[q*e*(b*c - a*d)   Subst[Int[x^(q*
(p + 1) - 1)*(((-a)*e + c*x^q)^m/(b*e - d*x^q)^(m + 2)), x], x, (e*((a + b*
x)/(c + d*x)))^(1/q)], x]] /; FreeQ[{a, b, c, d, e, m}, x] && FractionQ[p]
&& IntegerQ[m]
```

rule 2053
```
Int[(x_)^(m_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)
))^(p_), x_Symbol] :> Simp[1/n   Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(e*(
(a + b*x)/(c + d*x)))^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, e, m, n, p},
 x] && IntegerQ[Simplify[(m + 1)/n]]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{\left(b\,x^{2}+a\right)\left(5 a d\,x^{2}-7 b c\,x^{2}+2 a c\right)}{8 a^{3} x^{4} e \sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}+\frac{\left(\frac{\left(-3 a^{2} d^{2}+18 a b c d-15 b^{2} c^{2}\right) \ln\left(\frac{2 a c e+\left(a d e+b c e\right) x^{2}+2 \sqrt{a c e}\,\sqrt{b d e\,x^{4}+\left(a d e+b c e\right) x^{2}+a c e}}{x^{2}}\right)}{2 \sqrt{a c e}}-\frac{8 b\left(a \cdots\right.}{\left(a d-b c \cdots\right.} \cdots\right.}{8 a^{3} e \sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}\,\left(d\,x^{2}+c\right)}$
default	Expression too large to display

input
```
int(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b*x^2+a)*(5*a*d*x^2-7*b*c*x^2+2*a*c)/a^3/x^4/e/(e*(b*x^2+a)/(d*x^2+c
))^(1/2)+1/8/a^3*(1/2*(-3*a^2*d^2+18*a*b*c*d-15*b^2*c^2)/(a*c*e)^(1/2)*ln(
(2*a*c*e+(a*d*e+b*c*e)*x^2+2*(a*c*e)^(1/2)*(b*d*e*x^4+(a*d*e+b*c*e)*x^2+a*
c*e)^(1/2))/x^2)-8*b*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(d*x^2+c)/(a*d-b*c)/(b*d*
e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2))/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2)*((
d*x^2+c)*(b*x^2+a)*e)^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 3.43 (sec) , antiderivative size = 613, normalized size of antiderivative = 2.39

$$\int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \left[\frac{3\left((5\,b^3c^2 - 6\,ab^2cd + a^2bd^2)x^6 + (5\,ab^2c^2 - 6\,a^2bcd + a^3d^2)x^4\right)\sqrt{ace}\,\log\left(\frac{(b^2c^2+6}{}\right.}{}\right.$$

input
```
integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
[1/32*(3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^6 + (5*a*b^2*c^2 - 6*a^2
*b*c*d + a^3*d^2)*x^4)*sqrt(a*c*e)*log(((b^2*c^2 + 6*a*b*c*d + a^2*d^2)*e*
x^4 + 8*a^2*c^2*e + 8*(a*b*c^2 + a^2*c*d)*e*x^2 - 4*((b*c*d + a*d^2)*x^4 +
 2*a*c^2 + (b*c^2 + 3*a*c*d)*x^2)*sqrt(a*c*e)*sqrt((b*e*x^2 + a*e)/(d*x^2
+ c)))/x^4) + 4*((15*a*b^2*c^2*d - 13*a^2*b*c*d^2)*x^6 - 2*a^3*c^3 + (15*a
*b^2*c^3 - 8*a^2*b*c^2*d - 5*a^3*c*d^2)*x^4 + (5*a^2*b*c^3 - 7*a^3*c^2*d)*
x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a^4*b*c*e^2*x^6 + a^5*c*e^2*x^4),
 1/16*(3*((5*b^3*c^2 - 6*a*b^2*c*d + a^2*b*d^2)*x^6 + (5*a*b^2*c^2 - 6*a^2
*b*c*d + a^3*d^2)*x^4)*sqrt(-a*c*e)*arctan(1/2*sqrt(-a*c*e)*((b*c + a*d)*x
^2 + 2*a*c)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c))/(a*b*c*e*x^2 + a^2*c*e)) + 2
*((15*a*b^2*c^2*d - 13*a^2*b*c*d^2)*x^6 - 2*a^3*c^3 + (15*a*b^2*c^3 - 8*a^
2*b*c^2*d - 5*a^3*c*d^2)*x^4 + (5*a^2*b*c^3 - 7*a^3*c^2*d)*x^2)*sqrt((b*e*
x^2 + a*e)/(d*x^2 + c)))/(a^4*b*c*e^2*x^6 + a^5*c*e^2*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\, dx = \text{Timed out}$$

input
```
integrate(1/x**5/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(e>0)', see `assume?` for more de
tails)Is e
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{x^5 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Unable to divide, perhaps due to ro
unding error%%%{%%%{2,[1,4,4]%%%},[2,1,7,0]%%%}+%%%{%%%{-8,[2,3,4]%%%},[2,
1,6,1]%%%
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x^5\,{\left(\frac{e\,\left(b\,x^2+a\right)}{d\,x^2+c}\right)}^{3/2}}\,dx$$

input
```
int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^5*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 517, normalized size of antiderivative = 2.01

$$\int \frac{1}{x^5\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=\frac{\sqrt{e}\,\left(-2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^3c^2-5\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,a^3cd\,x^2+5\sqrt{d\,x^2+c}\,\sqrt{b}\right.}{}$$

input
```
int(1/x^5/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(e)*( - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**3*c**2 - 5*sqrt(c + d*
x**2)*sqrt(a + b*x**2)*a**3*c*d*x**2 + 5*sqrt(c + d*x**2)*sqrt(a + b*x**2)
*a**2*b*c**2*x**2 - 13*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a**2*b*c*d*x**4 +
 15*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*b**2*c**2*x**4 + 3*sqrt(c)*sqrt(a)
*log(sqrt(a)*sqrt(a + b*x**2)*c - sqrt(c)*sqrt(c + d*x**2)*a)*a**3*d**2*x*
*4 - 18*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c - sqrt(c)*sqrt(c +
d*x**2)*a)*a**2*b*c*d*x**4 + 3*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2
)*c - sqrt(c)*sqrt(c + d*x**2)*a)*a**2*b*d**2*x**6 + 15*sqrt(c)*sqrt(a)*lo
g(sqrt(a)*sqrt(a + b*x**2)*c - sqrt(c)*sqrt(c + d*x**2)*a)*a*b**2*c**2*x**
4 - 18*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2)*c - sqrt(c)*sqrt(c + d
*x**2)*a)*a*b**2*c*d*x**6 + 15*sqrt(c)*sqrt(a)*log(sqrt(a)*sqrt(a + b*x**2
)*c - sqrt(c)*sqrt(c + d*x**2)*a)*b**3*c**2*x**6 - 3*sqrt(c)*sqrt(a)*log(x
)*a**3*d**2*x**4 + 18*sqrt(c)*sqrt(a)*log(x)*a**2*b*c*d*x**4 - 3*sqrt(c)*s
qrt(a)*log(x)*a**2*b*d**2*x**6 - 15*sqrt(c)*sqrt(a)*log(x)*a*b**2*c**2*x**
4 + 18*sqrt(c)*sqrt(a)*log(x)*a*b**2*c*d*x**6 - 15*sqrt(c)*sqrt(a)*log(x)*
b**3*c**2*x**6))/(8*a**4*c*e**2*x**4*(a + b*x**2))
```

3.92 $\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx$

Optimal result

Integrand size = 26, antiderivative size = 477

$$\begin{aligned}\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx &= \frac{(b^2c^2-16abcd+16a^2d^2)\,x}{5b^3de\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} \\ &+ \frac{(7bc-8ad)x(a+bx^2)}{5b^3e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} + \frac{6dx^3(a+bx^2)}{5b^2e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} - \frac{x^3(c+dx^2)}{be\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} \\ &- \frac{\sqrt{a}(b^2c^2-16abcd+16a^2d^2)\,E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{5b^{7/2}de\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} \\ &- \frac{a^{3/2}(7bc-8ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{5b^{7/2}e\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}\end{aligned}$$

output
```
1/5*(16*a^2*d^2-16*a*b*c*d+b^2*c^2)*x/b^3/d/e/(b*e/d-(-a*d+b*c)*e/d/(d*x^2
+c))^(1/2)+1/5*(-8*a*d+7*b*c)*x*(b*x^2+a)/b^3/e/(b*e/d-(-a*d+b*c)*e/d/(d*x
^2+c))^(1/2)+6/5*d*x^3*(b*x^2+a)/b^2/e/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1
/2)-x^3*(d*x^2+c)/b/e/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-1/5*a^(1/2)*(
16*a^2*d^2-16*a*b*c*d+b^2*c^2)*EllipticE(b^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/
2),(1-a*d/b/c)^(1/2))/b^(7/2)/d/e/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(b*e/d-(
-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-1/5*a^(3/2)*(-8*a*d+7*b*c)*InverseJacobiAM(
arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(7/2)/e/(a*(d*x^2+c)/c/(b*x
^2+a))^(1/2)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.83 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.57

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}dx(c+dx^2)\left(-8a^2d+ab(7c-2dx^2)+b^2x^2(2c+dx^2)\right)-ic(b^2c^2-1\right.}{}$$

input
```
Integrate[x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*d*x*(c + d*x^2)*(-8*a^2*d +
a*b*(7*c - 2*d*x^2) + b^2*x^2*(2*c + d*x^2)) - I*c*(b^2*c^2 - 16*a*b*c*d +
 16*a^2*d^2)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[S
qrt[b/a]*x], (a*d)/(b*c)] + I*c*(b^2*c^2 - 9*a*b*c*d + 8*a^2*d^2)*Sqrt[1 +
 (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b
*c)]))/(5*b^3*Sqrt[b/a]*d*e^2*(a + b*x^2))
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 406, normalized size of antiderivative = 0.85, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {2058, 369, 27, 443, 444, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{2058}$$

$$\frac{\sqrt{a+bx^2}\int \frac{x^4(dx^2+c)^{3/2}}{(bx^2+a)^{3/2}}dx}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{369}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\int \frac{3x^2\sqrt{dx^2+c}(2dx^2+c)}{\sqrt{bx^2+a}}dx}{b}-\frac{x^3(c+dx^2)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{3\int \frac{x^2\sqrt{dx^2+c}(2dx^2+c)}{\sqrt{bx^2+a}}dx}{b}-\frac{x^3(c+dx^2)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{443}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{3\left(\frac{\int \frac{x^2(d(7bc-8ad)x^2+c(5bc-6ad))}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{5b}+\frac{2dx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}\right)}{b}-\frac{x^3(c+dx^2)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{444}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{3\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3b}-\frac{\int \frac{d\left(ac(7bc-8ad)-\left(b^2c^2-16abdc+16a^2d^2\right)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3bd}}{5b}+\frac{2dx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}\right)}{b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a+bx^2}\left(\frac{3\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3b}-\frac{\int \frac{ac(7bc-8ad)-\left(b^2c^2-16abdc+16a^2d^2\right)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}}{5b}+\frac{2dx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b}\right)}{b}-\frac{x^3\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 406$$

$$\frac{\sqrt{a+bx^2}\left(\frac{3\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3b}-\frac{ac(7bc-8ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-\left(16a^2d^2-16abcd+b^2c^2\right)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}}{5b}+\frac{2dx^3\sqrt{a+bx^2}\sqrt{c+dx^2}}{5b} \cdots\right.}{b}\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{a+bx^2}\left(\frac{3\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3b}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(7bc-8ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(16a^2d^2-16abcd+b^2c^2\right)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}\cdots}{5b}\cdots\right)}{b}\cdots\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\big\downarrow$ 388

$$\frac{\sqrt{a+bx^2}\left(\frac{3\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3b}-\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(7bc-8ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(16a^2d^2-16abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{b\cdots}}{\left(dx^2\cdots\right.}\cdots}{\cdots}\right)}{3b}\cdots}{5b}\cdots\right)}{b}\cdots\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\big\downarrow$ 313

$$\frac{\sqrt{a+bx^2}\left(\frac{3\left(\frac{\frac{x\sqrt{a+bx^2}\sqrt{c+dx^2}(7bc-8ad)}{3b}-\frac{\frac{c^{3/2}\sqrt{a+bx^2}(7bc-8ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-\left(16a^2d^2-16abcd+b^2c^2\right)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+b}}{b\sqrt{}}\right)}{3b}}{5b}\right)}{b}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Int[x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[a + b*x^2]*(-((x^3*(c + d*x^2)^(3/2))/(b*Sqrt[a + b*x^2])) + (3*((2*
d*x^3*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(5*b) + (((7*b*c - 8*a*d)*x*Sqrt[a
+ b*x^2]*Sqrt[c + d*x^2])/(3*b) - (-((b^2*c^2 - 16*a*b*c*d + 16*a^2*d^2)*(
(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*Ellipti
cE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a +
b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2)*(7*b*c - 8*a*d)*Sqr
t[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqr
t[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b))/(5*b))
)/b))/(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 443

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*(g*x)^(m + 1)*(a + b*x^2)^(p
 + 1)*((c + d*x^2)^q/(b*g*(m + 2*(p + q + 1) + 1))), x] + Simp[1/(b*(m + 2*
(p + q + 1) + 1))   Int[(g*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[c*((
b*e - a*f)*(m + 1) + b*e*2*(p + q + 1)) + (d*(b*e - a*f)*(m + 1) + f*2*q*(b
*c - a*d) + b*e*d*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f
, g, m, p}, x] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^2, c + d*x^
2])
```

rule 444

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 2058

```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 13.16 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.64

method	result
risch	$-\frac{x\left(-bd\,x^{2}+3ad-2bc\right)\left(b\,x^{2}+a\right)}{5b^{3}e\sqrt{\frac{e\left(b\,x^{2}+a\right)}{d\,x^{2}+c}}}+\frac{\left(-\frac{2\left(11a^{2}d^{2}-11abcd+b^{2}c^{2}\right)ace\sqrt{1+\frac{x^{2}b}{a}}\,\sqrt{1+\frac{x^{2}d}{c}}\,\left(\operatorname{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)-\operatorname{EllipticE}\left(\right.\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^{4}+ade\,x^{2}+bce\,x^{2}+ace}\,\left(ade+bce+e(ad-bc)\right)}\right.}{}$
default	$-\frac{\left(b\,x^{2}+a\right)\left(-\sqrt{(d\,x^{2}+c)(b\,x^{2}+a)}\,\sqrt{-\frac{b}{a}}\,b^{2}d^{3}x^{7}+2\sqrt{(d\,x^{2}+c)(b\,x^{2}+a)}\,\sqrt{-\frac{b}{a}}\,ab\,d^{3}x^{5}-3\sqrt{(d\,x^{2}+c)(b\,x^{2}+a)}\,\sqrt{-\frac{b}{a}}\,b^{2}c\,d^{2}x^{5}+3\sqrt{\right.}{}$

input
```
int(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*x*(-b*d*x^2+3*a*d-2*b*c)*(b*x^2+a)/b^3/e/(e*(b*x^2+a)/(d*x^2+c))^(1/2
)+1/5/b^3*(-2*(11*a^2*d^2-11*a*b*c*d+b^2*c^2)*a*c*e/(-b/a)^(1/2)*(1+1/a*x^
2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)
/(a*d*e+b*c*e+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b
/e)^(1/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))-a*(5*
a^2*d^2-13*a*b*c*d+7*b^2*c^2)/b/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^
2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^
(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2))+5*a^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b*
(-(b*d*e*x^2+b*c*e)/a/(a*d-b*c)*x/e/((x^2+a/b)*(b*d*e*x^2+b*c*e))^(1/2)+(1
/a+b*c/a/(a*d-b*c))/(-b/a)^(1/2)*(1+1/a*x^2*b)^(1/2)*(1+1/c*x^2*d)^(1/2)/(
b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)*EllipticF(x*(-b/a)^(1/2),(-1+(a
*d*e+b*c*e)/c/b/e)^(1/2))-2*d*b/(a*d-b*c)*c*e/(-b/a)^(1/2)*(1+1/a*x^2*b)^(
1/2)*(1+1/c*x^2*d)^(1/2)/(b*d*e*x^4+a*d*e*x^2+b*c*e*x^2+a*c*e)^(1/2)/(a*d*
e+b*c*e+e*(a*d-b*c))*(EllipticF(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1
/2))-EllipticE(x*(-b/a)^(1/2),(-1+(a*d*e+b*c*e)/c/b/e)^(1/2)))))/e/(e*(b*x
^2+a)/(d*x^2+c))^(1/2)*((d*x^2+c)*(b*x^2+a)*e)^(1/2)/(d*x^2+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 425, normalized size of antiderivative = 0.89

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx =$$

$$-\frac{((b^3c^3 - 16\,ab^2c^2d + 16\,a^2bcd^2)x^3 + (ab^2c^3 - 16\,a^2bc^2d + 16\,a^3cd^2)x)\sqrt{\frac{be}{d}}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right) \mid \frac{ad}{bc}) -}{}$$

input
```
integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/5*(((b^3*c^3 - 16*a*b^2*c^2*d + 16*a^2*b*c*d^2)*x^3 + (a*b^2*c^3 - 16*a
^2*b*c^2*d + 16*a^3*c*d^2)*x)*sqrt(b*e/d)*sqrt(-c/d)*elliptic_e(arcsin(sqr
t(-c/d)/x), a*d/(b*c)) - ((b^3*c^3 - 16*a*b^2*c^2*d + 8*a^2*b*d^3 + (16*a^
2*b - 7*a*b^2)*c*d^2)*x^3 + (a*b^2*c^3 - 16*a^2*b*c^2*d + 8*a^3*d^3 + (16*
a^3 - 7*a^2*b)*c*d^2)*x)*sqrt(b*e/d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/
d)/x), a*d/(b*c)) - (b^3*d^3*x^8 + (3*b^3*c*d^2 - 2*a*b^2*d^3)*x^6 + a*b^2
*c^3 - 16*a^2*b*c^2*d + 16*a^3*c*d^2 + (3*b^3*c^2*d - 11*a*b^2*c*d^2 + 8*a
^2*b*d^3)*x^4 + (b^3*c^3 - 8*a*b^2*c^2*d - 8*a^2*b*c*d^2 + 16*a^3*d^3)*x^2
)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(b^5*d*e^2*x^3 + a*b^4*d*e^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**4/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^4/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

Giac [F]

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^4/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^4}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^4/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^4}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output

```
(sqrt(e)*(3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*d*x - 2*sqrt(c + d*x**2)
*sqrt(a + b*x**2)*a*d**2*x**3 - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c**2
*x + 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*d*x**3 + sqrt(c + d*x**2)*sqr
t(a + b*x**2)*b*d**2*x**5 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)
/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*
d*x**6),x)*a**3*d**3 - 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a*
*2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x*
*6),x)*a**2*b*c*d**2 + 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**
2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**
6),x)*a**2*b*d**3*x**2 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a
**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x
**6),x)*a*b**2*c**2*d - 12*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a
**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x
**6),x)*a*b**2*c*d**2*x**2 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4
)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2
*d*x**6),x)*b**3*c**2*d*x**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(
a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*
x**6),x)*a**3*c**2*d + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c +
 a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)
*a**2*b*c**3 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2...
```

3.93 $\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 380

$$\begin{aligned}\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx &= \frac{(7bc-8ad)x}{3b^2e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}+\frac{4dx(a+bx^2)}{3b^2e\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} \\ &-\frac{x(c+dx^2)}{be\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}-\frac{\sqrt{a}(7bc-8ad)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{3b^{5/2}e\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} \\ &+\frac{\sqrt{a}(3bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{3b^{5/2}e\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}\end{aligned}$$

output

```
1/3*(-8*a*d+7*b*c)*x/b^2/e/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)+4/3*d*x*
(b*x^2+a)/b^2/e/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-x*(d*x^2+c)/b/e/(b*
e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-1/3*a^(1/2)*(-8*a*d+7*b*c)*EllipticE(b
^(1/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/b^(5/2)/e/(a*(d*x^2+
c)/c/(b*x^2+a))^(1/2)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)+1/3*a^(1/2)*(
-4*a*d+3*b*c)*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))
/b^(5/2)/e/(a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c)
)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.09 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.58

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(\sqrt{\frac{b}{a}}x(c+dx^2)\left(-3bc+4ad+bdx^2\right)+ic(-7bc+8ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}} \cdots\right.}{3a^2 \left(\cdots \right.}$$

input
```
Integrate[x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(Sqrt[b/a]*x*(c + d*x^2)*(-3*b*c + 4*a*
d + b*d*x^2) + I*c*(-7*b*c + 8*a*d)*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c
]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (4*I)*c*(-(b*c) + a*d)*
Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x],
(a*d)/(b*c)]))/(3*a^2*(b/a)^(5/2)*e^2*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 344, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {2058, 369, 403, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \ 2058$$

$$\frac{\sqrt{a+bx^2}\int \frac{x^2(dx^2+c)^{3/2}}{(bx^2+a)^{3/2}}dx}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \ 369$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\int \frac{\sqrt{dx^2+c}\left(4dx^2+c\right)}{\sqrt{bx^2+a}}dx}{b}-\frac{x\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\Big\downarrow$ 403

$$\frac{\sqrt{a+bx^2}\left(\frac{\frac{\int \frac{d(7bc-8ad)x^2+c(3bc-4ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{4dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}}{b}-\frac{x\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\Big\downarrow$ 406

$$\frac{\sqrt{a+bx^2}\left(\frac{\frac{c(3bc-4ad)\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+d(7bc-8ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3b}+\frac{4dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}}{b}-\frac{x\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\Big\downarrow$ 320

$$\frac{\sqrt{a+bx^2}\left(\frac{\frac{d(7bc-8ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3b}+\frac{4dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}}{b}-\frac{x\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\Big\downarrow$ 388

$$\frac{\sqrt{a+bx^2}\left(\frac{\frac{d(7bc-8ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}(3bc-4ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}}{3b}+\frac{4dx\sqrt{a+bx^2}\sqrt{c+dx^2}}{3b}}{b}-\frac{x\left(c+dx^2\right)^{3/2}}{b\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\Big\downarrow$ 313

$$\frac{\sqrt{a+bx^2}\left(\frac{\frac{\frac{c^{3/2}\sqrt{a+bx^2}(3bc-4ad)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+d(7bc-8ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{3b}+\frac{4dx\sqrt{a+bx^2}\sqrt{\cdots}}{3b}}{b}\cdots\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Int[x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x]
```

output
```
(Sqrt[a + b*x^2]*(-((x*(c + d*x^2)^(3/2))/(b*Sqrt[a + b*x^2])) + ((4*d*x*S
qrt[a + b*x^2]*Sqrt[c + d*x^2])/(3*b) + (d*(7*b*c - 8*a*d)*((x*Sqrt[a + b*
x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]*EllipticE[ArcTan[(Sqr
t[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c
+ d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*(3*b*c - 4*a*d)*Sqrt[a + b*x^2]*El
lipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(a*Sqrt[d]*Sqrt[(c*
(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))/(3*b))/b))/(e*Sqrt[(e*(a +
 b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
 + d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 13.03 (sec) , antiderivative size = 643, normalized size of antiderivative = 1.69

method	result
default	$\frac{(b\,x^2+a)\left(\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,b\,d^2x^5+\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,a\,d^2x^3+\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{-\frac{b}{a}}\,bcd\,x^3+3\sqrt{db\,x^4+ad\,x^2}\right.}{}$
risch	$\frac{dx(b\,x^2+a)}{3b^2e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}-\frac{\left(-\frac{2bd(5ad-4bc)ace\sqrt{1+\frac{x^2b}{a}}\,\sqrt{1+\frac{x^2d}{c}}\,\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)-\mathrm{EllipticE}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)\right)}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4+ade\,x^2+bce\,x^2+ace}\,(ade+bce+e(ad-bc))}-\right.}{}$

input
```
int(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(b*x^2+a)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b*d^2*x^5+((d*x^2+
c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*d^2*x^3+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b
/a)^(1/2)*b*c*d*x^3+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*d
^2*x^3-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c*d*x^3+4*((d*
x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d-4*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^
2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2)
)*b*c^2-8*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1
/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d+7*((d*x^2+c)*(b*x^2+a)
)^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(
a*d/b/c)^(1/2))*b*c^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*c*d*x+3*(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*c*d*x-3*(b*d*x^4+a*d*x^2
+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c^2*x)/b^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2
)/(d*x^2+c)^2/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.80

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx =$$

$$-\frac{((7\,b^2c^2-8\,abcd)x^3+(7\,abc^2-8\,a^2cd)x)\sqrt{\frac{be}{d}}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-((7\,b^2c^2-4\,abd^2-(8\,ab-}{}$$

input
```
integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(((7*b^2*c^2 - 8*a*b*c*d)*x^3 + (7*a*b*c^2 - 8*a^2*c*d)*x)*sqrt(b*e/d
)*sqrt(-c/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((7*b^2*c^2 - 4
*a*b*d^2 - (8*a*b - 3*b^2)*c*d)*x^3 + (7*a*b*c^2 - 4*a^2*d^2 - (8*a^2 - 3*
a*b)*c*d)*x)*sqrt(b*e/d)*sqrt(-c/d)*elliptic_f(arcsin(sqrt(-c/d)/x), a*d/(
b*c)) - (b^2*d^2*x^6 + (5*b^2*c*d - 4*a*b*d^2)*x^4 + 7*a*b*c^2 - 8*a^2*c*d
 + (4*b^2*c^2 + 3*a*b*c*d - 8*a^2*d^2)*x^2)*sqrt((b*e*x^2 + a*e)/(d*x^2 +
c)))/(b^4*e^2*x^3 + a*b^3*e^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**2/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(x^2/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

Giac [F]

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/((b*x^2 + a)*e/(d*x^2 + c))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{x^2}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(x^2/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(e)*( - 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c*d*x + 2*sqrt(c + d*x*
*2)*sqrt(a + b*x**2)*a*d**2*x**3 + 3*sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c
**2*x - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x*
*2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*d**3
 + 11*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 +
 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*c*d**2
 - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 +
2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*d**3*x*
*2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2
+ 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c**2*
d + 11*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2
+ 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c*d**
2*x**2 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x
**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**3*c**
2*d*x**2 + 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2
 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*c**2*d
 - 3*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b
*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*c**3 + 3*int
((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2
+ 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*c**2*d*x**2 - 3*i...
```

3.94 $\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx$

Optimal result

Integrand size = 22, antiderivative size = 251

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx = \frac{dx}{be\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} + \frac{(bc-2ad)E\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right)|1-\frac{ad}{bc}\right)}{\sqrt{a}b^{3/2}e\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} + \frac{\sqrt{a}d\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{b}x}{\sqrt{a}}\right),1-\frac{ad}{bc}\right)}{b^{3/2}e\sqrt{\frac{a(c+dx^2)}{c(a+bx^2)}}\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}$$

output
```
d*x/b/e/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)+(-2*a*d+b*c)*EllipticE(b^(1
/2)*x/a^(1/2)/(1+b*x^2/a)^(1/2),(1-a*d/b/c)^(1/2))/a^(1/2)/b^(3/2)/e/(a*(d
*x^2+c)/c/(b*x^2+a))^(1/2)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)+a^(1/2)*
d*InverseJacobiAM(arctan(b^(1/2)*x/a^(1/2)),(1-a*d/b/c)^(1/2))/b^(3/2)/e/(
a*(d*x^2+c)/c/(b*x^2+a))^(1/2)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.06 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.81

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-ic(-bc+2ad)\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\frac{dx^2}{c}}E\left(i\operatorname{arcsinh}\left(\sqrt{\frac{b}{a}}x\right)|\frac{ad}{bc}\right)+(bc-ad)\right.}{a^2\left(\frac{b}{a}\right)^{3/2}e^2(a}$$

input
```
Integrate[((e*(a + b*x^2))/(c + d*x^2))^(-3/2),x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*((-I)*c*(-(b*c) + 2*a*d)*Sqrt[1 + (b*x^
2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] +
 (b*c - a*d)*(Sqrt[b/a]*x*(c + d*x^2) - I*c*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (
d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)])))/(a^2*(b/a)^(3/
2)*e^2*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {2058, 315, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \; 2058$$

$$\frac{\sqrt{a+bx^2}\int \frac{(dx^2+c)^{3/2}}{(bx^2+a)^{3/2}}dx}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 315$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\int \frac{d\left(ac-(bc-2ad)x^2\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\int \frac{ac-(bc-2ad)x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 406$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 320$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 388$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int \frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)\right)}{ab}+\frac{x\sqrt{c+dx^2}(bc-ad)}{ab\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \; 313$$

$$\frac{\sqrt{a+bx^2}\left(\frac{d\left(\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}-(bc-2ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ab}+\frac{x\sqrt{c+dx^2}(bc-a}{ab\sqrt{a+bx^2}}\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Int[((e*(a + b*x^2))/(c + d*x^2))^(-3/2),x]
```

output
```
(Sqrt[a + b*x^2]*(((b*c - a*d)*x*Sqrt[c + d*x^2])/(a*b*Sqrt[a + b*x^2]) +
(d*(-((b*c - 2*a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sq
rt[a + b*x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*
Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2]))) + (c^(3/2
)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])
/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*b)))
/(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. $2(240) = 480$.

Time = 4.33 (sec) , antiderivative size = 514, normalized size of antiderivative = 2.05

method	result
default	$-\frac{(b\,x^2+a)\left(\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,a\,d^2x^3-\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,bcd\,x^3+\sqrt{(d\,x^2+c)(b\,x^2+a)}\,\sqrt{\frac{b\,x^2+a}{a}}\,\sqrt{\frac{d\,x^2}{c}}\right.}{}$

input
```
int(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^2+a)/b*((b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*d^2*x^3-(
b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c*d*x^3+((d*x^2+c)*(b*x^
2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/
2),(a*d/b/c)^(1/2))*a*c*d-((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*
((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2-2*((d*
x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(
x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c*d+((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+
a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*
b*c^2+(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*c*d*x-(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c^2*x)/(e*(b*x^2+a)/(d*x^2+c))^(3/
2)/(d*x^2+c)^2/a/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{((b^2c^2-2\,abcd)x^3+(abc^2-2\,a^2cd)x)\sqrt{\frac{be}{d}}\sqrt{-\frac{c}{d}}E(\arcsin\left(\frac{\sqrt{-\frac{c}{d}}}{x}\right)\mid\frac{ad}{bc})-((b^2c^2-2}{}$$

input
```
integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
(((b^2*c^2 - 2*a*b*c*d)*x^3 + (a*b*c^2 - 2*a^2*c*d)*x)*sqrt(b*e/d)*sqrt(-c
/d)*elliptic_e(arcsin(sqrt(-c/d)/x), a*d/(b*c)) - ((b^2*c^2 - 2*a*b*c*d -
a*b*d^2)*x^3 + (a*b*c^2 - 2*a^2*c*d - a^2*d^2)*x)*sqrt(b*e/d)*sqrt(-c/d)*e
lliptic_f(arcsin(sqrt(-c/d)/x), a*d/(b*c)) + (a*b*d^2*x^4 + 2*a^2*d^2*x^2
- a*b*c^2 + 2*a^2*c*d)*sqrt((b*e*x^2 + a*e)/(d*x^2 + c)))/(a*b^3*e^2*x^3 +
 a^2*b^2*e^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(((b*x^2 + a)*e/(d*x^2 + c))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/((e*(a + b*x^2))/(c + d*x^2))^(3/2),x)
```

output
```
int(1/((e*(a + b*x^2))/(c + d*x^2))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{e}\left(\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,cx+\left(\int \frac{\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,x^4}{b^2d\,x^6+2abd\,x^4+b^2c\,x^4+a^2d\,x^2+2abc\,x^2+a^2c}dx\right)a^2d^2-\left(\int \right.\right.}{}$$

input
```
int(1/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(e)*(sqrt(c + d*x**2)*sqrt(a + b*x**2)*c*x + int((sqrt(c + d*x**2)*sq
rt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 +
 b**2*c*x**4 + b**2*d*x**6),x)*a**2*d**2 - int((sqrt(c + d*x**2)*sqrt(a +
b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c
*x**4 + b**2*d*x**6),x)*a*b*c*d + int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x
**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b
**2*d*x**6),x)*a*b*d**2*x**2 - int((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4
)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2
*d*x**6),x)*b**2*c*d*x**2))/(a*e**2*(a + b*x**2))
```

3.95 $\int \frac{1}{x^2\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 379

$$
\begin{aligned}
\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx &= \frac{bc-ad}{abex\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}-\frac{c(2bc-ad)\left(a+bx^2\right)}{a^2bex\left(c+dx^2\right)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} \\
&\quad-\frac{\sqrt{c}\sqrt{d}(2bc-ad)\left(a+bx^2\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{a^2be\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\left(c+dx^2\right)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} \\
&\quad+\frac{c^{3/2}\sqrt{d}\left(a+bx^2\right)\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{a^2e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\left(c+dx^2\right)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}
\end{aligned}
$$

output
```
(-a*d+b*c)/a/b/e/x/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-c*(-a*d+2*b*c)*(
b*x^2+a)/a^2/b/e/x/(d*x^2+c)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-c^(1/2
)*d^(1/2)*(-a*d+2*b*c)*(b*x^2+a)*EllipticE(d^(1/2)*x/c^(1/2)/(1+d*x^2/c)^(
1/2),(1-b*c/a/d)^(1/2))/a^2/b/e/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)/
(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)+c^(3/2)*d^(1/2)*(b*x^2+a)*InverseJa
cobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c/a/d)^(1/2))/a^2/e/(c*(b*x^2+a)/a/(
d*x^2+c))^(1/2)/(d*x^2+c)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.25 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.59

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{\frac{b}{a}}(c+dx^2)\left(ac+2bcx^2-adx^2\right)+ic(-2bc+ad)x\sqrt{1+\frac{bx^2}{a}}\sqrt{1+\cdots\right.}{\cdots}$$

input
```
Integrate[1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[b/a]*(c + d*x^2)*(a*c + 2*b*c*x
^2 - a*d*x^2)) + I*c*(-2*b*c + a*d)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)
/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)] - (2*I)*c*(-(b*c) + a*d
)*x*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)]))/(a^2*Sqrt[b/a]*e^2*x*(a + b*x^2))
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.95, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.423$, Rules used = {2058, 370, 25, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{a+bx^2}\int \frac{(dx^2+c)^{3/2}}{x^2(bx^2+a)^{3/2}}dx}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 370}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\sqrt{c+dx^2}(bc-ad)}{abx\sqrt{a+bx^2}}-\frac{\int -\frac{c\left(bdx^2+2bc-ad\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\int \frac{c\left(bdx^2+2bc-ad\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{c\int \frac{bdx^2+2bc-ad}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{c\left(-\frac{\int -\frac{bd\left((2bc-ad)x^2+ac\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}\right)}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{c\left(\frac{\int \frac{bd\left((2bc-ad)x^2+ac\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}\right)}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{c\left(\frac{bd\int \frac{(2bc-ad)x^2+ac}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}\right)}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \ 406$$

$$\frac{\sqrt{a+bx^2}\left(\frac{c\left(\frac{bd\left(ac\int \frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+(2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}\right)}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \ 320$$

$$\frac{\sqrt{a+bx^2}\left(\frac{c\left(\frac{bd\left((2bc-ad)\int \frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{c^{3/2}\sqrt{a+bx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}\right)}{ab}+\frac{\sqrt{c+dx^2}(bc-a}{abx\sqrt{a+bx^2}}\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \ 388$$

$$\frac{\sqrt{a+bx^2}\left(\frac{c\left(\frac{bd\left((2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{c^{3/2}\sqrt{a+bx^2}\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(2bc-ad)}{acx}\right)}{ab}+\frac{\sqrt{c}}{}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 313

$$\frac{\sqrt{a+bx^2}\left(\frac{c\left(\frac{bd\left(\frac{c^{3/2}\sqrt{a+bx^2}\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+(2bc-ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(}{acx}\right)}{ab}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

input
```
Int[1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(Sqrt[a + b*x^2]*(((b*c - a*d)*Sqrt[c + d*x^2])/(a*b*x*Sqrt[a + b*x^2]) +
(c*(-(((2*b*c - a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*c*x)) + (b*d*((2*
b*c - a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*
x^2]*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*S
qrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (c^(3/2)*Sqrt[a +
 b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(Sqrt[d]*
Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/(a*c)))/(a*b)))/(
e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [A] (verified)

Time = 10.47 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.72

method	result
default	$-\frac{(b x^2+a)\left(\sqrt{(d x^2+c)(b x^2+a)}\sqrt{-\frac{b}{a}}\, bcd x^4-\sqrt{db x^4+ad x^2+bc x^2+ac}\sqrt{-\frac{b}{a}}\, a d^2 x^4+\sqrt{db x^4+ad x^2+bc x^2+ac}\sqrt{-\frac{b}{a}}\, bcd x^4-2\cdots\right.}{\cdots}$
risch	$-\frac{c(b x^2+a)}{a^2 x e\sqrt{\frac{e(b x^2+a)}{d x^2+c}}}+\frac{\left(\frac{a^2 d^2\sqrt{1+\frac{x^2 b}{a}}\sqrt{1+\frac{x^2 d}{c}}\,\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-1+\frac{ade+bce}{cbe}}\right)}{b\sqrt{-\frac{b}{a}}\sqrt{bde x^4+ade x^2+bce x^2+ace}}-\frac{2dbc^2ae\sqrt{1+\frac{x^2 b}{a}}\sqrt{1+\frac{x^2 d}{c}}\left(\mathrm{EllipticF}\left(x\sqrt{-\frac{b}{a}},\sqrt{-\cdots}\right.\right.}{\sqrt{-\frac{b}{a}}\sqrt{bde x^4+ade x^2+bce\cdots}}\cdots\right.}{\cdots}$

input
```
int(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-(b*x^2+a)*(((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b*c*d*x^4-(b*d*x^4+a*
d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*d^2*x^4+(b*d*x^4+a*d*x^2+b*c*x^2+a
*c)^(1/2)*(-b/a)^(1/2)*b*c*d*x^4-2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*c
*d*x+2*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)
*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b*c^2*x+((d*x^2+c)*(b*x^2+a))^(
1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d
/b/c)^(1/2))*a*c*d*x-2*b*c^2*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*Ellip
ticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*x*((d*x^2+c)*(b*x^2+a))^(1/2)+((d*x^2
+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*c*d*x^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-
b/a)^(1/2)*b*c^2*x^2-(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*c*
d*x^2+(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b*c^2*x^2+((d*x^2+c
)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*c^2)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(d*x^
2+c)^2/a^2/x/(-b/a)^(1/2)/(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.71

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{((2\,b^3cd-ab^2d^2)x^3+(2\,ab^2cd-a^2bd^2)x)\sqrt{\frac{ace}{d^2}}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right)\mid\frac{ad}{bc})-((\cdots}{\cdots}$$

input
```
integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
(((2*b^3*c*d - a*b^2*d^2)*x^3 + (2*a*b^2*c*d - a^2*b*d^2)*x)*sqrt(a*c*e/d^
2)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - ((2*b^3*c*d +
(a^2*b - a*b^2)*d^2)*x^3 + (2*a*b^2*c*d + (a^3 - a^2*b)*d^2)*x)*sqrt(a*c*e
/d^2)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) - (2*a*b^2*c^
2*x^2 + a^2*b*c^2 + (2*a*b^2*c*d - a^2*b*d^2)*x^4)*sqrt((b*e*x^2 + a*e)/(d
*x^2 + c)))/(a^3*b^2*e^2*x^3 + a^4*b*e^2*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**2/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2), x)
```

Giac [F]

$$\int \frac{1}{x^2\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{x^2 \left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^2*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^2 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{e}\left(-2\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ac+\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+a}\,ad\,x^2-\sqrt{d\,x^2+c}\,\sqrt{b\,x^2+\right.}{}$$

input
```
int(1/x^2/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(e)*( - 2*sqrt(c + d*x**2)*sqrt(a + b*x**2)*a*c + sqrt(c + d*x**2)*sq
rt(a + b*x**2)*a*d*x**2 - sqrt(c + d*x**2)*sqrt(a + b*x**2)*b*c*x**2 - int
((sqrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*
x**2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*d**2*x + int((s
qrt(c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**
2 + 2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*c*d*x - int((sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 +
2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a*b**2*d**2*x**3 + int((sqrt(
c + d*x**2)*sqrt(a + b*x**2)*x**4)/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 +
2*a*b*d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*b**3*c*d*x**3 + 3*int((sqrt(c
 + d*x**2)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*
d*x**4 + b**2*c*x**4 + b**2*d*x**6),x)*a**3*c*d*x - 3*int((sqrt(c + d*x**2
)*sqrt(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 +
b**2*c*x**4 + b**2*d*x**6),x)*a**2*b*c**2*x + 3*int((sqrt(c + d*x**2)*sqrt
(a + b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c
*x**4 + b**2*d*x**6),x)*a**2*b*c*d*x**3 - 3*int((sqrt(c + d*x**2)*sqrt(a +
 b*x**2))/(a**2*c + a**2*d*x**2 + 2*a*b*c*x**2 + 2*a*b*d*x**4 + b**2*c*x**
4 + b**2*d*x**6),x)*a*b**2*c**2*x**3))/(2*a**2*e**2*x*(a + b*x**2))
```

3.96 $\int \frac{1}{x^4\left(\frac{e\left(a+bx^2\right)}{c+dx^2}\right)^{3/2}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 455

$$
\begin{aligned}
\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx &= \frac{bc-ad}{abex^3\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} \\
&- \frac{(4bc-3ad)\left(a+bx^2\right)}{3a^2bex^3\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} + \frac{c(8bc-7ad)\left(a+bx^2\right)}{3a^3ex\left(c+dx^2\right)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} \\
&+ \frac{\sqrt{c}\sqrt{d}(8bc-7ad)\left(a+bx^2\right)E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{3a^3e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\left(c+dx^2\right)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}} \\
&- \frac{\sqrt{c}\sqrt{d}(4bc-3ad)\left(a+bx^2\right)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{3a^3e\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}\left(c+dx^2\right)\sqrt{\frac{be}{d}-\frac{(bc-ad)e}{d(c+dx^2)}}}
\end{aligned}
$$

output
```
(-a*d+b*c)/a/b/e/x^3/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-1/3*(-3*a*d+4*
b*c)*(b*x^2+a)/a^2/b/e/x^3/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)+1/3*c*(-
7*a*d+8*b*c)*(b*x^2+a)/a^3/e/x/(d*x^2+c)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^
(1/2)+1/3*c^(1/2)*d^(1/2)*(-7*a*d+8*b*c)*(b*x^2+a)*EllipticE(d^(1/2)*x/c^(
1/2)/(1+d*x^2/c)^(1/2),(1-b*c/a/d)^(1/2))/a^3/e/(c*(b*x^2+a)/a/(d*x^2+c))^
(1/2)/(d*x^2+c)/(b*e/d-(-a*d+b*c)*e/d/(d*x^2+c))^(1/2)-1/3*c^(1/2)*d^(1/2)
*(-3*a*d+4*b*c)*(b*x^2+a)*InverseJacobiAM(arctan(d^(1/2)*x/c^(1/2)),(1-b*c
/a/d)^(1/2))/a^3/e/(c*(b*x^2+a)/a/(d*x^2+c))^(1/2)/(d*x^2+c)/(b*e/d-(-a*d+
b*c)*e/d/(d*x^2+c))^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.84 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.58

$$\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \frac{\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\left(-\sqrt{\frac{b}{a}}(c+dx^2)\left(-8b^2cx^4+a^2(c+4dx^2)+ab(-4cx^2+7dx^4)\right)-ibc(-\right.}{}$$

input
```
Integrate[1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*(-(Sqrt[b/a]*(c + d*x^2)*(-8*b^2*c*x^4
+ a^2*(c + 4*d*x^2) + a*b*(-4*c*x^2 + 7*d*x^4))) - I*b*c*(-8*b*c + 7*a*d)*
x^3*Sqrt[1 + (b*x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticE[I*ArcSinh[Sqrt[b/a]*
x], (a*d)/(b*c)] - I*(8*b^2*c^2 - 11*a*b*c*d + 3*a^2*d^2)*x^3*Sqrt[1 + (b*
x^2)/a]*Sqrt[1 + (d*x^2)/c]*EllipticF[I*ArcSinh[Sqrt[b/a]*x], (a*d)/(b*c)]
))/(3*a^3*Sqrt[b/a]*e^2*x^3*(a + b*x^2))
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 412, normalized size of antiderivative = 0.91, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {2058, 370, 25, 445, 27, 445, 25, 27, 406, 320, 388, 313}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 2058}$$

$$\frac{\sqrt{a+bx^2}\int \frac{(dx^2+c)^{3/2}}{x^4(bx^2+a)^{3/2}}dx}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 370}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\sqrt{c+dx^2}(bc-ad)}{abx^3\sqrt{a+bx^2}}-\frac{\int -\frac{d(3bc-2ad)x^2+c(4bc-3ad)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{\int \frac{d(3bc-2ad)x^2+c(4bc-3ad)}{x^4\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx^3\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 445}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{-\frac{\int \frac{bc\left(d(4bc-3ad)x^2+c(8bc-7ad)\right)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3ax^3}}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx^3\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{a+bx^2}\left(\frac{-\frac{b\int \frac{d(4bc-3ad)x^2+c(8bc-7ad)}{x^2\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3ax^3}}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx^3\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \ 445$$

$$\frac{\sqrt{a+bx^2}\left(\frac{-\frac{b\left(-\frac{\int -\frac{cd\left(b(8bc-7ad)x^2+a(4bc-3ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{ax}\right)}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3ax^3}}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx^3\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \ 25$$

$$\frac{\sqrt{a+bx^2}\left(\frac{-\frac{b\left(\frac{\int \frac{cd\left(b(8bc-7ad)x^2+a(4bc-3ad)\right)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{ac}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{ax}\right)}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3ax^3}}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx^3\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \ 27$$

$$\frac{\sqrt{a+bx^2}\left(\frac{-\frac{b\left(\frac{d\int \frac{b(8bc-7ad)x^2+a(4bc-3ad)}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{ax}\right)}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3ax^3}}{ab}+\frac{\sqrt{c+dx^2}(bc-ad)}{abx^3\sqrt{a+bx^2}}\right)}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$$\Big\downarrow \ 406$$

$$\frac{\sqrt{a+bx^2}\left(\frac{-\frac{b\left(\frac{d\left(a(4bc-3ad)\int\frac{1}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+b(8bc-7ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx\right)}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{ax}\right)}{3a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(4bc-3ad)}{3ax^3}}{ab}\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 320

$$\frac{\sqrt{a+bx^2}\left(\frac{-\frac{b\left(\frac{d\left(b(8bc-7ad)\int\frac{x^2}{\sqrt{bx^2+a}\sqrt{dx^2+c}}dx+\frac{\sqrt{c}\sqrt{a+bx^2}(4bc-3ad)\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c(a+bx^2)}{a(c+dx^2)}}}\right)}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7ad)}{ax}\right)}{3a}-\sqrt{a+b}}{ab}\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}}$$

$\downarrow$ 388

$$\frac{\sqrt{a+bx^2}\left(-\frac{\frac{b\left(\frac{d\left(b(8bc-7ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{c\int\frac{\sqrt{bx^2+a}}{\left(dx^2+c\right)^{3/2}}dx}{b}\right)+\frac{\sqrt{c}\sqrt{a+bx^2}(4bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)}{a}-\frac{\sqrt{a+bx^2}\sqrt{c+dx^2}(8bc-7a\cdots}{ax}\cdots\right)}{3a}\cdots}{ab}\cdots\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\cdots}$$

$$\Big\downarrow \; 313$$

$$\frac{\sqrt{a+bx^2}\left(-\frac{\frac{b\left(\frac{d\left(\frac{\sqrt{c}\sqrt{a+bx^2}(4bc-3ad)\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right),1-\frac{bc}{ad}\right)}{\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}+b(8bc-7ad)\left(\frac{x\sqrt{a+bx^2}}{b\sqrt{c+dx^2}}-\frac{\sqrt{c}\sqrt{a+bx^2}E\left(\arctan\left(\frac{\sqrt{d}x}{\sqrt{c}}\right)|1-\frac{bc}{ad}\right)}{b\sqrt{d}\sqrt{c+dx^2}\sqrt{\frac{c\left(a+bx^2\right)}{a\left(c+dx^2\right)}}}\right)\right)}{a}-\frac{\sqrt{a+bx\cdots}}{\cdots}\right)}{3a}\cdots}{ab}\cdots\right.}{e\sqrt{c+dx^2}\sqrt{\frac{e(a+bx^2)}{c+dx^2}}\cdots}$$

input
```
Int[1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x]
```

output
```
(Sqrt[a + b*x^2]*(((b*c - a*d)*Sqrt[c + d*x^2])/(a*b*x^3*Sqrt[a + b*x^2])
+ (-1/3*((4*b*c - 3*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*x^3) - (b*(-(
((8*b*c - 7*a*d)*Sqrt[a + b*x^2]*Sqrt[c + d*x^2])/(a*x)) + (d*(b*(8*b*c -
7*a*d)*((x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]) - (Sqrt[c]*Sqrt[a + b*x^2]
*EllipticE[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)])/(b*Sqrt[d]*Sqrt[
(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])) + (Sqrt[c]*(4*b*c - 3*a
*d)*Sqrt[a + b*x^2]*EllipticF[ArcTan[(Sqrt[d]*x)/Sqrt[c]], 1 - (b*c)/(a*d)
])/(Sqrt[d]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]*Sqrt[c + d*x^2])))/a))/(
3*a))/(a*b)))/(e*Sqrt[(e*(a + b*x^2))/(c + d*x^2)]*Sqrt[c + d*x^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 313
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]/(c*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(c
+ d*x^2)))]))*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

rule 320
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]/(a*Rt[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[c*((a + b*x^2)/(a*(
c + d*x^2)))]))*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - b*(c/(a*d))], x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 370
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c +
 d*x^2)^(q - 1)/(a*b*e*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(e*x)
^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*Simp[c*(b*c*2*(p + 1) + (b*c - a
*d)*(m + 1)) + d*(b*c*2*(p + 1) + (b*c - a*d)*(m + 2*(q - 1) + 1))*x^2, x],
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
&& GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 388
```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[x*(Sqrt[a + b*x^2]/(b*Sqrt[c + d*x^2])), x] - Simp[c/b   Int[Sqrt[
a + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

rule 406
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[e   Int[(a + b*x^2)^p*(c + d*x^2)^q, x], x] + Sim
p[f   Int[x^2*(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, e,
f, p, q}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 2058
```
Int[(u_.)*((e_.)*((a_.) + (b_.)*(x_)^(n_.))^(q_.)*((c_) + (d_.)*(x_)^(n_))^
(r_.))^(p_), x_Symbol] :> Simp[Simp[(e*(a + b*x^n)^q*(c + d*x^n)^r)^p/((a +
 b*x^n)^(p*q)*(c + d*x^n)^(p*r))]   Int[u*(a + b*x^n)^(p*q)*(c + d*x^n)^(p*
r), x], x] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. 2(432) = 864.

Time = 11.31 (sec) , antiderivative size = 866, normalized size of antiderivative = 1.90

method	result
default	$-\frac{\left(b\,x^2+a\right)\left(4\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^6-5\sqrt{\left(d\,x^2+c\right)\left(b\,x^2+a\right)}\,\sqrt{-\frac{b}{a}}\,b^2cd\,x^6+3\sqrt{db\,x^4+ad\,x^2+bc\,x^2+ac}\,\sqrt{-\frac{b}{a}}\,ab\,d^2x^6-\right.}{}$
risch	$-\frac{\left(b\,x^2+a\right)\left(4ad\,x^2-5bc\,x^2+ac\right)}{3a^3x^3e\sqrt{\frac{e\left(b\,x^2+a\right)}{d\,x^2+c}}}+\frac{\left(3\left(a^2d^2-2abcd+b^2c^2\right)a\left(-\frac{\left(bd\,x^2e+bce\right)x}{a\left(ad-bc\right)e\sqrt{\left(x^2+\frac{a}{b}\right)\left(bd\,x^2e+bce\right)}}+\frac{\left(\frac{1}{a}+\frac{bc}{a\left(ad-bc\right)}\right)\sqrt{1+\frac{x^2b}{a}}\,\sqrt{1+\right.}{\sqrt{-\frac{b}{a}}\,\sqrt{bde\,x^4}}\right.\right.}{}$

input
```
int(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*x^2+a)*(4*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^6-5*(
(d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*b^2*c*d*x^6+3*(b*d*x^4+a*d*x^2+b*c
*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*b*d^2*x^6-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(
1/2)*(-b/a)^(1/2)*b^2*c*d*x^6-3*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^
(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a^2*d^
2*x^3+11*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/
2)*EllipticF(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b*c*d*x^3-8*((d*x^2+c)*(b*x
^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticF(x*(-b/a)^(1
/2),(a*d/b/c)^(1/2))*b^2*c^2*x^3-7*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/
a)^(1/2)*((d*x^2+c)/c)^(1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*a*b
*c*d*x^3+8*((d*x^2+c)*(b*x^2+a))^(1/2)*((b*x^2+a)/a)^(1/2)*((d*x^2+c)/c)^(
1/2)*EllipticE(x*(-b/a)^(1/2),(a*d/b/c)^(1/2))*b^2*c^2*x^3+4*((d*x^2+c)*(b
*x^2+a))^(1/2)*(-b/a)^(1/2)*a^2*d^2*x^4-5*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/
a)^(1/2)*b^2*c^2*x^4+3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*a*
b*c*d*x^4-3*(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)*(-b/a)^(1/2)*b^2*c^2*x^4+5
*((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2)*a^2*c*d*x^2-4*((d*x^2+c)*(b*x^2+
a))^(1/2)*(-b/a)^(1/2)*a*b*c^2*x^2+((d*x^2+c)*(b*x^2+a))^(1/2)*(-b/a)^(1/2
)*a^2*c^2)/(e*(b*x^2+a)/(d*x^2+c))^(3/2)/(d*x^2+c)^2/a^3/x^3/(-b/a)^(1/2)/
(b*d*x^4+a*d*x^2+b*c*x^2+a*c)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 374, normalized size of antiderivative = 0.82

$$\int \frac{1}{x^4 \left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}} dx =$$

$$-\frac{((8 b^4c^2d - 7 ab^3cd^2)x^5 + (8 ab^3c^2d - 7 a^2b^2cd^2)x^3)\sqrt{\frac{ace}{d^2}}\sqrt{-\frac{b}{a}}E(\arcsin\left(x\sqrt{-\frac{b}{a}}\right) \mid \frac{ad}{bc}) - ((8 b^4c^2d - 3 a^3}{}$$

input
```
integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*(((8*b^4*c^2*d - 7*a*b^3*c*d^2)*x^5 + (8*a*b^3*c^2*d - 7*a^2*b^2*c*d^
2)*x^3)*sqrt(a*c*e/d^2)*sqrt(-b/a)*elliptic_e(arcsin(x*sqrt(-b/a)), a*d/(b
*c)) - ((8*b^4*c^2*d - 3*a^3*b*d^3 + (4*a^2*b^2 - 7*a*b^3)*c*d^2)*x^5 + (8
*a*b^3*c^2*d - 3*a^4*d^3 + (4*a^3*b - 7*a^2*b^2)*c*d^2)*x^3)*sqrt(a*c*e/d^
2)*sqrt(-b/a)*elliptic_f(arcsin(x*sqrt(-b/a)), a*d/(b*c)) + (a^3*b*c^3 - (
8*a*b^3*c^2*d - 7*a^2*b^2*c*d^2)*x^6 - (8*a*b^3*c^3 - 3*a^2*b^2*c^2*d - 4*
a^3*b*c*d^2)*x^4 - (4*a^2*b^2*c^3 - 5*a^3*b*c^2*d)*x^2)*sqrt((b*e*x^2 + a*
e)/(d*x^2 + c)))/(a^4*b^2*c*e^2*x^5 + a^5*b*c*e^2*x^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/x**4/(e*(b*x**2+a)/(d*x**2+c))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx = \int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4), x)
```

Giac [F]

$$\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{\left(\frac{(bx^2+a)e}{dx^2+c}\right)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(((b*x^2 + a)*e/(d*x^2 + c))^(3/2)*x^4), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=\int \frac{1}{x^4\left(\frac{e\,(b\,x^2+a)}{d\,x^2+c}\right)^{3/2}}\,dx$$

input
```
int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)),x)
```

output
```
int(1/(x^4*((e*(a + b*x^2))/(c + d*x^2))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x^4\left(\frac{e(a+bx^2)}{c+dx^2}\right)^{3/2}}\,dx=\text{too large to display}$$

input
```
int(1/x^4/(e*(b*x^2+a)/(d*x^2+c))^(3/2),x)
```

output
```
(sqrt(e)*( - sqrt(c + d*x**2)*sqrt(a + b*x**2)*c + 4*int((sqrt(c + d*x**2)
*sqrt(a + b*x**2))/(a**3*c*d*x**2 + a**3*d**2*x**4 + 2*a**2*b*c**2*x**2 +
4*a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**6 + 4*a*b**2*c**2*x**4 + 5*a*b**2*c*d
*x**6 + a*b**2*d**2*x**8 + 2*b**3*c**2*x**6 + 2*b**3*c*d*x**8),x)*a**3*c*d
**2*x**3 + 4*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**3*c*d*x**2 + a**3
*d**2*x**4 + 2*a**2*b*c**2*x**2 + 4*a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**6 +
 4*a*b**2*c**2*x**4 + 5*a*b**2*c*d*x**6 + a*b**2*d**2*x**8 + 2*b**3*c**2*x
**6 + 2*b**3*c*d*x**8),x)*a**2*b*c**2*d*x**3 + 4*int((sqrt(c + d*x**2)*sqr
t(a + b*x**2))/(a**3*c*d*x**2 + a**3*d**2*x**4 + 2*a**2*b*c**2*x**2 + 4*a*
*2*b*c*d*x**4 + 2*a**2*b*d**2*x**6 + 4*a*b**2*c**2*x**4 + 5*a*b**2*c*d*x**
6 + a*b**2*d**2*x**8 + 2*b**3*c**2*x**6 + 2*b**3*c*d*x**8),x)*a**2*b*c*d**
2*x**5 - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**3*c*d*x**2 + a**3*d
**2*x**4 + 2*a**2*b*c**2*x**2 + 4*a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**6 + 4
*a*b**2*c**2*x**4 + 5*a*b**2*c*d*x**6 + a*b**2*d**2*x**8 + 2*b**3*c**2*x**
6 + 2*b**3*c*d*x**8),x)*a*b**2*c**3*x**3 + 4*int((sqrt(c + d*x**2)*sqrt(a
+ b*x**2))/(a**3*c*d*x**2 + a**3*d**2*x**4 + 2*a**2*b*c**2*x**2 + 4*a**2*b
*c*d*x**4 + 2*a**2*b*d**2*x**6 + 4*a*b**2*c**2*x**4 + 5*a*b**2*c*d*x**6 +
a*b**2*d**2*x**8 + 2*b**3*c**2*x**6 + 2*b**3*c*d*x**8),x)*a*b**2*c**2*d*x*
*5 - 8*int((sqrt(c + d*x**2)*sqrt(a + b*x**2))/(a**3*c*d*x**2 + a**3*d**2*
x**4 + 2*a**2*b*c**2*x**2 + 4*a**2*b*c*d*x**4 + 2*a**2*b*d**2*x**6 + 4*...
```

3.97 $\int \sqrt{\frac{x}{1+x}} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 22

$$\int \sqrt{\frac{x}{1+x}} \, dx = \sqrt{x}\sqrt{1+x} - \operatorname{arcsinh}\left(\sqrt{x}\right)$$

output
```
x^(1/2)*(1+x)^(1/2)-arcsinh(x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 51 vs. 2(22) = 44.

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \sqrt{\frac{x}{1+x}} \, dx = \frac{\sqrt{\frac{x}{1+x}}\left(\sqrt{x}(1+x) + \sqrt{1+x}\log\left(-\sqrt{x} + \sqrt{1+x}\right)\right)}{\sqrt{x}}$$

input
```
Integrate[Sqrt[x/(1 + x)],x]
```

output
```
(Sqrt[x/(1 + x)]*(Sqrt[x]*(1 + x) + Sqrt[1 + x]*Log[-Sqrt[x] + Sqrt[1 + x]
]))/Sqrt[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {2050, 60, 63, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x}{x+1}}dx$$

$$\Big\downarrow \text{ 2050}$$

$$\int \frac{\sqrt{x}}{\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 60}$$

$$\sqrt{x}\sqrt{x+1}-\frac{1}{2}\int \frac{1}{\sqrt{x}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{ 63}$$

$$\sqrt{x}\sqrt{x+1}-\int \frac{1}{\sqrt{x+1}}d\sqrt{x}$$

$$\Big\downarrow \text{ 222}$$

$$\sqrt{x}\sqrt{x+1}-\text{arcsinh}\left(\sqrt{x}\right)$$

input
```
Int[Sqrt[x/(1 + x)],x]
```

output
```
Sqrt[x]*Sqrt[1 + x] - ArcSinh[Sqrt[x]]
```

Defintions of rubi rules used

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(16) = 32$.

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

method	result	size
default	$\frac{\sqrt{\frac{x}{x+1}}\,(x+1)\left(2\sqrt{x^2+x}-\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\right)}{2\sqrt{(x+1)x}}$	45
risch	$(x+1)\sqrt{\frac{x}{x+1}}-\frac{\ln\left(\frac{1}{2}+x+\sqrt{x^2+x}\right)\sqrt{\frac{x}{x+1}}\,\sqrt{(x+1)x}}{2x}$	47
trager	$2\left(\frac{1}{2}+\frac{x}{2}\right)\sqrt{\frac{x}{x+1}}-\frac{\ln\left(2\sqrt{\frac{x}{x+1}}\,x+2\sqrt{\frac{x}{x+1}}+2x+1\right)}{2}$	49

input
```
int((x/(x+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(x/(x+1))^(1/2)*(x+1)*(2*(x^2+x)^(1/2)-ln(1/2+x+(x^2+x)^(1/2)))/((x+1)
*x)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(16) = 32$.

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.91

$$\int \sqrt{\frac{x}{1+x}}\,dx = (x+1)\sqrt{\frac{x}{x+1}} - \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

input
```
integrate((x/(1+x))^(1/2),x, algorithm="fricas")
```

output
```
(x + 1)*sqrt(x/(x + 1)) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(sqrt(x/(x
 + 1)) - 1)
```

Sympy [F]

$$\int \sqrt{\frac{x}{1+x}}\,dx = \int \sqrt{\frac{x}{x+1}}\,dx$$

input
```
integrate((x/(1+x))**(1/2),x)
```

output
```
Integral(sqrt(x/(x + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. $2(16) = 32$.

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.32

$$\int \sqrt{\frac{x}{1+x}}\,dx = -\frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1} - \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}+1\right) + \frac{1}{2}\log\left(\sqrt{\frac{x}{x+1}}-1\right)$$

input
```
integrate((x/(1+x))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(x/(x + 1))/(x/(x + 1) - 1) - 1/2*log(sqrt(x/(x + 1)) + 1) + 1/2*log(
sqrt(x/(x + 1)) - 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(16) = 32$.

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{x}{1+x}}\,dx = \frac{1}{2}\log\left(\left|-2\,x+2\,\sqrt{x^2+x}-1\right|\right)\operatorname{sgn}(x+1) + \sqrt{x^2+x}\operatorname{sgn}(x+1)$$

input
```
integrate((x/(1+x))^(1/2),x, algorithm="giac")
```

output
```
1/2*log(abs(-2*x + 2*sqrt(x^2 + x) - 1))*sgn(x + 1) + sqrt(x^2 + x)*sgn(x
+ 1)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.59

$$\int \sqrt{\frac{x}{1+x}}\,dx = -\mathrm{atanh}\left(\sqrt{\frac{x}{x+1}}\right) - \frac{\sqrt{\frac{x}{x+1}}}{\frac{x}{x+1}-1}$$

input
```
int((x/(x + 1))^(1/2),x)
```

output
```
- atanh((x/(x + 1))^(1/2)) - (x/(x + 1))^(1/2)/(x/(x + 1) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

$$\int \sqrt{\frac{x}{1+x}}\,dx = \sqrt{x}\,\sqrt{x+1} - \log\left(\sqrt{x+1}+\sqrt{x}\right)$$

input
```
int((x/(1+x))^(1/2),x)
```

output
```
sqrt(x)*sqrt(x + 1) - log(sqrt(x + 1) + sqrt(x))
```

3.98 $\int \sqrt{\frac{x^2}{1+x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \frac{\sqrt{x^2}\sqrt{1+x^2}}{x}$$

output
```
(x^2)^(1/2)*(x^2+1)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \frac{x}{\sqrt{\frac{x^2}{1+x^2}}}$$

input
```
Integrate[Sqrt[x^2/(1 + x^2)],x]
```

output
```
x/Sqrt[x^2/(1 + x^2)]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2050, 34, 241}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x^2}{x^2+1}}dx$$
$$\Big\downarrow \text{2050}$$
$$\int \frac{\sqrt{x^2}}{\sqrt{x^2+1}}dx$$
$$\Big\downarrow \text{34}$$
$$\frac{\sqrt{x^2}\int \frac{x}{\sqrt{x^2+1}}dx}{x}$$
$$\Big\downarrow \text{241}$$
$$\frac{\sqrt{x^2}\sqrt{x^2+1}}{x}$$

input
```
Int[Sqrt[x^2/(1 + x^2)],x]
```

output
```
(Sqrt[x^2]*Sqrt[1 + x^2])/x
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 241
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(a + b*x^2)^(p + 1)/
(2*b*(p + 1)), x] /; FreeQ[{a, b, p}, x] && NeQ[p, -1]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

method	result	size
gosper	$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23
default	$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23
trager	$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23
risch	$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23
orering	$\frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$	23

input
```
int((x^2/(x^2+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
(x^2+1)/x*(x^2/(x^2+1))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \frac{(x^2+1)\sqrt{\frac{x^2}{x^2+1}}}{x}$$

input
```
integrate((x^2/(x^2+1))^(1/2),x, algorithm="fricas")
```

output
```
(x^2 + 1)*sqrt(x^2/(x^2 + 1))/x
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = x\sqrt{\frac{x^2}{x^2+1}} + \frac{\sqrt{\frac{x^2}{x^2+1}}}{x}$$

input
```
integrate((x**2/(x**2+1))**(1/2),x)
```

output
```
x*sqrt(x**2/(x**2 + 1)) + sqrt(x**2/(x**2 + 1))/x
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.35

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \sqrt{x^2+1}$$

input
```
integrate((x^2/(x^2+1))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 + 1)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \sqrt{\frac{x^2}{1+x^2}}\,dx = \sqrt{x^2+1}\mathrm{sgn}(x) - \mathrm{sgn}(x)$$

input
```
integrate((x^2/(x^2+1))^(1/2),x, algorithm="giac")
```

output
```
sqrt(x^2 + 1)*sgn(x) - sgn(x)
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.65

$$\int \sqrt{\frac{x^2}{1+x^2}}\, dx = \frac{\sqrt{x^4+x^2}}{x}$$

input
```
int((x^2/(x^2 + 1))^(1/2),x)
```

output
```
(x^2 + x^4)^(1/2)/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.30

$$\int \sqrt{\frac{x^2}{1+x^2}}\, dx = \sqrt{x^2+1}$$

input
```
int((x^2/(x^2+1))^(1/2),x)
```

output
```
sqrt(x**2 + 1)
```

3.99 $\int \sqrt{\frac{x^3}{1+x^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 286

$$\int \sqrt{\frac{x^3}{1+x^3}}\,dx = \frac{\left(1+\sqrt{3}\right)\sqrt{x^3}\sqrt{1+x^3}}{x\left(1+\left(1+\sqrt{3}\right)x\right)} - \frac{\sqrt[4]{3}\sqrt{x^3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,E\left(\arccos\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{x\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}} - \frac{\left(1-\sqrt{3}\right)\sqrt{x^3}(1+x)\sqrt{\frac{1-x+x^2}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\,\text{EllipticF}\left(\arccos\left(\frac{1+\left(1-\sqrt{3}\right)x}{1+\left(1+\sqrt{3}\right)x}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[4]{3}x\sqrt{\frac{x(1+x)}{\left(1+\left(1+\sqrt{3}\right)x\right)^2}}\sqrt{1+x^3}}$$

output
```
(1+3^(1/2))*(x^3)^(1/2)*(x^3+1)^(1/2)/x/(1+(1+3^(1/2))*x)-3^(1/4)*(x^3)^(1
/2)*(1+x)*((x^2-x+1)/(1+(1+3^(1/2))*x)^2)^(1/2)*EllipticE((1-(1+(1-3^(1/2)
)*x)^2/(1+(1+3^(1/2))*x)^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))/x/(x*(1+x)/(1+(
1+3^(1/2))*x)^2)^(1/2)/(x^3+1)^(1/2)-1/6*(1-3^(1/2))*(x^3)^(1/2)*(1+x)*((x
^2-x+1)/(1+(1+3^(1/2))*x)^2)^(1/2)*InverseJacobiAM(arccos((1+(1-3^(1/2))*x
)/(1+(1+3^(1/2))*x)),1/4*6^(1/2)+1/4*2^(1/2))*3^(3/4)/x/(x*(1+x)/(1+(1+3^(
1/2))*x)^2)^(1/2)/(x^3+1)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.09

$$\int \sqrt{\frac{x^3}{1+x^3}}\,dx = \frac{2}{5}x\sqrt{x^3}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{5}{6},\frac{11}{6},-x^3\right)$$

input
```
Integrate[Sqrt[x^3/(1 + x^3)],x]
```

output
```
(2*x*Sqrt[x^3]*Hypergeometric2F1[1/2, 5/6, 11/6, -x^3])/5
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {2050, 34, 851, 837, 25, 766, 2420}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x^3}{x^3+1}}dx$$

$$\Big\downarrow \text{2050}$$

$$\int \frac{\sqrt{x^3}}{\sqrt{x^3+1}}dx$$

$$\Big\downarrow \text{34}$$

$$\frac{\sqrt{x^3}\int \frac{x^{3/2}}{\sqrt{x^3+1}}dx}{x^{3/2}}$$

$$\Big\downarrow \text{851}$$

$$\frac{2\sqrt{x^3}\int \frac{x^2}{\sqrt{x^3+1}}d\sqrt{x}}{x^{3/2}}$$

$$\Big\downarrow \text{837}$$

$$\frac{2\sqrt{x^3}\left(-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}d\sqrt{x}-\frac{1}{2}\int -\frac{2x^2-\sqrt{3}+1}{\sqrt{x^3+1}}d\sqrt{x}\right)}{x^{3/2}}$$

$$\Big\downarrow \; 25$$

$$\frac{2\sqrt{x^3}\left(\frac{1}{2}\int \frac{2x^2-\sqrt{3}+1}{\sqrt{x^3+1}}d\sqrt{x}-\frac{1}{2}\left(1-\sqrt{3}\right)\int \frac{1}{\sqrt{x^3+1}}d\sqrt{x}\right)}{x^{3/2}}$$

$$\Big\downarrow \; 766$$

$$\frac{2\sqrt{x^3}\left(\frac{1}{2}\int \frac{2x^2-\sqrt{3}+1}{\sqrt{x^3+1}}d\sqrt{x}-\frac{\left(1-\sqrt{3}\right)\sqrt{x}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\text{EllipticF}\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right),\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\sqrt[4]{3}\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}\right)}{x^{3/2}}$$

$$\Big\downarrow \; 2420$$

$$\frac{2\sqrt{x^3}\left(\frac{1}{2}\left(\frac{\left(1+\sqrt{3}\right)\sqrt{x}\sqrt{x^3+1}}{\left(1+\sqrt{3}\right)x+1}-\frac{\sqrt[4]{3}\sqrt{x}(x+1)\sqrt{\frac{x^2-x+1}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}E\left(\arccos\left(\frac{\left(1-\sqrt{3}\right)x+1}{\left(1+\sqrt{3}\right)x+1}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt{\frac{x(x+1)}{\left(\left(1+\sqrt{3}\right)x+1\right)^2}}\sqrt{x^3+1}}\right)-\frac{\left(1-\sqrt{3}\right)\sqrt{x}(x+1)\sqrt{\frac{x^2-}{((1+\sqrt{3}}}}{4}}{x^{3/2}}$$

input
```
Int[Sqrt[x^3/(1 + x^3)],x]
```

output
```
(2*Sqrt[x^3]*((((1 + Sqrt[3])*Sqrt[x]*Sqrt[1 + x^3])/(1 + (1 + Sqrt[3])*x)
 - (3^(1/4)*Sqrt[x]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*El
lipticE[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])
/4])/(Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3]))/2 - ((1 -
Sqrt[3])*Sqrt[x]*(1 + x)*Sqrt[(1 - x + x^2)/(1 + (1 + Sqrt[3])*x)^2]*Ellip
ticF[ArcCos[(1 + (1 - Sqrt[3])*x)/(1 + (1 + Sqrt[3])*x)], (2 + Sqrt[3])/4]
)/(4*3^(1/4)*Sqrt[(x*(1 + x))/(1 + (1 + Sqrt[3])*x)^2]*Sqrt[1 + x^3])))/x^
(3/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 766
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

rule 837
```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Simp[(Sqrt[3] - 1)*(s^2/(2*r^2))   Int[1/Sqrt[
a + b*x^6], x], x] - Simp[1/(2*r^2)   Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/S
qrt[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

rule 851
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(k*n)/c^
n))^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] &&
FractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

rule 2420
```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 1.69 (sec) , antiderivative size = 1522, normalized size of antiderivative = 5.32

method	result	size
default	Expression too large to display	1522

input
```
int((x^3/(x^3+1))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(x^3/(x^3+1))^(1/2)/x*(x^3+1)*(I*3^(1/2)*((I*3^(1/2)+3)*x/(1+I*3^(1/2))
/(x+1))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((I*3^(1/2)-2*
x+1)/(1+I*3^(1/2))/(x+1))^(1/2)*EllipticE(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(
x+1))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(I*3^(1/2)+3))^(1/2
))*x^2+2*I*3^(1/2)*((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(x+1))^(1/2)*((I*3^(1/2)
+2*x-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(x+1))
^(1/2)*EllipticE(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(x+1))^(1/2),((I*3^(1/2)-3
)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(I*3^(1/2)+3))^(1/2))*x+I*3^(1/2)*((I*3^(1/2
)+3)*x/(1+I*3^(1/2))/(x+1))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(x+1))^
(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(x+1))^(1/2)*EllipticE(((I*3^(1/2)+
3)*x/(1+I*3^(1/2))/(x+1))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)
/(I*3^(1/2)+3))^(1/2))-2*((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(x+1))^(1/2)*((I*3
^(1/2)+2*x-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/
(x+1))^(1/2)*EllipticF(((I*3^(1/2)+3)*x/(1+I*3^(1/2))/(x+1))^(1/2),((I*3^(
1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)/(I*3^(1/2)+3))^(1/2))*x^2+3*((I*3^(1/2
)+3)*x/(1+I*3^(1/2))/(x+1))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(x+1))^
(1/2)*((I*3^(1/2)-2*x+1)/(1+I*3^(1/2))/(x+1))^(1/2)*EllipticE(((I*3^(1/2)+
3)*x/(1+I*3^(1/2))/(x+1))^(1/2),((I*3^(1/2)-3)*(1+I*3^(1/2))/(I*3^(1/2)-1)
/(I*3^(1/2)+3))^(1/2))*x^2-I*3^(1/2)*x^3-4*((I*3^(1/2)+3)*x/(1+I*3^(1/2))/
(x+1))^(1/2)*((I*3^(1/2)+2*x-1)/(I*3^(1/2)-1)/(x+1))^(1/2)*((I*3^(1/2)-...
```

Fricas [F]

$$\int \sqrt{\frac{x^3}{1+x^3}}\,dx = \int \sqrt{\frac{x^3}{x^3+1}}\,dx$$

input
```
integrate((x^3/(x^3+1))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(x^3/(x^3 + 1)), x)
```

Sympy [F]

$$\int \sqrt{\frac{x^3}{1+x^3}}\,dx = \int \sqrt{\frac{x^3}{x^3+1}}\,dx$$

input
```
integrate((x**3/(x**3+1))**(1/2),x)
```

output
```
Integral(sqrt(x**3/(x**3 + 1)), x)
```

Maxima [F]

$$\int \sqrt{\frac{x^3}{1+x^3}}\,dx = \int \sqrt{\frac{x^3}{x^3+1}}\,dx$$

input
```
integrate((x^3/(x^3+1))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^3/(x^3 + 1)), x)
```

Giac [F]

$$\int \sqrt{\frac{x^3}{1+x^3}}\,dx = \int \sqrt{\frac{x^3}{x^3+1}}\,dx$$

input
```
integrate((x^3/(x^3+1))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^3/(x^3 + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{x^3}{1+x^3}}\,dx = \int \sqrt{\frac{x^3}{x^3+1}}\,dx$$

input
```
int((x^3/(x^3 + 1))^(1/2),x)
```

output
```
int((x^3/(x^3 + 1))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\frac{x^3}{1+x^3}}\,dx = \int \frac{\sqrt{x}\,\sqrt{x^3+1}\,x}{x^3+1}dx$$

input
```
int((x^3/(x^3+1))^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(x**3 + 1)*x)/(x**3 + 1),x)
```

3.100 $\int \sqrt{\frac{x^n}{1+x^n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 46

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx=\frac{2x\sqrt{x^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-x^n\right)}{2+n}$$

output
```
2*x*(x^n)^(1/2)*hypergeom([1/2, 1/2+1/n],[3/2+1/n],-x^n)/(2+n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx=\frac{2x\sqrt{x^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}+\frac{1}{n},\frac{3}{2}+\frac{1}{n},-x^n\right)}{2+n}$$

input
```
Integrate[Sqrt[x^n/(1 + x^n)],x]
```

output
```
(2*x*Sqrt[x^n]*Hypergeometric2F1[1/2, 1/2 + n^(-1), 3/2 + n^(-1), -x^n])/(
2 + n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2050, 34, 888}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\frac{x^n}{x^n+1}}dx$$

$$\Big\downarrow \text{2050}$$

$$\int \frac{\sqrt{x^n}}{\sqrt{x^n+1}}dx$$

$$\Big\downarrow \text{34}$$

$$x^{-n/2}\sqrt{x^n}\int \frac{x^{n/2}}{\sqrt{x^n+1}}dx$$

$$\Big\downarrow \text{888}$$

$$\frac{2x^{\frac{n+2}{2}-\frac{n}{2}}\sqrt{x^n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}\left(1+\frac{2}{n}\right),\frac{1}{2}\left(3+\frac{2}{n}\right),-x^n\right)}{n+2}$$

input
```
Int[Sqrt[x^n/(1 + x^n)],x]
```

output
```
(2*x^(-1/2*n + (2 + n)/2)*Sqrt[x^n]*Hypergeometric2F1[1/2, (1 + 2/n)/2, (3
 + 2/n)/2, -x^n])/(2 + n)
```

Defintions of rubi rules used

rule 34
```
Int[(u_.)*((a_.)*(x_)^(m_))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*x^m)^F
racPart[p]/x^(m*FracPart[p]))   Int[u*x^(m*p), x], x] /; FreeQ[{a, m, p}, x
] &&  !IntegerQ[p]
```

rule 888
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

rule 2050
```
Int[(u_.)*(((e_.)*((a_.) + (b_.)*(x_)^(n_.)))/((c_) + (d_.)*(x_)^(n_.)))^(p
_), x_Symbol] :> Int[u*((a*e + b*e*x^n)^p/(c + d*x^n)^p), x] /; FreeQ[{a, b
, c, d, e, n, p}, x] && GtQ[b*d*e, 0] && GtQ[c - a*(d/b), 0]
```

Maple [F]

$$\int \sqrt{\frac{x^n}{1+x^n}}dx$$

input
```
int((x^n/(1+x^n))^(1/2),x)
```

output
```
int((x^n/(1+x^n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((x^n/(1+x^n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

input
```
integrate((x**n/(1+x**n))**(1/2),x)
```

output
```
Integral(sqrt(x**n/(x**n + 1)), x)
```

Maxima [F]

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

input
```
integrate((x^n/(1+x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x^n/(x^n + 1)), x)
```

Giac [F]

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

input
```
integrate((x^n/(1+x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x^n/(x^n + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \int \sqrt{\frac{x^n}{x^n+1}}\,dx$$

input
```
int((x^n/(x^n + 1))^(1/2),x)
```

output
```
int((x^n/(x^n + 1))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\frac{x^n}{1+x^n}}\,dx = \int \frac{x^{\frac{n}{2}}\sqrt{x^n+1}}{x^n+1}dx$$

input
```
int((x^n/(1+x^n))^(1/2),x)
```

output
```
int((x**(n/2)*sqrt(x**n + 1))/(x**n + 1),x)
```

3.101 $\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = \frac{64}{25}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{5/2} - \frac{48}{35}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{7/2} + \frac{8}{45}\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{9/2}$$

output
```
64/25*(2-(4+(-9+5*x)^(1/2))^(1/2))^(5/2)-48/35*(2-(4+(-9+5*x)^(1/2))^(1/2)
)^(7/2)+8/45*(2-(4+(-9+5*x)^(1/2))^(1/2))^(9/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = -\frac{8\sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\left(443-175x-4\sqrt{-9+5x}-64\sqrt{4+\sqrt{-9+5x}}+10\sqrt{-9+5x}\sqrt{4+\sqrt{-9}}\right.}{1575}$$

input
```
Integrate[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

output
```
(-8*Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]]*(443 - 175*x - 4*Sqrt[-9 + 5*x] - 6
4*Sqrt[4 + Sqrt[-9 + 5*x]] + 10*Sqrt[-9 + 5*x]*Sqrt[4 + Sqrt[-9 + 5*x]]))/
1575
```

Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {7267, 896, 25, 1388, 900, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{2-\sqrt{\sqrt{5x-9}+4}}dx$$

$$\Big\downarrow \text{7267}$$

$$\frac{2}{5}\int \sqrt{5x-9}\sqrt{2-\sqrt{\sqrt{5x-9}+4}}d\sqrt{5x-9}$$

$$\Big\downarrow \text{896}$$

$$\frac{2}{5}\int \sqrt{5x-9}\sqrt{2-\sqrt[4]{5x-9}}d\left(\sqrt{5x-9}+4\right)$$

$$\Big\downarrow \text{25}$$

$$-\frac{2}{5}\int -\sqrt{5x-9}\sqrt{2-\sqrt[4]{5x-9}}d\left(\sqrt{5x-9}+4\right)$$

$$\Big\downarrow \text{1388}$$

$$-\frac{2}{5}\int \left(2-\sqrt[4]{5x-9}\right)^{3/2}\left(\sqrt[4]{5x-9}+2\right)d\left(\sqrt{5x-9}+4\right)$$

$$\Big\downarrow \text{900}$$

$$-\frac{4}{5}\int \sqrt[4]{5x-9}\left(-\sqrt{5x-9}-2\right)^{3/2}\left(\sqrt{5x-9}+6\right)d\sqrt[4]{5x-9}$$

$$\Big\downarrow \text{86}$$

$$-\frac{4}{5}\int \left(\left(-\sqrt{5x-9}-2\right)^{7/2}-6\left(-\sqrt{5x-9}-2\right)^{5/2}+8\left(-\sqrt{5x-9}-2\right)^{3/2}\right)d\sqrt[4]{5x-9}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4}{5}\left(-\frac{2}{9}\left(-\sqrt{5x-9}-2\right)^{9/2}+\frac{12}{7}\left(-\sqrt{5x-9}-2\right)^{7/2}-\frac{16}{5}\left(-\sqrt{5x-9}-2\right)^{5/2}\right)$$

input
```
Int[Sqrt[2 - Sqrt[4 + Sqrt[-9 + 5*x]]],x]
```

output
```
(-4*((-16*(-2 - Sqrt[-9 + 5*x])^(5/2))/5 + (12*(-2 - Sqrt[-9 + 5*x])^(7/2)
)/7 - (2*(-2 - Sqrt[-9 + 5*x])^(9/2))/9))/5
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 896
```
Int[((a_) + (b_.)*(v_)^(n_))^(p_.)*(x_)^(m_.), x_Symbol] :> With[{c = Coeff
icient[v, x, 0], d = Coefficient[v, x, 1]}, Simp[1/d^(m + 1)   Subst[Int[Si
mplifyIntegrand[(x - c)^m*(a + b*x^n)^p, x], x], x, v], x] /; NeQ[c, 0]] /;
 FreeQ[{a, b, n, p}, x] && LinearQ[v, x] && IntegerQ[m]
```

rule 900
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> With[{g = Denominator[n]}, Simp[g   Subst[Int[x^(g - 1)*(a + b*x^(g*n)
)^p*(c + d*x^(g*n))^q, x], x, x^(1/g)], x]] /; FreeQ[{a, b, c, d, p, q}, x]
 && NeQ[b*c - a*d, 0] && FractionQ[n]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.72

method	result	size
derivativedivides	$\frac{64\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{5}{2}}}{25}-\frac{48\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{7}{2}}}{35}+\frac{8\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{9}{2}}}{45}$	59
default	$\frac{64\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{5}{2}}}{25}-\frac{48\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{7}{2}}}{35}+\frac{8\left(2-\sqrt{4+\sqrt{-9+5x}}\right)^{\frac{9}{2}}}{45}$	59

input
```
int((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
64/25*(2-(4+(-9+5*x)^(1/2))^(1/2))^(5/2)-48/35*(2-(4+(-9+5*x)^(1/2))^(1/2)
)^(7/2)+8/45*(2-(4+(-9+5*x)^(1/2))^(1/2))^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

$$\int\sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx =$$
$$-\frac{8}{1575}\left(2\left(5\sqrt{5\,x-9}-32\right)\sqrt{\sqrt{5\,x-9}+4}-175\,x-4\sqrt{5\,x-9}+443\right)\sqrt{-\sqrt{\sqrt{5\,x-9}+4}+2}$$

input
```
integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-8/1575*(2*(5*sqrt(5*x - 9) - 32)*sqrt(sqrt(5*x - 9) + 4) - 175*x - 4*sqrt
(5*x - 9) + 443)*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2)
```

Sympy [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.79

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = \frac{8\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{\frac{9}{2}}}{45} - \frac{48\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{\frac{7}{2}}}{35} + \frac{64\left(2-\sqrt{\sqrt{5x-9}+4}\right)^{\frac{5}{2}}}{25}$$

input
```
integrate((2-(4+(-9+5*x)**(1/2))**(1/2))**(1/2),x)
```

output
```
8*(2 - sqrt(sqrt(5*x - 9) + 4))**(9/2)/45 - 48*(2 - sqrt(sqrt(5*x - 9) + 4
))**(7/2)/35 + 64*(2 - sqrt(sqrt(5*x - 9) + 4))**(5/2)/25
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.71

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = \frac{8}{45}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{9}{2}} - \frac{48}{35}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{7}{2}} + \frac{64}{25}\left(-\sqrt{\sqrt{5\,x-9}+4}+2\right)^{\frac{5}{2}}$$

input
```
integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
8/45*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(9/2) - 48/35*(-sqrt(sqrt(5*x - 9) + 4
) + 2)^(7/2) + 64/25*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(58) = 116$.

Time = 0.22 (sec) , antiderivative size = 474, normalized size of antiderivative = 5.78

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx =$$
$$-\frac{8}{1575}\left(\left(35\left(\sqrt{\sqrt{5\,x-9}+4}-2\right)^4\sqrt{-\sqrt{\sqrt{5\,x-9}+4}+2}+360\left(\sqrt{\sqrt{5\,x-9}+4}-2\right)^3\sqrt{-\sqrt{\sqrt{5\right.\right.$$
$$-51)$$

input
```
integrate((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

output
```
-8/1575*((35*(sqrt(sqrt(5*x - 9) + 4) - 2)^4*sqrt(-sqrt(sqrt(5*x - 9) + 4)
 + 2) + 360*(sqrt(sqrt(5*x - 9) + 4) - 2)^3*sqrt(-sqrt(sqrt(5*x - 9) + 4)
+ 2) + 1512*(sqrt(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4)
+ 2) - 3360*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(3/2) + 5040*sqrt(-sqrt(sqrt(5*
x - 9) + 4) + 2))*sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) -
18*(5*(sqrt(sqrt(5*x - 9) + 4) - 2)^3*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) +
 42*(sqrt(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) - 1
40*(-sqrt(sqrt(5*x - 9) + 4) + 2)^(3/2) + 280*sqrt(-sqrt(sqrt(5*x - 9) + 4
) + 2))*sgn(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 84*(3*(sqr
t(sqrt(5*x - 9) + 4) - 2)^2*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2) - 20*(-sqrt
(sqrt(5*x - 9) + 4) + 2)^(3/2) + 60*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sg
n(-4*(sqrt(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79) - 840*((-sqrt(sqrt(5*x
 - 9) + 4) + 2)^(3/2) - 6*sqrt(-sqrt(sqrt(5*x - 9) + 4) + 2))*sgn(-4*(sqrt
(5*x - 9) + 4)^2 + 32*sqrt(5*x - 9) + 79))*sgn(20*x - 51)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = \int \sqrt{2-\sqrt{\sqrt{5\,x-9}+4}}\,dx$$

input
```
int((2 - ((5*x - 9)^(1/2) + 4)^(1/2))^(1/2),x)
```

output
```
int((2 - ((5*x - 9)^(1/2) + 4)^(1/2))^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.70

$$\int \sqrt{2-\sqrt{4+\sqrt{-9+5x}}}\,dx = \frac{8\sqrt{-\sqrt{\sqrt{5x-9}+4}+2}\left(-10\sqrt{5x-9}\,\sqrt{\sqrt{5x-9}+4}+64\sqrt{\sqrt{5x-9}+4}+4\sqrt{5x-9}+175x-443\right)}{1575}$$

input
```
int((2-(4+(-9+5*x)^(1/2))^(1/2))^(1/2),x)
```

output
```
(8*sqrt( - sqrt(sqrt(5*x - 9) + 4) + 2)*( - 10*sqrt(5*x - 9)*sqrt(sqrt(5*x
 - 9) + 4) + 64*sqrt(sqrt(5*x - 9) + 4) + 4*sqrt(5*x - 9) + 175*x - 443))/
1575
```

3.102 $\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx$

Optimal result

Integrand size = 21, antiderivative size = 160

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx = \frac{16}{5}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{5/2} - \frac{24}{7}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{7/2} + 8\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{9/2} - \frac{160}{11}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{11/2} + \frac{144}{13}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{13/2} - \frac{56}{15}\left(1+\sqrt{1+\sqrt{-1+x}}\right)^{15/2} + \frac{8}{17}\left(1\right.$$

output

```
16/5*(1+(1+(-1+x)^(1/2))^(1/2))^(5/2)-24/7*(1+(1+(-1+x)^(1/2))^(1/2))^(7/2
)+8*(1+(1+(-1+x)^(1/2))^(1/2))^(9/2)-160/11*(1+(1+(-1+x)^(1/2))^(1/2))^(11
/2)+144/13*(1+(1+(-1+x)^(1/2))^(1/2))^(13/2)-56/15*(1+(1+(-1+x)^(1/2))^(1/
2))^(15/2)+8/17*(1+(1+(-1+x)^(1/2))^(1/2))^(17/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.59

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx$$
$$= \frac{8\sqrt{1+\sqrt{1+\sqrt{-1+x}}}\left(-8872+1109\sqrt{-1+x}+28231(-1+x)+77(-1+x)^{3/2}+15015(-1+x)^2\right.}{255255}$$

input
```
Integrate[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

output
```
(8*Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*(-8872 + 1109*Sqrt[-1 + x] + 28231*(-1
 + x) + 77*(-1 + x)^(3/2) + 15015*(-1 + x)^2 + Sqrt[1 + Sqrt[-1 + x]]*(-76
96 + 4544*Sqrt[-1 + x] + 7*(-168 + 143*Sqrt[-1 + x])*x)))/255255
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7267, 7267, 25, 2003, 2091, 2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{\sqrt{x-1}+1}+1}x\,dx$$
$$\Big\downarrow \text{ 7267}$$
$$2\int \sqrt{\sqrt{\sqrt{x-1}+1}+1}\sqrt{x-1}xd\sqrt{x-1}$$
$$\Big\downarrow \text{ 7267}$$
$$4\int -\sqrt{\sqrt{\sqrt{x-1}+1}+1}\left((x-2)^2+1\right)\sqrt{\sqrt{x-1}+1}(2-x)d\sqrt{\sqrt{x-1}+1}$$
$$\Big\downarrow \text{ 25}$$

$$-4\int \sqrt{\sqrt{\sqrt{x-1}+1}+1}\left((x-2)^2+1\right)\sqrt{\sqrt{x-1}+1}(2-x)d\sqrt{\sqrt{x-1}+1}$$

$\Big\downarrow$ 2003

$$-4\int \left(1-\sqrt{\sqrt{x-1}+1}\right)\left(\sqrt{\sqrt{x-1}+1}+1\right)^{3/2}\left((x-2)^2+1\right)\sqrt{\sqrt{x-1}+1}d\sqrt{\sqrt{x-1}+1}$$

$\Big\downarrow$ 2091

$$-4\int \left(1-\sqrt{\sqrt{x-1}+1}\right)\left(\sqrt{\sqrt{x-1}+1}+1\right)^{3/2}\sqrt{\sqrt{x-1}+1}\left((x-1)^2-2(x-1)+2\right)d\sqrt{\sqrt{x-1}+1}$$

$\Big\downarrow$ 2115

$$-4\int \left(-\left(\sqrt{\sqrt{x-1}+1}+1\right)^{15/2}+7\left(\sqrt{\sqrt{x-1}+1}+1\right)^{13/2}-18\left(\sqrt{\sqrt{x-1}+1}+1\right)^{11/2}+20\left(\sqrt{\sqrt{x-1}+\right.\right.$$

$\Big\downarrow$ 2009

$$4\left(\frac{2}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{17/2}-\frac{14}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{15/2}+\frac{36}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{13/2}-\frac{40}{11}\left(\sqrt{\sqrt{x-1}+\right.\right.$$

input
```
Int[Sqrt[1 + Sqrt[1 + Sqrt[-1 + x]]]*x,x]
```

output
```
4*((4*(1 + Sqrt[1 + Sqrt[-1 + x]])^(5/2))/5 - (6*(1 + Sqrt[1 + Sqrt[-1 + x
]])^(7/2))/7 + 2*(1 + Sqrt[1 + Sqrt[-1 + x]])^(9/2) - (40*(1 + Sqrt[1 + Sq
rt[-1 + x]])^(11/2))/11 + (36*(1 + Sqrt[1 + Sqrt[-1 + x]])^(13/2))/13 - (1
4*(1 + Sqrt[1 + Sqrt[-1 + x]])^(15/2))/15 + (2*(1 + Sqrt[1 + Sqrt[-1 + x]]
)^(17/2))/17)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2003
```
Int[(u_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :
> Int[u*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}
, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&
  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2091
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && PolyQ[Px, x] && BinomialQ[z, x
] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ[u, x])
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.67

method	result
derivativedivides	$\frac{16\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{5}{2}}}{5}-\frac{24\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{7}{2}}}{7}+8\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{9}{2}}-\frac{160\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{11}{2}}}{11}+$
default	$\frac{16\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{5}{2}}}{5}-\frac{24\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{7}{2}}}{7}+8\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{9}{2}}-\frac{160\left(1+\sqrt{1+\sqrt{x-1}}\right)^{\frac{11}{2}}}{11}+$

input
```
int((1+(1+(x-1)^(1/2))^(1/2))^(1/2)*x,x,method=_RETURNVERBOSE)
```

output
```
16/5*(1+(1+(x-1)^(1/2))^(1/2))^(5/2)-24/7*(1+(1+(x-1)^(1/2))^(1/2))^(7/2)+
8*(1+(1+(x-1)^(1/2))^(1/2))^(9/2)-160/11*(1+(1+(x-1)^(1/2))^(1/2))^(11/2)+
144/13*(1+(1+(x-1)^(1/2))^(1/2))^(13/2)-56/15*(1+(1+(x-1)^(1/2))^(1/2))^(1
5/2)+8/17*(1+(1+(x-1)^(1/2))^(1/2))^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.39

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx$$

$$= \frac{8}{255255}\left(15015\,x^2 + (77\,x + 1032)\sqrt{x-1} + \left((1001\,x + 4544)\sqrt{x-1} - 1176\,x - 7696\right)\sqrt{\sqrt{x-1}+1} -\right.$$

input
```
integrate((1+(1+(x-1)^(1/2))^(1/2))^(1/2)*x,x, algorithm="fricas")
```

output
```
8/255255*(15015*x^2 + (77*x + 1032)*sqrt(x - 1) + ((1001*x + 4544)*sqrt(x
- 1) - 1176*x - 7696)*sqrt(sqrt(x - 1) + 1) - 1799*x - 22088)*sqrt(sqrt(sq
rt(x - 1) + 1) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx = \frac{8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{17}{2}}}{17} - \frac{56\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{15}{2}}}{15} + \frac{144\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{13}{2}}}{13} - \frac{160\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{11}{2}}}{11} + 8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{9}{2}} - \frac{24\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{7}{2}}}{7} + \frac{16\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{5}{2}}}{5}$$

input
```
integrate((1+(1+(x-1)**(1/2))**(1/2))**(1/2)*x,x)
```

output
```
8*(sqrt(sqrt(x - 1) + 1) + 1)**(17/2)/17 - 56*(sqrt(sqrt(x - 1) + 1) + 1)*
*(15/2)/15 + 144*(sqrt(sqrt(x - 1) + 1) + 1)**(13/2)/13 - 160*(sqrt(sqrt(x
 - 1) + 1) + 1)**(11/2)/11 + 8*(sqrt(sqrt(x - 1) + 1) + 1)**(9/2) - 24*(sq
rt(sqrt(x - 1) + 1) + 1)**(7/2)/7 + 16*(sqrt(sqrt(x - 1) + 1) + 1)**(5/2)/
5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.66

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx = \frac{8}{17}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{17}{2}} - \frac{56}{15}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{15}{2}} + \frac{144}{13}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{13}{2}} - \frac{160}{11}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{11}{2}} + 8\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{9}{2}} - \frac{24}{7}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{7}{2}} + \frac{16}{5}\left(\sqrt{\sqrt{x-1}+1}+1\right)^{\frac{5}{2}}$$

input
```
integrate((1+(1+(x-1)^(1/2))^(1/2))^(1/2)*x,x, algorithm="maxima")
```

output
```
8/17*(sqrt(sqrt(x - 1) + 1) + 1)^(17/2) - 56/15*(sqrt(sqrt(x - 1) + 1) + 1
)^(15/2) + 144/13*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 160/11*(sqrt(sqrt(x
 - 1) + 1) + 1)^(11/2) + 8*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) - 24/7*(sqrt(
sqrt(x - 1) + 1) + 1)^(7/2) + 16/5*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(106) = 212$.

Time = 0.18 (sec) , antiderivative size = 859, normalized size of antiderivative = 5.37

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx = \text{Too large to display}$$

input
```
integrate((1+(1+(x-1)^(1/2))^(1/2))^(1/2)*x,x, algorithm="giac")
```

output
```
8/765765*(7*(6435*(sqrt(sqrt(x - 1) + 1) + 1)^(17/2) - 58344*(sqrt(sqrt(x
- 1) + 1) + 1)^(15/2) + 235620*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 556920
*(sqrt(sqrt(x - 1) + 1) + 1)^(11/2) + 850850*(sqrt(sqrt(x - 1) + 1) + 1)^(
9/2) - 875160*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) + 612612*(sqrt(sqrt(x - 1)
 + 1) + 1)^(5/2) - 291720*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 109395*sqrt(
sqrt(sqrt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7)
 + 119*(429*(sqrt(sqrt(x - 1) + 1) + 1)^(15/2) - 3465*(sqrt(sqrt(x - 1) +
1) + 1)^(13/2) + 12285*(sqrt(sqrt(x - 1) + 1) + 1)^(11/2) - 25025*(sqrt(sq
rt(x - 1) + 1) + 1)^(9/2) + 32175*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) - 2702
7*(sqrt(sqrt(x - 1) + 1) + 1)^(5/2) + 15015*(sqrt(sqrt(x - 1) + 1) + 1)^(3
/2) - 6435*sqrt(sqrt(sqrt(x - 1) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*
sqrt(x - 1) - 7) - 765*(231*(sqrt(sqrt(x - 1) + 1) + 1)^(13/2) - 1638*(sqr
t(sqrt(x - 1) + 1) + 1)^(11/2) + 5005*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) -
8580*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) + 9009*(sqrt(sqrt(x - 1) + 1) + 1)^
(5/2) - 6006*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) + 3003*sqrt(sqrt(sqrt(x - 1
) + 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) - 3315*(63*(sq
rt(sqrt(x - 1) + 1) + 1)^(11/2) - 385*(sqrt(sqrt(x - 1) + 1) + 1)^(9/2) +
990*(sqrt(sqrt(x - 1) + 1) + 1)^(7/2) - 1386*(sqrt(sqrt(x - 1) + 1) + 1)^(
5/2) + 1155*(sqrt(sqrt(x - 1) + 1) + 1)^(3/2) - 693*sqrt(sqrt(sqrt(x - 1)
+ 1) + 1))*sgn(4*(sqrt(x - 1) + 1)^2 - 8*sqrt(x - 1) - 7) + 9724*(35*(s...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx=\int x\,\sqrt{\sqrt{\sqrt{x-1}+1}+1}\,dx$$

input
```
int(x*(((x - 1)^(1/2) + 1)^(1/2) + 1)^(1/2),x)
```

output
```
int(x*(((x - 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.51

$$\int \sqrt{1+\sqrt{1+\sqrt{-1+x}}}x\,dx$$
$$= \frac{8\sqrt{\sqrt{\sqrt{x-1}+1}+1}\left(1001\sqrt{x-1}\,\sqrt{\sqrt{x-1}+1}\,x+4544\sqrt{x-1}\,\sqrt{\sqrt{x-1}+1}-1176\sqrt{\sqrt{x-1}+1}\right.}{255255}$$

input
```
int((1+(1+(x-1)^(1/2))^(1/2))^(1/2)*x,x)
```

output
```
(8*sqrt(sqrt(sqrt(x - 1) + 1) + 1)*(1001*sqrt(x - 1)*sqrt(sqrt(x - 1) + 1)
*x + 4544*sqrt(x - 1)*sqrt(sqrt(x - 1) + 1) - 1176*sqrt(sqrt(x - 1) + 1)*x
 - 7696*sqrt(sqrt(x - 1) + 1) + 77*sqrt(x - 1)*x + 1032*sqrt(x - 1) + 1501
5*x**2 - 1799*x - 22088))/255255
```

3.103 $\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 190

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = -\frac{32}{5}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{5/2} + \frac{48}{7}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{7/2} + \frac{112}{9}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{9/2} - \frac{320}{11}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{11/2} + \frac{288}{13}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{13/2} - \frac{112}{15}\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{15/2} + \frac{16}{17}$$

output
```
-32/5*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(5/2)+48/7*(1+(1+(1+x^(1/2))^(1/2))^
(1/2))^(7/2)+112/9*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(9/2)-320/11*(1+(1+(1+x
^(1/2))^(1/2))^(1/2))^(11/2)+288/13*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(13/2)
-112/15*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(15/2)+16/17*(1+(1+(1+x^(1/2))^(1/
2))^(1/2))^(17/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.88

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$$
$$= \frac{16\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\left(-8\left(3519-1094\sqrt{1+\sqrt{1+\sqrt{x}}}+163\sqrt{1+\sqrt{x}}+584\sqrt{1+\sqrt{1+\sqrt{x}}}\sqrt{1\right.\right.}{765}$$

input
```
Integrate[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

output
```
(16*Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]]*(-8*(3519 - 1094*Sqrt[1 + Sqrt[1
 + Sqrt[x]]] + 163*Sqrt[1 + Sqrt[x]] + 584*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*Sqr
t[1 + Sqrt[x]]) + 7*(659 - 504*Sqrt[1 + Sqrt[1 + Sqrt[x]]] + 33*Sqrt[1 + S
qrt[x]] + 429*Sqrt[1 + Sqrt[1 + Sqrt[x]]]*Sqrt[1 + Sqrt[x]])*Sqrt[x] + 450
45*x))/765765
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {7267, 7267, 25, 7267, 2003, 2115, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\,dx$$

$$\Big\downarrow \text{7267}$$

$$2\int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\sqrt{x}d\sqrt{x}$$

$$\Big\downarrow \text{7267}$$

$$4\int -\sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\sqrt{\sqrt{x}+1}(1-x)d\sqrt{\sqrt{x}+1}$$

$$\Big\downarrow \text{25}$$

$$-4\int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\sqrt{\sqrt{x}+1}(1-x)d\sqrt{\sqrt{x}+1}$$

$$\Big\downarrow \text{7267}$$

$$8\int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}(1-x)(2-x)x^{3/2}d\sqrt{\sqrt{\sqrt{x}+1}+1}$$

$$\Big\downarrow \text{2003}$$

$$8\int \left(1-\sqrt{\sqrt{\sqrt{x}+1}+1}\right)\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{3/2}(2-x)x^{3/2}d\sqrt{\sqrt{\sqrt{x}+1}+1}$$

$$\Big\downarrow \text{2115}$$

$$8\int \left(\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{15/2}-7\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{13/2}+18\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{11/2}-20\left(\sqrt{\sqrt{\sqrt{}}}\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$8\left(\frac{2}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{17/2}-\frac{14}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{15/2}+\frac{36}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{13/2}-\frac{40}{11}\left(\sqrt{\sqrt{}}\right.\right.$$

input

```
Int[Sqrt[1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]]],x]
```

output

```
8*((-4*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(5/2))/5 + (6*(1 + Sqrt[1 + Sqrt[
1 + Sqrt[x]]])^(7/2))/7 + (14*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(9/2))/9 -
 (40*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(11/2))/11 + (36*(1 + Sqrt[1 + Sqrt
[1 + Sqrt[x]]])^(13/2))/13 - (14*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(15/2))
/15 + (2*(1 + Sqrt[1 + Sqrt[1 + Sqrt[x]]])^(17/2))/17)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2003
```
Int[(u_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :
> Int[u*(c + d*x)^(n + p)*(a/c + (b/d)*x)^p, x] /; FreeQ[{a, b, c, d, n, p}
, x] && EqQ[b*c^2 + a*d^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0] &&
  !IntegerQ[n]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2115
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f
_.)*(x_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^
n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && PolyQ[Px,
 x] && IntegersQ[m, n]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.64

method	result
derivativedivides	$-\frac{32\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{5}{2}}}{5}+\frac{48\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{7}{2}}}{7}+\frac{112\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{9}{2}}}{9}-\frac{320\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{1}}{11}$
default	$-\frac{32\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{5}{2}}}{5}+\frac{48\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{7}{2}}}{7}+\frac{112\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{\frac{9}{2}}}{9}-\frac{320\left(1+\sqrt{1+\sqrt{1+\sqrt{x}}}\right)^{1}}{11}$

input
```
int((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-32/5*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(5/2)+48/7*(1+(1+(1+x^(1/2))^(1/2))^
(1/2))^(7/2)+112/9*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(9/2)-320/11*(1+(1+(1+x
^(1/2))^(1/2))^(1/2))^(11/2)+288/13*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(13/2)
-112/15*(1+(1+(1+x^(1/2))^(1/2))^(1/2))^(15/2)+16/17*(1+(1+(1+x^(1/2))^(1/
2))^(1/2))^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.40

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$$
$$= \frac{16}{765765}\left(\left(231\sqrt{x}-1304\right)\sqrt{\sqrt{x}+1}+\left(\left(3003\sqrt{x}-4672\right)\sqrt{\sqrt{x}+1}-3528\sqrt{x}+8752\right)\sqrt{\sqrt{\sqrt{x}+1}+\right.$$

input
```
integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
16/765765*((231*sqrt(x) - 1304)*sqrt(sqrt(x) + 1) + ((3003*sqrt(x) - 4672)
*sqrt(sqrt(x) + 1) - 3528*sqrt(x) + 8752)*sqrt(sqrt(sqrt(x) + 1) + 1) + 45
045*x + 4613*sqrt(x) - 28152)*sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.87

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = \frac{16\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{17}{2}}}{17} - \frac{112\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{15}{2}}}{15} + \frac{288\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{13}{2}}}{13} - \frac{320\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{11}{2}}}{11} + \frac{112\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{9}{2}}}{9} + \frac{48\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{7}{2}}}{7} - \frac{32\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{5}{2}}}{5}$$

input
```
integrate((1+(1+(1+x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

output
```
16*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(17/2)/17 - 112*(sqrt(sqrt(sqrt(x) +
 1) + 1) + 1)**(15/2)/15 + 288*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(13/2)/1
3 - 320*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(11/2)/11 + 112*(sqrt(sqrt(sqrt
(x) + 1) + 1) + 1)**(9/2)/9 + 48*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(7/2)/
7 - 32*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.63

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = \frac{16}{17}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{17}{2}} - \frac{112}{15}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{15}{2}} + \frac{288}{13}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{13}{2}} - \frac{320}{11}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{11}{2}} + \frac{112}{9}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{9}{2}} + \frac{48}{7}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{7}{2}} - \frac{32}{5}\left(\sqrt{\sqrt{\sqrt{x}+1}+1}+1\right)^{\frac{5}{2}}$$

input
```
integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
16/17*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(17/2) - 112/15*(sqrt(sqrt(sqrt(x)
 + 1) + 1) + 1)^(15/2) + 288/13*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(13/2) -
 320/11*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(11/2) + 112/9*(sqrt(sqrt(sqrt(x
) + 1) + 1) + 1)^(9/2) + 48/7*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(7/2) - 32
/5*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7916 vs. $2(120) = 240$.

Time = 39.83 (sec) , antiderivative size = 7916, normalized size of antiderivative = 41.66

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = \text{Too large to display}$$

input

```
integrate((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

output

```
16/765765*(7*(6435*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(17/2) - 58344*(sqrt(
sqrt(sqrt(x) + 1) + 1) + 1)^(15/2) + 235620*(sqrt(sqrt(sqrt(x) + 1) + 1) +
 1)^(13/2) - 556920*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(11/2) + 850850*(sqr
t(sqrt(sqrt(x) + 1) + 1) + 1)^(9/2) - 875160*(sqrt(sqrt(sqrt(x) + 1) + 1)
+ 1)^(7/2) + 612612*(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)^(5/2) - 291720*(sqrt
(sqrt(sqrt(x) + 1) + 1) + 1)^(3/2) + 109395*sqrt(sqrt(sqrt(sqrt(x) + 1) +
1) + 1))*sgn(70368744177664*(sqrt(sqrt(x) + 1) + 1)^92 - 6473924464345088*
(sqrt(sqrt(x) + 1) + 1)^91 + 291326600895528960*(sqrt(sqrt(x) + 1) + 1)^90
 - 8545580292935516160*(sqrt(sqrt(x) + 1) + 1)^89 + 183728762437276532736*
(sqrt(sqrt(x) + 1) + 1)^88 - 3086556782054646743040*(sqrt(sqrt(x) + 1) + 1
)^87 + 42179809308639429132288*(sqrt(sqrt(x) + 1) + 1)^86 - 48197884682284
1400164352*(sqrt(sqrt(x) + 1) + 1)^85 + 4697911198078384159588352*(sqrt(sq
rt(x) + 1) + 1)^84 - 39651330432185076620984320*(sqrt(sqrt(x) + 1) + 1)^83
 + 293183639716003233721745408*(sqrt(sqrt(x) + 1) + 1)^82 - 19166563364402
69370174734336*(sqrt(sqrt(x) + 1) + 1)^81 + 11160164453620451334571425792*
(sqrt(sqrt(x) + 1) + 1)^80 - 58223902019906429347317153792*(sqrt(sqrt(x) +
 1) + 1)^79 + 273479024956137655533112918016*(sqrt(sqrt(x) + 1) + 1)^78 -
1160956607882993155309408616448*(sqrt(sqrt(x) + 1) + 1)^77 + 4467886822469
532994953426239488*(sqrt(sqrt(x) + 1) + 1)^76 - 15624039803063454614788052
615168*(sqrt(sqrt(x) + 1) + 1)^75 + 49728771914087708805425247813632*(s...
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx = \int \sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\,dx$$

input
```
int(((((x^(1/2) + 1)^(1/2) + 1)^(1/2) + 1)^(1/2),x)
```

output
```
int(((((x^(1/2) + 1)^(1/2) + 1)^(1/2) + 1)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.48

$$\int \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{x}}}}\,dx$$
$$= \frac{16\sqrt{\sqrt{\sqrt{\sqrt{x}+1}+1}+1}\left(3003\sqrt{x}\,\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{\sqrt{x}+1}+1}-4672\sqrt{\sqrt{x}+1}\,\sqrt{\sqrt{\sqrt{x}+1}+1}-3528\right.}{}$$

input
```
int((1+(1+(1+x^(1/2))^(1/2))^(1/2))^(1/2),x)
```

output
```
(16*sqrt(sqrt(sqrt(sqrt(x) + 1) + 1) + 1)*(3003*sqrt(x)*sqrt(sqrt(x) + 1)*
sqrt(sqrt(sqrt(x) + 1) + 1) - 4672*sqrt(sqrt(x) + 1)*sqrt(sqrt(sqrt(x) + 1
) + 1) - 3528*sqrt(x)*sqrt(sqrt(sqrt(x) + 1) + 1) + 8752*sqrt(sqrt(sqrt(x)
 + 1) + 1) + 231*sqrt(x)*sqrt(sqrt(x) + 1) - 1304*sqrt(sqrt(x) + 1) + 4613
*sqrt(x) + 45045*x - 28152))/765765
```

3.104 $\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 233

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = -\frac{16}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{3/2} + \frac{136}{5}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{5/2} - \frac{480}{7}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{7/2} + \frac{304}{3}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{9/2} - \frac{760}{11}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{11/2} + \frac{300}{13}\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)$$

output

```
-16/3*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(3/2)+136/5*(2+(3+(-1+2*x^(1/2))^
(1/2))^(1/2))^(5/2)-480/7*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(7/2)+304/3*(
2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(9/2)-760/11*(2+(3+(-1+2*x^(1/2))^(1/2))
^(1/2))^(11/2)+300/13*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(13/2)-56/15*(2+(
3+(-1+2*x^(1/2))^(1/2))^(1/2))^(15/2)+4/17*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/
2))^(17/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.80

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx$$
$$= \frac{8\sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\left(8\left(-15510-7428\sqrt{3+\sqrt{-1+2\sqrt{x}}}+211\sqrt{-1+2\sqrt{x}}+1700\sqrt{3+\sqrt{-}\right.\right.}{}$$

input
```
Integrate[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

output
```
(8*Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]]*(8*(-15510 - 7428*Sqrt[3 + Sqr
t[-1 + 2*Sqrt[x]]] + 211*Sqrt[-1 + 2*Sqrt[x]] + 1700*Sqrt[3 + Sqrt[-1 + 2*
Sqrt[x]]]*Sqrt[-1 + 2*Sqrt[x]]) + 7*(-549 - 672*Sqrt[3 + Sqrt[-1 + 2*Sqrt[
x]]] - 121*Sqrt[-1 + 2*Sqrt[x]] + 286*Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]*Sqrt[
-1 + 2*Sqrt[x]])*Sqrt[x] + 30030*x))/255255
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {7267, 7267, 7267, 25, 2091, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\,dx$$
$$\Big\downarrow \text{7267}$$
$$2\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\sqrt{x}d\sqrt{x}$$
$$\Big\downarrow \text{7267}$$

$$\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\sqrt{2\sqrt{x}-1}(x+1)d\sqrt{2\sqrt{x}-1}$$

$\Big\downarrow$ 7267

$$2\int -\sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\sqrt{\sqrt{2\sqrt{x}-1}+3}\left((x-3)^2+1\right)(3-x)d\sqrt{\sqrt{2\sqrt{x}-1}+3}$$

$\Big\downarrow$ 25

$$-2\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\sqrt{\sqrt{2\sqrt{x}-1}+3}\left((x-3)^2+1\right)(3-x)d\sqrt{\sqrt{2\sqrt{x}-1}+3}$$

$\Big\downarrow$ 2091

$$-2\int \sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\sqrt{\sqrt{2\sqrt{x}-1}+3}(3-x)\left(x^2-6x+10\right)d\sqrt{\sqrt{2\sqrt{x}-1}+3}$$

$\Big\downarrow$ 2123

$$-2\int \left(-\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{15/2}+14\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{13/2}-75\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{11/2}+19$$

$\Big\downarrow$ 2009

$$2\left(\frac{2}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{17/2}-\frac{28}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{15/2}+\frac{150}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{13/2}-\frac{380}{11}$$

input
```
Int[Sqrt[2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]]],x]
```

output
```
2*((-8*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(3/2))/3 + (68*(2 + Sqrt[3 + S
qrt[-1 + 2*Sqrt[x]]])^(5/2))/5 - (240*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])
^(7/2))/7 + (152*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(9/2))/3 - (380*(2 +
 Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(11/2))/11 + (150*(2 + Sqrt[3 + Sqrt[-1 +
 2*Sqrt[x]]])^(13/2))/13 - (28*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(15/2)
)/15 + (2*(2 + Sqrt[3 + Sqrt[-1 + 2*Sqrt[x]]])^(17/2))/17)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2091
```
Int[(Px_)*(u_)^(p_.)*(z_)^(q_.), x_Symbol] :> Int[Px*ExpandToSum[z, x]^q*Ex
pandToSum[u, x]^p, x] /; FreeQ[{p, q}, x] && PolyQ[Px, x] && BinomialQ[z, x
] && TrinomialQ[u, x] &&  !(BinomialMatchQ[z, x] && TrinomialMatchQ[u, x])
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.66

method	result
derivativedivides	$-\frac{16\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{3}{2}}}{3}+\frac{136\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{5}{2}}}{5}-\frac{480\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{7}{2}}}{7}+\frac{304\left(2+\sqrt{3+\right.}}{3}$
default	$-\frac{16\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{3}{2}}}{3}+\frac{136\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{5}{2}}}{5}-\frac{480\left(2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}\right)^{\frac{7}{2}}}{7}+\frac{304\left(2+\sqrt{3+\right.}}{3}$

input
```
int((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-16/3*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(3/2)+136/5*(2+(3+(-1+2*x^(1/2))^
(1/2))^(1/2))^(5/2)-480/7*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(7/2)+304/3*(
2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(9/2)-760/11*(2+(3+(-1+2*x^(1/2))^(1/2))
^(1/2))^(11/2)+300/13*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(13/2)-56/15*(2+(
3+(-1+2*x^(1/2))^(1/2))^(1/2))^(15/2)+4/17*(2+(3+(-1+2*x^(1/2))^(1/2))^(1/
2))^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.36

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx =$$

$$-\frac{8}{255255}\left(\left(847\sqrt{x}-1688\right)\sqrt{2\sqrt{x}-1}-2\left(\left(1001\sqrt{x}+6800\right)\sqrt{2\sqrt{x}-1}-2352\sqrt{x}-29712\right)\right.$$

input
```
integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="fricas")
```

output
```
-8/255255*((847*sqrt(x) - 1688)*sqrt(2*sqrt(x) - 1) - 2*((1001*sqrt(x) + 6
800)*sqrt(2*sqrt(x) - 1) - 2352*sqrt(x) - 29712)*sqrt(sqrt(2*sqrt(x) - 1)
+ 3) - 30030*x + 3843*sqrt(x) + 124080)*sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3)
 + 2)
```

Sympy [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.87

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = \frac{4\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}}}{17} - \frac{56\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}}}{15} + \frac{300\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{13}{2}}}{13} - \frac{760\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{11}{2}}}{11} + \frac{304\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{9}{2}}}{3} - \frac{480\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{7}{2}}}{7} + \frac{136\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{5}{2}}}{5} - \frac{16\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{3}{2}}}{3}$$

input
```
integrate((2+(3+(-1+2*x**(1/2))**(1/2))**(1/2))**(1/2),x)
```

output
```
4*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)**(17/2)/17 - 56*(sqrt(sqrt(2*sqrt(x)
 - 1) + 3) + 2)**(15/2)/15 + 300*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)**(13/
2)/13 - 760*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)**(11/2)/11 + 304*(sqrt(sqr
t(2*sqrt(x) - 1) + 3) + 2)**(9/2)/3 - 480*(sqrt(sqrt(2*sqrt(x) - 1) + 3) +
 2)**(7/2)/7 + 136*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)**(5/2)/5 - 16*(sqrt
(sqrt(2*sqrt(x) - 1) + 3) + 2)**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.66

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = \frac{4}{17}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}} - \frac{56}{15}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}} + \frac{300}{13}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{13}{2}} - \frac{760}{11}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{11}{2}} + \frac{304}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{9}{2}} - \frac{480}{7}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{7}{2}} + \frac{136}{5}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{5}{2}} - \frac{16}{3}\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{3}{2}}$$

input
```
integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="maxima")
```

output
```
4/17*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(17/2) - 56/15*(sqrt(sqrt(2*sqrt(
x) - 1) + 3) + 2)^(15/2) + 300/13*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(13/
2) - 760/11*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(11/2) + 304/3*(sqrt(sqrt(
2*sqrt(x) - 1) + 3) + 2)^(9/2) - 480/7*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)
^(7/2) + 136/5*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(5/2) - 16/3*(sqrt(sqrt
(2*sqrt(x) - 1) + 3) + 2)^(3/2)
```

Giac [A] (verification not implemented)

Time = 3.52 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.16

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx$$

$$= \frac{4}{255255}\left(15015\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{17}{2}} - 238238\left(\sqrt{\sqrt{2\sqrt{x}-1}+3}+2\right)^{\frac{15}{2}} + 1472625\left(\sqrt{\sqrt{2\sqrt{}}}\right.\right.$$

$$+ 376832\,x^{22} + 8224768\,x^{21} + 113971200\,x^{20} + 1130782720\,x^{19} + 8582063104\,x^{18} + 51933387264\,x^{17} + 257575619584\,x^{16} + 1066188686592\,x^{15} + 3723204389632\,x^{14} + 11019822890016\,x^{13} + 27631512444352\,x^{12} + 58424530490176\,x^{11} + 103336828749760\,x^{10} + 151203890043312\,x^{9} + 180411181747936\,x^{8} + 172287199292960\,x^{7} + 128457231939048\,x^{6} + 72257964298210\,x^{5} + 29175203228012\,x^{4} + 7830371130072\,x^{3} + 1228114804752\,x^{2} + 87490886400\,x + 933120000\Big)$$

input
```
integrate((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x, algorithm="giac")
```

output
```
4/255255*(15015*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(17/2) - 238238*(sqrt(
sqrt(2*sqrt(x) - 1) + 3) + 2)^(15/2) + 1472625*(sqrt(sqrt(2*sqrt(x) - 1) +
 3) + 2)^(13/2) - 4408950*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(11/2) + 646
6460*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(9/2) - 4375800*(sqrt(sqrt(2*sqrt
(x) - 1) + 3) + 2)^(7/2) + 1735734*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(5/
2) - 340340*(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)^(3/2))*sgn(8192*x^23 + 376
832*x^22 + 8224768*x^21 + 113971200*x^20 + 1130782720*x^19 + 8582063104*x^
18 + 51933387264*x^17 + 257575619584*x^16 + 1066188686592*x^15 + 372320438
9632*x^14 + 11019822890016*x^13 + 27631512444352*x^12 + 58424530490176*x^1
1 + 103336828749760*x^10 + 151203890043312*x^9 + 180411181747936*x^8 + 172
287199292960*x^7 + 128457231939048*x^6 + 72257964298210*x^5 + 291752032280
12*x^4 + 7830371130072*x^3 + 1228114804752*x^2 + 87490886400*x + 933120000
)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx=\int \sqrt{\sqrt{\sqrt{2\,\sqrt{x}-1}+3}+2}\,dx$$

input
```
int((((2*x^(1/2) - 1)^(1/2) + 3)^(1/2) + 2)^(1/2),x)
```

output
```
int((((2*x^(1/2) - 1)^(1/2) + 3)^(1/2) + 2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.47

$$\int \sqrt{2+\sqrt{3+\sqrt{-1+2\sqrt{x}}}}\,dx = \frac{8\sqrt{\sqrt{\sqrt{2\sqrt{x}-1}+3}+2}\left(2002\sqrt{x}\,\sqrt{2\sqrt{x}-1}\,\sqrt{\sqrt{2\sqrt{x}-1}+3}+13600\sqrt{2\sqrt{x}-1}\,\sqrt{\sqrt{2\sqrt{x}-1}+3}\right.}{}$$

input
```
int((2+(3+(-1+2*x^(1/2))^(1/2))^(1/2))^(1/2),x)
```

output
```
(8*sqrt(sqrt(sqrt(2*sqrt(x) - 1) + 3) + 2)*(2002*sqrt(x)*sqrt(2*sqrt(x) -
1)*sqrt(sqrt(2*sqrt(x) - 1) + 3) + 13600*sqrt(2*sqrt(x) - 1)*sqrt(sqrt(2*s
qrt(x) - 1) + 3) - 4704*sqrt(x)*sqrt(sqrt(2*sqrt(x) - 1) + 3) - 59424*sqrt
(sqrt(2*sqrt(x) - 1) + 3) - 847*sqrt(x)*sqrt(2*sqrt(x) - 1) + 1688*sqrt(2*
sqrt(x) - 1) - 3843*sqrt(x) + 30030*x - 124080))/255255
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

def is_atom(expn):
  try:
    if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
      return True
    else:
      return False

  except AttributeError as error:
    return False

def expnType(expn):
  debug=False
  if debug:
    print("expn=",expn,"type(expn)=",type(expn))

  if is_atom(expn):
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])   #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file