Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.6-Improper-linear-binomial/79-1.1.6.1

Nasser M. Abbasi

May 19, 2024 Compiled on May 19, 2024 at 8:22am

Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**144**]. This is test number [79].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (144)	0.00 (0)
Mathematica	100.00 (144)	0.00 (0)
Mupad	99.31 (143)	0.69 (1)
Maple	65.97 (95)	34.03 (49)
Fricas	54.86 (79)	45.14 (65)
Giac	54.86 (79)	45.14 (65)
Reduce	52.08 (75)	47.92 (69)
Maxima	47.22 (68)	52.78 (76)
Sympy	35.42 (51)	64.58 (93)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

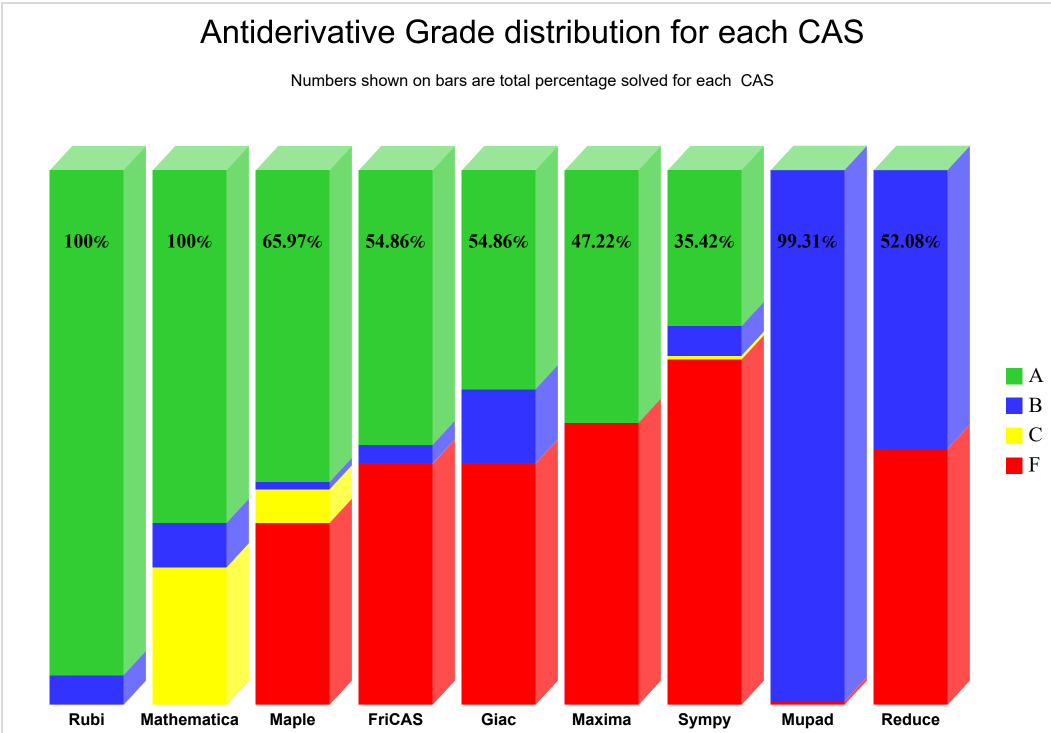
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

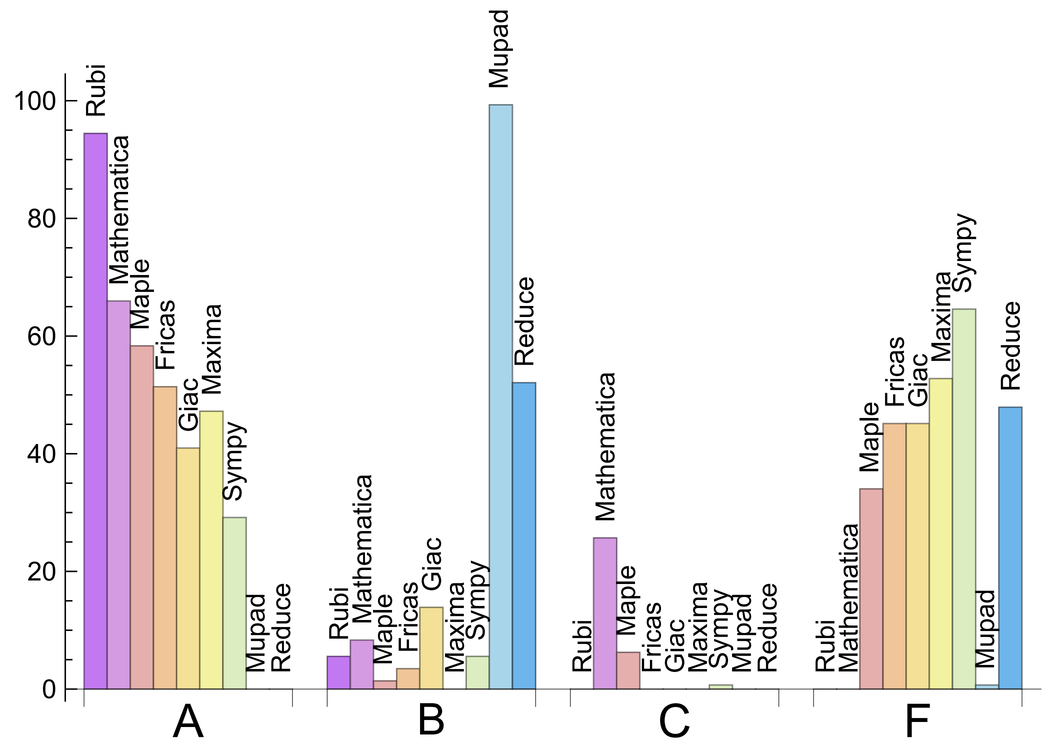
System	% A grade	% B grade	% C grade	% F grade
Rubi	94.444	5.556	0.000	0.000
Mathematica	65.972	8.333	25.694	0.000
Maple	58.333	1.389	6.250	34.028
Fricas	51.389	3.472	0.000	45.139
Maxima	47.222	0.000	0.000	52.778
Giac	40.972	13.889	0.000	45.139
Sympy	29.167	5.556	0.694	64.583
Mupad	0.000	99.306	0.000	0.694
Reduce	0.000	52.083	0.000	47.917

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Mupad	1	0.00	100.00	0.00
Maple	49	100.00	0.00	0.00
Fricas	65	90.77	0.00	9.23
Giac	65	100.00	0.00	0.00
Reduce	69	100.00	0.00	0.00
Maxima	76	100.00	0.00	0.00
Sympy	93	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.05
Fricas	0.08
Reduce	0.21
Sympy	0.30
Giac	0.36
Maple	0.37
Rubi	0.38
Mathematica	3.13
Mupad	7.97

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	42.23	0.69	31.00	0.61
Mupad	43.61	0.74	38.00	0.68
Maxima	55.00	0.84	43.00	0.80
Mathematica	56.11	1.02	47.50	0.88
Reduce	65.89	0.96	35.00	0.82
Giac	81.29	1.20	52.00	0.82
Fricas	84.87	1.08	46.00	0.89
Sympy	84.96	1.46	37.00	0.91
Rubi	101.59	1.61	63.00	1.00

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

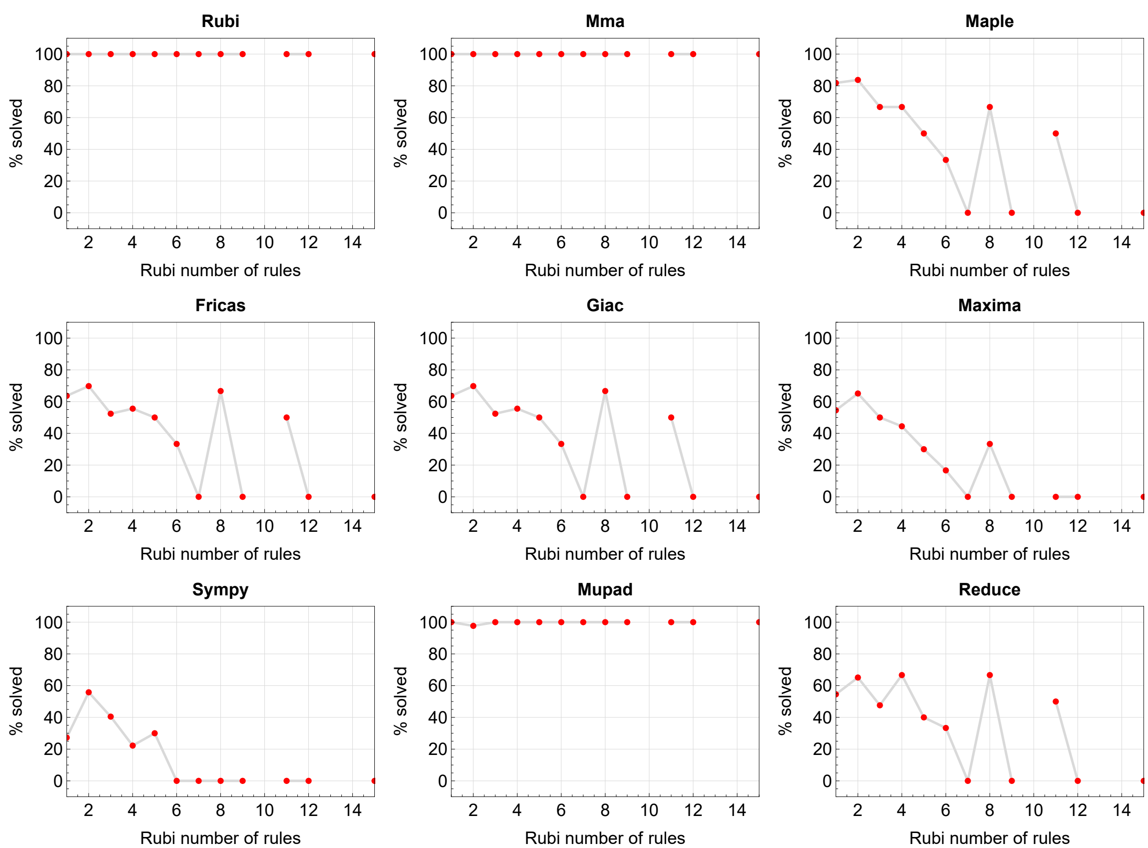


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

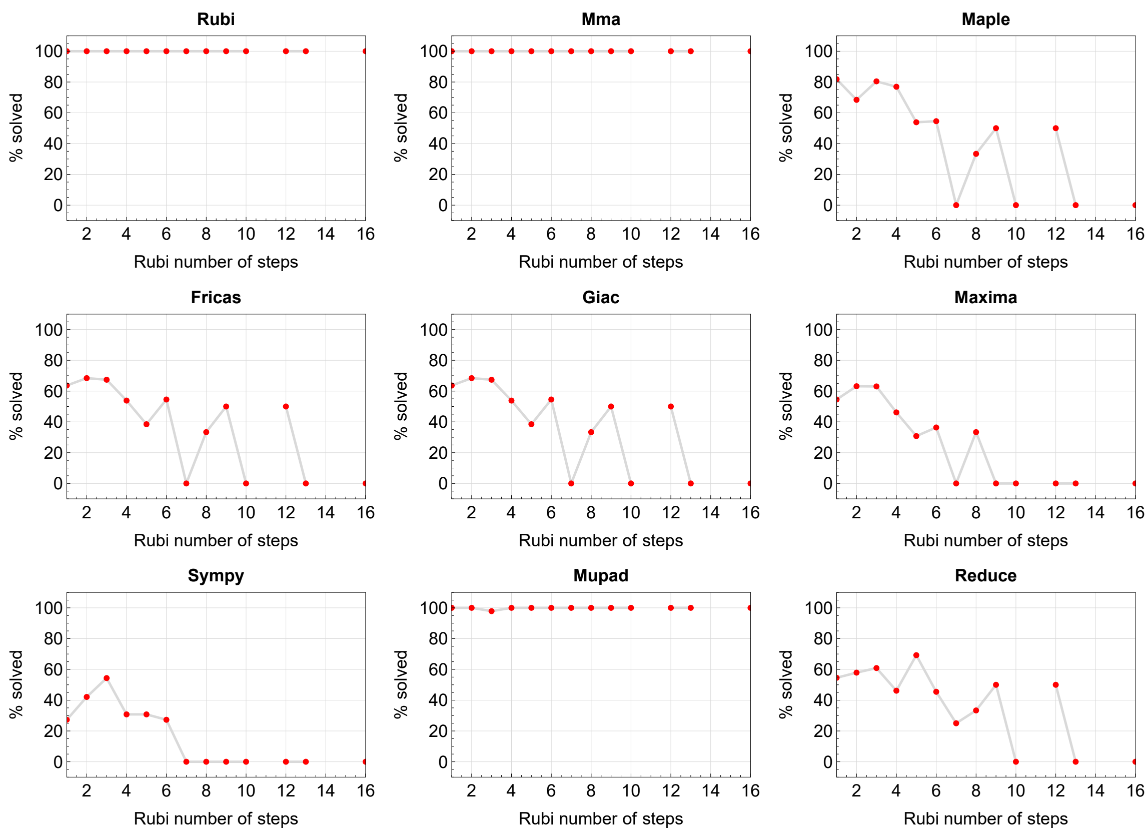


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

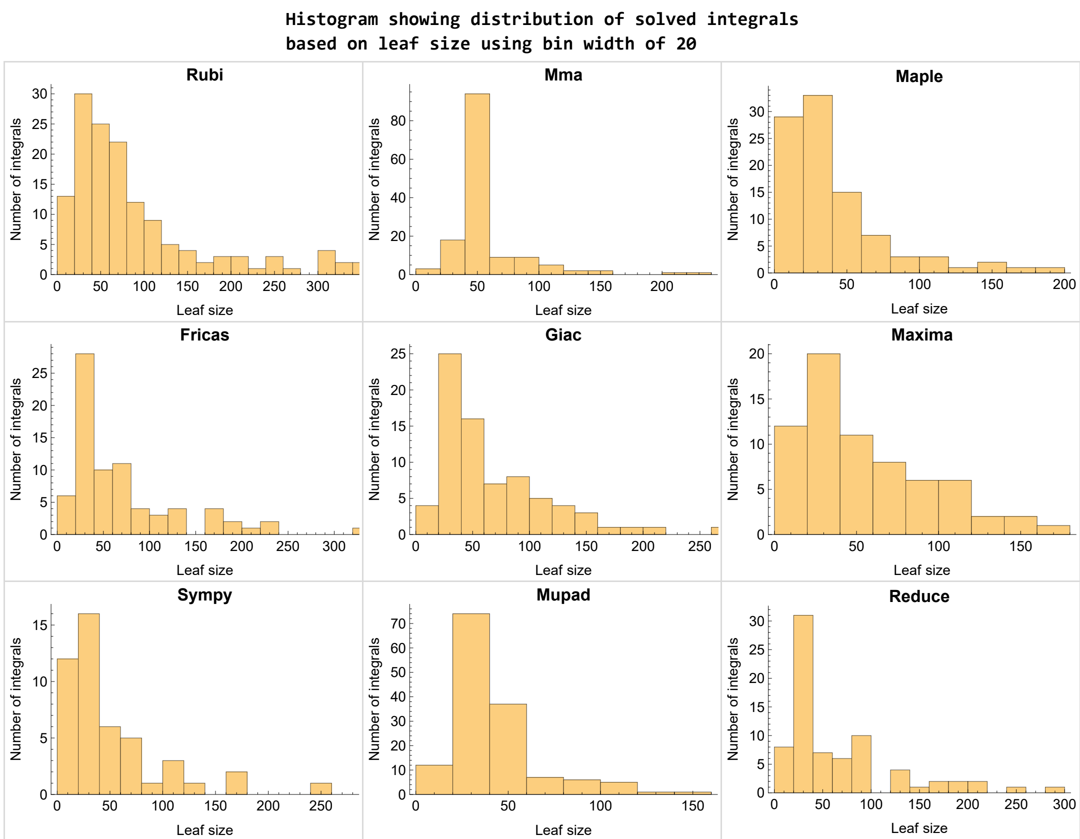


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

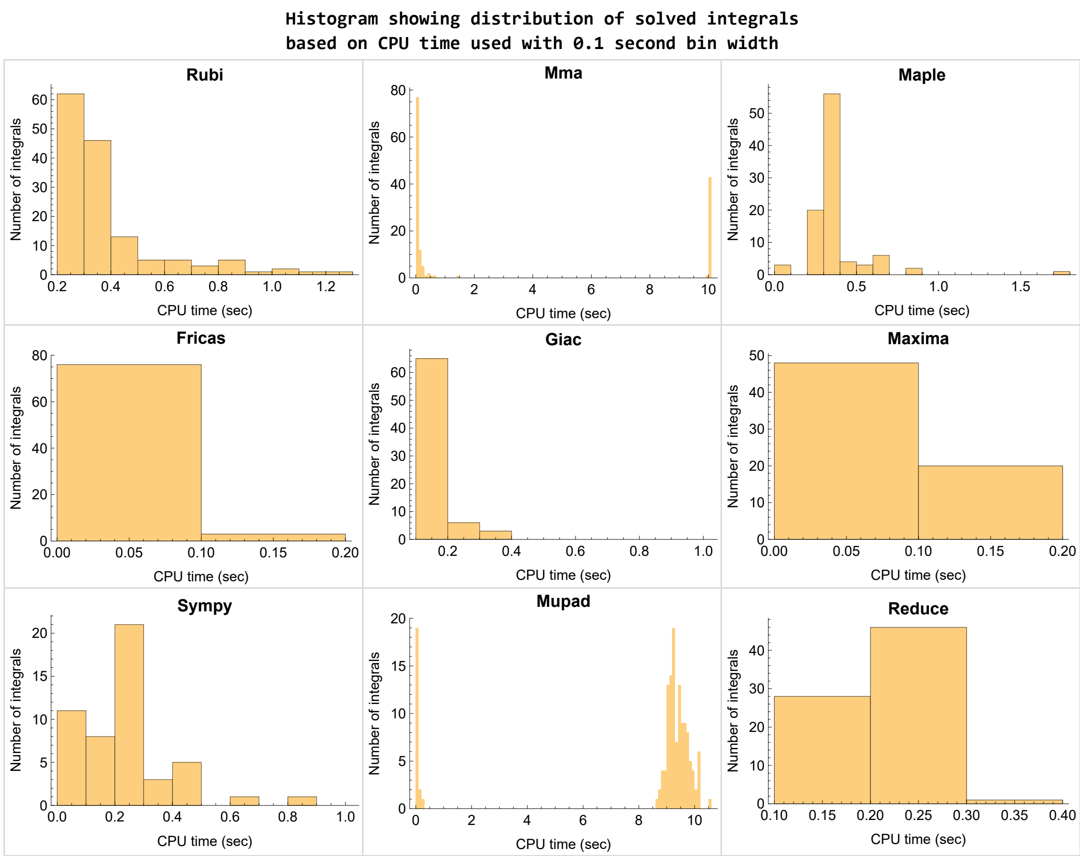


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

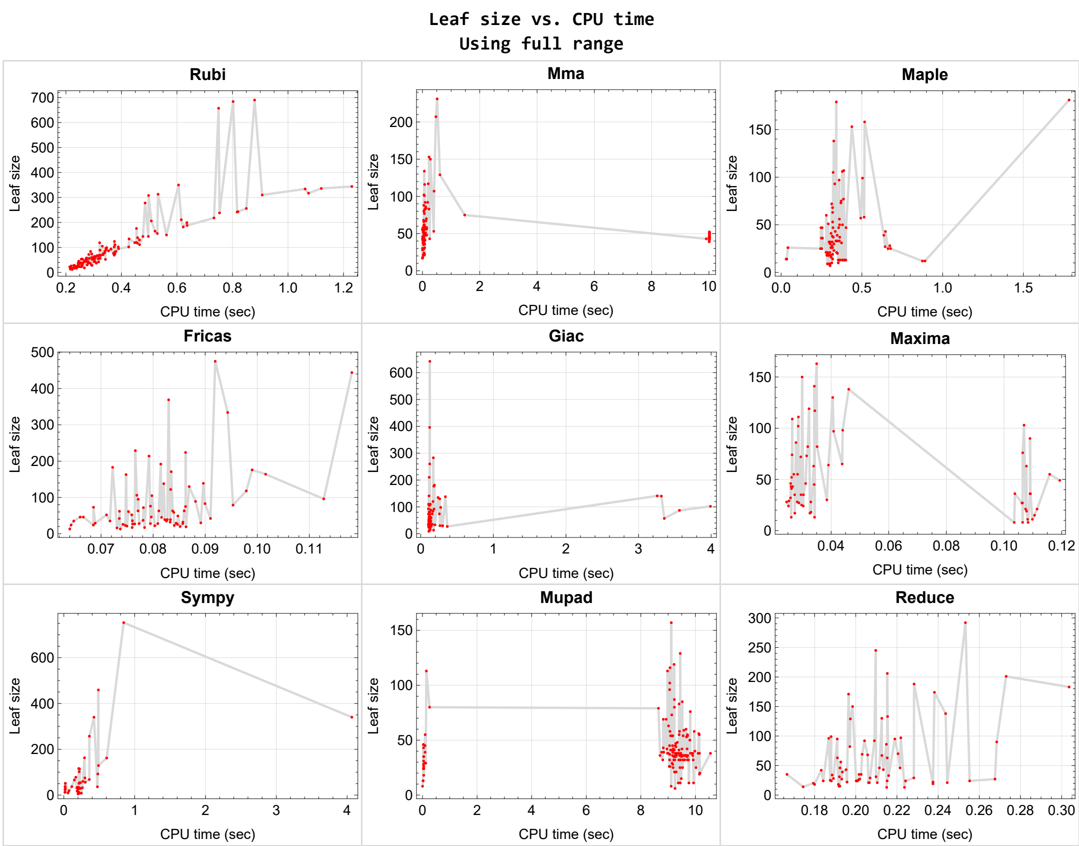


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{51, 52, 53, 54, 55, 56, 57\}$

Mathematica {}

Maple $\{67, 73, 79\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

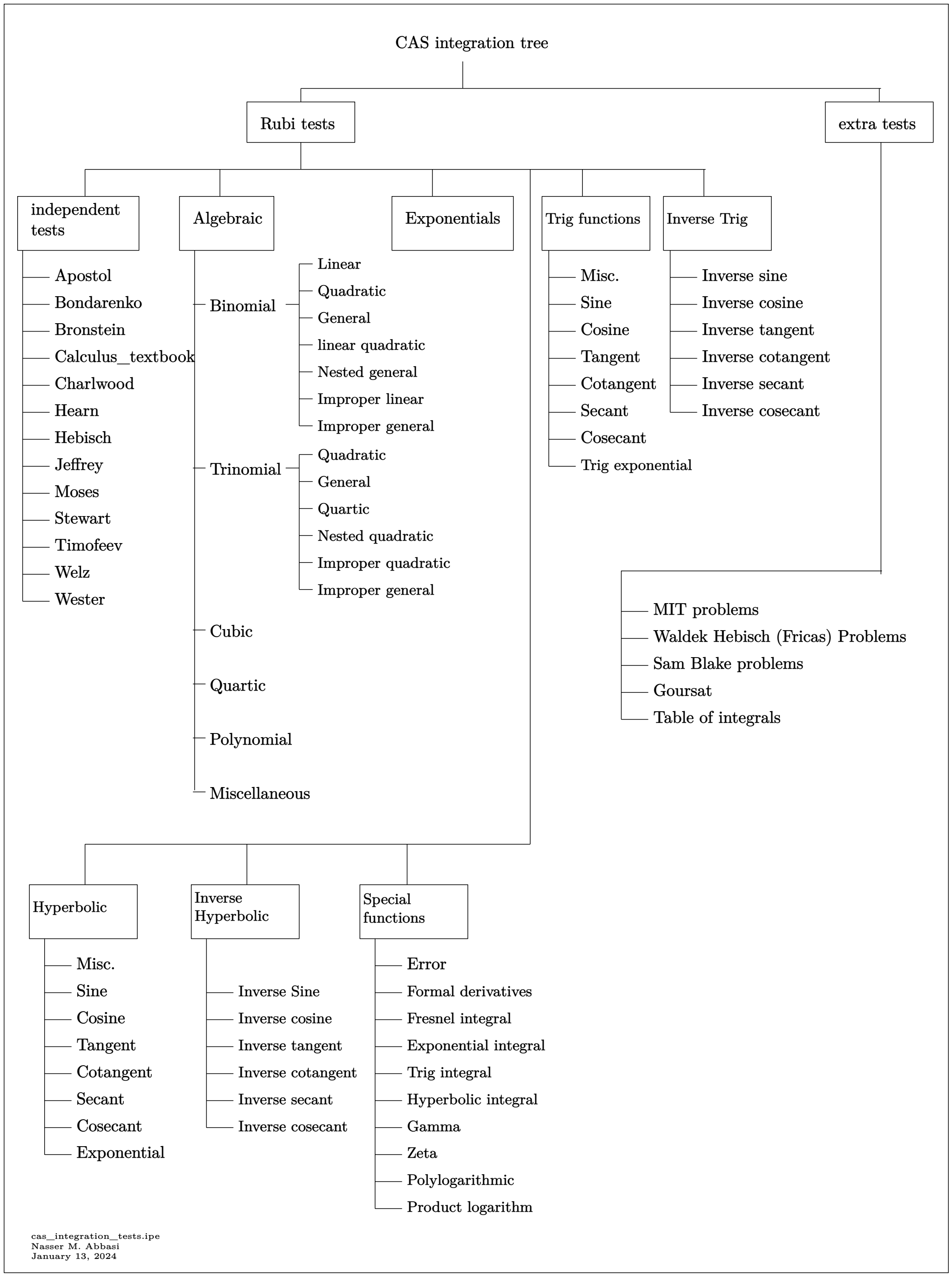
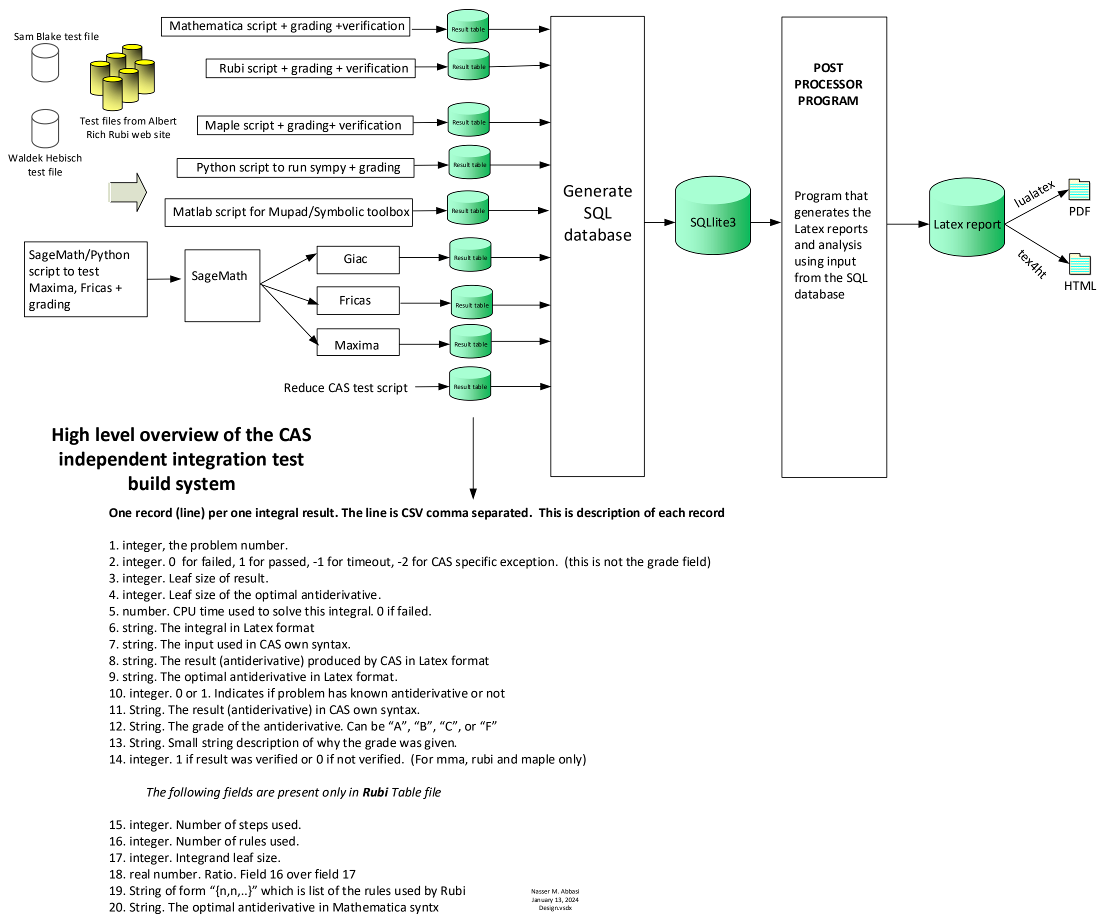


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144 }

B grade { 51, 52, 53, 54, 55, 56, 57, 123 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 33, 34, 35, 40, 41, 42, 43, 44, 48, 51, 52, 53, 54, 55, 56, 57, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144 }

B grade { 20, 28, 32, 36, 37, 38, 39, 45, 46, 47, 49, 50 }

C grade { 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 72, 73, 75, 76, 87, 88, 89, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 127, 128, 139, 140, 141 }

B grade { 36, 126 }

C grade { 66, 67, 69, 70, 78, 79, 81, 82, 90 }

F normal fail { 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 71, 74, 77, 80, 83, 84, 85, 86, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 142, 143, 144 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 127, 128, 139, 140, 141 }

B grade { 20, 36, 49, 50, 126 }

C grade { }

F normal fail { 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 138, 142, 143, 144 }

F(-1) timedout fail { }

F(-2) exception fail { 132, 133, 134, 135, 136, 137 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 126, 127, 128, 139, 140, 141 }

B grade { }

C grade { }

F normal fail { 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 142, 143, 144 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 21, 22, 23, 24, 25, 26, 30, 31, 33, 37, 40, 41, 42, 43, 44, 48, 49, 91, 92, 93, 94, 95, 96, 97, 98, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 128, 139, 140, 141 }

B grade { 12, 20, 27, 28, 29, 32, 34, 35, 36, 38, 39, 45, 46, 47, 50, 99, 100, 101, 126, 127 }

C grade { }

F normal fail { 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 142, 143, 144 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144 }

C grade { }

F normal fail { }

F(-1) timedout fail { 103 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 17, 18, 19, 20, 25, 26, 27, 28, 32, 33, 34, 35, 40, 41, 42, 43, 44, 45, 46, 47, 48, 50, 91, 92, 93, 94, 95, 96, 97, 98, 128 }

B grade { 12, 36, 37, 38, 39, 126, 127, 141 }

C grade { 49 }

F normal fail { 13, 14, 15, 16, 21, 22, 23, 24, 29, 30, 31, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 142, 143, 144 }

F(-1) timedout fail { }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 63, 64, 65, 78, 79, 80, 81, 82, 83, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 126, 127, 128 }

C grade { }

F normal fail { 25, 27, 31, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 84, 85, 86, 87, 88, 89, 90, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	46	46	51	46	46	46
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.91	0.82	0.82	0.82
time (sec)	N/A	0.319	0.003	0.247	0.026	0.066	0.022	0.111	0.211	0.033

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	0.81
time (sec)	N/A	0.302	0.002	0.249	0.030	0.065	0.018	0.104	0.202	0.043

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.276	0.002	0.244	0.030	0.064	0.018	0.112	0.188	0.038

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.76
time (sec)	N/A	0.238	0.000	0.031	0.034	0.074	0.018	0.176	0.224	0.022

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	18	16	10	20	15	15
N.S.	1	1.00	1.00	0.89	1.00	0.89	0.56	1.11	0.83	0.83
time (sec)	N/A	0.267	0.006	0.310	0.033	0.073	0.060	0.129	0.192	9.482

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	45	70	41
N.S.	1	1.00	0.83	1.02	1.07	1.50	0.88	1.07	1.67	0.98
time (sec)	N/A	0.318	0.039	0.335	0.034	0.077	0.112	0.137	0.221	9.228

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	72	86	130	78	73	138	79
N.S.	1	1.00	0.89	0.95	1.13	1.71	1.03	0.96	1.82	1.04
time (sec)	N/A	0.371	0.037	0.313	0.028	0.087	0.169	0.119	0.244	8.646

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	88	93	117	183	114	93	201	113
N.S.	1	1.00	0.86	0.91	1.15	1.79	1.12	0.91	1.97	1.11
time (sec)	N/A	0.426	0.044	0.332	0.034	0.072	0.222	0.105	0.273	0.141

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	134	129	106	141	214	459	105	133	119
N.S.	1	0.77	0.74	0.61	0.81	1.22	2.62	0.60	0.76	0.68
time (sec)	N/A	0.427	0.610	0.379	0.034	0.079	0.488	0.121	0.215	9.219

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	97	107	73	102	171	257	81	95	87
N.S.	1	0.79	0.87	0.59	0.83	1.39	2.09	0.66	0.77	0.71
time (sec)	N/A	0.344	0.406	0.370	0.029	0.083	0.361	0.126	0.219	9.229

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	60	83	56	63	122	102	59	56	55
N.S.	1	0.82	1.14	0.77	0.86	1.67	1.40	0.81	0.77	0.75
time (sec)	N/A	0.293	0.227	0.361	0.034	0.083	0.216	0.114	0.193	0.098

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	55	23	27	63	75	59	25	28
N.S.	1	1.00	1.96	0.82	0.96	2.25	2.68	2.11	0.89	1.00
time (sec)	N/A	0.244	0.046	0.327	0.029	0.081	0.297	0.130	0.188	9.169

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	24	22	21	35	35	0	24	39	24
N.S.	1	0.60	0.55	0.52	0.88	0.88	0.00	0.60	0.98	0.60
time (sec)	N/A	0.235	0.074	0.343	0.027	0.072	0.000	0.121	0.193	0.045

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	54	48	47	72	72	0	50	92	43
N.S.	1	0.61	0.54	0.53	0.81	0.81	0.00	0.56	1.03	0.48
time (sec)	N/A	0.272	0.149	0.356	0.029	0.078	0.000	0.126	0.204	0.038

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	89	70	75	111	105	0	74	150	96
N.S.	1	0.64	0.50	0.54	0.79	0.75	0.00	0.53	1.07	0.69
time (sec)	N/A	0.322	0.136	0.372	0.029	0.080	0.000	0.256	0.198	9.055

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	124	92	97	150	138	0	98	206	116
N.S.	1	0.66	0.49	0.52	0.80	0.74	0.00	0.52	1.10	0.62
time (sec)	N/A	0.375	0.181	0.359	0.030	0.082	0.000	0.274	0.215	9.070

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	91	92	53	90	62	129	47	95	63
N.S.	1	0.72	0.73	0.42	0.71	0.49	1.02	0.37	0.75	0.50
time (sec)	N/A	0.345	0.090	0.319	0.109	0.074	0.489	0.128	0.191	9.015

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	64	82	43	63	52	68	37	69	45
N.S.	1	0.73	0.93	0.49	0.72	0.59	0.77	0.42	0.78	0.51
time (sec)	N/A	0.303	0.073	0.646	0.108	0.071	0.360	0.133	0.203	0.093

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	37	55	28	36	42	29	27	43	26
N.S.	1	0.69	1.02	0.52	0.67	0.78	0.54	0.50	0.80	0.48
time (sec)	N/A	0.260	0.049	0.315	0.104	0.074	0.223	0.129	0.213	0.044

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	14	46	9	8	23	8	27	21	8
N.S.	1	1.08	3.54	0.69	0.62	1.77	0.62	2.08	1.62	0.62
time (sec)	N/A	0.216	0.036	0.297	0.104	0.075	0.215	0.365	0.210	9.113

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	22	21	19	28	29	0	29	33	18
N.S.	1	0.59	0.57	0.51	0.76	0.78	0.00	0.78	0.89	0.49
time (sec)	N/A	0.214	0.063	0.290	0.025	0.069	0.000	0.301	0.216	0.046

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	45	31	31	55	46	0	39	68	28
N.S.	1	0.62	0.42	0.42	0.75	0.63	0.00	0.53	0.93	0.38
time (sec)	N/A	0.246	0.078	0.302	0.028	0.080	0.000	0.113	0.206	0.036

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	72	41	41	82	61	0	49	99	73
N.S.	1	0.66	0.38	0.38	0.75	0.56	0.00	0.45	0.91	0.67
time (sec)	N/A	0.282	0.091	0.309	0.032	0.075	0.000	0.135	0.188	9.133

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	99	51	51	109	76	0	59	130	69
N.S.	1	0.68	0.35	0.35	0.75	0.52	0.00	0.41	0.90	0.48
time (sec)	N/A	0.347	0.107	0.292	0.027	0.079	0.000	0.133	0.213	8.930

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	105	86	58	103	59	162	130	14	80
N.S.	1	0.68	0.55	0.37	0.66	0.38	1.05	0.84	0.09	0.52
time (sec)	N/A	0.331	0.110	0.514	0.107	0.084	0.605	0.260	200.021	0.259

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	74	83	47	76	49	92	120	82	60
N.S.	1	0.69	0.77	0.44	0.70	0.45	0.85	1.11	0.76	0.56
time (sec)	N/A	0.293	0.084	0.402	0.106	0.085	0.480	0.168	0.197	9.657

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	43	62	31	49	39	32	110	13	39
N.S.	1	0.66	0.95	0.48	0.75	0.60	0.49	1.69	0.20	0.60
time (sec)	N/A	0.267	0.060	0.374	0.119	0.083	0.261	0.108	0.158	0.093

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	16	51	10	21	19	8	110	35	19
N.S.	1	0.84	2.68	0.53	1.11	1.00	0.42	5.79	1.84	1.00
time (sec)	N/A	0.216	0.034	0.354	0.111	0.086	0.248	0.132	0.167	10.147

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	26	24	19	28	39	0	64	67	20
N.S.	1	0.59	0.55	0.43	0.64	0.89	0.00	1.45	1.52	0.45
time (sec)	N/A	0.219	0.055	0.319	0.028	0.083	0.000	0.118	0.213	10.164

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	53	36	39	55	63	0	74	183	31
N.S.	1	0.59	0.40	0.43	0.61	0.70	0.00	0.82	2.03	0.34
time (sec)	N/A	0.255	0.079	0.347	0.027	0.086	0.000	0.158	0.304	0.040

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	84	48	50	82	83	0	84	14	40
N.S.	1	0.62	0.35	0.37	0.60	0.61	0.00	0.62	0.10	0.29
time (sec)	N/A	0.289	0.120	0.362	0.035	0.090	0.000	0.167	200.037	9.981

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	17	56	12	11	28	12	30	24	11
N.S.	1	0.81	2.67	0.57	0.52	1.33	0.57	1.43	1.14	0.52
time (sec)	N/A	0.220	0.049	0.875	0.108	0.077	0.209	0.118	0.200	9.762

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	17	56	12	11	28	12	30	24	11
N.S.	1	0.61	2.00	0.43	0.39	1.00	0.43	1.07	0.86	0.39
time (sec)	N/A	0.217	0.045	0.891	0.110	0.081	0.211	0.299	0.184	9.932

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	55	18	27	26	31	52	22	25
N.S.	1	1.00	1.96	0.64	0.96	0.93	1.11	1.86	0.79	0.89
time (sec)	N/A	0.231	0.008	0.296	0.106	0.074	0.212	0.113	0.196	10.021

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	28	55	26	27	27	31	52	22	25
N.S.	1	0.88	1.72	0.81	0.84	0.84	0.97	1.62	0.69	0.78
time (sec)	N/A	0.228	0.006	0.308	0.106	0.076	0.212	0.112	0.238	9.435

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	18	57	25	21	28	54	41	21	42
N.S.	1	1.50	4.75	2.08	1.75	2.33	4.50	3.42	1.75	3.50
time (sec)	N/A	0.228	0.045	0.662	0.107	0.079	0.221	0.120	0.244	8.956

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	20	59	25	19	29	60	40	19	43
N.S.	1	0.77	2.27	0.96	0.73	1.12	2.31	1.54	0.73	1.65
time (sec)	N/A	0.238	0.056	0.678	0.108	0.084	0.268	0.118	0.238	8.823

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	58	37	29	27	51	59	18	36
N.S.	1	1.00	2.42	1.54	1.21	1.12	2.12	2.46	0.75	1.50
time (sec)	N/A	0.238	0.015	0.323	0.025	0.076	0.214	0.140	0.180	8.705

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	56	38	32	28	54	61	18	39
N.S.	1	1.00	2.24	1.52	1.28	1.12	2.16	2.44	0.72	1.56
time (sec)	N/A	0.240	0.015	0.317	0.026	0.086	0.235	0.140	0.192	8.762

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	37	47	27	36	37	26	25	42	26
N.S.	1	0.73	0.92	0.53	0.71	0.73	0.51	0.49	0.82	0.51
time (sec)	N/A	0.260	0.050	0.644	0.109	0.078	0.201	0.110	0.183	9.041

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	37	55	28	36	42	29	27	43	26
N.S.	1	0.69	1.02	0.52	0.67	0.78	0.54	0.50	0.80	0.48
time (sec)	N/A	0.253	0.060	0.673	0.109	0.091	0.237	0.109	0.195	9.070

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	35	46	33	41	32	34	33	31	29
N.S.	1	0.66	0.87	0.62	0.77	0.60	0.64	0.62	0.58	0.55
time (sec)	N/A	0.251	0.046	0.339	0.026	0.083	0.192	0.107	0.192	0.119

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	37	46	33	43	32	34	33	34	29
N.S.	1	0.62	0.77	0.55	0.72	0.53	0.57	0.55	0.57	0.48
time (sec)	N/A	0.257	0.049	0.361	0.027	0.086	0.205	0.116	0.189	0.082

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	39	50	33	43	36	32	37	28	29
N.S.	1	0.70	0.89	0.59	0.77	0.64	0.57	0.66	0.50	0.52
time (sec)	N/A	0.252	0.078	0.349	0.026	0.076	0.182	0.115	0.189	9.146

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	12	40	7	8	20	5	25	20	6
N.S.	1	0.86	2.86	0.50	0.57	1.43	0.36	1.79	1.43	0.43
time (sec)	N/A	0.223	0.044	0.303	0.106	0.085	0.200	0.114	0.179	9.254

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	39	9	17	17	17	33	14	11
N.S.	1	1.00	2.44	0.56	1.06	1.06	1.06	2.06	0.88	0.69
time (sec)	N/A	0.221	0.045	0.287	0.033	0.076	0.187	0.130	0.174	9.371

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	39	14	17	17	17	33	15	11
N.S.	1	1.00	2.44	0.88	1.06	1.06	1.06	2.06	0.94	0.69
time (sec)	N/A	0.219	0.028	0.319	0.027	0.078	0.193	0.195	0.192	9.443

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	56	67	39	55	45	56	35	63	39
N.S.	1	0.67	0.81	0.47	0.66	0.54	0.67	0.42	0.76	0.47
time (sec)	N/A	0.284	0.119	0.636	0.116	0.082	0.302	0.159	0.191	9.169

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	29	10	15	22	36	9	21	21
N.S.	1	1.00	2.23	0.77	1.15	1.69	2.77	0.69	1.62	1.62
time (sec)	N/A	0.216	0.043	0.314	0.111	0.080	0.474	0.112	0.207	9.296

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	14	46	9	8	23	8	27	21	8
N.S.	1	1.17	3.83	0.75	0.67	1.92	0.67	2.25	1.75	0.67
time (sec)	N/A	0.221	0.007	0.303	0.108	0.084	0.219	0.142	0.206	0.003

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	350	48	0	0	0	0	0	115	36
N.S.	1	10.00	1.37	0.00	0.00	0.00	0.00	0.00	3.29	1.03
time (sec)	N/A	0.606	10.015	0.000	0.000	0.000	0.000	0.000	0.338	9.425

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	690	45	0	0	0	0	0	79	36
N.S.	1	19.71	1.29	0.00	0.00	0.00	0.00	0.00	2.26	1.03
time (sec)	N/A	0.880	10.024	0.000	0.000	0.000	0.000	0.000	0.322	9.527

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	313	45	0	0	0	0	0	79	36
N.S.	1	8.94	1.29	0.00	0.00	0.00	0.00	0.00	2.26	1.03
time (sec)	N/A	0.532	10.012	0.000	0.000	0.000	0.000	0.000	0.279	9.674

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	657	45	0	0	0	0	0	13	36
N.S.	1	18.77	1.29	0.00	0.00	0.00	0.00	0.00	0.37	1.03
time (sec)	N/A	0.750	10.011	0.000	0.000	0.000	0.000	0.000	0.212	9.557

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	278	43	0	0	0	0	0	13	36
N.S.	1	8.42	1.30	0.00	0.00	0.00	0.00	0.00	0.39	1.09
time (sec)	N/A	0.485	9.905	0.000	0.000	0.000	0.000	0.000	0.198	9.574

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	33	684	45	0	0	0	0	0	29	36
N.S.	1	20.73	1.36	0.00	0.00	0.00	0.00	0.00	0.88	1.09
time (sec)	N/A	0.802	10.013	0.000	0.000	0.000	0.000	0.000	0.232	9.438

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	A	F	F	F	F	F	F	B
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	35	308	47	0	0	0	0	0	29	36
N.S.	1	8.80	1.34	0.00	0.00	0.00	0.00	0.00	0.83	1.03
time (sec)	N/A	0.498	10.018	0.000	0.000	0.000	0.000	0.000	0.196	9.718

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	160	139	48	0	0	0	0	0	96	36
N.S.	1	0.87	0.30	0.00	0.00	0.00	0.00	0.00	0.60	0.22
time (sec)	N/A	0.458	10.017	0.000	0.000	0.000	0.000	0.000	0.296	9.487

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	107	102	45	0	0	0	0	0	60	36
N.S.	1	0.95	0.42	0.00	0.00	0.00	0.00	0.00	0.56	0.34
time (sec)	N/A	0.389	10.013	0.000	0.000	0.000	0.000	0.000	0.255	9.164

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	102	45	0	0	0	0	0	60	36
N.S.	1	0.90	0.40	0.00	0.00	0.00	0.00	0.00	0.53	0.32
time (sec)	N/A	0.374	10.013	0.000	0.000	0.000	0.000	0.000	0.252	9.648

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	71	45	0	0	0	0	0	13	36
N.S.	1	1.15	0.73	0.00	0.00	0.00	0.00	0.00	0.21	0.58
time (sec)	N/A	0.334	10.014	0.000	0.000	0.000	0.000	0.000	0.206	9.617

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	71	43	0	0	0	0	0	13	36
N.S.	1	1.04	0.63	0.00	0.00	0.00	0.00	0.00	0.19	0.53
time (sec)	N/A	0.333	10.011	0.000	0.000	0.000	0.000	0.000	0.230	9.306

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	95	45	0	0	0	0	0	27	36
N.S.	1	1.13	0.54	0.00	0.00	0.00	0.00	0.00	0.32	0.43
time (sec)	N/A	0.390	10.016	0.000	0.000	0.000	0.000	0.000	0.193	9.276

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	154	130	50	0	0	0	0	0	46	36
N.S.	1	0.84	0.32	0.00	0.00	0.00	0.00	0.00	0.30	0.23
time (sec)	N/A	0.465	10.013	0.000	0.000	0.000	0.000	0.000	0.222	9.638

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	207	165	50	0	0	0	0	0	86	36
N.S.	1	0.80	0.24	0.00	0.00	0.00	0.00	0.00	0.42	0.17
time (sec)	N/A	0.521	10.014	0.000	0.000	0.000	0.000	0.000	0.215	9.900

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	53	41	18	0	0	0	0	13	32
N.S.	1	1.18	0.91	0.40	0.00	0.00	0.00	0.00	0.29	0.71
time (sec)	N/A	0.296	10.014	0.294	0.000	0.000	0.000	0.000	0.232	9.545

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	45	53	44	32	0	0	0	0	13	32
N.S.	1	1.18	0.98	0.71	0.00	0.00	0.00	0.00	0.29	0.71
time (sec)	N/A	0.295	10.022	0.297	0.000	0.000	0.000	0.000	0.202	9.393

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	68	44	0	0	0	0	0	13	35
N.S.	1	1.15	0.75	0.00	0.00	0.00	0.00	0.00	0.22	0.59
time (sec)	N/A	0.325	10.012	0.000	0.000	0.000	0.000	0.000	0.202	9.488

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	23	45	18	0	0	0	0	13	32
N.S.	1	1.21	2.37	0.95	0.00	0.00	0.00	0.00	0.68	1.68
time (sec)	N/A	0.253	10.013	0.283	0.000	0.000	0.000	0.000	0.212	9.453

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	23	42	21	0	0	0	0	13	32
N.S.	1	1.21	2.21	1.11	0.00	0.00	0.00	0.00	0.68	1.68
time (sec)	N/A	0.240	10.010	0.278	0.000	0.000	0.000	0.000	0.198	9.741

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	67	44	0	0	0	0	0	13	35
N.S.	1	1.14	0.75	0.00	0.00	0.00	0.00	0.00	0.22	0.59
time (sec)	N/A	0.317	10.013	0.000	0.000	0.000	0.000	0.000	0.193	9.697

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	53	39	18	0	0	0	0	13	32
N.S.	1	1.18	0.87	0.40	0.00	0.00	0.00	0.00	0.29	0.71
time (sec)	N/A	0.296	10.011	0.306	0.000	0.000	0.000	0.000	0.191	9.918

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	59	53	42	32	0	0	0	0	13	32
N.S.	1	0.90	0.71	0.54	0.00	0.00	0.00	0.00	0.22	0.54
time (sec)	N/A	0.294	10.016	0.316	0.000	0.000	0.000	0.000	0.192	9.705

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	68	42	0	0	0	0	0	13	35
N.S.	1	0.99	0.61	0.00	0.00	0.00	0.00	0.00	0.19	0.51
time (sec)	N/A	0.323	10.011	0.000	0.000	0.000	0.000	0.000	0.205	9.217

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	23	43	18	0	0	0	0	13	32
N.S.	1	1.21	2.26	0.95	0.00	0.00	0.00	0.00	0.68	1.68
time (sec)	N/A	0.257	10.011	0.288	0.000	0.000	0.000	0.000	0.192	9.605

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	23	40	21	0	0	0	0	13	32
N.S.	1	1.21	2.11	1.11	0.00	0.00	0.00	0.00	0.68	1.68
time (sec)	N/A	0.248	10.009	0.285	0.000	0.000	0.000	0.000	0.242	9.744

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	67	42	0	0	0	0	0	13	35
N.S.	1	1.14	0.71	0.00	0.00	0.00	0.00	0.00	0.22	0.59
time (sec)	N/A	0.313	10.012	0.000	0.000	0.000	0.000	0.000	0.195	9.218

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	74	43	18	0	0	0	0	24	32
N.S.	1	0.96	0.56	0.23	0.00	0.00	0.00	0.00	0.31	0.42
time (sec)	N/A	0.334	10.012	0.292	0.000	0.000	0.000	0.000	0.201	9.206

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	74	45	32	0	0	0	0	24	32
N.S.	1	0.99	0.60	0.43	0.00	0.00	0.00	0.00	0.32	0.43
time (sec)	N/A	0.334	10.009	0.303	0.000	0.000	0.000	0.000	0.215	9.246

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	94	44	0	0	0	0	0	27	35
N.S.	1	1.07	0.50	0.00	0.00	0.00	0.00	0.00	0.31	0.40
time (sec)	N/A	0.363	10.013	0.000	0.000	0.000	0.000	0.000	0.202	9.264

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	44	45	18	0	0	0	0	24	32
N.S.	1	0.85	0.87	0.35	0.00	0.00	0.00	0.00	0.46	0.62
time (sec)	N/A	0.280	10.011	0.286	0.000	0.000	0.000	0.000	0.224	8.804

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	F	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	44	44	21	0	0	0	0	24	32
N.S.	1	0.83	0.83	0.40	0.00	0.00	0.00	0.00	0.45	0.60
time (sec)	N/A	0.274	10.009	0.301	0.000	0.000	0.000	0.000	0.255	9.058

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	93	44	0	0	0	0	0	27	35
N.S.	1	0.92	0.44	0.00	0.00	0.00	0.00	0.00	0.27	0.35
time (sec)	N/A	0.362	10.016	0.000	0.000	0.000	0.000	0.000	0.268	9.273

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	55	45	0	0	0	0	0	129	48
N.S.	1	1.41	1.15	0.00	0.00	0.00	0.00	0.00	3.31	1.23
time (sec)	N/A	0.276	0.025	0.000	0.000	0.000	0.000	0.000	0.214	9.338

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	44	41	0	0	0	0	0	133	45
N.S.	1	1.16	1.08	0.00	0.00	0.00	0.00	0.00	3.50	1.18
time (sec)	N/A	0.266	0.023	0.000	0.000	0.000	0.000	0.000	0.194	9.260

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	38	50	40	0	0	0	0	0	123	44
N.S.	1	1.32	1.05	0.00	0.00	0.00	0.00	0.00	3.24	1.16
time (sec)	N/A	0.271	0.022	0.000	0.000	0.000	0.000	0.000	0.198	9.284

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	37	43	30	0	0	0	0	114	41
N.S.	1	1.28	1.48	1.03	0.00	0.00	0.00	0.00	3.93	1.41
time (sec)	N/A	0.247	0.025	0.327	0.000	0.000	0.000	0.000	0.189	9.092

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	44	47	0	0	0	0	112	41
N.S.	1	1.00	1.19	1.27	0.00	0.00	0.00	0.00	3.03	1.11
time (sec)	N/A	0.244	0.035	0.321	0.000	0.000	0.000	0.000	0.197	9.163

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	58	43	30	0	0	0	0	112	41
N.S.	1	1.29	0.96	0.67	0.00	0.00	0.00	0.00	2.49	0.91
time (sec)	N/A	0.274	0.016	0.303	0.000	0.000	0.000	0.000	0.204	9.166

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	F	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	52	63	0	0	0	0	114	41
N.S.	1	1.00	0.98	1.19	0.00	0.00	0.00	0.00	2.15	0.77
time (sec)	N/A	0.260	0.033	0.319	0.000	0.000	0.000	0.000	0.199	9.398

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	56	47	46	46	51	46	46	46
N.S.	1	1.00	1.00	0.84	0.82	0.82	0.91	0.82	0.82	0.82
time (sec)	N/A	0.324	0.003	0.254	0.031	0.067	0.020	0.119	0.192	0.037

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	0.81
time (sec)	N/A	0.305	0.002	0.256	0.030	0.083	0.018	0.118	0.203	0.046

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.292	0.003	0.252	0.025	0.069	0.016	0.123	0.200	0.036

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.76
time (sec)	N/A	0.231	0.000	0.035	0.026	0.064	0.015	0.121	0.215	0.022

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	26	28	26	19	30	26	25
N.S.	1	1.00	1.00	0.93	1.00	0.93	0.68	1.07	0.93	0.89
time (sec)	N/A	0.294	0.004	0.318	0.033	0.085	0.073	0.264	0.201	9.527

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	68	73	95	66	73	97	69
N.S.	1	1.00	0.96	0.99	1.06	1.38	0.96	1.06	1.41	1.00
time (sec)	N/A	0.374	0.046	0.319	0.032	0.077	0.148	0.108	0.222	8.825

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	101	105	119	163	116	108	171	113
N.S.	1	1.00	0.91	0.95	1.07	1.47	1.05	0.97	1.54	1.02
time (sec)	N/A	0.465	0.067	0.322	0.032	0.075	0.210	0.111	0.196	8.978

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	134	138	163	229	163	141	245	157
N.S.	1	1.00	0.89	0.92	1.09	1.53	1.09	0.94	1.63	1.05
time (sec)	N/A	0.562	0.065	0.326	0.035	0.077	0.292	0.113	0.210	9.117

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	256	102	13	130	139	0	642	129	102
N.S.	1	1.16	0.46	0.06	0.59	0.63	0.00	2.92	0.59	0.46
time (sec)	N/A	0.850	0.052	0.385	0.041	0.090	0.000	0.125	0.197	9.059

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	188	80	13	97	106	0	396	96	80
N.S.	1	1.15	0.49	0.08	0.59	0.65	0.00	2.41	0.59	0.49
time (sec)	N/A	0.637	0.046	0.362	0.041	0.077	0.000	0.122	0.187	9.230

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	120	58	13	64	73	0	210	63	58
N.S.	1	1.11	0.54	0.12	0.59	0.68	0.00	1.94	0.58	0.54
time (sec)	N/A	0.454	0.032	0.354	0.039	0.069	0.000	0.116	0.215	9.067

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	41	13	30	39	0	81	29	39
N.S.	1	1.00	0.79	0.25	0.58	0.75	0.00	1.56	0.56	0.75
time (sec)	N/A	0.303	0.020	0.392	0.039	0.082	0.000	0.116	0.207	8.839

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	30	46	13	0	79	0	45	31	0
N.S.	1	0.94	1.44	0.41	0.00	2.47	0.00	1.41	0.97	0.00
time (sec)	N/A	0.259	0.025	0.366	0.000	0.095	0.000	0.121	0.210	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	117	84	13	0	224	0	92	92	42
N.S.	1	1.04	0.75	0.12	0.00	2.00	0.00	0.82	0.82	0.38
time (sec)	N/A	0.458	0.125	0.379	0.000	0.086	0.000	0.182	0.209	9.509

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	218	117	13	0	334	0	135	188	42
N.S.	1	1.12	0.60	0.07	0.00	1.72	0.00	0.70	0.97	0.22
time (sec)	N/A	0.733	0.200	0.395	0.000	0.094	0.000	0.238	0.228	9.191

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	278	317	150	13	0	444	0	181	292	42
N.S.	1	1.14	0.54	0.05	0.00	1.60	0.00	0.65	1.05	0.15
time (sec)	N/A	1.074	0.276	0.401	0.000	0.118	0.000	0.188	0.253	9.695

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	206	231	181	0	475	0	141	78	38
N.S.	1	0.80	0.89	0.70	0.00	1.83	0.00	0.54	0.30	0.15
time (sec)	N/A	0.508	0.513	1.783	0.000	0.092	0.000	3.257	0.315	9.467

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	176	207	153	0	192	0	140	60	38
N.S.	1	0.75	0.88	0.65	0.00	0.82	0.00	0.60	0.26	0.16
time (sec)	N/A	0.454	0.469	0.438	0.000	0.081	0.000	3.315	0.363	9.004

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	119	153	107	0	369	0	87	13	38
N.S.	1	0.67	0.86	0.60	0.00	2.08	0.00	0.49	0.07	0.21
time (sec)	N/A	0.322	0.229	0.388	0.000	0.083	0.000	3.564	0.248	9.288

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	21	20	0	21	0	14	13	21
N.S.	1	1.00	0.91	0.87	0.00	0.91	0.00	0.61	0.57	0.91
time (sec)	N/A	0.244	0.028	0.349	0.000	0.075	0.000	0.128	0.259	9.230

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	81	43	37	0	52	0	57	29	47
N.S.	1	1.08	0.57	0.49	0.00	0.69	0.00	0.76	0.39	0.63
time (sec)	N/A	0.376	0.251	0.363	0.000	0.076	0.000	3.355	0.286	9.281

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	119	53	50	0	63	0	70	29	58
N.S.	1	1.13	0.50	0.48	0.00	0.60	0.00	0.67	0.28	0.55
time (sec)	N/A	0.448	0.393	0.380	0.000	0.084	0.000	0.121	0.314	9.417

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	182	75	77	0	96	0	102	47	129
N.S.	1	1.15	0.47	0.49	0.00	0.61	0.00	0.65	0.30	0.82
time (sec)	N/A	0.623	1.473	0.386	0.000	0.113	0.000	3.991	0.272	9.443

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	273	334	52	0	0	0	0	0	179	38
N.S.	1	1.22	0.19	0.00	0.00	0.00	0.00	0.00	0.66	0.14
time (sec)	N/A	1.062	10.016	0.000	0.000	0.000	0.000	0.000	0.301	9.357

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	192	238	50	0	0	0	0	0	119	38
N.S.	1	1.24	0.26	0.00	0.00	0.00	0.00	0.00	0.62	0.20
time (sec)	N/A	0.753	10.013	0.000	0.000	0.000	0.000	0.000	0.283	10.098

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	113	144	47	0	0	0	0	0	59	38
N.S.	1	1.27	0.42	0.00	0.00	0.00	0.00	0.00	0.52	0.34
time (sec)	N/A	0.478	10.014	0.000	0.000	0.000	0.000	0.000	0.236	9.899

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	91	111	45	0	0	0	0	0	37	38
N.S.	1	1.22	0.49	0.00	0.00	0.00	0.00	0.00	0.41	0.42
time (sec)	N/A	0.377	10.014	0.000	0.000	0.000	0.000	0.000	0.186	10.140

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	211	52	0	0	0	0	0	54	38
N.S.	1	1.21	0.30	0.00	0.00	0.00	0.00	0.00	0.31	0.22
time (sec)	N/A	0.616	10.016	0.000	0.000	0.000	0.000	0.000	0.209	10.552

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	255	310	52	0	0	0	0	0	71	38
N.S.	1	1.22	0.20	0.00	0.00	0.00	0.00	0.00	0.28	0.15
time (sec)	N/A	0.907	10.012	0.000	0.000	0.000	0.000	0.000	0.241	9.818

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	249	336	50	0	0	0	0	0	139	38
N.S.	1	1.35	0.20	0.00	0.00	0.00	0.00	0.00	0.56	0.15
time (sec)	N/A	1.120	10.017	0.000	0.000	0.000	0.000	0.000	0.280	9.391

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	168	242	47	0	0	0	0	0	79	38
N.S.	1	1.44	0.28	0.00	0.00	0.00	0.00	0.00	0.47	0.23
time (sec)	N/A	0.817	10.012	0.000	0.000	0.000	0.000	0.000	0.272	8.974

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	156	45	0	0	0	0	0	64	38
N.S.	1	1.79	0.52	0.00	0.00	0.00	0.00	0.00	0.74	0.44
time (sec)	N/A	0.529	10.014	0.000	0.000	0.000	0.000	0.000	0.260	9.256

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	117	243	52	0	0	0	0	0	37	38
N.S.	1	2.08	0.44	0.00	0.00	0.00	0.00	0.00	0.32	0.32
time (sec)	N/A	0.820	10.014	0.000	0.000	0.000	0.000	0.000	0.253	10.148

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	200	344	52	0	0	0	0	0	54	38
N.S.	1	1.72	0.26	0.00	0.00	0.00	0.00	0.00	0.27	0.19
time (sec)	N/A	1.230	10.018	0.000	0.000	0.000	0.000	0.000	0.234	9.877

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	55	53	0	0	0	0	0	137	56
N.S.	1	1.15	1.10	0.00	0.00	0.00	0.00	0.00	2.85	1.17
time (sec)	N/A	0.316	0.028	0.000	0.000	0.000	0.000	0.000	0.232	10.124

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	61	179	74	164	753	260	174	76
N.S.	1	1.00	0.82	2.42	1.00	2.22	10.18	3.51	2.35	1.03
time (sec)	N/A	0.377	0.070	0.342	0.026	0.102	0.845	0.123	0.238	9.812

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	43	60	52	89	340	138	90	53
N.S.	1	1.00	0.83	1.15	1.00	1.71	6.54	2.65	1.73	1.02
time (sec)	N/A	0.342	0.057	0.280	0.026	0.088	0.425	0.339	0.268	9.675

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	22	26	25	30	29	25	29	25
N.S.	1	1.00	0.88	1.04	1.00	1.20	1.16	1.00	1.16	1.00
time (sec)	N/A	0.258	0.031	0.042	0.030	0.089	0.019	0.109	0.228	9.182

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0	40	37
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.08	1.00
time (sec)	N/A	0.280	0.030	0.000	0.000	0.000	0.000	0.000	0.182	9.063

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0	386	45
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	10.43	1.22
time (sec)	N/A	0.288	0.025	0.000	0.000	0.000	0.000	0.000	0.209	9.162

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0	790	45
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	21.35	1.22
time (sec)	N/A	0.310	0.038	0.000	0.000	0.000	0.000	0.000	0.191	9.053

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	69	58	0	0	0	0	0	59	54
N.S.	1	1.11	0.94	0.00	0.00	0.00	0.00	0.00	0.95	0.87
time (sec)	N/A	0.321	0.137	0.000	0.000	0.000	0.000	0.000	0.598	9.168

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	67	58	0	0	0	0	0	34	54
N.S.	1	1.08	0.94	0.00	0.00	0.00	0.00	0.00	0.55	0.87
time (sec)	N/A	0.312	0.116	0.000	0.000	0.000	0.000	0.000	0.334	9.139

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	58	0	0	0	0	0	14	54
N.S.	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	0.23	0.87
time (sec)	N/A	0.301	0.090	0.000	0.000	0.000	0.000	0.000	0.234	9.093

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	0	0	0	0	0	18	55
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.30	0.92
time (sec)	N/A	0.306	0.085	0.000	0.000	0.000	0.000	0.000	0.223	9.404

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	56	0	0	0	0	0	32	55
N.S.	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	0.53	0.92
time (sec)	N/A	0.300	0.145	0.000	0.000	0.000	0.000	0.000	0.202	9.708

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	62	69	58	0	0	0	0	0	54	55
N.S.	1	1.11	0.94	0.00	0.00	0.00	0.00	0.00	0.87	0.89
time (sec)	N/A	0.321	0.151	0.000	0.000	0.000	0.000	0.000	0.239	10.142

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	57	53	0	0	0	0	0	508	58
N.S.	1	1.04	0.96	0.00	0.00	0.00	0.00	0.00	9.24	1.05
time (sec)	N/A	0.321	0.026	0.000	0.000	0.000	0.000	0.000	0.211	9.956

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	199	116	158	138	176	0	283	286	54
N.S.	1	1.19	0.69	0.95	0.83	1.05	0.00	1.69	1.71	0.32
time (sec)	N/A	0.636	0.076	0.517	0.046	0.099	0.000	0.173	0.213	9.714

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	144	77	99	98	118	0	177	286	54
N.S.	1	1.18	0.63	0.81	0.80	0.97	0.00	1.45	2.34	0.44
time (sec)	N/A	0.497	0.067	0.504	0.044	0.098	0.000	0.180	0.202	9.551

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	84	49	57	65	75	340	89	268	50
N.S.	1	1.09	0.64	0.74	0.84	0.97	4.42	1.16	3.48	0.65
time (sec)	N/A	0.380	0.054	0.493	0.044	0.086	4.067	0.138	0.230	9.796

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	64	79	56	0	0	0	0	0	20	59
N.S.	1	1.23	0.88	0.00	0.00	0.00	0.00	0.00	0.31	0.92
time (sec)	N/A	0.340	0.037	0.000	0.000	0.000	0.000	0.000	0.229	9.575

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	85	58	0	0	0	0	0	22	83
N.S.	1	1.25	0.85	0.00	0.00	0.00	0.00	0.00	0.32	1.22
time (sec)	N/A	0.340	0.038	0.000	0.000	0.000	0.000	0.000	0.212	9.402

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	69	86	61	0	0	0	0	0	22	85
N.S.	1	1.25	0.88	0.00	0.00	0.00	0.00	0.00	0.32	1.23
time (sec)	N/A	0.354	0.045	0.000	0.000	0.000	0.000	0.000	0.220	9.518

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [120] had the largest ratio of [1]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	11	0.182
2	A	2	2	1.00	11	0.182
3	A	2	2	1.00	11	0.182
4	A	1	1	1.00	9	0.111
5	A	2	2	1.00	11	0.182
6	A	2	2	1.00	11	0.182
7	A	2	2	1.00	11	0.182
8	A	2	2	1.00	11	0.182
9	A	6	5	0.77	13	0.385
10	A	5	4	0.79	13	0.308
11	A	4	3	0.82	13	0.231
12	A	3	2	1.00	13	0.154
13	A	1	1	0.60	13	0.077
14	A	2	2	0.61	13	0.154
15	A	3	3	0.64	13	0.231
16	A	4	4	0.66	13	0.308
17	A	6	5	0.72	13	0.385
18	A	5	4	0.73	13	0.308
19	A	4	3	0.69	13	0.231
20	A	3	2	1.08	13	0.154
21	A	1	1	0.59	13	0.077

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	0.62	13	0.154
23	A	3	3	0.66	13	0.231
24	A	4	4	0.68	13	0.308
25	A	6	5	0.68	15	0.333
26	A	5	4	0.69	15	0.267
27	A	4	3	0.66	15	0.200
28	A	3	2	0.84	15	0.133
29	A	1	1	0.59	15	0.067
30	A	2	2	0.59	15	0.133
31	A	3	3	0.62	15	0.200
32	A	3	2	0.81	13	0.154
33	A	3	2	0.61	13	0.154
34	A	3	2	1.00	13	0.154
35	A	3	2	0.88	13	0.154
36	A	3	2	1.50	16	0.125
37	A	3	2	0.77	17	0.118
38	A	3	2	1.00	15	0.133
39	A	3	2	1.00	16	0.125
40	A	4	3	0.73	13	0.231
41	A	4	3	0.69	13	0.231
42	A	4	3	0.66	11	0.273
43	A	4	3	0.62	11	0.273
44	A	4	3	0.70	11	0.273
45	A	3	2	0.86	13	0.154
46	A	3	2	1.00	11	0.182
47	A	3	2	1.00	11	0.182
48	A	5	4	0.67	11	0.364
49	A	3	2	1.00	15	0.133
50	A	3	2	1.17	13	0.154
51	B	7	6	10.00	13	0.462
52	B	8	7	19.71	13	0.538
53	B	6	5	8.94	13	0.385

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	B	7	6	18.77	13	0.462
55	B	5	4	8.42	13	0.308
56	B	8	7	20.73	13	0.538
57	B	6	5	8.80	13	0.385
58	A	6	5	0.87	13	0.385
59	A	5	4	0.95	13	0.308
60	A	5	4	0.90	13	0.308
61	A	4	3	1.15	13	0.231
62	A	4	3	1.04	13	0.231
63	A	5	4	1.13	13	0.308
64	A	6	5	0.84	13	0.385
65	A	7	6	0.80	13	0.462
66	A	4	3	1.18	13	0.231
67	A	4	3	1.18	13	0.231
68	A	4	3	1.15	13	0.231
69	A	3	2	1.21	13	0.154
70	A	3	2	1.21	13	0.154
71	A	4	3	1.14	13	0.231
72	A	4	3	1.18	13	0.231
73	A	4	3	0.90	13	0.231
74	A	4	3	0.99	13	0.231
75	A	3	2	1.21	13	0.154
76	A	3	2	1.21	13	0.154
77	A	4	3	1.14	13	0.231
78	A	5	4	0.96	13	0.308
79	A	5	4	0.99	13	0.308
80	A	5	4	1.07	13	0.308
81	A	4	3	0.85	13	0.231
82	A	4	3	0.83	13	0.231
83	A	5	4	0.92	13	0.308
84	A	1	1	1.41	11	0.091
85	A	3	2	1.16	14	0.143

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	1	1	1.32	13	0.077
87	A	3	2	1.28	11	0.182
88	A	3	2	1.00	11	0.182
89	A	1	1	1.29	11	0.091
90	A	1	1	1.00	11	0.091
91	A	3	3	1.00	13	0.231
92	A	3	3	1.00	13	0.231
93	A	3	3	1.00	13	0.231
94	A	1	1	1.00	11	0.091
95	A	3	3	1.00	13	0.231
96	A	3	3	1.00	13	0.231
97	A	3	3	1.00	13	0.231
98	A	3	3	1.00	13	0.231
99	A	8	8	1.16	15	0.533
100	A	6	6	1.15	15	0.400
101	A	4	4	1.11	15	0.267
102	A	2	2	1.00	15	0.133
103	A	3	2	0.94	15	0.133
104	A	6	5	1.04	15	0.333
105	A	9	8	1.12	15	0.533
106	A	12	11	1.14	15	0.733
107	A	5	5	0.80	15	0.333
108	A	4	4	0.75	15	0.267
109	A	2	2	0.67	15	0.133
110	A	1	1	1.00	15	0.067
111	A	3	3	1.08	15	0.200
112	A	4	4	1.13	15	0.267
113	A	6	6	1.15	15	0.400
114	A	13	12	1.22	15	0.800
115	A	10	9	1.24	15	0.600
116	A	7	6	1.27	15	0.400
117	A	6	5	1.22	15	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	9	8	1.21	15	0.533
119	A	12	11	1.22	15	0.733
120	A	16	15	1.35	15	1.000
121	A	13	12	1.44	15	0.800
122	A	10	9	1.79	15	0.600
123	B	13	12	2.08	15	0.800
124	A	16	15	1.72	15	1.000
125	A	3	3	1.15	13	0.231
126	A	3	3	1.00	15	0.200
127	A	3	3	1.00	15	0.200
128	A	1	1	1.00	13	0.077
129	A	2	2	1.00	15	0.133
130	A	2	2	1.00	15	0.133
131	A	2	2	1.00	15	0.133
132	A	3	3	1.11	17	0.176
133	A	3	3	1.08	17	0.176
134	A	3	3	1.00	17	0.176
135	A	3	3	1.00	17	0.176
136	A	3	3	1.00	17	0.176
137	A	3	3	1.11	17	0.176
138	A	3	3	1.04	15	0.200
139	A	4	4	1.19	19	0.211
140	A	3	3	1.18	19	0.158
141	A	2	2	1.09	17	0.118
142	A	2	2	1.23	19	0.105
143	A	2	2	1.25	19	0.105
144	A	2	2	1.25	19	0.105

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (bx + cx^2)^4 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int (bx + cx^2)^4 \, dx = \frac{b^4x^5}{5} + \frac{2}{3}b^3cx^6 + \frac{6}{7}b^2c^2x^7 + \frac{1}{2}bc^3x^8 + \frac{c^4x^9}{9}$$

output
```
1/5*b^4*x^5+2/3*b^3*c*x^6+6/7*b^2*c^2*x^7+1/2*b*c^3*x^8+1/9*c^4*x^9
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (bx + cx^2)^4 \, dx = \frac{b^4x^5}{5} + \frac{2}{3}b^3cx^6 + \frac{6}{7}b^2c^2x^7 + \frac{1}{2}bc^3x^8 + \frac{c^4x^9}{9}$$

input
```
Integrate[(b*x + c*x^2)^4,x]
```

output
```
(b^4*x^5)/5 + (2*b^3*c*x^6)/3 + (6*b^2*c^2*x^7)/7 + (b*c^3*x^8)/2 + (c^4*x
^9)/9
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+cx^2)^4 \, dx$$

$$\Big\downarrow \text{1080}$$

$$\int (b^4x^4+4b^3cx^5+6b^2c^2x^6+4bc^3x^7+c^4x^8) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^4x^5}{5}+\frac{2}{3}b^3cx^6+\frac{6}{7}b^2c^2x^7+\frac{1}{2}bc^3x^8+\frac{c^4x^9}{9}$$

input
```
Int[(b*x + c*x^2)^4,x]
```

output
```
(b^4*x^5)/5 + (2*b^3*c*x^6)/3 + (6*b^2*c^2*x^7)/7 + (b*c^3*x^8)/2 + (c^4*x
^9)/9
```

Defintions of rubi rules used

rule 1080
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[x^p*
(b + c*x)^p, x], x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{x^{5}\left(70c^{4}x^{4}+315b\,c^{3}x^{3}+540b^{2}c^{2}x^{2}+420b^{3}cx+126b^{4}\right)}{630}$	47
default	$\frac{1}{5}b^{4}x^{5}+\frac{2}{3}b^{3}c\,x^{6}+\frac{6}{7}b^{2}c^{2}x^{7}+\frac{1}{2}b\,c^{3}x^{8}+\frac{1}{9}c^{4}x^{9}$	47
norman	$\frac{1}{5}b^{4}x^{5}+\frac{2}{3}b^{3}c\,x^{6}+\frac{6}{7}b^{2}c^{2}x^{7}+\frac{1}{2}b\,c^{3}x^{8}+\frac{1}{9}c^{4}x^{9}$	47
risch	$\frac{1}{5}b^{4}x^{5}+\frac{2}{3}b^{3}c\,x^{6}+\frac{6}{7}b^{2}c^{2}x^{7}+\frac{1}{2}b\,c^{3}x^{8}+\frac{1}{9}c^{4}x^{9}$	47
parallelrisch	$\frac{1}{5}b^{4}x^{5}+\frac{2}{3}b^{3}c\,x^{6}+\frac{6}{7}b^{2}c^{2}x^{7}+\frac{1}{2}b\,c^{3}x^{8}+\frac{1}{9}c^{4}x^{9}$	47
orering	$\frac{x\left(70c^{4}x^{4}+315b\,c^{3}x^{3}+540b^{2}c^{2}x^{2}+420b^{3}cx+126b^{4}\right)\left(c\,x^{2}+bx\right)^{4}}{630\left(cx+b\right)^{4}}$	63

input
```
int((c*x^2+b*x)^4,x,method=_RETURNVERBOSE)
```

output
```
1/630*x^5*(70*c^4*x^4+315*b*c^3*x^3+540*b^2*c^2*x^2+420*b^3*c*x+126*b^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \left(bx+cx^2\right)^4\,dx=\frac{1}{9}\,c^4x^9+\frac{1}{2}\,bc^3x^8+\frac{6}{7}\,b^2c^2x^7+\frac{2}{3}\,b^3cx^6+\frac{1}{5}\,b^4x^5$$

input
```
integrate((c*x^2+b*x)^4,x, algorithm="fricas")
```

output
```
1/9*c^4*x^9 + 1/2*b*c^3*x^8 + 6/7*b^2*c^2*x^7 + 2/3*b^3*c*x^6 + 1/5*b^4*x^
5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \left(bx+cx^2\right)^4\,dx=\frac{b^4x^5}{5}+\frac{2b^3cx^6}{3}+\frac{6b^2c^2x^7}{7}+\frac{bc^3x^8}{2}+\frac{c^4x^9}{9}$$

input
```
integrate((c*x**2+b*x)**4,x)
```

output
```
b**4*x**5/5 + 2*b**3*c*x**6/3 + 6*b**2*c**2*x**7/7 + b*c**3*x**8/2 + c**4*
x**9/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \left(bx+cx^2\right)^4\,dx=\frac{1}{9}\,c^4x^9+\frac{1}{2}\,bc^3x^8+\frac{6}{7}\,b^2c^2x^7+\frac{2}{3}\,b^3cx^6+\frac{1}{5}\,b^4x^5$$

input
```
integrate((c*x^2+b*x)^4,x, algorithm="maxima")
```

output
```
1/9*c^4*x^9 + 1/2*b*c^3*x^8 + 6/7*b^2*c^2*x^7 + 2/3*b^3*c*x^6 + 1/5*b^4*x^
5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \left(bx+cx^2\right)^4\,dx=\frac{1}{9}\,c^4x^9+\frac{1}{2}\,bc^3x^8+\frac{6}{7}\,b^2c^2x^7+\frac{2}{3}\,b^3cx^6+\frac{1}{5}\,b^4x^5$$

input
```
integrate((c*x^2+b*x)^4,x, algorithm="giac")
```

output
```
1/9*c^4*x^9 + 1/2*b*c^3*x^8 + 6/7*b^2*c^2*x^7 + 2/3*b^3*c*x^6 + 1/5*b^4*x^
5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \left(bx+cx^2\right)^4 \, dx = \frac{b^4\,x^5}{5}+\frac{2\,b^3\,c\,x^6}{3}+\frac{6\,b^2\,c^2\,x^7}{7}+\frac{b\,c^3\,x^8}{2}+\frac{c^4\,x^9}{9}$$

input
```
int((b*x + c*x^2)^4,x)
```

output
```
(b^4*x^5)/5 + (c^4*x^9)/9 + (2*b^3*c*x^6)/3 + (b*c^3*x^8)/2 + (6*b^2*c^2*x
^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \left(bx+cx^2\right)^4 \, dx = \frac{x^5(70c^4x^4+315b\,c^3x^3+540b^2c^2x^2+420b^3cx+126b^4)}{630}$$

input
```
int((c*x^2+b*x)^4,x)
```

output
```
(x**5*(126*b**4 + 420*b**3*c*x + 540*b**2*c**2*x**2 + 315*b*c**3*x**3 + 70
*c**4*x**4))/630
```

3.2 $\int (bx+cx^2)^3 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int (bx+cx^2)^3 \, dx = \frac{b^3x^4}{4} + \frac{3}{5}b^2cx^5 + \frac{1}{2}bc^2x^6 + \frac{c^3x^7}{7}$$

output
```
1/4*b^3*x^4+3/5*b^2*c*x^5+1/2*b*c^2*x^6+1/7*c^3*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int (bx+cx^2)^3 \, dx = \frac{b^3x^4}{4} + \frac{3}{5}b^2cx^5 + \frac{1}{2}bc^2x^6 + \frac{c^3x^7}{7}$$

input
```
Integrate[(b*x + c*x^2)^3,x]
```

output
```
(b^3*x^4)/4 + (3*b^2*c*x^5)/5 + (b*c^2*x^6)/2 + (c^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^3 \, dx$$

$$\Big\downarrow \text{ 1080}$$

$$\int \left(b^3x^3+3b^2cx^4+3bc^2x^5+c^3x^6\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b^3x^4}{4}+\frac{3}{5}b^2cx^5+\frac{1}{2}bc^2x^6+\frac{c^3x^7}{7}$$

input
```
Int[(b*x + c*x^2)^3,x]
```

output
```
(b^3*x^4)/4 + (3*b^2*c*x^5)/5 + (b*c^2*x^6)/2 + (c^3*x^7)/7
```

Defintions of rubi rules used

rule 1080
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[x^p*
(b + c*x)^p, x], x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{x^{4}\left(20c^{3}x^{3}+70b\,c^{2}x^{2}+84b^{2}cx+35b^{3}\right)}{140}$	36
default	$\frac{1}{4}b^{3}x^{4}+\frac{3}{5}x^{5}b^{2}c+\frac{1}{2}b\,c^{2}x^{6}+\frac{1}{7}c^{3}x^{7}$	36
norman	$\frac{1}{4}b^{3}x^{4}+\frac{3}{5}x^{5}b^{2}c+\frac{1}{2}b\,c^{2}x^{6}+\frac{1}{7}c^{3}x^{7}$	36
risch	$\frac{1}{4}b^{3}x^{4}+\frac{3}{5}x^{5}b^{2}c+\frac{1}{2}b\,c^{2}x^{6}+\frac{1}{7}c^{3}x^{7}$	36
parallelrisch	$\frac{1}{4}b^{3}x^{4}+\frac{3}{5}x^{5}b^{2}c+\frac{1}{2}b\,c^{2}x^{6}+\frac{1}{7}c^{3}x^{7}$	36
orering	$\frac{x\left(20c^{3}x^{3}+70b\,c^{2}x^{2}+84b^{2}cx+35b^{3}\right)\left(c\,x^{2}+bx\right)^{3}}{140\left(cx+b\right)^{3}}$	52

input
```
int((c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/140*x^4*(20*c^3*x^3+70*b*c^2*x^2+84*b^2*c*x+35*b^3)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(bx+cx^{2}\right)^{3}\,dx=\frac{1}{7}\,c^{3}x^{7}+\frac{1}{2}\,bc^{2}x^{6}+\frac{3}{5}\,b^{2}cx^{5}+\frac{1}{4}\,b^{3}x^{4}$$

input
```
integrate((c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*b^2*c*x^5 + 1/4*b^3*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \left(bx+cx^2\right)^3\,dx=\frac{b^3x^4}{4}+\frac{3b^2cx^5}{5}+\frac{bc^2x^6}{2}+\frac{c^3x^7}{7}$$

input
```
integrate((c*x**2+b*x)**3,x)
```

output
```
b**3*x**4/4 + 3*b**2*c*x**5/5 + b*c**2*x**6/2 + c**3*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(bx+cx^2\right)^3\,dx=\frac{1}{7}\,c^3x^7+\frac{1}{2}\,bc^2x^6+\frac{3}{5}\,b^2cx^5+\frac{1}{4}\,b^3x^4$$

input
```
integrate((c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*b^2*c*x^5 + 1/4*b^3*x^4
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(bx+cx^2\right)^3\,dx=\frac{1}{7}\,c^3x^7+\frac{1}{2}\,bc^2x^6+\frac{3}{5}\,b^2cx^5+\frac{1}{4}\,b^3x^4$$

input
```
integrate((c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*b^2*c*x^5 + 1/4*b^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(bx+cx^2\right)^3 \, dx = \frac{b^3\,x^4}{4}+\frac{3\,b^2\,c\,x^5}{5}+\frac{b\,c^2\,x^6}{2}+\frac{c^3\,x^7}{7}$$

input
```
int((b*x + c*x^2)^3,x)
```

output
```
(b^3*x^4)/4 + (c^3*x^7)/7 + (3*b^2*c*x^5)/5 + (b*c^2*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(bx+cx^2\right)^3 \, dx = \frac{x^4(20c^3x^3+70b\,c^2x^2+84b^2cx+35b^3)}{140}$$

input
```
int((c*x^2+b*x)^3,x)
```

output
```
(x**4*(35*b**3 + 84*b**2*c*x + 70*b*c**2*x**2 + 20*c**3*x**3))/140
```

3.3 $\int (bx + cx^2)^2 \, dx$

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int (bx + cx^2)^2 \, dx = \frac{b^2x^3}{3} + \frac{1}{2}bcx^4 + \frac{c^2x^5}{5}$$

output
```
1/3*b^2*x^3+1/2*b*c*x^4+1/5*c^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (bx + cx^2)^2 \, dx = \frac{b^2x^3}{3} + \frac{1}{2}bcx^4 + \frac{c^2x^5}{5}$$

input
```
Integrate[(b*x + c*x^2)^2,x]
```

output
```
(b^2*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^2 \, dx$$
$$\Big\downarrow \text{ 1080}$$
$$\int \left(b^2x^2+2bcx^3+c^2x^4\right) dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{b^2x^3}{3}+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

input
```
Int[(b*x + c*x^2)^2,x]
```

output
```
(b^2*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Defintions of rubi rules used

rule 1080
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[x^p*
(b + c*x)^p, x], x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^{3}\left(6c^{2}x^{2}+15cbx+10b^{2}\right)}{30}$	25
default	$\frac{1}{3}b^{2}x^{3}+\frac{1}{2}bc\,x^{4}+\frac{1}{5}c^{2}x^{5}$	25
norman	$\frac{1}{3}b^{2}x^{3}+\frac{1}{2}bc\,x^{4}+\frac{1}{5}c^{2}x^{5}$	25
risch	$\frac{1}{3}b^{2}x^{3}+\frac{1}{2}bc\,x^{4}+\frac{1}{5}c^{2}x^{5}$	25
parallelrisch	$\frac{1}{3}b^{2}x^{3}+\frac{1}{2}bc\,x^{4}+\frac{1}{5}c^{2}x^{5}$	25
orering	$\frac{x\left(6c^{2}x^{2}+15cbx+10b^{2}\right)\left(c\,x^{2}+bx\right)^{2}}{30\left(cx+b\right)^{2}}$	41

input
```
int((c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/30*x^3*(6*c^2*x^2+15*b*c*x+10*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(bx+cx^2\right)^2\,dx=\frac{1}{5}\,c^2x^5+\frac{1}{2}\,bcx^4+\frac{1}{3}\,b^2x^3$$

input
```
integrate((c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
1/5*c^2*x^5 + 1/2*b*c*x^4 + 1/3*b^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(bx + cx^2\right)^2 \, dx = \frac{b^2x^3}{3} + \frac{bcx^4}{2} + \frac{c^2x^5}{5}$$

input
```
integrate((c*x**2+b*x)**2,x)
```

output
```
b**2*x**3/3 + b*c*x**4/2 + c**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(bx + cx^2\right)^2 \, dx = \frac{1}{5} \, c^2x^5 + \frac{1}{2} \, bcx^4 + \frac{1}{3} \, b^2x^3$$

input
```
integrate((c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
1/5*c^2*x^5 + 1/2*b*c*x^4 + 1/3*b^2*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(bx + cx^2\right)^2 \, dx = \frac{1}{5} \, c^2x^5 + \frac{1}{2} \, bcx^4 + \frac{1}{3} \, b^2x^3$$

input
```
integrate((c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
1/5*c^2*x^5 + 1/2*b*c*x^4 + 1/3*b^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(bx+cx^2\right)^2\,dx=\frac{b^2\,x^3}{3}+\frac{b\,c\,x^4}{2}+\frac{c^2\,x^5}{5}$$

input
```
int((b*x + c*x^2)^2,x)
```

output
```
(b^2*x^3)/3 + (c^2*x^5)/5 + (b*c*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(bx+cx^2\right)^2\,dx=\frac{x^3(6c^2x^2+15bcx+10b^2)}{30}$$

input
```
int((c*x^2+b*x)^2,x)
```

output
```
(x**3*(10*b**2 + 15*b*c*x + 6*c**2*x**2))/30
```

3.4 $\int (bx + cx^2) \, dx$

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int (bx + cx^2) \, dx = \frac{bx^2}{2} + \frac{cx^3}{3}$$

output
```
1/2*b*x^2+1/3*c*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (bx + cx^2) \, dx = \frac{bx^2}{2} + \frac{cx^3}{3}$$

input
```
Integrate[b*x + c*x^2,x]
```

output
```
(b*x^2)/2 + (c*x^3)/3
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx + cx^2) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{bx^2}{2} + \frac{cx^3}{3}$$

input
```
Int[b*x + c*x^2,x]
```

output
```
(b*x^2)/2 + (c*x^3)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^2(2cx+3b)}{6}$	14
default	$\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3$	14
norman	$\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3$	14
risch	$\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3$	14
parallelrisch	$\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3$	14
parts	$\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3$	14
orering	$\frac{x(2cx+3b)(c\,x^2+bx)}{6cx+6b}$	28

input
```
int(c*x^2+b*x,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^2*(2*c*x+3*b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(bx+cx^2\right)\,dx=\frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2$$

input
```
integrate(c*x^2+b*x,x, algorithm="fricas")
```

output
```
1/3*c*x^3 + 1/2*b*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int \left(bx + cx^2\right) \, dx = \frac{bx^2}{2} + \frac{cx^3}{3}$$

input
```
integrate(c*x**2+b*x,x)
```

output
```
b*x**2/2 + c*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(bx + cx^2\right) \, dx = \frac{1}{3} \, cx^3 + \frac{1}{2} \, bx^2$$

input
```
integrate(c*x^2+b*x,x, algorithm="maxima")
```

output
```
1/3*c*x^3 + 1/2*b*x^2
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int \left(bx + cx^2\right) \, dx = \frac{1}{3} \, cx^3 + \frac{1}{2} \, bx^2$$

input
```
integrate(c*x^2+b*x,x, algorithm="giac")
```

output
```
1/3*c*x^3 + 1/2*b*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (bx + cx^2) \, dx = \frac{x^2 (3 b + 2 c x)}{6}$$

input
```
int(b*x + c*x^2,x)
```

output
```
(x^2*(3*b + 2*c*x))/6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (bx + cx^2) \, dx = \frac{x^2(2cx + 3b)}{6}$$

input
```
int(c*x^2+b*x,x)
```

output
```
(x**2*(3*b + 2*c*x))/6
```

3.5 $\int \frac{1}{bx+cx^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{bx+cx^2}\,dx=\frac{\log(x)}{b}-\frac{\log(b+cx)}{b}$$

output
```
ln(x)/b-ln(c*x+b)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{bx+cx^2}\,dx=\frac{\log(x)}{b}-\frac{\log(b+cx)}{b}$$

input
```
Integrate[(b*x + c*x^2)^(-1),x]
```

output
```
Log[x]/b - Log[b + c*x]/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{bx+cx^2}\,dx$$

$$\Big\downarrow \text{1080}$$

$$\int \left(\frac{1}{bx}-\frac{c}{b(b+cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\log(x)}{b}-\frac{\log(b+cx)}{b}$$

input
```
Int[(b*x + c*x^2)^(-1),x]
```

output
```
Log[x]/b - Log[b + c*x]/b
```

Defintions of rubi rules used

rule 1080
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[x^p*
(b + c*x)^p, x], x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{\ln(x)-\ln(cx+b)}{b}$	16
default	$\frac{\ln(x)}{b}-\frac{\ln(cx+b)}{b}$	19
norman	$\frac{\ln(x)}{b}-\frac{\ln(cx+b)}{b}$	19
risch	$\frac{\ln(-x)}{b}-\frac{\ln(cx+b)}{b}$	21

input
```
int(1/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
(ln(x)-ln(c*x+b))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{bx+cx^2}\,dx=-\frac{\log(cx+b)-\log(x)}{b}$$

input
```
integrate(1/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
-(log(c*x + b) - log(x))/b
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{1}{bx+cx^2}\,dx=\frac{\log(x)-\log\left(\frac{b}{c}+x\right)}{b}$$

input
```
integrate(1/(c*x**2+b*x),x)
```

output
```
(log(x) - log(b/c + x))/b
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{bx+cx^2}\,dx = -\frac{\log(cx+b)}{b} + \frac{\log(x)}{b}$$

input
```
integrate(1/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
-log(c*x + b)/b + log(x)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{bx+cx^2}\,dx = -\frac{\log(|cx+b|)}{b} + \frac{\log(|x|)}{b}$$

input
```
integrate(1/(c*x^2+b*x),x, algorithm="giac")
```

output
```
-log(abs(c*x + b))/b + log(abs(x))/b
```

Mupad [B] (verification not implemented)

Time = 9.48 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{bx+cx^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b}$$

input
```
int(1/(b*x + c*x^2),x)
```

output
```
-(2*atanh((2*c*x)/b + 1))/b
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{bx+cx^2}\,dx=\frac{-\log(cx+b)+\log(x)}{b}$$

input
```
int(1/(c*x^2+b*x),x)
```

output
```
( - log(b + c*x) + log(x))/b
```

3.6 $\int \frac{1}{(bx+cx^2)^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{1}{(bx+cx^2)^2} dx = -\frac{1}{b^2x} - \frac{c}{b^2(b+cx)} - \frac{2c\log(x)}{b^3} + \frac{2c\log(b+cx)}{b^3}$$

output
```
-1/b^2/x-c/b^2/(c*x+b)-2*c*ln(x)/b^3+2*c*ln(c*x+b)/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx+cx^2)^2} dx = -\frac{b\left(\frac{1}{x}+\frac{c}{b+cx}\right)+2c\log(x)-2c\log(b+cx)}{b^3}$$

input
```
Integrate[(b*x + c*x^2)^(-2),x]
```

output
```
-((b*(x^(-1) + c/(b + c*x)) + 2*c*Log[x] - 2*c*Log[b + c*x])/b^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{ 1080}$$

$$\int \left(\frac{2c^2}{b^3(b+cx)}-\frac{2c}{b^3x}+\frac{c^2}{b^2(b+cx)^2}+\frac{1}{b^2x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2c\log(x)}{b^3}+\frac{2c\log(b+cx)}{b^3}-\frac{c}{b^2(b+cx)}-\frac{1}{b^2x}$$

input
```
Int[(b*x + c*x^2)^(-2),x]
```

output
```
-(1/(b^2*x)) - c/(b^2*(b + c*x)) - (2*c*Log[x])/b^3 + (2*c*Log[b + c*x])/b
^3
```

Defintions of rubi rules used

rule 1080
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[x^p*
(b + c*x)^p, x], x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{1}{b^2 x}-\frac{c}{b^2(cx+b)}-\frac{2c\ln(x)}{b^3}+\frac{2c\ln(cx+b)}{b^3}$	43
risch	$\frac{-\frac{2cx}{b^2}-\frac{1}{b}}{x(cx+b)}-\frac{2c\ln(x)}{b^3}+\frac{2c\ln(-cx-b)}{b^3}$	49
norman	$\frac{\frac{2c^2x^2}{b^3}-\frac{1}{b}}{x(cx+b)}-\frac{2c\ln(x)}{b^3}+\frac{2c\ln(cx+b)}{b^3}$	50
parallelrisch	$-\frac{2c^2\ln(x)x^2-2\ln(cx+b)x^2c^2+2\ln(x)xbc-2\ln(cx+b)xbc-2c^2x^2+b^2}{b^3x(cx+b)}$	70

input
```
int(1/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^2/x-c/b^2/(c*x+b)-2*c*ln(x)/b^3+2*c*ln(c*x+b)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.50

$$\int \frac{1}{(bx+cx^2)^2}\,dx = -\frac{2\,bcx+b^2-2\,(c^2x^2+bcx)\log(cx+b)+2\,(c^2x^2+bcx)\log(x)}{b^3cx^2+b^4x}$$

input
```
integrate(1/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
-(2*b*c*x + b^2 - 2*(c^2*x^2 + b*c*x)*log(c*x + b) + 2*(c^2*x^2 + b*c*x)*l
og(x))/(b^3*c*x^2 + b^4*x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{1}{(bx+cx^2)^2}\,dx = \frac{-b-2cx}{b^3x+b^2cx^2} + \frac{2c\left(-\log(x)+\log\left(\frac{b}{c}+x\right)\right)}{b^3}$$

input
```
integrate(1/(c*x**2+b*x)**2,x)
```

output
```
(-b - 2*c*x)/(b**3*x + b**2*c*x**2) + 2*c*(-log(x) + log(b/c + x))/b**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{(bx+cx^2)^2}\,dx = -\frac{2\,cx+b}{b^2cx^2+b^3x} + \frac{2\,c\log(cx+b)}{b^3} - \frac{2\,c\log(x)}{b^3}$$

input
```
integrate(1/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-(2*c*x + b)/(b^2*c*x^2 + b^3*x) + 2*c*log(c*x + b)/b^3 - 2*c*log(x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{(bx+cx^2)^2}\,dx = \frac{2\,c\log(|cx+b|)}{b^3} - \frac{2\,c\log(|x|)}{b^3} - \frac{2\,cx+b}{(cx^2+bx)b^2}$$

input
```
integrate(1/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
2*c*log(abs(c*x + b))/b^3 - 2*c*log(abs(x))/b^3 - (2*c*x + b)/((c*x^2 + b*
x)*b^2)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{(bx+cx^2)^2}\,dx = \frac{4\,c\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b^3} - \frac{\frac{1}{b}+\frac{2\,c\,x}{b^2}}{c\,x^2+b\,x}$$

input
```
int(1/(b*x + c*x^2)^2,x)
```

output
```
(4*c*atanh((2*c*x)/b + 1))/b^3 - (1/b + (2*c*x)/b^2)/(b*x + c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67

$$\int \frac{1}{(bx+cx^2)^2}\,dx = \frac{2\log(cx+b)\,bcx+2\log(cx+b)\,c^2x^2-2\log(x)\,bcx-2\log(x)\,c^2x^2-b^2+2c^2x^2}{b^3x\,(cx+b)}$$

input
```
int(1/(c*x^2+b*x)^2,x)
```

output
```
(2*log(b + c*x)*b*c*x + 2*log(b + c*x)*c**2*x**2 - 2*log(x)*b*c*x - 2*log(
x)*c**2*x**2 - b**2 + 2*c**2*x**2)/(b**3*x*(b + c*x))
```

3.7 $\int \frac{1}{(bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 76

$$\int \frac{1}{(bx+cx^2)^3}\,dx = -\frac{1}{2b^3x^2}+\frac{3c}{b^4x}+\frac{c^2}{2b^3(b+cx)^2}+\frac{3c^2}{b^4(b+cx)}+\frac{6c^2\log(x)}{b^5}-\frac{6c^2\log(b+cx)}{b^5}$$

output
```
-1/2/b^3/x^2+3*c/b^4/x+1/2*c^2/b^3/(c*x+b)^2+3*c^2/b^4/(c*x+b)+6*c^2*ln(x)
/b^5-6*c^2*ln(c*x+b)/b^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{1}{(bx+cx^2)^3}\,dx = \frac{\frac{b(-b^3+4b^2cx+18bc^2x^2+12c^3x^3)}{x^2(b+cx)^2}+12c^2\log(x)-12c^2\log(b+cx)}{2b^5}$$

input
```
Integrate[(b*x + c*x^2)^(-3),x]
```

output
```
((b*(-b^3 + 4*b^2*c*x + 18*b*c^2*x^2 + 12*c^3*x^3))/(x^2*(b + c*x)^2) + 12
*c^2*Log[x] - 12*c^2*Log[b + c*x])/(2*b^5)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^3}\,dx$$
$$\Big\downarrow \text{1080}$$
$$\int \left(-\frac{6c^3}{b^5(b+cx)}+\frac{6c^2}{b^5x}-\frac{3c^3}{b^4(b+cx)^2}-\frac{3c}{b^4x^2}-\frac{c^3}{b^3(b+cx)^3}+\frac{1}{b^3x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{6c^2\log(x)}{b^5}-\frac{6c^2\log(b+cx)}{b^5}+\frac{3c^2}{b^4(b+cx)}+\frac{3c}{b^4x}+\frac{c^2}{2b^3(b+cx)^2}-\frac{1}{2b^3x^2}$$

input
```
Int[(b*x + c*x^2)^(-3),x]
```

output
```
-1/2*1/(b^3*x^2) + (3*c)/(b^4*x) + c^2/(2*b^3*(b + c*x)^2) + (3*c^2)/(b^4*
(b + c*x)) + (6*c^2*Log[x])/b^5 - (6*c^2*Log[b + c*x])/b^5
```

Defintions of rubi rules used

rule 1080
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[x^p*
(b + c*x)^p, x], x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

method	result
norman	$\frac{-\frac{9c^4x^4}{b^5}-\frac{1}{2b}+\frac{2cx}{b^2}-\frac{12c^3x^3}{b^4}}{x^2(cx+b)^2}+\frac{6c^2\ln(x)}{b^5}-\frac{6c^2\ln(cx+b)}{b^5}$
default	$-\frac{1}{2b^3x^2}+\frac{3c}{b^4x}+\frac{c^2}{2b^3(cx+b)^2}+\frac{3c^2}{b^4(cx+b)}+\frac{6c^2\ln(x)}{b^5}-\frac{6c^2\ln(cx+b)}{b^5}$
risch	$\frac{\frac{6c^3x^3}{b^4}+\frac{9c^2x^2}{b^3}+\frac{2cx}{b^2}-\frac{1}{2b}}{x^2(cx+b)^2}+\frac{6c^2\ln(-x)}{b^5}-\frac{6c^2\ln(cx+b)}{b^5}$
parallelrisch	$\frac{12\ln(x)x^4c^6-12\ln(cx+b)x^4c^6+24\ln(x)x^3b\,c^5-24\ln(cx+b)x^3b\,c^5+12\ln(x)x^2b^2c^4-12\ln(cx+b)x^2b^2c^4+12x^3b\,c^5+18b^2c^4x^2}{2b^5c^2x^2(cx+b)^2}$

input
```
int(1/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
(-9*c^4/b^5*x^4-1/2/b+2/b^2*c*x-12*c^3/b^4*x^3)/x^2/(c*x+b)^2+6*c^2*ln(x)/
b^5-6*c^2*ln(c*x+b)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.71

$$\int \frac{1}{(bx+cx^2)^3}\,dx = \frac{12\,bc^3x^3+18\,b^2c^2x^2+4\,b^3cx-b^4-12\,(c^4x^4+2\,bc^3x^3+b^2c^2x^2)\log(cx+b)+12\,(c^4x^4+2\,bc^3x^3+b^2c^2}{2\,(b^5c^2x^4+2\,b^6cx^3+b^7x^2)}$$

input
```
integrate(1/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/2*(12*b*c^3*x^3 + 18*b^2*c^2*x^2 + 4*b^3*c*x - b^4 - 12*(c^4*x^4 + 2*b*c
^3*x^3 + b^2*c^2*x^2)*log(c*x + b) + 12*(c^4*x^4 + 2*b*c^3*x^3 + b^2*c^2*x
^2)*log(x))/(b^5*c^2*x^4 + 2*b^6*c*x^3 + b^7*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{1}{(bx+cx^2)^3}\,dx = \frac{-b^3+4b^2cx+18bc^2x^2+12c^3x^3}{2b^6x^2+4b^5cx^3+2b^4c^2x^4}+\frac{6c^2\left(\log(x)-\log\left(\frac{b}{c}+x\right)\right)}{b^5}$$

input
```
integrate(1/(c*x**2+b*x)**3,x)
```

output
```
(-b**3 + 4*b**2*c*x + 18*b*c**2*x**2 + 12*c**3*x**3)/(2*b**6*x**2 + 4*b**5
*c*x**3 + 2*b**4*c**2*x**4) + 6*c**2*(log(x) - log(b/c + x))/b**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \frac{1}{(bx+cx^2)^3}\,dx = \frac{12\,c^3x^3+18\,bc^2x^2+4\,b^2cx-b^3}{2\,(b^4c^2x^4+2\,b^5cx^3+b^6x^2)}-\frac{6\,c^2\log(cx+b)}{b^5}+\frac{6\,c^2\log(x)}{b^5}$$

input
```
integrate(1/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/2*(12*c^3*x^3 + 18*b*c^2*x^2 + 4*b^2*c*x - b^3)/(b^4*c^2*x^4 + 2*b^5*c*x
^3 + b^6*x^2) - 6*c^2*log(c*x + b)/b^5 + 6*c^2*log(x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \frac{1}{(bx+cx^2)^3}\,dx = -\frac{6\,c^2\log(|cx+b|)}{b^5} + \frac{6\,c^2\log(|x|)}{b^5} + \frac{12\,c^3x^3+18\,bc^2x^2+4\,b^2cx-b^3}{2\,(cx^2+bx)^2b^4}$$

input
```
integrate(1/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
-6*c^2*log(abs(c*x + b))/b^5 + 6*c^2*log(abs(x))/b^5 + 1/2*(12*c^3*x^3 + 1
8*b*c^2*x^2 + 4*b^2*c*x - b^3)/((c*x^2 + b*x)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 8.65 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{1}{(bx+cx^2)^3}\,dx = \frac{\frac{9\,c^2\,x^2}{b^3}-\frac{1}{2\,b}+\frac{6\,c^3\,x^3}{b^4}+\frac{2\,c\,x}{b^2}}{b^2\,x^2+2\,b\,c\,x^3+c^2\,x^4} - \frac{12\,c^2\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b^5}$$

input
```
int(1/(b*x + c*x^2)^3,x)
```

output
```
((9*c^2*x^2)/b^3 - 1/(2*b) + (6*c^3*x^3)/b^4 + (2*c*x)/b^2)/(b^2*x^2 + c^2
*x^4 + 2*b*c*x^3) - (12*c^2*atanh((2*c*x)/b + 1))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.82

$$\int \frac{1}{(bx+cx^2)^3}\,dx = \frac{-12\log(cx+b)\,b^2c^2x^2-24\log(cx+b)\,b\,c^3x^3-12\log(cx+b)\,c^4x^4+12\log(x)\,b^2c^2x^2+24\log(x)\,b\,c^3x^3}{2b^5x^2\,(c^2x^2+2bcx+b^2)}$$

input
```
int(1/(c*x^2+b*x)^3,x)
```

output
```
( - 12*log(b + c*x)*b**2*c**2*x**2 - 24*log(b + c*x)*b*c**3*x**3 - 12*log(
b + c*x)*c**4*x**4 + 12*log(x)*b**2*c**2*x**2 + 24*log(x)*b*c**3*x**3 + 12
*log(x)*c**4*x**4 - b**4 + 4*b**3*c*x + 12*b**2*c**2*x**2 - 6*c**4*x**4)/(
2*b**5*x**2*(b**2 + 2*b*c*x + c**2*x**2))
```

3.8 $\int \frac{1}{\left(bx+cx^2\right)^4}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 102

$$\int \frac{1}{\left(bx+cx^2\right)^4}\,dx = -\frac{1}{3b^4x^3}+\frac{2c}{b^5x^2}-\frac{10c^2}{b^6x}-\frac{c^3}{3b^4(b+cx)^3}-\frac{2c^3}{b^5(b+cx)^2}-\frac{10c^3}{b^6(b+cx)}-\frac{20c^3\log(x)}{b^7}+\frac{20c^3\log(b+cx)}{b^7}$$

output
```
-1/3/b^4/x^3+2*c/b^5/x^2-10*c^2/b^6/x-1/3*c^3/b^4/(c*x+b)^3-2*c^3/b^5/(c*x
+b)^2-10*c^3/b^6/(c*x+b)-20*c^3*ln(x)/b^7+20*c^3*ln(c*x+b)/b^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.86

$$\int \frac{1}{\left(bx+cx^2\right)^4}\,dx = -\frac{\frac{b(b^5-3b^4cx+15b^3c^2x^2+110b^2c^3x^3+150bc^4x^4+60c^5x^5)}{x^3(b+cx)^3}+60c^3\log(x)-60c^3\log(b+cx)}{3b^7}$$

input
```
Integrate[(b*x + c*x^2)^(-4),x]
```

output
```
-1/3*((b*(b^5 - 3*b^4*c*x + 15*b^3*c^2*x^2 + 110*b^2*c^3*x^3 + 150*b*c^4*x
^4 + 60*c^5*x^5))/(x^3*(b + c*x)^3) + 60*c^3*Log[x] - 60*c^3*Log[b + c*x])
/b^7
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^4}\,dx$$

$$\Big\downarrow \text{1080}$$

$$\int \left(\frac{20c^4}{b^7(b+cx)}-\frac{20c^3}{b^7x}+\frac{10c^4}{b^6(b+cx)^2}+\frac{10c^2}{b^6x^2}+\frac{4c^4}{b^5(b+cx)^3}-\frac{4c}{b^5x^3}+\frac{c^4}{b^4(b+cx)^4}+\frac{1}{b^4x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{20c^3\log(x)}{b^7}+\frac{20c^3\log(b+cx)}{b^7}-\frac{10c^3}{b^6(b+cx)}-\frac{10c^2}{b^6x}-\frac{2c^3}{b^5(b+cx)^2}+\frac{2c}{b^5x^2}-\frac{c^3}{3b^4(b+cx)^3}-\frac{1}{3b^4x^3}$$

input
```
Int[(b*x + c*x^2)^(-4),x]
```

output
```
-1/3*1/(b^4*x^3) + (2*c)/(b^5*x^2) - (10*c^2)/(b^6*x) - c^3/(3*b^4*(b + c*
x)^3) - (2*c^3)/(b^5*(b + c*x)^2) - (10*c^3)/(b^6*(b + c*x)) - (20*c^3*Log
[x])/b^7 + (20*c^3*Log[b + c*x])/b^7
```

Defintions of rubi rules used

rule 1080
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[x^p*
(b + c*x)^p, x], x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

method	result
norman	$\frac{\frac{cx}{b^2}-\frac{1}{3b}-\frac{5c^2x^2}{b^3}+\frac{60c^4x^4}{b^5}+\frac{90c^5x^5}{b^6}+\frac{110c^6x^6}{3b^7}}{(cx+b)^3x^3}-\frac{20c^3\ln(x)}{b^7}+\frac{20c^3\ln(cx+b)}{b^7}$
risch	$\frac{-\frac{20c^5x^5}{b^6}-\frac{50c^4x^4}{b^5}-\frac{110c^3x^3}{3b^4}-\frac{5c^2x^2}{b^3}+\frac{cx}{b^2}-\frac{1}{3b}}{(cx+b)^3x^3}+\frac{20c^3\ln(-cx-b)}{b^7}-\frac{20c^3\ln(x)}{b^7}$
default	$-\frac{1}{3b^4x^3}+\frac{2c}{b^5x^2}-\frac{10c^2}{b^6x}-\frac{c^3}{3b^4(cx+b)^3}-\frac{2c^3}{b^5(cx+b)^2}-\frac{10c^3}{b^6(cx+b)}-\frac{20c^3\ln(x)}{b^7}+\frac{20c^3\ln(cx+b)}{b^7}$
parallelrisch	$-\frac{60\ln(x)x^6c^6-60\ln(cx+b)x^6c^6+180\ln(x)x^5bc^5-180\ln(cx+b)x^5bc^5-110x^6c^6+180\ln(x)x^4b^2c^4-180\ln(cx+b)x^4b^2c^4-2}{3b^7(cx+b)^3x^3}$

input
```
int(1/(c*x^2+b*x)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/b^2*c*x-1/3/b-5*c^2/b^3*x^2+60*c^4/b^5*x^4+90*c^5/b^6*x^5+110/3*c^6/b^7
*x^6)/(c*x+b)^3/x^3-20*c^3*ln(x)/b^7+20*c^3*ln(c*x+b)/b^7
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.79

$$\int \frac{1}{(bx+cx^2)^4}\,dx =$$
$$-\frac{60\,bc^5x^5+150\,b^2c^4x^4+110\,b^3c^3x^3+15\,b^4c^2x^2-3\,b^5cx+b^6-60\,(c^6x^6+3\,bc^5x^5+3\,b^2c^4x^4+b^3c^3x^3)}{3\,(b^7c^3x^6+3\,b^8c^2x^5+3\,b^9cx^4+b^{10}x^3)}$$

input
```
integrate(1/(c*x^2+b*x)^4,x, algorithm="fricas")
```

output
```
-1/3*(60*b*c^5*x^5 + 150*b^2*c^4*x^4 + 110*b^3*c^3*x^3 + 15*b^4*c^2*x^2 -
3*b^5*c*x + b^6 - 60*(c^6*x^6 + 3*b*c^5*x^5 + 3*b^2*c^4*x^4 + b^3*c^3*x^3)
*log(c*x + b) + 60*(c^6*x^6 + 3*b*c^5*x^5 + 3*b^2*c^4*x^4 + b^3*c^3*x^3)*l
og(x))/(b^7*c^3*x^6 + 3*b^8*c^2*x^5 + 3*b^9*c*x^4 + b^10*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int \frac{1}{(bx+cx^2)^4}\,dx = \frac{-b^5+3b^4cx-15b^3c^2x^2-110b^2c^3x^3-150bc^4x^4-60c^5x^5}{3b^9x^3+9b^8cx^4+9b^7c^2x^5+3b^6c^3x^6} + \frac{20c^3\left(-\log\left(x\right)+\log\left(\frac{b}{c}+x\right)\right)}{b^7}$$

input
```
integrate(1/(c*x**2+b*x)**4,x)
```

output
```
(-b**5 + 3*b**4*c*x - 15*b**3*c**2*x**2 - 110*b**2*c**3*x**3 - 150*b*c**4*
x**4 - 60*c**5*x**5)/(3*b**9*x**3 + 9*b**8*c*x**4 + 9*b**7*c**2*x**5 + 3*b
**6*c**3*x**6) + 20*c**3*(-log(x) + log(b/c + x))/b**7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.15

$$\int \frac{1}{(bx+cx^2)^4}\,dx = -\frac{60\,c^5x^5+150\,bc^4x^4+110\,b^2c^3x^3+15\,b^3c^2x^2-3\,b^4cx+b^5}{3\,(b^6c^3x^6+3\,b^7c^2x^5+3\,b^8cx^4+b^9x^3)} + \frac{20\,c^3\log\left(cx+b\right)}{b^7} - \frac{20\,c^3\log\left(x\right)}{b^7}$$

input
```
integrate(1/(c*x^2+b*x)^4,x, algorithm="maxima")
```

output
```
-1/3*(60*c^5*x^5 + 150*b*c^4*x^4 + 110*b^2*c^3*x^3 + 15*b^3*c^2*x^2 - 3*b^
4*c*x + b^5)/(b^6*c^3*x^6 + 3*b^7*c^2*x^5 + 3*b^8*c*x^4 + b^9*x^3) + 20*c^
3*log(c*x + b)/b^7 - 20*c^3*log(x)/b^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.91

$$\int \frac{1}{(bx+cx^2)^4}\,dx = \frac{20\,c^3\log(|cx+b|)}{b^7} - \frac{20\,c^3\log(|x|)}{b^7} - \frac{60\,c^5x^5+150\,bc^4x^4+110\,b^2c^3x^3+15\,b^3c^2x^2-3\,b^4cx+b^5}{3\,(cx^2+bx)^3b^6}$$

input
```
integrate(1/(c*x^2+b*x)^4,x, algorithm="giac")
```

output
```
20*c^3*log(abs(c*x + b))/b^7 - 20*c^3*log(abs(x))/b^7 - 1/3*(60*c^5*x^5 +
150*b*c^4*x^4 + 110*b^2*c^3*x^3 + 15*b^3*c^2*x^2 - 3*b^4*c*x + b^5)/((c*x^
2 + b*x)^3*b^6)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int \frac{1}{(bx+cx^2)^4}\,dx = \frac{40\,c^3\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b^7} - \frac{\frac{1}{3\,b}+\frac{5\,c^2\,x^2}{b^3}+\frac{110\,c^3\,x^3}{3\,b^4}+\frac{50\,c^4\,x^4}{b^5}+\frac{20\,c^5\,x^5}{b^6}-\frac{c\,x}{b^2}}{b^3\,x^3+3\,b^2\,c\,x^4+3\,b\,c^2\,x^5+c^3\,x^6}$$

input
```
int(1/(b*x + c*x^2)^4,x)
```

output
```
(40*c^3*atanh((2*c*x)/b + 1))/b^7 - (1/(3*b) + (5*c^2*x^2)/b^3 + (110*c^3*
x^3)/(3*b^4) + (50*c^4*x^4)/b^5 + (20*c^5*x^5)/b^6 - (c*x)/b^2)/(b^3*x^3 +
 c^3*x^6 + 3*b^2*c*x^4 + 3*b*c^2*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.97

$$\int \frac{1}{(bx+cx^2)^4}\,dx$$
$$= \frac{60\log(cx+b)\,b^3c^3x^3 + 180\log(cx+b)\,b^2c^4x^4 + 180\log(cx+b)\,b\,c^5x^5 + 60\log(cx+b)\,c^6x^6 - 60\log(x)\,b}{3b^7x^3\,(c^3x}$$

input
```
int(1/(c*x^2+b*x)^4,x)
```

output
```
(60*log(b + c*x)*b**3*c**3*x**3 + 180*log(b + c*x)*b**2*c**4*x**4 + 180*lo
g(b + c*x)*b*c**5*x**5 + 60*log(b + c*x)*c**6*x**6 - 60*log(x)*b**3*c**3*x
**3 - 180*log(x)*b**2*c**4*x**4 - 180*log(x)*b*c**5*x**5 - 60*log(x)*c**6*
x**6 - b**6 + 3*b**5*c*x - 15*b**4*c**2*x**2 - 90*b**3*c**3*x**3 - 90*b**2
*c**4*x**4 + 20*c**6*x**6)/(3*b**7*x**3*(b**3 + 3*b**2*c*x + 3*b*c**2*x**2
 + c**3*x**3))
```

3.9 $\int (bx + cx^2)^{5/2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 175

$$\int (bx + cx^2)^{5/2} \, dx = \frac{5b^5\sqrt{bx+cx^2}}{512c^3} - \frac{5b^4x\sqrt{bx+cx^2}}{768c^2} + \frac{b^3x^2\sqrt{bx+cx^2}}{192c} + \frac{9}{32}b^2x^3\sqrt{bx+cx^2} + \frac{5}{12}bcx^4\sqrt{bx+cx^2} + \frac{1}{6}c^2x^5\sqrt{bx+cx^2} - \frac{5b^6\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{512c^{7/2}}$$

output
```
5/512*b^5*(c*x^2+b*x)^(1/2)/c^3-5/768*b^4*x*(c*x^2+b*x)^(1/2)/c^2+1/192*b^
3*x^2*(c*x^2+b*x)^(1/2)/c+9/32*b^2*x^3*(c*x^2+b*x)^(1/2)+5/12*b*c*x^4*(c*x
^2+b*x)^(1/2)+1/6*c^2*x^5*(c*x^2+b*x)^(1/2)-5/512*b^6*arctanh(c^(1/2)*x/(c
*x^2+b*x)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.74

$$\int (bx + cx^2)^{5/2} \, dx = \frac{\sqrt{x(b+cx)}\left(\sqrt{c}(15b^5 - 10b^4cx + 8b^3c^2x^2 + 432b^2c^3x^3 + 640bc^4x^4 + 256c^5x^5) + \frac{30b^6\operatorname{arctanh}}{\sqrt{x}\sqrt{}}\right.}{1536c^{7/2}}$$

input
```
Integrate[(b*x + c*x^2)^(5/2),x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[c]*(15*b^5 - 10*b^4*c*x + 8*b^3*c^2*x^2 + 432*b^2
*c^3*x^3 + 640*b*c^4*x^4 + 256*c^5*x^5) + (30*b^6*ArcTanh[(Sqrt[c]*Sqrt[x]
)/(Sqrt[b] - Sqrt[b + c*x])])/(Sqrt[x]*Sqrt[b + c*x])))/(1536*c^(7/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1087, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\int \left(cx^2+bx\right)^{3/2}dx}{24c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int \sqrt{cx^2+bx}dx}{16c}\right)}{24c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)}{24c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(b+2cx)(bx+cx^2)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)}{24c}$$

$$\Big\downarrow 219$$

$$\frac{(b+2cx)(bx+cx^2)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)}{24c}$$

input
```
Int[(b*x + c*x^2)^(5/2),x]
```

output
```
((b + 2*c*x)*(b*x + c*x^2)^(5/2))/(12*c) - (5*b^2*(((b + 2*c*x)*(b*x + c*x
^2)^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*Ar
cTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))/(16*c)))/(24*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{\left(256c^{5}x^{5}+640b\,x^{4}c^{4}+432b^{2}c^{3}x^{3}+8c^{2}x^{2}b^{3}-10b^{4}cx+15b^{5}\right)x(cx+b)}{1536c^{3}\sqrt{x(cx+b)}}-\frac{5b^{6}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{1024c^{\frac{7}{2}}}$	106
default	$\frac{(2cx+b)\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{12c}-\frac{5b^{2}\left(\frac{(2cx+b)\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{8c}-\frac{3b^{2}\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx}}{4c}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{24c}$	118

input
```
int((c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/1536*(256*c^5*x^5+640*b*c^4*x^4+432*b^2*c^3*x^3+8*b^3*c^2*x^2-10*b^4*c*x
+15*b^5)*x*(c*x+b)/c^3/(x*(c*x+b))^(1/2)-5/1024*b^6/c^(7/2)*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.22

$$\int (bx+cx^2)^{5/2}\,dx = \left[\frac{15\,b^6\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,(256\,c^6x^5+640\,bc^5x^4+432\,b^2c^4x^3+8\,b^3c^3x^2}{3072\,c^4}\right.$$

input
```
integrate((c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/3072*(15*b^6*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(
256*c^6*x^5 + 640*b*c^5*x^4 + 432*b^2*c^4*x^3 + 8*b^3*c^3*x^2 - 10*b^4*c^2
*x + 15*b^5*c)*sqrt(c*x^2 + b*x))/c^4, 1/1536*(15*b^6*sqrt(-c)*arctan(sqrt
(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (256*c^6*x^5 + 640*b*c^5*x^4 + 432*b^2
*c^4*x^3 + 8*b^3*c^3*x^2 - 10*b^4*c^2*x + 15*b^5*c)*sqrt(c*x^2 + b*x))/c^4
]
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.62

$$\int \left(bx+cx^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x**2+b*x)**(5/2),x)
```

output
```
b**2*Piecewise((-5*b**4*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) +
2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/
(2*c) + x)**2), True))/(128*c**3) + sqrt(b*x + c*x**2)*(5*b**3/(64*c**3) -
 5*b**2*x/(96*c**2) + b*x**2/(24*c) + x**3/4), Ne(c, 0)), (2*(b*x)**(7/2)/
(7*b**3), Ne(b, 0)), (0, True)) + 2*b*c*Piecewise((7*b**5*Piecewise((log(b
 + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c
) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(256*c**4) + sqrt
(b*x + c*x**2)*(-7*b**4/(128*c**4) + 7*b**3*x/(192*c**3) - 7*b**2*x**2/(24
0*c**2) + b*x**3/(40*c) + x**4/5), Ne(c, 0)), (2*(b*x)**(9/2)/(9*b**4), Ne
(b, 0)), (0, True)) + c**2*Piecewise((-21*b**6*Piecewise((log(b + 2*sqrt(c
)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(
b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(1024*c**5) + sqrt(b*x + c*x
**2)*(21*b**5/(512*c**5) - 7*b**4*x/(256*c**4) + 7*b**3*x**2/(320*c**3) -
3*b**2*x**3/(160*c**2) + b*x**4/(60*c) + x**5/6), Ne(c, 0)), (2*(b*x)**(11
/2)/(11*b**5), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int \left(bx+cx^2\right)^{5/2} dx = \frac{1}{6}\left(cx^2+bx\right)^{\frac{5}{2}}x + \frac{5\sqrt{cx^2+bx}b^4x}{256\,c^2} - \frac{5\left(cx^2+bx\right)^{\frac{3}{2}}b^2x}{96\,c} - \frac{5\,b^6\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{1024\,c^{\frac{7}{2}}} + \frac{5\sqrt{cx^2+bx}b^5}{512\,c^3} - \frac{5\left(cx^2+bx\right)^{\frac{3}{2}}b^3}{192\,c^2} + \frac{\left(cx^2+bx\right)^{\frac{5}{2}}b}{12\,c}$$

input
```
integrate((c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(c*x^2 + b*x)^(5/2)*x + 5/256*sqrt(c*x^2 + b*x)*b^4*x/c^2 - 5/96*(c*x^
2 + b*x)^(3/2)*b^2*x/c - 5/1024*b^6*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sq
rt(c))/c^(7/2) + 5/512*sqrt(c*x^2 + b*x)*b^5/c^3 - 5/192*(c*x^2 + b*x)^(3/
2)*b^3/c^2 + 1/12*(c*x^2 + b*x)^(5/2)*b/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.60

$$\int \left(bx+cx^2\right)^{5/2} dx = \frac{5\,b^6\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{1024\,c^{\frac{7}{2}}} + \frac{1}{1536}\sqrt{cx^2+bx}\left(\frac{15\,b^5}{c^3}-2\left(\frac{5\,b^4}{c^2}-4\left(\frac{b^3}{c}+2\left(27\,b^2+8\left(2\,c^2x+5\,bc\right)x\right)x\right)x\right)x\right)$$

input
```
integrate((c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
5/1024*b^6*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(7/2)
 + 1/1536*sqrt(c*x^2 + b*x)*(15*b^5/c^3 - 2*(5*b^4/c^2 - 4*(b^3/c + 2*(27*
b^2 + 8*(2*c^2*x + 5*b*c)*x)*x)*x)*x)
```

Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.68

$$\int \left(bx+cx^2\right)^{5/2}\,dx=\frac{\left(c\,x^2+b\,x\right)^{5/2}\,\left(\frac{b}{2}+c\,x\right)}{6\,c}-\frac{5\,b^2\,\left(\frac{\left(c\,x^2+b\,x\right)^{3/2}\,\left(\frac{b}{2}+c\,x\right)}{4\,c}-\frac{3\,b^2\,\left(\sqrt{c\,x^2+b\,x}\,\left(\frac{x}{2}+\frac{b}{4\,c}\right)-\frac{b^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}\right)}{16\,c}\right)}{24\,c}$$

input
```
int((b*x + c*x^2)^(5/2),x)
```

output
```
((b*x + c*x^2)^(5/2)*(b/2 + c*x))/(6*c) - (5*b^2*(((b*x + c*x^2)^(3/2)*(b/
2 + c*x))/(4*c) - (3*b^2*((b*x + c*x^2)^(1/2)*(x/2 + b/(4*c)) - (b^2*log((
b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/(8*c^(3/2))))/(16*c)))/(24*c)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.76

$$\int \left(bx+cx^2\right)^{5/2}\,dx=\frac{15\sqrt{x}\,\sqrt{cx+b}\,b^5c-10\sqrt{x}\,\sqrt{cx+b}\,b^4c^2x+8\sqrt{x}\,\sqrt{cx+b}\,b^3c^3x^2+432\sqrt{x}\,\sqrt{cx+b}\,b^2c^4x^3+}{1536c^4}$$

input
```
int((c*x^2+b*x)^(5/2),x)
```

output
```
(15*sqrt(x)*sqrt(b + c*x)*b**5*c - 10*sqrt(x)*sqrt(b + c*x)*b**4*c**2*x +
8*sqrt(x)*sqrt(b + c*x)*b**3*c**3*x**2 + 432*sqrt(x)*sqrt(b + c*x)*b**2*c*
*4*x**3 + 640*sqrt(x)*sqrt(b + c*x)*b*c**5*x**4 + 256*sqrt(x)*sqrt(b + c*x
)*c**6*x**5 - 15*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b*
*6)/(1536*c**4)
```

3.10 $\int (bx + cx^2)^{3/2}\, dx$

Optimal result

Integrand size = 13, antiderivative size = 123

$$\int (bx+cx^2)^{3/2}\, dx = -\frac{3b^3\sqrt{bx+cx^2}}{64c^2}+\frac{b^2x\sqrt{bx+cx^2}}{32c} + \frac{3}{8}bx^2\sqrt{bx+cx^2}+\frac{1}{4}cx^3\sqrt{bx+cx^2}+\frac{3b^4\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{5/2}}$$

output
```
-3/64*b^3*(c*x^2+b*x)^(1/2)/c^2+1/32*b^2*x*(c*x^2+b*x)^(1/2)/c+3/8*b*x^2*(
c*x^2+b*x)^(1/2)+1/4*c*x^3*(c*x^2+b*x)^(1/2)+3/64*b^4*arctanh(c^(1/2)*x/(c
*x^2+b*x)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.87

$$\int (bx+cx^2)^{3/2}\, dx = \frac{\sqrt{x(b+cx)}\left(\sqrt{c}(-3b^3+2b^2cx+24bc^2x^2+16c^3x^3)+\frac{6b^4\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{x}}{-\sqrt{b}+\sqrt{b+cx}}\right)}{\sqrt{x}\sqrt{b+cx}}\right)}{64c^{5/2}}$$

input
```
Integrate[(b*x + c*x^2)^(3/2),x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[c]*(-3*b^3 + 2*b^2*c*x + 24*b*c^2*x^2 + 16*c^3*x^
3) + (6*b^4*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])])/(Sqrt[x
]*Sqrt[b + c*x])))/(64*c^(5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{3/2}dx$$

$$\Big\downarrow \ 1087$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int \sqrt{cx^2+bx}dx}{16c}$$

$$\Big\downarrow \ 1087$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}$$

$$\Big\downarrow \ 1091$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}$$

$$\Big\downarrow \ 219$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}$$

input
```
Int[(b*x + c*x^2)^(3/2),x]
```

output
```
((b + 2*c*x)*(b*x + c*x^2)^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x)*Sqrt[b*x +
c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))
/(16*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\frac{\frac{3\,\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)b^{4}}{64}-\frac{3\left(\sqrt{c}\,b^{3}-\frac{2c^{\frac{3}{2}}b^{2}x}{3}-8c^{\frac{5}{2}}b\,x^{2}-\frac{16c^{\frac{7}{2}}x^{3}}{3}\right)\sqrt{x(cx+b)}}{64}}{c^{\frac{5}{2}}}$	73
risch	$-\frac{\left(-16c^{3}x^{3}-24b\,c^{2}x^{2}-2b^{2}cx+3b^{3}\right)x(cx+b)}{64c^{2}\sqrt{x(cx+b)}}+\frac{3b^{4}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{128c^{\frac{5}{2}}}$	84
default	$\frac{(2cx+b)\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{8c}-\frac{3b^{2}\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx}}{4c}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}$	87

input
```
int((c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/64/c^(5/2)*(arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))*b^4-(c^(1/2)*b^3-2/3*c^
(3/2)*b^2*x-8*c^(5/2)*b*x^2-16/3*c^(7/2)*x^3)*(x*(c*x+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.39

$$\int (bx+cx^2)^{3/2}\,dx = \left[\frac{3\,b^4\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,(16\,c^4x^3+24\,bc^3x^2+2\,b^2c^2x-3\,b^3c)\sqrt{cx^2+bx}}{128\,c^3}, -\frac{3\,b^4\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(16\,c^4x^3+24\,bc^3x^2+2\,b^2c^2x-3\,b^3c)\sqrt{cx^2+bx}}{64\,c^3}\right]$$

input
```
integrate((c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[1/128*(3*b^4*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(16
*c^4*x^3 + 24*b*c^3*x^2 + 2*b^2*c^2*x - 3*b^3*c)*sqrt(c*x^2 + b*x))/c^3, -
1/64*(3*b^4*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) - (16*c^
4*x^3 + 24*b*c^3*x^2 + 2*b^2*c^2*x - 3*b^3*c)*sqrt(c*x^2 + b*x))/c^3]
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.09

$$\int (bx+cx^2)^{3/2}\,dx = b\left(\begin{cases} \frac{b^3\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{16c^2}+\sqrt{bx+cx^2}\left(-\frac{b^2}{8c^2}+\frac{bx}{12c}+\frac{x^2}{3}\right) & \text{for } c\neq 0\\ \frac{2(bx)^{\frac{5}{2}}}{5b^2} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right) + c\left(\begin{cases} -\frac{5b^4\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{128c^3}+\sqrt{bx+cx^2}\cdot\left(\frac{5b^3}{64c^3}-\frac{5b^2x}{96c^2}+\frac{bx^2}{24c}+\frac{x^3}{4}\right) & \text{for } c\neq 0\\ \frac{2(bx)^{\frac{7}{2}}}{7b^3} & \text{for } b\neq 0\\ 0 & \text{otherwise}\end{cases}\right)$$

input
```
integrate((c*x**2+b*x)**(3/2),x)
```

output
```
b*Piecewise((b**3*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)
/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c)
+ x)**2), True))/(16*c**2) + sqrt(b*x + c*x**2)*(-b**2/(8*c**2) + b*x/(12*
c) + x**2/3), Ne(c, 0)), (2*(b*x)**(5/2)/(5*b**2), Ne(b, 0)), (0, True)) +
 c*Piecewise((-5*b**4*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*
c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2
*c) + x)**2), True))/(128*c**3) + sqrt(b*x + c*x**2)*(5*b**3/(64*c**3) - 5
*b**2*x/(96*c**2) + b*x**2/(24*c) + x**3/4), Ne(c, 0)), (2*(b*x)**(7/2)/(7
*b**3), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.83

$$\int \left(bx+cx^2\right)^{3/2}\,dx=\frac{1}{4}\left(cx^2+bx\right)^{\frac{3}{2}}x-\frac{3\sqrt{cx^2+bx}b^2x}{32\,c}+\frac{3\,b^4\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{128\,c^{\frac{5}{2}}}-\frac{3\sqrt{cx^2+bx}b^3}{64\,c^2}+\frac{\left(cx^2+bx\right)^{\frac{3}{2}}b}{8\,c}$$

input
```
integrate((c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(c*x^2 + b*x)^(3/2)*x - 3/32*sqrt(c*x^2 + b*x)*b^2*x/c + 3/128*b^4*log
(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(5/2) - 3/64*sqrt(c*x^2 + b*x)
*b^3/c^2 + 1/8*(c*x^2 + b*x)^(3/2)*b/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.66

$$\int \left(bx+cx^2\right)^{3/2}\,dx=-\frac{3\,b^4\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{5}{2}}}+\frac{1}{64}\sqrt{cx^2+bx}\left(2\left(4\,(2\,cx+3\,b)x+\frac{b^2}{c}\right)x-\frac{3\,b^3}{c^2}\right)$$

input
```
integrate((c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
-3/128*b^4*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(5/2)
 + 1/64*sqrt(c*x^2 + b*x)*(2*(4*(2*c*x + 3*b)*x + b^2/c)*x - 3*b^3/c^2)
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

$$\int (bx+cx^2)^{3/2}\,dx = \frac{(c\,x^2+b\,x)^{3/2}\,\left(\frac{b}{2}+c\,x\right)}{4\,c} - \frac{3\,b^2\left(\sqrt{c\,x^2+b\,x}\,\left(\frac{x}{2}+\frac{b}{4\,c}\right)-\frac{b^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}\right)}{16\,c}$$

input

```
int((b*x + c*x^2)^(3/2),x)
```

output

```
((b*x + c*x^2)^(3/2)*(b/2 + c*x))/(4*c) - (3*b^2*((b*x + c*x^2)^(1/2)*(x/2
 + b/(4*c)) - (b^2*log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/(8*c^(3
/2))))/(16*c)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int (bx+cx^2)^{3/2}\,dx = \frac{-3\sqrt{x}\,\sqrt{cx+b}\,b^3c+2\sqrt{x}\,\sqrt{cx+b}\,b^2c^2x+24\sqrt{x}\,\sqrt{cx+b}\,b\,c^3x^2+16\sqrt{x}\,\sqrt{cx+b}\,c^4x^3+3\sqrt{}}{64c^3}$$

input

```
int((c*x^2+b*x)^(3/2),x)
```

output

```
( - 3*sqrt(x)*sqrt(b + c*x)*b**3*c + 2*sqrt(x)*sqrt(b + c*x)*b**2*c**2*x +
 24*sqrt(x)*sqrt(b + c*x)*b*c**3*x**2 + 16*sqrt(x)*sqrt(b + c*x)*c**4*x**3
 + 3*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**4)/(64*c**3
)
```

3.11 $\int \sqrt{bx+cx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \sqrt{bx+cx^2}\,dx=\frac{b\sqrt{bx+cx^2}}{4c}+\frac{1}{2}x\sqrt{bx+cx^2}-\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}$$

output
```
1/4*b*(c*x^2+b*x)^(1/2)/c+1/2*x*(c*x^2+b*x)^(1/2)-1/4*b^2*arctanh(c^(1/2)*
x/(c*x^2+b*x)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \sqrt{bx+cx^2}\,dx=\frac{\sqrt{x(b+cx)}\left(\sqrt{c}(b+2cx)+\frac{2b^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}-\sqrt{b+cx}}\right)}{\sqrt{x}\sqrt{b+cx}}\right)}{4c^{3/2}}$$

input
```
Integrate[Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[c]*(b + 2*c*x) + (2*b^2*ArcTanh[(Sqrt[c]*Sqrt[x])
/(Sqrt[b] - Sqrt[b + c*x])])/(Sqrt[x]*Sqrt[b + c*x])))/(4*c^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{bx+cx^2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^2+bx}}dx}{8c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}$$

$$\Big\downarrow \text{219}$$

$$\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}$$

input
```
Int[Sqrt[b*x + c*x^2],x]
```

output
```
((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x
+ c*x^2]])/(4*c^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{(2cx+b)\sqrt{c\,x^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{8c^{\frac{3}{2}}}$	56
pseudoelliptic	$\frac{2c^{\frac{3}{2}}\sqrt{x(cx+b)}\,x+b\sqrt{c}\,\sqrt{x(cx+b)}-\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)b^2}{4c^{\frac{3}{2}}}$	58
risch	$\frac{(2cx+b)x(cx+b)}{4c\sqrt{x(cx+b)}}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{8c^{\frac{3}{2}}}$	60

input
```
int((c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*x+b)/c*(c*x^2+b*x)^(1/2)-1/8*b^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.67

$$\int\sqrt{bx+cx^2}\,dx$$

$$=\left[\frac{b^2\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,(2\,c^2x+bc)\sqrt{cx^2+bx}}{8\,c^2},\frac{b^2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)+(2\,c^2x}{4\,c^2}\right.$$

input
```
integrate((c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(b^2*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(2*c^2*
x + b*c)*sqrt(c*x^2 + b*x))/c^2, 1/4*(b^2*sqrt(-c)*arctan(sqrt(c*x^2 + b*x
)*sqrt(-c)/(c*x + b)) + (2*c^2*x + b*c)*sqrt(c*x^2 + b*x))/c^2]
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \sqrt{bx + cx^2}\,dx = \begin{cases} -\frac{b^2\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c} \neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{8c} + \left(\frac{b}{4c}+\frac{x}{2}\right)\sqrt{bx+cx^2} & \text{for } c \neq 0 \\ \frac{2(bx)^{\frac{3}{2}}}{3b} & \text{for } b \neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((-b**2*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/
sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) +
 x)**2), True))/(8*c) + (b/(4*c) + x/2)*sqrt(b*x + c*x**2), Ne(c, 0)), (2*
(b*x)**(3/2)/(3*b), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \sqrt{bx+cx^2}\,dx = \frac{1}{2}\sqrt{cx^2+bx}x - \frac{b^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^2+bx}b}{4\,c}$$

input
```
integrate((c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^2 + b*x)*x - 1/8*b^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt
(c))/c^(3/2) + 1/4*sqrt(c*x^2 + b*x)*b/c
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \sqrt{bx+cx^2}\,dx = \frac{1}{4}\sqrt{cx^2+bx}\left(2\,x+\frac{b}{c}\right) + \frac{b^2\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{3}{2}}}$$

input
```
integrate((c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x)*(2*x + b/c) + 1/8*b^2*log(abs(2*(sqrt(c)*x - sqrt(c*
x^2 + b*x))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \sqrt{bx+cx^2}\,dx = \sqrt{c\,x^2+b\,x}\,\left(\frac{x}{2}+\frac{b}{4\,c}\right) - \frac{b^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}$$

input
```
int((b*x + c*x^2)^(1/2),x)
```

output
```
(b*x + c*x^2)^(1/2)*(x/2 + b/(4*c)) - (b^2*log((b/2 + c*x)/c^(1/2) + (b*x
+ c*x^2)^(1/2)))/(8*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \sqrt{bx + cx^2}\, dx = \frac{\sqrt{x}\,\sqrt{cx+b}\,bc + 2\sqrt{x}\,\sqrt{cx+b}\,c^2x - \sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^2}{4c^2}$$

input
```
int((c*x^2+b*x)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(b + c*x)*b*c + 2*sqrt(x)*sqrt(b + c*x)*c**2*x - sqrt(c)*log(
(sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**2)/(4*c**2)
```

3.12 $\int \frac{1}{\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}$$

output
```
2*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = -\frac{2\sqrt{x}\sqrt{b+cx}\log\left(-\sqrt{c}\sqrt{x}+\sqrt{b+cx}\right)}{\sqrt{c}\sqrt{x(b+cx)}}$$

input
```
Integrate[1/Sqrt[b*x + c*x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[b + c*x]*Log[-(Sqrt[c]*Sqrt[x]) + Sqrt[b + c*x]])/(Sqrt[c
]*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1091}$$

$$2\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}$$

input
```
Int[1/Sqrt[b*x + c*x^2],x]
```

output
```
(2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{\sqrt{c}}$	23
default	$\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{\sqrt{c}}$	29

input
```
int(1/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/c^(1/2)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = \left[\frac{\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{\sqrt{c}}, -\frac{2\,\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)}{c}\right]$$

input
```
integrate(1/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/sqrt(c), -2*sqrt(-c)*arctan(
sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b))/c]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(26) = 52$.

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = \begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } c \neq 0 \wedge \frac{b^2}{c} \neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{for } c \neq 0 \\ \frac{2\sqrt{bx}}{b} & \text{for } b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(c, 0)
 & Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2
), Ne(c, 0)), (2*sqrt(b*x)/b, Ne(b, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = \frac{\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{\sqrt{c}}$$

input
```
integrate(1/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/sqrt(c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(22) = 44$.

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = \frac{1}{4}\sqrt{cx^2+bx}\left(2\,x+\frac{b}{c}\right)+\frac{b^2\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{3}{2}}}$$

input
```
integrate(1/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x)*(2*x + b/c) + 1/8*b^2*log(abs(2*(sqrt(c)*x - sqrt(c*
x^2 + b*x))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = \frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{\sqrt{c}}$$

input
```
int(1/(b*x + c*x^2)^(1/2),x)
```

output
```
log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2))/c^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = \frac{2\sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)}{c}$$

input
```
int(1/(c*x^2+b*x)^(1/2),x)
```

output
```
(2*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b)))/c
```

3.13 $\int \frac{1}{(bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx = \frac{2}{b\sqrt{bx+cx^2}} - \frac{4\sqrt{bx+cx^2}}{b^2x}$$

output
```
2/b/(c*x^2+b*x)^(1/2)-4*(c*x^2+b*x)^(1/2)/b^2/x
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.55

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx = -\frac{2(b+2cx)}{b^2\sqrt{x(b+cx)}}$$

input
```
Integrate[(b*x + c*x^2)^(-3/2),x]
```

output
```
(-2*(b + 2*c*x))/(b^2*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1088}$$

$$-\frac{2(b+2cx)}{b^2\sqrt{bx+cx^2}}$$

input
```
Int[(b*x + c*x^2)^(-3/2),x]
```

output
```
(-2*(b + 2*c*x))/(b^2*Sqrt[b*x + c*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
pseudoelliptic	$-\frac{2(2cx+b)}{b^{2}\sqrt{x(cx+b)}}$	21
default	$-\frac{2(2cx+b)}{b^{2}\sqrt{c\,x^{2}+bx}}$	23
gosper	$-\frac{2x(2cx+b)(cx+b)}{b^{2}(c\,x^{2}+bx)^{\frac{3}{2}}}$	29
orering	$-\frac{2x(2cx+b)(cx+b)}{b^{2}(c\,x^{2}+bx)^{\frac{3}{2}}}$	29
trager	$-\frac{2(2cx+b)\sqrt{c\,x^{2}+bx}}{(cx+b)b^{2}x}$	33
risch	$-\frac{2(cx+b)}{b^{2}\sqrt{x(cx+b)}}-\frac{2cx}{\sqrt{x(cx+b)}\,b^{2}}$	37

input
```
int(1/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(2*c*x+b)/b^2/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx = -\frac{2\,\sqrt{cx^2+bx}(2\,cx+b)}{b^2cx^2+b^3x}$$

input
```
integrate(1/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(c*x^2 + b*x)*(2*c*x + b)/(b^2*c*x^2 + b^3*x)
```

Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((b*x + c*x**2)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx = -\frac{4\,cx}{\sqrt{cx^2+bx}b^2} - \frac{2}{\sqrt{cx^2+bx}b}$$

input
```
integrate(1/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
-4*c*x/(sqrt(c*x^2 + b*x)*b^2) - 2/(sqrt(c*x^2 + b*x)*b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx = -\frac{2\left(\frac{2\,cx}{b^2}+\frac{1}{b}\right)}{\sqrt{cx^2+bx}}$$

input
```
integrate(1/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
-2*(2*c*x/b^2 + 1/b)/sqrt(c*x^2 + b*x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx = -\frac{2\,b+4\,c\,x}{b^2\,\sqrt{c\,x^2+b\,x}}$$

input
```
int(1/(b*x + c*x^2)^(3/2),x)
```

output
```
-(2*b + 4*c*x)/(b^2*(b*x + c*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx = \frac{-4\sqrt{c}\,\sqrt{cx+b}\,x-2\sqrt{x}\,b-4\sqrt{x}\,cx}{\sqrt{cx+b}\,b^2x}$$

input
```
int(1/(c*x^2+b*x)^(3/2),x)
```

output
```
(2*( - 2*sqrt(c)*sqrt(b + c*x)*x - sqrt(x)*b - 2*sqrt(x)*c*x))/(sqrt(b + c
*x)*b**2*x)
```

3.14 $\int \frac{1}{(bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 89

$$\int \frac{1}{(bx+cx^2)^{5/2}}\,dx = \frac{2}{3b\,(bx+cx^2)^{3/2}} + \frac{4}{b^2x\sqrt{bx+cx^2}} - \frac{16\sqrt{bx+cx^2}}{3b^3x^2} + \frac{32c\sqrt{bx+cx^2}}{3b^4x}$$

output
```
2/3/b/(c*x^2+b*x)^(3/2)+4/b^2/x/(c*x^2+b*x)^(1/2)-16/3*(c*x^2+b*x)^(1/2)/b
^3/x^2+32/3*c*(c*x^2+b*x)^(1/2)/b^4/x
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.54

$$\int \frac{1}{(bx+cx^2)^{5/2}}\,dx = \frac{-2b^3+12b^2cx+48bc^2x^2+32c^3x^3}{3b^4(x(b+cx))^{3/2}}$$

input
```
Integrate[(b*x + c*x^2)^(-5/2),x]
```

output
```
(-2*b^3 + 12*b^2*c*x + 48*b*c^2*x^2 + 32*c^3*x^3)/(3*b^4*(x*(b + c*x))^(3/
2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1089}$$

$$-\frac{8c\int \frac{1}{(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{16c(b+2cx)}{3b^4\sqrt{bx+cx^2}}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}$$

input
```
Int[(b*x + c*x^2)^(-5/2),x]
```

output
```
(-2*(b + 2*c*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (16*c*(b + 2*c*x))/(3*b^4*S
qrt[b*x + c*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{2(2cx+b)}{3b^2(c\,x^2+bx)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{c\,x^2+bx}}$	47
pseudoelliptic	$-\frac{2(2cx+b)\left(-8c^2x^2-8cbx+b^2\right)}{3\sqrt{x(cx+b)}\,x(cx+b)b^4}$	48
gosper	$-\frac{2x(cx+b)\left(-16c^3x^3-24b\,c^2x^2-6b^2cx+b^3\right)}{3b^4(c\,x^2+bx)^{\frac{5}{2}}}$	51
orering	$-\frac{2x(cx+b)\left(-16c^3x^3-24b\,c^2x^2-6b^2cx+b^3\right)}{3b^4(c\,x^2+bx)^{\frac{5}{2}}}$	51
trager	$-\frac{2\left(-16c^3x^3-24b\,c^2x^2-6b^2cx+b^3\right)\sqrt{c\,x^2+bx}}{3b^4x^2(cx+b)^2}$	55
risch	$-\frac{2(cx+b)(-8cx+b)}{3b^4x\sqrt{x(cx+b)}}+\frac{2c^2(8cx+9b)x}{3\sqrt{x(cx+b)}\,(cx+b)b^4}$	63

input
```
int(1/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(2*c*x+b)/b^2/(c*x^2+b*x)^(3/2)+16/3*c/b^4*(2*c*x+b)/(c*x^2+b*x)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \frac{1}{(bx+cx^2)^{5/2}}\,dx=\frac{2\left(16\,c^3x^3+24\,bc^2x^2+6\,b^2cx-b^3\right)\sqrt{cx^2+bx}}{3\left(b^4c^2x^4+2\,b^5cx^3+b^6x^2\right)}$$

input
```
integrate(1/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(16*c^3*x^3 + 24*b*c^2*x^2 + 6*b^2*c*x - b^3)*sqrt(c*x^2 + b*x)/(b^4*c
^2*x^4 + 2*b^5*c*x^3 + b^6*x^2)
```

Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{5/2}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral((b*x + c*x**2)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \frac{1}{(bx+cx^2)^{5/2}}\,dx = -\frac{4\,cx}{3\,(cx^2+bx)^{\frac{3}{2}}b^2} + \frac{32\,c^2x}{3\,\sqrt{cx^2+bx}b^4} - \frac{2}{3\,(cx^2+bx)^{\frac{3}{2}}b} + \frac{16\,c}{3\,\sqrt{cx^2+bx}b^3}$$

input
```
integrate(1/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
-4/3*c*x/((c*x^2 + b*x)^(3/2)*b^2) + 32/3*c^2*x/(sqrt(c*x^2 + b*x)*b^4) -
2/3/((c*x^2 + b*x)^(3/2)*b) + 16/3*c/(sqrt(c*x^2 + b*x)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.56

$$\int \frac{1}{(bx+cx^2)^{5/2}}\,dx = \frac{2\left(2\left(4\,x\left(\frac{2\,c^3x}{b^4}+\frac{3\,c^2}{b^3}\right)+\frac{3\,c}{b^2}\right)x-\frac{1}{b}\right)}{3\,(cx^2+bx)^{\frac{3}{2}}}$$

input
```
integrate(1/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
2/3*(2*(4*x*(2*c^3*x/b^4 + 3*c^2/b^3) + 3*c/b^2)*x - 1/b)/(c*x^2 + b*x)^(3
/2)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.48

$$\int \frac{1}{\left(bx+cx^2\right)^{5/2}}\,dx=\frac{\left(2\,b+4\,c\,x\right)\,\left(-b^2+8\,b\,c\,x+8\,c^2\,x^2\right)}{3\,b^4\,{\left(c\,x^2+b\,x\right)}^{3/2}}$$

input
```
int(1/(b*x + c*x^2)^(5/2),x)
```

output
```
((2*b + 4*c*x)*(8*c^2*x^2 - b^2 + 8*b*c*x))/(3*b^4*(b*x + c*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{1}{\left(bx+cx^2\right)^{5/2}}\,dx=\frac{-\frac{32\sqrt{c}\,\sqrt{cx+b}\,bc\,x^2}{3}-\frac{32\sqrt{c}\,\sqrt{cx+b}\,c^2x^3}{3}-\frac{2\sqrt{x}\,b^3}{3}+4\sqrt{x}\,b^2cx+16\sqrt{x}\,b\,c^2x^2+\frac{32\sqrt{x}\,c^3x^3}{3}}{\sqrt{cx+b}\,b^4x^2\left(cx+b\right)}$$

input
```
int(1/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*( - 16*sqrt(c)*sqrt(b + c*x)*b*c*x**2 - 16*sqrt(c)*sqrt(b + c*x)*c**2*x
**3 - sqrt(x)*b**3 + 6*sqrt(x)*b**2*c*x + 24*sqrt(x)*b*c**2*x**2 + 16*sqrt
(x)*c**3*x**3))/(3*sqrt(b + c*x)*b**4*x**2*(b + c*x))
```

3.15 $\int \frac{1}{(bx+cx^2)^{7/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = \frac{2}{5b(bx+cx^2)^{5/2}} + \frac{4}{3b^2x(bx+cx^2)^{3/2}} + \frac{32}{3b^3x^2\sqrt{bx+cx^2}} - \frac{64\sqrt{bx+cx^2}}{5b^4x^3} + \frac{256c\sqrt{bx+cx^2}}{15b^5x^2} - \frac{512c^2\sqrt{bx+cx^2}}{15b^6x}$$

output
```
2/5/b/(c*x^2+b*x)^(5/2)+4/3/b^2/x/(c*x^2+b*x)^(3/2)+32/3/b^3/x^2/(c*x^2+b*
x)^(1/2)-64/5*(c*x^2+b*x)^(1/2)/b^4/x^3+256/15*c*(c*x^2+b*x)^(1/2)/b^5/x^2
-512/15*c^2*(c*x^2+b*x)^(1/2)/b^6/x
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.50

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = -\frac{2(3b^5-10b^4cx+80b^3c^2x^2+480b^2c^3x^3+640bc^4x^4+256c^5x^5)}{15b^6(x(b+cx))^{5/2}}$$

input
```
Integrate[(b*x + c*x^2)^(-7/2),x]
```

output
```
(-2*(3*b^5 - 10*b^4*c*x + 80*b^3*c^2*x^2 + 480*b^2*c^3*x^3 + 640*b*c^4*x^4
 + 256*c^5*x^5))/(15*b^6*(x*(b + c*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{16c\int \frac{1}{(cx^2+bx)^{5/2}}dx}{5b^2}-\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{16c\left(-\frac{8c\int \frac{1}{(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}\right)}{5b^2}-\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}}-\frac{16c\left(\frac{16c(b+2cx)}{3b^4\sqrt{bx+cx^2}}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}\right)}{5b^2}$$

input
```
Int[(b*x + c*x^2)^(-7/2),x]
```

output
```
(-2*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/2)) - (16*c*((-2*(b + 2*c*x))/(3*
b^2*(b*x + c*x^2)^(3/2)) + (16*c*(b + 2*c*x))/(3*b^4*Sqrt[b*x + c*x^2])))/
(5*b^2)
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{2x(cx+b)\left(256c^{5}x^{5}+640b\,x^{4}c^{4}+480b^{2}c^{3}x^{3}+80c^{2}x^{2}b^{3}-10b^{4}cx+3b^{5}\right)}{15b^{6}\left(c\,x^{2}+bx\right)^{\frac{7}{2}}}$	75
orering	$-\frac{2x(cx+b)\left(256c^{5}x^{5}+640b\,x^{4}c^{4}+480b^{2}c^{3}x^{3}+80c^{2}x^{2}b^{3}-10b^{4}cx+3b^{5}\right)}{15b^{6}\left(c\,x^{2}+bx\right)^{\frac{7}{2}}}$	75
default	$-\frac{2(2cx+b)}{5b^{2}\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}-\frac{16c\left(-\frac{2(2cx+b)}{3b^{2}\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^{4}\sqrt{c\,x^{2}+bx}}\right)}{5b^{2}}$	76
pseudoelliptic	$\frac{-\frac{512}{15}c^{5}x^{5}-\frac{256}{3}b\,x^{4}c^{4}-64b^{2}c^{3}x^{3}-\frac{32}{3}c^{2}x^{2}b^{3}+\frac{4}{3}b^{4}cx-\frac{2}{5}b^{5}}{x^{2}(cx+b)^{2}\sqrt{x(cx+b)}\,b^{6}}$	77
trager	$-\frac{2\left(256c^{5}x^{5}+640b\,x^{4}c^{4}+480b^{2}c^{3}x^{3}+80c^{2}x^{2}b^{3}-10b^{4}cx+3b^{5}\right)\sqrt{c\,x^{2}+bx}}{15b^{6}(cx+b)^{3}x^{3}}$	79
risch	$-\frac{2(cx+b)\left(128c^{2}x^{2}-19cbx+3b^{2}\right)}{15b^{6}x^{2}\sqrt{x(cx+b)}}-\frac{2c^{3}\left(128c^{2}x^{2}+275cbx+150b^{2}\right)x}{15\sqrt{x(cx+b)}\,\left(c^{2}x^{2}+2cbx+b^{2}\right)b^{6}}$	98

input
```
int(1/(c*x^2+b*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*x*(c*x+b)*(256*c^5*x^5+640*b*c^4*x^4+480*b^2*c^3*x^3+80*b^3*c^2*x^2-
10*b^4*c*x+3*b^5)/b^6/(c*x^2+b*x)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.75

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = -\frac{2\,(256\,c^5x^5+640\,bc^4x^4+480\,b^2c^3x^3+80\,b^3c^2x^2-10\,b^4cx+3\,b^5)\sqrt{cx^2+bx}}{15\,(b^6c^3x^6+3\,b^7c^2x^5+3\,b^8cx^4+b^9x^3)}$$

input
```
integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(256*c^5*x^5 + 640*b*c^4*x^4 + 480*b^2*c^3*x^3 + 80*b^3*c^2*x^2 - 10
*b^4*c*x + 3*b^5)*sqrt(c*x^2 + b*x)/(b^6*c^3*x^6 + 3*b^7*c^2*x^5 + 3*b^8*c
*x^4 + b^9*x^3)
```

Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(7/2),x)
```

output
```
Integral((b*x + c*x**2)**(-7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.79

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = -\frac{4\,cx}{5\,(cx^2+bx)^{\frac{5}{2}}b^2} + \frac{64\,c^2x}{15\,(cx^2+bx)^{\frac{3}{2}}b^4} - \frac{512\,c^3x}{15\,\sqrt{cx^2+bx}b^6} - \frac{2}{5\,(cx^2+bx)^{\frac{5}{2}}b} + \frac{32\,c}{15\,(cx^2+bx)^{\frac{3}{2}}b^3} - \frac{256\,c^2}{15\,\sqrt{cx^2+bx}b^5}$$

input
```
integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

output
```
-4/5*c*x/((c*x^2 + b*x)^(5/2)*b^2) + 64/15*c^2*x/((c*x^2 + b*x)^(3/2)*b^4)
 - 512/15*c^3*x/(sqrt(c*x^2 + b*x)*b^6) - 2/5/((c*x^2 + b*x)^(5/2)*b) + 32
/15*c/((c*x^2 + b*x)^(3/2)*b^3) - 256/15*c^2/(sqrt(c*x^2 + b*x)*b^5)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.53

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = -\frac{2\left(2\left(8\left(2\left(4\,x\left(\frac{2\,c^5x}{b^6}+\frac{5\,c^4}{b^5}\right)+\frac{15\,c^3}{b^4}\right)x+\frac{5\,c^2}{b^3}\right)x-\frac{5\,c}{b^2}\right)x+\frac{3}{b}\right)}{15\,(cx^2+bx)^{\frac{5}{2}}}$$

input
```
integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="giac")
```

output
```
-2/15*(2*(8*(2*(4*x*(2*c^5*x/b^6 + 5*c^4/b^5) + 15*c^3/b^4)*x + 5*c^2/b^3)
*x - 5*c/b^2)*x + 3/b)/(c*x^2 + b*x)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.69

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx =$$

$$-\frac{6\,b^5+256\,b\,c^2\,(c\,x^2+b\,x)^2+512\,c^3\,x\,(c\,x^2+b\,x)^2-32\,b^3\,c\,(c\,x^2+b\,x)+12\,b^4\,c\,x-64\,b^2\,c^2\,x\,(c\,x^2+b\,x}{15\,b^6\,(c\,x^2+b\,x)^{5/2}}$$

input
```
int(1/(b*x + c*x^2)^(7/2),x)
```

output
```
-(6*b^5 + 256*b*c^2*(b*x + c*x^2)^2 + 512*c^3*x*(b*x + c*x^2)^2 - 32*b^3*c
*(b*x + c*x^2) + 12*b^4*c*x - 64*b^2*c^2*x*(b*x + c*x^2))/(15*b^6*(b*x + c
*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = \frac{\frac{512\sqrt{c}\,\sqrt{cx+b}\,b^2c^2x^3}{15}+\frac{1024\sqrt{c}\,\sqrt{cx+b}\,b\,c^3x^4}{15}+\frac{512\sqrt{c}\,\sqrt{cx+b}\,c^4x^5}{15}-\frac{2\sqrt{x}\,b^5}{5}+\frac{4\sqrt{x}\,b^4cx}{3}-\frac{32\sqrt{x}\,b^3c^2x^2}{3}}{\sqrt{cx+b}\,b^6x^3\left(c^2x^2+2bcx+b^2\right)}$$

input
```
int(1/(c*x^2+b*x)^(7/2),x)
```

output
```
(2*(256*sqrt(c)*sqrt(b + c*x)*b**2*c**2*x**3 + 512*sqrt(c)*sqrt(b + c*x)*b
*c**3*x**4 + 256*sqrt(c)*sqrt(b + c*x)*c**4*x**5 - 3*sqrt(x)*b**5 + 10*sqr
t(x)*b**4*c*x - 80*sqrt(x)*b**3*c**2*x**2 - 480*sqrt(x)*b**2*c**3*x**3 - 6
40*sqrt(x)*b*c**4*x**4 - 256*sqrt(x)*c**5*x**5))/(15*sqrt(b + c*x)*b**6*x*
*3*(b**2 + 2*b*c*x + c**2*x**2))
```

3.16 $\int \frac{1}{(bx+cx^2)^{9/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 187

$$\begin{aligned}\int \frac{1}{(bx+cx^2)^{9/2}}\,dx &= \frac{2}{7b\,(bx+cx^2)^{7/2}}+\frac{4}{5b^2x\,(bx+cx^2)^{5/2}}\\&+\frac{16}{5b^3x^2\,(bx+cx^2)^{3/2}}+\frac{32}{b^4x^3\sqrt{bx+cx^2}}-\frac{256\sqrt{bx+cx^2}}{7b^5x^4}\\&+\frac{1536c\sqrt{bx+cx^2}}{35b^6x^3}-\frac{2048c^2\sqrt{bx+cx^2}}{35b^7x^2}+\frac{4096c^3\sqrt{bx+cx^2}}{35b^8x}\end{aligned}$$

output
```
2/7/b/(c*x^2+b*x)^(7/2)+4/5/b^2/x/(c*x^2+b*x)^(5/2)+16/5/b^3/x^2/(c*x^2+b*
x)^(3/2)+32/b^4/x^3/(c*x^2+b*x)^(1/2)-256/7*(c*x^2+b*x)^(1/2)/b^5/x^4+1536
/35*c*(c*x^2+b*x)^(1/2)/b^6/x^3-2048/35*c^2*(c*x^2+b*x)^(1/2)/b^7/x^2+4096
/35*c^3*(c*x^2+b*x)^(1/2)/b^8/x
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.49

$$\int \frac{1}{(bx+cx^2)^{9/2}}\,dx = -\frac{2(5b^7-14b^6cx+56b^5c^2x^2-560b^4c^3x^3-4480b^3c^4x^4-8960b^2c^5x^5-7168bc^6x^6-2048c^7x^7)}{35b^8(x(b+cx))^{7/2}}$$

input
```
Integrate[(b*x + c*x^2)^(-9/2),x]
```

output
```
(-2*(5*b^7 - 14*b^6*c*x + 56*b^5*c^2*x^2 - 560*b^4*c^3*x^3 - 4480*b^3*c^4*
x^4 - 8960*b^2*c^5*x^5 - 7168*b*c^6*x^6 - 2048*c^7*x^7))/(35*b^8*(x*(b + c
*x))^(7/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.66, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1089, 1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{9/2}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{24c\int \frac{1}{(cx^2+bx)^{7/2}}dx}{7b^2}-\frac{2(b+2cx)}{7b^2(bx+cx^2)^{7/2}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{24c\left(-\frac{16c\int \frac{1}{(cx^2+bx)^{5/2}}dx}{5b^2}-\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}}\right)}{7b^2}-\frac{2(b+2cx)}{7b^2(bx+cx^2)^{7/2}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{24c\left(-\frac{16c\left(-\frac{8c\int\frac{1}{\left(cx^2+bx\right)^{3/2}}dx}{3b^2}-\frac{2(b+2cx)}{3b^2\left(bx+cx^2\right)^{3/2}}\right)}{5b^2}-\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}}\right)}{7b^2}-\frac{2(b+2cx)}{7b^2\left(bx+cx^2\right)^{7/2}}$$

$$\Big\downarrow \text{ 1088}$$

$$-\frac{2(b+2cx)}{7b^2\left(bx+cx^2\right)^{7/2}}-\frac{24c\left(-\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}}-\frac{16c\left(\frac{16c(b+2cx)}{3b^4\sqrt{bx+cx^2}}-\frac{2(b+2cx)}{3b^2\left(bx+cx^2\right)^{3/2}}\right)}{5b^2}\right)}{7b^2}$$

input
```
Int[(b*x + c*x^2)^(-9/2),x]
```

output
```
(-2*(b + 2*c*x))/(7*b^2*(b*x + c*x^2)^(7/2)) - (24*c*((-2*(b + 2*c*x))/(5*
b^2*(b*x + c*x^2)^(5/2)) - (16*c*((-2*(b + 2*c*x))/(3*b^2*(b*x + c*x^2)^(3
/2)) + (16*c*(b + 2*c*x))/(3*b^4*Sqrt[b*x + c*x^2])))/(5*b^2)))/(7*b^2)
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.52

method	result	size
gosper	$-\frac{2x(cx+b)(-2048c^7x^7-7168x^6c^6b-8960c^5x^5b^2-4480c^4x^4b^3-560c^3x^3b^4+56c^2x^2b^5-14cx\,b^6+5b^7)}{35b^8(c\,x^2+bx)^{\frac{9}{2}}}$	97
orering	$-\frac{2x(cx+b)(-2048c^7x^7-7168x^6c^6b-8960c^5x^5b^2-4480c^4x^4b^3-560c^3x^3b^4+56c^2x^2b^5-14cx\,b^6+5b^7)}{35b^8(c\,x^2+bx)^{\frac{9}{2}}}$	97
trager	$-\frac{2(-2048c^7x^7-7168x^6c^6b-8960c^5x^5b^2-4480c^4x^4b^3-560c^3x^3b^4+56c^2x^2b^5-14cx\,b^6+5b^7)\sqrt{c\,x^2+bx}}{35b^8x^4(cx+b)^4}$	101
default	$-\frac{2(2cx+b)}{7b^2(c\,x^2+bx)^{\frac{7}{2}}}-\frac{24c\left(-\frac{2(2cx+b)}{5b^2(c\,x^2+bx)^{\frac{5}{2}}}-\frac{16c\left(-\frac{2(2cx+b)}{3b^2(c\,x^2+bx)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{c\,x^2+bx}}\right)}{5b^2}\right)}{7b^2}$	105
risch	$-\frac{2(cx+b)(-1024c^3x^3+162b\,c^2x^2-34b^2cx+5b^3)}{35b^8x^3\sqrt{x(cx+b)}}+\frac{2c^4(1024c^3x^3+3234b\,c^2x^2+3430b^2cx+1225b^3)x}{35\sqrt{x(cx+b)}\,(c^3x^3+3b\,c^2x^2+3b^2cx+b^3)b^8}$	131

input
```
int(1/(c*x^2+b*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*x*(c*x+b)*(-2048*c^7*x^7-7168*b*c^6*x^6-8960*b^2*c^5*x^5-4480*b^3*c^
4*x^4-560*b^4*c^3*x^3+56*b^5*c^2*x^2-14*b^6*c*x+5*b^7)/b^8/(c*x^2+b*x)^(9/
2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.74

$$\int \frac{1}{(bx+cx^2)^{9/2}}\,dx = \frac{2\,(2048\,c^7x^7+7168\,bc^6x^6+8960\,b^2c^5x^5+4480\,b^3c^4x^4+560\,b^4c^3x^3-56\,b^5c^2x^2+14}{35\,(b^8c^4x^8+4\,b^9c^3x^7+6\,b^{10}c^2x^6+4\,b^{11}cx^5+b^{12}x^4)}$$

input
```
integrate(1/(c*x^2+b*x)^(9/2),x, algorithm="fricas")
```

output
```
2/35*(2048*c^7*x^7 + 7168*b*c^6*x^6 + 8960*b^2*c^5*x^5 + 4480*b^3*c^4*x^4
+ 560*b^4*c^3*x^3 - 56*b^5*c^2*x^2 + 14*b^6*c*x - 5*b^7)*sqrt(c*x^2 + b*x)
/(b^8*c^4*x^8 + 4*b^9*c^3*x^7 + 6*b^10*c^2*x^6 + 4*b^11*c*x^5 + b^12*x^4)
```

Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{9/2}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(9/2),x)
```

output
```
Integral((b*x + c*x**2)**(-9/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.80

$$\int \frac{1}{(bx+cx^2)^{9/2}}\,dx = -\frac{4\,cx}{7\,(cx^2+bx)^{\frac{7}{2}}b^2} + \frac{96\,c^2x}{35\,(cx^2+bx)^{\frac{5}{2}}b^4} - \frac{512\,c^3x}{35\,(cx^2+bx)^{\frac{3}{2}}b^6} + \frac{4096\,c^4x}{35\,\sqrt{cx^2+bx}b^8} - \frac{2}{7\,(cx^2+bx)^{\frac{7}{2}}b} + \frac{48\,c}{35\,(cx^2+bx)^{\frac{5}{2}}b^3} - \frac{256\,c^2}{35\,(cx^2+bx)^{\frac{3}{2}}b^5} + \frac{2048\,c^3}{35\,\sqrt{cx^2+bx}b^7}$$

input
```
integrate(1/(c*x^2+b*x)^(9/2),x, algorithm="maxima")
```

output
```
-4/7*c*x/((c*x^2 + b*x)^(7/2)*b^2) + 96/35*c^2*x/((c*x^2 + b*x)^(5/2)*b^4)
 - 512/35*c^3*x/((c*x^2 + b*x)^(3/2)*b^6) + 4096/35*c^4*x/(sqrt(c*x^2 + b*
x)*b^8) - 2/7/((c*x^2 + b*x)^(7/2)*b) + 48/35*c/((c*x^2 + b*x)^(5/2)*b^3)
- 256/35*c^2/((c*x^2 + b*x)^(3/2)*b^5) + 2048/35*c^3/(sqrt(c*x^2 + b*x)*b^
7)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.52

$$\int \frac{1}{(bx+cx^2)^{9/2}}\,dx = \frac{2\left(2\left(4\left(2\left(8\left(2\left(4\,x\left(\frac{2\,c^7x}{b^8}+\frac{7\,c^6}{b^7}\right)+\frac{35\,c^5}{b^6}\right)x+\frac{35\,c^4}{b^5}\right)x+\frac{35\,c^3}{b^4}\right)x-\frac{7\,c^2}{b^3}\right)x+\frac{7\,c}{b^2}\right)x-}{35\,(cx^2+bx)^{\frac{7}{2}}}$$

input
```
integrate(1/(c*x^2+b*x)^(9/2),x, algorithm="giac")
```

output
```
2/35*(2*(4*(2*(8*(2*(4*x*(2*c^7*x/b^8 + 7*c^6/b^7) + 35*c^5/b^6)*x + 35*c^
4/b^5)*x + 35*c^3/b^4)*x - 7*c^2/b^3)*x + 7*c/b^2)*x - 5/b)/(c*x^2 + b*x)^
(7/2)
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.62

$$\int \frac{1}{(bx+cx^2)^{9/2}}\,dx = \frac{\frac{48\,c}{35\,b^3}+\frac{96\,c^2\,x}{35\,b^4}}{\left(c\,x^2+b\,x\right)^{5/2}}-\frac{\frac{2}{7\,b}+\frac{4\,c\,x}{7\,b^2}}{\left(c\,x^2+b\,x\right)^{7/2}}-\frac{\frac{256\,c^2}{35\,b^5}+\frac{512\,c^3\,x}{35\,b^6}}{\left(c\,x^2+b\,x\right)^{3/2}}+\frac{\frac{2048\,c^3}{35\,b^7}+\frac{4096\,c^4\,x}{35\,b^8}}{\sqrt{c\,x^2+b\,x}}$$

input
```
int(1/(b*x + c*x^2)^(9/2),x)
```

output
```
((48*c)/(35*b^3) + (96*c^2*x)/(35*b^4))/(b*x + c*x^2)^(5/2) - (2/(7*b) + (
4*c*x)/(7*b^2))/(b*x + c*x^2)^(7/2) - ((256*c^2)/(35*b^5) + (512*c^3*x)/(3
5*b^6))/(b*x + c*x^2)^(3/2) + ((2048*c^3)/(35*b^7) + (4096*c^4*x)/(35*b^8)
)/(b*x + c*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.10

$$\int \frac{1}{(bx+cx^2)^{9/2}}\,dx = \frac{-\frac{4096\sqrt{c}\,\sqrt{cx+b}\,b^3c^3x^4}{35} - \frac{12288\sqrt{c}\,\sqrt{cx+b}\,b^2c^4x^5}{35} - \frac{12288\sqrt{c}\,\sqrt{cx+b}\,b\,c^5x^6}{35} - \frac{4096\sqrt{c}\,\sqrt{cx+b}\,c^6x^7}{35} - \frac{2\sqrt{x}}{7}}{\sqrt{cx+b}\,b^8x^4}$$

input

```
int(1/(c*x^2+b*x)^(9/2),x)
```

output

```
(2*( - 2048*sqrt(c)*sqrt(b + c*x)*b**3*c**3*x**4 - 6144*sqrt(c)*sqrt(b + c
*x)*b**2*c**4*x**5 - 6144*sqrt(c)*sqrt(b + c*x)*b*c**5*x**6 - 2048*sqrt(c)
*sqrt(b + c*x)*c**6*x**7 - 5*sqrt(x)*b**7 + 14*sqrt(x)*b**6*c*x - 56*sqrt(
x)*b**5*c**2*x**2 + 560*sqrt(x)*b**4*c**3*x**3 + 4480*sqrt(x)*b**3*c**4*x*
*4 + 8960*sqrt(x)*b**2*c**5*x**5 + 7168*sqrt(x)*b*c**6*x**6 + 2048*sqrt(x)
*c**7*x**7))/(35*sqrt(b + c*x)*b**8*x**4*(b**3 + 3*b**2*c*x + 3*b*c**2*x**
2 + c**3*x**3))
```

3.17 $\int (3x-4x^2)^{5/2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 126

$$\int (3x-4x^2)^{5/2} \, dx = -\frac{1215\sqrt{3-4x}\sqrt{x}}{32768} - \frac{135\sqrt{3-4x}x^{3/2}}{4096} - \frac{9}{256}\sqrt{3-4x}x^{5/2} - \frac{39}{32}\sqrt{3-4x}x^{7/2} + \frac{5}{4}(3-4x)^{3/2}x^{7/2} + \frac{8}{3}\sqrt{3-4x}x^{11/2} + \frac{3645 \arcsin\left(\frac{2\sqrt{x}}{\sqrt{3}}\right)}{65536}$$

output
```
-1215/32768*(3-4*x)^(1/2)*x^(1/2)-135/4096*(3-4*x)^(1/2)*x^(3/2)-9/256*(3-
4*x)^(1/2)*x^(5/2)-39/32*(3-4*x)^(1/2)*x^(7/2)+5/4*(3-4*x)^(3/2)*x^(7/2)+8
/3*(3-4*x)^(1/2)*x^(11/2)+3645/65536*arcsin(2/3*x^(1/2)*3^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.73

$$\int (3x -4x^2)^{5/2} \, dx = \frac{\sqrt{-x(-3+4x)}\left(2\sqrt{x}\sqrt{-3+4x}(-3645-3240x-3456x^2+248832x^3-491520x^4+2621\right.}{196608\sqrt{x}\sqrt{-3+4x}}$$

input
```
Integrate[(3*x - 4*x^2)^(5/2),x]
```

output
```
(Sqrt[-(x*(-3 + 4*x))]*(2*Sqrt[x]*Sqrt[-3 + 4*x]*(-3645 - 3240*x - 3456*x^
2 + 248832*x^3 - 491520*x^4 + 262144*x^5) + 10935*Log[-2*Sqrt[x] + Sqrt[-3
 + 4*x]]))/(196608*Sqrt[x]*Sqrt[-3 + 4*x])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1087, 1087, 1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x-4x^2\right)^{5/2} dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{15}{32}\int \left(3x-4x^2\right)^{3/2} dx-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{15}{32}\left(\frac{27}{64}\int \sqrt{3x-4x^2}dx-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}\right)-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{15}{32}\left(\frac{27}{64}\left(\frac{9}{32}\int \frac{1}{\sqrt{3x-4x^2}}dx-\frac{1}{16}(3-8x)\sqrt{3x-4x^2}\right)-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}\right)-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{15}{32}\left(\frac{27}{64}\left(-\frac{3}{64}\int \frac{1}{\sqrt{1-\frac{1}{9}(3-8x)^2}}d(3-8x)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)\right)-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}\right)-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{15}{32}\left(\frac{27}{64}\left(-\frac{9}{64}\arcsin\left(\frac{1}{3}(3-8x)\right)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)\right)-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}\right)-\frac{1}{48}(3-8x)\left(3x-4x^2\right)^{5/2}$$

input
```
Int[(3*x - 4*x^2)^(5/2),x]
```

output
```
-1/48*((3 - 8*x)*(3*x - 4*x^2)^(5/2)) + (15*(-1/32*((3 - 8*x)*(3*x - 4*x^2
)^(3/2)) + (27*(-1/16*((3 - 8*x)*Sqrt[3*x - 4*x^2]) - (9*ArcSin[(3 - 8*x)/
3])/64))/64))/32
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.42

method	result
risch	$-\frac{(262144x^5-491520x^4+248832x^3-3456x^2-3240x-3645)x(4x-3)}{98304\sqrt{-x(4x-3)}}+\frac{3645\arcsin(-1+\frac{8x}{3})}{131072}$
default	$-\frac{(-8x+3)(-4x^2+3x)^{\frac{5}{2}}}{48}-\frac{15(-4x^2+3x)^{\frac{3}{2}}(-8x+3)}{1024}-\frac{405(-8x+3)\sqrt{-4x^2+3x}}{32768}+\frac{3645\arcsin(-1+\frac{8x}{3})}{131072}$
meijerg	$-\frac{10935i\left(-\frac{i\sqrt{\pi}\sqrt{x}\sqrt{3}\left(-\frac{1835008}{243}x^5+\frac{1146880}{81}x^4-7168x^3+\frac{896}{9}x^2+\frac{280}{3}x+105\right)\sqrt{-\frac{4x}{3}+1}}{30240}+\frac{i\sqrt{\pi}\arcsin\left(\frac{2\sqrt{x}\sqrt{3}}{3}\right)}{192}\right)}{1024\sqrt{\pi}}$
trager	$\left(\frac{8}{3}x^5-5x^4+\frac{81}{32}x^3-\frac{9}{256}x^2-\frac{135}{4096}x-\frac{1215}{32768}\right)\sqrt{-4x^2+3x}+\frac{3645\operatorname{RootOf}(\_Z^2+1)\ln(-8\operatorname{RootOf}(\_Z^2+$ … $}{13107}$

input
```
int((-4*x^2+3*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/98304*(262144*x^5-491520*x^4+248832*x^3-3456*x^2-3240*x-3645)*x*(4*x-3)
/(-x*(4*x-3))^(1/2)+3645/131072*arcsin(-1+8/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.49

$$\int (3x-4x^2)^{5/2}\,dx=\frac{1}{98304}\left(262144\,x^5-491520\,x^4+248832\,x^3-3456\,x^2-3240\,x-3645\right)\sqrt{-4\,x^2+3\,x}-\frac{3645}{65536}\arctan\left(\frac{2\sqrt{-4\,x^2+3\,x}}{4\,x-3}\right)$$

input
```
integrate((-4*x^2+3*x)^(5/2),x, algorithm="fricas")
```

output
```
1/98304*(262144*x^5 - 491520*x^4 + 248832*x^3 - 3456*x^2 - 3240*x - 3645)*
sqrt(-4*x^2 + 3*x) - 3645/65536*arctan(2*sqrt(-4*x^2 + 3*x)/(4*x - 3))
```

Sympy [A] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.02

$$\int \left(3x-4x^2\right)^{5/2} dx = 9\sqrt{-4x^2+3x}\left(\frac{x^3}{4}-\frac{x^2}{32}-\frac{15x}{512}-\frac{135}{4096}\right) - 24\sqrt{-4x^2+3x}\left(\frac{x^4}{5}-\frac{3x^3}{160}-\frac{21x^2}{1280}-\frac{63x}{4096}-\frac{567}{32768}\right) + 16\sqrt{-4x^2+3x}\left(\frac{x^5}{6}-\frac{x^4}{80}-\frac{27x^3}{2560}-\frac{189x^2}{20480}-\frac{567x}{65536}-\frac{5103}{524288}\right)+\frac{3645\operatorname{asin}\left(\frac{8x}{3}-1\right)}{131072}$$

input
```
integrate((-4*x**2+3*x)**(5/2),x)
```

output
```
9*sqrt(-4*x**2 + 3*x)*(x**3/4 - x**2/32 - 15*x/512 - 135/4096) - 24*sqrt(-
4*x**2 + 3*x)*(x**4/5 - 3*x**3/160 - 21*x**2/1280 - 63*x/4096 - 567/32768)
 + 16*sqrt(-4*x**2 + 3*x)*(x**5/6 - x**4/80 - 27*x**3/2560 - 189*x**2/2048
0 - 567*x/65536 - 5103/524288) + 3645*asin(8*x/3 - 1)/131072
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.71

$$\int \left(3x-4x^2\right)^{5/2} dx = \frac{1}{6}\left(-4x^2+3x\right)^{\frac{5}{2}}x-\frac{1}{16}\left(-4x^2+3x\right)^{\frac{5}{2}} + \frac{15}{128}\left(-4x^2+3x\right)^{\frac{3}{2}}x-\frac{45}{1024}\left(-4x^2+3x\right)^{\frac{3}{2}}+\frac{405}{4096}\sqrt{-4x^2+3x}x - \frac{1215}{32768}\sqrt{-4x^2+3x}-\frac{3645}{131072}\arcsin\left(-\frac{8}{3}x+1\right)$$

input
```
integrate((-4*x^2+3*x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(-4*x^2 + 3*x)^(5/2)*x - 1/16*(-4*x^2 + 3*x)^(5/2) + 15/128*(-4*x^2 +
3*x)^(3/2)*x - 45/1024*(-4*x^2 + 3*x)^(3/2) + 405/4096*sqrt(-4*x^2 + 3*x)*
x - 1215/32768*sqrt(-4*x^2 + 3*x) - 3645/131072*arcsin(-8/3*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.37

$$\int (3x-4x^2)^{5/2}\,dx = \frac{1}{98304}\left(8\left(16\left(8\left(32\left(8x-15\right)x+243\right)x-27\right)x-405\right)x-3645\right)\sqrt{-4x^2+3x} + \frac{3645}{131072}\arcsin\left(\frac{8}{3}x-1\right)$$

input
```
integrate((-4*x^2+3*x)^(5/2),x, algorithm="giac")
```

output
```
1/98304*(8*(16*(8*(32*(8*x - 15)*x + 243)*x - 27)*x - 405)*x - 3645)*sqrt(
-4*x^2 + 3*x) + 3645/131072*arcsin(8/3*x - 1)
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.50

$$\int (3x-4x^2)^{5/2}\,dx = \frac{3645\,\mathrm{asin}\left(\frac{8x}{3}-1\right)}{131072} + \frac{15\left(4x-\frac{3}{2}\right)\left(3x-4x^2\right)^{3/2}}{512} + \frac{\left(4x-\frac{3}{2}\right)\left(3x-4x^2\right)^{5/2}}{24} + \frac{405\left(\frac{x}{2}-\frac{3}{16}\right)\sqrt{3x-4x^2}}{2048}$$

input
```
int((3*x - 4*x^2)^(5/2),x)
```

output
```
(3645*asin((8*x)/3 - 1))/131072 + (15*(4*x - 3/2)*(3*x - 4*x^2)^(3/2))/512
 + ((4*x - 3/2)*(3*x - 4*x^2)^(5/2))/24 + (405*(x/2 - 3/16)*(3*x - 4*x^2)^
(1/2))/2048
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.75

$$\int \left(3x-4x^2\right)^{5/2}\,dx=\frac{8\sqrt{x}\,\sqrt{-4x+3}\,x^5}{3}-5\sqrt{x}\,\sqrt{-4x+3}\,x^4+\frac{81\sqrt{x}\,\sqrt{-4x+3}\,x^3}{32}-\frac{9\sqrt{x}\,\sqrt{-4x+3}\,x^2}{256}-\frac{135\sqrt{x}\,\sqrt{-4x+3}\,x}{4096}-\frac{1215\sqrt{x}\,\sqrt{-4x+3}}{32768}-\frac{3645\log\left(\frac{\sqrt{-4x+3}+2\sqrt{x}\,i}{\sqrt{3}}\right)i}{65536}$$

input
```
int((-4*x^2+3*x)^(5/2),x)
```

output
```
(524288*sqrt(x)*sqrt( - 4*x + 3)*x**5 - 983040*sqrt(x)*sqrt( - 4*x + 3)*x*
*4 + 497664*sqrt(x)*sqrt( - 4*x + 3)*x**3 - 6912*sqrt(x)*sqrt( - 4*x + 3)*
x**2 - 6480*sqrt(x)*sqrt( - 4*x + 3)*x - 7290*sqrt(x)*sqrt( - 4*x + 3) - 1
0935*log((sqrt( - 4*x + 3) + 2*sqrt(x)*i)/sqrt(3))*i)/196608
```

3.18 $\int (3x-4x^2)^{3/2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 88

$$\int (3x-4x^2)^{3/2}\,dx = -\frac{81\sqrt{3-4x}\sqrt{x}}{1024}-\frac{9}{128}\sqrt{3-4x}x^{3/2}+\frac{9}{8}\sqrt{3-4x}x^{5/2}-\sqrt{3-4x}x^{7/2}+\frac{243\arcsin\left(\frac{2\sqrt{x}}{\sqrt{3}}\right)}{2048}$$

output
```
-81/1024*(3-4*x)^(1/2)*x^(1/2)-9/128*(3-4*x)^(1/2)*x^(3/2)+9/8*(3-4*x)^(1/
2)*x^(5/2)-(3-4*x)^(1/2)*x^(7/2)+243/2048*arcsin(2/3*x^(1/2)*3^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.93

$$\int (3x-4x^2)^{3/2}\,dx = \frac{\sqrt{-x(-3+4x)}\left(-2\sqrt{x}\sqrt{-3+4x}(81+72x-1152x^2+1024x^3)+243\log\left(-2\sqrt{x}+\sqrt{-3\right.\right.}}{2048\sqrt{x}\sqrt{-3+4x}}$$

input
```
Integrate[(3*x - 4*x^2)^(3/2),x]
```

output
```
(Sqrt[-(x*(-3 + 4*x))]*(-2*Sqrt[x]*Sqrt[-3 + 4*x]*(81 + 72*x - 1152*x^2 +
1024*x^3) + 243*Log[-2*Sqrt[x] + Sqrt[-3 + 4*x]]))/(2048*Sqrt[x]*Sqrt[-3 +
 4*x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1087, 1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x-4x^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{27}{64}\int \sqrt{3x-4x^2}dx-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{27}{64}\left(\frac{9}{32}\int \frac{1}{\sqrt{3x-4x^2}}dx-\frac{1}{16}(3-8x)\sqrt{3x-4x^2}\right)-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{27}{64}\left(-\frac{3}{64}\int \frac{1}{\sqrt{1-\frac{1}{9}(3-8x)^2}}d(3-8x)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)\right)-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}$$

$$\Big\downarrow \text{ 223}$$

$$\frac{27}{64}\left(-\frac{9}{64}\arcsin\left(\frac{1}{3}(3-8x)\right)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)\right)-\frac{1}{32}(3-8x)\left(3x-4x^2\right)^{3/2}$$

input
```
Int[(3*x - 4*x^2)^(3/2),x]
```

output
```
-1/32*((3 - 8*x)*(3*x - 4*x^2)^(3/2)) + (27*(-1/16*((3 - 8*x)*Sqrt[3*x - 4
*x^2]) - (9*ArcSin[(3 - 8*x)/3])/64))/64
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.49

method	result
risch	$\frac{(1024x^3-1152x^2+72x+81)x(4x-3)}{1024\sqrt{-x(4x-3)}}+\frac{243\arcsin\left(-1+\frac{8x}{3}\right)}{4096}$
default	$-\frac{(-4x^2+3x)^{\frac{3}{2}}(-8x+3)}{32}-\frac{27(-8x+3)\sqrt{-4x^2+3x}}{1024}+\frac{243\arcsin\left(-1+\frac{8x}{3}\right)}{4096}$
pseudoelliptic	$-\frac{243\arctan\left(\frac{\sqrt{-4x^2+3x}}{2x}\right)}{2048}+\frac{(-1024x^3+1152x^2-72x-81)\sqrt{-4x^2+3x}}{1024}$
meijerg	$-\frac{243i\left(-\frac{i\sqrt{\pi}\sqrt{x}\sqrt{3}\left(\frac{5120}{27}x^3-\frac{640}{3}x^2+\frac{40}{3}x+15\right)\sqrt{-\frac{4x}{3}+1}}{360}+\frac{i\sqrt{\pi}\arcsin\left(\frac{2\sqrt{x}\sqrt{3}}{3}\right)}{16}\right)}{128\sqrt{\pi}}$
trager	$\left(-x^3+\frac{9}{8}x^2-\frac{9}{128}x-\frac{81}{1024}\right)\sqrt{-4x^2+3x}-\frac{243\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(8\,\mathrm{RootOf}\left(\_Z^2+1\right)x+4\sqrt{-4x^2+3}\right.}{4096}$

input
```
int((-4*x^2+3*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1024*(1024*x^3-1152*x^2+72*x+81)*x*(4*x-3)/(-x*(4*x-3))^(1/2)+243/4096*a
rcsin(-1+8/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \left(3x-4x^2\right)^{3/2} dx = -\frac{1}{1024}\left(1024\,x^3-1152\,x^2+72\,x+81\right)\sqrt{-4\,x^2+3\,x} - \frac{243}{2048}\arctan\left(\frac{2\sqrt{-4\,x^2+3\,x}}{4\,x-3}\right)$$

input
```
integrate((-4*x^2+3*x)^(3/2),x, algorithm="fricas")
```

output
```
-1/1024*(1024*x^3 - 1152*x^2 + 72*x + 81)*sqrt(-4*x^2 + 3*x) - 243/2048*ar
ctan(2*sqrt(-4*x^2 + 3*x)/(4*x - 3))
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.77

$$\int \left(3x-4x^2\right)^{3/2} dx = 3\sqrt{-4x^2+3x}\left(\frac{x^2}{3}-\frac{x}{16}-\frac{9}{128}\right) - 4\sqrt{-4x^2+3x}\left(\frac{x^3}{4}-\frac{x^2}{32}-\frac{15x}{512}-\frac{135}{4096}\right) + \frac{243\operatorname{asin}\left(\frac{8x}{3}-1\right)}{4096}$$

input
```
integrate((-4*x**2+3*x)**(3/2),x)
```

output
```
3*sqrt(-4*x**2 + 3*x)*(x**2/3 - x/16 - 9/128) - 4*sqrt(-4*x**2 + 3*x)*(x**
3/4 - x**2/32 - 15*x/512 - 135/4096) + 243*asin(8*x/3 - 1)/4096
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.72

$$\int \left(3x-4x^2\right)^{3/2}\,dx=\frac{1}{4}\left(-4x^2+3x\right)^{\frac{3}{2}}x-\frac{3}{32}\left(-4x^2+3x\right)^{\frac{3}{2}}+\frac{27}{128}\sqrt{-4x^2+3x}x-\frac{81}{1024}\sqrt{-4x^2+3x}-\frac{243}{4096}\arcsin\left(-\frac{8}{3}x+1\right)$$

input
```
integrate((-4*x^2+3*x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(-4*x^2 + 3*x)^(3/2)*x - 3/32*(-4*x^2 + 3*x)^(3/2) + 27/128*sqrt(-4*x^
2 + 3*x)*x - 81/1024*sqrt(-4*x^2 + 3*x) - 243/4096*arcsin(-8/3*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.42

$$\int \left(3x-4x^2\right)^{3/2}\,dx=-\frac{1}{1024}\left(8\left(16\left(8x-9\right)x+9\right)x+81\right)\sqrt{-4x^2+3x}+\frac{243}{4096}\arcsin\left(\frac{8}{3}x-1\right)$$

input
```
integrate((-4*x^2+3*x)^(3/2),x, algorithm="giac")
```

output
```
-1/1024*(8*(16*(8*x - 9)*x + 9)*x + 81)*sqrt(-4*x^2 + 3*x) + 243/4096*arcs
in(8/3*x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.51

$$\int \left(3x-4x^2\right)^{3/2}\,dx=\frac{243\,\mathrm{asin}\left(\frac{8\,x}{3}-1\right)}{4096}+\frac{\left(4\,x-\frac{3}{2}\right)\,\left(3\,x-4\,x^2\right)^{3/2}}{16}+\frac{27\,\left(\frac{x}{2}-\frac{3}{16}\right)\,\sqrt{3\,x-4\,x^2}}{64}$$

input
```
int((3*x - 4*x^2)^(3/2),x)
```

output
```
(243*asin((8*x)/3 - 1))/4096 + ((4*x - 3/2)*(3*x - 4*x^2)^(3/2))/16 + (27*
(x/2 - 3/16)*(3*x - 4*x^2)^(1/2))/64
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.78

$$\int \left(3x-4x^2\right)^{3/2}\,dx=-\sqrt{x}\,\sqrt{-4x+3}\,x^3+\frac{9\sqrt{x}\,\sqrt{-4x+3}\,x^2}{8}-\frac{9\sqrt{x}\,\sqrt{-4x+3}\,x}{128}-\frac{81\sqrt{x}\,\sqrt{-4x+3}}{1024}-\frac{243\log\left(\frac{\sqrt{-4x+3}+2\sqrt{x}\,i}{\sqrt{3}}\right)i}{2048}$$

input
```
int((-4*x^2+3*x)^(3/2),x)
```

output
```
( - 2048*sqrt(x)*sqrt( - 4*x + 3)*x**3 + 2304*sqrt(x)*sqrt( - 4*x + 3)*x**
2 - 144*sqrt(x)*sqrt( - 4*x + 3)*x - 162*sqrt(x)*sqrt( - 4*x + 3) - 243*lo
g((sqrt( - 4*x + 3) + 2*sqrt(x)*i)/sqrt(3))*i)/2048
```

3.19 $\int \sqrt{3x-4x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \sqrt{3x-4x^2}\,dx = \frac{3}{16}\sqrt{3-4x}\sqrt{x}-\frac{1}{8}(3-4x)^{3/2}\sqrt{x}+\frac{9}{32}\arcsin\left(\frac{2\sqrt{x}}{\sqrt{3}}\right)$$

output
```
3/16*(3-4*x)^(1/2)*x^(1/2)-1/8*(3-4*x)^(3/2)*x^(1/2)+9/32*arcsin(2/3*x^(1/
2)*3^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \sqrt{3x-4x^2}\,dx = \frac{1}{32}\sqrt{-x(-3+4x)}\left(-6+16x+\frac{9\log\left(-2\sqrt{x}+\sqrt{-3+4x}\right)}{\sqrt{x}\sqrt{-3+4x}}\right)$$

input
```
Integrate[Sqrt[3*x - 4*x^2],x]
```

output
```
(Sqrt[-(x*(-3 + 4*x))]*(-6 + 16*x + (9*Log[-2*Sqrt[x] + Sqrt[-3 + 4*x]])/(
Sqrt[x]*Sqrt[-3 + 4*x])))/32
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{3x-4x^2}\,dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{9}{32}\int \frac{1}{\sqrt{3x-4x^2}}dx-\frac{1}{16}(3-8x)\sqrt{3x-4x^2}$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{3}{64}\int \frac{1}{\sqrt{1-\frac{1}{9}(3-8x)^2}}d(3-8x)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{9}{64}\arcsin\left(\frac{1}{3}(3-8x)\right)-\frac{1}{16}\sqrt{3x-4x^2}(3-8x)$$

input
```
Int[Sqrt[3*x - 4*x^2],x]
```

output
```
-1/16*((3 - 8*x)*Sqrt[3*x - 4*x^2]) - (9*ArcSin[(3 - 8*x)/3])/64
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.52

method	result
default	$-\frac{(-8x+3)\sqrt{-4x^2+3x}}{16}+\frac{9\arcsin\left(-1+\frac{8x}{3}\right)}{64}$
risch	$-\frac{(8x-3)x(4x-3)}{16\sqrt{-x(4x-3)}}+\frac{9\arcsin\left(-1+\frac{8x}{3}\right)}{64}$
pseudoelliptic	$-\frac{9\arctan\left(\frac{\sqrt{-4x^2+3x}}{2x}\right)}{32}+\frac{\sqrt{-4x^2+3x}\,(8x-3)}{16}$
meijerg	$-\frac{9i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{3}\,(-8x+3)\sqrt{-\frac{4x}{3}+1}}{9}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{2\sqrt{x}\,\sqrt{3}}{3}\right)}{2}\right)}{16\sqrt{\pi}}$
trager	$\left(\frac{x}{2}-\frac{3}{16}\right)\sqrt{-4x^2+3x}-\frac{9\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(8\,\mathrm{RootOf}\left(\_Z^2+1\right)x+4\sqrt{-4x^2+3x}-3\,\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{64}$

input
```
int((-4*x^2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/16*(-8*x+3)*(-4*x^2+3*x)^(1/2)+9/64*arcsin(-1+8/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \sqrt{3x-4x^2}\,dx=\frac{1}{16}\,\sqrt{-4\,x^2+3\,x}(8\,x-3)-\frac{9}{32}\,\arctan\left(\frac{2\,\sqrt{-4\,x^2+3\,x}}{4\,x-3}\right)$$

input
```
integrate((-4*x^2+3*x)^(1/2),x, algorithm="fricas")
```

output
```
1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) - 9/32*arctan(2*sqrt(-4*x^2 + 3*x)/(4*x
- 3))
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.54

$$\int \sqrt{3x-4x^2}\,dx=\left(\frac{x}{2}-\frac{3}{16}\right)\sqrt{-4x^2+3x}+\frac{9\,\mathrm{asin}\left(\frac{8x}{3}-1\right)}{64}$$

input
```
integrate((-4*x**2+3*x)**(1/2),x)
```

output
```
(x/2 - 3/16)*sqrt(-4*x**2 + 3*x) + 9*asin(8*x/3 - 1)/64
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

$$\int \sqrt{3x-4x^2}\,dx=\frac{1}{2}\,\sqrt{-4\,x^2+3\,x}x-\frac{3}{16}\,\sqrt{-4\,x^2+3\,x}-\frac{9}{64}\,\arcsin\left(-\frac{8}{3}\,x+1\right)$$

input
```
integrate((-4*x^2+3*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-4*x^2 + 3*x)*x - 3/16*sqrt(-4*x^2 + 3*x) - 9/64*arcsin(-8/3*x +
1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.50

$$\int \sqrt{3x-4x^2}\,dx = \frac{1}{16}\sqrt{-4x^2+3x}(8x-3)+\frac{9}{64}\arcsin\left(\frac{8}{3}x-1\right)$$

input
```
integrate((-4*x^2+3*x)^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) + 9/64*arcsin(8/3*x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.48

$$\int \sqrt{3x-4x^2}\,dx = \frac{9\operatorname{asin}\left(\frac{8x}{3}-1\right)}{64}+\left(\frac{x}{2}-\frac{3}{16}\right)\sqrt{3x-4x^2}$$

input
```
int((3*x - 4*x^2)^(1/2),x)
```

output
```
(9*asin((8*x)/3 - 1))/64 + (x/2 - 3/16)*(3*x - 4*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \sqrt{3x-4x^2}\,dx = \frac{\sqrt{x}\,\sqrt{-4x+3}\,x}{2}-\frac{3\sqrt{x}\,\sqrt{-4x+3}}{16}-\frac{9\log\left(\frac{\sqrt{-4x+3}+2\sqrt{x}\,i}{\sqrt{3}}\right)i}{32}$$

input
```
int((-4*x^2+3*x)^(1/2),x)
```

output
```
(16*sqrt(x)*sqrt( - 4*x + 3)*x - 6*sqrt(x)*sqrt( - 4*x + 3) - 9*log((sqrt(
 - 4*x + 3) + 2*sqrt(x)*i)/sqrt(3))*i)/32
```

3.20 $\int \frac{1}{\sqrt{3x-4x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = \arcsin\left(\frac{2\sqrt{x}}{\sqrt{3}}\right)$$

output
```
arcsin(2/3*x^(1/2)*3^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(13) = 26.

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.54

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = -\frac{\sqrt{x}\sqrt{-3+4x}\log\left(-2\sqrt{x}+\sqrt{-3+4x}\right)}{\sqrt{-x(-3+4x)}}$$

input
```
Integrate[1/Sqrt[3*x - 4*x^2],x]
```

output
```
-((Sqrt[x]*Sqrt[-3 + 4*x]*Log[-2*Sqrt[x] + Sqrt[-3 + 4*x]])/Sqrt[-(x*(-3 + 
 4*x))])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x-4x^2}}dx$$
$$\Big\downarrow \text{1090}$$
$$-\frac{1}{6}\int \frac{1}{\sqrt{1-\frac{1}{9}(3-8x)^2}}d(3-8x)$$
$$\Big\downarrow \text{223}$$
$$-\frac{1}{2}\arcsin\left(\frac{1}{3}(3-8x)\right)$$

input
```
Int[1/Sqrt[3*x - 4*x^2],x]
```

output
```
-1/2*ArcSin[(3 - 8*x)/3]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{\arcsin\left(-1+\frac{8x}{3}\right)}{2}$	9
meijerg	$\arcsin\left(\frac{2\sqrt{x}\,\sqrt{3}}{3}\right)$	10
pseudoelliptic	$-\arctan\left(\frac{\sqrt{-4x^2+3x}}{2x}\right)$	20
trager	$\frac{\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-8\,\mathrm{RootOf}\left(\_Z^2+1\right)x+4\sqrt{-4x^2+3x}+3\,\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{2}$	41

input
```
int(1/(-4*x^2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(-1+8/3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(9) = 18$.

Time = 0.07 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = -\arctan\left(\frac{2\sqrt{-4\,x^2+3\,x}}{4\,x-3}\right)$$

input
```
integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(2*sqrt(-4*x^2 + 3*x)/(4*x - 3))
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx=\frac{\operatorname{asin}\left(\frac{8x}{3}-1\right)}{2}$$

input
```
integrate(1/(-4*x**2+3*x)**(1/2),x)
```

output
```
asin(8*x/3 - 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx=-\frac{1}{2}\arcsin\left(-\frac{8}{3}x+1\right)$$

input
```
integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arcsin(-8/3*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(9) = 18.

Time = 0.37 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.08

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx=\frac{1}{16}\sqrt{-4\,x^2+3\,x}(8\,x-3)+\frac{9}{64}\arcsin\left(\frac{8}{3}\,x-1\right)$$

input
```
integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) + 9/64*arcsin(8/3*x - 1)
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.62

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = \frac{\operatorname{asin}\left(\frac{8\,x}{3}-1\right)}{2}$$

input
```
int(1/(3*x - 4*x^2)^(1/2),x)
```

output
```
asin((8*x)/3 - 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = -\log\left(\frac{\sqrt{-4x+3}+2\sqrt{x}\,i}{\sqrt{3}}\right)i$$

input
```
int(1/(-4*x^2+3*x)^(1/2),x)
```

output
```
- log((sqrt( - 4*x + 3) + 2*sqrt(x)*i)/sqrt(3))*i
```

3.21 $\int \frac{1}{(3x-4x^2)^{3/2}} dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{1}{(3x-4x^2)^{3/2}} dx = \frac{2}{3\sqrt{3-4x}\sqrt{x}} - \frac{4\sqrt{3-4x}}{9\sqrt{x}}$$

output
```
2/3/(3-4*x)^(1/2)/x^(1/2)-4/9*(3-4*x)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.57

$$\int \frac{1}{(3x-4x^2)^{3/2}} dx = \frac{2(-3+8x)}{9\sqrt{-x(-3+4x)}}$$

input
```
Integrate[(3*x - 4*x^2)^(-3/2),x]
```

output
```
(2*(-3 + 8*x))/(9*Sqrt[-(x*(-3 + 4*x))])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.59, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1088}$$

$$-\frac{2(3-8x)}{9\sqrt{3x-4x^2}}$$

input
```
Int[(3*x - 4*x^2)^(-3/2),x]
```

output
```
(-2*(3 - 8*x))/(9*Sqrt[3*x - 4*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && 
 NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.51

method	result	size
default	$-\frac{2(-8x+3)}{9\sqrt{-4x^2+3x}}$	19
pseudoelliptic	$\frac{\frac{16x}{9}-\frac{2}{3}}{\sqrt{-4x^2+3x}}$	19
meijerg	$-\frac{2\sqrt{3}\,(1-\frac{8x}{3})}{9\sqrt{x}\,\sqrt{-\frac{4x}{3}+1}}$	21
gosper	$-\frac{2x(4x-3)(8x-3)}{9(-4x^2+3x)^{\frac{3}{2}}}$	25
orering	$-\frac{2x(4x-3)(8x-3)}{9(-4x^2+3x)^{\frac{3}{2}}}$	25
trager	$-\frac{2(8x-3)\sqrt{-4x^2+3x}}{9x(4x-3)}$	29

input
```
int(1/(-4*x^2+3*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(-8*x+3)/(-4*x^2+3*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx = -\frac{2\sqrt{-4\,x^2+3\,x}(8\,x-3)}{9\,(4\,x^2-3\,x)}$$

input
```
integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/9*sqrt(-4*x^2 + 3*x)*(8*x - 3)/(4*x^2 - 3*x)
```

Sympy [F]

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx=\int \frac{1}{(-4x^2+3x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(-4*x**2+3*x)**(3/2),x)
```

output
```
Integral((-4*x**2 + 3*x)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx=\frac{16\,x}{9\,\sqrt{-4\,x^2+3\,x}}-\frac{2}{3\,\sqrt{-4\,x^2+3\,x}}$$

input
```
integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="maxima")
```

output
```
16/9*x/sqrt(-4*x^2 + 3*x) - 2/3/sqrt(-4*x^2 + 3*x)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx=-\frac{2\,\sqrt{-4\,x^2+3\,x}(8\,x-3)}{9\,(4\,x^2-3\,x)}$$

input
```
integrate(1/(-4*x^2+3*x)^(3/2),x, algorithm="giac")
```

output
```
-2/9*sqrt(-4*x^2 + 3*x)*(8*x - 3)/(4*x^2 - 3*x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.49

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx = \frac{16\,x-6}{9\,\sqrt{3\,x-4\,x^2}}$$

input
```
int(1/(3*x - 4*x^2)^(3/2),x)
```

output
```
(16*x - 6)/(9*(3*x - 4*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{1}{(3x-4x^2)^{3/2}}\,dx = \frac{-\frac{8\sqrt{-4x+3}\,ix}{9}+\frac{16\sqrt{x}\,x}{9}-\frac{2\sqrt{x}}{3}}{\sqrt{-4x+3}\,x}$$

input
```
int(1/(-4*x^2+3*x)^(3/2),x)
```

output
```
(2*( - 4*sqrt( - 4*x + 3)*i*x + 8*sqrt(x)*x - 3*sqrt(x)))/(9*sqrt( - 4*x +
 3)*x)
```

3.22 $\int \frac{1}{(3x-4x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx = \frac{2}{9(3-4x)^{3/2}x^{3/2}} + \frac{4}{9\sqrt{3-4x}x^{3/2}} - \frac{16\sqrt{3-4x}}{81x^{3/2}} - \frac{128\sqrt{3-4x}}{243\sqrt{x}}$$

output
```
2/9/(3-4*x)^(3/2)/x^(3/2)+4/9/(3-4*x)^(1/2)/x^(3/2)-16/81*(3-4*x)^(1/2)/x^
(3/2)-128/243*(3-4*x)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.42

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx = -\frac{54+432x-2304x^2+2048x^3}{243(-x(-3+4x))^{3/2}}$$

input
```
Integrate[(3*x - 4*x^2)^(-5/2),x]
```

output
```
-1/243*(54 + 432*x - 2304*x^2 + 2048*x^3)/(-(x*(-3 + 4*x)))^(3/2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.62, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{32}{27}\int \frac{1}{(3x-4x^2)^{3/2}}dx-\frac{2(3-8x)}{27\,(3x-4x^2)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{64(3-8x)}{243\sqrt{3x-4x^2}}-\frac{2(3-8x)}{27\,(3x-4x^2)^{3/2}}$$

input
```
Int[(3*x - 4*x^2)^(-5/2),x]
```

output
```
(-2*(3 - 8*x))/(27*(3*x - 4*x^2)^(3/2)) - (64*(3 - 8*x))/(243*Sqrt[3*x - 4
*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.42

method	result	size
meijerg	$-\frac{2\sqrt{3}\,\left(\frac{1024}{27}x^{3}-\frac{128}{3}x^{2}+8x+1\right)}{81x^{\frac{3}{2}}\left(-\frac{4x}{3}+1\right)^{\frac{3}{2}}}$	31
gosper	$\frac{2x(4x-3)\left(1024x^{3}-1152x^{2}+216x+27\right)}{243\left(-4x^{2}+3x\right)^{\frac{5}{2}}}$	35
orering	$\frac{2x(4x-3)\left(1024x^{3}-1152x^{2}+216x+27\right)}{243\left(-4x^{2}+3x\right)^{\frac{5}{2}}}$	35
default	$-\frac{2(-8x+3)}{27\left(-4x^{2}+3x\right)^{\frac{3}{2}}}-\frac{64(-8x+3)}{243\sqrt{-4x^{2}+3x}}$	38
trager	$-\frac{2\left(1024x^{3}-1152x^{2}+216x+27\right)\sqrt{-4x^{2}+3x}}{243x^{2}\left(4x-3\right)^{2}}$	39
pseudoelliptic	$\frac{2048x^{3}-2304x^{2}+432x+54}{\sqrt{-4x^{2}+3x}\,\left(972x^{2}-729x\right)}$	39

input
```
int(1/(-4*x^2+3*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/81/x^(3/2)*3^(1/2)*(1024/27*x^3-128/3*x^2+8*x+1)/(-4/3*x+1)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx=-\frac{2\,(1024\,x^3-1152\,x^2+216\,x+27)\sqrt{-4\,x^2+3\,x}}{243\,(16\,x^4-24\,x^3+9\,x^2)}$$

input
```
integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/243*(1024*x^3 - 1152*x^2 + 216*x + 27)*sqrt(-4*x^2 + 3*x)/(16*x^4 - 24*
x^3 + 9*x^2)
```

Sympy [F]

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx = \int \frac{1}{(-4x^2+3x)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(-4*x**2+3*x)**(5/2),x)
```

output
```
Integral((-4*x**2 + 3*x)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx = \frac{512\,x}{243\,\sqrt{-4\,x^2+3\,x}} - \frac{64}{81\,\sqrt{-4\,x^2+3\,x}} + \frac{16\,x}{27\,(-4\,x^2+3\,x)^{\frac{3}{2}}} - \frac{2}{9\,(-4\,x^2+3\,x)^{\frac{3}{2}}}$$

input
```
integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="maxima")
```

output
```
512/243*x/sqrt(-4*x^2 + 3*x) - 64/81/sqrt(-4*x^2 + 3*x) + 16/27*x/(-4*x^2
+ 3*x)^(3/2) - 2/9/(-4*x^2 + 3*x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.53

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx = -\frac{2\,(8\,(16\,(8\,x-9)x+27)x+27)\sqrt{-4\,x^2+3\,x}}{243\,(4\,x^2-3\,x)^2}$$

input
```
integrate(1/(-4*x^2+3*x)^(5/2),x, algorithm="giac")
```

output
```
-2/243*(8*(16*(8*x - 9)*x + 27)*x + 27)*sqrt(-4*x^2 + 3*x)/(4*x^2 - 3*x)^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.38

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx = \frac{(16\,x-6)\,(-128\,x^2+96\,x+9)}{243\,(3\,x-4\,x^2)^{3/2}}$$

input
```
int(1/(3*x - 4*x^2)^(5/2),x)
```

output
```
((16*x - 6)*(96*x - 128*x^2 + 9))/(243*(3*x - 4*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

$$\int \frac{1}{(3x-4x^2)^{5/2}}\,dx = \frac{\frac{1024\sqrt{-4x+3}\,i\,x^3}{243} - \frac{256\sqrt{-4x+3}\,i\,x^2}{81} + \frac{2048\sqrt{x}\,x^3}{243} - \frac{256\sqrt{x}\,x^2}{27} + \frac{16\sqrt{x}\,x}{9} + \frac{2\sqrt{x}}{9}}{\sqrt{-4x+3}\,x^2\,(4x-3)}$$

input
```
int(1/(-4*x^2+3*x)^(5/2),x)
```

output
```
(2*(512*sqrt( - 4*x + 3)*i*x**3 - 384*sqrt( - 4*x + 3)*i*x**2 + 1024*sqrt(
x)*x**3 - 1152*sqrt(x)*x**2 + 216*sqrt(x)*x + 27*sqrt(x)))/(243*sqrt( - 4*
x + 3)*x**2*(4*x - 3))
```

3.23 $\int \frac{1}{(3x-4x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 109

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx = \frac{2}{15(3-4x)^{5/2}x^{5/2}} + \frac{4}{27(3-4x)^{3/2}x^{5/2}} + \frac{32}{81\sqrt{3-4x}\,x^{5/2}} - \frac{64\sqrt{3-4x}}{405x^{5/2}} - \frac{1024\sqrt{3-4x}}{3645x^{3/2}} - \frac{8192\sqrt{3-4x}}{10935\sqrt{x}}$$

output
```
2/15/(3-4*x)^(5/2)/x^(5/2)+4/27/(3-4*x)^(3/2)/x^(5/2)+32/81/(3-4*x)^(1/2)/
x^(5/2)-64/405*(3-4*x)^(1/2)/x^(5/2)-1024/3645*(3-4*x)^(1/2)/x^(3/2)-8192/
10935*(3-4*x)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.38

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx = \frac{2(-729-3240x-34560x^2+276480x^3-491520x^4+262144x^5)}{10935(-x(-3+4x))^{5/2}}$$

input
```
Integrate[(3*x - 4*x^2)^(-7/2),x]
```

output
```
(2*(-729 - 3240*x - 34560*x^2 + 276480*x^3 - 491520*x^4 + 262144*x^5))/(10
935*(-(x*(-3 + 4*x)))^(5/2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.66, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx$$
$$\Big\downarrow \text{1089}$$
$$\frac{64}{45}\int \frac{1}{(3x-4x^2)^{5/2}}dx-\frac{2(3-8x)}{45\,(3x-4x^2)^{5/2}}$$
$$\Big\downarrow \text{1089}$$
$$\frac{64}{45}\left(\frac{32}{27}\int \frac{1}{(3x-4x^2)^{3/2}}dx-\frac{2(3-8x)}{27\,(3x-4x^2)^{3/2}}\right)-\frac{2(3-8x)}{45\,(3x-4x^2)^{5/2}}$$
$$\Big\downarrow \text{1088}$$
$$\frac{64}{45}\left(-\frac{64(3-8x)}{243\sqrt{3x-4x^2}}-\frac{2(3-8x)}{27\,(3x-4x^2)^{3/2}}\right)-\frac{2(3-8x)}{45\,(3x-4x^2)^{5/2}}$$

input
```
Int[(3*x - 4*x^2)^(-7/2),x]
```

output
```
(-2*(3 - 8*x))/(45*(3*x - 4*x^2)^(5/2)) + (64*((-2*(3 - 8*x))/(27*(3*x - 4
*x^2)^(3/2)) - (64*(3 - 8*x))/(243*Sqrt[3*x - 4*x^2])))/45
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.38

method	result	size
meijerg	$-\frac{2\sqrt{3}\left(-\frac{262144}{243}x^{5}+\frac{163840}{81}x^{4}-\frac{10240}{9}x^{3}+\frac{1280}{9}x^{2}+\frac{40}{3}x+3\right)}{1215x^{\frac{5}{2}}\left(-\frac{4x}{3}+1\right)^{\frac{5}{2}}}$	41
gosper	$-\frac{2x(4x-3)\left(262144x^{5}-491520x^{4}+276480x^{3}-34560x^{2}-3240x-729\right)}{10935\left(-4x^{2}+3x\right)^{\frac{7}{2}}}$	45
orering	$-\frac{2x(4x-3)\left(262144x^{5}-491520x^{4}+276480x^{3}-34560x^{2}-3240x-729\right)}{10935\left(-4x^{2}+3x\right)^{\frac{7}{2}}}$	45
trager	$-\frac{2\left(262144x^{5}-491520x^{4}+276480x^{3}-34560x^{2}-3240x-729\right)\sqrt{-4x^{2}+3x}}{10935x^{3}(4x-3)^{3}}$	49
pseudoelliptic	$\frac{\frac{524288}{10935}x^{5}-\frac{65536}{729}x^{4}+\frac{4096}{81}x^{3}-\frac{512}{81}x^{2}-\frac{16}{27}x-\frac{2}{15}}{x^{2}(4x-3)^{2}\sqrt{-4x^{2}+3x}}$	49
default	$-\frac{2(-8x+3)}{45\left(-4x^{2}+3x\right)^{\frac{5}{2}}}-\frac{128(-8x+3)}{1215\left(-4x^{2}+3x\right)^{\frac{3}{2}}}-\frac{4096(-8x+3)}{10935\sqrt{-4x^{2}+3x}}$	56

input
```
int(1/(-4*x^2+3*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/1215/x^(5/2)*3^(1/2)*(-262144/243*x^5+163840/81*x^4-10240/9*x^3+1280/9*
x^2+40/3*x+3)/(-4/3*x+1)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.56

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx = -\frac{2\,(262144\,x^5-491520\,x^4+276480\,x^3-34560\,x^2-3240\,x-729)\sqrt{-4\,x^2+3\,x}}{10935\,(64\,x^6-144\,x^5+108\,x^4-27\,x^3)}$$

input
```
integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/10935*(262144*x^5 - 491520*x^4 + 276480*x^3 - 34560*x^2 - 3240*x - 729)
*sqrt(-4*x^2 + 3*x)/(64*x^6 - 144*x^5 + 108*x^4 - 27*x^3)
```

Sympy [F]

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx = \int \frac{1}{(-4x^2+3x)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(-4*x**2+3*x)**(7/2),x)
```

output
```
Integral((-4*x**2 + 3*x)**(-7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx = \frac{32768\,x}{10935\,\sqrt{-4\,x^2+3\,x}} - \frac{4096}{3645\,\sqrt{-4\,x^2+3\,x}} + \frac{1024\,x}{1215\,(-4\,x^2+3\,x)^{\frac{3}{2}}} - \frac{128}{405\,(-4\,x^2+3\,x)^{\frac{3}{2}}} + \frac{16\,x}{45\,(-4\,x^2+3\,x)^{\frac{5}{2}}} - \frac{2}{15\,(-4\,x^2+3\,x)^{\frac{5}{2}}}$$

input
```
integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="maxima")
```

output
```
32768/10935*x/sqrt(-4*x^2 + 3*x) - 4096/3645/sqrt(-4*x^2 + 3*x) + 1024/121
5*x/(-4*x^2 + 3*x)^(3/2) - 128/405/(-4*x^2 + 3*x)^(3/2) + 16/45*x/(-4*x^2
+ 3*x)^(5/2) - 2/15/(-4*x^2 + 3*x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.45

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx = -\frac{2\,(8\,(32\,(8\,(16\,(8\,x-15)x+135)x-135)x-405)x-729)\sqrt{-4\,x^2+3\,x}}{10935\,(4\,x^2-3\,x)^3}$$

input
```
integrate(1/(-4*x^2+3*x)^(7/2),x, algorithm="giac")
```

output
```
-2/10935*(8*(32*(8*(16*(8*x - 15)*x + 135)*x - 135)*x - 405)*x - 729)*sqrt
(-4*x^2 + 3*x)/(4*x^2 - 3*x)^3
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.67

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx = -\frac{6480\,x-9216\,x\,(3\,x-4\,x^2)-32768\,x\,(3\,x-4\,x^2)^2+12288\,(3\,x-4\,x^2)^2-13824\,x^2+1458}{(3\,x-4\,x^2)^{3/2}\,(32805\,x-43740\,x^2)}$$

input
```
int(1/(3*x - 4*x^2)^(7/2),x)
```

output
```
-(6480*x - 9216*x*(3*x - 4*x^2) - 32768*x*(3*x - 4*x^2)^2 + 12288*(3*x - 4
*x^2)^2 - 13824*x^2 + 1458)/((3*x - 4*x^2)^(3/2)*(32805*x - 43740*x^2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.91

$$\int \frac{1}{(3x-4x^2)^{7/2}}\,dx = \frac{\frac{262144\sqrt{-4x+3}\,i\,x^5}{10935} - \frac{131072\sqrt{-4x+3}\,i\,x^4}{3645} + \frac{16384\sqrt{-4x+3}\,i\,x^3}{1215} + \frac{524288\sqrt{x}\,x^5}{10935} - \frac{65536\sqrt{x}\,x^4}{729} + \frac{4096\sqrt{}}{81}}{\sqrt{-4x+3}\,x^3\,(16x^2-24x+9)}$$

input
```
int(1/(-4*x^2+3*x)^(7/2),x)
```

output
```
(2*(131072*sqrt( - 4*x + 3)*i*x**5 - 196608*sqrt( - 4*x + 3)*i*x**4 + 7372
8*sqrt( - 4*x + 3)*i*x**3 + 262144*sqrt(x)*x**5 - 491520*sqrt(x)*x**4 + 27
6480*sqrt(x)*x**3 - 34560*sqrt(x)*x**2 - 3240*sqrt(x)*x - 729*sqrt(x)))/(1
0935*sqrt( - 4*x + 3)*x**3*(16*x**2 - 24*x + 9))
```

3.24 $\int \frac{1}{(3x-4x^2)^{9/2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 145

$$\int \frac{1}{(3x-4x^2)^{9/2}}\,dx = \frac{2}{21(3-4x)^{7/2}x^{7/2}} + \frac{4}{45(3-4x)^{5/2}x^{7/2}} + \frac{16}{135(3-4x)^{3/2}x^{7/2}} + \frac{32}{81\sqrt{3-4x}x^{7/2}} - \frac{256\sqrt{3-4x}}{1701x^{7/2}} - \frac{2048\sqrt{3-4x}}{8505x^{5/2}} - \frac{32768\sqrt{3-4x}}{76545x^{3/2}} - \frac{262144\sqrt{3-4x}}{229635\sqrt{x}}$$

output
```
2/21/(3-4*x)^(7/2)/x^(7/2)+4/45/(3-4*x)^(5/2)/x^(7/2)+16/135/(3-4*x)^(3/2)
/x^(7/2)+32/81/(3-4*x)^(1/2)/x^(7/2)-256/1701*(3-4*x)^(1/2)/x^(7/2)-2048/8
505*(3-4*x)^(1/2)/x^(5/2)-32768/76545*(3-4*x)^(1/2)/x^(3/2)-262144/229635*
(3-4*x)^(1/2)/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.35

$$\int \frac{1}{(3x-4x^2)^{9/2}}\,dx = -\frac{2(10935+40824x+217728x^2+2903040x^3-30965760x^4+82575360x^5-88080384x^6+33554432x^7)}{229635(-x(-3+4x))^{7/2}}$$

input
```
Integrate[(3*x - 4*x^2)^(-9/2),x]
```

output
```
(-2*(10935 + 40824*x + 217728*x^2 + 2903040*x^3 - 30965760*x^4 + 82575360*
x^5 - 88080384*x^6 + 33554432*x^7))/(229635*(-(x*(-3 + 4*x)))^(7/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1089, 1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x-4x^2)^{9/2}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{32}{21}\int \frac{1}{(3x-4x^2)^{7/2}}dx-\frac{2(3-8x)}{63(3x-4x^2)^{7/2}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{32}{21}\left(\frac{64}{45}\int \frac{1}{(3x-4x^2)^{5/2}}dx-\frac{2(3-8x)}{45(3x-4x^2)^{5/2}}\right)-\frac{2(3-8x)}{63(3x-4x^2)^{7/2}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{32}{21}\left(\frac{64}{45}\left(\frac{32}{27}\int \frac{1}{(3x-4x^2)^{3/2}}dx-\frac{2(3-8x)}{27(3x-4x^2)^{3/2}}\right)-\frac{2(3-8x)}{45(3x-4x^2)^{5/2}}\right)-\frac{2(3-8x)}{63(3x-4x^2)^{7/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{32}{21}\left(\frac{64}{45}\left(-\frac{64(3-8x)}{243\sqrt{3x-4x^2}}-\frac{2(3-8x)}{27(3x-4x^2)^{3/2}}\right)-\frac{2(3-8x)}{45(3x-4x^2)^{5/2}}\right)-\frac{2(3-8x)}{63(3x-4x^2)^{7/2}}$$

input
```
Int[(3*x - 4*x^2)^(-9/2),x]
```

output
```
(-2*(3 - 8*x))/(63*(3*x - 4*x^2)^(7/2)) + (32*((-2*(3 - 8*x))/(45*(3*x - 4
*x^2)^(5/2)) + (64*((-2*(3 - 8*x))/(27*(3*x - 4*x^2)^(3/2)) - (64*(3 - 8*x
))/(243*Sqrt[3*x - 4*x^2])))/45))/21
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.35

method	result	size
meijerg	$-\frac{2\sqrt{3}\left(\frac{33554432}{2187}x^{7}-\frac{29360128}{729}x^{6}+\frac{9175040}{243}x^{5}-\frac{1146880}{81}x^{4}+\frac{35840}{27}x^{3}+\frac{896}{9}x^{2}+\frac{56}{3}x+5\right)}{8505x^{\frac{7}{2}}\left(-\frac{4x}{3}+1\right)^{\frac{7}{2}}}$	51
gosper	$\frac{2x(4x-3)(33554432x^{7}-88080384x^{6}+82575360x^{5}-30965760x^{4}+2903040x^{3}+217728x^{2}+40824x+10935)}{229635(-4x^{2}+3x)^{\frac{9}{2}}}$	55
orering	$\frac{2x(4x-3)(33554432x^{7}-88080384x^{6}+82575360x^{5}-30965760x^{4}+2903040x^{3}+217728x^{2}+40824x+10935)}{229635(-4x^{2}+3x)^{\frac{9}{2}}}$	55
trager	$-\frac{2(33554432x^{7}-88080384x^{6}+82575360x^{5}-30965760x^{4}+2903040x^{3}+217728x^{2}+40824x+10935)\sqrt{-4x^{2}+3x}}{229635x^{4}(4x-3)^{4}}$	59
default	$-\frac{2(-8x+3)}{63(-4x^{2}+3x)^{\frac{7}{2}}}-\frac{64(-8x+3)}{945(-4x^{2}+3x)^{\frac{5}{2}}}-\frac{4096(-8x+3)}{25515(-4x^{2}+3x)^{\frac{3}{2}}}-\frac{131072(-8x+3)}{229635\sqrt{-4x^{2}+3x}}$	74

input
```
int(1/(-4*x^2+3*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/8505/x^(7/2)*3^(1/2)*(33554432/2187*x^7-29360128/729*x^6+9175040/243*x^
5-1146880/81*x^4+35840/27*x^3+896/9*x^2+56/3*x+5)/(-4/3*x+1)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.52

$$\int \frac{1}{(3x-4x^2)^{9/2}}\,dx = -\frac{2\,(33554432\,x^7 - 88080384\,x^6 + 82575360\,x^5 - 30965760\,x^4 + 2903040\,x^3 + 217728\,x^2 + 40824\,x + 1093}{229635\,(256\,x^8 - 768\,x^7 + 864\,x^6 - 432\,x^5 + 81\,x^4)}$$

input
```
integrate(1/(-4*x^2+3*x)^(9/2),x, algorithm="fricas")
```

output
```
-2/229635*(33554432*x^7 - 88080384*x^6 + 82575360*x^5 - 30965760*x^4 + 290
3040*x^3 + 217728*x^2 + 40824*x + 10935)*sqrt(-4*x^2 + 3*x)/(256*x^8 - 768
*x^7 + 864*x^6 - 432*x^5 + 81*x^4)
```

Sympy [F]

$$\int \frac{1}{(3x-4x^2)^{9/2}}\,dx = \int \frac{1}{(-4x^2+3x)^{\frac{9}{2}}}\,dx$$

input
```
integrate(1/(-4*x**2+3*x)**(9/2),x)
```

output
```
Integral((-4*x**2 + 3*x)**(-9/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.75

$$\int \frac{1}{(3x-4x^2)^{9/2}}\,dx = \frac{1048576\,x}{229635\,\sqrt{-4\,x^2+3\,x}} - \frac{131072}{76545\,\sqrt{-4\,x^2+3\,x}} + \frac{32768\,x}{25515\,(-4\,x^2+3\,x)^{\frac{3}{2}}} - \frac{4096}{8505\,(-4\,x^2+3\,x)^{\frac{3}{2}}} + \frac{512\,x}{945\,(-4\,x^2+3\,x)^{\frac{5}{2}}} - \frac{64}{315\,(-4\,x^2+3\,x)^{\frac{5}{2}}} + \frac{16\,x}{63\,(-4\,x^2+3\,x)^{\frac{7}{2}}} - \frac{2}{21\,(-4\,x^2+3\,x)^{\frac{7}{2}}}$$

input
```
integrate(1/(-4*x^2+3*x)^(9/2),x, algorithm="maxima")
```

output
```
1048576/229635*x/sqrt(-4*x^2 + 3*x) - 131072/76545/sqrt(-4*x^2 + 3*x) + 32
768/25515*x/(-4*x^2 + 3*x)^(3/2) - 4096/8505/(-4*x^2 + 3*x)^(3/2) + 512/94
5*x/(-4*x^2 + 3*x)^(5/2) - 64/315/(-4*x^2 + 3*x)^(5/2) + 16/63*x/(-4*x^2 +
 3*x)^(7/2) - 2/21/(-4*x^2 + 3*x)^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.41

$$\int \frac{1}{(3x-4x^2)^{9/2}}\,dx = -\frac{2\,(8\,(16\,(8\,(32\,(8\,(16\,(8\,x-21)x+315)x-945)x+2835)x+1701)x+5103)x+10935)\sqrt{-4\,x^2+3\,x}}{229635\,(4\,x^2-3\,x)^4}$$

input
```
integrate(1/(-4*x^2+3*x)^(9/2),x, algorithm="giac")
```

output
```
-2/229635*(8*(16*(8*(32*(8*(16*(8*x - 21)*x + 315)*x - 945)*x + 2835)*x +
1701)*x + 5103)*x + 10935)*sqrt(-4*x^2 + 3*x)/(4*x^2 - 3*x)^4
```

Mupad [B] (verification not implemented)

Time = 8.93 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.48

$$\int \frac{1}{(3x-4x^2)^{9/2}}\,dx = \frac{\frac{16\,x}{63}-\frac{2}{21}}{(3\,x-4\,x^2)^{7/2}}+\frac{\frac{512\,x}{945}-\frac{64}{315}}{(3\,x-4\,x^2)^{5/2}}+\frac{\frac{32768\,x}{25515}-\frac{4096}{8505}}{(3\,x-4\,x^2)^{3/2}}+\frac{\frac{1048576\,x}{229635}-\frac{131072}{76545}}{\sqrt{3\,x-4\,x^2}}$$

input
```
int(1/(3*x - 4*x^2)^(9/2),x)
```

output
```
((16*x)/63 - 2/21)/(3*x - 4*x^2)^(7/2) + ((512*x)/945 - 64/315)/(3*x - 4*x
^2)^(5/2) + ((32768*x)/25515 - 4096/8505)/(3*x - 4*x^2)^(3/2) + ((1048576*
x)/229635 - 131072/76545)/(3*x - 4*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.90

$$\int \frac{1}{(3x-4x^2)^{9/2}}\,dx = \frac{\frac{33554432\sqrt{-4x+3}\,i\,x^7}{229635} - \frac{8388608\sqrt{-4x+3}\,i\,x^6}{25515} + \frac{2097152\sqrt{-4x+3}\,i\,x^5}{8505} - \frac{524288\sqrt{-4x+3}\,i\,x^4}{8505} + \frac{67108864}{22963}}{\sqrt{-4x+3}\,x^4\,(64x^3 -}$$

input
```
int(1/(-4*x^2+3*x)^(9/2),x)
```

output
```
(2*(16777216*sqrt( - 4*x + 3)*i*x**7 - 37748736*sqrt( - 4*x + 3)*i*x**6 +
28311552*sqrt( - 4*x + 3)*i*x**5 - 7077888*sqrt( - 4*x + 3)*i*x**4 + 33554
432*sqrt(x)*x**7 - 88080384*sqrt(x)*x**6 + 82575360*sqrt(x)*x**5 - 3096576
0*sqrt(x)*x**4 + 2903040*sqrt(x)*x**3 + 217728*sqrt(x)*x**2 + 40824*sqrt(x
)*x + 10935*sqrt(x)))/(229635*sqrt( - 4*x + 3)*x**4*(64*x**3 - 144*x**2 +
108*x - 27))
```

3.25 $\int (3ix+4x^2)^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 155

$$\int (3ix+4x^2)^{5/2}\,dx=\frac{1215i\sqrt{3ix+4x^2}}{32768}-\frac{135x\sqrt{3ix+4x^2}}{4096}-\frac{9}{256}ix^2\sqrt{3ix+4x^2}-\frac{81}{32}x^3\sqrt{3ix+4x^2}+5ix^4\sqrt{3ix+4x^2}+\frac{8}{3}x^5\sqrt{3ix+4x^2}+\frac{3645\operatorname{arctanh}\left(\frac{2x}{\sqrt{3ix+4x^2}}\right)}{65536}$$

output
```
1215/32768*I*(3*I*x+4*x^2)^(1/2)-135/4096*x*(3*I*x+4*x^2)^(1/2)-9/256*I*x^
2*(3*I*x+4*x^2)^(1/2)-81/32*x^3*(3*I*x+4*x^2)^(1/2)+5*I*x^4*(3*I*x+4*x^2)^
(1/2)+8/3*x^5*(3*I*x+4*x^2)^(1/2)+3645/65536*arctanh(2*x/(3*I*x+4*x^2)^(1/
2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.55

$$\int (3ix+4x^2)^{5/2}\,dx=\frac{\sqrt{x(3i+4x)}\left(7290i-6480x-6912ix^2-497664x^3+983040ix^4+524288x^5-\frac{10935\log(-2\sqrt{}}{\sqrt{x}\sqrt{3}}\right.}{196608}$$

input
```
Integrate[((3*I)*x + 4*x^2)^(5/2),x]
```

output
```
(Sqrt[x*(3*I + 4*x)]*(7290*I - 6480*x - (6912*I)*x^2 - 497664*x^3 + (98304
0*I)*x^4 + 524288*x^5 - (10935*Log[-2*Sqrt[x] + Sqrt[3*I + 4*x]])/(Sqrt[x]
*Sqrt[3*I + 4*x])))/196608
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1087, 1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4x^2+3ix\right)^{5/2}dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{15}{32}\int \left(4x^2+3ix\right)^{3/2}dx+\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{15}{32}\left(\frac{27}{64}\int \sqrt{4x^2+3ix}dx+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}\right)+\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{15}{32}\left(\frac{27}{64}\left(\frac{9}{32}\int \frac{1}{\sqrt{4x^2+3ix}}dx+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)\right)+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}\right)+\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{15}{32}\left(\frac{27}{64}\left(\frac{3}{64}\int \frac{1}{\sqrt{\frac{1}{9}(8x+3i)^2+1}}d(8x+3i)+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)\right)+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}\right)+\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}$$

$$\Big\downarrow \text{222}$$

$$\frac{15}{32}\left(\frac{27}{64}\left(\frac{9}{64}\text{arcsinh}\left(\frac{1}{3}(8x+3i)\right)+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)\right)+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}\right)+\frac{1}{48}(8x+3i)\left(4x^2+3ix\right)^{5/2}$$

input
```
Int[((3*I)*x + 4*x^2)^(5/2),x]
```

output
```
((3*I + 8*x)*((3*I)*x + 4*x^2)^(5/2))/48 + (15*(((3*I + 8*x)*((3*I)*x + 4*
x^2)^(3/2))/32 + (27*(((3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/16 + (9*ArcSinh[
(3*I + 8*x)/3])/64))/64))/32
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.37

method	result
risch	$\frac{(262144x^5+491520ix^4-248832x^3-3456ix^2-3240x+3645i)x(3i+4x)}{98304\sqrt{x(3i+4x)}}+\frac{3645\,\mathrm{arcsinh}\left(i+\frac{8x}{3}\right)}{131072}$
default	$\frac{(3i+8x)(4x^2+3ix)^{\frac{5}{2}}}{48}+\frac{15(4x^2+3ix)^{\frac{3}{2}}(3i+8x)}{1024}+\frac{405(3i+8x)\sqrt{4x^2+3ix}}{32768}+\frac{3645\,\mathrm{arcsinh}\left(i+\frac{8x}{3}\right)}{131072}$
trager	$\left(5ix^4+\frac{8}{3}x^5-\frac{9}{256}ix^2-\frac{81}{32}x^3+\frac{1215}{32768}i-\frac{135}{4096}x\right)\sqrt{4x^2+3ix}+\frac{3645\ln\left(440x+144+165i-192i\sqrt{4x^2+3ix}-38\right.}{131072}$

input
```
int((3*I*x+4*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/98304*(3645*I-3240*x-3456*I*x^2-248832*x^3+491520*I*x^4+262144*x^5)*x*(3
*I+4*x)/(x*(3*I+4*x))^(1/2)+3645/131072*arcsinh(I+8/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.38

$$\int (3ix+4x^2)^{5/2}\,dx=\frac{1}{98304}\left(262144\,x^5+491520i\,x^4-248832\,x^3-3456i\,x^2-3240\,x+3645i\right)\sqrt{4\,x^2+3i\,x}-\frac{3645}{131072}\log\left(-2\,x+\sqrt{4\,x^2+3i\,x}-\frac{3}{4}i\right)-\frac{8991}{1048576}$$

input
```
integrate((3*I*x+4*x^2)^(5/2),x, algorithm="fricas")
```

output
```
1/98304*(262144*x^5 + 491520*I*x^4 - 248832*x^3 - 3456*I*x^2 - 3240*x + 36
45*I)*sqrt(4*x^2 + 3*I*x) - 3645/131072*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3
/4*I) - 8991/1048576
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.05

$$\int \left(3ix+4x^2\right)^{5/2}\,dx = -9\sqrt{4x^2+3ix}\left(\frac{x^3}{4}+\frac{ix^2}{32}+\frac{15x}{512}-\frac{135i}{4096}\right) + 16\sqrt{4x^2+3ix}\left(\frac{x^5}{6}+\frac{ix^4}{80}+\frac{27x^3}{2560}-\frac{189ix^2}{20480}-\frac{567x}{65536}+\frac{5103i}{524288}\right) + 24i\left(\sqrt{4x^2+3ix}\left(\frac{x^4}{5}+\frac{3ix^3}{160}+\frac{21x^2}{1280}-\frac{63ix}{4096}-\frac{567}{32768}\right)+\frac{1701i\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{131072}\right) + \frac{44469\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{131072}$$

input
```
integrate((3*I*x+4*x**2)**(5/2),x)
```

output
```
-9*sqrt(4*x**2 + 3*I*x)*(x**3/4 + I*x**2/32 + 15*x/512 - 135*I/4096) + 16*
sqrt(4*x**2 + 3*I*x)*(x**5/6 + I*x**4/80 + 27*x**3/2560 - 189*I*x**2/20480
 - 567*x/65536 + 5103*I/524288) + 24*I*(sqrt(4*x**2 + 3*I*x)*(x**4/5 + 3*I
*x**3/160 + 21*x**2/1280 - 63*I*x/4096 - 567/32768) + 1701*I*asinh(8*x/3 +
 I)/131072) + 44469*asinh(8*x/3 + I)/131072
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.66

$$\int \left(3ix+4x^2\right)^{5/2}\,dx = \frac{1}{6}\left(4x^2+3ix\right)^{\frac{5}{2}}x+\frac{1}{16}i\left(4x^2+3ix\right)^{\frac{5}{2}} + \frac{15}{128}\left(4x^2+3ix\right)^{\frac{3}{2}}x+\frac{45}{1024}i\left(4x^2+3ix\right)^{\frac{3}{2}}+\frac{405}{4096}\sqrt{4x^2+3ix}\,x + \frac{1215}{32768}i\sqrt{4x^2+3ix}+\frac{3645}{131072}\log\left(8x+4\sqrt{4x^2+3ix}+3i\right)$$

input
```
integrate((3*I*x+4*x^2)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(4*x^2 + 3*I*x)^(5/2)*x + 1/16*I*(4*x^2 + 3*I*x)^(5/2) + 15/128*(4*x^2
 + 3*I*x)^(3/2)*x + 45/1024*I*(4*x^2 + 3*I*x)^(3/2) + 405/4096*sqrt(4*x^2
+ 3*I*x)*x + 1215/32768*I*sqrt(4*x^2 + 3*I*x) + 3645/131072*log(8*x + 4*sq
rt(4*x^2 + 3*I*x) + 3*I)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.84

$$\int (3ix+4x^2)^{5/2}\,dx = \frac{1}{196608}(8(16(8(32(8x+15i)x-243)x-27i)x-405)x+3645i)\sqrt{8x^2+2\sqrt{16x^2+9}x} - \frac{3645}{131072}\log\left(2\sqrt{8x^2+2\sqrt{16x^2+9}x}\left(\frac{3i\,x}{4x^2+\sqrt{16x^4+9x^2}}+1\right)-8x-3i\right)$$

input
```
integrate((3*I*x+4*x^2)^(5/2),x, algorithm="giac")
```

output
```
1/196608*(8*(16*(8*(32*(8*x + 15*I)*x - 243)*x - 27*I)*x - 405)*x + 3645*I
)*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2))
 + 1) - 3645/131072*log(2*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*x/(4*x^2
 + sqrt(16*x^4 + 9*x^2)) + 1) - 8*x - 3*I)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.52

$$\int (3ix+4x^2)^{5/2}\,dx = \frac{3645\ln\left(x+\frac{\sqrt{x\,(4\,x+3\mathrm{i})}}{2}+\frac{3}{8}\mathrm{i}\right)}{131072} + \frac{15\,\left(4\,x+\frac{3}{2}\mathrm{i}\right)\,{\left(4\,x^2+x\,3\mathrm{i}\right)}^{3/2}}{512} + \frac{\left(4\,x+\frac{3}{2}\mathrm{i}\right)\,{\left(4\,x^2+x\,3\mathrm{i}\right)}^{5/2}}{24} + \frac{405\,\left(\frac{x}{2}+\frac{3}{16}\mathrm{i}\right)\,\sqrt{4\,x^2+x\,3\mathrm{i}}}{2048}$$

input
```
int((x*3i + 4*x^2)^(5/2),x)
```

output
```
(3645*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/131072 + (15*(4*x + 3i/2)*(x
*3i + 4*x^2)^(3/2))/512 + ((4*x + 3i/2)*(x*3i + 4*x^2)^(5/2))/24 + (405*(x
/2 + 3i/16)*(x*3i + 4*x^2)^(1/2))/2048
```

Reduce [F]

$$\int \left(3ix+4x^2\right)^{5/2} dx = \int \left(4x^2+3ix\right)^{\frac{5}{2}} dx$$

input
```
int((3*I*x+4*x^2)^(5/2),x)
```

output
```
int((3*I*x+4*x^2)^(5/2),x)
```

3.26 $\int \left(3ix+4x^2\right)^{3/2} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \left(3ix+4x^2\right)^{3/2} \, dx = \frac{81i\sqrt{3ix+4x^2}}{1024} - \frac{9}{128}x\sqrt{3ix+4x^2} + \frac{9}{8}ix^2\sqrt{3ix+4x^2} + x^3\sqrt{3ix+4x^2} + \frac{243\text{arctanh}\left(\frac{2x}{\sqrt{3ix+4x^2}}\right)}{2048}$$

output
```
81/1024*I*(3*I*x+4*x^2)^(1/2)-9/128*x*(3*I*x+4*x^2)^(1/2)+9/8*I*x^2*(3*I*x
+4*x^2)^(1/2)+x^3*(3*I*x+4*x^2)^(1/2)+243/2048*arctanh(2*x/(3*I*x+4*x^2)^(
1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.77

$$\int \left(3ix+4x^2\right)^{3/2} \, dx = \frac{2x(-243+108ix-3744x^2+7680ix^3+4096x^4)-243\sqrt{x}\sqrt{3i+4x}\log\left(-2\sqrt{x}+\sqrt{3i+4x}\right)}{2048\sqrt{x(3i+4x)}}$$

input
```
Integrate[((3*I)*x + 4*x^2)^(3/2),x]
```

output
```
(2*x*(-243 + (108*I)*x - 3744*x^2 + (7680*I)*x^3 + 4096*x^4) - 243*Sqrt[x]
*Sqrt[3*I + 4*x]*Log[-2*Sqrt[x] + Sqrt[3*I + 4*x]])/(2048*Sqrt[x*(3*I + 4*
x)])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.69, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1087, 1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(4x^2+3ix\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{27}{64}\int \sqrt{4x^2+3ix}dx+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{27}{64}\left(\frac{9}{32}\int \frac{1}{\sqrt{4x^2+3ix}}dx+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)\right)+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{27}{64}\left(\frac{3}{64}\int \frac{1}{\sqrt{\frac{1}{9}(8x+3i)^2+1}}d(8x+3i)+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)\right)+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}$$

$$\Big\downarrow \text{ 222}$$

$$\frac{27}{64}\left(\frac{9}{64}\text{arcsinh}\left(\frac{1}{3}(8x+3i)\right)+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)\right)+\frac{1}{32}(8x+3i)\left(4x^2+3ix\right)^{3/2}$$

input
```
Int[((3*I)*x + 4*x^2)^(3/2),x]
```

output
```
((3*I + 8*x)*((3*I)*x + 4*x^2)^(3/2))/32 + (27*(((3*I + 8*x)*Sqrt[(3*I)*x
+ 4*x^2])/16 + (9*ArcSinh[(3*I + 8*x)/3])/64))/64
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.44

method	result
risch	$\frac{\left(1024 x^{3}+1152 i x^{2}-72 x +81 i\right) x \left(3 i +4 x \right)}{1024 \sqrt{x \left(3 i +4 x \right)}}+\frac{243 \,\operatorname{arcsinh}\left(i +\frac{8 x}{3}\right)}{4096}$
default	$\frac{\left(4 x^{2}+3 i x \right)^{\frac{3}{2}} \left(3 i +8 x \right)}{32}+\frac{27 \left(3 i +8 x \right) \sqrt{4 x^{2}+3 i x}}{1024}+\frac{243 \,\operatorname{arcsinh}\left(i +\frac{8 x}{3}\right)}{4096}$
trager	$\left(\frac{9}{8} i x^{2}+x^{3}+\frac{81}{1024} i -\frac{9}{128} x \right) \sqrt{4 x^{2}+3 i x}+\frac{243 \ln \left(440 x +144+165 i -192 i \sqrt{4 x^{2}+3 i x}-384 i x +220 \sqrt{4 x^{2}+3 i x}\right.}{4096}$
pseudoelliptic	$\frac{729 x^{4} \left(\frac{27 \ln \left(\frac{2 x +\sqrt{x \left(3 i +4 x \right)}}{x}\right)}{512}-\frac{27 \ln \left(\frac{\sqrt{x \left(3 i +4 x \right)}-2 x}{x}\right)}{512}+\left(i x^{2}+\frac{8}{9} x^{3}+\frac{9}{128} i -\frac{1}{16} x \right) \sqrt{x \left(3 i +4 x \right)}\right)}{8 \left(2 x +\sqrt{x \left(3 i +4 x \right)}\right)^{4} \left(-\sqrt{x \left(3 i +4 x \right)}+2 x \right)^{4}}$

input
```
int((3*I*x+4*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1024*(81*I-72*x+1152*I*x^2+1024*x^3)*x*(3*I+4*x)/(x*(3*I+4*x))^(1/2)+243
/4096*arcsinh(I+8/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.45

$$\int \left(3ix+4x^2\right)^{3/2} dx = \frac{1}{1024} \left(1024 x^3 + 1152i x^2 - 72 x + 81i\right)\sqrt{4 x^2 + 3i x} - \frac{243}{4096} \log\left(-2 x + \sqrt{4 x^2 + 3i x} - \frac{3}{4}i\right) - \frac{567}{32768}$$

input
```
integrate((3*I*x+4*x^2)^(3/2),x, algorithm="fricas")
```

output
```
1/1024*(1024*x^3 + 1152*I*x^2 - 72*x + 81*I)*sqrt(4*x^2 + 3*I*x) - 243/409
6*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I) - 567/32768
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.85

$$\int \left(3ix+4x^2\right)^{3/2} dx = 4\sqrt{4x^2+3ix}\left(\frac{x^3}{4}+\frac{ix^2}{32}+\frac{15x}{512}-\frac{135i}{4096}\right) + 3i\left(\sqrt{4x^2+3ix}\left(\frac{x^2}{3}+\frac{ix}{16}+\frac{9}{128}\right)-\frac{27i\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{512}\right)-\frac{405\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{4096}$$

input
```
integrate((3*I*x+4*x**2)**(3/2),x)
```

output
```
4*sqrt(4*x**2 + 3*I*x)*(x**3/4 + I*x**2/32 + 15*x/512 - 135*I/4096) + 3*I*
(sqrt(4*x**2 + 3*I*x)*(x**2/3 + I*x/16 + 9/128) - 27*I*asinh(8*x/3 + I)/51
2) - 405*asinh(8*x/3 + I)/4096
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.70

$$\int \left(3ix+4x^2\right)^{3/2}\,dx = \frac{1}{4}\left(4\,x^2+3i\,x\right)^{\frac{3}{2}}x+\frac{3}{32}i\left(4\,x^2+3i\,x\right)^{\frac{3}{2}} + \frac{27}{128}\sqrt{4\,x^2+3i\,x}x+\frac{81}{1024}i\sqrt{4\,x^2+3i\,x}+\frac{243}{4096}\log\left(8\,x+4\sqrt{4\,x^2+3i\,x}+3i\right)$$

input
```
integrate((3*I*x+4*x^2)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(4*x^2 + 3*I*x)^(3/2)*x + 3/32*I*(4*x^2 + 3*I*x)^(3/2) + 27/128*sqrt(4
*x^2 + 3*I*x)*x + 81/1024*I*sqrt(4*x^2 + 3*I*x) + 243/4096*log(8*x + 4*sqr
t(4*x^2 + 3*I*x) + 3*I)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11

$$\int \left(3ix+4x^2\right)^{3/2}\,dx = \frac{1}{2048}\left(8\left(16\left(8\,x+9i\right)x-9\right)x+81i\right)\sqrt{8\,x^2+2\sqrt{16\,x^2+9}x}\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right) - \frac{243}{4096}\log\left(2\sqrt{8\,x^2+2\sqrt{16\,x^2+9}x}\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right)-8\,x-3i\right)$$

input
```
integrate((3*I*x+4*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/2048*(8*(16*(8*x + 9*I)*x - 9)*x + 81*I)*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)
*x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 243/4096*log(2*sqrt(8*x^2
 + 2*sqrt(16*x^2 + 9)*x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 8*x
- 3*I)
```

Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.56

$$\int \left(3ix+4x^2\right)^{3/2}\,dx=\frac{243\,\ln\left(x+\frac{\sqrt{x\,\left(4\,x+3\mathrm{i}\right)}}{2}+\frac{3}{8}\mathrm{i}\right)}{4096}+\frac{\left(4\,x+\frac{3}{2}\mathrm{i}\right)\,{\left(4\,x^2+x\,3\mathrm{i}\right)}^{3/2}}{16}+\frac{27\,\left(\frac{x}{2}+\frac{3}{16}\mathrm{i}\right)\,\sqrt{4\,x^2+x\,3\mathrm{i}}}{64}$$

input
```
int((x*3i + 4*x^2)^(3/2),x)
```

output
```
(243*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/4096 + ((4*x + 3i/2)*(x*3i +
4*x^2)^(3/2))/16 + (27*(x/2 + 3i/16)*(x*3i + 4*x^2)^(1/2))/64
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.76

$$\int \left(3ix+4x^2\right)^{3/2}\,dx=\frac{9\sqrt{x}\,\sqrt{3i+4x}\,i\,x^2}{8}+\frac{81\sqrt{x}\,\sqrt{3i+4x}\,i}{1024}+\sqrt{x}\,\sqrt{3i+4x}\,x^3-\frac{9\sqrt{x}\,\sqrt{3i+4x}\,x}{128}+\frac{243\log\left(\frac{\sqrt{3i+4x}+2\sqrt{x}}{\sqrt{i}\,\sqrt{3}}\right)}{2048}$$

input
```
int((3*I*x+4*x^2)^(3/2),x)
```

output
```
(2304*sqrt(x)*sqrt(3*i + 4*x)*i*x**2 + 162*sqrt(x)*sqrt(3*i + 4*x)*i + 204
8*sqrt(x)*sqrt(3*i + 4*x)*x**3 - 144*sqrt(x)*sqrt(3*i + 4*x)*x + 243*log((
sqrt(3*i + 4*x) + 2*sqrt(x))/(sqrt(i)*sqrt(3))))/2048
```

3.27 $\int \sqrt{3ix+4x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \sqrt{3ix+4x^2}\,dx=\frac{3}{16}i\sqrt{3ix+4x^2}+\frac{1}{2}x\sqrt{3ix+4x^2}+\frac{9}{32}\text{arctanh}\left(\frac{2x}{\sqrt{3ix+4x^2}}\right)$$

output
```
3/16*I*(3*I*x+4*x^2)^(1/2)+1/2*x*(3*I*x+4*x^2)^(1/2)+9/32*arctanh(2*x/(3*I
*x+4*x^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.95

$$\int \sqrt{3ix+4x^2}\,dx=\frac{1}{32}\sqrt{x(3i+4x)}\left(6i+16x-\frac{9\log\left(-2\sqrt{x}+\sqrt{3i+4x}\right)}{\sqrt{x}\sqrt{3i+4x}}\right)$$

input
```
Integrate[Sqrt[(3*I)*x + 4*x^2],x]
```

output
```
(Sqrt[x*(3*I + 4*x)]*(6*I + 16*x - (9*Log[-2*Sqrt[x] + Sqrt[3*I + 4*x]])/(
Sqrt[x]*Sqrt[3*I + 4*x])))/32
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.66, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1087, 1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{4x^2+3ix}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{9}{32}\int \frac{1}{\sqrt{4x^2+3ix}}dx+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)$$

$$\Big\downarrow \text{1090}$$

$$\frac{3}{64}\int \frac{1}{\sqrt{\frac{1}{9}(8x+3i)^2+1}}d(8x+3i)+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)$$

$$\Big\downarrow \text{222}$$

$$\frac{9}{64}\text{arcsinh}\left(\frac{1}{3}(8x+3i)\right)+\frac{1}{16}\sqrt{4x^2+3ix}(8x+3i)$$

input
```
Int[Sqrt[(3*I)*x + 4*x^2],x]
```

output
```
((3*I + 8*x)*Sqrt[(3*I)*x + 4*x^2])/16 + (9*ArcSinh[(3*I + 8*x)/3])/64
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.48

method	result	size
default	$\frac{(3i+8x)\sqrt{4x^2+3ix}}{16}+\frac{9\ \mathrm{arcsinh}\left(i+\frac{8x}{3}\right)}{64}$	31
risch	$\frac{(3i+8x)x(3i+4x)}{16\sqrt{x(3i+4x)}}+\frac{9\ \mathrm{arcsinh}\left(i+\frac{8x}{3}\right)}{64}$	36
trager	$\left(\frac{3i}{16}+\frac{x}{2}\right)\sqrt{4x^2+3ix}+\frac{9\ln\left(440x+144+165i-192i\sqrt{4x^2+3ix}-384ix+220\sqrt{4x^2+3ix}\right)}{64}$	64
pseudoelliptic	$-\frac{27\left(\frac{3\ln\left(\frac{2x+\sqrt{x(3i+4x)}}{x}\right)}{4}-\frac{3\ln\left(\frac{\sqrt{x(3i+4x)}-2x}{x}\right)}{4}+\left(i+\frac{8x}{3}\right)\sqrt{x(3i+4x)}\right)x^2}{16\left(2x+\sqrt{x(3i+4x)}\right)^2\left(-\sqrt{x(3i+4x)}+2x\right)^2}$	100

input
```
int((3*I*x+4*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/16*(3*I+8*x)*(3*I*x+4*x^2)^(1/2)+9/64*arcsinh(I+8/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.60

$$\int \sqrt{3ix+4x^2}\,dx = \frac{1}{16}\sqrt{4\,x^2+3i\,x}(8\,x+3i) - \frac{9}{64}\,\log\left(-2\,x+\sqrt{4\,x^2+3i\,x}-\frac{3}{4}i\right) - \frac{9}{256}$$

input
```
integrate((3*I*x+4*x^2)^(1/2),x, algorithm="fricas")
```

output
```
1/16*sqrt(4*x^2 + 3*I*x)*(8*x + 3*I) - 9/64*log(-2*x + sqrt(4*x^2 + 3*I*x)
 - 3/4*I) - 9/256
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.49

$$\int \sqrt{3ix+4x^2}\,dx = \left(\frac{x}{2}+\frac{3i}{16}\right)\sqrt{4x^2+3ix} + \frac{9\,\text{asinh}\left(\frac{8x}{3}+i\right)}{64}$$

input
```
integrate((3*I*x+4*x**2)**(1/2),x)
```

output
```
(x/2 + 3*I/16)*sqrt(4*x**2 + 3*I*x) + 9*asinh(8*x/3 + I)/64
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \sqrt{3ix+4x^2}\,dx = \frac{1}{2}\sqrt{4\,x^2+3i\,x}x + \frac{3}{16}i\sqrt{4\,x^2+3i\,x} + \frac{9}{64}\,\log\left(8\,x+4\,\sqrt{4\,x^2+3i\,x}+3i\right)$$

input
```
integrate((3*I*x+4*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(4*x^2 + 3*I*x)*x + 3/16*I*sqrt(4*x^2 + 3*I*x) + 9/64*log(8*x + 4*
sqrt(4*x^2 + 3*I*x) + 3*I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(45) = 90$.

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.69

$$\int \sqrt{3ix + 4x^2}\, dx = \frac{1}{32}\sqrt{8\,x^2 + 2\,\sqrt{16\,x^2 + 9}x}(8\,x + 3i)\left(\frac{3i\,x}{4\,x^2 + \sqrt{16\,x^4 + 9\,x^2}} + 1\right) - \frac{9}{64}\log\left(2\,\sqrt{8\,x^2 + 2\,\sqrt{16\,x^2 + 9}x}\left(\frac{3i\,x}{4\,x^2 + \sqrt{16\,x^4 + 9\,x^2}} + 1\right) - 8\,x - 3i\right)$$

input
```
integrate((3*I*x+4*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/32*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(8*x + 3*I)*(3*I*x/(4*x^2 + sqrt(1
6*x^4 + 9*x^2)) + 1) - 9/64*log(2*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*
x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 8*x - 3*I)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.60

$$\int \sqrt{3ix + 4x^2}\, dx = \frac{9\,\ln\left(x + \frac{\sqrt{x\,(4\,x+3\mathrm{i})}}{2} + \frac{3}{8}\mathrm{i}\right)}{64} + \left(\frac{x}{2} + \frac{3}{16}\mathrm{i}\right)\,\sqrt{4\,x^2 + x\,3\mathrm{i}}$$

input
```
int((x*3i + 4*x^2)^(1/2),x)
```

output
```
(9*log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8))/64 + (x/2 + 3i/16)*(x*3i + 4*x^
2)^(1/2)
```

Reduce [F]

$$\int \sqrt{3ix+4x^2}\,dx = \int \sqrt{x}\,\sqrt{3i+4x}dx$$

input
```
int((3*I*x+4*x^2)^(1/2),x)
```

output
```
int(sqrt(x)*sqrt(3*i + 4*x),x)
```

3.28 $\int \frac{1}{\sqrt{3ix+4x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 19

$$\int \frac{1}{\sqrt{3ix+4x^2}}\,dx = \operatorname{arctanh}\left(\frac{2x}{\sqrt{3ix+4x^2}}\right)$$

output
```
arctanh(2*x/(3*I*x+4*x^2)^(1/2))
```

Mathematica [B] (verified)

Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 51 vs. 2(19) = 38.

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{3ix+4x^2}}\,dx = -\frac{\sqrt{x}\sqrt{3i+4x}\log\left(-2\sqrt{x}+\sqrt{3i+4x}\right)}{\sqrt{x(3i+4x)}}$$

input
```
Integrate[1/Sqrt[(3*I)*x + 4*x^2],x]
```

output
```
-((Sqrt[x]*Sqrt[3*I + 4*x]*Log[-2*Sqrt[x] + Sqrt[3*I + 4*x]])/Sqrt[x*(3*I
+ 4*x)])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1090, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{4x^2+3ix}}dx$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{1}{6}\int \frac{1}{\sqrt{\frac{1}{9}(8x+3i)^2+1}}d(8x+3i)$$

$$\Big\downarrow \text{ 222}$$

$$\frac{1}{2}\text{arcsinh}\left(\frac{1}{3}(8x+3i)\right)$$

input
```
Int[1/Sqrt[(3*I)*x + 4*x^2],x]
```

output
```
ArcSinh[(3*I + 8*x)/3]/2
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{\operatorname{arcsinh}\left(i+\frac{8x}{3}\right)}{2}$	10
trager	$-\frac{\ln\left(-440x-144-165i-192i\sqrt{4x^2+3ix}+384ix+220\sqrt{4x^2+3ix}\right)}{2}$	44
pseudoelliptic	$\frac{\ln\left(\frac{2x+\sqrt{x(3i+4x)}}{x}\right)}{2}-\frac{\ln\left(\frac{\sqrt{x(3i+4x)}-2x}{x}\right)}{2}$	44

input
```
int(1/(3*I*x+4*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsinh(I+8/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{3ix+4x^2}}\,dx = -\frac{1}{2}\log\left(-2\,x+\sqrt{4\,x^2+3i\,x}-\frac{3}{4}i\right)$$

input
```
integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/2*log(-2*x + sqrt(4*x^2 + 3*I*x) - 3/4*I)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.42

$$\int \frac{1}{\sqrt{3ix+4x^2}}\,dx = \frac{\operatorname{asinh}\left(\frac{8x}{3}+i\right)}{2}$$

input
```
integrate(1/(3*I*x+4*x**2)**(1/2),x)
```

output
```
asinh(8*x/3 + I)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{3ix+4x^2}}\,dx = \frac{1}{2}\log\left(8\,x+4\,\sqrt{4\,x^2+3i\,x}+3i\right)$$

input
```
integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*log(8*x + 4*sqrt(4*x^2 + 3*I*x) + 3*I)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(15) = 30.

Time = 0.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 5.79

$$\int \frac{1}{\sqrt{3ix+4x^2}}\,dx = \frac{1}{32}\sqrt{8\,x^2+2\,\sqrt{16\,x^2+9}x}(8\,x+3i)\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right) - \frac{9}{64}\log\left(2\,\sqrt{8\,x^2+2\,\sqrt{16\,x^2+9}x}\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right) - 8\,x-3i\right)$$

input
```
integrate(1/(3*I*x+4*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/32*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(8*x + 3*I)*(3*I*x/(4*x^2 + sqrt(1
6*x^4 + 9*x^2)) + 1) - 9/64*log(2*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*
x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1) - 8*x - 3*I)
```

Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{3ix+4x^2}}\,dx=\frac{\ln\left(x+\frac{\sqrt{x\,(4\,x+3\mathrm{i})}}{2}+\frac{3}{8}\mathrm{i}\right)}{2}$$

input
```
int(1/(x*3i + 4*x^2)^(1/2),x)
```

output
```
log(x + (x*(4*x + 3i))^(1/2)/2 + 3i/8)/2
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.84

$$\int \frac{1}{\sqrt{3ix+4x^2}}\,dx=-\frac{\log\left(-\sqrt{3i+4x}+2\sqrt{x}\right)}{2}+\frac{\log\left(\sqrt{3i+4x}+2\sqrt{x}\right)}{2}$$

input
```
int(1/(3*I*x+4*x^2)^(1/2),x)
```

output
```
( - log( - sqrt(3*i + 4*x) + 2*sqrt(x)) + log(sqrt(3*i + 4*x) + 2*sqrt(x))
)/2
```

3.29 $\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx = -\frac{2i}{3\sqrt{3ix+4x^2}} + \frac{4\sqrt{3ix+4x^2}}{9x}$$

output
```
-2/3*I/(3*I*x+4*x^2)^(1/2)+4/9*(3*I*x+4*x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.55

$$\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx = \frac{2(3i+8x)}{9\sqrt{x(3i+4x)}}$$

input
```
Integrate[((3*I)*x + 4*x^2)^(-3/2),x]
```

output
```
(2*(3*I + 8*x))/(9*Sqrt[x*(3*I + 4*x)])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.59, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4x^2+3ix)^{3/2}}\,dx$$

$$\Big\downarrow \text{1088}$$

$$\frac{2(8x+3i)}{9\sqrt{4x^2+3ix}}$$

input
```
Int[((3*I)*x + 4*x^2)^(-3/2),x]
```

output
```
(2*(3*I + 8*x))/(9*Sqrt[(3*I)*x + 4*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b + 
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] && 
 NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.43

method	result	size
risch	$\frac{\frac{2i}{3}+\frac{16x}{9}}{\sqrt{x(3i+4x)}}$	19
pseudoelliptic	$\frac{\frac{2i}{3}+\frac{16x}{9}}{\sqrt{x(3i+4x)}}$	19
default	$\frac{\frac{2i}{3}+\frac{16x}{9}}{\sqrt{4x^2+3ix}}$	21
gosper	$\frac{2x(3i+4x)(3i+8x)}{9(4x^2+3ix)^{\frac{3}{2}}}$	28
orering	$\frac{2x(3i+4x)(3i+8x)}{9(4x^2+3ix)^{\frac{3}{2}}}$	28
trager	$\frac{\left(-\frac{14}{225}+\frac{16i}{75}\right)(24ix+32x+12i-9)\sqrt{4x^2+3ix}}{x(12ix-16x-12i-9)}$	44

input
```
int(1/(3*I*x+4*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/9*(3*I+8*x)/(x*(3*I+4*x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx = \frac{2\left(16\,x^2+\sqrt{4\,x^2+3i\,x}(8\,x+3i)+12i\,x\right)}{9\,(4\,x^2+3i\,x)}$$

input
```
integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/9*(16*x^2 + sqrt(4*x^2 + 3*I*x)*(8*x + 3*I) + 12*I*x)/(4*x^2 + 3*I*x)
```

Sympy [F]

$$\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx=\int \frac{1}{(4x^2+3ix)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(3*I*x+4*x**2)**(3/2),x)
```

output
```
Integral((4*x**2 + 3*I*x)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.64

$$\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx=\frac{16\,x}{9\,\sqrt{4\,x^2+3i\,x}}+\frac{2i}{3\,\sqrt{4\,x^2+3i\,x}}$$

input
```
integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="maxima")
```

output
```
16/9*x/sqrt(4*x^2 + 3*I*x) + 2/3*I/sqrt(4*x^2 + 3*I*x)
```

Giac [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 64 vs. 2(30) = 60.

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.45

$$\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx=\frac{\sqrt{8\,x^2+2\,\sqrt{16\,x^2+9}x}(8\,x+3i)\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right)}{9\,(4\,x^2+3i\,x)}$$

input
```
integrate(1/(3*I*x+4*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/9*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(8*x + 3*I)*(3*I*x/(4*x^2 + sqrt(16
*x^4 + 9*x^2)) + 1)/(4*x^2 + 3*I*x)
```

Mupad [B] (verification not implemented)

Time = 10.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.45

$$\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx = \frac{16\,x+6\mathrm{i}}{9\,\sqrt{4\,x^2+x\,3\mathrm{i}}}$$

input
```
int(1/(x*3i + 4*x^2)^(3/2),x)
```

output
```
(16*x + 6i)/(9*(x*3i + 4*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.52

$$\int \frac{1}{(3ix+4x^2)^{3/2}}\,dx = \frac{\frac{8\sqrt{3i+4x}\,ix}{3}+\frac{16\sqrt{3i+4x}\,x^2}{9}+\frac{20\sqrt{x}\,ix}{3}+\frac{32\sqrt{x}\,x^2}{9}-2\sqrt{x}}{\sqrt{3i+4x}\,x\,(3i+2x)}$$

input
```
int(1/(3*I*x+4*x^2)^(3/2),x)
```

output
```
(2*(12*sqrt(3*i + 4*x)*i*x + 8*sqrt(3*i + 4*x)*x**2 + 30*sqrt(x)*i*x + 16*
sqrt(x)*x**2 - 9*sqrt(x)))/(9*sqrt(3*i + 4*x)*x*(3*i + 2*x))
```

3.30 $\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 90

$$\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx = -\frac{2i}{9(3ix+4x^2)^{3/2}} - \frac{4}{9x\sqrt{3ix+4x^2}} - \frac{16i\sqrt{3ix+4x^2}}{81x^2} + \frac{128\sqrt{3ix+4x^2}}{243x}$$

output
```
-2/9*I/(3*I*x+4*x^2)^(3/2)-4/9/x/(3*I*x+4*x^2)^(1/2)-16/81*I*(3*I*x+4*x^2)
^(1/2)/x^2+128/243*(3*I*x+4*x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.40

$$\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx = \frac{54i-432x+2304ix^2+2048x^3}{243(x(3i+4x))^{3/2}}$$

input
```
Integrate[((3*I)*x + 4*x^2)^(-5/2),x]
```

output
```
(54*I - 432*x + (2304*I)*x^2 + 2048*x^3)/(243*(x*(3*I + 4*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.59, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4x^2+3ix)^{5/2}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{32}{27}\int \frac{1}{(4x^2+3ix)^{3/2}}dx+\frac{2(8x+3i)}{27\,(4x^2+3ix)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{64(8x+3i)}{243\sqrt{4x^2+3ix}}+\frac{2(8x+3i)}{27\,(4x^2+3ix)^{3/2}}$$

input
```
Int[((3*I)*x + 4*x^2)^(-5/2),x]
```

output
```
(2*(3*I + 8*x))/(27*((3*I)*x + 4*x^2)^(3/2)) + (64*(3*I + 8*x))/(243*Sqrt[
(3*I)*x + 4*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.43

method	result	size
gosper	$\frac{2x(3i+4x)(1024x^3+1152ix^2-216x+27i)}{243(4x^2+3ix)^{\frac{5}{2}}}$	39
orering	$\frac{2x(3i+4x)(1024x^3+1152ix^2-216x+27i)}{243(4x^2+3ix)^{\frac{5}{2}}}$	39
risch	$\frac{\frac{2048}{243}x^3+\frac{256}{27}ix^2-\frac{16}{9}x+\frac{2}{9}i}{x(3i+4x)\sqrt{x(3i+4x)}}$	41
pseudoelliptic	$\frac{2048x^3+2304ix^2-432x+54i}{243x(3i+4x)\sqrt{x(3i+4x)}}$	41
default	$\frac{\frac{2i}{9}+\frac{16x}{27}}{(4x^2+3ix)^{\frac{3}{2}}}+\frac{\frac{64i}{81}+\frac{512x}{243}}{\sqrt{4x^2+3ix}}$	42
trager	$\frac{\left(\frac{88}{151875}+\frac{26i}{16875}\right)(-76800ix^3-102400x^3-115200ix^2+86400x^2+16200ix+21600x-2700i+2025)\sqrt{4x^2+3ix}}{(12ix-16x-12i-9)^2x^2}$	66

input
```
int(1/(3*I*x+4*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/243*x*(3*I+4*x)*(1152*I*x^2+1024*x^3+27*I-216*x)/(3*I*x+4*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.70

$$\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx = \frac{2\left(2048\,x^4+3072i\,x^3-1152\,x^2+(1024\,x^3+1152i\,x^2-216\,x+27i)\sqrt{4\,x^2+3i\,x}\right)}{243\,(16\,x^4+24i\,x^3-9\,x^2)}$$

input
```
integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="fricas")
```

output
```
2/243*(2048*x^4 + 3072*I*x^3 - 1152*x^2 + (1024*x^3 + 1152*I*x^2 - 216*x +
 27*I)*sqrt(4*x^2 + 3*I*x))/(16*x^4 + 24*I*x^3 - 9*x^2)
```

Sympy [F]

$$\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx = \int \frac{1}{(4x^2+3ix)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(3*I*x+4*x**2)**(5/2),x)
```

output
```
Integral((4*x**2 + 3*I*x)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.61

$$\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx = \frac{512\,x}{243\,\sqrt{4\,x^2+3i\,x}} + \frac{64i}{81\,\sqrt{4\,x^2+3i\,x}} + \frac{16\,x}{27\,(4\,x^2+3i\,x)^{\frac{3}{2}}} + \frac{2i}{9\,(4\,x^2+3i\,x)^{\frac{3}{2}}}$$

input
```
integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="maxima")
```

output
```
512/243*x/sqrt(4*x^2 + 3*I*x) + 64/81*I/sqrt(4*x^2 + 3*I*x) + 16/27*x/(4*x
^2 + 3*I*x)^(3/2) + 2/9*I/(4*x^2 + 3*I*x)^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.82

$$\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx = \frac{(8\,(16\,(8\,x+9i)x-27)x+27i)\sqrt{8\,x^2+2\,\sqrt{16\,x^2+9x}}\left(\frac{3i\,x}{4\,x^2+\sqrt{16\,x^4+9\,x^2}}+1\right)}{243\,(4\,x^2+3i\,x)^2}$$

input
```
integrate(1/(3*I*x+4*x^2)^(5/2),x, algorithm="giac")
```

output
```
1/243*(8*(16*(8*x + 9*I)*x - 27)*x + 27*I)*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)
*x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2)) + 1)/(4*x^2 + 3*I*x)^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.34

$$\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx = \frac{(16\,x+6\mathrm{i})\,\left(128\,x^2+x\,96\mathrm{i}+9\right)}{243\,{\left(4\,x^2+x\,3\mathrm{i}\right)}^{3/2}}$$

input
```
int(1/(x*3i + 4*x^2)^(5/2),x)
```

output
```
((16*x + 6i)*(x*96i + 128*x^2 + 9))/(243*(x*3i + 4*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.03

$$\int \frac{1}{(3ix+4x^2)^{5/2}}\,dx = \frac{-\frac{114688\sqrt{3i+4x}\,i\,x^6}{81}+\frac{5120\sqrt{3i+4x}\,i\,x^4}{9}+768\sqrt{3i+4x}\,i\,x^2-\frac{65536\sqrt{3i+4x}\,x^7}{243}+\frac{20480\sqrt{3i+4x}\,x^{5}}{9}}{\sqrt{3i+4x}\,x^2\,(134}$$

input
```
int(1/(3*I*x+4*x^2)^(5/2),x)
```

output
```
(2*( - 172032*sqrt(3*i + 4*x)*i*x**6 + 69120*sqrt(3*i + 4*x)*i*x**4 + 9331
2*sqrt(3*i + 4*x)*i*x**2 - 32768*sqrt(3*i + 4*x)*x**7 + 276480*sqrt(3*i +
4*x)*x**5 + 165888*sqrt(3*i + 4*x)*x**3 + 368640*sqrt(x)*i*x**6 - 323136*s
qrt(x)*i*x**4 - 312012*sqrt(x)*i*x**2 - 6561*sqrt(x)*i + 65536*sqrt(x)*x**
7 - 677376*sqrt(x)*x**5 - 311040*sqrt(x)*x**3 + 49572*sqrt(x)*x))/(243*sqr
t(3*i + 4*x)*x**2*(1344*i*x**4 - 540*i*x**2 - 729*i + 256*x**5 - 2160*x**3
 - 1296*x))
```

3.31 $\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 136

$$\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx = -\frac{2i}{15\,(3ix+4x^2)^{5/2}} - \frac{4}{27x\,(3ix+4x^2)^{3/2}} + \frac{32i}{81x^2\sqrt{3ix+4x^2}} - \frac{64\sqrt{3ix+4x^2}}{405x^3} - \frac{1024i\sqrt{3ix+4x^2}}{3645x^2} + \frac{8192\sqrt{3ix+4x^2}}{10935x}$$

output
```
-2/15*I/(3*I*x+4*x^2)^(5/2)-4/27/x/(3*I*x+4*x^2)^(3/2)+32/81*I/x^2/(3*I*x+
4*x^2)^(1/2)-64/405*(3*I*x+4*x^2)^(1/2)/x^3-1024/3645*I*(3*I*x+4*x^2)^(1/2
)/x^2+8192/10935*(3*I*x+4*x^2)^(1/2)/x
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.35

$$\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx = \frac{1458i - 6480x - 69120ix^2 - 552960x^3 + 983040ix^4 + 524288x^5}{10935(x(3i+4x))^{5/2}}$$

input
```
Integrate[((3*I)*x + 4*x^2)^(-7/2),x]
```

output
```
(1458*I - 6480*x - (69120*I)*x^2 - 552960*x^3 + (983040*I)*x^4 + 524288*x^
5)/(10935*(x*(3*I + 4*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.62, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(4x^2+3ix)^{7/2}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{64}{45}\int \frac{1}{(4x^2+3ix)^{5/2}}dx+\frac{2(8x+3i)}{45\,(4x^2+3ix)^{5/2}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{64}{45}\left(\frac{32}{27}\int \frac{1}{(4x^2+3ix)^{3/2}}dx+\frac{2(8x+3i)}{27\,(4x^2+3ix)^{3/2}}\right)+\frac{2(8x+3i)}{45\,(4x^2+3ix)^{5/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{2(8x+3i)}{45\,(4x^2+3ix)^{5/2}}+\frac{64}{45}\left(\frac{64(8x+3i)}{243\sqrt{4x^2+3ix}}+\frac{2(8x+3i)}{27\,(4x^2+3ix)^{3/2}}\right)$$

input
```
Int[((3*I)*x + 4*x^2)^(-7/2),x]
```

output
```
(2*(3*I + 8*x))/(45*((3*I)*x + 4*x^2)^(5/2)) + (64*((2*(3*I + 8*x))/(27*((
3*I)*x + 4*x^2)^(3/2)) + (64*(3*I + 8*x))/(243*Sqrt[(3*I)*x + 4*x^2])))/45
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.37

method	result
gosper	$\frac{2x(3i+4x)(262144x^5+491520ix^4-276480x^3-34560ix^2-3240x+729i)}{10935(4x^2+3ix)^{\frac{7}{2}}}$
orering	$\frac{2x(3i+4x)(262144x^5+491520ix^4-276480x^3-34560ix^2-3240x+729i)}{10935(4x^2+3ix)^{\frac{7}{2}}}$
risch	$\frac{\frac{524288}{10935}x^5+\frac{65536}{729}ix^4-\frac{4096}{81}x^3-\frac{512}{81}ix^2-\frac{16}{27}x+\frac{2}{15}i}{x^2(3i+4x)^2\sqrt{x(3i+4x)}}$
pseudoelliptic	$\frac{\frac{524288}{10935}x^5+\frac{65536}{729}ix^4-\frac{4096}{81}x^3-\frac{512}{81}ix^2-\frac{16}{27}x+\frac{2}{15}i}{x^2(3i+4x)^2\sqrt{x(3i+4x)}}$
default	$\frac{\frac{2i}{15}+\frac{16x}{45}}{(4x^2+3ix)^{\frac{5}{2}}}+\frac{\frac{128i}{405}+\frac{1024x}{1215}}{(4x^2+3ix)^{\frac{3}{2}}}+\frac{\frac{4096i}{3645}+\frac{32768x}{10935}}{\sqrt{4x^2+3ix}}$
trager	$\frac{(\frac{1054}{4271484375}+\frac{224i}{1423828125})(12288000000ix^5+16384000000x^5+30720000000ix^4-23040000000x^4-12960000000ix^3-17280}{(12ix-16x-12i-9)}$

input
```
int(1/(3*I*x+4*x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/10935*x*(3*I+4*x)*(491520*I*x^4+262144*x^5-34560*I*x^2-276480*x^3+729*I-
3240*x)/(3*I*x+4*x^2)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.61

$$\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx = \frac{2\,(524288\,x^6 + 1179648i\,x^5 - 884736\,x^4 - 221184i\,x^3 + (262144\,x^5 + 491520i\,x^4 - }{10935\,(64\,x^6 + 144i\,x^5 - 108\,x^4 - 2}$$

input
```
integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="fricas")
```

output
```
2/10935*(524288*x^6 + 1179648*I*x^5 - 884736*x^4 - 221184*I*x^3 + (262144*
x^5 + 491520*I*x^4 - 276480*x^3 - 34560*I*x^2 - 3240*x + 729*I)*sqrt(4*x^2
 + 3*I*x))/(64*x^6 + 144*I*x^5 - 108*x^4 - 27*I*x^3)
```

Sympy [F]

$$\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx = \int \frac{1}{(4x^2+3ix)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(3*I*x+4*x**2)**(7/2),x)
```

output
```
Integral((4*x**2 + 3*I*x)**(-7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.60

$$\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx = \frac{32768\,x}{10935\,\sqrt{4\,x^2+3i\,x}} + \frac{4096i}{3645\,\sqrt{4\,x^2+3i\,x}} + \frac{1024\,x}{1215\,(4\,x^2+3i\,x)^{\frac{3}{2}}} + \frac{128i}{405\,(4\,x^2+3i\,x)^{\frac{3}{2}}} + \frac{16\,x}{45\,(4\,x^2+3i\,x)^{\frac{5}{2}}} + \frac{2i}{15\,(4\,x^2+3i\,x)^{\frac{5}{2}}}$$

input
```
integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="maxima")
```

output
```
32768/10935*x/sqrt(4*x^2 + 3*I*x) + 4096/3645*I/sqrt(4*x^2 + 3*I*x) + 1024
/1215*x/(4*x^2 + 3*I*x)^(3/2) + 128/405*I/(4*x^2 + 3*I*x)^(3/2) + 16/45*x/
(4*x^2 + 3*I*x)^(5/2) + 2/15*I/(4*x^2 + 3*I*x)^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.62

$$\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx = \frac{(8\,(32\,(8\,(16\,(8\,x+15i)x-135)x-135i)x-405)x+729i)\sqrt{8\,x^2+2\,\sqrt{16\,x^2+9}x}}{10935\,(4\,x^2+3i\,x)^3}$$

input
```
integrate(1/(3*I*x+4*x^2)^(7/2),x, algorithm="giac")
```

output
```
1/10935*(8*(32*(8*(16*(8*x + 15*I)*x - 135)*x - 135*I)*x - 405)*x + 729*I)
*sqrt(8*x^2 + 2*sqrt(16*x^2 + 9)*x)*(3*I*x/(4*x^2 + sqrt(16*x^4 + 9*x^2))
+ 1)/(4*x^2 + 3*I*x)^3
```

Mupad [B] (verification not implemented)

Time = 9.98 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.29

$$\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx = -\frac{-524288\,x^5-x^4\,983040\mathrm{i}+552960\,x^3+x^2\,69120\mathrm{i}+6480\,x-1458\mathrm{i}}{10935\,\left(x\,\left(4\,x+3\mathrm{i}\right)\right)^{5/2}}$$

input
```
int(1/(x*3i + 4*x^2)^(7/2),x)
```

output
```
-(6480*x + x^2*69120i + 552960*x^3 - x^4*983040i - 524288*x^5 - 1458i)/(10
935*(x*(4*x + 3i))^(5/2))
```

Reduce [F]

$$\int \frac{1}{(3ix+4x^2)^{7/2}}\,dx = \int \frac{1}{(4x^2+3ix)^{\frac{7}{2}}}dx$$

input
```
int(1/(3*I*x+4*x^2)^(7/2),x)
```

output
```
int(1/(3*I*x+4*x^2)^(7/2),x)
```

3.32 $\int \frac{1}{\sqrt{2x-3x^2}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 21

$$\int \frac{1}{\sqrt{2x-3x^2}} \, dx = \frac{2 \arcsin \left(\sqrt{\frac{3}{2}}\sqrt{x}\right)}{\sqrt{3}}$$

output
```
2/3*arcsin(1/2*6^(1/2)*x^(1/2))*3^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 56 vs. 2(21) = 42.

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.67

$$\int \frac{1}{\sqrt{2x-3x^2}} \, dx = -\frac{2\sqrt{x}\sqrt{-2+3x}\log\left(-\sqrt{3}\sqrt{x}+\sqrt{-2+3x}\right)}{\sqrt{3}\sqrt{-x(-2+3x)}}$$

input
```
Integrate[1/Sqrt[2*x - 3*x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[-2 + 3*x]*Log[-(Sqrt[3]*Sqrt[x]) + Sqrt[-2 + 3*x]])/(Sqrt
[3]*Sqrt[-(x*(-2 + 3*x))])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x-3x^2}}dx$$
$$\Big\downarrow \ 1090$$
$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{4}(2-6x)^2}}d(2-6x)}{2\sqrt{3}}$$
$$\Big\downarrow \ 223$$
$$-\frac{\arcsin\left(\frac{1}{2}(2-6x)\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[2*x - 3*x^2],x]
```

output
```
-(ArcSin[(2 - 6*x)/2]/Sqrt[3])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{\sqrt{3}\,\arcsin(3x-1)}{3}$	12
meijerg	$\frac{2\sqrt{3}\,\arcsin\left(\frac{\sqrt{x}\,\sqrt{2}\,\sqrt{3}}{2}\right)}{3}$	18
pseudoelliptic	$-\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{-3x^2+2x}\,\sqrt{3}}{3x}\right)}{3}$	26
trager	$-\frac{\mathrm{RootOf}\left(\_Z^2+3\right)\ln\left(3\,\mathrm{RootOf}\left(\_Z^2+3\right)x+3\sqrt{-3x^2+2x}-\mathrm{RootOf}\left(\_Z^2+3\right)\right)}{3}$	41

input
```
int(1/(-3*x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arcsin(3*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{2x-3x^2}}\,dx = -\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2+2\,x}}{3\,x-2}\right)$$

input
```
integrate(1/(-3*x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(3)*arctan(sqrt(3)*sqrt(-3*x^2 + 2*x)/(3*x - 2))
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt{2x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}\,(3x-1)}{3}$$

input
```
integrate(1/(-3*x**2+2*x)**(1/2),x)
```

output
```
sqrt(3)*asin(3*x - 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{2x-3x^2}}\,dx = -\frac{1}{3}\,\sqrt{3}\arcsin\left(-3\,x+1\right)$$

input
```
integrate(1/(-3*x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arcsin(-3*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 30 vs. 2(14) = 28.

Time = 0.12 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{2x-3x^2}}\,dx = \frac{1}{6}\,\sqrt{-3\,x^2+2\,x}(3\,x-1)+\frac{1}{18}\,\sqrt{3}\arcsin\left(3\,x-1\right)$$

input
```
integrate(1/(-3*x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(-3*x^2 + 2*x)*(3*x - 1) + 1/18*sqrt(3)*arcsin(3*x - 1)
```

Mupad [B] (verification not implemented)

Time = 9.76 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt{2x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}(3\,x-1)}{3}$$

input
```
int(1/(2*x - 3*x^2)^(1/2),x)
```

output
```
(3^(1/2)*asin(3*x - 1))/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{1}{\sqrt{2x-3x^2}}\,dx = -\frac{2\sqrt{3}\log\left(\frac{\sqrt{-3x+2}+\sqrt{x}\,\sqrt{3}\,i}{\sqrt{2}}\right)i}{3}$$

input
```
int(1/(-3*x^2+2*x)^(1/2),x)
```

output
```
( - 2*sqrt(3)*log((sqrt( - 3*x + 2) + sqrt(x)*sqrt(3)*i)/sqrt(2))*i)/3
```

3.33 $\int \frac{1}{\sqrt{-2x-3x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{\sqrt{-2x-3x^2}}\,dx = \frac{2\arctan\left(\frac{\sqrt{3}x}{\sqrt{-2x-3x^2}}\right)}{\sqrt{3}}$$

output
```
2/3*arctan(3^(1/2)*x/(-3*x^2-2*x)^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{-2x-3x^2}}\,dx = -\frac{2\sqrt{x}\sqrt{2+3x}\log\left(-\sqrt{3}\sqrt{x}+\sqrt{2+3x}\right)}{\sqrt{3}\sqrt{-x(2+3x)}}$$

input
```
Integrate[1/Sqrt[-2*x - 3*x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[2 + 3*x]*Log[-(Sqrt[3]*Sqrt[x]) + Sqrt[2 + 3*x]])/(Sqrt[3
]*Sqrt[-(x*(2 + 3*x))])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.61, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^2-2x}}dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{1}{4}(-6x-2)^2}}d(-6x-2)}{2\sqrt{3}}$$

$$\Big\downarrow \text{223}$$

$$-\frac{\arcsin\left(\frac{1}{2}(-6x-2)\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[-2*x - 3*x^2],x]
```

output
```
-(ArcSin[(-2 - 6*x)/2]/Sqrt[3])
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.43

method	result	size
default	$\frac{\sqrt{3}\,\arcsin(3x+1)}{3}$	12
meijerg	$-\frac{2i\sqrt{3}\,\operatorname{arcsinh}\left(\frac{\sqrt{x}\,\sqrt{2}\,\sqrt{3}}{2}\right)}{3}$	19
pseudoelliptic	$-\frac{2\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\,\sqrt{-3x^2-2x}}{3x}\right)}{3}$	26
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+3\right)\ln\left(3\operatorname{RootOf}\left(\_Z^2+3\right)x+3\sqrt{-3x^2-2x}+\operatorname{RootOf}\left(\_Z^2+3\right)\right)}{3}$	39

input
```
int(1/(-3*x^2-2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*3^(1/2)*arcsin(3*x+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-2x-3x^2}}\,dx = -\frac{2}{3}\,\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt{-3\,x^2-2\,x}}{3\,x+2}\right)$$

input
```
integrate(1/(-3*x^2-2*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(3)*arctan(sqrt(3)*sqrt(-3*x^2 - 2*x)/(3*x + 2))
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{-2x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}\,(3x+1)}{3}$$

input
```
integrate(1/(-3*x**2-2*x)**(1/2),x)
```

output
```
sqrt(3)*asin(3*x + 1)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.39

$$\int \frac{1}{\sqrt{-2x-3x^2}}\,dx = -\frac{1}{3}\,\sqrt{3}\arcsin\left(-3\,x-1\right)$$

input
```
integrate(1/(-3*x^2-2*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arcsin(-3*x - 1)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{-2x-3x^2}}\,dx = \frac{1}{6}\,\sqrt{-3\,x^2-2\,x}(3\,x+1)+\frac{1}{18}\,\sqrt{3}\arcsin\left(3\,x+1\right)$$

input
```
integrate(1/(-3*x^2-2*x)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(-3*x^2 - 2*x)*(3*x + 1) + 1/18*sqrt(3)*arcsin(3*x + 1)
```

Mupad [B] (verification not implemented)

Time = 9.93 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.39

$$\int \frac{1}{\sqrt{-2x-3x^2}}\,dx = \frac{\sqrt{3}\,\mathrm{asin}(3\,x+1)}{3}$$

input
```
int(1/(- 2*x - 3*x^2)^(1/2),x)
```

output
```
(3^(1/2)*asin(3*x + 1))/3
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt{-2x-3x^2}}\,dx = -\frac{2\sqrt{3}\,\log\left(\frac{\sqrt{-3x-2}+\sqrt{x}\,\sqrt{3}\,i}{\sqrt{2}}\right)i}{3}$$

input
```
int(1/(-3*x^2-2*x)^(1/2),x)
```

output
```
( - 2*sqrt(3)*log((sqrt( - 3*x - 2) + sqrt(x)*sqrt(3)*i)/sqrt(2))*i)/3
```

3.34 $\int \frac{1}{\sqrt{2x+3x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{\sqrt{2x+3x^2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{2x+3x^2}}\right)}{\sqrt{3}}$$

output
```
2/3*arctanh(3^(1/2)*x/(3*x^2+2*x)^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{2x+3x^2}}\,dx = -\frac{2\sqrt{x}\sqrt{2+3x}\log\left(-\sqrt{3}\sqrt{x}+\sqrt{2+3x}\right)}{\sqrt{3}\sqrt{x(2+3x)}}$$

input
```
Integrate[1/Sqrt[2*x + 3*x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[2 + 3*x]*Log[-(Sqrt[3]*Sqrt[x]) + Sqrt[2 + 3*x]])/(Sqrt[3
]*Sqrt[x*(2 + 3*x)])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^2+2x}}dx$$

$$\Big\downarrow \text{1091}$$

$$2\int \frac{1}{1-\frac{3x^2}{3x^2+2x}}d\frac{x}{\sqrt{3x^2+2x}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{3x^2+2x}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[2*x + 3*x^2],x]
```

output
```
(2*ArcTanh[(Sqrt[3]*x)/Sqrt[2*x + 3*x^2]])/Sqrt[3]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.64

method	result	size
meijerg	$\frac{2\sqrt{3}\,\operatorname{arcsinh}\left(\frac{\sqrt{x}\,\sqrt{2}\,\sqrt{3}}{2}\right)}{3}$	18
pseudoelliptic	$\frac{2\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3x^{2}+2x}\,\sqrt{3}}{3x}\right)}{3}$	26
default	$\frac{\ln\left(\frac{(3x+1)\sqrt{3}}{3}+\sqrt{3x^{2}+2x}\right)\sqrt{3}}{3}$	29
trager	$\frac{\operatorname{RootOf}\left(\_Z^{2}-3\right)\ln\left(3\operatorname{RootOf}\left(\_Z^{2}-3\right)x+3\sqrt{3x^{2}+2x}+\operatorname{RootOf}\left(\_Z^{2}-3\right)\right)}{3}$	39

input
```
int(1/(3*x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)*arcsinh(1/2*x^(1/2)*2^(1/2)*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{2x+3x^2}}\,dx = \frac{1}{3}\,\sqrt{3}\log\left(\sqrt{3}\sqrt{3\,x^{2}+2\,x}+3\,x+1\right)$$

input
```
integrate(1/(3*x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*log(sqrt(3)*sqrt(3*x^2 + 2*x) + 3*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.11

$$\int \frac{1}{\sqrt{2x+3x^2}}\,dx = \frac{\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2+2x}+2\right)}{3}$$

input
```
integrate(1/(3*x**2+2*x)**(1/2),x)
```

output
```
sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*x**2 + 2*x) + 2)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{2x+3x^2}}\,dx = \frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2+2x}+6x+2\right)$$

input
```
integrate(1/(3*x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 + 2*x) + 6*x + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(22) = 44.

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

$$\int \frac{1}{\sqrt{2x+3x^2}}\,dx = \frac{1}{6}\sqrt{3x^2+2x}(3x+1) + \frac{1}{18}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2+2x}\right)-1\right|\right)$$

input
```
integrate(1/(3*x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3*x^2 + 2*x)*(3*x + 1) + 1/18*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)*x
 - sqrt(3*x^2 + 2*x)) - 1))
```

Mupad [B] (verification not implemented)

Time = 10.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{2x+3x^2}}\,dx = \frac{\sqrt{3}\,\ln\left(\sqrt{3}\,\left(x+\frac{1}{3}\right)+\sqrt{3\,x^2+2\,x}\right)}{3}$$

input
```
int(1/(2*x + 3*x^2)^(1/2),x)
```

output
```
(3^(1/2)*log(3^(1/2)*(x + 1/3) + (2*x + 3*x^2)^(1/2)))/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.79

$$\int \frac{1}{\sqrt{2x+3x^2}}\,dx = \frac{2\sqrt{3}\log\left(\frac{\sqrt{3x+2}+\sqrt{x}\,\sqrt{3}}{\sqrt{2}}\right)}{3}$$

input
```
int(1/(3*x^2+2*x)^(1/2),x)
```

output
```
(2*sqrt(3)*log((sqrt(3*x + 2) + sqrt(x)*sqrt(3))/sqrt(2)))/3
```

3.35 $\int \frac{1}{\sqrt{-2x+3x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 32

$$\int \frac{1}{\sqrt{-2x+3x^2}}\,dx = \frac{2\text{arcsinh}\left(\frac{\sqrt{-2x+3x^2}}{\sqrt{2}\sqrt{x}}\right)}{\sqrt{3}}$$

output
```
2/3*arcsinh(1/2*(3*x^2-2*x)^(1/2)*2^(1/2)/x^(1/2))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.72

$$\int \frac{1}{\sqrt{-2x+3x^2}}\,dx = -\frac{2\sqrt{x}\sqrt{-2+3x}\log\left(-\sqrt{3}\sqrt{x}+\sqrt{-2+3x}\right)}{\sqrt{3}\sqrt{x(-2+3x)}}$$

input
```
Integrate[1/Sqrt[-2*x + 3*x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[-2 + 3*x]*Log[-(Sqrt[3]*Sqrt[x]) + Sqrt[-2 + 3*x]])/(Sqrt
[3]*Sqrt[x*(-2 + 3*x)])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.88, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x^2-2x}}dx$$
$$\Big\downarrow \text{1091}$$
$$2\int \frac{1}{1-\frac{3x^2}{3x^2-2x}}d\frac{x}{\sqrt{3x^2-2x}}$$
$$\Big\downarrow \text{219}$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{3}x}{\sqrt{3x^2-2x}}\right)}{\sqrt{3}}$$

input
```
Int[1/Sqrt[-2*x + 3*x^2],x]
```

output
```
(2*ArcTanh[(Sqrt[3]*x)/Sqrt[-2*x + 3*x^2]])/Sqrt[3]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$\frac{2\sqrt{3}\,\operatorname{arctanh}\left(\frac{\sqrt{3x^2-2x}\,\sqrt{3}}{3x}\right)}{3}$	26
default	$\frac{\ln\left(\frac{(3x-1)\sqrt{3}}{3}+\sqrt{3x^2-2x}\right)\sqrt{3}}{3}$	29
meijerg	$\frac{2\sqrt{3}\,\sqrt{-\operatorname{signum}\left(x-\frac{2}{3}\right)}\,\arcsin\left(\frac{\sqrt{x}\,\sqrt{2}\,\sqrt{3}}{2}\right)}{3\sqrt{\operatorname{signum}\left(x-\frac{2}{3}\right)}}$	32
trager	$\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(3\operatorname{RootOf}\left(\_Z^2-3\right)x+3\sqrt{3x^2-2x}-\operatorname{RootOf}\left(\_Z^2-3\right)\right)}{3}$	41

input
```
int(1/(3*x^2-2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*3^(1/2)*arctanh(1/3*(3*x^2-2*x)^(1/2)/x*3^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{-2x+3x^2}}\,dx = \frac{1}{3}\sqrt{3}\log\left(-\sqrt{3}\sqrt{3\,x^2-2\,x}-3\,x+1\right)$$

input
```
integrate(1/(3*x^2-2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(3)*log(-sqrt(3)*sqrt(3*x^2 - 2*x) - 3*x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{-2x+3x^2}}\,dx = \frac{\sqrt{3}\log\left(6x+2\sqrt{3}\sqrt{3x^2-2x}-2\right)}{3}$$

input
```
integrate(1/(3*x**2-2*x)**(1/2),x)
```

output
```
sqrt(3)*log(6*x + 2*sqrt(3)*sqrt(3*x**2 - 2*x) - 2)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{1}{\sqrt{-2x+3x^2}}\,dx = \frac{1}{3}\sqrt{3}\log\left(2\sqrt{3}\sqrt{3x^2-2x}+6x-2\right)$$

input
```
integrate(1/(3*x^2-2*x)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(3)*log(2*sqrt(3)*sqrt(3*x^2 - 2*x) + 6*x - 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. 2(25) = 50.

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{-2x+3x^2}}\,dx = \frac{1}{6}\sqrt{3x^2-2x}(3x-1) + \frac{1}{18}\sqrt{3}\log\left(\left|-\sqrt{3}\left(\sqrt{3}x-\sqrt{3x^2-2x}\right)+1\right|\right)$$

input
```
integrate(1/(3*x^2-2*x)^(1/2),x, algorithm="giac")
```

output
```
1/6*sqrt(3*x^2 - 2*x)*(3*x - 1) + 1/18*sqrt(3)*log(abs(-sqrt(3)*(sqrt(3)*x
 - sqrt(3*x^2 - 2*x)) + 1))
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{-2x+3x^2}}\,dx = \frac{\sqrt{3}\,\ln\left(\sqrt{3}\left(x-\frac{1}{3}\right)+\sqrt{3\,x^2-2\,x}\right)}{3}$$

input
```
int(1/(3*x^2 - 2*x)^(1/2),x)
```

output
```
(3^(1/2)*log(3^(1/2)*(x - 1/3) + (3*x^2 - 2*x)^(1/2)))/3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{-2x+3x^2}}\,dx = \frac{2\sqrt{3}\log\left(\frac{\sqrt{3x-2}+\sqrt{x}\,\sqrt{3}}{\sqrt{2}}\right)}{3}$$

input
```
int(1/(3*x^2-2*x)^(1/2),x)
```

output
```
(2*sqrt(3)*log((sqrt(3*x - 2) + sqrt(x)*sqrt(3))/sqrt(2)))/3
```

3.36 $\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 12

$$\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx = -\frac{\arcsin(1-2bx)}{b}$$

output
```
arcsin(2*b*x-1)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 57 vs. 2(12) = 24.

Time = 0.04 (sec) , antiderivative size = 57, normalized size of antiderivative = 4.75

$$\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx = -\frac{2\sqrt{x}\sqrt{-1+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{-1+bx}\right)}{\sqrt{b}\sqrt{-bx(-1+bx)}}$$

input
```
Integrate[1/Sqrt[b*x - b^2*x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[-1 + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[-1 + b*x]])/(Sqrt
[b]*Sqrt[-(b*x*(-1 + b*x))])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx - b^2x^2}}\,dx$$

$$\Big\downarrow \; 1090$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{\left(b-2b^2x\right)^2}{b^2}}}d\left(b-2b^2x\right)}{b^2}$$

$$\Big\downarrow \; 223$$

$$-\frac{\arcsin\left(\frac{b-2b^2x}{b}\right)}{b}$$

input
```
Int[1/Sqrt[b*x - b^2*x^2],x]
```

output
```
-(ArcSin[(b - 2*b^2*x)/b]/b)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 24 vs. 2(11) = 22.

Time = 0.66 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

method	result	size
pseudoelliptic	$-\frac{2\arctan\left(\frac{\sqrt{-bx(bx-1)}}{xb}\right)}{b}$	25
default	$\frac{\arctan\left(\frac{\sqrt{b^2}\left(x-\frac{1}{2b}\right)}{\sqrt{-b^2x^2+bx}}\right)}{\sqrt{b^2}}$	35

input
```
int(1/(-b^2*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctan((-b*x*(b*x-1))^(1/2)/x/b)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. 2(11) = 22.

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 2.33

$$\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx = -\frac{2\arctan\left(\frac{\sqrt{-b^2x^2+bx}}{bx-1}\right)}{b}$$

input
```
integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(sqrt(-b^2*x^2 + b*x)/(b*x - 1))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(8) = 16$.

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.50

$$\int \frac{1}{\sqrt{bx - b^2x^2}}\,dx = \begin{cases} \frac{\log\left(-2b^2x+b+2\sqrt{-b^2}\sqrt{-b^2x^2+bx}\right)}{\sqrt{-b^2}} & \text{for } b^2 \neq 0 \\ \frac{2\sqrt{bx}}{b} & \text{for } b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-b**2*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((log(-2*b**2*x + b + 2*sqrt(-b**2)*sqrt(-b**2*x**2 + b*x))/sqrt(
-b**2), Ne(b**2, 0)), (2*sqrt(b*x)/b, Ne(b, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{bx - b^2x^2}}\,dx = -\frac{\arcsin\left(-\frac{2\,b^2x-b}{b}\right)}{b}$$

input
```
integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-(2*b^2*x - b)/b)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(11) = 22$.

Time = 0.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 3.42

$$\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx = \frac{1}{4}\sqrt{-b^2x^2+bx}\left(2\,x-\frac{1}{b}\right)-\frac{\arcsin\left(-2\,bx+1\right)\operatorname{sgn}(b)}{8\,|b|}$$

input
```
integrate(1/(-b^2*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-b^2*x^2 + b*x)*(2*x - 1/b) - 1/8*arcsin(-2*b*x + 1)*sgn(b)/abs(b
)
```

Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.50

$$\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx = \frac{\ln\left(\frac{\frac{b}{2}-b^2\,x}{\sqrt{-b^2}}+\sqrt{b\,x-b^2\,x^2}\right)}{\sqrt{-b^2}}$$

input
```
int(1/(b*x - b^2*x^2)^(1/2),x)
```

output
```
log((b/2 - b^2*x)/(-b^2)^(1/2) + (b*x - b^2*x^2)^(1/2))/(-b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{bx-b^2x^2}}\,dx = -\frac{2\log\left(\sqrt{-bx+1}+\sqrt{x}\,\sqrt{b}\,i\right)i}{b}$$

input
```
int(1/(-b^2*x^2+b*x)^(1/2),x)
```

output
```
( - 2*log(sqrt( - b*x + 1) + sqrt(x)*sqrt(b)*i)*i)/b
```

3.37 $\int \frac{1}{\sqrt{-bx-b^2x^2}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 26

$$\int \frac{1}{\sqrt{-bx-b^2x^2}} \, dx = \frac{2\arctan\left(\frac{bx}{\sqrt{-bx-b^2x^2}}\right)}{b}$$

output
```
2*arctan(b*x/(-b^2*x^2-b*x)^(1/2))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 59 vs. 2(26) = 52.

Time = 0.06 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.27

$$\int \frac{1}{\sqrt{-bx-b^2x^2}} \, dx = \frac{4\sqrt{x}\sqrt{1+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-1+\sqrt{1+bx}}\right)}{\sqrt{b}\sqrt{-bx(1+bx)}}$$

input
```
Integrate[1/Sqrt[-(b*x) - b^2*x^2],x]
```

output
```
(4*Sqrt[x]*Sqrt[1 + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-1 + Sqrt[1 + b*x])])/
(Sqrt[b]*Sqrt[-(b*x*(1 + b*x))])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.77, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-b^2x^2 - bx}}\,dx$$

$$\Big\downarrow \; 1090$$

$$-\frac{\int \frac{1}{\sqrt{1-\frac{(-2xb^2-b)^2}{b^2}}}d(-2xb^2-b)}{b^2}$$

$$\Big\downarrow \; 223$$

$$-\frac{\arcsin\left(\frac{-2b^2x-b}{b}\right)}{b}$$

input
```
Int[1/Sqrt[-(b*x) - b^2*x^2],x]
```

output
```
-(ArcSin[(-b - 2*b^2*x)/b]/b)
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

method	result	size
pseudoelliptic	$-\frac{2 \arctan\left(\frac{\sqrt{-b x (b x+1)}}{b x}\right)}{b}$	25
default	$\frac{\arctan\left(\frac{\sqrt{b^2}\left(x+\frac{1}{2b}\right)}{\sqrt{-b^2 x^2-b x}}\right)}{\sqrt{b^2}}$	36

input
```
int(1/(-b^2*x^2-b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*arctan(1/b/x*(-b*x*(b*x+1))^(1/2))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{-bx-b^2x^2}}\,dx = -\frac{2 \arctan\left(\frac{\sqrt{-b^2x^2-bx}}{bx+1}\right)}{b}$$

input
```
integrate(1/(-b^2*x^2-b*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(sqrt(-b^2*x^2 - b*x)/(b*x + 1))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(22) = 44.

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 2.31

$$\int \frac{1}{\sqrt{-bx-b^2x^2}}\,dx = \begin{cases} \frac{\log\left(-2b^2x-b+2\sqrt{-b^2}\sqrt{-b^2x^2-bx}\right)}{\sqrt{-b^2}} & \text{for } b^2 \neq 0 \\ -\frac{2\sqrt{-bx}}{b} & \text{for } b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/(-b**2*x**2-b*x)**(1/2),x)
```

output
```
Piecewise((log(-2*b**2*x - b + 2*sqrt(-b**2)*sqrt(-b**2*x**2 - b*x))/sqrt(
-b**2), Ne(b**2, 0)), (-2*sqrt(-b*x)/b, Ne(b, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{-bx-b^2x^2}}\,dx = -\frac{\arcsin\left(-\frac{2\,b^2x+b}{b}\right)}{b}$$

input
```
integrate(1/(-b^2*x^2-b*x)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-(2*b^2*x + b)/b)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int \frac{1}{\sqrt{-bx-b^2x^2}}\,dx = \frac{1}{4}\sqrt{-b^2x^2-bx}\left(2\,x+\frac{1}{b}\right)-\frac{\arcsin\left(-2\,bx-1\right)\mathrm{sgn}(b)}{8\,|b|}$$

input
```
integrate(1/(-b^2*x^2-b*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(-b^2*x^2 - b*x)*(2*x + 1/b) - 1/8*arcsin(-2*b*x - 1)*sgn(b)/abs(b
)
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

$$\int \frac{1}{\sqrt{-bx-b^2x^2}}\,dx=\frac{\ln\left(\sqrt{-b^2\,x^2-b\,x}-\frac{x\,b^2+\frac{b}{2}}{\sqrt{-b^2}}\right)}{\sqrt{-b^2}}$$

input
```
int(1/(- b*x - b^2*x^2)^(1/2),x)
```

output
```
log((- b*x - b^2*x^2)^(1/2) - (b/2 + b^2*x)/(-b^2)^(1/2))/(-b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt{-bx-b^2x^2}}\,dx=-\frac{2\log\left(\sqrt{bx+1}+\sqrt{x}\,\sqrt{b}\right)i}{b}$$

input
```
int(1/(-b^2*x^2-b*x)^(1/2),x)
```

output
```
( - 2*log(sqrt(b*x + 1) + sqrt(x)*sqrt(b))*i)/b
```

3.38 $\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx = \frac{2\text{arctanh}\left(\frac{bx}{\sqrt{bx+b^2x^2}}\right)}{b}$$

output
```
2*arctanh(b*x/(b^2*x^2+b*x)^(1/2))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 58 vs. 2(24) = 48.

Time = 0.02 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.42

$$\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx = \frac{4\sqrt{x}\sqrt{1+bx}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-1+\sqrt{1+bx}}\right)}{\sqrt{b}\sqrt{bx(1+bx)}}$$

input
```
Integrate[1/Sqrt[b*x + b^2*x^2],x]
```

output
```
(4*Sqrt[x]*Sqrt[1 + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-1 + Sqrt[1 + b*x])])/
(Sqrt[b]*Sqrt[b*x*(1 + b*x)])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b^2x^2+bx}}dx$$

$$\Big\downarrow \text{1091}$$

$$2\int \frac{1}{1-\frac{b^2x^2}{b^2x^2+bx}}d\frac{x}{\sqrt{b^2x^2+bx}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2\text{arctanh}\left(\frac{bx}{\sqrt{b^2x^2+bx}}\right)}{b}$$

input
```
Int[1/Sqrt[b*x + b^2*x^2],x]
```

output
```
(2*ArcTanh[(b*x)/Sqrt[b*x + b^2*x^2]])/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.54

method	result	size
default	$\frac{\ln\left(\frac{\frac{1}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{b^2x^2+bx}\right)}{\sqrt{b^2}}$	37
pseudoelliptic	$\frac{\ln\left(\frac{bx+\sqrt{bx(bx+1)}}{x}\right)-\ln\left(\frac{-bx+\sqrt{bx(bx+1)}}{x}\right)}{b}$	47

input
```
int(1/(b^2*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln((1/2*b+b^2*x)/(b^2)^(1/2)+(b^2*x^2+b*x)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx = -\frac{\log\left(-2\,bx+2\,\sqrt{b^2x^2+bx}-1\right)}{b}$$

input
```
integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
-log(-2*b*x + 2*sqrt(b^2*x^2 + b*x) - 1)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 51 vs. 2(20) = 40.

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.12

$$\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx = \begin{cases} \frac{\log\left(2b^2x+b+2\sqrt{b^2x^2+bx}\sqrt{b^2}\right)}{\sqrt{b^2}} & \text{for } b^2 \neq 0 \\ \frac{2\sqrt{bx}}{b} & \text{for } b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/(b**2*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((log(2*b**2*x + b + 2*sqrt(b**2*x**2 + b*x)*sqrt(b**2))/sqrt(b**
2), Ne(b**2, 0)), (2*sqrt(b*x)/b, Ne(b, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.21

$$\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx = \frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2+bx}b+b\right)}{b}$$

input
```
integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
log(2*b^2*x + 2*sqrt(b^2*x^2 + b*x)*b + b)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(22) = 44.

Time = 0.14 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx = \frac{1}{4}\,\sqrt{b^2x^2+bx}\left(2\,x+\frac{1}{b}\right)+\frac{\log\left(\left|-2\,\left(x|b|-\sqrt{b^2x^2+bx}\right)|b|-b\right|\right)}{8\,|b|}$$

input
```
integrate(1/(b^2*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b^2*x^2 + b*x)*(2*x + 1/b) + 1/8*log(abs(-2*(x*abs(b) - sqrt(b^2*
x^2 + b*x))*abs(b) - b))/abs(b)
```

Mupad [B] (verification not implemented)

Time = 8.70 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.50

$$\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx = \frac{\ln\left(\frac{x\,b^2+\frac{b}{2}}{\sqrt{b^2}}+\sqrt{b^2\,x^2+b\,x}\right)}{\sqrt{b^2}}$$

input
```
int(1/(b*x + b^2*x^2)^(1/2),x)
```

output
```
log((b/2 + b^2*x)/(b^2)^(1/2) + (b*x + b^2*x^2)^(1/2))/(b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt{bx+b^2x^2}}\,dx = \frac{2\log\left(\sqrt{bx+1}+\sqrt{x}\,\sqrt{b}\right)}{b}$$

input
```
int(1/(b^2*x^2+b*x)^(1/2),x)
```

output
```
(2*log(sqrt(b*x + 1) + sqrt(x)*sqrt(b)))/b
```

3.39 $\int \frac{1}{\sqrt{-bx+b^2x^2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 25

$$\int \frac{1}{\sqrt{-bx+b^2x^2}}\,dx = \frac{2\operatorname{arctanh}\left(\frac{bx}{\sqrt{-bx+b^2x^2}}\right)}{b}$$

output
```
2*arctanh(b*x/(b^2*x^2-b*x)^(1/2))/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 56 vs. 2(25) = 50.

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.24

$$\int \frac{1}{\sqrt{-bx+b^2x^2}}\,dx = -\frac{2\sqrt{x}\sqrt{-1+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{-1+bx}\right)}{\sqrt{b}\sqrt{bx(-1+bx)}}$$

input
```
Integrate[1/Sqrt[-(b*x) + b^2*x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[-1 + b*x]*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[-1 + b*x]])/(Sqrt
[b]*Sqrt[b*x*(-1 + b*x)])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{b^2x^2-bx}}dx$$

$$\Big\downarrow \text{ 1091}$$

$$2\int \frac{1}{1-\frac{b^2x^2}{b^2x^2-bx}}d\frac{x}{\sqrt{b^2x^2-bx}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\text{arctanh}\left(\frac{bx}{\sqrt{b^2x^2-bx}}\right)}{b}$$

input
```
Int[1/Sqrt[-(b*x) + b^2*x^2],x]
```

output
```
(2*ArcTanh[(b*x)/Sqrt[-(b*x) + b^2*x^2]])/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.52

method	result	size
default	$\frac{\ln\left(\frac{-\frac{1}{2}b+b^2x}{\sqrt{b^2}}+\sqrt{b^2x^2-bx}\right)}{\sqrt{b^2}}$	38
pseudoelliptic	$\frac{\ln\left(\frac{bx+\sqrt{bx(bx-1)}}{x}\right)-\ln\left(\frac{-bx+\sqrt{bx(bx-1)}}{x}\right)}{b}$	47

input
```
int(1/(b^2*x^2-b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln((-1/2*b+b^2*x)/(b^2)^(1/2)+(b^2*x^2-b*x)^(1/2))/(b^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{1}{\sqrt{-bx+b^2x^2}}\,dx = -\frac{\log\left(-2\,bx+2\,\sqrt{b^2x^2-bx}+1\right)}{b}$$

input
```
integrate(1/(b^2*x^2-b*x)^(1/2),x, algorithm="fricas")
```

output
```
-log(-2*b*x + 2*sqrt(b^2*x^2 - b*x) + 1)/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(20) = 40.

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \frac{1}{\sqrt{-bx+b^2x^2}}\,dx = \begin{cases} \frac{\log\left(2b^2x-b+2\sqrt{b^2x^2-bx}\sqrt{b^2}\right)}{\sqrt{b^2}} & \text{for } b^2 \neq 0 \\ -\frac{2\sqrt{-bx}}{b} & \text{for } b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/(b**2*x**2-b*x)**(1/2),x)
```

output
```
Piecewise((log(2*b**2*x - b + 2*sqrt(b**2*x**2 - b*x)*sqrt(b**2))/sqrt(b**
2), Ne(b**2, 0)), (-2*sqrt(-b*x)/b, Ne(b, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{1}{\sqrt{-bx+b^2x^2}}\,dx = \frac{\log\left(2\,b^2x+2\,\sqrt{b^2x^2-bx}b-b\right)}{b}$$

input
```
integrate(1/(b^2*x^2-b*x)^(1/2),x, algorithm="maxima")
```

output
```
log(2*b^2*x + 2*sqrt(b^2*x^2 - b*x)*b - b)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(23) = 46.

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.44

$$\int \frac{1}{\sqrt{-bx+b^2x^2}}\,dx = \frac{1}{4}\,\sqrt{b^2x^2-bx}\left(2\,x-\frac{1}{b}\right)+\frac{\log\left(\left|-2\,\left(x|b|-\sqrt{b^2x^2-bx}\right)|b|+b\right|\right)}{8\,|b|}$$

input
```
integrate(1/(b^2*x^2-b*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(b^2*x^2 - b*x)*(2*x - 1/b) + 1/8*log(abs(-2*(x*abs(b) - sqrt(b^2*
x^2 - b*x))*abs(b) + b))/abs(b)
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{1}{\sqrt{-bx+b^2x^2}}\,dx = \frac{\ln\left(\sqrt{b^2\,x^2-b\,x}-\frac{\frac{b}{2}-b^2\,x}{\sqrt{b^2}}\right)}{\sqrt{b^2}}$$

input
```
int(1/(b^2*x^2 - b*x)^(1/2),x)
```

output
```
log((b^2*x^2 - b*x)^(1/2) - (b/2 - b^2*x)/(b^2)^(1/2))/(b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \frac{1}{\sqrt{-bx+b^2x^2}}\,dx = \frac{2\log\left(\sqrt{bx-1}+\sqrt{x}\,\sqrt{b}\right)}{b}$$

input
```
int(1/(b^2*x^2-b*x)^(1/2),x)
```

output
```
(2*log(sqrt(b*x - 1) + sqrt(x)*sqrt(b)))/b
```

3.40 $\int \sqrt{6x-x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \sqrt{6x-x^2}\,dx=\frac{3}{2}\sqrt{6-x}\sqrt{x}-\frac{1}{2}(6-x)^{3/2}\sqrt{x}+9\arcsin\left(\frac{\sqrt{x}}{\sqrt{6}}\right)$$

output
```
3/2*(6-x)^(1/2)*x^(1/2)-1/2*(6-x)^(3/2)*x^(1/2)+9*arcsin(1/6*6^(1/2)*x^(1/
2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int \sqrt{6x-x^2}\,dx=\frac{1}{2}\sqrt{-((-6+x)x)}\left(-3+x+\frac{18\log\left(\sqrt{-6+x}-\sqrt{x}\right)}{\sqrt{-6+x}\sqrt{x}}\right)$$

input
```
Integrate[Sqrt[6*x - x^2],x]
```

output
```
(Sqrt[-((-6 + x)*x)]*(-3 + x + (18*Log[Sqrt[-6 + x] - Sqrt[x]])/(Sqrt[-6 +
 x]*Sqrt[x])))/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{6x-x^2}\,dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{9}{2}\int \frac{1}{\sqrt{6x-x^2}}dx-\frac{1}{2}(3-x)\sqrt{6x-x^2}$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{3}{4}\int \frac{1}{\sqrt{1-\frac{1}{36}(6-2x)^2}}d(6-2x)-\frac{1}{2}\sqrt{6x-x^2}(3-x)$$

$$\Big\downarrow \text{ 223}$$

$$-\frac{9}{2}\arcsin\left(\frac{1}{6}(6-2x)\right)-\frac{1}{2}\sqrt{6x-x^2}(3-x)$$

input
```
Int[Sqrt[6*x - x^2],x]
```

output
```
-1/2*((3 - x)*Sqrt[6*x - x^2]) - (9*ArcSin[(6 - 2*x)/6])/2
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.53

method	result
risch	$-\frac{(-3+x)x(x-6)}{2\sqrt{-x(x-6)}}+\frac{9\arcsin\left(-1+\frac{x}{3}\right)}{2}$
default	$-\frac{(-2x+6)\sqrt{-x^2+6x}}{4}+\frac{9\arcsin\left(-1+\frac{x}{3}\right)}{2}$
pseudoelliptic	$-9\arctan\left(\frac{\sqrt{-x(x-6)}}{x}\right)+\frac{(-3+x)\sqrt{-x(x-6)}}{2}$
meijerg	$-\frac{18i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{6}\,(3-x)\sqrt{-\frac{x}{6}+1}}{36}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{6}\,\sqrt{x}}{6}\right)}{2}\right)}{\sqrt{\pi}}$
trager	$\left(-\frac{3}{2}+\frac{x}{2}\right)\sqrt{-x^2+6x}+\frac{9\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(-\mathrm{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2+6x}+3\,\mathrm{RootOf}\left(\_Z^2+1\right)\right)}{2}$

input
```
int((-x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-3+x)*x*(x-6)/(-x*(x-6))^(1/2)+9/2*arcsin(-1+1/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \sqrt{6x - x^2}\, dx = \frac{1}{2}\sqrt{-x^2 + 6\,x}(x - 3) - 9\,\arctan\left(\frac{\sqrt{-x^2 + 6\,x}}{x - 6}\right)$$

input
```
integrate((-x^2+6*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(-x^2 + 6*x)*(x - 3) - 9*arctan(sqrt(-x^2 + 6*x)/(x - 6))
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.51

$$\int \sqrt{6x - x^2}\, dx = \left(\frac{x}{2} - \frac{3}{2}\right)\sqrt{-x^2 + 6x} + \frac{9\,\mathrm{asin}\left(\frac{x}{3} - 1\right)}{2}$$

input
```
integrate((-x**2+6*x)**(1/2),x)
```

output
```
(x/2 - 3/2)*sqrt(-x**2 + 6*x) + 9*asin(x/3 - 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.71

$$\int \sqrt{6x - x^2}\, dx = \frac{1}{2}\sqrt{-x^2 + 6\,x}x - \frac{3}{2}\sqrt{-x^2 + 6\,x} - \frac{9}{2}\,\arcsin\left(-\frac{1}{3}\,x + 1\right)$$

input
```
integrate((-x^2+6*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-x^2 + 6*x)*x - 3/2*sqrt(-x^2 + 6*x) - 9/2*arcsin(-1/3*x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.49

$$\int \sqrt{6x-x^2}\,dx = \frac{1}{2}\sqrt{-x^2+6x}(x-3) + \frac{9}{2}\arcsin\left(\frac{1}{3}x-1\right)$$

input
```
integrate((-x^2+6*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 6*x)*(x - 3) + 9/2*arcsin(1/3*x - 1)
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.51

$$\int \sqrt{6x-x^2}\,dx = \frac{9\,\mathrm{asin}\left(\frac{x}{3}-1\right)}{2} + \left(\frac{x}{2}-\frac{3}{2}\right)\sqrt{6\,x-x^2}$$

input
```
int((6*x - x^2)^(1/2),x)
```

output
```
(9*asin(x/3 - 1))/2 + (x/2 - 3/2)*(6*x - x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int \sqrt{6x-x^2}\,dx = \frac{\sqrt{x}\,\sqrt{-x+6}\,x}{2} - \frac{3\sqrt{x}\,\sqrt{-x+6}}{2} - 9\log\left(\frac{\sqrt{-x+6}+\sqrt{x}\,i}{\sqrt{6}}\right)i$$

input
```
int((-x^2+6*x)^(1/2),x)
```

output
```
(sqrt(x)*sqrt( - x + 6)*x - 3*sqrt(x)*sqrt( - x + 6) - 18*log((sqrt( - x +
 6) + sqrt(x)*i)/sqrt(6))*i)/2
```

3.41 $\int \sqrt{5x-9x^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \sqrt{5x-9x^2}\,dx=\frac{5}{36}\sqrt{5-9x}\sqrt{x}-\frac{1}{18}(5-9x)^{3/2}\sqrt{x}+\frac{25}{108}\arcsin\left(\frac{3\sqrt{x}}{\sqrt{5}}\right)$$

output
```
5/36*(5-9*x)^(1/2)*x^(1/2)-1/18*(5-9*x)^(3/2)*x^(1/2)+25/108*arcsin(3/5*x^
(1/2)*5^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.02

$$\int \sqrt{5x-9x^2}\,dx=\frac{1}{108}\sqrt{-x(-5+9x)}\left(-15+54x+\frac{25\log\left(-3\sqrt{x}+\sqrt{-5+9x}\right)}{\sqrt{x}\sqrt{-5+9x}}\right)$$

input
```
Integrate[Sqrt[5*x - 9*x^2],x]
```

output
```
(Sqrt[-(x*(-5 + 9*x))]*(-15 + 54*x + (25*Log[-3*Sqrt[x] + Sqrt[-5 + 9*x]])
/(Sqrt[x]*Sqrt[-5 + 9*x])))/108
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{5x-9x^2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{25}{72}\int \frac{1}{\sqrt{5x-9x^2}}dx-\frac{1}{36}(5-18x)\sqrt{5x-9x^2}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{5}{216}\int \frac{1}{\sqrt{1-\frac{1}{25}(5-18x)^2}}d(5-18x)-\frac{1}{36}\sqrt{5x-9x^2}(5-18x)$$

$$\Big\downarrow \text{223}$$

$$-\frac{25}{216}\arcsin\left(\frac{1}{5}(5-18x)\right)-\frac{1}{36}\sqrt{5x-9x^2}(5-18x)$$

input
```
Int[Sqrt[5*x - 9*x^2],x]
```

output
```
-1/36*((5 - 18*x)*Sqrt[5*x - 9*x^2]) - (25*ArcSin[(5 - 18*x)/5])/216
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.52

method	result
default	$-\frac{(-18x+5)\sqrt{-9x^2+5x}}{36}+\frac{25\arcsin\left(\frac{18x}{5}-1\right)}{216}$
risch	$-\frac{(18x-5)x(9x-5)}{36\sqrt{-x(9x-5)}}+\frac{25\arcsin\left(\frac{18x}{5}-1\right)}{216}$
pseudoelliptic	$-\frac{25\arctan\left(\frac{\sqrt{-9x^2+5x}}{3x}\right)}{108}+\frac{(18x-5)\sqrt{-9x^2+5x}}{36}$
meijerg	$-\frac{25i\left(-\frac{i\sqrt{\pi}\sqrt{x}\sqrt{5}\left(-\frac{54x}{5}+3\right)\sqrt{-\frac{9x}{5}+1}}{10}+\frac{i\sqrt{\pi}\arcsin\left(\frac{3\sqrt{x}\sqrt{5}}{5}\right)}{2}\right)}{54\sqrt{\pi}}$
trager	$\left(\frac{x}{2}-\frac{5}{36}\right)\sqrt{-9x^2+5x}+\frac{25\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-18\operatorname{RootOf}\left(\_Z^2+1\right)x+6\sqrt{-9x^2+5x}+5\operatorname{RootOf}\left(\_Z^2+1\right.\right.}{216}$

input
```
int((-9*x^2+5*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/36*(-18*x+5)*(-9*x^2+5*x)^(1/2)+25/216*arcsin(18/5*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

$$\int \sqrt{5x-9x^2}\,dx=\frac{1}{36}\sqrt{-9\,x^2+5\,x}(18\,x-5)-\frac{25}{108}\arctan\left(\frac{3\sqrt{-9\,x^2+5\,x}}{9\,x-5}\right)$$

input
```
integrate((-9*x^2+5*x)^(1/2),x, algorithm="fricas")
```

output
```
1/36*sqrt(-9*x^2 + 5*x)*(18*x - 5) - 25/108*arctan(3*sqrt(-9*x^2 + 5*x)/(9
*x - 5))
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.54

$$\int \sqrt{5x-9x^2}\,dx=\left(\frac{x}{2}-\frac{5}{36}\right)\sqrt{-9x^2+5x}+\frac{25\,\mathrm{asin}\left(\frac{18x}{5}-1\right)}{216}$$

input
```
integrate((-9*x**2+5*x)**(1/2),x)
```

output
```
(x/2 - 5/36)*sqrt(-9*x**2 + 5*x) + 25*asin(18*x/5 - 1)/216
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.67

$$\int \sqrt{5x-9x^2}\,dx=\frac{1}{2}\sqrt{-9\,x^2+5\,x}x-\frac{5}{36}\sqrt{-9\,x^2+5\,x}-\frac{25}{216}\arcsin\left(-\frac{18}{5}\,x+1\right)$$

input
```
integrate((-9*x^2+5*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(-9*x^2 + 5*x)*x - 5/36*sqrt(-9*x^2 + 5*x) - 25/216*arcsin(-18/5*x
 + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.50

$$\int \sqrt{5x-9x^2}\,dx = \frac{1}{36}\,\sqrt{-9\,x^2+5\,x}(18\,x-5)+\frac{25}{216}\,\arcsin\left(\frac{18}{5}\,x-1\right)$$

input
```
integrate((-9*x^2+5*x)^(1/2),x, algorithm="giac")
```

output
```
1/36*sqrt(-9*x^2 + 5*x)*(18*x - 5) + 25/216*arcsin(18/5*x - 1)
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.48

$$\int \sqrt{5x-9x^2}\,dx = \frac{25\,\mathrm{asin}\left(\frac{18\,x}{5}-1\right)}{216}+\left(\frac{x}{2}-\frac{5}{36}\right)\,\sqrt{5\,x-9\,x^2}$$

input
```
int((5*x - 9*x^2)^(1/2),x)
```

output
```
(25*asin((18*x)/5 - 1))/216 + (x/2 - 5/36)*(5*x - 9*x^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \sqrt{5x-9x^2}\,dx = \frac{\sqrt{x}\,\sqrt{-9x+5}\,x}{2}-\frac{5\sqrt{x}\,\sqrt{-9x+5}}{36}-\frac{25\log\left(\frac{\sqrt{-9x+5}+3\sqrt{x}\,i}{\sqrt{5}}\right)i}{108}$$

input
```
int((-9*x^2+5*x)^(1/2),x)
```

output
```
(54*sqrt(x)*sqrt( - 9*x + 5)*x - 15*sqrt(x)*sqrt( - 9*x + 5) - 25*log((sqr
t( - 9*x + 5) + 3*sqrt(x)*i)/sqrt(5))*i)/108
```

3.42 $\int \sqrt{4x+x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 53

$$\int \sqrt{4x+x^2}\,dx = \sqrt{4x+x^2} + \frac{1}{2}x\sqrt{4x+x^2} - 4\text{arcsinh}\left(\frac{\sqrt{4x+x^2}}{2\sqrt{4+x}}\right)$$

output
```
(x^2+4*x)^(1/2)+1/2*x*(x^2+4*x)^(1/2)-4*arcsinh(1/2*(x^2+4*x)^(1/2)/(4+x)^
(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.87

$$\int \sqrt{4x+x^2}\,dx = \frac{1}{2}\sqrt{x(4+x)}\left(2+x+\frac{8\log\left(-\sqrt{x}+\sqrt{4+x}\right)}{\sqrt{x}\sqrt{4+x}}\right)$$

input
```
Integrate[Sqrt[4*x + x^2],x]
```

output
```
(Sqrt[x*(4 + x)]*(2 + x + (8*Log[-Sqrt[x] + Sqrt[4 + x]])/(Sqrt[x]*Sqrt[4
+ x])))/2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.66, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2+4x}\,dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{1}{2}(x+2)\sqrt{x^2+4x}-2\int \frac{1}{\sqrt{x^2+4x}}dx$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{1}{2}(x+2)\sqrt{x^2+4x}-4\int \frac{1}{1-\frac{x^2}{x^2+4x}}d\frac{x}{\sqrt{x^2+4x}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}(x+2)\sqrt{x^2+4x}-4\text{arctanh}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

input
```
Int[Sqrt[4*x + x^2],x]
```

output
```
((2 + x)*Sqrt[4*x + x^2])/2 - 4*ArcTanh[x/Sqrt[4*x + x^2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{(2x+4)\sqrt{x^2+4x}}{4}-2\ln\left(2+x+\sqrt{x^2+4x}\right)$	33
risch	$\frac{(2+x)x(x+4)}{2\sqrt{x(x+4)}}-2\ln\left(2+x+\sqrt{x^2+4x}\right)$	33
trager	$\left(1+\frac{x}{2}\right)\sqrt{x^2+4x}+2\ln\left(\sqrt{x^2+4x}-2-x\right)$	34
meijerg	$-\frac{8\left(-\frac{\sqrt{\pi}\sqrt{x}\left(\frac{3x}{2}+3\right)\sqrt{\frac{x}{4}+1}}{12}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{\sqrt{x}}{2}\right)}{2}\right)}{\sqrt{\pi}}$	38
pseudoelliptic	$\frac{8\left((2+x)\sqrt{x(x+4)}-4\ln\left(\frac{x+\sqrt{x(x+4)}}{x}\right)+4\ln\left(\frac{\sqrt{x(x+4)}-x}{x}\right)\right)x^2}{\left(x+\sqrt{x(x+4)}\right)^2\left(-\sqrt{x(x+4)}+x\right)^2}$	76

input
```
int((x^2+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*x+4)*(x^2+4*x)^(1/2)-2*ln(2+x+(x^2+4*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

$$\int \sqrt{4x+x^2}\,dx = \frac{1}{2}\sqrt{x^2+4\,x}(x+2)+2\,\log\left(-x+\sqrt{x^2+4\,x}-2\right)$$

input
```
integrate((x^2+4*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*log(-x + sqrt(x^2 + 4*x) - 2)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64

$$\int \sqrt{4x+x^2}\,dx = \left(\frac{x}{2}+1\right)\sqrt{x^2+4x}-2\log\left(2x+2\sqrt{x^2+4x}+4\right)$$

input
```
integrate((x**2+4*x)**(1/2),x)
```

output
```
(x/2 + 1)*sqrt(x**2 + 4*x) - 2*log(2*x + 2*sqrt(x**2 + 4*x) + 4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \sqrt{4x+x^2}\,dx = \frac{1}{2}\sqrt{x^2+4\,x}x+\sqrt{x^2+4\,x}-2\,\log\left(2\,x+2\,\sqrt{x^2+4\,x}+4\right)$$

input
```
integrate((x^2+4*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^2 + 4*x)*x + sqrt(x^2 + 4*x) - 2*log(2*x + 2*sqrt(x^2 + 4*x) +
4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

$$\int \sqrt{4x+x^2}\,dx = \frac{1}{2}\sqrt{x^2+4\,x}(x+2)+2\,\log\left(\left|-x+\sqrt{x^2+4\,x}-2\right|\right)$$

input
```
integrate((x^2+4*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*log(abs(-x + sqrt(x^2 + 4*x) - 2))
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.55

$$\int \sqrt{4x+x^2}\,dx = \sqrt{x^2+4\,x}\left(\frac{x}{2}+1\right)-2\,\ln\left(x+\sqrt{x\,(x+4)}+2\right)$$

input
```
int((4*x + x^2)^(1/2),x)
```

output
```
(4*x + x^2)^(1/2)*(x/2 + 1) - 2*log(x + (x*(x + 4))^(1/2) + 2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int \sqrt{4x+x^2}\,dx = \frac{\sqrt{x}\,\sqrt{x+4}\,x}{2}+\sqrt{x}\,\sqrt{x+4}-4\log\left(\frac{\sqrt{x+4}}{2}+\frac{\sqrt{x}}{2}\right)$$

input
```
int((x^2+4*x)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(x + 4)*x + 2*sqrt(x)*sqrt(x + 4) - 8*log((sqrt(x + 4) + sqrt
(x))/2))/2
```

3.43 $\int \sqrt{-8x+x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 60

$$\int \sqrt{-8x+x^2}\,dx = 2\sqrt{-8x+x^2}+\frac{\left(-8x+x^2\right)^{3/2}}{2x}-16\text{arcsinh}\left(\frac{\sqrt{-8x+x^2}}{2\sqrt{2}\sqrt{x}}\right)$$

output
```
2*(x^2-8*x)^(1/2)+1/2*(x^2-8*x)^(3/2)/x-16*arcsinh(1/4*(x^2-8*x)^(1/2)*2^(
1/2)/x^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \sqrt{-8x+x^2}\,dx = \frac{1}{2}\sqrt{(-8+x)x}\left(-4+x+\frac{32\log\left(\sqrt{-8+x}-\sqrt{x}\right)}{\sqrt{-8+x}\sqrt{x}}\right)$$

input
```
Integrate[Sqrt[-8*x + x^2],x]
```

output
```
(Sqrt[(-8 + x)*x]*(-4 + x + (32*Log[Sqrt[-8 + x] - Sqrt[x]])/(Sqrt[-8 + x]
*Sqrt[x])))/2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2-8x}\,dx$$

$$\Big\downarrow \text{1087}$$

$$-8\int \frac{1}{\sqrt{x^2-8x}}dx-\frac{1}{2}\sqrt{x^2-8x}(4-x)$$

$$\Big\downarrow \text{1091}$$

$$-16\int \frac{1}{1-\frac{x^2}{x^2-8x}}d\frac{x}{\sqrt{x^2-8x}}-\frac{1}{2}\sqrt{x^2-8x}(4-x)$$

$$\Big\downarrow \text{219}$$

$$-16\text{arctanh}\left(\frac{x}{\sqrt{x^2-8x}}\right)-\frac{1}{2}\sqrt{x^2-8x}(4-x)$$

input
```
Int[Sqrt[-8*x + x^2],x]
```

output
```
-1/2*((4 - x)*Sqrt[-8*x + x^2]) - 16*ArcTanh[x/Sqrt[-8*x + x^2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.55

method	result	size
default	$\frac{(2x-8)\sqrt{x^2-8x}}{4}-8\ln\left(x-4+\sqrt{x^2-8x}\right)$	33
risch	$\frac{(x-4)x(-8+x)}{2\sqrt{x(-8+x)}}-8\ln\left(x-4+\sqrt{x^2-8x}\right)$	33
trager	$\left(\frac{x}{2}-2\right)\sqrt{x^2-8x}+8\ln\left(4-x+\sqrt{x^2-8x}\right)$	34
meijerg	$-\frac{32i\sqrt{\mathrm{signum}(-8+x)}\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\left(-\frac{3x}{4}+3\right)\sqrt{-\frac{x}{8}+1}}{24}+\frac{i\sqrt{\pi}\,\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{4}\right)}{2}\right)}{\sqrt{\pi}\,\sqrt{-\mathrm{signum}(-8+x)}}$	61
pseudoelliptic	$\frac{32\left((x-4)\sqrt{x(-8+x)}-16\ln\left(\frac{x+\sqrt{x(-8+x)}}{x}\right)+16\ln\left(\frac{\sqrt{x(-8+x)}-x}{x}\right)\right)x^2}{\left(x+\sqrt{x(-8+x)}\right)^2\left(-\sqrt{x(-8+x)}+x\right)^2}$	76

input
```
int((x^2-8*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*x-8)*(x^2-8*x)^(1/2)-8*ln(x-4+(x^2-8*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.53

$$\int \sqrt{-8x+x^2}\,dx = \frac{1}{2}\sqrt{x^2-8\,x}(x-4)+8\,\log\left(-x+\sqrt{x^2-8\,x}+4\right)$$

input
```
integrate((x^2-8*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*sqrt(x^2 - 8*x)*(x - 4) + 8*log(-x + sqrt(x^2 - 8*x) + 4)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \sqrt{-8x+x^2}\,dx = \left(\frac{x}{2}-2\right)\sqrt{x^2-8x}-8\log\left(2x+2\sqrt{x^2-8x}-8\right)$$

input
```
integrate((x**2-8*x)**(1/2),x)
```

output
```
(x/2 - 2)*sqrt(x**2 - 8*x) - 8*log(2*x + 2*sqrt(x**2 - 8*x) - 8)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.72

$$\int \sqrt{-8x+x^2}\,dx = \frac{1}{2}\sqrt{x^2-8\,x}x-2\sqrt{x^2-8\,x}-8\,\log\left(2\,x+2\sqrt{x^2-8\,x}-8\right)$$

input
```
integrate((x^2-8*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^2 - 8*x)*x - 2*sqrt(x^2 - 8*x) - 8*log(2*x + 2*sqrt(x^2 - 8*x)
- 8)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.55

$$\int \sqrt{-8x+x^2}\,dx = \frac{1}{2}\sqrt{x^2-8\,x}(x-4)+8\,\log\left(\left|-x+\sqrt{x^2-8\,x}+4\right|\right)$$

input
```
integrate((x^2-8*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 - 8*x)*(x - 4) + 8*log(abs(-x + sqrt(x^2 - 8*x) + 4))
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.48

$$\int \sqrt{-8x+x^2}\,dx = \left(\frac{x}{2}-2\right)\sqrt{x^2-8\,x}-8\,\ln\left(x+\sqrt{x\,(x-8)}-4\right)$$

input
```
int((x^2 - 8*x)^(1/2),x)
```

output
```
(x/2 - 2)*(x^2 - 8*x)^(1/2) - 8*log(x + (x*(x - 8))^(1/2) - 4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.57

$$\int \sqrt{-8x+x^2}\,dx = \frac{\sqrt{x}\,\sqrt{x-8}\,x}{2}-2\sqrt{x}\,\sqrt{x-8}-16\log\left(\frac{\sqrt{x-8}+\sqrt{x}}{2\sqrt{2}}\right)$$

input
```
int((x^2-8*x)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(x - 8)*x - 4*sqrt(x)*sqrt(x - 8) - 32*log((sqrt(x - 8) + sqr
t(x))/(2*sqrt(2))))/2
```

3.44 $\int \sqrt{-x+x^2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 56

$$\int \sqrt{-x+x^2}\,dx=\frac{1}{4}\sqrt{-x+x^2}+\frac{\left(-x+x^2\right)^{3/2}}{2x}-\frac{1}{4}\text{arcsinh}\left(\frac{\sqrt{-x+x^2}}{\sqrt{x}}\right)$$

output
```
1/4*(x^2-x)^(1/2)+1/2*(x^2-x)^(3/2)/x-1/4*arcsinh((x^2-x)^(1/2)/x^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \sqrt{-x+x^2}\,dx=\frac{1}{4}\sqrt{(-1+x)x}\left(-1+2x-\frac{2\text{arctanh}\left(\frac{\sqrt{-1+x}}{-1+\sqrt{x}}\right)}{\sqrt{-1+x}\sqrt{x}}\right)$$

input
```
Integrate[Sqrt[-x + x^2],x]
```

output
```
(Sqrt[(-1 + x)*x]*(-1 + 2*x - (2*ArcTanh[Sqrt[-1 + x]/(-1 + Sqrt[x])])/(Sq
rt[-1 + x]*Sqrt[x])))/4
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x^2-x}\,dx$$

$$\Big\downarrow \text{1087}$$

$$-\frac{1}{8}\int \frac{1}{\sqrt{x^2-x}}dx-\frac{1}{4}\sqrt{x^2-x}(1-2x)$$

$$\Big\downarrow \text{1091}$$

$$-\frac{1}{4}\int \frac{1}{1-\frac{x^2}{x^2-x}}d\frac{x}{\sqrt{x^2-x}}-\frac{1}{4}\sqrt{x^2-x}(1-2x)$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{4}\text{arctanh}\left(\frac{x}{\sqrt{x^2-x}}\right)-\frac{1}{4}\sqrt{x^2-x}(1-2x)$$

input
```
Int[Sqrt[-x + x^2],x]
```

output
```
-1/4*((1 - 2*x)*Sqrt[-x + x^2]) - ArcTanh[x/Sqrt[-x + x^2]]/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.59

method	result	size
default	$\frac{(2x-1)\sqrt{x^2-x}}{4}-\frac{\ln\left(x-\frac{1}{2}+\sqrt{x^2-x}\right)}{8}$	33
risch	$\frac{(2x-1)x(x-1)}{4\sqrt{x(x-1)}}-\frac{\ln\left(x-\frac{1}{2}+\sqrt{x^2-x}\right)}{8}$	35
trager	$\left(\frac{x}{2}-\frac{1}{4}\right)\sqrt{x^2-x}+\frac{\ln\left(2\sqrt{x^2-x}+1-2x\right)}{8}$	36
meijerg	$-\frac{i\sqrt{\operatorname{signum}(x-1)}\left(-\frac{i\sqrt{\pi}\sqrt{x}\,(-6x+3)\sqrt{1-x}}{6}+\frac{i\sqrt{\pi}\,\arcsin(\sqrt{x})}{2}\right)}{2\sqrt{\pi}\sqrt{-\operatorname{signum}(x-1)}}$	53
pseudoelliptic	$-\frac{x^2\left(-4\sqrt{x(x-1)}\,x+\ln\left(\frac{x+\sqrt{x(x-1)}}{x}\right)-\ln\left(\frac{\sqrt{x(x-1)}-x}{x}\right)+2\sqrt{x(x-1)}\right)}{8\left(x+\sqrt{x(x-1)}\right)^2\left(\sqrt{x(x-1)}-x\right)^2}$	82

input
```
int((x^2-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*x-1)*(x^2-x)^(1/2)-1/8*ln(x-1/2+(x^2-x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \sqrt{-x+x^2}\,dx = \frac{1}{4}\sqrt{x^2-x}(2\,x-1) + \frac{1}{8}\log\left(-2\,x+2\,\sqrt{x^2-x}+1\right)$$

input
```
integrate((x^2-x)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(x^2 - x)*(2*x - 1) + 1/8*log(-2*x + 2*sqrt(x^2 - x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.57

$$\int \sqrt{-x+x^2}\,dx = \left(\frac{x}{2}-\frac{1}{4}\right)\sqrt{x^2-x} - \frac{\log\left(2x+2\sqrt{x^2-x}-1\right)}{8}$$

input
```
integrate((x**2-x)**(1/2),x)
```

output
```
(x/2 - 1/4)*sqrt(x**2 - x) - log(2*x + 2*sqrt(x**2 - x) - 1)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.77

$$\int \sqrt{-x+x^2}\,dx = \frac{1}{2}\sqrt{x^2-x}x - \frac{1}{4}\sqrt{x^2-x} - \frac{1}{8}\log\left(2\,x+2\,\sqrt{x^2-x}-1\right)$$

input
```
integrate((x^2-x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(x^2 - x)*x - 1/4*sqrt(x^2 - x) - 1/8*log(2*x + 2*sqrt(x^2 - x) -
1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.66

$$\int \sqrt{-x+x^2}\,dx=\frac{1}{4}\,\sqrt{x^2-x}(2\,x-1)+\frac{1}{8}\,\log\left(\left|-2\,x+2\,\sqrt{x^2-x}+1\right|\right)$$

input
```
integrate((x^2-x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(x^2 - x)*(2*x - 1) + 1/8*log(abs(-2*x + 2*sqrt(x^2 - x) + 1))
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.52

$$\int \sqrt{-x+x^2}\,dx=\sqrt{x^2-x}\,\left(\frac{x}{2}-\frac{1}{4}\right)-\frac{\ln\left(x+\sqrt{x\,\left(x-1\right)}-\frac{1}{2}\right)}{8}$$

input
```
int((x^2 - x)^(1/2),x)
```

output
```
(x^2 - x)^(1/2)*(x/2 - 1/4) - log(x + (x*(x - 1))^(1/2) - 1/2)/8
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.50

$$\int \sqrt{-x+x^2}\,dx=\frac{\sqrt{x}\,\sqrt{x-1}\,x}{2}-\frac{\sqrt{x}\,\sqrt{x-1}}{4}-\frac{\log\left(\sqrt{x-1}+\sqrt{x}\right)}{4}$$

input
```
int((x^2-x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(x - 1)*x - sqrt(x)*sqrt(x - 1) - log(sqrt(x - 1) + sqrt(x)
))/4
```

3.45 $\int \frac{1}{\sqrt{6x-x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{1}{\sqrt{6x-x^2}}\,dx = 2\arcsin\left(\frac{\sqrt{x}}{\sqrt{6}}\right)$$

output
```
2*arcsin(1/6*6^(1/2)*x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 40 vs. $2(14) = 28$.

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.86

$$\int \frac{1}{\sqrt{6x-x^2}}\,dx = -\frac{2\sqrt{-6+x}\sqrt{x}\log\left(\sqrt{-6+x}-\sqrt{x}\right)}{\sqrt{-((-6+x)x)}}$$

input
```
Integrate[1/Sqrt[6*x - x^2],x]
```

output
```
(-2*Sqrt[-6 + x]*Sqrt[x]*Log[Sqrt[-6 + x] - Sqrt[x]])/Sqrt[-((-6 + x)*x)]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{6x-x^2}}dx$$
$$\Big\downarrow \; 1090$$
$$-\frac{1}{6}\int \frac{1}{\sqrt{1-\frac{1}{36}(6-2x)^2}}d(6-2x)$$
$$\Big\downarrow \; 223$$
$$-\arcsin\left(\frac{1}{6}(6-2x)\right)$$

input
```
Int[1/Sqrt[6*x - x^2],x]
```

output
```
-ArcSin[(6 - 2*x)/6]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

method	result	si
default	$\arcsin\left(-1+\frac{x}{3}\right)$	7
meijerg	$2\arcsin\left(\frac{\sqrt{6}\,\sqrt{x}}{6}\right)$	12
pseudoelliptic	$-2\arctan\left(\frac{\sqrt{-x(x-6)}}{x}\right)$	16
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)x+\sqrt{-x^2+6x}+3\operatorname{RootOf}\left(\_Z^2+1\right)\right)$	38

input
```
int(1/(-x^2+6*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(-1+1/3*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{6x-x^2}}\,dx = -2\,\arctan\left(\frac{\sqrt{-x^2+6\,x}}{x-6}\right)$$

input
```
integrate(1/(-x^2+6*x)^(1/2),x, algorithm="fricas")
```

output
```
-2*arctan(sqrt(-x^2 + 6*x)/(x - 6))
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.36

$$\int \frac{1}{\sqrt{6x-x^2}}\,dx = \operatorname{asin}\left(\frac{x}{3}-1\right)$$

input
```
integrate(1/(-x**2+6*x)**(1/2),x)
```

output
```
asin(x/3 - 1)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.57

$$\int \frac{1}{\sqrt{6x-x^2}}\,dx = -\arcsin\left(-\frac{1}{3}\,x+1\right)$$

input
```
integrate(1/(-x^2+6*x)^(1/2),x, algorithm="maxima")
```

output
```
-arcsin(-1/3*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. 2(11) = 22.

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.79

$$\int \frac{1}{\sqrt{6x-x^2}}\,dx = \frac{1}{2}\,\sqrt{-x^2+6\,x}(x-3)+\frac{9}{2}\,\arcsin\left(\frac{1}{3}\,x-1\right)$$

input
```
integrate(1/(-x^2+6*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 6*x)*(x - 3) + 9/2*arcsin(1/3*x - 1)
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.43

$$\int \frac{1}{\sqrt{6x-x^2}}\,dx = \operatorname{asin}\left(\frac{x}{3}-1\right)$$

input
```
int(1/(6*x - x^2)^(1/2),x)
```

output
```
asin(x/3 - 1)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.43

$$\int \frac{1}{\sqrt{6x-x^2}}\,dx = -2\log\left(\frac{\sqrt{-x+6}+\sqrt{x}\,i}{\sqrt{6}}\right)i$$

input
```
int(1/(-x^2+6*x)^(1/2),x)
```

output
```
- 2*log((sqrt( - x + 6) + sqrt(x)*i)/sqrt(6))*i
```

3.46 $\int \frac{1}{\sqrt{4x+x^2}} dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{1}{\sqrt{4x+x^2}} dx = 2\text{arctanh}\left(\frac{x}{\sqrt{4x+x^2}}\right)$$

output
```
2*arctanh(x/(x^2+4*x)^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(16) = 32.

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{1}{\sqrt{4x+x^2}} dx = -\frac{2\sqrt{x}\sqrt{4+x}\log\left(-\sqrt{x}+\sqrt{4+x}\right)}{\sqrt{x(4+x)}}$$

input
```
Integrate[1/Sqrt[4*x + x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[4 + x]*Log[-Sqrt[x] + Sqrt[4 + x]])/Sqrt[x*(4 + x)]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2+4x}}dx$$

$$\Big\downarrow \text{1091}$$

$$2\int \frac{1}{1-\frac{x^2}{x^2+4x}}d\frac{x}{\sqrt{x^2+4x}}$$

$$\Big\downarrow \text{219}$$

$$2\text{arctanh}\left(\frac{x}{\sqrt{x^2+4x}}\right)$$

input
```
Int[1/Sqrt[4*x + x^2],x]
```

output
```
2*ArcTanh[x/Sqrt[4*x + x^2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

method	result	size
meijerg	$2 \operatorname{arcsinh}\left(\frac{\sqrt{x}}{2}\right)$	9
default	$\ln\left(2+x+\sqrt{x^2+4x}\right)$	14
trager	$\ln\left(2+x+\sqrt{x^2+4x}\right)$	14
pseudoelliptic	$2 \operatorname{arctanh}\left(\frac{\sqrt{x(x+4)}}{x}\right)$	15

input
```
int(1/(x^2+4*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*arcsinh(1/2*x^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{4x+x^2}}\,dx = -\log\left(-x+\sqrt{x^2+4\,x}-2\right)$$

input
```
integrate(1/(x^2+4*x)^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 + 4*x) - 2)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{4x+x^2}}\,dx = \log\left(2x+2\sqrt{x^2+4x}+4\right)$$

input
```
integrate(1/(x**2+4*x)**(1/2),x)
```

output
```
log(2*x + 2*sqrt(x**2 + 4*x) + 4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{4x+x^2}}\,dx = \log\left(2\,x+2\,\sqrt{x^2+4\,x}+4\right)$$

input
```
integrate(1/(x^2+4*x)^(1/2),x, algorithm="maxima")
```

output
```
log(2*x + 2*sqrt(x^2 + 4*x) + 4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{4x+x^2}}\,dx = \frac{1}{2}\,\sqrt{x^2+4\,x}(x+2)+2\,\log\left(\left|-x+\sqrt{x^2+4\,x}-2\right|\right)$$

input
```
integrate(1/(x^2+4*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 + 4*x)*(x + 2) + 2*log(abs(-x + sqrt(x^2 + 4*x) - 2))
```

Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{4x+x^2}}\,dx = \ln\left(x+\sqrt{x\,(x+4)}+2\right)$$

input
```
int(1/(4*x + x^2)^(1/2),x)
```

output
```
log(x + (x*(x + 4))^(1/2) + 2)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{1}{\sqrt{4x+x^2}}\,dx = 2\log\left(\frac{\sqrt{x+4}}{2}+\frac{\sqrt{x}}{2}\right)$$

input
```
int(1/(x^2+4*x)^(1/2),x)
```

output
```
2*log((sqrt(x + 4) + sqrt(x))/2)
```

3.47 $\int \frac{1}{\sqrt{-2x+x^2}}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 16

$$\int \frac{1}{\sqrt{-2x+x^2}}\,dx = 2\text{arctanh}\left(\frac{x}{\sqrt{-2x+x^2}}\right)$$

output
```
2*arctanh(x/(x^2-2*x)^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 39 vs. 2(16) = 32.

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.44

$$\int \frac{1}{\sqrt{-2x+x^2}}\,dx = -\frac{2\sqrt{-2+x}\sqrt{x}\log\left(\sqrt{-2+x}-\sqrt{x}\right)}{\sqrt{(-2+x)x}}$$

input
```
Integrate[1/Sqrt[-2*x + x^2],x]
```

output
```
(-2*Sqrt[-2 + x]*Sqrt[x]*Log[Sqrt[-2 + x] - Sqrt[x]])/Sqrt[(-2 + x)*x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x^2-2x}}dx$$

$$\Big\downarrow \text{1091}$$

$$2\int \frac{1}{1-\frac{x^2}{x^2-2x}}d\frac{x}{\sqrt{x^2-2x}}$$

$$\Big\downarrow \text{219}$$

$$2\text{arctanh}\left(\frac{x}{\sqrt{x^2-2x}}\right)$$

input
```
Int[1/Sqrt[-2*x + x^2],x]
```

output
```
2*ArcTanh[x/Sqrt[-2*x + x^2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

method	result	size
default	$\ln\left(x-1+\sqrt{x^2-2x}\right)$	14
trager	$\ln\left(x-1+\sqrt{x^2-2x}\right)$	14
pseudoelliptic	$2\,\mathrm{arctanh}\left(\frac{\sqrt{x(x-2)}}{x}\right)$	15
meijerg	$\frac{2\sqrt{-\mathrm{signum}(x-2)}\,\arcsin\left(\frac{\sqrt{2}\,\sqrt{x}}{2}\right)}{\sqrt{\mathrm{signum}(x-2)}}$	26

input
```
int(1/(x^2-2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
ln(x-1+(x^2-2*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{-2x+x^2}}\,dx = -\log\left(-x+\sqrt{x^2-2\,x}+1\right)$$

input
```
integrate(1/(x^2-2*x)^(1/2),x, algorithm="fricas")
```

output
```
-log(-x + sqrt(x^2 - 2*x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{-2x+x^2}}\,dx = \log \left(2x + 2\sqrt{x^2-2x} - 2\right)$$

input
```
integrate(1/(x**2-2*x)**(1/2),x)
```

output
```
log(2*x + 2*sqrt(x**2 - 2*x) - 2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{-2x+x^2}}\,dx = \log \left(2\,x + 2\,\sqrt{x^2-2\,x} - 2\right)$$

input
```
integrate(1/(x^2-2*x)^(1/2),x, algorithm="maxima")
```

output
```
log(2*x + 2*sqrt(x^2 - 2*x) - 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(14) = 28.

Time = 0.20 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \frac{1}{\sqrt{-2x+x^2}}\,dx = \frac{1}{2}\,\sqrt{x^2-2\,x}(x-1) + \frac{1}{2}\,\log \left(\left|-x + \sqrt{x^2-2\,x} + 1\right|\right)$$

input
```
integrate(1/(x^2-2*x)^(1/2),x, algorithm="giac")
```

output
```
1/2*sqrt(x^2 - 2*x)*(x - 1) + 1/2*log(abs(-x + sqrt(x^2 - 2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{-2x+x^2}}\,dx = \ln\left(x+\sqrt{x\,(x-2)}-1\right)$$

input
```
int(1/(x^2 - 2*x)^(1/2),x)
```

output
```
log(x + (x*(x - 2))^(1/2) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{1}{\sqrt{-2x+x^2}}\,dx = 2\log\left(\frac{\sqrt{x-2}+\sqrt{x}}{\sqrt{2}}\right)$$

input
```
int(1/(x^2-2*x)^(1/2),x)
```

output
```
2*log((sqrt(x - 2) + sqrt(x))/sqrt(2))
```

3.48 $\int (x-x^2)^{3/2}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 83

$$\int (x-x^2)^{3/2}\,dx = -\frac{3}{64}\sqrt{1-x}\sqrt{x}-\frac{1}{32}\sqrt{1-x}x^{3/2}+\frac{3}{8}\sqrt{1-x}x^{5/2}-\frac{1}{4}\sqrt{1-x}x^{7/2}+\frac{3\arcsin\left(\sqrt{x}\right)}{64}$$

output
```
-3/64*(1-x)^(1/2)*x^(1/2)-1/32*(1-x)^(1/2)*x^(3/2)+3/8*(1-x)^(1/2)*x^(5/2)
-1/4*(1-x)^(1/2)*x^(7/2)+3/64*arcsin(x^(1/2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int (x-x^2)^{3/2}\,dx = \frac{x(-3+x+26x^2-40x^3+16x^4)+6\sqrt{-1+x}\sqrt{x}\,\text{arctanh}\left(\frac{\sqrt{-1+x}}{-1+\sqrt{x}}\right)}{64\sqrt{-((-1+x)x)}}$$

input
```
Integrate[(x - x^2)^(3/2),x]
```

output
```
(x*(-3 + x + 26*x^2 - 40*x^3 + 16*x^4) + 6*Sqrt[-1 + x]*Sqrt[x]*ArcTanh[Sq
rt[-1 + x]/(-1 + Sqrt[x])])/(64*Sqrt[-((-1 + x)*x)])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1087, 1087, 1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(x-x^2\right)^{3/2} dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{3}{16}\int \sqrt{x-x^2}dx-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{3}{16}\left(\frac{1}{8}\int \frac{1}{\sqrt{x-x^2}}dx-\frac{1}{4}(1-2x)\sqrt{x-x^2}\right)-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}$$

$$\Big\downarrow \text{1090}$$

$$\frac{3}{16}\left(-\frac{1}{8}\int \frac{1}{\sqrt{1-(1-2x)^2}}d(1-2x)-\frac{1}{4}\sqrt{x-x^2}(1-2x)\right)-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}$$

$$\Big\downarrow \text{223}$$

$$\frac{3}{16}\left(-\frac{1}{8}\arcsin(1-2x)-\frac{1}{4}\sqrt{x-x^2}(1-2x)\right)-\frac{1}{8}(1-2x)\left(x-x^2\right)^{3/2}$$

input
```
Int[(x - x^2)^(3/2),x]
```

output
```
-1/8*((1 - 2*x)*(x - x^2)^(3/2)) + (3*(-1/4*((1 - 2*x)*Sqrt[x - x^2]) - Ar
cSin[1 - 2*x]/8))/16
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.47

method	result
risch	$\frac{\left(16x^3-24x^2+2x+3\right)x(x-1)}{64\sqrt{-x(x-1)}}+\frac{3\arcsin(2x-1)}{128}$
default	$-\frac{(1-2x)\left(-x^2+x\right)^{\frac{3}{2}}}{8}-\frac{3\sqrt{-x^2+x}\,(1-2x)}{64}+\frac{3\arcsin(2x-1)}{128}$
pseudoelliptic	$-\frac{3\arctan\left(\frac{\sqrt{-x(x-1)}}{x}\right)}{64}+\frac{\left(-16x^3+24x^2-2x-3\right)\sqrt{-x(x-1)}}{64}$
meijerg	$-\frac{3i\left(-\frac{i\sqrt{\pi}\,\sqrt{x}\,\left(80x^3-120x^2+10x+15\right)\sqrt{1-x}}{240}+\frac{i\sqrt{\pi}\,\arcsin\left(\sqrt{x}\right)}{16}\right)}{4\sqrt{\pi}}$
trager	$\left(-\frac{1}{4}x^3+\frac{3}{8}x^2-\frac{1}{32}x-\frac{3}{64}\right)\sqrt{-x^2+x}-\frac{3\,\mathrm{RootOf}\left(\_Z^2+1\right)\ln\left(2\,\mathrm{RootOf}\left(\_Z^2+1\right)x+2\sqrt{-x^2+x}-\mathrm{RootO}\right.}{128}$

input
```
int((-x^2+x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/64*(16*x^3-24*x^2+2*x+3)*x*(x-1)/(-x*(x-1))^(1/2)+3/128*arcsin(2*x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \left(x-x^2\right)^{3/2} \, dx = -\frac{1}{64} \left(16 \, x^3 - 24 \, x^2 + 2 \, x + 3\right)\sqrt{-x^2+x} - \frac{3}{64} \, \arctan\left(\frac{\sqrt{-x^2+x}}{x-1}\right)$$

input
```
integrate((-x^2+x)^(3/2),x, algorithm="fricas")
```

output
```
-1/64*(16*x^3 - 24*x^2 + 2*x + 3)*sqrt(-x^2 + x) - 3/64*arctan(sqrt(-x^2 +
 x)/(x - 1))
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.67

$$\int \left(x-x^2\right)^{3/2} \, dx = \sqrt{-x^2+x}\left(\frac{x^2}{3} - \frac{x}{12} - \frac{1}{8}\right) - \sqrt{-x^2+x}\left(\frac{x^3}{4} - \frac{x^2}{24} - \frac{5x}{96} - \frac{5}{64}\right) + \frac{3 \operatorname{asin}(2x-1)}{128}$$

input
```
integrate((-x**2+x)**(3/2),x)
```

output
```
sqrt(-x**2 + x)*(x**2/3 - x/12 - 1/8) - sqrt(-x**2 + x)*(x**3/4 - x**2/24
- 5*x/96 - 5/64) + 3*asin(2*x - 1)/128
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.66

$$\int \left(x-x^2\right)^{3/2} \, dx = \frac{1}{4} \left(-x^2+x\right)^{\frac{3}{2}} x - \frac{1}{8} \left(-x^2+x\right)^{\frac{3}{2}} + \frac{3}{32} \sqrt{-x^2+x}\, x - \frac{3}{64} \sqrt{-x^2+x} + \frac{3}{128} \arcsin\left(2 \, x - 1\right)$$

input
```
integrate((-x^2+x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(-x^2 + x)^(3/2)*x - 1/8*(-x^2 + x)^(3/2) + 3/32*sqrt(-x^2 + x)*x - 3/
64*sqrt(-x^2 + x) + 3/128*arcsin(2*x - 1)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.42

$$\int \left(x-x^2\right)^{3/2}\,dx=-\frac{1}{64}\left(2\left(4\left(2\,x-3\right)x+1\right)x+3\right)\sqrt{-x^2+x}+\frac{3}{128}\,\arcsin\left(2\,x-1\right)$$

input
```
integrate((-x^2+x)^(3/2),x, algorithm="giac")
```

output
```
-1/64*(2*(4*(2*x - 3)*x + 1)*x + 3)*sqrt(-x^2 + x) + 3/128*arcsin(2*x - 1)
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.47

$$\int \left(x-x^2\right)^{3/2}\,dx=\frac{3\,\mathrm{asin}(2\,x-1)}{128}+\frac{3\,\sqrt{x-x^2}\,\left(\frac{x}{2}-\frac{1}{4}\right)}{16}+\frac{\left(x-x^2\right)^{3/2}\,\left(x-\frac{1}{2}\right)}{4}$$

input
```
int((x - x^2)^(3/2),x)
```

output
```
(3*asin(2*x - 1))/128 + (3*(x - x^2)^(1/2)*(x/2 - 1/4))/16 + ((x - x^2)^(3
/2)*(x - 1/2))/4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

$$\int \left(x-x^2\right)^{3/2}\,dx = -\frac{\sqrt{x}\,\sqrt{1-x}\,x^3}{4}+\frac{3\sqrt{x}\,\sqrt{1-x}\,x^2}{8} - \frac{\sqrt{x}\,\sqrt{1-x}\,x}{32}-\frac{3\sqrt{x}\,\sqrt{1-x}}{64}-\frac{3\log\left(\sqrt{1-x}+\sqrt{x}\,i\right)i}{64}$$

input
```
int((-x^2+x)^(3/2),x)
```

output
```
( - 16*sqrt(x)*sqrt( - x + 1)*x**3 + 24*sqrt(x)*sqrt( - x + 1)*x**2 - 2*sq
rt(x)*sqrt( - x + 1)*x - 3*sqrt(x)*sqrt( - x + 1) - 3*log(sqrt( - x + 1) +
 sqrt(x)*i)*i)/64
```

3.49 $\int \frac{1}{\sqrt{3-4x}\sqrt{x}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 13

$$\int \frac{1}{\sqrt{3-4x}\sqrt{x}}\,dx = \arcsin\left(\frac{2\sqrt{x}}{\sqrt{3}}\right)$$

output
```
arcsin(2/3*x^(1/2)*3^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.23

$$\int \frac{1}{\sqrt{3-4x}\sqrt{x}}\,dx = -2\arctan\left(\frac{2\sqrt{x}}{\sqrt{3}-\sqrt{3-4x}}\right)$$

input
```
Integrate[1/(Sqrt[3 - 4*x]*Sqrt[x]),x]
```

output
```
-2*ArcTan[(2*Sqrt[x])/(Sqrt[3] - Sqrt[3 - 4*x])]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {63, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3-4x}\sqrt{x}}dx$$

$$\Big\downarrow \text{63}$$

$$2\int \frac{1}{\sqrt{3-4x}}d\sqrt{x}$$

$$\Big\downarrow \text{223}$$

$$\arcsin\left(\frac{2\sqrt{x}}{\sqrt{3}}\right)$$

input
```
Int[1/(Sqrt[3 - 4*x]*Sqrt[x]),x]
```

output
```
ArcSin[(2*Sqrt[x])/Sqrt[3]]
```

Defintions of rubi rules used

rule 63
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_Symbol] :> Simp[2/b   S
ubst[Int[1/Sqrt[c + d*(x^2/b)], x], x, Sqrt[b*x]], x] /; FreeQ[{b, c, d}, x
] && GtQ[c, 0]
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
meijerg	$\arcsin\left(\frac{2\sqrt{x}\,\sqrt{3}}{3}\right)$	10
default	$\frac{\sqrt{(3-4x)x}\,\arcsin\left(-1+\frac{8x}{3}\right)}{2\sqrt{3-4x}\,\sqrt{x}}$	28

input
```
int(1/(3-4*x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(2/3*x^(1/2)*3^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 22 vs. 2(9) = 18.

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.69

$$\int \frac{1}{\sqrt{3-4x}\sqrt{x}}\,dx = -\arctan\left(\frac{2\sqrt{x}\sqrt{-4x+3}}{4x-3}\right)$$

input
```
integrate(1/(3-4*x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
-arctan(2*sqrt(x)*sqrt(-4*x + 3)/(4*x - 3))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.47 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.77

$$\int \frac{1}{\sqrt{3-4x}\sqrt{x}}\,dx = \begin{cases} -i\operatorname{acosh}\left(\frac{2\sqrt{3}\sqrt{x}}{3}\right) & \text{for } |x| > \frac{3}{4} \\ \operatorname{asin}\left(\frac{2\sqrt{3}\sqrt{x}}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(1/(3-4*x)**(1/2)/x**(1/2),x)
```

output
```
Piecewise((-I*acosh(2*sqrt(3)*sqrt(x)/3), Abs(x) > 3/4), (asin(2*sqrt(3)*s
qrt(x)/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{\sqrt{3-4x}\sqrt{x}}\,dx = -\arctan\left(\frac{\sqrt{-4\,x+3}}{2\,\sqrt{x}}\right)$$

input
```
integrate(1/(3-4*x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
-arctan(1/2*sqrt(-4*x + 3)/sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{3-4x}\sqrt{x}}\,dx = \arcsin\left(\frac{2}{3}\,\sqrt{3}\sqrt{x}\right)$$

input
```
integrate(1/(3-4*x)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
arcsin(2/3*sqrt(3)*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{3-4x}\sqrt{x}}\,dx = 2\,\mathrm{atan}\left(\frac{\sqrt{3}-\sqrt{3-4\,x}}{2\,\sqrt{x}}\right)$$

input
```
int(1/(x^(1/2)*(3 - 4*x)^(1/2)),x)
```

output
```
2*atan((3^(1/2) - (3 - 4*x)^(1/2))/(2*x^(1/2)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{3-4x}\sqrt{x}}\,dx = -\log\left(\frac{\sqrt{-4x+3}+2\sqrt{x}\,i}{\sqrt{3}}\right)i$$

input
```
int(1/(3-4*x)^(1/2)/x^(1/2),x)
```

output
```
- log((sqrt( - 4*x + 3) + 2*sqrt(x)*i)/sqrt(3))*i
```

3.50 $\int \frac{1}{\sqrt{3x-4x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = -\frac{1}{2}\arcsin\left(1-\frac{8x}{3}\right)$$

output
```
1/2*arcsin(-1+8/3*x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 46 vs. $2(12) = 24$.

Time = 0.01 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.83

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = -\frac{\sqrt{x}\sqrt{-3+4x}\log\left(-2\sqrt{x}+\sqrt{-3+4x}\right)}{\sqrt{-x(-3+4x)}}$$

input
```
Integrate[1/Sqrt[3*x - 4*x^2],x]
```

output
```
-((Sqrt[x]*Sqrt[-3 + 4*x]*Log[-2*Sqrt[x] + Sqrt[-3 + 4*x]])/Sqrt[-(x*(-3 +
 4*x))])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1090, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{3x-4x^2}}dx$$
$$\Big\downarrow \text{1090}$$
$$-\frac{1}{6}\int \frac{1}{\sqrt{1-\frac{1}{9}(3-8x)^2}}d(3-8x)$$
$$\Big\downarrow \text{223}$$
$$-\frac{1}{2}\arcsin\left(\frac{1}{3}(3-8x)\right)$$

input
```
Int[1/Sqrt[3*x - 4*x^2],x]
```

output
```
-1/2*ArcSin[(3 - 8*x)/3]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\arcsin\left(-1+\frac{8x}{3}\right)}{2}$	9
meijerg	$\arcsin\left(\frac{2\sqrt{x}\,\sqrt{3}}{3}\right)$	10
pseudoelliptic	$-\arctan\left(\frac{\sqrt{-4x^2+3x}}{2x}\right)$	20
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(8\operatorname{RootOf}\left(\_Z^2+1\right)x+4\sqrt{-4x^2+3x}-3\operatorname{RootOf}\left(\_Z^2+1\right)\right)}{2}$	41

input
```
int(1/(-4*x^2+3*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arcsin(-1+8/3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(8) = 16$.

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = -\arctan\left(\frac{2\sqrt{-4x^2+3x}}{4x-3}\right)$$

input
```
integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="fricas")
```

output
```
-arctan(2*sqrt(-4*x^2 + 3*x)/(4*x - 3))
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = \frac{\operatorname{asin}\left(\frac{8x}{3}-1\right)}{2}$$

input
```
integrate(1/(-4*x**2+3*x)**(1/2),x)
```

output
```
asin(8*x/3 - 1)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = -\frac{1}{2}\arcsin\left(-\frac{8}{3}x+1\right)$$

input
```
integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*arcsin(-8/3*x + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(8) = 16.

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = \frac{1}{16}\sqrt{-4\,x^2+3\,x}(8\,x-3)+\frac{9}{64}\arcsin\left(\frac{8}{3}\,x-1\right)$$

input
```
integrate(1/(-4*x^2+3*x)^(1/2),x, algorithm="giac")
```

output
```
1/16*sqrt(-4*x^2 + 3*x)*(8*x - 3) + 9/64*arcsin(8/3*x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = \frac{\operatorname{asin}\left(\frac{8\,x}{3}-1\right)}{2}$$

input
```
int(1/(3*x - 4*x^2)^(1/2),x)
```

output
```
asin((8*x)/3 - 1)/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.75

$$\int \frac{1}{\sqrt{3x-4x^2}}\,dx = -\log\left(\frac{\sqrt{-4x+3}+2\sqrt{x}\,i}{\sqrt{3}}\right)i$$

input
```
int(1/(-4*x^2+3*x)^(1/2),x)
```

output
```
 - log((sqrt( - 4*x + 3) + 2*sqrt(x)*i)/sqrt(3))*i
```

3.51 $\int (ax+bx^2)^{4/3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int (ax+bx^2)^{4/3} \, dx = \frac{3(ax+bx^2)^{7/3} \text{Hypergeometric2F1}\left(1, \frac{14}{3}, \frac{10}{3}, -\frac{bx}{a}\right)}{7a}$$

output
```
3/7*(b*x^2+a*x)^(7/3)*hypergeom([1, 14/3],[10/3],-b*x/a)/a
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.37

$$\int (ax+bx^2)^{4/3} \, dx = \frac{3ax^2 \sqrt[3]{x(a+bx)} \text{Hypergeometric2F1}\left(-\frac{4}{3}, \frac{7}{3}, \frac{10}{3}, -\frac{bx}{a}\right)}{7\sqrt[3]{1+\frac{bx}{a}}}$$

input
```
Integrate[(a*x + b*x^2)^(4/3),x]
```

output
```
(3*a*x^2*(x*(a + b*x))^(1/3)*Hypergeometric2F1[-4/3, 7/3, 10/3, -((b*x)/a)
])/(7*(1 + (b*x)/a)^(1/3))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 350 vs. $2(35) = 70$.

Time = 0.61 (sec) , antiderivative size = 350, normalized size of antiderivative = 10.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1087, 1087, 1093, 1090, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax+bx^2)^{4/3}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{3(a+2bx)(ax+bx^2)^{4/3}}{22b}-\frac{2a^2\int \sqrt[3]{bx^2+ax}dx}{11b}$$

$$\Big\downarrow \text{1087}$$

$$\frac{3(a+2bx)(ax+bx^2)^{4/3}}{22b}-\frac{2a^2\left(\frac{3(a+2bx)\sqrt[3]{ax+bx^2}}{10b}-\frac{a^2\int \frac{1}{(bx^2+ax)^{2/3}}dx}{10b}\right)}{11b}$$

$$\Big\downarrow \text{1093}$$

$$\frac{3(a+2bx)(ax+bx^2)^{4/3}}{22b}-\frac{2a^2\left(\frac{3(a+2bx)\sqrt[3]{ax+bx^2}}{10b}-\frac{a^2\left(-\frac{b(ax+bx^2)}{a^2}\right)^{2/3}\int \frac{1}{\left(-\frac{b^2x^2}{a^2}-\frac{bx}{a}\right)^{2/3}}dx}{10b(ax+bx^2)^{2/3}}\right)}{11b}$$

$$\Big\downarrow \text{1090}$$

$$\frac{3(a+2bx)(ax+bx^2)^{4/3}}{22b}-\frac{2a^2\left(\frac{a^4\left(-\frac{b(ax+bx^2)}{a^2}\right)^{2/3}\int \frac{1}{\left(1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}\right)^{2/3}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{5\ 2^{2/3}b^3(ax+bx^2)^{2/3}}+\frac{3(a+2bx)\sqrt[3]{ax+bx^2}}{10b}\right)}{11b}$$

$\downarrow$ 234

$$\frac{3(a+2bx)\left(ax+bx^2\right)^{4/3}}{22b}-\frac{2a^2\left(\frac{3(a+2bx)\sqrt[3]{ax+bx^2}}{10b}-\frac{3a^2\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\left(-\frac{b\left(ax+bx^2\right)}{a^2}\right)^{2/3}\int\frac{1}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}{10\ 2^{2/3}b\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)\left(ax+bx^2\right)^{2/3}}\right)}{11b}$$

$\downarrow$ 760

$$\frac{3(a+2bx)\left(ax+bx^2\right)^{4/3}}{22b}-\frac{2a^2\left(\frac{3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\frac{2b^2x}{a^2}+\frac{b}{a}+1\right)\left(-\frac{b\left(ax+bx^2\right)}{a^2}\right)^{2/3}\sqrt{\frac{\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2+\sqrt[3]{1-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}+1}{\left(\frac{2b^2x}{a^2}+\frac{b}{a}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\frac{\frac{2xb^2}{a^2}+\frac{b}{a}+\sqrt{3}+1}{\frac{2xb^2}{a^2}+\frac{b}{a}-\sqrt{3}+1}\right.\right.}{5\ 2^{2/3}b\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)\sqrt{-\frac{\frac{2b^2x}{a^2}+\frac{b}{a}+1}{\left(\frac{2b^2x}{a^2}+\frac{b}{a}-\sqrt{3}+1\right)^2}}\left(ax+bx^2\right)^{2/3}}\right.}{11b}$$

input

```
Int[(a*x + b*x^2)^(4/3),x]
```

output

```
(3*(a + 2*b*x)*(a*x + b*x^2)^(4/3))/(22*b) - (2*a^2*((3*(a + 2*b*x)*(a*x +
 b*x^2)^(1/3))/(10*b) + (3^(3/4)*Sqrt[2 - Sqrt[3]]*a^2*(1 + b/a + (2*b^2*x
)/a^2)*(-((b*(a*x + b*x^2))/a^2))^(2/3)*Sqrt[(1 + (-(b/a) - (2*b^2*x)/a^2)
^2 + (1 - (a^2*(-(b/a) - (2*b^2*x)/a^2)^2)/b^2)^(1/3))/(1 - Sqrt[3] + b/a
+ (2*b^2*x)/a^2)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + b/a + (2*b^2*x)/a^2)/(
1 - Sqrt[3] + b/a + (2*b^2*x)/a^2)], -7 + 4*Sqrt[3]])/(5*2^(2/3)*b*(-(b/a)
 - (2*b^2*x)/a^2)*Sqrt[-((1 + b/a + (2*b^2*x)/a^2)/(1 - Sqrt[3] + b/a + (2
*b^2*x)/a^2)^2)]*(a*x + b*x^2)^(2/3))))/(11*b)
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \left(b\,x^{2}+a x \right)^{\frac{4}{3}}d x$$

input
```
int((b*x^2+a*x)^(4/3),x)
```

output
```
int((b*x^2+a*x)^(4/3),x)
```

Fricas [F]

$$\int \left(ax+bx^2\right)^{4/3}\,dx=\int \left(bx^2+ax\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*x^2+a*x)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^(4/3), x)
```

Sympy [F]

$$\int \left(ax+bx^2\right)^{4/3}\,dx=\int \left(ax+bx^2\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*x**2+a*x)**(4/3),x)
```

output
```
Integral((a*x + b*x**2)**(4/3), x)
```

Maxima [F]

$$\int \left(ax+bx^2\right)^{4/3}\,dx=\int \left(bx^2+ax\right)^{\frac{4}{3}}\,dx$$

input
```
integrate((b*x^2+a*x)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(4/3), x)
```

Giac [F]

$$\int \left(ax+bx^2\right)^{4/3} \, dx=\int \left(bx^2+ax\right)^{\frac{4}{3}} \, dx$$

input
```
integrate((b*x^2+a*x)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^(4/3), x)
```

Mupad [B] (verification not implemented)

Time = 9.43 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \left(ax+bx^2\right)^{4/3} \, dx=\frac{3\,x\,{\left(b\,x^2+a\,x\right)}^{4/3}\,{}_2\mathrm{F}_1\left(-\frac{4}{3},\frac{7}{3};\ \frac{10}{3};\ -\frac{b\,x}{a}\right)}{7\,{\left(\frac{b\,x}{a}+1\right)}^{4/3}}$$

input
```
int((a*x + b*x^2)^(4/3),x)
```

output
```
(3*x*(a*x + b*x^2)^(4/3)*hypergeom([-4/3, 7/3], 10/3, -(b*x)/a))/(7*((b*x)
/a + 1)^(4/3))
```

Reduce [F]

$$\int (ax +bx^2)^{4/3} \, dx = \frac{-6(bx+a)^{\frac{1}{3}} a^4-6(bx+a)^{\frac{1}{3}} a^3bx+3(bx+a)^{\frac{1}{3}} a^2b^2x^2+45(bx+a)^{\frac{1}{3}} a\, b^3x^3+30(bx+a)^{\frac{1}{3}}}{110x^{\frac{2}{3}}b^3}$$

input
```
int((b*x^2+a*x)^(4/3),x)
```

output
```
( - 6*(a + b*x)**(1/3)*a**4 - 6*(a + b*x)**(1/3)*a**3*b*x + 3*(a + b*x)**(
1/3)*a**2*b**2*x**2 + 45*(a + b*x)**(1/3)*a*b**3*x**3 + 30*(a + b*x)**(1/3
)*b**4*x**4 - 4*x**(2/3)*int((a + b*x)**(1/3)/(x**(2/3)*a*x + x**(2/3)*b*x
**2),x)*a**5)/(110*x**(2/3)*b**3)
```

3.52 $\int (ax + bx^2)^{2/3} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int (ax + bx^2)^{2/3} \, dx = \frac{3(ax + bx^2)^{5/3} \text{Hypergeometric2F1}\left(1, \frac{10}{3}, \frac{8}{3}, -\frac{bx}{a}\right)}{5a}$$

output
```
3/5*(b*x^2+a*x)^(5/3)*hypergeom([1, 10/3],[8/3],-b*x/a)/a
```

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int (ax + bx^2)^{2/3} \, dx = \frac{3x(x(a + bx))^{2/3} \text{Hypergeometric2F1}\left(-\frac{2}{3}, \frac{5}{3}, \frac{8}{3}, -\frac{bx}{a}\right)}{5\left(1 + \frac{bx}{a}\right)^{2/3}}$$

input
```
Integrate[(a*x + b*x^2)^(2/3),x]
```

output
```
(3*x*(x*(a + b*x))^(2/3)*Hypergeometric2F1[-2/3, 5/3, 8/3, -((b*x)/a)])/(5
*(1 + (b*x)/a)^(2/3))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 690 vs. $2(35) = 70$.

Time = 0.88 (sec) , antiderivative size = 690, normalized size of antiderivative = 19.71, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {1087, 1093, 1090, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)^{2/3} dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{3(a+2bx)\left(ax+bx^2\right)^{2/3}}{14b}-\frac{a^2\int \frac{1}{\sqrt[3]{bx^2+ax}}dx}{7b}$$

$$\Big\downarrow \text{1093}$$

$$\frac{3(a+2bx)\left(ax+bx^2\right)^{2/3}}{14b}-\frac{a^2\sqrt[3]{-\frac{b\left(ax+bx^2\right)}{a^2}}\int \frac{1}{\sqrt[3]{-\frac{b^2x^2}{a^2}-\frac{bx}{a}}}dx}{7b\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{a^4\sqrt[3]{-\frac{b\left(ax+bx^2\right)}{a^2}}\int \frac{1}{\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{7\sqrt[3]{2}b^3\sqrt[3]{ax+bx^2}}+\frac{3(a+2bx)\left(ax+bx^2\right)^{2/3}}{14b}$$

$$\Big\downarrow \text{233}$$

$$\frac{3(a+2bx)\left(ax+bx^2\right)^{2/3}}{14b}-\frac{3a^2\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\sqrt[3]{-\frac{b\left(ax+bx^2\right)}{a^2}}\int \frac{\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}{14\sqrt[3]{2}b\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)\sqrt[3]{ax+bx^2}}$$

$\big\downarrow$ 833

$$\frac{3(a+2bx)\left(ax+bx^2\right)^{2/3}}{14b}-\frac{3a^2\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\sqrt[3]{-\frac{b\left(ax+bx^2\right)}{a^2}}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}-\int\frac{\frac{2xb^2}{a^2}+\frac{b}{a}+\sqrt{3}+\ldots}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\ldots\right.}{b^2}}}\ldots\right.}{14\sqrt[3]{2}b\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)\sqrt[3]{ax+bx^2}}$$

$\big\downarrow$ 760

$$\frac{3(a+2bx)\left(ax+bx^2\right)^{2/3}}{14b}-\frac{3a^2\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\sqrt[3]{-\frac{b\left(ax+bx^2\right)}{a^2}}\left(-\int\frac{\frac{2xb^2}{a^2}+\frac{b}{a}+\sqrt{3}+1}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{2b^2x}{a^2}+\frac{b}{a}\ldots\right.}{\ldots}\right.}{14\sqrt[3]{2}b\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)\sqrt[3]{ax+b\ldots}}$$

$\big\downarrow$ 2418

$$\frac{3(a+2bx)\left(ax+bx^2\right)^{2/3}}{14b}-\frac{3a^2\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\sqrt[3]{-\frac{b\left(ax+bx^2\right)}{a^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{2b^2x}{a^2}+\frac{b}{a}+1\right)\sqrt{\frac{\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2+\sqrt[3]{1-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}+1}{\left(\frac{2b^2x}{a^2}+\frac{b}{a}-\sqrt{3}+1\right)^2}}\text{Ellip}\ldots}{\sqrt[4]{3}\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\sqrt{-\frac{\frac{2b^2x}{a^2}+\frac{b}{a}+1}{\left(\frac{2b^2x}{a^2}+\frac{b}{a}-\sqrt{3}+1\right)\ldots}}}\right.}{\ldots}$$

input

```
Int[(a*x + b*x^2)^(2/3),x]
```

output
```
(3*(a + 2*b*x)*(a*x + b*x^2)^(2/3))/(14*b) - (3*a^2*Sqrt[-((a^2*(-(b/a) -
(2*b^2*x)/a^2)^2)/b^2)]*(-((b*(a*x + b*x^2))/a^2))^(1/3)*((-2*Sqrt[-((a^2*
(-(b/a) - (2*b^2*x)/a^2)^2)/b^2)])/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^2) + (
3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + b/a + (2*b^2*x)/a^2)*Sqrt[(1 + (-(b/a) - (2
*b^2*x)/a^2)^2 + (1 - (a^2*(-(b/a) - (2*b^2*x)/a^2)^2)/b^2)^(1/3))/(1 - Sq
rt[3] + b/a + (2*b^2*x)/a^2)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + b/a + (2*b
^2*x)/a^2)/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^2)], -7 + 4*Sqrt[3]])/(Sqrt[-(
(a^2*(-(b/a) - (2*b^2*x)/a^2)^2)/b^2)]*Sqrt[-((1 + b/a + (2*b^2*x)/a^2)/(1
 - Sqrt[3] + b/a + (2*b^2*x)/a^2)^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3]
)*(1 + b/a + (2*b^2*x)/a^2)*Sqrt[(1 + (-(b/a) - (2*b^2*x)/a^2)^2 + (1 - (a
^2*(-(b/a) - (2*b^2*x)/a^2)^2)/b^2)^(1/3))/(1 - Sqrt[3] + b/a + (2*b^2*x)/
a^2)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + b/a + (2*b^2*x)/a^2)/(1 - Sqrt[3]
+ b/a + (2*b^2*x)/a^2)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((a^2*(-(b/a) - (
2*b^2*x)/a^2)^2)/b^2)]*Sqrt[-((1 + b/a + (2*b^2*x)/a^2)/(1 - Sqrt[3] + b/a
 + (2*b^2*x)/a^2)^2)])))/(14*2^(1/3)*b*(-(b/a) - (2*b^2*x)/a^2)*(a*x + b*x
^2)^(1/3))
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \left(b\,x^{2}+ax\right)^{\frac{2}{3}}dx$$

input
```
int((b*x^2+a*x)^(2/3),x)
```

output
```
int((b*x^2+a*x)^(2/3),x)
```

Fricas [F]

$$\int \left(ax+bx^2\right)^{2/3} \, dx = \int \left(bx^2+ax\right)^{\frac{2}{3}} \, dx$$

input
```
integrate((b*x^2+a*x)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^(2/3), x)
```

Sympy [F]

$$\int \left(ax+bx^2\right)^{2/3} \, dx = \int \left(ax+bx^2\right)^{\frac{2}{3}} \, dx$$

input
```
integrate((b*x**2+a*x)**(2/3),x)
```

output
```
Integral((a*x + b*x**2)**(2/3), x)
```

Maxima [F]

$$\int \left(ax+bx^2\right)^{2/3} \, dx = \int \left(bx^2+ax\right)^{\frac{2}{3}} \, dx$$

input
```
integrate((b*x^2+a*x)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(2/3), x)
```

Giac [F]

$$\int \left(ax+bx^2\right)^{2/3}\,dx=\int \left(bx^2+ax\right)^{\frac{2}{3}}\,dx$$

input
```
integrate((b*x^2+a*x)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^(2/3), x)
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \left(ax+bx^2\right)^{2/3}\,dx=\frac{3\,x\,\left(b\,x^2+a\,x\right)^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{5}{3};\ \frac{8}{3};\ -\frac{b\,x}{a}\right)}{5\,\left(\frac{b\,x}{a}+1\right)^{2/3}}$$

input
```
int((a*x + b*x^2)^(2/3),x)
```

output
```
(3*x*(a*x + b*x^2)^(2/3)*hypergeom([-2/3, 5/3], 8/3, -(b*x)/a))/(5*((b*x)/
a + 1)^(2/3))
```

Reduce [F]

$$\int (ax+bx^2)^{2/3}\,dx=\frac{-6(bx+a)^{\frac{2}{3}}a^2+3(bx+a)^{\frac{2}{3}}abx+6(bx+a)^{\frac{2}{3}}b^2x^2-2x^{\frac{1}{3}}\left(\int \frac{(bx+a)^{\frac{2}{3}}}{x^{\frac{4}{3}}a+x^{\frac{7}{3}}b}dx\right)a^3}{14x^{\frac{1}{3}}b^2}$$

input
```
int((b*x^2+a*x)^(2/3),x)
```

output
```
( - 6*(a + b*x)**(2/3)*a**2 + 3*(a + b*x)**(2/3)*a*b*x + 6*(a + b*x)**(2/3
)*b**2*x**2 - 2*x**(1/3)*int((a + b*x)**(2/3)/(x**(1/3)*a*x + x**(1/3)*b*x
**2),x)*a**3)/(14*x**(1/3)*b**2)
```

3.53 $\int \sqrt[3]{ax+bx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \sqrt[3]{ax+bx^2}\,dx=\frac{3(ax+bx^2)^{4/3}\operatorname{Hypergeometric2F1}\left(1,\frac{8}{3},\frac{7}{3},-\frac{bx}{a}\right)}{4a}$$

output
```
3/4*(b*x^2+a*x)^(4/3)*hypergeom([1, 8/3],[7/3],-b*x/a)/a
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \sqrt[3]{ax+bx^2}\,dx=\frac{3x\sqrt[3]{x(a+bx)}\operatorname{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{7}{3},-\frac{bx}{a}\right)}{4\sqrt[3]{1+\frac{bx}{a}}}$$

input
```
Integrate[(a*x + b*x^2)^(1/3),x]
```

output
```
(3*x*(x*(a + b*x))^(1/3)*Hypergeometric2F1[-1/3, 4/3, 7/3, -((b*x)/a)])/(4
*(1 + (b*x)/a)^(1/3))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 313 vs. 2(35) = 70.

Time = 0.53 (sec) , antiderivative size = 313, normalized size of antiderivative = 8.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1087, 1093, 1090, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{ax+bx^2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{3(a+2bx)\sqrt[3]{ax+bx^2}}{10b}-\frac{a^2\int \frac{1}{(bx^2+ax)^{2/3}}dx}{10b}$$

$$\Big\downarrow \text{1093}$$

$$\frac{3(a+2bx)\sqrt[3]{ax+bx^2}}{10b}-\frac{a^2\left(-\frac{b(ax+bx^2)}{a^2}\right)^{2/3}\int \frac{1}{\left(-\frac{b^2x^2}{a^2}-\frac{bx}{a}\right)^{2/3}}dx}{10b\,(ax+bx^2)^{2/3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{a^4\left(-\frac{b(ax+bx^2)}{a^2}\right)^{2/3}\int \frac{1}{\left(1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}\right)^{2/3}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{5\ 2^{2/3}b^3\,(ax+bx^2)^{2/3}}+\frac{3(a+2bx)\sqrt[3]{ax+bx^2}}{10b}$$

$$\Big\downarrow \text{234}$$

$$\frac{3(a+2bx)\sqrt[3]{ax+bx^2}}{10b}-$$

$$\frac{3a^2\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\left(-\frac{b(ax+bx^2)}{a^2}\right)^{2/3}\int \frac{1}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}{10\ 2^{2/3}b\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)(ax+bx^2)^{2/3}}$$

$$\Big\downarrow \text{760}$$

$$\frac{3^{3/4}\sqrt{2-\sqrt{3}}a^2\left(\frac{2b^2x}{a^2}+\frac{b}{a}+1\right)\left(-\frac{b(ax+bx^2)}{a^2}\right)^{2/3}\sqrt{\frac{\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2+\sqrt[3]{1-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}+1}{\left(\frac{2b^2x}{a^2}+\frac{b}{a}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\left(\ldots\right.\right.}{5\,2^{2/3}b\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)\sqrt{-\frac{\frac{2b^2x}{a^2}+\frac{b}{a}+1}{\left(\frac{2b^2x}{a^2}+\frac{b}{a}-\sqrt{3}+1\right)^2}}\,(ax+bx^2)^{2/3}}$$

$$\frac{3(a+2bx)\sqrt[3]{ax+bx^2}}{10b}$$

input
```
Int[(a*x + b*x^2)^(1/3),x]
```

output
```
(3*(a + 2*b*x)*(a*x + b*x^2)^(1/3))/(10*b) + (3^(3/4)*Sqrt[2 - Sqrt[3]]*a^
2*(1 + b/a + (2*b^2*x)/a^2)*(-((b*(a*x + b*x^2))/a^2))^(2/3)*Sqrt[(1 + (-(
b/a) - (2*b^2*x)/a^2)^2 + (1 - (a^2*(-(b/a) - (2*b^2*x)/a^2)^2)/b^2)^(1/3)
)/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^2)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + b
/a + (2*b^2*x)/a^2)/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^2)], -7 + 4*Sqrt[3]])
/(5*2^(2/3)*b*(-(b/a) - (2*b^2*x)/a^2)*Sqrt[-((1 + b/a + (2*b^2*x)/a^2)/(1
 - Sqrt[3] + b/a + (2*b^2*x)/a^2)^2)]*(a*x + b*x^2)^(2/3))
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \left(b\,x^{2}+ax\right)^{\frac{1}{3}}dx$$

input
```
int((b*x^2+a*x)^(1/3),x)
```

output
```
int((b*x^2+a*x)^(1/3),x)
```

Fricas [F]

$$\int \sqrt[3]{ax+bx^2}\,dx=\int \left(bx^2+ax\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a*x)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^(1/3), x)
```

Sympy [F]

$$\int \sqrt[3]{ax+bx^2}\,dx = \int \sqrt[3]{ax+bx^2}\,dx$$

input
```
integrate((b*x**2+a*x)**(1/3),x)
```

output
```
Integral((a*x + b*x**2)**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{ax+bx^2}\,dx = \int (bx^2+ax)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a*x)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(1/3), x)
```

Giac [F]

$$\int \sqrt[3]{ax+bx^2}\,dx = \int (bx^2+ax)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^2+a*x)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^(1/3), x)
```

Mupad [B] (verification not implemented)

Time = 9.67 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \sqrt[3]{ax+bx^2}\,dx = \frac{3\,x\,{\left(b\,x^2+a\,x\right)}^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{4}{3};\ \frac{7}{3};\ -\frac{b\,x}{a}\right)}{4\,{\left(\frac{b\,x}{a}+1\right)}^{1/3}}$$

input
```
int((a*x + b*x^2)^(1/3),x)
```

output
```
(3*x*(a*x + b*x^2)^(1/3)*hypergeom([-1/3, 4/3], 7/3, -(b*x)/a))/(4*((b*x)/
a + 1)^(1/3))
```

Reduce [F]

$$\int \sqrt[3]{ax+bx^2}\,dx = \frac{3(bx+a)^{\frac{1}{3}}a^2+3(bx+a)^{\frac{1}{3}}abx+6(bx+a)^{\frac{1}{3}}b^2x^2+2x^{\frac{2}{3}}\left(\int \frac{(bx+a)^{\frac{1}{3}}}{x^{\frac{5}{3}}a+x^{\frac{8}{3}}b}dx\right)a^3}{10x^{\frac{2}{3}}b^2}$$

input
```
int((b*x^2+a*x)^(1/3),x)
```

output
```
(3*(a + b*x)**(1/3)*a**2 + 3*(a + b*x)**(1/3)*a*b*x + 6*(a + b*x)**(1/3)*b
**2*x**2 + 2*x**(2/3)*int((a + b*x)**(1/3)/(x**(2/3)*a*x + x**(2/3)*b*x**2
),x)*a**3)/(10*x**(2/3)*b**2)
```

3.54 $\int \frac{1}{\sqrt[3]{ax+bx^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{1}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3(ax+bx^2)^{2/3}\,\text{Hypergeometric2F1}\left(1,\frac{4}{3},\frac{5}{3},-\frac{bx}{a}\right)}{2a}$$

output
```
3/2*(b*x^2+a*x)^(2/3)*hypergeom([1, 4/3],[5/3],-b*x/a)/a
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \frac{1}{\sqrt[3]{ax+bx^2}}\,dx = \frac{3x\sqrt[3]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{5}{3},-\frac{bx}{a}\right)}{2\sqrt[3]{x(a+bx)}}$$

input
```
Integrate[(a*x + b*x^2)^(-1/3),x]
```

output
```
(3*x*(1 + (b*x)/a)^(1/3)*Hypergeometric2F1[1/3, 2/3, 5/3, -((b*x)/a)])/(2*
(x*(a + b*x))^(1/3))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 657 vs. $2(35) = 70$.

Time = 0.75 (sec) , antiderivative size = 657, normalized size of antiderivative = 18.77, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {1093, 1090, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{ax+bx^2}}dx$$

$$\Big\downarrow \text{1093}$$

$$\frac{\sqrt[3]{-\frac{b(ax+bx^2)}{a^2}}\int \frac{1}{\sqrt[3]{-\frac{b^2x^2}{a^2}-\frac{bx}{a}}}dx}{\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{a^2\sqrt[3]{-\frac{b(ax+bx^2)}{a^2}}\int \frac{1}{\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{\sqrt[3]{2}b^2\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{233}$$

$$\frac{3\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\sqrt[3]{-\frac{b(ax+bx^2)}{a^2}}\int \frac{\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}{2\sqrt[3]{2}\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{833}$$

$$\frac{3\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\sqrt[3]{-\frac{b\left(ax+bx^2\right)}{a^2}}\left(\left(1+\sqrt{3}\right)\int\frac{1}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}-\int\frac{\frac{2xb^2}{a^2}+\frac{b}{a}+\sqrt{3}+1}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b^2}}}\right.}{2\sqrt[3]{2}\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)\sqrt[3]{ax+bx^2}}$$

$\downarrow$ 760

$$\frac{3\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\sqrt[3]{-\frac{b\left(ax+bx^2\right)}{a^2}}\left(-\int\frac{\frac{2xb^2}{a^2}+\frac{b}{a}+\sqrt{3}+1}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{2b^2x}{a^2}+\frac{b}{a}+1\right)}{}\right.}{2\sqrt[3]{2}\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)\sqrt[3]{ax+bx^2}}$$

$\downarrow$ 2418

$$3\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\sqrt[3]{-\frac{b\left(ax+bx^2\right)}{a^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{2b^2x}{a^2}+\frac{b}{a}+1\right)\sqrt{\frac{\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2+\sqrt[3]{1-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}+1}{\left(\frac{2b^2x}{a^2}+\frac{b}{a}-\sqrt{3}+1\right)^2}}\text{Ellipti}}{\sqrt[4]{3}\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\sqrt{-\frac{\frac{2b^2x}{a^2}+\frac{b}{a}+1}{\left(\frac{2b^2x}{a^2}+\frac{b}{a}-\sqrt{3}+1\right)^2}}}\right.$$

input
```
Int[(a*x + b*x^2)^(-1/3),x]
```

output
```
(3*Sqrt[-((a^2*(-(b/a) - (2*b^2*x)/a^2)^2)/b^2)]*(-((b*(a*x + b*x^2))/a^2)
)^(1/3)*((-2*Sqrt[-((a^2*(-(b/a) - (2*b^2*x)/a^2)^2)/b^2)])/(1 - Sqrt[3] +
 b/a + (2*b^2*x)/a^2) + (3^(1/4)*Sqrt[2 + Sqrt[3]]*(1 + b/a + (2*b^2*x)/a^
2)*Sqrt[(1 + (-(b/a) - (2*b^2*x)/a^2)^2 + (1 - (a^2*(-(b/a) - (2*b^2*x)/a^
2)^2)/b^2)^(1/3))/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^2)^2]*EllipticE[ArcSin[
(1 + Sqrt[3] + b/a + (2*b^2*x)/a^2)/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^2)],
-7 + 4*Sqrt[3]])/(Sqrt[-((a^2*(-(b/a) - (2*b^2*x)/a^2)^2)/b^2)]*Sqrt[-((1
+ b/a + (2*b^2*x)/a^2)/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^2)^2)]) - (2*Sqrt[
2 - Sqrt[3]]*(1 + Sqrt[3])*(1 + b/a + (2*b^2*x)/a^2)*Sqrt[(1 + (-(b/a) - (
2*b^2*x)/a^2)^2 + (1 - (a^2*(-(b/a) - (2*b^2*x)/a^2)^2)/b^2)^(1/3))/(1 - S
qrt[3] + b/a + (2*b^2*x)/a^2)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + b/a + (2*
b^2*x)/a^2)/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^2)], -7 + 4*Sqrt[3]])/(3^(1/4
)*Sqrt[-((a^2*(-(b/a) - (2*b^2*x)/a^2)^2)/b^2)]*Sqrt[-((1 + b/a + (2*b^2*x
)/a^2)/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^2)^2)])))/(2*2^(1/3)*(-(b/a) - (2*
b^2*x)/a^2)*(a*x + b*x^2)^(1/3))
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^2+ax)^{\frac{1}{3}}}dx$$

input
```
int(1/(b*x^2+a*x)^(1/3),x)
```

output
```
int(1/(b*x^2+a*x)^(1/3),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a*x)^(1/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^(-1/3), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{\sqrt[3]{ax+bx^2}}\,dx$$

input
```
integrate(1/(b*x**2+a*x)**(1/3),x)
```

output
```
Integral((a*x + b*x**2)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a*x)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(-1/3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[3]{ax+bx^2}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a*x)^(1/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^(-1/3), x)
```

Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{1}{\sqrt[3]{ax+bx^2}}\,dx=\frac{3\,x\,{\left(\frac{b\,x}{a}+1\right)}^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{2}{3};\ \frac{5}{3};\ -\frac{b\,x}{a}\right)}{2\,{\left(b\,x^2+a\,x\right)}^{1/3}}$$

input

```
int(1/(a*x + b*x^2)^(1/3),x)
```

output

```
(3*x*((b*x)/a + 1)^(1/3)*hypergeom([1/3, 2/3], 5/3, -(b*x)/a))/(2*(a*x + b
*x^2)^(1/3))
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{ax+bx^2}}\,dx=\int \frac{1}{x^{\frac{1}{3}}\left(bx+a\right)^{\frac{1}{3}}}dx$$

input

```
int(1/(b*x^2+a*x)^(1/3),x)
```

output

```
int(1/(x**(1/3)*(a + b*x)**(1/3)),x)
```

3.55 $\int \frac{1}{\left(ax+bx^2\right)^{2/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{1}{\left(ax+bx^2\right)^{2/3}}\,dx=\frac{3\sqrt[3]{ax+bx^2}\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{4}{3},-\frac{bx}{a}\right)}{a}$$

output
```
3*(b*x^2+a*x)^(1/3)*hypergeom([2/3, 1],[4/3],-b*x/a)/a
```

Mathematica [A] (verified)

Time = 9.91 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.30

$$\int \frac{1}{\left(ax+bx^2\right)^{2/3}}\,dx=\frac{3x\left(1+\frac{bx}{a}\right)^{2/3}\text{Hypergeometric2F1}\left(\frac{1}{3},\frac{2}{3},\frac{4}{3},-\frac{bx}{a}\right)}{(x(a+bx))^{2/3}}$$

input
```
Integrate[(a*x + b*x^2)^(-2/3),x]
```

output
```
(3*x*(1 + (b*x)/a)^(2/3)*Hypergeometric2F1[1/3, 2/3, 4/3, -((b*x)/a)])/(x*
(a + b*x))^(2/3)
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 278 vs. $2(33) = 66$.

Time = 0.49 (sec) , antiderivative size = 278, normalized size of antiderivative = 8.42, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1093, 1090, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx^2)^{2/3}}\,dx$$

$$\Big\downarrow \ 1093$$

$$\frac{\left(-\frac{b(ax+bx^2)}{a^2}\right)^{2/3}\int \frac{1}{\left(-\frac{b^2x^2}{a^2}-\frac{bx}{a}\right)^{2/3}}dx}{(ax+bx^2)^{2/3}}$$

$$\Big\downarrow \ 1090$$

$$-\frac{\sqrt[3]{2}a^2\left(-\frac{b(ax+bx^2)}{a^2}\right)^{2/3}\int \frac{1}{\left(1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}\right)^{2/3}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b^2\,(ax+bx^2)^{2/3}}$$

$$\Big\downarrow \ 234$$

$$\frac{3\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\left(-\frac{b(ax+bx^2)}{a^2}\right)^{2/3}\int \frac{1}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}{2^{2/3}\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)(ax+bx^2)^{2/3}}$$

$$\Big\downarrow \ 760$$

$$-\frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}\left(\frac{2b^2x}{a^2}+\frac{b}{a}+1\right)\left(-\frac{b(ax+bx^2)}{a^2}\right)^{2/3}\sqrt{\frac{\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2+\sqrt[3]{1-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}+1}{\left(\frac{2b^2x}{a^2}+\frac{b}{a}-\sqrt{3}+1\right)^2}}\,\text{EllipticF}\left(\arcsin\right.}{\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)\sqrt{-\frac{\frac{2b^2x}{a^2}+\frac{b}{a}+1}{\left(\frac{2b^2x}{a^2}+\frac{b}{a}-\sqrt{3}+1\right)^2}}\,(ax+bx^2)^{2/3}}$$

input
```
Int[(a*x + b*x^2)^(-2/3),x]
```

output
```
-((2^(1/3)*3^(3/4)*Sqrt[2 - Sqrt[3]]*(1 + b/a + (2*b^2*x)/a^2)*(-((b*(a*x
+ b*x^2))/a^2))^(2/3)*Sqrt[(1 + (-(b/a) - (2*b^2*x)/a^2)^2 + (1 - (a^2*(-(
b/a) - (2*b^2*x)/a^2)^2)/b^2)^(1/3))/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^2)^2
]*EllipticF[ArcSin[(1 + Sqrt[3] + b/a + (2*b^2*x)/a^2)/(1 - Sqrt[3] + b/a
+ (2*b^2*x)/a^2)], -7 + 4*Sqrt[3]])/((-(b/a) - (2*b^2*x)/a^2)*Sqrt[-((1 +
b/a + (2*b^2*x)/a^2)/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^2)^2)]*(a*x + b*x^2)
^(2/3)))
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+ax\right)^{\frac{2}{3}}}dx$$

input
```
int(1/(b*x^2+a*x)^(2/3),x)
```

output
```
int(1/(b*x^2+a*x)^(2/3),x)
```

Fricas [F]

$$\int \frac{1}{(ax+bx^2)^{2/3}}\,dx = \int \frac{1}{\left(bx^{2}+ax\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a*x)^(2/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^(-2/3), x)
```

Sympy [F]

$$\int \frac{1}{(ax+bx^2)^{2/3}}\,dx = \int \frac{1}{\left(ax+bx^{2}\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x**2+a*x)**(2/3),x)
```

output
```
Integral((a*x + b*x**2)**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{(ax+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a*x)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(-2/3), x)
```

Giac [F]

$$\int \frac{1}{(ax+bx^2)^{2/3}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a*x)^(2/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^(-2/3), x)
```

Mupad [B] (verification not implemented)

Time = 9.57 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ax+bx^2)^{2/3}}\,dx = \frac{3\,x\,\left(\frac{b\,x}{a}+1\right)^{2/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{2}{3};\ \frac{4}{3};\ -\frac{b\,x}{a}\right)}{\left(b\,x^2+a\,x\right)^{2/3}}$$

input
```
int(1/(a*x + b*x^2)^(2/3),x)
```

output
```
(3*x*((b*x)/a + 1)^(2/3)*hypergeom([1/3, 2/3], 4/3, -(b*x)/a))/(a*x + b*x^
2)^(2/3)
```

Reduce [F]

$$\int \frac{1}{(ax+bx^2)^{2/3}}\,dx = \int \frac{1}{x^{\frac{2}{3}}\,(bx+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(b*x^2+a*x)^(2/3),x)
```

output
```
int(1/(x**(2/3)*(a + b*x)**(2/3)),x)
```

3.56 $\int \frac{1}{(ax+bx^2)^{4/3}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{1}{(ax+bx^2)^{4/3}}\,dx = -\frac{3\,\text{Hypergeometric2F1}\left(-\frac{2}{3},1,\frac{2}{3},-\frac{bx}{a}\right)}{a\sqrt[3]{ax+bx^2}}$$

output
```
-3*hypergeom([-2/3, 1],[2/3],-b*x/a)/a/(b*x^2+a*x)^(1/3)
```

Mathematica [A] (verified)

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.36

$$\int \frac{1}{(ax+bx^2)^{4/3}}\,dx = -\frac{3\sqrt[3]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{3},\frac{4}{3},\frac{2}{3},-\frac{bx}{a}\right)}{a\sqrt[3]{x(a+bx)}}$$

input
```
Integrate[(a*x + b*x^2)^(-4/3),x]
```

output
```
(-3*(1 + (b*x)/a)^(1/3)*Hypergeometric2F1[-1/3, 4/3, 2/3, -((b*x)/a)])/(a*
(x*(a + b*x))^(1/3))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 684 vs. $2(33) = 66$.

Time = 0.80 (sec) , antiderivative size = 684, normalized size of antiderivative = 20.73, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {1089, 1093, 1090, 233, 833, 760, 2418}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx^2)^{4/3}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{2b\int \frac{1}{\sqrt[3]{bx^2+ax}}dx}{a^2}-\frac{3(a+2bx)}{a^2\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{1093}$$

$$\frac{2b\sqrt[3]{-\frac{b(ax+bx^2)}{a^2}}\int \frac{1}{\sqrt[3]{-\frac{b^2x^2}{a^2}-\frac{bx}{a}}}dx}{a^2\sqrt[3]{ax+bx^2}}-\frac{3(a+2bx)}{a^2\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{2^{2/3}\sqrt[3]{-\frac{b(ax+bx^2)}{a^2}}\int \frac{1}{\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b\sqrt[3]{ax+bx^2}}-\frac{3(a+2bx)}{a^2\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{233}$$

$$\frac{3b\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\sqrt[3]{-\frac{b(ax+bx^2)}{a^2}}\int \frac{\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}{\sqrt[3]{2}a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)\sqrt[3]{ax+bx^2}}-\frac{3(a+2bx)}{a^2\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{833}$$

$$\frac{3b\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\sqrt[3]{-\frac{b(ax+bx^2)}{a^2}}\left((1+\sqrt{3})\int\frac{1}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}-\int\frac{\frac{2xb^2}{a^2}+\frac{b}{a}+\sqrt{3}+1}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b^2}}}\right.}{\sqrt[3]{2}a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)\sqrt[3]{ax+bx^2}}$$

$$\frac{3(a+2bx)}{a^2\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{760}$$

$$\frac{3b\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\sqrt[3]{-\frac{b(ax+bx^2)}{a^2}}\left(-\int\frac{\frac{2xb^2}{a^2}+\frac{b}{a}+\sqrt{3}+1}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{2b^2x}{a^2}+\frac{b}{a}+\right.}{}\right.}{\sqrt[3]{2}a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)\sqrt[3]{ax+bx}}$$

$$\frac{3(a+2bx)}{a^2\sqrt[3]{ax+bx^2}}$$

$$\Big\downarrow \text{2418}$$

$$\frac{3b\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\sqrt[3]{-\frac{b(ax+bx^2)}{a^2}}\left(-\frac{2\sqrt{2-\sqrt{3}}\left(1+\sqrt{3}\right)\left(\frac{2b^2x}{a^2}+\frac{b}{a}+1\right)\sqrt{\frac{\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2+\sqrt[3]{1-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}+1}{\left(\frac{2b^2x}{a^2}+\frac{b}{a}-\sqrt{3}+1\right)^2}}\text{Ellipt}}{\sqrt[4]{3}\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\sqrt{-\frac{\frac{2b^2x}{a^2}+\frac{b}{a}+1}{\left(\frac{2b^2x}{a^2}+\frac{b}{a}-\sqrt{3}+1\right)^2}}}\right.}{}$$

$$\frac{3(a+2bx)}{a^2\sqrt[3]{ax+bx^2}}$$

input
```
Int[(a*x + b*x^2)^(-4/3),x]
```

output
```
(-3*(a + 2*b*x))/(a^2*(a*x + b*x^2)^(1/3)) + (3*b*Sqrt[-((a^2*(-(b/a) - (2
*b^2*x)/a^2)^2)/b^2)]*(-((b*(a*x + b*x^2))/a^2))^(1/3)*((-2*Sqrt[-((a^2*(-
(b/a) - (2*b^2*x)/a^2)^2)/b^2)])/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^2) + (3^
(1/4)*Sqrt[2 + Sqrt[3]]*(1 + b/a + (2*b^2*x)/a^2)*Sqrt[(1 + (-(b/a) - (2*b
^2*x)/a^2)^2 + (1 - (a^2*(-(b/a) - (2*b^2*x)/a^2)^2)/b^2)^(1/3))/(1 - Sqrt
[3] + b/a + (2*b^2*x)/a^2)^2]*EllipticE[ArcSin[(1 + Sqrt[3] + b/a + (2*b^2
*x)/a^2)/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^2)], -7 + 4*Sqrt[3]])/(Sqrt[-((a
^2*(-(b/a) - (2*b^2*x)/a^2)^2)/b^2)]*Sqrt[-((1 + b/a + (2*b^2*x)/a^2)/(1 -
 Sqrt[3] + b/a + (2*b^2*x)/a^2)^2)]) - (2*Sqrt[2 - Sqrt[3]]*(1 + Sqrt[3])*
(1 + b/a + (2*b^2*x)/a^2)*Sqrt[(1 + (-(b/a) - (2*b^2*x)/a^2)^2 + (1 - (a^2
*(-(b/a) - (2*b^2*x)/a^2)^2)/b^2)^(1/3))/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^
2)^2]*EllipticF[ArcSin[(1 + Sqrt[3] + b/a + (2*b^2*x)/a^2)/(1 - Sqrt[3] +
b/a + (2*b^2*x)/a^2)], -7 + 4*Sqrt[3]])/(3^(1/4)*Sqrt[-((a^2*(-(b/a) - (2*
b^2*x)/a^2)^2)/b^2)]*Sqrt[-((1 + b/a + (2*b^2*x)/a^2)/(1 - Sqrt[3] + b/a +
 (2*b^2*x)/a^2)^2)])))/(2^(1/3)*a^2*(-(b/a) - (2*b^2*x)/a^2)*(a*x + b*x^2)
^(1/3))
```

Defintions of rubi rules used

rule 233
```
Int[((a_) + (b_.)*(x_)^2)^(-1/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[x/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 833
```
Int[(x_)/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]
], s = Denom[Rt[b/a, 3]]}, Simp[(-(1 + Sqrt[3]))*(s/r)   Int[1/Sqrt[a + b*x
^3], x], x] + Simp[1/r   Int[((1 + Sqrt[3])*s + r*x)/Sqrt[a + b*x^3], x], x
]] /; FreeQ[{a, b}, x] && NegQ[a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 2418
```
Int[((c_) + (d_.)*(x_))/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = N
umer[Simplify[(1 + Sqrt[3])*(d/c)]], s = Denom[Simplify[(1 + Sqrt[3])*(d/c)
]]}, Simp[2*d*s^3*(Sqrt[a + b*x^3]/(a*r^2*((1 - Sqrt[3])*s + r*x))), x] + S
imp[3^(1/4)*Sqrt[2 + Sqrt[3]]*d*s*(s + r*x)*(Sqrt[(s^2 - r*s*x + r^2*x^2)/(
(1 - Sqrt[3])*s + r*x)^2]/(r^2*Sqrt[a + b*x^3]*Sqrt[(-s)*((s + r*x)/((1 - S
qrt[3])*s + r*x)^2)]))*EllipticE[ArcSin[((1 + Sqrt[3])*s + r*x)/((1 - Sqrt[
3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b, c, d}, x] && NegQ[a] &&
 EqQ[b*c^3 - 2*(5 + 3*Sqrt[3])*a*d^3, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{2}+ax\right)^{\frac{4}{3}}}dx$$

input
```
int(1/(b*x^2+a*x)^(4/3),x)
```

output
```
int(1/(b*x^2+a*x)^(4/3),x)
```

Fricas [F]

$$\int \frac{1}{(ax+bx^2)^{4/3}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a*x)^(4/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^(2/3)/(b^2*x^4 + 2*a*b*x^3 + a^2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(ax+bx^2)^{4/3}}\,dx = \int \frac{1}{(ax+bx^2)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*x**2+a*x)**(4/3),x)
```

output
```
Integral((a*x + b*x**2)**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(ax+bx^2)^{4/3}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a*x)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(-4/3), x)
```

Giac [F]

$$\int \frac{1}{(ax+bx^2)^{4/3}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a*x)^(4/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^(-4/3), x)
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ax+bx^2)^{4/3}}\,dx = -\frac{3\,x\,\left(\frac{b\,x}{a}+1\right)^{4/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{4}{3};\ \frac{2}{3};\ -\frac{b\,x}{a}\right)}{(b\,x^2+a\,x)^{4/3}}$$

input
```
int(1/(a*x + b*x^2)^(4/3),x)
```

output
```
-(3*x*((b*x)/a + 1)^(4/3)*hypergeom([-1/3, 4/3], 2/3, -(b*x)/a))/(a*x + b*
x^2)^(4/3)
```

Reduce [F]

$$\int \frac{1}{(ax+bx^2)^{4/3}}\,dx = \int \frac{1}{x^{\frac{4}{3}}\,(bx+a)^{\frac{1}{3}}\,a+x^{\frac{7}{3}}\,(bx+a)^{\frac{1}{3}}\,b}dx$$

input
```
int(1/(b*x^2+a*x)^(4/3),x)
```

output
```
int(1/(x**(1/3)*(a + b*x)**(1/3)*a*x + x**(1/3)*(a + b*x)**(1/3)*b*x**2),x
)
```

3.57 $\int \frac{1}{\left(ax+bx^2\right)^{5/3}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{1}{\left(ax+bx^2\right)^{5/3}} \, dx = -\frac{3 \,\text{Hypergeometric2F1}\left(-\frac{4}{3},1,\frac{1}{3},-\frac{bx}{a}\right)}{2a\left(ax+bx^2\right)^{2/3}}$$

output
```
-3/2*hypergeom([-4/3, 1],[1/3],-b*x/a)/a/(b*x^2+a*x)^(2/3)
```

Mathematica [A] (verified)

Time = 10.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.34

$$\int \frac{1}{\left(ax+bx^2\right)^{5/3}} \, dx = -\frac{3\left(1+\frac{bx}{a}\right)^{2/3} \text{Hypergeometric2F1}\left(-\frac{2}{3},\frac{5}{3},\frac{1}{3},-\frac{bx}{a}\right)}{2a(x(a+bx))^{2/3}}$$

input
```
Integrate[(a*x + b*x^2)^(-5/3),x]
```

output
```
(-3*(1 + (b*x)/a)^(2/3)*Hypergeometric2F1[-2/3, 5/3, 1/3, -((b*x)/a)])/(2*
a*(x*(a + b*x))^(2/3))
```

Rubi [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 308 vs. $2(35) = 70$.

Time = 0.50 (sec) , antiderivative size = 308, normalized size of antiderivative = 8.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1089, 1093, 1090, 234, 760}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+bx^2)^{5/3}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{b\int \frac{1}{(bx^2+ax)^{2/3}}dx}{a^2}-\frac{3(a+2bx)}{2a^2(ax+bx^2)^{2/3}}$$

$$\Big\downarrow \text{1093}$$

$$-\frac{b\left(-\frac{b(ax+bx^2)}{a^2}\right)^{2/3}\int \frac{1}{\left(-\frac{b^2x^2}{a^2}-\frac{bx}{a}\right)^{2/3}}dx}{a^2(ax+bx^2)^{2/3}}-\frac{3(a+2bx)}{2a^2(ax+bx^2)^{2/3}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{\sqrt[3]{2}\left(-\frac{b(ax+bx^2)}{a^2}\right)^{2/3}\int \frac{1}{\left(1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}\right)^{2/3}}d\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)}{b(ax+bx^2)^{2/3}}-\frac{3(a+2bx)}{2a^2(ax+bx^2)^{2/3}}$$

$$\Big\downarrow \text{234}$$

$$-\frac{3b\sqrt{-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}\left(-\frac{b(ax+bx^2)}{a^2}\right)^{2/3}\int \frac{1}{\sqrt{-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}d\sqrt[3]{1-\frac{a^2\left(-\frac{2xb^2}{a^2}-\frac{b}{a}\right)^2}{b^2}}}{2^{2/3}a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)(ax+bx^2)^{2/3}}-\frac{3(a+2bx)}{2a^2(ax+bx^2)^{2/3}}$$

$$\Big\downarrow \text{760}$$

$$\frac{\sqrt[3]{2}3^{3/4}\sqrt{2-\sqrt{3}}b\left(\frac{2b^2x}{a^2}+\frac{b}{a}+1\right)\left(-\frac{b(ax+bx^2)}{a^2}\right)^{2/3}\sqrt{\frac{\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2+\sqrt[3]{1-\frac{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)^2}{b^2}}+1}{\left(\frac{2b^2x}{a^2}+\frac{b}{a}-\sqrt{3}+1\right)^2}}\text{EllipticF}\left(\arcsin\left(\right.\right.}{a^2\left(-\frac{2b^2x}{a^2}-\frac{b}{a}\right)\sqrt{-\frac{\frac{2b^2x}{a^2}+\frac{b}{a}+1}{\left(\frac{2b^2x}{a^2}+\frac{b}{a}-\sqrt{3}+1\right)^2}}(ax+bx^2)^{2/3}}$$
$$\frac{3(a+2bx)}{2a^2\left(ax+bx^2\right)^{2/3}}$$

input
```
Int[(a*x + b*x^2)^(-5/3),x]
```

output
```
(-3*(a + 2*b*x))/(2*a^2*(a*x + b*x^2)^(2/3)) + (2^(1/3)*3^(3/4)*Sqrt[2 - S
qrt[3]]*b*(1 + b/a + (2*b^2*x)/a^2)*(-((b*(a*x + b*x^2))/a^2))^(2/3)*Sqrt[
(1 + (-(b/a) - (2*b^2*x)/a^2)^2 + (1 - (a^2*(-(b/a) - (2*b^2*x)/a^2)^2)/b^
2)^(1/3))/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^2)^2]*EllipticF[ArcSin[(1 + Sqr
t[3] + b/a + (2*b^2*x)/a^2)/(1 - Sqrt[3] + b/a + (2*b^2*x)/a^2)], -7 + 4*S
qrt[3]])/(a^2*(-(b/a) - (2*b^2*x)/a^2)*Sqrt[-((1 + b/a + (2*b^2*x)/a^2)/(1
 - Sqrt[3] + b/a + (2*b^2*x)/a^2)^2)]*(a*x + b*x^2)^(2/3))
```

Defintions of rubi rules used

rule 234
```
Int[((a_) + (b_.)*(x_)^2)^(-2/3), x_Symbol] :> Simp[3*(Sqrt[b*x^2]/(2*b*x))
   Subst[Int[1/Sqrt[-a + x^3], x], x, (a + b*x^2)^(1/3)], x] /; FreeQ[{a, b
}, x]
```

rule 760
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^3], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[2*Sqrt[2 - Sqrt[3]]*(s + r*x)*(Sqrt[(s^2 - r*s
*x + r^2*x^2)/((1 - Sqrt[3])*s + r*x)^2]/(3^(1/4)*r*Sqrt[a + b*x^3]*Sqrt[(-
s)*((s + r*x)/((1 - Sqrt[3])*s + r*x)^2)]))*EllipticF[ArcSin[((1 + Sqrt[3])
*s + r*x)/((1 - Sqrt[3])*s + r*x)], -7 + 4*Sqrt[3]], x]] /; FreeQ[{a, b}, x
] && NegQ[a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{(b\,x^2+ax)^{\frac{5}{3}}}dx$$

input
```
int(1/(b*x^2+a*x)^(5/3),x)
```

output
```
int(1/(b*x^2+a*x)^(5/3),x)
```

Fricas [F]

$$\int \frac{1}{(ax+bx^2)^{5/3}}\,dx=\int \frac{1}{(bx^2+ax)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a*x)^(5/3),x, algorithm="fricas")
```

output
```
integral((b*x^2 + a*x)^(1/3)/(b^2*x^4 + 2*a*b*x^3 + a^2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(ax+bx^2)^{5/3}}\,dx = \int \frac{1}{(ax+bx^2)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(b*x**2+a*x)**(5/3),x)
```

output
```
Integral((a*x + b*x**2)**(-5/3), x)
```

Maxima [F]

$$\int \frac{1}{(ax+bx^2)^{5/3}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a*x)^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*x^2 + a*x)^(-5/3), x)
```

Giac [F]

$$\int \frac{1}{(ax+bx^2)^{5/3}}\,dx = \int \frac{1}{(bx^2+ax)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(b*x^2+a*x)^(5/3),x, algorithm="giac")
```

output
```
integrate((b*x^2 + a*x)^(-5/3), x)
```

Mupad [B] (verification not implemented)

Time = 9.72 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{1}{(ax+bx^2)^{5/3}}\,dx = -\frac{3\,x\,\left(\frac{b\,x}{a}+1\right)^{5/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{5}{3};\ \frac{1}{3};\ -\frac{b\,x}{a}\right)}{2\,\left(b\,x^2+a\,x\right)^{5/3}}$$

input
```
int(1/(a*x + b*x^2)^(5/3),x)
```

output
```
-(3*x*((b*x)/a + 1)^(5/3)*hypergeom([-2/3, 5/3], 1/3, -(b*x)/a))/(2*(a*x +
 b*x^2)^(5/3))
```

Reduce [F]

$$\int \frac{1}{(ax+bx^2)^{5/3}}\,dx = \int \frac{1}{x^{\frac{5}{3}}\,(bx+a)^{\frac{2}{3}}\,a+x^{\frac{8}{3}}\,(bx+a)^{\frac{2}{3}}\,b}dx$$

input
```
int(1/(b*x^2+a*x)^(5/3),x)
```

output
```
int(1/(x**(2/3)*(a + b*x)**(2/3)*a*x + x**(2/3)*(a + b*x)**(2/3)*b*x**2),x
)
```

3.58 $\int \left(bx+cx^2\right)^{5/4} dx$

Optimal result

Integrand size = 13, antiderivative size = 160

$$\int \left(bx+cx^2\right)^{5/4} dx = -\frac{5b^3\sqrt[4]{bx+cx^2}}{84c^2} + \frac{b^2x\sqrt[4]{bx+cx^2}}{42c} + \frac{1}{7}bx^2\sqrt[4]{bx+cx^2} + \frac{2}{7}x\left(bx+cx^2\right)^{5/4} - \frac{5b^{7/2}\left(\frac{cx}{b+cx}\right)^{3/4}\sqrt{b+cx}\sqrt[4]{bx+cx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}}{\sqrt{b+cx}}\right),2\right)}{84c^3x}$$

output
```
-5/84*b^3*(c*x^2+b*x)^(1/4)/c^2+1/42*b^2*x*(c*x^2+b*x)^(1/4)/c+1/7*b*x^2*(
c*x^2+b*x)^(1/4)+2/7*x*(c*x^2+b*x)^(5/4)-5/84*b^(7/2)*(c*x/(c*x+b))^(3/4)*
(c*x+b)^(1/2)*(c*x^2+b*x)^(1/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)/(c*x+b)
^(1/2)),2^(1/2))/c^3/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.30

$$\int \left(bx+cx^2\right)^{5/4}\,dx=\frac{4bx^2\sqrt[4]{x(b+cx)}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4},\frac{13}{4},-\frac{cx}{b}\right)}{9\sqrt[4]{1+\frac{cx}{b}}}$$

input
```
Integrate[(b*x + c*x^2)^(5/4),x]
```

output
```
(4*b*x^2*(x*(b + c*x))^(1/4)*Hypergeometric2F1[-5/4, 9/4, 13/4, -((c*x)/b)
])/(9*(1 + (c*x)/b)^(1/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {1087, 1087, 1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{5/4}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}-\frac{5b^2\int \sqrt[4]{cx^2+bx}dx}{28c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}-\frac{5b^2\left(\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{b^2\int \frac{1}{\left(cx^2+bx\right)^{3/4}}dx}{12c}\right)}{28c}$$

$$\Big\downarrow \text{1093}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}-\frac{5b^2\left(\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{b^2\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4}\int \frac{1}{\left(-\frac{c^2x^2}{b^2}-\frac{cx}{b}\right)^{3/4}}dx}{12c(bx+cx^2)^{3/4}}\right)}{28c}$$

$$\Big\downarrow \; 1090$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}-\frac{5b^2\left(\frac{b^4\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}\right)^{3/4}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{6\sqrt{2}c^3(bx+cx^2)^{3/4}}+\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}\right)}{28c}$$

$$\Big\downarrow \; 230$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/4}}{7c}-\frac{5b^2\left(\frac{b^3\left(-\frac{c\left(bx+cx^2\right)}{b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right),2\right)}{3\sqrt{2}c^2(bx+cx^2)^{3/4}}+\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}\right)}{28c}$$

input
```
Int[(b*x + c*x^2)^(5/4),x]
```

output
```
((b + 2*c*x)*(b*x + c*x^2)^(5/4))/(7*c) - (5*b^2*(((b + 2*c*x)*(b*x + c*x^
2)^(1/4))/(3*c) + (b^3*(-((c*(b*x + c*x^2))/b^2))^(3/4)*EllipticF[ArcSin[(
b*(-(c/b) - (2*c^2*x)/b^2))/c]/2, 2])/(3*Sqrt[2]*c^2*(b*x + c*x^2)^(3/4)))
)/(28*c)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \left(c\,x^2+bx\right)^{\frac{5}{4}}dx$$

input
```
int((c*x^2+b*x)^(5/4),x)
```

output
```
int((c*x^2+b*x)^(5/4),x)
```

Fricas [F]

$$\int \left(bx+cx^2\right)^{5/4}\,dx=\int \left(cx^2+bx\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(5/4), x)
```

Sympy [F]

$$\int \left(bx+cx^2\right)^{5/4}\,dx=\int \left(bx+cx^2\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((c*x**2+b*x)**(5/4),x)
```

output
```
Integral((b*x + c*x**2)**(5/4), x)
```

Maxima [F]

$$\int \left(bx+cx^2\right)^{5/4}\,dx=\int \left(cx^2+bx\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(5/4), x)
```

Giac [F]

$$\int \left(bx+cx^2\right)^{5/4}\,dx=\int \left(cx^2+bx\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(5/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.22

$$\int \left(bx+cx^2\right)^{5/4}\,dx=\frac{4\,x\,{\left(c\,x^2+b\,x\right)}^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{9}{4};\ \frac{13}{4};\ -\frac{c\,x}{b}\right)}{9\,{\left(\frac{c\,x}{b}+1\right)}^{5/4}}$$

input
```
int((b*x + c*x^2)^(5/4),x)
```

output
```
(4*x*(b*x + c*x^2)^(5/4)*hypergeom([-5/4, 9/4], 13/4, -(c*x)/b))/(9*((c*x)
/b + 1)^(5/4))
```

Reduce [F]

$$\int (bx+cx^2)^{5/4}\,dx=\frac{-20x^{\frac{1}{4}}(cx+b)^{\frac{1}{4}}b^3+8x^{\frac{5}{4}}(cx+b)^{\frac{1}{4}}b^2c+144x^{\frac{9}{4}}(cx+b)^{\frac{1}{4}}bc^2+96x^{\frac{13}{4}}(cx+b)^{\frac{1}{4}}c^3+5\left(\int \frac{(c}{x^{\frac{3}{4}}}\right.}{336c^2}$$

input
```
int((c*x^2+b*x)^(5/4),x)
```

output
```
( - 20*x**(1/4)*(b + c*x)**(1/4)*b**3 + 8*x**(1/4)*(b + c*x)**(1/4)*b**2*c
*x + 144*x**(1/4)*(b + c*x)**(1/4)*b*c**2*x**2 + 96*x**(1/4)*(b + c*x)**(1
/4)*c**3*x**3 + 5*int((b + c*x)**(1/4)/(x**(3/4)*b + x**(3/4)*c*x),x)*b**4
)/(336*c**2)
```

3.59 $\int (bx+cx^2)^{3/4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 107

$$\int (bx+cx^2)^{3/4}\,dx=\frac{b(bx+cx^2)^{3/4}}{5c}+\frac{2}{5}x(bx+cx^2)^{3/4}-\frac{3b^3\sqrt[4]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}E\left(\frac{1}{2}\arcsin\left(1+\frac{2cx}{b}\right)\middle|2\right)}{10\sqrt{2}c^2\sqrt[4]{bx+cx^2}}$$

output
```
1/5*b*(c*x^2+b*x)^(3/4)/c+2/5*x*(c*x^2+b*x)^(3/4)-3/20*b^3*(-c*x/b-c^2*x^2
/b^2)^(1/4)*EllipticE(sin(1/2*arcsin(1+2*c*x/b)),2^(1/2))*2^(1/2)/c^2/(c*x
^2+b*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.42

$$\int (bx+cx^2)^{3/4}\,dx=\frac{4x(x(b+cx))^{3/4}\,\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{7}{4},\frac{11}{4},-\frac{cx}{b}\right)}{7\left(1+\frac{cx}{b}\right)^{3/4}}$$

input
```
Integrate[(b*x + c*x^2)^(3/4),x]
```

output
```
(4*x*(x*(b + c*x))^(3/4)*Hypergeometric2F1[-3/4, 7/4, 11/4, -((c*x)/b)])/(
7*(1 + (c*x)/b)^(3/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1087, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{3/4}dx$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}-\frac{3b^2\int \frac{1}{\sqrt[4]{cx^2+bx}}dx}{20c}$$

$$\Big\downarrow \text{ 1093}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}-\frac{3b^2\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}\int \frac{1}{\sqrt[4]{-\frac{c^2x^2}{b^2}-\frac{cx}{b}}}dx}{20c\sqrt[4]{bx+cx^2}}$$

$$\Big\downarrow \text{ 1090}$$

$$\frac{3b^4\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}\int \frac{1}{\sqrt[4]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{20\sqrt{2}c^3\sqrt[4]{bx+cx^2}}+\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}$$

$$\Big\downarrow \text{ 226}$$

$$\frac{3b^3\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}E\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right)\middle|2\right)}{10\sqrt{2}c^2\sqrt[4]{bx+cx^2}}+\frac{(b+2cx)\left(bx+cx^2\right)^{3/4}}{5c}$$

input
```
Int[(b*x + c*x^2)^(3/4),x]
```

output
```
((b + 2*c*x)*(b*x + c*x^2)^(3/4))/(5*c) + (3*b^3*(-((c*(b*x + c*x^2))/b^2)
)^(1/4)*EllipticE[ArcSin[(b*(-(c/b) - (2*c^2*x)/b^2))/c]/2, 2])/(10*Sqrt[2
]*c^2*(b*x + c*x^2)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \left(c\,x^{2}+b x \right)^{\frac{3}{4}}d x$$

input
```
int((c*x^2+b*x)^(3/4),x)
```

output
```
int((c*x^2+b*x)^(3/4),x)
```

Fricas [F]

$$\int \left(bx+cx^2\right)^{3/4}\,dx=\int \left(cx^2+bx\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(3/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(3/4), x)
```

Sympy [F]

$$\int \left(bx+cx^2\right)^{3/4}\,dx=\int \left(bx+cx^2\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((c*x**2+b*x)**(3/4),x)
```

output
```
Integral((b*x + c*x**2)**(3/4), x)
```

Maxima [F]

$$\int \left(bx+cx^2\right)^{3/4}\,dx=\int \left(cx^2+bx\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(3/4), x)
```

Giac [F]

$$\int \left(bx + cx^2\right)^{3/4} dx = \int \left(cx^2 + bx\right)^{\frac{3}{4}} dx$$

input
```
integrate((c*x^2+b*x)^(3/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(3/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.34

$$\int \left(bx + cx^2\right)^{3/4} dx = \frac{4\,x\,\left(c\,x^2 + b\,x\right)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{4}, \frac{7}{4};\ \frac{11}{4};\ -\frac{c\,x}{b}\right)}{7\,\left(\frac{c\,x}{b} + 1\right)^{3/4}}$$

input
```
int((b*x + c*x^2)^(3/4),x)
```

output
```
(4*x*(b*x + c*x^2)^(3/4)*hypergeom([-3/4, 7/4], 11/4, -(c*x)/b))/(7*((c*x)
/b + 1)^(3/4))
```

Reduce [F]

$$\int \left(bx + cx^2\right)^{3/4} dx = \frac{4x^{\frac{3}{4}}(cx+b)^{\frac{3}{4}} b + 8x^{\frac{7}{4}}(cx+b)^{\frac{3}{4}} c - 3\left(\int \frac{(cx+b)^{\frac{3}{4}}}{x^{\frac{1}{4}} b + x^{\frac{5}{4}} c} dx\right) b^2}{20c}$$

input
```
int((c*x^2+b*x)^(3/4),x)
```

output
```
(4*x**(3/4)*(b + c*x)**(3/4)*b + 8*x**(3/4)*(b + c*x)**(3/4)*c*x - 3*int((
b + c*x)**(3/4)/(x**(1/4)*b + x**(1/4)*c*x),x)*b**2)/(20*c)
```

3.60 $\int \sqrt[4]{bx+cx^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 113

$$\int \sqrt[4]{bx+cx^2}\,dx = \frac{b\sqrt[4]{bx+cx^2}}{3c}+\frac{2}{3}x\sqrt[4]{bx+cx^2}+\frac{b^{3/2}\left(\frac{cx}{b+cx}\right)^{3/4}\sqrt{b+cx}\sqrt[4]{bx+cx^2}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}}{\sqrt{b+cx}}\right),2\right)}{3c^2x}$$

output
```
1/3*b*(c*x^2+b*x)^(1/4)/c+2/3*x*(c*x^2+b*x)^(1/4)+1/3*b^(3/2)*(c*x/(c*x+b)
)^(3/4)*(c*x+b)^(1/2)*(c*x^2+b*x)^(1/4)*InverseJacobiAM(1/2*arcsin(b^(1/2)
/(c*x+b)^(1/2)),2^(1/2))/c^2/x
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.40

$$\int \sqrt[4]{bx+cx^2}\,dx = \frac{4x\sqrt[4]{x(b+cx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{9}{4},-\frac{cx}{b}\right)}{5\sqrt[4]{1+\frac{cx}{b}}}$$

input
```
Integrate[(b*x + c*x^2)^(1/4),x]
```

output
```
(4*x*(x*(b + c*x))^(1/4)*Hypergeometric2F1[-1/4, 5/4, 9/4, -((c*x)/b)])/(5
*(1 + (c*x)/b)^(1/4))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1087, 1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{bx+cx^2}\,dx$$

$$\Big\downarrow \; 1087$$

$$\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{b^2\int \frac{1}{(cx^2+bx)^{3/4}}dx}{12c}$$

$$\Big\downarrow \; 1093$$

$$\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}-\frac{b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\int \frac{1}{\left(-\frac{c^2x^2}{b^2}-\frac{cx}{b}\right)^{3/4}}dx}{12c\,(bx+cx^2)^{3/4}}$$

$$\Big\downarrow \; 1090$$

$$\frac{b^4\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}\right)^{3/4}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{6\sqrt{2}c^3\,(bx+cx^2)^{3/4}}+\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}$$

$$\Big\downarrow \; 230$$

$$\frac{b^3\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right),2\right)}{3\sqrt{2}c^2\,(bx+cx^2)^{3/4}}+\frac{(b+2cx)\sqrt[4]{bx+cx^2}}{3c}$$

input
```
Int[(b*x + c*x^2)^(1/4),x]
```

output
```
((b + 2*c*x)*(b*x + c*x^2)^(1/4))/(3*c) + (b^3*(-((c*(b*x + c*x^2))/b^2))^
(3/4)*EllipticF[ArcSin[(b*(-(c/b) - (2*c^2*x)/b^2))/c]/2, 2])/(3*Sqrt[2]*c
^2*(b*x + c*x^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \left(c\,x^{2}+bx\right)^{\frac{1}{4}}dx$$

input
```
int((c*x^2+b*x)^(1/4),x)
```

output
```
int((c*x^2+b*x)^(1/4),x)
```

Fricas [F]

$$\int \sqrt[4]{bx+cx^2}\,dx = \int (cx^2+bx)^{\frac{1}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(1/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(1/4), x)
```

Sympy [F]

$$\int \sqrt[4]{bx+cx^2}\,dx = \int \sqrt[4]{bx+cx^2}\,dx$$

input
```
integrate((c*x**2+b*x)**(1/4),x)
```

output
```
Integral((b*x + c*x**2)**(1/4), x)
```

Maxima [F]

$$\int \sqrt[4]{bx+cx^2}\,dx = \int (cx^2+bx)^{\frac{1}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{bx+cx^2}\,dx = \int \left(cx^2+bx\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((c*x^2+b*x)^(1/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(1/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.65 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.32

$$\int \sqrt[4]{bx+cx^2}\,dx = \frac{4\,x\,\left(c\,x^2+b\,x\right)^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{5}{4};\ \frac{9}{4};\ -\frac{c\,x}{b}\right)}{5\,\left(\frac{c\,x}{b}+1\right)^{1/4}}$$

input
```
int((b*x + c*x^2)^(1/4),x)
```

output
```
(4*x*(b*x + c*x^2)^(1/4)*hypergeom([-1/4, 5/4], 9/4, -(c*x)/b))/(5*((c*x)/
b + 1)^(1/4))
```

Reduce [F]

$$\int \sqrt[4]{bx+cx^2}\,dx = \frac{4x^{\frac{1}{4}}(cx+b)^{\frac{1}{4}}b+8x^{\frac{5}{4}}(cx+b)^{\frac{1}{4}}c-\left(\int \frac{(cx+b)^{\frac{1}{4}}}{x^{\frac{3}{4}}b+x^{\frac{7}{4}}c}dx\right)b^2}{12c}$$

input
```
int((c*x^2+b*x)^(1/4),x)
```

output
```
(4*x**(1/4)*(b + c*x)**(1/4)*b + 8*x**(1/4)*(b + c*x)**(1/4)*c*x - int((b
+ c*x)**(1/4)/(x**(3/4)*b + x**(3/4)*c*x),x)*b**2)/(12*c)
```

3.61 $\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx=\frac{\sqrt{2}b\sqrt[4]{-\frac{cx}{b}-\frac{c^2x^2}{b^2}}E\left(\frac{1}{2}\arcsin\left(1+\frac{2cx}{b}\right)\middle|2\right)}{c\sqrt[4]{bx+cx^2}}$$

output
```
2^(1/2)*b*(-c*x/b-c^2*x^2/b^2)^(1/4)*EllipticE(sin(1/2*arcsin(1+2*c*x/b)),
2^(1/2))/c/(c*x^2+b*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx=\frac{4x\sqrt[4]{1+\frac{cx}{b}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{cx}{b}\right)}{3\sqrt[4]{x(b+cx)}}$$

input
```
Integrate[(b*x + c*x^2)^(-1/4),x]
```

output
```
(4*x*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, -((c*x)/b)])/(3*
(x*(b + c*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}dx$$

$$\Big\downarrow \text{1093}$$

$$\frac{\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{-\frac{c^2x^2}{b^2}-\frac{cx}{b}}}dx}{\sqrt[4]{bx+cx^2}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{b^2\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{\sqrt{2}c^2\sqrt[4]{bx+cx^2}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{\sqrt{2}b\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right)\middle|2\right)}{c\sqrt[4]{bx+cx^2}}$$

input
```
Int[(b*x + c*x^2)^(-1/4),x]
```

output
```
-((Sqrt[2]*b*(-((c*(b*x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[(b*(-(c/b)
- (2*c^2*x)/b^2))/c]/2, 2])/(c*(b*x + c*x^2)^(1/4)))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{(c\,x^2+bx)^{\frac{1}{4}}}dx$$

input
```
int(1/(c*x^2+b*x)^(1/4),x)
```

output
```
int(1/(c*x^2+b*x)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(1/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(-1/4), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx = \int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(1/4),x)
```

output
```
Integral((b*x + c*x**2)**(-1/4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(1/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.62 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.58

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx = \frac{4\,x\,\left(\frac{c\,x}{b}+1\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{c\,x}{b}\right)}{3\,\left(c\,x^2+b\,x\right)^{1/4}}$$

input
```
int(1/(b*x + c*x^2)^(1/4),x)
```

output
```
(4*x*((c*x)/b + 1)^(1/4)*hypergeom([1/4, 3/4], 7/4, -(c*x)/b))/(3*(b*x + c
*x^2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{bx+cx^2}}\,dx = \int \frac{1}{x^{\frac{1}{4}}\,(cx+b)^{\frac{1}{4}}}\,dx$$

input
```
int(1/(c*x^2+b*x)^(1/4),x)
```

output
```
int(1/(x**(1/4)*(b + c*x)**(1/4)),x)
```

3.62 $\int \frac{1}{(bx+cx^2)^{3/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 68

$$\int \frac{1}{(bx+cx^2)^{3/4}}\,dx = -\frac{4\left(\frac{cx}{b+cx}\right)^{3/4}(b+cx)^{3/2}\,\mathrm{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}}{\sqrt{b+cx}}\right),2\right)}{\sqrt{b}c\,(bx+cx^2)^{3/4}}$$

output
```
-4*(c*x/(c*x+b))^(3/4)*(c*x+b)^(3/2)*InverseJacobiAM(1/2*arcsin(b^(1/2)/(c
*x+b)^(1/2)),2^(1/2))/b^(1/2)/c/(c*x^2+b*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.63

$$\int \frac{1}{(bx+cx^2)^{3/4}}\,dx = \frac{4x\left(1+\frac{cx}{b}\right)^{3/4}\mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{cx}{b}\right)}{(x(b+cx))^{3/4}}$$

input
```
Integrate[(b*x + c*x^2)^(-3/4),x]
```

output
```
(4*x*(1 + (c*x)/b)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, -((c*x)/b)])/(x*
(b + c*x))^(3/4)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{3/4}}dx$$

$$\Big\downarrow \text{1093}$$

$$\frac{\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\int \frac{1}{\left(-\frac{c^2x^2}{b^2}-\frac{cx}{b}\right)^{3/4}}dx}{(bx+cx^2)^{3/4}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{\sqrt{2}b^2\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}\right)^{3/4}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c^2\,(bx+cx^2)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$-\frac{2\sqrt{2}b\left(-\frac{c(bx+cx^2)}{b^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right),2\right)}{c\,(bx+cx^2)^{3/4}}$$

input
```
Int[(b*x + c*x^2)^(-3/4),x]
```

output
```
(-2*Sqrt[2]*b*(-((c*(b*x + c*x^2))/b^2))^(3/4)*EllipticF[ArcSin[(b*(-(c/b)
 - (2*c^2*x)/b^2))/c]/2, 2])/(c*(b*x + c*x^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{(c\,x^2+bx)^{\frac{3}{4}}}dx$$

input
```
int(1/(c*x^2+b*x)^(3/4),x)
```

output
```
int(1/(c*x^2+b*x)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(bx+cx^2)^{3/4}}\,dx=\int \frac{1}{(cx^2+bx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(3/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(-3/4), x)
```

Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{3/4}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(3/4),x)
```

output
```
Integral((b*x + c*x**2)**(-3/4), x)
```

Maxima [F]

$$\int \frac{1}{(bx+cx^2)^{3/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(bx+cx^2)^{3/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(3/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.53

$$\int \frac{1}{(bx+cx^2)^{3/4}}\,dx = \frac{4\,x\,\left(\frac{c\,x}{b}+1\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{5}{4};\ -\frac{c\,x}{b}\right)}{\left(c\,x^2+b\,x\right)^{3/4}}$$

input
```
int(1/(b*x + c*x^2)^(3/4),x)
```

output
```
(4*x*((c*x)/b + 1)^(3/4)*hypergeom([1/4, 3/4], 5/4, -(c*x)/b))/(b*x + c*x^
2)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(bx+cx^2)^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}\,(cx+b)^{\frac{3}{4}}}dx$$

input
```
int(1/(c*x^2+b*x)^(3/4),x)
```

output
```
int(1/(x**(3/4)*(b + c*x)**(3/4)),x)
```

3.63 $\int \frac{1}{(bx+cx^2)^{5/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 84

$$\int \frac{1}{(bx+cx^2)^{5/4}}\,dx = -\frac{4}{b\sqrt[4]{bx+cx^2}} + \frac{8\sqrt[4]{\frac{cx}{b+cx}}\sqrt{b+cx}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}}{\sqrt{b+cx}}\right)\middle|2\right)}{b^{3/2}\sqrt[4]{bx+cx^2}}$$

output
```
-4/b/(c*x^2+b*x)^(1/4)+8*(c*x/(c*x+b))^(1/4)*(c*x+b)^(1/2)*EllipticE(sin(1
/2*arcsin(b^(1/2)/(c*x+b)^(1/2))),2^(1/2))/b^(3/2)/(c*x^2+b*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

$$\int \frac{1}{(bx+cx^2)^{5/4}}\,dx = -\frac{4\sqrt[4]{1+\frac{cx}{b}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},-\frac{cx}{b}\right)}{b\sqrt[4]{x(b+cx)}}$$

input
```
Integrate[(b*x + c*x^2)^(-5/4),x]
```

output
```
(-4*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, -((c*x)/b)])/(b*
(x*(b + c*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1089, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{5/4}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{4c\int \frac{1}{\sqrt[4]{cx^2+bx}}dx}{b^2}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

$$\Big\downarrow \text{1093}$$

$$\frac{4c\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{-\frac{c^2x^2}{b^2}-\frac{cx}{b}}}dx}{b^2\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{2\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{4\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right)\middle|2\right)}{b\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}$$

input
```
Int[(b*x + c*x^2)^(-5/4),x]
```

output
```
(-4*(b + 2*c*x))/(b^2*(b*x + c*x^2)^(1/4)) - (4*Sqrt[2]*(-((c*(b*x + c*x^2
))/b^2))^(1/4)*EllipticE[ArcSin[(b*(-(c/b) - (2*c^2*x)/b^2))/c]/2, 2])/(b*
(b*x + c*x^2)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{(c\,x^2+bx)^{\frac{5}{4}}}dx$$

input
```
int(1/(c*x^2+b*x)^(5/4),x)
```

output
```
int(1/(c*x^2+b*x)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{(bx + cx^2)^{5/4}}\,dx = \int \frac{1}{(cx^2 + bx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(5/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(3/4)/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(bx + cx^2)^{5/4}}\,dx = \int \frac{1}{(bx + cx^2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(5/4),x)
```

output
```
Integral((b*x + c*x**2)**(-5/4), x)
```

Maxima [F]

$$\int \frac{1}{(bx + cx^2)^{5/4}}\,dx = \int \frac{1}{(cx^2 + bx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(bx+cx^2)^{5/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(5/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.43

$$\int \frac{1}{(bx+cx^2)^{5/4}}\,dx = -\frac{4\,x\,\left(\frac{c\,x}{b}+1\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{5}{4};\ \frac{3}{4};\ -\frac{c\,x}{b}\right)}{(c\,x^2+b\,x)^{5/4}}$$

input
```
int(1/(b*x + c*x^2)^(5/4),x)
```

output
```
-(4*x*((c*x)/b + 1)^(5/4)*hypergeom([-1/4, 5/4], 3/4, -(c*x)/b))/(b*x + c*
x^2)^(5/4)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.32

$$\int \frac{1}{(bx+cx^2)^{5/4}}\,dx = \frac{4x^{\frac{1}{4}}(cx+b)^{\frac{1}{4}}}{\sqrt{x}\,\sqrt{cx+b}\,b}$$

input
```
int(1/(c*x^2+b*x)^(5/4),x)
```

output
```
(4*x**(1/4)*(b + c*x)**(1/4))/(sqrt(x)*sqrt(b + c*x)*b)
```

3.64 $\int \frac{1}{(bx+cx^2)^{9/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 154

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx = \frac{4}{5b\,(bx+cx^2)^{5/4}} + \frac{96c}{5b^3\sqrt[4]{bx+cx^2}} + \frac{8}{b^2x\sqrt[4]{bx+cx^2}} - \frac{48(bx+cx^2)^{3/4}}{5b^3x^2} - \frac{96c\sqrt[4]{\frac{cx}{b+cx}}\sqrt{b+cx}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}}{\sqrt{b+cx}}\right)\middle|2\right)}{5b^{7/2}\sqrt[4]{bx+cx^2}}$$

output
```
4/5/b/(c*x^2+b*x)^(5/4)+96/5*c/b^3/(c*x^2+b*x)^(1/4)+8/b^2/x/(c*x^2+b*x)^(
1/4)-48/5*(c*x^2+b*x)^(3/4)/b^3/x^2-96/5*c*(c*x/(c*x+b))^(1/4)*(c*x+b)^(1/
2)*EllipticE(sin(1/2*arcsin(b^(1/2)/(c*x+b)^(1/2))),2^(1/2))/b^(7/2)/(c*x^
2+b*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.32

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx = -\frac{4\sqrt[4]{1+\frac{cx}{b}}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{9}{4},-\frac{1}{4},-\frac{cx}{b}\right)}{5b^2x\sqrt[4]{x(b+cx)}}$$

input
```
Integrate[(b*x + c*x^2)^(-9/4),x]
```

output
```
(-4*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[-5/4, 9/4, -1/4, -((c*x)/b)])/(5
*b^2*x*(x*(b + c*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.84, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1089, 1089, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{9/4}}dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{12c\int \frac{1}{(cx^2+bx)^{5/4}}dx}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{12c\left(\frac{4c\int \frac{1}{\sqrt[4]{cx^2+bx}}dx}{b^2}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}$$

$$\Big\downarrow \text{1093}$$

$$-\frac{12c\left(\frac{4c\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{-\frac{c^2x^2}{b^2}-\frac{cx}{b}}}dx}{b^2\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{12c\left(-\frac{2\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/4}}$$

$\Big\downarrow$ 226

$$-\frac{12c\left(-\frac{4\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right)\middle|2\right)}{b\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2\left(bx+cx^2\right)^{5/4}}$$

input
```
Int[(b*x + c*x^2)^(-9/4),x]
```

output
```
(-4*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/4)) - (12*c*((-4*(b + 2*c*x))/(b^
2*(b*x + c*x^2)^(1/4)) - (4*Sqrt[2]*(-((c*(b*x + c*x^2))/b^2))^(1/4)*Ellip
ticE[ArcSin[(b*(-(c/b) - (2*c^2*x)/b^2))/c]/2, 2])/(b*(b*x + c*x^2)^(1/4))
))/(5*b^2)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{(c\,x^2+bx)^{\frac{9}{4}}}dx$$

input
```
int(1/(c*x^2+b*x)^(9/4),x)
```

output
```
int(1/(c*x^2+b*x)^(9/4),x)
```

Fricas [F]

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx=\int \frac{1}{(cx^2+bx)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(9/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(3/4)/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^
3), x)
```

Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx=\int \frac{1}{(bx+cx^2)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(9/4),x)
```

output
```
Integral((b*x + c*x**2)**(-9/4), x)
```

Maxima [F]

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx=\int \frac{1}{(cx^2+bx)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(9/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-9/4), x)
```

Giac [F]

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx=\int \frac{1}{(cx^2+bx)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(9/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-9/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.64 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.23

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx=-\frac{4\,x\,\left(\frac{c\,x}{b}+1\right)^{9/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{9}{4};\ -\frac{1}{4};\ -\frac{c\,x}{b}\right)}{5\,(c\,x^2+b\,x)^{9/4}}$$

input
```
int(1/(b*x + c*x^2)^(9/4),x)
```

output
```
-(4*x*((c*x)/b + 1)^(9/4)*hypergeom([-5/4, 9/4], -1/4, -(c*x)/b))/(5*(b*x
+ c*x^2)^(9/4))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.30

$$\int \frac{1}{(bx+cx^2)^{9/4}}\,dx=\frac{-\frac{128}{15}c^2x^2-\frac{32}{3}bcx-\frac{4}{3}b^2}{x^{\frac{3}{4}}\,(cx+b)^{\frac{3}{4}}\,\sqrt{x}\,\sqrt{cx+b}\,b^3}$$

input
```
int(1/(c*x^2+b*x)^(9/4),x)
```

output
```
(4*x**(1/4)*(b + c*x)**(1/4)*( - 5*b**2 - 40*b*c*x - 32*c**2*x**2))/(15*sq
rt(x)*sqrt(b + c*x)*b**3*x*(b + c*x))
```

3.65 $\int \frac{1}{(bx+cx^2)^{13/4}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 207

$$\begin{aligned}\int \frac{1}{(bx+cx^2)^{13/4}}\,dx &= \frac{4}{9b(bx+cx^2)^{9/4}}+\frac{8}{5b^2x(bx+cx^2)^{5/4}} \\ &\quad -\frac{896c^2}{15b^5\sqrt[4]{bx+cx^2}}+\frac{112}{5b^3x^2\sqrt[4]{bx+cx^2}}-\frac{224(bx+cx^2)^{3/4}}{9b^4x^3} \\ &\quad +\frac{448c(bx+cx^2)^{3/4}}{15b^5x^2}+\frac{896c^2\sqrt[4]{\frac{cx}{b+cx}}\sqrt{b+cx}\,E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{b}}{\sqrt{b+cx}}\right)\middle|2\right)}{15b^{11/2}\sqrt[4]{bx+cx^2}}\end{aligned}$$

output

```
4/9/b/(c*x^2+b*x)^(9/4)+8/5/b^2/x/(c*x^2+b*x)^(5/4)-896/15*c^2/b^5/(c*x^2+
b*x)^(1/4)+112/5/b^3/x^2/(c*x^2+b*x)^(1/4)-224/9*(c*x^2+b*x)^(3/4)/b^4/x^3
+448/15*c*(c*x^2+b*x)^(3/4)/b^5/x^2+896/15*c^2*(c*x/(c*x+b))^(1/4)*(c*x+b)
^(1/2)*EllipticE(sin(1/2*arcsin(b^(1/2)/(c*x+b)^(1/2))),2^(1/2))/b^(11/2)/
(c*x^2+b*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.24

$$\int \frac{1}{(bx+cx^2)^{13/4}}\,dx = -\frac{4\sqrt[4]{1+\frac{cx}{b}}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{13}{4},-\frac{5}{4},-\frac{cx}{b}\right)}{9b^3x^2\sqrt[4]{x(b+cx)}}$$

input
```
Integrate[(b*x + c*x^2)^(-13/4),x]
```

output
```
(-4*(1 + (c*x)/b)^(1/4)*Hypergeometric2F1[-9/4, 13/4, -5/4, -((c*x)/b)])/(
9*b^3*x^2*(x*(b + c*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.462, Rules used = {1089, 1089, 1089, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{13/4}}dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{28c\int \frac{1}{(cx^2+bx)^{9/4}}dx}{9b^2}-\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{28c\left(-\frac{12c\int \frac{1}{(cx^2+bx)^{5/4}}dx}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}\right)}{9b^2}-\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{28c\left(-\frac{12c\left(\frac{4c\int \frac{1}{\sqrt[4]{cx^2+bx}}dx}{b^2}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}\right)}{9b^2}-\frac{4(b+2cx)}{9b^2\left(bx+cx^2\right)^{9/4}}$$

$$\Big\downarrow \text{ 1093}$$

$$-\frac{28c\left(-\frac{12c\left(\frac{4c\sqrt[4]{-\frac{c\left(bx+cx^2\right)}{b^2}}\int \frac{1}{\sqrt[4]{-\frac{c^2x^2}{b^2}-\frac{cx}{b}}}dx}{b^2\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}\right)}{9b^2}-\frac{4(b+2cx)}{9b^2\left(bx+cx^2\right)^{9/4}}$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{28c\left(-\frac{12c\left(-\frac{2\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}\int \frac{1}{\sqrt[4]{1-\frac{b^2\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)^2}{c^2}}}d\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}\right)}{9b^2}-\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}}$$

$$\Big\downarrow \; 226$$

$$-\frac{28c\left(-\frac{12c\left(-\frac{4\sqrt{2}\sqrt[4]{-\frac{c(bx+cx^2)}{b^2}}E\left(\frac{1}{2}\arcsin\left(\frac{b\left(-\frac{2xc^2}{b^2}-\frac{c}{b}\right)}{c}\right)\middle|2\right)}{b\sqrt[4]{bx+cx^2}}-\frac{4(b+2cx)}{b^2\sqrt[4]{bx+cx^2}}\right)}{5b^2}-\frac{4(b+2cx)}{5b^2(bx+cx^2)^{5/4}}\right)}{9b^2}-\frac{4(b+2cx)}{9b^2(bx+cx^2)^{9/4}}$$

input
```
Int[(b*x + c*x^2)^(-13/4),x]
```

output
```
(-4*(b + 2*c*x))/(9*b^2*(b*x + c*x^2)^(9/4)) - (28*c*((-4*(b + 2*c*x))/(5*
b^2*(b*x + c*x^2)^(5/4)) - (12*c*((-4*(b + 2*c*x))/(b^2*(b*x + c*x^2)^(1/4
)) - (4*Sqrt[2]*(-((c*(b*x + c*x^2))/b^2))^(1/4)*EllipticE[ArcSin[(b*(-(c/
b) - (2*c^2*x)/b^2))/c]/2, 2])/(b*(b*x + c*x^2)^(1/4))))/(5*b^2)))/(9*b^2)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{(c\,x^2+bx)^{\frac{13}{4}}}dx$$

input
```
int(1/(c*x^2+b*x)^(13/4),x)
```

output
```
int(1/(c*x^2+b*x)^(13/4),x)
```

Fricas [F]

$$\int \frac{1}{(bx+cx^2)^{13/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(13/4),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(3/4)/(c^4*x^8 + 4*b*c^3*x^7 + 6*b^2*c^2*x^6 + 4*b^
3*c*x^5 + b^4*x^4), x)
```

Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{13/4}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(13/4),x)
```

output
```
Integral((b*x + c*x**2)**(-13/4), x)
```

Maxima [F]

$$\int \frac{1}{(bx+cx^2)^{13/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(13/4),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(-13/4), x)
```

Giac [F]

$$\int \frac{1}{(bx+cx^2)^{13/4}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{13}{4}}}\,dx$$

input
```
integrate(1/(c*x^2+b*x)^(13/4),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(-13/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.17

$$\int \frac{1}{(bx+cx^2)^{13/4}}\,dx = -\frac{4\,x\,\left(\frac{c\,x}{b}+1\right)^{13/4}\,{}_2\mathrm{F}_1\left(-\frac{9}{4},\frac{13}{4};\ -\frac{5}{4};\ -\frac{c\,x}{b}\right)}{9\,(c\,x^2+b\,x)^{13/4}}$$

input
```
int(1/(b*x + c*x^2)^(13/4),x)
```

output
```
-(4*x*((c*x)/b + 1)^(13/4)*hypergeom([-9/4, 13/4], -5/4, -(c*x)/b))/(9*(b*
x + c*x^2)^(13/4))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.42

$$\int \frac{1}{(bx+cx^2)^{13/4}}\,dx = \frac{4(cx+b)^{\frac{1}{4}}\,(2048c^4x^4+4608b\,c^3x^3+2880b^2c^2x^2+240b^3cx-45b^4)}{315x^{\frac{7}{4}}\sqrt{x}\,\sqrt{cx+b}\,b^5\,(c^2x^2+2bcx+b^2)}$$

input
```
int(1/(c*x^2+b*x)^(13/4),x)
```

output
```
(4*x**(1/4)*(b + c*x)**(1/4)*( - 45*b**4 + 240*b**3*c*x + 2880*b**2*c**2*x
**2 + 4608*b*c**3*x**3 + 2048*c**4*x**4))/(315*sqrt(x)*sqrt(b + c*x)*b**5*
x**2*(b**2 + 2*b*c*x + c**2*x**2))
```

3.66 $\int \frac{1}{\sqrt[4]{2x+3x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{1}{\sqrt[4]{2x+3x^2}}\,dx = \frac{2\sqrt[4]{-2x-3x^2}\,E\left(\frac{1}{2}\arcsin(1+3x)\middle|\,2\right)}{3^{3/4}\sqrt[4]{2x+3x^2}}$$

output
```
2/3*(-3*x^2-2*x)^(1/4)*EllipticE(sin(1/2*arcsin(1+3*x)),2^(1/2))*3^(1/4)/(
3*x^2+2*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt[4]{2x+3x^2}}\,dx = \frac{2(x(2+3x))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{3x}{2}\right)}{3\left(1+\frac{3x}{2}\right)^{3/4}}$$

input
```
Integrate[(2*x + 3*x^2)^(-1/4),x]
```

output
```
(2*(x*(2 + 3*x))^(3/4)*Hypergeometric2F1[1/4, 3/4, 7/4, (-3*x)/2])/(3*(1 +
 (3*x)/2)^(3/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{3x^2+2x}}dx$$

$$\Big\downarrow \text{1093}$$

$$\frac{\sqrt[4]{3}\sqrt[4]{-3x^2-2x}\int \frac{1}{\sqrt[4]{-\frac{9x^2}{4}-\frac{3x}{2}}}dx}{\sqrt{2}\sqrt[4]{3x^2+2x}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{2\sqrt[4]{-3x^2-2x}\int \frac{1}{\sqrt[4]{1-\frac{4}{9}\left(-\frac{9x}{2}-\frac{3}{2}\right)^2}}d\left(-\frac{9x}{2}-\frac{3}{2}\right)}{3\ 3^{3/4}\sqrt[4]{3x^2+2x}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{2\sqrt[4]{-3x^2-2x}E\left(\frac{1}{2}\arcsin\left(\frac{2}{3}\left(-\frac{9x}{2}-\frac{3}{2}\right)\right)|2\right)}{3^{3/4}\sqrt[4]{3x^2+2x}}$$

input
```
Int[(2*x + 3*x^2)^(-1/4),x]
```

output
```
(-2*(-2*x - 3*x^2)^(1/4)*EllipticE[ArcSin[(2*(-3/2 - (9*x)/2))/3]/2, 2])/(
3^(3/4)*(2*x + 3*x^2)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.40

method	result	size
meijerg	$\frac{2\,2^{\frac{3}{4}} x^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],-\frac{3x}{2}\right)}{3}$	18

input
```
int(1/(3*x^2+2*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
2/3*2^(3/4)*x^(3/4)*hypergeom([1/4,3/4],[7/4],-3/2*x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{2x+3x^2}}\,dx = \int \frac{1}{\left(3\,x^2+2\,x\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2*x)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2*x)^(-1/4), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{2x+3x^2}}\,dx = \int \frac{1}{\sqrt[4]{3x^2+2x}}\,dx$$

input
```
integrate(1/(3*x**2+2*x)**(1/4),x)
```

output
```
Integral((3*x**2 + 2*x)**(-1/4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{2x+3x^2}}\,dx = \int \frac{1}{\left(3\,x^2+2\,x\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2*x)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{2x+3x^2}}\,dx=\int \frac{1}{(3\,x^2+2\,x)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2*x)^(1/4),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2*x)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt[4]{2x+3x^2}}\,dx=\frac{2\,2^{3/4}\,x\,(3\,x+2)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{3\,x}{2}\right)}{3\,(3\,x^2+2\,x)^{1/4}}$$

input
```
int(1/(2*x + 3*x^2)^(1/4),x)
```

output
```
(2*2^(3/4)*x*(3*x + 2)^(1/4)*hypergeom([1/4, 3/4], 7/4, -(3*x)/2))/(3*(2*x
 + 3*x^2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{2x+3x^2}}\,dx=\int \frac{1}{x^{\frac{1}{4}}\,(3x+2)^{\frac{1}{4}}}dx$$

input
```
int(1/(3*x^2+2*x)^(1/4),x)
```

output
```
int(1/(x**(1/4)*(3*x + 2)**(1/4)),x)
```

3.67 $\int \frac{1}{\sqrt[4]{-2x+3x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{1}{\sqrt[4]{-2x+3x^2}}\,dx = -\frac{2\sqrt[4]{2x-3x^2}\,E\left(\frac{1}{2}\arcsin(1-3x)\middle|2\right)}{3^{3/4}\sqrt[4]{-2x+3x^2}}$$

output
```
2/3*(-3*x^2+2*x)^(1/4)*EllipticE(sin(1/2*arcsin(-1+3*x)),2^(1/2))*3^(1/4)/
(3*x^2-2*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{1}{\sqrt[4]{-2x+3x^2}}\,dx = \frac{2\left(\frac{2}{3}\right)^{3/4}(x(-2+3x))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},1-\frac{3x}{2}\right)}{3x^{3/4}}$$

input
```
Integrate[(-2*x + 3*x^2)^(-1/4),x]
```

output
```
(2*(2/3)^(3/4)*(x*(-2 + 3*x))^(3/4)*Hypergeometric2F1[1/4, 3/4, 7/4, 1 - (
3*x)/2])/(3*x^(3/4))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{3x^2-2x}}dx$$

$$\Big\downarrow \text{1093}$$

$$\frac{\sqrt[4]{3}\sqrt[4]{2x-3x^2}\int \frac{1}{\sqrt[4]{\frac{3x}{2}-\frac{9x^2}{4}}}dx}{\sqrt{2}\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{2\sqrt[4]{2x-3x^2}\int \frac{1}{\sqrt[4]{1-\frac{4}{9}\left(\frac{3}{2}-\frac{9x}{2}\right)^2}}d\left(\frac{3}{2}-\frac{9x}{2}\right)}{3\ 3^{3/4}\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{2\sqrt[4]{2x-3x^2}E\left(\frac{1}{2}\arcsin\left(\frac{2}{3}\left(\frac{3}{2}-\frac{9x}{2}\right)\right)\middle|2\right)}{3^{3/4}\sqrt[4]{3x^2-2x}}$$

input
```
Int[(-2*x + 3*x^2)^(-1/4),x]
```

output
```
(-2*(2*x - 3*x^2)^(1/4)*EllipticE[ArcSin[(2*(3/2 - (9*x)/2))/3]/2, 2])/(3^
(3/4)*(-2*x + 3*x^2)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

method	result	size
meijerg	$\frac{2\,2^{\frac{3}{4}} \left(-\operatorname{signum}\left(x-\frac{2}{3}\right)\right)^{\frac{1}{4}} x^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],\frac{3x}{2}\right)}{3 \operatorname{signum}\left(x-\frac{2}{3}\right)^{\frac{1}{4}}}$	32

input
```
int(1/(3*x^2-2*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
2/3*2^(3/4)/signum(x-2/3)^(1/4)*(-signum(x-2/3))^(1/4)*x^(3/4)*hypergeom([
1/4,3/4],[7/4],3/2*x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{-2x+3x^2}}\,dx = \int \frac{1}{\left(3\,x^2-2\,x\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2-2*x)^(1/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 2*x)^(-1/4), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{-2x+3x^2}}\,dx = \int \frac{1}{\sqrt[4]{3x^2-2x}}\,dx$$

input
```
integrate(1/(3*x**2-2*x)**(1/4),x)
```

output
```
Integral((3*x**2 - 2*x)**(-1/4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{-2x+3x^2}}\,dx = \int \frac{1}{\left(3\,x^2-2\,x\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2-2*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate((3*x^2 - 2*x)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{-2x+3x^2}}\,dx=\int \frac{1}{(3\,x^2-2\,x)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(3*x^2-2*x)^(1/4),x, algorithm="giac")
```

output
```
integrate((3*x^2 - 2*x)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

$$\int \frac{1}{\sqrt[4]{-2x+3x^2}}\,dx=\frac{2\,2^{3/4}\,x\,(2-3\,x)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ \frac{3\,x}{2}\right)}{3\,(3\,x^2-2\,x)^{1/4}}$$

input
```
int(1/(3*x^2 - 2*x)^(1/4),x)
```

output
```
(2*2^(3/4)*x*(2 - 3*x)^(1/4)*hypergeom([1/4, 3/4], 7/4, (3*x)/2))/(3*(3*x^
2 - 2*x)^(1/4))
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{-2x+3x^2}}\,dx=\int \frac{1}{x^{\frac{1}{4}}\,(3x-2)^{\frac{1}{4}}}dx$$

input
```
int(1/(3*x^2-2*x)^(1/4),x)
```

output
```
int(1/(x**(1/4)*(3*x - 2)**(1/4)),x)
```

3.68 $\int \frac{1}{\sqrt[4]{ax+3x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{1}{\sqrt[4]{ax+3x^2}}\,dx=\frac{\sqrt{2}a\sqrt[4]{-\frac{x}{a}-\frac{3x^2}{a^2}}E\left(\frac{1}{2}\arcsin\left(1+\frac{6x}{a}\right)\middle|2\right)}{3^{3/4}\sqrt[4]{ax+3x^2}}$$

output
```
1/3*2^(1/2)*a*(-x/a-3*x^2/a^2)^(1/4)*EllipticE(sin(1/2*arcsin(1+6*x/a)),2^
(1/2))*3^(1/4)/(a*x+3*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[4]{ax+3x^2}}\,dx=\frac{4x\sqrt[4]{1+\frac{3x}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{3x}{a}\right)}{3\sqrt[4]{x(a+3x)}}$$

input
```
Integrate[(a*x + 3*x^2)^(-1/4),x]
```

output
```
(4*x*(1 + (3*x)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, (-3*x)/a])/(3*(x
*(a + 3*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{ax+3x^2}}dx$$

$$\Big\downarrow \text{1093}$$

$$\frac{\sqrt[4]{3}\sqrt[4]{-\frac{ax+3x^2}{a^2}}\int \frac{1}{\sqrt[4]{-\frac{9x^2}{a^2}-\frac{3x}{a}}}dx}{\sqrt[4]{ax+3x^2}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{a^2\sqrt[4]{-\frac{ax+3x^2}{a^2}}\int \frac{1}{\sqrt[4]{1-\frac{1}{9}a^2\left(-\frac{18x}{a^2}-\frac{3}{a}\right)^2}}d\left(-\frac{18x}{a^2}-\frac{3}{a}\right)}{3\sqrt{2}3^{3/4}\sqrt[4]{ax+3x^2}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{\sqrt{2}a\sqrt[4]{-\frac{ax+3x^2}{a^2}}E\left(\frac{1}{2}\arcsin\left(\frac{1}{3}a\left(-\frac{18x}{a^2}-\frac{3}{a}\right)\right)|2\right)}{3^{3/4}\sqrt[4]{ax+3x^2}}$$

input
```
Int[(a*x + 3*x^2)^(-1/4),x]
```

output
```
-((Sqrt[2]*a*(-((a*x + 3*x^2)/a^2))^(1/4)*EllipticE[ArcSin[(a*(-3/a - (18*
x)/a^2))/3]/2, 2])/(3^(3/4)*(a*x + 3*x^2)^(1/4)))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{(ax + 3x^2)^{\frac{1}{4}}} dx$$

input
```
int(1/(a*x+3*x^2)^(1/4),x)
```

output
```
int(1/(a*x+3*x^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{ax + 3x^2}} dx = \int \frac{1}{(ax + 3\,x^2)^{\frac{1}{4}}} dx$$

input
```
integrate(1/(a*x+3*x^2)^(1/4),x, algorithm="fricas")
```

output
```
integral((a*x + 3*x^2)^(-1/4), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{ax+3x^2}}\,dx = \int \frac{1}{\sqrt[4]{ax+3x^2}}\,dx$$

input
```
integrate(1/(a*x+3*x**2)**(1/4),x)
```

output
```
Integral((a*x + 3*x**2)**(-1/4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{ax+3x^2}}\,dx = \int \frac{1}{(ax+3\,x^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a*x+3*x^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((a*x + 3*x^2)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{ax+3x^2}}\,dx = \int \frac{1}{(ax+3\,x^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a*x+3*x^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((a*x + 3*x^2)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt[4]{ax+3x^2}}\,dx=\frac{4\,x\,{\left(\frac{3\,x}{a}+1\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{3\,x}{a}\right)}{3\,{\left(3\,x^2+a\,x\right)}^{1/4}}$$

input
```
int(1/(a*x + 3*x^2)^(1/4),x)
```

output
```
(4*x*((3*x)/a + 1)^(1/4)*hypergeom([1/4, 3/4], 7/4, -(3*x)/a))/(3*(a*x + 3
*x^2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{ax+3x^2}}\,dx=\int \frac{1}{x^{\frac{1}{4}}\,(a+3x)^{\frac{1}{4}}}dx$$

input
```
int(1/(a*x+3*x^2)^(1/4),x)
```

output
```
int(1/(x**(1/4)*(a + 3*x)**(1/4)),x)
```

3.69 $\int \frac{1}{\sqrt[4]{2x-3x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{\sqrt[4]{2x-3x^2}}\,dx = -\frac{2E\left(\frac{1}{2}\arcsin(1-3x)\middle|2\right)}{3^{3/4}}$$

output
```
2/3*EllipticE(sin(1/2*arcsin(-1+3*x)),2^(1/2))*3^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.37

$$\int \frac{1}{\sqrt[4]{2x-3x^2}}\,dx = -\frac{2\left(\frac{2}{3}\right)^{3/4}(-x(-2+3x))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},1-\frac{3x}{2}\right)}{3x^{3/4}}$$

input
```
Integrate[(2*x - 3*x^2)^(-1/4),x]
```

output
```
(-2*(2/3)^(3/4)*(-(x*(-2 + 3*x)))^(3/4)*Hypergeometric2F1[1/4, 3/4, 7/4, 1
 - (3*x)/2])/(3*x^(3/4))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{2x-3x^2}}dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{\int \frac{1}{\sqrt[4]{1-\frac{1}{4}(2-6x)^2}}d(2-6x)}{2\ 3^{3/4}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{2E\left(\frac{1}{2}\arcsin\left(\frac{1}{2}(2-6x)\right)\middle|2\right)}{3^{3/4}}$$

input
```
Int[(2*x - 3*x^2)^(-1/4),x]
```

output
```
(-2*EllipticE[ArcSin[(2 - 6*x)/2]/2, 2])/3^(3/4)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
meijerg	$\frac{2\,2^{\frac{3}{4}} x^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],\frac{3x}{2}\right)}{3}$	18

input
```
int(1/(-3*x^2+2*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
2/3*2^(3/4)*x^(3/4)*hypergeom([1/4,3/4],[7/4],3/2*x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{2x-3x^2}}\,dx = \int \frac{1}{(-3\,x^2+2\,x)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2*x)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2*x)^(3/4)/(3*x^2 - 2*x), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{2x-3x^2}}\,dx = \int \frac{1}{\sqrt[4]{-3x^2+2x}}\,dx$$

input
```
integrate(1/(-3*x**2+2*x)**(1/4),x)
```

output
```
Integral((-3*x**2 + 2*x)**(-1/4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{2x-3x^2}}\,dx = \int \frac{1}{\left(-3\,x^2+2\,x\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-3*x^2 + 2*x)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{2x-3x^2}}\,dx = \int \frac{1}{\left(-3\,x^2+2\,x\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2*x)^(1/4),x, algorithm="giac")
```

output
```
integrate((-3*x^2 + 2*x)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{1}{\sqrt[4]{2x-3x^2}}\,dx = \frac{2\,2^{3/4}\,x\,(2-3\,x)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ \frac{3\,x}{2}\right)}{3\,(2\,x-3\,x^2)^{1/4}}$$

input
```
int(1/(2*x - 3*x^2)^(1/4),x)
```

output
```
(2*2^(3/4)*x*(2 - 3*x)^(1/4)*hypergeom([1/4, 3/4], 7/4, (3*x)/2))/(3*(2*x
- 3*x^2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{2x-3x^2}}\,dx = \int \frac{1}{x^{\frac{1}{4}}\left(-3x+2\right)^{\frac{1}{4}}}dx$$

input

```
int(1/(-3*x^2+2*x)^(1/4),x)
```

output

```
int(1/(x**(1/4)*( - 3*x + 2)**(1/4)),x)
```

3.70 $\int \frac{1}{\sqrt[4]{-2x-3x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{\sqrt[4]{-2x-3x^2}}\,dx = \frac{2E\left(\frac{1}{2}\arcsin(1+3x)\middle|\,2\right)}{3^{3/4}}$$

output
```
2/3*EllipticE(sin(1/2*arcsin(1+3*x)),2^(1/2))*3^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.21

$$\int \frac{1}{\sqrt[4]{-2x-3x^2}}\,dx = -\frac{2(-x(2+3x))^{3/4}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},-\frac{3x}{2}\right)}{3\left(1+\frac{3x}{2}\right)^{3/4}}$$

input
```
Integrate[(-2*x - 3*x^2)^(-1/4),x]
```

output
```
(-2*(-(x*(2 + 3*x)))^(3/4)*Hypergeometric2F1[1/4, 3/4, 7/4, (-3*x)/2])/(3*
(1 + (3*x)/2)^(3/4))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{-3x^2-2x}}dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{\int \frac{1}{\sqrt[4]{1-\frac{1}{4}(-6x-2)^2}}d(-6x-2)}{2\ 3^{3/4}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{2E\left(\frac{1}{2}\arcsin\left(\frac{1}{2}(-6x-2)\right)\middle|2\right)}{3^{3/4}}$$

input
```
Int[(-2*x - 3*x^2)^(-1/4),x]
```

output
```
(-2*EllipticE[ArcSin[(-2 - 6*x)/2]/2, 2])/3^(3/4)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

method	result	size
meijerg	$-\frac{2(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} x^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{7}{4}\right],-\frac{3x}{2}\right)}{3}$	21

input
```
int(1/(-3*x^2-2*x)^(1/4),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-1)^(3/4)*2^(3/4)*x^(3/4)*hypergeom([1/4,3/4],[7/4],-3/2*x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{-2x-3x^2}}\,dx = \int \frac{1}{\left(-3\,x^2-2\,x\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2*x)^(1/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 - 2*x)^(3/4)/(3*x^2 + 2*x), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{-2x-3x^2}}\,dx = \int \frac{1}{\sqrt[4]{-3x^2-2x}}\,dx$$

input
```
integrate(1/(-3*x**2-2*x)**(1/4),x)
```

output
```
Integral((-3*x**2 - 2*x)**(-1/4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{-2x-3x^2}}\,dx = \int \frac{1}{\left(-3\,x^2-2\,x\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2*x)^(1/4),x, algorithm="maxima")
```

output
```
integrate((-3*x^2 - 2*x)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{-2x-3x^2}}\,dx = \int \frac{1}{\left(-3\,x^2-2\,x\right)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2*x)^(1/4),x, algorithm="giac")
```

output
```
integrate((-3*x^2 - 2*x)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{1}{\sqrt[4]{-2x-3x^2}}\,dx = \frac{2\,2^{3/4}\,x\,{\left(3\,x+2\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ -\frac{3\,x}{2}\right)}{3\,{\left(-3\,x^2-2\,x\right)}^{1/4}}$$

input
```
int(1/(- 2*x - 3*x^2)^(1/4),x)
```

output
```
(2*2^(3/4)*x*(3*x + 2)^(1/4)*hypergeom([1/4, 3/4], 7/4, -(3*x)/2))/(3*(- 2
*x - 3*x^2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{-2x-3x^2}}\,dx=\int \frac{1}{x^{\frac{1}{4}}\left(-3x-2\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(-3*x^2-2*x)^(1/4),x)
```

output
```
int(1/(x**(1/4)*( - 3*x - 2)**(1/4)),x)
```

3.71 $\int \frac{1}{\sqrt[4]{ax-3x^2}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{1}{\sqrt[4]{ax-3x^2}}\,dx = -\frac{\sqrt{2}a\sqrt[4]{\frac{x}{a}-\frac{3x^2}{a^2}}E\left(\frac{1}{2}\arcsin\left(1-\frac{6x}{a}\right)\middle|2\right)}{3^{3/4}\sqrt[4]{ax-3x^2}}$$

output
```
-1/3*2^(1/2)*a*(x/a-3*x^2/a^2)^(1/4)*EllipticE(sin(1/2*arcsin(1-6*x/a)),2^
(1/2))*3^(1/4)/(a*x-3*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.75

$$\int \frac{1}{\sqrt[4]{ax-3x^2}}\,dx = \frac{4x\sqrt[4]{1-\frac{3x}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{7}{4},\frac{3x}{a}\right)}{3\sqrt[4]{(a-3x)x}}$$

input
```
Integrate[(a*x - 3*x^2)^(-1/4),x]
```

output
```
(4*x*(1 - (3*x)/a)^(1/4)*Hypergeometric2F1[1/4, 3/4, 7/4, (3*x)/a])/(3*((a
 - 3*x)*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{ax-3x^2}}dx$$

$$\Big\downarrow \text{1093}$$

$$\frac{\sqrt[4]{3}\sqrt[4]{\frac{ax-3x^2}{a^2}}\int \frac{1}{\sqrt[4]{\frac{3x}{a}-\frac{9x^2}{a^2}}}dx}{\sqrt[4]{ax-3x^2}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{a^2\sqrt[4]{\frac{ax-3x^2}{a^2}}\int \frac{1}{\sqrt[4]{1-\frac{1}{9}a^2\left(\frac{3}{a}-\frac{18x}{a^2}\right)^2}}d\left(\frac{3}{a}-\frac{18x}{a^2}\right)}{3\sqrt{2}3^{3/4}\sqrt[4]{ax-3x^2}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{\sqrt{2}a\sqrt[4]{\frac{ax-3x^2}{a^2}}E\left(\frac{1}{2}\arcsin\left(\frac{1}{3}a\left(\frac{3}{a}-\frac{18x}{a^2}\right)\right)\middle|2\right)}{3^{3/4}\sqrt[4]{ax-3x^2}}$$

input
```
Int[(a*x - 3*x^2)^(-1/4),x]
```

output
```
-((Sqrt[2]*a*((a*x - 3*x^2)/a^2)^(1/4)*EllipticE[ArcSin[(a*(3/a - (18*x)/a
^2))/3]/2, 2])/(3^(3/4)*(a*x - 3*x^2)^(1/4)))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{(ax-3x^2)^{\frac{1}{4}}}dx$$

input
```
int(1/(a*x-3*x^2)^(1/4),x)
```

output
```
int(1/(a*x-3*x^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{ax-3x^2}}\,dx = \int \frac{1}{(ax-3\,x^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a*x-3*x^2)^(1/4),x, algorithm="fricas")
```

output
```
integral((a*x - 3*x^2)^(-1/4), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{ax-3x^2}}\,dx=\int \frac{1}{\sqrt[4]{ax-3x^2}}\,dx$$

input
```
integrate(1/(a*x-3*x**2)**(1/4),x)
```

output
```
Integral((a*x - 3*x**2)**(-1/4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{ax-3x^2}}\,dx=\int \frac{1}{(ax-3\,x^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a*x-3*x^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((a*x - 3*x^2)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{ax-3x^2}}\,dx=\int \frac{1}{(ax-3\,x^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(a*x-3*x^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((a*x - 3*x^2)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt[4]{ax-3x^2}}\,dx = \frac{4\,x\,\left(1-\frac{3\,x}{a}\right)^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{7}{4};\ \frac{3\,x}{a}\right)}{3\,\left(a\,x-3\,x^2\right)^{1/4}}$$

input
```
int(1/(a*x - 3*x^2)^(1/4),x)
```

output
```
(4*x*(1 - (3*x)/a)^(1/4)*hypergeom([1/4, 3/4], 7/4, (3*x)/a))/(3*(a*x - 3*
x^2)^(1/4))
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{ax-3x^2}}\,dx = \int \frac{1}{x^{\frac{1}{4}}\,(a-3x)^{\frac{1}{4}}}dx$$

input
```
int(1/(a*x-3*x^2)^(1/4),x)
```

output
```
int(1/(x**(1/4)*(a - 3*x)**(1/4)),x)
```

3.72 $\int \frac{1}{(2x+3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 45

$$\int \frac{1}{(2x+3x^2)^{3/4}}\,dx = \frac{2(-2x-3x^2)^{3/4}\,\mathrm{EllipticF}\left(\frac{1}{2}\arcsin(1+3x),2\right)}{\sqrt[4]{3}\,(2x+3x^2)^{3/4}}$$

output
```
2/3*(-3*x^2-2*x)^(3/4)*InverseJacobiAM(1/2*arcsin(1+3*x),2^(1/2))*3^(3/4)/
(3*x^2+2*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{1}{(2x+3x^2)^{3/4}}\,dx = \frac{2\sqrt[4]{x(2+3x)}\,\mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{3x}{2}\right)}{\sqrt[4]{1+\frac{3x}{2}}}$$

input
```
Integrate[(2*x + 3*x^2)^(-3/4),x]
```

output
```
(2*(x*(2 + 3*x))^(1/4)*Hypergeometric2F1[1/4, 3/4, 5/4, (-3*x)/2])/(1 + (3
*x)/2)^(1/4)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2+2x)^{3/4}}dx$$

$$\Big\downarrow \text{1093}$$

$$\frac{3^{3/4}\left(-3x^2-2x\right)^{3/4}\int \frac{1}{\left(-\frac{9x^2}{4}-\frac{3x}{2}\right)^{3/4}}dx}{2\sqrt{2}\,(3x^2+2x)^{3/4}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{2\left(-3x^2-2x\right)^{3/4}\int \frac{1}{\left(1-\frac{4}{9}\left(-\frac{9x}{2}-\frac{3}{2}\right)^2\right)^{3/4}}d\left(-\frac{9x}{2}-\frac{3}{2}\right)}{3\sqrt[4]{3}\,(3x^2+2x)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$-\frac{2\left(-3x^2-2x\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{2}{3}\left(-\frac{9x}{2}-\frac{3}{2}\right)\right),2\right)}{\sqrt[4]{3}\,(3x^2+2x)^{3/4}}$$

input
```
Int[(2*x + 3*x^2)^(-3/4),x]
```

output
```
(-2*(-2*x - 3*x^2)^(3/4)*EllipticF[ArcSin[(2*(-3/2 - (9*x)/2))/3]/2, 2])/(
3^(1/4)*(2*x + 3*x^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.40

method	result	size
meijerg	$2\,2^{\frac{1}{4}} x^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{4}\right], \left[\frac{5}{4}\right], -\frac{3x}{2}\right)$	18

input
```
int(1/(3*x^2+2*x)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
2*2^(1/4)*x^(1/4)*hypergeom([1/4,3/4],[5/4],-3/2*x)
```

Fricas [F]

$$\int \frac{1}{(2x+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2+2\,x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2*x)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2*x)^(-3/4), x)
```

Sympy [F]

$$\int \frac{1}{(2x+3x^2)^{3/4}}\,dx = \int \frac{1}{(3x^2+2x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(3*x**2+2*x)**(3/4),x)
```

output
```
Integral((3*x**2 + 2*x)**(-3/4), x)
```

Maxima [F]

$$\int \frac{1}{(2x+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2+2\,x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2*x)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(2x+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2+2\,x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2*x)^(3/4),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2*x)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.92 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.71

$$\int \frac{1}{(2x+3x^2)^{3/4}}\,dx = \frac{2\,2^{1/4}\,x\,(3\,x+2)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\ \frac{5}{4};\ -\frac{3\,x}{2}\right)}{(3\,x^2+2\,x)^{3/4}}$$

input
```
int(1/(2*x + 3*x^2)^(3/4),x)
```

output
```
(2*2^(1/4)*x*(3*x + 2)^(3/4)*hypergeom([1/4, 3/4], 5/4, -(3*x)/2))/(2*x +
3*x^2)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(2x+3x^2)^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}\,(3x+2)^{\frac{3}{4}}}dx$$

input
```
int(1/(3*x^2+2*x)^(3/4),x)
```

output
```
int(1/(x**(3/4)*(3*x + 2)**(3/4)),x)
```

3.73 $\int \frac{1}{\left(-2x+3x^2\right)^{3/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{1}{\left(-2x+3x^2\right)^{3/4}}\,dx=-\frac{2\sqrt{2}x^{3/4}(-2+3x)^{3/4}\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right),2\right)}{\sqrt[4]{3}\left(-2x+3x^2\right)^{3/4}}$$

output
```
-2/3*2^(1/2)*x^(3/4)*(-2+3*x)^(3/4)*InverseJacobiAM(1/2*arcsin(1/3*6^(1/2)
/x^(1/2)),2^(1/2))*3^(3/4)/(3*x^2-2*x)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{1}{\left(-2x+3x^2\right)^{3/4}}\,dx=\frac{2\sqrt[4]{\frac{2}{3}}\sqrt[4]{x(-2+3x)}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},1-\frac{3x}{2}\right)}{\sqrt[4]{x}}$$

input
```
Integrate[(-2*x + 3*x^2)^(-3/4),x]
```

output
```
(2*(2/3)^(1/4)*(x*(-2 + 3*x))^(1/4)*Hypergeometric2F1[1/4, 3/4, 5/4, 1 - (
3*x)/2])/x^(1/4)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2-2x)^{3/4}}dx$$

$$\Big\downarrow \text{1093}$$

$$\frac{3^{3/4}(2x-3x^2)^{3/4}\int \frac{1}{\left(\frac{3x}{2}-\frac{9x^2}{4}\right)^{3/4}}dx}{2\sqrt{2}(3x^2-2x)^{3/4}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{2(2x-3x^2)^{3/4}\int \frac{1}{\left(1-\frac{4}{9}\left(\frac{3}{2}-\frac{9x}{2}\right)^2\right)^{3/4}}d\left(\frac{3}{2}-\frac{9x}{2}\right)}{3\sqrt[4]{3}(3x^2-2x)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$-\frac{2(2x-3x^2)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{2}{3}\left(\frac{3}{2}-\frac{9x}{2}\right)\right),2\right)}{\sqrt[4]{3}(3x^2-2x)^{3/4}}$$

input
```
Int[(-2*x + 3*x^2)^(-3/4),x]
```

output
```
(-2*(2*x - 3*x^2)^(3/4)*EllipticF[ArcSin[(2*(3/2 - (9*x)/2))/3]/2, 2])/(3^
(1/4)*(-2*x + 3*x^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

method	result	size
meijerg	$\frac{2\,2^{\frac{1}{4}}\left(-\operatorname{signum}\left(x-\frac{2}{3}\right)\right)^{\frac{3}{4}} x^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],\left[\frac{5}{4}\right],\frac{3x}{2}\right)}{\operatorname{signum}\left(x-\frac{2}{3}\right)^{\frac{3}{4}}}$	32

input
```
int(1/(3*x^2-2*x)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
2*2^(1/4)/signum(x-2/3)^(3/4)*(-signum(x-2/3))^(3/4)*x^(1/4)*hypergeom([1/
4,3/4],[5/4],3/2*x)
```

Fricas [F]

$$\int \frac{1}{(-2x+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2-2\,x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(3*x^2-2*x)^(3/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 2*x)^(-3/4), x)
```

Sympy [F]

$$\int \frac{1}{(-2x+3x^2)^{3/4}}\,dx = \int \frac{1}{(3x^2-2x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(3*x**2-2*x)**(3/4),x)
```

output
```
Integral((3*x**2 - 2*x)**(-3/4), x)
```

Maxima [F]

$$\int \frac{1}{(-2x+3x^2)^{3/4}}\,dx = \int \frac{1}{(3\,x^2-2\,x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(3*x^2-2*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate((3*x^2 - 2*x)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(-2x+3x^2)^{3/4}}\,dx=\int \frac{1}{(3\,x^2-2\,x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(3*x^2-2*x)^(3/4),x, algorithm="giac")
```

output
```
integrate((3*x^2 - 2*x)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.54

$$\int \frac{1}{(-2x+3x^2)^{3/4}}\,dx=\frac{2\,2^{1/4}\,x\,(2-3\,x)^{3/4}\,{}_2F_1\left(\frac{1}{4},\frac{3}{4};\ \frac{5}{4};\ \frac{3\,x}{2}\right)}{(3\,x^2-2\,x)^{3/4}}$$

input
```
int(1/(3*x^2 - 2*x)^(3/4),x)
```

output
```
(2*2^(1/4)*x*(2 - 3*x)^(3/4)*hypergeom([1/4, 3/4], 5/4, (3*x)/2))/(3*x^2 -
 2*x)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(-2x+3x^2)^{3/4}}\,dx=\int \frac{1}{x^{\frac{3}{4}}\,(3x-2)^{\frac{3}{4}}}dx$$

input
```
int(1/(3*x^2-2*x)^(3/4),x)
```

output
```
int(1/(x**(3/4)*(3*x - 2)**(3/4)),x)
```

3.74 $\int \frac{1}{(ax+3x^2)^{3/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{1}{(ax+3x^2)^{3/4}}\,dx = -\frac{4\left(\frac{x}{a+3x}\right)^{3/4}(a+3x)^{3/2}\operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+3x}}\right),2\right)}{\sqrt[4]{3}\sqrt{a}\,(ax+3x^2)^{3/4}}$$

output
```
-4/3*(x/(a+3*x))^(3/4)*(a+3*x)^(3/2)*InverseJacobiAM(1/2*arcsin(a^(1/2)/(a
+3*x)^(1/2)),2^(1/2))*3^(3/4)/a^(1/2)/(a*x+3*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.61

$$\int \frac{1}{(ax+3x^2)^{3/4}}\,dx = \frac{4x\left(1+\frac{3x}{a}\right)^{3/4}\operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{3x}{a}\right)}{(x(a+3x))^{3/4}}$$

input
```
Integrate[(a*x + 3*x^2)^(-3/4),x]
```

output
```
(4*x*(1 + (3*x)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, (-3*x)/a])/(x*(a
 + 3*x))^(3/4)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+3x^2)^{3/4}}\,dx$$

$$\Big\downarrow \ 1093$$

$$\frac{3^{3/4}\left(-\frac{ax+3x^2}{a^2}\right)^{3/4}\int \frac{1}{\left(-\frac{9x^2}{a^2}-\frac{3x}{a}\right)^{3/4}}dx}{(ax+3x^2)^{3/4}}$$

$$\Big\downarrow \ 1090$$

$$-\frac{\sqrt{2}a^2\left(-\frac{ax+3x^2}{a^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{1}{9}a^2\left(-\frac{18x}{a^2}-\frac{3}{a}\right)^2\right)^{3/4}}d\left(-\frac{18x}{a^2}-\frac{3}{a}\right)}{3\sqrt[4]{3}\,(ax+3x^2)^{3/4}}$$

$$\Big\downarrow \ 230$$

$$-\frac{2\sqrt{2}a\left(-\frac{ax+3x^2}{a^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{1}{3}a\left(-\frac{18x}{a^2}-\frac{3}{a}\right)\right),2\right)}{\sqrt[4]{3}\,(ax+3x^2)^{3/4}}$$

input
```
Int[(a*x + 3*x^2)^(-3/4),x]
```

output
```
(-2*Sqrt[2]*a*(-((a*x + 3*x^2)/a^2))^(3/4)*EllipticF[ArcSin[(a*(-3/a - (18
*x)/a^2))/3]/2, 2])/(3^(1/4)*(a*x + 3*x^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{(ax+3x^2)^{\frac{3}{4}}} dx$$

input
```
int(1/(a*x+3*x^2)^(3/4),x)
```

output
```
int(1/(a*x+3*x^2)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(ax+3x^2)^{3/4}} dx = \int \frac{1}{(ax+3\,x^2)^{\frac{3}{4}}} dx$$

input
```
integrate(1/(a*x+3*x^2)^(3/4),x, algorithm="fricas")
```

output
```
integral((a*x + 3*x^2)^(-3/4), x)
```

Sympy [F]

$$\int \frac{1}{(ax+3x^2)^{3/4}}\,dx = \int \frac{1}{(ax+3x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a*x+3*x**2)**(3/4),x)
```

output
```
Integral((a*x + 3*x**2)**(-3/4), x)
```

Maxima [F]

$$\int \frac{1}{(ax+3x^2)^{3/4}}\,dx = \int \frac{1}{(ax+3\,x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a*x+3*x^2)^(3/4),x, algorithm="maxima")
```

output
```
integrate((a*x + 3*x^2)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(ax+3x^2)^{3/4}}\,dx = \int \frac{1}{(ax+3\,x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a*x+3*x^2)^(3/4),x, algorithm="giac")
```

output
```
integrate((a*x + 3*x^2)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.51

$$\int \frac{1}{(ax+3x^2)^{3/4}}\,dx = \frac{4\,x\,\left(\frac{3\,x}{a}+1\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{5}{4};\ -\frac{3\,x}{a}\right)}{\left(3\,x^2+a\,x\right)^{3/4}}$$

input
```
int(1/(a*x + 3*x^2)^(3/4),x)
```

output
```
(4*x*((3*x)/a + 1)^(3/4)*hypergeom([1/4, 3/4], 5/4, -(3*x)/a))/(a*x + 3*x^
2)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(ax+3x^2)^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}\,(a+3x)^{\frac{3}{4}}}dx$$

input
```
int(1/(a*x+3*x^2)^(3/4),x)
```

output
```
int(1/(x**(3/4)*(a + 3*x)**(3/4)),x)
```

3.75 $\int \frac{1}{(2x-3x^2)^{3/4}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{(2x-3x^2)^{3/4}} \, dx = -\frac{2 \operatorname{EllipticF}\left(\frac{1}{2}\arcsin(1-3x),2\right)}{\sqrt[4]{3}}$$

output

```
2/3*InverseJacobiAM(1/2*arcsin(-1+3*x),2^(1/2))*3^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.26

$$\int \frac{1}{(2x-3x^2)^{3/4}} \, dx = -\frac{2\sqrt[4]{\frac{2}{3}}\sqrt[4]{-x(-2+3x)} \operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},1-\frac{3x}{2}\right)}{\sqrt[4]{x}}$$

input

```
Integrate[(2*x - 3*x^2)^(-3/4),x]
```

output

```
(-2*(2/3)^(1/4)*(-(x*(-2 + 3*x)))^(1/4)*Hypergeometric2F1[1/4, 3/4, 5/4, 1
 - (3*x)/2])/x^(1/4)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x-3x^2)^{3/4}}\,dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{\int \frac{1}{\left(1-\frac{1}{4}(2-6x)^2\right)^{3/4}}d(2-6x)}{2\sqrt[4]{3}}$$

$$\Big\downarrow \text{230}$$

$$-\frac{2\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{1}{2}(2-6x)\right),2\right)}{\sqrt[4]{3}}$$

input
```
Int[(2*x - 3*x^2)^(-3/4),x]
```

output
```
(-2*EllipticF[ArcSin[(2 - 6*x)/2]/2, 2])/3^(1/4)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

method	result	size
meijerg	$2\,2^{\frac{1}{4}} x^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{4}\right], \left[\frac{5}{4}\right], \frac{3x}{2}\right)$	18

input
```
int(1/(-3*x^2+2*x)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
2*2^(1/4)*x^(1/4)*hypergeom([1/4,3/4],[5/4],3/2*x)
```

Fricas [F]

$$\int \frac{1}{(2x-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2+2\,x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2*x)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 + 2*x)^(1/4)/(3*x^2 - 2*x), x)
```

Sympy [F]

$$\int \frac{1}{(2x-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3x^2+2x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x**2+2*x)**(3/4),x)
```

output
```
Integral((-3*x**2 + 2*x)**(-3/4), x)
```

Maxima [F]

$$\int \frac{1}{(2x-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2+2\,x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-3*x^2 + 2*x)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(2x-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2+2\,x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2*x)^(3/4),x, algorithm="giac")
```

output
```
integrate((-3*x^2 + 2*x)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.61 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{1}{(2x-3x^2)^{3/4}}\,dx = \frac{2\,2^{1/4}\,x\,(2-3\,x)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{5}{4};\ \frac{3\,x}{2}\right)}{(2\,x-3\,x^2)^{3/4}}$$

input
```
int(1/(2*x - 3*x^2)^(3/4),x)
```

output
```
(2*2^(1/4)*x*(2 - 3*x)^(3/4)*hypergeom([1/4, 3/4], 5/4, (3*x)/2))/(2*x - 3
*x^2)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(2x-3x^2)^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}\,(-3x+2)^{\frac{3}{4}}}dx$$

input
```
int(1/(-3*x^2+2*x)^(3/4),x)
```

output
```
int(1/(x**(3/4)*( - 3*x + 2)**(3/4)),x)
```

3.76 $\int \frac{1}{(-2x-3x^2)^{3/4}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{1}{(-2x-3x^2)^{3/4}} \, dx = \frac{2 \operatorname{EllipticF}\left(\frac{1}{2}\arcsin(1+3x),2\right)}{\sqrt[4]{3}}$$

output
```
2/3*InverseJacobiAM(1/2*arcsin(1+3*x),2^(1/2))*3^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.11

$$\int \frac{1}{(-2x-3x^2)^{3/4}} \, dx = -\frac{2\sqrt[4]{-x(2+3x)} \operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},-\frac{3x}{2}\right)}{\sqrt[4]{1+\frac{3x}{2}}}$$

input
```
Integrate[(-2*x - 3*x^2)^(-3/4),x]
```

output
```
(-2*(-(x*(2 + 3*x)))^(1/4)*Hypergeometric2F1[1/4, 3/4, 5/4, (-3*x)/2])/(1
+ (3*x)/2)^(1/4)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^2-2x)^{3/4}}\,dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{\int \frac{1}{\left(1-\frac{1}{4}(-6x-2)^2\right)^{3/4}}d(-6x-2)}{2\sqrt[4]{3}}$$

$$\Big\downarrow \text{ 230}$$

$$-\frac{2\,\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{1}{2}(-6x-2)\right),2\right)}{\sqrt[4]{3}}$$

input
```
Int[(-2*x - 3*x^2)^(-3/4),x]
```

output
```
(-2*EllipticF[ArcSin[(-2 - 6*x)/2]/2, 2])/3^(1/4)
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11

method	result	size
meijerg	$-2(-1)^{\frac{1}{4}} 2^{\frac{1}{4}} x^{\frac{1}{4}} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{3}{4}\right], \left[\frac{5}{4}\right], -\frac{3x}{2}\right)$	21

input
```
int(1/(-3*x^2-2*x)^(3/4),x,method=_RETURNVERBOSE)
```

output
```
-2*(-1)^(1/4)*2^(1/4)*x^(1/4)*hypergeom([1/4,3/4],[5/4],-3/2*x)
```

Fricas [F]

$$\int \frac{1}{(-2x-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2-2\,x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2*x)^(3/4),x, algorithm="fricas")
```

output
```
integral(-(-3*x^2 - 2*x)^(1/4)/(3*x^2 + 2*x), x)
```

Sympy [F]

$$\int \frac{1}{(-2x-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3x^2-2x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x**2-2*x)**(3/4),x)
```

output
```
Integral((-3*x**2 - 2*x)**(-3/4), x)
```

Maxima [F]

$$\int \frac{1}{(-2x-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2-2\,x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2*x)^(3/4),x, algorithm="maxima")
```

output
```
integrate((-3*x^2 - 2*x)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(-2x-3x^2)^{3/4}}\,dx = \int \frac{1}{(-3\,x^2-2\,x)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2*x)^(3/4),x, algorithm="giac")
```

output
```
integrate((-3*x^2 - 2*x)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.74 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.68

$$\int \frac{1}{(-2x-3x^2)^{3/4}}\,dx = \frac{2\,2^{1/4}\,x\,(3\,x+2)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{5}{4};\ -\frac{3\,x}{2}\right)}{(-3\,x^2-2\,x)^{3/4}}$$

input
```
int(1/(- 2*x - 3*x^2)^(3/4),x)
```

output
```
(2*2^(1/4)*x*(3*x + 2)^(3/4)*hypergeom([1/4, 3/4], 5/4, -(3*x)/2))/(- 2*x
- 3*x^2)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(-2x-3x^2)^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}(-3x-2)^{\frac{3}{4}}}dx$$

input
```
int(1/(-3*x^2-2*x)^(3/4),x)
```

output
```
int(1/(x**(3/4)*( - 3*x - 2)**(3/4)),x)
```

3.77 $\int \frac{1}{(ax-3x^2)^{3/4}} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{1}{(ax-3x^2)^{3/4}} \, dx = -\frac{2\sqrt{2}a\left(\frac{x}{a}-\frac{3x^2}{a^2}\right)^{3/4} \operatorname{EllipticF}\left(\frac{1}{2}\arcsin\left(1-\frac{6x}{a}\right),2\right)}{\sqrt[4]{3}\,(ax-3x^2)^{3/4}}$$

output
```
-2/3*2^(1/2)*a*(x/a-3*x^2/a^2)^(3/4)*InverseJacobiAM(1/2*arcsin(1-6*x/a),2
^(1/2))*3^(3/4)/(a*x-3*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.71

$$\int \frac{1}{(ax-3x^2)^{3/4}} \, dx = \frac{4x\left(1-\frac{3x}{a}\right)^{3/4} \operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{3}{4},\frac{5}{4},\frac{3x}{a}\right)}{((a-3x)x)^{3/4}}$$

input
```
Integrate[(a*x - 3*x^2)^(-3/4),x]
```

output
```
(4*x*(1 - (3*x)/a)^(3/4)*Hypergeometric2F1[1/4, 3/4, 5/4, (3*x)/a])/((a -
3*x)*x)^(3/4)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1093, 1090, 230}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax-3x^2)^{3/4}}\,dx$$

$$\Big\downarrow \text{1093}$$

$$\frac{3^{3/4}\left(\frac{ax-3x^2}{a^2}\right)^{3/4}\int \frac{1}{\left(\frac{3x}{a}-\frac{9x^2}{a^2}\right)^{3/4}}dx}{(ax-3x^2)^{3/4}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{\sqrt{2}a^2\left(\frac{ax-3x^2}{a^2}\right)^{3/4}\int \frac{1}{\left(1-\frac{1}{9}a^2\left(\frac{3}{a}-\frac{18x}{a^2}\right)^2\right)^{3/4}}d\left(\frac{3}{a}-\frac{18x}{a^2}\right)}{3\sqrt[4]{3}\,(ax-3x^2)^{3/4}}$$

$$\Big\downarrow \text{230}$$

$$-\frac{2\sqrt{2}a\left(\frac{ax-3x^2}{a^2}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arcsin\left(\frac{1}{3}a\left(\frac{3}{a}-\frac{18x}{a^2}\right)\right),2\right)}{\sqrt[4]{3}\,(ax-3x^2)^{3/4}}$$

input
```
Int[(a*x - 3*x^2)^(-3/4),x]
```

output
```
(-2*Sqrt[2]*a*((a*x - 3*x^2)/a^2)^(3/4)*EllipticF[ArcSin[(a*(3/a - (18*x)/
a^2))/3]/2, 2])/(3^(1/4)*(a*x - 3*x^2)^(3/4))
```

Defintions of rubi rules used

rule 230
```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2/(a^(3/4)*Rt[-b/a, 2]
))*EllipticF[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{(ax-3x^2)^{\frac{3}{4}}}dx$$

input
```
int(1/(a*x-3*x^2)^(3/4),x)
```

output
```
int(1/(a*x-3*x^2)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(ax-3x^2)^{3/4}}\,dx = \int \frac{1}{(ax-3\,x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a*x-3*x^2)^(3/4),x, algorithm="fricas")
```

output
```
integral((a*x - 3*x^2)^(-3/4), x)
```

Sympy [F]

$$\int \frac{1}{(ax-3x^2)^{3/4}}\,dx = \int \frac{1}{(ax-3x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a*x-3*x**2)**(3/4),x)
```

output
```
Integral((a*x - 3*x**2)**(-3/4), x)
```

Maxima [F]

$$\int \frac{1}{(ax-3x^2)^{3/4}}\,dx = \int \frac{1}{(ax-3\,x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a*x-3*x^2)^(3/4),x, algorithm="maxima")
```

output
```
integrate((a*x - 3*x^2)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(ax-3x^2)^{3/4}}\,dx = \int \frac{1}{(ax-3\,x^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(a*x-3*x^2)^(3/4),x, algorithm="giac")
```

output
```
integrate((a*x - 3*x^2)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.22 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.59

$$\int \frac{1}{(ax-3x^2)^{3/4}}\,dx = \frac{4\,x\,\left(1-\frac{3\,x}{a}\right)^{3/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{4};\ \frac{5}{4};\ \frac{3\,x}{a}\right)}{\left(a\,x-3\,x^2\right)^{3/4}}$$

input
```
int(1/(a*x - 3*x^2)^(3/4),x)
```

output
```
(4*x*(1 - (3*x)/a)^(3/4)*hypergeom([1/4, 3/4], 5/4, (3*x)/a))/(a*x - 3*x^2
)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(ax-3x^2)^{3/4}}\,dx = \int \frac{1}{x^{\frac{3}{4}}\,(a-3x)^{\frac{3}{4}}}dx$$

input
```
int(1/(a*x-3*x^2)^(3/4),x)
```

output
```
int(1/(x**(3/4)*(a - 3*x)**(3/4)),x)
```

3.78 $\int \frac{1}{(2x+3x^2)^{5/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 77

$$\int \frac{1}{(2x+3x^2)^{5/4}}\,dx = -\frac{2}{\sqrt[4]{2x+3x^2}} + \frac{2\sqrt{2}\sqrt[4]{3}\sqrt[4]{x}\sqrt[4]{2+3x}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{2}}{\sqrt{2+3x}}\right)\middle|2\right)}{\sqrt[4]{2x+3x^2}}$$

output
```
-2/(3*x^2+2*x)^(1/4)+2*2^(1/2)*3^(1/4)*x^(1/4)*(2+3*x)^(1/4)*EllipticE(sin
(1/2*arcsin(2^(1/2)/(2+3*x)^(1/2)),2^(1/2))/(3*x^2+2*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.56

$$\int \frac{1}{(2x+3x^2)^{5/4}}\,dx = -\frac{2^{3/4}x(2+3x)^{5/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},-\frac{3x}{2}\right)}{(x(2+3x))^{5/4}}$$

input
```
Integrate[(2*x + 3*x^2)^(-5/4),x]
```

output
```
-((2^(3/4)*x*(2 + 3*x)^(5/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, (-3*x)/2])/
(x*(2 + 3*x))^(5/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1089, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2+2x)^{5/4}}dx$$

$$\Big\downarrow \text{ 1089}$$

$$3\int \frac{1}{\sqrt[4]{3x^2+2x}}dx-\frac{2(3x+1)}{\sqrt[4]{3x^2+2x}}$$

$$\Big\downarrow \text{ 1093}$$

$$\frac{3\sqrt[4]{3}\sqrt[4]{-3x^2-2x}\int \frac{1}{\sqrt[4]{-\frac{9x^2}{4}-\frac{3x}{2}}}dx}{\sqrt{2}\sqrt[4]{3x^2+2x}}-\frac{2(3x+1)}{\sqrt[4]{3x^2+2x}}$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{2\sqrt[4]{-3x^2-2x}\int \frac{1}{\sqrt[4]{1-\frac{4}{9}\left(-\frac{9x}{2}-\frac{3}{2}\right)^2}}d\left(-\frac{9x}{2}-\frac{3}{2}\right)}{3^{3/4}\sqrt[4]{3x^2+2x}}-\frac{2(3x+1)}{\sqrt[4]{3x^2+2x}}$$

$$\Big\downarrow \text{ 226}$$

$$-\frac{2\sqrt[4]{3}\sqrt[4]{-3x^2-2x}E\left(\frac{1}{2}\arcsin\left(\frac{2}{3}\left(-\frac{9x}{2}-\frac{3}{2}\right)\right)\middle|2\right)}{\sqrt[4]{3x^2+2x}}-\frac{2(3x+1)}{\sqrt[4]{3x^2+2x}}$$

input
```
Int[(2*x + 3*x^2)^(-5/4),x]
```

output
```
(-2*(1 + 3*x))/(2*x + 3*x^2)^(1/4) - (2*3^(1/4)*(-2*x - 3*x^2)^(1/4)*Ellip
ticE[ArcSin[(2*(-3/2 - (9*x)/2))/3]/2, 2])/(2*x + 3*x^2)^(1/4)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.23

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{4}, \frac{5}{4}\right], \left[\frac{3}{4}\right], -\frac{3x}{2}\right)}{x^{\frac{1}{4}}}$	18

input
```
int(1/(3*x^2+2*x)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-2^(3/4)/x^(1/4)*hypergeom([-1/4,5/4],[3/4],-3/2*x)
```

Fricas [F]

$$\int \frac{1}{(2x+3x^2)^{5/4}}\,dx = \int \frac{1}{(3\,x^2+2\,x)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2*x)^(5/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2*x)^(3/4)/(9*x^4 + 12*x^3 + 4*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(2x+3x^2)^{5/4}}\,dx = \int \frac{1}{(3x^2+2x)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(3*x**2+2*x)**(5/4),x)
```

output
```
Integral((3*x**2 + 2*x)**(-5/4), x)
```

Maxima [F]

$$\int \frac{1}{(2x+3x^2)^{5/4}}\,dx = \int \frac{1}{(3\,x^2+2\,x)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2*x)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(2x+3x^2)^{5/4}}\,dx = \int \frac{1}{(3\,x^2+2\,x)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(3*x^2+2*x)^(5/4),x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2*x)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.42

$$\int \frac{1}{(2x+3x^2)^{5/4}}\,dx = -\frac{2^{3/4}\,x\,(3\,x+2)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{5}{4};\ \frac{3}{4};\ -\frac{3\,x}{2}\right)}{(3\,x^2+2\,x)^{5/4}}$$

input
```
int(1/(2*x + 3*x^2)^(5/4),x)
```

output
```
-(2^(3/4)*x*(3*x + 2)^(5/4)*hypergeom([-1/4, 5/4], 3/4, -(3*x)/2))/(2*x +
3*x^2)^(5/4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.31

$$\int \frac{1}{(2x+3x^2)^{5/4}}\,dx = \frac{2x^{\frac{1}{4}}(3x+2)^{\frac{1}{4}}}{\sqrt{x}\,\sqrt{3x+2}}$$

input
```
int(1/(3*x^2+2*x)^(5/4),x)
```

output
```
(2*x**(1/4)*(3*x + 2)**(1/4))/(sqrt(x)*sqrt(3*x + 2))
```

3.79 $\int \frac{1}{(-2x+3x^2)^{5/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 75

$$\int \frac{1}{(-2x+3x^2)^{5/4}}\,dx = -\frac{2}{\sqrt[4]{-2x+3x^2}} + \frac{2\sqrt{2}\sqrt[4]{3}\sqrt[4]{x}\sqrt[4]{-2+3x}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right)\middle|2\right)}{\sqrt[4]{-2x+3x^2}}$$

output
```
-2/(3*x^2-2*x)^(1/4)+2*2^(1/2)*3^(1/4)*x^(1/4)*(-2+3*x)^(1/4)*EllipticE(si
n(1/2*arcsin(1/3*6^(1/2)/x^(1/2))),2^(1/2))/(3*x^2-2*x)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.60

$$\int \frac{1}{(-2x+3x^2)^{5/4}}\,dx = -\frac{2^{3/4}\sqrt[4]{3}\sqrt[4]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},1-\frac{3x}{2}\right)}{\sqrt[4]{x(-2+3x)}}$$

input
```
Integrate[(-2*x + 3*x^2)^(-5/4),x]
```

output
```
-((2^(3/4)*3^(1/4)*x^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, 1 - (3*x)/2])
/(x*(-2 + 3*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1089, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3x^2-2x)^{5/4}}dx$$

$$\Big\downarrow \text{1089}$$

$$3\int \frac{1}{\sqrt[4]{3x^2-2x}}dx+\frac{2(1-3x)}{\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \text{1093}$$

$$\frac{3\sqrt[4]{3}\sqrt[4]{2x-3x^2}\int \frac{1}{\sqrt[4]{\frac{3x}{2}-\frac{9x^2}{4}}}dx}{\sqrt{2}\sqrt[4]{3x^2-2x}}+\frac{2(1-3x)}{\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{2(1-3x)}{\sqrt[4]{3x^2-2x}}-\frac{2\sqrt[4]{2x-3x^2}\int \frac{1}{\sqrt[4]{1-\frac{4}{9}\left(\frac{3}{2}-\frac{9x}{2}\right)^2}}d\left(\frac{3}{2}-\frac{9x}{2}\right)}{3^{3/4}\sqrt[4]{3x^2-2x}}$$

$$\Big\downarrow \text{226}$$

$$\frac{2(1-3x)}{\sqrt[4]{3x^2-2x}}-\frac{2\sqrt[4]{3}\sqrt[4]{2x-3x^2}E\left(\frac{1}{2}\arcsin\left(\frac{2}{3}\left(\frac{3}{2}-\frac{9x}{2}\right)\right)\middle|2\right)}{\sqrt[4]{3x^2-2x}}$$

input
```
Int[(-2*x + 3*x^2)^(-5/4),x]
```

output
```
(2*(1 - 3*x))/(-2*x + 3*x^2)^(1/4) - (2*3^(1/4)*(2*x - 3*x^2)^(1/4)*Ellipt
icE[ArcSin[(2*(3/2 - (9*x)/2))/3]/2, 2])/(-2*x + 3*x^2)^(1/4)
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.43

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}} \left(-\operatorname{signum}\left(x-\frac{2}{3}\right)\right)^{\frac{5}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{4},\frac{5}{4}\right],\left[\frac{3}{4}\right],\frac{3x}{2}\right)}{\operatorname{signum}\left(x-\frac{2}{3}\right)^{\frac{5}{4}} x^{\frac{1}{4}}}$	32

input
```
int(1/(3*x^2-2*x)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-2^(3/4)/signum(x-2/3)^(5/4)*(-signum(x-2/3))^(5/4)/x^(1/4)*hypergeom([-1/
4,5/4],[3/4],3/2*x)
```

Fricas [F]

$$\int \frac{1}{\left(-2x+3x^2\right)^{5/4}}\,dx=\int \frac{1}{\left(3\,x^{2}-2\,x\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(3*x^2-2*x)^(5/4),x, algorithm="fricas")
```

output
```
integral((3*x^2 - 2*x)^(3/4)/(9*x^4 - 12*x^3 + 4*x^2), x)
```

Sympy [F]

$$\int \frac{1}{\left(-2x+3x^2\right)^{5/4}}\,dx=\int \frac{1}{\left(3x^{2}-2x\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(3*x**2-2*x)**(5/4),x)
```

output
```
Integral((3*x**2 - 2*x)**(-5/4), x)
```

Maxima [F]

$$\int \frac{1}{\left(-2x+3x^2\right)^{5/4}}\,dx=\int \frac{1}{\left(3\,x^{2}-2\,x\right)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(3*x^2-2*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate((3*x^2 - 2*x)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(-2x+3x^2)^{5/4}}\,dx = \int \frac{1}{(3\,x^2-2\,x)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(3*x^2-2*x)^(5/4),x, algorithm="giac")
```

output
```
integrate((3*x^2 - 2*x)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.43

$$\int \frac{1}{(-2x+3x^2)^{5/4}}\,dx = -\frac{2^{3/4}\,x\,(2-3\,x)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{5}{4};\ \frac{3}{4};\ \frac{3\,x}{2}\right)}{(3\,x^2-2\,x)^{5/4}}$$

input
```
int(1/(3*x^2 - 2*x)^(5/4),x)
```

output
```
-(2^(3/4)*x*(2 - 3*x)^(5/4)*hypergeom([-1/4, 5/4], 3/4, (3*x)/2))/(3*x^2 -
 2*x)^(5/4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.32

$$\int \frac{1}{(-2x+3x^2)^{5/4}}\,dx = -\frac{2x^{\frac{1}{4}}(3x-2)^{\frac{1}{4}}}{\sqrt{x}\,\sqrt{3x-2}}$$

input
```
int(1/(3*x^2-2*x)^(5/4),x)
```

output
```
( - 2*x**(1/4)*(3*x - 2)**(1/4))/(sqrt(x)*sqrt(3*x - 2))
```

3.80 $\int \frac{1}{(ax+3x^2)^{5/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 88

$$\int \frac{1}{(ax+3x^2)^{5/4}}\,dx = -\frac{4}{a\sqrt[4]{ax+3x^2}} + \frac{8\sqrt[4]{3}\sqrt[4]{\frac{x}{a+3x}}\sqrt{a+3x}E\left(\frac{1}{2}\arcsin\left(\frac{\sqrt{a}}{\sqrt{a+3x}}\right)\middle|2\right)}{a^{3/2}\sqrt[4]{ax+3x^2}}$$

output
```
-4/a/(a*x+3*x^2)^(1/4)+8*3^(1/4)*(x/(a+3*x))^(1/4)*(a+3*x)^(1/2)*EllipticE
(sin(1/2*arcsin(a^(1/2)/(a+3*x)^(1/2))),2^(1/2))/a^(3/2)/(a*x+3*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.50

$$\int \frac{1}{(ax+3x^2)^{5/4}}\,dx = -\frac{4\sqrt[4]{1+\frac{3x}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},-\frac{3x}{a}\right)}{a\sqrt[4]{x(a+3x)}}$$

input
```
Integrate[(a*x + 3*x^2)^(-5/4),x]
```

output
```
(-4*(1 + (3*x)/a)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, (-3*x)/a])/(a*(x
*(a + 3*x))^(1/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {1089, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax+3x^2)^{5/4}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{12\int \frac{1}{\sqrt[4]{3x^2+ax}}dx}{a^2}-\frac{4(a+6x)}{a^2\sqrt[4]{ax+3x^2}}$$

$$\Big\downarrow \text{1093}$$

$$\frac{12\sqrt[4]{3}\sqrt[4]{-\frac{ax+3x^2}{a^2}}\int \frac{1}{\sqrt[4]{-\frac{9x^2}{a^2}-\frac{3x}{a}}}dx}{a^2\sqrt[4]{ax+3x^2}}-\frac{4(a+6x)}{a^2\sqrt[4]{ax+3x^2}}$$

$$\Big\downarrow \text{1090}$$

$$-\frac{2\sqrt{2}\sqrt[4]{-\frac{ax+3x^2}{a^2}}\int \frac{1}{\sqrt[4]{1-\frac{1}{9}a^2\left(-\frac{18x}{a^2}-\frac{3}{a}\right)^2}}d\left(-\frac{18x}{a^2}-\frac{3}{a}\right)}{3^{3/4}\sqrt[4]{ax+3x^2}}-\frac{4(a+6x)}{a^2\sqrt[4]{ax+3x^2}}$$

$$\Big\downarrow \text{226}$$

$$-\frac{4\sqrt{2}\sqrt[4]{3}\sqrt[4]{-\frac{ax+3x^2}{a^2}}E\left(\frac{1}{2}\arcsin\left(\frac{1}{3}a\left(-\frac{18x}{a^2}-\frac{3}{a}\right)\right)|2\right)}{a\sqrt[4]{ax+3x^2}}-\frac{4(a+6x)}{a^2\sqrt[4]{ax+3x^2}}$$

input
```
Int[(a*x + 3*x^2)^(-5/4),x]
```

output
```
(-4*(a + 6*x))/(a^2*(a*x + 3*x^2)^(1/4)) - (4*Sqrt[2]*3^(1/4)*(-((a*x + 3*
x^2)/a^2))^(1/4)*EllipticE[ArcSin[(a*(-3/a - (18*x)/a^2))/3]/2, 2])/(a*(a*
x + 3*x^2)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{(ax+3x^2)^{\frac{5}{4}}}dx$$

input
```
int(1/(a*x+3*x^2)^(5/4),x)
```

output
```
int(1/(a*x+3*x^2)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{(ax+3x^2)^{5/4}}\,dx = \int \frac{1}{(ax+3\,x^2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a*x+3*x^2)^(5/4),x, algorithm="fricas")
```

output
```
integral((a*x + 3*x^2)^(3/4)/(a^2*x^2 + 6*a*x^3 + 9*x^4), x)
```

Sympy [F]

$$\int \frac{1}{(ax+3x^2)^{5/4}}\,dx = \int \frac{1}{(ax+3x^2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a*x+3*x**2)**(5/4),x)
```

output
```
Integral((a*x + 3*x**2)**(-5/4), x)
```

Maxima [F]

$$\int \frac{1}{(ax+3x^2)^{5/4}}\,dx = \int \frac{1}{(ax+3\,x^2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a*x+3*x^2)^(5/4),x, algorithm="maxima")
```

output
```
integrate((a*x + 3*x^2)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(ax+3x^2)^{5/4}}\,dx = \int \frac{1}{(ax+3\,x^2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a*x+3*x^2)^(5/4),x, algorithm="giac")
```

output
```
integrate((a*x + 3*x^2)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.40

$$\int \frac{1}{(ax+3x^2)^{5/4}}\,dx = -\frac{4\,x\,\left(\frac{3\,x}{a}+1\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{5}{4};\ \frac{3}{4};\ -\frac{3\,x}{a}\right)}{(3\,x^2+a\,x)^{5/4}}$$

input
```
int(1/(a*x + 3*x^2)^(5/4),x)
```

output
```
-(4*x*((3*x)/a + 1)^(5/4)*hypergeom([-1/4, 5/4], 3/4, -(3*x)/a))/(a*x + 3*
x^2)^(5/4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.31

$$\int \frac{1}{(ax+3x^2)^{5/4}}\,dx = \frac{4x^{\frac{1}{4}}(a+3x)^{\frac{1}{4}}}{\sqrt{x}\,\sqrt{a+3x}\,a}$$

input
```
int(1/(a*x+3*x^2)^(5/4),x)
```

output
```
(4*x**(1/4)*(a + 3*x)**(1/4))/(sqrt(x)*sqrt(a + 3*x)*a)
```

3.81 $\int \frac{1}{(2x-3x^2)^{5/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 52

$$\int \frac{1}{(2x-3x^2)^{5/4}}\,dx = \frac{2}{\sqrt[4]{2-3x}\sqrt[4]{x}} - \frac{2(2-3x)^{3/4}}{\sqrt[4]{x}} + 2\sqrt[4]{3}E\left(\frac{1}{2}\arcsin(1-3x)\middle|2\right)$$

output
```
2/(2-3*x)^(1/4)/x^(1/4)-2*(2-3*x)^(3/4)/x^(1/4)-2*EllipticE(sin(1/2*arcsin
(-1+3*x)),2^(1/2))*3^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{1}{(2x-3x^2)^{5/4}}\,dx = \frac{2^{3/4}\sqrt[4]{3}\sqrt[4]{x}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},1-\frac{3x}{2}\right)}{\sqrt[4]{-x(-2+3x)}}$$

input
```
Integrate[(2*x - 3*x^2)^(-5/4),x]
```

output
```
(2^(3/4)*3^(1/4)*x^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, 1 - (3*x)/2])/(
-(x*(-2 + 3*x)))^(1/4)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1089, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2x-3x^2)^{5/4}} \, dx$$

$$\Big\downarrow \text{1089}$$

$$-3\int \frac{1}{\sqrt[4]{2x-3x^2}}dx-\frac{2(1-3x)}{\sqrt[4]{2x-3x^2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2}\sqrt[4]{3}\int \frac{1}{\sqrt[4]{1-\frac{1}{4}(2-6x)^2}}d(2-6x)-\frac{2(1-3x)}{\sqrt[4]{2x-3x^2}}$$

$$\Big\downarrow \text{226}$$

$$2\sqrt[4]{3}E\left(\frac{1}{2}\arcsin\left(\frac{1}{2}(2-6x)\right)\middle|2\right)-\frac{2(1-3x)}{\sqrt[4]{2x-3x^2}}$$

input
```
Int[(2*x - 3*x^2)^(-5/4),x]
```

output
```
(-2*(1 - 3*x))/(2*x - 3*x^2)^(1/4) + 2*3^(1/4)*EllipticE[ArcSin[(2 - 6*x)/
2]/2, 2]
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.29 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.35

method	result	size
meijerg	$-\frac{2^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{4}, \frac{5}{4}\right], \left[\frac{3}{4}\right], \frac{3x}{2}\right)}{x^{\frac{1}{4}}}$	18

input
```
int(1/(-3*x^2+2*x)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-2^(3/4)/x^(1/4)*hypergeom([-1/4,5/4],[3/4],3/2*x)
```

Fricas [F]

$$\int \frac{1}{(2x-3x^2)^{5/4}}\,dx = \int \frac{1}{(-3\,x^2+2\,x)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2*x)^(5/4),x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2*x)^(3/4)/(9*x^4 - 12*x^3 + 4*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(2x-3x^2)^{5/4}}\,dx = \int \frac{1}{(-3x^2+2x)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-3*x**2+2*x)**(5/4),x)
```

output
```
Integral((-3*x**2 + 2*x)**(-5/4), x)
```

Maxima [F]

$$\int \frac{1}{(2x-3x^2)^{5/4}}\,dx = \int \frac{1}{(-3\,x^2+2\,x)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate((-3*x^2 + 2*x)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(2x-3x^2)^{5/4}}\,dx = \int \frac{1}{(-3\,x^2+2\,x)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2+2*x)^(5/4),x, algorithm="giac")
```

output
```
integrate((-3*x^2 + 2*x)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 8.80 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.62

$$\int \frac{1}{(2x-3x^2)^{5/4}}\,dx = -\frac{2^{3/4}\,x\,(2-3\,x)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{5}{4};\ \frac{3}{4};\ \frac{3\,x}{2}\right)}{(2\,x-3\,x^2)^{5/4}}$$

input
```
int(1/(2*x - 3*x^2)^(5/4),x)
```

output
```
-(2^(3/4)*x*(2 - 3*x)^(5/4)*hypergeom([-1/4, 5/4], 3/4, (3*x)/2))/(2*x - 3
*x^2)^(5/4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.46

$$\int \frac{1}{(2x-3x^2)^{5/4}}\,dx = \frac{2x^{\frac{1}{4}}(-3x+2)^{\frac{1}{4}}}{\sqrt{x}\,\sqrt{-3x+2}}$$

input
```
int(1/(-3*x^2+2*x)^(5/4),x)
```

output
```
(2*x**(1/4)*( - 3*x + 2)**(1/4))/(sqrt(x)*sqrt( - 3*x + 2))
```

3.82 $\int \frac{1}{(-2x-3x^2)^{5/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 53

$$\int \frac{1}{(-2x-3x^2)^{5/4}}\,dx = -\frac{2}{\sqrt[4]{-2x-3x^2}}-\frac{2(-2x-3x^2)^{3/4}}{x}-2\sqrt[4]{3}E\left(\frac{1}{2}\arcsin(1+3x)\middle|2\right)$$

output
```
-2/(-3*x^2-2*x)^(1/4)-2*(-3*x^2-2*x)^(3/4)/x-2*EllipticE(sin(1/2*arcsin(1+
3*x)),2^(1/2))*3^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{1}{(-2x-3x^2)^{5/4}}\,dx = -\frac{2^{3/4}x(2+3x)^{5/4}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},-\frac{3x}{2}\right)}{(-x(2+3x))^{5/4}}$$

input
```
Integrate[(-2*x - 3*x^2)^(-5/4),x]
```

output
```
-((2^(3/4)*x*(2 + 3*x)^(5/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, (-3*x)/2])/
(-(x*(2 + 3*x)))^(5/4))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1089, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(-3x^2-2x)^{5/4}}dx$$

$$\Big\downarrow \text{1089}$$

$$\frac{2(3x+1)}{\sqrt[4]{-3x^2-2x}}-3\int \frac{1}{\sqrt[4]{-3x^2-2x}}dx$$

$$\Big\downarrow \text{1090}$$

$$\frac{1}{2}\sqrt[4]{3}\int \frac{1}{\sqrt[4]{1-\frac{1}{4}(-6x-2)^2}}d(-6x-2)+\frac{2(3x+1)}{\sqrt[4]{-3x^2-2x}}$$

$$\Big\downarrow \text{226}$$

$$2\sqrt[4]{3}E\left(\frac{1}{2}\arcsin\left(\frac{1}{2}(-6x-2)\right)\middle|2\right)+\frac{2(3x+1)}{\sqrt[4]{-3x^2-2x}}$$

input
```
Int[(-2*x - 3*x^2)^(-5/4),x]
```

output
```
(2*(1 + 3*x))/(-2*x - 3*x^2)^(1/4) + 2*3^(1/4)*EllipticE[ArcSin[(-2 - 6*x)
/2]/2, 2]
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.40

method	result	size
meijerg	$-\frac{(-1)^{\frac{3}{4}} 2^{\frac{3}{4}} \operatorname{hypergeom}\left(\left[-\frac{1}{4}, \frac{5}{4}\right], \left[\frac{3}{4}\right], -\frac{3x}{2}\right)}{x^{\frac{1}{4}}}$	21

input
```
int(1/(-3*x^2-2*x)^(5/4),x,method=_RETURNVERBOSE)
```

output
```
-(-1)^(3/4)*2^(3/4)/x^(1/4)*hypergeom([-1/4,5/4],[3/4],-3/2*x)
```

Fricas [F]

$$\int \frac{1}{(-2x-3x^2)^{5/4}}\,dx = \int \frac{1}{(-3\,x^2-2\,x)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2*x)^(5/4),x, algorithm="fricas")
```

output
```
integral((-3*x^2 - 2*x)^(3/4)/(9*x^4 + 12*x^3 + 4*x^2), x)
```

Sympy [F]

$$\int \frac{1}{(-2x-3x^2)^{5/4}}\,dx = \int \frac{1}{(-3x^2-2x)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-3*x**2-2*x)**(5/4),x)
```

output
```
Integral((-3*x**2 - 2*x)**(-5/4), x)
```

Maxima [F]

$$\int \frac{1}{(-2x-3x^2)^{5/4}}\,dx = \int \frac{1}{(-3\,x^2-2\,x)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2*x)^(5/4),x, algorithm="maxima")
```

output
```
integrate((-3*x^2 - 2*x)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(-2x-3x^2)^{5/4}}\,dx = \int \frac{1}{(-3\,x^2-2\,x)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(-3*x^2-2*x)^(5/4),x, algorithm="giac")
```

output
```
integrate((-3*x^2 - 2*x)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

$$\int \frac{1}{(-2x-3x^2)^{5/4}}\,dx = -\frac{2^{3/4}\,x\,(3\,x+2)^{5/4}\,{}_2F_1\left(-\frac{1}{4},\frac{5}{4};\ \frac{3}{4};\ -\frac{3\,x}{2}\right)}{(-3\,x^2-2\,x)^{5/4}}$$

input
```
int(1/(- 2*x - 3*x^2)^(5/4),x)
```

output
```
-(2^(3/4)*x*(3*x + 2)^(5/4)*hypergeom([-1/4, 5/4], 3/4, -(3*x)/2))/(- 2*x
- 3*x^2)^(5/4)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.45

$$\int \frac{1}{(-2x-3x^2)^{5/4}}\,dx = -\frac{2x^{\frac{1}{4}}(-3x-2)^{\frac{1}{4}}}{\sqrt{x}\,\sqrt{-3x-2}}$$

input
```
int(1/(-3*x^2-2*x)^(5/4),x)
```

output
```
( - 2*x**(1/4)*( - 3*x - 2)**(1/4))/(sqrt(x)*sqrt( - 3*x - 2))
```

3.83 $\int \frac{1}{(ax-3x^2)^{5/4}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 101

$$\int \frac{1}{(ax-3x^2)^{5/4}}\,dx = \frac{4}{a\sqrt[4]{ax-3x^2}} - \frac{8(ax-3x^2)^{3/4}}{a^2x} + \frac{4\sqrt{2}\sqrt[4]{3}\sqrt[4]{\frac{x}{a}-\frac{3x^2}{a^2}}\,E\left(\frac{1}{2}\arcsin\left(1-\frac{6x}{a}\right)\middle|2\right)}{a\sqrt[4]{ax-3x^2}}$$

output
```
4/a/(a*x-3*x^2)^(1/4)-8*(a*x-3*x^2)^(3/4)/a^2/x+4*2^(1/2)*3^(1/4)*(x/a-3*x
^2/a^2)^(1/4)*EllipticE(sin(1/2*arcsin(1-6*x/a)),2^(1/2))/a/(a*x-3*x^2)^(1
/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.44

$$\int \frac{1}{(ax-3x^2)^{5/4}}\,dx = -\frac{4\sqrt[4]{1-\frac{3x}{a}}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{5}{4},\frac{3}{4},\frac{3x}{a}\right)}{a\sqrt[4]{(a-3x)x}}$$

input
```
Integrate[(a*x - 3*x^2)^(-5/4),x]
```

output
```
(-4*(1 - (3*x)/a)^(1/4)*Hypergeometric2F1[-1/4, 5/4, 3/4, (3*x)/a])/(a*((a
 - 3*x)*x)^(1/4))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1089, 1093, 1090, 226}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax-3x^2)^{5/4}}dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{12\int \frac{1}{\sqrt[4]{ax-3x^2}}dx}{a^2}-\frac{4(a-6x)}{a^2\sqrt[4]{ax-3x^2}}$$

$$\Big\downarrow \text{1093}$$

$$-\frac{12\sqrt[4]{3}\sqrt[4]{\frac{ax-3x^2}{a^2}}\int \frac{1}{\sqrt[4]{\frac{3x}{a}-\frac{9x^2}{a^2}}}dx}{a^2\sqrt[4]{ax-3x^2}}-\frac{4(a-6x)}{a^2\sqrt[4]{ax-3x^2}}$$

$$\Big\downarrow \text{1090}$$

$$\frac{2\sqrt{2}\sqrt[4]{\frac{ax-3x^2}{a^2}}\int \frac{1}{\sqrt[4]{1-\frac{1}{9}a^2\left(\frac{3}{a}-\frac{18x}{a^2}\right)^2}}d\left(\frac{3}{a}-\frac{18x}{a^2}\right)}{3^{3/4}\sqrt[4]{ax-3x^2}}-\frac{4(a-6x)}{a^2\sqrt[4]{ax-3x^2}}$$

$$\Big\downarrow \text{226}$$

$$\frac{4\sqrt{2}\sqrt[4]{3}\sqrt[4]{\frac{ax-3x^2}{a^2}}E\left(\frac{1}{2}\arcsin\left(\frac{1}{3}a\left(\frac{3}{a}-\frac{18x}{a^2}\right)\right)\middle|2\right)}{a\sqrt[4]{ax-3x^2}}-\frac{4(a-6x)}{a^2\sqrt[4]{ax-3x^2}}$$

input
```
Int[(a*x - 3*x^2)^(-5/4),x]
```

output
```
(-4*(a - 6*x))/(a^2*(a*x - 3*x^2)^(1/4)) + (4*Sqrt[2]*3^(1/4)*((a*x - 3*x^
2)/a^2)^(1/4)*EllipticE[ArcSin[(a*(3/a - (18*x)/a^2))/3]/2, 2])/(a*(a*x -
3*x^2)^(1/4))
```

Defintions of rubi rules used

rule 226
```
Int[((a_) + (b_.)*(x_)^2)^(-1/4), x_Symbol] :> Simp[(2/(a^(1/4)*Rt[-b/a, 2]
))*EllipticE[(1/2)*ArcSin[Rt[-b/a, 2]*x], 2], x] /; FreeQ[{a, b}, x] && GtQ
[a, 0] && NegQ[b/a]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

rule 1093
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b*x + c*x^2)^p/((-
c)*((b*x + c*x^2)/b^2))^p   Int[((-c)*(x/b) - c^2*(x^2/b^2))^p, x], x] /; F
reeQ[{b, c}, x] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [F]

$$\int \frac{1}{(ax - 3x^2)^{\frac{5}{4}}}dx$$

input
```
int(1/(a*x-3*x^2)^(5/4),x)
```

output
```
int(1/(a*x-3*x^2)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{(ax - 3x^2)^{5/4}}\, dx = \int \frac{1}{(ax - 3\,x^2)^{\frac{5}{4}}}\, dx$$

input
```
integrate(1/(a*x-3*x^2)^(5/4),x, algorithm="fricas")
```

output
```
integral((a*x - 3*x^2)^(3/4)/(a^2*x^2 - 6*a*x^3 + 9*x^4), x)
```

Sympy [F]

$$\int \frac{1}{(ax - 3x^2)^{5/4}}\, dx = \int \frac{1}{(ax - 3x^2)^{\frac{5}{4}}}\, dx$$

input
```
integrate(1/(a*x-3*x**2)**(5/4),x)
```

output
```
Integral((a*x - 3*x**2)**(-5/4), x)
```

Maxima [F]

$$\int \frac{1}{(ax - 3x^2)^{5/4}}\, dx = \int \frac{1}{(ax - 3\,x^2)^{\frac{5}{4}}}\, dx$$

input
```
integrate(1/(a*x-3*x^2)^(5/4),x, algorithm="maxima")
```

output
```
integrate((a*x - 3*x^2)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(ax-3x^2)^{5/4}}\,dx = \int \frac{1}{(ax-3\,x^2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(a*x-3*x^2)^(5/4),x, algorithm="giac")
```

output
```
integrate((a*x - 3*x^2)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.35

$$\int \frac{1}{(ax-3x^2)^{5/4}}\,dx = -\frac{4\,x\,\left(1-\frac{3\,x}{a}\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{5}{4};\ \frac{3}{4};\ \frac{3\,x}{a}\right)}{(a\,x-3\,x^2)^{5/4}}$$

input
```
int(1/(a*x - 3*x^2)^(5/4),x)
```

output
```
-(4*x*(1 - (3*x)/a)^(5/4)*hypergeom([-1/4, 5/4], 3/4, (3*x)/a))/(a*x - 3*x
^2)^(5/4)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.27

$$\int \frac{1}{(ax-3x^2)^{5/4}}\,dx = \frac{4x^{\frac{1}{4}}(a-3x)^{\frac{1}{4}}}{\sqrt{x}\,\sqrt{a-3x}\,a}$$

input
```
int(1/(a*x-3*x^2)^(5/4),x)
```

output
```
(4*x**(1/4)*(a - 3*x)**(1/4))/(sqrt(x)*sqrt(a - 3*x)*a)
```

3.84 $\int (bx + cx^2)^p \, dx$

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int (bx + cx^2)^p \, dx = \frac{(bx + cx^2)^{1+p} \text{Hypergeometric2F1}\left(1, 2(1+p), 2+p, -\frac{cx}{b}\right)}{b(1+p)}$$

output
```
(c*x^2+b*x)^(p+1)*hypergeom([1, 2*p+2],[2+p],-c*x/b)/b/(p+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int (bx+cx^2)^p \, dx = \frac{x(x(b+cx))^p \left(1 + \frac{cx}{b}\right)^{-p} \text{Hypergeometric2F1}\left(-p, 1+p, 2+p, -\frac{cx}{b}\right)}{1+p}$$

input
```
Integrate[(b*x + c*x^2)^p,x]
```

output
```
(x*(x*(b + c*x))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x)/b)])/((1 + 
p)*(1 + (c*x)/b)^p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx + cx^2)^p \, dx$$

$$\Big\downarrow \text{1096}$$

$$-\frac{\left(-\frac{cx}{b}\right)^{-p-1} (bx + cx^2)^{p+1} \text{Hypergeometric2F1}\left(-p, p+1, p+2, \frac{b+cx}{b}\right)}{b(p+1)}$$

input
```
Int[(b*x + c*x^2)^p,x]
```

output
```
-(((-((c*x)/b))^(-1 - p)*(b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p
, 2 + p, (b + c*x)/b])/(b*(1 + p)))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

Maple [F]

$$\int (c\,x^2 + bx)^p \, dx$$

input
```
int((c*x^2+b*x)^p,x)
```

output
```
int((c*x^2+b*x)^p,x)
```

Fricas [F]

$$\int \left(bx+cx^2\right)^p\,dx=\int \left(cx^2+bx\right)^p\,dx$$

input
```
integrate((c*x^2+b*x)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^p, x)
```

Sympy [F]

$$\int \left(bx+cx^2\right)^p\,dx=\int \left(bx+cx^2\right)^p\,dx$$

input
```
integrate((c*x**2+b*x)**p,x)
```

output
```
Integral((b*x + c*x**2)**p, x)
```

Maxima [F]

$$\int \left(bx+cx^2\right)^p\,dx=\int \left(cx^2+bx\right)^p\,dx$$

input
```
integrate((c*x^2+b*x)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^p, x)
```

Giac [F]

$$\int \left(bx + cx^2\right)^p \, dx = \int \left(cx^2 + bx\right)^p \, dx$$

input
```
integrate((c*x^2+b*x)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 9.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int \left(bx + cx^2\right)^p \, dx = \frac{x\left(c\,x^2 + b\,x\right)^p {}_2F_1\left(-p, p+1;\ p+2;\ -\frac{c\,x}{b}\right)}{\left(\frac{c\,x}{b} + 1\right)^p \,(p+1)}$$

input
```
int((b*x + c*x^2)^p,x)
```

output
```
(x*(b*x + c*x^2)^p*hypergeom([-p, p + 1], p + 2, -(c*x)/b))/((((c*x)/b + 1)
^p*(p + 1))
```

Reduce [F]

$$\int \left(bx + cx^2\right)^p \, dx = \frac{\left(c\,x^2 + bx\right)^p b + 2\left(c\,x^2 + bx\right)^p cx - 2\left(\int \frac{\left(c\,x^2 + bx\right)^p}{2cp\,x^2 + 2bpx + c\,x^2 + bx} dx\right) b^2 p^2 - \left(\int \frac{\left(c\,x^2 + bx\right)^p}{2cp\,x^2 + 2bpx + c\,x^2 + bx} dx\right) b^2 p}{2c\,(2p+1)}$$

input
```
int((c*x^2+b*x)^p,x)
```

output
```
((b*x + c*x**2)**p*b + 2*(b*x + c*x**2)**p*c*x - 2*int((b*x + c*x**2)**p/(
2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**2*p**2 - int((b*x + c*x**2)**p/
(2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**2*p)/(2*c*(2*p + 1))
```

3.85 $\int (bx - b^2x^2)^p \, dx$

Optimal result

Integrand size = 14, antiderivative size = 38

$$\int (bx - b^2x^2)^p \, dx = \frac{(bx - b^2x^2)^{1+p} \, \text{Hypergeometric2F1}(1, 2(1+p), 2+p, bx)}{b(1+p)}$$

output
```
(-b^2*x^2+b*x)^(p+1)*hypergeom([1, 2*p+2],[2+p],b*x)/b/(p+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

$$\int (bx - b^2x^2)^p \, dx = \frac{x(1-bx)^{-p}(bx(1-bx))^p \, \text{Hypergeometric2F1}(-p, 1+p, 2+p, bx)}{1+p}$$

input
```
Integrate[(b*x - b^2*x^2)^p,x]
```

output
```
(x*(b*x*(1 - b*x))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, b*x])/((1 + p)*(1
 - b*x)^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx - b^2x^2)^p \, dx$$

$$\Big\downarrow \; 1090$$

$$-\frac{2^{-2p-1} \int \left(1 - \frac{(b-2b^2x)^2}{b^2}\right)^p d(b - 2b^2x)}{b^2}$$

$$\Big\downarrow \; 237$$

$$-\frac{2^{-2p-1}(b - 2b^2x) \text{Hypergeometric2F1}\left(\frac{1}{2}, -p, \frac{3}{2}, \frac{(b-2b^2x)^2}{b^2}\right)}{b^2}$$

input
```
Int[(b*x - b^2*x^2)^p,x]
```

output
```
-((2^(-1 - 2*p)*(b - 2*b^2*x)*Hypergeometric2F1[1/2, -p, 3/2, (b - 2*b^2*x
)^2/b^2])/b^2)
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [F]

$$\int \left(-b^2x^2+bx\right)^p dx$$

input
```
int((-b^2*x^2+b*x)^p,x)
```

output
```
int((-b^2*x^2+b*x)^p,x)
```

Fricas [F]

$$\int \left(bx-b^2x^2\right)^p \, dx = \int \left(-b^2x^2+bx\right)^p dx$$

input
```
integrate((-b^2*x^2+b*x)^p,x, algorithm="fricas")
```

output
```
integral((-b^2*x^2 + b*x)^p, x)
```

Sympy [F]

$$\int \left(bx-b^2x^2\right)^p \, dx = \int \left(-b^2x^2+bx\right)^p \, dx$$

input
```
integrate((-b**2*x**2+b*x)**p,x)
```

output
```
Integral((-b**2*x**2 + b*x)**p, x)
```

Maxima [F]

$$\int \left(bx-b^2x^2\right)^p\,dx=\int \left(-b^2x^2+bx\right)^p\,dx$$

input
```
integrate((-b^2*x^2+b*x)^p,x, algorithm="maxima")
```

output
```
integrate((-b^2*x^2 + b*x)^p, x)
```

Giac [F]

$$\int \left(bx-b^2x^2\right)^p\,dx=\int \left(-b^2x^2+bx\right)^p\,dx$$

input
```
integrate((-b^2*x^2+b*x)^p,x, algorithm="giac")
```

output
```
integrate((-b^2*x^2 + b*x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.18

$$\int \left(bx-b^2x^2\right)^p\,dx=\frac{x\,\left(b\,x-b^2\,x^2\right)^p\,{}_2\mathrm{F}_1\left(-p,p+1;\ p+2;\ b\,x\right)}{{\left(1-b\,x\right)}^p\,\left(p+1\right)}$$

input
```
int((b*x - b^2*x^2)^p,x)
```

output
```
(x*(b*x - b^2*x^2)^p*hypergeom([-p, p + 1], p + 2, b*x))/((1 - b*x)^p*(p +
 1))
```

Reduce [F]

$$\int \left(bx-b^2x^2\right)^p\,dx$$
$$=\frac{2(-b^2x^2+bx)^p\,bx-(-b^2x^2+bx)^p-2\left(\int\frac{(-b^2x^2+bx)^p}{2bp\,x^2+b\,x^2-2px-x}dx\right)p^2-\left(\int\frac{(-b^2x^2+bx)^p}{2bp\,x^2+b\,x^2-2px-x}dx\right)p}{2b\,(2p+1)}$$

input
```
int((-b^2*x^2+b*x)^p,x)
```

output
```
(2*( - b**2*x**2 + b*x)**p*b*x - ( - b**2*x**2 + b*x)**p - 2*int(( - b**2*
x**2 + b*x)**p/(2*b*p*x**2 + b*x**2 - 2*p*x - x),x)*p**2 - int(( - b**2*x*
*2 + b*x)**p/(2*b*p*x**2 + b*x**2 - 2*p*x - x),x)*p)/(2*b*(2*p + 1))
```

3.86 $\int (bx + b^2x^2)^p \, dx$

Optimal result

Integrand size = 13, antiderivative size = 38

$$\int (bx + b^2x^2)^p \, dx = \frac{(bx + b^2x^2)^{1+p} \text{Hypergeometric2F1}(1, 2(1+p), 2+p, -bx)}{b(1+p)}$$

output
```
(b^2*x^2+b*x)^(p+1)*hypergeom([1, 2*p+2],[2+p],-b*x)/b/(p+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int (bx + b^2x^2)^p \, dx = \frac{x(1+bx)^{-p}(bx(1+bx))^p \text{Hypergeometric2F1}(-p, 1+p, 2+p, -bx)}{1+p}$$

input
```
Integrate[(b*x + b^2*x^2)^p,x]
```

output
```
(x*(b*x*(1 + b*x))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -(b*x)])/((1 + p)
*(1 + b*x)^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.32, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(b^2x^2+bx\right)^p\,dx$$

$$\Big\downarrow \text{ 1096}$$

$$-\frac{(-bx)^{-p-1}\left(b^2x^2+bx\right)^{p+1}\text{Hypergeometric2F1}(-p,p+1,p+2,bx+1)}{b(p+1)}$$

input
```
Int[(b*x + b^2*x^2)^p,x]
```

output
```
-(((-(b*x))^(-1 - p)*(b*x + b^2*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p,
2 + p, 1 + b*x])/(b*(1 + p)))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

Maple [F]

$$\int \left(b^2x^2+bx\right)^p dx$$

input
```
int((b^2*x^2+b*x)^p,x)
```

output
```
int((b^2*x^2+b*x)^p,x)
```

Fricas [F]

$$\int \left(bx+b^2x^2\right)^p\,dx=\int \left(b^2x^2+bx\right)^p\,dx$$

input
```
integrate((b^2*x^2+b*x)^p,x, algorithm="fricas")
```

output
```
integral((b^2*x^2 + b*x)^p, x)
```

Sympy [F]

$$\int \left(bx+b^2x^2\right)^p\,dx=\int \left(b^2x^2+bx\right)^p\,dx$$

input
```
integrate((b**2*x**2+b*x)**p,x)
```

output
```
Integral((b**2*x**2 + b*x)**p, x)
```

Maxima [F]

$$\int \left(bx+b^2x^2\right)^p\,dx=\int \left(b^2x^2+bx\right)^p\,dx$$

input
```
integrate((b^2*x^2+b*x)^p,x, algorithm="maxima")
```

output
```
integrate((b^2*x^2 + b*x)^p, x)
```

Giac [F]

$$\int \left(bx+b^2x^2\right)^p \, dx = \int \left(b^2x^2+bx\right)^p \, dx$$

input
```
integrate((b^2*x^2+b*x)^p,x, algorithm="giac")
```

output
```
integrate((b^2*x^2 + b*x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.16

$$\int \left(bx+b^2x^2\right)^p \, dx = \frac{x\,(b^2\,x^2+b\,x)^p\,{}_2\mathrm{F}_1(-p,p+1;\ p+2;\ -b\,x)}{(b\,x+1)^p\,(p+1)}$$

input
```
int((b*x + b^2*x^2)^p,x)
```

output
```
(x*(b*x + b^2*x^2)^p*hypergeom([-p, p + 1], p + 2, -b*x))/((b*x + 1)^p*(p
+ 1))
```

Reduce [F]

$$\int \left(bx+b^2x^2\right)^p \, dx = \frac{2(b^2x^2+bx)^p bx+(b^2x^2+bx)^p-2\left(\int \frac{(b^2x^2+bx)^p}{2bp\,x^2+b\,x^2+2px+x}dx\right)p^2-\left(\int \frac{(b^2x^2+bx)^p}{2bp\,x^2+b\,x^2+2px+x}dx\right)p}{2b\,(2p+1)}$$

input
```
int((b^2*x^2+b*x)^p,x)
```

output
```
(2*(b**2*x**2 + b*x)**p*b*x + (b**2*x**2 + b*x)**p - 2*int((b**2*x**2 + b*
x)**p/(2*b*p*x**2 + b*x**2 + 2*p*x + x),x)*p**2 - int((b**2*x**2 + b*x)**p
/(2*b*p*x**2 + b*x**2 + 2*p*x + x),x)*p)/(2*b*(2*p + 1))
```

3.87 $\int (2x-3x^2)^p \, dx$

Optimal result

Integrand size = 11, antiderivative size = 29

$$\int \left(2x-3x^2\right)^p \, dx=\frac{2^p x^{1+p} \text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{3x}{2}\right)}{1+p}$$

output
```
2^p*x^(p+1)*hypergeom([-p, p+1],[2+p],3/2*x)/(p+1)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

$$\int \left(2x-3x^2\right)^p \, dx =\frac{2^p (2-3x)^{-p} x ((2-3x) x)^p \text{Hypergeometric2F1}\left(-p,1+p,2+p,\frac{3x}{2}\right)}{1+p}$$

input
```
Integrate[(2*x - 3*x^2)^p,x]
```

output
```
(2^p*x*((2 - 3*x)*x)^p*Hypergeometric2F1[-p, 1 + p, 2 + p, (3*x)/2])/((1 +
 p)*(2 - 3*x)^p)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.28, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2x-3x^2)^p \, dx$$

$$\Big\downarrow \text{1090}$$

$$-\frac{1}{2}3^{-p-1}\int \left(1-\frac{1}{4}(2-6x)^2\right)^p d(2-6x)$$

$$\Big\downarrow \text{237}$$

$$-\frac{1}{2}3^{-p-1}(2-6x)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{4}(2-6x)^2\right)$$

input
```
Int[(2*x - 3*x^2)^p,x]
```

output
```
-1/2*(3^(-1 - p)*(2 - 6*x)*Hypergeometric2F1[1/2, -p, 3/2, (2 - 6*x)^2/4])
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.03

method	result	size
meijerg	$\frac{2^{p} x^{p+1} \operatorname{hypergeom}\left(\left[-p,p+1\right],\left[2+p\right],\frac{3x}{2}\right)}{p+1}$	30

input
```
int((-3*x^2+2*x)^p,x,method=_RETURNVERBOSE)
```

output
```
2^p*x^(p+1)*hypergeom([-p,p+1],[2+p],3/2*x)/(p+1)
```

Fricas [F]

$$\int (2x-3x^2)^p \, dx = \int (-3\,x^2+2\,x)^p \, dx$$

input
```
integrate((-3*x^2+2*x)^p,x, algorithm="fricas")
```

output
```
integral((-3*x^2 + 2*x)^p, x)
```

Sympy [F]

$$\int (2x-3x^2)^p \, dx = \int (-3x^2+2x)^p \, dx$$

input
```
integrate((-3*x**2+2*x)**p,x)
```

output
```
Integral((-3*x**2 + 2*x)**p, x)
```

Maxima [F]

$$\int \left(2x-3x^2\right)^p \, dx = \int \left(-3\,x^2+2\,x\right)^p \, dx$$

input
```
integrate((-3*x^2+2*x)^p,x, algorithm="maxima")
```

output
```
integrate((-3*x^2 + 2*x)^p, x)
```

Giac [F]

$$\int \left(2x-3x^2\right)^p \, dx = \int \left(-3\,x^2+2\,x\right)^p \, dx$$

input
```
integrate((-3*x^2+2*x)^p,x, algorithm="giac")
```

output
```
integrate((-3*x^2 + 2*x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int \left(2x-3x^2\right)^p \, dx = \frac{x\,\left(2\,x-3\,x^2\right)^p\,{}_2\mathrm{F}_1\left(-p,p+1;\ p+2;\ \frac{3\,x}{2}\right)}{\left(1-\frac{3\,x}{2}\right)^p\,\left(p+1\right)}$$

input
```
int((2*x - 3*x^2)^p,x)
```

output
```
(x*(2*x - 3*x^2)^p*hypergeom([-p, p + 1], p + 2, (3*x)/2))/((1 - (3*x)/2)^
p*(p + 1))
```

Reduce [F]

$$\int \left(2x-3x^2\right)^p\,dx$$
$$=\frac{3(-3x^2+2x)^p\,x-(-3x^2+2x)^p-4\left(\int\frac{(-3x^2+2x)^p}{6p\,x^2-4px+3x^2-2x}dx\right)p^2-2\left(\int\frac{(-3x^2+2x)^p}{6p\,x^2-4px+3x^2-2x}dx\right)p}{6p+3}$$

input
```
int((-3*x^2+2*x)^p,x)
```

output
```
(3*( - 3*x**2 + 2*x)**p*x - ( - 3*x**2 + 2*x)**p - 4*int(( - 3*x**2 + 2*x)
**p/(6*p*x**2 - 4*p*x + 3*x**2 - 2*x),x)*p**2 - 2*int(( - 3*x**2 + 2*x)**p
/(6*p*x**2 - 4*p*x + 3*x**2 - 2*x),x)*p)/(3*(2*p + 1))
```

3.88 $\int (-2x-3x^2)^p \, dx$

Optimal result

Integrand size = 11, antiderivative size = 37

$$\int (-2x-3x^2)^p \, dx = -\frac{(-2x-3x^2)^{1+p} \text{Hypergeometric2F1}\left(1,2(1+p),2+p,-\frac{3x}{2}\right)}{2(1+p)}$$

output
```
-1/2*(-3*x^2-2*x)^(p+1)*hypergeom([1, 2*p+2],[2+p],-3/2*x)/(p+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int (-2x-3x^2)^p \, dx = \frac{2^p x(2+3x)^{-p}(-x(2+3x))^p \text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{3x}{2}\right)}{1+p}$$

input
```
Integrate[(-2*x - 3*x^2)^p,x]
```

output
```
(2^p*x*(-(x*(2 + 3*x)))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, (-3*x)/2])/(
(1 + p)*(2 + 3*x)^p)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1090, 237}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-3x^2-2x\right)^p \, dx$$

$$\Big\downarrow \text{ 1090}$$

$$-\frac{1}{2}3^{-p-1}\int \left(1-\frac{1}{4}(-6x-2)^2\right)^p d(-6x-2)$$

$$\Big\downarrow \text{ 237}$$

$$-\frac{1}{2}3^{-p-1}(-6x-2)\,\text{Hypergeometric2F1}\left(\frac{1}{2},-p,\frac{3}{2},\frac{1}{4}(-6x-2)^2\right)$$

input
```
Int[(-2*x - 3*x^2)^p,x]
```

output
```
-1/2*(3^(-1 - p)*(-2 - 6*x)*Hypergeometric2F1[1/2, -p, 3/2, (-2 - 6*x)^2/4
])
```

Defintions of rubi rules used

rule 237
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F1[-
p, 1/2, 1/2 + 1, (-b)*(x^2/a)], x] /; FreeQ[{a, b, p}, x] &&  !IntegerQ[2*p
] && GtQ[a, 0]
```

rule 1090
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/(2*c*(-4*
(c/(b^2 - 4*a*c)))^p)   Subst[Int[Simp[1 - x^2/(b^2 - 4*a*c), x]^p, x], x,
b + 2*c*x], x] /; FreeQ[{a, b, c, p}, x] && GtQ[4*a - b^2/c, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.27

method	result	size
meijerg	$\frac{(-1)^{2p} 2^{2p} (-1)^{-p} x^{p+1} 2^{-p} \operatorname{hypergeom}\left([-p,p+1],[2+p],-\frac{3x}{2}\right)}{p+1}$	47

input
```
int((-3*x^2-2*x)^p,x,method=_RETURNVERBOSE)
```

output
```
((-1)^p)^2*(2^p)^2*(-1)^(-p)/(p+1)*x^(p+1)*2^(-p)*hypergeom([-p,p+1],[2+p]
,-3/2*x)
```

Fricas [F]

$$\int (-2x - 3x^2)^p \, dx = \int (-3\,x^2 - 2\,x)^p \, dx$$

input
```
integrate((-3*x^2-2*x)^p,x, algorithm="fricas")
```

output
```
integral((-3*x^2 - 2*x)^p, x)
```

Sympy [F]

$$\int (-2x - 3x^2)^p \, dx = \int (-3x^2 - 2x)^p \, dx$$

input
```
integrate((-3*x**2-2*x)**p,x)
```

output
```
Integral((-3*x**2 - 2*x)**p, x)
```

Maxima [F]

$$\int \left(-2x-3x^2\right)^p\,dx=\int \left(-3\,x^2-2\,x\right)^p\,dx$$

input
```
integrate((-3*x^2-2*x)^p,x, algorithm="maxima")
```

output
```
integrate((-3*x^2 - 2*x)^p, x)
```

Giac [F]

$$\int \left(-2x-3x^2\right)^p\,dx=\int \left(-3\,x^2-2\,x\right)^p\,dx$$

input
```
integrate((-3*x^2-2*x)^p,x, algorithm="giac")
```

output
```
integrate((-3*x^2 - 2*x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.11

$$\int \left(-2x-3x^2\right)^p\,dx=\frac{x\left(-3\,x^2-2\,x\right)^p\,{}_2\mathrm{F}_1\left(-p,p+1;\ p+2;\ -\frac{3\,x}{2}\right)}{\left(\frac{3\,x}{2}+1\right)^p\,\left(p+1\right)}$$

input
```
int((- 2*x - 3*x^2)^p,x)
```

output
```
(x*(- 2*x - 3*x^2)^p*hypergeom([-p, p + 1], p + 2, -(3*x)/2))/(((3*x)/2 +
1)^p*(p + 1))
```

Reduce [F]

$$\int (-2x-3x^2)^p\, dx$$
$$= \frac{3(-3x^2-2x)^p x + (-3x^2-2x)^p - 4\left(\int \frac{(-3x^2-2x)^p}{6p\,x^2+4px+3x^2+2x}dx\right)p^2 - 2\left(\int \frac{(-3x^2-2x)^p}{6p\,x^2+4px+3x^2+2x}dx\right)p}{6p+3}$$

input
```
int((-3*x^2-2*x)^p,x)
```

output
```
(3*( - 3*x**2 - 2*x)**p*x + ( - 3*x**2 - 2*x)**p - 4*int(( - 3*x**2 - 2*x)
**p/(6*p*x**2 + 4*p*x + 3*x**2 + 2*x),x)*p**2 - 2*int(( - 3*x**2 - 2*x)**p
/(6*p*x**2 + 4*p*x + 3*x**2 + 2*x),x)*p)/(3*(2*p + 1))
```

3.89 $\int (2x+3x^2)^p \, dx$

Optimal result

Integrand size = 11, antiderivative size = 45

$$\int (2x+3x^2)^p \, dx = \frac{2^p x (2+3x)^{-p} (2x+3x^2)^p \text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{3x}{2}\right)}{1+p}$$

output
```
2^p*x*(3*x^2+2*x)^p*hypergeom([-p, p+1],[2+p],-3/2*x)/(p+1)/((2+3*x)^p)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int (2x+3x^2)^p \, dx = \frac{2^p x (2+3x)^{-p} (x(2+3x))^p \text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{3x}{2}\right)}{1+p}$$

input
```
Integrate[(2*x + 3*x^2)^p,x]
```

output
```
(2^p*x*(x*(2 + 3*x))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, (-3*x)/2])/((1
+ p)*(2 + 3*x)^p)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.29, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (3x^2+2x)^p \, dx$$

$$\Big\downarrow \text{1096}$$

$$-\frac{2^p 3^{-p-1}(-x)^{-p-1}\left(3x^2+2x\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{1}{2}(3x+2)\right)}{p+1}$$

input
```
Int[(2*x + 3*x^2)^p,x]
```

output
```
-((2^p*3^(-1 - p)*(-x)^(-1 - p)*(2*x + 3*x^2)^(1 + p)*Hypergeometric2F1[-p
, 1 + p, 2 + p, (2 + 3*x)/2])/(1 + p))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.67

method	result	size
meijerg	$\frac{2^{p} x^{p+1} \operatorname{hypergeom}\left(\left[-p,p+1\right],\left[2+p\right],-\frac{3x}{2}\right)}{p+1}$	30

input
```
int((3*x^2+2*x)^p,x,method=_RETURNVERBOSE)
```

output
```
2^p/(p+1)*x^(p+1)*hypergeom([-p,p+1],[2+p],-3/2*x)
```

Fricas [F]

$$\int \left(2x+3x^2\right)^p\,dx=\int \left(3\,x^2+2\,x\right)^p\,dx$$

input
```
integrate((3*x^2+2*x)^p,x, algorithm="fricas")
```

output
```
integral((3*x^2 + 2*x)^p, x)
```

Sympy [F]

$$\int \left(2x+3x^2\right)^p\,dx=\int \left(3x^2+2x\right)^p\,dx$$

input
```
integrate((3*x**2+2*x)**p,x)
```

output
```
Integral((3*x**2 + 2*x)**p, x)
```

Maxima [F]

$$\int \left(2x+3x^2\right)^p\,dx=\int \left(3\,x^2+2\,x\right)^p\,dx$$

input
```
integrate((3*x^2+2*x)^p,x, algorithm="maxima")
```

output
```
integrate((3*x^2 + 2*x)^p, x)
```

Giac [F]

$$\int \left(2x+3x^2\right)^p\,dx=\int \left(3\,x^2+2\,x\right)^p\,dx$$

input
```
integrate((3*x^2+2*x)^p,x, algorithm="giac")
```

output
```
integrate((3*x^2 + 2*x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \left(2x+3x^2\right)^p\,dx=\frac{x\left(3\,x^2+2\,x\right)^p\,{}_2\mathrm{F}_1\left(-p,p+1;\ p+2;\ -\frac{3\,x}{2}\right)}{\left(\frac{3\,x}{2}+1\right)^p\,\left(p+1\right)}$$

input
```
int((2*x + 3*x^2)^p,x)
```

output
```
(x*(2*x + 3*x^2)^p*hypergeom([-p, p + 1], p + 2, -(3*x)/2))/(((3*x)/2 + 1)
^p*(p + 1))
```

Reduce [F]

$$\int \left(2x+3x^2\right)^p\,dx$$
$$=\frac{3(3x^2+2x)^p x+(3x^2+2x)^p-4\left(\int \frac{(3x^2+2x)^p}{6p\,x^2+4px+3x^2+2x}dx\right)p^2-2\left(\int \frac{(3x^2+2x)^p}{6p\,x^2+4px+3x^2+2x}dx\right)p}{6p+3}$$

input
```
int((3*x^2+2*x)^p,x)
```

output
```
(3*(3*x**2 + 2*x)**p*x + (3*x**2 + 2*x)**p - 4*int((3*x**2 + 2*x)**p/(6*p*
x**2 + 4*p*x + 3*x**2 + 2*x),x)*p**2 - 2*int((3*x**2 + 2*x)**p/(6*p*x**2 +
 4*p*x + 3*x**2 + 2*x),x)*p)/(3*(2*p + 1))
```

3.90 $\int \left(-2x+3x^2\right)^p\,dx$

Optimal result

Integrand size = 11, antiderivative size = 53

$$\int \left(-2x+3x^2\right)^p\,dx = \frac{2^p 3^{-1-p} x^{-1-p}\left(-2x+3x^2\right)^{1+p}\text{Hypergeometric2F1}\left(-p,1+p,2+p,1-\frac{3x}{2}\right)}{1+p}$$

output
```
2^p*3^(-1-p)*x^(-1-p)*(3*x^2-2*x)^(p+1)*hypergeom([-p, p+1],[2+p],1-3/2*x)
/(p+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

$$\int \left(-2x+3x^2\right)^p\,dx = \frac{2^p 3^{-1-p} x^{-p}(-2+3x)(x(-2+3x))^p\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,1-\frac{3x}{2}\right)}{1+p}$$

input
```
Integrate[(-2*x + 3*x^2)^p,x]
```

output
```
(2^p*3^(-1 - p)*(-2 + 3*x)*(x*(-2 + 3*x))^p*Hypergeometric2F1[-p, 1 + p, 2
 + p, 1 - (3*x)/2])/((1 + p)*x^p)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = $\{1096\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(3x^2-2x\right)^p\,dx$$

$$\Big\downarrow \ 1096$$

$$-\frac{2^{2p+1}(4-6x)^{-p-1}\left(3x^2-2x\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{3x}{2}\right)}{p+1}$$

input
```
Int[(-2*x + 3*x^2)^p,x]
```

output
```
-((2^(1 + 2*p)*(4 - 6*x)^(-1 - p)*(-2*x + 3*x^2)^(1 + p)*Hypergeometric2F1
[-p, 1 + p, 2 + p, (3*x)/2])/(1 + p))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] && !IntegerQ[4*p] && !IntegerQ[3*p]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 5.

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.19

method	result	size
meijerg	$\frac{(-1)^{2p}2^{2p}(-1)^{-2p}\operatorname{signum}(x-\frac{2}{3})^{p}(-\operatorname{signum}(x-\frac{2}{3}))^{-p}x^{p+1}2^{-p}\operatorname{hypergeom}([-p,p+1],[2+p],\frac{3x}{2})}{p+1}$	63

input
```
int((3*x^2-2*x)^p,x,method=_RETURNVERBOSE)
```

output
```
((-1)^p)^2*(2^p)^2*(-1)^(-2*p)*signum(x-2/3)^p*(-signum(x-2/3))^(-p)/(p+1)
*x^(p+1)*2^(-p)*hypergeom([-p,p+1],[2+p],3/2*x)
```

Fricas [F]

$$\int (-2x+3x^2)^p\,dx = \int (3\,x^2-2\,x)^p\,dx$$

input
```
integrate((3*x^2-2*x)^p,x, algorithm="fricas")
```

output
```
integral((3*x^2 - 2*x)^p, x)
```

Sympy [F]

$$\int (-2x+3x^2)^p\,dx = \int (3x^2-2x)^p\,dx$$

input
```
integrate((3*x**2-2*x)**p,x)
```

output
```
Integral((3*x**2 - 2*x)**p, x)
```

Maxima [F]

$$\int \left(-2x+3x^2\right)^p \, dx = \int \left(3\,x^2-2\,x\right)^p \, dx$$

input
```
integrate((3*x^2-2*x)^p,x, algorithm="maxima")
```

output
```
integrate((3*x^2 - 2*x)^p, x)
```

Giac [F]

$$\int \left(-2x+3x^2\right)^p \, dx = \int \left(3\,x^2-2\,x\right)^p \, dx$$

input
```
integrate((3*x^2-2*x)^p,x, algorithm="giac")
```

output
```
integrate((3*x^2 - 2*x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \left(-2x+3x^2\right)^p \, dx = \frac{x\,\left(3\,x^2-2\,x\right)^p\,{}_2\mathrm{F}_1\left(-p,p+1;\ p+2;\ \frac{3\,x}{2}\right)}{\left(1-\frac{3\,x}{2}\right)^p\,\left(p+1\right)}$$

input
```
int((3*x^2 - 2*x)^p,x)
```

output
```
(x*(3*x^2 - 2*x)^p*hypergeom([-p, p + 1], p + 2, (3*x)/2))/((1 - (3*x)/2)^
p*(p + 1))
```

Reduce [F]

$$\int (-2x+3x^2)^p \, dx$$
$$= \frac{3(3x^2-2x)^p x-(3x^2-2x)^p-4\left(\int \frac{(3x^2-2x)^p}{6p\,x^2-4px+3x^2-2x}dx\right)p^2-2\left(\int \frac{(3x^2-2x)^p}{6p\,x^2-4px+3x^2-2x}dx\right)p}{6p+3}$$

input
```
int((3*x^2-2*x)^p,x)
```

output
```
(3*(3*x**2 - 2*x)**p*x - (3*x**2 - 2*x)**p - 4*int((3*x**2 - 2*x)**p/(6*p*
x**2 - 4*p*x + 3*x**2 - 2*x),x)*p**2 - 2*int((3*x**2 - 2*x)**p/(6*p*x**2 -
 4*p*x + 3*x**2 - 2*x),x)*p)/(3*(2*p + 1))
```

3.91 $\int (ax^2+bx^3)^4\,dx$

Optimal result

Integrand size = 13, antiderivative size = 56

$$\int (ax^2+bx^3)^4\,dx=\frac{a^4x^9}{9}+\frac{2}{5}a^3bx^{10}+\frac{6}{11}a^2b^2x^{11}+\frac{1}{3}ab^3x^{12}+\frac{b^4x^{13}}{13}$$

output
```
1/9*a^4*x^9+2/5*a^3*b*x^10+6/11*a^2*b^2*x^11+1/3*a*b^3*x^12+1/13*b^4*x^13
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int (ax^2+bx^3)^4\,dx=\frac{a^4x^9}{9}+\frac{2}{5}a^3bx^{10}+\frac{6}{11}a^2b^2x^{11}+\frac{1}{3}ab^3x^{12}+\frac{b^4x^{13}}{13}$$

input
```
Integrate[(a*x^2 + b*x^3)^4,x]
```

output
```
(a^4*x^9)/9 + (2*a^3*b*x^10)/5 + (6*a^2*b^2*x^11)/11 + (a*b^3*x^12)/3 + (b
^4*x^13)/13
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2027, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^4 dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int x^8(a+bx)^4dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(a^4x^8+4a^3bx^9+6a^2b^2x^{10}+4ab^3x^{11}+b^4x^{12}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^4x^9}{9}+\frac{2}{5}a^3bx^{10}+\frac{6}{11}a^2b^2x^{11}+\frac{1}{3}ab^3x^{12}+\frac{b^4x^{13}}{13}$$

input
```
Int[(a*x^2 + b*x^3)^4,x]
```

output
```
(a^4*x^9)/9 + (2*a^3*b*x^10)/5 + (6*a^2*b^2*x^11)/11 + (a*b^3*x^12)/3 + (b
^4*x^13)/13
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{x^{9} \left(495 b^{4} x^{4}+2145 a \,b^{3} x^{3}+3510 a^{2} b^{2} x^{2}+2574 a^{3} b x +715 a^{4}\right)}{6435}$	47
default	$\frac{1}{9} a^{4} x^{9}+\frac{2}{5} a^{3} b \,x^{10}+\frac{6}{11} a^{2} b^{2} x^{11}+\frac{1}{3} a \,b^{3} x^{12}+\frac{1}{13} b^{4} x^{13}$	47
norman	$\frac{1}{9} a^{4} x^{9}+\frac{2}{5} a^{3} b \,x^{10}+\frac{6}{11} a^{2} b^{2} x^{11}+\frac{1}{3} a \,b^{3} x^{12}+\frac{1}{13} b^{4} x^{13}$	47
risch	$\frac{1}{9} a^{4} x^{9}+\frac{2}{5} a^{3} b \,x^{10}+\frac{6}{11} a^{2} b^{2} x^{11}+\frac{1}{3} a \,b^{3} x^{12}+\frac{1}{13} b^{4} x^{13}$	47
parallelrisch	$\frac{1}{9} a^{4} x^{9}+\frac{2}{5} a^{3} b \,x^{10}+\frac{6}{11} a^{2} b^{2} x^{11}+\frac{1}{3} a \,b^{3} x^{12}+\frac{1}{13} b^{4} x^{13}$	47
orering	$\frac{x \left(495 b^{4} x^{4}+2145 a \,b^{3} x^{3}+3510 a^{2} b^{2} x^{2}+2574 a^{3} b x +715 a^{4}\right) \left(b \,x^{3}+a \,x^{2}\right)^{4}}{6435 \left(b x +a \right)^{4}}$	65

input
```
int((b*x^3+a*x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/6435*x^9*(495*b^4*x^4+2145*a*b^3*x^3+3510*a^2*b^2*x^2+2574*a^3*b*x+715*a
^4)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \left(ax^2+bx^3\right)^4 \, dx=\frac{1}{13} \, b^{4} x^{13} + \frac{1}{3} \, a b^{3} x^{12} + \frac{6}{11} \, a^{2} b^{2} x^{11} + \frac{2}{5} \, a^{3} b x^{10} + \frac{1}{9} \, a^{4} x^{9}$$

input
```
integrate((b*x^3+a*x^2)^4,x, algorithm="fricas")
```

output
```
1/13*b^4*x^13 + 1/3*a*b^3*x^12 + 6/11*a^2*b^2*x^11 + 2/5*a^3*b*x^10 + 1/9*
a^4*x^9
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \left(ax^2+bx^3\right)^4\,dx=\frac{a^4x^9}{9}+\frac{2a^3bx^{10}}{5}+\frac{6a^2b^2x^{11}}{11}+\frac{ab^3x^{12}}{3}+\frac{b^4x^{13}}{13}$$

input
```
integrate((b*x**3+a*x**2)**4,x)
```

output
```
a**4*x**9/9 + 2*a**3*b*x**10/5 + 6*a**2*b**2*x**11/11 + a*b**3*x**12/3 + b
**4*x**13/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \left(ax^2+bx^3\right)^4\,dx=\frac{1}{13}\,b^4x^{13}+\frac{1}{3}\,ab^3x^{12}+\frac{6}{11}\,a^2b^2x^{11}+\frac{2}{5}\,a^3bx^{10}+\frac{1}{9}\,a^4x^9$$

input
```
integrate((b*x^3+a*x^2)^4,x, algorithm="maxima")
```

output
```
1/13*b^4*x^13 + 1/3*a*b^3*x^12 + 6/11*a^2*b^2*x^11 + 2/5*a^3*b*x^10 + 1/9*
a^4*x^9
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \left(ax^2+bx^3\right)^4\,dx=\frac{1}{13}\,b^4x^{13}+\frac{1}{3}\,ab^3x^{12}+\frac{6}{11}\,a^2b^2x^{11}+\frac{2}{5}\,a^3bx^{10}+\frac{1}{9}\,a^4x^9$$

input
```
integrate((b*x^3+a*x^2)^4,x, algorithm="giac")
```

output
```
1/13*b^4*x^13 + 1/3*a*b^3*x^12 + 6/11*a^2*b^2*x^11 + 2/5*a^3*b*x^10 + 1/9*
a^4*x^9
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \left(ax^2+bx^3\right)^4\,dx=\frac{a^4\,x^9}{9}+\frac{2\,a^3\,b\,x^{10}}{5}+\frac{6\,a^2\,b^2\,x^{11}}{11}+\frac{a\,b^3\,x^{12}}{3}+\frac{b^4\,x^{13}}{13}$$

input
```
int((a*x^2 + b*x^3)^4,x)
```

output
```
(a^4*x^9)/9 + (b^4*x^13)/13 + (2*a^3*b*x^10)/5 + (a*b^3*x^12)/3 + (6*a^2*b
^2*x^11)/11
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \left(ax^2+bx^3\right)^4\,dx=\frac{x^9(495b^4x^4+2145a\,b^3x^3+3510a^2b^2x^2+2574a^3bx+715a^4)}{6435}$$

input
```
int((b*x^3+a*x^2)^4,x)
```

output
```
(x**9*(715*a**4 + 2574*a**3*b*x + 3510*a**2*b**2*x**2 + 2145*a*b**3*x**3 +
 495*b**4*x**4))/6435
```

3.92 $\int \left(ax^2+bx^3\right)^3\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \left(ax^2+bx^3\right)^3\,dx=\frac{a^3x^7}{7}+\frac{3}{8}a^2bx^8+\frac{1}{3}ab^2x^9+\frac{b^3x^{10}}{10}$$

output
```
1/7*a^3*x^7+3/8*a^2*b*x^8+1/3*a*b^2*x^9+1/10*b^3*x^10
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \left(ax^2+bx^3\right)^3\,dx=\frac{a^3x^7}{7}+\frac{3}{8}a^2bx^8+\frac{1}{3}ab^2x^9+\frac{b^3x^{10}}{10}$$

input
```
Integrate[(a*x^2 + b*x^3)^3,x]
```

output
```
(a^3*x^7)/7 + (3*a^2*b*x^8)/8 + (a*b^2*x^9)/3 + (b^3*x^10)/10
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2027, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^3\,dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int x^6(a+bx)^3dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(a^3x^6+3a^2bx^7+3ab^2x^8+b^3x^9\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^3x^7}{7}+\frac{3}{8}a^2bx^8+\frac{1}{3}ab^2x^9+\frac{b^3x^{10}}{10}$$

input
```
Int[(a*x^2 + b*x^3)^3,x]
```

output
```
(a^3*x^7)/7 + (3*a^2*b*x^8)/8 + (a*b^2*x^9)/3 + (b^3*x^10)/10
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{x^{7} \left(84 b^{3} x^{3}+280 a \,b^{2} x^{2}+315 a^{2} b x +120 a^{3}\right)}{840}$	36
default	$\frac{1}{7} a^{3} x^{7}+\frac{3}{8} a^{2} b \,x^{8}+\frac{1}{3} a \,b^{2} x^{9}+\frac{1}{10} b^{3} x^{10}$	36
norman	$\frac{1}{7} a^{3} x^{7}+\frac{3}{8} a^{2} b \,x^{8}+\frac{1}{3} a \,b^{2} x^{9}+\frac{1}{10} b^{3} x^{10}$	36
risch	$\frac{1}{7} a^{3} x^{7}+\frac{3}{8} a^{2} b \,x^{8}+\frac{1}{3} a \,b^{2} x^{9}+\frac{1}{10} b^{3} x^{10}$	36
parallelrisch	$\frac{1}{7} a^{3} x^{7}+\frac{3}{8} a^{2} b \,x^{8}+\frac{1}{3} a \,b^{2} x^{9}+\frac{1}{10} b^{3} x^{10}$	36
orering	$\frac{x \left(84 b^{3} x^{3}+280 a \,b^{2} x^{2}+315 a^{2} b x +120 a^{3}\right) \left(b \,x^{3}+a \,x^{2}\right)^{3}}{840 \left(b x +a \right)^{3}}$	54

input
```
int((b*x^3+a*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/840*x^7*(84*b^3*x^3+280*a*b^2*x^2+315*a^2*b*x+120*a^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(ax^2+bx^3\right)^3\,dx=\frac{1}{10}\,b^3x^{10}+\frac{1}{3}\,ab^2x^9+\frac{3}{8}\,a^2bx^8+\frac{1}{7}\,a^3x^7$$

input
```
integrate((b*x^3+a*x^2)^3,x, algorithm="fricas")
```

output
```
1/10*b^3*x^10 + 1/3*a*b^2*x^9 + 3/8*a^2*b*x^8 + 1/7*a^3*x^7
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \left(ax^2+bx^3\right)^3\,dx=\frac{a^3x^7}{7}+\frac{3a^2bx^8}{8}+\frac{ab^2x^9}{3}+\frac{b^3x^{10}}{10}$$

input
```
integrate((b*x**3+a*x**2)**3,x)
```

output
```
a**3*x**7/7 + 3*a**2*b*x**8/8 + a*b**2*x**9/3 + b**3*x**10/10
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(ax^2+bx^3\right)^3\,dx=\frac{1}{10}\,b^3x^{10}+\frac{1}{3}\,ab^2x^9+\frac{3}{8}\,a^2bx^8+\frac{1}{7}\,a^3x^7$$

input
```
integrate((b*x^3+a*x^2)^3,x, algorithm="maxima")
```

output
```
1/10*b^3*x^10 + 1/3*a*b^2*x^9 + 3/8*a^2*b*x^8 + 1/7*a^3*x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(ax^2+bx^3\right)^3\,dx=\frac{1}{10}\,b^3x^{10}+\frac{1}{3}\,ab^2x^9+\frac{3}{8}\,a^2bx^8+\frac{1}{7}\,a^3x^7$$

input
```
integrate((b*x^3+a*x^2)^3,x, algorithm="giac")
```

output
```
1/10*b^3*x^10 + 1/3*a*b^2*x^9 + 3/8*a^2*b*x^8 + 1/7*a^3*x^7
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(ax^2+bx^3\right)^3\,dx=\frac{a^3\,x^7}{7}+\frac{3\,a^2\,b\,x^8}{8}+\frac{a\,b^2\,x^9}{3}+\frac{b^3\,x^{10}}{10}$$

input
```
int((a*x^2 + b*x^3)^3,x)
```

output
```
(a^3*x^7)/7 + (b^3*x^10)/10 + (3*a^2*b*x^8)/8 + (a*b^2*x^9)/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(ax^2+bx^3\right)^3\,dx=\frac{x^7(84b^3x^3+280a\,b^2x^2+315a^2bx+120a^3)}{840}$$

input
```
int((b*x^3+a*x^2)^3,x)
```

output
```
(x**7*(120*a**3 + 315*a**2*b*x + 280*a*b**2*x**2 + 84*b**3*x**3))/840
```

3.93 $\int (ax^2+bx^3)^2 \, dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int (ax^2+bx^3)^2 \, dx = \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

output
```
1/5*a^2*x^5+1/3*a*b*x^6+1/7*b^2*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (ax^2+bx^3)^2 \, dx = \frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

input
```
Integrate[(a*x^2 + b*x^3)^2,x]
```

output
```
(a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2027, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^2+bx^3)^2\,dx$$

$$\Big\downarrow \text{2027}$$

$$\int x^4(a+bx)^2dx$$

$$\Big\downarrow \text{49}$$

$$\int (a^2x^4+2abx^5+b^2x^6)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2x^5}{5}+\frac{1}{3}abx^6+\frac{b^2x^7}{7}$$

input
```
Int[(a*x^2 + b*x^3)^2,x]
```

output
```
(a^2*x^5)/5 + (a*b*x^6)/3 + (b^2*x^7)/7
```

Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^{5}\left(15 b^{2} x^{2}+35 a b x+21 a^{2}\right)}{105}$	25
default	$\frac{1}{5} a^{2} x^{5}+\frac{1}{3} a b\, x^{6}+\frac{1}{7} b^{2} x^{7}$	25
norman	$\frac{1}{5} a^{2} x^{5}+\frac{1}{3} a b\, x^{6}+\frac{1}{7} b^{2} x^{7}$	25
risch	$\frac{1}{5} a^{2} x^{5}+\frac{1}{3} a b\, x^{6}+\frac{1}{7} b^{2} x^{7}$	25
parallelrisch	$\frac{1}{5} a^{2} x^{5}+\frac{1}{3} a b\, x^{6}+\frac{1}{7} b^{2} x^{7}$	25
orering	$\frac{x\left(15 b^{2} x^{2}+35 a b x+21 a^{2}\right)\left(b\, x^{3}+a\, x^{2}\right)^{2}}{105\left(b x+a\right)^{2}}$	43

input
```
int((b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/105*x^5*(15*b^2*x^2+35*a*b*x+21*a^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2+bx^3\right)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{1}{3}\,abx^6+\frac{1}{5}\,a^2x^5$$

input
```
integrate((b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2+bx^3\right)^2\,dx=\frac{a^2x^5}{5}+\frac{abx^6}{3}+\frac{b^2x^7}{7}$$

input
```
integrate((b*x**3+a*x**2)**2,x)
```

output
```
a**2*x**5/5 + a*b*x**6/3 + b**2*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2+bx^3\right)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{1}{3}\,abx^6+\frac{1}{5}\,a^2x^5$$

input
```
integrate((b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(ax^2+bx^3\right)^2\,dx=\frac{1}{7}\,b^2x^7+\frac{1}{3}\,abx^6+\frac{1}{5}\,a^2x^5$$

input
```
integrate((b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
1/7*b^2*x^7 + 1/3*a*b*x^6 + 1/5*a^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (ax^2+bx^3)^2 \, dx = \frac{a^2\,x^5}{5}+\frac{a\,b\,x^6}{3}+\frac{b^2\,x^7}{7}$$

input
```
int((a*x^2 + b*x^3)^2,x)
```

output
```
(a^2*x^5)/5 + (b^2*x^7)/7 + (a*b*x^6)/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (ax^2+bx^3)^2 \, dx = \frac{x^5(15b^2x^2+35abx+21a^2)}{105}$$

input
```
int((b*x^3+a*x^2)^2,x)
```

output
```
(x**5*(21*a**2 + 35*a*b*x + 15*b**2*x**2))/105
```

3.94 $\int (ax^2 + bx^3) \, dx$

Optimal result

Integrand size = 11, antiderivative size = 17

$$\int (ax^2 + bx^3) \, dx = \frac{ax^3}{3} + \frac{bx^4}{4}$$

output
```
1/3*a*x^3+1/4*b*x^4
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (ax^2 + bx^3) \, dx = \frac{ax^3}{3} + \frac{bx^4}{4}$$

input
```
Integrate[a*x^2 + b*x^3,x]
```

output
```
(a*x^3)/3 + (b*x^4)/4
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^2 + bx^3) \, dx$$

$$\Big\downarrow \ 2009$$

$$\frac{ax^3}{3} + \frac{bx^4}{4}$$

input
```
Int[a*x^2 + b*x^3,x]
```

output
```
(a*x^3)/3 + (b*x^4)/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^3(3bx+4a)}{12}$	14
default	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
norman	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
risch	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
parallelrisch	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
parts	$\frac{1}{3}a\,x^3+\frac{1}{4}b\,x^4$	14
orering	$\frac{x(3bx+4a)(b\,x^3+a\,x^2)}{12bx+12a}$	30

input
```
int(b*x^3+a*x^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*x^3*(3*b*x+4*a)
```

Fricas [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax^2+bx^3)\ dx=\frac{1}{4}\,bx^4+\frac{1}{3}\,ax^3$$

input
```
integrate(b*x^3+a*x^2,x, algorithm="fricas")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

Sympy [A] (verification not implemented)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int (ax^2 + bx^3) \, dx = \frac{ax^3}{3} + \frac{bx^4}{4}$$

input
```
integrate(b*x**3+a*x**2,x)
```

output
```
a*x**3/3 + b*x**4/4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax^2 + bx^3) \, dx = \frac{1}{4} bx^4 + \frac{1}{3} ax^3$$

input
```
integrate(b*x^3+a*x^2,x, algorithm="maxima")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax^2 + bx^3) \, dx = \frac{1}{4} bx^4 + \frac{1}{3} ax^3$$

input
```
integrate(b*x^3+a*x^2,x, algorithm="giac")
```

output
```
1/4*b*x^4 + 1/3*a*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax^2 + bx^3) \, dx = \frac{x^3 \, (4\,a + 3\,b\,x)}{12}$$

input
```
int(a*x^2 + b*x^3,x)
```

output
```
(x^3*(4*a + 3*b*x))/12
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (ax^2 + bx^3) \, dx = \frac{x^3(3bx + 4a)}{12}$$

input
```
int(b*x^3+a*x^2,x)
```

output
```
(x**3*(4*a + 3*b*x))/12
```

3.95 $\int \frac{1}{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{ax^2+bx^3}\,dx=-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

output
```
-1/a/x-b*ln(x)/a^2+b*ln(b*x+a)/a^2
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2+bx^3}\,dx=-\frac{1}{ax}-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-1),x]
```

output
```
-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2026, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^2+bx^3}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x^2(a+bx)}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{b^2}{a^2(a+bx)}-\frac{b}{a^2x}+\frac{1}{ax^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\log(x)}{a^2}+\frac{b\log(a+bx)}{a^2}-\frac{1}{ax}$$

input
```
Int[(a*x^2 + b*x^3)^(-1),x]
```

output
```
-(1/(a*x)) - (b*Log[x])/a^2 + (b*Log[a + b*x])/a^2
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$-\frac{b\ln(x)x-b\ln(bx+a)x+a}{a^2x}$	26
default	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
norman	$-\frac{1}{ax}-\frac{b\ln(x)}{a^2}+\frac{b\ln(bx+a)}{a^2}$	29
risch	$-\frac{1}{ax}+\frac{b\ln(-bx-a)}{a^2}-\frac{b\ln(x)}{a^2}$	32

input
```
int(1/(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
-(b*ln(x)*x-b*ln(b*x+a)*x+a)/a^2/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{ax^2+bx^3}\,dx=\frac{bx\log(bx+a)-bx\log(x)-a}{a^2x}$$

input
```
integrate(1/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
(b*x*log(b*x + a) - b*x*log(x) - a)/(a^2*x)
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.68

$$\int \frac{1}{ax^2 + bx^3} dx = -\frac{1}{ax} + \frac{b\left(-\log\left(x\right) + \log\left(\frac{a}{b} + x\right)\right)}{a^2}$$

input
```
integrate(1/(b*x**3+a*x**2),x)
```

output
```
-1/(a*x) + b*(-log(x) + log(a/b + x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^2 + bx^3} dx = \frac{b\log\left(bx + a\right)}{a^2} - \frac{b\log\left(x\right)}{a^2} - \frac{1}{ax}$$

input
```
integrate(1/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
b*log(b*x + a)/a^2 - b*log(x)/a^2 - 1/(a*x)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int \frac{1}{ax^2 + bx^3} dx = \frac{b\log\left(|bx + a|\right)}{a^2} - \frac{b\log\left(|x|\right)}{a^2} - \frac{1}{ax}$$

input
```
integrate(1/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
b*log(abs(b*x + a))/a^2 - b*log(abs(x))/a^2 - 1/(a*x)
```

Mupad [B] (verification not implemented)

Time = 9.53 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{ax^2+bx^3}\,dx = \frac{2\,b\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^2}-\frac{1}{a\,x}$$

input
```
int(1/(a*x^2 + b*x^3),x)
```

output
```
(2*b*atanh((2*b*x)/a + 1))/a^2 - 1/(a*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{1}{ax^2+bx^3}\,dx = \frac{\log(bx+a)\,bx-\log(x)\,bx-a}{a^2x}$$

input
```
int(1/(b*x^3+a*x^2),x)
```

output
```
(log(a + b*x)*b*x - log(x)*b*x - a)/(a**2*x)
```

3.96 $\int \frac{1}{\left(ax^2+bx^3\right)^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 69

$$\int \frac{1}{\left(ax^2+bx^3\right)^2}\,dx = -\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(a+bx)}-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}$$

output
```
-1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*l
n(b*x+a)/a^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{\left(ax^2+bx^3\right)^2}\,dx = -\frac{\frac{a(a^3-2a^2bx+6ab^2x^2+12b^3x^3)}{x^3(a+bx)}+12b^3\log(x)-12b^3\log(a+bx)}{3a^5}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-2),x]
```

output
```
-1/3*((a*(a^3 - 2*a^2*b*x + 6*a*b^2*x^2 + 12*b^3*x^3))/(x^3*(a + b*x)) + 1
2*b^3*Log[x] - 12*b^3*Log[a + b*x])/a^5
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {2026, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax^2+bx^3\right)^2} \, dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x^4(a+bx)^2}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{4b^4}{a^5(a+bx)}-\frac{4b^3}{a^5x}+\frac{b^4}{a^4(a+bx)^2}+\frac{3b^2}{a^4x^2}-\frac{2b}{a^3x^3}+\frac{1}{a^2x^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{4b^3\log(x)}{a^5}+\frac{4b^3\log(a+bx)}{a^5}-\frac{b^3}{a^4(a+bx)}-\frac{3b^2}{a^4x}+\frac{b}{a^3x^2}-\frac{1}{3a^2x^3}$$

input
```
Int[(a*x^2 + b*x^3)^(-2),x]
```

output
```
-1/3*1/(a^2*x^3) + b/(a^3*x^2) - (3*b^2)/(a^4*x) - b^3/(a^4*(a + b*x)) - (
4*b^3*Log[x])/a^5 + (4*b^3*Log[a + b*x])/a^5
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

method	result	size
default	$-\frac{1}{3a^2x^3}+\frac{b}{a^3x^2}-\frac{3b^2}{a^4x}-\frac{b^3}{a^4(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(bx+a)}{a^5}$	68
norman	$\frac{\frac{4b^4x^4}{a^5}-\frac{1}{3a}+\frac{2bx}{3a^2}-\frac{2b^2x^2}{a^3}}{x^3(bx+a)}-\frac{4b^3\ln(x)}{a^5}+\frac{4b^3\ln(bx+a)}{a^5}$	72
risch	$\frac{-\frac{4b^3x^3}{a^4}-\frac{2b^2x^2}{a^3}+\frac{2bx}{3a^2}-\frac{1}{3a}}{x^3(bx+a)}+\frac{4b^3\ln(-bx-a)}{a^5}-\frac{4b^3\ln(x)}{a^5}$	75
parallelrisch	$-\frac{12\ln(x)x^4b^4-12\ln(bx+a)x^4b^4+12\ln(x)x^3a\,b^3-12\ln(bx+a)x^3a\,b^3-12b^4x^4+6a^2b^2x^2-2a^3bx+a^4}{3a^5(bx+a)x^3}$	96

input
```
int(1/(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3/a^2/x^3+b/a^3/x^2-3*b^2/a^4/x-b^3/a^4/(b*x+a)-4*b^3*ln(x)/a^5+4*b^3*l
n(b*x+a)/a^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.38

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = -\frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4-12\,(b^4x^4+ab^3x^3)\log(bx+a)+12\,(b^4x^4+ab^3x^3)\log(x)}{3\,(a^5bx^4+a^6x^3)}$$

input
```
integrate(1/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(12*a*b^3*x^3 + 6*a^2*b^2*x^2 - 2*a^3*b*x + a^4 - 12*(b^4*x^4 + a*b^3
*x^3)*log(b*x + a) + 12*(b^4*x^4 + a*b^3*x^3)*log(x))/(a^5*b*x^4 + a^6*x^3
)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = \frac{-a^3+2a^2bx-6ab^2x^2-12b^3x^3}{3a^5x^3+3a^4bx^4} + \frac{4b^3\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^5}$$

input
```
integrate(1/(b*x**3+a*x**2)**2,x)
```

output
```
(-a**3 + 2*a**2*b*x - 6*a*b**2*x**2 - 12*b**3*x**3)/(3*a**5*x**3 + 3*a**4*
b*x**4) + 4*b**3*(-log(x) + log(a/b + x))/a**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = -\frac{12\,b^3x^3+6\,ab^2x^2-2\,a^2bx+a^3}{3\,(a^4bx^4+a^5x^3)} + \frac{4\,b^3\log(bx+a)}{a^5} - \frac{4\,b^3\log(x)}{a^5}$$

input
```
integrate(1/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
-1/3*(12*b^3*x^3 + 6*a*b^2*x^2 - 2*a^2*b*x + a^3)/(a^4*b*x^4 + a^5*x^3) +
4*b^3*log(b*x + a)/a^5 - 4*b^3*log(x)/a^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = \frac{4\,b^3\log\left(|bx+a|\right)}{a^5} - \frac{4\,b^3\log\left(|x|\right)}{a^5} - \frac{12\,ab^3x^3+6\,a^2b^2x^2-2\,a^3bx+a^4}{3\,(bx+a)a^5x^3}$$

input
```
integrate(1/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
4*b^3*log(abs(b*x + a))/a^5 - 4*b^3*log(abs(x))/a^5 - 1/3*(12*a*b^3*x^3 +
6*a^2*b^2*x^2 - 2*a^3*b*x + a^4)/((b*x + a)*a^5*x^3)
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = \frac{8\,b^3\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^5} - \frac{\frac{1}{3\,a}+\frac{2\,b^2\,x^2}{a^3}+\frac{4\,b^3\,x^3}{a^4}-\frac{2\,b\,x}{3\,a^2}}{b\,x^4+a\,x^3}$$

input
```
int(1/(a*x^2 + b*x^3)^2,x)
```

output
```
(8*b^3*atanh((2*b*x)/a + 1))/a^5 - (1/(3*a) + (2*b^2*x^2)/a^3 + (4*b^3*x^3
)/a^4 - (2*b*x)/(3*a^2))/(a*x^3 + b*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.41

$$\int \frac{1}{(ax^2+bx^3)^2}\,dx = \frac{12\log(bx+a)\,a\,b^3x^3+12\log(bx+a)\,b^4x^4-12\log(x)\,a\,b^3x^3-12\log(x)\,b^4x^4-a^4+2a^3bx-6a^2b^2x^2+}{3a^5x^3\,(bx+a)}$$

input
```
int(1/(b*x^3+a*x^2)^2,x)
```

output
```
(12*log(a + b*x)*a*b**3*x**3 + 12*log(a + b*x)*b**4*x**4 - 12*log(x)*a*b**
3*x**3 - 12*log(x)*b**4*x**4 - a**4 + 2*a**3*b*x - 6*a**2*b**2*x**2 + 12*b
**4*x**4)/(3*a**5*x**3*(a + b*x))
```

3.97 $\int \frac{1}{(ax^2+bx^3)^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 111

$$\int \frac{1}{(ax^2+bx^3)^3}\,dx = -\frac{1}{5a^3x^5}+\frac{3b}{4a^4x^4}-\frac{2b^2}{a^5x^3}+\frac{5b^3}{a^6x^2}-\frac{15b^4}{a^7x}-\frac{b^5}{2a^6(a+bx)^2}-\frac{6b^5}{a^7(a+bx)}-\frac{21b^5\log(x)}{a^8}+\frac{21b^5\log(a+bx)}{a^8}$$

output
```
-1/5/a^3/x^5+3/4*b/a^4/x^4-2*b^2/a^5/x^3+5*b^3/a^6/x^2-15*b^4/a^7/x-1/2*b^
5/a^6/(b*x+a)^2-6*b^5/a^7/(b*x+a)-21*b^5*ln(x)/a^8+21*b^5*ln(b*x+a)/a^8
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91

$$\int \frac{1}{(ax^2+bx^3)^3}\,dx = -\frac{\frac{a(4a^6-7a^5bx+14a^4b^2x^2-35a^3b^3x^3+140a^2b^4x^4+630ab^5x^5+420b^6x^6)}{x^5(a+bx)^2}+420b^5\log(x)-420b^5\log(a+bx)}{20a^8}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-3),x]
```

output
```
-1/20*((a*(4*a^6 - 7*a^5*b*x + 14*a^4*b^2*x^2 - 35*a^3*b^3*x^3 + 140*a^2*b
^4*x^4 + 630*a*b^5*x^5 + 420*b^6*x^6))/(x^5*(a + b*x)^2) + 420*b^5*Log[x]
- 420*b^5*Log[a + b*x])/a^8
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2026, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^3}dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x^6(a+bx)^3}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{21b^6}{a^8(a+bx)}-\frac{21b^5}{a^8x}+\frac{6b^6}{a^7(a+bx)^2}+\frac{15b^4}{a^7x^2}+\frac{b^6}{a^6(a+bx)^3}-\frac{10b^3}{a^6x^3}+\frac{6b^2}{a^5x^4}-\frac{3b}{a^4x^5}+\frac{1}{a^3x^6}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{21b^5\log(x)}{a^8}+\frac{21b^5\log(a+bx)}{a^8}-\frac{6b^5}{a^7(a+bx)}-\frac{15b^4}{a^7x}-\frac{b^5}{2a^6(a+bx)^2}+\frac{5b^3}{a^6x^2}-\frac{2b^2}{a^5x^3}+\frac{3b}{4a^4x^4}-\frac{1}{5a^3x^5}$$

input
```
Int[(a*x^2 + b*x^3)^(-3),x]
```

output
```
-1/5*1/(a^3*x^5) + (3*b)/(4*a^4*x^4) - (2*b^2)/(a^5*x^3) + (5*b^3)/(a^6*x^
2) - (15*b^4)/(a^7*x) - b^5/(2*a^6*(a + b*x)^2) - (6*b^5)/(a^7*(a + b*x))
- (21*b^5*Log[x])/a^8 + (21*b^5*Log[a + b*x])/a^8
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.95

method	result
norman	$\frac{-\frac{1}{5a}+\frac{7bx}{20a^{2}}-\frac{7b^{2}x^{2}}{10a^{3}}+\frac{7b^{3}x^{3}}{4a^{4}}-\frac{7b^{4}x^{4}}{a^{5}}+\frac{42b^{6}x^{6}}{a^{7}}+\frac{63b^{7}x^{7}}{2a^{8}}}{x^{5}\left(bx+a\right)^{2}}-\frac{21b^{5}\ln\left(x\right)}{a^{8}}+\frac{21b^{5}\ln\left(bx+a\right)}{a^{8}}$
default	$-\frac{1}{5a^{3}x^{5}}+\frac{3b}{4a^{4}x^{4}}-\frac{2b^{2}}{a^{5}x^{3}}+\frac{5b^{3}}{a^{6}x^{2}}-\frac{15b^{4}}{a^{7}x}-\frac{b^{5}}{2a^{6}\left(bx+a\right)^{2}}-\frac{6b^{5}}{a^{7}\left(bx+a\right)}-\frac{21b^{5}\ln\left(x\right)}{a^{8}}+\frac{21b^{5}\ln\left(bx+a\right)}{a^{8}}$
risch	$\frac{-\frac{21b^{6}x^{6}}{a^{7}}-\frac{63b^{5}x^{5}}{2a^{6}}-\frac{7b^{4}x^{4}}{a^{5}}+\frac{7b^{3}x^{3}}{4a^{4}}-\frac{7b^{2}x^{2}}{10a^{3}}+\frac{7bx}{20a^{2}}-\frac{1}{5a}}{x^{5}\left(bx+a\right)^{2}}-\frac{21b^{5}\ln\left(x\right)}{a^{8}}+\frac{21b^{5}\ln\left(-bx-a\right)}{a^{8}}$
parallelrisch	$-\frac{420\ln\left(x\right)x^{7}b^{7}-420\ln\left(bx+a\right)x^{7}b^{7}+840\ln\left(x\right)x^{6}a\,b^{6}-840\ln\left(bx+a\right)x^{6}a\,b^{6}-630x^{7}b^{7}+420\ln\left(x\right)x^{5}a^{2}b^{5}-420\ln\left(bx+a\right)x^{5}a^{2}b^{5}}{20a^{8}\left(bx+a\right)^{2}x^{5}}$

input
```
int(1/(b*x^3+a*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/5/a+7/20*b/a^2*x-7/10*b^2/a^3*x^2+7/4*b^3/a^4*x^3-7*b^4/a^5*x^4+42*b^6
/a^7*x^6+63/2*b^7/a^8*x^7)/x^5/(b*x+a)^2-21*b^5*ln(x)/a^8+21*b^5*ln(b*x+a)
/a^8
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.47

$$\int \frac{1}{(ax^2+bx^3)^3}\,dx = -\frac{420\,ab^6x^6+630\,a^2b^5x^5+140\,a^3b^4x^4-35\,a^4b^3x^3+14\,a^5b^2x^2-7\,a^6bx+4\,a^7-420\,(b^7x^7+2\,ab^6x^6+}{20\,(a^8b^2x^7+2\,a^9bx^6+a^{10}x^5)}$$

input
```
integrate(1/(b*x^3+a*x^2)^3,x, algorithm="fricas")
```

output
```
-1/20*(420*a*b^6*x^6 + 630*a^2*b^5*x^5 + 140*a^3*b^4*x^4 - 35*a^4*b^3*x^3
+ 14*a^5*b^2*x^2 - 7*a^6*b*x + 4*a^7 - 420*(b^7*x^7 + 2*a*b^6*x^6 + a^2*b^
5*x^5)*log(b*x + a) + 420*(b^7*x^7 + 2*a*b^6*x^6 + a^2*b^5*x^5)*log(x))/(a
^8*b^2*x^7 + 2*a^9*b*x^6 + a^10*x^5)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.05

$$\int \frac{1}{(ax^2+bx^3)^3}\,dx = \frac{-4a^6+7a^5bx-14a^4b^2x^2+35a^3b^3x^3-140a^2b^4x^4-630ab^5x^5-420b^6x^6}{20a^9x^5+40a^8bx^6+20a^7b^2x^7} + \frac{21b^5\left(-\log(x)+\log\left(\frac{a}{b}+x\right)\right)}{a^8}$$

input
```
integrate(1/(b*x**3+a*x**2)**3,x)
```

output
```
(-4*a**6 + 7*a**5*b*x - 14*a**4*b**2*x**2 + 35*a**3*b**3*x**3 - 140*a**2*b
**4*x**4 - 630*a*b**5*x**5 - 420*b**6*x**6)/(20*a**9*x**5 + 40*a**8*b*x**6
 + 20*a**7*b**2*x**7) + 21*b**5*(-log(x) + log(a/b + x))/a**8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.07

$$\int \frac{1}{(ax^2+bx^3)^3}\,dx = -\frac{420\,b^6x^6+630\,ab^5x^5+140\,a^2b^4x^4-35\,a^3b^3x^3+14\,a^4b^2x^2-7\,a^5bx+4\,a^6}{20\,(a^7b^2x^7+2\,a^8bx^6+a^9x^5)} + \frac{21\,b^5\log(bx+a)}{a^8} - \frac{21\,b^5\log(x)}{a^8}$$

input
```
integrate(1/(b*x^3+a*x^2)^3,x, algorithm="maxima")
```

output
```
-1/20*(420*b^6*x^6 + 630*a*b^5*x^5 + 140*a^2*b^4*x^4 - 35*a^3*b^3*x^3 + 14
*a^4*b^2*x^2 - 7*a^5*b*x + 4*a^6)/(a^7*b^2*x^7 + 2*a^8*b*x^6 + a^9*x^5) +
21*b^5*log(b*x + a)/a^8 - 21*b^5*log(x)/a^8
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.97

$$\int \frac{1}{(ax^2+bx^3)^3}\,dx = \frac{21\,b^5\log(|bx+a|)}{a^8} - \frac{21\,b^5\log(|x|)}{a^8} - \frac{420\,ab^6x^6+630\,a^2b^5x^5+140\,a^3b^4x^4-35\,a^4b^3x^3+14\,a^5b^2x^2-7\,a^6bx+4\,a^7}{20\,(bx+a)^2a^8x^5}$$

input
```
integrate(1/(b*x^3+a*x^2)^3,x, algorithm="giac")
```

output
```
21*b^5*log(abs(b*x + a))/a^8 - 21*b^5*log(abs(x))/a^8 - 1/20*(420*a*b^6*x^
6 + 630*a^2*b^5*x^5 + 140*a^3*b^4*x^4 - 35*a^4*b^3*x^3 + 14*a^5*b^2*x^2 -
7*a^6*b*x + 4*a^7)/((b*x + a)^2*a^8*x^5)
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.02

$$\int \frac{1}{(ax^2+bx^3)^3}\,dx = \frac{42\,b^5\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^8} - \frac{\frac{1}{5\,a}+\frac{7\,b^2\,x^2}{10\,a^3}-\frac{7\,b^3\,x^3}{4\,a^4}+\frac{7\,b^4\,x^4}{a^5}+\frac{63\,b^5\,x^5}{2\,a^6}+\frac{21\,b^6\,x^6}{a^7}-\frac{7\,b\,x}{20\,a^2}}{a^2\,x^5+2\,a\,b\,x^6+b^2\,x^7}$$

input
```
int(1/(a*x^2 + b*x^3)^3,x)
```

output
```
(42*b^5*atanh((2*b*x)/a + 1))/a^8 - (1/(5*a) + (7*b^2*x^2)/(10*a^3) - (7*b
^3*x^3)/(4*a^4) + (7*b^4*x^4)/a^5 + (63*b^5*x^5)/(2*a^6) + (21*b^6*x^6)/a^
7 - (7*b*x)/(20*a^2))/(a^2*x^5 + b^2*x^7 + 2*a*b*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.54

$$\int \frac{1}{(ax^2+bx^3)^3}\,dx = \frac{420\log(bx+a)\,a^2b^5x^5+840\log(bx+a)\,a\,b^6x^6+420\log(bx+a)\,b^7x^7-420\log(x)\,a^2b^5x^5-840\log(x)\,a}{20a^8x^5\,(b^2x^2+2}$$

input
```
int(1/(b*x^3+a*x^2)^3,x)
```

output
```
(420*log(a + b*x)*a**2*b**5*x**5 + 840*log(a + b*x)*a*b**6*x**6 + 420*log(
a + b*x)*b**7*x**7 - 420*log(x)*a**2*b**5*x**5 - 840*log(x)*a*b**6*x**6 -
420*log(x)*b**7*x**7 - 4*a**7 + 7*a**6*b*x - 14*a**5*b**2*x**2 + 35*a**4*b
**3*x**3 - 140*a**3*b**4*x**4 - 420*a**2*b**5*x**5 + 210*b**7*x**7)/(20*a*
*8*x**5*(a**2 + 2*a*b*x + b**2*x**2))
```

3.98 $\int \frac{1}{\left(ax^2+bx^3\right)^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 150

$$\int \frac{1}{\left(ax^2+bx^3\right)^4}\,dx = -\frac{1}{7a^4x^7}+\frac{2b}{3a^5x^6}-\frac{2b^2}{a^6x^5}+\frac{5b^3}{a^7x^4}-\frac{35b^4}{3a^8x^3}+\frac{28b^5}{a^9x^2}-\frac{84b^6}{a^{10}x}-\frac{b^7}{3a^8(a+bx)^3}-\frac{4b^7}{a^9(a+bx)^2}-\frac{36b^7}{a^{10}(a+bx)}-\frac{120b^7\log(x)}{a^{11}}+\frac{120b^7\log(a+bx)}{a^{11}}$$

output
```
-1/7/a^4/x^7+2/3*b/a^5/x^6-2*b^2/a^6/x^5+5*b^3/a^7/x^4-35/3*b^4/a^8/x^3+28
*b^5/a^9/x^2-84*b^6/a^10/x-1/3*b^7/a^8/(b*x+a)^3-4*b^7/a^9/(b*x+a)^2-36*b^
7/a^10/(b*x+a)-120*b^7*ln(x)/a^11+120*b^7*ln(b*x+a)/a^11
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.89

$$\int \frac{1}{\left(ax^2+bx^3\right)^4}\,dx = -\frac{\frac{a\left(3a^9-5a^8bx+9a^7b^2x^2-18a^6b^3x^3+42a^5b^4x^4-126a^4b^5x^5+630a^3b^6x^6+4620a^2b^7x^7+6300ab^8x^8+2520b^9x^9\right)}{x^7(a+bx)^3}+2520b^7\log(x)-25}{21a^{11}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-4),x]
```

output
```
-1/21*((a*(3*a^9 - 5*a^8*b*x + 9*a^7*b^2*x^2 - 18*a^6*b^3*x^3 + 42*a^5*b^4
*x^4 - 126*a^4*b^5*x^5 + 630*a^3*b^6*x^6 + 4620*a^2*b^7*x^7 + 6300*a*b^8*x
^8 + 2520*b^9*x^9))/(x^7*(a + b*x)^3) + 2520*b^7*Log[x] - 2520*b^7*Log[a +
 b*x])/a^11
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {2026, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^4}\,dx$$

$$\Big\downarrow \text{2026}$$

$$\int \frac{1}{x^8(a+bx)^4}dx$$

$$\Big\downarrow \text{54}$$

$$\int \left(\frac{120b^8}{a^{11}(a+bx)}-\frac{120b^7}{a^{11}x}+\frac{36b^8}{a^{10}(a+bx)^2}+\frac{84b^6}{a^{10}x^2}+\frac{8b^8}{a^9(a+bx)^3}-\frac{56b^5}{a^9x^3}+\frac{b^8}{a^8(a+bx)^4}+\frac{35b^4}{a^8x^4}-\frac{20b^3}{a^7x^5}+\frac{10b^2}{a^6x^6}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{120b^7\log(x)}{a^{11}}+\frac{120b^7\log(a+bx)}{a^{11}}-\frac{36b^7}{a^{10}(a+bx)}-\frac{84b^6}{a^{10}x}-\frac{4b^7}{a^9(a+bx)^2}+\frac{28b^5}{a^9x^2}-\frac{b^7}{3a^8(a+bx)^3}-\frac{35b^4}{3a^8x^3}+\frac{5b^3}{a^7x^4}-\frac{2b^2}{a^6x^5}+\frac{2b}{3a^5x^6}-\frac{1}{7a^4x^7}$$

input
```
Int[(a*x^2 + b*x^3)^(-4),x]
```

output
```
-1/7*1/(a^4*x^7) + (2*b)/(3*a^5*x^6) - (2*b^2)/(a^6*x^5) + (5*b^3)/(a^7*x^
4) - (35*b^4)/(3*a^8*x^3) + (28*b^5)/(a^9*x^2) - (84*b^6)/(a^10*x) - b^7/(
3*a^8*(a + b*x)^3) - (4*b^7)/(a^9*(a + b*x)^2) - (36*b^7)/(a^10*(a + b*x))
 - (120*b^7*Log[x])/a^11 + (120*b^7*Log[a + b*x])/a^11
```

Defintions of rubi rules used

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

method	result
norman	$\frac{\frac{220b^{10}x^{10}}{a^{11}}-\frac{1}{7a}+\frac{5bx}{21a^2}-\frac{3b^2x^2}{7a^3}+\frac{6b^3x^3}{7a^4}-\frac{2b^4x^4}{a^5}+\frac{6b^5x^5}{a^6}-\frac{30b^6x^6}{a^7}+\frac{360b^8x^8}{a^9}+\frac{540b^9x^9}{a^{10}}}{x^7(bx+a)^3}-\frac{120b^7\ln(x)}{a^{11}}+\frac{120b^7\ln(bx+a)}{a^{11}}$
risch	$\frac{-\frac{120b^9x^9}{a^{10}}-\frac{300b^8x^8}{a^9}-\frac{220b^7x^7}{a^8}-\frac{30b^6x^6}{a^7}+\frac{6b^5x^5}{a^6}-\frac{2b^4x^4}{a^5}+\frac{6b^3x^3}{7a^4}-\frac{3b^2x^2}{7a^3}+\frac{5bx}{21a^2}-\frac{1}{7a}}{x^7(bx+a)^3}-\frac{120b^7\ln(x)}{a^{11}}+\frac{120b^7\ln(-bx-a)}{a^{11}}$
default	$-\frac{1}{7a^4x^7}+\frac{2b}{3a^5x^6}-\frac{2b^2}{a^6x^5}+\frac{5b^3}{a^7x^4}-\frac{35b^4}{3a^8x^3}+\frac{28b^5}{a^9x^2}-\frac{84b^6}{a^{10}x}-\frac{b^7}{3a^8(bx+a)^3}-\frac{4b^7}{a^9(bx+a)^2}-\frac{36b^7}{a^{10}(bx+a)}-$ 120
parallelrisch	$-\frac{2520\ln(x)x^{10}b^{10}-2520\ln(bx+a)x^{10}b^{10}+7560\ln(x)x^9ab^9-7560\ln(bx+a)x^9ab^9-4620x^{10}b^{10}+7560\ln(x)x^8a^2b^8-7560\ \mathrm{ln}}{}$

input
```
int(1/(b*x^3+a*x^2)^4,x,method=_RETURNVERBOSE)
```

output
```
(220*b^10/a^11*x^10-1/7/a+5/21*b/a^2*x-3/7*b^2/a^3*x^2+6/7*b^3/a^4*x^3-2*b
^4/a^5*x^4+6*b^5/a^6*x^5-30*b^6/a^7*x^6+360*b^8/a^9*x^8+540*b^9/a^10*x^9)/
x^7/(b*x+a)^3-120*b^7*ln(x)/a^11+120*b^7*ln(b*x+a)/a^11
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.53

$$\int \frac{1}{\left(ax^2+bx^3\right)^4}\,dx = -\frac{2520\,ab^9x^9+6300\,a^2b^8x^8+4620\,a^3b^7x^7+630\,a^4b^6x^6-126\,a^5b^5x^5+42\,a^6b^4x^4-18\,a^7b^3x^3+9\,a^8b^2x^2}{21\,(a^{11}b^3}$$

input
```
integrate(1/(b*x^3+a*x^2)^4,x, algorithm="fricas")
```

output
```
-1/21*(2520*a*b^9*x^9 + 6300*a^2*b^8*x^8 + 4620*a^3*b^7*x^7 + 630*a^4*b^6*
x^6 - 126*a^5*b^5*x^5 + 42*a^6*b^4*x^4 - 18*a^7*b^3*x^3 + 9*a^8*b^2*x^2 -
5*a^9*b*x + 3*a^10 - 2520*(b^10*x^10 + 3*a*b^9*x^9 + 3*a^2*b^8*x^8 + a^3*b
^7*x^7)*log(b*x + a) + 2520*(b^10*x^10 + 3*a*b^9*x^9 + 3*a^2*b^8*x^8 + a^3
*b^7*x^7)*log(x))/(a^11*b^3*x^10 + 3*a^12*b^2*x^9 + 3*a^13*b*x^8 + a^14*x^
7)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(ax^2+bx^3\right)^4}\,dx = \frac{-3a^9+5a^8bx-9a^7b^2x^2+18a^6b^3x^3-42a^5b^4x^4+126a^4b^5x^5-630a^3b^6x^6-4620a^2b^7x^7-6300ab^8x^8-}{21a^{13}x^7+63a^{12}bx^8+63a^{11}b^2x^9+21a^{10}b^3x^{10}} + \frac{120b^7\left(-\log\left(x\right)+\log\left(\frac{a}{b}+x\right)\right)}{a^{11}}$$

input
```
integrate(1/(b*x**3+a*x**2)**4,x)
```

output
```
(-3*a**9 + 5*a**8*b*x - 9*a**7*b**2*x**2 + 18*a**6*b**3*x**3 - 42*a**5*b**
4*x**4 + 126*a**4*b**5*x**5 - 630*a**3*b**6*x**6 - 4620*a**2*b**7*x**7 - 6
300*a*b**8*x**8 - 2520*b**9*x**9)/(21*a**13*x**7 + 63*a**12*b*x**8 + 63*a*
*11*b**2*x**9 + 21*a**10*b**3*x**10) + 120*b**7*(-log(x) + log(a/b + x))/a
**11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.09

$$\int \frac{1}{(ax^2+bx^3)^4}\,dx = -\frac{2520\,b^9x^9+6300\,ab^8x^8+4620\,a^2b^7x^7+630\,a^3b^6x^6-126\,a^4b^5x^5+42\,a^5b^4x^4-18\,a^6b^3x^3+9\,a^7b^2x^2-}{21\,(a^{10}b^3x^{10}+3\,a^{11}b^2x^9+3\,a^{12}bx^8+a^{13}x^7)} + \frac{120\,b^7\log(bx+a)}{a^{11}} - \frac{120\,b^7\log(x)}{a^{11}}$$

input
```
integrate(1/(b*x^3+a*x^2)^4,x, algorithm="maxima")
```

output
```
-1/21*(2520*b^9*x^9 + 6300*a*b^8*x^8 + 4620*a^2*b^7*x^7 + 630*a^3*b^6*x^6
- 126*a^4*b^5*x^5 + 42*a^5*b^4*x^4 - 18*a^6*b^3*x^3 + 9*a^7*b^2*x^2 - 5*a^
8*b*x + 3*a^9)/(a^10*b^3*x^10 + 3*a^11*b^2*x^9 + 3*a^12*b*x^8 + a^13*x^7)
+ 120*b^7*log(b*x + a)/a^11 - 120*b^7*log(x)/a^11
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.94

$$\int \frac{1}{(ax^2+bx^3)^4}\,dx = \frac{120\,b^7\log(|bx+a|)}{a^{11}} - \frac{120\,b^7\log(|x|)}{a^{11}} - \frac{2520\,ab^9x^9+6300\,a^2b^8x^8+4620\,a^3b^7x^7+630\,a^4b^6x^6-126\,a^5b^5x^5+42\,a^6b^4x^4-18\,a^7b^3x^3+9\,a^8b^2x^2}{21\,(bx+a)^3a^{11}x^7}$$

input
```
integrate(1/(b*x^3+a*x^2)^4,x, algorithm="giac")
```

output
```
120*b^7*log(abs(b*x + a))/a^11 - 120*b^7*log(abs(x))/a^11 - 1/21*(2520*a*b
^9*x^9 + 6300*a^2*b^8*x^8 + 4620*a^3*b^7*x^7 + 630*a^4*b^6*x^6 - 126*a^5*b
^5*x^5 + 42*a^6*b^4*x^4 - 18*a^7*b^3*x^3 + 9*a^8*b^2*x^2 - 5*a^9*b*x + 3*a
^10)/((b*x + a)^3*a^11*x^7)
```

Mupad [B] (verification not implemented)

Time = 9.12 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.05

$$\int \frac{1}{(ax^2+bx^3)^4}\,dx$$
$$= \frac{240\,b^7\,\mathrm{atanh}\left(\frac{2\,b\,x}{a}+1\right)}{a^{11}} - \frac{\frac{1}{7\,a}+\frac{3\,b^2\,x^2}{7\,a^3}-\frac{6\,b^3\,x^3}{7\,a^4}+\frac{2\,b^4\,x^4}{a^5}-\frac{6\,b^5\,x^5}{a^6}+\frac{30\,b^6\,x^6}{a^7}+\frac{220\,b^7\,x^7}{a^8}+\frac{300\,b^8\,x^8}{a^9}+\frac{120\,b^9\,x^9}{a^{10}}-\frac{5\,b\,x}{21\,a^2}}{a^3\,x^7+3\,a^2\,b\,x^8+3\,a\,b^2\,x^9+b^3\,x^{10}}$$

input
```
int(1/(a*x^2 + b*x^3)^4,x)
```

output
```
(240*b^7*atanh((2*b*x)/a + 1))/a^11 - (1/(7*a) + (3*b^2*x^2)/(7*a^3) - (6*
b^3*x^3)/(7*a^4) + (2*b^4*x^4)/a^5 - (6*b^5*x^5)/a^6 + (30*b^6*x^6)/a^7 +
(220*b^7*x^7)/a^8 + (300*b^8*x^8)/a^9 + (120*b^9*x^9)/a^10 - (5*b*x)/(21*a
^2))/(a^3*x^7 + b^3*x^10 + 3*a^2*b*x^8 + 3*a*b^2*x^9)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.63

$$\int \frac{1}{(ax^2+bx^3)^4}\,dx$$
$$= \frac{2520\log(bx+a)\,a^3b^7x^7+7560\log(bx+a)\,a^2b^8x^8+7560\log(bx+a)\,a\,b^9x^9+2520\log(bx+a)\,b^{10}x^{10}-}{}$$

input
```
int(1/(b*x^3+a*x^2)^4,x)
```

output

```
(2520*log(a + b*x)*a**3*b**7*x**7 + 7560*log(a + b*x)*a**2*b**8*x**8 + 756
0*log(a + b*x)*a*b**9*x**9 + 2520*log(a + b*x)*b**10*x**10 - 2520*log(x)*a
**3*b**7*x**7 - 7560*log(x)*a**2*b**8*x**8 - 7560*log(x)*a*b**9*x**9 - 252
0*log(x)*b**10*x**10 - 3*a**10 + 5*a**9*b*x - 9*a**8*b**2*x**2 + 18*a**7*b
**3*x**3 - 42*a**6*b**4*x**4 + 126*a**5*b**5*x**5 - 630*a**4*b**6*x**6 - 3
780*a**3*b**7*x**7 - 3780*a**2*b**8*x**8 + 840*b**10*x**10)/(21*a**11*x**7
*(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3))
```

3.99 $\int (ax^2+bx^3)^{7/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 220

$$\int (ax^2+bx^3)^{7/2}\,dx = -\frac{2a^7(ax^2+bx^3)^{9/2}}{9b^8x^9}+\frac{14a^6(ax^2+bx^3)^{11/2}}{11b^8x^{11}} -\frac{42a^5(ax^2+bx^3)^{13/2}}{13b^8x^{13}}+\frac{14a^4(ax^2+bx^3)^{15/2}}{3b^8x^{15}}-\frac{70a^3(ax^2+bx^3)^{17/2}}{17b^8x^{17}} +\frac{42a^2(ax^2+bx^3)^{19/2}}{19b^8x^{19}}-\frac{2a(ax^2+bx^3)^{21/2}}{3b^8x^{21}}+\frac{2(ax^2+bx^3)^{23/2}}{23b^8x^{23}}$$

output
```
-2/9*a^7*(b*x^3+a*x^2)^(9/2)/b^8/x^9+14/11*a^6*(b*x^3+a*x^2)^(11/2)/b^8/x^
11-42/13*a^5*(b*x^3+a*x^2)^(13/2)/b^8/x^13+14/3*a^4*(b*x^3+a*x^2)^(15/2)/b
^8/x^15-70/17*a^3*(b*x^3+a*x^2)^(17/2)/b^8/x^17+42/19*a^2*(b*x^3+a*x^2)^(1
9/2)/b^8/x^19-2/3*a*(b*x^3+a*x^2)^(21/2)/b^8/x^21+2/23*(b*x^3+a*x^2)^(23/2
)/b^8/x^23
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.46

$$\int (ax^2+bx^3)^{7/2}\,dx=\frac{2x(a+bx)^5(-2048a^7+9216a^6bx-25344a^5b^2x^2+54912a^4b^3x^3-102960a^3b^4x^4+175032}{9561123b^8\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(7/2),x]
```

output
```
(2*x*(a + b*x)^5*(-2048*a^7 + 9216*a^6*b*x - 25344*a^5*b^2*x^2 + 54912*a^4
*b^3*x^3 - 102960*a^3*b^4*x^4 + 175032*a^2*b^5*x^5 - 277134*a*b^6*x^6 + 41
5701*b^7*x^7))/(9561123*b^8*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}=0.533$, Rules used = {1908, 1922, 1922, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^2+bx^3)^{7/2}\,dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{2(ax^2+bx^3)^{9/2}}{23bx^2}-\frac{14a\int \frac{(bx^3+ax^2)^{7/2}}{x}dx}{23b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2(ax^2+bx^3)^{9/2}}{23bx^2}-\frac{14a\left(\frac{2(ax^2+bx^3)^{9/2}}{21bx^3}-\frac{4a\int \frac{(bx^3+ax^2)^{7/2}}{x^2}dx}{7b}\right)}{23b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2(ax^2+bx^3)^{9/2}}{23bx^2}-\frac{14a\left(\frac{2(ax^2+bx^3)^{9/2}}{21bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{19bx^4}-\frac{10a\int\frac{\left(bx^3+ax^2\right)^{7/2}}{x^3}dx}{19b}\right)}{7b}\right)}{23b}$$

$\downarrow$ 1922

$$\frac{2(ax^2+bx^3)^{9/2}}{23bx^2}-\frac{14a\left(\frac{2(ax^2+bx^3)^{9/2}}{21bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{19bx^4}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{17bx^5}-\frac{8a\int\frac{\left(bx^3+ax^2\right)^{7/2}}{x^4}dx}{17b}\right)}{19b}\right)}{7b}\right)}{23b}$$

$\downarrow$ 1922

$$\frac{2\left(ax^2+bx^3\right)^{9/2}}{23bx^2}-\frac{14a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{21bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{19bx^4}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{17bx^5}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{15bx^6}-\frac{2a\int\frac{\left(bx^3+ax^2\right)^{7/2}}{x^5}dx}{5b}\right)}{17b}\right)}{19b}\right)}{7b}\right)}{23b}$$

$\downarrow$ 1922

$$\frac{2\left(ax^2+bx^3\right)^{9/2}}{23bx^2}-\frac{14a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{21bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{19bx^4}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{17bx^5}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{15bx^6}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{13bx^7}-\frac{4a\int\frac{\left(bx^3+ax^2\right)^{7/2}}{x^6}dx}{13b}\right)}{5b}\right)}{17b}\right)}{19b}\right)}{7b}\right)}{23b}$$

↓ 1922

$$\frac{2\left(ax^2+bx^3\right)^{9/2}}{23bx^2} -$$

$$4a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{11bx^8} - \frac{2a\int \frac{\left(bx^3 \cdots\right.}{\cdots}}{\cdots}\right.$$

$$2a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{13bx^7} - \frac{\cdots}{13b}\right.$$

$$8a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{15bx^6} - \frac{\cdots}{5b}\right.$$

$$10a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{17bx^5} - \frac{\cdots}{17b}\right.$$

$$4a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{19bx^4} - \frac{\cdots}{19b}\right.$$

$$14a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{21bx^3} - \frac{\cdots}{7b}\right.$$

↓ 1920

$$\frac{2\left(ax^2+bx^3\right)^{9/2}}{23bx^2}-$$

$$2a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{13bx^7}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{11bx^8}-\frac{4a\left(ax^2+b\right.}{99b^2}\right.}{13b}\right.$$

$$8a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{15bx^6}-\frac{\quad}{5b}\right.$$

$$10a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{17bx^5}-\frac{\quad}{17b}\right.$$

$$4a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{19bx^4}-\frac{\quad}{19b}\right.$$

$$14a\left(\frac{2\left(ax^2+bx^3\right)^{9/2}}{21bx^3}-\frac{\quad}{7b}\right.$$

input
```
Int[(a*x^2 + b*x^3)^(7/2),x]
```

output
```
(2*(a*x^2 + b*x^3)^(9/2))/(23*b*x^2) - (14*a*((2*(a*x^2 + b*x^3)^(9/2))/(2
1*b*x^3) - (4*a*((2*(a*x^2 + b*x^3)^(9/2))/(19*b*x^4) - (10*a*((2*(a*x^2 +
 b*x^3)^(9/2))/(17*b*x^5) - (8*a*((2*(a*x^2 + b*x^3)^(9/2))/(15*b*x^6) - (
2*a*((2*(a*x^2 + b*x^3)^(9/2))/(13*b*x^7) - (4*a*((-4*a*(a*x^2 + b*x^3)^(9
/2))/(99*b^2*x^9) + (2*(a*x^2 + b*x^3)^(9/2))/(11*b*x^8)))/(13*b)))/(5*b))
)/(17*b)))/(19*b)))/(7*b)))/(23*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.06

method	result
pseudoelliptic	$\frac{2(bx+a)^{\frac{9}{2}}}{9b}$
gosper	$-\frac{2(bx+a)\left(-415701x^7b^7+277134b^6a\,x^6-175032a^2x^5b^5+102960b^4x^4a^3-54912b^3x^3a^4+25344x^2b^2a^5-9216xb\,a^6+2048a\right.}{9561123b^8x^7}$
default	$-\frac{2(bx+a)\left(-415701x^7b^7+277134b^6a\,x^6-175032a^2x^5b^5+102960b^4x^4a^3-54912b^3x^3a^4+25344x^2b^2a^5-9216xb\,a^6+2048a\right.}{9561123b^8x^7}$
orering	$-\frac{2(bx+a)\left(-415701x^7b^7+277134b^6a\,x^6-175032a^2x^5b^5+102960b^4x^4a^3-54912b^3x^3a^4+25344x^2b^2a^5-9216xb\,a^6+2048a\right.}{9561123b^8x^7}$
risch	$-\frac{2\sqrt{x^2(bx+a)}\left(-415701b^{11}x^{11}-1385670a\,b^{10}x^{10}-1560702a^2b^9x^9-597168a^3b^8x^8-429a^4b^7x^7+462a^5b^6x^6-504a^6x^5b^5\right.}{9561123x\,b^8}$
trager	$-\frac{2\left(-415701b^{11}x^{11}-1385670a\,b^{10}x^{10}-1560702a^2b^9x^9-597168a^3b^8x^8-429a^4b^7x^7+462a^5b^6x^6-504a^6x^5b^5+560a^7x^4b^4\right.}{9561123b^8x}$

input
```
int((b*x^3+a*x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/9/b*(b*x+a)^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.63

$$\int (ax^2+bx^3)^{7/2}\,dx = \frac{2\left(415701\,b^{11}x^{11}+1385670\,ab^{10}x^{10}+1560702\,a^2b^9x^9+597168\,a^3b^8x^8+429\,a^4b^7x^7-462\,a^5\right.}{9561}$$

input
```
integrate((b*x^3+a*x^2)^(7/2),x, algorithm="fricas")
```

output
```
2/9561123*(415701*b^11*x^11 + 1385670*a*b^10*x^10 + 1560702*a^2*b^9*x^9 +
597168*a^3*b^8*x^8 + 429*a^4*b^7*x^7 - 462*a^5*b^6*x^6 + 504*a^6*b^5*x^5 -
 560*a^7*b^4*x^4 + 640*a^8*b^3*x^3 - 768*a^9*b^2*x^2 + 1024*a^10*b*x - 204
8*a^11)*sqrt(b*x^3 + a*x^2)/(b^8*x)
```

Sympy [F]

$$\int \left(ax^2+bx^3\right)^{7/2} dx = \int \left(ax^2+bx^3\right)^{\frac{7}{2}} dx$$

input
```
integrate((b*x**3+a*x**2)**(7/2),x)
```

output
```
Integral((a*x**2 + b*x**3)**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.59

$$\int \left(ax^2 +bx^3\right)^{7/2} dx = \frac{2\left(415701\,b^{11}x^{11}+1385670\,ab^{10}x^{10}+1560702\,a^2b^9x^9+597168\,a^3b^8x^8+429\,a^4b^7x^7-462\,a^5\right.}{9561}$$

input
```
integrate((b*x^3+a*x^2)^(7/2),x, algorithm="maxima")
```

output
```
2/9561123*(415701*b^11*x^11 + 1385670*a*b^10*x^10 + 1560702*a^2*b^9*x^9 +
597168*a^3*b^8*x^8 + 429*a^4*b^7*x^7 - 462*a^5*b^6*x^6 + 504*a^6*b^5*x^5 -
 560*a^7*b^4*x^4 + 640*a^8*b^3*x^3 - 768*a^9*b^2*x^2 + 1024*a^10*b*x - 204
8*a^11)*sqrt(b*x + a)/b^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. 2(188) = 376.

Time = 0.12 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.92

$$\int \left(ax^2+bx^3\right)^{7/2} dx = \text{Too large to display}$$

input
```
integrate((b*x^3+a*x^2)^(7/2),x, algorithm="giac")
```

output
```
4096/9561123*a^(23/2)*sgn(x)/b^8 + 2/334639305*(52003*(429*(b*x + a)^(15/2
) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)
^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015
*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^4*sgn(x)/b^7 + 12236*(643
5*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^
2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x
 + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7
+ 109395*sqrt(b*x + a)*a^8)*a^3*sgn(x)/b^7 + 8694*(12155*(b*x + a)^(19/2)
- 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x +
 a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^
5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*
x + a)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*a^2*sgn(x)/b^7 + 1380*(46189*
(b*x + a)^(21/2) - 510510*(b*x + a)^(19/2)*a + 2567565*(b*x + a)^(17/2)*a^
2 - 7759752*(b*x + a)^(15/2)*a^3 + 15668730*(b*x + a)^(13/2)*a^4 - 2222110
8*(b*x + a)^(11/2)*a^5 + 22632610*(b*x + a)^(9/2)*a^6 - 16628040*(b*x + a)
^(7/2)*a^7 + 8729721*(b*x + a)^(5/2)*a^8 - 3233230*(b*x + a)^(3/2)*a^9 + 9
69969*sqrt(b*x + a)*a^10)*a*sgn(x)/b^7 + 165*(88179*(b*x + a)^(23/2) - 106
2347*(b*x + a)^(21/2)*a + 5870865*(b*x + a)^(19/2)*a^2 - 19684665*(b*x + a
)^(17/2)*a^3 + 44618574*(b*x + a)^(15/2)*a^4 - 72076158*(b*x + a)^(13/2)*a
^5 + 85180914*(b*x + a)^(11/2)*a^6 - 74364290*(b*x + a)^(9/2)*a^7 + 478...
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.46

$$\int \left(ax^2+bx^3\right)^{7/2}\,dx =$$
$$-\frac{2\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^4\,(2048\,a^7-9216\,a^6\,b\,x+25344\,a^5\,b^2\,x^2-54912\,a^4\,b^3\,x^3+102960\,a^3\,b^4\,x^4-1750}{9561123\,b^8\,x}$$

input
```
int((a*x^2 + b*x^3)^(7/2),x)
```

output
```
-(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^4*(2048*a^7 - 415701*b^7*x^7 + 277134*
a*b^6*x^6 + 25344*a^5*b^2*x^2 - 54912*a^4*b^3*x^3 + 102960*a^3*b^4*x^4 - 1
75032*a^2*b^5*x^5 - 9216*a^6*b*x))/(9561123*b^8*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.59

$$\int (ax^2 + bx^3)^{7/2}\,dx = \frac{2\sqrt{bx+a}\,(415701b^{11}x^{11} + 1385670a\,b^{10}x^{10} + 1560702a^2b^9x^9 + 597168a^3b^8x^8 + 429a^4b^7x^7 -}{956112}$$

input
```
int((b*x^3+a*x^2)^(7/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 2048*a**11 + 1024*a**10*b*x - 768*a**9*b**2*x**2 + 64
0*a**8*b**3*x**3 - 560*a**7*b**4*x**4 + 504*a**6*b**5*x**5 - 462*a**5*b**6
*x**6 + 429*a**4*b**7*x**7 + 597168*a**3*b**8*x**8 + 1560702*a**2*b**9*x**
9 + 1385670*a*b**10*x**10 + 415701*b**11*x**11))/(9561123*b**8)
```

3.100 $\int \left(ax^2+bx^3\right)^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 164

$$\int \left(ax^2+bx^3\right)^{5/2}\,dx=-\frac{2a^5\left(ax^2+bx^3\right)^{7/2}}{7b^6x^7}+\frac{10a^4\left(ax^2+bx^3\right)^{9/2}}{9b^6x^9}-\frac{20a^3\left(ax^2+bx^3\right)^{11/2}}{11b^6x^{11}}+\frac{20a^2\left(ax^2+bx^3\right)^{13/2}}{13b^6x^{13}}-\frac{2a\left(ax^2+bx^3\right)^{15/2}}{3b^6x^{15}}+\frac{2\left(ax^2+bx^3\right)^{17/2}}{17b^6x^{17}}$$

output
```
-2/7*a^5*(b*x^3+a*x^2)^(7/2)/b^6/x^7+10/9*a^4*(b*x^3+a*x^2)^(9/2)/b^6/x^9-
20/11*a^3*(b*x^3+a*x^2)^(11/2)/b^6/x^11+20/13*a^2*(b*x^3+a*x^2)^(13/2)/b^6
/x^13-2/3*a*(b*x^3+a*x^2)^(15/2)/b^6/x^15+2/17*(b*x^3+a*x^2)^(17/2)/b^6/x^
17
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

$$\int \left(ax^2+bx^3\right)^{5/2}\,dx=\frac{2x(a+bx)^4\left(-256a^5+896a^4bx-2016a^3b^2x^2+3696a^2b^3x^3-6006ab^4x^4+9009b^5x^5\right)}{153153b^6\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*x*(a + b*x)^4*(-256*a^5 + 896*a^4*b*x - 2016*a^3*b^2*x^2 + 3696*a^2*b^3
*x^3 - 6006*a*b^4*x^4 + 9009*b^5*x^5))/(153153*b^6*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {1908, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^{5/2} dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x}dx}{17b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^2}dx}{15b}\right)}{17b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^3}dx}{13b}\right)}{15b}\right)}{17b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x^4}dx}{11b}\right)}{13b}\right)}{15b}\right)}{17b}$$

$\downarrow$ 1922

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{9bx^6}-\frac{2a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x^5}dx}{9b}\right)}{11b}\right)}{13b}\right)}{15b}\right)}{17b}$$

$\downarrow$ 1920

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{9bx^6}-\frac{4a\left(ax^2+bx^3\right)^{7/2}}{63b^2x^7}\right)}{11b}\right)}{13b}\right)}{15b}\right)}{17b}$$

input
```
Int[(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*(a*x^2 + b*x^3)^(7/2))/(17*b*x^2) - (10*a*((2*(a*x^2 + b*x^3)^(7/2))/(1
5*b*x^3) - (8*a*((2*(a*x^2 + b*x^3)^(7/2))/(13*b*x^4) - (6*a*((2*(a*x^2 +
b*x^3)^(7/2))/(11*b*x^5) - (4*a*((-4*a*(a*x^2 + b*x^3)^(7/2))/(63*b^2*x^7)
 + (2*(a*x^2 + b*x^3)^(7/2))/(9*b*x^6)))/(11*b)))/(13*b)))/(15*b)))/(17*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.08

method	result
pseudoelliptic	$\frac{2(bx+a)^{\frac{7}{2}}}{7b}$
gosper	$-\frac{2(bx+a)\left(-9009b^5x^5+6006a\,b^4x^4-3696a^2b^3x^3+2016a^3b^2x^2-896a^4bx+256a^5\right)\left(b\,x^3+a\,x^2\right)^{\frac{5}{2}}}{153153b^6x^5}$
default	$-\frac{2(bx+a)\left(-9009b^5x^5+6006a\,b^4x^4-3696a^2b^3x^3+2016a^3b^2x^2-896a^4bx+256a^5\right)\left(b\,x^3+a\,x^2\right)^{\frac{5}{2}}}{153153b^6x^5}$
orering	$-\frac{2(bx+a)\left(-9009b^5x^5+6006a\,b^4x^4-3696a^2b^3x^3+2016a^3b^2x^2-896a^4bx+256a^5\right)\left(b\,x^3+a\,x^2\right)^{\frac{5}{2}}}{153153b^6x^5}$
risch	$-\frac{2\sqrt{x^2(bx+a)}\,\left(-9009b^8x^8-21021a\,b^7x^7-12705a^2b^6x^6-63a^3b^5x^5+70a^4b^4x^4-80a^5x^3b^3+96a^6x^2b^2-128a^7bx+256a^8\right)}{153153x\,b^6}$
trager	$-\frac{2\left(-9009b^8x^8-21021a\,b^7x^7-12705a^2b^6x^6-63a^3b^5x^5+70a^4b^4x^4-80a^5x^3b^3+96a^6x^2b^2-128a^7bx+256a^8\right)\sqrt{b\,x^3+a\,x^2}}{153153b^6x}$

input
```
int((b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/7*(b*x+a)^(7/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.65

$$\int (ax^2 +bx^3)^{5/2}\,dx = \frac{2\left(9009\,b^8x^8 + 21021\,ab^7x^7 + 12705\,a^2b^6x^6 + 63\,a^3b^5x^5 - 70\,a^4b^4x^4 + 80\,a^5b^3x^3 - 96\,a^6b^2x^2\right.}{153153\,b^6x}$$

input
```
integrate((b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
2/153153*(9009*b^8*x^8 + 21021*a*b^7*x^7 + 12705*a^2*b^6*x^6 + 63*a^3*b^5*
x^5 - 70*a^4*b^4*x^4 + 80*a^5*b^3*x^3 - 96*a^6*b^2*x^2 + 128*a^7*b*x - 256
*a^8)*sqrt(b*x^3 + a*x^2)/(b^6*x)
```

Sympy [F]

$$\int \left(ax^2 + bx^3\right)^{5/2} \, dx = \int \left(ax^2 + bx^3\right)^{\frac{5}{2}} \, dx$$

input
```
integrate((b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral((a*x**2 + b*x**3)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.59

$$\int \left(ax^2 + bx^3\right)^{5/2} \, dx = \frac{2\,(9009\,b^8x^8 + 21021\,ab^7x^7 + 12705\,a^2b^6x^6 + 63\,a^3b^5x^5 - 70\,a^4b^4x^4 + 80\,a^5b^3x^3 - 96\,a^6b^2x^2}{153153\,b^6}$$

input
```
integrate((b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
2/153153*(9009*b^8*x^8 + 21021*a*b^7*x^7 + 12705*a^2*b^6*x^6 + 63*a^3*b^5*
x^5 - 70*a^4*b^4*x^4 + 80*a^5*b^3*x^3 - 96*a^6*b^2*x^2 + 128*a^7*b*x - 256
*a^8)*sqrt(b*x + a)/b^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(140) = 280$.

Time = 0.12 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.41

$$\int \left(ax^2+bx^3\right)^{5/2} dx = \frac{512\,a^{\frac{17}{2}}\operatorname{sgn}(x)}{153153\,b^6} + \frac{2\left(\frac{1105\left(63\,(bx+a)^{\frac{11}{2}}-385\,(bx+a)^{\frac{9}{2}}a+990\,(bx+a)^{\frac{7}{2}}a^2-1386\,(bx+a)^{\frac{5}{2}}a^3+1155\,(bx+a)^{\frac{3}{2}}a^4-693\,\sqrt{bx+a}a^5\right)a^3\operatorname{sgn}(x)}{b^5}+\frac{765\left(231\,(bx+a)^{\frac{13}{2}}\right.}{}\right.}{}$$

input
```
integrate((b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
512/153153*a^(17/2)*sgn(x)/b^6 + 2/765765*(1105*(63*(b*x + a)^(11/2) - 385
*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 +
1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^3*sgn(x)/b^5 + 765*(23
1*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 -
8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)
*a^5 + 3003*sqrt(b*x + a)*a^6)*a^2*sgn(x)/b^5 + 357*(429*(b*x + a)^(15/2)
- 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(
9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(
b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a*sgn(x)/b^5 + 7*(6435*(b*x +
 a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 5569
20*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7
/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395
*sqrt(b*x + a)*a^8)*sgn(x)/b^5)/b
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.49

$$\int \left(ax^2+bx^3\right)^{5/2} dx = -\frac{2\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^3\,(256\,a^5-896\,a^4\,b\,x+2016\,a^3\,b^2\,x^2-3696\,a^2\,b^3\,x^3+6006\,a\,b^4\,x^4-9009\,b^5\,x^5)}{153153\,b^6\,x}$$

input
```
int((a*x^2 + b*x^3)^(5/2),x)
```

output
```
-(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^3*(256*a^5 - 9009*b^5*x^5 + 6006*a*b^4
*x^4 + 2016*a^3*b^2*x^2 - 3696*a^2*b^3*x^3 - 896*a^4*b*x))/(153153*b^6*x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.59

$$\int (ax^2 + bx^3)^{5/2} \, dx = \frac{2\sqrt{bx+a}\,(9009b^8x^8 + 21021a\,b^7x^7 + 12705a^2b^6x^6 + 63a^3b^5x^5 - 70a^4b^4x^4 + 80a^5b^3x^3 - 96a}{153153b^6}$$

input
```
int((b*x^3+a*x^2)^(5/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 256*a**8 + 128*a**7*b*x - 96*a**6*b**2*x**2 + 80*a**5
*b**3*x**3 - 70*a**4*b**4*x**4 + 63*a**3*b**5*x**5 + 12705*a**2*b**6*x**6
+ 21021*a*b**7*x**7 + 9009*b**8*x**8))/(153153*b**6)
```

3.101 $\int (ax^2+bx^3)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int (ax^2+bx^3)^{3/2}\,dx = -\frac{2a^3(ax^2+bx^3)^{5/2}}{5b^4x^5} + \frac{6a^2(ax^2+bx^3)^{7/2}}{7b^4x^7} - \frac{2a(ax^2+bx^3)^{9/2}}{3b^4x^9} + \frac{2(ax^2+bx^3)^{11/2}}{11b^4x^{11}}$$

output
```
-2/5*a^3*(b*x^3+a*x^2)^(5/2)/b^4/x^5+6/7*a^2*(b*x^3+a*x^2)^(7/2)/b^4/x^7-2
/3*a*(b*x^3+a*x^2)^(9/2)/b^4/x^9+2/11*(b*x^3+a*x^2)^(11/2)/b^4/x^11
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int (ax^2+bx^3)^{3/2}\,dx = \frac{2x(a+bx)^3\left(-16a^3+40a^2bx-70ab^2x^2+105b^3x^3\right)}{1155b^4\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(a + b*x)^3*(-16*a^3 + 40*a^2*b*x - 70*a*b^2*x^2 + 105*b^3*x^3))/(115
5*b^4*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^{3/2}dx$$

$$\Big\downarrow \ 1908$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x}dx}{11b}$$

$$\Big\downarrow \ 1922$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^2}dx}{9b}\right)}{11b}$$

$$\Big\downarrow \ 1922$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{2a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^3}dx}{7b}\right)}{9b}\right)}{11b}$$

$$\Big\downarrow \ 1920$$

$$\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}\right)}{9b}\right)}{11b}$$

input
```
Int[(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*(a*x^2 + b*x^3)^(5/2))/(11*b*x^2) - (6*a*((2*(a*x^2 + b*x^3)^(5/2))/(9*
b*x^3) - (4*a*((-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x
^3)^(5/2))/(7*b*x^4)))/(9*b)))/(11*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.12

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{5}{2}}}{5b}$	13
gosper	$-\frac{2(bx+a)\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{1155b^{4}x^{3}}$	57
default	$-\frac{2(bx+a)\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{1155b^{4}x^{3}}$	57
orering	$-\frac{2(bx+a)\left(-105b^{3}x^{3}+70a\,b^{2}x^{2}-40a^{2}bx+16a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{1155b^{4}x^{3}}$	57
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-105b^{5}x^{5}-140a\,b^{4}x^{4}-5a^{2}b^{3}x^{3}+6a^{3}b^{2}x^{2}-8a^{4}bx+16a^{5}\right)}{1155x\,b^{4}}$	72
trager	$-\frac{2\left(-105b^{5}x^{5}-140a\,b^{4}x^{4}-5a^{2}b^{3}x^{3}+6a^{3}b^{2}x^{2}-8a^{4}bx+16a^{5}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{1155b^{4}x}$	74

input
```
int((b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*(b*x+a)^(5/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx^3+ax^2}}{1155\,b^4x}$$

input
```
integrate((b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^
4*b*x - 16*a^5)*sqrt(b*x^3 + a*x^2)/(b^4*x)
```

Sympy [F]

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=\int \left(ax^2+bx^3\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((a*x**2 + b*x**3)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.59

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\,(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5)\sqrt{bx+a}}{1155\,b^4}$$

input
```
integrate((b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^
4*b*x - 16*a^5)*sqrt(b*x + a)/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(92) = 184.

Time = 0.12 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.94

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx=\frac{32\,a^{\frac{11}{2}}\operatorname{sgn}(x)}{1155\,b^4}+\frac{2\left(\frac{99\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)a^2\operatorname{sgn}(x)}{b^3}+\frac{22\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}\right.}{b^3}\right.}{346}$$

input
```
integrate((b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
32/1155*a^(11/2)*sgn(x)/b^4 + 2/3465*(99*(5*(b*x + a)^(7/2) - 21*(b*x + a)
^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2*sgn(x)/b^3 +
 22*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2
- 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a*sgn(x)/b^3 + 5*(63*(b
*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b
*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*sgn(
x)/b^3)/b
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.54

$$\int \left(ax^2+bx^3\right)^{3/2}\,dx = -\frac{2\,\sqrt{b\,x^3+a\,x^2}\,{\left(a+b\,x\right)}^2\,\left(16\,a^3-40\,a^2\,b\,x+70\,a\,b^2\,x^2-105\,b^3\,x^3\right)}{1155\,b^4\,x}$$

input
```
int((a*x^2 + b*x^3)^(3/2),x)
```

output
```
-(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(16*a^3 - 105*b^3*x^3 + 70*a*b^2*x^2
 - 40*a^2*b*x))/(1155*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.58

$$\int \left(ax^2 + bx^3\right)^{3/2}\,dx = \frac{2\sqrt{bx+a}\,(105b^5x^5+140a\,b^4x^4+5a^2b^3x^3-6a^3b^2x^2+8a^4bx-16a^5)}{1155b^4}$$

input
```
int((b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 16*a**5 + 8*a**4*b*x - 6*a**3*b**2*x**2 + 5*a**2*b**3
*x**3 + 140*a*b**4*x**4 + 105*b**5*x**5))/(1155*b**4)
```

3.102 $\int \sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \sqrt{ax^2+bx^3}\,dx = -\frac{2a(ax^2+bx^3)^{3/2}}{3b^2x^3}+\frac{2(ax^2+bx^3)^{5/2}}{5b^2x^5}$$

output
```
-2/3*a*(b*x^3+a*x^2)^(3/2)/b^2/x^3+2/5*(b*x^3+a*x^2)^(5/2)/b^2/x^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.79

$$\int \sqrt{ax^2+bx^3}\,dx = \frac{2\sqrt{x^2(a+bx)}(-2a^2+abx+3b^2x^2)}{15b^2x}$$

input
```
Integrate[Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(-2*a^2 + a*b*x + 3*b^2*x^2))/(15*b^2*x)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax^2+bx^3}\,dx$$
$$\Big\downarrow \text{ 1908}$$
$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{2a\int \frac{\sqrt{bx^3+ax^2}}{x}dx}{5b}$$
$$\Big\downarrow \text{ 1920}$$
$$\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}$$

input
```
Int[Sqrt[a*x^2 + b*x^3],x]
```

output
```
(-4*a*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/2))/(5*b
*x^2)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.25

method	result	size
pseudoelliptic	$\frac{2(bx+a)^{\frac{3}{2}}}{3b}$	13
gosper	$-\frac{2(bx+a)(-3bx+2a)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{2}x}$	35
default	$-\frac{2(bx+a)(-3bx+2a)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{2}x}$	35
orering	$-\frac{2(bx+a)(-3bx+2a)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{2}x}$	35
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,(-3b^{2}x^{2}-abx+2a^{2})}{15x\,b^{2}}$	39
trager	$-\frac{2(-3b^{2}x^{2}-abx+2a^{2})\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{2}x}$	41

input
```
int((b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \sqrt{ax^2+bx^3}\,dx=\frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx^3+ax^2}}{15\,b^2x}$$

input
```
integrate((b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x^3 + a*x^2)/(b^2*x)
```

Sympy [F]

$$\int \sqrt{ax^2+bx^3}\,dx = \int \sqrt{ax^2+bx^3}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(sqrt(a*x**2 + b*x**3), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.58

$$\int \sqrt{ax^2+bx^3}\,dx = \frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}}{15\,b^2}$$

input
```
integrate((b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.56

$$\int \sqrt{ax^2+bx^3}\,dx$$
$$= \frac{4\,a^{\frac{5}{2}}\mathrm{sgn}(x)}{15\,b^2} + \frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)a\mathrm{sgn}(x)}{b} + \frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)\mathrm{sgn}(x)}{b}\right)}{15\,b}$$

input
```
integrate((b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
4/15*a^(5/2)*sgn(x)/b^2 + 2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a*
sgn(x)/b + (3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^
2)*sgn(x)/b)/b
```

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.75

$$\int \sqrt{ax^2+bx^3}\,dx=\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(-2\,a^2+a\,b\,x+3\,b^2\,x^2\right)}{15\,b^2\,x}$$

input
```
int((a*x^2 + b*x^3)^(1/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(3*b^2*x^2 - 2*a^2 + a*b*x))/(15*b^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.56

$$\int \sqrt{ax^2+bx^3}\,dx=\frac{2\sqrt{bx+a}\,(3b^2x^2+abx-2a^2)}{15b^2}$$

input
```
int((b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a**2 + a*b*x + 3*b**2*x**2))/(15*b**2)
```

3.103 $\int \frac{1}{\sqrt{ax^2+bx^3}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 32

$$\int \frac{1}{\sqrt{ax^2+bx^3}} \, dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{\sqrt{a}}$$

output
```
-2*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{ax^2+bx^3}} \, dx = -\frac{2x\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[1/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(-2*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(Sqrt[a]*Sqrt[x^2*(a +
 b*x)])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{1914}$$

$$-2\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

input
```
Int[1/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(-2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.41

method	result	size
pseudoelliptic	$\frac{2\sqrt{bx+a}}{b}$	13
default	$-\frac{2x\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,\sqrt{a}}$	39

input
```
int(1/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b*x+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.47

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \left[\frac{\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)}{\sqrt{a}}, \frac{2\,\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)}{a}\right]$$

input
```
integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2)/sqrt(a), 2*sqrt(
-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x))/a]
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt{ax^2+bx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*x**2 + b*x**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax^2 + bx^3}}\, dx = \int \frac{1}{\sqrt{bx^3 + ax^2}}\, dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.41

$$\int \frac{1}{\sqrt{ax^2 + bx^3}}\, dx = -\frac{2 \arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right) \operatorname{sgn}(x)}{\sqrt{-a}} + \frac{2 \arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\operatorname{sgn}(x)}$$

input
```
integrate(1/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2*arctan(sqrt(a)/sqrt(-a))*sgn(x)/sqrt(-a) + 2*arctan(sqrt(b*x + a)/sqrt(
-a))/(sqrt(-a)*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{ax^2 + bx^3}}\, dx = \int \frac{1}{\sqrt{b\,x^3 + a\,x^2}}\, dx$$

input
```
int(1/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int(1/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{1}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{a}\,\left(\log\left(\sqrt{bx+a}-\sqrt{a}\right)-\log\left(\sqrt{bx+a}+\sqrt{a}\right)\right)}{a}$$

input
```
int(1/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(sqrt(a)*(log(sqrt(a + b*x) - sqrt(a)) - log(sqrt(a + b*x) + sqrt(a))))/a
```

3.104 $\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 112

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx=\frac{5b}{4a^2\sqrt{ax^2+bx^3}}-\frac{1}{2ax\sqrt{ax^2+bx^3}}+\frac{15b^2x}{4a^3\sqrt{ax^2+bx^3}}-\frac{15b^2\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{4a^{7/2}}$$

output
```
5/4*b/a^2/(b*x^3+a*x^2)^(1/2)-1/2/a/x/(b*x^3+a*x^2)^(1/2)+15/4*b^2*x/a^3/(
b*x^3+a*x^2)^(1/2)-15/4*b^2*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.75

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx=\frac{\sqrt{a}(-2a^2+5abx+15b^2x^2)-15b^2x^2\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{7/2}x\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-3/2),x]
```

output
```
(Sqrt[a]*(-2*a^2 + 5*a*b*x + 15*b^2*x^2) - 15*b^2*x^2*Sqrt[a + b*x]*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2)*x*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1912, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{5\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{5\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{5\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{5\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{5\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}$$

input
```
Int[(a*x^2 + b*x^3)^(-3/2),x]
```

output
```
2/(a*x*Sqrt[a*x^2 + b*x^3]) + (5*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*
(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^
3]])/a^(3/2)))/(4*a)))/a
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.12

method	result	size
pseudoelliptic	$-\frac{2}{\sqrt{bx+a}\,b}$	13
default	$-\frac{x(bx+a)\left(15\sqrt{bx+a}\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{2}x^{2}-5a^{\frac{3}{2}}bx-15b^{2}x^{2}\sqrt{a}+2a^{\frac{5}{2}}\right)}{4(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}a^{\frac{7}{2}}}$	76
risch	$-\frac{(bx+a)(-7bx+2a)}{4a^{3}x\sqrt{x^{2}(bx+a)}}+\frac{b^{2}\left(-\frac{30\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{16}{\sqrt{bx+a}}\right)\sqrt{bx+a}\,x}{8a^{3}\sqrt{x^{2}(bx+a)}}$	88

input
```
int(1/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(b*x+a)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.00

$$\int \frac{1}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\left[\frac{15\left(b^3x^4+ab^2x^3\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(15\,ab^2x^2+5\,a^2bx-2\,a^3\right)\sqrt{b}}{8\left(a^4bx^4+a^5x^3\right)}\right.$$

input
```
integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/8*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 +
 a*x^2)*sqrt(a))/x^2) + 2*(15*a*b^2*x^2 + 5*a^2*b*x - 2*a^3)*sqrt(b*x^3 +
a*x^2))/(a^4*b*x^4 + a^5*x^3), 1/4*(15*(b^3*x^4 + a*b^2*x^3)*sqrt(-a)*arct
an(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (15*a*b^2*x^2 + 5*a^2*b*x
 - 2*a^3)*sqrt(b*x^3 + a*x^2))/(a^4*b*x^4 + a^5*x^3)]
```

Sympy [F]

$$\int \frac{1}{(ax^2 + bx^3)^{3/2}}\,dx = \int \frac{1}{(ax^2 + bx^3)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2 + bx^3)^{3/2}}\,dx = \int \frac{1}{(bx^3 + ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = \frac{15\,b^2\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3\mathrm{sgn}\left(x\right)} + \frac{2\,b^2}{\sqrt{bx+a}a^3\mathrm{sgn}\left(x\right)} + \frac{7\,(bx+a)^{\frac{3}{2}}b^2 - 9\,\sqrt{bx+a}ab^2}{4\,a^3b^2x^2\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
15/4*b^2*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3*sgn(x)) + 2*b^2/(sqr
t(b*x + a)*a^3*sgn(x)) + 1/4*(7*(b*x + a)^(3/2)*b^2 - 9*sqrt(b*x + a)*a*b^
2)/(a^3*b^2*x^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.38

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\,x\,\left(\frac{a}{b\,x}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{2};\ \frac{9}{2};\ -\frac{a}{b\,x}\right)}{7\,\left(b\,x^3+a\,x^2\right)^{3/2}}$$

input
```
int(1/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
-(2*x*(a/(b*x) + 1)^(3/2)*hypergeom([3/2, 7/2], 9/2, -a/(b*x)))/(7*(a*x^2
+ b*x^3)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.82

$$\int \frac{1}{(ax^2+bx^3)^{3/2}}\,dx = \frac{15\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2x^2-15\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)b^2x^2}{8\sqrt{bx+a}\,a^4x^2}$$

input
```
int(1/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**2*x**2 - 15*sqrt
(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**2*x**2 - 4*a**3 + 10*a**
2*b*x + 30*a*b**2*x**2)/(8*sqrt(a + b*x)*a**4*x**2)
```

3.105 $\int \frac{1}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 194

$$\int \frac{1}{(ax^2+bx^3)^{5/2}}\,dx = \frac{11b}{24a^2(ax^2+bx^3)^{3/2}} - \frac{1}{4ax(ax^2+bx^3)^{3/2}} - \frac{33b^2x}{32a^3(ax^2+bx^3)^{3/2}} + \frac{231b^3x^2}{64a^4(ax^2+bx^3)^{3/2}} + \frac{385b^4x^3}{64a^5(ax^2+bx^3)^{3/2}} + \frac{1155b^4x}{64a^6\sqrt{ax^2+bx^3}} - \frac{1155b^4\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{64a^{13/2}}$$

output

```
11/24*b/a^2/(b*x^3+a*x^2)^(3/2)-1/4/a/x/(b*x^3+a*x^2)^(3/2)-33/32*b^2*x/a^
3/(b*x^3+a*x^2)^(3/2)+231/64*b^3*x^2/a^4/(b*x^3+a*x^2)^(3/2)+385/64*b^4*x^
3/a^5/(b*x^3+a*x^2)^(3/2)+1155/64*b^4*x/a^6/(b*x^3+a*x^2)^(1/2)-1155/64*b^
4*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(13/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.60

$$\int \frac{1}{(ax^2+bx^3)^{5/2}}\,dx = \frac{\sqrt{a}(-48a^5+88a^4bx-198a^3b^2x^2+693a^2b^3x^3+4620ab^4x^4+3465b^5x^5)-3465b^4x}{192a^{13/2}x\,(x^2(a+bx))^{3/2}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-5/2),x]
```

output
```
(Sqrt[a]*(-48*a^5 + 88*a^4*b*x - 198*a^3*b^2*x^2 + 693*a^2*b^3*x^3 + 4620*
a*b^4*x^4 + 3465*b^5*x^5) - 3465*b^4*x^4*(a + b*x)^(3/2)*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/(192*a^(13/2)*x*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1912, 1929, 1931, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{11\int \frac{1}{x^2(bx^3+ax^2)^{3/2}}dx}{3a}+\frac{2}{3ax\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{11\left(\frac{9\int \frac{1}{x^4\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2}{3ax\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{11\left(\frac{9\left(-\frac{7b\int\frac{1}{x^3\sqrt{bx^3+ax^2}}dx}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2}{3ax\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{11\left(\frac{9\left(-\frac{7b\left(-\frac{5b\int\frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2}{3ax\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{11\left(\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\int\frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2}{3ax\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{11\left(\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int\frac{1}{\sqrt{bx^3+ax^2}}\,dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2}{3ax\left(ax^2+bx^3\right)^{3/2}}$$

$$\downarrow\ 1914$$

$$\frac{11\left(\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int\frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2}{3ax\left(ax^2+bx^3\right)^{3/2}}$$

$\Big\downarrow$ 219

$$\frac{11\left(\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2}{3ax\left(ax^2+bx^3\right)^{3/2}}$$

input

```
Int[(a*x^2 + b*x^3)^(-5/2),x]
```

output
```
2/(3*a*x*(a*x^2 + b*x^3)^(3/2)) + (11*(2/(a*x^3*Sqrt[a*x^2 + b*x^3]) + (9*
(-1/4*Sqrt[a*x^2 + b*x^3]/(a*x^5) - (7*b*(-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4)
 - (5*b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a
*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/(6
*a)))/(8*a)))/a))/(3*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.07

method	result
pseudoelliptic	$-\frac{2}{3(bx+a)^{\frac{3}{2}} b}$
default	$-\frac{x(bx+a)\left(3465(bx+a)^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right) b^4x^4+198a^{\frac{7}{2}}b^2x^2-693a^{\frac{5}{2}}b^3x^3-4620a^{\frac{3}{2}}b^4x^4-3465\sqrt{a}\,b^5x^5-88a^{\frac{9}{2}}bx+48a^{\frac{11}{2}}}{192(b\,x^3+a\,x^2)^{\frac{5}{2}}a^{\frac{13}{2}}}$
risch	$-\frac{(bx+a)(-1545b^3x^3+518a\,b^2x^2-184a^2bx+48a^3)}{192a^6x^3\sqrt{x^2(bx+a)}}+\frac{b^4\left(-\frac{2310 \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{1280}{\sqrt{bx+a}}+\frac{256a}{3(bx+a)^{\frac{3}{2}}}\right)\sqrt{bx+a}\,x}{128a^6\sqrt{x^2(bx+a)}}$

input
```
int(1/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(b*x+a)^(3/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.72

$$\int \frac{1}{(ax^2+bx^3)^{5/2}}\,dx = \left[\frac{3465\,(b^6x^7+2\,ab^5x^6+a^2b^4x^5)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(3465\,ab^5x^5+4}{384\,(a^7b^2x^7+2\,a^8bx^6}\right.$$

input
```
integrate(1/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/384*(3465*(b^6*x^7 + 2*a*b^5*x^6 + a^2*b^4*x^5)*sqrt(a)*log((b*x^2 + 2*
a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(3465*a*b^5*x^5 + 4620*a^2*b
^4*x^4 + 693*a^3*b^3*x^3 - 198*a^4*b^2*x^2 + 88*a^5*b*x - 48*a^6)*sqrt(b*x
^3 + a*x^2))/(a^7*b^2*x^7 + 2*a^8*b*x^6 + a^9*x^5), 1/192*(3465*(b^6*x^7 +
 2*a*b^5*x^6 + a^2*b^4*x^5)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(
b*x^2 + a*x)) + (3465*a*b^5*x^5 + 4620*a^2*b^4*x^4 + 693*a^3*b^3*x^3 - 198
*a^4*b^2*x^2 + 88*a^5*b*x - 48*a^6)*sqrt(b*x^3 + a*x^2))/(a^7*b^2*x^7 + 2*
a^8*b*x^6 + a^9*x^5)]
```

Sympy [F]

$$\int \frac{1}{(ax^2+bx^3)^{5/2}}\,dx=\int \frac{1}{(ax^2+bx^3)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2+bx^3)^{5/2}}\,dx=\int \frac{1}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.70

$$\int \frac{1}{(ax^2+bx^3)^{5/2}}\,dx=\frac{1155\,b^4\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{64\,\sqrt{-a}a^6\mathrm{sgn}\left(x\right)}+\frac{2\,(15\,(bx+a)b^4+ab^4)}{3\,(bx+a)^{\frac{3}{2}}a^6\mathrm{sgn}\left(x\right)}+\frac{1545\,(bx+a)^{\frac{7}{2}}b^4-5153\,(bx+a)^{\frac{5}{2}}ab^4+5855\,(bx+a)^{\frac{3}{2}}a^2b^4-2295\,\sqrt{bx+a}a^3b^4}{192\,a^6b^4x^4\mathrm{sgn}\left(x\right)}$$

input
```
integrate(1/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
1155/64*b^4*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^6*sgn(x)) + 2/3*(15
*(b*x + a)*b^4 + a*b^4)/((b*x + a)^(3/2)*a^6*sgn(x)) + 1/192*(1545*(b*x +
a)^(7/2)*b^4 - 5153*(b*x + a)^(5/2)*a*b^4 + 5855*(b*x + a)^(3/2)*a^2*b^4 -
 2295*sqrt(b*x + a)*a^3*b^4)/(a^6*b^4*x^4*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.22

$$\int \frac{1}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\,x\,\left(\frac{a}{b\,x}+1\right)^{5/2}\,{}_2\mathrm{F}_1\left(\frac{5}{2},\frac{13}{2};\ \frac{15}{2};\ -\frac{a}{b\,x}\right)}{13\,\left(b\,x^3+a\,x^2\right)^{5/2}}$$

input
```
int(1/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
-(2*x*(a/(b*x) + 1)^(5/2)*hypergeom([5/2, 13/2], 15/2, -a/(b*x)))/(13*(a*x
^2 + b*x^3)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.97

$$\int \frac{1}{(ax^2+bx^3)^{5/2}}\,dx = \frac{3465\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a\,b^4x^4+3465\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right.}{}$$

input
```
int(1/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(3465*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**4*x**4 + 346
5*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**5*x**5 - 3465*sqrt
(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**4*x**4 - 3465*sqrt(a)*
sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**5*x**5 - 96*a**6 + 176*a**5*
b*x - 396*a**4*b**2*x**2 + 1386*a**3*b**3*x**3 + 9240*a**2*b**4*x**4 + 693
0*a*b**5*x**5)/(384*sqrt(a + b*x)*a**7*x**4*(a + b*x))
```

3.106 $\int \frac{1}{\left(ax^2+bx^3\right)^{7/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 278

$$\int \frac{1}{\left(ax^2+bx^3\right)^{7/2}}\,dx = \frac{17b}{60a^2\left(ax^2+bx^3\right)^{5/2}} - \frac{1}{6ax\left(ax^2+bx^3\right)^{5/2}} - \frac{17b^2x}{32a^3\left(ax^2+bx^3\right)^{5/2}} + \frac{221b^3x^2}{192a^4\left(ax^2+bx^3\right)^{5/2}} - \frac{2431b^4x^3}{768a^5\left(ax^2+bx^3\right)^{5/2}} + \frac{7293b^5x^4}{512a^6\left(ax^2+bx^3\right)^{5/2}} + \frac{51051b^6x^5}{2560a^7\left(ax^2+bx^3\right)^{5/2}} + \frac{17017b^6x^3}{512a^8\left(ax^2+bx^3\right)^{3/2}} + \frac{51051b^6x}{512a^9\sqrt{ax^2+bx^3}} - \frac{51051b^6\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{512a^{19/2}}$$

output
```
17/60*b/a^2/(b*x^3+a*x^2)^(5/2)-1/6/a/x/(b*x^3+a*x^2)^(5/2)-17/32*b^2*x/a^
3/(b*x^3+a*x^2)^(5/2)+221/192*b^3*x^2/a^4/(b*x^3+a*x^2)^(5/2)-2431/768*b^4
*x^3/a^5/(b*x^3+a*x^2)^(5/2)+7293/512*b^5*x^4/a^6/(b*x^3+a*x^2)^(5/2)+5105
1/2560*b^6*x^5/a^7/(b*x^3+a*x^2)^(5/2)+17017/512*b^6*x^3/a^8/(b*x^3+a*x^2)
^(3/2)+51051/512*b^6*x/a^9/(b*x^3+a*x^2)^(1/2)-51051/512*b^6*arctanh((b*x^
3+a*x^2)^(1/2)/a^(1/2)/x)/a^(19/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.54

$$\int \frac{1}{(ax^2+bx^3)^{7/2}}\,dx = \frac{\sqrt{a}(-1280a^8+2176a^7bx-4080a^6b^2x^2+8840a^5b^3x^3-24310a^4b^4x^4+109395a^3b^5}{7680a^{19}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-7/2),x]
```

output
```
(Sqrt[a]*(-1280*a^8 + 2176*a^7*b*x - 4080*a^6*b^2*x^2 + 8840*a^5*b^3*x^3 -
 24310*a^4*b^4*x^4 + 109395*a^3*b^5*x^5 + 1174173*a^2*b^6*x^6 + 1786785*a*
b^7*x^7 + 765765*b^8*x^8) - 765765*b^6*x^6*(a + b*x)^(5/2)*ArcTanh[Sqrt[a
+ b*x]/Sqrt[a]])/(7680*a^(19/2)*x*(x^2*(a + b*x))^(5/2))
```

Rubi [A] (verified)

Time = 1.07 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {1912, 1929, 1929, 1931, 1931, 1931, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{7/2}}\,dx$$

$$\Big\downarrow \; 1912$$

$$\frac{17\int \frac{1}{x^2(bx^3+ax^2)^{5/2}}dx}{5a}+\frac{2}{5ax\,(ax^2+bx^3)^{5/2}}$$

$$\Big\downarrow \; 1929$$

$$\frac{17\left(\frac{5\int \frac{1}{x^4\left(bx^3+ax^2\right)^{3/2}}dx}{a}+\frac{2}{3ax^3(ax^2+bx^3)^{3/2}}\right)}{5a}+\frac{2}{5ax\,(ax^2+bx^3)^{5/2}}$$

$$\Big\downarrow \; 1929$$

$$\frac{17\left(\frac{5\left(\frac{13\int \frac{1}{x^6\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax^5\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2}{3ax^3(ax^2+bx^3)^{3/2}}\right)}{5a}+\frac{2}{5ax\left(ax^2+bx^3\right)^{5/2}}$$

$\downarrow$ 1931

$$\frac{17\left(\frac{5\left(\frac{13\left(-\frac{11b\int \frac{1}{x^5\sqrt{bx^3+ax^2}}dx}{12a}-\frac{\sqrt{ax^2+bx^3}}{6ax^7}\right)}{a}+\frac{2}{ax^5\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2}{3ax^3(ax^2+bx^3)^{3/2}}\right)}{5a}+\frac{2}{5ax\left(ax^2+bx^3\right)^{5/2}}$$

$\downarrow$ 1931

$$\frac{17\left(\frac{5\left(\frac{13\left(-\frac{11b\left(-\frac{9b\int \frac{1}{x^4\sqrt{bx^3+ax^2}}dx}{10a}-\frac{\sqrt{ax^2+bx^3}}{5ax^6}\right)}{12a}-\frac{\sqrt{ax^2+bx^3}}{6ax^7}\right)}{a}+\frac{2}{ax^5\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2}{3ax^3(ax^2+bx^3)^{3/2}}\right)}{5a}+\frac{2}{5ax\left(ax^2+bx^3\right)^{5/2}}$$

$\downarrow$ 1931

$$\frac{17\left(\frac{5\left(\frac{13\left(-\frac{11b\left(-\frac{9b\left(-\frac{7b\int\frac{1}{x^3\sqrt{bx^3+ax^2}}dx}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{10a}-\frac{\sqrt{ax^2+bx^3}}{5ax^6}\right)}{12a}-\frac{\sqrt{ax^2+bx^3}}{6ax^7}\right)}{a}+\frac{2}{ax^5\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2}{3ax^3(ax^2+bx^3)^{3/2}}\right)}{5a}+\frac{2}{5ax\left(ax^2+bx^3\right)^{5/2}}$$

$$\Big\downarrow \; 1931$$

$$17\left(\frac{5\left(\frac{13\left(-\frac{11b\left(-\frac{9b\left(-\frac{7b\left(-\frac{5b\int\frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{10a}-\frac{\sqrt{ax^2+bx^3}}{5ax^6}\right)}{12a}-\frac{\sqrt{ax^2+bx^3}}{6ax^7}\right)}{a}+\frac{2}{ax^5\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2}{3ax^3(ax^2+b\cdots}\right.$$

↓ 1931

$$\left(5\left(\frac{13\left(-\frac{11b\left(-\frac{9b\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\int\frac{1}{x\sqrt{bx^3+ax^2}}\,dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{10a}-\frac{\sqrt{ax^2+bx^3}}{5ax^6}\right)}{12a}-\frac{\sqrt{ax^2+bx^3}}{6ax^7}\right)}{a}+\frac{2}{ax^5\sqrt{ax^2+bx^3}}\right.\right.$$

↓ 1931

$$\left(\left(\left(13\left(-\frac{11b\left(-\frac{9b\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int\frac{1}{\sqrt{bx^3+ax^2}}\,dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{10a}-\frac{\sqrt{ax^2+bx^3}}{5ax^6}\right)}{12a}-\frac{\sqrt{ax^2+bx^3}}{6ax^7}\right.\right.\right.\right.$$

↓ 1914

$$\Bigg(\Bigg(\Bigg(13\left(-\frac{11b\left(-\frac{9b\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{10a}-\frac{\sqrt{ax^2+bx^3}}{5ax^6}\right)}{12a}-\sqrt{\cdots}\right.$$

↓ 219

$$\left(\left(\left(13\left(-\frac{11b\left(-\frac{9b\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{10a}-\frac{\sqrt{ax^2+bx^3}}{5ax^6}\right)}{12a}-\frac{\sqrt{a\cdots}}{6\cdots}\right.\right.\right.\right.$$

input
```
Int[(a*x^2 + b*x^3)^(-7/2),x]
```

output
```
2/(5*a*x*(a*x^2 + b*x^3)^(5/2)) + (17*(2/(3*a*x^3*(a*x^2 + b*x^3)^(3/2)) +
 (5*(2/(a*x^5*Sqrt[a*x^2 + b*x^3]) + (13*(-1/6*Sqrt[a*x^2 + b*x^3]/(a*x^7)
 - (11*b*(-1/5*Sqrt[a*x^2 + b*x^3]/(a*x^6) - (9*b*(-1/4*Sqrt[a*x^2 + b*x^3
]/(a*x^5) - (7*b*(-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*b*(-1/2*Sqrt[a*x^2
 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqr
t[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)))/(8*a)))/(10*a)))/(
12*a)))/a))/a))/(5*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.05

method	result
pseudoelliptic	$-\frac{2}{5(bx+a)^{\frac{5}{2}}b}$
default	$-\frac{x(bx+a)\left(4080a^{\frac{13}{2}}b^2x^2-2176a^{\frac{15}{2}}bx-1174173a^{\frac{5}{2}}b^6x^6+765765(bx+a)^{\frac{5}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^6x^6+1280a^{\frac{17}{2}}-8840a^{\frac{11}{2}}b^3}{7680(b\,x^3+a\,x^2)^{\frac{7}{2}}a^{\frac{19}{2}}}$
risch	$-\frac{(bx+a)(-335685b^5x^5+116270a\,b^4x^4-46936a^2b^3x^3+18288a^3b^2x^2-6016a^4bx+1280a^5)}{7680a^9x^5\sqrt{x^2(bx+a)}}+\frac{b^6\left(-\frac{102102\operatorname{arctanh}\left(\frac{\sqrt{bx}}{\sqrt{a}}\right)}{\sqrt{a}}\right.}{}$

input
```
int(1/(b*x^3+a*x^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(b*x+a)^(5/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.60

$$\int \frac{1}{(ax^2+bx^3)^{7/2}}\,dx = \left[\frac{765765\,(b^9x^{10}+3\,ab^8x^9+3\,a^2b^7x^8+a^3b^6x^7)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(7}{}\right.$$

input
```
integrate(1/(b*x^3+a*x^2)^(7/2),x, algorithm="fricas")
```

output
```
[1/15360*(765765*(b^9*x^10 + 3*a*b^8*x^9 + 3*a^2*b^7*x^8 + a^3*b^6*x^7)*sq
rt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(765765
*a*b^8*x^8 + 1786785*a^2*b^7*x^7 + 1174173*a^3*b^6*x^6 + 109395*a^4*b^5*x^
5 - 24310*a^5*b^4*x^4 + 8840*a^6*b^3*x^3 - 4080*a^7*b^2*x^2 + 2176*a^8*b*x
 - 1280*a^9)*sqrt(b*x^3 + a*x^2))/(a^10*b^3*x^10 + 3*a^11*b^2*x^9 + 3*a^12
*b*x^8 + a^13*x^7), 1/7680*(765765*(b^9*x^10 + 3*a*b^8*x^9 + 3*a^2*b^7*x^8
 + a^3*b^6*x^7)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)
) + (765765*a*b^8*x^8 + 1786785*a^2*b^7*x^7 + 1174173*a^3*b^6*x^6 + 109395
*a^4*b^5*x^5 - 24310*a^5*b^4*x^4 + 8840*a^6*b^3*x^3 - 4080*a^7*b^2*x^2 + 2
176*a^8*b*x - 1280*a^9)*sqrt(b*x^3 + a*x^2))/(a^10*b^3*x^10 + 3*a^11*b^2*x
^9 + 3*a^12*b*x^8 + a^13*x^7)]
```

Sympy [F]

$$\int \frac{1}{(ax^2+bx^3)^{7/2}}\,dx = \int \frac{1}{(ax^2+bx^3)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(7/2),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-7/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2+bx^3)^{7/2}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.65

$$\int \frac{1}{(ax^2+bx^3)^{7/2}}\,dx = \frac{51051\,b^6\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{512\sqrt{-a}a^9\text{sgn}\,(x)} + \frac{2\left(420\,(bx+a)^2b^6+35\,(bx+a)ab^6+3\,a^2b^6\right)}{15\,(bx+a)^{\frac{5}{2}}a^9\text{sgn}\,(x)} + \frac{335685\,(bx+a)^{\frac{11}{2}}b^6-1794695\,(bx+a)^{\frac{9}{2}}ab^6+3868866\,(bx+a)^{\frac{7}{2}}a^2b^6-4213566\,(bx+a)^{\frac{5}{2}}a^3b^6+23269}{7680\,a^9b^6x^6\text{sgn}\,(x)}$$

input
```
integrate(1/(b*x^3+a*x^2)^(7/2),x, algorithm="giac")
```

output
```
51051/512*b^6*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^9*sgn(x)) + 2/15*
(420*(b*x + a)^2*b^6 + 35*(b*x + a)*a*b^6 + 3*a^2*b^6)/((b*x + a)^(5/2)*a^
9*sgn(x)) + 1/7680*(335685*(b*x + a)^(11/2)*b^6 - 1794695*(b*x + a)^(9/2)*
a*b^6 + 3868866*(b*x + a)^(7/2)*a^2*b^6 - 4213566*(b*x + a)^(5/2)*a^3*b^6
+ 2326905*(b*x + a)^(3/2)*a^4*b^6 - 524475*sqrt(b*x + a)*a^5*b^6)/(a^9*b^6
*x^6*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.15

$$\int \frac{1}{(ax^2+bx^3)^{7/2}}\,dx = -\frac{2\,x\,\left(\frac{a}{b\,x}+1\right)^{7/2}{}_2\mathrm{F}_1\left(\frac{7}{2},\frac{19}{2};\ \frac{21}{2};\ -\frac{a}{b\,x}\right)}{19\,(b\,x^3+a\,x^2)^{7/2}}$$

input
```
int(1/(a*x^2 + b*x^3)^(7/2),x)
```

output
```
-(2*x*(a/(b*x) + 1)^(7/2)*hypergeom([7/2, 19/2], 21/2, -a/(b*x)))/(19*(a*x
^2 + b*x^3)^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.05

$$\int \frac{1}{(ax^2+bx^3)^{7/2}}\,dx = \frac{765765\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a^2b^6x^6+1531530\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}\right.}{}$$

input
```
int(1/(b*x^3+a*x^2)^(7/2),x)
```

output
```
(765765*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a**2*b**6*x**6
+ 1531530*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**7*x**7 +
 765765*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**8*x**8 - 765
765*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a**2*b**6*x**6 - 15
31530*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**7*x**7 - 765
765*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**8*x**8 - 2560*a*
*9 + 4352*a**8*b*x - 8160*a**7*b**2*x**2 + 17680*a**6*b**3*x**3 - 48620*a*
*5*b**4*x**4 + 218790*a**4*b**5*x**5 + 2348346*a**3*b**6*x**6 + 3573570*a*
*2*b**7*x**7 + 1531530*a*b**8*x**8)/(15360*sqrt(a + b*x)*a**10*x**6*(a**2
+ 2*a*b*x + b**2*x**2))
```

3.107 $\int (ax^2+bx^3)^{2/3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 259

$$\int (ax^2+bx^3)^{2/3}\,dx = \frac{a(ax^2+bx^3)^{2/3}}{9b} - \frac{4a^2(ax^2+bx^3)^{2/3}}{27b^2x} + \frac{1}{3}x(ax^2+bx^3)^{2/3} - \frac{4a^3(ax^2+bx^3)^{2/3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}\right)}{27\sqrt{3}b^{7/3}x^{4/3}(a+bx)^{2/3}} - \frac{2a^3(ax^2+bx^3)^{2/3}\log(x)}{81b^{7/3}x^{4/3}(a+bx)^{2/3}} - \frac{2a^3(ax^2+b}{27b}$$

output
```
1/9*a*(b*x^3+a*x^2)^(2/3)/b-4/27*a^2*(b*x^3+a*x^2)^(2/3)/b^2/x+1/3*x*(b*x^
3+a*x^2)^(2/3)-4/81*a^3*(b*x^3+a*x^2)^(2/3)*arctan(1/3*3^(1/2)+2/3*(b*x+a)
^(1/3)*3^(1/2)/b^(1/3)/x^(1/3))*3^(1/2)/b^(7/3)/x^(4/3)/(b*x+a)^(2/3)-2/81
*a^3*(b*x^3+a*x^2)^(2/3)*ln(x)/b^(7/3)/x^(4/3)/(b*x+a)^(2/3)-2/27*a^3*(b*x
^3+a*x^2)^(2/3)*ln(1-(b*x+a)^(1/3)/b^(1/3)/x^(1/3))/b^(7/3)/x^(4/3)/(b*x+a
)^(2/3)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.89

$$\int (ax^2+bx^3)^{2/3}\,dx = \frac{x^{2/3}\sqrt[3]{a+bx}\Big(-12a^2\sqrt[3]{b}\sqrt[3]{x}(a+bx)^{2/3}+9ab^{4/3}x^{4/3}(a+bx)^{2/3}+27b^{7/3}x^{7/3}(a+bx)^{2/3}+4\sqrt{\ }$$

input
```
Integrate[(a*x^2 + b*x^3)^(2/3),x]
```

output
```
(x^(2/3)*(a + b*x)^(1/3)*(-12*a^2*b^(1/3)*x^(1/3)*(a + b*x)^(2/3) + 9*a*b^
(4/3)*x^(4/3)*(a + b*x)^(2/3) + 27*b^(7/3)*x^(7/3)*(a + b*x)^(2/3) + 4*Sqr
t[3]*a^3*ArcTan[(Sqrt[3]*b^(1/3)*x^(1/3))/(b^(1/3)*x^(1/3) + 2*(a + b*x)^(
1/3))] - 4*a^3*Log[-(b^(1/3)*x^(1/3)) + (a + b*x)^(1/3)] + 2*a^3*Log[b^(2/
3)*x^(2/3) + b^(1/3)*x^(1/3)*(a + b*x)^(1/3) + (a + b*x)^(2/3)]))/(81*b^(7
/3)*(x^2*(a + b*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1910, 1930, 1930, 1917, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^2+bx^3)^{2/3}\,dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{2}{9}a\int \frac{x^2}{\sqrt[3]{bx^3+ax^2}}dx+\frac{1}{3}x(ax^2+bx^3)^{2/3}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2}{9}a\left(\frac{\left(ax^2+bx^3\right)^{2/3}}{2b}-\frac{2a\int \frac{x}{\sqrt[3]{bx^3+ax^2}}dx}{3b}\right)+\frac{1}{3}x\left(ax^2+bx^3\right)^{2/3}$$

$\Big\downarrow$ 1930

$$\frac{2}{9}a\left(\frac{\left(ax^2+bx^3\right)^{2/3}}{2b}-\frac{2a\left(\frac{\left(ax^2+bx^3\right)^{2/3}}{bx}-\frac{a\int \frac{1}{\sqrt[3]{bx^3+ax^2}}dx}{3b}\right)}{3b}\right)+\frac{1}{3}x\left(ax^2+bx^3\right)^{2/3}$$

$\Big\downarrow$ 1917

$$\frac{2}{9}a\left(\frac{\left(ax^2+bx^3\right)^{2/3}}{2b}-\frac{2a\left(\frac{\left(ax^2+bx^3\right)^{2/3}}{bx}-\frac{ax^{2/3}\sqrt[3]{a+bx}\int \frac{1}{x^{2/3}\sqrt[3]{a+bx}}dx}{3b\sqrt[3]{ax^2+bx^3}}\right)}{3b}\right)+\frac{1}{3}x\left(ax^2+bx^3\right)^{2/3}$$

$\Big\downarrow$ 71

$$\frac{2}{9}a\left(\frac{\left(ax^2+bx^3\right)^{2/3}}{2b}-\frac{2a\left(\frac{\left(ax^2+bx^3\right)^{2/3}}{bx}-\frac{ax^{2/3}\sqrt[3]{a+bx}\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{x}}-1\right)}{2\sqrt[3]{b}}-\frac{\log(x)}{2\sqrt[3]{b}}\right)}{3b\sqrt[3]{ax^2+bx^3}}\right)}{3b}\right)+\frac{1}{3}x\left(ax^2+bx^3\right)^{2/3}$$

input
```
Int[(a*x^2 + b*x^3)^(2/3),x]
```

output
```
(x*(a*x^2 + b*x^3)^(2/3))/3 + (2*a*((a*x^2 + b*x^3)^(2/3)/(2*b) - (2*a*((a
*x^2 + b*x^3)^(2/3)/(b*x) - (a*x^(2/3)*(a + b*x)^(1/3)*(-((Sqrt[3]*ArcTan[
1/Sqrt[3] + (2*(a + b*x)^(1/3))/(Sqrt[3]*b^(1/3)*x^(1/3))])/b^(1/3)) - Log
[x]/(2*b^(1/3)) - (3*Log[-1 + (a + b*x)^(1/3)/(b^(1/3)*x^(1/3))])/(2*b^(1/
3))))/(3*b*(a*x^2 + b*x^3)^(1/3))))/(3*b)))/9
```

Defintions of rubi rules used

rule 71
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{27b^{\frac{7}{3}}\left(x^{2}(bx+a)\right)^{\frac{2}{3}}x^{2}+9a\,b^{\frac{4}{3}}x\left(x^{2}(bx+a)\right)^{\frac{2}{3}}-4\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(x^{2}(bx+a)\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)x\,a^{3}-12a^{2}b^{\frac{1}{3}}\left(x^{2}(bx+a)\right)^{\frac{2}{3}}-4\ln\left(\right.}{81b^{\frac{7}{3}}x}$

input
```
int((b*x^3+a*x^2)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
1/81*(27*b^(7/3)*(x^2*(b*x+a))^(2/3)*x^2+9*a*b^(4/3)*x*(x^2*(b*x+a))^(2/3)
-4*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(x^2*(b*x+a))^(1/3))/b^(1/3)/x)
*x*a^3-12*a^2*b^(1/3)*(x^2*(b*x+a))^(2/3)-4*ln((-b^(1/3)*x+(x^2*(b*x+a))^(
1/3))/x)*x*a^3+2*ln((b^(2/3)*x^2+b^(1/3)*(x^2*(b*x+a))^(1/3)*x+(x^2*(b*x+a
))^(2/3))/x^2)*x*a^3)/b^(7/3)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.83

$$\int (ax^2+bx^3)^{2/3}\,dx = \left[\frac{6\sqrt{\frac{1}{3}}a^3bx\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(\frac{3\,bx^2+2\,ax-3\,(bx^3+ax^2)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x-3\,\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}bx^2-(bx^3+ax^2)^{\frac{1}{3}}bx+2\,(bx^3+ax^2)^{\frac{2}{3}}(-\right.}{x}\right.}{}\right.$$

$$-\frac{12\sqrt{\frac{1}{3}}a^3bx\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{\frac{1}{3}}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+ax^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)+4\,a^3(-b)^{\frac{2}{3}}\,x\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+ax^2)^{\frac{1}{3}}}{x}\right)}{81\,b^3x}$$

input
```
integrate((b*x^3+a*x^2)^(2/3),x, algorithm="fricas")
```

output
```
[1/81*(6*sqrt(1/3)*a^3*b*x*sqrt((-b)^(1/3)/b)*log((3*b*x^2 + 2*a*x - 3*(b*
x^3 + a*x^2)^(1/3)*(-b)^(2/3)*x - 3*sqrt(1/3)*((-b)^(1/3)*b*x^2 - (b*x^3 +
 a*x^2)^(1/3)*b*x + 2*(b*x^3 + a*x^2)^(2/3)*(-b)^(2/3))*sqrt((-b)^(1/3)/b)
)/x) - 4*a^3*(-b)^(2/3)*x*log(((-b)^(1/3)*x + (b*x^3 + a*x^2)^(1/3))/x) +
2*a^3*(-b)^(2/3)*x*log(((-b)^(2/3)*x^2 - (b*x^3 + a*x^2)^(1/3)*(-b)^(1/3)*
x + (b*x^3 + a*x^2)^(2/3))/x^2) + 3*(9*b^3*x^2 + 3*a*b^2*x - 4*a^2*b)*(b*x
^3 + a*x^2)^(2/3))/(b^3*x), -1/81*(12*sqrt(1/3)*a^3*b*x*sqrt(-(-b)^(1/3)/b
)*arctan(-sqrt(1/3)*((-b)^(1/3)*x - 2*(b*x^3 + a*x^2)^(1/3))*sqrt(-(-b)^(1
/3)/b)/x) + 4*a^3*(-b)^(2/3)*x*log(((-b)^(1/3)*x + (b*x^3 + a*x^2)^(1/3))/
x) - 2*a^3*(-b)^(2/3)*x*log(((-b)^(2/3)*x^2 - (b*x^3 + a*x^2)^(1/3)*(-b)^(
1/3)*x + (b*x^3 + a*x^2)^(2/3))/x^2) - 3*(9*b^3*x^2 + 3*a*b^2*x - 4*a^2*b)
*(b*x^3 + a*x^2)^(2/3))/(b^3*x)]
```

Sympy [F]

$$\int \left(ax^2+bx^3\right)^{2/3} \, dx = \int \left(ax^2+bx^3\right)^{\frac{2}{3}} \, dx$$

input
```
integrate((b*x**3+a*x**2)**(2/3),x)
```

output
```
Integral((a*x**2 + b*x**3)**(2/3), x)
```

Maxima [F]

$$\int \left(ax^2+bx^3\right)^{2/3} \, dx = \int \left(bx^3+ax^2\right)^{\frac{2}{3}} \, dx$$

input
```
integrate((b*x^3+a*x^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(2/3), x)
```

Giac [A] (verification not implemented)

Time = 3.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.54

$$\int \left(ax^2+bx^3\right)^{2/3}\,dx = -\frac{1}{81}a^3\left(\frac{4\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(b+\frac{a}{x}\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{7}{3}}}+\frac{3\left(4\left(b+\frac{a}{x}\right)^{\frac{8}{3}}-11\left(b+\frac{a}{x}\right)^{\frac{5}{3}}b-2\left(b+\frac{a}{x}\right)^{\frac{2}{3}}b^2\right)x^3}{a^3b^2}-\frac{2\log\left(\left(b\right.\right.}{}$$

input
```
integrate((b*x^3+a*x^2)^(2/3),x, algorithm="giac")
```

output
```
-1/81*a^3*(4*sqrt(3)*arctan(1/3*sqrt(3)*(2*(b + a/x)^(1/3) + b^(1/3))/b^(1
/3))/b^(7/3) + 3*(4*(b + a/x)^(8/3) - 11*(b + a/x)^(5/3)*b - 2*(b + a/x)^(
2/3)*b^2)*x^3/(a^3*b^2) - 2*log((b + a/x)^(2/3) + (b + a/x)^(1/3)*b^(1/3)
+ b^(2/3))/b^(7/3) + 4*log(abs((b + a/x)^(1/3) - b^(1/3)))/b^(7/3))
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.15

$$\int \left(ax^2+bx^3\right)^{2/3}\,dx = \frac{3\,x\,\left(b\,x^3+a\,x^2\right)^{2/3}\,{}_2\mathrm{F}_1\left(-\frac{2}{3},\frac{7}{3};\ \frac{10}{3};\ -\frac{b\,x}{a}\right)}{7\,\left(\frac{b\,x}{a}+1\right)^{2/3}}$$

input
```
int((a*x^2 + b*x^3)^(2/3),x)
```

output
```
(3*x*(a*x^2 + b*x^3)^(2/3)*hypergeom([-2/3, 7/3], 10/3, -(b*x)/a))/(7*((b*
x)/a + 1)^(2/3))
```

Reduce [F]

$$\int \left(ax^2+bx^3\right)^{2/3} dx = \frac{-12x^{\frac{1}{3}}(bx+a)^{\frac{2}{3}} a^2 + 9x^{\frac{4}{3}}(bx+a)^{\frac{2}{3}} ab + 27x^{\frac{7}{3}}(bx+a)^{\frac{2}{3}} b^2 + 4\left(\int \frac{(bx+a)^{\frac{2}{3}}}{x^{\frac{2}{3}}a+x^{\frac{5}{3}}b}dx\right) a^3}{81b^2}$$

input
```
int((b*x^3+a*x^2)^(2/3),x)
```

output
```
( - 12*x**(1/3)*(a + b*x)**(2/3)*a**2 + 9*x**(1/3)*(a + b*x)**(2/3)*a*b*x
+ 27*x**(1/3)*(a + b*x)**(2/3)*b**2*x**2 + 4*int((a + b*x)**(2/3)/(x**(2/3
)*a + x**(2/3)*b*x),x)*a**3)/(81*b**2)
```

3.108 $\int \sqrt[3]{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 235

$$\int \sqrt[3]{ax^2+bx^3}\,dx = \frac{a\sqrt[3]{ax^2+bx^3}}{6b}+\frac{1}{2}x\sqrt[3]{ax^2+bx^3}+\frac{a^2\sqrt[3]{ax^2+bx^3}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a+bx}}\right)}{3\sqrt{3}b^{5/3}x^{2/3}\sqrt[3]{a+bx}}+\frac{a^2\sqrt[3]{ax^2+bx^3}\log(a+bx)}{18b^{5/3}x^{2/3}\sqrt[3]{a+bx}}+\frac{a^2\sqrt[3]{ax^2+bx^3}\log\left(1-\frac{\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a+bx}}\right)}{6b^{5/3}x^{2/3}\sqrt[3]{a+bx}}$$

output

```
1/6*a*(b*x^3+a*x^2)^(1/3)/b+1/2*x*(b*x^3+a*x^2)^(1/3)+1/9*a^2*(b*x^3+a*x^2
)^(1/3)*arctan(1/3*3^(1/2)+2/3*b^(1/3)*x^(1/3)*3^(1/2)/(b*x+a)^(1/3))*3^(1
/2)/b^(5/3)/x^(2/3)/(b*x+a)^(1/3)+1/18*a^2*(b*x^3+a*x^2)^(1/3)*ln(b*x+a)/b
^(5/3)/x^(2/3)/(b*x+a)^(1/3)+1/6*a^2*(b*x^3+a*x^2)^(1/3)*ln(1-b^(1/3)*x^(1
/3)/(b*x+a)^(1/3))/b^(5/3)/x^(2/3)/(b*x+a)^(1/3)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.88

$$\int \sqrt[3]{ax^2+bx^3}\,dx$$
$$=\frac{x^{4/3}(a+bx)^{2/3}\left(3ab^{2/3}x^{2/3}\sqrt[3]{a+bx}+9b^{5/3}x^{5/3}\sqrt[3]{a+bx}+2\sqrt{3}a^2\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{b}\sqrt[3]{x}+2\sqrt[3]{a+bx}}\right)+2a^2\log\right.}{18b^{5/3}\left(x^2(a+bx)\right)^{2/3}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(1/3),x]
```

output
```
(x^(4/3)*(a + b*x)^(2/3)*(3*a*b^(2/3)*x^(2/3)*(a + b*x)^(1/3) + 9*b^(5/3)*
x^(5/3)*(a + b*x)^(1/3) + 2*Sqrt[3]*a^2*ArcTan[(Sqrt[3]*b^(1/3)*x^(1/3))/(
b^(1/3)*x^(1/3) + 2*(a + b*x)^(1/3))] + 2*a^2*Log[-(b^(1/3)*x^(1/3)) + (a
+ b*x)^(1/3)] - a^2*Log[b^(2/3)*x^(2/3) + b^(1/3)*x^(1/3)*(a + b*x)^(1/3)
+ (a + b*x)^(2/3)]))/(18*b^(5/3)*(x^2*(a + b*x))^(2/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1910, 1930, 1938, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[3]{ax^2+bx^3}\,dx$$
$$\Big\downarrow \; 1910$$
$$\frac{1}{6}a\int \frac{x^2}{(bx^3+ax^2)^{2/3}}dx+\frac{1}{2}x\sqrt[3]{ax^2+bx^3}$$
$$\Big\downarrow \; 1930$$
$$\frac{1}{6}a\left(\frac{\sqrt[3]{ax^2+bx^3}}{b}-\frac{2a\int \frac{x}{(bx^3+ax^2)^{2/3}}dx}{3b}\right)+\frac{1}{2}x\sqrt[3]{ax^2+bx^3}$$

$$\Big\downarrow \text{1938}$$

$$\frac{1}{6}a\left(\frac{\sqrt[3]{ax^2+bx^3}}{b}-\frac{2ax^{4/3}(a+bx)^{2/3}\int \frac{1}{\sqrt[3]{x}(a+bx)^{2/3}}dx}{3b\left(ax^2+bx^3\right)^{2/3}}\right)+\frac{1}{2}x\sqrt[3]{ax^2+bx^3}$$

$$\Big\downarrow \text{71}$$

$$\frac{1}{6}a\left(\frac{\sqrt[3]{ax^2+bx^3}}{b}-\frac{2ax^{4/3}(a+bx)^{2/3}\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{b}\sqrt[3]{x}}{\sqrt{3}\sqrt[3]{a+bx}}+\frac{1}{\sqrt{3}}\right)}{b^{2/3}}-\frac{\log(a+bx)}{2b^{2/3}}-\frac{3\log\left(\frac{\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{a+bx}}-1\right)}{2b^{2/3}}\right)}{3b\left(ax^2+bx^3\right)^{2/3}}\right)+\frac{1}{2}x\sqrt[3]{ax^2+bx^3}$$

input

```
Int[(a*x^2 + b*x^3)^(1/3),x]
```

output

```
(x*(a*x^2 + b*x^3)^(1/3))/2 + (a*((a*x^2 + b*x^3)^(1/3)/b - (2*a*x^(4/3)*(
a + b*x)^(2/3)*(-((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*b^(1/3)*x^(1/3))/(Sqrt[3]
*(a + b*x)^(1/3))])/b^(2/3)) - Log[a + b*x]/(2*b^(2/3)) - (3*Log[-1 + (b^(
1/3)*x^(1/3))/(a + b*x)^(1/3)])/(2*b^(2/3))))/(3*b*(a*x^2 + b*x^3)^(2/3)))
)/6
```

Defintions of rubi rules used

rule 71

```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{9x\left(x^{2}(bx+a)\right)^{\frac{1}{3}} b^{\frac{5}{3}}+3a\left(x^{2}(bx+a)\right)^{\frac{1}{3}} b^{\frac{2}{3}}-2a^{2}\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(x^{2}(bx+a)\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+2a^{2}\ln\left(\frac{-b^{\frac{1}{3}}x+\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}{x}\right)-a}{18b^{\frac{5}{3}}}$

input
```
int((b*x^3+a*x^2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
1/18*(9*x*(x^2*(b*x+a))^(1/3)*b^(5/3)+3*a*(x^2*(b*x+a))^(1/3)*b^(2/3)-2*a^
2*3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(x^2*(b*x+a))^(1/3))/b^(1/3)/x)+
2*a^2*ln((-b^(1/3)*x+(x^2*(b*x+a))^(1/3))/x)-a^2*ln((b^(2/3)*x^2+b^(1/3)*(
x^2*(b*x+a))^(1/3)*x+(x^2*(b*x+a))^(2/3))/x^2))/b^(5/3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.82

$$\int \sqrt[3]{ax^2+bx^3}\,dx = -\frac{6\sqrt{\frac{1}{3}}a^2(b^2)^{\frac{1}{6}}b\arctan\left(\frac{\sqrt{\frac{1}{3}}\left((b^2)^{\frac{1}{3}}bx+2\,(bx^3+ax^2)^{\frac{1}{3}}(b^2)^{\frac{2}{3}}\right)(b^2)^{\frac{1}{6}}}{b^2x}\right)-2\,a^2(b^2)^{\frac{2}{3}}\log\left(-\frac{(b^2)^{\frac{2}{3}}x-(bx^3+ax^2)^{\frac{1}{3}}b}{x}\right)+a^2}{18\,b^3}$$

input
```
integrate((b*x^3+a*x^2)^(1/3),x, algorithm="fricas")
```

output
```
-1/18*(6*sqrt(1/3)*a^2*(b^2)^(1/6)*b*arctan(sqrt(1/3)*((b^2)^(1/3)*b*x + 2
*(b*x^3 + a*x^2)^(1/3)*(b^2)^(2/3))*(b^2)^(1/6)/(b^2*x)) - 2*a^2*(b^2)^(2/
3)*log(-((b^2)^(2/3)*x - (b*x^3 + a*x^2)^(1/3)*b)/x) + a^2*(b^2)^(2/3)*log
(((b^2)^(1/3)*b*x^2 + (b*x^3 + a*x^2)^(1/3)*(b^2)^(2/3)*x + (b*x^3 + a*x^2
)^(2/3)*b)/x^2) - 3*(3*b^3*x + a*b^2)*(b*x^3 + a*x^2)^(1/3))/b^3
```

Sympy [F]

$$\int \sqrt[3]{ax^2+bx^3}\,dx = \int \sqrt[3]{ax^2+bx^3}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(1/3),x)
```

output
```
Integral((a*x**2 + b*x**3)**(1/3), x)
```

Maxima [F]

$$\int \sqrt[3]{ax^2+bx^3}\,dx=\int \left(bx^3+ax^2\right)^{\frac{1}{3}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(1/3), x)
```

Giac [A] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.60

$$\int \sqrt[3]{ax^2+bx^3}\,dx = -\frac{\frac{2\sqrt{3}a^3\arctan\left(\frac{\sqrt{3}\left(2\left(b+\frac{a}{x}\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}}\right)}{b^{\frac{5}{3}}}+\frac{a^3\log\left(\left(b+\frac{a}{x}\right)^{\frac{2}{3}}+\left(b+\frac{a}{x}\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{b^{\frac{5}{3}}}-\frac{2a^3\log\left(\left|\left(b+\frac{a}{x}\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right|\right)}{b^{\frac{5}{3}}}-\frac{3\left(a^3\left(b+\frac{a}{x}\right)^{\frac{4}{3}}+2a^3\left(b+\cdots\right.\right.}{a^2b}}{18a}$$

input
```
integrate((b*x^3+a*x^2)^(1/3),x, algorithm="giac")
```

output
```
-1/18*(2*sqrt(3)*a^3*arctan(1/3*sqrt(3)*(2*(b + a/x)^(1/3) + b^(1/3))/b^(1
/3))/b^(5/3) + a^3*log((b + a/x)^(2/3) + (b + a/x)^(1/3)*b^(1/3) + b^(2/3)
)/b^(5/3) - 2*a^3*log(abs((b + a/x)^(1/3) - b^(1/3)))/b^(5/3) - 3*(a^3*(b
+ a/x)^(4/3) + 2*a^3*(b + a/x)^(1/3)*b)*x^2/(a^2*b))/a
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.16

$$\int \sqrt[3]{ax^2+bx^3}\,dx = \frac{3\,x\,\left(b\,x^3+a\,x^2\right)^{1/3}\,{}_2\mathrm{F}_1\left(-\frac{1}{3},\frac{5}{3};\ \frac{8}{3};\ -\frac{b\,x}{a}\right)}{5\,\left(\frac{b\,x}{a}+1\right)^{1/3}}$$

input
```
int((a*x^2 + b*x^3)^(1/3),x)
```

output
```
(3*x*(a*x^2 + b*x^3)^(1/3)*hypergeom([-1/3, 5/3], 8/3, -(b*x)/a))/(5*((b*x
)/a + 1)^(1/3))
```

Reduce [F]

$$\int \sqrt[3]{ax^2+bx^3}\,dx = \frac{3x^{\frac{2}{3}}(bx+a)^{\frac{1}{3}}\,a+9x^{\frac{5}{3}}(bx+a)^{\frac{1}{3}}\,b-2\left(\int \frac{(bx+a)^{\frac{1}{3}}}{x^{\frac{1}{3}}a+x^{\frac{4}{3}}b}dx\right)a^2}{18b}$$

input
```
int((b*x^3+a*x^2)^(1/3),x)
```

output
```
(3*x**(2/3)*(a + b*x)**(1/3)*a + 9*x**(2/3)*(a + b*x)**(1/3)*b*x - 2*int((
a + b*x)**(1/3)/(x**(1/3)*a + x**(1/3)*b*x),x)*a**2)/(18*b)
```

3.109 $\int \frac{1}{\sqrt[3]{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 177

$$\int \frac{1}{\sqrt[3]{ax^2+bx^3}}\,dx = -\frac{\sqrt{3}x^{2/3}\sqrt[3]{a+bx}\arctan\left(\frac{1}{\sqrt{3}}+\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}\right)}{\sqrt[3]{b}\sqrt[3]{ax^2+bx^3}} - \frac{x^{2/3}\sqrt[3]{a+bx}\log(x)}{2\sqrt[3]{b}\sqrt[3]{ax^2+bx^3}} - \frac{3x^{2/3}\sqrt[3]{a+bx}\log\left(1-\frac{\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{x}}\right)}{2\sqrt[3]{b}\sqrt[3]{ax^2+bx^3}}$$

output
```
-3^(1/2)*x^(2/3)*(b*x+a)^(1/3)*arctan(1/3*3^(1/2)+2/3*(b*x+a)^(1/3)*3^(1/2
)/b^(1/3)/x^(1/3))/b^(1/3)/(b*x^3+a*x^2)^(1/3)-1/2*x^(2/3)*(b*x+a)^(1/3)*l
n(x)/b^(1/3)/(b*x^3+a*x^2)^(1/3)-3/2*x^(2/3)*(b*x+a)^(1/3)*ln(1-(b*x+a)^(1
/3)/b^(1/3)/x^(1/3))/b^(1/3)/(b*x^3+a*x^2)^(1/3)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.86

$$\int \frac{1}{\sqrt[3]{ax^2+bx^3}}\,dx$$
$$=\frac{x^{2/3}\sqrt[3]{a+bx}\left(2\sqrt{3}\arctan\left(\frac{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}{\sqrt[3]{b}\sqrt[3]{x}+2\sqrt[3]{a+bx}}\right)-2\log\left(-\sqrt[3]{b}\sqrt[3]{x}+\sqrt[3]{a+bx}\right)+\log\left(b^{2/3}x^{2/3}+\sqrt[3]{b}\sqrt[3]{x}\right.\right.}{2\sqrt[3]{b}\sqrt[3]{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-1/3),x]
```

output
```
(x^(2/3)*(a + b*x)^(1/3)*(2*Sqrt[3]*ArcTan[(Sqrt[3]*b^(1/3)*x^(1/3))/(b^(1
/3)*x^(1/3) + 2*(a + b*x)^(1/3))] - 2*Log[-(b^(1/3)*x^(1/3)) + (a + b*x)^(
1/3)] + Log[b^(2/3)*x^(2/3) + b^(1/3)*x^(1/3)*(a + b*x)^(1/3) + (a + b*x)^
(2/3)]))/(2*b^(1/3)*(x^2*(a + b*x))^(1/3))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.67, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1917, 71}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[3]{ax^2+bx^3}}\,dx$$
$$\Big\downarrow \; 1917$$
$$\frac{x^{2/3}\sqrt[3]{a+bx}\int \frac{1}{x^{2/3}\sqrt[3]{a+bx}}dx}{\sqrt[3]{ax^2+bx^3}}$$
$$\Big\downarrow \; 71$$

$$\frac{x^{2/3}\sqrt[3]{a+bx}\left(-\frac{\sqrt{3}\arctan\left(\frac{2\sqrt[3]{a+bx}}{\sqrt{3}\sqrt[3]{b}\sqrt[3]{x}}+\frac{1}{\sqrt{3}}\right)}{\sqrt[3]{b}}-\frac{3\log\left(\frac{\sqrt[3]{a+bx}}{\sqrt[3]{b}\sqrt[3]{x}}-1\right)}{2\sqrt[3]{b}}-\frac{\log(x)}{2\sqrt[3]{b}}\right)}{\sqrt[3]{ax^2+bx^3}}$$

input
```
Int[(a*x^2 + b*x^3)^(-1/3),x]
```

output
```
(x^(2/3)*(a + b*x)^(1/3)*(-((Sqrt[3]*ArcTan[1/Sqrt[3] + (2*(a + b*x)^(1/3)
)/(Sqrt[3]*b^(1/3)*x^(1/3))])/b^(1/3)) - Log[x]/(2*b^(1/3)) - (3*Log[-1 +
(a + b*x)^(1/3)/(b^(1/3)*x^(1/3))])/(2*b^(1/3))))/(a*x^2 + b*x^3)^(1/3)
```

Defintions of rubi rules used

rule 71
```
Int[1/(((a_.) + (b_.)*(x_))^(1/3)*((c_.) + (d_.)*(x_))^(2/3)), x_Symbol] :>
 With[{q = Rt[d/b, 3]}, Simp[(-Sqrt[3])*(q/d)*ArcTan[2*q*((a + b*x)^(1/3)/(
Sqrt[3]*(c + d*x)^(1/3))) + 1/Sqrt[3]], x] + (-Simp[3*(q/(2*d))*Log[q*((a +
 b*x)^(1/3)/(c + d*x)^(1/3)) - 1], x] - Simp[(q/(2*d))*Log[c + d*x], x])] /
; FreeQ[{a, b, c, d}, x] && PosQ[d/b]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.60

method	result
pseudoelliptic	$-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(b^{\frac{1}{3}}x+2\left(x^{2}(bx+a)\right)^{\frac{1}{3}}\right)}{3b^{\frac{1}{3}}x}\right)+\ln\left(\frac{-b^{\frac{1}{3}}x+\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}{x}\right)-\frac{\ln\left(\frac{b^{\frac{2}{3}}x^{2}+b^{\frac{1}{3}}\left(x^{2}(bx+a)\right)^{\frac{1}{3}}x+\left(x^{2}(bx+a)\right)^{\frac{2}{3}}}{x^{2}}\right)}{2}}{b^{\frac{1}{3}}}$

input
```
int(1/(b*x^3+a*x^2)^(1/3),x,method=_RETURNVERBOSE)
```

output
```
-1/b^(1/3)*(3^(1/2)*arctan(1/3*3^(1/2)*(b^(1/3)*x+2*(x^2*(b*x+a))^(1/3))/b
^(1/3)/x)+ln((-b^(1/3)*x+(x^2*(b*x+a))^(1/3))/x)-1/2*ln((b^(2/3)*x^2+b^(1/
3)*(x^2*(b*x+a))^(1/3)*x+(x^2*(b*x+a))^(2/3))/x^2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 369, normalized size of antiderivative = 2.08

$$\int \frac{1}{\sqrt[3]{ax^2+bx^3}}\,dx$$

$$= \left[\frac{\sqrt{3}b\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}\log\left(\frac{3\,bx^2+2\,ax-3\,(bx^3+ax^2)^{\frac{1}{3}}(-b)^{\frac{2}{3}}x-\sqrt{3}\left((-b)^{\frac{1}{3}}bx^2-(bx^3+ax^2)^{\frac{1}{3}}bx+2\,(bx^3+ax^2)^{\frac{2}{3}}(-b)^{\frac{2}{3}}\right)\sqrt{\frac{(-b)^{\frac{1}{3}}}{b}}}{x}\right)-2\,(-}{2\,b}\right.$$

$$-\frac{2\sqrt{3}b\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}\arctan\left(-\frac{\sqrt{3}\left((-b)^{\frac{1}{3}}x-2\,(bx^3+ax^2)^{\frac{1}{3}}\right)\sqrt{-\frac{(-b)^{\frac{1}{3}}}{b}}}{3\,x}\right)+2\,(-b)^{\frac{2}{3}}\log\left(\frac{(-b)^{\frac{1}{3}}x+(bx^3+ax^2)^{\frac{1}{3}}}{x}\right)-(-b}{2\,b}$$

input
```
integrate(1/(b*x^3+a*x^2)^(1/3),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(3)*b*sqrt((-b)^(1/3)/b)*log((3*b*x^2 + 2*a*x - 3*(b*x^3 + a*x^2
)^(1/3)*(-b)^(2/3)*x - sqrt(3)*((-b)^(1/3)*b*x^2 - (b*x^3 + a*x^2)^(1/3)*b
*x + 2*(b*x^3 + a*x^2)^(2/3)*(-b)^(2/3))*sqrt((-b)^(1/3)/b))/x) - 2*(-b)^(
2/3)*log(((-b)^(1/3)*x + (b*x^3 + a*x^2)^(1/3))/x) + (-b)^(2/3)*log(((-b)^
(2/3)*x^2 - (b*x^3 + a*x^2)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a*x^2)^(2/3))/x^
2))/b, -1/2*(2*sqrt(3)*b*sqrt(-(-b)^(1/3)/b)*arctan(-1/3*sqrt(3)*((-b)^(1/
3)*x - 2*(b*x^3 + a*x^2)^(1/3))*sqrt(-(-b)^(1/3)/b)/x) + 2*(-b)^(2/3)*log(
((-b)^(1/3)*x + (b*x^3 + a*x^2)^(1/3))/x) - (-b)^(2/3)*log(((-b)^(2/3)*x^2
 - (b*x^3 + a*x^2)^(1/3)*(-b)^(1/3)*x + (b*x^3 + a*x^2)^(2/3))/x^2))/b]
```

Sympy [F]

$$\int \frac{1}{\sqrt[3]{ax^2+bx^3}}\,dx=\int \frac{1}{\sqrt[3]{ax^2+bx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(1/3),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-1/3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[3]{ax^2+bx^3}}\,dx=\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{1}{3}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(1/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-1/3), x)
```

Giac [A] (verification not implemented)

Time = 3.56 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.49

$$\int \frac{1}{\sqrt[3]{ax^2+bx^3}}\,dx=-\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(2\left(b+\frac{a}{x}\right)^{\frac{1}{3}}+b^{\frac{1}{3}}\right)}{3\,b^{\frac{1}{3}}}\right)}{b^{\frac{1}{3}}}+\frac{\log\left(\left(b+\frac{a}{x}\right)^{\frac{2}{3}}+\left(b+\frac{a}{x}\right)^{\frac{1}{3}}b^{\frac{1}{3}}+b^{\frac{2}{3}}\right)}{2\,b^{\frac{1}{3}}}-\frac{\log\left(\left|\left(b+\frac{a}{x}\right)^{\frac{1}{3}}-b^{\frac{1}{3}}\right|\right)}{b^{\frac{1}{3}}}$$

input
```
integrate(1/(b*x^3+a*x^2)^(1/3),x, algorithm="giac")
```

output
```
-sqrt(3)*arctan(1/3*sqrt(3)*(2*(b + a/x)^(1/3) + b^(1/3))/b^(1/3))/b^(1/3)
 + 1/2*log((b + a/x)^(2/3) + (b + a/x)^(1/3)*b^(1/3) + b^(2/3))/b^(1/3) -
log(abs((b + a/x)^(1/3) - b^(1/3)))/b^(1/3)
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.21

$$\int \frac{1}{\sqrt[3]{ax^2+bx^3}}\,dx = \frac{3\,x\,\left(\frac{b\,x}{a}+1\right)^{1/3}\,{}_2\mathrm{F}_1\left(\frac{1}{3},\frac{1}{3};\ \frac{4}{3};\ -\frac{b\,x}{a}\right)}{\left(b\,x^3+a\,x^2\right)^{1/3}}$$

input
```
int(1/(a*x^2 + b*x^3)^(1/3),x)
```

output
```
(3*x*((b*x)/a + 1)^(1/3)*hypergeom([1/3, 1/3], 4/3, -(b*x)/a))/(a*x^2 + b*
x^3)^(1/3)
```

Reduce [F]

$$\int \frac{1}{\sqrt[3]{ax^2+bx^3}}\,dx = \int \frac{1}{x^{\frac{2}{3}}\,(bx+a)^{\frac{1}{3}}}dx$$

input
```
int(1/(b*x^3+a*x^2)^(1/3),x)
```

output
```
int(1/(x**(2/3)*(a + b*x)**(1/3)),x)
```

3.110 $\int \frac{1}{(ax^2+bx^3)^{2/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 23

$$\int \frac{1}{(ax^2+bx^3)^{2/3}}\,dx = -\frac{3\sqrt[3]{ax^2+bx^3}}{ax}$$

output
```
-3*(b*x^3+a*x^2)^(1/3)/a/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(ax^2+bx^3)^{2/3}}\,dx = -\frac{3\sqrt[3]{x^2(a+bx)}}{ax}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-2/3),x]
```

output
```
(-3*(x^2*(a + b*x))^(1/3))/(a*x)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = $\{1906\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{2/3}}\,dx$$

$$\Big\downarrow \text{1906}$$

$$-\frac{3\sqrt[3]{ax^2+bx^3}}{ax}$$

input
```
Int[(a*x^2 + b*x^3)^(-2/3),x]
```

output
```
(-3*(a*x^2 + b*x^3)^(1/3))/(a*x)
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$-\frac{3\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}{ax}$	20
trager	$-\frac{3\left(b\,x^{3}+a\,x^{2}\right)^{\frac{1}{3}}}{ax}$	22
risch	$-\frac{3x(bx+a)}{\left(x^{2}(bx+a)\right)^{\frac{2}{3}}a}$	23
gosper	$-\frac{3x(bx+a)}{a\left(b\,x^{3}+a\,x^{2}\right)^{\frac{2}{3}}}$	25
orering	$-\frac{3x(bx+a)}{a\left(b\,x^{3}+a\,x^{2}\right)^{\frac{2}{3}}}$	25

input
```
int(1/(b*x^3+a*x^2)^(2/3),x,method=_RETURNVERBOSE)
```

output
```
-3*(x^2*(b*x+a))^(1/3)/a/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{\left(ax^2+bx^3\right)^{2/3}}\,dx=-\frac{3\left(bx^3+ax^2\right)^{\frac{1}{3}}}{ax}$$

input
```
integrate(1/(b*x^3+a*x^2)^(2/3),x, algorithm="fricas")
```

output
```
-3*(b*x^3 + a*x^2)^(1/3)/(a*x)
```

Sympy [F]

$$\int \frac{1}{\left(ax^2+bx^3\right)^{2/3}}\,dx=\int \frac{1}{\left(ax^2+bx^3\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(2/3),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-2/3), x)
```

Maxima [F]

$$\int \frac{1}{\left(ax^2+bx^3\right)^{2/3}}\,dx=\int \frac{1}{\left(bx^3+ax^2\right)^{\frac{2}{3}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(2/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-2/3), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{1}{\left(ax^2+bx^3\right)^{2/3}}\,dx=-\frac{3\left(b+\frac{a}{x}\right)^{\frac{1}{3}}}{a}$$

input
```
integrate(1/(b*x^3+a*x^2)^(2/3),x, algorithm="giac")
```

output
```
-3*(b + a/x)^(1/3)/a
```

Mupad [B] (verification not implemented)

Time = 9.23 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.91

$$\int \frac{1}{(ax^2+bx^3)^{2/3}}\,dx = -\frac{3\,{\left(b\,x^3+a\,x^2\right)}^{1/3}}{a\,x}$$

input
```
int(1/(a*x^2 + b*x^3)^(2/3),x)
```

output
```
-(3*(a*x^2 + b*x^3)^(1/3))/(a*x)
```

Reduce [F]

$$\int \frac{1}{(ax^2+bx^3)^{2/3}}\,dx = \int \frac{1}{x^{\frac{4}{3}}\,(bx+a)^{\frac{2}{3}}}dx$$

input
```
int(1/(b*x^3+a*x^2)^(2/3),x)
```

output
```
int(1/(x**(1/3)*(a + b*x)**(2/3)*x),x)
```

3.111 $\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 75

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx=\frac{3}{ax\sqrt[3]{ax^2+bx^3}}-\frac{18(ax^2+bx^3)^{2/3}}{5a^2x^3}+\frac{27b(ax^2+bx^3)^{2/3}}{5a^3x^2}$$

output
```
3/a/x/(b*x^3+a*x^2)^(1/3)-18/5*(b*x^3+a*x^2)^(2/3)/a^2/x^3+27/5*b*(b*x^3+a
*x^2)^(2/3)/a^3/x^2
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.57

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx=-\frac{3x(a+bx)\left(a^2-3abx-9b^2x^2\right)}{5a^3\left(x^2(a+bx)\right)^{4/3}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-4/3),x]
```

output
```
(-3*x*(a + b*x)*(a^2 - 3*a*b*x - 9*b^2*x^2))/(5*a^3*(x^2*(a + b*x))^(4/3))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1907, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{4/3}} \, dx$$

$$\Big\downarrow \text{1907}$$

$$\frac{6 \int \frac{1}{x^2 \sqrt[3]{bx^3+ax^2}}dx}{a} + \frac{3}{ax\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{6 \left(-\frac{3b\int \frac{1}{x\sqrt[3]{bx^3+ax^2}}dx}{5a} - \frac{3(ax^2+bx^3)^{2/3}}{5ax^3}\right)}{a} + \frac{3}{ax\sqrt[3]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{6 \left(\frac{9b(ax^2+bx^3)^{2/3}}{10a^2x^2} - \frac{3(ax^2+bx^3)^{2/3}}{5ax^3}\right)}{a} + \frac{3}{ax\sqrt[3]{ax^2+bx^3}}$$

input
```
Int[(a*x^2 + b*x^3)^(-4/3),x]
```

output
```
3/(a*x*(a*x^2 + b*x^3)^(1/3)) + (6*((-3*(a*x^2 + b*x^3)^(2/3))/(5*a*x^3) +
 (9*b*(a*x^2 + b*x^3)^(2/3))/(10*a^2*x^2)))/a
```

Defintions of rubi rules used

rule 1907
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b, j, n}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j +
1)/(n - j)], 0] && LtQ[p, -1]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

method	result	size
pseudoelliptic	$-\frac{3\left(-9 b^{2} x^{2}-3 a b x+a^{2}\right)}{5 x \left(x^{2} \left(b x+a\right)\right)^{\frac{1}{3}} a^{3}}$	37
gosper	$-\frac{3 x \left(b x+a\right) \left(-9 b^{2} x^{2}-3 a b x+a^{2}\right)}{5 a^{3} \left(b \,x^{3}+a \,x^{2}\right)^{\frac{4}{3}}}$	42
orering	$-\frac{3 x \left(b x+a\right) \left(-9 b^{2} x^{2}-3 a b x+a^{2}\right)}{5 a^{3} \left(b \,x^{3}+a \,x^{2}\right)^{\frac{4}{3}}}$	42
trager	$-\frac{3 \left(-9 b^{2} x^{2}-3 a b x+a^{2}\right) \left(b \,x^{3}+a \,x^{2}\right)^{\frac{2}{3}}}{5 \left(b x+a\right) a^{3} x^{3}}$	46
risch	$-\frac{3 \left(b x+a\right) \left(-4 b x+a\right)}{5 a^{3} x \left(x^{2} \left(b x+a\right)\right)^{\frac{1}{3}}}+\frac{3 b^{2} x}{\left(x^{2} \left(b x+a\right)\right)^{\frac{1}{3}} a^{3}}$	52

input
```
int(1/(b*x^3+a*x^2)^(4/3),x,method=_RETURNVERBOSE)
```

output
```
-3/5/x*(-9*b^2*x^2-3*a*b*x+a^2)/(x^2*(b*x+a))^(1/3)/a^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx = \frac{3\,(9\,b^2x^2+3\,abx-a^2)(bx^3+ax^2)^{\frac{2}{3}}}{5\,(a^3bx^4+a^4x^3)}$$

input
```
integrate(1/(b*x^3+a*x^2)^(4/3),x, algorithm="fricas")
```

output
```
3/5*(9*b^2*x^2 + 3*a*b*x - a^2)*(b*x^3 + a*x^2)^(2/3)/(a^3*b*x^4 + a^4*x^3
)
```

Sympy [F]

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{1}{(ax^2+bx^3)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(4/3),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-4/3), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{4}{3}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(4/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-4/3), x)
```

Giac [A] (verification not implemented)

Time = 3.36 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.76

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx = \frac{3\left(\frac{5\,b^2}{a\left(b+\frac{a}{x}\right)^{\frac{1}{3}}} - \frac{a^4\left(b+\frac{a}{x}\right)^{\frac{5}{3}} - 5\,a^4\left(b+\frac{a}{x}\right)^{\frac{2}{3}} b}{a^5}\right)}{5\,a^2}$$

input
```
integrate(1/(b*x^3+a*x^2)^(4/3),x, algorithm="giac")
```

output
```
3/5*(5*b^2/(a*(b + a/x)^(1/3)) - (a^4*(b + a/x)^(5/3) - 5*a^4*(b + a/x)^(2
/3)*b)/a^5)/a^2
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.63

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx = \frac{3\left(b\,x^3+a\,x^2\right)^{2/3}\left(-a^2+3\,a\,b\,x+9\,b^2\,x^2\right)}{5\,a^3\,x^3\,\left(a+b\,x\right)}$$

input
```
int(1/(a*x^2 + b*x^3)^(4/3),x)
```

output
```
(3*(a*x^2 + b*x^3)^(2/3)*(9*b^2*x^2 - a^2 + 3*a*b*x))/(5*a^3*x^3*(a + b*x)
)
```

Reduce [F]

$$\int \frac{1}{(ax^2+bx^3)^{4/3}}\,dx = \int \frac{1}{x^{\frac{8}{3}}\,(bx+a)^{\frac{1}{3}}\,a + x^{\frac{11}{3}}\,(bx+a)^{\frac{1}{3}}\,b}dx$$

input
```
int(1/(b*x^3+a*x^2)^(4/3),x)
```

output
```
int(1/(x**(2/3)*(a + b*x)**(1/3)*a*x**2 + x**(2/3)*(a + b*x)**(1/3)*b*x**3
),x)
```

3.112 $\int \frac{1}{(ax^2+bx^3)^{5/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 105

$$\int \frac{1}{(ax^2+bx^3)^{5/3}}\,dx = \frac{3}{2ax\,(ax^2+bx^3)^{2/3}} - \frac{27\sqrt[3]{ax^2+bx^3}}{14a^2x^3} + \frac{81b\sqrt[3]{ax^2+bx^3}}{28a^3x^2} - \frac{243b^2\sqrt[3]{ax^2+bx^3}}{28a^4x}$$

output
```
3/2/a/x/(b*x^3+a*x^2)^(2/3)-27/14*(b*x^3+a*x^2)^(1/3)/a^2/x^3+81/28*b*(b*x
^3+a*x^2)^(1/3)/a^3/x^2-243/28*b^2*(b*x^3+a*x^2)^(1/3)/a^4/x
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.50

$$\int \frac{1}{(ax^2+bx^3)^{5/3}}\,dx = -\frac{3(4a^3-9a^2bx+54ab^2x^2+81b^3x^3)}{28a^4x\,(x^2(a+bx))^{2/3}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-5/3),x]
```

output
```
(-3*(4*a^3 - 9*a^2*b*x + 54*a*b^2*x^2 + 81*b^3*x^3))/(28*a^4*x*(x^2*(a + b
*x))^(2/3))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {1907, 1922, 1922, 1906}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{5/3}}dx$$

$$\Big\downarrow \text{1907}$$

$$\frac{9\int \frac{1}{x^2(bx^3+ax^2)^{2/3}}dx}{2a}+\frac{3}{2ax(ax^2+bx^3)^{2/3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{9\left(-\frac{6b\int \frac{1}{x\left(bx^3+ax^2\right)^{2/3}}dx}{7a}-\frac{3\sqrt[3]{ax^2+bx^3}}{7ax^3}\right)}{2a}+\frac{3}{2ax(ax^2+bx^3)^{2/3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{9\left(-\frac{6b\left(-\frac{3b\int \frac{1}{\left(bx^3+ax^2\right)^{2/3}}dx}{4a}-\frac{3\sqrt[3]{ax^2+bx^3}}{4ax^2}\right)}{7a}-\frac{3\sqrt[3]{ax^2+bx^3}}{7ax^3}\right)}{2a}+\frac{3}{2ax(ax^2+bx^3)^{2/3}}$$

$$\Big\downarrow \text{1906}$$

$$\frac{9\left(-\frac{6b\left(\frac{9b\sqrt[3]{ax^2+bx^3}}{4a^2x}-\frac{3\sqrt[3]{ax^2+bx^3}}{4ax^2}\right)}{7a}-\frac{3\sqrt[3]{ax^2+bx^3}}{7ax^3}\right)}{2a}+\frac{3}{2ax(ax^2+bx^3)^{2/3}}$$

input
```
Int[(a*x^2 + b*x^3)^(-5/3),x]
```

output
```
3/(2*a*x*(a*x^2 + b*x^3)^(2/3)) + (9*((-3*(a*x^2 + b*x^3)^(1/3))/(7*a*x^3)
 - (6*b*((-3*(a*x^2 + b*x^3)^(1/3))/(4*a*x^2) + (9*b*(a*x^2 + b*x^3)^(1/3)
)/(4*a^2*x)))/(7*a)))/(2*a)
```

Defintions of rubi rules used

rule 1906
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(b*(n - j)*(p + 1)*x^(n - 1)), x] /; FreeQ[{a, b, j, n, p},
x] &&  !IntegerQ[p] && NeQ[n, j] && EqQ[j*p - n + j + 1, 0]
```

rule 1907
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b, j, n}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j +
1)/(n - j)], 0] && LtQ[p, -1]
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.48

method	result	size
pseudoelliptic	$\frac{-\frac{243}{28} b^{3} x^{3}-\frac{81}{14} a \,b^{2} x^{2}+\frac{27}{28} a^{2} b x -\frac{3}{7} a^{3}}{x \left(x^{2} \left(b x +a \right)\right)^{\frac{2}{3}} a^{4}}$	50
gosper	$-\frac{3 x \left(b x +a \right) \left(81 b^{3} x^{3}+54 a \,b^{2} x^{2}-9 a^{2} b x +4 a^{3}\right)}{28 a^{4} \left(b \,x^{3}+a \,x^{2}\right)^{\frac{5}{3}}}$	55
orering	$-\frac{3 x \left(b x +a \right) \left(81 b^{3} x^{3}+54 a \,b^{2} x^{2}-9 a^{2} b x +4 a^{3}\right)}{28 a^{4} \left(b \,x^{3}+a \,x^{2}\right)^{\frac{5}{3}}}$	55
trager	$-\frac{3 \left(81 b^{3} x^{3}+54 a \,b^{2} x^{2}-9 a^{2} b x +4 a^{3}\right) \left(b \,x^{3}+a \,x^{2}\right)^{\frac{1}{3}}}{28 \left(b x +a \right) a^{4} x^{3}}$	59
risch	$-\frac{3 \left(b x +a \right) \left(67 b^{2} x^{2}-13 a b x +4 a^{2}\right)}{28 a^{4} x \left(x^{2} \left(b x +a \right)\right)^{\frac{2}{3}}}-\frac{3 b^{3} x^{2}}{2 a^{4} \left(x^{2} \left(b x +a \right)\right)^{\frac{2}{3}}}$	67

input
```
int(1/(b*x^3+a*x^2)^(5/3),x,method=_RETURNVERBOSE)
```

output
```
3/28*(-81*b^3*x^3-54*a*b^2*x^2+9*a^2*b*x-4*a^3)/x/(x^2*(b*x+a))^(2/3)/a^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.60

$$\int \frac{1}{(ax^2+bx^3)^{5/3}}\,dx = -\frac{3\,(81\,b^3x^3+54\,ab^2x^2-9\,a^2bx+4\,a^3)(bx^3+ax^2)^{\frac{1}{3}}}{28\,(a^4bx^4+a^5x^3)}$$

input
```
integrate(1/(b*x^3+a*x^2)^(5/3),x, algorithm="fricas")
```

output
```
-3/28*(81*b^3*x^3 + 54*a*b^2*x^2 - 9*a^2*b*x + 4*a^3)*(b*x^3 + a*x^2)^(1/3
)/(a^4*b*x^4 + a^5*x^3)
```

Sympy [F]

$$\int \frac{1}{(ax^2+bx^3)^{5/3}}\,dx=\int \frac{1}{(ax^2+bx^3)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(5/3),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-5/3), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2+bx^3)^{5/3}}\,dx=\int \frac{1}{(bx^3+ax^2)^{\frac{5}{3}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(5/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-5/3), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.67

$$\int \frac{1}{(ax^2+bx^3)^{5/3}}\,dx=-\frac{3\,b^3}{2\,a^4\left(b+\frac{a}{x}\right)^{\frac{2}{3}}} - \frac{3\left(4\,a^{24}\left(b+\frac{a}{x}\right)^{\frac{7}{3}}-21\,a^{24}\left(b+\frac{a}{x}\right)^{\frac{4}{3}}b+84\,a^{24}\left(b+\frac{a}{x}\right)^{\frac{1}{3}}b^2\right)}{28\,a^{28}}$$

input
```
integrate(1/(b*x^3+a*x^2)^(5/3),x, algorithm="giac")
```

output
```
-3/2*b^3/(a^4*(b + a/x)^(2/3)) - 3/28*(4*a^24*(b + a/x)^(7/3) - 21*a^24*(b
 + a/x)^(4/3)*b + 84*a^24*(b + a/x)^(1/3)*b^2)/a^28
```

Mupad [B] (verification not implemented)

Time = 9.42 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.55

$$\int \frac{1}{(ax^2+bx^3)^{5/3}}\,dx = -\frac{3\,(b\,x^3+a\,x^2)^{1/3}\,(4\,a^3-9\,a^2\,b\,x+54\,a\,b^2\,x^2+81\,b^3\,x^3)}{28\,a^4\,x^3\,(a+b\,x)}$$

input
```
int(1/(a*x^2 + b*x^3)^(5/3),x)
```

output
```
-(3*(a*x^2 + b*x^3)^(1/3)*(4*a^3 + 81*b^3*x^3 + 54*a*b^2*x^2 - 9*a^2*b*x))
/(28*a^4*x^3*(a + b*x))
```

Reduce [F]

$$\int \frac{1}{(ax^2+bx^3)^{5/3}}\,dx = \int \frac{1}{x^{\frac{10}{3}}\,(bx+a)^{\frac{2}{3}}\,a+x^{\frac{13}{3}}\,(bx+a)^{\frac{2}{3}}\,b}dx$$

input
```
int(1/(b*x^3+a*x^2)^(5/3),x)
```

output
```
int(1/(x**(1/3)*(a + b*x)**(2/3)*a*x**3 + x**(1/3)*(a + b*x)**(2/3)*b*x**4
),x)
```

3.113 $\int \frac{1}{(ax^2+bx^3)^{7/3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 158

$$\int \frac{1}{(ax^2+bx^3)^{7/3}}\,dx = \frac{3}{4ax\,(ax^2+bx^3)^{4/3}} + \frac{45}{4a^2x^3\sqrt[3]{ax^2+bx^3}} - \frac{135(ax^2+bx^3)^{2/3}}{11a^3x^5} + \frac{1215b(ax^2+bx^3)^{2/3}}{88a^4x^4} - \frac{729b^2(ax^2+bx^3)^{2/3}}{44a^5x^3} + \frac{2187b^3(ax^2+bx^3)^{2/3}}{88a^6x^2}$$

output
```
3/4/a/x/(b*x^3+a*x^2)^(4/3)+45/4/a^2/x^3/(b*x^3+a*x^2)^(1/3)-135/11*(b*x^3
+a*x^2)^(2/3)/a^3/x^5+1215/88*b*(b*x^3+a*x^2)^(2/3)/a^4/x^4-729/44*b^2*(b*
x^3+a*x^2)^(2/3)/a^5/x^3+2187/88*b^3*(b*x^3+a*x^2)^(2/3)/a^6/x^2
```

Mathematica [A] (verified)

Time = 1.47 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.47

$$\int \frac{1}{(ax^2+bx^3)^{7/3}}\,dx = \frac{3(-8a^5+15a^4bx-36a^3b^2x^2+162a^2b^3x^3+972ab^4x^4+729b^5x^5)}{88a^6x\,(x^2(a+bx))^{4/3}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-7/3),x]
```

output
```
(3*(-8*a^5 + 15*a^4*b*x - 36*a^3*b^2*x^2 + 162*a^2*b^3*x^3 + 972*a*b^4*x^4
 + 729*b^5*x^5))/(88*a^6*x*(x^2*(a + b*x))^(4/3))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1907, 1921, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{7/3}}dx$$

$$\Big\downarrow \text{1907}$$

$$\frac{15\int \frac{1}{x^2(bx^3+ax^2)^{4/3}}dx}{4a}+\frac{3}{4ax(ax^2+bx^3)^{4/3}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{15\left(\frac{12\int \frac{1}{x^4\sqrt[3]{bx^3+ax^2}}dx}{a}+\frac{3}{ax^3\sqrt[3]{ax^2+bx^3}}\right)}{4a}+\frac{3}{4ax(ax^2+bx^3)^{4/3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{15\left(\frac{12\left(-\frac{9b\int \frac{1}{x^3\sqrt[3]{bx^3+ax^2}}dx}{11a}-\frac{3(ax^2+bx^3)^{2/3}}{11ax^5}\right)}{a}+\frac{3}{ax^3\sqrt[3]{ax^2+bx^3}}\right)}{4a}+\frac{3}{4ax(ax^2+bx^3)^{4/3}}$$

$$\Big\downarrow \text{1922}$$

$$\frac{15\left(\frac{12\left(-\frac{9b\left(-\frac{3b\int \frac{1}{x^2\sqrt[3]{bx^3+ax^2}}dx}{4a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{8ax^4}\right)}{11a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{11ax^5}\right)}{a}+\frac{3}{ax^3\sqrt[3]{ax^2+bx^3}}\right)}{4a}+\frac{3}{4ax\left(ax^2+bx^3\right)^{4/3}}$$

$\downarrow$ 1922

$$\frac{15\left(\frac{12\left(-\frac{9b\left(-\frac{3b\left(-\frac{3b\int \frac{1}{x\sqrt[3]{bx^3+ax^2}}dx}{5a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{5ax^3}\right)}{4a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{8ax^4}\right)}{11a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{11ax^5}\right)}{a}+\frac{3}{ax^3\sqrt[3]{ax^2+bx^3}}\right)}{4a}+\frac{3}{4ax\left(ax^2+bx^3\right)^{4/3}}$$

$\downarrow$ 1920

$$\frac{15\left(\frac{12\left(-\frac{9b\left(-\frac{3b\left(\frac{9b\left(ax^2+bx^3\right)^{2/3}}{10a^2x^2}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{5ax^3}\right)}{4a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{8ax^4}\right)}{11a}-\frac{3\left(ax^2+bx^3\right)^{2/3}}{11ax^5}\right)}{a}+\frac{3}{ax^3\sqrt[3]{ax^2+bx^3}}\right)}{4a}+\frac{3}{4ax\left(ax^2+bx^3\right)^{4/3}}$$

input
```
Int[(a*x^2 + b*x^3)^(-7/3),x]
```

output
```
3/(4*a*x*(a*x^2 + b*x^3)^(4/3)) + (15*(3/(a*x^3*(a*x^2 + b*x^3)^(1/3)) + (
12*((-3*(a*x^2 + b*x^3)^(2/3))/(11*a*x^5) - (9*b*((-3*(a*x^2 + b*x^3)^(2/3
))/(8*a*x^4) - (3*b*((-3*(a*x^2 + b*x^3)^(2/3))/(5*a*x^3) + (9*b*(a*x^2 +
b*x^3)^(2/3))/(10*a^2*x^2)))/(4*a)))/(11*a)))/a))/(4*a)
```

Defintions of rubi rules used

rule 1907
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b, j, n}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j +
1)/(n - j)], 0] && LtQ[p, -1]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.49

method	result	size
gosper	$-\frac{3x(bx+a)\left(-729b^{5}x^{5}-972a\,b^{4}x^{4}-162a^{2}b^{3}x^{3}+36a^{3}b^{2}x^{2}-15a^{4}bx+8a^{5}\right)}{88a^{6}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{7}{3}}}$	77
orering	$-\frac{3x(bx+a)\left(-729b^{5}x^{5}-972a\,b^{4}x^{4}-162a^{2}b^{3}x^{3}+36a^{3}b^{2}x^{2}-15a^{4}bx+8a^{5}\right)}{88a^{6}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{7}{3}}}$	77
pseudoelliptic	$\frac{\frac{2187}{88}b^{5}x^{5}+\frac{729}{22}a\,b^{4}x^{4}+\frac{243}{44}a^{2}b^{3}x^{3}-\frac{27}{22}a^{3}b^{2}x^{2}+\frac{45}{88}a^{4}bx-\frac{3}{11}a^{5}}{x^{3}(bx+a)\left(x^{2}(bx+a)\right)^{\frac{1}{3}}a^{6}}$	79
trager	$-\frac{3\left(-729b^{5}x^{5}-972a\,b^{4}x^{4}-162a^{2}b^{3}x^{3}+36a^{3}b^{2}x^{2}-15a^{4}bx+8a^{5}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{2}{3}}}{88x^{5}a^{6}(bx+a)^{2}}$	81
risch	$-\frac{3(bx+a)\left(-311b^{3}x^{3}+90a\,b^{2}x^{2}-31a^{2}bx+8a^{3}\right)}{88a^{6}x^{3}\left(x^{2}(bx+a)\right)^{\frac{1}{3}}}+\frac{3b^{4}(19bx+20a)x}{4\left(x^{2}(bx+a)\right)^{\frac{1}{3}}(bx+a)a^{6}}$	91

input
```
int(1/(b*x^3+a*x^2)^(7/3),x,method=_RETURNVERBOSE)
```

output
```
-3/88*x*(b*x+a)*(-729*b^5*x^5-972*a*b^4*x^4-162*a^2*b^3*x^3+36*a^3*b^2*x^2
-15*a^4*b*x+8*a^5)/a^6/(b*x^3+a*x^2)^(7/3)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.61

$$\int \frac{1}{(ax^2+bx^3)^{7/3}}\,dx = \frac{3\,(729\,b^5x^5+972\,ab^4x^4+162\,a^2b^3x^3-36\,a^3b^2x^2+15\,a^4bx-8\,a^5)(bx^3+ax^2)^{\frac{2}{3}}}{88\,(a^6b^2x^7+2\,a^7bx^6+a^8x^5)}$$

input
```
integrate(1/(b*x^3+a*x^2)^(7/3),x, algorithm="fricas")
```

output
```
3/88*(729*b^5*x^5 + 972*a*b^4*x^4 + 162*a^2*b^3*x^3 - 36*a^3*b^2*x^2 + 15*
a^4*b*x - 8*a^5)*(b*x^3 + a*x^2)^(2/3)/(a^6*b^2*x^7 + 2*a^7*b*x^6 + a^8*x^
5)
```

Sympy [F]

$$\int \frac{1}{(ax^2+bx^3)^{7/3}}\,dx = \int \frac{1}{(ax^2+bx^3)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(7/3),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-7/3), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2+bx^3)^{7/3}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{7}{3}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(7/3),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-7/3), x)
```

Giac [A] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.65

$$\int \frac{1}{(ax^2+bx^3)^{7/3}}\,dx = \frac{3\left(20\left(b+\frac{a}{x}\right)b^4-b^5\right)}{4\,a^6\left(b+\frac{a}{x}\right)^{\frac{4}{3}}} - \frac{3\left(8\,a^{60}\left(b+\frac{a}{x}\right)^{\frac{11}{3}}-55\,a^{60}\left(b+\frac{a}{x}\right)^{\frac{8}{3}}b+176\,a^{60}\left(b+\frac{a}{x}\right)^{\frac{5}{3}}b^2-440\,a^{60}\left(b+\frac{a}{x}\right)^{\frac{2}{3}}b^3\right)}{88\,a^{66}}$$

input
```
integrate(1/(b*x^3+a*x^2)^(7/3),x, algorithm="giac")
```

output
```
3/4*(20*(b + a/x)*b^4 - b^5)/(a^6*(b + a/x)^(4/3)) - 3/88*(8*a^60*(b + a/x
)^(11/3) - 55*a^60*(b + a/x)^(8/3)*b + 176*a^60*(b + a/x)^(5/3)*b^2 - 440*
a^60*(b + a/x)^(2/3)*b^3)/a^66
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.82

$$\int \frac{1}{(ax^2+bx^3)^{7/3}}\,dx = \frac{\left(b\,x^3+a\,x^2\right)^{2/3}\left(\frac{729\,b^3}{88\,a^5}+\frac{2187\,b^4\,x}{88\,a^6}\right)}{x^2\,\left(a+b\,x\right)} - \frac{\left(b\,x^3+a\,x^2\right)^{2/3}\left(\frac{135\,b^2}{44\,a^3}+\frac{42\,b^3\,x}{11\,a^4}\right)}{x^3\,{\left(a+b\,x\right)}^2} - \frac{3\,\left(b\,x^3+a\,x^2\right)^{2/3}}{11\,a^3\,x^5} + \frac{93\,b\,\left(b\,x^3+a\,x^2\right)^{2/3}}{88\,a^4\,x^4}$$

input
```
int(1/(a*x^2 + b*x^3)^(7/3),x)
```

output
```
((a*x^2 + b*x^3)^(2/3)*((729*b^3)/(88*a^5) + (2187*b^4*x)/(88*a^6)))/(x^2*
(a + b*x)) - ((a*x^2 + b*x^3)^(2/3)*((135*b^2)/(44*a^3) + (42*b^3*x)/(11*a
^4)))/(x^3*(a + b*x)^2) - (3*(a*x^2 + b*x^3)^(2/3))/(11*a^3*x^5) + (93*b*(
a*x^2 + b*x^3)^(2/3))/(88*a^4*x^4)
```

Reduce [F]

$$\int \frac{1}{(ax^2+bx^3)^{7/3}}\,dx = \int \frac{1}{x^{\frac{14}{3}}(bx+a)^{\frac{1}{3}}a^2+2x^{\frac{17}{3}}(bx+a)^{\frac{1}{3}}ab+x^{\frac{20}{3}}(bx+a)^{\frac{1}{3}}b^2}dx$$

input
```
int(1/(b*x^3+a*x^2)^(7/3),x)
```

output
```
int(1/(x**(2/3)*(a + b*x)**(1/3)*a**2*x**4 + 2*x**(2/3)*(a + b*x)**(1/3)*a
*b*x**5 + x**(2/3)*(a + b*x)**(1/3)*b**2*x**6),x)
```

3.114 $\int (ax^2+bx^3)^{9/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 273

$$\int (ax^2+bx^3)^{9/4}\,dx=\frac{320a^7\sqrt[4]{ax^2+bx^3}}{149017b^5}-\frac{160a^6x\sqrt[4]{ax^2+bx^3}}{149017b^4}+\frac{112a^5x^2\sqrt[4]{ax^2+bx^3}}{149017b^3}-\frac{8a^4x^3\sqrt[4]{ax^2+bx^3}}{13547b^2}+\frac{20a^3x^4\sqrt[4]{ax^2+bx^3}}{40641b}+\frac{20a^2x^5\sqrt[4]{ax^2+bx^3}}{2139}+\frac{4}{93}ax^3(ax^2+bx^3)^{5/4}+\frac{4}{31}x(ax^2+bx^3)^{9/4}-\frac{640a^{17/2}x^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{149017b^{11/2}(ax^2+bx^3)^{3/4}}$$

output
```
320/149017*a^7*(b*x^3+a*x^2)^(1/4)/b^5-160/149017*a^6*x*(b*x^3+a*x^2)^(1/4
)/b^4+112/149017*a^5*x^2*(b*x^3+a*x^2)^(1/4)/b^3-8/13547*a^4*x^3*(b*x^3+a*
x^2)^(1/4)/b^2+20/40641*a^3*x^4*(b*x^3+a*x^2)^(1/4)/b+20/2139*a^2*x^5*(b*x
^3+a*x^2)^(1/4)+4/93*a*x^3*(b*x^3+a*x^2)^(5/4)+4/31*x*(b*x^3+a*x^2)^(9/4)-
640/149017*a^(17/2)*x^(3/2)*(1+b*x/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(
1/2)*x^(1/2)/a^(1/2)),2^(1/2))/b^(11/2)/(b*x^3+a*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.19

$$\int (ax^2+bx^3)^{9/4}\,dx=\frac{2a^2x^5\sqrt[4]{x^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{9}{4},\frac{11}{2},\frac{13}{2},-\frac{bx}{a}\right)}{11\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(9/4),x]
```

output
```
(2*a^2*x^5*(x^2*(a + b*x))^(1/4)*Hypergeometric2F1[-9/4, 11/2, 13/2, -((b*
x)/a)])/(11*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {1910, 1927, 1927, 1930, 1930, 1930, 1930, 1930, 1938, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^2+bx^3)^{9/4}\,dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{9}{31}a\int x^2(bx^3+ax^2)^{5/4}dx+\frac{4}{31}x(ax^2+bx^3)^{9/4}$$

$$\Big\downarrow \text{1927}$$

$$\frac{9}{31}a\left(\frac{5}{27}a\int x^4\sqrt[4]{bx^3+ax^2}dx+\frac{4}{27}x^3(ax^2+bx^3)^{5/4}\right)+\frac{4}{31}x(ax^2+bx^3)^{9/4}$$

$$\Big\downarrow \text{1927}$$

$$\frac{9}{31}a\left(\frac{5}{27}a\left(\frac{1}{23}a\int\frac{x^6}{(bx^3+ax^2)^{3/4}}dx+\frac{4}{23}x^5\sqrt[4]{ax^2+bx^3}\right)+\frac{4}{27}x^3\left(ax^2+bx^3\right)^{5/4}\right)+\frac{4}{31}x\left(ax^2+bx^3\right)^{9/4}$$

$$\Big\downarrow \; 1930$$

$$\frac{9}{31}a\left(\frac{5}{27}a\left(\frac{1}{23}a\left(\frac{4x^4\sqrt[4]{ax^2+bx^3}}{19b}-\frac{18a\int\frac{x^5}{(bx^3+ax^2)^{3/4}}dx}{19b}\right)+\frac{4}{23}x^5\sqrt[4]{ax^2+bx^3}\right)+\frac{4}{27}x^3\left(ax^2+bx^3\right)^{5/4}\right)+\frac{4}{31}x\left(ax^2+bx^3\right)^{9/4}$$

$$\Big\downarrow \; 1930$$

$$\frac{9}{31}a\left(\frac{5}{27}a\left(\frac{1}{23}a\left(\frac{4x^4\sqrt[4]{ax^2+bx^3}}{19b}-\frac{18a\left(\frac{4x^3\sqrt[4]{ax^2+bx^3}}{15b}-\frac{14a\int\frac{x^4}{\left(bx^3+ax^2\right)^{3/4}}dx}{15b}\right)}{19b}\right)+\frac{4}{23}x^5\sqrt[4]{ax^2+bx^3}\right)+\cdots\right)+\frac{4}{31}x\left(ax^2+bx^3\right)^{9/4}$$

$$\Big\downarrow \; 1930$$

$$\frac{9}{31}a\left(\frac{5}{27}a\left(\frac{1}{23}a\left(\frac{4x^4\sqrt[4]{ax^2+bx^3}}{19b}-\frac{18a\left(\frac{4x^3\sqrt[4]{ax^2+bx^3}}{15b}-\frac{14a\left(\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\int\frac{x^3}{\left(bx^3+ax^2\right)^{3/4}}dx}{11b}\right)}{15b}\right)}{19b}\right)+\cdots\right)\right)+\frac{4}{31}x\left(ax^2+bx^3\right)^{9/4}$$

$$\Big\downarrow \; 1930$$

$$\frac{9}{31}a\left(\frac{5}{27}a\left(\frac{1}{23}a\left(\frac{4x^4\sqrt[4]{ax^2+bx^3}}{19b}-\frac{18a\left(\frac{4x^3\sqrt[4]{ax^2+bx^3}}{15b}-\frac{14a\left(\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\int\frac{}{\left(bx^3+\right.}}{7}\right.}{11b}\right.}{15b}\right.}{19b}\right.\right.\right.$$

$$\frac{4}{31}x\left(ax^2+bx^3\right)^{9/4}$$

$\downarrow$ 1930

$$\frac{9}{31}a\left(\frac{5}{27}a\left(\frac{1}{23}a\left(\frac{4x^4\sqrt[4]{ax^2+bx^3}}{19b}-\frac{18a\left(\frac{4x^3\sqrt[4]{ax^2+bx^3}}{15b}-\frac{14a\left(\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(4\sqrt[4]{a\cdots\right.}{\cdots}\right.}{\cdots}\right.}{15b}\right.}{19b}\right.\right.\right.$$

$$\frac{4}{31}x\left(ax^2+bx^3\right)^{9/4}$$

$\downarrow$ 1938

$$\frac{9}{31}a\left(\frac{5}{27}a\left(\frac{1}{23}a\left(\frac{4x^4\sqrt[4]{ax^2+bx^3}}{19b}-\frac{18a\left(\frac{4x^3\sqrt[4]{ax^2+bx^3}}{15b}-\frac{14a\left(\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{a}}{}\right.}{}\right.}{}\right)}{15b}\right)}{19b}\right.\right.\right.$$

$$\frac{4}{31}x\left(ax^2+bx^3\right)^{9/4}$$

$\downarrow$ 73

$$\frac{9}{31}a\left(\frac{5}{27}a\left(\frac{1}{23}a\left(\frac{4x^4\sqrt[4]{ax^2+bx^3}}{19b}-\frac{18a\left(\frac{4x^3\sqrt[4]{ax^2+bx^3}}{15b}-\frac{14a\left(\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(4\sqrt[4]{a\ldots}\right.}{\ldots}\right.}{\ldots}\right.}{15\ldots}\right.}{19b}\right.\right.\right.$$

$$\frac{4}{31}x\left(ax^2+bx^3\right)^{9/4}$$

↓ 765

$$\frac{9}{31}a\left(\frac{5}{27}a\left(\frac{1}{23}a\left(\frac{4x^4\sqrt[4]{ax^2+bx^3}}{19b}-\frac{18a\left(\frac{4x^3\sqrt[4]{ax^2+bx^3}}{15b}-\frac{14a\left(\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{a}\cdots}{}\right.}{}\right.}{}\right.}{19b}\right.\right.\right.$$

$$\frac{4}{31}x\left(ax^2+bx^3\right)^{9/4}$$

↓ 762

$$\frac{9}{31}a\left(\frac{5}{27}a\left(\frac{1}{23}a\left(\frac{4x^4\sqrt[4]{ax^2+bx^3}}{19b}-\frac{18a\left(\frac{4x^3\sqrt[4]{ax^2+bx^3}}{15b}-\frac{14a\left(\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{a\ldots}}{}\right.}{}\right.}{}\right.}{}\right.}{19b}\right.\right.\right.$$

input
```
Int[(a*x^2 + b*x^3)^(9/4),x]
```

output
```
(4*x*(a*x^2 + b*x^3)^(9/4))/31 + (9*a*((4*x^3*(a*x^2 + b*x^3)^(5/4))/27 +
(5*a*((4*x^5*(a*x^2 + b*x^3)^(1/4))/23 + (a*((4*x^4*(a*x^2 + b*x^3)^(1/4))
/(19*b) - (18*a*((4*x^3*(a*x^2 + b*x^3)^(1/4))/(15*b) - (14*a*((4*x^2*(a*x
^2 + b*x^3)^(1/4))/(11*b) - (10*a*((4*x*(a*x^2 + b*x^3)^(1/4))/(7*b) - (6*
a*((4*(a*x^2 + b*x^3)^(1/4))/(3*b) - (8*a^(5/4)*x^(3/2)*(a + b*x)^(3/4)*Sq
rt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/(3*b^2
*(a*x^2 + b*x^3)^(3/4)*Sqrt[-(a/b) + (a + b*x)/b])))/(7*b)))/(11*b)))/(15*
b)))/(19*b)))/23))/27))/31
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \left(b\,x^{3}+a\,x^{2}\right)^{\frac{9}{4}}dx$$

input
```
int((b*x^3+a*x^2)^(9/4),x)
```

output
```
int((b*x^3+a*x^2)^(9/4),x)
```

Fricas [F]

$$\int \left(ax^2+bx^3\right)^{9/4} dx = \int \left(bx^3+ax^2\right)^{\frac{9}{4}} dx$$

input
```
integrate((b*x^3+a*x^2)^(9/4),x, algorithm="fricas")
```

output
```
integral((b^2*x^6 + 2*a*b*x^5 + a^2*x^4)*(b*x^3 + a*x^2)^(1/4), x)
```

Sympy [F]

$$\int \left(ax^2+bx^3\right)^{9/4} dx = \int \left(ax^2+bx^3\right)^{\frac{9}{4}} dx$$

input
```
integrate((b*x**3+a*x**2)**(9/4),x)
```

output
```
Integral((a*x**2 + b*x**3)**(9/4), x)
```

Maxima [F]

$$\int \left(ax^2+bx^3\right)^{9/4} dx = \int \left(bx^3+ax^2\right)^{\frac{9}{4}} dx$$

input
```
integrate((b*x^3+a*x^2)^(9/4),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(9/4), x)
```

Giac [F]

$$\int \left(ax^2+bx^3\right)^{9/4}\,dx=\int \left(bx^3+ax^2\right)^{\frac{9}{4}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(9/4),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^(9/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.14

$$\int \left(ax^2+bx^3\right)^{9/4}\,dx=\frac{2\,x\,\left(b\,x^3+a\,x^2\right)^{9/4}\,{}_2\mathrm{F}_1\left(-\frac{9}{4},\frac{11}{2};\ \frac{13}{2};\ -\frac{b\,x}{a}\right)}{11\,\left(\frac{b\,x}{a}+1\right)^{9/4}}$$

input
```
int((a*x^2 + b*x^3)^(9/4),x)
```

output
```
(2*x*(a*x^2 + b*x^3)^(9/4)*hypergeom([-9/4, 11/2], 13/2, -(b*x)/a))/(11*((
b*x)/a + 1)^(9/4))
```

Reduce [F]

$$\int \left(ax^2+bx^3\right)^{9/4}\,dx=\frac{\frac{320\sqrt{x}\,(bx+a)^{\frac{1}{4}}a^7}{149017}-\frac{160\sqrt{x}\,(bx+a)^{\frac{1}{4}}a^6bx}{149017}+\frac{112\sqrt{x}\,(bx+a)^{\frac{1}{4}}a^5b^2x^2}{149017}-\frac{8\sqrt{x}\,(bx+a)^{\frac{1}{4}}a^4b^3x^3}{13547}+\frac{20\sqrt{x}\,(bx+a)^{\frac{1}{4}}a^3b^4x^4}{40641}}{b^5}$$

input
```
int((b*x^3+a*x^2)^(9/4),x)
```

output
```
(4*(240*sqrt(x)*(a + b*x)**(1/4)*a**7 - 120*sqrt(x)*(a + b*x)**(1/4)*a**6*
b*x + 84*sqrt(x)*(a + b*x)**(1/4)*a**5*b**2*x**2 - 66*sqrt(x)*(a + b*x)**(
1/4)*a**4*b**3*x**3 + 55*sqrt(x)*(a + b*x)**(1/4)*a**3*b**4*x**4 + 20273*s
qrt(x)*(a + b*x)**(1/4)*a**2*b**5*x**5 + 33649*sqrt(x)*(a + b*x)**(1/4)*a*
b**6*x**6 + 14421*sqrt(x)*(a + b*x)**(1/4)*b**7*x**7 - 120*int((sqrt(x)*(a
 + b*x)**(1/4))/(a*x + b*x**2),x)*a**8))/(447051*b**5)
```

3.115 $\int (ax^2+bx^3)^{5/4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 192

$$\int (ax^2+bx^3)^{5/4}\,dx = \frac{80a^4\sqrt[4]{ax^2+bx^3}}{4389b^3} - \frac{40a^3x\sqrt[4]{ax^2+bx^3}}{4389b^2} + \frac{4a^2x^2\sqrt[4]{ax^2+bx^3}}{627b} + \frac{4}{57}ax^3\sqrt[4]{ax^2+bx^3} + \frac{4}{19}x(ax^2+bx^3)^{5/4} - \frac{160a^{11/2}x^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{4389b^{7/2}(ax^2+bx^3)^{3/4}}$$

output

```
80/4389*a^4*(b*x^3+a*x^2)^(1/4)/b^3-40/4389*a^3*x*(b*x^3+a*x^2)^(1/4)/b^2+
4/627*a^2*x^2*(b*x^3+a*x^2)^(1/4)/b+4/57*a*x^3*(b*x^3+a*x^2)^(1/4)+4/19*x*
(b*x^3+a*x^2)^(5/4)-160/4389*a^(11/2)*x^(3/2)*(1+b*x/a)^(3/4)*InverseJacob
iAM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/b^(7/2)/(b*x^3+a*x^2)^(3/
4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.26

$$\int \left(ax^2+bx^3\right)^{5/4} dx = \frac{2ax^3\sqrt[4]{x^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{5}{4},\frac{7}{2},\frac{9}{2},-\frac{bx}{a}\right)}{7\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(5/4),x]
```

output
```
(2*a*x^3*(x^2*(a + b*x))^(1/4)*Hypergeometric2F1[-5/4, 7/2, 9/2, -((b*x)/a
)])/(7*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1910, 1927, 1930, 1930, 1930, 1938, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^{5/4} dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{5}{19}a\int x^2\sqrt[4]{bx^3+ax^2}dx+\frac{4}{19}x\left(ax^2+bx^3\right)^{5/4}$$

$$\Big\downarrow \text{1927}$$

$$\frac{5}{19}a\left(\frac{1}{15}a\int \frac{x^4}{\left(bx^3+ax^2\right)^{3/4}}dx+\frac{4}{15}x^3\sqrt[4]{ax^2+bx^3}\right)+\frac{4}{19}x\left(ax^2+bx^3\right)^{5/4}$$

$$\Big\downarrow \text{1930}$$

$$\frac{5}{19}a\left(\frac{1}{15}a\left(\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\int\frac{x^3}{(bx^3+ax^2)^{3/4}}dx}{11b}\right)+\frac{4}{15}x^3\sqrt[4]{ax^2+bx^3}\right)+\frac{4}{19}x\left(ax^2+bx^3\right)^{5/4}$$

$\Big\downarrow$ 1930

$$\frac{5}{19}a\left(\frac{1}{15}a\left(\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\int\frac{x^2}{\left(bx^3+ax^2\right)^{3/4}}dx}{7b}\right)}{11b}\right)+\frac{4}{15}x^3\sqrt[4]{ax^2+bx^3}\right)+\frac{4}{19}x\left(ax^2+bx^3\right)^{5/4}$$

$\Big\downarrow$ 1930

$$\frac{5}{19}a\left(\frac{1}{15}a\left(\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{2a\int\frac{x}{\left(bx^3+ax^2\right)^{3/4}}dx}{3b}\right)}{7b}\right)}{11b}\right)+\frac{4}{15}x^3\sqrt[4]{ax^2\right.$$

$$\frac{4}{19}x\left(ax^2+bx^3\right)^{5/4}$$

$\Big\downarrow$ 1938

$$\frac{5}{19}a\left(\frac{1}{15}a\left(\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{2ax^{3/2}(a+bx)^{3/4}\int\frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{3b\left(ax^2+bx^3\right)^{3/4}}\right)}{7b}\right)}{11b}\right)\right)+\frac{4}{15}$$

$$\frac{4}{19}x\left(ax^2+bx^3\right)^{5/4}$$

$$\Big\downarrow \; 73$$

$$\frac{5}{19}a\left(\frac{1}{15}a\left(\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{8ax^{3/2}(a+bx)^{3/4}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{3b^2\left(ax^2+bx^3\right)^{3/4}}\right)}{7b}\right)}{11b}\right)\right.$$

$$\frac{4}{19}x\left(ax^2+bx^3\right)^{5/4}$$

$$\Big\downarrow \; 765$$

$$\frac{5}{19}a\left(\frac{1}{15}a\left(\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{8ax^{3/2}(a+bx)^{3/4}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+b}}{3b^2\left(ax^2+bx^3\right)^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right.}{7b}\right.}{11b}\right.\right.$$

$$\frac{4}{19}x\left(ax^2+bx^3\right)^{5/4}$$

$$\Big\downarrow \; 762$$

$$\frac{5}{19}a\left(\frac{1}{15}a\left(\frac{4x^2\sqrt[4]{ax^2+bx^3}}{11b}-\frac{10a\left(\frac{4x\sqrt[4]{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{8a^{5/4}x^{3/2}(a+bx)^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\sqrt[4]{\cdots}\right.\right.}{3b^2\left(ax^2+bx^3\right)^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\cdots\right.}{7b}\cdots\right.}{11b}\cdots\right.\right.$$

$$\frac{4}{19}x\left(ax^2+bx^3\right)^{5/4}$$

input
```
Int[(a*x^2 + b*x^3)^(5/4),x]
```

output
```
(4*x*(a*x^2 + b*x^3)^(5/4))/19 + (5*a*((4*x^3*(a*x^2 + b*x^3)^(1/4))/15 +
(a*((4*x^2*(a*x^2 + b*x^3)^(1/4))/(11*b) - (10*a*((4*x*(a*x^2 + b*x^3)^(1/
4))/(7*b) - (6*a*((4*(a*x^2 + b*x^3)^(1/4))/(3*b) - (8*a^(5/4)*x^(3/2)*(a
+ b*x)^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4
)], -1])/(3*b^2*(a*x^2 + b*x^3)^(3/4)*Sqrt[-(a/b) + (a + b*x)/b])))/(7*b))
)/(11*b)))/15))/19
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{4}}d x$$

input
```
int((b*x^3+a*x^2)^(5/4),x)
```

output
```
int((b*x^3+a*x^2)^(5/4),x)
```

Fricas [F]

$$\int \left(ax^2+bx^3\right)^{5/4}\,dx=\int \left(bx^{3}+ax^{2}\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^(5/4), x)
```

Sympy [F]

$$\int \left(ax^2+bx^3\right)^{5/4}\,dx=\int \left(ax^{2}+bx^{3}\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(5/4),x)
```

output
```
Integral((a*x**2 + b*x**3)**(5/4), x)
```

Maxima [F]

$$\int \left(ax^2+bx^3\right)^{5/4}\,dx=\int \left(bx^3+ax^2\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/4), x)
```

Giac [F]

$$\int \left(ax^2+bx^3\right)^{5/4}\,dx=\int \left(bx^3+ax^2\right)^{\frac{5}{4}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^(5/4), x)
```

Mupad [B] (verification not implemented)

Time = 10.10 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.20

$$\int \left(ax^2+bx^3\right)^{5/4}\,dx=\frac{2\,x\,\left(b\,x^3+a\,x^2\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{7}{2};\ \frac{9}{2};\ -\frac{b\,x}{a}\right)}{7\,\left(\frac{b\,x}{a}+1\right)^{5/4}}$$

input
```
int((a*x^2 + b*x^3)^(5/4),x)
```

output
```
(2*x*(a*x^2 + b*x^3)^(5/4)*hypergeom([-5/4, 7/2], 9/2, -(b*x)/a))/(7*((b*x
)/a + 1)^(5/4))
```

Reduce [F]

$$\int \left(ax^2+bx^3\right)^{5/4} dx = \frac{\frac{80\sqrt{x}\,(bx+a)^{\frac{1}{4}}a^4}{4389} - \frac{40\sqrt{x}\,(bx+a)^{\frac{1}{4}}a^3bx}{4389} + \frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}a^2b^2x^2}{627} + \frac{16\sqrt{x}\,(bx+a)^{\frac{1}{4}}a\,b^3x^3}{57} + \frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}b^4x^4}{19} - \frac{40\left(\right.}{}}{b^3}$$

input
```
int((b*x^3+a*x^2)^(5/4),x)
```

output
```
(4*(20*sqrt(x)*(a + b*x)**(1/4)*a**4 - 10*sqrt(x)*(a + b*x)**(1/4)*a**3*b*
x + 7*sqrt(x)*(a + b*x)**(1/4)*a**2*b**2*x**2 + 308*sqrt(x)*(a + b*x)**(1/
4)*a*b**3*x**3 + 231*sqrt(x)*(a + b*x)**(1/4)*b**4*x**4 - 10*int((sqrt(x)*
(a + b*x)**(1/4))/(a*x + b*x**2),x)*a**5))/(4389*b**3)
```

3.116 $\int \sqrt[4]{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 113

$$\int \sqrt[4]{ax^2+bx^3}\,dx=\frac{4a\sqrt[4]{ax^2+bx^3}}{21b}+\frac{4}{7}x\sqrt[4]{ax^2+bx^3}-\frac{8a^{5/2}x^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{21b^{3/2}\left(ax^2+bx^3\right)^{3/4}}$$

output
```
4/21*a*(b*x^3+a*x^2)^(1/4)/b+4/7*x*(b*x^3+a*x^2)^(1/4)-8/21*a^(5/2)*x^(3/2
)*(1+b*x/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1
/2))/b^(3/2)/(b*x^3+a*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.42

$$\int \sqrt[4]{ax^2+bx^3}\,dx=\frac{2x\sqrt[4]{x^2(a+bx)}\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{3}{2},\frac{5}{2},-\frac{bx}{a}\right)}{3\sqrt[4]{1+\frac{bx}{a}}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(1/4),x]
```

output
```
(2*x*(x^2*(a + b*x))^(1/4)*Hypergeometric2F1[-1/4, 3/2, 5/2, -((b*x)/a)])/
(3*(1 + (b*x)/a)^(1/4))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.27, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1910, 1930, 1938, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt[4]{ax^2+bx^3}\,dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{1}{7}a\int \frac{x^2}{(bx^3+ax^2)^{3/4}}dx+\frac{4}{7}x\sqrt[4]{ax^2+bx^3}$$

$$\Big\downarrow \text{1930}$$

$$\frac{1}{7}a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{(bx^3+ax^2)^{3/4}}dx}{3b}\right)+\frac{4}{7}x\sqrt[4]{ax^2+bx^3}$$

$$\Big\downarrow \text{1938}$$

$$\frac{1}{7}a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{2ax^{3/2}(a+bx)^{3/4}\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{3b\left(ax^2+bx^3\right)^{3/4}}\right)+\frac{4}{7}x\sqrt[4]{ax^2+bx^3}$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{7}a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{8ax^{3/2}(a+bx)^{3/4}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{3b^2\left(ax^2+bx^3\right)^{3/4}}\right)+\frac{4}{7}x\sqrt[4]{ax^2+bx^3}$$

$$\Big\downarrow \text{765}$$

$$\frac{1}{7}a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{8ax^{3/2}(a+bx)^{3/4}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{3b^2\left(ax^2+bx^3\right)^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)+\frac{4}{7}x\sqrt[4]{ax^2+bx^3}$$

$$\Big\downarrow \; 762$$

$$\frac{1}{7}a\left(\frac{4\sqrt[4]{ax^2+bx^3}}{3b}-\frac{8a^{5/4}x^{3/2}(a+bx)^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{3b^2\left(ax^2+bx^3\right)^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)+\frac{4}{7}x\sqrt[4]{ax^2+bx^3}$$

input
```
Int[(a*x^2 + b*x^3)^(1/4),x]
```

output
```
(4*x*(a*x^2 + b*x^3)^(1/4))/7 + (a*((4*(a*x^2 + b*x^3)^(1/4))/(3*b) - (8*a
^(5/4)*x^(3/2)*(a + b*x)^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a +
 b*x)^(1/4)/a^(1/4)], -1])/(3*b^2*(a*x^2 + b*x^3)^(3/4)*Sqrt[-(a/b) + (a +
 b*x)/b])))/7
```

Defintions of rubi rules used

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \left(b\,x^{3}+a\,x^{2}\right)^{\frac{1}{4}}d x$$

input
```
int((b*x^3+a*x^2)^(1/4),x)
```

output
```
int((b*x^3+a*x^2)^(1/4),x)
```

Fricas [F]

$$\int \sqrt[4]{ax^2+bx^3}\,dx=\int \left(bx^{3}+ax^{2}\right)^{\frac{1}{4}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^(1/4), x)
```

Sympy [F]

$$\int \sqrt[4]{ax^2 + bx^3}\, dx = \int \sqrt[4]{ax^2 + bx^3}\, dx$$

input
```
integrate((b*x**3+a*x**2)**(1/4),x)
```

output
```
Integral((a*x**2 + b*x**3)**(1/4), x)
```

Maxima [F]

$$\int \sqrt[4]{ax^2 + bx^3}\, dx = \int (bx^3 + ax^2)^{\frac{1}{4}}\, dx$$

input
```
integrate((b*x^3+a*x^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(1/4), x)
```

Giac [F]

$$\int \sqrt[4]{ax^2 + bx^3}\, dx = \int (bx^3 + ax^2)^{\frac{1}{4}}\, dx$$

input
```
integrate((b*x^3+a*x^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^(1/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.90 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.34

$$\int \sqrt[4]{ax^2+bx^3}\,dx=\frac{2\,x\,{\left(b\,x^3+a\,x^2\right)}^{1/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{3}{2};\ \frac{5}{2};\ -\frac{b\,x}{a}\right)}{3\,{\left(\frac{b\,x}{a}+1\right)}^{1/4}}$$

input
```
int((a*x^2 + b*x^3)^(1/4),x)
```

output
```
(2*x*(a*x^2 + b*x^3)^(1/4)*hypergeom([-1/4, 3/2], 5/2, -(b*x)/a))/(3*((b*x
)/a + 1)^(1/4))
```

Reduce [F]

$$\int \sqrt[4]{ax^2+bx^3}\,dx=\frac{\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}a}{21}+\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}bx}{7}-\frac{2\left(\int\frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{b\,x^2+ax}dx\right)a^2}{21}}{b}$$

input
```
int((b*x^3+a*x^2)^(1/4),x)
```

output
```
(2*(2*sqrt(x)*(a + b*x)**(1/4)*a + 6*sqrt(x)*(a + b*x)**(1/4)*b*x - int((s
qrt(x)*(a + b*x)**(1/4))/(a*x + b*x**2),x)*a**2))/(21*b)
```

3.117 $\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 91

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = -\frac{2\sqrt[4]{ax^2+bx^3}}{ax} - \frac{2\sqrt{b}x^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{\sqrt{a}\,(ax^2+bx^3)^{3/4}}$$

output
```
-2*(b*x^3+a*x^2)^(1/4)/a/x-2*b^(1/2)*x^(3/2)*(1+b*x/a)^(3/4)*InverseJacobi
AM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/a^(1/2)/(b*x^3+a*x^2)^(3/4
)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.49

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = -\frac{2x\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3}{4},\frac{1}{2},-\frac{bx}{a}\right)}{(x^2(a+bx))^{3/4}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-3/4),x]
```

output
```
(-2*x*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-1/2, 3/4, 1/2, -((b*x)/a)])/(
x^2*(a + b*x))^(3/4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.22, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1917, 61, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax^2+bx^3\right)^{3/4}}dx$$

$$\Big\downarrow \text{ 1917}$$

$$\frac{x^{3/2}(a+bx)^{3/4}\int \frac{1}{x^{3/2}(a+bx)^{3/4}}dx}{\left(ax^2+bx^3\right)^{3/4}}$$

$$\Big\downarrow \text{ 61}$$

$$\frac{x^{3/2}(a+bx)^{3/4}\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{2a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{\left(ax^2+bx^3\right)^{3/4}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{x^{3/2}(a+bx)^{3/4}\left(-\frac{2\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{\left(ax^2+bx^3\right)^{3/4}}$$

$$\Big\downarrow \text{ 765}$$

$$\frac{x^{3/2}(a+bx)^{3/4}\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{a\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{\left(ax^2+bx^3\right)^{3/4}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{x^{3/2}(a+bx)^{3/4}\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{a^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{(ax^2+bx^3)^{3/4}}$$

input
```
Int[(a*x^2 + b*x^3)^(-3/4),x]
```

output
```
(x^(3/2)*(a + b*x)^(3/4)*((-2*(a + b*x)^(1/4))/(a*Sqrt[x]) - (2*Sqrt[1 - (
a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/(a^(3/4)*Sqrt[
-(a/b) + (a + b*x)/b])))/(a*x^2 + b*x^3)^(3/4)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{4}}}d x$$

input
```
int(1/(b*x^3+a*x^2)^(3/4),x)
```

output
```
int(1/(b*x^3+a*x^2)^(3/4),x)
```

Fricas [F]

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{1}{\left(bx^3+ax^2\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^(-3/4), x)
```

Sympy [F]

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{1}{\left(ax^2+bx^3\right)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(3/4),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-3/4), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-3/4), x)
```

Giac [F]

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{3}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^(-3/4), x)
```

Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.42

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = -\frac{2\,x\,\left(\frac{b\,x}{a}+1\right)^{3/4}\,{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{3}{4};\ \frac{1}{2};\ -\frac{b\,x}{a}\right)}{\left(b\,x^3+a\,x^2\right)^{3/4}}$$

input
```
int(1/(a*x^2 + b*x^3)^(3/4),x)
```

output
```
-(2*x*((b*x)/a + 1)^(3/4)*hypergeom([-1/2, 3/4], 1/2, -(b*x)/a))/(a*x^2 + 
b*x^3)^(3/4)
```

Reduce [F]

$$\int \frac{1}{(ax^2+bx^3)^{3/4}}\,dx = \int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{\sqrt{bx+a}\,a\,x^2+\sqrt{bx+a}\,b\,x^3}dx$$

input
```
int(1/(b*x^3+a*x^2)^(3/4),x)
```

output
```
int((sqrt(x)*(a + b*x)**(3/4))/(sqrt(a + b*x)*a*x**2 + sqrt(a + b*x)*b*x**
3),x)
```

3.118 $\int \frac{1}{(ax^2+bx^3)^{7/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 174

$$\int \frac{1}{(ax^2+bx^3)^{7/4}}\,dx = \frac{4}{3ax(ax^2+bx^3)^{3/4}} - \frac{26\sqrt[4]{ax^2+bx^3}}{15a^2x^3} + \frac{13b\sqrt[4]{ax^2+bx^3}}{5a^3x^2} - \frac{13b^2\sqrt[4]{ax^2+bx^3}}{2a^4x} - \frac{13b^{5/2}x^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{2a^{7/2}(ax^2+bx^3)^{3/4}}$$

output

```
4/3/a/x/(b*x^3+a*x^2)^(3/4)-26/15*(b*x^3+a*x^2)^(1/4)/a^2/x^3+13/5*b*(b*x^
3+a*x^2)^(1/4)/a^3/x^2-13/2*b^2*(b*x^3+a*x^2)^(1/4)/a^4/x-13/2*b^(5/2)*x^(
3/2)*(1+b*x/a)^(3/4)*InverseJacobiAM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2
^(1/2))/a^(7/2)/(b*x^3+a*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.30

$$\int \frac{1}{(ax^2+bx^3)^{7/4}}\,dx = -\frac{2\left(1+\frac{bx}{a}\right)^{3/4}\operatorname{Hypergeometric2F1}\left(-\frac{5}{2},\frac{7}{4},-\frac{3}{2},-\frac{bx}{a}\right)}{5ax(x^2(a+bx))^{3/4}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-7/4),x]
```

output
```
(-2*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-5/2, 7/4, -3/2, -((b*x)/a)])/(5
*a*x*(x^2*(a + b*x))^(3/4))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {1912, 1931, 1931, 1917, 61, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{7/4}}dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{13\int \frac{1}{x^2(bx^3+ax^2)^{3/4}}dx}{3a}+\frac{4}{3ax(ax^2+bx^3)^{3/4}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{13\left(-\frac{9b\int \frac{1}{x\left(bx^3+ax^2\right)^{3/4}}dx}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}\right)}{3a}+\frac{4}{3ax(ax^2+bx^3)^{3/4}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{13\left(-\frac{9b\left(-\frac{5b\int \frac{1}{\left(bx^3+ax^2\right)^{3/4}}dx}{6a}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}\right)}{3a}+\frac{4}{3ax(ax^2+bx^3)^{3/4}}$$

$$\Big\downarrow \text{1917}$$

$$\frac{13\left(-\frac{9b\left(-\frac{5bx^{3/2}(a+bx)^{3/4}\int \frac{1}{x^{3/2}(a+bx)^{3/4}}dx}{6a\left(ax^2+bx^3\right)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}\right)}{3a}+\frac{4}{3ax\left(ax^2+bx^3\right)^{3/4}}$$

$\Big\downarrow$ 61

$$\frac{13\left(-\frac{9b\left(-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{b\int \frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{2a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a\left(ax^2+bx^3\right)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}\right)}{3a}+\frac{4}{3ax\left(ax^2+bx^3\right)^{3/4}}$$

$\Big\downarrow$ 73

$$\frac{13\left(-\frac{9b\left(-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{2\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a\left(ax^2+bx^3\right)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}\right)}{3a}+\frac{4}{3ax\left(ax^2+bx^3\right)^{3/4}}$$

$\Big\downarrow$ 765

$$\frac{13\left(-\frac{9b\left(-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{a\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a\left(ax^2+bx^3\right)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}\right)}{3a}+\frac{4}{3ax\left(ax^2+bx^3\right)^{3/4}}$$

$$\Big\downarrow \text{762}$$

$$\frac{13\left(-\frac{9b\left(-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{a^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a\left(ax^2+bx^3\right)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}\right)}{3a}+\frac{4}{3ax\left(ax^2+bx^3\right)^{3/4}}$$

input
```
Int[(a*x^2 + b*x^3)^(-7/4),x]
```

output
```
4/(3*a*x*(a*x^2 + b*x^3)^(3/4)) + (13*((-2*(a*x^2 + b*x^3)^(1/4))/(5*a*x^3
) - (9*b*((-2*(a*x^2 + b*x^3)^(1/4))/(3*a*x^2) - (5*b*x^(3/2)*(a + b*x)^(3
/4)*((-2*(a + b*x)^(1/4))/(a*Sqrt[x]) - (2*Sqrt[1 - (a + b*x)/a]*EllipticF
[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/(a^(3/4)*Sqrt[-(a/b) + (a + b*x)/b]
)))/(6*a*(a*x^2 + b*x^3)^(3/4))))/(10*a)))/(3*a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{7}{4}}}d x$$

input
```
int(1/(b*x^3+a*x^2)^(7/4),x)
```

output
```
int(1/(b*x^3+a*x^2)^(7/4),x)
```

Fricas [F]

$$\int \frac{1}{(ax^2+bx^3)^{7/4}}\,dx=\int \frac{1}{\left(bx^{3}+ax^{2}\right)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^(1/4)/(b^2*x^6 + 2*a*b*x^5 + a^2*x^4), x)
```

Sympy [F]

$$\int \frac{1}{(ax^2+bx^3)^{7/4}}\,dx = \int \frac{1}{(ax^2+bx^3)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(7/4),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-7/4), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2+bx^3)^{7/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(7/4),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-7/4), x)
```

Giac [F]

$$\int \frac{1}{(ax^2+bx^3)^{7/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{7}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(7/4),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^(-7/4), x)
```

Mupad [B] (verification not implemented)

Time = 10.55 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.22

$$\int \frac{1}{\left(ax^2+bx^3\right)^{7/4}}\,dx=-\frac{2\,x\,{\left(\frac{b\,x}{a}+1\right)}^{7/4}\,{}_2\mathrm{F}_1\left(-\frac{5}{2},\frac{7}{4};\ -\frac{3}{2};\ -\frac{b\,x}{a}\right)}{5\,{\left(b\,x^3+a\,x^2\right)}^{7/4}}$$

input
```
int(1/(a*x^2 + b*x^3)^(7/4),x)
```

output
```
-(2*x*((b*x)/a + 1)^(7/4)*hypergeom([-5/2, 7/4], -3/2, -(b*x)/a))/(5*(a*x^
2 + b*x^3)^(7/4))
```

Reduce [F]

$$\int \frac{1}{\left(ax^2+bx^3\right)^{7/4}}\,dx=\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{\sqrt{bx+a}\,a^2x^4+2\sqrt{bx+a}\,ab\,x^5+\sqrt{bx+a}\,b^2x^6}dx$$

input
```
int(1/(b*x^3+a*x^2)^(7/4),x)
```

output
```
int((sqrt(x)*(a + b*x)**(3/4))/(sqrt(a + b*x)*a**2*x**4 + 2*sqrt(a + b*x)*
a*b*x**5 + sqrt(a + b*x)*b**2*x**6),x)
```

3.119 $\int \frac{1}{(ax^2+bx^3)^{11/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 255

$$\int \frac{1}{(ax^2+bx^3)^{11/4}}\,dx = \frac{4}{7ax(ax^2+bx^3)^{7/4}} + \frac{100}{21a^2x^3(ax^2+bx^3)^{3/4}} - \frac{50\sqrt[4]{ax^2+bx^3}}{9a^3x^5} + \frac{425b\sqrt[4]{ax^2+bx^3}}{63a^4x^4} - \frac{1105b^2\sqrt[4]{ax^2+bx^3}}{126a^5x^3} + \frac{1105b^3\sqrt[4]{ax^2+bx^3}}{84a^6x^2} - \frac{5525b^4\sqrt[4]{ax^2+bx^3}}{168a^7x} - \frac{5525b^{9/2}x^{3/2}\left(1+\frac{bx}{a}\right)^{3/4}\text{EllipticF}\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right),2\right)}{168a^{13/2}(ax^2+bx^3)^{3/4}}$$

output

```
4/7/a/x/(b*x^3+a*x^2)^(7/4)+100/21/a^2/x^3/(b*x^3+a*x^2)^(3/4)-50/9*(b*x^3
+a*x^2)^(1/4)/a^3/x^5+425/63*b*(b*x^3+a*x^2)^(1/4)/a^4/x^4-1105/126*b^2*(b
*x^3+a*x^2)^(1/4)/a^5/x^3+1105/84*b^3*(b*x^3+a*x^2)^(1/4)/a^6/x^2-5525/168
*b^4*(b*x^3+a*x^2)^(1/4)/a^7/x-5525/168*b^(9/2)*x^(3/2)*(1+b*x/a)^(3/4)*In
verseJacobiAM(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2)),2^(1/2))/a^(13/2)/(b*x^3
+a*x^2)^(3/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.20

$$\int \frac{1}{(ax^2+bx^3)^{11/4}}\,dx = -\frac{2\left(1+\frac{bx}{a}\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{9}{2},\frac{11}{4},-\frac{7}{2},-\frac{bx}{a}\right)}{9a^2x^3\left(x^2(a+bx)\right)^{3/4}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-11/4),x]
```

output
```
(-2*(1 + (b*x)/a)^(3/4)*Hypergeometric2F1[-9/2, 11/4, -7/2, -((b*x)/a)])/(
9*a^2*x^3*(x^2*(a + b*x))^(3/4))
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 310, normalized size of antiderivative = 1.22, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {1912, 1929, 1931, 1931, 1931, 1931, 1917, 61, 73, 765, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{11/4}}\,dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{25\int \frac{1}{x^2(bx^3+ax^2)^{7/4}}dx}{7a}+\frac{4}{7ax\left(ax^2+bx^3\right)^{7/4}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{25\left(\frac{7\int \frac{1}{x^4\left(bx^3+ax^2\right)^{3/4}}dx}{a}+\frac{4}{3ax^3(ax^2+bx^3)^{3/4}}\right)}{7a}+\frac{4}{7ax\left(ax^2+bx^3\right)^{7/4}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{25\left(\frac{7\left(-\frac{17b\int \frac{1}{x^3\left(bx^3+ax^2\right)^{3/4}}dx}{18a}-\frac{2\sqrt[4]{ax^2+bx^3}}{9ax^5}\right)}{a}+\frac{4}{3ax^3(ax^2+bx^3)^{3/4}}\right)}{7a}+\frac{4}{7ax\left(ax^2+bx^3\right)^{7/4}}$$

$$\Big\downarrow \ 1931$$

$$\frac{25\left(\frac{7\left(-\frac{17b\left(-\frac{13b\int \frac{1}{x^2\left(bx^3+ax^2\right)^{3/4}}dx}{14a}-\frac{2\sqrt[4]{ax^2+bx^3}}{7ax^4}\right)}{18a}-\frac{2\sqrt[4]{ax^2+bx^3}}{9ax^5}\right)}{a}+\frac{4}{3ax^3(ax^2+bx^3)^{3/4}}\right)}{7a}+\frac{4}{7ax\left(ax^2+bx^3\right)^{7/4}}$$

$$\Big\downarrow \ 1931$$

$$\frac{25\left(\frac{7\left(-\frac{17b\left(-\frac{13b\left(-\frac{9b\int\frac{1}{x\left(bx^3+ax^2\right)^{3/4}}dx}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}\right)}{14a}-\frac{2\sqrt[4]{ax^2+bx^3}}{7ax^4}\right)}{18a}-\frac{2\sqrt[4]{ax^2+bx^3}}{9ax^5}\right)}{a}+\frac{4}{3ax^3(ax^2+bx^3)^{3/4}}\right)}{7a}+\frac{4}{7ax\left(ax^2+bx^3\right)^{7/4}}$$

$$\Big\downarrow \text{ 1931}$$

$$\frac{25\left(\frac{7\left(-\frac{17b\left(-\frac{13b\left(-\frac{9b\left(-\frac{5b\int\frac{1}{\left(bx^3+ax^2\right)^{3/4}}dx}{6a}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}\right)}{14a}-\frac{2\sqrt[4]{ax^2+bx^3}}{7ax^4}\right)}{18a}-\frac{2\sqrt[4]{ax^2+bx^3}}{9ax^5}\right)}{a}+\frac{}{3a\cdots}\right.}{7a}$$

$$\frac{4}{7ax\left(ax^2+bx^3\right)^{7/4}}$$

$$\Big\downarrow \; 1917$$

$$\frac{25\left(\frac{7\left(-\frac{17b\left(-\frac{13b\left(-\frac{9b\left(-\frac{5bx^{3/2}(a+bx)^{3/4}\int\frac{1}{x^{3/2}(a+bx)^{3/4}}dx}{6a\left(ax^2+bx^3\right)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}\right)}{14a}-\frac{2\sqrt[4]{ax^2+bx^3}}{7ax^4}\right)}{18a}-\frac{2\sqrt[4]{ax^2+\cdots}}{9ax^5}\cdots}{a}\cdots}{7a}$$

$$\frac{4}{7ax\left(ax^2+bx^3\right)^{7/4}}$$

$$\Big\downarrow \; 61$$

$$25\left(\frac{7\left(-\frac{17b\left(-\frac{13b\left(-\frac{9b\left(-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{b\int\frac{1}{\sqrt{x}(a+bx)^{3/4}}dx}{2a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a\left(ax^2+bx^3\right)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}\right)}{14a}-\frac{2\sqrt[4]{ax^2+bx^3 \cdots}}{7ax^4}\cdots}{18a}\cdots}{a}\cdots$$

$\downarrow$ 73

$$25\left(\frac{\left(7\left(-\frac{17b\left(-\frac{13b\left(-\frac{9b\left(-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{2\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a\left(ax^2+bx^3\right)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}\right)}{14a}-\frac{2\sqrt[4]{a\cdots}}{\cdots}\cdots\right)}{18a}\cdots\right)\cdots\right)}{a}\cdots\right.$$

↓ 765

$$25\left(\frac{7\left(-\frac{17b\left(-\frac{13b\left(-\frac{9b\left(-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{a\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a\left(ax^2+bx^3\right)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^2+bx^3}}{5ax^3}\right)}{14a}\right)}{18a}\right)}{a}$$

↓ 762

$$7\left(-\frac{17b\left(-\frac{13b\left(-\frac{9b\left(-\frac{5bx^{3/2}(a+bx)^{3/4}\left(-\frac{2\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{a^{3/4}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{2\sqrt[4]{a+bx}}{a\sqrt{x}}\right)}{6a\left(ax^2+bx^3\right)^{3/4}}-\frac{2\sqrt[4]{ax^2+bx^3}}{3ax^2}\right)}{10a}-\frac{2\sqrt[4]{ax^{2}\cdots}}{5a\cdots}\right)}{14a}\cdots\right)}{18a}\cdots\right)$$

input
```
Int[(a*x^2 + b*x^3)^(-11/4),x]
```

output
```
4/(7*a*x*(a*x^2 + b*x^3)^(7/4)) + (25*(4/(3*a*x^3*(a*x^2 + b*x^3)^(3/4)) +
 (7*((-2*(a*x^2 + b*x^3)^(1/4))/(9*a*x^5) - (17*b*((-2*(a*x^2 + b*x^3)^(1/
4))/(7*a*x^4) - (13*b*((-2*(a*x^2 + b*x^3)^(1/4))/(5*a*x^3) - (9*b*((-2*(a
*x^2 + b*x^3)^(1/4))/(3*a*x^2) - (5*b*x^(3/2)*(a + b*x)^(3/4)*((-2*(a + b*
x)^(1/4))/(a*Sqrt[x]) - (2*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x
)^(1/4)/a^(1/4)], -1])/(a^(3/4)*Sqrt[-(a/b) + (a + b*x)/b])))/(6*a*(a*x^2
+ b*x^3)^(3/4))))/(10*a)))/(14*a)))/(18*a)))/a))/(7*a)
```

Defintions of rubi rules used

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^{3}+a\,x^{2})^{\frac{11}{4}}}dx$$

input
```
int(1/(b*x^3+a*x^2)^(11/4),x)
```

output
```
int(1/(b*x^3+a*x^2)^(11/4),x)
```

Fricas [F]

$$\int \frac{1}{(ax^2+bx^3)^{11/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(11/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^(1/4)/(b^3*x^9 + 3*a*b^2*x^8 + 3*a^2*b*x^7 + a^3*
x^6), x)
```

Sympy [F]

$$\int \frac{1}{(ax^2+bx^3)^{11/4}}\,dx = \int \frac{1}{(ax^2+bx^3)^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(11/4),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-11/4), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2+bx^3)^{11/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(11/4),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-11/4), x)
```

Giac [F]

$$\int \frac{1}{(ax^2+bx^3)^{11/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{11}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(11/4),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^(-11/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.82 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.15

$$\int \frac{1}{(ax^2+bx^3)^{11/4}}\,dx = -\frac{2\,x\,\left(\frac{b\,x}{a}+1\right)^{11/4}\,{}_2\mathrm{F}_1\left(-\frac{9}{2},\frac{11}{4};\ -\frac{7}{2};\ -\frac{b\,x}{a}\right)}{9\,(b\,x^3+a\,x^2)^{11/4}}$$

input
```
int(1/(a*x^2 + b*x^3)^(11/4),x)
```

output
```
-(2*x*((b*x)/a + 1)^(11/4)*hypergeom([-9/2, 11/4], -7/2, -(b*x)/a))/(9*(a*
x^2 + b*x^3)^(11/4))
```

Reduce [F]

$$\int \frac{1}{(ax^2+bx^3)^{11/4}}\,dx = \int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{\sqrt{bx+a}\,a^3x^6+3\sqrt{bx+a}\,a^2b\,x^7+3\sqrt{bx+a}\,a\,b^2x^8+\sqrt{bx+a}\,b^3x^9}dx$$

input
```
int(1/(b*x^3+a*x^2)^(11/4),x)
```

output
```
int((sqrt(x)*(a + b*x)**(3/4))/(sqrt(a + b*x)*a**3*x**6 + 3*sqrt(a + b*x)*
a**2*b*x**7 + 3*sqrt(a + b*x)*a*b**2*x**8 + sqrt(a + b*x)*b**3*x**9),x)
```

3.120 $\int \left(ax^2+bx^3\right)^{7/4} dx$

Optimal result

Integrand size = 15, antiderivative size = 249

$$\int \left(ax^2+bx^3\right)^{7/4} dx = \frac{224a^6x}{16575b^4\sqrt[4]{ax^2+bx^3}} - \frac{112a^5x^2}{49725b^3\sqrt[4]{ax^2+bx^3}} + \frac{56a^4x^3}{49725b^2\sqrt[4]{ax^2+bx^3}} - \frac{4a^3x^4}{5525b\sqrt[4]{ax^2+bx^3}} + \frac{4a^2x^5}{425\sqrt[4]{ax^2+bx^3}} + \frac{4}{75}ax^3\left(ax^2+bx^3\right)^{3/4} + \frac{4}{25}x\left(ax^2+bx^3\right)^{7/4} - \frac{448a^{13/2}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{16575b^{9/2}\sqrt[4]{ax^2+bx^3}}$$

output

```
224/16575*a^6*x/b^4/(b*x^3+a*x^2)^(1/4)-112/49725*a^5*x^2/b^3/(b*x^3+a*x^2
)^(1/4)+56/49725*a^4*x^3/b^2/(b*x^3+a*x^2)^(1/4)-4/5525*a^3*x^4/b/(b*x^3+a
*x^2)^(1/4)+4/425*a^2*x^5/(b*x^3+a*x^2)^(1/4)+4/75*a*x^3*(b*x^3+a*x^2)^(3/
4)+4/25*x*(b*x^3+a*x^2)^(7/4)-448/16575*a^(13/2)*x^(1/2)*((b*x+a)/a)^(1/4)
*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/b^(9/2)/(b*x^
3+a*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.20

$$\int \left(ax^2+bx^3\right)^{7/4} dx = \frac{2ax^3(x^2(a+bx))^{3/4}\text{Hypergeometric2F1}\left(-\frac{7}{4},\frac{9}{2},\frac{11}{2},-\frac{bx}{a}\right)}{9\left(1+\frac{bx}{a}\right)^{3/4}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(7/4),x]
```

output
```
(2*a*x^3*(x^2*(a + b*x))^(3/4)*Hypergeometric2F1[-7/4, 9/2, 11/2, -((b*x)/
a)])/(9*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.35, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {1910, 1927, 1930, 1930, 1930, 1930, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^{7/4} dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{7}{25}a\int x^2\left(bx^3+ax^2\right)^{3/4}dx+\frac{4}{25}x\left(ax^2+bx^3\right)^{7/4}$$

$$\Big\downarrow \text{1927}$$

$$\frac{7}{25}a\left(\frac{1}{7}a\int \frac{x^4}{\sqrt[4]{bx^3+ax^2}}dx+\frac{4}{21}x^3\left(ax^2+bx^3\right)^{3/4}\right)+\frac{4}{25}x\left(ax^2+bx^3\right)^{7/4}$$

$$\Big\downarrow \text{1930}$$

$$\frac{7}{25}a\left(\frac{1}{7}a\left(\frac{4x^2\left(ax^2+bx^3\right)^{3/4}}{17b}-\frac{14a\int\frac{x^3}{\sqrt[4]{bx^3+ax^2}}dx}{17b}\right)+\frac{4}{21}x^3\left(ax^2+bx^3\right)^{3/4}\right)+\frac{4}{25}x\left(ax^2+bx^3\right)^{7/4}$$

$\Big\downarrow$ 1930

$$\frac{7}{25}a\left(\frac{1}{7}a\left(\frac{4x^2\left(ax^2+bx^3\right)^{3/4}}{17b}-\frac{14a\left(\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\int\frac{x^2}{\sqrt[4]{bx^3+ax^2}}dx}{13b}\right)}{17b}\right)+\frac{4}{21}x^3\left(ax^2+bx^3\right)^{3/4}\right)+\frac{4}{25}x\left(ax^2+bx^3\right)^{7/4}$$

$\Big\downarrow$ 1930

$$\frac{7}{25}a\left(\frac{1}{7}a\left(\frac{4x^2\left(ax^2+bx^3\right)^{3/4}}{17b}-\frac{14a\left(\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\int\frac{x}{\sqrt[4]{bx^3+ax^2}}dx}{3b}\right)}{13b}\right)}{17b}\right)+\frac{4}{21}x^3\left(ax^2 -\right.\right.$$

$$\frac{4}{25}x\left(ax^2+bx^3\right)^{7/4}$$

$\Big\downarrow$ 1930

$$\frac{7}{25}a\left(\frac{1}{7}a\left(\frac{4x^2(ax^2+bx^3)^{3/4}}{17b}-\frac{14a\left(\frac{4x(ax^2+bx^3)^{3/4}}{13b}-\frac{10a\left(\frac{4(ax^2+bx^3)^{3/4}}{9b}-\frac{2a\left(\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{2a\int\frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{5b}\right)}{3b}\right)}{13b}\right)}{17b}\right.\right.$$

$$\frac{4}{25}x(ax^2+bx^3)^{7/4}$$

$\downarrow$ 1917

$$\frac{7}{25}a\left(\frac{1}{7}a\left(\frac{4x^2\left(ax^2+bx^3\right)^{3/4}}{17b}-\frac{14a\left(\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{2a\sqrt{x}\sqrt[4]{a+bx}\int\frac{1}{\sqrt{x}\sqrt[4]{a+}}}{5b\sqrt[4]{ax^2+bx^3}}\right.}{3b}\right.}{13b}\right.}{17b}\right.\right.$$

$$\frac{4}{25}x\left(ax^2+bx^3\right)^{7/4}$$

$$\Big\downarrow \; 73$$

$$\frac{7}{25}a\left(\frac{1}{7}a\left(\frac{4x^2\left(ax^2+bx^3\right)^{3/4}}{17b}-\frac{14a\left(\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\int\frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\cdots}}\cdots}{5b^2\sqrt[4]{ax^2+b\cdots}}\right)}{3b}\cdots\right)}{13b}\cdots\right)}{17b}\cdots\right)\cdots\right)$$

$$\frac{4}{25}x\left(ax^2+bx^3\right)^{7/4}$$

$\Big\downarrow$ 836

$$\frac{7}{25}a\left(\frac{1}{7}a\left(\frac{4x^2\left(ax^2+bx^3\right)^{3/4}}{17b}-\frac{14a\left(\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int\frac{\sqrt{a}+\cdots}{\sqrt{a}\sqrt{\cdots}}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{13b}\right.}{17b}\right.\right.$$

$$\frac{4}{25}x\left(ax^2+bx^3\right)^{7/4}$$

$$\Big\downarrow \; 27$$

$$\frac{7}{25}a\left(\frac{1}{7}a\left(\frac{4x^2\left(ax^2+bx^3\right)^{3/4}}{17b}-\frac{14a\left(\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+}}{\sqrt{\frac{a+bx}{b}-}}\right.}{5}\right.}{3}\right)}{13b}\right)}{17b}\right.\right.$$

$$\frac{4}{25}x\left(ax^2+bx^3\right)^{7/4}$$

$\downarrow$ 765

$$\frac{7}{25}a\left(\frac{1}{7}a\left(\frac{4x^2\left(ax^2+bx^3\right)^{3/4}}{17b}-\frac{14a\left(\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+}}{\sqrt{\frac{a+bx}{b}-}}\right.}{}\right)}{}\right)}{13b}\right)}{17b}\right.\right.$$

$$\frac{4}{25}x\left(ax^2+bx^3\right)^{7/4}$$

↓ 762

$$\frac{7}{25}a\left(\frac{1}{7}a\left(\frac{4x^2\left(ax^2+bx^3\right)^{3/4}}{17b}-\frac{14a\left(\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+}}{\sqrt{\frac{a+bx}{b}-}}\right.}{}\right.}{}\right.}{13}\right.}{17b}\right.\right.$$

↓ 1390

$$8a\sqrt{x}\,\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}}{\quad}\right.$$

$$2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\right.$$

$$10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\right.$$

$$14a\left(\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\right.$$

$$\frac{7}{25}a\left(\frac{1}{7}a\left(\frac{4x^2\left(ax^2+bx^3\right)^{3/4}}{17b}-\frac{\quad}{17b}\right.\right.$$

↓ 1389

$$8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+}{a}}}{}\right.$$

$$2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\right.$$

$$10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\right.$$

$$14a\left(\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\right.$$

$$\frac{7}{25}a\left(\frac{1}{7}a\left(\frac{4x^2\left(ax^2+bx^3\right)^{3/4}}{17b}-\frac{}{17b}\right.\right.$$

↓ 327

$$8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\cdots}}{}\right.$$

$$2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{}{}\right.$$

$$10a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{}{}\right.$$

$$14a\left(\frac{4x\left(ax^2+bx^3\right)^{3/4}}{13b}-\frac{}{}\right.$$

$$\frac{7}{25}a\left(\frac{1}{7}a\left(\frac{4x^2\left(ax^2+bx^3\right)^{3/4}}{17b}-\frac{}{17b}\right.\right.$$

input
```
Int[(a*x^2 + b*x^3)^(7/4),x]
```

output
```
(4*x*(a*x^2 + b*x^3)^(7/4))/25 + (7*a*((4*x^3*(a*x^2 + b*x^3)^(3/4))/21 +
(a*((4*x^2*(a*x^2 + b*x^3)^(3/4))/(17*b) - (14*a*((4*x*(a*x^2 + b*x^3)^(3/
4))/(13*b) - (10*a*((4*(a*x^2 + b*x^3)^(3/4))/(9*b) - (2*a*((4*(a*x^2 + b*
x^3)^(3/4))/(5*b*x) - (8*a*Sqrt[x]*(a + b*x)^(1/4)*((a^(3/4)*Sqrt[1 - (a +
 b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a
+ b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4
)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(5*b^2*(a*x^2 + b*x^3)^(1/4)
)))/(3*b)))/(13*b)))/(17*b)))/7))/25
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{3}+a\,x^{2}\right)^{\frac{7}{4}}d x$$

input
```
int((b*x^3+a*x^2)^(7/4),x)
```

output
```
int((b*x^3+a*x^2)^(7/4),x)
```

Fricas [F]

$$\int (ax^2+bx^3)^{7/4}\,dx=\int (bx^3+ax^2)^{\frac{7}{4}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(7/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^(7/4), x)
```

Sympy [F]

$$\int (ax^2+bx^3)^{7/4}\,dx=\int \left(ax^2+bx^3\right)^{\frac{7}{4}}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(7/4),x)
```

output
```
Integral((a*x**2 + b*x**3)**(7/4), x)
```

Maxima [F]

$$\int \left(ax^2+bx^3\right)^{7/4}\,dx=\int \left(bx^3+ax^2\right)^{\frac{7}{4}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(7/4),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(7/4), x)
```

Giac [F]

$$\int \left(ax^2+bx^3\right)^{7/4}\,dx=\int \left(bx^3+ax^2\right)^{\frac{7}{4}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(7/4),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^(7/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.15

$$\int \left(ax^2+bx^3\right)^{7/4}\,dx=\frac{2\,x\,\left(b\,x^3+a\,x^2\right)^{7/4}\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{9}{2};\ \frac{11}{2};\ -\frac{b\,x}{a}\right)}{9\,\left(\frac{b\,x}{a}+1\right)^{7/4}}$$

input
```
int((a*x^2 + b*x^3)^(7/4),x)
```

output
```
(2*x*(a*x^2 + b*x^3)^(7/4)*hypergeom([-7/4, 9/2], 11/2, -(b*x)/a))/(9*((b*
x)/a + 1)^(7/4))
```

Reduce [F]

$$\int \left(ax^2+bx^3\right)^{7/4} dx = \frac{-\frac{224\sqrt{x}\,(bx+a)^{\frac{3}{4}}a^5}{16575} + \frac{112\sqrt{x}\,(bx+a)^{\frac{3}{4}}a^4bx}{9945} - \frac{56\sqrt{x}\,(bx+a)^{\frac{3}{4}}a^3b^2x^2}{5525} + \frac{4\sqrt{x}\,(bx+a)^{\frac{3}{4}}a^2b^3x^3}{425} + \frac{16\sqrt{x}\,(bx+a)^{\frac{3}{4}}a\,b^4x^4}{75} \cdots}{b^4}$$

input
```
int((b*x^3+a*x^2)^(7/4),x)
```

output
```
(4*( - 168*sqrt(x)*(a + b*x)**(3/4)*a**5 + 140*sqrt(x)*(a + b*x)**(3/4)*a*
*4*b*x - 126*sqrt(x)*(a + b*x)**(3/4)*a**3*b**2*x**2 + 117*sqrt(x)*(a + b*
x)**(3/4)*a**2*b**3*x**3 + 2652*sqrt(x)*(a + b*x)**(3/4)*a*b**4*x**4 + 198
9*sqrt(x)*(a + b*x)**(3/4)*b**5*x**5 + 84*int((sqrt(x)*(a + b*x)**(3/4))/(
a*x + b*x**2),x)*a**6))/(49725*b**4)
```

3.121 $\int \left(ax^2+bx^3\right)^{3/4} dx$

Optimal result

Integrand size = 15, antiderivative size = 168

$$\int \left(ax^2+bx^3\right)^{3/4} dx = \frac{8a^3x}{65b^2\sqrt[4]{ax^2+bx^3}} - \frac{4a^2x^2}{195b\sqrt[4]{ax^2+bx^3}} + \frac{4ax^3}{39\sqrt[4]{ax^2+bx^3}} + \frac{4}{13}x\left(ax^2+bx^3\right)^{3/4} - \frac{16a^{7/2}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{65b^{5/2}\sqrt[4]{ax^2+bx^3}}$$

output
```
8/65*a^3*x/b^2/(b*x^3+a*x^2)^(1/4)-4/195*a^2*x^2/b/(b*x^3+a*x^2)^(1/4)+4/3
9*a*x^3/(b*x^3+a*x^2)^(1/4)+4/13*x*(b*x^3+a*x^2)^(3/4)-16/65*a^(7/2)*x^(1/
2)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^
(1/2))/b^(5/2)/(b*x^3+a*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.28

$$\int \left(ax^2+bx^3\right)^{3/4} dx = \frac{2x\left(x^2(a+bx)\right)^{3/4}\text{Hypergeometric2F1}\left(-\frac{3}{4},\frac{5}{2},\frac{7}{2},-\frac{bx}{a}\right)}{5\left(1+\frac{bx}{a}\right)^{3/4}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(3/4),x]
```

output
```
(2*x*(x^2*(a + b*x))^(3/4)*Hypergeometric2F1[-3/4, 5/2, 7/2, -((b*x)/a)])/
(5*(1 + (b*x)/a)^(3/4))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.44, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {1910, 1930, 1930, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^{3/4} dx$$

$$\Big\downarrow \text{1910}$$

$$\frac{3}{13}a\int \frac{x^2}{\sqrt[4]{bx^3+ax^2}}dx+\frac{4}{13}x\left(ax^2+bx^3\right)^{3/4}$$

$$\Big\downarrow \text{1930}$$

$$\frac{3}{13}a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\int \frac{x}{\sqrt[4]{bx^3+ax^2}}dx}{3b}\right)+\frac{4}{13}x\left(ax^2+bx^3\right)^{3/4}$$

$$\Big\downarrow \text{1930}$$

$$\frac{3}{13}a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{2a\int \frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{5b}\right)}{3b}\right)+\frac{4}{13}x\left(ax^2+bx^3\right)^{3/4}$$

$$\Big\downarrow \text{1917}$$

$$\frac{3}{13}a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{2a\sqrt{x}\sqrt[4]{a+bx}\int\frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{5b\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)+\frac{4}{13}x\left(ax^2+bx^3\right)^{3/4}$$

$\Big\downarrow$ 73

$$\frac{3}{13}a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\int\frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)+\frac{4}{13}x\left(ax^2+bx^3\right)^{3/4}$$

$\Big\downarrow$ 836

$$\frac{3}{13}a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)+\frac{4}{13}x\left(ax^2+bx^3\right)^{3/4}$$

$\Big\downarrow$ 27

$$\frac{3}{13}a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{5b^2\sqrt[4]{ax^2+bx^3}}\right)}{3b}\right)+\frac{4}{13}x\left(ax^2+bx^3\right)^{3/4}$$

$\Big\downarrow$ 765

$$\frac{3}{13}a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right.}{5b^2\sqrt[4]{ax^2+bx^3}}\right.}{3b}\right.$$

$$\frac{4}{13}x\left(ax^2+bx^3\right)^{3/4}$$

$\downarrow$ 762

$$\frac{3}{13}a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+\cdots}}{\sqrt[4]{\cdots}}\right.\right.}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right.}{5b^2\sqrt[4]{ax^2+bx^3}}\right.}{3b}\right.$$

$$\frac{4}{13}x\left(ax^2+bx^3\right)^{3/4}$$

$\downarrow$ 1390

$$\frac{3}{13}a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\text{arcs}\right.}{\sqrt{\frac{a+bx}{b}-}}\right.}{5b^2\sqrt[4]{ax^2+bx^3}}\right.}{3b}\right.$$

$$\frac{4}{13}x\left(ax^2+bx^3\right)^{3/4}$$

$\downarrow$ 1389

$$\frac{3}{13}a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{9b}-\frac{2a\left(\frac{4\left(ax^2+bx^3\right)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\text{EllipticF}\left(\right.}{\sqrt{\frac{a+}{b}}}\right.}{5b^2\sqrt[4]{ax^2+bx^3}}\right.}{3b}\right.$$

$$\frac{4}{13}x\left(ax^2+bx^3\right)^{3/4}$$

$\downarrow$ 327

$$\frac{3}{13}a\left(\frac{4(ax^2+bx^3)^{3/4}}{9b}-\frac{2a\left(\frac{4(ax^2+bx^3)^{3/4}}{5bx}-\frac{8a\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{Ellipti}\cdots}{\cdots}\right.}{5b^2\sqrt[4]{ax^2+bx^3}}\right.}{3b}\right.$$

$$\frac{4}{13}x\left(ax^2+bx^3\right)^{3/4}$$

input
```
Int[(a*x^2 + b*x^3)^(3/4),x]
```

output
```
(4*x*(a*x^2 + b*x^3)^(3/4))/13 + (3*a*((4*(a*x^2 + b*x^3)^(3/4))/(9*b) - (
2*a*((4*(a*x^2 + b*x^3)^(3/4))/(5*b*x) - (8*a*Sqrt[x]*(a + b*x)^(1/4)*((a^
(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1]
)/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[Ar
cSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(5*b^2*(a
*x^2 + b*x^3)^(1/4))))/(3*b)))/13
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1910
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[x*((a*x^j
 + b*x^n)^p/(n*p + 1)), x] + Simp[a*(n - j)*(p/(n*p + 1))   Int[x^j*(a*x^j
+ b*x^n)^(p - 1), x], x] /; FreeQ[{a, b}, x] &&  !IntegerQ[p] && LtQ[0, j,
n] && GtQ[p, 0] && NeQ[n*p + 1, 0]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

Maple [F]

$$\int \left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{4}}d x$$

input
```
int((b*x^3+a*x^2)^(3/4),x)
```

output
```
int((b*x^3+a*x^2)^(3/4),x)
```

Fricas [F]

$$\int \left(ax^2+bx^3\right)^{3/4}\,dx=\int \left(bx^{3}+ax^{2}\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(3/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^(3/4), x)
```

Sympy [F]

$$\int \left(ax^2+bx^3\right)^{3/4}\,dx=\int \left(ax^2+bx^3\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((b*x**3+a*x**2)**(3/4),x)
```

output
```
Integral((a*x**2 + b*x**3)**(3/4), x)
```

Maxima [F]

$$\int \left(ax^2+bx^3\right)^{3/4}\,dx=\int \left(bx^3+ax^2\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(3/4),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/4), x)
```

Giac [F]

$$\int \left(ax^2+bx^3\right)^{3/4}\,dx=\int \left(bx^3+ax^2\right)^{\frac{3}{4}}\,dx$$

input
```
integrate((b*x^3+a*x^2)^(3/4),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^(3/4), x)
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.23

$$\int \left(ax^2+bx^3\right)^{3/4} dx = \frac{2x\left(bx^3+ax^2\right)^{3/4} {}_2F_1\left(-\frac{3}{4},\frac{5}{2};\ \frac{7}{2};\ -\frac{bx}{a}\right)}{5\left(\frac{bx}{a}+1\right)^{3/4}}$$

input
```
int((a*x^2 + b*x^3)^(3/4),x)
```

output
```
(2*x*(a*x^2 + b*x^3)^(3/4)*hypergeom([-3/4, 5/2], 7/2, -(b*x)/a))/(5*((b*x
)/a + 1)^(3/4))
```

Reduce [F]

$$\int \left(ax^2+bx^3\right)^{3/4} dx = \frac{-\frac{8\sqrt{x}\,(bx+a)^{\frac{3}{4}}a^2}{65}+\frac{4\sqrt{x}\,(bx+a)^{\frac{3}{4}}abx}{39}+\frac{4\sqrt{x}\,(bx+a)^{\frac{3}{4}}b^2x^2}{13}+\frac{4\left(\int\frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b\,x^2+ax}dx\right)a^3}{65}}{b^2}$$

input
```
int((b*x^3+a*x^2)^(3/4),x)
```

output
```
(4*( - 6*sqrt(x)*(a + b*x)**(3/4)*a**2 + 5*sqrt(x)*(a + b*x)**(3/4)*a*b*x
+ 15*sqrt(x)*(a + b*x)**(3/4)*b**2*x**2 + 3*int((sqrt(x)*(a + b*x)**(3/4))
/(a*x + b*x**2),x)*a**3))/(195*b**2)
```

3.122 $\int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 87

$$\int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx=\frac{4x}{\sqrt[4]{ax^2+bx^3}}-\frac{4\sqrt{a}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{\sqrt{b}\sqrt[4]{ax^2+bx^3}}$$

output
```
4*x/(b*x^3+a*x^2)^(1/4)-4*a^(1/2)*x^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(sin(
1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/b^(1/2)/(b*x^3+a*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.52

$$\int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx=\frac{2x\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{3}{2},-\frac{bx}{a}\right)}{\sqrt[4]{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-1/4),x]
```

output
```
(2*x*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[1/4, 1/2, 3/2, -((b*x)/a)])/(x^
2*(a + b*x))^(1/4)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.79, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt[4]{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{\sqrt{x}\sqrt[4]{a+bx}\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{73}$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{836}$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{765}$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 762}$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1390}$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1389}$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 327}$$

$$\frac{4\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{b\sqrt[4]{ax^2+bx^3}}$$

input
```
Int[(a*x^2 + b*x^3)^(-1/4),x]
```

output
```
(4*Sqrt[x]*(a + b*x)^(1/4)*((a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSi
n[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqr
t[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(
a/b) + (a + b*x)/b]))/(b*(a*x^2 + b*x^3)^(1/4))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{1}{4}}}dx$$

input
```
int(1/(b*x^3+a*x^2)^(1/4),x)
```

output
```
int(1/(b*x^3+a*x^2)^(1/4),x)
```

Fricas [F]

$$\int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx=\int \frac{1}{(bx^3+ax^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(1/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^(-1/4), x)
```

Sympy [F]

$$\int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(1/4),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-1/4), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(1/4),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-1/4), x)
```

Giac [F]

$$\int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{1}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(1/4),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^(-1/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.44

$$\int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx=\frac{2\,x\,{\left(\frac{b\,x}{a}+1\right)}^{1/4}\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\,x}{a}\right)}{{\left(b\,x^3+a\,x^2\right)}^{1/4}}$$

input
```
int(1/(a*x^2 + b*x^3)^(1/4),x)
```

output
```
(2*x*((b*x)/a + 1)^(1/4)*hypergeom([1/4, 1/2], 3/2, -(b*x)/a))/(a*x^2 + b*
x^3)^(1/4)
```

Reduce [F]

$$\int \frac{1}{\sqrt[4]{ax^2+bx^3}}\,dx=\frac{4\sqrt{x}\,(bx+a)^{\frac{1}{4}}+\sqrt{bx+a}\,\left(\int \frac{\sqrt{x}\,(bx+a)^{\frac{3}{4}}}{b^2x^3+2ab\,x^2+a^2x}dx\right)a}{3\sqrt{bx+a}}$$

input
```
int(1/(b*x^3+a*x^2)^(1/4),x)
```

output
```
(4*sqrt(x)*(a + b*x)**(1/4) + sqrt(a + b*x)*int((sqrt(x)*(a + b*x)**(3/4))
/(a**2*x + 2*a*b*x**2 + b**2*x**3),x)*a)/(3*sqrt(a + b*x))
```

3.123 $\int \frac{1}{\left(ax^2+bx^3\right)^{5/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 117

$$\int \frac{1}{\left(ax^2+bx^3\right)^{5/4}}\,dx=\frac{7b}{3a^2\sqrt[4]{ax^2+bx^3}}-\frac{2}{3ax\sqrt[4]{ax^2+bx^3}}+\frac{7b^{3/2}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{a^{5/2}\sqrt[4]{ax^2+bx^3}}$$

output
```
7/3*b/a^2/(b*x^3+a*x^2)^(1/4)-2/3/a/x/(b*x^3+a*x^2)^(1/4)+7*b^(3/2)*x^(1/2
)*((b*x+a)/a)^(1/4)*EllipticE(sin(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(
1/2))/a^(5/2)/(b*x^3+a*x^2)^(1/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.44

$$\int \frac{1}{\left(ax^2+bx^3\right)^{5/4}}\,dx=-\frac{2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5}{4},-\frac{1}{2},-\frac{bx}{a}\right)}{3ax\sqrt[4]{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-5/4),x]
```

output
```
(-2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-3/2, 5/4, -1/2, -((b*x)/a)])/(3
*a*x*(x^2*(a + b*x))^(1/4))
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 243 vs. 2(117) = 234.

Time = 0.82 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.08, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {1912, 1931, 1931, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(ax^2+bx^3\right)^{5/4}}dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{7\int \frac{1}{x^2\sqrt[4]{bx^3+ax^2}}dx}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{b\int \frac{1}{x\sqrt[4]{bx^3+ax^2}}dx}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{7\left(-\frac{b\left(\frac{b\int \frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$$\Big\downarrow \text{1917}$$

$$\frac{7\left(-\frac{b\left(\frac{b\sqrt{x}\sqrt[4]{a+bx}\int \frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$\downarrow$ 73

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\int \frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$\downarrow$ 836

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$\downarrow$ 27

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int \frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$\downarrow$ 765

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int \frac{1}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$\Big\downarrow$ 762

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int \frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{a}+\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$\Big\downarrow$ 1390

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2(a}{}\right.}{a}$$

$$\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$\downarrow$ 1389

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\right.}{a}$$

$$\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

$\downarrow$ 327

$$\frac{7\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}\right.}{a}$$

$$\frac{4}{ax\sqrt[4]{ax^2+bx^3}}$$

input
```
Int[(a*x^2 + b*x^3)^(-5/4),x]
```

output
```
4/(a*x*(a*x^2 + b*x^3)^(1/4)) + (7*((-2*(a*x^2 + b*x^3)^(3/4))/(3*a*x^3) -
 (b*((-2*(a*x^2 + b*x^3)^(3/4))/(a*x^2) + (2*Sqrt[x]*(a + b*x)^(1/4)*((a^(
3/4)*Sqrt[1 - (a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])
/Sqrt[-(a/b) + (a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[Arc
Sin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(a*(a*x^2
+ b*x^3)^(1/4))))/(2*a)))/a
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [F]

$$\int \frac{1}{(b\,x^{3}+a\,x^{2})^{\frac{5}{4}}}dx$$

input
```
int(1/(b*x^3+a*x^2)^(5/4),x)
```

output
```
int(1/(b*x^3+a*x^2)^(5/4),x)
```

Fricas [F]

$$\int \frac{1}{(ax^2+bx^3)^{5/4}}\,dx=\int \frac{1}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(5/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^(3/4)/(b^2*x^6 + 2*a*b*x^5 + a^2*x^4), x)
```

Sympy [F]

$$\int \frac{1}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{1}{(ax^2+bx^3)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(5/4),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-5/4), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(5/4),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-5/4), x)
```

Giac [F]

$$\int \frac{1}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{5}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(5/4),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^(-5/4), x)
```

Mupad [B] (verification not implemented)

Time = 10.15 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.32

$$\int \frac{1}{(ax^2+bx^3)^{5/4}}\,dx = -\frac{2\,x\,\left(\frac{b\,x}{a}+1\right)^{5/4}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{5}{4};\ -\frac{1}{2};\ -\frac{b\,x}{a}\right)}{3\,\left(b\,x^3+a\,x^2\right)^{5/4}}$$

input
```
int(1/(a*x^2 + b*x^3)^(5/4),x)
```

output
```
-(2*x*((b*x)/a + 1)^(5/4)*hypergeom([-3/2, 5/4], -1/2, -(b*x)/a))/(3*(a*x^
2 + b*x^3)^(5/4))
```

Reduce [F]

$$\int \frac{1}{(ax^2+bx^3)^{5/4}}\,dx = \int \frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{\sqrt{bx+a}\,a\,x^3+\sqrt{bx+a}\,b\,x^4}dx$$

input
```
int(1/(b*x^3+a*x^2)^(5/4),x)
```

output
```
int((sqrt(x)*(a + b*x)**(1/4))/(sqrt(a + b*x)*a*x**3 + sqrt(a + b*x)*b*x**
4),x)
```

3.124 $\int \frac{1}{(ax^2+bx^3)^{9/4}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 200

$$\int \frac{1}{(ax^2+bx^3)^{9/4}}\,dx=\frac{4}{5ax\,(ax^2+bx^3)^{5/4}}+\frac{209b^3}{20a^5\sqrt[4]{ax^2+bx^3}}-\frac{38}{35a^2x^3\sqrt[4]{ax^2+bx^3}}+\frac{57b}{35a^3x^2\sqrt[4]{ax^2+bx^3}}-\frac{209b^2}{70a^4x\sqrt[4]{ax^2+bx^3}}+\frac{627b^{7/2}\sqrt{x}\sqrt[4]{\frac{a+bx}{a}}E\left(\frac{1}{2}\arctan\left(\frac{\sqrt{b}\sqrt{x}}{\sqrt{a}}\right)\middle|2\right)}{20a^{11/2}\sqrt[4]{ax^2+bx^3}}$$

output
```
4/5/a/x/(b*x^3+a*x^2)^(5/4)+209/20*b^3/a^5/(b*x^3+a*x^2)^(1/4)-38/35/a^2/x
^3/(b*x^3+a*x^2)^(1/4)+57/35*b/a^3/x^2/(b*x^3+a*x^2)^(1/4)-209/70*b^2/a^4/
x/(b*x^3+a*x^2)^(1/4)+627/20*b^(7/2)*x^(1/2)*((b*x+a)/a)^(1/4)*EllipticE(s
in(1/2*arctan(b^(1/2)*x^(1/2)/a^(1/2))),2^(1/2))/a^(11/2)/(b*x^3+a*x^2)^(1
/4)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.02 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.26

$$\int \frac{1}{(ax^2+bx^3)^{9/4}}\,dx = -\frac{2\sqrt[4]{1+\frac{bx}{a}}\,\text{Hypergeometric2F1}\left(-\frac{7}{2},\frac{9}{4},-\frac{5}{2},-\frac{bx}{a}\right)}{7a^2x^3\sqrt[4]{x^2(a+bx)}}$$

input
```
Integrate[(a*x^2 + b*x^3)^(-9/4),x]
```

output
```
(-2*(1 + (b*x)/a)^(1/4)*Hypergeometric2F1[-7/2, 9/4, -5/2, -((b*x)/a)])/(7
*a^2*x^3*(x^2*(a + b*x))^(1/4))
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.72, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {1912, 1929, 1931, 1931, 1931, 1931, 1917, 73, 836, 27, 765, 762, 1390, 1389, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^2+bx^3)^{9/4}}\,dx$$

$$\Big\downarrow \text{1912}$$

$$\frac{19\int \frac{1}{x^2(bx^3+ax^2)^{5/4}}dx}{5a}+\frac{4}{5ax\left(ax^2+bx^3\right)^{5/4}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{19\left(\frac{15\int \frac{1}{x^4\sqrt[4]{bx^3+ax^2}}dx}{a}+\frac{4}{ax^3\sqrt[4]{ax^2+bx^3}}\right)}{5a}+\frac{4}{5ax\left(ax^2+bx^3\right)^{5/4}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{19\left(\frac{15\left(-\frac{11b\int \frac{1}{x^3\sqrt[4]{bx^3+ax^2}}dx}{14a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{7ax^5}\right)}{a}+\frac{4}{ax^3\sqrt[4]{ax^2+bx^3}}\right)}{5a}+\frac{4}{5ax\left(ax^2+bx^3\right)^{5/4}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{19\left(\frac{15\left(-\frac{11b\left(-\frac{7b\int \frac{1}{x^2\sqrt[4]{bx^3+ax^2}}dx}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}\right)}{14a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{7ax^5}\right)}{a}+\frac{4}{ax^3\sqrt[4]{ax^2+bx^3}}\right)}{5a}+\frac{4}{5ax\left(ax^2+bx^3\right)^{5/4}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{19\left(\frac{15\left(-\frac{11b\left(-\frac{7b\left(-\frac{b\int\frac{1}{x\sqrt[4]{bx^3+ax^2}}dx}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}\right)}{14a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{7ax^5}\right)}{a}+\frac{4}{ax^3\sqrt[4]{ax^2+bx^3}}\right)}{5a}+\frac{4}{5ax\left(ax^2+bx^3\right)^{5/4}}$$

$\downarrow$ 1931

$$\frac{19\left(\frac{15\left(-\frac{11b\left(-\frac{7b\left(-\frac{b\left(\frac{b\int\frac{1}{\sqrt[4]{bx^3+ax^2}}dx}{2a}-\frac{2(ax^2+bx^3)^{3/4}}{ax^2}\right)}{2a}-\frac{2(ax^2+bx^3)^{3/4}}{3ax^3}\right)}{10a}-\frac{2(ax^2+bx^3)^{3/4}}{5ax^4}\right)}{14a}-\frac{2(ax^2+bx^3)^{3/4}}{7ax^5}\right)}{a}+\frac{4}{ax^3\sqrt[4]{ax^2-}}}{5a}$$

$$\frac{4}{5ax\left(ax^2+bx^3\right)^{5/4}}$$

↓ 1917

$$\frac{19\left(\frac{15\left(-\frac{11b\left(-\frac{7b\left(-\frac{b\left(\frac{b\sqrt{x}\sqrt[4]{a+bx}\int\frac{1}{\sqrt{x}\sqrt[4]{a+bx}}dx}{2a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}\right)}{14a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{7ax^5}\right)}{a}+ \cdots\right)}{5a}$$

$$\frac{4}{5ax\left(ax^2+bx^3\right)^{5/4}}$$

↓ 73

$$\frac{19\left(\frac{15\left(-\frac{11b\left(-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\int\frac{\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{5ax^4}\right)}{14a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{7ax^5}\right.}{a}\right.}{5a}$$

$$\frac{4}{5ax\left(ax^2+bx^3\right)^{5/4}}$$

↓ 836

$$19\left(\frac{\left(15\left(-\frac{11b\left(-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\sqrt{a}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{a}\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\cdots\right)}{10a}\cdots\right)}{14a}\cdots\right)\cdots\right)}{a}\cdots\right.$$

↓ 27

$$19\left(\frac{15\left(-\frac{11b\left(-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\,d\sqrt[4]{a+bx}-\sqrt{a}\int\frac{1}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\,d\sqrt[4]{a+bx}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{3ax^3}\right)}{10a}-\frac{2\left(\cdots\right.}{\cdots}\cdots\right)}{14a}\cdots\right)}{a}\cdots\right.$$

↓ 765

$$19\left(-\frac{15\left(-\frac{11b\left(-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\,d\sqrt[4]{a+bx}-\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{1}{\sqrt{1-\frac{a+bx}{a}}}\,d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\frac{2\left(ax^2+bx^3\right)\cdots}{3ax^3}\cdots\right)}{10a}\cdots\right)}{14a}\cdots\right)}{a}\cdots\right)$$

↓ 762

$$\Bigg(15\Bigg(-\frac{11b\Bigg(-\frac{7b\Bigg(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}d\sqrt[4]{a+bx}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+bx^3\right)^{3/4}}{ax^2}\right)}{2a}-\cdots}{10a}}{14a}$$

↓ 1390

$$\left(15\left(-\frac{11b\left(-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{a}+\sqrt{a+bx}}{\sqrt{1-\frac{a+bx}{a}}}\,d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax^2+b\right.}{ax}\right.}{2a}\right.}{10a}\right.}{14a}\right.\right.$$

↓ 1389

$$15\left(-\frac{11b\left(-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{\sqrt{a}\sqrt{1-\frac{a+bx}{a}}\int\frac{\sqrt{\frac{\sqrt{a+bx}}{\sqrt{a}}+1}}{\sqrt{1-\frac{\sqrt{a+bx}}{\sqrt{a}}}}d\sqrt[4]{a+bx}}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\frac{2\left(ax\right.}{}\right.}{2a}\right.}{10a}\right.}{14a}\right.$$

↓ 327

$$\left(15\left(-\frac{11b\left(-\frac{7b\left(-\frac{b\left(\frac{2\sqrt{x}\sqrt[4]{a+bx}\left(\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}E\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right)\middle|-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}-\frac{a^{3/4}\sqrt{1-\frac{a+bx}{a}}\,\mathrm{EllipticF}\left(\arcsin\left(\frac{\sqrt[4]{a+bx}}{\sqrt[4]{a}}\right),-1\right)}{\sqrt{\frac{a+bx}{b}-\frac{a}{b}}}\right)}{a\sqrt[4]{ax^2+bx^3}}-\cdots}{2a}\cdots\right)}{10a}\cdots\right)}{14a}\cdots\right)\cdots$$

input
```
Int[(a*x^2 + b*x^3)^(-9/4),x]
```

output
```
4/(5*a*x*(a*x^2 + b*x^3)^(5/4)) + (19*(4/(a*x^3*(a*x^2 + b*x^3)^(1/4)) + (
15*((-2*(a*x^2 + b*x^3)^(3/4))/(7*a*x^5) - (11*b*((-2*(a*x^2 + b*x^3)^(3/4
))/(5*a*x^4) - (7*b*((-2*(a*x^2 + b*x^3)^(3/4))/(3*a*x^3) - (b*((-2*(a*x^2
 + b*x^3)^(3/4))/(a*x^2) + (2*Sqrt[x]*(a + b*x)^(1/4)*((a^(3/4)*Sqrt[1 - (
a + b*x)/a]*EllipticE[ArcSin[(a + b*x)^(1/4)/a^(1/4)], -1])/Sqrt[-(a/b) +
(a + b*x)/b] - (a^(3/4)*Sqrt[1 - (a + b*x)/a]*EllipticF[ArcSin[(a + b*x)^(
1/4)/a^(1/4)], -1])/Sqrt[-(a/b) + (a + b*x)/b]))/(a*(a*x^2 + b*x^3)^(1/4))
))/(2*a)))/(10*a)))/(14*a)))/a))/(5*a)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 765
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[Sqrt[1 + b*(x^4/a)]/Sqrt
[a + b*x^4]   Int[1/Sqrt[1 + b*(x^4/a)], x], x] /; FreeQ[{a, b}, x] && NegQ
[b/a] &&  !GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1389
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[d/Sq
rt[a]   Int[Sqrt[1 + e*(x^2/d)]/Sqrt[1 - e*(x^2/d)], x], x] /; FreeQ[{a, c,
 d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] && GtQ[a, 0]
```

rule 1390
```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> Simp[Sqrt
[1 + c*(x^4/a)]/Sqrt[a + c*x^4]   Int[(d + e*x^2)/Sqrt[1 + c*(x^4/a)], x],
x] /; FreeQ[{a, c, d, e}, x] && EqQ[c*d^2 + a*e^2, 0] && NegQ[c/a] &&  !GtQ
[a, 0] &&  !(LtQ[a, 0] && GtQ[c, 0])
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

Maple [F]

$$\int \frac{1}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{9}{4}}}dx$$

input
```
int(1/(b*x^3+a*x^2)^(9/4),x)
```

output
```
int(1/(b*x^3+a*x^2)^(9/4),x)
```

Fricas [F]

$$\int \frac{1}{(ax^2+bx^3)^{9/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(9/4),x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^(3/4)/(b^3*x^9 + 3*a*b^2*x^8 + 3*a^2*b*x^7 + a^3*
x^6), x)
```

Sympy [F]

$$\int \frac{1}{(ax^2+bx^3)^{9/4}}\,dx = \int \frac{1}{(ax^2+bx^3)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(b*x**3+a*x**2)**(9/4),x)
```

output
```
Integral((a*x**2 + b*x**3)**(-9/4), x)
```

Maxima [F]

$$\int \frac{1}{(ax^2+bx^3)^{9/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(9/4),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(-9/4), x)
```

Giac [F]

$$\int \frac{1}{(ax^2+bx^3)^{9/4}}\,dx = \int \frac{1}{(bx^3+ax^2)^{\frac{9}{4}}}\,dx$$

input
```
integrate(1/(b*x^3+a*x^2)^(9/4),x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^(-9/4), x)
```

Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.19

$$\int \frac{1}{(ax^2+bx^3)^{9/4}}\,dx = -\frac{2\,x\,\left(\frac{b\,x}{a}+1\right)^{9/4}\,{}_2\mathrm{F}_1\left(-\frac{7}{2},\frac{9}{4};\ -\frac{5}{2};\ -\frac{b\,x}{a}\right)}{7\,\left(b\,x^3+a\,x^2\right)^{9/4}}$$

input
```
int(1/(a*x^2 + b*x^3)^(9/4),x)
```

output
```
-(2*x*((b*x)/a + 1)^(9/4)*hypergeom([-7/2, 9/4], -5/2, -(b*x)/a))/(7*(a*x^
2 + b*x^3)^(9/4))
```

Reduce [F]

$$\int \frac{1}{(ax^2+bx^3)^{9/4}}\,dx = \int \frac{\sqrt{x}\,(bx+a)^{\frac{1}{4}}}{\sqrt{bx+a}\,a^2x^5+2\sqrt{bx+a}\,ab\,x^6+\sqrt{bx+a}\,b^2x^7}dx$$

input
```
int(1/(b*x^3+a*x^2)^(9/4),x)
```

output
```
int((sqrt(x)*(a + b*x)**(1/4))/(sqrt(a + b*x)*a**2*x**5 + 2*sqrt(a + b*x)*
a*b*x**6 + sqrt(a + b*x)*b**2*x**7),x)
```

3.125 $\int (ax^2+bx^3)^p \, dx$

Optimal result

Integrand size = 13, antiderivative size = 48

$$\int (ax^2+bx^3)^p \, dx = \frac{(ax^2+bx^3)^{1+p} \text{Hypergeometric2F1}\left(1,2+3p,2(1+p),-\frac{bx}{a}\right)}{a(1+2p)x}$$

output
```
(b*x^3+a*x^2)^(p+1)*hypergeom([1, 2+3*p],[2*p+2],-b*x/a)/a/(1+2*p)/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int (ax^2+bx^3)^p \, dx = \frac{x(x^2(a+bx))^p \left(1+\frac{bx}{a}\right)^{-p} \text{Hypergeometric2F1}\left(-p,1+2p,2+2p,-\frac{bx}{a}\right)}{1+2p}$$

input
```
Integrate[(a*x^2 + b*x^3)^p,x]
```

output
```
(x*(x^2*(a + b*x))^p*Hypergeometric2F1[-p, 1 + 2*p, 2 + 2*p, -((b*x)/a)])/
((1 + 2*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {1917, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ax^2+bx^3)^p\,dx$$

$$\Big\downarrow \ 1917$$

$$x^{-2p}(a+bx)^{-p}\left(ax^2+bx^3\right)^p\int x^{2p}(a+bx)^p dx$$

$$\Big\downarrow \ 76$$

$$x^{-2p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\int x^{2p}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \ 74$$

$$\frac{x\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p \text{Hypergeometric2F1}\left(-p,2p+1,2(p+1),-\frac{bx}{a}\right)}{2p+1}$$

input
```
Int[(a*x^2 + b*x^3)^p,x]
```

output
```
(x*(a*x^2 + b*x^3)^p*Hypergeometric2F1[-p, 1 + 2*p, 2*(1 + p), -((b*x)/a)]
)/((1 + 2*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \left(b\,x^{3}+a\,x^{2}\right)^{p}dx$$

input
```
int((b*x^3+a*x^2)^p,x)
```

output
```
int((b*x^3+a*x^2)^p,x)
```

Fricas [F]

$$\int \left(ax^2+bx^3\right)^p\,dx=\int \left(bx^3+ax^2\right)^p\,dx$$

input
```
integrate((b*x^3+a*x^2)^p,x, algorithm="fricas")
```

output
```
integral((b*x^3 + a*x^2)^p, x)
```

Sympy [F]

$$\int \left(ax^2+bx^3\right)^p\,dx=\int \left(ax^2+bx^3\right)^p\,dx$$

input
```
integrate((b*x**3+a*x**2)**p,x)
```

output
```
Integral((a*x**2 + b*x**3)**p, x)
```

Maxima [F]

$$\int \left(ax^2+bx^3\right)^p\,dx=\int \left(bx^3+ax^2\right)^p\,dx$$

input
```
integrate((b*x^3+a*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^p, x)
```

Giac [F]

$$\int \left(ax^2+bx^3\right)^p\,dx=\int \left(bx^3+ax^2\right)^p\,dx$$

input
```
integrate((b*x^3+a*x^2)^p,x, algorithm="giac")
```

output
```
integrate((b*x^3 + a*x^2)^p, x)
```

Mupad [B] (verification not implemented)

Time = 10.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.17

$$\int \left(ax^2+bx^3\right)^p\,dx=\frac{x\,\left(b\,x^3+a\,x^2\right)^p\,{}_2\mathrm{F}_1\left(2\,p+1,-p;\ 2\,p+2;\ -\frac{b\,x}{a}\right)}{\left(2\,p+1\right)\,\left(\frac{b\,x}{a}+1\right)^p}$$

input
```
int((a*x^2 + b*x^3)^p,x)
```

output
```
(x*(a*x^2 + b*x^3)^p*hypergeom([2*p + 1, -p], 2*p + 2, -(b*x)/a))/((2*p +
1)*((b*x)/a + 1)^p)
```

Reduce [F]

$$\int \left(ax^2+bx^3\right)^p\,dx$$
$$=\frac{\left(b\,x^3+a\,x^2\right)^p a+3\left(b\,x^3+a\,x^2\right)^p bx-6\left(\int \frac{\left(b\,x^3+a\,x^2\right)^p}{3bp\,x^2+3apx+b\,x^2+ax}dx\right)a^2p^2-2\left(\int \frac{\left(b\,x^3+a\,x^2\right)^p}{3bp\,x^2+3apx+b\,x^2+ax}dx\right)a^2p}{3b\left(3p+1\right)}$$

input
```
int((b*x^3+a*x^2)^p,x)
```

output
```
((a*x**2 + b*x**3)**p*a + 3*(a*x**2 + b*x**3)**p*b*x - 6*int((a*x**2 + b*x
**3)**p/(3*a*p*x + a*x + 3*b*p*x**2 + b*x**2),x)*a**2*p**2 - 2*int((a*x**2
 + b*x**3)**p/(3*a*p*x + a*x + 3*b*p*x**2 + b*x**2),x)*a**2*p)/(3*b*(3*p +
 1))
```

3.126 $\int (ax^n + bx^{1+n})^3 \, dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int (ax^n + bx^{1+n})^3 \, dx = \frac{ab^2x^{3(1+n)}}{1+n} + \frac{a^3x^{1+3n}}{1+3n} + \frac{3a^2bx^{2+3n}}{2+3n} + \frac{b^3x^{4+3n}}{4+3n}$$

output
```
a*b^2*x^(3+3*n)/(1+n)+a^3*x^(1+3*n)/(1+3*n)+3*a^2*b*x^(2+3*n)/(2+3*n)+b^3*
x^(4+3*n)/(4+3*n)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.82

$$\int (ax^n + bx^{1+n})^3 \, dx = x^{1+3n}\left(\frac{a^3}{1+3n} + \frac{3a^2bx}{2+3n} + \frac{ab^2x^2}{1+n} + \frac{b^3x^3}{4+3n}\right)$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^3,x]
```

output
```
x^(1 + 3*n)*(a^3/(1 + 3*n) + (3*a^2*b*x)/(2 + 3*n) + (a*b^2*x^2)/(1 + n) +
 (b^3*x^3)/(4 + 3*n))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2027, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^n+bx^{n+1}\right)^3\,dx$$

$$\Big\downarrow \ 2027$$

$$\int x^{3n}(a+bx)^3dx$$

$$\Big\downarrow \ 53$$

$$\int \left(a^3x^{3n}+3a^2bx^{3n+1}+3ab^2x^{3n+2}+b^3x^{3(n+1)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{a^3x^{3n+1}}{3n+1}+\frac{3a^2bx^{3n+2}}{3n+2}+\frac{ab^2x^{3(n+1)}}{n+1}+\frac{b^3x^{3n+4}}{3n+4}$$

input
```
Int[(a*x^n + b*x^(1 + n))^3,x]
```

output
```
(a*b^2*x^(3*(1 + n)))/(1 + n) + (a^3*x^(1 + 3*n))/(1 + 3*n) + (3*a^2*b*x^(
2 + 3*n))/(2 + 3*n) + (b^3*x^(4 + 3*n))/(4 + 3*n)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(74) = 148$.

Time = 0.34 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.42

method	result
risch	$\frac{x\left(9b^3n^3x^3+27a\,b^2n^3x^2+18b^3n^2x^3+27a^2b\,n^3x+63a\,b^2n^2x^2+11n\,x^3b^3+9a^3n^3+72a^2b\,n^2x+42n\,x^2a\,b^2+2b^3x^3+27a^3n^2+57\ldots\right.}{(1+3n)(2+3n)(1+n)(4+3n)}$
orering	$\frac{\left(9b^3n^3x^3+27a\,b^2n^3x^2+18b^3n^2x^3+27a^2b\,n^3x+63a\,b^2n^2x^2+11n\,x^3b^3+9a^3n^3+72a^2b\,n^2x+42n\,x^2a\,b^2+2b^3x^3+27a^3n^2+57n\ldots\right.}{(1+3n)(2+3n)(1+n)(4+3n)(bx+a)^3}$
parallelrisch	$\frac{9x\,x^{3n}a^3n^3+27x\,x^{2n}x^{1+n}a^2b\,n^3+27x\,x^nx^{2+2n}a\,b^2n^3+9x\,x^{3+3n}b^3n^3+27x\,x^{3n}a^3n^2+72x\,x^{2n}x^{1+n}a^2b\,n^2+63x\,x^nx^{2+2n}a\,b\ldots}{(\ldots}$

input
```
int((a*x^n+b*x^(1+n))^3,x,method=_RETURNVERBOSE)
```

output
```
x*(9*b^3*n^3*x^3+27*a*b^2*n^3*x^2+18*b^3*n^2*x^3+27*a^2*b*n^3*x+63*a*b^2*n
^2*x^2+11*b^3*n*x^3+9*a^3*n^3+72*a^2*b*n^2*x+42*a*b^2*n*x^2+2*b^3*x^3+27*a
^3*n^2+57*a^2*b*n*x+8*a*b^2*x^2+26*a^3*n+12*a^2*b*x+8*a^3)/(1+3*n)/(2+3*n)
/(1+n)/(4+3*n)*(x^n)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(74) = 148$.

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.22

$$\int \left(ax^n+bx^{1+n}\right)^3\,dx$$
$$=\frac{(9\,a^3n^3+27\,a^3n^2+26\,a^3n+(9\,b^3n^3+18\,b^3n^2+11\,b^3n+2\,b^3)x^3+8\,a^3+(27\,ab^2n^3+63\,ab^2n^2+42\,ab^2\ldots}{(27\,n^4+90\,n^3+105\,n^2+50\,n+8)x^2\ldots}$$

input
```
integrate((a*x^n+b*x^(1+n))^3,x, algorithm="fricas")
```

output
```
(9*a^3*n^3 + 27*a^3*n^2 + 26*a^3*n + (9*b^3*n^3 + 18*b^3*n^2 + 11*b^3*n +
2*b^3)*x^3 + 8*a^3 + (27*a*b^2*n^3 + 63*a*b^2*n^2 + 42*a*b^2*n + 8*a*b^2)*
x^2 + 3*(9*a^2*b*n^3 + 24*a^2*b*n^2 + 19*a^2*b*n + 4*a^2*b)*x)*x^(3*n + 3)
/((27*n^4 + 90*n^3 + 105*n^2 + 50*n + 8)*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 753 vs. $2(63) = 126$.

Time = 0.84 (sec) , antiderivative size = 753, normalized size of antiderivative = 10.18

$$\int \left(ax^n + bx^{1+n}\right)^3 dx$$
$$= \begin{cases} -\frac{a^3}{3x^3} - \frac{3a^2b}{2x^2} - \frac{3ab^2}{x} + b^3\log(x) \\ -\frac{a^3}{2x^2} - \frac{3a^2b}{x} + 3ab^2\log(x) + b^3x \\ -\frac{a^3}{x} + 3a^2b\log(x) + 3ab^2x + \frac{b^3x^2}{2} \\ a^3\log(x) + 3a^2bx + \frac{3ab^2x^2}{2} + \frac{b^3x^3}{3} \\ \frac{9a^3n^3xx^{3n}}{27n^4+90n^3+105n^2+50n+8} + \frac{27a^3n^2xx^{3n}}{27n^4+90n^3+105n^2+50n+8} + \frac{26a^3nxx^{3n}}{27n^4+90n^3+105n^2+50n+8} + \frac{8a^3xx^{3n}}{27n^4+90n^3+105n^2+50n+8} + \frac{27a}{27n^4+9} \end{cases}$$

input
```
integrate((a*x**n+b*x**(1+n))**3,x)
```

output
```
Piecewise((-a**3/(3*x**3) - 3*a**2*b/(2*x**2) - 3*a*b**2/x + b**3*log(x),
Eq(n, -4/3)), (-a**3/(2*x**2) - 3*a**2*b/x + 3*a*b**2*log(x) + b**3*x, Eq(
n, -1)), (-a**3/x + 3*a**2*b*log(x) + 3*a*b**2*x + b**3*x**2/2, Eq(n, -2/3
)), (a**3*log(x) + 3*a**2*b*x + 3*a*b**2*x**2/2 + b**3*x**3/3, Eq(n, -1/3)
), (9*a**3*n**3*x*x**(3*n)/(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8) + 27*
a**3*n**2*x*x**(3*n)/(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8) + 26*a**3*n
*x*x**(3*n)/(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8) + 8*a**3*x*x**(3*n)/
(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8) + 27*a**2*b*n**3*x*x**(2*n)*x**(
n + 1)/(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8) + 72*a**2*b*n**2*x*x**(2*
n)*x**(n + 1)/(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8) + 57*a**2*b*n*x*x*
*(2*n)*x**(n + 1)/(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8) + 12*a**2*b*x*
x**(2*n)*x**(n + 1)/(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8) + 27*a*b**2*
n**3*x*x**n*x**(2*n + 2)/(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8) + 63*a*
b**2*n**2*x*x**n*x**(2*n + 2)/(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8) +
42*a*b**2*n*x*x**n*x**(2*n + 2)/(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8)
+ 8*a*b**2*x*x**n*x**(2*n + 2)/(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8) +
 9*b**3*n**3*x*x**(3*n + 3)/(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8) + 18
*b**3*n**2*x*x**(3*n + 3)/(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8) + 11*b
**3*n*x*x**(3*n + 3)/(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8) + 2*b**3*x*
x**(3*n + 3)/(27*n**4 + 90*n**3 + 105*n**2 + 50*n + 8), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \left(ax^n+bx^{1+n}\right)^3\,dx=\frac{b^3x^{3\,n+4}}{3\,n+4}+\frac{ab^2x^{3\,n+3}}{n+1}+\frac{3\,a^2bx^{3\,n+2}}{3\,n+2}+\frac{a^3x^{3\,n+1}}{3\,n+1}$$

input
```
integrate((a*x^n+b*x^(1+n))^3,x, algorithm="maxima")
```

output
```
b^3*x^(3*n + 4)/(3*n + 4) + a*b^2*x^(3*n + 3)/(n + 1) + 3*a^2*b*x^(3*n + 2
)/(3*n + 2) + a^3*x^(3*n + 1)/(3*n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(74) = 148$.

Time = 0.12 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.51

$$\int (ax^n + bx^{1+n})^3 \, dx$$
$$= \frac{9 b^3n^3x^4x^{3n} + 27 ab^2n^3x^3x^{3n} + 18 b^3n^2x^4x^{3n} + 27 a^2bn^3x^2x^{3n} + 63 ab^2n^2x^3x^{3n} + 11 b^3nx^4x^{3n} + 9 a^3n^3 \cdots}{2 \cdots}$$

input
```
integrate((a*x^n+b*x^(1+n))^3,x, algorithm="giac")
```

output
```
(9*b^3*n^3*x^4*x^(3*n) + 27*a*b^2*n^3*x^3*x^(3*n) + 18*b^3*n^2*x^4*x^(3*n)
 + 27*a^2*b*n^3*x^2*x^(3*n) + 63*a*b^2*n^2*x^3*x^(3*n) + 11*b^3*n*x^4*x^(3
*n) + 9*a^3*n^3*x*x^(3*n) + 72*a^2*b*n^2*x^2*x^(3*n) + 42*a*b^2*n*x^3*x^(3
*n) + 2*b^3*x^4*x^(3*n) + 27*a^3*n^2*x*x^(3*n) + 57*a^2*b*n*x^2*x^(3*n) +
8*a*b^2*x^3*x^(3*n) + 26*a^3*n*x*x^(3*n) + 12*a^2*b*x^2*x^(3*n) + 8*a^3*x*
x^(3*n))/(27*n^4 + 90*n^3 + 105*n^2 + 50*n + 8)
```

Mupad [B] (verification not implemented)

Time = 9.81 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int (ax^n + bx^{1+n})^3 \, dx = \frac{b^3 x^{3n} x^4}{3n+4} + \frac{a^3 x x^{3n}}{3n+1} + \frac{a b^2 x^{3n} x^3}{n+1} + \frac{3 a^2 b x^{3n} x^2}{3n+2}$$

input
```
int((a*x^n + b*x^(n + 1))^3,x)
```

output
```
(b^3*x^(3*n)*x^4)/(3*n + 4) + (a^3*x*x^(3*n))/(3*n + 1) + (a*b^2*x^(3*n)*x
^3)/(n + 1) + (3*a^2*b*x^(3*n)*x^2)/(3*n + 2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.35

$$\int \left(ax^n + bx^{1+n}\right)^3 \, dx$$
$$= \frac{x^{3n}x(9b^3n^3x^3 + 27a\,b^2n^3x^2 + 18b^3n^2x^3 + 27a^2b\,n^3x + 63a\,b^2n^2x^2 + 11b^3n\,x^3 + 9a^3n^3 + 72a^2b\,n^2x + 42a}{27n^4 + 90n^3 + 105n^2 + 50n + 8}$$

input
```
int((a*x^n+b*x^(1+n))^3,x)
```

output
```
(x**(3*n)*x*(9*a**3*n**3 + 27*a**3*n**2 + 26*a**3*n + 8*a**3 + 27*a**2*b*n
**3*x + 72*a**2*b*n**2*x + 57*a**2*b*n*x + 12*a**2*b*x + 27*a*b**2*n**3*x*
*2 + 63*a*b**2*n**2*x**2 + 42*a*b**2*n*x**2 + 8*a*b**2*x**2 + 9*b**3*n**3*
x**3 + 18*b**3*n**2*x**3 + 11*b**3*n*x**3 + 2*b**3*x**3))/(27*n**4 + 90*n*
*3 + 105*n**2 + 50*n + 8)
```

3.127 $\int \left(ax^n+bx^{1+n}\right)^2\,dx$

Optimal result

Integrand size = 15, antiderivative size = 52

$$\int \left(ax^n+bx^{1+n}\right)^2\,dx=\frac{abx^{2(1+n)}}{1+n}+\frac{a^2x^{1+2n}}{1+2n}+\frac{b^2x^{3+2n}}{3+2n}$$

output
```
a*b*x^(2+2*n)/(1+n)+a^2*x^(1+2*n)/(1+2*n)+b^2*x^(3+2*n)/(3+2*n)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.83

$$\int \left(ax^n+bx^{1+n}\right)^2\,dx=x^{1+2n}\left(\frac{a^2}{1+2n}+\frac{abx}{1+n}+\frac{b^2x^2}{3+2n}\right)$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^2,x]
```

output
```
x^(1 + 2*n)*(a^2/(1 + 2*n) + (a*b*x)/(1 + n) + (b^2*x^2)/(3 + 2*n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {2027, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^n + bx^{n+1}\right)^2 \, dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int x^{2n}(a+bx)^2dx$$

$$\Big\downarrow \text{ 53}$$

$$\int \left(a^2x^{2n} + 2abx^{2n+1} + b^2x^{2(n+1)}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2x^{2n+1}}{2n+1} + \frac{abx^{2(n+1)}}{n+1} + \frac{b^2x^{2n+3}}{2n+3}$$

input
```
Int[(a*x^n + b*x^(1 + n))^2,x]
```

output
```
(a*b*x^(2*(1 + n)))/(1 + n) + (a^2*x^(1 + 2*n))/(1 + 2*n) + (b^2*x^(3 + 2*
n))/(3 + 2*n)
```

Defintions of rubi rules used

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.15

method	result
norman	$\frac{a^2 x \,e^{2n\ln(x)}}{1+2n}+\frac{b^2 x^3 e^{2n\ln(x)}}{3+2n}+\frac{ab\,x^2 e^{2n\ln(x)}}{1+n}$
risch	$\frac{x\left(2b^2n^2x^2+4ab\,n^2x+3n\,x^2b^2+2a^2n^2+8nxab+b^2x^2+5n\,a^2+3abx+3a^2\right)x^{2n}}{(1+2n)(1+n)(3+2n)}$
orering	$\frac{\left(2b^2n^2x^2+4ab\,n^2x+3n\,x^2b^2+2a^2n^2+8nxab+b^2x^2+5n\,a^2+3abx+3a^2\right)x\left(a\,x^n+b\,x^{1+n}\right)^2}{(1+2n)(1+n)(3+2n)(bx+a)^2}$
parallelrisch	$\frac{2x\,x^{2n}a^2n^2+4x\,x^n x^{1+n}ab\,n^2+2x\,x^{2+2n}b^2n^2+5x\,x^{2n}a^2n+8x\,x^n x^{1+n}abn+3x\,x^{2+2n}b^2n+3x\,x^{2n}a^2+3x\,x^n x^{1+n}ab+x\,x^{2+2}\cdots}{(1+2n)(1+n)(3+2n)}$

input
```
int((a*x^n+b*x^(1+n))^2,x,method=_RETURNVERBOSE)
```

output
```
a^2/(1+2*n)*x*exp(n*ln(x))^2+b^2/(3+2*n)*x^3*exp(n*ln(x))^2+a*b/(1+n)*x^2*
exp(n*ln(x))^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.71

$$\int \left(ax^n+bx^{1+n}\right)^2\,dx = \frac{\left(2\,a^2n^2+5\,a^2n+\left(2\,b^2n^2+3\,b^2n+b^2\right)x^2+3\,a^2+\left(4\,abn^2+8\,abn+3\,ab\right)x\right)x^{2\,n+2}}{\left(4\,n^3+12\,n^2+11\,n+3\right)x}$$

input
```
integrate((a*x^n+b*x^(1+n))^2,x, algorithm="fricas")
```

output
```
(2*a^2*n^2 + 5*a^2*n + (2*b^2*n^2 + 3*b^2*n + b^2)*x^2 + 3*a^2 + (4*a*b*n^
2 + 8*a*b*n + 3*a*b)*x)*x^(2*n + 2)/((4*n^3 + 12*n^2 + 11*n + 3)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(42) = 84$.

Time = 0.42 (sec) , antiderivative size = 340, normalized size of antiderivative = 6.54

$$\int \left(ax^n + bx^{1+n}\right)^2 \, dx$$
$$= \begin{cases} -\frac{a^2}{2x^2} - \frac{2ab}{x} + b^2 \log\left(x\right) \\ -\frac{a^2}{x} + 2ab \log\left(x\right) + b^2 x \\ a^2 \log\left(x\right) + 2abx + \frac{b^2 x^2}{2} \\ \frac{2a^2 n^2 x x^{2n}}{4n^3+12n^2+11n+3} + \frac{5a^2 n x x^{2n}}{4n^3+12n^2+11n+3} + \frac{3a^2 x x^{2n}}{4n^3+12n^2+11n+3} + \frac{4abn^2 x x^n x^{n+1}}{4n^3+12n^2+11n+3} + \frac{8abn x x^n x^{n+1}}{4n^3+12n^2+11n+3} + \frac{3ab x x^n x^{n+1}}{4n^3+12n^2+11n+3} \end{cases}$$

input
```
integrate((a*x**n+b*x**(1+n))**2,x)
```

output
```
Piecewise((-a**2/(2*x**2) - 2*a*b/x + b**2*log(x), Eq(n, -3/2)), (-a**2/x
+ 2*a*b*log(x) + b**2*x, Eq(n, -1)), (a**2*log(x) + 2*a*b*x + b**2*x**2/2,
 Eq(n, -1/2)), (2*a**2*n**2*x*x**(2*n)/(4*n**3 + 12*n**2 + 11*n + 3) + 5*a
**2*n*x*x**(2*n)/(4*n**3 + 12*n**2 + 11*n + 3) + 3*a**2*x*x**(2*n)/(4*n**3
 + 12*n**2 + 11*n + 3) + 4*a*b*n**2*x*x**n*x**(n + 1)/(4*n**3 + 12*n**2 +
11*n + 3) + 8*a*b*n*x*x**n*x**(n + 1)/(4*n**3 + 12*n**2 + 11*n + 3) + 3*a*
b*x*x**n*x**(n + 1)/(4*n**3 + 12*n**2 + 11*n + 3) + 2*b**2*n**2*x*x**(2*n
+ 2)/(4*n**3 + 12*n**2 + 11*n + 3) + 3*b**2*n*x*x**(2*n + 2)/(4*n**3 + 12*
n**2 + 11*n + 3) + b**2*x*x**(2*n + 2)/(4*n**3 + 12*n**2 + 11*n + 3), True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \left(ax^n + bx^{1+n}\right)^2 \, dx = \frac{b^2 x^{2n+3}}{2n+3} + \frac{abx^{2n+2}}{n+1} + \frac{a^2 x^{2n+1}}{2n+1}$$

input
```
integrate((a*x^n+b*x^(1+n))^2,x, algorithm="maxima")
```

output
```
b^2*x^(2*n + 3)/(2*n + 3) + a*b*x^(2*n + 2)/(n + 1) + a^2*x^(2*n + 1)/(2*n
 + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(52) = 104$.

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.65

$$\int \left(ax^n + bx^{1+n}\right)^2\,dx$$
$$= \frac{2\,b^2n^2x^3x^{2\,n} + 4\,abn^2x^2x^{2\,n} + 3\,b^2nx^3x^{2\,n} + 2\,a^2n^2xx^{2\,n} + 8\,abnx^2x^{2\,n} + b^2x^3x^{2\,n} + 5\,a^2nxx^{2\,n} + 3\,abx^2x^2}{4\,n^3 + 12\,n^2 + 11\,n + 3}$$

input
```
integrate((a*x^n+b*x^(1+n))^2,x, algorithm="giac")
```

output
```
(2*b^2*n^2*x^3*x^(2*n) + 4*a*b*n^2*x^2*x^(2*n) + 3*b^2*n*x^3*x^(2*n) + 2*a
^2*n^2*x*x^(2*n) + 8*a*b*n*x^2*x^(2*n) + b^2*x^3*x^(2*n) + 5*a^2*n*x*x^(2*
n) + 3*a*b*x^2*x^(2*n) + 3*a^2*x*x^(2*n))/(4*n^3 + 12*n^2 + 11*n + 3)
```

Mupad [B] (verification not implemented)

Time = 9.68 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.02

$$\int \left(ax^n + bx^{1+n}\right)^2\,dx = \frac{b^2\,x^{2\,n}\,x^3}{2\,n+3} + \frac{a^2\,x\,x^{2\,n}}{2\,n+1} + \frac{a\,b\,x^{2\,n}\,x^2}{n+1}$$

input
```
int((a*x^n + b*x^(n + 1))^2,x)
```

output
```
(b^2*x^(2*n)*x^3)/(2*n + 3) + (a^2*x*x^(2*n))/(2*n + 1) + (a*b*x^(2*n)*x^2
)/(n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.73

$$\int (ax^n + bx^{1+n})^2 \, dx = \frac{x^{2n}x(2b^2n^2x^2 + 4ab\,n^2x + 3b^2n\,x^2 + 2a^2n^2 + 8abnx + b^2x^2 + 5a^2n + 3abx + 3a^2)}{4n^3 + 12n^2 + 11n + 3}$$

input
```
int((a*x^n+b*x^(1+n))^2,x)
```

output
```
(x**(2*n)*x*(2*a**2*n**2 + 5*a**2*n + 3*a**2 + 4*a*b*n**2*x + 8*a*b*n*x +
3*a*b*x + 2*b**2*n**2*x**2 + 3*b**2*n*x**2 + b**2*x**2))/(4*n**3 + 12*n**2
 + 11*n + 3)
```

3.128 $\int (ax^n + bx^{1+n}) \, dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int (ax^n + bx^{1+n}) \, dx = \frac{ax^{1+n}}{1+n} + \frac{bx^{2+n}}{2+n}$$

output
```
a*x^(1+n)/(1+n)+b*x^(2+n)/(2+n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int (ax^n + bx^{1+n}) \, dx = x^{1+n}\left(\frac{a}{1+n} + \frac{bx}{2+n}\right)$$

input
```
Integrate[a*x^n + b*x^(1 + n),x]
```

output
```
x^(1 + n)*(a/(1 + n) + (b*x)/(2 + n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^n+bx^{n+1}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{ax^{n+1}}{n+1}+\frac{bx^{n+2}}{n+2}$$

input
```
Int[a*x^n + b*x^(1 + n),x]
```

output
```
(a*x^(1 + n))/(1 + n) + (b*x^(2 + n))/(2 + n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{a\,x^{1+n}}{1+n}+\frac{b\,x^{2+n}}{2+n}$	26
risch	$\frac{ax\,x^{n}}{1+n}+\frac{bx\,x^{1+n}}{2+n}$	26
parts	$\frac{a\,x^{1+n}}{1+n}+\frac{b\,x^{2+n}}{2+n}$	26
norman	$\frac{ax\,\mathrm{e}^{n\ln(x)}}{1+n}+\frac{b\,x^{2}\mathrm{e}^{n\ln(x)}}{2+n}$	30
parallelrisch	$\frac{a\,x^{n}xn+b\,x^{1+n}xn+2a\,x^{n}x+b\,x^{1+n}x}{(1+n)(2+n)}$	44
orering	$\frac{(nxb+an+bx+2a)x(a\,x^{n}+b\,x^{1+n})}{(1+n)(2+n)(bx+a)}$	47

input
```
int(a*x^n+b*x^(1+n),x,method=_RETURNVERBOSE)
```

output
```
a*x^(1+n)/(1+n)+b*x^(2+n)/(2+n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \left(ax^n+bx^{1+n}\right)\,dx=\frac{(an+(bn+b)x+2\,a)x^{n+1}}{n^2+3\,n+2}$$

input
```
integrate(a*x^n+b*x^(1+n),x, algorithm="fricas")
```

output
```
(a*n + (b*n + b)*x + 2*a)*x^(n + 1)/(n^2 + 3*n + 2)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \left(ax^n + bx^{1+n}\right) dx = a\left(\begin{cases} \frac{x^{n+1}}{n+1} & \text{for } n \neq -1 \\ \log\left(x\right) & \text{otherwise} \end{cases}\right) + b\left(\begin{cases} \frac{x^{n+2}}{n+2} & \text{for } n \neq -2 \\ \log\left(x\right) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a*x**n+b*x**(1+n),x)
```

output
```
a*Piecewise((x**(n + 1)/(n + 1), Ne(n, -1)), (log(x), True)) + b*Piecewise
((x**(n + 2)/(n + 2), Ne(n, -2)), (log(x), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(ax^n + bx^{1+n}\right) dx = \frac{bx^{n+2}}{n+2} + \frac{ax^{n+1}}{n+1}$$

input
```
integrate(a*x^n+b*x^(1+n),x, algorithm="maxima")
```

output
```
b*x^(n + 2)/(n + 2) + a*x^(n + 1)/(n + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(ax^n + bx^{1+n}\right) dx = \frac{bx^{n+2}}{n+2} + \frac{ax^{n+1}}{n+1}$$

input
```
integrate(a*x^n+b*x^(1+n),x, algorithm="giac")
```

output
```
b*x^(n + 2)/(n + 2) + a*x^(n + 1)/(n + 1)
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \left(ax^n+bx^{1+n}\right)\,dx=\frac{b\,x^n\,x^2}{n+2}+\frac{a\,x\,x^n}{n+1}$$

input
```
int(a*x^n + b*x^(n + 1),x)
```

output
```
(b*x^n*x^2)/(n + 2) + (a*x*x^n)/(n + 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.16

$$\int \left(ax^n+bx^{1+n}\right)\,dx=\frac{x^n x(bnx+an+bx+2a)}{n^2+3n+2}$$

input
```
int(a*x^n+b*x^(1+n),x)
```

output
```
(x**n*x*(a*n + 2*a + b*n*x + b*x))/(n**2 + 3*n + 2)
```

3.129 $\int \frac{1}{ax^n+bx^{1+n}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{ax^n+bx^{1+n}}\,dx=\frac{x^{1-n}\,\text{Hypergeometric2F1}\left(1,1-n,2-n,-\frac{bx}{a}\right)}{a(1-n)}$$

output
```
x^(1-n)*hypergeom([1, 1-n],[2-n],-b*x/a)/a/(1-n)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^n+bx^{1+n}}\,dx=\frac{x^{1-n}\,\text{Hypergeometric2F1}\left(1,1-n,2-n,-\frac{bx}{a}\right)}{a(1-n)}$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^(-1),x]
```

output
```
(x^(1 - n)*Hypergeometric2F1[1, 1 - n, 2 - n, -((b*x)/a)])/(a*(1 - n))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2027, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{ax^n + bx^{n+1}} \, dx$$

$$\Big\downarrow \; 2027$$

$$\int \frac{x^{-n}}{a + bx} dx$$

$$\Big\downarrow \; 74$$

$$\frac{x^{1-n} \text{Hypergeometric2F1}\left(1, 1-n, 2-n, -\frac{bx}{a}\right)}{a(1-n)}$$

input
```
Int[(a*x^n + b*x^(1 + n))^(-1),x]
```

output
```
(x^(1 - n)*Hypergeometric2F1[1, 1 - n, 2 - n, -((b*x)/a)])/(a*(1 - n))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [F]

$$\int \frac{1}{a\,x^{n}+b\,x^{1+n}}dx$$

input
```
int(1/(a*x^n+b*x^(1+n)),x)
```

output
```
int(1/(a*x^n+b*x^(1+n)),x)
```

Fricas [F]

$$\int \frac{1}{ax^n + bx^{1+n}}\,dx = \int \frac{1}{bx^{n+1} + ax^n}\,dx$$

input
```
integrate(1/(a*x^n+b*x^(1+n)),x, algorithm="fricas")
```

output
```
integral(1/(b*x^(n + 1) + a*x^n), x)
```

Sympy [F]

$$\int \frac{1}{ax^n + bx^{1+n}}\,dx = \int \frac{1}{ax^n + bx^{n+1}}\,dx$$

input
```
integrate(1/(a*x**n+b*x**(1+n)),x)
```

output
```
Integral(1/(a*x**n + b*x**(n + 1)), x)
```

Maxima [F]

$$\int \frac{1}{ax^n + bx^{1+n}}\,dx = \int \frac{1}{bx^{n+1} + ax^n}\,dx$$

input
```
integrate(1/(a*x^n+b*x^(1+n)),x, algorithm="maxima")
```

output
```
integrate(1/(b*x^(n + 1) + a*x^n), x)
```

Giac [F]

$$\int \frac{1}{ax^n + bx^{1+n}}\,dx = \int \frac{1}{bx^{n+1} + ax^n}\,dx$$

input
```
integrate(1/(a*x^n+b*x^(1+n)),x, algorithm="giac")
```

output
```
integrate(1/(b*x^(n + 1) + a*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.06 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{1}{ax^n + bx^{1+n}}\,dx = \frac{x\,{}_2\mathrm{F}_1\left(1, 1-n;\ 2-n;\ -\frac{b\,x}{a}\right)}{a\,x^n - a\,n\,x^n}$$

input
```
int(1/(a*x^n + b*x^(n + 1)),x)
```

output
```
(x*hypergeom([1, 1 - n], 2 - n, -(b*x)/a))/(a*x^n - a*n*x^n)
```

Reduce [F]

$$\int \frac{1}{ax^n + bx^{1+n}}\,dx = \frac{-x^n\left(\int \frac{1}{x^n ax + x^n b\,x^2}dx\right)an - 1}{x^n bn}$$

input
```
int(1/(a*x^n+b*x^(1+n)),x)
```

output
```
( - (x**n*int(1/(x**n*a*x + x**n*b*x**2),x)*a*n + 1))/(x**n*b*n)
```

3.130 $\int \frac{1}{(ax^n+bx^{1+n})^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{(ax^n+bx^{1+n})^2}\,dx = \frac{x^{1-2n}\,\text{Hypergeometric2F1}\left(2,1-2n,2-2n,-\frac{bx}{a}\right)}{a^2(1-2n)}$$

output
```
x^(1-2*n)*hypergeom([2, 1-2*n],[2-2*n],-b*x/a)/a^2/(1-2*n)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax^n+bx^{1+n})^2}\,dx = \frac{x^{1-2n}\,\text{Hypergeometric2F1}\left(2,1-2n,2-2n,-\frac{bx}{a}\right)}{a^2(1-2n)}$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^(-2),x]
```

output
```
(x^(1 - 2*n)*Hypergeometric2F1[2, 1 - 2*n, 2 - 2*n, -((b*x)/a)])/(a^2*(1 - 
 2*n))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2027, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^n + bx^{n+1})^2} \, dx$$
$$\Big\downarrow \ 2027$$
$$\int \frac{x^{-2n}}{(a+bx)^2} dx$$
$$\Big\downarrow \ 74$$
$$\frac{x^{1-2n} \text{Hypergeometric2F1}\left(2, 1-2n, 2-2n, -\frac{bx}{a}\right)}{a^2(1-2n)}$$

input
```
Int[(a*x^n + b*x^(1 + n))^(-2),x]
```

output
```
(x^(1 - 2*n)*Hypergeometric2F1[2, 1 - 2*n, 2 - 2*n, -((b*x)/a)])/(a^2*(1 -
 2*n))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [F]

$$\int \frac{1}{\left(a\,x^{n}+b\,x^{1+n}\right)^{2}}dx$$

input
```
int(1/(a*x^n+b*x^(1+n))^2,x)
```

output
```
int(1/(a*x^n+b*x^(1+n))^2,x)
```

Fricas [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^2}\,dx = \int \frac{1}{(bx^{n+1} + ax^n)^2}\,dx$$

input
```
integrate(1/(a*x^n+b*x^(1+n))^2,x, algorithm="fricas")
```

output
```
integral(1/(2*a*b*x^(n + 1)*x^n + a^2*x^(2*n) + b^2*x^(2*n + 2)), x)
```

Sympy [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^2}\,dx = \int \frac{1}{(ax^n + bx^{n+1})^2}\,dx$$

input
```
integrate(1/(a*x**n+b*x**(1+n))**2,x)
```

output
```
Integral((a*x**n + b*x**(n + 1))**(-2), x)
```

Maxima [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^2}\,dx = \int \frac{1}{(bx^{n+1} + ax^n)^2}\,dx$$

input
```
integrate(1/(a*x^n+b*x^(1+n))^2,x, algorithm="maxima")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(-2), x)
```

Giac [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^2}\,dx = \int \frac{1}{(bx^{n+1} + ax^n)^2}\,dx$$

input
```
integrate(1/(a*x^n+b*x^(1+n))^2,x, algorithm="giac")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(-2), x)
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{1}{(ax^n + bx^{1+n})^2}\,dx = \frac{x\,{}_2F_1\left(2, 1 - 2\,n;\ 2 - 2\,n;\ -\frac{b\,x}{a}\right)}{a^2\,x^{2\,n} - 2\,a^2\,n\,x^{2\,n}}$$

input
```
int(1/(a*x^n + b*x^(n + 1))^2,x)
```

output
```
(x*hypergeom([2, 1 - 2*n], 2 - 2*n, -(b*x)/a))/(a^2*x^(2*n) - 2*a^2*n*x^(2
*n))
```

Reduce [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^2}\,dx$$
$$= \frac{-4x^{2n}\left(\int \frac{x}{2x^{2n}a^2n - x^{2n}a^2 + 4x^{2n}abnx - 2x^{2n}abx + 2x^{2n}b^2n\,x^2 - x^{2n}b^2x^2}dx\right) ab\,n^2 + 2x^{2n}\left(\int \frac{x}{2x^{2n}a^2n - x^{2n}a^2 + 4x^{2n}abnx - 2x^{2n}abx}\right.}{}$$

input
```
int(1/(a*x^n+b*x^(1+n))^2,x)
```

output
```
( - 4*x**(2*n)*int(x/(2*x**(2*n)*a**2*n - x**(2*n)*a**2 + 4*x**(2*n)*a*b*n
*x - 2*x**(2*n)*a*b*x + 2*x**(2*n)*b**2*n*x**2 - x**(2*n)*b**2*x**2),x)*a*
b*n**2 + 2*x**(2*n)*int(x/(2*x**(2*n)*a**2*n - x**(2*n)*a**2 + 4*x**(2*n)*
a*b*n*x - 2*x**(2*n)*a*b*x + 2*x**(2*n)*b**2*n*x**2 - x**(2*n)*b**2*x**2),
x)*a*b*n - 4*x**(2*n)*int(x/(2*x**(2*n)*a**2*n - x**(2*n)*a**2 + 4*x**(2*n
)*a*b*n*x - 2*x**(2*n)*a*b*x + 2*x**(2*n)*b**2*n*x**2 - x**(2*n)*b**2*x**2
),x)*b**2*n**2*x + 2*x**(2*n)*int(x/(2*x**(2*n)*a**2*n - x**(2*n)*a**2 + 4
*x**(2*n)*a*b*n*x - 2*x**(2*n)*a*b*x + 2*x**(2*n)*b**2*n*x**2 - x**(2*n)*b
**2*x**2),x)*b**2*n*x - x)/(x**(2*n)*a*(2*a*n - a + 2*b*n*x - b*x))
```

3.131 $\int \frac{1}{(ax^n+bx^{1+n})^3} dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{1}{(ax^n + bx^{1+n})^3} dx = \frac{x^{1-3n} \text{Hypergeometric2F1}\left(3, 1-3n, 2-3n, -\frac{bx}{a}\right)}{a^3(1-3n)}$$

output
```
x^(1-3*n)*hypergeom([3, 1-3*n],[2-3*n],-b*x/a)/a^3/(1-3*n)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \frac{1}{(ax^n + bx^{1+n})^3} dx = \frac{x^{1-3n} \text{Hypergeometric2F1}\left(3, 1-3n, 2-3n, -\frac{bx}{a}\right)}{a^3(1-3n)}$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^(-3),x]
```

output
```
(x^(1 - 3*n)*Hypergeometric2F1[3, 1 - 3*n, 2 - 3*n, -((b*x)/a)])/(a^3*(1 - 
 3*n))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{2027, 74\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^n + bx^{n+1})^3} dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^{-3n}}{(a+bx)^3} dx$$

$$\Big\downarrow \text{74}$$

$$\frac{x^{1-3n}\text{Hypergeometric2F1}\left(3, 1-3n, 2-3n, -\frac{bx}{a}\right)}{a^3(1-3n)}$$

input
```
Int[(a*x^n + b*x^(1 + n))^(-3),x]
```

output
```
(x^(1 - 3*n)*Hypergeometric2F1[3, 1 - 3*n, 2 - 3*n, -((b*x)/a)])/(a^3*(1 -
 3*n))
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [F]

$$\int \frac{1}{(a\,x^n + b\,x^{1+n})^3}dx$$

input
```
int(1/(a*x^n+b*x^(1+n))^3,x)
```

output
```
int(1/(a*x^n+b*x^(1+n))^3,x)
```

Fricas [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^3}\,dx = \int \frac{1}{(bx^{n+1} + ax^n)^3}\,dx$$

input
```
integrate(1/(a*x^n+b*x^(1+n))^3,x, algorithm="fricas")
```

output
```
integral(1/(3*a^2*b*x^(2*n)*x^(n + 1) + a^3*x^(3*n) + (b^3*x^(n + 1) + 3*a
*b^2*x^n)*x^(2*n + 2)), x)
```

Sympy [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^3}\,dx = \int \frac{1}{(ax^n + bx^{n+1})^3}\,dx$$

input
```
integrate(1/(a*x**n+b*x**(1+n))**3,x)
```

output
```
Integral((a*x**n + b*x**(n + 1))**(-3), x)
```

Maxima [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^3}\,dx = \int \frac{1}{(bx^{n+1} + ax^n)^3}\,dx$$

input
```
integrate(1/(a*x^n+b*x^(1+n))^3,x, algorithm="maxima")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(-3), x)
```

Giac [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^3}\,dx = \int \frac{1}{(bx^{n+1} + ax^n)^3}\,dx$$

input
```
integrate(1/(a*x^n+b*x^(1+n))^3,x, algorithm="giac")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(-3), x)
```

Mupad [B] (verification not implemented)

Time = 9.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.22

$$\int \frac{1}{(ax^n + bx^{1+n})^3}\,dx = \frac{x\,{}_2F_1\left(3, 1-3\,n;\ 2-3\,n;\ -\frac{b\,x}{a}\right)}{a^3\,x^{3\,n} - 3\,a^3\,n\,x^{3\,n}}$$

input
```
int(1/(a*x^n + b*x^(n + 1))^3,x)
```

output
```
(x*hypergeom([3, 1 - 3*n], 2 - 3*n, -(b*x)/a))/(a^3*x^(3*n) - 3*a^3*n*x^(3
*n))
```

Reduce [F]

$$
\int \frac{1}{\left(ax^n + bx^{1+n}\right)^3}\,dx
$$
$$
= \frac{-9x^{3n}\left(\int \frac{x}{3x^{3n}a^3n - x^{3n}a^3 + 9x^{3n}a^2bnx - 3x^{3n}a^2bx + 9x^{3n}a\,b^2n\,x^2 - 3x^{3n}a\,b^2x^2 + 3x^{3n}b^3n\,x^3 - x^{3n}b^3x^3}dx\right)a^2b\,n^2 + x^{3n}\left(\int \frac{}{3x^{3n}a^3n-}\right.}{}
$$

input
```
int(1/(a*x^n+b*x^(1+n))^3,x)
```

output
```
( - 9*x**(3*n)*int(x/(3*x**(3*n)*a**3*n - x**(3*n)*a**3 + 9*x**(3*n)*a**2*
b*n*x - 3*x**(3*n)*a**2*b*x + 9*x**(3*n)*a*b**2*n*x**2 - 3*x**(3*n)*a*b**2
*x**2 + 3*x**(3*n)*b**3*n*x**3 - x**(3*n)*b**3*x**3),x)*a**2*b*n**2 + x**(
3*n)*int(x/(3*x**(3*n)*a**3*n - x**(3*n)*a**3 + 9*x**(3*n)*a**2*b*n*x - 3*
x**(3*n)*a**2*b*x + 9*x**(3*n)*a*b**2*n*x**2 - 3*x**(3*n)*a*b**2*x**2 + 3*
x**(3*n)*b**3*n*x**3 - x**(3*n)*b**3*x**3),x)*a**2*b - 18*x**(3*n)*int(x/(
3*x**(3*n)*a**3*n - x**(3*n)*a**3 + 9*x**(3*n)*a**2*b*n*x - 3*x**(3*n)*a**
2*b*x + 9*x**(3*n)*a*b**2*n*x**2 - 3*x**(3*n)*a*b**2*x**2 + 3*x**(3*n)*b**
3*n*x**3 - x**(3*n)*b**3*x**3),x)*a*b**2*n**2*x + 2*x**(3*n)*int(x/(3*x**(
3*n)*a**3*n - x**(3*n)*a**3 + 9*x**(3*n)*a**2*b*n*x - 3*x**(3*n)*a**2*b*x
+ 9*x**(3*n)*a*b**2*n*x**2 - 3*x**(3*n)*a*b**2*x**2 + 3*x**(3*n)*b**3*n*x*
*3 - x**(3*n)*b**3*x**3),x)*a*b**2*x - 9*x**(3*n)*int(x/(3*x**(3*n)*a**3*n
 - x**(3*n)*a**3 + 9*x**(3*n)*a**2*b*n*x - 3*x**(3*n)*a**2*b*x + 9*x**(3*n
)*a*b**2*n*x**2 - 3*x**(3*n)*a*b**2*x**2 + 3*x**(3*n)*b**3*n*x**3 - x**(3*
n)*b**3*x**3),x)*b**3*n**2*x**2 + x**(3*n)*int(x/(3*x**(3*n)*a**3*n - x**(
3*n)*a**3 + 9*x**(3*n)*a**2*b*n*x - 3*x**(3*n)*a**2*b*x + 9*x**(3*n)*a*b**
2*n*x**2 - 3*x**(3*n)*a*b**2*x**2 + 3*x**(3*n)*b**3*n*x**3 - x**(3*n)*b**3
*x**3),x)*b**3*x**2 - x)/(x**(3*n)*a*(3*a**2*n - a**2 + 6*a*b*n*x - 2*a*b*
x + 3*b**2*n*x**2 - b**2*x**2))
```

3.132 $\int (ax^n + bx^{1+n})^{5/2}\, dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int (ax^n + bx^{1+n})^{5/2}\, dx = \frac{2x^{-n}\left(-\frac{bx}{a}\right)^{-5n/2}(ax^n + bx^{1+n})^{7/2}\,\text{Hypergeometric2F1}\left(\frac{7}{2}, -\frac{5n}{2}, \frac{9}{2}, 1 + \frac{bx}{a}\right)}{7b}$$

output
```
2/7*(a*x^n+b*x^(1+n))^(7/2)*hypergeom([7/2, -5/2*n],[9/2],1+b*x/a)/b/(x^n)
/((-b*x/a)^(5/2*n))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int (ax^n + bx^{1+n})^{5/2}\, dx = \frac{2\left(-\frac{bx}{a}\right)^{-5n/2}(a + bx)\,(x^n(a + bx))^{5/2}\,\text{Hypergeometric2F1}\left(\frac{7}{2}, -\frac{5n}{2}, \frac{9}{2}, 1 + \frac{bx}{a}\right)}{7b}$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^(5/2),x]
```

output
```
(2*(a + b*x)*(x^n*(a + b*x))^(5/2)*Hypergeometric2F1[7/2, (-5*n)/2, 9/2, 1
 + (b*x)/a])/(7*b*(-((b*x)/a))^((5*n)/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1917, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^n+bx^{n+1}\right)^{5/2}\,dx$$

$$\Big\downarrow \text{ 1917}$$

$$\frac{x^{-n/2}\sqrt{ax^n+bx^{n+1}}\int x^{5n/2}(a+bx)^{5/2}dx}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 77}$$

$$\frac{x^{2n}\left(-\frac{bx}{a}\right)^{-5n/2}\sqrt{ax^n+bx^{n+1}}\int \left(-\frac{bx}{a}\right)^{5n/2}(a+bx)^{5/2}dx}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 75}$$

$$\frac{2x^{2n}(a+bx)^3\left(-\frac{bx}{a}\right)^{-5n/2}\sqrt{ax^n+bx^{n+1}}\,\text{Hypergeometric2F1}\left(\frac{7}{2},-\frac{5n}{2},\frac{9}{2},\frac{bx}{a}+1\right)}{7b}$$

input
```
Int[(a*x^n + b*x^(1 + n))^(5/2),x]
```

output
```
(2*x^(2*n)*(a + b*x)^3*Sqrt[a*x^n + b*x^(1 + n)]*Hypergeometric2F1[7/2, (-
5*n)/2, 9/2, 1 + (b*x)/a])/(7*b*(-((b*x)/a))^((5*n)/2))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \left(a\,x^{n}+b\,x^{1+n}\right)^{\frac{5}{2}}dx$$

input
```
int((a*x^n+b*x^(1+n))^(5/2),x)
```

output
```
int((a*x^n+b*x^(1+n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(ax^n+bx^{1+n}\right)^{5/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*x^n+b*x^(1+n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (ax^n + bx^{1+n})^{5/2} \, dx = \int (ax^n + bx^{n+1})^{\frac{5}{2}} \, dx$$

input
```
integrate((a*x**n+b*x**(1+n))**(5/2),x)
```

output
```
Integral((a*x**n + b*x**(n + 1))**(5/2), x)
```

Maxima [F]

$$\int (ax^n + bx^{1+n})^{5/2} \, dx = \int (bx^{n+1} + ax^n)^{\frac{5}{2}} \, dx$$

input
```
integrate((a*x^n+b*x^(1+n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(5/2), x)
```

Giac [F]

$$\int (ax^n + bx^{1+n})^{5/2} \, dx = \int (bx^{n+1} + ax^n)^{\frac{5}{2}} \, dx$$

input
```
integrate((a*x^n+b*x^(1+n))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.17 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \left(ax^n + bx^{1+n}\right)^{5/2} dx = \frac{x\,(a\,x^n + b\,x^{n+1})^{5/2}\,{}_2\mathrm{F}_1\left(-\frac{5}{2}, \frac{5\,n}{2} + 1;\ \frac{5\,n}{2} + 2;\ -\frac{b\,x}{a}\right)}{\left(\frac{5\,n}{2} + 1\right)\,\left(\frac{b\,x}{a} + 1\right)^{5/2}}$$

input
```
int((a*x^n + b*x^(n + 1))^(5/2),x)
```

output
```
(x*(a*x^n + b*x^(n + 1))^(5/2)*hypergeom([-5/2, (5*n)/2 + 1], (5*n)/2 + 2,
 -(b*x)/a))/(((5*n)/2 + 1)*((b*x)/a + 1)^(5/2))
```

Reduce [F]

$$\int \left(ax^n + bx^{1+n}\right)^{5/2} dx = \left(\int x^{\frac{5n}{2}} \sqrt{bx + a}\, x^2 dx\right) b^2 + 2\left(\int x^{\frac{5n}{2}} \sqrt{bx + a}\, x dx\right) ab + \left(\int x^{\frac{5n}{2}} \sqrt{bx + a} dx\right) a^2$$

input
```
int((a*x^n+b*x^(1+n))^(5/2),x)
```

output
```
int(x**((5*n)/2)*sqrt(a + b*x)*x**2,x)*b**2 + 2*int(x**((5*n)/2)*sqrt(a +
b*x)*x,x)*a*b + int(x**((5*n)/2)*sqrt(a + b*x),x)*a**2
```

3.133 $\int (ax^n+bx^{1+n})^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int (ax^n+bx^{1+n})^{3/2}\,dx=\frac{2x^{-n}\left(-\frac{bx}{a}\right)^{-3n/2}(ax^n+bx^{1+n})^{5/2}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{3n}{2},\frac{7}{2},1+\frac{bx}{a}\right)}{5b}$$

output
```
2/5*(a*x^n+b*x^(1+n))^(5/2)*hypergeom([5/2, -3/2*n],[7/2],1+b*x/a)/b/(x^n)
/((-b*x/a)^(3/2*n))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int (ax^n+bx^{1+n})^{3/2}\,dx=\frac{2\left(-\frac{bx}{a}\right)^{-3n/2}(a+bx)\,(x^n(a+bx))^{3/2}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{3n}{2},\frac{7}{2},1+\frac{bx}{a}\right)}{5b}$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^(3/2),x]
```

output
```
(2*(a + b*x)*(x^n*(a + b*x))^(3/2)*Hypergeometric2F1[5/2, (-3*n)/2, 7/2, 1
 + (b*x)/a])/(5*b*(-((b*x)/a))^((3*n)/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1917, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^n+bx^{n+1}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 1917}$$

$$\frac{x^{-n/2}\sqrt{ax^n+bx^{n+1}}\int x^{3n/2}(a+bx)^{3/2}dx}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 77}$$

$$\frac{x^n\left(-\frac{bx}{a}\right)^{-3n/2}\sqrt{ax^n+bx^{n+1}}\int \left(-\frac{bx}{a}\right)^{3n/2}(a+bx)^{3/2}dx}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 75}$$

$$\frac{2x^n(a+bx)^2\left(-\frac{bx}{a}\right)^{-3n/2}\sqrt{ax^n+bx^{n+1}}\,\text{Hypergeometric2F1}\left(\frac{5}{2},-\frac{3n}{2},\frac{7}{2},\frac{bx}{a}+1\right)}{5b}$$

input
```
Int[(a*x^n + b*x^(1 + n))^(3/2),x]
```

output
```
(2*x^n*(a + b*x)^2*Sqrt[a*x^n + b*x^(1 + n)]*Hypergeometric2F1[5/2, (-3*n)
/2, 7/2, 1 + (b*x)/a])/(5*b*(-((b*x)/a))^((3*n)/2))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \left(a\,x^{n}+b\,x^{1+n}\right)^{\frac{3}{2}}\,dx$$

input
```
int((a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
int((a*x^n+b*x^(1+n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(ax^n+bx^{1+n}\right)^{3/2}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*x^n+b*x^(1+n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(ax^n + bx^{1+n}\right)^{3/2} dx = \int \left(ax^n + bx^{n+1}\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*x**n+b*x**(1+n))**(3/2),x)
```

output
```
Integral((a*x**n + b*x**(n + 1))**(3/2), x)
```

Maxima [F]

$$\int \left(ax^n + bx^{1+n}\right)^{3/2} dx = \int \left(bx^{n+1} + ax^n\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*x^n+b*x^(1+n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(3/2), x)
```

Giac [F]

$$\int \left(ax^n + bx^{1+n}\right)^{3/2} dx = \int \left(bx^{n+1} + ax^n\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*x^n+b*x^(1+n))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \left(ax^n+bx^{1+n}\right)^{3/2}\,dx=\frac{x\,\left(a\,x^n+b\,x^{n+1}\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{3\,n}{2}+1;\ \frac{3\,n}{2}+2;\ -\frac{b\,x}{a}\right)}{\left(\frac{3\,n}{2}+1\right)\,\left(\frac{b\,x}{a}+1\right)^{3/2}}$$

input
```
int((a*x^n + b*x^(n + 1))^(3/2),x)
```

output
```
(x*(a*x^n + b*x^(n + 1))^(3/2)*hypergeom([-3/2, (3*n)/2 + 1], (3*n)/2 + 2,
 -(b*x)/a))/(((3*n)/2 + 1)*((b*x)/a + 1)^(3/2))
```

Reduce [F]

$$\int \left(ax^n+bx^{1+n}\right)^{3/2}\,dx=\left(\int x^{\frac{3n}{2}}\sqrt{bx+a}\,xdx\right)b+\left(\int x^{\frac{3n}{2}}\sqrt{bx+a}dx\right)a$$

input
```
int((a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
int(x**((3*n)/2)*sqrt(a + b*x)*x,x)*b + int(x**((3*n)/2)*sqrt(a + b*x),x)*
a
```

3.134 $\int \sqrt{ax^n+bx^{1+n}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \sqrt{ax^n+bx^{1+n}}\,dx = \frac{2x^{-n}\left(-\frac{bx}{a}\right)^{-n/2}\left(ax^n+bx^{1+n}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{n}{2},\frac{5}{2},1+\frac{bx}{a}\right)}{3b}$$

output
```
2/3*(a*x^n+b*x^(1+n))^(3/2)*hypergeom([3/2, -1/2*n],[5/2],1+b*x/a)/b/(x^n)
/((-b*x/a)^(1/2*n))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \sqrt{ax^n+bx^{1+n}}\,dx = \frac{2\left(-\frac{bx}{a}\right)^{-n/2}(a+bx)\sqrt{x^n(a+bx)}\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{n}{2},\frac{5}{2},1+\frac{bx}{a}\right)}{3b}$$

input
```
Integrate[Sqrt[a*x^n + b*x^(1 + n)],x]
```

output
```
(2*(a + b*x)*Sqrt[x^n*(a + b*x)]*Hypergeometric2F1[3/2, -1/2*n, 5/2, 1 + (
b*x)/a])/(3*b*(-((b*x)/a))^(n/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1917, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax^n + bx^{n+1}}\,dx$$

$$\Big\downarrow \text{1917}$$

$$\frac{x^{-n/2}\sqrt{ax^n + bx^{n+1}}\int x^{n/2}\sqrt{a+bx}dx}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{77}$$

$$\frac{\left(-\frac{bx}{a}\right)^{-n/2}\sqrt{ax^n + bx^{n+1}}\int \left(-\frac{bx}{a}\right)^{n/2}\sqrt{a+bx}dx}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{75}$$

$$\frac{2(a+bx)\left(-\frac{bx}{a}\right)^{-n/2}\sqrt{ax^n + bx^{n+1}}\,\text{Hypergeometric2F1}\left(\frac{3}{2},-\frac{n}{2},\frac{5}{2},\frac{bx}{a}+1\right)}{3b}$$

input
```
Int[Sqrt[a*x^n + b*x^(1 + n)],x]
```

output
```
(2*(a + b*x)*Sqrt[a*x^n + b*x^(1 + n)]*Hypergeometric2F1[3/2, -1/2*n, 5/2,
 1 + (b*x)/a])/(3*b*(-((b*x)/a))^(n/2))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \sqrt{a\,x^{n}+b\,x^{1+n}}dx$$

input
```
int((a*x^n+b*x^(1+n))^(1/2),x)
```

output
```
int((a*x^n+b*x^(1+n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \sqrt{ax^n+bx^{1+n}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a*x^n+b*x^(1+n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \sqrt{ax^n + bx^{1+n}}\,dx = \int \sqrt{ax^n + bx^{n+1}}\,dx$$

input
```
integrate((a*x**n+b*x**(1+n))**(1/2),x)
```

output
```
Integral(sqrt(a*x**n + b*x**(n + 1)), x)
```

Maxima [F]

$$\int \sqrt{ax^n + bx^{1+n}}\,dx = \int \sqrt{bx^{n+1} + ax^n}\,dx$$

input
```
integrate((a*x^n+b*x^(1+n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^(n + 1) + a*x^n), x)
```

Giac [F]

$$\int \sqrt{ax^n + bx^{1+n}}\,dx = \int \sqrt{bx^{n+1} + ax^n}\,dx$$

input
```
integrate((a*x^n+b*x^(1+n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^(n + 1) + a*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.87

$$\int \sqrt{ax^n + bx^{1+n}}\,dx = \frac{x\sqrt{a\,x^n + b\,x^{n+1}}\,{}_2\mathrm{F}_1\left(-\frac{1}{2}, \frac{n}{2}+1;\ \frac{n}{2}+2;\ -\frac{b\,x}{a}\right)}{\left(\frac{n}{2}+1\right)\sqrt{\frac{b\,x}{a}+1}}$$

input
```
int((a*x^n + b*x^(n + 1))^(1/2),x)
```

output
```
(x*(a*x^n + b*x^(n + 1))^(1/2)*hypergeom([-1/2, n/2 + 1], n/2 + 2, -(b*x)/
a))/((n/2 + 1)*((b*x)/a + 1)^(1/2))
```

Reduce [F]

$$\int \sqrt{ax^n + bx^{1+n}}\,dx = \int x^{\frac{n}{2}}\sqrt{bx+a}dx$$

input
```
int((a*x^n+b*x^(1+n))^(1/2),x)
```

output
```
int(x**(n/2)*sqrt(a + b*x),x)
```

3.135 $\int \frac{1}{\sqrt{ax^n+bx^{1+n}}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \frac{1}{\sqrt{ax^n+bx^{1+n}}}\,dx=\frac{2x^{-n}\left(-\frac{bx}{a}\right)^{n/2}\sqrt{ax^n+bx^{1+n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{3}{2},1+\frac{bx}{a}\right)}{b}$$

output
```
2*(-b*x/a)^(1/2*n)*(a*x^n+b*x^(1+n))^(1/2)*hypergeom([1/2, 1/2*n],[3/2],1+
b*x/a)/b/(x^n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{1}{\sqrt{ax^n+bx^{1+n}}}\,dx=\frac{2\left(-\frac{bx}{a}\right)^{n/2}(a+bx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{3}{2},1+\frac{bx}{a}\right)}{b\sqrt{x^n(a+bx)}}$$

input
```
Integrate[1/Sqrt[a*x^n + b*x^(1 + n)],x]
```

output
```
(2*(-((b*x)/a))^(n/2)*(a + b*x)*Hypergeometric2F1[1/2, n/2, 3/2, 1 + (b*x)
/a])/(b*Sqrt[x^n*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1917, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{ax^n+bx^{n+1}}}dx$$

$$\Big\downarrow \text{ 1917}$$

$$\frac{x^{n/2}\sqrt{a+bx}\int \frac{x^{-n/2}}{\sqrt{a+bx}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{ 77}$$

$$\frac{\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{n/2}\int \frac{\left(-\frac{bx}{a}\right)^{-n/2}}{\sqrt{a+bx}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{ 75}$$

$$\frac{2(a+bx)\left(-\frac{bx}{a}\right)^{n/2}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n}{2},\frac{3}{2},\frac{bx}{a}+1\right)}{b\sqrt{ax^n+bx^{n+1}}}$$

input
```
Int[1/Sqrt[a*x^n + b*x^(1 + n)],x]
```

output
```
(2*(-((b*x)/a))^(n/2)*(a + b*x)*Hypergeometric2F1[1/2, n/2, 3/2, 1 + (b*x)
/a])/(b*Sqrt[a*x^n + b*x^(1 + n)])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{1}{\sqrt{a\,x^{n}+b\,x^{1+n}}}dx$$

input
```
int(1/(a*x^n+b*x^(1+n))^(1/2),x)
```

output
```
int(1/(a*x^n+b*x^(1+n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{ax^n+bx^{1+n}}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x^n+b*x^(1+n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{\sqrt{ax^n + bx^{1+n}}}\,dx = \int \frac{1}{\sqrt{ax^n + bx^{n+1}}}\,dx$$

input
```
integrate(1/(a*x**n+b*x**(1+n))**(1/2),x)
```

output
```
Integral(1/sqrt(a*x**n + b*x**(n + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{ax^n + bx^{1+n}}}\,dx = \int \frac{1}{\sqrt{bx^{n+1} + ax^n}}\,dx$$

input
```
integrate(1/(a*x^n+b*x^(1+n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*x^(n + 1) + a*x^n), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{ax^n + bx^{1+n}}}\,dx = \int \frac{1}{\sqrt{bx^{n+1} + ax^n}}\,dx$$

input
```
integrate(1/(a*x^n+b*x^(1+n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*x^(n + 1) + a*x^n), x)
```

Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{ax^n + bx^{1+n}}} \, dx = -\frac{x \sqrt{\frac{b\,x}{a} + 1}\,{}_2\mathrm{F}_1\left(\frac{1}{2}, 1 - \frac{n}{2};\ 2 - \frac{n}{2};\ -\frac{b\,x}{a}\right)}{\left(\frac{n}{2} - 1\right)\sqrt{a\,x^n + b\,x^{n+1}}}$$

input
```
int(1/(a*x^n + b*x^(n + 1))^(1/2),x)
```

output
```
-(x*((b*x)/a + 1)^(1/2)*hypergeom([1/2, 1 - n/2], 2 - n/2, -(b*x)/a))/((n/
2 - 1)*(a*x^n + b*x^(n + 1))^(1/2))
```

Reduce [F]

$$\int \frac{1}{\sqrt{ax^n + bx^{1+n}}} \, dx = \int \frac{1}{x^{\frac{n}{2}} \sqrt{bx + a}} dx$$

input
```
int(1/(a*x^n+b*x^(1+n))^(1/2),x)
```

output
```
int(1/(x**(n/2)*sqrt(a + b*x)),x)
```

3.136 $\int \frac{1}{(ax^n+bx^{1+n})^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \frac{1}{(ax^n+bx^{1+n})^{3/2}}\,dx=-\frac{2x^{-n}\left(-\frac{bx}{a}\right)^{3n/2}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3n}{2},\frac{1}{2},1+\frac{bx}{a}\right)}{b\sqrt{ax^n+bx^{1+n}}}$$

output
```
-2*(-b*x/a)^(3/2*n)*hypergeom([-1/2, 3/2*n],[1/2],1+b*x/a)/b/(x^n)/(a*x^n+
b*x^(1+n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.93

$$\int \frac{1}{(ax^n+bx^{1+n})^{3/2}}\,dx=-\frac{2\left(-\frac{bx}{a}\right)^{3n/2}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3n}{2},\frac{1}{2},1+\frac{bx}{a}\right)}{b\left(x^n(a+bx)\right)^{3/2}}$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^(-3/2),x]
```

output
```
(-2*(-((b*x)/a))^((3*n)/2)*(a + b*x)*Hypergeometric2F1[-1/2, (3*n)/2, 1/2,
 1 + (b*x)/a])/(b*(x^n*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {1917, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^n+bx^{n+1})^{3/2}}dx$$

$$\Big\downarrow \ 1917$$

$$\frac{x^{n/2}\sqrt{a+bx}\int \frac{x^{-3n/2}}{(a+bx)^{3/2}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \ 77$$

$$\frac{x^{-n}\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{3n/2}\int \frac{\left(-\frac{bx}{a}\right)^{-3n/2}}{(a+bx)^{3/2}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \ 75$$

$$-\frac{2x^{-n}\left(-\frac{bx}{a}\right)^{3n/2}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{3n}{2},\frac{1}{2},\frac{bx}{a}+1\right)}{b\sqrt{ax^n+bx^{n+1}}}$$

input
```
Int[(a*x^n + b*x^(1 + n))^(-3/2),x]
```

output
```
(-2*(-((b*x)/a))^((3*n)/2)*Hypergeometric2F1[-1/2, (3*n)/2, 1/2, 1 + (b*x)
/a])/(b*x^n*Sqrt[a*x^n + b*x^(1 + n)])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{1}{(a\,x^{n}+b\,x^{1+n})^{\frac{3}{2}}}dx$$

input
```
int(1/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
int(1/(a*x^n+b*x^(1+n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ax^n+bx^{1+n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^{3/2}}\,dx = \int \frac{1}{(ax^n + bx^{n+1})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*x**n+b*x**(1+n))**(3/2),x)
```

output
```
Integral((a*x**n + b*x**(n + 1))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^{3/2}}\,dx = \int \frac{1}{(bx^{n+1} + ax^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^{3/2}}\,dx = \int \frac{1}{(bx^{n+1} + ax^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(-3/2), x)
```

Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{1}{(ax^n+bx^{1+n})^{3/2}}\,dx=-\frac{x\,\left(\frac{b\,x}{a}+1\right)^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},1-\frac{3\,n}{2};\ 2-\frac{3\,n}{2};\ -\frac{b\,x}{a}\right)}{\left(\frac{3\,n}{2}-1\right)\,\left(a\,x^n+b\,x^{n+1}\right)^{3/2}}$$

input
```
int(1/(a*x^n + b*x^(n + 1))^(3/2),x)
```

output
```
-(x*((b*x)/a + 1)^(3/2)*hypergeom([3/2, 1 - (3*n)/2], 2 - (3*n)/2, -(b*x)/
a))/((((3*n)/2 - 1)*(a*x^n + b*x^(n + 1))^(3/2))
```

Reduce [F]

$$\int \frac{1}{(ax^n+bx^{1+n})^{3/2}}\,dx=\int \frac{1}{x^{\frac{3n}{2}}\sqrt{bx+a}\,a+x^{\frac{3n}{2}}\sqrt{bx+a}\,bx}dx$$

input
```
int(1/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
int(1/(x**((3*n)/2)*sqrt(a + b*x)*a + x**((3*n)/2)*sqrt(a + b*x)*b*x),x)
```

3.137 $\int \frac{1}{(ax^n+bx^{1+n})^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \frac{1}{(ax^n+bx^{1+n})^{5/2}}\,dx=-\frac{2x^{-n}\left(-\frac{bx}{a}\right)^{5n/2}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5n}{2},-\frac{1}{2},1+\frac{bx}{a}\right)}{3b\,(ax^n+bx^{1+n})^{3/2}}$$

output
```
-2/3*(-b*x/a)^(5/2*n)*hypergeom([-3/2, 5/2*n],[-1/2],1+b*x/a)/b/(x^n)/(a*x
^n+b*x^(1+n))^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{1}{(ax^n+bx^{1+n})^{5/2}}\,dx=-\frac{2\left(-\frac{bx}{a}\right)^{5n/2}(a+bx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5n}{2},-\frac{1}{2},1+\frac{bx}{a}\right)}{3b\,(x^n(a+bx))^{5/2}}$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^(-5/2),x]
```

output
```
(-2*(-((b*x)/a))^((5*n)/2)*(a + b*x)*Hypergeometric2F1[-3/2, (5*n)/2, -1/2
, 1 + (b*x)/a])/(3*b*(x^n*(a + b*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1917, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(ax^n+bx^{n+1})^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1917}$$

$$\frac{x^{n/2}\sqrt{a+bx}\int \frac{x^{-5n/2}}{(a+bx)^{5/2}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{ 77}$$

$$\frac{x^{-2n}\sqrt{a+bx}\left(-\frac{bx}{a}\right)^{5n/2}\int \frac{\left(-\frac{bx}{a}\right)^{-5n/2}}{(a+bx)^{5/2}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{ 75}$$

$$-\frac{2x^{-2n}\left(-\frac{bx}{a}\right)^{5n/2}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{5n}{2},-\frac{1}{2},\frac{bx}{a}+1\right)}{3b(a+bx)\sqrt{ax^n+bx^{n+1}}}$$

input
```
Int[(a*x^n + b*x^(1 + n))^(-5/2),x]
```

output
```
(-2*(-((b*x)/a))^((5*n)/2)*Hypergeometric2F1[-3/2, (5*n)/2, -1/2, 1 + (b*x
)/a])/(3*b*x^(2*n)*(a + b*x)*Sqrt[a*x^n + b*x^(1 + n)])
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \frac{1}{\left(a\,x^{n}+b\,x^{1+n}\right)^{\frac{5}{2}}}dx$$

input
```
int(1/(a*x^n+b*x^(1+n))^(5/2),x)
```

output
```
int(1/(a*x^n+b*x^(1+n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(ax^n+bx^{1+n})^{5/2}}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(1/(a*x^n+b*x^(1+n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^{5/2}}\,dx = \int \frac{1}{(ax^n + bx^{n+1})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*x**n+b*x**(1+n))**(5/2),x)
```

output
```
Integral((a*x**n + b*x**(n + 1))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^{5/2}}\,dx = \int \frac{1}{(bx^{n+1} + ax^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*x^n+b*x^(1+n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^{5/2}}\,dx = \int \frac{1}{(bx^{n+1} + ax^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*x^n+b*x^(1+n))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(-5/2), x)
```

Mupad [B] (verification not implemented)

Time = 10.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.89

$$\int \frac{1}{(ax^n + bx^{1+n})^{5/2}}\,dx = -\frac{x\left(\frac{b\,x}{a}+1\right)^{5/2}\,{}_2\mathrm{F}_1\left(\frac{5}{2},1-\frac{5\,n}{2};\ 2-\frac{5\,n}{2};\ -\frac{b\,x}{a}\right)}{\left(\frac{5\,n}{2}-1\right)\,\left(a\,x^n+b\,x^{n+1}\right)^{5/2}}$$

input
```
int(1/(a*x^n + b*x^(n + 1))^(5/2),x)
```

output
```
-(x*((b*x)/a + 1)^(5/2)*hypergeom([5/2, 1 - (5*n)/2], 2 - (5*n)/2, -(b*x)/
a))/(((5*n)/2 - 1)*(a*x^n + b*x^(n + 1))^(5/2))
```

Reduce [F]

$$\int \frac{1}{(ax^n + bx^{1+n})^{5/2}}\,dx = \int \frac{1}{x^{\frac{5n}{2}}\sqrt{bx+a}\,a^2 + 2x^{\frac{5n}{2}}\sqrt{bx+a}\,abx + x^{\frac{5n}{2}}\sqrt{bx+a}\,b^2x^2}dx$$

input
```
int(1/(a*x^n+b*x^(1+n))^(5/2),x)
```

output
```
int(1/(x**((5*n)/2)*sqrt(a + b*x)*a**2 + 2*x**((5*n)/2)*sqrt(a + b*x)*a*b*
x + x**((5*n)/2)*sqrt(a + b*x)*b**2*x**2),x)
```

3.138 $\int \left(ax^n+bx^{1+n}\right)^p\,dx$

Optimal result

Integrand size = 15, antiderivative size = 55

$$\int \left(ax^n+bx^{1+n}\right)^p\,dx = \frac{x^{1-n}\left(ax^n+bx^{1+n}\right)^{1+p}\,\text{Hypergeometric2F1}\left(1,2+p+np,2+np,-\frac{bx}{a}\right)}{a(1+np)}$$

output
```
x^(1-n)*(a*x^n+b*x^(1+n))^(p+1)*hypergeom([1, n*p+p+2],[n*p+2],-b*x/a)/a/(
n*p+1)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \left(ax^n+bx^{1+n}\right)^p\,dx = \frac{x(x^n(a+bx))^p\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,1+np,2+np,-\frac{bx}{a}\right)}{1+np}$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^p,x]
```

output
```
(x*(x^n*(a + b*x))^p*Hypergeometric2F1[-p, 1 + n*p, 2 + n*p, -((b*x)/a)])/
((1 + n*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1917, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^n+bx^{n+1}\right)^p\,dx$$

$$\Big\downarrow \ 1917$$

$$x^{-np}(a+bx)^{-p}\left(ax^n+bx^{n+1}\right)^p\int x^{np}(a+bx)^pdx$$

$$\Big\downarrow \ 76$$

$$x^{-np}\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p\int x^{np}\left(\frac{bx}{a}+1\right)^p dx$$

$$\Big\downarrow \ 74$$

$$\frac{x\left(\frac{bx}{a}+1\right)^{-p}\left(ax^n+bx^{n+1}\right)^p \text{Hypergeometric2F1}\left(-p,np+1,np+2,-\frac{bx}{a}\right)}{np+1}$$

input
```
Int[(a*x^n + b*x^(1 + n))^p,x]
```

output
```
(x*(a*x^n + b*x^(1 + n))^p*Hypergeometric2F1[-p, 1 + n*p, 2 + n*p, -((b*x)
/a)])/((1 + n*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int (a\,x^n + b\,x^{1+n})^p\,dx$$

input
```
int((a*x^n+b*x^(1+n))^p,x)
```

output
```
int((a*x^n+b*x^(1+n))^p,x)
```

Fricas [F]

$$\int (ax^n + bx^{1+n})^p\,dx = \int (bx^{n+1} + ax^n)^p\,dx$$

input
```
integrate((a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

output
```
integral((b*x^(n + 1) + a*x^n)^p, x)
```

Sympy [F]

$$\int \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(ax^n + bx^{n+1}\right)^p \, dx$$

input
```
integrate((a*x**n+b*x**(1+n))**p,x)
```

output
```
Integral((a*x**n + b*x**(n + 1))**p, x)
```

Maxima [F]

$$\int \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p \, dx$$

input
```
integrate((a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^p, x)
```

Giac [F]

$$\int \left(ax^n + bx^{1+n}\right)^p \, dx = \int \left(bx^{n+1} + ax^n\right)^p \, dx$$

input
```
integrate((a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^p, x)
```

Mupad [B] (verification not implemented)

Time = 9.96 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \left(ax^n + bx^{1+n}\right)^p \, dx = \frac{x\,\left(a\,x^n + b\,x^{n+1}\right)^p\,{}_2\mathrm{F}_1\left(n\,p+1,-p;\ n\,p+2;\ -\frac{b\,x}{a}\right)}{\left(\frac{b\,x}{a}+1\right)^p\,\left(n\,p+1\right)}$$

input
```
int((a*x^n + b*x^(n + 1))^p,x)
```

output
```
(x*(a*x^n + b*x^(n + 1))^p*hypergeom([n*p + 1, -p], n*p + 2, -(b*x)/a))/((
(b*x)/a + 1)^p*(n*p + 1))
```

Reduce [F]

$$\int \left(ax^n + bx^{1+n}\right)^p \, dx = \frac{(x^n a + x^n bx)^p\, a + (x^n a + x^n bx)^p\, bnx + (x^n a + x^n bx)^p\, bx - \left(\int \frac{(x^n a+x^n bx)^p}{b\,n^2p\,x^2+a\,n^2px+2bnp\,x^2+2anpx+bn\,x^2+bp\,x^2+anx\cdots}\right.}{}$$

input
```
int((a*x^n+b*x^(1+n))^p,x)
```

output
```
((x**n*a + x**n*b*x)**p*a + (x**n*a + x**n*b*x)**p*b*n*x + (x**n*a + x**n*
b*x)**p*b*x - int((x**n*a + x**n*b*x)**p/(a*n**2*p*x + 2*a*n*p*x + a*n*x +
 a*p*x + a*x + b*n**2*p*x**2 + 2*b*n*p*x**2 + b*n*x**2 + b*p*x**2 + b*x**2
),x)*a**2*n**3*p**2 - 2*int((x**n*a + x**n*b*x)**p/(a*n**2*p*x + 2*a*n*p*x
 + a*n*x + a*p*x + a*x + b*n**2*p*x**2 + 2*b*n*p*x**2 + b*n*x**2 + b*p*x**
2 + b*x**2),x)*a**2*n**2*p**2 - int((x**n*a + x**n*b*x)**p/(a*n**2*p*x + 2
*a*n*p*x + a*n*x + a*p*x + a*x + b*n**2*p*x**2 + 2*b*n*p*x**2 + b*n*x**2 +
 b*p*x**2 + b*x**2),x)*a**2*n**2*p - int((x**n*a + x**n*b*x)**p/(a*n**2*p*
x + 2*a*n*p*x + a*n*x + a*p*x + a*x + b*n**2*p*x**2 + 2*b*n*p*x**2 + b*n*x
**2 + b*p*x**2 + b*x**2),x)*a**2*n*p**2 - int((x**n*a + x**n*b*x)**p/(a*n*
*2*p*x + 2*a*n*p*x + a*n*x + a*p*x + a*x + b*n**2*p*x**2 + 2*b*n*p*x**2 +
b*n*x**2 + b*p*x**2 + b*x**2),x)*a**2*n*p)/(b*(n**2*p + 2*n*p + n + p + 1)
)
```

3.139 $\int (ax^n + bx^{1+n})^{3/n} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 167

$$\int (ax^n + bx^{1+n})^{3/n} \, dx = \frac{3a^2nx^{-3-2n}(ax^n + bx^{1+n})^{2+\frac{3}{n}}}{b^4(3+2n)} + \frac{nx^{-3-4n}(ax^n + bx^{1+n})^{4+\frac{3}{n}}}{b^4(3+4n)} - \frac{a^3n(a+bx)(ax^n + bx^{1+n})^{3/n}}{b^4(3+n)x^3} - \frac{an(a+bx)^3(ax^n + bx^{1+n})^{3/n}}{b^4(1+n)x^3}$$

output
```
3*a^2*n*x^(-3-2*n)*(a*x^n+b*x^(1+n))^(2+3/n)/b^4/(3+2*n)+n*x^(-3-4*n)*(a*x
^n+b*x^(1+n))^(4+3/n)/b^4/(3+4*n)-a^3*n*(b*x+a)*(a*x^n+b*x^(1+n))^(3/n)/b^
4/(3+n)/x^3-a*n*(b*x+a)^3*(a*x^n+b*x^(1+n))^(3/n)/b^4/(1+n)/x^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.69

$$\int (ax^n + bx^{1+n})^{3/n} \, dx = \frac{n(a+bx)(x^n(a+bx))^{3/n}\left(-2a^3n^3 + 2a^2bn^2(3+n)x - ab^2n(9+9n+2n^2)x^2 + b^3(9+1\right.}{b^4(1+n)(3+n)(3+2n)(3+4n)x^3}$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^(3/n),x]
```

output
```
(n*(a + b*x)*(x^n*(a + b*x))^(3/n)*(-2*a^3*n^3 + 2*a^2*b*n^2*(3 + n)*x - a
*b^2*n*(9 + 9*n + 2*n^2)*x^2 + b^3*(9 + 18*n + 11*n^2 + 2*n^3)*x^3))/(b^4*
(1 + n)*(3 + n)*(3 + 2*n)*(3 + 4*n)*x^3)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^n+bx^{n+1}\right)^{3/n}dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{nx^{-n}\left(ax^n+bx^{n+1}\right)^{\frac{n+3}{n}}}{b(4n+3)}-\frac{3an\int \frac{\left(ax^n+bx^{n+1}\right)^{3/n}}{x}dx}{b(4n+3)}$$

$$\Big\downarrow \text{1922}$$

$$\frac{nx^{-n}\left(ax^n+bx^{n+1}\right)^{\frac{n+3}{n}}}{b(4n+3)}-\frac{3an\left(\frac{nx^{-n-1}\left(ax^n+bx^{n+1}\right)^{\frac{n+3}{n}}}{3b(n+1)}-\frac{2an\int \frac{\left(ax^n+bx^{n+1}\right)^{3/n}}{x^2}dx}{3b(n+1)}\right)}{b(4n+3)}$$

$$\Big\downarrow \text{1922}$$

$$\frac{nx^{-n}\left(ax^n+bx^{n+1}\right)^{\frac{n+3}{n}}}{b(4n+3)}-\frac{3an\left(\frac{nx^{-n-1}\left(ax^n+bx^{n+1}\right)^{\frac{n+3}{n}}}{3b(n+1)}-\frac{2an\left(\frac{nx^{-n-2}\left(ax^n+bx^{n+1}\right)^{\frac{n+3}{n}}}{b(2n+3)}-\frac{an\int \frac{\left(ax^n+bx^{n+1}\right)^{3/n}}{x^3}dx}{b(2n+3)}\right)}{3b(n+1)}\right)}{b(4n+3)}$$

$$\Big\downarrow \textcolor{blue}{1920}$$

$$\frac{nx^{-n}\left(ax^n+bx^{n+1}\right)^{\frac{n+3}{n}}}{b(4n+3)}-\frac{3an\left(\frac{nx^{-n-1}\left(ax^n+bx^{n+1}\right)^{\frac{n+3}{n}}}{3b(n+1)}-\frac{2an\left(\frac{nx^{-n-2}\left(ax^n+bx^{n+1}\right)^{\frac{n+3}{n}}}{b(2n+3)}-\frac{an^2x^{-n-3}\left(ax^n+bx^{n+1}\right)^{\frac{n+3}{n}}}{b^2(n+3)(2n+3)}\right)}{3b(n+1)}\right)}{b(4n+3)}$$

input
```
Int[(a*x^n + b*x^(1 + n))^(3/n),x]
```

output
```
(n*(a*x^n + b*x^(1 + n))^((3 + n)/n))/(b*(3 + 4*n)*x^n) - (3*a*n*((n*x^(-1
 - n)*(a*x^n + b*x^(1 + n))^((3 + n)/n))/(3*b*(1 + n)) - (2*a*n*(-((a*n^2*
x^(-3 - n)*(a*x^n + b*x^(1 + n))^((3 + n)/n))/(b^2*(3 + n)*(3 + 2*n))) + (
n*x^(-2 - n)*(a*x^n + b*x^(1 + n))^((3 + n)/n))/(b*(3 + 2*n))))/(3*b*(1 +
n))))/(b*(3 + 4*n))
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95

method	result
orering	$-\frac{\left(-2 b^{3} n^{3} x^{3}+2 a\,b^{2} n^{3} x^{2}-11 b^{3} n^{2} x^{3}-2 a^{2} b\,n^{3} x+9 a\,b^{2} n^{2} x^{2}-18 n\,x^{3} b^{3}+2 a^{3} n^{3}-6 a^{2} b\,n^{2} x+9 n\,x^{2} a\,b^{2}-9 b^{3} x^{3}\right) n \left(b x+a\right) \left(a\,x^{n}+b\,x^{1+\right.}{b^{4} \left(8 n^{4}+50 n^{3}+105 n^{2}+90 n+27\right) x^{3}}$

input
```
int((a*x^n+b*x^(1+n))^(3/n),x,method=_RETURNVERBOSE)
```

output
```
-(-2*b^3*n^3*x^3+2*a*b^2*n^3*x^2-11*b^3*n^2*x^3-2*a^2*b*n^3*x+9*a*b^2*n^2*
x^2-18*b^3*n*x^3+2*a^3*n^3-6*a^2*b*n^2*x+9*a*b^2*n*x^2-9*b^3*x^3)*n/b^4/(8
*n^4+50*n^3+105*n^2+90*n+27)*(b*x+a)/x^3*(a*x^n+b*x^(1+n))^(3/n)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05

$$\int \left(ax^n+bx^{1+n}\right)^{3/n}\,dx =$$

$$-\frac{(2\,a^4n^4-6\,a^3bn^3x-(2\,b^4n^4+11\,b^4n^3+18\,b^4n^2+9\,b^4n)x^4-(2\,ab^3n^3+9\,ab^3n^2+9\,ab^3n)x^3+3\,(a^2b^2n^{}}{(8\,b^4n^4+50\,b^4n^3+105\,b^4n^2+90\,b^4n+27\,b^4)x^3}$$

input
```
integrate((a*x^n+b*x^(1+n))^(3/n),x, algorithm="fricas")
```

output
```
-(2*a^4*n^4 - 6*a^3*b*n^3*x - (2*b^4*n^4 + 11*b^4*n^3 + 18*b^4*n^2 + 9*b^4
*n)*x^4 - (2*a*b^3*n^3 + 9*a*b^3*n^2 + 9*a*b^3*n)*x^3 + 3*(a^2*b^2*n^3 + 3
*a^2*b^2*n^2)*x^2)*((b*x + a)*x^(n + 1)/x)^(3/n)/((8*b^4*n^4 + 50*b^4*n^3
+ 105*b^4*n^2 + 90*b^4*n + 27*b^4)*x^3)
```

Sympy [F]

$$\int \left(ax^n+bx^{1+n}\right)^{3/n}\,dx=\int \left(ax^n+bx^{n+1}\right)^{\frac{3}{n}}\,dx$$

input
```
integrate((a*x**n+b*x**(1+n))**(3/n),x)
```

output
```
Integral((a*x**n + b*x**(n + 1))**(3/n), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.83

$$\int \left(ax^n+bx^{1+n}\right)^{3/n}\,dx=\frac{((2\,n^4+11\,n^3+18\,n^2+9\,n)b^4x^4-2\,a^4n^4+6\,a^3bn^3x+(2\,n^3+9\,n^2+9\,n)ab^3x^3-3\,(n^3}{(8\,n^4+50\,n^3+105\,n^2+90\,n+27)b^4x^3}$$

input
```
integrate((a*x^n+b*x^(1+n))^(3/n),x, algorithm="maxima")
```

output
```
((2*n^4 + 11*n^3 + 18*n^2 + 9*n)*b^4*x^4 - 2*a^4*n^4 + 6*a^3*b*n^3*x + (2*
n^3 + 9*n^2 + 9*n)*a*b^3*x^3 - 3*(n^3 + 3*n^2)*a^2*b^2*x^2)*e^(3*log(b*x +
 a)/n + 3*log(x^n)/n)/((8*n^4 + 50*n^3 + 105*n^2 + 90*n + 27)*b^4*x^3)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.69

$$\int (ax^n + bx^{1+n})^{3/n} \, dx = \frac{2\,(bx+a)^{\frac{3}{n}}b^4n^4x^4 + 11\,(bx+a)^{\frac{3}{n}}b^4n^3x^4 + 2\,(bx+a)^{\frac{3}{n}}ab^3n^3x^3 + 18\,(bx+a)^{\frac{3}{n}}b^4n^2x^4 - 3}{}$$

input
```
integrate((a*x^n+b*x^(1+n))^(3/n),x, algorithm="giac")
```

output
```
(2*(b*x + a)^(3/n)*b^4*n^4*x^4 + 11*(b*x + a)^(3/n)*b^4*n^3*x^4 + 2*(b*x +
 a)^(3/n)*a*b^3*n^3*x^3 + 18*(b*x + a)^(3/n)*b^4*n^2*x^4 - 3*(b*x + a)^(3/
n)*a^2*b^2*n^3*x^2 + 9*(b*x + a)^(3/n)*a*b^3*n^2*x^3 + 9*(b*x + a)^(3/n)*b
^4*n*x^4 - 2*(b*x + a)^(3/n)*a^4*n^4 + 6*(b*x + a)^(3/n)*a^3*b*n^3*x - 9*(
b*x + a)^(3/n)*a^2*b^2*n^2*x^2 + 9*(b*x + a)^(3/n)*a*b^3*n*x^3)/(8*b^4*n^4
 + 50*b^4*n^3 + 105*b^4*n^2 + 90*b^4*n + 27*b^4)
```

Mupad [B] (verification not implemented)

Time = 9.71 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.32

$$\int (ax^n + bx^{1+n})^{3/n} \, dx = \frac{x\,(a\,x^n + b\,x^{n+1})^{3/n}\,{}_2F_1\left(4, -\frac{3}{n};\ 5;\ -\frac{b\,x}{a}\right)}{4\,\left(\frac{b\,x}{a}+1\right)^{3/n}}$$

input
```
int((a*x^n + b*x^(n + 1))^(3/n),x)
```

output
```
(x*(a*x^n + b*x^(n + 1))^(3/n)*hypergeom([4, -3/n], 5, -(b*x)/a))/(4*((b*x
)/a + 1)^(3/n))
```

Reduce [F]

$$\int (ax^n + bx^{1+n})^{3/n}\, dx = \frac{n\left((x^n a + x^n bx)^{\frac{3}{n}} a + (x^n a + x^n bx)^{\frac{3}{n}} bnx + (x^n a + x^n bx)^{\frac{3}{n}} bx - 12\left(\int \frac{(x^n a+x^n}{4bn^2x^2+4an^2x+7bnx^2}\right.\right.}{}$$

input
```
int((a*x^n+b*x^(1+n))^(3/n),x)
```

output
```
(n*((x**n*a + x**n*b*x)**(3/n)*a + (x**n*a + x**n*b*x)**(3/n)*b*n*x + (x**
n*a + x**n*b*x)**(3/n)*b*x - 12*int((x**n*a + x**n*b*x)**(3/n)/(4*a*n**2*x
 + 7*a*n*x + 3*a*x + 4*b*n**2*x**2 + 7*b*n*x**2 + 3*b*x**2),x)*a**2*n**2 -
 21*int((x**n*a + x**n*b*x)**(3/n)/(4*a*n**2*x + 7*a*n*x + 3*a*x + 4*b*n**
2*x**2 + 7*b*n*x**2 + 3*b*x**2),x)*a**2*n - 9*int((x**n*a + x**n*b*x)**(3/
n)/(4*a*n**2*x + 7*a*n*x + 3*a*x + 4*b*n**2*x**2 + 7*b*n*x**2 + 3*b*x**2),
x)*a**2))/(b*(4*n**2 + 7*n + 3))
```

3.140 $\int \left(ax^n+bx^{1+n}\right)^{2/n} dx$

Optimal result

Integrand size = 19, antiderivative size = 122

$$\int \left(ax^n+bx^{1+n}\right)^{2/n} dx=\frac{nx^{-2-3n}\left(ax^n+bx^{1+n}\right)^{3+\frac{2}{n}}}{b^3(2+3n)}+\frac{a^2n(a+bx)\left(ax^n+bx^{1+n}\right)^{2/n}}{b^3(2+n)x^2}-\frac{an(a+bx)^2\left(ax^n+bx^{1+n}\right)^{2/n}}{b^3(1+n)x^2}$$

output
```
n*x^(-2-3*n)*(a*x^n+b*x^(1+n))^(3+2/n)/b^3/(2+3*n)+a^2*n*(b*x+a)*(a*x^n+b*
x^(1+n))^(2/n)/b^3/(2+n)/x^2-a*n*(b*x+a)^2*(a*x^n+b*x^(1+n))^(2/n)/b^3/(1+
n)/x^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.63

$$\int \left(ax^n+bx^{1+n}\right)^{2/n} dx=\frac{n(a+bx)\left(x^n(a+bx)\right)^{2/n}\left(a^2n^2-abn(2+n)x+b^2\left(2+3n+n^2\right)x^2\right)}{b^3(1+n)(2+n)(2+3n)x^2}$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^(2/n),x]
```

output
```
(n*(a + b*x)*(x^n*(a + b*x))^(2/n)*(a^2*n^2 - a*b*n*(2 + n)*x + b^2*(2 + 3
*n + n^2)*x^2))/(b^3*(1 + n)*(2 + n)*(2 + 3*n)*x^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1908, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^n+bx^{n+1}\right)^{2/n}dx$$

$$\Big\downarrow \text{1908}$$

$$\frac{nx^{-n}\left(ax^n+bx^{n+1}\right)^{\frac{n+2}{n}}}{b(3n+2)}-\frac{2an\int \frac{\left(ax^n+bx^{n+1}\right)^{2/n}}{x}dx}{b(3n+2)}$$

$$\Big\downarrow \text{1922}$$

$$\frac{nx^{-n}\left(ax^n+bx^{n+1}\right)^{\frac{n+2}{n}}}{b(3n+2)}-\frac{2an\left(\frac{nx^{-n-1}\left(ax^n+bx^{n+1}\right)^{\frac{n+2}{n}}}{2b(n+1)}-\frac{an\int \frac{\left(ax^n+bx^{n+1}\right)^{2/n}}{x^2}dx}{2b(n+1)}\right)}{b(3n+2)}$$

$$\Big\downarrow \text{1920}$$

$$\frac{nx^{-n}\left(ax^n+bx^{n+1}\right)^{\frac{n+2}{n}}}{b(3n+2)}-\frac{2an\left(\frac{nx^{-n-1}\left(ax^n+bx^{n+1}\right)^{\frac{n+2}{n}}}{2b(n+1)}-\frac{an^2x^{-n-2}\left(ax^n+bx^{n+1}\right)^{\frac{n+2}{n}}}{2b^2(n+1)(n+2)}\right)}{b(3n+2)}$$

input
```
Int[(a*x^n + b*x^(1 + n))^(2/n),x]
```

output
```
(n*(a*x^n + b*x^(1 + n))^((2 + n)/n))/(b*(2 + 3*n)*x^n) - (2*a*n*(-1/2*(a*
n^2*x^(-2 - n)*(a*x^n + b*x^(1 + n))^((2 + n)/n))/(b^2*(1 + n)*(2 + n)) +
(n*x^(-1 - n)*(a*x^n + b*x^(1 + n))^((2 + n)/n))/(2*b*(1 + n))))/(b*(2 + 3
*n))
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.81

method	result	size
orering	$\frac{n\left(b^{2} n^{2} x^{2}-a b\, n^{2} x+3 n\, x^{2} b^{2}+a^{2} n^{2}-2 n x a b+2 b^{2} x^{2}\right)\left(b x+a\right)\left(a\, x^{n}+b\, x^{1+n}\right)^{\frac{2}{n}}}{b^{3}\left(3 n^{3}+11 n^{2}+12 n+4\right) x^{2}}$	99

input
```
int((a*x^n+b*x^(1+n))^(2/n),x,method=_RETURNVERBOSE)
```

output
```
n*(b^2*n^2*x^2-a*b*n^2*x+3*b^2*n*x^2+a^2*n^2-2*a*b*n*x+2*b^2*x^2)/b^3/(3*n
^3+11*n^2+12*n+4)*(b*x+a)/x^2*(a*x^n+b*x^(1+n))^(2/n)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.97

$$\int (ax^n + bx^{1+n})^{2/n}\, dx = \frac{(a^3n^3 - 2\,a^2bn^2x + (b^3n^3 + 3\,b^3n^2 + 2\,b^3n)x^3 + (ab^2n^2 + 2\,ab^2n)x^2)\left(\frac{(bx+a)x^{n+1}}{x}\right)^{\frac{2}{n}}}{(3\,b^3n^3 + 11\,b^3n^2 + 12\,b^3n + 4\,b^3)x^2}$$

input
```
integrate((a*x^n+b*x^(1+n))^(2/n),x, algorithm="fricas")
```

output
```
(a^3*n^3 - 2*a^2*b*n^2*x + (b^3*n^3 + 3*b^3*n^2 + 2*b^3*n)*x^3 + (a*b^2*n^
2 + 2*a*b^2*n)*x^2)*((b*x + a)*x^(n + 1)/x)^(2/n)/((3*b^3*n^3 + 11*b^3*n^2
 + 12*b^3*n + 4*b^3)*x^2)
```

Sympy [F]

$$\int (ax^n + bx^{1+n})^{2/n}\, dx = \int (ax^n + bx^{n+1})^{\frac{2}{n}}\, dx$$

input
```
integrate((a*x**n+b*x**(1+n))**(2/n),x)
```

output
```
Integral((a*x**n + b*x**(n + 1))**(2/n), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int (ax^n + bx^{1+n})^{2/n}\, dx = \frac{((n^3 + 3\,n^2 + 2\,n)b^3x^3 + a^3n^3 - 2\,a^2bn^2x + (n^2 + 2\,n)ab^2x^2)e^{\left(\frac{2\,\log(bx+a)}{n} + \frac{2\,\log(x^n)}{n}\right)}}{(3\,n^3 + 11\,n^2 + 12\,n + 4)b^3x^2}$$

input
```
integrate((a*x^n+b*x^(1+n))^(2/n),x, algorithm="maxima")
```

output
```
((n^3 + 3*n^2 + 2*n)*b^3*x^3 + a^3*n^3 - 2*a^2*b*n^2*x + (n^2 + 2*n)*a*b^2
*x^2)*e^(2*log(b*x + a)/n + 2*log(x^n)/n)/((3*n^3 + 11*n^2 + 12*n + 4)*b^3
*x^2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.45

$$\int (ax^n + bx^{1+n})^{2/n} \, dx = \frac{(bx+a)^{\frac{2}{n}}b^3n^3x^3 + 3\,(bx+a)^{\frac{2}{n}}b^3n^2x^3 + (bx+a)^{\frac{2}{n}}ab^2n^2x^2 + 2\,(bx+a)^{\frac{2}{n}}b^3nx^3 + (bx+a)}{3\,b^3n^3 + 11\,b^3n^2 + 12\,b^3n + 4\,b^3}$$

input
```
integrate((a*x^n+b*x^(1+n))^(2/n),x, algorithm="giac")
```

output
```
((b*x + a)^(2/n)*b^3*n^3*x^3 + 3*(b*x + a)^(2/n)*b^3*n^2*x^3 + (b*x + a)^(
2/n)*a*b^2*n^2*x^2 + 2*(b*x + a)^(2/n)*b^3*n*x^3 + (b*x + a)^(2/n)*a^3*n^3
 - 2*(b*x + a)^(2/n)*a^2*b*n^2*x + 2*(b*x + a)^(2/n)*a*b^2*n*x^2)/(3*b^3*n
^3 + 11*b^3*n^2 + 12*b^3*n + 4*b^3)
```

Mupad [B] (verification not implemented)

Time = 9.55 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.44

$$\int (ax^n + bx^{1+n})^{2/n} \, dx = \frac{x\,(a\,x^n + b\,x^{n+1})^{2/n}\,{}_2F_1\left(3, -\frac{2}{n};\ 4;\ -\frac{b\,x}{a}\right)}{3\,\left(\frac{b\,x}{a}+1\right)^{2/n}}$$

input
```
int((a*x^n + b*x^(n + 1))^(2/n),x)
```

output
```
(x*(a*x^n + b*x^(n + 1))^(2/n)*hypergeom([3, -2/n], 4, -(b*x)/a))/(3*((b*x
)/a + 1)^(2/n))
```

Reduce [F]

$$\int (ax^n+bx^{1+n})^{2/n}\,dx = \frac{n\left((x^na+x^nbx)^{\frac{2}{n}}a+(x^na+x^nbx)^{\frac{2}{n}}bnx+(x^na+x^nbx)^{\frac{2}{n}}bx-6\left(\int\frac{(x^na+x^nbx}{3b\,n^2x^2+3a\,n^2x+5bn\,x^2+}\right.\right.}{}$$

input
```
int((a*x^n+b*x^(1+n))^(2/n),x)
```

output
```
(n*((x**n*a + x**n*b*x)**(2/n)*a + (x**n*a + x**n*b*x)**(2/n)*b*n*x + (x**
n*a + x**n*b*x)**(2/n)*b*x - 6*int((x**n*a + x**n*b*x)**(2/n)/(3*a*n**2*x
+ 5*a*n*x + 2*a*x + 3*b*n**2*x**2 + 5*b*n*x**2 + 2*b*x**2),x)*a**2*n**2 -
10*int((x**n*a + x**n*b*x)**(2/n)/(3*a*n**2*x + 5*a*n*x + 2*a*x + 3*b*n**2
*x**2 + 5*b*n*x**2 + 2*b*x**2),x)*a**2*n - 4*int((x**n*a + x**n*b*x)**(2/n
)/(3*a*n**2*x + 5*a*n*x + 2*a*x + 3*b*n**2*x**2 + 5*b*n*x**2 + 2*b*x**2),x
)*a**2))/(b*(3*n**2 + 5*n + 2))
```

3.141 $\int \left(ax^n+bx^{1+n}\right)^{\frac{1}{n}} dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int \left(ax^n+bx^{1+n}\right)^{\frac{1}{n}} dx=-\frac{anx^{-1-n}\left(ax^n+bx^{1+n}\right)^{1+\frac{1}{n}}}{b^2(1+n)}+\frac{nx^{-1-2n}\left(ax^n+bx^{1+n}\right)^{2+\frac{1}{n}}}{b^2(1+2n)}$$

output
```
-a*n*x^(-1-n)*(a*x^n+b*x^(1+n))^(1+1/n)/b^2/(1+n)+n*x^(-1-2*n)*(a*x^n+b*x^
(1+n))^(2+1/n)/b^2/(1+2*n)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.64

$$\int \left(ax^n+bx^{1+n}\right)^{\frac{1}{n}} dx=\frac{n(a+bx)\left(x^n(a+bx)\right)^{\frac{1}{n}}(-an+b(1+n)x)}{b^2(1+n)(1+2n)x}$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^n^(-1),x]
```

output
```
(n*(a + b*x)*(x^n*(a + b*x))^n^(-1)*(-(a*n) + b*(1 + n)*x))/(b^2*(1 + n)*(
1 + 2*n)*x)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^n+bx^{n+1}\right)^{\frac{1}{n}}dx$$

$$\Big\downarrow \text{ 1908}$$

$$\frac{nx^{-n}\left(ax^n+bx^{n+1}\right)^{\frac{1}{n}+1}}{b(2n+1)}-\frac{an\int \frac{\left(ax^n+bx^{n+1}\right)^{\frac{1}{n}}}{x}dx}{b(2n+1)}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{nx^{-n}\left(ax^n+bx^{n+1}\right)^{\frac{1}{n}+1}}{b(2n+1)}-\frac{an^2x^{-n-1}\left(ax^n+bx^{n+1}\right)^{\frac{1}{n}+1}}{b^2(n+1)(2n+1)}$$

input
```
Int[(a*x^n + b*x^(1 + n))^n^(-1),x]
```

output
```
-((a*n^2*x^(-1 - n)*(a*x^n + b*x^(1 + n))^(1 + n^(-1)))/(b^2*(1 + n)*(1 +
2*n))) + (n*(a*x^n + b*x^(1 + n))^(1 + n^(-1)))/(b*(1 + 2*n)*x^n)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.74

method	result	size
orering	$-\frac{(-nxb+an-bx)n(bx+a)\left(a\,x^{n}+b\,x^{1+n}\right)^{\frac{1}{n}}}{b^{2}(2n^{2}+3n+1)x}$	57

input
```
int((a*x^n+b*x^(1+n))^(1/n),x,method=_RETURNVERBOSE)
```

output
```
-(-b*n*x+a*n-b*x)*n/b^2/(2*n^2+3*n+1)*(b*x+a)/x*(a*x^n+b*x^(1+n))^(1/n)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \left(ax^n+bx^{1+n}\right)^{\frac{1}{n}}\,dx=-\frac{(a^2n^2-abnx-(b^2n^2+b^2n)x^2)\left(\frac{(bx+a)x^{n+1}}{x}\right)^{\left(\frac{1}{n}\right)}}{(2\,b^2n^2+3\,b^2n+b^2)x}$$

input
```
integrate((a*x^n+b*x^(1+n))^(1/n),x, algorithm="fricas")
```

output
```
-(a^2*n^2 - a*b*n*x - (b^2*n^2 + b^2*n)*x^2)*((b*x + a)*x^(n + 1)/x)^(1/n)
/((2*b^2*n^2 + 3*b^2*n + b^2)*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(68) = 136$.

Time = 4.07 (sec) , antiderivative size = 340, normalized size of antiderivative = 4.42

$$\int \left(ax^n + bx^{1+n}\right)^{\frac{1}{n}} dx$$
$$= \begin{cases} \frac{x(ax^n)^{\frac{1}{n}}}{2} & \text{for } b = 0 \\ -\frac{a\log\left(\frac{a}{b}+x\right)}{b^2} + \frac{x}{b} & \text{for } n = -1 \\ \frac{a\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{ab^2+b^3x} + \frac{a\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{ab^2+b^3x} + \frac{a}{ab^2+b^3x} + \frac{bx\log\left(\sqrt{x}-\sqrt{-\frac{a}{b}}\right)}{ab^2+b^3x} + \frac{bx\log\left(\sqrt{x}+\sqrt{-\frac{a}{b}}\right)}{ab^2+b^3x} & \text{for } n = -\frac{1}{2} \\ -\frac{a^2n^2(ax^n+bx^{n+1})^{\frac{1}{n}}}{2b^2n^2x+3b^2nx+b^2x} + \frac{abnx(ax^n+bx^{n+1})^{\frac{1}{n}}}{2b^2n^2x+3b^2nx+b^2x} + \frac{b^2n^2x^2(ax^n+bx^{n+1})^{\frac{1}{n}}}{2b^2n^2x+3b^2nx+b^2x} + \frac{b^2nx^2(ax^n+bx^{n+1})^{\frac{1}{n}}}{2b^2n^2x+3b^2nx+b^2x} & \text{otherwise} \end{cases}$$

input
```
integrate((a*x**n+b*x**(1+n))**(1/n),x)
```

output
```
Piecewise((x*(a*x**n)**(1/n)/2, Eq(b, 0)), (-a*log(a/b + x)/b**2 + x/b, Eq
(n, -1)), (a*log(sqrt(x) - sqrt(-a/b))/(a*b**2 + b**3*x) + a*log(sqrt(x) +
 sqrt(-a/b))/(a*b**2 + b**3*x) + a/(a*b**2 + b**3*x) + b*x*log(sqrt(x) - s
qrt(-a/b))/(a*b**2 + b**3*x) + b*x*log(sqrt(x) + sqrt(-a/b))/(a*b**2 + b**
3*x), Eq(n, -1/2)), (-a**2*n**2*(a*x**n + b*x**(n + 1))**(1/n)/(2*b**2*n**
2*x + 3*b**2*n*x + b**2*x) + a*b*n*x*(a*x**n + b*x**(n + 1))**(1/n)/(2*b**
2*n**2*x + 3*b**2*n*x + b**2*x) + b**2*n**2*x**2*(a*x**n + b*x**(n + 1))**
(1/n)/(2*b**2*n**2*x + 3*b**2*n*x + b**2*x) + b**2*n*x**2*(a*x**n + b*x**(
n + 1))**(1/n)/(2*b**2*n**2*x + 3*b**2*n*x + b**2*x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.84

$$\int \left(ax^n + bx^{1+n}\right)^{\frac{1}{n}} dx = \frac{((n^2+n)b^2x^2 - a^2n^2 + abnx)e^{\left(\frac{\log(bx+a)}{n} + \frac{\log(x^n)}{n}\right)}}{(2\,n^2 + 3\,n + 1)b^2x}$$

input
```
integrate((a*x^n+b*x^(1+n))^(1/n),x, algorithm="maxima")
```

output
```
((n^2 + n)*b^2*x^2 - a^2*n^2 + a*b*n*x)*e^(log(b*x + a)/n + log(x^n)/n)/((
2*n^2 + 3*n + 1)*b^2*x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.16

$$\int \left(ax^n + bx^{1+n}\right)^{\frac{1}{n}} dx$$
$$= \frac{(bx+a)^{\left(\frac{1}{n}\right)}b^2n^2x^2 + (bx+a)^{\left(\frac{1}{n}\right)}b^2nx^2 - (bx+a)^{\left(\frac{1}{n}\right)}a^2n^2 + (bx+a)^{\left(\frac{1}{n}\right)}abnx}{2\,b^2n^2 + 3\,b^2n + b^2}$$

input
```
integrate((a*x^n+b*x^(1+n))^(1/n),x, algorithm="giac")
```

output
```
((b*x + a)^(1/n)*b^2*n^2*x^2 + (b*x + a)^(1/n)*b^2*n*x^2 - (b*x + a)^(1/n)
*a^2*n^2 + (b*x + a)^(1/n)*a*b*n*x)/(2*b^2*n^2 + 3*b^2*n + b^2)
```

Mupad [B] (verification not implemented)

Time = 9.80 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.65

$$\int \left(ax^n + bx^{1+n}\right)^{\frac{1}{n}} dx = \frac{x\left(a\,x^n + b\,x^{n+1}\right)^{1/n}{}_2\mathrm{F}_1\left(2, -\frac{1}{n};\ 3;\ -\frac{b\,x}{a}\right)}{2\left(\frac{b\,x}{a}+1\right)^{1/n}}$$

input
```
int((a*x^n + b*x^(n + 1))^(1/n),x)
```

output
```
(x*(a*x^n + b*x^(n + 1))^(1/n)*hypergeom([2, -1/n], 3, -(b*x)/a))/(2*((b*x
)/a + 1)^(1/n))
```

Reduce [F]

$$\int \left(ax^n + bx^{1+n}\right)^{\frac{1}{n}} dx$$
$$= \frac{n\left((x^n a + x^n bx)^{\frac{1}{n}} a + (x^n a + x^n bx)^{\frac{1}{n}} bnx + (x^n a + x^n bx)^{\frac{1}{n}} bx - 2\left(\int \frac{(x^n a+x^n bx)^{\frac{1}{n}}}{2b\,n^2x^2+2a\,n^2x+3bn\,x^2+3anx+b\,x^2+ax}dx\right.\right.}{b\left(2n^2+3\right.}$$

input
```
int((a*x^n+b*x^(1+n))^(1/n),x)
```

output
```
(n*((x**n*a + x**n*b*x)**(1/n)*a + (x**n*a + x**n*b*x)**(1/n)*b*n*x + (x**
n*a + x**n*b*x)**(1/n)*b*x - 2*int((x**n*a + x**n*b*x)**(1/n)/(2*a*n**2*x
+ 3*a*n*x + a*x + 2*b*n**2*x**2 + 3*b*n*x**2 + b*x**2),x)*a**2*n**2 - 3*in
t((x**n*a + x**n*b*x)**(1/n)/(2*a*n**2*x + 3*a*n*x + a*x + 2*b*n**2*x**2 +
 3*b*n*x**2 + b*x**2),x)*a**2*n - int((x**n*a + x**n*b*x)**(1/n)/(2*a*n**2
*x + 3*a*n*x + a*x + 2*b*n**2*x**2 + 3*b*n*x**2 + b*x**2),x)*a**2))/(b*(2*
n**2 + 3*n + 1))
```

3.142 $\int \left(ax^n+bx^{1+n}\right)^{-1/n} dx$

Optimal result

Integrand size = 19, antiderivative size = 64

$$\int \left(ax^n+bx^{1+n}\right)^{-1/n} dx=\frac{nx(a+bx)\left(ax^n+bx^{1+n}\right)^{-1/n}\text{Hypergeometric2F1}\left(1,-\frac{1-n}{n},2-\frac{1}{n},1+\frac{bx}{a}\right)}{a(1-n)}$$

output
```
n*x*(b*x+a)*hypergeom([1, -(1-n)/n],[2-1/n],1+b*x/a)/a/(1-n)/((a*x^n+b*x^(
1+n))^(1/n))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88

$$\int \left(ax^n+bx^{1+n}\right)^{-1/n} dx = -\frac{nx(a+bx)\left(x^n(a+bx)\right)^{-1/n}\text{Hypergeometric2F1}\left(1,\frac{-1+n}{n},2-\frac{1}{n},1+\frac{bx}{a}\right)}{a(-1+n)}$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^(-n^(-1)),x]
```

output
```
-((n*x*(a + b*x)*Hypergeometric2F1[1, (-1 + n)/n, 2 - n^(-1), 1 + (b*x)/a]
)/(a*(-1 + n)*(x^n*(a + b*x))^n^(-1)))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1917, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^n+bx^{n+1}\right)^{-1/n} dx$$

$$\Big\downarrow \text{1917}$$

$$x(a+bx)^{\frac{1}{n}}\left(ax^n+bx^{n+1}\right)^{-1/n}\int \frac{(a+bx)^{-1/n}}{x}dx$$

$$\Big\downarrow \text{75}$$

$$\frac{nx(a+bx)^{\frac{1}{n}-\frac{1-n}{n}}\left(ax^n+bx^{n+1}\right)^{-1/n}\text{Hypergeometric2F1}\left(1,-\frac{1-n}{n},2-\frac{1}{n},\frac{bx}{a}+1\right)}{a(1-n)}$$

input
```
Int[(a*x^n + b*x^(1 + n))^(-n^(-1)),x]
```

output
```
(n*x*(a + b*x)^(n^(-1) - (1 - n)/n)*Hypergeometric2F1[1, -((1 - n)/n), 2 -
 n^(-1), 1 + (b*x)/a])/(a*(1 - n)*(a*x^n + b*x^(1 + n))^n^(-1))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \left(a\,x^{n}+b\,x^{1+n}\right)^{-\frac{1}{n}}dx$$

input
```
int((a*x^n+b*x^(1+n))^(-1/n),x)
```

output
```
int((a*x^n+b*x^(1+n))^(-1/n),x)
```

Fricas [F]

$$\int \left(ax^{n}+bx^{1+n}\right)^{-1/n}\,dx=\int \frac{1}{\left(bx^{n+1}+ax^{n}\right)^{\left(\frac{1}{n}\right)}}\,dx$$

input
```
integrate((a*x^n+b*x^(1+n))^(-1/n),x, algorithm="fricas")
```

output
```
integral(1/((b*x^(n + 1) + a*x^n)^(1/n)), x)
```

Sympy [F]

$$\int \left(ax^n + bx^{1+n}\right)^{-1/n} dx = \int \left(ax^n + bx^{n+1}\right)^{-\frac{1}{n}} dx$$

input
```
integrate((a*x**n+b*x**(1+n))**(-1/n),x)
```

output
```
Integral((a*x**n + b*x**(n + 1))**(-1/n), x)
```

Maxima [F]

$$\int \left(ax^n + bx^{1+n}\right)^{-1/n} dx = \int \frac{1}{\left(bx^{n+1} + ax^n\right)^{\left(\frac{1}{n}\right)}} dx$$

input
```
integrate((a*x^n+b*x^(1+n))^(-1/n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^(n + 1) + a*x^n)^(1/n)), x)
```

Giac [F]

$$\int \left(ax^n + bx^{1+n}\right)^{-1/n} dx = \int \frac{1}{\left(bx^{n+1} + ax^n\right)^{\left(\frac{1}{n}\right)}} dx$$

input
```
integrate((a*x^n+b*x^(1+n))^(-1/n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^(n + 1) + a*x^n)^(1/n)), x)
```

Mupad [B] (verification not implemented)

Time = 9.58 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \left(ax^n+bx^{1+n}\right)^{-1/n}\,dx=-\frac{n\,x\,\left(\frac{a}{b\,x}+1\right)^{1/n}\,{}_2\mathrm{F}_1\left(\frac{1}{n},\frac{1}{n};\ \frac{1}{n}+1;\ -\frac{a}{b\,x}\right)}{\left(a\,x^n+b\,x^{n+1}\right)^{1/n}}$$

input
```
int(1/(a*x^n + b*x^(n + 1))^(1/n),x)
```

output
```
-(n*x*(a/(b*x) + 1)^(1/n)*hypergeom([1/n, 1/n], 1/n + 1, -a/(b*x)))/(a*x^n
 + b*x^(n + 1))^(1/n)
```

Reduce [F]

$$\int \left(ax^n+bx^{1+n}\right)^{-1/n}\,dx=\int \frac{1}{\left(x^na+x^nbx\right)^{\frac{1}{n}}}dx$$

input
```
int((a*x^n+b*x^(1+n))^(-1/n),x)
```

output
```
int(1/(x**n*a + x**n*b*x)**(1/n),x)
```

3.143 $\int \left(ax^n+bx^{1+n}\right)^{-2/n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \left(ax^n+bx^{1+n}\right)^{-2/n}\,dx = -\frac{bnx^2(a+bx)\left(ax^n+bx^{1+n}\right)^{-2/n}\text{Hypergeometric2F1}\left(2,-\frac{2-n}{n},2-\frac{2}{n},1+\frac{bx}{a}\right)}{a^2(2-n)}$$

output
```
-b*n*x^2*(b*x+a)*hypergeom([2, -(2-n)/n],[2-2/n],1+b*x/a)/a^2/(2-n)/((a*x^
n+b*x^(1+n))^(2/n))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.85

$$\int \left(ax^n+bx^{1+n}\right)^{-2/n}\,dx = \frac{bnx^2(a+bx)\left(x^n(a+bx)\right)^{-2/n}\text{Hypergeometric2F1}\left(2,\frac{-2+n}{n},2-\frac{2}{n},1+\frac{bx}{a}\right)}{a^2(-2+n)}$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^(-2/n),x]
```

output
```
(b*n*x^2*(a + b*x)*Hypergeometric2F1[2, (-2 + n)/n, 2 - 2/n, 1 + (b*x)/a])
/(a^2*(-2 + n)*(x^n*(a + b*x))^(2/n))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1917, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^n+bx^{n+1}\right)^{-2/n}dx$$

$$\Big\downarrow \text{1917}$$

$$x^2(a+bx)^{2/n}\left(ax^n+bx^{n+1}\right)^{-2/n}\int \frac{(a+bx)^{-2/n}}{x^2}dx$$

$$\Big\downarrow \text{75}$$

$$-\frac{bnx^2(a+bx)^{\frac{2}{n}-\frac{2-n}{n}}\left(ax^n+bx^{n+1}\right)^{-2/n}\text{Hypergeometric2F1}\left(2,-\frac{2-n}{n},2-\frac{2}{n},\frac{bx}{a}+1\right)}{a^2(2-n)}$$

input
```
Int[(a*x^n + b*x^(1 + n))^(-2/n),x]
```

output
```
-((b*n*x^2*(a + b*x)^(2/n - (2 - n)/n)*Hypergeometric2F1[2, -((2 - n)/n),
2 - 2/n, 1 + (b*x)/a])/(a^2*(2 - n)*(a*x^n + b*x^(1 + n))^(2/n)))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \left(a\,x^{n}+b\,x^{1+n}\right)^{-\frac{2}{n}}dx$$

input
```
int((a*x^n+b*x^(1+n))^(-2/n),x)
```

output
```
int((a*x^n+b*x^(1+n))^(-2/n),x)
```

Fricas [F]

$$\int \left(ax^n+bx^{1+n}\right)^{-2/n}\,dx=\int \frac{1}{\left(bx^{n+1}+ax^n\right)^{\frac{2}{n}}}\,dx$$

input
```
integrate((a*x^n+b*x^(1+n))^(-2/n),x, algorithm="fricas")
```

output
```
integral(1/((b*x^(n + 1) + a*x^n)^(2/n)), x)
```

Sympy [F]

$$\int \left(ax^n+bx^{1+n}\right)^{-2/n}\,dx=\int \left(ax^n+bx^{n+1}\right)^{-\frac{2}{n}}\,dx$$

input
```
integrate((a*x**n+b*x**(1+n))**(-2/n),x)
```

output
```
Integral((a*x**n + b*x**(n + 1))**(-2/n), x)
```

Maxima [F]

$$\int \left(ax^n+bx^{1+n}\right)^{-2/n}\,dx=\int \frac{1}{\left(bx^{n+1}+ax^n\right)^{\frac{2}{n}}}\,dx$$

input
```
integrate((a*x^n+b*x^(1+n))^(-2/n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^(n + 1) + a*x^n)^(2/n)), x)
```

Giac [F]

$$\int \left(ax^n+bx^{1+n}\right)^{-2/n}\,dx=\int \frac{1}{\left(bx^{n+1}+ax^n\right)^{\frac{2}{n}}}\,dx$$

input
```
integrate((a*x^n+b*x^(1+n))^(-2/n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^(n + 1) + a*x^n)^(2/n)), x)
```

Mupad [B] (verification not implemented)

Time = 9.40 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int \left(ax^n+bx^{1+n}\right)^{-2/n}\,dx=-\frac{x\,\left(\frac{a}{b\,x}+1\right)^{2/n}\,{}_2\mathrm{F}_1\left(\frac{2}{n},\frac{n+2}{n};\ \frac{2\,(n+1)}{n};\ -\frac{a}{b\,x}\right)}{\left(\frac{2\,(n+1)}{n}-1\right)\,\left(a\,x^n+b\,x^{n+1}\right)^{2/n}}$$

input
```
int(1/(a*x^n + b*x^(n + 1))^(2/n),x)
```

output
```
-(x*(a/(b*x) + 1)^(2/n)*hypergeom([2/n, (n + 2)/n], (2*(n + 1))/n, -a/(b*x
)))/((((2*(n + 1))/n - 1)*(a*x^n + b*x^(n + 1))^(2/n))
```

Reduce [F]

$$\int \left(ax^n+bx^{1+n}\right)^{-2/n}\,dx=\int \frac{1}{(x^na+x^nbx)^{\frac{2}{n}}}dx$$

input
```
int((a*x^n+b*x^(1+n))^(-2/n),x)
```

output
```
int(1/(x**n*a + x**n*b*x)**(2/n),x)
```

3.144 $\int \left(ax^n+bx^{1+n}\right)^{-3/n}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 69

$$\int \left(ax^n+bx^{1+n}\right)^{-3/n}\,dx=\frac{b^2nx^3(a+bx)\left(ax^n+bx^{1+n}\right)^{-3/n}\text{Hypergeometric2F1}\left(3,-\frac{3-n}{n},2-\frac{3}{n},1+\frac{bx}{a}\right)}{a^3(3-n)}$$

output
```
b^2*n*x^3*(b*x+a)*hypergeom([3, -(3-n)/n],[2-3/n],1+b*x/a)/a^3/(3-n)/((a*x
^n+b*x^(1+n))^(3/n))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.88

$$\int \left(ax^n+bx^{1+n}\right)^{-3/n}\,dx=-\frac{b^2nx^3(a+bx)\left(x^n(a+bx)\right)^{-3/n}\text{Hypergeometric2F1}\left(3,\frac{-3+n}{n},2-\frac{3}{n},1+\frac{bx}{a}\right)}{a^3(-3+n)}$$

input
```
Integrate[(a*x^n + b*x^(1 + n))^(-3/n),x]
```

output
```
-((b^2*n*x^3*(a + b*x)*Hypergeometric2F1[3, (-3 + n)/n, 2 - 3/n, 1 + (b*x)
/a])/(a^3*(-3 + n)*(x^n*(a + b*x))^(3/n)))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1917, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^n+bx^{n+1}\right)^{-3/n}dx$$

$$\Big\downarrow \text{ 1917}$$

$$x^3(a+bx)^{3/n}\left(ax^n+bx^{n+1}\right)^{-3/n}\int \frac{(a+bx)^{-3/n}}{x^3}dx$$

$$\Big\downarrow \text{ 75}$$

$$\frac{b^2nx^3(a+bx)^{\frac{3}{n}-\frac{3-n}{n}}\left(ax^n+bx^{n+1}\right)^{-3/n}\text{Hypergeometric2F1}\left(3,-\frac{3-n}{n},2-\frac{3}{n},\frac{bx}{a}+1\right)}{a^3(3-n)}$$

input
```
Int[(a*x^n + b*x^(1 + n))^(-3/n),x]
```

output
```
(b^2*n*x^3*(a + b*x)^(3/n - (3 - n)/n)*Hypergeometric2F1[3, -((3 - n)/n),
2 - 3/n, 1 + (b*x)/a])/(a^3*(3 - n)*(a*x^n + b*x^(1 + n))^(3/n))
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 1917
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^FracPart[p]/(x^(j*FracPart[p])*(a + b*x^(n - j))^FracPart[p])   Int[
x^(j*p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, j, n, p}, x] &&  !Integ
erQ[p] && NeQ[n, j] && PosQ[n - j]
```

Maple [F]

$$\int \left(a\,x^{n}+b\,x^{1+n}\right)^{-\frac{3}{n}}d x$$

input
```
int((a*x^n+b*x^(1+n))^(-3/n),x)
```

output
```
int((a*x^n+b*x^(1+n))^(-3/n),x)
```

Fricas [F]

$$\int \left(ax^n+bx^{1+n}\right)^{-3/n}\,dx=\int \frac{1}{\left(bx^{n+1}+ax^n\right)^{\frac{3}{n}}}\,dx$$

input
```
integrate((a*x^n+b*x^(1+n))^(-3/n),x, algorithm="fricas")
```

output
```
integral(1/((b*x^(n + 1) + a*x^n)^(3/n)), x)
```

Sympy [F]

$$\int \left(ax^n + bx^{1+n}\right)^{-3/n} dx = \int \left(ax^n + bx^{n+1}\right)^{-\frac{3}{n}} dx$$

input
```
integrate((a*x**n+b*x**(1+n))**(-3/n),x)
```

output
```
Integral((a*x**n + b*x**(n + 1))**(-3/n), x)
```

Maxima [F]

$$\int \left(ax^n + bx^{1+n}\right)^{-3/n} dx = \int \frac{1}{\left(bx^{n+1} + ax^n\right)^{\frac{3}{n}}} dx$$

input
```
integrate((a*x^n+b*x^(1+n))^(-3/n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x^(n + 1) + a*x^n)^(3/n)), x)
```

Giac [F]

$$\int \left(ax^n + bx^{1+n}\right)^{-3/n} dx = \int \frac{1}{\left(bx^{n+1} + ax^n\right)^{\frac{3}{n}}} dx$$

input
```
integrate((a*x^n+b*x^(1+n))^(-3/n),x, algorithm="giac")
```

output
```
integrate(1/((b*x^(n + 1) + a*x^n)^(3/n)), x)
```

Mupad [B] (verification not implemented)

Time = 9.52 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.23

$$\int \left(ax^n+bx^{1+n}\right)^{-3/n}\,dx=-\frac{x\,{\left(\frac{a}{b\,x}+1\right)}^{3/n}\,{}_2\mathrm{F}_1\left(\frac{3}{n},\frac{2\,n+3}{n};\ \frac{3\,\left(n+1\right)}{n};\ -\frac{a}{b\,x}\right)}{\left(\frac{3\,\left(n+1\right)}{n}-1\right)\,{\left(a\,x^n+b\,x^{n+1}\right)}^{3/n}}$$

input
```
int(1/(a*x^n + b*x^(n + 1))^(3/n),x)
```

output
```
-(x*(a/(b*x) + 1)^(3/n)*hypergeom([3/n, (2*n + 3)/n], (3*(n + 1))/n, -a/(b
*x)))/(((3*(n + 1))/n - 1)*(a*x^n + b*x^(n + 1))^(3/n))
```

Reduce [F]

$$\int \left(ax^n+bx^{1+n}\right)^{-3/n}\,dx=\int \frac{1}{\left(x^n a+x^n b x\right)^{\frac{3}{n}}}dx$$

input
```
int((a*x^n+b*x^(1+n))^(-3/n),x)
```

output
```
int(1/(x**n*a + x**n*b*x)**(3/n),x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#            Port of original Maple grading function by
#            Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#            added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file