Computer Algebra
Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.6-Improper-linear-binomial/81-1.1.6.3

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3.249 $\int (d+ex)^2\,(bx+cx^2)^p\,dx$ . . . 2038
3.250 $\int (d+ex)\,(bx+cx^2)^p\,dx$ . . . 2045
3.251 $\int (bx+cx^2)^p\,dx$ . . . 2050
3.252 $\int \frac{(bx+cx^2)^p}{d+ex}\,dx$ . . . 2054
3.253 $\int \frac{(bx+cx^2)^p}{(d+ex)^2}\,dx$ . . . 2059
3.254 $\int (d+ex)^{3/2}\,(bx+cx^2)^p\,dx$ . . . 2065
3.255 $\int \sqrt{d+ex}(bx+cx^2)^p\,dx$ . . . 2071
3.256 $\int \frac{(bx+cx^2)^p}{\sqrt{d+ex}}\,dx$ . . . 2077
3.257 $\int \frac{(bx+cx^2)^p}{(d+ex)^{3/2}}\,dx$ . . . 2083
3.258 $\int (3+ex)^m\,(2x+cx^2)^p\,dx$ . . . 2089
3.259 $\int (3+ex)^m\,(bx+cx^2)^p\,dx$ . . . 2095
3.260 $\int (d+ex)^m\,(2x+cx^2)^p\,dx$ . . . 2101
3.261 $\int (d+ex)^m\,(bx+cx^2)^p\,dx$ . . . 2107

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CHAPTER 1

Introduction

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**261**]. This is test number [81].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (261)	0.00 (0)
Mathematica	100.00 (261)	0.00 (0)
Maple	91.19 (238)	8.81 (23)
Fricas	91.19 (238)	8.81 (23)
Giac	67.82 (177)	32.18 (84)
Reduce	65.90 (172)	34.10 (89)
Mupad	55.94 (146)	44.06 (115)
Maxima	51.72 (135)	48.28 (126)
Sympy	44.44 (116)	55.56 (145)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

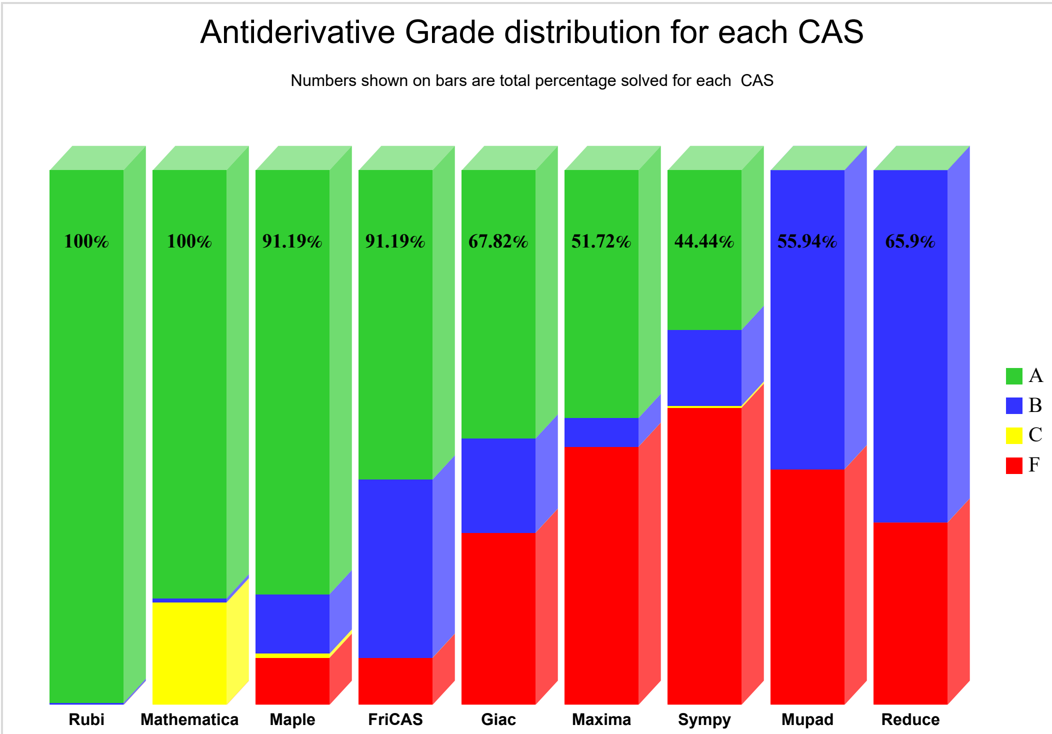
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

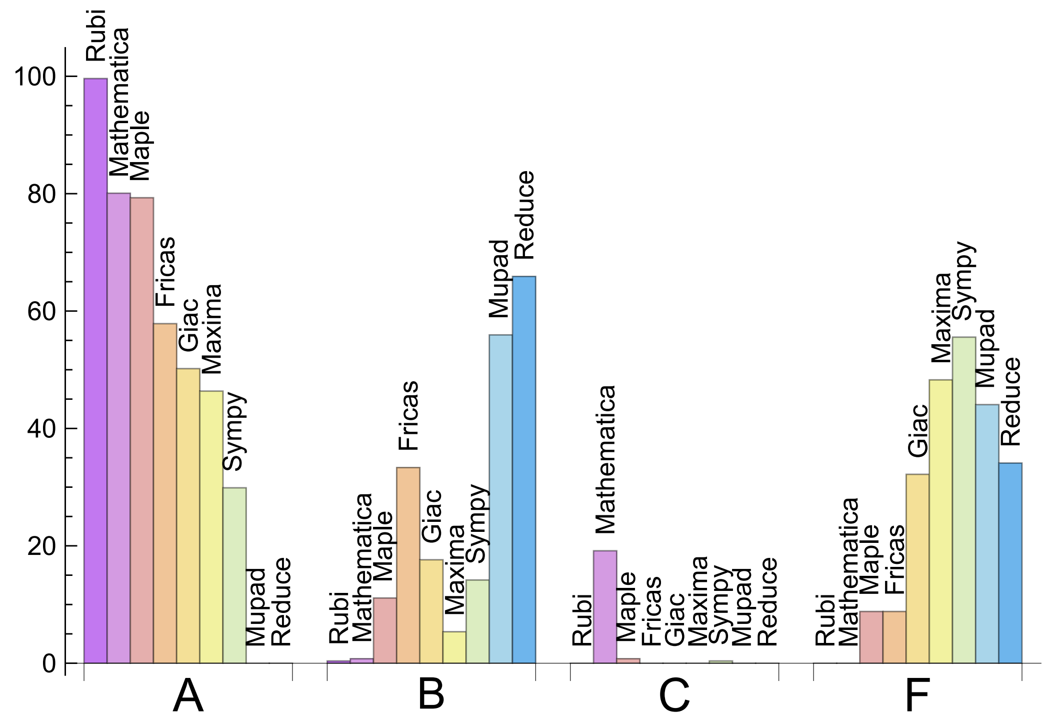
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.617	0.383	0.000	0.000
Mathematica	80.077	0.766	19.157	0.000
Maple	79.310	11.111	0.766	8.812
Fricas	57.854	33.333	0.000	8.812
Giac	50.192	17.625	0.000	32.184
Maxima	46.360	5.364	0.000	48.276
Sympy	29.885	14.176	0.383	55.556
Mupad	0.000	55.939	0.000	44.061
Reduce	0.000	65.900	0.000	34.100

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	23	100.00	0.00	0.00
Maple	23	100.00	0.00	0.00
Giac	84	90.48	3.57	5.95
Reduce	89	100.00	0.00	0.00
Mupad	115	0.00	100.00	0.00
Maxima	126	60.32	0.00	39.68
Sympy	145	87.59	12.41	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.04
Giac	0.18
Fricas	0.48
Rubi	0.62
Reduce	1.00
Maple	1.00
Mathematica	3.19
Sympy	3.96
Mupad	4.90

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mathematica	176.83	1.07	130.00	0.95
Rubi	181.84	1.03	145.00	1.00
Maxima	189.16	1.37	142.00	1.18
Maple	243.26	1.18	152.50	0.96
Giac	331.71	1.89	189.00	1.28
Reduce	420.24	2.42	238.50	1.67
Sympy	504.04	3.25	204.00	1.50
Fricas	590.95	2.71	344.00	1.77
Mupad	638.18	3.36	149.00	1.16

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

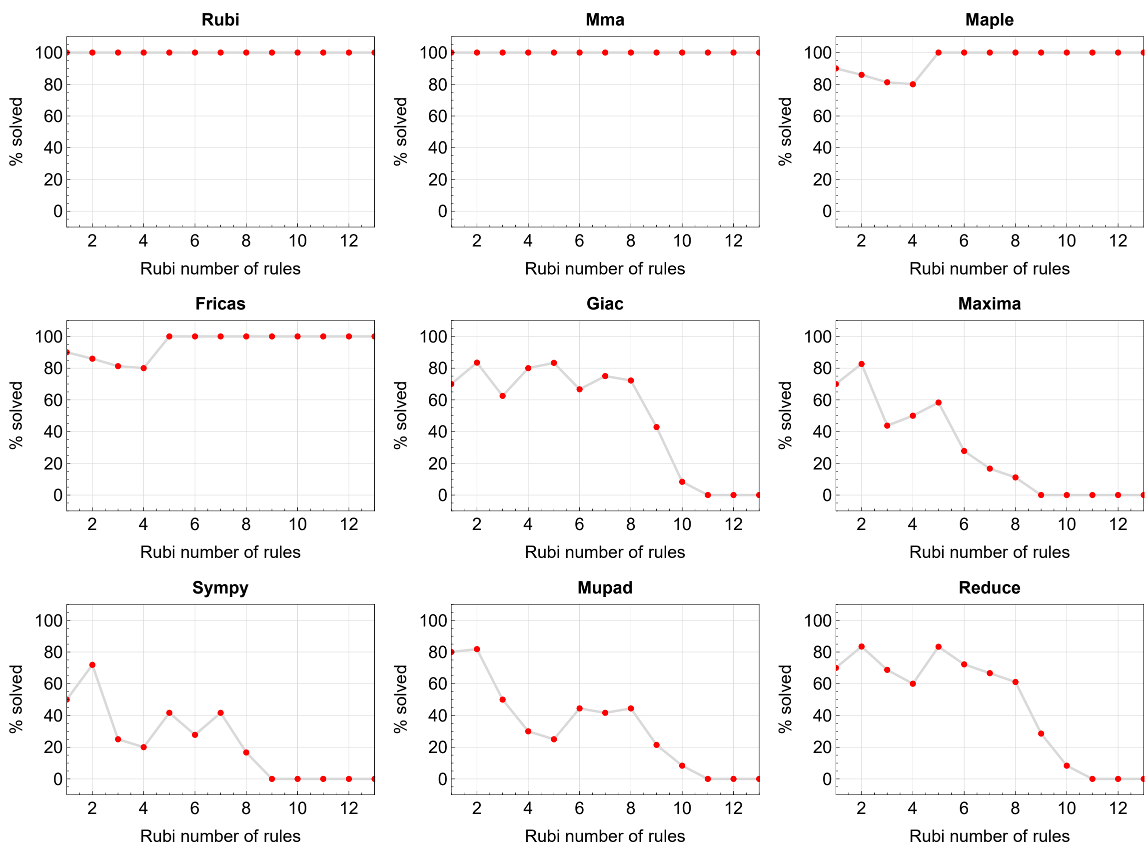


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

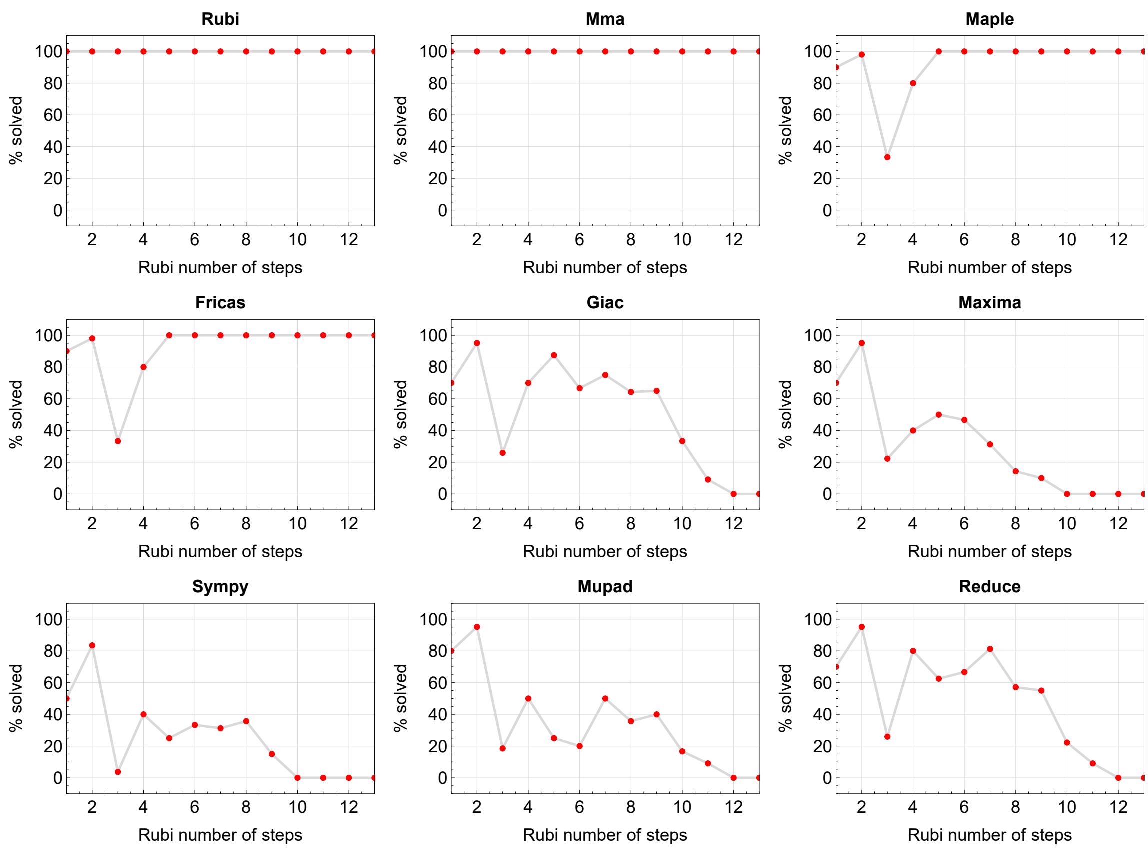


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

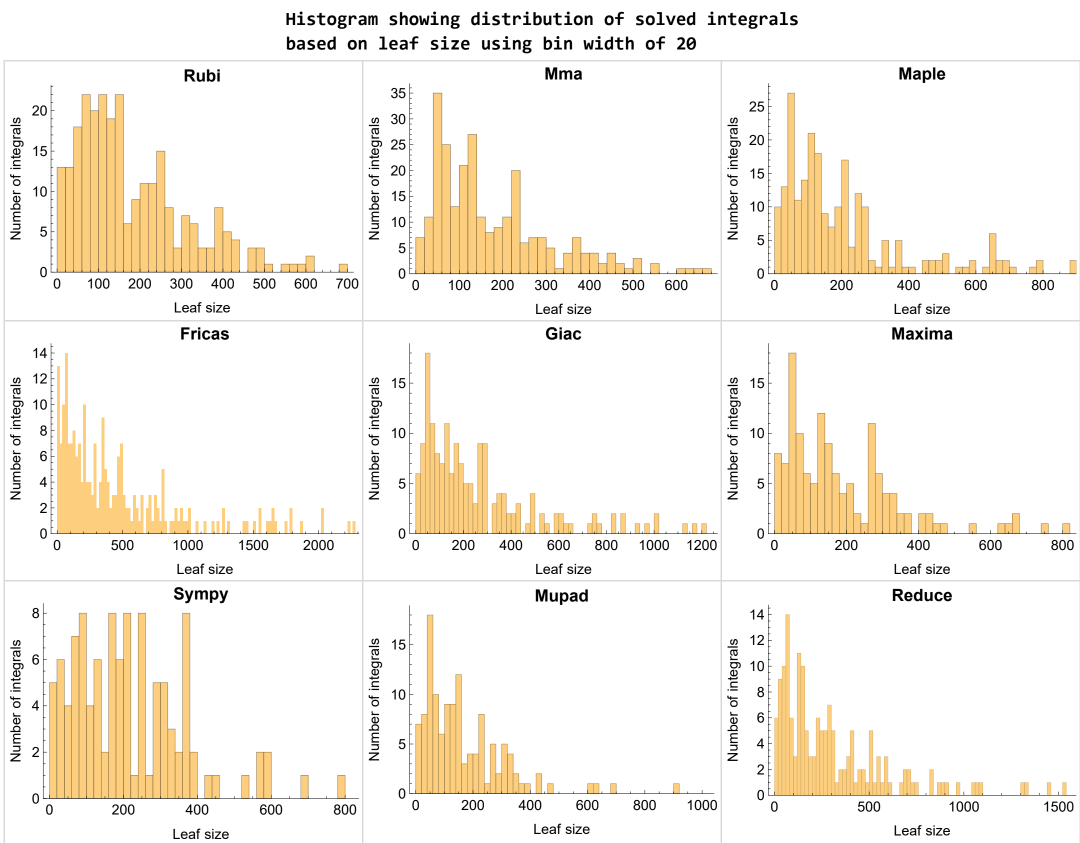


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

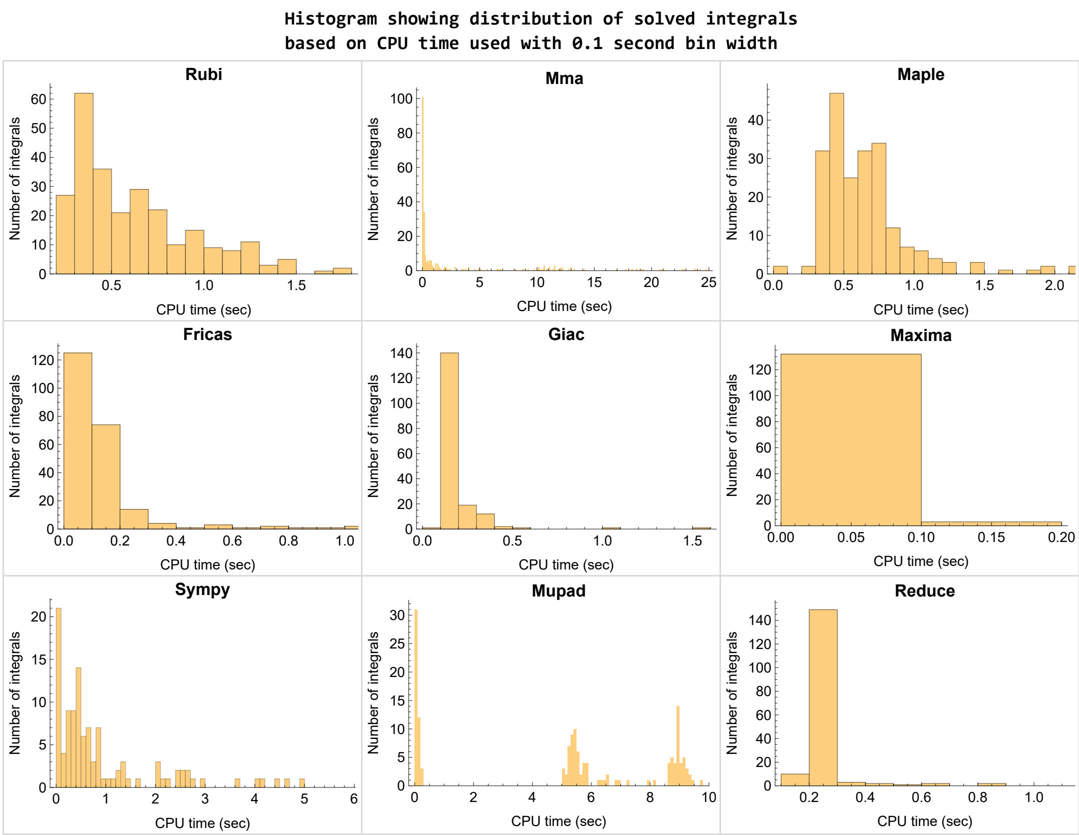


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

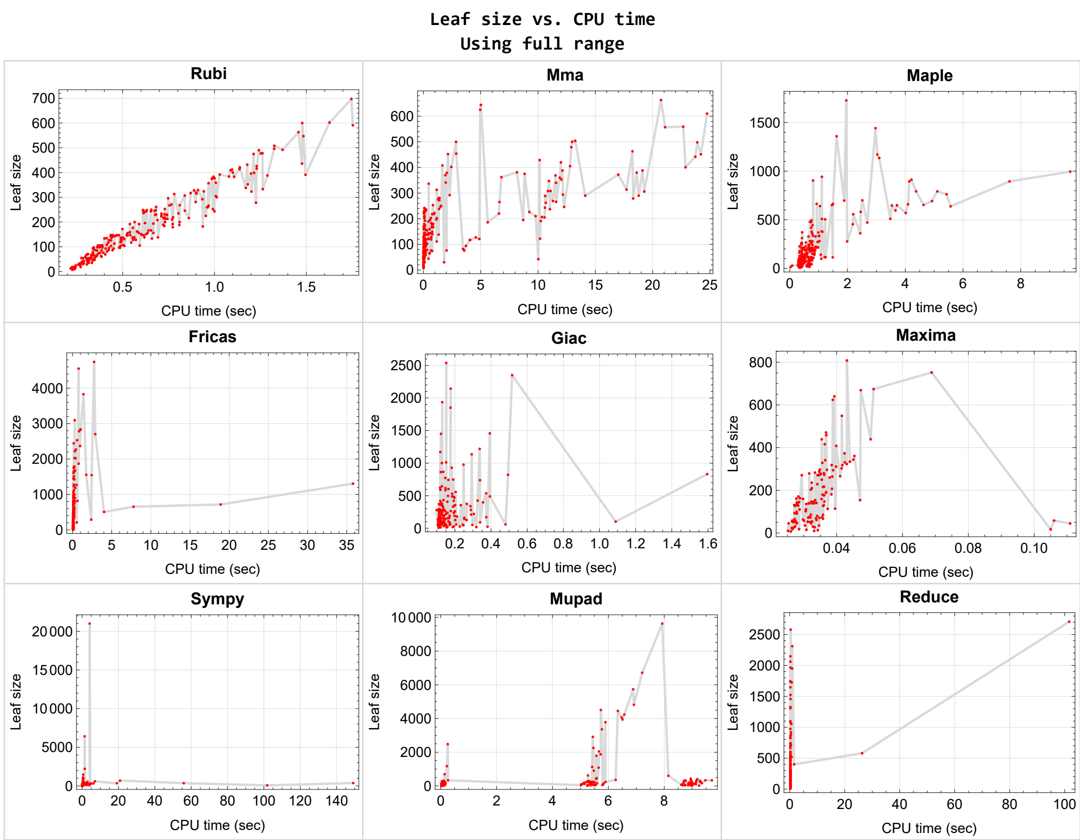


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

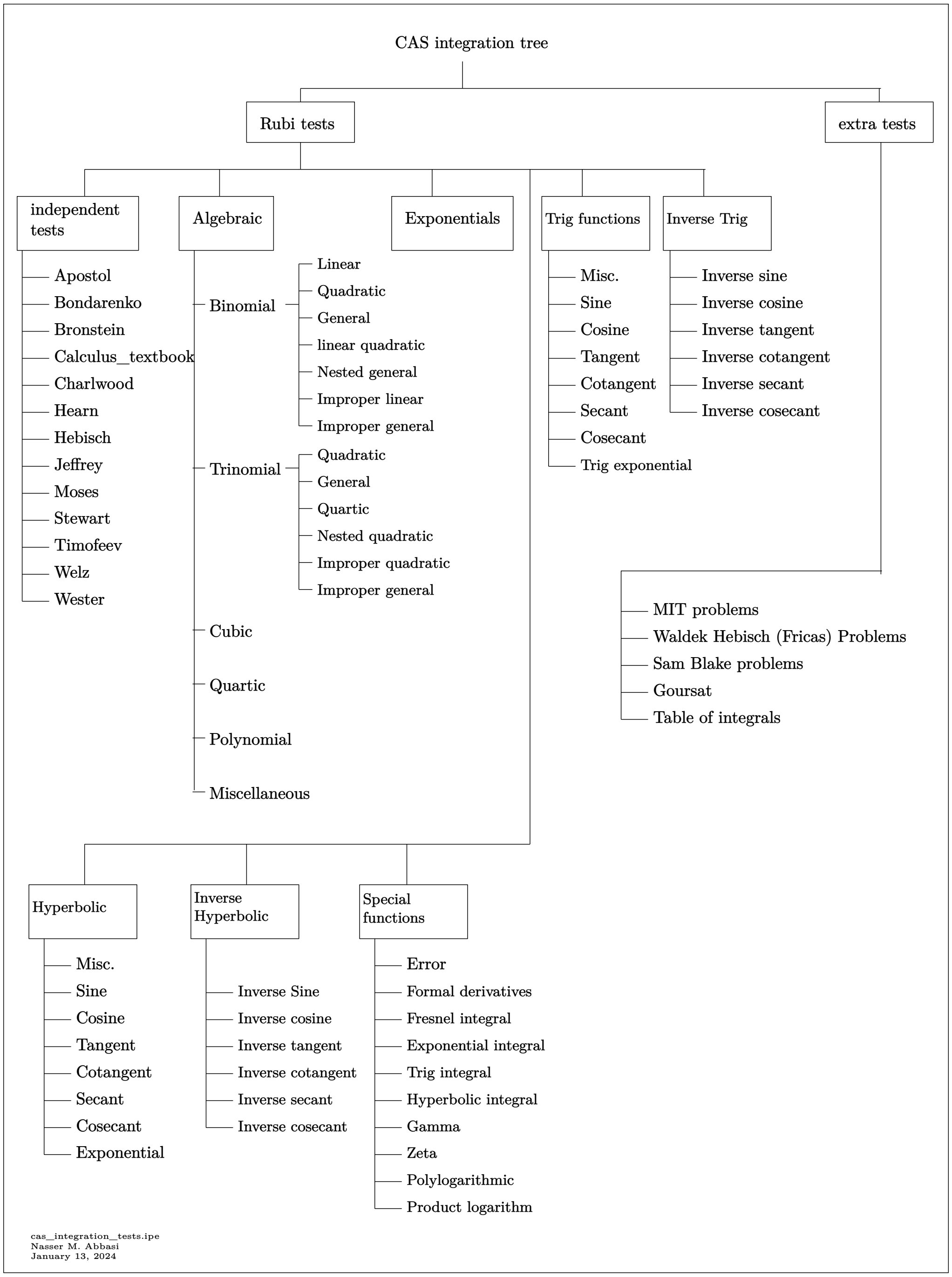
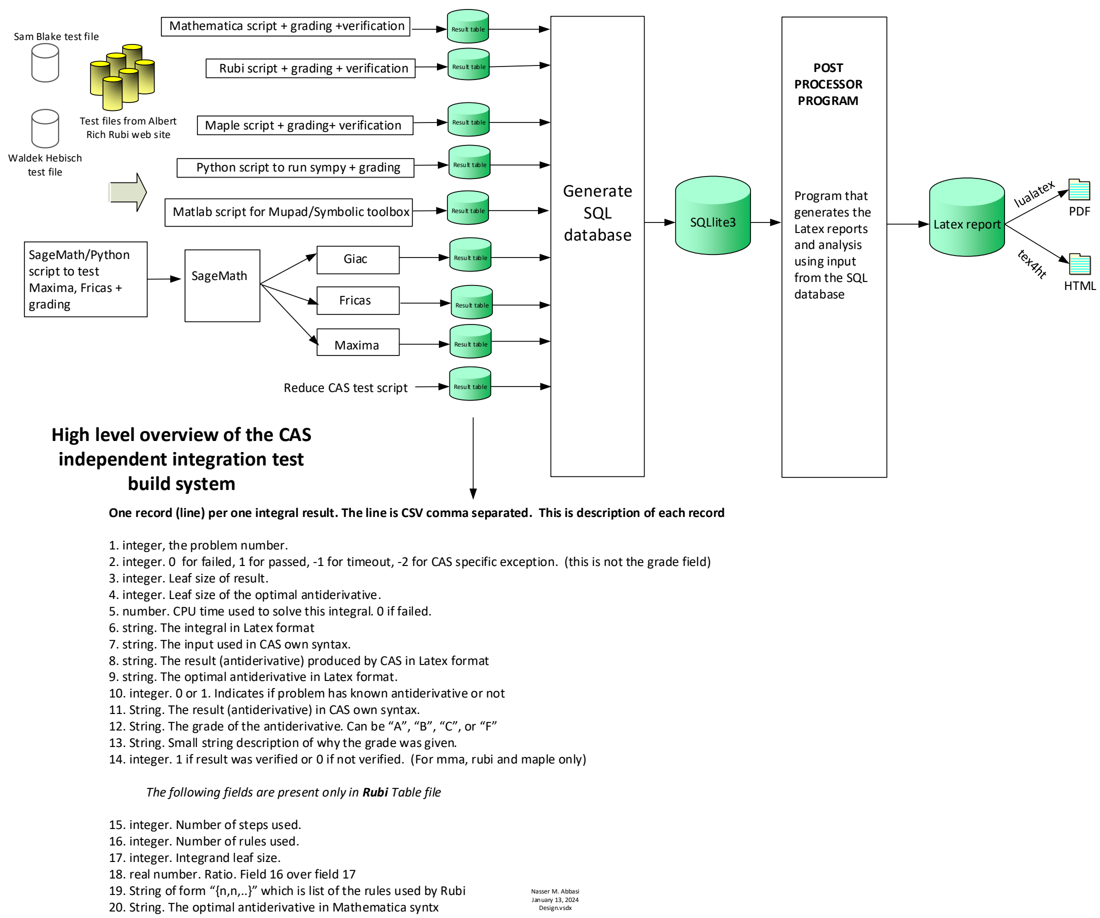


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

Rubi 31
Mma 32
Maple 32
Fricas 33
Maxima 33
Giac 34
Mupad 35
Sympy 35
Reduce 36

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261 }

B grade { 234 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 208, 209, 210, 228, 229, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261 }

B grade { 6, 227 }

C grade { 133, 143, 152, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 230, 231, 232, 233, 234, 235, 236, 237 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 192, 193, 204, 205, 206, 207, 208, 209, 210, 213, 214, 215, 216, 219, 224, 225, 226, 229, 230, 233, 240 }

B grade { 189, 190, 191, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 211, 212, 217, 218, 220, 221, 222, 223, 227, 228, 231, 232, 234, 235, 238, 239 }

C grade { 236, 237 }

F normal fail { 15, 16, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 9, 10, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 67, 68, 78, 83, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 113, 114, 115, 116, 117, 118, 121, 123, 124, 125, 129, 130, 131, 132, 133, 134, 139, 140, 141, 142, 143, 144, 148, 149, 150, 151, 152, 153, 154, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 171, 172, 173, 174, 175, 178, 179, 180, 181, 182, 185, 186, 187, 188, 191, 192, 193, 194, 198, 199, 200, 205, 206, 207, 208, 209, 213, 214, 215, 228, 229, 230, 232, 233, 234, 235, 236, 237 }

B grade { 5, 6, 7, 8, 11, 12, 34, 35, 46, 47, 48, 49, 50, 61, 62, 63, 64, 65, 66, 69, 70, 71, 72, 73, 74, 75, 76, 77, 79, 80, 81, 82, 89, 97, 111, 112, 119, 120, 122, 126, 127, 128, 135, 136, 137, 138, 145, 146, 147, 155, 156, 163, 168, 169, 170, 176, 177, 183, 184, 189, 190, 195, 196, 197, 201, 202, 203, 204, 210, 211, 212, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 231, 238, 239, 240 }

C grade { }

F normal fail { 15, 16, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 2, 3, 4, 5, 7, 8, 9, 10, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 129, 130, 131, 132, 139, 140, 141, 142, 148, 149, 150, 151, 157, 158, 159, 160, 164, 165, 166, 167, 173, 174, 175, 180, 181, 182, 239, 240 }

B grade { 6, 11, 12, 13, 62, 70, 71, 79, 80, 171, 172, 178, 179, 238 }

C grade { }

F normal fail { 15, 16, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261 }

F(-1) timedout fail { }

F(-2) exception fail { 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 133, 134, 135, 136, 137, 138, 143, 144, 145, 146, 147, 152, 153, 154, 155, 156, 161, 162, 163, 168, 169, 170, 176, 177, 183, 184 }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 62, 63, 64, 65, 66, 67, 68, 69, 72, 73, 74, 75, 76, 77, 78, 85, 86, 87, 88, 93, 94, 95, 96, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 123, 124, 125, 126, 129, 130, 131, 132, 139, 140, 141, 142, 148, 149, 150, 151, 157, 158, 159, 161, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 178, 179, 180, 181, 182 }

B grade { 11, 12, 13, 45, 46, 61, 70, 71, 79, 80, 81, 82, 83, 84, 89, 90, 91, 92, 97, 98, 99, 100, 121, 122, 127, 128, 135, 136, 137, 138, 145, 146, 147, 154, 155, 160, 163, 169, 170, 176, 177, 183, 184, 238, 239, 240 }

C grade { }

F normal fail { 15, 16, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261 }

F(-1) timedout fail { 144, 153, 156 }

F(-2) exception fail { 133, 134, 143, 152, 162 }

Mupad

A grade { }

B grade { 1, 2, 3, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 141, 142, 151, 159, 160, 165, 166, 167, 172, 173, 174, 175, 178, 179, 180, 181, 182, 234, 238, 239, 240, 251 }

C grade { }

F normal fail { }

F(-1) timedout fail { 4, 5, 6, 7, 8, 9, 15, 16, 133, 134, 135, 136, 137, 138, 139, 140, 143, 144, 145, 146, 147, 148, 149, 150, 152, 153, 154, 155, 156, 157, 158, 161, 162, 163, 164, 168, 169, 170, 171, 176, 177, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 235, 236, 237, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 14, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 58, 59, 68, 78, 83, 84, 85, 86, 90, 91, 92, 93, 94, 95, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 110, 111, 112, 129, 130, 131, 132, 140, 141, 142, 151, 157, 158 }

B grade { 11, 12, 13, 17, 56, 57, 63, 64, 65, 66, 67, 71, 72, 73, 74, 75, 76, 77, 81, 82, 87, 88, 89, 96, 107, 108, 109, 139, 148, 149, 150, 159, 160, 234, 238, 239, 240 }

C grade { 236 }

F normal fail { 4, 5, 6, 7, 8, 9, 10, 15, 16, 116, 117, 118, 119, 120, 126, 127, 128, 133, 134, 135, 136, 137, 138, 143, 144, 145, 146, 147, 152, 153, 154, 155, 156, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 235, 237, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261 }

F(-1) timedout fail { 54, 60, 61, 62, 69, 70, 79, 80, 113, 114, 115, 121, 122, 123, 124, 125, 220, 221 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 139, 140, 141, 142, 143, 144, 145, 149, 150, 151, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 178, 179, 180, 181, 182, 183, 238, 239, 240 }

C grade { }

F normal fail { 15, 16, 136, 137, 138, 146, 147, 148, 152, 153, 154, 155, 156, 177, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	7	8	7	12	5	8	9	7
N.S.	1	1.00	1.00	1.14	1.00	1.71	0.71	1.14	1.29	1.00
time (sec)	N/A	0.227	0.006	0.339	0.026	0.104	0.028	0.110	0.214	0.031

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	9	9	10	10	8	11	9	8
N.S.	1	1.00	0.90	0.90	1.00	1.00	0.80	1.10	0.90	0.80
time (sec)	N/A	0.226	0.005	0.368	0.033	0.116	0.051	0.111	0.209	0.066

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	15	9	10	10	8	13	11	8
N.S.	1	1.00	1.25	0.75	0.83	0.83	0.67	1.08	0.92	0.67
time (sec)	N/A	0.227	0.004	0.323	0.025	0.097	0.031	0.121	0.244	0.064

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	52	55	38	58	65	0	47	66	0
N.S.	1	0.91	0.96	0.67	1.02	1.14	0.00	0.82	1.16	0.00
time (sec)	N/A	0.321	0.111	0.434	0.106	0.127	0.000	0.248	0.210	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	36	49	29	45	57	0	40	42	0
N.S.	1	0.92	1.26	0.74	1.15	1.46	0.00	1.03	1.08	0.00
time (sec)	N/A	0.281	0.077	0.415	0.111	0.109	0.000	0.119	0.205	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	18	43	12	31	44	0	26	31	0
N.S.	1	0.90	2.15	0.60	1.55	2.20	0.00	1.30	1.55	0.00
time (sec)	N/A	0.243	0.051	0.409	0.027	0.094	0.000	0.114	0.230	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	36	49	26	45	79	0	49	64	0
N.S.	1	0.92	1.26	0.67	1.15	2.03	0.00	1.26	1.64	0.00
time (sec)	N/A	0.278	0.100	0.438	0.032	0.086	0.000	0.118	0.222	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	52	57	42	58	112	0	57	133	0
N.S.	1	0.91	1.00	0.74	1.02	1.96	0.00	1.00	2.33	0.00
time (sec)	N/A	0.326	0.122	0.437	0.027	0.093	0.000	0.173	0.211	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	44	19	17	27	0	27	30	0
N.S.	1	1.00	1.69	0.73	0.65	1.04	0.00	1.04	1.15	0.00
time (sec)	N/A	0.269	0.060	0.646	0.105	0.087	0.000	0.227	0.226	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	17	11	10	15	18	0	18	14	15
N.S.	1	1.31	0.85	0.77	1.15	1.38	0.00	1.38	1.08	1.15
time (sec)	N/A	0.239	0.017	0.390	0.031	0.081	0.000	0.189	0.209	8.985

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	70	129	674	343	2218	530	296	333
N.S.	1	1.00	0.74	1.36	7.09	3.61	23.35	5.58	3.12	3.51
time (sec)	N/A	0.426	0.070	0.515	0.051	0.103	1.442	0.185	0.254	9.474

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	60	76	323	201	993	293	162	197
N.S.	1	1.00	0.87	1.10	4.68	2.91	14.39	4.25	2.35	2.86
time (sec)	N/A	0.363	0.050	0.459	0.043	0.092	0.732	0.112	0.207	8.875

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	34	37	114	81	299	118	70	88
N.S.	1	1.00	0.83	0.90	2.78	1.98	7.29	2.88	1.71	2.15
time (sec)	N/A	0.307	0.032	0.384	0.040	0.117	0.379	0.141	0.214	8.945

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	20	20	18	21	18
N.S.	1	1.00	1.00	1.06	1.00	1.11	1.11	1.00	1.17	1.00
time (sec)	N/A	0.226	0.004	0.351	0.027	0.092	0.018	0.319	0.247	9.248

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	0	0	0	0	0	25	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.78	0.00
time (sec)	N/A	0.268	0.032	0.000	0.000	0.000	0.000	0.000	0.225	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	36	0	0	0	0	0	181	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	4.64	0.00
time (sec)	N/A	0.279	0.035	0.000	0.000	0.000	0.000	0.000	0.220	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	99	97	99	99	107	100	99	91
N.S.	1	1.00	1.60	1.56	1.60	1.60	1.73	1.61	1.60	1.47
time (sec)	N/A	0.383	0.012	0.306	0.028	0.071	0.027	0.116	0.200	8.644

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	67	74	73	73	80	76	75	68
N.S.	1	1.00	1.08	1.19	1.18	1.18	1.29	1.23	1.21	1.10
time (sec)	N/A	0.358	0.013	0.300	0.036	0.078	0.024	0.113	0.215	0.038

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	52	51	51	54	53	51	51
N.S.	1	1.00	0.89	0.95	0.93	0.93	0.98	0.96	0.93	0.93
time (sec)	N/A	0.329	0.009	0.303	0.027	0.083	0.021	0.171	0.206	8.749

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	28	27	27	29	29	27	28
N.S.	1	1.00	0.88	0.85	0.82	0.82	0.88	0.88	0.82	0.85
time (sec)	N/A	0.288	0.005	0.082	0.033	0.079	0.017	0.194	0.211	0.039

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	14	13	13	12	13	13	13
N.S.	1	1.00	1.00	0.82	0.76	0.76	0.71	0.76	0.76	0.76
time (sec)	N/A	0.233	0.000	0.026	0.034	0.073	0.015	0.154	0.192	0.021

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	43	45	47	37	45	50	46
N.S.	1	1.00	0.91	0.96	1.00	1.04	0.82	1.00	1.11	1.02
time (sec)	N/A	0.337	0.013	0.365	0.025	0.078	0.089	0.110	0.234	8.852

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	46	53	72	44	96	79	54
N.S.	1	1.00	0.85	0.96	1.10	1.50	0.92	2.00	1.65	1.12
time (sec)	N/A	0.342	0.020	0.365	0.032	0.082	0.133	0.116	0.197	0.070

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	52	50	65	81	63	54	92	63
N.S.	1	1.00	0.95	0.91	1.18	1.47	1.15	0.98	1.67	1.15
time (sec)	N/A	0.347	0.015	0.358	0.027	0.094	0.243	0.243	0.214	8.743

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	44	46	71	71	75	45	59	68
N.S.	1	1.00	0.73	0.77	1.18	1.18	1.25	0.75	0.98	1.13
time (sec)	N/A	0.352	0.013	0.358	0.035	0.080	0.216	0.145	0.201	0.049

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	43	45	80	80	85	76	79	78
N.S.	1	1.00	0.69	0.73	1.29	1.29	1.37	1.23	1.27	1.26
time (sec)	N/A	0.352	0.013	0.358	0.034	0.081	0.266	0.120	0.231	8.980

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	159	162	161	161	178	175	174	149
N.S.	1	1.00	1.16	1.18	1.18	1.18	1.30	1.28	1.27	1.09
time (sec)	N/A	0.554	0.019	0.333	0.028	0.087	0.031	0.112	0.219	0.071

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	125	127	127	138	134	133	118
N.S.	1	1.00	1.00	0.98	1.00	1.00	1.09	1.06	1.05	0.93
time (sec)	N/A	0.491	0.013	0.335	0.036	0.072	0.028	0.116	0.208	9.012

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	87	89	85	85	94	94	92	78
N.S.	1	1.00	1.00	1.02	0.98	0.98	1.08	1.08	1.06	0.90
time (sec)	N/A	0.424	0.011	0.332	0.036	0.075	0.022	0.120	0.195	0.038

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	50	52	51	51	54	53	51	51
N.S.	1	1.00	0.91	0.95	0.93	0.93	0.98	0.96	0.93	0.93
time (sec)	N/A	0.347	0.011	0.301	0.032	0.078	0.024	0.111	0.225	0.054

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	30	25	24	24	24	24	24	24
N.S.	1	1.00	1.00	0.83	0.80	0.80	0.80	0.80	0.80	0.80
time (sec)	N/A	0.283	0.001	0.289	0.026	0.073	0.018	0.113	0.224	0.034

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	106	122	131	133	116	139	151	141
N.S.	1	1.00	1.14	1.31	1.41	1.43	1.25	1.49	1.62	1.52
time (sec)	N/A	0.451	0.033	0.454	0.027	0.084	0.153	0.115	0.203	0.058

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	114	131	138	203	126	189	206	158
N.S.	1	1.00	1.07	1.22	1.29	1.90	1.18	1.77	1.93	1.48
time (sec)	N/A	0.502	0.080	0.401	0.028	0.084	0.271	0.199	0.210	9.348

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	116	128	147	238	155	139	266	151
N.S.	1	1.00	0.97	1.08	1.24	2.00	1.30	1.17	2.24	1.27
time (sec)	N/A	0.489	0.059	0.402	0.036	0.092	0.434	0.120	0.223	8.870

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	134	130	159	245	163	135	259	158
N.S.	1	1.00	1.12	1.08	1.32	2.04	1.36	1.12	2.16	1.32
time (sec)	N/A	0.492	0.043	0.404	0.035	0.085	0.677	0.121	0.214	0.118

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	126	130	177	225	180	216	226	167
N.S.	1	1.00	0.96	0.99	1.35	1.72	1.37	1.65	1.73	1.27
time (sec)	N/A	0.509	0.035	0.395	0.036	0.082	0.863	0.112	0.237	0.117

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	116	127	181	181	196	139	144	169
N.S.	1	1.00	0.88	0.96	1.37	1.37	1.48	1.05	1.09	1.28
time (sec)	N/A	0.491	0.034	0.395	0.038	0.079	1.280	0.115	0.225	0.088

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	116	126	191	191	207	139	195	181
N.S.	1	1.00	0.85	0.92	1.39	1.39	1.51	1.01	1.42	1.32
time (sec)	N/A	0.502	0.027	0.402	0.037	0.086	2.015	0.108	0.262	9.011

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	117	131	207	207	221	140	206	197
N.S.	1	1.00	0.85	0.96	1.51	1.51	1.61	1.02	1.50	1.44
time (sec)	N/A	0.491	0.038	0.401	0.039	0.077	3.690	0.106	0.249	8.693

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	225	227	229	229	257	250	249	213
N.S.	1	1.00	1.00	1.01	1.02	1.02	1.14	1.11	1.11	0.95
time (sec)	N/A	0.711	0.033	0.339	0.034	0.083	0.034	0.113	0.264	8.966

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	169	177	171	171	199	193	191	156
N.S.	1	1.00	1.04	1.09	1.06	1.06	1.23	1.19	1.18	0.96
time (sec)	N/A	0.579	0.019	0.340	0.028	0.071	0.029	0.171	0.284	8.915

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	127	125	127	127	138	134	133	118
N.S.	1	1.00	1.00	0.98	1.00	1.00	1.09	1.06	1.05	0.93
time (sec)	N/A	0.494	0.026	0.338	0.027	0.086	0.045	0.113	0.260	0.053

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	75	73	73	80	77	75	69
N.S.	1	1.00	1.00	1.00	0.97	0.97	1.07	1.03	1.00	0.92
time (sec)	N/A	0.376	0.013	0.300	0.032	0.073	0.024	0.111	0.217	8.612

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	43	36	35	35	37	35	35	35
N.S.	1	1.00	1.00	0.84	0.81	0.81	0.86	0.81	0.81	0.81
time (sec)	N/A	0.288	0.002	0.287	0.035	0.072	0.019	0.112	0.214	0.042

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	144	247	264	266	243	285	302	294
N.S.	1	1.00	0.95	1.64	1.75	1.76	1.61	1.89	2.00	1.95
time (sec)	N/A	0.600	0.081	0.408	0.039	0.076	0.233	0.135	0.221	0.062

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	160	260	273	370	257	346	381	435
N.S.	1	1.00	0.96	1.57	1.64	2.23	1.55	2.08	2.30	2.62
time (sec)	N/A	0.655	0.043	0.411	0.036	0.087	0.428	0.131	0.239	8.951

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	207	260	280	428	284	279	464	352
N.S.	1	1.00	1.04	1.30	1.40	2.14	1.42	1.40	2.32	1.76
time (sec)	N/A	0.731	0.089	0.405	0.036	0.089	0.838	0.100	0.238	8.951

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	210	256	294	480	301	276	545	307
N.S.	1	1.00	0.99	1.20	1.38	2.25	1.41	1.30	2.56	1.44
time (sec)	N/A	0.744	0.085	0.403	0.039	0.086	1.327	0.116	0.219	8.710

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	210	253	303	481	316	389	536	302
N.S.	1	1.00	0.99	1.19	1.42	2.26	1.48	1.83	2.52	1.42
time (sec)	N/A	0.733	0.095	0.402	0.041	0.083	2.406	0.125	0.221	0.120

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	242	255	311	462	326	265	486	312
N.S.	1	1.00	1.11	1.17	1.43	2.12	1.50	1.22	2.23	1.43
time (sec)	N/A	0.728	0.082	0.405	0.040	0.094	6.198	0.129	0.204	8.959

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	231	252	331	407	343	274	400	268
N.S.	1	1.00	1.01	1.11	1.45	1.79	1.50	1.20	1.75	1.18
time (sec)	N/A	0.742	0.074	0.401	0.042	0.088	19.116	0.131	0.218	8.980

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	221	250	334	334	362	284	286	315
N.S.	1	1.00	0.96	1.09	1.45	1.45	1.57	1.23	1.24	1.37
time (sec)	N/A	0.740	0.052	0.401	0.044	0.088	55.917	0.118	0.224	8.765

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	221	249	344	344	374	284	362	325
N.S.	1	1.00	0.96	1.08	1.49	1.49	1.62	1.23	1.57	1.41
time (sec)	N/A	0.682	0.061	0.401	0.045	0.096	149.152	0.109	0.199	8.763

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	222	255	361	361	0	285	373	343
N.S.	1	1.00	0.95	1.09	1.54	1.54	0.00	1.22	1.59	1.47
time (sec)	N/A	0.711	0.059	0.405	0.045	0.074	0.000	0.108	0.200	9.158

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	104	90	134	142	151	165	144	174	106
N.S.	1	1.05	0.91	1.35	1.43	1.53	1.67	1.45	1.76	1.07
time (sec)	N/A	0.489	0.042	0.408	0.028	0.091	0.842	0.252	0.199	0.152

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	69	59	85	91	97	112	91	111	65
N.S.	1	1.08	0.92	1.33	1.42	1.52	1.75	1.42	1.73	1.02
time (sec)	N/A	0.406	0.024	0.400	0.028	0.090	0.608	0.339	0.195	9.023

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	47	42	54	53	53	73	55	64	49
N.S.	1	1.12	1.00	1.29	1.26	1.26	1.74	1.31	1.52	1.17
time (sec)	N/A	0.347	0.014	0.407	0.027	0.083	0.409	0.105	0.203	0.154

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	35	29	30	30	29	41	32	32	28
N.S.	1	1.17	0.97	1.00	1.00	0.97	1.37	1.07	1.07	0.93
time (sec)	N/A	0.315	0.008	0.362	0.033	0.071	0.212	0.109	0.204	0.104

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	16	18	16	10	20	15	15
N.S.	1	1.00	1.00	0.89	1.00	0.89	0.56	1.11	0.83	0.83
time (sec)	N/A	0.271	0.004	0.409	0.029	0.078	0.070	0.158	0.228	0.041

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	60	48	49	53	50	0	66	48	93
N.S.	1	1.13	0.91	0.92	1.00	0.94	0.00	1.25	0.91	1.75
time (sec)	N/A	0.385	0.022	0.479	0.031	0.135	0.000	0.122	0.224	8.892

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	96	83	87	128	207	0	286	227	116
N.S.	1	1.10	0.95	1.00	1.47	2.38	0.00	3.29	2.61	1.33
time (sec)	N/A	0.485	0.071	0.494	0.028	0.535	0.000	0.141	0.223	9.020

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	148	116	131	266	506	0	234	598	235
N.S.	1	1.10	0.87	0.98	1.99	3.78	0.00	1.75	4.46	1.75
time (sec)	N/A	0.606	0.176	0.504	0.041	4.019	0.000	0.126	0.232	9.310

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	128	116	200	216	349	381	216	398	218
N.S.	1	1.08	0.98	1.69	1.83	2.96	3.23	1.83	3.37	1.85
time (sec)	N/A	0.572	0.040	0.418	0.036	0.090	1.681	0.150	0.244	8.969

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	104	95	154	163	257	306	163	307	166
N.S.	1	1.11	1.01	1.64	1.73	2.73	3.26	1.73	3.27	1.77
time (sec)	N/A	0.509	0.068	0.415	0.029	0.096	1.203	0.121	0.260	0.170

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	97	79	121	132	198	250	129	237	118
N.S.	1	1.11	0.91	1.39	1.52	2.28	2.87	1.48	2.72	1.36
time (sec)	N/A	0.480	0.059	0.408	0.028	0.087	0.699	0.115	0.264	8.955

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	86	67	85	93	149	173	99	157	101
N.S.	1	1.18	0.92	1.16	1.27	2.04	2.37	1.36	2.15	1.38
time (sec)	N/A	0.445	0.054	0.415	0.027	0.090	0.340	0.109	0.257	9.133

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	81	56	63	69	111	128	74	128	57
N.S.	1	1.25	0.86	0.97	1.06	1.71	1.97	1.14	1.97	0.88
time (sec)	N/A	0.415	0.029	0.382	0.034	0.081	0.239	0.128	0.224	8.938

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	35	43	45	63	37	45	70	41
N.S.	1	1.00	0.83	1.02	1.07	1.50	0.88	1.07	1.67	0.98
time (sec)	N/A	0.316	0.033	0.352	0.028	0.086	0.120	0.107	0.243	8.674

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	117	111	111	177	288	0	200	313	143
N.S.	1	1.06	1.01	1.01	1.61	2.62	0.00	1.82	2.85	1.30
time (sec)	N/A	0.547	0.068	0.497	0.036	2.389	0.000	0.134	0.215	8.994

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	156	145	146	373	653	0	557	974	304
N.S.	1	1.08	1.01	1.01	2.59	4.53	0.00	3.87	6.76	2.11
time (sec)	N/A	0.668	0.127	0.513	0.042	7.783	0.000	0.204	0.219	9.287

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	216	202	371	408	694	687	397	835	399
N.S.	1	1.06	1.00	1.83	2.01	3.42	3.38	1.96	4.11	1.97
time (sec)	N/A	0.861	0.084	0.428	0.040	0.107	20.872	0.356	0.221	9.186

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	192	179	312	341	579	597	325	704	333
N.S.	1	1.07	1.00	1.74	1.91	3.23	3.34	1.82	3.93	1.86
time (sec)	N/A	0.773	0.069	0.424	0.037	0.104	7.038	0.135	0.213	0.262

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	184	165	262	296	493	524	280	598	268
N.S.	1	1.08	0.96	1.53	1.73	2.88	3.06	1.64	3.50	1.57
time (sec)	N/A	0.727	0.091	0.415	0.037	0.099	2.913	0.129	0.216	9.274

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	152	130	212	250	426	389	256	500	238
N.S.	1	1.12	0.96	1.56	1.84	3.13	2.86	1.88	3.68	1.75
time (sec)	N/A	0.631	0.062	0.405	0.035	0.087	1.311	0.128	0.218	9.139

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	156	138	181	217	385	371	219	472	211
N.S.	1	1.14	1.01	1.32	1.58	2.81	2.71	1.60	3.45	1.54
time (sec)	N/A	0.636	0.118	0.421	0.036	0.118	0.823	0.124	0.254	0.154

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	166	144	158	180	327	345	181	415	149
N.S.	1	1.15	1.00	1.10	1.25	2.27	2.40	1.26	2.88	1.03
time (sec)	N/A	0.641	0.066	0.415	0.036	0.088	0.513	0.113	0.232	8.911

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	128	102	106	136	234	219	127	263	132
N.S.	1	1.16	0.93	0.96	1.24	2.13	1.99	1.15	2.39	1.20
time (sec)	N/A	0.558	0.058	0.383	0.031	0.091	0.341	0.113	0.238	9.108

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	68	72	86	130	78	73	138	79
N.S.	1	1.00	0.89	0.95	1.13	1.71	1.03	0.96	1.82	1.04
time (sec)	N/A	0.395	0.035	0.356	0.032	0.092	0.168	0.146	0.205	0.103

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	193	200	192	193	439	716	0	422	825	331
N.S.	1	1.04	0.99	1.00	2.27	3.71	0.00	2.19	4.27	1.72
time (sec)	N/A	0.811	0.135	0.527	0.050	18.899	0.000	0.194	0.266	9.705

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	243	230	226	752	1305	0	867	2061	603
N.S.	1	1.06	1.00	0.98	3.27	5.67	0.00	3.77	8.96	2.62
time (sec)	N/A	1.000	0.195	0.545	0.069	35.795	0.000	0.136	0.233	8.148

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	41	54	143	292	584	141	52
N.S.	1	1.00	0.74	0.60	0.79	2.10	4.29	8.59	2.07	0.76
time (sec)	N/A	0.343	0.047	0.671	0.027	0.124	0.489	0.148	0.204	5.469

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	41	54	118	245	418	117	52
N.S.	1	1.00	0.74	0.60	0.79	1.74	3.60	6.15	1.72	0.76
time (sec)	N/A	0.330	0.052	0.644	0.032	0.092	0.349	0.119	0.209	0.032

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	41	54	95	85	275	93	52
N.S.	1	1.00	0.74	0.60	0.79	1.40	1.25	4.04	1.37	0.76
time (sec)	N/A	0.332	0.042	0.645	0.026	0.097	0.582	0.109	0.239	5.486

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	50	41	54	71	85	158	69	52
N.S.	1	1.00	0.74	0.60	0.79	1.04	1.25	2.32	1.01	0.76
time (sec)	N/A	0.336	0.037	0.650	0.027	0.095	0.543	0.112	0.237	0.033

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	49	41	67	48	87	67	45	52
N.S.	1	1.00	0.74	0.62	1.02	0.73	1.32	1.02	0.68	0.79
time (sec)	N/A	0.337	0.035	0.650	0.036	0.092	0.405	0.173	0.217	5.787

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	48	41	61	57	80	69	46	52
N.S.	1	1.00	0.75	0.64	0.95	0.89	1.25	1.08	0.72	0.81
time (sec)	N/A	0.332	0.039	0.395	0.034	0.095	0.875	0.122	0.217	5.549

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	50	41	58	69	211	55	54	52
N.S.	1	1.00	0.78	0.64	0.91	1.08	3.30	0.86	0.84	0.81
time (sec)	N/A	0.331	0.045	0.402	0.030	0.120	0.303	0.120	0.256	0.029

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	49	42	50	79	314	52	65	49
N.S.	1	1.00	0.74	0.64	0.76	1.20	4.76	0.79	0.98	0.74
time (sec)	N/A	0.331	0.050	0.394	0.028	0.090	0.440	0.297	0.296	0.038

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	124	108	139	290	590	1171	303	138
N.S.	1	1.00	0.84	0.73	0.95	1.97	4.01	7.97	2.06	0.94
time (sec)	N/A	0.480	0.091	0.750	0.032	0.091	0.631	0.119	0.246	5.821

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	124	108	139	251	204	863	262	138
N.S.	1	1.00	0.84	0.73	0.95	1.71	1.39	5.87	1.78	0.94
time (sec)	N/A	0.468	0.089	0.748	0.033	0.090	0.825	0.122	0.278	0.026

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	125	108	139	214	204	591	221	138
N.S.	1	1.00	0.85	0.73	0.95	1.46	1.39	4.02	1.50	0.94
time (sec)	N/A	0.447	0.081	0.721	0.032	0.083	0.716	0.120	0.245	0.025

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	124	108	139	175	204	355	180	138
N.S.	1	1.00	0.84	0.73	0.95	1.19	1.39	2.41	1.22	0.94
time (sec)	N/A	0.436	0.068	0.710	0.032	0.103	0.674	0.233	0.208	0.027

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	124	108	160	138	202	160	139	138
N.S.	1	1.00	0.86	0.74	1.10	0.95	1.39	1.10	0.96	0.95
time (sec)	N/A	0.442	0.080	0.728	0.030	0.088	0.607	0.169	0.216	0.024

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	123	112	147	147	185	189	141	153
N.S.	1	1.00	0.86	0.78	1.03	1.03	1.29	1.32	0.99	1.07
time (sec)	N/A	0.458	0.084	0.448	0.032	0.089	2.011	0.117	0.207	5.598

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	123	110	145	159	173	180	148	145
N.S.	1	1.00	0.86	0.77	1.01	1.11	1.21	1.26	1.03	1.01
time (sec)	N/A	0.449	0.087	0.464	0.029	0.093	2.063	0.212	0.217	0.047

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	123	108	147	168	787	174	159	140
N.S.	1	1.00	0.86	0.76	1.03	1.17	5.50	1.22	1.11	0.98
time (sec)	N/A	0.435	0.086	0.466	0.033	0.104	0.475	0.112	0.224	5.391

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	232	208	271	478	371	1936	516	239
N.S.	1	1.00	0.94	0.84	1.09	1.93	1.50	7.81	2.08	0.96
time (sec)	N/A	0.680	0.143	0.793	0.035	0.095	1.163	0.130	0.204	5.246

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	232	208	271	426	371	1450	458	239
N.S.	1	1.00	0.94	0.84	1.09	1.72	1.50	5.85	1.85	0.96
time (sec)	N/A	0.612	0.133	0.776	0.034	0.092	1.040	0.123	0.208	5.312

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	231	208	271	373	371	1012	400	239
N.S.	1	1.00	0.93	0.84	1.09	1.50	1.50	4.08	1.61	0.96
time (sec)	N/A	0.632	0.156	0.741	0.029	0.091	0.917	0.151	0.220	5.309

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	231	208	271	321	371	622	342	239
N.S.	1	1.00	0.93	0.84	1.09	1.29	1.50	2.51	1.38	0.96
time (sec)	N/A	0.611	0.130	0.743	0.035	0.104	0.833	0.118	0.261	5.456

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	232	208	288	270	369	288	284	239
N.S.	1	1.00	0.95	0.85	1.18	1.11	1.51	1.18	1.16	0.98
time (sec)	N/A	0.614	0.124	0.741	0.035	0.084	0.761	0.117	0.256	5.405

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	231	212	279	280	337	373	286	268
N.S.	1	1.00	0.95	0.88	1.15	1.16	1.39	1.54	1.18	1.11
time (sec)	N/A	0.618	0.123	0.471	0.032	0.087	4.489	0.112	0.266	0.039

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	231	212	277	291	311	360	293	281
N.S.	1	1.00	0.95	0.87	1.14	1.19	1.27	1.48	1.20	1.15
time (sec)	N/A	0.654	0.120	0.487	0.035	0.091	4.644	0.118	0.232	5.269

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	232	207	277	302	298	355	304	278
N.S.	1	1.00	0.97	0.86	1.15	1.26	1.24	1.48	1.27	1.16
time (sec)	N/A	0.617	0.119	0.481	0.035	0.091	4.931	0.201	0.230	0.045

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	182	138	141	0	816	241	221	329	2482
N.S.	1	1.16	0.88	0.90	0.00	5.20	1.54	1.41	2.10	15.81
time (sec)	N/A	0.936	0.256	1.073	0.000	0.625	2.623	0.305	0.244	0.251

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	138	107	114	0	592	197	154	223	2048
N.S.	1	1.17	0.91	0.97	0.00	5.02	1.67	1.31	1.89	17.36
time (sec)	N/A	0.692	0.179	0.540	0.000	0.239	2.541	0.160	0.241	5.665

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	105	98	98	0	441	168	106	134	697
N.S.	1	1.14	1.07	1.07	0.00	4.79	1.83	1.15	1.46	7.58
time (sec)	N/A	0.593	0.212	0.523	0.000	0.138	2.268	0.164	0.237	0.136

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	85	75	67	0	346	148	75	76	100
N.S.	1	1.10	0.97	0.87	0.00	4.49	1.92	0.97	0.99	1.30
time (sec)	N/A	0.385	0.107	0.512	0.000	0.109	2.180	0.132	0.274	5.391

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	85	76	61	0	355	138	67	122	625
N.S.	1	1.10	0.99	0.79	0.00	4.61	1.79	0.87	1.58	8.12
time (sec)	N/A	0.361	0.131	0.510	0.000	0.109	2.729	0.125	0.226	5.270

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	133	102	97	0	681	170	105	259	2258
N.S.	1	1.30	1.00	0.95	0.00	6.68	1.67	1.03	2.54	22.14
time (sec)	N/A	0.660	0.315	0.524	0.000	0.130	2.512	0.107	0.228	5.466

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	188	128	134	0	1424	211	167	737	4509
N.S.	1	1.36	0.93	0.97	0.00	10.32	1.53	1.21	5.34	32.67
time (sec)	N/A	0.809	0.682	0.549	0.000	0.226	2.412	0.115	0.234	5.732

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	252	182	177	0	2532	262	282	1458	4068
N.S.	1	1.35	0.97	0.95	0.00	13.54	1.40	1.51	7.80	21.75
time (sec)	N/A	0.992	0.686	0.744	0.000	0.587	2.673	0.289	0.273	6.476

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	278	203	224	0	1555	0	425	862	3360
N.S.	1	1.11	0.81	0.89	0.00	6.20	0.00	1.69	3.43	13.39
time (sec)	N/A	1.226	0.649	0.661	0.000	1.745	0.000	0.326	0.262	5.772

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	225	167	195	0	1267	0	332	698	2913
N.S.	1	1.12	0.84	0.98	0.00	6.34	0.00	1.66	3.49	14.56
time (sec)	N/A	0.953	0.736	0.645	0.000	0.503	0.000	0.190	0.281	5.445

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	179	143	154	0	996	0	260	547	1127
N.S.	1	1.13	0.90	0.97	0.00	6.26	0.00	1.64	3.44	7.09
time (sec)	N/A	0.773	0.610	0.607	0.000	0.234	0.000	0.147	0.270	5.420

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	147	139	144	0	764	0	197	404	429
N.S.	1	0.99	0.93	0.97	0.00	5.13	0.00	1.32	2.71	2.88
time (sec)	N/A	0.617	0.785	0.585	0.000	0.115	0.000	0.134	0.224	5.561

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	138	119	133	0	756	0	166	550	1174
N.S.	1	0.99	0.85	0.95	0.00	5.40	0.00	1.19	3.93	8.39
time (sec)	N/A	0.618	0.665	0.579	0.000	0.118	0.000	0.138	0.254	0.221

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	189	149	160	0	1120	0	236	743	3784
N.S.	1	1.15	0.91	0.98	0.00	6.83	0.00	1.44	4.53	23.07
time (sec)	N/A	0.721	0.773	0.595	0.000	0.170	0.000	0.194	0.239	5.896

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	260	202	174	0	2268	0	338	1098	4234
N.S.	1	1.20	0.94	0.81	0.00	10.50	0.00	1.56	5.08	19.60
time (sec)	N/A	0.960	1.170	0.693	0.000	0.426	0.000	0.286	0.228	6.571

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	333	272	204	0	3829	0	470	2148	5736
N.S.	1	1.20	0.98	0.74	0.00	13.82	0.00	1.70	7.75	20.71
time (sec)	N/A	1.264	1.382	0.704	0.000	1.393	0.000	0.198	0.255	6.888

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	378	391	313	336	0	2702	0	749	1744	4450
N.S.	1	1.03	0.83	0.89	0.00	7.15	0.00	1.98	4.61	11.77
time (sec)	N/A	1.496	1.159	0.806	0.000	2.899	0.000	0.191	0.226	6.340

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	323	275	273	0	2367	0	617	1522	3946
N.S.	1	1.04	0.88	0.88	0.00	7.59	0.00	1.98	4.88	12.65
time (sec)	N/A	1.202	1.280	0.753	0.000	0.959	0.000	0.148	0.253	6.508

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	266	250	273	0	2021	0	537	1308	1792
N.S.	1	0.87	0.82	0.90	0.00	6.63	0.00	1.76	4.29	5.88
time (sec)	N/A	0.969	1.537	0.757	0.000	0.388	0.000	0.373	0.234	5.572

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	246	222	259	0	1655	0	432	1078	910
N.S.	1	0.88	0.79	0.92	0.00	5.91	0.00	1.54	3.85	3.25
time (sec)	N/A	0.839	0.957	0.740	0.000	0.163	0.000	0.137	0.242	5.478

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	260	197	209	0	1628	0	375	1327	1880
N.S.	1	1.03	0.78	0.83	0.00	6.46	0.00	1.49	5.27	7.46
time (sec)	N/A	0.964	1.123	0.629	0.000	0.177	0.000	0.266	0.250	5.725

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	281	247	241	0	2220	0	489	1647	4815
N.S.	1	0.98	0.86	0.84	0.00	7.76	0.00	1.71	5.76	16.84
time (sec)	N/A	0.898	1.495	0.649	0.000	0.290	0.000	0.394	0.256	6.915

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	352	292	281	0	2784	0	602	1966	6715
N.S.	1	1.09	0.91	0.87	0.00	8.65	0.00	1.87	6.11	20.85
time (sec)	N/A	1.182	2.291	0.696	0.000	0.843	0.000	0.164	0.240	7.211

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	398	436	408	279	0	4744	0	773	2580	9635
N.S.	1	1.10	1.03	0.70	0.00	11.92	0.00	1.94	6.48	24.21
time (sec)	N/A	1.477	1.661	0.756	0.000	2.755	0.000	0.147	0.320	7.930

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	194	281	188	439	498	434	255	400	361
N.S.	1	0.71	1.02	0.68	1.60	1.81	1.58	0.93	1.45	1.31
time (sec)	N/A	0.667	1.345	0.520	0.035	0.092	0.417	0.131	1.597	6.263

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	155	173	132	283	338	284	171	256	231
N.S.	1	0.79	0.89	0.68	1.45	1.73	1.46	0.88	1.31	1.18
time (sec)	N/A	0.527	0.770	0.473	0.033	0.094	0.398	0.374	0.201	5.898

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	98	122	85	154	206	167	102	137	127
N.S.	1	0.79	0.98	0.69	1.24	1.66	1.35	0.82	1.10	1.02
time (sec)	N/A	0.372	0.461	0.754	0.033	0.130	0.398	1.089	0.187	5.828

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	60	83	56	63	122	102	59	56	55
N.S.	1	0.82	1.14	0.77	0.86	1.67	1.40	0.81	0.77	0.75
time (sec)	N/A	0.296	0.050	0.369	0.030	0.126	0.210	0.480	0.191	0.057

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	138	402	99	0	482	0	0	168	0
N.S.	1	1.22	3.56	0.88	0.00	4.27	0.00	0.00	1.49	0.00
time (sec)	N/A	0.564	2.435	1.093	0.000	0.100	0.000	0.000	0.206	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	146	159	120	0	839	0	0	603	0
N.S.	1	1.21	1.31	0.99	0.00	6.93	0.00	0.00	4.98	0.00
time (sec)	N/A	0.526	0.539	0.559	0.000	0.117	0.000	0.000	0.228	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	127	122	94	0	474	0	407	1056	0
N.S.	1	0.95	0.92	0.71	0.00	3.56	0.00	3.06	7.94	0.00
time (sec)	N/A	0.425	10.173	0.656	0.000	0.100	0.000	0.159	0.560	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	192	191	167	0	964	0	831	21	0
N.S.	1	0.93	0.93	0.81	0.00	4.68	0.00	4.03	0.10	0.00
time (sec)	N/A	0.540	10.234	0.585	0.000	0.099	0.000	1.596	200.048	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	271	243	243	0	1606	0	1219	21	0
N.S.	1	0.93	0.83	0.83	0.00	5.50	0.00	4.17	0.07	0.00
time (sec)	N/A	0.766	10.605	0.667	0.000	0.118	0.000	0.337	200.033	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	359	308	357	0	2444	0	2144	21	0
N.S.	1	0.92	0.79	0.91	0.00	6.23	0.00	5.47	0.05	0.00
time (sec)	N/A	1.008	10.870	0.779	0.000	0.153	0.000	0.177	200.030	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	231	370	266	624	707	864	372	580	0
N.S.	1	0.59	0.95	0.68	1.60	1.81	2.21	0.95	1.48	0.00
time (sec)	N/A	0.728	1.915	0.559	0.039	0.102	0.489	0.129	26.309	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	192	280	188	416	491	570	261	386	0
N.S.	1	0.65	0.94	0.63	1.40	1.65	1.92	0.88	1.30	0.00
time (sec)	N/A	0.590	1.402	0.684	0.036	0.089	0.476	0.124	0.279	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	135	167	116	236	303	326	163	217	208
N.S.	1	0.71	0.87	0.61	1.24	1.59	1.71	0.85	1.14	1.09
time (sec)	N/A	0.413	0.861	0.644	0.032	0.090	0.441	0.122	0.217	5.511

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	97	107	73	102	171	257	81	95	87
N.S.	1	0.79	0.87	0.59	0.83	1.39	2.09	0.66	0.77	0.71
time (sec)	N/A	0.343	0.046	0.581	0.031	0.087	0.358	0.130	0.244	0.120

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	235	500	184	0	896	0	0	456	0
N.S.	1	1.10	2.34	0.86	0.00	4.19	0.00	0.00	2.13	0.00
time (sec)	N/A	0.804	2.849	0.778	0.000	0.304	0.000	0.000	0.241	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-1)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	210	205	185	0	1008	0	0	719	0
N.S.	1	1.07	1.05	0.94	0.00	5.14	0.00	0.00	3.67	0.00
time (sec)	N/A	0.769	10.571	0.875	0.000	0.144	0.000	0.000	0.397	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	216	237	221	0	1745	0	497	2312	0
N.S.	1	0.99	1.08	1.01	0.00	7.97	0.00	2.27	10.56	0.00
time (sec)	N/A	0.724	11.042	0.891	0.000	0.144	0.000	0.162	0.890	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	292	269	240	0	2837	0	929	21	0
N.S.	1	1.17	1.08	0.96	0.00	11.39	0.00	3.73	0.08	0.00
time (sec)	N/A	0.934	11.308	0.789	0.000	1.009	0.000	0.183	200.032	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	200	207	156	0	1066	0	1136	21	0
N.S.	1	0.93	0.96	0.72	0.00	4.94	0.00	5.26	0.10	0.00
time (sec)	N/A	0.565	10.333	0.857	0.000	0.107	0.000	0.293	200.064	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	507	268	454	480	808	916	1472	489	21	0
N.S.	1	0.53	0.90	0.95	1.59	1.81	2.90	0.96	0.04	0.00
time (sec)	N/A	0.811	2.865	0.697	0.043	0.106	0.601	0.185	200.044	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	229	341	328	549	644	986	349	516	0
N.S.	1	0.58	0.87	0.84	1.40	1.64	2.52	0.89	1.32	0.00
time (sec)	N/A	0.677	1.947	0.646	0.042	0.092	0.556	0.198	0.657	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	172	238	192	318	398	570	223	297	0
N.S.	1	0.66	0.92	0.74	1.23	1.54	2.20	0.86	1.15	0.00
time (sec)	N/A	0.499	1.368	0.570	0.038	0.085	0.525	0.127	0.279	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	134	129	106	141	214	459	105	133	119
N.S.	1	0.77	0.74	0.61	0.81	1.22	2.62	0.60	0.76	0.68
time (sec)	N/A	0.401	0.078	0.510	0.034	0.089	0.480	0.127	0.184	0.139

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F(-2)	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	388	625	476	0	1544	0	0	21	0
N.S.	1	1.02	1.64	1.25	0.00	4.06	0.00	0.00	0.06	0.00
time (sec)	N/A	1.289	4.957	0.740	0.000	2.438	0.000	0.000	200.040	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	338	347	328	0	1868	0	0	21	0
N.S.	1	1.02	1.04	0.98	0.00	5.61	0.00	0.00	0.06	0.00
time (sec)	N/A	1.171	10.957	0.893	0.000	0.784	0.000	0.000	200.029	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	300	308	333	0	2028	0	743	21	0
N.S.	1	0.90	0.92	1.00	0.00	6.09	0.00	2.23	0.06	0.00
time (sec)	N/A	1.013	11.093	1.020	0.000	0.252	0.000	0.164	200.030	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	305	340	280	0	3097	0	977	21	0
N.S.	1	0.86	0.96	0.79	0.00	8.77	0.00	2.77	0.06	0.00
time (sec)	N/A	1.007	11.577	1.026	0.000	0.275	0.000	0.248	200.031	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F(-1)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	413	416	388	377	0	4553	0	0	21	0
N.S.	1	1.01	0.94	0.91	0.00	11.02	0.00	0.00	0.05	0.00
time (sec)	N/A	1.230	12.135	1.130	0.000	0.772	0.000	0.000	200.037	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	157	156	113	260	297	258	150	229	0
N.S.	1	0.95	0.94	0.68	1.57	1.79	1.55	0.90	1.38	0.00
time (sec)	N/A	0.587	0.228	0.713	0.038	0.090	0.519	0.136	0.251	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	113	109	79	154	191	185	96	135	0
N.S.	1	1.05	1.01	0.73	1.43	1.77	1.71	0.89	1.25	0.00
time (sec)	N/A	0.455	0.130	0.690	0.047	0.095	0.463	0.133	0.258	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	91	46	75	119	122	59	64	77
N.S.	1	1.00	1.65	0.84	1.36	2.16	2.22	1.07	1.16	1.40
time (sec)	N/A	0.324	0.197	0.652	0.032	0.085	0.336	0.126	0.230	5.482

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	55	23	27	63	75	59	25	28
N.S.	1	1.00	1.96	0.82	0.96	2.25	2.68	2.11	0.89	1.00
time (sec)	N/A	0.257	0.007	0.580	0.027	0.077	0.286	0.115	0.206	5.359

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	68	92	42	0	129	0	58	101	0
N.S.	1	1.31	1.77	0.81	0.00	2.48	0.00	1.12	1.94	0.00
time (sec)	N/A	0.303	0.150	0.756	0.000	0.080	0.000	0.207	0.223	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	117	137	83	0	344	0	0	518	0
N.S.	1	1.21	1.41	0.86	0.00	3.55	0.00	0.00	5.34	0.00
time (sec)	N/A	0.422	0.399	0.773	0.000	0.087	0.000	0.000	0.257	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	196	180	140	0	738	0	483	1950	0
N.S.	1	1.20	1.10	0.86	0.00	4.53	0.00	2.96	11.96	0.00
time (sec)	N/A	0.622	0.851	0.852	0.000	0.102	0.000	0.195	0.856	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	148	142	115	189	363	0	127	240	0
N.S.	1	1.10	1.06	0.86	1.41	2.71	0.00	0.95	1.79	0.00
time (sec)	N/A	0.522	0.470	0.754	0.037	0.090	0.000	0.265	0.302	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	103	100	82	110	243	0	88	148	96
N.S.	1	1.13	1.10	0.90	1.21	2.67	0.00	0.97	1.63	1.05
time (sec)	N/A	0.436	0.173	0.688	0.029	0.090	0.000	0.172	0.244	5.460

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	33	30	30	55	44	0	33	68	31
N.S.	1	0.69	0.62	0.62	1.15	0.92	0.00	0.69	1.42	0.65
time (sec)	N/A	0.266	0.108	0.599	0.028	0.077	0.000	0.138	0.242	5.264

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	24	22	21	35	35	0	24	39	24
N.S.	1	0.60	0.55	0.52	0.88	0.88	0.00	0.60	0.98	0.60
time (sec)	N/A	0.243	0.004	0.595	0.030	0.071	0.000	0.381	0.228	5.247

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	126	144	107	0	448	0	160	290	0
N.S.	1	1.06	1.21	0.90	0.00	3.76	0.00	1.34	2.44	0.00
time (sec)	N/A	0.436	0.440	0.818	0.000	0.087	0.000	0.252	0.288	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	223	218	187	0	910	0	821	919	0
N.S.	1	1.13	1.11	0.95	0.00	4.62	0.00	4.17	4.66	0.00
time (sec)	N/A	0.712	0.901	0.876	0.000	0.101	0.000	0.494	0.442	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	324	289	272	0	1652	0	738	2708	0
N.S.	1	1.12	1.00	0.94	0.00	5.72	0.00	2.55	9.37	0.00
time (sec)	N/A	0.942	10.617	0.964	0.000	0.169	0.000	0.339	101.651	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	219	195	179	458	526	0	210	504	0
N.S.	1	0.94	0.83	0.76	1.96	2.25	0.00	0.90	2.15	0.00
time (sec)	N/A	0.775	0.579	0.841	0.037	0.093	0.000	0.290	0.233	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	87	105	83	297	147	0	127	343	129
N.S.	1	0.62	0.75	0.59	2.12	1.05	0.00	0.91	2.45	0.92
time (sec)	N/A	0.354	0.251	0.763	0.035	0.078	0.000	0.202	0.208	5.151

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	78	95	94	203	129	0	110	264	111
N.S.	1	0.46	0.56	0.56	1.20	0.76	0.00	0.65	1.56	0.66
time (sec)	N/A	0.337	0.213	0.750	0.032	0.080	0.000	0.140	0.217	5.348

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	71	67	73	130	104	0	86	164	76
N.S.	1	0.63	0.59	0.65	1.15	0.92	0.00	0.76	1.45	0.67
time (sec)	N/A	0.311	0.200	0.671	0.036	0.092	0.000	0.129	0.216	5.385

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	54	48	47	72	72	0	50	92	43
N.S.	1	0.61	0.54	0.53	0.81	0.81	0.00	0.56	1.03	0.48
time (sec)	N/A	0.284	0.006	0.631	0.031	0.097	0.000	0.152	0.215	5.023

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	253	253	218	0	1012	0	645	880	0
N.S.	1	1.07	1.07	0.92	0.00	4.29	0.00	2.73	3.73	0.00
time (sec)	N/A	0.730	0.831	0.931	0.000	0.149	0.000	0.128	0.401	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	383	379	334	0	1784	0	1457	21	0
N.S.	1	1.10	1.09	0.96	0.00	5.13	0.00	4.19	0.06	0.00
time (sec)	N/A	1.107	2.038	1.007	0.000	0.247	0.000	0.393	200.044	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	137	218	216	640	289	0	264	695	278
N.S.	1	0.51	0.81	0.80	2.38	1.07	0.00	0.98	2.58	1.03
time (sec)	N/A	0.440	0.761	0.928	0.039	0.090	0.000	0.177	0.224	5.327

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	147	189	192	471	262	0	238	569	244
N.S.	1	0.64	0.83	0.84	2.07	1.15	0.00	1.04	2.50	1.07
time (sec)	N/A	0.541	0.435	0.820	0.037	0.091	0.000	0.160	0.258	5.223

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	146	155	151	329	219	0	196	422	193
N.S.	1	0.69	0.73	0.71	1.54	1.03	0.00	0.92	1.98	0.91
time (sec)	N/A	0.468	0.304	0.780	0.036	0.098	0.000	0.140	0.218	5.134

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	106	105	109	210	164	0	142	276	125
N.S.	1	0.60	0.59	0.61	1.18	0.92	0.00	0.80	1.55	0.70
time (sec)	N/A	0.388	0.236	0.704	0.032	0.104	0.000	0.147	0.226	5.086

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	89	70	75	111	105	0	74	150	96
N.S.	1	0.64	0.50	0.54	0.79	0.75	0.00	0.53	1.07	0.69
time (sec)	N/A	0.342	0.007	0.669	0.031	0.086	0.000	0.140	0.218	5.030

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	426	452	369	0	1782	0	1851	1726	0
N.S.	1	1.02	1.08	0.88	0.00	4.27	0.00	4.44	4.14	0.00
time (sec)	N/A	1.230	2.144	0.990	0.000	0.195	0.000	0.176	0.698	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	548	591	644	509	0	2832	0	2349	21	0
N.S.	1	1.08	1.18	0.93	0.00	5.17	0.00	4.29	0.04	0.00
time (sec)	N/A	1.754	5.017	1.116	0.000	1.034	0.000	0.517	200.029	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	377	372	664	0	466	0	0	867	0
N.S.	1	0.88	0.87	1.55	0.00	1.09	0.00	0.00	2.03	0.00
time (sec)	N/A	1.001	17.000	0.931	0.000	0.212	0.000	0.000	3.496	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	333	317	294	444	0	397	0	0	411	0
N.S.	1	0.95	0.88	1.33	0.00	1.19	0.00	0.00	1.23	0.00
time (sec)	N/A	0.870	12.176	0.789	0.000	0.210	0.000	0.000	2.584	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	250	226	377	0	351	0	0	454	0
N.S.	1	0.93	0.84	1.41	0.00	1.31	0.00	0.00	1.69	0.00
time (sec)	N/A	0.657	9.241	0.759	0.000	0.196	0.000	0.000	1.584	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	236	195	276	0	357	0	0	469	0
N.S.	1	1.13	0.94	1.33	0.00	1.72	0.00	0.00	2.25	0.00
time (sec)	N/A	0.614	8.705	0.787	0.000	0.176	0.000	0.000	1.613	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	313	265	495	0	557	0	0	0	0
N.S.	1	1.28	1.09	2.03	0.00	2.28	0.00	0.00	0.00	0.00
time (sec)	N/A	0.781	6.638	0.750	0.000	0.147	0.000	0.000	3.219	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	412	362	641	0	868	0	0	0	0
N.S.	1	1.31	1.15	2.04	0.00	2.76	0.00	0.00	0.00	0.00
time (sec)	N/A	1.096	6.802	1.043	0.000	0.150	0.000	0.000	5.098	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	620	547	559	1359	0	637	0	0	729	0
N.S.	1	0.88	0.90	2.19	0.00	1.03	0.00	0.00	1.18	0.00
time (sec)	N/A	1.485	22.672	1.622	0.000	0.207	0.000	0.000	6.778	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	477	463	942	0	546	0	0	1193	0
N.S.	1	0.86	0.84	1.71	0.00	0.99	0.00	0.00	2.16	0.00
time (sec)	N/A	1.253	18.240	1.113	0.000	0.179	0.000	0.000	5.338	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	375	380	650	0	465	0	0	867	0
N.S.	1	0.86	0.87	1.49	0.00	1.07	0.00	0.00	1.99	0.00
time (sec)	N/A	0.986	18.611	1.459	0.000	0.179	0.000	0.000	4.070	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	370	319	306	663	0	516	0	0	873	0
N.S.	1	0.86	0.83	1.79	0.00	1.39	0.00	0.00	2.36	0.00
time (sec)	N/A	0.900	19.271	1.496	0.000	0.179	0.000	0.000	2.951	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	307	279	581	0	531	0	0	0	0
N.S.	1	1.02	0.93	1.93	0.00	1.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.838	18.300	2.457	0.000	0.168	0.000	0.000	7.143	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	376	369	646	0	807	0	0	0	0
N.S.	1	1.12	1.10	1.93	0.00	2.41	0.00	0.00	0.00	0.00
time (sec)	N/A	0.987	11.528	3.538	0.000	0.277	0.000	0.000	9.004	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	496	479	792	0	1230	0	0	0	0
N.S.	1	1.18	1.14	1.88	0.00	2.91	0.00	0.00	0.00	0.00
time (sec)	N/A	1.325	12.908	5.112	0.000	0.166	0.000	0.000	11.485	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	817	697	663	1728	0	747	0	0	0	0
N.S.	1	0.85	0.81	2.12	0.00	0.91	0.00	0.00	0.00	0.00
time (sec)	N/A	1.746	20.732	1.956	0.000	0.226	0.000	0.000	10.526	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	678	563	557	1442	0	640	0	0	0	0
N.S.	1	0.83	0.82	2.13	0.00	0.94	0.00	0.00	0.00	0.00
time (sec)	N/A	1.458	21.088	2.969	0.000	0.162	0.000	0.000	8.594	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	580	478	498	1171	0	750	0	0	0	0
N.S.	1	0.82	0.86	2.02	0.00	1.29	0.00	0.00	0.00	0.00
time (sec)	N/A	1.262	23.892	3.030	0.000	0.202	0.000	0.000	6.969	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	455	421	442	1135	0	806	0	0	0	0
N.S.	1	0.93	0.97	2.49	0.00	1.77	0.00	0.00	0.00	0.00
time (sec)	N/A	1.138	23.725	3.098	0.000	0.191	0.000	0.000	10.264	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	409	401	912	0	813	0	0	0	0
N.S.	1	0.93	0.92	2.08	0.00	1.86	0.00	0.00	0.00	0.00
time (sec)	N/A	1.088	22.872	4.214	0.000	0.223	0.000	0.000	21.812	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	490	500	894	0	1184	0	0	0	0
N.S.	1	1.05	1.07	1.91	0.00	2.53	0.00	0.00	0.00	0.00
time (sec)	N/A	1.242	12.973	7.609	0.000	0.342	0.000	0.000	24.394	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	575	600	610	995	0	1675	0	0	0	0
N.S.	1	1.04	1.06	1.73	0.00	2.91	0.00	0.00	0.00	0.00
time (sec)	N/A	1.478	24.747	9.714	0.000	0.326	0.000	0.000	34.735	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	433	399	388	697	0	467	0	0	1001	0
N.S.	1	0.92	0.90	1.61	0.00	1.08	0.00	0.00	2.31	0.00
time (sec)	N/A	1.216	19.118	1.885	0.000	0.176	0.000	0.000	4.545	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	353	315	314	505	0	401	0	0	716	0
N.S.	1	0.89	0.89	1.43	0.00	1.14	0.00	0.00	2.03	0.00
time (sec)	N/A	0.903	17.720	1.224	0.000	0.143	0.000	0.000	3.299	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	245	246	381	0	351	0	0	453	0
N.S.	1	0.90	0.90	1.40	0.00	1.29	0.00	0.00	1.67	0.00
time (sec)	N/A	0.674	12.277	1.106	0.000	0.155	0.000	0.000	1.873	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	94	121	215	0	299	0	0	27	0
N.S.	1	0.46	0.59	1.04	0.00	1.45	0.00	0.00	0.13	0.00
time (sec)	N/A	0.339	4.880	0.980	0.000	0.151	0.000	0.000	0.922	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	94	94	113	0	108	0	0	42	0
N.S.	1	1.01	1.01	1.22	0.00	1.16	0.00	0.00	0.45	0.00
time (sec)	N/A	0.339	3.713	1.487	0.000	0.116	0.000	0.000	0.308	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	146	127	361	0	385	0	0	71	0
N.S.	1	0.71	0.62	1.76	0.00	1.88	0.00	0.00	0.35	0.00
time (sec)	N/A	0.438	4.577	2.440	0.000	0.133	0.000	0.000	0.800	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	328	290	510	0	606	0	0	99	0
N.S.	1	1.23	1.09	1.91	0.00	2.27	0.00	0.00	0.37	0.00
time (sec)	N/A	0.890	14.117	3.483	0.000	0.135	0.000	0.000	2.983	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	432	381	638	0	950	0	0	127	0
N.S.	1	1.26	1.11	1.85	0.00	2.76	0.00	0.00	0.37	0.00
time (sec)	N/A	1.179	8.164	5.570	0.000	0.142	0.000	0.000	1.982	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	414	360	700	0	609	0	0	0	0
N.S.	1	0.93	0.81	1.58	0.00	1.37	0.00	0.00	0.00	0.00
time (sec)	N/A	1.136	11.893	2.512	0.000	0.192	0.000	0.000	6.754	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	329	262	557	0	519	0	0	1348	0
N.S.	1	0.96	0.76	1.62	0.00	1.51	0.00	0.00	3.93	0.00
time (sec)	N/A	0.911	10.685	2.199	0.000	0.175	0.000	0.000	4.668	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	261	210	454	0	452	0	0	683	0
N.S.	1	0.97	0.78	1.69	0.00	1.69	0.00	0.00	2.55	0.00
time (sec)	N/A	0.682	9.791	2.185	0.000	0.136	0.000	0.000	2.938	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	243	186	277	0	379	0	0	43	0
N.S.	1	1.15	0.88	1.31	0.00	1.79	0.00	0.00	0.20	0.00
time (sec)	N/A	0.628	5.614	1.987	0.000	0.116	0.000	0.000	2.033	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	296	220	471	0	493	0	0	329	0
N.S.	1	1.20	0.89	1.91	0.00	2.00	0.00	0.00	1.33	0.00
time (sec)	N/A	0.748	6.593	2.685	0.000	0.095	0.000	0.000	1.764	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	392	266	649	0	797	0	0	1386	0
N.S.	1	1.20	0.82	1.99	0.00	2.44	0.00	0.00	4.25	0.00
time (sec)	N/A	1.029	11.588	3.713	0.000	0.103	0.000	0.000	6.638	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	508	420	692	0	1277	0	0	0	0
N.S.	1	1.19	0.98	1.62	0.00	2.99	0.00	0.00	0.00	0.00
time (sec)	N/A	1.326	11.999	4.924	0.000	0.155	0.000	0.000	5.276	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	485	492	451	895	0	933	0	0	0	0
N.S.	1	1.01	0.93	1.85	0.00	1.92	0.00	0.00	0.00	0.00
time (sec)	N/A	1.371	24.215	4.139	0.000	0.162	0.000	0.000	18.503	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	400	405	792	0	816	0	0	0	0
N.S.	1	1.05	1.07	2.08	0.00	2.15	0.00	0.00	0.00	0.00
time (sec)	N/A	1.100	12.798	4.374	0.000	0.098	0.000	0.000	9.817	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	351	357	353	659	0	700	0	0	0	0
N.S.	1	1.02	1.01	1.88	0.00	1.99	0.00	0.00	0.00	0.00
time (sec)	N/A	0.968	12.038	4.095	0.000	0.118	0.000	0.000	8.044	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	384	290	570	0	581	0	0	0	0
N.S.	1	1.17	0.88	1.73	0.00	1.77	0.00	0.00	0.00	0.00
time (sec)	N/A	1.079	18.785	4.017	0.000	0.100	0.000	0.000	7.913	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	383	375	596	0	767	0	0	58	0
N.S.	1	1.11	1.09	1.73	0.00	2.22	0.00	0.00	0.17	0.00
time (sec)	N/A	0.995	8.829	3.659	0.000	0.114	0.000	0.000	4.457	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	475	429	653	0	953	0	0	0	0
N.S.	1	1.12	1.01	1.54	0.00	2.24	0.00	0.00	0.00	0.00
time (sec)	N/A	1.214	10.142	4.632	0.000	0.110	0.000	0.000	4.746	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	527	602	504	762	0	1510	0	0	0	0
N.S.	1	1.14	0.96	1.45	0.00	2.87	0.00	0.00	0.00	0.00
time (sec)	N/A	1.627	13.246	5.431	0.000	0.170	0.000	0.000	9.487	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	117	215	0	211	0	0	29	0
N.S.	1	1.00	2.29	4.22	0.00	4.14	0.00	0.00	0.57	0.00
time (sec)	N/A	0.314	4.081	0.880	0.000	0.110	0.000	0.000	1.172	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	76	115	0	78	0	0	44	0
N.S.	1	1.00	1.49	2.25	0.00	1.53	0.00	0.00	0.86	0.00
time (sec)	N/A	0.306	3.558	1.241	0.000	0.081	0.000	0.000	0.353	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	53	117	215	0	213	0	0	29	0
N.S.	1	0.29	0.64	1.17	0.00	1.16	0.00	0.00	0.16	0.00
time (sec)	N/A	0.303	4.046	0.925	0.000	0.091	0.000	0.000	1.106	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	53	82	115	0	78	0	0	44	0
N.S.	1	0.65	1.00	1.40	0.00	0.95	0.00	0.00	0.54	0.00
time (sec)	N/A	0.314	3.459	1.204	0.000	0.082	0.000	0.000	0.362	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	64	56	0	16	0	0	26	0
N.S.	1	1.00	6.40	5.60	0.00	1.60	0.00	0.00	2.60	0.00
time (sec)	N/A	0.231	0.535	0.685	0.000	0.087	0.000	0.000	0.311	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	44	42	0	6	0	0	26	0
N.S.	1	1.00	4.40	4.20	0.00	0.60	0.00	0.00	2.60	0.00
time (sec)	N/A	0.236	0.442	0.688	0.000	0.080	0.000	0.000	0.287	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	66	17	0	16	0	0	25	0
N.S.	1	1.00	5.50	1.42	0.00	1.33	0.00	0.00	2.08	0.00
time (sec)	N/A	0.217	0.547	0.685	0.000	0.091	0.000	0.000	0.347	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	B	F	A	B	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	35	42	35	0	16	70	0	23	25
N.S.	1	2.92	3.50	2.92	0.00	1.33	5.83	0.00	1.92	2.08
time (sec)	N/A	0.332	10.028	0.596	0.000	0.086	4.077	0.000	0.254	0.019

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	66	50	0	16	0	0	25	0
N.S.	1	1.00	5.50	4.17	0.00	1.33	0.00	0.00	2.08	0.00
time (sec)	N/A	0.241	0.002	0.661	0.000	0.078	0.000	0.000	0.397	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	30	26	0	11	76	0	28	0
N.S.	1	1.00	1.88	1.62	0.00	0.69	4.75	0.00	1.75	0.00
time (sec)	N/A	0.222	1.806	0.681	0.000	0.080	101.821	0.000	0.542	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	29	30	44	0	11	0	0	28	0
N.S.	1	1.81	1.88	2.75	0.00	0.69	0.00	0.00	1.75	0.00
time (sec)	N/A	0.305	0.006	0.690	0.000	0.084	0.000	0.000	0.475	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	236	904	669	1449	21005	2539	1739	1085
N.S.	1	1.00	0.88	3.39	2.51	5.43	78.67	9.51	6.51	4.06
time (sec)	N/A	0.755	0.230	0.806	0.047	0.115	4.136	0.152	0.219	5.654

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	138	419	318	584	6418	1001	662	464
N.S.	1	1.00	0.87	2.64	2.00	3.67	40.36	6.30	4.16	2.92
time (sec)	N/A	0.539	0.123	0.741	0.042	0.098	1.386	0.128	0.198	5.583

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	116	113	160	1095	260	158	146
N.S.	1	1.00	1.00	1.55	1.51	2.13	14.60	3.47	2.11	1.95
time (sec)	N/A	0.372	0.067	0.638	0.037	0.083	0.497	0.200	0.222	5.362

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	86	0	0	0	0	0	21	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.23	0.00
time (sec)	N/A	0.407	0.076	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	190	208	174	0	0	0	0	0	34	0
N.S.	1	1.09	0.92	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.690	0.143	0.000	0.000	0.000	0.000	0.000	0.221	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	378	396	337	0	0	0	0	0	45	0
N.S.	1	1.05	0.89	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	1.199	0.464	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	105	111	0	0	0	0	0	21	0
N.S.	1	1.33	1.41	0.00	0.00	0.00	0.00	0.00	0.27	0.00
time (sec)	N/A	0.375	1.158	0.000	0.000	0.000	0.000	0.000	200.037	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	105	76	0	0	0	0	0	18	0
N.S.	1	1.38	1.00	0.00	0.00	0.00	0.00	0.00	0.24	0.00
time (sec)	N/A	0.388	0.678	0.000	0.000	0.000	0.000	0.000	0.248	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	74	105	74	0	0	0	0	0	22	0
N.S.	1	1.42	1.00	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.366	0.766	0.000	0.000	0.000	0.000	0.000	0.210	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	105	138	0	0	0	0	0	37	0
N.S.	1	1.38	1.82	0.00	0.00	0.00	0.00	0.00	0.49	0.00
time (sec)	N/A	0.366	1.329	0.000	0.000	0.000	0.000	0.000	0.428	0.000

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	105	76	0	0	0	0	0	58	0
N.S.	1	1.30	0.94	0.00	0.00	0.00	0.00	0.00	0.72	0.00
time (sec)	N/A	0.381	2.021	0.000	0.000	0.000	0.000	0.000	0.473	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	165	177	137	0	0	0	0	0	1548	0
N.S.	1	1.07	0.83	0.00	0.00	0.00	0.00	0.00	9.38	0.00
time (sec)	N/A	0.555	0.122	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	96	79	0	0	0	0	0	456	0
N.S.	1	1.19	0.98	0.00	0.00	0.00	0.00	0.00	5.63	0.00
time (sec)	N/A	0.342	0.051	0.000	0.000	0.000	0.000	0.000	0.258	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	39	55	45	0	0	0	0	0	129	48
N.S.	1	1.41	1.15	0.00	0.00	0.00	0.00	0.00	3.31	1.23
time (sec)	N/A	0.276	0.003	0.000	0.000	0.000	0.000	0.000	0.231	5.260

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	97	57	0	0	0	0	0	375	0
N.S.	1	1.67	0.98	0.00	0.00	0.00	0.00	0.00	6.47	0.00
time (sec)	N/A	0.389	0.156	0.000	0.000	0.000	0.000	0.000	0.317	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	108	57	0	0	0	0	0	0	0
N.S.	1	1.86	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.414	0.150	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	101	120	0	0	0	0	0	0	0
N.S.	1	1.28	1.52	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.384	0.329	0.000	0.000	0.000	0.000	0.000	1.206	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	101	78	0	0	0	0	0	0	0
N.S.	1	1.29	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.365	0.251	0.000	0.000	0.000	0.000	0.000	0.561	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	99	78	0	0	0	0	0	979	0
N.S.	1	1.27	1.00	0.00	0.00	0.00	0.00	0.00	12.55	0.00
time (sec)	N/A	0.374	0.213	0.000	0.000	0.000	0.000	0.000	0.342	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	81	99	78	0	0	0	0	0	0	0
N.S.	1	1.22	0.96	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.362	0.309	0.000	0.000	0.000	0.000	0.000	0.569	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	101	56	0	0	0	0	0	0	0
N.S.	1	1.74	0.97	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.370	0.132	0.000	0.000	0.000	0.000	0.000	0.242	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	59	103	58	0	0	0	0	0	0	0
N.S.	1	1.75	0.98	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.390	0.183	0.000	0.000	0.000	0.000	0.000	0.266	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	101	74	0	0	0	0	0	0	0
N.S.	1	1.35	0.99	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.372	0.143	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	103	76	0	0	0	0	0	0	0
N.S.	1	1.36	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.375	0.142	0.000	0.000	0.000	0.000	0.000	0.281	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [198] had the largest ratio of [.5652169999999999969]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	13	0.154
2	A	1	1	1.00	18	0.056
3	A	1	1	1.00	13	0.077
4	A	6	5	0.91	21	0.238
5	A	5	4	0.92	21	0.190
6	A	3	2	0.90	21	0.095
7	A	5	4	0.92	21	0.190
8	A	6	5	0.91	21	0.238
9	A	4	3	1.00	17	0.176
10	A	1	1	1.31	17	0.059
11	A	2	2	1.00	21	0.095
12	A	2	2	1.00	21	0.095
13	A	2	2	1.00	19	0.105
14	A	1	1	1.00	7	0.143
15	A	3	3	1.00	21	0.143
16	A	3	3	1.00	21	0.143
17	A	2	2	1.00	17	0.118
18	A	2	2	1.00	17	0.118
19	A	2	2	1.00	17	0.118
20	A	2	2	1.00	15	0.133
21	A	1	1	1.00	9	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	2	2	1.00	17	0.118
23	A	2	2	1.00	17	0.118
24	A	2	2	1.00	17	0.118
25	A	2	2	1.00	17	0.118
26	A	2	2	1.00	17	0.118
27	A	2	2	1.00	19	0.105
28	A	2	2	1.00	19	0.105
29	A	2	2	1.00	19	0.105
30	A	2	2	1.00	17	0.118
31	A	2	2	1.00	11	0.182
32	A	2	2	1.00	19	0.105
33	A	2	2	1.00	19	0.105
34	A	2	2	1.00	19	0.105
35	A	2	2	1.00	19	0.105
36	A	2	2	1.00	19	0.105
37	A	2	2	1.00	19	0.105
38	A	2	2	1.00	19	0.105
39	A	2	2	1.00	19	0.105
40	A	2	2	1.00	19	0.105
41	A	2	2	1.00	19	0.105
42	A	2	2	1.00	19	0.105
43	A	2	2	1.00	17	0.118
44	A	2	2	1.00	11	0.182
45	A	2	2	1.00	19	0.105
46	A	2	2	1.00	19	0.105
47	A	2	2	1.00	19	0.105
48	A	2	2	1.00	19	0.105
49	A	2	2	1.00	19	0.105
50	A	2	2	1.00	19	0.105
51	A	2	2	1.00	19	0.105
52	A	2	2	1.00	19	0.105
53	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
54	A	2	2	1.00	19	0.105
55	A	2	2	1.05	19	0.105
56	A	2	2	1.08	19	0.105
57	A	2	2	1.12	19	0.105
58	A	2	2	1.17	17	0.118
59	A	2	2	1.00	11	0.182
60	A	2	2	1.13	19	0.105
61	A	2	2	1.10	19	0.105
62	A	2	2	1.10	19	0.105
63	A	2	2	1.08	19	0.105
64	A	2	2	1.11	19	0.105
65	A	2	2	1.11	19	0.105
66	A	2	2	1.18	19	0.105
67	A	2	2	1.25	17	0.118
68	A	2	2	1.00	11	0.182
69	A	2	2	1.06	19	0.105
70	A	2	2	1.08	19	0.105
71	A	2	2	1.06	19	0.105
72	A	2	2	1.07	19	0.105
73	A	2	2	1.08	19	0.105
74	A	2	2	1.12	19	0.105
75	A	2	2	1.14	19	0.105
76	A	2	2	1.15	19	0.105
77	A	2	2	1.16	17	0.118
78	A	2	2	1.00	11	0.182
79	A	2	2	1.04	19	0.105
80	A	2	2	1.06	19	0.105
81	A	2	2	1.00	19	0.105
82	A	2	2	1.00	19	0.105
83	A	2	2	1.00	19	0.105
84	A	2	2	1.00	19	0.105
85	A	2	2	1.00	19	0.105

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.00	19	0.105
87	A	2	2	1.00	19	0.105
88	A	2	2	1.00	19	0.105
89	A	2	2	1.00	21	0.095
90	A	2	2	1.00	21	0.095
91	A	2	2	1.00	21	0.095
92	A	2	2	1.00	21	0.095
93	A	2	2	1.00	21	0.095
94	A	2	2	1.00	21	0.095
95	A	2	2	1.00	21	0.095
96	A	2	2	1.00	21	0.095
97	A	2	2	1.00	21	0.095
98	A	2	2	1.00	21	0.095
99	A	2	2	1.00	21	0.095
100	A	2	2	1.00	21	0.095
101	A	2	2	1.00	21	0.095
102	A	2	2	1.00	21	0.095
103	A	2	2	1.00	21	0.095
104	A	2	2	1.00	21	0.095
105	A	9	8	1.16	21	0.381
106	A	8	7	1.17	21	0.333
107	A	7	6	1.14	21	0.286
108	A	4	3	1.10	21	0.143
109	A	4	3	1.10	21	0.143
110	A	6	5	1.30	21	0.238
111	A	7	6	1.36	21	0.286
112	A	8	7	1.35	21	0.333
113	A	10	9	1.11	21	0.429
114	A	9	8	1.12	21	0.381
115	A	8	7	1.13	21	0.333
116	A	7	6	0.99	21	0.286
117	A	7	6	0.99	21	0.286

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	7	6	1.15	21	0.286
119	A	8	7	1.20	21	0.333
120	A	9	8	1.20	21	0.381
121	A	11	10	1.03	21	0.476
122	A	10	9	1.04	21	0.429
123	A	9	8	0.87	21	0.381
124	A	9	8	0.88	21	0.381
125	A	9	8	1.03	21	0.381
126	A	9	8	0.98	21	0.381
127	A	9	8	1.09	21	0.381
128	A	10	9	1.10	21	0.429
129	A	7	6	0.71	21	0.286
130	A	7	6	0.79	21	0.286
131	A	5	4	0.79	19	0.211
132	A	4	3	0.82	13	0.231
133	A	7	6	1.22	21	0.286
134	A	7	6	1.21	21	0.286
135	A	4	3	0.95	21	0.143
136	A	5	4	0.93	21	0.190
137	A	7	6	0.93	21	0.286
138	A	9	8	0.92	21	0.381
139	A	8	7	0.59	21	0.333
140	A	8	7	0.65	21	0.333
141	A	6	5	0.71	19	0.263
142	A	5	4	0.79	13	0.308
143	A	9	8	1.10	21	0.381
144	A	10	9	1.07	21	0.429
145	A	8	7	0.99	21	0.333
146	A	9	8	1.17	21	0.381
147	A	5	4	0.93	21	0.190
148	A	9	8	0.53	21	0.381
149	A	9	8	0.58	21	0.381

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	7	6	0.66	19	0.316
151	A	6	5	0.77	13	0.385
152	A	11	10	1.02	21	0.476
153	A	12	11	1.02	21	0.524
154	A	10	9	0.90	21	0.429
155	A	10	9	0.86	21	0.429
156	A	10	9	1.01	21	0.429
157	A	6	5	0.95	21	0.238
158	A	6	5	1.05	21	0.238
159	A	4	3	1.00	19	0.158
160	A	3	2	1.00	13	0.154
161	A	3	2	1.31	21	0.095
162	A	4	3	1.21	21	0.143
163	A	6	5	1.20	21	0.238
164	A	6	5	1.10	21	0.238
165	A	7	6	1.13	21	0.286
166	A	1	1	0.69	19	0.053
167	A	1	1	0.60	13	0.077
168	A	5	4	1.06	21	0.190
169	A	6	5	1.13	21	0.238
170	A	8	7	1.12	21	0.333
171	A	7	6	0.94	21	0.286
172	A	2	2	0.62	21	0.095
173	A	2	2	0.46	21	0.095
174	A	2	2	0.63	19	0.105
175	A	2	2	0.61	13	0.154
176	A	7	6	1.07	21	0.286
177	A	8	7	1.10	21	0.333
178	A	3	3	0.51	21	0.143
179	A	4	4	0.64	21	0.190
180	A	3	3	0.69	21	0.143
181	A	3	3	0.60	19	0.158

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	3	3	0.64	13	0.231
183	A	9	8	1.02	21	0.381
184	A	10	9	1.08	21	0.429
185	A	10	10	0.88	23	0.435
186	A	9	9	0.95	23	0.391
187	A	7	7	0.93	23	0.304
188	A	7	7	1.13	23	0.304
189	A	9	9	1.28	23	0.391
190	A	11	11	1.31	23	0.478
191	A	12	12	0.88	23	0.522
192	A	11	11	0.86	23	0.478
193	A	9	9	0.86	23	0.391
194	A	9	9	0.86	23	0.391
195	A	9	9	1.02	23	0.391
196	A	9	9	1.12	23	0.391
197	A	11	11	1.18	23	0.478
198	A	13	13	0.85	23	0.565
199	A	11	11	0.83	23	0.478
200	A	11	11	0.82	23	0.478
201	A	11	11	0.93	23	0.478
202	A	11	11	0.93	23	0.478
203	A	11	11	1.05	23	0.478
204	A	11	11	1.04	23	0.478
205	A	12	12	0.92	23	0.522
206	A	10	10	0.89	23	0.435
207	A	8	8	0.90	23	0.348
208	A	3	3	0.46	23	0.130
209	A	3	3	1.01	23	0.130
210	A	5	5	0.71	23	0.217
211	A	10	10	1.23	23	0.435
212	A	12	12	1.26	23	0.522
213	A	12	12	0.93	23	0.522

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
214	A	10	10	0.96	23	0.435
215	A	8	8	0.97	23	0.348
216	A	8	8	1.15	23	0.348
217	A	8	8	1.20	23	0.348
218	A	10	10	1.20	23	0.435
219	A	12	12	1.19	23	0.522
220	A	12	12	1.01	23	0.522
221	A	10	10	1.05	23	0.435
222	A	10	10	1.02	23	0.435
223	A	10	10	1.17	23	0.435
224	A	10	10	1.11	23	0.435
225	A	10	10	1.12	23	0.435
226	A	12	12	1.14	23	0.522
227	A	6	6	1.00	23	0.261
228	A	6	6	1.00	23	0.261
229	A	6	6	0.29	23	0.261
230	A	6	6	0.65	23	0.261
231	A	2	2	1.00	19	0.105
232	A	2	2	1.00	19	0.105
233	A	1	1	1.00	24	0.042
234	B	8	7	2.92	24	0.292
235	A	2	2	1.00	23	0.087
236	A	1	1	1.00	24	0.042
237	A	6	5	1.81	23	0.217
238	A	2	2	1.00	19	0.105
239	A	2	2	1.00	19	0.105
240	A	2	2	1.00	17	0.118
241	A	2	2	1.00	19	0.105
242	A	3	3	1.09	19	0.158
243	A	4	4	1.05	19	0.211
244	A	3	2	1.33	21	0.095
245	A	3	2	1.38	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	3	2	1.42	21	0.095
247	A	3	2	1.38	21	0.095
248	A	3	2	1.30	21	0.095
249	A	4	4	1.07	19	0.211
250	A	2	2	1.19	17	0.118
251	A	1	1	1.41	11	0.091
252	A	3	2	1.67	19	0.105
253	A	3	2	1.86	19	0.105
254	A	3	2	1.28	21	0.095
255	A	3	2	1.29	21	0.095
256	A	3	2	1.27	21	0.095
257	A	3	2	1.22	21	0.095
258	A	3	2	1.74	19	0.105
259	A	3	2	1.75	19	0.105
260	A	3	2	1.35	19	0.105
261	A	3	2	1.36	19	0.105

CHAPTER 3

Listing of integrals

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3.37 $\int \frac{\left(bx+cx^2\right)^2}{(d+ex)^6}\,dx$ . . . 322
3.38 $\int \frac{\left(bx+cx^2\right)^2}{(d+ex)^7}\,dx$ . . . 328
3.39 $\int \frac{\left(bx+cx^2\right)^2}{(d+ex)^8}\,dx$ . . . 334
3.40 $\int (d+ex)^4 \left(bx+cx^2\right)^3\,dx$ . . . 340
3.41 $\int (d+ex)^3 \left(bx+cx^2\right)^3\,dx$ . . . 348
3.42 $\int (d+ex)^2 \left(bx+cx^2\right)^3\,dx$ . . . 355
3.43 $\int (d+ex) \left(bx+cx^2\right)^3\,dx$ . . . 361
3.44 $\int \left(bx+cx^2\right)^3\,dx$ . . . 367
3.45 $\int \frac{\left(bx+cx^2\right)^3}{d+ex}\,dx$ . . . 372
3.46 $\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^2}\,dx$ . . . 379
3.47 $\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^3}\,dx$ . . . 387
3.48 $\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^4}\,dx$ . . . 395
3.49 $\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^5}\,dx$ . . . 403
3.50 $\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^6}\,dx$ . . . 411
3.51 $\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^7}\,dx$ . . . 418
3.52 $\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^8}\,dx$ . . . 426
3.53 $\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^9}\,dx$ . . . 433
3.54 $\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^{10}}\,dx$ . . . 441
3.55 $\int \frac{(d+ex)^4}{bx+cx^2}\,dx$ . . . 448
3.56 $\int \frac{(d+ex)^3}{bx+cx^2}\,dx$ . . . 454
3.57 $\int \frac{(d+ex)^2}{bx+cx^2}\,dx$ . . . 460

3.58 $\int \frac{d+ex}{bx+cx^2}\,dx$ . . . 465
3.59 $\int \frac{1}{bx+cx^2}\,dx$ . . . 470
3.60 $\int \frac{1}{(d+ex)(bx+cx^2)}\,dx$ . . . 475
3.61 $\int \frac{1}{(d+ex)^2(bx+cx^2)}\,dx$ . . . 480
3.62 $\int \frac{1}{(d+ex)^3(bx+cx^2)}\,dx$ . . . 486
3.63 $\int \frac{(d+ex)^5}{(bx+cx^2)^2}\,dx$ . . . 493
3.64 $\int \frac{(d+ex)^4}{(bx+cx^2)^2}\,dx$ . . . 500
3.65 $\int \frac{(d+ex)^3}{(bx+cx^2)^2}\,dx$ . . . 507
3.66 $\int \frac{(d+ex)^2}{(bx+cx^2)^2}\,dx$ . . . 513
3.67 $\int \frac{d+ex}{(bx+cx^2)^2}\,dx$ . . . 519
3.68 $\int \frac{1}{(bx+cx^2)^2}\,dx$ . . . 525
3.69 $\int \frac{1}{(d+ex)(bx+cx^2)^2}\,dx$ . . . 530
3.70 $\int \frac{1}{(d+ex)^2(bx+cx^2)^2}\,dx$ . . . 536
3.71 $\int \frac{(d+ex)^7}{(bx+cx^2)^3}\,dx$ . . . 544
3.72 $\int \frac{(d+ex)^6}{(bx+cx^2)^3}\,dx$ . . . 553
3.73 $\int \frac{(d+ex)^5}{(bx+cx^2)^3}\,dx$ . . . 561
3.74 $\int \frac{(d+ex)^4}{(bx+cx^2)^3}\,dx$ . . . 569
3.75 $\int \frac{(d+ex)^3}{(bx+cx^2)^3}\,dx$ . . . 576
3.76 $\int \frac{(d+ex)^2}{(bx+cx^2)^3}\,dx$ . . . 583
3.77 $\int \frac{d+ex}{(bx+cx^2)^3}\,dx$ . . . 590
3.78 $\int \frac{1}{(bx+cx^2)^3}\,dx$ . . . 597
3.79 $\int \frac{1}{(d+ex)(bx+cx^2)^3}\,dx$ . . . 603
3.80 $\int \frac{1}{(d+ex)^2(bx+cx^2)^3}\,dx$ . . . 611
3.81 $\int (d+ex)^{7/2}\left(bx+cx^2\right)\,dx$ . . . 620
3.82 $\int (d+ex)^{5/2}\left(bx+cx^2\right)\,dx$ . . . 626
3.83 $\int (d+ex)^{3/2}\left(bx+cx^2\right)\,dx$ . . . 632
3.84 $\int \sqrt{d+ex}(bx+cx^2)\,dx$ . . . 638
3.85 $\int \frac{bx+cx^2}{\sqrt{d+ex}}\,dx$ . . . 644
3.86 $\int \frac{bx+cx^2}{(d+ex)^{3/2}}\,dx$ . . . 650
3.87 $\int \frac{bx+cx^2}{(d+ex)^{5/2}}\,dx$ . . . 655
3.88 $\int \frac{bx+cx^2}{(d+ex)^{7/2}}\,dx$ . . . 660
3.89 $\int (d+ex)^{7/2}\left(bx+cx^2\right)^2\,dx$ . . . 665
3.90 $\int (d+ex)^{5/2}\left(bx+cx^2\right)^2\,dx$ . . . 673
3.91 $\int (d+ex)^{3/2}\left(bx+cx^2\right)^2\,dx$ . . . 680

3.92 $\int \sqrt{d+ex}\left(bx+cx^2\right)^2\,dx$ . . . 687
3.93 $\int \frac{\left(bx+cx^2\right)^2}{\sqrt{d+ex}}\,dx$ . . . 694
3.94 $\int \frac{\left(bx+cx^2\right)^2}{(d+ex)^{3/2}}\,dx$ . . . 700
3.95 $\int \frac{\left(bx+cx^2\right)^2}{(d+ex)^{5/2}}\,dx$ . . . 706
3.96 $\int \frac{\left(bx+cx^2\right)^2}{(d+ex)^{7/2}}\,dx$ . . . 712
3.97 $\int (d+ex)^{7/2}\left(bx+cx^2\right)^3\,dx$ . . . 719
3.98 $\int (d+ex)^{5/2}\left(bx+cx^2\right)^3\,dx$ . . . 728
3.99 $\int (d+ex)^{3/2}\left(bx+cx^2\right)^3\,dx$ . . . 736
3.100 $\int \sqrt{d+ex}\left(bx+cx^2\right)^3\,dx$ . . . 744
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3.102 $\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^{3/2}}\,dx$ . . . 760
3.103 $\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^{5/2}}\,dx$ . . . 768
3.104 $\int \frac{\left(bx+cx^2\right)^3}{(d+ex)^{7/2}}\,dx$ . . . 776
3.105 $\int \frac{(d+ex)^{7/2}}{bx+cx^2}\,dx$ . . . 784
3.106 $\int \frac{(d+ex)^{5/2}}{bx+cx^2}\,dx$ . . . 793
3.107 $\int \frac{(d+ex)^{3/2}}{bx+cx^2}\,dx$ . . . 801
3.108 $\int \frac{\sqrt{d+ex}}{bx+cx^2}\,dx$ . . . 809
3.109 $\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)}\,dx$ . . . 816
3.110 $\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)}\,dx$ . . . 823
3.111 $\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)}\,dx$ . . . 831
3.112 $\int \frac{1}{(d+ex)^{7/2}\left(bx+cx^2\right)}\,dx$ . . . 840
3.113 $\int \frac{(d+ex)^{9/2}}{\left(bx+cx^2\right)^2}\,dx$ . . . 849
3.114 $\int \frac{(d+ex)^{7/2}}{\left(bx+cx^2\right)^2}\,dx$ . . . 860
3.115 $\int \frac{(d+ex)^{5/2}}{\left(bx+cx^2\right)^2}\,dx$ . . . 870
3.116 $\int \frac{(d+ex)^{3/2}}{\left(bx+cx^2\right)^2}\,dx$ . . . 879
3.117 $\int \frac{\sqrt{d+ex}}{\left(bx+cx^2\right)^2}\,dx$ . . . 887
3.118 $\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^2}\,dx$ . . . 895
3.119 $\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^2}\,dx$ . . . 904
3.120 $\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^2}\,dx$ . . . 914
3.121 $\int \frac{(d+ex)^{11/2}}{\left(bx+cx^2\right)^3}\,dx$ . . . 924
3.122 $\int \frac{(d+ex)^{9/2}}{\left(bx+cx^2\right)^3}\,dx$ . . . 936
3.123 $\int \frac{(d+ex)^{7/2}}{\left(bx+cx^2\right)^3}\,dx$ . . . 948

3.124 $\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^3}\,dx$ 959
3.125 $\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^3}\,dx$ 970
3.126 $\int \frac{\sqrt{d+ex}}{(bx+cx^2)^3}\,dx$ 981
3.127 $\int \frac{1}{\sqrt{d+ex}(bx+cx^2)^3}\,dx$ 992
3.128 $\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)^3}\,dx$ 1003
3.129 $\int (d+ex)^3\sqrt{bx+cx^2}\,dx$ 1014
3.130 $\int (d+ex)^2\sqrt{bx+cx^2}\,dx$ 1025
3.131 $\int (d+ex)\sqrt{bx+cx^2}\,dx$ 1034
3.132 $\int \sqrt{bx+cx^2}\,dx$ 1041
3.133 $\int \frac{\sqrt{bx+cx^2}}{d+ex}\,dx$ 1047
3.134 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^2}\,dx$ 1054
3.135 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^3}\,dx$ 1061
3.136 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^4}\,dx$ 1068
3.137 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^5}\,dx$ 1076
3.138 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^6}\,dx$ 1085
3.139 $\int (d+ex)^3\left(bx+cx^2\right)^{3/2}\,dx$ 1095
3.140 $\int (d+ex)^2\left(bx+cx^2\right)^{3/2}\,dx$ 1108
3.141 $\int (d+ex)\left(bx+cx^2\right)^{3/2}\,dx$ 1119
3.142 $\int \left(bx+cx^2\right)^{3/2}\,dx$ 1128
3.143 $\int \frac{\left(bx+cx^2\right)^{3/2}}{d+ex}\,dx$ 1135
3.144 $\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^2}\,dx$ 1144
3.145 $\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^3}\,dx$ 1153
3.146 $\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^4}\,dx$ 1163
3.147 $\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^5}\,dx$ 1173
3.148 $\int (d+ex)^3\left(bx+cx^2\right)^{5/2}\,dx$ 1181
3.149 $\int (d+ex)^2\left(bx+cx^2\right)^{5/2}\,dx$ 1193
3.150 $\int (d+ex)\left(bx+cx^2\right)^{5/2}\,dx$ 1206
3.151 $\int \left(bx+cx^2\right)^{5/2}\,dx$ 1216
3.152 $\int \frac{\left(bx+cx^2\right)^{5/2}}{d+ex}\,dx$ 1223
3.153 $\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^2}\,dx$ 1233
3.154 $\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^3}\,dx$ 1243
3.155 $\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^4}\,dx$ 1254

3.156 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^5}\,dx$ 1264
3.157 $\int \frac{(d+ex)^3}{\sqrt{bx+cx^2}}\,dx$ 1274
3.158 $\int \frac{(d+ex)^2}{\sqrt{bx+cx^2}}\,dx$ 1283
3.159 $\int \frac{d+ex}{\sqrt{bx+cx^2}}\,dx$ 1290
3.160 $\int \frac{1}{\sqrt{bx+cx^2}}\,dx$ 1296
3.161 $\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx$ 1301
3.162 $\int \frac{1}{(d+ex)^2\sqrt{bx+cx^2}}\,dx$ 1306
3.163 $\int \frac{1}{(d+ex)^3\sqrt{bx+cx^2}}\,dx$ 1312
3.164 $\int \frac{(d+ex)^3}{(bx+cx^2)^{3/2}}\,dx$ 1320
3.165 $\int \frac{(d+ex)^2}{(bx+cx^2)^{3/2}}\,dx$ 1327
3.166 $\int \frac{d+ex}{(bx+cx^2)^{3/2}}\,dx$ 1334
3.167 $\int \frac{1}{(bx+cx^2)^{3/2}}\,dx$ 1339
3.168 $\int \frac{1}{(d+ex)(bx+cx^2)^{3/2}}\,dx$ 1344
3.169 $\int \frac{1}{(d+ex)^2(bx+cx^2)^{3/2}}\,dx$ 1351
3.170 $\int \frac{1}{(d+ex)^3(bx+cx^2)^{3/2}}\,dx$ 1360
3.171 $\int \frac{(d+ex)^4}{(bx+cx^2)^{5/2}}\,dx$ 1370
3.172 $\int \frac{(d+ex)^3}{(bx+cx^2)^{5/2}}\,dx$ 1380
3.173 $\int \frac{(d+ex)^2}{(bx+cx^2)^{5/2}}\,dx$ 1387
3.174 $\int \frac{d+ex}{(bx+cx^2)^{5/2}}\,dx$ 1393
3.175 $\int \frac{1}{(bx+cx^2)^{5/2}}\,dx$ 1399
3.176 $\int \frac{1}{(d+ex)(bx+cx^2)^{5/2}}\,dx$ 1404
3.177 $\int \frac{1}{(d+ex)^2(bx+cx^2)^{5/2}}\,dx$ 1413
3.178 $\int \frac{(d+ex)^4}{(bx+cx^2)^{7/2}}\,dx$ 1423
3.179 $\int \frac{(d+ex)^3}{(bx+cx^2)^{7/2}}\,dx$ 1432
3.180 $\int \frac{(d+ex)^2}{(bx+cx^2)^{7/2}}\,dx$ 1441
3.181 $\int \frac{d+ex}{(bx+cx^2)^{7/2}}\,dx$ 1449
3.182 $\int \frac{1}{(bx+cx^2)^{7/2}}\,dx$ 1456
3.183 $\int \frac{1}{(d+ex)(bx+cx^2)^{7/2}}\,dx$ 1462
3.184 $\int \frac{1}{(d+ex)^2(bx+cx^2)^{7/2}}\,dx$ 1472
3.185 $\int (d+ex)^{3/2}\sqrt{bx+cx^2}\,dx$ 1483
3.186 $\int \sqrt{d+ex}\sqrt{bx+cx^2}\,dx$ 1493
3.187 $\int \frac{\sqrt{bx+cx^2}}{\sqrt{d+ex}}\,dx$ 1502

3.188 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{3/2}}\,dx$ . . . 1510
3.189 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{5/2}}\,dx$ . . . 1518
3.190 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{7/2}}\,dx$ . . . 1527
3.191 $\int (d+ex)^{3/2}\left(bx+cx^2\right)^{3/2}\,dx$ . . . 1537
3.192 $\int \sqrt{d+ex}\left(bx+cx^2\right)^{3/2}\,dx$ . . . 1548
3.193 $\int \frac{\left(bx+cx^2\right)^{3/2}}{\sqrt{d+ex}}\,dx$ . . . 1560
3.194 $\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^{3/2}}\,dx$ . . . 1570
3.195 $\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^{5/2}}\,dx$ . . . 1580
3.196 $\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^{7/2}}\,dx$ . . . 1590
3.197 $\int \frac{\left(bx+cx^2\right)^{3/2}}{(d+ex)^{9/2}}\,dx$ . . . 1600
3.198 $\int \sqrt{d+ex}\left(bx+cx^2\right)^{5/2}\,dx$ . . . 1612
3.199 $\int \frac{\left(bx+cx^2\right)^{5/2}}{\sqrt{d+ex}}\,dx$ . . . 1625
3.200 $\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^{3/2}}\,dx$ . . . 1636
3.201 $\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^{5/2}}\,dx$ . . . 1647
3.202 $\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^{7/2}}\,dx$ . . . 1659
3.203 $\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^{9/2}}\,dx$ . . . 1670
3.204 $\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^{11/2}}\,dx$ . . . 1683
3.205 $\int \frac{(d+ex)^{7/2}}{\sqrt{bx+cx^2}}\,dx$ . . . 1697
3.206 $\int \frac{(d+ex)^{5/2}}{\sqrt{bx+cx^2}}\,dx$ . . . 1708
3.207 $\int \frac{(d+ex)^{3/2}}{\sqrt{bx+cx^2}}\,dx$ . . . 1718
3.208 $\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx$ . . . 1726
3.209 $\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx$ . . . 1732
3.210 $\int \frac{1}{(d+ex)^{3/2}\sqrt{bx+cx^2}}\,dx$ . . . 1737
3.211 $\int \frac{1}{(d+ex)^{5/2}\sqrt{bx+cx^2}}\,dx$ . . . 1744
3.212 $\int \frac{1}{(d+ex)^{7/2}\sqrt{bx+cx^2}}\,dx$ . . . 1753
3.213 $\int \frac{(d+ex)^{7/2}}{\left(bx+cx^2\right)^{3/2}}\,dx$ . . . 1763
3.214 $\int \frac{(d+ex)^{5/2}}{\left(bx+cx^2\right)^{3/2}}\,dx$ . . . 1774
3.215 $\int \frac{(d+ex)^{3/2}}{\left(bx+cx^2\right)^{3/2}}\,dx$ . . . 1784
3.216 $\int \frac{\sqrt{d+ex}}{\left(bx+cx^2\right)^{3/2}}\,dx$ . . . 1793
3.217 $\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}}\,dx$ . . . 1801

3.218 $\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)^{3/2}}\,dx$ . . . 1810
3.219 $\int \frac{1}{(d+ex)^{5/2}(bx+cx^2)^{3/2}}\,dx$ . . . 1820
3.220 $\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^{5/2}}\,dx$ . . . 1832
3.221 $\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{5/2}}\,dx$ . . . 1844
3.222 $\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{5/2}}\,dx$ . . . 1854
3.223 $\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{5/2}}\,dx$ . . . 1864
3.224 $\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{5/2}}\,dx$ . . . 1874
3.225 $\int \frac{1}{\sqrt{d+ex}(bx+cx^2)^{5/2}}\,dx$ . . . 1884
3.226 $\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)^{5/2}}\,dx$ . . . 1895
3.227 $\int \frac{\sqrt{d+ex}}{\sqrt{2x-3x^2}}\,dx$ . . . 1907
3.228 $\int \frac{1}{\sqrt{d+ex}\sqrt{2x-3x^2}}\,dx$ . . . 1913
3.229 $\int \frac{\sqrt{d+ex}}{\sqrt{-2x-3x^2}}\,dx$ . . . 1919
3.230 $\int \frac{1}{\sqrt{d+ex}\sqrt{-2x-3x^2}}\,dx$ . . . 1926
3.231 $\int \frac{\sqrt{1+x}}{\sqrt{x-x^2}}\,dx$ . . . 1932
3.232 $\int \frac{1}{\sqrt{1+x}\sqrt{x-x^2}}\,dx$ . . . 1937
3.233 $\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{1+x}}\,dx$ . . . 1942
3.234 $\int \frac{1-x}{\sqrt{-x}\sqrt{1-x^2}}\,dx$ . . . 1947
3.235 $\int \frac{\sqrt{1-x}}{\sqrt{-x-x^2}}\,dx$ . . . 1954
3.236 $\int \frac{\sqrt{1-x}}{\sqrt{2-x}\sqrt{x}}\,dx$ . . . 1959
3.237 $\int \frac{\sqrt{1-x}}{\sqrt{2x-x^2}}\,dx$ . . . 1964
3.238 $\int (d+ex)^m \left(bx+cx^2\right)^3\,dx$ . . . 1970
3.239 $\int (d+ex)^m \left(bx+cx^2\right)^2\,dx$ . . . 1980
3.240 $\int (d+ex)^m \left(bx+cx^2\right)\,dx$ . . . 1989
3.241 $\int \frac{(d+ex)^m}{bx+cx^2}\,dx$ . . . 1995
3.242 $\int \frac{(d+ex)^m}{(bx+cx^2)^2}\,dx$ . . . 2000
3.243 $\int \frac{(d+ex)^m}{(bx+cx^2)^3}\,dx$ . . . 2006
3.244 $\int (d+ex)^m \left(bx+cx^2\right)^{3/2}\,dx$ . . . 2013
3.245 $\int (d+ex)^m \sqrt{bx+cx^2}\,dx$ . . . 2018
3.246 $\int \frac{(d+ex)^m}{\sqrt{bx+cx^2}}\,dx$ . . . 2023
3.247 $\int \frac{(d+ex)^m}{(bx+cx^2)^{3/2}}\,dx$ . . . 2028
3.248 $\int \frac{(d+ex)^m}{(bx+cx^2)^{5/2}}\,dx$ . . . 2033
3.249 $\int (d+ex)^2 \left(bx+cx^2\right)^p\,dx$ . . . 2038
3.250 $\int (d+ex) \left(bx+cx^2\right)^p\,dx$ . . . 2045

3.251 $\int (bx+cx^2)^p \, dx$ . . . 2050
3.252 $\int \frac{(bx+cx^2)^p}{d+ex} \, dx$ . . . 2054
3.253 $\int \frac{(bx+cx^2)^p}{(d+ex)^2} \, dx$ . . . 2059
3.254 $\int (d+ex)^{3/2} (bx+cx^2)^p \, dx$ . . . 2065
3.255 $\int \sqrt{d+ex}(bx+cx^2)^p \, dx$ . . . 2071
3.256 $\int \frac{(bx+cx^2)^p}{\sqrt{d+ex}} \, dx$ . . . 2077
3.257 $\int \frac{(bx+cx^2)^p}{(d+ex)^{3/2}} \, dx$ . . . 2083
3.258 $\int (3+ex)^m (2x+cx^2)^p \, dx$ . . . 2089
3.259 $\int (3+ex)^m (bx+cx^2)^p \, dx$ . . . 2095
3.260 $\int (d+ex)^m (2x+cx^2)^p \, dx$ . . . 2101
3.261 $\int (d+ex)^m (bx+cx^2)^p \, dx$ . . . 2107

3.1 $\int \frac{2x+x^2}{(1+x)^2}\,dx$

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Rubi [A] (verified) 122
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Optimal result

Integrand size = 13, antiderivative size = 7

$$\int \frac{2x+x^2}{(1+x)^2}\,dx=x+\frac{1}{1+x}$$

output
```
x+1/(1+x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{2x+x^2}{(1+x)^2}\,dx=x+\frac{1}{1+x}$$

input
```
Integrate[(2*x + x^2)/(1 + x)^2,x]
```

output
```
x + (1 + x)^(-1)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1107, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2+2x}{(x+1)^2}\,dx$$

$$\Big\downarrow \ 1107$$

$$\int \left(1-\frac{1}{(x+1)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$x+\frac{1}{x+1}$$

input
```
Int[(2*x + x^2)/(1 + x)^2,x]
```

output
```
x + (1 + x)^(-1)
```

Defintions of rubi rules used

rule 1107
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && EqQ[2*c*d - b*e, 0] && IGtQ[p, 0] &&  !(EqQ[
m, 3] && NeQ[p, 1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

method	result	size
default	$x+\frac{1}{x+1}$	8
risch	$x+\frac{1}{x+1}$	8
gosper	$\frac{x^2}{x+1}$	10
norman	$\frac{x^2}{x+1}$	10
parallelrisch	$\frac{x^2}{x+1}$	10
orering	$\frac{x(x^2+2x)}{(x+1)(2+x)}$	20
meijerg	$\frac{x(3x+6)}{3+3x}-\frac{2x}{x+1}$	23

input
```
int((x^2+2*x)/(x+1)^2,x,method=_RETURNVERBOSE)
```

output
```
x+1/(x+1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.71

$$\int \frac{2x+x^2}{(1+x)^2}\,dx=\frac{x^2+x+1}{x+1}$$

input
```
integrate((x^2+2*x)/(1+x)^2,x, algorithm="fricas")
```

output
```
(x^2 + x + 1)/(x + 1)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.71

$$\int \frac{2x+x^2}{(1+x)^2}\,dx = x+\frac{1}{x+1}$$

input
```
integrate((x**2+2*x)/(1+x)**2,x)
```

output
```
x + 1/(x + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{2x+x^2}{(1+x)^2}\,dx = x+\frac{1}{x+1}$$

input
```
integrate((x^2+2*x)/(1+x)^2,x, algorithm="maxima")
```

output
```
x + 1/(x + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.14

$$\int \frac{2x+x^2}{(1+x)^2}\,dx = x+\frac{1}{x+1}+1$$

input
```
integrate((x^2+2*x)/(1+x)^2,x, algorithm="giac")
```

output
```
x + 1/(x + 1) + 1
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.00

$$\int \frac{2x+x^2}{(1+x)^2}\,dx = x+\frac{1}{x+1}$$

input
```
int((2*x + x^2)/(x + 1)^2,x)
```

output
```
x + 1/(x + 1)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.29

$$\int \frac{2x+x^2}{(1+x)^2}\,dx = \frac{x^2}{x+1}$$

input
```
int((x^2+2*x)/(1+x)^2,x)
```

output
```
x**2/(x + 1)
```

3.2 $\int \frac{b+2cx}{bx+cx^2} \, dx$

Optimal result . . . 126
Mathematica [A] (verified) . . . 126
Rubi [A] (verified) . . . 127
Maple [A] (verified) . . . 127
Fricas [A] (verification not implemented) . . . 128
Sympy [A] (verification not implemented) . . . 128
Maxima [A] (verification not implemented) . . . 129
Giac [A] (verification not implemented) . . . 129
Mupad [B] (verification not implemented) . . . 129
Reduce [B] (verification not implemented) . . . 130

Optimal result

Integrand size = 18, antiderivative size = 10

$$\int \frac{b+2cx}{bx+cx^2} \, dx = \log \left(bx+cx^2\right)$$

output
```
ln(c*x^2+b*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{b+2cx}{bx+cx^2} \, dx = \log(x)+\log(b+cx)$$

input
```
Integrate[(b + 2*c*x)/(b*x + c*x^2),x]
```

output
```
Log[x] + Log[b + c*x]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.056, Rules used = {1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{b+2cx}{bx+cx^2}\,dx$$

$$\Big\downarrow \text{ 1103}$$

$$\log\left(bx+cx^2\right)$$

input
```
Int[(b + 2*c*x)/(b*x + c*x^2),x]
```

output
```
Log[b*x + c*x^2]
```

Defintions of rubi rules used

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
default	$\ln\left(x(cx+b)\right)$	9
norman	$\ln\left(x\right)+\ln\left(cx+b\right)$	10
parallelrisch	$\ln\left(x\right)+\ln\left(cx+b\right)$	10
derivativedivides	$\ln\left(c\,x^{2}+bx\right)$	11
risch	$\ln\left(c\,x^{2}+bx\right)$	11

input
```
int((2*c*x+b)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
ln(x*(c*x+b))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(cx^2+bx\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
log(c*x^2 + b*x)
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(bx+cx^2\right)$$

input
```
integrate((2*c*x+b)/(c*x**2+b*x),x)
```

output
```
log(b*x + c*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(cx^2+bx\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
log(c*x^2 + b*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log\left(\left|cx^2+bx\right|\right)$$

input
```
integrate((2*c*x+b)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
log(abs(c*x^2 + b*x))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \ln\left(x\left(b+c\,x\right)\right)$$

input
```
int((b + 2*c*x)/(b*x + c*x^2),x)
```

output
```
log(x*(b + c*x))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

$$\int \frac{b+2cx}{bx+cx^2}\,dx = \log(cx+b)+\log(x)$$

input
```
int((2*c*x+b)/(c*x^2+b*x),x)
```

output
```
log(b + c*x) + log(x)
```

3.3 $\int \frac{1+x}{2x+x^2} \, dx$

Optimal result . . . 131
Mathematica [A] (verified) . . . 131
Rubi [A] (verified) . . . 132
Maple [A] (verified) . . . 133
Fricas [A] (verification not implemented) . . . 133
Sympy [A] (verification not implemented) . . . 134
Maxima [A] (verification not implemented) . . . 134
Giac [A] (verification not implemented) . . . 134
Mupad [B] (verification not implemented) . . . 135
Reduce [B] (verification not implemented) . . . 135

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{1+x}{2x+x^2} \, dx = \frac{1}{2}\log\left(2x+x^2\right)$$

output
```
1/2*ln(x^2+2*x)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.25

$$\int \frac{1+x}{2x+x^2} \, dx = \frac{\log(x)}{2} + \frac{1}{2}\log(2+x)$$

input
```
Integrate[(1 + x)/(2*x + x^2),x]
```

output
```
Log[x]/2 + Log[2 + x]/2
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = $\{1103\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x+1}{x^2+2x} \, dx$$

$$\Big\downarrow \; 1103$$

$$\frac{1}{2} \log \left(x^2+2x\right)$$

input
```
Int[(1 + x)/(2*x + x^2),x]
```

output
```
Log[2*x + x^2]/2
```

Defintions of rubi rules used

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\ln(x(2+x))}{2}$	9
risch	$\frac{\ln(x^2+2x)}{2}$	11
norman	$\frac{\ln(x)}{2}+\frac{\ln(2+x)}{2}$	12
parallelrisch	$\frac{\ln(x)}{2}+\frac{\ln(2+x)}{2}$	12
meijerg	$\frac{\ln(1+\frac{x}{2})}{2}+\frac{\ln(x)}{2}-\frac{\ln(2)}{2}$	18

input
```
int((x+1)/(x^2+2*x),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(x*(2+x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x}{2x+x^2}\,dx=\frac{1}{2}\log\left(x^2+2\,x\right)$$

input
```
integrate((1+x)/(x^2+2*x),x, algorithm="fricas")
```

output
```
1/2*log(x^2 + 2*x)
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1+x}{2x+x^2} \, dx = \frac{\log(x^2+2x)}{2}$$

input
```
integrate((1+x)/(x**2+2*x),x)
```

output
```
log(x**2 + 2*x)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{1+x}{2x+x^2} \, dx = \frac{1}{2} \log\left(x^2+2\,x\right)$$

input
```
integrate((1+x)/(x^2+2*x),x, algorithm="maxima")
```

output
```
1/2*log(x^2 + 2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{1+x}{2x+x^2} \, dx = \frac{1}{2} \log\left(2\left|\frac{1}{2}\,x^2+x\right|\right)$$

input
```
integrate((1+x)/(x^2+2*x),x, algorithm="giac")
```

output
```
1/2*log(2*abs(1/2*x^2 + x))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{1+x}{2x+x^2}\,dx = \frac{\ln\left(x\left(x+2\right)\right)}{2}$$

input
```
int((x + 1)/(2*x + x^2),x)
```

output
```
log(x*(x + 2))/2
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

$$\int \frac{1+x}{2x+x^2}\,dx = \frac{\log(x+2)}{2} + \frac{\log(x)}{2}$$

input
```
int((1+x)/(x^2+2*x),x)
```

output
```
(log(x + 2) + log(x))/2
```

3.4 $\int \frac{(2x-x^2)^{3/2}}{2-2x} \, dx$

Optimal result 136
Mathematica [A] (verified) 136
Rubi [A] (verified) 137
Maple [A] (verified) 138
Fricas [A] (verification not implemented) 139
Sympy [F] 139
Maxima [A] (verification not implemented) 140
Giac [A] (verification not implemented) 140
Mupad [F(-1)] 140
Reduce [B] (verification not implemented) 141

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{(2x-x^2)^{3/2}}{2-2x} \, dx = -\frac{1}{2}\sqrt{2-x}\sqrt{x} - \frac{1}{6}(2-x)^{3/2}x^{3/2} + \frac{1}{2}\text{arctanh}\left(\sqrt{2-x}\sqrt{x}\right)$$

output
```
-1/2*(2-x)^(1/2)*x^(1/2)-1/6*(2-x)^(3/2)*x^(3/2)+1/2*arctanh((2-x)^(1/2)*x
^(1/2))
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{(2x-x^2)^{3/2}}{2-2x} \, dx = \frac{1}{6}\sqrt{-((-2+x)x)}\left(-3-2x+x^2+\frac{6\arctan\left(1+\sqrt{-2+x}\sqrt{x}-x\right)}{\sqrt{-2+x}\sqrt{x}}\right)$$

input
```
Integrate[(2*x - x^2)^(3/2)/(2 - 2*x),x]
```

output
```
(Sqrt[-((-2 + x)*x)]*(-3 - 2*x + x^2 + (6*ArcTan[1 + Sqrt[-2 + x]*Sqrt[x]
- x])/(Sqrt[-2 + x]*Sqrt[x])))/6
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1109, 27, 1109, 1112, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(2x-x^2)^{3/2}}{2-2x}dx$$

$$\Big\downarrow \text{1109}$$

$$\int \frac{\sqrt{2x-x^2}}{2(1-x)}dx-\frac{1}{6}(2x-x^2)^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{\sqrt{2x-x^2}}{1-x}dx-\frac{1}{6}(2x-x^2)^{3/2}$$

$$\Big\downarrow \text{1109}$$

$$\frac{1}{2}\left(\int \frac{1}{(1-x)\sqrt{2x-x^2}}dx-\sqrt{2x-x^2}\right)-\frac{1}{6}(2x-x^2)^{3/2}$$

$$\Big\downarrow \text{1112}$$

$$\frac{1}{2}\left(-4\int \frac{1}{4(2x-x^2)-4}d\sqrt{2x-x^2}-\sqrt{2x-x^2}\right)-\frac{1}{6}(2x-x^2)^{3/2}$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\left(\text{arctanh}\left(\sqrt{2x-x^2}\right)-\sqrt{2x-x^2}\right)-\frac{1}{6}(2x-x^2)^{3/2}$$

input
```
Int[(2*x - x^2)^(3/2)/(2 - 2*x),x]
```

output
```
-1/6*(2*x - x^2)^(3/2) + (-Sqrt[2*x - x^2] + ArcTanh[Sqrt[2*x - x^2]])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1109
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[d*p*((b^2 - 4*a*c)/(b*e*(m + 2*p + 1)))   Int[(d + e*x)^m*(a + b*x
 + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[2*c*d - b*
e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m, -1] &&  !(IGtQ[(m - 1
)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && RationalQ[m] && IntegerQ[2*p]
```

rule 1112
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Simp[4*c   Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a
+ b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{(x^2-2x-3)x(x-2)}{6\sqrt{-x(x-2)}}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-(x-1)^2}}\right)}{2}$	38
default	$-\frac{\left(1-(x-1)^2\right)^{\frac{3}{2}}}{6}-\frac{\sqrt{1-(x-1)^2}}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-(x-1)^2}}\right)}{2}$	42
trager	$\frac{(x+1)(-3+x)\sqrt{-x^2+2x}}{6}+\frac{\ln\left(\frac{\sqrt{-x^2+2x}+1}{x-1}\right)}{2}$	43
pseudoelliptic	$-\frac{\ln\left(\sqrt{-x(x-2)}-1\right)}{4}+\frac{\ln\left(\sqrt{-x(x-2)}+1\right)}{4}+\frac{(x^2-2x-3)\sqrt{-x(x-2)}}{6}$	46

input
```
int((-x^2+2*x)^(3/2)/(-2*x+2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(x^2-2*x-3)*x*(x-2)/(-x*(x-2))^(1/2)+1/2*arctanh(1/(1-(x-1)^2)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.14

$$\int \frac{(2x-x^2)^{3/2}}{2-2x}\,dx = \frac{1}{6}\left(x^2-2\,x-3\right)\sqrt{-x^2+2\,x} + \frac{1}{2}\log\left(\frac{x+\sqrt{-x^2+2\,x}}{x}\right) - \frac{1}{2}\log\left(-\frac{x-\sqrt{-x^2+2\,x}}{x}\right)$$

input
```
integrate((-x^2+2*x)^(3/2)/(2-2*x),x, algorithm="fricas")
```

output
```
1/6*(x^2 - 2*x - 3)*sqrt(-x^2 + 2*x) + 1/2*log((x + sqrt(-x^2 + 2*x))/x) -
 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

Sympy [F]

$$\int \frac{(2x-x^2)^{3/2}}{2-2x}\,dx = -\frac{\int \frac{2x\sqrt{-x^2+2x}}{x-1}\,dx + \int \left(-\frac{x^2\sqrt{-x^2+2x}}{x-1}\right)dx}{2}$$

input
```
integrate((-x**2+2*x)**(3/2)/(2-2*x),x)
```

output
```
-(Integral(2*x*sqrt(-x**2 + 2*x)/(x - 1), x) + Integral(-x**2*sqrt(-x**2 +
 2*x)/(x - 1), x))/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{(2x-x^2)^{3/2}}{2-2x}\,dx = -\frac{1}{6}\left(-x^2+2\,x\right)^{\frac{3}{2}} - \frac{1}{2}\sqrt{-x^2+2\,x} + \frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2\,x}}{|x-1|} + \frac{2}{|x-1|}\right)$$

input
```
integrate((-x^2+2*x)^(3/2)/(2-2*x),x, algorithm="maxima")
```

output
```
-1/6*(-x^2 + 2*x)^(3/2) - 1/2*sqrt(-x^2 + 2*x) + 1/2*log(2*sqrt(-x^2 + 2*x
)/abs(x - 1) + 2/abs(x - 1))
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

$$\int \frac{(2x-x^2)^{3/2}}{2-2x}\,dx = \frac{1}{6}((x-2)x-3)\sqrt{-x^2+2\,x} - \frac{1}{2}\log\left(-\frac{2\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right)$$

input
```
integrate((-x^2+2*x)^(3/2)/(2-2*x),x, algorithm="giac")
```

output
```
1/6*((x - 2)*x - 3)*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/a
bs(-2*x + 2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2x-x^2)^{3/2}}{2-2x}\,dx = -\int \frac{(2\,x-x^2)^{3/2}}{2\,x-2}\,dx$$

input
```
int(-(2*x - x^2)^(3/2)/(2*x - 2),x)
```

output
```
-int((2*x - x^2)^(3/2)/(2*x - 2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int \frac{(2x-x^2)^{3/2}}{2-2x}\,dx = -atan\left(\sqrt{-x+2}+\sqrt{x}\,i-1\right)i+atan\left(\sqrt{-x+2}+\sqrt{x}\,i+1\right)i + \frac{\sqrt{x}\,\sqrt{-x+2}\,x^2}{6}-\frac{\sqrt{x}\,\sqrt{-x+2}\,x}{3}-\frac{\sqrt{x}\,\sqrt{-x+2}}{2}$$

input
```
int((-x^2+2*x)^(3/2)/(2-2*x),x)
```

output
```
( - 6*atan(sqrt( - x + 2) + sqrt(x)*i - 1)*i + 6*atan(sqrt( - x + 2) + sqr
t(x)*i + 1)*i + sqrt(x)*sqrt( - x + 2)*x**2 - 2*sqrt(x)*sqrt( - x + 2)*x -
 3*sqrt(x)*sqrt( - x + 2))/6
```

3.5 $\int \frac{\sqrt{2x-x^2}}{2-2x} \, dx$

Optimal result 142
Mathematica [A] (verified) 142
Rubi [A] (verified) 143
Maple [A] (verified) 144
Fricas [B] (verification not implemented) 145
Sympy [F] 145
Maxima [A] (verification not implemented) 146
Giac [A] (verification not implemented) 146
Mupad [F(-1)] 146
Reduce [B] (verification not implemented) 147

Optimal result

Integrand size = 21, antiderivative size = 39

$$\int \frac{\sqrt{2x-x^2}}{2-2x} \, dx = -\frac{1}{2}\sqrt{2-x}\sqrt{x} + \frac{1}{2}\operatorname{arctanh}\left(\sqrt{2-x}\sqrt{x}\right)$$

output
```
-1/2*(2-x)^(1/2)*x^(1/2)+1/2*arctanh((2-x)^(1/2)*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{\sqrt{2x-x^2}}{2-2x} \, dx = \frac{1}{2}\sqrt{-((-2+x)x)}\left(-1 + \frac{2\arctan\left(1+\sqrt{-2+x}\sqrt{x}-x\right)}{\sqrt{-2+x}\sqrt{x}}\right)$$

input
```
Integrate[Sqrt[2*x - x^2]/(2 - 2*x),x]
```

output
```
(Sqrt[-((-2 + x)*x)]*(-1 + (2*ArcTan[1 + Sqrt[-2 + x]*Sqrt[x] - x])/(Sqrt[
-2 + x]*Sqrt[x])))/2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1109, 27, 1112, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{2x-x^2}}{2-2x}\,dx$$

$$\Big\downarrow \text{1109}$$

$$\int \frac{1}{2(1-x)\sqrt{2x-x^2}}dx-\frac{1}{2}\sqrt{2x-x^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}dx-\frac{1}{2}\sqrt{2x-x^2}$$

$$\Big\downarrow \text{1112}$$

$$-2\int \frac{1}{4\left(2x-x^2\right)-4}d\sqrt{2x-x^2}-\frac{1}{2}\sqrt{2x-x^2}$$

$$\Big\downarrow \text{220}$$

$$\frac{1}{2}\text{arctanh}\left(\sqrt{2x-x^2}\right)-\frac{1}{2}\sqrt{2x-x^2}$$

input
```
Int[Sqrt[2*x - x^2]/(2 - 2*x),x]
```

output
```
-1/2*Sqrt[2*x - x^2] + ArcTanh[Sqrt[2*x - x^2]]/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1109
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[d*p*((b^2 - 4*a*c)/(b*e*(m + 2*p + 1)))   Int[(d + e*x)^m*(a + b*x
 + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[2*c*d - b*
e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m, -1] &&  !(IGtQ[(m - 1
)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && RationalQ[m] && IntegerQ[2*p]
```

rule 1112
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Simp[4*c   Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a
 + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{\sqrt{1-(x-1)^{2}}}{2}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-(x-1)^{2}}}\right)}{2}$	29
risch	$\frac{x(x-2)}{2\sqrt{-x(x-2)}}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-(x-1)^{2}}}\right)}{2}$	30
trager	$-\frac{\sqrt{-x^{2}+2x}}{2}-\frac{\ln\left(\frac{\sqrt{-x^{2}+2x}-1}{x-1}\right)}{2}$	37
pseudoelliptic	$-\frac{\sqrt{-x(x-2)}}{2}-\frac{\ln\left(\sqrt{-x(x-2)}-1\right)}{4}+\frac{\ln\left(\sqrt{-x(x-2)}+1\right)}{4}$	38

input
```
int((-x^2+2*x)^(1/2)/(-2*x+2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(1-(x-1)^2)^(1/2)+1/2*arctanh(1/(1-(x-1)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(27) = 54$.

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.46

$$\int \frac{\sqrt{2x-x^2}}{2-2x}\,dx = -\frac{1}{2}\sqrt{-x^2+2\,x}+\frac{1}{2}\log\left(\frac{x+\sqrt{-x^2+2\,x}}{x}\right) - \frac{1}{2}\log\left(-\frac{x-\sqrt{-x^2+2\,x}}{x}\right)$$

input
```
integrate((-x^2+2*x)^(1/2)/(2-2*x),x, algorithm="fricas")
```

output
```
-1/2*sqrt(-x^2 + 2*x) + 1/2*log((x + sqrt(-x^2 + 2*x))/x) - 1/2*log(-(x -
sqrt(-x^2 + 2*x))/x)
```

Sympy [F]

$$\int \frac{\sqrt{2x-x^2}}{2-2x}\,dx = -\frac{\int \frac{\sqrt{-x^2+2x}}{x-1}\,dx}{2}$$

input
```
integrate((-x**2+2*x)**(1/2)/(2-2*x),x)
```

output
```
-Integral(sqrt(-x**2 + 2*x)/(x - 1), x)/2
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{2x-x^2}}{2-2x}\,dx = -\frac{1}{2}\sqrt{-x^2+2\,x}+\frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2\,x}}{|x-1|}+\frac{2}{|x-1|}\right)$$

input
```
integrate((-x^2+2*x)^(1/2)/(2-2*x),x, algorithm="maxima")
```

output
```
-1/2*sqrt(-x^2 + 2*x) + 1/2*log(2*sqrt(-x^2 + 2*x)/abs(x - 1) + 2/abs(x -
1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.03

$$\int \frac{\sqrt{2x-x^2}}{2-2x}\,dx = -\frac{1}{2}\sqrt{-x^2+2\,x}-\frac{1}{2}\log\left(-\frac{2\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right)$$

input
```
integrate((-x^2+2*x)^(1/2)/(2-2*x),x, algorithm="giac")
```

output
```
-1/2*sqrt(-x^2 + 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x + 2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2x-x^2}}{2-2x}\,dx = -\int \frac{\sqrt{2\,x-x^2}}{2\,x-2}\,dx$$

input
```
int(-(2*x - x^2)^(1/2)/(2*x - 2),x)
```

output
```
-int((2*x - x^2)^(1/2)/(2*x - 2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.08

$$\int \frac{\sqrt{2x-x^2}}{2-2x}\,dx = -atan\left(\sqrt{-x+2}+\sqrt{x}\,i-1\right)i + atan\left(\sqrt{-x+2}+\sqrt{x}\,i+1\right)i - \frac{\sqrt{x}\,\sqrt{-x+2}}{2}$$

input
```
int((-x^2+2*x)^(1/2)/(2-2*x),x)
```

output
```
( - 2*atan(sqrt( - x + 2) + sqrt(x)*i - 1)*i + 2*atan(sqrt( - x + 2) + sqr
t(x)*i + 1)*i - sqrt(x)*sqrt( - x + 2))/2
```

3.6 $\int \frac{1}{(2-2x)\sqrt{2x-x^2}}\,dx$

Optimal result 148
Mathematica [B] (verified) 148
Rubi [A] (verified) 149
Maple [A] (verified) 150
Fricas [B] (verification not implemented) 150
Sympy [F] 151
Maxima [B] (verification not implemented) 151
Giac [A] (verification not implemented) 151
Mupad [F(-1)] 152
Reduce [B] (verification not implemented) 152

Optimal result

Integrand size = 21, antiderivative size = 20

$$\int \frac{1}{(2-2x)\sqrt{2x-x^2}}\,dx = \frac{1}{2}\text{arctanh}\left(\sqrt{2-x}\sqrt{x}\right)$$

output
```
1/2*arctanh((2-x)^(1/2)*x^(1/2))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 43 vs. 2(20) = 40.

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.15

$$\int \frac{1}{(2-2x)\sqrt{2x-x^2}}\,dx = -\frac{\sqrt{-2+x}\sqrt{x}\arctan\left(1+\sqrt{-2+x}\sqrt{x}-x\right)}{\sqrt{-((-2+x)x)}}$$

input
```
Integrate[1/((2 - 2*x)*Sqrt[2*x - x^2]),x]
```

output
```
-((Sqrt[-2 + x]*Sqrt[x]*ArcTan[1 + Sqrt[-2 + x]*Sqrt[x] - x])/Sqrt[-((-2 +
 x)*x)])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1112, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2-2x)\sqrt{2x-x^2}}dx$$
$$\Big\downarrow \text{1112}$$
$$-4\int \frac{1}{8(2x-x^2)-8}d\sqrt{2x-x^2}$$
$$\Big\downarrow \text{220}$$
$$\frac{1}{2}\text{arctanh}\left(\sqrt{2x-x^2}\right)$$

input
```
Int[1/((2 - 2*x)*Sqrt[2*x - x^2]),x]
```

output
```
ArcTanh[Sqrt[2*x - x^2]]/2
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1112
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Simp[4*c   Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a
+ b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.60

method	result	size
pseudoelliptic	$\frac{\operatorname{arctanh}\left(\sqrt{-x(x-2)}\right)}{2}$	12
default	$\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-(x-1)^2}}\right)}{2}$	15
trager	$\frac{\ln\left(\frac{\sqrt{-x^2+2x}+1}{x-1}\right)}{2}$	23

input
```
int(1/(-2*x+2)/(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctanh((-x*(x-2))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. 2(14) = 28.

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int \frac{1}{(2-2x)\sqrt{2x-x^2}}\,dx = \frac{1}{2}\log\left(\frac{x+\sqrt{-x^2+2\,x}}{x}\right) - \frac{1}{2}\log\left(-\frac{x-\sqrt{-x^2+2\,x}}{x}\right)$$

input
```
integrate(1/(2-2*x)/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*log((x + sqrt(-x^2 + 2*x))/x) - 1/2*log(-(x - sqrt(-x^2 + 2*x))/x)
```

Sympy [F]

$$\int \frac{1}{(2-2x)\sqrt{2x-x^2}}\,dx = -\frac{\int \frac{1}{x\sqrt{-x^2+2x}-\sqrt{-x^2+2x}}\,dx}{2}$$

input
```
integrate(1/(2-2*x)/(-x**2+2*x)**(1/2),x)
```

output
```
-Integral(1/(x*sqrt(-x**2 + 2*x) - sqrt(-x**2 + 2*x)), x)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(14) = 28.

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{(2-2x)\sqrt{2x-x^2}}\,dx = \frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2x}}{|x-1|}+\frac{2}{|x-1|}\right)$$

input
```
integrate(1/(2-2*x)/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*log(2*sqrt(-x^2 + 2*x)/abs(x - 1) + 2/abs(x - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.30

$$\int \frac{1}{(2-2x)\sqrt{2x-x^2}}\,dx = -\frac{1}{2}\log\left(-\frac{2\left(\sqrt{-x^2+2x}-1\right)}{|-2x+2|}\right)$$

input
```
integrate(1/(2-2*x)/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-2*x + 2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-2x)\sqrt{2x-x^2}}\,dx = -\int \frac{1}{(2\,x-2)\,\sqrt{2\,x-x^2}}\,dx$$

input
```
int(-1/((2*x - 2)*(2*x - x^2)^(1/2)),x)
```

output
```
-int(1/((2*x - 2)*(2*x - x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{(2-2x)\sqrt{2x-x^2}}\,dx = i\left(-atan\left(\sqrt{-x+2}+\sqrt{x}\,i-1\right)+atan\left(\sqrt{-x+2}+\sqrt{x}\,i+1\right)\right)$$

input
```
int(1/(2-2*x)/(-x^2+2*x)^(1/2),x)
```

output
```
i*( - atan(sqrt( - x + 2) + sqrt(x)*i - 1) + atan(sqrt( - x + 2) + sqrt(x)
*i + 1))
```

3.7 $\int \frac{1}{(2-2x)(2x-x^2)^{3/2}} \, dx$

Optimal result . . . 153
Mathematica [A] (verified) . . . 153
Rubi [A] (verified) . . . 154
Maple [A] (verified) . . . 155
Fricas [B] (verification not implemented) . . . 156
Sympy [F] . . . 156
Maxima [A] (verification not implemented) . . . 157
Giac [A] (verification not implemented) . . . 157
Mupad [F(-1)] . . . 157
Reduce [B] (verification not implemented) . . . 158

Optimal result

Integrand size = 21, antiderivative size = 39

$$\int \frac{1}{(2-2x)\left(2x-x^2\right)^{3/2}} \, dx = -\frac{1}{2\sqrt{2-x}\sqrt{x}} + \frac{1}{2}\text{arctanh}\left(\sqrt{2-x}\sqrt{x}\right)$$

output
```
-1/2/(2-x)^(1/2)/x^(1/2)+1/2*arctanh((2-x)^(1/2)*x^(1/2))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{(2-2x)\left(2x-x^2\right)^{3/2}} \, dx = -\frac{1+2\sqrt{-2+x}\sqrt{x}\arctan\left(1+\sqrt{-2+x}\sqrt{x}-x\right)}{2\sqrt{-((-2+x)x)}}$$

input
```
Integrate[1/((2 - 2*x)*(2*x - x^2)^(3/2)),x]
```

output
```
-1/2*(1 + 2*Sqrt[-2 + x]*Sqrt[x]*ArcTan[1 + Sqrt[-2 + x]*Sqrt[x] - x])/Sqr
t[-((-2 + x)*x)]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1111, 27, 1112, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2-2x)(2x-x^2)^{3/2}}\,dx$$
$$\Big\downarrow \text{1111}$$
$$\int \frac{1}{2(1-x)\sqrt{2x-x^2}}dx-\frac{1}{2\sqrt{2x-x^2}}$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{2}\int \frac{1}{(1-x)\sqrt{2x-x^2}}dx-\frac{1}{2\sqrt{2x-x^2}}$$
$$\Big\downarrow \text{1112}$$
$$-2\int \frac{1}{4(2x-x^2)-4}d\sqrt{2x-x^2}-\frac{1}{2\sqrt{2x-x^2}}$$
$$\Big\downarrow \text{220}$$
$$\frac{1}{2}\text{arctanh}\left(\sqrt{2x-x^2}\right)-\frac{1}{2\sqrt{2x-x^2}}$$

input
```
Int[1/((2 - 2*x)*(2*x - x^2)^(3/2)),x]
```

output
```
-1/2*1/Sqrt[2*x - x^2] + ArcTanh[Sqrt[2*x - x^2]]/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1111
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[2*c*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(e*(p + 1)*
(b^2 - 4*a*c))), x] - Simp[2*c*e*((m + 2*p + 3)/(e*(p + 1)*(b^2 - 4*a*c)))
  Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, m}, x] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

rule 1112
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Simp[4*c   Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a
+ b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.67

method	result	size
risch	$-\frac{1}{2\sqrt{-x(x-2)}}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-(x-1)^{2}}}\right)}{2}$	26
default	$-\frac{1}{2\sqrt{1-(x-1)^{2}}}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-(x-1)^{2}}}\right)}{2}$	29
trager	$\frac{\sqrt{-x^{2}+2x}}{2x(x-2)}+\frac{\ln\left(\frac{\sqrt{-x^{2}+2x}+1}{x-1}\right)}{2}$	45
pseudoelliptic	$\frac{\ln\left(\sqrt{-x(x-2)}+1\right)\sqrt{-x(x-2)}-\ln\left(\sqrt{-x(x-2)}-1\right)\sqrt{-x(x-2)}-2}{4\sqrt{-x(x-2)}}$	54

input
```
int(1/(-2*x+2)/(-x^2+2*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-x*(x-2))^(1/2)+1/2*arctanh(1/(1-(x-1)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(27) = 54$.

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.03

$$\int \frac{1}{(2-2x)\left(2x-x^2\right)^{3/2}}\,dx = \frac{(x^2-2\,x)\log\left(\frac{x+\sqrt{-x^2+2\,x}}{x}\right)-(x^2-2\,x)\log\left(-\frac{x-\sqrt{-x^2+2\,x}}{x}\right)+\sqrt{-x^2+2\,x}}{2\,(x^2-2\,x)}$$

input
```
integrate(1/(2-2*x)/(-x^2+2*x)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((x^2 - 2*x)*log((x + sqrt(-x^2 + 2*x))/x) - (x^2 - 2*x)*log(-(x - sqr
t(-x^2 + 2*x))/x) + sqrt(-x^2 + 2*x))/(x^2 - 2*x)
```

Sympy [F]

$$\int \frac{1}{(2-2x)\left(2x-x^2\right)^{3/2}}\,dx = -\frac{\int \frac{1}{-x^3\sqrt{-x^2+2x}+3x^2\sqrt{-x^2+2x}-2x\sqrt{-x^2+2x}}\,dx}{2}$$

input
```
integrate(1/(2-2*x)/(-x**2+2*x)**(3/2),x)
```

output
```
-Integral(1/(-x**3*sqrt(-x**2 + 2*x) + 3*x**2*sqrt(-x**2 + 2*x) - 2*x*sqrt
(-x**2 + 2*x)), x)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int \frac{1}{(2-2x)\left(2x-x^2\right)^{3/2}}\,dx = -\frac{1}{2\sqrt{-x^2+2\,x}} + \frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2\,x}}{|x-1|} + \frac{2}{|x-1|}\right)$$

input
```
integrate(1/(2-2*x)/(-x^2+2*x)^(3/2),x, algorithm="maxima")
```

output
```
-1/2/sqrt(-x^2 + 2*x) + 1/2*log(2*sqrt(-x^2 + 2*x)/abs(x - 1) + 2/abs(x -
1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{1}{(2-2x)\left(2x-x^2\right)^{3/2}}\,dx = \frac{\sqrt{-x^2+2\,x}}{2\left(x^2-2\,x\right)} - \frac{1}{2}\log\left(-\frac{2\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right)$$

input
```
integrate(1/(2-2*x)/(-x^2+2*x)^(3/2),x, algorithm="giac")
```

output
```
1/2*sqrt(-x^2 + 2*x)/(x^2 - 2*x) - 1/2*log(-2*(sqrt(-x^2 + 2*x) - 1)/abs(-
2*x + 2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-2x)\left(2x-x^2\right)^{3/2}}\,dx = -\int \frac{1}{(2\,x-2)\,\left(2\,x-x^2\right)^{3/2}}\,dx$$

input
```
int(-1/((2*x - 2)*(2*x - x^2)^(3/2)),x)
```

output
```
-int(1/((2*x - 2)*(2*x - x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.64

$$\int \frac{1}{(2-2x)\left(2x-x^2\right)^{3/2}}\,dx = \frac{-2\sqrt{-x+2}\,atan\left(\sqrt{-x+2}+\sqrt{x}\,i-1\right)ix+2\sqrt{-x+2}\,atan\left(\sqrt{-x+2}+\sqrt{}\right.}{2\sqrt{-x+2}\,x}$$

input

```
int(1/(2-2*x)/(-x^2+2*x)^(3/2),x)
```

output

```
( - 2*sqrt( - x + 2)*atan(sqrt( - x + 2) + sqrt(x)*i - 1)*i*x + 2*sqrt( -
x + 2)*atan(sqrt( - x + 2) + sqrt(x)*i + 1)*i*x - sqrt(x))/(2*sqrt( - x +
2)*x)
```

3.8 $\int \frac{1}{(2-2x)(2x-x^2)^{5/2}}\,dx$

Optimal result 159
Mathematica [A] (verified) 159
Rubi [A] (verified) 160
Maple [A] (verified) 161
Fricas [B] (verification not implemented) 162
Sympy [F] 162
Maxima [A] (verification not implemented) 163
Giac [A] (verification not implemented) 163
Mupad [F(-1)] 164
Reduce [B] (verification not implemented) 164

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \frac{1}{(2-2x)\left(2x-x^2\right)^{5/2}}\,dx=-\frac{1}{6(2-x)^{3/2}x^{3/2}}-\frac{1}{2\sqrt{2-x}\sqrt{x}}+\frac{1}{2}\text{arctanh}\left(\sqrt{2-x}\sqrt{x}\right)$$

output
```
-1/6/(2-x)^(3/2)/x^(3/2)-1/2/(2-x)^(1/2)/x^(1/2)+1/2*arctanh((2-x)^(1/2)*x
^(1/2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2-2x)\left(2x-x^2\right)^{5/2}}\,dx=\frac{-1-6x+3x^2+6(-2+x)^{3/2}x^{3/2}\arctan\left(1+\sqrt{-2+x}\sqrt{x}-x\right)}{6(-((-2+x)x))^{3/2}}$$

input
```
Integrate[1/((2 - 2*x)*(2*x - x^2)^(5/2)),x]
```

output
```
(-1 - 6*x + 3*x^2 + 6*(-2 + x)^(3/2)*x^(3/2)*ArcTan[1 + Sqrt[-2 + x]*Sqrt[
x] - x])/(6*(-((-2 + x)*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1111, 27, 1111, 1112, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(2-2x)(2x-x^2)^{5/2}}dx$$
$$\Big\downarrow \text{1111}$$
$$\int \frac{1}{2(1-x)(2x-x^2)^{3/2}}dx-\frac{1}{6(2x-x^2)^{3/2}}$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{2}\int \frac{1}{(1-x)(2x-x^2)^{3/2}}dx-\frac{1}{6(2x-x^2)^{3/2}}$$
$$\Big\downarrow \text{1111}$$
$$\frac{1}{2}\left(\int \frac{1}{(1-x)\sqrt{2x-x^2}}dx-\frac{1}{\sqrt{2x-x^2}}\right)-\frac{1}{6(2x-x^2)^{3/2}}$$
$$\Big\downarrow \text{1112}$$
$$\frac{1}{2}\left(-4\int \frac{1}{4(2x-x^2)-4}d\sqrt{2x-x^2}-\frac{1}{\sqrt{2x-x^2}}\right)-\frac{1}{6(2x-x^2)^{3/2}}$$
$$\Big\downarrow \text{220}$$
$$\frac{1}{2}\left(\text{arctanh}\left(\sqrt{2x-x^2}\right)-\frac{1}{\sqrt{2x-x^2}}\right)-\frac{1}{6(2x-x^2)^{3/2}}$$

input
```
Int[1/((2 - 2*x)*(2*x - x^2)^(5/2)),x]
```

output
```
-1/6*1/(2*x - x^2)^(3/2) + (-(1/Sqrt[2*x - x^2]) + ArcTanh[Sqrt[2*x - x^2]
])/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1111
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[2*c*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/(e*(p + 1)*
(b^2 - 4*a*c))), x] - Simp[2*c*e*((m + 2*p + 3)/(e*(p + 1)*(b^2 - 4*a*c)))
  Int[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, m}, x] && EqQ[2*c*d - b*e, 0] && NeQ[m + 2*p + 3, 0] && LtQ[p, -1] &&  !G
tQ[m, 1] && RationalQ[m] && IntegerQ[2*p]
```

rule 1112
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Simp[4*c   Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a
+ b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

method	result	size
default	$-\frac{1}{6\left(1-(x-1)^{2}\right)^{\frac{3}{2}}}-\frac{1}{2\sqrt{1-(x-1)^{2}}}+\frac{\operatorname{arctanh}\left(\frac{1}{\sqrt{1-(x-1)^{2}}}\right)}{2}$	42
trager	$\frac{\left(3x^{2}-6x-1\right)\sqrt{-x^{2}+2x}}{6\left(x-2\right)^{2}x^{2}}+\frac{\ln\left(\frac{\sqrt{-x^{2}+2x}+1}{x-1}\right)}{2}$	55
pseudoelliptic	$-\frac{x\sqrt{-x\left(x-2\right)}\,\left(x-2\right)\ln\left(\sqrt{-x\left(x-2\right)}-1\right)-x\sqrt{-x\left(x-2\right)}\,\left(x-2\right)\ln\left(\sqrt{-x\left(x-2\right)}+1\right)+2x^{2}-4x-\frac{2}{3}}{4\sqrt{-x\left(x-2\right)}\,x\left(x-2\right)}$	78

input
```
int(1/(-2*x+2)/(-x^2+2*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(1-(x-1)^2)^(3/2)-1/2/(1-(x-1)^2)^(1/2)+1/2*arctanh(1/(1-(x-1)^2)^(1/
2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(39) = 78$.

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.96

$$\int \frac{1}{(2-2x)\left(2x-x^2\right)^{5/2}}\,dx = \frac{3\left(x^4-4\,x^3+4\,x^2\right)\log\left(\frac{x+\sqrt{-x^2+2\,x}}{x}\right)-3\left(x^4-4\,x^3+4\,x^2\right)\log\left(-\frac{x-\sqrt{-x^2+2}}{x}\right.}{6\left(x^4-4\,x^3+4\,x^2\right)}$$

input
```
integrate(1/(2-2*x)/(-x^2+2*x)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(3*(x^4 - 4*x^3 + 4*x^2)*log((x + sqrt(-x^2 + 2*x))/x) - 3*(x^4 - 4*x^
3 + 4*x^2)*log(-(x - sqrt(-x^2 + 2*x))/x) + (3*x^2 - 6*x - 1)*sqrt(-x^2 +
2*x))/(x^4 - 4*x^3 + 4*x^2)
```

Sympy [F]

$$\int \frac{1}{(2-2x)\left(2x-x^2\right)^{5/2}}\,dx = -\frac{\int \frac{1}{x^5\sqrt{-x^2+2x}-5x^4\sqrt{-x^2+2x}+8x^3\sqrt{-x^2+2x}-4x^2\sqrt{-x^2+2x}}\,dx}{2}$$

input
```
integrate(1/(2-2*x)/(-x**2+2*x)**(5/2),x)
```

output
```
-Integral(1/(x**5*sqrt(-x**2 + 2*x) - 5*x**4*sqrt(-x**2 + 2*x) + 8*x**3*sq
rt(-x**2 + 2*x) - 4*x**2*sqrt(-x**2 + 2*x)), x)/2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.02

$$\int \frac{1}{(2-2x)\,(2x-x^2)^{5/2}}\,dx = -\frac{1}{2\sqrt{-x^2+2\,x}} - \frac{1}{6\,(-x^2+2\,x)^{\frac{3}{2}}} + \frac{1}{2}\log\left(\frac{2\sqrt{-x^2+2\,x}}{|x-1|} + \frac{2}{|x-1|}\right)$$

input
```
integrate(1/(2-2*x)/(-x^2+2*x)^(5/2),x, algorithm="maxima")
```

output
```
-1/2/sqrt(-x^2 + 2*x) - 1/6/(-x^2 + 2*x)^(3/2) + 1/2*log(2*sqrt(-x^2 + 2*x
)/abs(x - 1) + 2/abs(x - 1))
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int \frac{1}{(2-2x)\,(2x-x^2)^{5/2}}\,dx = \frac{(3\,(x-2)x-1)\sqrt{-x^2+2\,x}}{6\,(x^2-2\,x)^2} - \frac{1}{2}\log\left(-\frac{2\left(\sqrt{-x^2+2\,x}-1\right)}{|-2\,x+2|}\right)$$

input
```
integrate(1/(2-2*x)/(-x^2+2*x)^(5/2),x, algorithm="giac")
```

output
```
1/6*(3*(x - 2)*x - 1)*sqrt(-x^2 + 2*x)/(x^2 - 2*x)^2 - 1/2*log(-2*(sqrt(-x
^2 + 2*x) - 1)/abs(-2*x + 2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(2-2x)(2x-x^2)^{5/2}}\,dx = -\int \frac{1}{(2\,x-2)\,(2\,x-x^2)^{5/2}}\,dx$$

input
```
int(-1/((2*x - 2)*(2*x - x^2)^(5/2)),x)
```

output
```
-int(1/((2*x - 2)*(2*x - x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.33

$$\int \frac{1}{(2-2x)(2x-x^2)^{5/2}}\,dx = \frac{-6\sqrt{-x+2}\,atan\left(\sqrt{-x+2}+\sqrt{x}\,i-1\right)i\,x^3+12\sqrt{-x+2}\,atan\left(\sqrt{-x+2}\right.}{}$$

input
```
int(1/(2-2*x)/(-x^2+2*x)^(5/2),x)
```

output
```
( - 6*sqrt( - x + 2)*atan(sqrt( - x + 2) + sqrt(x)*i - 1)*i*x**3 + 12*sqrt
( - x + 2)*atan(sqrt( - x + 2) + sqrt(x)*i - 1)*i*x**2 + 6*sqrt( - x + 2)*
atan(sqrt( - x + 2) + sqrt(x)*i + 1)*i*x**3 - 12*sqrt( - x + 2)*atan(sqrt(
 - x + 2) + sqrt(x)*i + 1)*i*x**2 - 3*sqrt(x)*x**2 + 6*sqrt(x)*x + sqrt(x)
)/(6*sqrt( - x + 2)*x**2*(x - 2))
```

3.9 $\int \frac{\sqrt{2x+x^2}}{1+x}\,dx$

Optimal result 165
Mathematica [A] (verified) 165
Rubi [A] (verified) 166
Maple [A] (verified) 167
Fricas [A] (verification not implemented) 168
Sympy [F] 168
Maxima [A] (verification not implemented) 168
Giac [A] (verification not implemented) 169
Mupad [F(-1)] 169
Reduce [B] (verification not implemented) 169

Optimal result

Integrand size = 17, antiderivative size = 26

$$\int \frac{\sqrt{2x+x^2}}{1+x}\,dx = \sqrt{2x+x^2} - \arctan\left(\sqrt{2x+x^2}\right)$$

output
```
(x^2+2*x)^(1/2)-arctan((x^2+2*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.69

$$\int \frac{\sqrt{2x+x^2}}{1+x}\,dx = \sqrt{x(2+x)}\left(1 + \frac{2\arctan\left(1+x-\sqrt{x}\sqrt{2+x}\right)}{\sqrt{x}\sqrt{2+x}}\right)$$

input
```
Integrate[Sqrt[2*x + x^2]/(1 + x),x]
```

output
```
Sqrt[x*(2 + x)]*(1 + (2*ArcTan[1 + x - Sqrt[x]*Sqrt[2 + x]])/(Sqrt[x]*Sqrt
[2 + x]))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1109, 1112, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x^2+2x}}{x+1} \, dx$$
$$\Big\downarrow \text{ 1109}$$
$$\sqrt{x^2+2x}-\int \frac{1}{(x+1)\sqrt{x^2+2x}}dx$$
$$\Big\downarrow \text{ 1112}$$
$$\sqrt{x^2+2x}-4\int \frac{1}{4\left(x^2+2x\right)+4}d\sqrt{x^2+2x}$$
$$\Big\downarrow \text{ 216}$$
$$\sqrt{x^2+2x}-\arctan\left(\sqrt{x^2+2x}\right)$$

input
```
Int[Sqrt[2*x + x^2]/(1 + x),x]
```

output
```
Sqrt[2*x + x^2] - ArcTan[Sqrt[2*x + x^2]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 1109
```
Int[((d_) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[d*p*((b^2 - 4*a*c)/(b*e*(m + 2*p + 1)))   Int[(d + e*x)^m*(a + b*x
 + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && EqQ[2*c*d - b*
e, 0] && NeQ[m + 2*p + 3, 0] && GtQ[p, 0] &&  !LtQ[m, -1] &&  !(IGtQ[(m - 1
)/2, 0] && ( !IntegerQ[p] || LtQ[m, 2*p])) && RationalQ[m] && IntegerQ[2*p]
```

rule 1112
```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symb
ol] :> Simp[4*c   Subst[Int[1/(b^2*e - 4*a*c*e + 4*c*e*x^2), x], x, Sqrt[a
+ b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[2*c*d - b*e, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.73

method	result	size
pseudoelliptic	$\sqrt{x\left(2+x\right)}-\arctan\left(\sqrt{x\left(2+x\right)}\right)$	19
default	$\sqrt{\left(x+1\right)^{2}-1}+\arctan\left(\frac{1}{\sqrt{\left(x+1\right)^{2}-1}}\right)$	21
risch	$\frac{x\left(2+x\right)}{\sqrt{x\left(2+x\right)}}+\arctan\left(\frac{1}{\sqrt{\left(x+1\right)^{2}-1}}\right)$	24
trager	$\sqrt{x^{2}+2x}+\operatorname{RootOf}\left(\_Z^{2}+1\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^{2}+1\right)-\sqrt{x^{2}+2x}}{x+1}\right)$	44

input
```
int((x^2+2*x)^(1/2)/(x+1),x,method=_RETURNVERBOSE)
```

output
```
(x*(2+x))^(1/2)-arctan((x*(2+x))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{2x+x^2}}{1+x}\,dx = \sqrt{x^2+2\,x} - 2\,\arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

input
```
integrate((x^2+2*x)^(1/2)/(1+x),x, algorithm="fricas")
```

output
```
sqrt(x^2 + 2*x) - 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Sympy [F]

$$\int \frac{\sqrt{2x+x^2}}{1+x}\,dx = \int \frac{\sqrt{x\,(x+2)}}{x+1}\,dx$$

input
```
integrate((x**2+2*x)**(1/2)/(1+x),x)
```

output
```
Integral(sqrt(x*(x + 2))/(x + 1), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{2x+x^2}}{1+x}\,dx = \sqrt{x^2+2\,x} + \arcsin\left(\frac{1}{|x+1|}\right)$$

input
```
integrate((x^2+2*x)^(1/2)/(1+x),x, algorithm="maxima")
```

output
```
sqrt(x^2 + 2*x) + arcsin(1/abs(x + 1))
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{2x+x^2}}{1+x}\,dx = \sqrt{x^2+2\,x} - 2\,\arctan\left(-x+\sqrt{x^2+2\,x}-1\right)$$

input
```
integrate((x^2+2*x)^(1/2)/(1+x),x, algorithm="giac")
```

output
```
sqrt(x^2 + 2*x) - 2*arctan(-x + sqrt(x^2 + 2*x) - 1)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{2x+x^2}}{1+x}\,dx = \int \frac{\sqrt{x^2+2\,x}}{x+1}\,dx$$

input
```
int((2*x + x^2)^(1/2)/(x + 1),x)
```

output
```
int((2*x + x^2)^(1/2)/(x + 1), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{2x+x^2}}{1+x}\,dx = -2atan\left(\sqrt{x+2}+\sqrt{x}-1\right)+2atan\left(\sqrt{x+2}+\sqrt{x}+1\right)+\sqrt{x}\,\sqrt{x+2}$$

input
```
int((x^2+2*x)^(1/2)/(1+x),x)
```

output
```
- 2*atan(sqrt(x + 2) + sqrt(x) - 1) + 2*atan(sqrt(x + 2) + sqrt(x) + 1) +
sqrt(x)*sqrt(x + 2)
```

3.10 $\int \frac{1}{(2+x)\sqrt{2x+x^2}} dx$

Optimal result 170
Mathematica [A] (verified) 170
Rubi [A] (verified) 171
Maple [A] (verified) 171
Fricas [A] (verification not implemented) 172
Sympy [F] 173
Maxima [A] (verification not implemented) 173
Giac [A] (verification not implemented) 173
Mupad [B] (verification not implemented) 174
Reduce [B] (verification not implemented) 174

Optimal result

Integrand size = 17, antiderivative size = 13

$$\int \frac{1}{(2+x)\sqrt{2x+x^2}} dx = \frac{x}{\sqrt{2x+x^2}}$$

output
```
x/(x^2+2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{1}{(2+x)\sqrt{2x+x^2}} dx = \frac{x}{\sqrt{x(2+x)}}$$

input
```
Integrate[1/((2 + x)*Sqrt[2*x + x^2]),x]
```

output
```
x/Sqrt[x*(2 + x)]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(x+2)\sqrt{x^2+2x}}dx$$

$$\Big\downarrow \ 1123$$

$$\frac{\sqrt{x^2+2x}}{x+2}$$

input
```
Int[1/((2 + x)*Sqrt[2*x + x^2]),x]
```

output
```
Sqrt[2*x + x^2]/(2 + x)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{x}{\sqrt{x(2+x)}}$	10
pseudoelliptic	$\frac{x}{\sqrt{x(2+x)}}$	10
gosper	$\frac{x}{\sqrt{x^2+2x}}$	12
orering	$\frac{x}{\sqrt{x^2+2x}}$	12
trager	$\frac{\sqrt{x^2+2x}}{2+x}$	16
meijerg	$\frac{\sqrt{2}\,\sqrt{x}}{2\sqrt{1+\frac{x}{2}}}$	16
default	$\frac{\sqrt{(2+x)^2-4-2x}}{2+x}$	19

input
```
int(1/(2+x)/(x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
x/(x*(2+x))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{1}{(2+x)\sqrt{2x+x^2}}\,dx = \frac{x+\sqrt{x^2+2\,x}+2}{x+2}$$

input
```
integrate(1/(2+x)/(x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
(x + sqrt(x^2 + 2*x) + 2)/(x + 2)
```

Sympy [F]

$$\int \frac{1}{(2+x)\sqrt{2x+x^2}}\,dx = \int \frac{1}{\sqrt{x\,(x+2)}\,(x+2)}\,dx$$

input
```
integrate(1/(2+x)/(x**2+2*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x*(x + 2))*(x + 2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{(2+x)\sqrt{2x+x^2}}\,dx = \frac{\sqrt{x^2+2\,x}}{x+2}$$

input
```
integrate(1/(2+x)/(x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
sqrt(x^2 + 2*x)/(x + 2)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.38

$$\int \frac{1}{(2+x)\sqrt{2x+x^2}}\,dx = \frac{2}{x-\sqrt{x^2+2\,x}+2}$$

input
```
integrate(1/(2+x)/(x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
2/(x - sqrt(x^2 + 2*x) + 2)
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{1}{(2+x)\sqrt{2x+x^2}}\,dx = \frac{\sqrt{x^2+2\,x}}{x+2}$$

input
```
int(1/((2*x + x^2)^(1/2)*(x + 2)),x)
```

output
```
(2*x + x^2)^(1/2)/(x + 2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{(2+x)\sqrt{2x+x^2}}\,dx = \frac{\sqrt{x+2}+\sqrt{x}}{\sqrt{x+2}}$$

input
```
int(1/(2+x)/(x^2+2*x)^(1/2),x)
```

output
```
(sqrt(x + 2) + sqrt(x))/sqrt(x + 2)
```

3.11 $\int (d+ex)^m \left(cdx+cex^2\right)^3 \, dx$

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Optimal result

Integrand size = 21, antiderivative size = 95

$$\int (d+ex)^m \left(cdx+cex^2\right)^3 \, dx = -\frac{c^3d^3(d+ex)^{4+m}}{e^4(4+m)} + \frac{3c^3d^2(d+ex)^{5+m}}{e^4(5+m)} - \frac{3c^3d(d+ex)^{6+m}}{e^4(6+m)} + \frac{c^3(d+ex)^{7+m}}{e^4(7+m)}$$

output
```
-c^3*d^3*(e*x+d)^(4+m)/e^4/(4+m)+3*c^3*d^2*(e*x+d)^(5+m)/e^4/(5+m)-3*c^3*d
*(e*x+d)^(6+m)/e^4/(6+m)+c^3*(e*x+d)^(7+m)/e^4/(7+m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.74

$$\int (d+ex)^m \left(cdx+cex^2\right)^3 \, dx = \frac{c^3(d+ex)^{4+m}\left(-\frac{d^3}{4+m}+\frac{3d^2(d+ex)}{5+m}-\frac{3d(d+ex)^2}{6+m}+\frac{(d+ex)^3}{7+m}\right)}{e^4}$$

input
```
Integrate[(d + e*x)^m*(c*d*x + c*e*x^2)^3,x]
```

output
```
(c^3*(d + e*x)^(4 + m)*(-(d^3/(4 + m)) + (3*d^2*(d + e*x))/(5 + m) - (3*d*
(d + e*x)^2)/(6 + m) + (d + e*x)^3/(7 + m)))/e^4
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(cdx+cex^2\right)^3 (d+ex)^m \, dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(-\frac{c^3d^3(d+ex)^{m+3}}{e^3}+\frac{3c^3d^2(d+ex)^{m+4}}{e^3}-\frac{3c^3d(d+ex)^{m+5}}{e^3}+\frac{c^3(d+ex)^{m+6}}{e^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^3d^3(d+ex)^{m+4}}{e^4(m+4)}+\frac{3c^3d^2(d+ex)^{m+5}}{e^4(m+5)}-\frac{3c^3d(d+ex)^{m+6}}{e^4(m+6)}+\frac{c^3(d+ex)^{m+7}}{e^4(m+7)}$$

input
```
Int[(d + e*x)^m*(c*d*x + c*e*x^2)^3,x]
```

output
```
-((c^3*d^3*(d + e*x)^(4 + m))/(e^4*(4 + m))) + (3*c^3*d^2*(d + e*x)^(5 + m
))/(e^4*(5 + m)) - (3*c^3*d*(d + e*x)^(6 + m))/(e^4*(6 + m)) + (c^3*(d + e
*x)^(7 + m))/(e^4*(7 + m))
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.36

method	result
gosper	$-\frac{c^3(ex+d)^{4+m}(-e^3m^3x^3-15e^3m^2x^3+3d\,e^2m^2x^2-74e^3m\,x^3+27d\,e^2m\,x^2-120e^3x^3-6d^2emx+60d\,e^2x^2-24d^2ex+6d^3)}{e^4(m^4+22m^3+179m^2+638m+840)}$
orering	$-\frac{(-e^3m^3x^3-15e^3m^2x^3+3d\,e^2m^2x^2-74e^3m\,x^3+27d\,e^2m\,x^2-120e^3x^3-6d^2emx+60d\,e^2x^2-24d^2ex+6d^3)(ex+d)(ex+d)^{}}{e^4(m^4+22m^3+179m^2+638m+840)x^3}$
risch	$-\frac{c^3(-e^7m^3x^7-4d\,e^6m^3x^6-15e^7m^2x^7-6d^2e^5m^3x^5-57d\,e^6m^2x^6-74e^7m\,x^7-4d^3e^4m^3x^4-78d^2e^5m^2x^5-269d\,e^6m\,x^6-1}{}$
norman	$\frac{c^3e^3x^7e^{m\ln(ex+d)}}{7+m}+\frac{c^3d\,e^2(4m+21)x^6e^{m\ln(ex+d)}}{m^2+13m+42}+\frac{(2+m)m(1+m)c^3d^4x^3e^{m\ln(ex+d)}}{e(m^4+22m^3+179m^2+638m+840)}-\frac{6c^3d^7e^{m\ln(ex+d)}}{e^4(m^4+22m^3+179m^2+63}$
parallelrisch	$x^3(ex+d)^m c^3d^5e^3m^3+158x^4(ex+d)^m c^3d^4e^4m+3x^3(ex+d)^m c^3d^5e^3m^2+2x^3(ex+d)^m c^3d^5e^3m-3x^2(ex+d)^m c^3d^6e^2m^2+4$

input
```
int((e*x+d)^m*(c*e*x^2+c*d*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-c^3/e^4*(e*x+d)^(4+m)/(m^4+22*m^3+179*m^2+638*m+840)*(-e^3*m^3*x^3-15*e^3
*m^2*x^3+3*d*e^2*m^2*x^2-74*e^3*m*x^3+27*d*e^2*m*x^2-120*e^3*x^3-6*d^2*e*m
*x+60*d*e^2*x^2-24*d^2*e*x+6*d^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(95) = 190$.

Time = 0.10 (sec) , antiderivative size = 343, normalized size of antiderivative = 3.61

$$\int (d+ex)^m\left(cdx+cex^2\right)^3\,dx$$
$$=\frac{(6\,c^3d^6emx-6\,c^3d^7+(c^3e^7m^3+15\,c^3e^7m^2+74\,c^3e^7m+120\,c^3e^7)x^7+(4\,c^3de^6m^3+57\,c^3de^6m^2+269}{}$$

input
```
integrate((e*x+d)^m*(c*e*x^2+c*d*x)^3,x, algorithm="fricas")
```

output
```
(6*c^3*d^6*e*m*x - 6*c^3*d^7 + (c^3*e^7*m^3 + 15*c^3*e^7*m^2 + 74*c^3*e^7*
m + 120*c^3*e^7)*x^7 + (4*c^3*d*e^6*m^3 + 57*c^3*d*e^6*m^2 + 269*c^3*d*e^6
*m + 420*c^3*d*e^6)*x^6 + 6*(c^3*d^2*e^5*m^3 + 13*c^3*d^2*e^5*m^2 + 57*c^3
*d^2*e^5*m + 84*c^3*d^2*e^5)*x^5 + 2*(2*c^3*d^3*e^4*m^3 + 21*c^3*d^3*e^4*m
^2 + 79*c^3*d^3*e^4*m + 105*c^3*d^3*e^4)*x^4 + (c^3*d^4*e^3*m^3 + 3*c^3*d^
4*e^3*m^2 + 2*c^3*d^4*e^3*m)*x^3 - 3*(c^3*d^5*e^2*m^2 + c^3*d^5*e^2*m)*x^2
)*(e*x + d)^m/(e^4*m^4 + 22*e^4*m^3 + 179*e^4*m^2 + 638*e^4*m + 840*e^4)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2218 vs. $2(85) = 170$.

Time = 1.44 (sec) , antiderivative size = 2218, normalized size of antiderivative = 23.35

$$\int (d+ex)^m \left(cdx+cex^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(c*e*x**2+c*d*x)**3,x)
```

output
```
Piecewise((c**3*d**3*d**m*x**4/4, Eq(e, 0)), (6*c**3*d**3*log(d/e + x)/(6*
d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 11*c**3*d**3/
(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 18*c**3*d*
*2*e*x*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**
7*x**3) + 27*c**3*d**2*e*x/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2
+ 6*e**7*x**3) + 18*c**3*d*e**2*x**2*log(d/e + x)/(6*d**3*e**4 + 18*d**2*e
**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 18*c**3*d*e**2*x**2/(6*d**3*e**4 +
 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3) + 6*c**3*e**3*x**3*log(d/e
 + x)/(6*d**3*e**4 + 18*d**2*e**5*x + 18*d*e**6*x**2 + 6*e**7*x**3), Eq(m,
 -7)), (-6*c**3*d**3*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2)
 - 15*c**3*d**3/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 12*c**3*d**2*e*
x*log(d/e + x)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 24*c**3*d**2*e*x
/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 6*c**3*d*e**2*x**2*log(d/e + x
)/(2*d**2*e**4 + 4*d*e**5*x + 2*e**6*x**2) - 6*c**3*d*e**2*x**2/(2*d**2*e*
*4 + 4*d*e**5*x + 2*e**6*x**2) + 2*c**3*e**3*x**3/(2*d**2*e**4 + 4*d*e**5*
x + 2*e**6*x**2), Eq(m, -6)), (6*c**3*d**3*log(d/e + x)/(2*d*e**4 + 2*e**5
*x) + 12*c**3*d**3/(2*d*e**4 + 2*e**5*x) + 6*c**3*d**2*e*x*log(d/e + x)/(2
*d*e**4 + 2*e**5*x) + 6*c**3*d**2*e*x/(2*d*e**4 + 2*e**5*x) - 3*c**3*d*e**
2*x**2/(2*d*e**4 + 2*e**5*x) + c**3*e**3*x**3/(2*d*e**4 + 2*e**5*x), Eq(m,
 -5)), (-c**3*d**3*log(d/e + x)/e**4 + c**3*d**2*x/e**3 - c**3*d*x**2/(...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 674 vs. $2(95) = 190$.

Time = 0.05 (sec) , antiderivative size = 674, normalized size of antiderivative = 7.09

$$\int (d+ex)^m \left(cdx+cex^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(c*e*x^2+c*d*x)^3,x, algorithm="maxima")
```

output
```
((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2
 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*c^3*d^3/((m^4 + 10*m^
3 + 35*m^2 + 50*m + 24)*e^4) + 3*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*
x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e
^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*c^3
*d^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^4) + 3*((m^5 + 15*
m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 5
0*m^2 + 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*
(m^3 + 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x
 - 120*d^6)*(e*x + d)^m*c^3*d/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^
2 + 1764*m + 720)*e^4) + ((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1
764*m + 720)*e^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)
*d*e^6*x^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m
^4 + 6*m^3 + 11*m^2 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x
^3 + 360*(m^2 + m)*d^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m*c^3/
((m^7 + 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 504
0)*e^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(95) = 190$.

Time = 0.19 (sec) , antiderivative size = 530, normalized size of antiderivative = 5.58

$$\int (d+ex)^m \left(cdx+cex^2\right)^3 dx$$
$$= \frac{(ex+d)^m c^3 e^7 m^3 x^7 + 4\,(ex+d)^m c^3 d e^6 m^3 x^6 + 15\,(ex+d)^m c^3 e^7 m^2 x^7 + 6\,(ex+d)^m c^3 d^2 e^5 m^3 x^5 + 57\,(e}{}$$

input
```
integrate((e*x+d)^m*(c*e*x^2+c*d*x)^3,x, algorithm="giac")
```

output
```
((e*x + d)^m*c^3*e^7*m^3*x^7 + 4*(e*x + d)^m*c^3*d*e^6*m^3*x^6 + 15*(e*x +
 d)^m*c^3*e^7*m^2*x^7 + 6*(e*x + d)^m*c^3*d^2*e^5*m^3*x^5 + 57*(e*x + d)^m
*c^3*d*e^6*m^2*x^6 + 74*(e*x + d)^m*c^3*e^7*m*x^7 + 4*(e*x + d)^m*c^3*d^3*
e^4*m^3*x^4 + 78*(e*x + d)^m*c^3*d^2*e^5*m^2*x^5 + 269*(e*x + d)^m*c^3*d*e
^6*m*x^6 + 120*(e*x + d)^m*c^3*e^7*x^7 + (e*x + d)^m*c^3*d^4*e^3*m^3*x^3 +
 42*(e*x + d)^m*c^3*d^3*e^4*m^2*x^4 + 342*(e*x + d)^m*c^3*d^2*e^5*m*x^5 +
420*(e*x + d)^m*c^3*d*e^6*x^6 + 3*(e*x + d)^m*c^3*d^4*e^3*m^2*x^3 + 158*(e
*x + d)^m*c^3*d^3*e^4*m*x^4 + 504*(e*x + d)^m*c^3*d^2*e^5*x^5 - 3*(e*x + d
)^m*c^3*d^5*e^2*m^2*x^2 + 2*(e*x + d)^m*c^3*d^4*e^3*m*x^3 + 210*(e*x + d)^
m*c^3*d^3*e^4*x^4 - 3*(e*x + d)^m*c^3*d^5*e^2*m*x^2 + 6*(e*x + d)^m*c^3*d^
6*e*m*x - 6*(e*x + d)^m*c^3*d^7)/(e^4*m^4 + 22*e^4*m^3 + 179*e^4*m^2 + 638
*e^4*m + 840*e^4)
```

Mupad [B] (verification not implemented)

Time = 9.47 (sec) , antiderivative size = 333, normalized size of antiderivative = 3.51

$$\int (d+ex)^m \left(cdx+cex^2\right)^3 \, dx = (d+e\,x)^m \left(\frac{c^3\,e^3\,x^7\,(m^3+15\,m^2+74\,m+120)}{m^4+22\,m^3+179\,m^2+638\,m+840} - \frac{6\,c^3\,d^7}{e^4\,(m^4+22\,m^3+179\,m^2+638\,m+840)} + \frac{2\,c^3\,d^3\,x^4\,(2\,m^3+21\,m^2+79\,m+105)}{m^4+22\,m^3+179\,m^2+638\,m+840} + \frac{6\,c^3\,d^6\,m\,x}{e^3\,(m^4+22\,m^3+179\,m^2+638\,m+840)} + \frac{c^3\,d\,e^2\,x^6\,(4\,m^3+57\,m^2+269\,m+420)}{m^4+22\,m^3+179\,m^2+638\,m+840} + \frac{6\,c^3\,d^2\,e\,x^5\,(m^3+13\,m^2+57\,m+84)}{m^4+22\,m^3+179\,m^2+638\,m+840} - \frac{3\,c^3\,d^5\,m\,x^2\,(m+1)}{e^2\,(m^4+22\,m^3+179\,m^2+638\,m+840)} + \frac{c^3\,d^4\,m\,x^3\,(m^2+3\,m+2)}{e\,(m^4+22\,m^3+179\,m^2+638\,m+840)}\right)$$

input
```
int((c*d*x + c*e*x^2)^3*(d + e*x)^m,x)
```

output
```
(d + e*x)^m*((c^3*e^3*x^7*(74*m + 15*m^2 + m^3 + 120))/(638*m + 179*m^2 +
22*m^3 + m^4 + 840) - (6*c^3*d^7)/(e^4*(638*m + 179*m^2 + 22*m^3 + m^4 + 8
40)) + (2*c^3*d^3*x^4*(79*m + 21*m^2 + 2*m^3 + 105))/(638*m + 179*m^2 + 22
*m^3 + m^4 + 840) + (6*c^3*d^6*m*x)/(e^3*(638*m + 179*m^2 + 22*m^3 + m^4 +
 840)) + (c^3*d*e^2*x^6*(269*m + 57*m^2 + 4*m^3 + 420))/(638*m + 179*m^2 +
 22*m^3 + m^4 + 840) + (6*c^3*d^2*e*x^5*(57*m + 13*m^2 + m^3 + 84))/(638*m
 + 179*m^2 + 22*m^3 + m^4 + 840) - (3*c^3*d^5*m*x^2*(m + 1))/(e^2*(638*m +
 179*m^2 + 22*m^3 + m^4 + 840)) + (c^3*d^4*m*x^3*(3*m + m^2 + 2))/(e*(638*
m + 179*m^2 + 22*m^3 + m^4 + 840)))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 296, normalized size of antiderivative = 3.12

$$\int (d+ex)^m \left(cdx+cex^2\right)^3 \, dx$$
$$= \frac{(ex+d)^m c^3(e^7m^3x^7 + 4d\,e^6m^3x^6 + 15e^7m^2x^7 + 6d^2e^5m^3x^5 + 57d\,e^6m^2x^6 + 74e^7m\,x^7 + 4d^3e^4m^3x^4 + 7}{}$$

input
```
int((e*x+d)^m*(c*e*x^2+c*d*x)^3,x)
```

output
```
((d + e*x)**m*c**3*( - 6*d**7 + 6*d**6*e*m*x - 3*d**5*e**2*m**2*x**2 - 3*d
**5*e**2*m*x**2 + d**4*e**3*m**3*x**3 + 3*d**4*e**3*m**2*x**3 + 2*d**4*e**
3*m*x**3 + 4*d**3*e**4*m**3*x**4 + 42*d**3*e**4*m**2*x**4 + 158*d**3*e**4*
m*x**4 + 210*d**3*e**4*x**4 + 6*d**2*e**5*m**3*x**5 + 78*d**2*e**5*m**2*x*
*5 + 342*d**2*e**5*m*x**5 + 504*d**2*e**5*x**5 + 4*d*e**6*m**3*x**6 + 57*d
*e**6*m**2*x**6 + 269*d*e**6*m*x**6 + 420*d*e**6*x**6 + e**7*m**3*x**7 + 1
5*e**7*m**2*x**7 + 74*e**7*m*x**7 + 120*e**7*x**7))/(e**4*(m**4 + 22*m**3
+ 179*m**2 + 638*m + 840))
```

3.12 $\int (d+ex)^m \left(cdx+cex^2\right)^2 \, dx$

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Optimal result

Integrand size = 21, antiderivative size = 69

$$\int (d+ex)^m \left(cdx+cex^2\right)^2 \, dx = \frac{c^2d^2(d+ex)^{3+m}}{e^3(3+m)} - \frac{2c^2d(d+ex)^{4+m}}{e^3(4+m)} + \frac{c^2(d+ex)^{5+m}}{e^3(5+m)}$$

output
```
c^2*d^2*(e*x+d)^(3+m)/e^3/(3+m)-2*c^2*d*(e*x+d)^(4+m)/e^3/(4+m)+c^2*(e*x+d
)^(5+m)/e^3/(5+m)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.87

$$\int (d+ex)^m \left(cdx+cex^2\right)^2 \, dx = \frac{c^2(d+ex)^{3+m}\left(2d^2-2de(3+m)x+e^2(12+7m+m^2)\,x^2\right)}{e^3(3+m)(4+m)(5+m)}$$

input
```
Integrate[(d + e*x)^m*(c*d*x + c*e*x^2)^2,x]
```

output
```
(c^2*(d + e*x)^(3 + m)*(2*d^2 - 2*d*e*(3 + m)*x + e^2*(12 + 7*m + m^2)*x^2
))/(e^3*(3 + m)*(4 + m)*(5 + m))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(cdx+cex^2\right)^2 (d+ex)^m\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{c^2d^2(d+ex)^{m+2}}{e^2}-\frac{2c^2d(d+ex)^{m+3}}{e^2}+\frac{c^2(d+ex)^{m+4}}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c^2d^2(d+ex)^{m+3}}{e^3(m+3)}-\frac{2c^2d(d+ex)^{m+4}}{e^3(m+4)}+\frac{c^2(d+ex)^{m+5}}{e^3(m+5)}$$

input
```
Int[(d + e*x)^m*(c*d*x + c*e*x^2)^2,x]
```

output
```
(c^2*d^2*(d + e*x)^(3 + m))/(e^3*(3 + m)) - (2*c^2*d*(d + e*x)^(4 + m))/(e
^3*(4 + m)) + (c^2*(d + e*x)^(5 + m))/(e^3*(5 + m))
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

method	result
gosper	$\frac{c^2(ex+d)^{3+m}(e^2m^2x^2+7e^2m\,x^2-2demx+12e^2x^2-6dex+2d^2)}{e^3(m^3+12m^2+47m+60)}$
orering	$\frac{(e^2m^2x^2+7e^2m\,x^2-2demx+12e^2x^2-6dex+2d^2)(ex+d)(ex+d)^m(ce\,x^2+cdx)^2}{e^3(m^3+12m^2+47m+60)x^2}$
risch	$\frac{c^2(e^5m^2x^5+3d\,e^4m^2x^4+7e^5m\,x^5+3d^2e^3m^2x^3+19d\,e^4m\,x^4+12e^5x^5+d^3e^2m^2x^2+15d^2e^3m\,x^3+30d\,e^4x^4+d^3e^2m\,x^2+20d^2}{(4+m)(5+m)(3+m)e^3}$
norman	$\frac{c^2e^2x^5e^{m\ln(ex+d)}}{5+m}+\frac{c^2d^2(3m^2+15m+20)x^3e^{m\ln(ex+d)}}{m^3+12m^2+47m+60}+\frac{(3m+10)c^2de\,x^4e^{m\ln(ex+d)}}{m^2+9m+20}+\frac{m(1+m)c^2d^3x^2e^{m\ln(ex+d)}}{e(m^3+12m^2+47m+60)}$
parallelrisch	$x^5(ex+d)^m c^2d\,e^5m^2+7x^5(ex+d)^m c^2d\,e^5m+3x^4(ex+d)^m c^2d^2e^4m^2+12x^5(ex+d)^m c^2d\,e^5+19x^4(ex+d)^m c^2d^2e^4m+3x^3(ex$

input
```
int((e*x+d)^m*(c*e*x^2+c*d*x)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2/e^3*(e*x+d)^(3+m)/(m^3+12*m^2+47*m+60)*(e^2*m^2*x^2+7*e^2*m*x^2-2*d*e*
m*x+12*e^2*x^2-6*d*e*x+2*d^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. 2(69) = 138.

Time = 0.09 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.91

$$\int (d+ex)^m\left(cdx+cex^2\right)^2\,dx = -\frac{(2\,c^2d^4emx-2\,c^2d^5-(c^2e^5m^2+7\,c^2e^5m+12\,c^2e^5)x^5-(3\,c^2de^4m^2+19\,c^2de^4m+30\,c^2de^4)x^4-(3\,c}{e^3m^3+12\,e^3m^2+47\,e^3m+6}$$

input
```
integrate((e*x+d)^m*(c*e*x^2+c*d*x)^2,x, algorithm="fricas")
```

output
```
-(2*c^2*d^4*e*m*x - 2*c^2*d^5 - (c^2*e^5*m^2 + 7*c^2*e^5*m + 12*c^2*e^5)*x
^5 - (3*c^2*d*e^4*m^2 + 19*c^2*d*e^4*m + 30*c^2*d*e^4)*x^4 - (3*c^2*d^2*e^
3*m^2 + 15*c^2*d^2*e^3*m + 20*c^2*d^2*e^3)*x^3 - (c^2*d^3*e^2*m^2 + c^2*d^
3*e^2*m)*x^2)*(e*x + d)^m/(e^3*m^3 + 12*e^3*m^2 + 47*e^3*m + 60*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 993 vs. $2(61) = 122$.

Time = 0.73 (sec) , antiderivative size = 993, normalized size of antiderivative = 14.39

$$\int (d+ex)^m \left(cdx+cex^2\right)^2 dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(c*e*x**2+c*d*x)**2,x)
```

output
```
Piecewise((c**2*d**2*d**m*x**3/3, Eq(e, 0)), (2*c**2*d**2*log(d/e + x)/(2*
d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 3*c**2*d**2/(2*d**2*e**3 + 4*d*e**
4*x + 2*e**5*x**2) + 4*c**2*d*e*x*log(d/e + x)/(2*d**2*e**3 + 4*d*e**4*x +
 2*e**5*x**2) + 4*c**2*d*e*x/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 2*
c**2*e**2*x**2*log(d/e + x)/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2), Eq(m
, -5)), (-2*c**2*d**2*log(d/e + x)/(d*e**3 + e**4*x) - 4*c**2*d**2/(d*e**3
 + e**4*x) - 2*c**2*d*e*x*log(d/e + x)/(d*e**3 + e**4*x) - 2*c**2*d*e*x/(d
*e**3 + e**4*x) + c**2*e**2*x**2/(d*e**3 + e**4*x), Eq(m, -4)), (c**2*d**2
*log(d/e + x)/e**3 - c**2*d*x/e**2 + c**2*x**2/(2*e), Eq(m, -3)), (2*c**2*
d**5*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) - 2*c**
2*d**4*e*m*x*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3)
 + c**2*d**3*e**2*m**2*x**2*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e*
*3*m + 60*e**3) + c**2*d**3*e**2*m*x**2*(d + e*x)**m/(e**3*m**3 + 12*e**3*
m**2 + 47*e**3*m + 60*e**3) + 3*c**2*d**2*e**3*m**2*x**3*(d + e*x)**m/(e**
3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 15*c**2*d**2*e**3*m*x**3*(d
 + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) + 20*c**2*d**2
*e**3*x**3*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*m + 60*e**3) +
 3*c**2*d*e**4*m**2*x**4*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**2 + 47*e**3*
m + 60*e**3) + 19*c**2*d*e**4*m*x**4*(d + e*x)**m/(e**3*m**3 + 12*e**3*m**
2 + 47*e**3*m + 60*e**3) + 30*c**2*d*e**4*x**4*(d + e*x)**m/(e**3*m**3 ...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(69) = 138$.

Time = 0.04 (sec) , antiderivative size = 323, normalized size of antiderivative = 4.68

$$\int (d+ex)^m \left(cdx+cex^2\right)^2 \, dx$$
$$= \frac{((m^2+3\,m+2)e^3x^3+(m^2+m)de^2x^2-2\,d^2emx+2\,d^3)(ex+d)^m c^2d^2}{(m^3+6\,m^2+11\,m+6)e^3}$$
$$+ \frac{2\,((m^3+6\,m^2+11\,m+6)e^4x^4+(m^3+3\,m^2+2\,m)de^3x^3-3\,(m^2+m)d^2e^2x^2+6\,d^3emx-6\,d^4)(ex}{(m^4+10\,m^3+35\,m^2+50\,m+24)e^3}$$
$$+ \frac{((m^4+10\,m^3+35\,m^2+50\,m+24)e^5x^5+(m^4+6\,m^3+11\,m^2+6\,m)de^4x^4-4\,(m^3+3\,m^2+2\,m)d}{(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+12}$$

input
```
integrate((e*x+d)^m*(c*e*x^2+c*d*x)^2,x, algorithm="maxima")
```

output
```
((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x
 + d)^m*c^2*d^2/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m +
6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d
^3*e*m*x - 6*d^4)*(e*x + d)^m*c^2*d/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e
^3) + ((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2
 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e
^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*c^2/((m^5 + 15*m^4 + 85*m^3 +
225*m^2 + 274*m + 120)*e^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(69) = 138$.

Time = 0.11 (sec) , antiderivative size = 293, normalized size of antiderivative = 4.25

$$\int (d+ex)^m \left(cdx+cex^2\right)^2 \, dx$$
$$= \frac{(ex+d)^m c^2e^5m^2x^5+3\,(ex+d)^m c^2de^4m^2x^4+7\,(ex+d)^m c^2e^5mx^5+3\,(ex+d)^m c^2d^2e^3m^2x^3+19\,(ex}{}$$

input
```
integrate((e*x+d)^m*(c*e*x^2+c*d*x)^2,x, algorithm="giac")
```

output
```
((e*x + d)^m*c^2*e^5*m^2*x^5 + 3*(e*x + d)^m*c^2*d*e^4*m^2*x^4 + 7*(e*x +
d)^m*c^2*e^5*m*x^5 + 3*(e*x + d)^m*c^2*d^2*e^3*m^2*x^3 + 19*(e*x + d)^m*c^
2*d*e^4*m*x^4 + 12*(e*x + d)^m*c^2*e^5*x^5 + (e*x + d)^m*c^2*d^3*e^2*m^2*x
^2 + 15*(e*x + d)^m*c^2*d^2*e^3*m*x^3 + 30*(e*x + d)^m*c^2*d*e^4*x^4 + (e*
x + d)^m*c^2*d^3*e^2*m*x^2 + 20*(e*x + d)^m*c^2*d^2*e^3*x^3 - 2*(e*x + d)^
m*c^2*d^4*e*m*x + 2*(e*x + d)^m*c^2*d^5)/(e^3*m^3 + 12*e^3*m^2 + 47*e^3*m
+ 60*e^3)
```

Mupad [B] (verification not implemented)

Time = 8.88 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.86

$$\int (d+ex)^m \left(cdx+cex^2\right)^2 dx = (d+e\,x)^m \left(\frac{2\,c^2\,d^5}{e^3\,\left(m^3+12\,m^2+47\,m+60\right)} + \frac{c^2\,e^2\,x^5\,\left(m^2+7\,m+12\right)}{m^3+12\,m^2+47\,m+60} + \frac{c^2\,d^2\,x^3\,\left(3\,m^2+15\,m+20\right)}{m^3+12\,m^2+47\,m+60} - \frac{2\,c^2\,d^4\,m\,x}{e^2\,\left(m^3+12\,m^2+47\,m+60\right)} + \frac{c^2\,d\,e\,x^4\,\left(3\,m^2+19\,m+30\right)}{m^3+12\,m^2+47\,m+60} + \frac{c^2\,d^3\,m\,x^2\,\left(m+1\right)}{e\,\left(m^3+12\,m^2+47\,m+60\right)}\right)$$

input
```
int((c*d*x + c*e*x^2)^2*(d + e*x)^m,x)
```

output
```
(d + e*x)^m*((2*c^2*d^5)/(e^3*(47*m + 12*m^2 + m^3 + 60)) + (c^2*e^2*x^5*(
7*m + m^2 + 12))/(47*m + 12*m^2 + m^3 + 60) + (c^2*d^2*x^3*(15*m + 3*m^2 +
 20))/(47*m + 12*m^2 + m^3 + 60) - (2*c^2*d^4*m*x)/(e^2*(47*m + 12*m^2 + m
^3 + 60)) + (c^2*d*e*x^4*(19*m + 3*m^2 + 30))/(47*m + 12*m^2 + m^3 + 60) +
 (c^2*d^3*m*x^2*(m + 1))/(e*(47*m + 12*m^2 + m^3 + 60)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.35

$$\int (d+ex)^m \left(cdx+cex^2\right)^2 \, dx$$
$$= \frac{(ex+d)^m c^2(e^5m^2x^5+3d\,e^4m^2x^4+7e^5m\,x^5+3d^2e^3m^2x^3+19d\,e^4m\,x^4+12e^5x^5+d^3e^2m^2x^2+15d^2e^3}{e^3\,(m^3+12m^2+47m+60)}$$

input
```
int((e*x+d)^m*(c*e*x^2+c*d*x)^2,x)
```

output
```
((d + e*x)**m*c**2*(2*d**5 - 2*d**4*e*m*x + d**3*e**2*m**2*x**2 + d**3*e**
2*m*x**2 + 3*d**2*e**3*m**2*x**3 + 15*d**2*e**3*m*x**3 + 20*d**2*e**3*x**3
 + 3*d*e**4*m**2*x**4 + 19*d*e**4*m*x**4 + 30*d*e**4*x**4 + e**5*m**2*x**5
 + 7*e**5*m*x**5 + 12*e**5*x**5))/(e**3*(m**3 + 12*m**2 + 47*m + 60))
```

3.13 $\int (d+ex)^m \left(cdx+cex^2\right) \, dx$

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Optimal result

Integrand size = 19, antiderivative size = 41

$$\int (d+ex)^m \left(cdx+cex^2\right) \, dx=-\frac{cd(d+ex)^{2+m}}{e^2(2+m)}+\frac{c(d+ex)^{3+m}}{e^2(3+m)}$$

output
```
-c*d*(e*x+d)^(2+m)/e^2/(2+m)+c*(e*x+d)^(3+m)/e^2/(3+m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.83

$$\int (d+ex)^m \left(cdx+cex^2\right) \, dx=\frac{c(d+ex)^{2+m}(-d+e(2+m)x)}{e^2(2+m)(3+m)}$$

input
```
Integrate[(d + e*x)^m*(c*d*x + c*e*x^2),x]
```

output
```
(c*(d + e*x)^(2 + m)*(-d + e*(2 + m)*x))/(e^2*(2 + m)*(3 + m))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1121, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cdx + cex^2)(d+ex)^m\,dx$$

$$\Big\downarrow \text{1121}$$

$$\int \left(\frac{c(d+ex)^{m+2}}{e} - \frac{cd(d+ex)^{m+1}}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(d+ex)^{m+3}}{e^2(m+3)} - \frac{cd(d+ex)^{m+2}}{e^2(m+2)}$$

input
```
Int[(d + e*x)^m*(c*d*x + c*e*x^2),x]
```

output
```
-((c*d*(d + e*x)^(2 + m))/(e^2*(2 + m))) + (c*(d + e*x)^(3 + m))/(e^2*(3 +
 m))
```

Defintions of rubi rules used

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.90

method	result	siz
gosper	$-\frac{c(ex+d)^{2+m}(-mex-2ex+d)}{e^{2}(m^{2}+5m+6)}$	37
orering	$-\frac{(-mex-2ex+d)(ex+d)(ex+d)^{m}(ce\,x^{2}+cdx)}{e^{2}(m^{2}+5m+6)x}$	53
risch	$-\frac{c(-e^{3}m\,x^{3}-2d\,e^{2}m\,x^{2}-2e^{3}x^{3}-d^{2}emx-3d\,e^{2}x^{2}+d^{3})(ex+d)^{m}}{(2+m)(3+m)e^{2}}$	72
norman	$\frac{ce\,x^{3}\mathrm{e}^{m\ln(ex+d)}}{3+m}+\frac{dc(3+2m)x^{2}\mathrm{e}^{m\ln(ex+d)}}{m^{2}+5m+6}+\frac{mc\,d^{2}x\,\mathrm{e}^{m\ln(ex+d)}}{e(m^{2}+5m+6)}-\frac{c\,d^{3}\mathrm{e}^{m\ln(ex+d)}}{e^{2}(m^{2}+5m+6)}$	109
parallelrisch	$\frac{x^{3}(ex+d)^{m}cd\,e^{3}m+2x^{3}(ex+d)^{m}cd\,e^{3}+2x^{2}(ex+d)^{m}c\,d^{2}e^{2}m+3x^{2}(ex+d)^{m}c\,d^{2}e^{2}+x(ex+d)^{m}c\,d^{3}em-c\,d^{4}(ex+d)^{m}}{e^{2}(3+m)(2+m)d}$	120

input
```
int((e*x+d)^m*(c*e*x^2+c*d*x),x,method=_RETURNVERBOSE)
```

output
```
-c/e^2*(e*x+d)^(2+m)/(m^2+5*m+6)*(-e*m*x-2*e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.98

$$\int (d+ex)^m\left(cdx+cex^2\right)\,dx$$
$$=\frac{(cd^2emx-cd^3+(ce^3m+2\,ce^3)x^3+(2\,cde^2m+3\,cde^2)x^2)(ex+d)^m}{e^2m^2+5\,e^2m+6\,e^2}$$

input
```
integrate((e*x+d)^m*(c*e*x^2+c*d*x),x, algorithm="fricas")
```

output
```
(c*d^2*e*m*x - c*d^3 + (c*e^3*m + 2*c*e^3)*x^3 + (2*c*d*e^2*m + 3*c*d*e^2)
*x^2)*(e*x + d)^m/(e^2*m^2 + 5*e^2*m + 6*e^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(34) = 68$.

Time = 0.38 (sec) , antiderivative size = 299, normalized size of antiderivative = 7.29

$$\int (d+ex)^m \left(cdx+cex^2\right)\,dx$$
$$= \begin{cases} \frac{cdd^m x^2}{2} \\ \frac{cd\log\left(\frac{d}{e}+x\right)}{de^2+e^3x} + \frac{cd}{de^2+e^3x} + \frac{cex\log\left(\frac{d}{e}+x\right)}{de^2+e^3x} \\ -\frac{cd\log\left(\frac{d}{e}+x\right)}{e^2} + \frac{cx}{e} \\ -\frac{cd^3(d+ex)^m}{e^2m^2+5e^2m+6e^2} + \frac{cd^2emx(d+ex)^m}{e^2m^2+5e^2m+6e^2} + \frac{2cde^2mx^2(d+ex)^m}{e^2m^2+5e^2m+6e^2} + \frac{3cde^2x^2(d+ex)^m}{e^2m^2+5e^2m+6e^2} + \frac{ce^3mx^3(d+ex)^m}{e^2m^2+5e^2m+6e^2} + \frac{2ce^3x^3(d+ex)^m}{e^2m^2+5e^2m+6e^2} \end{cases}$$

input
```
integrate((e*x+d)**m*(c*e*x**2+c*d*x),x)
```

output
```
Piecewise((c*d*d**m*x**2/2, Eq(e, 0)), (c*d*log(d/e + x)/(d*e**2 + e**3*x)
 + c*d/(d*e**2 + e**3*x) + c*e*x*log(d/e + x)/(d*e**2 + e**3*x), Eq(m, -3)
), (-c*d*log(d/e + x)/e**2 + c*x/e, Eq(m, -2)), (-c*d**3*(d + e*x)**m/(e**
2*m**2 + 5*e**2*m + 6*e**2) + c*d**2*e*m*x*(d + e*x)**m/(e**2*m**2 + 5*e**
2*m + 6*e**2) + 2*c*d*e**2*m*x**2*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e
**2) + 3*c*d*e**2*x**2*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + c*e*
*3*m*x**3*(d + e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2) + 2*c*e**3*x**3*(d
+ e*x)**m/(e**2*m**2 + 5*e**2*m + 6*e**2), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(41) = 82$.

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.78

$$\int (d+ex)^m \left(cdx+cex^2\right)\,dx$$
$$= \frac{(e^2(m+1)x^2 + demx - d^2)(ex+d)^m cd}{(m^2+3\,m+2)e^2} + \frac{((m^2+3\,m+2)e^3x^3 + (m^2+m)de^2x^2 - 2\,d^2emx + 2\,d^3)(ex+d)^m c}{(m^3+6\,m^2+11\,m+6)e^2}$$

input
```
integrate((e*x+d)^m*(c*e*x^2+c*d*x),x, algorithm="maxima")
```

output
```
(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*c*d/((m^2 + 3*m + 2)*e^2) +
((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x
 + d)^m*c/((m^3 + 6*m^2 + 11*m + 6)*e^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(41) = 82$.

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.88

$$\int (d+ex)^m \left(cdx+cex^2\right) dx = \frac{(ex+d)^m ce^3mx^3 + 2\,(ex+d)^m cde^2mx^2 + 2\,(ex+d)^m ce^3x^3 + (ex+d)^m cd^2emx + 3\,(ex+d)^m cde^2x^2 -}{e^2m^2 + 5\,e^2m + 6\,e^2}$$

input
```
integrate((e*x+d)^m*(c*e*x^2+c*d*x),x, algorithm="giac")
```

output
```
((e*x + d)^m*c*e^3*m*x^3 + 2*(e*x + d)^m*c*d*e^2*m*x^2 + 2*(e*x + d)^m*c*e
^3*x^3 + (e*x + d)^m*c*d^2*e*m*x + 3*(e*x + d)^m*c*d*e^2*x^2 - (e*x + d)^m
*c*d^3)/(e^2*m^2 + 5*e^2*m + 6*e^2)
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.15

$$\int (d+ex)^m \left(cdx+cex^2\right) dx = (d+e\,x)^m \left(\frac{c\,e\,x^3\,(m+2)}{m^2+5\,m+6} - \frac{c\,d^3}{e^2\,(m^2+5\,m+6)} + \frac{c\,d\,x^2\,(2\,m+3)}{m^2+5\,m+6} + \frac{c\,d^2\,m\,x}{e\,(m^2+5\,m+6)}\right)$$

input
```
int((c*d*x + c*e*x^2)*(d + e*x)^m,x)
```

output
```
(d + e*x)^m*((c*e*x^3*(m + 2))/(5*m + m^2 + 6) - (c*d^3)/(e^2*(5*m + m^2 +
 6)) + (c*d*x^2*(2*m + 3))/(5*m + m^2 + 6) + (c*d^2*m*x)/(e*(5*m + m^2 + 6
)))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.71

$$\int (d+ex)^m \left(cdx+cex^2\right) \, dx = \frac{(ex+d)^m c(e^3m\,x^3+2d\,e^2m\,x^2+2e^3x^3+d^2emx+3d\,e^2x^2-d^3)}{e^2\,(m^2+5m+6)}$$

input
```
int((e*x+d)^m*(c*e*x^2+c*d*x),x)
```

output
```
((d + e*x)**m*c*( - d**3 + d**2*e*m*x + 2*d*e**2*m*x**2 + 3*d*e**2*x**2 +
e**3*m*x**3 + 2*e**3*x**3))/(e**2*(m**2 + 5*m + 6))
```

3.14 $\int (d+ex)^m \, dx$

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Reduce [B] (verification not implemented) . . . 200

Optimal result

Integrand size = 7, antiderivative size = 18

$$\int (d+ex)^m \, dx = \frac{(d+ex)^{1+m}}{e(1+m)}$$

output
```
(e*x+d)^(1+m)/e/(1+m)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (d+ex)^m \, dx = \frac{(d+ex)^{1+m}}{e(1+m)}$$

input
```
Integrate[(d + e*x)^m,x]
```

output
```
(d + e*x)^(1 + m)/(e*(1 + m))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^m\,dx$$

$$\Big\downarrow \; 17$$

$$\frac{(d+ex)^{m+1}}{e(m+1)}$$

input
```
Int[(d + e*x)^m,x]
```

output
```
(d + e*x)^(1 + m)/(e*(1 + m))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
gosper	$\frac{(ex+d)^{1+m}}{e(1+m)}$	19
default	$\frac{(ex+d)^{1+m}}{e(1+m)}$	19
risch	$\frac{(ex+d)(ex+d)^{m}}{e(1+m)}$	22
orering	$\frac{(ex+d)(ex+d)^{m}}{e(1+m)}$	22
parallelrisch	$\frac{(ex+d)^{m} dex+(ex+d)^{m} d^{2}}{(1+m)de}$	36
norman	$\frac{x\,e^{m\ln(ex+d)}}{1+m}+\frac{d\,e^{m\ln(ex+d)}}{e(1+m)}$	37

input
```
int((e*x+d)^m,x,method=_RETURNVERBOSE)
```

output
```
(e*x+d)^(1+m)/e/(1+m)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (d+ex)^m\,dx=\frac{(ex+d)(ex+d)^m}{em+e}$$

input
```
integrate((e*x+d)^m,x, algorithm="fricas")
```

output
```
(e*x + d)*(e*x + d)^m/(e*m + e)
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (d+ex)^m\,dx = \frac{\begin{cases} \frac{(d+ex)^{m+1}}{m+1} & \text{for } m \neq -1 \\ \log{(d+ex)} & \text{otherwise} \end{cases}}{e}$$

input
```
integrate((e*x+d)**m,x)
```

output
```
Piecewise(((d + e*x)**(m + 1)/(m + 1), Ne(m, -1)), (log(d + e*x), True))/e
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (d+ex)^m\,dx = \frac{(ex+d)^{m+1}}{e(m+1)}$$

input
```
integrate((e*x+d)^m,x, algorithm="maxima")
```

output
```
(e*x + d)^(m + 1)/(e*(m + 1))
```

Giac [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (d+ex)^m\,dx = \frac{(ex+d)^{m+1}}{e(m+1)}$$

input
```
integrate((e*x+d)^m,x, algorithm="giac")
```

output
```
(e*x + d)^(m + 1)/(e*(m + 1))
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (d+ex)^m\,dx = \frac{(d+e\,x)^{m+1}}{e\,(m+1)}$$

input
```
int((d + e*x)^m,x)
```

output
```
(d + e*x)^(m + 1)/(e*(m + 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int (d+ex)^m\,dx = \frac{(ex+d)^m\,(ex+d)}{e\,(m+1)}$$

input
```
int((e*x+d)^m,x)
```

output
```
((d + e*x)**m*(d + e*x))/(e*(m + 1))
```

3.15 $\int \frac{(d+ex)^m}{cdx+cex^2}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 32

$$\int \frac{(d+ex)^m}{cdx+cex^2}\,dx = -\frac{(d+ex)^m \operatorname{Hypergeometric2F1}\left(1,m,1+m,1+\frac{ex}{d}\right)}{cdm}$$

output
```
-(e*x+d)^m*hypergeom([1, m],[1+m],1+e*x/d)/c/d/m
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m}{cdx+cex^2}\,dx = -\frac{(d+ex)^m \operatorname{Hypergeometric2F1}\left(1,m,1+m,1+\frac{ex}{d}\right)}{cdm}$$

input
```
Integrate[(d + e*x)^m/(c*d*x + c*e*x^2),x]
```

output
```
-(((d + e*x)^m*Hypergeometric2F1[1, m, 1 + m, 1 + (e*x)/d])/(c*d*m))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1121, 27, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{cdx+cex^2} dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{(d+ex)^{m-1}}{cx} dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d+ex)^{m-1}}{x} dx}{c}$$

$$\Big\downarrow \text{75}$$

$$-\frac{(d+ex)^m \text{Hypergeometric2F1}\left(1,m,m+1,\frac{ex}{d}+1\right)}{cdm}$$

input
```
Int[(d + e*x)^m/(c*d*x + c*e*x^2),x]
```

output
```
-(((d + e*x)^m*Hypergeometric2F1[1, m, 1 + m, 1 + (e*x)/d])/(c*d*m))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [F]

$$\int \frac{(ex+d)^m}{ce\,x^2+cdx}dx$$

input
```
int((e*x+d)^m/(c*e*x^2+c*d*x),x)
```

output
```
int((e*x+d)^m/(c*e*x^2+c*d*x),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{cdx+cex^2}\,dx = \int \frac{(ex+d)^m}{cex^2+cdx}\,dx$$

input
```
integrate((e*x+d)^m/(c*e*x^2+c*d*x),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m/(c*e*x^2 + c*d*x), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m}{cdx+cex^2}\,dx = \frac{\int \frac{(d+ex)^m}{dx+ex^2}\,dx}{c}$$

input
```
integrate((e*x+d)**m/(c*e*x**2+c*d*x),x)
```

output
```
Integral((d + e*x)**m/(d*x + e*x**2), x)/c
```

Maxima [F]

$$\int \frac{(d+ex)^m}{cdx+cex^2}\,dx=\int \frac{(ex+d)^m}{cex^2+cdx}\,dx$$

input
```
integrate((e*x+d)^m/(c*e*x^2+c*d*x),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/(c*e*x^2 + c*d*x), x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{cdx+cex^2}\,dx=\int \frac{(ex+d)^m}{cex^2+cdx}\,dx$$

input
```
integrate((e*x+d)^m/(c*e*x^2+c*d*x),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/(c*e*x^2 + c*d*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{cdx+cex^2}\,dx=\int \frac{(d+e\,x)^m}{c\,e\,x^2+c\,d\,x}\,dx$$

input
```
int((d + e*x)^m/(c*d*x + c*e*x^2),x)
```

output
```
int((d + e*x)^m/(c*d*x + c*e*x^2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{cdx+cex^2}\,dx = \frac{\int \frac{(ex+d)^m}{e\,x^2+dx}dx}{c}$$

input
```
int((e*x+d)^m/(c*e*x^2+c*d*x),x)
```

output
```
int((d + e*x)**m/(d*x + e*x**2),x)/c
```

3.16 $\int \frac{(d+ex)^m}{(cdx+cex^2)^2} \, dx$

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Optimal result

Integrand size = 21, antiderivative size = 39

$$\int \frac{(d+ex)^m}{(cdx+cex^2)^2} \, dx = -\frac{e(d+ex)^{-1+m} \operatorname{Hypergeometric2F1}\left(2,-1+m,m,1+\frac{ex}{d}\right)}{c^2d^2(1-m)}$$

output
```
-e*(e*x+d)^(-1+m)*hypergeom([2, -1+m],[m],1+e*x/d)/c^2/d^2/(1-m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^m}{(cdx+cex^2)^2} \, dx = \frac{e(d+ex)^{-1+m} \operatorname{Hypergeometric2F1}\left(2,-1+m,m,1+\frac{ex}{d}\right)}{c^2d^2(-1+m)}$$

input
```
Integrate[(d + e*x)^m/(c*d*x + c*e*x^2)^2,x]
```

output
```
(e*(d + e*x)^(-1 + m)*Hypergeometric2F1[2, -1 + m, m, 1 + (e*x)/d])/(c^2*d
^2*(-1 + m))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1121, 27, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{(cdx+cex^2)^2}dx$$

$$\Big\downarrow \text{1121}$$

$$\int \frac{(d+ex)^{m-2}}{c^2x^2}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d+ex)^{m-2}}{x^2}dx}{c^2}$$

$$\Big\downarrow \text{75}$$

$$-\frac{e(d+ex)^{m-1}\text{Hypergeometric2F1}\left(2,m-1,m,\frac{ex}{d}+1\right)}{c^2d^2(1-m)}$$

input
```
Int[(d + e*x)^m/(c*d*x + c*e*x^2)^2,x]
```

output
```
-((e*(d + e*x)^(-1 + m)*Hypergeometric2F1[2, -1 + m, m, 1 + (e*x)/d])/(c^2
*d^2*(1 - m)))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 1121
```
Int[((d_) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^(m + p)*(a/d + (c/e)*x)^p, x], x]
/; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && (Int
egerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && LtQ[c, 0]))
```

Maple [F]

$$\int \frac{(ex+d)^m}{(ce\,x^2+cdx)^2}dx$$

input
```
int((e*x+d)^m/(c*e*x^2+c*d*x)^2,x)
```

output
```
int((e*x+d)^m/(c*e*x^2+c*d*x)^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{(cdx+cex^2)^2}\,dx=\int \frac{(ex+d)^m}{(cex^2+cdx)^2}\,dx$$

input
```
integrate((e*x+d)^m/(c*e*x^2+c*d*x)^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)^m/(c^2*e^2*x^4 + 2*c^2*d*e*x^3 + c^2*d^2*x^2), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m}{(cdx+cex^2)^2}\,dx = \frac{\int \frac{(d+ex)^m}{d^2x^2+2dex^3+e^2x^4}\,dx}{c^2}$$

input
```
integrate((e*x+d)**m/(c*e*x**2+c*d*x)**2,x)
```

output
```
Integral((d + e*x)**m/(d**2*x**2 + 2*d*e*x**3 + e**2*x**4), x)/c**2
```

Maxima [F]

$$\int \frac{(d+ex)^m}{(cdx+cex^2)^2}\,dx = \int \frac{(ex+d)^m}{(cex^2+cdx)^2}\,dx$$

input
```
integrate((e*x+d)^m/(c*e*x^2+c*d*x)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/(c*e*x^2 + c*d*x)^2, x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{(cdx+cex^2)^2}\,dx = \int \frac{(ex+d)^m}{(cex^2+cdx)^2}\,dx$$

input
```
integrate((e*x+d)^m/(c*e*x^2+c*d*x)^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/(c*e*x^2 + c*d*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{(cdx+cex^2)^2}\,dx=\int \frac{(d+e\,x)^m}{(c\,e\,x^2+c\,d\,x)^2}\,dx$$

input
```
int((d + e*x)^m/(c*d*x + c*e*x^2)^2,x)
```

output
```
int((d + e*x)^m/(c*d*x + c*e*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{(cdx+cex^2)^2}\,dx = \frac{-(ex+d)^m+\left(\int \frac{(ex+d)^m}{e^2x^3+2de\,x^2+d^2x}dx\right)demx-2\left(\int \frac{(ex+d)^m}{e^2x^3+2de\,x^2+d^2x}dx\right)dex+\left(\int \frac{(ex+d)^m}{e^2x^3+2de\,x^2+d^2x}dx\right)e^2m\,x^2}{c^2dx\,(ex+d)}$$

input
```
int((e*x+d)^m/(c*e*x^2+c*d*x)^2,x)
```

output
```
( - (d + e*x)**m + int((d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)*d
*e*m*x - 2*int((d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)*d*e*x + i
nt((d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)*e**2*m*x**2 - 2*int((
d + e*x)**m/(d**2*x + 2*d*e*x**2 + e**2*x**3),x)*e**2*x**2)/(c**2*d*x*(d +
 e*x))
```

3.17 $\int (d+ex)^4 (bx+cx^2)\, dx$

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Giac [A] (verification not implemented) 215
Mupad [B] (verification not implemented) 215
Reduce [B] (verification not implemented) 216

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int (d+ex)^4 (bx+cx^2)\, dx = \frac{d(cd-be)(d+ex)^5}{5e^3} - \frac{(2cd-be)(d+ex)^6}{6e^3} + \frac{c(d+ex)^7}{7e^3}$$

output
```
1/5*d*(-b*e+c*d)*(e*x+d)^5/e^3-1/6*(-b*e+2*c*d)*(e*x+d)^6/e^3+1/7*c*(e*x+d
)^7/e^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.60

$$\int (d+ex)^4 (bx+cx^2)\, dx = \frac{1}{2}bd^4x^2 + \frac{1}{3}d^3(cd+4be)x^3 + \frac{1}{2}d^2e(2cd+3be)x^4 + \frac{2}{5}de^2(3cd+2be)x^5 + \frac{1}{6}e^3(4cd+be)x^6 + \frac{1}{7}ce^4x^7$$

input
```
Integrate[(d + e*x)^4*(b*x + c*x^2),x]
```

output
```
(b*d^4*x^2)/2 + (d^3*(c*d + 4*b*e)*x^3)/3 + (d^2*e*(2*c*d + 3*b*e)*x^4)/2
+ (2*d*e^2*(3*c*d + 2*b*e)*x^5)/5 + (e^3*(4*c*d + b*e)*x^6)/6 + (c*e^4*x^7
)/7
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+cx^2)\,(d+ex)^4\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{(d+ex)^5(be-2cd)}{e^2}+\frac{d(d+ex)^4(cd-be)}{e^2}+\frac{c(d+ex)^6}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(d+ex)^6(2cd-be)}{6e^3}+\frac{d(d+ex)^5(cd-be)}{5e^3}+\frac{c(d+ex)^7}{7e^3}$$

input
```
Int[(d + e*x)^4*(b*x + c*x^2),x]
```

output
```
(d*(c*d - b*e)*(d + e*x)^5)/(5*e^3) - ((2*c*d - b*e)*(d + e*x)^6)/(6*e^3)
+ (c*(d + e*x)^7)/(7*e^3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.56

method	result
norman	$\frac{e^4 c x^7}{7}+\left(\frac{1}{6} b e^4+\frac{2}{3} d e^3 c\right) x^6+\left(\frac{4}{5} b d e^3+\frac{6}{5} d^2 e^2 c\right) x^5+\left(\frac{3}{2} d^2 e^2 b+c d^3 e\right) x^4+\left(\frac{4}{3} d^3 e b+\frac{1}{3} c d^4\right)$
gosper	$\frac{x^2\left(30 e^4 c x^5+35 x^4 b e^4+140 x^4 d e^3 c+168 x^3 b d e^3+252 x^3 d^2 e^2 c+315 x^2 d^2 e^2 b+210 x^2 c d^3 e+280 x d^3 e b+70 x c d^4+105 b d^4\right)}{210}$
default	$\frac{e^4 c x^7}{7}+\frac{\left(b e^4+4 d e^3 c\right) x^6}{6}+\frac{\left(4 b d e^3+6 d^2 e^2 c\right) x^5}{5}+\frac{\left(6 d^2 e^2 b+4 c d^3 e\right) x^4}{4}+\frac{\left(4 d^3 e b+c d^4\right) x^3}{3}+\frac{b d^4 x^2}{2}$
risch	$\frac{1}{7} e^4 c x^7+\frac{1}{6} x^6 b e^4+\frac{2}{3} d e^3 c x^6+\frac{4}{5} x^5 b d e^3+\frac{6}{5} x^5 d^2 e^2 c+\frac{3}{2} x^4 d^2 e^2 b+x^4 c d^3 e+\frac{4}{3} x^3 d^3 e b+\frac{1}{3} c$
parallelrisch	$\frac{1}{7} e^4 c x^7+\frac{1}{6} x^6 b e^4+\frac{2}{3} d e^3 c x^6+\frac{4}{5} x^5 b d e^3+\frac{6}{5} x^5 d^2 e^2 c+\frac{3}{2} x^4 d^2 e^2 b+x^4 c d^3 e+\frac{4}{3} x^3 d^3 e b+\frac{1}{3} c$
orering	$\frac{x\left(30 e^4 c x^5+35 x^4 b e^4+140 x^4 d e^3 c+168 x^3 b d e^3+252 x^3 d^2 e^2 c+315 x^2 d^2 e^2 b+210 x^2 c d^3 e+280 x d^3 e b+70 x c d^4+105 b d^4\right)\left(c x^2+\right.}{210 c x+210 b}$

input
```
int((e*x+d)^4*(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
1/7*e^4*c*x^7+(1/6*b*e^4+2/3*d*e^3*c)*x^6+(4/5*b*d*e^3+6/5*d^2*e^2*c)*x^5+
(3/2*d^2*e^2*b+c*d^3*e)*x^4+(4/3*d^3*e*b+1/3*c*d^4)*x^3+1/2*b*d^4*x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.60

$$\int (d+ex)^4 \left(bx+cx^2\right) dx = \frac{1}{7} c e^4 x^7 + \frac{1}{2} b d^4 x^2 + \frac{1}{6} \left(4 c d e^3 + b e^4\right) x^6 + \frac{2}{5} \left(3 c d^2 e^2 + 2 b d e^3\right) x^5 + \frac{1}{2} \left(2 c d^3 e + 3 b d^2 e^2\right) x^4 + \frac{1}{3} \left(c d^4 + 4 b d^3 e\right) x^3$$

input
```
integrate((e*x+d)^4*(c*x^2+b*x),x, algorithm="fricas")
```

output
```
1/7*c*e^4*x^7 + 1/2*b*d^4*x^2 + 1/6*(4*c*d*e^3 + b*e^4)*x^6 + 2/5*(3*c*d^2
*e^2 + 2*b*d*e^3)*x^5 + 1/2*(2*c*d^3*e + 3*b*d^2*e^2)*x^4 + 1/3*(c*d^4 + 4
*b*d^3*e)*x^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(53) = 106$.

Time = 0.03 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.73

$$\int (d+ex)^4 \left(bx+cx^2\right) dx = \frac{bd^4x^2}{2} + \frac{ce^4x^7}{7} + x^6\left(\frac{be^4}{6} + \frac{2cde^3}{3}\right) + x^5\cdot\left(\frac{4bde^3}{5} + \frac{6cd^2e^2}{5}\right) + x^4\cdot\left(\frac{3bd^2e^2}{2} + cd^3e\right) + x^3\cdot\left(\frac{4bd^3e}{3} + \frac{cd^4}{3}\right)$$

input
```
integrate((e*x+d)**4*(c*x**2+b*x),x)
```

output
```
b*d**4*x**2/2 + c*e**4*x**7/7 + x**6*(b*e**4/6 + 2*c*d*e**3/3) + x**5*(4*b
*d*e**3/5 + 6*c*d**2*e**2/5) + x**4*(3*b*d**2*e**2/2 + c*d**3*e) + x**3*(4
*b*d**3*e/3 + c*d**4/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.60

$$\int (d+ex)^4 \left(bx+cx^2\right) dx = \frac{1}{7}ce^4x^7 + \frac{1}{2}bd^4x^2 + \frac{1}{6}\left(4\,cde^3 + be^4\right)x^6 + \frac{2}{5}\left(3\,cd^2e^2 + 2\,bde^3\right)x^5 + \frac{1}{2}\left(2\,cd^3e + 3\,bd^2e^2\right)x^4 + \frac{1}{3}\left(cd^4 + 4\,bd^3e\right)x^3$$

input
```
integrate((e*x+d)^4*(c*x^2+b*x),x, algorithm="maxima")
```

output
```
1/7*c*e^4*x^7 + 1/2*b*d^4*x^2 + 1/6*(4*c*d*e^3 + b*e^4)*x^6 + 2/5*(3*c*d^2
*e^2 + 2*b*d*e^3)*x^5 + 1/2*(2*c*d^3*e + 3*b*d^2*e^2)*x^4 + 1/3*(c*d^4 + 4
*b*d^3*e)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.61

$$\int (d+ex)^4 \left(bx+cx^2\right)\, dx = \frac{1}{7}\, ce^4x^7 + \frac{2}{3}\, cde^3x^6 + \frac{1}{6}\, be^4x^6 + \frac{6}{5}\, cd^2e^2x^5 + \frac{4}{5}\, bde^3x^5 + cd^3ex^4 + \frac{3}{2}\, bd^2e^2x^4 + \frac{1}{3}\, cd^4x^3 + \frac{4}{3}\, bd^3ex^3 + \frac{1}{2}\, bd^4x^2$$

input
```
integrate((e*x+d)^4*(c*x^2+b*x),x, algorithm="giac")
```

output
```
1/7*c*e^4*x^7 + 2/3*c*d*e^3*x^6 + 1/6*b*e^4*x^6 + 6/5*c*d^2*e^2*x^5 + 4/5*
b*d*e^3*x^5 + c*d^3*e*x^4 + 3/2*b*d^2*e^2*x^4 + 1/3*c*d^4*x^3 + 4/3*b*d^3*
e*x^3 + 1/2*b*d^4*x^2
```

Mupad [B] (verification not implemented)

Time = 8.64 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.47

$$\int (d+ex)^4 \left(bx+cx^2\right)\, dx = x^3 \left(\frac{c\,d^4}{3}+\frac{4\,b\,e\,d^3}{3}\right)+x^6 \left(\frac{b\,e^4}{6}+\frac{2\,c\,d\,e^3}{3}\right)+\frac{b\,d^4\,x^2}{2} + \frac{c\,e^4\,x^7}{7}+\frac{d^2\,e\,x^4\,(3\,b\,e+2\,c\,d)}{2}+\frac{2\,d\,e^2\,x^5\,(2\,b\,e+3\,c\,d)}{5}$$

input
```
int((b*x + c*x^2)*(d + e*x)^4,x)
```

output
```
x^3*((c*d^4)/3 + (4*b*d^3*e)/3) + x^6*((b*e^4)/6 + (2*c*d*e^3)/3) + (b*d^4
*x^2)/2 + (c*e^4*x^7)/7 + (d^2*e*x^4*(3*b*e + 2*c*d))/2 + (2*d*e^2*x^5*(2*
b*e + 3*c*d))/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.60

$$\int (d+ex)^4 \left(bx+cx^2\right) \, dx$$
$$= \frac{x^2(30c\, e^4x^5 + 35b\, e^4x^4 + 140cd\, e^3x^4 + 168bd\, e^3x^3 + 252c\, d^2e^2x^3 + 315b\, d^2e^2x^2 + 210c\, d^3e\, x^2 + 280b\, d^3ex}{210}$$

input
```
int((e*x+d)^4*(c*x^2+b*x),x)
```

output
```
(x**2*(105*b*d**4 + 280*b*d**3*e*x + 315*b*d**2*e**2*x**2 + 168*b*d*e**3*x
**3 + 35*b*e**4*x**4 + 70*c*d**4*x + 210*c*d**3*e*x**2 + 252*c*d**2*e**2*x
**3 + 140*c*d*e**3*x**4 + 30*c*e**4*x**5))/210
```

3.18 $\int (d+ex)^3 (bx+cx^2)\, dx$

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Reduce [B] (verification not implemented) 222

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int (d+ex)^3 (bx+cx^2)\, dx = \frac{d(cd-be)(d+ex)^4}{4e^3} - \frac{(2cd-be)(d+ex)^5}{5e^3} + \frac{c(d+ex)^6}{6e^3}$$

output
```
1/4*d*(-b*e+c*d)*(e*x+d)^4/e^3-1/5*(-b*e+2*c*d)*(e*x+d)^5/e^3+1/6*c*(e*x+d
)^6/e^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int (d+ex)^3 (bx+cx^2)\, dx = \frac{1}{60}x^2(30bd^3 + 20d^2(cd+3be)x + 45de(cd+be)x^2 + 12e^2(3cd+be)x^3 + 10ce^3x^4)$$

input
```
Integrate[(d + e*x)^3*(b*x + c*x^2),x]
```

output
```
(x^2*(30*b*d^3 + 20*d^2*(c*d + 3*b*e)*x + 45*d*e*(c*d + b*e)*x^2 + 12*e^2*
(3*c*d + b*e)*x^3 + 10*c*e^3*x^4))/60
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+cx^2)(d+ex)^3\,dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(\frac{(d+ex)^4(be-2cd)}{e^2}+\frac{d(d+ex)^3(cd-be)}{e^2}+\frac{c(d+ex)^5}{e^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{(d+ex)^5(2cd-be)}{5e^3}+\frac{d(d+ex)^4(cd-be)}{4e^3}+\frac{c(d+ex)^6}{6e^3}$$

input
```
Int[(d + e*x)^3*(b*x + c*x^2),x]
```

output
```
(d*(c*d - b*e)*(d + e*x)^4)/(4*e^3) - ((2*c*d - b*e)*(d + e*x)^5)/(5*e^3)
+ (c*(d + e*x)^6)/(6*e^3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

method	result	size
norman	$\frac{c\,e^3x^6}{6}+\left(\frac{1}{5}b\,e^3+\frac{3}{5}cd\,e^2\right)x^5+\left(\frac{3}{4}bd\,e^2+\frac{3}{4}d^2ec\right)x^4+\left(b\,d^2e+\frac{1}{3}c\,d^3\right)x^3+\frac{b\,d^3x^2}{2}$	74
gosper	$\frac{x^2(10c\,x^4e^3+12x^3b\,e^3+36cd\,x^3e^2+45x^2bd\,e^2+45c\,d^2e\,x^2+60b\,d^2ex+20d^3cx+30b\,d^3)}{60}$	76
default	$\frac{c\,e^3x^6}{6}+\frac{(b\,e^3+3cd\,e^2)x^5}{5}+\frac{(3bd\,e^2+3d^2ec)x^4}{4}+\frac{(3b\,d^2e+c\,d^3)x^3}{3}+\frac{b\,d^3x^2}{2}$	76
risch	$\frac{1}{6}c\,e^3x^6+\frac{1}{5}x^5b\,e^3+\frac{3}{5}d\,e^2c\,x^5+\frac{3}{4}x^4bd\,e^2+\frac{3}{4}x^4d^2ec+x^3b\,d^2e+\frac{1}{3}c\,d^3x^3+\frac{1}{2}b\,d^3x^2$	77
parallelrisch	$\frac{1}{6}c\,e^3x^6+\frac{1}{5}x^5b\,e^3+\frac{3}{5}d\,e^2c\,x^5+\frac{3}{4}x^4bd\,e^2+\frac{3}{4}x^4d^2ec+x^3b\,d^2e+\frac{1}{3}c\,d^3x^3+\frac{1}{2}b\,d^3x^2$	77
orering	$\frac{x(10c\,x^4e^3+12x^3b\,e^3+36cd\,x^3e^2+45x^2bd\,e^2+45c\,d^2e\,x^2+60b\,d^2ex+20d^3cx+30b\,d^3)(c\,x^2+bx)}{60cx+60b}$	90

input
```
int((e*x+d)^3*(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*c*e^3*x^6+(1/5*b*e^3+3/5*c*d*e^2)*x^5+(3/4*b*d*e^2+3/4*d^2*e*c)*x^4+(b
*d^2*e+1/3*c*d^3)*x^3+1/2*b*d^3*x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int (d+ex)^3\left(bx+cx^2\right)\,dx=\frac{1}{6}\,ce^3x^6+\frac{1}{2}\,bd^3x^2+\frac{1}{5}\,\left(3\,cde^2+be^3\right)x^5+\frac{3}{4}\,\left(cd^2e+bde^2\right)x^4+\frac{1}{3}\,\left(cd^3+3\,bd^2e\right)x^3$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x),x, algorithm="fricas")
```

output
```
1/6*c*e^3*x^6 + 1/2*b*d^3*x^2 + 1/5*(3*c*d*e^2 + b*e^3)*x^5 + 3/4*(c*d^2*e
 + b*d*e^2)*x^4 + 1/3*(c*d^3 + 3*b*d^2*e)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29

$$\int (d+ex)^3 \left(bx+cx^2\right) dx = \frac{bd^3x^2}{2} + \frac{ce^3x^6}{6} + x^5\left(\frac{be^3}{5} + \frac{3cde^2}{5}\right) + x^4 \cdot \left(\frac{3bde^2}{4} + \frac{3cd^2e}{4}\right) + x^3\left(bd^2e + \frac{cd^3}{3}\right)$$

input
```
integrate((e*x+d)**3*(c*x**2+b*x),x)
```

output
```
b*d**3*x**2/2 + c*e**3*x**6/6 + x**5*(b*e**3/5 + 3*c*d*e**2/5) + x**4*(3*b
*d*e**2/4 + 3*c*d**2*e/4) + x**3*(b*d**2*e + c*d**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int (d+ex)^3 \left(bx+cx^2\right) dx = \frac{1}{6}\,ce^3x^6 + \frac{1}{2}\,bd^3x^2 + \frac{1}{5}\,(3\,cde^2 + be^3)x^5 + \frac{3}{4}\,(cd^2e + bde^2)x^4 + \frac{1}{3}\,(cd^3 + 3\,bd^2e)x^3$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x),x, algorithm="maxima")
```

output
```
1/6*c*e^3*x^6 + 1/2*b*d^3*x^2 + 1/5*(3*c*d*e^2 + b*e^3)*x^5 + 3/4*(c*d^2*e
 + b*d*e^2)*x^4 + 1/3*(c*d^3 + 3*b*d^2*e)*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int (d+ex)^3 \left(bx+cx^2\right) \, dx = \frac{1}{6} ce^3x^6 + \frac{3}{5} cde^2x^5 + \frac{1}{5} be^3x^5 + \frac{3}{4} cd^2ex^4 + \frac{3}{4} bde^2x^4 + \frac{1}{3} cd^3x^3 + bd^2ex^3 + \frac{1}{2} bd^3x^2$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x),x, algorithm="giac")
```

output
```
1/6*c*e^3*x^6 + 3/5*c*d*e^2*x^5 + 1/5*b*e^3*x^5 + 3/4*c*d^2*e*x^4 + 3/4*b*
d*e^2*x^4 + 1/3*c*d^3*x^3 + b*d^2*e*x^3 + 1/2*b*d^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int (d+ex)^3 \left(bx+cx^2\right) \, dx = x^3 \left(\frac{c\,d^3}{3} + b\,e\,d^2\right) + x^5 \left(\frac{b\,e^3}{5} + \frac{3\,c\,d\,e^2}{5}\right) + \frac{b\,d^3\,x^2}{2} + \frac{c\,e^3\,x^6}{6} + \frac{3\,d\,e\,x^4\,(b\,e+c\,d)}{4}$$

input
```
int((b*x + c*x^2)*(d + e*x)^3,x)
```

output
```
x^3*((c*d^3)/3 + b*d^2*e) + x^5*((b*e^3)/5 + (3*c*d*e^2)/5) + (b*d^3*x^2)/
2 + (c*e^3*x^6)/6 + (3*d*e*x^4*(b*e + c*d))/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.21

$$\int (d+ex)^3 \left(bx+cx^2\right) \, dx$$
$$= \frac{x^2(10c\, e^3x^4 + 12b\, e^3x^3 + 36cd\, e^2x^3 + 45bd\, e^2x^2 + 45c\, d^2e\, x^2 + 60b\, d^2ex + 20c\, d^3x + 30b\, d^3)}{60}$$

input
```
int((e*x+d)^3*(c*x^2+b*x),x)
```

output
```
(x**2*(30*b*d**3 + 60*b*d**2*e*x + 45*b*d*e**2*x**2 + 12*b*e**3*x**3 + 20*
c*d**3*x + 45*c*d**2*e*x**2 + 36*c*d*e**2*x**3 + 10*c*e**3*x**4))/60
```

3.19 $\int (d+ex)^2 \left(bx+cx^2\right) \, dx$

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Optimal result

Integrand size = 17, antiderivative size = 55

$$\int (d+ex)^2 \left(bx+cx^2\right) \, dx = \frac{1}{2}bd^2x^2 + \frac{1}{3}d(cd+2be)x^3 + \frac{1}{4}e(2cd+be)x^4 + \frac{1}{5}ce^2x^5$$

output
```
1/2*b*d^2*x^2+1/3*d*(2*b*e+c*d)*x^3+1/4*e*(b*e+2*c*d)*x^4+1/5*c*e^2*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int (d+ex)^2 \left(bx+cx^2\right) \, dx = \frac{1}{60}x^2\left(30bd^2 + 20d(cd+2be)x + 15e(2cd+be)x^2 + 12ce^2x^3\right)$$

input
```
Integrate[(d + e*x)^2*(b*x + c*x^2),x]
```

output
```
(x^2*(30*b*d^2 + 20*d*(c*d + 2*b*e)*x + 15*e*(2*c*d + b*e)*x^2 + 12*c*e^2*
x^3))/60
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx + cx^2)(d + ex)^2 \, dx$$

$$\Big\downarrow \; 1140$$

$$\int \left(ex^3(be + 2cd) + dx^2(2be + cd) + bd^2x + ce^2x^4\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}ex^4(be + 2cd) + \frac{1}{3}dx^3(2be + cd) + \frac{1}{2}bd^2x^2 + \frac{1}{5}ce^2x^5$$

input
```
Int[(d + e*x)^2*(b*x + c*x^2),x]
```

output
```
(b*d^2*x^2)/2 + (d*(c*d + 2*b*e)*x^3)/3 + (e*(2*c*d + b*e)*x^4)/4 + (c*e^2
*x^5)/5
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{x^{2}\left(12c\,e^{2}x^{3}+15b\,e^{2}x^{2}+30cd\,x^{2}e+40bdxe+20c\,d^{2}x+30b\,d^{2}\right)}{60}$	52
default	$\frac{c\,e^{2}x^{5}}{5}+\frac{\left(b\,e^{2}+2dec\right)x^{4}}{4}+\frac{\left(2bde+c\,d^{2}\right)x^{3}}{3}+\frac{b\,x^{2}d^{2}}{2}$	52
norman	$\frac{c\,e^{2}x^{5}}{5}+\left(\frac{1}{4}b\,e^{2}+\frac{1}{2}dec\right)x^{4}+\left(\frac{2}{3}bde+\frac{1}{3}c\,d^{2}\right)x^{3}+\frac{b\,x^{2}d^{2}}{2}$	52
risch	$\frac{1}{5}c\,e^{2}x^{5}+\frac{1}{4}x^{4}b\,e^{2}+\frac{1}{2}dec\,x^{4}+\frac{2}{3}bde\,x^{3}+\frac{1}{3}c\,d^{2}x^{3}+\frac{1}{2}b\,x^{2}d^{2}$	54
parallelrisch	$\frac{1}{5}c\,e^{2}x^{5}+\frac{1}{4}x^{4}b\,e^{2}+\frac{1}{2}dec\,x^{4}+\frac{2}{3}bde\,x^{3}+\frac{1}{3}c\,d^{2}x^{3}+\frac{1}{2}b\,x^{2}d^{2}$	54
orering	$\frac{x\left(12c\,e^{2}x^{3}+15b\,e^{2}x^{2}+30cd\,x^{2}e+40bdxe+20c\,d^{2}x+30b\,d^{2}\right)\left(c\,x^{2}+bx\right)}{60cx+60b}$	66

input
```
int((e*x+d)^2*(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
1/60*x^2*(12*c*e^2*x^3+15*b*e^2*x^2+30*c*d*e*x^2+40*b*d*e*x+20*c*d^2*x+30*
b*d^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (d+ex)^2\left(bx+cx^2\right)\,dx=\frac{1}{5}\,ce^2x^5+\frac{1}{2}\,bd^2x^2+\frac{1}{4}\,(2\,cde+be^2)x^4+\frac{1}{3}\,(cd^2+2\,bde)x^3$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x),x, algorithm="fricas")
```

output
```
1/5*c*e^2*x^5 + 1/2*b*d^2*x^2 + 1/4*(2*c*d*e + b*e^2)*x^4 + 1/3*(c*d^2 + 2
*b*d*e)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int (d+ex)^2 \left(bx+cx^2\right) \, dx = \frac{bd^2x^2}{2} + \frac{ce^2x^5}{5} + x^4\left(\frac{be^2}{4} + \frac{cde}{2}\right) + x^3 \cdot \left(\frac{2bde}{3} + \frac{cd^2}{3}\right)$$

input
```
integrate((e*x+d)**2*(c*x**2+b*x),x)
```

output
```
b*d**2*x**2/2 + c*e**2*x**5/5 + x**4*(b*e**2/4 + c*d*e/2) + x**3*(2*b*d*e/
3 + c*d**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (d+ex)^2 \left(bx+cx^2\right) \, dx = \frac{1}{5} \, ce^2x^5 + \frac{1}{2} \, bd^2x^2 + \frac{1}{4} \, \left(2 \, cde + be^2\right)x^4 + \frac{1}{3} \, \left(cd^2 + 2 \, bde\right)x^3$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x),x, algorithm="maxima")
```

output
```
1/5*c*e^2*x^5 + 1/2*b*d^2*x^2 + 1/4*(2*c*d*e + b*e^2)*x^4 + 1/3*(c*d^2 + 2
*b*d*e)*x^3
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int (d+ex)^2 \left(bx+cx^2\right) \, dx = \frac{1}{5} \, ce^2x^5 + \frac{1}{2} \, cdex^4 + \frac{1}{4} \, be^2x^4 + \frac{1}{3} \, cd^2x^3 + \frac{2}{3} \, bdex^3 + \frac{1}{2} \, bd^2x^2$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x),x, algorithm="giac")
```

output
```
1/5*c*e^2*x^5 + 1/2*c*d*e*x^4 + 1/4*b*e^2*x^4 + 1/3*c*d^2*x^3 + 2/3*b*d*e*
x^3 + 1/2*b*d^2*x^2
```

Mupad [B] (verification not implemented)

Time = 8.75 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (d+ex)^2 \left(bx+cx^2\right) \, dx = x^3 \left(\frac{c\,d^2}{3}+\frac{2\,b\,e\,d}{3}\right)+x^4 \left(\frac{b\,e^2}{4}+\frac{c\,d\,e}{2}\right)+\frac{b\,d^2\,x^2}{2}+\frac{c\,e^2\,x^5}{5}$$

input
```
int((b*x + c*x^2)*(d + e*x)^2,x)
```

output
```
x^3*((c*d^2)/3 + (2*b*d*e)/3) + x^4*((b*e^2)/4 + (c*d*e)/2) + (b*d^2*x^2)/
2 + (c*e^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (d+ex)^2 \left(bx+cx^2\right) \, dx = \frac{x^2(12c\,e^2x^3+15b\,e^2x^2+30cde\,x^2+40bdex+20c\,d^2x+30b\,d^2)}{60}$$

input
```
int((e*x+d)^2*(c*x^2+b*x),x)
```

output
```
(x**2*(30*b*d**2 + 40*b*d*e*x + 15*b*e**2*x**2 + 20*c*d**2*x + 30*c*d*e*x*
*2 + 12*c*e**2*x**3))/60
```

3.20 $\int (d+ex)(bx+cx^2)\,dx$

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Reduce [B] (verification not implemented) 232

Optimal result

Integrand size = 15, antiderivative size = 33

$$\int (d+ex)\left(bx+cx^2\right)\,dx=\frac{1}{2}bdx^2+\frac{1}{3}(cd+be)x^3+\frac{1}{4}cex^4$$

output
```
1/2*b*d*x^2+1/3*(b*e+c*d)*x^3+1/4*c*e*x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (d+ex)\left(bx+cx^2\right)\,dx=\frac{1}{12}x^2(cx(4d+3ex)+b(6d+4ex))$$

input
```
Integrate[(d + e*x)*(b*x + c*x^2),x]
```

output
```
(x^2*(c*x*(4*d + 3*e*x) + b*(6*d + 4*e*x)))/12
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+cx^2)(d+ex)\,dx$$

$$\Big\downarrow \ 1140$$

$$\int (x^2(be+cd)+bdx+cex^3)\,dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{3}x^3(be+cd)+\frac{1}{2}bdx^2+\frac{1}{4}cex^4$$

input
```
Int[(d + e*x)*(b*x + c*x^2),x]
```

output
```
(b*d*x^2)/2 + ((c*d + b*e)*x^3)/3 + (c*e*x^4)/4
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{x^{2}\left(3ce\,x^{2}+4bex+4cdx+6bd\right)}{12}$	28
default	$\frac{bd\,x^{2}}{2}+\frac{\left(be+cd\right)x^{3}}{3}+\frac{ce\,x^{4}}{4}$	28
norman	$\frac{ce\,x^{4}}{4}+\left(\frac{be}{3}+\frac{cd}{3}\right)x^{3}+\frac{bd\,x^{2}}{2}$	29
risch	$\frac{1}{2}bd\,x^{2}+\frac{1}{3}be\,x^{3}+\frac{1}{3}cd\,x^{3}+\frac{1}{4}ce\,x^{4}$	30
parallelrisch	$\frac{1}{2}bd\,x^{2}+\frac{1}{3}be\,x^{3}+\frac{1}{3}cd\,x^{3}+\frac{1}{4}ce\,x^{4}$	30
orering	$\frac{x\left(3ce\,x^{2}+4bex+4cdx+6bd\right)\left(c\,x^{2}+bx\right)}{12cx+12b}$	42

input
```
int((e*x+d)*(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2*(3*c*e*x^2+4*b*e*x+4*c*d*x+6*b*d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int (d+ex)\left(bx+cx^2\right)\,dx=\frac{1}{4}\,cex^4+\frac{1}{2}\,bdx^2+\frac{1}{3}\,(cd+be)x^3$$

input
```
integrate((e*x+d)*(c*x^2+b*x),x, algorithm="fricas")
```

output
```
1/4*c*e*x^4 + 1/2*b*d*x^2 + 1/3*(c*d + b*e)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (d+ex)\left(bx+cx^2\right)\,dx=\frac{bdx^2}{2}+\frac{cex^4}{4}+x^3\left(\frac{be}{3}+\frac{cd}{3}\right)$$

input
```
integrate((e*x+d)*(c*x**2+b*x),x)
```

output
```
b*d*x**2/2 + c*e*x**4/4 + x**3*(b*e/3 + c*d/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int (d+ex)\left(bx+cx^2\right)\,dx=\frac{1}{4}\,cex^4+\frac{1}{2}\,bdx^2+\frac{1}{3}\,(cd+be)x^3$$

input
```
integrate((e*x+d)*(c*x^2+b*x),x, algorithm="maxima")
```

output
```
1/4*c*e*x^4 + 1/2*b*d*x^2 + 1/3*(c*d + b*e)*x^3
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (d+ex)\left(bx+cx^2\right)\,dx=\frac{1}{4}\,cex^4+\frac{1}{3}\,cdx^3+\frac{1}{3}\,bex^3+\frac{1}{2}\,bdx^2$$

input
```
integrate((e*x+d)*(c*x^2+b*x),x, algorithm="giac")
```

output
```
1/4*c*e*x^4 + 1/3*c*d*x^3 + 1/3*b*e*x^3 + 1/2*b*d*x^2
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int (d+ex)\left(bx+cx^2\right)\,dx=\frac{c\,e\,x^4}{4}+\left(\frac{b\,e}{3}+\frac{c\,d}{3}\right)\,x^3+\frac{b\,d\,x^2}{2}$$

input
```
int((b*x + c*x^2)*(d + e*x),x)
```

output
```
x^3*((b*e)/3 + (c*d)/3) + (b*d*x^2)/2 + (c*e*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int (d+ex)\left(bx+cx^2\right)\,dx=\frac{x^2(3ce\,x^2+4bex+4cdx+6bd)}{12}$$

input
```
int((e*x+d)*(c*x^2+b*x),x)
```

output
```
(x**2*(6*b*d + 4*b*e*x + 4*c*d*x + 3*c*e*x**2))/12
```

3.21 $\int (bx + cx^2) \, dx$

Optimal result 233
Mathematica [A] (verified) 233
Rubi [A] (verified) 234
Maple [A] (verified) 235
Fricas [A] (verification not implemented) 235
Sympy [A] (verification not implemented) 236
Maxima [A] (verification not implemented) 236
Giac [A] (verification not implemented) 236
Mupad [B] (verification not implemented) 237
Reduce [B] (verification not implemented) 237

Optimal result

Integrand size = 9, antiderivative size = 17

$$\int (bx + cx^2) \, dx = \frac{bx^2}{2} + \frac{cx^3}{3}$$

output
```
1/2*b*x^2+1/3*c*x^3
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int (bx + cx^2) \, dx = \frac{bx^2}{2} + \frac{cx^3}{3}$$

input
```
Integrate[b*x + c*x^2,x]
```

output
```
(b*x^2)/2 + (c*x^3)/3
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx + cx^2)\, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{bx^2}{2} + \frac{cx^3}{3}$$

input
```
Int[b*x + c*x^2,x]
```

output
```
(b*x^2)/2 + (c*x^3)/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.82

method	result	size
gosper	$\frac{x^2(2cx+3b)}{6}$	14
default	$\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3$	14
norman	$\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3$	14
risch	$\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3$	14
parallelrisch	$\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3$	14
parts	$\frac{1}{2}b\,x^2+\frac{1}{3}c\,x^3$	14
orering	$\frac{x(2cx+3b)(c\,x^2+bx)}{6cx+6b}$	28

input
```
int(c*x^2+b*x,x,method=_RETURNVERBOSE)
```

output
```
1/6*x^2*(2*c*x+3*b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (bx+cx^2)\ dx = \frac{1}{3}\,cx^3+\frac{1}{2}\,bx^2$$

input
```
integrate(c*x^2+b*x,x, algorithm="fricas")
```

output
```
1/3*c*x^3 + 1/2*b*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.71

$$\int (bx + cx^2) \, dx = \frac{bx^2}{2} + \frac{cx^3}{3}$$

input
```
integrate(c*x**2+b*x,x)
```

output
```
b*x**2/2 + c*x**3/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (bx + cx^2) \, dx = \frac{1}{3} cx^3 + \frac{1}{2} bx^2$$

input
```
integrate(c*x^2+b*x,x, algorithm="maxima")
```

output
```
1/3*c*x^3 + 1/2*b*x^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (bx + cx^2) \, dx = \frac{1}{3} cx^3 + \frac{1}{2} bx^2$$

input
```
integrate(c*x^2+b*x,x, algorithm="giac")
```

output
```
1/3*c*x^3 + 1/2*b*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (bx + cx^2)\ dx = \frac{x^2\,(3\,b + 2\,c\,x)}{6}$$

input
```
int(b*x + c*x^2,x)
```

output
```
(x^2*(3*b + 2*c*x))/6
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.76

$$\int (bx + cx^2)\ dx = \frac{x^2(2cx + 3b)}{6}$$

input
```
int(c*x^2+b*x,x)
```

output
```
(x**2*(3*b + 2*c*x))/6
```

3.22 $\int \frac{bx+cx^2}{d+ex}\,dx$

Optimal result 238
Mathematica [A] (verified) 238
Rubi [A] (verified) 239
Maple [A] (verified) 240
Fricas [A] (verification not implemented) 240
Sympy [A] (verification not implemented) 240
Maxima [A] (verification not implemented) 241
Giac [A] (verification not implemented) 241
Mupad [B] (verification not implemented) 241
Reduce [B] (verification not implemented) 242

Optimal result

Integrand size = 17, antiderivative size = 45

$$\int \frac{bx+cx^2}{d+ex}\,dx = -\frac{(cd-be)x}{e^2}+\frac{cx^2}{2e}+\frac{d(cd-be)\log(d+ex)}{e^3}$$

output
```
-(-b*e+c*d)*x/e^2+1/2*c*x^2/e+d*(-b*e+c*d)*ln(e*x+d)/e^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{bx+cx^2}{d+ex}\,dx = \frac{ex(-2cd+2be+cex)+2d(cd-be)\log(d+ex)}{2e^3}$$

input
```
Integrate[(b*x + c*x^2)/(d + e*x),x]
```

output
```
(e*x*(-2*c*d + 2*b*e + c*e*x) + 2*d*(c*d - b*e)*Log[d + e*x])/(2*e^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+cx^2}{d+ex} \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{d(cd-be)}{e^2(d+ex)}+\frac{be-cd}{e^2}+\frac{cx}{e}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d(cd-be)\log(d+ex)}{e^3}-\frac{x(cd-be)}{e^2}+\frac{cx^2}{2e}$$

input
```
Int[(b*x + c*x^2)/(d + e*x),x]
```

output
```
-(((c*d - b*e)*x)/e^2) + (c*x^2)/(2*e) + (d*(c*d - b*e)*Log[d + e*x])/e^3
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{\frac{1}{2}ce\,x^2+bex-cdx}{e^2}-\frac{d(be-cd)\ln(ex+d)}{e^3}$	43
norman	$\frac{(be-cd)x}{e^2}+\frac{c\,x^2}{2e}-\frac{d(be-cd)\ln(ex+d)}{e^3}$	44
risch	$\frac{c\,x^2}{2e}+\frac{bx}{e}-\frac{cdx}{e^2}-\frac{d\ln(ex+d)b}{e^2}+\frac{d^2\ln(ex+d)c}{e^3}$	52
parallelrisch	$-\frac{-x^2c\,e^2+2\ln(ex+d)bde-2\ln(ex+d)c\,d^2-2xb\,e^2+2cdxe}{2e^3}$	52

input
```
int((c*x^2+b*x)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/e^2*(1/2*c*e*x^2+b*e*x-c*d*x)-d*(b*e-c*d)/e^3*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{bx+cx^2}{d+ex}\,dx=\frac{ce^2x^2-2\,(cde-be^2)x+2\,(cd^2-bde)\log{(ex+d)}}{2\,e^3}$$

input
```
integrate((c*x^2+b*x)/(e*x+d),x, algorithm="fricas")
```

output
```
1/2*(c*e^2*x^2 - 2*(c*d*e - b*e^2)*x + 2*(c*d^2 - b*d*e)*log(e*x + d))/e^3
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{bx+cx^2}{d+ex}\,dx=\frac{cx^2}{2e}-\frac{d(be-cd)\log{(d+ex)}}{e^3}+x\left(\frac{b}{e}-\frac{cd}{e^2}\right)$$

input
```
integrate((c*x**2+b*x)/(e*x+d),x)
```

output
```
c*x**2/(2*e) - d*(b*e - c*d)*log(d + e*x)/e**3 + x*(b/e - c*d/e**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{bx + cx^2}{d + ex}\,dx = \frac{cex^2 - 2\,(cd - be)x}{2\,e^2} + \frac{(cd^2 - bde)\log{(ex + d)}}{e^3}$$

input
```
integrate((c*x^2+b*x)/(e*x+d),x, algorithm="maxima")
```

output
```
1/2*(c*e*x^2 - 2*(c*d - b*e)*x)/e^2 + (c*d^2 - b*d*e)*log(e*x + d)/e^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00

$$\int \frac{bx + cx^2}{d + ex}\,dx = \frac{cex^2 - 2\,cdx + 2\,bex}{2\,e^2} + \frac{(cd^2 - bde)\log{(|ex + d|)}}{e^3}$$

input
```
integrate((c*x^2+b*x)/(e*x+d),x, algorithm="giac")
```

output
```
1/2*(c*e*x^2 - 2*c*d*x + 2*b*e*x)/e^2 + (c*d^2 - b*d*e)*log(abs(e*x + d))/
e^3
```

Mupad [B] (verification not implemented)

Time = 8.85 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{bx + cx^2}{d + ex}\,dx = x\left(\frac{b}{e} - \frac{c\,d}{e^2}\right) + \frac{c\,x^2}{2\,e} + \frac{\ln{(d + e\,x)}\,\left(c\,d^2 - b\,d\,e\right)}{e^3}$$

input
```
int((b*x + c*x^2)/(d + e*x),x)
```

output
```
x*(b/e - (c*d)/e^2) + (c*x^2)/(2*e) + (log(d + e*x)*(c*d^2 - b*d*e))/e^3
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \frac{bx + cx^2}{d + ex}\,dx = \frac{-2\log(ex + d)\,bde + 2\log(ex + d)\,c\,d^2 + 2b\,e^2x - 2cdex + c\,e^2x^2}{2e^3}$$

input
```
int((c*x^2+b*x)/(e*x+d),x)
```

output
```
( - 2*log(d + e*x)*b*d*e + 2*log(d + e*x)*c*d**2 + 2*b*e**2*x - 2*c*d*e*x
+ c*e**2*x**2)/(2*e**3)
```

3.23 $\int \frac{bx+cx^2}{(d+ex)^2} \, dx$

Optimal result 243
Mathematica [A] (verified) 243
Rubi [A] (verified) 244
Maple [A] (verified) 245
Fricas [A] (verification not implemented) 245
Sympy [A] (verification not implemented) 246
Maxima [A] (verification not implemented) 246
Giac [A] (verification not implemented) 246
Mupad [B] (verification not implemented) 247
Reduce [B] (verification not implemented) 247

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \frac{bx+cx^2}{(d+ex)^2} \, dx = \frac{cx}{e^2} - \frac{d(cd-be)}{e^3(d+ex)} - \frac{(2cd-be)\log(d+ex)}{e^3}$$

output
```
c*x/e^2-d*(-b*e+c*d)/e^3/(e*x+d)-(-b*e+2*c*d)*ln(e*x+d)/e^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{bx+cx^2}{(d+ex)^2} \, dx = \frac{cex + \frac{d(-cd+be)}{d+ex} + (-2cd+be)\log(d+ex)}{e^3}$$

input
```
Integrate[(b*x + c*x^2)/(d + e*x)^2,x]
```

output
```
(c*e*x + (d*(-(c*d) + b*e))/(d + e*x) + (-2*c*d + b*e)*Log[d + e*x])/e^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+cx^2}{(d+ex)^2}dx$$
$$\Big\downarrow \text{1140}$$
$$\int \left(\frac{be-2cd}{e^2(d+ex)}+\frac{d(cd-be)}{e^2(d+ex)^2}+\frac{c}{e^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{d(cd-be)}{e^3(d+ex)}-\frac{(2cd-be)\log(d+ex)}{e^3}+\frac{cx}{e^2}$$

input
```
Int[(b*x + c*x^2)/(d + e*x)^2,x]
```

output
```
(c*x)/e^2 - (d*(c*d - b*e))/(e^3*(d + e*x)) - ((2*c*d - b*e)*Log[d + e*x])
/e^3
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

method	result	size
default	$\frac{cx}{e^2}+\frac{(be-2cd)\ln(ex+d)}{e^3}+\frac{d(be-cd)}{e^3(ex+d)}$	46
norman	$\frac{\frac{c\,x^2}{e}+\frac{d(be-2cd)}{e^3}}{ex+d}+\frac{(be-2cd)\ln(ex+d)}{e^3}$	50
risch	$\frac{cx}{e^2}+\frac{\ln(ex+d)b}{e^2}-\frac{2cd\ln(ex+d)}{e^3}+\frac{db}{e^2(ex+d)}-\frac{d^2c}{e^3(ex+d)}$	61
parallelrisch	$\frac{\ln(ex+d)xb\,e^2-2\ln(ex+d)xcde+x^2c\,e^2+\ln(ex+d)bde-2\ln(ex+d)c\,d^2+bde-2c\,d^2}{e^3(ex+d)}$	77

input
```
int((c*x^2+b*x)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
c*x/e^2+1/e^3*(b*e-2*c*d)*ln(e*x+d)+d*(b*e-c*d)/e^3/(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.50

$$\int\frac{bx+cx^2}{(d+ex)^2}\,dx=\frac{ce^2x^2+cdex-cd^2+bde-(2\,cd^2-bde+(2\,cde-be^2)x)\log(ex+d)}{e^4x+de^3}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
(c*e^2*x^2 + c*d*e*x - c*d^2 + b*d*e - (2*c*d^2 - b*d*e + (2*c*d*e - b*e^2
)*x)*log(e*x + d))/(e^4*x + d*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{bx+cx^2}{(d+ex)^2}\,dx=\frac{cx}{e^2}+\frac{bde-cd^2}{de^3+e^4x}+\frac{(be-2cd)\log{(d+ex)}}{e^3}$$

input
```
integrate((c*x**2+b*x)/(e*x+d)**2,x)
```

output
```
c*x/e**2 + (b*d*e - c*d**2)/(d*e**3 + e**4*x) + (b*e - 2*c*d)*log(d + e*x)
/e**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.10

$$\int \frac{bx+cx^2}{(d+ex)^2}\,dx=-\frac{cd^2-bde}{e^4x+de^3}+\frac{cx}{e^2}-\frac{(2\,cd-be)\log{(ex+d)}}{e^3}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(c*d^2 - b*d*e)/(e^4*x + d*e^3) + c*x/e^2 - (2*c*d - b*e)*log(e*x + d)/e^
3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.00

$$\int \frac{bx+cx^2}{(d+ex)^2}\,dx=c\left(\frac{2\,d\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^3}+\frac{ex+d}{e^3}-\frac{d^2}{(ex+d)e^3}\right)-\frac{b\left(\frac{\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e}-\frac{d}{(ex+d)e}\right)}{e}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^2,x, algorithm="giac")
```

output
```
c*(2*d*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^3 + (e*x + d)/e^3 - d^2/((
e*x + d)*e^3)) - b*(log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e - d/((e*x + d
)*e))/e
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{bx+cx^2}{(d+ex)^2}\,dx = \frac{\ln(d+e\,x)\,(b\,e-2\,c\,d)}{e^3} - \frac{c\,d^2-b\,d\,e}{e\,(x\,e^3+d\,e^2)} + \frac{c\,x}{e^2}$$

input
```
int((b*x + c*x^2)/(d + e*x)^2,x)
```

output
```
(log(d + e*x)*(b*e - 2*c*d))/e^3 - (c*d^2 - b*d*e)/(e*(d*e^2 + e^3*x)) + (
c*x)/e^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.65

$$\int \frac{bx+cx^2}{(d+ex)^2}\,dx = \frac{\log(ex+d)\,bde+\log(ex+d)\,b\,e^2x-2\log(ex+d)\,c\,d^2-2\log(ex+d)\,cdex-b\,e^2x+2cdex+c\,e^2x^2}{e^3\,(ex+d)}$$

input
```
int((c*x^2+b*x)/(e*x+d)^2,x)
```

output
```
(log(d + e*x)*b*d*e + log(d + e*x)*b*e**2*x - 2*log(d + e*x)*c*d**2 - 2*lo
g(d + e*x)*c*d*e*x - b*e**2*x + 2*c*d*e*x + c*e**2*x**2)/(e**3*(d + e*x))
```

3.24 $\int \frac{bx+cx^2}{(d+ex)^3} \, dx$

Optimal result 248
Mathematica [A] (verified) 248
Rubi [A] (verified) 249
Maple [A] (verified) 250
Fricas [A] (verification not implemented) 250
Sympy [A] (verification not implemented) 251
Maxima [A] (verification not implemented) 251
Giac [A] (verification not implemented) 251
Mupad [B] (verification not implemented) 252
Reduce [B] (verification not implemented) 252

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{bx+cx^2}{(d+ex)^3} \, dx = -\frac{d(cd-be)}{2e^3(d+ex)^2} + \frac{2cd-be}{e^3(d+ex)} + \frac{c\log(d+ex)}{e^3}$$

output
```
-1/2*d*(-b*e+c*d)/e^3/(e*x+d)^2+(-b*e+2*c*d)/e^3/(e*x+d)+c*ln(e*x+d)/e^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{bx+cx^2}{(d+ex)^3} \, dx = \frac{-be(d+2ex)+cd(3d+4ex)+2c(d+ex)^2\log(d+ex)}{2e^3(d+ex)^2}$$

input
```
Integrate[(b*x + c*x^2)/(d + e*x)^3,x]
```

output
```
(-(b*e*(d + 2*e*x)) + c*d*(3*d + 4*e*x) + 2*c*(d + e*x)^2*Log[d + e*x])/(2
*e^3*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+cx^2}{(d+ex)^3}\,dx$$

$$\Big\downarrow \ 1140$$

$$\int \left(\frac{be-2cd}{e^2(d+ex)^2}+\frac{d(cd-be)}{e^2(d+ex)^3}+\frac{c}{e^2(d+ex)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{d(cd-be)}{2e^3(d+ex)^2}+\frac{2cd-be}{e^3(d+ex)}+\frac{c\log(d+ex)}{e^3}$$

input
```
Int[(b*x + c*x^2)/(d + e*x)^3,x]
```

output
```
-1/2*(d*(c*d - b*e))/(e^3*(d + e*x)^2) + (2*c*d - b*e)/(e^3*(d + e*x)) + (
c*Log[d + e*x])/e^3
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

method	result	size
norman	$\frac{-\frac{d(be-3cd)}{2e^3}-\frac{(be-2cd)x}{e^2}}{(ex+d)^2}+\frac{c\ln(ex+d)}{e^3}$	50
risch	$\frac{-\frac{d(be-3cd)}{2e^3}-\frac{(be-2cd)x}{e^2}}{(ex+d)^2}+\frac{c\ln(ex+d)}{e^3}$	50
default	$\frac{c\ln(ex+d)}{e^3}-\frac{be-2cd}{e^3(ex+d)}+\frac{d(be-cd)}{2e^3(ex+d)^2}$	54
parallelrisch	$\frac{2\ln(ex+d)x^2c\,e^2+4\ln(ex+d)xcde+2\ln(ex+d)c\,d^2-2xb\,e^2+4cdxe-bde+3c\,d^2}{2e^3(ex+d)^2}$	77

input
```
int((c*x^2+b*x)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*d*(b*e-3*c*d)/e^3-(b*e-2*c*d)/e^2*x)/(e*x+d)^2+c*ln(e*x+d)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.47

$$\int \frac{bx+cx^2}{(d+ex)^3}\,dx=\frac{3\,cd^2-bde+2\,(2\,cde-be^2)x+2\,(ce^2x^2+2\,cdex+cd^2)\log(ex+d)}{2\,(e^5x^2+2\,de^4x+d^2e^3)}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*(3*c*d^2 - b*d*e + 2*(2*c*d*e - b*e^2)*x + 2*(c*e^2*x^2 + 2*c*d*e*x +
c*d^2)*log(e*x + d))/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{bx+cx^2}{(d+ex)^3}\,dx = \frac{c\log{(d+ex)}}{e^3} + \frac{-bde+3cd^2+x(-2be^2+4cde)}{2d^2e^3+4de^4x+2e^5x^2}$$

input
```
integrate((c*x**2+b*x)/(e*x+d)**3,x)
```

output
```
c*log(d + e*x)/e**3 + (-b*d*e + 3*c*d**2 + x*(-2*b*e**2 + 4*c*d*e))/(2*d**
2*e**3 + 4*d*e**4*x + 2*e**5*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \frac{bx+cx^2}{(d+ex)^3}\,dx = \frac{3\,cd^2-bde+2\,(2\,cde-be^2)x}{2\,(e^5x^2+2\,de^4x+d^2e^3)} + \frac{c\log{(ex+d)}}{e^3}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*(3*c*d^2 - b*d*e + 2*(2*c*d*e - b*e^2)*x)/(e^5*x^2 + 2*d*e^4*x + d^2*e
^3) + c*log(e*x + d)/e^3
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{bx+cx^2}{(d+ex)^3}\,dx = \frac{c\log{(|ex+d|)}}{e^3} + \frac{2\,(2\,cd-be)x+\frac{3\,cd^2-bde}{e}}{2\,(ex+d)^2e^2}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^3,x, algorithm="giac")
```

output
```
c*log(abs(e*x + d))/e^3 + 1/2*(2*(2*c*d - b*e)*x + (3*c*d^2 - b*d*e)/e)/((
e*x + d)^2*e^2)
```

Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{bx+cx^2}{(d+ex)^3}\,dx = \frac{\frac{3\,c\,d^2-b\,d\,e}{2\,e^3}-\frac{x\,(b\,e-2\,c\,d)}{e^2}}{d^2+2\,d\,e\,x+e^2\,x^2}+\frac{c\,\ln\left(d+e\,x\right)}{e^3}$$

input
```
int((b*x + c*x^2)/(d + e*x)^3,x)
```

output
```
((3*c*d^2 - b*d*e)/(2*e^3) - (x*(b*e - 2*c*d))/e^2)/(d^2 + e^2*x^2 + 2*d*e
*x) + (c*log(d + e*x))/e^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.67

$$\int \frac{bx+cx^2}{(d+ex)^3}\,dx = \frac{2\log(ex+d)\,c\,d^3+4\log(ex+d)\,c\,d^2ex+2\log(ex+d)\,cd\,e^2x^2+b\,e^3x^2+c\,d^3-2cd\,e^2x^2}{2d\,e^3\left(e^2x^2+2dex+d^2\right)}$$

input
```
int((c*x^2+b*x)/(e*x+d)^3,x)
```

output
```
(2*log(d + e*x)*c*d**3 + 4*log(d + e*x)*c*d**2*e*x + 2*log(d + e*x)*c*d*e*
*2*x**2 + b*e**3*x**2 + c*d**3 - 2*c*d*e**2*x**2)/(2*d*e**3*(d**2 + 2*d*e*
x + e**2*x**2))
```

3.25 $\int \frac{bx+cx^2}{(d+ex)^4} \, dx$

Optimal result 253
Mathematica [A] (verified) 253
Rubi [A] (verified) 254
Maple [A] (verified) 255
Fricas [A] (verification not implemented) 255
Sympy [A] (verification not implemented) 256
Maxima [A] (verification not implemented) 256
Giac [A] (verification not implemented) 256
Mupad [B] (verification not implemented) 257
Reduce [B] (verification not implemented) 257

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \frac{bx+cx^2}{(d+ex)^4} \, dx = -\frac{d(cd-be)}{3e^3(d+ex)^3} + \frac{2cd-be}{2e^3(d+ex)^2} - \frac{c}{e^3(d+ex)}$$

output
```
-1/3*d*(-b*e+c*d)/e^3/(e*x+d)^3+1/2*(-b*e+2*c*d)/e^3/(e*x+d)^2-c/e^3/(e*x+
d)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \frac{bx+cx^2}{(d+ex)^4} \, dx = -\frac{be(d+3ex)+2c(d^2+3dex+3e^2x^2)}{6e^3(d+ex)^3}$$

input
```
Integrate[(b*x + c*x^2)/(d + e*x)^4,x]
```

output
```
-1/6*(b*e*(d + 3*e*x) + 2*c*(d^2 + 3*d*e*x + 3*e^2*x^2))/(e^3*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+cx^2}{(d+ex)^4}\,dx$$
$$\Big\downarrow \text{ 1140}$$
$$\int \left(\frac{be-2cd}{e^2(d+ex)^3}+\frac{d(cd-be)}{e^2(d+ex)^4}+\frac{c}{e^2(d+ex)^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{2cd-be}{2e^3(d+ex)^2}-\frac{d(cd-be)}{3e^3(d+ex)^3}-\frac{c}{e^3(d+ex)}$$

input
```
Int[(b*x + c*x^2)/(d + e*x)^4,x]
```

output
```
-1/3*(d*(c*d - b*e))/(e^3*(d + e*x)^3) + (2*c*d - b*e)/(2*e^3*(d + e*x)^2)
 - c/(e^3*(d + e*x))
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{6x^2c\,e^2+3xb\,e^2+6cdxe+bde+2c\,d^2}{6e^3(ex+d)^3}$	46
norman	$\frac{-\frac{c\,x^2}{e}-\frac{(be+2cd)x}{2e^2}-\frac{d(be+2cd)}{6e^3}}{(ex+d)^3}$	47
risch	$\frac{-\frac{c\,x^2}{e}-\frac{(be+2cd)x}{2e^2}-\frac{d(be+2cd)}{6e^3}}{(ex+d)^3}$	47
parallelrisch	$\frac{-6x^2c\,e^2-3xb\,e^2-6cdxe-bde-2c\,d^2}{6e^3(ex+d)^3}$	47
default	$\frac{d(be-cd)}{3e^3(ex+d)^3}-\frac{c}{e^3(ex+d)}-\frac{be-2cd}{2e^3(ex+d)^2}$	56
orering	$-\frac{(6x^2c\,e^2+3xb\,e^2+6cdxe+bde+2c\,d^2)(c\,x^2+bx)}{6e^3(cx+b)(ex+d)^3x}$	65

input
```
int((c*x^2+b*x)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(6*c*e^2*x^2+3*b*e^2*x+6*c*d*e*x+b*d*e+2*c*d^2)/e^3/(e*x+d)^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{bx+cx^2}{(d+ex)^4}\,dx = -\frac{6\,ce^2x^2+2\,cd^2+bde+3\,(2\,cde+be^2)x}{6\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
-1/6*(6*c*e^2*x^2 + 2*c*d^2 + b*d*e + 3*(2*c*d*e + b*e^2)*x)/(e^6*x^3 + 3*
d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.25

$$\int \frac{bx+cx^2}{(d+ex)^4}\,dx = \frac{-bde-2cd^2-6ce^2x^2+x(-3be^2-6cde)}{6d^3e^3+18d^2e^4x+18de^5x^2+6e^6x^3}$$

input
```
integrate((c*x**2+b*x)/(e*x+d)**4,x)
```

output
```
(-b*d*e - 2*c*d**2 - 6*c*e**2*x**2 + x*(-3*b*e**2 - 6*c*d*e))/(6*d**3*e**3
 + 18*d**2*e**4*x + 18*d*e**5*x**2 + 6*e**6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18

$$\int \frac{bx+cx^2}{(d+ex)^4}\,dx = -\frac{6\,ce^2x^2+2\,cd^2+bde+3\,(2\,cde+be^2)x}{6\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
-1/6*(6*c*e^2*x^2 + 2*c*d^2 + b*d*e + 3*(2*c*d*e + b*e^2)*x)/(e^6*x^3 + 3*
d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int \frac{bx+cx^2}{(d+ex)^4}\,dx = -\frac{6\,ce^2x^2+6\,cdex+3\,be^2x+2\,cd^2+bde}{6\,(ex+d)^3e^3}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^4,x, algorithm="giac")
```

output
```
-1/6*(6*c*e^2*x^2 + 6*c*d*e*x + 3*b*e^2*x + 2*c*d^2 + b*d*e)/((e*x + d)^3*
e^3)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int \frac{bx+cx^2}{(d+ex)^4}\,dx = -\frac{\frac{d\,(b\,e+2\,c\,d)}{6\,e^3}+\frac{x\,(b\,e+2\,c\,d)}{2\,e^2}+\frac{c\,x^2}{e}}{d^3+3\,d^2\,e\,x+3\,d\,e^2\,x^2+e^3\,x^3}$$

input
```
int((b*x + c*x^2)/(d + e*x)^4,x)
```

output
```
-((d*(b*e + 2*c*d))/(6*e^3) + (x*(b*e + 2*c*d))/(2*e^2) + (c*x^2)/e)/(d^3
+ e^3*x^3 + 3*d*e^2*x^2 + 3*d^2*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{bx+cx^2}{(d+ex)^4}\,dx = \frac{2c\,e^2x^3-3bdex-b\,d^2}{6d\,e^2\,(e^3x^3+3d\,e^2x^2+3d^2ex+d^3)}$$

input
```
int((c*x^2+b*x)/(e*x+d)^4,x)
```

output
```
( - b*d**2 - 3*b*d*e*x + 2*c*e**2*x**3)/(6*d*e**2*(d**3 + 3*d**2*e*x + 3*d
*e**2*x**2 + e**3*x**3))
```

3.26 $\int \frac{bx+cx^2}{(d+ex)^5} \, dx$

Optimal result 258
Mathematica [A] (verified) 258
Rubi [A] (verified) 259
Maple [A] (verified) 260
Fricas [A] (verification not implemented) 260
Sympy [A] (verification not implemented) 261
Maxima [A] (verification not implemented) 261
Giac [A] (verification not implemented) 261
Mupad [B] (verification not implemented) 262
Reduce [B] (verification not implemented) 262

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \frac{bx+cx^2}{(d+ex)^5} \, dx = -\frac{d(cd-be)}{4e^3(d+ex)^4} + \frac{2cd-be}{3e^3(d+ex)^3} - \frac{c}{2e^3(d+ex)^2}$$

output
```
-1/4*d*(-b*e+c*d)/e^3/(e*x+d)^4+1/3*(-b*e+2*c*d)/e^3/(e*x+d)^3-1/2*c/e^3/(
e*x+d)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.69

$$\int \frac{bx+cx^2}{(d+ex)^5} \, dx = -\frac{be(d+4ex)+c(d^2+4dex+6e^2x^2)}{12e^3(d+ex)^4}$$

input
```
Integrate[(b*x + c*x^2)/(d + e*x)^5,x]
```

output
```
-1/12*(b*e*(d + 4*e*x) + c*(d^2 + 4*d*e*x + 6*e^2*x^2))/(e^3*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+cx^2}{(d+ex)^5} \, dx$$

$$\Big\downarrow \; 1140$$

$$\int \left(\frac{be-2cd}{e^2(d+ex)^4}+\frac{d(cd-be)}{e^2(d+ex)^5}+\frac{c}{e^2(d+ex)^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2cd-be}{3e^3(d+ex)^3}-\frac{d(cd-be)}{4e^3(d+ex)^4}-\frac{c}{2e^3(d+ex)^2}$$

input
```
Int[(b*x + c*x^2)/(d + e*x)^5,x]
```

output
```
-1/4*(d*(c*d - b*e))/(e^3*(d + e*x)^4) + (2*c*d - b*e)/(3*e^3*(d + e*x)^3)
 - c/(2*e^3*(d + e*x)^2)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

method	result	size
gosper	$-\frac{6x^{2}c\,e^{2}+4xb\,e^{2}+4cdxe+bde+c\,d^{2}}{12e^{3}(ex+d)^{4}}$	45
risch	$\frac{-\frac{c\,x^{2}}{2e}-\frac{(be+cd)x}{3e^{2}}-\frac{d(be+cd)}{12e^{3}}}{(ex+d)^{4}}$	45
norman	$\frac{-\frac{c\,x^{2}}{2e}-\frac{\left(b\,e^{2}+dec\right)x}{3e^{3}}-\frac{d\left(b\,e^{2}+dec\right)}{12e^{4}}}{(ex+d)^{4}}$	51
parallelrisch	$\frac{-6c\,x^{2}e^{3}-4b\,e^{3}x-4cd\,e^{2}x-bd\,e^{2}-d^{2}ec}{12e^{4}(ex+d)^{4}}$	52
default	$-\frac{be-2cd}{3e^{3}(ex+d)^{3}}+\frac{d(be-cd)}{4e^{3}(ex+d)^{4}}-\frac{c}{2e^{3}(ex+d)^{2}}$	56
orering	$-\frac{\left(6x^{2}c\,e^{2}+4xb\,e^{2}+4cdxe+bde+c\,d^{2}\right)\left(c\,x^{2}+bx\right)}{12e^{3}(cx+b)(ex+d)^{4}x}$	64

input
```
int((c*x^2+b*x)/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/12/e^3*(6*c*e^2*x^2+4*b*e^2*x+4*c*d*e*x+b*d*e+c*d^2)/(e*x+d)^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29

$$\int \frac{bx+cx^2}{(d+ex)^5}\,dx=-\frac{6\,ce^2x^2+cd^2+bde+4\,(cde+be^2)x}{12\,(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3)}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^5,x, algorithm="fricas")
```

output
```
-1/12*(6*c*e^2*x^2 + c*d^2 + b*d*e + 4*(c*d*e + b*e^2)*x)/(e^7*x^4 + 4*d*e
^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.37

$$\int \frac{bx+cx^2}{(d+ex)^5}\,dx = \frac{-bde-cd^2-6ce^2x^2+x(-4be^2-4cde)}{12d^4e^3+48d^3e^4x+72d^2e^5x^2+48de^6x^3+12e^7x^4}$$

input
```
integrate((c*x**2+b*x)/(e*x+d)**5,x)
```

output
```
(-b*d*e - c*d**2 - 6*c*e**2*x**2 + x*(-4*b*e**2 - 4*c*d*e))/(12*d**4*e**3
+ 48*d**3*e**4*x + 72*d**2*e**5*x**2 + 48*d*e**6*x**3 + 12*e**7*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29

$$\int \frac{bx+cx^2}{(d+ex)^5}\,dx = -\frac{6\,ce^2x^2+cd^2+bde+4\,(cde+be^2)x}{12\,(e^7x^4+4\,de^6x^3+6\,d^2e^5x^2+4\,d^3e^4x+d^4e^3)}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^5,x, algorithm="maxima")
```

output
```
-1/12*(6*c*e^2*x^2 + c*d^2 + b*d*e + 4*(c*d*e + b*e^2)*x)/(e^7*x^4 + 4*d*e
^6*x^3 + 6*d^2*e^5*x^2 + 4*d^3*e^4*x + d^4*e^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int \frac{bx+cx^2}{(d+ex)^5}\,dx = -\frac{\frac{6\,c}{(ex+d)^2e^2}-\frac{8\,cd}{(ex+d)^3e^2}+\frac{3\,cd^2}{(ex+d)^4e^2}+\frac{4\,b}{(ex+d)^3e}-\frac{3\,bd}{(ex+d)^4e}}{12\,e}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^5,x, algorithm="giac")
```

output
```
-1/12*(6*c/((e*x + d)^2*e^2) - 8*c*d/((e*x + d)^3*e^2) + 3*c*d^2/((e*x + d
)^4*e^2) + 4*b/((e*x + d)^3*e) - 3*b*d/((e*x + d)^4*e))/e
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.26

$$\int \frac{bx+cx^2}{(d+ex)^5}\,dx = -\frac{\frac{d\,(b\,e+c\,d)}{12\,e^3}+\frac{x\,(b\,e+c\,d)}{3\,e^2}+\frac{c\,x^2}{2\,e}}{d^4+4\,d^3\,e\,x+6\,d^2\,e^2\,x^2+4\,d\,e^3\,x^3+e^4\,x^4}$$

input
```
int((b*x + c*x^2)/(d + e*x)^5,x)
```

output
```
-((d*(b*e + c*d))/(12*e^3) + (x*(b*e + c*d))/(3*e^2) + (c*x^2)/(2*e))/(d^4
 + e^4*x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.27

$$\int \frac{bx+cx^2}{(d+ex)^5}\,dx = \frac{-6c\,e^2x^2-4b\,e^2x-4cdex-bde-c\,d^2}{12e^3\,(e^4x^4+4d\,e^3x^3+6d^2e^2x^2+4d^3ex+d^4)}$$

input
```
int((c*x^2+b*x)/(e*x+d)^5,x)
```

output
```
( - b*d*e - 4*b*e**2*x - c*d**2 - 4*c*d*e*x - 6*c*e**2*x**2)/(12*e**3*(d**
4 + 4*d**3*e*x + 6*d**2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))
```

3.27 $\int (d+ex)^4 (bx+cx^2)^2 \, dx$

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Optimal result

Integrand size = 19, antiderivative size = 137

$$\int (d+ex)^4 (bx+cx^2)^2 \, dx = \frac{d^2(cd-be)^2(d+ex)^5}{5e^5} - \frac{d(cd-be)(2cd-be)(d+ex)^6}{3e^5} + \frac{(6c^2d^2-6bcde+b^2e^2)(d+ex)^7}{7e^5} - \frac{c(2cd-be)(d+ex)^8}{4e^5} + \frac{c^2(d+ex)^9}{9e^5}$$

output
```
1/5*d^2*(-b*e+c*d)^2*(e*x+d)^5/e^5-1/3*d*(-b*e+c*d)*(-b*e+2*c*d)*(e*x+d)^6
/e^5+1/7*(b^2*e^2-6*b*c*d*e+6*c^2*d^2)*(e*x+d)^7/e^5-1/4*c*(-b*e+2*c*d)*(e
*x+d)^8/e^5+1/9*c^2*(e*x+d)^9/e^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.16

$$\int (d+ex)^4 (bx+cx^2)^2 \, dx = \frac{1}{3}b^2d^4x^3 + \frac{1}{2}bd^3(cd+2be)x^4 + \frac{1}{5}d^2(c^2d^2+8bcde+6b^2e^2)x^5 + \frac{2}{3}de(c^2d^2+3bcde+b^2e^2)x^6 + \frac{1}{7}e^2(6c^2d^2+8bcde+b^2e^2)x^7 + \frac{1}{4}ce^3(2cd+be)x^8 + \frac{1}{9}c^2e^4x^9$$

input
```
Integrate[(d + e*x)^4*(b*x + c*x^2)^2,x]
```

output
```
(b^2*d^4*x^3)/3 + (b*d^3*(c*d + 2*b*e)*x^4)/2 + (d^2*(c^2*d^2 + 8*b*c*d*e
+ 6*b^2*e^2)*x^5)/5 + (2*d*e*(c^2*d^2 + 3*b*c*d*e + b^2*e^2)*x^6)/3 + (e^2
*(6*c^2*d^2 + 8*b*c*d*e + b^2*e^2)*x^7)/7 + (c*e^3*(2*c*d + b*e)*x^8)/4 +
(c^2*e^4*x^9)/9
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^2 (d+ex)^4 \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{(d+ex)^6 \left(b^2e^2-6bcde+6c^2d^2\right)}{e^4}+\frac{d^2(d+ex)^4(cd-be)^2}{e^4}-\frac{2c(d+ex)^7(2cd-be)}{e^4}+\frac{2d(d+ex)^5(cd-be)}{e^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(d+ex)^7 \left(b^2e^2-6bcde+6c^2d^2\right)}{7e^5}+\frac{d^2(d+ex)^5(cd-be)^2}{5e^5}-\frac{c(d+ex)^8(2cd-be)}{4e^5}-\frac{d(d+ex)^6(cd-be)(2cd-be)}{3e^5}+\frac{c^2(d+ex)^9}{9e^5}$$

input
```
Int[(d + e*x)^4*(b*x + c*x^2)^2,x]
```

output
```
(d^2*(c*d - b*e)^2*(d + e*x)^5)/(5*e^5) - (d*(c*d - b*e)*(2*c*d - b*e)*(d
+ e*x)^6)/(3*e^5) + ((6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*x)^7)/(7*e^5
) - (c*(2*c*d - b*e)*(d + e*x)^8)/(4*e^5) + (c^2*(d + e*x)^9)/(9*e^5)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.18

method	result
norman	$\frac{e^4c^2x^9}{9} + \left(\frac{1}{4}e^4bc + \frac{1}{2}d\,e^3c^2\right)x^8 + \left(\frac{1}{7}e^4b^2 + \frac{8}{7}d\,e^3bc + \frac{6}{7}d^2e^2c^2\right)x^7 + \big(\frac{2}{3}d\,e^3b^2 + 2d^2e^2bc + \frac{2}{3}d^3$
default	$\frac{e^4c^2x^9}{9} + \frac{(2e^4bc+4d\,e^3c^2)x^8}{8} + \frac{(e^4b^2+8d\,e^3bc+6d^2e^2c^2)x^7}{7} + \frac{(4d\,e^3b^2+12d^2e^2bc+4d^3e\,c^2)x^6}{6} + \frac{(6d^2e^2b^2+8d^3ebc+}{5}$
gosper	$\frac{x^3\big(140e^4c^2x^6+315x^5e^4bc+630x^5d\,e^3c^2+180x^4e^4b^2+1440x^4d\,e^3bc+1080x^4d^2e^2c^2+840x^3d\,e^3b^2+2520x^3d^2e^2bc+840x^3d^3e}{1260}$
risch	$\frac{1}{9}e^4c^2x^9 + \frac{1}{4}x^8e^4bc + \frac{1}{2}d\,e^3c^2x^8 + \frac{1}{7}x^7e^4b^2 + \frac{8}{7}x^7d\,e^3bc + \frac{6}{7}x^7d^2e^2c^2 + \frac{2}{3}x^6d\,e^3b^2 + 2x^6d^2e^2b$
parallelrisch	$\frac{1}{9}e^4c^2x^9 + \frac{1}{4}x^8e^4bc + \frac{1}{2}d\,e^3c^2x^8 + \frac{1}{7}x^7e^4b^2 + \frac{8}{7}x^7d\,e^3bc + \frac{6}{7}x^7d^2e^2c^2 + \frac{2}{3}x^6d\,e^3b^2 + 2x^6d^2e^2b$
orering	$\frac{x\big(140e^4c^2x^6+315x^5e^4bc+630x^5d\,e^3c^2+180x^4e^4b^2+1440x^4d\,e^3bc+1080x^4d^2e^2c^2+840x^3d\,e^3b^2+2520x^3d^2e^2bc+840x^3d^3e\,c}{1260(cx+b)^2}$

input
```
int((e*x+d)^4*(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/9*e^4*c^2*x^9+(1/4*e^4*b*c+1/2*d*e^3*c^2)*x^8+(1/7*e^4*b^2+8/7*d*e^3*b*c
+6/7*d^2*e^2*c^2)*x^7+(2/3*d*e^3*b^2+2*d^2*e^2*b*c+2/3*d^3*e*c^2)*x^6+(6/5
*d^2*e^2*b^2+8/5*d^3*e*b*c+1/5*c^2*d^4)*x^5+(d^3*e*b^2+1/2*d^4*b*c)*x^4+1/
3*b^2*d^4*x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

$$\int (d+ex)^4 \left(bx+cx^2\right)^2 \, dx = \frac{1}{9}\,c^2e^4x^9 + \frac{1}{3}\,b^2d^4x^3 + \frac{1}{4}\,\left(2\,c^2de^3 + bce^4\right)x^8 + \frac{1}{7}\,\left(6\,c^2d^2e^2 + 8\,bcde^3 + b^2e^4\right)x^7 + \frac{2}{3}\,\left(c^2d^3e + 3\,bcd^2e^2 + b^2de^3\right)x^6 + \frac{1}{5}\,\left(c^2d^4 + 8\,bcd^3e + 6\,b^2d^2e^2\right)x^5 + \frac{1}{2}\,\left(bcd^4 + 2\,b^2d^3e\right)x^4$$

input
```
integrate((e*x+d)^4*(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
1/9*c^2*e^4*x^9 + 1/3*b^2*d^4*x^3 + 1/4*(2*c^2*d*e^3 + b*c*e^4)*x^8 + 1/7*
(6*c^2*d^2*e^2 + 8*b*c*d*e^3 + b^2*e^4)*x^7 + 2/3*(c^2*d^3*e + 3*b*c*d^2*e
^2 + b^2*d*e^3)*x^6 + 1/5*(c^2*d^4 + 8*b*c*d^3*e + 6*b^2*d^2*e^2)*x^5 + 1/
2*(b*c*d^4 + 2*b^2*d^3*e)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.30

$$\int (d+ex)^4 \left(bx+cx^2\right)^2 \, dx = \frac{b^2d^4x^3}{3} + \frac{c^2e^4x^9}{9} + x^8\left(\frac{bce^4}{4} + \frac{c^2de^3}{2}\right) + x^7\left(\frac{b^2e^4}{7} + \frac{8bcde^3}{7} + \frac{6c^2d^2e^2}{7}\right) + x^6 \cdot \left(\frac{2b^2de^3}{3} + 2bcd^2e^2 + \frac{2c^2d^3e}{3}\right) + x^5 \cdot \left(\frac{6b^2d^2e^2}{5} + \frac{8bcd^3e}{5} + \frac{c^2d^4}{5}\right) + x^4\left(b^2d^3e + \frac{bcd^4}{2}\right)$$

input
```
integrate((e*x+d)**4*(c*x**2+b*x)**2,x)
```

output
```
b**2*d**4*x**3/3 + c**2*e**4*x**9/9 + x**8*(b*c*e**4/4 + c**2*d*e**3/2) +
x**7*(b**2*e**4/7 + 8*b*c*d*e**3/7 + 6*c**2*d**2*e**2/7) + x**6*(2*b**2*d*
e**3/3 + 2*b*c*d**2*e**2 + 2*c**2*d**3*e/3) + x**5*(6*b**2*d**2*e**2/5 + 8
*b*c*d**3*e/5 + c**2*d**4/5) + x**4*(b**2*d**3*e + b*c*d**4/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

$$\int (d+ex)^4 \left(bx+cx^2\right)^2 \, dx = \frac{1}{9}\,c^2e^4x^9 + \frac{1}{3}\,b^2d^4x^3 + \frac{1}{4}\,\left(2\,c^2de^3 + bce^4\right)x^8 + \frac{1}{7}\,\left(6\,c^2d^2e^2 + 8\,bcde^3 + b^2e^4\right)x^7 + \frac{2}{3}\,\left(c^2d^3e + 3\,bcd^2e^2 + b^2de^3\right)x^6 + \frac{1}{5}\,\left(c^2d^4 + 8\,bcd^3e + 6\,b^2d^2e^2\right)x^5 + \frac{1}{2}\,\left(bcd^4 + 2\,b^2d^3e\right)x^4$$

input
```
integrate((e*x+d)^4*(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
1/9*c^2*e^4*x^9 + 1/3*b^2*d^4*x^3 + 1/4*(2*c^2*d*e^3 + b*c*e^4)*x^8 + 1/7*
(6*c^2*d^2*e^2 + 8*b*c*d*e^3 + b^2*e^4)*x^7 + 2/3*(c^2*d^3*e + 3*b*c*d^2*e
^2 + b^2*d*e^3)*x^6 + 1/5*(c^2*d^4 + 8*b*c*d^3*e + 6*b^2*d^2*e^2)*x^5 + 1/
2*(b*c*d^4 + 2*b^2*d^3*e)*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.28

$$\int (d+ex)^4 \left(bx+cx^2\right)^2 \, dx = \frac{1}{9}\,c^2e^4x^9 + \frac{1}{2}\,c^2de^3x^8 + \frac{1}{4}\,bce^4x^8 + \frac{6}{7}\,c^2d^2e^2x^7 + \frac{8}{7}\,bcde^3x^7 + \frac{1}{7}\,b^2e^4x^7 + \frac{2}{3}\,c^2d^3ex^6 + 2\,bcd^2e^2x^6 + \frac{2}{3}\,b^2de^3x^6 + \frac{1}{5}\,c^2d^4x^5 + \frac{8}{5}\,bcd^3ex^5 + \frac{6}{5}\,b^2d^2e^2x^5 + \frac{1}{2}\,bcd^4x^4 + b^2d^3ex^4 + \frac{1}{3}\,b^2d^4x^3$$

input
```
integrate((e*x+d)^4*(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
1/9*c^2*e^4*x^9 + 1/2*c^2*d*e^3*x^8 + 1/4*b*c*e^4*x^8 + 6/7*c^2*d^2*e^2*x^
7 + 8/7*b*c*d*e^3*x^7 + 1/7*b^2*e^4*x^7 + 2/3*c^2*d^3*e*x^6 + 2*b*c*d^2*e^
2*x^6 + 2/3*b^2*d*e^3*x^6 + 1/5*c^2*d^4*x^5 + 8/5*b*c*d^3*e*x^5 + 6/5*b^2*
d^2*e^2*x^5 + 1/2*b*c*d^4*x^4 + b^2*d^3*e*x^4 + 1/3*b^2*d^4*x^3
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.09

$$\int (d+ex)^4 \left(bx+cx^2\right)^2 \, dx = x^5 \left(\frac{6\,b^2\,d^2\,e^2}{5}+\frac{8\,b\,c\,d^3\,e}{5}+\frac{c^2\,d^4}{5}\right) + x^7 \left(\frac{b^2\,e^4}{7}+\frac{8\,b\,c\,d\,e^3}{7}+\frac{6\,c^2\,d^2\,e^2}{7}\right) + \frac{b^2\,d^4\,x^3}{3}+\frac{c^2\,e^4\,x^9}{9}+\frac{b\,d^3\,x^4\,(2\,b\,e+c\,d)}{2} + \frac{c\,e^3\,x^8\,(b\,e+2\,c\,d)}{4}+\frac{2\,d\,e\,x^6\,(b^2\,e^2+3\,b\,c\,d\,e+c^2\,d^2)}{3}$$

input
```
int((b*x + c*x^2)^2*(d + e*x)^4,x)
```

output
```
x^5*((c^2*d^4)/5 + (6*b^2*d^2*e^2)/5 + (8*b*c*d^3*e)/5) + x^7*((b^2*e^4)/7
 + (6*c^2*d^2*e^2)/7 + (8*b*c*d*e^3)/7) + (b^2*d^4*x^3)/3 + (c^2*e^4*x^9)/
9 + (b*d^3*x^4*(2*b*e + c*d))/2 + (c*e^3*x^8*(b*e + 2*c*d))/4 + (2*d*e*x^6
*(b^2*e^2 + c^2*d^2 + 3*b*c*d*e))/3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.27

$$\int (d+ex)^4 \left(bx+cx^2\right)^2 \, dx = \frac{x^3(140c^2e^4x^6+315bc\,e^4x^5+630c^2d\,e^3x^5+180b^2e^4x^4+1440bcd\,e^3x^4+1080c^2d^2e^2x^4+840b^2d\,e^3x^3+2}{}$$

input
```
int((e*x+d)^4*(c*x^2+b*x)^2,x)
```

output

```
(x**3*(420*b**2*d**4 + 1260*b**2*d**3*e*x + 1512*b**2*d**2*e**2*x**2 + 840
*b**2*d*e**3*x**3 + 180*b**2*e**4*x**4 + 630*b*c*d**4*x + 2016*b*c*d**3*e*
x**2 + 2520*b*c*d**2*e**2*x**3 + 1440*b*c*d*e**3*x**4 + 315*b*c*e**4*x**5
+ 252*c**2*d**4*x**2 + 840*c**2*d**3*e*x**3 + 1080*c**2*d**2*e**2*x**4 + 6
30*c**2*d*e**3*x**5 + 140*c**2*e**4*x**6))/1260
```

3.28 $\int (d+ex)^3 \left(bx+cx^2\right)^2 \, dx$

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Reduce [B] (verification not implemented) 275

Optimal result

Integrand size = 19, antiderivative size = 127

$$\int (d+ex)^3 \left(bx+cx^2\right)^2 \, dx = \frac{1}{3}b^2d^3x^3 + \frac{1}{4}bd^2(2cd+3be)x^4 + \frac{1}{5}d\left(c^2d^2+6bcde+3b^2e^2\right)x^5 + \frac{1}{6}e\left(3c^2d^2+6bcde+b^2e^2\right)x^6 + \frac{1}{7}ce^2(3cd+2be)x^7 + \frac{1}{8}c^2e^3x^8$$

output
```
1/3*b^2*d^3*x^3+1/4*b*d^2*(3*b*e+2*c*d)*x^4+1/5*d*(3*b^2*e^2+6*b*c*d*e+c^2
*d^2)*x^5+1/6*e*(b^2*e^2+6*b*c*d*e+3*c^2*d^2)*x^6+1/7*c*e^2*(2*b*e+3*c*d)*
x^7+1/8*c^2*e^3*x^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int (d+ex)^3 \left(bx+cx^2\right)^2 \, dx = \frac{1}{3}b^2d^3x^3 + \frac{1}{4}bd^2(2cd+3be)x^4 + \frac{1}{5}d\left(c^2d^2+6bcde+3b^2e^2\right)x^5 + \frac{1}{6}e\left(3c^2d^2+6bcde+b^2e^2\right)x^6 + \frac{1}{7}ce^2(3cd+2be)x^7 + \frac{1}{8}c^2e^3x^8$$

input
```
Integrate[(d + e*x)^3*(b*x + c*x^2)^2,x]
```

output
```
(b^2*d^3*x^3)/3 + (b*d^2*(2*c*d + 3*b*e)*x^4)/4 + (d*(c^2*d^2 + 6*b*c*d*e
+ 3*b^2*e^2)*x^5)/5 + (e*(3*c^2*d^2 + 6*b*c*d*e + b^2*e^2)*x^6)/6 + (c*e^2
*(3*c*d + 2*b*e)*x^7)/7 + (c^2*e^3*x^8)/8
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^2 (d+ex)^3\,dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(ex^5\left(b^2e^2+6bcde+3c^2d^2\right)+dx^4\left(3b^2e^2+6bcde+c^2d^2\right)+b^2d^3x^2+bd^2x^3(3be+2cd)+ce^2x^6(2be+3cd)+\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6}ex^6\left(b^2e^2+6bcde+3c^2d^2\right)+\frac{1}{5}dx^5\left(3b^2e^2+6bcde+c^2d^2\right)+\frac{1}{3}b^2d^3x^3+\frac{1}{4}bd^2x^4(3be+2cd)+\frac{1}{7}ce^2x^7(2be+3cd)+\frac{1}{8}c^2e^3x^8$$

input
```
Int[(d + e*x)^3*(b*x + c*x^2)^2,x]
```

output
```
(b^2*d^3*x^3)/3 + (b*d^2*(2*c*d + 3*b*e)*x^4)/4 + (d*(c^2*d^2 + 6*b*c*d*e
+ 3*b^2*e^2)*x^5)/5 + (e*(3*c^2*d^2 + 6*b*c*d*e + b^2*e^2)*x^6)/6 + (c*e^2
*(3*c*d + 2*b*e)*x^7)/7 + (c^2*e^3*x^8)/8
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

method	result
norman	$\frac{e^3 c^2 x^8}{8} + \left(\frac{2}{7} e^3 b c + \frac{3}{7} d e^2 c^2\right) x^7 + \left(\frac{1}{6} e^3 b^2 + d e^2 b c + \frac{1}{2} d^2 e c^2\right) x^6 + \left(\frac{3}{5} d e^2 b^2 + \frac{6}{5} d^2 e b c + \frac{1}{5} c^2 d^3\right)$
default	$\frac{e^3 c^2 x^8}{8} + \frac{(2 e^3 b c + 3 d e^2 c^2) x^7}{7} + \frac{(e^3 b^2 + 6 d e^2 b c + 3 d^2 e c^2) x^6}{6} + \frac{(3 d e^2 b^2 + 6 d^2 e b c + c^2 d^3) x^5}{5} + \frac{(3 d^2 e b^2 + 2 b c d^3) x^4}{4} + b$
gosper	$\frac{x^3 (105 e^3 c^2 x^5 + 240 x^4 e^3 b c + 360 x^4 d e^2 c^2 + 140 x^3 e^3 b^2 + 840 x^3 d e^2 b c + 420 x^3 d^2 e c^2 + 504 x^2 d e^2 b^2 + 1008 x^2 d^2 e b c + 168 c^2 d^3 x^2 + 6}{840}$
risch	$\frac{1}{8} e^3 c^2 x^8 + \frac{2}{7} x^7 e^3 b c + \frac{3}{7} d e^2 c^2 x^7 + \frac{1}{6} x^6 e^3 b^2 + x^6 d e^2 b c + \frac{1}{2} x^6 d^2 e c^2 + \frac{3}{5} x^5 d e^2 b^2 + \frac{6}{5} x^5 d^2 e b c +$
parallelrisch	$\frac{1}{8} e^3 c^2 x^8 + \frac{2}{7} x^7 e^3 b c + \frac{3}{7} d e^2 c^2 x^7 + \frac{1}{6} x^6 e^3 b^2 + x^6 d e^2 b c + \frac{1}{2} x^6 d^2 e c^2 + \frac{3}{5} x^5 d e^2 b^2 + \frac{6}{5} x^5 d^2 e b c +$
orering	$\frac{x (105 e^3 c^2 x^5 + 240 x^4 e^3 b c + 360 x^4 d e^2 c^2 + 140 x^3 e^3 b^2 + 840 x^3 d e^2 b c + 420 x^3 d^2 e c^2 + 504 x^2 d e^2 b^2 + 1008 x^2 d^2 e b c + 168 c^2 d^3 x^2 + 63}{840 (c x + b)^2}$

input
```
int((e*x+d)^3*(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*e^3*c^2*x^8+(2/7*e^3*b*c+3/7*d*e^2*c^2)*x^7+(1/6*e^3*b^2+d*e^2*b*c+1/2
*d^2*e*c^2)*x^6+(3/5*d*e^2*b^2+6/5*d^2*e*b*c+1/5*c^2*d^3)*x^5+(3/4*d^2*e*b
^2+1/2*b*c*d^3)*x^4+1/3*b^2*d^3*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int (d+ex)^3 \left(bx+cx^2\right)^2 \, dx = \frac{1}{8}\, c^2 e^3 x^8 + \frac{1}{3}\, b^2 d^3 x^3 + \frac{1}{7}\, \left(3\, c^2 d e^2 + 2\, b c e^3\right) x^7 + \frac{1}{6}\, \left(3\, c^2 d^2 e + 6\, b c d e^2 + b^2 e^3\right) x^6 + \frac{1}{5}\, \left(c^2 d^3 + 6\, b c d^2 e + 3\, b^2 d e^2\right) x^5 + \frac{1}{4}\, \left(2\, b c d^3 + 3\, b^2 d^2 e\right) x^4$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
1/8*c^2*e^3*x^8 + 1/3*b^2*d^3*x^3 + 1/7*(3*c^2*d*e^2 + 2*b*c*e^3)*x^7 + 1/
6*(3*c^2*d^2*e + 6*b*c*d*e^2 + b^2*e^3)*x^6 + 1/5*(c^2*d^3 + 6*b*c*d^2*e +
 3*b^2*d*e^2)*x^5 + 1/4*(2*b*c*d^3 + 3*b^2*d^2*e)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.09

$$\int (d+ex)^3 \left(bx+cx^2\right)^2 \, dx = \frac{b^2 d^3 x^3}{3} + \frac{c^2 e^3 x^8}{8} + x^7 \cdot \left(\frac{2bce^3}{7} + \frac{3c^2 de^2}{7}\right) + x^6 \left(\frac{b^2 e^3}{6} + bcde^2 + \frac{c^2 d^2 e}{2}\right) + x^5 \cdot \left(\frac{3b^2 de^2}{5} + \frac{6bcd^2 e}{5} + \frac{c^2 d^3}{5}\right) + x^4 \cdot \left(\frac{3b^2 d^2 e}{4} + \frac{bcd^3}{2}\right)$$

input
```
integrate((e*x+d)**3*(c*x**2+b*x)**2,x)
```

output
```
b**2*d**3*x**3/3 + c**2*e**3*x**8/8 + x**7*(2*b*c*e**3/7 + 3*c**2*d*e**2/7
) + x**6*(b**2*e**3/6 + b*c*d*e**2 + c**2*d**2*e/2) + x**5*(3*b**2*d*e**2/
5 + 6*b*c*d**2*e/5 + c**2*d**3/5) + x**4*(3*b**2*d**2*e/4 + b*c*d**3/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int (d+ex)^3 \left(bx+cx^2\right)^2 \, dx = \frac{1}{8}c^2e^3x^8 + \frac{1}{3}b^2d^3x^3 + \frac{1}{7}\left(3\,c^2de^2 + 2\,bce^3\right)x^7 + \frac{1}{6}\left(3\,c^2d^2e + 6\,bcde^2 + b^2e^3\right)x^6 + \frac{1}{5}\left(c^2d^3 + 6\,bcd^2e + 3\,b^2de^2\right)x^5 + \frac{1}{4}\left(2\,bcd^3 + 3\,b^2d^2e\right)x^4$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
1/8*c^2*e^3*x^8 + 1/3*b^2*d^3*x^3 + 1/7*(3*c^2*d*e^2 + 2*b*c*e^3)*x^7 + 1/
6*(3*c^2*d^2*e + 6*b*c*d*e^2 + b^2*e^3)*x^6 + 1/5*(c^2*d^3 + 6*b*c*d^2*e +
 3*b^2*d*e^2)*x^5 + 1/4*(2*b*c*d^3 + 3*b^2*d^2*e)*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

$$\int (d+ex)^3 \left(bx+cx^2\right)^2 \, dx = \frac{1}{8}c^2e^3x^8 + \frac{3}{7}c^2de^2x^7 + \frac{2}{7}bce^3x^7 + \frac{1}{2}c^2d^2ex^6 + bcde^2x^6 + \frac{1}{6}b^2e^3x^6 + \frac{1}{5}c^2d^3x^5 + \frac{6}{5}bcd^2ex^5 + \frac{3}{5}b^2de^2x^5 + \frac{1}{2}bcd^3x^4 + \frac{3}{4}b^2d^2ex^4 + \frac{1}{3}b^2d^3x^3$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
1/8*c^2*e^3*x^8 + 3/7*c^2*d*e^2*x^7 + 2/7*b*c*e^3*x^7 + 1/2*c^2*d^2*e*x^6
+ b*c*d*e^2*x^6 + 1/6*b^2*e^3*x^6 + 1/5*c^2*d^3*x^5 + 6/5*b*c*d^2*e*x^5 +
3/5*b^2*d*e^2*x^5 + 1/2*b*c*d^3*x^4 + 3/4*b^2*d^2*e*x^4 + 1/3*b^2*d^3*x^3
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93

$$\int (d+ex)^3 \left(bx+cx^2\right)^2 \, dx = x^5 \left(\frac{3\,b^2\,d\,e^2}{5}+\frac{6\,b\,c\,d^2\,e}{5}+\frac{c^2\,d^3}{5}\right) + x^6 \left(\frac{b^2\,e^3}{6}+b\,c\,d\,e^2+\frac{c^2\,d^2\,e}{2}\right)+\frac{b^2\,d^3\,x^3}{3}+\frac{c^2\,e^3\,x^8}{8} + \frac{b\,d^2\,x^4\,(3\,b\,e+2\,c\,d)}{4}+\frac{c\,e^2\,x^7\,(2\,b\,e+3\,c\,d)}{7}$$

input
```
int((b*x + c*x^2)^2*(d + e*x)^3,x)
```

output
```
x^5*((c^2*d^3)/5 + (3*b^2*d*e^2)/5 + (6*b*c*d^2*e)/5) + x^6*((b^2*e^3)/6 +
 (c^2*d^2*e)/2 + b*c*d*e^2) + (b^2*d^3*x^3)/3 + (c^2*e^3*x^8)/8 + (b*d^2*x
^4*(3*b*e + 2*c*d))/4 + (c*e^2*x^7*(2*b*e + 3*c*d))/7
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int (d+ex)^3 \left(bx+cx^2\right)^2 \, dx = \frac{x^3(105c^2e^3x^5+240bc\,e^3x^4+360c^2d\,e^2x^4+140b^2e^3x^3+840bcd\,e^2x^3+420c^2d^2e\,x^3+504b^2d\,e^2x^2+1008}{840}$$

input
```
int((e*x+d)^3*(c*x^2+b*x)^2,x)
```

output
```
(x**3*(280*b**2*d**3 + 630*b**2*d**2*e*x + 504*b**2*d*e**2*x**2 + 140*b**2
*e**3*x**3 + 420*b*c*d**3*x + 1008*b*c*d**2*e*x**2 + 840*b*c*d*e**2*x**3 +
 240*b*c*e**3*x**4 + 168*c**2*d**3*x**2 + 420*c**2*d**2*e*x**3 + 360*c**2*
d*e**2*x**4 + 105*c**2*e**3*x**5))/840
```

3.29 $\int (d+ex)^2 \left(bx+cx^2\right)^2 \, dx$

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Optimal result

Integrand size = 19, antiderivative size = 87

$$\int (d+ex)^2 \left(bx+cx^2\right)^2 \, dx = \frac{1}{3}b^2d^2x^3 + \frac{1}{2}bd(cd+be)x^4 + \frac{1}{5}\left(c^2d^2+4bcde+b^2e^2\right)x^5 + \frac{1}{3}ce(cd+be)x^6 + \frac{1}{7}c^2e^2x^7$$

output
```
1/3*b^2*d^2*x^3+1/2*b*d*(b*e+c*d)*x^4+1/5*(b^2*e^2+4*b*c*d*e+c^2*d^2)*x^5+
1/3*c*e*(b*e+c*d)*x^6+1/7*c^2*e^2*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int (d+ex)^2 \left(bx+cx^2\right)^2 \, dx = \frac{1}{3}b^2d^2x^3 + \frac{1}{2}bd(cd+be)x^4 + \frac{1}{5}\left(c^2d^2+4bcde+b^2e^2\right)x^5 + \frac{1}{3}ce(cd+be)x^6 + \frac{1}{7}c^2e^2x^7$$

input
```
Integrate[(d + e*x)^2*(b*x + c*x^2)^2,x]
```

output
```
(b^2*d^2*x^3)/3 + (b*d*(c*d + b*e)*x^4)/2 + ((c^2*d^2 + 4*b*c*d*e + b^2*e^
2)*x^5)/5 + (c*e*(c*d + b*e)*x^6)/3 + (c^2*e^2*x^7)/7
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+cx^2)^2 (d+ex)^2 \, dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(x^4 \left(b^2e^2+4bcde+c^2d^2\right)+b^2d^2x^2+2cex^5(be+cd)+2bdx^3(be+cd)+c^2e^2x^6\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{5}x^5\left(b^2e^2+4bcde+c^2d^2\right)+\frac{1}{3}b^2d^2x^3+\frac{1}{3}cex^6(be+cd)+\frac{1}{2}bdx^4(be+cd)+\frac{1}{7}c^2e^2x^7$$

input
```
Int[(d + e*x)^2*(b*x + c*x^2)^2,x]
```

output
```
(b^2*d^2*x^3)/3 + (b*d*(c*d + b*e)*x^4)/2 + ((c^2*d^2 + 4*b*c*d*e + b^2*e^
2)*x^5)/5 + (c*e*(c*d + b*e)*x^6)/3 + (c^2*e^2*x^7)/7
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

method	result
norman	$\frac{c^{2}e^{2}x^{7}}{7}+\left(\frac{1}{3}e^{2}bc+\frac{1}{3}de\,c^{2}\right)x^{6}+\left(\frac{1}{5}b^{2}e^{2}+\frac{4}{5}bcde+\frac{1}{5}c^{2}d^{2}\right)x^{5}+\left(\frac{1}{2}de\,b^{2}+\frac{1}{2}bc\,d^{2}\right)x^{4}+\frac{x^{3}b^{2}d^{2}}{3}$
default	$\frac{c^{2}e^{2}x^{7}}{7}+\frac{(2e^{2}bc+2de\,c^{2})x^{6}}{6}+\frac{(b^{2}e^{2}+4bcde+c^{2}d^{2})x^{5}}{5}+\frac{(2de\,b^{2}+2bc\,d^{2})x^{4}}{4}+\frac{x^{3}b^{2}d^{2}}{3}$
gosper	$\frac{x^{3}\left(30c^{2}e^{2}x^{4}+70x^{3}e^{2}bc+70x^{3}de\,c^{2}+42x^{2}b^{2}e^{2}+168x^{2}bcde+42d^{2}c^{2}x^{2}+105xde\,b^{2}+105xbc\,d^{2}+70b^{2}d^{2}\right)}{210}$
risch	$\frac{1}{7}c^{2}e^{2}x^{7}+\frac{1}{3}x^{6}e^{2}bc+\frac{1}{3}de\,c^{2}x^{6}+\frac{1}{5}x^{5}b^{2}e^{2}+\frac{4}{5}x^{5}bcde+\frac{1}{5}x^{5}c^{2}d^{2}+\frac{1}{2}x^{4}de\,b^{2}+\frac{1}{2}bc\,d^{2}x^{4}+\frac{1}{3}x^{3}$
parallelrisch	$\frac{1}{7}c^{2}e^{2}x^{7}+\frac{1}{3}x^{6}e^{2}bc+\frac{1}{3}de\,c^{2}x^{6}+\frac{1}{5}x^{5}b^{2}e^{2}+\frac{4}{5}x^{5}bcde+\frac{1}{5}x^{5}c^{2}d^{2}+\frac{1}{2}x^{4}de\,b^{2}+\frac{1}{2}bc\,d^{2}x^{4}+\frac{1}{3}x^{3}$
orering	$\frac{x\left(30c^{2}e^{2}x^{4}+70x^{3}e^{2}bc+70x^{3}de\,c^{2}+42x^{2}b^{2}e^{2}+168x^{2}bcde+42d^{2}c^{2}x^{2}+105xde\,b^{2}+105xbc\,d^{2}+70b^{2}d^{2}\right)\left(c\,x^{2}+bx\right)^{2}}{210(cx+b)^{2}}$

input
```
int((e*x+d)^2*(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/7*c^2*e^2*x^7+(1/3*e^2*b*c+1/3*d*e*c^2)*x^6+(1/5*b^2*e^2+4/5*b*c*d*e+1/5
*c^2*d^2)*x^5+(1/2*d*e*b^2+1/2*b*c*d^2)*x^4+1/3*x^3*b^2*d^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int (d+ex)^{2}\left(bx+cx^{2}\right)^{2}\,dx=\frac{1}{7}\,c^{2}e^{2}x^{7}+\frac{1}{3}\,b^{2}d^{2}x^{3}+\frac{1}{3}\,\left(c^{2}de+bce^{2}\right)x^{6}+\frac{1}{5}\,\left(c^{2}d^{2}+4\,bcde+b^{2}e^{2}\right)x^{5}+\frac{1}{2}\,\left(bcd^{2}+b^{2}de\right)x^{4}$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
1/7*c^2*e^2*x^7 + 1/3*b^2*d^2*x^3 + 1/3*(c^2*d*e + b*c*e^2)*x^6 + 1/5*(c^2
*d^2 + 4*b*c*d*e + b^2*e^2)*x^5 + 1/2*(b*c*d^2 + b^2*d*e)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int (d+ex)^2 \left(bx+cx^2\right)^2 \, dx = \frac{b^2d^2x^3}{3} + \frac{c^2e^2x^7}{7} + x^6\left(\frac{bce^2}{3} + \frac{c^2de}{3}\right) + x^5\left(\frac{b^2e^2}{5} + \frac{4bcde}{5} + \frac{c^2d^2}{5}\right) + x^4\left(\frac{b^2de}{2} + \frac{bcd^2}{2}\right)$$

input
```
integrate((e*x+d)**2*(c*x**2+b*x)**2,x)
```

output
```
b**2*d**2*x**3/3 + c**2*e**2*x**7/7 + x**6*(b*c*e**2/3 + c**2*d*e/3) + x**
5*(b**2*e**2/5 + 4*b*c*d*e/5 + c**2*d**2/5) + x**4*(b**2*d*e/2 + b*c*d**2/
2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int (d+ex)^2 \left(bx+cx^2\right)^2 \, dx = \frac{1}{7}c^2e^2x^7 + \frac{1}{3}b^2d^2x^3 + \frac{1}{3}\left(c^2de + bce^2\right)x^6 + \frac{1}{5}\left(c^2d^2 + 4\,bcde + b^2e^2\right)x^5 + \frac{1}{2}\left(bcd^2 + b^2de\right)x^4$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
1/7*c^2*e^2*x^7 + 1/3*b^2*d^2*x^3 + 1/3*(c^2*d*e + b*c*e^2)*x^6 + 1/5*(c^2
*d^2 + 4*b*c*d*e + b^2*e^2)*x^5 + 1/2*(b*c*d^2 + b^2*d*e)*x^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int (d+ex)^2 \left(bx+cx^2\right)^2 \, dx = \frac{1}{7}c^2e^2x^7 + \frac{1}{3}c^2dex^6 + \frac{1}{3}bce^2x^6 + \frac{1}{5}c^2d^2x^5 + \frac{4}{5}bcdex^5 + \frac{1}{5}b^2e^2x^5 + \frac{1}{2}bcd^2x^4 + \frac{1}{2}b^2dex^4 + \frac{1}{3}b^2d^2x^3$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
1/7*c^2*e^2*x^7 + 1/3*c^2*d*e*x^6 + 1/3*b*c*e^2*x^6 + 1/5*c^2*d^2*x^5 + 4/
5*b*c*d*e*x^5 + 1/5*b^2*e^2*x^5 + 1/2*b*c*d^2*x^4 + 1/2*b^2*d*e*x^4 + 1/3*
b^2*d^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.90

$$\int (d+ex)^2 \left(bx+cx^2\right)^2 \, dx = x^5 \left(\frac{b^2 e^2}{5} + \frac{4 b c d e}{5} + \frac{c^2 d^2}{5}\right) + \frac{b^2 d^2 x^3}{3} + \frac{c^2 e^2 x^7}{7} + \frac{b d x^4 (b e + c d)}{2} + \frac{c e x^6 (b e + c d)}{3}$$

input
```
int((b*x + c*x^2)^2*(d + e*x)^2,x)
```

output
```
x^5*((b^2*e^2)/5 + (c^2*d^2)/5 + (4*b*c*d*e)/5) + (b^2*d^2*x^3)/3 + (c^2*e
^2*x^7)/7 + (b*d*x^4*(b*e + c*d))/2 + (c*e*x^6*(b*e + c*d))/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06

$$\int (d+ex)^2 \left(bx+cx^2\right)^2 \, dx$$
$$= \frac{x^3(30c^2e^2x^4 + 70bc\,e^2x^3 + 70c^2de\,x^3 + 42b^2e^2x^2 + 168bcde\,x^2 + 42c^2d^2x^2 + 105b^2dex + 105bc\,d^2x + 70b}{210}$$

input
```
int((e*x+d)^2*(c*x^2+b*x)^2,x)
```

output
```
(x**3*(70*b**2*d**2 + 105*b**2*d*e*x + 42*b**2*e**2*x**2 + 105*b*c*d**2*x
+ 168*b*c*d*e*x**2 + 70*b*c*e**2*x**3 + 42*c**2*d**2*x**2 + 70*c**2*d*e*x*
*3 + 30*c**2*e**2*x**4))/210
```

3.30 $\int (d+ex) (bx+cx^2)^2 \, dx$

Optimal result . . . 282
Mathematica [A] (verified) . . . 282
Rubi [A] (verified) . . . 283
Maple [A] (verified) . . . 284
Fricas [A] (verification not implemented) . . . 284
Sympy [A] (verification not implemented) . . . 285
Maxima [A] (verification not implemented) . . . 285
Giac [A] (verification not implemented) . . . 285
Mupad [B] (verification not implemented) . . . 286
Reduce [B] (verification not implemented) . . . 286

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int (d+ex) \left(bx+cx^2\right)^2 \, dx = \frac{1}{3}b^2dx^3 + \frac{1}{4}b(2cd+be)x^4 + \frac{1}{5}c(cd+2be)x^5 + \frac{1}{6}c^2ex^6$$

output
```
1/3*b^2*d*x^3+1/4*b*(b*e+2*c*d)*x^4+1/5*c*(2*b*e+c*d)*x^5+1/6*c^2*e*x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int (d+ex) \left(bx+cx^2\right)^2 \, dx = \frac{1}{60}x^3\left(5b^2(4d+3ex)+6bcx(5d+4ex)+2c^2x^2(6d+5ex)\right)$$

input
```
Integrate[(d + e*x)*(b*x + c*x^2)^2,x]
```

output
```
(x^3*(5*b^2*(4*d + 3*e*x) + 6*b*c*x*(5*d + 4*e*x) + 2*c^2*x^2*(6*d + 5*e*x
)))/60
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx + cx^2)^2 (d + ex)\, dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(b^2dx^2 + cx^4(2be + cd) + bx^3(be + 2cd) + c^2ex^5\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}b^2dx^3 + \frac{1}{5}cx^5(2be + cd) + \frac{1}{4}bx^4(be + 2cd) + \frac{1}{6}c^2ex^6$$

input
```
Int[(d + e*x)*(b*x + c*x^2)^2,x]
```

output
```
(b^2*d*x^3)/3 + (b*(2*c*d + b*e)*x^4)/4 + (c*(c*d + 2*b*e)*x^5)/5 + (c^2*e
*x^6)/6
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{x^3(10c^2e\,x^3+24bce\,x^2+12c^2d\,x^2+15xe\,b^2+30bcdx+20b^2d)}{60}$	52
default	$\frac{c^2e\,x^6}{6}+\frac{(2bce+c^2d)x^5}{5}+\frac{(e\,b^2+2dbc)x^4}{4}+\frac{b^2d\,x^3}{3}$	52
norman	$\frac{c^2e\,x^6}{6}+\left(\frac{2}{5}bce+\frac{1}{5}c^2d\right)x^5+\left(\frac{1}{4}e\,b^2+\frac{1}{2}dbc\right)x^4+\frac{b^2d\,x^3}{3}$	52
risch	$\frac{1}{6}c^2e\,x^6+\frac{2}{5}x^5bce+\frac{1}{5}c^2d\,x^5+\frac{1}{4}b^2e\,x^4+\frac{1}{2}dbc\,x^4+\frac{1}{3}b^2d\,x^3$	54
parallelrisch	$\frac{1}{6}c^2e\,x^6+\frac{2}{5}x^5bce+\frac{1}{5}c^2d\,x^5+\frac{1}{4}b^2e\,x^4+\frac{1}{2}dbc\,x^4+\frac{1}{3}b^2d\,x^3$	54
orering	$\frac{x(10c^2e\,x^3+24bce\,x^2+12c^2d\,x^2+15xe\,b^2+30bcdx+20b^2d)(c\,x^2+bx)^2}{60(cx+b)^2}$	68

input
```
int((e*x+d)*(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*x^3*(10*c^2*e*x^3+24*b*c*e*x^2+12*c^2*d*x^2+15*b^2*e*x+30*b*c*d*x+20*
b^2*d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(bx+cx^2\right)^2\,dx=\frac{1}{6}\,c^2ex^6+\frac{1}{3}\,b^2dx^3+\frac{1}{5}\,\left(c^2d+2\,bce\right)x^5+\frac{1}{4}\,\left(2\,bcd+b^2e\right)x^4$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
1/6*c^2*e*x^6 + 1/3*b^2*d*x^3 + 1/5*(c^2*d + 2*b*c*e)*x^5 + 1/4*(2*b*c*d +
 b^2*e)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int (d+ex)\left(bx+cx^2\right)^2\,dx=\frac{b^2dx^3}{3}+\frac{c^2ex^6}{6}+x^5\cdot\left(\frac{2bce}{5}+\frac{c^2d}{5}\right)+x^4\left(\frac{b^2e}{4}+\frac{bcd}{2}\right)$$

input
```
integrate((e*x+d)*(c*x**2+b*x)**2,x)
```

output
```
b**2*d*x**3/3 + c**2*e*x**6/6 + x**5*(2*b*c*e/5 + c**2*d/5) + x**4*(b**2*e
/4 + b*c*d/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(bx+cx^2\right)^2\,dx=\frac{1}{6}\,c^2ex^6+\frac{1}{3}\,b^2dx^3+\frac{1}{5}\,\left(c^2d+2\,bce\right)x^5+\frac{1}{4}\,\left(2\,bcd+b^2e\right)x^4$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
1/6*c^2*e*x^6 + 1/3*b^2*d*x^3 + 1/5*(c^2*d + 2*b*c*e)*x^5 + 1/4*(2*b*c*d +
 b^2*e)*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int (d+ex)\left(bx+cx^2\right)^2\,dx=\frac{1}{6}\,c^2ex^6+\frac{1}{5}\,c^2dx^5+\frac{2}{5}\,bcex^5+\frac{1}{2}\,bcdx^4+\frac{1}{4}\,b^2ex^4+\frac{1}{3}\,b^2dx^3$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
1/6*c^2*e*x^6 + 1/5*c^2*d*x^5 + 2/5*b*c*e*x^5 + 1/2*b*c*d*x^4 + 1/4*b^2*e*
x^4 + 1/3*b^2*d*x^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(bx+cx^2\right)^2\,dx=x^4\left(\frac{e\,b^2}{4}+\frac{c\,d\,b}{2}\right)+x^5\left(\frac{d\,c^2}{5}+\frac{2\,b\,e\,c}{5}\right)+\frac{b^2\,d\,x^3}{3}+\frac{c^2\,e\,x^6}{6}$$

input
```
int((b*x + c*x^2)^2*(d + e*x),x)
```

output
```
x^4*((b^2*e)/4 + (b*c*d)/2) + x^5*((c^2*d)/5 + (2*b*c*e)/5) + (b^2*d*x^3)/
3 + (c^2*e*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (d+ex)\left(bx+cx^2\right)^2\,dx = \frac{x^3(10c^2e\,x^3+24bce\,x^2+12c^2d\,x^2+15b^2ex+30bcdx+20b^2d)}{60}$$

input
```
int((e*x+d)*(c*x^2+b*x)^2,x)
```

output
```
(x**3*(20*b**2*d + 15*b**2*e*x + 30*b*c*d*x + 24*b*c*e*x**2 + 12*c**2*d*x*
*2 + 10*c**2*e*x**3))/60
```

3.31 $\int (bx+cx^2)^2\,dx$

Optimal result 287
Mathematica [A] (verified) 287
Rubi [A] (verified) 288
Maple [A] (verified) 289
Fricas [A] (verification not implemented) 289
Sympy [A] (verification not implemented) 290
Maxima [A] (verification not implemented) 290
Giac [A] (verification not implemented) 290
Mupad [B] (verification not implemented) 291
Reduce [B] (verification not implemented) 291

Optimal result

Integrand size = 11, antiderivative size = 30

$$\int (bx+cx^2)^2\,dx=\frac{b^2x^3}{3}+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

output
```
1/3*b^2*x^3+1/2*b*c*x^4+1/5*c^2*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int (bx+cx^2)^2\,dx=\frac{b^2x^3}{3}+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

input
```
Integrate[(b*x + c*x^2)^2,x]
```

output
```
(b^2*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^2 dx$$
$$\Big\downarrow \ 1080$$
$$\int \left(b^2x^2+2bcx^3+c^2x^4\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{b^2x^3}{3}+\frac{1}{2}bcx^4+\frac{c^2x^5}{5}$$

input
```
Int[(b*x + c*x^2)^2,x]
```

output
```
(b^2*x^3)/3 + (b*c*x^4)/2 + (c^2*x^5)/5
```

Defintions of rubi rules used

rule 1080
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[x^p*
(b + c*x)^p, x], x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{x^3(6c^2x^2+15cbx+10b^2)}{30}$	25
default	$\frac{1}{3}b^2x^3+\frac{1}{2}bc\,x^4+\frac{1}{5}c^2x^5$	25
norman	$\frac{1}{3}b^2x^3+\frac{1}{2}bc\,x^4+\frac{1}{5}c^2x^5$	25
risch	$\frac{1}{3}b^2x^3+\frac{1}{2}bc\,x^4+\frac{1}{5}c^2x^5$	25
parallelrisch	$\frac{1}{3}b^2x^3+\frac{1}{2}bc\,x^4+\frac{1}{5}c^2x^5$	25
orering	$\frac{x(6c^2x^2+15cbx+10b^2)(c\,x^2+bx)^2}{30(cx+b)^2}$	41

input
```
int((c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/30*x^3*(6*c^2*x^2+15*b*c*x+10*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (bx+cx^2)^2\,dx=\frac{1}{5}\,c^2x^5+\frac{1}{2}\,bcx^4+\frac{1}{3}\,b^2x^3$$

input
```
integrate((c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
1/5*c^2*x^5 + 1/2*b*c*x^4 + 1/3*b^2*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(bx + cx^2\right)^2 \, dx = \frac{b^2x^3}{3} + \frac{bcx^4}{2} + \frac{c^2x^5}{5}$$

input
```
integrate((c*x**2+b*x)**2,x)
```

output
```
b**2*x**3/3 + b*c*x**4/2 + c**2*x**5/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(bx + cx^2\right)^2 \, dx = \frac{1}{5}\,c^2x^5 + \frac{1}{2}\,bcx^4 + \frac{1}{3}\,b^2x^3$$

input
```
integrate((c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
1/5*c^2*x^5 + 1/2*b*c*x^4 + 1/3*b^2*x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(bx + cx^2\right)^2 \, dx = \frac{1}{5}\,c^2x^5 + \frac{1}{2}\,bcx^4 + \frac{1}{3}\,b^2x^3$$

input
```
integrate((c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
1/5*c^2*x^5 + 1/2*b*c*x^4 + 1/3*b^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(bx+cx^2\right)^2\,dx=\frac{b^2\,x^3}{3}+\frac{b\,c\,x^4}{2}+\frac{c^2\,x^5}{5}$$

input
```
int((b*x + c*x^2)^2,x)
```

output
```
(b^2*x^3)/3 + (c^2*x^5)/5 + (b*c*x^4)/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int \left(bx+cx^2\right)^2\,dx=\frac{x^3(6c^2x^2+15bcx+10b^2)}{30}$$

input
```
int((c*x^2+b*x)^2,x)
```

output
```
(x**3*(10*b**2 + 15*b*c*x + 6*c**2*x**2))/30
```

3.32 $\int \frac{(bx+cx^2)^2}{d+ex} dx$

Optimal result 292
Mathematica [A] (verified) 292
Rubi [A] (verified) 293
Maple [A] (verified) 294
Fricas [A] (verification not implemented) 294
Sympy [A] (verification not implemented) 295
Maxima [A] (verification not implemented) 295
Giac [A] (verification not implemented) 296
Mupad [B] (verification not implemented) 296
Reduce [B] (verification not implemented) 297

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{(bx+cx^2)^2}{d+ex} dx = -\frac{d(cd-be)^2x}{e^4} + \frac{(cd-be)^2x^2}{2e^3} - \frac{c(cd-2be)x^3}{3e^2} + \frac{c^2x^4}{4e} + \frac{d^2(cd-be)^2\log(d+ex)}{e^5}$$

output

```
-d*(-b*e+c*d)^2*x/e^4+1/2*(-b*e+c*d)^2*x^2/e^3-1/3*c*(-2*b*e+c*d)*x^3/e^2+
1/4*c^2*x^4/e+d^2*(-b*e+c*d)^2*ln(e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14

$$\int \frac{(bx+cx^2)^2}{d+ex} dx = -\frac{d(cd-be)^2x}{e^4} + \frac{(-cd+be)^2x^2}{2e^3} - \frac{c(cd-2be)x^3}{3e^2} + \frac{c^2x^4}{4e} + \frac{(c^2d^4-2bcd^3e+b^2d^2e^2)\log(d+ex)}{e^5}$$

input

```
Integrate[(b*x + c*x^2)^2/(d + e*x),x]
```

output
```
-((d*(c*d - b*e)^2*x)/e^4) + ((-(c*d) + b*e)^2*x^2)/(2*e^3) - (c*(c*d - 2*
b*e)*x^3)/(3*e^2) + (c^2*x^4)/(4*e) + ((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^
2)*Log[d + e*x])/e^5
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^2}{d+ex}\,dx$$

$$\Big\downarrow \; 1140$$

$$\int \left(\frac{d^2(cd-be)^2}{e^4(d+ex)}-\frac{d(cd-be)^2}{e^4}+\frac{x(be-cd)^2}{e^3}-\frac{cx^2(cd-2be)}{e^2}+\frac{c^2x^3}{e}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{d^2(cd-be)^2\log(d+ex)}{e^5}-\frac{dx(cd-be)^2}{e^4}+\frac{x^2(cd-be)^2}{2e^3}-\frac{cx^3(cd-2be)}{3e^2}+\frac{c^2x^4}{4e}$$

input
```
Int[(b*x + c*x^2)^2/(d + e*x),x]
```

output
```
-((d*(c*d - b*e)^2*x)/e^4) + ((c*d - b*e)^2*x^2)/(2*e^3) - (c*(c*d - 2*b*e
)*x^3)/(3*e^2) + (c^2*x^4)/(4*e) + (d^2*(c*d - b*e)^2*Log[d + e*x])/e^5
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.31

method	result
norman	$\frac{c^2x^4}{4e}+\frac{(b^2e^2-2bcde+c^2d^2)x^2}{2e^3}+\frac{c(2be-cd)x^3}{3e^2}-\frac{d(b^2e^2-2bcde+c^2d^2)x}{e^4}+\frac{d^2(b^2e^2-2bcde+c^2d^2)\ln(ex+d)}{e^5}$
default	$-\frac{-\frac{c^2x^4e^3}{4}+\frac{\left(-(be-cd)ce^2-e^3bc\right)x^3}{3}+\frac{\left(-(be-cd)be^2+ce\left(bde-cd^2\right)\right)x^2}{2}+(be-cd)\left(bde-cd^2\right)x}{e^4}+\frac{d^2\left(b^2e^2-2bcde+c^2d^2\right)\ln(e$
risch	$\frac{c^2x^4}{4e}+\frac{2x^3bc}{3e}-\frac{c^2dx^3}{3e^2}+\frac{x^2b^2}{2e}-\frac{x^2dbc}{e^2}+\frac{x^2d^2c^2}{2e^3}-\frac{b^2dx}{e^2}+\frac{2bcd^2x}{e^3}-\frac{c^2d^3x}{e^4}+\frac{d^2\ln(ex+d)b^2}{e^3}-\frac{2d^3\ln(ex+}{e^4}$
parallelrisch	$\frac{3c^2x^4e^4+8x^3bce^4-4dc^2x^3e^3+6x^2b^2e^4-12x^2bcde^3+6x^2c^2d^2e^2+12\ln(ex+d)b^2d^2e^2-24\ln(ex+d)bcd^3e+12\ln(ex+d)c^2d^4}{12e^5}$

input
```
int((c*x^2+b*x)^2/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*c^2*x^4/e+1/2/e^3*(b^2*e^2-2*b*c*d*e+c^2*d^2)*x^2+1/3*c/e^2*(2*b*e-c*d
)*x^3-d*(b^2*e^2-2*b*c*d*e+c^2*d^2)/e^4*x+d^2*(b^2*e^2-2*b*c*d*e+c^2*d^2)/
e^5*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.43

$$\int \frac{(bx+cx^2)^2}{d+ex}\,dx$$
$$=\frac{3c^2e^4x^4-4(c^2de^3-2bce^4)x^3+6(c^2d^2e^2-2bcde^3+b^2e^4)x^2-12(c^2d^3e-2bcd^2e^2+b^2de^3)x+12(c^2}{12e^5}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d),x, algorithm="fricas")
```

output
```
1/12*(3*c^2*e^4*x^4 - 4*(c^2*d*e^3 - 2*b*c*e^4)*x^3 + 6*(c^2*d^2*e^2 - 2*b
*c*d*e^3 + b^2*e^4)*x^2 - 12*(c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x + 1
2*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*log(e*x + d))/e^5
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.25

$$\int \frac{(bx+cx^2)^2}{d+ex}\,dx = \frac{c^2x^4}{4e} + \frac{d^2(be-cd)^2\log(d+ex)}{e^5} + x^3\cdot\left(\frac{2bc}{3e} - \frac{c^2d}{3e^2}\right) + x^2\left(\frac{b^2}{2e} - \frac{bcd}{e^2} + \frac{c^2d^2}{2e^3}\right) + x\left(-\frac{b^2d}{e^2} + \frac{2bcd^2}{e^3} - \frac{c^2d^3}{e^4}\right)$$

input
```
integrate((c*x**2+b*x)**2/(e*x+d),x)
```

output
```
c**2*x**4/(4*e) + d**2*(b*e - c*d)**2*log(d + e*x)/e**5 + x**3*(2*b*c/(3*e
) - c**2*d/(3*e**2)) + x**2*(b**2/(2*e) - b*c*d/e**2 + c**2*d**2/(2*e**3))
 + x*(-b**2*d/e**2 + 2*b*c*d**2/e**3 - c**2*d**3/e**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.41

$$\int \frac{(bx+cx^2)^2}{d+ex}\,dx = \frac{3\,c^2e^3x^4 - 4\,(c^2de^2 - 2\,bce^3)x^3 + 6\,(c^2d^2e - 2\,bcde^2 + b^2e^3)x^2 - 12\,(c^2d^3 - 2\,bcd^2e + b^2de^2)x}{12\,e^4} + \frac{(c^2d^4 - 2\,bcd^3e + b^2d^2e^2)\log(ex+d)}{e^5}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d),x, algorithm="maxima")
```

output
```
1/12*(3*c^2*e^3*x^4 - 4*(c^2*d*e^2 - 2*b*c*e^3)*x^3 + 6*(c^2*d^2*e - 2*b*c
*d*e^2 + b^2*e^3)*x^2 - 12*(c^2*d^3 - 2*b*c*d^2*e + b^2*d*e^2)*x)/e^4 + (c
^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.49

$$\int \frac{(bx+cx^2)^2}{d+ex}\,dx = \frac{3\,c^2e^3x^4 - 4\,c^2de^2x^3 + 8\,bce^3x^3 + 6\,c^2d^2ex^2 - 12\,bcde^2x^2 + 6\,b^2e^3x^2 - 12\,c^2d^3x + 24\,bcd^2ex - 12\,b^2de^2x}{12\,e^4} + \frac{(c^2d^4 - 2\,bcd^3e + b^2d^2e^2)\log(|ex+d|)}{e^5}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d),x, algorithm="giac")
```

output
```
1/12*(3*c^2*e^3*x^4 - 4*c^2*d*e^2*x^3 + 8*b*c*e^3*x^3 + 6*c^2*d^2*e*x^2 -
12*b*c*d*e^2*x^2 + 6*b^2*e^3*x^2 - 12*c^2*d^3*x + 24*b*c*d^2*e*x - 12*b^2*
d*e^2*x)/e^4 + (c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*log(abs(e*x + d))/e^5
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.52

$$\int \frac{(bx+cx^2)^2}{d+ex}\,dx = x^2\left(\frac{b^2}{2\,e} + \frac{d\left(\frac{c^2\,d}{e^2} - \frac{2\,b\,c}{e}\right)}{2\,e}\right) - x^3\left(\frac{c^2\,d}{3\,e^2} - \frac{2\,b\,c}{3\,e}\right) + \frac{c^2\,x^4}{4\,e} + \frac{\ln(d+e\,x)\,\left(b^2\,d^2\,e^2 - 2\,b\,c\,d^3\,e + c^2\,d^4\right)}{e^5} - \frac{d\,x\left(\frac{b^2}{e} + \frac{d\left(\frac{c^2\,d}{e^2} - \frac{2\,b\,c}{e}\right)}{e}\right)}{e}$$

input
```
int((b*x + c*x^2)^2/(d + e*x),x)
```

output
```
x^2*(b^2/(2*e) + (d*((c^2*d)/e^2 - (2*b*c)/e))/(2*e)) - x^3*((c^2*d)/(3*e^
2) - (2*b*c)/(3*e)) + (c^2*x^4)/(4*e) + (log(d + e*x)*(c^2*d^4 + b^2*d^2*e
^2 - 2*b*c*d^3*e))/e^5 - (d*x*(b^2/e + (d*((c^2*d)/e^2 - (2*b*c)/e))/e))/e
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.62

$$\int \frac{(bx+cx^2)^2}{d+ex}\,dx = \frac{12\log(ex+d)\,b^2d^2e^2 - 24\log(ex+d)\,bc\,d^3e + 12\log(ex+d)\,c^2d^4 - 12b^2d\,e^3x + 6b^2e^4x^2 + 24bc\,d^2e^2x - }{12e^5}$$

input
```
int((c*x^2+b*x)^2/(e*x+d),x)
```

output
```
(12*log(d + e*x)*b**2*d**2*e**2 - 24*log(d + e*x)*b*c*d**3*e + 12*log(d +
e*x)*c**2*d**4 - 12*b**2*d*e**3*x + 6*b**2*e**4*x**2 + 24*b*c*d**2*e**2*x
- 12*b*c*d*e**3*x**2 + 8*b*c*e**4*x**3 - 12*c**2*d**3*e*x + 6*c**2*d**2*e*
*2*x**2 - 4*c**2*d*e**3*x**3 + 3*c**2*e**4*x**4)/(12*e**5)
```

3.33 $\int \frac{(bx+cx^2)^2}{(d+ex)^2} \, dx$

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Optimal result

Integrand size = 19, antiderivative size = 107

$$\int \frac{(bx+cx^2)^2}{(d+ex)^2} \, dx = \frac{(cd-be)(3cd-be)x}{e^4} - \frac{c(cd-be)x^2}{e^3} + \frac{c^2x^3}{3e^2} - \frac{d^2(cd-be)^2}{e^5(d+ex)} - \frac{2d(cd-be)(2cd-be)\log(d+ex)}{e^5}$$

output
```
(-b*e+c*d)*(-b*e+3*c*d)*x/e^4-c*(-b*e+c*d)*x^2/e^3+1/3*c^2*x^3/e^2-d^2*(-b
*e+c*d)^2/e^5/(e*x+d)-2*d*(-b*e+c*d)*(-b*e+2*c*d)*ln(e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07

$$\int \frac{(bx+cx^2)^2}{(d+ex)^2} \, dx = \frac{3e(3c^2d^2-4bcde+b^2e^2)\,x - 3ce^2(cd-be)x^2 + c^2e^3x^3 - \frac{3d^2(cd-be)^2}{d+ex} - 6d(2c^2d^2-3bcde+b^2e^2)\log(d+}{3e^5}$$

input
```
Integrate[(b*x + c*x^2)^2/(d + e*x)^2,x]
```

output
```
(3*e*(3*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*x - 3*c*e^2*(c*d - b*e)*x^2 + c^2*e
^3*x^3 - (3*d^2*(c*d - b*e)^2)/(d + e*x) - 6*d*(2*c^2*d^2 - 3*b*c*d*e + b^
2*e^2)*Log[d + e*x])/(3*e^5)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^2}{(d+ex)^2} \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{d^2(cd-be)^2}{e^4(d+ex)^2} + \frac{2d(be-2cd)(cd-be)}{e^4(d+ex)} + \frac{(3cd-be)(cd-be)}{e^4} - \frac{2cx(cd-be)}{e^3} + \frac{c^2x^2}{e^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{d^2(cd-be)^2}{e^5(d+ex)} - \frac{2d(cd-be)(2cd-be)\log(d+ex)}{e^5} + \frac{x(cd-be)(3cd-be)}{e^4} - \frac{cx^2(cd-be)}{e^3} + \frac{c^2x^3}{3e^2}$$

input
```
Int[(b*x + c*x^2)^2/(d + e*x)^2,x]
```

output
```
((c*d - b*e)*(3*c*d - b*e)*x)/e^4 - (c*(c*d - b*e)*x^2)/e^3 + (c^2*x^3)/(3
*e^2) - (d^2*(c*d - b*e)^2)/(e^5*(d + e*x)) - (2*d*(c*d - b*e)*(2*c*d - b*
e)*Log[d + e*x])/e^5
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.22

method	result
default	$\frac{\frac{1}{3}c^2e^2x^3+bc\,e^2x^2-c^2de\,x^2+b^2e^2x-4bcdex+3c^2d^2x}{e^4}-\frac{2d\left(b^2e^2-3bcde+2c^2d^2\right)\ln\left(ex+d\right)}{e^5}-\frac{d^2\left(b^2e^2-2bcde+c^2d^2\right)}{e^5\left(ex+d\right)}$
norman	$\frac{\frac{\left(b^2e^2-3bcde+2c^2d^2\right)x^2}{e^3}+\frac{\left(2d^2e^2b^2-6d^3ebc+4c^2d^4\right)x}{d\,e^4}+\frac{c^2x^4}{3e}+\frac{c(3be-2cd)x^3}{3e^2}}{ex+d}-\frac{2d\left(b^2e^2-3bcde+2c^2d^2\right)\ln\left(ex+d\right)}{e^5}$
risch	$\frac{c^2x^3}{3e^2}+\frac{bc\,x^2}{e^2}-\frac{c^2d\,x^2}{e^3}+\frac{b^2x}{e^2}-\frac{4bcdx}{e^3}+\frac{3c^2d^2x}{e^4}-\frac{2d\ln\left(ex+d\right)b^2}{e^3}+\frac{6d^2\ln\left(ex+d\right)bc}{e^4}-\frac{4d^3\ln\left(ex+d\right)c^2}{e^5}-\frac{d^2}{e^3(e}$
parallelrisch	$-\frac{-c^2x^4e^4-3x^3bc\,e^4+2d\,c^2x^3e^3+6\ln\left(ex+d\right)x\,b^2d\,e^3-18\ln\left(ex+d\right)xbc\,d^2e^2+12\ln\left(ex+d\right)x\,c^2d^3e-3x^2b^2e^4+9x^2bcd\,e^3-6x^2}{3e^5\left(ex+d\right)}$

input
```
int((c*x^2+b*x)^2/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/e^4*(1/3*c^2*e^2*x^3+b*c*e^2*x^2-c^2*d*e*x^2+b^2*e^2*x-4*b*c*d*e*x+3*c^2
*d^2*x)-2*d/e^5*(b^2*e^2-3*b*c*d*e+2*c^2*d^2)*ln(e*x+d)-d^2*(b^2*e^2-2*b*c
*d*e+c^2*d^2)/e^5/(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.90

$$\int \frac{\left(bx+cx^2\right)^2}{(d+ex)^2}\,dx$$
$$=\frac{c^2e^4x^4-3\,c^2d^4+6\,bcd^3e-3\,b^2d^2e^2-\left(2\,c^2de^3-3\,bce^4\right)x^3+3\left(2\,c^2d^2e^2-3\,bcde^3+b^2e^4\right)x^2+3\left(3\,c^2d^3e\right.}{3\left(e^6x+d\right.}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/3*(c^2*e^4*x^4 - 3*c^2*d^4 + 6*b*c*d^3*e - 3*b^2*d^2*e^2 - (2*c^2*d*e^3
- 3*b*c*e^4)*x^3 + 3*(2*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*x^2 + 3*(3*c^
2*d^3*e - 4*b*c*d^2*e^2 + b^2*d*e^3)*x - 6*(2*c^2*d^4 - 3*b*c*d^3*e + b^2*
d^2*e^2 + (2*c^2*d^3*e - 3*b*c*d^2*e^2 + b^2*d*e^3)*x)*log(e*x + d))/(e^6*
x + d*e^5)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.18

$$\int \frac{(bx+cx^2)^2}{(d+ex)^2}\,dx = \frac{c^2x^3}{3e^2} - \frac{2d(be-2cd)\,(be-cd)\log{(d+ex)}}{e^5} + x^2\left(\frac{bc}{e^2} - \frac{c^2d}{e^3}\right) + x\left(\frac{b^2}{e^2} - \frac{4bcd}{e^3} + \frac{3c^2d^2}{e^4}\right) + \frac{-b^2d^2e^2 + 2bcd^3e - c^2d^4}{de^5 + e^6x}$$

input
```
integrate((c*x**2+b*x)**2/(e*x+d)**2,x)
```

output
```
c**2*x**3/(3*e**2) - 2*d*(b*e - 2*c*d)*(b*e - c*d)*log(d + e*x)/e**5 + x**
2*(b*c/e**2 - c**2*d/e**3) + x*(b**2/e**2 - 4*b*c*d/e**3 + 3*c**2*d**2/e**
4) + (-b**2*d**2*e**2 + 2*b*c*d**3*e - c**2*d**4)/(d*e**5 + e**6*x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.29

$$\int \frac{(bx+cx^2)^2}{(d+ex)^2}\,dx = -\frac{c^2d^4 - 2\,bcd^3e + b^2d^2e^2}{e^6x + de^5} + \frac{c^2e^2x^3 - 3\,(c^2de - bce^2)x^2 + 3\,(3\,c^2d^2 - 4\,bcde + b^2e^2)x}{3\,e^4} - \frac{2\,(2\,c^2d^3 - 3\,bcd^2e + b^2de^2)\log{(ex+d)}}{e^5}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)/(e^6*x + d*e^5) + 1/3*(c^2*e^2*x^3
- 3*(c^2*d*e - b*c*e^2)*x^2 + 3*(3*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*x)/e^4 -
 2*(2*c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.77

$$\int \frac{(bx+cx^2)^2}{(d+ex)^2}\,dx = \frac{\left(c^2 - \frac{3\,(2\,c^2de - bce^2)}{(ex+d)e} + \frac{3\,(6\,c^2d^2e^2 - 6\,bcde^3 + b^2e^4)}{(ex+d)^2e^2}\right)(ex+d)^3}{3\,e^5} + \frac{2\,(2\,c^2d^3 - 3\,bcd^2e + b^2de^2)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^5} - \frac{\frac{c^2d^4e^3}{ex+d} - \frac{2\,bcd^3e^4}{ex+d} + \frac{b^2d^2e^5}{ex+d}}{e^8}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/3*(c^2 - 3*(2*c^2*d*e - b*c*e^2)/((e*x + d)*e) + 3*(6*c^2*d^2*e^2 - 6*b*
c*d*e^3 + b^2*e^4)/((e*x + d)^2*e^2))*(e*x + d)^3/e^5 + 2*(2*c^2*d^3 - 3*b
*c*d^2*e + b^2*d*e^2)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^5 - (c^2*d^
4*e^3/(e*x + d) - 2*b*c*d^3*e^4/(e*x + d) + b^2*d^2*e^5/(e*x + d))/e^8
```

Mupad [B] (verification not implemented)

Time = 9.35 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.48

$$\int \frac{(bx+cx^2)^2}{(d+ex)^2}\,dx = x\left(\frac{b^2}{e^2} + \frac{2\,d\left(\frac{2\,c^2\,d}{e^3} - \frac{2\,b\,c}{e^2}\right)}{e} - \frac{c^2\,d^2}{e^4}\right) - x^2\left(\frac{c^2\,d}{e^3} - \frac{b\,c}{e^2}\right) - \frac{b^2\,d^2\,e^2 - 2\,b\,c\,d^3\,e + c^2\,d^4}{e\,(x\,e^5 + d\,e^4)} - \frac{\ln(d+e\,x)\,(2\,b^2\,d\,e^2 - 6\,b\,c\,d^2\,e + 4\,c^2\,d^3)}{e^5} + \frac{c^2\,x^3}{3\,e^2}$$

input
```
int((b*x + c*x^2)^2/(d + e*x)^2,x)
```

output
```
x*(b^2/e^2 + (2*d*((2*c^2*d)/e^3 - (2*b*c)/e^2))/e - (c^2*d^2)/e^4) - x^2*
((c^2*d)/e^3 - (b*c)/e^2) - (c^2*d^4 + b^2*d^2*e^2 - 2*b*c*d^3*e)/(e*(d*e^
4 + e^5*x)) - (log(d + e*x)*(4*c^2*d^3 + 2*b^2*d*e^2 - 6*b*c*d^2*e))/e^5 +
 (c^2*x^3)/(3*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.93

$$\int \frac{(bx+cx^2)^2}{(d+ex)^2}\,dx = \frac{-6\log(ex+d)\,b^2d^2e^2 - 6\log(ex+d)\,b^2d\,e^3x + 18\log(ex+d)\,bc\,d^3e + 18\log(ex+d)\,bc\,d^2e^2x - 12\log(e}{}$$

input
```
int((c*x^2+b*x)^2/(e*x+d)^2,x)
```

output
```
( - 6*log(d + e*x)*b**2*d**2*e**2 - 6*log(d + e*x)*b**2*d*e**3*x + 18*log(
d + e*x)*b*c*d**3*e + 18*log(d + e*x)*b*c*d**2*e**2*x - 12*log(d + e*x)*c*
*2*d**4 - 12*log(d + e*x)*c**2*d**3*e*x + 6*b**2*d*e**3*x + 3*b**2*e**4*x*
*2 - 18*b*c*d**2*e**2*x - 9*b*c*d*e**3*x**2 + 3*b*c*e**4*x**3 + 12*c**2*d*
*3*e*x + 6*c**2*d**2*e**2*x**2 - 2*c**2*d*e**3*x**3 + c**2*e**4*x**4)/(3*e
**5*(d + e*x))
```

3.34 $\int \frac{(bx+cx^2)^2}{(d+ex)^3}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 119

$$\int \frac{(bx+cx^2)^2}{(d+ex)^3}\,dx = -\frac{c(3cd-2be)x}{e^4}+\frac{c^2x^2}{2e^3}-\frac{d^2(cd-be)^2}{2e^5(d+ex)^2}+\frac{2d(cd-be)(2cd-be)}{e^5(d+ex)}+\frac{(6c^2d^2-6bcde+b^2e^2)\log(d+ex)}{e^5}$$

output
```
-c*(-2*b*e+3*c*d)*x/e^4+1/2*c^2*x^2/e^3-1/2*d^2*(-b*e+c*d)^2/e^5/(e*x+d)^2
+2*d*(-b*e+c*d)*(-b*e+2*c*d)/e^5/(e*x+d)+(b^2*e^2-6*b*c*d*e+6*c^2*d^2)*ln(
e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.97

$$\int \frac{(bx+cx^2)^2}{(d+ex)^3}\,dx = \frac{-2ce(3cd-2be)x+c^2e^2x^2-\frac{d^2(cd-be)^2}{(d+ex)^2}+\frac{4d(2c^2d^2-3bcde+b^2e^2)}{d+ex}+2(6c^2d^2-6bcde+b^2e^2)\log(d+ex)}{2e^5}$$

input
```
Integrate[(b*x + c*x^2)^2/(d + e*x)^3,x]
```

output
```
(-2*c*e*(3*c*d - 2*b*e)*x + c^2*e^2*x^2 - (d^2*(c*d - b*e)^2)/(d + e*x)^2
+ (4*d*(2*c^2*d^2 - 3*b*c*d*e + b^2*e^2))/(d + e*x) + 2*(6*c^2*d^2 - 6*b*c
*d*e + b^2*e^2)*Log[d + e*x])/(2*e^5)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^2}{(d+ex)^3}\,dx$$
$$\Big\downarrow \text{1140}$$
$$\int \left(\frac{b^2e^2-6bcde+6c^2d^2}{e^4(d+ex)}+\frac{d^2(cd-be)^2}{e^4(d+ex)^3}+\frac{2d(cd-be)(be-2cd)}{e^4(d+ex)^2}-\frac{c(3cd-2be)}{e^4}+\frac{c^2x}{e^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{(b^2e^2-6bcde+6c^2d^2)\log(d+ex)}{e^5}-\frac{d^2(cd-be)^2}{2e^5(d+ex)^2}+\frac{2d(2cd-be)(cd-be)}{e^5(d+ex)}-\frac{cx(3cd-2be)}{e^4}+\frac{c^2x^2}{2e^3}$$

input
```
Int[(b*x + c*x^2)^2/(d + e*x)^3,x]
```

output
```
-((c*(3*c*d - 2*b*e)*x)/e^4) + (c^2*x^2)/(2*e^3) - (d^2*(c*d - b*e)^2)/(2*
e^5*(d + e*x)^2) + (2*d*(c*d - b*e)*(2*c*d - b*e))/(e^5*(d + e*x)) + ((6*c
^2*d^2 - 6*b*c*d*e + b^2*e^2)*Log[d + e*x])/e^5
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08

method	result
default	$\frac{c\left(\frac{1}{2}ce\,x^{2}+2bex-3cdx\right)}{e^{4}}+\frac{\left(b^{2}e^{2}-6bcde+6c^{2}d^{2}\right)\ln\left(ex+d\right)}{e^{5}}+\frac{2d\left(b^{2}e^{2}-3bcde+2c^{2}d^{2}\right)}{e^{5}\left(ex+d\right)}-\frac{d^{2}\left(b^{2}e^{2}-2bcde+c^{2}d^{2}\right)}{2e^{5}\left(ex+d\right)^{2}}$
norman	$\frac{\frac{c^{2}x^{4}}{2e}+\frac{d^{2}\left(3b^{2}e^{2}-18bcde+18c^{2}d^{2}\right)}{2e^{5}}+\frac{2c(be-cd)x^{3}}{e^{2}}+\frac{2d\left(b^{2}e^{2}-6bcde+6c^{2}d^{2}\right)x}{e^{4}}}{\left(ex+d\right)^{2}}+\frac{\left(b^{2}e^{2}-6bcde+6c^{2}d^{2}\right)\ln\left(ex+d\right)}{e^{5}}$
risch	$\frac{c^{2}x^{2}}{2e^{3}}+\frac{2cbx}{e^{3}}-\frac{3c^{2}dx}{e^{4}}+\frac{\left(2d\,e^{2}b^{2}-6d^{2}ebc+4c^{2}d^{3}\right)x+\frac{d^{2}\left(3b^{2}e^{2}-10bcde+7c^{2}d^{2}\right)}{2e}}{e^{4}\left(ex+d\right)^{2}}+\frac{\ln\left(ex+d\right)b^{2}}{e^{3}}-\frac{6\ln\left(ex+d\right)bcd}{e^{4}}+6$
parallelrisch	$c^{2}x^{4}e^{4}+2\ln\left(ex+d\right)x^{2}b^{2}e^{4}-12\ln\left(ex+d\right)x^{2}bcd\,e^{3}+12\ln\left(ex+d\right)x^{2}c^{2}d^{2}e^{2}+4x^{3}bc\,e^{4}-4d\,c^{2}x^{3}e^{3}+4\ln\left(ex+d\right)x\,b^{2}d\,e^{3}-24\ln(ex$

input
```
int((c*x^2+b*x)^2/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
c/e^4*(1/2*c*e*x^2+2*b*e*x-3*c*d*x)+(b^2*e^2-6*b*c*d*e+6*c^2*d^2)*ln(e*x+d
)/e^5+2*d/e^5*(b^2*e^2-3*b*c*d*e+2*c^2*d^2)/(e*x+d)-1/2*d^2*(b^2*e^2-2*b*c
*d*e+c^2*d^2)/e^5/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(115) = 230$.

Time = 0.09 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.00

$$\int \frac{(bx+cx^2)^2}{(d+ex)^3}\,dx$$
$$= \frac{c^2e^4x^4 + 7\,c^2d^4 - 10\,bcd^3e + 3\,b^2d^2e^2 - 4\,(c^2de^3 - bce^4)x^3 - (11\,c^2d^2e^2 - 8\,bcde^3)x^2 + 2\,(c^2d^3e - 4\,bcd^2e}{2\,(e}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/2*(c^2*e^4*x^4 + 7*c^2*d^4 - 10*b*c*d^3*e + 3*b^2*d^2*e^2 - 4*(c^2*d*e^3
 - b*c*e^4)*x^3 - (11*c^2*d^2*e^2 - 8*b*c*d*e^3)*x^2 + 2*(c^2*d^3*e - 4*b*
c*d^2*e^2 + 2*b^2*d*e^3)*x + 2*(6*c^2*d^4 - 6*b*c*d^3*e + b^2*d^2*e^2 + (6
*c^2*d^2*e^2 - 6*b*c*d*e^3 + b^2*e^4)*x^2 + 2*(6*c^2*d^3*e - 6*b*c*d^2*e^2
 + b^2*d*e^3)*x)*log(e*x + d))/(e^7*x^2 + 2*d*e^6*x + d^2*e^5)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.30

$$\int \frac{(bx+cx^2)^2}{(d+ex)^3}\,dx = \frac{c^2x^2}{2e^3} + x\left(\frac{2bc}{e^3} - \frac{3c^2d}{e^4}\right) + \frac{3b^2d^2e^2 - 10bcd^3e + 7c^2d^4 + x(4b^2de^3 - 12bcd^2e^2 + 8c^2d^3e)}{2d^2e^5 + 4de^6x + 2e^7x^2} + \frac{(b^2e^2 - 6bcde + 6c^2d^2)\log(d+ex)}{e^5}$$

input
```
integrate((c*x**2+b*x)**2/(e*x+d)**3,x)
```

output
```
c**2*x**2/(2*e**3) + x*(2*b*c/e**3 - 3*c**2*d/e**4) + (3*b**2*d**2*e**2 -
10*b*c*d**3*e + 7*c**2*d**4 + x*(4*b**2*d*e**3 - 12*b*c*d**2*e**2 + 8*c**2
*d**3*e))/(2*d**2*e**5 + 4*d*e**6*x + 2*e**7*x**2) + (b**2*e**2 - 6*b*c*d*
e + 6*c**2*d**2)*log(d + e*x)/e**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.24

$$\int \frac{(bx+cx^2)^2}{(d+ex)^3}\,dx = \frac{7\,c^2d^4-10\,bcd^3e+3\,b^2d^2e^2+4\,(2\,c^2d^3e-3\,bcd^2e^2+b^2de^3)x}{2\,(e^7x^2+2\,de^6x+d^2e^5)} + \frac{c^2ex^2-2\,(3\,c^2d-2\,bce)x}{2\,e^4} + \frac{(6\,c^2d^2-6\,bcde+b^2e^2)\log{(ex+d)}}{e^5}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*(7*c^2*d^4 - 10*b*c*d^3*e + 3*b^2*d^2*e^2 + 4*(2*c^2*d^3*e - 3*b*c*d^2
*e^2 + b^2*d*e^3)*x)/(e^7*x^2 + 2*d*e^6*x + d^2*e^5) + 1/2*(c^2*e*x^2 - 2*
(3*c^2*d - 2*b*c*e)*x)/e^4 + (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*log(e*x + d
)/e^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.17

$$\int \frac{(bx+cx^2)^2}{(d+ex)^3}\,dx = \frac{(6\,c^2d^2-6\,bcde+b^2e^2)\log{(|ex+d|)}}{e^5} + \frac{c^2e^3x^2-6\,c^2de^2x+4\,bce^3x}{2\,e^6} + \frac{7\,c^2d^4-10\,bcd^3e+3\,b^2d^2e^2+4\,(2\,c^2d^3e-3\,bcd^2e^2+b^2de^3)x}{2\,(ex+d)^2e^5}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^3,x, algorithm="giac")
```

output
```
(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*log(abs(e*x + d))/e^5 + 1/2*(c^2*e^3*x^2
 - 6*c^2*d*e^2*x + 4*b*c*e^3*x)/e^6 + 1/2*(7*c^2*d^4 - 10*b*c*d^3*e + 3*b^
2*d^2*e^2 + 4*(2*c^2*d^3*e - 3*b*c*d^2*e^2 + b^2*d*e^3)*x)/((e*x + d)^2*e^
5)
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.27

$$\int \frac{(bx+cx^2)^2}{(d+ex)^3}\,dx = \frac{\frac{3\,b^2\,d^2\,e^2-10\,b\,c\,d^3\,e+7\,c^2\,d^4}{2\,e}+x\,\left(2\,b^2\,d\,e^2-6\,b\,c\,d^2\,e+4\,c^2\,d^3\right)}{d^2\,e^4+2\,d\,e^5\,x+e^6\,x^2} - x\,\left(\frac{3\,c^2\,d}{e^4}-\frac{2\,b\,c}{e^3}\right)+\frac{\ln\left(d+e\,x\right)\,\left(b^2\,e^2-6\,b\,c\,d\,e+6\,c^2\,d^2\right)}{e^5}+\frac{c^2\,x^2}{2\,e^3}$$

input
```
int((b*x + c*x^2)^2/(d + e*x)^3,x)
```

output
```
((7*c^2*d^4 + 3*b^2*d^2*e^2 - 10*b*c*d^3*e)/(2*e) + x*(4*c^2*d^3 + 2*b^2*d
*e^2 - 6*b*c*d^2*e))/(d^2*e^4 + e^6*x^2 + 2*d*e^5*x) - x*((3*c^2*d)/e^4 -
(2*b*c)/e^3) + (log(d + e*x)*(b^2*e^2 + 6*c^2*d^2 - 6*b*c*d*e))/e^5 + (c^2
*x^2)/(2*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 266, normalized size of antiderivative = 2.24

$$\int \frac{(bx+cx^2)^2}{(d+ex)^3}\,dx = \frac{2\log(ex+d)\,b^2d^2e^2+4\log(ex+d)\,b^2d\,e^3x+2\log(ex+d)\,b^2e^4x^2-12\log(ex+d)\,bc\,d^3e-24\log(ex+$$

input
```
int((c*x^2+b*x)^2/(e*x+d)^3,x)
```

output
```
(2*log(d + e*x)*b**2*d**2*e**2 + 4*log(d + e*x)*b**2*d*e**3*x + 2*log(d +
e*x)*b**2*e**4*x**2 - 12*log(d + e*x)*b*c*d**3*e - 24*log(d + e*x)*b*c*d**
2*e**2*x - 12*log(d + e*x)*b*c*d*e**3*x**2 + 12*log(d + e*x)*c**2*d**4 + 2
4*log(d + e*x)*c**2*d**3*e*x + 12*log(d + e*x)*c**2*d**2*e**2*x**2 + b**2*
d**2*e**2 - 2*b**2*e**4*x**2 - 6*b*c*d**3*e + 12*b*c*d*e**3*x**2 + 4*b*c*e
**4*x**3 + 6*c**2*d**4 - 12*c**2*d**2*e**2*x**2 - 4*c**2*d*e**3*x**3 + c**
2*e**4*x**4)/(2*e**5*(d**2 + 2*d*e*x + e**2*x**2))
```

3.35 $\int \frac{(bx+cx^2)^2}{(d+ex)^4} dx$

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Giac [A] (verification not implemented) 314
Mupad [B] (verification not implemented) 315
Reduce [B] (verification not implemented) 315

Optimal result

Integrand size = 19, antiderivative size = 120

$$\int \frac{(bx+cx^2)^2}{(d+ex)^4} dx = \frac{c^2x}{e^4} - \frac{d^2(cd-be)^2}{3e^5(d+ex)^3} + \frac{d(cd-be)(2cd-be)}{e^5(d+ex)^2} - \frac{6c^2d^2-6bcde+b^2e^2}{e^5(d+ex)} - \frac{2c(2cd-be)\log(d+ex)}{e^5}$$

output
```
c^2*x/e^4-1/3*d^2*(-b*e+c*d)^2/e^5/(e*x+d)^3+d*(-b*e+c*d)*(-b*e+2*c*d)/e^5
/(e*x+d)^2-(b^2*e^2-6*b*c*d*e+6*c^2*d^2)/e^5/(e*x+d)-2*c*(-b*e+2*c*d)*ln(e
*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.12

$$\int \frac{(bx+cx^2)^2}{(d+ex)^4} dx = \frac{-b^2e^2(d^2+3dex+3e^2x^2)+bcde(11d^2+27dex+18e^2x^2)+c^2(-13d^4-27d^3ex-9d^2e^2x^2+9de^3x^3+}{3e^5(d+ex)^3}$$

input
```
Integrate[(b*x + c*x^2)^2/(d + e*x)^4,x]
```

output
```
(-(b^2*e^2*(d^2 + 3*d*e*x + 3*e^2*x^2)) + b*c*d*e*(11*d^2 + 27*d*e*x + 18*
e^2*x^2) + c^2*(-13*d^4 - 27*d^3*e*x - 9*d^2*e^2*x^2 + 9*d*e^3*x^3 + 3*e^4
*x^4) - 6*c*(2*c*d - b*e)*(d + e*x)^3*Log[d + e*x])/(3*e^5*(d + e*x)^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^2}{(d+ex)^4} \, dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(\frac{b^2e^2-6bcde+6c^2d^2}{e^4(d+ex)^2}+\frac{d^2(cd-be)^2}{e^4(d+ex)^4}-\frac{2c(2cd-be)}{e^4(d+ex)}+\frac{2d(cd-be)(be-2cd)}{e^4(d+ex)^3}+\frac{c^2}{e^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^2e^2-6bcde+6c^2d^2}{e^5(d+ex)}-\frac{d^2(cd-be)^2}{3e^5(d+ex)^3}+\frac{d(cd-be)(2cd-be)}{e^5(d+ex)^2}-\frac{2c(2cd-be)\log(d+ex)}{e^5}+\frac{c^2x}{e^4}$$

input
```
Int[(b*x + c*x^2)^2/(d + e*x)^4,x]
```

output
```
(c^2*x)/e^4 - (d^2*(c*d - b*e)^2)/(3*e^5*(d + e*x)^3) + (d*(c*d - b*e)*(2*
c*d - b*e))/(e^5*(d + e*x)^2) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)/(e^5*(d
+ e*x)) - (2*c*(2*c*d - b*e)*Log[d + e*x])/e^5
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08

method	result
norman	$\frac{\frac{c^2 x^4}{e}-\frac{d^2\left(b^2e^2-11bcde+22c^2d^2\right)}{3e^5}-\frac{\left(b^2e^2-6bcde+12c^2d^2\right)x^2}{e^3}-\frac{d\left(b^2e^2-9bcde+18c^2d^2\right)x}{e^4}}{(ex+d)^3}+\frac{2c(be-2cd)\ln(ex+d)}{e^5}$
default	$\frac{c^2x}{e^4}-\frac{d^2(b^2e^2-2bcde+c^2d^2)}{3e^5(ex+d)^3}+\frac{2c(be-2cd)\ln(ex+d)}{e^5}-\frac{b^2e^2-6bcde+6c^2d^2}{e^5(ex+d)}+\frac{d(b^2e^2-3bcde+2c^2d^2)}{e^5(ex+d)^2}$
risch	$\frac{c^2x}{e^4}+\frac{(-e^3b^2+6d\,e^2bc-6d^2e\,c^2)x^2-d(b^2e^2-9bcde+10c^2d^2)x-\frac{d^2\left(b^2e^2-11bcde+13c^2d^2\right)}{3e}}{e^4(ex+d)^3}+\frac{2c\ln(ex+d)b}{e^4}-\frac{4c^2d\ln(ex$
parallelrisch	$6\ln(ex+d)x^3bc\,e^4-12\ln(ex+d)x^3c^2d\,e^3+3c^2x^4e^4+18\ln(ex+d)x^2bcd\,e^3-36\ln(ex+d)x^2c^2d^2e^2+18\ln(ex+d)xbc\,d^2e^2-36\ln$

input
```
int((c*x^2+b*x)^2/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
(c^2*x^4/e-1/3*d^2*(b^2*e^2-11*b*c*d*e+22*c^2*d^2)/e^5-(b^2*e^2-6*b*c*d*e+
12*c^2*d^2)/e^3*x^2-d*(b^2*e^2-9*b*c*d*e+18*c^2*d^2)/e^4*x)/(e*x+d)^3+2/e^
5*c*(b*e-2*c*d)*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(118) = 236$.

Time = 0.09 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.04

$$\int \frac{(bx+cx^2)^2}{(d+ex)^4}\,dx = \frac{3\,c^2e^4x^4 + 9\,c^2de^3x^3 - 13\,c^2d^4 + 11\,bcd^3e - b^2d^2e^2 - 3\,(3\,c^2d^2e^2 - 6\,bcde^3 + b^2e^4)x^2 - 3\,(9\,c^2d^3e - 9\,bcd^2 \ldots}{3\,(e^8x^3 + 3\,d \ldots}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/3*(3*c^2*e^4*x^4 + 9*c^2*d*e^3*x^3 - 13*c^2*d^4 + 11*b*c*d^3*e - b^2*d^2
*e^2 - 3*(3*c^2*d^2*e^2 - 6*b*c*d*e^3 + b^2*e^4)*x^2 - 3*(9*c^2*d^3*e - 9*
b*c*d^2*e^2 + b^2*d*e^3)*x - 6*(2*c^2*d^4 - b*c*d^3*e + (2*c^2*d*e^3 - b*c
*e^4)*x^3 + 3*(2*c^2*d^2*e^2 - b*c*d*e^3)*x^2 + 3*(2*c^2*d^3*e - b*c*d^2*e
^2)*x)*log(e*x + d))/(e^8*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [A] (verification not implemented)

Time = 0.68 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.36

$$\int \frac{(bx+cx^2)^2}{(d+ex)^4}\,dx = \frac{c^2x}{e^4} + \frac{2c(be-2cd)\log{(d+ex)}}{e^5} + \frac{-b^2d^2e^2 + 11bcd^3e - 13c^2d^4 + x^2(-3b^2e^4 + 18bcde^3 - 18c^2d^2e^2) + x(-3b^2de^3 + 27bcd^2e^2 - 30c^2d^3e)}{3d^3e^5 + 9d^2e^6x + 9de^7x^2 + 3e^8x^3}$$

input
```
integrate((c*x**2+b*x)**2/(e*x+d)**4,x)
```

output
```
c**2*x/e**4 + 2*c*(b*e - 2*c*d)*log(d + e*x)/e**5 + (-b**2*d**2*e**2 + 11*
b*c*d**3*e - 13*c**2*d**4 + x**2*(-3*b**2*e**4 + 18*b*c*d*e**3 - 18*c**2*d
**2*e**2) + x*(-3*b**2*d*e**3 + 27*b*c*d**2*e**2 - 30*c**2*d**3*e))/(3*d**
3*e**5 + 9*d**2*e**6*x + 9*d*e**7*x**2 + 3*e**8*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.32

$$\int \frac{(bx+cx^2)^2}{(d+ex)^4}\,dx = -\frac{13\,c^2d^4-11\,bcd^3e+b^2d^2e^2+3\,(6\,c^2d^2e^2-6\,bcde^3+b^2e^4)x^2+3\,(10\,c^2d^3e-9\,bcd^2e^2+b^2de^3)x}{3\,(e^8x^3+3\,de^7x^2+3\,d^2e^6x+d^3e^5)} + \frac{c^2x}{e^4} - \frac{2\,(2\,c^2d-bce)\log(ex+d)}{e^5}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^4,x, algorithm="maxima")
```

output
```
-1/3*(13*c^2*d^4 - 11*b*c*d^3*e + b^2*d^2*e^2 + 3*(6*c^2*d^2*e^2 - 6*b*c*d
*e^3 + b^2*e^4)*x^2 + 3*(10*c^2*d^3*e - 9*b*c*d^2*e^2 + b^2*d*e^3)*x)/(e^8
*x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5) + c^2*x/e^4 - 2*(2*c^2*d - b*c
*e)*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.12

$$\int \frac{(bx+cx^2)^2}{(d+ex)^4}\,dx = \frac{c^2x}{e^4} - \frac{2\,(2\,c^2d-bce)\log(|ex+d|)}{e^5} - \frac{13\,c^2d^4-11\,bcd^3e+b^2d^2e^2+3\,(6\,c^2d^2e^2-6\,bcde^3+b^2e^4)x^2+3\,(10\,c^2d^3e-9\,bcd^2e^2+b^2de^3)x}{3\,(ex+d)^3e^5}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^4,x, algorithm="giac")
```

output
```
c^2*x/e^4 - 2*(2*c^2*d - b*c*e)*log(abs(e*x + d))/e^5 - 1/3*(13*c^2*d^4 -
11*b*c*d^3*e + b^2*d^2*e^2 + 3*(6*c^2*d^2*e^2 - 6*b*c*d*e^3 + b^2*e^4)*x^2
 + 3*(10*c^2*d^3*e - 9*b*c*d^2*e^2 + b^2*d*e^3)*x)/((e*x + d)^3*e^5)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.32

$$\int \frac{(bx+cx^2)^2}{(d+ex)^4}\,dx = \frac{c^2\,x}{e^4} - \frac{x^2\,(b^2\,e^3-6\,b\,c\,d\,e^2+6\,c^2\,d^2\,e)+\frac{b^2\,d^2\,e^2-11\,b\,c\,d^3\,e+13\,c^2\,d^4}{3\,e}+x\,(b^2\,d\,e^2-9\,b\,c\,d^2\,e+10\,c^2\,d^3)}{d^3\,e^4+3\,d^2\,e^5\,x+3\,d\,e^6\,x^2+e^7\,x^3} - \frac{\ln(d+e\,x)\,\left(4\,c^2\,d-2\,b\,c\,e\right)}{e^5}$$

input
```
int((b*x + c*x^2)^2/(d + e*x)^4,x)
```

output
```
(c^2*x)/e^4 - (x^2*(b^2*e^3 + 6*c^2*d^2*e - 6*b*c*d*e^2) + (13*c^2*d^4 + b
^2*d^2*e^2 - 11*b*c*d^3*e)/(3*e) + x*(10*c^2*d^3 + b^2*d*e^2 - 9*b*c*d^2*e
))/(d^3*e^4 + e^7*x^3 + 3*d^2*e^5*x + 3*d*e^6*x^2) - (log(d + e*x)*(4*c^2*
d - 2*b*c*e))/e^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.16

$$\int \frac{(bx+cx^2)^2}{(d+ex)^4}\,dx = \frac{6\log(ex+d)\,bc\,d^4e+18\log(ex+d)\,bc\,d^3e^2x+18\log(ex+d)\,bc\,d^2e^3x^2+6\log(ex+d)\,bcd\,e^4x^3-12\log}{}$$

input
```
int((c*x^2+b*x)^2/(e*x+d)^4,x)
```

output
```
(6*log(d + e*x)*b*c*d**4*e + 18*log(d + e*x)*b*c*d**3*e**2*x + 18*log(d +
e*x)*b*c*d**2*e**3*x**2 + 6*log(d + e*x)*b*c*d*e**4*x**3 - 12*log(d + e*x)
*c**2*d**5 - 36*log(d + e*x)*c**2*d**4*e*x - 36*log(d + e*x)*c**2*d**3*e**
2*x**2 - 12*log(d + e*x)*c**2*d**2*e**3*x**3 + b**2*e**5*x**3 + 5*b*c*d**4
*e + 9*b*c*d**3*e**2*x - 6*b*c*d*e**4*x**3 - 10*c**2*d**5 - 18*c**2*d**4*e
*x + 12*c**2*d**2*e**3*x**3 + 3*c**2*d*e**4*x**4)/(3*d*e**5*(d**3 + 3*d**2
*e*x + 3*d*e**2*x**2 + e**3*x**3))
```

3.36 $\int \frac{(bx+cx^2)^2}{(d+ex)^5} \, dx$

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Optimal result

Integrand size = 19, antiderivative size = 131

$$\int \frac{(bx+cx^2)^2}{(d+ex)^5} \, dx = -\frac{d^2(cd-be)^2}{4e^5(d+ex)^4} + \frac{2d(cd-be)(2cd-be)}{3e^5(d+ex)^3} - \frac{6c^2d^2-6bcde+b^2e^2}{2e^5(d+ex)^2} + \frac{2c(2cd-be)}{e^5(d+ex)} + \frac{c^2\log(d+ex)}{e^5}$$

output
```
-1/4*d^2*(-b*e+c*d)^2/e^5/(e*x+d)^4+2/3*d*(-b*e+c*d)*(-b*e+2*c*d)/e^5/(e*x
+d)^3-1/2*(b^2*e^2-6*b*c*d*e+6*c^2*d^2)/e^5/(e*x+d)^2+2*c*(-b*e+2*c*d)/e^5
/(e*x+d)+c^2*ln(e*x+d)/e^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.96

$$\int \frac{(bx+cx^2)^2}{(d+ex)^5} \, dx = \frac{-b^2e^2(d^2+4dex+6e^2x^2)-6bce(d^3+4d^2ex+6de^2x^2+4e^3x^3)+c^2d(25d^3+88d^2ex+108de^2x^2+48e}{12e^5(d+ex)^4}$$

input
```
Integrate[(b*x + c*x^2)^2/(d + e*x)^5,x]
```

output
```
(-(b^2*e^2*(d^2 + 4*d*e*x + 6*e^2*x^2)) - 6*b*c*e*(d^3 + 4*d^2*e*x + 6*d*e
^2*x^2 + 4*e^3*x^3) + c^2*d*(25*d^3 + 88*d^2*e*x + 108*d*e^2*x^2 + 48*e^3*
x^3) + 12*c^2*(d + e*x)^4*Log[d + e*x])/(12*e^5*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^2}{(d+ex)^5}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{b^2e^2-6bcde+6c^2d^2}{e^4(d+ex)^3}+\frac{d^2(cd-be)^2}{e^4(d+ex)^5}-\frac{2c(2cd-be)}{e^4(d+ex)^2}+\frac{2d(cd-be)(be-2cd)}{e^4(d+ex)^4}+\frac{c^2}{e^4(d+ex)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2e^2-6bcde+6c^2d^2}{2e^5(d+ex)^2}-\frac{d^2(cd-be)^2}{4e^5(d+ex)^4}+\frac{2c(2cd-be)}{e^5(d+ex)}+\frac{2d(cd-be)(2cd-be)}{3e^5(d+ex)^3}+\frac{c^2\log(d+ex)}{e^5}$$

input
```
Int[(b*x + c*x^2)^2/(d + e*x)^5,x]
```

output
```
-1/4*(d^2*(c*d - b*e)^2)/(e^5*(d + e*x)^4) + (2*d*(c*d - b*e)*(2*c*d - b*e
))/(3*e^5*(d + e*x)^3) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)/(2*e^5*(d + e*x
)^2) + (2*c*(2*c*d - b*e))/(e^5*(d + e*x)) + (c^2*Log[d + e*x])/e^5
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.99

method	result
risch	$\frac{-\frac{2c(be-2cd)x^3}{e^2}-\frac{\left(b^2e^2+6bcde-18c^2d^2\right)x^2}{2e^3}-\frac{d\left(b^2e^2+6bcde-22c^2d^2\right)x}{3e^4}-\frac{d^2\left(b^2e^2+6bcde-25c^2d^2\right)}{12e^5}}{(ex+d)^4}+\frac{c^2\ln(ex+d)}{e^5}$
norman	$\frac{-\frac{d^2\left(b^2e^2+6bcde-25c^2d^2\right)}{12e^5}-\frac{2\left(bce-2c^2d\right)x^3}{e^2}-\frac{\left(b^2e^2+6bcde-18c^2d^2\right)x^2}{2e^3}-\frac{d\left(b^2e^2+6bcde-22c^2d^2\right)x}{3e^4}}{(ex+d)^4}+\frac{c^2\ln(ex+d)}{e^5}$
default	$\frac{2d(b^2e^2-3bcde+2c^2d^2)}{3e^5(ex+d)^3}-\frac{d^2(b^2e^2-2bcde+c^2d^2)}{4e^5(ex+d)^4}+\frac{c^2\ln(ex+d)}{e^5}-\frac{2c(be-2cd)}{e^5(ex+d)}-\frac{b^2e^2-6bcde+6c^2d^2}{2e^5(ex+d)^2}$
parallelrisch	$\frac{12\ln(ex+d)x^4c^2e^4+48\ln(ex+d)x^3c^2d\,e^3+72\ln(ex+d)x^2c^2d^2e^2-24x^3bc\,e^4+48d\,c^2x^3e^3+48\ln(ex+d)x\,c^2d^3e-6x^2b^2e^4-36}{12e^5(ex+d)^4}$

input
```
int((c*x^2+b*x)^2/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
(-2*c*(b*e-2*c*d)/e^2*x^3-1/2*(b^2*e^2+6*b*c*d*e-18*c^2*d^2)/e^3*x^2-1/3*d
*(b^2*e^2+6*b*c*d*e-22*c^2*d^2)/e^4*x-1/12*d^2*(b^2*e^2+6*b*c*d*e-25*c^2*d
^2)/e^5)/(e*x+d)^4+c^2*ln(e*x+d)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.72

$$\int \frac{(bx + cx^2)^2}{(d + ex)^5} dx$$
$$= \frac{25 c^2d^4 - 6 bcd^3e - b^2d^2e^2 + 24 (2 c^2de^3 - bce^4)x^3 + 6 (18 c^2d^2e^2 - 6 bcde^3 - b^2e^4)x^2 + 4 (22 c^2d^3e - 6 bc}{12 (e^9x^4 + 4 de^8x^3 + 6 d^2e^7x^2 + 4}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^5,x, algorithm="fricas")
```

output
```
1/12*(25*c^2*d^4 - 6*b*c*d^3*e - b^2*d^2*e^2 + 24*(2*c^2*d*e^3 - b*c*e^4)*
x^3 + 6*(18*c^2*d^2*e^2 - 6*b*c*d*e^3 - b^2*e^4)*x^2 + 4*(22*c^2*d^3*e - 6
*b*c*d^2*e^2 - b^2*d*e^3)*x + 12*(c^2*e^4*x^4 + 4*c^2*d*e^3*x^3 + 6*c^2*d^
2*e^2*x^2 + 4*c^2*d^3*e*x + c^2*d^4)*log(e*x + d))/(e^9*x^4 + 4*d*e^8*x^3
+ 6*d^2*e^7*x^2 + 4*d^3*e^6*x + d^4*e^5)
```

Sympy [A] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.37

$$\int \frac{(bx + cx^2)^2}{(d + ex)^5} dx = \frac{c^2 \log(d + ex)}{e^5}$$
$$+ \frac{-b^2d^2e^2 - 6bcd^3e + 25c^2d^4 + x^3(-24bce^4 + 48c^2de^3) + x^2(-6b^2e^4 - 36bcde^3 + 108c^2d^2e^2) + x(-4b^2d}{12d^4e^5 + 48d^3e^6x + 72d^2e^7x^2 + 48de^8x^3 + 12e^9x^4}$$

input
```
integrate((c*x**2+b*x)**2/(e*x+d)**5,x)
```

output
```
c**2*log(d + e*x)/e**5 + (-b**2*d**2*e**2 - 6*b*c*d**3*e + 25*c**2*d**4 +
x**3*(-24*b*c*e**4 + 48*c**2*d*e**3) + x**2*(-6*b**2*e**4 - 36*b*c*d*e**3
+ 108*c**2*d**2*e**2) + x*(-4*b**2*d*e**3 - 24*b*c*d**2*e**2 + 88*c**2*d**
3*e))/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3
+ 12*e**9*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.35

$$\int \frac{(bx+cx^2)^2}{(d+ex)^5}\,dx$$
$$= \frac{25\,c^2d^4 - 6\,bcd^3e - b^2d^2e^2 + 24\,(2\,c^2de^3 - bce^4)x^3 + 6\,(18\,c^2d^2e^2 - 6\,bcde^3 - b^2e^4)x^2 + 4\,(22\,c^2d^3e - 6\,bc}{12\,(e^9x^4 + 4\,de^8x^3 + 6\,d^2e^7x^2 + 4\,d^3e^6x + d^4e^5)}$$
$$+ \frac{c^2\log(ex+d)}{e^5}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^5,x, algorithm="maxima")
```

output
```
1/12*(25*c^2*d^4 - 6*b*c*d^3*e - b^2*d^2*e^2 + 24*(2*c^2*d*e^3 - b*c*e^4)*
x^3 + 6*(18*c^2*d^2*e^2 - 6*b*c*d*e^3 - b^2*e^4)*x^2 + 4*(22*c^2*d^3*e - 6
*b*c*d^2*e^2 - b^2*d*e^3)*x)/(e^9*x^4 + 4*d*e^8*x^3 + 6*d^2*e^7*x^2 + 4*d^
3*e^6*x + d^4*e^5) + c^2*log(e*x + d)/e^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.65

$$\int \frac{(bx+cx^2)^2}{(d+ex)^5}\,dx = -\frac{c^2\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^5}$$
$$+ \frac{\frac{48\,c^2de^{15}}{ex+d} - \frac{36\,c^2d^2e^{15}}{(ex+d)^2} + \frac{16\,c^2d^3e^{15}}{(ex+d)^3} - \frac{3\,c^2d^4e^{15}}{(ex+d)^4} - \frac{24\,bce^{16}}{ex+d} + \frac{36\,bcde^{16}}{(ex+d)^2} - \frac{24\,bcd^2e^{16}}{(ex+d)^3} + \frac{6\,bcd^3e^{16}}{(ex+d)^4} - \frac{6\,b^2e^{17}}{(ex+d)^2} + \frac{8\,b^2de^{17}}{(ex+d)^3} -}{12\,e^{20}}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^5,x, algorithm="giac")
```

output
```
-c^2*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^5 + 1/12*(48*c^2*d*e^15/(e*x
 + d) - 36*c^2*d^2*e^15/(e*x + d)^2 + 16*c^2*d^3*e^15/(e*x + d)^3 - 3*c^2*
d^4*e^15/(e*x + d)^4 - 24*b*c*e^16/(e*x + d) + 36*b*c*d*e^16/(e*x + d)^2 -
 24*b*c*d^2*e^16/(e*x + d)^3 + 6*b*c*d^3*e^16/(e*x + d)^4 - 6*b^2*e^17/(e*
x + d)^2 + 8*b^2*d*e^17/(e*x + d)^3 - 3*b^2*d^2*e^17/(e*x + d)^4)/e^20
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.27

$$\int \frac{(bx+cx^2)^2}{(d+ex)^5}\,dx = \frac{c^2\,\ln(d+e\,x)}{e^5} - \frac{\frac{b^2\,d^2\,e^2+6\,b\,c\,d^3\,e-25\,c^2\,d^4}{12\,e^5}+\frac{x^2\,(b^2\,e^2+6\,b\,c\,d\,e-18\,c^2\,d^2)}{2\,e^3}+\frac{x\,(b^2\,d\,e^2+6\,b\,c\,d^2\,e-22\,c^2\,d^3)}{3\,e^4}+\frac{2\,c\,x^3\,(b\,e-2\,c\,d)}{e^2}}{d^4+4\,d^3\,e\,x+6\,d^2\,e^2\,x^2+4\,d\,e^3\,x^3+e^4\,x^4}$$

input
```
int((b*x + c*x^2)^2/(d + e*x)^5,x)
```

output
```
(c^2*log(d + e*x))/e^5 - ((b^2*d^2*e^2 - 25*c^2*d^4 + 6*b*c*d^3*e)/(12*e^5
) + (x^2*(b^2*e^2 - 18*c^2*d^2 + 6*b*c*d*e))/(2*e^3) + (x*(b^2*d*e^2 - 22*
c^2*d^3 + 6*b*c*d^2*e))/(3*e^4) + (2*c*x^3*(b*e - 2*c*d))/e^2)/(d^4 + e^4*
x^4 + 4*d*e^3*x^3 + 6*d^2*e^2*x^2 + 4*d^3*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.73

$$\int \frac{(bx+cx^2)^2}{(d+ex)^5}\,dx = \frac{12\log(ex+d)\,c^2d^5+48\log(ex+d)\,c^2d^4ex+72\log(ex+d)\,c^2d^3e^2x^2+48\log(ex+d)\,c^2d^2e^3x^3+12\log}{12d\,e^5\,(e^4x^4+4d\,e^3x}$$

input
```
int((c*x^2+b*x)^2/(e*x+d)^5,x)
```

output
```
(12*log(d + e*x)*c**2*d**5 + 48*log(d + e*x)*c**2*d**4*e*x + 72*log(d + e*
x)*c**2*d**3*e**2*x**2 + 48*log(d + e*x)*c**2*d**2*e**3*x**3 + 12*log(d +
e*x)*c**2*d*e**4*x**4 - b**2*d**3*e**2 - 4*b**2*d**2*e**3*x - 6*b**2*d*e**
4*x**2 + 6*b*c*e**5*x**4 + 13*c**2*d**5 + 40*c**2*d**4*e*x + 36*c**2*d**3*
e**2*x**2 - 12*c**2*d*e**4*x**4)/(12*d*e**5*(d**4 + 4*d**3*e*x + 6*d**2*e*
*2*x**2 + 4*d*e**3*x**3 + e**4*x**4))
```

3.37 $\int \frac{(bx+cx^2)^2}{(d+ex)^6} dx$

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Rubi [A] (verified) 323
Maple [A] (verified) 324
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Reduce [B] (verification not implemented) 327

Optimal result

Integrand size = 19, antiderivative size = 132

$$\int \frac{(bx+cx^2)^2}{(d+ex)^6} dx = -\frac{d^2(cd-be)^2}{5e^5(d+ex)^5} + \frac{d(cd-be)(2cd-be)}{2e^5(d+ex)^4} - \frac{6c^2d^2-6bcde+b^2e^2}{3e^5(d+ex)^3} + \frac{c(2cd-be)}{e^5(d+ex)^2} - \frac{c^2}{e^5(d+ex)}$$

output
```
-1/5*d^2*(-b*e+c*d)^2/e^5/(e*x+d)^5+1/2*d*(-b*e+c*d)*(-b*e+2*c*d)/e^5/(e*x
+d)^4-1/3*(b^2*e^2-6*b*c*d*e+6*c^2*d^2)/e^5/(e*x+d)^3+c*(-b*e+2*c*d)/e^5/(
e*x+d)^2-c^2/e^5/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.88

$$\int \frac{(bx+cx^2)^2}{(d+ex)^6} dx = -\frac{b^2e^2(d^2+5dex+10e^2x^2)+3bce(d^3+5d^2ex+10de^2x^2+10e^3x^3)+6c^2(d^4+5d^3ex+10d^2e^2x^2+10}{30e^5(d+ex)^5}$$

input
```
Integrate[(b*x + c*x^2)^2/(d + e*x)^6,x]
```

output
```
-1/30*(b^2*e^2*(d^2 + 5*d*e*x + 10*e^2*x^2) + 3*b*c*e*(d^3 + 5*d^2*e*x + 1
0*d*e^2*x^2 + 10*e^3*x^3) + 6*c^2*(d^4 + 5*d^3*e*x + 10*d^2*e^2*x^2 + 10*d
*e^3*x^3 + 5*e^4*x^4))/(e^5*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^2}{(d+ex)^6}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{b^2e^2-6bcde+6c^2d^2}{e^4(d+ex)^4}+\frac{d^2(cd-be)^2}{e^4(d+ex)^6}-\frac{2c(2cd-be)}{e^4(d+ex)^3}+\frac{2d(cd-be)(be-2cd)}{e^4(d+ex)^5}+\frac{c^2}{e^4(d+ex)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2e^2-6bcde+6c^2d^2}{3e^5(d+ex)^3}-\frac{d^2(cd-be)^2}{5e^5(d+ex)^5}+\frac{c(2cd-be)}{e^5(d+ex)^2}+\frac{d(cd-be)(2cd-be)}{2e^5(d+ex)^4}-\frac{c^2}{e^5(d+ex)}$$

input
```
Int[(b*x + c*x^2)^2/(d + e*x)^6,x]
```

output
```
-1/5*(d^2*(c*d - b*e)^2)/(e^5*(d + e*x)^5) + (d*(c*d - b*e)*(2*c*d - b*e))
/(2*e^5*(d + e*x)^4) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)/(3*e^5*(d + e*x)^
3) + (c*(2*c*d - b*e))/(e^5*(d + e*x)^2) - c^2/(e^5*(d + e*x))
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.96

method	result
risch	$\frac{-\frac{c^{2}x^{4}}{e}-\frac{c(be+2cd)x^{3}}{e^{2}}-\frac{\left(b^{2}e^{2}+3bcde+6c^{2}d^{2}\right)x^{2}}{3e^{3}}-\frac{d\left(b^{2}e^{2}+3bcde+6c^{2}d^{2}\right)x}{6e^{4}}-\frac{d^{2}\left(b^{2}e^{2}+3bcde+6c^{2}d^{2}\right)}{30e^{5}}}{(ex+d)^{5}}$
norman	$\frac{-\frac{c^{2}x^{4}}{e}-\frac{\left(bce+2c^{2}d\right)x^{3}}{e^{2}}-\frac{\left(b^{2}e^{2}+3bcde+6c^{2}d^{2}\right)x^{2}}{3e^{3}}-\frac{d\left(b^{2}e^{2}+3bcde+6c^{2}d^{2}\right)x}{6e^{4}}-\frac{d^{2}\left(b^{2}e^{2}+3bcde+6c^{2}d^{2}\right)}{30e^{5}}}{(ex+d)^{5}}$
gosper	$-\frac{30c^{2}x^{4}e^{4}+30x^{3}bc\,e^{4}+60d\,c^{2}x^{3}e^{3}+10x^{2}b^{2}e^{4}+30x^{2}bcd\,e^{3}+60x^{2}c^{2}d^{2}e^{2}+5x\,b^{2}d\,e^{3}+15xbc\,d^{2}e^{2}+30x\,c^{2}d^{3}e+d^{2}e^{2}b^{2}+3d^{3}eb}{30e^{5}(ex+d)^{5}}$
parallelrisch	$\frac{-30c^{2}x^{4}e^{4}-30x^{3}bc\,e^{4}-60d\,c^{2}x^{3}e^{3}-10x^{2}b^{2}e^{4}-30x^{2}bcd\,e^{3}-60x^{2}c^{2}d^{2}e^{2}-5x\,b^{2}d\,e^{3}-15xbc\,d^{2}e^{2}-30x\,c^{2}d^{3}e-d^{2}e^{2}b^{2}-3d^{3}ebc}{30e^{5}(ex+d)^{5}}$
default	$-\frac{b^{2}e^{2}-6bcde+6c^{2}d^{2}}{3e^{5}(ex+d)^{3}}+\frac{d(b^{2}e^{2}-3bcde+2c^{2}d^{2})}{2e^{5}(ex+d)^{4}}-\frac{d^{2}\left(b^{2}e^{2}-2bcde+c^{2}d^{2}\right)}{5e^{5}(ex+d)^{5}}-\frac{c^{2}}{e^{5}(ex+d)}-\frac{c(be-2cd)}{e^{5}(ex+d)^{2}}$
orering	$-\frac{(30c^{2}x^{4}e^{4}+30x^{3}bc\,e^{4}+60d\,c^{2}x^{3}e^{3}+10x^{2}b^{2}e^{4}+30x^{2}bcd\,e^{3}+60x^{2}c^{2}d^{2}e^{2}+5x\,b^{2}d\,e^{3}+15xbc\,d^{2}e^{2}+30x\,c^{2}d^{3}e+d^{2}e^{2}b^{2}+3d^{3}e}{30e^{5}(cx+b)^{2}(ex+d)^{5}x^{2}}$

input
```
int((c*x^2+b*x)^2/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

output
```
(-c^2*x^4/e-c*(b*e+2*c*d)/e^2*x^3-1/3*(b^2*e^2+3*b*c*d*e+6*c^2*d^2)/e^3*x^
2-1/6*d*(b^2*e^2+3*b*c*d*e+6*c^2*d^2)/e^4*x-1/30*d^2*(b^2*e^2+3*b*c*d*e+6*
c^2*d^2)/e^5)/(e*x+d)^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.37

$$\int \frac{(bx+cx^2)^2}{(d+ex)^6}\,dx = -\frac{30\,c^2e^4x^4+6\,c^2d^4+3\,bcd^3e+b^2d^2e^2+30\,(2\,c^2de^3+bce^4)x^3+10\,(6\,c^2d^2e^2+3\,bcde^3+b^2e^4)x^2+5\,(6}{30\,(e^{10}x^5+5\,de^9x^4+10\,d^2e^8x^3+10\,d^3e^7x^2+5\,d^4e^6x+d^5e^5)}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^6,x, algorithm="fricas")
```

output
```
-1/30*(30*c^2*e^4*x^4 + 6*c^2*d^4 + 3*b*c*d^3*e + b^2*d^2*e^2 + 30*(2*c^2*
d*e^3 + b*c*e^4)*x^3 + 10*(6*c^2*d^2*e^2 + 3*b*c*d*e^3 + b^2*e^4)*x^2 + 5*
(6*c^2*d^3*e + 3*b*c*d^2*e^2 + b^2*d*e^3)*x)/(e^10*x^5 + 5*d*e^9*x^4 + 10*
d^2*e^8*x^3 + 10*d^3*e^7*x^2 + 5*d^4*e^6*x + d^5*e^5)
```

Sympy [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.48

$$\int \frac{(bx+cx^2)^2}{(d+ex)^6}\,dx = \frac{-b^2d^2e^2-3bcd^3e-6c^2d^4-30c^2e^4x^4+x^3(-30bce^4-60c^2de^3)+x^2(-10b^2e^4-30bcde^3-60c^2d^2e^2)+}{30d^5e^5+150d^4e^6x+300d^3e^7x^2+300d^2e^8x^3+150de^9x^4+30e^{10}x^5}$$

input
```
integrate((c*x**2+b*x)**2/(e*x+d)**6,x)
```

output
```
(-b**2*d**2*e**2 - 3*b*c*d**3*e - 6*c**2*d**4 - 30*c**2*e**4*x**4 + x**3*(
-30*b*c*e**4 - 60*c**2*d*e**3) + x**2*(-10*b**2*e**4 - 30*b*c*d*e**3 - 60*
c**2*d**2*e**2) + x*(-5*b**2*d*e**3 - 15*b*c*d**2*e**2 - 30*c**2*d**3*e))/
(30*d**5*e**5 + 150*d**4*e**6*x + 300*d**3*e**7*x**2 + 300*d**2*e**8*x**3
+ 150*d*e**9*x**4 + 30*e**10*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.37

$$\int \frac{(bx+cx^2)^2}{(d+ex)^6}\,dx = -\frac{30\,c^2e^4x^4+6\,c^2d^4+3\,bcd^3e+b^2d^2e^2+30\,(2\,c^2de^3+bce^4)x^3+10\,(6\,c^2d^2e^2+3\,bcde^3+b^2e^4)x^2+5\,(6}{30\,(e^{10}x^5+5\,de^9x^4+10\,d^2e^8x^3+10\,d^3e^7x^2+5\,d^4e^6x+d^5e^5)}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^6,x, algorithm="maxima")
```

output
```
-1/30*(30*c^2*e^4*x^4 + 6*c^2*d^4 + 3*b*c*d^3*e + b^2*d^2*e^2 + 30*(2*c^2*
d*e^3 + b*c*e^4)*x^3 + 10*(6*c^2*d^2*e^2 + 3*b*c*d*e^3 + b^2*e^4)*x^2 + 5*
(6*c^2*d^3*e + 3*b*c*d^2*e^2 + b^2*d*e^3)*x)/(e^10*x^5 + 5*d*e^9*x^4 + 10*
d^2*e^8*x^3 + 10*d^3*e^7*x^2 + 5*d^4*e^6*x + d^5*e^5)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.05

$$\int \frac{(bx+cx^2)^2}{(d+ex)^6}\,dx = -\frac{30\,c^2e^4x^4+60\,c^2de^3x^3+30\,bce^4x^3+60\,c^2d^2e^2x^2+30\,bcde^3x^2+10\,b^2e^4x^2+30\,c^2d^3ex+15\,bcd^2e^2x+}{30\,(ex+d)^5e^5}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^6,x, algorithm="giac")
```

output
```
-1/30*(30*c^2*e^4*x^4 + 60*c^2*d*e^3*x^3 + 30*b*c*e^4*x^3 + 60*c^2*d^2*e^2
*x^2 + 30*b*c*d*e^3*x^2 + 10*b^2*e^4*x^2 + 30*c^2*d^3*e*x + 15*b*c*d^2*e^2
*x + 5*b^2*d*e^3*x + 6*c^2*d^4 + 3*b*c*d^3*e + b^2*d^2*e^2)/((e*x + d)^5*e
^5)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.28

$$\int \frac{(bx+cx^2)^2}{(d+ex)^6}\,dx = -\frac{\frac{x^2\,(b^2\,e^2+3\,b\,c\,d\,e+6\,c^2\,d^2)}{3\,e^3}+\frac{c^2\,x^4}{e}+\frac{d^2\,(b^2\,e^2+3\,b\,c\,d\,e+6\,c^2\,d^2)}{30\,e^5}+\frac{c\,x^3\,(b\,e+2\,c\,d)}{e^2}+\frac{d\,x\,(b^2\,e^2+3\,b\,c\,d\,e+6\,c^2\,d^2)}{6\,e^4}}{d^5+5\,d^4\,e\,x+10\,d^3\,e^2\,x^2+10\,d^2\,e^3\,x^3+5\,d\,e^4\,x^4+e^5\,x^5}$$

input
```
int((b*x + c*x^2)^2/(d + e*x)^6,x)
```

output
```
-((x^2*(b^2*e^2 + 6*c^2*d^2 + 3*b*c*d*e))/(3*e^3) + (c^2*x^4)/e + (d^2*(b^
2*e^2 + 6*c^2*d^2 + 3*b*c*d*e))/(30*e^5) + (c*x^3*(b*e + 2*c*d))/e^2 + (d*
x*(b^2*e^2 + 6*c^2*d^2 + 3*b*c*d*e))/(6*e^4))/(d^5 + e^5*x^5 + 5*d*e^4*x^4
 + 10*d^3*e^2*x^2 + 10*d^2*e^3*x^3 + 5*d^4*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.09

$$\int \frac{(bx+cx^2)^2}{(d+ex)^6}\,dx = \frac{6c^2e^4x^5-30bcd\,e^3x^3-10b^2d\,e^3x^2-30bc\,d^2e^2x^2-5b^2d^2e^2x-15bc\,d^3ex-b^2d^3e-3bc\,d^4}{30d\,e^4\,(e^5x^5+5d\,e^4x^4+10d^2e^3x^3+10d^3e^2x^2+5d^4ex+d^5)}$$

input
```
int((c*x^2+b*x)^2/(e*x+d)^6,x)
```

output
```
( - b**2*d**3*e - 5*b**2*d**2*e**2*x - 10*b**2*d*e**3*x**2 - 3*b*c*d**4 -
15*b*c*d**3*e*x - 30*b*c*d**2*e**2*x**2 - 30*b*c*d*e**3*x**3 + 6*c**2*e**4
*x**5)/(30*d*e**4*(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*e**3*x*
*3 + 5*d*e**4*x**4 + e**5*x**5))
```

3.38 $\int \frac{(bx+cx^2)^2}{(d+ex)^7}\,dx$

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Maple [A] (verified) 330
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Maxima [A] (verification not implemented) 332
Giac [A] (verification not implemented) 332
Mupad [B] (verification not implemented) 333
Reduce [B] (verification not implemented) 333

Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{(bx+cx^2)^2}{(d+ex)^7}\,dx = -\frac{d^2(cd-be)^2}{6e^5(d+ex)^6}+\frac{2d(cd-be)(2cd-be)}{5e^5(d+ex)^5} - \frac{6c^2d^2-6bcde+b^2e^2}{4e^5(d+ex)^4}+\frac{2c(2cd-be)}{3e^5(d+ex)^3}-\frac{c^2}{2e^5(d+ex)^2}$$

output
```
-1/6*d^2*(-b*e+c*d)^2/e^5/(e*x+d)^6+2/5*d*(-b*e+c*d)*(-b*e+2*c*d)/e^5/(e*x
+d)^5-1/4*(b^2*e^2-6*b*c*d*e+6*c^2*d^2)/e^5/(e*x+d)^4+2/3*c*(-b*e+2*c*d)/e
^5/(e*x+d)^3-1/2*c^2/e^5/(e*x+d)^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int \frac{(bx+cx^2)^2}{(d+ex)^7}\,dx = -\frac{b^2e^2(d^2+6dex+15e^2x^2)+2bce(d^3+6d^2ex+15de^2x^2+20e^3x^3)+2c^2(d^4+6d^3ex+15d^2e^2x^2+20e}{60e^5(d+ex)^6}$$

input
```
Integrate[(b*x + c*x^2)^2/(d + e*x)^7,x]
```

output
```
-1/60*(b^2*e^2*(d^2 + 6*d*e*x + 15*e^2*x^2) + 2*b*c*e*(d^3 + 6*d^2*e*x + 1
5*d*e^2*x^2 + 20*e^3*x^3) + 2*c^2*(d^4 + 6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d
*e^3*x^3 + 15*e^4*x^4))/(e^5*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^2}{(d+ex)^7}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{b^2e^2-6bcde+6c^2d^2}{e^4(d+ex)^5}+\frac{d^2(cd-be)^2}{e^4(d+ex)^7}-\frac{2c(2cd-be)}{e^4(d+ex)^4}+\frac{2d(cd-be)(be-2cd)}{e^4(d+ex)^6}+\frac{c^2}{e^4(d+ex)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2e^2-6bcde+6c^2d^2}{4e^5(d+ex)^4}-\frac{d^2(cd-be)^2}{6e^5(d+ex)^6}+\frac{2c(2cd-be)}{3e^5(d+ex)^3}+\frac{2d(cd-be)(2cd-be)}{5e^5(d+ex)^5}-\frac{c^2}{2e^5(d+ex)^2}$$

input
```
Int[(b*x + c*x^2)^2/(d + e*x)^7,x]
```

output
```
-1/6*(d^2*(c*d - b*e)^2)/(e^5*(d + e*x)^6) + (2*d*(c*d - b*e)*(2*c*d - b*e
))/(5*e^5*(d + e*x)^5) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)/(4*e^5*(d + e*x
)^4) + (2*c*(2*c*d - b*e))/(3*e^5*(d + e*x)^3) - c^2/(2*e^5*(d + e*x)^2)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.92

method	result
risch	$\frac{-\frac{c^2x^4}{2e}-\frac{2c(be+cd)x^3}{3e^2}-\frac{\left(b^2e^2+2bcde+2c^2d^2\right)x^2}{4e^3}-\frac{d\left(b^2e^2+2bcde+2c^2d^2\right)x}{10e^4}-\frac{d^2\left(b^2e^2+2bcde+2c^2d^2\right)}{60e^5}}{(ex+d)^6}$
gosper	$-\frac{30c^2x^4e^4+40x^3bc\,e^4+40d\,c^2x^3e^3+15x^2b^2e^4+30x^2bcd\,e^3+30x^2c^2d^2e^2+6x\,b^2d\,e^3+12xbc\,d^2e^2+12x\,c^2d^3e+d^2e^2b^2+2d^3eb}{60e^5(ex+d)^6}$
norman	$\frac{-\frac{c^2x^4}{2e}-\frac{2\left(e^2bc+de\,c^2\right)x^3}{3e^3}-\frac{\left(e^3b^2+2d\,e^2bc+2d^2e\,c^2\right)x^2}{4e^4}-\frac{d\left(e^3b^2+2d\,e^2bc+2d^2e\,c^2\right)x}{10e^5}-\frac{d^2\left(e^3b^2+2d\,e^2bc+2d^2e\,c^2\right)}{60e^6}}{(ex+d)^6}$
default	$-\frac{2c(be-2cd)}{3e^5(ex+d)^3}-\frac{b^2e^2-6bcde+6c^2d^2}{4e^5(ex+d)^4}+\frac{2d(b^2e^2-3bcde+2c^2d^2)}{5e^5(ex+d)^5}-\frac{c^2}{2e^5(ex+d)^2}-\frac{d^2(b^2e^2-2bcde+c^2d^2)}{6e^5(ex+d)^6}$
parallelrisch	$\frac{-30c^2x^4e^5-40bc\,e^5x^3-40c^2d\,e^4x^3-15b^2e^5x^2-30bcd\,e^4x^2-30c^2d^2e^3x^2-6b^2d\,e^4x-12bc\,d^2e^3x-12c^2d^3e^2x-b^2d^2e^3-2bc\,d^3}{60e^6(ex+d)^6}$
orering	$-\frac{(30c^2x^4e^4+40x^3bc\,e^4+40d\,c^2x^3e^3+15x^2b^2e^4+30x^2bcd\,e^3+30x^2c^2d^2e^2+6x\,b^2d\,e^3+12xbc\,d^2e^2+12x\,c^2d^3e+d^2e^2b^2+2d^3e}{60e^5(cx+b)^2(ex+d)^6x^2}$

input
```
int((c*x^2+b*x)^2/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*c^2*x^4/e-2/3*c/e^2*(b*e+c*d)*x^3-1/4/e^3*(b^2*e^2+2*b*c*d*e+2*c^2*d
^2)*x^2-1/10*d/e^4*(b^2*e^2+2*b*c*d*e+2*c^2*d^2)*x-1/60*d^2/e^5*(b^2*e^2+2
*b*c*d*e+2*c^2*d^2))/(e*x+d)^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.39

$$\int \frac{(bx+cx^2)^2}{(d+ex)^7}\,dx = -\frac{30\,c^2e^4x^4 + 2\,c^2d^4 + 2\,bcd^3e + b^2d^2e^2 + 40\,(c^2de^3 + bce^4)x^3 + 15\,(2\,c^2d^2e^2 + 2\,bcde^3 + b^2e^4)x^2 + 6\,(2\,c}{60\,(e^{11}x^6 + 6\,de^{10}x^5 + 15\,d^2e^9x^4 + 20\,d^3e^8x^3 + 15\,d^4e^7x^2 + 6\,d^5e^6x + d^6e^5)}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^7,x, algorithm="fricas")
```

output
```
-1/60*(30*c^2*e^4*x^4 + 2*c^2*d^4 + 2*b*c*d^3*e + b^2*d^2*e^2 + 40*(c^2*d*
e^3 + b*c*e^4)*x^3 + 15*(2*c^2*d^2*e^2 + 2*b*c*d*e^3 + b^2*e^4)*x^2 + 6*(2
*c^2*d^3*e + 2*b*c*d^2*e^2 + b^2*d*e^3)*x)/(e^11*x^6 + 6*d*e^10*x^5 + 15*d
^2*e^9*x^4 + 20*d^3*e^8*x^3 + 15*d^4*e^7*x^2 + 6*d^5*e^6*x + d^6*e^5)
```

Sympy [A] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.51

$$\int \frac{(bx+cx^2)^2}{(d+ex)^7}\,dx = \frac{-b^2d^2e^2 - 2bcd^3e - 2c^2d^4 - 30c^2e^4x^4 + x^3(-40bce^4 - 40c^2de^3) + x^2(-15b^2e^4 - 30bcde^3 - 30c^2d^2e^2) +}{60d^6e^5 + 360d^5e^6x + 900d^4e^7x^2 + 1200d^3e^8x^3 + 900d^2e^9x^4 + 360de^{10}x^5 + 60}$$

input
```
integrate((c*x**2+b*x)**2/(e*x+d)**7,x)
```

output
```
(-b**2*d**2*e**2 - 2*b*c*d**3*e - 2*c**2*d**4 - 30*c**2*e**4*x**4 + x**3*(
-40*b*c*e**4 - 40*c**2*d*e**3) + x**2*(-15*b**2*e**4 - 30*b*c*d*e**3 - 30*
c**2*d**2*e**2) + x*(-6*b**2*d*e**3 - 12*b*c*d**2*e**2 - 12*c**2*d**3*e))/
(60*d**6*e**5 + 360*d**5*e**6*x + 900*d**4*e**7*x**2 + 1200*d**3*e**8*x**3
 + 900*d**2*e**9*x**4 + 360*d*e**10*x**5 + 60*e**11*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.39

$$\int \frac{(bx+cx^2)^2}{(d+ex)^7}\,dx = -\frac{30\,c^2e^4x^4+2\,c^2d^4+2\,bcd^3e+b^2d^2e^2+40\,(c^2de^3+bce^4)x^3+15\,(2\,c^2d^2e^2+2\,bcde^3+b^2e^4)x^2+6\,(2\,c}{60\,(e^{11}x^6+6\,de^{10}x^5+15\,d^2e^9x^4+20\,d^3e^8x^3+15\,d^4e^7x^2+6\,d^5e^6x+d^6e^5)}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^7,x, algorithm="maxima")
```

output
```
-1/60*(30*c^2*e^4*x^4 + 2*c^2*d^4 + 2*b*c*d^3*e + b^2*d^2*e^2 + 40*(c^2*d*
e^3 + b*c*e^4)*x^3 + 15*(2*c^2*d^2*e^2 + 2*b*c*d*e^3 + b^2*e^4)*x^2 + 6*(2
*c^2*d^3*e + 2*b*c*d^2*e^2 + b^2*d*e^3)*x)/(e^11*x^6 + 6*d*e^10*x^5 + 15*d
^2*e^9*x^4 + 20*d^3*e^8*x^3 + 15*d^4*e^7*x^2 + 6*d^5*e^6*x + d^6*e^5)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01

$$\int \frac{(bx+cx^2)^2}{(d+ex)^7}\,dx = -\frac{30\,c^2e^4x^4+40\,c^2de^3x^3+40\,bce^4x^3+30\,c^2d^2e^2x^2+30\,bcde^3x^2+15\,b^2e^4x^2+12\,c^2d^3ex+12\,bcd^2e^2x+}{60\,(ex+d)^6e^5}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^7,x, algorithm="giac")
```

output
```
-1/60*(30*c^2*e^4*x^4 + 40*c^2*d*e^3*x^3 + 40*b*c*e^4*x^3 + 30*c^2*d^2*e^2
*x^2 + 30*b*c*d*e^3*x^2 + 15*b^2*e^4*x^2 + 12*c^2*d^3*e*x + 12*b*c*d^2*e^2
*x + 6*b^2*d*e^3*x + 2*c^2*d^4 + 2*b*c*d^3*e + b^2*d^2*e^2)/((e*x + d)^6*e
^5)
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.32

$$\int \frac{(bx+cx^2)^2}{(d+ex)^7}\,dx = -\frac{\frac{x^2\,(b^2\,e^2+2\,b\,c\,d\,e+2\,c^2\,d^2)}{4\,e^3}+\frac{c^2\,x^4}{2\,e}+\frac{d^2\,(b^2\,e^2+2\,b\,c\,d\,e+2\,c^2\,d^2)}{60\,e^5}+\frac{2\,c\,x^3\,(b\,e+c\,d)}{3\,e^2}+\frac{d\,x\,(b^2\,e^2+2\,b\,c\,d\,e+2\,c^2\,d^2)}{10\,e^4}}{d^6+6\,d^5\,e\,x+15\,d^4\,e^2\,x^2+20\,d^3\,e^3\,x^3+15\,d^2\,e^4\,x^4+6\,d\,e^5\,x^5+e^6\,x^6}$$

input
```
int((b*x + c*x^2)^2/(d + e*x)^7,x)
```

output
```
-((x^2*(b^2*e^2 + 2*c^2*d^2 + 2*b*c*d*e))/(4*e^3) + (c^2*x^4)/(2*e) + (d^2
*(b^2*e^2 + 2*c^2*d^2 + 2*b*c*d*e))/(60*e^5) + (2*c*x^3*(b*e + c*d))/(3*e^
2) + (d*x*(b^2*e^2 + 2*c^2*d^2 + 2*b*c*d*e))/(10*e^4))/(d^6 + e^6*x^6 + 6*
d*e^5*x^5 + 15*d^4*e^2*x^2 + 20*d^3*e^3*x^3 + 15*d^2*e^4*x^4 + 6*d^5*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.42

$$\int \frac{(bx+cx^2)^2}{(d+ex)^7}\,dx = \frac{-30c^2e^4x^4-40bc\,e^4x^3-40c^2d\,e^3x^3-15b^2e^4x^2-30bcd\,e^3x^2-30c^2d^2e^2x^2-6b^2d\,e^3x-12bc\,d^2e^2x-1}{60e^5\,(e^6x^6+6d\,e^5x^5+15d^2e^4x^4+20d^3e^3x^3+15d^4e^2x^2+6d^5ex+d^6}$$

input
```
int((c*x^2+b*x)^2/(e*x+d)^7,x)
```

output
```
( - b**2*d**2*e**2 - 6*b**2*d*e**3*x - 15*b**2*e**4*x**2 - 2*b*c*d**3*e -
12*b*c*d**2*e**2*x - 30*b*c*d*e**3*x**2 - 40*b*c*e**4*x**3 - 2*c**2*d**4 -
 12*c**2*d**3*e*x - 30*c**2*d**2*e**2*x**2 - 40*c**2*d*e**3*x**3 - 30*c**2
*e**4*x**4)/(60*e**5*(d**6 + 6*d**5*e*x + 15*d**4*e**2*x**2 + 20*d**3*e**3
*x**3 + 15*d**2*e**4*x**4 + 6*d*e**5*x**5 + e**6*x**6))
```

3.39 $\int \frac{(bx+cx^2)^2}{(d+ex)^8}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{(bx+cx^2)^2}{(d+ex)^8}\,dx = -\frac{d^2(cd-be)^2}{7e^5(d+ex)^7} + \frac{d(cd-be)(2cd-be)}{3e^5(d+ex)^6} - \frac{6c^2d^2-6bcde+b^2e^2}{5e^5(d+ex)^5} + \frac{c(2cd-be)}{2e^5(d+ex)^4} - \frac{c^2}{3e^5(d+ex)^3}$$

output
```
-1/7*d^2*(-b*e+c*d)^2/e^5/(e*x+d)^7+1/3*d*(-b*e+c*d)*(-b*e+2*c*d)/e^5/(e*x
+d)^6-1/5*(b^2*e^2-6*b*c*d*e+6*c^2*d^2)/e^5/(e*x+d)^5+1/2*c*(-b*e+2*c*d)/e
^5/(e*x+d)^4-1/3*c^2/e^5/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{(bx+cx^2)^2}{(d+ex)^8}\,dx = -\frac{2b^2e^2(d^2+7dex+21e^2x^2)+3bce(d^3+7d^2ex+21de^2x^2+35e^3x^3)+2c^2(d^4+7d^3ex+21d^2e^2x^2+35}{210e^5(d+ex)^7}$$

input
```
Integrate[(b*x + c*x^2)^2/(d + e*x)^8,x]
```

output
```
-1/210*(2*b^2*e^2*(d^2 + 7*d*e*x + 21*e^2*x^2) + 3*b*c*e*(d^3 + 7*d^2*e*x
+ 21*d*e^2*x^2 + 35*e^3*x^3) + 2*c^2*(d^4 + 7*d^3*e*x + 21*d^2*e^2*x^2 + 3
5*d*e^3*x^3 + 35*e^4*x^4))/(e^5*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^2}{(d+ex)^8}\,dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(\frac{b^2e^2-6bcde+6c^2d^2}{e^4(d+ex)^6}+\frac{d^2(cd-be)^2}{e^4(d+ex)^8}-\frac{2c(2cd-be)}{e^4(d+ex)^5}+\frac{2d(cd-be)(be-2cd)}{e^4(d+ex)^7}+\frac{c^2}{e^4(d+ex)^4}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^2e^2-6bcde+6c^2d^2}{5e^5(d+ex)^5}-\frac{d^2(cd-be)^2}{7e^5(d+ex)^7}+\frac{c(2cd-be)}{2e^5(d+ex)^4}+\frac{d(cd-be)(2cd-be)}{3e^5(d+ex)^6}-\frac{c^2}{3e^5(d+ex)^3}$$

input
```
Int[(b*x + c*x^2)^2/(d + e*x)^8,x]
```

output
```
-1/7*(d^2*(c*d - b*e)^2)/(e^5*(d + e*x)^7) + (d*(c*d - b*e)*(2*c*d - b*e))
/(3*e^5*(d + e*x)^6) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)/(5*e^5*(d + e*x)^
5) + (c*(2*c*d - b*e))/(2*e^5*(d + e*x)^4) - c^2/(3*e^5*(d + e*x)^3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

method	result
risch	$\frac{-\frac{c^2x^4}{3e}-\frac{c(3be+2cd)x^3}{6e^2}-\frac{\left(2b^2e^2+3bcde+2c^2d^2\right)x^2}{10e^3}-\frac{d\left(2b^2e^2+3bcde+2c^2d^2\right)x}{30e^4}-\frac{d^2\left(2b^2e^2+3bcde+2c^2d^2\right)}{210e^5}}{(ex+d)^7}$
gosper	$-\frac{70c^2x^4e^4+105x^3bc\,e^4+70d\,c^2x^3e^3+42x^2b^2e^4+63x^2bcd\,e^3+42x^2c^2d^2e^2+14x\,b^2d\,e^3+21xbc\,d^2e^2+14x\,c^2d^3e+2d^2e^2b^2+3c}{210e^5(ex+d)^7}$
default	$-\frac{c^2}{3e^5(ex+d)^3}-\frac{c(be-2cd)}{2e^5(ex+d)^4}-\frac{d^2(b^2e^2-2bcde+c^2d^2)}{7e^5(ex+d)^7}-\frac{b^2e^2-6bcde+6c^2d^2}{5e^5(ex+d)^5}+\frac{d(b^2e^2-3bcde+2c^2d^2)}{3e^5(ex+d)^6}$
parallelrisch	$\frac{-70c^2x^4e^6-105bc\,e^6x^3-70c^2d\,e^5x^3-42b^2e^6x^2-63bcd\,e^5x^2-42c^2d^2e^4x^2-14b^2d\,e^5x-21bc\,d^2e^4x-14c^2d^3e^3x-2b^2d^2e^4-3b}{210e^7(ex+d)^7}$
norman	$\frac{-\frac{c^2x^4}{3e}-\frac{\left(3e^3bc+2d\,e^2c^2\right)x^3}{6e^4}-\frac{\left(2e^4b^2+3d\,e^3bc+2d^2e^2c^2\right)x^2}{10e^5}-\frac{d\left(2e^4b^2+3d\,e^3bc+2d^2e^2c^2\right)x}{30e^6}-\frac{d^2\left(2e^4b^2+3d\,e^3bc+2d^2e^2c^2\right)}{210e^7}}{(ex+d)^7}$
orering	$-\frac{(70c^2x^4e^4+105x^3bc\,e^4+70d\,c^2x^3e^3+42x^2b^2e^4+63x^2bcd\,e^3+42x^2c^2d^2e^2+14x\,b^2d\,e^3+21xbc\,d^2e^2+14x\,c^2d^3e+2d^2e^2b^2+3}{210e^5(cx+b)^2(ex+d)^7x^2}$

input
```
int((c*x^2+b*x)^2/(e*x+d)^8,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*c^2*x^4/e-1/6*c/e^2*(3*b*e+2*c*d)*x^3-1/10/e^3*(2*b^2*e^2+3*b*c*d*e+
2*c^2*d^2)*x^2-1/30*d/e^4*(2*b^2*e^2+3*b*c*d*e+2*c^2*d^2)*x-1/210*d^2/e^5*
(2*b^2*e^2+3*b*c*d*e+2*c^2*d^2))/(e*x+d)^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.51

$$\int \frac{(bx + cx^2)^2}{(d + ex)^8} \, dx = -\frac{70 c^2 e^4 x^4 + 2 c^2 d^4 + 3 bcd^3 e + 2 b^2 d^2 e^2 + 35 (2 c^2 de^3 + 3 bce^4) x^3 + 21 (2 c^2 d^2 e^2 + 3 bcde^3 + 2 b^2 e^4) x^2 +}{210 (e^{12} x^7 + 7 de^{11} x^6 + 21 d^2 e^{10} x^5 + 35 d^3 e^9 x^4 + 35 d^4 e^8 x^3 + 21 d^5 e^7 x^2 + 7 d^6 e}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^8,x, algorithm="fricas")
```

output
```
-1/210*(70*c^2*e^4*x^4 + 2*c^2*d^4 + 3*b*c*d^3*e + 2*b^2*d^2*e^2 + 35*(2*c
^2*d*e^3 + 3*b*c*e^4)*x^3 + 21*(2*c^2*d^2*e^2 + 3*b*c*d*e^3 + 2*b^2*e^4)*x
^2 + 7*(2*c^2*d^3*e + 3*b*c*d^2*e^2 + 2*b^2*d*e^3)*x)/(e^12*x^7 + 7*d*e^11
*x^6 + 21*d^2*e^10*x^5 + 35*d^3*e^9*x^4 + 35*d^4*e^8*x^3 + 21*d^5*e^7*x^2
+ 7*d^6*e^6*x + d^7*e^5)
```

Sympy [A] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.61

$$\int \frac{(bx + cx^2)^2}{(d + ex)^8} \, dx = \frac{-2b^2 d^2 e^2 - 3bcd^3 e - 2c^2 d^4 - 70c^2 e^4 x^4 + x^3(-105bce^4 - 70c^2 de^3) + x^2(-42b^2 e^4 - 63bcde^3 - 42c^2 d^2 e^2)}{210d^7 e^5 + 1470d^6 e^6 x + 4410d^5 e^7 x^2 + 7350d^4 e^8 x^3 + 7350d^3 e^9 x^4 + 4410d^2 e^{10} x^5 + 147}$$

input
```
integrate((c*x**2+b*x)**2/(e*x+d)**8,x)
```

output
```
(-2*b**2*d**2*e**2 - 3*b*c*d**3*e - 2*c**2*d**4 - 70*c**2*e**4*x**4 + x**3
*(-105*b*c*e**4 - 70*c**2*d*e**3) + x**2*(-42*b**2*e**4 - 63*b*c*d*e**3 -
42*c**2*d**2*e**2) + x*(-14*b**2*d*e**3 - 21*b*c*d**2*e**2 - 14*c**2*d**3*
e))/(210*d**7*e**5 + 1470*d**6*e**6*x + 4410*d**5*e**7*x**2 + 7350*d**4*e*
*8*x**3 + 7350*d**3*e**9*x**4 + 4410*d**2*e**10*x**5 + 1470*d*e**11*x**6 +
 210*e**12*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.51

$$\int \frac{(bx+cx^2)^2}{(d+ex)^8}\,dx = -\frac{70\,c^2e^4x^4+2\,c^2d^4+3\,bcd^3e+2\,b^2d^2e^2+35\,(2\,c^2de^3+3\,bce^4)x^3+21\,(2\,c^2d^2e^2+3\,bcde^3+2\,b^2e^4)x^2+}{210\,(e^{12}x^7+7\,de^{11}x^6+21\,d^2e^{10}x^5+35\,d^3e^9x^4+35\,d^4e^8x^3+21\,d^5e^7x^2+7\,d^6e}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^8,x, algorithm="maxima")
```

output
```
-1/210*(70*c^2*e^4*x^4 + 2*c^2*d^4 + 3*b*c*d^3*e + 2*b^2*d^2*e^2 + 35*(2*c
^2*d*e^3 + 3*b*c*e^4)*x^3 + 21*(2*c^2*d^2*e^2 + 3*b*c*d*e^3 + 2*b^2*e^4)*x
^2 + 7*(2*c^2*d^3*e + 3*b*c*d^2*e^2 + 2*b^2*d*e^3)*x)/(e^12*x^7 + 7*d*e^11
*x^6 + 21*d^2*e^10*x^5 + 35*d^3*e^9*x^4 + 35*d^4*e^8*x^3 + 21*d^5*e^7*x^2
+ 7*d^6*e^6*x + d^7*e^5)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.02

$$\int \frac{(bx+cx^2)^2}{(d+ex)^8}\,dx = -\frac{70\,c^2e^4x^4+70\,c^2de^3x^3+105\,bce^4x^3+42\,c^2d^2e^2x^2+63\,bcde^3x^2+42\,b^2e^4x^2+14\,c^2d^3ex+21\,bcd^2e^2x}{210\,(ex+d)^7e^5}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^8,x, algorithm="giac")
```

output
```
-1/210*(70*c^2*e^4*x^4 + 70*c^2*d*e^3*x^3 + 105*b*c*e^4*x^3 + 42*c^2*d^2*e
^2*x^2 + 63*b*c*d*e^3*x^2 + 42*b^2*e^4*x^2 + 14*c^2*d^3*e*x + 21*b*c*d^2*e
^2*x + 14*b^2*d*e^3*x + 2*c^2*d^4 + 3*b*c*d^3*e + 2*b^2*d^2*e^2)/((e*x + d
)^7*e^5)
```

Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.44

$$\int \frac{(bx+cx^2)^2}{(d+ex)^8}\,dx = -\frac{\frac{x^2\,(2\,b^2\,e^2+3\,b\,c\,d\,e+2\,c^2\,d^2)}{10\,e^3}+\frac{c^2\,x^4}{3\,e}+\frac{d^2\,(2\,b^2\,e^2+3\,b\,c\,d\,e+2\,c^2\,d^2)}{210\,e^5}+\frac{c\,x^3\,(3\,b\,e+2\,c\,d)}{6\,e^2}+\frac{d\,x\,(2\,b^2\,e^2+3\,b\,c\,d\,e+2\,c^2\,d^2)}{30\,e^4}}{d^7+7\,d^6\,e\,x+21\,d^5\,e^2\,x^2+35\,d^4\,e^3\,x^3+35\,d^3\,e^4\,x^4+21\,d^2\,e^5\,x^5+7\,d\,e^6\,x^6+e^7\,x^7}$$

input
```
int((b*x + c*x^2)^2/(d + e*x)^8,x)
```

output
```
-((x^2*(2*b^2*e^2 + 2*c^2*d^2 + 3*b*c*d*e))/(10*e^3) + (c^2*x^4)/(3*e) + (
d^2*(2*b^2*e^2 + 2*c^2*d^2 + 3*b*c*d*e))/(210*e^5) + (c*x^3*(3*b*e + 2*c*d
))/(6*e^2) + (d*x*(2*b^2*e^2 + 2*c^2*d^2 + 3*b*c*d*e))/(30*e^4))/(d^7 + e^
7*x^7 + 7*d*e^6*x^6 + 21*d^5*e^2*x^2 + 35*d^4*e^3*x^3 + 35*d^3*e^4*x^4 + 2
1*d^2*e^5*x^5 + 7*d^6*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.50

$$\int \frac{(bx+cx^2)^2}{(d+ex)^8}\,dx = \frac{-70c^2e^4x^4-105bc\,e^4x^3-70c^2d\,e^3x^3-42b^2e^4x^2-63bcd\,e^3x^2-42c^2d^2e^2x^2-14b^2d\,e^3x-21bc\,d^2e^2x-}{210e^5\,(e^7x^7+7d\,e^6x^6+21d^2e^5x^5+35d^3e^4x^4+35d^4e^3x^3+21d^5e^2x^2+7d^6}$$

input
```
int((c*x^2+b*x)^2/(e*x+d)^8,x)
```

output
```
( - 2*b**2*d**2*e**2 - 14*b**2*d*e**3*x - 42*b**2*e**4*x**2 - 3*b*c*d**3*e
 - 21*b*c*d**2*e**2*x - 63*b*c*d*e**3*x**2 - 105*b*c*e**4*x**3 - 2*c**2*d*
*4 - 14*c**2*d**3*e*x - 42*c**2*d**2*e**2*x**2 - 70*c**2*d*e**3*x**3 - 70*
c**2*e**4*x**4)/(210*e**5*(d**7 + 7*d**6*e*x + 21*d**5*e**2*x**2 + 35*d**4
*e**3*x**3 + 35*d**3*e**4*x**4 + 21*d**2*e**5*x**5 + 7*d*e**6*x**6 + e**7*
x**7))
```

3.40 $\int (d+ex)^4 (bx+cx^2)^3 \, dx$

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Optimal result

Integrand size = 19, antiderivative size = 225

$$\int (d+ex)^4 (bx+cx^2)^3 \, dx = \frac{1}{4}b^3d^4x^4+\frac{1}{5}b^2d^3(3cd+4be)x^5+\frac{1}{2}bd^2(c^2d^2+4bcde+2b^2e^2)x^6 + \frac{1}{7}d(c^3d^3+12bc^2d^2e+18b^2cde^2+4b^3e^3)x^7 + \frac{1}{8}e(4c^3d^3+18bc^2d^2e+12b^2cde^2+b^3e^3)x^8 + \frac{1}{3}ce^2(2c^2d^2+4bcde+b^2e^2)x^9 + \frac{1}{10}c^2e^3(4cd+3be)x^{10}+\frac{1}{11}c^3e^4x^{11}$$

output
```
1/4*b^3*d^4*x^4+1/5*b^2*d^3*(4*b*e+3*c*d)*x^5+1/2*b*d^2*(2*b^2*e^2+4*b*c*d
*e+c^2*d^2)*x^6+1/7*d*(4*b^3*e^3+18*b^2*c*d*e^2+12*b*c^2*d^2*e+c^3*d^3)*x^
7+1/8*e*(b^3*e^3+12*b^2*c*d*e^2+18*b*c^2*d^2*e+4*c^3*d^3)*x^8+1/3*c*e^2*(b
^2*e^2+4*b*c*d*e+2*c^2*d^2)*x^9+1/10*c^2*e^3*(3*b*e+4*c*d)*x^10+1/11*c^3*e
^4*x^11
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00

$$\int (d+ex)^4 \left(bx+cx^2\right)^3 \, dx = \frac{1}{4}b^3d^4x^4+\frac{1}{5}b^2d^3(3cd+4be)x^5+\frac{1}{2}bd^2\left(c^2d^2+4bcde+2b^2e^2\right)x^6 + \frac{1}{7}d\left(c^3d^3+12bc^2d^2e+18b^2cde^2+4b^3e^3\right)x^7 + \frac{1}{8}e\left(4c^3d^3+18bc^2d^2e+12b^2cde^2+b^3e^3\right)x^8 + \frac{1}{3}ce^2\left(2c^2d^2+4bcde+b^2e^2\right)x^9 + \frac{1}{10}c^2e^3(4cd+3be)x^{10}+\frac{1}{11}c^3e^4x^{11}$$

input
```
Integrate[(d + e*x)^4*(b*x + c*x^2)^3,x]
```

output
```
(b^3*d^4*x^4)/4 + (b^2*d^3*(3*c*d + 4*b*e)*x^5)/5 + (b*d^2*(c^2*d^2 + 4*b*
c*d*e + 2*b^2*e^2)*x^6)/2 + (d*(c^3*d^3 + 12*b*c^2*d^2*e + 18*b^2*c*d*e^2
+ 4*b^3*e^3)*x^7)/7 + (e*(4*c^3*d^3 + 18*b*c^2*d^2*e + 12*b^2*c*d*e^2 + b^
3*e^3)*x^8)/8 + (c*e^2*(2*c^2*d^2 + 4*b*c*d*e + b^2*e^2)*x^9)/3 + (c^2*e^3
*(4*c*d + 3*b*e)*x^10)/10 + (c^3*e^4*x^11)/11
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^3 (d+ex)^4 \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(b^3d^4x^3+3ce^2x^8\left(b^2e^2+4bcde+2c^2d^2\right)+3bd^2x^5\left(2b^2e^2+4bcde+c^2d^2\right)+b^2d^3x^4(4be+3cd)+ex^7\left(b^3e^3+1\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}b^3d^4x^4 + \frac{1}{3}ce^2x^9\left(b^2e^2 + 4bcde + 2c^2d^2\right) + \frac{1}{2}bd^2x^6\left(2b^2e^2 + 4bcde + c^2d^2\right) + \frac{1}{5}b^2d^3x^5(4be + 3cd) + \frac{1}{8}ex^8\left(b^3e^3 + 12b^2cde^2 + 18bc^2d^2e + 4c^3d^3\right) + \frac{1}{7}dx^7\left(4b^3e^3 + 18b^2cde^2 + 12bc^2d^2e + c^3d^3\right) + \frac{1}{10}c^2e^3x^{10}(3be + 4cd) + \frac{1}{11}c^3e^4x^{11}$$

input
```
Int[(d + e*x)^4*(b*x + c*x^2)^3,x]
```

output
```
(b^3*d^4*x^4)/4 + (b^2*d^3*(3*c*d + 4*b*e)*x^5)/5 + (b*d^2*(c^2*d^2 + 4*b*
c*d*e + 2*b^2*e^2)*x^6)/2 + (d*(c^3*d^3 + 12*b*c^2*d^2*e + 18*b^2*c*d*e^2
+ 4*b^3*e^3)*x^7)/7 + (e*(4*c^3*d^3 + 18*b*c^2*d^2*e + 12*b^2*c*d*e^2 + b^
3*e^3)*x^8)/8 + (c*e^2*(2*c^2*d^2 + 4*b*c*d*e + b^2*e^2)*x^9)/3 + (c^2*e^3
*(4*c*d + 3*b*e)*x^10)/10 + (c^3*e^4*x^11)/11
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.01

method	result
norman	$\frac{e^{4} c^{3} x^{11}}{11}+\left(\frac{3}{10} e^{4} b\,c^{2}+\frac{2}{5} d\,e^{3} c^{3}\right) x^{10}+\left(\frac{1}{3} e^{4} b^{2} c+\frac{4}{3} d\,e^{3} b\,c^{2}+\frac{2}{3} d^{2} e^{2} c^{3}\right) x^{9}+\left(\frac{1}{8} e^{4} b^{3}+\frac{3}{2} d\,e^{3} b^{2} c\right.$
default	$\frac{e^{4} c^{3} x^{11}}{11}+\frac{\left(3 e^{4} b\,c^{2}+4 d\,e^{3} c^{3}\right) x^{10}}{10}+\frac{\left(3 e^{4} b^{2} c+12 d\,e^{3} b\,c^{2}+6 d^{2} e^{2} c^{3}\right) x^{9}}{9}+\frac{\left(e^{4} b^{3}+12 d\,e^{3} b^{2} c+18 d^{2} e^{2} b\,c^{2}+4 d^{3} e\,c^{3}\right) x^{8}}{8}+$
gosper	$x^{4}\left(840 e^{4} c^{3} x^{7}+2772 x^{6} e^{4} b\,c^{2}+3696 x^{6} d\,e^{3} c^{3}+3080 x^{5} e^{4} b^{2} c+12320 x^{5} d\,e^{3} b\,c^{2}+6160 x^{5} d^{2} e^{2} c^{3}+1155 x^{4} e^{4} b^{3}+13860 x^{4} d\,e^{3} b^{2} c+\right.$
risch	$\frac{1}{11} e^{4} c^{3} x^{11}+\frac{3}{10} x^{10} e^{4} b\,c^{2}+\frac{2}{5} d\,e^{3} c^{3} x^{10}+\frac{1}{3} x^{9} e^{4} b^{2} c+\frac{4}{3} x^{9} d\,e^{3} b\,c^{2}+\frac{2}{3} x^{9} d^{2} e^{2} c^{3}+\frac{1}{8} x^{8} e^{4} b^{3}+\frac{3}{2} x$
parallelrisch	$\frac{1}{11} e^{4} c^{3} x^{11}+\frac{3}{10} x^{10} e^{4} b\,c^{2}+\frac{2}{5} d\,e^{3} c^{3} x^{10}+\frac{1}{3} x^{9} e^{4} b^{2} c+\frac{4}{3} x^{9} d\,e^{3} b\,c^{2}+\frac{2}{3} x^{9} d^{2} e^{2} c^{3}+\frac{1}{8} x^{8} e^{4} b^{3}+\frac{3}{2} x$
orering	$x\left(840 e^{4} c^{3} x^{7}+2772 x^{6} e^{4} b\,c^{2}+3696 x^{6} d\,e^{3} c^{3}+3080 x^{5} e^{4} b^{2} c+12320 x^{5} d\,e^{3} b\,c^{2}+6160 x^{5} d^{2} e^{2} c^{3}+1155 x^{4} e^{4} b^{3}+13860 x^{4} d\,e^{3} b^{2} c+\right.$

input
```
int((e*x+d)^4*(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/11*e^4*c^3*x^11+(3/10*e^4*b*c^2+2/5*d*e^3*c^3)*x^10+(1/3*e^4*b^2*c+4/3*d
*e^3*b*c^2+2/3*d^2*e^2*c^3)*x^9+(1/8*e^4*b^3+3/2*d*e^3*b^2*c+9/4*d^2*e^2*b
*c^2+1/2*d^3*e*c^3)*x^8+(4/7*d*e^3*b^3+18/7*d^2*e^2*b^2*c+12/7*d^3*e*b*c^2
+1/7*d^4*c^3)*x^7+(d^2*e^2*b^3+2*d^3*e*b^2*c+1/2*b*c^2*d^4)*x^6+(4/5*d^3*e
*b^3+3/5*b^2*c*d^4)*x^5+1/4*b^3*d^4*x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.02

$$\begin{aligned}\int (d+ex)^4\left(bx+cx^2\right)^3\,dx=&\ \frac{1}{11}\,c^3e^4x^{11}+\frac{1}{4}\,b^3d^4x^4+\frac{1}{10}\,\left(4\,c^3de^3+3\,bc^2e^4\right)x^{10}\\&+\frac{1}{3}\,\left(2\,c^3d^2e^2+4\,bc^2de^3+b^2ce^4\right)x^9\\&+\frac{1}{8}\,\left(4\,c^3d^3e+18\,bc^2d^2e^2+12\,b^2cde^3+b^3e^4\right)x^8\\&+\frac{1}{7}\,\left(c^3d^4+12\,bc^2d^3e+18\,b^2cd^2e^2+4\,b^3de^3\right)x^7\\&+\frac{1}{2}\,\left(bc^2d^4+4\,b^2cd^3e+2\,b^3d^2e^2\right)x^6\\&+\frac{1}{5}\,\left(3\,b^2cd^4+4\,b^3d^3e\right)x^5\end{aligned}$$

input
```
integrate((e*x+d)^4*(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/11*c^3*e^4*x^11 + 1/4*b^3*d^4*x^4 + 1/10*(4*c^3*d*e^3 + 3*b*c^2*e^4)*x^1
0 + 1/3*(2*c^3*d^2*e^2 + 4*b*c^2*d*e^3 + b^2*c*e^4)*x^9 + 1/8*(4*c^3*d^3*e
 + 18*b*c^2*d^2*e^2 + 12*b^2*c*d*e^3 + b^3*e^4)*x^8 + 1/7*(c^3*d^4 + 12*b*
c^2*d^3*e + 18*b^2*c*d^2*e^2 + 4*b^3*d*e^3)*x^7 + 1/2*(b*c^2*d^4 + 4*b^2*c
*d^3*e + 2*b^3*d^2*e^2)*x^6 + 1/5*(3*b^2*c*d^4 + 4*b^3*d^3*e)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.14

$$\int (d+ex)^4\left(bx+cx^2\right)^3\,dx = \frac{b^3d^4x^4}{4} + \frac{c^3e^4x^{11}}{11} + x^{10}\cdot\left(\frac{3bc^2e^4}{10} + \frac{2c^3de^3}{5}\right) + x^9\left(\frac{b^2ce^4}{3} + \frac{4bc^2de^3}{3} + \frac{2c^3d^2e^2}{3}\right) + x^8\left(\frac{b^3e^4}{8} + \frac{3b^2cde^3}{2} + \frac{9bc^2d^2e^2}{4} + \frac{c^3d^3e}{2}\right) + x^7\cdot\left(\frac{4b^3de^3}{7} + \frac{18b^2cd^2e^2}{7} + \frac{12bc^2d^3e}{7} + \frac{c^3d^4}{7}\right) + x^6\left(b^3d^2e^2 + 2b^2cd^3e + \frac{bc^2d^4}{2}\right) + x^5\cdot\left(\frac{4b^3d^3e}{5} + \frac{3b^2cd^4}{5}\right)$$

input
```
integrate((e*x+d)**4*(c*x**2+b*x)**3,x)
```

output
```
b**3*d**4*x**4/4 + c**3*e**4*x**11/11 + x**10*(3*b*c**2*e**4/10 + 2*c**3*d
*e**3/5) + x**9*(b**2*c*e**4/3 + 4*b*c**2*d*e**3/3 + 2*c**3*d**2*e**2/3) +
 x**8*(b**3*e**4/8 + 3*b**2*c*d*e**3/2 + 9*b*c**2*d**2*e**2/4 + c**3*d**3*
e/2) + x**7*(4*b**3*d*e**3/7 + 18*b**2*c*d**2*e**2/7 + 12*b*c**2*d**3*e/7
+ c**3*d**4/7) + x**6*(b**3*d**2*e**2 + 2*b**2*c*d**3*e + b*c**2*d**4/2) +
 x**5*(4*b**3*d**3*e/5 + 3*b**2*c*d**4/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.02

$$\int (d+ex)^4 \left(bx+cx^2\right)^3 \, dx = \frac{1}{11} c^3 e^4 x^{11} + \frac{1}{4} b^3 d^4 x^4 + \frac{1}{10} \left(4 c^3 d e^3 + 3 b c^2 e^4\right) x^{10} + \frac{1}{3} \left(2 c^3 d^2 e^2 + 4 b c^2 d e^3 + b^2 c e^4\right) x^9 + \frac{1}{8} \left(4 c^3 d^3 e + 18 b c^2 d^2 e^2 + 12 b^2 c d e^3 + b^3 e^4\right) x^8 + \frac{1}{7} \left(c^3 d^4 + 12 b c^2 d^3 e + 18 b^2 c d^2 e^2 + 4 b^3 d e^3\right) x^7 + \frac{1}{2} \left(b c^2 d^4 + 4 b^2 c d^3 e + 2 b^3 d^2 e^2\right) x^6 + \frac{1}{5} \left(3 b^2 c d^4 + 4 b^3 d^3 e\right) x^5$$

input
```
integrate((e*x+d)^4*(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/11*c^3*e^4*x^11 + 1/4*b^3*d^4*x^4 + 1/10*(4*c^3*d*e^3 + 3*b*c^2*e^4)*x^1
0 + 1/3*(2*c^3*d^2*e^2 + 4*b*c^2*d*e^3 + b^2*c*e^4)*x^9 + 1/8*(4*c^3*d^3*e
 + 18*b*c^2*d^2*e^2 + 12*b^2*c*d*e^3 + b^3*e^4)*x^8 + 1/7*(c^3*d^4 + 12*b*
c^2*d^3*e + 18*b^2*c*d^2*e^2 + 4*b^3*d*e^3)*x^7 + 1/2*(b*c^2*d^4 + 4*b^2*c
*d^3*e + 2*b^3*d^2*e^2)*x^6 + 1/5*(3*b^2*c*d^4 + 4*b^3*d^3*e)*x^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.11

$$\int (d+ex)^4 \left(bx+cx^2\right)^3 \, dx = \frac{1}{11} c^3 e^4 x^{11} + \frac{2}{5} c^3 d e^3 x^{10} + \frac{3}{10} b c^2 e^4 x^{10} + \frac{2}{3} c^3 d^2 e^2 x^9 + \frac{4}{3} b c^2 d e^3 x^9 + \frac{1}{3} b^2 c e^4 x^9 + \frac{1}{2} c^3 d^3 e x^8 + \frac{9}{4} b c^2 d^2 e^2 x^8 + \frac{3}{2} b^2 c d e^3 x^8 + \frac{1}{8} b^3 e^4 x^8 + \frac{1}{7} c^3 d^4 x^7 + \frac{12}{7} b c^2 d^3 e x^7 + \frac{18}{7} b^2 c d^2 e^2 x^7 + \frac{4}{7} b^3 d e^3 x^7 + \frac{1}{2} b c^2 d^4 x^6 + 2 b^2 c d^3 e x^6 + b^3 d^2 e^2 x^6 + \frac{3}{5} b^2 c d^4 x^5 + \frac{4}{5} b^3 d^3 e x^5 + \frac{1}{4} b^3 d^4 x^4$$

input
```
integrate((e*x+d)^4*(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
1/11*c^3*e^4*x^11 + 2/5*c^3*d*e^3*x^10 + 3/10*b*c^2*e^4*x^10 + 2/3*c^3*d^2
*e^2*x^9 + 4/3*b*c^2*d*e^3*x^9 + 1/3*b^2*c*e^4*x^9 + 1/2*c^3*d^3*e*x^8 + 9
/4*b*c^2*d^2*e^2*x^8 + 3/2*b^2*c*d*e^3*x^8 + 1/8*b^3*e^4*x^8 + 1/7*c^3*d^4
*x^7 + 12/7*b*c^2*d^3*e*x^7 + 18/7*b^2*c*d^2*e^2*x^7 + 4/7*b^3*d*e^3*x^7 +
 1/2*b*c^2*d^4*x^6 + 2*b^2*c*d^3*e*x^6 + b^3*d^2*e^2*x^6 + 3/5*b^2*c*d^4*x
^5 + 4/5*b^3*d^3*e*x^5 + 1/4*b^3*d^4*x^4
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.95

$$\int (d+ex)^4 \left(bx+cx^2\right)^3 \, dx = x^7 \left(\frac{4\,b^3\,d\,e^3}{7}+\frac{18\,b^2\,c\,d^2\,e^2}{7}+\frac{12\,b\,c^2\,d^3\,e}{7}+\frac{c^3\,d^4}{7}\right) + x^8 \left(\frac{b^3\,e^4}{8}+\frac{3\,b^2\,c\,d\,e^3}{2}+\frac{9\,b\,c^2\,d^2\,e^2}{4}+\frac{c^3\,d^3\,e}{2}\right)+\frac{b^3\,d^4\,x^4}{4} + \frac{c^3\,e^4\,x^{11}}{11}+\frac{b^2\,d^3\,x^5\,(4\,b\,e+3\,c\,d)}{5}+\frac{c^2\,e^3\,x^{10}\,(3\,b\,e+4\,c\,d)}{10} + \frac{b\,d^2\,x^6\,(2\,b^2\,e^2+4\,b\,c\,d\,e+c^2\,d^2)}{2} + \frac{c\,e^2\,x^9\,(b^2\,e^2+4\,b\,c\,d\,e+2\,c^2\,d^2)}{3}$$

input
```
int((b*x + c*x^2)^3*(d + e*x)^4,x)
```

output
```
x^7*((c^3*d^4)/7 + (4*b^3*d*e^3)/7 + (18*b^2*c*d^2*e^2)/7 + (12*b*c^2*d^3*
e)/7) + x^8*((b^3*e^4)/8 + (c^3*d^3*e)/2 + (9*b*c^2*d^2*e^2)/4 + (3*b^2*c*
d*e^3)/2) + (b^3*d^4*x^4)/4 + (c^3*e^4*x^11)/11 + (b^2*d^3*x^5*(4*b*e + 3*
c*d))/5 + (c^2*e^3*x^10*(3*b*e + 4*c*d))/10 + (b*d^2*x^6*(2*b^2*e^2 + c^2*
d^2 + 4*b*c*d*e))/2 + (c*e^2*x^9*(b^2*e^2 + 2*c^2*d^2 + 4*b*c*d*e))/3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.11

$$\int (d+ex)^4 \left(bx+cx^2\right)^3 \, dx$$
$$= \frac{x^4(840c^3e^4x^7 + 2772b\,c^2e^4x^6 + 3696c^3d\,e^3x^6 + 3080b^2c\,e^4x^5 + 12320b\,c^2d\,e^3x^5 + 6160c^3d^2e^2x^5 + 1155b^3e}{}$$

input
```
int((e*x+d)^4*(c*x^2+b*x)^3,x)
```

output
```
(x**4*(2310*b**3*d**4 + 7392*b**3*d**3*e*x + 9240*b**3*d**2*e**2*x**2 + 52
80*b**3*d*e**3*x**3 + 1155*b**3*e**4*x**4 + 5544*b**2*c*d**4*x + 18480*b**
2*c*d**3*e*x**2 + 23760*b**2*c*d**2*e**2*x**3 + 13860*b**2*c*d*e**3*x**4 +
 3080*b**2*c*e**4*x**5 + 4620*b*c**2*d**4*x**2 + 15840*b*c**2*d**3*e*x**3
+ 20790*b*c**2*d**2*e**2*x**4 + 12320*b*c**2*d*e**3*x**5 + 2772*b*c**2*e**
4*x**6 + 1320*c**3*d**4*x**3 + 4620*c**3*d**3*e*x**4 + 6160*c**3*d**2*e**2
*x**5 + 3696*c**3*d*e**3*x**6 + 840*c**3*e**4*x**7))/9240
```

3.41 $\int (d+ex)^3 (bx+cx^2)^3 \, dx$

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Optimal result

Integrand size = 19, antiderivative size = 162

$$\begin{aligned}\int (d+ex)^3 (bx+cx^2)^3 \, dx = {} & \frac{1}{4}b^3d^3x^4 + \frac{3}{5}b^2d^2(cd+be)x^5 + \frac{1}{2}bd(c^2d^2+3bcde+b^2e^2)\,x^6 \\ & + \frac{1}{7}(cd+be)\left(c^2d^2+8bcde+b^2e^2\right)x^7 \\ & + \frac{3}{8}ce(c^2d^2+3bcde+b^2e^2)\,x^8 \\ & + \frac{1}{3}c^2e^2(cd+be)x^9 + \frac{1}{10}c^3e^3x^{10}\end{aligned}$$

output
```
1/4*b^3*d^3*x^4+3/5*b^2*d^2*(b*e+c*d)*x^5+1/2*b*d*(b^2*e^2+3*b*c*d*e+c^2*d
^2)*x^6+1/7*(b*e+c*d)*(b^2*e^2+8*b*c*d*e+c^2*d^2)*x^7+3/8*c*e*(b^2*e^2+3*b
*c*d*e+c^2*d^2)*x^8+1/3*c^2*e^2*(b*e+c*d)*x^9+1/10*c^3*e^3*x^10
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.04

$$\int (d+ex)^3 \left(bx+cx^2\right)^3 \, dx = \frac{1}{4}b^3d^3x^4 + \frac{3}{5}b^2d^2(cd+be)x^5 + \frac{1}{2}bd\left(c^2d^2+3bcde+b^2e^2\right)x^6 + \frac{1}{7}\left(c^3d^3+9bc^2d^2e+9b^2cde^2+b^3e^3\right)x^7 + \frac{3}{8}ce\left(c^2d^2+3bcde+b^2e^2\right)x^8 + \frac{1}{3}c^2e^2(cd+be)x^9 + \frac{1}{10}c^3e^3x^{10}$$

input

```
Integrate[(d + e*x)^3*(b*x + c*x^2)^3,x]
```

output

```
(b^3*d^3*x^4)/4 + (3*b^2*d^2*(c*d + b*e)*x^5)/5 + (b*d*(c^2*d^2 + 3*b*c*d*
e + b^2*e^2)*x^6)/2 + ((c^3*d^3 + 9*b*c^2*d^2*e + 9*b^2*c*d*e^2 + b^3*e^3)
*x^7)/7 + (3*c*e*(c^2*d^2 + 3*b*c*d*e + b^2*e^2)*x^8)/8 + (c^2*e^2*(c*d +
b*e)*x^9)/3 + (c^3*e^3*x^10)/10
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^3 (d+ex)^3 \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(b^3d^3x^3 + 3cex^7\left(b^2e^2+3bcde+c^2d^2\right) + x^6(be+cd)\left(b^2e^2+8bcde+c^2d^2\right) + 3bdx^5\left(b^2e^2+3bcde+c^2d^2\right) + 3$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}b^3d^3x^4+\frac{3}{8}cex^8\left(b^2e^2+3bcde+c^2d^2\right)+\frac{1}{7}x^7(be+cd)\left(b^2e^2+8bcde+c^2d^2\right)+\frac{1}{2}bdx^6\left(b^2e^2+3bcde+c^2d^2\right)+\frac{3}{5}b^2d^2x^5(be+cd)+\frac{1}{3}c^2e^2x^9(be+cd)+\frac{1}{10}c^3e^3x^{10}$$

input
```
Int[(d + e*x)^3*(b*x + c*x^2)^3,x]
```

output
```
(b^3*d^3*x^4)/4 + (3*b^2*d^2*(c*d + b*e)*x^5)/5 + (b*d*(c^2*d^2 + 3*b*c*d*
e + b^2*e^2)*x^6)/2 + ((c*d + b*e)*(c^2*d^2 + 8*b*c*d*e + b^2*e^2)*x^7)/7
+ (3*c*e*(c^2*d^2 + 3*b*c*d*e + b^2*e^2)*x^8)/8 + (c^2*e^2*(c*d + b*e)*x^9
)/3 + (c^3*e^3*x^10)/10
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.09

method	result
norman	$\frac{c^3e^3x^{10}}{10}+\left(\frac{1}{3}e^3bc^2+\frac{1}{3}de^2c^3\right)x^9+\left(\frac{3}{8}e^3b^2c+\frac{9}{8}de^2bc^2+\frac{3}{8}d^2ec^3\right)x^8+\left(\frac{1}{7}b^3e^3+\frac{9}{7}de^2b^2c+\right.$
default	$\frac{c^3e^3x^{10}}{10}+\frac{\left(3e^3bc^2+3de^2c^3\right)x^9}{9}+\frac{\left(3e^3b^2c+9de^2bc^2+3d^2ec^3\right)x^8}{8}+\frac{\left(b^3e^3+9de^2b^2c+9d^2ebc^2+d^3c^3\right)x^7}{7}+$
gosper	$\frac{x^4\left(84c^3e^3x^6+280x^5e^3bc^2+280x^5de^2c^3+315x^4e^3b^2c+945x^4de^2bc^2+315x^4d^2ec^3+120x^3b^3e^3+1080x^3de^2b^2c+1080x^3d^2e\right.}{840}$
risch	$\frac{1}{10}c^3e^3x^{10}+\frac{1}{3}x^9e^3bc^2+\frac{1}{3}c^3de^2x^9+\frac{3}{8}x^8e^3b^2c+\frac{9}{8}x^8de^2bc^2+\frac{3}{8}x^8d^2ec^3+\frac{1}{7}x^7b^3e^3+\frac{9}{7}x^7d$
parallelrisch	$\frac{1}{10}c^3e^3x^{10}+\frac{1}{3}x^9e^3bc^2+\frac{1}{3}c^3de^2x^9+\frac{3}{8}x^8e^3b^2c+\frac{9}{8}x^8de^2bc^2+\frac{3}{8}x^8d^2ec^3+\frac{1}{7}x^7b^3e^3+\frac{9}{7}x^7d$
orering	$\frac{x\left(84c^3e^3x^6+280x^5e^3bc^2+280x^5de^2c^3+315x^4e^3b^2c+945x^4de^2bc^2+315x^4d^2ec^3+120x^3b^3e^3+1080x^3de^2b^2c+1080x^3d^2eb\right.}{840(cx+b)^3}$

input
```
int((e*x+d)^3*(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*c^3*e^3*x^10+(1/3*e^3*b*c^2+1/3*d*e^2*c^3)*x^9+(3/8*e^3*b^2*c+9/8*d*e
^2*b*c^2+3/8*d^2*e*c^3)*x^8+(1/7*b^3*e^3+9/7*d*e^2*b^2*c+9/7*d^2*e*b*c^2+1
/7*d^3*c^3)*x^7+(1/2*d*e^2*b^3+3/2*d^2*e*b^2*c+1/2*d^3*b*c^2)*x^6+(3/5*d^2
*e*b^3+3/5*b^2*c*d^3)*x^5+1/4*b^3*d^3*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06

$$\int (d+ex)^3 \left(bx+cx^2\right)^3 \, dx = \frac{1}{10} c^3 e^3 x^{10} + \frac{1}{4} b^3 d^3 x^4 + \frac{1}{3} \left(c^3 d e^2 + b c^2 e^3\right) x^9 + \frac{3}{8} \left(c^3 d^2 e + 3 \, b c^2 d e^2 + b^2 c e^3\right) x^8 + \frac{1}{7} \left(c^3 d^3 + 9 \, b c^2 d^2 e + 9 \, b^2 c d e^2 + b^3 e^3\right) x^7 + \frac{1}{2} \left(b c^2 d^3 + 3 \, b^2 c d^2 e + b^3 d e^2\right) x^6 + \frac{3}{5} \left(b^2 c d^3 + b^3 d^2 e\right) x^5$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/10*c^3*e^3*x^10 + 1/4*b^3*d^3*x^4 + 1/3*(c^3*d*e^2 + b*c^2*e^3)*x^9 + 3/
8*(c^3*d^2*e + 3*b*c^2*d*e^2 + b^2*c*e^3)*x^8 + 1/7*(c^3*d^3 + 9*b*c^2*d^2
*e + 9*b^2*c*d*e^2 + b^3*e^3)*x^7 + 1/2*(b*c^2*d^3 + 3*b^2*c*d^2*e + b^3*d
*e^2)*x^6 + 3/5*(b^2*c*d^3 + b^3*d^2*e)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.23

$$\int (d+ex)^3\left(bx+cx^2\right)^3\,dx = \frac{b^3d^3x^4}{4} + \frac{c^3e^3x^{10}}{10} + x^9\left(\frac{bc^2e^3}{3} + \frac{c^3de^2}{3}\right) + x^8\cdot\left(\frac{3b^2ce^3}{8} + \frac{9bc^2de^2}{8} + \frac{3c^3d^2e}{8}\right) + x^7\left(\frac{b^3e^3}{7} + \frac{9b^2cde^2}{7} + \frac{9bc^2d^2e}{7} + \frac{c^3d^3}{7}\right) + x^6\left(\frac{b^3de^2}{2} + \frac{3b^2cd^2e}{2} + \frac{bc^2d^3}{2}\right) + x^5\cdot\left(\frac{3b^3d^2e}{5} + \frac{3b^2cd^3}{5}\right)$$

input
```
integrate((e*x+d)**3*(c*x**2+b*x)**3,x)
```

output
```
b**3*d**3*x**4/4 + c**3*e**3*x**10/10 + x**9*(b*c**2*e**3/3 + c**3*d*e**2/
3) + x**8*(3*b**2*c*e**3/8 + 9*b*c**2*d*e**2/8 + 3*c**3*d**2*e/8) + x**7*(
b**3*e**3/7 + 9*b**2*c*d*e**2/7 + 9*b*c**2*d**2*e/7 + c**3*d**3/7) + x**6*
(b**3*d*e**2/2 + 3*b**2*c*d**2*e/2 + b*c**2*d**3/2) + x**5*(3*b**3*d**2*e/
5 + 3*b**2*c*d**3/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06

$$\int (d+ex)^3\left(bx+cx^2\right)^3\,dx = \frac{1}{10}\,c^3e^3x^{10} + \frac{1}{4}\,b^3d^3x^4 + \frac{1}{3}\,\left(c^3de^2 + bc^2e^3\right)x^9 + \frac{3}{8}\,\left(c^3d^2e + 3\,bc^2de^2 + b^2ce^3\right)x^8 + \frac{1}{7}\,\left(c^3d^3 + 9\,bc^2d^2e + 9\,b^2cde^2 + b^3e^3\right)x^7 + \frac{1}{2}\,\left(bc^2d^3 + 3\,b^2cd^2e + b^3de^2\right)x^6 + \frac{3}{5}\,\left(b^2cd^3 + b^3d^2e\right)x^5$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/10*c^3*e^3*x^10 + 1/4*b^3*d^3*x^4 + 1/3*(c^3*d*e^2 + b*c^2*e^3)*x^9 + 3/
8*(c^3*d^2*e + 3*b*c^2*d*e^2 + b^2*c*e^3)*x^8 + 1/7*(c^3*d^3 + 9*b*c^2*d^2
*e + 9*b^2*c*d*e^2 + b^3*e^3)*x^7 + 1/2*(b*c^2*d^3 + 3*b^2*c*d^2*e + b^3*d
*e^2)*x^6 + 3/5*(b^2*c*d^3 + b^3*d^2*e)*x^5
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.19

$$\int (d+ex)^3 \left(bx+cx^2\right)^3 \, dx = \frac{1}{10}c^3e^3x^{10} + \frac{1}{3}c^3de^2x^9 + \frac{1}{3}bc^2e^3x^9 + \frac{3}{8}c^3d^2ex^8 + \frac{9}{8}bc^2de^2x^8 + \frac{3}{8}b^2ce^3x^8 + \frac{1}{7}c^3d^3x^7 + \frac{9}{7}bc^2d^2ex^7 + \frac{9}{7}b^2cde^2x^7 + \frac{1}{7}b^3e^3x^7 + \frac{1}{2}bc^2d^3x^6 + \frac{3}{2}b^2cd^2ex^6 + \frac{1}{2}b^3de^2x^6 + \frac{3}{5}b^2cd^3x^5 + \frac{3}{5}b^3d^2ex^5 + \frac{1}{4}b^3d^3x^4$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
1/10*c^3*e^3*x^10 + 1/3*c^3*d*e^2*x^9 + 1/3*b*c^2*e^3*x^9 + 3/8*c^3*d^2*e*
x^8 + 9/8*b*c^2*d*e^2*x^8 + 3/8*b^2*c*e^3*x^8 + 1/7*c^3*d^3*x^7 + 9/7*b*c^
2*d^2*e*x^7 + 9/7*b^2*c*d*e^2*x^7 + 1/7*b^3*e^3*x^7 + 1/2*b*c^2*d^3*x^6 +
3/2*b^2*c*d^2*e*x^6 + 1/2*b^3*d*e^2*x^6 + 3/5*b^2*c*d^3*x^5 + 3/5*b^3*d^2*
e*x^5 + 1/4*b^3*d^3*x^4
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.96

$$\int (d+ex)^3 \left(bx+cx^2\right)^3 \, dx = x^7 \left(\frac{b^3\,e^3}{7} + \frac{9\,b^2\,c\,d\,e^2}{7} + \frac{9\,b\,c^2\,d^2\,e}{7} + \frac{c^3\,d^3}{7}\right) + \frac{b^3\,d^3\,x^4}{4} + \frac{c^3\,e^3\,x^{10}}{10} + \frac{b\,d\,x^6\,\left(b^2\,e^2+3\,b\,c\,d\,e+c^2\,d^2\right)}{2} + \frac{3\,c\,e\,x^8\,\left(b^2\,e^2+3\,b\,c\,d\,e+c^2\,d^2\right)}{8} + \frac{3\,b^2\,d^2\,x^5\,\left(b\,e+c\,d\right)}{5} + \frac{c^2\,e^2\,x^9\,\left(b\,e+c\,d\right)}{3}$$

input
```
int((b*x + c*x^2)^3*(d + e*x)^3,x)
```

output
```
x^7*((b^3*e^3)/7 + (c^3*d^3)/7 + (9*b*c^2*d^2*e)/7 + (9*b^2*c*d*e^2)/7) +
(b^3*d^3*x^4)/4 + (c^3*e^3*x^10)/10 + (b*d*x^6*(b^2*e^2 + c^2*d^2 + 3*b*c*
d*e))/2 + (3*c*e*x^8*(b^2*e^2 + c^2*d^2 + 3*b*c*d*e))/8 + (3*b^2*d^2*x^5*(
b*e + c*d))/5 + (c^2*e^2*x^9*(b*e + c*d))/3
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.18

$$\int (d+ex)^3 \left(bx+cx^2\right)^3 dx$$
$$= \frac{x^4(84c^3e^3x^6 + 280b\,c^2e^3x^5 + 280c^3d\,e^2x^5 + 315b^2c\,e^3x^4 + 945b\,c^2d\,e^2x^4 + 315c^3d^2e\,x^4 + 120b^3e^3x^3 + 108}{}$$

input
```
int((e*x+d)^3*(c*x^2+b*x)^3,x)
```

output
```
(x**4*(210*b**3*d**3 + 504*b**3*d**2*e*x + 420*b**3*d*e**2*x**2 + 120*b**3
*e**3*x**3 + 504*b**2*c*d**3*x + 1260*b**2*c*d**2*e*x**2 + 1080*b**2*c*d*e
**2*x**3 + 315*b**2*c*e**3*x**4 + 420*b*c**2*d**3*x**2 + 1080*b*c**2*d**2*
e*x**3 + 945*b*c**2*d*e**2*x**4 + 280*b*c**2*e**3*x**5 + 120*c**3*d**3*x**
3 + 315*c**3*d**2*e*x**4 + 280*c**3*d*e**2*x**5 + 84*c**3*e**3*x**6))/840
```

3.42 $\int (d+ex)^2 (bx+cx^2)^3 \, dx$

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Reduce [B] (verification not implemented) 360

Optimal result

Integrand size = 19, antiderivative size = 127

$$\int (d+ex)^2 (bx+cx^2)^3 \, dx = \frac{1}{4}b^3d^2x^4 + \frac{1}{5}b^2d(3cd+2be)x^5 + \frac{1}{6}b(3c^2d^2+6bcde+b^2e^2)x^6 + \frac{1}{7}c(c^2d^2+6bcde+3b^2e^2)x^7 + \frac{1}{8}c^2e(2cd+3be)x^8 + \frac{1}{9}c^3e^2x^9$$

output
```
1/4*b^3*d^2*x^4+1/5*b^2*d*(2*b*e+3*c*d)*x^5+1/6*b*(b^2*e^2+6*b*c*d*e+3*c^2
*d^2)*x^6+1/7*c*(3*b^2*e^2+6*b*c*d*e+c^2*d^2)*x^7+1/8*c^2*e*(3*b*e+2*c*d)*
x^8+1/9*c^3*e^2*x^9
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int (d+ex)^2 (bx+cx^2)^3 \, dx = \frac{1}{4}b^3d^2x^4 + \frac{1}{5}b^2d(3cd+2be)x^5 + \frac{1}{6}b(3c^2d^2+6bcde+b^2e^2)x^6 + \frac{1}{7}c(c^2d^2+6bcde+3b^2e^2)x^7 + \frac{1}{8}c^2e(2cd+3be)x^8 + \frac{1}{9}c^3e^2x^9$$

input
```
Integrate[(d + e*x)^2*(b*x + c*x^2)^3,x]
```

output
```
(b^3*d^2*x^4)/4 + (b^2*d*(3*c*d + 2*b*e)*x^5)/5 + (b*(3*c^2*d^2 + 6*b*c*d*
e + b^2*e^2)*x^6)/6 + (c*(c^2*d^2 + 6*b*c*d*e + 3*b^2*e^2)*x^7)/7 + (c^2*e
*(2*c*d + 3*b*e)*x^8)/8 + (c^3*e^2*x^9)/9
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^3 (d+ex)^2 \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(b^3d^2x^3+cx^6\left(3b^2e^2+6bcde+c^2d^2\right)+bx^5\left(b^2e^2+6bcde+3c^2d^2\right)+b^2dx^4(2be+3cd)+c^2ex^7(3be+2cd)+c$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}b^3d^2x^4+\frac{1}{7}cx^7\left(3b^2e^2+6bcde+c^2d^2\right)+\frac{1}{6}bx^6\left(b^2e^2+6bcde+3c^2d^2\right)+\frac{1}{5}b^2dx^5(2be+3cd)+\frac{1}{8}c^2ex^8(3be+2cd)+\frac{1}{9}c^3e^2x^9$$

input
```
Int[(d + e*x)^2*(b*x + c*x^2)^3,x]
```

output
```
(b^3*d^2*x^4)/4 + (b^2*d*(3*c*d + 2*b*e)*x^5)/5 + (b*(3*c^2*d^2 + 6*b*c*d*
e + b^2*e^2)*x^6)/6 + (c*(c^2*d^2 + 6*b*c*d*e + 3*b^2*e^2)*x^7)/7 + (c^2*e
*(2*c*d + 3*b*e)*x^8)/8 + (c^3*e^2*x^9)/9
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

method	result
norman	$\frac{c^3e^2x^9}{9}+\left(\frac{3}{8}b\,c^2e^2+\frac{1}{4}de\,c^3\right)x^8+\left(\frac{3}{7}e^2b^2c+\frac{6}{7}c^2deb+\frac{1}{7}c^3d^2\right)x^7+\left(\frac{1}{6}e^2b^3+de\,b^2c+\frac{1}{2}b\,c^2d^2\right)$
default	$\frac{c^3e^2x^9}{9}+\frac{(3b\,c^2e^2+2de\,c^3)x^8}{8}+\frac{(3e^2b^2c+6c^2deb+c^3d^2)x^7}{7}+\frac{(e^2b^3+6de\,b^2c+3b\,c^2d^2)x^6}{6}+\frac{(2b^3de+3b^2c\,d^2)x^5}{5}+$
gosper	$\frac{x^4(280c^3e^2x^5+945x^4b\,c^2e^2+630x^4de\,c^3+1080x^3e^2b^2c+2160x^3c^2deb+360c^3d^2x^3+420x^2e^2b^3+2520x^2de\,b^2c+1260b\,c^2d^2x^2}{2520}$
risch	$\frac{1}{9}c^3e^2x^9+\frac{3}{8}x^8b\,c^2e^2+\frac{1}{4}de\,c^3x^8+\frac{3}{7}x^7e^2b^2c+\frac{6}{7}x^7c^2deb+\frac{1}{7}x^7c^3d^2+\frac{1}{6}x^6e^2b^3+x^6de\,b^2c+$
parallelrisch	$\frac{1}{9}c^3e^2x^9+\frac{3}{8}x^8b\,c^2e^2+\frac{1}{4}de\,c^3x^8+\frac{3}{7}x^7e^2b^2c+\frac{6}{7}x^7c^2deb+\frac{1}{7}x^7c^3d^2+\frac{1}{6}x^6e^2b^3+x^6de\,b^2c+$
orering	$\frac{x(280c^3e^2x^5+945x^4b\,c^2e^2+630x^4de\,c^3+1080x^3e^2b^2c+2160x^3c^2deb+360c^3d^2x^3+420x^2e^2b^3+2520x^2de\,b^2c+1260b\,c^2d^2x^2}{2520(cx+b)^3}$

input
```
int((e*x+d)^2*(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*c^3*e^2*x^9+(3/8*b*c^2*e^2+1/4*d*e*c^3)*x^8+(3/7*e^2*b^2*c+6/7*c^2*d*e
*b+1/7*c^3*d^2)*x^7+(1/6*e^2*b^3+d*e*b^2*c+1/2*b*c^2*d^2)*x^6+(2/5*b^3*d*e
+3/5*b^2*c*d^2)*x^5+1/4*b^3*d^2*x^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int (d+ex)^2 \left(bx+cx^2\right)^3 \, dx = \frac{1}{9}c^3e^2x^9 + \frac{1}{4}b^3d^2x^4 + \frac{1}{8}\left(2c^3de + 3bc^2e^2\right)x^8 + \frac{1}{7}\left(c^3d^2 + 6bc^2de + 3b^2ce^2\right)x^7 + \frac{1}{6}\left(3bc^2d^2 + 6b^2cde + b^3e^2\right)x^6 + \frac{1}{5}\left(3b^2cd^2 + 2b^3de\right)x^5$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/9*c^3*e^2*x^9 + 1/4*b^3*d^2*x^4 + 1/8*(2*c^3*d*e + 3*b*c^2*e^2)*x^8 + 1/
7*(c^3*d^2 + 6*b*c^2*d*e + 3*b^2*c*e^2)*x^7 + 1/6*(3*b*c^2*d^2 + 6*b^2*c*d
*e + b^3*e^2)*x^6 + 1/5*(3*b^2*c*d^2 + 2*b^3*d*e)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.09

$$\int (d+ex)^2 \left(bx+cx^2\right)^3 \, dx = \frac{b^3d^2x^4}{4} + \frac{c^3e^2x^9}{9} + x^8 \cdot \left(\frac{3bc^2e^2}{8} + \frac{c^3de}{4}\right) + x^7 \cdot \left(\frac{3b^2ce^2}{7} + \frac{6bc^2de}{7} + \frac{c^3d^2}{7}\right) + x^6\left(\frac{b^3e^2}{6} + b^2cde + \frac{bc^2d^2}{2}\right) + x^5 \cdot \left(\frac{2b^3de}{5} + \frac{3b^2cd^2}{5}\right)$$

input
```
integrate((e*x+d)**2*(c*x**2+b*x)**3,x)
```

output
```
b**3*d**2*x**4/4 + c**3*e**2*x**9/9 + x**8*(3*b*c**2*e**2/8 + c**3*d*e/4)
+ x**7*(3*b**2*c*e**2/7 + 6*b*c**2*d*e/7 + c**3*d**2/7) + x**6*(b**3*e**2/
6 + b**2*c*d*e + b*c**2*d**2/2) + x**5*(2*b**3*d*e/5 + 3*b**2*c*d**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00

$$\int (d+ex)^2 \left(bx+cx^2\right)^3 \, dx = \frac{1}{9} c^3 e^2 x^9 + \frac{1}{4} b^3 d^2 x^4 + \frac{1}{8} \left(2 c^3 de + 3 bc^2 e^2\right) x^8 + \frac{1}{7} \left(c^3 d^2 + 6 bc^2 de + 3 b^2 ce^2\right) x^7 + \frac{1}{6} \left(3 bc^2 d^2 + 6 b^2 cde + b^3 e^2\right) x^6 + \frac{1}{5} \left(3 b^2 cd^2 + 2 b^3 de\right) x^5$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/9*c^3*e^2*x^9 + 1/4*b^3*d^2*x^4 + 1/8*(2*c^3*d*e + 3*b*c^2*e^2)*x^8 + 1/
7*(c^3*d^2 + 6*b*c^2*d*e + 3*b^2*c*e^2)*x^7 + 1/6*(3*b*c^2*d^2 + 6*b^2*c*d
*e + b^3*e^2)*x^6 + 1/5*(3*b^2*c*d^2 + 2*b^3*d*e)*x^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.06

$$\int (d+ex)^2 \left(bx+cx^2\right)^3 \, dx = \frac{1}{9} c^3 e^2 x^9 + \frac{1}{4} c^3 dex^8 + \frac{3}{8} bc^2 e^2 x^8 + \frac{1}{7} c^3 d^2 x^7 + \frac{6}{7} bc^2 dex^7 + \frac{3}{7} b^2 ce^2 x^7 + \frac{1}{2} bc^2 d^2 x^6 + b^2 cdex^6 + \frac{1}{6} b^3 e^2 x^6 + \frac{3}{5} b^2 cd^2 x^5 + \frac{2}{5} b^3 dex^5 + \frac{1}{4} b^3 d^2 x^4$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
1/9*c^3*e^2*x^9 + 1/4*c^3*d*e*x^8 + 3/8*b*c^2*e^2*x^8 + 1/7*c^3*d^2*x^7 +
6/7*b*c^2*d*e*x^7 + 3/7*b^2*c*e^2*x^7 + 1/2*b*c^2*d^2*x^6 + b^2*c*d*e*x^6
+ 1/6*b^3*e^2*x^6 + 3/5*b^2*c*d^2*x^5 + 2/5*b^3*d*e*x^5 + 1/4*b^3*d^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93

$$\int (d+ex)^2\left(bx+cx^2\right)^3\,dx = x^6\left(\frac{b^3\,e^2}{6}+b^2\,c\,d\,e+\frac{b\,c^2\,d^2}{2}\right) + x^7\left(\frac{3\,b^2\,c\,e^2}{7}+\frac{6\,b\,c^2\,d\,e}{7}+\frac{c^3\,d^2}{7}\right)+\frac{b^3\,d^2\,x^4}{4} + \frac{c^3\,e^2\,x^9}{9}+\frac{b^2\,d\,x^5\,(2\,b\,e+3\,c\,d)}{5}+\frac{c^2\,e\,x^8\,(3\,b\,e+2\,c\,d)}{8}$$

input
```
int((b*x + c*x^2)^3*(d + e*x)^2,x)
```

output
```
x^6*((b^3*e^2)/6 + (b*c^2*d^2)/2 + b^2*c*d*e) + x^7*((c^3*d^2)/7 + (3*b^2*
c*e^2)/7 + (6*b*c^2*d*e)/7) + (b^3*d^2*x^4)/4 + (c^3*e^2*x^9)/9 + (b^2*d*x
^5*(2*b*e + 3*c*d))/5 + (c^2*e*x^8*(3*b*e + 2*c*d))/8
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.05

$$\int (d+ex)^2\left(bx+cx^2\right)^3\,dx = \frac{x^4(280c^3e^2x^5+945b\,c^2e^2x^4+630c^3de\,x^4+1080b^2c\,e^2x^3+2160b\,c^2de\,x^3+360c^3d^2x^3+420b^3e^2x^2+252}{2520}$$

input
```
int((e*x+d)^2*(c*x^2+b*x)^3,x)
```

output
```
(x**4*(630*b**3*d**2 + 1008*b**3*d*e*x + 420*b**3*e**2*x**2 + 1512*b**2*c*
d**2*x + 2520*b**2*c*d*e*x**2 + 1080*b**2*c*e**2*x**3 + 1260*b*c**2*d**2*x
**2 + 2160*b*c**2*d*e*x**3 + 945*b*c**2*e**2*x**4 + 360*c**3*d**2*x**3 + 6
30*c**3*d*e*x**4 + 280*c**3*e**2*x**5))/2520
```

3.43 $\int (d+ex)\left(bx+cx^2\right)^3\,dx$

Optimal result . . . 361
Mathematica [A] (verified) . . . 361
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Giac [A] (verification not implemented) . . . 365
Mupad [B] (verification not implemented) . . . 365
Reduce [B] (verification not implemented) . . . 366

Optimal result

Integrand size = 17, antiderivative size = 75

$$\int (d+ex)\left(bx+cx^2\right)^3\,dx=\frac{1}{4}b^3dx^4+\frac{1}{5}b^2(3cd+be)x^5+\frac{1}{2}bc(cd+be)x^6+\frac{1}{7}c^2(cd+3be)x^7+\frac{1}{8}c^3ex^8$$

output
```
1/4*b^3*d*x^4+1/5*b^2*(b*e+3*c*d)*x^5+1/2*b*c*(b*e+c*d)*x^6+1/7*c^2*(3*b*e
+c*d)*x^7+1/8*c^3*e*x^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int (d+ex)\left(bx+cx^2\right)^3\,dx=\frac{1}{4}b^3dx^4+\frac{1}{5}b^2(3cd+be)x^5+\frac{1}{2}bc(cd+be)x^6+\frac{1}{7}c^2(cd+3be)x^7+\frac{1}{8}c^3ex^8$$

input
```
Integrate[(d + e*x)*(b*x + c*x^2)^3,x]
```

output
```
(b^3*d*x^4)/4 + (b^2*(3*c*d + b*e)*x^5)/5 + (b*c*(c*d + b*e)*x^6)/2 + (c^2
*(c*d + 3*b*e)*x^7)/7 + (c^3*e*x^8)/8
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^3 (d+ex)\,dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(b^3dx^3+b^2x^4(be+3cd)+c^2x^6(3be+cd)+3bcx^5(be+cd)+c^3ex^7\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}b^3dx^4+\frac{1}{5}b^2x^5(be+3cd)+\frac{1}{7}c^2x^7(3be+cd)+\frac{1}{2}bcx^6(be+cd)+\frac{1}{8}c^3ex^8$$

input
```
Int[(d + e*x)*(b*x + c*x^2)^3,x]
```

output

```
(b^3*d*x^4)/4 + (b^2*(3*c*d + b*e)*x^5)/5 + (b*c*(c*d + b*e)*x^6)/2 + (c^2
*(c*d + 3*b*e)*x^7)/7 + (c^3*e*x^8)/8
```


Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

method	result	size
norman	$\frac{c^3 e x^8}{8}+\left(\frac{3}{7} b c^2 e+\frac{1}{7} d c^3\right) x^7+\left(\frac{1}{2} c e b^2+\frac{1}{2} b c^2 d\right) x^6+\left(\frac{1}{5} e b^3+\frac{3}{5} c d b^2\right) x^5+\frac{b^3 d x^4}{4}$	75
gosper	$\frac{x^4\left(35 c^3 x^4 e+120 b c^2 x^3 e+40 c^3 d x^3+140 b^2 c e x^2+140 b c^2 d x^2+56 b^3 e x+168 b^2 c x d+70 b^3 d\right)}{280}$	76
default	$\frac{c^3 e x^8}{8}+\frac{\left(3 b c^2 e+d c^3\right) x^7}{7}+\frac{\left(3 c e b^2+3 b c^2 d\right) x^6}{6}+\frac{\left(e b^3+3 c d b^2\right) x^5}{5}+\frac{b^3 d x^4}{4}$	76
risch	$\frac{1}{8} c^3 e x^8+\frac{3}{7} x^7 b c^2 e+\frac{1}{7} c^3 d x^7+\frac{1}{2} x^6 c e b^2+\frac{1}{2} x^6 b c^2 d+\frac{1}{5} x^5 e b^3+\frac{3}{5} b^2 c d x^5+\frac{1}{4} b^3 d x^4$	78
parallelrisch	$\frac{1}{8} c^3 e x^8+\frac{3}{7} x^7 b c^2 e+\frac{1}{7} c^3 d x^7+\frac{1}{2} x^6 c e b^2+\frac{1}{2} x^6 b c^2 d+\frac{1}{5} x^5 e b^3+\frac{3}{5} b^2 c d x^5+\frac{1}{4} b^3 d x^4$	78
orering	$\frac{x\left(35 c^3 x^4 e+120 b c^2 x^3 e+40 c^3 d x^3+140 b^2 c e x^2+140 b c^2 d x^2+56 b^3 e x+168 b^2 c x d+70 b^3 d\right)\left(c x^2+b x\right)^3}{280(c x+b)^3}$	92

input
```
int((e*x+d)*(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*c^3*e*x^8+(3/7*b*c^2*e+1/7*d*c^3)*x^7+(1/2*c*e*b^2+1/2*b*c^2*d)*x^6+(1
/5*e*b^3+3/5*c*d*b^2)*x^5+1/4*b^3*d*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int (d+ex)\left(bx+cx^2\right)^3\,dx=\frac{1}{8}c^3ex^8+\frac{1}{4}b^3dx^4+\frac{1}{7}\left(c^3d+3\,bc^2e\right)x^7+\frac{1}{2}\left(bc^2d+b^2ce\right)x^6+\frac{1}{5}\left(3\,b^2cd+b^3e\right)x^5$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/8*c^3*e*x^8 + 1/4*b^3*d*x^4 + 1/7*(c^3*d + 3*b*c^2*e)*x^7 + 1/2*(b*c^2*d
 + b^2*c*e)*x^6 + 1/5*(3*b^2*c*d + b^3*e)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int (d+ex)\left(bx+cx^2\right)^3\,dx=\frac{b^3dx^4}{4}+\frac{c^3ex^8}{8}+x^7\cdot\left(\frac{3bc^2e}{7}+\frac{c^3d}{7}\right)+x^6\left(\frac{b^2ce}{2}+\frac{bc^2d}{2}\right)+x^5\left(\frac{b^3e}{5}+\frac{3b^2cd}{5}\right)$$

input
```
integrate((e*x+d)*(c*x**2+b*x)**3,x)
```

output
```
b**3*d*x**4/4 + c**3*e*x**8/8 + x**7*(3*b*c**2*e/7 + c**3*d/7) + x**6*(b**
2*c*e/2 + b*c**2*d/2) + x**5*(b**3*e/5 + 3*b**2*c*d/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int (d+ex)\left(bx+cx^2\right)^3\,dx=\frac{1}{8}\,c^3ex^8+\frac{1}{4}\,b^3dx^4+\frac{1}{7}\,\left(c^3d+3\,bc^2e\right)x^7+\frac{1}{2}\,\left(bc^2d+b^2ce\right)x^6+\frac{1}{5}\,\left(3\,b^2cd+b^3e\right)x^5$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/8*c^3*e*x^8 + 1/4*b^3*d*x^4 + 1/7*(c^3*d + 3*b*c^2*e)*x^7 + 1/2*(b*c^2*d
 + b^2*c*e)*x^6 + 1/5*(3*b^2*c*d + b^3*e)*x^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int (d+ex)\left(bx+cx^2\right)^3\,dx=\frac{1}{8}c^3ex^8+\frac{1}{7}c^3dx^7+\frac{3}{7}bc^2ex^7+\frac{1}{2}bc^2dx^6+\frac{1}{2}b^2cex^6+\frac{3}{5}b^2cdx^5+\frac{1}{5}b^3ex^5+\frac{1}{4}b^3dx^4$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
1/8*c^3*e*x^8 + 1/7*c^3*d*x^7 + 3/7*b*c^2*e*x^7 + 1/2*b*c^2*d*x^6 + 1/2*b^
2*c*e*x^6 + 3/5*b^2*c*d*x^5 + 1/5*b^3*e*x^5 + 1/4*b^3*d*x^4
```

Mupad [B] (verification not implemented)

Time = 8.61 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int (d+ex)\left(bx+cx^2\right)^3\,dx=x^5\left(\frac{e\,b^3}{5}+\frac{3\,c\,d\,b^2}{5}\right)+x^7\left(\frac{d\,c^3}{7}+\frac{3\,b\,e\,c^2}{7}\right)+\frac{b^3\,d\,x^4}{4}+\frac{c^3\,e\,x^8}{8}+\frac{b\,c\,x^6\,(b\,e+c\,d)}{2}$$

input
```
int((b*x + c*x^2)^3*(d + e*x),x)
```

output
```
x^5*((b^3*e)/5 + (3*b^2*c*d)/5) + x^7*((c^3*d)/7 + (3*b*c^2*e)/7) + (b^3*d
*x^4)/4 + (c^3*e*x^8)/8 + (b*c*x^6*(b*e + c*d))/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int (d+ex) \left(bx+cx^2\right)^3 \, dx$$
$$= \frac{x^4(35c^3e\,x^4+120b\,c^2e\,x^3+40c^3d\,x^3+140b^2ce\,x^2+140b\,c^2d\,x^2+56b^3ex+168b^2cdx+70b^3d)}{280}$$

input
```
int((e*x+d)*(c*x^2+b*x)^3,x)
```

output
```
(x**4*(70*b**3*d + 56*b**3*e*x + 168*b**2*c*d*x + 140*b**2*c*e*x**2 + 140*
b*c**2*d*x**2 + 120*b*c**2*e*x**3 + 40*c**3*d*x**3 + 35*c**3*e*x**4))/280
```

3.44 $\int (bx + cx^2)^3 \, dx$

Optimal result . . . 367
Mathematica [A] (verified) . . . 367
Rubi [A] (verified) . . . 368
Maple [A] (verified) . . . 369
Fricas [A] (verification not implemented) . . . 369
Sympy [A] (verification not implemented) . . . 370
Maxima [A] (verification not implemented) . . . 370
Giac [A] (verification not implemented) . . . 370
Mupad [B] (verification not implemented) . . . 371
Reduce [B] (verification not implemented) . . . 371

Optimal result

Integrand size = 11, antiderivative size = 43

$$\int (bx + cx^2)^3 \, dx = \frac{b^3x^4}{4} + \frac{3}{5}b^2cx^5 + \frac{1}{2}bc^2x^6 + \frac{c^3x^7}{7}$$

output
```
1/4*b^3*x^4+3/5*b^2*c*x^5+1/2*b*c^2*x^6+1/7*c^3*x^7
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int (bx + cx^2)^3 \, dx = \frac{b^3x^4}{4} + \frac{3}{5}b^2cx^5 + \frac{1}{2}bc^2x^6 + \frac{c^3x^7}{7}$$

input
```
Integrate[(b*x + c*x^2)^3,x]
```

output
```
(b^3*x^4)/4 + (3*b^2*c*x^5)/5 + (b*c^2*x^6)/2 + (c^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^3 dx$$

$$\Big\downarrow \text{1080}$$

$$\int \left(b^3x^3+3b^2cx^4+3bc^2x^5+c^3x^6\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3x^4}{4}+\frac{3}{5}b^2cx^5+\frac{1}{2}bc^2x^6+\frac{c^3x^7}{7}$$

input
```
Int[(b*x + c*x^2)^3,x]
```

output
```
(b^3*x^4)/4 + (3*b^2*c*x^5)/5 + (b*c^2*x^6)/2 + (c^3*x^7)/7
```

Defintions of rubi rules used

rule 1080
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[x^p*
(b + c*x)^p, x], x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

method	result	size
gosper	$\frac{x^{4}\left(20c^{3}x^{3}+70b\,c^{2}x^{2}+84b^{2}cx+35b^{3}\right)}{140}$	36
default	$\frac{1}{4}b^{3}x^{4}+\frac{3}{5}x^{5}b^{2}c+\frac{1}{2}b\,c^{2}x^{6}+\frac{1}{7}c^{3}x^{7}$	36
norman	$\frac{1}{4}b^{3}x^{4}+\frac{3}{5}x^{5}b^{2}c+\frac{1}{2}b\,c^{2}x^{6}+\frac{1}{7}c^{3}x^{7}$	36
risch	$\frac{1}{4}b^{3}x^{4}+\frac{3}{5}x^{5}b^{2}c+\frac{1}{2}b\,c^{2}x^{6}+\frac{1}{7}c^{3}x^{7}$	36
parallelrisch	$\frac{1}{4}b^{3}x^{4}+\frac{3}{5}x^{5}b^{2}c+\frac{1}{2}b\,c^{2}x^{6}+\frac{1}{7}c^{3}x^{7}$	36
orering	$\frac{x\left(20c^{3}x^{3}+70b\,c^{2}x^{2}+84b^{2}cx+35b^{3}\right)\left(c\,x^{2}+bx\right)^{3}}{140\left(cx+b\right)^{3}}$	52

input
```
int((c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/140*x^4*(20*c^3*x^3+70*b*c^2*x^2+84*b^2*c*x+35*b^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(bx+cx^{2}\right)^{3}\,dx=\frac{1}{7}\,c^{3}x^{7}+\frac{1}{2}\,bc^{2}x^{6}+\frac{3}{5}\,b^{2}cx^{5}+\frac{1}{4}\,b^{3}x^{4}$$

input
```
integrate((c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*b^2*c*x^5 + 1/4*b^3*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \left(bx + cx^2\right)^3 \, dx = \frac{b^3x^4}{4} + \frac{3b^2cx^5}{5} + \frac{bc^2x^6}{2} + \frac{c^3x^7}{7}$$

input
```
integrate((c*x**2+b*x)**3,x)
```

output
```
b**3*x**4/4 + 3*b**2*c*x**5/5 + b*c**2*x**6/2 + c**3*x**7/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(bx + cx^2\right)^3 \, dx = \frac{1}{7}\,c^3x^7 + \frac{1}{2}\,bc^2x^6 + \frac{3}{5}\,b^2cx^5 + \frac{1}{4}\,b^3x^4$$

input
```
integrate((c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*b^2*c*x^5 + 1/4*b^3*x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(bx + cx^2\right)^3 \, dx = \frac{1}{7}\,c^3x^7 + \frac{1}{2}\,bc^2x^6 + \frac{3}{5}\,b^2cx^5 + \frac{1}{4}\,b^3x^4$$

input
```
integrate((c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
1/7*c^3*x^7 + 1/2*b*c^2*x^6 + 3/5*b^2*c*x^5 + 1/4*b^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(bx+cx^2\right)^3 \, dx = \frac{b^3\,x^4}{4}+\frac{3\,b^2\,c\,x^5}{5}+\frac{b\,c^2\,x^6}{2}+\frac{c^3\,x^7}{7}$$

input
```
int((b*x + c*x^2)^3,x)
```

output
```
(b^3*x^4)/4 + (c^3*x^7)/7 + (3*b^2*c*x^5)/5 + (b*c^2*x^6)/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.81

$$\int \left(bx+cx^2\right)^3 \, dx = \frac{x^4(20c^3x^3+70b\,c^2x^2+84b^2cx+35b^3)}{140}$$

input
```
int((c*x^2+b*x)^3,x)
```

output
```
(x**4*(35*b**3 + 84*b**2*c*x + 70*b*c**2*x**2 + 20*c**3*x**3))/140
```

3.45 $\int \frac{(bx+cx^2)^3}{d+ex}\,dx$

Optimal result 372
Mathematica [A] (verified) 372
Rubi [A] (verified) 373
Maple [A] (verified) 374
Fricas [A] (verification not implemented) 375
Sympy [A] (verification not implemented) 375
Maxima [A] (verification not implemented) 376
Giac [B] (verification not implemented) 376
Mupad [B] (verification not implemented) 377
Reduce [B] (verification not implemented) 378

Optimal result

Integrand size = 19, antiderivative size = 151

$$\int \frac{(bx+cx^2)^3}{d+ex}\,dx = -\frac{d^2(cd-be)^3x}{e^6} + \frac{d(cd-be)^3x^2}{2e^5} - \frac{(cd-be)^3x^3}{3e^4} + \frac{c(c^2d^2-3bcde+3b^2e^2)\,x^4}{4e^3} - \frac{c^2(cd-3be)x^5}{5e^2} + \frac{c^3x^6}{6e} + \frac{d^3(cd-be)^3\log(d+ex)}{e^7}$$

output
```
-d^2*(-b*e+c*d)^3*x/e^6+1/2*d*(-b*e+c*d)^3*x^2/e^5-1/3*(-b*e+c*d)^3*x^3/e^
4+1/4*c*(3*b^2*e^2-3*b*c*d*e+c^2*d^2)*x^4/e^3-1/5*c^2*(-3*b*e+c*d)*x^5/e^2
+1/6*c^3*x^6/e+d^3*(-b*e+c*d)^3*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.95

$$\int \frac{(bx+cx^2)^3}{d+ex}\,dx = \frac{-60d^2e(cd-be)^3x + 30de^2(cd-be)^3x^2 + 20e^3(-cd+be)^3x^3 + 15ce^4(c^2d^2-3bcde+3b^2e^2)\,x^4 - 12c^2e}{60e^7}$$

input
```
Integrate[(b*x + c*x^2)^3/(d + e*x),x]
```

output
```
(-60*d^2*e*(c*d - b*e)^3*x + 30*d*e^2*(c*d - b*e)^3*x^2 + 20*e^3*(-(c*d) +
 b*e)^3*x^3 + 15*c*e^4*(c^2*d^2 - 3*b*c*d*e + 3*b^2*e^2)*x^4 - 12*c^2*e^5*
(c*d - 3*b*e)*x^5 + 10*c^3*e^6*x^6 + 60*d^3*(c*d - b*e)^3*Log[d + e*x])/(6
0*e^7)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx+cx^2\right)^3}{d+ex}\,dx$$

$$\Big\downarrow \ 1140$$

$$\int \left(\frac{cx^3\left(3b^2e^2-3bcde+c^2d^2\right)}{e^3}-\frac{c^2x^4(cd-3be)}{e^2}+\frac{d^3(cd-be)^3}{e^6(d+ex)}-\frac{d^2(cd-be)^3}{e^6}+\frac{dx(cd-be)^3}{e^5}+\frac{x^2(be-cd)^3}{e^4}\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{cx^4\left(3b^2e^2-3bcde+c^2d^2\right)}{4e^3}-\frac{c^2x^5(cd-3be)}{5e^2}+\frac{d^3(cd-be)^3\log(d+ex)}{e^7}-\frac{d^2x(cd-be)^3}{e^6}+\frac{dx^2(cd-be)^3}{2e^5}-\frac{x^3(cd-be)^3}{3e^4}+\frac{c^3x^6}{6e}$$

input
```
Int[(b*x + c*x^2)^3/(d + e*x),x]
```

output
```
-((d^2*(c*d - b*e)^3*x)/e^6) + (d*(c*d - b*e)^3*x^2)/(2*e^5) - ((c*d - b*e
)^3*x^3)/(3*e^4) + (c*(c^2*d^2 - 3*b*c*d*e + 3*b^2*e^2)*x^4)/(4*e^3) - (c^
2*(c*d - 3*b*e)*x^5)/(5*e^2) + (c^3*x^6)/(6*e) + (d^3*(c*d - b*e)^3*Log[d
+ e*x])/e^7
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.64

method	result
norman	$\frac{d^2(b^3e^3-3d\,e^2b^2c+3d^2eb\,c^2-d^3c^3)x}{e^6}+\frac{c^3x^6}{6e}+\frac{(b^3e^3-3d\,e^2b^2c+3d^2eb\,c^2-d^3c^3)x^3}{3e^4}+\frac{c(3b^2e^2-3bcde+c^2d^2)x^4}{4e^3}-\frac{d(b$
risch	$\frac{c^3x^6}{6e}+\frac{3x^5c^2b}{5e}-\frac{c^3d\,x^5}{5e^2}+\frac{3x^4b^2c}{4e}-\frac{3x^4b\,c^2d}{4e^2}+\frac{x^4c^3d^2}{4e^3}+\frac{x^3b^3}{3e}-\frac{x^3b^2cd}{e^2}+\frac{x^3b\,c^2d^2}{e^3}-\frac{x^3c^3d^3}{3e^4}-\frac{x^2b^3d}{2e^2}+$
parallelrisch	$-\frac{-10x^6c^3e^6-36x^5b\,c^2e^6+12x^5c^3d\,e^5-45x^4b^2c\,e^6+45x^4b\,c^2d\,e^5-15x^4c^3d^2e^4-20x^3b^3e^6+60x^3b^2cd\,e^5-60x^3b\,c^2d^2e^4+20x$
default	$\frac{\frac{c^3x^6e^5}{6}+\frac{\left((be-cd)e^4c^2+2c^2e^5b\right)x^5}{5}+\frac{\left(2(be-cd)e^4bc+ce\left(e^4b^2-d\,e^3bc+d^2e^2c^2\right)\right)x^4}{4}+\frac{\left((be-cd)\left(e^4b^2-d\,e^3bc+d^2e^2c^2\right)+ce\left(-d\,e^3b^2+d\right.\right.}{3}}{e^6}$

input
```
int((c*x^2+b*x)^3/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
d^2*(b^3*e^3-3*b^2*c*d*e^2+3*b*c^2*d^2*e-c^3*d^3)/e^6*x+1/6*c^3*x^6/e+1/3/
e^4*(b^3*e^3-3*b^2*c*d*e^2+3*b*c^2*d^2*e-c^3*d^3)*x^3+1/4*c*(3*b^2*e^2-3*b
*c*d*e+c^2*d^2)*x^4/e^3-1/2*d/e^5*(b^3*e^3-3*b^2*c*d*e^2+3*b*c^2*d^2*e-c^3
*d^3)*x^2+1/5/e^2*c^2*(3*b*e-c*d)*x^5-d^3*(b^3*e^3-3*b^2*c*d*e^2+3*b*c^2*d
^2*e-c^3*d^3)/e^7*ln(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.76

$$\int \frac{(bx+cx^2)^3}{d+ex}\,dx$$
$$= \frac{10\,c^3e^6x^6 - 12\,(c^3de^5 - 3\,bc^2e^6)x^5 + 15\,(c^3d^2e^4 - 3\,bc^2de^5 + 3\,b^2ce^6)x^4 - 20\,(c^3d^3e^3 - 3\,bc^2d^2e^4 + 3\,b^2cd}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d),x, algorithm="fricas")
```

output
```
1/60*(10*c^3*e^6*x^6 - 12*(c^3*d*e^5 - 3*b*c^2*e^6)*x^5 + 15*(c^3*d^2*e^4
- 3*b*c^2*d*e^5 + 3*b^2*c*e^6)*x^4 - 20*(c^3*d^3*e^3 - 3*b*c^2*d^2*e^4 + 3
*b^2*c*d*e^5 - b^3*e^6)*x^3 + 30*(c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*b^2*c*
d^2*e^4 - b^3*d*e^5)*x^2 - 60*(c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*b^2*c*d^3*e
^3 - b^3*d^2*e^4)*x + 60*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*
d^3*e^3)*log(e*x + d))/e^7
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.61

$$\int \frac{(bx+cx^2)^3}{d+ex}\,dx = \frac{c^3x^6}{6e} - \frac{d^3(be-cd)^3\log(d+ex)}{e^7} + x^5\cdot\left(\frac{3bc^2}{5e} - \frac{c^3d}{5e^2}\right) + x^4\cdot\left(\frac{3b^2c}{4e} - \frac{3bc^2d}{4e^2} + \frac{c^3d^2}{4e^3}\right) + x^3\left(\frac{b^3}{3e} - \frac{b^2cd}{e^2} + \frac{bc^2d^2}{e^3} - \frac{c^3d^3}{3e^4}\right) + x^2\left(-\frac{b^3d}{2e^2} + \frac{3b^2cd^2}{2e^3} - \frac{3bc^2d^3}{2e^4} + \frac{c^3d^4}{2e^5}\right) + x\left(\frac{b^3d^2}{e^3} - \frac{3b^2cd^3}{e^4} + \frac{3bc^2d^4}{e^5} - \frac{c^3d^5}{e^6}\right)$$

input
```
integrate((c*x**2+b*x)**3/(e*x+d),x)
```

output
```
c**3*x**6/(6*e) - d**3*(b*e - c*d)**3*log(d + e*x)/e**7 + x**5*(3*b*c**2/(
5*e) - c**3*d/(5*e**2)) + x**4*(3*b**2*c/(4*e) - 3*b*c**2*d/(4*e**2) + c**
3*d**2/(4*e**3)) + x**3*(b**3/(3*e) - b**2*c*d/e**2 + b*c**2*d**2/e**3 - c
**3*d**3/(3*e**4)) + x**2*(-b**3*d/(2*e**2) + 3*b**2*c*d**2/(2*e**3) - 3*b
*c**2*d**3/(2*e**4) + c**3*d**4/(2*e**5)) + x*(b**3*d**2/e**3 - 3*b**2*c*d
**3/e**4 + 3*b*c**2*d**4/e**5 - c**3*d**5/e**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.75

$$\int \frac{(bx+cx^2)^3}{d+ex}\,dx = \frac{10\,c^3e^5x^6 - 12\,(c^3de^4 - 3\,bc^2e^5)x^5 + 15\,(c^3d^2e^3 - 3\,bc^2de^4 + 3\,b^2ce^5)x^4 - 20\,(c^3d^3e^2 - 3\,bc^2d^2e^3 + 3\,b^2cd}{60} + \frac{(c^3d^6 - 3\,bc^2d^5e + 3\,b^2cd^4e^2 - b^3d^3e^3)\log(ex+d)}{e^7}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d),x, algorithm="maxima")
```

output
```
1/60*(10*c^3*e^5*x^6 - 12*(c^3*d*e^4 - 3*b*c^2*e^5)*x^5 + 15*(c^3*d^2*e^3
- 3*b*c^2*d*e^4 + 3*b^2*c*e^5)*x^4 - 20*(c^3*d^3*e^2 - 3*b*c^2*d^2*e^3 + 3
*b^2*c*d*e^4 - b^3*e^5)*x^3 + 30*(c^3*d^4*e - 3*b*c^2*d^3*e^2 + 3*b^2*c*d^
2*e^3 - b^3*d*e^4)*x^2 - 60*(c^3*d^5 - 3*b*c^2*d^4*e + 3*b^2*c*d^3*e^2 - b
^3*d^2*e^3)*x)/e^6 + (c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*
e^3)*log(e*x + d)/e^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. 2(141) = 282.

Time = 0.14 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.89

$$\int \frac{(bx+cx^2)^3}{d+ex}\,dx = \frac{10\,c^3e^5x^6 - 12\,c^3de^4x^5 + 36\,bc^2e^5x^5 + 15\,c^3d^2e^3x^4 - 45\,bc^2de^4x^4 + 45\,b^2ce^5x^4 - 20\,c^3d^3e^2x^3 + 60\,bc^2d^2e^3}{} + \frac{(c^3d^6 - 3\,bc^2d^5e + 3\,b^2cd^4e^2 - b^3d^3e^3)\log(|ex+d|)}{e^7}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d),x, algorithm="giac")
```

output
```
1/60*(10*c^3*e^5*x^6 - 12*c^3*d*e^4*x^5 + 36*b*c^2*e^5*x^5 + 15*c^3*d^2*e^
3*x^4 - 45*b*c^2*d*e^4*x^4 + 45*b^2*c*e^5*x^4 - 20*c^3*d^3*e^2*x^3 + 60*b*
c^2*d^2*e^3*x^3 - 60*b^2*c*d*e^4*x^3 + 20*b^3*e^5*x^3 + 30*c^3*d^4*e*x^2 -
 90*b*c^2*d^3*e^2*x^2 + 90*b^2*c*d^2*e^3*x^2 - 30*b^3*d*e^4*x^2 - 60*c^3*d
^5*x + 180*b*c^2*d^4*e*x - 180*b^2*c*d^3*e^2*x + 60*b^3*d^2*e^3*x)/e^6 + (
c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)*log(abs(e*x + d))
/e^7
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.95

$$\int \frac{(bx+cx^2)^3}{d+ex}\,dx = x^3\left(\frac{b^3}{3\,e}-\frac{d\left(\frac{3\,b^2\,c}{e}-\frac{d\left(\frac{3\,b\,c^2}{e}-\frac{c^3\,d}{e^2}\right)}{e}\right)}{3\,e}\right) + x^5\left(\frac{3\,b\,c^2}{5\,e}-\frac{c^3\,d}{5\,e^2}\right) + x^4\left(\frac{3\,b^2\,c}{4\,e}-\frac{d\left(\frac{3\,b\,c^2}{e}-\frac{c^3\,d}{e^2}\right)}{4\,e}\right) + \frac{\ln\left(d+e\,x\right)\,\left(-b^3\,d^3\,e^3+3\,b^2\,c\,d^4\,e^2-3\,b\,c^2\,d^5\,e+c^3\,d^6\right)}{e^7} + \frac{c^3\,x^6}{6\,e} - \frac{d\,x^2\left(\frac{b^3}{e}-\frac{d\left(\frac{3\,b^2\,c}{e}-\frac{d\left(\frac{3\,b\,c^2}{e}-\frac{c^3\,d}{e^2}\right)}{e}\right)}{e}\right)}{2\,e} + \frac{d^2\,x\left(\frac{b^3}{e}-\frac{d\left(\frac{3\,b^2\,c}{e}-\frac{d\left(\frac{3\,b\,c^2}{e}-\frac{c^3\,d}{e^2}\right)}{e}\right)}{e}\right)}{e^2}$$

input
```
int((b*x + c*x^2)^3/(d + e*x),x)
```

output
```
x^3*(b^3/(3*e) - (d*((3*b^2*c)/e - (d*((3*b*c^2)/e - (c^3*d)/e^2))/e))/(3*
e)) + x^5*((3*b*c^2)/(5*e) - (c^3*d)/(5*e^2)) + x^4*((3*b^2*c)/(4*e) - (d*
((3*b*c^2)/e - (c^3*d)/e^2))/(4*e)) + (log(d + e*x)*(c^3*d^6 - b^3*d^3*e^3
 + 3*b^2*c*d^4*e^2 - 3*b*c^2*d^5*e))/e^7 + (c^3*x^6)/(6*e) - (d*x^2*(b^3/e
 - (d*((3*b^2*c)/e - (d*((3*b*c^2)/e - (c^3*d)/e^2))/e))/e))/(2*e) + (d^2*
x*(b^3/e - (d*((3*b^2*c)/e - (d*((3*b*c^2)/e - (c^3*d)/e^2))/e))/e))/e^2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.00

$$\int \frac{(bx + cx^2)^3}{d + ex} \, dx$$
$$= \frac{-60 \log(ex + d) \, b^3 d^3 e^3 + 180 \log(ex + d) \, b^2 c \, d^4 e^2 - 180 \log(ex + d) \, b \, c^2 d^5 e + 60 \log(ex + d) \, c^3 d^6 + 60 b^3}{}$$

input
```
int((c*x^2+b*x)^3/(e*x+d),x)
```

output
```
( - 60*log(d + e*x)*b**3*d**3*e**3 + 180*log(d + e*x)*b**2*c*d**4*e**2 - 1
80*log(d + e*x)*b*c**2*d**5*e + 60*log(d + e*x)*c**3*d**6 + 60*b**3*d**2*e
**4*x - 30*b**3*d*e**5*x**2 + 20*b**3*e**6*x**3 - 180*b**2*c*d**3*e**3*x +
 90*b**2*c*d**2*e**4*x**2 - 60*b**2*c*d*e**5*x**3 + 45*b**2*c*e**6*x**4 +
180*b*c**2*d**4*e**2*x - 90*b*c**2*d**3*e**3*x**2 + 60*b*c**2*d**2*e**4*x*
*3 - 45*b*c**2*d*e**5*x**4 + 36*b*c**2*e**6*x**5 - 60*c**3*d**5*e*x + 30*c
**3*d**4*e**2*x**2 - 20*c**3*d**3*e**3*x**3 + 15*c**3*d**2*e**4*x**4 - 12*
c**3*d*e**5*x**5 + 10*c**3*e**6*x**6)/(60*e**7)
```

3.46 $\int \frac{(bx+cx^2)^3}{(d+ex)^2}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 166

$$\int \frac{(bx+cx^2)^3}{(d+ex)^2}\,dx = \frac{d(5cd-2be)(cd-be)^2x}{e^6} - \frac{(cd-be)^2(4cd-be)x^2}{2e^5} + \frac{c(cd-be)^2x^3}{e^4} - \frac{c^2(2cd-3be)x^4}{4e^3} + \frac{c^3x^5}{5e^2} - \frac{d^3(cd-be)^3}{e^7(d+ex)} - \frac{3d^2(cd-be)^2(2cd-be)\log(d+ex)}{e^7}$$

output
```
d*(-2*b*e+5*c*d)*(-b*e+c*d)^2*x/e^6-1/2*(-b*e+c*d)^2*(-b*e+4*c*d)*x^2/e^5+
c*(-b*e+c*d)^2*x^3/e^4-1/4*c^2*(-3*b*e+2*c*d)*x^4/e^3+1/5*c^3*x^5/e^2-d^3*
(-b*e+c*d)^3/e^7/(e*x+d)-3*d^2*(-b*e+c*d)^2*(-b*e+2*c*d)*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.96

$$\int \frac{(bx+cx^2)^3}{(d+ex)^2}\,dx = \frac{20de(5cd-2be)(cd-be)^2x + 10e^2(cd-be)^2(-4cd+be)x^2 + 20ce^3(cd-be)^2x^3 - 5c^2e^4(2cd-3be)x^4 +}{20e^7}$$

input
```
Integrate[(b*x + c*x^2)^3/(d + e*x)^2,x]
```

output
```
(20*d*e*(5*c*d - 2*b*e)*(c*d - b*e)^2*x + 10*e^2*(c*d - b*e)^2*(-4*c*d + b
*e)*x^2 + 20*c*e^3*(c*d - b*e)^2*x^3 - 5*c^2*e^4*(2*c*d - 3*b*e)*x^4 + 4*c
^3*e^5*x^5 - (20*d^3*(c*d - b*e)^3)/(d + e*x) - 60*d^2*(c*d - b*e)^2*(2*c*
d - b*e)*Log[d + e*x])/(20*e^7)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^3}{(d+ex)^2}dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(-\frac{c^2x^3(2cd-3be)}{e^3}+\frac{d^3(cd-be)^3}{e^6(d+ex)^2}-\frac{3d^2(cd-be)^2(2cd-be)}{e^6(d+ex)}+\frac{d(5cd-2be)(cd-be)^2}{e^6}+\frac{x(be-4cd)(be-}{e^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2x^4(2cd-3be)}{4e^3}-\frac{d^3(cd-be)^3}{e^7(d+ex)}-\frac{3d^2(cd-be)^2(2cd-be)\log(d+ex)}{e^7}+\frac{dx(5cd-2be)(cd-be)^2}{e^6}-\frac{x^2(cd-be)^2(4cd-be)}{2e^5}+\frac{cx^3(cd-be)^2}{e^4}+\frac{c^3x^5}{5e^2}$$

input
```
Int[(b*x + c*x^2)^3/(d + e*x)^2,x]
```

output
```
(d*(5*c*d - 2*b*e)*(c*d - b*e)^2*x)/e^6 - ((c*d - b*e)^2*(4*c*d - b*e)*x^2
)/(2*e^5) + (c*(c*d - b*e)^2*x^3)/e^4 - (c^2*(2*c*d - 3*b*e)*x^4)/(4*e^3)
+ (c^3*x^5)/(5*e^2) - (d^3*(c*d - b*e)^3)/(e^7*(d + e*x)) - (3*d^2*(c*d -
b*e)^2*(2*c*d - b*e)*Log[d + e*x])/e^7
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.57

method	result
norman	$\frac{\frac{d\left(3b^3d^2e^3-12b^2c\,d^3e^2+15b\,c^2d^4e-6d^5c^3\right)}{e^7}+\frac{c^3x^6}{5e}+\frac{\left(b^3e^3-4d\,e^2b^2c+5d^2eb\,c^2-2d^3c^3\right)x^3}{2e^4}+\frac{c\left(4b^2e^2-5bcde+2c^2d^2\right)x^4}{4e^3}+\frac{3c^2(5be-2cd}{20e^2}}{ex+d}$
default	$-\frac{-\frac{1}{5}c^3x^5e^4-\frac{3}{4}b\,c^2e^4x^4+\frac{1}{2}c^3d\,e^3x^4-b^2c\,e^4x^3+2b\,c^2d\,e^3x^3-c^3d^2e^2x^3-\frac{1}{2}b^3e^4x^2+3b^2cd\,e^3x^2-\frac{9}{2}b\,c^2d^2e^2x^2+2c^3d^3e\,x^2+2d}{e^6}$
risch	$\frac{c^3x^5}{5e^2}+\frac{3b\,c^2x^4}{4e^2}-\frac{c^3d\,x^4}{2e^3}+\frac{b^2c\,x^3}{e^2}-\frac{2b\,c^2d\,x^3}{e^3}+\frac{c^3d^2x^3}{e^4}+\frac{b^3x^2}{2e^2}-\frac{3b^2cd\,x^2}{e^3}+\frac{9b\,c^2d^2x^2}{2e^4}-\frac{2c^3d^3x^2}{e^5}-\frac{2d\,b^3x}{e^3}$
parallelrisch	$\frac{10x^4c^3d^2e^4-20x^3c^3d^3e^3-30x^2b^3d\,e^5-240b^2c\,d^4e^2+60\ln(ex+d)x\,b^3d^2e^4-120\ln(ex+d)x\,c^3d^5e+300b\,c^2d^5e-120d^6c^3-25x}{}$

input
```
int((c*x^2+b*x)^3/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
(d*(3*b^3*d^2*e^3-12*b^2*c*d^3*e^2+15*b*c^2*d^4*e-6*c^3*d^5)/e^7+1/5*c^3*x
^6/e+1/2*(b^3*e^3-4*b^2*c*d*e^2+5*b*c^2*d^2*e-2*c^3*d^3)/e^4*x^3+1/4*c*(4*
b^2*e^2-5*b*c*d*e+2*c^2*d^2)/e^3*x^4+3/20*c^2*(5*b*e-2*c*d)/e^2*x^5-3/2*d*
(b^3*e^3-4*b^2*c*d*e^2+5*b*c^2*d^2*e-2*c^3*d^3)/e^5*x^2)/(e*x+d)+3*d^2/e^7
*(b^3*e^3-4*b^2*c*d*e^2+5*b*c^2*d^2*e-2*c^3*d^3)*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(160) = 320$.

Time = 0.09 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.23

$$\int \frac{(bx + cx^2)^3}{(d + ex)^2}\,dx$$
$$= \frac{4\,c^3e^6x^6 - 20\,c^3d^6 + 60\,bc^2d^5e - 60\,b^2cd^4e^2 + 20\,b^3d^3e^3 - 3\,(2\,c^3de^5 - 5\,bc^2e^6)x^5 + 5\,(2\,c^3d^2e^4 - 5\,bc^2de^5}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/20*(4*c^3*e^6*x^6 - 20*c^3*d^6 + 60*b*c^2*d^5*e - 60*b^2*c*d^4*e^2 + 20*
b^3*d^3*e^3 - 3*(2*c^3*d*e^5 - 5*b*c^2*e^6)*x^5 + 5*(2*c^3*d^2*e^4 - 5*b*c
^2*d*e^5 + 4*b^2*c*e^6)*x^4 - 10*(2*c^3*d^3*e^3 - 5*b*c^2*d^2*e^4 + 4*b^2*
c*d*e^5 - b^3*e^6)*x^3 + 30*(2*c^3*d^4*e^2 - 5*b*c^2*d^3*e^3 + 4*b^2*c*d^2
*e^4 - b^3*d*e^5)*x^2 + 20*(5*c^3*d^5*e - 12*b*c^2*d^4*e^2 + 9*b^2*c*d^3*e
^3 - 2*b^3*d^2*e^4)*x - 60*(2*c^3*d^6 - 5*b*c^2*d^5*e + 4*b^2*c*d^4*e^2 -
b^3*d^3*e^3 + (2*c^3*d^5*e - 5*b*c^2*d^4*e^2 + 4*b^2*c*d^3*e^3 - b^3*d^2*e
^4)*x)*log(e*x + d))/(e^8*x + d*e^7)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.55

$$\int \frac{(bx + cx^2)^3}{(d + ex)^2}\,dx = \frac{c^3x^5}{5e^2} + \frac{3d^2(be - 2cd)\,(be - cd)^2\log{(d + ex)}}{e^7} + x^4\cdot\left(\frac{3bc^2}{4e^2} - \frac{c^3d}{2e^3}\right) + x^3\left(\frac{b^2c}{e^2} - \frac{2bc^2d}{e^3} + \frac{c^3d^2}{e^4}\right) + x^2\left(\frac{b^3}{2e^2} - \frac{3b^2cd}{e^3} + \frac{9bc^2d^2}{2e^4} - \frac{2c^3d^3}{e^5}\right) + x\left(-\frac{2b^3d}{e^3} + \frac{9b^2cd^2}{e^4} - \frac{12bc^2d^3}{e^5} + \frac{5c^3d^4}{e^6}\right) + \frac{b^3d^3e^3 - 3b^2cd^4e^2 + 3bc^2d^5e - c^3d^6}{de^7 + e^8x}$$

input
```
integrate((c*x**2+b*x)**3/(e*x+d)**2,x)
```

output
```
c**3*x**5/(5*e**2) + 3*d**2*(b*e - 2*c*d)*(b*e - c*d)**2*log(d + e*x)/e**7
 + x**4*(3*b*c**2/(4*e**2) - c**3*d/(2*e**3)) + x**3*(b**2*c/e**2 - 2*b*c*
*2*d/e**3 + c**3*d**2/e**4) + x**2*(b**3/(2*e**2) - 3*b**2*c*d/e**3 + 9*b*
c**2*d**2/(2*e**4) - 2*c**3*d**3/e**5) + x*(-2*b**3*d/e**3 + 9*b**2*c*d**2
/e**4 - 12*b*c**2*d**3/e**5 + 5*c**3*d**4/e**6) + (b**3*d**3*e**3 - 3*b**2
*c*d**4*e**2 + 3*b*c**2*d**5*e - c**3*d**6)/(d*e**7 + e**8*x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.64

$$\int \frac{(bx+cx^2)^3}{(d+ex)^2}\,dx = -\frac{c^3d^6-3\,bc^2d^5e+3\,b^2cd^4e^2-b^3d^3e^3}{e^8x+de^7}$$
$$+\frac{4\,c^3e^4x^5-5\,(2\,c^3de^3-3\,bc^2e^4)x^4+20\,(c^3d^2e^2-2\,bc^2de^3+b^2ce^4)x^3-10\,(4\,c^3d^3e-9\,bc^2d^2e^2+6\,b^2cd}{20\,e^6}$$
$$-\frac{3\,(2\,c^3d^5-5\,bc^2d^4e+4\,b^2cd^3e^2-b^3d^2e^3)\log(ex+d)}{e^7}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^2,x, algorithm="maxima")
```

output
```
-(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)/(e^8*x + d*e^7)
 + 1/20*(4*c^3*e^4*x^5 - 5*(2*c^3*d*e^3 - 3*b*c^2*e^4)*x^4 + 20*(c^3*d^2*e
^2 - 2*b*c^2*d*e^3 + b^2*c*e^4)*x^3 - 10*(4*c^3*d^3*e - 9*b*c^2*d^2*e^2 +
6*b^2*c*d*e^3 - b^3*e^4)*x^2 + 20*(5*c^3*d^4 - 12*b*c^2*d^3*e + 9*b^2*c*d^
2*e^2 - 2*b^3*d*e^3)*x)/e^6 - 3*(2*c^3*d^5 - 5*b*c^2*d^4*e + 4*b^2*c*d^3*e
^2 - b^3*d^2*e^3)*log(e*x + d)/e^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(160) = 320$.

Time = 0.13 (sec) , antiderivative size = 346, normalized size of antiderivative = 2.08

$$\int \frac{(bx+cx^2)^3}{(d+ex)^2}\,dx$$
$$= \frac{\left(4c^3 - \frac{15(2c^3de-bc^2e^2)}{(ex+d)e} + \frac{20(5c^3d^2e^2-5bc^2de^3+b^2ce^4)}{(ex+d)^2e^2} - \frac{10(20c^3d^3e^3-30bc^2d^2e^4+12b^2cde^5-b^3e^6)}{(ex+d)^3e^3} + \frac{60(5c^3d^4e^4-10bc^2d^3e^5}{(ex+d)}\right.}{20e^7}$$
$$+ \frac{3(2c^3d^5-5bc^2d^4e+4b^2cd^3e^2-b^3d^2e^3)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^7}$$
$$- \frac{\frac{c^3d^6e^5}{ex+d} - \frac{3bc^2d^5e^6}{ex+d} + \frac{3b^2cd^4e^7}{ex+d} - \frac{b^3d^3e^8}{ex+d}}{e^{12}}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^2,x, algorithm="giac")
```

output
```
1/20*(4*c^3 - 15*(2*c^3*d*e - b*c^2*e^2)/((e*x + d)*e) + 20*(5*c^3*d^2*e^2
 - 5*b*c^2*d*e^3 + b^2*c*e^4)/((e*x + d)^2*e^2) - 10*(20*c^3*d^3*e^3 - 30*
b*c^2*d^2*e^4 + 12*b^2*c*d*e^5 - b^3*e^6)/((e*x + d)^3*e^3) + 60*(5*c^3*d^
4*e^4 - 10*b*c^2*d^3*e^5 + 6*b^2*c*d^2*e^6 - b^3*d*e^7)/((e*x + d)^4*e^4))
*(e*x + d)^5/e^7 + 3*(2*c^3*d^5 - 5*b*c^2*d^4*e + 4*b^2*c*d^3*e^2 - b^3*d^
2*e^3)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))/e^7 - (c^3*d^6*e^5/(e*x + d)
 - 3*b*c^2*d^5*e^6/(e*x + d) + 3*b^2*c*d^4*e^7/(e*x + d) - b^3*d^3*e^8/(e*
x + d))/e^12
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.62

$$\int \frac{(bx+cx^2)^3}{(d+ex)^2}\,dx = x^4\left(\frac{3\,b\,c^2}{4\,e^2}-\frac{c^3\,d}{2\,e^3}\right)-x^3\left(\frac{2\,d\left(\frac{3\,b\,c^2}{e^2}-\frac{2\,c^3\,d}{e^3}\right)}{3\,e}-\frac{b^2\,c}{e^2}+\frac{c^3\,d^2}{3\,e^4}\right)+x^2\left(\frac{b^3}{2\,e^2}+\frac{d\left(\frac{2\,d\left(\frac{3\,b\,c^2}{e^2}-\frac{2\,c^3\,d}{e^3}\right)}{e}-\frac{3\,b^2\,c}{e^2}+\frac{c^3\,d^2}{e^4}\right)}{e}-\frac{d^2\left(\frac{3\,b\,c^2}{e^2}-\frac{2\,c^3\,d}{e^3}\right)}{2\,e^2}\right)+x\left(\frac{d^2\left(\frac{2\,d\left(\frac{3\,b\,c^2}{e^2}-\frac{2\,c^3\,d}{e^3}\right)}{e}-\frac{3\,b^2\,c}{e^2}+\frac{c^3\,d^2}{e^4}\right)}{e^2}-\frac{2\,d\left(\frac{b^3}{e^2}+\frac{2\,d\left(\frac{2\,d\left(\frac{3\,b\,c^2}{e^2}-\frac{2\,c^3\,d}{e^3}\right)}{e}-\frac{3\,b^2\,c}{e^2}+\frac{c^3\,d^2}{e^4}\right)}{e}-\frac{d^2\left(\frac{3\,b\,c^2}{e^2}-\frac{2\,c^3\,d}{e^3}\right)}{e^2}\right)}{e}\right)-\frac{\ln(d+e\,x)\,\left(-3\,b^3\,d^2\,e^3+12\,b^2\,c\,d^3\,e^2-15\,b\,c^2\,d^4\,e+6\,c^3\,d^5\right)}{e^7}-\frac{-b^3\,d^3\,e^3+3\,b^2\,c\,d^4\,e^2-3\,b\,c^2\,d^5\,e+c^3\,d^6}{e\,\left(x\,e^7+d\,e^6\right)}+\frac{c^3\,x^5}{5\,e^2}$$

input
```
int((b*x + c*x^2)^3/(d + e*x)^2,x)
```

output
```
x^4*((3*b*c^2)/(4*e^2) - (c^3*d)/(2*e^3)) - x^3*((2*d*((3*b*c^2)/e^2 - (2*
c^3*d)/e^3))/(3*e) - (b^2*c)/e^2 + (c^3*d^2)/(3*e^4)) + x^2*(b^3/(2*e^2) +
 (d*((2*d*((3*b*c^2)/e^2 - (2*c^3*d)/e^3))/e - (3*b^2*c)/e^2 + (c^3*d^2)/e
^4))/e - (d^2*((3*b*c^2)/e^2 - (2*c^3*d)/e^3))/(2*e^2)) + x*((d^2*((2*d*((
3*b*c^2)/e^2 - (2*c^3*d)/e^3))/e - (3*b^2*c)/e^2 + (c^3*d^2)/e^4))/e^2 - (
2*d*(b^3/e^2 + (2*d*((2*d*((3*b*c^2)/e^2 - (2*c^3*d)/e^3))/e - (3*b^2*c)/e
^2 + (c^3*d^2)/e^4))/e - (d^2*((3*b*c^2)/e^2 - (2*c^3*d)/e^3))/e^2))/e) -
(log(d + e*x)*(6*c^3*d^5 - 3*b^3*d^2*e^3 + 12*b^2*c*d^3*e^2 - 15*b*c^2*d^4
*e))/e^7 - (c^3*d^6 - b^3*d^3*e^3 + 3*b^2*c*d^4*e^2 - 3*b*c^2*d^5*e)/(e*(d
*e^6 + e^7*x)) + (c^3*x^5)/(5*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.30

$$\int \frac{(bx+cx^2)^3}{(d+ex)^2}\,dx$$
$$= \frac{-240\log(ex+d)\,b^2c\,d^4e^2 + 300\log(ex+d)\,b\,c^2d^5e + 240b^2c\,d^3e^3x + 120b^2c\,d^2e^4x^2 - 40b^2cd\,e^5x^3 - 300b}{}$$

input
```
int((c*x^2+b*x)^3/(e*x+d)^2,x)
```

output
```
(60*log(d + e*x)*b**3*d**3*e**3 + 60*log(d + e*x)*b**3*d**2*e**4*x - 240*l
og(d + e*x)*b**2*c*d**4*e**2 - 240*log(d + e*x)*b**2*c*d**3*e**3*x + 300*l
og(d + e*x)*b*c**2*d**5*e + 300*log(d + e*x)*b*c**2*d**4*e**2*x - 120*log(
d + e*x)*c**3*d**6 - 120*log(d + e*x)*c**3*d**5*e*x - 60*b**3*d**2*e**4*x
- 30*b**3*d*e**5*x**2 + 10*b**3*e**6*x**3 + 240*b**2*c*d**3*e**3*x + 120*b
**2*c*d**2*e**4*x**2 - 40*b**2*c*d*e**5*x**3 + 20*b**2*c*e**6*x**4 - 300*b
*c**2*d**4*e**2*x - 150*b*c**2*d**3*e**3*x**2 + 50*b*c**2*d**2*e**4*x**3 -
 25*b*c**2*d*e**5*x**4 + 15*b*c**2*e**6*x**5 + 120*c**3*d**5*e*x + 60*c**3
*d**4*e**2*x**2 - 20*c**3*d**3*e**3*x**3 + 10*c**3*d**2*e**4*x**4 - 6*c**3
*d*e**5*x**5 + 4*c**3*e**6*x**6)/(20*e**7*(d + e*x))
```

3.47 $\int \frac{(bx+cx^2)^3}{(d+ex)^3}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 200

$$\int \frac{(bx+cx^2)^3}{(d+ex)^3}\,dx = -\frac{(cd-be)\left(10c^2d^2-8bcde+b^2e^2\right)x}{e^6}+\frac{3c(cd-be)(2cd-be)x^2}{2e^5} -\frac{c^2(cd-be)x^3}{e^4}+\frac{c^3x^4}{4e^3}-\frac{d^3(cd-be)^3}{2e^7(d+ex)^2}+\frac{3d^2(cd-be)^2(2cd-be)}{e^7(d+ex)} +\frac{3d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)\log(d+ex)}{e^7}$$

output
```
-(-b*e+c*d)*(b^2*e^2-8*b*c*d*e+10*c^2*d^2)*x/e^6+3/2*c*(-b*e+c*d)*(-b*e+2*
c*d)*x^2/e^5-c^2*(-b*e+c*d)*x^3/e^4+1/4*c^3*x^4/e^3-1/2*d^3*(-b*e+c*d)^3/e
^7/(e*x+d)^2+3*d^2*(-b*e+c*d)^2*(-b*e+2*c*d)/e^7/(e*x+d)+3*d*(-b*e+c*d)*(b
^2*e^2-5*b*c*d*e+5*c^2*d^2)*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.04

$$\int \frac{(bx+cx^2)^3}{(d+ex)^3}\,dx$$
$$= \frac{4e(-10c^3d^3+18bc^2d^2e-9b^2cde^2+b^3e^3)\,x+6ce^2(2c^2d^2-3bcde+b^2e^2)\,x^2-4c^2e^3(cd-be)x^3+c^3e^4x^4 \cdots}{4e^7}$$

input
```
Integrate[(b*x + c*x^2)^3/(d + e*x)^3,x]
```

output
```
(4*e*(-10*c^3*d^3 + 18*b*c^2*d^2*e - 9*b^2*c*d*e^2 + b^3*e^3)*x + 6*c*e^2*
(2*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*x^2 - 4*c^2*e^3*(c*d - b*e)*x^3 + c^3*e^
4*x^4 - (2*d^3*(c*d - b*e)^3)/(d + e*x)^2 + (12*d^2*(c*d - b*e)^2*(2*c*d -
 b*e))/(d + e*x) + 12*d*(5*c^3*d^3 - 10*b*c^2*d^2*e + 6*b^2*c*d*e^2 - b^3*
e^3)*Log[d + e*x])/(4*e^7)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^3}{(d+ex)^3}\,dx$$
$$\Big\downarrow \text{1140}$$
$$\int \left(\frac{3d(b^2e^2-5bcde+5c^2d^2)\,(cd-be)}{e^6(d+ex)}+\frac{(-b^2e^2+8bcde-10c^2d^2)\,(cd-be)}{e^6}-\frac{3c^2x^2(cd-be)}{e^4}+\frac{d^3(cd-be)^3}{e^6(d+ex)^3} \cdots\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)\log(d+ex)}{e^7}-\frac{x(cd-be)\left(b^2e^2-8bcde+10c^2d^2\right)}{e^6}-\frac{c^2x^3(cd-be)}{e^4}-\frac{d^3(cd-be)^3}{2e^7(d+ex)^2}+\frac{3d^2(cd-be)^2(2cd-be)}{e^7(d+ex)}+\frac{3cx^2(cd-be)(2cd-be)}{2e^5}+\frac{c^3x^4}{4e^3}$$

input
```
Int[(b*x + c*x^2)^3/(d + e*x)^3,x]
```

output
```
-(((c*d - b*e)*(10*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*x)/e^6) + (3*c*(c*d - b*
e)*(2*c*d - b*e)*x^2)/(2*e^5) - (c^2*(c*d - b*e)*x^3)/e^4 + (c^3*x^4)/(4*e
^3) - (d^3*(c*d - b*e)^3)/(2*e^7*(d + e*x)^2) + (3*d^2*(c*d - b*e)^2*(2*c*
d - b*e))/(e^7*(d + e*x)) + (3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*
e^2)*Log[d + e*x])/e^7
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.30

method	result
norman	$\frac{\frac{\left(b^3e^3-6d\,e^2b^2c+10d^2eb\,c^2-5d^3c^3\right)x^3}{e^4}+\frac{c^3x^6}{4e}-\frac{d^2\left(9d\,e^3b^3-54d^2e^2b^2c+90d^3eb\,c^2-45d^4c^3\right)}{2e^7}+\frac{c\left(6b^2e^2-10bcde+5c^2d^2\right)x^4}{4e^3}+\frac{c^2(2be-c}{2e^2}}{(ex+d)^2}$
default	$\frac{\frac{1}{4}c^3x^4e^3+b\,c^2e^3x^3-c^3d\,e^2x^3+\frac{3}{2}b^2c\,e^3x^2-\frac{9}{2}b\,c^2d\,e^2x^2+3c^3d^2e\,x^2+b^3e^3x-9d\,e^2b^2cx+18d^2eb\,c^2x-10d^3c^3x}{e^6}-\frac{3d(b^3e^3-6d}{}$
risch	$\frac{c^3x^4}{4e^3}+\frac{b\,c^2x^3}{e^3}-\frac{c^3d\,x^3}{e^4}+\frac{3b^2c\,x^2}{2e^3}-\frac{9b\,c^2d\,x^2}{2e^4}+\frac{3c^3d^2x^2}{e^5}+\frac{b^3x}{e^3}-\frac{9d\,b^2cx}{e^4}+\frac{18d^2b\,c^2x}{e^5}-\frac{10d^3c^3x}{e^6}+\frac{(-3b^3}{}$
parallelrisch	$-\frac{-5x^4c^3d^2e^4+20x^3c^3d^3e^3-108b^2c\,d^4e^2+24\ln(ex+d)x\,b^3d^2e^4-120\ln(ex+d)x\,c^3d^5e+180b\,c^2d^5e-90d^6c^3+10x^4b\,c^2d\,e^5-}{}$

input
```
int((c*x^2+b*x)^3/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/e^4*(b^3*e^3-6*b^2*c*d*e^2+10*b*c^2*d^2*e-5*c^3*d^3)*x^3+1/4*c^3*x^6/e-
1/2*d^2*(9*b^3*d*e^3-54*b^2*c*d^2*e^2+90*b*c^2*d^3*e-45*c^3*d^4)/e^7+1/4*c
*(6*b^2*e^2-10*b*c*d*e+5*c^2*d^2)/e^3*x^4+1/2*c^2*(2*b*e-c*d)/e^2*x^5-2*d*
(3*b^3*d*e^3-18*b^2*c*d^2*e^2+30*b*c^2*d^3*e-15*c^3*d^4)/e^6*x)/(e*x+d)^2-
3*d/e^7*(b^3*e^3-6*b^2*c*d*e^2+10*b*c^2*d^2*e-5*c^3*d^3)*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(194) = 388$.

Time = 0.09 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.14

$$\int \frac{(bx+cx^2)^3}{(d+ex)^3}\,dx = \frac{c^3e^6x^6 + 22\,c^3d^6 - 54\,bc^2d^5e + 42\,b^2cd^4e^2 - 10\,b^3d^3e^3 - 2\,(c^3de^5 - 2\,bc^2e^6)x^5 + (5\,c^3d^2e^4 - 10\,bc^2de^5 + 6}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/4*(c^3*e^6*x^6 + 22*c^3*d^6 - 54*b*c^2*d^5*e + 42*b^2*c*d^4*e^2 - 10*b^3
*d^3*e^3 - 2*(c^3*d*e^5 - 2*b*c^2*e^6)*x^5 + (5*c^3*d^2*e^4 - 10*b*c^2*d*e
^5 + 6*b^2*c*e^6)*x^4 - 4*(5*c^3*d^3*e^3 - 10*b*c^2*d^2*e^4 + 6*b^2*c*d*e^
5 - b^3*e^6)*x^3 - 2*(34*c^3*d^4*e^2 - 63*b*c^2*d^3*e^3 + 33*b^2*c*d^2*e^4
 - 4*b^3*d*e^5)*x^2 - 4*(4*c^3*d^5*e - 3*b*c^2*d^4*e^2 - 3*b^2*c*d^3*e^3 +
 2*b^3*d^2*e^4)*x + 12*(5*c^3*d^6 - 10*b*c^2*d^5*e + 6*b^2*c*d^4*e^2 - b^3
*d^3*e^3 + (5*c^3*d^4*e^2 - 10*b*c^2*d^3*e^3 + 6*b^2*c*d^2*e^4 - b^3*d*e^5
)*x^2 + 2*(5*c^3*d^5*e - 10*b*c^2*d^4*e^2 + 6*b^2*c*d^3*e^3 - b^3*d^2*e^4)
*x)*log(e*x + d))/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.42

$$\int \frac{(bx+cx^2)^3}{(d+ex)^3}\,dx = \frac{c^3x^4}{4e^3} - \frac{3d(be-cd)\left(b^2e^2-5bcde+5c^2d^2\right)\log(d+ex)}{e^7} + x^3\left(\frac{bc^2}{e^3}-\frac{c^3d}{e^4}\right) + x^2\cdot\left(\frac{3b^2c}{2e^3}-\frac{9bc^2d}{2e^4}+\frac{3c^3d^2}{e^5}\right) + x\left(\frac{b^3}{e^3}-\frac{9b^2cd}{e^4}+\frac{18bc^2d^2}{e^5}-\frac{10c^3d^3}{e^6}\right) + \frac{-5b^3d^3e^3+21b^2cd^4e^2-27bc^2d^5e+11c^3d^6+x(-6b^3d^2e^4+24b^2cd^3e^3-30bc^2d^4e^2+12c^3d^5e)}{2d^2e^7+4de^8x+2e^9x^2}$$

input
```
integrate((c*x**2+b*x)**3/(e*x+d)**3,x)
```

output
```
c**3*x**4/(4*e**3) - 3*d*(b*e - c*d)*(b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2)
*log(d + e*x)/e**7 + x**3*(b*c**2/e**3 - c**3*d/e**4) + x**2*(3*b**2*c/(2*
e**3) - 9*b*c**2*d/(2*e**4) + 3*c**3*d**2/e**5) + x*(b**3/e**3 - 9*b**2*c*
d/e**4 + 18*b*c**2*d**2/e**5 - 10*c**3*d**3/e**6) + (-5*b**3*d**3*e**3 + 2
1*b**2*c*d**4*e**2 - 27*b*c**2*d**5*e + 11*c**3*d**6 + x*(-6*b**3*d**2*e**
4 + 24*b**2*c*d**3*e**3 - 30*b*c**2*d**4*e**2 + 12*c**3*d**5*e))/(2*d**2*e
**7 + 4*d*e**8*x + 2*e**9*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.40

$$\int \frac{(bx+cx^2)^3}{(d+ex)^3}\,dx = \frac{11\,c^3d^6-27\,bc^2d^5e+21\,b^2cd^4e^2-5\,b^3d^3e^3+6\,(2\,c^3d^5e-5\,bc^2d^4e^2+4\,b^2cd^3e^3-b^3d^2e^4)x}{2\,(e^9x^2+2\,de^8x+d^2e^7)} + \frac{c^3e^3x^4-4\,(c^3de^2-bc^2e^3)x^3+6\,(2\,c^3d^2e-3\,bc^2de^2+b^2ce^3)x^2-4\,(10\,c^3d^3-18\,bc^2d^2e+9\,b^2cde^2-b}{4\,e^6} + \frac{3\,(5\,c^3d^4-10\,bc^2d^3e+6\,b^2cd^2e^2-b^3de^3)\log(ex+d)}{e^7}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/2*(11*c^3*d^6 - 27*b*c^2*d^5*e + 21*b^2*c*d^4*e^2 - 5*b^3*d^3*e^3 + 6*(2
*c^3*d^5*e - 5*b*c^2*d^4*e^2 + 4*b^2*c*d^3*e^3 - b^3*d^2*e^4)*x)/(e^9*x^2
+ 2*d*e^8*x + d^2*e^7) + 1/4*(c^3*e^3*x^4 - 4*(c^3*d*e^2 - b*c^2*e^3)*x^3
+ 6*(2*c^3*d^2*e - 3*b*c^2*d*e^2 + b^2*c*e^3)*x^2 - 4*(10*c^3*d^3 - 18*b*c
^2*d^2*e + 9*b^2*c*d*e^2 - b^3*e^3)*x)/e^6 + 3*(5*c^3*d^4 - 10*b*c^2*d^3*e
 + 6*b^2*c*d^2*e^2 - b^3*d*e^3)*log(e*x + d)/e^7
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.40

$$\int \frac{(bx+cx^2)^3}{(d+ex)^3}\,dx = \frac{3\,(5\,c^3d^4-10\,bc^2d^3e+6\,b^2cd^2e^2-b^3de^3)\log(|ex+d|)}{e^7} + \frac{11\,c^3d^6-27\,bc^2d^5e+21\,b^2cd^4e^2-5\,b^3d^3e^3+6\,(2\,c^3d^5e-5\,bc^2d^4e^2+4\,b^2cd^3e^3-b^3d^2e^4)x}{2\,(ex+d)^2e^7} + \frac{c^3e^9x^4-4\,c^3de^8x^3+4\,bc^2e^9x^3+12\,c^3d^2e^7x^2-18\,bc^2de^8x^2+6\,b^2ce^9x^2-40\,c^3d^3e^6x+72\,bc^2d^2e^7x-3}{4\,e^{12}}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^3,x, algorithm="giac")
```

output
```
3*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 - b^3*d*e^3)*log(abs(e*x +
 d))/e^7 + 1/2*(11*c^3*d^6 - 27*b*c^2*d^5*e + 21*b^2*c*d^4*e^2 - 5*b^3*d^3
*e^3 + 6*(2*c^3*d^5*e - 5*b*c^2*d^4*e^2 + 4*b^2*c*d^3*e^3 - b^3*d^2*e^4)*x
)/((e*x + d)^2*e^7) + 1/4*(c^3*e^9*x^4 - 4*c^3*d*e^8*x^3 + 4*b*c^2*e^9*x^3
 + 12*c^3*d^2*e^7*x^2 - 18*b*c^2*d*e^8*x^2 + 6*b^2*c*e^9*x^2 - 40*c^3*d^3*
e^6*x + 72*b*c^2*d^2*e^7*x - 36*b^2*c*d*e^8*x + 4*b^3*e^9*x)/e^12
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.76

$$\int \frac{(bx+cx^2)^3}{(d+ex)^3}\,dx = x^3\left(\frac{b\,c^2}{e^3}-\frac{c^3\,d}{e^4}\right) - x^2\left(\frac{3\,d\left(\frac{3\,b\,c^2}{e^3}-\frac{3\,c^3\,d}{e^4}\right)}{2\,e}-\frac{3\,b^2\,c}{2\,e^3}+\frac{3\,c^3\,d^2}{2\,e^5}\right) + \frac{x\left(-3\,b^3\,d^2\,e^3+12\,b^2\,c\,d^3\,e^2-15\,b\,c^2\,d^4\,e+6\,c^3\,d^5\right)+\frac{-5\,b^3\,d^3\,e^3+21\,b^2\,c\,d^4\,e^2-27\,b\,c^2\,d^5\,e+11\,c^3\,d^6}{2\,e}}{d^2\,e^6+2\,d\,e^7\,x+e^8\,x^2} + x\left(\frac{b^3}{e^3}+\frac{3\,d\left(\frac{3\,d\left(\frac{3\,b\,c^2}{e^3}-\frac{3\,c^3\,d}{e^4}\right)}{e}-\frac{3\,b^2\,c}{e^3}+\frac{3\,c^3\,d^2}{e^5}\right)}{e}-\frac{c^3\,d^3}{e^6}-\frac{3\,d^2\left(\frac{3\,b\,c^2}{e^3}-\frac{3\,c^3\,d}{e^4}\right)}{e^2}\right) + \frac{c^3\,x^4}{4\,e^3}+\frac{\ln\left(d+e\,x\right)\left(-3\,b^3\,d\,e^3+18\,b^2\,c\,d^2\,e^2-30\,b\,c^2\,d^3\,e+15\,c^3\,d^4\right)}{e^7}$$

input
```
int((b*x + c*x^2)^3/(d + e*x)^3,x)
```

output
```
x^3*((b*c^2)/e^3 - (c^3*d)/e^4) - x^2*((3*d*((3*b*c^2)/e^3 - (3*c^3*d)/e^4
))/(2*e) - (3*b^2*c)/(2*e^3) + (3*c^3*d^2)/(2*e^5)) + (x*(6*c^3*d^5 - 3*b^
3*d^2*e^3 + 12*b^2*c*d^3*e^2 - 15*b*c^2*d^4*e) + (11*c^3*d^6 - 5*b^3*d^3*e
^3 + 21*b^2*c*d^4*e^2 - 27*b*c^2*d^5*e)/(2*e))/(d^2*e^6 + e^8*x^2 + 2*d*e^
7*x) + x*(b^3/e^3 + (3*d*((3*d*((3*b*c^2)/e^3 - (3*c^3*d)/e^4))/e - (3*b^2
*c)/e^3 + (3*c^3*d^2)/e^5))/e - (c^3*d^3)/e^6 - (3*d^2*((3*b*c^2)/e^3 - (3
*c^3*d)/e^4))/e^2) + (c^3*x^4)/(4*e^3) + (log(d + e*x)*(15*c^3*d^4 - 3*b^3
*d*e^3 + 18*b^2*c*d^2*e^2 - 30*b*c^2*d^3*e))/e^7
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.32

$$\int \frac{(bx+cx^2)^3}{(d+ex)^3}\,dx$$

$$= 30c^3d^6+36b^2c\,d^4e^2-60b\,c^2d^5e-6b^3d^3e^3+72\log(ex+d)\,b^2c\,d^2e^4x^2-120\log(ex+d)\,b\,c^2d^3e^3x^2+72$$

input
```
int((c*x^2+b*x)^3/(e*x+d)^3,x)
```

output
```
( - 12*log(d + e*x)*b**3*d**3*e**3 - 24*log(d + e*x)*b**3*d**2*e**4*x - 12
*log(d + e*x)*b**3*d*e**5*x**2 + 72*log(d + e*x)*b**2*c*d**4*e**2 + 144*lo
g(d + e*x)*b**2*c*d**3*e**3*x + 72*log(d + e*x)*b**2*c*d**2*e**4*x**2 - 12
0*log(d + e*x)*b*c**2*d**5*e - 240*log(d + e*x)*b*c**2*d**4*e**2*x - 120*l
og(d + e*x)*b*c**2*d**3*e**3*x**2 + 60*log(d + e*x)*c**3*d**6 + 120*log(d
+ e*x)*c**3*d**5*e*x + 60*log(d + e*x)*c**3*d**4*e**2*x**2 - 6*b**3*d**3*e
**3 + 12*b**3*d*e**5*x**2 + 4*b**3*e**6*x**3 + 36*b**2*c*d**4*e**2 - 72*b*
*2*c*d**2*e**4*x**2 - 24*b**2*c*d*e**5*x**3 + 6*b**2*c*e**6*x**4 - 60*b*c*
*2*d**5*e + 120*b*c**2*d**3*e**3*x**2 + 40*b*c**2*d**2*e**4*x**3 - 10*b*c*
*2*d*e**5*x**4 + 4*b*c**2*e**6*x**5 + 30*c**3*d**6 - 60*c**3*d**4*e**2*x**
2 - 20*c**3*d**3*e**3*x**3 + 5*c**3*d**2*e**4*x**4 - 2*c**3*d*e**5*x**5 +
c**3*e**6*x**6)/(4*e**7*(d**2 + 2*d*e*x + e**2*x**2))
```

3.48 $\int \frac{(bx+cx^2)^3}{(d+ex)^4}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 213

$$\int \frac{(bx+cx^2)^3}{(d+ex)^4}\,dx = \frac{c(10c^2d^2-12bcde+3b^2e^2)\,x}{e^6} - \frac{c^2(4cd-3be)x^2}{2e^5} + \frac{c^3x^3}{3e^4} - \frac{d^3(cd-be)^3}{3e^7(d+ex)^3} + \frac{3d^2(cd-be)^2(2cd-be)}{2e^7(d+ex)^2} - \frac{3d(cd-be)\,(5c^2d^2-5bcde+b^2e^2)}{e^7(d+ex)} - \frac{(2cd-be)\,(10c^2d^2-10bcde+b^2e^2)\log(d+ex)}{e^7}$$

output
```
c*(3*b^2*e^2-12*b*c*d*e+10*c^2*d^2)*x/e^6-1/2*c^2*(-3*b*e+4*c*d)*x^2/e^5+1
/3*c^3*x^3/e^4-1/3*d^3*(-b*e+c*d)^3/e^7/(e*x+d)^3+3/2*d^2*(-b*e+c*d)^2*(-b
*e+2*c*d)/e^7/(e*x+d)^2-3*d*(-b*e+c*d)*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)/e^7/(
e*x+d)-(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*d^2)*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.99

$$\int \frac{(bx+cx^2)^3}{(d+ex)^4}\,dx$$
$$= \frac{6ce(10c^2d^2-12bcde+3b^2e^2)\,x-3c^2e^2(4cd-3be)x^2+2c^3e^3x^3-\frac{2d^3(cd-be)^3}{(d+ex)^3}+\frac{9d^2(cd-be)^2(2cd-be)}{(d+ex)^2}+\frac{18d(-5}{}}{6e^7}$$

input
```
Integrate[(b*x + c*x^2)^3/(d + e*x)^4,x]
```

output
```
(6*c*e*(10*c^2*d^2 - 12*b*c*d*e + 3*b^2*e^2)*x - 3*c^2*e^2*(4*c*d - 3*b*e)
*x^2 + 2*c^3*e^3*x^3 - (2*d^3*(c*d - b*e)^3)/(d + e*x)^3 + (9*d^2*(c*d - b
*e)^2*(2*c*d - b*e))/(d + e*x)^2 + (18*d*(-5*c^3*d^3 + 10*b*c^2*d^2*e - 6*
b^2*c*d*e^2 + b^3*e^3))/(d + e*x) + 6*(-20*c^3*d^3 + 30*b*c^2*d^2*e - 12*b
^2*c*d*e^2 + b^3*e^3)*Log[d + e*x])/(6*e^7)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^3}{(d+ex)^4}\,dx$$
$$\Big\downarrow \text{1140}$$
$$\int \left(\frac{(2cd-be)\left(-b^2e^2+10bcde-10c^2d^2\right)}{e^6(d+ex)}+\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{e^6(d+ex)^2}+\frac{c\left(3b^2e^2-12bcde+10c^2d^2\right)}{e^6}\right.$$
$$\Big\downarrow \text{2009}$$

$$-\frac{3d(b^2e^2-5bcde+5c^2d^2)(cd-be)}{e^7(d+ex)}-\frac{(2cd-be)(b^2e^2-10bcde+10c^2d^2)\log(d+ex)}{e^7}+\frac{cx(3b^2e^2-12bcde+10c^2d^2)}{e^6}-\frac{c^2x^2(4cd-3be)}{2e^5}-\frac{d^3(cd-be)^3}{3e^7(d+ex)^3}+\frac{3d^2(2cd-be)(cd-be)^2}{2e^7(d+ex)^2}+\frac{c^3x^3}{3e^4}$$

input
```
Int[(b*x + c*x^2)^3/(d + e*x)^4,x]
```

output
```
(c*(10*c^2*d^2 - 12*b*c*d*e + 3*b^2*e^2)*x)/e^6 - (c^2*(4*c*d - 3*b*e)*x^2
)/(2*e^5) + (c^3*x^3)/(3*e^4) - (d^3*(c*d - b*e)^3)/(3*e^7*(d + e*x)^3) +
(3*d^2*(c*d - b*e)^2*(2*c*d - b*e))/(2*e^7*(d + e*x)^2) - (3*d*(c*d - b*e)
*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(e^7*(d + e*x)) - ((2*c*d - b*e)*(10*c
^2*d^2 - 10*b*c*d*e + b^2*e^2)*Log[d + e*x])/e^7
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.20

method	result
norman	$\frac{\frac{c^3x^6}{3e}+\frac{d^3\left(11b^3e^3-132d\,e^2b^2c+330d^2eb\,c^2-220d^3c^3\right)}{6e^7}+\frac{c\left(6b^2e^2-15bcde+10c^2d^2\right)x^4}{2e^3}+\frac{c^2(3be-2cd)x^5}{2e^2}+\frac{3d\left(b^3e^3-12d\,e^2b^2c+30d^2eb\,c^2 \cdots\right.}{e^5}}{(ex+d)^3}$
default	$\frac{c\left(\frac{1}{3}c^2e^2x^3+\frac{3}{2}bc\,e^2x^2-2c^2de\,x^2+3b^2e^2x-12bcdex+10c^2d^2x\right)}{e^6}+\frac{d^3\left(b^3e^3-3d\,e^2b^2c+3d^2eb\,c^2-d^3c^3\right)}{3e^7(ex+d)^3}+\frac{\left(b^3e^3-12d\,e^2b^2c \cdots\right.}{}$
risch	$\frac{c^3x^3}{3e^4}+\frac{3c^2b\,x^2}{2e^4}-\frac{2c^3d\,x^2}{e^5}+\frac{3c\,b^2x}{e^4}-\frac{12c^2bdx}{e^5}+\frac{10c^3d^2x}{e^6}+\frac{\left(3b^3d\,e^4-18b^2c\,d^2e^3+30d^3b\,c^2e^2-15c^3d^4e\right)x^2+\frac{3d^2(\cdots}{}}{}$
parallelrisch	$\frac{30x^4c^3d^2e^4+18x^2b^3d\,e^5-132b^2c\,d^4e^2+18\ln(ex+d)x\,b^3d^2e^4-360\ln(ex+d)x\,c^3d^5e+330b\,c^2d^5e-220d^6c^3-45x^4b\,c^2d\,e^5-21 \cdots}{}$

input
```
int((c*x^2+b*x)^3/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
(1/3*c^3*x^6/e+1/6*d^3*(11*b^3*e^3-132*b^2*c*d*e^2+330*b*c^2*d^2*e-220*c^3
*d^3)/e^7+1/2*c*(6*b^2*e^2-15*b*c*d*e+10*c^2*d^2)/e^3*x^4+1/2*c^2*(3*b*e-2
*c*d)/e^2*x^5+3*d*(b^3*e^3-12*b^2*c*d*e^2+30*b*c^2*d^2*e-20*c^3*d^3)/e^5*x
^2+3/2*d^2*(3*b^3*e^3-36*b^2*c*d*e^2+90*b*c^2*d^2*e-60*c^3*d^3)/e^6*x)/(e*
x+d)^3+1/e^7*(b^3*e^3-12*b^2*c*d*e^2+30*b*c^2*d^2*e-20*c^3*d^3)*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(205) = 410$.

Time = 0.09 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.25

$$\int \frac{(bx+cx^2)^3}{(d+ex)^4}\,dx = \frac{2\,c^3e^6x^6 - 74\,c^3d^6 + 141\,bc^2d^5e - 78\,b^2cd^4e^2 + 11\,b^3d^3e^3 - 3\,(2\,c^3de^5 - 3\,bc^2e^6)x^5 + 3\,(10\,c^3d^2e^4 - 15\,bc^2}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^4,x, algorithm="fricas")
```

output
```
1/6*(2*c^3*e^6*x^6 - 74*c^3*d^6 + 141*b*c^2*d^5*e - 78*b^2*c*d^4*e^2 + 11*
b^3*d^3*e^3 - 3*(2*c^3*d*e^5 - 3*b*c^2*e^6)*x^5 + 3*(10*c^3*d^2*e^4 - 15*b
*c^2*d*e^5 + 6*b^2*c*e^6)*x^4 + (146*c^3*d^3*e^3 - 189*b*c^2*d^2*e^4 + 54*
b^2*c*d*e^5)*x^3 + 3*(26*c^3*d^4*e^2 - 9*b*c^2*d^3*e^3 - 18*b^2*c*d^2*e^4
+ 6*b^3*d*e^5)*x^2 - 3*(34*c^3*d^5*e - 81*b*c^2*d^4*e^2 + 54*b^2*c*d^3*e^3
 - 9*b^3*d^2*e^4)*x - 6*(20*c^3*d^6 - 30*b*c^2*d^5*e + 12*b^2*c*d^4*e^2 -
b^3*d^3*e^3 + (20*c^3*d^3*e^3 - 30*b*c^2*d^2*e^4 + 12*b^2*c*d*e^5 - b^3*e^
6)*x^3 + 3*(20*c^3*d^4*e^2 - 30*b*c^2*d^3*e^3 + 12*b^2*c*d^2*e^4 - b^3*d*e
^5)*x^2 + 3*(20*c^3*d^5*e - 30*b*c^2*d^4*e^2 + 12*b^2*c*d^3*e^3 - b^3*d^2*
e^4)*x)*log(e*x + d))/(e^10*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x + d^3*e^7)
```

Sympy [A] (verification not implemented)

Time = 1.33 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.41

$$\int \frac{(bx+cx^2)^3}{(d+ex)^4}\,dx = \frac{c^3x^3}{3e^4} + x^2\cdot\left(\frac{3bc^2}{2e^4} - \frac{2c^3d}{e^5}\right) + x\left(\frac{3b^2c}{e^4} - \frac{12bc^2d}{e^5} + \frac{10c^3d^2}{e^6}\right) + \frac{11b^3d^3e^3 - 78b^2cd^4e^2 + 141bc^2d^5e - 74c^3d^6 + x^2\cdot(18b^3de^5 - 108b^2cd^2e^4 + 180bc^2d^3e^3 - 90c^3d^4e^2) + \cdots}{6d^3e^7 + 18d^2e^8x + 18de^9x^2 + 6e^{10}x^3} + \frac{(be-2cd)(b^2e^2 - 10bcde + 10c^2d^2)\log(d+ex)}{e^7}$$

input
```
integrate((c*x**2+b*x)**3/(e*x+d)**4,x)
```

output
```
c**3*x**3/(3*e**4) + x**2*(3*b*c**2/(2*e**4) - 2*c**3*d/e**5) + x*(3*b**2*
c/e**4 - 12*b*c**2*d/e**5 + 10*c**3*d**2/e**6) + (11*b**3*d**3*e**3 - 78*b
**2*c*d**4*e**2 + 141*b*c**2*d**5*e - 74*c**3*d**6 + x**2*(18*b**3*d*e**5
- 108*b**2*c*d**2*e**4 + 180*b*c**2*d**3*e**3 - 90*c**3*d**4*e**2) + x*(27
*b**3*d**2*e**4 - 180*b**2*c*d**3*e**3 + 315*b*c**2*d**4*e**2 - 162*c**3*d
**5*e))/(6*d**3*e**7 + 18*d**2*e**8*x + 18*d*e**9*x**2 + 6*e**10*x**3) + (
b*e - 2*c*d)*(b**2*e**2 - 10*b*c*d*e + 10*c**2*d**2)*log(d + e*x)/e**7
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.38

$$\int \frac{(bx+cx^2)^3}{(d+ex)^4}\,dx = -\frac{74\,c^3d^6 - 141\,bc^2d^5e + 78\,b^2cd^4e^2 - 11\,b^3d^3e^3 + 18\,(5\,c^3d^4e^2 - 10\,bc^2d^3e^3 + 6\,b^2cd^2e^4 - b^3de^5)x^2 + 9\,(\cdots}{6\,(e^{10}x^3 + 3\,de^9x^2 + 3\,d^2e^8x + d^3e^7)} + \frac{2\,c^3e^2x^3 - 3\,(4\,c^3de - 3\,bc^2e^2)x^2 + 6\,(10\,c^3d^2 - 12\,bc^2de + 3\,b^2ce^2)x}{6\,e^6} - \frac{(20\,c^3d^3 - 30\,bc^2d^2e + 12\,b^2cde^2 - b^3e^3)\log(ex+d)}{e^7}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^4,x, algorithm="maxima")
```

output
```
-1/6*(74*c^3*d^6 - 141*b*c^2*d^5*e + 78*b^2*c*d^4*e^2 - 11*b^3*d^3*e^3 + 1
8*(5*c^3*d^4*e^2 - 10*b*c^2*d^3*e^3 + 6*b^2*c*d^2*e^4 - b^3*d*e^5)*x^2 + 9
*(18*c^3*d^5*e - 35*b*c^2*d^4*e^2 + 20*b^2*c*d^3*e^3 - 3*b^3*d^2*e^4)*x)/(
e^10*x^3 + 3*d*e^9*x^2 + 3*d^2*e^8*x + d^3*e^7) + 1/6*(2*c^3*e^2*x^3 - 3*(
4*c^3*d*e - 3*b*c^2*e^2)*x^2 + 6*(10*c^3*d^2 - 12*b*c^2*d*e + 3*b^2*c*e^2)
*x)/e^6 - (20*c^3*d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*log(e*x
 + d)/e^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.30

$$\int \frac{(bx+cx^2)^3}{(d+ex)^4}\,dx = -\frac{(20\,c^3d^3-30\,bc^2d^2e+12\,b^2cde^2-b^3e^3)\log(|ex+d|)}{e^7} - \frac{74\,c^3d^6-141\,bc^2d^5e+78\,b^2cd^4e^2-11\,b^3d^3e^3+18\,(5\,c^3d^4e^2-10\,bc^2d^3e^3+6\,b^2cd^2e^4-b^3de^5)x^2+9\,(1}{6\,(ex+d)^3e^7} + \frac{2\,c^3e^8x^3-12\,c^3de^7x^2+9\,bc^2e^8x^2+60\,c^3d^2e^6x-72\,bc^2de^7x+18\,b^2ce^8x}{6\,e^{12}}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^4,x, algorithm="giac")
```

output
```
-(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*log(abs(e*x + d)
)/e^7 - 1/6*(74*c^3*d^6 - 141*b*c^2*d^5*e + 78*b^2*c*d^4*e^2 - 11*b^3*d^3*
e^3 + 18*(5*c^3*d^4*e^2 - 10*b*c^2*d^3*e^3 + 6*b^2*c*d^2*e^4 - b^3*d*e^5)*
x^2 + 9*(18*c^3*d^5*e - 35*b*c^2*d^4*e^2 + 20*b^2*c*d^3*e^3 - 3*b^3*d^2*e^
4)*x)/((e*x + d)^3*e^7) + 1/6*(2*c^3*e^8*x^3 - 12*c^3*d*e^7*x^2 + 9*b*c^2*
e^8*x^2 + 60*c^3*d^2*e^6*x - 72*b*c^2*d*e^7*x + 18*b^2*c*e^8*x)/e^12
```

Mupad [B] (verification not implemented)

Time = 8.71 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.44

$$\int \frac{(bx+cx^2)^3}{(d+ex)^4}\,dx = x^2\left(\frac{3\,b\,c^2}{2\,e^4}-\frac{2\,c^3\,d}{e^5}\right)-x\left(\frac{4\,d\left(\frac{3\,b\,c^2}{e^4}-\frac{4\,c^3\,d}{e^5}\right)}{e}-\frac{3\,b^2\,c}{e^4}+\frac{6\,c^3\,d^2}{e^6}\right)$$

$$-\frac{x\left(-\frac{9\,b^3\,d^2\,e^3}{2}+30\,b^2\,c\,d^3\,e^2-\frac{105\,b\,c^2\,d^4\,e}{2}+27\,c^3\,d^5\right)-x^2\left(3\,b^3\,d\,e^4-18\,b^2\,c\,d^2\,e^3+30\,b\,c^2\,d^3\,e^2-15\,c^3\,d}{d^3\,e^6+3\,d^2\,e^7\,x+3\,d\,e^8\,x^2+e^9\,x^3}$$

$$+\frac{\ln\left(d+e\,x\right)\,\left(b^3\,e^3-12\,b^2\,c\,d\,e^2+30\,b\,c^2\,d^2\,e-20\,c^3\,d^3\right)}{e^7}+\frac{c^3\,x^3}{3\,e^4}$$

input
```
int((b*x + c*x^2)^3/(d + e*x)^4,x)
```

output
```
x^2*((3*b*c^2)/(2*e^4) - (2*c^3*d)/e^5) - x*((4*d*((3*b*c^2)/e^4 - (4*c^3*
d)/e^5))/e - (3*b^2*c)/e^4 + (6*c^3*d^2)/e^6) - (x*(27*c^3*d^5 - (9*b^3*d^
2*e^3)/2 + 30*b^2*c*d^3*e^2 - (105*b*c^2*d^4*e)/2) - x^2*(3*b^3*d*e^4 - 15
*c^3*d^4*e + 30*b*c^2*d^3*e^2 - 18*b^2*c*d^2*e^3) + (74*c^3*d^6 - 11*b^3*d
^3*e^3 + 78*b^2*c*d^4*e^2 - 141*b*c^2*d^5*e)/(6*e))/(d^3*e^6 + e^9*x^3 + 3
*d^2*e^7*x + 3*d*e^8*x^2) + (log(d + e*x)*(b^3*e^3 - 20*c^3*d^3 + 30*b*c^2
*d^2*e - 12*b^2*c*d*e^2))/e^7 + (c^3*x^3)/(3*e^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 545, normalized size of antiderivative = 2.56

$$\int \frac{(bx+cx^2)^3}{(d+ex)^4}\,dx$$

$$= \frac{-100c^3d^6-120\log(ex+d)\,c^3d^3e^3x^3+6\log(ex+d)\,b^3e^6x^3-60b^2c\,d^4e^2+150b\,c^2d^5e+5b^3d^3e^3-216\,\mathrm{l}}{}$$

input
```
int((c*x^2+b*x)^3/(e*x+d)^4,x)
```

output
```
(6*log(d + e*x)*b**3*d**3*e**3 + 18*log(d + e*x)*b**3*d**2*e**4*x + 18*log
(d + e*x)*b**3*d*e**5*x**2 + 6*log(d + e*x)*b**3*e**6*x**3 - 72*log(d + e*
x)*b**2*c*d**4*e**2 - 216*log(d + e*x)*b**2*c*d**3*e**3*x - 216*log(d + e*
x)*b**2*c*d**2*e**4*x**2 - 72*log(d + e*x)*b**2*c*d*e**5*x**3 + 180*log(d
+ e*x)*b*c**2*d**5*e + 540*log(d + e*x)*b*c**2*d**4*e**2*x + 540*log(d + e
*x)*b*c**2*d**3*e**3*x**2 + 180*log(d + e*x)*b*c**2*d**2*e**4*x**3 - 120*l
og(d + e*x)*c**3*d**6 - 360*log(d + e*x)*c**3*d**5*e*x - 360*log(d + e*x)*
c**3*d**4*e**2*x**2 - 120*log(d + e*x)*c**3*d**3*e**3*x**3 + 5*b**3*d**3*e
**3 + 9*b**3*d**2*e**4*x - 6*b**3*e**6*x**3 - 60*b**2*c*d**4*e**2 - 108*b*
*2*c*d**3*e**3*x + 72*b**2*c*d*e**5*x**3 + 18*b**2*c*e**6*x**4 + 150*b*c**
2*d**5*e + 270*b*c**2*d**4*e**2*x - 180*b*c**2*d**2*e**4*x**3 - 45*b*c**2*
d*e**5*x**4 + 9*b*c**2*e**6*x**5 - 100*c**3*d**6 - 180*c**3*d**5*e*x + 120
*c**3*d**3*e**3*x**3 + 30*c**3*d**2*e**4*x**4 - 6*c**3*d*e**5*x**5 + 2*c**
3*e**6*x**6)/(6*e**7*(d**3 + 3*d**2*e*x + 3*d*e**2*x**2 + e**3*x**3))
```

3.49 $\int \frac{(bx+cx^2)^3}{(d+ex)^5}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 213

$$\begin{aligned}\int \frac{(bx+cx^2)^3}{(d+ex)^5}\,dx = &-\frac{c^2(5cd-3be)x}{e^6}+\frac{c^3x^2}{2e^5}-\frac{d^3(cd-be)^3}{4e^7(d+ex)^4}\\ &+\frac{d^2(cd-be)^2(2cd-be)}{e^7(d+ex)^3}-\frac{3d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)}{2e^7(d+ex)^2}\\ &+\frac{(2cd-be)\left(10c^2d^2-10bcde+b^2e^2\right)}{e^7(d+ex)}\\ &+\frac{3c(5c^2d^2-5bcde+b^2e^2)\log(d+ex)}{e^7}\end{aligned}$$

output

```
-c^2*(-3*b*e+5*c*d)*x/e^6+1/2*c^3*x^2/e^5-1/4*d^3*(-b*e+c*d)^3/e^7/(e*x+d)
^4+d^2*(-b*e+c*d)^2*(-b*e+2*c*d)/e^7/(e*x+d)^3-3/2*d*(-b*e+c*d)*(b^2*e^2-5
*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^2+(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*
d^2)/e^7/(e*x+d)+3*c*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.99

$$\int \frac{(bx+cx^2)^3}{(d+ex)^5}\,dx = \frac{-4c^2e(5cd-3be)x+2c^3e^2x^2-\frac{d^3(cd-be)^3}{(d+ex)^4}+\frac{4d^2(cd-be)^2(2cd-be)}{(d+ex)^3}+\frac{6d(-5c^3d^3+10bc^2d^2e-6b^2cde^2+b^3e^3)}{(d+ex)^2}+\frac{80c^3d^3-12\ldots}{\ldots}}{4e^7}$$

input
```
Integrate[(b*x + c*x^2)^3/(d + e*x)^5,x]
```

output
```
(-4*c^2*e*(5*c*d - 3*b*e)*x + 2*c^3*e^2*x^2 - (d^3*(c*d - b*e)^3)/(d + e*x
)^4 + (4*d^2*(c*d - b*e)^2*(2*c*d - b*e))/(d + e*x)^3 + (6*d*(-5*c^3*d^3 +
 10*b*c^2*d^2*e - 6*b^2*c*d*e^2 + b^3*e^3))/(d + e*x)^2 + (80*c^3*d^3 - 12
0*b*c^2*d^2*e + 48*b^2*c*d*e^2 - 4*b^3*e^3)/(d + e*x) + 12*c*(5*c^2*d^2 -
5*b*c*d*e + b^2*e^2)*Log[d + e*x])/(4*e^7)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^3}{(d+ex)^5}\,dx$$

$$\Big\downarrow \; 1140$$

$$\int \left(\frac{3c(b^2e^2-5bcde+5c^2d^2)}{e^6(d+ex)}+\frac{(2cd-be)(-b^2e^2+10bcde-10c^2d^2)}{e^6(d+ex)^2}+\frac{3d(cd-be)(b^2e^2-5bcde+5c^2d^2)}{e^6(d+ex)^3}-\ldots\right.$$

$$\Big\downarrow \; 2009$$

$$\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{e^7(d+ex)}-\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{2e^7(d+ex)^2}+\frac{3c\left(b^2e^2-5bcde+5c^2d^2\right)\log(d+ex)}{e^7}-\frac{c^2x(5cd-3be)}{e^6}-\frac{d^3(cd-be)^3}{4e^7(d+ex)^4}+\frac{d^2(cd-be)^2(2cd-be)}{e^7(d+ex)^3}+\frac{c^3x^2}{2e^5}$$

input
```
Int[(b*x + c*x^2)^3/(d + e*x)^5,x]
```

output
```
-((c^2*(5*c*d - 3*b*e)*x)/e^6) + (c^3*x^2)/(2*e^5) - (d^3*(c*d - b*e)^3)/(
4*e^7*(d + e*x)^4) + (d^2*(c*d - b*e)^2*(2*c*d - b*e))/(e^7*(d + e*x)^3) -
 (3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(2*e^7*(d + e*x)^2) +
 ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2))/(e^7*(d + e*x)) + (3*
c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*Log[d + e*x])/e^7
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.19

method	result
norman	$\frac{\frac{c^3x^6}{2e}-\frac{d^3\left(b^3e^3-25d\,e^2b^2c+125d^2eb\,c^2-125d^3c^3\right)}{4e^7}-\frac{\left(b^3e^3-12d\,e^2b^2c+60d^2eb\,c^2-60d^3c^3\right)x^3}{e^4}+\frac{3c^2(be-cd)x^5}{e^2}-\frac{3d\left(b^3e^3-18d\,e^2b^2c+9 \ldots\right)}{2e \ldots}}{(ex+d)^4}$
default	$\frac{c^2\left(\frac{1}{2}ce\,x^2+3bex-5cdx\right)}{e^6}-\frac{d^2\left(b^3e^3-4d\,e^2b^2c+5d^2eb\,c^2-2d^3c^3\right)}{e^7(ex+d)^3}+\frac{d^3\left(b^3e^3-3d\,e^2b^2c+3d^2eb\,c^2-d^3c^3\right)}{4e^7(ex+d)^4}+\frac{3c\left(b^2e^2-5bcd \ldots\right.}{\ldots}$
risch	$\frac{c^3x^2}{2e^5}+\frac{3c^2bx}{e^5}-\frac{5c^3dx}{e^6}+\frac{\left(-b^3e^5+12b^2cd\,e^4-30b\,c^2d^2e^3+20c^3d^3e^2\right)x^3-\frac{3de\left(b^3e^3-18d\,e^2b^2c+50d^2eb\,c^2-35d^3c^3\right)x^2}{2}-d^2\left(\ldots\right.}{e^6(ex+d)^4}$
parallelrisch	$\frac{240x^3c^3d^3e^3-6x^2b^3d\,e^5+25b^2c\,d^4e^2+240\ln(ex+d)x\,c^3d^5e+12\ln(ex+d)x^4b^2c\,e^6-125b\,c^2d^5e+125d^6c^3+72\ln(ex+d)x^2b^2c \ldots}{\ldots}$

input
```
int((c*x^2+b*x)^3/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
(1/2*c^3*x^6/e-1/4*d^3*(b^3*e^3-25*b^2*c*d*e^2+125*b*c^2*d^2*e-125*c^3*d^3
)/e^7-(b^3*e^3-12*b^2*c*d*e^2+60*b*c^2*d^2*e-60*c^3*d^3)/e^4*x^3+3*c^2*(b*
e-c*d)/e^2*x^5-3/2*d*(b^3*e^3-18*b^2*c*d*e^2+90*b*c^2*d^2*e-90*c^3*d^3)/e^
5*x^2-d^2*(b^3*e^3-22*b^2*c*d*e^2+110*b*c^2*d^2*e-110*c^3*d^3)/e^6*x)/(e*x
+d)^4+3*c*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*ln(e*x+d)/e^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(207) = 414$.

Time = 0.08 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.26

$$\int \frac{(bx + cx^2)^3}{(d + ex)^5} dx = \frac{2 c^3 e^6 x^6 + 57 c^3 d^6 - 77 bc^2 d^5 e + 25 b^2 c d^4 e^2 - b^3 d^3 e^3 - 12 (c^3 d e^5 - bc^2 e^6) x^5 - 4 (17 c^3 d^2 e^4 - 12 bc^2 d e^5) x}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^5,x, algorithm="fricas")
```

output
```
1/4*(2*c^3*e^6*x^6 + 57*c^3*d^6 - 77*b*c^2*d^5*e + 25*b^2*c*d^4*e^2 - b^3*
d^3*e^3 - 12*(c^3*d*e^5 - b*c^2*e^6)*x^5 - 4*(17*c^3*d^2*e^4 - 12*b*c^2*d*
e^5)*x^4 - 4*(8*c^3*d^3*e^3 + 12*b*c^2*d^2*e^4 - 12*b^2*c*d*e^5 + b^3*e^6)
*x^3 + 6*(22*c^3*d^4*e^2 - 42*b*c^2*d^3*e^3 + 18*b^2*c*d^2*e^4 - b^3*d*e^5
)*x^2 + 4*(42*c^3*d^5*e - 62*b*c^2*d^4*e^2 + 22*b^2*c*d^3*e^3 - b^3*d^2*e^
4)*x + 12*(5*c^3*d^6 - 5*b*c^2*d^5*e + b^2*c*d^4*e^2 + (5*c^3*d^2*e^4 - 5*
b*c^2*d*e^5 + b^2*c*e^6)*x^4 + 4*(5*c^3*d^3*e^3 - 5*b*c^2*d^2*e^4 + b^2*c*
d*e^5)*x^3 + 6*(5*c^3*d^4*e^2 - 5*b*c^2*d^3*e^3 + b^2*c*d^2*e^4)*x^2 + 4*(
5*c^3*d^5*e - 5*b*c^2*d^4*e^2 + b^2*c*d^3*e^3)*x)*log(e*x + d))/(e^11*x^4
+ 4*d*e^10*x^3 + 6*d^2*e^9*x^2 + 4*d^3*e^8*x + d^4*e^7)
```

Sympy [A] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.48

$$\int \frac{(bx+cx^2)^3}{(d+ex)^5}\,dx = \frac{c^3x^2}{2e^5} + \frac{3c(b^2e^2-5bcde+5c^2d^2)\log{(d+ex)}}{e^7} + x\left(\frac{3bc^2}{e^5}-\frac{5c^3d}{e^6}\right)$$
$$+ \frac{-b^3d^3e^3+25b^2cd^4e^2-77bc^2d^5e+57c^3d^6+x^3(-4b^3e^6+48b^2cde^5-120bc^2d^2e^4+80c^3d^3e^3)+x^2(-6}{4d^4e^7+16d^3e^8x+24d^2e^9}$$

input
```
integrate((c*x**2+b*x)**3/(e*x+d)**5,x)
```

output
```
c**3*x**2/(2*e**5) + 3*c*(b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2)*log(d + e*x
)/e**7 + x*(3*b*c**2/e**5 - 5*c**3*d/e**6) + (-b**3*d**3*e**3 + 25*b**2*c*
d**4*e**2 - 77*b*c**2*d**5*e + 57*c**3*d**6 + x**3*(-4*b**3*e**6 + 48*b**2
*c*d*e**5 - 120*b*c**2*d**2*e**4 + 80*c**3*d**3*e**3) + x**2*(-6*b**3*d*e*
*5 + 108*b**2*c*d**2*e**4 - 300*b*c**2*d**3*e**3 + 210*c**3*d**4*e**2) + x
*(-4*b**3*d**2*e**4 + 88*b**2*c*d**3*e**3 - 260*b*c**2*d**4*e**2 + 188*c**
3*d**5*e))/(4*d**4*e**7 + 16*d**3*e**8*x + 24*d**2*e**9*x**2 + 16*d*e**10*
x**3 + 4*e**11*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.42

$$\int \frac{(bx+cx^2)^3}{(d+ex)^5}\,dx$$
$$= \frac{57\,c^3d^6-77\,bc^2d^5e+25\,b^2cd^4e^2-b^3d^3e^3+4\,(20\,c^3d^3e^3-30\,bc^2d^2e^4+12\,b^2cde^5-b^3e^6)x^3+6\,(35\,c^3d^4e}{4\,(e^{11}x^4+4\,de^{10}x^3+6\,d^2e^9x^2+}$$
$$+ \frac{c^3ex^2-2\,(5\,c^3d-3\,bc^2e)x}{2\,e^6} + \frac{3\,(5\,c^3d^2-5\,bc^2de+b^2ce^2)\log{(ex+d)}}{e^7}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^5,x, algorithm="maxima")
```

output
```
1/4*(57*c^3*d^6 - 77*b*c^2*d^5*e + 25*b^2*c*d^4*e^2 - b^3*d^3*e^3 + 4*(20*
c^3*d^3*e^3 - 30*b*c^2*d^2*e^4 + 12*b^2*c*d*e^5 - b^3*e^6)*x^3 + 6*(35*c^3
*d^4*e^2 - 50*b*c^2*d^3*e^3 + 18*b^2*c*d^2*e^4 - b^3*d*e^5)*x^2 + 4*(47*c^
3*d^5*e - 65*b*c^2*d^4*e^2 + 22*b^2*c*d^3*e^3 - b^3*d^2*e^4)*x)/(e^11*x^4
+ 4*d*e^10*x^3 + 6*d^2*e^9*x^2 + 4*d^3*e^8*x + d^4*e^7) + 1/2*(c^3*e*x^2 -
 2*(5*c^3*d - 3*b*c^2*e)*x)/e^6 + 3*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*
log(e*x + d)/e^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.83

$$\int \frac{(bx+cx^2)^3}{(d+ex)^5}\,dx$$
$$= \frac{\left(c^3 - \frac{6\,(2\,c^3de - bc^2e^2)}{(ex+d)e}\right)(ex+d)^2}{2\,e^7} - \frac{3\,(5\,c^3d^2 - 5\,bc^2de + b^2ce^2)\log\left(\frac{|ex+d|}{(ex+d)^2|e|}\right)}{e^7}$$
$$+ \frac{\frac{80\,c^3d^3e^{29}}{ex+d} - \frac{30\,c^3d^4e^{29}}{(ex+d)^2} + \frac{8\,c^3d^5e^{29}}{(ex+d)^3} - \frac{c^3d^6e^{29}}{(ex+d)^4} - \frac{120\,bc^2d^2e^{30}}{ex+d} + \frac{60\,bc^2d^3e^{30}}{(ex+d)^2} - \frac{20\,bc^2d^4e^{30}}{(ex+d)^3} + \frac{3\,bc^2d^5e^{30}}{(ex+d)^4} + \frac{48\,b^2cde^{31}}{ex+d} - 3\ldots}{4\,e^{36}}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^5,x, algorithm="giac")
```

output
```
1/2*(c^3 - 6*(2*c^3*d*e - b*c^2*e^2)/((e*x + d)*e))*(e*x + d)^2/e^7 - 3*(5
*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*log(abs(e*x + d)/((e*x + d)^2*abs(e)))
/e^7 + 1/4*(80*c^3*d^3*e^29/(e*x + d) - 30*c^3*d^4*e^29/(e*x + d)^2 + 8*c^
3*d^5*e^29/(e*x + d)^3 - c^3*d^6*e^29/(e*x + d)^4 - 120*b*c^2*d^2*e^30/(e*
x + d) + 60*b*c^2*d^3*e^30/(e*x + d)^2 - 20*b*c^2*d^4*e^30/(e*x + d)^3 + 3
*b*c^2*d^5*e^30/(e*x + d)^4 + 48*b^2*c*d*e^31/(e*x + d) - 36*b^2*c*d^2*e^3
1/(e*x + d)^2 + 16*b^2*c*d^3*e^31/(e*x + d)^3 - 3*b^2*c*d^4*e^31/(e*x + d)
^4 - 4*b^3*e^32/(e*x + d) + 6*b^3*d*e^32/(e*x + d)^2 - 4*b^3*d^2*e^32/(e*x
 + d)^3 + b^3*d^3*e^32/(e*x + d)^4)/e^36
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.42

$$\int \frac{(bx+cx^2)^3}{(d+ex)^5}\,dx = x\left(\frac{3\,b\,c^2}{e^5}-\frac{5\,c^3\,d}{e^6}\right)$$
$$-\frac{x^2\left(\frac{3\,b^3\,d\,e^4}{2}-27\,b^2\,c\,d^2\,e^3+75\,b\,c^2\,d^3\,e^2-\frac{105\,c^3\,d^4\,e}{2}\right)-x\left(-b^3\,d^2\,e^3+22\,b^2\,c\,d^3\,e^2-65\,b\,c^2\,d^4\,e+47\,c^3\,d\right.}{d^4\,e^6+4\,d^3\,e^7\,x+6\,d^2\,e^8\,x^2+}$$
$$+\frac{\ln(d+e\,x)\,\left(3\,b^2\,c\,e^2-15\,b\,c^2\,d\,e+15\,c^3\,d^2\right)}{e^7}+\frac{c^3\,x^2}{2\,e^5}$$

input
```
int((b*x + c*x^2)^3/(d + e*x)^5,x)
```

output
```
x*((3*b*c^2)/e^5 - (5*c^3*d)/e^6) - (x^2*((3*b^3*d*e^4)/2 - (105*c^3*d^4*e
)/2 + 75*b*c^2*d^3*e^2 - 27*b^2*c*d^2*e^3) - x*(47*c^3*d^5 - b^3*d^2*e^3 +
 22*b^2*c*d^3*e^2 - 65*b*c^2*d^4*e) - (57*c^3*d^6 - b^3*d^3*e^3 + 25*b^2*c
*d^4*e^2 - 77*b*c^2*d^5*e)/(4*e) + x^3*(b^3*e^5 - 20*c^3*d^3*e^2 + 30*b*c^
2*d^2*e^3 - 12*b^2*c*d*e^4))/(d^4*e^6 + e^10*x^4 + 4*d^3*e^7*x + 4*d*e^9*x
^3 + 6*d^2*e^8*x^2) + (log(d + e*x)*(15*c^3*d^2 + 3*b^2*c*e^2 - 15*b*c^2*d
*e))/e^7 + (c^3*x^2)/(2*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 536, normalized size of antiderivative = 2.52

$$\int \frac{(bx+cx^2)^3}{(d+ex)^5}\,dx$$
$$=\frac{12\log(ex+d)\,b^2c\,d^5e^2-60\log(ex+d)\,b\,c^2d^6e+240\log(ex+d)\,c^3d^6ex+360\log(ex+d)\,c^3d^5e^2x^2+24}{}$$

input
```
int((c*x^2+b*x)^3/(e*x+d)^5,x)
```

output

```
(12*log(d + e*x)*b**2*c*d**5*e**2 + 48*log(d + e*x)*b**2*c*d**4*e**3*x + 7
2*log(d + e*x)*b**2*c*d**3*e**4*x**2 + 48*log(d + e*x)*b**2*c*d**2*e**5*x*
*3 + 12*log(d + e*x)*b**2*c*d*e**6*x**4 - 60*log(d + e*x)*b*c**2*d**6*e -
240*log(d + e*x)*b*c**2*d**5*e**2*x - 360*log(d + e*x)*b*c**2*d**4*e**3*x*
*2 - 240*log(d + e*x)*b*c**2*d**3*e**4*x**3 - 60*log(d + e*x)*b*c**2*d**2*
e**5*x**4 + 60*log(d + e*x)*c**3*d**7 + 240*log(d + e*x)*c**3*d**6*e*x + 3
60*log(d + e*x)*c**3*d**5*e**2*x**2 + 240*log(d + e*x)*c**3*d**4*e**3*x**3
 + 60*log(d + e*x)*c**3*d**3*e**4*x**4 + b**3*e**7*x**4 + 13*b**2*c*d**5*e
**2 + 40*b**2*c*d**4*e**3*x + 36*b**2*c*d**3*e**4*x**2 - 12*b**2*c*d*e**6*
x**4 - 65*b*c**2*d**6*e - 200*b*c**2*d**5*e**2*x - 180*b*c**2*d**4*e**3*x*
*2 + 60*b*c**2*d**2*e**5*x**4 + 12*b*c**2*d*e**6*x**5 + 65*c**3*d**7 + 200
*c**3*d**6*e*x + 180*c**3*d**5*e**2*x**2 - 60*c**3*d**3*e**4*x**4 - 12*c**
3*d**2*e**5*x**5 + 2*c**3*d*e**6*x**6)/(4*d*e**7*(d**4 + 4*d**3*e*x + 6*d*
*2*e**2*x**2 + 4*d*e**3*x**3 + e**4*x**4))
```

3.50 $\int \frac{(bx+cx^2)^3}{(d+ex)^6}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 218

$$\int \frac{(bx+cx^2)^3}{(d+ex)^6}\,dx = \frac{c^3x}{e^6} - \frac{d^3(cd-be)^3}{5e^7(d+ex)^5} + \frac{3d^2(cd-be)^2(2cd-be)}{4e^7(d+ex)^4} - \frac{d(cd-be)\,(5c^2d^2-5bcde+b^2e^2)}{e^7(d+ex)^3} + \frac{(2cd-be)\,(10c^2d^2-10bcde+b^2e^2)}{2e^7(d+ex)^2} - \frac{3c(5c^2d^2-5bcde+b^2e^2)}{e^7(d+ex)} - \frac{3c^2(2cd-be)\log(d+ex)}{e^7}$$

output
```
c^3*x/e^6-1/5*d^3*(-b*e+c*d)^3/e^7/(e*x+d)^5+3/4*d^2*(-b*e+c*d)^2*(-b*e+2*
c*d)/e^7/(e*x+d)^4-d*(-b*e+c*d)*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^
3+1/2*(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*d^2)/e^7/(e*x+d)^2-3*c*(b^2*
e^2-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)-3*c^2*(-b*e+2*c*d)*ln(e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.11

$$\int \frac{(bx+cx^2)^3}{(d+ex)^6}\,dx =$$
$$-\frac{b^3e^3(d^3+5d^2ex+10de^2x^2+10e^3x^3)+12b^2ce^2(d^4+5d^3ex+10d^2e^2x^2+10de^3x^3+5e^4x^4)-bc^2de(1}{}$$

input
```
Integrate[(b*x + c*x^2)^3/(d + e*x)^6,x]
```

output
```
-1/20*(b^3*e^3*(d^3 + 5*d^2*e*x + 10*d*e^2*x^2 + 10*e^3*x^3) + 12*b^2*c*e^
2*(d^4 + 5*d^3*e*x + 10*d^2*e^2*x^2 + 10*d*e^3*x^3 + 5*e^4*x^4) - b*c^2*d*
e*(137*d^4 + 625*d^3*e*x + 1100*d^2*e^2*x^2 + 900*d*e^3*x^3 + 300*e^4*x^4)
 + 2*c^3*(87*d^6 + 375*d^5*e*x + 600*d^4*e^2*x^2 + 400*d^3*e^3*x^3 + 50*d^
2*e^4*x^4 - 50*d*e^5*x^5 - 10*e^6*x^6) + 60*c^2*(2*c*d - b*e)*(d + e*x)^5*
Log[d + e*x])/(e^7*(d + e*x)^5)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^3}{(d+ex)^6}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{3c(b^2e^2-5bcde+5c^2d^2)}{e^6(d+ex)^2}+\frac{(2cd-be)(-b^2e^2+10bcde-10c^2d^2)}{e^6(d+ex)^3}+\frac{3d(cd-be)(b^2e^2-5bcde+5c^2d^2)}{e^6(d+ex)^4}-\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3c(b^2e^2-5bcde+5c^2d^2)}{e^7(d+ex)}+\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{2e^7(d+ex)^2}-\frac{d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{e^7(d+ex)^3}-\frac{3c^2(2cd-be)\log(d+ex)}{e^7}-\frac{d^3(cd-be)^3}{5e^7(d+ex)^5}+\frac{3d^2(cd-be)^2(2cd-be)}{4e^7(d+ex)^4}+\frac{c^3x}{e^6}$$

input
```
Int[(b*x + c*x^2)^3/(d + e*x)^6,x]
```

output
```
(c^3*x)/e^6 - (d^3*(c*d - b*e)^3)/(5*e^7*(d + e*x)^5) + (3*d^2*(c*d - b*e)
^2*(2*c*d - b*e))/(4*e^7*(d + e*x)^4) - (d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*
d*e + b^2*e^2))/(e^7*(d + e*x)^3) + ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*
e + b^2*e^2))/(2*e^7*(d + e*x)^2) - (3*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)
)/(e^7*(d + e*x)) - (3*c^2*(2*c*d - b*e)*Log[d + e*x])/e^7
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.17

method	result
norman	$\frac{\frac{c^3x^6}{e}-\frac{d^3\left(b^3e^3+12d\,e^2b^2c-137d^2eb\,c^2+274d^3c^3\right)}{20e^7}-\frac{\left(3e^2b^2c-15c^2deb+30c^3d^2\right)x^4}{e^3}-\frac{\left(b^3e^3+12d\,e^2b^2c-90d^2eb\,c^2+180d^3c^3\right)x^3}{2e^4}-\cdots}{(ex+d)^5}$
risch	$\frac{c^3x}{e^6}+\frac{\left(-3b^2c\,e^5+15b\,c^2d\,e^4-15c^3d^2e^3\right)x^4-\frac{e^2\left(b^3e^3+12d\,e^2b^2c-90d^2eb\,c^2+100d^3c^3\right)x^3}{2}-\frac{de\left(b^3e^3+12d\,e^2b^2c-110d^2eb\,c^2+130d\cdots\right)}{2}\cdots}{e^6(ex+d)^5}$
default	$\frac{c^3x}{e^6}+\frac{d\left(b^3e^3-6d\,e^2b^2c+10d^2eb\,c^2-5d^3c^3\right)}{e^7(ex+d)^3}-\frac{3d^2\left(b^3e^3-4d\,e^2b^2c+5d^2eb\,c^2-2d^3c^3\right)}{4e^7(ex+d)^4}+\frac{3c^2(be-2cd)\ln(ex+d)}{e^7}+\frac{d^3(b\cdots}{\cdots}$
parallelrisch	$\frac{-600x^4c^3d^2e^4-1800x^3c^3d^3e^3-10x^2b^3d\,e^5-12b^2c\,d^4e^2-600\ln(ex+d)x\,c^3d^5e+137b\,c^2d^5e-274d^6c^3+300x^4b\,c^2d\,e^5+900x^3\cdots}{\cdots}$

input
```
int((c*x^2+b*x)^3/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

output
```
(c^3*x^6/e-1/20*d^3*(b^3*e^3+12*b^2*c*d*e^2-137*b*c^2*d^2*e+274*c^3*d^3)/e
^7-(3*b^2*c*e^2-15*b*c^2*d*e+30*c^3*d^2)/e^3*x^4-1/2*(b^3*e^3+12*b^2*c*d*e
^2-90*b*c^2*d^2*e+180*c^3*d^3)/e^4*x^3-1/2*d*(b^3*e^3+12*b^2*c*d*e^2-110*b
*c^2*d^2*e+220*c^3*d^3)/e^5*x^2-1/4*d^2*(b^3*e^3+12*b^2*c*d*e^2-125*b*c^2*
d^2*e+250*c^3*d^3)/e^6*x)/(e*x+d)^5+3*c^2/e^7*(b*e-2*c*d)*ln(e*x+d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(212) = 424$.

Time = 0.09 (sec) , antiderivative size = 462, normalized size of antiderivative = 2.12

$$\int \frac{(bx + cx^2)^3}{(d + ex)^6}\, dx = \frac{20\, c^3 e^6 x^6 + 100\, c^3 d e^5 x^5 - 174\, c^3 d^6 + 137\, b c^2 d^5 e - 12\, b^2 c d^4 e^2 - b^3 d^3 e^3 - 20\, (5\, c^3 d^2 e^4 - 15\, b c^2 d e^5 + 3\, b^2 c}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^6,x, algorithm="fricas")
```

output
```
1/20*(20*c^3*e^6*x^6 + 100*c^3*d*e^5*x^5 - 174*c^3*d^6 + 137*b*c^2*d^5*e -
 12*b^2*c*d^4*e^2 - b^3*d^3*e^3 - 20*(5*c^3*d^2*e^4 - 15*b*c^2*d*e^5 + 3*b
^2*c*e^6)*x^4 - 10*(80*c^3*d^3*e^3 - 90*b*c^2*d^2*e^4 + 12*b^2*c*d*e^5 + b
^3*e^6)*x^3 - 10*(120*c^3*d^4*e^2 - 110*b*c^2*d^3*e^3 + 12*b^2*c*d^2*e^4 +
 b^3*d*e^5)*x^2 - 5*(150*c^3*d^5*e - 125*b*c^2*d^4*e^2 + 12*b^2*c*d^3*e^3
+ b^3*d^2*e^4)*x - 60*(2*c^3*d^6 - b*c^2*d^5*e + (2*c^3*d*e^5 - b*c^2*e^6)
*x^5 + 5*(2*c^3*d^2*e^4 - b*c^2*d*e^5)*x^4 + 10*(2*c^3*d^3*e^3 - b*c^2*d^2
*e^4)*x^3 + 10*(2*c^3*d^4*e^2 - b*c^2*d^3*e^3)*x^2 + 5*(2*c^3*d^5*e - b*c^
2*d^4*e^2)*x)*log(e*x + d))/(e^12*x^5 + 5*d*e^11*x^4 + 10*d^2*e^10*x^3 + 1
0*d^3*e^9*x^2 + 5*d^4*e^8*x + d^5*e^7)
```

Sympy [A] (verification not implemented)

Time = 6.20 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.50

$$\int \frac{(bx+cx^2)^3}{(d+ex)^6}\,dx = \frac{c^3x}{e^6} + \frac{3c^2(be-2cd)\log(d+ex)}{e^7} + \frac{-b^3d^3e^3 - 12b^2cd^4e^2 + 137bc^2d^5e - 174c^3d^6 + x^4(-60b^2ce^6 + 300bc^2de^5 - 300c^3d^2e^4) + x^3(-10b^3e^6 - \ldots}{20d^5e^7 + \ldots}$$

input
```
integrate((c*x**2+b*x)**3/(e*x+d)**6,x)
```

output
```
c**3*x/e**6 + 3*c**2*(b*e - 2*c*d)*log(d + e*x)/e**7 + (-b**3*d**3*e**3 -
12*b**2*c*d**4*e**2 + 137*b*c**2*d**5*e - 174*c**3*d**6 + x**4*(-60*b**2*c
*e**6 + 300*b*c**2*d*e**5 - 300*c**3*d**2*e**4) + x**3*(-10*b**3*e**6 - 12
0*b**2*c*d*e**5 + 900*b*c**2*d**2*e**4 - 1000*c**3*d**3*e**3) + x**2*(-10*
b**3*d*e**5 - 120*b**2*c*d**2*e**4 + 1100*b*c**2*d**3*e**3 - 1300*c**3*d**
4*e**2) + x*(-5*b**3*d**2*e**4 - 60*b**2*c*d**3*e**3 + 625*b*c**2*d**4*e**
2 - 770*c**3*d**5*e))/(20*d**5*e**7 + 100*d**4*e**8*x + 200*d**3*e**9*x**2
 + 200*d**2*e**10*x**3 + 100*d*e**11*x**4 + 20*e**12*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.43

$$\int \frac{(bx+cx^2)^3}{(d+ex)^6}\,dx = -\frac{174\,c^3d^6 - 137\,bc^2d^5e + 12\,b^2cd^4e^2 + b^3d^3e^3 + 60\,(5\,c^3d^2e^4 - 5\,bc^2de^5 + b^2ce^6)x^4 + 10\,(100\,c^3d^3e^3 - 90\ldots}{20\,(e^{12}x^5 + 5\ldots} + \frac{c^3x}{e^6} - \frac{3\,(2\,c^3d - bc^2e)\log(ex+d)}{e^7}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^6,x, algorithm="maxima")
```

output
```
-1/20*(174*c^3*d^6 - 137*b*c^2*d^5*e + 12*b^2*c*d^4*e^2 + b^3*d^3*e^3 + 60
*(5*c^3*d^2*e^4 - 5*b*c^2*d*e^5 + b^2*c*e^6)*x^4 + 10*(100*c^3*d^3*e^3 - 9
0*b*c^2*d^2*e^4 + 12*b^2*c*d*e^5 + b^3*e^6)*x^3 + 10*(130*c^3*d^4*e^2 - 11
0*b*c^2*d^3*e^3 + 12*b^2*c*d^2*e^4 + b^3*d*e^5)*x^2 + 5*(154*c^3*d^5*e - 1
25*b*c^2*d^4*e^2 + 12*b^2*c*d^3*e^3 + b^3*d^2*e^4)*x)/(e^12*x^5 + 5*d*e^11
*x^4 + 10*d^2*e^10*x^3 + 10*d^3*e^9*x^2 + 5*d^4*e^8*x + d^5*e^7) + c^3*x/e
^6 - 3*(2*c^3*d - b*c^2*e)*log(e*x + d)/e^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.22

$$\int \frac{(bx+cx^2)^3}{(d+ex)^6}\,dx = \frac{c^3x}{e^6} - \frac{3\,(2\,c^3d-bc^2e)\log\left(|ex+d|\right)}{e^7} - \frac{174\,c^3d^6 - 137\,bc^2d^5e + 12\,b^2cd^4e^2 + b^3d^3e^3 + 60\,(5\,c^3d^2e^4 - 5\,bc^2de^5 + b^2ce^6)x^4 + 10\,(100\,c^3d^3e^3 - 90}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^6,x, algorithm="giac")
```

output
```
c^3*x/e^6 - 3*(2*c^3*d - b*c^2*e)*log(abs(e*x + d))/e^7 - 1/20*(174*c^3*d^
6 - 137*b*c^2*d^5*e + 12*b^2*c*d^4*e^2 + b^3*d^3*e^3 + 60*(5*c^3*d^2*e^4 -
 5*b*c^2*d*e^5 + b^2*c*e^6)*x^4 + 10*(100*c^3*d^3*e^3 - 90*b*c^2*d^2*e^4 +
 12*b^2*c*d*e^5 + b^3*e^6)*x^3 + 10*(130*c^3*d^4*e^2 - 110*b*c^2*d^3*e^3 +
 12*b^2*c*d^2*e^4 + b^3*d*e^5)*x^2 + 5*(154*c^3*d^5*e - 125*b*c^2*d^4*e^2
+ 12*b^2*c*d^3*e^3 + b^3*d^2*e^4)*x)/((e*x + d)^5*e^7)
```

Mupad [B] (verification not implemented)

Time = 8.96 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.43

$$\int \frac{(bx+cx^2)^3}{(d+ex)^6}\,dx = \frac{c^3\,x}{e^6} - \frac{x^4\,(3\,b^2\,c\,e^5 - 15\,b\,c^2\,d\,e^4 + 15\,c^3\,d^2\,e^3) + x^2\left(\frac{b^3\,d\,e^4}{2} + 6\,b^2\,c\,d^2\,e^3 - 55\,b\,c^2\,d^3\,e^2 + 65\,c^3\,d^4\,e\right) + x\left(\frac{b^3\,d^2\,e}{4}\right.}{d^5\,e^6 + 5\,d^4\,e^7\,x + 1} - \frac{\ln(d+e\,x)\,(6\,c^3\,d - 3\,b\,c^2\,e)}{e^7}$$

input
```
int((b*x + c*x^2)^3/(d + e*x)^6,x)
```

output
```
(c^3*x)/e^6 - (x^4*(3*b^2*c*e^5 + 15*c^3*d^2*e^3 - 15*b*c^2*d*e^4) + x^2*(
(b^3*d*e^4)/2 + 65*c^3*d^4*e - 55*b*c^2*d^3*e^2 + 6*b^2*c*d^2*e^3) + x*((7
7*c^3*d^5)/2 + (b^3*d^2*e^3)/4 + 3*b^2*c*d^3*e^2 - (125*b*c^2*d^4*e)/4) +
(174*c^3*d^6 + b^3*d^3*e^3 + 12*b^2*c*d^4*e^2 - 137*b*c^2*d^5*e)/(20*e) +
x^3*((b^3*e^5)/2 + 50*c^3*d^3*e^2 - 45*b*c^2*d^2*e^3 + 6*b^2*c*d*e^4))/(d^
5*e^6 + e^11*x^5 + 5*d^4*e^7*x + 5*d*e^10*x^4 + 10*d^3*e^8*x^2 + 10*d^2*e^
9*x^3) - (log(d + e*x)*(6*c^3*d - 3*b*c^2*e))/e^7
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 486, normalized size of antiderivative = 2.23

$$\int \frac{(bx+cx^2)^3}{(d+ex)^6}\,dx$$
$$= \frac{60\log(ex+d)\,b\,c^2d\,e^6x^5 + 60\log(ex+d)\,b\,c^2d^6e - 600\log(ex+d)\,c^3d^6ex - 1200\log(ex+d)\,c^3d^5e^2x^2 -}{}$$

input
```
int((c*x^2+b*x)^3/(e*x+d)^6,x)
```

output
```
(60*log(d + e*x)*b*c**2*d**6*e + 300*log(d + e*x)*b*c**2*d**5*e**2*x + 600
*log(d + e*x)*b*c**2*d**4*e**3*x**2 + 600*log(d + e*x)*b*c**2*d**3*e**4*x*
*3 + 300*log(d + e*x)*b*c**2*d**2*e**5*x**4 + 60*log(d + e*x)*b*c**2*d*e**
6*x**5 - 120*log(d + e*x)*c**3*d**7 - 600*log(d + e*x)*c**3*d**6*e*x - 120
0*log(d + e*x)*c**3*d**5*e**2*x**2 - 1200*log(d + e*x)*c**3*d**4*e**3*x**3
 - 600*log(d + e*x)*c**3*d**3*e**4*x**4 - 120*log(d + e*x)*c**3*d**2*e**5*
x**5 - b**3*d**4*e**3 - 5*b**3*d**3*e**4*x - 10*b**3*d**2*e**5*x**2 - 10*b
**3*d*e**6*x**3 + 12*b**2*c*e**7*x**5 + 77*b*c**2*d**6*e + 325*b*c**2*d**5
*e**2*x + 500*b*c**2*d**4*e**3*x**2 + 300*b*c**2*d**3*e**4*x**3 - 60*b*c**
2*d*e**6*x**5 - 154*c**3*d**7 - 650*c**3*d**6*e*x - 1000*c**3*d**5*e**2*x*
*2 - 600*c**3*d**4*e**3*x**3 + 120*c**3*d**2*e**5*x**5 + 20*c**3*d*e**6*x*
*6)/(20*d*e**7*(d**5 + 5*d**4*e*x + 10*d**3*e**2*x**2 + 10*d**2*e**3*x**3
+ 5*d*e**4*x**4 + e**5*x**5))
```

3.51 $\int \frac{(bx+cx^2)^3}{(d+ex)^7} dx$

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Optimal result

Integrand size = 19, antiderivative size = 228

$$\begin{aligned}\int \frac{(bx+cx^2)^3}{(d+ex)^7} dx = & -\frac{d^3(cd-be)^3}{6e^7(d+ex)^6} + \frac{3d^2(cd-be)^2(2cd-be)}{5e^7(d+ex)^5} \\ & - \frac{3d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)}{4e^7(d+ex)^4} \\ & + \frac{(2cd-be)\left(10c^2d^2-10bcde+b^2e^2\right)}{3e^7(d+ex)^3} \\ & - \frac{3c(5c^2d^2-5bcde+b^2e^2)}{2e^7(d+ex)^2} + \frac{3c^2(2cd-be)}{e^7(d+ex)} + \frac{c^3\log(d+ex)}{e^7}\end{aligned}$$

output
```
-1/6*d^3*(-b*e+c*d)^3/e^7/(e*x+d)^6+3/5*d^2*(-b*e+c*d)^2*(-b*e+2*c*d)/e^7/
(e*x+d)^5-3/4*d*(-b*e+c*d)*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^4+1/3
*(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*d^2)/e^7/(e*x+d)^3-3/2*c*(b^2*e^2
-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^2+3*c^2*(-b*e+2*c*d)/e^7/(e*x+d)+c^3*ln(
e*x+d)/e^7
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.01

$$\int \frac{(bx + cx^2)^3}{(d + ex)^7} \, dx$$
$$= \frac{-b^3e^3(d^3 + 6d^2ex + 15de^2x^2 + 20e^3x^3) - 6b^2ce^2(d^4 + 6d^3ex + 15d^2e^2x^2 + 20de^3x^3 + 15e^4x^4) - 30bc^2e(}{}$$

input
```
Integrate[(b*x + c*x^2)^3/(d + e*x)^7,x]
```

output
```
(-(b^3*e^3*(d^3 + 6*d^2*e*x + 15*d*e^2*x^2 + 20*e^3*x^3)) - 6*b^2*c*e^2*(d
^4 + 6*d^3*e*x + 15*d^2*e^2*x^2 + 20*d*e^3*x^3 + 15*e^4*x^4) - 30*b*c^2*e*
(d^5 + 6*d^4*e*x + 15*d^3*e^2*x^2 + 20*d^2*e^3*x^3 + 15*d*e^4*x^4 + 6*e^5*
x^5) + c^3*d*(147*d^5 + 822*d^4*e*x + 1875*d^3*e^2*x^2 + 2200*d^2*e^3*x^3
+ 1350*d*e^4*x^4 + 360*e^5*x^5) + 60*c^3*(d + e*x)^6*Log[d + e*x])/(60*e^7
*(d + e*x)^6)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx + cx^2)^3}{(d + ex)^7} \, dx$$
$$\Big\downarrow \; 1140$$
$$\int \left(\frac{3c(b^2e^2 - 5bcde + 5c^2d^2)}{e^6(d + ex)^3} + \frac{(2cd - be)(-b^2e^2 + 10bcde - 10c^2d^2)}{e^6(d + ex)^4} + \frac{3d(cd - be)(b^2e^2 - 5bcde + 5c^2d^2)}{e^6(d + ex)^5} - \right.$$
$$\Big\downarrow \; 2009$$

$$-\frac{3c(b^2e^2-5bcde+5c^2d^2)}{2e^7(d+ex)^2}+\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{3e^7(d+ex)^3}-\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{4e^7(d+ex)^4}+\frac{3c^2(2cd-be)}{e^7(d+ex)}-\frac{d^3(cd-be)^3}{6e^7(d+ex)^6}+\frac{3d^2(cd-be)^2(2cd-be)}{5e^7(d+ex)^5}+\frac{c^3\log(d+ex)}{e^7}$$

input
```
Int[(b*x + c*x^2)^3/(d + e*x)^7,x]
```

output
```
-1/6*(d^3*(c*d - b*e)^3)/(e^7*(d + e*x)^6) + (3*d^2*(c*d - b*e)^2*(2*c*d -
 b*e))/(5*e^7*(d + e*x)^5) - (3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2
*e^2))/(4*e^7*(d + e*x)^4) + ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2
*e^2))/(3*e^7*(d + e*x)^3) - (3*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(2*e^
7*(d + e*x)^2) + (3*c^2*(2*c*d - b*e))/(e^7*(d + e*x)) + (c^3*Log[d + e*x]
)/e^7
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.11

method	result
risch	$\frac{-\frac{3c^2(be-2cd)x^5}{e^2}-\frac{3c\left(b^2e^2+5bcde-15c^2d^2\right)x^4}{2e^3}-\frac{\left(b^3e^3+6d\,e^2b^2c+30d^2eb\,c^2-110d^3c^3\right)x^3}{3e^4}-\frac{d\left(b^3e^3+6d\,e^2b^2c+30d^2eb\,c^2-125d^3c^3\right)x^2}{4e^5}}{(ex+d)^6}$
norman	$\frac{-\frac{d^3\left(b^3e^3+6d\,e^2b^2c+30d^2eb\,c^2-147d^3c^3\right)}{60e^7}-\frac{3\left(b\,c^2e-2d\,c^3\right)x^5}{e^2}-\frac{3\left(e^2b^2c+5c^2deb-15c^3d^2\right)x^4}{2e^3}-\frac{\left(b^3e^3+6d\,e^2b^2c+30d^2eb\,c^2-110d^3c^3\right)}{3e^4}}{(ex+d)^6}$
default	$-\frac{b^3e^3-12d\,e^2b^2c+30d^2eb\,c^2-20d^3c^3}{3e^7(ex+d)^3}+\frac{3d\left(b^3e^3-6d\,e^2b^2c+10d^2eb\,c^2-5d^3c^3\right)}{4e^7(ex+d)^4}+\frac{c^3\ln(ex+d)}{e^7}-\frac{3d^2\left(b^3e^3-4d\,e^2b^2c+5\right.}{5e^7(ex+}$
parallelrisch	$1350x^4c^3d^2e^4+2200x^3c^3d^3e^3-15x^2b^3d\,e^5-6b^2c\,d^4e^2+360\ln(ex+d)x\,c^3d^5e-30b\,c^2d^5e+147d^6c^3+60\ln(ex+d)x^6c^3e^6-450$

input
```
int((c*x^2+b*x)^3/(e*x+d)^7,x,method=_RETURNVERBOSE)
```

output
```
(-3*c^2*(b*e-2*c*d)/e^2*x^5-3/2*c*(b^2*e^2+5*b*c*d*e-15*c^2*d^2)/e^3*x^4-1
/3*(b^3*e^3+6*b^2*c*d*e^2+30*b*c^2*d^2*e-110*c^3*d^3)/e^4*x^3-1/4*d*(b^3*e
^3+6*b^2*c*d*e^2+30*b*c^2*d^2*e-125*c^3*d^3)/e^5*x^2-1/10*d^2*(b^3*e^3+6*b
^2*c*d*e^2+30*b*c^2*d^2*e-137*c^3*d^3)/e^6*x-1/60*d^3*(b^3*e^3+6*b^2*c*d*e
^2+30*b*c^2*d^2*e-147*c^3*d^3)/e^7)/(e*x+d)^6+c^3*ln(e*x+d)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.79

$$\int \frac{(bx+cx^2)^3}{(d+ex)^7}\,dx$$
$$= \frac{147\,c^3d^6-30\,bc^2d^5e-6\,b^2cd^4e^2-b^3d^3e^3+180\,(2\,c^3de^5-bc^2e^6)x^5+90\,(15\,c^3d^2e^4-5\,bc^2de^5-b^2ce^6)x^4}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^7,x, algorithm="fricas")
```

output
```
1/60*(147*c^3*d^6 - 30*b*c^2*d^5*e - 6*b^2*c*d^4*e^2 - b^3*d^3*e^3 + 180*(
2*c^3*d*e^5 - b*c^2*e^6)*x^5 + 90*(15*c^3*d^2*e^4 - 5*b*c^2*d*e^5 - b^2*c*
e^6)*x^4 + 20*(110*c^3*d^3*e^3 - 30*b*c^2*d^2*e^4 - 6*b^2*c*d*e^5 - b^3*e^
6)*x^3 + 15*(125*c^3*d^4*e^2 - 30*b*c^2*d^3*e^3 - 6*b^2*c*d^2*e^4 - b^3*d*
e^5)*x^2 + 6*(137*c^3*d^5*e - 30*b*c^2*d^4*e^2 - 6*b^2*c*d^3*e^3 - b^3*d^2
*e^4)*x + 60*(c^3*e^6*x^6 + 6*c^3*d*e^5*x^5 + 15*c^3*d^2*e^4*x^4 + 20*c^3*
d^3*e^3*x^3 + 15*c^3*d^4*e^2*x^2 + 6*c^3*d^5*e*x + c^3*d^6)*log(e*x + d))/
(e^13*x^6 + 6*d*e^12*x^5 + 15*d^2*e^11*x^4 + 20*d^3*e^10*x^3 + 15*d^4*e^9*
x^2 + 6*d^5*e^8*x + d^6*e^7)
```

Sympy [A] (verification not implemented)

Time = 19.12 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.50

$$\int \frac{(bx+cx^2)^3}{(d+ex)^7}\,dx = \frac{c^3\log(d+ex)}{e^7} + \frac{-b^3d^3e^3 - 6b^2cd^4e^2 - 30bc^2d^5e + 147c^3d^6 + x^5(-180bc^2e^6 + 360c^3de^5) + x^4(-90b^2ce^6 - 450bc^2de^5 + \cdots}{60d\cdots}$$

input
```
integrate((c*x**2+b*x)**3/(e*x+d)**7,x)
```

output
```
c**3*log(d + e*x)/e**7 + (-b**3*d**3*e**3 - 6*b**2*c*d**4*e**2 - 30*b*c**2
*d**5*e + 147*c**3*d**6 + x**5*(-180*b*c**2*e**6 + 360*c**3*d*e**5) + x**4
*(-90*b**2*c*e**6 - 450*b*c**2*d*e**5 + 1350*c**3*d**2*e**4) + x**3*(-20*b
**3*e**6 - 120*b**2*c*d*e**5 - 600*b*c**2*d**2*e**4 + 2200*c**3*d**3*e**3)
 + x**2*(-15*b**3*d*e**5 - 90*b**2*c*d**2*e**4 - 450*b*c**2*d**3*e**3 + 18
75*c**3*d**4*e**2) + x*(-6*b**3*d**2*e**4 - 36*b**2*c*d**3*e**3 - 180*b*c*
*2*d**4*e**2 + 822*c**3*d**5*e))/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**
4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**
5 + 60*e**13*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.45

$$\int \frac{(bx+cx^2)^3}{(d+ex)^7}\,dx$$
$$= \frac{147\,c^3d^6 - 30\,bc^2d^5e - 6\,b^2cd^4e^2 - b^3d^3e^3 + 180\,(2\,c^3de^5 - bc^2e^6)x^5 + 90\,(15\,c^3d^2e^4 - 5\,bc^2de^5 - b^2ce^6)x^{}}{60\,(e^{13}x^6 + 6}$$
$$+ \frac{c^3\log(ex+d)}{e^7}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^7,x, algorithm="maxima")
```

output
```
1/60*(147*c^3*d^6 - 30*b*c^2*d^5*e - 6*b^2*c*d^4*e^2 - b^3*d^3*e^3 + 180*(
2*c^3*d*e^5 - b*c^2*e^6)*x^5 + 90*(15*c^3*d^2*e^4 - 5*b*c^2*d*e^5 - b^2*c*
e^6)*x^4 + 20*(110*c^3*d^3*e^3 - 30*b*c^2*d^2*e^4 - 6*b^2*c*d*e^5 - b^3*e^
6)*x^3 + 15*(125*c^3*d^4*e^2 - 30*b*c^2*d^3*e^3 - 6*b^2*c*d^2*e^4 - b^3*d*
e^5)*x^2 + 6*(137*c^3*d^5*e - 30*b*c^2*d^4*e^2 - 6*b^2*c*d^3*e^3 - b^3*d^2
*e^4)*x)/(e^13*x^6 + 6*d*e^12*x^5 + 15*d^2*e^11*x^4 + 20*d^3*e^10*x^3 + 15
*d^4*e^9*x^2 + 6*d^5*e^8*x + d^6*e^7) + c^3*log(e*x + d)/e^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.20

$$\int \frac{(bx+cx^2)^3}{(d+ex)^7}\,dx = \frac{c^3\log(|ex+d|)}{e^7}$$
$$+ \frac{180\,(2\,c^3de^4 - bc^2e^5)x^5 + 90\,(15\,c^3d^2e^3 - 5\,bc^2de^4 - b^2ce^5)x^4 + 20\,(110\,c^3d^3e^2 - 30\,bc^2d^2e^3 - 6\,b^2cde^4}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^7,x, algorithm="giac")
```

output
```
c^3*log(abs(e*x + d))/e^7 + 1/60*(180*(2*c^3*d*e^4 - b*c^2*e^5)*x^5 + 90*(
15*c^3*d^2*e^3 - 5*b*c^2*d*e^4 - b^2*c*e^5)*x^4 + 20*(110*c^3*d^3*e^2 - 30
*b*c^2*d^2*e^3 - 6*b^2*c*d*e^4 - b^3*e^5)*x^3 + 15*(125*c^3*d^4*e - 30*b*c
^2*d^3*e^2 - 6*b^2*c*d^2*e^3 - b^3*d*e^4)*x^2 + 6*(137*c^3*d^5 - 30*b*c^2*
d^4*e - 6*b^2*c*d^3*e^2 - b^3*d^2*e^3)*x + (147*c^3*d^6 - 30*b*c^2*d^5*e -
 6*b^2*c*d^4*e^2 - b^3*d^3*e^3)/e)/((e*x + d)^6*e^6)
```

Mupad [B] (verification not implemented)

Time = 8.98 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.18

$$\int \frac{(bx+cx^2)^3}{(d+ex)^7}\,dx = \frac{c^3\ln(d+e\,x)}{e^7} - \frac{x^5\,(3\,b\,c^2\,e^6-6\,c^3\,d\,e^5)+x^4\left(\frac{3\,b^2\,c\,e^6}{2}+\frac{15\,b\,c^2\,d\,e^5}{2}-\frac{45\,c^3\,d^2\,e^4}{2}\right)+x\left(\frac{b^3\,d^2\,e^4}{10}+\frac{3\,b^2\,c\,d^3\,e^3}{5}+3\,b\,c^2\,d^4\,e^2-137\ldots\right)}{}$$

input
```
int((b*x + c*x^2)^3/(d + e*x)^7,x)
```

output
```
(c^3*log(d + e*x))/e^7 - (x^5*(3*b*c^2*e^6 - 6*c^3*d*e^5) + x^4*((3*b^2*c*
e^6)/2 - (45*c^3*d^2*e^4)/2 + (15*b*c^2*d*e^5)/2) + x*((b^3*d^2*e^4)/10 -
(137*c^3*d^5*e)/10 + 3*b*c^2*d^4*e^2 + (3*b^2*c*d^3*e^3)/5) + x^2*((b^3*d*
e^5)/4 - (125*c^3*d^4*e^2)/4 + (15*b*c^2*d^3*e^3)/2 + (3*b^2*c*d^2*e^4)/2)
 + x^3*((b^3*e^6)/3 - (110*c^3*d^3*e^3)/3 + 10*b*c^2*d^2*e^4 + 2*b^2*c*d*e
^5) - (49*c^3*d^6)/20 + (b^3*d^3*e^3)/60 + (b^2*c*d^4*e^2)/10 + (b*c^2*d^5
*e)/2)/(e^7*(d + e*x)^6)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.75

$$\int \frac{(bx+cx^2)^3}{(d+ex)^7}\,dx = \frac{60\log(ex+d)\,c^3d^7+360\log(ex+d)\,c^3d^6ex+900\log(ex+d)\,c^3d^5e^2x^2+1200\log(ex+d)\,c^3d^4e^3x^3+9\ldots}{}$$

input
```
int((c*x^2+b*x)^3/(e*x+d)^7,x)
```

output

```
(60*log(d + e*x)*c**3*d**7 + 360*log(d + e*x)*c**3*d**6*e*x + 900*log(d +
e*x)*c**3*d**5*e**2*x**2 + 1200*log(d + e*x)*c**3*d**4*e**3*x**3 + 900*log
(d + e*x)*c**3*d**3*e**4*x**4 + 360*log(d + e*x)*c**3*d**2*e**5*x**5 + 60*
log(d + e*x)*c**3*d*e**6*x**6 - b**3*d**4*e**3 - 6*b**3*d**3*e**4*x - 15*b
**3*d**2*e**5*x**2 - 20*b**3*d*e**6*x**3 - 6*b**2*c*d**5*e**2 - 36*b**2*c*
d**4*e**3*x - 90*b**2*c*d**3*e**4*x**2 - 120*b**2*c*d**2*e**5*x**3 - 90*b*
*2*c*d*e**6*x**4 + 30*b*c**2*e**7*x**6 + 87*c**3*d**7 + 462*c**3*d**6*e*x
+ 975*c**3*d**5*e**2*x**2 + 1000*c**3*d**4*e**3*x**3 + 450*c**3*d**3*e**4*
x**4 - 60*c**3*d*e**6*x**6)/(60*d*e**7*(d**6 + 6*d**5*e*x + 15*d**4*e**2*x
**2 + 20*d**3*e**3*x**3 + 15*d**2*e**4*x**4 + 6*d*e**5*x**5 + e**6*x**6))
```

3.52 $\int \frac{(bx+cx^2)^3}{(d+ex)^8}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 230

$$\begin{aligned}\int \frac{(bx+cx^2)^3}{(d+ex)^8}\,dx = &-\frac{d^3(cd-be)^3}{7e^7(d+ex)^7}+\frac{d^2(cd-be)^2(2cd-be)}{2e^7(d+ex)^6}\\ &-\frac{3d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)}{5e^7(d+ex)^5}\\ &+\frac{(2cd-be)\left(10c^2d^2-10bcde+b^2e^2\right)}{4e^7(d+ex)^4}\\ &-\frac{c(5c^2d^2-5bcde+b^2e^2)}{e^7(d+ex)^3}+\frac{3c^2(2cd-be)}{2e^7(d+ex)^2}-\frac{c^3}{e^7(d+ex)}\end{aligned}$$

output
```
-1/7*d^3*(-b*e+c*d)^3/e^7/(e*x+d)^7+1/2*d^2*(-b*e+c*d)^2*(-b*e+2*c*d)/e^7/
(e*x+d)^6-3/5*d*(-b*e+c*d)*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^5+1/4
*(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*d^2)/e^7/(e*x+d)^4-c*(b^2*e^2-5*b
*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^3+3/2*c^2*(-b*e+2*c*d)/e^7/(e*x+d)^2-c^3/e^7
/(e*x+d)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.96

$$\int \frac{(bx+cx^2)^3}{(d+ex)^8}\,dx = -\frac{b^3e^3(d^3+7d^2ex+21de^2x^2+35e^3x^3)+4b^2ce^2(d^4+7d^3ex+21d^2e^2x^2+35de^3x^3+35e^4x^4)+10bc^2e(}{}$$

input
```
Integrate[(b*x + c*x^2)^3/(d + e*x)^8,x]
```

output
```
-1/140*(b^3*e^3*(d^3 + 7*d^2*e*x + 21*d*e^2*x^2 + 35*e^3*x^3) + 4*b^2*c*e^
2*(d^4 + 7*d^3*e*x + 21*d^2*e^2*x^2 + 35*d*e^3*x^3 + 35*e^4*x^4) + 10*b*c^
2*e*(d^5 + 7*d^4*e*x + 21*d^3*e^2*x^2 + 35*d^2*e^3*x^3 + 35*d*e^4*x^4 + 21
*e^5*x^5) + 20*c^3*(d^6 + 7*d^5*e*x + 21*d^4*e^2*x^2 + 35*d^3*e^3*x^3 + 35
*d^2*e^4*x^4 + 21*d*e^5*x^5 + 7*e^6*x^6))/(e^7*(d + e*x)^7)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^3}{(d+ex)^8}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{3c(b^2e^2-5bcde+5c^2d^2)}{e^6(d+ex)^4}+\frac{(2cd-be)(-b^2e^2+10bcde-10c^2d^2)}{e^6(d+ex)^5}+\frac{3d(cd-be)(b^2e^2-5bcde+5c^2d^2)}{e^6(d+ex)^6}-\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c(b^2e^2-5bcde+5c^2d^2)}{e^7(d+ex)^3}+\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{4e^7(d+ex)^4}-\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{5e^7(d+ex)^5}+\frac{3c^2(2cd-be)}{2e^7(d+ex)^2}-\frac{d^3(cd-be)^3}{7e^7(d+ex)^7}+\frac{d^2(cd-be)^2(2cd-be)}{2e^7(d+ex)^6}-\frac{c^3}{e^7(d+ex)}$$

input
```
Int[(b*x + c*x^2)^3/(d + e*x)^8,x]
```

output
```
-1/7*(d^3*(c*d - b*e)^3)/(e^7*(d + e*x)^7) + (d^2*(c*d - b*e)^2*(2*c*d - b
*e))/(2*e^7*(d + e*x)^6) - (3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e
^2))/(5*e^7*(d + e*x)^5) + ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e
^2))/(4*e^7*(d + e*x)^4) - (c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(e^7*(d +
 e*x)^3) + (3*c^2*(2*c*d - b*e))/(2*e^7*(d + e*x)^2) - c^3/(e^7*(d + e*x))
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.09

method	result
risch	$\frac{-\frac{c^3x^6}{e}-\frac{3c^2(be+2cd)x^5}{2e^2}-\frac{c\left(2b^2e^2+5bcde+10c^2d^2\right)x^4}{2e^3}-\frac{\left(b^3e^3+4d\,e^2b^2c+10d^2eb\,c^2+20d^3c^3\right)x^3}{4e^4}-\frac{3d\left(b^3e^3+4d\,e^2b^2c+10d^2eb\,c^2+20d^3\right.}{20e^5}}{(ex+d)^7}$
norman	$\frac{-\frac{c^3x^6}{e}-\frac{3\left(b\,c^2e+2d\,c^3\right)x^5}{2e^2}-\frac{\left(2e^2b^2c+5c^2deb+10c^3d^2\right)x^4}{2e^3}-\frac{\left(b^3e^3+4d\,e^2b^2c+10d^2eb\,c^2+20d^3c^3\right)x^3}{4e^4}-\frac{3d\left(b^3e^3+4d\,e^2b^2c+10d^2eb\,c^2+\right.}{20e^5}}{(ex+d)^7}$
default	$-\frac{c(b^2e^2-5bcde+5c^2d^2)}{e^7(ex+d)^3}-\frac{b^3e^3-12d\,e^2b^2c+30d^2eb\,c^2-20d^3c^3}{4e^7(ex+d)^4}+\frac{d^3(b^3e^3-3d\,e^2b^2c+3d^2eb\,c^2-d^3c^3)}{7e^7(ex+d)^7}+\frac{3d\left(b^3e^3-6d\,e\right.}{5}$
gosper	$-\frac{140x^6c^3e^6+210x^5b\,c^2e^6+420x^5c^3d\,e^5+140x^4b^2c\,e^6+350x^4b\,c^2d\,e^5+700x^4c^3d^2e^4+35x^3b^3e^6+140x^3b^2cd\,e^5+350x^3b\,c^2d^2}{}$
parallelrisch	$\frac{-140x^6c^3e^6-210x^5b\,c^2e^6-420x^5c^3d\,e^5-140x^4b^2c\,e^6-350x^4b\,c^2d\,e^5-700x^4c^3d^2e^4-35x^3b^3e^6-140x^3b^2cd\,e^5-350x^3b\,c^2d^2e}{}$
orering	$-\frac{(140x^6c^3e^6+210x^5b\,c^2e^6+420x^5c^3d\,e^5+140x^4b^2c\,e^6+350x^4b\,c^2d\,e^5+700x^4c^3d^2e^4+35x^3b^3e^6+140x^3b^2cd\,e^5+350x^3b\,c^2d}{}$

input
```
int((c*x^2+b*x)^3/(e*x+d)^8,x,method=_RETURNVERBOSE)
```

output
```
(-c^3*x^6/e-3/2*c^2*(b*e+2*c*d)/e^2*x^5-1/2*c*(2*b^2*e^2+5*b*c*d*e+10*c^2*
d^2)/e^3*x^4-1/4*(b^3*e^3+4*b^2*c*d*e^2+10*b*c^2*d^2*e+20*c^3*d^3)/e^4*x^3
-3/20*d*(b^3*e^3+4*b^2*c*d*e^2+10*b*c^2*d^2*e+20*c^3*d^3)/e^5*x^2-1/20*d^2
*(b^3*e^3+4*b^2*c*d*e^2+10*b*c^2*d^2*e+20*c^3*d^3)/e^6*x-1/140*d^3*(b^3*e^
3+4*b^2*c*d*e^2+10*b*c^2*d^2*e+20*c^3*d^3)/e^7)/(e*x+d)^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.45

$$\int \frac{(bx+cx^2)^3}{(d+ex)^8}\,dx =$$
$$-\frac{140\,c^3e^6x^6+20\,c^3d^6+10\,bc^2d^5e+4\,b^2cd^4e^2+b^3d^3e^3+210\,(2\,c^3de^5+bc^2e^6)x^5+70\,(10\,c^3d^2e^4+5\,bc^2}{140\,(e^{14}x^7+}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^8,x, algorithm="fricas")
```

output
```
-1/140*(140*c^3*e^6*x^6 + 20*c^3*d^6 + 10*b*c^2*d^5*e + 4*b^2*c*d^4*e^2 +
b^3*d^3*e^3 + 210*(2*c^3*d*e^5 + b*c^2*e^6)*x^5 + 70*(10*c^3*d^2*e^4 + 5*b
*c^2*d*e^5 + 2*b^2*c*e^6)*x^4 + 35*(20*c^3*d^3*e^3 + 10*b*c^2*d^2*e^4 + 4*
b^2*c*d*e^5 + b^3*e^6)*x^3 + 21*(20*c^3*d^4*e^2 + 10*b*c^2*d^3*e^3 + 4*b^2
*c*d^2*e^4 + b^3*d*e^5)*x^2 + 7*(20*c^3*d^5*e + 10*b*c^2*d^4*e^2 + 4*b^2*c
*d^3*e^3 + b^3*d^2*e^4)*x)/(e^14*x^7 + 7*d*e^13*x^6 + 21*d^2*e^12*x^5 + 35
*d^3*e^11*x^4 + 35*d^4*e^10*x^3 + 21*d^5*e^9*x^2 + 7*d^6*e^8*x + d^7*e^7)
```

Sympy [A] (verification not implemented)

Time = 55.92 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.57

$$\int \frac{(bx+cx^2)^3}{(d+ex)^8}\,dx$$
$$= \frac{-b^3d^3e^3 - 4b^2cd^4e^2 - 10bc^2d^5e - 20c^3d^6 - 140c^3e^6x^6 + x^5(-210bc^2e^6 - 420c^3de^5) + x^4(-140b^2ce^6 - 35}{140d^7e^7 + 9}$$

input
```
integrate((c*x**2+b*x)**3/(e*x+d)**8,x)
```

output
```
(-b**3*d**3*e**3 - 4*b**2*c*d**4*e**2 - 10*b*c**2*d**5*e - 20*c**3*d**6 -
140*c**3*e**6*x**6 + x**5*(-210*b*c**2*e**6 - 420*c**3*d*e**5) + x**4*(-14
0*b**2*c*e**6 - 350*b*c**2*d*e**5 - 700*c**3*d**2*e**4) + x**3*(-35*b**3*e
**6 - 140*b**2*c*d*e**5 - 350*b*c**2*d**2*e**4 - 700*c**3*d**3*e**3) + x**
2*(-21*b**3*d*e**5 - 84*b**2*c*d**2*e**4 - 210*b*c**2*d**3*e**3 - 420*c**3
*d**4*e**2) + x*(-7*b**3*d**2*e**4 - 28*b**2*c*d**3*e**3 - 70*b*c**2*d**4*
e**2 - 140*c**3*d**5*e))/(140*d**7*e**7 + 980*d**6*e**8*x + 2940*d**5*e**9
*x**2 + 4900*d**4*e**10*x**3 + 4900*d**3*e**11*x**4 + 2940*d**2*e**12*x**5
 + 980*d*e**13*x**6 + 140*e**14*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.45

$$\int \frac{(bx+cx^2)^3}{(d+ex)^8}\,dx = -\frac{140\,c^3e^6x^6+20\,c^3d^6+10\,bc^2d^5e+4\,b^2cd^4e^2+b^3d^3e^3+210\,(2\,c^3de^5+bc^2e^6)x^5+70\,(10\,c^3d^2e^4+5\,bc^2}{140\,(e^{14}x^7+}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^8,x, algorithm="maxima")
```

output
```
-1/140*(140*c^3*e^6*x^6 + 20*c^3*d^6 + 10*b*c^2*d^5*e + 4*b^2*c*d^4*e^2 +
b^3*d^3*e^3 + 210*(2*c^3*d*e^5 + b*c^2*e^6)*x^5 + 70*(10*c^3*d^2*e^4 + 5*b
*c^2*d*e^5 + 2*b^2*c*e^6)*x^4 + 35*(20*c^3*d^3*e^3 + 10*b*c^2*d^2*e^4 + 4*
b^2*c*d*e^5 + b^3*e^6)*x^3 + 21*(20*c^3*d^4*e^2 + 10*b*c^2*d^3*e^3 + 4*b^2
*c*d^2*e^4 + b^3*d*e^5)*x^2 + 7*(20*c^3*d^5*e + 10*b*c^2*d^4*e^2 + 4*b^2*c
*d^3*e^3 + b^3*d^2*e^4)*x)/(e^14*x^7 + 7*d*e^13*x^6 + 21*d^2*e^12*x^5 + 35
*d^3*e^11*x^4 + 35*d^4*e^10*x^3 + 21*d^5*e^9*x^2 + 7*d^6*e^8*x + d^7*e^7)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.23

$$\int \frac{(bx+cx^2)^3}{(d+ex)^8}\,dx = -\frac{140\,c^3e^6x^6+420\,c^3de^5x^5+210\,bc^2e^6x^5+700\,c^3d^2e^4x^4+350\,bc^2de^5x^4+140\,b^2ce^6x^4+700\,c^3d^3e^3x^3+}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^8,x, algorithm="giac")
```

output
```
-1/140*(140*c^3*e^6*x^6 + 420*c^3*d*e^5*x^5 + 210*b*c^2*e^6*x^5 + 700*c^3*
d^2*e^4*x^4 + 350*b*c^2*d*e^5*x^4 + 140*b^2*c*e^6*x^4 + 700*c^3*d^3*e^3*x^
3 + 350*b*c^2*d^2*e^4*x^3 + 140*b^2*c*d*e^5*x^3 + 35*b^3*e^6*x^3 + 420*c^3
*d^4*e^2*x^2 + 210*b*c^2*d^3*e^3*x^2 + 84*b^2*c*d^2*e^4*x^2 + 21*b^3*d*e^5
*x^2 + 140*c^3*d^5*e*x + 70*b*c^2*d^4*e^2*x + 28*b^2*c*d^3*e^3*x + 7*b^3*d
^2*e^4*x + 20*c^3*d^6 + 10*b*c^2*d^5*e + 4*b^2*c*d^4*e^2 + b^3*d^3*e^3)/((
e*x + d)^7*e^7)
```

Mupad [B] (verification not implemented)

Time = 8.77 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.37

$$\int \frac{(bx+cx^2)^3}{(d+ex)^8}\,dx =$$
$$-\frac{\frac{d^3\,\left(b^3\,e^3+4\,b^2\,c\,d\,e^2+10\,b\,c^2\,d^2\,e+20\,c^3\,d^3\right)}{140\,e^7}+\frac{x^3\,\left(b^3\,e^3+4\,b^2\,c\,d\,e^2+10\,b\,c^2\,d^2\,e+20\,c^3\,d^3\right)}{4\,e^4}+\frac{c^3\,x^6}{e}+\frac{3\,c^2\,x^5\,(b\,e+2\,c\,d)}{2\,e^2}+\frac{c\,x^4\,\left(2\,b^2\,e^2\cdots\right.}{\cdots}}{d^7+7\,d^6\,e\,x+21\,d^5\,e^2\,x^2+35\,d^4\,e^3\,x^3+35\,d^3\,e^4\cdots}$$

input
```
int((b*x + c*x^2)^3/(d + e*x)^8,x)
```

output
```
-((d^3*(b^3*e^3 + 20*c^3*d^3 + 10*b*c^2*d^2*e + 4*b^2*c*d*e^2))/(140*e^7)
+ (x^3*(b^3*e^3 + 20*c^3*d^3 + 10*b*c^2*d^2*e + 4*b^2*c*d*e^2))/(4*e^4) +
(c^3*x^6)/e + (3*c^2*x^5*(b*e + 2*c*d))/(2*e^2) + (c*x^4*(2*b^2*e^2 + 10*c
^2*d^2 + 5*b*c*d*e))/(2*e^3) + (3*d*x^2*(b^3*e^3 + 20*c^3*d^3 + 10*b*c^2*d
^2*e + 4*b^2*c*d*e^2))/(20*e^5) + (d^2*x*(b^3*e^3 + 20*c^3*d^3 + 10*b*c^2*
d^2*e + 4*b^2*c*d*e^2))/(20*e^6))/(d^7 + e^7*x^7 + 7*d*e^6*x^6 + 21*d^5*e^
2*x^2 + 35*d^4*e^3*x^3 + 35*d^3*e^4*x^4 + 21*d^2*e^5*x^5 + 7*d^6*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.24

$$\int \frac{(bx+cx^2)^3}{(d+ex)^8}\,dx$$
$$=\frac{20c^3e^6x^7-210b\,c^2d\,e^5x^5-140b^2cd\,e^5x^4-350b\,c^2d^2e^4x^4-35b^3d\,e^5x^3-140b^2c\,d^2e^4x^3-350b\,c^2d^3e^3x^3\cdots}{140d\,e^6\left(e^7x^7+7d\,e^6x^6+21d^2e^5x^5+\cdots\right.}$$

input
```
int((c*x^2+b*x)^3/(e*x+d)^8,x)
```

output
```
( - b**3*d**4*e**2 - 7*b**3*d**3*e**3*x - 21*b**3*d**2*e**4*x**2 - 35*b**3
*d*e**5*x**3 - 4*b**2*c*d**5*e - 28*b**2*c*d**4*e**2*x - 84*b**2*c*d**3*e*
*3*x**2 - 140*b**2*c*d**2*e**4*x**3 - 140*b**2*c*d*e**5*x**4 - 10*b*c**2*d
**6 - 70*b*c**2*d**5*e*x - 210*b*c**2*d**4*e**2*x**2 - 350*b*c**2*d**3*e**
3*x**3 - 350*b*c**2*d**2*e**4*x**4 - 210*b*c**2*d*e**5*x**5 + 20*c**3*e**6
*x**7)/(140*d*e**6*(d**7 + 7*d**6*e*x + 21*d**5*e**2*x**2 + 35*d**4*e**3*x
**3 + 35*d**3*e**4*x**4 + 21*d**2*e**5*x**5 + 7*d*e**6*x**6 + e**7*x**7))
```

3.53 $\int \frac{(bx+cx^2)^3}{(d+ex)^9} dx$

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Optimal result

Integrand size = 19, antiderivative size = 231

$$\begin{aligned}\int \frac{(bx+cx^2)^3}{(d+ex)^9} dx = & -\frac{d^3(cd-be)^3}{8e^7(d+ex)^8} + \frac{3d^2(cd-be)^2(2cd-be)}{7e^7(d+ex)^7} \\ & - \frac{d(cd-be)\,(5c^2d^2-5bcde+b^2e^2)}{2e^7(d+ex)^6} \\ & + \frac{(2cd-be)\,(10c^2d^2-10bcde+b^2e^2)}{5e^7(d+ex)^5} \\ & - \frac{3c(5c^2d^2-5bcde+b^2e^2)}{4e^7(d+ex)^4} + \frac{c^2(2cd-be)}{e^7(d+ex)^3} - \frac{c^3}{2e^7(d+ex)^2}\end{aligned}$$

output
```
-1/8*d^3*(-b*e+c*d)^3/e^7/(e*x+d)^8+3/7*d^2*(-b*e+c*d)^2*(-b*e+2*c*d)/e^7/
(e*x+d)^7-1/2*d*(-b*e+c*d)*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^6+1/5
*(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*d^2)/e^7/(e*x+d)^5-3/4*c*(b^2*e^2
-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^4+c^2*(-b*e+2*c*d)/e^7/(e*x+d)^3-1/2*c^3
/e^7/(e*x+d)^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.96

$$\int \frac{(bx+cx^2)^3}{(d+ex)^9}\,dx =$$
$$-\frac{b^3e^3(d^3+8d^2ex+28de^2x^2+56e^3x^3)+3b^2ce^2(d^4+8d^3ex+28d^2e^2x^2+56de^3x^3+70e^4x^4)+5bc^2e(d}{}$$

input
```
Integrate[(b*x + c*x^2)^3/(d + e*x)^9,x]
```

output
```
-1/280*(b^3*e^3*(d^3 + 8*d^2*e*x + 28*d*e^2*x^2 + 56*e^3*x^3) + 3*b^2*c*e^
2*(d^4 + 8*d^3*e*x + 28*d^2*e^2*x^2 + 56*d*e^3*x^3 + 70*e^4*x^4) + 5*b*c^2
*e*(d^5 + 8*d^4*e*x + 28*d^3*e^2*x^2 + 56*d^2*e^3*x^3 + 70*d*e^4*x^4 + 56*
e^5*x^5) + 5*c^3*(d^6 + 8*d^5*e*x + 28*d^4*e^2*x^2 + 56*d^3*e^3*x^3 + 70*d
^2*e^4*x^4 + 56*d*e^5*x^5 + 28*e^6*x^6))/(e^7*(d + e*x)^8)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^3}{(d+ex)^9}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{3c(b^2e^2-5bcde+5c^2d^2)}{e^6(d+ex)^5}+\frac{(2cd-be)(-b^2e^2+10bcde-10c^2d^2)}{e^6(d+ex)^6}+\frac{3d(cd-be)(b^2e^2-5bcde+5c^2d^2)}{e^6(d+ex)^7}-\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3c(b^2e^2-5bcde+5c^2d^2)}{4e^7(d+ex)^4}+\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{5e^7(d+ex)^5}-\frac{d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{2e^7(d+ex)^6}+\frac{c^2(2cd-be)}{e^7(d+ex)^3}-\frac{d^3(cd-be)^3}{8e^7(d+ex)^8}+\frac{3d^2(cd-be)^2(2cd-be)}{7e^7(d+ex)^7}-\frac{c^3}{2e^7(d+ex)^2}$$

input
```
Int[(b*x + c*x^2)^3/(d + e*x)^9,x]
```

output
```
-1/8*(d^3*(c*d - b*e)^3)/(e^7*(d + e*x)^8) + (3*d^2*(c*d - b*e)^2*(2*c*d -
 b*e))/(7*e^7*(d + e*x)^7) - (d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e
^2))/(2*e^7*(d + e*x)^6) + ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e
^2))/(5*e^7*(d + e*x)^5) - (3*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(4*e^7*
(d + e*x)^4) + (c^2*(2*c*d - b*e))/(e^7*(d + e*x)^3) - c^3/(2*e^7*(d + e*x
)^2)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.08

method	result
risch	$\frac{-\frac{c^3x^6}{2e}-\frac{c^2(be+cd)x^5}{e^2}-\frac{c\left(3b^2e^2+5bcde+5c^2d^2\right)x^4}{4e^3}-\frac{\left(b^3e^3+3d\,e^2b^2c+5d^2eb\,c^2+5d^3c^3\right)x^3}{5e^4}-\frac{d\left(b^3e^3+3d\,e^2b^2c+5d^2eb\,c^2+5d^3c^3\right)x^2}{10e^5}-}{(ex+d)^8}$
norman	$\frac{-\frac{c^3x^6}{2e}-\frac{\left(b\,c^2e^2+de\,c^3\right)x^5}{e^3}-\frac{\left(3e^3b^2c+5d\,e^2b\,c^2+5d^2e\,c^3\right)x^4}{4e^4}-\frac{\left(e^4b^3+3d\,e^3b^2c+5d^2e^2b\,c^2+5d^3e\,c^3\right)x^3}{5e^5}-\frac{d\left(e^4b^3+3d\,e^3b^2c+5d^2e^2b\,c^2\right.}{10e^6}}{(ex+d)^8}$
default	$-\frac{c^2(be-2cd)}{e^7(ex+d)^3}-\frac{3c(b^2e^2-5bcde+5c^2d^2)}{4e^7(ex+d)^4}-\frac{3d^2(b^3e^3-4d\,e^2b^2c+5d^2eb\,c^2-2d^3c^3)}{7e^7(ex+d)^7}-\frac{b^3e^3-12d\,e^2b^2c+30d^2eb\,c^2-20d^3c}{5e^7(ex+d)^5}$
gosper	$-\frac{140x^6c^3e^6+280x^5b\,c^2e^6+280x^5c^3d\,e^5+210x^4b^2c\,e^6+350x^4b\,c^2d\,e^5+350x^4c^3d^2e^4+56x^3b^3e^6+168x^3b^2cd\,e^5+280x^3b\,c^2d^2}{}$
parallelrisch	$\frac{-140c^3x^6e^7-280b\,c^2e^7x^5-280c^3d\,e^6x^5-210b^2c\,e^7x^4-350b\,c^2d\,e^6x^4-350c^3d^2e^5x^4-56b^3e^7x^3-168b^2cd\,e^6x^3-280b\,c^2d^2e^5x}{}$
orering	$-\frac{(140x^6c^3e^6+280x^5b\,c^2e^6+280x^5c^3d\,e^5+210x^4b^2c\,e^6+350x^4b\,c^2d\,e^5+350x^4c^3d^2e^4+56x^3b^3e^6+168x^3b^2cd\,e^5+280x^3b\,c^2d}{}$

input
```
int((c*x^2+b*x)^3/(e*x+d)^9,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*c^3*x^6/e-1/e^2*c^2*(b*e+c*d)*x^5-1/4*c/e^3*(3*b^2*e^2+5*b*c*d*e+5*c
^2*d^2)*x^4-1/5/e^4*(b^3*e^3+3*b^2*c*d*e^2+5*b*c^2*d^2*e+5*c^3*d^3)*x^3-1/
10*d/e^5*(b^3*e^3+3*b^2*c*d*e^2+5*b*c^2*d^2*e+5*c^3*d^3)*x^2-1/35*d^2/e^6*
(b^3*e^3+3*b^2*c*d*e^2+5*b*c^2*d^2*e+5*c^3*d^3)*x-1/280*d^3/e^7*(b^3*e^3+3
*b^2*c*d*e^2+5*b*c^2*d^2*e+5*c^3*d^3))/(e*x+d)^8
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.49

$$\int \frac{(bx+cx^2)^3}{(d+ex)^9}\,dx =$$
$$-\frac{140\,c^3e^6x^6+5\,c^3d^6+5\,bc^2d^5e+3\,b^2cd^4e^2+b^3d^3e^3+280\,(c^3de^5+bc^2e^6)x^5+70\,(5\,c^3d^2e^4+5\,bc^2de^5-}{280\,(e^{15}x^8+8\,de^{14}x^7+2}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^9,x, algorithm="fricas")
```

output
```
-1/280*(140*c^3*e^6*x^6 + 5*c^3*d^6 + 5*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + b^
3*d^3*e^3 + 280*(c^3*d*e^5 + b*c^2*e^6)*x^5 + 70*(5*c^3*d^2*e^4 + 5*b*c^2*
d*e^5 + 3*b^2*c*e^6)*x^4 + 56*(5*c^3*d^3*e^3 + 5*b*c^2*d^2*e^4 + 3*b^2*c*d
*e^5 + b^3*e^6)*x^3 + 28*(5*c^3*d^4*e^2 + 5*b*c^2*d^3*e^3 + 3*b^2*c*d^2*e^
4 + b^3*d*e^5)*x^2 + 8*(5*c^3*d^5*e + 5*b*c^2*d^4*e^2 + 3*b^2*c*d^3*e^3 +
b^3*d^2*e^4)*x)/(e^15*x^8 + 8*d*e^14*x^7 + 28*d^2*e^13*x^6 + 56*d^3*e^12*x
^5 + 70*d^4*e^11*x^4 + 56*d^5*e^10*x^3 + 28*d^6*e^9*x^2 + 8*d^7*e^8*x + d^
8*e^7)
```

Sympy [A] (verification not implemented)

Time = 149.15 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.62

$$\int \frac{(bx + cx^2)^3}{(d + ex)^9}\,dx$$
$$= \frac{-b^3d^3e^3 - 3b^2cd^4e^2 - 5bc^2d^5e - 5c^3d^6 - 140c^3e^6x^6 + x^5(-280bc^2e^6 - 280c^3de^5) + x^4(-210b^2ce^6 - 350b}{280d^8e^7 + 2240d^7e^8x +}$$

input
```
integrate((c*x**2+b*x)**3/(e*x+d)**9,x)
```

output
```
(-b**3*d**3*e**3 - 3*b**2*c*d**4*e**2 - 5*b*c**2*d**5*e - 5*c**3*d**6 - 14
0*c**3*e**6*x**6 + x**5*(-280*b*c**2*e**6 - 280*c**3*d*e**5) + x**4*(-210*
b**2*c*e**6 - 350*b*c**2*d*e**5 - 350*c**3*d**2*e**4) + x**3*(-56*b**3*e**
6 - 168*b**2*c*d*e**5 - 280*b*c**2*d**2*e**4 - 280*c**3*d**3*e**3) + x**2*
(-28*b**3*d*e**5 - 84*b**2*c*d**2*e**4 - 140*b*c**2*d**3*e**3 - 140*c**3*d
**4*e**2) + x*(-8*b**3*d**2*e**4 - 24*b**2*c*d**3*e**3 - 40*b*c**2*d**4*e*
*2 - 40*c**3*d**5*e))/(280*d**8*e**7 + 2240*d**7*e**8*x + 7840*d**6*e**9*x
**2 + 15680*d**5*e**10*x**3 + 19600*d**4*e**11*x**4 + 15680*d**3*e**12*x**
5 + 7840*d**2*e**13*x**6 + 2240*d*e**14*x**7 + 280*e**15*x**8)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.49

$$\int \frac{(bx+cx^2)^3}{(d+ex)^9}\,dx =$$
$$-\frac{140\,c^3e^6x^6+5\,c^3d^6+5\,bc^2d^5e+3\,b^2cd^4e^2+b^3d^3e^3+280\,(c^3de^5+bc^2e^6)x^5+70\,(5\,c^3d^2e^4+5\,bc^2de^5 -}{280\,(e^{15}x^8+8\,de^{14}x^7+2}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^9,x, algorithm="maxima")
```

output
```
-1/280*(140*c^3*e^6*x^6 + 5*c^3*d^6 + 5*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + b^
3*d^3*e^3 + 280*(c^3*d*e^5 + b*c^2*e^6)*x^5 + 70*(5*c^3*d^2*e^4 + 5*b*c^2*
d*e^5 + 3*b^2*c*e^6)*x^4 + 56*(5*c^3*d^3*e^3 + 5*b*c^2*d^2*e^4 + 3*b^2*c*d
*e^5 + b^3*e^6)*x^3 + 28*(5*c^3*d^4*e^2 + 5*b*c^2*d^3*e^3 + 3*b^2*c*d^2*e^
4 + b^3*d*e^5)*x^2 + 8*(5*c^3*d^5*e + 5*b*c^2*d^4*e^2 + 3*b^2*c*d^3*e^3 +
b^3*d^2*e^4)*x)/(e^15*x^8 + 8*d*e^14*x^7 + 28*d^2*e^13*x^6 + 56*d^3*e^12*x
^5 + 70*d^4*e^11*x^4 + 56*d^5*e^10*x^3 + 28*d^6*e^9*x^2 + 8*d^7*e^8*x + d^
8*e^7)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.23

$$\int \frac{(bx+cx^2)^3}{(d+ex)^9}\,dx =$$
$$-\frac{140\,c^3e^6x^6+280\,c^3de^5x^5+280\,bc^2e^6x^5+350\,c^3d^2e^4x^4+350\,bc^2de^5x^4+210\,b^2ce^6x^4+280\,c^3d^3e^3x^3+}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^9,x, algorithm="giac")
```

output
```
-1/280*(140*c^3*e^6*x^6 + 280*c^3*d*e^5*x^5 + 280*b*c^2*e^6*x^5 + 350*c^3*
d^2*e^4*x^4 + 350*b*c^2*d*e^5*x^4 + 210*b^2*c*e^6*x^4 + 280*c^3*d^3*e^3*x^
3 + 280*b*c^2*d^2*e^4*x^3 + 168*b^2*c*d*e^5*x^3 + 56*b^3*e^6*x^3 + 140*c^3
*d^4*e^2*x^2 + 140*b*c^2*d^3*e^3*x^2 + 84*b^2*c*d^2*e^4*x^2 + 28*b^3*d*e^5
*x^2 + 40*c^3*d^5*e*x + 40*b*c^2*d^4*e^2*x + 24*b^2*c*d^3*e^3*x + 8*b^3*d^
2*e^4*x + 5*c^3*d^6 + 5*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 + b^3*d^3*e^3)/((e*x
 + d)^8*e^7)
```

Mupad [B] (verification not implemented)

Time = 8.76 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.41

$$\int \frac{(bx+cx^2)^3}{(d+ex)^9}\,dx =$$
$$-\frac{\frac{d^3\,\left(b^3\,e^3+3\,b^2\,c\,d\,e^2+5\,b\,c^2\,d^2\,e+5\,c^3\,d^3\right)}{280\,e^7}+\frac{x^3\,\left(b^3\,e^3+3\,b^2\,c\,d\,e^2+5\,b\,c^2\,d^2\,e+5\,c^3\,d^3\right)}{5\,e^4}+\frac{c^3\,x^6}{2\,e}+\frac{c^2\,x^5\,\left(b\,e+c\,d\right)}{e^2}+\frac{c\,x^4\,\left(3\,b^2\,e^2+5\,b\,c\,d\,e\dots\right.}{4\,e^3}}{d^8+8\,d^7\,e\,x+28\,d^6\,e^2\,x^2+56\,d^5\,e^3\,x^3+70\,d^4\,e^4\,x^4+56\,d^3\,e\dots}$$

input
```
int((b*x + c*x^2)^3/(d + e*x)^9,x)
```

output
```
-((d^3*(b^3*e^3 + 5*c^3*d^3 + 5*b*c^2*d^2*e + 3*b^2*c*d*e^2))/(280*e^7) +
(x^3*(b^3*e^3 + 5*c^3*d^3 + 5*b*c^2*d^2*e + 3*b^2*c*d*e^2))/(5*e^4) + (c^3
*x^6)/(2*e) + (c^2*x^5*(b*e + c*d))/e^2 + (c*x^4*(3*b^2*e^2 + 5*c^2*d^2 +
5*b*c*d*e))/(4*e^3) + (d*x^2*(b^3*e^3 + 5*c^3*d^3 + 5*b*c^2*d^2*e + 3*b^2*
c*d*e^2))/(10*e^5) + (d^2*x*(b^3*e^3 + 5*c^3*d^3 + 5*b*c^2*d^2*e + 3*b^2*c
*d*e^2))/(35*e^6))/(d^8 + e^8*x^8 + 8*d*e^7*x^7 + 28*d^6*e^2*x^2 + 56*d^5*
e^3*x^3 + 70*d^4*e^4*x^4 + 56*d^3*e^5*x^5 + 28*d^2*e^6*x^6 + 8*d^7*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.57

$$\int \frac{(bx+cx^2)^3}{(d+ex)^9}\,dx$$
$$=\frac{-140c^3e^6x^6-280b\,c^2e^6x^5-280c^3d\,e^5x^5-210b^2c\,e^6x^4-350b\,c^2d\,e^5x^4-350c^3d^2e^4x^4-56b^3e^6x^3-168\dots}{280\dots}$$

input
```
int((c*x^2+b*x)^3/(e*x+d)^9,x)
```

output
```
( - b**3*d**3*e**3 - 8*b**3*d**2*e**4*x - 28*b**3*d*e**5*x**2 - 56*b**3*e*
*6*x**3 - 3*b**2*c*d**4*e**2 - 24*b**2*c*d**3*e**3*x - 84*b**2*c*d**2*e**4
*x**2 - 168*b**2*c*d*e**5*x**3 - 210*b**2*c*e**6*x**4 - 5*b*c**2*d**5*e -
40*b*c**2*d**4*e**2*x - 140*b*c**2*d**3*e**3*x**2 - 280*b*c**2*d**2*e**4*x
**3 - 350*b*c**2*d*e**5*x**4 - 280*b*c**2*e**6*x**5 - 5*c**3*d**6 - 40*c**
3*d**5*e*x - 140*c**3*d**4*e**2*x**2 - 280*c**3*d**3*e**3*x**3 - 350*c**3*
d**2*e**4*x**4 - 280*c**3*d*e**5*x**5 - 140*c**3*e**6*x**6)/(280*e**7*(d**
8 + 8*d**7*e*x + 28*d**6*e**2*x**2 + 56*d**5*e**3*x**3 + 70*d**4*e**4*x**4
 + 56*d**3*e**5*x**5 + 28*d**2*e**6*x**6 + 8*d*e**7*x**7 + e**8*x**8))
```

3.54 $\int \frac{(bx+cx^2)^3}{(d+ex)^{10}}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 234

$$\begin{aligned}\int \frac{(bx+cx^2)^3}{(d+ex)^{10}}\,dx = &-\frac{d^3(cd-be)^3}{9e^7(d+ex)^9}+\frac{3d^2(cd-be)^2(2cd-be)}{8e^7(d+ex)^8}\\&-\frac{3d(cd-be)\,(5c^2d^2-5bcde+b^2e^2)}{7e^7(d+ex)^7}\\&+\frac{(2cd-be)\,(10c^2d^2-10bcde+b^2e^2)}{6e^7(d+ex)^6}\\&-\frac{3c(5c^2d^2-5bcde+b^2e^2)}{5e^7(d+ex)^5}+\frac{3c^2(2cd-be)}{4e^7(d+ex)^4}-\frac{c^3}{3e^7(d+ex)^3}\end{aligned}$$

output
```
-1/9*d^3*(-b*e+c*d)^3/e^7/(e*x+d)^9+3/8*d^2*(-b*e+c*d)^2*(-b*e+2*c*d)/e^7/
(e*x+d)^8-3/7*d*(-b*e+c*d)*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^7+1/6
*(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*d^2)/e^7/(e*x+d)^6-3/5*c*(b^2*e^2
-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^5+3/4*c^2*(-b*e+2*c*d)/e^7/(e*x+d)^4-1/3
*c^3/e^7/(e*x+d)^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.95

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{10}}\,dx = -\frac{5b^3e^3(d^3+9d^2ex+36de^2x^2+84e^3x^3)+12b^2ce^2(d^4+9d^3ex+36d^2e^2x^2+84de^3x^3+126e^4x^4)+15b}{}$$

input
```
Integrate[(b*x + c*x^2)^3/(d + e*x)^10,x]
```

output
```
-1/2520*(5*b^3*e^3*(d^3 + 9*d^2*e*x + 36*d*e^2*x^2 + 84*e^3*x^3) + 12*b^2*
c*e^2*(d^4 + 9*d^3*e*x + 36*d^2*e^2*x^2 + 84*d*e^3*x^3 + 126*e^4*x^4) + 15
*b*c^2*e*(d^5 + 9*d^4*e*x + 36*d^3*e^2*x^2 + 84*d^2*e^3*x^3 + 126*d*e^4*x^
4 + 126*e^5*x^5) + 10*c^3*(d^6 + 9*d^5*e*x + 36*d^4*e^2*x^2 + 84*d^3*e^3*x
^3 + 126*d^2*e^4*x^4 + 126*d*e^5*x^5 + 84*e^6*x^6))/(e^7*(d + e*x)^9)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{10}}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{3c(b^2e^2-5bcde+5c^2d^2)}{e^6(d+ex)^6}+\frac{(2cd-be)\,(-b^2e^2+10bcde-10c^2d^2)}{e^6(d+ex)^7}+\frac{3d(cd-be)\,(b^2e^2-5bcde+5c^2d^2)}{e^6(d+ex)^8}-\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3c(b^2e^2-5bcde+5c^2d^2)}{5e^7(d+ex)^5}+\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{6e^7(d+ex)^6}-\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{7e^7(d+ex)^7}+\frac{3c^2(2cd-be)}{4e^7(d+ex)^4}-\frac{d^3(cd-be)^3}{9e^7(d+ex)^9}+\frac{3d^2(cd-be)^2(2cd-be)}{8e^7(d+ex)^8}-\frac{c^3}{3e^7(d+ex)^3}$$

input
```
Int[(b*x + c*x^2)^3/(d + e*x)^10,x]
```

output
```
-1/9*(d^3*(c*d - b*e)^3)/(e^7*(d + e*x)^9) + (3*d^2*(c*d - b*e)^2*(2*c*d -
 b*e))/(8*e^7*(d + e*x)^8) - (3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2
*e^2))/(7*e^7*(d + e*x)^7) + ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2
*e^2))/(6*e^7*(d + e*x)^6) - (3*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2))/(5*e^
7*(d + e*x)^5) + (3*c^2*(2*c*d - b*e))/(4*e^7*(d + e*x)^4) - c^3/(3*e^7*(d
 + e*x)^3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.09

method	result
risch	$\frac{-\frac{c^3x^6}{3e}-\frac{c^2(3be+2cd)x^5}{4e^2}-\frac{c\left(12b^2e^2+15bcde+10c^2d^2\right)x^4}{20e^3}-\frac{\left(5b^3e^3+12d\,e^2b^2c+15d^2eb\,c^2+10d^3c^3\right)x^3}{30e^4}-\frac{d\left(5b^3e^3+12d\,e^2b^2c+15d^2eb\,c^2-\right.}{70e^5}}{(ex+d)^9}$
default	$-\frac{c^3}{3e^7(ex+d)^3}-\frac{3c^2(be-2cd)}{4e^7(ex+d)^4}+\frac{d^3(b^3e^3-3d\,e^2b^2c+3d^2eb\,c^2-d^3c^3)}{9e^7(ex+d)^9}+\frac{3d(b^3e^3-6d\,e^2b^2c+10d^2eb\,c^2-5d^3c^3)}{7e^7(ex+d)^7}-\frac{3c(b}{}$
gosper	$-\frac{840x^6c^3e^6+1890x^5b\,c^2e^6+1260x^5c^3d\,e^5+1512x^4b^2c\,e^6+1890x^4b\,c^2d\,e^5+1260x^4c^3d^2e^4+420x^3b^3e^6+1008x^3b^2cd\,e^5+126}{}$
norman	$\frac{-\frac{c^3x^6}{3e}-\frac{\left(3e^3b\,c^2+2d\,e^2c^3\right)x^5}{4e^4}-\frac{\left(12e^4b^2c+15d\,e^3b\,c^2+10d^2e^2c^3\right)x^4}{20e^5}-\frac{\left(5b^3e^5+12b^2cd\,e^4+15b\,c^2d^2e^3+10c^3d^3e^2\right)x^3}{30e^6}-\frac{d\left(5b^3e^5+12b^2c\right.}{}}{(ex+d)}$
parallelrisch	$\frac{-840c^3x^6e^8-1890b\,c^2e^8x^5-1260c^3d\,e^7x^5-1512b^2c\,e^8x^4-1890b\,c^2d\,e^7x^4-1260c^3d^2e^6x^4-420b^3e^8x^3-1008b^2cd\,e^7x^3-1260}{}$
orering	$-\frac{(840x^6c^3e^6+1890x^5b\,c^2e^6+1260x^5c^3d\,e^5+1512x^4b^2c\,e^6+1890x^4b\,c^2d\,e^5+1260x^4c^3d^2e^4+420x^3b^3e^6+1008x^3b^2cd\,e^5+12}{}$

input
```
int((c*x^2+b*x)^3/(e*x+d)^10,x,method=_RETURNVERBOSE)
```

output
```
(-1/3*c^3*x^6/e-1/4*c^2/e^2*(3*b*e+2*c*d)*x^5-1/20*c/e^3*(12*b^2*e^2+15*b*
c*d*e+10*c^2*d^2)*x^4-1/30/e^4*(5*b^3*e^3+12*b^2*c*d*e^2+15*b*c^2*d^2*e+10
*c^3*d^3)*x^3-1/70*d/e^5*(5*b^3*e^3+12*b^2*c*d*e^2+15*b*c^2*d^2*e+10*c^3*d
^3)*x^2-1/280*d^2/e^6*(5*b^3*e^3+12*b^2*c*d*e^2+15*b*c^2*d^2*e+10*c^3*d^3)
*x-1/2520*d^3/e^7*(5*b^3*e^3+12*b^2*c*d*e^2+15*b*c^2*d^2*e+10*c^3*d^3))/(e
*x+d)^9
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.54

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{10}}\,dx =$$
$$-\frac{840\,c^3e^6x^6+10\,c^3d^6+15\,bc^2d^5e+12\,b^2cd^4e^2+5\,b^3d^3e^3+630\,(2\,c^3de^5+3\,bc^2e^6)x^5+126\,(10\,c^3d^2e^4+}{2520\,(e^{16}x^9+9\,de^{15}}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^10,x, algorithm="fricas")
```

output
```
-1/2520*(840*c^3*e^6*x^6 + 10*c^3*d^6 + 15*b*c^2*d^5*e + 12*b^2*c*d^4*e^2
+ 5*b^3*d^3*e^3 + 630*(2*c^3*d*e^5 + 3*b*c^2*e^6)*x^5 + 126*(10*c^3*d^2*e^
4 + 15*b*c^2*d*e^5 + 12*b^2*c*e^6)*x^4 + 84*(10*c^3*d^3*e^3 + 15*b*c^2*d^2
*e^4 + 12*b^2*c*d*e^5 + 5*b^3*e^6)*x^3 + 36*(10*c^3*d^4*e^2 + 15*b*c^2*d^3
*e^3 + 12*b^2*c*d^2*e^4 + 5*b^3*d*e^5)*x^2 + 9*(10*c^3*d^5*e + 15*b*c^2*d^
4*e^2 + 12*b^2*c*d^3*e^3 + 5*b^3*d^2*e^4)*x)/(e^16*x^9 + 9*d*e^15*x^8 + 36
*d^2*e^14*x^7 + 84*d^3*e^13*x^6 + 126*d^4*e^12*x^5 + 126*d^5*e^11*x^4 + 84
*d^6*e^10*x^3 + 36*d^7*e^9*x^2 + 9*d^8*e^8*x + d^9*e^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{10}}\,dx = \text{Timed out}$$

input
```
integrate((c*x**2+b*x)**3/(e*x+d)**10,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.54

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{10}}\,dx = -\frac{840\,c^3e^6x^6+10\,c^3d^6+15\,bc^2d^5e+12\,b^2cd^4e^2+5\,b^3d^3e^3+630\,(2\,c^3de^5+3\,bc^2e^6)x^5+126\,(10\,c^3d^2e^4+\cdots}{2520\,(e^{16}x^9+9\,de^{15}\cdots}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^10,x, algorithm="maxima")
```

output
```
-1/2520*(840*c^3*e^6*x^6 + 10*c^3*d^6 + 15*b*c^2*d^5*e + 12*b^2*c*d^4*e^2
+ 5*b^3*d^3*e^3 + 630*(2*c^3*d*e^5 + 3*b*c^2*e^6)*x^5 + 126*(10*c^3*d^2*e^
4 + 15*b*c^2*d*e^5 + 12*b^2*c*e^6)*x^4 + 84*(10*c^3*d^3*e^3 + 15*b*c^2*d^2
*e^4 + 12*b^2*c*d*e^5 + 5*b^3*e^6)*x^3 + 36*(10*c^3*d^4*e^2 + 15*b*c^2*d^3
*e^3 + 12*b^2*c*d^2*e^4 + 5*b^3*d*e^5)*x^2 + 9*(10*c^3*d^5*e + 15*b*c^2*d^
4*e^2 + 12*b^2*c*d^3*e^3 + 5*b^3*d^2*e^4)*x)/(e^16*x^9 + 9*d*e^15*x^8 + 36
*d^2*e^14*x^7 + 84*d^3*e^13*x^6 + 126*d^4*e^12*x^5 + 126*d^5*e^11*x^4 + 84
*d^6*e^10*x^3 + 36*d^7*e^9*x^2 + 9*d^8*e^8*x + d^9*e^7)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.22

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{10}}\,dx = -\frac{840\,c^3e^6x^6+1260\,c^3de^5x^5+1890\,bc^2e^6x^5+1260\,c^3d^2e^4x^4+1890\,bc^2de^5x^4+1512\,b^2ce^6x^4+840\,c^3d^3e}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^10,x, algorithm="giac")
```

output
```
-1/2520*(840*c^3*e^6*x^6 + 1260*c^3*d*e^5*x^5 + 1890*b*c^2*e^6*x^5 + 1260*
c^3*d^2*e^4*x^4 + 1890*b*c^2*d*e^5*x^4 + 1512*b^2*c*e^6*x^4 + 840*c^3*d^3*
e^3*x^3 + 1260*b*c^2*d^2*e^4*x^3 + 1008*b^2*c*d*e^5*x^3 + 420*b^3*e^6*x^3
+ 360*c^3*d^4*e^2*x^2 + 540*b*c^2*d^3*e^3*x^2 + 432*b^2*c*d^2*e^4*x^2 + 18
0*b^3*d*e^5*x^2 + 90*c^3*d^5*e*x + 135*b*c^2*d^4*e^2*x + 108*b^2*c*d^3*e^3
*x + 45*b^3*d^2*e^4*x + 10*c^3*d^6 + 15*b*c^2*d^5*e + 12*b^2*c*d^4*e^2 + 5
*b^3*d^3*e^3)/((e*x + d)^9*e^7)
```

Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.47

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{10}}\,dx = -\frac{\frac{d^3\,\left(5\,b^3\,e^3+12\,b^2\,c\,d\,e^2+15\,b\,c^2\,d^2\,e+10\,c^3\,d^3\right)}{2520\,e^7}+\frac{x^3\,\left(5\,b^3\,e^3+12\,b^2\,c\,d\,e^2+15\,b\,c^2\,d^2\,e+10\,c^3\,d^3\right)}{30\,e^4}+\frac{c^3\,x^6}{3\,e}+\frac{c^2\,x^5\,\left(3\,b\,e+2\,c\,d\right)}{4\,e^2}+\frac{c\,x^4\,(1}{}}{d^9+9\,d^8\,e\,x+36\,d^7\,e^2\,x^2+84\,d^6\,e^3\,x^3+126\,d^5\,e^4\,x^4+126}$$

input
```
int((b*x + c*x^2)^3/(d + e*x)^10,x)
```

output
```
-((d^3*(5*b^3*e^3 + 10*c^3*d^3 + 15*b*c^2*d^2*e + 12*b^2*c*d*e^2))/(2520*e
^7) + (x^3*(5*b^3*e^3 + 10*c^3*d^3 + 15*b*c^2*d^2*e + 12*b^2*c*d*e^2))/(30
*e^4) + (c^3*x^6)/(3*e) + (c^2*x^5*(3*b*e + 2*c*d))/(4*e^2) + (c*x^4*(12*b
^2*e^2 + 10*c^2*d^2 + 15*b*c*d*e))/(20*e^3) + (d*x^2*(5*b^3*e^3 + 10*c^3*d
^3 + 15*b*c^2*d^2*e + 12*b^2*c*d*e^2))/(70*e^5) + (d^2*x*(5*b^3*e^3 + 10*c
^3*d^3 + 15*b*c^2*d^2*e + 12*b^2*c*d*e^2))/(280*e^6))/(d^9 + e^9*x^9 + 9*d
*e^8*x^8 + 36*d^7*e^2*x^2 + 84*d^6*e^3*x^3 + 126*d^5*e^4*x^4 + 126*d^4*e^5
*x^5 + 84*d^3*e^6*x^6 + 36*d^2*e^7*x^7 + 9*d^8*e*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.59

$$\int \frac{(bx + cx^2)^3}{(d + ex)^{10}}\, dx$$
$$= \frac{-840c^3e^6x^6 - 1890b\,c^2e^6x^5 - 1260c^3d\,e^5x^5 - 1512b^2c\,e^6x^4 - 1890b\,c^2d\,e^5x^4 - 1260c^3d^2e^4x^4 - 420b^3e^6x^3}{2}$$

input
```
int((c*x^2+b*x)^3/(e*x+d)^10,x)
```

output
```
( - 5*b**3*d**3*e**3 - 45*b**3*d**2*e**4*x - 180*b**3*d*e**5*x**2 - 420*b*
*3*e**6*x**3 - 12*b**2*c*d**4*e**2 - 108*b**2*c*d**3*e**3*x - 432*b**2*c*d
**2*e**4*x**2 - 1008*b**2*c*d*e**5*x**3 - 1512*b**2*c*e**6*x**4 - 15*b*c**
2*d**5*e - 135*b*c**2*d**4*e**2*x - 540*b*c**2*d**3*e**3*x**2 - 1260*b*c**
2*d**2*e**4*x**3 - 1890*b*c**2*d*e**5*x**4 - 1890*b*c**2*e**6*x**5 - 10*c*
*3*d**6 - 90*c**3*d**5*e*x - 360*c**3*d**4*e**2*x**2 - 840*c**3*d**3*e**3*
x**3 - 1260*c**3*d**2*e**4*x**4 - 1260*c**3*d*e**5*x**5 - 840*c**3*e**6*x*
*6)/(2520*e**7*(d**9 + 9*d**8*e*x + 36*d**7*e**2*x**2 + 84*d**6*e**3*x**3
+ 126*d**5*e**4*x**4 + 126*d**4*e**5*x**5 + 84*d**3*e**6*x**6 + 36*d**2*e*
*7*x**7 + 9*d*e**8*x**8 + e**9*x**9))
```

3.55 $\int \frac{(d+ex)^4}{bx+cx^2}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 99

$$\int \frac{(d+ex)^4}{bx+cx^2}\,dx = \frac{e^2(6c^2d^2-4bcde+b^2e^2)\,x}{c^3}+\frac{e^3(4cd-be)x^2}{2c^2}+\frac{e^4x^3}{3c}+\frac{d^4\log(x)}{b}-\frac{(cd-be)^4\log(b+cx)}{bc^4}$$

output
```
e^2*(b^2*e^2-4*b*c*d*e+6*c^2*d^2)*x/c^3+1/2*e^3*(-b*e+4*c*d)*x^2/c^2+1/3*e
^4*x^3/c+d^4*ln(x)/b-(-b*e+c*d)^4*ln(c*x+b)/b/c^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^4}{bx+cx^2}\,dx = \frac{bce^2x(6b^2e^2-3bce(8d+ex)+2c^2(18d^2+6dex+e^2x^2))+6c^4d^4\log(x)-6(cd-be)^4\log(b+cx)}{6bc^4}$$

input
```
Integrate[(d + e*x)^4/(b*x + c*x^2),x]
```

output
```
(b*c*e^2*x*(6*b^2*e^2 - 3*b*c*e*(8*d + e*x) + 2*c^2*(18*d^2 + 6*d*e*x + e^
2*x^2)) + 6*c^4*d^4*Log[x] - 6*(c*d - b*e)^4*Log[b + c*x])/(6*b*c^4)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{bx+cx^2}\,dx$$

$$\Big\downarrow \text{1141}$$

$$c\int \left(\frac{d^4}{bcx}+\frac{e^4x^2}{c^2}+\frac{e^2(6c^2d^2-4bced+b^2e^2)}{c^4}+\frac{e^3(4cd-be)x}{c^3}-\frac{(cd-be)^4}{bc^4(b+cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c\left(\frac{e^2x(b^2e^2-4bcde+6c^2d^2)}{c^4}-\frac{(cd-be)^4\log(b+cx)}{bc^5}+\frac{e^3x^2(4cd-be)}{2c^3}+\frac{d^4\log(x)}{bc}+\frac{e^4x^3}{3c^2}\right)$$

input
```
Int[(d + e*x)^4/(b*x + c*x^2),x]
```

output
```
c*((e^2*(6*c^2*d^2 - 4*b*c*d*e + b^2*e^2)*x)/c^4 + (e^3*(4*c*d - b*e)*x^2)
/(2*c^3) + (e^4*x^3)/(3*c^2) + (d^4*Log[x])/(b*c) - ((c*d - b*e)^4*Log[b +
 c*x])/(b*c^5))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.35

method	result
norman	$\frac{e^2(b^2e^2-4bcde+6c^2d^2)x}{c^3}+\frac{e^4x^3}{3c}-\frac{e^3(be-4cd)x^2}{2c^2}+\frac{d^4\ln(x)}{b}-\frac{(e^4b^4-4d\,e^3b^3c+6d^2e^2b^2c^2-4d^3eb\,c^3+d^4c^4)\ln(cx+b}{b\,c^4}$
default	$\frac{e^2(\frac{1}{3}c^2e^2x^3-\frac{1}{2}bc\,e^2x^2+2c^2de\,x^2+b^2e^2x-4bcdex+6c^2d^2x)}{c^3}+\frac{(-e^4b^4+4d\,e^3b^3c-6d^2e^2b^2c^2+4d^3eb\,c^3-d^4c^4)\ln(cx+b)}{b\,c^4}+$
risch	$\frac{e^4x^3}{3c}-\frac{e^4b\,x^2}{2c^2}+\frac{2d\,e^3x^2}{c}+\frac{e^4b^2x}{c^3}-\frac{4e^3bdx}{c^2}+\frac{6e^2d^2x}{c}-\frac{b^3\ln(cx+b)e^4}{c^4}+\frac{4b^2\ln(cx+b)d\,e^3}{c^3}-\frac{6b\ln(cx+b)d^2e^2}{c^2}$
parallelrisch	$\frac{2e^4x^3b\,c^3-3x^2b^2c^2e^4+12x^2b\,c^3d\,e^3+6d^4\ln(x)c^4-6\ln(cx+b)b^4e^4+24\ln(cx+b)b^3cd\,e^3-36\ln(cx+b)b^2c^2d^2e^2+24\ln(cx+b)b}{6b\,c^4}$

input
```
int((e*x+d)^4/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
e^2*(b^2*e^2-4*b*c*d*e+6*c^2*d^2)*x/c^3+1/3*e^4*x^3/c-1/2*e^3/c^2*(b*e-4*c
*d)*x^2+d^4*ln(x)/b-(b^4*e^4-4*b^3*c*d*e^3+6*b^2*c^2*d^2*e^2-4*b*c^3*d^3*e
+c^4*d^4)/b/c^4*ln(c*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.53

$$\int \frac{(d+ex)^4}{bx+cx^2}\,dx = \frac{2\,bc^3e^4x^3 + 6\,c^4d^4\log(x) + 3\,(4\,bc^3de^3 - b^2c^2e^4)x^2 + 6\,(6\,bc^3d^2e^2 - 4\,b^2c^2de^3 + b^3ce^4)x - 6\,(c^4d^4 - 4\,bc^3 \cdots}{6\,bc^4}$$

input
```
integrate((e*x+d)^4/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
1/6*(2*b*c^3*e^4*x^3 + 6*c^4*d^4*log(x) + 3*(4*b*c^3*d*e^3 - b^2*c^2*e^4)*
x^2 + 6*(6*b*c^3*d^2*e^2 - 4*b^2*c^2*d*e^3 + b^3*c*e^4)*x - 6*(c^4*d^4 - 4
*b*c^3*d^3*e + 6*b^2*c^2*d^2*e^2 - 4*b^3*c*d*e^3 + b^4*e^4)*log(c*x + b))/
(b*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex)^4}{bx+cx^2}\,dx = x^2\left(-\frac{be^4}{2c^2} + \frac{2de^3}{c}\right) + x\left(\frac{b^2e^4}{c^3} - \frac{4bde^3}{c^2} + \frac{6d^2e^2}{c}\right) + \frac{e^4x^3}{3c} + \frac{d^4\log(x)}{b} - \frac{(be-cd)^4\log\left(x + \frac{bc^3d^4 + \frac{b(be-cd)^4}{c}}{b^4e^4 - 4b^3cde^3 + 6b^2c^2d^2e^2 - 4bc^3d^3e + 2c^4d^4}\right)}{bc^4}$$

input
```
integrate((e*x+d)**4/(c*x**2+b*x),x)
```

output
```
x**2*(-b*e**4/(2*c**2) + 2*d*e**3/c) + x*(b**2*e**4/c**3 - 4*b*d*e**3/c**2
 + 6*d**2*e**2/c) + e**4*x**3/(3*c) + d**4*log(x)/b - (b*e - c*d)**4*log(x
 + (b*c**3*d**4 + b*(b*e - c*d)**4/c)/(b**4*e**4 - 4*b**3*c*d*e**3 + 6*b**
2*c**2*d**2*e**2 - 4*b*c**3*d**3*e + 2*c**4*d**4))/(b*c**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)^4}{bx+cx^2}\,dx = \frac{d^4\log(x)}{b} + \frac{2\,c^2e^4x^3+3\,(4\,c^2de^3-bce^4)x^2+6\,(6\,c^2d^2e^2-4\,bcde^3+b^2e^4)x}{6\,c^3} - \frac{(c^4d^4-4\,bc^3d^3e+6\,b^2c^2d^2e^2-4\,b^3cde^3+b^4e^4)\log(cx+b)}{bc^4}$$

input
```
integrate((e*x+d)^4/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
d^4*log(x)/b + 1/6*(2*c^2*e^4*x^3 + 3*(4*c^2*d*e^3 - b*c*e^4)*x^2 + 6*(6*c
^2*d^2*e^2 - 4*b*c*d*e^3 + b^2*e^4)*x)/c^3 - (c^4*d^4 - 4*b*c^3*d^3*e + 6*
b^2*c^2*d^2*e^2 - 4*b^3*c*d*e^3 + b^4*e^4)*log(c*x + b)/(b*c^4)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.45

$$\int \frac{(d+ex)^4}{bx+cx^2}\,dx = \frac{d^4\log(|x|)}{b} + \frac{2\,c^2e^4x^3+12\,c^2de^3x^2-3\,bce^4x^2+36\,c^2d^2e^2x-24\,bcde^3x+6\,b^2e^4x}{6\,c^3} - \frac{(c^4d^4-4\,bc^3d^3e+6\,b^2c^2d^2e^2-4\,b^3cde^3+b^4e^4)\log(|cx+b|)}{bc^4}$$

input
```
integrate((e*x+d)^4/(c*x^2+b*x),x, algorithm="giac")
```

output
```
d^4*log(abs(x))/b + 1/6*(2*c^2*e^4*x^3 + 12*c^2*d*e^3*x^2 - 3*b*c*e^4*x^2
+ 36*c^2*d^2*e^2*x - 24*b*c*d*e^3*x + 6*b^2*e^4*x)/c^3 - (c^4*d^4 - 4*b*c^
3*d^3*e + 6*b^2*c^2*d^2*e^2 - 4*b^3*c*d*e^3 + b^4*e^4)*log(abs(c*x + b))/(
b*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^4}{bx+cx^2}\,dx = x\left(\frac{b\left(\frac{b\,e^4}{c^2}-\frac{4\,d\,e^3}{c}\right)}{c}+\frac{6\,d^2\,e^2}{c}\right)-x^2\left(\frac{b\,e^4}{2\,c^2}-\frac{2\,d\,e^3}{c}\right)+\frac{e^4\,x^3}{3\,c}+\frac{d^4\,\ln(x)}{b}-\frac{\ln(b+c\,x)\,(b\,e-c\,d)^4}{b\,c^4}$$

input
```
int((d + e*x)^4/(b*x + c*x^2),x)
```

output
```
x*((b*((b*e^4)/c^2 - (4*d*e^3)/c))/c + (6*d^2*e^2)/c) - x^2*((b*e^4)/(2*c^
2) - (2*d*e^3)/c) + (e^4*x^3)/(3*c) + (d^4*log(x))/b - (log(b + c*x)*(b*e
- c*d)^4)/(b*c^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex)^4}{bx+cx^2}\,dx = \frac{-6\log(cx+b)\,b^4e^4+24\log(cx+b)\,b^3cd\,e^3-36\log(cx+b)\,b^2c^2d^2e^2+24\log(cx+b)\,b\,c^3d^3e-6\log(cx \cdots}{6b \cdots}$$

input
```
int((e*x+d)^4/(c*x^2+b*x),x)
```

output
```
( - 6*log(b + c*x)*b**4*e**4 + 24*log(b + c*x)*b**3*c*d*e**3 - 36*log(b +
c*x)*b**2*c**2*d**2*e**2 + 24*log(b + c*x)*b*c**3*d**3*e - 6*log(b + c*x)*
c**4*d**4 + 6*log(x)*c**4*d**4 + 6*b**3*c*e**4*x - 24*b**2*c**2*d*e**3*x -
 3*b**2*c**2*e**4*x**2 + 36*b*c**3*d**2*e**2*x + 12*b*c**3*d*e**3*x**2 + 2
*b*c**3*e**4*x**3)/(6*b*c**4)
```

3.56 $\int \frac{(d+ex)^3}{bx+cx^2} dx$

Optimal result 454
Mathematica [A] (verified) 454
Rubi [A] (verified) 455
Maple [A] (verified) 456
Fricas [A] (verification not implemented) 456
Sympy [B] (verification not implemented) 457
Maxima [A] (verification not implemented) 457
Giac [A] (verification not implemented) 458
Mupad [B] (verification not implemented) 458
Reduce [B] (verification not implemented) 458

Optimal result

Integrand size = 19, antiderivative size = 64

$$\int \frac{(d+ex)^3}{bx+cx^2} dx = \frac{e^2(3cd-be)x}{c^2} + \frac{e^3x^2}{2c} + \frac{d^3\log(x)}{b} - \frac{(cd-be)^3\log(b+cx)}{bc^3}$$

output
```
e^2*(-b*e+3*c*d)*x/c^2+1/2*e^3*x^2/c+d^3*ln(x)/b-(-b*e+c*d)^3*ln(c*x+b)/b/
c^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^3}{bx+cx^2} dx = \frac{bce^2x(6cd-2be+cex)+2c^3d^3\log(x)-2(cd-be)^3\log(b+cx)}{2bc^3}$$

input
```
Integrate[(d + e*x)^3/(b*x + c*x^2),x]
```

output
```
(b*c*e^2*x*(6*c*d - 2*b*e + c*e*x) + 2*c^3*d^3*Log[x] - 2*(c*d - b*e)^3*Lo
g[b + c*x])/(2*b*c^3)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{bx+cx^2}\,dx$$

$$\Big\downarrow \ 1141$$

$$c\int \left(\frac{d^3}{bcx}+\frac{e^2(3cd-be)}{c^3}+\frac{e^3x}{c^2}-\frac{(cd-be)^3}{bc^3(b+cx)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$c\left(-\frac{(cd-be)^3\log(b+cx)}{bc^4}+\frac{e^2x(3cd-be)}{c^3}+\frac{d^3\log(x)}{bc}+\frac{e^3x^2}{2c^2}\right)$$

input
```
Int[(d + e*x)^3/(b*x + c*x^2),x]
```

output
```
c*((e^2*(3*c*d - b*e)*x)/c^3 + (e^3*x^2)/(2*c^2) + (d^3*Log[x])/(b*c) - ((
c*d - b*e)^3*Log[b + c*x])/(b*c^4))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.33

method	result	s
default	$-\frac{e^2\left(-\frac{1}{2}ce\,x^2+bex-3cdx\right)}{c^2}+\frac{\left(b^3e^3-3d\,e^2b^2c+3d^2eb\,c^2-d^3c^3\right)\ln(cx+b)}{c^3b}+\frac{d^3\ln(x)}{b}$	8
norman	$\frac{e^3x^2}{2c}-\frac{e^2(be-3cd)x}{c^2}+\frac{d^3\ln(x)}{b}+\frac{\left(b^3e^3-3d\,e^2b^2c+3d^2eb\,c^2-d^3c^3\right)\ln(cx+b)}{c^3b}$	8
parallelrisch	$\frac{e^3x^2b\,c^2+2d^3\ln(x)c^3+2\ln(cx+b)b^3e^3-6\ln(cx+b)b^2cd\,e^2+6\ln(cx+b)b\,c^2d^2e-2\ln(cx+b)c^3d^3-2x\,b^2c\,e^3+6xb\,c^2d\,e^2}{2b\,c^3}$	1
risch	$\frac{e^3x^2}{2c}-\frac{e^3bx}{c^2}+\frac{3d\,e^2x}{c}+\frac{b^2\ln(-cx-b)e^3}{c^3}-\frac{3b\ln(-cx-b)d\,e^2}{c^2}+\frac{3\ln(-cx-b)d^2e}{c}-\frac{\ln(-cx-b)d^3}{b}+\frac{d^3\ln(x)}{b}$	1

input
```
int((e*x+d)^3/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-e^2/c^2*(-1/2*c*e*x^2+b*e*x-3*c*d*x)+1/c^3*(b^3*e^3-3*b^2*c*d*e^2+3*b*c^2
*d^2*e-c^3*d^3)/b*ln(c*x+b)+d^3*ln(x)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex)^3}{bx+cx^2}\,dx = \frac{bc^2e^3x^2+2\,c^3d^3\log(x)+2\,(3\,bc^2de^2-b^2ce^3)x-2\,(c^3d^3-3\,bc^2d^2e+3\,b^2cde^2-b^3e^3)\log(cx+b)}{2\,bc^3}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
1/2*(b*c^2*e^3*x^2 + 2*c^3*d^3*log(x) + 2*(3*b*c^2*d*e^2 - b^2*c*e^3)*x -
2*(c^3*d^3 - 3*b*c^2*d^2*e + 3*b^2*c*d*e^2 - b^3*e^3)*log(c*x + b))/(b*c^3
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(54) = 108$.

Time = 0.61 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.75

$$\int \frac{(d+ex)^3}{bx+cx^2}\,dx = x\left(-\frac{be^3}{c^2}+\frac{3de^2}{c}\right)+\frac{e^3x^2}{2c}+\frac{d^3\log(x)}{b} + \frac{(be-cd)^3\log\left(x+\frac{-bc^2d^3+\frac{b(be-cd)^3}{c}}{b^3e^3-3b^2cde^2+3bc^2d^2e-2c^3d^3}\right)}{bc^3}$$

input
```
integrate((e*x+d)**3/(c*x**2+b*x),x)
```

output
```
x*(-b*e**3/c**2 + 3*d*e**2/c) + e**3*x**2/(2*c) + d**3*log(x)/b + (b*e - c
*d)**3*log(x + (-b*c**2*d**3 + b*(b*e - c*d)**3/c)/(b**3*e**3 - 3*b**2*c*d
*e**2 + 3*b*c**2*d**2*e - 2*c**3*d**3))/(b*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex)^3}{bx+cx^2}\,dx = \frac{d^3\log(x)}{b}+\frac{ce^3x^2+2\,(3\,cde^2-be^3)x}{2\,c^2} - \frac{(c^3d^3-3\,bc^2d^2e+3\,b^2cde^2-b^3e^3)\log(cx+b)}{bc^3}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
d^3*log(x)/b + 1/2*(c*e^3*x^2 + 2*(3*c*d*e^2 - b*e^3)*x)/c^2 - (c^3*d^3 -
3*b*c^2*d^2*e + 3*b^2*c*d*e^2 - b^3*e^3)*log(c*x + b)/(b*c^3)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.42

$$\int \frac{(d+ex)^3}{bx+cx^2}\,dx = \frac{d^3\log(|x|)}{b} + \frac{ce^3x^2 + 6\,cde^2x - 2\,be^3x}{2\,c^2} - \frac{(c^3d^3 - 3\,bc^2d^2e + 3\,b^2cde^2 - b^3e^3)\log(|cx+b|)}{bc^3}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x),x, algorithm="giac")
```

output
```
d^3*log(abs(x))/b + 1/2*(c*e^3*x^2 + 6*c*d*e^2*x - 2*b*e^3*x)/c^2 - (c^3*d
^3 - 3*b*c^2*d^2*e + 3*b^2*c*d*e^2 - b^3*e^3)*log(abs(c*x + b))/(b*c^3)
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.02

$$\int \frac{(d+ex)^3}{bx+cx^2}\,dx = \frac{e^3\,x^2}{2\,c} - x\left(\frac{b\,e^3}{c^2} - \frac{3\,d\,e^2}{c}\right) + \frac{d^3\,\ln(x)}{b} + \frac{\ln(b+c\,x)\,(b\,e-c\,d)^3}{b\,c^3}$$

input
```
int((d + e*x)^3/(b*x + c*x^2),x)
```

output
```
(e^3*x^2)/(2*c) - x*((b*e^3)/c^2 - (3*d*e^2)/c) + (d^3*log(x))/b + (log(b
+ c*x)*(b*e - c*d)^3)/(b*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.73

$$\int \frac{(d+ex)^3}{bx+cx^2}\,dx = \frac{2\log(cx+b)\,b^3e^3 - 6\log(cx+b)\,b^2cd\,e^2 + 6\log(cx+b)\,b\,c^2d^2e - 2\log(cx+b)\,c^3d^3 + 2\log(x)\,c^3d^3 - 2b^2}{2b\,c^3}$$

input
```
int((e*x+d)^3/(c*x^2+b*x),x)
```

output
```
(2*log(b + c*x)*b**3*e**3 - 6*log(b + c*x)*b**2*c*d*e**2 + 6*log(b + c*x)*
b*c**2*d**2*e - 2*log(b + c*x)*c**3*d**3 + 2*log(x)*c**3*d**3 - 2*b**2*c*e
**3*x + 6*b*c**2*d*e**2*x + b*c**2*e**3*x**2)/(2*b*c**3)
```

3.57 $\int \frac{(d+ex)^2}{bx+cx^2} \, dx$

Optimal result 460
Mathematica [A] (verified) 460
Rubi [A] (verified) 461
Maple [A] (verified) 462
Fricas [A] (verification not implemented) 462
Sympy [B] (verification not implemented) 463
Maxima [A] (verification not implemented) 463
Giac [A] (verification not implemented) 463
Mupad [B] (verification not implemented) 464
Reduce [B] (verification not implemented) 464

Optimal result

Integrand size = 19, antiderivative size = 42

$$\int \frac{(d+ex)^2}{bx+cx^2} \, dx = \frac{e^2x}{c} + \frac{d^2\log(x)}{b} - \frac{(cd-be)^2\log(b+cx)}{bc^2}$$

output
```
e^2*x/c+d^2*ln(x)/b-(-b*e+c*d)^2*ln(c*x+b)/b/c^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{bx+cx^2} \, dx = \frac{bce^2x + c^2d^2\log(x) - (cd-be)^2\log(b+cx)}{bc^2}$$

input
```
Integrate[(d + e*x)^2/(b*x + c*x^2),x]
```

output
```
(b*c*e^2*x + c^2*d^2*Log[x] - (c*d - b*e)^2*Log[b + c*x])/(b*c^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{bx+cx^2}\,dx$$

$$\Big\downarrow \text{1141}$$

$$c\int \left(\frac{d^2}{bcx}+\frac{e^2}{c^2}-\frac{(cd-be)^2}{bc^2(b+cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c\left(-\frac{(cd-be)^2\log(b+cx)}{bc^3}+\frac{d^2\log(x)}{bc}+\frac{e^2x}{c^2}\right)$$

input
```
Int[(d + e*x)^2/(b*x + c*x^2),x]
```

output
```
c*((e^2*x)/c^2 + (d^2*Log[x])/(b*c) - ((c*d - b*e)^2*Log[b + c*x])/(b*c^3)
)
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.29

method	result	size
norman	$\frac{e^2 x}{c}+\frac{d^2 \ln(x)}{b}-\frac{(b^2 e^2-2bcde+c^2 d^2) \ln(cx+b)}{b\,c^2}$	54
default	$\frac{e^2 x}{c}+\frac{(-b^2 e^2+2bcde-c^2 d^2) \ln(cx+b)}{b\,c^2}+\frac{d^2 \ln(x)}{b}$	55
risch	$\frac{e^2 x}{c}-\frac{b \ln(cx+b) e^2}{c^2}+\frac{2 \ln(cx+b) de}{c}-\frac{\ln(cx+b) d^2}{b}+\frac{d^2 \ln(-x)}{b}$	63
parallelrisch	$\frac{d^2 \ln(x) c^2-\ln(cx+b) b^2 e^2+2 \ln(cx+b) bcde-\ln(cx+b) c^2 d^2+e^2 xbc}{b\,c^2}$	65

input
```
int((e*x+d)^2/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
e^2*x/c+d^2*ln(x)/b-(b^2*e^2-2*b*c*d*e+c^2*d^2)/b/c^2*ln(c*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^2}{bx+cx^2}\,dx=\frac{bce^2x+c^2d^2\log(x)-(c^2d^2-2\,bcde+b^2e^2)\log(cx+b)}{bc^2}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
(b*c*e^2*x + c^2*d^2*log(x) - (c^2*d^2 - 2*b*c*d*e + b^2*e^2)*log(c*x + b)
)/(b*c^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(34) = 68$.

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.74

$$\int \frac{(d+ex)^2}{bx+cx^2}\,dx = \frac{e^2x}{c} + \frac{d^2\log{(x)}}{b} - \frac{(be-cd)^2\log{\left(x + \frac{bcd^2+\frac{b(be-cd)^2}{c}}{b^2e^2-2bcde+2c^2d^2}\right)}}{bc^2}$$

input
```
integrate((e*x+d)**2/(c*x**2+b*x),x)
```

output
```
e**2*x/c + d**2*log(x)/b - (b*e - c*d)**2*log(x + (b*c*d**2 + b*(b*e - c*d
)**2/c)/(b**2*e**2 - 2*b*c*d*e + 2*c**2*d**2))/(b*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^2}{bx+cx^2}\,dx = \frac{e^2x}{c} + \frac{d^2\log{(x)}}{b} - \frac{(c^2d^2-2\,bcde+b^2e^2)\log{(cx+b)}}{bc^2}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
e^2*x/c + d^2*log(x)/b - (c^2*d^2 - 2*b*c*d*e + b^2*e^2)*log(c*x + b)/(b*c
^2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^2}{bx+cx^2}\,dx = \frac{e^2x}{c} + \frac{d^2\log{(|x|)}}{b} - \frac{(c^2d^2-2\,bcde+b^2e^2)\log{(|cx+b|)}}{bc^2}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x),x, algorithm="giac")
```

output
```
e^2*x/c + d^2*log(abs(x))/b - (c^2*d^2 - 2*b*c*d*e + b^2*e^2)*log(abs(c*x
+ b))/(b*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.17

$$\int \frac{(d+ex)^2}{bx+cx^2}\,dx = \frac{e^2\,x}{c} - \ln\left(b+c\,x\right)\,\left(\frac{d^2}{b}+\frac{b\,e^2}{c^2}-\frac{2\,d\,e}{c}\right)+\frac{d^2\,\ln\left(x\right)}{b}$$

input
```
int((d + e*x)^2/(b*x + c*x^2),x)
```

output
```
(e^2*x)/c - log(b + c*x)*(d^2/b + (b*e^2)/c^2 - (2*d*e)/c) + (d^2*log(x))/
b
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex)^2}{bx+cx^2}\,dx = \frac{-\log(cx+b)\,b^2e^2+2\log(cx+b)\,bcde-\log(cx+b)\,c^2d^2+\log(x)\,c^2d^2+bc\,e^2x}{b\,c^2}$$

input
```
int((e*x+d)^2/(c*x^2+b*x),x)
```

output
```
( - log(b + c*x)*b**2*e**2 + 2*log(b + c*x)*b*c*d*e - log(b + c*x)*c**2*d*
*2 + log(x)*c**2*d**2 + b*c*e**2*x)/(b*c**2)
```

3.58 $\int \frac{d+ex}{bx+cx^2} dx$

Optimal result . . . 465
Mathematica [A] (verified) . . . 465
Rubi [A] (verified) . . . 466
Maple [A] (verified) . . . 467
Fricas [A] (verification not implemented) . . . 467
Sympy [A] (verification not implemented) . . . 467
Maxima [A] (verification not implemented) . . . 468
Giac [A] (verification not implemented) . . . 468
Mupad [B] (verification not implemented) . . . 468
Reduce [B] (verification not implemented) . . . 469

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{d+ex}{bx+cx^2} dx = \frac{d\log(x)}{b} - \frac{(cd-be)\log(b+cx)}{bc}$$

output
```
d*ln(x)/b-(-b*e+c*d)*ln(c*x+b)/b/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{d+ex}{bx+cx^2} dx = \frac{d\log(x)}{b} + \frac{(-cd+be)\log(b+cx)}{bc}$$

input
```
Integrate[(d + e*x)/(b*x + c*x^2),x]
```

output
```
(d*Log[x])/b + ((-(c*d) + b*e)*Log[b + c*x])/(b*c)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{bx+cx^2} \, dx$$

$$\Big\downarrow \text{ 1141}$$

$$c\int \left(\frac{d}{bcx}-\frac{cd-be}{bc(b+cx)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$c\left(\frac{d\log(x)}{bc}-\frac{(cd-be)\log(b+cx)}{bc^2}\right)$$

input
```
Int[(d + e*x)/(b*x + c*x^2),x]
```

output
```
c*((d*Log[x])/(b*c) - ((c*d - b*e)*Log[b + c*x])/(b*c^2))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{(be-cd)\ln(cx+b)}{bc}+\frac{d\ln(x)}{b}$	30
norman	$\frac{(be-cd)\ln(cx+b)}{bc}+\frac{d\ln(x)}{b}$	30
parallelrisch	$\frac{d\ln(x)c+\ln(cx+b)be-\ln(cx+b)cd}{bc}$	33
risch	$\frac{d\ln(x)}{b}+\frac{\ln(-cx-b)e}{c}-\frac{\ln(-cx-b)d}{b}$	38

input
```
int((e*x+d)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
(b*e-c*d)/b/c*ln(c*x+b)+d/b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{d+ex}{bx+cx^2}\,dx=\frac{cd\log(x)-(cd-be)\log(cx+b)}{bc}$$

input
```
integrate((e*x+d)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
(c*d*log(x) - (c*d - b*e)*log(c*x + b))/(b*c)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \frac{d+ex}{bx+cx^2}\,dx=\frac{d\log(x)}{b}+\frac{(be-cd)\log\left(x+\frac{-bd+\frac{b(be-cd)}{c}}{be-2cd}\right)}{bc}$$

input
```
integrate((e*x+d)/(c*x**2+b*x),x)
```

output
```
d*log(x)/b + (b*e - c*d)*log(x + (-b*d + b*(b*e - c*d)/c)/(b*e - 2*c*d))/(
b*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{bx+cx^2}\,dx = \frac{d\log(x)}{b} - \frac{(cd-be)\log(cx+b)}{bc}$$

input
```
integrate((e*x+d)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
d*log(x)/b - (c*d - b*e)*log(c*x + b)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{d+ex}{bx+cx^2}\,dx = \frac{d\log(|x|)}{b} - \frac{(cd-be)\log(|cx+b|)}{bc}$$

input
```
integrate((e*x+d)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
d*log(abs(x))/b - (c*d - b*e)*log(abs(c*x + b))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{d+ex}{bx+cx^2}\,dx = \frac{d\,\ln(x)}{b} - \ln(b+c\,x)\,\left(\frac{d}{b}-\frac{e}{c}\right)$$

input
```
int((d + e*x)/(b*x + c*x^2),x)
```

output
```
(d*log(x))/b - log(b + c*x)*(d/b - e/c)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{d+ex}{bx+cx^2}\,dx=\frac{\log(cx+b)\,be-\log(cx+b)\,cd+\log(x)\,cd}{bc}$$

input
```
int((e*x+d)/(c*x^2+b*x),x)
```

output
```
(log(b + c*x)*b*e - log(b + c*x)*c*d + log(x)*c*d)/(b*c)
```

3.59 $\int \frac{1}{bx+cx^2} \, dx$

Optimal result 470
Mathematica [A] (verified) 470
Rubi [A] (verified) 471
Maple [A] (verified) 472
Fricas [A] (verification not implemented) 472
Sympy [A] (verification not implemented) 472
Maxima [A] (verification not implemented) 473
Giac [A] (verification not implemented) 473
Mupad [B] (verification not implemented) 473
Reduce [B] (verification not implemented) 474

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{1}{bx + cx^2} \, dx = \frac{\log(x)}{b} - \frac{\log(b + cx)}{b}$$

output
```
ln(x)/b-ln(c*x+b)/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{bx + cx^2} \, dx = \frac{\log(x)}{b} - \frac{\log(b + cx)}{b}$$

input
```
Integrate[(b*x + c*x^2)^(-1),x]
```

output
```
Log[x]/b - Log[b + c*x]/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{bx+cx^2}\,dx$$
$$\Big\downarrow \; 1080$$
$$\int \left(\frac{1}{bx}-\frac{c}{b(b+cx)}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\log(x)}{b}-\frac{\log(b+cx)}{b}$$

input
```
Int[(b*x + c*x^2)^(-1),x]
```

output
```
Log[x]/b - Log[b + c*x]/b
```

Defintions of rubi rules used

rule 1080
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[x^p*
(b + c*x)^p, x], x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

method	result	size
parallelrisch	$\frac{\ln(x)-\ln(cx+b)}{b}$	16
default	$\frac{\ln(x)}{b}-\frac{\ln(cx+b)}{b}$	19
norman	$\frac{\ln(x)}{b}-\frac{\ln(cx+b)}{b}$	19
risch	$\frac{\ln(-x)}{b}-\frac{\ln(cx+b)}{b}$	21

input
```
int(1/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
(ln(x)-ln(c*x+b))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{bx+cx^2}\,dx = -\frac{\log(cx+b)-\log(x)}{b}$$

input
```
integrate(1/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
-(log(c*x + b) - log(x))/b
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.56

$$\int \frac{1}{bx+cx^2}\,dx = \frac{\log(x)-\log\left(\frac{b}{c}+x\right)}{b}$$

input
```
integrate(1/(c*x**2+b*x),x)
```

output
```
(log(x) - log(b/c + x))/b
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{bx+cx^2}\,dx = -\frac{\log(cx+b)}{b} + \frac{\log(x)}{b}$$

input
```
integrate(1/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
-log(c*x + b)/b + log(x)/b
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{bx+cx^2}\,dx = -\frac{\log(|cx+b|)}{b} + \frac{\log(|x|)}{b}$$

input
```
integrate(1/(c*x^2+b*x),x, algorithm="giac")
```

output
```
-log(abs(c*x + b))/b + log(abs(x))/b
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{bx+cx^2}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b}$$

input
```
int(1/(b*x + c*x^2),x)
```

output
```
-(2*atanh((2*c*x)/b + 1))/b
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{1}{bx+cx^2}\,dx=\frac{-\log(cx+b)+\log(x)}{b}$$

input
```
int(1/(c*x^2+b*x),x)
```

output
```
( - log(b + c*x) + log(x))/b
```

3.60 $\int \frac{1}{(d+ex)(bx+cx^2)}\,dx$

Optimal result 475
Mathematica [A] (verified) 475
Rubi [A] (verified) 476
Maple [A] (verified) 477
Fricas [A] (verification not implemented) 477
Sympy [F(-1)] 478
Maxima [A] (verification not implemented) 478
Giac [A] (verification not implemented) 478
Mupad [B] (verification not implemented) 479
Reduce [B] (verification not implemented) 479

Optimal result

Integrand size = 19, antiderivative size = 53

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)}\,dx=\frac{\log(x)}{bd}-\frac{c\log(b+cx)}{b(cd-be)}+\frac{e\log(d+ex)}{d(cd-be)}$$

output
```
ln(x)/b/d-c*ln(c*x+b)/b/(-b*e+c*d)+e*ln(e*x+d)/d/(-b*e+c*d)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)}\,dx=\frac{cd\log(x)-be\log(x)-cd\log(b+cx)+be\log(d+ex)}{bcd^2-b^2de}$$

input
```
Integrate[1/((d + e*x)*(b*x + c*x^2)),x]
```

output
```
(c*d*Log[x] - b*e*Log[x] - c*d*Log[b + c*x] + b*e*Log[d + e*x])/(b*c*d^2 -
 b^2*d*e)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)(d+ex)}dx$$

$$\Big\downarrow \text{1141}$$

$$c\int \left(\frac{e^2}{cd(cd-be)(d+ex)}+\frac{1}{bcdx}-\frac{c}{b(cd-be)(b+cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c\left(-\frac{\log(b+cx)}{b(cd-be)}+\frac{e\log(d+ex)}{cd(cd-be)}+\frac{\log(x)}{bcd}\right)$$

input
```
Int[1/((d + e*x)*(b*x + c*x^2)),x]
```

output
```
c*(Log[x]/(b*c*d) - Log[b + c*x]/(b*(c*d - b*e)) + (e*Log[d + e*x])/(c*d*(
c*d - b*e)))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

method	result	size
parallelrisch	$\frac{\ln(x)be-d\ln(x)c+\ln(cx+b)cd-e\ln(ex+d)b}{bd(be-cd)}$	49
default	$\frac{c\ln(cx+b)}{b(be-cd)}-\frac{e\ln(ex+d)}{d(be-cd)}+\frac{\ln(x)}{db}$	54
norman	$\frac{c\ln(cx+b)}{b(be-cd)}-\frac{e\ln(ex+d)}{d(be-cd)}+\frac{\ln(x)}{db}$	54
risch	$-\frac{e\ln(ex+d)}{d(be-cd)}+\frac{\ln(-x)}{db}+\frac{c\ln(cx+b)}{b(be-cd)}$	56

input
```
int(1/(e*x+d)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
(ln(x)*b*e-d*ln(x)*c+ln(c*x+b)*c*d-e*ln(e*x+d)*b)/b/d/(b*e-c*d)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)(bx+cx^2)}\,dx=-\frac{cd\log(cx+b)-be\log(ex+d)-(cd-be)\log(x)}{bcd^2-b^2de}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
-(c*d*log(c*x + b) - b*e*log(e*x + d) - (c*d - b*e)*log(x))/(b*c*d^2 - b^2
*d*e)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)(bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x**2+b*x),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)(bx+cx^2)}\,dx = -\frac{c\log(cx+b)}{bcd-b^2e} + \frac{e\log(ex+d)}{cd^2-bde} + \frac{\log(x)}{bd}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
-c*log(c*x + b)/(b*c*d - b^2*e) + e*log(e*x + d)/(c*d^2 - b*d*e) + log(x)/
(b*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \frac{1}{(d+ex)(bx+cx^2)}\,dx = -\frac{c^2\log(|cx+b|)}{bc^2d-b^2ce} + \frac{e^2\log(|ex+d|)}{cd^2e-bde^2} + \frac{\log(|x|)}{bd}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
-c^2*log(abs(c*x + b))/(b*c^2*d - b^2*c*e) + e^2*log(abs(e*x + d))/(c*d^2*
e - b*d*e^2) + log(abs(x))/(b*d)
```

Mupad [B] (verification not implemented)

Time = 8.89 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.75

$$\int \frac{1}{(d+ex)(bx+cx^2)}\,dx = \frac{e\,\ln\left(\frac{(d+e\,x)^2}{x\,(b+c\,x)}\right)}{2\,c\,d^2-2\,b\,d\,e} - \frac{\ln\left(\frac{b-\sqrt{b^2}+2\,c\,x}{b+\sqrt{b^2}+2\,c\,x}\right)\,(b\,e-2\,c\,d)}{(2\,c\,d^2-2\,b\,d\,e)\,\sqrt{b^2}}$$

input
```
int(1/((b*x + c*x^2)*(d + e*x)),x)
```

output
```
(e*log((d + e*x)^2/(x*(b + c*x))))/(2*c*d^2 - 2*b*d*e) - (log((b - (b^2)^(
1/2) + 2*c*x)/(b + (b^2)^(1/2) + 2*c*x))*(b*e - 2*c*d))/((2*c*d^2 - 2*b*d*
e)*(b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.91

$$\int \frac{1}{(d+ex)(bx+cx^2)}\,dx = \frac{\log(cx+b)\,cd-\log(ex+d)\,be+\log(x)\,be-\log(x)\,cd}{bd\,(be-cd)}$$

input
```
int(1/(e*x+d)/(c*x^2+b*x),x)
```

output
```
(log(b + c*x)*c*d - log(d + e*x)*b*e + log(x)*b*e - log(x)*c*d)/(b*d*(b*e
- c*d))
```

3.61 $\int \frac{1}{(d+ex)^2(bx+cx^2)}\,dx$

Optimal result 480
Mathematica [A] (verified) 480
Rubi [A] (verified) 481
Maple [A] (verified) 482
Fricas [B] (verification not implemented) 482
Sympy [F(-1)] 483
Maxima [A] (verification not implemented) 483
Giac [B] (verification not implemented) 484
Mupad [B] (verification not implemented) 484
Reduce [B] (verification not implemented) 485

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{1}{(d+ex)^2(bx+cx^2)}\,dx = -\frac{e}{d(cd-be)(d+ex)}+\frac{\log(x)}{bd^2}-\frac{c^2\log(b+cx)}{b(cd-be)^2}+\frac{e(2cd-be)\log(d+ex)}{d^2(cd-be)^2}$$

output
```
-e/d/(-b*e+c*d)/(e*x+d)+ln(x)/b/d^2-c^2*ln(c*x+b)/b/(-b*e+c*d)^2+e*(-b*e+2
*c*d)*ln(e*x+d)/d^2/(-b*e+c*d)^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.95

$$\int \frac{1}{(d+ex)^2(bx+cx^2)}\,dx = \frac{\log(x)+\frac{-c^2d^2(d+ex)\log(b+cx)+be(d(-cd+be)+(2cd-be)(d+ex)\log(d+ex))}{(cd-be)^2(d+ex)}}{bd^2}$$

input
```
Integrate[1/((d + e*x)^2*(b*x + c*x^2)),x]
```

output
```
(Log[x] + (-(c^2*d^2*(d + e*x)*Log[b + c*x]) + b*e*(d*(-(c*d) + b*e) + (2*
c*d - b*e)*(d + e*x)*Log[d + e*x]))/((c*d - b*e)^2*(d + e*x)))/(b*d^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)(d+ex)^2}dx$$
$$\Big\downarrow \text{1141}$$
$$c\int \left(-\frac{c^2}{b(cd-be)^2(b+cx)}+\frac{1}{bd^2xc}+\frac{e^2(2cd-be)}{d^2(cd-be)^2(d+ex)c}+\frac{e^2}{d(cd-be)(d+ex)^2c}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$c\left(\frac{e(2cd-be)\log(d+ex)}{cd^2(cd-be)^2}+\frac{\log(x)}{bcd^2}-\frac{e}{cd(d+ex)(cd-be)}-\frac{c\log(b+cx)}{b(cd-be)^2}\right)$$

input
```
Int[1/((d + e*x)^2*(b*x + c*x^2)),x]
```

output
```
c*(-(e/(c*d*(c*d - b*e)*(d + e*x))) + Log[x]/(b*c*d^2) - (c*Log[b + c*x])/
(b*(c*d - b*e)^2) + (e*(2*c*d - b*e)*Log[d + e*x])/(c*d^2*(c*d - b*e)^2))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

method	result
default	$-\frac{c^2 \ln(cx+b)}{(be-cd)^2 b}+\frac{e}{d(be-cd)(ex+d)}-\frac{e(be-2cd)\ln(ex+d)}{d^2(be-cd)^2}+\frac{\ln(x)}{b\,d^2}$
norman	$-\frac{e^2 x}{d^2(be-cd)(ex+d)}+\frac{\ln(x)}{b\,d^2}-\frac{c^2\ln(cx+b)}{b(b^2e^2-2bcde+c^2d^2)}-\frac{e(be-2cd)\ln(ex+d)}{d^2(b^2e^2-2bcde+c^2d^2)}$
risch	$\frac{e}{d(be-cd)(ex+d)}-\frac{e^2\ln(-ex-d)b}{d^2(b^2e^2-2bcde+c^2d^2)}+\frac{2e\ln(-ex-d)c}{d(b^2e^2-2bcde+c^2d^2)}+\frac{\ln(-x)}{b\,d^2}-\frac{c^2\ln(cx+b)}{b(b^2e^2-2bcde+c^2d^2)}$
parallelrisch	$\frac{\ln(x)x\,b^2e^4-2\ln(x)xbcd\,e^3+\ln(x)x\,c^2d^2e^2-\ln(cx+b)x\,c^2d^2e^2-\ln(ex+d)x\,b^2e^4+2\ln(ex+d)xbcd\,e^3+\ln(x)b^2d\,e^3-2\ln(x)bc\,d}{(b^2e^2-2bcde+c^2d^2)b(ex+d)d^2e}$

input
```
int(1/(e*x+d)^2/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-c^2/(b*e-c*d)^2/b*ln(c*x+b)+e/d/(b*e-c*d)/(e*x+d)-e*(b*e-2*c*d)/d^2/(b*e-
c*d)^2*ln(e*x+d)+ln(x)/b/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(87) = 174.

Time = 0.53 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.38

$$\int \frac{1}{(d+ex)^2(bx+cx^2)}\,dx = -\frac{bcd^2e-b^2de^2+(c^2d^2ex+c^2d^3)\log(cx+b)-(2\,bcd^2e-b^2de^2+(2\,bcde^2-b^2e^3)x)\log(ex+d)-(c}{bc^2d^5-2\,b^2cd^4e+b^3d^3e^2+(bc^2d^4e-2\,b^2cd^3e^2+b^3}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
-(b*c*d^2*e - b^2*d*e^2 + (c^2*d^2*e*x + c^2*d^3)*log(c*x + b) - (2*b*c*d^
2*e - b^2*d*e^2 + (2*b*c*d*e^2 - b^2*e^3)*x)*log(e*x + d) - (c^2*d^3 - 2*b
*c*d^2*e + b^2*d*e^2 + (c^2*d^2*e - 2*b*c*d*e^2 + b^2*e^3)*x)*log(x))/(b*c
^2*d^5 - 2*b^2*c*d^4*e + b^3*d^3*e^2 + (b*c^2*d^4*e - 2*b^2*c*d^3*e^2 + b^
3*d^2*e^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(c*x**2+b*x),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.47

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)}\,dx = -\frac{c^2 \log(cx+b)}{bc^2d^2 - 2\,b^2cde + b^3e^2} + \frac{(2\,cde - be^2)\log(ex+d)}{c^2d^4 - 2\,bcd^3e + b^2d^2e^2} - \frac{e}{cd^3 - bd^2e + (cd^2e - bde^2)x} + \frac{\log(x)}{bd^2}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
-c^2*log(c*x + b)/(b*c^2*d^2 - 2*b^2*c*d*e + b^3*e^2) + (2*c*d*e - b*e^2)*
log(e*x + d)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2) - e/(c*d^3 - b*d^2*e +
(c*d^2*e - b*d*e^2)*x) + log(x)/(b*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(87) = 174$.

Time = 0.14 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.29

$$\int \frac{1}{(d+ex)^2(bx+cx^2)}\,dx$$
$$= -\frac{e^3}{(cd^2e^2 - bde^3)(ex+d)} - \frac{(2\,cde - be^2)\log\left(\left|-c + \frac{2\,cd}{ex+d} - \frac{cd^2}{(ex+d)^2} - \frac{be}{ex+d} + \frac{bde}{(ex+d)^2}\right|\right)}{2\,(c^2d^4 - 2\,bcd^3e + b^2d^2e^2)} - \frac{(2\,c^2d^2e^2 - 2\,bcde^3 + b^2e^4)\log\left(\frac{\left|-2\,cde + \frac{2\,cd^2e}{ex+d} + be^2 - \frac{2\,bde^2}{ex+d} - e^2|b|\right|}{\left|-2\,cde + \frac{2\,cd^2e}{ex+d} + be^2 - \frac{2\,bde^2}{ex+d} + e^2|b|\right|}\right)}{2\,(c^2d^4 - 2\,bcd^3e + b^2d^2e^2)e^2|b|}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x),x, algorithm="giac")
```

output
```
-e^3/((c*d^2*e^2 - b*d*e^3)*(e*x + d)) - 1/2*(2*c*d*e - b*e^2)*log(abs(-c
+ 2*c*d/(e*x + d) - c*d^2/(e*x + d)^2 - b*e/(e*x + d) + b*d*e/(e*x + d)^2)
)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2) - 1/2*(2*c^2*d^2*e^2 - 2*b*c*d*e^3
 + b^2*e^4)*log(abs(-2*c*d*e + 2*c*d^2*e/(e*x + d) + b*e^2 - 2*b*d*e^2/(e*
x + d) - e^2*abs(b))/abs(-2*c*d*e + 2*c*d^2*e/(e*x + d) + b*e^2 - 2*b*d*e^
2/(e*x + d) + e^2*abs(b)))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*e^2*abs(
b))
```

Mupad [B] (verification not implemented)

Time = 9.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.33

$$\int \frac{1}{(d+ex)^2(bx+cx^2)}\,dx = \frac{\ln(x)}{b\,d^2} - \frac{c^2\,\ln(b+c\,x)}{b^3\,e^2 - 2\,b^2\,c\,d\,e + b\,c^2\,d^2} - \frac{\ln(d+e\,x)\,(b\,e^2 - 2\,c\,d\,e)}{b^2\,d^2\,e^2 - 2\,b\,c\,d^3\,e + c^2\,d^4} + \frac{e}{d\,(b\,e - c\,d)\,(d+e\,x)}$$

input
```
int(1/((b*x + c*x^2)*(d + e*x)^2),x)
```

output
```
log(x)/(b*d^2) - (c^2*log(b + c*x))/(b^3*e^2 + b*c^2*d^2 - 2*b^2*c*d*e) -
(log(d + e*x)*(b*e^2 - 2*c*d*e))/(c^2*d^4 + b^2*d^2*e^2 - 2*b*c*d^3*e) + e
/(d*(b*e - c*d)*(d + e*x))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.61

$$\int \frac{1}{(d+ex)^2\,(bx+cx^2)}\,dx$$
$$= \frac{-\log(cx+b)\,c^2d^3-\log(cx+b)\,c^2d^2ex-\log(ex+d)\,b^2d\,e^2-\log(ex+d)\,b^2e^3x+2\log(ex+d)\,bc\,d^2e+}{b\,d^2\,(b^2e^3x-}$$

input
```
int(1/(e*x+d)^2/(c*x^2+b*x),x)
```

output
```
( - log(b + c*x)*c**2*d**3 - log(b + c*x)*c**2*d**2*e*x - log(d + e*x)*b**
2*d*e**2 - log(d + e*x)*b**2*e**3*x + 2*log(d + e*x)*b*c*d**2*e + 2*log(d
+ e*x)*b*c*d*e**2*x + log(x)*b**2*d*e**2 + log(x)*b**2*e**3*x - 2*log(x)*b
*c*d**2*e - 2*log(x)*b*c*d*e**2*x + log(x)*c**2*d**3 + log(x)*c**2*d**2*e*
x - b**2*e**3*x + b*c*d*e**2*x)/(b*d**2*(b**2*d*e**2 + b**2*e**3*x - 2*b*c
*d**2*e - 2*b*c*d*e**2*x + c**2*d**3 + c**2*d**2*e*x))
```

3.62 $\int \frac{1}{(d+ex)^3(bx+cx^2)}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 134

$$\int \frac{1}{(d+ex)^3\,(bx+cx^2)}\,dx=-\frac{e}{2d(cd-be)(d+ex)^2}-\frac{e(2cd-be)}{d^2(cd-be)^2(d+ex)}+\frac{\log(x)}{bd^3}-\frac{c^3\log(b+cx)}{b(cd-be)^3}+\frac{e(3c^2d^2-3bcde+b^2e^2)\log(d+ex)}{d^3(cd-be)^3}$$

output
```
-1/2*e/d/(-b*e+c*d)/(e*x+d)^2-e*(-b*e+2*c*d)/d^2/(-b*e+c*d)^2/(e*x+d)+ln(x
)/b/d^3-c^3*ln(c*x+b)/b/(-b*e+c*d)^3+e*(b^2*e^2-3*b*c*d*e+3*c^2*d^2)*ln(e*
x+d)/d^3/(-b*e+c*d)^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.87

$$\int \frac{1}{(d+ex)^3\,(bx+cx^2)}\,dx=\frac{\log(x)}{bd^3}+\frac{\frac{2c^3\log(b+cx)}{b}+\frac{e\left(\frac{d(cd-be)(-be(3d+2ex)+cd(5d+4ex))}{(d+ex)^2}-2(3c^2d^2-3bcde+b^2e^2)\log(d+ex)\right)}{d^3}}{2(-cd+be)^3}$$

input
```
Integrate[1/((d + e*x)^3*(b*x + c*x^2)),x]
```

output
```
Log[x]/(b*d^3) + ((2*c^3*Log[b + c*x])/b + (e*((d*(c*d - b*e)*(-(b*e*(3*d
+ 2*e*x)) + c*d*(5*d + 4*e*x)))/(d + e*x)^2 - 2*(3*c^2*d^2 - 3*b*c*d*e + b
^2*e^2)*Log[d + e*x]))/d^3)/(2*(-(c*d) + b*e)^3)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)(d+ex)^3}dx$$

$$\Big\downarrow \text{1141}$$

$$c\int \left(-\frac{c^3}{b(cd-be)^3(b+cx)}+\frac{1}{bd^3xc}+\frac{e^2(3c^2d^2-3bced+b^2e^2)}{d^3(cd-be)^3(d+ex)c}+\frac{e^2(2cd-be)}{d^2(cd-be)^2(d+ex)^2c}+\frac{e^2}{d(cd-be)(d+ex\right.$$

$$\Big\downarrow \text{2009}$$

$$c\left(\frac{e(b^2e^2-3bcde+3c^2d^2)\log(d+ex)}{cd^3(cd-be)^3}-\frac{c^2\log(b+cx)}{b(cd-be)^3}+\frac{\log(x)}{bcd^3}-\frac{e(2cd-be)}{cd^2(d+ex)(cd-be)^2}-\frac{e}{2cd(d+ex)^2(cd-\right.$$

input
```
Int[1/((d + e*x)^3*(b*x + c*x^2)),x]
```

output
```
c*(-1/2*e/(c*d*(c*d - b*e)*(d + e*x)^2) - (e*(2*c*d - b*e))/(c*d^2*(c*d -
b*e)^2*(d + e*x)) + Log[x]/(b*c*d^3) - (c^2*Log[b + c*x])/(b*(c*d - b*e)^3
) + (e*(3*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*Log[d + e*x])/(c*d^3*(c*d - b*e)^
3))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

method	result
default	$\frac{c^3 \ln(cx+b)}{(be-cd)^3 b}+\frac{e}{2d(be-cd)(ex+d)^2}+\frac{e(be-2cd)}{d^2(be-cd)^2(ex+d)}-\frac{e\left(b^2e^2-3bcde+3c^2d^2\right)\ln(ex+d)}{d^3(be-cd)^3}+\frac{\ln(x)}{b\,d^3}$
norman	$\frac{\frac{\left(-2b\,e^2+3dec\right)ex}{d^2\left(b^2e^2-2bcde+c^2d^2\right)}+\frac{\left(-3b\,e^2+5dec\right)e^2x^2}{2d^3\left(b^2e^2-2bcde+c^2d^2\right)}}{(ex+d)^2}+\frac{\ln(x)}{b\,d^3}+\frac{c^3\ln(cx+b)}{b(b^3e^3-3d\,e^2b^2c+3d^2eb\,c^2-d^3c^3)}-\frac{e\left(b^2e^2-3bcde+3c^2d^2\right)}{d^3(b^3e^3-3d\,e^2b^2c+3d^2e}$
risch	$\frac{\frac{e^2(be-2cd)x}{d^2\left(b^2e^2-2bcde+c^2d^2\right)}+\frac{e(3be-5cd)}{2d\left(b^2e^2-2bcde+c^2d^2\right)}}{(ex+d)^2}+\frac{c^3\ln(cx+b)}{b(b^3e^3-3d\,e^2b^2c+3d^2eb\,c^2-d^3c^3)}-\frac{e^3\ln(-ex-d)b^2}{d^3(b^3e^3-3d\,e^2b^2c+3d^2eb\,c^2-d^3c^3)}$
parallelrisch	$\frac{-6\ln(x)x^2b^2cd\,e^4+6\ln(x)x^2b\,c^2d^2e^3+6\ln(ex+d)x^2b^2cd\,e^4-6\ln(ex+d)x^2b\,c^2d^2e^3-12\ln(x)x\,b^2c\,d^2e^3+12\ln(x)xb\,c^2d^3e^2+}{}$

input
```
int(1/(e*x+d)^3/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
c^3/(b*e-c*d)^3/b*ln(c*x+b)+1/2*e/d/(b*e-c*d)/(e*x+d)^2+e*(b*e-2*c*d)/d^2/
(b*e-c*d)^2/(e*x+d)-e*(b^2*e^2-3*b*c*d*e+3*c^2*d^2)/d^3/(b*e-c*d)^3*ln(e*x
+d)+ln(x)/b/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(132) = 264$.

Time = 4.02 (sec) , antiderivative size = 506, normalized size of antiderivative = 3.78

$$\int \frac{1}{(d+ex)^3\,(bx+cx^2)}\,dx =$$
$$-\frac{5\,bc^2d^4e - 8\,b^2cd^3e^2 + 3\,b^3d^2e^3 + 2\,(2\,bc^2d^3e^2 - 3\,b^2cd^2e^3 + b^3de^4)x + 2\,(c^3d^3e^2x^2 + 2\,c^3d^4ex + c^3d^5)\,\mathrm{lo}}{}$$

input
```
integrate(1/(e*x+d)^3/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
-1/2*(5*b*c^2*d^4*e - 8*b^2*c*d^3*e^2 + 3*b^3*d^2*e^3 + 2*(2*b*c^2*d^3*e^2
 - 3*b^2*c*d^2*e^3 + b^3*d*e^4)*x + 2*(c^3*d^3*e^2*x^2 + 2*c^3*d^4*e*x + c
^3*d^5)*log(c*x + b) - 2*(3*b*c^2*d^4*e - 3*b^2*c*d^3*e^2 + b^3*d^2*e^3 +
(3*b*c^2*d^2*e^3 - 3*b^2*c*d*e^4 + b^3*e^5)*x^2 + 2*(3*b*c^2*d^3*e^2 - 3*b
^2*c*d^2*e^3 + b^3*d*e^4)*x)*log(e*x + d) - 2*(c^3*d^5 - 3*b*c^2*d^4*e + 3
*b^2*c*d^3*e^2 - b^3*d^2*e^3 + (c^3*d^3*e^2 - 3*b*c^2*d^2*e^3 + 3*b^2*c*d*
e^4 - b^3*e^5)*x^2 + 2*(c^3*d^4*e - 3*b*c^2*d^3*e^2 + 3*b^2*c*d^2*e^3 - b^
3*d*e^4)*x)*log(x))/(b*c^3*d^8 - 3*b^2*c^2*d^7*e + 3*b^3*c*d^6*e^2 - b^4*d
^5*e^3 + (b*c^3*d^6*e^2 - 3*b^2*c^2*d^5*e^3 + 3*b^3*c*d^4*e^4 - b^4*d^3*e^
5)*x^2 + 2*(b*c^3*d^7*e - 3*b^2*c^2*d^6*e^2 + 3*b^3*c*d^5*e^3 - b^4*d^4*e^
4)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\,(bx+cx^2)}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**3/(c*x**2+b*x),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. $2(132) = 264$.

Time = 0.04 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.99

$$\int \frac{1}{(d+ex)^3(bx+cx^2)}\,dx = -\frac{c^3\log(cx+b)}{bc^3d^3-3b^2c^2d^2e+3b^3cde^2-b^4e^3} + \frac{(3c^2d^2e-3bcde^2+b^2e^3)\log(ex+d)}{c^3d^6-3bc^2d^5e+3b^2cd^4e^2-b^3d^3e^3} - \frac{5cd^2e-3bde^2+2(2cde^2-be^3)x}{2(c^2d^6-2bcd^5e+b^2d^4e^2+(c^2d^4e^2-2bcd^3e^3+b^2d^2e^4)x^2+2(c^2d^5e-2bcd^4e^2+b^2d^3e^3)x)} + \frac{\log(x)}{bd^3}$$

input
```
integrate(1/(e*x+d)^3/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
-c^3*log(c*x + b)/(b*c^3*d^3 - 3*b^2*c^2*d^2*e + 3*b^3*c*d*e^2 - b^4*e^3)
+ (3*c^2*d^2*e - 3*b*c*d*e^2 + b^2*e^3)*log(e*x + d)/(c^3*d^6 - 3*b*c^2*d^
5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3) - 1/2*(5*c*d^2*e - 3*b*d*e^2 + 2*(2*c
*d*e^2 - b*e^3)*x)/(c^2*d^6 - 2*b*c*d^5*e + b^2*d^4*e^2 + (c^2*d^4*e^2 - 2
*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2 + 2*(c^2*d^5*e - 2*b*c*d^4*e^2 + b^2*d^3*e
^3)*x) + log(x)/(b*d^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.75

$$\int \frac{1}{(d+ex)^3(bx+cx^2)}\,dx = -\frac{c^4\log(|cx+b|)}{bc^4d^3-3b^2c^3d^2e+3b^3c^2de^2-b^4ce^3} + \frac{(3c^2d^2e^2-3bcde^3+b^2e^4)\log(|ex+d|)}{c^3d^6e-3bc^2d^5e^2+3b^2cd^4e^3-b^3d^3e^4} + \frac{\log(|x|)}{bd^3} - \frac{5c^2d^4e-8bcd^3e^2+3b^2d^2e^3+2(2c^2d^3e^2-3bcd^2e^3+b^2de^4)x}{2(cd-be)^3(ex+d)^2d^3}$$

input
```
integrate(1/(e*x+d)^3/(c*x^2+b*x),x, algorithm="giac")
```

output
```
-c^4*log(abs(c*x + b))/(b*c^4*d^3 - 3*b^2*c^3*d^2*e + 3*b^3*c^2*d*e^2 - b^
4*c*e^3) + (3*c^2*d^2*e^2 - 3*b*c*d*e^3 + b^2*e^4)*log(abs(e*x + d))/(c^3*
d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 - b^3*d^3*e^4) + log(abs(x))/(b*
d^3) - 1/2*(5*c^2*d^4*e - 8*b*c*d^3*e^2 + 3*b^2*d^2*e^3 + 2*(2*c^2*d^3*e^2
 - 3*b*c*d^2*e^3 + b^2*d*e^4)*x)/((c*d - b*e)^3*(e*x + d)^2*d^3)
```

Mupad [B] (verification not implemented)

Time = 9.31 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.75

$$\int \frac{1}{(d+ex)^3 (bx+cx^2)} \, dx = \frac{\frac{3\,b\,e^2-5\,c\,d\,e}{2\,d\,\left(b^2\,e^2-2\,b\,c\,d\,e+c^2\,d^2\right)}+\frac{e^2\,x\,\left(b\,e-2\,c\,d\right)}{d^2\,\left(b^2\,e^2-2\,b\,c\,d\,e+c^2\,d^2\right)}}{d^2+2\,d\,e\,x+e^2\,x^2} + \frac{c^3\,\ln\left(b+c\,x\right)}{b^4\,e^3-3\,b^3\,c\,d\,e^2+3\,b^2\,c^2\,d^2\,e-b\,c^3\,d^3} + \frac{\ln\left(d+e\,x\right)\,\left(b^2\,e^3-3\,b\,c\,d\,e^2+3\,c^2\,d^2\,e\right)}{-b^3\,d^3\,e^3+3\,b^2\,c\,d^4\,e^2-3\,b\,c^2\,d^5\,e+c^3\,d^6} + \frac{\ln\left(x\right)}{b\,d^3}$$

input
```
int(1/((b*x + c*x^2)*(d + e*x)^3),x)
```

output
```
((3*b*e^2 - 5*c*d*e)/(2*d*(b^2*e^2 + c^2*d^2 - 2*b*c*d*e)) + (e^2*x*(b*e -
 2*c*d))/(d^2*(b^2*e^2 + c^2*d^2 - 2*b*c*d*e)))/(d^2 + e^2*x^2 + 2*d*e*x)
+ (c^3*log(b + c*x))/(b^4*e^3 - b*c^3*d^3 + 3*b^2*c^2*d^2*e - 3*b^3*c*d*e^
2) + (log(d + e*x)*(b^2*e^3 + 3*c^2*d^2*e - 3*b*c*d*e^2))/(c^3*d^6 - b^3*d
^3*e^3 + 3*b^2*c*d^4*e^2 - 3*b*c^2*d^5*e) + log(x)/(b*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 598, normalized size of antiderivative = 4.46

$$\int \frac{1}{(d+ex)^3 (bx+cx^2)} \, dx = \frac{-6\log(ex+d)\,b\,c^2d^2e^3x^2-12\log(x)\,b^2c\,d^2e^3x-6\log(x)\,b^2cd\,e^4x^2+12\log(x)\,b\,c^2d^3e^2x+6\log(x)\,b\,c^2d}{}$$

input
```
int(1/(e*x+d)^3/(c*x^2+b*x),x)
```

output
```
(2*log(b + c*x)*c**3*d**5 + 4*log(b + c*x)*c**3*d**4*e*x + 2*log(b + c*x)*
c**3*d**3*e**2*x**2 - 2*log(d + e*x)*b**3*d**2*e**3 - 4*log(d + e*x)*b**3*
d*e**4*x - 2*log(d + e*x)*b**3*e**5*x**2 + 6*log(d + e*x)*b**2*c*d**3*e**2
 + 12*log(d + e*x)*b**2*c*d**2*e**3*x + 6*log(d + e*x)*b**2*c*d*e**4*x**2
- 6*log(d + e*x)*b*c**2*d**4*e - 12*log(d + e*x)*b*c**2*d**3*e**2*x - 6*lo
g(d + e*x)*b*c**2*d**2*e**3*x**2 + 2*log(x)*b**3*d**2*e**3 + 4*log(x)*b**3
*d*e**4*x + 2*log(x)*b**3*e**5*x**2 - 6*log(x)*b**2*c*d**3*e**2 - 12*log(x
)*b**2*c*d**2*e**3*x - 6*log(x)*b**2*c*d*e**4*x**2 + 6*log(x)*b*c**2*d**4*
e + 12*log(x)*b*c**2*d**3*e**2*x + 6*log(x)*b*c**2*d**2*e**3*x**2 - 2*log(
x)*c**3*d**5 - 4*log(x)*c**3*d**4*e*x - 2*log(x)*c**3*d**3*e**2*x**2 + 2*b
**3*d**2*e**3 - b**3*e**5*x**2 - 5*b**2*c*d**3*e**2 + 3*b**2*c*d*e**4*x**2
 + 3*b*c**2*d**4*e - 2*b*c**2*d**2*e**3*x**2)/(2*b*d**3*(b**3*d**2*e**3 +
2*b**3*d*e**4*x + b**3*e**5*x**2 - 3*b**2*c*d**3*e**2 - 6*b**2*c*d**2*e**3
*x - 3*b**2*c*d*e**4*x**2 + 3*b*c**2*d**4*e + 6*b*c**2*d**3*e**2*x + 3*b*c
**2*d**2*e**3*x**2 - c**3*d**5 - 2*c**3*d**4*e*x - c**3*d**3*e**2*x**2))
```

3.63 $\int \frac{(d+ex)^5}{(bx+cx^2)^2} \, dx$

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Optimal result

Integrand size = 19, antiderivative size = 118

$$\int \frac{(d+ex)^5}{(bx+cx^2)^2} \, dx = -\frac{d^5}{b^2x} + \frac{e^4(5cd-2be)x}{c^3} + \frac{e^5x^2}{2c^2} - \frac{(cd-be)^5}{b^2c^4(b+cx)} - \frac{d^4(2cd-5be)\log(x)}{b^3} + \frac{(cd-be)^4(2cd+3be)\log(b+cx)}{b^3c^4}$$

output
```
-d^5/b^2/x+e^4*(-2*b*e+5*c*d)*x/c^3+1/2*e^5*x^2/c^2-(-b*e+c*d)^5/b^2/c^4/(
c*x+b)-d^4*(-5*b*e+2*c*d)*ln(x)/b^3+(-b*e+c*d)^4*(3*b*e+2*c*d)*ln(c*x+b)/b
^3/c^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^5}{(bx+cx^2)^2} \, dx = -\frac{d^5}{b^2x} + \frac{e^4(5cd-2be)x}{c^3} + \frac{e^5x^2}{2c^2} + \frac{(-cd+be)^5}{b^2c^4(b+cx)} + \frac{d^4(-2cd+5be)\log(x)}{b^3} + \frac{(cd-be)^4(2cd+3be)\log(b+cx)}{b^3c^4}$$

input
```
Integrate[(d + e*x)^5/(b*x + c*x^2)^2,x]
```

output
```
-(d^5/(b^2*x)) + (e^4*(5*c*d - 2*b*e)*x)/c^3 + (e^5*x^2)/(2*c^2) + (-(c*d)
 + b*e)^5/(b^2*c^4*(b + c*x)) + (d^4*(-2*c*d + 5*b*e)*Log[x])/b^3 + ((c*d
- b*e)^4*(2*c*d + 3*b*e)*Log[b + c*x])/(b^3*c^4)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5}{(bx+cx^2)^2}\,dx$$

$$\Big\downarrow \; 1141$$

$$c^2\int \left(\frac{d^5}{b^2c^2x^2}-\frac{(2cd-5be)d^4}{b^3c^2x}+\frac{e^4(5cd-2be)}{c^5}+\frac{e^5x}{c^4}+\frac{(cd-be)^4(2cd+3be)}{b^3c^5(b+cx)}+\frac{(cd-be)^5}{b^2c^5(b+cx)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$c^2\left(\frac{(cd-be)^4(3be+2cd)\log(b+cx)}{b^3c^6}-\frac{d^4\log(x)(2cd-5be)}{b^3c^2}-\frac{(cd-be)^5}{b^2c^6(b+cx)}-\frac{d^5}{b^2c^2x}+\frac{e^4x(5cd-2be)}{c^5}+\frac{e^5x^2}{2c^4}\right)$$

input
```
Int[(d + e*x)^5/(b*x + c*x^2)^2,x]
```

output
```
c^2*(-(d^5/(b^2*c^2*x)) + (e^4*(5*c*d - 2*b*e)*x)/c^5 + (e^5*x^2)/(2*c^4)
- (c*d - b*e)^5/(b^2*c^6*(b + c*x)) - (d^4*(2*c*d - 5*b*e)*Log[x])/(b^3*c^
2) + ((c*d - b*e)^4*(2*c*d + 3*b*e)*Log[b + c*x])/(b^3*c^6))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.69

method	result
default	$-\frac{e^4\left(-\frac{1}{2}ce\,x^2+2bex-5cdx\right)}{c^3}+\frac{\left(3b^5e^5-10b^4d\,e^4c+10b^3d^2e^3c^2-5c^4d^4eb+2d^5c^5\right)\ln(cx+b)}{c^4b^3}-\frac{-b^5e^5+5b^4d\,e^4c-10b^3d^2e^3}{c^4b}$
norman	$\frac{-\frac{d^5}{b}+\frac{e^5x^4}{2c}-\frac{e^4(3be-10cd)x^3}{2c^2}-\frac{\left(3b^5e^5-10b^4d\,e^4c+10b^3d^2e^3c^2-10b^2c^3d^3e^2+5c^4d^4eb-2d^5c^5\right)x^2}{b^3c^3}}{x(cx+b)}+\left(3b^5e^5-10b^4d\,e^4c+10b^3d^2e\right.$
risch	$\frac{e^5x^2}{2c^2}-\frac{2e^5bx}{c^3}+\frac{5d\,e^4x}{c^2}+\frac{\frac{\left(b^5e^5-5b^4d\,e^4c+10b^3d^2e^3c^2-10b^2c^3d^3e^2+5c^4d^4eb-2d^5c^5\right)x}{b^2c}-\frac{d^5c^3}{b}}{c^3x(cx+b)}+\frac{5d^4\ln(x)e}{b^2}-\frac{2d^5\ln(x)}{b^3}$
parallelrisch	$4\ln(cx+b)xb\,c^5d^5+10x^3b^3c^3d\,e^4+20x^2b^4c^2d\,e^4-10x^2b\,c^5d^4e-20x^2b^3c^3d^2e^3+20x^2b^2c^4d^3e^2-4\ln(x)x^2c^6d^5-10\ln(cx+b)x^2$

input
```
int((e*x+d)^5/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-e^4/c^3*(-1/2*c*e*x^2+2*b*e*x-5*c*d*x)+1/c^4*(3*b^5*e^5-10*b^4*c*d*e^4+10
*b^3*c^2*d^2*e^3-5*b*c^4*d^4*e+2*c^5*d^5)/b^3*ln(c*x+b)-(-b^5*e^5+5*b^4*c*
d*e^4-10*b^3*c^2*d^2*e^3+10*b^2*c^3*d^3*e^2-5*b*c^4*d^4*e+c^5*d^5)/c^4/b^2
/(c*x+b)-d^5/b^2/x+d^4*(5*b*e-2*c*d)/b^3*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(116) = 232$.

Time = 0.09 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.96

$$\int \frac{(d+ex)^5}{(bx+cx^2)^2}\,dx$$
$$= \frac{b^3c^3e^5x^4 - 2\,b^2c^4d^5 + (10\,b^3c^3de^4 - 3\,b^4c^2e^5)x^3 + 2\,(5\,b^4c^2de^4 - 2\,b^5ce^5)x^2 - 2\,(2\,bc^5d^5 - 5\,b^2c^4d^4e + 10\,b}{}$$

input
```
integrate((e*x+d)^5/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
1/2*(b^3*c^3*e^5*x^4 - 2*b^2*c^4*d^5 + (10*b^3*c^3*d*e^4 - 3*b^4*c^2*e^5)*
x^3 + 2*(5*b^4*c^2*d*e^4 - 2*b^5*c*e^5)*x^2 - 2*(2*b*c^5*d^5 - 5*b^2*c^4*d
^4*e + 10*b^3*c^3*d^3*e^2 - 10*b^4*c^2*d^2*e^3 + 5*b^5*c*d*e^4 - b^6*e^5)*
x + 2*((2*c^6*d^5 - 5*b*c^5*d^4*e + 10*b^3*c^3*d^2*e^3 - 10*b^4*c^2*d*e^4
+ 3*b^5*c*e^5)*x^2 + (2*b*c^5*d^5 - 5*b^2*c^4*d^4*e + 10*b^4*c^2*d^2*e^3 -
 10*b^5*c*d*e^4 + 3*b^6*e^5)*x)*log(c*x + b) - 2*((2*c^6*d^5 - 5*b*c^5*d^4
*e)*x^2 + (2*b*c^5*d^5 - 5*b^2*c^4*d^4*e)*x)*log(x))/(b^3*c^5*x^2 + b^4*c^
4*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(110) = 220$.

Time = 1.68 (sec) , antiderivative size = 381, normalized size of antiderivative = 3.23

$$\begin{aligned}
&\int \frac{(d+ex)^5}{(bx+cx^2)^2}\,dx \\
&= x\left(-\frac{2be^5}{c^3}+\frac{5de^4}{c^2}\right) \\
&\quad + \frac{-bc^4d^5 + x(b^5e^5 - 5b^4cde^4 + 10b^3c^2d^2e^3 - 10b^2c^3d^3e^2 + 5bc^4d^4e - 2c^5d^5)}{b^3c^4x + b^2c^5x^2} \\
&\quad + \frac{e^5x^2}{2c^2} + \frac{d^4\cdot(5be-2cd)\log\left(x + \frac{-5b^2c^3d^4e+2bc^4d^5+bc^3d^4\cdot(5be-2cd)}{3b^5e^5-10b^4cde^4+10b^3c^2d^2e^3-10bc^4d^4e+4c^5d^5}\right)}{b^3} \\
&\quad + \frac{(be-cd)^4\cdot(3be+2cd)\log\left(x + \frac{-5b^2c^3d^4e+2bc^4d^5+\frac{b(be-cd)^4\cdot(3be+2cd)}{c}}{3b^5e^5-10b^4cde^4+10b^3c^2d^2e^3-10bc^4d^4e+4c^5d^5}\right)}{b^3c^4}
\end{aligned}$$

input
```
integrate((e*x+d)**5/(c*x**2+b*x)**2,x)
```

output
```
x*(-2*b*e**5/c**3 + 5*d*e**4/c**2) + (-b*c**4*d**5 + x*(b**5*e**5 - 5*b**4
*c*d*e**4 + 10*b**3*c**2*d**2*e**3 - 10*b**2*c**3*d**3*e**2 + 5*b*c**4*d**
4*e - 2*c**5*d**5))/(b**3*c**4*x + b**2*c**5*x**2) + e**5*x**2/(2*c**2) +
d**4*(5*b*e - 2*c*d)*log(x + (-5*b**2*c**3*d**4*e + 2*b*c**4*d**5 + b*c**3
*d**4*(5*b*e - 2*c*d))/(3*b**5*e**5 - 10*b**4*c*d*e**4 + 10*b**3*c**2*d**2
*e**3 - 10*b*c**4*d**4*e + 4*c**5*d**5))/b**3 + (b*e - c*d)**4*(3*b*e + 2*
c*d)*log(x + (-5*b**2*c**3*d**4*e + 2*b*c**4*d**5 + b*(b*e - c*d)**4*(3*b*
e + 2*c*d)/c)/(3*b**5*e**5 - 10*b**4*c*d*e**4 + 10*b**3*c**2*d**2*e**3 - 1
0*b*c**4*d**4*e + 4*c**5*d**5))/(b**3*c**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.83

$$\int \frac{(d+ex)^5}{(bx+cx^2)^2}\,dx = -\frac{bc^4d^5+(2\,c^5d^5-5\,bc^4d^4e+10\,b^2c^3d^3e^2-10\,b^3c^2d^2e^3+5\,b^4cde^4-b^5e^5)x}{b^2c^5x^2+b^3c^4x} - \frac{(2\,cd^5-5\,bd^4e)\log(x)}{b^3} + \frac{ce^5x^2+2\,(5\,cde^4-2\,be^5)x}{2\,c^3} + \frac{(2\,c^5d^5-5\,bc^4d^4e+10\,b^3c^2d^2e^3-10\,b^4cde^4+3\,b^5e^5)\log(cx+b)}{b^3c^4}$$

input
```
integrate((e*x+d)^5/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-(b*c^4*d^5 + (2*c^5*d^5 - 5*b*c^4*d^4*e + 10*b^2*c^3*d^3*e^2 - 10*b^3*c^2
*d^2*e^3 + 5*b^4*c*d*e^4 - b^5*e^5)*x)/(b^2*c^5*x^2 + b^3*c^4*x) - (2*c*d^
5 - 5*b*d^4*e)*log(x)/b^3 + 1/2*(c*e^5*x^2 + 2*(5*c*d*e^4 - 2*b*e^5)*x)/c^
3 + (2*c^5*d^5 - 5*b*c^4*d^4*e + 10*b^3*c^2*d^2*e^3 - 10*b^4*c*d*e^4 + 3*b
^5*e^5)*log(c*x + b)/(b^3*c^4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.83

$$\int \frac{(d+ex)^5}{(bx+cx^2)^2}\,dx = -\frac{(2\,cd^5-5\,bd^4e)\log(|x|)}{b^3}+\frac{c^2e^5x^2+10\,c^2de^4x-4\,bce^5x}{2\,c^4}+\frac{(2\,c^5d^5-5\,bc^4d^4e+10\,b^3c^2d^2e^3-10\,b^4cde^4+3\,b^5e^5)\log(|cx+b|)}{b^3c^4}-\frac{bc^4d^5+(2\,c^5d^5-5\,bc^4d^4e+10\,b^2c^3d^3e^2-10\,b^3c^2d^2e^3+5\,b^4cde^4-b^5e^5)x}{(cx+b)b^2c^4x}$$

input
```
integrate((e*x+d)^5/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
-(2*c*d^5 - 5*b*d^4*e)*log(abs(x))/b^3 + 1/2*(c^2*e^5*x^2 + 10*c^2*d*e^4*x
 - 4*b*c*e^5*x)/c^4 + (2*c^5*d^5 - 5*b*c^4*d^4*e + 10*b^3*c^2*d^2*e^3 - 10
*b^4*c*d*e^4 + 3*b^5*e^5)*log(abs(c*x + b))/(b^3*c^4) - (b*c^4*d^5 + (2*c^
5*d^5 - 5*b*c^4*d^4*e + 10*b^2*c^3*d^3*e^2 - 10*b^3*c^2*d^2*e^3 + 5*b^4*c*
d*e^4 - b^5*e^5)*x)/((c*x + b)*b^2*c^4*x)
```

Mupad [B] (verification not implemented)

Time = 8.97 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.85

$$\int \frac{(d+ex)^5}{(bx+cx^2)^2}\,dx = \frac{e^5\,x^2}{2\,c^2}-\frac{\frac{c^3\,d^5}{b}-\frac{x\,\left(b^5\,e^5-5\,b^4\,c\,d\,e^4+10\,b^3\,c^2\,d^2\,e^3-10\,b^2\,c^3\,d^3\,e^2+5\,b\,c^4\,d^4\,e-2\,c^5\,d^5\right)}{b^2\,c}}{c^4\,x^2+b\,c^3\,x}-x\left(\frac{2\,b\,e^5}{c^3}-\frac{5\,d\,e^4}{c^2}\right)+\frac{\ln(b+c\,x)\,\left(3\,b^5\,e^5-10\,b^4\,c\,d\,e^4+10\,b^3\,c^2\,d^2\,e^3-5\,b\,c^4\,d^4\,e+2\,c^5\,d^5\right)}{b^3\,c^4}+\frac{d^4\,\ln(x)\,\left(5\,b\,e-2\,c\,d\right)}{b^3}$$

input
```
int((d + e*x)^5/(b*x + c*x^2)^2,x)
```

output
```
(e^5*x^2)/(2*c^2) - ((c^3*d^5)/b - (x*(b^5*e^5 - 2*c^5*d^5 - 10*b^2*c^3*d^
3*e^2 + 10*b^3*c^2*d^2*e^3 + 5*b*c^4*d^4*e - 5*b^4*c*d*e^4))/(b^2*c))/(c^4
*x^2 + b*c^3*x) - x*((2*b*e^5)/c^3 - (5*d*e^4)/c^2) + (log(b + c*x)*(3*b^5
*e^5 + 2*c^5*d^5 + 10*b^3*c^2*d^2*e^3 - 5*b*c^4*d^4*e - 10*b^4*c*d*e^4))/(
b^3*c^4) + (d^4*log(x)*(5*b*e - 2*c*d))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 398, normalized size of antiderivative = 3.37

$$\int \frac{(d+ex)^5}{(bx+cx^2)^2}\,dx$$
$$= \frac{6\log(cx+b)\,b^6e^5x - 20\log(cx+b)\,b^5cd\,e^4x + 6\log(cx+b)\,b^5c\,e^5x^2 + 20\log(cx+b)\,b^4c^2d^2e^3x - 20\log(}{}$$

input
```
int((e*x+d)^5/(c*x^2+b*x)^2,x)
```

output
```
(6*log(b + c*x)*b**6*e**5*x - 20*log(b + c*x)*b**5*c*d*e**4*x + 6*log(b +
c*x)*b**5*c*e**5*x**2 + 20*log(b + c*x)*b**4*c**2*d**2*e**3*x - 20*log(b +
 c*x)*b**4*c**2*d*e**4*x**2 + 20*log(b + c*x)*b**3*c**3*d**2*e**3*x**2 - 1
0*log(b + c*x)*b**2*c**4*d**4*e*x + 4*log(b + c*x)*b*c**5*d**5*x - 10*log(
b + c*x)*b*c**5*d**4*e*x**2 + 4*log(b + c*x)*c**6*d**5*x**2 + 10*log(x)*b*
*2*c**4*d**4*e*x - 4*log(x)*b*c**5*d**5*x + 10*log(x)*b*c**5*d**4*e*x**2 -
 4*log(x)*c**6*d**5*x**2 - 6*b**5*c*e**5*x**2 + 20*b**4*c**2*d*e**4*x**2 -
 3*b**4*c**2*e**5*x**3 - 20*b**3*c**3*d**2*e**3*x**2 + 10*b**3*c**3*d*e**4
*x**3 + b**3*c**3*e**5*x**4 - 2*b**2*c**4*d**5 + 20*b**2*c**4*d**3*e**2*x*
*2 - 10*b*c**5*d**4*e*x**2 + 4*c**6*d**5*x**2)/(2*b**3*c**4*x*(b + c*x))
```

3.64 $\int \frac{(d+ex)^4}{(bx+cx^2)^2} dx$

Optimal result 500
Mathematica [A] (verified) 500
Rubi [A] (verified) 501
Maple [A] (verified) 502
Fricas [B] (verification not implemented) 503
Sympy [B] (verification not implemented) 503
Maxima [A] (verification not implemented) 504
Giac [A] (verification not implemented) 504
Mupad [B] (verification not implemented) 505
Reduce [B] (verification not implemented) 505

Optimal result

Integrand size = 19, antiderivative size = 94

$$\int \frac{(d+ex)^4}{(bx+cx^2)^2} dx = -\frac{d^4}{b^2x} + \frac{e^4x}{c^2} - \frac{(cd-be)^4}{b^2c^3(b+cx)} - \frac{2d^3(cd-2be)\log(x)}{b^3} + \frac{2(cd-be)^3(cd+be)\log(b+cx)}{b^3c^3}$$

output
```
-d^4/b^2/x+e^4*x/c^2-(-b*e+c*d)^4/b^2/c^3/(c*x+b)-2*d^3*(-2*b*e+c*d)*ln(x)
/b^3+2*(-b*e+c*d)^3*(b*e+c*d)*ln(c*x+b)/b^3/c^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^4}{(bx+cx^2)^2} dx = -\frac{d^4}{b^2x} + \frac{e^4x}{c^2} - \frac{(cd-be)^4}{b^2c^3(b+cx)} + \frac{2d^3(-cd+2be)\log(x)}{b^3} + \frac{2(cd-be)^3(cd+be)\log(b+cx)}{b^3c^3}$$

input
```
Integrate[(d + e*x)^4/(b*x + c*x^2)^2,x]
```

output
```
-(d^4/(b^2*x)) + (e^4*x)/c^2 - (c*d - b*e)^4/(b^2*c^3*(b + c*x)) + (2*d^3*
(-(c*d) + 2*b*e)*Log[x])/b^3 + (2*(c*d - b*e)^3*(c*d + b*e)*Log[b + c*x])/
(b^3*c^3)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(bx+cx^2)^2}\,dx$$

$$\Big\downarrow \; 1141$$

$$c^2\int \left(\frac{d^4}{b^2c^2x^2}-\frac{2(cd-2be)d^3}{b^3c^2x}+\frac{e^4}{c^4}+\frac{2(cd-be)^3(cd+be)}{b^3c^4(b+cx)}+\frac{(cd-be)^4}{b^2c^4(b+cx)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$c^2\left(\frac{2(cd-be)^3(be+cd)\log(b+cx)}{b^3c^5}-\frac{2d^3\log(x)(cd-2be)}{b^3c^2}-\frac{(cd-be)^4}{b^2c^5(b+cx)}-\frac{d^4}{b^2c^2x}+\frac{e^4x}{c^4}\right)$$

input
```
Int[(d + e*x)^4/(b*x + c*x^2)^2,x]
```

output
```
c^2*(-(d^4/(b^2*c^2*x)) + (e^4*x)/c^4 - (c*d - b*e)^4/(b^2*c^5*(b + c*x))
- (2*d^3*(c*d - 2*b*e)*Log[x])/(b^3*c^2) + (2*(c*d - b*e)^3*(c*d + b*e)*Lo
g[b + c*x])/(b^3*c^5))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.64

method	result
default	$\frac{e^4x}{c^2}+\frac{\left(-2e^4b^4+4d\,e^3b^3c-4d^3eb\,c^3+2c^4d^4\right)\ln(cx+b)}{b^3c^3}-\frac{e^4b^4-4d\,e^3b^3c+6d^2e^2b^2c^2-4d^3eb\,c^3+c^4d^4}{c^3b^2(cx+b)}-\frac{d^4}{b^2x}+\frac{2d^3(2be-$
norman	$\frac{\frac{e^4x^3}{c}+\frac{\left(2e^4b^4-4d\,e^3b^3c+6d^2e^2b^2c^2-4d^3eb\,c^3+2c^4d^4\right)x^2}{b^3c^2}-\frac{d^4}{b}}{x(cx+b)}+\frac{2d^3(2be-cd)\ln(x)}{b^3}-\frac{2\left(e^4b^4-2d\,e^3b^3c+2d^3eb\,c^3-c^4d^4\right)\ln(}{b^3c^3}$
risch	$\frac{e^4x}{c^2}+\frac{-\frac{\left(e^4b^4-4d\,e^3b^3c+6d^2e^2b^2c^2-4d^3eb\,c^3+2c^4d^4\right)x}{b^2c}-\frac{c^2d^4}{b}}{c^2x(cx+b)}-\frac{2b\ln(cx+b)e^4}{c^3}+\frac{4\ln(cx+b)d\,e^3}{c^2}-\frac{4\ln(cx+b)d^3e}{b^2}+2$
parallelrisch	$4\ln(x)x^2b\,c^4d^3e-2\ln(x)x^2c^5d^4-2\ln(cx+b)x^2b^4c\,e^4+4\ln(cx+b)x^2b^3c^2d\,e^3-4\ln(cx+b)x^2b\,c^4d^3e+2\ln(cx+b)x^2c^5d^4+x^3b^3$

input
```
int((e*x+d)^4/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
e^4*x/c^2+(-2*b^4*e^4+4*b^3*c*d*e^3-4*b*c^3*d^3*e+2*c^4*d^4)/b^3/c^3*ln(c*
x+b)-1/c^3*(b^4*e^4-4*b^3*c*d*e^3+6*b^2*c^2*d^2*e^2-4*b*c^3*d^3*e+c^4*d^4)
/b^2/(c*x+b)-d^4/b^2/x+2*d^3*(2*b*e-c*d)/b^3*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(94) = 188$.

Time = 0.10 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.73

$$\int \frac{(d+ex)^4}{(bx+cx^2)^2}\,dx$$
$$= \frac{b^3c^2e^4x^3 + b^4ce^4x^2 - b^2c^3d^4 - (2\,bc^4d^4 - 4\,b^2c^3d^3e + 6\,b^3c^2d^2e^2 - 4\,b^4cde^3 + b^5e^4)x + 2\,((c^5d^4 - 2\,bc^4d^3e}{}$$

input
```
integrate((e*x+d)^4/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
(b^3*c^2*e^4*x^3 + b^4*c*e^4*x^2 - b^2*c^3*d^4 - (2*b*c^4*d^4 - 4*b^2*c^3*
d^3*e + 6*b^3*c^2*d^2*e^2 - 4*b^4*c*d*e^3 + b^5*e^4)*x + 2*((c^5*d^4 - 2*b
*c^4*d^3*e + 2*b^3*c^2*d*e^3 - b^4*c*e^4)*x^2 + (b*c^4*d^4 - 2*b^2*c^3*d^3
*e + 2*b^4*c*d*e^3 - b^5*e^4)*x)*log(c*x + b) - 2*((c^5*d^4 - 2*b*c^4*d^3*
e)*x^2 + (b*c^4*d^4 - 2*b^2*c^3*d^3*e)*x)*log(x))/(b^3*c^4*x^2 + b^4*c^3*x
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(87) = 174$.

Time = 1.20 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.26

$$\int \frac{(d+ex)^4}{(bx+cx^2)^2}\,dx = \frac{-bc^3d^4 + x(-b^4e^4 + 4b^3cde^3 - 6b^2c^2d^2e^2 + 4bc^3d^3e - 2c^4d^4)}{b^3c^3x + b^2c^4x^2} + \frac{e^4x}{c^2} + \frac{2d^3\cdot(2be-cd)\log\left(x + \frac{4b^2c^2d^3e - 2bc^3d^4 - 2bc^2d^3\cdot(2be-cd)}{2b^4e^4 - 4b^3cde^3 + 8bc^3d^3e - 4c^4d^4}\right)}{b^3} - \frac{2(be-cd)^3\,(be+cd)\log\left(x + \frac{4b^2c^2d^3e - 2bc^3d^4 + \frac{2b(be-cd)^3(be+cd)}{c}}{2b^4e^4 - 4b^3cde^3 + 8bc^3d^3e - 4c^4d^4}\right)}{b^3c^3}$$

input
```
integrate((e*x+d)**4/(c*x**2+b*x)**2,x)
```

output
```
(-b*c**3*d**4 + x*(-b**4*e**4 + 4*b**3*c*d*e**3 - 6*b**2*c**2*d**2*e**2 +
4*b*c**3*d**3*e - 2*c**4*d**4))/(b**3*c**3*x + b**2*c**4*x**2) + e**4*x/c*
*2 + 2*d**3*(2*b*e - c*d)*log(x + (4*b**2*c**2*d**3*e - 2*b*c**3*d**4 - 2*
b*c**2*d**3*(2*b*e - c*d))/(2*b**4*e**4 - 4*b**3*c*d*e**3 + 8*b*c**3*d**3*
e - 4*c**4*d**4))/b**3 - 2*(b*e - c*d)**3*(b*e + c*d)*log(x + (4*b**2*c**2
*d**3*e - 2*b*c**3*d**4 + 2*b*(b*e - c*d)**3*(b*e + c*d)/c)/(2*b**4*e**4 -
 4*b**3*c*d*e**3 + 8*b*c**3*d**3*e - 4*c**4*d**4))/(b**3*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.73

$$\int \frac{(d+ex)^4}{(bx+cx^2)^2}\,dx = \frac{e^4x}{c^2} - \frac{bc^3d^4 + (2\,c^4d^4 - 4\,bc^3d^3e + 6\,b^2c^2d^2e^2 - 4\,b^3cde^3 + b^4e^4)x}{b^2c^4x^2 + b^3c^3x} - \frac{2\,(cd^4 - 2\,bd^3e)\log(x)}{b^3} + \frac{2\,(c^4d^4 - 2\,bc^3d^3e + 2\,b^3cde^3 - b^4e^4)\log(cx+b)}{b^3c^3}$$

input
```
integrate((e*x+d)^4/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
e^4*x/c^2 - (b*c^3*d^4 + (2*c^4*d^4 - 4*b*c^3*d^3*e + 6*b^2*c^2*d^2*e^2 -
4*b^3*c*d*e^3 + b^4*e^4)*x)/(b^2*c^4*x^2 + b^3*c^3*x) - 2*(c*d^4 - 2*b*d^3
*e)*log(x)/b^3 + 2*(c^4*d^4 - 2*b*c^3*d^3*e + 2*b^3*c*d*e^3 - b^4*e^4)*log
(c*x + b)/(b^3*c^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.73

$$\int \frac{(d+ex)^4}{(bx+cx^2)^2}\,dx = \frac{e^4x}{c^2} - \frac{2\,(cd^4 - 2\,bd^3e)\log(|x|)}{b^3} + \frac{2\,(c^4d^4 - 2\,bc^3d^3e + 2\,b^3cde^3 - b^4e^4)\log(|cx+b|)}{b^3c^3} - \frac{bc^2d^4 + \frac{(2\,c^4d^4 - 4\,bc^3d^3e + 6\,b^2c^2d^2e^2 - 4\,b^3cde^3 + b^4e^4)x}{c}}{(cx+b)b^2c^2x}$$

input
```
integrate((e*x+d)^4/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
e^4*x/c^2 - 2*(c*d^4 - 2*b*d^3*e)*log(abs(x))/b^3 + 2*(c^4*d^4 - 2*b*c^3*d
^3*e + 2*b^3*c*d*e^3 - b^4*e^4)*log(abs(c*x + b))/(b^3*c^3) - (b*c^2*d^4 +
 (2*c^4*d^4 - 4*b*c^3*d^3*e + 6*b^2*c^2*d^2*e^2 - 4*b^3*c*d*e^3 + b^4*e^4)
*x/c)/((c*x + b)*b^2*c^2*x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.77

$$\int \frac{(d+ex)^4}{(bx+cx^2)^2}\,dx = \frac{e^4\,x}{c^2} - \frac{\frac{c^2\,d^4}{b} + \frac{x\,\left(b^4\,e^4-4\,b^3\,c\,d\,e^3+6\,b^2\,c^2\,d^2\,e^2-4\,b\,c^3\,d^3\,e+2\,c^4\,d^4\right)}{b^2\,c}}{c^3\,x^2+b\,c^2\,x} + \frac{2\,d^3\,\ln\left(x\right)\,\left(2\,b\,e-c\,d\right)}{b^3} - \frac{\ln\left(b+c\,x\right)\,\left(2\,b^4\,e^4-4\,b^3\,c\,d\,e^3+4\,b\,c^3\,d^3\,e-2\,c^4\,d^4\right)}{b^3\,c^3}$$

input
```
int((d + e*x)^4/(b*x + c*x^2)^2,x)
```

output
```
(e^4*x)/c^2 - ((c^2*d^4)/b + (x*(b^4*e^4 + 2*c^4*d^4 + 6*b^2*c^2*d^2*e^2 -
 4*b*c^3*d^3*e - 4*b^3*c*d*e^3))/(b^2*c))/(c^3*x^2 + b*c^2*x) + (2*d^3*log
(x)*(2*b*e - c*d))/b^3 - (log(b + c*x)*(2*b^4*e^4 - 2*c^4*d^4 + 4*b*c^3*d^
3*e - 4*b^3*c*d*e^3))/(b^3*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 307, normalized size of antiderivative = 3.27

$$\int \frac{(d+ex)^4}{(bx+cx^2)^2}\,dx = \frac{-2\log(cx+b)\,b^5e^4x+4\log(cx+b)\,b^4cd\,e^3x-2\log(cx+b)\,b^4c\,e^4x^2+4\log(cx+b)\,b^3c^2d\,e^3x^2-4\log(c}{}$$

input
```
int((e*x+d)^4/(c*x^2+b*x)^2,x)
```

output
```
( - 2*log(b + c*x)*b**5*e**4*x + 4*log(b + c*x)*b**4*c*d*e**3*x - 2*log(b
+ c*x)*b**4*c*e**4*x**2 + 4*log(b + c*x)*b**3*c**2*d*e**3*x**2 - 4*log(b +
 c*x)*b**2*c**3*d**3*e*x + 2*log(b + c*x)*b*c**4*d**4*x - 4*log(b + c*x)*b
*c**4*d**3*e*x**2 + 2*log(b + c*x)*c**5*d**4*x**2 + 4*log(x)*b**2*c**3*d**
3*e*x - 2*log(x)*b*c**4*d**4*x + 4*log(x)*b*c**4*d**3*e*x**2 - 2*log(x)*c*
*5*d**4*x**2 + 2*b**4*c*e**4*x**2 - 4*b**3*c**2*d*e**3*x**2 + b**3*c**2*e*
*4*x**3 - b**2*c**3*d**4 + 6*b**2*c**3*d**2*e**2*x**2 - 4*b*c**4*d**3*e*x*
*2 + 2*c**5*d**4*x**2)/(b**3*c**3*x*(b + c*x))
```

3.65 $\int \frac{(d+ex)^3}{(bx+cx^2)^2} dx$

Optimal result 507
Mathematica [A] (verified) 507
Rubi [A] (verified) 508
Maple [A] (verified) 509
Fricas [B] (verification not implemented) 510
Sympy [B] (verification not implemented) 510
Maxima [A] (verification not implemented) 511
Giac [A] (verification not implemented) 511
Mupad [B] (verification not implemented) 512
Reduce [B] (verification not implemented) 512

Optimal result

Integrand size = 19, antiderivative size = 87

$$\int \frac{(d+ex)^3}{(bx+cx^2)^2} dx = -\frac{d^3}{b^2x} - \frac{(cd-be)^3}{b^2c^2(b+cx)} - \frac{d^2(2cd-3be)\log(x)}{b^3} + \frac{(cd-be)^2(2cd+be)\log(b+cx)}{b^3c^2}$$

output
```
-d^3/b^2/x-(-b*e+c*d)^3/b^2/c^2/(c*x+b)-d^2*(-3*b*e+2*c*d)*ln(x)/b^3+(-b*e
+c*d)^2*(b*e+2*c*d)*ln(c*x+b)/b^3/c^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^3}{(bx+cx^2)^2} dx = \frac{-\frac{bd^3}{x} + \frac{b(-cd+be)^3}{c^2(b+cx)} + d^2(-2cd+3be)\log(x) + \frac{(cd-be)^2(2cd+be)\log(b+cx)}{c^2}}{b^3}$$

input
```
Integrate[(d + e*x)^3/(b*x + c*x^2)^2,x]
```

output
```
(-((b*d^3)/x) + (b*(-(c*d) + b*e)^3)/(c^2*(b + c*x)) + d^2*(-2*c*d + 3*b*e
)*Log[x] + ((c*d - b*e)^2*(2*c*d + b*e)*Log[b + c*x])/c^2)/b^3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(bx+cx^2)^2}\,dx$$

$$\Big\downarrow \; 1141$$

$$c^2\int \left(\frac{d^3}{b^2c^2x^2}-\frac{(2cd-3be)d^2}{b^3c^2x}+\frac{(cd-be)^2(2cd+be)}{b^3c^3(b+cx)}+\frac{(cd-be)^3}{b^2c^3(b+cx)^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$c^2\left(\frac{(cd-be)^2(be+2cd)\log(b+cx)}{b^3c^4}-\frac{d^2\log(x)(2cd-3be)}{b^3c^2}-\frac{(cd-be)^3}{b^2c^4(b+cx)}-\frac{d^3}{b^2c^2x}\right)$$

input
```
Int[(d + e*x)^3/(b*x + c*x^2)^2,x]
```

output
```
c^2*(-(d^3/(b^2*c^2*x)) - (c*d - b*e)^3/(b^2*c^4*(b + c*x)) - (d^2*(2*c*d
- 3*b*e)*Log[x])/(b^3*c^2) + ((c*d - b*e)^2*(2*c*d + b*e)*Log[b + c*x])/(b
^3*c^4))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.39

method	result
default	$\frac{\left(b^{3} e^{3}-3 d^{2} e b\,c^{2}+2 d^{3} c^{3}\right) \ln \left(c x +b \right)}{b^{3} c^{2}}-\frac{-b^{3} e^{3}+3 d\,e^{2} b^{2} c -3 d^{2} e b\,c^{2}+d^{3} c^{3}}{b^{2} c^{2} \left(c x +b \right)}-\frac{d^{3}}{b^{2} x}+\frac{d^{2} \left(3 b e -2 c d \right) \ln \left(x \right)}{b^{3}}$
norman	$\frac{-\frac{d^{3}}{b}-\frac{\left(b^{3} e^{3}-3 d\,e^{2} b^{2} c +3 d^{2} e b\,c^{2}-2 d^{3} c^{3}\right) x^{2}}{b^{3} c}}{x \left(c x +b \right)}+\frac{d^{2} \left(3 b e -2 c d \right) \ln \left(x \right)}{b^{3}}+\frac{\left(b^{3} e^{3}-3 d^{2} e b\,c^{2}+2 d^{3} c^{3}\right) \ln \left(c x +b \right)}{b^{3} c^{2}}$
risch	$\frac{\frac{\left(b^{3} e^{3}-3 d\,e^{2} b^{2} c +3 d^{2} e b\,c^{2}-2 d^{3} c^{3}\right) x}{b^{2} c^{2}}-\frac{d^{3}}{b}}{x \left(c x +b \right)}+\frac{\ln \left(-c x -b \right) e^{3}}{c^{2}}-\frac{3 \ln \left(-c x -b \right) d^{2} e}{b^{2}}+\frac{2 c \ln \left(-c x -b \right) d^{3}}{b^{3}}+\frac{3 d^{2} \ln \left(x \right) e}{b^{2}}-\frac{2 d^{3} \ln (}{b^{3}}$
parallelrisch	$\frac{3 \ln \left(x \right) x^{2} b\,c^{3} d^{2} e -2 \ln \left(x \right) x^{2} c^{4} d^{3}+\ln \left(c x +b \right) x^{2} b^{3} c\,e^{3}-3 \ln \left(c x +b \right) x^{2} b\,c^{3} d^{2} e +2 \ln \left(c x +b \right) x^{2} c^{4} d^{3}+3 \ln \left(x \right) x\,b^{2} c^{2} d^{2} e -2 \ln \left(x \right) x b\,c^{3}}{b^{3} c^{2} x \left(c x +b \right)}$

input
```
int((e*x+d)^3/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(b^3*e^3-3*b*c^2*d^2*e+2*c^3*d^3)/b^3/c^2*ln(c*x+b)-(-b^3*e^3+3*b^2*c*d*e^
2-3*b*c^2*d^2*e+c^3*d^3)/b^2/c^2/(c*x+b)-d^3/b^2/x+d^2*(3*b*e-2*c*d)/b^3*l
n(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 198 vs. $2(87) = 174$.

Time = 0.09 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.28

$$\int \frac{(d+ex)^3}{(bx+cx^2)^2}\,dx = -\frac{b^2c^2d^3 + (2\,bc^3d^3 - 3\,b^2c^2d^2e + 3\,b^3cde^2 - b^4e^3)x - ((2\,c^4d^3 - 3\,bc^3d^2e + b^3ce^3)x^2 + (2\,bc^3d^3 - 3\,b^2c^2d}{b^3c^3x^2 + b^4c^2x}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
-(b^2*c^2*d^3 + (2*b*c^3*d^3 - 3*b^2*c^2*d^2*e + 3*b^3*c*d*e^2 - b^4*e^3)*
x - ((2*c^4*d^3 - 3*b*c^3*d^2*e + b^3*c*e^3)*x^2 + (2*b*c^3*d^3 - 3*b^2*c^
2*d^2*e + b^4*e^3)*x)*log(c*x + b) + ((2*c^4*d^3 - 3*b*c^3*d^2*e)*x^2 + (2
*b*c^3*d^3 - 3*b^2*c^2*d^2*e)*x)*log(x))/(b^3*c^3*x^2 + b^4*c^2*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(78) = 156$.

Time = 0.70 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.87

$$\int \frac{(d+ex)^3}{(bx+cx^2)^2}\,dx = \frac{-bc^2d^3 + x(b^3e^3 - 3b^2cde^2 + 3bc^2d^2e - 2c^3d^3)}{b^3c^2x + b^2c^3x^2} + \frac{d^2\cdot(3be-2cd)\log\left(x + \frac{-3b^2cd^2e+2bc^2d^3+bcd^2\cdot(3be-2cd)}{b^3e^3-6bc^2d^2e+4c^3d^3}\right)}{b^3} + \frac{(be-cd)^2\,(be+2cd)\log\left(x + \frac{-3b^2cd^2e+2bc^2d^3+\frac{b(be-cd)^2(be+2cd)}{c}}{b^3e^3-6bc^2d^2e+4c^3d^3}\right)}{b^3c^2}$$

input
```
integrate((e*x+d)**3/(c*x**2+b*x)**2,x)
```

output
```
(-b*c**2*d**3 + x*(b**3*e**3 - 3*b**2*c*d*e**2 + 3*b*c**2*d**2*e - 2*c**3*
d**3))/(b**3*c**2*x + b**2*c**3*x**2) + d**2*(3*b*e - 2*c*d)*log(x + (-3*b
**2*c*d**2*e + 2*b*c**2*d**3 + b*c*d**2*(3*b*e - 2*c*d))/(b**3*e**3 - 6*b*
c**2*d**2*e + 4*c**3*d**3))/b**3 + (b*e - c*d)**2*(b*e + 2*c*d)*log(x + (-
3*b**2*c*d**2*e + 2*b*c**2*d**3 + b*(b*e - c*d)**2*(b*e + 2*c*d)/c)/(b**3*
e**3 - 6*b*c**2*d**2*e + 4*c**3*d**3))/(b**3*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.52

$$\int \frac{(d+ex)^3}{(bx+cx^2)^2}\,dx = -\frac{bc^2d^3 + (2\,c^3d^3 - 3\,bc^2d^2e + 3\,b^2cde^2 - b^3e^3)x}{b^2c^3x^2 + b^3c^2x} - \frac{(2\,cd^3 - 3\,bd^2e)\log(x)}{b^3} + \frac{(2\,c^3d^3 - 3\,bc^2d^2e + b^3e^3)\log(cx+b)}{b^3c^2}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-(b*c^2*d^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e + 3*b^2*c*d*e^2 - b^3*e^3)*x)/(b^
2*c^3*x^2 + b^3*c^2*x) - (2*c*d^3 - 3*b*d^2*e)*log(x)/b^3 + (2*c^3*d^3 - 3
*b*c^2*d^2*e + b^3*e^3)*log(c*x + b)/(b^3*c^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.48

$$\int \frac{(d+ex)^3}{(bx+cx^2)^2}\,dx = -\frac{(2\,cd^3 - 3\,bd^2e)\log(|x|)}{b^3} + \frac{(2\,c^3d^3 - 3\,bc^2d^2e + b^3e^3)\log(|cx+b|)}{b^3c^2} - \frac{bc^2d^3 + (2\,c^3d^3 - 3\,bc^2d^2e + 3\,b^2cde^2 - b^3e^3)x}{(cx+b)b^2c^2x}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
-(2*c*d^3 - 3*b*d^2*e)*log(abs(x))/b^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e + b^3*
e^3)*log(abs(c*x + b))/(b^3*c^2) - (b*c^2*d^3 + (2*c^3*d^3 - 3*b*c^2*d^2*e
 + 3*b^2*c*d*e^2 - b^3*e^3)*x)/((c*x + b)*b^2*c^2*x)
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^3}{(bx+cx^2)^2}\,dx = \ln(b+c\,x)\left(\frac{e^3}{c^2}+\frac{2\,c\,d^3}{b^3}-\frac{3\,d^2\,e}{b^2}\right) - \frac{\frac{d^3}{b}-\frac{x\,(b^3\,e^3-3\,b^2\,c\,d\,e^2+3\,b\,c^2\,d^2\,e-2\,c^3\,d^3)}{b^2\,c^2}}{c\,x^2+b\,x} + \frac{d^2\,\ln(x)\,(3\,b\,e-2\,c\,d)}{b^3}$$

input
```
int((d + e*x)^3/(b*x + c*x^2)^2,x)
```

output
```
log(b + c*x)*(e^3/c^2 + (2*c*d^3)/b^3 - (3*d^2*e)/b^2) - (d^3/b - (x*(b^3*
e^3 - 2*c^3*d^3 + 3*b*c^2*d^2*e - 3*b^2*c*d*e^2))/(b^2*c^2))/(b*x + c*x^2)
 + (d^2*log(x)*(3*b*e - 2*c*d))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.72

$$\int \frac{(d+ex)^3}{(bx+cx^2)^2}\,dx$$
$$= \frac{\log(cx+b)\,b^4e^3x+\log(cx+b)\,b^3ce^3x^2-3\log(cx+b)\,b^2c^2d^2ex+2\log(cx+b)\,bc^3d^3x-3\log(cx+b)\,b}{}$$

input
```
int((e*x+d)^3/(c*x^2+b*x)^2,x)
```

output
```
(log(b + c*x)*b**4*e**3*x + log(b + c*x)*b**3*c*e**3*x**2 - 3*log(b + c*x)
*b**2*c**2*d**2*e*x + 2*log(b + c*x)*b*c**3*d**3*x - 3*log(b + c*x)*b*c**3
*d**2*e*x**2 + 2*log(b + c*x)*c**4*d**3*x**2 + 3*log(x)*b**2*c**2*d**2*e*x
 - 2*log(x)*b*c**3*d**3*x + 3*log(x)*b*c**3*d**2*e*x**2 - 2*log(x)*c**4*d*
*3*x**2 - b**3*c*e**3*x**2 - b**2*c**2*d**3 + 3*b**2*c**2*d*e**2*x**2 - 3*
b*c**3*d**2*e*x**2 + 2*c**4*d**3*x**2)/(b**3*c**2*x*(b + c*x))
```

3.66 $\int \frac{(d+ex)^2}{(bx+cx^2)^2} \, dx$

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Optimal result

Integrand size = 19, antiderivative size = 73

$$\int \frac{(d+ex)^2}{(bx+cx^2)^2} \, dx = -\frac{d^2}{b^2x} - \frac{(cd-be)^2}{b^2c(b+cx)} - \frac{2d(cd-be)\log(x)}{b^3} + \frac{2d(cd-be)\log(b+cx)}{b^3}$$

output
```
-d^2/b^2/x-(-b*e+c*d)^2/b^2/c/(c*x+b)-2*d*(-b*e+c*d)*ln(x)/b^3+2*d*(-b*e+c
*d)*ln(c*x+b)/b^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2}{(bx+cx^2)^2} \, dx = \frac{-\frac{bd^2}{x} - \frac{b(cd-be)^2}{c(b+cx)} + 2d(-cd+be)\log(x) + 2d(cd-be)\log(b+cx)}{b^3}$$

input
```
Integrate[(d + e*x)^2/(b*x + c*x^2)^2,x]
```

output
```
(-((b*d^2)/x) - (b*(c*d - b*e)^2)/(c*(b + c*x)) + 2*d*(-(c*d) + b*e)*Log[x
] + 2*d*(c*d - b*e)*Log[b + c*x])/b^3
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.18, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(bx+cx^2)^2}dx$$

$$\Big\downarrow \ 1141$$

$$c^2\int \left(\frac{d^2}{b^2c^2x^2}-\frac{2(cd-be)d}{b^3c^2x}+\frac{2(cd-be)d}{b^3c(b+cx)}+\frac{(cd-be)^2}{b^2c^2(b+cx)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$c^2\left(-\frac{2d\log(x)(cd-be)}{b^3c^2}+\frac{2d(cd-be)\log(b+cx)}{b^3c^2}-\frac{(cd-be)^2}{b^2c^3(b+cx)}-\frac{d^2}{b^2c^2x}\right)$$

input
```
Int[(d + e*x)^2/(b*x + c*x^2)^2,x]
```

output
```
c^2*(-(d^2/(b^2*c^2*x)) - (c*d - b*e)^2/(b^2*c^3*(b + c*x)) - (2*d*(c*d -
b*e)*Log[x])/(b^3*c^2) + (2*d*(c*d - b*e)*Log[b + c*x])/(b^3*c^2))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.16

method	result
default	$-\frac{b^2e^2-2bcde+c^2d^2}{b^2c(cx+b)}-\frac{2d(be-cd)\ln(cx+b)}{b^3}-\frac{d^2}{b^2x}+\frac{2d(be-cd)\ln(x)}{b^3}$
norman	$\frac{\frac{\left(b^2e^2-2bcde+2c^2d^2\right)x^2}{b^3}-\frac{d^2}{b}}{x(cx+b)}+\frac{2d(be-cd)\ln(x)}{b^3}-\frac{2d(be-cd)\ln(cx+b)}{b^3}$
risch	$\frac{-\frac{\left(b^2e^2-2bcde+2c^2d^2\right)x}{b^2c}-\frac{d^2}{b}}{x(cx+b)}-\frac{2d\ln(cx+b)e}{b^2}+\frac{2d^2\ln(cx+b)c}{b^3}+\frac{2d\ln(-x)e}{b^2}-\frac{2d^2\ln(-x)c}{b^3}$
parallelrisch	$\frac{2\ln(x)x^2b\,c^2de-2\ln(x)x^2c^3d^2-2\ln(cx+b)x^2b\,c^2de+2\ln(cx+b)x^2c^3d^2+2\ln(x)x\,b^2cde-2\ln(x)xb\,c^2d^2-2\ln(cx+b)x\,b^2cde+}{b^3cx(cx+b)}$

input
```
int((e*x+d)^2/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-(b^2*e^2-2*b*c*d*e+c^2*d^2)/b^2/c/(c*x+b)-2*d*(b*e-c*d)/b^3*ln(c*x+b)-d^2
/b^2/x+2*d*(b*e-c*d)/b^3*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. 2(73) = 146.

Time = 0.09 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.04

$$\int \frac{(d+ex)^2}{(bx+cx^2)^2}\,dx =$$
$$-\frac{b^2cd^2+(2\,bc^2d^2-2\,b^2cde+b^3e^2)x-2\,((c^3d^2-bc^2de)x^2+(bc^2d^2-b^2cde)x)\log{(cx+b)}+2\,((c^3d^2-}{b^3c^2x^2+b^4cx}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
-(b^2*c*d^2 + (2*b*c^2*d^2 - 2*b^2*c*d*e + b^3*e^2)*x - 2*((c^3*d^2 - b*c^
2*d*e)*x^2 + (b*c^2*d^2 - b^2*c*d*e)*x)*log(c*x + b) + 2*((c^3*d^2 - b*c^2
*d*e)*x^2 + (b*c^2*d^2 - b^2*c*d*e)*x)*log(x))/(b^3*c^2*x^2 + b^4*c*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(63) = 126$.

Time = 0.34 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.37

$$\int \frac{(d+ex)^2}{(bx+cx^2)^2}\,dx = \frac{-bcd^2 + x(-b^2e^2 + 2bcde - 2c^2d^2)}{b^3cx + b^2c^2x^2} + \frac{2d(be-cd)\log\left(x + \frac{2b^2de - 2bcd^2 - 2bd(be-cd)}{4bcde - 4c^2d^2}\right)}{b^3} - \frac{2d(be-cd)\log\left(x + \frac{2b^2de - 2bcd^2 + 2bd(be-cd)}{4bcde - 4c^2d^2}\right)}{b^3}$$

input
```
integrate((e*x+d)**2/(c*x**2+b*x)**2,x)
```

output
```
(-b*c*d**2 + x*(-b**2*e**2 + 2*b*c*d*e - 2*c**2*d**2))/(b**3*c*x + b**2*c*
*2*x**2) + 2*d*(b*e - c*d)*log(x + (2*b**2*d*e - 2*b*c*d**2 - 2*b*d*(b*e -
 c*d))/(4*b*c*d*e - 4*c**2*d**2))/b**3 - 2*d*(b*e - c*d)*log(x + (2*b**2*d
*e - 2*b*c*d**2 + 2*b*d*(b*e - c*d))/(4*b*c*d*e - 4*c**2*d**2))/b**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \frac{(d+ex)^2}{(bx+cx^2)^2}\,dx = -\frac{bcd^2 + (2\,c^2d^2 - 2\,bcde + b^2e^2)x}{b^2c^2x^2 + b^3cx} + \frac{2\,(cd^2 - bde)\log(cx+b)}{b^3} - \frac{2\,(cd^2 - bde)\log(x)}{b^3}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-(b*c*d^2 + (2*c^2*d^2 - 2*b*c*d*e + b^2*e^2)*x)/(b^2*c^2*x^2 + b^3*c*x) +
 2*(c*d^2 - b*d*e)*log(c*x + b)/b^3 - 2*(c*d^2 - b*d*e)*log(x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int \frac{(d+ex)^2}{(bx+cx^2)^2}\,dx = -\frac{2\,(cd^2-bde)\log(|x|)}{b^3} + \frac{2\,(c^2d^2-bcde)\log(|cx+b|)}{b^3c} - \frac{2\,c^2d^2x-2\,bcdex+b^2e^2x+bcd^2}{(cx^2+bx)b^2c}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
-2*(c*d^2 - b*d*e)*log(abs(x))/b^3 + 2*(c^2*d^2 - b*c*d*e)*log(abs(c*x + b
))/(b^3*c) - (2*c^2*d^2*x - 2*b*c*d*e*x + b^2*e^2*x + b*c*d^2)/((c*x^2 + b
*x)*b^2*c)
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^2}{(bx+cx^2)^2}\,dx = \frac{4\,d\,\mathrm{atanh}\left(\frac{2\,d\,(b\,e-c\,d)\,(b+2\,c\,x)}{b\,(2\,c\,d^2-2\,b\,d\,e)}\right)\,(b\,e-c\,d)}{b^3} - \frac{\frac{d^2}{b}+\frac{x\,(b^2\,e^2-2\,b\,c\,d\,e+2\,c^2\,d^2)}{b^2\,c}}{c\,x^2+b\,x}$$

input
```
int((d + e*x)^2/(b*x + c*x^2)^2,x)
```

output
```
(4*d*atanh((2*d*(b*e - c*d)*(b + 2*c*x))/(b*(2*c*d^2 - 2*b*d*e)))*(b*e - c
*d))/b^3 - (d^2/b + (x*(b^2*e^2 + 2*c^2*d^2 - 2*b*c*d*e))/(b^2*c))/(b*x +
c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.15

$$\int \frac{(d+ex)^2}{(bx+cx^2)^2}\,dx$$
$$= \frac{-2\log(cx+b)\,b^2dex+2\log(cx+b)\,bc\,d^2x-2\log(cx+b)\,bcde\,x^2+2\log(cx+b)\,c^2d^2x^2+2\log(x)\,b^2de}{b^3x\,(cx+b)}$$

input
```
int((e*x+d)^2/(c*x^2+b*x)^2,x)
```

output
```
( - 2*log(b + c*x)*b**2*d*e*x + 2*log(b + c*x)*b*c*d**2*x - 2*log(b + c*x)
*b*c*d*e*x**2 + 2*log(b + c*x)*c**2*d**2*x**2 + 2*log(x)*b**2*d*e*x - 2*lo
g(x)*b*c*d**2*x + 2*log(x)*b*c*d*e*x**2 - 2*log(x)*c**2*d**2*x**2 - b**2*d
**2 + b**2*e**2*x**2 - 2*b*c*d*e*x**2 + 2*c**2*d**2*x**2)/(b**3*x*(b + c*x
))
```

3.67 $\int \frac{d+ex}{(bx+cx^2)^2} dx$

Optimal result 519
Mathematica [A] (verified) 519
Rubi [A] (verified) 520
Maple [A] (verified) 521
Fricas [A] (verification not implemented) 521
Sympy [B] (verification not implemented) 522
Maxima [A] (verification not implemented) 522
Giac [A] (verification not implemented) 523
Mupad [B] (verification not implemented) 523
Reduce [B] (verification not implemented) 523

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{d+ex}{(bx+cx^2)^2} dx = -\frac{d}{b^2x} - \frac{cd-be}{b^2(b+cx)} - \frac{(2cd-be)\log(x)}{b^3} + \frac{(2cd-be)\log(b+cx)}{b^3}$$

output
```
-d/b^2/x-(-b*e+c*d)/b^2/(c*x+b)-(-b*e+2*c*d)*ln(x)/b^3+(-b*e+2*c*d)*ln(c*x
+b)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{d+ex}{(bx+cx^2)^2} dx = \frac{-\frac{bd}{x} + \frac{b(-cd+be)}{b+cx} + (-2cd+be)\log(x) + (2cd-be)\log(b+cx)}{b^3}$$

input
```
Integrate[(d + e*x)/(b*x + c*x^2)^2,x]
```

output
```
(-((b*d)/x) + (b*(-(c*d) + b*e))/(b + c*x) + (-2*c*d + b*e)*Log[x] + (2*c*
d - b*e)*Log[b + c*x])/b^3
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(bx+cx^2)^2}dx$$

$$\Big\downarrow \text{1141}$$

$$c^2\int \left(\frac{d}{b^2c^2x^2}-\frac{2cd-be}{b^3c^2x}+\frac{2cd-be}{b^3c(b+cx)}+\frac{cd-be}{b^2c(b+cx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c^2\left(-\frac{\log(x)(2cd-be)}{b^3c^2}+\frac{(2cd-be)\log(b+cx)}{b^3c^2}-\frac{cd-be}{b^2c^2(b+cx)}-\frac{d}{b^2c^2x}\right)$$

input
```
Int[(d + e*x)/(b*x + c*x^2)^2,x]
```

output
```
c^2*(-(d/(b^2*c^2*x)) - (c*d - b*e)/(b^2*c^2*(b + c*x)) - ((2*c*d - b*e)*L
og[x])/(b^3*c^2) + ((2*c*d - b*e)*Log[b + c*x])/(b^3*c^2))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

method	result
default	$-\frac{(be-2cd)\ln(cx+b)}{b^3}+\frac{be-cd}{b^2(cx+b)}-\frac{d}{b^2x}+\frac{(be-2cd)\ln(x)}{b^3}$
norman	$\frac{\frac{c(-be+2cd)x^2}{b^3}-\frac{d}{b}}{x(cx+b)}+\frac{(be-2cd)\ln(x)}{b^3}-\frac{(be-2cd)\ln(cx+b)}{b^3}$
risch	$\frac{\frac{(be-2cd)x}{b^2}-\frac{d}{b}}{x(cx+b)}-\frac{\ln(cx+b)e}{b^2}+\frac{2\ln(cx+b)cd}{b^3}+\frac{\ln(-x)e}{b^2}-\frac{2\ln(-x)cd}{b^3}$
parallelrisch	$\frac{\ln(x)x^2b\,c^2e-2\ln(x)x^2c^3d-\ln(cx+b)x^2b\,c^2e+2\ln(cx+b)x^2c^3d+\ln(x)x\,b^2ce-2\ln(x)xb\,c^2d-\ln(cx+b)x\,b^2ce+2\ln(cx+b)xb\,c}{b^3cx(cx+b)}$

input
```
int((e*x+d)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-(b*e-2*c*d)/b^3*ln(c*x+b)+(b*e-c*d)/b^2/(c*x+b)-d/b^2/x+(b*e-2*c*d)/b^3*l
n(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.71

$$\int \frac{d+ex}{(bx+cx^2)^2}\,dx =$$
$$-\frac{b^2d+(2\,bcd-b^2e)x-((2\,c^2d-bce)x^2+(2\,bcd-b^2e)x)\log{(cx+b)}+((2\,c^2d-bce)x^2+(2\,bcd-b^2}{b^3cx^2+b^4x}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
-(b^2*d + (2*b*c*d - b^2*e)*x - ((2*c^2*d - b*c*e)*x^2 + (2*b*c*d - b^2*e)
*x)*log(c*x + b) + ((2*c^2*d - b*c*e)*x^2 + (2*b*c*d - b^2*e)*x)*log(x))/(
b^3*c*x^2 + b^4*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(54) = 108$.

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.97

$$\int \frac{d+ex}{(bx+cx^2)^2}\,dx = \frac{-bd+x(be-2cd)}{b^3x+b^2cx^2} + \frac{(be-2cd)\log\left(x+\frac{b^2e-2bcd-b(be-2cd)}{2bce-4c^2d}\right)}{b^3} - \frac{(be-2cd)\log\left(x+\frac{b^2e-2bcd+b(be-2cd)}{2bce-4c^2d}\right)}{b^3}$$

input
```
integrate((e*x+d)/(c*x**2+b*x)**2,x)
```

output
```
(-b*d + x*(b*e - 2*c*d))/(b**3*x + b**2*c*x**2) + (b*e - 2*c*d)*log(x + (b
**2*e - 2*b*c*d - b*(b*e - 2*c*d))/(2*b*c*e - 4*c**2*d))/b**3 - (b*e - 2*c
*d)*log(x + (b**2*e - 2*b*c*d + b*(b*e - 2*c*d))/(2*b*c*e - 4*c**2*d))/b**
3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{d+ex}{(bx+cx^2)^2}\,dx = -\frac{bd+(2\,cd-be)x}{b^2cx^2+b^3x} + \frac{(2\,cd-be)\log(cx+b)}{b^3} - \frac{(2\,cd-be)\log(x)}{b^3}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-(b*d + (2*c*d - b*e)*x)/(b^2*c*x^2 + b^3*x) + (2*c*d - b*e)*log(c*x + b)/
b^3 - (2*c*d - b*e)*log(x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14

$$\int \frac{d+ex}{(bx+cx^2)^2}\,dx = -\frac{(2\,cd-be)\log(|x|)}{b^3} - \frac{2\,cdx-bex+bd}{(cx^2+bx)b^2} + \frac{(2\,c^2d-bce)\log(|cx+b|)}{b^3c}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
-(2*c*d - b*e)*log(abs(x))/b^3 - (2*c*d*x - b*e*x + b*d)/((c*x^2 + b*x)*b^
2) + (2*c^2*d - b*c*e)*log(abs(c*x + b))/(b^3*c)
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{d+ex}{(bx+cx^2)^2}\,dx = -\frac{\frac{d}{b}-\frac{x\,(b\,e-2\,c\,d)}{b^2}}{c\,x^2+b\,x} - \frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)\,(b\,e-2\,c\,d)}{b^3}$$

input
```
int((d + e*x)/(b*x + c*x^2)^2,x)
```

output
```
- (d/b - (x*(b*e - 2*c*d))/b^2)/(b*x + c*x^2) - (2*atanh((2*c*x)/b + 1)*(b
*e - 2*c*d))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.97

$$\int \frac{d+ex}{(bx+cx^2)^2}\,dx = \frac{-\log(cx+b)\,b^2ex+2\log(cx+b)\,bcdx-\log(cx+b)\,bce\,x^2+2\log(cx+b)\,c^2d\,x^2+\log(x)\,b^2ex-2\log(\cdots}{b^3x\,(cx+b)}$$

input
```
int((e*x+d)/(c*x^2+b*x)^2,x)
```

output
```
( - log(b + c*x)*b**2*e*x + 2*log(b + c*x)*b*c*d*x - log(b + c*x)*b*c*e*x*
*2 + 2*log(b + c*x)*c**2*d*x**2 + log(x)*b**2*e*x - 2*log(x)*b*c*d*x + log
(x)*b*c*e*x**2 - 2*log(x)*c**2*d*x**2 - b**2*d - b*c*e*x**2 + 2*c**2*d*x**
2)/(b**3*x*(b + c*x))
```

3.68 $\int \frac{1}{(bx+cx^2)^2} dx$

Optimal result 525
Mathematica [A] (verified) 525
Rubi [A] (verified) 526
Maple [A] (verified) 527
Fricas [A] (verification not implemented) 527
Sympy [A] (verification not implemented) 528
Maxima [A] (verification not implemented) 528
Giac [A] (verification not implemented) 528
Mupad [B] (verification not implemented) 529
Reduce [B] (verification not implemented) 529

Optimal result

Integrand size = 11, antiderivative size = 42

$$\int \frac{1}{(bx+cx^2)^2} dx = -\frac{1}{b^2x} - \frac{c}{b^2(b+cx)} - \frac{2c\log(x)}{b^3} + \frac{2c\log(b+cx)}{b^3}$$

output
```
-1/b^2/x-c/b^2/(c*x+b)-2*c*ln(x)/b^3+2*c*ln(c*x+b)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{1}{(bx+cx^2)^2} dx = -\frac{b\left(\frac{1}{x} + \frac{c}{b+cx}\right) + 2c\log(x) - 2c\log(b+cx)}{b^3}$$

input
```
Integrate[(b*x + c*x^2)^(-2),x]
```

output
```
-((b*(x^(-1) + c/(b + c*x)) + 2*c*Log[x] - 2*c*Log[b + c*x])/b^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^2}dx$$

$$\Big\downarrow \text{ 1080}$$

$$\int \left(\frac{2c^2}{b^3(b+cx)}-\frac{2c}{b^3x}+\frac{c^2}{b^2(b+cx)^2}+\frac{1}{b^2x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2c\log(x)}{b^3}+\frac{2c\log(b+cx)}{b^3}-\frac{c}{b^2(b+cx)}-\frac{1}{b^2x}$$

input
```
Int[(b*x + c*x^2)^(-2),x]
```

output
```
-(1/(b^2*x)) - c/(b^2*(b + c*x)) - (2*c*Log[x])/b^3 + (2*c*Log[b + c*x])/b
^3
```

Defintions of rubi rules used

rule 1080
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[x^p*
(b + c*x)^p, x], x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{1}{b^{2}x}-\frac{c}{b^{2}(cx+b)}-\frac{2c\ln(x)}{b^{3}}+\frac{2c\ln(cx+b)}{b^{3}}$	43
risch	$\frac{-\frac{2cx}{b^{2}}-\frac{1}{b}}{x(cx+b)}-\frac{2c\ln(x)}{b^{3}}+\frac{2c\ln(-cx-b)}{b^{3}}$	49
norman	$\frac{\frac{2c^{2}x^{2}}{b^{3}}-\frac{1}{b}}{x(cx+b)}-\frac{2c\ln(x)}{b^{3}}+\frac{2c\ln(cx+b)}{b^{3}}$	50
parallelrisch	$-\frac{2c^{2}\ln(x)x^{2}-2\ln(cx+b)x^{2}c^{2}+2\ln(x)xbc-2\ln(cx+b)xbc-2c^{2}x^{2}+b^{2}}{b^{3}x(cx+b)}$	70

input
```
int(1/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/b^2/x-c/b^2/(c*x+b)-2*c*ln(x)/b^3+2*c*ln(c*x+b)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.50

$$\int\frac{1}{(bx+cx^2)^2}\,dx=-\frac{2\,bcx+b^2-2\,(c^2x^2+bcx)\log(cx+b)+2\,(c^2x^2+bcx)\log(x)}{b^3cx^2+b^4x}$$

input
```
integrate(1/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
-(2*b*c*x + b^2 - 2*(c^2*x^2 + b*c*x)*log(c*x + b) + 2*(c^2*x^2 + b*c*x)*l
og(x))/(b^3*c*x^2 + b^4*x)
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.88

$$\int \frac{1}{(bx+cx^2)^2}\,dx = \frac{-b-2cx}{b^3x+b^2cx^2} + \frac{2c\left(-\log(x)+\log\left(\frac{b}{c}+x\right)\right)}{b^3}$$

input
```
integrate(1/(c*x**2+b*x)**2,x)
```

output
```
(-b - 2*c*x)/(b**3*x + b**2*c*x**2) + 2*c*(-log(x) + log(b/c + x))/b**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{(bx+cx^2)^2}\,dx = -\frac{2\,cx+b}{b^2cx^2+b^3x} + \frac{2\,c\log(cx+b)}{b^3} - \frac{2\,c\log(x)}{b^3}$$

input
```
integrate(1/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-(2*c*x + b)/(b^2*c*x^2 + b^3*x) + 2*c*log(c*x + b)/b^3 - 2*c*log(x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07

$$\int \frac{1}{(bx+cx^2)^2}\,dx = \frac{2\,c\log(|cx+b|)}{b^3} - \frac{2\,c\log(|x|)}{b^3} - \frac{2\,cx+b}{(cx^2+bx)b^2}$$

input
```
integrate(1/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
2*c*log(abs(c*x + b))/b^3 - 2*c*log(abs(x))/b^3 - (2*c*x + b)/((c*x^2 + b*
x)*b^2)
```

Mupad [B] (verification not implemented)

Time = 8.67 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{(bx+cx^2)^2}\,dx = \frac{4\,c\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b^3} - \frac{\frac{1}{b}+\frac{2\,c\,x}{b^2}}{c\,x^2+b\,x}$$

input
```
int(1/(b*x + c*x^2)^2,x)
```

output
```
(4*c*atanh((2*c*x)/b + 1))/b^3 - (1/b + (2*c*x)/b^2)/(b*x + c*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.67

$$\int \frac{1}{(bx+cx^2)^2}\,dx = \frac{2\log(cx+b)\,bcx+2\log(cx+b)\,c^2x^2-2\log(x)\,bcx-2\log(x)\,c^2x^2-b^2+2c^2x^2}{b^3x\,(cx+b)}$$

input
```
int(1/(c*x^2+b*x)^2,x)
```

output
```
(2*log(b + c*x)*b*c*x + 2*log(b + c*x)*c**2*x**2 - 2*log(x)*b*c*x - 2*log(
x)*c**2*x**2 - b**2 + 2*c**2*x**2)/(b**3*x*(b + c*x))
```

3.69 $\int \frac{1}{(d+ex)(bx+cx^2)^2}\,dx$

Optimal result . . . 530
Mathematica [A] (verified) . . . 530
Rubi [A] (verified) . . . 531
Maple [A] (verified) . . . 532
Fricas [B] (verification not implemented) . . . 533
Sympy [F(-1)] . . . 533
Maxima [A] (verification not implemented) . . . 534
Giac [A] (verification not implemented) . . . 534
Mupad [B] (verification not implemented) . . . 535
Reduce [B] (verification not implemented) . . . 535

Optimal result

Integrand size = 19, antiderivative size = 110

$$\int \frac{1}{(d+ex)\,(bx+cx^2)^2}\,dx = -\frac{1}{b^2dx}-\frac{c^2}{b^2(cd-be)(b+cx)}-\frac{(2cd+be)\log(x)}{b^3d^2}+\frac{c^2(2cd-3be)\log(b+cx)}{b^3(cd-be)^2}+\frac{e^3\log(d+ex)}{d^2(cd-be)^2}$$

output
```
-1/b^2/d/x-c^2/b^2/(-b*e+c*d)/(c*x+b)-(b*e+2*c*d)*ln(x)/b^3/d^2+c^2*(-3*b*
e+2*c*d)*ln(c*x+b)/b^3/(-b*e+c*d)^2+e^3*ln(e*x+d)/d^2/(-b*e+c*d)^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

$$\int \frac{1}{(d+ex)\,(bx+cx^2)^2}\,dx = -\frac{1}{b^2dx}+\frac{c^2}{b^2(-cd+be)(b+cx)}+\frac{(-2cd-be)\log(x)}{b^3d^2}+\frac{(2c^3d-3bc^2e)\log(b+cx)}{b^3(-cd+be)^2}+\frac{e^3\log(d+ex)}{d^2(cd-be)^2}$$

input
```
Integrate[1/((d + e*x)*(b*x + c*x^2)^2),x]
```

output
```
-(1/(b^2*d*x)) + c^2/(b^2*(-(c*d) + b*e)*(b + c*x)) + ((-2*c*d - b*e)*Log[
x])/(b^3*d^2) + ((2*c^3*d - 3*b*c^2*e)*Log[b + c*x])/(b^3*(-(c*d) + b*e)^2
) + (e^3*Log[d + e*x])/(d^2*(c*d - b*e)^2)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^2 (d+ex)} dx$$

$$\Big\downarrow \text{1141}$$

$$c^2 \int \left(\frac{e^4}{c^2d^2(cd-be)^2(d+ex)} - \frac{2cd+be}{b^3c^2d^2x} + \frac{c(2cd-3be)}{b^3(cd-be)^2(b+cx)} + \frac{1}{b^2c^2dx^2} + \frac{c}{b^2(cd-be)(b+cx)^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$c^2 \left(-\frac{\log(x)(be+2cd)}{b^3c^2d^2} + \frac{(2cd-3be)\log(b+cx)}{b^3(cd-be)^2} - \frac{1}{b^2c^2dx} - \frac{1}{b^2(b+cx)(cd-be)} + \frac{e^3\log(d+ex)}{c^2d^2(cd-be)^2}\right)$$

input
```
Int[1/((d + e*x)*(b*x + c*x^2)^2),x]
```

output
```
c^2*(-(1/(b^2*c^2*d*x)) - 1/(b^2*(c*d - b*e)*(b + c*x)) - ((2*c*d + b*e)*L
og[x])/(b^3*c^2*d^2) + ((2*c*d - 3*b*e)*Log[b + c*x])/(b^3*(c*d - b*e)^2)
+ (e^3*Log[d + e*x])/(c^2*d^2*(c*d - b*e)^2))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.01

method	result
default	$\frac{c^2}{(be-cd)b^2(cx+b)}-\frac{c^2(3be-2cd)\ln(cx+b)}{(be-cd)^2b^3}+\frac{e^3\ln(ex+d)}{d^2(be-cd)^2}-\frac{1}{b^2dx}+\frac{(-be-2cd)\ln(x)}{b^3d^2}$
norman	$\frac{\frac{(bce-2c^2d)cx^2}{db^3(be-cd)}-\frac{1}{db}}{x(cx+b)}+\frac{e^3\ln(ex+d)}{d^2(b^2e^2-2bcde+c^2d^2)}-\frac{(be+2cd)\ln(x)}{b^3d^2}-\frac{c^2(3be-2cd)\ln(cx+b)}{b^3(b^2e^2-2bcde+c^2d^2)}$
risch	$\frac{-\frac{c(be-2cd)x}{b^2d(be-cd)}-\frac{1}{db}}{x(cx+b)}-\frac{3c^2\ln(cx+b)e}{b^2(b^2e^2-2bcde+c^2d^2)}+\frac{2c^3\ln(cx+b)d}{b^3(b^2e^2-2bcde+c^2d^2)}-\frac{\ln(-x)e}{b^2d^2}-\frac{2\ln(-x)c}{b^3d}+\frac{e^3\ln(-ex-d)}{d^2(b^2e^2-2bcde+c^2d^2}$
parallelrisch	$-\frac{\ln(x)x^2b^3c^2e^3-3\ln(x)x^2bc^4d^2e+2\ln(x)x^2c^5d^3+3\ln(cx+b)x^2bc^4d^2e-2\ln(cx+b)x^2c^5d^3-\ln(ex+d)x^2b^3c^2e^3+\ln(x)xb^4c}{}$

input
```
int(1/(e*x+d)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2/(b*e-c*d)/b^2/(c*x+b)-c^2*(3*b*e-2*c*d)/(b*e-c*d)^2/b^3*ln(c*x+b)+e^3/
d^2/(b*e-c*d)^2*ln(e*x+d)-1/b^2/d/x+1/b^3/d^2*(-b*e-2*c*d)*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. 2(110) = 220.

Time = 2.39 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.62

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^2}\,dx = -\frac{b^2c^2d^3 - 2\,b^3cd^2e + b^4de^2 + (2\,bc^3d^3 - 3\,b^2c^2d^2e + b^3cde^2)x - ((2\,c^4d^3 - 3\,bc^3d^2e)x^2 + (2\,bc^3d^3 - 3\,b^2c}{(b^3c^3d^4 - 2\,b^4c^2d^3e +}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
-(b^2*c^2*d^3 - 2*b^3*c*d^2*e + b^4*d*e^2 + (2*b*c^3*d^3 - 3*b^2*c^2*d^2*e
 + b^3*c*d*e^2)*x - ((2*c^4*d^3 - 3*b*c^3*d^2*e)*x^2 + (2*b*c^3*d^3 - 3*b^
2*c^2*d^2*e)*x)*log(c*x + b) - (b^3*c*e^3*x^2 + b^4*e^3*x)*log(e*x + d) +
((2*c^4*d^3 - 3*b*c^3*d^2*e + b^3*c*e^3)*x^2 + (2*b*c^3*d^3 - 3*b^2*c^2*d^
2*e + b^4*e^3)*x)*log(x))/((b^3*c^3*d^4 - 2*b^4*c^2*d^3*e + b^5*c*d^2*e^2)
*x^2 + (b^4*c^2*d^4 - 2*b^5*c*d^3*e + b^6*d^2*e^2)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x**2+b*x)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.61

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^2}\,dx = \frac{e^3\log\left(ex+d\right)}{c^2d^4-2\,bcd^3e+b^2d^2e^2}+\frac{\left(2\,c^3d-3\,bc^2e\right)\log\left(cx+b\right)}{b^3c^2d^2-2\,b^4cde+b^5e^2} - \frac{bcd-b^2e+\left(2\,c^2d-bce\right)x}{\left(b^2c^2d^2-b^3cde\right)x^2+\left(b^3cd^2-b^4de\right)x} - \frac{\left(2\,cd+be\right)\log\left(x\right)}{b^3d^2}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
e^3*log(e*x + d)/(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2) + (2*c^3*d - 3*b*c^
2*e)*log(c*x + b)/(b^3*c^2*d^2 - 2*b^4*c*d*e + b^5*e^2) - (b*c*d - b^2*e +
 (2*c^2*d - b*c*e)*x)/((b^2*c^2*d^2 - b^3*c*d*e)*x^2 + (b^3*c*d^2 - b^4*d*
e)*x) - (2*c*d + b*e)*log(x)/(b^3*d^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.82

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^2}\,dx = \frac{e^4\log\left(|ex+d|\right)}{c^2d^4e-2\,bcd^3e^2+b^2d^2e^3} + \frac{\left(2\,c^4d-3\,bc^3e\right)\log\left(|cx+b|\right)}{b^3c^3d^2-2\,b^4c^2de+b^5ce^2} - \frac{\left(2\,cd+be\right)\log\left(|x|\right)}{b^3d^2} - \frac{bc^2d^3-2\,b^2cd^2e+b^3de^2+\left(2\,c^3d^3-3\,bc^2d^2e+b^2cde^2\right)x}{\left(cd-be\right)^2\left(cx+b\right)b^2d^2x}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
e^4*log(abs(e*x + d))/(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3) + (2*c^4*d
 - 3*b*c^3*e)*log(abs(c*x + b))/(b^3*c^3*d^2 - 2*b^4*c^2*d*e + b^5*c*e^2)
- (2*c*d + b*e)*log(abs(x))/(b^3*d^2) - (b*c^2*d^3 - 2*b^2*c*d^2*e + b^3*d
*e^2 + (2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2)*x)/((c*d - b*e)^2*(c*x +
b)*b^2*d^2*x)
```

Mupad [B] (verification not implemented)

Time = 8.99 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.30

$$\int \frac{1}{(d+ex)(bx+cx^2)^2}\,dx = \frac{\ln(b+c\,x)\ (2\,c^3\,d-3\,b\,c^2\,e)}{b^5\,e^2-2\,b^4\,c\,d\,e+b^3\,c^2\,d^2}-\frac{\frac{1}{b\,d}-\frac{x\,(2\,c^2\,d-b\,c\,e)}{b^2\,d\,(b\,e-c\,d)}}{c\,x^2+b\,x}+\frac{e^3\,\ln(d+e\,x)}{d^2\,(b\,e-c\,d)^2}-\frac{\ln(x)\ (b\,e+2\,c\,d)}{b^3\,d^2}$$

input
```
int(1/((b*x + c*x^2)^2*(d + e*x)),x)
```

output
```
(log(b + c*x)*(2*c^3*d - 3*b*c^2*e))/(b^5*e^2 + b^3*c^2*d^2 - 2*b^4*c*d*e)
 - (1/(b*d) - (x*(2*c^2*d - b*c*e))/(b^2*d*(b*e - c*d)))/(b*x + c*x^2) + (
e^3*log(d + e*x))/(d^2*(b*e - c*d)^2) - (log(x)*(b*e + 2*c*d))/(b^3*d^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.85

$$\int \frac{1}{(d+ex)(bx+cx^2)^2}\,dx = \frac{-3\log(cx+b)\,b^2c^2d^2ex+2\log(cx+b)\,bc^3d^3x-3\log(cx+b)\,bc^3d^2e\,x^2+2\log(cx+b)\,c^4d^3x^2+\log(ex}{}$$

input
```
int(1/(e*x+d)/(c*x^2+b*x)^2,x)
```

output
```
( - 3*log(b + c*x)*b**2*c**2*d**2*e*x + 2*log(b + c*x)*b*c**3*d**3*x - 3*l
og(b + c*x)*b*c**3*d**2*e*x**2 + 2*log(b + c*x)*c**4*d**3*x**2 + log(d + e
*x)*b**4*e**3*x + log(d + e*x)*b**3*c*e**3*x**2 - log(x)*b**4*e**3*x - log
(x)*b**3*c*e**3*x**2 + 3*log(x)*b**2*c**2*d**2*e*x - 2*log(x)*b*c**3*d**3*
x + 3*log(x)*b*c**3*d**2*e*x**2 - 2*log(x)*c**4*d**3*x**2 - b**4*d*e**2 +
2*b**3*c*d**2*e - b**2*c**2*d**3 + b**2*c**2*d*e**2*x**2 - 3*b*c**3*d**2*e
*x**2 + 2*c**4*d**3*x**2)/(b**3*d**2*x*(b**3*e**2 - 2*b**2*c*d*e + b**2*c*
e**2*x + b*c**2*d**2 - 2*b*c**2*d*e*x + c**3*d**2*x))
```

3.70 $\int \frac{1}{(d+ex)^2(bx+cx^2)^2}\,dx$

Optimal result 536
Mathematica [A] (verified) 536
Rubi [A] (verified) 537
Maple [A] (verified) 538
Fricas [B] (verification not implemented) 539
Sympy [F(-1)] 539
Maxima [B] (verification not implemented) 540
Giac [B] (verification not implemented) 541
Mupad [B] (verification not implemented) 542
Reduce [B] (verification not implemented) 542

Optimal result

Integrand size = 19, antiderivative size = 144

$$\int \frac{1}{(d+ex)^2(bx+cx^2)^2}\,dx = -\frac{1}{b^2d^2x} - \frac{c^3}{b^2(cd-be)^2(b+cx)} - \frac{e^3}{d^2(cd-be)^2(d+ex)} - \frac{2(cd+be)\log(x)}{b^3d^3} + \frac{2c^3(cd-2be)\log(b+cx)}{b^3(cd-be)^3} + \frac{2e^3(2cd-be)\log(d+ex)}{d^3(cd-be)^3}$$

output
```
-1/b^2/d^2/x-c^3/b^2/(-b*e+c*d)^2/(c*x+b)-e^3/d^2/(-b*e+c*d)^2/(e*x+d)-2*(
b*e+c*d)*ln(x)/b^3/d^3+2*c^3*(-2*b*e+c*d)*ln(c*x+b)/b^3/(-b*e+c*d)^3+2*e^3
*(-b*e+2*c*d)*ln(e*x+d)/d^3/(-b*e+c*d)^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01

$$\int \frac{1}{(d+ex)^2(bx+cx^2)^2}\,dx = -\frac{1}{b^2d^2x} - \frac{c^3}{b^2(cd-be)^2(b+cx)} - \frac{e^3}{d^2(cd-be)^2(d+ex)} - \frac{2(cd+be)\log(x)}{b^3d^3} + \frac{2c^3(-cd+2be)\log(b+cx)}{b^3(-cd+be)^3} + \frac{2e^3(2cd-be)\log(d+ex)}{d^3(cd-be)^3}$$

input
```
Integrate[1/((d + e*x)^2*(b*x + c*x^2)^2),x]
```

output
```
-(1/(b^2*d^2*x)) - c^3/(b^2*(c*d - b*e)^2*(b + c*x)) - e^3/(d^2*(c*d - b*e
)^2*(d + e*x)) - (2*(c*d + b*e)*Log[x])/(b^3*d^3) + (2*c^3*(-(c*d) + 2*b*e
)*Log[b + c*x])/(b^3*(-(c*d) + b*e)^3) + (2*e^3*(2*c*d - b*e)*Log[d + e*x]
)/(d^3*(c*d - b*e)^3)
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^2 (d+ex)^2} dx$$

$$\Big\downarrow \text{1141}$$

$$c^2 \int \left(\frac{2(2cd-be)e^4}{c^2d^3(cd-be)^3(d+ex)} + \frac{e^4}{c^2d^2(cd-be)^2(d+ex)^2} - \frac{2(cd+be)}{b^3c^2d^3x} + \frac{2c^2(cd-2be)}{b^3(cd-be)^3(b+cx)} + \frac{1}{b^2c^2d^2x^2} + \frac{1}{b^2(\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$c^2 \left(-\frac{2\log(x)(be+cd)}{b^3c^2d^3} + \frac{2c(cd-2be)\log(b+cx)}{b^3(cd-be)^3} - \frac{1}{b^2c^2d^2x} - \frac{c}{b^2(b+cx)(cd-be)^2} + \frac{2e^3(2cd-be)\log(d+ex)}{c^2d^3(cd-be)^3}\ldots\right.$$

input
```
Int[1/((d + e*x)^2*(b*x + c*x^2)^2),x]
```

output
```
c^2*(-(1/(b^2*c^2*d^2*x)) - c/(b^2*(c*d - b*e)^2*(b + c*x)) - e^3/(c^2*d^2
*(c*d - b*e)^2*(d + e*x)) - (2*(c*d + b*e)*Log[x])/(b^3*c^2*d^3) + (2*c*(c
*d - 2*b*e)*Log[b + c*x])/(b^3*(c*d - b*e)^3) + (2*e^3*(2*c*d - b*e)*Log[d
 + e*x])/(c^2*d^3*(c*d - b*e)^3))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.01

method	result
default	$-\frac{c^{3}}{b^{2}\left(be-cd\right)^{2}\left(cx+b\right)}+\frac{2c^{3}\left(2be-cd\right)\ln\left(cx+b\right)}{b^{3}\left(be-cd\right)^{3}}-\frac{e^{3}}{d^{2}\left(be-cd\right)^{2}\left(ex+d\right)}+\frac{2e^{3}\left(be-2cd\right)\ln\left(ex+d\right)}{d^{3}\left(be-cd\right)^{3}}-\frac{1}{b^{2}d^{2}x}+\frac{\left(-2be-2c\right.}{b^{3}d^{3}}$
norman	$\frac{\frac{\left(2e^{4}b^{4}-d\,e^{3}b^{3}c-d^{3}eb\,c^{3}+2c^{4}d^{4}\right)x^{2}}{d^{3}b^{3}\left(b^{2}e^{2}-2bcde+c^{2}d^{2}\right)}+\frac{\left(2b^{3}e^{3}-d\,e^{2}b^{2}c-d^{2}eb\,c^{2}+2d^{3}c^{3}\right)ce\,x^{3}}{d^{3}b^{3}\left(b^{2}e^{2}-2bcde+c^{2}d^{2}\right)}-\frac{1}{db}}{\left(cx+b\right)x\left(ex+d\right)}-\frac{2\left(be+cd\right)\ln\left(x\right)}{b^{3}d^{3}}+\frac{2c^{3}\left(2be-cd\right)\ln\left(c\right.}{b^{3}\left(b^{3}e^{3}-3d\,e^{2}b^{2}c+3d^{2}e\right.}$
risch	$\frac{-\frac{2ce\left(b^{2}e^{2}-bcde+c^{2}d^{2}\right)x^{2}}{b^{2}d^{2}\left(b^{2}e^{2}-2bcde+c^{2}d^{2}\right)}-\frac{\left(2b^{3}e^{3}-d\,e^{2}b^{2}c-d^{2}eb\,c^{2}+2d^{3}c^{3}\right)x}{d^{2}b^{2}\left(b^{2}e^{2}-2bcde+c^{2}d^{2}\right)}-\frac{1}{db}}{\left(cx+b\right)x\left(ex+d\right)}+\frac{4c^{3}\ln\left(cx+b\right)e}{b^{2}\left(b^{3}e^{3}-3d\,e^{2}b^{2}c+3d^{2}eb\,c^{2}-d^{3}c^{3}\right)}-\frac{2c^{4}\,\mathrm{l}}{b^{3}\left(b^{3}e^{3}-3d\,e^{2}b\right.}$
parallelrisch	$-\frac{-4\ln\left(cx+b\right)x\,b^{2}c^{3}d^{4}e-2\ln\left(cx+b\right)x^{2}b\,c^{4}d^{4}e+2\ln\left(ex+d\right)x^{2}b^{4}cd\,e^{4}+2\ln\left(x\right)x\,b^{5}d\,e^{4}-2\ln\left(x\right)xb\,c^{4}d^{5}+2\ln\left(cx+b\right)xb\,c^{4}d^{5}-2}{}$

input
```
int(1/(e*x+d)^2/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-c^3/b^2/(b*e-c*d)^2/(c*x+b)+2*c^3*(2*b*e-c*d)/b^3/(b*e-c*d)^3*ln(c*x+b)-e
^3/d^2/(b*e-c*d)^2/(e*x+d)+2*e^3*(b*e-2*c*d)/d^3/(b*e-c*d)^3*ln(e*x+d)-1/b
^2/d^2/x+(-2*b*e-2*c*d)/b^3/d^3*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. 2(144) = 288.

Time = 7.78 (sec) , antiderivative size = 653, normalized size of antiderivative = 4.53

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^2} \, dx = -\frac{b^2c^3d^5 - 3\,b^3c^2d^4e + 3\,b^4cd^3e^2 - b^5d^2e^3 + 2\,(bc^4d^4e - 2\,b^2c^3d^3e^2 + 2\,b^3c^2d^2e^3 - b^4cde^4)x^2 + (2\,bc^4d^5 -}{}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
-(b^2*c^3*d^5 - 3*b^3*c^2*d^4*e + 3*b^4*c*d^3*e^2 - b^5*d^2*e^3 + 2*(b*c^4
*d^4*e - 2*b^2*c^3*d^3*e^2 + 2*b^3*c^2*d^2*e^3 - b^4*c*d*e^4)*x^2 + (2*b*c
^4*d^5 - 3*b^2*c^3*d^4*e + 3*b^4*c*d^2*e^3 - 2*b^5*d*e^4)*x - 2*((c^5*d^4*
e - 2*b*c^4*d^3*e^2)*x^3 + (c^5*d^5 - b*c^4*d^4*e - 2*b^2*c^3*d^3*e^2)*x^2
 + (b*c^4*d^5 - 2*b^2*c^3*d^4*e)*x)*log(c*x + b) - 2*((2*b^3*c^2*d*e^4 - b
^4*c*e^5)*x^3 + (2*b^3*c^2*d^2*e^3 + b^4*c*d*e^4 - b^5*e^5)*x^2 + (2*b^4*c
*d^2*e^3 - b^5*d*e^4)*x)*log(e*x + d) + 2*((c^5*d^4*e - 2*b*c^4*d^3*e^2 +
2*b^3*c^2*d*e^4 - b^4*c*e^5)*x^3 + (c^5*d^5 - b*c^4*d^4*e - 2*b^2*c^3*d^3*
e^2 + 2*b^3*c^2*d^2*e^3 + b^4*c*d*e^4 - b^5*e^5)*x^2 + (b*c^4*d^5 - 2*b^2*
c^3*d^4*e + 2*b^4*c*d^2*e^3 - b^5*d*e^4)*x)*log(x))/((b^3*c^4*d^6*e - 3*b^
4*c^3*d^5*e^2 + 3*b^5*c^2*d^4*e^3 - b^6*c*d^3*e^4)*x^3 + (b^3*c^4*d^7 - 2*
b^4*c^3*d^6*e + 2*b^6*c*d^4*e^3 - b^7*d^3*e^4)*x^2 + (b^4*c^3*d^7 - 3*b^5*
c^2*d^6*e + 3*b^6*c*d^5*e^2 - b^7*d^4*e^3)*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^2} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(c*x**2+b*x)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(144) = 288$.

Time = 0.04 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.59

$$\int \frac{1}{(d+ex)^2\,(bx+cx^2)^2}\,dx$$
$$= \frac{2\,(c^4d - 2\,bc^3e)\log(cx+b)}{b^3c^3d^3 - 3\,b^4c^2d^2e + 3\,b^5cde^2 - b^6e^3} + \frac{2\,(2\,cde^3 - be^4)\log(ex+d)}{c^3d^6 - 3\,bc^2d^5e + 3\,b^2cd^4e^2 - b^3d^3e^3}$$
$$- \frac{bc^2d^3 - 2\,b^2cd^2e + b^3de^2 + 2\,(c^3d^2e - bc^2de^2 + b^2ce^3)x^2 + (2\,c^3d^3 - bc^2d^2e - b^2cde^2 + 2\,b^3e^3)x}{(b^2c^3d^4e - 2\,b^3c^2d^3e^2 + b^4cd^2e^3)x^3 + (b^2c^3d^5 - b^3c^2d^4e - b^4cd^3e^2 + b^5d^2e^3)x^2 + (b^3c^2d^5 - 2\,b^4cd^4e + b}$$
$$- \frac{2\,(cd+be)\log(x)}{b^3d^3}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
2*(c^4*d - 2*b*c^3*e)*log(c*x + b)/(b^3*c^3*d^3 - 3*b^4*c^2*d^2*e + 3*b^5*
c*d*e^2 - b^6*e^3) + 2*(2*c*d*e^3 - b*e^4)*log(e*x + d)/(c^3*d^6 - 3*b*c^2
*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3) - (b*c^2*d^3 - 2*b^2*c*d^2*e + b^3
*d*e^2 + 2*(c^3*d^2*e - b*c^2*d*e^2 + b^2*c*e^3)*x^2 + (2*c^3*d^3 - b*c^2*
d^2*e - b^2*c*d*e^2 + 2*b^3*e^3)*x)/((b^2*c^3*d^4*e - 2*b^3*c^2*d^3*e^2 +
b^4*c*d^2*e^3)*x^3 + (b^2*c^3*d^5 - b^3*c^2*d^4*e - b^4*c*d^3*e^2 + b^5*d^
2*e^3)*x^2 + (b^3*c^2*d^5 - 2*b^4*c*d^4*e + b^5*d^3*e^2)*x) - 2*(c*d + b*e
)*log(x)/(b^3*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(144) = 288$.

Time = 0.20 (sec) , antiderivative size = 557, normalized size of antiderivative = 3.87

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^2}\,dx$$
$$= -\frac{e^7}{(c^2d^4e^4 - 2\,bcd^3e^5 + b^2d^2e^6)(ex+d)} - \frac{(2\,cde^3 - be^4)\log\left(\left|-c + \frac{2\,cd}{ex+d} - \frac{cd^2}{(ex+d)^2} - \frac{be}{ex+d} + \frac{bde}{(ex+d)^2}\right|\right)}{c^3d^6 - 3\,bc^2d^5e + 3\,b^2cd^4e^2 - b^3d^3e^3} + \frac{(2\,c^4d^4e^2 - 4\,bc^3d^3e^3 + 2\,b^3cde^5 - b^4e^6)\log\left(\frac{\left|-2\,cde + \frac{2\,cd^2e}{ex+d} + be^2 - \frac{2\,bde^2}{ex+d} - e^2|b|\right|}{\left|-2\,cde + \frac{2\,cd^2e}{ex+d} + be^2 - \frac{2\,bde^2}{ex+d} + e^2|b|\right|}\right)}{(b^2c^3d^6 - 3\,b^3c^2d^5e + 3\,b^4cd^4e^2 - b^5d^3e^3)e^2|b|} - \frac{\frac{2\,c^4d^3e - 3\,bc^3d^2e^2 + 3\,b^2c^2de^3 - b^3ce^4}{cd^2 - bde} - \frac{2\,c^4d^4e^2 - 4\,bc^3d^3e^3 + 6\,b^2c^2d^2e^4 - 4\,b^3cde^5 + b^4e^6}{(cd^2 - bde)(ex+d)e}}{(cd-be)^2b^2\left(c - \frac{2\,cd}{ex+d} + \frac{cd^2}{(ex+d)^2} + \frac{be}{ex+d} - \frac{bde}{(ex+d)^2}\right)d^2}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
-e^7/((c^2*d^4*e^4 - 2*b*c*d^3*e^5 + b^2*d^2*e^6)*(e*x + d)) - (2*c*d*e^3
- b*e^4)*log(abs(-c + 2*c*d/(e*x + d) - c*d^2/(e*x + d)^2 - b*e/(e*x + d)
+ b*d*e/(e*x + d)^2))/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3
*e^3) + (2*c^4*d^4*e^2 - 4*b*c^3*d^3*e^3 + 2*b^3*c*d*e^5 - b^4*e^6)*log(ab
s(-2*c*d*e + 2*c*d^2*e/(e*x + d) + b*e^2 - 2*b*d*e^2/(e*x + d) - e^2*abs(b
))/abs(-2*c*d*e + 2*c*d^2*e/(e*x + d) + b*e^2 - 2*b*d*e^2/(e*x + d) + e^2*
abs(b)))/((b^2*c^3*d^6 - 3*b^3*c^2*d^5*e + 3*b^4*c*d^4*e^2 - b^5*d^3*e^3)*
e^2*abs(b)) - ((2*c^4*d^3*e - 3*b*c^3*d^2*e^2 + 3*b^2*c^2*d*e^3 - b^3*c*e^
4)/(c*d^2 - b*d*e) - (2*c^4*d^4*e^2 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4
- 4*b^3*c*d*e^5 + b^4*e^6)/((c*d^2 - b*d*e)*(e*x + d)*e))/((c*d - b*e)^2*b
^2*(c - 2*c*d/(e*x + d) + c*d^2/(e*x + d)^2 + b*e/(e*x + d) - b*d*e/(e*x +
 d)^2)*d^2)
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.11

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^2} \, dx$$
$$= -\frac{\frac{1}{b\,d}+\frac{2\,x^2\,\left(b^2\,c\,e^3-b\,c^2\,d\,e^2+c^3\,d^2\,e\right)}{b^2\,d^2\,\left(b^2\,e^2-2\,b\,c\,d\,e+c^2\,d^2\right)}+\frac{x\,\left(b\,e+c\,d\right)\,\left(2\,b^2\,e^2-3\,b\,c\,d\,e+2\,c^2\,d^2\right)}{b^2\,d^2\,\left(b^2\,e^2-2\,b\,c\,d\,e+c^2\,d^2\right)}}{c\,e\,x^3+\left(b\,e+c\,d\right)\,x^2+b\,d\,x} - \frac{\ln\left(b+c\,x\right)\,\left(2\,c^4\,d-4\,b\,c^3\,e\right)}{b^6\,e^3-3\,b^5\,c\,d\,e^2+3\,b^4\,c^2\,d^2\,e-b^3\,c^3\,d^3} - \frac{\ln\left(d+e\,x\right)\,\left(2\,b\,e^4-4\,c\,d\,e^3\right)}{-b^3\,d^3\,e^3+3\,b^2\,c\,d^4\,e^2-3\,b\,c^2\,d^5\,e+c^3\,d^6} - \frac{2\,\ln\left(x\right)\,\left(b\,e+c\,d\right)}{b^3\,d^3}$$

input
```
int(1/((b*x + c*x^2)^2*(d + e*x)^2),x)
```

output
```
- (1/(b*d) + (2*x^2*(b^2*c*e^3 + c^3*d^2*e - b*c^2*d*e^2))/(b^2*d^2*(b^2*e
^2 + c^2*d^2 - 2*b*c*d*e)) + (x*(b*e + c*d)*(2*b^2*e^2 + 2*c^2*d^2 - 3*b*c
*d*e))/(b^2*d^2*(b^2*e^2 + c^2*d^2 - 2*b*c*d*e)))/(x^2*(b*e + c*d) + b*d*x
 + c*e*x^3) - (log(b + c*x)*(2*c^4*d - 4*b*c^3*e))/(b^6*e^3 - b^3*c^3*d^3
+ 3*b^4*c^2*d^2*e - 3*b^5*c*d*e^2) - (log(d + e*x)*(2*b*e^4 - 4*c*d*e^3))/
(c^3*d^6 - b^3*d^3*e^3 + 3*b^2*c*d^4*e^2 - 3*b*c^2*d^5*e) - (2*log(x)*(b*e
 + c*d))/(b^3*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 974, normalized size of antiderivative = 6.76

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^2/(c*x^2+b*x)^2,x)
```

output

```
(4*log(b + c*x)*b**3*c**3*d**4*e**2*x + 4*log(b + c*x)*b**3*c**3*d**3*e**3
*x**2 + 2*log(b + c*x)*b**2*c**4*d**5*e*x + 6*log(b + c*x)*b**2*c**4*d**4*
e**2*x**2 + 4*log(b + c*x)*b**2*c**4*d**3*e**3*x**3 - 2*log(b + c*x)*b*c**
5*d**6*x + 2*log(b + c*x)*b*c**5*d**4*e**2*x**3 - 2*log(b + c*x)*c**6*d**6
*x**2 - 2*log(b + c*x)*c**6*d**5*e*x**3 + 2*log(d + e*x)*b**6*d*e**5*x + 2
*log(d + e*x)*b**6*e**6*x**2 - 2*log(d + e*x)*b**5*c*d**2*e**4*x + 2*log(d
 + e*x)*b**5*c*e**6*x**3 - 4*log(d + e*x)*b**4*c**2*d**3*e**3*x - 6*log(d
+ e*x)*b**4*c**2*d**2*e**4*x**2 - 2*log(d + e*x)*b**4*c**2*d*e**5*x**3 - 4
*log(d + e*x)*b**3*c**3*d**3*e**3*x**2 - 4*log(d + e*x)*b**3*c**3*d**2*e**
4*x**3 - 2*log(x)*b**6*d*e**5*x - 2*log(x)*b**6*e**6*x**2 + 2*log(x)*b**5*
c*d**2*e**4*x - 2*log(x)*b**5*c*e**6*x**3 + 4*log(x)*b**4*c**2*d**3*e**3*x
 + 6*log(x)*b**4*c**2*d**2*e**4*x**2 + 2*log(x)*b**4*c**2*d*e**5*x**3 - 4*
log(x)*b**3*c**3*d**4*e**2*x + 4*log(x)*b**3*c**3*d**2*e**4*x**3 - 2*log(x
)*b**2*c**4*d**5*e*x - 6*log(x)*b**2*c**4*d**4*e**2*x**2 - 4*log(x)*b**2*c
**4*d**3*e**3*x**3 + 2*log(x)*b*c**5*d**6*x - 2*log(x)*b*c**5*d**4*e**2*x*
*3 + 2*log(x)*c**6*d**6*x**2 + 2*log(x)*c**6*d**5*e*x**3 - b**6*d**2*e**4
- 2*b**6*d*e**5*x + 2*b**5*c*d**3*e**3 + 3*b**5*c*d**2*e**4*x - b**4*c**2*
d**3*e**3*x + 2*b**4*c**2*d*e**5*x**3 - 2*b**3*c**3*d**5*e + b**3*c**3*d**
4*e**2*x - 4*b**3*c**3*d**2*e**4*x**3 + b**2*c**4*d**6 - 3*b**2*c**4*d**5*
e*x + 4*b**2*c**4*d**3*e**3*x**3 + 2*b*c**5*d**6*x - 2*b*c**5*d**4*e**2...
```

3.71 $\int \frac{(d+ex)^7}{(bx+cx^2)^3}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 203

$$\int \frac{(d+ex)^7}{(bx+cx^2)^3}\,dx = -\frac{d^7}{2b^3x^2}+\frac{d^6(3cd-7be)}{b^4x}+\frac{e^6(7cd-3be)x}{c^4}+\frac{e^7x^2}{2c^3}+\frac{(cd-be)^7}{2b^3c^5(b+cx)^2} + \frac{(cd-be)^6(3cd+4be)}{b^4c^5(b+cx)}+\frac{3d^5(2c^2d^2-7bcde+7b^2e^2)\log(x)}{b^5} - \frac{3(cd-be)^5\left(2c^2d^2+3bcde+2b^2e^2\right)\log(b+cx)}{b^5c^5}$$

output

```
-1/2*d^7/b^3/x^2+d^6*(-7*b*e+3*c*d)/b^4/x+e^6*(-3*b*e+7*c*d)*x/c^4+1/2*e^7
*x^2/c^3+1/2*(-b*e+c*d)^7/b^3/c^5/(c*x+b)^2+(-b*e+c*d)^6*(4*b*e+3*c*d)/b^4
/c^5/(c*x+b)+3*d^5*(7*b^2*e^2-7*b*c*d*e+2*c^2*d^2)*ln(x)/b^5-3*(-b*e+c*d)^
5*(2*b^2*e^2+3*b*c*d*e+2*c^2*d^2)*ln(c*x+b)/b^5/c^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^7}{(bx+cx^2)^3}\,dx = \frac{1}{2}\left(-\frac{d^7}{b^3x^2}+\frac{2d^6(3cd-7be)}{b^4x}+\frac{2e^6(7cd-3be)x}{c^4}+\frac{e^7x^2}{c^3}+\frac{(cd-be)^7}{b^3c^5(b+cx)^2}+\frac{2(cd-be)^6(3cd+4be)}{b^4c^5(b+cx)}+\frac{6d^5(2c^2d^2-7bcde+7b^2e^2)\log(x)}{b^5}+\frac{6(-cd+be)^5(2c^2d^2+3bcde+2b^2e^2)\log(b+cx)}{b^5c^5}\right)$$

input
```
Integrate[(d + e*x)^7/(b*x + c*x^2)^3,x]
```

output
```
(-(d^7/(b^3*x^2)) + (2*d^6*(3*c*d - 7*b*e))/(b^4*x) + (2*e^6*(7*c*d - 3*b*
e)*x)/c^4 + (e^7*x^2)/c^3 + (c*d - b*e)^7/(b^3*c^5*(b + c*x)^2) + (2*(c*d
- b*e)^6*(3*c*d + 4*b*e))/(b^4*c^5*(b + c*x)) + (6*d^5*(2*c^2*d^2 - 7*b*c*
d*e + 7*b^2*e^2)*Log[x])/b^5 + (6*(-(c*d) + b*e)^5*(2*c^2*d^2 + 3*b*c*d*e
+ 2*b^2*e^2)*Log[b + c*x])/(b^5*c^5))/2
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^7}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{1141}$$

$$c^3\int \left(\frac{d^7}{b^3c^3x^3}-\frac{(3cd-7be)d^6}{b^4c^3x^2}+\frac{3(2c^2d^2-7bced+7b^2e^2)\,d^5}{b^5c^3x}+\frac{e^6(7cd-3be)}{c^7}+\frac{e^7x}{c^6}-\frac{3(cd-be)^5\,(2c^2d^2+3b}{b^5c^7(b+cx}\right.$$

$$\Big\downarrow \text{2009}$$

$$c^3\left(\frac{(cd-be)^6(4be+3cd)}{b^4c^8(b+cx)}+\frac{d^6(3cd-7be)}{b^4c^3x}+\frac{(cd-be)^7}{2b^3c^8(b+cx)^2}-\frac{d^7}{2b^3c^3x^2}-\frac{3(cd-be)^5\left(2b^2e^2+3bcde+2c^2d^2\right)\log}{b^5c^8}\right.$$

input
```
Int[(d + e*x)^7/(b*x + c*x^2)^3,x]
```

output
```
c^3*(-1/2*d^7/(b^3*c^3*x^2) + (d^6*(3*c*d - 7*b*e))/(b^4*c^3*x) + (e^6*(7*
c*d - 3*b*e)*x)/c^7 + (e^7*x^2)/(2*c^6) + (c*d - b*e)^7/(2*b^3*c^8*(b + c*
x)^2) + ((c*d - b*e)^6*(3*c*d + 4*b*e))/(b^4*c^8*(b + c*x)) + (3*d^5*(2*c^
2*d^2 - 7*b*c*d*e + 7*b^2*e^2)*Log[x])/(b^5*c^3) - (3*(c*d - b*e)^5*(2*c^2
*d^2 + 3*b*c*d*e + 2*b^2*e^2)*Log[b + c*x])/(b^5*c^8))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.83

method	result
default	$-\frac{e^6\left(-\frac{1}{2}ce\,x^2+3bex-7cdx\right)}{c^4}+\frac{\left(6b^7e^7-21b^6d\,e^6c+21d^2e^5b^5c^2-21c^5d^5e^2b^2+21c^6d^6eb-6d^7c^7\right)\ln(cx+b)}{b^5c^5}-\frac{-4b^7e^7+21b\cdots}{\cdots}$
norman	$\frac{\frac{\left(12b^7e^7-42b^6d\,e^6c+42d^2e^5b^5c^2-35d^3e^4b^4c^3+21c^5d^5e^2b^2-21c^6d^6eb+6d^7c^7\right)x^3}{c^4b^4}-\frac{d^7}{2b}+\frac{e^7x^6}{2c}-\frac{d^6(7be-2cd)x}{b^2}-\frac{e^6(2be-7cd)x^5}{c^2}+\frac{(18b\cdots}{\cdots}}{x^2(cx+b)^2}$
risch	$\frac{e^7x^2}{2c^3}-\frac{3e^7bx}{c^4}+\frac{7e^6dx}{c^3}+\frac{\frac{\left(4b^7e^7-21b^6d\,e^6c+42d^2e^5b^5c^2-35d^3e^4b^4c^3+21c^5d^5e^2b^2-21c^6d^6eb+6d^7c^7\right)x^3}{b^4}+\frac{(7b^7e^7-35b^6d\,e^6c\cdots}{\cdots}}{c^4x^2(c\cdots}$
parallelrisch	$\frac{14x^5b^5c^4d\,e^6-84x^3b^7c^2d\,e^6+84x^3b^6c^3d^2e^5-70x^3b^5c^4d^3e^4+42x^3b^3c^6d^5e^2-42x^3b^2c^7d^6e-63x^2b^8cd\,e^6+63x^2b^7c^2d^2e^5-35x\cdots}{\cdots}$

input
```
int((e*x+d)^7/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-e^6/c^4*(-1/2*c*e*x^2+3*b*e*x-7*c*d*x)+(6*b^7*e^7-21*b^6*c*d*e^6+21*b^5*c
^2*d^2*e^5-21*b^2*c^5*d^5*e^2+21*b*c^6*d^6*e-6*c^7*d^7)/b^5/c^5*ln(c*x+b)-
(-4*b^7*e^7+21*b^6*c*d*e^6-42*b^5*c^2*d^2*e^5+35*b^4*c^3*d^3*e^4-21*b^2*c^
5*d^5*e^2+14*b*c^6*d^6*e-3*c^7*d^7)/c^5/b^4/(c*x+b)-1/2/c^5*(b^7*e^7-7*b^6
*c*d*e^6+21*b^5*c^2*d^2*e^5-35*b^4*c^3*d^3*e^4+35*b^3*c^4*d^4*e^3-21*b^2*c
^5*d^5*e^2+7*b*c^6*d^6*e-c^7*d^7)/b^3/(c*x+b)^2-1/2*d^7/b^3/x^2-d^6*(7*b*e
-3*c*d)/b^4/x+3*d^5*(7*b^2*e^2-7*b*c*d*e+2*c^2*d^2)*ln(x)/b^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. 2(197) = 394.

Time = 0.11 (sec) , antiderivative size = 694, normalized size of antiderivative = 3.42

$$\int \frac{(d+ex)^7}{(bx+cx^2)^3}\,dx = \frac{b^5c^4e^7x^6-b^4c^5d^7+2\left(7\,b^5c^4de^6-2\,b^6c^3e^7\right)x^5+\left(28\,b^6c^3de^6-11\,b^7c^2e^7\right)x^4+2\left(6\,bc^8d^7-21\,b^2c^7d^6e+2\cdots\right.}{\cdots}$$

input
```
integrate((e*x+d)^7/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/2*(b^5*c^4*e^7*x^6 - b^4*c^5*d^7 + 2*(7*b^5*c^4*d*e^6 - 2*b^6*c^3*e^7)*x
^5 + (28*b^6*c^3*d*e^6 - 11*b^7*c^2*e^7)*x^4 + 2*(6*b*c^8*d^7 - 21*b^2*c^7
*d^6*e + 21*b^3*c^6*d^5*e^2 - 35*b^5*c^4*d^3*e^4 + 42*b^6*c^3*d^2*e^5 - 14
*b^7*c^2*d*e^6 + b^8*c*e^7)*x^3 + (18*b^2*c^7*d^7 - 63*b^3*c^6*d^6*e + 63*
b^4*c^5*d^5*e^2 - 35*b^5*c^4*d^4*e^3 - 35*b^6*c^3*d^3*e^4 + 63*b^7*c^2*d^2
*e^5 - 35*b^8*c*d*e^6 + 7*b^9*e^7)*x^2 + 2*(2*b^3*c^6*d^7 - 7*b^4*c^5*d^6*
e)*x - 6*((2*c^9*d^7 - 7*b*c^8*d^6*e + 7*b^2*c^7*d^5*e^2 - 7*b^5*c^4*d^2*e
^5 + 7*b^6*c^3*d*e^6 - 2*b^7*c^2*e^7)*x^4 + 2*(2*b*c^8*d^7 - 7*b^2*c^7*d^6
*e + 7*b^3*c^6*d^5*e^2 - 7*b^6*c^3*d^2*e^5 + 7*b^7*c^2*d*e^6 - 2*b^8*c*e^7
)*x^3 + (2*b^2*c^7*d^7 - 7*b^3*c^6*d^6*e + 7*b^4*c^5*d^5*e^2 - 7*b^7*c^2*d
^2*e^5 + 7*b^8*c*d*e^6 - 2*b^9*e^7)*x^2)*log(c*x + b) + 6*((2*c^9*d^7 - 7*
b*c^8*d^6*e + 7*b^2*c^7*d^5*e^2)*x^4 + 2*(2*b*c^8*d^7 - 7*b^2*c^7*d^6*e +
7*b^3*c^6*d^5*e^2)*x^3 + (2*b^2*c^7*d^7 - 7*b^3*c^6*d^6*e + 7*b^4*c^5*d^5*
e^2)*x^2)*log(x))/(b^5*c^7*x^4 + 2*b^6*c^6*x^3 + b^7*c^5*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 687 vs. $2(201) = 402$.

Time = 20.87 (sec) , antiderivative size = 687, normalized size of antiderivative = 3.38

$$\int \frac{(d+ex)^7}{(bx+cx^2)^3}\,dx = x\left(-\frac{3be^7}{c^4}+\frac{7de^6}{c^3}\right) + \frac{-b^3c^5d^7 + x^3\cdot(8b^7ce^7-42b^6c^2de^6+84b^5c^3d^2e^5-70b^4c^4d^3e^4+42b^2c^6d^5e^2-42bc^7d^6e+12c^8d^7)+x^{}}{2b} + \frac{e^7x^2}{2c^3} + \frac{3d^5\cdot(7b^2e^2-7bcde+2c^2d^2)\log\left(x+\frac{-21b^3c^4d^5e^2+21b^2c^5d^6e-6bc^6d^7+3bc^4d^5\cdot(7b^2e^2-7bcde+2c^2d^2)}{6b^7e^7-21b^6cde^6+21b^5c^2d^2e^5-42b^2c^5d^5e^2+42bc^6d^6e-12c^7d^7}\right)}{b^5} + \frac{3(be-cd)^5\cdot(2b^2e^2+3bcde+2c^2d^2)\log\left(x+\frac{-21b^3c^4d^5e^2+21b^2c^5d^6e-6bc^6d^7+\frac{3b(be-cd)^5\cdot(2b^2e^2+3bcde+2c^2d^2)}{c}}{6b^7e^7-21b^6cde^6+21b^5c^2d^2e^5-42b^2c^5d^5e^2+42bc^6d^6e-12c^7d^7}\right)}{b^5c^5}$$

input
```
integrate((e*x+d)**7/(c*x**2+b*x)**3,x)
```

output
```
x*(-3*b*e**7/c**4 + 7*d*e**6/c**3) + (-b**3*c**5*d**7 + x**3*(8*b**7*c*e**
7 - 42*b**6*c**2*d*e**6 + 84*b**5*c**3*d**2*e**5 - 70*b**4*c**4*d**3*e**4
+ 42*b**2*c**6*d**5*e**2 - 42*b*c**7*d**6*e + 12*c**8*d**7) + x**2*(7*b**8
*e**7 - 35*b**7*c*d*e**6 + 63*b**6*c**2*d**2*e**5 - 35*b**5*c**3*d**3*e**4
 - 35*b**4*c**4*d**4*e**3 + 63*b**3*c**5*d**5*e**2 - 63*b**2*c**6*d**6*e +
 18*b*c**7*d**7) + x*(-14*b**3*c**5*d**6*e + 4*b**2*c**6*d**7))/(2*b**6*c*
*5*x**2 + 4*b**5*c**6*x**3 + 2*b**4*c**7*x**4) + e**7*x**2/(2*c**3) + 3*d*
*5*(7*b**2*e**2 - 7*b*c*d*e + 2*c**2*d**2)*log(x + (-21*b**3*c**4*d**5*e**
2 + 21*b**2*c**5*d**6*e - 6*b*c**6*d**7 + 3*b*c**4*d**5*(7*b**2*e**2 - 7*b
*c*d*e + 2*c**2*d**2))/(6*b**7*e**7 - 21*b**6*c*d*e**6 + 21*b**5*c**2*d**2
*e**5 - 42*b**2*c**5*d**5*e**2 + 42*b*c**6*d**6*e - 12*c**7*d**7))/b**5 +
3*(b*e - c*d)**5*(2*b**2*e**2 + 3*b*c*d*e + 2*c**2*d**2)*log(x + (-21*b**3
*c**4*d**5*e**2 + 21*b**2*c**5*d**6*e - 6*b*c**6*d**7 + 3*b*(b*e - c*d)**5
*(2*b**2*e**2 + 3*b*c*d*e + 2*c**2*d**2)/c)/(6*b**7*e**7 - 21*b**6*c*d*e**
6 + 21*b**5*c**2*d**2*e**5 - 42*b**2*c**5*d**5*e**2 + 42*b*c**6*d**6*e - 1
2*c**7*d**7))/(b**5*c**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(197) = 394$.

Time = 0.04 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.01

$$\int \frac{(d+ex)^7}{(bx+cx^2)^3}\,dx =$$

$$-\frac{b^3c^5d^7 - 2\left(6\,c^8d^7 - 21\,bc^7d^6e + 21\,b^2c^6d^5e^2 - 35\,b^4c^4d^3e^4 + 42\,b^5c^3d^2e^5 - 21\,b^6c^2de^6 + 4\,b^7ce^7\right)x^3 - (1}{2(}$$

$$+\frac{ce^7x^2 + 2\left(7\,cde^6 - 3\,be^7\right)x}{2\,c^4} + \frac{3\left(2\,c^2d^7 - 7\,bcd^6e + 7\,b^2d^5e^2\right)\log(x)}{b^5}$$

$$-\frac{3\left(2\,c^7d^7 - 7\,bc^6d^6e + 7\,b^2c^5d^5e^2 - 7\,b^5c^2d^2e^5 + 7\,b^6cde^6 - 2\,b^7e^7\right)\log(cx+b)}{b^5c^5}$$

input
```
integrate((e*x+d)^7/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(b^3*c^5*d^7 - 2*(6*c^8*d^7 - 21*b*c^7*d^6*e + 21*b^2*c^6*d^5*e^2 - 3
5*b^4*c^4*d^3*e^4 + 42*b^5*c^3*d^2*e^5 - 21*b^6*c^2*d*e^6 + 4*b^7*c*e^7)*x
^3 - (18*b*c^7*d^7 - 63*b^2*c^6*d^6*e + 63*b^3*c^5*d^5*e^2 - 35*b^4*c^4*d^
4*e^3 - 35*b^5*c^3*d^3*e^4 + 63*b^6*c^2*d^2*e^5 - 35*b^7*c*d*e^6 + 7*b^8*e
^7)*x^2 - 2*(2*b^2*c^6*d^7 - 7*b^3*c^5*d^6*e)*x)/(b^4*c^7*x^4 + 2*b^5*c^6*
x^3 + b^6*c^5*x^2) + 1/2*(c*e^7*x^2 + 2*(7*c*d*e^6 - 3*b*e^7)*x)/c^4 + 3*(
2*c^2*d^7 - 7*b*c*d^6*e + 7*b^2*d^5*e^2)*log(x)/b^5 - 3*(2*c^7*d^7 - 7*b*c
^6*d^6*e + 7*b^2*c^5*d^5*e^2 - 7*b^5*c^2*d^2*e^5 + 7*b^6*c*d*e^6 - 2*b^7*e
^7)*log(c*x + b)/(b^5*c^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(197) = 394$.

Time = 0.36 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.96

$$\int \frac{(d+ex)^7}{(bx+cx^2)^3}\,dx = \frac{3\,(2\,c^2d^7-7\,bcd^6e+7\,b^2d^5e^2)\log(|x|)}{b^5} + \frac{c^3e^7x^2+14\,c^3de^6x-6\,bc^2e^7x}{2\,c^6} - \frac{3\,(2\,c^7d^7-7\,bc^6d^6e+7\,b^2c^5d^5e^2-7\,b^5c^2d^2e^5+7\,b^6cde^6-2\,b^7e^7)\log(|cx+b|)}{b^5c^5} - \frac{b^3c^5d^7-2\,(6\,c^8d^7-21\,bc^7d^6e+21\,b^2c^6d^5e^2-35\,b^4c^4d^3e^4+42\,b^5c^3d^2e^5-21\,b^6c^2de^6+4\,b^7ce^7)x^3-(1}{}$$

input
```
integrate((e*x+d)^7/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
3*(2*c^2*d^7 - 7*b*c*d^6*e + 7*b^2*d^5*e^2)*log(abs(x))/b^5 + 1/2*(c^3*e^7
*x^2 + 14*c^3*d*e^6*x - 6*b*c^2*e^7*x)/c^6 - 3*(2*c^7*d^7 - 7*b*c^6*d^6*e
+ 7*b^2*c^5*d^5*e^2 - 7*b^5*c^2*d^2*e^5 + 7*b^6*c*d*e^6 - 2*b^7*e^7)*log(a
bs(c*x + b))/(b^5*c^5) - 1/2*(b^3*c^5*d^7 - 2*(6*c^8*d^7 - 21*b*c^7*d^6*e
+ 21*b^2*c^6*d^5*e^2 - 35*b^4*c^4*d^3*e^4 + 42*b^5*c^3*d^2*e^5 - 21*b^6*c^
2*d*e^6 + 4*b^7*c*e^7)*x^3 - (18*b*c^7*d^7 - 63*b^2*c^6*d^6*e + 63*b^3*c^5
*d^5*e^2 - 35*b^4*c^4*d^4*e^3 - 35*b^5*c^3*d^3*e^4 + 63*b^6*c^2*d^2*e^5 -
35*b^7*c*d*e^6 + 7*b^8*e^7)*x^2 - 2*(2*b^2*c^6*d^7 - 7*b^3*c^5*d^6*e)*x)/(
(c*x + b)^2*b^4*c^5*x^2)
```

Mupad [B] (verification not implemented)

Time = 9.19 (sec) , antiderivative size = 399, normalized size of antiderivative = 1.97

$$\int \frac{(d+ex)^7}{(bx+cx^2)^3}\,dx = \frac{e^7\,x^2}{2\,c^3} - x\left(\frac{3\,b\,e^7}{c^4} - \frac{7\,d\,e^6}{c^3}\right) - \frac{\frac{c^4\,d^7}{2\,b} - \frac{x^3\,(4\,b^7\,e^7-21\,b^6\,c\,d\,e^6+42\,b^5\,c^2\,d^2\,e^5-35\,b^4\,c^3\,d^3\,e^4+21\,b^2\,c^5\,d^5\,e^2-21\,b\,c^6\,d^6\,e+6\,c^7\,d^7)}{b^4} - \frac{x^2\,(7\,b^7\,e^7-35\,b^6\,c\,d\,e^6+63\,b^5\,c^2\,d^2\,e}{}}{b^2\,c^4\,x^2+2\,b\,c^5\,x^3+c^6\,x^4} + \frac{\ln(b+c\,x)\,(6\,b^7\,e^7-21\,b^6\,c\,d\,e^6+21\,b^5\,c^2\,d^2\,e^5-21\,b^2\,c^5\,d^5\,e^2+21\,b\,c^6\,d^6\,e-6\,c^7\,d^7)}{b^5\,c^5} + \frac{3\,d^5\,\ln(x)\,(7\,b^2\,e^2-7\,b\,c\,d\,e+2\,c^2\,d^2)}{b^5}$$

input
```
int((d + e*x)^7/(b*x + c*x^2)^3,x)
```

output
```
(e^7*x^2)/(2*c^3) - x*((3*b*e^7)/c^4 - (7*d*e^6)/c^3) - ((c^4*d^7)/(2*b) -
 (x^3*(4*b^7*e^7 + 6*c^7*d^7 + 21*b^2*c^5*d^5*e^2 - 35*b^4*c^3*d^3*e^4 + 4
2*b^5*c^2*d^2*e^5 - 21*b*c^6*d^6*e - 21*b^6*c*d*e^6))/b^4 - (x^2*(7*b^7*e^
7 + 18*c^7*d^7 + 63*b^2*c^5*d^5*e^2 - 35*b^3*c^4*d^4*e^3 - 35*b^4*c^3*d^3*
e^4 + 63*b^5*c^2*d^2*e^5 - 63*b*c^6*d^6*e - 35*b^6*c*d*e^6))/(2*b^3*c) + (
c^4*d^6*x*(7*b*e - 2*c*d))/b^2)/(c^6*x^4 + 2*b*c^5*x^3 + b^2*c^4*x^2) + (l
og(b + c*x)*(6*b^7*e^7 - 6*c^7*d^7 - 21*b^2*c^5*d^5*e^2 + 21*b^5*c^2*d^2*e
^5 + 21*b*c^6*d^6*e - 21*b^6*c*d*e^6))/(b^5*c^5) + (3*d^5*log(x)*(7*b^2*e^
2 + 2*c^2*d^2 - 7*b*c*d*e))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 835, normalized size of antiderivative = 4.11

$$\int \frac{(d+ex)^7}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^7/(c*x^2+b*x)^3,x)
```

output
```
(12*log(b + c*x)*b**9*e**7*x**2 - 42*log(b + c*x)*b**8*c*d*e**6*x**2 + 24*
log(b + c*x)*b**8*c*e**7*x**3 + 42*log(b + c*x)*b**7*c**2*d**2*e**5*x**2 -
 84*log(b + c*x)*b**7*c**2*d*e**6*x**3 + 12*log(b + c*x)*b**7*c**2*e**7*x*
*4 + 84*log(b + c*x)*b**6*c**3*d**2*e**5*x**3 - 42*log(b + c*x)*b**6*c**3*
d*e**6*x**4 + 42*log(b + c*x)*b**5*c**4*d**2*e**5*x**4 - 42*log(b + c*x)*b
**4*c**5*d**5*e**2*x**2 + 42*log(b + c*x)*b**3*c**6*d**6*e*x**2 - 84*log(b
 + c*x)*b**3*c**6*d**5*e**2*x**3 - 12*log(b + c*x)*b**2*c**7*d**7*x**2 + 8
4*log(b + c*x)*b**2*c**7*d**6*e*x**3 - 42*log(b + c*x)*b**2*c**7*d**5*e**2
*x**4 - 24*log(b + c*x)*b*c**8*d**7*x**3 + 42*log(b + c*x)*b*c**8*d**6*e*x
**4 - 12*log(b + c*x)*c**9*d**7*x**4 + 42*log(x)*b**4*c**5*d**5*e**2*x**2
- 42*log(x)*b**3*c**6*d**6*e*x**2 + 84*log(x)*b**3*c**6*d**5*e**2*x**3 + 1
2*log(x)*b**2*c**7*d**7*x**2 - 84*log(x)*b**2*c**7*d**6*e*x**3 + 42*log(x)
*b**2*c**7*d**5*e**2*x**4 + 24*log(x)*b*c**8*d**7*x**3 - 42*log(x)*b*c**8*
d**6*e*x**4 + 12*log(x)*c**9*d**7*x**4 + 6*b**9*e**7*x**2 - 21*b**8*c*d*e*
*6*x**2 + 21*b**7*c**2*d**2*e**5*x**2 - 12*b**7*c**2*e**7*x**4 + 42*b**6*c
**3*d*e**6*x**4 - 4*b**6*c**3*e**7*x**5 - 35*b**5*c**4*d**4*e**3*x**2 - 42
*b**5*c**4*d**2*e**5*x**4 + 14*b**5*c**4*d*e**6*x**5 + b**5*c**4*e**7*x**6
 - b**4*c**5*d**7 - 14*b**4*c**5*d**6*e*x + 42*b**4*c**5*d**5*e**2*x**2 +
35*b**4*c**5*d**3*e**4*x**4 + 4*b**3*c**6*d**7*x - 42*b**3*c**6*d**6*e*x**
2 + 12*b**2*c**7*d**7*x**2 - 21*b**2*c**7*d**5*e**2*x**4 + 21*b*c**8*d*...
```

3.72 $\int \frac{(d+ex)^6}{\left(bx+cx^2\right)^3} dx$

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Optimal result

Integrand size = 19, antiderivative size = 179

$$\begin{aligned}\int \frac{(d+ex)^6}{\left(bx+cx^2\right)^3} dx = & -\frac{d^6}{2b^3x^2} + \frac{3d^5(cd-2be)}{b^4x} + \frac{e^6x}{c^3} + \frac{(cd-be)^6}{2b^3c^4(b+cx)^2} \\ & + \frac{3(cd-be)^5(cd+be)}{b^4c^4(b+cx)} + \frac{3d^4(2c^2d^2-6bcde+5b^2e^2)\log(x)}{b^5} \\ & - \frac{3(cd-be)^4\left(2c^2d^2+2bcde+b^2e^2\right)\log(b+cx)}{b^5c^4}\end{aligned}$$

output
```
-1/2*d^6/b^3/x^2+3*d^5*(-2*b*e+c*d)/b^4/x+e^6*x/c^3+1/2*(-b*e+c*d)^6/b^3/c
^4/(c*x+b)^2+3*(-b*e+c*d)^5*(b*e+c*d)/b^4/c^4/(c*x+b)+3*d^4*(5*b^2*e^2-6*b
*c*d*e+2*c^2*d^2)*ln(x)/b^5-3*(-b*e+c*d)^4*(b^2*e^2+2*b*c*d*e+2*c^2*d^2)*l
n(c*x+b)/b^5/c^4
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^6}{(bx+cx^2)^3}\,dx = -\frac{d^6}{2b^3x^2} + \frac{3d^5(cd-2be)}{b^4x} + \frac{e^6x}{c^3} + \frac{(cd-be)^6}{2b^3c^4(b+cx)^2} + \frac{3(cd-be)^5(cd+be)}{b^4c^4(b+cx)} + \frac{3d^4(2c^2d^2-6bcde+5b^2e^2)\log(x)}{b^5} - \frac{3(cd-be)^4\left(2c^2d^2+2bcde+b^2e^2\right)\log(b+cx)}{b^5c^4}$$

input
```
Integrate[(d + e*x)^6/(b*x + c*x^2)^3,x]
```

output
```
-1/2*d^6/(b^3*x^2) + (3*d^5*(c*d - 2*b*e))/(b^4*x) + (e^6*x)/c^3 + (c*d -
b*e)^6/(2*b^3*c^4*(b + c*x)^2) + (3*(c*d - b*e)^5*(c*d + b*e))/(b^4*c^4*(b
 + c*x)) + (3*d^4*(2*c^2*d^2 - 6*b*c*d*e + 5*b^2*e^2)*Log[x])/b^5 - (3*(c*
d - b*e)^4*(2*c^2*d^2 + 2*b*c*d*e + b^2*e^2)*Log[b + c*x])/(b^5*c^4)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.07, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^6}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{1141}$$

$$c^3\int \left(\frac{d^6}{b^3c^3x^3} - \frac{3(cd-2be)d^5}{b^4c^3x^2} + \frac{3\left(2c^2d^2-6bced+5b^2e^2\right)d^4}{b^5c^3x} + \frac{e^6}{c^6} - \frac{3(cd-be)^4\left(2c^2d^2+2bced+b^2e^2\right)}{b^5c^6(b+cx)} - 3(c\right.$$

$$\Big\downarrow \text{2009}$$

$$c^3\left(\frac{3(cd-be)^5(be+cd)}{b^4c^7(b+cx)}+\frac{3d^5(cd-2be)}{b^4c^3x}+\frac{(cd-be)^6}{2b^3c^7(b+cx)^2}-\frac{d^6}{2b^3c^3x^2}-\frac{3(cd-be)^4\left(b^2e^2+2bcde+2c^2d^2\right)\log(}{b^5c^7}\right.$$

input
```
Int[(d + e*x)^6/(b*x + c*x^2)^3,x]
```

output
```
c^3*(-1/2*d^6/(b^3*c^3*x^2) + (3*d^5*(c*d - 2*b*e))/(b^4*c^3*x) + (e^6*x)/
c^6 + (c*d - b*e)^6/(2*b^3*c^7*(b + c*x)^2) + (3*(c*d - b*e)^5*(c*d + b*e)
)/(b^4*c^7*(b + c*x)) + (3*d^4*(2*c^2*d^2 - 6*b*c*d*e + 5*b^2*e^2)*Log[x])
/(b^5*c^3) - (3*(c*d - b*e)^4*(2*c^2*d^2 + 2*b*c*d*e + b^2*e^2)*Log[b + c*
x])/(b^5*c^7))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.74

method	result
default	$\frac{e^6x}{c^3}+\frac{\left(-3b^6e^6+6b^5d\,e^5c-15b^2c^4d^4e^2+18b\,c^5d^5e-6d^6c^6\right)\ln(cx+b)}{b^5c^4}-\frac{3b^6e^6-12b^5d\,e^5c+15b^4d^2e^4c^2-15b^2c^4d^4e^2+12b\,c}{b^4c^4(cx+b)}$
norman	$\frac{\frac{e^6x^5}{c}-\frac{d^6}{2b}-\frac{2d^5(3be-cd)x}{b^2}-\frac{\left(6b^6e^6-12b^5d\,e^5c+15b^4d^2e^4c^2-15b^2c^4d^4e^2+18b\,c^5d^5e-6d^6c^6\right)x^3}{b^4c^3}-\frac{(9b^6e^6-18b^5d\,e^5c+15b^4d^2e^4c^2+20b}{}}{x^2(cx+b)^2}$
risch	$\frac{e^6x}{c^3}+\frac{-\frac{3\left(b^6e^6-4b^5d\,e^5c+5b^4d^2e^4c^2-5b^2c^4d^4e^2+6b\,c^5d^5e-2d^6c^6\right)x^3}{b^4}-\frac{(5b^6e^6-18b^5d\,e^5c+15b^4d^2e^4c^2+20b^3d^3e^3c^3-45b^2c^4d^4e^2}{2b^3c}}{c^3x^2(cx+b)^2}$
parallelrisch	$\frac{-9x^2b^8e^6+12\ln(cx+b)x^4b^5c^3d\,e^5-36\ln(x)x^2b^3c^5d^5e+12\ln(cx+b)x^2b^7cd\,e^5-30\ln(cx+b)x^2b^4c^4d^4e^2+36\ln(cx+b)x^2b^3c^5d}{}$

input
```
int((e*x+d)^6/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
e^6*x/c^3+(-3*b^6*e^6+6*b^5*c*d*e^5-15*b^2*c^4*d^4*e^2+18*b*c^5*d^5*e-6*c^
6*d^6)/b^5/c^4*ln(c*x+b)-(3*b^6*e^6-12*b^5*c*d*e^5+15*b^4*c^2*d^2*e^4-15*b
^2*c^4*d^4*e^2+12*b*c^5*d^5*e-3*c^6*d^6)/b^4/c^4/(c*x+b)-1/2*(-b^6*e^6+6*b
^5*c*d*e^5-15*b^4*c^2*d^2*e^4+20*b^3*c^3*d^3*e^3-15*b^2*c^4*d^4*e^2+6*b*c^
5*d^5*e-c^6*d^6)/b^3/c^4/(c*x+b)^2-1/2*d^6/b^3/x^2+3*d^4*(5*b^2*e^2-6*b*c*
d*e+2*c^2*d^2)*ln(x)/b^5-3*d^5*(2*b*e-c*d)/b^4/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(175) = 350$.

Time = 0.10 (sec) , antiderivative size = 579, normalized size of antiderivative = 3.23

$$\int \frac{(d+ex)^6}{(bx+cx^2)^3}\,dx$$
$$= \frac{2\,b^5c^3e^6x^5 + 4\,b^6c^2e^6x^4 - b^4c^4d^6 + 2\,(6\,bc^7d^6 - 18\,b^2c^6d^5e + 15\,b^3c^5d^4e^2 - 15\,b^5c^3d^2e^4 + 12\,b^6c^2de^5 - 2\,b^7}{}$$

input
```
integrate((e*x+d)^6/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/2*(2*b^5*c^3*e^6*x^5 + 4*b^6*c^2*e^6*x^4 - b^4*c^4*d^6 + 2*(6*b*c^7*d^6
- 18*b^2*c^6*d^5*e + 15*b^3*c^5*d^4*e^2 - 15*b^5*c^3*d^2*e^4 + 12*b^6*c^2*
d*e^5 - 2*b^7*c*e^6)*x^3 + (18*b^2*c^6*d^6 - 54*b^3*c^5*d^5*e + 45*b^4*c^4
*d^4*e^2 - 20*b^5*c^3*d^3*e^3 - 15*b^6*c^2*d^2*e^4 + 18*b^7*c*d*e^5 - 5*b^
8*e^6)*x^2 + 4*(b^3*c^5*d^6 - 3*b^4*c^4*d^5*e)*x - 6*((2*c^8*d^6 - 6*b*c^7
*d^5*e + 5*b^2*c^6*d^4*e^2 - 2*b^5*c^3*d*e^5 + b^6*c^2*e^6)*x^4 + 2*(2*b*c
^7*d^6 - 6*b^2*c^6*d^5*e + 5*b^3*c^5*d^4*e^2 - 2*b^6*c^2*d*e^5 + b^7*c*e^6
)*x^3 + (2*b^2*c^6*d^6 - 6*b^3*c^5*d^5*e + 5*b^4*c^4*d^4*e^2 - 2*b^7*c*d*e
^5 + b^8*e^6)*x^2)*log(c*x + b) + 6*((2*c^8*d^6 - 6*b*c^7*d^5*e + 5*b^2*c^
6*d^4*e^2)*x^4 + 2*(2*b*c^7*d^6 - 6*b^2*c^6*d^5*e + 5*b^3*c^5*d^4*e^2)*x^3
 + (2*b^2*c^6*d^6 - 6*b^3*c^5*d^5*e + 5*b^4*c^4*d^4*e^2)*x^2)*log(x))/(b^5
*c^6*x^4 + 2*b^6*c^5*x^3 + b^7*c^4*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 597 vs. $2(175) = 350$.

Time = 7.04 (sec) , antiderivative size = 597, normalized size of antiderivative = 3.34

$$\int \frac{(d+ex)^6}{(bx+cx^2)^3}\,dx$$
$$= \frac{-b^3c^4d^6 + x^3(-6b^6ce^6 + 24b^5c^2de^5 - 30b^4c^3d^2e^4 + 30b^2c^5d^4e^2 - 36bc^6d^5e + 12c^7d^6) + x^2(-5b^7e^6 + 18b^6}{2b^6c^4x^2 + 4b^5c^5x^3}$$
$$+ \frac{e^6x}{c^3}$$
$$+ \frac{3d^4\cdot(5b^2e^2 - 6bcde + 2c^2d^2)\log\left(x + \frac{15b^3c^3d^4e^2 - 18b^2c^4d^5e + 6bc^5d^6 - 3bc^3d^4\cdot(5b^2e^2 - 6bcde + 2c^2d^2)}{3b^6e^6 - 6b^5cde^5 + 30b^2c^4d^4e^2 - 36bc^5d^5e + 12c^6d^6}\right)}{b^5}$$
$$- \frac{3(be-cd)^4\,(b^2e^2 + 2bcde + 2c^2d^2)\log\left(x + \frac{15b^3c^3d^4e^2 - 18b^2c^4d^5e + 6bc^5d^6 + \frac{3b(be-cd)^4\left(b^2e^2 + 2bcde + 2c^2d^2\right)}{c}}{3b^6e^6 - 6b^5cde^5 + 30b^2c^4d^4e^2 - 36bc^5d^5e + 12c^6d^6}\right)}{b^5c^4}$$

input
```
integrate((e*x+d)**6/(c*x**2+b*x)**3,x)
```

output
```
(-b**3*c**4*d**6 + x**3*(-6*b**6*c*e**6 + 24*b**5*c**2*d*e**5 - 30*b**4*c*
*3*d**2*e**4 + 30*b**2*c**5*d**4*e**2 - 36*b*c**6*d**5*e + 12*c**7*d**6) +
 x**2*(-5*b**7*e**6 + 18*b**6*c*d*e**5 - 15*b**5*c**2*d**2*e**4 - 20*b**4*
c**3*d**3*e**3 + 45*b**3*c**4*d**4*e**2 - 54*b**2*c**5*d**5*e + 18*b*c**6*
d**6) + x*(-12*b**3*c**4*d**5*e + 4*b**2*c**5*d**6))/(2*b**6*c**4*x**2 + 4
*b**5*c**5*x**3 + 2*b**4*c**6*x**4) + e**6*x/c**3 + 3*d**4*(5*b**2*e**2 -
6*b*c*d*e + 2*c**2*d**2)*log(x + (15*b**3*c**3*d**4*e**2 - 18*b**2*c**4*d*
*5*e + 6*b*c**5*d**6 - 3*b*c**3*d**4*(5*b**2*e**2 - 6*b*c*d*e + 2*c**2*d**
2))/(3*b**6*e**6 - 6*b**5*c*d*e**5 + 30*b**2*c**4*d**4*e**2 - 36*b*c**5*d*
*5*e + 12*c**6*d**6))/b**5 - 3*(b*e - c*d)**4*(b**2*e**2 + 2*b*c*d*e + 2*c
**2*d**2)*log(x + (15*b**3*c**3*d**4*e**2 - 18*b**2*c**4*d**5*e + 6*b*c**5
*d**6 + 3*b*(b*e - c*d)**4*(b**2*e**2 + 2*b*c*d*e + 2*c**2*d**2)/c)/(3*b**
6*e**6 - 6*b**5*c*d*e**5 + 30*b**2*c**4*d**4*e**2 - 36*b*c**5*d**5*e + 12*
c**6*d**6))/(b**5*c**4)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.91

$$\int \frac{(d+ex)^6}{(bx+cx^2)^3}\,dx = \frac{e^6x}{c^3}$$
$$-\frac{b^3c^4d^6 - 6\,(2\,c^7d^6 - 6\,bc^6d^5e + 5\,b^2c^5d^4e^2 - 5\,b^4c^3d^2e^4 + 4\,b^5c^2de^5 - b^6ce^6)x^3 - (18\,bc^6d^6 - 54\,b^2c^5d^5e}{2\,(b^4c^6x^4 + 2\,b^5c^5x^3}$$
$$+\frac{3\,(2\,c^2d^6 - 6\,bcd^5e + 5\,b^2d^4e^2)\log(x)}{b^5}$$
$$-\frac{3\,(2\,c^6d^6 - 6\,bc^5d^5e + 5\,b^2c^4d^4e^2 - 2\,b^5cde^5 + b^6e^6)\log(cx+b)}{b^5c^4}$$

input
```
integrate((e*x+d)^6/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
e^6*x/c^3 - 1/2*(b^3*c^4*d^6 - 6*(2*c^7*d^6 - 6*b*c^6*d^5*e + 5*b^2*c^5*d^
4*e^2 - 5*b^4*c^3*d^2*e^4 + 4*b^5*c^2*d*e^5 - b^6*c*e^6)*x^3 - (18*b*c^6*d
^6 - 54*b^2*c^5*d^5*e + 45*b^3*c^4*d^4*e^2 - 20*b^4*c^3*d^3*e^3 - 15*b^5*c
^2*d^2*e^4 + 18*b^6*c*d*e^5 - 5*b^7*e^6)*x^2 - 4*(b^2*c^5*d^6 - 3*b^3*c^4*
d^5*e)*x)/(b^4*c^6*x^4 + 2*b^5*c^5*x^3 + b^6*c^4*x^2) + 3*(2*c^2*d^6 - 6*b
*c*d^5*e + 5*b^2*d^4*e^2)*log(x)/b^5 - 3*(2*c^6*d^6 - 6*b*c^5*d^5*e + 5*b^
2*c^4*d^4*e^2 - 2*b^5*c*d*e^5 + b^6*e^6)*log(c*x + b)/(b^5*c^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 325, normalized size of antiderivative = 1.82

$$\int \frac{(d+ex)^6}{(bx+cx^2)^3}\,dx = \frac{e^6x}{c^3} + \frac{3\,(2\,c^2d^6 - 6\,bcd^5e + 5\,b^2d^4e^2)\log(|x|)}{b^5}$$
$$-\frac{3\,(2\,c^6d^6 - 6\,bc^5d^5e + 5\,b^2c^4d^4e^2 - 2\,b^5cde^5 + b^6e^6)\log(|cx+b|)}{b^5c^4}$$
$$-\frac{b^3c^4d^6 - 6\,(2\,c^7d^6 - 6\,bc^6d^5e + 5\,b^2c^5d^4e^2 - 5\,b^4c^3d^2e^4 + 4\,b^5c^2de^5 - b^6ce^6)x^3 - (18\,bc^6d^6 - 54\,b^2c^5d^5e}{2\,(cx+b)^2b^4c}$$

input
```
integrate((e*x+d)^6/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
e^6*x/c^3 + 3*(2*c^2*d^6 - 6*b*c*d^5*e + 5*b^2*d^4*e^2)*log(abs(x))/b^5 -
3*(2*c^6*d^6 - 6*b*c^5*d^5*e + 5*b^2*c^4*d^4*e^2 - 2*b^5*c*d*e^5 + b^6*e^6
)*log(abs(c*x + b))/(b^5*c^4) - 1/2*(b^3*c^4*d^6 - 6*(2*c^7*d^6 - 6*b*c^6*
d^5*e + 5*b^2*c^5*d^4*e^2 - 5*b^4*c^3*d^2*e^4 + 4*b^5*c^2*d*e^5 - b^6*c*e^
6)*x^3 - (18*b*c^6*d^6 - 54*b^2*c^5*d^5*e + 45*b^3*c^4*d^4*e^2 - 20*b^4*c^
3*d^3*e^3 - 15*b^5*c^2*d^2*e^4 + 18*b^6*c*d*e^5 - 5*b^7*e^6)*x^2 - 4*(b^2*
c^5*d^6 - 3*b^3*c^4*d^5*e)*x)/((c*x + b)^2*b^4*c^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.86

$$\int \frac{(d+ex)^6}{(bx+cx^2)^3}\,dx = \frac{e^6\,x}{c^3} - \frac{\frac{3\,x^3\,\left(b^6\,e^6-4\,b^5\,c\,d\,e^5+5\,b^4\,c^2\,d^2\,e^4-5\,b^2\,c^4\,d^4\,e^2+6\,b\,c^5\,d^5\,e-2\,c^6\,d^6\right)}{b^4}+\frac{c^3\,d^6}{2\,b}+\frac{x^2\,\left(5\,b^6\,e^6-18\,b^5\,c\,d\,e^5+15\,b^4\,c^2\,d^2\,e^4+20\,b^3\,c^3\,d^3\,e^3-45\right)}{2\,b^3\,c}}{b^2\,c^3\,x^2+2\,b\,c^4\,x^3+c^5\,x^4} - \frac{\ln\left(b+c\,x\right)\,\left(3\,b^6\,e^6-6\,b^5\,c\,d\,e^5+15\,b^2\,c^4\,d^4\,e^2-18\,b\,c^5\,d^5\,e+6\,c^6\,d^6\right)}{b^5\,c^4} + \frac{3\,d^4\,\ln\left(x\right)\,\left(5\,b^2\,e^2-6\,b\,c\,d\,e+2\,c^2\,d^2\right)}{b^5}$$

input
```
int((d + e*x)^6/(b*x + c*x^2)^3,x)
```

output
```
(e^6*x)/c^3 - ((3*x^3*(b^6*e^6 - 2*c^6*d^6 - 5*b^2*c^4*d^4*e^2 + 5*b^4*c^2
*d^2*e^4 + 6*b*c^5*d^5*e - 4*b^5*c*d*e^5))/b^4 + (c^3*d^6)/(2*b) + (x^2*(5
*b^6*e^6 - 18*c^6*d^6 - 45*b^2*c^4*d^4*e^2 + 20*b^3*c^3*d^3*e^3 + 15*b^4*c
^2*d^2*e^4 + 54*b*c^5*d^5*e - 18*b^5*c*d*e^5))/(2*b^3*c) + (2*c^3*d^5*x*(3
*b*e - c*d))/b^2)/(c^5*x^4 + 2*b*c^4*x^3 + b^2*c^3*x^2) - (log(b + c*x)*(3
*b^6*e^6 + 6*c^6*d^6 + 15*b^2*c^4*d^4*e^2 - 18*b*c^5*d^5*e - 6*b^5*c*d*e^5
))/(b^5*c^4) + (3*d^4*log(x)*(5*b^2*e^2 + 2*c^2*d^2 - 6*b*c*d*e))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 704, normalized size of antiderivative = 3.93

$$\int \frac{(d+ex)^6}{(bx+cx^2)^3}\,dx$$
$$= \frac{60\log(x)\,b^3c^5d^4e^2x^3 - 72\log(x)\,b^2c^6d^5e\,x^3 + 30\log(x)\,b^2c^6d^4e^2x^4 - 36\log(x)\,b\,c^7d^5e\,x^4 - 3b^8e^6x^2 - 6c^8d}{}$$

input
```
int((e*x+d)^6/(c*x^2+b*x)^3,x)
```

output
```
( - 6*log(b + c*x)*b**8*e**6*x**2 + 12*log(b + c*x)*b**7*c*d*e**5*x**2 - 1
2*log(b + c*x)*b**7*c*e**6*x**3 + 24*log(b + c*x)*b**6*c**2*d*e**5*x**3 -
6*log(b + c*x)*b**6*c**2*e**6*x**4 + 12*log(b + c*x)*b**5*c**3*d*e**5*x**4
 - 30*log(b + c*x)*b**4*c**4*d**4*e**2*x**2 + 36*log(b + c*x)*b**3*c**5*d*
*5*e*x**2 - 60*log(b + c*x)*b**3*c**5*d**4*e**2*x**3 - 12*log(b + c*x)*b**
2*c**6*d**6*x**2 + 72*log(b + c*x)*b**2*c**6*d**5*e*x**3 - 30*log(b + c*x)
*b**2*c**6*d**4*e**2*x**4 - 24*log(b + c*x)*b*c**7*d**6*x**3 + 36*log(b +
c*x)*b*c**7*d**5*e*x**4 - 12*log(b + c*x)*c**8*d**6*x**4 + 30*log(x)*b**4*
c**4*d**4*e**2*x**2 - 36*log(x)*b**3*c**5*d**5*e*x**2 + 60*log(x)*b**3*c**
5*d**4*e**2*x**3 + 12*log(x)*b**2*c**6*d**6*x**2 - 72*log(x)*b**2*c**6*d**
5*e*x**3 + 30*log(x)*b**2*c**6*d**4*e**2*x**4 + 24*log(x)*b*c**7*d**6*x**3
 - 36*log(x)*b*c**7*d**5*e*x**4 + 12*log(x)*c**8*d**6*x**4 - 3*b**8*e**6*x
**2 + 6*b**7*c*d*e**5*x**2 + 6*b**6*c**2*e**6*x**4 - 20*b**5*c**3*d**3*e**
3*x**2 - 12*b**5*c**3*d*e**5*x**4 + 2*b**5*c**3*e**6*x**5 - b**4*c**4*d**6
 - 12*b**4*c**4*d**5*e*x + 30*b**4*c**4*d**4*e**2*x**2 + 15*b**4*c**4*d**2
*e**4*x**4 + 4*b**3*c**5*d**6*x - 36*b**3*c**5*d**5*e*x**2 + 12*b**2*c**6*
d**6*x**2 - 15*b**2*c**6*d**4*e**2*x**4 + 18*b*c**7*d**5*e*x**4 - 6*c**8*d
**6*x**4)/(2*b**5*c**4*x**2*(b**2 + 2*b*c*x + c**2*x**2))
```

3.73 $\int \frac{(d+ex)^5}{(bx+cx^2)^3}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 171

$$\int \frac{(d+ex)^5}{(bx+cx^2)^3}\,dx = -\frac{d^5}{2b^3x^2}+\frac{d^4(3cd-5be)}{b^4x}+\frac{(cd-be)^5}{2b^3c^3(b+cx)^2}+\frac{(cd-be)^4(3cd+2be)}{b^4c^3(b+cx)}+\frac{d^3(6c^2d^2-15bcde+10b^2e^2)\log(x)}{b^5}-\frac{(cd-be)^3(6c^2d^2+3bcde+b^2e^2)\log(b+cx)}{b^5c^3}$$

output

```
-1/2*d^5/b^3/x^2+d^4*(-5*b*e+3*c*d)/b^4/x+1/2*(-b*e+c*d)^5/b^3/c^3/(c*x+b)
^2+(-b*e+c*d)^4*(2*b*e+3*c*d)/b^4/c^3/(c*x+b)+d^3*(10*b^2*e^2-15*b*c*d*e+6
*c^2*d^2)*ln(x)/b^5-(-b*e+c*d)^3*(b^2*e^2+3*b*c*d*e+6*c^2*d^2)*ln(c*x+b)/b
^5/c^3
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^5}{(bx+cx^2)^3}\,dx = -\frac{\frac{b^2d^5}{x^2}+\frac{2bd^4(-3cd+5be)}{x}+\frac{b^2(-cd+be)^5}{c^3(b+cx)^2}-\frac{2b(cd-be)^4(3cd+2be)}{c^3(b+cx)}-2d^3(6c^2d^2-15bcde+10b^2e^2)\log(x)+\frac{2(cd-be)^3}{}}{2b^5}$$

input
```
Integrate[(d + e*x)^5/(b*x + c*x^2)^3,x]
```

output
```
-1/2*((b^2*d^5)/x^2 + (2*b*d^4*(-3*c*d + 5*b*e))/x + (b^2*(-(c*d) + b*e)^5
)/(c^3*(b + c*x)^2) - (2*b*(c*d - b*e)^4*(3*c*d + 2*b*e))/(c^3*(b + c*x))
- 2*d^3*(6*c^2*d^2 - 15*b*c*d*e + 10*b^2*e^2)*Log[x] + (2*(c*d - b*e)^3*(6
*c^2*d^2 + 3*b*c*d*e + b^2*e^2)*Log[b + c*x])/c^3)/b^5
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^5}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{1141}$$

$$c^3\int \left(\frac{d^5}{b^3c^3x^3}-\frac{(3cd-5be)d^4}{b^4c^3x^2}+\frac{(6c^2d^2-15bced+10b^2e^2)\,d^3}{b^5c^3x}-\frac{(cd-be)^3\,(6c^2d^2+3bced+b^2e^2)}{b^5c^5(b+cx)}-\frac{(cd-be}{b^4c^5}\right.$$

$$\Big\downarrow \text{2009}$$

$$c^3\left(\frac{(cd-be)^4(2be+3cd)}{b^4c^6(b+cx)}+\frac{d^4(3cd-5be)}{b^4c^3x}+\frac{(cd-be)^5}{2b^3c^6(b+cx)^2}-\frac{d^5}{2b^3c^3x^2}-\frac{(cd-be)^3\,(b^2e^2+3bcde+6c^2d^2)\log(}{b^5c^6}\right.$$

input
```
Int[(d + e*x)^5/(b*x + c*x^2)^3,x]
```

output
```
c^3*(-1/2*d^5/(b^3*c^3*x^2) + (d^4*(3*c*d - 5*b*e))/(b^4*c^3*x) + (c*d - b
*e)^5/(2*b^3*c^6*(b + c*x)^2) + ((c*d - b*e)^4*(3*c*d + 2*b*e))/(b^4*c^6*(
b + c*x)) + (d^3*(6*c^2*d^2 - 15*b*c*d*e + 10*b^2*e^2)*Log[x])/(b^5*c^3) -
 ((c*d - b*e)^3*(6*c^2*d^2 + 3*b*c*d*e + b^2*e^2)*Log[b + c*x])/(b^5*c^6))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.53

method	result
norman	$\frac{\frac{\left(2b^{5}e^{5}-5d\,e^{4}b^{4}c+10b^{2}c^{3}d^{3}e^{2}-15d^{4}eb\,c^{4}+6d^{5}c^{5}\right)x^{3}}{b^{4}c^{2}}-\frac{d^{5}}{2b}-\frac{d^{4}(5be-2cd)x}{b^{2}}+\frac{\left(3b^{5}e^{5}-5d\,e^{4}b^{4}c-10b^{3}d^{2}e^{3}c^{2}+30b^{2}c^{3}d^{3}e^{2}-45d^{4}eb\,c^{4}+1}{2b^{3}c^{3}}}{x^{2}(cx+b)^{2}}$
default	$\frac{\left(b^{5}e^{5}-10b^{2}c^{3}d^{3}e^{2}+15d^{4}eb\,c^{4}-6d^{5}c^{5}\right)\ln(cx+b)}{b^{5}c^{3}}-\frac{-2b^{5}e^{5}+5d\,e^{4}b^{4}c-10b^{2}c^{3}d^{3}e^{2}+10d^{4}eb\,c^{4}-3d^{5}c^{5}}{b^{4}c^{3}(cx+b)}-b^{5}e^{5}-5d\,e^{4}b^{4}c+1$
risch	$\frac{\frac{\left(2b^{5}e^{5}-5d\,e^{4}b^{4}c+10b^{2}c^{3}d^{3}e^{2}-15d^{4}eb\,c^{4}+6d^{5}c^{5}\right)x^{3}}{b^{4}c^{2}}-\frac{d^{5}}{2b}-\frac{d^{4}(5be-2cd)x}{b^{2}}+\frac{\left(3b^{5}e^{5}-5d\,e^{4}b^{4}c-10b^{3}d^{2}e^{3}c^{2}+30b^{2}c^{3}d^{3}e^{2}-45d^{4}eb\,c^{4}+1}{2b^{3}c^{3}}}{x^{2}(cx+b)^{2}}$
parallelrisch	$12\ln(x)x^{4}c^{7}d^{5}-12\ln(cx+b)x^{4}c^{7}d^{5}+2\ln(cx+b)x^{2}b^{7}e^{5}+4x^{3}b^{6}c\,e^{5}+12x^{3}b\,c^{6}d^{5}+18x^{2}b^{2}c^{5}d^{5}-30\ln(x)x^{2}b^{3}c^{4}d^{4}e+4x\,b^{3}c^{4}d^{5}$

input
```
int((e*x+d)^5/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
((2*b^5*e^5-5*b^4*c*d*e^4+10*b^2*c^3*d^3*e^2-15*b*c^4*d^4*e+6*c^5*d^5)/b^4
/c^2*x^3-1/2*d^5/b-d^4*(5*b*e-2*c*d)/b^2*x+1/2*(3*b^5*e^5-5*b^4*c*d*e^4-10
*b^3*c^2*d^2*e^3+30*b^2*c^3*d^3*e^2-45*b*c^4*d^4*e+18*c^5*d^5)/b^3/c^3*x^2
)/x^2/(c*x+b)^2+d^3*(10*b^2*e^2-15*b*c*d*e+6*c^2*d^2)*ln(x)/b^5+(b^5*e^5-1
0*b^2*c^3*d^3*e^2+15*b*c^4*d^4*e-6*c^5*d^5)/b^5/c^3*ln(c*x+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(167) = 334$.

Time = 0.10 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.88

$$\int \frac{(d+ex)^5}{(bx+cx^2)^3}\,dx =$$
$$-\frac{b^4c^3d^5 - 2\,(6\,bc^6d^5 - 15\,b^2c^5d^4e + 10\,b^3c^4d^3e^2 - 5\,b^5c^2de^4 + 2\,b^6ce^5)x^3 - (18\,b^2c^5d^5 - 45\,b^3c^4d^4e + 30\,}{}$$

input
```
integrate((e*x+d)^5/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(b^4*c^3*d^5 - 2*(6*b*c^6*d^5 - 15*b^2*c^5*d^4*e + 10*b^3*c^4*d^3*e^2
 - 5*b^5*c^2*d*e^4 + 2*b^6*c*e^5)*x^3 - (18*b^2*c^5*d^5 - 45*b^3*c^4*d^4*e
 + 30*b^4*c^3*d^3*e^2 - 10*b^5*c^2*d^2*e^3 - 5*b^6*c*d*e^4 + 3*b^7*e^5)*x^
2 - 2*(2*b^3*c^4*d^5 - 5*b^4*c^3*d^4*e)*x + 2*((6*c^7*d^5 - 15*b*c^6*d^4*e
 + 10*b^2*c^5*d^3*e^2 - b^5*c^2*e^5)*x^4 + 2*(6*b*c^6*d^5 - 15*b^2*c^5*d^4
*e + 10*b^3*c^4*d^3*e^2 - b^6*c*e^5)*x^3 + (6*b^2*c^5*d^5 - 15*b^3*c^4*d^4
*e + 10*b^4*c^3*d^3*e^2 - b^7*e^5)*x^2)*log(c*x + b) - 2*((6*c^7*d^5 - 15*
b*c^6*d^4*e + 10*b^2*c^5*d^3*e^2)*x^4 + 2*(6*b*c^6*d^5 - 15*b^2*c^5*d^4*e
+ 10*b^3*c^4*d^3*e^2)*x^3 + (6*b^2*c^5*d^5 - 15*b^3*c^4*d^4*e + 10*b^4*c^3
*d^3*e^2)*x^2)*log(x))/(b^5*c^5*x^4 + 2*b^6*c^4*x^3 + b^7*c^3*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 524 vs. $2(165) = 330$.

Time = 2.91 (sec) , antiderivative size = 524, normalized size of antiderivative = 3.06

$$\int \frac{(d+ex)^5}{(bx+cx^2)^3}\,dx$$
$$= \frac{-b^3c^3d^5 + x^3\cdot(4b^5ce^5 - 10b^4c^2de^4 + 20b^2c^4d^3e^2 - 30bc^5d^4e + 12c^6d^5) + x^2\cdot(3b^6e^5 - 5b^5cde^4 - 10b^4c^2d}{2b^6c^3x^2 + 4b^5c^4x^3 + 2b^4c^5x^4}$$
$$+ \frac{d^3\cdot(10b^2e^2 - 15bcde + 6c^2d^2)\log\left(x + \frac{-10b^3c^2d^3e^2 + 15b^2c^3d^4e - 6bc^4d^5 + bc^2d^3\cdot(10b^2e^2 - 15bcde + 6c^2d^2)}{b^5e^5 - 20b^2c^3d^3e^2 + 30bc^4d^4e - 12c^5d^5}\right)}{b^5}$$
$$+ \frac{(be-cd)^3\,(b^2e^2 + 3bcde + 6c^2d^2)\log\left(x + \frac{-10b^3c^2d^3e^2 + 15b^2c^3d^4e - 6bc^4d^5 + \frac{b(be-cd)^3(b^2e^2+3bcde+6c^2d^2)}{c}}{b^5e^5 - 20b^2c^3d^3e^2 + 30bc^4d^4e - 12c^5d^5}\right)}{b^5c^3}$$

input
```
integrate((e*x+d)**5/(c*x**2+b*x)**3,x)
```

output
```
(-b**3*c**3*d**5 + x**3*(4*b**5*c*e**5 - 10*b**4*c**2*d*e**4 + 20*b**2*c**
4*d**3*e**2 - 30*b*c**5*d**4*e + 12*c**6*d**5) + x**2*(3*b**6*e**5 - 5*b**
5*c*d*e**4 - 10*b**4*c**2*d**2*e**3 + 30*b**3*c**3*d**3*e**2 - 45*b**2*c**
4*d**4*e + 18*b*c**5*d**5) + x*(-10*b**3*c**3*d**4*e + 4*b**2*c**4*d**5))/
(2*b**6*c**3*x**2 + 4*b**5*c**4*x**3 + 2*b**4*c**5*x**4) + d**3*(10*b**2*e
**2 - 15*b*c*d*e + 6*c**2*d**2)*log(x + (-10*b**3*c**2*d**3*e**2 + 15*b**2
*c**3*d**4*e - 6*b*c**4*d**5 + b*c**2*d**3*(10*b**2*e**2 - 15*b*c*d*e + 6*
c**2*d**2))/(b**5*e**5 - 20*b**2*c**3*d**3*e**2 + 30*b*c**4*d**4*e - 12*c*
*5*d**5))/b**5 + (b*e - c*d)**3*(b**2*e**2 + 3*b*c*d*e + 6*c**2*d**2)*log(
x + (-10*b**3*c**2*d**3*e**2 + 15*b**2*c**3*d**4*e - 6*b*c**4*d**5 + b*(b*
e - c*d)**3*(b**2*e**2 + 3*b*c*d*e + 6*c**2*d**2)/c)/(b**5*e**5 - 20*b**2*
c**3*d**3*e**2 + 30*b*c**4*d**4*e - 12*c**5*d**5))/(b**5*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.73

$$\int \frac{(d+ex)^5}{(bx+cx^2)^3}\,dx = -\frac{b^3c^3d^5 - 2\,(6\,c^6d^5 - 15\,bc^5d^4e + 10\,b^2c^4d^3e^2 - 5\,b^4c^2de^4 + 2\,b^5ce^5)x^3 - (18\,bc^5d^5 - 45\,b^2c^4d^4e + 30\,b^3c}{2\,(b^4c^5x^4 + 2\,b^5c^4x^3 + b^6c^3x^2)} + \frac{(6\,c^2d^5 - 15\,bcd^4e + 10\,b^2d^3e^2)\log{(x)}}{b^5} - \frac{(6\,c^5d^5 - 15\,bc^4d^4e + 10\,b^2c^3d^3e^2 - b^5e^5)\log{(cx+b)}}{b^5c^3}$$

input
```
integrate((e*x+d)^5/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(b^3*c^3*d^5 - 2*(6*c^6*d^5 - 15*b*c^5*d^4*e + 10*b^2*c^4*d^3*e^2 - 5
*b^4*c^2*d*e^4 + 2*b^5*c*e^5)*x^3 - (18*b*c^5*d^5 - 45*b^2*c^4*d^4*e + 30*
b^3*c^3*d^3*e^2 - 10*b^4*c^2*d^2*e^3 - 5*b^5*c*d*e^4 + 3*b^6*e^5)*x^2 - 2*
(2*b^2*c^4*d^5 - 5*b^3*c^3*d^4*e)*x)/(b^4*c^5*x^4 + 2*b^5*c^4*x^3 + b^6*c^
3*x^2) + (6*c^2*d^5 - 15*b*c*d^4*e + 10*b^2*d^3*e^2)*log(x)/b^5 - (6*c^5*d
^5 - 15*b*c^4*d^4*e + 10*b^2*c^3*d^3*e^2 - b^5*e^5)*log(c*x + b)/(b^5*c^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)^5}{(bx+cx^2)^3}\,dx = \frac{(6\,c^2d^5 - 15\,bcd^4e + 10\,b^2d^3e^2)\log{(|x|)}}{b^5} - \frac{(6\,c^5d^5 - 15\,bc^4d^4e + 10\,b^2c^3d^3e^2 - b^5e^5)\log{(|cx+b|)}}{b^5c^3} - \frac{b^3c^3d^5 - 2\,(6\,c^6d^5 - 15\,bc^5d^4e + 10\,b^2c^4d^3e^2 - 5\,b^4c^2de^4 + 2\,b^5ce^5)x^3 - (18\,bc^5d^5 - 45\,b^2c^4d^4e + 30\,b^3c}{2\,(cx+b)^2b^4c^3x^2}$$

input
```
integrate((e*x+d)^5/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
(6*c^2*d^5 - 15*b*c*d^4*e + 10*b^2*d^3*e^2)*log(abs(x))/b^5 - (6*c^5*d^5 -
 15*b*c^4*d^4*e + 10*b^2*c^3*d^3*e^2 - b^5*e^5)*log(abs(c*x + b))/(b^5*c^3
) - 1/2*(b^3*c^3*d^5 - 2*(6*c^6*d^5 - 15*b*c^5*d^4*e + 10*b^2*c^4*d^3*e^2
- 5*b^4*c^2*d*e^4 + 2*b^5*c*e^5)*x^3 - (18*b*c^5*d^5 - 45*b^2*c^4*d^4*e +
30*b^3*c^3*d^3*e^2 - 10*b^4*c^2*d^2*e^3 - 5*b^5*c*d*e^4 + 3*b^6*e^5)*x^2 -
 2*(2*b^2*c^4*d^5 - 5*b^3*c^3*d^4*e)*x)/((c*x + b)^2*b^4*c^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 9.27 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.57

$$\int \frac{(d+ex)^5}{(bx+cx^2)^3}\,dx = \frac{d^3\,\ln(x)\,\left(10\,b^2\,e^2-15\,b\,c\,d\,e+6\,c^2\,d^2\right)}{b^5} - \frac{\frac{d^5}{2\,b}+\frac{d^4\,x\,(5\,b\,e-2\,c\,d)}{b^2}-\frac{x^2\,\left(3\,b^5\,e^5-5\,b^4\,c\,d\,e^4-10\,b^3\,c^2\,d^2\,e^3+30\,b^2\,c^3\,d^3\,e^2-45\,b\,c^4\,d^4\,e+18\,c^5\,d^5\right)}{2\,b^3\,c^3}-\frac{x^3\,\left(2\,b^5\,e^5-5\,b^4\,c\,d\,e^4+10\,b^2\,c^3\,d\right.}{b^4\,c^2}}{b^2\,x^2+2\,b\,c\,x^3+c^2\,x^4} + \frac{\ln(b+c\,x)\,\left(b\,e-c\,d\right)^3\,\left(b^2\,e^2+3\,b\,c\,d\,e+6\,c^2\,d^2\right)}{b^5\,c^3}$$

input
```
int((d + e*x)^5/(b*x + c*x^2)^3,x)
```

output
```
(d^3*log(x)*(10*b^2*e^2 + 6*c^2*d^2 - 15*b*c*d*e))/b^5 - (d^5/(2*b) + (d^4
*x*(5*b*e - 2*c*d))/b^2 - (x^2*(3*b^5*e^5 + 18*c^5*d^5 + 30*b^2*c^3*d^3*e^
2 - 10*b^3*c^2*d^2*e^3 - 45*b*c^4*d^4*e - 5*b^4*c*d*e^4))/(2*b^3*c^3) - (x
^3*(2*b^5*e^5 + 6*c^5*d^5 + 10*b^2*c^3*d^3*e^2 - 15*b*c^4*d^4*e - 5*b^4*c*
d*e^4))/(b^4*c^2))/(b^2*x^2 + c^2*x^4 + 2*b*c*x^3) + (log(b + c*x)*(b*e -
c*d)^3*(b^2*e^2 + 6*c^2*d^2 + 3*b*c*d*e))/(b^5*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 598, normalized size of antiderivative = 3.50

$$\int \frac{(d+ex)^5}{(bx+cx^2)^3}\,dx = \frac{4\log(cx+b)\,b^6ce^5x^3+2\log(cx+b)\,b^5c^2e^5x^4-12\log(cx+b)\,b^2c^5d^5x^2-24\log(cx+b)\,bc^6d^5x^3+12\log}{}$$

input
```
int((e*x+d)^5/(c*x^2+b*x)^3,x)
```

output
```
(2*log(b + c*x)*b**7*e**5*x**2 + 4*log(b + c*x)*b**6*c*e**5*x**3 + 2*log(b
 + c*x)*b**5*c**2*e**5*x**4 - 20*log(b + c*x)*b**4*c**3*d**3*e**2*x**2 + 3
0*log(b + c*x)*b**3*c**4*d**4*e*x**2 - 40*log(b + c*x)*b**3*c**4*d**3*e**2
*x**3 - 12*log(b + c*x)*b**2*c**5*d**5*x**2 + 60*log(b + c*x)*b**2*c**5*d*
*4*e*x**3 - 20*log(b + c*x)*b**2*c**5*d**3*e**2*x**4 - 24*log(b + c*x)*b*c
**6*d**5*x**3 + 30*log(b + c*x)*b*c**6*d**4*e*x**4 - 12*log(b + c*x)*c**7*
d**5*x**4 + 20*log(x)*b**4*c**3*d**3*e**2*x**2 - 30*log(x)*b**3*c**4*d**4*
e*x**2 + 40*log(x)*b**3*c**4*d**3*e**2*x**3 + 12*log(x)*b**2*c**5*d**5*x**
2 - 60*log(x)*b**2*c**5*d**4*e*x**3 + 20*log(x)*b**2*c**5*d**3*e**2*x**4 +
 24*log(x)*b*c**6*d**5*x**3 - 30*log(x)*b*c**6*d**4*e*x**4 + 12*log(x)*c**
7*d**5*x**4 + b**7*e**5*x**2 - 10*b**5*c**2*d**2*e**3*x**2 - 2*b**5*c**2*e
**5*x**4 - b**4*c**3*d**5 - 10*b**4*c**3*d**4*e*x + 20*b**4*c**3*d**3*e**2
*x**2 + 5*b**4*c**3*d*e**4*x**4 + 4*b**3*c**4*d**5*x - 30*b**3*c**4*d**4*e
*x**2 + 12*b**2*c**5*d**5*x**2 - 10*b**2*c**5*d**3*e**2*x**4 + 15*b*c**6*d
**4*e*x**4 - 6*c**7*d**5*x**4)/(2*b**5*c**3*x**2*(b**2 + 2*b*c*x + c**2*x*
*2))
```

3.74 $\int \frac{(d+ex)^4}{\left(bx+cx^2\right)^3}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 136

$$\int \frac{(d+ex)^4}{(bx+cx^2)^3}\,dx = -\frac{d^4}{2b^3x^2}+\frac{d^3(3cd-4be)}{b^4x}+\frac{(cd-be)^4}{2b^3c^2(b+cx)^2}+\frac{(cd-be)^3(3cd+be)}{b^4c^2(b+cx)} + \frac{6d^2(cd-be)^2\log(x)}{b^5}-\frac{6d^2(cd-be)^2\log(b+cx)}{b^5}$$

output
```
-1/2*d^4/b^3/x^2+d^3*(-4*b*e+3*c*d)/b^4/x+1/2*(-b*e+c*d)^4/b^3/c^2/(c*x+b)
^2+(-b*e+c*d)^3*(b*e+3*c*d)/b^4/c^2/(c*x+b)+6*d^2*(-b*e+c*d)^2*ln(x)/b^5-6
*d^2*(-b*e+c*d)^2*ln(c*x+b)/b^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.96

$$\int \frac{(d+ex)^4}{(bx+cx^2)^3}\,dx = -\frac{\frac{b^2d^4}{x^2}+\frac{2bd^3(-3cd+4be)}{x}-\frac{b^2(cd-be)^4}{c^2(b+cx)^2}+\frac{2b(-cd+be)^3(3cd+be)}{c^2(b+cx)}-12d^2(cd-be)^2\log(x)+12d^2(cd-be)^2\log(b+c}{2b^5}$$

input
```
Integrate[(d + e*x)^4/(b*x + c*x^2)^3,x]
```

output
```
-1/2*((b^2*d^4)/x^2 + (2*b*d^3*(-3*c*d + 4*b*e))/x - (b^2*(c*d - b*e)^4)/(
c^2*(b + c*x)^2) + (2*b*(-(c*d) + b*e)^3*(3*c*d + b*e))/(c^2*(b + c*x)) -
12*d^2*(c*d - b*e)^2*Log[x] + 12*d^2*(c*d - b*e)^2*Log[b + c*x])/b^5
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(bx+cx^2)^3}dx$$

$$\Big\downarrow \text{ 1141}$$

$$c^3\int \left(\frac{d^4}{b^3c^3x^3}-\frac{(3cd-4be)d^3}{b^4c^3x^2}+\frac{6(cd-be)^2d^2}{b^5c^3x}-\frac{6(cd-be)^2d^2}{b^5c^2(b+cx)}-\frac{(cd-be)^3(3cd+be)}{b^4c^4(b+cx)^2}-\frac{(cd-be)^4}{b^3c^4(b+cx)^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$c^3\left(\frac{6d^2\log(x)(cd-be)^2}{b^5c^3}-\frac{6d^2(cd-be)^2\log(b+cx)}{b^5c^3}+\frac{(cd-be)^3(be+3cd)}{b^4c^5(b+cx)}+\frac{d^3(3cd-4be)}{b^4c^3x}+\frac{(cd-be)^4}{2b^3c^5(b+cx)^2}\right.$$

input
```
Int[(d + e*x)^4/(b*x + c*x^2)^3,x]
```

output
```
c^3*(-1/2*d^4/(b^3*c^3*x^2) + (d^3*(3*c*d - 4*b*e))/(b^4*c^3*x) + (c*d - b
*e)^4/(2*b^3*c^5*(b + c*x)^2) + ((c*d - b*e)^3*(3*c*d + b*e))/(b^4*c^5*(b
+ c*x)) + (6*d^2*(c*d - b*e)^2*Log[x])/(b^5*c^3) - (6*d^2*(c*d - b*e)^2*Lo
g[b + c*x])/(b^5*c^3))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.56

method	result
norman	$\frac{-\frac{d^{4}}{2b}+\frac{2\left(2d\,e^{3}b^{3}-6d^{2}e^{2}b^{2}c+12d^{3}eb\,c^{2}-6d^{4}c^{3}\right)x^{3}}{b^{4}}+\frac{\left(e^{4}b^{4}+4d\,e^{3}b^{3}c-18d^{2}e^{2}b^{2}c^{2}+36d^{3}eb\,c^{3}-18d^{4}c^{4}\right)x^{4}}{2b^{5}}-\frac{2d^{3}(2be-cd)x}{b^{2}}}{x^{2}(cx+b)^{2}}+\frac{6d^{2}(b^{2}$
default	$-\frac{e^{4}b^{4}-6d^{2}e^{2}b^{2}c^{2}+8d^{3}eb\,c^{3}-3d^{4}c^{4}}{b^{4}c^{2}(cx+b)}-\frac{-e^{4}b^{4}+4d\,e^{3}b^{3}c-6d^{2}e^{2}b^{2}c^{2}+4d^{3}eb\,c^{3}-d^{4}c^{4}}{2b^{3}c^{2}(cx+b)^{2}}-\frac{6d^{2}\left(b^{2}e^{2}-2bcde+c^{2}d^{2}\right)\ln(cx+b)}{b^{5}}$
risch	$\frac{-\frac{\left(e^{4}b^{4}-6d^{2}e^{2}b^{2}c^{2}+12d^{3}eb\,c^{3}-6d^{4}c^{4}\right)x^{3}}{b^{4}c}-\frac{\left(e^{4}b^{4}+4d\,e^{3}b^{3}c-18d^{2}e^{2}b^{2}c^{2}+36d^{3}eb\,c^{3}-18d^{4}c^{4}\right)x^{2}}{2b^{3}c^{2}}-\frac{2d^{3}(2be-cd)x}{b^{2}}-\frac{d^{4}}{2b}}{x^{2}(cx+b)^{2}}+\frac{6d^{2}\ln(-}{b^{3}}$
parallelrisch	$-24\ln(x)x^{2}b^{3}c\,d^{3}e+24\ln(cx+b)x^{2}b^{3}c\,d^{3}e+48\ln(cx+b)x^{3}b^{2}c^{2}d^{3}e+12\ln(x)x^{4}b^{2}c^{2}d^{2}e^{2}-24\ln(x)x^{4}b\,c^{3}d^{3}e-12\ln(cx+b)x^{4}b$

input
```
int((e*x+d)^4/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
(-1/2*d^4/b+2*(2*b^3*d*e^3-6*b^2*c*d^2*e^2+12*b*c^2*d^3*e-6*c^3*d^4)/b^4*x
^3+1/2*(b^4*e^4+4*b^3*c*d*e^3-18*b^2*c^2*d^2*e^2+36*b*c^3*d^3*e-18*c^4*d^4
)/b^5*x^4-2*d^3*(2*b*e-c*d)/b^2*x)/x^2/(c*x+b)^2+6*d^2*(b^2*e^2-2*b*c*d*e+
c^2*d^2)/b^5*ln(x)-6*d^2*(b^2*e^2-2*b*c*d*e+c^2*d^2)/b^5*ln(c*x+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(132) = 264$.

Time = 0.09 (sec) , antiderivative size = 426, normalized size of antiderivative = 3.13

$$\int \frac{(d+ex)^4}{(bx+cx^2)^3}\,dx =$$
$$-\frac{b^4c^2d^4 - 2\,(6\,bc^5d^4 - 12\,b^2c^4d^3e + 6\,b^3c^3d^2e^2 - b^5ce^4)x^3 - (18\,b^2c^4d^4 - 36\,b^3c^3d^3e + 18\,b^4c^2d^2e^2 - 4\,b^5}{}$$

input
```
integrate((e*x+d)^4/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(b^4*c^2*d^4 - 2*(6*b*c^5*d^4 - 12*b^2*c^4*d^3*e + 6*b^3*c^3*d^2*e^2
- b^5*c*e^4)*x^3 - (18*b^2*c^4*d^4 - 36*b^3*c^3*d^3*e + 18*b^4*c^2*d^2*e^2
 - 4*b^5*c*d*e^3 - b^6*e^4)*x^2 - 4*(b^3*c^3*d^4 - 2*b^4*c^2*d^3*e)*x + 12
*((c^6*d^4 - 2*b*c^5*d^3*e + b^2*c^4*d^2*e^2)*x^4 + 2*(b*c^5*d^4 - 2*b^2*c
^4*d^3*e + b^3*c^3*d^2*e^2)*x^3 + (b^2*c^4*d^4 - 2*b^3*c^3*d^3*e + b^4*c^2
*d^2*e^2)*x^2)*log(c*x + b) - 12*((c^6*d^4 - 2*b*c^5*d^3*e + b^2*c^4*d^2*e
^2)*x^4 + 2*(b*c^5*d^4 - 2*b^2*c^4*d^3*e + b^3*c^3*d^2*e^2)*x^3 + (b^2*c^4
*d^4 - 2*b^3*c^3*d^3*e + b^4*c^2*d^2*e^2)*x^2)*log(x))/(b^5*c^4*x^4 + 2*b^
6*c^3*x^3 + b^7*c^2*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 389 vs. $2(126) = 252$.

Time = 1.31 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.86

$$\int \frac{(d+ex)^4}{(bx+cx^2)^3}\,dx$$
$$= \frac{-b^3c^2d^4 + x^3(-2b^4ce^4 + 12b^2c^3d^2e^2 - 24bc^4d^3e + 12c^5d^4) + x^2(-b^5e^4 - 4b^4cde^3 + 18b^3c^2d^2e^2 - 36b^2c^3d}{2b^6c^2x^2 + 4b^5c^3x^3 + 2b^4c^4x^4}$$
$$+ \frac{6d^2(be-cd)^2\log\left(x + \frac{6b^3d^2e^2 - 12b^2cd^3e + 6bc^2d^4 - 6bd^2(be-cd)^2}{12b^2cd^2e^2 - 24bc^2d^3e + 12c^3d^4}\right)}{b^5}$$
$$- \frac{6d^2(be-cd)^2\log\left(x + \frac{6b^3d^2e^2 - 12b^2cd^3e + 6bc^2d^4 + 6bd^2(be-cd)^2}{12b^2cd^2e^2 - 24bc^2d^3e + 12c^3d^4}\right)}{b^5}$$

input
```
integrate((e*x+d)**4/(c*x**2+b*x)**3,x)
```

output
```
(-b**3*c**2*d**4 + x**3*(-2*b**4*c*e**4 + 12*b**2*c**3*d**2*e**2 - 24*b*c*
*4*d**3*e + 12*c**5*d**4) + x**2*(-b**5*e**4 - 4*b**4*c*d*e**3 + 18*b**3*c
**2*d**2*e**2 - 36*b**2*c**3*d**3*e + 18*b*c**4*d**4) + x*(-8*b**3*c**2*d*
*3*e + 4*b**2*c**3*d**4))/(2*b**6*c**2*x**2 + 4*b**5*c**3*x**3 + 2*b**4*c*
*4*x**4) + 6*d**2*(b*e - c*d)**2*log(x + (6*b**3*d**2*e**2 - 12*b**2*c*d**
3*e + 6*b*c**2*d**4 - 6*b*d**2*(b*e - c*d)**2)/(12*b**2*c*d**2*e**2 - 24*b
*c**2*d**3*e + 12*c**3*d**4))/b**5 - 6*d**2*(b*e - c*d)**2*log(x + (6*b**3
*d**2*e**2 - 12*b**2*c*d**3*e + 6*b*c**2*d**4 + 6*b*d**2*(b*e - c*d)**2)/(
12*b**2*c*d**2*e**2 - 24*b*c**2*d**3*e + 12*c**3*d**4))/b**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.84

$$\int \frac{(d+ex)^4}{(bx+cx^2)^3}\,dx = -\frac{b^3c^2d^4 - 2\,(6\,c^5d^4 - 12\,bc^4d^3e + 6\,b^2c^3d^2e^2 - b^4ce^4)x^3 - (18\,bc^4d^4 - 36\,b^2c^3d^3e + 18\,b^3c^2d^2e^2 - 4\,b^4cde}{2\,(b^4c^4x^4 + 2\,b^5c^3x^3 + b^6c^2x^2)} - \frac{6\,(c^2d^4 - 2\,bcd^3e + b^2d^2e^2)\log(cx+b)}{b^5} + \frac{6\,(c^2d^4 - 2\,bcd^3e + b^2d^2e^2)\log(x)}{b^5}$$

input
```
integrate((e*x+d)^4/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(b^3*c^2*d^4 - 2*(6*c^5*d^4 - 12*b*c^4*d^3*e + 6*b^2*c^3*d^2*e^2 - b^
4*c*e^4)*x^3 - (18*b*c^4*d^4 - 36*b^2*c^3*d^3*e + 18*b^3*c^2*d^2*e^2 - 4*b
^4*c*d*e^3 - b^5*e^4)*x^2 - 4*(b^2*c^3*d^4 - 2*b^3*c^2*d^3*e)*x)/(b^4*c^4*
x^4 + 2*b^5*c^3*x^3 + b^6*c^2*x^2) - 6*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^
2)*log(c*x + b)/b^5 + 6*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*log(x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.88

$$\int \frac{(d+ex)^4}{(bx+cx^2)^3}\,dx$$
$$= \frac{6\left(c^2d^4-2\,bcd^3e+b^2d^2e^2\right)\log\left(|x|\right)}{b^5} - \frac{6\left(c^3d^4-2\,bc^2d^3e+b^2cd^2e^2\right)\log\left(|cx+b|\right)}{b^5c}$$
$$+ \frac{12\,c^5d^4x^3-24\,bc^4d^3ex^3+12\,b^2c^3d^2e^2x^3-2\,b^4ce^4x^3+18\,bc^4d^4x^2-36\,b^2c^3d^3ex^2+18\,b^3c^2d^2e^2x^2-4\,b}{2\left(cx^2+bx\right)^2b^4c^2}$$

input
```
integrate((e*x+d)^4/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
6*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*log(abs(x))/b^5 - 6*(c^3*d^4 - 2*b
*c^2*d^3*e + b^2*c*d^2*e^2)*log(abs(c*x + b))/(b^5*c) + 1/2*(12*c^5*d^4*x^
3 - 24*b*c^4*d^3*e*x^3 + 12*b^2*c^3*d^2*e^2*x^3 - 2*b^4*c*e^4*x^3 + 18*b*c
^4*d^4*x^2 - 36*b^2*c^3*d^3*e*x^2 + 18*b^3*c^2*d^2*e^2*x^2 - 4*b^4*c*d*e^3
*x^2 - b^5*e^4*x^2 + 4*b^2*c^3*d^4*x - 8*b^3*c^2*d^3*e*x - b^3*c^2*d^4)/((
c*x^2 + b*x)^2*b^4*c^2)
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.75

$$\int \frac{(d+ex)^4}{(bx+cx^2)^3}\,dx =$$
$$-\frac{\frac{d^4}{2\,b}+\frac{2\,d^3\,x\,(2\,b\,e-c\,d)}{b^2}+\frac{x^2\,\left(b^4\,e^4+4\,b^3\,c\,d\,e^3-18\,b^2\,c^2\,d^2\,e^2+36\,b\,c^3\,d^3\,e-18\,c^4\,d^4\right)}{2\,b^3\,c^2}+\frac{x^3\,\left(b^4\,e^4-6\,b^2\,c^2\,d^2\,e^2+12\,b\,c^3\,d^3\,e-6\,c^4\,d^4\right)}{b^4\,c}}{b^2\,x^2+2\,b\,c\,x^3+c^2\,x^4}$$
$$-\frac{12\,d^2\,\mathrm{atanh}\left(\frac{6\,d^2\,{(b\,e-c\,d)}^2\,(b+2\,c\,x)}{b\,\left(6\,b^2\,d^2\,e^2-12\,b\,c\,d^3\,e+6\,c^2\,d^4\right)}\right)\,{(b\,e-c\,d)}^2}{b^5}$$

input
```
int((d + e*x)^4/(b*x + c*x^2)^3,x)
```

output
```
- (d^4/(2*b) + (2*d^3*x*(2*b*e - c*d))/b^2 + (x^2*(b^4*e^4 - 18*c^4*d^4 -
18*b^2*c^2*d^2*e^2 + 36*b*c^3*d^3*e + 4*b^3*c*d*e^3))/(2*b^3*c^2) + (x^3*(
b^4*e^4 - 6*c^4*d^4 - 6*b^2*c^2*d^2*e^2 + 12*b*c^3*d^3*e))/(b^4*c))/(b^2*x
^2 + c^2*x^4 + 2*b*c*x^3) - (12*d^2*atanh((6*d^2*(b*e - c*d)^2*(b + 2*c*x)
)/(b*(6*c^2*d^4 + 6*b^2*d^2*e^2 - 12*b*c*d^3*e)))*(b*e - c*d)^2)/b^5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 500, normalized size of antiderivative = 3.68

$$\int \frac{(d+ex)^4}{(bx+cx^2)^3}\,dx$$
$$= \frac{-12\log(cx+b)\,b^4c\,d^2e^2x^2 + 24\log(cx+b)\,b^3c^2d^3e\,x^2 - 24\log(cx+b)\,b^3c^2d^2e^2x^3 + 48\log(cx+b)\,b^2c^3d^3}{}$$

input
```
int((e*x+d)^4/(c*x^2+b*x)^3,x)
```

output
```
( - 12*log(b + c*x)*b**4*c*d**2*e**2*x**2 + 24*log(b + c*x)*b**3*c**2*d**3
*e*x**2 - 24*log(b + c*x)*b**3*c**2*d**2*e**2*x**3 - 12*log(b + c*x)*b**2*
c**3*d**4*x**2 + 48*log(b + c*x)*b**2*c**3*d**3*e*x**3 - 12*log(b + c*x)*b
**2*c**3*d**2*e**2*x**4 - 24*log(b + c*x)*b*c**4*d**4*x**3 + 24*log(b + c*
x)*b*c**4*d**3*e*x**4 - 12*log(b + c*x)*c**5*d**4*x**4 + 12*log(x)*b**4*c*
d**2*e**2*x**2 - 24*log(x)*b**3*c**2*d**3*e*x**2 + 24*log(x)*b**3*c**2*d**
2*e**2*x**3 + 12*log(x)*b**2*c**3*d**4*x**2 - 48*log(x)*b**2*c**3*d**3*e*x
**3 + 12*log(x)*b**2*c**3*d**2*e**2*x**4 + 24*log(x)*b*c**4*d**4*x**3 - 24
*log(x)*b*c**4*d**3*e*x**4 + 12*log(x)*c**5*d**4*x**4 - 4*b**5*d*e**3*x**2
 - b**4*c*d**4 - 8*b**4*c*d**3*e*x + 12*b**4*c*d**2*e**2*x**2 + b**4*c*e**
4*x**4 + 4*b**3*c**2*d**4*x - 24*b**3*c**2*d**3*e*x**2 + 12*b**2*c**3*d**4
*x**2 - 6*b**2*c**3*d**2*e**2*x**4 + 12*b*c**4*d**3*e*x**4 - 6*c**5*d**4*x
**4)/(2*b**5*c*x**2*(b**2 + 2*b*c*x + c**2*x**2))
```

3.75 $\int \frac{(d+ex)^3}{(bx+cx^2)^3}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 137

$$\int \frac{(d+ex)^3}{(bx+cx^2)^3}\,dx = -\frac{d^3}{2b^3x^2} + \frac{3d^2(cd-be)}{b^4x} + \frac{(cd-be)^3}{2b^3c(b+cx)^2} + \frac{3d(cd-be)^2}{b^4(b+cx)} + \frac{3d(cd-be)(2cd-be)\log(x)}{b^5} - \frac{3d(cd-be)(2cd-be)\log(b+cx)}{b^5}$$

output
```
-1/2*d^3/b^3/x^2+3*d^2*(-b*e+c*d)/b^4/x+1/2*(-b*e+c*d)^3/b^3/c/(c*x+b)^2+3
*d*(-b*e+c*d)^2/b^4/(c*x+b)+3*d*(-b*e+c*d)*(-b*e+2*c*d)*ln(x)/b^5-3*d*(-b*
e+c*d)*(-b*e+2*c*d)*ln(c*x+b)/b^5
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^3}{(bx+cx^2)^3}\,dx = -\frac{\frac{b^2d^3}{x^2} + \frac{6bd^2(-cd+be)}{x} + \frac{b^2(-cd+be)^3}{c(b+cx)^2} - \frac{6bd(cd-be)^2}{b+cx} - 6d(2c^2d^2-3bcde+b^2e^2)\log(x) + 6d(2c^2d^2-3bcde+b}{2b^5}$$

input
```
Integrate[(d + e*x)^3/(b*x + c*x^2)^3,x]
```

output
```
-1/2*((b^2*d^3)/x^2 + (6*b*d^2*(-(c*d) + b*e))/x + (b^2*(-(c*d) + b*e)^3)/
(c*(b + c*x)^2) - (6*b*d*(c*d - b*e)^2)/(b + c*x) - 6*d*(2*c^2*d^2 - 3*b*c
*d*e + b^2*e^2)*Log[x] + 6*d*(2*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*Log[b + c*x
])/b^5
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.14, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \ 1141$$

$$c^3\int \left(\frac{d^3}{b^3c^3x^3}-\frac{3(cd-be)d^2}{b^4c^3x^2}+\frac{3(cd-be)(2cd-be)d}{b^5c^3x}-\frac{3(cd-be)(2cd-be)d}{b^5c^2(b+cx)}-\frac{3(cd-be)^2d}{b^4c^2(b+cx)^2}-\frac{(cd-be)^3}{b^3c^3(b+cx)^3}\right.$$

$$\Big\downarrow \ 2009$$

$$c^3\left(\frac{3d\log(x)(cd-be)(2cd-be)}{b^5c^3}-\frac{3d(cd-be)(2cd-be)\log(b+cx)}{b^5c^3}+\frac{3d^2(cd-be)}{b^4c^3x}+\frac{3d(cd-be)^2}{b^4c^3(b+cx)}+\frac{(cd-}{2b^3c^4(b}\right.$$

input
```
Int[(d + e*x)^3/(b*x + c*x^2)^3,x]
```

output
```
c^3*(-1/2*d^3/(b^3*c^3*x^2) + (3*d^2*(c*d - b*e))/(b^4*c^3*x) + (c*d - b*e
)^3/(2*b^3*c^4*(b + c*x)^2) + (3*d*(c*d - b*e)^2)/(b^4*c^3*(b + c*x)) + (3
*d*(c*d - b*e)*(2*c*d - b*e)*Log[x])/(b^5*c^3) - (3*d*(c*d - b*e)*(2*c*d -
 b*e)*Log[b + c*x])/(b^5*c^3))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.32

method	result
default	$-\frac{b^{3}e^{3}-3d\,e^{2}b^{2}c+3d^{2}eb\,c^{2}-d^{3}c^{3}}{2b^{3}c\left(cx+b\right)^{2}}-\frac{3d\left(b^{2}e^{2}-3bcde+2c^{2}d^{2}\right)\ln\left(cx+b\right)}{b^{5}}+\frac{3d\left(b^{2}e^{2}-2bcde+c^{2}d^{2}\right)}{b^{4}\left(cx+b\right)}-\frac{d^{3}}{2b^{3}x^{2}}+\frac{3d\left(b^{2}e^{2}-\right.}{}$
norman	$\frac{\frac{\left(b^{3}e^{3}-6d\,e^{2}b^{2}c+18d^{2}eb\,c^{2}-12d^{3}c^{3}\right)x^{3}}{b^{4}}-\frac{d^{3}}{2b}+\frac{c\left(b^{3}e^{3}-9d\,e^{2}b^{2}c+27d^{2}eb\,c^{2}-18d^{3}c^{3}\right)x^{4}}{2b^{5}}-\frac{d^{2}\left(3be-2cd\right)x}{b^{2}}}{x^{2}\left(cx+b\right)^{2}}+\frac{3d\left(b^{2}e^{2}-3bcde+2c^{2}d^{2}\right.}{b^{5}}$
risch	$\frac{\frac{3cd\left(b^{2}e^{2}-3bcde+2c^{2}d^{2}\right)x^{3}}{b^{4}}-\frac{\left(b^{3}e^{3}-9d\,e^{2}b^{2}c+27d^{2}eb\,c^{2}-18d^{3}c^{3}\right)x^{2}}{2b^{3}c}-\frac{d^{2}\left(3be-2cd\right)x}{b^{2}}-\frac{d^{3}}{2b}}{x^{2}\left(cx+b\right)^{2}}-\frac{3d\ln\left(cx+b\right)e^{2}}{b^{3}}+\frac{9d^{2}\ln\left(cx+b\right)ce}{b^{4}}$
parallelrisch	$\frac{12\ln\left(x\right)x^{4}c^{4}d^{3}-b^{4}d^{3}-12x^{3}b^{3}cd\,e^{2}+36x^{3}b^{2}c^{2}d^{2}e+24\ln\left(x\right)x^{3}b\,c^{3}d^{3}-24\ln\left(cx+b\right)x^{3}b\,c^{3}d^{3}+6\ln\left(x\right)x^{2}b^{4}d\,e^{2}+12\ln\left(x\right)x^{2}b^{2}c^{2}}{}$

input
```
int((e*x+d)^3/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b^3*e^3-3*b^2*c*d*e^2+3*b*c^2*d^2*e-c^3*d^3)/b^3/c/(c*x+b)^2-3*d*(b^
2*e^2-3*b*c*d*e+2*c^2*d^2)/b^5*ln(c*x+b)+3*d*(b^2*e^2-2*b*c*d*e+c^2*d^2)/b
^4/(c*x+b)-1/2*d^3/b^3/x^2+3*d*(b^2*e^2-3*b*c*d*e+2*c^2*d^2)/b^5*ln(x)-3*d
^2*(b*e-c*d)/b^4/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(133) = 266$.

Time = 0.12 (sec) , antiderivative size = 385, normalized size of antiderivative = 2.81

$$\int \frac{(d+ex)^3}{(bx+cx^2)^3}\,dx = -\frac{b^4cd^3 - 6\,(2\,bc^4d^3 - 3\,b^2c^3d^2e + b^3c^2de^2)x^3 - (18\,b^2c^3d^3 - 27\,b^3c^2d^2e + 9\,b^4cde^2 - b^5e^3)x^2 - 2\,(2\,b^3c^2d}{}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(b^4*c*d^3 - 6*(2*b*c^4*d^3 - 3*b^2*c^3*d^2*e + b^3*c^2*d*e^2)*x^3 -
(18*b^2*c^3*d^3 - 27*b^3*c^2*d^2*e + 9*b^4*c*d*e^2 - b^5*e^3)*x^2 - 2*(2*b
^3*c^2*d^3 - 3*b^4*c*d^2*e)*x + 6*((2*c^5*d^3 - 3*b*c^4*d^2*e + b^2*c^3*d*
e^2)*x^4 + 2*(2*b*c^4*d^3 - 3*b^2*c^3*d^2*e + b^3*c^2*d*e^2)*x^3 + (2*b^2*
c^3*d^3 - 3*b^3*c^2*d^2*e + b^4*c*d*e^2)*x^2)*log(c*x + b) - 6*((2*c^5*d^3
 - 3*b*c^4*d^2*e + b^2*c^3*d*e^2)*x^4 + 2*(2*b*c^4*d^3 - 3*b^2*c^3*d^2*e +
 b^3*c^2*d*e^2)*x^3 + (2*b^2*c^3*d^3 - 3*b^3*c^2*d^2*e + b^4*c*d*e^2)*x^2)
*log(x))/(b^5*c^3*x^4 + 2*b^6*c^2*x^3 + b^7*c*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(124) = 248$.

Time = 0.82 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.71

$$\int \frac{(d+ex)^3}{(bx+cx^2)^3}\,dx = \frac{-b^3cd^3 + x^3\cdot(6b^2c^2de^2 - 18bc^3d^2e + 12c^4d^3) + x^2(-b^4e^3 + 9b^3cde^2 - 27b^2c^2d^2e + 18bc^3d^3) + x(-6b^3cd}{2b^6cx^2 + 4b^5c^2x^3 + 2b^4c^3x^4} + \frac{3d(be-2cd)\,(be-cd)\log\left(x + \frac{3b^3de^2 - 9b^2cd^2e + 6bc^2d^3 - 3bd(be-2cd)(be-cd)}{6b^2cde^2 - 18bc^2d^2e + 12c^3d^3}\right)}{b^5} - \frac{3d(be-2cd)\,(be-cd)\log\left(x + \frac{3b^3de^2 - 9b^2cd^2e + 6bc^2d^3 + 3bd(be-2cd)(be-cd)}{6b^2cde^2 - 18bc^2d^2e + 12c^3d^3}\right)}{b^5}$$

input
```
integrate((e*x+d)**3/(c*x**2+b*x)**3,x)
```

output
```
(-b**3*c*d**3 + x**3*(6*b**2*c**2*d*e**2 - 18*b*c**3*d**2*e + 12*c**4*d**3
) + x**2*(-b**4*e**3 + 9*b**3*c*d*e**2 - 27*b**2*c**2*d**2*e + 18*b*c**3*d
**3) + x*(-6*b**3*c*d**2*e + 4*b**2*c**2*d**3))/(2*b**6*c*x**2 + 4*b**5*c*
*2*x**3 + 2*b**4*c**3*x**4) + 3*d*(b*e - 2*c*d)*(b*e - c*d)*log(x + (3*b**
3*d*e**2 - 9*b**2*c*d**2*e + 6*b*c**2*d**3 - 3*b*d*(b*e - 2*c*d)*(b*e - c*
d))/(6*b**2*c*d*e**2 - 18*b*c**2*d**2*e + 12*c**3*d**3))/b**5 - 3*d*(b*e -
 2*c*d)*(b*e - c*d)*log(x + (3*b**3*d*e**2 - 9*b**2*c*d**2*e + 6*b*c**2*d*
*3 + 3*b*d*(b*e - 2*c*d)*(b*e - c*d))/(6*b**2*c*d*e**2 - 18*b*c**2*d**2*e
+ 12*c**3*d**3))/b**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.58

$$\int \frac{(d+ex)^3}{(bx+cx^2)^3}\,dx = -\frac{b^3cd^3 - 6\,(2\,c^4d^3 - 3\,bc^3d^2e + b^2c^2de^2)x^3 - (18\,bc^3d^3 - 27\,b^2c^2d^2e + 9\,b^3cde^2 - b^4e^3)x^2 - 2\,(2\,b^2c^2d^3 -}{2\,(b^4c^3x^4 + 2\,b^5c^2x^3 + b^6cx^2)} - \frac{3\,(2\,c^2d^3 - 3\,bcd^2e + b^2de^2)\log(cx+b)}{b^5} + \frac{3\,(2\,c^2d^3 - 3\,bcd^2e + b^2de^2)\log(x)}{b^5}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(b^3*c*d^3 - 6*(2*c^4*d^3 - 3*b*c^3*d^2*e + b^2*c^2*d*e^2)*x^3 - (18*
b*c^3*d^3 - 27*b^2*c^2*d^2*e + 9*b^3*c*d*e^2 - b^4*e^3)*x^2 - 2*(2*b^2*c^2
*d^3 - 3*b^3*c*d^2*e)*x)/(b^4*c^3*x^4 + 2*b^5*c^2*x^3 + b^6*c*x^2) - 3*(2*
c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*log(c*x + b)/b^5 + 3*(2*c^2*d^3 - 3*b*c
*d^2*e + b^2*d*e^2)*log(x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.60

$$\int \frac{(d+ex)^3}{(bx+cx^2)^3}\,dx$$
$$= \frac{3\left(2\,c^2d^3-3\,bcd^2e+b^2de^2\right)\log\left(|x|\right)}{b^5}-\frac{3\left(2\,c^3d^3-3\,bc^2d^2e+b^2cde^2\right)\log\left(|cx+b|\right)}{b^5c}$$
$$+\frac{12\,c^4d^3x^3-18\,bc^3d^2ex^3+6\,b^2c^2de^2x^3+18\,bc^3d^3x^2-27\,b^2c^2d^2ex^2+9\,b^3cde^2x^2-b^4e^3x^2+4\,b^2c^2d^3x-}{2\,(cx^2+bx)^2b^4c}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
3*(2*c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*log(abs(x))/b^5 - 3*(2*c^3*d^3 - 3
*b*c^2*d^2*e + b^2*c*d*e^2)*log(abs(c*x + b))/(b^5*c) + 1/2*(12*c^4*d^3*x^
3 - 18*b*c^3*d^2*e*x^3 + 6*b^2*c^2*d*e^2*x^3 + 18*b*c^3*d^3*x^2 - 27*b^2*c
^2*d^2*e*x^2 + 9*b^3*c*d*e^2*x^2 - b^4*e^3*x^2 + 4*b^2*c^2*d^3*x - 6*b^3*c
*d^2*e*x - b^3*c*d^3)/((c*x^2 + b*x)^2*b^4*c)
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)^3}{(bx+cx^2)^3}\,dx$$
$$= -\frac{\frac{d^3}{2\,b}+\frac{d^2\,x\,(3\,b\,e-2\,c\,d)}{b^2}+\frac{x^2\,\left(b^3\,e^3-9\,b^2\,c\,d\,e^2+27\,b\,c^2\,d^2\,e-18\,c^3\,d^3\right)}{2\,b^3\,c}-\frac{3\,c\,d\,x^3\,\left(b^2\,e^2-3\,b\,c\,d\,e+2\,c^2\,d^2\right)}{b^4}}{b^2\,x^2+2\,b\,c\,x^3+c^2\,x^4}$$
$$-\frac{6\,d\,\mathrm{atanh}\left(\frac{3\,d\,(b\,e-c\,d)\,(b\,e-2\,c\,d)\,(b+2\,c\,x)}{b\,\left(3\,b^2\,d\,e^2-9\,b\,c\,d^2\,e+6\,c^2\,d^3\right)}\right)\,(b\,e-c\,d)\,(b\,e-2\,c\,d)}{b^5}$$

input
```
int((d + e*x)^3/(b*x + c*x^2)^3,x)
```

output
```
- (d^3/(2*b) + (d^2*x*(3*b*e - 2*c*d))/b^2 + (x^2*(b^3*e^3 - 18*c^3*d^3 +
27*b*c^2*d^2*e - 9*b^2*c*d*e^2))/(2*b^3*c) - (3*c*d*x^3*(b^2*e^2 + 2*c^2*d
^2 - 3*b*c*d*e))/b^4)/(b^2*x^2 + c^2*x^4 + 2*b*c*x^3) - (6*d*atanh((3*d*(b
*e - c*d)*(b*e - 2*c*d)*(b + 2*c*x))/(b*(6*c^2*d^3 + 3*b^2*d*e^2 - 9*b*c*d
^2*e)))*(b*e - c*d)*(b*e - 2*c*d))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 472, normalized size of antiderivative = 3.45

$$\int \frac{(d+ex)^3}{(bx+cx^2)^3}\,dx$$
$$= \frac{-6c^5d^3x^4 - 12\log(cx+b)\,b^3c^2d\,e^2x^3 + 36\log(cx+b)\,b^2c^3d^2e\,x^3 - 6\log(cx+b)\,b^2c^3d\,e^2x^4 + 18\log(cx+}{}$$

input
```
int((e*x+d)^3/(c*x^2+b*x)^3,x)
```

output
```
( - 6*log(b + c*x)*b**4*c*d*e**2*x**2 + 18*log(b + c*x)*b**3*c**2*d**2*e*x
**2 - 12*log(b + c*x)*b**3*c**2*d*e**2*x**3 - 12*log(b + c*x)*b**2*c**3*d*
*3*x**2 + 36*log(b + c*x)*b**2*c**3*d**2*e*x**3 - 6*log(b + c*x)*b**2*c**3
*d*e**2*x**4 - 24*log(b + c*x)*b*c**4*d**3*x**3 + 18*log(b + c*x)*b*c**4*d
**2*e*x**4 - 12*log(b + c*x)*c**5*d**3*x**4 + 6*log(x)*b**4*c*d*e**2*x**2
- 18*log(x)*b**3*c**2*d**2*e*x**2 + 12*log(x)*b**3*c**2*d*e**2*x**3 + 12*l
og(x)*b**2*c**3*d**3*x**2 - 36*log(x)*b**2*c**3*d**2*e*x**3 + 6*log(x)*b**
2*c**3*d*e**2*x**4 + 24*log(x)*b*c**4*d**3*x**3 - 18*log(x)*b*c**4*d**2*e*
x**4 + 12*log(x)*c**5*d**3*x**4 - b**5*e**3*x**2 - b**4*c*d**3 - 6*b**4*c*
d**2*e*x + 6*b**4*c*d*e**2*x**2 + 4*b**3*c**2*d**3*x - 18*b**3*c**2*d**2*e
*x**2 + 12*b**2*c**3*d**3*x**2 - 3*b**2*c**3*d*e**2*x**4 + 9*b*c**4*d**2*e
*x**4 - 6*c**5*d**3*x**4)/(2*b**5*c*x**2*(b**2 + 2*b*c*x + c**2*x**2))
```

3.76 $\int \frac{(d+ex)^2}{\left(bx+cx^2\right)^3} \, dx$

Optimal result 583
Mathematica [A] (verified) 583
Rubi [A] (verified) 584
Maple [A] (verified) 585
Fricas [B] (verification not implemented) 586
Sympy [B] (verification not implemented) 586
Maxima [A] (verification not implemented) 587
Giac [A] (verification not implemented) 587
Mupad [B] (verification not implemented) 588
Reduce [B] (verification not implemented) 588

Optimal result

Integrand size = 19, antiderivative size = 144

$$\int \frac{(d+ex)^2}{\left(bx+cx^2\right)^3} \, dx = -\frac{d^2}{2b^3x^2} + \frac{d(3cd-2be)}{b^4x} + \frac{(cd-be)^2}{2b^3(b+cx)^2} + \frac{(cd-be)(3cd-be)}{b^4(b+cx)} + \frac{(6c^2d^2-6bcde+b^2e^2)\log(x)}{b^5} - \frac{(6c^2d^2-6bcde+b^2e^2)\log(b+cx)}{b^5}$$

output
```
-1/2*d^2/b^3/x^2+d*(-2*b*e+3*c*d)/b^4/x+1/2*(-b*e+c*d)^2/b^3/(c*x+b)^2+(-b
*e+c*d)*(-b*e+3*c*d)/b^4/(c*x+b)+(b^2*e^2-6*b*c*d*e+6*c^2*d^2)*ln(x)/b^5-(
b^2*e^2-6*b*c*d*e+6*c^2*d^2)*ln(c*x+b)/b^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^2}{\left(bx+cx^2\right)^3} \, dx = \frac{-\frac{b^2d^2}{x^2} - \frac{2bd(-3cd+2be)}{x} + \frac{b^2(cd-be)^2}{(b+cx)^2} + \frac{2b(3c^2d^2-4bcde+b^2e^2)}{b+cx} + 2(6c^2d^2-6bcde+b^2e^2)\log(x) - 2(6c^2d^2-6bcd}{2b^5}$$

input
```
Integrate[(d + e*x)^2/(b*x + c*x^2)^3,x]
```

output
```
(-((b^2*d^2)/x^2) - (2*b*d*(-3*c*d + 2*b*e))/x + (b^2*(c*d - b*e)^2)/(b +
c*x)^2 + (2*b*(3*c^2*d^2 - 4*b*c*d*e + b^2*e^2))/(b + c*x) + 2*(6*c^2*d^2
- 6*b*c*d*e + b^2*e^2)*Log[x] - 2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*Log[b
+ c*x])/(2*b^5)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.15, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{1141}$$

$$c^3\int \left(\frac{d^2}{b^3c^3x^3}-\frac{(3cd-2be)d}{b^4c^3x^2}+\frac{6c^2d^2-6bced+b^2e^2}{b^5c^3x}-\frac{6c^2d^2-6bced+b^2e^2}{b^5c^2(b+cx)}-\frac{(cd-be)(3cd-be)}{b^4c^2(b+cx)^2}-\frac{(cd-\right.$$

$$\Big\downarrow \text{2009}$$

$$c^3\left(\frac{d(3cd-2be)}{b^4c^3x}+\frac{(cd-be)(3cd-be)}{b^4c^3(b+cx)}-\frac{d^2}{2b^3c^3x^2}+\frac{(cd-be)^2}{2b^3c^3(b+cx)^2}+\frac{\log(x)\left(b^2e^2-6bcde+6c^2d^2\right)}{b^5c^3}-\frac{(b^2e^2-\right.$$

input
```
Int[(d + e*x)^2/(b*x + c*x^2)^3,x]
```

output
```
c^3*(-1/2*d^2/(b^3*c^3*x^2) + (d*(3*c*d - 2*b*e))/(b^4*c^3*x) + (c*d - b*e
)^2/(2*b^3*c^3*(b + c*x)^2) + ((c*d - b*e)*(3*c*d - b*e))/(b^4*c^3*(b + c*
x)) + ((6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*Log[x])/(b^5*c^3) - ((6*c^2*d^2 -
 6*b*c*d*e + b^2*e^2)*Log[b + c*x])/(b^5*c^3))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.10

method	result
default	$-\frac{\left(b^{2} e^{2}-6 b c d e+6 c^{2} d^{2}\right) \ln \left(c x+b\right)}{b^{5}}+\frac{b^{2} e^{2}-4 b c d e+3 c^{2} d^{2}}{b^{4} \left(c x+b\right)}+\frac{b^{2} e^{2}-2 b c d e+c^{2} d^{2}}{2 b^{3} \left(c x+b\right)^{2}}-\frac{d^{2}}{2 b^{3} x^{2}}+\frac{\left(b^{2} e^{2}-6 b c d e+6 c^{2} d^{2}\right) \ln \left(x\right)}{b^{5}}$
norman	$\frac{\frac{\left(b^{2} c^{2} e^{2}-6 c^{3} d e b+6 c^{4} d^{2}\right) x^{3}}{b^{4} c}-\frac{d^{2}}{2 b}-\frac{2 d \left(b e-c d\right) x}{b^{2}}+\frac{\left(3 b^{2} c^{2} e^{2}-18 c^{3} d e b+18 c^{4} d^{2}\right) x^{2}}{2 b^{3} c^{2}}}{x^{2} \left(c x+b\right)^{2}}+\frac{\left(b^{2} e^{2}-6 b c d e+6 c^{2} d^{2}\right) \ln \left(x\right)}{b^{5}}-\frac{\left(b^{2} e^{2}-6 b c d e\right.}{}$
risch	$\frac{\frac{c \left(b^{2} e^{2}-6 b c d e+6 c^{2} d^{2}\right) x^{3}}{b^{4}}+\frac{3 \left(b^{2} e^{2}-6 b c d e+6 c^{2} d^{2}\right) x^{2}}{2 b^{3}}-\frac{2 d \left(b e-c d\right) x}{b^{2}}-\frac{d^{2}}{2 b}}{x^{2} \left(c x+b\right)^{2}}-\frac{\ln \left(c x+b\right) e^{2}}{b^{3}}+\frac{6 \ln \left(c x+b\right) c d e}{b^{4}}-\frac{6 \ln \left(c x+b\right) c^{2} d^{2}}{b^{5}}-$
parallelrisch	$\frac{-24 \ln \left(x\right) x^{3} b^{2} c^{4} d e+24 \ln \left(c x+b\right) x^{3} b^{2} c^{4} d e+12 \ln \left(x\right) x^{4} c^{6} d^{2}-12 \ln \left(c x+b\right) x^{4} c^{6} d^{2}-12 \ln \left(x\right) x^{4} b c^{5} d e-4 \ln \left(c x+b\right) x^{3} b^{3} c^{3} e^{2}-12 \ln}{}$

input
```
int((e*x+d)^2/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-(b^2*e^2-6*b*c*d*e+6*c^2*d^2)*ln(c*x+b)/b^5+(b^2*e^2-4*b*c*d*e+3*c^2*d^2)
/b^4/(c*x+b)+1/2*(b^2*e^2-2*b*c*d*e+c^2*d^2)/b^3/(c*x+b)^2-1/2*d^2/b^3/x^2
+(b^2*e^2-6*b*c*d*e+6*c^2*d^2)*ln(x)/b^5-d*(2*b*e-3*c*d)/b^4/x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(140) = 280$.

Time = 0.09 (sec) , antiderivative size = 327, normalized size of antiderivative = 2.27

$$\int \frac{(d+ex)^2}{(bx+cx^2)^3}\,dx = -\frac{b^4d^2 - 2\,(6\,bc^3d^2 - 6\,b^2c^2de + b^3ce^2)x^3 - 3\,(6\,b^2c^2d^2 - 6\,b^3cde + b^4e^2)x^2 - 4\,(b^3cd^2 - b^4de)x + 2\,((6\,c^4}{}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(b^4*d^2 - 2*(6*b*c^3*d^2 - 6*b^2*c^2*d*e + b^3*c*e^2)*x^3 - 3*(6*b^2
*c^2*d^2 - 6*b^3*c*d*e + b^4*e^2)*x^2 - 4*(b^3*c*d^2 - b^4*d*e)*x + 2*((6*
c^4*d^2 - 6*b*c^3*d*e + b^2*c^2*e^2)*x^4 + 2*(6*b*c^3*d^2 - 6*b^2*c^2*d*e
+ b^3*c*e^2)*x^3 + (6*b^2*c^2*d^2 - 6*b^3*c*d*e + b^4*e^2)*x^2)*log(c*x +
b) - 2*((6*c^4*d^2 - 6*b*c^3*d*e + b^2*c^2*e^2)*x^4 + 2*(6*b*c^3*d^2 - 6*b
^2*c^2*d*e + b^3*c*e^2)*x^3 + (6*b^2*c^2*d^2 - 6*b^3*c*d*e + b^4*e^2)*x^2)
*log(x))/(b^5*c^2*x^4 + 2*b^6*c*x^3 + b^7*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(136) = 272$.

Time = 0.51 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.40

$$\int \frac{(d+ex)^2}{(bx+cx^2)^3}\,dx = \frac{-b^3d^2 + x^3\cdot(2b^2ce^2 - 12bc^2de + 12c^3d^2) + x^2\cdot(3b^3e^2 - 18b^2cde + 18bc^2d^2) + x(-4b^3de + 4b^2cd^2)}{2b^6x^2 + 4b^5cx^3 + 2b^4c^2x^4} + \frac{(b^2e^2 - 6bcde + 6c^2d^2)\log\left(x + \frac{b^3e^2 - 6b^2cde + 6bc^2d^2 - b(b^2e^2 - 6bcde + 6c^2d^2)}{2b^2ce^2 - 12bc^2de + 12c^3d^2}\right)}{b^5} - \frac{(b^2e^2 - 6bcde + 6c^2d^2)\log\left(x + \frac{b^3e^2 - 6b^2cde + 6bc^2d^2 + b(b^2e^2 - 6bcde + 6c^2d^2)}{2b^2ce^2 - 12bc^2de + 12c^3d^2}\right)}{b^5}$$

input
```
integrate((e*x+d)**2/(c*x**2+b*x)**3,x)
```

output
```
(-b**3*d**2 + x**3*(2*b**2*c*e**2 - 12*b*c**2*d*e + 12*c**3*d**2) + x**2*(
3*b**3*e**2 - 18*b**2*c*d*e + 18*b*c**2*d**2) + x*(-4*b**3*d*e + 4*b**2*c*
d**2))/(2*b**6*x**2 + 4*b**5*c*x**3 + 2*b**4*c**2*x**4) + (b**2*e**2 - 6*b
*c*d*e + 6*c**2*d**2)*log(x + (b**3*e**2 - 6*b**2*c*d*e + 6*b*c**2*d**2 -
b*(b**2*e**2 - 6*b*c*d*e + 6*c**2*d**2))/(2*b**2*c*e**2 - 12*b*c**2*d*e +
12*c**3*d**2))/b**5 - (b**2*e**2 - 6*b*c*d*e + 6*c**2*d**2)*log(x + (b**3*
e**2 - 6*b**2*c*d*e + 6*b*c**2*d**2 + b*(b**2*e**2 - 6*b*c*d*e + 6*c**2*d*
*2))/(2*b**2*c*e**2 - 12*b*c**2*d*e + 12*c**3*d**2))/b**5
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^2}{(bx+cx^2)^3}\,dx = -\frac{b^3d^2 - 2\,(6\,c^3d^2 - 6\,bc^2de + b^2ce^2)x^3 - 3\,(6\,bc^2d^2 - 6\,b^2cde + b^3e^2)x^2 - 4\,(b^2cd^2 - b^3de)x}{2\,(b^4c^2x^4 + 2\,b^5cx^3 + b^6x^2)} - \frac{(6\,c^2d^2 - 6\,bcde + b^2e^2)\log(cx+b)}{b^5} + \frac{(6\,c^2d^2 - 6\,bcde + b^2e^2)\log(x)}{b^5}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(b^3*d^2 - 2*(6*c^3*d^2 - 6*b*c^2*d*e + b^2*c*e^2)*x^3 - 3*(6*b*c^2*d
^2 - 6*b^2*c*d*e + b^3*e^2)*x^2 - 4*(b^2*c*d^2 - b^3*d*e)*x)/(b^4*c^2*x^4
+ 2*b^5*c*x^3 + b^6*x^2) - (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*log(c*x + b)/
b^5 + (6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*log(x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.26

$$\int \frac{(d+ex)^2}{(bx+cx^2)^3}\,dx = \frac{(6\,c^2d^2 - 6\,bcde + b^2e^2)\log(|x|)}{b^5} - \frac{(6\,c^3d^2 - 6\,bc^2de + b^2ce^2)\log(|cx+b|)}{b^5c} + \frac{12\,c^3d^2x^3 - 12\,bc^2dex^3 + 2\,b^2ce^2x^3 + 18\,bc^2d^2x^2 - 18\,b^2cdex^2 + 3\,b^3e^2x^2 + 4\,b^2cd^2x - 4\,b^3dex - b^3d^2}{2\,(cx^2+bx)^2b^4}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*log(abs(x))/b^5 - (6*c^3*d^2 - 6*b*c^2*d
*e + b^2*c*e^2)*log(abs(c*x + b))/(b^5*c) + 1/2*(12*c^3*d^2*x^3 - 12*b*c^2
*d*e*x^3 + 2*b^2*c*e^2*x^3 + 18*b*c^2*d^2*x^2 - 18*b^2*c*d*e*x^2 + 3*b^3*e
^2*x^2 + 4*b^2*c*d^2*x - 4*b^3*d*e*x - b^3*d^2)/((c*x^2 + b*x)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 8.91 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^2}{(bx+cx^2)^3}\,dx = -\frac{\frac{d^2}{2\,b}-\frac{3\,x^2\,\left(b^2\,e^2-6\,b\,c\,d\,e+6\,c^2\,d^2\right)}{2\,b^3}-\frac{c\,x^3\,\left(b^2\,e^2-6\,b\,c\,d\,e+6\,c^2\,d^2\right)}{b^4}+\frac{2\,d\,x\,\left(b\,e-c\,d\right)}{b^2}}{b^2\,x^2+2\,b\,c\,x^3+c^2\,x^4} - \frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)\,\left(b^2\,e^2-6\,b\,c\,d\,e+6\,c^2\,d^2\right)}{b^5}$$

input
```
int((d + e*x)^2/(b*x + c*x^2)^3,x)
```

output
```
- (d^2/(2*b) - (3*x^2*(b^2*e^2 + 6*c^2*d^2 - 6*b*c*d*e))/(2*b^3) - (c*x^3*
(b^2*e^2 + 6*c^2*d^2 - 6*b*c*d*e))/b^4 + (2*d*x*(b*e - c*d))/b^2)/(b^2*x^2
 + c^2*x^4 + 2*b*c*x^3) - (2*atanh((2*c*x)/b + 1)*(b^2*e^2 + 6*c^2*d^2 - 6
*b*c*d*e))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 415, normalized size of antiderivative = 2.88

$$\int \frac{(d+ex)^2}{(bx+cx^2)^3}\,dx = \frac{2b^4e^2x^2-6c^4d^2x^4-4\log(cx+b)\,b^3c\,e^2x^3-12\log(cx+b)\,b^2c^2d^2x^2-2\log(cx+b)\,b^2c^2e^2x^4-24\log(cx}{}$$

input
```
int((e*x+d)^2/(c*x^2+b*x)^3,x)
```

output
```
( - 2*log(b + c*x)*b**4*e**2*x**2 + 12*log(b + c*x)*b**3*c*d*e*x**2 - 4*lo
g(b + c*x)*b**3*c*e**2*x**3 - 12*log(b + c*x)*b**2*c**2*d**2*x**2 + 24*log
(b + c*x)*b**2*c**2*d*e*x**3 - 2*log(b + c*x)*b**2*c**2*e**2*x**4 - 24*log
(b + c*x)*b*c**3*d**2*x**3 + 12*log(b + c*x)*b*c**3*d*e*x**4 - 12*log(b +
c*x)*c**4*d**2*x**4 + 2*log(x)*b**4*e**2*x**2 - 12*log(x)*b**3*c*d*e*x**2
+ 4*log(x)*b**3*c*e**2*x**3 + 12*log(x)*b**2*c**2*d**2*x**2 - 24*log(x)*b*
*2*c**2*d*e*x**3 + 2*log(x)*b**2*c**2*e**2*x**4 + 24*log(x)*b*c**3*d**2*x*
*3 - 12*log(x)*b*c**3*d*e*x**4 + 12*log(x)*c**4*d**2*x**4 - b**4*d**2 - 4*
b**4*d*e*x + 2*b**4*e**2*x**2 + 4*b**3*c*d**2*x - 12*b**3*c*d*e*x**2 + 12*
b**2*c**2*d**2*x**2 - b**2*c**2*e**2*x**4 + 6*b*c**3*d*e*x**4 - 6*c**4*d**
2*x**4)/(2*b**5*x**2*(b**2 + 2*b*c*x + c**2*x**2))
```

3.77 $\int \frac{d+ex}{(bx+cx^2)^3} dx$

Optimal result 590
Mathematica [A] (verified) 590
Rubi [A] (verified) 591
Maple [A] (verified) 592
Fricas [B] (verification not implemented) 593
Sympy [B] (verification not implemented) 593
Maxima [A] (verification not implemented) 594
Giac [A] (verification not implemented) 594
Mupad [B] (verification not implemented) 595
Reduce [B] (verification not implemented) 595

Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{d+ex}{(bx+cx^2)^3} dx = -\frac{d}{2b^3x^2} + \frac{3cd-be}{b^4x} + \frac{c(cd-be)}{2b^3(b+cx)^2} + \frac{c(3cd-2be)}{b^4(b+cx)} + \frac{3c(2cd-be)\log(x)}{b^5} - \frac{3c(2cd-be)\log(b+cx)}{b^5}$$

output
```
-1/2*d/b^3/x^2+(-b*e+3*c*d)/b^4/x+1/2*c*(-b*e+c*d)/b^3/(c*x+b)^2+c*(-2*b*e
+3*c*d)/b^4/(c*x+b)+3*c*(-b*e+2*c*d)*ln(x)/b^5-3*c*(-b*e+2*c*d)*ln(c*x+b)/
b^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \frac{d+ex}{(bx+cx^2)^3} dx = \frac{-\frac{b(-12c^3dx^3+6bc^2x^2(-3d+ex)+b^3(d+2ex)+b^2cx(-4d+9ex))}{x^2(b+cx)^2} + 6c(2cd-be)\log(x) + 6c(-2cd+be)\log(b+cx)}{2b^5}$$

input
```
Integrate[(d + e*x)/(b*x + c*x^2)^3,x]
```

output
```
(-((b*(-12*c^3*d*x^3 + 6*b*c^2*x^2*(-3*d + e*x) + b^3*(d + 2*e*x) + b^2*c*
x*(-4*d + 9*e*x)))/(x^2*(b + c*x)^2)) + 6*c*(2*c*d - b*e)*Log[x] + 6*c*(-2
*c*d + b*e)*Log[b + c*x])/(2*b^5)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{1141}$$

$$c^3\int \left(\frac{d}{b^3c^3x^3}+\frac{3(2cd-be)}{b^5c^2x}-\frac{3(2cd-be)}{b^5c(b+cx)}-\frac{3cd-be}{b^4c^3x^2}-\frac{3cd-2be}{b^4c(b+cx)^2}-\frac{cd-be}{b^3c(b+cx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c^3\left(\frac{3\log(x)(2cd-be)}{b^5c^2}-\frac{3(2cd-be)\log(b+cx)}{b^5c^2}+\frac{3cd-be}{b^4c^3x}+\frac{3cd-2be}{b^4c^2(b+cx)}-\frac{d}{2b^3c^3x^2}+\frac{cd-be}{2b^3c^2(b+cx)^2}\right)$$

input
```
Int[(d + e*x)/(b*x + c*x^2)^3,x]
```

output
```
c^3*(-1/2*d/(b^3*c^3*x^2) + (3*c*d - b*e)/(b^4*c^3*x) + (c*d - b*e)/(2*b^3
*c^2*(b + c*x)^2) + (3*c*d - 2*b*e)/(b^4*c^2*(b + c*x)) + (3*(2*c*d - b*e)
*Log[x])/(b^5*c^2) - (3*(2*c*d - b*e)*Log[b + c*x])/(b^5*c^2))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96

method	result
default	$-\frac{c(2be-3cd)}{b^4(cx+b)}-\frac{(be-cd)c}{2b^3(cx+b)^2}+\frac{3c(be-2cd)\ln(cx+b)}{b^5}-\frac{d}{2b^3x^2}-\frac{be-3cd}{b^4x}-\frac{3c(be-2cd)\ln(x)}{b^5}$
norman	$\frac{-\frac{d}{2b}-\frac{(be-2cd)x}{b^2}+\frac{2c\left(3bce-6c^2d\right)x^3}{b^4}+\frac{c^2\left(9bce-18c^2d\right)x^4}{2b^5}}{x^2(cx+b)^2}-\frac{3c(be-2cd)\ln(x)}{b^5}+\frac{3c(be-2cd)\ln(cx+b)}{b^5}$
risch	$\frac{-\frac{3c^2(be-2cd)x^3}{b^4}-\frac{9c(be-2cd)x^2}{2b^3}-\frac{(be-2cd)x}{b^2}-\frac{d}{2b}}{x^2(cx+b)^2}-\frac{3c\ln(x)e}{b^4}+\frac{6c^2\ln(x)d}{b^5}+\frac{3c\ln(-cx-b)e}{b^4}-\frac{6c^2\ln(-cx-b)d}{b^5}$
parallelrisch	$-\frac{6\ln(x)x^4bc^3e-12\ln(x)x^4c^4d-6\ln(cx+b)x^4bc^3e+12\ln(cx+b)x^4c^4d+12\ln(x)x^3b^2c^2e-24\ln(x)x^3bc^3d-12\ln(cx+b)x^3b^2}{}$

input
```
int((e*x+d)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-c*(2*b*e-3*c*d)/b^4/(c*x+b)-1/2*(b*e-c*d)*c/b^3/(c*x+b)^2+3*c*(b*e-2*c*d)
/b^5*ln(c*x+b)-1/2*d/b^3/x^2-(b*e-3*c*d)/b^4/x-3*c*(b*e-2*c*d)/b^5*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(106) = 212$.

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.13

$$\int \frac{d+ex}{(bx+cx^2)^3}\,dx = -\frac{b^4d-6\,(2\,bc^3d-b^2c^2e)x^3-9\,(2\,b^2c^2d-b^3ce)x^2-2\,(2\,b^3cd-b^4e)x+6\,((2\,c^4d-bc^3e)x^4+2\,(2\,bc^3d}{2\,(b^5c}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(b^4*d - 6*(2*b*c^3*d - b^2*c^2*e)*x^3 - 9*(2*b^2*c^2*d - b^3*c*e)*x^
2 - 2*(2*b^3*c*d - b^4*e)*x + 6*((2*c^4*d - b*c^3*e)*x^4 + 2*(2*b*c^3*d -
b^2*c^2*e)*x^3 + (2*b^2*c^2*d - b^3*c*e)*x^2)*log(c*x + b) - 6*((2*c^4*d -
 b*c^3*e)*x^4 + 2*(2*b*c^3*d - b^2*c^2*e)*x^3 + (2*b^2*c^2*d - b^3*c*e)*x^
2)*log(x))/(b^5*c^2*x^4 + 2*b^6*c*x^3 + b^7*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(104) = 208$.

Time = 0.34 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.99

$$\begin{aligned}\int \frac{d+ex}{(bx+cx^2)^3}\,dx &= \frac{-b^3d+x^3(-6bc^2e+12c^3d)+x^2(-9b^2ce+18bc^2d)+x(-2b^3e+4b^2cd)}{2b^6x^2+4b^5cx^3+2b^4c^2x^4}\\ &\quad-\frac{3c(be-2cd)\log\left(x+\frac{3b^2ce-6bc^2d-3bc(be-2cd)}{6bc^2e-12c^3d}\right)}{b^5}\\ &\quad+\frac{3c(be-2cd)\log\left(x+\frac{3b^2ce-6bc^2d+3bc(be-2cd)}{6bc^2e-12c^3d}\right)}{b^5}\end{aligned}$$

input
```
integrate((e*x+d)/(c*x**2+b*x)**3,x)
```

output
```
(-b**3*d + x**3*(-6*b*c**2*e + 12*c**3*d) + x**2*(-9*b**2*c*e + 18*b*c**2*
d) + x*(-2*b**3*e + 4*b**2*c*d))/(2*b**6*x**2 + 4*b**5*c*x**3 + 2*b**4*c**
2*x**4) - 3*c*(b*e - 2*c*d)*log(x + (3*b**2*c*e - 6*b*c**2*d - 3*b*c*(b*e
- 2*c*d))/(6*b*c**2*e - 12*c**3*d))/b**5 + 3*c*(b*e - 2*c*d)*log(x + (3*b*
*2*c*e - 6*b*c**2*d + 3*b*c*(b*e - 2*c*d))/(6*b*c**2*e - 12*c**3*d))/b**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.24

$$\int \frac{d+ex}{(bx+cx^2)^3}\,dx = -\frac{b^3d-6\,(2\,c^3d-bc^2e)x^3-9\,(2\,bc^2d-b^2ce)x^2-2\,(2\,b^2cd-b^3e)x}{2\,(b^4c^2x^4+2\,b^5cx^3+b^6x^2)} - \frac{3\,(2\,c^2d-bce)\log{(cx+b)}}{b^5} + \frac{3\,(2\,c^2d-bce)\log{(x)}}{b^5}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(b^3*d - 6*(2*c^3*d - b*c^2*e)*x^3 - 9*(2*b*c^2*d - b^2*c*e)*x^2 - 2*
(2*b^2*c*d - b^3*e)*x)/(b^4*c^2*x^4 + 2*b^5*c*x^3 + b^6*x^2) - 3*(2*c^2*d
- b*c*e)*log(c*x + b)/b^5 + 3*(2*c^2*d - b*c*e)*log(x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

$$\int \frac{d+ex}{(bx+cx^2)^3}\,dx = \frac{3\,(2\,c^2d-bce)\log{(|x|)}}{b^5} - \frac{3\,(2\,c^3d-bc^2e)\log{(|cx+b|)}}{b^5c} + \frac{12\,c^3dx^3-6\,bc^2ex^3+18\,bc^2dx^2-9\,b^2cex^2+4\,b^2cdx-2\,b^3ex-b^3d}{2\,(cx^2+bx)^2b^4}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
3*(2*c^2*d - b*c*e)*log(abs(x))/b^5 - 3*(2*c^3*d - b*c^2*e)*log(abs(c*x +
b))/(b^5*c) + 1/2*(12*c^3*d*x^3 - 6*b*c^2*e*x^3 + 18*b*c^2*d*x^2 - 9*b^2*c
*e*x^2 + 4*b^2*c*d*x - 2*b^3*e*x - b^3*d)/((c*x^2 + b*x)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.20

$$\int \frac{d+ex}{(bx+cx^2)^3}\,dx = -\frac{\frac{d}{2\,b}+\frac{x\,(b\,e-2\,c\,d)}{b^2}+\frac{9\,c\,x^2\,(b\,e-2\,c\,d)}{2\,b^3}+\frac{3\,c^2\,x^3\,(b\,e-2\,c\,d)}{b^4}}{b^2\,x^2+2\,b\,c\,x^3+c^2\,x^4} - \frac{6\,c\,\mathrm{atanh}\left(\frac{3\,c\,(b\,e-2\,c\,d)\,(b+2\,c\,x)}{b\,(6\,c^2\,d-3\,b\,c\,e)}\right)\,(b\,e-2\,c\,d)}{b^5}$$

input
```
int((d + e*x)/(b*x + c*x^2)^3,x)
```

output
```
- (d/(2*b) + (x*(b*e - 2*c*d))/b^2 + (9*c*x^2*(b*e - 2*c*d))/(2*b^3) + (3*
c^2*x^3*(b*e - 2*c*d))/b^4)/(b^2*x^2 + c^2*x^4 + 2*b*c*x^3) - (6*c*atanh((
3*c*(b*e - 2*c*d)*(b + 2*c*x))/(b*(6*c^2*d - 3*b*c*e)))*(b*e - 2*c*d))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.39

$$\int \frac{d+ex}{(bx+cx^2)^3}\,dx = \frac{6\log(cx+b)\,b^3ce\,x^2-12\log(cx+b)\,b^2c^2d\,x^2+12\log(cx+b)\,b^2c^2e\,x^3-24\log(cx+b)\,b\,c^3d\,x^3+6\log(c}{}$$

input
```
int((e*x+d)/(c*x^2+b*x)^3,x)
```

output
```
(6*log(b + c*x)*b**3*c*e*x**2 - 12*log(b + c*x)*b**2*c**2*d*x**2 + 12*log(
b + c*x)*b**2*c**2*e*x**3 - 24*log(b + c*x)*b*c**3*d*x**3 + 6*log(b + c*x)
*b*c**3*e*x**4 - 12*log(b + c*x)*c**4*d*x**4 - 6*log(x)*b**3*c*e*x**2 + 12
*log(x)*b**2*c**2*d*x**2 - 12*log(x)*b**2*c**2*e*x**3 + 24*log(x)*b*c**3*d
*x**3 - 6*log(x)*b*c**3*e*x**4 + 12*log(x)*c**4*d*x**4 - b**4*d - 2*b**4*e
*x + 4*b**3*c*d*x - 6*b**3*c*e*x**2 + 12*b**2*c**2*d*x**2 + 3*b*c**3*e*x**
4 - 6*c**4*d*x**4)/(2*b**5*x**2*(b**2 + 2*b*c*x + c**2*x**2))
```

3.78 $\int \frac{1}{(bx+cx^2)^3} \, dx$

Optimal result 597
Mathematica [A] (verified) 597
Rubi [A] (verified) 598
Maple [A] (verified) 599
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Giac [A] (verification not implemented) 601
Mupad [B] (verification not implemented) 601
Reduce [B] (verification not implemented) 601

Optimal result

Integrand size = 11, antiderivative size = 76

$$\int \frac{1}{(bx+cx^2)^3} \, dx = -\frac{1}{2b^3x^2} + \frac{3c}{b^4x} + \frac{c^2}{2b^3(b+cx)^2} + \frac{3c^2}{b^4(b+cx)} + \frac{6c^2\log(x)}{b^5} - \frac{6c^2\log(b+cx)}{b^5}$$

output
```
-1/2/b^3/x^2+3*c/b^4/x+1/2*c^2/b^3/(c*x+b)^2+3*c^2/b^4/(c*x+b)+6*c^2*ln(x)
/b^5-6*c^2*ln(c*x+b)/b^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89

$$\int \frac{1}{(bx+cx^2)^3} \, dx = \frac{\frac{b(-b^3+4b^2cx+18bc^2x^2+12c^3x^3)}{x^2(b+cx)^2} + 12c^2\log(x) - 12c^2\log(b+cx)}{2b^5}$$

input
```
Integrate[(b*x + c*x^2)^(-3),x]
```

output
```
((b*(-b^3 + 4*b^2*c*x + 18*b*c^2*x^2 + 12*c^3*x^3))/(x^2*(b + c*x)^2) + 12
*c^2*Log[x] - 12*c^2*Log[b + c*x])/(2*b^5)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1080, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^3}\,dx$$
$$\Big\downarrow \text{1080}$$
$$\int \left(-\frac{6c^3}{b^5(b+cx)}+\frac{6c^2}{b^5x}-\frac{3c^3}{b^4(b+cx)^2}-\frac{3c}{b^4x^2}-\frac{c^3}{b^3(b+cx)^3}+\frac{1}{b^3x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{6c^2\log(x)}{b^5}-\frac{6c^2\log(b+cx)}{b^5}+\frac{3c^2}{b^4(b+cx)}+\frac{3c}{b^4x}+\frac{c^2}{2b^3(b+cx)^2}-\frac{1}{2b^3x^2}$$

input
```
Int[(b*x + c*x^2)^(-3),x]
```

output
```
-1/2*1/(b^3*x^2) + (3*c)/(b^4*x) + c^2/(2*b^3*(b + c*x)^2) + (3*c^2)/(b^4*
(b + c*x)) + (6*c^2*Log[x])/b^5 - (6*c^2*Log[b + c*x])/b^5
```

Defintions of rubi rules used

rule 1080
```
Int[((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[x^p*
(b + c*x)^p, x], x] /; FreeQ[{b, c}, x] && IntegerQ[p]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.95

method	result
norman	$\frac{-\frac{9c^4x^4}{b^5}-\frac{1}{2b}+\frac{2cx}{b^2}-\frac{12c^3x^3}{b^4}}{x^2(cx+b)^2}+\frac{6c^2\ln(x)}{b^5}-\frac{6c^2\ln(cx+b)}{b^5}$
default	$-\frac{1}{2b^3x^2}+\frac{3c}{b^4x}+\frac{c^2}{2b^3(cx+b)^2}+\frac{3c^2}{b^4(cx+b)}+\frac{6c^2\ln(x)}{b^5}-\frac{6c^2\ln(cx+b)}{b^5}$
risch	$\frac{\frac{6c^3x^3}{b^4}+\frac{9c^2x^2}{b^3}+\frac{2cx}{b^2}-\frac{1}{2b}}{x^2(cx+b)^2}+\frac{6c^2\ln(-x)}{b^5}-\frac{6c^2\ln(cx+b)}{b^5}$
parallelrisch	$\frac{12\ln(x)x^4c^6-12\ln(cx+b)x^4c^6+24\ln(x)x^3b\,c^5-24\ln(cx+b)x^3b\,c^5+12\ln(x)x^2b^2c^4-12\ln(cx+b)x^2b^2c^4+12x^3b\,c^5+18b^2c^4x}{2b^5c^2x^2(cx+b)^2}$

input
```
int(1/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
(-9*c^4/b^5*x^4-1/2/b+2/b^2*c*x-12*c^3/b^4*x^3)/x^2/(c*x+b)^2+6*c^2*ln(x)/
b^5-6*c^2*ln(c*x+b)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.71

$$\int \frac{1}{(bx+cx^2)^3}\,dx$$
$$= \frac{12\,bc^3x^3+18\,b^2c^2x^2+4\,b^3cx-b^4-12\,(c^4x^4+2\,bc^3x^3+b^2c^2x^2)\log(cx+b)+12\,(c^4x^4+2\,bc^3x^3+b^2c^2}{2\,(b^5c^2x^4+2\,b^6cx^3+b^7x^2)}$$

input
```
integrate(1/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/2*(12*b*c^3*x^3 + 18*b^2*c^2*x^2 + 4*b^3*c*x - b^4 - 12*(c^4*x^4 + 2*b*c
^3*x^3 + b^2*c^2*x^2)*log(c*x + b) + 12*(c^4*x^4 + 2*b*c^3*x^3 + b^2*c^2*x
^2)*log(x))/(b^5*c^2*x^4 + 2*b^6*c*x^3 + b^7*x^2)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.03

$$\int \frac{1}{(bx+cx^2)^3}\,dx = \frac{-b^3+4b^2cx+18bc^2x^2+12c^3x^3}{2b^6x^2+4b^5cx^3+2b^4c^2x^4}+\frac{6c^2\left(\log(x)-\log\left(\frac{b}{c}+x\right)\right)}{b^5}$$

input
```
integrate(1/(c*x**2+b*x)**3,x)
```

output
```
(-b**3 + 4*b**2*c*x + 18*b*c**2*x**2 + 12*c**3*x**3)/(2*b**6*x**2 + 4*b**5
*c*x**3 + 2*b**4*c**2*x**4) + 6*c**2*(log(x) - log(b/c + x))/b**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.13

$$\int \frac{1}{(bx+cx^2)^3}\,dx = \frac{12\,c^3x^3+18\,bc^2x^2+4\,b^2cx-b^3}{2\,(b^4c^2x^4+2\,b^5cx^3+b^6x^2)}-\frac{6\,c^2\log(cx+b)}{b^5}+\frac{6\,c^2\log(x)}{b^5}$$

input
```
integrate(1/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/2*(12*c^3*x^3 + 18*b*c^2*x^2 + 4*b^2*c*x - b^3)/(b^4*c^2*x^4 + 2*b^5*c*x
^3 + b^6*x^2) - 6*c^2*log(c*x + b)/b^5 + 6*c^2*log(x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.96

$$\int \frac{1}{(bx+cx^2)^3}\,dx = -\frac{6\,c^2\log\left(|cx+b|\right)}{b^5} + \frac{6\,c^2\log\left(|x|\right)}{b^5} + \frac{12\,c^3x^3+18\,bc^2x^2+4\,b^2cx-b^3}{2\,(cx^2+bx)^2b^4}$$

input
```
integrate(1/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
-6*c^2*log(abs(c*x + b))/b^5 + 6*c^2*log(abs(x))/b^5 + 1/2*(12*c^3*x^3 + 1
8*b*c^2*x^2 + 4*b^2*c*x - b^3)/((c*x^2 + b*x)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.04

$$\int \frac{1}{(bx+cx^2)^3}\,dx = \frac{\frac{9\,c^2\,x^2}{b^3}-\frac{1}{2\,b}+\frac{6\,c^3\,x^3}{b^4}+\frac{2\,c\,x}{b^2}}{b^2\,x^2+2\,b\,c\,x^3+c^2\,x^4} - \frac{12\,c^2\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b^5}$$

input
```
int(1/(b*x + c*x^2)^3,x)
```

output
```
((9*c^2*x^2)/b^3 - 1/(2*b) + (6*c^3*x^3)/b^4 + (2*c*x)/b^2)/(b^2*x^2 + c^2
*x^4 + 2*b*c*x^3) - (12*c^2*atanh((2*c*x)/b + 1))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.82

$$\int \frac{1}{(bx+cx^2)^3}\,dx = \frac{-12\log(cx+b)\,b^2c^2x^2-24\log(cx+b)\,b\,c^3x^3-12\log(cx+b)\,c^4x^4+12\log(x)\,b^2c^2x^2+24\log(x)\,b\,c^3x^3}{2b^5x^2\,(c^2x^2+2bcx+b^2)}$$

input
```
int(1/(c*x^2+b*x)^3,x)
```

output
```
( - 12*log(b + c*x)*b**2*c**2*x**2 - 24*log(b + c*x)*b*c**3*x**3 - 12*log(
b + c*x)*c**4*x**4 + 12*log(x)*b**2*c**2*x**2 + 24*log(x)*b*c**3*x**3 + 12
*log(x)*c**4*x**4 - b**4 + 4*b**3*c*x + 12*b**2*c**2*x**2 - 6*c**4*x**4)/(
2*b**5*x**2*(b**2 + 2*b*c*x + c**2*x**2))
```

3.79 $\int \frac{1}{(d+ex)(bx+cx^2)^3}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 193

$$\int \frac{1}{(d+ex)(bx+cx^2)^3}\,dx = -\frac{1}{2b^3dx^2}+\frac{3cd+be}{b^4d^2x}+\frac{c^3}{2b^3(cd-be)(b+cx)^2} + \frac{c^3(3cd-4be)}{b^4(cd-be)^2(b+cx)}+\frac{(6c^2d^2+3bcde+b^2e^2)\log(x)}{b^5d^3} - \frac{c^3(6c^2d^2-15bcde+10b^2e^2)\log(b+cx)}{b^5(cd-be)^3}+\frac{e^5\log(d+ex)}{d^3(cd-be)^3}$$

output
```
-1/2/b^3/d/x^2+(b*e+3*c*d)/b^4/d^2/x+1/2*c^3/b^3/(-b*e+c*d)/(c*x+b)^2+c^3*
(-4*b*e+3*c*d)/b^4/(-b*e+c*d)^2/(c*x+b)+(b^2*e^2+3*b*c*d*e+6*c^2*d^2)*ln(x
)/b^5/d^3-c^3*(10*b^2*e^2-15*b*c*d*e+6*c^2*d^2)*ln(c*x+b)/b^5/(-b*e+c*d)^3
+e^5*ln(e*x+d)/d^3/(-b*e+c*d)^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.99

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^3}\,dx = -\frac{1}{2b^3dx^2}+\frac{3cd+be}{b^4d^2x}-\frac{c^3}{2b^3(-cd+be)(b+cx)^2}+\frac{c^3(3cd-4be)}{b^4(cd-be)^2(b+cx)}+\frac{\left(6c^2d^2+3bcde+b^2e^2\right)\log(x)}{b^5d^3}+\frac{c^3\left(6c^2d^2-15bcde+10b^2e^2\right)\log(b+cx)}{b^5(-cd+be)^3}+\frac{e^5\log(d+ex)}{d^3(cd-be)^3}$$

input
```
Integrate[1/((d + e*x)*(b*x + c*x^2)^3),x]
```

output
```
-1/2*1/(b^3*d*x^2) + (3*c*d + b*e)/(b^4*d^2*x) - c^3/(2*b^3*(-(c*d) + b*e)
*(b + c*x)^2) + (c^3*(3*c*d - 4*b*e))/(b^4*(c*d - b*e)^2*(b + c*x)) + ((6*
c^2*d^2 + 3*b*c*d*e + b^2*e^2)*Log[x])/(b^5*d^3) + (c^3*(6*c^2*d^2 - 15*b*
c*d*e + 10*b^2*e^2)*Log[b + c*x])/(b^5*(-(c*d) + b*e)^3) + (e^5*Log[d + e*
x])/(d^3*(c*d - b*e)^3)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(bx+cx^2\right)^3(d+ex)}dx$$

$$\Big\downarrow \; 1141$$

$$c^3\int \left(\frac{e^6}{c^3d^3(cd-be)^3(d+ex)}+\frac{6c^2d^2+3bced+b^2e^2}{b^5c^3d^3x}-\frac{c\left(6c^2d^2-15bced+10b^2e^2\right)}{b^5(cd-be)^3(b+cx)}-\frac{3cd+be}{b^4c^3d^2x^2}-\frac{c(3cd-\cdots}{b^4(cd-be)^{\cdots}}\right.$$

$$\Big\downarrow \; 2009$$

$$c^3\left(\frac{be+3cd}{b^4c^3d^2x}+\frac{3cd-4be}{b^4(b+cx)(cd-be)^2}-\frac{1}{2b^3c^3dx^2}+\frac{1}{2b^3(b+cx)^2(cd-be)}-\frac{(10b^2e^2-15bcde+6c^2d^2)\log(b+c\cdots}{b^5(cd-be)^3}\right.$$

input
```
Int[1/((d + e*x)*(b*x + c*x^2)^3),x]
```

output
```
c^3*(-1/2*1/(b^3*c^3*d*x^2) + (3*c*d + b*e)/(b^4*c^3*d^2*x) + 1/(2*b^3*(c*
d - b*e)*(b + c*x)^2) + (3*c*d - 4*b*e)/(b^4*(c*d - b*e)^2*(b + c*x)) + ((
6*c^2*d^2 + 3*b*c*d*e + b^2*e^2)*Log[x])/(b^5*c^3*d^3) - ((6*c^2*d^2 - 15*
b*c*d*e + 10*b^2*e^2)*Log[b + c*x])/(b^5*(c*d - b*e)^3) + (e^5*Log[d + e*x
])/(c^3*d^3*(c*d - b*e)^3))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.00

method	result
default	$-\frac{c^3}{2(be-cd)b^3(cx+b)^2}-\frac{c^3(4be-3cd)}{(be-cd)^2b^4(cx+b)}+\frac{c^3(10b^2e^2-15bcde+6c^2d^2)\ln(cx+b)}{(be-cd)^3b^5}-\frac{e^5\ln(ex+d)}{(be-cd)^3d^3}-\frac{1}{2b^3d\,x^2}-\frac{-be\cdots}{d^2\cdots}$
norman	$\frac{\frac{(be+2cd)x}{b^2d^2}+\frac{(-3b^3ce^3-2b^2c^2de^2+18bc^3d^2e-12c^4d^3)cx^3}{d^2b^4(b^2e^2-2bcde+c^2d^2)}-\frac{1}{2db}+\frac{c^2(-4b^3ce^3-3b^2c^2de^2+27bc^3d^2e-18c^4d^3)x^4}{2d^2b^5(b^2e^2-2bcde+c^2d^2)}}{x^2(cx+b)^2}+\frac{(b^2e^2+3bcde\cdots}{b^5\cdots}$
risch	$\frac{\frac{c^2(b^3e^3+de^2b^2c-9d^2ebc^2+6d^3c^3)x^3}{d^2b^4(b^2e^2-2bcde+c^2d^2)}+\frac{c(4b^3e^3+3de^2b^2c-27d^2ebc^2+18d^3c^3)x^2}{2b^3d^2(b^2e^2-2bcde+c^2d^2)}+\frac{(be+2cd)x}{b^2d^2}-\frac{1}{2db}}{x^2(cx+b)^2}+\frac{\ln(-x)e^2}{b^3d^3}+\frac{3\ln(-x)ce}{b^4d^2}\cdots$
parallelrisch	$\frac{-12\ln(x)x^4c^7d^5+12\ln(cx+b)x^4c^7d^5+24x^3bc^6d^5+30\ln(x)x^2b^3c^4d^4e-45x^4bc^6d^4e-4xb^3c^4d^5-4x^4b^4c^3de^4+x^4b^3c^4d^2e^3+\cdots}{\cdots}$

input
```
int(1/(e*x+d)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*c^3/(b*e-c*d)/b^3/(c*x+b)^2-c^3*(4*b*e-3*c*d)/(b*e-c*d)^2/b^4/(c*x+b)
+c^3*(10*b^2*e^2-15*b*c*d*e+6*c^2*d^2)/(b*e-c*d)^3/b^5*ln(c*x+b)-e^5/(b*e-
c*d)^3/d^3*ln(e*x+d)-1/2/b^3/d/x^2-(-b*e-3*c*d)/d^2/b^4/x+(b^2*e^2+3*b*c*d
*e+6*c^2*d^2)*ln(x)/b^5/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(189) = 378$.

Time = 18.90 (sec) , antiderivative size = 716, normalized size of antiderivative = 3.71

$$\int \frac{1}{(d+ex)(bx+cx^2)^3}\,dx = -\frac{b^4c^3d^5 - 3\,b^5c^2d^4e + 3\,b^6cd^3e^2 - b^7d^2e^3 - 2\,(6\,bc^6d^5 - 15\,b^2c^5d^4e + 10\,b^3c^4d^3e^2 - b^5c^2de^4)x^3 - (18\,b^2c^5}{}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(b^4*c^3*d^5 - 3*b^5*c^2*d^4*e + 3*b^6*c*d^3*e^2 - b^7*d^2*e^3 - 2*(6
*b*c^6*d^5 - 15*b^2*c^5*d^4*e + 10*b^3*c^4*d^3*e^2 - b^5*c^2*d*e^4)*x^3 -
(18*b^2*c^5*d^5 - 45*b^3*c^4*d^4*e + 30*b^4*c^3*d^3*e^2 + b^5*c^2*d^2*e^3
- 4*b^6*c*d*e^4)*x^2 - 2*(2*b^3*c^4*d^5 - 5*b^4*c^3*d^4*e + 3*b^5*c^2*d^3*
e^2 + b^6*c*d^2*e^3 - b^7*d*e^4)*x + 2*((6*c^7*d^5 - 15*b*c^6*d^4*e + 10*b
^2*c^5*d^3*e^2)*x^4 + 2*(6*b*c^6*d^5 - 15*b^2*c^5*d^4*e + 10*b^3*c^4*d^3*e
^2)*x^3 + (6*b^2*c^5*d^5 - 15*b^3*c^4*d^4*e + 10*b^4*c^3*d^3*e^2)*x^2)*log
(c*x + b) - 2*(b^5*c^2*e^5*x^4 + 2*b^6*c*e^5*x^3 + b^7*e^5*x^2)*log(e*x +
d) - 2*((6*c^7*d^5 - 15*b*c^6*d^4*e + 10*b^2*c^5*d^3*e^2 - b^5*c^2*e^5)*x^
4 + 2*(6*b*c^6*d^5 - 15*b^2*c^5*d^4*e + 10*b^3*c^4*d^3*e^2 - b^6*c*e^5)*x^
3 + (6*b^2*c^5*d^5 - 15*b^3*c^4*d^4*e + 10*b^4*c^3*d^3*e^2 - b^7*e^5)*x^2)
*log(x))/((b^5*c^5*d^6 - 3*b^6*c^4*d^5*e + 3*b^7*c^3*d^4*e^2 - b^8*c^2*d^3
*e^3)*x^4 + 2*(b^6*c^4*d^6 - 3*b^7*c^3*d^5*e + 3*b^8*c^2*d^4*e^2 - b^9*c*d
^3*e^3)*x^3 + (b^7*c^3*d^6 - 3*b^8*c^2*d^5*e + 3*b^9*c*d^4*e^2 - b^10*d^3*
e^3)*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)/(c*x**2+b*x)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(189) = 378$.

Time = 0.05 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.27

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^3}\,dx = \frac{e^5\log(ex+d)}{c^3d^6-3\,bc^2d^5e+3\,b^2cd^4e^2-b^3d^3e^3} - \frac{\left(6\,c^5d^2-15\,bc^4de+10\,b^2c^3e^2\right)\log(cx+b)}{b^5c^3d^3-3\,b^6c^2d^2e+3\,b^7cde^2-b^8e^3} - \frac{b^3c^2d^3-2\,b^4cd^2e+b^5de^2-2\left(6\,c^5d^3-9\,bc^4d^2e+b^2c^3de^2+b^3c^2e^3\right)x^3-\left(18\,bc^4d^3-27\,b^2c^3d^2e+3\,b^3c\right.}{2\left(\left(b^4c^4d^4-2\,b^5c^3d^3e+b^6c^2d^2e^2\right)x^4+2\left(b^5c^3d^4-2\,b^6c^2d^3e+b^7cd^2e^2\right)x^3+\left(b^6\right.\right.} + \frac{\left(6\,c^2d^2+3\,bcde+b^2e^2\right)\log(x)}{b^5d^3}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
e^5*log(e*x + d)/(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)
 - (6*c^5*d^2 - 15*b*c^4*d*e + 10*b^2*c^3*e^2)*log(c*x + b)/(b^5*c^3*d^3 -
 3*b^6*c^2*d^2*e + 3*b^7*c*d*e^2 - b^8*e^3) - 1/2*(b^3*c^2*d^3 - 2*b^4*c*d
^2*e + b^5*d*e^2 - 2*(6*c^5*d^3 - 9*b*c^4*d^2*e + b^2*c^3*d*e^2 + b^3*c^2*
e^3)*x^3 - (18*b*c^4*d^3 - 27*b^2*c^3*d^2*e + 3*b^3*c^2*d*e^2 + 4*b^4*c*e^
3)*x^2 - 2*(2*b^2*c^3*d^3 - 3*b^3*c^2*d^2*e + b^5*e^3)*x)/((b^4*c^4*d^4 -
2*b^5*c^3*d^3*e + b^6*c^2*d^2*e^2)*x^4 + 2*(b^5*c^3*d^4 - 2*b^6*c^2*d^3*e
+ b^7*c*d^2*e^2)*x^3 + (b^6*c^2*d^4 - 2*b^7*c*d^3*e + b^8*d^2*e^2)*x^2) +
(6*c^2*d^2 + 3*b*c*d*e + b^2*e^2)*log(x)/(b^5*d^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(189) = 378$.

Time = 0.19 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.19

$$\int \frac{1}{(d+ex)\,(bx+cx^2)^3}\,dx = \frac{e^6\log\left(|ex+d|\right)}{c^3d^6e-3\,bc^2d^5e^2+3\,b^2cd^4e^3-b^3d^3e^4} - \frac{\left(6\,c^6d^2-15\,bc^5de+10\,b^2c^4e^2\right)\log\left(|cx+b|\right)}{b^5c^4d^3-3\,b^6c^3d^2e+3\,b^7c^2de^2-b^8ce^3} + \frac{\left(6\,c^2d^2+3\,bcde+b^2e^2\right)\log\left(|x|\right)}{b^5d^3} - \frac{b^3c^3d^5-3\,b^4c^2d^4e+3\,b^5cd^3e^2-b^6d^2e^3-2\left(6\,c^6d^5-15\,bc^5d^4e+10\,b^2c^4d^3e^2-b^4c^2de^4\right)x^3-(18\,bc^5d^5}{2\,(cd-be}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
e^6*log(abs(e*x + d))/(c^3*d^6*e - 3*b*c^2*d^5*e^2 + 3*b^2*c*d^4*e^3 - b^3
*d^3*e^4) - (6*c^6*d^2 - 15*b*c^5*d*e + 10*b^2*c^4*e^2)*log(abs(c*x + b))/
(b^5*c^4*d^3 - 3*b^6*c^3*d^2*e + 3*b^7*c^2*d*e^2 - b^8*c*e^3) + (6*c^2*d^2
 + 3*b*c*d*e + b^2*e^2)*log(abs(x))/(b^5*d^3) - 1/2*(b^3*c^3*d^5 - 3*b^4*c
^2*d^4*e + 3*b^5*c*d^3*e^2 - b^6*d^2*e^3 - 2*(6*c^6*d^5 - 15*b*c^5*d^4*e +
 10*b^2*c^4*d^3*e^2 - b^4*c^2*d*e^4)*x^3 - (18*b*c^5*d^5 - 45*b^2*c^4*d^4*
e + 30*b^3*c^3*d^3*e^2 + b^4*c^2*d^2*e^3 - 4*b^5*c*d*e^4)*x^2 - 2*(2*b^2*c
^4*d^5 - 5*b^3*c^3*d^4*e + 3*b^4*c^2*d^3*e^2 + b^5*c*d^2*e^3 - b^6*d*e^4)*
x)/((c*d - b*e)^3*(c*x + b)^2*b^4*d^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 9.70 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.72

$$\int \frac{1}{(d+ex)\,(bx+cx^2)^3}\,dx = \frac{\frac{x\,(b\,e+2\,c\,d)}{b^2\,d^2}-\frac{1}{2\,b\,d}+\frac{x^2\,\left(4\,b^3\,c\,e^3+3\,b^2\,c^2\,d\,e^2-27\,b\,c^3\,d^2\,e+18\,c^4\,d^3\right)}{2\,b^3\,d^2\,\left(b^2\,e^2-2\,b\,c\,d\,e+c^2\,d^2\right)}+\frac{x^3\,\left(b^3\,c^2\,e^3+b^2\,c^3\,d\,e^2-9\,b\,c^4\,d^2\,e+6\,c^5\,d^3\right)}{b^4\,d^2\,\left(b^2\,e^2-2\,b\,c\,d\,e+c^2\,d^2\right)}}{b^2\,x^2+2\,b\,c\,x^3+c^2\,x^4} + \frac{\ln\left(b+c\,x\right)\,\left(10\,b^2\,c^3\,e^2-15\,b\,c^4\,d\,e+6\,c^5\,d^2\right)}{b^8\,e^3-3\,b^7\,c\,d\,e^2+3\,b^6\,c^2\,d^2\,e-b^5\,c^3\,d^3} - \frac{e^5\,\ln\left(d+e\,x\right)}{d^3\,\left(b\,e-c\,d\right)^3} + \frac{\ln\left(x\right)\,\left(b^2\,e^2+3\,b\,c\,d\,e+6\,c^2\,d^2\right)}{b^5\,d^3}$$

input
```
int(1/((b*x + c*x^2)^3*(d + e*x)),x)
```

output
```
((x*(b*e + 2*c*d))/(b^2*d^2) - 1/(2*b*d) + (x^2*(18*c^4*d^3 + 4*b^3*c*e^3
+ 3*b^2*c^2*d*e^2 - 27*b*c^3*d^2*e))/(2*b^3*d^2*(b^2*e^2 + c^2*d^2 - 2*b*c
*d*e)) + (x^3*(6*c^5*d^3 + b^3*c^2*e^3 + b^2*c^3*d*e^2 - 9*b*c^4*d^2*e))/(
b^4*d^2*(b^2*e^2 + c^2*d^2 - 2*b*c*d*e)))/(b^2*x^2 + c^2*x^4 + 2*b*c*x^3)
+ (log(b + c*x)*(6*c^5*d^2 + 10*b^2*c^3*e^2 - 15*b*c^4*d*e))/(b^8*e^3 - b^
5*c^3*d^3 + 3*b^6*c^2*d^2*e - 3*b^7*c*d*e^2) - (e^5*log(d + e*x))/(d^3*(b*
e - c*d)^3) + (log(x)*(b^2*e^2 + 6*c^2*d^2 + 3*b*c*d*e))/(b^5*d^3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 825, normalized size of antiderivative = 4.27

$$\int \frac{1}{(d+ex)\,(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)/(c*x^2+b*x)^3,x)
```

output
```
(20*log(b + c*x)*b**4*c**3*d**3*e**2*x**2 - 30*log(b + c*x)*b**3*c**4*d**4
*e*x**2 + 40*log(b + c*x)*b**3*c**4*d**3*e**2*x**3 + 12*log(b + c*x)*b**2*
c**5*d**5*x**2 - 60*log(b + c*x)*b**2*c**5*d**4*e*x**3 + 20*log(b + c*x)*b
**2*c**5*d**3*e**2*x**4 + 24*log(b + c*x)*b*c**6*d**5*x**3 - 30*log(b + c*
x)*b*c**6*d**4*e*x**4 + 12*log(b + c*x)*c**7*d**5*x**4 - 2*log(d + e*x)*b*
*7*e**5*x**2 - 4*log(d + e*x)*b**6*c*e**5*x**3 - 2*log(d + e*x)*b**5*c**2*
e**5*x**4 + 2*log(x)*b**7*e**5*x**2 + 4*log(x)*b**6*c*e**5*x**3 + 2*log(x)
*b**5*c**2*e**5*x**4 - 20*log(x)*b**4*c**3*d**3*e**2*x**2 + 30*log(x)*b**3
*c**4*d**4*e*x**2 - 40*log(x)*b**3*c**4*d**3*e**2*x**3 - 12*log(x)*b**2*c*
*5*d**5*x**2 + 60*log(x)*b**2*c**5*d**4*e*x**3 - 20*log(x)*b**2*c**5*d**3*
e**2*x**4 - 24*log(x)*b*c**6*d**5*x**3 + 30*log(x)*b*c**6*d**4*e*x**4 - 12
*log(x)*c**7*d**5*x**4 - b**7*d**2*e**3 + 2*b**7*d*e**4*x + 3*b**6*c*d**3*
e**2 - 2*b**6*c*d**2*e**3*x + 3*b**6*c*d*e**4*x**2 - 3*b**5*c**2*d**4*e -
6*b**5*c**2*d**3*e**2*x - b**5*c**2*d**2*e**3*x**2 + b**4*c**3*d**5 + 10*b
**4*c**3*d**4*e*x - 20*b**4*c**3*d**3*e**2*x**2 - b**4*c**3*d*e**4*x**4 -
4*b**3*c**4*d**5*x + 30*b**3*c**4*d**4*e*x**2 - 12*b**2*c**5*d**5*x**2 + 1
0*b**2*c**5*d**3*e**2*x**4 - 15*b*c**6*d**4*e*x**4 + 6*c**7*d**5*x**4)/(2*
b**5*d**3*x**2*(b**5*e**3 - 3*b**4*c*d*e**2 + 2*b**4*c*e**3*x + 3*b**3*c**
2*d**2*e - 6*b**3*c**2*d*e**2*x + b**3*c**2*e**3*x**2 - b**2*c**3*d**3 + 6
*b**2*c**3*d**2*e*x - 3*b**2*c**3*d*e**2*x**2 - 2*b*c**4*d**3*x + 3*b*c...
```

3.80 $\int \frac{1}{(d+ex)^2(bx+cx^2)^3}\,dx$

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Optimal result

Integrand size = 19, antiderivative size = 230

$$\int \frac{1}{(d+ex)^2(bx+cx^2)^3}\,dx = -\frac{1}{2b^3d^2x^2}+\frac{3cd+2be}{b^4d^3x}+\frac{c^4}{2b^3(cd-be)^2(b+cx)^2} + \frac{c^4(3cd-5be)}{b^4(cd-be)^3(b+cx)}-\frac{e^5}{d^3(cd-be)^3(d+ex)} + \frac{3(2c^2d^2+2bcde+b^2e^2)\log(x)}{b^5d^4} - \frac{3c^4(2c^2d^2-6bcde+5b^2e^2)\log(b+cx)}{b^5(cd-be)^4} + \frac{3e^5(2cd-be)\log(d+ex)}{d^4(cd-be)^4}$$

output
```
-1/2/b^3/d^2/x^2+(2*b*e+3*c*d)/b^4/d^3/x+1/2*c^4/b^3/(-b*e+c*d)^2/(c*x+b)^
2+c^4*(-5*b*e+3*c*d)/b^4/(-b*e+c*d)^3/(c*x+b)-e^5/d^3/(-b*e+c*d)^3/(e*x+d)
+3*(b^2*e^2+2*b*c*d*e+2*c^2*d^2)*ln(x)/b^5/d^4-3*c^4*(5*b^2*e^2-6*b*c*d*e+
2*c^2*d^2)*ln(c*x+b)/b^5/(-b*e+c*d)^4+3*e^5*(-b*e+2*c*d)*ln(e*x+d)/d^4/(-b
*e+c*d)^4
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^3} dx = -\frac{1}{2b^3d^2x^2} + \frac{3cd+2be}{b^4d^3x} + \frac{c^4}{2b^3(cd-be)^2(b+cx)^2} + \frac{c^4(-3cd+5be)}{b^4(-cd+be)^3(b+cx)} - \frac{e^5}{d^3(cd-be)^3(d+ex)} + \frac{3(2c^2d^2+2bcde+b^2e^2)\log(x)}{b^5d^4} - \frac{3c^4(2c^2d^2-6bcde+5b^2e^2)\log(b+cx)}{b^5(cd-be)^4} + \frac{3e^5(2cd-be)\log(d+ex)}{d^4(cd-be)^4}$$

input
```
Integrate[1/((d + e*x)^2*(b*x + c*x^2)^3),x]
```

output
```
-1/2*1/(b^3*d^2*x^2) + (3*c*d + 2*b*e)/(b^4*d^3*x) + c^4/(2*b^3*(c*d - b*e
)^2*(b + c*x)^2) + (c^4*(-3*c*d + 5*b*e))/(b^4*(-(c*d) + b*e)^3*(b + c*x))
 - e^5/(d^3*(c*d - b*e)^3*(d + e*x)) + (3*(2*c^2*d^2 + 2*b*c*d*e + b^2*e^2
)*Log[x])/(b^5*d^4) - (3*c^4*(2*c^2*d^2 - 6*b*c*d*e + 5*b^2*e^2)*Log[b + c
*x])/(b^5*(c*d - b*e)^4) + (3*e^5*(2*c*d - b*e)*Log[d + e*x])/(d^4*(c*d -
b*e)^4)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^3 (d+ex)^2} dx$$

$$\Big\downarrow \text{ 1141}$$

$$c^3 \int \left(\frac{3(2cd-be)e^6}{c^3d^4(cd-be)^4(d+ex)} + \frac{e^6}{c^3d^3(cd-be)^3(d+ex)^2} + \frac{3(2c^2d^2+2bced+b^2e^2)}{b^5c^3d^4x} - \frac{3c^2(2c^2d^2-6bced+5b^2}{b^5(cd-be)^4(b+cx)}\right.$$

$$\Big\downarrow \text{2009}$$

$$c^3 \left(\frac{2be+3cd}{b^4c^3d^3x} + \frac{c(3cd-5be)}{b^4(b+cx)(cd-be)^3} - \frac{1}{2b^3c^3d^2x^2} + \frac{c}{2b^3(b+cx)^2(cd-be)^2} - \frac{3c(5b^2e^2-6bcde+2c^2d^2)\log(b}{b^5(cd-be)^4}\right.$$

input
```
Int[1/((d + e*x)^2*(b*x + c*x^2)^3),x]
```

output
```
c^3*(-1/2*1/(b^3*c^3*d^2*x^2) + (3*c*d + 2*b*e)/(b^4*c^3*d^3*x) + c/(2*b^3
*(c*d - b*e)^2*(b + c*x)^2) + (c*(3*c*d - 5*b*e))/(b^4*(c*d - b*e)^3*(b +
c*x)) - e^5/(c^3*d^3*(c*d - b*e)^3*(d + e*x)) + (3*(2*c^2*d^2 + 2*b*c*d*e
+ b^2*e^2)*Log[x])/(b^5*c^3*d^4) - (3*c*(2*c^2*d^2 - 6*b*c*d*e + 5*b^2*e^2
)*Log[b + c*x])/(b^5*(c*d - b*e)^4) + (3*e^5*(2*c*d - b*e)*Log[d + e*x])/(
c^3*d^4*(c*d - b*e)^4))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.98

method	result
default	$\frac{c^4}{2(be-cd)^2 b^3 (cx+b)^2}+\frac{c^4(5be-3cd)}{(be-cd)^3 b^4 (cx+b)}-\frac{3c^4(5b^2e^2-6bcde+2c^2d^2)\ln(cx+b)}{(be-cd)^4 b^5}+\frac{e^5}{d^3(be-cd)^3(ex+d)}-\frac{3e^5(be-2cd}{d^4(be-}$
norman	$\frac{\frac{(-3b^6e^6+2b^4d^2e^4c^2+6b^3d^3e^3c^3-20b c^5d^5e+12d^6c^6)x^3}{d^4b^4(b^3e^3-3d e^2b^2c+3d^2eb c^2-d^3c^3)}-\frac{1}{2db}+\frac{(3be+4cd)x}{2b^2d^2}+\frac{c(-12b^6e^6+20b^4d^2e^4c^2+9b^3d^3e^3c^3-39b^2c^4d^4e^2-8b c^5}{2d^4b^5(b^3e^3-3d e^2b^2c+3d^2eb c^2-d^3c^3)}}{(ex+d)x^2(cx+b)^2}$
risch	$\frac{\frac{3c^2e(e^4b^4-d e^3b^3c-d^2e^2b^2c^2+4d^3eb c^3-2d^4c^4)x^4}{b^4d^3(b^3e^3-3d e^2b^2c+3d^2eb c^2-d^3c^3)}+\frac{3c(4b^5e^5-3d e^4b^4c-5b^3d^2e^3c^2+10b^2c^3d^3e^2+2d^4eb c^4-4d^5c^5)x^3}{2b^4d^3(b^3e^3-3d e^2b^2c+3d^2eb c^2-d^3c^3)}+\frac{(6b^5e^5-13b}{2b^3c}}{(ex+d)x^2(cx+b)^2}$
parallelrisch	Expression too large to display

input
```
int(1/(e*x+d)^2/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*c^4/(b*e-c*d)^2/b^3/(c*x+b)^2+c^4*(5*b*e-3*c*d)/(b*e-c*d)^3/b^4/(c*x+b
)-3*c^4*(5*b^2*e^2-6*b*c*d*e+2*c^2*d^2)/(b*e-c*d)^4/b^5*ln(c*x+b)+e^5/d^3/
(b*e-c*d)^3/(e*x+d)-3*e^5*(b*e-2*c*d)/d^4/(b*e-c*d)^4*ln(e*x+d)-1/2/b^3/d^
2/x^2-(-2*b*e-3*c*d)/b^4/d^3/x+(3*b^2*e^2+6*b*c*d*e+6*c^2*d^2)/d^4/b^5*ln(
x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1305 vs. 2(226) = 452.

Time = 35.79 (sec) , antiderivative size = 1305, normalized size of antiderivative = 5.67

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(b^4*c^4*d^7 - 4*b^5*c^3*d^6*e + 6*b^6*c^2*d^5*e^2 - 4*b^7*c*d^4*e^3
+ b^8*d^3*e^4 - 6*(2*b*c^7*d^6*e - 6*b^2*c^6*d^5*e^2 + 5*b^3*c^5*d^4*e^3 -
 2*b^5*c^3*d^2*e^5 + b^6*c^2*d*e^6)*x^4 - 3*(4*b*c^7*d^7 - 6*b^2*c^6*d^6*e
 - 8*b^3*c^5*d^5*e^2 + 15*b^4*c^4*d^4*e^3 - 2*b^5*c^3*d^3*e^4 - 7*b^6*c^2*
d^2*e^5 + 4*b^7*c*d*e^6)*x^3 - (18*b^2*c^6*d^7 - 50*b^3*c^5*d^6*e + 33*b^4
*c^4*d^5*e^2 + 12*b^5*c^3*d^4*e^3 - 13*b^6*c^2*d^3*e^4 - 6*b^7*c*d^2*e^5 +
 6*b^8*d*e^6)*x^2 - (4*b^3*c^5*d^7 - 13*b^4*c^4*d^6*e + 12*b^5*c^3*d^5*e^2
 + 2*b^6*c^2*d^4*e^3 - 8*b^7*c*d^3*e^4 + 3*b^8*d^2*e^5)*x + 6*((2*c^8*d^6*
e - 6*b*c^7*d^5*e^2 + 5*b^2*c^6*d^4*e^3)*x^5 + (2*c^8*d^7 - 2*b*c^7*d^6*e
- 7*b^2*c^6*d^5*e^2 + 10*b^3*c^5*d^4*e^3)*x^4 + (4*b*c^7*d^7 - 10*b^2*c^6*
d^6*e + 4*b^3*c^5*d^5*e^2 + 5*b^4*c^4*d^4*e^3)*x^3 + (2*b^2*c^6*d^7 - 6*b^
3*c^5*d^6*e + 5*b^4*c^4*d^5*e^2)*x^2)*log(c*x + b) - 6*((2*b^5*c^3*d*e^6 -
 b^6*c^2*e^7)*x^5 + (2*b^5*c^3*d^2*e^5 + 3*b^6*c^2*d*e^6 - 2*b^7*c*e^7)*x^
4 + (4*b^6*c^2*d^2*e^5 - b^8*e^7)*x^3 + (2*b^7*c*d^2*e^5 - b^8*d*e^6)*x^2)
*log(e*x + d) - 6*((2*c^8*d^6*e - 6*b*c^7*d^5*e^2 + 5*b^2*c^6*d^4*e^3 - 2*
b^5*c^3*d*e^6 + b^6*c^2*e^7)*x^5 + (2*c^8*d^7 - 2*b*c^7*d^6*e - 7*b^2*c^6*
d^5*e^2 + 10*b^3*c^5*d^4*e^3 - 2*b^5*c^3*d^2*e^5 - 3*b^6*c^2*d*e^6 + 2*b^7
*c*e^7)*x^4 + (4*b*c^7*d^7 - 10*b^2*c^6*d^6*e + 4*b^3*c^5*d^5*e^2 + 5*b^4*
c^4*d^4*e^3 - 4*b^6*c^2*d^2*e^5 + b^8*e^7)*x^3 + (2*b^2*c^6*d^7 - 6*b^3*c^
5*d^6*e + 5*b^4*c^4*d^5*e^2 - 2*b^7*c*d^2*e^5 + b^8*d*e^6)*x^2)*log(x))...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(1/(e*x+d)**2/(c*x**2+b*x)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 752 vs. $2(226) = 452$.

Time = 0.07 (sec) , antiderivative size = 752, normalized size of antiderivative = 3.27

$$\int \frac{1}{(d+ex)^2\,(bx+cx^2)^3}\,dx = -\frac{3\,(2\,c^6d^2-6\,bc^5de+5\,b^2c^4e^2)\log\left(cx+b\right)}{b^5c^4d^4-4\,b^6c^3d^3e+6\,b^7c^2d^2e^2-4\,b^8cde^3+b^9e^4} + \frac{3\,(2\,cde^5-be^6)\log\left(ex+d\right)}{c^4d^8-4\,bc^3d^7e+6\,b^2c^2d^6e^2-4\,b^3cd^5e^3+b^4d^4e^4} - \frac{b^3c^3d^5-3\,b^4c^2d^4e+3\,b^5cd^3e^2-b^6d^2e^3-6\,(2\,c^6d^4e-4\,bc^5d^3e^2+b^2c^4d^2e^3+b^3c^3de^4-b^4c^2e^5)x^4-3\,(}{2\,((b^4c^5d^6e-3\,b^5c^4d^5e^2+3\,b^6c^3d^4e^3-b^7c^2d^3e^4)x^5+(b^4c^5d^7-b} + \frac{3\,(2\,c^2d^2+2\,bcde+b^2e^2)\log\left(x\right)}{b^5d^4}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-3*(2*c^6*d^2 - 6*b*c^5*d*e + 5*b^2*c^4*e^2)*log(c*x + b)/(b^5*c^4*d^4 - 4
*b^6*c^3*d^3*e + 6*b^7*c^2*d^2*e^2 - 4*b^8*c*d*e^3 + b^9*e^4) + 3*(2*c*d*e
^5 - b*e^6)*log(e*x + d)/(c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 - 4*
b^3*c*d^5*e^3 + b^4*d^4*e^4) - 1/2*(b^3*c^3*d^5 - 3*b^4*c^2*d^4*e + 3*b^5*
c*d^3*e^2 - b^6*d^2*e^3 - 6*(2*c^6*d^4*e - 4*b*c^5*d^3*e^2 + b^2*c^4*d^2*e
^3 + b^3*c^3*d*e^4 - b^4*c^2*e^5)*x^4 - 3*(4*c^6*d^5 - 2*b*c^5*d^4*e - 10*
b^2*c^4*d^3*e^2 + 5*b^3*c^3*d^2*e^3 + 3*b^4*c^2*d*e^4 - 4*b^5*c*e^5)*x^3 -
 (18*b*c^5*d^5 - 32*b^2*c^4*d^4*e + b^3*c^3*d^3*e^2 + 13*b^4*c^2*d^2*e^3 -
 6*b^6*e^5)*x^2 - (4*b^2*c^4*d^5 - 9*b^3*c^3*d^4*e + 3*b^4*c^2*d^3*e^2 + 5
*b^5*c*d^2*e^3 - 3*b^6*d*e^4)*x)/((b^4*c^5*d^6*e - 3*b^5*c^4*d^5*e^2 + 3*b
^6*c^3*d^4*e^3 - b^7*c^2*d^3*e^4)*x^5 + (b^4*c^5*d^7 - b^5*c^4*d^6*e - 3*b
^6*c^3*d^5*e^2 + 5*b^7*c^2*d^4*e^3 - 2*b^8*c*d^3*e^4)*x^4 + (2*b^5*c^4*d^7
 - 5*b^6*c^3*d^6*e + 3*b^7*c^2*d^5*e^2 + b^8*c*d^4*e^3 - b^9*d^3*e^4)*x^3
+ (b^6*c^3*d^7 - 3*b^7*c^2*d^6*e + 3*b^8*c*d^5*e^2 - b^9*d^4*e^3)*x^2) + 3
*(2*c^2*d^2 + 2*b*c*d*e + b^2*e^2)*log(x)/(b^5*d^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(226) = 452$.

Time = 0.14 (sec) , antiderivative size = 867, normalized size of antiderivative = 3.77

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^3} dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
-e^11/((c^3*d^6*e^6 - 3*b*c^2*d^5*e^7 + 3*b^2*c*d^4*e^8 - b^3*d^3*e^9)*(e*
x + d)) - 3/2*(2*c*d*e^5 - b*e^6)*log(abs(-c + 2*c*d/(e*x + d) - c*d^2/(e*
x + d)^2 - b*e/(e*x + d) + b*d*e/(e*x + d)^2))/(c^4*d^8 - 4*b*c^3*d^7*e +
6*b^2*c^2*d^6*e^2 - 4*b^3*c*d^5*e^3 + b^4*d^4*e^4) - 3/2*(4*c^6*d^6*e^2 -
12*b*c^5*d^5*e^3 + 10*b^2*c^4*d^4*e^4 - 2*b^5*c*d*e^7 + b^6*e^8)*log(abs(-
2*c*d*e + 2*c*d^2*e/(e*x + d) + b*e^2 - 2*b*d*e^2/(e*x + d) - e^2*abs(b))/
abs(-2*c*d*e + 2*c*d^2*e/(e*x + d) + b*e^2 - 2*b*d*e^2/(e*x + d) + e^2*abs
(b)))/((b^4*c^4*d^8 - 4*b^5*c^3*d^7*e + 6*b^6*c^2*d^6*e^2 - 4*b^7*c*d^5*e^
3 + b^8*d^4*e^4)*e^2*abs(b)) + 1/2*(12*c^7*d^5*e - 30*b*c^6*d^4*e^2 + 16*b
^2*c^5*d^3*e^3 + 6*b^3*c^4*d^2*e^4 - 14*b^4*c^3*d*e^5 + 5*b^5*c^2*e^6 - 2*
(18*c^7*d^6*e^2 - 54*b*c^6*d^5*e^3 + 47*b^2*c^5*d^4*e^4 - 4*b^3*c^4*d^3*e^
5 - 29*b^4*c^3*d^2*e^6 + 22*b^5*c^2*d*e^7 - 5*b^6*c*e^8)/((e*x + d)*e) + (
36*c^7*d^7*e^3 - 126*b*c^6*d^6*e^4 + 144*b^2*c^5*d^5*e^5 - 45*b^3*c^4*d^4*
e^6 - 70*b^4*c^3*d^3*e^7 + 87*b^5*c^2*d^2*e^8 - 36*b^6*c*d*e^9 + 5*b^7*e^1
0)/((e*x + d)^2*e^2) - 6*(2*c^7*d^8*e^4 - 8*b*c^6*d^7*e^5 + 11*b^2*c^5*d^6
*e^6 - 5*b^3*c^4*d^5*e^7 - 5*b^4*c^3*d^4*e^8 + 9*b^5*c^2*d^3*e^9 - 5*b^6*c
*d^2*e^10 + b^7*d*e^11)/((e*x + d)^3*e^3))/((c*d - b*e)^4*b^4*(c - 2*c*d/(
e*x + d) + c*d^2/(e*x + d)^2 + b*e/(e*x + d) - b*d*e/(e*x + d)^2)^2*d^4)
```

Mupad [B] (verification not implemented)

Time = 8.15 (sec) , antiderivative size = 603, normalized size of antiderivative = 2.62

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^3} \, dx = \frac{\ln(x) \left(3 b^2 e^2 + 6 b c d e + 6 c^2 d^2\right)}{b^5 d^4} - \frac{\ln(d+e x) \left(3 b e^6 - 6 c d e^5\right)}{b^4 d^4 e^4 - 4 b^3 c d^5 e^3 + 6 b^2 c^2 d^6 e^2 - 4 b c^3 d^7 e + c^4 d^8} - \frac{\ln(b+c x) \left(15 b^2 c^4 e^2 - 18 b c^5 d e + 6 c^6 d^2\right)}{b^9 e^4 - 4 b^8 c d e^3 + 6 b^7 c^2 d^2 e^2 - 4 b^6 c^3 d^3 e + b^5 c^4 d^4} - \frac{\frac{1}{2 b d} - \frac{x (3 b e + 4 c d)}{2 b^2 d^2} + \frac{x^2 \left(-6 b^5 e^5 + 13 b^3 c^2 d^2 e^3 + b^2 c^3 d^3 e^2 - 32 b c^4 d^4 e + 18 c^5 d^5\right)}{2 b^3 d^3 \left(b^3 e^3 - 3 b^2 c d e^2 + 3 b c^2 d^2 e - c^3 d^3\right)} + \frac{3 x^3 \left(-4 b^5 c e^5 + 3 b^4 c^2 d e^4 + 5 b^3 c^3 d^2 e^3 - 10\right.}{2 b^4 d^3 \left(b^3 e^3 - 3 b^2 c d e^2 + 3 b\right.}}{x^3 \left(e b^2 + 2 c d b\right) + x^4 \left(d c^2 + 2 b e c\right) + b^2 d}$$

input
```
int(1/((b*x + c*x^2)^3*(d + e*x)^2),x)
```

output
```
(log(x)*(3*b^2*e^2 + 6*c^2*d^2 + 6*b*c*d*e))/(b^5*d^4) - (log(d + e*x)*(3*
b*e^6 - 6*c*d*e^5))/(c^4*d^8 + b^4*d^4*e^4 - 4*b^3*c*d^5*e^3 + 6*b^2*c^2*d
^6*e^2 - 4*b*c^3*d^7*e) - (log(b + c*x)*(6*c^6*d^2 + 15*b^2*c^4*e^2 - 18*b
*c^5*d*e))/(b^9*e^4 + b^5*c^4*d^4 - 4*b^6*c^3*d^3*e + 6*b^7*c^2*d^2*e^2 -
4*b^8*c*d*e^3) - (1/(2*b*d) - (x*(3*b*e + 4*c*d))/(2*b^2*d^2) + (x^2*(18*c
^5*d^5 - 6*b^5*e^5 + b^2*c^3*d^3*e^2 + 13*b^3*c^2*d^2*e^3 - 32*b*c^4*d^4*e
))/(2*b^3*d^3*(b^3*e^3 - c^3*d^3 + 3*b*c^2*d^2*e - 3*b^2*c*d*e^2)) + (3*x^
3*(4*c^6*d^5 - 4*b^5*c*e^5 + 3*b^4*c^2*d*e^4 - 10*b^2*c^4*d^3*e^2 + 5*b^3*
c^3*d^2*e^3 - 2*b*c^5*d^4*e))/(2*b^4*d^3*(b^3*e^3 - c^3*d^3 + 3*b*c^2*d^2*
e - 3*b^2*c*d*e^2)) + (3*c^2*e*x^4*(2*c^4*d^4 - b^4*e^4 + b^2*c^2*d^2*e^2
- 4*b*c^3*d^3*e + b^3*c*d*e^3))/(b^4*d^3*(b^3*e^3 - c^3*d^3 + 3*b*c^2*d^2*
e - 3*b^2*c*d*e^2)))/(x^3*(b^2*e + 2*b*c*d) + x^4*(c^2*d + 2*b*c*e) + b^2*
d*x^2 + c^2*e*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 2061, normalized size of antiderivative = 8.96

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^3} \, dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^2/(c*x^2+b*x)^3,x)
```

output
```
( - 60*log(b + c*x)*b**5*c**4*d**5*e**3*x**2 - 60*log(b + c*x)*b**5*c**4*d
**4*e**4*x**3 + 42*log(b + c*x)*b**4*c**5*d**6*e**2*x**2 - 78*log(b + c*x)
*b**4*c**5*d**5*e**3*x**3 - 120*log(b + c*x)*b**4*c**5*d**4*e**4*x**4 + 12
*log(b + c*x)*b**3*c**6*d**7*e*x**2 + 96*log(b + c*x)*b**3*c**6*d**6*e**2*
x**3 + 24*log(b + c*x)*b**3*c**6*d**5*e**3*x**4 - 60*log(b + c*x)*b**3*c**
6*d**4*e**4*x**5 - 12*log(b + c*x)*b**2*c**7*d**8*x**2 + 12*log(b + c*x)*b
**2*c**7*d**7*e*x**3 + 66*log(b + c*x)*b**2*c**7*d**6*e**2*x**4 + 42*log(b
 + c*x)*b**2*c**7*d**5*e**3*x**5 - 24*log(b + c*x)*b*c**8*d**8*x**3 - 12*l
og(b + c*x)*b*c**8*d**7*e*x**4 + 12*log(b + c*x)*b*c**8*d**6*e**2*x**5 - 1
2*log(b + c*x)*c**9*d**8*x**4 - 12*log(b + c*x)*c**9*d**7*e*x**5 - 12*log(
d + e*x)*b**9*d*e**7*x**2 - 12*log(d + e*x)*b**9*e**8*x**3 + 18*log(d + e*
x)*b**8*c*d**2*e**6*x**2 - 6*log(d + e*x)*b**8*c*d*e**7*x**3 - 24*log(d +
e*x)*b**8*c*e**8*x**4 + 12*log(d + e*x)*b**7*c**2*d**3*e**5*x**2 + 48*log(
d + e*x)*b**7*c**2*d**2*e**6*x**3 + 24*log(d + e*x)*b**7*c**2*d*e**7*x**4
- 12*log(d + e*x)*b**7*c**2*e**8*x**5 + 24*log(d + e*x)*b**6*c**3*d**3*e**
5*x**3 + 42*log(d + e*x)*b**6*c**3*d**2*e**6*x**4 + 18*log(d + e*x)*b**6*c
**3*d*e**7*x**5 + 12*log(d + e*x)*b**5*c**4*d**3*e**5*x**4 + 12*log(d + e*
x)*b**5*c**4*d**2*e**6*x**5 + 12*log(x)*b**9*d*e**7*x**2 + 12*log(x)*b**9*
e**8*x**3 - 18*log(x)*b**8*c*d**2*e**6*x**2 + 6*log(x)*b**8*c*d*e**7*x**3
+ 24*log(x)*b**8*c*e**8*x**4 - 12*log(x)*b**7*c**2*d**3*e**5*x**2 - 48*...
```

3.81 $\int (d+ex)^{7/2} (bx+cx^2) \, dx$

Optimal result 620
Mathematica [A] (verified) 620
Rubi [A] (verified) 621
Maple [A] (verified) 622
Fricas [B] (verification not implemented) 622
Sympy [B] (verification not implemented) 623
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Reduce [B] (verification not implemented) 625

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int (d+ex)^{7/2} (bx+cx^2) \, dx = \frac{2d(cd-be)(d+ex)^{9/2}}{9e^3} - \frac{2(2cd-be)(d+ex)^{11/2}}{11e^3} + \frac{2c(d+ex)^{13/2}}{13e^3}$$

output
```
2/9*d*(-b*e+c*d)*(e*x+d)^(9/2)/e^3-2/11*(-b*e+2*c*d)*(e*x+d)^(11/2)/e^3+2/
13*c*(e*x+d)^(13/2)/e^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int (d+ex)^{7/2} (bx+cx^2) \, dx = \frac{2(d+ex)^{9/2} (13be(-2d+9ex)+c(8d^2-36dex+99e^2x^2))}{1287e^3}$$

input
```
Integrate[(d + e*x)^(7/2)*(b*x + c*x^2),x]
```

output
```
(2*(d + e*x)^(9/2)*(13*b*e*(-2*d + 9*e*x) + c*(8*d^2 - 36*d*e*x + 99*e^2*x
^2)))/(1287*e^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)(d+ex)^{7/2}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{(d+ex)^{9/2}(be-2cd)}{e^2}+\frac{d(d+ex)^{7/2}(cd-be)}{e^2}+\frac{c(d+ex)^{11/2}}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(d+ex)^{11/2}(2cd-be)}{11e^3}+\frac{2d(d+ex)^{9/2}(cd-be)}{9e^3}+\frac{2c(d+ex)^{13/2}}{13e^3}$$

input
```
Int[(d + e*x)^(7/2)*(b*x + c*x^2),x]
```

output
```
(2*d*(c*d - b*e)*(d + e*x)^(9/2))/(9*e^3) - (2*(2*c*d - b*e)*(d + e*x)^(11
/2))/(11*e^3) + (2*c*(d + e*x)^(13/2))/(13*e^3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60

method	result
pseudoelliptic	$-\frac{4(ex+d)^{\frac{9}{2}}\left(-\frac{9x\left(\frac{11cx}{13}+b\right)e^{2}}{2}+d\left(\frac{18cx}{13}+b\right)e-\frac{4c\,d^{2}}{13}\right)}{99e^{3}}$
gosper	$-\frac{2(ex+d)^{\frac{9}{2}}\left(-99x^{2}c\,e^{2}-117xb\,e^{2}+36cdxe+26bde-8c\,d^{2}\right)}{1287e^{3}}$
derivativedivides	$\frac{\frac{2c(ex+d)^{\frac{13}{2}}}{13}+\frac{2(be-2cd)(ex+d)^{\frac{11}{2}}}{11}-\frac{2d(be-cd)(ex+d)^{\frac{9}{2}}}{9}}{e^{3}}$
default	$\frac{\frac{2c(ex+d)^{\frac{13}{2}}}{13}-\frac{2(-be+2cd)(ex+d)^{\frac{11}{2}}}{11}-\frac{2d(be-cd)(ex+d)^{\frac{9}{2}}}{9}}{e^{3}}$
orering	$-\frac{2\left(-99x^{2}c\,e^{2}-117xb\,e^{2}+36cdxe+26bde-8c\,d^{2}\right)(ex+d)^{\frac{9}{2}}\left(c\,x^{2}+bx\right)}{1287e^{3}x(cx+b)}$
trager	$-\frac{2\left(-99e^{6}c\,x^{6}-117b\,e^{6}x^{5}-360cd\,e^{5}x^{5}-442bd\,e^{5}x^{4}-458c\,d^{2}e^{4}x^{4}-598b\,d^{2}e^{4}x^{3}-212c\,d^{3}e^{3}x^{3}-312b\,d^{3}e^{3}x^{2}-3c\,d^{4}e^{2}\right.}{1287e^{3}}$
risch	$-\frac{2\left(-99e^{6}c\,x^{6}-117b\,e^{6}x^{5}-360cd\,e^{5}x^{5}-442bd\,e^{5}x^{4}-458c\,d^{2}e^{4}x^{4}-598b\,d^{2}e^{4}x^{3}-212c\,d^{3}e^{3}x^{3}-312b\,d^{3}e^{3}x^{2}-3c\,d^{4}e^{2}\right.}{1287e^{3}}$

input
```
int((e*x+d)^(7/2)*(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-4/99*(e*x+d)^(9/2)*(-9/2*x*(11/13*c*x+b)*e^2+d*(18/13*c*x+b)*e-4/13*c*d^2
)/e^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(56) = 112.

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.10

$$\int (d+ex)^{7/2}\left(bx+cx^2\right)\,dx=\frac{2\left(99\,ce^6x^6+8\,cd^6-26\,bd^5e+9\left(40\,cde^5+13\,be^6\right)x^5+2\left(229\,cd^2e^4+221\,bde^5\right)x^4+2\left(106\,cd^3\right.\right.}{1287\,e^3}$$

input
```
integrate((e*x+d)^(7/2)*(c*x^2+b*x),x, algorithm="fricas")
```

output
```
2/1287*(99*c*e^6*x^6 + 8*c*d^6 - 26*b*d^5*e + 9*(40*c*d*e^5 + 13*b*e^6)*x^
5 + 2*(229*c*d^2*e^4 + 221*b*d*e^5)*x^4 + 2*(106*c*d^3*e^3 + 299*b*d^2*e^4
)*x^3 + 3*(c*d^4*e^2 + 104*b*d^3*e^3)*x^2 - (4*c*d^5*e - 13*b*d^4*e^2)*x)*
sqrt(e*x + d)/e^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(65) = 130$.

Time = 0.49 (sec) , antiderivative size = 292, normalized size of antiderivative = 4.29

$$\int (d+ex)^{7/2} (bx + cx^2) \, dx = \begin{cases} -\frac{4bd^5\sqrt{d+ex}}{99e^2} + \frac{2bd^4x\sqrt{d+ex}}{99e} + \frac{16bd^3x^2\sqrt{d+ex}}{33} + \frac{92bd^2ex^3\sqrt{d+ex}}{99} + \frac{68bde^2x^4\sqrt{d+ex}}{99} + \frac{2be^3x^5\sqrt{d+ex}}{11} + \frac{16cd^6\sqrt{}}{1287} \\ d^{\frac{7}{2}}\left(\frac{bx^2}{2} + \frac{cx^3}{3}\right) \end{cases}$$

input
```
integrate((e*x+d)**(7/2)*(c*x**2+b*x),x)
```

output
```
Piecewise((-4*b*d**5*sqrt(d + e*x)/(99*e**2) + 2*b*d**4*x*sqrt(d + e*x)/(9
9*e) + 16*b*d**3*x**2*sqrt(d + e*x)/33 + 92*b*d**2*e*x**3*sqrt(d + e*x)/99
 + 68*b*d*e**2*x**4*sqrt(d + e*x)/99 + 2*b*e**3*x**5*sqrt(d + e*x)/11 + 16
*c*d**6*sqrt(d + e*x)/(1287*e**3) - 8*c*d**5*x*sqrt(d + e*x)/(1287*e**2) +
 2*c*d**4*x**2*sqrt(d + e*x)/(429*e) + 424*c*d**3*x**3*sqrt(d + e*x)/1287
+ 916*c*d**2*e*x**4*sqrt(d + e*x)/1287 + 80*c*d*e**2*x**5*sqrt(d + e*x)/14
3 + 2*c*e**3*x**6*sqrt(d + e*x)/13, Ne(e, 0)), (d**(7/2)*(b*x**2/2 + c*x**
3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (d+ex)^{7/2} (bx + cx^2) \, dx = \frac{2\left(99\,(ex+d)^{\frac{13}{2}}c - 117\,(2\,cd - be)(ex+d)^{\frac{11}{2}} + 143\,(cd^2 - bde)(ex+d)^{\frac{9}{2}}\right)}{1287\,e^3}$$

input
```
integrate((e*x+d)^(7/2)*(c*x^2+b*x),x, algorithm="maxima")
```

output
```
2/1287*(99*(e*x + d)^(13/2)*c - 117*(2*c*d - b*e)*(e*x + d)^(11/2) + 143*(
c*d^2 - b*d*e)*(e*x + d)^(9/2))/e^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(56) = 112$.

Time = 0.15 (sec) , antiderivative size = 584, normalized size of antiderivative = 8.59

$$\int (d+ex)^{7/2}\left(bx+cx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(7/2)*(c*x^2+b*x),x, algorithm="giac")
```

output
```
2/45045*(15015*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*b*d^4/e + 3003*(3*(e*
x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c*d^4/e^2 + 12
012*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*b*d^
3/e + 5148*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*
d^2 - 35*sqrt(e*x + d)*d^3)*c*d^3/e^2 + 7722*(5*(e*x + d)^(7/2) - 21*(e*x
+ d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b*d^2/e + 85
8*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 -
420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*c*d^2/e^2 + 572*(35*(e*x
+ d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x +
d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b*d/e + 260*(63*(e*x + d)^(11/2) - 3
85*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3
+ 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*c*d/e^2 + 65*(63*(e*x
+ d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x
+ d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b/e + 1
5*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d
^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^
(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*c/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int (d+ex)^{7/2} \left(bx+cx^2\right) \, dx = \frac{2\,(d+e\,x)^{9/2}\,\left(99\,c\,(d+e\,x)^2+143\,c\,d^2+117\,b\,e\,(d+e\,x)-234\,c\,d\,(d+e\,x)-143\,b\,d\,e\right)}{1287\,e^3}$$

input
```
int((b*x + c*x^2)*(d + e*x)^(7/2),x)
```

output
```
(2*(d + e*x)^(9/2)*(99*c*(d + e*x)^2 + 143*c*d^2 + 117*b*e*(d + e*x) - 234
*c*d*(d + e*x) - 143*b*d*e))/(1287*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.07

$$\int (d+ex)^{7/2} \left(bx+cx^2\right) \, dx = \frac{2\sqrt{ex+d}\,(99c\,e^6x^6+117b\,e^6x^5+360cd\,e^5x^5+442bd\,e^5x^4+458c\,d^2e^4x^4+598b\,d^2e^4x^3+212c}{1287e^3}$$

input
```
int((e*x+d)^(7/2)*(c*x^2+b*x),x)
```

output
```
(2*sqrt(d + e*x)*( - 26*b*d**5*e + 13*b*d**4*e**2*x + 312*b*d**3*e**3*x**2
 + 598*b*d**2*e**4*x**3 + 442*b*d*e**5*x**4 + 117*b*e**6*x**5 + 8*c*d**6 -
 4*c*d**5*e*x + 3*c*d**4*e**2*x**2 + 212*c*d**3*e**3*x**3 + 458*c*d**2*e**
4*x**4 + 360*c*d*e**5*x**5 + 99*c*e**6*x**6))/(1287*e**3)
```

3.82 $\int (d+ex)^{5/2} (bx+cx^2) \, dx$

Optimal result . . . 626
Mathematica [A] (verified) . . . 626
Rubi [A] (verified) . . . 627
Maple [A] (verified) . . . 628
Fricas [B] (verification not implemented) . . . 628
Sympy [B] (verification not implemented) . . . 629
Maxima [A] (verification not implemented) . . . 629
Giac [B] (verification not implemented) . . . 630
Mupad [B] (verification not implemented) . . . 631
Reduce [B] (verification not implemented) . . . 631

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int (d+ex)^{5/2} (bx+cx^2) \, dx = \frac{2d(cd-be)(d+ex)^{7/2}}{7e^3} - \frac{2(2cd-be)(d+ex)^{9/2}}{9e^3} + \frac{2c(d+ex)^{11/2}}{11e^3}$$

output
```
2/7*d*(-b*e+c*d)*(e*x+d)^(7/2)/e^3-2/9*(-b*e+2*c*d)*(e*x+d)^(9/2)/e^3+2/11
*c*(e*x+d)^(11/2)/e^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int (d+ex)^{5/2} (bx+cx^2) \, dx = \frac{2(d+ex)^{7/2} (11be(-2d+7ex) + c(8d^2 - 28dex + 63e^2x^2))}{693e^3}$$

input
```
Integrate[(d + e*x)^(5/2)*(b*x + c*x^2),x]
```

output
```
(2*(d + e*x)^(7/2)*(11*b*e*(-2*d + 7*e*x) + c*(8*d^2 - 28*d*e*x + 63*e^2*x
^2)))/(693*e^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right) (d+ex)^{5/2} \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{(d+ex)^{7/2}(be-2cd)}{e^2}+\frac{d(d+ex)^{5/2}(cd-be)}{e^2}+\frac{c(d+ex)^{9/2}}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(d+ex)^{9/2}(2cd-be)}{9e^3}+\frac{2d(d+ex)^{7/2}(cd-be)}{7e^3}+\frac{2c(d+ex)^{11/2}}{11e^3}$$

input
```
Int[(d + e*x)^(5/2)*(b*x + c*x^2),x]
```

output
```
(2*d*(c*d - b*e)*(d + e*x)^(7/2))/(7*e^3) - (2*(2*c*d - b*e)*(d + e*x)^(9/
2))/(9*e^3) + (2*c*(d + e*x)^(11/2))/(11*e^3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60

method	result
pseudoelliptic	$-\frac{4(ex+d)^{\frac{7}{2}}\left(-\frac{7x\left(\frac{9cx}{11}+b\right)e^{2}}{2}+d\left(\frac{14cx}{11}+b\right)e-\frac{4c\,d^{2}}{11}\right)}{63e^{3}}$
gosper	$-\frac{2(ex+d)^{\frac{7}{2}}\left(-63x^{2}c\,e^{2}-77xb\,e^{2}+28cdxe+22bde-8c\,d^{2}\right)}{693e^{3}}$
derivativedivides	$\frac{\frac{2c(ex+d)^{\frac{11}{2}}}{11}+\frac{2(be-2cd)(ex+d)^{\frac{9}{2}}}{9}-\frac{2d(be-cd)(ex+d)^{\frac{7}{2}}}{7}}{e^{3}}$
default	$\frac{\frac{2c(ex+d)^{\frac{11}{2}}}{11}-\frac{2(-be+2cd)(ex+d)^{\frac{9}{2}}}{9}-\frac{2d(be-cd)(ex+d)^{\frac{7}{2}}}{7}}{e^{3}}$
orering	$-\frac{2\left(-63x^{2}c\,e^{2}-77xb\,e^{2}+28cdxe+22bde-8c\,d^{2}\right)(ex+d)^{\frac{7}{2}}\left(c\,x^{2}+bx\right)}{693e^{3}x(cx+b)}$
trager	$-\frac{2\left(-63c\,e^{5}x^{5}-77b\,e^{5}x^{4}-161cd\,e^{4}x^{4}-209bd\,e^{4}x^{3}-113c\,d^{2}e^{3}x^{3}-165b\,d^{2}e^{3}x^{2}-3d^{3}e^{2}c\,x^{2}-11b\,d^{3}e^{2}x+4c\,d^{4}ex+22b\,d\right.}{693e^{3}}$
risch	$-\frac{2\left(-63c\,e^{5}x^{5}-77b\,e^{5}x^{4}-161cd\,e^{4}x^{4}-209bd\,e^{4}x^{3}-113c\,d^{2}e^{3}x^{3}-165b\,d^{2}e^{3}x^{2}-3d^{3}e^{2}c\,x^{2}-11b\,d^{3}e^{2}x+4c\,d^{4}ex+22b\,d\right.}{693e^{3}}$

input
```
int((e*x+d)^(5/2)*(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-4/63*(e*x+d)^(7/2)*(-7/2*x*(9/11*c*x+b)*e^2+d*(14/11*c*x+b)*e-4/11*c*d^2)
/e^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(56) = 112$.

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.74

$$\int (d+ex)^{5/2}\left(bx+cx^2\right)\,dx=\frac{2\left(63\,ce^5x^5+8\,cd^5-22\,bd^4e+7\left(23\,cde^4+11\,be^5\right)x^4+\left(113\,cd^2e^3+209\,bde^4\right)x^3+3\left(cd^3e^2+5\right.\right.}{693\,e^3}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+b*x),x, algorithm="fricas")
```

output
```
2/693*(63*c*e^5*x^5 + 8*c*d^5 - 22*b*d^4*e + 7*(23*c*d*e^4 + 11*b*e^5)*x^4
 + (113*c*d^2*e^3 + 209*b*d*e^4)*x^3 + 3*(c*d^3*e^2 + 55*b*d^2*e^3)*x^2 -
(4*c*d^4*e - 11*b*d^3*e^2)*x)*sqrt(e*x + d)/e^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(65) = 130$.

Time = 0.35 (sec) , antiderivative size = 245, normalized size of antiderivative = 3.60

$$\int (d+ex)^{5/2} (bx+cx^2)\, dx = \begin{cases} -\frac{4bd^4\sqrt{d+ex}}{63e^2} + \frac{2bd^3x\sqrt{d+ex}}{63e} + \frac{10bd^2x^2\sqrt{d+ex}}{21} + \frac{38bdex^3\sqrt{d+ex}}{63} + \frac{2be^2x^4\sqrt{d+ex}}{9} + \frac{16cd^5\sqrt{d+ex}}{693e^3} - \frac{8cd^4x\sqrt{d+e}}{693e^2} \\ d^{\frac{5}{2}}\left(\frac{bx^2}{2} + \frac{cx^3}{3}\right) \end{cases}$$

input
```
integrate((e*x+d)**(5/2)*(c*x**2+b*x),x)
```

output
```
Piecewise((-4*b*d**4*sqrt(d + e*x)/(63*e**2) + 2*b*d**3*x*sqrt(d + e*x)/(6
3*e) + 10*b*d**2*x**2*sqrt(d + e*x)/21 + 38*b*d*e*x**3*sqrt(d + e*x)/63 +
2*b*e**2*x**4*sqrt(d + e*x)/9 + 16*c*d**5*sqrt(d + e*x)/(693*e**3) - 8*c*d
**4*x*sqrt(d + e*x)/(693*e**2) + 2*c*d**3*x**2*sqrt(d + e*x)/(231*e) + 226
*c*d**2*x**3*sqrt(d + e*x)/693 + 46*c*d*e*x**4*sqrt(d + e*x)/99 + 2*c*e**2
*x**5*sqrt(d + e*x)/11, Ne(e, 0)), (d**(5/2)*(b*x**2/2 + c*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (d+ex)^{5/2} (bx+cx^2)\, dx = \frac{2\left(63\,(ex+d)^{\frac{11}{2}}c - 77\,(2\,cd-be)(ex+d)^{\frac{9}{2}} + 99\,(cd^2-bde)(ex+d)^{\frac{7}{2}}\right)}{693\,e^3}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+b*x),x, algorithm="maxima")
```

output
```
2/693*(63*(e*x + d)^(11/2)*c - 77*(2*c*d - b*e)*(e*x + d)^(9/2) + 99*(c*d^
2 - b*d*e)*(e*x + d)^(7/2))/e^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(56) = 112$.

Time = 0.12 (sec) , antiderivative size = 418, normalized size of antiderivative = 6.15

$$\int (d+ex)^{5/2}\left(bx+cx^2\right)\,dx = \frac{2\left(\frac{1155\left((ex+d)^{\frac{3}{2}}-3\sqrt{ex+d}\,d\right)bd^3}{e} + \frac{231\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}\,d^2\right)cd^3}{e^2} + \frac{693\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+\right.}{e}\right.}{}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+b*x),x, algorithm="giac")
```

output
```
2/3465*(1155*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*b*d^3/e + 231*(3*(e*x +
 d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c*d^3/e^2 + 693*(
3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*b*d^2/e +
 297*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 -
35*sqrt(e*x + d)*d^3)*c*d^2/e^2 + 297*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5
/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b*d/e + 33*(35*(e*x
 + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x +
 d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*c*d/e^2 + 11*(35*(e*x + d)^(9/2) -
180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3
+ 315*sqrt(e*x + d)*d^4)*b/e + 5*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2
)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^
(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*c/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int (d+ex)^{5/2} \left(bx+cx^2\right) dx = \frac{2\,{\left(d+e\,x\right)}^{7/2}\,\left(63\,c\,{\left(d+e\,x\right)}^2+99\,c\,d^2+77\,b\,e\,\left(d+e\,x\right)-154\,c\,d\,\left(d+e\,x\right)-99\,b\,d\,e\right)}{693\,e^3}$$

input
```
int((b*x + c*x^2)*(d + e*x)^(5/2),x)
```

output
```
(2*(d + e*x)^(7/2)*(63*c*(d + e*x)^2 + 99*c*d^2 + 77*b*e*(d + e*x) - 154*c
*d*(d + e*x) - 99*b*d*e))/(693*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.72

$$\int (d+ex)^{5/2} \left(bx+cx^2\right) dx = \frac{2\sqrt{ex+d}\,(63c\,e^5x^5+77b\,e^5x^4+161cd\,e^4x^4+209bd\,e^4x^3+113c\,d^2e^3x^3+165b\,d^2e^3x^2+3c\,d^3e}{693e^3}$$

input
```
int((e*x+d)^(5/2)*(c*x^2+b*x),x)
```

output
```
(2*sqrt(d + e*x)*( - 22*b*d**4*e + 11*b*d**3*e**2*x + 165*b*d**2*e**3*x**2
 + 209*b*d*e**4*x**3 + 77*b*e**5*x**4 + 8*c*d**5 - 4*c*d**4*e*x + 3*c*d**3
*e**2*x**2 + 113*c*d**2*e**3*x**3 + 161*c*d*e**4*x**4 + 63*c*e**5*x**5))/(
693*e**3)
```

3.83 $\int (d+ex)^{3/2} \left(bx+cx^2\right) \, dx$

Optimal result . . . 632
Mathematica [A] (verified) . . . 632
Rubi [A] (verified) . . . 633
Maple [A] (verified) . . . 634
Fricas [A] (verification not implemented) . . . 634
Sympy [A] (verification not implemented) . . . 635
Maxima [A] (verification not implemented) . . . 635
Giac [B] (verification not implemented) . . . 636
Mupad [B] (verification not implemented) . . . 636
Reduce [B] (verification not implemented) . . . 637

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int (d+ex)^{3/2} \left(bx+cx^2\right) \, dx = \frac{2d(cd-be)(d+ex)^{5/2}}{5e^3} - \frac{2(2cd-be)(d+ex)^{7/2}}{7e^3} + \frac{2c(d+ex)^{9/2}}{9e^3}$$

output
```
2/5*d*(-b*e+c*d)*(e*x+d)^(5/2)/e^3-2/7*(-b*e+2*c*d)*(e*x+d)^(7/2)/e^3+2/9*
c*(e*x+d)^(9/2)/e^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int (d+ex)^{3/2} \left(bx+cx^2\right) \, dx = \frac{2(d+ex)^{5/2} (9be(-2d+5ex)+c(8d^2-20dex+35e^2x^2))}{315e^3}$$

input
```
Integrate[(d + e*x)^(3/2)*(b*x + c*x^2),x]
```

output
```
(2*(d + e*x)^(5/2)*(9*b*e*(-2*d + 5*e*x) + c*(8*d^2 - 20*d*e*x + 35*e^2*x^
2)))/(315*e^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+cx^2)(d+ex)^{3/2}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{(d+ex)^{5/2}(be-2cd)}{e^2}+\frac{d(d+ex)^{3/2}(cd-be)}{e^2}+\frac{c(d+ex)^{7/2}}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(d+ex)^{7/2}(2cd-be)}{7e^3}+\frac{2d(d+ex)^{5/2}(cd-be)}{5e^3}+\frac{2c(d+ex)^{9/2}}{9e^3}$$

input
```
Int[(d + e*x)^(3/2)*(b*x + c*x^2),x]
```

output
```
(2*d*(c*d - b*e)*(d + e*x)^(5/2))/(5*e^3) - (2*(2*c*d - b*e)*(d + e*x)^(7/
2))/(7*e^3) + (2*c*(d + e*x)^(9/2))/(9*e^3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60

method	result	size
pseudoelliptic	$-\frac{4(ex+d)^{\frac{5}{2}}\left(-\frac{5x\left(\frac{7cx}{9}+b\right)e^{2}}{2}+d\left(\frac{10cx}{9}+b\right)e-\frac{4c\,d^{2}}{9}\right)}{35e^{3}}$	41
gosper	$-\frac{2(ex+d)^{\frac{5}{2}}\left(-35x^{2}c\,e^{2}-45xb\,e^{2}+20cdxe+18bde-8c\,d^{2}\right)}{315e^{3}}$	47
derivativedivides	$\frac{\frac{2c(ex+d)^{\frac{9}{2}}}{9}+\frac{2(be-2cd)(ex+d)^{\frac{7}{2}}}{7}-\frac{2d(be-cd)(ex+d)^{\frac{5}{2}}}{5}}{e^{3}}$	52
default	$\frac{\frac{2c(ex+d)^{\frac{9}{2}}}{9}-\frac{2(-be+2cd)(ex+d)^{\frac{7}{2}}}{7}-\frac{2d(be-cd)(ex+d)^{\frac{5}{2}}}{5}}{e^{3}}$	53
orering	$-\frac{2\left(-35x^{2}c\,e^{2}-45xb\,e^{2}+20cdxe+18bde-8c\,d^{2}\right)(ex+d)^{\frac{5}{2}}\left(c\,x^{2}+bx\right)}{315e^{3}x(cx+b)}$	66
trager	$-\frac{2\left(-35c\,e^{4}x^{4}-45b\,e^{4}x^{3}-50d\,e^{3}c\,x^{3}-72bd\,e^{3}x^{2}-3c\,d^{2}e^{2}x^{2}-9b\,d^{2}e^{2}x+4c\,d^{3}ex+18d^{3}eb-8c\,d^{4}\right)\sqrt{ex+d}}{315e^{3}}$	95
risch	$-\frac{2\left(-35c\,e^{4}x^{4}-45b\,e^{4}x^{3}-50d\,e^{3}c\,x^{3}-72bd\,e^{3}x^{2}-3c\,d^{2}e^{2}x^{2}-9b\,d^{2}e^{2}x+4c\,d^{3}ex+18d^{3}eb-8c\,d^{4}\right)\sqrt{ex+d}}{315e^{3}}$	95

input
```
int((e*x+d)^(3/2)*(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-4/35*(e*x+d)^(5/2)*(-5/2*x*(7/9*c*x+b)*e^2+d*(10/9*c*x+b)*e-4/9*c*d^2)/e^
3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.40

$$\int (d+ex)^{3/2}\left(bx+cx^2\right)\,dx=\frac{2\left(35\,ce^4x^4+8\,cd^4-18\,bd^3e+5\left(10\,cde^3+9\,be^4\right)x^3+3\left(cd^2e^2+24\,bde^3\right)x^2-\left(4\,cd^3e-9\,bd^2e\right.}{315\,e^3}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x),x, algorithm="fricas")
```

output
```
2/315*(35*c*e^4*x^4 + 8*c*d^4 - 18*b*d^3*e + 5*(10*c*d*e^3 + 9*b*e^4)*x^3
+ 3*(c*d^2*e^2 + 24*b*d*e^3)*x^2 - (4*c*d^3*e - 9*b*d^2*e^2)*x)*sqrt(e*x +
 d)/e^3
```

Sympy [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25

$$\int (d+ex)^{3/2} \left(bx+cx^2\right) \, dx = \begin{cases} \frac{2\left(\frac{c(d+ex)^{\frac{9}{2}}}{9e^2}+\frac{(d+ex)^{\frac{7}{2}}(be-2cd)}{7e^2}+\frac{(d+ex)^{\frac{5}{2}}\left(-bde+cd^2\right)}{5e^2}\right)}{e} & \text{for } e\neq 0 \\ d^{\frac{3}{2}}\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((e*x+d)**(3/2)*(c*x**2+b*x),x)
```

output
```
Piecewise((2*(c*(d + e*x)**(9/2)/(9*e**2) + (d + e*x)**(7/2)*(b*e - 2*c*d)
/(7*e**2) + (d + e*x)**(5/2)*(-b*d*e + c*d**2)/(5*e**2))/e, Ne(e, 0)), (d*
*(3/2)*(b*x**2/2 + c*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int (d+ex)^{3/2} \left(bx+cx^2\right) \, dx = \frac{2\left(35\,(ex+d)^{\frac{9}{2}}c-45\,(2\,cd-be)(ex+d)^{\frac{7}{2}}+63\,(cd^2-bde)(ex+d)^{\frac{5}{2}}\right)}{315\,e^3}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x),x, algorithm="maxima")
```

output
```
2/315*(35*(e*x + d)^(9/2)*c - 45*(2*c*d - b*e)*(e*x + d)^(7/2) + 63*(c*d^2
 - b*d*e)*(e*x + d)^(5/2))/e^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(56) = 112$.

Time = 0.11 (sec) , antiderivative size = 275, normalized size of antiderivative = 4.04

$$\int (d+ex)^{3/2} \left(bx+cx^2\right) dx = \frac{2\left(\frac{105\left((ex+d)^{\frac{3}{2}}-3\sqrt{ex+d}d\right)bd^2}{e}+\frac{21\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}d^2\right)cd^2}{e^2}+\frac{42\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\right.}{e}\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x),x, algorithm="giac")
```

output
```
2/315*(105*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*b*d^2/e + 21*(3*(e*x + d)
^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c*d^2/e^2 + 42*(3*(e
*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*b*d/e + 18*(5
*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt
(e*x + d)*d^3)*c*d/e^2 + 9*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*
(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b/e + (35*(e*x + d)^(9/2) - 18
0*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 +
315*sqrt(e*x + d)*d^4)*c/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int (d+ex)^{3/2} \left(bx+cx^2\right) dx = \frac{2\,{\left(d+e\,x\right)}^{5/2}\,\left(35\,c\,{\left(d+e\,x\right)}^2+63\,c\,d^2+45\,b\,e\,\left(d+e\,x\right)-90\,c\,d\,\left(d+e\,x\right)-63\,b\,d\,e\right)}{315\,e^3}$$

input
```
int((b*x + c*x^2)*(d + e*x)^(3/2),x)
```

output
```
(2*(d + e*x)^(5/2)*(35*c*(d + e*x)^2 + 63*c*d^2 + 45*b*e*(d + e*x) - 90*c*
d*(d + e*x) - 63*b*d*e))/(315*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.37

$$\int (d+ex)^{3/2} (bx+cx^2) \, dx = \frac{2\sqrt{ex+d}\,(35c\,e^4x^4 + 45b\,e^4x^3 + 50cd\,e^3x^3 + 72bd\,e^3x^2 + 3c\,d^2e^2x^2 + 9b\,d^2e^2x - 4c\,d^3ex - 18b}{315e^3}$$

input
```
int((e*x+d)^(3/2)*(c*x^2+b*x),x)
```

output
```
(2*sqrt(d + e*x)*( - 18*b*d**3*e + 9*b*d**2*e**2*x + 72*b*d*e**3*x**2 + 45
*b*e**4*x**3 + 8*c*d**4 - 4*c*d**3*e*x + 3*c*d**2*e**2*x**2 + 50*c*d*e**3*
x**3 + 35*c*e**4*x**4))/(315*e**3)
```

3.84 $\int \sqrt{d+ex}(bx+cx^2)\,dx$

Optimal result 638
Mathematica [A] (verified) 638
Rubi [A] (verified) 639
Maple [A] (verified) 640
Fricas [A] (verification not implemented) 640
Sympy [A] (verification not implemented) 641
Maxima [A] (verification not implemented) 641
Giac [B] (verification not implemented) 642
Mupad [B] (verification not implemented) 642
Reduce [B] (verification not implemented) 643

Optimal result

Integrand size = 19, antiderivative size = 68

$$\int \sqrt{d+ex}(bx+cx^2)\,dx = \frac{2d(cd-be)(d+ex)^{3/2}}{3e^3} - \frac{2(2cd-be)(d+ex)^{5/2}}{5e^3} + \frac{2c(d+ex)^{7/2}}{7e^3}$$

output
```
2/3*d*(-b*e+c*d)*(e*x+d)^(3/2)/e^3-2/5*(-b*e+2*c*d)*(e*x+d)^(5/2)/e^3+2/7*
c*(e*x+d)^(7/2)/e^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.74

$$\int \sqrt{d+ex}(bx+cx^2)\,dx = \frac{2(d+ex)^{3/2}\left(7be(-2d+3ex)+c(8d^2-12dex+15e^2x^2)\right)}{105e^3}$$

input
```
Integrate[Sqrt[d + e*x]*(b*x + c*x^2),x]
```

output
```
(2*(d + e*x)^(3/2)*(7*b*e*(-2*d + 3*e*x) + c*(8*d^2 - 12*d*e*x + 15*e^2*x^
2)))/(105*e^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx + cx^2) \sqrt{d+ex} \, dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(\frac{(d+ex)^{3/2}(be-2cd)}{e^2} + \frac{d\sqrt{d+ex}(cd-be)}{e^2} + \frac{c(d+ex)^{5/2}}{e^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2(d+ex)^{5/2}(2cd-be)}{5e^3} + \frac{2d(d+ex)^{3/2}(cd-be)}{3e^3} + \frac{2c(d+ex)^{7/2}}{7e^3}$$

input
```
Int[Sqrt[d + e*x]*(b*x + c*x^2),x]
```

output
```
(2*d*(c*d - b*e)*(d + e*x)^(3/2))/(3*e^3) - (2*(2*c*d - b*e)*(d + e*x)^(5/
2))/(5*e^3) + (2*c*(d + e*x)^(7/2))/(7*e^3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.60

method	result	size
pseudoelliptic	$-\frac{4(ex+d)^{\frac{3}{2}}\left(-\frac{3x\left(\frac{5cx}{7}+b\right)e^{2}}{2}+d\left(\frac{6cx}{7}+b\right)e-\frac{4c\,d^{2}}{7}\right)}{15e^{3}}$	41
gosper	$-\frac{2(ex+d)^{\frac{3}{2}}\left(-15x^{2}c\,e^{2}-21xb\,e^{2}+12cdxe+14bde-8c\,d^{2}\right)}{105e^{3}}$	47
derivativedivides	$\frac{\frac{2c(ex+d)^{\frac{7}{2}}}{7}+\frac{2(be-2cd)(ex+d)^{\frac{5}{2}}}{5}-\frac{2d(be-cd)(ex+d)^{\frac{3}{2}}}{3}}{e^{3}}$	52
default	$\frac{\frac{2c(ex+d)^{\frac{7}{2}}}{7}-\frac{2(-be+2cd)(ex+d)^{\frac{5}{2}}}{5}-\frac{2d(be-cd)(ex+d)^{\frac{3}{2}}}{3}}{e^{3}}$	53
orering	$-\frac{2\left(-15x^{2}c\,e^{2}-21xb\,e^{2}+12cdxe+14bde-8c\,d^{2}\right)(ex+d)^{\frac{3}{2}}\left(c\,x^{2}+bx\right)}{105e^{3}x(cx+b)}$	66
trager	$-\frac{2\left(-15c\,e^{3}x^{3}-21b\,e^{3}x^{2}-3cd\,e^{2}x^{2}-7bd\,e^{2}x+4c\,d^{2}ex+14b\,d^{2}e-8c\,d^{3}\right)\sqrt{ex+d}}{105e^{3}}$	71
risch	$-\frac{2\left(-15c\,e^{3}x^{3}-21b\,e^{3}x^{2}-3cd\,e^{2}x^{2}-7bd\,e^{2}x+4c\,d^{2}ex+14b\,d^{2}e-8c\,d^{3}\right)\sqrt{ex+d}}{105e^{3}}$	71

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-4/15*(e*x+d)^(3/2)*(-3/2*x*(5/7*c*x+b)*e^2+d*(6/7*c*x+b)*e-4/7*c*d^2)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \sqrt{d+ex}\left(bx+cx^2\right)\,dx = \frac{2\left(15\,ce^3x^3+8\,cd^3-14\,bd^2e+3\,(cde^2+7\,be^3)x^2-(4\,cd^2e-7\,bde^2)x\right)\sqrt{ex+d}}{105\,e^3}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x),x, algorithm="fricas")
```

output
```
2/105*(15*c*e^3*x^3 + 8*c*d^3 - 14*b*d^2*e + 3*(c*d*e^2 + 7*b*e^3)*x^2 - (
4*c*d^2*e - 7*b*d*e^2)*x)*sqrt(e*x + d)/e^3
```

Sympy [A] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.25

$$\int \sqrt{d+ex}\left(bx+cx^2\right)\,dx=\begin{cases}\frac{2\left(\frac{c(d+ex)^{\frac{7}{2}}}{7e^2}+\frac{(d+ex)^{\frac{5}{2}}(be-2cd)}{5e^2}+\frac{(d+ex)^{\frac{3}{2}}\left(-bde+cd^2\right)}{3e^2}\right)}{e} & \text{for } e\neq 0\\ \sqrt{d}\left(\frac{bx^2}{2}+\frac{cx^3}{3}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((e*x+d)**(1/2)*(c*x**2+b*x),x)
```

output
```
Piecewise((2*(c*(d + e*x)**(7/2)/(7*e**2) + (d + e*x)**(5/2)*(b*e - 2*c*d)
/(5*e**2) + (d + e*x)**(3/2)*(-b*d*e + c*d**2)/(3*e**2))/e, Ne(e, 0)), (sq
rt(d)*(b*x**2/2 + c*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.79

$$\int \sqrt{d+ex}\left(bx+cx^2\right)\,dx = \frac{2\left(15\,(ex+d)^{\frac{7}{2}}c-21\,(2\,cd-be)(ex+d)^{\frac{5}{2}}+35\,(cd^2-bde)(ex+d)^{\frac{3}{2}}\right)}{105\,e^3}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x),x, algorithm="maxima")
```

output
```
2/105*(15*(e*x + d)^(7/2)*c - 21*(2*c*d - b*e)*(e*x + d)^(5/2) + 35*(c*d^2
 - b*d*e)*(e*x + d)^(3/2))/e^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(56) = 112$.

Time = 0.11 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.32

$$\int \sqrt{d+ex}\left(bx+cx^2\right)\,dx$$
$$= \frac{2\left(\frac{35\left((ex+d)^{\frac{3}{2}}-3\sqrt{ex+d}d\right)bd}{e}+\frac{7\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}d^2\right)cd}{e^2}+\frac{7\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}d^2\right)b}{e}+\frac{3\left(\right.}{}\right.}{105\,e}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x),x, algorithm="giac")
```

output
```
2/105*(35*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*b*d/e + 7*(3*(e*x + d)^(5/
2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c*d/e^2 + 7*(3*(e*x + d)
^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*b/e + 3*(5*(e*x + d)
^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*
d^3)*c/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.76

$$\int \sqrt{d+ex}\left(bx+cx^2\right)\,dx$$
$$= \frac{2\,{\left(d+e\,x\right)}^{3/2}\,\left(15\,c\,{\left(d+e\,x\right)}^2+35\,c\,d^2+21\,b\,e\,\left(d+e\,x\right)-42\,c\,d\,\left(d+e\,x\right)-35\,b\,d\,e\right)}{105\,e^3}$$

input
```
int((b*x + c*x^2)*(d + e*x)^(1/2),x)
```

output
```
(2*(d + e*x)^(3/2)*(15*c*(d + e*x)^2 + 35*c*d^2 + 21*b*e*(d + e*x) - 42*c*
d*(d + e*x) - 35*b*d*e))/(105*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \sqrt{d+ex}\left(bx+cx^2\right)\,dx$$
$$=\frac{2\sqrt{ex+d}\,(15c\,e^3x^3+21b\,e^3x^2+3cd\,e^2x^2+7bd\,e^2x-4c\,d^2ex-14b\,d^2e+8c\,d^3)}{105e^3}$$

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x),x)
```

output
```
(2*sqrt(d + e*x)*( - 14*b*d**2*e + 7*b*d*e**2*x + 21*b*e**3*x**2 + 8*c*d**
3 - 4*c*d**2*e*x + 3*c*d*e**2*x**2 + 15*c*e**3*x**3))/(105*e**3)
```

3.85 $\int \frac{bx+cx^2}{\sqrt{d+ex}} \, dx$

Optimal result 644
Mathematica [A] (verified) 644
Rubi [A] (verified) 645
Maple [A] (verified) 646
Fricas [A] (verification not implemented) 646
Sympy [A] (verification not implemented) 647
Maxima [A] (verification not implemented) 647
Giac [A] (verification not implemented) 648
Mupad [B] (verification not implemented) 648
Reduce [B] (verification not implemented) 648

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int \frac{bx+cx^2}{\sqrt{d+ex}} \, dx = \frac{2d(cd-be)\sqrt{d+ex}}{e^3} - \frac{2(2cd-be)(d+ex)^{3/2}}{3e^3} + \frac{2c(d+ex)^{5/2}}{5e^3}$$

output
```
2*d*(-b*e+c*d)*(e*x+d)^(1/2)/e^3-2/3*(-b*e+2*c*d)*(e*x+d)^(3/2)/e^3+2/5*c*
(e*x+d)^(5/2)/e^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \frac{bx+cx^2}{\sqrt{d+ex}} \, dx = \frac{2\sqrt{d+ex}(5be(-2d+ex)+c(8d^2-4dex+3e^2x^2))}{15e^3}$$

input
```
Integrate[(b*x + c*x^2)/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(5*b*e*(-2*d + e*x) + c*(8*d^2 - 4*d*e*x + 3*e^2*x^2)))/(
15*e^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+cx^2}{\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{\sqrt{d+ex}(be-2cd)}{e^2}+\frac{d(cd-be)}{e^2\sqrt{d+ex}}+\frac{c(d+ex)^{3/2}}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(d+ex)^{3/2}(2cd-be)}{3e^3}+\frac{2d\sqrt{d+ex}(cd-be)}{e^3}+\frac{2c(d+ex)^{5/2}}{5e^3}$$

input
```
Int[(b*x + c*x^2)/Sqrt[d + e*x],x]
```

output
```
(2*d*(c*d - b*e)*Sqrt[d + e*x])/e^3 - (2*(2*c*d - b*e)*(d + e*x)^(3/2))/(3
*e^3) + (2*c*(d + e*x)^(5/2))/(5*e^3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.62

method	result	size
pseudoelliptic	$-\frac{4\left(-\frac{\left(\frac{3cx}{5}+b\right)x\,e^{2}}{2}+d\left(\frac{2cx}{5}+b\right)e-\frac{4c\,d^{2}}{5}\right)\sqrt{ex+d}}{3e^{3}}$	41
gosper	$-\frac{2\left(-3x^{2}c\,e^{2}-5xb\,e^{2}+4cdxe+10bde-8c\,d^{2}\right)\sqrt{ex+d}}{15e^{3}}$	47
trager	$-\frac{2\left(-3x^{2}c\,e^{2}-5xb\,e^{2}+4cdxe+10bde-8c\,d^{2}\right)\sqrt{ex+d}}{15e^{3}}$	47
risch	$-\frac{2\left(-3x^{2}c\,e^{2}-5xb\,e^{2}+4cdxe+10bde-8c\,d^{2}\right)\sqrt{ex+d}}{15e^{3}}$	47
derivativedivides	$\frac{\frac{2c(ex+d)^{\frac{5}{2}}}{5}+\frac{2(be-2cd)(ex+d)^{\frac{3}{2}}}{3}-2d(be-cd)\sqrt{ex+d}}{e^{3}}$	52
default	$\frac{\frac{2c(ex+d)^{\frac{5}{2}}}{5}+\frac{2(be-2cd)(ex+d)^{\frac{3}{2}}}{3}-2d(be-cd)\sqrt{ex+d}}{e^{3}}$	52
orering	$-\frac{2\left(-3x^{2}c\,e^{2}-5xb\,e^{2}+4cdxe+10bde-8c\,d^{2}\right)\sqrt{ex+d}\,\left(c\,x^{2}+bx\right)}{15e^{3}x(cx+b)}$	66

input
```
int((c*x^2+b*x)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(-1/2*(3/5*c*x+b)*x*e^2+d*(2/5*c*x+b)*e-4/5*c*d^2)*(e*x+d)^(1/2)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.73

$$\int \frac{bx+cx^2}{\sqrt{d+ex}}\,dx=\frac{2\left(3\,ce^2x^2+8\,cd^2-10\,bde-(4\,cde-5\,be^2)x\right)\sqrt{ex+d}}{15\,e^3}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*c*e^2*x^2 + 8*c*d^2 - 10*b*d*e - (4*c*d*e - 5*b*e^2)*x)*sqrt(e*x +
 d)/e^3
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.32

$$\int \frac{bx+cx^2}{\sqrt{d+ex}}\,dx = \begin{cases} \frac{\frac{2b\left(-d\sqrt{d+ex}+\frac{(d+ex)^{\frac{3}{2}}}{3}\right)}{e}+\frac{2c\left(d^2\sqrt{d+ex}-\frac{2d(d+ex)^{\frac{3}{2}}}{3}+\frac{(d+ex)^{\frac{5}{2}}}{5}\right)}{e^2}}{e} & \text{for } e\neq 0 \\ \frac{\frac{bx^2}{2}+\frac{cx^3}{3}}{\sqrt{d}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+b*x)/(e*x+d)**(1/2),x)
```

output
```
Piecewise(((2*b*(-d*sqrt(d + e*x) + (d + e*x)**(3/2)/3)/e + 2*c*(d**2*sqrt
(d + e*x) - 2*d*(d + e*x)**(3/2)/3 + (d + e*x)**(5/2)/5)/e**2)/e, Ne(e, 0)
), ((b*x**2/2 + c*x**3/3)/sqrt(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{bx+cx^2}{\sqrt{d+ex}}\,dx = \frac{2\left(\frac{5\left((ex+d)^{\frac{3}{2}}-3\sqrt{ex+d}d\right)b}{e}+\frac{\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}d^2\right)c}{e^2}\right)}{15e}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(5*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*b/e + (3*(e*x + d)^(5/2) - 1
0*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c/e^2)/e
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{bx+cx^2}{\sqrt{d+ex}}\,dx = \frac{2\left(\frac{5\left((ex+d)^{\frac{3}{2}}-3\sqrt{ex+d}d\right)b}{e}+\frac{\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}d^2\right)c}{e^2}\right)}{15e}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
2/15*(5*((e*x + d)^(3/2) - 3*sqrt(e*x + d)*d)*b/e + (3*(e*x + d)^(5/2) - 1
0*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2)*c/e^2)/e
```

Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int \frac{bx+cx^2}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+e\,x}\left(3\,c\,(d+e\,x)^2+15\,c\,d^2+5\,b\,e\,(d+e\,x)-10\,c\,d\,(d+e\,x)-15\,b\,d\,e\right)}{15\,e^3}$$

input
```
int((b*x + c*x^2)/(d + e*x)^(1/2),x)
```

output
```
(2*(d + e*x)^(1/2)*(3*c*(d + e*x)^2 + 15*c*d^2 + 5*b*e*(d + e*x) - 10*c*d*
(d + e*x) - 15*b*d*e))/(15*e^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.68

$$\int \frac{bx+cx^2}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{ex+d}\,(3c\,e^2x^2+5b\,e^2x-4cdex-10bde+8c\,d^2)}{15e^3}$$

input
```
int((c*x^2+b*x)/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(d + e*x)*( - 10*b*d*e + 5*b*e**2*x + 8*c*d**2 - 4*c*d*e*x + 3*c*e*
*2*x**2))/(15*e**3)
```

3.86 $\int \frac{bx+cx^2}{(d+ex)^{3/2}}\,dx$

Optimal result 650
Mathematica [A] (verified) 650
Rubi [A] (verified) 651
Maple [A] (verified) 652
Fricas [A] (verification not implemented) 652
Sympy [A] (verification not implemented) 653
Maxima [A] (verification not implemented) 653
Giac [A] (verification not implemented) 654
Mupad [B] (verification not implemented) 654
Reduce [B] (verification not implemented) 654

Optimal result

Integrand size = 19, antiderivative size = 64

$$\int \frac{bx+cx^2}{(d+ex)^{3/2}}\,dx=-\frac{2d(cd-be)}{e^3\sqrt{d+ex}}-\frac{2(2cd-be)\sqrt{d+ex}}{e^3}+\frac{2c(d+ex)^{3/2}}{3e^3}$$

output
```
-2*d*(-b*e+c*d)/e^3/(e*x+d)^(1/2)-2*(-b*e+2*c*d)*(e*x+d)^(1/2)/e^3+2/3*c*(
e*x+d)^(3/2)/e^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int \frac{bx+cx^2}{(d+ex)^{3/2}}\,dx=\frac{2(3be(2d+ex)+c(-8d^2-4dex+e^2x^2))}{3e^3\sqrt{d+ex}}$$

input
```
Integrate[(b*x + c*x^2)/(d + e*x)^(3/2),x]
```

output
```
(2*(3*b*e*(2*d + e*x) + c*(-8*d^2 - 4*d*e*x + e^2*x^2)))/(3*e^3*Sqrt[d + e
*x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+cx^2}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{be-2cd}{e^2\sqrt{d+ex}}+\frac{d(cd-be)}{e^2(d+ex)^{3/2}}+\frac{c\sqrt{d+ex}}{e^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\sqrt{d+ex}(2cd-be)}{e^3}-\frac{2d(cd-be)}{e^3\sqrt{d+ex}}+\frac{2c(d+ex)^{3/2}}{3e^3}$$

input
```
Int[(b*x + c*x^2)/(d + e*x)^(3/2),x]
```

output
```
(-2*d*(c*d - b*e))/(e^3*Sqrt[d + e*x]) - (2*(2*c*d - b*e)*Sqrt[d + e*x])/e
^3 + (2*c*(d + e*x)^(3/2))/(3*e^3)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{2\left(\frac{cx}{3}+b\right)x\,e^{2}+4d\left(-\frac{2cx}{3}+b\right)e-\frac{16c\,d^{2}}{3}}{\sqrt{ex+d}\,e^{3}}$	41
gosper	$\frac{\frac{2}{3}x^{2}c\,e^{2}+2xb\,e^{2}-\frac{8}{3}cdxe+4bde-\frac{16}{3}c\,d^{2}}{\sqrt{ex+d}\,e^{3}}$	46
trager	$\frac{\frac{2}{3}x^{2}c\,e^{2}+2xb\,e^{2}-\frac{8}{3}cdxe+4bde-\frac{16}{3}c\,d^{2}}{\sqrt{ex+d}\,e^{3}}$	46
risch	$\frac{2(cex+3be-5cd)\sqrt{ex+d}}{3e^{3}}+\frac{2d(be-cd)}{e^{3}\sqrt{ex+d}}$	48
derivativedivides	$\frac{\frac{2c(ex+d)^{\frac{3}{2}}}{3}+2be\sqrt{ex+d}-4cd\sqrt{ex+d}+\frac{2d(be-cd)}{\sqrt{ex+d}}}{e^{3}}$	55
default	$\frac{\frac{2c(ex+d)^{\frac{3}{2}}}{3}+2be\sqrt{ex+d}-4cd\sqrt{ex+d}+\frac{2d(be-cd)}{\sqrt{ex+d}}}{e^{3}}$	55
orering	$\frac{2\left(x^{2}c\,e^{2}+3xb\,e^{2}-4cdxe+6bde-8c\,d^{2}\right)\left(c\,x^{2}+bx\right)}{3e^{3}\sqrt{ex+d}\,x(cx+b)}$	65

input
```
int((c*x^2+b*x)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/(e*x+d)^(1/2)*(1/2*(1/3*c*x+b)*x*e^2+d*(-2/3*c*x+b)*e-4/3*c*d^2)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{bx+cx^2}{(d+ex)^{3/2}}\,dx=\frac{2\,(ce^2x^2-8\,cd^2+6\,bde-(4\,cde-3\,be^2)x)\sqrt{ex+d}}{3\,(e^4x+de^3)}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(c*e^2*x^2 - 8*c*d^2 + 6*b*d*e - (4*c*d*e - 3*b*e^2)*x)*sqrt(e*x + d)/
(e^4*x + d*e^3)
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.25

$$\int \frac{bx + cx^2}{(d + ex)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c(d+ex)^{\frac{3}{2}}}{3e^2} + \frac{d(be-cd)}{e^2\sqrt{d+ex}} + \frac{\sqrt{d+ex}(be-2cd)}{e^2}\right)}{e} & \text{for } e \neq 0 \\ \frac{\frac{bx^2}{2} + \frac{cx^3}{3}}{d^{\frac{3}{2}}} & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+b*x)/(e*x+d)**(3/2),x)
```

output
```
Piecewise((2*(c*(d + e*x)**(3/2)/(3*e**2) + d*(b*e - c*d)/(e**2*sqrt(d + e
*x)) + sqrt(d + e*x)*(b*e - 2*c*d)/e**2)/e, Ne(e, 0)), ((b*x**2/2 + c*x**3
/3)/d**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{bx + cx^2}{(d + ex)^{3/2}}\,dx = \frac{2\left(\frac{(ex+d)^{\frac{3}{2}}c - 3(2\,cd - be)\sqrt{ex+d}}{e^2} - \frac{3\,(cd^2 - bde)}{\sqrt{ex+d}e^2}\right)}{3\,e}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
2/3*(((e*x + d)^(3/2)*c - 3*(2*c*d - b*e)*sqrt(e*x + d))/e^2 - 3*(c*d^2 -
b*d*e)/(sqrt(e*x + d)*e^2))/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{bx+cx^2}{(d+ex)^{3/2}}\,dx = -\frac{2\,(cd^2-bde)}{\sqrt{ex+d}e^3} + \frac{2\left((ex+d)^{\frac{3}{2}}ce^6 - 6\sqrt{ex+d}cde^6 + 3\sqrt{ex+d}be^7\right)}{3\,e^9}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
-2*(c*d^2 - b*d*e)/(sqrt(e*x + d)*e^3) + 2/3*((e*x + d)^(3/2)*c*e^6 - 6*sq
rt(e*x + d)*c*d*e^6 + 3*sqrt(e*x + d)*b*e^7)/e^9
```

Mupad [B] (verification not implemented)

Time = 5.55 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{bx+cx^2}{(d+ex)^{3/2}}\,dx = \frac{2\,c\,(d+e\,x)^2-6\,c\,d^2+6\,b\,e\,(d+e\,x)-12\,c\,d\,(d+e\,x)+6\,b\,d\,e}{3\,e^3\,\sqrt{d+e\,x}}$$

input
```
int((b*x + c*x^2)/(d + e*x)^(3/2),x)
```

output
```
(2*c*(d + e*x)^2 - 6*c*d^2 + 6*b*e*(d + e*x) - 12*c*d*(d + e*x) + 6*b*d*e)
/(3*e^3*(d + e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.72

$$\int \frac{bx+cx^2}{(d+ex)^{3/2}}\,dx = \frac{\frac{2}{3}c\,e^2x^2+2b\,e^2x-\frac{8}{3}cdex+4bde-\frac{16}{3}c\,d^2}{\sqrt{ex+d}\,e^3}$$

input
```
int((c*x^2+b*x)/(e*x+d)^(3/2),x)
```

output
```
(2*(6*b*d*e + 3*b*e**2*x - 8*c*d**2 - 4*c*d*e*x + c*e**2*x**2))/(3*sqrt(d
+ e*x)*e**3)
```

3.87 $\int \frac{bx+cx^2}{(d+ex)^{5/2}}\,dx$

Optimal result 655
Mathematica [A] (verified) 655
Rubi [A] (verified) 656
Maple [A] (verified) 657
Fricas [A] (verification not implemented) 657
Sympy [B] (verification not implemented) 658
Maxima [A] (verification not implemented) 658
Giac [A] (verification not implemented) 659
Mupad [B] (verification not implemented) 659
Reduce [B] (verification not implemented) 659

Optimal result

Integrand size = 19, antiderivative size = 64

$$\int \frac{bx+cx^2}{(d+ex)^{5/2}}\,dx = -\frac{2d(cd-be)}{3e^3(d+ex)^{3/2}}+\frac{2(2cd-be)}{e^3\sqrt{d+ex}}+\frac{2c\sqrt{d+ex}}{e^3}$$

output
```
-2/3*d*(-b*e+c*d)/e^3/(e*x+d)^(3/2)+2*(-b*e+2*c*d)/e^3/(e*x+d)^(1/2)+2*c*(
e*x+d)^(1/2)/e^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.78

$$\int \frac{bx+cx^2}{(d+ex)^{5/2}}\,dx = \frac{2(-be(2d+3ex)+c(8d^2+12dex+3e^2x^2))}{3e^3(d+ex)^{3/2}}$$

input
```
Integrate[(b*x + c*x^2)/(d + e*x)^(5/2),x]
```

output
```
(2*(-(b*e*(2*d + 3*e*x)) + c*(8*d^2 + 12*d*e*x + 3*e^2*x^2)))/(3*e^3*(d +
e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+cx^2}{(d+ex)^{5/2}}\,dx$$
$$\Big\downarrow \text{1140}$$
$$\int \left(\frac{be-2cd}{e^2(d+ex)^{3/2}}+\frac{d(cd-be)}{e^2(d+ex)^{5/2}}+\frac{c}{e^2\sqrt{d+ex}}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2(2cd-be)}{e^3\sqrt{d+ex}}-\frac{2d(cd-be)}{3e^3(d+ex)^{3/2}}+\frac{2c\sqrt{d+ex}}{e^3}$$

input
```
Int[(b*x + c*x^2)/(d + e*x)^(5/2),x]
```

output
```
(-2*d*(c*d - b*e))/(3*e^3*(d + e*x)^(3/2)) + (2*(2*c*d - b*e))/(e^3*Sqrt[d
 + e*x]) + (2*c*Sqrt[d + e*x])/e^3
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$-\frac{4\left(\frac{3x(-cx+b)e^{2}}{2}+d(-6cx+b)e-4c\,d^{2}\right)}{3(ex+d)^{\frac{3}{2}}e^{3}}$	41
gosper	$-\frac{2\left(-3x^{2}c\,e^{2}+3xb\,e^{2}-12cdxe+2bde-8c\,d^{2}\right)}{3(ex+d)^{\frac{3}{2}}e^{3}}$	47
trager	$-\frac{2\left(-3x^{2}c\,e^{2}+3xb\,e^{2}-12cdxe+2bde-8c\,d^{2}\right)}{3(ex+d)^{\frac{3}{2}}e^{3}}$	47
derivativedivides	$\frac{2c\sqrt{ex+d}-\frac{2(be-2cd)}{\sqrt{ex+d}}+\frac{2d(be-cd)}{3(ex+d)^{\frac{3}{2}}}}{e^{3}}$	51
default	$\frac{2c\sqrt{ex+d}-\frac{2(be-2cd)}{\sqrt{ex+d}}+\frac{2d(be-cd)}{3(ex+d)^{\frac{3}{2}}}}{e^{3}}$	51
risch	$\frac{2c\sqrt{ex+d}}{e^{3}}-\frac{2\left(3xb\,e^{2}-6cdxe+2bde-5c\,d^{2}\right)}{3e^{3}(ex+d)^{\frac{3}{2}}}$	52
orering	$-\frac{2\left(-3x^{2}c\,e^{2}+3xb\,e^{2}-12cdxe+2bde-8c\,d^{2}\right)\left(c\,x^{2}+bx\right)}{3e^{3}(ex+d)^{\frac{3}{2}}x(cx+b)}$	66

input
```
int((c*x^2+b*x)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3/(e*x+d)^(3/2)*(3/2*x*(-c*x+b)*e^2+d*(-6*c*x+b)*e-4*c*d^2)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{bx+cx^2}{(d+ex)^{5/2}}\,dx=\frac{2\,(3\,ce^2x^2+8\,cd^2-2\,bde+3\,(4\,cde-be^2)x)\sqrt{ex+d}}{3\,(e^5x^2+2\,de^4x+d^2e^3)}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*c*e^2*x^2 + 8*c*d^2 - 2*b*d*e + 3*(4*c*d*e - b*e^2)*x)*sqrt(e*x + d
)/(e^5*x^2 + 2*d*e^4*x + d^2*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(60) = 120$.

Time = 0.30 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.30

$$\int \frac{bx + cx^2}{(d+ex)^{5/2}}\,dx = \begin{cases} -\frac{4bde}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} - \frac{6be^2x}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} + \frac{16cd^2}{3de^3\sqrt{d+ex}+3e^4x\sqrt{d+ex}} + \frac{24cdex}{3de^3\sqrt{d+ex}+3e^4x\sqrt{}} \\ \frac{\frac{bx^2}{2}+\frac{cx^3}{3}}{d^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((c*x**2+b*x)/(e*x+d)**(5/2),x)
```

output
```
Piecewise((-4*b*d*e/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) - 6*
b*e**2*x/(3*d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 16*c*d**2/(3*
d*e**3*sqrt(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 24*c*d*e*x/(3*d*e**3*sqrt
(d + e*x) + 3*e**4*x*sqrt(d + e*x)) + 6*c*e**2*x**2/(3*d*e**3*sqrt(d + e*x
) + 3*e**4*x*sqrt(d + e*x)), Ne(e, 0)), ((b*x**2/2 + c*x**3/3)/d**(5/2), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{bx + cx^2}{(d+ex)^{5/2}}\,dx = \frac{2\left(\frac{3\sqrt{ex+d}c}{e^2} - \frac{cd^2-bde-3(2cd-be)(ex+d)}{(ex+d)^{\frac{3}{2}}e^2}\right)}{3e}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(3*sqrt(e*x + d)*c/e^2 - (c*d^2 - b*d*e - 3*(2*c*d - b*e)*(e*x + d))/(
(e*x + d)^(3/2)*e^2))/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.86

$$\int \frac{bx + cx^2}{(d+ex)^{5/2}}\,dx = \frac{2\sqrt{ex+d}c}{e^3} + \frac{2\left(6\,(ex+d)cd - cd^2 - 3\,(ex+d)be + bde\right)}{3\,(ex+d)^{\frac{3}{2}}e^3}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
2*sqrt(e*x + d)*c/e^3 + 2/3*(6*(e*x + d)*c*d - c*d^2 - 3*(e*x + d)*b*e + b
*d*e)/((e*x + d)^(3/2)*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.81

$$\int \frac{bx + cx^2}{(d+ex)^{5/2}}\,dx = \frac{6\,c\,(d+e\,x)^2 - 2\,c\,d^2 - 6\,b\,e\,(d+e\,x) + 12\,c\,d\,(d+e\,x) + 2\,b\,d\,e}{3\,e^3\,(d+e\,x)^{3/2}}$$

input
```
int((b*x + c*x^2)/(d + e*x)^(5/2),x)
```

output
```
(6*c*(d + e*x)^2 - 2*c*d^2 - 6*b*e*(d + e*x) + 12*c*d*(d + e*x) + 2*b*d*e)
/(3*e^3*(d + e*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{bx + cx^2}{(d+ex)^{5/2}}\,dx = \frac{2c\,e^2x^2 - 2b\,e^2x + 8cdex - \frac{4}{3}bde + \frac{16}{3}c\,d^2}{\sqrt{ex+d}\,e^3\,(ex+d)}$$

input
```
int((c*x^2+b*x)/(e*x+d)^(5/2),x)
```

output
```
(2*( - 2*b*d*e - 3*b*e**2*x + 8*c*d**2 + 12*c*d*e*x + 3*c*e**2*x**2))/(3*s
qrt(d + e*x)*e**3*(d + e*x))
```

3.88 $\int \frac{bx+cx^2}{(d+ex)^{7/2}} dx$

Optimal result 660
Mathematica [A] (verified) 660
Rubi [A] (verified) 661
Maple [A] (verified) 662
Fricas [A] (verification not implemented) 662
Sympy [B] (verification not implemented) 663
Maxima [A] (verification not implemented) 663
Giac [A] (verification not implemented) 664
Mupad [B] (verification not implemented) 664
Reduce [B] (verification not implemented) 664

Optimal result

Integrand size = 19, antiderivative size = 66

$$\int \frac{bx+cx^2}{(d+ex)^{7/2}} dx = -\frac{2d(cd-be)}{5e^3(d+ex)^{5/2}} + \frac{2(2cd-be)}{3e^3(d+ex)^{3/2}} - \frac{2c}{e^3\sqrt{d+ex}}$$

output
```
-2/5*d*(-b*e+c*d)/e^3/(e*x+d)^(5/2)+2/3*(-b*e+2*c*d)/e^3/(e*x+d)^(3/2)-2*c
/e^3/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \frac{bx+cx^2}{(d+ex)^{7/2}} dx = -\frac{2(be(2d+5ex)+c(8d^2+20dex+15e^2x^2))}{15e^3(d+ex)^{5/2}}$$

input
```
Integrate[(b*x + c*x^2)/(d + e*x)^(7/2),x]
```

output
```
(-2*(b*e*(2*d + 5*e*x) + c*(8*d^2 + 20*d*e*x + 15*e^2*x^2)))/(15*e^3*(d +
e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{bx+cx^2}{(d+ex)^{7/2}}\,dx$$

$$\Big\downarrow \ 1140$$

$$\int \left(\frac{be-2cd}{e^2(d+ex)^{5/2}}+\frac{d(cd-be)}{e^2(d+ex)^{7/2}}+\frac{c}{e^2(d+ex)^{3/2}}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{2(2cd-be)}{3e^3(d+ex)^{3/2}}-\frac{2d(cd-be)}{5e^3(d+ex)^{5/2}}-\frac{2c}{e^3\sqrt{d+ex}}$$

input
```
Int[(b*x + c*x^2)/(d + e*x)^(7/2),x]
```

output
```
(-2*d*(c*d - b*e))/(5*e^3*(d + e*x)^(5/2)) + (2*(2*c*d - b*e))/(3*e^3*(d +
 e*x)^(3/2)) - (2*c)/(e^3*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{-\frac{2x(3cx+b)e^{2}}{3}-\frac{4d(10cx+b)e}{15}-\frac{16c\,d^{2}}{15}}{(ex+d)^{\frac{5}{2}}e^{3}}$	42
gosper	$-\frac{2(15x^{2}c\,e^{2}+5xb\,e^{2}+20cdxe+2bde+8c\,d^{2})}{15(ex+d)^{\frac{5}{2}}e^{3}}$	47
trager	$-\frac{2(15x^{2}c\,e^{2}+5xb\,e^{2}+20cdxe+2bde+8c\,d^{2})}{15(ex+d)^{\frac{5}{2}}e^{3}}$	47
derivativedivides	$\frac{\frac{2d(be-cd)}{5(ex+d)^{\frac{5}{2}}}-\frac{2c}{\sqrt{ex+d}}-\frac{2(be-2cd)}{3(ex+d)^{\frac{3}{2}}}}{e^{3}}$	52
default	$\frac{\frac{2d(be-cd)}{5(ex+d)^{\frac{5}{2}}}-\frac{2c}{\sqrt{ex+d}}-\frac{2(be-2cd)}{3(ex+d)^{\frac{3}{2}}}}{e^{3}}$	52
orering	$-\frac{2(15x^{2}c\,e^{2}+5xb\,e^{2}+20cdxe+2bde+8c\,d^{2})(c\,x^{2}+bx)}{15e^{3}(ex+d)^{\frac{5}{2}}x(cx+b)}$	66

input
```
int((c*x^2+b*x)/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(-5*x*(3*c*x+b)*e^2-2*d*(10*c*x+b)*e-8*c*d^2)/(e*x+d)^(5/2)/e^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

$$\int \frac{bx+cx^2}{(d+ex)^{7/2}}\,dx = -\frac{2\,(15\,ce^2x^2+8\,cd^2+2\,bde+5\,(4\,cde+be^2)x)\sqrt{ex+d}}{15\,(e^6x^3+3\,de^5x^2+3\,d^2e^4x+d^3e^3)}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(15*c*e^2*x^2 + 8*c*d^2 + 2*b*d*e + 5*(4*c*d*e + b*e^2)*x)*sqrt(e*x
+ d)/(e^6*x^3 + 3*d*e^5*x^2 + 3*d^2*e^4*x + d^3*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(63) = 126$.

Time = 0.44 (sec) , antiderivative size = 314, normalized size of antiderivative = 4.76

$$\int \frac{bx + cx^2}{(d+ex)^{7/2}}\,dx = \begin{cases} -\frac{4bde}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}} - \frac{10be^2x}{15d^2e^3\sqrt{d+ex}+30de^4x\sqrt{d+ex}+15e^5x^2\sqrt{d+ex}} - \frac{}{15d^2e^3\sqrt{}} \\ \frac{\frac{bx^2}{2}+\frac{cx^3}{3}}{d^{\frac{7}{2}}} \end{cases}$$

input
```
integrate((c*x**2+b*x)/(e*x+d)**(7/2),x)
```

output
```
Piecewise((-4*b*d*e/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x
) + 15*e**5*x**2*sqrt(d + e*x)) - 10*b*e**2*x/(15*d**2*e**3*sqrt(d + e*x)
+ 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)) - 16*c*d**2/(15*
d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d
+ e*x)) - 40*c*d*e*x/(15*d**2*e**3*sqrt(d + e*x) + 30*d*e**4*x*sqrt(d + e*
x) + 15*e**5*x**2*sqrt(d + e*x)) - 30*c*e**2*x**2/(15*d**2*e**3*sqrt(d + e
*x) + 30*d*e**4*x*sqrt(d + e*x) + 15*e**5*x**2*sqrt(d + e*x)), Ne(e, 0)),
((b*x**2/2 + c*x**3/3)/d**(7/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.76

$$\int \frac{bx + cx^2}{(d+ex)^{7/2}}\,dx = -\frac{2\left(15\,(ex+d)^2c + 3\,cd^2 - 3\,bde - 5\,(2\,cd - be)(ex+d)\right)}{15\,(ex+d)^{\frac{5}{2}}e^3}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
-2/15*(15*(e*x + d)^2*c + 3*c*d^2 - 3*b*d*e - 5*(2*c*d - b*e)*(e*x + d))/(
(e*x + d)^(5/2)*e^3)
```

Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int \frac{bx + cx^2}{(d + ex)^{7/2}}\,dx = -\frac{2\left(15\,(ex + d)^2c - 10\,(ex + d)cd + 3\,cd^2 + 5\,(ex + d)be - 3\,bde\right)}{15\,(ex + d)^{\frac{5}{2}}e^3}$$

input
```
integrate((c*x^2+b*x)/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
-2/15*(15*(e*x + d)^2*c - 10*(e*x + d)*c*d + 3*c*d^2 + 5*(e*x + d)*b*e - 3
*b*d*e)/((e*x + d)^(5/2)*e^3)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \frac{bx + cx^2}{(d + ex)^{7/2}}\,dx = -\frac{\left(\frac{2\,b\,e}{3} - \frac{4\,c\,d}{3}\right)\,(d + e\,x) + 2\,c\,(d + e\,x)^2 + \frac{2\,c\,d^2}{5} - \frac{2\,b\,d\,e}{5}}{e^3\,(d + e\,x)^{5/2}}$$

input
```
int((b*x + c*x^2)/(d + e*x)^(7/2),x)
```

output
```
-(((2*b*e)/3 - (4*c*d)/3)*(d + e*x) + 2*c*(d + e*x)^2 + (2*c*d^2)/5 - (2*b
*d*e)/5)/(e^3*(d + e*x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int \frac{bx + cx^2}{(d + ex)^{7/2}}\,dx = \frac{-2c\,e^2x^2 - \frac{2}{3}b\,e^2x - \frac{8}{3}cdex - \frac{4}{15}bde - \frac{16}{15}c\,d^2}{\sqrt{ex + d}\,e^3\,(e^2x^2 + 2dex + d^2)}$$

input
```
int((c*x^2+b*x)/(e*x+d)^(7/2),x)
```

output
```
(2*( - 2*b*d*e - 5*b*e**2*x - 8*c*d**2 - 20*c*d*e*x - 15*c*e**2*x**2))/(15
*sqrt(d + e*x)*e**3*(d**2 + 2*d*e*x + e**2*x**2))
```

3.89 $\int (d+ex)^{7/2} \left(bx+cx^2\right)^2 \, dx$

Optimal result 665
Mathematica [A] (verified) 665
Rubi [A] (verified) 666
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Reduce [B] (verification not implemented) 671

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int (d+ex)^{7/2} \left(bx+cx^2\right)^2 \, dx = \frac{2d^2(cd-be)^2(d+ex)^{9/2}}{9e^5} - \frac{4d(cd-be)(2cd-be)(d+ex)^{11/2}}{11e^5} + \frac{2\left(6c^2d^2-6bcde+b^2e^2\right)(d+ex)^{13/2}}{13e^5} - \frac{4c(2cd-be)(d+ex)^{15/2}}{15e^5} + \frac{2c^2(d+ex)^{17/2}}{17e^5}$$

output
```
2/9*d^2*(-b*e+c*d)^2*(e*x+d)^(9/2)/e^5-4/11*d*(-b*e+c*d)*(-b*e+2*c*d)*(e*x
+d)^(11/2)/e^5+2/13*(b^2*e^2-6*b*c*d*e+6*c^2*d^2)*(e*x+d)^(13/2)/e^5-4/15*
c*(-b*e+2*c*d)*(e*x+d)^(15/2)/e^5+2/17*c^2*(e*x+d)^(17/2)/e^5
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int (d+ex)^{7/2} (bx+cx^2)^2 \, dx = \frac{2(d+ex)^{9/2} \left(85b^2e^2(8d^2-36dex+99e^2x^2)+34bce(-16d^3+72d^2ex-198de^2x^2+429e^3x^3)\right.}{109395e^5}$$

input
```
Integrate[(d + e*x)^(7/2)*(b*x + c*x^2)^2,x]
```

output
```
(2*(d + e*x)^(9/2)*(85*b^2*e^2*(8*d^2 - 36*d*e*x + 99*e^2*x^2) + 34*b*c*e*
(-16*d^3 + 72*d^2*e*x - 198*d*e^2*x^2 + 429*e^3*x^3) + c^2*(128*d^4 - 576*
d^3*e*x + 1584*d^2*e^2*x^2 - 3432*d*e^3*x^3 + 6435*e^4*x^4)))/(109395*e^5)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^2 (d+ex)^{7/2} \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{(d+ex)^{11/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{e^4}+\frac{d^2(d+ex)^{7/2}(cd-be)^2}{e^4}-\frac{2c(d+ex)^{13/2}(2cd-be)}{e^4}+\frac{2d(d+ex)^{9/2}}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(d+ex)^{13/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{13e^5}+\frac{2d^2(d+ex)^{9/2}(cd-be)^2}{9e^5}-\frac{4c(d+ex)^{15/2}(2cd-be)}{15e^5}-\frac{4d(d+ex)^{11/2}(cd-be)(2cd-be)}{11e^5}+\frac{2c^2(d+ex)^{17/2}}{17e^5}$$

input
```
Int[(d + e*x)^(7/2)*(b*x + c*x^2)^2,x]
```

output
```
(2*d^2*(c*d - b*e)^2*(d + e*x)^(9/2))/(9*e^5) - (4*d*(c*d - b*e)*(2*c*d -
b*e)*(d + e*x)^(11/2))/(11*e^5) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d
+ e*x)^(13/2))/(13*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(15/2))/(15*e^5) +
(2*c^2*(d + e*x)^(17/2))/(17*e^5)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{16(ex+d)^{\frac{9}{2}}\left(\frac{99x^2\left(\frac{13}{17}c^2x^2+\frac{26}{15}cbx+b^2\right)e^4}{8}-\frac{9x\left(\frac{286}{255}c^2x^2+\frac{11}{5}cbx+b^2\right)d\,e^3}{2}+d^2\left(\frac{198}{85}c^2x^2+\frac{18}{5}cbx+b^2\right)e^2-\frac{4c\,d^3\left(\frac{18cx}{17}+b\right)e}{5}}{1287e^5}$
gosper	$\frac{2(ex+d)^{\frac{9}{2}}\left(6435c^2x^4e^4+14586x^3bc\,e^4-3432d\,c^2x^3e^3+8415x^2b^2e^4-6732x^2bcd\,e^3+1584x^2c^2d^2e^2-3060x\,b^2d\,e^3+244\right.}{109395e^5}$
derivativedivides	$\frac{\frac{2c^2(ex+d)^{\frac{17}{2}}}{17}+\frac{2\left(-2c^2d+2c(be-cd)\right)(ex+d)^{\frac{15}{2}}}{15}+\frac{2\left(c^2d^2-4dc(be-cd)+(be-cd)^2\right)(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left(2d^2c(be-cd)-2d(be-cd)^2\right)(ex}{11}}{e^5}$
default	$\frac{\frac{2c^2(ex+d)^{\frac{17}{2}}}{17}+\frac{2\left(-2c^2d+2c(be-cd)\right)(ex+d)^{\frac{15}{2}}}{15}+\frac{2\left(c^2d^2-4dc(be-cd)+(be-cd)^2\right)(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left(2d^2c(be-cd)-2d(be-cd)^2\right)(ex}{11}}{e^5}$
orering	$\frac{2\left(6435c^2x^4e^4+14586x^3bc\,e^4-3432d\,c^2x^3e^3+8415x^2b^2e^4-6732x^2bcd\,e^3+1584x^2c^2d^2e^2-3060x\,b^2d\,e^3+2448xbc\,d^2e^2\right.}{109395e^5(cx+b)^2x^2}$
trager	$2\left(6435c^2e^8x^8+14586bc\,e^8x^7+22308c^2d\,e^7x^7+8415b^2e^8x^6+51612bcd\,e^7x^6+26466c^2d^2e^6x^6+30600b^2d\,e^7x^5+63036bc\right.$
risch	$2\left(6435c^2e^8x^8+14586bc\,e^8x^7+22308c^2d\,e^7x^7+8415b^2e^8x^6+51612bcd\,e^7x^6+26466c^2d^2e^6x^6+30600b^2d\,e^7x^5+63036bc\right.$

input
```
int((e*x+d)^(7/2)*(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
16/1287*(e*x+d)^(9/2)*(99/8*x^2*(13/17*c^2*x^2+26/15*c*b*x+b^2)*e^4-9/2*x*
(286/255*c^2*x^2+11/5*c*b*x+b^2)*d*e^3+d^2*(198/85*c^2*x^2+18/5*c*b*x+b^2)
*e^2-4/5*c*d^3*(18/17*c*x+b)*e+16/85*c^2*d^4)/e^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 290 vs. 2(127) = 254.

Time = 0.09 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.97

$$\int (d+ex)^{7/2} (bx + cx^2)^2 \, dx = \frac{2 (6435 c^2e^8x^8 + 128 c^2d^8 - 544 bcd^7e + 680 b^2d^6e^2 + 858 (26 c^2de^7 + 17 bce^8)x^7 + 33 (802 c^2d^2}{}$$

input
```
integrate((e*x+d)^(7/2)*(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
2/109395*(6435*c^2*e^8*x^8 + 128*c^2*d^8 - 544*b*c*d^7*e + 680*b^2*d^6*e^2
 + 858*(26*c^2*d*e^7 + 17*b*c*e^8)*x^7 + 33*(802*c^2*d^2*e^6 + 1564*b*c*d*
e^7 + 255*b^2*e^8)*x^6 + 36*(303*c^2*d^3*e^5 + 1751*b*c*d^2*e^6 + 850*b^2*
d*e^7)*x^5 + 5*(7*c^2*d^4*e^4 + 5440*b*c*d^3*e^5 + 7786*b^2*d^2*e^6)*x^4 -
 10*(4*c^2*d^5*e^3 - 17*b*c*d^4*e^4 - 1802*b^2*d^3*e^5)*x^3 + 3*(16*c^2*d^
6*e^2 - 68*b*c*d^5*e^3 + 85*b^2*d^4*e^4)*x^2 - 4*(16*c^2*d^7*e - 68*b*c*d^
6*e^2 + 85*b^2*d^5*e^3)*x)*sqrt(e*x + d)/e^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. 2(144) = 288.

Time = 0.63 (sec) , antiderivative size = 590, normalized size of antiderivative = 4.01

$$\int (d+ex)^{7/2} (bx + cx^2)^2 \, dx = \begin{cases} \frac{16b^2d^6\sqrt{d+ex}}{1287e^3} - \frac{8b^2d^5x\sqrt{d+ex}}{1287e^2} + \frac{2b^2d^4x^2\sqrt{d+ex}}{429e} + \frac{424b^2d^3x^3\sqrt{d+ex}}{1287} + \frac{916b^2d^2ex^4\sqrt{d+ex}}{1287} + \frac{80b^2de^2x^5\sqrt{d+ex}}{143} + \\ d^{\frac{7}{2}} \left(\frac{b^2x^3}{3} + \frac{bcx^4}{2} + \frac{c^2x^5}{5} \right) \end{cases}$$

input
```
integrate((e*x+d)**(7/2)*(c*x**2+b*x)**2,x)
```

output
```
Piecewise((16*b**2*d**6*sqrt(d + e*x)/(1287*e**3) - 8*b**2*d**5*x*sqrt(d +
 e*x)/(1287*e**2) + 2*b**2*d**4*x**2*sqrt(d + e*x)/(429*e) + 424*b**2*d**3
*x**3*sqrt(d + e*x)/1287 + 916*b**2*d**2*e*x**4*sqrt(d + e*x)/1287 + 80*b*
*2*d*e**2*x**5*sqrt(d + e*x)/143 + 2*b**2*e**3*x**6*sqrt(d + e*x)/13 - 64*
b*c*d**7*sqrt(d + e*x)/(6435*e**4) + 32*b*c*d**6*x*sqrt(d + e*x)/(6435*e**
3) - 8*b*c*d**5*x**2*sqrt(d + e*x)/(2145*e**2) + 4*b*c*d**4*x**3*sqrt(d +
e*x)/(1287*e) + 640*b*c*d**3*x**4*sqrt(d + e*x)/1287 + 824*b*c*d**2*e*x**5
*sqrt(d + e*x)/715 + 184*b*c*d*e**2*x**6*sqrt(d + e*x)/195 + 4*b*c*e**3*x*
*7*sqrt(d + e*x)/15 + 256*c**2*d**8*sqrt(d + e*x)/(109395*e**5) - 128*c**2
*d**7*x*sqrt(d + e*x)/(109395*e**4) + 32*c**2*d**6*x**2*sqrt(d + e*x)/(364
65*e**3) - 16*c**2*d**5*x**3*sqrt(d + e*x)/(21879*e**2) + 14*c**2*d**4*x**
4*sqrt(d + e*x)/(21879*e) + 2424*c**2*d**3*x**5*sqrt(d + e*x)/12155 + 1604
*c**2*d**2*e*x**6*sqrt(d + e*x)/3315 + 104*c**2*d*e**2*x**7*sqrt(d + e*x)/
255 + 2*c**2*e**3*x**8*sqrt(d + e*x)/17, Ne(e, 0)), (d**(7/2)*(b**2*x**3/3
 + b*c*x**4/2 + c**2*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

$$\int (d+ex)^{7/2} (bx+cx^2)^2\, dx = \frac{2\left(6435\,(ex+d)^{\frac{17}{2}}c^2 - 14586\,(2\,c^2d-bce)(ex+d)^{\frac{15}{2}} + 8415\,(6\,c^2d^2-6\,bcde+b^2e^2)(ex+d)\right.}{10939}$$

input
```
integrate((e*x+d)^(7/2)*(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
2/109395*(6435*(e*x + d)^(17/2)*c^2 - 14586*(2*c^2*d - b*c*e)*(e*x + d)^(1
5/2) + 8415*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(e*x + d)^(13/2) - 19890*(2*
c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*(e*x + d)^(11/2) + 12155*(c^2*d^4 - 2*b
*c*d^3*e + b^2*d^2*e^2)*(e*x + d)^(9/2))/e^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1171 vs. $2(127) = 254$.

Time = 0.12 (sec) , antiderivative size = 1171, normalized size of antiderivative = 7.97

$$\int (d+ex)^{7/2} \left(bx+cx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(7/2)*(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
2/765765*(51051*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x +
d)*d^2)*b^2*d^4/e^2 + 43758*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35
*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b*c*d^4/e^3 + 87516*(5*(e*x +
 d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x +
d)*d^3)*b^2*d^3/e^2 + 2431*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 3
78*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*
c^2*d^4/e^4 + 19448*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x
 + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b*c*d^3
/e^3 + 14586*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(
5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b^2*d^2/e^2 +
4420*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^
2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d
)*d^5)*c^2*d^3/e^4 + 13260*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d +
990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*
d^4 - 693*sqrt(e*x + d)*d^5)*b*c*d^2/e^3 + 4420*(63*(e*x + d)^(11/2) - 385
*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 +
1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b^2*d/e^2 + 1530*(231*(e
*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580
*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5
 + 3003*sqrt(e*x + d)*d^6)*c^2*d^2/e^4 + 2040*(231*(e*x + d)^(13/2) - 1...
```

Mupad [B] (verification not implemented)

Time = 5.82 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

$$\int (d+ex)^{7/2} \left(bx+cx^2\right)^2 \, dx = \frac{2\,c^2\,{\left(d+e\,x\right)}^{17/2}}{17\,e^5} - \frac{{\left(d+e\,x\right)}^{11/2}\,\left(4\,b^2\,d\,e^2-12\,b\,c\,d^2\,e+8\,c^2\,d^3\right)}{11\,e^5} + \frac{{\left(d+e\,x\right)}^{13/2}\,\left(2\,b^2\,e^2-12\,b\,c\,d\,e+12\,c^2\,d^2\right)}{13\,e^5} - \frac{\left(8\,c^2\,d-4\,b\,c\,e\right)\,{\left(d+e\,x\right)}^{15/2}}{15\,e^5} + \frac{2\,d^2\,{\left(b\,e-c\,d\right)}^2\,{\left(d+e\,x\right)}^{9/2}}{9\,e^5}$$

input
```
int((b*x + c*x^2)^2*(d + e*x)^(7/2),x)
```

output
```
(2*c^2*(d + e*x)^(17/2))/(17*e^5) - ((d + e*x)^(11/2)*(8*c^2*d^3 + 4*b^2*d
*e^2 - 12*b*c*d^2*e))/(11*e^5) + ((d + e*x)^(13/2)*(2*b^2*e^2 + 12*c^2*d^2
 - 12*b*c*d*e))/(13*e^5) - ((8*c^2*d - 4*b*c*e)*(d + e*x)^(15/2))/(15*e^5)
 + (2*d^2*(b*e - c*d)^2*(d + e*x)^(9/2))/(9*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.06

$$\int (d+ex)^{7/2} \left(bx+cx^2\right)^2 \, dx = \frac{2\sqrt{ex+d}\,(6435c^2e^8x^8 + 14586bc\,e^8x^7 + 22308c^2d\,e^7x^7 + 8415b^2e^8x^6 + 51612bcd\,e^7x^6 + 2646}{}$$

input
```
int((e*x+d)^(7/2)*(c*x^2+b*x)^2,x)
```

output

```
(2*sqrt(d + e*x)*(680*b**2*d**6*e**2 - 340*b**2*d**5*e**3*x + 255*b**2*d**
4*e**4*x**2 + 18020*b**2*d**3*e**5*x**3 + 38930*b**2*d**2*e**6*x**4 + 3060
0*b**2*d*e**7*x**5 + 8415*b**2*e**8*x**6 - 544*b*c*d**7*e + 272*b*c*d**6*e
**2*x - 204*b*c*d**5*e**3*x**2 + 170*b*c*d**4*e**4*x**3 + 27200*b*c*d**3*e
**5*x**4 + 63036*b*c*d**2*e**6*x**5 + 51612*b*c*d*e**7*x**6 + 14586*b*c*e*
*8*x**7 + 128*c**2*d**8 - 64*c**2*d**7*e*x + 48*c**2*d**6*e**2*x**2 - 40*c
**2*d**5*e**3*x**3 + 35*c**2*d**4*e**4*x**4 + 10908*c**2*d**3*e**5*x**5 +
26466*c**2*d**2*e**6*x**6 + 22308*c**2*d*e**7*x**7 + 6435*c**2*e**8*x**8))
/(109395*e**5)
```

3.90 $\int (d+ex)^{5/2} \left(bx+cx^2\right)^2 \, dx$

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Optimal result

Integrand size = 21, antiderivative size = 147

$$\int (d+ex)^{5/2} \left(bx+cx^2\right)^2 \, dx = \frac{2d^2(cd-be)^2(d+ex)^{7/2}}{7e^5} - \frac{4d(cd-be)(2cd-be)(d+ex)^{9/2}}{9e^5} + \frac{2\left(6c^2d^2-6bcde+b^2e^2\right)(d+ex)^{11/2}}{11e^5} - \frac{4c(2cd-be)(d+ex)^{13/2}}{13e^5} + \frac{2c^2(d+ex)^{15/2}}{15e^5}$$

output
```
2/7*d^2*(-b*e+c*d)^2*(e*x+d)^(7/2)/e^5-4/9*d*(-b*e+c*d)*(-b*e+2*c*d)*(e*x+
d)^(9/2)/e^5+2/11*(b^2*e^2-6*b*c*d*e+6*c^2*d^2)*(e*x+d)^(11/2)/e^5-4/13*c*
(-b*e+2*c*d)*(e*x+d)^(13/2)/e^5+2/15*c^2*(e*x+d)^(15/2)/e^5
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int (d+ex)^{5/2} \left(bx+cx^2\right)^2 \, dx = \frac{2(d+ex)^{7/2} \left(65b^2e^2(8d^2-28dex+63e^2x^2)+30bce(-16d^3+56d^2ex-126de^2x^2+231e^3x^3)\right.}{45045e^5}$$

input
```
Integrate[(d + e*x)^(5/2)*(b*x + c*x^2)^2,x]
```

output
```
(2*(d + e*x)^(7/2)*(65*b^2*e^2*(8*d^2 - 28*d*e*x + 63*e^2*x^2) + 30*b*c*e*
(-16*d^3 + 56*d^2*e*x - 126*d*e^2*x^2 + 231*e^3*x^3) + c^2*(128*d^4 - 448*
d^3*e*x + 1008*d^2*e^2*x^2 - 1848*d*e^3*x^3 + 3003*e^4*x^4)))/(45045*e^5)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^2 (d+ex)^{5/2} \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{(d+ex)^{9/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{e^4}+\frac{d^2(d+ex)^{5/2}(cd-be)^2}{e^4}-\frac{2c(d+ex)^{11/2}(2cd-be)}{e^4}+\frac{2d(d+ex)^{7/2}(\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(d+ex)^{11/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{11e^5}+\frac{2d^2(d+ex)^{7/2}(cd-be)^2}{7e^5}-\frac{4c(d+ex)^{13/2}(2cd-be)}{13e^5}-\frac{4d(d+ex)^{9/2}(cd-be)(2cd-be)}{9e^5}+\frac{2c^2(d+ex)^{15/2}}{15e^5}$$

input
```
Int[(d + e*x)^(5/2)*(b*x + c*x^2)^2,x]
```

output
```
(2*d^2*(c*d - b*e)^2*(d + e*x)^(7/2))/(7*e^5) - (4*d*(c*d - b*e)*(2*c*d -
b*e)*(d + e*x)^(9/2))/(9*e^5) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d +
e*x)^(11/2))/(11*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(13/2))/(13*e^5) + (2
*c^2*(d + e*x)^(15/2))/(15*e^5)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{16(ex+d)^{\frac{7}{2}}\left(\frac{63x^2\left(\frac{11}{15}c^2x^2+\frac{22}{13}cbx+b^2\right)e^4}{8}-\frac{7x\left(\frac{66}{65}c^2x^2+\frac{27}{13}cbx+b^2\right)d\,e^3}{2}+d^2\left(\frac{126}{65}c^2x^2+\frac{42}{13}cbx+b^2\right)e^2-\frac{12\left(\frac{14cx}{15}+b\right)c\,d^3e}{13}\right.}{693e^5}$
gosper	$\frac{2(ex+d)^{\frac{7}{2}}\left(3003c^2x^4e^4+6930x^3bc\,e^4-1848d\,c^2x^3e^3+4095x^2b^2e^4-3780x^2bcd\,e^3+1008x^2c^2d^2e^2-1820x\,b^2d\,e^3+1680\right.}{45045e^5}$
derivativedivides	$\frac{\frac{2c^2(ex+d)^{\frac{15}{2}}}{15}+\frac{2\left(-2c^2d+2c(be-cd)\right)(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left(c^2d^2-4dc(be-cd)+(be-cd)^2\right)(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(2d^2c(be-cd)-2d(be-cd)^2\right)(ex}{9}}{e^5}$
default	$\frac{\frac{2c^2(ex+d)^{\frac{15}{2}}}{15}+\frac{2\left(-2c^2d+2c(be-cd)\right)(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left(c^2d^2-4dc(be-cd)+(be-cd)^2\right)(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(2d^2c(be-cd)-2d(be-cd)^2\right)(ex}{9}}{e^5}$
orering	$\frac{2\left(3003c^2x^4e^4+6930x^3bc\,e^4-1848d\,c^2x^3e^3+4095x^2b^2e^4-3780x^2bcd\,e^3+1008x^2c^2d^2e^2-1820x\,b^2d\,e^3+1680xbc\,d^2e^2-\right.}{45045e^5(cx+b)^2x^2}$
trager	$\frac{2\left(3003c^2e^7x^7+6930bc\,e^7x^6+7161c^2d\,e^6x^6+4095b^2e^7x^5+17010bcd\,e^6x^5+4473c^2d^2e^5x^5+10465b^2d\,e^6x^4+11130bc\,d^2\right.}{}$
risch	$\frac{2\left(3003c^2e^7x^7+6930bc\,e^7x^6+7161c^2d\,e^6x^6+4095b^2e^7x^5+17010bcd\,e^6x^5+4473c^2d^2e^5x^5+10465b^2d\,e^6x^4+11130bc\,d^2\right.}{}$

input
```
int((e*x+d)^(5/2)*(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
16/693*(e*x+d)^(7/2)*(63/8*x^2*(11/15*c^2*x^2+22/13*c*b*x+b^2)*e^4-7/2*x*(
66/65*c^2*x^2+27/13*c*b*x+b^2)*d*e^3+d^2*(126/65*c^2*x^2+42/13*c*b*x+b^2)*
e^2-12/13*(14/15*c*x+b)*c*d^3*e+16/65*c^2*d^4)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.71

$$\int (d+ex)^{5/2} (bx+cx^2)^2 \, dx = \frac{2 (3003 c^2e^7x^7 + 128 c^2d^7 - 480 bcd^6e + 520 b^2d^5e^2 + 231 (31 c^2de^6 + 30 bce^7)x^6 + 63 (71 c^2d^2e}{}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
2/45045*(3003*c^2*e^7*x^7 + 128*c^2*d^7 - 480*b*c*d^6*e + 520*b^2*d^5*e^2
+ 231*(31*c^2*d*e^6 + 30*b*c*e^7)*x^6 + 63*(71*c^2*d^2*e^5 + 270*b*c*d*e^6
 + 65*b^2*e^7)*x^5 + 35*(c^2*d^3*e^4 + 318*b*c*d^2*e^5 + 299*b^2*d*e^6)*x^
4 - 5*(8*c^2*d^4*e^3 - 30*b*c*d^3*e^4 - 1469*b^2*d^2*e^5)*x^3 + 3*(16*c^2*
d^5*e^2 - 60*b*c*d^4*e^3 + 65*b^2*d^3*e^4)*x^2 - 4*(16*c^2*d^6*e - 60*b*c*
d^5*e^2 + 65*b^2*d^4*e^3)*x)*sqrt(e*x + d)/e^5
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.39

$$\int (d+ex)^{5/2} (bx+cx^2)^2 \, dx = \begin{cases} \frac{2 \left(\frac{c^2(d+ex)^{\frac{15}{2}}}{15e^4} + \frac{(d+ex)^{\frac{13}{2}} \cdot (2bce-4c^2d)}{13e^4} + \frac{(d+ex)^{\frac{11}{2}} (b^2e^2-6bcde+6c^2d^2)}{11e^4} + \frac{(d+ex)^{\frac{9}{2}} (-2b^2de^2+6bcd^2e-4c^2d^3)}{9e^4} + \frac{(d+ex)^{\frac{7}{2}} (b^2d^2e^2-}{7e^4} \right.}{e} \\ d^{\frac{5}{2}} \left(\frac{b^2x^3}{3} + \frac{bcx^4}{2} + \frac{c^2x^5}{5} \right) \end{cases}$$

input
```
integrate((e*x+d)**(5/2)*(c*x**2+b*x)**2,x)
```

output
```
Piecewise((2*(c**2*(d + e*x)**(15/2)/(15*e**4) + (d + e*x)**(13/2)*(2*b*c*
e - 4*c**2*d)/(13*e**4) + (d + e*x)**(11/2)*(b**2*e**2 - 6*b*c*d*e + 6*c**
2*d**2)/(11*e**4) + (d + e*x)**(9/2)*(-2*b**2*d*e**2 + 6*b*c*d**2*e - 4*c*
*2*d**3)/(9*e**4) + (d + e*x)**(7/2)*(b**2*d**2*e**2 - 2*b*c*d**3*e + c**2
*d**4)/(7*e**4))/e, Ne(e, 0)), (d**(5/2)*(b**2*x**3/3 + b*c*x**4/2 + c**2*
x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

$$\int (d+ex)^{5/2} \left(bx + cx^2\right)^2 dx = \frac{2\left(3003\,(ex+d)^{\frac{15}{2}}c^2 - 6930\,(2\,c^2d - bce)(ex+d)^{\frac{13}{2}} + 4095\,(6\,c^2d^2 - 6\,bcde + b^2e^2)(ex+d)^{\frac{11}{2}}\right.}{45045}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
2/45045*(3003*(e*x + d)^(15/2)*c^2 - 6930*(2*c^2*d - b*c*e)*(e*x + d)^(13/
2) + 4095*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(e*x + d)^(11/2) - 10010*(2*c^
2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*(e*x + d)^(9/2) + 6435*(c^2*d^4 - 2*b*c*d
^3*e + b^2*d^2*e^2)*(e*x + d)^(7/2))/e^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 863 vs. 2(127) = 254.

Time = 0.12 (sec) , antiderivative size = 863, normalized size of antiderivative = 5.87

$$\int (d+ex)^{5/2} \left(bx + cx^2\right)^2 dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
2/45045*(3003*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)
*d^2)*b^2*d^3/e^2 + 2574*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e
*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b*c*d^3/e^3 + 3861*(5*(e*x + d)^
(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d
^3)*b^2*d^2/e^2 + 143*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e
*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*c^2*d
^3/e^4 + 858*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(
5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b*c*d^2/e^3 +
429*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2
- 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b^2*d/e^2 + 195*(63*(e*
x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*
x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*c^2*d
^2/e^4 + 390*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^
(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt
(e*x + d)*d^5)*b*c*d/e^3 + 65*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d
 + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/
2)*d^4 - 693*sqrt(e*x + d)*d^5)*b^2/e^2 + 45*(231*(e*x + d)^(13/2) - 1638*
(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 +
 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*
d^6)*c^2*d/e^4 + 30*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5...
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

$$\int (d+ex)^{5/2}\left(bx+cx^2\right)^2\,dx=\frac{2\,c^2\,{\left(d+e\,x\right)}^{15/2}}{15\,e^5}-\frac{{\left(d+e\,x\right)}^{9/2}\,\left(4\,b^2\,d\,e^2-12\,b\,c\,d^2\,e+8\,c^2\,d^3\right)}{9\,e^5}+\frac{{\left(d+e\,x\right)}^{11/2}\,\left(2\,b^2\,e^2-12\,b\,c\,d\,e+12\,c^2\,d^2\right)}{11\,e^5}-\frac{\left(8\,c^2\,d-4\,b\,c\,e\right)\,{\left(d+e\,x\right)}^{13/2}}{13\,e^5}+\frac{2\,d^2\,{\left(b\,e-c\,d\right)}^2\,{\left(d+e\,x\right)}^{7/2}}{7\,e^5}$$

input
```
int((b*x + c*x^2)^2*(d + e*x)^(5/2),x)
```

output
```
(2*c^2*(d + e*x)^(15/2))/(15*e^5) - ((d + e*x)^(9/2)*(8*c^2*d^3 + 4*b^2*d*
e^2 - 12*b*c*d^2*e))/(9*e^5) + ((d + e*x)^(11/2)*(2*b^2*e^2 + 12*c^2*d^2 -
 12*b*c*d*e))/(11*e^5) - ((8*c^2*d - 4*b*c*e)*(d + e*x)^(13/2))/(13*e^5) +
 (2*d^2*(b*e - c*d)^2*(d + e*x)^(7/2))/(7*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.78

$$\int (d+ex)^{5/2} \left(bx +cx^2\right)^2 \,dx = \frac{2\sqrt{ex+d}\,(3003c^2e^7x^7 + 6930bc\,e^7x^6 + 7161c^2d\,e^6x^6 + 4095b^2e^7x^5 + 17010bcd\,e^6x^5 + 4473c^2d$$

input
```
int((e*x+d)^(5/2)*(c*x^2+b*x)^2,x)
```

output
```
(2*sqrt(d + e*x)*(520*b**2*d**5*e**2 - 260*b**2*d**4*e**3*x + 195*b**2*d**
3*e**4*x**2 + 7345*b**2*d**2*e**5*x**3 + 10465*b**2*d*e**6*x**4 + 4095*b**
2*e**7*x**5 - 480*b*c*d**6*e + 240*b*c*d**5*e**2*x - 180*b*c*d**4*e**3*x**
2 + 150*b*c*d**3*e**4*x**3 + 11130*b*c*d**2*e**5*x**4 + 17010*b*c*d*e**6*x
**5 + 6930*b*c*e**7*x**6 + 128*c**2*d**7 - 64*c**2*d**6*e*x + 48*c**2*d**5
*e**2*x**2 - 40*c**2*d**4*e**3*x**3 + 35*c**2*d**3*e**4*x**4 + 4473*c**2*d
**2*e**5*x**5 + 7161*c**2*d*e**6*x**6 + 3003*c**2*e**7*x**7))/(45045*e**5)
```

3.91 $\int (d+ex)^{3/2} (bx+cx^2)^2 \, dx$

Optimal result 680
Mathematica [A] (verified) 680
Rubi [A] (verified) 681
Maple [A] (verified) 682
Fricas [A] (verification not implemented) 683
Sympy [A] (verification not implemented) 683
Maxima [A] (verification not implemented) 684
Giac [B] (verification not implemented) 684
Mupad [B] (verification not implemented) 685
Reduce [B] (verification not implemented) 686

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int (d+ex)^{3/2} (bx+cx^2)^2 \, dx = \frac{2d^2(cd-be)^2(d+ex)^{5/2}}{5e^5} - \frac{4d(cd-be)(2cd-be)(d+ex)^{7/2}}{7e^5} + \frac{2(6c^2d^2-6bcde+b^2e^2)(d+ex)^{9/2}}{9e^5} - \frac{4c(2cd-be)(d+ex)^{11/2}}{11e^5} + \frac{2c^2(d+ex)^{13/2}}{13e^5}$$

output
```
2/5*d^2*(-b*e+c*d)^2*(e*x+d)^(5/2)/e^5-4/7*d*(-b*e+c*d)*(-b*e+2*c*d)*(e*x+
d)^(7/2)/e^5+2/9*(b^2*e^2-6*b*c*d*e+6*c^2*d^2)*(e*x+d)^(9/2)/e^5-4/11*c*(-
b*e+2*c*d)*(e*x+d)^(11/2)/e^5+2/13*c^2*(e*x+d)^(13/2)/e^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.85

$$\int (d+ex)^{3/2} (bx+cx^2)^2 \, dx = \frac{2(d+ex)^{5/2} (143b^2e^2(8d^2-20dex+35e^2x^2)+78bce(-16d^3+40d^2ex-70de^2x^2+105e^3x^3)}{45045e^5}$$

input
```
Integrate[(d + e*x)^(3/2)*(b*x + c*x^2)^2,x]
```

output
```
(2*(d + e*x)^(5/2)*(143*b^2*e^2*(8*d^2 - 20*d*e*x + 35*e^2*x^2) + 78*b*c*e
*(-16*d^3 + 40*d^2*e*x - 70*d*e^2*x^2 + 105*e^3*x^3) + 3*c^2*(128*d^4 - 32
0*d^3*e*x + 560*d^2*e^2*x^2 - 840*d*e^3*x^3 + 1155*e^4*x^4)))/(45045*e^5)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^2 (d+ex)^{3/2}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{(d+ex)^{7/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{e^4}+\frac{d^2(d+ex)^{3/2}(cd-be)^2}{e^4}-\frac{2c(d+ex)^{9/2}(2cd-be)}{e^4}+\frac{2d(d+ex)^{5/2}(c$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(d+ex)^{9/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{9e^5}+\frac{2d^2(d+ex)^{5/2}(cd-be)^2}{5e^5}-\frac{4c(d+ex)^{11/2}(2cd-be)}{11e^5}-\frac{4d(d+ex)^{7/2}(cd-be)(2cd-be)}{7e^5}+\frac{2c^2(d+ex)^{13/2}}{13e^5}$$

input
```
Int[(d + e*x)^(3/2)*(b*x + c*x^2)^2,x]
```

output
```
(2*d^2*(c*d - b*e)^2*(d + e*x)^(5/2))/(5*e^5) - (4*d*(c*d - b*e)*(2*c*d -
b*e)*(d + e*x)^(7/2))/(7*e^5) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d +
e*x)^(9/2))/(9*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(11/2))/(11*e^5) + (2*c
^2*(d + e*x)^(13/2))/(13*e^5)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{16(ex+d)^{\frac{5}{2}}\left(\frac{35x^{2}\left(\frac{9}{13}c^{2}x^{2}+\frac{18}{11}cbx+b^{2}\right)e^{4}}{8}-\frac{5\left(\frac{126}{143}c^{2}x^{2}+\frac{21}{11}cbx+b^{2}\right)xd\,e^{3}}{2}+d^{2}\left(\frac{210}{143}c^{2}x^{2}+\frac{30}{11}cbx+b^{2}\right)e^{2}-\frac{12\left(\frac{10cx}{13}+b\right)c\,d^{3}e}{11}\right.}{315e^{5}}$
gosper	$\frac{2(ex+d)^{\frac{5}{2}}\left(3465c^{2}x^{4}e^{4}+8190x^{3}bc\,e^{4}-2520d\,c^{2}x^{3}e^{3}+5005x^{2}b^{2}e^{4}-5460x^{2}bcd\,e^{3}+1680x^{2}c^{2}d^{2}e^{2}-2860x\,b^{2}d\,e^{3}+3120\right.}{45045e^{5}}$
derivativedivides	$\frac{\frac{2c^{2}(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left(-2c^{2}d+2c(be-cd)\right)(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(c^{2}d^{2}-4dc(be-cd)+(be-cd)^{2}\right)(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(2d^{2}c(be-cd)-2d(be-cd)^{2}\right)(ex+}{7}}{e^{5}}$
default	$\frac{\frac{2c^{2}(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left(-2c^{2}d+2c(be-cd)\right)(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(c^{2}d^{2}-4dc(be-cd)+(be-cd)^{2}\right)(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(2d^{2}c(be-cd)-2d(be-cd)^{2}\right)(ex+}{7}}{e^{5}}$
orering	$\frac{2\left(3465c^{2}x^{4}e^{4}+8190x^{3}bc\,e^{4}-2520d\,c^{2}x^{3}e^{3}+5005x^{2}b^{2}e^{4}-5460x^{2}bcd\,e^{3}+1680x^{2}c^{2}d^{2}e^{2}-2860x\,b^{2}d\,e^{3}+3120xbc\,d^{2}e^{2}-\right.}{45045e^{5}(cx+b)^{2}x^{2}}$
trager	$2\left(3465e^{6}c^{2}x^{6}+8190bc\,e^{6}x^{5}+4410c^{2}d\,e^{5}x^{5}+5005b^{2}e^{6}x^{4}+10920bcd\,e^{5}x^{4}+105c^{2}d^{2}e^{4}x^{4}+7150b^{2}d\,e^{5}x^{3}+390bc\,d^{2}e^{4}x^{3}\right.$
risch	$2\left(3465e^{6}c^{2}x^{6}+8190bc\,e^{6}x^{5}+4410c^{2}d\,e^{5}x^{5}+5005b^{2}e^{6}x^{4}+10920bcd\,e^{5}x^{4}+105c^{2}d^{2}e^{4}x^{4}+7150b^{2}d\,e^{5}x^{3}+390bc\,d^{2}e^{4}x^{3}\right.$

input
```
int((e*x+d)^(3/2)*(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
16/315*(e*x+d)^(5/2)*(35/8*x^2*(9/13*c^2*x^2+18/11*c*b*x+b^2)*e^4-5/2*(126
/143*c^2*x^2+21/11*c*b*x+b^2)*x*d*e^3+d^2*(210/143*c^2*x^2+30/11*c*b*x+b^2
)*e^2-12/11*(10/13*c*x+b)*c*d^3*e+48/143*c^2*d^4)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.46

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^2 \, dx = \frac{2\,(3465\,c^2e^6x^6 + 384\,c^2d^6 - 1248\,bcd^5e + 1144\,b^2d^4e^2 + 630\,(7\,c^2de^5 + 13\,bce^6)x^5 + 35\,(3\,c^2d^2e \ldots}{}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
2/45045*(3465*c^2*e^6*x^6 + 384*c^2*d^6 - 1248*b*c*d^5*e + 1144*b^2*d^4*e^
2 + 630*(7*c^2*d*e^5 + 13*b*c*e^6)*x^5 + 35*(3*c^2*d^2*e^4 + 312*b*c*d*e^5
 + 143*b^2*e^6)*x^4 - 10*(12*c^2*d^3*e^3 - 39*b*c*d^2*e^4 - 715*b^2*d*e^5)
*x^3 + 3*(48*c^2*d^4*e^2 - 156*b*c*d^3*e^3 + 143*b^2*d^2*e^4)*x^2 - 4*(48*
c^2*d^5*e - 156*b*c*d^4*e^2 + 143*b^2*d^3*e^3)*x)*sqrt(e*x + d)/e^5
```

Sympy [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.39

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^2 \, dx = \begin{cases} \frac{2\left(\frac{c^2(d+ex)^{\frac{13}{2}}}{13e^4} + \frac{(d+ex)^{\frac{11}{2}}\cdot\left(2bce-4c^2d\right)}{11e^4} + \frac{(d+ex)^{\frac{9}{2}}\left(b^2e^2-6bcde+6c^2d^2\right)}{9e^4} + \frac{(d+ex)^{\frac{7}{2}}\left(-2b^2de^2+6bcd^2e-4c^2d^3\right)}{7e^4} + \frac{(d+ex)^{\frac{5}{2}}\left(b^2d^2e^2 - \ldots\right.}{5e^4} \ldots}{e} \\ d^{\frac{3}{2}}\left(\frac{b^2x^3}{3} + \frac{bcx^4}{2} + \frac{c^2x^5}{5}\right) \end{cases}$$

input
```
integrate((e*x+d)**(3/2)*(c*x**2+b*x)**2,x)
```

output
```
Piecewise((2*(c**2*(d + e*x)**(13/2)/(13*e**4) + (d + e*x)**(11/2)*(2*b*c*
e - 4*c**2*d)/(11*e**4) + (d + e*x)**(9/2)*(b**2*e**2 - 6*b*c*d*e + 6*c**2
*d**2)/(9*e**4) + (d + e*x)**(7/2)*(-2*b**2*d*e**2 + 6*b*c*d**2*e - 4*c**2
*d**3)/(7*e**4) + (d + e*x)**(5/2)*(b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d
**4)/(5*e**4))/e, Ne(e, 0)), (d**(3/2)*(b**2*x**3/3 + b*c*x**4/2 + c**2*x*
*5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^2 \, dx = \frac{2 \left(3465 \, (ex+d)^{\frac{13}{2}} c^2 - 8190 \, (2 \, c^2 d - bce)(ex+d)^{\frac{11}{2}} + 5005 \, (6 \, c^2 d^2 - 6 \, bcde + b^2 e^2)(ex+d)^{\frac{9}{2}}}{45045}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
2/45045*(3465*(e*x + d)^(13/2)*c^2 - 8190*(2*c^2*d - b*c*e)*(e*x + d)^(11/
2) + 5005*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(e*x + d)^(9/2) - 12870*(2*c^2
*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*(e*x + d)^(7/2) + 9009*(c^2*d^4 - 2*b*c*d^
3*e + b^2*d^2*e^2)*(e*x + d)^(5/2))/e^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 591 vs. $2(127) = 254$.

Time = 0.12 (sec) , antiderivative size = 591, normalized size of antiderivative = 4.02

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
2/45045*(3003*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)
*d^2)*b^2*d^2/e^2 + 2574*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e
*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b*c*d^2/e^3 + 2574*(5*(e*x + d)^
(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d
^3)*b^2*d/e^2 + 143*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x
 + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*c^2*d^2
/e^4 + 572*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/
2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b*c*d/e^3 + 143*
(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 42
0*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b^2/e^2 + 130*(63*(e*x + d)
^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)
^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*c^2*d/e^4 +
 130*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^
2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d
)*d^5)*b*c/e^3 + 15*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005
*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4
 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*c^2/e^4)/e
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^2 \, dx = \frac{2\,c^2\,{\left(d+e\,x\right)}^{13/2}}{13\,e^5} - \frac{{\left(d+e\,x\right)}^{7/2}\,\left(4\,b^2\,d\,e^2-12\,b\,c\,d^2\,e+8\,c^2\,d^3\right)}{7\,e^5} + \frac{{\left(d+e\,x\right)}^{9/2}\,\left(2\,b^2\,e^2-12\,b\,c\,d\,e+12\,c^2\,d^2\right)}{9\,e^5} - \frac{\left(8\,c^2\,d-4\,b\,c\,e\right)\,{\left(d+e\,x\right)}^{11/2}}{11\,e^5} + \frac{2\,d^2\,{\left(b\,e-c\,d\right)}^2\,{\left(d+e\,x\right)}^{5/2}}{5\,e^5}$$

input
```
int((b*x + c*x^2)^2*(d + e*x)^(3/2),x)
```

output
```
(2*c^2*(d + e*x)^(13/2))/(13*e^5) - ((d + e*x)^(7/2)*(8*c^2*d^3 + 4*b^2*d*
e^2 - 12*b*c*d^2*e))/(7*e^5) + ((d + e*x)^(9/2)*(2*b^2*e^2 + 12*c^2*d^2 -
12*b*c*d*e))/(9*e^5) - ((8*c^2*d - 4*b*c*e)*(d + e*x)^(11/2))/(11*e^5) + (
2*d^2*(b*e - c*d)^2*(d + e*x)^(5/2))/(5*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.50

$$\int (d+ex)^{3/2} (bx+cx^2)^2 \, dx = \frac{2\sqrt{ex+d}\,(3465c^2e^6x^6 + 8190bc\,e^6x^5 + 4410c^2d\,e^5x^5 + 5005b^2e^6x^4 + 10920bcd\,e^5x^4 + 105c^2d^2}{}$$

input
```
int((e*x+d)^(3/2)*(c*x^2+b*x)^2,x)
```

output
```
(2*sqrt(d + e*x)*(1144*b**2*d**4*e**2 - 572*b**2*d**3*e**3*x + 429*b**2*d*
*2*e**4*x**2 + 7150*b**2*d*e**5*x**3 + 5005*b**2*e**6*x**4 - 1248*b*c*d**5
*e + 624*b*c*d**4*e**2*x - 468*b*c*d**3*e**3*x**2 + 390*b*c*d**2*e**4*x**3
 + 10920*b*c*d*e**5*x**4 + 8190*b*c*e**6*x**5 + 384*c**2*d**6 - 192*c**2*d
**5*e*x + 144*c**2*d**4*e**2*x**2 - 120*c**2*d**3*e**3*x**3 + 105*c**2*d**
2*e**4*x**4 + 4410*c**2*d*e**5*x**5 + 3465*c**2*e**6*x**6))/(45045*e**5)
```

3.92 $\int \sqrt{d+ex}(bx+cx^2)^2\,dx$

Optimal result 687
Mathematica [A] (verified) 688
Rubi [A] (verified) 688
Maple [A] (verified) 689
Fricas [A] (verification not implemented) 690
Sympy [A] (verification not implemented) 690
Maxima [A] (verification not implemented) 691
Giac [B] (verification not implemented) 691
Mupad [B] (verification not implemented) 692
Reduce [B] (verification not implemented) 693

Optimal result

Integrand size = 21, antiderivative size = 147

$$\int \sqrt{d+ex}(bx+cx^2)^2\,dx = \frac{2d^2(cd-be)^2(d+ex)^{3/2}}{3e^5} - \frac{4d(cd-be)(2cd-be)(d+ex)^{5/2}}{5e^5} + \frac{2(6c^2d^2-6bcde+b^2e^2)(d+ex)^{7/2}}{7e^5} - \frac{4c(2cd-be)(d+ex)^{9/2}}{9e^5} + \frac{2c^2(d+ex)^{11/2}}{11e^5}$$

output
```
2/3*d^2*(-b*e+c*d)^2*(e*x+d)^(3/2)/e^5-4/5*d*(-b*e+c*d)*(-b*e+2*c*d)*(e*x+
d)^(5/2)/e^5+2/7*(b^2*e^2-6*b*c*d*e+6*c^2*d^2)*(e*x+d)^(7/2)/e^5-4/9*c*(-b
*e+2*c*d)*(e*x+d)^(9/2)/e^5+2/11*c^2*(e*x+d)^(11/2)/e^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.84

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^2\,dx$$
$$=\frac{2(d+ex)^{3/2}\left(33b^2e^2(8d^2-12dex+15e^2x^2)+22bce(-16d^3+24d^2ex-30de^2x^2+35e^3x^3)+c^2(128d^4-\ldots\right.}{3465e^5}$$

input
```
Integrate[Sqrt[d + e*x]*(b*x + c*x^2)^2,x]
```

output
```
(2*(d + e*x)^(3/2)*(33*b^2*e^2*(8*d^2 - 12*d*e*x + 15*e^2*x^2) + 22*b*c*e*
(-16*d^3 + 24*d^2*e*x - 30*d*e^2*x^2 + 35*e^3*x^3) + c^2*(128*d^4 - 192*d^
3*e*x + 240*d^2*e^2*x^2 - 280*d*e^3*x^3 + 315*e^4*x^4)))/(3465*e^5)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^2\sqrt{d+ex}\,dx$$
$$\Big\downarrow \text{ 1140}$$
$$\int \left(\frac{(d+ex)^{5/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{e^4}+\frac{d^2\sqrt{d+ex}(cd-be)^2}{e^4}-\frac{2c(d+ex)^{7/2}(2cd-be)}{e^4}+\frac{2d(d+ex)^{3/2}(cd-\ldots}{e^4}\right.$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{2(d+ex)^{7/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{7e^5}+\frac{2d^2(d+ex)^{3/2}(cd-be)^2}{3e^5}-\frac{4c(d+ex)^{9/2}(2cd-be)}{9e^5}-\frac{4d(d+ex)^{5/2}(cd-be)(2cd-be)}{5e^5}+\frac{2c^2(d+ex)^{11/2}}{11e^5}$$

input
```
Int[Sqrt[d + e*x]*(b*x + c*x^2)^2,x]
```

output
```
(2*d^2*(c*d - b*e)^2*(d + e*x)^(3/2))/(3*e^5) - (4*d*(c*d - b*e)*(2*c*d -
b*e)*(d + e*x)^(5/2))/(5*e^5) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d +
e*x)^(7/2))/(7*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(9/2))/(9*e^5) + (2*c^2
*(d + e*x)^(11/2))/(11*e^5)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{16\left(\frac{15\left(\frac{7}{11}c^2x^2+\frac{14}{9}cbx+b^2\right)x^2e^4}{8}-\frac{3xd\left(\frac{70}{99}c^2x^2+\frac{5}{3}cbx+b^2\right)e^3}{2}+d^2\left(\frac{10}{11}c^2x^2+2cbx+b^2\right)e^2-\frac{4\left(\frac{6cx}{11}+b\right)c\,d^3e}{3}+\frac{16c^2d^4}{33}\right)(ex+}{105e^5}$
gosper	$\frac{2(ex+d)^{\frac{3}{2}}(315c^2x^4e^4+770x^3bc\,e^4-280d\,c^2x^3e^3+495x^2b^2e^4-660x^2bcd\,e^3+240x^2c^2d^2e^2-396x\,b^2d\,e^3+528xbc\,d^2e^2-}{3465e^5}$
derivativedivides	$\frac{\frac{2c^2(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(-2c^2d+2c(be-cd)\right)(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(c^2d^2-4dc(be-cd)+(be-cd)^2\right)(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(2d^2c(be-cd)-2d(be-cd)^2\right)(ex+c}{5}}{e^5}$
default	$\frac{\frac{2c^2(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(-2c^2d+2c(be-cd)\right)(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(c^2d^2-4dc(be-cd)+(be-cd)^2\right)(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(2d^2c(be-cd)-2d(be-cd)^2\right)(ex+c}{5}}{e^5}$
orering	$\frac{2(315c^2x^4e^4+770x^3bc\,e^4-280d\,c^2x^3e^3+495x^2b^2e^4-660x^2bcd\,e^3+240x^2c^2d^2e^2-396x\,b^2d\,e^3+528xbc\,d^2e^2-192x\,c^2d}{3465e^5(cx+b)^2x^2}$
trager	$\frac{2(315c^2e^5x^5+770bc\,e^5x^4+35c^2d\,e^4x^4+495b^2e^5x^3+110bcd\,e^4x^3-40c^2d^2e^3x^3+99b^2d\,e^4x^2-132bc\,d^2e^3x^2+48c^2d^3e^2x}{3465e^5}$
risch	$\frac{2(315c^2e^5x^5+770bc\,e^5x^4+35c^2d\,e^4x^4+495b^2e^5x^3+110bcd\,e^4x^3-40c^2d^2e^3x^3+99b^2d\,e^4x^2-132bc\,d^2e^3x^2+48c^2d^3e^2x}{3465e^5}$

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
16/105*(15/8*(7/11*c^2*x^2+14/9*c*b*x+b^2)*x^2*e^4-3/2*x*d*(70/99*c^2*x^2+
5/3*c*b*x+b^2)*e^3+d^2*(10/11*c^2*x^2+2*c*b*x+b^2)*e^2-4/3*(6/11*c*x+b)*c*
d^3*e+16/33*c^2*d^4)*(e*x+d)^(3/2)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.19

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^2\,dx$$
$$= \frac{2\,(315\,c^2e^5x^5 + 128\,c^2d^5 - 352\,bcd^4e + 264\,b^2d^3e^2 + 35\,(c^2de^4 + 22\,bce^5)x^4 - 5\,(8\,c^2d^2e^3 - 22\,bcde^4 - 99}{3465\,e^5}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
2/3465*(315*c^2*e^5*x^5 + 128*c^2*d^5 - 352*b*c*d^4*e + 264*b^2*d^3*e^2 +
35*(c^2*d*e^4 + 22*b*c*e^5)*x^4 - 5*(8*c^2*d^2*e^3 - 22*b*c*d*e^4 - 99*b^2
*e^5)*x^3 + 3*(16*c^2*d^3*e^2 - 44*b*c*d^2*e^3 + 33*b^2*d*e^4)*x^2 - 4*(16
*c^2*d^4*e - 44*b*c*d^3*e^2 + 33*b^2*d^2*e^3)*x)*sqrt(e*x + d)/e^5
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.39

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^2\,dx$$
$$= \begin{cases} \frac{2\left(\frac{c^2(d+ex)^{\frac{11}{2}}}{11e^4} + \frac{(d+ex)^{\frac{9}{2}}\cdot(2bce-4c^2d)}{9e^4} + \frac{(d+ex)^{\frac{7}{2}}\left(b^2e^2-6bcde+6c^2d^2\right)}{7e^4} + \frac{(d+ex)^{\frac{5}{2}}\left(-2b^2de^2+6bcd^2e-4c^2d^3\right)}{5e^4} + \frac{(d+ex)^{\frac{3}{2}}\left(b^2d^2e^2-2bcd^3e+c^2d^4\right)}{3e^4}\right)}{e} \\ \sqrt{d}\left(\frac{b^2x^3}{3} + \frac{bcx^4}{2} + \frac{c^2x^5}{5}\right) \end{cases}$$

input
```
integrate((e*x+d)**(1/2)*(c*x**2+b*x)**2,x)
```

output
```
Piecewise((2*(c**2*(d + e*x)**(11/2)/(11*e**4) + (d + e*x)**(9/2)*(2*b*c*e
 - 4*c**2*d)/(9*e**4) + (d + e*x)**(7/2)*(b**2*e**2 - 6*b*c*d*e + 6*c**2*d
**2)/(7*e**4) + (d + e*x)**(5/2)*(-2*b**2*d*e**2 + 6*b*c*d**2*e - 4*c**2*d
**3)/(5*e**4) + (d + e*x)**(3/2)*(b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**
4)/(3*e**4))/e, Ne(e, 0)), (sqrt(d)*(b**2*x**3/3 + b*c*x**4/2 + c**2*x**5/
5), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.95

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^2\,dx = \frac{2\left(315\,(ex+d)^{\frac{11}{2}}c^2-770\,(2\,c^2d-bce)(ex+d)^{\frac{9}{2}}+495\,(6\,c^2d^2-6\,bcde+b^2e^2)(ex+d)^{\frac{7}{2}}-1386\,(2\,c^2d^3\right.}{3465\,e^5}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
2/3465*(315*(e*x + d)^(11/2)*c^2 - 770*(2*c^2*d - b*c*e)*(e*x + d)^(9/2) +
 495*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(e*x + d)^(7/2) - 1386*(2*c^2*d^3 -
 3*b*c*d^2*e + b^2*d*e^2)*(e*x + d)^(5/2) + 1155*(c^2*d^4 - 2*b*c*d^3*e +
b^2*d^2*e^2)*(e*x + d)^(3/2))/e^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. 2(127) = 254.

Time = 0.23 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.41

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^2\,dx = \frac{2\left(\frac{231\left(3\,(ex+d)^{\frac{5}{2}}-10\,(ex+d)^{\frac{3}{2}}d+15\,\sqrt{ex+d}d^2\right)b^2d}{e^2}+\frac{198\left(5\,(ex+d)^{\frac{7}{2}}-21\,(ex+d)^{\frac{5}{2}}d+35\,(ex+d)^{\frac{3}{2}}d^2-35\,\sqrt{ex+d}d^3\right)bcd}{e^3}+\frac{99\left(5\,(ex+d\right.}{}\right.}{}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
2/3465*(231*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d
^2)*b^2*d/e^2 + 198*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x +
d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b*c*d/e^3 + 99*(5*(e*x + d)^(7/2) - 2
1*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b^2/e
^2 + 11*(35*(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*
d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*c^2*d/e^4 + 22*(35*
(e*x + d)^(9/2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e
*x + d)^(3/2)*d^3 + 315*sqrt(e*x + d)*d^4)*b*c/e^3 + 5*(63*(e*x + d)^(11/2
) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)
*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*c^2/e^4)/e
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.94

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^2\,dx=\frac{2\,c^2\,{\left(d+e\,x\right)}^{11/2}}{11\,e^5}-\frac{{\left(d+e\,x\right)}^{5/2}\,\left(4\,b^2\,d\,e^2-12\,b\,c\,d^2\,e+8\,c^2\,d^3\right)}{5\,e^5}+\frac{{\left(d+e\,x\right)}^{7/2}\,\left(2\,b^2\,e^2-12\,b\,c\,d\,e+12\,c^2\,d^2\right)}{7\,e^5}-\frac{\left(8\,c^2\,d-4\,b\,c\,e\right)\,{\left(d+e\,x\right)}^{9/2}}{9\,e^5}+\frac{2\,d^2\,{\left(b\,e-c\,d\right)}^2\,{\left(d+e\,x\right)}^{3/2}}{3\,e^5}$$

input
```
int((b*x + c*x^2)^2*(d + e*x)^(1/2),x)
```

output
```
(2*c^2*(d + e*x)^(11/2))/(11*e^5) - ((d + e*x)^(5/2)*(8*c^2*d^3 + 4*b^2*d*
e^2 - 12*b*c*d^2*e))/(5*e^5) + ((d + e*x)^(7/2)*(2*b^2*e^2 + 12*c^2*d^2 -
12*b*c*d*e))/(7*e^5) - ((8*c^2*d - 4*b*c*e)*(d + e*x)^(9/2))/(9*e^5) + (2*
d^2*(b*e - c*d)^2*(d + e*x)^(3/2))/(3*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.22

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^2 \, dx$$
$$= \frac{2\sqrt{ex+d}\,(315c^2e^5x^5 + 770bc\,e^5x^4 + 35c^2d\,e^4x^4 + 495b^2e^5x^3 + 110bcd\,e^4x^3 - 40c^2d^2e^3x^3 + 99b^2d\,e^4x^2 -}{346}$$

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x)^2,x)
```

output
```
(2*sqrt(d + e*x)*(264*b**2*d**3*e**2 - 132*b**2*d**2*e**3*x + 99*b**2*d*e*
*4*x**2 + 495*b**2*e**5*x**3 - 352*b*c*d**4*e + 176*b*c*d**3*e**2*x - 132*
b*c*d**2*e**3*x**2 + 110*b*c*d*e**4*x**3 + 770*b*c*e**5*x**4 + 128*c**2*d*
*5 - 64*c**2*d**4*e*x + 48*c**2*d**3*e**2*x**2 - 40*c**2*d**2*e**3*x**3 +
35*c**2*d*e**4*x**4 + 315*c**2*e**5*x**5))/(3465*e**5)
```

3.93 $\int \frac{(bx+cx^2)^2}{\sqrt{d+ex}}\,dx$

Optimal result 694
Mathematica [A] (verified) 694
Rubi [A] (verified) 695
Maple [A] (verified) 696
Fricas [A] (verification not implemented) 697
Sympy [A] (verification not implemented) 697
Maxima [A] (verification not implemented) 698
Giac [A] (verification not implemented) 698
Mupad [B] (verification not implemented) 699
Reduce [B] (verification not implemented) 699

Optimal result

Integrand size = 21, antiderivative size = 145

$$\int \frac{(bx+cx^2)^2}{\sqrt{d+ex}}\,dx = \frac{2d^2(cd-be)^2\sqrt{d+ex}}{e^5} - \frac{4d(cd-be)(2cd-be)(d+ex)^{3/2}}{3e^5} + \frac{2(6c^2d^2-6bcde+b^2e^2)\,(d+ex)^{5/2}}{5e^5} - \frac{4c(2cd-be)(d+ex)^{7/2}}{7e^5} + \frac{2c^2(d+ex)^{9/2}}{9e^5}$$

output
```
2*d^2*(-b*e+c*d)^2*(e*x+d)^(1/2)/e^5-4/3*d*(-b*e+c*d)*(-b*e+2*c*d)*(e*x+d)
^(3/2)/e^5+2/5*(b^2*e^2-6*b*c*d*e+6*c^2*d^2)*(e*x+d)^(5/2)/e^5-4/7*c*(-b*e
+2*c*d)*(e*x+d)^(7/2)/e^5+2/9*c^2*(e*x+d)^(9/2)/e^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.86

$$\int \frac{(bx+cx^2)^2}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{d+ex}(21b^2e^2(8d^2-4dex+3e^2x^2)+18bce(-16d^3+8d^2ex-6de^2x^2+5e^3x^3)+c^2(128d^4-64d^3ex}{315e^5}$$

input
```
Integrate[(b*x + c*x^2)^2/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(21*b^2*e^2*(8*d^2 - 4*d*e*x + 3*e^2*x^2) + 18*b*c*e*(-16
*d^3 + 8*d^2*e*x - 6*d*e^2*x^2 + 5*e^3*x^3) + c^2*(128*d^4 - 64*d^3*e*x +
48*d^2*e^2*x^2 - 40*d*e^3*x^3 + 35*e^4*x^4)))/(315*e^5)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^2}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{(d+ex)^{3/2}(b^2e^2-6bcde+6c^2d^2)}{e^4}+\frac{d^2(cd-be)^2}{e^4\sqrt{d+ex}}-\frac{2c(d+ex)^{5/2}(2cd-be)}{e^4}+\frac{2d\sqrt{d+ex}(cd-be)(be-2}{e^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(d+ex)^{5/2}(b^2e^2-6bcde+6c^2d^2)}{5e^5}+\frac{2d^2\sqrt{d+ex}(cd-be)^2}{e^5}-\frac{4c(d+ex)^{7/2}(2cd-be)}{7e^5}-\frac{4d(d+ex)^{3/2}(cd-be)(2cd-be)}{3e^5}+\frac{2c^2(d+ex)^{9/2}}{9e^5}$$

input
```
Int[(b*x + c*x^2)^2/Sqrt[d + e*x],x]
```

output
```
(2*d^2*(c*d - b*e)^2*Sqrt[d + e*x])/e^5 - (4*d*(c*d - b*e)*(2*c*d - b*e)*(
d + e*x)^(3/2))/(3*e^5) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*x)^(
5/2))/(5*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(7/2))/(7*e^5) + (2*c^2*(d +
e*x)^(9/2))/(9*e^5)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$\frac{16\sqrt{ex+d}\left(\frac{3\left(\frac{5}{9}c^2x^2+\frac{10}{7}cbx+b^2\right)x^2e^4}{8}-\frac{xd\left(\frac{10}{21}c^2x^2+\frac{9}{7}cbx+b^2\right)e^3}{2}+d^2\left(\frac{2}{7}c^2x^2+\frac{6}{7}cbx+b^2\right)e^2-\frac{12\left(\frac{2cx}{9}+b\right)c\,d^3e}{7}+\frac{16c^2d^4}{21}\right)}{15e^5}$
gosper	$\frac{2(35c^2x^4e^4+90x^3bc\,e^4-40d\,c^2x^3e^3+63x^2b^2e^4-108x^2bcd\,e^3+48x^2c^2d^2e^2-84x\,b^2d\,e^3+144xbc\,d^2e^2-64x\,c^2d^3e+168c}{315e^5}$
trager	$\frac{2(35c^2x^4e^4+90x^3bc\,e^4-40d\,c^2x^3e^3+63x^2b^2e^4-108x^2bcd\,e^3+48x^2c^2d^2e^2-84x\,b^2d\,e^3+144xbc\,d^2e^2-64x\,c^2d^3e+168c}{315e^5}$
risch	$\frac{2(35c^2x^4e^4+90x^3bc\,e^4-40d\,c^2x^3e^3+63x^2b^2e^4-108x^2bcd\,e^3+48x^2c^2d^2e^2-84x\,b^2d\,e^3+144xbc\,d^2e^2-64x\,c^2d^3e+168c}{315e^5}$
derivativedivides	$\frac{\frac{2c^2(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(-2c^2d+2c(be-cd)\right)(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(c^2d^2-4dc(be-cd)+(be-cd)^2\right)(ex+d)^{\frac{5}{2}}}{5}+\frac{2\left(2d^2c(be-cd)-2d(be-cd)^2\right)(ex+d)}{3}}{e^5}$
default	$\frac{\frac{2c^2(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(-2c^2d+2c(be-cd)\right)(ex+d)^{\frac{7}{2}}}{7}+\frac{2\left(c^2d^2-4dc(be-cd)+(be-cd)^2\right)(ex+d)^{\frac{5}{2}}}{5}+\frac{2\left(2d^2c(be-cd)-2d(be-cd)^2\right)(ex+d)}{3}}{e^5}$
orering	$\frac{2(35c^2x^4e^4+90x^3bc\,e^4-40d\,c^2x^3e^3+63x^2b^2e^4-108x^2bcd\,e^3+48x^2c^2d^2e^2-84x\,b^2d\,e^3+144xbc\,d^2e^2-64x\,c^2d^3e+168c}{315e^5(cx+b)^2x^2}$

input
```
int((c*x^2+b*x)^2/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
16/15*(e*x+d)^(1/2)*(3/8*(5/9*c^2*x^2+10/7*c*b*x+b^2)*x^2*e^4-1/2*x*d*(10/
21*c^2*x^2+9/7*c*b*x+b^2)*e^3+d^2*(2/7*c^2*x^2+6/7*c*b*x+b^2)*e^2-12/7*(2/
9*c*x+b)*c*d^3*e+16/21*c^2*d^4)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \frac{(bx+cx^2)^2}{\sqrt{d+ex}}\,dx = \frac{2\left(35\,c^2e^4x^4+128\,c^2d^4-288\,bcd^3e+168\,b^2d^2e^2-10\,(4\,c^2de^3-9\,bce^4)x^3+3\,(16\,c^2d^2e^2-36\,bcde^3+21\right.}{315\,e^5}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*c^2*e^4*x^4 + 128*c^2*d^4 - 288*b*c*d^3*e + 168*b^2*d^2*e^2 - 10
*(4*c^2*d*e^3 - 9*b*c*e^4)*x^3 + 3*(16*c^2*d^2*e^2 - 36*b*c*d*e^3 + 21*b^2
*e^4)*x^2 - 4*(16*c^2*d^3*e - 36*b*c*d^2*e^2 + 21*b^2*d*e^3)*x)*sqrt(e*x +
 d)/e^5
```

Sympy [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.39

$$\int \frac{(bx+cx^2)^2}{\sqrt{d+ex}}\,dx = \begin{cases} \frac{2\left(\frac{c^2(d+ex)^{\frac{9}{2}}}{9e^4}+\frac{(d+ex)^{\frac{7}{2}}\cdot(2bce-4c^2d)}{7e^4}+\frac{(d+ex)^{\frac{5}{2}}\left(b^2e^2-6bcde+6c^2d^2\right)}{5e^4}+\frac{(d+ex)^{\frac{3}{2}}\left(-2b^2de^2+6bcd^2e-4c^2d^3\right)}{3e^4}+\frac{\sqrt{d+ex}\left(b^2d^2e^2-2bcd^3e+c^2d^4\right)}{e^4}\right)}{e} \\ \frac{\frac{b^2x^3}{3}+\frac{bcx^4}{2}+\frac{c^2x^5}{5}}{\sqrt{d}} \end{cases}$$

input
```
integrate((c*x**2+b*x)**2/(e*x+d)**(1/2),x)
```

output
```
Piecewise((2*(c**2*(d + e*x)**(9/2)/(9*e**4) + (d + e*x)**(7/2)*(2*b*c*e -
 4*c**2*d)/(7*e**4) + (d + e*x)**(5/2)*(b**2*e**2 - 6*b*c*d*e + 6*c**2*d**
2)/(5*e**4) + (d + e*x)**(3/2)*(-2*b**2*d*e**2 + 6*b*c*d**2*e - 4*c**2*d**
3)/(3*e**4) + sqrt(d + e*x)*(b**2*d**2*e**2 - 2*b*c*d**3*e + c**2*d**4)/e*
*4)/e, Ne(e, 0)), ((b**2*x**3/3 + b*c*x**4/2 + c**2*x**5/5)/sqrt(d), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10

$$\int \frac{(bx+cx^2)^2}{\sqrt{d+ex}}\,dx = \frac{2\left(\frac{21\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}d^2\right)b^2}{e^2}+\frac{18\left(5(ex+d)^{\frac{7}{2}}-21(ex+d)^{\frac{5}{2}}d+35(ex+d)^{\frac{3}{2}}d^2-35\sqrt{ex+d}d^3\right)bc}{e^3}+\frac{\left(35(ex+d)^{\frac{9}{2}}-18\right.}{}\right.}{315\,e}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(21*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2
)*b^2/e^2 + 18*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3
/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b*c/e^3 + (35*(e*x + d)^(9/2) - 180*(e*x +
 d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt
(e*x + d)*d^4)*c^2/e^4)/e
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.10

$$\int \frac{(bx+cx^2)^2}{\sqrt{d+ex}}\,dx = \frac{2\left(\frac{21\left(3(ex+d)^{\frac{5}{2}}-10(ex+d)^{\frac{3}{2}}d+15\sqrt{ex+d}d^2\right)b^2}{e^2}+\frac{18\left(5(ex+d)^{\frac{7}{2}}-21(ex+d)^{\frac{5}{2}}d+35(ex+d)^{\frac{3}{2}}d^2-35\sqrt{ex+d}d^3\right)bc}{e^3}+\frac{\left(35(ex+d)^{\frac{9}{2}}-18\right.}{}\right.}{315\,e}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
2/315*(21*(3*(e*x + d)^(5/2) - 10*(e*x + d)^(3/2)*d + 15*sqrt(e*x + d)*d^2
)*b^2/e^2 + 18*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3
/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b*c/e^3 + (35*(e*x + d)^(9/2) - 180*(e*x +
 d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt
(e*x + d)*d^4)*c^2/e^4)/e
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.95

$$\int \frac{(bx+cx^2)^2}{\sqrt{d+ex}}\,dx = \frac{2\,c^2\,(d+e\,x)^{9/2}}{9\,e^5} - \frac{(d+e\,x)^{3/2}\,(4\,b^2\,d\,e^2-12\,b\,c\,d^2\,e+8\,c^2\,d^3)}{3\,e^5} + \frac{(d+e\,x)^{5/2}\,(2\,b^2\,e^2-12\,b\,c\,d\,e+12\,c^2\,d^2)}{5\,e^5} - \frac{(8\,c^2\,d-4\,b\,c\,e)\,(d+e\,x)^{7/2}}{7\,e^5} + \frac{2\,d^2\,(b\,e-c\,d)^2\,\sqrt{d+e\,x}}{e^5}$$

input
```
int((b*x + c*x^2)^2/(d + e*x)^(1/2),x)
```

output
```
(2*c^2*(d + e*x)^(9/2))/(9*e^5) - ((d + e*x)^(3/2)*(8*c^2*d^3 + 4*b^2*d*e^
2 - 12*b*c*d^2*e))/(3*e^5) + ((d + e*x)^(5/2)*(2*b^2*e^2 + 12*c^2*d^2 - 12
*b*c*d*e))/(5*e^5) - ((8*c^2*d - 4*b*c*e)*(d + e*x)^(7/2))/(7*e^5) + (2*d^
2*(b*e - c*d)^2*(d + e*x)^(1/2))/e^5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96

$$\int \frac{(bx+cx^2)^2}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{ex+d}\,(35c^2e^4x^4+90bc\,e^4x^3-40c^2d\,e^3x^3+63b^2e^4x^2-108bcd\,e^3x^2+48c^2d^2e^2x^2-84b^2d\,e^3x+144}{315e^5}$$

input
```
int((c*x^2+b*x)^2/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(d + e*x)*(168*b**2*d**2*e**2 - 84*b**2*d*e**3*x + 63*b**2*e**4*x**
2 - 288*b*c*d**3*e + 144*b*c*d**2*e**2*x - 108*b*c*d*e**3*x**2 + 90*b*c*e*
*4*x**3 + 128*c**2*d**4 - 64*c**2*d**3*e*x + 48*c**2*d**2*e**2*x**2 - 40*c
**2*d*e**3*x**3 + 35*c**2*e**4*x**4))/(315*e**5)
```

3.94 $\int \frac{(bx+cx^2)^2}{(d+ex)^{3/2}}\, dx$

Optimal result . . . 700
Mathematica [A] (verified) . . . 700
Rubi [A] (verified) . . . 701
Maple [A] (verified) . . . 702
Fricas [A] (verification not implemented) . . . 703
Sympy [A] (verification not implemented) . . . 703
Maxima [A] (verification not implemented) . . . 704
Giac [A] (verification not implemented) . . . 704
Mupad [B] (verification not implemented) . . . 705
Reduce [B] (verification not implemented) . . . 705

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{3/2}}\, dx = -\frac{2d^2(cd-be)^2}{e^5\sqrt{d+ex}} - \frac{4d(cd-be)(2cd-be)\sqrt{d+ex}}{e^5} + \frac{2(6c^2d^2-6bcde+b^2e^2)\,(d+ex)^{3/2}}{3e^5} - \frac{4c(2cd-be)(d+ex)^{5/2}}{5e^5} + \frac{2c^2(d+ex)^{7/2}}{7e^5}$$

output
```
-2*d^2*(-b*e+c*d)^2/e^5/(e*x+d)^(1/2)-4*d*(-b*e+c*d)*(-b*e+2*c*d)*(e*x+d)^
(1/2)/e^5+2/3*(b^2*e^2-6*b*c*d*e+6*c^2*d^2)*(e*x+d)^(3/2)/e^5-4/5*c*(-b*e+
2*c*d)*(e*x+d)^(5/2)/e^5+2/7*c^2*(e*x+d)^(7/2)/e^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.86

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{3/2}}\, dx = \frac{70b^2e^2(-8d^2-4dex+e^2x^2)+84bce(16d^3+8d^2ex-2de^2x^2+e^3x^3)-6c^2(128d^4+6}{105e^5\sqrt{d+ex}}$$

input
```
Integrate[(b*x + c*x^2)^2/(d + e*x)^(3/2),x]
```

output
```
(70*b^2*e^2*(-8*d^2 - 4*d*e*x + e^2*x^2) + 84*b*c*e*(16*d^3 + 8*d^2*e*x -
2*d*e^2*x^2 + e^3*x^3) - 6*c^2*(128*d^4 + 64*d^3*e*x - 16*d^2*e^2*x^2 + 8*
d*e^3*x^3 - 5*e^4*x^4))/(105*e^5*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{\sqrt{d+ex}(b^2e^2-6bcde+6c^2d^2)}{e^4}+\frac{d^2(cd-be)^2}{e^4(d+ex)^{3/2}}-\frac{2c(d+ex)^{3/2}(2cd-be)}{e^4}+\frac{2d(cd-be)(be-2cd)}{e^4\sqrt{d+ex}}+\frac{c^2(\cdots$$

$$\Big\downarrow \text{2009}$$

$$\frac{2(d+ex)^{3/2}\left(b^2e^2-6bcde+6c^2d^2\right)}{3e^5}-\frac{2d^2(cd-be)^2}{e^5\sqrt{d+ex}}-\frac{4c(d+ex)^{5/2}(2cd-be)}{5e^5}-\frac{4d\sqrt{d+ex}(cd-be)(2cd-be)}{e^5}+\frac{2c^2(d+ex)^{7/2}}{7e^5}$$

input
```
Int[(b*x + c*x^2)^2/(d + e*x)^(3/2),x]
```

output
```
(-2*d^2*(c*d - b*e)^2)/(e^5*Sqrt[d + e*x]) - (4*d*(c*d - b*e)*(2*c*d - b*e
)*Sqrt[d + e*x])/e^5 + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*x)^(3/2
))/(3*e^5) - (4*c*(2*c*d - b*e)*(d + e*x)^(5/2))/(5*e^5) + (2*c^2*(d + e*x
)^(7/2))/(7*e^5)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.78

method	result
pseudoelliptic	$\frac{(30c^2x^4+84bc\,x^3+70b^2x^2)e^4-280\left(\frac{6}{35}c^2x^2+\frac{3}{5}cbx+b^2\right)xd\,e^3-560d^2\left(-\frac{6}{35}c^2x^2-\frac{6}{5}cbx+b^2\right)e^2+1344\left(-\frac{2cx}{7}+b\right)c\,d^3e-7}{105\sqrt{ex+d}\,e^5}$
risch	$-\frac{2(-15e^3x^3c^2-42e^3x^2cb+39d\,e^2c^2x^2-35e^3x\,b^2+126d\,e^2cbx-87d^2e\,c^2x+175d\,e^2b^2-462d^2ebc+279c^2d^3)\sqrt{ex+d}}{105e^5}$ −
gosper	$-\frac{2(-15c^2x^4e^4-42x^3bc\,e^4+24d\,c^2x^3e^3-35x^2b^2e^4+84x^2bcd\,e^3-48x^2c^2d^2e^2+140x\,b^2d\,e^3-336xbc\,d^2e^2+192x\,c^2d^3e}{105\sqrt{ex+d}\,e^5}$
trager	$-\frac{2(-15c^2x^4e^4-42x^3bc\,e^4+24d\,c^2x^3e^3-35x^2b^2e^4+84x^2bcd\,e^3-48x^2c^2d^2e^2+140x\,b^2d\,e^3-336xbc\,d^2e^2+192x\,c^2d^3e}{105\sqrt{ex+d}\,e^5}$
orering	$-\frac{2(-15c^2x^4e^4-42x^3bc\,e^4+24d\,c^2x^3e^3-35x^2b^2e^4+84x^2bcd\,e^3-48x^2c^2d^2e^2+140x\,b^2d\,e^3-336xbc\,d^2e^2+192x\,c^2d^3e}{105e^5(cx+b)^2\sqrt{ex+d}\,x^2}$
derivativedivides	$\frac{\frac{2c^2(ex+d)^{\frac{7}{2}}}{7}+\frac{4bce(ex+d)^{\frac{5}{2}}}{5}-\frac{8c^2d(ex+d)^{\frac{5}{2}}}{5}+\frac{2b^2e^2(ex+d)^{\frac{3}{2}}}{3}-4bcde(ex+d)^{\frac{3}{2}}+4c^2d^2(ex+d)^{\frac{3}{2}}-4b^2d\,e^2\sqrt{ex+d}+12bc\,d^2e}{e^5}$
default	$\frac{\frac{2c^2(ex+d)^{\frac{7}{2}}}{7}+\frac{4bce(ex+d)^{\frac{5}{2}}}{5}-\frac{8c^2d(ex+d)^{\frac{5}{2}}}{5}+\frac{2b^2e^2(ex+d)^{\frac{3}{2}}}{3}-4bcde(ex+d)^{\frac{3}{2}}+4c^2d^2(ex+d)^{\frac{3}{2}}-4b^2d\,e^2\sqrt{ex+d}+12bc\,d^2e}{e^5}$

input
```
int((c*x^2+b*x)^2/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*((30*c^2*x^4+84*b*c*x^3+70*b^2*x^2)*e^4-280*(6/35*c^2*x^2+3/5*c*b*x+
b^2)*x*d*e^3-560*d^2*(-6/35*c^2*x^2-6/5*c*b*x+b^2)*e^2+1344*(-2/7*c*x+b)*c
*d^3*e-768*c^2*d^4)/(e*x+d)^(1/2)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{3/2}}\,dx = \frac{2\,(15\,c^2e^4x^4 - 384\,c^2d^4 + 672\,bcd^3e - 280\,b^2d^2e^2 - 6\,(4\,c^2de^3 - 7\,bce^4)x^3 + (48\,c^2d^2e^2}{105\,(e^6x + de^5)}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
2/105*(15*c^2*e^4*x^4 - 384*c^2*d^4 + 672*b*c*d^3*e - 280*b^2*d^2*e^2 - 6*
(4*c^2*d*e^3 - 7*b*c*e^4)*x^3 + (48*c^2*d^2*e^2 - 84*b*c*d*e^3 + 35*b^2*e^
4)*x^2 - 4*(48*c^2*d^3*e - 84*b*c*d^2*e^2 + 35*b^2*d*e^3)*x)*sqrt(e*x + d)
/(e^6*x + d*e^5)
```

Sympy [A] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.29

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^2(d+ex)^{\frac{7}{2}}}{7e^4} - \frac{d^2(be-cd)^2}{e^4\sqrt{d+ex}} + \frac{(d+ex)^{\frac{5}{2}}\cdot\left(2bce-4c^2d\right)}{5e^4} + \frac{(d+ex)^{\frac{3}{2}}\left(b^2e^2-6bcde+6c^2d^2\right)}{3e^4} + \frac{\sqrt{d+ex}\left(-2b^2de^2+6bcd^2e-4c^2d^3\right)}{e^4}\right)}{e} \\ \frac{\frac{b^2x^3}{3}+\frac{bcx^4}{2}+\frac{c^2x^5}{5}}{d^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((c*x**2+b*x)**2/(e*x+d)**(3/2),x)
```

output
```
Piecewise((2*(c**2*(d + e*x)**(7/2)/(7*e**4) - d**2*(b*e - c*d)**2/(e**4*s
qrt(d + e*x)) + (d + e*x)**(5/2)*(2*b*c*e - 4*c**2*d)/(5*e**4) + (d + e*x)
**(3/2)*(b**2*e**2 - 6*b*c*d*e + 6*c**2*d**2)/(3*e**4) + sqrt(d + e*x)*(-2
*b**2*d*e**2 + 6*b*c*d**2*e - 4*c**2*d**3)/e**4)/e, Ne(e, 0)), ((b**2*x**3
/3 + b*c*x**4/2 + c**2*x**5/5)/d**(3/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{15\,(ex+d)^{\frac{7}{2}}c^2-42\,(2\,c^2d-bce)(ex+d)^{\frac{5}{2}}+35\,(6\,c^2d^2-6\,bcde+b^2e^2)(ex+d)^{\frac{3}{2}}-210\,(2\,c^2d^3-3\,bcd^2e+b^2de^2)\sqrt{ex+d}}{e^4}\right.}{105\,e}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
2/105*((15*(e*x + d)^(7/2)*c^2 - 42*(2*c^2*d - b*c*e)*(e*x + d)^(5/2) + 35
*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(e*x + d)^(3/2) - 210*(2*c^2*d^3 - 3*b*
c*d^2*e + b^2*d*e^2)*sqrt(e*x + d))/e^4 - 105*(c^2*d^4 - 2*b*c*d^3*e + b^2
*d^2*e^2)/(sqrt(e*x + d)*e^4))/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.32

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{3/2}}\,dx = -\frac{2\,(c^2d^4-2\,bcd^3e+b^2d^2e^2)}{\sqrt{ex+d}e^5} + \frac{2\left(15\,(ex+d)^{\frac{7}{2}}c^2e^{30}-84\,(ex+d)^{\frac{5}{2}}c^2de^{30}+210\,(ex+d)^{\frac{3}{2}}c^2d^2e^{30}-420\,\sqrt{ex+d}c^2d^3e^{30}+42\,(ex+d)^{\frac{5}{2}}b\right.}{105\,e^{35}}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
-2*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)/(sqrt(e*x + d)*e^5) + 2/105*(15*(
e*x + d)^(7/2)*c^2*e^30 - 84*(e*x + d)^(5/2)*c^2*d*e^30 + 210*(e*x + d)^(3
/2)*c^2*d^2*e^30 - 420*sqrt(e*x + d)*c^2*d^3*e^30 + 42*(e*x + d)^(5/2)*b*c
*e^31 - 210*(e*x + d)^(3/2)*b*c*d*e^31 + 630*sqrt(e*x + d)*b*c*d^2*e^31 +
35*(e*x + d)^(3/2)*b^2*e^32 - 210*sqrt(e*x + d)*b^2*d*e^32)/e^35
```

Mupad [B] (verification not implemented)

Time = 5.60 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.07

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{3/2}}\,dx = \frac{2\,c^2\,{(d+e\,x)}^{7/2}}{7\,e^5}-\frac{\sqrt{d+e\,x}\,\left(4\,b^2\,d\,e^2-12\,b\,c\,d^2\,e+8\,c^2\,d^3\right)}{e^5}+\frac{{(d+e\,x)}^{3/2}\,\left(2\,b^2\,e^2-12\,b\,c\,d\,e+12\,c^2\,d^2\right)}{3\,e^5}-\frac{2\,b^2\,d^2\,e^2-4\,b\,c\,d^3\,e+2\,c^2\,d^4}{e^5\,\sqrt{d+e\,x}}-\frac{\left(8\,c^2\,d-4\,b\,c\,e\right)\,{(d+e\,x)}^{5/2}}{5\,e^5}$$

input
```
int((b*x + c*x^2)^2/(d + e*x)^(3/2),x)
```

output
```
(2*c^2*(d + e*x)^(7/2))/(7*e^5) - ((d + e*x)^(1/2)*(8*c^2*d^3 + 4*b^2*d*e^
2 - 12*b*c*d^2*e))/e^5 + ((d + e*x)^(3/2)*(2*b^2*e^2 + 12*c^2*d^2 - 12*b*c
*d*e))/(3*e^5) - (2*c^2*d^4 + 2*b^2*d^2*e^2 - 4*b*c*d^3*e)/(e^5*(d + e*x)^
(1/2)) - ((8*c^2*d - 4*b*c*e)*(d + e*x)^(5/2))/(5*e^5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.99

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{3/2}}\,dx = \frac{\frac{2}{7}c^2e^4x^4+\frac{4}{5}bc\,e^4x^3-\frac{16}{35}c^2d\,e^3x^3+\frac{2}{3}b^2e^4x^2-\frac{8}{5}bcd\,e^3x^2+\frac{32}{35}c^2d^2e^2x^2-\frac{8}{3}b^2d\,e^3x+\frac{32}{5}b}{\sqrt{ex+d}\,e^5}$$

input
```
int((c*x^2+b*x)^2/(e*x+d)^(3/2),x)
```

output
```
(2*( - 280*b**2*d**2*e**2 - 140*b**2*d*e**3*x + 35*b**2*e**4*x**2 + 672*b*
c*d**3*e + 336*b*c*d**2*e**2*x - 84*b*c*d*e**3*x**2 + 42*b*c*e**4*x**3 - 3
84*c**2*d**4 - 192*c**2*d**3*e*x + 48*c**2*d**2*e**2*x**2 - 24*c**2*d*e**3
*x**3 + 15*c**2*e**4*x**4))/(105*sqrt(d + e*x)*e**5)
```

3.95 $\int \frac{(bx+cx^2)^2}{(d+ex)^{5/2}}\,dx$

Optimal result 706
Mathematica [A] (verified) 706
Rubi [A] (verified) 707
Maple [A] (verified) 708
Fricas [A] (verification not implemented) 709
Sympy [A] (verification not implemented) 709
Maxima [A] (verification not implemented) 710
Giac [A] (verification not implemented) 710
Mupad [B] (verification not implemented) 711
Reduce [B] (verification not implemented) 711

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{5/2}}\,dx = -\frac{2d^2(cd-be)^2}{3e^5(d+ex)^{3/2}} + \frac{4d(cd-be)(2cd-be)}{e^5\sqrt{d+ex}} + \frac{2(6c^2d^2-6bcde+b^2e^2)\sqrt{d+ex}}{e^5} - \frac{4c(2cd-be)(d+ex)^{3/2}}{3e^5} + \frac{2c^2(d+ex)^{5/2}}{5e^5}$$

output
```
-2/3*d^2*(-b*e+c*d)^2/e^5/(e*x+d)^(3/2)+4*d*(-b*e+c*d)*(-b*e+2*c*d)/e^5/(e
*x+d)^(1/2)+2*(b^2*e^2-6*b*c*d*e+6*c^2*d^2)*(e*x+d)^(1/2)/e^5-4/3*c*(-b*e+
2*c*d)*(e*x+d)^(3/2)/e^5+2/5*c^2*(e*x+d)^(5/2)/e^5
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.86

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{5/2}}\,dx = \frac{2(5b^2e^2(8d^2+12dex+3e^2x^2)+10bce(-16d^3-24d^2ex-6de^2x^2+e^3x^3)+c^2(128d^4}{15e^5(d+ex)^{3/2}}$$

input
```
Integrate[(b*x + c*x^2)^2/(d + e*x)^(5/2),x]
```

output
```
(2*(5*b^2*e^2*(8*d^2 + 12*d*e*x + 3*e^2*x^2) + 10*b*c*e*(-16*d^3 - 24*d^2*
e*x - 6*d*e^2*x^2 + e^3*x^3) + c^2*(128*d^4 + 192*d^3*e*x + 48*d^2*e^2*x^2
 - 8*d*e^3*x^3 + 3*e^4*x^4)))/(15*e^5*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{b^2e^2-6bcde+6c^2d^2}{e^4\sqrt{d+ex}}+\frac{d^2(cd-be)^2}{e^4(d+ex)^{5/2}}-\frac{2c\sqrt{d+ex}(2cd-be)}{e^4}+\frac{2d(cd-be)(be-2cd)}{e^4(d+ex)^{3/2}}+\frac{c^2(d+ex)^{3/2}}{e^4}\right)d$$

$$\Big\downarrow \text{2009}$$

$$\frac{2\sqrt{d+ex}(b^2e^2-6bcde+6c^2d^2)}{e^5}-\frac{2d^2(cd-be)^2}{3e^5(d+ex)^{3/2}}-\frac{4c(d+ex)^{3/2}(2cd-be)}{3e^5}+\frac{4d(cd-be)(2cd-be)}{e^5\sqrt{d+ex}}+\frac{2c^2(d+ex)^{5/2}}{5e^5}$$

input
```
Int[(b*x + c*x^2)^2/(d + e*x)^(5/2),x]
```

output
```
(-2*d^2*(c*d - b*e)^2)/(3*e^5*(d + e*x)^(3/2)) + (4*d*(c*d - b*e)*(2*c*d -
 b*e))/(e^5*Sqrt[d + e*x]) + (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*Sqrt[d +
 e*x])/e^5 - (4*c*(2*c*d - b*e)*(d + e*x)^(3/2))/(3*e^5) + (2*c^2*(d + e*x
)^(5/2))/(5*e^5)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.77

method	result
risch	$\frac{2\left(3c^2e^2x^2+10e^2xbc-14c^2dex+15b^2e^2-80bcde+73c^2d^2\right)\sqrt{ex+d}}{15e^5}+\frac{2\left(6xb\,e^2-12cdxe+5bde-11c\,d^2\right)d(be-cd)}{3e^5(ex+d)^{\frac{3}{2}}}$
pseudoelliptic	$\frac{(6c^2x^4+20bc\,x^3+30b^2x^2)e^4+120x\left(-\frac{2}{15}c^2x^2-cbx+b^2\right)d\,e^3+80d^2\left(\frac{6}{5}c^2x^2-6cbx+b^2\right)e^2-320\left(-\frac{6cx}{5}+b\right)c\,d^3e+256c^2d^4}{15(ex+d)^{\frac{3}{2}}e^5}$
gosper	$\frac{\frac{2}{5}c^2x^4e^4+\frac{4}{3}x^3bc\,e^4-\frac{16}{15}d\,c^2x^3e^3+2x^2b^2e^4-8x^2bcd\,e^3+\frac{32}{5}x^2c^2d^2e^2+8x\,b^2d\,e^3-32xbc\,d^2e^2+\frac{128}{5}x\,c^2d^3e+\frac{16}{3}d^2e^2b^2-}{(ex+d)^{\frac{3}{2}}e^5}$
trager	$\frac{\frac{2}{5}c^2x^4e^4+\frac{4}{3}x^3bc\,e^4-\frac{16}{15}d\,c^2x^3e^3+2x^2b^2e^4-8x^2bcd\,e^3+\frac{32}{5}x^2c^2d^2e^2+8x\,b^2d\,e^3-32xbc\,d^2e^2+\frac{128}{5}x\,c^2d^3e+\frac{16}{3}d^2e^2b^2-}{(ex+d)^{\frac{3}{2}}e^5}$
derivativedivides	$\frac{\frac{2c^2(ex+d)^{\frac{5}{2}}}{5}+\frac{4bce(ex+d)^{\frac{3}{2}}}{3}-\frac{8c^2d(ex+d)^{\frac{3}{2}}}{3}+2b^2e^2\sqrt{ex+d}-12bcde\sqrt{ex+d}+12c^2d^2\sqrt{ex+d}+\frac{4d\left(b^2e^2-3bcde+2c^2d^2\right)}{\sqrt{ex+d}}-\frac{2d}{}}{e^5}$
default	$\frac{\frac{2c^2(ex+d)^{\frac{5}{2}}}{5}+\frac{4bce(ex+d)^{\frac{3}{2}}}{3}-\frac{8c^2d(ex+d)^{\frac{3}{2}}}{3}+2b^2e^2\sqrt{ex+d}-12bcde\sqrt{ex+d}+12c^2d^2\sqrt{ex+d}+\frac{4d\left(b^2e^2-3bcde+2c^2d^2\right)}{\sqrt{ex+d}}-\frac{2d}{}}{e^5}$
orering	$\frac{2\left(3c^2x^4e^4+10x^3bc\,e^4-8d\,c^2x^3e^3+15x^2b^2e^4-60x^2bcd\,e^3+48x^2c^2d^2e^2+60x\,b^2d\,e^3-240xbc\,d^2e^2+192x\,c^2d^3e+40d^2e^2\right.}{15e^5(cx+b)^2(ex+d)^{\frac{3}{2}}x^2}$

input
```
int((c*x^2+b*x)^2/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*c^2*e^2*x^2+10*b*c*e^2*x-14*c^2*d*e*x+15*b^2*e^2-80*b*c*d*e+73*c^2
*d^2)*(e*x+d)^(1/2)/e^5+2/3*(6*b*e^2*x-12*c*d*e*x+5*b*d*e-11*c*d^2)*d*(b*e
-c*d)/e^5/(e*x+d)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.11

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{5/2}}\,dx = \frac{2\,(3\,c^2e^4x^4+128\,c^2d^4-160\,bcd^3e+40\,b^2d^2e^2-2\,(4\,c^2de^3-5\,bce^4)x^3+3\,(16\,c^2d^2e^2-}{15\,(e^7x^2+2\,de^6x+d}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*c^2*e^4*x^4 + 128*c^2*d^4 - 160*b*c*d^3*e + 40*b^2*d^2*e^2 - 2*(4*
c^2*d*e^3 - 5*b*c*e^4)*x^3 + 3*(16*c^2*d^2*e^2 - 20*b*c*d*e^3 + 5*b^2*e^4)
*x^2 + 12*(16*c^2*d^3*e - 20*b*c*d^2*e^2 + 5*b^2*d*e^3)*x)*sqrt(e*x + d)/(
e^7*x^2 + 2*d*e^6*x + d^2*e^5)
```

Sympy [A] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.21

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^2(d+ex)^{\frac{5}{2}}}{5e^4}-\frac{d^2(be-cd)^2}{3e^4(d+ex)^{\frac{3}{2}}}+\frac{2d(be-2cd)(be-cd)}{e^4\sqrt{d+ex}}+\frac{(d+ex)^{\frac{3}{2}}\cdot\left(2bce-4c^2d\right)}{3e^4}+\frac{\sqrt{d+ex}\left(b^2e^2-6bcde+6c^2d^2\right)}{e^4}\right)}{e} & \text{for } e \neq \\ \frac{\frac{b^2x^3}{3}+\frac{bcx^4}{2}+\frac{c^2x^5}{5}}{d^{\frac{5}{2}}} & \text{otherw} \end{cases}$$

input
```
integrate((c*x**2+b*x)**2/(e*x+d)**(5/2),x)
```

output
```
Piecewise((2*(c**2*(d + e*x)**(5/2)/(5*e**4) - d**2*(b*e - c*d)**2/(3*e**4
*(d + e*x)**(3/2)) + 2*d*(b*e - 2*c*d)*(b*e - c*d)/(e**4*sqrt(d + e*x)) +
(d + e*x)**(3/2)*(2*b*c*e - 4*c**2*d)/(3*e**4) + sqrt(d + e*x)*(b**2*e**2
- 6*b*c*d*e + 6*c**2*d**2)/e**4)/e, Ne(e, 0)), ((b**2*x**3/3 + b*c*x**4/2
+ c**2*x**5/5)/d**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{5/2}}\,dx = \frac{2\left(\frac{3\,(ex+d)^{\frac{5}{2}}c^2-10\,(2\,c^2d-bce)(ex+d)^{\frac{3}{2}}+15\,(6\,c^2d^2-6\,bcde+b^2e^2)\sqrt{ex+d}}{e^4}-\frac{5\,(c^2d^4-2\,bcd^3e+b^2d^2e^2-6\,(2\,c^2d^3}{(ex+d)^{\frac{3}{2}}\,e^{}}\right.}{15\,e}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
2/15*((3*(e*x + d)^(5/2)*c^2 - 10*(2*c^2*d - b*c*e)*(e*x + d)^(3/2) + 15*(
6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*sqrt(e*x + d))/e^4 - 5*(c^2*d^4 - 2*b*c*d
^3*e + b^2*d^2*e^2 - 6*(2*c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*(e*x + d))/((
e*x + d)^(3/2)*e^4))/e
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.26

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{5/2}}\,dx = \frac{2\,(12\,(ex+d)c^2d^3-c^2d^4-18\,(ex+d)bcd^2e+2\,bcd^3e+6\,(ex+d)b^2de^2-b^2d^2e^2)}{3\,(ex+d)^{\frac{3}{2}}e^5}$$
$$+\frac{2\left(3\,(ex+d)^{\frac{5}{2}}c^2e^{20}-20\,(ex+d)^{\frac{3}{2}}c^2de^{20}+90\,\sqrt{ex+d}c^2d^2e^{20}+10\,(ex+d)^{\frac{3}{2}}bce^{21}-90\,\sqrt{ex+d}bcde^{21}+\right.}{15\,e^{25}}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
2/3*(12*(e*x + d)*c^2*d^3 - c^2*d^4 - 18*(e*x + d)*b*c*d^2*e + 2*b*c*d^3*e
 + 6*(e*x + d)*b^2*d*e^2 - b^2*d^2*e^2)/((e*x + d)^(3/2)*e^5) + 2/15*(3*(e
*x + d)^(5/2)*c^2*e^20 - 20*(e*x + d)^(3/2)*c^2*d*e^20 + 90*sqrt(e*x + d)*
c^2*d^2*e^20 + 10*(e*x + d)^(3/2)*b*c*e^21 - 90*sqrt(e*x + d)*b*c*d*e^21 +
 15*sqrt(e*x + d)*b^2*e^22)/e^25
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.01

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{5/2}}\,dx = \frac{2\,c^2\,(d+e\,x)^{5/2}}{5\,e^5} + \frac{\sqrt{d+e\,x}\,(2\,b^2\,e^2-12\,b\,c\,d\,e+12\,c^2\,d^2)}{e^5} - \frac{(8\,c^2\,d-4\,b\,c\,e)\,(d+e\,x)^{3/2}}{3\,e^5} + \frac{(d+e\,x)\,(4\,b^2\,d\,e^2-12\,b\,c\,d^2\,e+8\,c^2\,d^3)-\frac{2\,c^2\,d^4}{3}-\frac{2\,b^2\,d^2\,e^2}{3}+\frac{4\,b\,c\,d^3\,e}{3}}{e^5\,(d+e\,x)^{3/2}}$$

input
```
int((b*x + c*x^2)^2/(d + e*x)^(5/2),x)
```

output
```
(2*c^2*(d + e*x)^(5/2))/(5*e^5) + ((d + e*x)^(1/2)*(2*b^2*e^2 + 12*c^2*d^2
 - 12*b*c*d*e))/e^5 - ((8*c^2*d - 4*b*c*e)*(d + e*x)^(3/2))/(3*e^5) + ((d
+ e*x)*(8*c^2*d^3 + 4*b^2*d*e^2 - 12*b*c*d^2*e) - (2*c^2*d^4)/3 - (2*b^2*d
^2*e^2)/3 + (4*b*c*d^3*e)/3)/(e^5*(d + e*x)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.03

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{5/2}}\,dx = \frac{\frac{2}{5}c^2e^4x^4+\frac{4}{3}bc\,e^4x^3-\frac{16}{15}c^2d\,e^3x^3+2b^2e^4x^2-8bcd\,e^3x^2+\frac{32}{5}c^2d^2e^2x^2+8b^2d\,e^3x-32b}{\sqrt{ex+d}\,e^5\,(ex+d)}$$

input
```
int((c*x^2+b*x)^2/(e*x+d)^(5/2),x)
```

output
```
(2*(40*b**2*d**2*e**2 + 60*b**2*d*e**3*x + 15*b**2*e**4*x**2 - 160*b*c*d**
3*e - 240*b*c*d**2*e**2*x - 60*b*c*d*e**3*x**2 + 10*b*c*e**4*x**3 + 128*c*
*2*d**4 + 192*c**2*d**3*e*x + 48*c**2*d**2*e**2*x**2 - 8*c**2*d*e**3*x**3
+ 3*c**2*e**4*x**4))/(15*sqrt(d + e*x)*e**5*(d + e*x))
```

3.96 $\int \frac{(bx+cx^2)^2}{(d+ex)^{7/2}}\,dx$

Optimal result 712
Mathematica [A] (verified) 712
Rubi [A] (verified) 713
Maple [A] (verified) 714
Fricas [A] (verification not implemented) 715
Sympy [B] (verification not implemented) 715
Maxima [A] (verification not implemented) 716
Giac [A] (verification not implemented) 717
Mupad [B] (verification not implemented) 717
Reduce [B] (verification not implemented) 718

Optimal result

Integrand size = 21, antiderivative size = 143

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{7/2}}\,dx = -\frac{2d^2(cd-be)^2}{5e^5(d+ex)^{5/2}}+\frac{4d(cd-be)(2cd-be)}{3e^5(d+ex)^{3/2}} - \frac{2(6c^2d^2-6bcde+b^2e^2)}{e^5\sqrt{d+ex}}-\frac{4c(2cd-be)\sqrt{d+ex}}{e^5}+\frac{2c^2(d+ex)^{3/2}}{3e^5}$$

output
```
-2/5*d^2*(-b*e+c*d)^2/e^5/(e*x+d)^(5/2)+4/3*d*(-b*e+c*d)*(-b*e+2*c*d)/e^5/
(e*x+d)^(3/2)-2*(b^2*e^2-6*b*c*d*e+6*c^2*d^2)/e^5/(e*x+d)^(1/2)-4*c*(-b*e+
2*c*d)*(e*x+d)^(1/2)/e^5+2/3*c^2*(e*x+d)^(3/2)/e^5
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.86

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{7/2}}\,dx = -\frac{2(b^2e^2(8d^2+20dex+15e^2x^2)-6bce(16d^3+40d^2ex+30de^2x^2+5e^3x^3)+c^2(128d^4+320d^3ex+240d^2}{15e^5(d+ex)^{5/2}}$$

input
```
Integrate[(b*x + c*x^2)^2/(d + e*x)^(7/2),x]
```

output
```
(-2*(b^2*e^2*(8*d^2 + 20*d*e*x + 15*e^2*x^2) - 6*b*c*e*(16*d^3 + 40*d^2*e*
x + 30*d*e^2*x^2 + 5*e^3*x^3) + c^2*(128*d^4 + 320*d^3*e*x + 240*d^2*e^2*x
^2 + 40*d*e^3*x^3 - 5*e^4*x^4)))/(15*e^5*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{7/2}}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{b^2e^2-6bcde+6c^2d^2}{e^4(d+ex)^{3/2}}+\frac{d^2(cd-be)^2}{e^4(d+ex)^{7/2}}-\frac{2c(2cd-be)}{e^4\sqrt{d+ex}}+\frac{2d(cd-be)(be-2cd)}{e^4(d+ex)^{5/2}}+\frac{c^2\sqrt{d+ex}}{e^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(b^2e^2-6bcde+6c^2d^2)}{e^5\sqrt{d+ex}}-\frac{2d^2(cd-be)^2}{5e^5(d+ex)^{5/2}}-\frac{4c\sqrt{d+ex}(2cd-be)}{e^5}+\frac{4d(cd-be)(2cd-be)}{3e^5(d+ex)^{3/2}}+\frac{2c^2(d+ex)^{3/2}}{3e^5}$$

input
```
Int[(b*x + c*x^2)^2/(d + e*x)^(7/2),x]
```

output
```
(-2*d^2*(c*d - b*e)^2)/(5*e^5*(d + e*x)^(5/2)) + (4*d*(c*d - b*e)*(2*c*d -
 b*e))/(3*e^5*(d + e*x)^(3/2)) - (2*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2))/(e^
5*Sqrt[d + e*x]) - (4*c*(2*c*d - b*e)*Sqrt[d + e*x])/e^5 + (2*c^2*(d + e*x
)^(3/2))/(3*e^5)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$-\frac{16\left(\frac{15x^2\left(-\frac{1}{3}c^2x^2-2cbx+b^2\right)e^4}{8}+\frac{5dx\left(2c^2x^2-9cbx+b^2\right)e^3}{2}+d^2\left(30c^2x^2-30cbx+b^2\right)e^2-12c\,d^3\left(-\frac{10cx}{3}+b\right)e+16c^2d^4\right)}{15(ex+d)^{\frac{5}{2}}e^5}$
derivativedivides	$\frac{\frac{2c^2(ex+d)^{\frac{3}{2}}}{3}+4bce\sqrt{ex+d}-8c^2d\sqrt{ex+d}-\frac{2d^2\left(b^2e^2-2bcde+c^2d^2\right)}{5(ex+d)^{\frac{5}{2}}}-\frac{2\left(b^2e^2-6bcde+6c^2d^2\right)}{\sqrt{ex+d}}+\frac{4d\left(b^2e^2-3bcde+2c^2d^2\right)}{3(ex+d)^{\frac{3}{2}}}}{e^5}$
default	$\frac{\frac{2c^2(ex+d)^{\frac{3}{2}}}{3}+4bce\sqrt{ex+d}-8c^2d\sqrt{ex+d}-\frac{2d^2\left(b^2e^2-2bcde+c^2d^2\right)}{5(ex+d)^{\frac{5}{2}}}-\frac{2\left(b^2e^2-6bcde+6c^2d^2\right)}{\sqrt{ex+d}}+\frac{4d\left(b^2e^2-3bcde+2c^2d^2\right)}{3(ex+d)^{\frac{3}{2}}}}{e^5}$
gosper	$-\frac{2(-5c^2x^4e^4-30x^3bc\,e^4+40d\,c^2x^3e^3+15x^2b^2e^4-180x^2bcd\,e^3+240x^2c^2d^2e^2+20x\,b^2d\,e^3-240xbc\,d^2e^2+320x\,c^2d^3e}{15(ex+d)^{\frac{5}{2}}e^5}$
trager	$-\frac{2(-5c^2x^4e^4-30x^3bc\,e^4+40d\,c^2x^3e^3+15x^2b^2e^4-180x^2bcd\,e^3+240x^2c^2d^2e^2+20x\,b^2d\,e^3-240xbc\,d^2e^2+320x\,c^2d^3e}{15(ex+d)^{\frac{5}{2}}e^5}$
risch	$\frac{2c(cex+6be-11cd)\sqrt{ex+d}}{3e^5}-\frac{2(15x^2b^2e^4-90x^2bcd\,e^3+90x^2c^2d^2e^2+20x\,b^2d\,e^3-150xbc\,d^2e^2+160x\,c^2d^3e+8d^2e^2b^2}{15e^5\sqrt{ex+d}\,(e^2x^2+2dex+d^2)}$
orering	$-\frac{2(-5c^2x^4e^4-30x^3bc\,e^4+40d\,c^2x^3e^3+15x^2b^2e^4-180x^2bcd\,e^3+240x^2c^2d^2e^2+20x\,b^2d\,e^3-240xbc\,d^2e^2+320x\,c^2d^3e}{15e^5(cx+b)^2(ex+d)^{\frac{5}{2}}x^2}$

input
```
int((c*x^2+b*x)^2/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-16/15*(15/8*x^2*(-1/3*c^2*x^2-2*c*b*x+b^2)*e^4+5/2*d*x*(2*c^2*x^2-9*b*c*x
+b^2)*e^3+d^2*(30*c^2*x^2-30*b*c*x+b^2)*e^2-12*c*d^3*(-10/3*c*x+b)*e+16*c^
2*d^4)/(e*x+d)^(5/2)/e^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.17

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{7/2}}\,dx = \frac{2\,(5\,c^2e^4x^4 - 128\,c^2d^4 + 96\,bcd^3e - 8\,b^2d^2e^2 - 10\,(4\,c^2de^3 - 3\,bce^4)x^3 - 15\,(16\,c^2d^2e^2 -}{15\,(e^8x^3 + 3\,de^7x^2 + 3\,d^2e^6x}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
2/15*(5*c^2*e^4*x^4 - 128*c^2*d^4 + 96*b*c*d^3*e - 8*b^2*d^2*e^2 - 10*(4*c
^2*d*e^3 - 3*b*c*e^4)*x^3 - 15*(16*c^2*d^2*e^2 - 12*b*c*d*e^3 + b^2*e^4)*x
^2 - 20*(16*c^2*d^3*e - 12*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(e*x + d)/(e^8*
x^3 + 3*d*e^7*x^2 + 3*d^2*e^6*x + d^3*e^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 787 vs. 2(138) = 276.

Time = 0.47 (sec) , antiderivative size = 787, normalized size of antiderivative = 5.50

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{7/2}}\,dx = \begin{cases} -\frac{16b^2d^2e^2}{15d^2e^5\sqrt{d+ex}+30de^6x\sqrt{d+ex}+15e^7x^2\sqrt{d+ex}} - \frac{40b^2de^3x}{15d^2e^5\sqrt{d+ex}+30de^6x\sqrt{d+ex}+15e^7x^2\sqrt{d+ex}} - \frac{}{15d^2e^5\sqrt{}} \\ \frac{\frac{b^2x^3}{3}+\frac{bcx^4}{2}+\frac{c^2x^5}{5}}{d^{\frac{7}{2}}} \end{cases}$$

input
```
integrate((c*x**2+b*x)**2/(e*x+d)**(7/2),x)
```

output
```
Piecewise((-16*b**2*d**2*e**2/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sq
rt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 40*b**2*d*e**3*x/(15*d**2*e**5
*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) -
 30*b**2*e**4*x**2/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x)
 + 15*e**7*x**2*sqrt(d + e*x)) + 192*b*c*d**3*e/(15*d**2*e**5*sqrt(d + e*x
) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) + 480*b*c*d**2
*e**2*x/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*
x**2*sqrt(d + e*x)) + 360*b*c*d*e**3*x**2/(15*d**2*e**5*sqrt(d + e*x) + 30
*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) + 60*b*c*e**4*x**3/(
15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt
(d + e*x)) - 256*c**2*d**4/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(
d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 640*c**2*d**3*e*x/(15*d**2*e**5*s
qrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 4
80*c**2*d**2*e**2*x**2/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d +
e*x) + 15*e**7*x**2*sqrt(d + e*x)) - 80*c**2*d*e**3*x**3/(15*d**2*e**5*sqr
t(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 15*e**7*x**2*sqrt(d + e*x)) + 10*
c**2*e**4*x**4/(15*d**2*e**5*sqrt(d + e*x) + 30*d*e**6*x*sqrt(d + e*x) + 1
5*e**7*x**2*sqrt(d + e*x)), Ne(e, 0)), ((b**2*x**3/3 + b*c*x**4/2 + c**2*x
**5/5)/d**(7/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.03

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{7/2}}\,dx = \frac{2\left(\frac{5\left((ex+d)^{\frac{3}{2}}c^2-6\,(2\,c^2d-bce)\sqrt{ex+d}\right)}{e^4}-\frac{3\,c^2d^4-6\,bcd^3e+3\,b^2d^2e^2+15\,(6\,c^2d^2-6\,bcde+b^2e^2)(ex+d)^2-10\,(2\,c^2\cdots}{(ex+d)^{\frac{5}{2}}e^4}\cdots\right.}{15\,e}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
2/15*(5*((e*x + d)^(3/2)*c^2 - 6*(2*c^2*d - b*c*e)*sqrt(e*x + d))/e^4 - (3
*c^2*d^4 - 6*b*c*d^3*e + 3*b^2*d^2*e^2 + 15*(6*c^2*d^2 - 6*b*c*d*e + b^2*e
^2)*(e*x + d)^2 - 10*(2*c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*(e*x + d))/((e*
x + d)^(5/2)*e^4))/e
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.22

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{7/2}}\,dx = -\frac{2\left(90\,(ex+d)^2c^2d^2-20\,(ex+d)c^2d^3+3\,c^2d^4-90\,(ex+d)^2bcde+30\,(ex+d)bcd^2e-6\,bcd^3e+15\,(ex\right.}{15\,(ex+d)^{\frac{5}{2}}e^5} + \frac{2\left((ex+d)^{\frac{3}{2}}c^2e^{10}-12\,\sqrt{ex+d}c^2de^{10}+6\,\sqrt{ex+d}bce^{11}\right)}{3\,e^{15}}$$

input
```
integrate((c*x^2+b*x)^2/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
-2/15*(90*(e*x + d)^2*c^2*d^2 - 20*(e*x + d)*c^2*d^3 + 3*c^2*d^4 - 90*(e*x
 + d)^2*b*c*d*e + 30*(e*x + d)*b*c*d^2*e - 6*b*c*d^3*e + 15*(e*x + d)^2*b^
2*e^2 - 10*(e*x + d)*b^2*d*e^2 + 3*b^2*d^2*e^2)/((e*x + d)^(5/2)*e^5) + 2/
3*((e*x + d)^(3/2)*c^2*e^10 - 12*sqrt(e*x + d)*c^2*d*e^10 + 6*sqrt(e*x + d
)*b*c*e^11)/e^15
```

Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.98

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{7/2}}\,dx = -\frac{2\,(8\,b^2\,d^2\,e^2+20\,b^2\,d\,e^3\,x+15\,b^2\,e^4\,x^2-96\,b\,c\,d^3\,e-240\,b\,c\,d^2\,e^2\,x-180\,b\,c\,d\,e^3\,x^2-30\,b\,c\,e^4\,x^3+128\,c^2}{15\,e^5\,(d+e\,x)^{5/2}}$$

input
```
int((b*x + c*x^2)^2/(d + e*x)^(7/2),x)
```

output
```
-(2*(128*c^2*d^4 + 8*b^2*d^2*e^2 + 15*b^2*e^4*x^2 - 5*c^2*e^4*x^4 + 40*c^2
*d*e^3*x^3 - 96*b*c*d^3*e + 240*c^2*d^2*e^2*x^2 - 30*b*c*e^4*x^3 + 20*b^2*
d*e^3*x + 320*c^2*d^3*e*x - 240*b*c*d^2*e^2*x - 180*b*c*d*e^3*x^2))/(15*e^
5*(d + e*x)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.11

$$\int \frac{(bx+cx^2)^2}{(d+ex)^{7/2}}\,dx = \frac{\frac{2}{3}c^2e^4x^4+4bc\,e^4x^3-\frac{16}{3}c^2d\,e^3x^3-2b^2e^4x^2+24bcd\,e^3x^2-32c^2d^2e^2x^2-\frac{8}{3}b^2d\,e^3x+32}{\sqrt{ex+d}\,e^5\left(e^2x^2+2dex+d^2\right)}$$

input
```
int((c*x^2+b*x)^2/(e*x+d)^(7/2),x)
```

output
```
(2*( - 8*b**2*d**2*e**2 - 20*b**2*d*e**3*x - 15*b**2*e**4*x**2 + 96*b*c*d*
*3*e + 240*b*c*d**2*e**2*x + 180*b*c*d*e**3*x**2 + 30*b*c*e**4*x**3 - 128*
c**2*d**4 - 320*c**2*d**3*e*x - 240*c**2*d**2*e**2*x**2 - 40*c**2*d*e**3*x
**3 + 5*c**2*e**4*x**4))/(15*sqrt(d + e*x)*e**5*(d**2 + 2*d*e*x + e**2*x**
2))
```

3.97 $\int (d+ex)^{7/2} (bx+cx^2)^3 \, dx$

Optimal result . . . 719
Mathematica [A] (verified) . . . 720
Rubi [A] (verified) . . . 720
Maple [A] (verified) . . . 722
Fricas [B] (verification not implemented) . . . 722
Sympy [A] (verification not implemented) . . . 723
Maxima [A] (verification not implemented) . . . 724
Giac [B] (verification not implemented) . . . 725
Mupad [B] (verification not implemented) . . . 726
Reduce [B] (verification not implemented) . . . 726

Optimal result

Integrand size = 21, antiderivative size = 248

$$\int (d+ex)^{7/2} (bx+cx^2)^3 \, dx = \frac{2d^3(cd-be)^3(d+ex)^{9/2}}{9e^7} - \frac{6d^2(cd-be)^2(2cd-be)(d+ex)^{11/2}}{11e^7} + \frac{6d(cd-be)(5c^2d^2-5bcde+b^2e^2)(d+ex)^{13/2}}{13e^7} - \frac{2(2cd-be)(10c^2d^2-10bcde+b^2e^2)(d+ex)^{15/2}}{15e^7} + \frac{6c(5c^2d^2-5bcde+b^2e^2)(d+ex)^{17/2}}{17e^7} - \frac{6c^2(2cd-be)(d+ex)^{19/2}}{19e^7} + \frac{2c^3(d+ex)^{21/2}}{21e^7}$$

output

```
2/9*d^3*(-b*e+c*d)^3*(e*x+d)^(9/2)/e^7-6/11*d^2*(-b*e+c*d)^2*(-b*e+2*c*d)*
(e*x+d)^(11/2)/e^7+6/13*d*(-b*e+c*d)*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)
^(13/2)/e^7-2/15*(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*d^2)*(e*x+d)^(15/
2)/e^7+6/17*c*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)^(17/2)/e^7-6/19*c^2*(-
b*e+2*c*d)*(e*x+d)^(19/2)/e^7+2/21*c^3*(e*x+d)^(21/2)/e^7
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.94

$$\int (d+ex)^{7/2} (bx +cx^2)^3 \, dx = \frac{2(d+ex)^{9/2} (2261b^3e^3(-16d^3+72d^2ex-198de^2x^2+429e^3x^3)+399b^2ce^2(128d^4-576d^3ex$$

input
```
Integrate[(d + e*x)^(7/2)*(b*x + c*x^2)^3,x]
```

output
```
(2*(d + e*x)^(9/2)*(2261*b^3*e^3*(-16*d^3 + 72*d^2*e*x - 198*d*e^2*x^2 + 4
29*e^3*x^3) + 399*b^2*c*e^2*(128*d^4 - 576*d^3*e*x + 1584*d^2*e^2*x^2 - 34
32*d*e^3*x^3 + 6435*e^4*x^4) + 105*b*c^2*e*(-256*d^5 + 1152*d^4*e*x - 3168
*d^3*e^2*x^2 + 6864*d^2*e^3*x^3 - 12870*d*e^4*x^4 + 21879*e^5*x^5) + 5*c^3
*(1024*d^6 - 4608*d^5*e*x + 12672*d^4*e^2*x^2 - 27456*d^3*e^3*x^3 + 51480*
d^2*e^4*x^4 - 87516*d*e^5*x^5 + 138567*e^6*x^6)))/(14549535*e^7)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+cx^2)^3 (d+ex)^{7/2} \, dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(\frac{3c(d+ex)^{15/2}(b^2e^2-5bcde+5c^2d^2)}{e^6}+\frac{(d+ex)^{13/2}(2cd-be)(-b^2e^2+10bcde-10c^2d^2)}{e^6}+\frac{3d(d+ex)^{11}}{}\right.$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{6c(d+ex)^{17/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{17e^7}-\frac{2(d+ex)^{15/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{15e^7}+\frac{6d(d+ex)^{13/2}(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{13e^7}-\frac{6c^2(d+ex)^{19/2}(2cd-be)}{19e^7}+\frac{2d^3(d+ex)^{9/2}(cd-be)^3}{9e^7}-\frac{6d^2(d+ex)^{11/2}(cd-be)^2(2cd-be)}{11e^7}+\frac{2c^3(d+ex)^{21/2}}{21e^7}$$

input
```
Int[(d + e*x)^(7/2)*(b*x + c*x^2)^3,x]
```

output
```
(2*d^3*(c*d - b*e)^3*(d + e*x)^(9/2))/(9*e^7) - (6*d^2*(c*d - b*e)^2*(2*c*
d - b*e)*(d + e*x)^(11/2))/(11*e^7) + (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*
d*e + b^2*e^2)*(d + e*x)^(13/2))/(13*e^7) - (2*(2*c*d - b*e)*(10*c^2*d^2 -
 10*b*c*d*e + b^2*e^2)*(d + e*x)^(15/2))/(15*e^7) + (6*c*(5*c^2*d^2 - 5*b*
c*d*e + b^2*e^2)*(d + e*x)^(17/2))/(17*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*
x)^(19/2))/(19*e^7) + (2*c^3*(d + e*x)^(21/2))/(21*e^7)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$-\frac{32\left(-\frac{429\left(\frac{5}{7}c^3x^3+\frac{45}{19}bc^2x^2+\frac{45}{17}b^2cx+b^3\right)x^3e^6}{16}+\frac{99\left(\frac{130}{133}c^3x^3+\frac{975}{323}bc^2x^2+\frac{52}{17}b^2cx+b^3\right)x^2de^5}{8}-\frac{9x\left(\frac{3575}{2261}c^3x^3+\frac{1430}{323}bc^2x^2\right.}{2}\right.}{}$
derivativedivides	$\frac{2c^3(ex+d)^{\frac{21}{2}}}{21}+\frac{2\left(-3dc^3+3(be-cd)c^2\right)(ex+d)^{\frac{19}{2}}}{19}+\frac{2\left(3c^3d^2-9d(be-cd)c^2+3(be-cd)^2c\right)(ex+d)^{\frac{17}{2}}}{17}+\frac{2\left(-d^3c^3+9d^2(be-cd)c^2-\right.}{}$
default	$\frac{2c^3(ex+d)^{\frac{21}{2}}}{21}-\frac{2\left(3dc^3-3(be-cd)c^2\right)(ex+d)^{\frac{19}{2}}}{19}-\frac{2\left(-3c^3d^2+9d(be-cd)c^2-3(be-cd)^2c\right)(ex+d)^{\frac{17}{2}}}{17}-\frac{2\left(d^3c^3-9d^2(be-cd)c^2+9d\right.}{}$
gosper	$-\frac{2(ex+d)^{\frac{9}{2}}\left(-692835x^6c^3e^6-2297295x^5bc^2e^6+437580x^5c^3de^5-2567565x^4b^2ce^6+1351350x^4bc^2de^5-257400x^4c^3\right.}{}$
orering	$-\frac{2\left(-692835x^6c^3e^6-2297295x^5bc^2e^6+437580x^5c^3de^5-2567565x^4b^2ce^6+1351350x^4bc^2de^5-257400x^4c^3d^2e^4-96\right.}{}$
trager	$-\frac{2\left(-692835c^3e^{10}x^{10}-2297295bc^2e^{10}x^9-2333760c^3de^9x^9-2567565b^2ce^{10}x^8-7837830bc^2de^9x^8-2664090c^3d^2e^8\right.}{}$
risch	$-\frac{2\left(-692835c^3e^{10}x^{10}-2297295bc^2e^{10}x^9-2333760c^3de^9x^9-2567565b^2ce^{10}x^8-7837830bc^2de^9x^8-2664090c^3d^2e^8\right.}{}$

input
```
int((e*x+d)^(7/2)*(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-32/6435*(-429/16*(5/7*c^3*x^3+45/19*b*c^2*x^2+45/17*b^2*c*x+b^3)*x^3*e^6+
99/8*(130/133*c^3*x^3+975/323*b*c^2*x^2+52/17*b^2*c*x+b^3)*x^2*d*e^5-9/2*x
*(3575/2261*c^3*x^3+1430/323*b*c^2*x^2+66/17*b^2*c*x+b^3)*d^2*e^4+d^3*(858
0/2261*c^3*x^3+2970/323*b*c^2*x^2+108/17*b^2*c*x+b^3)*e^3-24/17*(165/133*c
^2*x^2+45/19*c*b*x+b^2)*c*d^4*e^2+240/323*c^2*d^5*(6/7*c*x+b)*e-320/2261*d
^6*c^3)*(e*x+d)^(9/2)/e^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(220) = 440$.

Time = 0.10 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.93

$$\int (d+ex)^{7/2}\left(bx+cx^2\right)^3\,dx = \frac{2\left(692835\,c^3e^{10}x^{10}+5120\,c^3d^{10}-26880\,bc^2d^9e+51072\,b^2cd^8e^2-36176\,b^3d^7e^3+36465\,(64\,c^3\right.}{}$$

input
```
integrate((e*x+d)^(7/2)*(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
2/14549535*(692835*c^3*e^10*x^10 + 5120*c^3*d^10 - 26880*b*c^2*d^9*e + 510
72*b^2*c*d^8*e^2 - 36176*b^3*d^7*e^3 + 36465*(64*c^3*d*e^9 + 63*b*c^2*e^10
)*x^9 + 19305*(138*c^3*d^2*e^8 + 406*b*c^2*d*e^9 + 133*b^2*c*e^10)*x^8 + 4
29*(2420*c^3*d^3*e^7 + 21210*b*c^2*d^2*e^8 + 20748*b^2*c*d*e^9 + 2261*b^3*
e^10)*x^7 + 231*(5*c^3*d^4*e^6 + 15720*b*c^2*d^3*e^7 + 45714*b^2*c*d^2*e^8
 + 14858*b^3*d*e^9)*x^6 - 63*(20*c^3*d^5*e^5 - 105*b*c^2*d^4*e^6 - 69084*b
^2*c*d^3*e^7 - 66538*b^3*d^2*e^8)*x^5 + 35*(40*c^3*d^6*e^4 - 210*b*c^2*d^5
*e^5 + 399*b^2*c*d^4*e^6 + 51680*b^3*d^3*e^7)*x^4 - 5*(320*c^3*d^7*e^3 - 1
680*b*c^2*d^6*e^4 + 3192*b^2*c*d^5*e^5 - 2261*b^3*d^4*e^6)*x^3 + 6*(320*c^
3*d^8*e^2 - 1680*b*c^2*d^7*e^3 + 3192*b^2*c*d^6*e^4 - 2261*b^3*d^5*e^5)*x^
2 - 8*(320*c^3*d^9*e - 1680*b*c^2*d^8*e^2 + 3192*b^2*c*d^7*e^3 - 2261*b^3*
d^6*e^4)*x)*sqrt(e*x + d)/e^7
```

Sympy [A] (verification not implemented)

Time = 1.16 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.50

$$\int (d+ex)^{7/2} \left(bx+cx^2\right)^3 \, dx = \begin{cases} \frac{2\left(\frac{c^3(d+ex)^{\frac{21}{2}}}{21e^6} + \frac{(d+ex)^{\frac{19}{2}}\cdot\left(3bc^2e-6c^3d\right)}{19e^6} + \frac{(d+ex)^{\frac{17}{2}}\cdot\left(3b^2ce^2-15bc^2de+15c^3d^2\right)}{17e^6} + \frac{(d+ex)^{\frac{15}{2}}\left(b^3e^3-12b^2cde^2+30bc^2d^2e-20c^3d^3\right)}{15e^6} + \cdots}{\cdots} \\ d^{\frac{7}{2}}\left(\frac{b^3x^4}{4} + \frac{3b^2cx^5}{5} + \frac{bc^2x^6}{2} + \frac{c^3x^7}{7}\right) \end{cases}$$

input
```
integrate((e*x+d)**(7/2)*(c*x**2+b*x)**3,x)
```

output
```
Piecewise((2*(c**3*(d + e*x)**(21/2)/(21*e**6) + (d + e*x)**(19/2)*(3*b*c*
*2*e - 6*c**3*d)/(19*e**6) + (d + e*x)**(17/2)*(3*b**2*c*e**2 - 15*b*c**2*
d*e + 15*c**3*d**2)/(17*e**6) + (d + e*x)**(15/2)*(b**3*e**3 - 12*b**2*c*d
*e**2 + 30*b*c**2*d**2*e - 20*c**3*d**3)/(15*e**6) + (d + e*x)**(13/2)*(-3
*b**3*d*e**3 + 18*b**2*c*d**2*e**2 - 30*b*c**2*d**3*e + 15*c**3*d**4)/(13*
e**6) + (d + e*x)**(11/2)*(3*b**3*d**2*e**3 - 12*b**2*c*d**3*e**2 + 15*b*c
**2*d**4*e - 6*c**3*d**5)/(11*e**6) + (d + e*x)**(9/2)*(-b**3*d**3*e**3 +
3*b**2*c*d**4*e**2 - 3*b*c**2*d**5*e + c**3*d**6)/(9*e**6))/e, Ne(e, 0)),
(d**(7/2)*(b**3*x**4/4 + 3*b**2*c*x**5/5 + b*c**2*x**6/2 + c**3*x**7/7), T
rue))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.09

$$\int (d+ex)^{7/2} \left(bx+cx^2\right)^3 dx = \frac{2 \left(692835 \, (ex+d)^{\frac{21}{2}} c^3 - 2297295 \, (2 \, c^3 d - bc^2 e)(ex+d)^{\frac{19}{2}} + 2567565 \, (5 \, c^3 d^2 - 5 \, bc^2 de + b^2 c$$

input
```
integrate((e*x+d)^(7/2)*(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
2/14549535*(692835*(e*x + d)^(21/2)*c^3 - 2297295*(2*c^3*d - b*c^2*e)*(e*x
 + d)^(19/2) + 2567565*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*(e*x + d)^(17
/2) - 969969*(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*(e*x
 + d)^(15/2) + 3357585*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 - b^3
*d*e^3)*(e*x + d)^(13/2) - 3968055*(2*c^3*d^5 - 5*b*c^2*d^4*e + 4*b^2*c*d^
3*e^2 - b^3*d^2*e^3)*(e*x + d)^(11/2) + 1616615*(c^3*d^6 - 3*b*c^2*d^5*e +
 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)*(e*x + d)^(9/2))/e^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1936 vs. $2(220) = 440$.

Time = 0.13 (sec) , antiderivative size = 1936, normalized size of antiderivative = 7.81

$$\int (d+ex)^{7/2} \left(bx+cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(7/2)*(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
2/14549535*(415701*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d
)^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b^3*d^4/e^3 + 138567*(35*(e*x + d)^(9/
2) - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)
*d^3 + 315*sqrt(e*x + d)*d^4)*b^2*c*d^4/e^4 + 184756*(35*(e*x + d)^(9/2) -
 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3
 + 315*sqrt(e*x + d)*d^4)*b^3*d^3/e^3 + 62985*(63*(e*x + d)^(11/2) - 385*(
e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 11
55*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b*c^2*d^4/e^5 + 251940*(63
*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386
*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b
^2*c*d^3/e^4 + 125970*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(
e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 -
 693*sqrt(e*x + d)*d^5)*b^3*d^2/e^3 + 4845*(231*(e*x + d)^(13/2) - 1638*(e
*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9
009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^
6)*c^3*d^4/e^6 + 58140*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5
005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*
d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*b*c^2*d^3/e^5 + 8
7210*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2
)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x...
```

Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.96

$$\int (d+ex)^{7/2} \left(bx+cx^2\right)^3 dx = \frac{(d+e\,x)^{15/2}\,\left(2\,b^3\,e^3-24\,b^2\,c\,d\,e^2+60\,b\,c^2\,d^2\,e-40\,c^3\,d^3\right)}{15\,e^7}+\frac{2\,c^3\,(d+e\,x)^{21/2}}{21\,e^7}-\frac{\left(12\,c^3\,d-6\,b\,c^2\,e\right)\,(d+e\,x)^{19/2}}{19\,e^7}+\frac{(d+e\,x)^{17/2}\,\left(6\,b^2\,c\,e^2-30\,b\,c^2\,d\,e+30\,c^3\,d^2\right)}{17\,e^7}+\frac{(d+e\,x)^{13/2}\,\left(-6\,b^3\,d\,e^3+36\,b^2\,c\,d^2\,e^2-60\,b\,c^2\,d^3\,e+30\,c^3\,d^4\right)}{13\,e^7}-\frac{2\,d^3\,(b\,e-c\,d)^3\,(d+e\,x)^{9/2}}{9\,e^7}+\frac{6\,d^2\,(b\,e-c\,d)^2\,(b\,e-2\,c\,d)\,(d+e\,x)^{11/2}}{11\,e^7}$$

input
```
int((b*x + c*x^2)^3*(d + e*x)^(7/2),x)
```

output
```
((d + e*x)^(15/2)*(2*b^3*e^3 - 40*c^3*d^3 + 60*b*c^2*d^2*e - 24*b^2*c*d*e^
2))/(15*e^7) + (2*c^3*(d + e*x)^(21/2))/(21*e^7) - ((12*c^3*d - 6*b*c^2*e)
*(d + e*x)^(19/2))/(19*e^7) + ((d + e*x)^(17/2)*(30*c^3*d^2 + 6*b^2*c*e^2
- 30*b*c^2*d*e))/(17*e^7) + ((d + e*x)^(13/2)*(30*c^3*d^4 - 6*b^3*d*e^3 +
36*b^2*c*d^2*e^2 - 60*b*c^2*d^3*e))/(13*e^7) - (2*d^3*(b*e - c*d)^3*(d + e
*x)^(9/2))/(9*e^7) + (6*d^2*(b*e - c*d)^2*(b*e - 2*c*d)*(d + e*x)^(11/2))/
(11*e^7)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.08

$$\int (d+ex)^{7/2} \left(bx+cx^2\right)^3 dx = \frac{2\sqrt{ex+d}\,(692835c^3e^{10}x^{10}+2297295b\,c^2e^{10}x^9+2333760c^3d\,e^9x^9+2567565b^2c\,e^{10}x^8+78378}{}$$

input
```
int((e*x+d)^(7/2)*(c*x^2+b*x)^3,x)
```

output

```
(2*sqrt(d + e*x)*( - 36176*b**3*d**7*e**3 + 18088*b**3*d**6*e**4*x - 13566
*b**3*d**5*e**5*x**2 + 11305*b**3*d**4*e**6*x**3 + 1808800*b**3*d**3*e**7*
x**4 + 4191894*b**3*d**2*e**8*x**5 + 3432198*b**3*d*e**9*x**6 + 969969*b**
3*e**10*x**7 + 51072*b**2*c*d**8*e**2 - 25536*b**2*c*d**7*e**3*x + 19152*b
**2*c*d**6*e**4*x**2 - 15960*b**2*c*d**5*e**5*x**3 + 13965*b**2*c*d**4*e**
6*x**4 + 4352292*b**2*c*d**3*e**7*x**5 + 10559934*b**2*c*d**2*e**8*x**6 +
8900892*b**2*c*d*e**9*x**7 + 2567565*b**2*c*e**10*x**8 - 26880*b*c**2*d**9
*e + 13440*b*c**2*d**8*e**2*x - 10080*b*c**2*d**7*e**3*x**2 + 8400*b*c**2*
d**6*e**4*x**3 - 7350*b*c**2*d**5*e**5*x**4 + 6615*b*c**2*d**4*e**6*x**5 +
 3631320*b*c**2*d**3*e**7*x**6 + 9099090*b*c**2*d**2*e**8*x**7 + 7837830*b
*c**2*d*e**9*x**8 + 2297295*b*c**2*e**10*x**9 + 5120*c**3*d**10 - 2560*c**
3*d**9*e*x + 1920*c**3*d**8*e**2*x**2 - 1600*c**3*d**7*e**3*x**3 + 1400*c*
*3*d**6*e**4*x**4 - 1260*c**3*d**5*e**5*x**5 + 1155*c**3*d**4*e**6*x**6 +
1038180*c**3*d**3*e**7*x**7 + 2664090*c**3*d**2*e**8*x**8 + 2333760*c**3*d
*e**9*x**9 + 692835*c**3*e**10*x**10))/(14549535*e**7)
```

3.98 $\int (d+ex)^{5/2} (bx+cx^2)^3 \, dx$

Optimal result 728
Mathematica [A] (verified) 729
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Optimal result

Integrand size = 21, antiderivative size = 248

$$\int (d+ex)^{5/2} (bx+cx^2)^3 \, dx = \frac{2d^3(cd-be)^3(d+ex)^{7/2}}{7e^7} - \frac{2d^2(cd-be)^2(2cd-be)(d+ex)^{9/2}}{3e^7} + \frac{6d(cd-be)(5c^2d^2-5bcde+b^2e^2)(d+ex)^{11/2}}{11e^7} - \frac{2(2cd-be)(10c^2d^2-10bcde+b^2e^2)(d+ex)^{13/2}}{13e^7} + \frac{2c(5c^2d^2-5bcde+b^2e^2)(d+ex)^{15/2}}{5e^7} - \frac{6c^2(2cd-be)(d+ex)^{17/2}}{17e^7} + \frac{2c^3(d+ex)^{19/2}}{19e^7}$$

output

```
2/7*d^3*(-b*e+c*d)^3*(e*x+d)^(7/2)/e^7-2/3*d^2*(-b*e+c*d)^2*(-b*e+2*c*d)*(
e*x+d)^(9/2)/e^7+6/11*d*(-b*e+c*d)*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)^(
11/2)/e^7-2/13*(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*d^2)*(e*x+d)^(13/2)
/e^7+2/5*c*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)^(15/2)/e^7-6/17*c^2*(-b*e
+2*c*d)*(e*x+d)^(17/2)/e^7+2/19*c^3*(e*x+d)^(19/2)/e^7
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.94

$$\int (d+ex)^{5/2} \left(bx+cx^2\right)^3 \, dx = \frac{2(d+ex)^{7/2} \left(1615b^3e^3(-16d^3+56d^2ex-126de^2x^2+231e^3x^3)+323b^2ce^2(128d^4-448d^3ex\right.}{}$$

input
```
Integrate[(d + e*x)^(5/2)*(b*x + c*x^2)^3,x]
```

output
```
(2*(d + e*x)^(7/2)*(1615*b^3*e^3*(-16*d^3 + 56*d^2*e*x - 126*d*e^2*x^2 + 2
31*e^3*x^3) + 323*b^2*c*e^2*(128*d^4 - 448*d^3*e*x + 1008*d^2*e^2*x^2 - 18
48*d*e^3*x^3 + 3003*e^4*x^4) + 95*b*c^2*e*(-256*d^5 + 896*d^4*e*x - 2016*d
^3*e^2*x^2 + 3696*d^2*e^3*x^3 - 6006*d*e^4*x^4 + 9009*e^5*x^5) + 5*c^3*(10
24*d^6 - 3584*d^5*e*x + 8064*d^4*e^2*x^2 - 14784*d^3*e^3*x^3 + 24024*d^2*e
^4*x^4 - 36036*d*e^5*x^5 + 51051*e^6*x^6)))/(4849845*e^7)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^3 (d+ex)^{5/2} \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{3c(d+ex)^{13/2} \left(b^2e^2-5bcde+5c^2d^2\right)}{e^6}+\frac{(d+ex)^{11/2}(2cd-be) \left(-b^2e^2+10bcde-10c^2d^2\right)}{e^6}+\frac{3d(d+ex)^{9/}}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2c(d+ex)^{15/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{5e^7}-\frac{2(d+ex)^{13/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{13e^7}+\frac{6d(d+ex)^{11/2}(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{11e^7}-\frac{6c^2(d+ex)^{17/2}(2cd-be)}{17e^7}+\frac{2d^3(d+ex)^{7/2}(cd-be)^3}{7e^7}-\frac{2d^2(d+ex)^{9/2}(cd-be)^2(2cd-be)}{3e^7}+\frac{2c^3(d+ex)^{19/2}}{19e^7}$$

input
```
Int[(d + e*x)^(5/2)*(b*x + c*x^2)^3,x]
```

output
```
(2*d^3*(c*d - b*e)^3*(d + e*x)^(7/2))/(7*e^7) - (2*d^2*(c*d - b*e)^2*(2*c*
d - b*e)*(d + e*x)^(9/2))/(3*e^7) + (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*
e + b^2*e^2)*(d + e*x)^(11/2))/(11*e^7) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 1
0*b*c*d*e + b^2*e^2)*(d + e*x)^(13/2))/(13*e^7) + (2*c*(5*c^2*d^2 - 5*b*c*
d*e + b^2*e^2)*(d + e*x)^(15/2))/(5*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^
(17/2))/(17*e^7) + (2*c^3*(d + e*x)^(19/2))/(19*e^7)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$-\frac{32(ex+d)^{\frac{7}{2}} (-\frac{231x^3 (\frac{13}{19}c^3x^3+\frac{39}{17}b\,c^2x^2+\frac{13}{5}b^2cx+b^3) e^6}{16}+\frac{63x^2 (\frac{286}{323}c^3x^3+\frac{143}{51}b\,c^2x^2+\frac{44}{15}b^2cx+b^3) d\,e^5}{8}-\frac{7x (\frac{429}{323}c^3x^3+ \cdots}{\cdots}}{\cdots}$
derivativedivides	$\frac{2c^3(ex+d)^{\frac{19}{2}}}{19}+\frac{2 (-3d\,c^3+3(be-cd)c^2)(ex+d)^{\frac{17}{2}}}{17}+\frac{2 (3c^3d^2-9d(be-cd)c^2+3(be-cd)^2c)(ex+d)^{\frac{15}{2}}}{15}+\frac{2 (-d^3c^3+9d^2(be-cd)c^2- \cdots}{\cdots}$
default	$\frac{2c^3(ex+d)^{\frac{19}{2}}}{19}-\frac{2 (3d\,c^3-3(be-cd)c^2)(ex+d)^{\frac{17}{2}}}{17}-\frac{2 (-3c^3d^2+9d(be-cd)c^2-3(be-cd)^2c)(ex+d)^{\frac{15}{2}}}{15}-\frac{2 (d^3c^3-9d^2(be-cd)c^2+9d \cdots}{\cdots}$
gosper	$-\frac{2(ex+d)^{\frac{7}{2}}(-255255x^6c^3e^6-855855x^5b\,c^2e^6+180180x^5c^3d\,e^5-969969x^4b^2c\,e^6+570570x^4b\,c^2d\,e^5-120120x^4c^3d^2e \cdots}{\cdots}$
orering	$-\frac{2(-255255x^6c^3e^6-855855x^5b\,c^2e^6+180180x^5c^3d\,e^5-969969x^4b^2c\,e^6+570570x^4b\,c^2d\,e^5-120120x^4c^3d^2e^4-37306 \cdots}{\cdots}$
trager	$-\frac{2(-255255e^9c^3x^9-855855b\,c^2e^9x^8-585585c^3d\,e^8x^8-969969b^2c\,e^9x^7-1996995b\,c^2d\,e^8x^7-345345c^3d^2e^7x^7-3730 \cdots}{\cdots}$
risch	$-\frac{2(-255255e^9c^3x^9-855855b\,c^2e^9x^8-585585c^3d\,e^8x^8-969969b^2c\,e^9x^7-1996995b\,c^2d\,e^8x^7-345345c^3d^2e^7x^7-3730 \cdots}{\cdots}$

input
```
int((e*x+d)^(5/2)*(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-32/3003*(e*x+d)^(7/2)*(-231/16*x^3*(13/19*c^3*x^3+39/17*b*c^2*x^2+13/5*b^
2*c*x+b^3)*e^6+63/8*x^2*(286/323*c^3*x^3+143/51*b*c^2*x^2+44/15*b^2*c*x+b^
3)*d*e^5-7/2*x*(429/323*c^3*x^3+66/17*b*c^2*x^2+18/5*b^2*c*x+b^3)*d^2*e^4+
d^3*(924/323*c^3*x^3+126/17*b*c^2*x^2+28/5*b^2*c*x+b^3)*e^3-8/5*c*d^4*(315
/323*c^2*x^2+35/17*c*b*x+b^2)*e^2+16/17*(14/19*c*x+b)*c^2*d^5*e-64/323*d^6
*c^3)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.72

$$\int (d+ex)^{5/2} (bx+cx^2)^3 \,dx = \frac{2 (255255\,c^3e^9x^9+5120\,c^3d^9-24320\,bc^2d^8e+41344\,b^2cd^7e^2-25840\,b^3d^6e^3+45045\,(13\,c^3de \cdots}{\cdots}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
2/4849845*(255255*c^3*e^9*x^9 + 5120*c^3*d^9 - 24320*b*c^2*d^8*e + 41344*b
^2*c*d^7*e^2 - 25840*b^3*d^6*e^3 + 45045*(13*c^3*d*e^8 + 19*b*c^2*e^9)*x^8
 + 3003*(115*c^3*d^2*e^7 + 665*b*c^2*d*e^8 + 323*b^2*c*e^9)*x^7 + 231*(5*c
^3*d^3*e^6 + 5225*b*c^2*d^2*e^7 + 10013*b^2*c*d*e^8 + 1615*b^3*e^9)*x^6 -
63*(20*c^3*d^4*e^5 - 95*b*c^2*d^3*e^6 - 22933*b^2*c*d^2*e^7 - 14535*b^3*d*
e^8)*x^5 + 35*(40*c^3*d^5*e^4 - 190*b*c^2*d^4*e^5 + 323*b^2*c*d^3*e^6 + 17
119*b^3*d^2*e^7)*x^4 - 5*(320*c^3*d^6*e^3 - 1520*b*c^2*d^5*e^4 + 2584*b^2*
c*d^4*e^5 - 1615*b^3*d^3*e^6)*x^3 + 6*(320*c^3*d^7*e^2 - 1520*b*c^2*d^6*e^
3 + 2584*b^2*c*d^5*e^4 - 1615*b^3*d^4*e^5)*x^2 - 8*(320*c^3*d^8*e - 1520*b
*c^2*d^7*e^2 + 2584*b^2*c*d^6*e^3 - 1615*b^3*d^5*e^4)*x)*sqrt(e*x + d)/e^7
```

Sympy [A] (verification not implemented)

Time = 1.04 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.50

$$\int (d+ex)^{5/2}\left(bx+cx^2\right)^3\,dx=\begin{cases}\frac{2\left(\frac{c^3(d+ex)^{\frac{19}{2}}}{19e^6}+\frac{(d+ex)^{\frac{17}{2}}\cdot\left(3bc^2e-6c^3d\right)}{17e^6}+\frac{(d+ex)^{\frac{15}{2}}\cdot\left(3b^2ce^2-15bc^2de+15c^3d^2\right)}{15e^6}+\frac{(d+ex)^{\frac{13}{2}}\left(b^3e^3-12b^2cde^2+30bc^2d^2e-20c^3d^3\right)}{13e^6}+\cdots\right.}{\cdots}\\ d^{\frac{5}{2}}\left(\frac{b^3x^4}{4}+\frac{3b^2cx^5}{5}+\frac{bc^2x^6}{2}+\frac{c^3x^7}{7}\right)\end{cases}$$

input
```
integrate((e*x+d)**(5/2)*(c*x**2+b*x)**3,x)
```

output
```
Piecewise((2*(c**3*(d + e*x)**(19/2)/(19*e**6) + (d + e*x)**(17/2)*(3*b*c*
*2*e - 6*c**3*d)/(17*e**6) + (d + e*x)**(15/2)*(3*b**2*c*e**2 - 15*b*c**2*
d*e + 15*c**3*d**2)/(15*e**6) + (d + e*x)**(13/2)*(b**3*e**3 - 12*b**2*c*d
*e**2 + 30*b*c**2*d**2*e - 20*c**3*d**3)/(13*e**6) + (d + e*x)**(11/2)*(-3
*b**3*d*e**3 + 18*b**2*c*d**2*e**2 - 30*b*c**2*d**3*e + 15*c**3*d**4)/(11*
e**6) + (d + e*x)**(9/2)*(3*b**3*d**2*e**3 - 12*b**2*c*d**3*e**2 + 15*b*c*
*2*d**4*e - 6*c**3*d**5)/(9*e**6) + (d + e*x)**(7/2)*(-b**3*d**3*e**3 + 3*
b**2*c*d**4*e**2 - 3*b*c**2*d**5*e + c**3*d**6)/(7*e**6))/e, Ne(e, 0)), (d
**(5/2)*(b**3*x**4/4 + 3*b**2*c*x**5/5 + b*c**2*x**6/2 + c**3*x**7/7), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.09

$$\int (d+ex)^{5/2} \left(bx +cx^2\right)^3 \, dx = \frac{2\left(255255\,(ex+d)^{\frac{19}{2}}c^3 - 855855\,(2\,c^3d - bc^2e)(ex+d)^{\frac{17}{2}} + 969969\,(5\,c^3d^2 - 5\,bc^2de + b^2ce^2)\right.}{}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
2/4849845*(255255*(e*x + d)^(19/2)*c^3 - 855855*(2*c^3*d - b*c^2*e)*(e*x +
 d)^(17/2) + 969969*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*(e*x + d)^(15/2)
 - 373065*(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*(e*x +
d)^(13/2) + 1322685*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 - b^3*d*
e^3)*(e*x + d)^(11/2) - 1616615*(2*c^3*d^5 - 5*b*c^2*d^4*e + 4*b^2*c*d^3*e
^2 - b^3*d^2*e^3)*(e*x + d)^(9/2) + 692835*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^
2*c*d^4*e^2 - b^3*d^3*e^3)*(e*x + d)^(7/2))/e^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1450 vs. $2(220) = 440$.

Time = 0.12 (sec) , antiderivative size = 1450, normalized size of antiderivative = 5.85

$$\int (d+ex)^{5/2} \left(bx+cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)*(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
2/4849845*(138567*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)
^(3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b^3*d^3/e^3 + 46189*(35*(e*x + d)^(9/2)
 - 180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d
^3 + 315*sqrt(e*x + d)*d^4)*b^2*c*d^3/e^4 + 46189*(35*(e*x + d)^(9/2) - 18
0*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 +
315*sqrt(e*x + d)*d^4)*b^3*d^2/e^3 + 20995*(63*(e*x + d)^(11/2) - 385*(e*x
 + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*
(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b*c^2*d^3/e^5 + 62985*(63*(e*
x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*
x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b^2*c
*d^2/e^4 + 20995*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x +
 d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*
sqrt(e*x + d)*d^5)*b^3*d/e^3 + 1615*(231*(e*x + d)^(13/2) - 1638*(e*x + d)
^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*
x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*c^3*
d^3/e^6 + 14535*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*
x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6
006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*b*c^2*d^2/e^5 + 14535*(2
31*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 -
 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(...
```

Mupad [B] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.96

$$\int (d+ex)^{5/2} \left(bx+cx^2\right)^3 \, dx = \frac{(d+e\,x)^{13/2}\,(2\,b^3\,e^3-24\,b^2\,c\,d\,e^2+60\,b\,c^2\,d^2\,e-40\,c^3\,d^3)}{13\,e^7} + \frac{2\,c^3\,(d+e\,x)^{19/2}}{19\,e^7} - \frac{(12\,c^3\,d-6\,b\,c^2\,e)\,(d+e\,x)^{17/2}}{17\,e^7} + \frac{(d+e\,x)^{15/2}\,(6\,b^2\,c\,e^2-30\,b\,c^2\,d\,e+30\,c^3\,d^2)}{15\,e^7} + \frac{(d+e\,x)^{11/2}\,(-6\,b^3\,d\,e^3+36\,b^2\,c\,d^2\,e^2-60\,b\,c^2\,d^3\,e+30\,c^3\,d^4)}{11\,e^7} - \frac{2\,d^3\,(b\,e-c\,d)^3\,(d+e\,x)^{7/2}}{7\,e^7} + \frac{2\,d^2\,(b\,e-c\,d)^2\,(b\,e-2\,c\,d)\,(d+e\,x)^{9/2}}{3\,e^7}$$

input
```
int((b*x + c*x^2)^3*(d + e*x)^(5/2),x)
```

output
```
((d + e*x)^(13/2)*(2*b^3*e^3 - 40*c^3*d^3 + 60*b*c^2*d^2*e - 24*b^2*c*d*e^
2))/(13*e^7) + (2*c^3*(d + e*x)^(19/2))/(19*e^7) - ((12*c^3*d - 6*b*c^2*e)
*(d + e*x)^(17/2))/(17*e^7) + ((d + e*x)^(15/2)*(30*c^3*d^2 + 6*b^2*c*e^2
- 30*b*c^2*d*e))/(15*e^7) + ((d + e*x)^(11/2)*(30*c^3*d^4 - 6*b^3*d*e^3 +
36*b^2*c*d^2*e^2 - 60*b*c^2*d^3*e))/(11*e^7) - (2*d^3*(b*e - c*d)^3*(d + e
*x)^(7/2))/(7*e^7) + (2*d^2*(b*e - c*d)^2*(b*e - 2*c*d)*(d + e*x)^(9/2))/(
3*e^7)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.85

$$\int (d+ex)^{5/2} (bx +cx^2)^3 \, dx = \frac{2\sqrt{ex+d}\,(255255c^3e^9x^9 + 855855b\,c^2e^9x^8 + 585585c^3d\,e^8x^8 + 969969b^2c\,e^9x^7 + 1996995b\,c^2 d}{}$$

input
```
int((e*x+d)^(5/2)*(c*x^2+b*x)^3,x)
```

output
```
(2*sqrt(d + e*x)*( - 25840*b**3*d**6*e**3 + 12920*b**3*d**5*e**4*x - 9690*
b**3*d**4*e**5*x**2 + 8075*b**3*d**3*e**6*x**3 + 599165*b**3*d**2*e**7*x**
4 + 915705*b**3*d*e**8*x**5 + 373065*b**3*e**9*x**6 + 41344*b**2*c*d**7*e*
*2 - 20672*b**2*c*d**6*e**3*x + 15504*b**2*c*d**5*e**4*x**2 - 12920*b**2*c
*d**4*e**5*x**3 + 11305*b**2*c*d**3*e**6*x**4 + 1444779*b**2*c*d**2*e**7*x
**5 + 2313003*b**2*c*d*e**8*x**6 + 969969*b**2*c*e**9*x**7 - 24320*b*c**2*
d**8*e + 12160*b*c**2*d**7*e**2*x - 9120*b*c**2*d**6*e**3*x**2 + 7600*b*c*
*2*d**5*e**4*x**3 - 6650*b*c**2*d**4*e**5*x**4 + 5985*b*c**2*d**3*e**6*x**
5 + 1206975*b*c**2*d**2*e**7*x**6 + 1996995*b*c**2*d*e**8*x**7 + 855855*b*
c**2*e**9*x**8 + 5120*c**3*d**9 - 2560*c**3*d**8*e*x + 1920*c**3*d**7*e**2
*x**2 - 1600*c**3*d**6*e**3*x**3 + 1400*c**3*d**5*e**4*x**4 - 1260*c**3*d*
*4*e**5*x**5 + 1155*c**3*d**3*e**6*x**6 + 345345*c**3*d**2*e**7*x**7 + 585
585*c**3*d*e**8*x**8 + 255255*c**3*e**9*x**9))/(4849845*e**7)
```

3.99 $\int (d+ex)^{3/2} (bx+cx^2)^3 \, dx$

Optimal result . . . 736
Mathematica [A] (verified) . . . 737
Rubi [A] (verified) . . . 737
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Optimal result

Integrand size = 21, antiderivative size = 248

$$\int (d+ex)^{3/2} (bx+cx^2)^3 \, dx = \frac{2d^3(cd-be)^3(d+ex)^{5/2}}{5e^7} - \frac{6d^2(cd-be)^2(2cd-be)(d+ex)^{7/2}}{7e^7} + \frac{2d(cd-be)(5c^2d^2-5bcde+b^2e^2)(d+ex)^{9/2}}{3e^7} - \frac{2(2cd-be)(10c^2d^2-10bcde+b^2e^2)(d+ex)^{11/2}}{11e^7} + \frac{6c(5c^2d^2-5bcde+b^2e^2)(d+ex)^{13/2}}{13e^7} - \frac{2c^2(2cd-be)(d+ex)^{15/2}}{5e^7} + \frac{2c^3(d+ex)^{17/2}}{17e^7}$$

output
```
2/5*d^3*(-b*e+c*d)^3*(e*x+d)^(5/2)/e^7-6/7*d^2*(-b*e+c*d)^2*(-b*e+2*c*d)*(
e*x+d)^(7/2)/e^7+2/3*d*(-b*e+c*d)*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)^(9
/2)/e^7-2/11*(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*d^2)*(e*x+d)^(11/2)/e
^7+6/13*c*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)^(13/2)/e^7-2/5*c^2*(-b*e+2
*c*d)*(e*x+d)^(15/2)/e^7+2/17*c^3*(e*x+d)^(17/2)/e^7
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.93

$$\int (d+ex)^{3/2} (bx+cx^2)^3 \, dx = \frac{2(d+ex)^{5/2} (221b^3e^3(-16d^3+40d^2ex-70de^2x^2+105e^3x^3)+51b^2ce^2(128d^4-320d^3ex+5$$

input
```
Integrate[(d + e*x)^(3/2)*(b*x + c*x^2)^3,x]
```

output
```
(2*(d + e*x)^(5/2)*(221*b^3*e^3*(-16*d^3 + 40*d^2*e*x - 70*d*e^2*x^2 + 105
*e^3*x^3) + 51*b^2*c*e^2*(128*d^4 - 320*d^3*e*x + 560*d^2*e^2*x^2 - 840*d*
e^3*x^3 + 1155*e^4*x^4) + 17*b*c^2*e*(-256*d^5 + 640*d^4*e*x - 1120*d^3*e^
2*x^2 + 1680*d^2*e^3*x^3 - 2310*d*e^4*x^4 + 3003*e^5*x^5) + c^3*(1024*d^6
- 2560*d^5*e*x + 4480*d^4*e^2*x^2 - 6720*d^3*e^3*x^3 + 9240*d^2*e^4*x^4 -
12012*d*e^5*x^5 + 15015*e^6*x^6)))/(255255*e^7)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+cx^2)^3 (d+ex)^{3/2} \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{3c(d+ex)^{11/2} (b^2e^2-5bcde+5c^2d^2)}{e^6} + \frac{(d+ex)^{9/2}(2cd-be) (-b^2e^2+10bcde-10c^2d^2)}{e^6} + \frac{3d(d+ex)^{7/2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{6c(d+ex)^{13/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{13e^7}-\frac{2(d+ex)^{11/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{11e^7}+\frac{2d(d+ex)^{9/2}(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{3e^7}-\frac{2c^2(d+ex)^{15/2}(2cd-be)}{5e^7}+\frac{2d^3(d+ex)^{5/2}(cd-be)^3}{5e^7}-\frac{6d^2(d+ex)^{7/2}(cd-be)^2(2cd-be)}{7e^7}+\frac{2c^3(d+ex)^{17/2}}{17e^7}$$

input
```
Int[(d + e*x)^(3/2)*(b*x + c*x^2)^3,x]
```

output
```
(2*d^3*(c*d - b*e)^3*(d + e*x)^(5/2))/(5*e^7) - (6*d^2*(c*d - b*e)^2*(2*c*
d - b*e)*(d + e*x)^(7/2))/(7*e^7) + (2*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*
e + b^2*e^2)*(d + e*x)^(9/2))/(3*e^7) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*
b*c*d*e + b^2*e^2)*(d + e*x)^(11/2))/(11*e^7) + (6*c*(5*c^2*d^2 - 5*b*c*d*
e + b^2*e^2)*(d + e*x)^(13/2))/(13*e^7) - (2*c^2*(2*c*d - b*e)*(d + e*x)^(
15/2))/(5*e^7) + (2*c^3*(d + e*x)^(17/2))/(17*e^7)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$-\frac{32(ex+d)^{\frac{5}{2}}\left(-\frac{105x^3\left(\frac{11}{17}c^3x^3+\frac{11}{5}b\,c^2x^2+\frac{33}{13}b^2cx+b^3\right)e^6}{16}+\frac{35x^2d\left(\frac{66}{85}c^3x^3+\frac{33}{13}b\,c^2x^2+\frac{36}{13}b^2cx+b^3\right)e^5}{8}-\frac{5x\left(\frac{231}{221}c^3x^3+\frac{42}{13}\right.}{}\right.}{}$
derivativedivides	$\frac{2c^3(ex+d)^{\frac{17}{2}}}{17}+\frac{2\left(-3d\,c^3+3(be-cd)c^2\right)(ex+d)^{\frac{15}{2}}}{15}+\frac{2\left(3c^3d^2-9d(be-cd)c^2+3(be-cd)^2c\right)(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left(-d^3c^3+9d^2(be-cd)c^2-\right.}{}$
default	$\frac{2c^3(ex+d)^{\frac{17}{2}}}{17}-\frac{2\left(3d\,c^3-3(be-cd)c^2\right)(ex+d)^{\frac{15}{2}}}{15}-\frac{2\left(-3c^3d^2+9d(be-cd)c^2-3(be-cd)^2c\right)(ex+d)^{\frac{13}{2}}}{13}-\frac{2\left(d^3c^3-9d^2(be-cd)c^2+9d\right.}{}$
gosper	$-\frac{2(ex+d)^{\frac{5}{2}}\left(-15015x^6c^3e^6-51051x^5b\,c^2e^6+12012x^5c^3d\,e^5-58905x^4b^2c\,e^6+39270x^4b\,c^2d\,e^5-9240x^4c^3d^2e^4-2320\right.}{}$
orering	$-\frac{2\left(-15015x^6c^3e^6-51051x^5b\,c^2e^6+12012x^5c^3d\,e^5-58905x^4b^2c\,e^6+39270x^4b\,c^2d\,e^5-9240x^4c^3d^2e^4-23205x^3b^3e^6+\right.}{}$
trager	$-\frac{2\left(-15015c^3e^8x^8-51051b\,c^2e^8x^7-18018c^3d\,e^7x^7-58905b^2c\,e^8x^6-62832b\,c^2d\,e^7x^6-231c^3d^2e^6x^6-23205b^3e^8x^5-\right.}{}$
risch	$-\frac{2\left(-15015c^3e^8x^8-51051b\,c^2e^8x^7-18018c^3d\,e^7x^7-58905b^2c\,e^8x^6-62832b\,c^2d\,e^7x^6-231c^3d^2e^6x^6-23205b^3e^8x^5-\right.}{}$

input
```
int((e*x+d)^(3/2)*(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-32/1155*(e*x+d)^(5/2)*(-105/16*x^3*(11/17*c^3*x^3+11/5*b*c^2*x^2+33/13*b^
2*c*x+b^3)*e^6+35/8*x^2*d*(66/85*c^3*x^3+33/13*b*c^2*x^2+36/13*b^2*c*x+b^3
)*e^5-5/2*x*(231/221*c^3*x^3+42/13*b*c^2*x^2+42/13*b^2*c*x+b^3)*d^2*e^4+d^
3*(420/221*c^3*x^3+70/13*b*c^2*x^2+60/13*b^2*c*x+b^3)*e^3-24/13*c*d^4*(35/
51*c^2*x^2+5/3*c*b*x+b^2)*e^2+16/13*c^2*d^5*(10/17*c*x+b)*e-64/221*d^6*c^3
)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.50

$$\int (d+ex)^{3/2}\left(bx+cx^2\right)^3\,dx=\frac{2\left(15015\,c^3e^8x^8+1024\,c^3d^8-4352\,bc^2d^7e+6528\,b^2cd^6e^2-3536\,b^3d^5e^3+3003\left(6\,c^3de^7+17\,b\right.\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
2/255255*(15015*c^3*e^8*x^8 + 1024*c^3*d^8 - 4352*b*c^2*d^7*e + 6528*b^2*c
*d^6*e^2 - 3536*b^3*d^5*e^3 + 3003*(6*c^3*d*e^7 + 17*b*c^2*e^8)*x^7 + 231*
(c^3*d^2*e^6 + 272*b*c^2*d*e^7 + 255*b^2*c*e^8)*x^6 - 21*(12*c^3*d^3*e^5 -
 51*b*c^2*d^2*e^6 - 3570*b^2*c*d*e^7 - 1105*b^3*e^8)*x^5 + 35*(8*c^3*d^4*e
^4 - 34*b*c^2*d^3*e^5 + 51*b^2*c*d^2*e^6 + 884*b^3*d*e^7)*x^4 - 5*(64*c^3*
d^5*e^3 - 272*b*c^2*d^4*e^4 + 408*b^2*c*d^3*e^5 - 221*b^3*d^2*e^6)*x^3 + 6
*(64*c^3*d^6*e^2 - 272*b*c^2*d^5*e^3 + 408*b^2*c*d^4*e^4 - 221*b^3*d^3*e^5
)*x^2 - 8*(64*c^3*d^7*e - 272*b*c^2*d^6*e^2 + 408*b^2*c*d^5*e^3 - 221*b^3*
d^4*e^4)*x)*sqrt(e*x + d)/e^7
```

Sympy [A] (verification not implemented)

Time = 0.92 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.50

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^3 dx = \begin{cases} \frac{2\left(\frac{c^3(d+ex)^{\frac{17}{2}}}{17e^6} + \frac{(d+ex)^{\frac{15}{2}}\cdot\left(3bc^2e-6c^3d\right)}{15e^6} + \frac{(d+ex)^{\frac{13}{2}}\cdot\left(3b^2ce^2-15bc^2de+15c^3d^2\right)}{13e^6} + \frac{(d+ex)^{\frac{11}{2}}\left(b^3e^3-12b^2cde^2+30bc^2d^2e-20c^3d^3\right)}{11e^6} + \cdots}{\cdots} \\ d^{\frac{3}{2}}\left(\frac{b^3x^4}{4} + \frac{3b^2cx^5}{5} + \frac{bc^2x^6}{2} + \frac{c^3x^7}{7}\right) \end{cases}$$

input
```
integrate((e*x+d)**(3/2)*(c*x**2+b*x)**3,x)
```

output
```
Piecewise((2*(c**3*(d + e*x)**(17/2)/(17*e**6) + (d + e*x)**(15/2)*(3*b*c*
*2*e - 6*c**3*d)/(15*e**6) + (d + e*x)**(13/2)*(3*b**2*c*e**2 - 15*b*c**2*
d*e + 15*c**3*d**2)/(13*e**6) + (d + e*x)**(11/2)*(b**3*e**3 - 12*b**2*c*d
*e**2 + 30*b*c**2*d**2*e - 20*c**3*d**3)/(11*e**6) + (d + e*x)**(9/2)*(-3*
b**3*d*e**3 + 18*b**2*c*d**2*e**2 - 30*b*c**2*d**3*e + 15*c**3*d**4)/(9*e*
*6) + (d + e*x)**(7/2)*(3*b**3*d**2*e**3 - 12*b**2*c*d**3*e**2 + 15*b*c**2
*d**4*e - 6*c**3*d**5)/(7*e**6) + (d + e*x)**(5/2)*(-b**3*d**3*e**3 + 3*b*
*2*c*d**4*e**2 - 3*b*c**2*d**5*e + c**3*d**6)/(5*e**6))/e, Ne(e, 0)), (d**
(3/2)*(b**3*x**4/4 + 3*b**2*c*x**5/5 + b*c**2*x**6/2 + c**3*x**7/7), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.09

$$\int (d+ex)^{3/2} (bx+cx^2)^3 \, dx = \frac{2 \left(15015 (ex+d)^{\frac{17}{2}} c^3 - 51051 (2 c^3 d - bc^2 e)(ex+d)^{\frac{15}{2}} + 58905 (5 c^3 d^2 - 5 bc^2 de + b^2 ce^2)(ex\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
2/255255*(15015*(e*x + d)^(17/2)*c^3 - 51051*(2*c^3*d - b*c^2*e)*(e*x + d)
^(15/2) + 58905*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*(e*x + d)^(13/2) - 2
3205*(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*(e*x + d)^(1
1/2) + 85085*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 - b^3*d*e^3)*(e
*x + d)^(9/2) - 109395*(2*c^3*d^5 - 5*b*c^2*d^4*e + 4*b^2*c*d^3*e^2 - b^3*
d^2*e^3)*(e*x + d)^(7/2) + 51051*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^
2 - b^3*d^3*e^3)*(e*x + d)^(5/2))/e^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. 2(220) = 440.

Time = 0.15 (sec) , antiderivative size = 1012, normalized size of antiderivative = 4.08

$$\int (d+ex)^{3/2} (bx+cx^2)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
2/765765*(21879*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(
3/2)*d^2 - 35*sqrt(e*x + d)*d^3)*b^3*d^2/e^3 + 7293*(35*(e*x + d)^(9/2) -
180*(e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3
+ 315*sqrt(e*x + d)*d^4)*b^2*c*d^2/e^4 + 4862*(35*(e*x + d)^(9/2) - 180*(e
*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*
sqrt(e*x + d)*d^4)*b^3*d/e^3 + 3315*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(
9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x +
d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b*c^2*d^2/e^5 + 6630*(63*(e*x + d)^(
11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(
5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b^2*c*d/e^4 +
 1105*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d
^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x +
d)*d^5)*b^3/e^3 + 255*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 50
05*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d
^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*c^3*d^2/e^6 + 1530
*(231*(e*x + d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^
2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(
3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*b*c^2*d/e^5 + 765*(231*(e*x + d)^(13/2)
 - 1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/
2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqr...
```

Mupad [B] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.96

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^3 \, dx = \frac{(d+e\,x)^{11/2}\,\left(2\,b^3\,e^3-24\,b^2\,c\,d\,e^2+60\,b\,c^2\,d^2\,e-40\,c^3\,d^3\right)}{11\,e^7} + \frac{2\,c^3\,(d+e\,x)^{17/2}}{17\,e^7} - \frac{\left(12\,c^3\,d-6\,b\,c^2\,e\right)\,(d+e\,x)^{15/2}}{15\,e^7} + \frac{(d+e\,x)^{13/2}\,\left(6\,b^2\,c\,e^2-30\,b\,c^2\,d\,e+30\,c^3\,d^2\right)}{13\,e^7} + \frac{(d+e\,x)^{9/2}\,\left(-6\,b^3\,d\,e^3+36\,b^2\,c\,d^2\,e^2-60\,b\,c^2\,d^3\,e+30\,c^3\,d^4\right)}{9\,e^7} - \frac{2\,d^3\,(b\,e-c\,d)^3\,(d+e\,x)^{5/2}}{5\,e^7} + \frac{6\,d^2\,(b\,e-c\,d)^2\,(b\,e-2\,c\,d)\,(d+e\,x)^{7/2}}{7\,e^7}$$

input
```
int((b*x + c*x^2)^3*(d + e*x)^(3/2),x)
```

output
```
((d + e*x)^(11/2)*(2*b^3*e^3 - 40*c^3*d^3 + 60*b*c^2*d^2*e - 24*b^2*c*d*e^
2))/(11*e^7) + (2*c^3*(d + e*x)^(17/2))/(17*e^7) - ((12*c^3*d - 6*b*c^2*e)
*(d + e*x)^(15/2))/(15*e^7) + ((d + e*x)^(13/2)*(30*c^3*d^2 + 6*b^2*c*e^2
- 30*b*c^2*d*e))/(13*e^7) + ((d + e*x)^(9/2)*(30*c^3*d^4 - 6*b^3*d*e^3 + 3
6*b^2*c*d^2*e^2 - 60*b*c^2*d^3*e))/(9*e^7) - (2*d^3*(b*e - c*d)^3*(d + e*x
)^(5/2))/(5*e^7) + (6*d^2*(b*e - c*d)^2*(b*e - 2*c*d)*(d + e*x)^(7/2))/(7*
e^7)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.61

$$\int (d+ex)^{3/2} (bx+cx^2)^3 \, dx = \frac{2\sqrt{ex+d}\,(15015c^3e^8x^8 + 51051b\,c^2e^8x^7 + 18018c^3d\,e^7x^7 + 58905b^2c\,e^8x^6 + 62832b\,c^2d\,e^7x^6 + \cdots}{\cdots}$$

input
```
int((e*x+d)^(3/2)*(c*x^2+b*x)^3,x)
```

output
```
(2*sqrt(d + e*x)*( - 3536*b**3*d**5*e**3 + 1768*b**3*d**4*e**4*x - 1326*b*
*3*d**3*e**5*x**2 + 1105*b**3*d**2*e**6*x**3 + 30940*b**3*d*e**7*x**4 + 23
205*b**3*e**8*x**5 + 6528*b**2*c*d**6*e**2 - 3264*b**2*c*d**5*e**3*x + 244
8*b**2*c*d**4*e**4*x**2 - 2040*b**2*c*d**3*e**5*x**3 + 1785*b**2*c*d**2*e*
*6*x**4 + 74970*b**2*c*d*e**7*x**5 + 58905*b**2*c*e**8*x**6 - 4352*b*c**2*
d**7*e + 2176*b*c**2*d**6*e**2*x - 1632*b*c**2*d**5*e**3*x**2 + 1360*b*c**
2*d**4*e**4*x**3 - 1190*b*c**2*d**3*e**5*x**4 + 1071*b*c**2*d**2*e**6*x**5
 + 62832*b*c**2*d*e**7*x**6 + 51051*b*c**2*e**8*x**7 + 1024*c**3*d**8 - 51
2*c**3*d**7*e*x + 384*c**3*d**6*e**2*x**2 - 320*c**3*d**5*e**3*x**3 + 280*
c**3*d**4*e**4*x**4 - 252*c**3*d**3*e**5*x**5 + 231*c**3*d**2*e**6*x**6 +
18018*c**3*d*e**7*x**7 + 15015*c**3*e**8*x**8))/(255255*e**7)
```

3.100 $\int \sqrt{d+ex}\left(bx+cx^2\right)^3\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 248

$$\begin{aligned}\int \sqrt{d+ex}\left(bx+cx^2\right)^3\,dx &= \frac{2d^3(cd-be)^3(d+ex)^{3/2}}{3e^7} \\ &\quad - \frac{6d^2(cd-be)^2(2cd-be)(d+ex)^{5/2}}{5e^7} \\ &\quad + \frac{6d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)(d+ex)^{7/2}}{7e^7} \\ &\quad - \frac{2(2cd-be)\left(10c^2d^2-10bcde+b^2e^2\right)(d+ex)^{9/2}}{9e^7} \\ &\quad + \frac{6c\left(5c^2d^2-5bcde+b^2e^2\right)(d+ex)^{11/2}}{11e^7} \\ &\quad - \frac{6c^2(2cd-be)(d+ex)^{13/2}}{13e^7} + \frac{2c^3(d+ex)^{15/2}}{15e^7}\end{aligned}$$

output
```
2/3*d^3*(-b*e+c*d)^3*(e*x+d)^(3/2)/e^7-6/5*d^2*(-b*e+c*d)^2*(-b*e+2*c*d)*(
e*x+d)^(5/2)/e^7+6/7*d*(-b*e+c*d)*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)^(7
/2)/e^7-2/9*(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*d^2)*(e*x+d)^(9/2)/e^7
+6/11*c*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)^(11/2)/e^7-6/13*c^2*(-b*e+2*
c*d)*(e*x+d)^(13/2)/e^7+2/15*c^3*(e*x+d)^(15/2)/e^7
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.93

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^3\,dx$$
$$= \frac{2(d+ex)^{3/2}\left(143b^3e^3(-16d^3+24d^2ex-30de^2x^2+35e^3x^3)+39b^2ce^2(128d^4-192d^3ex+240d^2e^2x^2-\right.}{}$$

input
```
Integrate[Sqrt[d + e*x]*(b*x + c*x^2)^3,x]
```

output
```
(2*(d + e*x)^(3/2)*(143*b^3*e^3*(-16*d^3 + 24*d^2*e*x - 30*d*e^2*x^2 + 35*
e^3*x^3) + 39*b^2*c*e^2*(128*d^4 - 192*d^3*e*x + 240*d^2*e^2*x^2 - 280*d*e
^3*x^3 + 315*e^4*x^4) + 15*b*c^2*e*(-256*d^5 + 384*d^4*e*x - 480*d^3*e^2*x
^2 + 560*d^2*e^3*x^3 - 630*d*e^4*x^4 + 693*e^5*x^5) + c^3*(1024*d^6 - 1536
*d^5*e*x + 1920*d^4*e^2*x^2 - 2240*d^3*e^3*x^3 + 2520*d^2*e^4*x^4 - 2772*d
*e^5*x^5 + 3003*e^6*x^6)))/(45045*e^7)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^3\sqrt{d+ex}\,dx$$

$$\Big\downarrow \ 1140$$

$$\int \left(\frac{3c(d+ex)^{9/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{e^6}+\frac{(d+ex)^{7/2}(2cd-be)\left(-b^2e^2+10bcde-10c^2d^2\right)}{e^6}+\frac{3d(d+ex)^{5/2}(\right.$$

$$\Big\downarrow \ 2009$$

$$\frac{6c(d+ex)^{11/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{11e^7}-\frac{2(d+ex)^{9/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{9e^7}+\frac{6d(d+ex)^{7/2}(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{7e^7}-\frac{6c^2(d+ex)^{13/2}(2cd-be)}{13e^7}+\frac{2d^3(d+ex)^{3/2}(cd-be)^3}{3e^7}-\frac{6d^2(d+ex)^{5/2}(cd-be)^2(2cd-be)}{5e^7}+\frac{2c^3(d+ex)^{15/2}}{15e^7}$$

input
```
Int[Sqrt[d + e*x]*(b*x + c*x^2)^3,x]
```

output
```
(2*d^3*(c*d - b*e)^3*(d + e*x)^(3/2))/(3*e^7) - (6*d^2*(c*d - b*e)^2*(2*c*
d - b*e)*(d + e*x)^(5/2))/(5*e^7) + (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*
e + b^2*e^2)*(d + e*x)^(7/2))/(7*e^7) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*
b*c*d*e + b^2*e^2)*(d + e*x)^(9/2))/(9*e^7) + (6*c*(5*c^2*d^2 - 5*b*c*d*e
+ b^2*e^2)*(d + e*x)^(11/2))/(11*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(13
/2))/(13*e^7) + (2*c^3*(d + e*x)^(15/2))/(15*e^7)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$-\frac{32\left(-\frac{35x^3\left(\frac{3}{5}c^3x^3+\frac{27}{13}bc^2x^2+\frac{27}{11}b^2cx+b^3\right)e^6}{16}+\frac{15x^2\left(\frac{42}{65}c^3x^3+\frac{315}{143}bc^2x^2+\frac{28}{11}b^2cx+b^3\right)de^5}{8}-\frac{3x\left(\frac{105}{143}c^3x^3+\frac{350}{143}bc^2x^2+\frac{30}{11}\cdots\right)}{2}\cdots\right)}{\cdots}$
derivativedivides	$\frac{2c^3(ex+d)^{\frac{15}{2}}}{15}+\frac{2\left(-3dc^3+3(be-cd)c^2\right)(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left(3c^3d^2-9d(be-cd)c^2+3(be-cd)^2c\right)(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(-d^3c^3+9d^2(be-cd)c^2-\cdots\right)}{\cdots}$
default	$\frac{2c^3(ex+d)^{\frac{15}{2}}}{15}-\frac{2\left(3dc^3-3(be-cd)c^2\right)(ex+d)^{\frac{13}{2}}}{13}-\frac{2\left(-3c^3d^2+9d(be-cd)c^2-3(be-cd)^2c\right)(ex+d)^{\frac{11}{2}}}{11}-\frac{2\left(d^3c^3-9d^2(be-cd)c^2+9d\cdots\right)}{\cdots}$
gosper	$-\frac{2(ex+d)^{\frac{3}{2}}\left(-3003x^6c^3e^6-10395x^5bc^2e^6+2772x^5c^3de^5-12285x^4b^2ce^6+9450x^4bc^2de^5-2520x^4c^3d^2e^4-5005x^3b\cdots\right)}{\cdots}$
orering	$-\frac{2\left(-3003x^6c^3e^6-10395x^5bc^2e^6+2772x^5c^3de^5-12285x^4b^2ce^6+9450x^4bc^2de^5-2520x^4c^3d^2e^4-5005x^3b^3e^6+1092\cdots\right)}{\cdots}$
trager	$-\frac{2\left(-3003c^3e^7x^7-10395bc^2e^7x^6-231c^3de^6x^6-12285b^2ce^7x^5-945bc^2de^6x^5+252c^3d^2e^5x^5-5005b^3e^7x^4-1365b^2c\cdots\right)}{\cdots}$
risch	$-\frac{2\left(-3003c^3e^7x^7-10395bc^2e^7x^6-231c^3de^6x^6-12285b^2ce^7x^5-945bc^2de^6x^5+252c^3d^2e^5x^5-5005b^3e^7x^4-1365b^2c\cdots\right)}{\cdots}$

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-32/315*(-35/16*x^3*(3/5*c^3*x^3+27/13*b*c^2*x^2+27/11*b^2*c*x+b^3)*e^6+15
/8*x^2*(42/65*c^3*x^3+315/143*b*c^2*x^2+28/11*b^2*c*x+b^3)*d*e^5-3/2*x*(10
5/143*c^3*x^3+350/143*b*c^2*x^2+30/11*b^2*c*x+b^3)*d^2*e^4+d^3*(140/143*c^
3*x^3+450/143*b*c^2*x^2+36/11*b^2*c*x+b^3)*e^3-24/11*c*d^4*(5/13*c^2*x^2+1
5/13*c*b*x+b^2)*e^2+240/143*(2/5*c*x+b)*c^2*d^5*e-64/143*d^6*c^3)*(e*x+d)^
(3/2)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.29

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^3\,dx$$

$$=\frac{2\left(3003\,c^3e^7x^7+1024\,c^3d^7-3840\,bc^2d^6e+4992\,b^2cd^5e^2-2288\,b^3d^4e^3+231\left(c^3de^6+45\,bc^2e^7\right)x^6-63\,(\cdots\right)}{\cdots}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
2/45045*(3003*c^3*e^7*x^7 + 1024*c^3*d^7 - 3840*b*c^2*d^6*e + 4992*b^2*c*d
^5*e^2 - 2288*b^3*d^4*e^3 + 231*(c^3*d*e^6 + 45*b*c^2*e^7)*x^6 - 63*(4*c^3
*d^2*e^5 - 15*b*c^2*d*e^6 - 195*b^2*c*e^7)*x^5 + 35*(8*c^3*d^3*e^4 - 30*b*
c^2*d^2*e^5 + 39*b^2*c*d*e^6 + 143*b^3*e^7)*x^4 - 5*(64*c^3*d^4*e^3 - 240*
b*c^2*d^3*e^4 + 312*b^2*c*d^2*e^5 - 143*b^3*d*e^6)*x^3 + 6*(64*c^3*d^5*e^2
 - 240*b*c^2*d^4*e^3 + 312*b^2*c*d^3*e^4 - 143*b^3*d^2*e^5)*x^2 - 8*(64*c^
3*d^6*e - 240*b*c^2*d^5*e^2 + 312*b^2*c*d^4*e^3 - 143*b^3*d^3*e^4)*x)*sqrt
(e*x + d)/e^7
```

Sympy [A] (verification not implemented)

Time = 0.83 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.50

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^3\,dx = \begin{cases} \frac{2\left(\frac{c^3(d+ex)^{\frac{15}{2}}}{15e^6}+\frac{(d+ex)^{\frac{13}{2}}\cdot\left(3bc^2e-6c^3d\right)}{13e^6}+\frac{(d+ex)^{\frac{11}{2}}\cdot\left(3b^2ce^2-15bc^2de+15c^3d^2\right)}{11e^6}+\frac{(d+ex)^{\frac{9}{2}}\left(b^3e^3-12b^2cde^2+30bc^2d^2e-20c^3d^3\right)}{9e^6}+\frac{(d+ex)^{\frac{7}{2}}\left(-3b^3d\cdots\right.}{}\cdots\right)}{e} \\ \sqrt{d}\left(\frac{b^3x^4}{4}+\frac{3b^2cx^5}{5}+\frac{bc^2x^6}{2}+\frac{c^3x^7}{7}\right) \end{cases}$$

input
```
integrate((e*x+d)**(1/2)*(c*x**2+b*x)**3,x)
```

output
```
Piecewise((2*(c**3*(d + e*x)**(15/2)/(15*e**6) + (d + e*x)**(13/2)*(3*b*c*
*2*e - 6*c**3*d)/(13*e**6) + (d + e*x)**(11/2)*(3*b**2*c*e**2 - 15*b*c**2*
d*e + 15*c**3*d**2)/(11*e**6) + (d + e*x)**(9/2)*(b**3*e**3 - 12*b**2*c*d*
e**2 + 30*b*c**2*d**2*e - 20*c**3*d**3)/(9*e**6) + (d + e*x)**(7/2)*(-3*b*
*3*d*e**3 + 18*b**2*c*d**2*e**2 - 30*b*c**2*d**3*e + 15*c**3*d**4)/(7*e**6
) + (d + e*x)**(5/2)*(3*b**3*d**2*e**3 - 12*b**2*c*d**3*e**2 + 15*b*c**2*d
**4*e - 6*c**3*d**5)/(5*e**6) + (d + e*x)**(3/2)*(-b**3*d**3*e**3 + 3*b**2
*c*d**4*e**2 - 3*b*c**2*d**5*e + c**3*d**6)/(3*e**6))/e, Ne(e, 0)), (sqrt(
d)*(b**3*x**4/4 + 3*b**2*c*x**5/5 + b*c**2*x**6/2 + c**3*x**7/7), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.09

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^3\,dx$$
$$= \frac{2\left(3003\,(ex+d)^{\frac{15}{2}}c^3 - 10395\,(2\,c^3d - bc^2e)(ex+d)^{\frac{13}{2}} + 12285\,(5\,c^3d^2 - 5\,bc^2de + b^2ce^2)(ex+d)^{\frac{11}{2}} - 50\right.}{}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
2/45045*(3003*(e*x + d)^(15/2)*c^3 - 10395*(2*c^3*d - b*c^2*e)*(e*x + d)^(
13/2) + 12285*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*(e*x + d)^(11/2) - 500
5*(20*c^3*d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*(e*x + d)^(9/2)
 + 19305*(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 - b^3*d*e^3)*(e*x +
 d)^(7/2) - 27027*(2*c^3*d^5 - 5*b*c^2*d^4*e + 4*b^2*c*d^3*e^2 - b^3*d^2*e
^3)*(e*x + d)^(5/2) + 15015*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b
^3*d^3*e^3)*(e*x + d)^(3/2))/e^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 622 vs. 2(220) = 440.

Time = 0.12 (sec) , antiderivative size = 622, normalized size of antiderivative = 2.51

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
2/45045*(1287*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/
2)*d^2 - 35*sqrt(e*x + d)*d^3)*b^3*d/e^3 + 429*(35*(e*x + d)^(9/2) - 180*(
e*x + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315
*sqrt(e*x + d)*d^4)*b^2*c*d/e^4 + 143*(35*(e*x + d)^(9/2) - 180*(e*x + d)^
(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sqrt(e*x
 + d)*d^4)*b^3/e^3 + 195*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)*d + 99
0*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(3/2)*d^
4 - 693*sqrt(e*x + d)*d^5)*b*c^2*d/e^5 + 195*(63*(e*x + d)^(11/2) - 385*(e
*x + d)^(9/2)*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 115
5*(e*x + d)^(3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b^2*c/e^4 + 15*(231*(e*x +
d)^(13/2) - 1638*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x
 + d)^(7/2)*d^3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 30
03*sqrt(e*x + d)*d^6)*c^3*d/e^6 + 45*(231*(e*x + d)^(13/2) - 1638*(e*x + d
)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^3 + 9009*(e
*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x + d)*d^6)*b*c
^2/e^5 + 7*(429*(e*x + d)^(15/2) - 3465*(e*x + d)^(13/2)*d + 12285*(e*x +
d)^(11/2)*d^2 - 25025*(e*x + d)^(9/2)*d^3 + 32175*(e*x + d)^(7/2)*d^4 - 27
027*(e*x + d)^(5/2)*d^5 + 15015*(e*x + d)^(3/2)*d^6 - 6435*sqrt(e*x + d)*d
^7)*c^3/e^6)/e
```

Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.96

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^3\,dx = \frac{(d+e\,x)^{9/2}\,(2\,b^3\,e^3-24\,b^2\,c\,d\,e^2+60\,b\,c^2\,d^2\,e-40\,c^3\,d^3)}{9\,e^7}+\frac{2\,c^3\,(d+e\,x)^{15/2}}{15\,e^7}-\frac{(12\,c^3\,d-6\,b\,c^2\,e)\,(d+e\,x)^{13/2}}{13\,e^7}+\frac{(d+e\,x)^{11/2}\,(6\,b^2\,c\,e^2-30\,b\,c^2\,d\,e+30\,c^3\,d^2)}{11\,e^7}+\frac{(d+e\,x)^{7/2}\,(-6\,b^3\,d\,e^3+36\,b^2\,c\,d^2\,e^2-60\,b\,c^2\,d^3\,e+30\,c^3\,d^4)}{7\,e^7}-\frac{2\,d^3\,(b\,e-c\,d)^3\,(d+e\,x)^{3/2}}{3\,e^7}+\frac{6\,d^2\,(b\,e-c\,d)^2\,(b\,e-2\,c\,d)\,(d+e\,x)^{5/2}}{5\,e^7}$$

input
```
int((b*x + c*x^2)^3*(d + e*x)^(1/2),x)
```

output
```
((d + e*x)^(9/2)*(2*b^3*e^3 - 40*c^3*d^3 + 60*b*c^2*d^2*e - 24*b^2*c*d*e^2
))/(9*e^7) + (2*c^3*(d + e*x)^(15/2))/(15*e^7) - ((12*c^3*d - 6*b*c^2*e)*(
d + e*x)^(13/2))/(13*e^7) + ((d + e*x)^(11/2)*(30*c^3*d^2 + 6*b^2*c*e^2 -
30*b*c^2*d*e))/(11*e^7) + ((d + e*x)^(7/2)*(30*c^3*d^4 - 6*b^3*d*e^3 + 36*
b^2*c*d^2*e^2 - 60*b*c^2*d^3*e))/(7*e^7) - (2*d^3*(b*e - c*d)^3*(d + e*x)^
(3/2))/(3*e^7) + (6*d^2*(b*e - c*d)^2*(b*e - 2*c*d)*(d + e*x)^(5/2))/(5*e^
7)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.38

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^3\,dx$$
$$= \frac{2\sqrt{ex+d}\,(3003c^3e^7x^7+10395b\,c^2e^7x^6+231c^3d\,e^6x^6+12285b^2c\,e^7x^5+945b\,c^2d\,e^6x^5-252c^3d^2e^5x^5+\cdots}{}$$

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x)^3,x)
```

output
```
(2*sqrt(d + e*x)*( - 2288*b**3*d**4*e**3 + 1144*b**3*d**3*e**4*x - 858*b**
3*d**2*e**5*x**2 + 715*b**3*d*e**6*x**3 + 5005*b**3*e**7*x**4 + 4992*b**2*
c*d**5*e**2 - 2496*b**2*c*d**4*e**3*x + 1872*b**2*c*d**3*e**4*x**2 - 1560*
b**2*c*d**2*e**5*x**3 + 1365*b**2*c*d*e**6*x**4 + 12285*b**2*c*e**7*x**5 -
 3840*b*c**2*d**6*e + 1920*b*c**2*d**5*e**2*x - 1440*b*c**2*d**4*e**3*x**2
 + 1200*b*c**2*d**3*e**4*x**3 - 1050*b*c**2*d**2*e**5*x**4 + 945*b*c**2*d*
e**6*x**5 + 10395*b*c**2*e**7*x**6 + 1024*c**3*d**7 - 512*c**3*d**6*e*x +
384*c**3*d**5*e**2*x**2 - 320*c**3*d**4*e**3*x**3 + 280*c**3*d**3*e**4*x**
4 - 252*c**3*d**2*e**5*x**5 + 231*c**3*d*e**6*x**6 + 3003*c**3*e**7*x**7))
/(45045*e**7)
```

3.101 $\int \frac{(bx+cx^2)^3}{\sqrt{d+ex}}\,dx$

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Mathematica [A] (verified) 753
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Optimal result

Integrand size = 21, antiderivative size = 244

$$\begin{aligned}\int \frac{(bx+cx^2)^3}{\sqrt{d+ex}}\,dx &= \frac{2d^3(cd-be)^3\sqrt{d+ex}}{e^7}-\frac{2d^2(cd-be)^2(2cd-be)(d+ex)^{3/2}}{e^7}\\ &+\frac{6d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)(d+ex)^{5/2}}{5e^7}\\ &-\frac{2(2cd-be)\left(10c^2d^2-10bcde+b^2e^2\right)(d+ex)^{7/2}}{7e^7}\\ &+\frac{2c\left(5c^2d^2-5bcde+b^2e^2\right)(d+ex)^{9/2}}{3e^7}\\ &-\frac{6c^2(2cd-be)(d+ex)^{11/2}}{11e^7}+\frac{2c^3(d+ex)^{13/2}}{13e^7}\end{aligned}$$

output
```
2*d^3*(-b*e+c*d)^3*(e*x+d)^(1/2)/e^7-2*d^2*(-b*e+c*d)^2*(-b*e+2*c*d)*(e*x+
d)^(3/2)/e^7+6/5*d*(-b*e+c*d)*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)^(5/2)/
e^7-2/7*(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*d^2)*(e*x+d)^(7/2)/e^7+2/3
*c*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)^(9/2)/e^7-6/11*c^2*(-b*e+2*c*d)*(
e*x+d)^(11/2)/e^7+2/13*c^3*(e*x+d)^(13/2)/e^7
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.95

$$\int \frac{(bx + cx^2)^3}{\sqrt{d + ex}} dx$$
$$= \frac{2\sqrt{d + ex}(429b^3e^3(-16d^3 + 8d^2ex - 6de^2x^2 + 5e^3x^3) + 143b^2ce^2(128d^4 - 64d^3ex + 48d^2e^2x^2 - 40de^3x^3$$

input
```
Integrate[(b*x + c*x^2)^3/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(429*b^3*e^3*(-16*d^3 + 8*d^2*e*x - 6*d*e^2*x^2 + 5*e^3*x
^3) + 143*b^2*c*e^2*(128*d^4 - 64*d^3*e*x + 48*d^2*e^2*x^2 - 40*d*e^3*x^3
+ 35*e^4*x^4) + 65*b*c^2*e*(-256*d^5 + 128*d^4*e*x - 96*d^3*e^2*x^2 + 80*d
^2*e^3*x^3 - 70*d*e^4*x^4 + 63*e^5*x^5) + 5*c^3*(1024*d^6 - 512*d^5*e*x +
384*d^4*e^2*x^2 - 320*d^3*e^3*x^3 + 280*d^2*e^4*x^4 - 252*d*e^5*x^5 + 231*
e^6*x^6)))/(15015*e^7)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx + cx^2)^3}{\sqrt{d + ex}} dx$$
$$\Big\downarrow \text{1140}$$
$$\int \left(\frac{3c(d + ex)^{7/2}(b^2e^2 - 5bcde + 5c^2d^2)}{e^6} + \frac{(d + ex)^{5/2}(2cd - be)(-b^2e^2 + 10bcde - 10c^2d^2)}{e^6} + \frac{3d(d + ex)^{3/2}(\right.$$
$$\Big\downarrow \text{2009}$$

$$\frac{2c(d+ex)^{9/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{3e^7}-\frac{2(d+ex)^{7/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{7e^7}+\frac{6d(d+ex)^{5/2}(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{5e^7}-\frac{6c^2(d+ex)^{11/2}(2cd-be)}{11e^7}+\frac{2d^3\sqrt{d+ex}(cd-be)^3}{e^7}-\frac{2d^2(d+ex)^{3/2}(cd-be)^2(2cd-be)}{e^7}+\frac{2c^3(d+ex)^{13/2}}{13e^7}$$

input
```
Int[(b*x + c*x^2)^3/Sqrt[d + e*x],x]
```

output
```
(2*d^3*(c*d - b*e)^3*Sqrt[d + e*x])/e^7 - (2*d^2*(c*d - b*e)^2*(2*c*d - b*
e)*(d + e*x)^(3/2))/e^7 + (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^
2)*(d + e*x)^(5/2))/(5*e^7) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e +
b^2*e^2)*(d + e*x)^(7/2))/(7*e^7) + (2*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)
*(d + e*x)^(9/2))/(3*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(11/2))/(11*e^7
) + (2*c^3*(d + e*x)^(13/2))/(13*e^7)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.85

method	result
pseudoelliptic	$-\frac{32\left(-\frac{5\left(\frac{7}{13}c^3x^3+\frac{21}{11}b\,c^2x^2+\frac{7}{3}b^2cx+b^3\right)x^3e^6}{16}+\frac{3x^2d\left(\frac{70}{143}c^3x^3+\frac{175}{99}b\,c^2x^2+\frac{20}{9}b^2cx+b^3\right)e^5}{8}-\frac{x\left(\frac{175}{429}c^3x^3+\frac{50}{33}b\,c^2x^2+2b^2cx\right.}{2}\right.}{3}$
derivativedivides	$\frac{\frac{2c^3(ex+d)^{\frac{13}{2}}}{13}+\frac{2\left(-3d\,c^3+3(be-cd)c^2\right)(ex+d)^{\frac{11}{2}}}{11}+\frac{2\left(3c^3d^2-9d(be-cd)c^2+3(be-cd)^2c\right)(ex+d)^{\frac{9}{2}}}{9}+\frac{2\left(-d^3c^3+9d^2(be-cd)c^2-9d\right.}{}}{}$
default	$\frac{\frac{2c^3(ex+d)^{\frac{13}{2}}}{13}-\frac{2\left(3d\,c^3-3(be-cd)c^2\right)(ex+d)^{\frac{11}{2}}}{11}-\frac{2\left(-3c^3d^2+9d(be-cd)c^2-3(be-cd)^2c\right)(ex+d)^{\frac{9}{2}}}{9}-\frac{2\left(d^3c^3-9d^2(be-cd)c^2+9d(\right.}{}}{}$
gosper	$-\frac{2(-1155x^6c^3e^6-4095x^5b\,c^2e^6+1260x^5c^3d\,e^5-5005x^4b^2c\,e^6+4550x^4b\,c^2d\,e^5-1400x^4c^3d^2e^4-2145x^3b^3e^6+5720x^3}{}$
trager	$-\frac{2(-1155x^6c^3e^6-4095x^5b\,c^2e^6+1260x^5c^3d\,e^5-5005x^4b^2c\,e^6+4550x^4b\,c^2d\,e^5-1400x^4c^3d^2e^4-2145x^3b^3e^6+5720x^3}{}$
risch	$-\frac{2(-1155x^6c^3e^6-4095x^5b\,c^2e^6+1260x^5c^3d\,e^5-5005x^4b^2c\,e^6+4550x^4b\,c^2d\,e^5-1400x^4c^3d^2e^4-2145x^3b^3e^6+5720x^3}{}$
orering	$-\frac{2(-1155x^6c^3e^6-4095x^5b\,c^2e^6+1260x^5c^3d\,e^5-5005x^4b^2c\,e^6+4550x^4b\,c^2d\,e^5-1400x^4c^3d^2e^4-2145x^3b^3e^6+5720x^3}{}$

input
```
int((c*x^2+b*x)^3/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-32/35*(-5/16*(7/13*c^3*x^3+21/11*b*c^2*x^2+7/3*b^2*c*x+b^3)*x^3*e^6+3/8*x
^2*d*(70/143*c^3*x^3+175/99*b*c^2*x^2+20/9*b^2*c*x+b^3)*e^5-1/2*x*(175/429
*c^3*x^3+50/33*b*c^2*x^2+2*b^2*c*x+b^3)*d^2*e^4+d^3*(100/429*c^3*x^3+10/11
*b*c^2*x^2+4/3*b^2*c*x+b^3)*e^3-8/3*c*d^4*(15/143*c^2*x^2+5/11*c*b*x+b^2)*
e^2+80/33*c^2*(2/13*c*x+b)*d^5*e-320/429*d^6*c^3)*(e*x+d)^(1/2)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.11

$$\int \frac{(bx+cx^2)^3}{\sqrt{d+ex}}\,dx$$
$$=\frac{2\,(1155\,c^3e^6x^6+5120\,c^3d^6-16640\,bc^2d^5e+18304\,b^2cd^4e^2-6864\,b^3d^3e^3-315\,(4\,c^3de^5-13\,bc^2e^6)x^5+}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/15015*(1155*c^3*e^6*x^6 + 5120*c^3*d^6 - 16640*b*c^2*d^5*e + 18304*b^2*c
*d^4*e^2 - 6864*b^3*d^3*e^3 - 315*(4*c^3*d*e^5 - 13*b*c^2*e^6)*x^5 + 35*(4
0*c^3*d^2*e^4 - 130*b*c^2*d*e^5 + 143*b^2*c*e^6)*x^4 - 5*(320*c^3*d^3*e^3
- 1040*b*c^2*d^2*e^4 + 1144*b^2*c*d*e^5 - 429*b^3*e^6)*x^3 + 6*(320*c^3*d^
4*e^2 - 1040*b*c^2*d^3*e^3 + 1144*b^2*c*d^2*e^4 - 429*b^3*d*e^5)*x^2 - 8*(
320*c^3*d^5*e - 1040*b*c^2*d^4*e^2 + 1144*b^2*c*d^3*e^3 - 429*b^3*d^2*e^4)
*x)*sqrt(e*x + d)/e^7
```

Sympy [A] (verification not implemented)

Time = 0.76 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.51

$$\int \frac{(bx+cx^2)^3}{\sqrt{d+ex}}\,dx$$
$$= \begin{cases} \frac{2\left(\frac{c^3(d+ex)^{\frac{13}{2}}}{13e^6}+\frac{(d+ex)^{\frac{11}{2}}\cdot\left(3bc^2e-6c^3d\right)}{11e^6}+\frac{(d+ex)^{\frac{9}{2}}\cdot\left(3b^2ce^2-15bc^2de+15c^3d^2\right)}{9e^6}+\frac{(d+ex)^{\frac{7}{2}}\left(b^3e^3-12b^2cde^2+30bc^2d^2e-20c^3d^3\right)}{7e^6}+\frac{(d+ex)^{\frac{5}{2}}\left(-3b^3de\right.}{}\right.}{e} \\ \frac{\frac{b^3x^4}{4}+\frac{3b^2cx^5}{5}+\frac{bc^2x^6}{2}+\frac{c^3x^7}{7}}{\sqrt{d}} \end{cases}$$

input
```
integrate((c*x**2+b*x)**3/(e*x+d)**(1/2),x)
```

output
```
Piecewise((2*(c**3*(d + e*x)**(13/2)/(13*e**6) + (d + e*x)**(11/2)*(3*b*c*
*2*e - 6*c**3*d)/(11*e**6) + (d + e*x)**(9/2)*(3*b**2*c*e**2 - 15*b*c**2*d
*e + 15*c**3*d**2)/(9*e**6) + (d + e*x)**(7/2)*(b**3*e**3 - 12*b**2*c*d*e*
*2 + 30*b*c**2*d**2*e - 20*c**3*d**3)/(7*e**6) + (d + e*x)**(5/2)*(-3*b**3
*d*e**3 + 18*b**2*c*d**2*e**2 - 30*b*c**2*d**3*e + 15*c**3*d**4)/(5*e**6)
+ (d + e*x)**(3/2)*(3*b**3*d**2*e**3 - 12*b**2*c*d**3*e**2 + 15*b*c**2*d**
4*e - 6*c**3*d**5)/(3*e**6) + sqrt(d + e*x)*(-b**3*d**3*e**3 + 3*b**2*c*d*
*4*e**2 - 3*b*c**2*d**5*e + c**3*d**6)/e**6)/e, Ne(e, 0)), ((b**3*x**4/4 +
 3*b**2*c*x**5/5 + b*c**2*x**6/2 + c**3*x**7/7)/sqrt(d), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.18

$$\int \frac{(bx+cx^2)^3}{\sqrt{d+ex}}\,dx = \frac{2\left(\frac{429\left(5(ex+d)^{\frac{7}{2}}-21(ex+d)^{\frac{5}{2}}d+35(ex+d)^{\frac{3}{2}}d^2-35\sqrt{ex+d}d^3\right)b^3}{e^3}+\frac{143\left(35(ex+d)^{\frac{9}{2}}-180(ex+d)^{\frac{7}{2}}d+378(ex+d)^{\frac{5}{2}}d^2-420(ex+d)^{\frac{3}{2}}d^3\cdots\right.}{e^4}\cdots\right.}{\cdots}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
2/15015*(429*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2
)*d^2 - 35*sqrt(e*x + d)*d^3)*b^3/e^3 + 143*(35*(e*x + d)^(9/2) - 180*(e*x
 + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sq
rt(e*x + d)*d^4)*b^2*c/e^4 + 65*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)
*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(
3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b*c^2/e^5 + 5*(231*(e*x + d)^(13/2) - 16
38*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^
3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x +
d)*d^6)*c^3/e^6)/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.18

$$\int \frac{(bx+cx^2)^3}{\sqrt{d+ex}}\,dx = \frac{2\left(\frac{429\left(5(ex+d)^{\frac{7}{2}}-21(ex+d)^{\frac{5}{2}}d+35(ex+d)^{\frac{3}{2}}d^2-35\sqrt{ex+d}d^3\right)b^3}{e^3}+\frac{143\left(35(ex+d)^{\frac{9}{2}}-180(ex+d)^{\frac{7}{2}}d+378(ex+d)^{\frac{5}{2}}d^2-420(ex+d)^{\frac{3}{2}}d^3\cdots\right.}{e^4}\cdots\right.}{\cdots}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
2/15015*(429*(5*(e*x + d)^(7/2) - 21*(e*x + d)^(5/2)*d + 35*(e*x + d)^(3/2
)*d^2 - 35*sqrt(e*x + d)*d^3)*b^3/e^3 + 143*(35*(e*x + d)^(9/2) - 180*(e*x
 + d)^(7/2)*d + 378*(e*x + d)^(5/2)*d^2 - 420*(e*x + d)^(3/2)*d^3 + 315*sq
rt(e*x + d)*d^4)*b^2*c/e^4 + 65*(63*(e*x + d)^(11/2) - 385*(e*x + d)^(9/2)
*d + 990*(e*x + d)^(7/2)*d^2 - 1386*(e*x + d)^(5/2)*d^3 + 1155*(e*x + d)^(
3/2)*d^4 - 693*sqrt(e*x + d)*d^5)*b*c^2/e^5 + 5*(231*(e*x + d)^(13/2) - 16
38*(e*x + d)^(11/2)*d + 5005*(e*x + d)^(9/2)*d^2 - 8580*(e*x + d)^(7/2)*d^
3 + 9009*(e*x + d)^(5/2)*d^4 - 6006*(e*x + d)^(3/2)*d^5 + 3003*sqrt(e*x +
d)*d^6)*c^3/e^6)/e
```

Mupad [B] (verification not implemented)

Time = 5.40 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.98

$$\int \frac{(bx+cx^2)^3}{\sqrt{d+ex}}\,dx = \frac{(d+e\,x)^{7/2}\,(2\,b^3\,e^3-24\,b^2\,c\,d\,e^2+60\,b\,c^2\,d^2\,e-40\,c^3\,d^3)}{7\,e^7} + \frac{2\,c^3\,(d+e\,x)^{13/2}}{13\,e^7} - \frac{(12\,c^3\,d-6\,b\,c^2\,e)\,(d+e\,x)^{11/2}}{11\,e^7} + \frac{(d+e\,x)^{9/2}\,(6\,b^2\,c\,e^2-30\,b\,c^2\,d\,e+30\,c^3\,d^2)}{9\,e^7} + \frac{(d+e\,x)^{5/2}\,(-6\,b^3\,d\,e^3+36\,b^2\,c\,d^2\,e^2-60\,b\,c^2\,d^3\,e+30\,c^3\,d^4)}{5\,e^7} - \frac{2\,d^3\,(b\,e-c\,d)^3\,\sqrt{d+e\,x}}{e^7} + \frac{2\,d^2\,(b\,e-c\,d)^2\,(b\,e-2\,c\,d)\,(d+e\,x)^{3/2}}{e^7}$$

input
```
int((b*x + c*x^2)^3/(d + e*x)^(1/2),x)
```

output
```
((d + e*x)^(7/2)*(2*b^3*e^3 - 40*c^3*d^3 + 60*b*c^2*d^2*e - 24*b^2*c*d*e^2
))/(7*e^7) + (2*c^3*(d + e*x)^(13/2))/(13*e^7) - ((12*c^3*d - 6*b*c^2*e)*(
d + e*x)^(11/2))/(11*e^7) + ((d + e*x)^(9/2)*(30*c^3*d^2 + 6*b^2*c*e^2 - 3
0*b*c^2*d*e))/(9*e^7) + ((d + e*x)^(5/2)*(30*c^3*d^4 - 6*b^3*d*e^3 + 36*b^
2*c*d^2*e^2 - 60*b*c^2*d^3*e))/(5*e^7) - (2*d^3*(b*e - c*d)^3*(d + e*x)^(1
/2))/e^7 + (2*d^2*(b*e - c*d)^2*(b*e - 2*c*d)*(d + e*x)^(3/2))/e^7
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.16

$$\int \frac{(bx+cx^2)^3}{\sqrt{d+ex}}\,dx$$
$$= \frac{2\sqrt{ex+d}\,(1155c^3e^6x^6+4095b\,c^2e^6x^5-1260c^3d\,e^5x^5+5005b^2c\,e^6x^4-4550b\,c^2d\,e^5x^4+1400c^3d^2e^4x^4+}{}$$

input
```
int((c*x^2+b*x)^3/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(d + e*x)*( - 6864*b**3*d**3*e**3 + 3432*b**3*d**2*e**4*x - 2574*b*
*3*d*e**5*x**2 + 2145*b**3*e**6*x**3 + 18304*b**2*c*d**4*e**2 - 9152*b**2*
c*d**3*e**3*x + 6864*b**2*c*d**2*e**4*x**2 - 5720*b**2*c*d*e**5*x**3 + 500
5*b**2*c*e**6*x**4 - 16640*b*c**2*d**5*e + 8320*b*c**2*d**4*e**2*x - 6240*
b*c**2*d**3*e**3*x**2 + 5200*b*c**2*d**2*e**4*x**3 - 4550*b*c**2*d*e**5*x*
*4 + 4095*b*c**2*e**6*x**5 + 5120*c**3*d**6 - 2560*c**3*d**5*e*x + 1920*c*
*3*d**4*e**2*x**2 - 1600*c**3*d**3*e**3*x**3 + 1400*c**3*d**2*e**4*x**4 -
1260*c**3*d*e**5*x**5 + 1155*c**3*e**6*x**6))/(15015*e**7)
```

3.102 $\int \frac{(bx+cx^2)^3}{(d+ex)^{3/2}}\,dx$

Optimal result 760
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Rubi [A] (verified) 761
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Optimal result

Integrand size = 21, antiderivative size = 242

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{3/2}}\,dx = -\frac{2d^3(cd-be)^3}{e^7\sqrt{d+ex}} - \frac{6d^2(cd-be)^2(2cd-be)\sqrt{d+ex}}{e^7} + \frac{2d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)(d+ex)^{3/2}}{e^7} - \frac{2(2cd-be)\left(10c^2d^2-10bcde+b^2e^2\right)(d+ex)^{5/2}}{5e^7} + \frac{6c\left(5c^2d^2-5bcde+b^2e^2\right)(d+ex)^{7/2}}{7e^7} - \frac{2c^2(2cd-be)(d+ex)^{9/2}}{3e^7} + \frac{2c^3(d+ex)^{11/2}}{11e^7}$$

output
```
-2*d^3*(-b*e+c*d)^3/e^7/(e*x+d)^(1/2)-6*d^2*(-b*e+c*d)^2*(-b*e+2*c*d)*(e*x
+d)^(1/2)/e^7+2*d*(-b*e+c*d)*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)^(3/2)/e
^7-2/5*(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*d^2)*(e*x+d)^(5/2)/e^7+6/7*
c*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)^(7/2)/e^7-2/3*c^2*(-b*e+2*c*d)*(e*
x+d)^(9/2)/e^7+2/11*c^3*(e*x+d)^(11/2)/e^7
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.95

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{3/2}}\,dx = \frac{2(231b^3e^3(16d^3+8d^2ex-2de^2x^2+e^3x^3)+99b^2ce^2(-128d^4-64d^3ex+16d^2e^2x^2-8$$

input
```
Integrate[(b*x + c*x^2)^3/(d + e*x)^(3/2),x]
```

output
```
(2*(231*b^3*e^3*(16*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3) + 99*b^2*c*e^
2*(-128*d^4 - 64*d^3*e*x + 16*d^2*e^2*x^2 - 8*d*e^3*x^3 + 5*e^4*x^4) + 55*
b*c^2*e*(256*d^5 + 128*d^4*e*x - 32*d^3*e^2*x^2 + 16*d^2*e^3*x^3 - 10*d*e^
4*x^4 + 7*e^5*x^5) - 5*c^3*(1024*d^6 + 512*d^5*e*x - 128*d^4*e^2*x^2 + 64*
d^3*e^3*x^3 - 40*d^2*e^4*x^4 + 28*d*e^5*x^5 - 21*e^6*x^6)))/(1155*e^7*Sqrt
[d + e*x])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{3c(d+ex)^{5/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{e^6}+\frac{(d+ex)^{3/2}(2cd-be)\left(-b^2e^2+10bcde-10c^2d^2\right)}{e^6}+\frac{3d\sqrt{d+ex}(cd$$

$$\Big\downarrow \text{2009}$$

$$\frac{6c(d+ex)^{7/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{7e^7}-\frac{2(d+ex)^{5/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{5e^7}+\frac{2d(d+ex)^{3/2}(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{e^7}-\frac{2c^2(d+ex)^{9/2}(2cd-be)}{3e^7}-\frac{2d^3(cd-be)^3}{e^7\sqrt{d+ex}}-\frac{6d^2\sqrt{d+ex}(cd-be)^2(2cd-be)}{e^7}+\frac{2c^3(d+ex)^{11/2}}{11e^7}$$

input
```
Int[(b*x + c*x^2)^3/(d + e*x)^(3/2),x]
```

output
```
(-2*d^3*(c*d - b*e)^3)/(e^7*Sqrt[d + e*x]) - (6*d^2*(c*d - b*e)^2*(2*c*d -
 b*e)*Sqrt[d + e*x])/e^7 + (2*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e + b^2*e
^2)*(d + e*x)^(3/2))/e^7 - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e + b^2
*e^2)*(d + e*x)^(5/2))/(5*e^7) + (6*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d
 + e*x)^(7/2))/(7*e^7) - (2*c^2*(2*c*d - b*e)*(d + e*x)^(9/2))/(3*e^7) + (
2*c^3*(d + e*x)^(11/2))/(11*e^7)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{(210c^3x^6+770b\,c^2x^5+990b^2c\,x^4+462b^3x^3)e^6-924x^2d\left(\frac{10}{33}c^3x^3+\frac{25}{21}b\,c^2x^2+\frac{12}{7}b^2cx+b^3\right)e^5+3696\left(\frac{25}{231}c^3x^3+\frac{10}{21}b\,c^2x^2\right.}{}$
risch	$\frac{2(105e^5x^5c^3+385e^5x^4c^2b-245x^4d\,e^4c^3+495e^5x^3b^2c-935x^3d\,e^4c^2b+445d^2e^3c^3x^3+231x^2e^5b^3-1287x^2d\,e^4b^2c+1815}{}$
gosper	$\frac{\frac{2}{11}x^6c^3e^6+\frac{2}{3}x^5b\,c^2e^6-\frac{8}{33}x^5c^3d\,e^5+\frac{6}{7}x^4b^2c\,e^6-\frac{20}{21}x^4b\,c^2d\,e^5+\frac{80}{231}x^4c^3d^2e^4+\frac{2}{5}x^3b^3e^6-\frac{48}{35}x^3b^2cd\,e^5+\frac{32}{21}x^3b\,c^2d^2e^4-}{}$
trager	$\frac{\frac{2}{11}x^6c^3e^6+\frac{2}{3}x^5b\,c^2e^6-\frac{8}{33}x^5c^3d\,e^5+\frac{6}{7}x^4b^2c\,e^6-\frac{20}{21}x^4b\,c^2d\,e^5+\frac{80}{231}x^4c^3d^2e^4+\frac{2}{5}x^3b^3e^6-\frac{48}{35}x^3b^2cd\,e^5+\frac{32}{21}x^3b\,c^2d^2e^4-}{}$
orering	$\frac{2(105x^6c^3e^6+385x^5b\,c^2e^6-140x^5c^3d\,e^5+495x^4b^2c\,e^6-550x^4b\,c^2d\,e^5+200x^4c^3d^2e^4+231x^3b^3e^6-792x^3b^2cd\,e^5+880x}{}$
derivativedivides	$\frac{\frac{2c^3(ex+d)^{\frac{11}{2}}}{11}+\frac{2b\,c^2e(ex+d)^{\frac{9}{2}}}{3}-\frac{4c^3d(ex+d)^{\frac{9}{2}}}{3}+\frac{6b^2c\,e^2(ex+d)^{\frac{7}{2}}}{7}-\frac{30b\,c^2de(ex+d)^{\frac{7}{2}}}{7}+\frac{30c^3d^2(ex+d)^{\frac{7}{2}}}{7}+\frac{2b^3e^3(ex+d)^{\frac{5}{2}}}{5}-\frac{24b}{}}{}$
default	$\frac{\frac{2c^3(ex+d)^{\frac{11}{2}}}{11}+\frac{2b\,c^2e(ex+d)^{\frac{9}{2}}}{3}-\frac{4c^3d(ex+d)^{\frac{9}{2}}}{3}+\frac{6b^2c\,e^2(ex+d)^{\frac{7}{2}}}{7}-\frac{30b\,c^2de(ex+d)^{\frac{7}{2}}}{7}+\frac{30c^3d^2(ex+d)^{\frac{7}{2}}}{7}+\frac{2b^3e^3(ex+d)^{\frac{5}{2}}}{5}-\frac{24b}{}}{}$

input
```
int((c*x^2+b*x)^3/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/1155*((210*c^3*x^6+770*b*c^2*x^5+990*b^2*c*x^4+462*b^3*x^3)*e^6-924*x^2*
d*(10/33*c^3*x^3+25/21*b*c^2*x^2+12/7*b^2*c*x+b^3)*e^5+3696*(25/231*c^3*x^
3+10/21*b*c^2*x^2+6/7*b^2*c*x+b^3)*x*d^2*e^4+7392*d^3*(-20/231*c^3*x^3-10/
21*b*c^2*x^2-12/7*b^2*c*x+b^3)*e^3-25344*(-5/99*c^2*x^2-5/9*c*b*x+b^2)*c*d
^4*e^2+28160*(-2/11*c*x+b)*c^2*d^5*e-10240*d^6*c^3)/(e*x+d)^(1/2)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.16

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{3/2}}\,dx = \frac{2\,(105\,c^3e^6x^6-5120\,c^3d^6+14080\,bc^2d^5e-12672\,b^2cd^4e^2+3696\,b^3d^3e^3-35\,(4\,c^3de^5}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
2/1155*(105*c^3*e^6*x^6 - 5120*c^3*d^6 + 14080*b*c^2*d^5*e - 12672*b^2*c*d
^4*e^2 + 3696*b^3*d^3*e^3 - 35*(4*c^3*d*e^5 - 11*b*c^2*e^6)*x^5 + 5*(40*c^
3*d^2*e^4 - 110*b*c^2*d*e^5 + 99*b^2*c*e^6)*x^4 - (320*c^3*d^3*e^3 - 880*b
*c^2*d^2*e^4 + 792*b^2*c*d*e^5 - 231*b^3*e^6)*x^3 + 2*(320*c^3*d^4*e^2 - 8
80*b*c^2*d^3*e^3 + 792*b^2*c*d^2*e^4 - 231*b^3*d*e^5)*x^2 - 8*(320*c^3*d^5
*e - 880*b*c^2*d^4*e^2 + 792*b^2*c*d^3*e^3 - 231*b^3*d^2*e^4)*x)*sqrt(e*x
+ d)/(e^8*x + d*e^7)
```

Sympy [A] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.39

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{3/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^3(d+ex)^{\frac{11}{2}}}{11e^6}+\frac{d^3(be-cd)^3}{e^6\sqrt{d+ex}}+\frac{(d+ex)^{\frac{9}{2}}\cdot\left(3bc^2e-6c^3d\right)}{9e^6}+\frac{(d+ex)^{\frac{7}{2}}\cdot\left(3b^2ce^2-15bc^2de+15c^3d^2\right)}{7e^6}+\frac{(d+ex)^{\frac{5}{2}}\left(b^3e^3-12b^2cde^2-\dots\right.}{5e^6}\dots}{\dots} \\ \frac{\frac{b^3x^4}{4}+\frac{3b^2cx^5}{5}+\frac{bc^2x^6}{2}+\frac{c^3x^7}{7}}{d^{\frac{3}{2}}} \end{cases}$$

input
```
integrate((c*x**2+b*x)**3/(e*x+d)**(3/2),x)
```

output
```
Piecewise((2*(c**3*(d + e*x)**(11/2)/(11*e**6) + d**3*(b*e - c*d)**3/(e**6
*sqrt(d + e*x)) + (d + e*x)**(9/2)*(3*b*c**2*e - 6*c**3*d)/(9*e**6) + (d +
 e*x)**(7/2)*(3*b**2*c*e**2 - 15*b*c**2*d*e + 15*c**3*d**2)/(7*e**6) + (d
+ e*x)**(5/2)*(b**3*e**3 - 12*b**2*c*d*e**2 + 30*b*c**2*d**2*e - 20*c**3*d
**3)/(5*e**6) + (d + e*x)**(3/2)*(-3*b**3*d*e**3 + 18*b**2*c*d**2*e**2 - 3
0*b*c**2*d**3*e + 15*c**3*d**4)/(3*e**6) + sqrt(d + e*x)*(3*b**3*d**2*e**3
 - 12*b**2*c*d**3*e**2 + 15*b*c**2*d**4*e - 6*c**3*d**5)/e**6)/e, Ne(e, 0)
), ((b**3*x**4/4 + 3*b**2*c*x**5/5 + b*c**2*x**6/2 + c**3*x**7/7)/d**(3/2)
, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.15

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{3/2}}\,dx = \frac{2\left(\frac{105\,(ex+d)^{\frac{11}{2}}c^3-385\,(2\,c^3d-bc^2e)(ex+d)^{\frac{9}{2}}+495\,(5\,c^3d^2-5\,bc^2de+b^2ce^2)(ex+d)^{\frac{7}{2}}-231\,(20\,c^3d^3-30\,bc^2d^2e+12}{}\right.}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
2/1155*((105*(e*x + d)^(11/2)*c^3 - 385*(2*c^3*d - b*c^2*e)*(e*x + d)^(9/2
) + 495*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*(e*x + d)^(7/2) - 231*(20*c^
3*d^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*(e*x + d)^(5/2) + 1155*
(5*c^3*d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 - b^3*d*e^3)*(e*x + d)^(3/2)
 - 3465*(2*c^3*d^5 - 5*b*c^2*d^4*e + 4*b^2*c*d^3*e^2 - b^3*d^2*e^3)*sqrt(e
*x + d))/e^6 - 1155*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e
^3)/(sqrt(e*x + d)*e^6))/e
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.54

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{3/2}}\,dx = -\frac{2\,(c^3d^6-3\,bc^2d^5e+3\,b^2cd^4e^2-b^3d^3e^3)}{\sqrt{ex+d}e^7} + \frac{2\left(105\,(ex+d)^{\frac{11}{2}}c^3e^{70}-770\,(ex+d)^{\frac{9}{2}}c^3de^{70}+2475\,(ex+d)^{\frac{7}{2}}c^3d^2e^{70}-4620\,(ex+d)^{\frac{5}{2}}c^3d^3e^{70}+5775\,(e\right.}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
-2*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)/(sqrt(e*x + d
)*e^7) + 2/1155*(105*(e*x + d)^(11/2)*c^3*e^70 - 770*(e*x + d)^(9/2)*c^3*d
*e^70 + 2475*(e*x + d)^(7/2)*c^3*d^2*e^70 - 4620*(e*x + d)^(5/2)*c^3*d^3*e
^70 + 5775*(e*x + d)^(3/2)*c^3*d^4*e^70 - 6930*sqrt(e*x + d)*c^3*d^5*e^70
+ 385*(e*x + d)^(9/2)*b*c^2*e^71 - 2475*(e*x + d)^(7/2)*b*c^2*d*e^71 + 693
0*(e*x + d)^(5/2)*b*c^2*d^2*e^71 - 11550*(e*x + d)^(3/2)*b*c^2*d^3*e^71 +
17325*sqrt(e*x + d)*b*c^2*d^4*e^71 + 495*(e*x + d)^(7/2)*b^2*c*e^72 - 2772
*(e*x + d)^(5/2)*b^2*c*d*e^72 + 6930*(e*x + d)^(3/2)*b^2*c*d^2*e^72 - 1386
0*sqrt(e*x + d)*b^2*c*d^3*e^72 + 231*(e*x + d)^(5/2)*b^3*e^73 - 1155*(e*x
+ d)^(3/2)*b^3*d*e^73 + 3465*sqrt(e*x + d)*b^3*d^2*e^73)/e^77
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.11

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{3/2}}\,dx = \frac{(d+e\,x)^{5/2}\,(2\,b^3\,e^3-24\,b^2\,c\,d\,e^2+60\,b\,c^2\,d^2\,e-40\,c^3\,d^3)}{5\,e^7} - \frac{-2\,b^3\,d^3\,e^3+6\,b^2\,c\,d^4\,e^2-6\,b\,c^2\,d^5\,e+2\,c^3\,d^6}{e^7\,\sqrt{d+e\,x}} + \frac{2\,c^3\,(d+e\,x)^{11/2}}{11\,e^7} - \frac{(12\,c^3\,d-6\,b\,c^2\,e)\,(d+e\,x)^{9/2}}{9\,e^7} + \frac{(d+e\,x)^{7/2}\,(6\,b^2\,c\,e^2-30\,b\,c^2\,d\,e+30\,c^3\,d^2)}{7\,e^7} + \frac{(d+e\,x)^{3/2}\,(-6\,b^3\,d\,e^3+36\,b^2\,c\,d^2\,e^2-60\,b\,c^2\,d^3\,e+30\,c^3\,d^4)}{3\,e^7} + \frac{6\,d^2\,(b\,e-c\,d)^2\,(b\,e-2\,c\,d)\,\sqrt{d+e\,x}}{e^7}$$

input
```
int((b*x + c*x^2)^3/(d + e*x)^(3/2),x)
```

output
```
((d + e*x)^(5/2)*(2*b^3*e^3 - 40*c^3*d^3 + 60*b*c^2*d^2*e - 24*b^2*c*d*e^2
))/(5*e^7) - (2*c^3*d^6 - 2*b^3*d^3*e^3 + 6*b^2*c*d^4*e^2 - 6*b*c^2*d^5*e)
/(e^7*(d + e*x)^(1/2)) + (2*c^3*(d + e*x)^(11/2))/(11*e^7) - ((12*c^3*d -
6*b*c^2*e)*(d + e*x)^(9/2))/(9*e^7) + ((d + e*x)^(7/2)*(30*c^3*d^2 + 6*b^2
*c*e^2 - 30*b*c^2*d*e))/(7*e^7) + ((d + e*x)^(3/2)*(30*c^3*d^4 - 6*b^3*d*e
^3 + 36*b^2*c*d^2*e^2 - 60*b*c^2*d^3*e))/(3*e^7) + (6*d^2*(b*e - c*d)^2*(b
*e - 2*c*d)*(d + e*x)^(1/2))/e^7
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.18

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{3/2}}\,dx = \frac{\frac{2}{11}c^3e^6x^6+\frac{2}{3}b\,c^2e^6x^5-\frac{8}{33}c^3d\,e^5x^5+\frac{6}{7}b^2c\,e^6x^4-\frac{20}{21}b\,c^2d\,e^5x^4+\frac{80}{231}c^3d^2e^4x^4+\frac{2}{5}b^3e^6x^3}{}$$

input

```
int((c*x^2+b*x)^3/(e*x+d)^(3/2),x)
```

output

```
(2*(3696*b**3*d**3*e**3 + 1848*b**3*d**2*e**4*x - 462*b**3*d*e**5*x**2 + 2
31*b**3*e**6*x**3 - 12672*b**2*c*d**4*e**2 - 6336*b**2*c*d**3*e**3*x + 158
4*b**2*c*d**2*e**4*x**2 - 792*b**2*c*d*e**5*x**3 + 495*b**2*c*e**6*x**4 +
14080*b*c**2*d**5*e + 7040*b*c**2*d**4*e**2*x - 1760*b*c**2*d**3*e**3*x**2
 + 880*b*c**2*d**2*e**4*x**3 - 550*b*c**2*d*e**5*x**4 + 385*b*c**2*e**6*x*
*5 - 5120*c**3*d**6 - 2560*c**3*d**5*e*x + 640*c**3*d**4*e**2*x**2 - 320*c
**3*d**3*e**3*x**3 + 200*c**3*d**2*e**4*x**4 - 140*c**3*d*e**5*x**5 + 105*
c**3*e**6*x**6))/(1155*sqrt(d + e*x)*e**7)
```

3.103 $\int \frac{(bx+cx^2)^3}{(d+ex)^{5/2}}\,dx$

Optimal result 768
Mathematica [A] (verified) 769
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Optimal result

Integrand size = 21, antiderivative size = 244

$$\begin{aligned}\int \frac{(bx+cx^2)^3}{(d+ex)^{5/2}}\,dx &= -\frac{2d^3(cd-be)^3}{3e^7(d+ex)^{3/2}}+\frac{6d^2(cd-be)^2(2cd-be)}{e^7\sqrt{d+ex}}\\ &+\frac{6d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)\sqrt{d+ex}}{e^7}\\ &-\frac{2(2cd-be)\left(10c^2d^2-10bcde+b^2e^2\right)(d+ex)^{3/2}}{3e^7}\\ &+\frac{6c(5c^2d^2-5bcde+b^2e^2)\,(d+ex)^{5/2}}{5e^7}-\frac{6c^2(2cd-be)(d+ex)^{7/2}}{7e^7}+\frac{2c^3(d+ex)^{9/2}}{9e^7}\end{aligned}$$

output
```
-2/3*d^3*(-b*e+c*d)^3/e^7/(e*x+d)^(3/2)+6*d^2*(-b*e+c*d)^2*(-b*e+2*c*d)/e^
7/(e*x+d)^(1/2)+6*d*(-b*e+c*d)*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)^(1/2)
/e^7-2/3*(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*d^2)*(e*x+d)^(3/2)/e^7+6/
5*c*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)^(5/2)/e^7-6/7*c^2*(-b*e+2*c*d)*(
e*x+d)^(7/2)/e^7+2/9*c^3*(e*x+d)^(9/2)/e^7
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.95

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{5/2}}\,dx = \frac{2(105b^3e^3(-16d^3-24d^2ex-6de^2x^2+e^3x^3)+63b^2ce^2(128d^4+192d^3ex+48d^2e^2x^2}{}$$

input
```
Integrate[(b*x + c*x^2)^3/(d + e*x)^(5/2),x]
```

output
```
(2*(105*b^3*e^3*(-16*d^3 - 24*d^2*e*x - 6*d*e^2*x^2 + e^3*x^3) + 63*b^2*c*
e^2*(128*d^4 + 192*d^3*e*x + 48*d^2*e^2*x^2 - 8*d*e^3*x^3 + 3*e^4*x^4) - 4
5*b*c^2*e*(256*d^5 + 384*d^4*e*x + 96*d^3*e^2*x^2 - 16*d^2*e^3*x^3 + 6*d*e
^4*x^4 - 3*e^5*x^5) + 5*c^3*(1024*d^6 + 1536*d^5*e*x + 384*d^4*e^2*x^2 - 6
4*d^3*e^3*x^3 + 24*d^2*e^4*x^4 - 12*d*e^5*x^5 + 7*e^6*x^6)))/(315*e^7*(d +
 e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{3c(d+ex)^{3/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{e^6}+\frac{\sqrt{d+ex}(2cd-be)\left(-b^2e^2+10bcde-10c^2d^2\right)}{e^6}+\frac{3d(cd-be)\left(b^2e^2\right.}{e^6\sqrt{d}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{6c(d+ex)^{5/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{5e^7}-\frac{2(d+ex)^{3/2}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{3e^7}+\frac{6d\sqrt{d+ex}(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{e^7}-\frac{6c^2(d+ex)^{7/2}(2cd-be)}{7e^7}-\frac{2d^3(cd-be)^3}{3e^7(d+ex)^{3/2}}+\frac{6d^2(cd-be)^2(2cd-be)}{e^7\sqrt{d+ex}}+\frac{2c^3(d+ex)^{9/2}}{9e^7}$$

input
```
Int[(b*x + c*x^2)^3/(d + e*x)^(5/2),x]
```

output
```
(-2*d^3*(c*d - b*e)^3)/(3*e^7*(d + e*x)^(3/2)) + (6*d^2*(c*d - b*e)^2*(2*c
*d - b*e))/(e^7*Sqrt[d + e*x]) + (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e +
 b^2*e^2)*Sqrt[d + e*x])/e^7 - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e +
 b^2*e^2)*(d + e*x)^(3/2))/(3*e^7) + (6*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2
)*(d + e*x)^(5/2))/(5*e^7) - (6*c^2*(2*c*d - b*e)*(d + e*x)^(7/2))/(7*e^7)
 + (2*c^3*(d + e*x)^(9/2))/(9*e^7)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.87

method	result
pseudoelliptic	$\frac{(70c^3x^6+270b\,c^2x^5+378b^2c\,x^4+210b^3x^3)e^6-1260\left(\frac{2}{21}c^3x^3+\frac{3}{7}b\,c^2x^2+\frac{4}{5}b^2cx+b^3\right)x^2d\,e^5-5040\left(-\frac{1}{21}c^3x^3-\frac{2}{7}b\,c^2x^2-\right.}{}$
risch	$-\frac{2(-35c^3x^4e^4-135b\,c^2e^4x^3+130c^3d\,e^3x^3-189b^2c\,e^4x^2+540b\,c^2d\,e^3x^2-345c^3d^2e^2x^2-105b^3e^4x+882b^2cd\,e^3x-166}{315e^7}$
gosper	$-\frac{2(-35x^6c^3e^6-135x^5b\,c^2e^6+60x^5c^3d\,e^5-189x^4b^2c\,e^6+270x^4b\,c^2d\,e^5-120x^4c^3d^2e^4-105x^3b^3e^6+504x^3b^2cd\,e^5-72}{}$
trager	$-\frac{2(-35x^6c^3e^6-135x^5b\,c^2e^6+60x^5c^3d\,e^5-189x^4b^2c\,e^6+270x^4b\,c^2d\,e^5-120x^4c^3d^2e^4-105x^3b^3e^6+504x^3b^2cd\,e^5-72}{}$
orering	$-\frac{2(-35x^6c^3e^6-135x^5b\,c^2e^6+60x^5c^3d\,e^5-189x^4b^2c\,e^6+270x^4b\,c^2d\,e^5-120x^4c^3d^2e^4-105x^3b^3e^6+504x^3b^2cd\,e^5-72}{}$
derivativedivides	$\frac{\frac{2c^3(ex+d)^{\frac{9}{2}}}{9}+\frac{6b\,c^2e(ex+d)^{\frac{7}{2}}}{7}-\frac{12c^3d(ex+d)^{\frac{7}{2}}}{7}+\frac{6b^2c\,e^2(ex+d)^{\frac{5}{2}}}{5}-6b\,c^2de(ex+d)^{\frac{5}{2}}+6c^3d^2(ex+d)^{\frac{5}{2}}+\frac{2b^3e^3(ex+d)^{\frac{3}{2}}}{3}-8b^2}{}$
default	$\frac{\frac{2c^3(ex+d)^{\frac{9}{2}}}{9}+\frac{6b\,c^2e(ex+d)^{\frac{7}{2}}}{7}-\frac{12c^3d(ex+d)^{\frac{7}{2}}}{7}+\frac{6b^2c\,e^2(ex+d)^{\frac{5}{2}}}{5}-6b\,c^2de(ex+d)^{\frac{5}{2}}+6c^3d^2(ex+d)^{\frac{5}{2}}+\frac{2b^3e^3(ex+d)^{\frac{3}{2}}}{3}-8b^2}{}$

input
```
int((c*x^2+b*x)^3/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*((70*c^3*x^6+270*b*c^2*x^5+378*b^2*c*x^4+210*b^3*x^3)*e^6-1260*(2/21
*c^3*x^3+3/7*b*c^2*x^2+4/5*b^2*c*x+b^3)*x^2*d*e^5-5040*(-1/21*c^3*x^3-2/7*
b*c^2*x^2-6/5*b^2*c*x+b^3)*x*d^2*e^4-3360*d^3*(4/21*c^3*x^3+18/7*b*c^2*x^2
-36/5*b^2*c*x+b^3)*e^3+16128*(5/21*c^2*x^2-15/7*c*b*x+b^2)*c*d^4*e^2-23040
*(-2/3*c*x+b)*c^2*d^5*e+10240*d^6*c^3)/(e*x+d)^(3/2)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.19

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{5/2}}\,dx = \frac{2\,(35\,c^3e^6x^6+5120\,c^3d^6-11520\,bc^2d^5e+8064\,b^2cd^4e^2-1680\,b^3d^3e^3-15\,(4\,c^3de^5-9}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
2/315*(35*c^3*e^6*x^6 + 5120*c^3*d^6 - 11520*b*c^2*d^5*e + 8064*b^2*c*d^4*
e^2 - 1680*b^3*d^3*e^3 - 15*(4*c^3*d*e^5 - 9*b*c^2*e^6)*x^5 + 3*(40*c^3*d^
2*e^4 - 90*b*c^2*d*e^5 + 63*b^2*c*e^6)*x^4 - (320*c^3*d^3*e^3 - 720*b*c^2*
d^2*e^4 + 504*b^2*c*d*e^5 - 105*b^3*e^6)*x^3 + 6*(320*c^3*d^4*e^2 - 720*b*
c^2*d^3*e^3 + 504*b^2*c*d^2*e^4 - 105*b^3*d*e^5)*x^2 + 24*(320*c^3*d^5*e -
 720*b*c^2*d^4*e^2 + 504*b^2*c*d^3*e^3 - 105*b^3*d^2*e^4)*x)*sqrt(e*x + d)
/(e^9*x^2 + 2*d*e^8*x + d^2*e^7)
```

Sympy [A] (verification not implemented)

Time = 4.64 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.27

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{5/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^3(d+ex)^{\frac{9}{2}}}{9e^6} + \frac{d^3(be-cd)^3}{3e^6(d+ex)^{\frac{3}{2}}} - \frac{3d^2(be-2cd)(be-cd)^2}{e^6\sqrt{d+ex}} + \frac{(d+ex)^{\frac{7}{2}}\cdot\left(3bc^2e-6c^3d\right)}{7e^6} + \frac{(d+ex)^{\frac{5}{2}}\cdot\left(3b^2ce^2-15bc^2de+15c^3d^2\right)}{5e^6} + \frac{(d \cdots}{\cdots}\right.}{e} \\ \frac{\frac{b^3x^4}{4}+\frac{3b^2cx^5}{5}+\frac{bc^2x^6}{2}+\frac{c^3x^7}{7}}{d^{\frac{5}{2}}} \end{cases}$$

input
```
integrate((c*x**2+b*x)**3/(e*x+d)**(5/2),x)
```

output
```
Piecewise((2*(c**3*(d + e*x)**(9/2)/(9*e**6) + d**3*(b*e - c*d)**3/(3*e**6
*(d + e*x)**(3/2)) - 3*d**2*(b*e - 2*c*d)*(b*e - c*d)**2/(e**6*sqrt(d + e*
x)) + (d + e*x)**(7/2)*(3*b*c**2*e - 6*c**3*d)/(7*e**6) + (d + e*x)**(5/2)
*(3*b**2*c*e**2 - 15*b*c**2*d*e + 15*c**3*d**2)/(5*e**6) + (d + e*x)**(3/2
)*(b**3*e**3 - 12*b**2*c*d*e**2 + 30*b*c**2*d**2*e - 20*c**3*d**3)/(3*e**6
) + sqrt(d + e*x)*(-3*b**3*d*e**3 + 18*b**2*c*d**2*e**2 - 30*b*c**2*d**3*e
 + 15*c**3*d**4)/e**6)/e, Ne(e, 0)), ((b**3*x**4/4 + 3*b**2*c*x**5/5 + b*c
**2*x**6/2 + c**3*x**7/7)/d**(5/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.14

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{5/2}}\,dx = \frac{2\left(\frac{35\,(ex+d)^{\frac{9}{2}}c^3-135\,(2\,c^3d-bc^2e)(ex+d)^{\frac{7}{2}}+189\,(5\,c^3d^2-5\,bc^2de+b^2ce^2)(ex+d)^{\frac{5}{2}}-105\,(20\,c^3d^3-30\,bc^2d^2e+12\,b^2}{e^6}\right.}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
2/315*((35*(e*x + d)^(9/2)*c^3 - 135*(2*c^3*d - b*c^2*e)*(e*x + d)^(7/2) +
 189*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*(e*x + d)^(5/2) - 105*(20*c^3*d
^3 - 30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*(e*x + d)^(3/2) + 945*(5*c
^3*d^4 - 10*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 - b^3*d*e^3)*sqrt(e*x + d))/e^6
- 105*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3 - 9*(2*c^3*
d^5 - 5*b*c^2*d^4*e + 4*b^2*c*d^3*e^2 - b^3*d^2*e^3)*(e*x + d))/((e*x + d)
^(3/2)*e^6))/e
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.48

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{5/2}}\,dx = \frac{2\,(18\,(ex+d)c^3d^5-c^3d^6-45\,(ex+d)bc^2d^4e+3\,bc^2d^5e+36\,(ex+d)b^2cd^3e^2-3\,b^2cd}{3\,(ex+d)^{\frac{3}{2}}e^7}$$
$$+\frac{2\left(35\,(ex+d)^{\frac{9}{2}}c^3e^{56}-270\,(ex+d)^{\frac{7}{2}}c^3de^{56}+945\,(ex+d)^{\frac{5}{2}}c^3d^2e^{56}-2100\,(ex+d)^{\frac{3}{2}}c^3d^3e^{56}+4725\,\sqrt{ex}\right.}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
2/3*(18*(e*x + d)*c^3*d^5 - c^3*d^6 - 45*(e*x + d)*b*c^2*d^4*e + 3*b*c^2*d
^5*e + 36*(e*x + d)*b^2*c*d^3*e^2 - 3*b^2*c*d^4*e^2 - 9*(e*x + d)*b^3*d^2*
e^3 + b^3*d^3*e^3)/((e*x + d)^(3/2)*e^7) + 2/315*(35*(e*x + d)^(9/2)*c^3*e
^56 - 270*(e*x + d)^(7/2)*c^3*d*e^56 + 945*(e*x + d)^(5/2)*c^3*d^2*e^56 -
2100*(e*x + d)^(3/2)*c^3*d^3*e^56 + 4725*sqrt(e*x + d)*c^3*d^4*e^56 + 135*
(e*x + d)^(7/2)*b*c^2*e^57 - 945*(e*x + d)^(5/2)*b*c^2*d*e^57 + 3150*(e*x
+ d)^(3/2)*b*c^2*d^2*e^57 - 9450*sqrt(e*x + d)*b*c^2*d^3*e^57 + 189*(e*x +
 d)^(5/2)*b^2*c*e^58 - 1260*(e*x + d)^(3/2)*b^2*c*d*e^58 + 5670*sqrt(e*x +
 d)*b^2*c*d^2*e^58 + 105*(e*x + d)^(3/2)*b^3*e^59 - 945*sqrt(e*x + d)*b^3*
d*e^59)/e^63
```

Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.15

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{5/2}}\,dx = \frac{(d+e\,x)^{3/2}\,(2\,b^3\,e^3-24\,b^2\,c\,d\,e^2+60\,b\,c^2\,d^2\,e-40\,c^3\,d^3)}{3\,e^7} + \frac{(d+e\,x)\,(-6\,b^3\,d^2\,e^3+24\,b^2\,c\,d^3\,e^2-30\,b\,c^2\,d^4\,e+12\,c^3\,d^5)-\frac{2\,c^3\,d^6}{3}+\frac{2\,b^3\,d^3\,e^3}{3}-2\,b^2\,c\,d^4\,e^2+2\,b\,c^2\,d^5\,e}{e^7\,(d+e\,x)^{3/2}} + \frac{2\,c^3\,(d+e\,x)^{9/2}}{9\,e^7} - \frac{(12\,c^3\,d-6\,b\,c^2\,e)\,(d+e\,x)^{7/2}}{7\,e^7} + \frac{(d+e\,x)^{5/2}\,(6\,b^2\,c\,e^2-30\,b\,c^2\,d\,e+30\,c^3\,d^2)}{5\,e^7} + \frac{\sqrt{d+e\,x}\,(-6\,b^3\,d\,e^3+36\,b^2\,c\,d^2\,e^2-60\,b\,c^2\,d^3\,e+30\,c^3\,d^4)}{e^7}$$

input
```
int((b*x + c*x^2)^3/(d + e*x)^(5/2),x)
```

output
```
((d + e*x)^(3/2)*(2*b^3*e^3 - 40*c^3*d^3 + 60*b*c^2*d^2*e - 24*b^2*c*d*e^2
))/(3*e^7) + ((d + e*x)*(12*c^3*d^5 - 6*b^3*d^2*e^3 + 24*b^2*c*d^3*e^2 - 3
0*b*c^2*d^4*e) - (2*c^3*d^6)/3 + (2*b^3*d^3*e^3)/3 - 2*b^2*c*d^4*e^2 + 2*b
*c^2*d^5*e)/(e^7*(d + e*x)^(3/2)) + (2*c^3*(d + e*x)^(9/2))/(9*e^7) - ((12
*c^3*d - 6*b*c^2*e)*(d + e*x)^(7/2))/(7*e^7) + ((d + e*x)^(5/2)*(30*c^3*d^
2 + 6*b^2*c*e^2 - 30*b*c^2*d*e))/(5*e^7) + ((d + e*x)^(1/2)*(30*c^3*d^4 -
6*b^3*d*e^3 + 36*b^2*c*d^2*e^2 - 60*b*c^2*d^3*e))/e^7
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.20

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{5/2}}\,dx = \frac{\frac{2}{9}c^3e^6x^6+\frac{6}{7}b\,c^2e^6x^5-\frac{8}{21}c^3d\,e^5x^5+\frac{6}{5}b^2c\,e^6x^4-\frac{12}{7}b\,c^2d\,e^5x^4+\frac{16}{21}c^3d^2e^4x^4+\frac{2}{3}b^3e^6x^3-}{}$$

input
```
int((c*x^2+b*x)^3/(e*x+d)^(5/2),x)
```

output
```
(2*( - 1680*b**3*d**3*e**3 - 2520*b**3*d**2*e**4*x - 630*b**3*d*e**5*x**2
+ 105*b**3*e**6*x**3 + 8064*b**2*c*d**4*e**2 + 12096*b**2*c*d**3*e**3*x +
3024*b**2*c*d**2*e**4*x**2 - 504*b**2*c*d*e**5*x**3 + 189*b**2*c*e**6*x**4
 - 11520*b*c**2*d**5*e - 17280*b*c**2*d**4*e**2*x - 4320*b*c**2*d**3*e**3*
x**2 + 720*b*c**2*d**2*e**4*x**3 - 270*b*c**2*d*e**5*x**4 + 135*b*c**2*e**
6*x**5 + 5120*c**3*d**6 + 7680*c**3*d**5*e*x + 1920*c**3*d**4*e**2*x**2 -
320*c**3*d**3*e**3*x**3 + 120*c**3*d**2*e**4*x**4 - 60*c**3*d*e**5*x**5 +
35*c**3*e**6*x**6))/(315*sqrt(d + e*x)*e**7*(d + e*x))
```

3.104 $\int \frac{(bx+cx^2)^3}{(d+ex)^{7/2}}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 240

$$\begin{aligned}\int \frac{(bx+cx^2)^3}{(d+ex)^{7/2}}\,dx = &-\frac{2d^3(cd-be)^3}{5e^7(d+ex)^{5/2}}\\ &+\frac{2d^2(cd-be)^2(2cd-be)}{e^7(d+ex)^{3/2}}-\frac{6d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)}{e^7\sqrt{d+ex}}\\ &-\frac{2(2cd-be)\left(10c^2d^2-10bcde+b^2e^2\right)\sqrt{d+ex}}{e^7}\\ &+\frac{2c\left(5c^2d^2-5bcde+b^2e^2\right)(d+ex)^{3/2}}{e^7}-\frac{6c^2(2cd-be)(d+ex)^{5/2}}{5e^7}+\frac{2c^3(d+ex)^{7/2}}{7e^7}\end{aligned}$$

output
```
-2/5*d^3*(-b*e+c*d)^3/e^7/(e*x+d)^(5/2)+2*d^2*(-b*e+c*d)^2*(-b*e+2*c*d)/e^
7/(e*x+d)^(3/2)-6*d*(-b*e+c*d)*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)/e^7/(e*x+d)^(
1/2)-2*(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*d^2)*(e*x+d)^(1/2)/e^7+2*c*
(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)^(3/2)/e^7-6/5*c^2*(-b*e+2*c*d)*(e*x+
d)^(5/2)/e^7+2/7*c^3*(e*x+d)^(7/2)/e^7
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.97

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{7/2}}\,dx = \frac{2(7b^3e^3(16d^3+40d^2ex+30de^2x^2+5e^3x^3)-7b^2ce^2(128d^4+320d^3ex+240d^2e^2x^2+}{}$$

input
```
Integrate[(b*x + c*x^2)^3/(d + e*x)^(7/2),x]
```

output
```
(2*(7*b^3*e^3*(16*d^3 + 40*d^2*e*x + 30*d*e^2*x^2 + 5*e^3*x^3) - 7*b^2*c*e
^2*(128*d^4 + 320*d^3*e*x + 240*d^2*e^2*x^2 + 40*d*e^3*x^3 - 5*e^4*x^4) +
7*b*c^2*e*(256*d^5 + 640*d^4*e*x + 480*d^3*e^2*x^2 + 80*d^2*e^3*x^3 - 10*d
*e^4*x^4 + 3*e^5*x^5) - c^3*(1024*d^6 + 2560*d^5*e*x + 1920*d^4*e^2*x^2 +
320*d^3*e^3*x^3 - 40*d^2*e^4*x^4 + 12*d*e^5*x^5 - 5*e^6*x^6)))/(35*e^7*(d
+ e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{7/2}}\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{3c\sqrt{d+ex}(b^2e^2-5bcde+5c^2d^2)}{e^6} + \frac{(2cd-be)(-b^2e^2+10bcde-10c^2d^2)}{e^6\sqrt{d+ex}} + \frac{3d(cd-be)(b^2e^2-5bcde+5}{e^6(d+ex)^{3/2}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{2c(d+ex)^{3/2}\left(b^2e^2-5bcde+5c^2d^2\right)}{e^7}-\frac{2\sqrt{d+ex}(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)}{e^7}-\frac{6d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)}{e^7\sqrt{d+ex}}-\frac{6c^2(d+ex)^{5/2}(2cd-be)}{5e^7}-\frac{2d^3(cd-be)^3}{5e^7(d+ex)^{5/2}}+\frac{2d^2(cd-be)^2(2cd-be)}{e^7(d+ex)^{3/2}}+\frac{2c^3(d+ex)^{7/2}}{7e^7}$$

input
```
Int[(b*x + c*x^2)^3/(d + e*x)^(7/2),x]
```

output
```
(-2*d^3*(c*d - b*e)^3)/(5*e^7*(d + e*x)^(5/2)) + (2*d^2*(c*d - b*e)^2*(2*c
*d - b*e))/(e^7*(d + e*x)^(3/2)) - (6*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e
 + b^2*e^2))/(e^7*Sqrt[d + e*x]) - (2*(2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d
*e + b^2*e^2)*Sqrt[d + e*x])/e^7 + (2*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*
(d + e*x)^(3/2))/e^7 - (6*c^2*(2*c*d - b*e)*(d + e*x)^(5/2))/(5*e^7) + (2*
c^3*(d + e*x)^(7/2))/(7*e^7)
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{2x^3\left(\frac{1}{7}c^3x^3+\frac{3}{5}b\,c^2x^2+b^2cx+b^3\right)e^6+12\left(-\frac{2}{35}c^3x^3-\frac{1}{3}b\,c^2x^2-\frac{4}{3}b^2cx+b^3\right)x^2d\,e^5+16x\,d^2\left(\frac{1}{7}c^3x^3+2b\,c^2x^2-6b^2cx+b^3\right)e^4}{(ex+d)^{\frac{5}{2}}e^7}$
risch	$\frac{2\left(5c^3x^3e^3+21e^3x^2b\,c^2-27c^3d\,e^2x^2+35x\,b^2c\,e^3-133xb\,c^2d\,e^2+106c^3d^2ex+35b^3e^3-385d\,e^2b^2c+896d^2eb\,c^2-562d^3c^3\right.}{35e^7}$
gosper	$\frac{2}{7}x^6c^3e^6+\frac{6}{5}x^5b\,c^2e^6-\frac{24}{35}x^5c^3d\,e^5+2x^4b^2c\,e^6-4x^4b\,c^2d\,e^5+\frac{16}{7}x^4c^3d^2e^4+2x^3b^3e^6-16x^3b^2cd\,e^5+32x^3b\,c^2d^2e^4-\frac{128}{7}x$
trager	$\frac{2}{7}x^6c^3e^6+\frac{6}{5}x^5b\,c^2e^6-\frac{24}{35}x^5c^3d\,e^5+2x^4b^2c\,e^6-4x^4b\,c^2d\,e^5+\frac{16}{7}x^4c^3d^2e^4+2x^3b^3e^6-16x^3b^2cd\,e^5+32x^3b\,c^2d^2e^4-\frac{128}{7}x$
derivativedivides	$\frac{2c^3(ex+d)^{\frac{7}{2}}}{7}+\frac{6b\,c^2e(ex+d)^{\frac{5}{2}}}{5}-\frac{12c^3d(ex+d)^{\frac{5}{2}}}{5}+2b^2c\,e^2(ex+d)^{\frac{3}{2}}-10b\,c^2de(ex+d)^{\frac{3}{2}}+10c^3d^2(ex+d)^{\frac{3}{2}}+2b^3e^3\sqrt{ex+d}-2$
default	$\frac{2c^3(ex+d)^{\frac{7}{2}}}{7}+\frac{6b\,c^2e(ex+d)^{\frac{5}{2}}}{5}-\frac{12c^3d(ex+d)^{\frac{5}{2}}}{5}+2b^2c\,e^2(ex+d)^{\frac{3}{2}}-10b\,c^2de(ex+d)^{\frac{3}{2}}+10c^3d^2(ex+d)^{\frac{3}{2}}+2b^3e^3\sqrt{ex+d}-2$
orering	$2\left(5x^6c^3e^6+21x^5b\,c^2e^6-12x^5c^3d\,e^5+35x^4b^2c\,e^6-70x^4b\,c^2d\,e^5+40x^4c^3d^2e^4+35x^3b^3e^6-280x^3b^2cd\,e^5+560x^3b\,c^2d^2e^4\right.$

input
```
int((c*x^2+b*x)^3/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
32/5/(e*x+d)^(5/2)*(5/16*x^3*(1/7*c^3*x^3+3/5*b*c^2*x^2+b^2*c*x+b^3)*e^6+1
5/8*(-2/35*c^3*x^3-1/3*b*c^2*x^2-4/3*b^2*c*x+b^3)*x^2*d*e^5+5/2*x*d^2*(1/7
*c^3*x^3+2*b*c^2*x^2-6*b^2*c*x+b^3)*e^4+d^3*(-20/7*c^3*x^3+30*b*c^2*x^2-20
*b^2*c*x+b^3)*e^3-8*c*d^4*(15/7*c^2*x^2-5*c*b*x+b^2)*e^2+16*(-10/7*c*x+b)*
c^2*d^5*e-64/7*d^6*c^3)/e^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.26

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{7/2}}\,dx = \frac{2\left(5\,c^3e^6x^6-1024\,c^3d^6+1792\,bc^2d^5e-896\,b^2cd^4e^2+112\,b^3d^3e^3-3\left(4\,c^3de^5-7\,bc^2e^6\right.\right.}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
2/35*(5*c^3*e^6*x^6 - 1024*c^3*d^6 + 1792*b*c^2*d^5*e - 896*b^2*c*d^4*e^2
+ 112*b^3*d^3*e^3 - 3*(4*c^3*d*e^5 - 7*b*c^2*e^6)*x^5 + 5*(8*c^3*d^2*e^4 -
 14*b*c^2*d*e^5 + 7*b^2*c*e^6)*x^4 - 5*(64*c^3*d^3*e^3 - 112*b*c^2*d^2*e^4
 + 56*b^2*c*d*e^5 - 7*b^3*e^6)*x^3 - 30*(64*c^3*d^4*e^2 - 112*b*c^2*d^3*e^
3 + 56*b^2*c*d^2*e^4 - 7*b^3*d*e^5)*x^2 - 40*(64*c^3*d^5*e - 112*b*c^2*d^4
*e^2 + 56*b^2*c*d^3*e^3 - 7*b^3*d^2*e^4)*x)*sqrt(e*x + d)/(e^10*x^3 + 3*d*
e^9*x^2 + 3*d^2*e^8*x + d^3*e^7)
```

Sympy [A] (verification not implemented)

Time = 4.93 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.24

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{7/2}}\,dx = \begin{cases} \frac{2\left(\frac{c^3(d+ex)^{\frac{7}{2}}}{7e^6} + \frac{d^3(be-cd)^3}{5e^6(d+ex)^{\frac{5}{2}}} - \frac{d^2(be-2cd)(be-cd)^2}{e^6(d+ex)^{\frac{3}{2}}} + \frac{3d(be-cd)\left(b^2e^2-5bcde+5c^2d^2\right)}{e^6\sqrt{d+ex}} + \frac{(d+ex)^{\frac{5}{2}}\cdot\left(3bc^2e-6c^3d\right)}{5e^6} + \frac{(d+ex)^{\frac{3}{2}}}{} \right.}{e} \\ \frac{\frac{b^3x^4}{4} + \frac{3b^2cx^5}{5} + \frac{bc^2x^6}{2} + \frac{c^3x^7}{7}}{d^{\frac{7}{2}}} \end{cases}$$

input
```
integrate((c*x**2+b*x)**3/(e*x+d)**(7/2),x)
```

output
```
Piecewise((2*(c**3*(d + e*x)**(7/2)/(7*e**6) + d**3*(b*e - c*d)**3/(5*e**6
*(d + e*x)**(5/2)) - d**2*(b*e - 2*c*d)*(b*e - c*d)**2/(e**6*(d + e*x)**(3
/2)) + 3*d*(b*e - c*d)*(b**2*e**2 - 5*b*c*d*e + 5*c**2*d**2)/(e**6*sqrt(d
+ e*x)) + (d + e*x)**(5/2)*(3*b*c**2*e - 6*c**3*d)/(5*e**6) + (d + e*x)**(
3/2)*(3*b**2*c*e**2 - 15*b*c**2*d*e + 15*c**3*d**2)/(3*e**6) + sqrt(d + e*
x)*(b**3*e**3 - 12*b**2*c*d*e**2 + 30*b*c**2*d**2*e - 20*c**3*d**3)/e**6)/
e, Ne(e, 0)), ((b**3*x**4/4 + 3*b**2*c*x**5/5 + b*c**2*x**6/2 + c**3*x**7/
7)/d**(7/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.15

$$\int \frac{(bx + cx^2)^3}{(d + ex)^{7/2}}\, dx = \frac{2\left(\frac{5\,(ex+d)^{\frac{7}{2}}c^3-21\,(2\,c^3d-bc^2e)(ex+d)^{\frac{5}{2}}+35\,(5\,c^3d^2-5\,bc^2de+b^2ce^2)(ex+d)^{\frac{3}{2}}-35\,(20\,c^3d^3-30\,bc^2d^2e+12\,b^2cde^2}{e^6}\right.}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
2/35*((5*(e*x + d)^(7/2)*c^3 - 21*(2*c^3*d - b*c^2*e)*(e*x + d)^(5/2) + 35
*(5*c^3*d^2 - 5*b*c^2*d*e + b^2*c*e^2)*(e*x + d)^(3/2) - 35*(20*c^3*d^3 -
30*b*c^2*d^2*e + 12*b^2*c*d*e^2 - b^3*e^3)*sqrt(e*x + d))/e^6 - 7*(c^3*d^6
 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3 + 15*(5*c^3*d^4 - 10*b*c^
2*d^3*e + 6*b^2*c*d^2*e^2 - b^3*d*e^3)*(e*x + d)^2 - 5*(2*c^3*d^5 - 5*b*c^
2*d^4*e + 4*b^2*c*d^3*e^2 - b^3*d^2*e^3)*(e*x + d))/((e*x + d)^(5/2)*e^6))
/e
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.48

$$\int \frac{(bx + cx^2)^3}{(d + ex)^{7/2}}\, dx =$$
$$-\frac{2\left(75\,(ex+d)^2c^3d^4 - 10\,(ex+d)c^3d^5 + c^3d^6 - 150\,(ex+d)^2bc^2d^3e + 25\,(ex+d)bc^2d^4e - 3\,bc^2d^5e + 90\,(\right.}{5\,(ex+d}$$
$$+\frac{2\left(5\,(ex+d)^{\frac{7}{2}}c^3e^{42} - 42\,(ex+d)^{\frac{5}{2}}c^3de^{42} + 175\,(ex+d)^{\frac{3}{2}}c^3d^2e^{42} - 700\,\sqrt{ex+d}c^3d^3e^{42} + 21\,(ex+d)^{\frac{5}{2}}bc\right.}{}$$

input
```
integrate((c*x^2+b*x)^3/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
-2/5*(75*(e*x + d)^2*c^3*d^4 - 10*(e*x + d)*c^3*d^5 + c^3*d^6 - 150*(e*x +
 d)^2*b*c^2*d^3*e + 25*(e*x + d)*b*c^2*d^4*e - 3*b*c^2*d^5*e + 90*(e*x + d
)^2*b^2*c*d^2*e^2 - 20*(e*x + d)*b^2*c*d^3*e^2 + 3*b^2*c*d^4*e^2 - 15*(e*x
 + d)^2*b^3*d*e^3 + 5*(e*x + d)*b^3*d^2*e^3 - b^3*d^3*e^3)/((e*x + d)^(5/2
)*e^7) + 2/35*(5*(e*x + d)^(7/2)*c^3*e^42 - 42*(e*x + d)^(5/2)*c^3*d*e^42
+ 175*(e*x + d)^(3/2)*c^3*d^2*e^42 - 700*sqrt(e*x + d)*c^3*d^3*e^42 + 21*(
e*x + d)^(5/2)*b*c^2*e^43 - 175*(e*x + d)^(3/2)*b*c^2*d*e^43 + 1050*sqrt(e
*x + d)*b*c^2*d^2*e^43 + 35*(e*x + d)^(3/2)*b^2*c*e^44 - 420*sqrt(e*x + d)
*b^2*c*d*e^44 + 35*sqrt(e*x + d)*b^3*e^45)/e^49
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.16

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{7/2}}\,dx = \frac{\sqrt{d+e\,x}\,\left(2\,b^3\,e^3-24\,b^2\,c\,d\,e^2+60\,b\,c^2\,d^2\,e-40\,c^3\,d^3\right)}{e^7}$$
$$+\frac{(d+e\,x)\,\left(-2\,b^3\,d^2\,e^3+8\,b^2\,c\,d^3\,e^2-10\,b\,c^2\,d^4\,e+4\,c^3\,d^5\right)-(d+e\,x)^2\,\left(-6\,b^3\,d\,e^3+36\,b^2\,c\,d^2\,e^2-60\,b\,c^2 \cdots\right.}{e^7\,(d+e\,x)^{5/2}}$$
$$+\frac{2\,c^3\,(d+e\,x)^{7/2}}{7\,e^7}-\frac{\left(12\,c^3\,d-6\,b\,c^2\,e\right)\,(d+e\,x)^{5/2}}{5\,e^7}$$
$$+\frac{(d+e\,x)^{3/2}\,\left(6\,b^2\,c\,e^2-30\,b\,c^2\,d\,e+30\,c^3\,d^2\right)}{3\,e^7}$$

input
```
int((b*x + c*x^2)^3/(d + e*x)^(7/2),x)
```

output
```
((d + e*x)^(1/2)*(2*b^3*e^3 - 40*c^3*d^3 + 60*b*c^2*d^2*e - 24*b^2*c*d*e^2
))/e^7 + ((d + e*x)*(4*c^3*d^5 - 2*b^3*d^2*e^3 + 8*b^2*c*d^3*e^2 - 10*b*c^
2*d^4*e) - (d + e*x)^2*(30*c^3*d^4 - 6*b^3*d*e^3 + 36*b^2*c*d^2*e^2 - 60*b
*c^2*d^3*e) - (2*c^3*d^6)/5 + (2*b^3*d^3*e^3)/5 - (6*b^2*c*d^4*e^2)/5 + (6
*b*c^2*d^5*e)/5)/(e^7*(d + e*x)^(5/2)) + (2*c^3*(d + e*x)^(7/2))/(7*e^7) -
 ((12*c^3*d - 6*b*c^2*e)*(d + e*x)^(5/2))/(5*e^7) + ((d + e*x)^(3/2)*(30*c
^3*d^2 + 6*b^2*c*e^2 - 30*b*c^2*d*e))/(3*e^7)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.27

$$\int \frac{(bx+cx^2)^3}{(d+ex)^{7/2}}\,dx = \frac{\frac{2}{7}c^3e^6x^6+\frac{6}{5}b\,c^2e^6x^5-\frac{24}{35}c^3d\,e^5x^5+2b^2c\,e^6x^4-4b\,c^2d\,e^5x^4+\frac{16}{7}c^3d^2e^4x^4+2b^3e^6x^3-1}{}$$

input
```
int((c*x^2+b*x)^3/(e*x+d)^(7/2),x)
```

output
```
(2*(112*b**3*d**3*e**3 + 280*b**3*d**2*e**4*x + 210*b**3*d*e**5*x**2 + 35*
b**3*e**6*x**3 - 896*b**2*c*d**4*e**2 - 2240*b**2*c*d**3*e**3*x - 1680*b**
2*c*d**2*e**4*x**2 - 280*b**2*c*d*e**5*x**3 + 35*b**2*c*e**6*x**4 + 1792*b
*c**2*d**5*e + 4480*b*c**2*d**4*e**2*x + 3360*b*c**2*d**3*e**3*x**2 + 560*
b*c**2*d**2*e**4*x**3 - 70*b*c**2*d*e**5*x**4 + 21*b*c**2*e**6*x**5 - 1024
*c**3*d**6 - 2560*c**3*d**5*e*x - 1920*c**3*d**4*e**2*x**2 - 320*c**3*d**3
*e**3*x**3 + 40*c**3*d**2*e**4*x**4 - 12*c**3*d*e**5*x**5 + 5*c**3*e**6*x*
*6))/(35*sqrt(d + e*x)*e**7*(d**2 + 2*d*e*x + e**2*x**2))
```

3.105 $\int \frac{(d+ex)^{7/2}}{bx+cx^2}\,dx$

Optimal result 784
Mathematica [A] (verified) 784
Rubi [A] (verified) 785
Maple [A] (verified) 788
Fricas [A] (verification not implemented) 788
Sympy [A] (verification not implemented) 789
Maxima [F(-2)] 790
Giac [A] (verification not implemented) 790
Mupad [B] (verification not implemented) 791
Reduce [B] (verification not implemented) 792

Optimal result

Integrand size = 21, antiderivative size = 157

$$\int \frac{(d+ex)^{7/2}}{bx+cx^2}\,dx = \frac{2e(3c^2d^2-3bcde+b^2e^2)\sqrt{d+ex}}{c^3}+\frac{2e(2cd-be)(d+ex)^{3/2}}{3c^2}+\frac{2e(d+ex)^{5/2}}{5c}-\frac{2d^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}+\frac{2(cd-be)^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{bc^{7/2}}$$

output
```
2*e*(b^2*e^2-3*b*c*d*e+3*c^2*d^2)*(e*x+d)^(1/2)/c^3+2/3*e*(-b*e+2*c*d)*(e*
x+d)^(3/2)/c^2+2/5*e*(e*x+d)^(5/2)/c-2*d^(7/2)*arctanh((e*x+d)^(1/2)/d^(1/
2))/b+2*(-b*e+c*d)^(7/2)*arctanh(c^(1/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b
/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^{7/2}}{bx+cx^2}\,dx = \frac{2e\sqrt{d+ex}(15b^2e^2-5bce(10d+ex)+c^2(58d^2+16dex+3e^2x^2))}{15c^3}-\frac{2(-cd+be)^{7/2}\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{-cd+be}}\right)}{bc^{7/2}}-\frac{2d^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}$$

input
```
Integrate[(d + e*x)^(7/2)/(b*x + c*x^2),x]
```

output
```
(2*e*Sqrt[d + e*x]*(15*b^2*e^2 - 5*b*c*e*(10*d + e*x) + c^2*(58*d^2 + 16*d
*e*x + 3*e^2*x^2)))/(15*c^3) - (2*(-(c*d) + b*e)^(7/2)*ArcTan[(Sqrt[c]*Sqr
t[d + e*x])/Sqrt[-(c*d) + b*e]])/(b*c^(7/2)) - (2*d^(7/2)*ArcTanh[Sqrt[d +
 e*x]/Sqrt[d]])/b
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1146, 1196, 1196, 1197, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{bx+cx^2}dx$$

$$\Big\downarrow \text{1146}$$

$$\frac{\int \frac{(d+ex)^{3/2}(cd^2+e(2cd-be)x)}{cx^2+bx}dx}{c}+\frac{2e(d+ex)^{5/2}}{5c}$$

$$\Big\downarrow \text{1196}$$

$$\frac{\frac{\int \frac{\sqrt{d+ex}\left(c^2d^3+e\left(3c^2d^2-3bced+b^2e^2\right)x\right)}{cx^2+bx}dx}{c}+\frac{2e(d+ex)^{3/2}(2cd-be)}{3c}}{c}+\frac{2e(d+ex)^{5/2}}{5c}$$

$$\Big\downarrow \text{1196}$$

$$\frac{\frac{\frac{\int \frac{c^3d^4+e(2cd-be)\left(2c^2d^2-2bced+b^2e^2\right)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{c}+\frac{2e\sqrt{d+ex}\left(b^2e^2-3bcde+3c^2d^2\right)}{c}}{c}+\frac{2e(d+ex)^{3/2}(2cd-be)}{3c}}{c}+\frac{2e(d+ex)^{5/2}}{5c}$$

$$\Big\downarrow \text{1197}$$

$$\frac{\frac{\frac{2\int -\frac{e\left(d(cd-be)\left(3c^2d^2-3bced+b^2e^2\right)-(2cd-be)\left(2c^2d^2-2bced+b^2e^2\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}+\frac{2e\sqrt{d+ex}\left(b^2e^2-3bcde+3c^2d^2\right)}{c}}{c}+\frac{2e(d+ex)^{3/2}(2cd-be)}{3c}}{c}+\frac{2e(d+ex)^{5/2}}{5c}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{2e\sqrt{d+ex}\left(b^2e^2-3bcde+3c^2d^2\right)}{c}-\frac{2\int \frac{e\left(d(cd-be)\left(3c^2d^2-3bced+b^2e^2\right)-(2cd-be)\left(2c^2d^2-2bced+b^2e^2\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}}{c}+\frac{2e(d+ex)^{3/2}(2cd-be)}{3c}}{c}+\frac{2e(d+ex)^{5/2}}{5c}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{2e\sqrt{d+ex}\left(b^2e^2-3bcde+3c^2d^2\right)}{c}-\frac{2e\int \frac{d(cd-be)\left(3c^2d^2-3bced+b^2e^2\right)-(2cd-be)\left(2c^2d^2-2bced+b^2e^2\right)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}}{c}+\frac{2e(d+ex)^{3/2}(2cd-be)}{3c}}{c}+\frac{2e(d+ex)^{5/2}}{5c}$$

$\Big\downarrow$ 1480

$$\frac{\frac{\frac{2e\sqrt{d+ex}\left(b^2e^2-3bcde+3c^2d^2\right)}{c}-\frac{2e\left(\frac{(cd-be)^4\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}-\frac{c^4d^4\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}\right)}{c}}{c}+\frac{2e(d+ex)^{3/2}(2cd-be)}{3c}}{c}+\frac{2e(d+ex)^{5/2}}{5c}$$

$\Big\downarrow$ 221

$$\frac{\frac{\frac{2e\sqrt{d+ex}\left(b^2e^2-3bcde+3c^2d^2\right)}{c}-\frac{2e\left(\frac{c^3d^{7/2}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{be}-\frac{(cd-be)^{7/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{c}e}\right)}{c}}{c}+\frac{2e(d+ex)^{3/2}(2cd-be)}{3c}}{c}+\frac{2e(d+ex)^{5/2}}{5c}$$

input
```
Int[(d + e*x)^(7/2)/(b*x + c*x^2),x]
```

output
```
(2*e*(d + e*x)^(5/2))/(5*c) + ((2*e*(2*c*d - b*e)*(d + e*x)^(3/2))/(3*c) +
 ((2*e*(3*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*Sqrt[d + e*x])/c - (2*e*((c^3*d^(
7/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*e) - ((c*d - b*e)^(7/2)*ArcTanh[(S
qrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*Sqrt[c]*e)))/c)/c)/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1146
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Simp[1/c   Int[(d + e*x)^
(m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
 x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{2\left((be-cd)^4 \arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)-\left(-\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)d^{\frac{7}{2}}c^3+e\sqrt{ex+d}\left(b^2e^2-\frac{10ec\left(\frac{ex}{10}+d\right)b}{3}+\frac{58c^2\left(\frac{3}{58}e^2x^2+\frac{8}{29}d\right.}{15}\right.\right.\right.}{\sqrt{c(be-cd)}\,b\,c^3}$
derivativedivides	$2e\left(\frac{\frac{(ex+d)^{\frac{5}{2}}c^2}{5}-\frac{bce(ex+d)^{\frac{3}{2}}}{3}+\frac{2c^2d(ex+d)^{\frac{3}{2}}}{3}+b^2e^2\sqrt{ex+d}-3bcde\sqrt{ex+d}+3c^2d^2\sqrt{ex+d}}{c^3}+\frac{(-b^4e^4+4d\,e^3b^3c-6d^2e}{}\right.$
default	$2e\left(\frac{\frac{(ex+d)^{\frac{5}{2}}c^2}{5}-\frac{bce(ex+d)^{\frac{3}{2}}}{3}+\frac{2c^2d(ex+d)^{\frac{3}{2}}}{3}+b^2e^2\sqrt{ex+d}-3bcde\sqrt{ex+d}+3c^2d^2\sqrt{ex+d}}{c^3}+\frac{(-b^4e^4+4d\,e^3b^3c-6d^2e}{}\right.$

input
```
int((e*x+d)^(7/2)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-2*((b*e-c*d)^4*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2))-(-arctanh((e*x
+d)^(1/2)/d^(1/2))*d^(7/2)*c^3+e*(e*x+d)^(1/2)*(b^2*e^2-10/3*e*c*(1/10*e*x
+d)*b+58/15*c^2*(3/58*e^2*x^2+8/29*d*e*x+d^2))*b)*(c*(b*e-c*d))^(1/2))/(c*
(b*e-c*d))^(1/2)/b/c^3
```

Fricas [A] (verification not implemented)

Time = 0.62 (sec) , antiderivative size = 816, normalized size of antiderivative = 5.20

$$\int \frac{(d+ex)^{7/2}}{bx+cx^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
[1/15*(15*c^3*d^(7/2)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 15*(c
^3*d^3 - 3*b*c^2*d^2*e + 3*b^2*c*d*e^2 - b^3*e^3)*sqrt((c*d - b*e)/c)*log(
(c*e*x + 2*c*d - b*e - 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) +
 2*(3*b*c^2*e^3*x^2 + 58*b*c^2*d^2*e - 50*b^2*c*d*e^2 + 15*b^3*e^3 + (16*b
*c^2*d*e^2 - 5*b^2*c*e^3)*x)*sqrt(e*x + d))/(b*c^3), 1/15*(15*c^3*d^(7/2)*
log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 30*(c^3*d^3 - 3*b*c^2*d^2*e
 + 3*b^2*c*d*e^2 - b^3*e^3)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*s
qrt(-(c*d - b*e)/c)/(c*d - b*e)) + 2*(3*b*c^2*e^3*x^2 + 58*b*c^2*d^2*e - 5
0*b^2*c*d*e^2 + 15*b^3*e^3 + (16*b*c^2*d*e^2 - 5*b^2*c*e^3)*x)*sqrt(e*x +
d))/(b*c^3), 1/15*(30*c^3*sqrt(-d)*d^3*arctan(sqrt(-d)/sqrt(e*x + d)) - 15
*(c^3*d^3 - 3*b*c^2*d^2*e + 3*b^2*c*d*e^2 - b^3*e^3)*sqrt((c*d - b*e)/c)*l
og((c*e*x + 2*c*d - b*e - 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)
) + 2*(3*b*c^2*e^3*x^2 + 58*b*c^2*d^2*e - 50*b^2*c*d*e^2 + 15*b^3*e^3 + (1
6*b*c^2*d*e^2 - 5*b^2*c*e^3)*x)*sqrt(e*x + d))/(b*c^3), 2/15*(15*c^3*sqrt(
-d)*d^3*arctan(sqrt(-d)/sqrt(e*x + d)) + 15*(c^3*d^3 - 3*b*c^2*d^2*e + 3*b
^2*c*d*e^2 - b^3*e^3)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(
c*d - b*e)/c)/(c*d - b*e)) + (3*b*c^2*e^3*x^2 + 58*b*c^2*d^2*e - 50*b^2*c*
d*e^2 + 15*b^3*e^3 + (16*b*c^2*d*e^2 - 5*b^2*c*e^3)*x)*sqrt(e*x + d))/(b*c
^3)]
```

Sympy [A] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)^{7/2}}{bx+cx^2}\,dx = \begin{cases} \frac{2\left(\frac{e^2(d+ex)^{\frac{5}{2}}}{5c} + \frac{(d+ex)^{\frac{3}{2}}\left(-be^3+2cde^2\right)}{3c^2} + \frac{\sqrt{d+ex}\left(b^2e^4-3bcde^3+3c^2d^2e^2\right)}{c^3} + \frac{d^4 e \operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d}}\right)}{b\sqrt{-d}} - \frac{e(be-cd)^4 \operatorname{atan}\left(\frac{\sqrt{d+e}}{\sqrt{\frac{be-c}{c}}}\right.}{bc^4\sqrt{\frac{be-cd}{c}}}\right.}{e} \\ d^{\frac{7}{2}}\left(-\frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for}\ c=0 \\ -\frac{\log\left(b-2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b} - \frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for}\ c=0 \\ \frac{\log\left(b+2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b}\right) \end{cases}$$

input
```
integrate((e*x+d)**(7/2)/(c*x**2+b*x),x)
```

output
```
Piecewise((2*(e**2*(d + e*x)**(5/2)/(5*c) + (d + e*x)**(3/2)*(-b*e**3 + 2*
c*d*e**2)/(3*c**2) + sqrt(d + e*x)*(b**2*e**4 - 3*b*c*d*e**3 + 3*c**2*d**2
*e**2)/c**3 + d**4*e*atan(sqrt(d + e*x)/sqrt(-d))/(b*sqrt(-d)) - e*(b*e -
c*d)**4*atan(sqrt(d + e*x)/sqrt((b*e - c*d)/c))/(b*c**4*sqrt((b*e - c*d)/c
)))/e, Ne(e, 0)), (d**(7/2)*(-2*c*Piecewise(((b/(2*c) + x)/b, Eq(c, 0)), (
-log(b - 2*c*(b/(2*c) + x))/(2*c), True))/b - 2*c*Piecewise(((b/(2*c) + x)
/b, Eq(c, 0)), (log(b + 2*c*(b/(2*c) + x))/(2*c), True))/b), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{7/2}}{bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex)^{7/2}}{bx+cx^2}\,dx = \frac{2\,d^4\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b\sqrt{-d}} - \frac{2\,(c^4d^4-4\,bc^3d^3e+6\,b^2c^2d^2e^2-4\,b^3cde^3+b^4e^4)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}bc^3} + \frac{2\left(3\,(ex+d)^{\frac{5}{2}}c^4e+10\,(ex+d)^{\frac{3}{2}}c^4de+45\,\sqrt{ex+d}c^4d^2e-5\,(ex+d)^{\frac{3}{2}}bc^3e^2-45\,\sqrt{ex+d}bc^3de^2+15\,\sqrt{e}\right.}{15\,c^5}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
2*d^4*arctan(sqrt(e*x + d)/sqrt(-d))/(b*sqrt(-d)) - 2*(c^4*d^4 - 4*b*c^3*d
^3*e + 6*b^2*c^2*d^2*e^2 - 4*b^3*c*d*e^3 + b^4*e^4)*arctan(sqrt(e*x + d)*c
/sqrt(-c^2*d + b*c*e))/(sqrt(-c^2*d + b*c*e)*b*c^3) + 2/15*(3*(e*x + d)^(5
/2)*c^4*e + 10*(e*x + d)^(3/2)*c^4*d*e + 45*sqrt(e*x + d)*c^4*d^2*e - 5*(e
*x + d)^(3/2)*b*c^3*e^2 - 45*sqrt(e*x + d)*b*c^3*d*e^2 + 15*sqrt(e*x + d)*
b^2*c^2*e^3)/c^5
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 2482, normalized size of antiderivative = 15.81

$$\int \frac{(d+ex)^{7/2}}{bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(7/2)/(b*x + c*x^2),x)
```

output
```
((2*e*(b*e - 2*c*d)^2)/c^3 - (2*e*(c*d^2 - b*d*e))/c^2)*(d + e*x)^(1/2) +
(2*e*(d + e*x)^(5/2))/(5*c) + (atan(((((8*(d + e*x)^(1/2)*(b^8*e^10 + 2*c^
8*d^8*e^2 - 8*b*c^7*d^7*e^3 + 28*b^2*c^6*d^6*e^4 - 56*b^3*c^5*d^5*e^5 + 70
*b^4*c^4*d^4*e^6 - 56*b^5*c^3*d^3*e^7 + 28*b^6*c^2*d^2*e^8 - 8*b^7*c*d*e^9
))/c^5 + (((8*(b^5*c^4*d*e^6 - 3*b^2*c^7*d^4*e^3 + 6*b^3*c^6*d^3*e^4 - 4*b
^4*c^5*d^2*e^5))/c^5 + (8*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2)*(d^7)^(1/2)*(d +
 e*x)^(1/2))/(b*c^5))*(d^7)^(1/2))/b)*(d^7)^(1/2)*1i)/b + (((8*(d + e*x)^(
1/2)*(b^8*e^10 + 2*c^8*d^8*e^2 - 8*b*c^7*d^7*e^3 + 28*b^2*c^6*d^6*e^4 - 56
*b^3*c^5*d^5*e^5 + 70*b^4*c^4*d^4*e^6 - 56*b^5*c^3*d^3*e^7 + 28*b^6*c^2*d^
2*e^8 - 8*b^7*c*d*e^9))/c^5 - (((8*(b^5*c^4*d*e^6 - 3*b^2*c^7*d^4*e^3 + 6*
b^3*c^6*d^3*e^4 - 4*b^4*c^5*d^2*e^5))/c^5 - (8*(b^3*c^7*e^3 - 2*b^2*c^8*d*
e^2)*(d^7)^(1/2)*(d + e*x)^(1/2))/(b*c^5))*(d^7)^(1/2))/b)*(d^7)^(1/2)*1i)
/b)/((16*(b^7*d^4*e^10 - 4*c^7*d^11*e^3 + 22*b*c^6*d^10*e^4 - 8*b^6*c*d^5*
e^9 - 52*b^2*c^5*d^9*e^5 + 69*b^3*c^4*d^8*e^6 - 56*b^4*c^3*d^7*e^7 + 28*b^
5*c^2*d^6*e^8))/c^5 + (((8*(d + e*x)^(1/2)*(b^8*e^10 + 2*c^8*d^8*e^2 - 8*b
*c^7*d^7*e^3 + 28*b^2*c^6*d^6*e^4 - 56*b^3*c^5*d^5*e^5 + 70*b^4*c^4*d^4*e^
6 - 56*b^5*c^3*d^3*e^7 + 28*b^6*c^2*d^2*e^8 - 8*b^7*c*d*e^9))/c^5 + (((8*(
b^5*c^4*d*e^6 - 3*b^2*c^7*d^4*e^3 + 6*b^3*c^6*d^3*e^4 - 4*b^4*c^5*d^2*e^5)
)/c^5 + (8*(b^3*c^7*e^3 - 2*b^2*c^8*d*e^2)*(d^7)^(1/2)*(d + e*x)^(1/2))/(b
*c^5))*(d^7)^(1/2))/b)*(d^7)^(1/2))/b - (((8*(d + e*x)^(1/2)*(b^8*e^10 ...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 329, normalized size of antiderivative = 2.10

$$\int \frac{(d+ex)^{7/2}}{bx+cx^2}\,dx = \frac{-30\sqrt{c}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{be-cd}}\right)b^3e^3+90\sqrt{c}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{be-cd}}\right)b^2cd\,e^2-90}{}$$

input
```
int((e*x+d)^(7/2)/(c*x^2+b*x),x)
```

output
```
( - 30*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e -
c*d)))*b**3*e**3 + 90*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt
(c)*sqrt(b*e - c*d)))*b**2*c*d*e**2 - 90*sqrt(c)*sqrt(b*e - c*d)*atan((sqr
t(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**2*d**2*e + 30*sqrt(c)*sqrt(b
*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*c**3*d**3 + 30
*sqrt(d + e*x)*b**3*c*e**3 - 100*sqrt(d + e*x)*b**2*c**2*d*e**2 - 10*sqrt(
d + e*x)*b**2*c**2*e**3*x + 116*sqrt(d + e*x)*b*c**3*d**2*e + 32*sqrt(d +
e*x)*b*c**3*d*e**2*x + 6*sqrt(d + e*x)*b*c**3*e**3*x**2 + 15*sqrt(d)*log(s
qrt(d + e*x) - sqrt(d))*c**4*d**3 - 15*sqrt(d)*log(sqrt(d + e*x) + sqrt(d)
)*c**4*d**3)/(15*b*c**4)
```

3.106 $\int \frac{(d+ex)^{5/2}}{bx+cx^2}\,dx$

Optimal result 793
Mathematica [A] (verified) 793
Rubi [A] (verified) 794
Maple [A] (verified) 796
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Reduce [B] (verification not implemented) 800

Optimal result

Integrand size = 21, antiderivative size = 118

$$\int \frac{(d+ex)^{5/2}}{bx+cx^2}\,dx = \frac{2e(2cd-be)\sqrt{d+ex}}{c^2}+\frac{2e(d+ex)^{3/2}}{3c} - \frac{2d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}+\frac{2(cd-be)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{bc^{5/2}}$$

output
```
2*e*(-b*e+2*c*d)*(e*x+d)^(1/2)/c^2+2/3*e*(e*x+d)^(3/2)/c-2*d^(5/2)*arctanh
((e*x+d)^(1/2)/d^(1/2))/b+2*(-b*e+c*d)^(5/2)*arctanh(c^(1/2)*(e*x+d)^(1/2)
/(-b*e+c*d)^(1/2))/b/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^{5/2}}{bx+cx^2}\,dx = \frac{2e\sqrt{d+ex}(7cd-3be+cex)}{3c^2} + \frac{2(-cd+be)^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{-cd+be}}\right)}{bc^{5/2}}-\frac{2d^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}$$

input
```
Integrate[(d + e*x)^(5/2)/(b*x + c*x^2),x]
```

output
```
(2*e*Sqrt[d + e*x]*(7*c*d - 3*b*e + c*e*x))/(3*c^2) + (2*(-(c*d) + b*e)^(5
/2)*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[-(c*d) + b*e]])/(b*c^(5/2)) - (2*d
^(5/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/b
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1146, 1196, 1197, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{bx+cx^2}dx$$

$$\Big\downarrow \text{1146}$$

$$\frac{\int \frac{\sqrt{d+ex}(cd^2+e(2cd-be)x)}{cx^2+bx}dx}{c}+\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{1196}$$

$$\frac{\frac{\int \frac{c^2d^3+e\left(3c^2d^2-3bced+b^2e^2\right)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{c}+\frac{2e\sqrt{d+ex}(2cd-be)}{c}}{c}+\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{1197}$$

$$\frac{\frac{2\int -\frac{e\left(d(cd-be)(2cd-be)-\left(3c^2d^2-3bced+b^2e^2\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}+\frac{2e\sqrt{d+ex}(2cd-be)}{c}}{c}+\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{2e\sqrt{d+ex}(2cd-be)}{c}-\frac{2\int \frac{e\left(d(cd-be)(2cd-be)-\left(3c^2d^2-3bced+b^2e^2\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}}{c}+\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2e\sqrt{d+ex}(2cd-be)}{c}-\frac{2e\int \frac{d(cd-be)(2cd-be)-\left(3c^2d^2-3bced+b^2e^2\right)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}}{c}+\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{2e\sqrt{d+ex}(2cd-be)}{c}-\frac{2e\left(\frac{(cd-be)^3\int\frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}-\frac{c^3d^3\int\frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}\right)}{c}}{c}+\frac{2e(d+ex)^{3/2}}{3c}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{2e\sqrt{d+ex}(2cd-be)}{c}-\frac{2e\left(\frac{c^2d^{5/2}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{be}-\frac{(cd-be)^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{c}e}\right)}{c}}{c}+\frac{2e(d+ex)^{3/2}}{3c}$$

input
```
Int[(d + e*x)^(5/2)/(b*x + c*x^2),x]
```

output
```
(2*e*(d + e*x)^(3/2))/(3*c) + ((2*e*(2*c*d - b*e)*Sqrt[d + e*x])/c - (2*e*
((c^2*d^(5/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*e) - ((c*d - b*e)^(5/2)*A
rcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*Sqrt[c]*e)))/c)/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1146
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Simp[1/c   Int[(d + e*x)^
(m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
 x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.97

method	result
pseudoelliptic	$-\frac{2\left(-(be-cd)^3\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)+\left(\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)d^{\frac{5}{2}}c^2+e\sqrt{ex+d}\left(\frac{(-ex-7d)c}{3}+be\right)b\right)\sqrt{c(be-cd)}\right)}{\sqrt{c(be-cd)}\,c^2b}$
derivativedivides	$2e\left(-\frac{-\frac{c(ex+d)^{\frac{3}{2}}}{3}+be\sqrt{ex+d}-2cd\sqrt{ex+d}}{c^2}+\frac{\left(b^3e^3-3de^2b^2c+3d^2ebc^2-d^3c^3\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{c^2be\sqrt{c(be-cd)}}-d^{\frac{5}{2}}\arctan\ldots\right.$
default	$2e\left(-\frac{-\frac{c(ex+d)^{\frac{3}{2}}}{3}+be\sqrt{ex+d}-2cd\sqrt{ex+d}}{c^2}+\frac{\left(b^3e^3-3de^2b^2c+3d^2ebc^2-d^3c^3\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{c^2be\sqrt{c(be-cd)}}-d^{\frac{5}{2}}\arctan\ldots\right.$

input
```
int((e*x+d)^(5/2)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-2*(-(b*e-c*d)^3*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2))+(arctanh((e*x
+d)^(1/2)/d^(1/2))*d^(5/2)*c^2+e*(e*x+d)^(1/2)*(1/3*(-e*x-7*d)*c+b*e)*b)*(
c*(b*e-c*d))^(1/2))/(c*(b*e-c*d))^(1/2)/c^2/b
```

Fricas [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 592, normalized size of antiderivative = 5.02

$$\int \frac{(d+ex)^{5/2}}{bx+cx^2}\,dx = \left[\frac{3\,c^2 d^{\frac{5}{2}}\log\left(\frac{ex-2\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)+3\left(c^2d^2-2\,bcde+b^2e^2\right)\sqrt{\frac{cd-be}{c}}\log\left(\frac{cex+2\,cd-be+2\,\sqrt{ex+}}{cx+b}\right.}{3\,bc^2}\right.$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
[1/3*(3*c^2*d^(5/2)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 3*(c^2*
d^2 - 2*b*c*d*e + b^2*e^2)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e +
2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) + 2*(b*c*e^2*x + 7*b*c*d
*e - 3*b^2*e^2)*sqrt(e*x + d))/(b*c^2), 1/3*(3*c^2*d^(5/2)*log((e*x - 2*sq
rt(e*x + d)*sqrt(d) + 2*d)/x) + 6*(c^2*d^2 - 2*b*c*d*e + b^2*e^2)*sqrt(-(c
*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + 2
*(b*c*e^2*x + 7*b*c*d*e - 3*b^2*e^2)*sqrt(e*x + d))/(b*c^2), 1/3*(6*c^2*sq
rt(-d)*d^2*arctan(sqrt(-d)/sqrt(e*x + d)) + 3*(c^2*d^2 - 2*b*c*d*e + b^2*e
^2)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*sqrt(
(c*d - b*e)/c))/(c*x + b)) + 2*(b*c*e^2*x + 7*b*c*d*e - 3*b^2*e^2)*sqrt(e*
x + d))/(b*c^2), 2/3*(3*c^2*sqrt(-d)*d^2*arctan(sqrt(-d)/sqrt(e*x + d)) +
3*(c^2*d^2 - 2*b*c*d*e + b^2*e^2)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x +
d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + (b*c*e^2*x + 7*b*c*d*e - 3*b^2*e^
2)*sqrt(e*x + d))/(b*c^2)]
```

Sympy [A] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.67

$$\int \frac{(d+ex)^{5/2}}{bx+cx^2}\,dx = \begin{cases} \frac{2\left(\frac{e^2(d+ex)^{\frac{3}{2}}}{3c}+\frac{\sqrt{d+ex}\left(-be^3+2cde^2\right)}{c^2}+\frac{d^3e\,\mathrm{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d}}\right)}{b\sqrt{-d}}+\frac{e(be-cd)^3\,\mathrm{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{be-cd}{c}}}\right)}{bc^3\sqrt{\frac{be-cd}{c}}}\right)}{e} & \text{f} \\ d^{\frac{5}{2}}\left(-\frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for } c=0\\ -\frac{\log\left(b-2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b}-\frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for } c=0\\ \frac{\log\left(b+2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b}\right) & \text{o} \end{cases}$$

input
```
integrate((e*x+d)**(5/2)/(c*x**2+b*x),x)
```

output
```
Piecewise((2*(e**2*(d + e*x)**(3/2)/(3*c) + sqrt(d + e*x)*(-b*e**3 + 2*c*d
*e**2)/c**2 + d**3*e*atan(sqrt(d + e*x)/sqrt(-d))/(b*sqrt(-d)) + e*(b*e -
c*d)**3*atan(sqrt(d + e*x)/sqrt((b*e - c*d)/c))/(b*c**3*sqrt((b*e - c*d)/c
)))/e, Ne(e, 0)), (d**(5/2)*(-2*c*Piecewise(((b/(2*c) + x)/b, Eq(c, 0)), (
-log(b - 2*c*(b/(2*c) + x))/(2*c), True))/b - 2*c*Piecewise(((b/(2*c) + x)
/b, Eq(c, 0)), (log(b + 2*c*(b/(2*c) + x))/(2*c), True))/b), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{5/2}}{bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.31

$$\int \frac{(d+ex)^{5/2}}{bx+cx^2}\,dx = \frac{2\,d^3\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b\sqrt{-d}} - \frac{2\,(c^3d^3-3\,bc^2d^2e+3\,b^2cde^2-b^3e^3)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}bc^2} + \frac{2\left((ex+d)^{\frac{3}{2}}c^2e+6\sqrt{ex+d}c^2de-3\sqrt{ex+d}bce^2\right)}{3\,c^3}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
2*d^3*arctan(sqrt(e*x + d)/sqrt(-d))/(b*sqrt(-d)) - 2*(c^3*d^3 - 3*b*c^2*d
^2*e + 3*b^2*c*d*e^2 - b^3*e^3)*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c*e
))/(sqrt(-c^2*d + b*c*e)*b*c^2) + 2/3*((e*x + d)^(3/2)*c^2*e + 6*sqrt(e*x
+ d)*c^2*d*e - 3*sqrt(e*x + d)*b*c*e^2)/c^3
```

Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 2048, normalized size of antiderivative = 17.36

$$\int \frac{(d+ex)^{5/2}}{bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(5/2)/(b*x + c*x^2),x)
```

output
```
(atan((((8*(d + e*x)^(1/2)*(b^6*e^8 + 2*c^6*d^6*e^2 - 6*b*c^5*d^5*e^3 + 1
5*b^2*c^4*d^4*e^4 - 20*b^3*c^3*d^3*e^5 + 15*b^4*c^2*d^2*e^6 - 6*b^5*c*d*e^
7))/c^3 + (((8*(b^4*c^3*d*e^5 + 2*b^2*c^5*d^3*e^3 - 3*b^3*c^4*d^2*e^4))/c^
3 + (8*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2)*(d^5)^(1/2)*(d + e*x)^(1/2))/(b*c^3
))*(d^5)^(1/2))/b)*(d^5)^(1/2)*1i)/b + (((8*(d + e*x)^(1/2)*(b^6*e^8 + 2*c
^6*d^6*e^2 - 6*b*c^5*d^5*e^3 + 15*b^2*c^4*d^4*e^4 - 20*b^3*c^3*d^3*e^5 + 1
5*b^4*c^2*d^2*e^6 - 6*b^5*c*d*e^7))/c^3 - (((8*(b^4*c^3*d*e^5 + 2*b^2*c^5*
d^3*e^3 - 3*b^3*c^4*d^2*e^4))/c^3 - (8*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2)*(d^
5)^(1/2)*(d + e*x)^(1/2))/(b*c^3))*(d^5)^(1/2))/b)*(d^5)^(1/2)*1i)/b)/((16
*(b^5*d^3*e^8 - 3*c^5*d^8*e^3 + 12*b*c^4*d^7*e^4 - 6*b^4*c*d^4*e^7 - 19*b^
2*c^3*d^6*e^5 + 15*b^3*c^2*d^5*e^6))/c^3 - (((8*(d + e*x)^(1/2)*(b^6*e^8 +
 2*c^6*d^6*e^2 - 6*b*c^5*d^5*e^3 + 15*b^2*c^4*d^4*e^4 - 20*b^3*c^3*d^3*e^5
 + 15*b^4*c^2*d^2*e^6 - 6*b^5*c*d*e^7))/c^3 + (((8*(b^4*c^3*d*e^5 + 2*b^2*
c^5*d^3*e^3 - 3*b^3*c^4*d^2*e^4))/c^3 + (8*(b^3*c^5*e^3 - 2*b^2*c^6*d*e^2)
*(d^5)^(1/2)*(d + e*x)^(1/2))/(b*c^3))*(d^5)^(1/2))/b)*(d^5)^(1/2))/b + ((
(8*(d + e*x)^(1/2)*(b^6*e^8 + 2*c^6*d^6*e^2 - 6*b*c^5*d^5*e^3 + 15*b^2*c^4
*d^4*e^4 - 20*b^3*c^3*d^3*e^5 + 15*b^4*c^2*d^2*e^6 - 6*b^5*c*d*e^7))/c^3 -
 (((8*(b^4*c^3*d*e^5 + 2*b^2*c^5*d^3*e^3 - 3*b^3*c^4*d^2*e^4))/c^3 - (8*(b
^3*c^5*e^3 - 2*b^2*c^6*d*e^2)*(d^5)^(1/2)*(d + e*x)^(1/2))/(b*c^3))*(d^5)^
(1/2))/b)*(d^5)^(1/2))/b))*(d^5)^(1/2)*2i)/b + (2*e*(d + e*x)^(3/2))/(3...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.89

$$\int \frac{(d+ex)^{5/2}}{bx+cx^2}\,dx = \frac{6\sqrt{c}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{be-cd}}\right)b^2e^2 - 12\sqrt{c}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{be-cd}}\right)bcde + 6\sqrt{c}\,\sqrt{be}$$

input
```
int((e*x+d)^(5/2)/(c*x^2+b*x),x)
```

output
```
(6*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)
))*b**2*e**2 - 12*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*
sqrt(b*e - c*d)))*b*c*d*e + 6*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*
c)/(sqrt(c)*sqrt(b*e - c*d)))*c**2*d**2 - 6*sqrt(d + e*x)*b**2*c*e**2 + 14
*sqrt(d + e*x)*b*c**2*d*e + 2*sqrt(d + e*x)*b*c**2*e**2*x + 3*sqrt(d)*log(
sqrt(d + e*x) - sqrt(d))*c**3*d**2 - 3*sqrt(d)*log(sqrt(d + e*x) + sqrt(d)
)*c**3*d**2)/(3*b*c**3)
```

3.107 $\int \frac{(d+ex)^{3/2}}{bx+cx^2}\,dx$

Optimal result . . . 801
Mathematica [A] (verified) . . . 801
Rubi [A] (verified) . . . 802
Maple [A] (verified) . . . 804
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Giac [A] (verification not implemented) . . . 806
Mupad [B] (verification not implemented) . . . 807
Reduce [B] (verification not implemented) . . . 808

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \frac{(d+ex)^{3/2}}{bx+cx^2}\,dx=\frac{2e\sqrt{d+ex}}{c}-\frac{2d^{3/2}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}+\frac{2(cd-be)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{bc^{3/2}}$$

output
```
2*e*(e*x+d)^(1/2)/c-2*d^(3/2)*arctanh((e*x+d)^(1/2)/d^(1/2))/b+2*(-b*e+c*d
)^(3/2)*arctanh(c^(1/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^{3/2}}{bx+cx^2}\,dx=\frac{2\left(b\sqrt{c}e\sqrt{d+ex}-(-cd+be)^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{-cd+be}}\right)-c^{3/2}d^{3/2}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\right)}{bc^{3/2}}$$

input
```
Integrate[(d + e*x)^(3/2)/(b*x + c*x^2),x]
```

output
```
(2*(b*Sqrt[c]*e*Sqrt[d + e*x] - (-(c*d) + b*e)^(3/2)*ArcTan[(Sqrt[c]*Sqrt[
d + e*x])/Sqrt[-(c*d) + b*e]] - c^(3/2)*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt
[d]]))/(b*c^(3/2))
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1146, 1197, 25, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{bx+cx^2}dx$$

$$\Big\downarrow \text{ 1146}$$

$$\frac{\int \frac{cd^2+e(2cd-be)x}{\sqrt{d+ex}(cx^2+bx)}dx}{c}+\frac{2e\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{ 1197}$$

$$\frac{2\int -\frac{e(d(cd-be)-(2cd-be)(d+ex))}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}+\frac{2e\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{2e\sqrt{d+ex}}{c}-\frac{2\int \frac{e(d(cd-be)-(2cd-be)(d+ex))}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2e\sqrt{d+ex}}{c}-\frac{2e\int \frac{d(cd-be)-(2cd-be)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{2e\sqrt{d+ex}}{c}-\frac{2e\left(\frac{(cd-be)^2\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}-\frac{c^2d^2\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}\right)}{c}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2e\sqrt{d+ex}}{c}-\frac{2e\left(\frac{cd^{3/2}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{be}-\frac{(cd-be)^{3/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{c}e}\right)}{c}$$

input
```
Int[(d + e*x)^(3/2)/(b*x + c*x^2),x]
```

output
```
(2*e*Sqrt[d + e*x])/c - (2*e*((c*d^(3/2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(
b*e) - ((c*d - b*e)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]]
)/(b*Sqrt[c]*e)))/c
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1146
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Simp[1/c   Int[(d + e*x)^
(m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
 x] /; FreeQ[{a, b, c, d, e}, x] && GtQ[m, 1]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07

method	result	size
pseudoelliptic	$\frac{-2(be-cd)^2 \arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)+2\left(-d^{\frac{3}{2}} \operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)c+be\sqrt{ex+d}\right)\sqrt{c(be-cd)}}{bc\sqrt{c(be-cd)}}$	98
derivativedivides	$2e\left(\frac{\sqrt{ex+d}}{c}+\frac{(-b^2e^2+2bcde-c^2d^2)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{cbe\sqrt{c(be-cd)}}-\frac{d^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be}\right)$	106
default	$2e\left(\frac{\sqrt{ex+d}}{c}+\frac{(-b^2e^2+2bcde-c^2d^2)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{cbe\sqrt{c(be-cd)}}-\frac{d^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be}\right)$	106

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
2*(-(b*e-c*d)^2*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2))+(-d^(3/2)*arct
anh((e*x+d)^(1/2)/d^(1/2))*c+b*e*(e*x+d)^(1/2))*(c*(b*e-c*d))^(1/2))/(c*(b
*e-c*d))^(1/2)/c/b
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 441, normalized size of antiderivative = 4.79

$$\int \frac{(d+ex)^{3/2}}{bx+cx^2}\,dx = \left[\frac{cd^{\frac{3}{2}}\log\left(\frac{ex-2\sqrt{ex+d}\sqrt{d}+2d}{x}\right)+2\sqrt{ex+d}be-(cd-be)\sqrt{\frac{cd-be}{c}}\log\left(\frac{cex+2cd-be-2\sqrt{ex+d}c}{cx+b}\right.}{bc}\right.$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
[(c*d^(3/2)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*sqrt(e*x + d)
*b*e - (c*d - b*e)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e - 2*sqrt(e
*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)))/(b*c), (c*d^(3/2)*log((e*x - 2*
sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*sqrt(e*x + d)*b*e + 2*(c*d - b*e)*sqrt
(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e))
)/(b*c), (2*c*sqrt(-d)*d*arctan(sqrt(-d)/sqrt(e*x + d)) + 2*sqrt(e*x + d)*
b*e - (c*d - b*e)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e - 2*sqrt(e*
x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)))/(b*c), 2*(c*sqrt(-d)*d*arctan(sq
rt(-d)/sqrt(e*x + d)) + sqrt(e*x + d)*b*e + (c*d - b*e)*sqrt(-(c*d - b*e)/
c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)))/(b*c)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(80) = 160$.

Time = 2.27 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.83

$$\int \frac{(d+ex)^{3/2}}{bx+cx^2}\,dx = \begin{cases} \frac{2\left(\frac{e^2\sqrt{d+ex}}{c} + \frac{d^2 e \operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d}}\right)}{b\sqrt{-d}} - \frac{e(be-cd)^2 \operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{be-cd}{c}}}\right)}{bc^2\sqrt{\frac{be-cd}{c}}}\right)}{e} & \\ d^{\frac{3}{2}}\left(-\frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for } c=0 \\ -\frac{\log\left(b-2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b} - \frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for } c=0 \\ \frac{\log\left(b+2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b}\right) & \end{cases}$$

input
```
integrate((e*x+d)**(3/2)/(c*x**2+b*x),x)
```

output
```
Piecewise((2*(e**2*sqrt(d + e*x)/c + d**2*e*atan(sqrt(d + e*x)/sqrt(-d))/(
b*sqrt(-d)) - e*(b*e - c*d)**2*atan(sqrt(d + e*x)/sqrt((b*e - c*d)/c))/(b*
c**2*sqrt((b*e - c*d)/c)))/e, Ne(e, 0)), (d**(3/2)*(-2*c*Piecewise(((b/(2*
c) + x)/b, Eq(c, 0)), (-log(b - 2*c*(b/(2*c) + x))/(2*c), True))/b - 2*c*P
iecewise(((b/(2*c) + x)/b, Eq(c, 0)), (log(b + 2*c*(b/(2*c) + x))/(2*c), T
rue))/b), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}}{bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^{3/2}}{bx+cx^2}\,dx = \frac{2\,d^2\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b\sqrt{-d}} + \frac{2\,\sqrt{ex+d}e}{c} - \frac{2\,(c^2d^2-2\,bcde+b^2e^2)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}bc}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
2*d^2*arctan(sqrt(e*x + d)/sqrt(-d))/(b*sqrt(-d)) + 2*sqrt(e*x + d)*e/c -
2*(c^2*d^2 - 2*b*c*d*e + b^2*e^2)*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c
*e))/(sqrt(-c^2*d + b*c*e)*b*c)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 697, normalized size of antiderivative = 7.58

$$\int \frac{(d+ex)^{3/2}}{bx+cx^2}\,dx = \frac{2\,e\,\sqrt{d+e\,x}}{c}$$

$$-\frac{2\,\mathrm{atanh}\left(\frac{16\,b^3\,e^6\,\sqrt{d^3}\,\sqrt{d+e\,x}}{16\,b^3\,d^2\,e^6-64\,b^2\,c\,d^3\,e^5+96\,b\,c^2\,d^4\,e^4-48\,c^3\,d^5\,e^3}+\frac{48\,c^2\,d^3\,e^3\,\sqrt{d^3}\,\sqrt{d+e\,x}}{64\,b^2\,d^3\,e^5+48\,c^2\,d^5\,e^3-\frac{16\,b^3\,d^2\,e^6}{c}-96\,b\,c\,d^4\,e^4}+\frac{64\,b^2\,d\,e^5\,\sqrt{\cdots}}{64\,b^2\,d^3\,e^5+48\,c^2\,d^5\,e^3-\cdots}\cdots\right)}{b}$$

$$+\frac{2\,\mathrm{atanh}\left(\frac{48\,d^3\,e^3\,\sqrt{d+e\,x}\,\sqrt{-b^3\,c^3\,e^3+3\,b^2\,c^4\,d\,e^2-3\,b\,c^5\,d^2\,e+c^6\,d^3}}{48\,c^3\,d^5\,e^3-80\,b^3\,d^2\,e^6-144\,b\,c^2\,d^4\,e^4+160\,b^2\,c\,d^3\,e^5+\frac{16\,b^4\,d\,e^7}{c}}+\frac{16\,b^2\,d\,e^5\,\sqrt{d+e\,x}\,\sqrt{-b^3\,c^3\,e^3+3\,b^2\,c^4\,d\,e^2-3\,b\,c^5\,d^2\,e+c^6\,d^3}}{16\,b^4\,c\,d\,e^7-80\,b^3\,c^2\,d^2\,e^6+160\,b^2\,c^3\,d^3\,e^5-144\,b\,c^4\,d^4\,e^4+48\,c^5\,\cdots}\cdots\right)}{b\,c^3}$$

input
```
int((d + e*x)^(3/2)/(b*x + c*x^2),x)
```

output
```
(2*e*(d + e*x)^(1/2))/c - (2*atanh((16*b^3*e^6*(d^3)^(1/2)*(d + e*x)^(1/2)
)/(16*b^3*d^2*e^6 - 48*c^3*d^5*e^3 + 96*b*c^2*d^4*e^4 - 64*b^2*c*d^3*e^5)
+ (48*c^2*d^3*e^3*(d^3)^(1/2)*(d + e*x)^(1/2))/(64*b^2*d^3*e^5 + 48*c^2*d^
5*e^3 - (16*b^3*d^2*e^6)/c - 96*b*c*d^4*e^4) + (64*b^2*d*e^5*(d^3)^(1/2)*(
d + e*x)^(1/2))/(64*b^2*d^3*e^5 + 48*c^2*d^5*e^3 - (16*b^3*d^2*e^6)/c - 96
*b*c*d^4*e^4) - (96*b*c*d^2*e^4*(d^3)^(1/2)*(d + e*x)^(1/2))/(64*b^2*d^3*e
^5 + 48*c^2*d^5*e^3 - (16*b^3*d^2*e^6)/c - 96*b*c*d^4*e^4))*(d^3)^(1/2))/b
 + (2*atanh((48*d^3*e^3*(d + e*x)^(1/2)*(c^6*d^3 - b^3*c^3*e^3 + 3*b^2*c^4
*d*e^2 - 3*b*c^5*d^2*e)^(1/2))/(48*c^3*d^5*e^3 - 80*b^3*d^2*e^6 - 144*b*c^
2*d^4*e^4 + 160*b^2*c*d^3*e^5 + (16*b^4*d*e^7)/c) + (16*b^2*d*e^5*(d + e*x
)^(1/2)*(c^6*d^3 - b^3*c^3*e^3 + 3*b^2*c^4*d*e^2 - 3*b*c^5*d^2*e)^(1/2))/(
48*c^5*d^5*e^3 - 144*b*c^4*d^4*e^4 + 160*b^2*c^3*d^3*e^5 - 80*b^3*c^2*d^2*
e^6 + 16*b^4*c*d*e^7) - (48*b*d^2*e^4*(d + e*x)^(1/2)*(c^6*d^3 - b^3*c^3*e
^3 + 3*b^2*c^4*d*e^2 - 3*b*c^5*d^2*e)^(1/2))/(16*b^4*d*e^7 + 48*c^4*d^5*e^
3 - 144*b*c^3*d^4*e^4 - 80*b^3*c*d^2*e^6 + 160*b^2*c^2*d^3*e^5))*(-c^3*(b*
e - c*d)^3)^(1/2))/(b*c^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.46

$$\int \frac{(d+ex)^{3/2}}{bx+cx^2}\,dx = \frac{-2\sqrt{c}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{be-cd}}\right)be+2\sqrt{c}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{be-cd}}\right)cd+2\sqrt{ex+d}\,bc}{b\,c^2}$$

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x),x)
```

output
```
( - 2*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c
*d)))*b*e + 2*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt
(b*e - c*d)))*c*d + 2*sqrt(d + e*x)*b*c*e + sqrt(d)*log(sqrt(d + e*x) - sq
rt(d))*c**2*d - sqrt(d)*log(sqrt(d + e*x) + sqrt(d))*c**2*d)/(b*c**2)
```

3.108 $\int \frac{\sqrt{d+ex}}{bx+cx^2}\,dx$

Optimal result . . . 809
Mathematica [A] (verified) . . . 809
Rubi [A] (verified) . . . 810
Maple [A] (verified) . . . 811
Fricas [A] (verification not implemented) . . . 812
Sympy [B] (verification not implemented) . . . 812
Maxima [F(-2)] . . . 813
Giac [A] (verification not implemented) . . . 814
Mupad [B] (verification not implemented) . . . 814
Reduce [B] (verification not implemented) . . . 815

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{\sqrt{d+ex}}{bx+cx^2}\,dx=-\frac{2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}+\frac{2\sqrt{cd-be}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{c}}$$

output
```
-2*d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))/b+2*(-b*e+c*d)^(1/2)*arctanh(c^(
1/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{d+ex}}{bx+cx^2}\,dx=\frac{\frac{2\sqrt{-cd+be}\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{-cd+be}}\right)}{\sqrt{c}}-2\sqrt{d}\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b}$$

input
```
Integrate[Sqrt[d + e*x]/(b*x + c*x^2),x]
```

output
```
((2*Sqrt[-(c*d) + b*e]*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[-(c*d) + b*e]])
/Sqrt[c] - 2*Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/b
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1148, 1450, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{bx+cx^2}dx$$

$$\Big\downarrow \text{ 1148}$$

$$2e\int \frac{d+ex}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}$$

$$\Big\downarrow \text{ 1450}$$

$$2e\left(\frac{cd\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{(cd-be)\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)$$

$$\Big\downarrow \text{ 221}$$

$$2e\left(\frac{\sqrt{cd-be}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{ce}}-\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{be}\right)$$

input
```
Int[Sqrt[d + e*x]/(b*x + c*x^2),x]
```

output
```
2*e*(-((Sqrt[d]*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*e)) + (Sqrt[c*d - b*e]*
ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*Sqrt[c]*e))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1148
```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Simp[2*e   Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c
*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1450
```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(d^2/2)*(b/q + 1)   Int[(d*x)^(m - 2)/(b/
2 + q/2 + c*x^2), x], x] - Simp[(d^2/2)*(b/q - 1)   Int[(d*x)^(m - 2)/(b/2
- q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] &&
 GeQ[m, 2]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

method	result	size
pseudoelliptic	$\frac{-2\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)+\frac{2(be-cd)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{\sqrt{c(be-cd)}}}{b}$	67
derivativedivides	$2e\left(-\frac{\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be}+\frac{(be-cd)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{be\sqrt{c(be-cd)}}\right)$	77
default	$2e\left(-\frac{\sqrt{d}\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be}+\frac{(be-cd)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{be\sqrt{c(be-cd)}}\right)$	77

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
2/b*(-d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))+(b*e-c*d)/(c*(b*e-c*d))^(1/2)
*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 346, normalized size of antiderivative = 4.49

$$\int \frac{\sqrt{d+ex}}{bx+cx^2}\,dx$$

$$= \left[\frac{\sqrt{\frac{cd-be}{c}}\log\left(\frac{cex+2\,cd-be+2\,\sqrt{ex+d}c\sqrt{\frac{cd-be}{c}}}{cx+b}\right)+\sqrt{d}\log\left(\frac{ex-2\,\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)}{b}, \frac{2\,\sqrt{-\frac{cd-be}{c}}\arctan\left(-\frac{\sqrt{ex+d}c\sqrt{-c}}{cd-be}\right.}{}\right.$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
[(sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*sqrt((c
*d - b*e)/c))/(c*x + b)) + sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*
d)/x))/b, (2*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e
)/c)/(c*d - b*e)) + sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x))/
b, (2*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x + d)) + sqrt((c*d - b*e)/c)*log((c
*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)))/b,
 2*(sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d
 - b*e)) + sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x + d)))/b]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(66) = 132$.

Time = 2.18 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.92

$$\int \frac{\sqrt{d+ex}}{bx+cx^2}\,dx = \begin{cases} \frac{2\left(\frac{de\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d}}\right)}{b\sqrt{-d}}+\frac{e(be-cd)\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{be-cd}{c}}}\right)}{bc\sqrt{\frac{be-cd}{c}}}\right)}{e} & \text{for } e\neq 0 \\ \sqrt{d}\left(-\frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for } c=0\\ -\frac{\log\left(b-2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b}-\frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for } c=0\\ \frac{\log\left(b+2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b}\right) & \text{otherwise}\end{cases}$$

input
```
integrate((e*x+d)**(1/2)/(c*x**2+b*x),x)
```

output
```
Piecewise((2*(d*e*atan(sqrt(d + e*x)/sqrt(-d))/(b*sqrt(-d)) + e*(b*e - c*d
)*atan(sqrt(d + e*x)/sqrt((b*e - c*d)/c))/(b*c*sqrt((b*e - c*d)/c)))/e, Ne
(e, 0)), (sqrt(d)*(-2*c*Piecewise(((b/(2*c) + x)/b, Eq(c, 0)), (-log(b - 2
*c*(b/(2*c) + x))/(2*c), True))/b - 2*c*Piecewise(((b/(2*c) + x)/b, Eq(c,
0)), (log(b + 2*c*(b/(2*c) + x))/(2*c), True))/b), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}}{bx+cx^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97

$$\int \frac{\sqrt{d+ex}}{bx+cx^2}\,dx=-\frac{2\,(cd-be)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}b}+\frac{2\,d\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b\sqrt{-d}}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
-2*(c*d - b*e)*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c*e))/(sqrt(-c^2*d +
 b*c*e)*b) + 2*d*arctan(sqrt(e*x + d)/sqrt(-d))/(b*sqrt(-d))
```

Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.30

$$\int \frac{\sqrt{d+ex}}{bx+cx^2}\,dx=\frac{2\,\mathrm{atanh}\left(\frac{16\,b\,c^2\,d\,e^3\,\sqrt{c^2\,d-b\,c\,e}\,\sqrt{d+e\,x}}{16\,b\,c^3\,d^2\,e^3-16\,b^2\,c^2\,d\,e^4}\right)\,\sqrt{c^2\,d-b\,c\,e}}{b\,c}-\frac{2\,\sqrt{d}\,\mathrm{atanh}\left(\frac{\sqrt{d+e\,x}}{\sqrt{d}}\right)}{b}$$

input
```
int((d + e*x)^(1/2)/(b*x + c*x^2),x)
```

output
```
(2*atanh((16*b*c^2*d*e^3*(c^2*d - b*c*e)^(1/2)*(d + e*x)^(1/2))/(16*b*c^3*
d^2*e^3 - 16*b^2*c^2*d*e^4))*(c^2*d - b*c*e)^(1/2))/(b*c) - (2*d^(1/2)*ata
nh((d + e*x)^(1/2)/d^(1/2)))/b
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{d+ex}}{bx+cx^2}\,dx$$
$$= \frac{2\sqrt{c}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{be-cd}}\right)+\sqrt{d}\log\left(\sqrt{ex+d}-\sqrt{d}\right)c-\sqrt{d}\log\left(\sqrt{ex+d}+\sqrt{d}\right)c}{bc}$$

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x),x)
```

output
```
(2*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)
)) + sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*c - sqrt(d)*log(sqrt(d + e*x) +
sqrt(d))*c)/(b*c)
```

3.109 $\int \frac{1}{\sqrt{d+ex}(bx+cx^2)}\,dx$

Optimal result 816
Mathematica [A] (verified) 816
Rubi [A] (verified) 817
Maple [A] (verified) 818
Fricas [A] (verification not implemented) 819
Sympy [B] (verification not implemented) 819
Maxima [F(-2)] 820
Giac [A] (verification not implemented) 821
Mupad [B] (verification not implemented) 821
Reduce [B] (verification not implemented) 822

Optimal result

Integrand size = 21, antiderivative size = 77

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)}\,dx = -\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b\sqrt{d}} + \frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{cd-be}}$$

output
```
-2*arctanh((e*x+d)^(1/2)/d^(1/2))/b/d^(1/2)+2*c^(1/2)*arctanh(c^(1/2)*(e*x
+d)^(1/2)/(-b*e+c*d)^(1/2))/b/(-b*e+c*d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)}\,dx = -\frac{\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{-cd+be}}\right)}{\sqrt{-cd+be}} + \frac{2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{b}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(b*x + c*x^2)),x]
```

output
```
-(((2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[-(c*d) + b*e]])/Sqrt[-(c
*d) + b*e] + (2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d])/b)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1149, 1406, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{1149}$$

$$2e\int \frac{1}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}$$

$$\Big\downarrow \text{1406}$$

$$2e\left(\frac{c\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{c\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)$$

$$\Big\downarrow \text{221}$$

$$2e\left(\frac{\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{be\sqrt{cd-be}}-\frac{\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b\sqrt{d}e}\right)$$

input
```
Int[1/(Sqrt[d + e*x]*(b*x + c*x^2)),x]
```

output
```
2*e*(-(ArcTanh[Sqrt[d + e*x]/Sqrt[d]]/(b*Sqrt[d]*e)) + (Sqrt[c]*ArcTanh[(S
qrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*e*Sqrt[c*d - b*e]))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1149
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Sym
bol] :> Simp[2*e   Subst[Int[1/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 +
 c*x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1406
```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Simp[c/q   Int[1/(b/2 - q/2 + c*x^2), x], x] - Simp[c/q   I
nt[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c
, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.79

method	result	size
pseudoelliptic	$\frac{-\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{\sqrt{d}}-\frac{2c\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{\sqrt{c(be-cd)}}}{b}$	61
derivativedivides	$2e\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be\sqrt{d}}-\frac{c\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{be\sqrt{c(be-cd)}}\right)$	71
default	$2e\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be\sqrt{d}}-\frac{c\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{be\sqrt{c(be-cd)}}\right)$	71

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
2/b*(-arctanh((e*x+d)^(1/2)/d^(1/2))/d^(1/2)-c/(c*(b*e-c*d))^(1/2)*arctan(
c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 355, normalized size of antiderivative = 4.61

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)}\,dx$$
$$= \left[\frac{d\sqrt{\frac{c}{cd-be}}\log\left(\frac{cex+2\,cd-be+2\,(cd-be)\sqrt{ex+d}\sqrt{\frac{c}{cd-be}}}{cx+b}\right)+\sqrt{d}\log\left(\frac{ex-2\,\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)}{bd},\right.$$
$$-\frac{2\,d\sqrt{-\frac{c}{cd-be}}\arctan\left(\sqrt{ex+d}\sqrt{-\frac{c}{cd-be}}\right)-\sqrt{d}\log\left(\frac{ex-2\,\sqrt{ex+d}\sqrt{d}+2\,d}{x}\right)}{bd}, \frac{d\sqrt{\frac{c}{cd-be}}\log\left(\frac{cex+2\,cd-be+2\,(cd-b}{cx+b}\right.}{}$$
$$\left.-\frac{2\left(d\sqrt{-\frac{c}{cd-be}}\arctan\left(\sqrt{ex+d}\sqrt{-\frac{c}{cd-be}}\right)-\sqrt{-d}\arctan\left(\frac{\sqrt{-d}}{\sqrt{ex+d}}\right)\right)}{bd}\right]$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
[(d*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x
+ d)*sqrt(c/(c*d - b*e)))/(c*x + b)) + sqrt(d)*log((e*x - 2*sqrt(e*x + d)*
sqrt(d) + 2*d)/x))/(b*d), -(2*d*sqrt(-c/(c*d - b*e))*arctan(sqrt(e*x + d)*
sqrt(-c/(c*d - b*e))) - sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/
x))/(b*d), (d*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e + 2*(c*d - b*e)
*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)) + 2*sqrt(-d)*arctan(sqrt(-d
)/sqrt(e*x + d)))/(b*d), -2*(d*sqrt(-c/(c*d - b*e))*arctan(sqrt(e*x + d)*s
qrt(-c/(c*d - b*e))) - sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x + d)))/(b*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(66) = 132$.

Time = 2.73 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.79

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)}\,dx = \begin{cases} \frac{2\left(-\frac{e\,\mathrm{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{be-cd}{c}}}\right)}{b\sqrt{\frac{be-cd}{c}}}+\frac{e\,\mathrm{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d}}\right)}{b\sqrt{-d}}\right)}{e} & \text{for } e\neq 0 \\ \frac{-\frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for } c=0\\ -\frac{\log\left(b-2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b}-\frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for } c=0\\ \frac{\log\left(b+2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b}}{\sqrt{d}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x),x)
```

output
```
Piecewise((2*(-e*atan(sqrt(d + e*x)/sqrt((b*e - c*d)/c))/(b*sqrt((b*e - c*
d)/c)) + e*atan(sqrt(d + e*x)/sqrt(-d))/(b*sqrt(-d)))/e, Ne(e, 0)), ((-2*c
*Piecewise(((b/(2*c) + x)/b, Eq(c, 0)), (-log(b - 2*c*(b/(2*c) + x))/(2*c)
, True))/b - 2*c*Piecewise(((b/(2*c) + x)/b, Eq(c, 0)), (log(b + 2*c*(b/(2
*c) + x))/(2*c), True))/b)/sqrt(d), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)}\,dx = -\frac{2\,c\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}b} + \frac{2\,\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b\sqrt{-d}}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
-2*c*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c*e))/(sqrt(-c^2*d + b*c*e)*b)
 + 2*arctan(sqrt(e*x + d)/sqrt(-d))/(b*sqrt(-d))
```

Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 625, normalized size of antiderivative = 8.12

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)}\,dx = -\frac{2\,\mathrm{atanh}\left(\frac{\sqrt{d+e\,x}}{\sqrt{d}}\right)}{b\,\sqrt{d}} + \frac{\mathrm{atan}\left(\frac{\frac{\left(16\,c^3\,e^2\,\sqrt{d+e\,x}+\frac{\sqrt{c^2\,d-b\,c\,e}\,\left(8\,b^2\,c^2\,e^3+\frac{\left(8\,b^3\,c^2\,e^3-16\,b^2\,c^3\,d\,e^2\right)\,\sqrt{c^2\,d-b\,c\,e}\,\sqrt{d+e\,x}}{b^2\,e-b\,c\,d}\right)}{b^2\,e-b\,c\,d}\right)\,\sqrt{c^2\,d-b\,c\,e}\,1\mathrm{i}}{b^2\,e-b\,c\,d}+\frac{\left(16\,c^3\,e^2\,\sqrt{d+e\,x}-\frac{\sqrt{c^2\,d-b\,c\,e}\,\cdots}{\cdots}\right)\cdots}{\cdots}}{\frac{\left(16\,c^3\,e^2\,\sqrt{d+e\,x}+\frac{\sqrt{c^2\,d-b\,c\,e}\,\left(8\,b^2\,c^2\,e^3+\frac{\left(8\,b^3\,c^2\,e^3-16\,b^2\,c^3\,d\,e^2\right)\,\sqrt{c^2\,d-b\,c\,e}\,\sqrt{d+e\,x}}{b^2\,e-b\,c\,d}\right)}{b^2\,e-b\,c\,d}\right)\,\sqrt{c^2\,d-b\,c\,e}}{b^2\,e-b\,c\,d}-\frac{\left(16\,c^3\,e^2\,\sqrt{d+e\,x}-\frac{\sqrt{c^2\,d-b\,c\,e}\,\cdots}{\cdots}\right)\cdots}{\cdots}}\right)\cdots}{b^2\,e-b\,c\,d\,\cdots}$$

input
```
int(1/((b*x + c*x^2)*(d + e*x)^(1/2)),x)
```

output
```
(atan((((16*c^3*e^2*(d + e*x)^(1/2) + ((c^2*d - b*c*e)^(1/2)*(8*b^2*c^2*e^
3 + ((8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2)*(c^2*d - b*c*e)^(1/2)*(d + e*x)^(1
/2))/(b^2*e - b*c*d)))/(b^2*e - b*c*d))*(c^2*d - b*c*e)^(1/2)*1i)/(b^2*e -
 b*c*d) + ((16*c^3*e^2*(d + e*x)^(1/2) - ((c^2*d - b*c*e)^(1/2)*(8*b^2*c^2
*e^3 - ((8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2)*(c^2*d - b*c*e)^(1/2)*(d + e*x)
^(1/2))/(b^2*e - b*c*d)))/(b^2*e - b*c*d))*(c^2*d - b*c*e)^(1/2)*1i)/(b^2*
e - b*c*d))/(((16*c^3*e^2*(d + e*x)^(1/2) + ((c^2*d - b*c*e)^(1/2)*(8*b^2*
c^2*e^3 + ((8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2)*(c^2*d - b*c*e)^(1/2)*(d + e
*x)^(1/2))/(b^2*e - b*c*d)))/(b^2*e - b*c*d))*(c^2*d - b*c*e)^(1/2))/(b^2*
e - b*c*d) - ((16*c^3*e^2*(d + e*x)^(1/2) - ((c^2*d - b*c*e)^(1/2)*(8*b^2*
c^2*e^3 - ((8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2)*(c^2*d - b*c*e)^(1/2)*(d + e
*x)^(1/2))/(b^2*e - b*c*d)))/(b^2*e - b*c*d))*(c^2*d - b*c*e)^(1/2))/(b^2*
e - b*c*d)))*(c^2*d - b*c*e)^(1/2)*2i)/(b^2*e - b*c*d) - (2*atanh((d + e*x
)^(1/2)/d^(1/2)))/(b*d^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.58

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)}\,dx$$
$$= \frac{-2\sqrt{c}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{be-cd}}\right)d+\sqrt{d}\log\left(\sqrt{ex+d}-\sqrt{d}\right)be-\sqrt{d}\log\left(\sqrt{ex+d}-\sqrt{d}\right)cd-\sqrt{d}\log}{bd\,(be-cd)}$$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x),x)
```

output
```
( - 2*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c
*d)))*d + sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*b*e - sqrt(d)*log(sqrt(d +
e*x) - sqrt(d))*c*d - sqrt(d)*log(sqrt(d + e*x) + sqrt(d))*b*e + sqrt(d)*l
og(sqrt(d + e*x) + sqrt(d))*c*d)/(b*d*(b*e - c*d))
```

3.110 $\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)}\,dx$

Optimal result 823
Mathematica [A] (verified) 823
Rubi [A] (verified) 824
Maple [A] (verified) 826
Fricas [A] (verification not implemented) 826
Sympy [A] (verification not implemented) 827
Maxima [F(-2)] 828
Giac [A] (verification not implemented) 828
Mupad [B] (verification not implemented) 829
Reduce [B] (verification not implemented) 829

Optimal result

Integrand size = 21, antiderivative size = 102

$$\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)}\,dx = -\frac{2e}{d(cd-be)\sqrt{d+ex}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{bd^{3/2}} + \frac{2c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b(cd-be)^{3/2}}$$

output
```
-2*e/d/(-b*e+c*d)/(e*x+d)^(1/2)-2*arctanh((e*x+d)^(1/2)/d^(1/2))/b/d^(3/2)
+2*c^(3/2)*arctanh(c^(1/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b/(-b*e+c*d)^(3
/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)}\,dx = \frac{2e}{d(-cd+be)\sqrt{d+ex}} + \frac{2c^{3/2}\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{-cd+be}}\right)}{b(-cd+be)^{3/2}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{bd^{3/2}}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(b*x + c*x^2)),x]
```

output
```
(2*e)/(d*(-(c*d) + b*e)*Sqrt[d + e*x]) + (2*c^(3/2)*ArcTan[(Sqrt[c]*Sqrt[d
 + e*x])/Sqrt[-(c*d) + b*e]])/(b*(-(c*d) + b*e)^(3/2)) - (2*ArcTanh[Sqrt[d
 + e*x]/Sqrt[d]])/(b*d^(3/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.30, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1147, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{1147}$$

$$\frac{\int \frac{cd-be-cex}{\sqrt{d+ex}(cx^2+bx)}dx}{d(cd-be)}-\frac{2e}{d\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{1197}$$

$$\frac{2\int \frac{e(2cd-be-c(d+ex))}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{d(cd-be)}-\frac{2e}{d\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2e\int \frac{2cd-be-c(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{d(cd-be)}-\frac{2e}{d\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{1480}$$

$$\frac{2e\left(\frac{c(cd-be)\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{c^2d\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{d(cd-be)}-\frac{2e}{d\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{221}$$

$$\frac{2e\left(\frac{c^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{be\sqrt{cd-be}}-\frac{\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(cd-be)}{b\sqrt{d}e}\right)}{d(cd-be)}-\frac{2e}{d\sqrt{d+ex}(cd-be)}$$

input
```
Int[1/((d + e*x)^(3/2)*(b*x + c*x^2)),x]
```

output
```
(-2*e)/(d*(c*d - b*e)*Sqrt[d + e*x]) + (2*e*(-(((c*d - b*e)*ArcTanh[Sqrt[d
 + e*x]/Sqrt[d]])/(b*Sqrt[d]*e)) + (c^(3/2)*d*ArcTanh[(Sqrt[c]*Sqrt[d + e*
x])/Sqrt[c*d - b*e]])/(b*e*Sqrt[c*d - b*e])))/(d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1147
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Sim
p[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x
, x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[m, -
1]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.95

method	result	size
pseudoelliptic	$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{b\,d^{\frac{3}{2}}}+\frac{2e}{d(be-cd)\sqrt{ex+d}}+\frac{2c^{2}\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{(be-cd)b\sqrt{c(be-cd)}}$	97
derivativedivides	$2e\left(\frac{1}{d(be-cd)\sqrt{ex+d}}+\frac{c^{2}\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{(be-cd)be\sqrt{c(be-cd)}}-\frac{\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{d^{\frac{3}{2}}\,be}\right)$	103
default	$2e\left(\frac{1}{d(be-cd)\sqrt{ex+d}}+\frac{c^{2}\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{(be-cd)be\sqrt{c(be-cd)}}-\frac{\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{d^{\frac{3}{2}}\,be}\right)$	103

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-2*arctanh((e*x+d)^(1/2)/d^(1/2))/b/d^(3/2)+2*e/d/(b*e-c*d)/(e*x+d)^(1/2)+
2/(b*e-c*d)*c^2/b/(c*(b*e-c*d))^(1/2)*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))
^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 681, normalized size of antiderivative = 6.68

$$\int\frac{1}{(d+ex)^{3/2}\,(bx+cx^{2})}\,dx=\left[-\frac{2\sqrt{ex+d}bde+(cd^{2}ex+cd^{3})\sqrt{\frac{c}{cd-be}}\log\left(\frac{cex+2\,cd-be-2\,(cd-be)\sqrt{ex+d}\sqrt{\frac{c}{cd-b}}}{cx+b}\right.}{bcd^{4}-b^{2}d^{3}e+(bcd^{3}e-}\right.$$

$$-\frac{2\sqrt{ex+d}bde+2\,(cd^{2}ex+cd^{3})\sqrt{-\frac{c}{cd-be}}\arctan\left(\sqrt{ex+d}\sqrt{-\frac{c}{cd-be}}\right)-(cd^{2}-bde+(cde-be^{2})x)\sqrt{d}\,\mathrm{lo}}{bcd^{4}-b^{2}d^{3}e+(bcd^{3}e-b^{2}d^{2}e^{2})x}$$

$$-\frac{2\sqrt{ex+d}bde-2\,(cd^{2}-bde+(cde-be^{2})x)\sqrt{-d}\arctan\left(\frac{\sqrt{-d}}{\sqrt{ex+d}}\right)+(cd^{2}ex+cd^{3})\sqrt{\frac{c}{cd-be}}\log\left(\frac{cex+2\,cd-}{\right.}{bcd^{4}-b^{2}d^{3}e+(bcd^{3}e-b^{2}d^{2}e^{2})x}$$

$$-\frac{2\left(\sqrt{ex+d}bde+(cd^{2}ex+cd^{3})\sqrt{-\frac{c}{cd-be}}\arctan\left(\sqrt{ex+d}\sqrt{-\frac{c}{cd-be}}\right)-(cd^{2}-bde+(cde-be^{2})x)\sqrt{-d}\right.}{bcd^{4}-b^{2}d^{3}e+(bcd^{3}e-b^{2}d^{2}e^{2})x}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
[-(2*sqrt(e*x + d)*b*d*e + (c*d^2*e*x + c*d^3)*sqrt(c/(c*d - b*e))*log((c*
e*x + 2*c*d - b*e - 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x
+ b)) - (c*d^2 - b*d*e + (c*d*e - b*e^2)*x)*sqrt(d)*log((e*x - 2*sqrt(e*x
+ d)*sqrt(d) + 2*d)/x))/(b*c*d^4 - b^2*d^3*e + (b*c*d^3*e - b^2*d^2*e^2)*x
), -(2*sqrt(e*x + d)*b*d*e + 2*(c*d^2*e*x + c*d^3)*sqrt(-c/(c*d - b*e))*ar
ctan(sqrt(e*x + d)*sqrt(-c/(c*d - b*e))) - (c*d^2 - b*d*e + (c*d*e - b*e^2
)*x)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x))/(b*c*d^4 - b^2*
d^3*e + (b*c*d^3*e - b^2*d^2*e^2)*x), -(2*sqrt(e*x + d)*b*d*e - 2*(c*d^2 -
 b*d*e + (c*d*e - b*e^2)*x)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x + d)) + (c*d
^2*e*x + c*d^3)*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e - 2*(c*d - b*
e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)))/(b*c*d^4 - b^2*d^3*e + (
b*c*d^3*e - b^2*d^2*e^2)*x), -2*(sqrt(e*x + d)*b*d*e + (c*d^2*e*x + c*d^3)
*sqrt(-c/(c*d - b*e))*arctan(sqrt(e*x + d)*sqrt(-c/(c*d - b*e))) - (c*d^2
- b*d*e + (c*d*e - b*e^2)*x)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x + d)))/(b*c
*d^4 - b^2*d^3*e + (b*c*d^3*e - b^2*d^2*e^2)*x)]
```

Sympy [A] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.67

$$\int \frac{1}{(d+ex)^{3/2}\,(bx+cx^2)}\,dx = \begin{cases} \frac{2\left(\frac{e^2}{d\sqrt{d+ex}(be-cd)} + \frac{ce\,\mathrm{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{be-cd}{c}}}\right)}{b\sqrt{\frac{be-cd}{c}}(be-cd)} + \frac{e\,\mathrm{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d}}\right)}{bd\sqrt{-d}}\right)}{e} & \\ \frac{-\frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for}\ c=0\\ -\frac{\log\left(b-2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b} - \frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for}\ c=0\\ \frac{\log\left(b+2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b}}{d^{\frac{3}{2}}} & \end{cases}$$

input
```
integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x),x)
```

output
```
Piecewise((2*(e**2/(d*sqrt(d + e*x)*(b*e - c*d)) + c*e*atan(sqrt(d + e*x)/
sqrt((b*e - c*d)/c))/(b*sqrt((b*e - c*d)/c)*(b*e - c*d)) + e*atan(sqrt(d +
 e*x)/sqrt(-d))/(b*d*sqrt(-d)))/e, Ne(e, 0)), ((-2*c*Piecewise(((b/(2*c) +
 x)/b, Eq(c, 0)), (-log(b - 2*c*(b/(2*c) + x))/(2*c), True))/b - 2*c*Piece
wise(((b/(2*c) + x)/b, Eq(c, 0)), (log(b + 2*c*(b/(2*c) + x))/(2*c), True)
)/b)/d**(3/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{3/2}\,(bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d+ex)^{3/2}\,(bx+cx^2)}\,dx = -\frac{2\,c^2\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{(bcd-b^2e)\sqrt{-c^2d+bce}} - \frac{2\,e}{(cd^2-bde)\sqrt{ex+d}} + \frac{2\,\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b\sqrt{-d}d}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
-2*c^2*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c*e))/((b*c*d - b^2*e)*sqrt(
-c^2*d + b*c*e)) - 2*e/((c*d^2 - b*d*e)*sqrt(e*x + d)) + 2*arctan(sqrt(e*x
 + d)/sqrt(-d))/(b*sqrt(-d)*d)
```

Mupad [B] (verification not implemented)

Time = 5.47 (sec) , antiderivative size = 2258, normalized size of antiderivative = 22.14

$$\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((b*x + c*x^2)*(d + e*x)^(3/2)),x)
```

output
```
(atan((((-c^3*(b*e - c*d)^3)^(1/2)*((d + e*x)^(1/2)*(16*c^8*d^8*e^2 - 64*b
*c^7*d^7*e^3 + 104*b^2*c^6*d^6*e^4 - 88*b^3*c^5*d^5*e^5 + 40*b^4*c^4*d^4*e
^6 - 8*b^5*c^3*d^3*e^7) - ((-c^3*(b*e - c*d)^3)^(1/2)*(72*b^3*c^6*d^8*e^4
- 16*b^2*c^7*d^9*e^3 - 128*b^4*c^5*d^7*e^5 + 112*b^5*c^4*d^6*e^6 - 48*b^6*
c^3*d^5*e^7 + 8*b^7*c^2*d^4*e^8 + ((-c^3*(b*e - c*d)^3)^(1/2)*(d + e*x)^(1
/2)*(16*b^2*c^8*d^11*e^2 - 88*b^3*c^7*d^10*e^3 + 200*b^4*c^6*d^9*e^4 - 240
*b^5*c^5*d^8*e^5 + 160*b^6*c^4*d^7*e^6 - 56*b^7*c^3*d^6*e^7 + 8*b^8*c^2*d^
5*e^8))/(b*(b*e - c*d)^3)))/(b*(b*e - c*d)^3))*1i)/(b*(b*e - c*d)^3) + ((-
c^3*(b*e - c*d)^3)^(1/2)*((d + e*x)^(1/2)*(16*c^8*d^8*e^2 - 64*b*c^7*d^7*e
^3 + 104*b^2*c^6*d^6*e^4 - 88*b^3*c^5*d^5*e^5 + 40*b^4*c^4*d^4*e^6 - 8*b^5
*c^3*d^3*e^7) - ((-c^3*(b*e - c*d)^3)^(1/2)*(16*b^2*c^7*d^9*e^3 - 72*b^3*c
^6*d^8*e^4 + 128*b^4*c^5*d^7*e^5 - 112*b^5*c^4*d^6*e^6 + 48*b^6*c^3*d^5*e^
7 - 8*b^7*c^2*d^4*e^8 + ((-c^3*(b*e - c*d)^3)^(1/2)*(d + e*x)^(1/2)*(16*b^
2*c^8*d^11*e^2 - 88*b^3*c^7*d^10*e^3 + 200*b^4*c^6*d^9*e^4 - 240*b^5*c^5*d
^8*e^5 + 160*b^6*c^4*d^7*e^6 - 56*b^7*c^3*d^6*e^7 + 8*b^8*c^2*d^5*e^8))/(b
*(b*e - c*d)^3)))/(b*(b*e - c*d)^3))*1i)/(b*(b*e - c*d)^3))/(16*c^7*d^6*e^
3 - 48*b*c^6*d^5*e^4 + 48*b^2*c^5*d^4*e^5 - 16*b^3*c^4*d^3*e^6 + ((-c^3*(b
*e - c*d)^3)^(1/2)*((d + e*x)^(1/2)*(16*c^8*d^8*e^2 - 64*b*c^7*d^7*e^3 + 1
04*b^2*c^6*d^6*e^4 - 88*b^3*c^5*d^5*e^5 + 40*b^4*c^4*d^4*e^6 - 8*b^5*c^3*d
^3*e^7) - ((-c^3*(b*e - c*d)^3)^(1/2)*(72*b^3*c^6*d^8*e^4 - 16*b^2*c^7*...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.54

$$\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)}\,dx = \frac{2\sqrt{c}\,\sqrt{ex+d}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{be-cd}}\right)c\,d^2+\sqrt{d}\,\sqrt{ex+d}\log\left(\sqrt{ex+d}-\right.}{}$$

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+b*x),x)
```

output
```
(2*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*s
qrt(b*e - c*d)))*c*d**2 + sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d
))*b**2*e**2 - 2*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b*c*d*
e + sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*c**2*d**2 - sqrt(d)
*sqrt(d + e*x)*log(sqrt(d + e*x) + sqrt(d))*b**2*e**2 + 2*sqrt(d)*sqrt(d +
 e*x)*log(sqrt(d + e*x) + sqrt(d))*b*c*d*e - sqrt(d)*sqrt(d + e*x)*log(sqr
t(d + e*x) + sqrt(d))*c**2*d**2 + 2*b**2*d*e**2 - 2*b*c*d**2*e)/(sqrt(d +
e*x)*b*d**2*(b**2*e**2 - 2*b*c*d*e + c**2*d**2))
```

3.111 $\int \frac{1}{(d+ex)^{5/2}(bx+cx^2)}\,dx$

Optimal result 831
Mathematica [A] (verified) 831
Rubi [A] (verified) 832
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Sympy [A] (verification not implemented) 836
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Reduce [B] (verification not implemented) 838

Optimal result

Integrand size = 21, antiderivative size = 138

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)}\,dx=-\frac{2e}{3d(cd-be)(d+ex)^{3/2}}-\frac{2e(2cd-be)}{d^2(cd-be)^2\sqrt{d+ex}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{bd^{5/2}}+\frac{2c^{5/2}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b(cd-be)^{5/2}}$$

output
```
-2/3*e/d/(-b*e+c*d)/(e*x+d)^(3/2)-2*e*(-b*e+2*c*d)/d^2/(-b*e+c*d)^2/(e*x+d
)^(1/2)-2*arctanh((e*x+d)^(1/2)/d^(1/2))/b/d^(5/2)+2*c^(5/2)*arctanh(c^(1/
2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b/(-b*e+c*d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)}\,dx=\frac{2e(be(4d+3ex)-cd(7d+6ex))}{3d^2(cd-be)^2(d+ex)^{3/2}}-\frac{2c^{5/2}\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{-cd+be}}\right)}{b(-cd+be)^{5/2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{bd^{5/2}}$$

input
```
Integrate[1/((d + e*x)^(5/2)*(b*x + c*x^2)),x]
```

output
```
(2*e*(b*e*(4*d + 3*e*x) - c*d*(7*d + 6*e*x)))/(3*d^2*(c*d - b*e)^2*(d + e*
x)^(3/2)) - (2*c^(5/2)*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[-(c*d) + b*e]])
/(b*(-(c*d) + b*e)^(5/2)) - (2*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*d^(5/2))
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1147, 1198, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)(d+ex)^{5/2}} \, dx$$

$$\Big\downarrow \text{1147}$$

$$\frac{\int \frac{cd-be-cex}{(d+ex)^{3/2}(cx^2+bx)}dx}{d(cd-be)} - \frac{2e}{3d(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \text{1198}$$

$$\frac{\frac{\int \frac{(cd-be)^2-ce(2cd-be)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{d(cd-be)} - \frac{2e(2cd-be)}{d\sqrt{d+ex}(cd-be)}}{d(cd-be)} - \frac{2e}{3d(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \text{1197}$$

$$\frac{\frac{2\int \frac{e\left(3c^2d^2-3bced+b^2e^2-c(2cd-be)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{d(cd-be)} - \frac{2e(2cd-be)}{d\sqrt{d+ex}(cd-be)}}{d(cd-be)} - \frac{2e}{3d(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2e\int \frac{3c^2d^2-3bced+b^2e^2-c(2cd-be)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{d(cd-be)} - \frac{2e(2cd-be)}{d\sqrt{d+ex}(cd-be)}}{d(cd-be)} - \frac{2e}{3d(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \text{1480}$$

$$\frac{\frac{2e\left(\frac{\frac{c(cd-be)^2\int\frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{c^3d^2\int\frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}}{d(cd-be)}\right)}{d(cd-be)}-\frac{2e(2cd-be)}{d\sqrt{d+ex}(cd-be)}}{d(cd-be)}-\frac{2e}{3d(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{2e\left(\frac{c^{5/2}d^2\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{be\sqrt{cd-be}}-\frac{\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(cd-be)^2}{b\sqrt{d}e}\right)}{d(cd-be)}-\frac{2e(2cd-be)}{d\sqrt{d+ex}(cd-be)}}{d(cd-be)}-\frac{2e}{3d(d+ex)^{3/2}(cd-be)}$$

input
```
Int[1/((d + e*x)^(5/2)*(b*x + c*x^2)),x]
```

output
```
(-2*e)/(3*d*(c*d - b*e)*(d + e*x)^(3/2)) + ((-2*e*(2*c*d - b*e))/(d*(c*d -
 b*e)*Sqrt[d + e*x]) + (2*e*(-(((c*d - b*e)^2*ArcTanh[Sqrt[d + e*x]/Sqrt[d
]])/(b*Sqrt[d]*e)) + (c^(5/2)*d^2*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d
 - b*e]])/(b*e*Sqrt[c*d - b*e])))/(d*(c*d - b*e)))/(d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1147
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Sim
p[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x
, x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[m, -
1]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.97

method	result	size
pseudoelliptic	$2e\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{eb\,d^{\frac{5}{2}}}+\frac{be-2cd}{d^{2}\left(be-cd\right)^{2}\sqrt{ex+d}}+\frac{1}{3d\left(be-cd\right)\left(ex+d\right)^{\frac{3}{2}}}-\frac{c^{3}\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c\left(be-cd\right)}}\right)}{\left(be-cd\right)^{2}be\sqrt{c\left(be-cd\right)}}\right)$	134
derivativedivides	$2e\left(-\frac{-be+2cd}{d^{2}\left(be-cd\right)^{2}\sqrt{ex+d}}+\frac{1}{3d\left(be-cd\right)\left(ex+d\right)^{\frac{3}{2}}}-\frac{c^{3}\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c\left(be-cd\right)}}\right)}{\left(be-cd\right)^{2}be\sqrt{c\left(be-cd\right)}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{eb\,d^{\frac{5}{2}}}\right)$	136
default	$2e\left(-\frac{-be+2cd}{d^{2}\left(be-cd\right)^{2}\sqrt{ex+d}}+\frac{1}{3d\left(be-cd\right)\left(ex+d\right)^{\frac{3}{2}}}-\frac{c^{3}\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c\left(be-cd\right)}}\right)}{\left(be-cd\right)^{2}be\sqrt{c\left(be-cd\right)}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{eb\,d^{\frac{5}{2}}}\right)$	136

input
```
int(1/(e*x+d)^(5/2)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
2*e*(-1/e/b/d^(5/2)*arctanh((e*x+d)^(1/2)/d^(1/2))+(b*e-2*c*d)/d^2/(b*e-c*
d)^2/(e*x+d)^(1/2)+1/3/d/(b*e-c*d)/(e*x+d)^(3/2)-1/(b*e-c*d)^2*c^3/b/e/(c*
(b*e-c*d))^(1/2)*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(116) = 232$.

Time = 0.23 (sec) , antiderivative size = 1424, normalized size of antiderivative = 10.32

$$\int \frac{1}{(d+ex)^{5/2}(bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
[1/3*(3*(c^2*d^3*e^2*x^2 + 2*c^2*d^4*e*x + c^2*d^5)*sqrt(c/(c*d - b*e))*lo
g((c*e*x + 2*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/
(c*x + b)) + 3*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (c^2*d^2*e^2 - 2*b*c
*d*e^3 + b^2*e^4)*x^2 + 2*(c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(
d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(7*b*c*d^3*e - 4*b^2*d
^2*e^2 + 3*(2*b*c*d^2*e^2 - b^2*d*e^3)*x)*sqrt(e*x + d))/(b*c^2*d^7 - 2*b^
2*c*d^6*e + b^3*d^5*e^2 + (b*c^2*d^5*e^2 - 2*b^2*c*d^4*e^3 + b^3*d^3*e^4)*
x^2 + 2*(b*c^2*d^6*e - 2*b^2*c*d^5*e^2 + b^3*d^4*e^3)*x), -1/3*(6*(c^2*d^3
*e^2*x^2 + 2*c^2*d^4*e*x + c^2*d^5)*sqrt(-c/(c*d - b*e))*arctan(sqrt(e*x +
 d)*sqrt(-c/(c*d - b*e))) - 3*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (c^2*
d^2*e^2 - 2*b*c*d*e^3 + b^2*e^4)*x^2 + 2*(c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*
d*e^3)*x)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(7*b*c*
d^3*e - 4*b^2*d^2*e^2 + 3*(2*b*c*d^2*e^2 - b^2*d*e^3)*x)*sqrt(e*x + d))/(b
*c^2*d^7 - 2*b^2*c*d^6*e + b^3*d^5*e^2 + (b*c^2*d^5*e^2 - 2*b^2*c*d^4*e^3
+ b^3*d^3*e^4)*x^2 + 2*(b*c^2*d^6*e - 2*b^2*c*d^5*e^2 + b^3*d^4*e^3)*x), 1
/3*(6*(c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2 + (c^2*d^2*e^2 - 2*b*c*d*e^3 +
b^2*e^4)*x^2 + 2*(c^2*d^3*e - 2*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(-d)*arcta
n(sqrt(-d)/sqrt(e*x + d)) + 3*(c^2*d^3*e^2*x^2 + 2*c^2*d^4*e*x + c^2*d^5)*
sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x + d)
*sqrt(c/(c*d - b*e)))/(c*x + b)) - 2*(7*b*c*d^3*e - 4*b^2*d^2*e^2 + 3*(...
```

Sympy [A] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.53

$$\int \frac{1}{(d+ex)^{5/2}\,(bx+cx^2)}\,dx = \begin{cases} \frac{2\left(\frac{e^2}{3d(d+ex)^{\frac{3}{2}}(be-cd)} + \frac{e^2(be-2cd)}{d^2\sqrt{d+ex}(be-cd)^2} - \frac{c^2e\,\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{be-cd}{c}}}\right)}{b\sqrt{\frac{be-cd}{c}}(be-cd)^2} + \frac{e\,\operatorname{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{-d}}\right)}{bd^2\sqrt{-d}}\right)}{e} & \\ \frac{-\frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for } c=0\\ -\frac{\log\left(b-2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b} - \frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for } c=0\\ \frac{\log\left(b+2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b}}{d^{\frac{5}{2}}} & \end{cases}$$

input
```
integrate(1/(e*x+d)**(5/2)/(c*x**2+b*x),x)
```

output
```
Piecewise((2*(e**2/(3*d*(d + e*x)**(3/2)*(b*e - c*d)) + e**2*(b*e - 2*c*d)
/(d**2*sqrt(d + e*x)*(b*e - c*d)**2) - c**2*e*atan(sqrt(d + e*x)/sqrt((b*e
 - c*d)/c))/(b*sqrt((b*e - c*d)/c)*(b*e - c*d)**2) + e*atan(sqrt(d + e*x)/
sqrt(-d))/(b*d**2*sqrt(-d)))/e, Ne(e, 0)), ((-2*c*Piecewise(((b/(2*c) + x)
/b, Eq(c, 0)), (-log(b - 2*c*(b/(2*c) + x))/(2*c), True))/b - 2*c*Piecewis
e(((b/(2*c) + x)/b, Eq(c, 0)), (log(b + 2*c*(b/(2*c) + x))/(2*c), True))/b
)/d**(5/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{5/2}\,(bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.21

$$\int \frac{1}{(d+ex)^{5/2}(bx+cx^2)}\,dx = -\frac{2\,c^3\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{(bc^2d^2-2\,b^2cde+b^3e^2)\sqrt{-c^2d+bce}} - \frac{2\,(6\,(ex+d)cde+cd^2e-3\,(ex+d)be^2-bde^2)}{3\,(c^2d^4-2\,bcd^3e+b^2d^2e^2)(ex+d)^{\frac{3}{2}}} + \frac{2\,\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b\sqrt{-d}d^2}$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
-2*c^3*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c*e))/((b*c^2*d^2 - 2*b^2*c*
d*e + b^3*e^2)*sqrt(-c^2*d + b*c*e)) - 2/3*(6*(e*x + d)*c*d*e + c*d^2*e -
3*(e*x + d)*b*e^2 - b*d*e^2)/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*(e*x +
 d)^(3/2)) + 2*arctan(sqrt(e*x + d)/sqrt(-d))/(b*sqrt(-d)*d^2)
```

Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 4509, normalized size of antiderivative = 32.67

$$\int \frac{1}{(d+ex)^{5/2}(bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((b*x + c*x^2)*(d + e*x)^(5/2)),x)
```

output
```
(atan(((((d + e*x)^(1/2)*(16*c^13*d^16*e^2 - 128*b*c^12*d^15*e^3 + 480*b^2
*c^11*d^14*e^4 - 1120*b^3*c^10*d^13*e^5 + 1800*b^4*c^9*d^12*e^6 - 2064*b^5
*c^8*d^11*e^7 + 1688*b^6*c^7*d^10*e^8 - 960*b^7*c^6*d^9*e^9 + 360*b^8*c^5*
d^8*e^10 - 80*b^9*c^4*d^7*e^11 + 8*b^10*c^3*d^6*e^12) + ((-c^5*(b*e - c*d)
^5)^(1/2)*(24*b^2*c^12*d^18*e^3 - 216*b^3*c^11*d^17*e^4 + 872*b^4*c^10*d^1
6*e^5 - 2080*b^5*c^9*d^15*e^6 + 3248*b^6*c^8*d^14*e^7 - 3472*b^7*c^7*d^13*
e^8 + 2576*b^8*c^6*d^12*e^9 - 1312*b^9*c^5*d^11*e^10 + 440*b^10*c^4*d^10*e
^11 - 88*b^11*c^3*d^9*e^12 + 8*b^12*c^2*d^8*e^13 - ((-c^5*(b*e - c*d)^5)^(
1/2)*(d + e*x)^(1/2)*(16*b^2*c^13*d^21*e^2 - 168*b^3*c^12*d^20*e^3 + 800*b
^4*c^11*d^19*e^4 - 2280*b^5*c^10*d^18*e^5 + 4320*b^6*c^9*d^17*e^6 - 5712*b
^7*c^8*d^16*e^7 + 5376*b^8*c^7*d^15*e^8 - 3600*b^9*c^6*d^14*e^9 + 1680*b^1
0*c^5*d^13*e^10 - 520*b^11*c^4*d^12*e^11 + 96*b^12*c^3*d^11*e^12 - 8*b^13*
c^2*d^10*e^13))/(b*(b*e - c*d)^5)))/(b*(b*e - c*d)^5))*(-c^5*(b*e - c*d)^5
)^(1/2)*1i)/(b*(b*e - c*d)^5) + (((d + e*x)^(1/2)*(16*c^13*d^16*e^2 - 128*
b*c^12*d^15*e^3 + 480*b^2*c^11*d^14*e^4 - 1120*b^3*c^10*d^13*e^5 + 1800*b^
4*c^9*d^12*e^6 - 2064*b^5*c^8*d^11*e^7 + 1688*b^6*c^7*d^10*e^8 - 960*b^7*c
^6*d^9*e^9 + 360*b^8*c^5*d^8*e^10 - 80*b^9*c^4*d^7*e^11 + 8*b^10*c^3*d^6*e
^12) - ((-c^5*(b*e - c*d)^5)^(1/2)*(24*b^2*c^12*d^18*e^3 - 216*b^3*c^11*d^
17*e^4 + 872*b^4*c^10*d^16*e^5 - 2080*b^5*c^9*d^15*e^6 + 3248*b^6*c^8*d^14
*e^7 - 3472*b^7*c^7*d^13*e^8 + 2576*b^8*c^6*d^12*e^9 - 1312*b^9*c^5*d^1...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 737, normalized size of antiderivative = 5.34

$$\int \frac{1}{(d+ex)^{5/2}\,(bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(5/2)/(c*x^2+b*x),x)
```

output
```
( - 6*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c
)*sqrt(b*e - c*d)))*c**2*d**4 - 6*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*at
an((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*c**2*d**3*e*x + 3*sqrt(d)*
sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b**3*d*e**3 + 3*sqrt(d)*sqrt(d
+ e*x)*log(sqrt(d + e*x) - sqrt(d))*b**3*e**4*x - 9*sqrt(d)*sqrt(d + e*x)*
log(sqrt(d + e*x) - sqrt(d))*b**2*c*d**2*e**2 - 9*sqrt(d)*sqrt(d + e*x)*lo
g(sqrt(d + e*x) - sqrt(d))*b**2*c*d*e**3*x + 9*sqrt(d)*sqrt(d + e*x)*log(s
qrt(d + e*x) - sqrt(d))*b*c**2*d**3*e + 9*sqrt(d)*sqrt(d + e*x)*log(sqrt(d
 + e*x) - sqrt(d))*b*c**2*d**2*e**2*x - 3*sqrt(d)*sqrt(d + e*x)*log(sqrt(d
 + e*x) - sqrt(d))*c**3*d**4 - 3*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) -
 sqrt(d))*c**3*d**3*e*x - 3*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) + sqrt
(d))*b**3*d*e**3 - 3*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) + sqrt(d))*b*
*3*e**4*x + 9*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) + sqrt(d))*b**2*c*d*
*2*e**2 + 9*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) + sqrt(d))*b**2*c*d*e*
*3*x - 9*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) + sqrt(d))*b*c**2*d**3*e
- 9*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) + sqrt(d))*b*c**2*d**2*e**2*x
+ 3*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) + sqrt(d))*c**3*d**4 + 3*sqrt(
d)*sqrt(d + e*x)*log(sqrt(d + e*x) + sqrt(d))*c**3*d**3*e*x + 8*b**3*d**2*
e**3 + 6*b**3*d*e**4*x - 22*b**2*c*d**3*e**2 - 18*b**2*c*d**2*e**3*x + 14*
b*c**2*d**4*e + 12*b*c**2*d**3*e**2*x)/(3*sqrt(d + e*x)*b*d**3*(b**3*d*...
```

3.112 $\int \frac{1}{(d+ex)^{7/2}(bx+cx^2)}\,dx$

Optimal result 840
Mathematica [A] (verified) 840
Rubi [A] (verified) 841
Maple [A] (verified) 844
Fricas [B] (verification not implemented) 844
Sympy [A] (verification not implemented) 845
Maxima [F(-2)] 846
Giac [A] (verification not implemented) 846
Mupad [B] (verification not implemented) 847
Reduce [B] (verification not implemented) 848

Optimal result

Integrand size = 21, antiderivative size = 187

$$\int \frac{1}{(d+ex)^{7/2}(bx+cx^2)}\,dx = -\frac{2e}{5d(cd-be)(d+ex)^{5/2}} - \frac{2e(2cd-be)}{3d^2(cd-be)^2(d+ex)^{3/2}} - \frac{2e(3c^2d^2-3bcde+b^2e^2)}{d^3(cd-be)^3\sqrt{d+ex}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{bd^{7/2}} + \frac{2c^{7/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b(cd-be)^{7/2}}$$

output
```
-2/5*e/d/(-b*e+c*d)/(e*x+d)^(5/2)-2/3*e*(-b*e+2*c*d)/d^2/(-b*e+c*d)^2/(e*x
+d)^(3/2)-2*e*(b^2*e^2-3*b*c*d*e+3*c^2*d^2)/d^3/(-b*e+c*d)^3/(e*x+d)^(1/2)
-2*arctanh((e*x+d)^(1/2)/d^(1/2))/b/d^(7/2)+2*c^(7/2)*arctanh(c^(1/2)*(e*x
+d)^(1/2)/(-b*e+c*d)^(1/2))/b/(-b*e+c*d)^(7/2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.97

$$\int \frac{1}{(d+ex)^{7/2}(bx+cx^2)}\,dx = \frac{2e(-3bcde(22d^2+35dex+15e^2x^2)+b^2e^2(23d^2+35dex+15e^2x^2)+c^2d^2(}{15d^3(-cd+be)^3(d+ex)^{5/2}} + \frac{2c^{7/2}\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{-cd+be}}\right)}{b(-cd+be)^{7/2}} - \frac{2\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{bd^{7/2}}$$

input
```
Integrate[1/((d + e*x)^(7/2)*(b*x + c*x^2)),x]
```

output
```
(2*e*(-3*b*c*d*e*(22*d^2 + 35*d*e*x + 15*e^2*x^2) + b^2*e^2*(23*d^2 + 35*d
*e*x + 15*e^2*x^2) + c^2*d^2*(58*d^2 + 100*d*e*x + 45*e^2*x^2)))/(15*d^3*(
-(c*d) + b*e)^3*(d + e*x)^(5/2)) + (2*c^(7/2)*ArcTan[(Sqrt[c]*Sqrt[d + e*x
])/Sqrt[-(c*d) + b*e]])/(b*(-(c*d) + b*e)^(7/2)) - (2*ArcTanh[Sqrt[d + e*x
]/Sqrt[d]])/(b*d^(7/2))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.35, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1147, 1198, 1198, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)(d+ex)^{7/2}}dx$$

$$\Big\downarrow \text{1147}$$

$$\frac{\int \frac{cd-be-cex}{(d+ex)^{5/2}(cx^2+bx)}dx}{d(cd-be)}-\frac{2e}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \text{1198}$$

$$\frac{\frac{\int \frac{(cd-be)^2-ce(2cd-be)x}{(d+ex)^{3/2}\left(cx^2+bx\right)}dx}{d(cd-be)}-\frac{2e(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{d(cd-be)}-\frac{2e}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \text{1198}$$

$$\frac{\frac{\frac{\int \frac{(cd-be)^3-ce\left(3c^2d^2-3bced+b^2e^2\right)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{d(cd-be)}-\frac{2e\left(b^2e^2-3bcde+3c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{d(cd-be)}-\frac{2e(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{d(cd-be)}-\frac{2e}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \text{1197}$$

$$\frac{\frac{\frac{2\int \frac{e\left((2cd-be)\left(2c^2d^2-2bced+b^2e^2\right)-c\left(3c^2d^2-3bced+b^2e^2\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{d(cd-be)}-\frac{2e\left(b^2e^2-3bcde+3c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{d(cd-be)}-\frac{2e(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{d(cd-be)}-\frac{2e}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\frac{2e\int \frac{(2cd-be)\left(2c^2d^2-2bced+b^2e^2\right)-c\left(3c^2d^2-3bced+b^2e^2\right)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{d(cd-be)}-\frac{2e\left(b^2e^2-3bcde+3c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{d(cd-be)}-\frac{2e(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{d(cd-be)}-\frac{2e}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \text{ 1480}$$

$$\frac{\frac{\frac{2e\left(\frac{c(cd-be)^3\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{c^4d^3\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{d(cd-be)}-\frac{2e\left(b^2e^2-3bcde+3c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{d(cd-be)}-\frac{2e(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{d(cd-be)}-\frac{2e}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{\frac{2e\left(\frac{c^{7/2}d^3\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{be\sqrt{cd-be}}-\frac{\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(cd-be)^3}{b\sqrt{d}e}\right)}{d(cd-be)}-\frac{2e\left(b^2e^2-3bcde+3c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{d(cd-be)}-\frac{2e(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{d(cd-be)}-\frac{2e}{5d(d+ex)^{5/2}(cd-be)}$$

input
```
Int[1/((d + e*x)^(7/2)*(b*x + c*x^2)),x]
```

output
```
(-2*e)/(5*d*(c*d - b*e)*(d + e*x)^(5/2)) + ((-2*e*(2*c*d - b*e))/(3*d*(c*d
 - b*e)*(d + e*x)^(3/2)) + ((-2*e*(3*c^2*d^2 - 3*b*c*d*e + b^2*e^2))/(d*(c
*d - b*e)*Sqrt[d + e*x]) + (2*e*(-(((c*d - b*e)^3*ArcTanh[Sqrt[d + e*x]/Sq
rt[d]])/(b*Sqrt[d]*e)) + (c^(7/2)*d^3*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt
[c*d - b*e]])/(b*e*Sqrt[c*d - b*e])))/(d*(c*d - b*e)))/(d*(c*d - b*e)))/(d
*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1147
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Sim
p[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x)^(m + 1)*(Simp[c*d - b*e - c*e*x
, x]/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[m, -
1]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$2e\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{eb\,d^{\frac{7}{2}}}+\frac{be-2cd}{3d^2(be-cd)^2(ex+d)^{\frac{3}{2}}}+\frac{b^2e^2-3bcde+3c^2d^2}{d^3(be-cd)^3\sqrt{ex+d}}+\frac{1}{5d(be-cd)(ex+d)^{\frac{5}{2}}}+\frac{c^4\arctan(\cdots)}{(be-cd)^3be\cdots}\right.$
derivativedivides	$2e\left(\frac{c^4\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{(be-cd)^3be\sqrt{c(be-cd)}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{eb\,d^{\frac{7}{2}}}-\frac{-be+2cd}{3d^2(be-cd)^2(ex+d)^{\frac{3}{2}}}-\frac{-b^2e^2+3bcde-3c^2d^2}{d^3(be-cd)^3\sqrt{ex+d}}+\frac{\cdots}{5d(be-c\cdots}\right.$
default	$2e\left(\frac{c^4\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{(be-cd)^3be\sqrt{c(be-cd)}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{eb\,d^{\frac{7}{2}}}-\frac{-be+2cd}{3d^2(be-cd)^2(ex+d)^{\frac{3}{2}}}-\frac{-b^2e^2+3bcde-3c^2d^2}{d^3(be-cd)^3\sqrt{ex+d}}+\frac{\cdots}{5d(be-c\cdots}\right.$

input
```
int(1/(e*x+d)^(7/2)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
2*e*(-1/e/b/d^(7/2)*arctanh((e*x+d)^(1/2)/d^(1/2))+1/3*(b*e-2*c*d)/d^2/(b*
e-c*d)^2/(e*x+d)^(3/2)+(b^2*e^2-3*b*c*d*e+3*c^2*d^2)/d^3/(b*e-c*d)^3/(e*x+
d)^(1/2)+1/5/d/(b*e-c*d)/(e*x+d)^(5/2)+1/(b*e-c*d)^3*c^4/b/e/(c*(b*e-c*d))
^(1/2)*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(161) = 322$.

Time = 0.59 (sec) , antiderivative size = 2532, normalized size of antiderivative = 13.54

$$\int \frac{1}{(d+ex)^{7/2}\,(bx+cx^2)}\,dx = \text{Too large to display}$$

input

```
integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x),x, algorithm="fricas")
```

output

```
[-1/15*(15*(c^3*d^4*e^3*x^3 + 3*c^3*d^5*e^2*x^2 + 3*c^3*d^6*e*x + c^3*d^7)
*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e - 2*(c*d - b*e)*sqrt(e*x + d
)*sqrt(c/(c*d - b*e)))/(c*x + b)) - 15*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*
d^4*e^2 - b^3*d^3*e^3 + (c^3*d^3*e^3 - 3*b*c^2*d^2*e^4 + 3*b^2*c*d*e^5 - b
^3*e^6)*x^3 + 3*(c^3*d^4*e^2 - 3*b*c^2*d^3*e^3 + 3*b^2*c*d^2*e^4 - b^3*d*e
^5)*x^2 + 3*(c^3*d^5*e - 3*b*c^2*d^4*e^2 + 3*b^2*c*d^3*e^3 - b^3*d^2*e^4)*
x)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(58*b*c^2*d^5*
e - 66*b^2*c*d^4*e^2 + 23*b^3*d^3*e^3 + 15*(3*b*c^2*d^3*e^3 - 3*b^2*c*d^2*
e^4 + b^3*d*e^5)*x^2 + 5*(20*b*c^2*d^4*e^2 - 21*b^2*c*d^3*e^3 + 7*b^3*d^2*
e^4)*x)*sqrt(e*x + d))/(b*c^3*d^10 - 3*b^2*c^2*d^9*e + 3*b^3*c*d^8*e^2 - b
^4*d^7*e^3 + (b*c^3*d^7*e^3 - 3*b^2*c^2*d^6*e^4 + 3*b^3*c*d^5*e^5 - b^4*d^
4*e^6)*x^3 + 3*(b*c^3*d^8*e^2 - 3*b^2*c^2*d^7*e^3 + 3*b^3*c*d^6*e^4 - b^4*
d^5*e^5)*x^2 + 3*(b*c^3*d^9*e - 3*b^2*c^2*d^8*e^2 + 3*b^3*c*d^7*e^3 - b^4*
d^6*e^4)*x), -1/15*(30*(c^3*d^4*e^3*x^3 + 3*c^3*d^5*e^2*x^2 + 3*c^3*d^6*e*
x + c^3*d^7)*sqrt(-c/(c*d - b*e))*arctan(sqrt(e*x + d)*sqrt(-c/(c*d - b*e)
)) - 15*(c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3 + (c^3*d^
3*e^3 - 3*b*c^2*d^2*e^4 + 3*b^2*c*d*e^5 - b^3*e^6)*x^3 + 3*(c^3*d^4*e^2 -
3*b*c^2*d^3*e^3 + 3*b^2*c*d^2*e^4 - b^3*d*e^5)*x^2 + 3*(c^3*d^5*e - 3*b*c^
2*d^4*e^2 + 3*b^2*c*d^3*e^3 - b^3*d^2*e^4)*x)*sqrt(d)*log((e*x - 2*sqrt(e*
x + d)*sqrt(d) + 2*d)/x) + 2*(58*b*c^2*d^5*e - 66*b^2*c*d^4*e^2 + 23*b^...
```

Sympy [A] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.40

$$\int \frac{1}{(d+ex)^{7/2}\,(bx+cx^2)}\,dx = \begin{cases} \frac{2\left(\frac{e^2}{5d(d+ex)^{\frac{5}{2}}(be-cd)} + \frac{e^2(be-2cd)}{3d^2(d+ex)^{\frac{3}{2}}(be-cd)^2} + \frac{e^2\left(b^2e^2-3bcde+3c^2d^2\right)}{d^3\sqrt{d+ex}(be-cd)^3} + \frac{c^3e\,\mathrm{atan}\left(\frac{\sqrt{d+ex}}{\sqrt{\frac{be-cd}{c}}}\right)}{b\sqrt{\frac{be-cd}{c}}(be-cd)^3} + \frac{e\,\mathrm{atan}\left(\cdots\right)}{bd^3\cdots} \cdots\right)}{e} \\ \frac{-\frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for } c=0\\ -\frac{\log\left(b-2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b} - \frac{2c\left(\begin{cases}\frac{\frac{b}{2c}+x}{b} & \text{for } c=0\\ \frac{\log\left(b+2c\left(\frac{b}{2c}+x\right)\right)}{2c} & \text{otherwise}\end{cases}\right)}{b}}{d^{\frac{7}{2}}} \end{cases}$$

input

```
integrate(1/(e*x+d)**(7/2)/(c*x**2+b*x),x)
```

output
```
Piecewise((2*(e**2/(5*d*(d + e*x)**(5/2)*(b*e - c*d)) + e**2*(b*e - 2*c*d)
/(3*d**2*(d + e*x)**(3/2)*(b*e - c*d)**2) + e**2*(b**2*e**2 - 3*b*c*d*e +
3*c**2*d**2)/(d**3*sqrt(d + e*x)*(b*e - c*d)**3) + c**3*e*atan(sqrt(d + e*
x)/sqrt((b*e - c*d)/c))/(b*sqrt((b*e - c*d)/c)*(b*e - c*d)**3) + e*atan(sq
rt(d + e*x)/sqrt(-d))/(b*d**3*sqrt(-d)))/e, Ne(e, 0)), ((-2*c*Piecewise(((
b/(2*c) + x)/b, Eq(c, 0)), (-log(b - 2*c*(b/(2*c) + x))/(2*c), True))/b -
2*c*Piecewise(((b/(2*c) + x)/b, Eq(c, 0)), (log(b + 2*c*(b/(2*c) + x))/(2*
c), True))/b)/d**(7/2), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{7/2}\,(bx+cx^2)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.51

$$\int \frac{1}{(d+ex)^{7/2}\,(bx+cx^2)}\,dx = -\frac{2\,c^4\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{(bc^3d^3-3\,b^2c^2d^2e+3\,b^3cde^2-b^4e^3)\sqrt{-c^2d+bce}}$$
$$-\frac{2\left(45\,(ex+d)^2c^2d^2e+10\,(ex+d)c^2d^3e+3\,c^2d^4e-45\,(ex+d)^2bcde^2-15\,(ex+d)bcd^2e^2-6\,bcd^3e^2+\right.}{15\,(c^3d^6-3\,bc^2d^5e+3\,b^2cd^4e^2-b^3d^3e^3)(ex+d)^{\frac{5}{2}}}$$
$$+\frac{2\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b\sqrt{-d}d^3}$$

input
```
integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
-2*c^4*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c*e))/((b*c^3*d^3 - 3*b^2*c^
2*d^2*e + 3*b^3*c*d*e^2 - b^4*e^3)*sqrt(-c^2*d + b*c*e)) - 2/15*(45*(e*x +
 d)^2*c^2*d^2*e + 10*(e*x + d)*c^2*d^3*e + 3*c^2*d^4*e - 45*(e*x + d)^2*b*
c*d*e^2 - 15*(e*x + d)*b*c*d^2*e^2 - 6*b*c*d^3*e^2 + 15*(e*x + d)^2*b^2*e^
3 + 5*(e*x + d)*b^2*d*e^3 + 3*b^2*d^2*e^3)/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b
^2*c*d^4*e^2 - b^3*d^3*e^3)*(e*x + d)^(5/2)) + 2*arctan(sqrt(e*x + d)/sqrt
(-d))/(b*sqrt(-d)*d^3)
```

Mupad [B] (verification not implemented)

Time = 6.48 (sec) , antiderivative size = 4068, normalized size of antiderivative = 21.75

$$\int \frac{1}{(d+ex)^{7/2}\,(bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/((b*x + c*x^2)*(d + e*x)^(7/2)),x)
```

output
```
(atan((((-c^7*(b*e - c*d)^7)^(1/2)*((d + e*x)^(1/2)*(16*c^18*d^24*e^2 - 19
2*b*c^17*d^23*e^3 + 1128*b^2*c^16*d^22*e^4 - 4312*b^3*c^15*d^21*e^5 + 1192
8*b^4*c^14*d^20*e^6 - 25032*b^5*c^13*d^19*e^7 + 40712*b^6*c^12*d^18*e^8 -
51768*b^7*c^11*d^17*e^9 + 51552*b^8*c^10*d^16*e^10 - 40048*b^9*c^9*d^15*e^
11 + 24024*b^10*c^8*d^14*e^12 - 10920*b^11*c^7*d^13*e^13 + 3640*b^12*c^6*d
^12*e^14 - 840*b^13*c^5*d^11*e^15 + 120*b^14*c^4*d^10*e^16 - 8*b^15*c^3*d^
9*e^17) - ((-c^7*(b*e - c*d)^7)^(1/2)*(432*b^3*c^16*d^26*e^4 - 32*b^2*c^17
*d^27*e^3 - 2720*b^4*c^15*d^25*e^5 + 10600*b^5*c^14*d^24*e^6 - 28608*b^6*c
^13*d^23*e^7 + 56672*b^7*c^12*d^22*e^8 - 85184*b^8*c^11*d^21*e^9 + 99000*b
^9*c^10*d^20*e^10 - 89760*b^10*c^9*d^19*e^11 + 63536*b^11*c^8*d^18*e^12 -
34848*b^12*c^7*d^17*e^13 + 14552*b^13*c^6*d^16*e^14 - 4480*b^14*c^5*d^15*e
^15 + 960*b^15*c^4*d^14*e^16 - 128*b^16*c^3*d^13*e^17 + 8*b^17*c^2*d^12*e^
18 + ((-c^7*(b*e - c*d)^7)^(1/2)*(d + e*x)^(1/2)*(16*b^2*c^18*d^31*e^2 - 2
48*b^3*c^17*d^30*e^3 + 1800*b^4*c^16*d^29*e^4 - 8120*b^5*c^15*d^28*e^5 + 2
5480*b^6*c^14*d^27*e^6 - 58968*b^7*c^13*d^26*e^7 + 104104*b^8*c^12*d^25*e^
8 - 143000*b^9*c^11*d^24*e^9 + 154440*b^10*c^10*d^23*e^10 - 131560*b^11*c^
9*d^22*e^11 + 88088*b^12*c^8*d^21*e^12 - 45864*b^13*c^7*d^20*e^13 + 18200*
b^14*c^6*d^19*e^14 - 5320*b^15*c^5*d^18*e^15 + 1080*b^16*c^4*d^17*e^16 - 1
36*b^17*c^3*d^16*e^17 + 8*b^18*c^2*d^15*e^18))/(b*(b*e - c*d)^7)))/(b*(b*e
 - c*d)^7))*1i)/(b*(b*e - c*d)^7) + ((-c^7*(b*e - c*d)^7)^(1/2)*((d + e...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1458, normalized size of antiderivative = 7.80

$$\int \frac{1}{(d+ex)^{7/2}(bx+cx^2)}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(7/2)/(c*x^2+b*x),x)
```

output
```
(30*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*
sqrt(b*e - c*d)))*c**3*d**6 + 60*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*ata
n((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*c**3*d**5*e*x + 30*sqrt(c)*
sqrt(d + e*x)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c
*d)))*c**3*d**4*e**2*x**2 + 15*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - s
qrt(d))*b**4*d**2*e**4 + 30*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt
(d))*b**4*d*e**5*x + 15*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))
*b**4*e**6*x**2 - 60*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b*
*3*c*d**3*e**3 - 120*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b*
*3*c*d**2*e**4*x - 60*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b
**3*c*d*e**5*x**2 + 90*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*
b**2*c**2*d**4*e**2 + 180*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d
))*b**2*c**2*d**3*e**3*x + 90*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sq
rt(d))*b**2*c**2*d**2*e**4*x**2 - 60*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*
x) - sqrt(d))*b*c**3*d**5*e - 120*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x)
- sqrt(d))*b*c**3*d**4*e**2*x - 60*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x)
 - sqrt(d))*b*c**3*d**3*e**3*x**2 + 15*sqrt(d)*sqrt(d + e*x)*log(sqrt(d +
e*x) - sqrt(d))*c**4*d**6 + 30*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - s
qrt(d))*c**4*d**5*e*x + 15*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(
d))*c**4*d**4*e**2*x**2 - 15*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) + ...
```

3.113 $\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^2} dx$

Optimal result 849
Mathematica [A] (verified) 850
Rubi [A] (verified) 850
Maple [A] (verified) 853
Fricas [A] (verification not implemented) 854
Sympy [F(-1)] 855
Maxima [F(-2)] 856
Giac [A] (verification not implemented) 856
Mupad [B] (verification not implemented) 857
Reduce [B] (verification not implemented) 858

Optimal result

Integrand size = 21, antiderivative size = 251

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^2} dx = \frac{e(2cd-be)\left(c^2d^2-bcde+5b^2e^2\right)\sqrt{d+ex}}{b^2c^3} + \frac{e(6c^2d^2-6bcde+5b^2e^2)\,(d+ex)^{3/2}}{3b^2c^2} - \frac{(cd-be)(2cd-be)(d+ex)^{5/2}}{b^2c(b+cx)} - \frac{d(d+ex)^{7/2}}{bx(b+cx)} + \frac{d^{7/2}(4cd-9be)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3} - \frac{(cd-be)^{7/2}(4cd+5be)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3c^{7/2}}$$

output

```
e*(-b*e+2*c*d)*(5*b^2*e^2-b*c*d*e+c^2*d^2)*(e*x+d)^(1/2)/b^2/c^3+1/3*e*(5*
b^2*e^2-6*b*c*d*e+6*c^2*d^2)*(e*x+d)^(3/2)/b^2/c^2-(-b*e+c*d)*(-b*e+2*c*d)
*(e*x+d)^(5/2)/b^2/c/(c*x+b)-d*(e*x+d)^(7/2)/b/x/(c*x+b)+d^(7/2)*(-9*b*e+4
*c*d)*arctanh((e*x+d)^(1/2)/d^(1/2))/b^3-(-b*e+c*d)^(7/2)*(5*b*e+4*c*d)*ar
ctanh(c^(1/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b^3/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^2}\,dx = \frac{-\frac{b\sqrt{d+ex}(6c^4d^4x+15b^4e^4x+3bc^3d^3(d-4ex)+2b^3ce^3x(-19d+5ex)-2b^2c^2e^2x(-9d^2+13dex+e^2x^2))}{c^3x(b+cx)}+\frac{3(-cd+be)}{}}{3b^3}$$

input
```
Integrate[(d + e*x)^(9/2)/(b*x + c*x^2)^2,x]
```

output
```
(-((b*Sqrt[d + e*x]*(6*c^4*d^4*x + 15*b^4*e^4*x + 3*b*c^3*d^3*(d - 4*e*x)
+ 2*b^3*c*e^3*x*(-19*d + 5*e*x) - 2*b^2*c^2*e^2*x*(-9*d^2 + 13*d*e*x + e^2
*x^2)))/(c^3*x*(b + c*x))) + (3*(-(c*d) + b*e)^(7/2)*(4*c*d + 5*b*e)*ArcTa
n[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[-(c*d) + b*e]])/c^(7/2) + 3*d^(7/2)*(4*c*d
- 9*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(3*b^3)
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1164, 27, 1196, 1196, 1196, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{ 1164}$$

$$-\frac{\int \frac{(d+ex)^{5/2}(d(4cd-9be)-5e(2cd-be)x)}{2(cx^2+bx)}dx}{b^2}-\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{b^2\,(bx+cx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{(d+ex)^{5/2}(d(4cd-9be)-5e(2cd-be)x)}{cx^2+bx}dx}{2b^2}-\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{b^2\,(bx+cx^2)}$$

$$\Big\downarrow \text{ 1196}$$

$$-\frac{\frac{\int \frac{(d+ex)^{3/2}\left(cd^2(4cd-9be)-e\left(6c^2d^2-6bced+5b^2e^2\right)x\right)}{cx^2+bx}dx}{c}-\frac{2e(d+ex)^{5/2}(2cd-be)}{c}}{2b^2}-\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

$\Big\downarrow$ 1196

$$-\frac{\frac{\frac{\int \frac{\sqrt{d+ex}\left(c^2d^3(4cd-9be)-e(2cd-be)\left(c^2d^2-bced+5b^2e^2\right)x\right)}{cx^2+bx}dx}{c}-\frac{2e(d+ex)^{3/2}\left(5b^2e^2-6bcde+6c^2d^2\right)}{3c}}{c}-\frac{2e(d+ex)^{5/2}(2cd-be)}{c}}{2b^2}-\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

$\Big\downarrow$ 1196

$$-\frac{\frac{\frac{\frac{\int \frac{c^3(4cd-9be)d^4+e\left(2c^4d^4-4bc^3ed^3-14b^2c^2e^2d^2+16b^3ce^3d-5b^4e^4\right)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{c}-\frac{2e\sqrt{d+ex}(2cd-be)\left(5b^2e^2-bcde+c^2d^2\right)}{c}}{c}-\frac{2e(d+ex)^{3/2}\left(5b^2e^2-6bcde+6c^2d^2\right)}{3c}}{c}}{2b^2}-\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

$\Big\downarrow$ 1197

$$-\frac{\frac{\frac{\frac{2\int \frac{e\left(d(cd-be)(2cd-be)\left(c^2d^2-bced+5b^2e^2\right)+\left(2c^4d^4-4bc^3ed^3-14b^2c^2e^2d^2+16b^3ce^3d-5b^4e^4\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}-\frac{2e\sqrt{d+ex}(2cd-be)\left(5b^2e^2-bcde+c^2d^2\right)}{c}}{c}}{c}}{2b^2}-\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{\frac{2e\int \frac{d(cd-be)(2cd-be)\left(c^2d^2-bced+5b^2e^2\right)+\left(2c^4d^4-4bc^3ed^3-14b^2c^2e^2d^2+16b^3ce^3d-5b^4e^4\right)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}-\frac{2e\sqrt{d+ex}(2cd-be)\left(5b^2e^2-bcde+c^2d^2\right)}{c}-}{c}}{c}}{2b^2}-\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

$\Big\downarrow$ 1480

$$-\frac{\frac{\frac{2e\left(\frac{c^4d^4(4cd-9be)\int \frac{1}{\frac{c(d+ex)-cd}{be}}d\sqrt{d+ex}}{c}-\frac{(cd-be)^4(5be+4cd)\int \frac{1}{\frac{-cd+be+c(d+ex)}{be}}d\sqrt{d+ex}}{c}\right)}{c}-\frac{2e\sqrt{d+ex}(2cd-be)\left(5b^2e^2-bcde+c^2d^2\right)}{c}-\frac{2e(d+ex)^{3/2}\left(5b\cdots\right.}{\cdots}}{c}}{2b^2}\cdots$$

$$\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

$\Big\downarrow$ 221

$$-\frac{\frac{\frac{2e\left(\frac{(cd-be)^{7/2}(5be+4cd)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{c}e}-\frac{c^3d^{7/2}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(4cd-9be)}{be}\right)}{c}-\frac{2e\sqrt{d+ex}(2cd-be)\left(5b^2e^2-bcde+c^2d^2\right)}{c}-\frac{2e(d+ex)^{3/2}\left(5b\cdots\right.}{\cdots}}{c}}{2b^2}\cdots$$

$$\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

input
```
Int[(d + e*x)^(9/2)/(b*x + c*x^2)^2,x]
```

output
```
-(((d + e*x)^(7/2)*(b*d + (2*c*d - b*e)*x))/(b^2*(b*x + c*x^2))) - ((-2*e*
(2*c*d - b*e)*(d + e*x)^(5/2))/c + ((-2*e*(6*c^2*d^2 - 6*b*c*d*e + 5*b^2*e
^2)*(d + e*x)^(3/2))/(3*c) + ((-2*e*(2*c*d - b*e)*(c^2*d^2 - b*c*d*e + 5*b
^2*e^2)*Sqrt[d + e*x])/c + (2*e*(-((c^3*d^(7/2)*(4*c*d - 9*b*e)*ArcTanh[Sq
rt[d + e*x]/Sqrt[d]])/(b*e)) + ((c*d - b*e)^(7/2)*(4*c*d + 5*b*e)*ArcTanh[
(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*Sqrt[c]*e)))/c)/c)/c)/(2*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$\frac{4(-be+cd)^4x(cx+b)\left(cd+\frac{5be}{4}\right)\sqrt{d}\,\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)-\left(9x(cx+b)c^3\left(be-\frac{4cd}{9}\right)d^4\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)+\left(2c^4d^4x+b\,d\right.\right.}{c^3b^3x\sqrt{d}\,(cx+b)\sqrt{c(}}$
derivativedivides	$2e^3\left(-\frac{-\frac{c(ex+d)^{\frac{3}{2}}}{3}+2be\sqrt{ex+d}-4cd\sqrt{ex+d}}{c^3}+\frac{\left(-\frac{1}{2}b^5e^5+2b^4cd\,e^4-3b^3d^2e^3c^2+2b^2c^3d^3e^2-\frac{1}{2}b\,c^4d^4e\right)\sqrt{ex+d}}{(ex+d)c+be-cd}+\frac{(5b^5e}{}\right.$
default	$2e^3\left(-\frac{-\frac{c(ex+d)^{\frac{3}{2}}}{3}+2be\sqrt{ex+d}-4cd\sqrt{ex+d}}{c^3}+\frac{\left(-\frac{1}{2}b^5e^5+2b^4cd\,e^4-3b^3d^2e^3c^2+2b^2c^3d^3e^2-\frac{1}{2}b\,c^4d^4e\right)\sqrt{ex+d}}{(ex+d)c+be-cd}+\frac{(5b^5e}{}\right.$
risch	$-\frac{d^4\sqrt{ex+d}}{b^2x}+\frac{e\left(\frac{2b^2e^2\left(\frac{c(ex+d)^{\frac{3}{2}}}{3}-2be\sqrt{ex+d}+4cd\sqrt{ex+d}\right)}{c^3}-\frac{d^{\frac{7}{2}}(9be-4cd)\,\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be}+\frac{2\left(-\frac{1}{2}b^5e^5+2b^4cd\,e^4-3b\right.}{}\right.}{}$

input
```
int((e*x+d)^(9/2)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
4/d^(1/2)/(c*(b*e-c*d))^(1/2)*((-b*e+c*d)^4*x*(c*x+b)*(c*d+5/4*b*e)*d^(1/2
)*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2))-1/4*(9*x*(c*x+b)*c^3*(b*e-4/
9*c*d)*d^4*arctanh((e*x+d)^(1/2)/d^(1/2))+(2*c^4*d^4*x+b*d^3*(-4*e*x+d)*c^
3+6*e^2*x*(-1/9*e^2*x^2-13/9*d*e*x+d^2)*b^2*c^2-38/3*e^3*x*(-5/19*e*x+d)*b
^3*c+5*b^4*e^4*x)*(e*x+d)^(1/2)*b*d^(1/2))*(c*(b*e-c*d))^(1/2))/c^3/b^3/x/
(c*x+b)
```

Fricas [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 1555, normalized size of antiderivative = 6.20

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(9/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
[-1/6*(3*((4*c^5*d^4 - 7*b*c^4*d^3*e - 3*b^2*c^3*d^2*e^2 + 11*b^3*c^2*d*e^
3 - 5*b^4*c*e^4)*x^2 + (4*b*c^4*d^4 - 7*b^2*c^3*d^3*e - 3*b^3*c^2*d^2*e^2
+ 11*b^4*c*d*e^3 - 5*b^5*e^4)*x)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d -
b*e + 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) + 3*((4*c^5*d^4 -
9*b*c^4*d^3*e)*x^2 + (4*b*c^4*d^4 - 9*b^2*c^3*d^3*e)*x)*sqrt(d)*log((e*x -
 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(2*b^3*c^2*e^4*x^3 - 3*b^2*c^3*d^4
+ 2*(13*b^3*c^2*d*e^3 - 5*b^4*c*e^4)*x^2 - (6*b*c^4*d^4 - 12*b^2*c^3*d^3*e
 + 18*b^3*c^2*d^2*e^2 - 38*b^4*c*d*e^3 + 15*b^5*e^4)*x)*sqrt(e*x + d))/(b^
3*c^4*x^2 + b^4*c^3*x), -1/6*(6*((4*c^5*d^4 - 7*b*c^4*d^3*e - 3*b^2*c^3*d^
2*e^2 + 11*b^3*c^2*d*e^3 - 5*b^4*c*e^4)*x^2 + (4*b*c^4*d^4 - 7*b^2*c^3*d^3
*e - 3*b^3*c^2*d^2*e^2 + 11*b^4*c*d*e^3 - 5*b^5*e^4)*x)*sqrt(-(c*d - b*e)/
c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + 3*((4*c^5*d
^4 - 9*b*c^4*d^3*e)*x^2 + (4*b*c^4*d^4 - 9*b^2*c^3*d^3*e)*x)*sqrt(d)*log((
e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(2*b^3*c^2*e^4*x^3 - 3*b^2*c^3
*d^4 + 2*(13*b^3*c^2*d*e^3 - 5*b^4*c*e^4)*x^2 - (6*b*c^4*d^4 - 12*b^2*c^3*
d^3*e + 18*b^3*c^2*d^2*e^2 - 38*b^4*c*d*e^3 + 15*b^5*e^4)*x)*sqrt(e*x + d)
)/(b^3*c^4*x^2 + b^4*c^3*x), -1/6*(6*((4*c^5*d^4 - 9*b*c^4*d^3*e)*x^2 + (4
*b*c^4*d^4 - 9*b^2*c^3*d^3*e)*x)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x + d)) +
 3*((4*c^5*d^4 - 7*b*c^4*d^3*e - 3*b^2*c^3*d^2*e^2 + 11*b^3*c^2*d*e^3 - 5*
b^4*c*e^4)*x^2 + (4*b*c^4*d^4 - 7*b^2*c^3*d^3*e - 3*b^3*c^2*d^2*e^2 + 1...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(9/2)/(c*x**2+b*x)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(9/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 425, normalized size of antiderivative = 1.69

$$\begin{aligned}\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^2}\,dx = &-\frac{(4\,cd^5-9\,bd^4e)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b^3\sqrt{-d}} \\ &+\frac{(4\,c^5d^5-11\,bc^4d^4e+4\,b^2c^3d^3e^2+14\,b^3c^2d^2e^3-16\,b^4cde^4+5\,b^5e^5)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}b^3c^3} \\ &+\frac{2\left((ex+d)^{\frac{3}{2}}c^4e^3+12\sqrt{ex+d}c^4de^3-6\sqrt{ex+d}bc^3e^4\right)}{3\,c^6} \\ &-\frac{2\,(ex+d)^{\frac{3}{2}}c^4d^4e-2\sqrt{ex+d}c^4d^5e-4\,(ex+d)^{\frac{3}{2}}bc^3d^3e^2+5\sqrt{ex+d}bc^3d^4e^2+6\,(ex+d)^{\frac{3}{2}}b^2c^2d^2e^3-6}{((ex+d)^2c-2\,(ex+d)cd+cd^2+}\end{aligned}$$

input
```
integrate((e*x+d)^(9/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
-(4*c*d^5 - 9*b*d^4*e)*arctan(sqrt(e*x + d)/sqrt(-d))/(b^3*sqrt(-d)) + (4*
c^5*d^5 - 11*b*c^4*d^4*e + 4*b^2*c^3*d^3*e^2 + 14*b^3*c^2*d^2*e^3 - 16*b^4
*c*d*e^4 + 5*b^5*e^5)*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c*e))/(sqrt(-
c^2*d + b*c*e)*b^3*c^3) + 2/3*((e*x + d)^(3/2)*c^4*e^3 + 12*sqrt(e*x + d)*
c^4*d*e^3 - 6*sqrt(e*x + d)*b*c^3*e^4)/c^6 - (2*(e*x + d)^(3/2)*c^4*d^4*e
- 2*sqrt(e*x + d)*c^4*d^5*e - 4*(e*x + d)^(3/2)*b*c^3*d^3*e^2 + 5*sqrt(e*x
 + d)*b*c^3*d^4*e^2 + 6*(e*x + d)^(3/2)*b^2*c^2*d^2*e^3 - 6*sqrt(e*x + d)*
b^2*c^2*d^3*e^3 - 4*(e*x + d)^(3/2)*b^3*c*d*e^4 + 4*sqrt(e*x + d)*b^3*c*d^
2*e^4 + (e*x + d)^(3/2)*b^4*e^5 - sqrt(e*x + d)*b^4*d*e^5)/(((e*x + d)^2*c
 - 2*(e*x + d)*c*d + c*d^2 + (e*x + d)*b*e - b*d*e)*b^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 5.77 (sec) , antiderivative size = 3360, normalized size of antiderivative = 13.39

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(9/2)/(b*x + c*x^2)^2,x)
```

output
```
(((d + e*x)^(3/2)*(b^4*e^5 + 2*c^4*d^4*e - 4*b*c^3*d^3*e^2 + 6*b^2*c^2*d^2
*e^3 - 4*b^3*c*d*e^4))/b^2 - ((d + e*x)^(1/2)*(b^4*d*e^5 + 2*c^4*d^5*e - 5
*b*c^3*d^4*e^2 - 4*b^3*c*d^2*e^4 + 6*b^2*c^2*d^3*e^3))/b^2)/((2*c^4*d - b*
c^3*e)*(d + e*x) - c^4*(d + e*x)^2 - c^4*d^2 + b*c^3*d*e) + (2*e^3*(d + e*
x)^(3/2))/(3*c^2) + (2*e^3*(4*c^2*d - 2*b*c*e)*(d + e*x)^(1/2))/c^4 - (ata
n((((((20*b^10*c^4*d*e^7 + 8*b^6*c^8*d^5*e^3 - 20*b^7*c^7*d^4*e^4 + 56*b^
8*c^6*d^3*e^5 - 64*b^9*c^5*d^2*e^6)/(b^6*c^5) + ((4*b^7*c^7*e^3 - 8*b^6*c^
8*d*e^2)*(9*b*e - 4*c*d)*(d^7)^(1/2)*(d + e*x)^(1/2))/(b^7*c^5))*(9*b*e -
4*c*d)*(d^7)^(1/2))/(2*b^3) + (2*(d + e*x)^(1/2)*(25*b^10*e^12 + 32*c^10*d
^10*e^2 - 160*b*c^9*d^9*e^3 + 234*b^2*c^8*d^8*e^4 + 24*b^3*c^7*d^7*e^5 - 4
20*b^4*c^6*d^6*e^6 + 504*b^5*c^5*d^5*e^7 - 42*b^6*c^4*d^4*e^8 - 408*b^7*c^
3*d^3*e^9 + 396*b^8*c^2*d^2*e^10 - 160*b^9*c*d*e^11))/(b^4*c^5))*(9*b*e -
4*c*d)*(d^7)^(1/2)*1i)/(2*b^3) - (((((20*b^10*c^4*d*e^7 + 8*b^6*c^8*d^5*e^
3 - 20*b^7*c^7*d^4*e^4 + 56*b^8*c^6*d^3*e^5 - 64*b^9*c^5*d^2*e^6)/(b^6*c^5
) - ((4*b^7*c^7*e^3 - 8*b^6*c^8*d*e^2)*(9*b*e - 4*c*d)*(d^7)^(1/2)*(d + e*
x)^(1/2))/(b^7*c^5))*(9*b*e - 4*c*d)*(d^7)^(1/2))/(2*b^3) - (2*(d + e*x)^(
1/2)*(25*b^10*e^12 + 32*c^10*d^10*e^2 - 160*b*c^9*d^9*e^3 + 234*b^2*c^8*d^
8*e^4 + 24*b^3*c^7*d^7*e^5 - 420*b^4*c^6*d^6*e^6 + 504*b^5*c^5*d^5*e^7 - 4
2*b^6*c^4*d^4*e^8 - 408*b^7*c^3*d^3*e^9 + 396*b^8*c^2*d^2*e^10 - 160*b^9*c
*d*e^11))/(b^4*c^5))*(9*b*e - 4*c*d)*(d^7)^(1/2)*1i)/(2*b^3))/((((((20*...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 862, normalized size of antiderivative = 3.43

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(9/2)/(c*x^2+b*x)^2,x)
```

output
```
(30*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d
)))*b**5*e**4*x - 66*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(
c)*sqrt(b*e - c*d)))*b**4*c*d*e**3*x + 30*sqrt(c)*sqrt(b*e - c*d)*atan((sq
rt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**4*c*e**4*x**2 + 18*sqrt(c)*sq
rt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*c**2*
d**2*e**2*x - 66*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*s
qrt(b*e - c*d)))*b**3*c**2*d*e**3*x**2 + 42*sqrt(c)*sqrt(b*e - c*d)*atan((
sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c**3*d**3*e*x + 18*sqrt(c
)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c
**3*d**2*e**2*x**2 - 24*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sq
rt(c)*sqrt(b*e - c*d)))*b*c**4*d**4*x + 42*sqrt(c)*sqrt(b*e - c*d)*atan((s
qrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**4*d**3*e*x**2 - 24*sqrt(c)
*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*c**5*d*
*4*x**2 - 30*sqrt(d + e*x)*b**5*c*e**4*x + 76*sqrt(d + e*x)*b**4*c**2*d*e*
*3*x - 20*sqrt(d + e*x)*b**4*c**2*e**4*x**2 - 36*sqrt(d + e*x)*b**3*c**3*d
**2*e**2*x + 52*sqrt(d + e*x)*b**3*c**3*d*e**3*x**2 + 4*sqrt(d + e*x)*b**3
*c**3*e**4*x**3 - 6*sqrt(d + e*x)*b**2*c**4*d**4 + 24*sqrt(d + e*x)*b**2*c
**4*d**3*e*x - 12*sqrt(d + e*x)*b*c**5*d**4*x + 27*sqrt(d)*log(sqrt(d + e*
x) - sqrt(d))*b**2*c**4*d**3*e*x - 12*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))
*b*c**5*d**4*x + 27*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*b*c**5*d**3*e*...
```

3.114 $\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^2}\,dx$

Optimal result . . . 860
Mathematica [A] (verified) . . . 861
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Fricas [A] (verification not implemented) . . . 865
Sympy [F(-1)] . . . 866
Maxima [F(-2)] . . . 867
Giac [A] (verification not implemented) . . . 867
Mupad [B] (verification not implemented) . . . 868
Reduce [B] (verification not implemented) . . . 868

Optimal result

Integrand size = 21, antiderivative size = 200

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^2}\,dx = \frac{e(2c^2d^2-2bcde+3b^2e^2)\sqrt{d+ex}}{b^2c^2} - \frac{(cd-be)(2cd-be)(d+ex)^{3/2}}{b^2c(b+cx)} - \frac{d(d+ex)^{5/2}}{bx(b+cx)} + \frac{d^{5/2}(4cd-7be)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3} - \frac{(cd-be)^{5/2}(4cd+3be)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3c^{5/2}}$$

output

```
e*(3*b^2*e^2-2*b*c*d*e+2*c^2*d^2)*(e*x+d)^(1/2)/b^2/c^2-(-b*e+c*d)*(-b*e+2
*c*d)*(e*x+d)^(3/2)/b^2/c/(c*x+b)-d*(e*x+d)^(5/2)/b/x/(c*x+b)+d^(5/2)*(-7*
b*e+4*c*d)*arctanh((e*x+d)^(1/2)/d^(1/2))/b^3-(-b*e+c*d)^(5/2)*(3*b*e+4*c*
d)*arctanh(c^(1/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b^3/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.84

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^2}\,dx = \frac{-\frac{b\sqrt{d+ex}(2c^3d^3x-3b^3e^3x+bc^2d^2(d-3ex)+b^2ce^2x(3d-2ex))}{c^2x(b+cx)} - \frac{(-cd+be)^{5/2}(4cd+3be)\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{-cd+be}}\right)}{c^{5/2}} + d^5}{b^3}$$

input
```
Integrate[(d + e*x)^(7/2)/(b*x + c*x^2)^2,x]
```

output
```
(-((b*Sqrt[d + e*x]*(2*c^3*d^3*x - 3*b^3*e^3*x + b*c^2*d^2*(d - 3*e*x) + b
^2*c*e^2*x*(3*d - 2*e*x)))/(c^2*x*(b + c*x))) - ((-(c*d) + b*e)^(5/2)*(4*c
*d + 3*b*e)*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[-(c*d) + b*e]])/c^(5/2) +
d^(5/2)*(4*c*d - 7*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/b^3
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1164, 27, 1196, 1196, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{1164}$$

$$-\frac{\int \frac{(d+ex)^{3/2}(d(4cd-7be)-3e(2cd-be)x)}{2(cx^2+bx)}dx}{b^2} - \frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2(bx+cx^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(d+ex)^{3/2}(d(4cd-7be)-3e(2cd-be)x)}{cx^2+bx}dx}{2b^2} - \frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2(bx+cx^2)}$$

$$\Big\downarrow \text{1196}$$

$$-\frac{\frac{\int \frac{\sqrt{d+ex}\left(cd^2(4cd-7be)-e\left(2c^2d^2-2bced+3b^2e^2\right)x\right)}{cx^2+bx}dx}{c}-\frac{2e(d+ex)^{3/2}(2cd-be)}{c}}{2b^2}-\frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

$$\Big\downarrow \; 1196$$

$$-\frac{\frac{\frac{\int \frac{c^2(4cd-7be)d^3+e(2cd-be)\left(c^2d^2-bced-3b^2e^2\right)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{c}-\frac{2e\sqrt{d+ex}\left(3b^2e^2-2bcde+2c^2d^2\right)}{c}}{c}-\frac{2e(d+ex)^{3/2}(2cd-be)}{c}}{2b^2}-\frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

$$\Big\downarrow \; 1197$$

$$-\frac{\frac{\frac{2\int \frac{e\left(d(cd-be)\left(2c^2d^2-2bced+3b^2e^2\right)+(2cd-be)\left(c^2d^2-bced-3b^2e^2\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}-\frac{2e\sqrt{d+ex}\left(3b^2e^2-2bcde+2c^2d^2\right)}{c}}{c}-\frac{2e(d+ex)^{3/2}(2cd-be)}{c}}{2b^2}-\frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\frac{2e\int \frac{d(cd-be)\left(2c^2d^2-2bced+3b^2e^2\right)+(2cd-be)\left(c^2d^2-bced-3b^2e^2\right)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}-\frac{2e\sqrt{d+ex}\left(3b^2e^2-2bcde+2c^2d^2\right)}{c}}{c}-\frac{2e(d+ex)^{3/2}(2cd-be)}{c}}{2b^2}-\frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

$$\Big\downarrow \; 1480$$

$$-\frac{\frac{\frac{2e\left(\frac{c^3d^3(4cd-7be)\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{(cd-be)^3(3be+4cd)\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{c}-\frac{2e\sqrt{d+ex}\left(3b^2e^2-2bcde+2c^2d^2\right)}{c}}{c}-\frac{2e(d+ex)^{3/2}(2cd-}{c}}{2b^2}-\frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{\frac{2e\left(\frac{(cd-be)^{5/2}(3be+4cd)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{ce}}-\frac{c^2d^{5/2}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(4cd-7be)}{be}\right)}{c}-\frac{2e\sqrt{d+ex}\left(3b^2e^2-2bcde+2c^2d^2\right)}{c}}{c}-\frac{2e(d+ex)^{3/2}(2cd-}{c}}{2b^2}$$
$$\frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

input
```
Int[(d + e*x)^(7/2)/(b*x + c*x^2)^2,x]
```

output
```
-(((d + e*x)^(5/2)*(b*d + (2*c*d - b*e)*x))/(b^2*(b*x + c*x^2))) - ((-2*e*
(2*c*d - b*e)*(d + e*x)^(3/2))/c + ((-2*e*(2*c^2*d^2 - 2*b*c*d*e + 3*b^2*e
^2)*Sqrt[d + e*x])/c + (2*e*(-((c^2*d^(5/2)*(4*c*d - 7*b*e)*ArcTanh[Sqrt[d
 + e*x]/Sqrt[d]])/(b*e)) + ((c*d - b*e)^(5/2)*(4*c*d + 3*b*e)*ArcTanh[(Sqr
t[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*Sqrt[c]*e)))/c)/c)/(2*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$-\frac{-4\left(cd+\frac{3be}{4}\right)x(cx+b)\sqrt{d}\,(-be+cd)^3\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)+\sqrt{c(be-cd)}\,\left(7\left(be-\frac{4cd}{7}\right)x(cx+b)c^2d^3\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)\right.}{\sqrt{d}\,\sqrt{c(be-cd)}\,c^2b^3x(cx+b)}$
derivativedivides	$2e^3\left(\frac{\sqrt{ex+d}}{c^2}-\frac{d^3\left(\frac{b\sqrt{ex+d}}{2x}+\frac{(7be-4cd)\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{2\sqrt{d}}\right)}{b^3e^3}-\frac{\left(-\frac{1}{2}b^4e^4+\frac{3}{2}d\,e^3b^3c-\frac{3}{2}d^2e^2b^2c^2+\frac{1}{2}d^3eb\,c^3\right)\sqrt{ex+d}}{(ex+d)c+be-cd}+\right.$
default	$2e^3\left(\frac{\sqrt{ex+d}}{c^2}-\frac{d^3\left(\frac{b\sqrt{ex+d}}{2x}+\frac{(7be-4cd)\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{2\sqrt{d}}\right)}{b^3e^3}-\frac{\left(-\frac{1}{2}b^4e^4+\frac{3}{2}d\,e^3b^3c-\frac{3}{2}d^2e^2b^2c^2+\frac{1}{2}d^3eb\,c^3\right)\sqrt{ex+d}}{(ex+d)c+be-cd}+\right.$
risch	$-\frac{d^3\sqrt{ex+d}}{b^2x}+\frac{e\left(\frac{2b^2e^2\sqrt{ex+d}}{c^2}-\frac{d^{\frac{5}{2}}(7be-4cd)\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{eb}-\frac{2\left(\frac{\left(-\frac{1}{2}b^4e^4+\frac{3}{2}d\,e^3b^3c-\frac{3}{2}d^2e^2b^2c^2+\frac{1}{2}d^3eb\,c^3\right)\sqrt{ex+d}}{(ex+d)c+be-cd}\right.}{}\right.}{b^2}$

input
```
int((e*x+d)^(7/2)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d^(1/2)/(c*(b*e-c*d))^(1/2)*(-4*(c*d+3/4*b*e)*x*(c*x+b)*d^(1/2)*(-b*e+c
*d)^3*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2))+(c*(b*e-c*d))^(1/2)*(7*(
b*e-4/7*c*d)*x*(c*x+b)*c^2*d^3*arctanh((e*x+d)^(1/2)/d^(1/2))+(2*d^3*c^3*x
+d^2*b*(-3*e*x+d)*c^2+3*e^2*x*b^2*(-2/3*e*x+d)*c-3*b^3*e^3*x)*(e*x+d)^(1/2
)*b*d^(1/2)))/c^2/b^3/x/(c*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 1267, normalized size of antiderivative = 6.34

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
[1/2*(((4*c^4*d^3 - 5*b*c^3*d^2*e - 2*b^2*c^2*d*e^2 + 3*b^3*c*e^3)*x^2 + (
4*b*c^3*d^3 - 5*b^2*c^2*d^2*e - 2*b^3*c*d*e^2 + 3*b^4*e^3)*x)*sqrt((c*d -
b*e)/c)*log((c*e*x + 2*c*d - b*e - 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/
(c*x + b)) - ((4*c^4*d^3 - 7*b*c^3*d^2*e)*x^2 + (4*b*c^3*d^3 - 7*b^2*c^2*d
^2*e)*x)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(2*b^3*c
*e^3*x^2 - b^2*c^2*d^3 - (2*b*c^3*d^3 - 3*b^2*c^2*d^2*e + 3*b^3*c*d*e^2 -
3*b^4*e^3)*x)*sqrt(e*x + d))/(b^3*c^3*x^2 + b^4*c^2*x), -1/2*(2*((4*c^4*d^
3 - 5*b*c^3*d^2*e - 2*b^2*c^2*d*e^2 + 3*b^3*c*e^3)*x^2 + (4*b*c^3*d^3 - 5*
b^2*c^2*d^2*e - 2*b^3*c*d*e^2 + 3*b^4*e^3)*x)*sqrt(-(c*d - b*e)/c)*arctan(
-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + ((4*c^4*d^3 - 7*b*c^3
*d^2*e)*x^2 + (4*b*c^3*d^3 - 7*b^2*c^2*d^2*e)*x)*sqrt(d)*log((e*x - 2*sqrt
(e*x + d)*sqrt(d) + 2*d)/x) - 2*(2*b^3*c*e^3*x^2 - b^2*c^2*d^3 - (2*b*c^3*
d^3 - 3*b^2*c^2*d^2*e + 3*b^3*c*d*e^2 - 3*b^4*e^3)*x)*sqrt(e*x + d))/(b^3*
c^3*x^2 + b^4*c^2*x), -1/2*(2*((4*c^4*d^3 - 7*b*c^3*d^2*e)*x^2 + (4*b*c^3*
d^3 - 7*b^2*c^2*d^2*e)*x)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x + d)) - ((4*c^
4*d^3 - 5*b*c^3*d^2*e - 2*b^2*c^2*d*e^2 + 3*b^3*c*e^3)*x^2 + (4*b*c^3*d^3
- 5*b^2*c^2*d^2*e - 2*b^3*c*d*e^2 + 3*b^4*e^3)*x)*sqrt((c*d - b*e)/c)*log(
(c*e*x + 2*c*d - b*e - 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) -
 2*(2*b^3*c*e^3*x^2 - b^2*c^2*d^3 - (2*b*c^3*d^3 - 3*b^2*c^2*d^2*e + 3*b^3
*c*d*e^2 - 3*b^4*e^3)*x)*sqrt(e*x + d))/(b^3*c^3*x^2 + b^4*c^2*x), -(((...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)/(c*x**2+b*x)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 332, normalized size of antiderivative = 1.66

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^2}\,dx = \frac{2\sqrt{ex+d}\,e^3}{c^2} - \frac{(4\,cd^4-7\,bd^3e)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b^3\sqrt{-d}} + \frac{(4\,c^4d^4-9\,bc^3d^3e+3\,b^2c^2d^2e^2+5\,b^3cde^3-3\,b^4e^4)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}\,b^3c^2} - \frac{2\,(ex+d)^{\frac{3}{2}}c^3d^3e-2\sqrt{ex+d}\,c^3d^4e-3\,(ex+d)^{\frac{3}{2}}bc^2d^2e^2+4\sqrt{ex+d}\,bc^2d^3e^2+3\,(ex+d)^{\frac{3}{2}}b^2cde^3-3\sqrt{\ }}{\left((ex+d)^2c-2\,(ex+d)cd+cd^2+(ex+d)be-bde\right)b^2c^2}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
2*sqrt(e*x + d)*e^3/c^2 - (4*c*d^4 - 7*b*d^3*e)*arctan(sqrt(e*x + d)/sqrt(
-d))/(b^3*sqrt(-d)) + (4*c^4*d^4 - 9*b*c^3*d^3*e + 3*b^2*c^2*d^2*e^2 + 5*b
^3*c*d*e^3 - 3*b^4*e^4)*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c*e))/(sqrt
(-c^2*d + b*c*e)*b^3*c^2) - (2*(e*x + d)^(3/2)*c^3*d^3*e - 2*sqrt(e*x + d)
*c^3*d^4*e - 3*(e*x + d)^(3/2)*b*c^2*d^2*e^2 + 4*sqrt(e*x + d)*b*c^2*d^3*e
^2 + 3*(e*x + d)^(3/2)*b^2*c*d*e^3 - 3*sqrt(e*x + d)*b^2*c*d^2*e^3 - (e*x
+ d)^(3/2)*b^3*e^4 + sqrt(e*x + d)*b^3*d*e^4)/(((e*x + d)^2*c - 2*(e*x + d
)*c*d + c*d^2 + (e*x + d)*b*e - b*d*e)*b^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 2913, normalized size of antiderivative = 14.56

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(7/2)/(b*x + c*x^2)^2,x)
```

output
```
(((d + e*x)^(1/2)*(b^3*d*e^4 - 2*c^3*d^4*e + 4*b*c^2*d^3*e^2 - 3*b^2*c*d^2
*e^3))/b^2 - ((d + e*x)^(3/2)*(b^3*e^4 - 2*c^3*d^3*e + 3*b*c^2*d^2*e^2 - 3
*b^2*c*d*e^3))/b^2)/((2*c^3*d - b*c^2*e)*(d + e*x) - c^3*(d + e*x)^2 - c^3
*d^2 + b*c^2*d*e) + (2*e^3*(d + e*x)^(1/2))/c^2 + (atan(((((2*(d + e*x)^(1
/2)*(9*b^8*e^10 + 32*c^8*d^8*e^2 - 128*b*c^7*d^7*e^3 + 154*b^2*c^6*d^6*e^4
 - 14*b^3*c^5*d^5*e^5 - 105*b^4*c^4*d^4*e^6 + 84*b^5*c^3*d^3*e^7 + 7*b^6*c
^2*d^2*e^8 - 30*b^7*c*d*e^9))/(b^4*c^3) + (((12*b^9*c^3*d*e^6 - 8*b^6*c^6*
d^4*e^3 + 16*b^7*c^5*d^3*e^4 - 20*b^8*c^4*d^2*e^5)/(b^6*c^3) + ((4*b^7*c^5
*e^3 - 8*b^6*c^6*d*e^2)*(7*b*e - 4*c*d)*(d^5)^(1/2)*(d + e*x)^(1/2))/(b^7*
c^3))*(7*b*e - 4*c*d)*(d^5)^(1/2))/(2*b^3))*(7*b*e - 4*c*d)*(d^5)^(1/2)*1i
)/(2*b^3) + (((2*(d + e*x)^(1/2)*(9*b^8*e^10 + 32*c^8*d^8*e^2 - 128*b*c^7*
d^7*e^3 + 154*b^2*c^6*d^6*e^4 - 14*b^3*c^5*d^5*e^5 - 105*b^4*c^4*d^4*e^6 +
 84*b^5*c^3*d^3*e^7 + 7*b^6*c^2*d^2*e^8 - 30*b^7*c*d*e^9))/(b^4*c^3) - (((
12*b^9*c^3*d*e^6 - 8*b^6*c^6*d^4*e^3 + 16*b^7*c^5*d^3*e^4 - 20*b^8*c^4*d^2
*e^5)/(b^6*c^3) - ((4*b^7*c^5*e^3 - 8*b^6*c^6*d*e^2)*(7*b*e - 4*c*d)*(d^5)
^(1/2)*(d + e*x)^(1/2))/(b^7*c^3))*(7*b*e - 4*c*d)*(d^5)^(1/2))/(2*b^3))*(
7*b*e - 4*c*d)*(d^5)^(1/2)*1i)/(2*b^3))/((2*(63*b^8*d^3*e^11 + 32*c^8*d^11
*e^3 - 176*b*c^7*d^10*e^4 - 246*b^7*c*d^4*e^10 + 262*b^2*c^6*d^9*e^5 + 141
*b^3*c^5*d^8*e^6 - 658*b^4*c^4*d^7*e^7 + 413*b^5*c^3*d^6*e^8 + 169*b^6*c^2
*d^5*e^9))/(b^6*c^3) + (((2*(d + e*x)^(1/2)*(9*b^8*e^10 + 32*c^8*d^8*e^...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 698, normalized size of antiderivative = 3.49

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(7/2)/(c*x^2+b*x)^2,x)
```

output

```
( - 6*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c
*d)))*b**4*e**3*x + 4*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt
(c)*sqrt(b*e - c*d)))*b**3*c*d*e**2*x - 6*sqrt(c)*sqrt(b*e - c*d)*atan((sq
rt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*c*e**3*x**2 + 10*sqrt(c)*sq
rt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c**2*
d**2*e*x + 4*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(
b*e - c*d)))*b**2*c**2*d*e**2*x**2 - 8*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(
d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**3*d**3*x + 10*sqrt(c)*sqrt(b*e
 - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**3*d**2*e*x*
*2 - 8*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e -
c*d)))*c**4*d**3*x**2 + 6*sqrt(d + e*x)*b**4*c*e**3*x - 6*sqrt(d + e*x)*b*
*3*c**2*d*e**2*x + 4*sqrt(d + e*x)*b**3*c**2*e**3*x**2 - 2*sqrt(d + e*x)*b
**2*c**3*d**3 + 6*sqrt(d + e*x)*b**2*c**3*d**2*e*x - 4*sqrt(d + e*x)*b*c**
4*d**3*x + 7*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*b**2*c**3*d**2*e*x - 4*s
qrt(d)*log(sqrt(d + e*x) - sqrt(d))*b*c**4*d**3*x + 7*sqrt(d)*log(sqrt(d +
 e*x) - sqrt(d))*b*c**4*d**2*e*x**2 - 4*sqrt(d)*log(sqrt(d + e*x) - sqrt(d
))*c**5*d**3*x**2 - 7*sqrt(d)*log(sqrt(d + e*x) + sqrt(d))*b**2*c**3*d**2*
e*x + 4*sqrt(d)*log(sqrt(d + e*x) + sqrt(d))*b*c**4*d**3*x - 7*sqrt(d)*log
(sqrt(d + e*x) + sqrt(d))*b*c**4*d**2*e*x**2 + 4*sqrt(d)*log(sqrt(d + e*x)
 + sqrt(d))*c**5*d**3*x**2)/(2*b**3*c**3*x*(b + c*x))
```

3.115 $\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^2}\,dx$

Optimal result . . . 870
Mathematica [A] (verified) . . . 870
Rubi [A] (verified) . . . 871
Maple [A] (verified) . . . 873
Fricas [A] (verification not implemented) . . . 874
Sympy [F(-1)] . . . 875
Maxima [F(-2)] . . . 876
Giac [A] (verification not implemented) . . . 876
Mupad [B] (verification not implemented) . . . 877
Reduce [B] (verification not implemented) . . . 877

Optimal result

Integrand size = 21, antiderivative size = 159

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^2}\,dx = -\frac{(cd-be)(2cd-be)\sqrt{d+ex}}{b^2c(b+cx)} - \frac{d(d+ex)^{3/2}}{bx(b+cx)} + \frac{d^{3/2}(4cd-5be)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3} - \frac{(cd-be)^{3/2}(4cd+be)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3c^{3/2}}$$

output
```
-(-b*e+c*d)*(-b*e+2*c*d)*(e*x+d)^(1/2)/b^2/c/(c*x+b)-d*(e*x+d)^(3/2)/b/x/(
c*x+b)+d^(3/2)*(-5*b*e+4*c*d)*arctanh((e*x+d)^(1/2)/d^(1/2))/b^3-(-b*e+c*d
)^(3/2)*(b*e+4*c*d)*arctanh(c^(1/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b^3/c^
(3/2)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^2}\,dx = \frac{-\frac{b\sqrt{d+ex}(2c^2d^2x+b^2e^2x+bcd(d-2ex))}{cx(b+cx)} + \frac{(-cd+be)^{3/2}(4cd+be)\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{-cd+be}}\right)}{c^{3/2}} + d^{3/2}(4cd-5be)\text{arc}}{b^3}$$

input
```
Integrate[(d + e*x)^(5/2)/(b*x + c*x^2)^2,x]
```

output
```
(-((b*Sqrt[d + e*x]*(2*c^2*d^2*x + b^2*e^2*x + b*c*d*(d - 2*e*x)))/(c*x*(b
 + c*x))) + ((-(c*d) + b*e)^(3/2)*(4*c*d + b*e)*ArcTan[(Sqrt[c]*Sqrt[d + e
*x])/Sqrt[-(c*d) + b*e]])/c^(3/2) + d^(3/2)*(4*c*d - 5*b*e)*ArcTanh[Sqrt[d
 + e*x]/Sqrt[d]])/b^3
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1164, 27, 1196, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^2}dx$$

$$\Big\downarrow \text{1164}$$

$$-\frac{\int \frac{\sqrt{d+ex}(d(4cd-5be)-e(2cd-be)x)}{2(cx^2+bx)}dx}{b^2}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2(bx+cx^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\sqrt{d+ex}(d(4cd-5be)-e(2cd-be)x)}{cx^2+bx}dx}{2b^2}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2(bx+cx^2)}$$

$$\Big\downarrow \text{1196}$$

$$-\frac{\frac{\int \frac{c(4cd-5be)d^2+e\left(2c^2d^2-2bced-b^2e^2\right)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{c}-\frac{2e\sqrt{d+ex}(2cd-be)}{c}}{2b^2}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2(bx+cx^2)}$$

$$\Big\downarrow \text{1197}$$

$$-\frac{\frac{2\int \frac{e\left(d(cd-be)(2cd-be)+\left(2c^2d^2-2bced-b^2e^2\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}-\frac{2e\sqrt{d+ex}(2cd-be)}{c}}{2b^2}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2(bx+cx^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{2e\int \frac{d(cd-be)(2cd-be)+\left(2c^2d^2-2bced-b^2e^2\right)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}-\frac{2e\sqrt{d+ex}(2cd-be)}{c}}{2b^2}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{\frac{2e\left(\frac{c^2d^2(4cd-5be)\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{(cd-be)^2(be+4cd)\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{c}-\frac{2e\sqrt{d+ex}(2cd-be)}{c}}{2b^2}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{2e\left(\frac{(cd-be)^{3/2}(be+4cd)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{c}e}-\frac{cd^{3/2}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(4cd-5be)}{be}\right)}{c}-\frac{2e\sqrt{d+ex}(2cd-be)}{c}}{2b^2}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

input
```
Int[(d + e*x)^(5/2)/(b*x + c*x^2)^2,x]
```

output
```
-(((d + e*x)^(3/2)*(b*d + (2*c*d - b*e)*x))/(b^2*(b*x + c*x^2))) - ((-2*e*
(2*c*d - b*e)*Sqrt[d + e*x])/c + (2*e*(-((c*d^(3/2)*(4*c*d - 5*b*e)*ArcTan
h[Sqrt[d + e*x]/Sqrt[d]])/(b*e)) + ((c*d - b*e)^(3/2)*(4*c*d + b*e)*ArcTan
h[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*Sqrt[c]*e)))/c)/(2*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.97

method	result
derivativedivides	$2e^3 \left(\frac{(be-cd)^2 \left(-\frac{be\sqrt{ex+d}}{2c((ex+d)c+be-cd)} + \frac{(be+4cd)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{2c\sqrt{c(be-cd)}} \right)}{b^3e^3} - \frac{d^2 \left(\frac{b\sqrt{ex+d}}{2x} + \frac{(5be-4cd)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{2\sqrt{d}} \right.}{b^3e^3} \right.$
default	$2e^3 \left(\frac{(be-cd)^2 \left(-\frac{be\sqrt{ex+d}}{2c((ex+d)c+be-cd)} + \frac{(be+4cd)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{2c\sqrt{c(be-cd)}} \right)}{b^3e^3} - \frac{d^2 \left(\frac{b\sqrt{ex+d}}{2x} + \frac{(5be-4cd)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{2\sqrt{d}} \right.}{b^3e^3} \right.$
pseudoelliptic	$-\frac{-4\sqrt{d}\,(-be+cd)^2 x(cx+b)\left(cd+\frac{be}{4}\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right) + \sqrt{c(be-cd)}\left(5x(cx+b)\left(be-\frac{4cd}{5}\right)c\,d^2 \operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right) + \right.}{\sqrt{d}\,\sqrt{c(be-cd)}\,b^3xc(cx+b)}$
risch	$-\frac{d^2\sqrt{ex+d}}{b^2x} + \frac{e\left(-\frac{d^{\frac{3}{2}}(5be-4cd)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be} + \frac{-\frac{be\left(b^2e^2-2bcde+c^2d^2\right)\sqrt{ex+d}}{c((ex+d)c+be-cd)} + \frac{\left(b^3e^3+2d\,e^2b^2c-7d^2eb\,c^2+4d^3c^3\right)}{c\sqrt{c(be-cd)}}}{be} \right.}{b^2}$

input
```
int((e*x+d)^(5/2)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
2*e^3*((b*e-c*d)^2/b^3/e^3*(-1/2*b*e/c*(e*x+d)^(1/2)/((e*x+d)*c+b*e-c*d)+1
/2*(b*e+4*c*d)/c/(c*(b*e-c*d))^(1/2)*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^
(1/2)))-d^2/b^3/e^3*(1/2*b*(e*x+d)^(1/2)/x+1/2*(5*b*e-4*c*d)/d^(1/2)*arcta
nh((e*x+d)^(1/2)/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 996, normalized size of antiderivative = 6.26

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
[-1/2*(((4*c^3*d^2 - 3*b*c^2*d*e - b^2*c*e^2)*x^2 + (4*b*c^2*d^2 - 3*b^2*c
*d*e - b^3*e^2)*x)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*sqrt(e
*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) + ((4*c^3*d^2 - 5*b*c^2*d*e)*x^2
 + (4*b*c^2*d^2 - 5*b^2*c*d*e)*x)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(
d) + 2*d)/x) + 2*(b^2*c*d^2 + (2*b*c^2*d^2 - 2*b^2*c*d*e + b^3*e^2)*x)*sqr
t(e*x + d))/(b^3*c^2*x^2 + b^4*c*x), -1/2*(2*((4*c^3*d^2 - 3*b*c^2*d*e - b
^2*c*e^2)*x^2 + (4*b*c^2*d^2 - 3*b^2*c*d*e - b^3*e^2)*x)*sqrt(-(c*d - b*e)
/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + ((4*c^3*d^
2 - 5*b*c^2*d*e)*x^2 + (4*b*c^2*d^2 - 5*b^2*c*d*e)*x)*sqrt(d)*log((e*x - 2
*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(b^2*c*d^2 + (2*b*c^2*d^2 - 2*b^2*c*d
*e + b^3*e^2)*x)*sqrt(e*x + d))/(b^3*c^2*x^2 + b^4*c*x), -1/2*(2*((4*c^3*d
^2 - 5*b*c^2*d*e)*x^2 + (4*b*c^2*d^2 - 5*b^2*c*d*e)*x)*sqrt(-d)*arctan(sqr
t(-d)/sqrt(e*x + d)) + ((4*c^3*d^2 - 3*b*c^2*d*e - b^2*c*e^2)*x^2 + (4*b*c
^2*d^2 - 3*b^2*c*d*e - b^3*e^2)*x)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d
- b*e + 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) + 2*(b^2*c*d^2 +
 (2*b*c^2*d^2 - 2*b^2*c*d*e + b^3*e^2)*x)*sqrt(e*x + d))/(b^3*c^2*x^2 + b^
4*c*x), -(((4*c^3*d^2 - 3*b*c^2*d*e - b^2*c*e^2)*x^2 + (4*b*c^2*d^2 - 3*b^
2*c*d*e - b^3*e^2)*x)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(
c*d - b*e)/c)/(c*d - b*e)) + ((4*c^3*d^2 - 5*b*c^2*d*e)*x^2 + (4*b*c^2*d^2
 - 5*b^2*c*d*e)*x)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x + d)) + (b^2*c*d^2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(c*x**2+b*x)**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.64

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^2}\,dx = -\frac{(4\,cd^3 - 5\,bd^2e)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b^3\sqrt{-d}} + \frac{(4\,c^3d^3 - 7\,bc^2d^2e + 2\,b^2cde^2 + b^3e^3)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}b^3c} - \frac{2\,(ex+d)^{\frac{3}{2}}c^2d^2e - 2\,\sqrt{ex+d}c^2d^3e - 2\,(ex+d)^{\frac{3}{2}}bcde^2 + 3\,\sqrt{ex+d}bcd^2e^2 + (ex+d)^{\frac{3}{2}}b^2e^3 - \sqrt{ex+d}b^2}{\left((ex+d)^2c - 2\,(ex+d)cd + cd^2 + (ex+d)be - bde\right)b^2c}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
-(4*c*d^3 - 5*b*d^2*e)*arctan(sqrt(e*x + d)/sqrt(-d))/(b^3*sqrt(-d)) + (4*
c^3*d^3 - 7*b*c^2*d^2*e + 2*b^2*c*d*e^2 + b^3*e^3)*arctan(sqrt(e*x + d)*c/
sqrt(-c^2*d + b*c*e))/(sqrt(-c^2*d + b*c*e)*b^3*c) - (2*(e*x + d)^(3/2)*c^
2*d^2*e - 2*sqrt(e*x + d)*c^2*d^3*e - 2*(e*x + d)^(3/2)*b*c*d*e^2 + 3*sqrt
(e*x + d)*b*c*d^2*e^2 + (e*x + d)^(3/2)*b^2*e^3 - sqrt(e*x + d)*b^2*d*e^3)
/(((e*x + d)^2*c - 2*(e*x + d)*c*d + c*d^2 + (e*x + d)*b*e - b*d*e)*b^2*c)
```

Mupad [B] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 1127, normalized size of antiderivative = 7.09

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(5/2)/(b*x + c*x^2)^2,x)
```

output
```
(((d + e*x)^(1/2)*(b^2*d*e^3 + 2*c^2*d^3*e - 3*b*c*d^2*e^2))/(b^2*c) - (e*
(d + e*x)^(3/2)*(b^2*e^2 + 2*c^2*d^2 - 2*b*c*d*e))/(b^2*c))/((b*e - 2*c*d)
*(d + e*x) + c*(d + e*x)^2 + c*d^2 - b*d*e) - (atanh((10*e^9*(d^3)^(1/2)*(
d + e*x)^(1/2))/(10*d^2*e^9 + (32*c*d^3*e^8)/b - (132*c^2*d^4*e^7)/b^2 + (
130*c^3*d^5*e^6)/b^3 - (40*c^4*d^6*e^5)/b^4) + (32*d*e^8*(d^3)^(1/2)*(d +
e*x)^(1/2))/(32*d^3*e^8 + (10*b*d^2*e^9)/c - (132*c*d^4*e^7)/b + (130*c^2*
d^5*e^6)/b^2 - (40*c^3*d^6*e^5)/b^3) - (132*c*d^2*e^7*(d^3)^(1/2)*(d + e*x
)^(1/2))/(32*b*d^3*e^8 - 132*c*d^4*e^7 + (130*c^2*d^5*e^6)/b + (10*b^2*d^2
*e^9)/c - (40*c^3*d^6*e^5)/b^2) + (130*c^2*d^3*e^6*(d^3)^(1/2)*(d + e*x)^(
1/2))/(32*b^2*d^3*e^8 + 130*c^2*d^5*e^6 - (40*c^3*d^6*e^5)/b + (10*b^3*d^2
*e^9)/c - 132*b*c*d^4*e^7) - (40*c^3*d^4*e^5*(d^3)^(1/2)*(d + e*x)^(1/2))/
(32*b^3*d^3*e^8 - 40*c^3*d^6*e^5 + 130*b*c^2*d^5*e^6 - 132*b^2*c*d^4*e^7 +
 (10*b^4*d^2*e^9)/c))*(5*b*e - 4*c*d)*(d^3)^(1/2))/b^3 - (atanh((30*d^3*e^
6*(d + e*x)^(1/2)*(c^6*d^3 - b^3*c^3*e^3 + 3*b^2*c^4*d*e^2 - 3*b*c^5*d^2*e
)^(1/2))/(14*b^3*d^2*e^9 + 110*c^3*d^5*e^6 - 82*b*c^2*d^4*e^7 - 4*b^2*c*d^
3*e^8 + (2*b^4*d*e^10)/c - (40*c^4*d^6*e^5)/b) - (2*d*e^8*(d + e*x)^(1/2)*
(c^6*d^3 - b^3*c^3*e^3 + 3*b^2*c^4*d*e^2 - 3*b*c^5*d^2*e)^(1/2))/(4*c^3*d^
3*e^8 - 14*b*c^2*d^2*e^9 + (82*c^4*d^4*e^7)/b - (110*c^5*d^5*e^6)/b^2 + (4
0*c^6*d^6*e^5)/b^3 - 2*b^2*c*d*e^10) + (18*d^2*e^7*(d + e*x)^(1/2)*(c^6*d^
3 - b^3*c^3*e^3 + 3*b^2*c^4*d*e^2 - 3*b*c^5*d^2*e)^(1/2))/(2*b^3*d*e^10...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 547, normalized size of antiderivative = 3.44

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^2}\,dx = \frac{2\sqrt{c}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{be-cd}}\right) b^3e^2x + 6\sqrt{c}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{be-cd}}\right) b^2cdex + 2\sqrt{c}\,\sqrt{}}{}$$

input
```
int((e*x+d)^(5/2)/(c*x^2+b*x)^2,x)
```

output
```
(2*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)
))*b**3*e**2*x + 6*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)
*sqrt(b*e - c*d)))*b**2*c*d*e*x + 2*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d +
 e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c*e**2*x**2 - 8*sqrt(c)*sqrt(b*e
- c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**2*d**2*x + 6
*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))
*b*c**2*d*e*x**2 - 8*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(
c)*sqrt(b*e - c*d)))*c**3*d**2*x**2 - 2*sqrt(d + e*x)*b**3*c*e**2*x - 2*sq
rt(d + e*x)*b**2*c**2*d**2 + 4*sqrt(d + e*x)*b**2*c**2*d*e*x - 4*sqrt(d +
e*x)*b*c**3*d**2*x + 5*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*b**2*c**2*d*e*
x - 4*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*b*c**3*d**2*x + 5*sqrt(d)*log(s
qrt(d + e*x) - sqrt(d))*b*c**3*d*e*x**2 - 4*sqrt(d)*log(sqrt(d + e*x) - sq
rt(d))*c**4*d**2*x**2 - 5*sqrt(d)*log(sqrt(d + e*x) + sqrt(d))*b**2*c**2*d
*e*x + 4*sqrt(d)*log(sqrt(d + e*x) + sqrt(d))*b*c**3*d**2*x - 5*sqrt(d)*lo
g(sqrt(d + e*x) + sqrt(d))*b*c**3*d*e*x**2 + 4*sqrt(d)*log(sqrt(d + e*x) +
 sqrt(d))*c**4*d**2*x**2)/(2*b**3*c**2*x*(b + c*x))
```

3.116 $\int \frac{(d+ex)^{3/2}}{\left(bx+cx^2\right)^2} \, dx$

Optimal result 879
Mathematica [A] (verified) 879
Rubi [A] (verified) 880
Maple [A] (verified) 882
Fricas [A] (verification not implemented) 883
Sympy [F] 884
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Giac [A] (verification not implemented) 884
Mupad [B] (verification not implemented) 885
Reduce [B] (verification not implemented) 886

Optimal result

Integrand size = 21, antiderivative size = 149

$$\int \frac{(d+ex)^{3/2}}{\left(bx+cx^2\right)^2} \, dx = -\frac{(2cd-be)\sqrt{d+ex}}{b^2(b+cx)} - \frac{d\sqrt{d+ex}}{bx(b+cx)} + \frac{\sqrt{d}(4cd-3be)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3} - \frac{\sqrt{cd-be}(4cd-be)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3\sqrt{c}}$$

output
```
-(-b*e+2*c*d)*(e*x+d)^(1/2)/b^2/(c*x+b)-d*(e*x+d)^(1/2)/b/x/(c*x+b)+d^(1/2
)*(-3*b*e+4*c*d)*arctanh((e*x+d)^(1/2)/d^(1/2))/b^3-(-b*e+c*d)^(1/2)*(-b*e
+4*c*d)*arctanh(c^(1/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b^3/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^{3/2}}{\left(bx+cx^2\right)^2} \, dx = \frac{\frac{b\sqrt{d+ex}(-bd-2cdx+bex)}{x(b+cx)} + \frac{\left(4c^2d^2-5bcde+b^2e^2\right)\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{-cd+be}}\right)}{\sqrt{c}\sqrt{-cd+be}} + \sqrt{d}(4cd-3be)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3}$$

input
```
Integrate[(d + e*x)^(3/2)/(b*x + c*x^2)^2,x]
```

output
```
((b*Sqrt[d + e*x]*(-(b*d) - 2*c*d*x + b*e*x))/(x*(b + c*x)) + ((4*c^2*d^2
- 5*b*c*d*e + b^2*e^2)*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[-(c*d) + b*e]])
/(Sqrt[c]*Sqrt[-(c*d) + b*e]) + Sqrt[d]*(4*c*d - 3*b*e)*ArcTanh[Sqrt[d + e
*x]/Sqrt[d]])/b^3
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1164, 27, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^2}dx$$

$$\Big\downarrow \text{1164}$$

$$-\frac{\int \frac{d(4cd-3be)+e(2cd-be)x}{2\sqrt{d+ex}(cx^2+bx)}dx}{b^2}-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{b^2(bx+cx^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{d(4cd-3be)+e(2cd-be)x}{\sqrt{d+ex}(cx^2+bx)}dx}{2b^2}-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{b^2(bx+cx^2)}$$

$$\Big\downarrow \text{1197}$$

$$-\frac{\int \frac{e(2d(cd-be)+(2cd-be)(d+ex))}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2}-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{b^2(bx+cx^2)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e\int \frac{2d(cd-be)+(2cd-be)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2}-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{b^2(bx+cx^2)}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{e\left(\frac{cd(4cd-3be)\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{(cd-be)(4cd-be)\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{b^2}-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

$\Big\downarrow$ 221

$$-\frac{e\left(\frac{\sqrt{cd-be}(4cd-be)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{c}e}-\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(4cd-3be)}{be}\right)}{b^2}-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{b^2\left(bx+cx^2\right)}$$

input
```
Int[(d + e*x)^(3/2)/(b*x + c*x^2)^2,x]
```

output
```
-((Sqrt[d + e*x]*(b*d + (2*c*d - b*e)*x))/(b^2*(b*x + c*x^2))) - (e*(-((Sq
rt[d]*(4*c*d - 3*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*e)) + (Sqrt[c*d -
 b*e]*(4*c*d - b*e)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*S
qrt[c]*e)))/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.97

method	result
derivativedivides	$2e^3\left(\frac{(be-cd)\left(\frac{be\sqrt{ex+d}}{2(ex+d)c+2be-2cd}+\frac{(be-4cd)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{2\sqrt{c(be-cd)}}\right)}{b^3e^3}-\frac{d\left(\frac{b\sqrt{ex+d}}{2x}+\frac{(3be-4cd)\,\text{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{2\sqrt{d}}\right)}{b^3e^3}\right)$
default	$2e^3\left(\frac{(be-cd)\left(\frac{be\sqrt{ex+d}}{2(ex+d)c+2be-2cd}+\frac{(be-4cd)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{2\sqrt{c(be-cd)}}\right)}{b^3e^3}-\frac{d\left(\frac{b\sqrt{ex+d}}{2x}+\frac{(3be-4cd)\,\text{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{2\sqrt{d}}\right)}{b^3e^3}\right)$
pseudoelliptic	$-\frac{-4x(cx+b)\left(cd-\frac{be}{4}\right)(-be+cd)\sqrt{d}\,\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)+\sqrt{c(be-cd)}\left(3x(cx+b)d\left(be-\frac{4cd}{3}\right)\text{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)+(2c\right.}{\sqrt{d}\,\sqrt{c(be-cd)}\,b^3x(cx+b)}$
risch	$-\frac{d\sqrt{ex+d}}{b^2x}-\frac{e\left(\frac{\sqrt{d}\,(3be-4cd)\,\text{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be}+\frac{\frac{2\left(-\frac{1}{2}b^2e^2+\frac{1}{2}bcde\right)\sqrt{ex+d}}{(ex+d)c+be-cd}-\frac{\left(b^2e^2-5bcde+4c^2d^2\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{\sqrt{c(be-cd)}}}{be}\right.}{b^2}$

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
2*e^3*((b*e-c*d)/b^3/e^3*(1/2*b*e*(e*x+d)^(1/2)/((e*x+d)*c+b*e-c*d)+1/2*(b
*e-4*c*d)/(c*(b*e-c*d))^(1/2)*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2)))
-d/b^3/e^3*(1/2*b*(e*x+d)^(1/2)/x+1/2*(3*b*e-4*c*d)/d^(1/2)*arctanh((e*x+d
)^(1/2)/d^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 764, normalized size of antiderivative = 5.13

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
[-1/2*(((4*c^2*d - b*c*e)*x^2 + (4*b*c*d - b^2*e)*x)*sqrt((c*d - b*e)/c)*l
og((c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)
) + ((4*c^2*d - 3*b*c*e)*x^2 + (4*b*c*d - 3*b^2*e)*x)*sqrt(d)*log((e*x - 2
*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(b^2*d + (2*b*c*d - b^2*e)*x)*sqrt(e*
x + d))/(b^3*c*x^2 + b^4*x), -1/2*(2*((4*c^2*d - b*c*e)*x^2 + (4*b*c*d - b
^2*e)*x)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)
/(c*d - b*e)) + ((4*c^2*d - 3*b*c*e)*x^2 + (4*b*c*d - 3*b^2*e)*x)*sqrt(d)*
log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(b^2*d + (2*b*c*d - b^2*e
)*x)*sqrt(e*x + d))/(b^3*c*x^2 + b^4*x), -1/2*(2*((4*c^2*d - 3*b*c*e)*x^2
+ (4*b*c*d - 3*b^2*e)*x)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x + d)) + ((4*c^2
*d - b*c*e)*x^2 + (4*b*c*d - b^2*e)*x)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*
c*d - b*e + 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) + 2*(b^2*d +
 (2*b*c*d - b^2*e)*x)*sqrt(e*x + d))/(b^3*c*x^2 + b^4*x), -(((4*c^2*d - b*
c*e)*x^2 + (4*b*c*d - b^2*e)*x)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)
*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + ((4*c^2*d - 3*b*c*e)*x^2 + (4*b*c*d
 - 3*b^2*e)*x)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x + d)) + (b^2*d + (2*b*c*d
 - b^2*e)*x)*sqrt(e*x + d))/(b^3*c*x^2 + b^4*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^2}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{x^2\,(b+cx)^2}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(c*x**2+b*x)**2,x)
```

output
```
Integral((d + e*x)**(3/2)/(x**2*(b + c*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.32

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^2}\,dx = \frac{(4\,c^2d^2 - 5\,bcde + b^2e^2)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}b^3} - \frac{(4\,cd^2 - 3\,bde)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b^3\sqrt{-d}} - \frac{2\,(ex+d)^{\frac{3}{2}}cde - 2\,\sqrt{ex+d}cd^2e - (ex+d)^{\frac{3}{2}}be^2 + 2\,\sqrt{ex+d}bde^2}{\left((ex+d)^2c - 2\,(ex+d)cd + cd^2 + (ex+d)be - bde\right)b^2}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
(4*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c
*e))/(sqrt(-c^2*d + b*c*e)*b^3) - (4*c*d^2 - 3*b*d*e)*arctan(sqrt(e*x + d)
/sqrt(-d))/(b^3*sqrt(-d)) - (2*(e*x + d)^(3/2)*c*d*e - 2*sqrt(e*x + d)*c*d
^2*e - (e*x + d)^(3/2)*b*e^2 + 2*sqrt(e*x + d)*b*d*e^2)/(((e*x + d)^2*c -
2*(e*x + d)*c*d + c*d^2 + (e*x + d)*b*e - b*d*e)*b^2)
```

Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.88

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^2}\,dx = -\frac{\frac{2\,(b\,d\,e^2-c\,d^2\,e)\,\sqrt{d+e\,x}}{b^2}-\frac{e\,(b\,e-2\,c\,d)\,(d+e\,x)^{3/2}}{b^2}}{(b\,e-2\,c\,d)\,(d+e\,x)+c\,(d+e\,x)^2+c\,d^2-b\,d\,e}$$
$$-\frac{\sqrt{d}\,\mathrm{atanh}\left(\frac{6\,c\,\sqrt{d}\,e^7\,\sqrt{d+e\,x}}{6\,c\,d\,e^7-\frac{14\,c^2\,d^2\,e^6}{b}+\frac{8\,c^3\,d^3\,e^5}{b^2}}-\frac{14\,c^2\,d^{3/2}\,e^6\,\sqrt{d+e\,x}}{6\,b\,c\,d\,e^7-14\,c^2\,d^2\,e^6+\frac{8\,c^3\,d^3\,e^5}{b}}+\frac{8\,c^3\,d^{5/2}\,e^5\,\sqrt{d+e\,x}}{6\,b^2\,c\,d\,e^7-14\,b\,c^2\,d^2\,e^6+8\,c^3\,d^3\,e^5}\right)\,(3\,b\,e-4\,c\,d)}{b^3}$$
$$-\frac{\mathrm{atanh}\left(\frac{2\,c\,d\,e^6\,\sqrt{c^2\,d-b\,c\,e}\,\sqrt{d+e\,x}}{2\,b\,c\,d\,e^7-10\,c^2\,d^2\,e^6+\frac{8\,c^3\,d^3\,e^5}{b}}-\frac{8\,c^2\,d^2\,e^5\,\sqrt{c^2\,d-b\,c\,e}\,\sqrt{d+e\,x}}{2\,b^2\,c\,d\,e^7-10\,b\,c^2\,d^2\,e^6+8\,c^3\,d^3\,e^5}\right)\,\sqrt{-c\,(b\,e-c\,d)}\,(b\,e-4\,c\,d)}{b^3\,c}$$

input
```
int((d + e*x)^(3/2)/(b*x + c*x^2)^2,x)
```

output
```
- ((2*(b*d*e^2 - c*d^2*e)*(d + e*x)^(1/2))/b^2 - (e*(b*e - 2*c*d)*(d + e*x
)^(3/2))/b^2)/((b*e - 2*c*d)*(d + e*x) + c*(d + e*x)^2 + c*d^2 - b*d*e) -
(d^(1/2)*atanh((6*c*d^(1/2)*e^7*(d + e*x)^(1/2))/(6*c*d*e^7 - (14*c^2*d^2*
e^6)/b + (8*c^3*d^3*e^5)/b^2) - (14*c^2*d^(3/2)*e^6*(d + e*x)^(1/2))/(6*b*
c*d*e^7 - 14*c^2*d^2*e^6 + (8*c^3*d^3*e^5)/b) + (8*c^3*d^(5/2)*e^5*(d + e*
x)^(1/2))/(8*c^3*d^3*e^5 - 14*b*c^2*d^2*e^6 + 6*b^2*c*d*e^7))*(3*b*e - 4*c
*d))/b^3 - (atanh((2*c*d*e^6*(c^2*d - b*c*e)^(1/2)*(d + e*x)^(1/2))/(2*b*c
*d*e^7 - 10*c^2*d^2*e^6 + (8*c^3*d^3*e^5)/b) - (8*c^2*d^2*e^5*(c^2*d - b*c
*e)^(1/2)*(d + e*x)^(1/2))/(8*c^3*d^3*e^5 - 10*b*c^2*d^2*e^6 + 2*b^2*c*d*e
^7))*(-c*(b*e - c*d))^(1/2)*(b*e - 4*c*d))/(b^3*c)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 404, normalized size of antiderivative = 2.71

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^2}\,dx = \frac{2\sqrt{c}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{be-cd}}\right)b^2ex - 8\sqrt{c}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{be-cd}}\right)bcdx + 2\sqrt{c}\,\sqrt{be}$$

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x)^2,x)
```

output
```
(2*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)
))*b**2*e*x - 8*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sq
rt(b*e - c*d)))*b*c*d*x + 2*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)
/(sqrt(c)*sqrt(b*e - c*d)))*b*c*e*x**2 - 8*sqrt(c)*sqrt(b*e - c*d)*atan((s
qrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*c**2*d*x**2 - 2*sqrt(d + e*x)*b
**2*c*d + 2*sqrt(d + e*x)*b**2*c*e*x - 4*sqrt(d + e*x)*b*c**2*d*x + 3*sqrt
(d)*log(sqrt(d + e*x) - sqrt(d))*b**2*c*e*x - 4*sqrt(d)*log(sqrt(d + e*x)
- sqrt(d))*b*c**2*d*x + 3*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*b*c**2*e*x*
*2 - 4*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*c**3*d*x**2 - 3*sqrt(d)*log(sq
rt(d + e*x) + sqrt(d))*b**2*c*e*x + 4*sqrt(d)*log(sqrt(d + e*x) + sqrt(d))
*b*c**2*d*x - 3*sqrt(d)*log(sqrt(d + e*x) + sqrt(d))*b*c**2*e*x**2 + 4*sqr
t(d)*log(sqrt(d + e*x) + sqrt(d))*c**3*d*x**2)/(2*b**3*c*x*(b + c*x))
```

3.117 $\int \frac{\sqrt{d+ex}}{(bx+cx^2)^2}\,dx$

Optimal result 887
Mathematica [A] (verified) 887
Rubi [A] (verified) 888
Maple [A] (verified) 890
Fricas [A] (verification not implemented) 891
Sympy [F] 891
Maxima [F(-2)] 892
Giac [A] (verification not implemented) 892
Mupad [B] (verification not implemented) 893
Reduce [B] (verification not implemented) 893

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^2}\,dx = -\frac{2c\sqrt{d+ex}}{b^2(b+cx)} - \frac{\sqrt{d+ex}}{bx(b+cx)} + \frac{(4cd-be)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3\sqrt{d}} - \frac{\sqrt{c}(4cd-3be)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3\sqrt{cd-be}}$$

output
```
-2*c*(e*x+d)^(1/2)/b^2/(c*x+b)-(e*x+d)^(1/2)/b/x/(c*x+b)+(-b*e+4*c*d)*arct
anh((e*x+d)^(1/2)/d^(1/2))/b^3/d^(1/2)-c^(1/2)*(-3*b*e+4*c*d)*arctanh(c^(1
/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b^3/(-b*e+c*d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^2}\,dx = \frac{-\frac{b(b+2cx)\sqrt{d+ex}}{x(b+cx)} + \frac{\sqrt{c}(4cd-3be)\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{-cd+be}}\right)}{\sqrt{-cd+be}} + \frac{(4cd-be)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{\sqrt{d}}}{b^3}$$

input
```
Integrate[Sqrt[d + e*x]/(b*x + c*x^2)^2,x]
```

output
```
(-((b*(b + 2*c*x)*Sqrt[d + e*x])/(x*(b + c*x))) + (Sqrt[c]*(4*c*d - 3*b*e)
*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[-(c*d) + b*e]])/Sqrt[-(c*d) + b*e] +
((4*c*d - b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d])/b^3
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1163, 27, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{ 1163}$$

$$\frac{\int -\frac{4cd-be+2cex}{2\sqrt{d+ex}(cx^2+bx)}dx}{b^2}-\frac{(b+2cx)\sqrt{d+ex}}{b^2(bx+cx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{4cd-be+2cex}{\sqrt{d+ex}(cx^2+bx)}dx}{2b^2}-\frac{(b+2cx)\sqrt{d+ex}}{b^2(bx+cx^2)}$$

$$\Big\downarrow \text{ 1197}$$

$$-\frac{\int \frac{e(2cd-be+2c(d+ex))}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2}-\frac{(b+2cx)\sqrt{d+ex}}{b^2(bx+cx^2)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{e\int \frac{2cd-be+2c(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2}-\frac{(b+2cx)\sqrt{d+ex}}{b^2(bx+cx^2)}$$

$$\Big\downarrow \text{ 1480}$$

$$-\frac{e\left(\frac{c(4cd-be)\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{c(4cd-3be)\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{b^2}-\frac{(b+2cx)\sqrt{d+ex}}{b^2(bx+cx^2)}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{e\left(\frac{\sqrt{c}(4cd-3be)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{be\sqrt{cd-be}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(4cd-be)}{b\sqrt{d}e}\right)}{b^2}-\frac{(b+2cx)\sqrt{d+ex}}{b^2\left(bx+cx^2\right)}$$

input
```
Int[Sqrt[d + e*x]/(b*x + c*x^2)^2,x]
```

output
```
-(((b + 2*c*x)*Sqrt[d + e*x])/(b^2*(b*x + c*x^2))) - (e*(-(((4*c*d - b*e)*
ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*Sqrt[d]*e)) + (Sqrt[c]*(4*c*d - 3*b*e)*
ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*e*Sqrt[c*d - b*e])))/
b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1163
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] - Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1
)*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p +
1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 0] && (LtQ[
m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c, d,
e, m, p, x]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{-4\left(cd-\frac{3be}{4}\right)x(cx+b)c\sqrt{d}\,\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)+\sqrt{c(be-cd)}\,\left(x(cx+b)(be-4cd)\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)+\sqrt{d}\,\sqrt{ex+d}\,b(2}{\sqrt{d}\,\sqrt{c(be-cd)}\,b^3x(cx+b)}$
derivativedivides	$2e^3\left(\frac{-\frac{b\sqrt{ex+d}}{2x}-\frac{(be-4cd)\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{2\sqrt{d}}}{b^3e^3}-\frac{c\left(\frac{be\sqrt{ex+d}}{2(ex+d)c+2be-2cd}+\frac{(3be-4cd)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{2\sqrt{c(be-cd)}}\right)}{b^3e^3}\right)$
default	$2e^3\left(\frac{-\frac{b\sqrt{ex+d}}{2x}-\frac{(be-4cd)\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{2\sqrt{d}}}{b^3e^3}-\frac{c\left(\frac{be\sqrt{ex+d}}{2(ex+d)c+2be-2cd}+\frac{(3be-4cd)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{2\sqrt{c(be-cd)}}\right)}{b^3e^3}\right)$
risch	$-\frac{\sqrt{ex+d}}{b^2x}-\frac{e\left(-\frac{(-be+4cd)\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be\sqrt{d}}+\frac{2c\left(\frac{be\sqrt{ex+d}}{2(ex+d)c+2be-2cd}+\frac{(3be-4cd)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{2\sqrt{c(be-cd)}}\right)}{be}\right)}{b^2}$

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d^(1/2)/(c*(b*e-c*d))^(1/2)*(-4*(c*d-3/4*b*e)*x*(c*x+b)*c*d^(1/2)*arcta
n(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2))+(c*(b*e-c*d))^(1/2)*(x*(c*x+b)*(b*e
-4*c*d)*arctanh((e*x+d)^(1/2)/d^(1/2))+d^(1/2)*(e*x+d)^(1/2)*b*(2*c*x+b)))
/b^3/x/(c*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 756, normalized size of antiderivative = 5.40

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
[-1/2*(((4*c^2*d^2 - 3*b*c*d*e)*x^2 + (4*b*c*d^2 - 3*b^2*d*e)*x)*sqrt(c/(c
*d - b*e))*log((c*e*x + 2*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(
c*d - b*e)))/(c*x + b)) + ((4*c^2*d - b*c*e)*x^2 + (4*b*c*d - b^2*e)*x)*sq
rt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(2*b*c*d*x + b^2*d)
*sqrt(e*x + d))/(b^3*c*d*x^2 + b^4*d*x), 1/2*(2*((4*c^2*d^2 - 3*b*c*d*e)*x
^2 + (4*b*c*d^2 - 3*b^2*d*e)*x)*sqrt(-c/(c*d - b*e))*arctan(sqrt(e*x + d)*
sqrt(-c/(c*d - b*e))) - ((4*c^2*d - b*c*e)*x^2 + (4*b*c*d - b^2*e)*x)*sqrt
(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(2*b*c*d*x + b^2*d)*s
qrt(e*x + d))/(b^3*c*d*x^2 + b^4*d*x), -1/2*(2*((4*c^2*d - b*c*e)*x^2 + (4
*b*c*d - b^2*e)*x)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x + d)) + ((4*c^2*d^2 -
 3*b*c*d*e)*x^2 + (4*b*c*d^2 - 3*b^2*d*e)*x)*sqrt(c/(c*d - b*e))*log((c*e*
x + 2*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x +
b)) + 2*(2*b*c*d*x + b^2*d)*sqrt(e*x + d))/(b^3*c*d*x^2 + b^4*d*x), (((4*c
^2*d^2 - 3*b*c*d*e)*x^2 + (4*b*c*d^2 - 3*b^2*d*e)*x)*sqrt(-c/(c*d - b*e))*
arctan(sqrt(e*x + d)*sqrt(-c/(c*d - b*e))) - ((4*c^2*d - b*c*e)*x^2 + (4*b
*c*d - b^2*e)*x)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x + d)) - (2*b*c*d*x + b^
2*d)*sqrt(e*x + d))/(b^3*c*d*x^2 + b^4*d*x)]
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^2}\,dx = \int \frac{\sqrt{d+ex}}{x^2\,(b+cx)^2}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(c*x**2+b*x)**2,x)
```

output
```
Integral(sqrt(d + e*x)/(x**2*(b + c*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.19

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^2}\,dx = \frac{(4\,c^2d-3\,bce)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{\sqrt{-c^2d+bce}b^3} - \frac{(4\,cd-be)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b^3\sqrt{-d}} - \frac{2\,(ex+d)^{\frac{3}{2}}ce-2\,\sqrt{ex+d}cde+\sqrt{ex+d}be^2}{\left((ex+d)^2c-2\,(ex+d)cd+cd^2+(ex+d)be-bde\right)b^2}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
(4*c^2*d - 3*b*c*e)*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c*e))/(sqrt(-c^
2*d + b*c*e)*b^3) - (4*c*d - b*e)*arctan(sqrt(e*x + d)/sqrt(-d))/(b^3*sqrt
(-d)) - (2*(e*x + d)^(3/2)*c*e - 2*sqrt(e*x + d)*c*d*e + sqrt(e*x + d)*b*e
^2)/(((e*x + d)^2*c - 2*(e*x + d)*c*d + c*d^2 + (e*x + d)*b*e - b*d*e)*b^2
)
```

Mupad [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1174, normalized size of antiderivative = 8.39

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(1/2)/(b*x + c*x^2)^2,x)
```

output
```
(atanh((2*c^2*e^6*(d + e*x)^(1/2))/(d^(3/2)*((8*c^3*e^5)/b - (2*c^2*e^6)/d
)) - (8*c^3*e^5*(d + e*x)^(1/2))/(d^(1/2)*(8*c^3*e^5 - (2*b*c^2*e^6)/d))*
(b*e - 4*c*d))/(b^3*d^(1/2)) - ((2*c*e*(d + e*x)^(3/2))/b^2 + (e*(b*e - 2*
c*d)*(d + e*x)^(1/2))/b^2)/((b*e - 2*c*d)*(d + e*x) + c*(d + e*x)^2 + c*d^
2 - b*d*e) + (atan(((((4*(d + e*x)^(1/2)*(5*b^2*c^3*e^4 + 16*c^5*d^2*e^2 -
 16*b*c^4*d*e^3))/b^4 - ((-c*(b*e - c*d))^(1/2)*(3*b*e - 4*c*d)*((2*(2*b^7
*c^2*e^4 - 4*b^6*c^3*d*e^3))/b^6 - (2*(2*b^7*c^2*e^3 - 4*b^6*c^3*d*e^2)*(-
c*(b*e - c*d))^(1/2)*(3*b*e - 4*c*d)*(d + e*x)^(1/2))/(b^4*(b^4*e - b^3*c*
d))))/(2*(b^4*e - b^3*c*d)))*(-c*(b*e - c*d))^(1/2)*(3*b*e - 4*c*d)*1i)/(2
*(b^4*e - b^3*c*d)) + (((4*(d + e*x)^(1/2)*(5*b^2*c^3*e^4 + 16*c^5*d^2*e^2
 - 16*b*c^4*d*e^3))/b^4 + ((-c*(b*e - c*d))^(1/2)*(3*b*e - 4*c*d)*((2*(2*b
^7*c^2*e^4 - 4*b^6*c^3*d*e^3))/b^6 + (2*(2*b^7*c^2*e^3 - 4*b^6*c^3*d*e^2)*
(-c*(b*e - c*d))^(1/2)*(3*b*e - 4*c*d)*(d + e*x)^(1/2))/(b^4*(b^4*e - b^3*
c*d))))/(2*(b^4*e - b^3*c*d)))*(-c*(b*e - c*d))^(1/2)*(3*b*e - 4*c*d)*1i)/
(2*(b^4*e - b^3*c*d)))/((4*(3*b^2*c^3*e^5 + 16*c^5*d^2*e^3 - 16*b*c^4*d*e^
4))/b^6 - (((4*(d + e*x)^(1/2)*(5*b^2*c^3*e^4 + 16*c^5*d^2*e^2 - 16*b*c^4*
d*e^3))/b^4 - ((-c*(b*e - c*d))^(1/2)*(3*b*e - 4*c*d)*((2*(2*b^7*c^2*e^4 -
 4*b^6*c^3*d*e^3))/b^6 - (2*(2*b^7*c^2*e^3 - 4*b^6*c^3*d*e^2)*(-c*(b*e - c
*d))^(1/2)*(3*b*e - 4*c*d)*(d + e*x)^(1/2))/(b^4*(b^4*e - b^3*c*d))))/(2*(
b^4*e - b^3*c*d)))*(-c*(b*e - c*d))^(1/2)*(3*b*e - 4*c*d))/(2*(b^4*e - ...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 550, normalized size of antiderivative = 3.93

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^2}\,dx$$

$$= \frac{-6\sqrt{c}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{be-cd}}\right)b^2dex + 8\sqrt{c}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{ex+d}\,c}{\sqrt{c}\,\sqrt{be-cd}}\right)bc\,d^2x - 6\sqrt{c}\,\sqrt{be-cd}\,atan\Big(}{}$$

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x)^2,x)
```

output
```
( - 6*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c
*d)))*b**2*d*e*x + 8*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(
c)*sqrt(b*e - c*d)))*b*c*d**2*x - 6*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d +
 e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c*d*e*x**2 + 8*sqrt(c)*sqrt(b*e - c*
d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*c**2*d**2*x**2 - 2*sq
rt(d + e*x)*b**3*d*e + 2*sqrt(d + e*x)*b**2*c*d**2 - 4*sqrt(d + e*x)*b**2*
c*d*e*x + 4*sqrt(d + e*x)*b*c**2*d**2*x + sqrt(d)*log(sqrt(d + e*x) - sqrt
(d))*b**3*e**2*x - 5*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*b**2*c*d*e*x + s
qrt(d)*log(sqrt(d + e*x) - sqrt(d))*b**2*c*e**2*x**2 + 4*sqrt(d)*log(sqrt(
d + e*x) - sqrt(d))*b*c**2*d**2*x - 5*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))
*b*c**2*d*e*x**2 + 4*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*c**3*d**2*x**2 -
 sqrt(d)*log(sqrt(d + e*x) + sqrt(d))*b**3*e**2*x + 5*sqrt(d)*log(sqrt(d +
 e*x) + sqrt(d))*b**2*c*d*e*x - sqrt(d)*log(sqrt(d + e*x) + sqrt(d))*b**2*
c*e**2*x**2 - 4*sqrt(d)*log(sqrt(d + e*x) + sqrt(d))*b*c**2*d**2*x + 5*sqr
t(d)*log(sqrt(d + e*x) + sqrt(d))*b*c**2*d*e*x**2 - 4*sqrt(d)*log(sqrt(d +
 e*x) + sqrt(d))*c**3*d**2*x**2)/(2*b**3*d*x*(b**2*e - b*c*d + b*c*e*x - c
**2*d*x))
```

3.118 $\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^2}\,dx$

Optimal result 895
Mathematica [A] (verified) 896
Rubi [A] (verified) 896
Maple [A] (verified) 898
Fricas [A] (verification not implemented) 899
Sympy [F] 900
Maxima [F(-2)] 901
Giac [A] (verification not implemented) 901
Mupad [B] (verification not implemented) 902
Reduce [B] (verification not implemented) 902

Optimal result

Integrand size = 21, antiderivative size = 164

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^2}\,dx = -\frac{c(2cd-be)\sqrt{d+ex}}{b^2d(cd-be)(b+cx)} - \frac{\sqrt{d+ex}}{bdx(b+cx)} + \frac{(4cd+be)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3d^{3/2}} - \frac{c^{3/2}(4cd-5be)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3(cd-be)^{3/2}}$$

output
```
-c*(-b*e+2*c*d)*(e*x+d)^(1/2)/b^2/d/(-b*e+c*d)/(c*x+b)-(e*x+d)^(1/2)/b/d/x
/(c*x+b)+(b*e+4*c*d)*arctanh((e*x+d)^(1/2)/d^(1/2))/b^3/d^(3/2)-c^(3/2)*(-
5*b*e+4*c*d)*arctanh(c^(1/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b^3/(-b*e+c*d
)^(3/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^2}\,dx = \frac{-\frac{b\sqrt{d+ex}(-bcd+b^2e-2c^2dx+bcex)}{d(-cd+be)x(b+cx)} - \frac{c^{3/2}(4cd-5be)\arctan\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{-cd+be}}\right)}{(-cd+be)^{3/2}} + \frac{(4cd+be)\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{d^{3/2}}}{b^3}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(b*x + c*x^2)^2),x]
```

output
```
(-((b*Sqrt[d + e*x]*(-(b*c*d) + b^2*e - 2*c^2*d*x + b*c*e*x))/(d*(-(c*d) +
 b*e)*x*(b + c*x))) - (c^(3/2)*(4*c*d - 5*b*e)*ArcTan[(Sqrt[c]*Sqrt[d + e*
x])/Sqrt[-(c*d) + b*e]])/(-(c*d) + b*e)^(3/2) + ((4*c*d + b*e)*ArcTanh[Sqr
t[d + e*x]/Sqrt[d]])/d^(3/2))/b^3
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1165, 27, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^2\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{1165}$$

$$-\frac{\int \frac{(cd-be)(4cd+be)+ce(2cd-be)x}{2\sqrt{d+ex}(cx^2+bx)}dx}{b^2d(cd-be)} - \frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d\,(bx+cx^2)\,(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(cd-be)(4cd+be)+ce(2cd-be)x}{\sqrt{d+ex}(cx^2+bx)}dx}{2b^2d(cd-be)} - \frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d\,(bx+cx^2)\,(cd-be)}$$

$$\Big\downarrow \text{1197}$$

$$-\frac{\int \frac{e(2c^2d^2-2bced-b^2e^2+c(2cd-be)(d+ex))}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2d(cd-be)}-\frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d(bx+cx^2)(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e\int \frac{2c^2d^2-2bced-b^2e^2+c(2cd-be)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2d(cd-be)}-\frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d(bx+cx^2)(cd-be)}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{e\left(\frac{c(cd-be)(be+4cd)\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{c^2d(4cd-5be)\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{b^2d(cd-be)}-\frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d(bx+cx^2)(cd-be)}$$

$$\Big\downarrow \text{221}$$

$$-\frac{e\left(\frac{c^{3/2}d(4cd-5be)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{be\sqrt{cd-be}}-\frac{\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(cd-be)(be+4cd)}{b\sqrt{de}}\right)}{b^2d(cd-be)}-\frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d(bx+cx^2)(cd-be)}$$

input
```
Int[1/(Sqrt[d + e*x]*(b*x + c*x^2)^2),x]
```

output
```
-((Sqrt[d + e*x]*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*(
b*x + c*x^2))) - (e*(-(((c*d - b*e)*(4*c*d + b*e)*ArcTanh[Sqrt[d + e*x]/Sq
rt[d]])/(b*Sqrt[d]*e)) + (c^(3/2)*d*(4*c*d - 5*b*e)*ArcTanh[(Sqrt[c]*Sqrt[
d + e*x])/Sqrt[c*d - b*e]])/(b*e*Sqrt[c*d - b*e])))/(b^2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.98

method	result
derivativedivides	$2 e^{3} \left(\frac{c^{2} \left(\frac{b e \sqrt{e x +d}}{2 \left(b e -c d \right) \left(\left(e x +d \right) c +b e -c d \right)}+\frac{\left(5 b e -4 c d \right) \arctan \left(\frac{c \sqrt{e x +d}}{\sqrt{c \left(b e -c d \right)}}\right)}{2 \left(b e -c d \right) \sqrt{c \left(b e -c d \right)}}\right)}{b^{3} e^{3}}+\frac{-\frac{b \sqrt{e x +d}}{2 d x}+\frac{\left(b e +4 c d \right) \operatorname{arctanh}\left(\frac{\sqrt{e x +d}}{\sqrt{d}}\right)}{2 d^{\frac{3}{2}}}}{b^{3} e^{3}}\right)$
default	$2 e^{3} \left(\frac{c^{2} \left(\frac{b e \sqrt{e x +d}}{2 \left(b e -c d \right) \left(\left(e x +d \right) c +b e -c d \right)}+\frac{\left(5 b e -4 c d \right) \arctan \left(\frac{c \sqrt{e x +d}}{\sqrt{c \left(b e -c d \right)}}\right)}{2 \left(b e -c d \right) \sqrt{c \left(b e -c d \right)}}\right)}{b^{3} e^{3}}+\frac{-\frac{b \sqrt{e x +d}}{2 d x}+\frac{\left(b e +4 c d \right) \operatorname{arctanh}\left(\frac{\sqrt{e x +d}}{\sqrt{d}}\right)}{2 d^{\frac{3}{2}}}}{b^{3} e^{3}}\right)$
risch	$-\frac{\sqrt{e x +d}}{d b^{2} x}-\frac{e \left(-\frac{\left(b e +4 c d \right) \operatorname{arctanh}\left(\frac{\sqrt{e x +d}}{\sqrt{d}}\right)}{b e \sqrt{d}}-\frac{2 d c^{2} \left(\frac{b e \sqrt{e x +d}}{2 \left(b e -c d \right) \left(\left(e x +d \right) c +b e -c d \right)}+\frac{\left(5 b e -4 c d \right) \arctan \left(\frac{c \sqrt{e x +d}}{\sqrt{c \left(b e -c d \right)}}\right)}{2 \left(b e -c d \right) \sqrt{c \left(b e -c d \right)}}\right)}{b e}\right)}{b^{2} d}$
pseudoelliptic	$-\frac{4 \left(\left(c d -\frac{5 b e}{4}\right) x \left(c x +b \right) c^{2} d^{\frac{5}{2}} \arctan \left(\frac{c \sqrt{e x +d}}{\sqrt{c \left(b e -c d \right)}}\right)-\frac{\sqrt{c \left(b e -c d \right)}\, \left(d x \left(b e +4 c d \right) \left(b e -c d \right) \left(c x +b \right) \operatorname{arctanh}\left(\frac{\sqrt{e x +d}}{\sqrt{d}}\right)+\sqrt{e x +d}\, b\, d \cdots\right.}{4}\right.}{\sqrt{c \left(b e -c d \right)}\, d^{\frac{5}{2}} x b^{3} \left(b e -c d \right) \left(c x +b \right)}$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
2*e^3*(c^2/b^3/e^3*(1/2*b*e/(b*e-c*d)*(e*x+d)^(1/2)/((e*x+d)*c+b*e-c*d)+1/
2*(5*b*e-4*c*d)/(b*e-c*d)/(c*(b*e-c*d))^(1/2)*arctan(c*(e*x+d)^(1/2)/(c*(b
*e-c*d))^(1/2)))+1/b^3/e^3*(-1/2*b/d*(e*x+d)^(1/2)/x+1/2*(b*e+4*c*d)/d^(3/
2)*arctanh((e*x+d)^(1/2)/d^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 1120, normalized size of antiderivative = 6.83

$$\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
[1/2*(((4*c^3*d^3 - 5*b*c^2*d^2*e)*x^2 + (4*b*c^2*d^3 - 5*b^2*c*d^2*e)*x)*
sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e - 2*(c*d - b*e)*sqrt(e*x + d)
*sqrt(c/(c*d - b*e)))/(c*x + b)) + ((4*c^3*d^2 - 3*b*c^2*d*e - b^2*c*e^2)*
x^2 + (4*b*c^2*d^2 - 3*b^2*c*d*e - b^3*e^2)*x)*sqrt(d)*log((e*x + 2*sqrt(e
*x + d)*sqrt(d) + 2*d)/x) - 2*(b^2*c*d^2 - b^3*d*e + (2*b*c^2*d^2 - b^2*c*
d*e)*x)*sqrt(e*x + d))/((b^3*c^2*d^3 - b^4*c*d^2*e)*x^2 + (b^4*c*d^3 - b^5
*d^2*e)*x), 1/2*(2*((4*c^3*d^3 - 5*b*c^2*d^2*e)*x^2 + (4*b*c^2*d^3 - 5*b^2
*c*d^2*e)*x)*sqrt(-c/(c*d - b*e))*arctan(sqrt(e*x + d)*sqrt(-c/(c*d - b*e)
)) + ((4*c^3*d^2 - 3*b*c^2*d*e - b^2*c*e^2)*x^2 + (4*b*c^2*d^2 - 3*b^2*c*d
*e - b^3*e^2)*x)*sqrt(d)*log((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*
(b^2*c*d^2 - b^3*d*e + (2*b*c^2*d^2 - b^2*c*d*e)*x)*sqrt(e*x + d))/((b^3*c
^2*d^3 - b^4*c*d^2*e)*x^2 + (b^4*c*d^3 - b^5*d^2*e)*x), -1/2*(2*((4*c^3*d^
2 - 3*b*c^2*d*e - b^2*c*e^2)*x^2 + (4*b*c^2*d^2 - 3*b^2*c*d*e - b^3*e^2)*x
)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e*x + d)) - ((4*c^3*d^3 - 5*b*c^2*d^2*e)*x
^2 + (4*b*c^2*d^3 - 5*b^2*c*d^2*e)*x)*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c
*d - b*e - 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)) + 2
*(b^2*c*d^2 - b^3*d*e + (2*b*c^2*d^2 - b^2*c*d*e)*x)*sqrt(e*x + d))/((b^3*
c^2*d^3 - b^4*c*d^2*e)*x^2 + (b^4*c*d^3 - b^5*d^2*e)*x), (((4*c^3*d^3 - 5*
b*c^2*d^2*e)*x^2 + (4*b*c^2*d^3 - 5*b^2*c*d^2*e)*x)*sqrt(-c/(c*d - b*e))*a
rctan(sqrt(e*x + d)*sqrt(-c/(c*d - b*e))) - ((4*c^3*d^2 - 3*b*c^2*d*e -...
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^2}\,dx = \int \frac{1}{x^2\,(b+cx)^2\,\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x)**2,x)
```

output
```
Integral(1/(x**2*(b + c*x)**2*sqrt(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.44

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^2}\,dx = \frac{(4\,c^3d-5\,bc^2e)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{(b^3cd-b^4e)\sqrt{-c^2d+bce}} - \frac{2\,(ex+d)^{\frac{3}{2}}c^2de-2\,\sqrt{ex+d}c^2d^2e-(ex+d)^{\frac{3}{2}}bce^2+2\,\sqrt{ex+d}bcde^2-\sqrt{ex+d}b^2e^3}{(b^2cd^2-b^3de)\left((ex+d)^2c-2\,(ex+d)cd+cd^2+(ex+d)be-bde\right)} - \frac{(4\,cd+be)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b^3\sqrt{-d}d}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
(4*c^3*d - 5*b*c^2*e)*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c*e))/((b^3*c
*d - b^4*e)*sqrt(-c^2*d + b*c*e)) - (2*(e*x + d)^(3/2)*c^2*d*e - 2*sqrt(e*
x + d)*c^2*d^2*e - (e*x + d)^(3/2)*b*c*e^2 + 2*sqrt(e*x + d)*b*c*d*e^2 - s
qrt(e*x + d)*b^2*e^3)/((b^2*c*d^2 - b^3*d*e)*((e*x + d)^2*c - 2*(e*x + d)*
c*d + c*d^2 + (e*x + d)*b*e - b*d*e)) - (4*c*d + b*e)*arctan(sqrt(e*x + d)
/sqrt(-d))/(b^3*sqrt(-d)*d)
```

Mupad [B] (verification not implemented)

Time = 5.90 (sec) , antiderivative size = 3784, normalized size of antiderivative = 23.07

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((b*x + c*x^2)^2*(d + e*x)^(1/2)),x)
```

output
```
(((d + e*x)^(1/2)*(b^2*e^3 + 2*c^2*d^2*e - 2*b*c*d*e^2))/(b^2*(c*d^2 - b*d
*e)) + (c*(b*e^2 - 2*c*d*e)*(d + e*x)^(3/2))/(b^2*(c*d^2 - b*d*e)))/((b*e
- 2*c*d)*(d + e*x) + c*(d + e*x)^2 + c*d^2 - b*d*e) + (atan((((-c^3*(b*e -
 c*d)^3)^(1/2)*((2*(d + e*x)^(1/2)*(b^4*c^3*e^6 + 32*c^7*d^4*e^2 - 64*b*c^
6*d^3*e^3 + 6*b^3*c^4*d*e^5 + 26*b^2*c^5*d^2*e^4))/(b^4*c^2*d^4 + b^6*d^2*
e^2 - 2*b^5*c*d^3*e) - ((-c^3*(b*e - c*d)^3)^(1/2)*(5*b*e - 4*c*d)*((4*b^9
*c^2*d*e^6 + 8*b^6*c^5*d^4*e^3 - 16*b^7*c^4*d^3*e^4 + 4*b^8*c^3*d^2*e^5)/(
b^6*c^2*d^4 + b^8*d^2*e^2 - 2*b^7*c*d^3*e) + ((-c^3*(b*e - c*d)^3)^(1/2)*(
5*b*e - 4*c*d)*(d + e*x)^(1/2)*(8*b^6*c^5*d^5*e^2 - 20*b^7*c^4*d^4*e^3 + 1
6*b^8*c^3*d^3*e^4 - 4*b^9*c^2*d^2*e^5))/((b^4*c^2*d^4 + b^6*d^2*e^2 - 2*b^
5*c*d^3*e)*(b^6*e^3 - b^3*c^3*d^3 + 3*b^4*c^2*d^2*e - 3*b^5*c*d*e^2))))/(2
*(b^6*e^3 - b^3*c^3*d^3 + 3*b^4*c^2*d^2*e - 3*b^5*c*d*e^2)))*(5*b*e - 4*c*
d)*1i)/(2*(b^6*e^3 - b^3*c^3*d^3 + 3*b^4*c^2*d^2*e - 3*b^5*c*d*e^2)) + ((-
c^3*(b*e - c*d)^3)^(1/2)*((2*(d + e*x)^(1/2)*(b^4*c^3*e^6 + 32*c^7*d^4*e^2
 - 64*b*c^6*d^3*e^3 + 6*b^3*c^4*d*e^5 + 26*b^2*c^5*d^2*e^4))/(b^4*c^2*d^4
+ b^6*d^2*e^2 - 2*b^5*c*d^3*e) + ((-c^3*(b*e - c*d)^3)^(1/2)*(5*b*e - 4*c*
d)*((4*b^9*c^2*d*e^6 + 8*b^6*c^5*d^4*e^3 - 16*b^7*c^4*d^3*e^4 + 4*b^8*c^3*
d^2*e^5)/(b^6*c^2*d^4 + b^8*d^2*e^2 - 2*b^7*c*d^3*e) - ((-c^3*(b*e - c*d)^
3)^(1/2)*(5*b*e - 4*c*d)*(d + e*x)^(1/2)*(8*b^6*c^5*d^5*e^2 - 20*b^7*c^4*d
^4*e^3 + 16*b^8*c^3*d^3*e^4 - 4*b^9*c^2*d^2*e^5))/((b^4*c^2*d^4 + b^6*d...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 743, normalized size of antiderivative = 4.53

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x)^2,x)
```

output

```
(10*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d
)))*b**2*c*d**2*e*x - 8*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sq
rt(c)*sqrt(b*e - c*d)))*b*c**2*d**3*x + 10*sqrt(c)*sqrt(b*e - c*d)*atan((s
qrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**2*d**2*e*x**2 - 8*sqrt(c)*
sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*c**3*d**
3*x**2 - 2*sqrt(d + e*x)*b**4*d*e**2 + 4*sqrt(d + e*x)*b**3*c*d**2*e - 2*s
qrt(d + e*x)*b**3*c*d*e**2*x - 2*sqrt(d + e*x)*b**2*c**2*d**3 + 6*sqrt(d +
 e*x)*b**2*c**2*d**2*e*x - 4*sqrt(d + e*x)*b*c**3*d**3*x - sqrt(d)*log(sqr
t(d + e*x) - sqrt(d))*b**4*e**3*x - 2*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))
*b**3*c*d*e**2*x - sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*b**3*c*e**3*x**2 +
 7*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*b**2*c**2*d**2*e*x - 2*sqrt(d)*log
(sqrt(d + e*x) - sqrt(d))*b**2*c**2*d*e**2*x**2 - 4*sqrt(d)*log(sqrt(d + e
*x) - sqrt(d))*b*c**3*d**3*x + 7*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*b*c*
*3*d**2*e*x**2 - 4*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*c**4*d**3*x**2 + s
qrt(d)*log(sqrt(d + e*x) + sqrt(d))*b**4*e**3*x + 2*sqrt(d)*log(sqrt(d + e
*x) + sqrt(d))*b**3*c*d*e**2*x + sqrt(d)*log(sqrt(d + e*x) + sqrt(d))*b**3
*c*e**3*x**2 - 7*sqrt(d)*log(sqrt(d + e*x) + sqrt(d))*b**2*c**2*d**2*e*x +
 2*sqrt(d)*log(sqrt(d + e*x) + sqrt(d))*b**2*c**2*d*e**2*x**2 + 4*sqrt(d)*
log(sqrt(d + e*x) + sqrt(d))*b*c**3*d**3*x - 7*sqrt(d)*log(sqrt(d + e*x) +
 sqrt(d))*b*c**3*d**2*e*x**2 + 4*sqrt(d)*log(sqrt(d + e*x) + sqrt(d))*c...
```

3.119 $\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)^2}\,dx$

Optimal result 904
Mathematica [A] (verified) 905
Rubi [A] (verified) 905
Maple [A] (verified) 908
Fricas [B] (verification not implemented) 909
Sympy [F] 910
Maxima [F(-2)] 910
Giac [A] (verification not implemented) 910
Mupad [B] (verification not implemented) 911
Reduce [B] (verification not implemented) 912

Optimal result

Integrand size = 21, antiderivative size = 216

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^2}\,dx = -\frac{e(2c^2d^2-2bcde+3b^2e^2)}{b^2d^2(cd-be)^2\sqrt{d+ex}} - \frac{c(2cd-be)}{b^2d(cd-be)(b+cx)\sqrt{d+ex}} - \frac{1}{bdx(b+cx)\sqrt{d+ex}} + \frac{(4cd+3be)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3d^{5/2}} - \frac{c^{5/2}(4cd-7be)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3(cd-be)^{5/2}}$$

output
```
-e*(3*b^2*e^2-2*b*c*d*e+2*c^2*d^2)/b^2/d^2/(-b*e+c*d)^2/(e*x+d)^(1/2)-c*(-
b*e+2*c*d)/b^2/d/(-b*e+c*d)/(c*x+b)/(e*x+d)^(1/2)-1/b/d/x/(c*x+b)/(e*x+d)^
(1/2)+(3*b*e+4*c*d)*arctanh((e*x+d)^(1/2)/d^(1/2))/b^3/d^(5/2)-c^(5/2)*(-7
*b*e+4*c*d)*arctanh(c^(1/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b^3/(-b*e+c*d)
^(5/2)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.94

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^2}\,dx=\frac{-\frac{b\left(2c^3d^2x(d+ex)+b^3e^2(d+3ex)+bc^2d\left(d^2-dex-2e^2x^2\right)+b^2ce\left(-2d^2-dex+3e^2x^2\right)\right)}{d^2(cd-be)^2x(b+cx)\sqrt{d+ex}}+\frac{c^{5/2}(4cd-7\cdots}{(\cdots}}{b^3}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(b*x + c*x^2)^2),x]
```

output
```
(-((b*(2*c^3*d^2*x*(d + e*x) + b^3*e^2*(d + 3*e*x) + b*c^2*d*(d^2 - d*e*x
- 2*e^2*x^2) + b^2*c*e*(-2*d^2 - d*e*x + 3*e^2*x^2)))/(d^2*(c*d - b*e)^2*x
*(b + c*x)*Sqrt[d + e*x])) + (c^(5/2)*(4*c*d - 7*b*e)*ArcTan[(Sqrt[c]*Sqrt
[d + e*x])/Sqrt[-(c*d) + b*e]])/(-(c*d) + b*e)^(5/2) + ((4*c*d + 3*b*e)*Ar
cTanh[Sqrt[d + e*x]/Sqrt[d]])/d^(5/2))/b^3
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {1165, 27, 1198, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(bx+cx^2\right)^2(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1165}$$

$$-\frac{\int \frac{(cd-be)(4cd+3be)+3ce(2cd-be)x}{2(d+ex)^{3/2}\left(cx^2+bx\right)}dx}{b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\left(bx+cx^2\right)\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(cd-be)(4cd+3be)+3ce(2cd-be)x}{(d+ex)^{3/2}\left(cx^2+bx\right)}dx}{2b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\left(bx+cx^2\right)\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{1198}$$

$$-\frac{\frac{\int \frac{(4cd+3be)(cd-be)^2+ce\left(2c^2d^2-2bced+3b^2e^2\right)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{d(cd-be)}+\frac{2e\left(3b^2e^2-2bcde+2c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{2b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\left(bx+cx^2\right)\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{ 1197}$$

$$-\frac{\frac{2\int \frac{e\left((2cd-be)\left(c^2d^2-bced-3b^2e^2\right)+c\left(2c^2d^2-2bced+3b^2e^2\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{d(cd-be)}+\frac{2e\left(3b^2e^2-2bcde+2c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{2b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\left(bx+cx^2\right)\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{2e\int \frac{(2cd-be)\left(c^2d^2-bced-3b^2e^2\right)+c\left(2c^2d^2-2bced+3b^2e^2\right)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{d(cd-be)}+\frac{2e\left(3b^2e^2-2bcde+2c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{2b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\left(bx+cx^2\right)\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{ 1480}$$

$$-\frac{\frac{2e\left(\frac{c(cd-be)^2(3be+4cd)\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{c^3d^2(4cd-7be)\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{d(cd-be)}+\frac{2e\left(3b^2e^2-2bcde+2c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{2b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\left(bx+cx^2\right)\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{2e\left(\frac{c^{5/2}d^2(4cd-7be)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{be\sqrt{cd-be}}-\frac{\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(cd-be)^2(3be+4cd)}{b\sqrt{d}e}\right)}{d(cd-be)}+\frac{2e\left(3b^2e^2-2bcde+2c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{2b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\left(bx+cx^2\right)\sqrt{d+ex}(cd-be)}$$

input
```
Int[1/((d + e*x)^(3/2)*(b*x + c*x^2)^2),x]
```

output
```
-((b*(c*d - b*e) + c*(2*c*d - b*e)*x)/(b^2*d*(c*d - b*e)*Sqrt[d + e*x]*(b*
x + c*x^2))) - ((2*e*(2*c^2*d^2 - 2*b*c*d*e + 3*b^2*e^2))/(d*(c*d - b*e)*S
qrt[d + e*x]) + (2*e*(-(((c*d - b*e)^2*(4*c*d + 3*b*e)*ArcTanh[Sqrt[d + e*
x]/Sqrt[d]])/(b*Sqrt[d]*e)) + (c^(5/2)*d^2*(4*c*d - 7*b*e)*ArcTanh[(Sqrt[c
]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*e*Sqrt[c*d - b*e])))/(d*(c*d - b*e))
)/(2*b^2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.81

method	result
derivativedivides	$2e^{3}\left(\frac{-\frac{b\sqrt{ex+d}}{2x}+\frac{(3be+4cd)\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{2\sqrt{d}}}{b^{3}d^{2}e^{3}}-\frac{1}{d^{2}(be-cd)^{2}\sqrt{ex+d}}-\frac{c^{3}\left(\frac{be\sqrt{ex+d}}{2(ex+d)c+2be-2cd}+\frac{(7be-4cd)\arctan\left(\cdots\right.}{2\sqrt{c(be-cd)}\cdots}\right.}{b^{3}e^{3}(be-cd)^{2}}\cdots\right.$
default	$2e^{3}\left(\frac{-\frac{b\sqrt{ex+d}}{2x}+\frac{(3be+4cd)\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{2\sqrt{d}}}{b^{3}d^{2}e^{3}}-\frac{1}{d^{2}(be-cd)^{2}\sqrt{ex+d}}-\frac{c^{3}\left(\frac{be\sqrt{ex+d}}{2(ex+d)c+2be-2cd}+\frac{(7be-4cd)\arctan\left(\cdots\right.}{2\sqrt{c(be-cd)}\cdots}\right.}{b^{3}e^{3}(be-cd)^{2}}\cdots\right.$
risch	$-\frac{\sqrt{ex+d}}{d^{2}b^{2}x}-\frac{e\left(-\frac{(3be+4cd)\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be\sqrt{d}}+\frac{2b^{2}e^{2}}{(be-cd)^{2}\sqrt{ex+d}}+\frac{2c^{3}d^{2}\left(\frac{be\sqrt{ex+d}}{2(ex+d)c+2be-2cd}+\frac{(7be-4cd)\arctan\left(\frac{c\sqrt{ex+\cdots}}{\sqrt{c(be-\cdots}}\right)}{2\sqrt{c(be-cd)}}\cdots\right.}{(be-cd)^{2}be}\cdots\right.}{b^{2}d^{2}}\cdots$
pseudoelliptic	$e^{3}\left(\frac{-\frac{b\sqrt{ex+d}}{x}+\frac{(3be+4cd)\,\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{\sqrt{d}}}{b^{3}d^{2}e^{3}}-\frac{2}{d^{2}(be-cd)^{2}\sqrt{ex+d}}-\frac{\sqrt{ex+d}\,c^{3}}{b^{2}e^{3}(cx+b)(be-cd)^{2}}-\frac{7\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-c\cdots}}\right.}{\sqrt{c(be-cd)}\,b^{2}e^{2}(be\cdots}\cdots\right.$

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
2*e^3*(1/b^3/d^2/e^3*(-1/2*b*(e*x+d)^(1/2)/x+1/2*(3*b*e+4*c*d)/d^(1/2)*arc
tanh((e*x+d)^(1/2)/d^(1/2)))-1/d^2/(b*e-c*d)^2/(e*x+d)^(1/2)-c^3/b^3/e^3/(
b*e-c*d)^2*(1/2*b*e*(e*x+d)^(1/2)/((e*x+d)*c+b*e-c*d)+1/2*(7*b*e-4*c*d)/(c
*(b*e-c*d))^(1/2)*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(194) = 388$.

Time = 0.43 (sec) , antiderivative size = 2268, normalized size of antiderivative = 10.50

$$\int \frac{1}{(d+ex)^{3/2}\,(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
[-1/2*(((4*c^4*d^4*e - 7*b*c^3*d^3*e^2)*x^3 + (4*c^4*d^5 - 3*b*c^3*d^4*e -
 7*b^2*c^2*d^3*e^2)*x^2 + (4*b*c^3*d^5 - 7*b^2*c^2*d^4*e)*x)*sqrt(c/(c*d -
 b*e))*log((c*e*x + 2*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d
- b*e)))/(c*x + b)) - ((4*c^4*d^3*e - 5*b*c^3*d^2*e^2 - 2*b^2*c^2*d*e^3 +
3*b^3*c*e^4)*x^3 + (4*c^4*d^4 - b*c^3*d^3*e - 7*b^2*c^2*d^2*e^2 + b^3*c*d*
e^3 + 3*b^4*e^4)*x^2 + (4*b*c^3*d^4 - 5*b^2*c^2*d^3*e - 2*b^3*c*d^2*e^2 +
3*b^4*d*e^3)*x)*sqrt(d)*log((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(
b^2*c^2*d^4 - 2*b^3*c*d^3*e + b^4*d^2*e^2 + (2*b*c^3*d^3*e - 2*b^2*c^2*d^2
*e^2 + 3*b^3*c*d*e^3)*x^2 + (2*b*c^3*d^4 - b^2*c^2*d^3*e - b^3*c*d^2*e^2 +
 3*b^4*d*e^3)*x)*sqrt(e*x + d))/((b^3*c^3*d^5*e - 2*b^4*c^2*d^4*e^2 + b^5*
c*d^3*e^3)*x^3 + (b^3*c^3*d^6 - b^4*c^2*d^5*e - b^5*c*d^4*e^2 + b^6*d^3*e^
3)*x^2 + (b^4*c^2*d^6 - 2*b^5*c*d^5*e + b^6*d^4*e^2)*x), 1/2*(2*((4*c^4*d^
4*e - 7*b*c^3*d^3*e^2)*x^3 + (4*c^4*d^5 - 3*b*c^3*d^4*e - 7*b^2*c^2*d^3*e^
2)*x^2 + (4*b*c^3*d^5 - 7*b^2*c^2*d^4*e)*x)*sqrt(-c/(c*d - b*e))*arctan(sq
rt(e*x + d)*sqrt(-c/(c*d - b*e))) + ((4*c^4*d^3*e - 5*b*c^3*d^2*e^2 - 2*b^
2*c^2*d*e^3 + 3*b^3*c*e^4)*x^3 + (4*c^4*d^4 - b*c^3*d^3*e - 7*b^2*c^2*d^2*
e^2 + b^3*c*d*e^3 + 3*b^4*e^4)*x^2 + (4*b*c^3*d^4 - 5*b^2*c^2*d^3*e - 2*b^
3*c*d^2*e^2 + 3*b^4*d*e^3)*x)*sqrt(d)*log((e*x + 2*sqrt(e*x + d)*sqrt(d) +
 2*d)/x) - 2*(b^2*c^2*d^4 - 2*b^3*c*d^3*e + b^4*d^2*e^2 + (2*b*c^3*d^3*e -
 2*b^2*c^2*d^2*e^2 + 3*b^3*c*d*e^3)*x^2 + (2*b*c^3*d^4 - b^2*c^2*d^3*e ...
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^2}\,dx = \int \frac{1}{x^2\left(b+cx\right)^2\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x)**2,x)
```

output
```
Integral(1/(x**2*(b + c*x)**2*(d + e*x)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.56

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^2}\,dx = \frac{\left(4\,c^4d-7\,bc^3e\right)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{\left(b^3c^2d^2-2\,b^4cde+b^5e^2\right)\sqrt{-c^2d+bce}} - \frac{2\left(ex+d\right)^2c^3d^2e-2\left(ex+d\right)c^3d^3e-2\left(ex+d\right)^2bc^2de^2+3\left(ex+d\right)bc^2d^2e^2+3\left(ex+d\right)^2b^2ce^3-7\left(ex+\right.}{\left(b^2c^2d^4-2\,b^3cd^3e+b^4d^2e^2\right)\left(\left(ex+d\right)^{\frac{5}{2}}c-2\left(ex+d\right)^{\frac{3}{2}}cd+\sqrt{ex+d}cd^2+\left(ex+\right.\right.} - \frac{\left(4\,cd+3\,be\right)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b^3\sqrt{-d}d^2}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
(4*c^4*d - 7*b*c^3*e)*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c*e))/((b^3*c
^2*d^2 - 2*b^4*c*d*e + b^5*e^2)*sqrt(-c^2*d + b*c*e)) - (2*(e*x + d)^2*c^3
*d^2*e - 2*(e*x + d)*c^3*d^3*e - 2*(e*x + d)^2*b*c^2*d*e^2 + 3*(e*x + d)*b
*c^2*d^2*e^2 + 3*(e*x + d)^2*b^2*c*e^3 - 7*(e*x + d)*b^2*c*d*e^3 + 2*b^2*c
*d^2*e^3 + 3*(e*x + d)*b^3*e^4 - 2*b^3*d*e^4)/((b^2*c^2*d^4 - 2*b^3*c*d^3*
e + b^4*d^2*e^2)*((e*x + d)^(5/2)*c - 2*(e*x + d)^(3/2)*c*d + sqrt(e*x + d
)*c*d^2 + (e*x + d)^(3/2)*b*e - sqrt(e*x + d)*b*d*e)) - (4*c*d + 3*b*e)*ar
ctan(sqrt(e*x + d)/sqrt(-d))/(b^3*sqrt(-d)*d^2)
```

Mupad [B] (verification not implemented)

Time = 6.57 (sec) , antiderivative size = 4234, normalized size of antiderivative = 19.60

$$\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((b*x + c*x^2)^2*(d + e*x)^(3/2)),x)
```

output
```
(atan((((-c^5*(b*e - c*d)^5)^(1/2)*(7*b*e - 4*c*d)*((d + e*x)^(1/2)*(64*b^
6*c^15*d^18*e^2 - 576*b^7*c^14*d^17*e^3 + 2228*b^8*c^13*d^16*e^4 - 4768*b^
9*c^12*d^15*e^5 + 5960*b^10*c^11*d^14*e^6 - 3976*b^11*c^10*d^13*e^7 + 578*
b^12*c^9*d^12*e^8 + 1004*b^13*c^8*d^11*e^9 - 442*b^14*c^7*d^10*e^10 - 320*
b^15*c^6*d^9*e^11 + 362*b^16*c^5*d^8*e^12 - 132*b^17*c^4*d^7*e^13 + 18*b^1
8*c^3*d^6*e^14) + ((-c^5*(b*e - c*d)^5)^(1/2)*(7*b*e - 4*c*d)*(8*b^10*c^13
*d^19*e^3 - 76*b^11*c^12*d^18*e^4 + 300*b^12*c^11*d^17*e^5 - 612*b^13*c^10
*d^16*e^6 + 576*b^14*c^9*d^15*e^7 + 168*b^15*c^8*d^14*e^8 - 1176*b^16*c^7*
d^13*e^9 + 1560*b^17*c^6*d^12*e^10 - 1128*b^18*c^5*d^11*e^11 + 484*b^19*c^
4*d^10*e^12 - 116*b^20*c^3*d^9*e^13 + 12*b^21*c^2*d^8*e^14 - ((-c^5*(b*e -
 c*d)^5)^(1/2)*(7*b*e - 4*c*d)*(d + e*x)^(1/2)*(16*b^12*c^13*d^21*e^2 - 16
8*b^13*c^12*d^20*e^3 + 800*b^14*c^11*d^19*e^4 - 2280*b^15*c^10*d^18*e^5 +
4320*b^16*c^9*d^17*e^6 - 5712*b^17*c^8*d^16*e^7 + 5376*b^18*c^7*d^15*e^8 -
 3600*b^19*c^6*d^14*e^9 + 1680*b^20*c^5*d^13*e^10 - 520*b^21*c^4*d^12*e^11
 + 96*b^22*c^3*d^11*e^12 - 8*b^23*c^2*d^10*e^13))/(2*(b^8*e^5 - b^3*c^5*d^
5 + 5*b^4*c^4*d^4*e - 10*b^5*c^3*d^3*e^2 + 10*b^6*c^2*d^2*e^3 - 5*b^7*c*d*
e^4))))/(2*(b^8*e^5 - b^3*c^5*d^5 + 5*b^4*c^4*d^4*e - 10*b^5*c^3*d^3*e^2 +
 10*b^6*c^2*d^2*e^3 - 5*b^7*c*d*e^4)))*1i)/(2*(b^8*e^5 - b^3*c^5*d^5 + 5*b
^4*c^4*d^4*e - 10*b^5*c^3*d^3*e^2 + 10*b^6*c^2*d^2*e^3 - 5*b^7*c*d*e^4)) +
 ((-c^5*(b*e - c*d)^5)^(1/2)*(7*b*e - 4*c*d)*((d + e*x)^(1/2)*(64*b^6*c...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1098, normalized size of antiderivative = 5.08

$$\int \frac{1}{(d+ex)^{3/2} (bx+cx^2)^2} \, dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+b*x)^2,x)
```

output
```
( - 14*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(
c)*sqrt(b*e - c*d)))*b**2*c**2*d**3*e*x + 8*sqrt(c)*sqrt(d + e*x)*sqrt(b*e
 - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**3*d**4*x -
14*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*s
qrt(b*e - c*d)))*b*c**3*d**3*e*x**2 + 8*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c
*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*c**4*d**4*x**2 - 3*s
qrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b**5*e**4*x + 5*sqrt(d)*
sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b**4*c*d*e**3*x - 3*sqrt(d)*sqr
t(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b**4*c*e**4*x**2 + 3*sqrt(d)*sqrt(
d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b**3*c**2*d**2*e**2*x + 5*sqrt(d)*sq
rt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b**3*c**2*d*e**3*x**2 - 9*sqrt(d)
*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b**2*c**3*d**3*e*x + 3*sqrt(d)
*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b**2*c**3*d**2*e**2*x**2 + 4*s
qrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b*c**4*d**4*x - 9*sqrt(d
)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b*c**4*d**3*e*x**2 + 4*sqrt(d
)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*c**5*d**4*x**2 + 3*sqrt(d)*sq
rt(d + e*x)*log(sqrt(d + e*x) + sqrt(d))*b**5*e**4*x - 5*sqrt(d)*sqrt(d +
e*x)*log(sqrt(d + e*x) + sqrt(d))*b**4*c*d*e**3*x + 3*sqrt(d)*sqrt(d + e*x
)*log(sqrt(d + e*x) + sqrt(d))*b**4*c*e**4*x**2 - 3*sqrt(d)*sqrt(d + e*x)*
log(sqrt(d + e*x) + sqrt(d))*b**3*c**2*d**2*e**2*x - 5*sqrt(d)*sqrt(d +...
```

3.120 $\int \frac{1}{(d+ex)^{5/2}(bx+cx^2)^2}\,dx$

Optimal result 914
Mathematica [A] (verified) 915
Rubi [A] (verified) 915
Maple [A] (verified) 918
Fricas [B] (verification not implemented) 919
Sympy [F] 920
Maxima [F(-2)] 920
Giac [A] (verification not implemented) 921
Mupad [B] (verification not implemented) 921
Reduce [B] (verification not implemented) 922

Optimal result

Integrand size = 21, antiderivative size = 277

$$\int \frac{1}{(d+ex)^{5/2}\,(bx+cx^2)^2}\,dx = -\frac{e(6c^2d^2-6bcde+5b^2e^2)}{3b^2d^2(cd-be)^2(d+ex)^{3/2}} - \frac{c(2cd-be)}{b^2d(cd-be)(b+cx)(d+ex)^{3/2}} - \frac{1}{bdx(b+cx)(d+ex)^{3/2}} - \frac{e(2cd-be)\,(c^2d^2-bcde+5b^2e^2)}{b^2d^3(cd-be)^3\sqrt{d+ex}} + \frac{(4cd+5be)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{b^3d^{7/2}} - \frac{c^{7/2}(4cd-9be)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b^3(cd-be)^{7/2}}$$

output
```
-1/3*e*(5*b^2*e^2-6*b*c*d*e+6*c^2*d^2)/b^2/d^2/(-b*e+c*d)^2/(e*x+d)^(3/2)-
c*(-b*e+2*c*d)/b^2/d/(-b*e+c*d)/(c*x+b)/(e*x+d)^(3/2)-1/b/d/x/(c*x+b)/(e*x
+d)^(3/2)-e*(-b*e+2*c*d)*(5*b^2*e^2-b*c*d*e+c^2*d^2)/b^2/d^3/(-b*e+c*d)^3/
(e*x+d)^(1/2)+(5*b*e+4*c*d)*arctanh((e*x+d)^(1/2)/d^(1/2))/b^3/d^(7/2)-c^(
7/2)*(-9*b*e+4*c*d)*arctanh(c^(1/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b^3/(-
b*e+c*d)^(7/2)
```

Mathematica [A] (verified)

Time = 1.38 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.98

$$\int \frac{1}{(d+ex)^{5/2}\,(bx+cx^2)^2}\,dx = \frac{\frac{b\left(-6c^4d^3x(d+ex)^2-3bc^3d^2(d-3ex)(d+ex)^2+b^4e^3\left(3d^2+20dex+15e^2x^2\right)+b^2c^2de\left(9d^3+9d^2ex-35de^2\right.}{d^3(cd-be)^3x(b+cx)(d+ex)^{3/2}}}{}$$

input
```
Integrate[1/((d + e*x)^(5/2)*(b*x + c*x^2)^2),x]
```

output
```
((b*(-6*c^4*d^3*x*(d + e*x)^2 - 3*b*c^3*d^2*(d - 3*e*x)*(d + e*x)^2 + b^4*
e^3*(3*d^2 + 20*d*e*x + 15*e^2*x^2) + b^2*c^2*d*e*(9*d^3 + 9*d^2*e*x - 35*
d*e^2*x^2 - 33*e^3*x^3) + b^3*c*e^2*(-9*d^3 - 41*d^2*e*x - 13*d*e^2*x^2 +
15*e^3*x^3)))/(d^3*(c*d - b*e)^3*x*(b + c*x)*(d + e*x)^(3/2)) - (3*c^(7/2)
*(4*c*d - 9*b*e)*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[-(c*d) + b*e]])/(-(c*
d) + b*e)^(7/2) + (3*(4*c*d + 5*b*e)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/d^(7/
2))/(3*b^3)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1165, 27, 1198, 1198, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^2\,(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{1165}$$

$$-\frac{\int \frac{(cd-be)(4cd+5be)+5ce(2cd-be)x}{2(d+ex)^{5/2}(cx^2+bx)}dx}{b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\,(bx+cx^2)\,(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(cd-be)(4cd+5be)+5ce(2cd-be)x}{(d+ex)^{5/2}(cx^2+bx)}dx}{2b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\,(bx+cx^2)\,(d+ex)^{3/2}(cd-be)}$$

$\Big\downarrow$ 1198

$$-\frac{\frac{\int \frac{(4cd+5be)(cd-be)^2+ce\left(6c^2d^2-6bced+5b^2e^2\right)x}{(d+ex)^{3/2}\left(cx^2+bx\right)}dx}{d(cd-be)}+\frac{2e\left(5b^2e^2-6bcde+6c^2d^2\right)}{3d(d+ex)^{3/2}(cd-be)}}{2b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\left(bx+cx^2\right)(d+ex)^{3/2}(cd-be)}$$

$\Big\downarrow$ 1198

$$-\frac{\frac{\frac{\int \frac{(4cd+5be)(cd-be)^3+ce(2cd-be)\left(c^2d^2-bced+5b^2e^2\right)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{d(cd-be)}+\frac{2e(2cd-be)\left(5b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{d(cd-be)}+\frac{2e\left(5b^2e^2-6bcde+6c^2d^2\right)}{3d(d+ex)^{3/2}(cd-be)}}{2b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\left(bx+cx^2\right)(d+ex)^{3/2}(cd-be)}$$

$\Big\downarrow$ 1197

$$-\frac{\frac{\frac{2\int \frac{e\left(2c^4d^4-4bc^3ed^3-14b^2c^2e^2d^2+16b^3ce^3d-5b^4e^4+c(2cd-be)\left(c^2d^2-bced+5b^2e^2\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{d(cd-be)}+\frac{2e(2cd-be)\left(5b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{d(cd-be)}+\frac{2e\left(5b^2e^2-6bcde+6c^2d^2\right)}{3d(d+ex)^{3/2}(cd-be)}}{2b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\left(bx+cx^2\right)(d+ex)^{3/2}(cd-be)}$$

$\Big\downarrow$ 27

$$-\frac{\frac{\frac{2e\int \frac{2c^4d^4-4bc^3ed^3-14b^2c^2e^2d^2+16b^3ce^3d-5b^4e^4+c(2cd-be)\left(c^2d^2-bced+5b^2e^2\right)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{d(cd-be)}+\frac{2e(2cd-be)\left(5b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{d(cd-be)}+\frac{2e\left(5b^2e^2-6bcde+6c^2d^2\right)}{3d(d+ex)^{3/2}(cd-be)}}{2b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\left(bx+cx^2\right)(d+ex)^{3/2}(cd-be)}$$

$\Big\downarrow$ 1480

$$-\frac{\frac{\frac{2e\left(\frac{c(cd-be)^3(5be+4cd)\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{c^4d^3(4cd-9be)\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{d(cd-be)}+\frac{2e(2cd-be)\left(5b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{d(cd-be)}+\frac{2e\left(5b^2e^2-6bcde+6c^2d^2\right)}{3d(d+ex)^{3/2}(cd-be)}}{2b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{b^2d\left(bx+cx^2\right)(d+ex)^{3/2}(cd-be)}$$

$\Big\downarrow$ 221

$$-\frac{\frac{\frac{2e\left(\frac{c^{7/2}d^3(4cd-9be)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{be\sqrt{cd-be}}-\frac{\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(cd-be)^3(5be+4cd)}{b\sqrt{de}}\right)}{d(cd-be)}+\frac{2e(2cd-be)\left(5b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{d(cd-be)}+\frac{2e\left(5b^2e^2-6bcde+6c^2\right.}{3d(d+ex)^{3/2}(cd-b}}{2b^2d(cd-be)}$$
$$\frac{cx(2cd-be)+b(cd-be)}{b^2d\left(bx+cx^2\right)(d+ex)^{3/2}(cd-be)}$$

input
```
Int[1/((d + e*x)^(5/2)*(b*x + c*x^2)^2),x]
```

output
```
-((b*(c*d - b*e) + c*(2*c*d - b*e)*x)/(b^2*d*(c*d - b*e)*(d + e*x)^(3/2)*(
b*x + c*x^2))) - ((2*e*(6*c^2*d^2 - 6*b*c*d*e + 5*b^2*e^2))/(3*d*(c*d - b*
e)*(d + e*x)^(3/2)) + ((2*e*(2*c*d - b*e)*(c^2*d^2 - b*c*d*e + 5*b^2*e^2))
/(d*(c*d - b*e)*Sqrt[d + e*x]) + (2*e*(-(((c*d - b*e)^3*(4*c*d + 5*b*e)*Ar
cTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*Sqrt[d]*e)) + (c^(7/2)*d^3*(4*c*d - 9*b*e
)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*e*Sqrt[c*d - b*e]))
)/(d*(c*d - b*e)))/(d*(c*d - b*e)))/(2*b^2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
 + e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
 + 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.74

method	result
derivativedivides	$2e^3\left(\frac{c^4\left(\frac{be\sqrt{ex+d}}{2(ex+d)c+2be-2cd}+\frac{(9be-4cd)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{2\sqrt{c(be-cd)}}\right)}{b^3e^3(be-cd)^3}-\frac{1}{3d^2(be-cd)^2(ex+d)^{\frac{3}{2}}}-\frac{2be-4cd}{d^3(be-cd)^3\sqrt{ex+d}}+\right.$
default	$2e^3\left(\frac{c^4\left(\frac{be\sqrt{ex+d}}{2(ex+d)c+2be-2cd}+\frac{(9be-4cd)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{2\sqrt{c(be-cd)}}\right)}{b^3e^3(be-cd)^3}-\frac{1}{3d^2(be-cd)^2(ex+d)^{\frac{3}{2}}}-\frac{2be-4cd}{d^3(be-cd)^3\sqrt{ex+d}}+\right.$
risch	$-\frac{\sqrt{ex+d}}{d^3b^2x}-\frac{e\left(-\frac{(5be+4cd)\,\text{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be\sqrt{d}}+\frac{4b^2e^2(be-2cd)}{(be-cd)^3\sqrt{ex+d}}+\frac{2d\,e^2b^2}{3(be-cd)^2(ex+d)^{\frac{3}{2}}}-\frac{2c^4d^3\left(\frac{be\sqrt{ex+d}}{2(ex+d)c+2be-2cd}+\frac{(9be-}{}\right.}{(be-cd)^3}\right.}{b^2d^3}$
pseudoelliptic	$e^3\left(\frac{-\frac{b\sqrt{ex+d}}{x}+\frac{(5be+4cd)\,\text{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{\sqrt{d}}}{b^3d^3e^3}-\frac{2}{3d^2(be-cd)^2(ex+d)^{\frac{3}{2}}}-\frac{4(be-2cd)}{d^3(be-cd)^3\sqrt{ex+d}}+\frac{\sqrt{ex+d}\,c^4}{b^2e^3(cx+b)(be-cd)}\right.$

input
```
int(1/(e*x+d)^(5/2)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
2*e^3*(c^4/b^3/e^3/(b*e-c*d)^3*(1/2*b*e*(e*x+d)^(1/2)/((e*x+d)*c+b*e-c*d)+
1/2*(9*b*e-4*c*d)/(c*(b*e-c*d))^(1/2)*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))
^(1/2)))-1/3/d^2/(b*e-c*d)^2/(e*x+d)^(3/2)-(2*b*e-4*c*d)/d^3/(b*e-c*d)^3/(
e*x+d)^(1/2)+1/b^3/d^3/e^3*(-1/2*b*(e*x+d)^(1/2)/x+1/2*(5*b*e+4*c*d)/d^(1/
2)*arctanh((e*x+d)^(1/2)/d^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 939 vs. 2(251) = 502.

Time = 1.39 (sec) , antiderivative size = 3829, normalized size of antiderivative = 13.82

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^2}\,dx = \int \frac{1}{x^2\left(b+cx\right)^2\left(d+ex\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(5/2)/(c*x**2+b*x)**2,x)
```

output
```
Integral(1/(x**2*(b + c*x)**2*(d + e*x)**(5/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 470, normalized size of antiderivative = 1.70

$$\int \frac{1}{(d+ex)^{5/2}(bx+cx^2)^2}\,dx = \frac{(4\,c^5d-9\,bc^4e)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{(b^3c^3d^3-3\,b^4c^2d^2e+3\,b^5cde^2-b^6e^3)\sqrt{-c^2d+bce}}$$
$$-\frac{2\,(ex+d)^{\frac{3}{2}}c^4d^3e-2\,\sqrt{ex+d}c^4d^4e-3\,(ex+d)^{\frac{3}{2}}bc^3d^2e^2+4\,\sqrt{ex+d}bc^3d^3e^2+3\,(ex+d)^{\frac{3}{2}}b^2c^2de^3-6\,\sqrt{}}{(b^2c^3d^6-3\,b^3c^2d^5e+3\,b^4cd^4e^2-b^5d^3e^3)\left((ex+d)^2c-2\,(ex+d)\right.}$$
$$-\frac{2\,(12\,(ex+d)cde^3+cd^2e^3-6\,(ex+d)be^4-bde^4)}{3\,(c^3d^6-3\,bc^2d^5e+3\,b^2cd^4e^2-b^3d^3e^3)(ex+d)^{\frac{3}{2}}}-\frac{(4\,cd+5\,be)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{b^3\sqrt{-d}d^3}$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
(4*c^5*d - 9*b*c^4*e)*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c*e))/((b^3*c
^3*d^3 - 3*b^4*c^2*d^2*e + 3*b^5*c*d*e^2 - b^6*e^3)*sqrt(-c^2*d + b*c*e))
- (2*(e*x + d)^(3/2)*c^4*d^3*e - 2*sqrt(e*x + d)*c^4*d^4*e - 3*(e*x + d)^(
3/2)*b*c^3*d^2*e^2 + 4*sqrt(e*x + d)*b*c^3*d^3*e^2 + 3*(e*x + d)^(3/2)*b^2
*c^2*d*e^3 - 6*sqrt(e*x + d)*b^2*c^2*d^2*e^3 - (e*x + d)^(3/2)*b^3*c*e^4 +
 4*sqrt(e*x + d)*b^3*c*d*e^4 - sqrt(e*x + d)*b^4*e^5)/((b^2*c^3*d^6 - 3*b^
3*c^2*d^5*e + 3*b^4*c*d^4*e^2 - b^5*d^3*e^3)*((e*x + d)^2*c - 2*(e*x + d)*
c*d + c*d^2 + (e*x + d)*b*e - b*d*e)) - 2/3*(12*(e*x + d)*c*d*e^3 + c*d^2*
e^3 - 6*(e*x + d)*b*e^4 - b*d*e^4)/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4
*e^2 - b^3*d^3*e^3)*(e*x + d)^(3/2)) - (4*c*d + 5*b*e)*arctan(sqrt(e*x + d
)/sqrt(-d))/(b^3*sqrt(-d)*d^3)
```

Mupad [B] (verification not implemented)

Time = 6.89 (sec) , antiderivative size = 5736, normalized size of antiderivative = 20.71

$$\int \frac{1}{(d+ex)^{5/2}(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/((b*x + c*x^2)^2*(d + e*x)^(5/2)),x)
```

output
```
((10*e^3*(b*e - 2*c*d)*(d + e*x))/(3*(c*d^2 - b*d*e)^2) - (2*e^3)/(3*(c*d^
2 - b*d*e)) + (e*(d + e*x)^2*(15*b^4*e^4 + 6*c^4*d^4 + 64*b^2*c^2*d^2*e^2
- 12*b*c^3*d^3*e - 58*b^3*c*d*e^3))/(3*b^2*(c*d^2 - b*d*e)^3) + (e*(b*e -
2*c*d)*(d + e*x)^3*(c^3*d^2 + 5*b^2*c*e^2 - b*c^2*d*e))/(b^2*(c*d^2 - b*d*
e)^3))/(c*(d + e*x)^(7/2) + (c*d^2 - b*d*e)*(d + e*x)^(3/2) + (b*e - 2*c*d
)*(d + e*x)^(5/2)) + (atan((((-c^7*(b*e - c*d)^7)^(1/2)*(9*b*e - 4*c*d)*((
d + e*x)^(1/2)*(64*b^6*c^20*d^26*e^2 - 832*b^7*c^19*d^25*e^3 + 4820*b^8*c^
18*d^24*e^4 - 16240*b^9*c^17*d^23*e^5 + 34490*b^10*c^16*d^22*e^6 - 45430*b
^11*c^15*d^21*e^7 + 29414*b^12*c^14*d^20*e^8 + 10670*b^13*c^13*d^19*e^9 -
39550*b^14*c^12*d^18*e^10 + 25730*b^15*c^11*d^17*e^11 + 19048*b^16*c^10*d^
16*e^12 - 53852*b^17*c^9*d^15*e^13 + 55510*b^18*c^8*d^14*e^14 - 35210*b^19
*c^7*d^13*e^15 + 14830*b^20*c^6*d^12*e^16 - 4082*b^21*c^5*d^11*e^17 + 670*
b^22*c^4*d^10*e^18 - 50*b^23*c^3*d^9*e^19) + ((-c^7*(b*e - c*d)^7)^(1/2)*(
9*b*e - 4*c*d)*(8*b^10*c^18*d^28*e^3 - 112*b^11*c^17*d^27*e^4 + 664*b^12*c
^16*d^26*e^5 - 2080*b^13*c^15*d^25*e^6 + 2996*b^14*c^14*d^24*e^7 + 2528*b^
15*c^13*d^23*e^8 - 23056*b^16*c^12*d^22*e^9 + 59312*b^17*c^11*d^21*e^10 -
95700*b^18*c^10*d^20*e^11 + 109648*b^19*c^9*d^19*e^12 - 92840*b^20*c^8*d^1
8*e^13 + 58688*b^21*c^7*d^17*e^14 - 27476*b^22*c^6*d^16*e^15 + 9280*b^23*c
^5*d^15*e^16 - 2144*b^24*c^4*d^14*e^17 + 304*b^25*c^3*d^13*e^18 - 20*b^26*
c^2*d^12*e^19 - ((-c^7*(b*e - c*d)^7)^(1/2)*(9*b*e - 4*c*d)*(d + e*x)^(...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 2148, normalized size of antiderivative = 7.75

$$\int \frac{1}{(d+ex)^{5/2}\,(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(5/2)/(c*x^2+b*x)^2,x)
```

output

```
(54*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*
sqrt(b*e - c*d)))*b**2*c**3*d**5*e*x + 54*sqrt(c)*sqrt(d + e*x)*sqrt(b*e -
 c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c**3*d**4*e**
2*x**2 - 24*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(
sqrt(c)*sqrt(b*e - c*d)))*b*c**4*d**6*x + 30*sqrt(c)*sqrt(d + e*x)*sqrt(b*
e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**4*d**5*e*x
**2 + 54*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqr
t(c)*sqrt(b*e - c*d)))*b*c**4*d**4*e**2*x**3 - 24*sqrt(c)*sqrt(d + e*x)*sq
rt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*c**5*d**6*
x**2 - 24*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sq
rt(c)*sqrt(b*e - c*d)))*c**5*d**5*e*x**3 - 15*sqrt(d)*sqrt(d + e*x)*log(sq
rt(d + e*x) - sqrt(d))*b**6*d*e**5*x - 15*sqrt(d)*sqrt(d + e*x)*log(sqrt(d
 + e*x) - sqrt(d))*b**6*e**6*x**2 + 48*sqrt(d)*sqrt(d + e*x)*log(sqrt(d +
e*x) - sqrt(d))*b**5*c*d**2*e**4*x + 33*sqrt(d)*sqrt(d + e*x)*log(sqrt(d +
 e*x) - sqrt(d))*b**5*c*d*e**5*x**2 - 15*sqrt(d)*sqrt(d + e*x)*log(sqrt(d
+ e*x) - sqrt(d))*b**5*c*e**6*x**3 - 42*sqrt(d)*sqrt(d + e*x)*log(sqrt(d +
 e*x) - sqrt(d))*b**4*c**2*d**3*e**3*x + 6*sqrt(d)*sqrt(d + e*x)*log(sqrt(
d + e*x) - sqrt(d))*b**4*c**2*d**2*e**4*x**2 + 48*sqrt(d)*sqrt(d + e*x)*lo
g(sqrt(d + e*x) - sqrt(d))*b**4*c**2*d*e**5*x**3 - 12*sqrt(d)*sqrt(d + e*x
)*log(sqrt(d + e*x) - sqrt(d))*b**3*c**3*d**4*e**2*x - 54*sqrt(d)*sqrt(...
```

3.121 $\int \frac{(d+ex)^{11/2}}{(bx+cx^2)^3} dx$

Optimal result 924
Mathematica [A] (verified) 925
Rubi [A] (verified) 925
Maple [A] (verified) 929
Fricas [A] (verification not implemented) 930
Sympy [F(-1)] 931
Maxima [F(-2)] 932
Giac [B] (verification not implemented) 932
Mupad [B] (verification not implemented) 933
Reduce [B] (verification not implemented) 934

Optimal result

Integrand size = 21, antiderivative size = 378

$$\int \frac{(d+ex)^{11/2}}{(bx+cx^2)^3} dx = -\frac{3e(c^2d^2-bcde-b^2e^2)(8c^2d^2-8bcde+5b^2e^2)\sqrt{d+ex}}{4b^4c^3} + \frac{(cd-be)(2cd-be)(12c^2d^2-12bcde-5b^2e^2)(d+ex)^{3/2}}{4b^4c^2(b+cx)} + \frac{(cd-be)(12c^2d^2-19bcde+2b^2e^2)(d+ex)^{5/2}}{4b^3c(b+cx)^2} + \frac{d(8cd-13be)(d+ex)^{7/2}}{4b^2x(b+cx)^2} - \frac{d(d+ex)^{9/2}}{2bx^2(b+cx)^2} - \frac{3d^{7/2}(16c^2d^2-44bcde+33b^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4b^5} + \frac{3(cd-be)^{7/2}(16c^2d^2+12bcde+5b^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{4b^5c^{7/2}}$$

output
```
-3/4*e*(-b^2*e^2-b*c*d*e+c^2*d^2)*(5*b^2*e^2-8*b*c*d*e+8*c^2*d^2)*(e*x+d)^
(1/2)/b^4/c^3+1/4*(-b*e+c*d)*(-b*e+2*c*d)*(-5*b^2*e^2-12*b*c*d*e+12*c^2*d^
2)*(e*x+d)^(3/2)/b^4/c^2/(c*x+b)+1/4*(-b*e+c*d)*(2*b^2*e^2-19*b*c*d*e+12*c
^2*d^2)*(e*x+d)^(5/2)/b^3/c/(c*x+b)^2+1/4*d*(-13*b*e+8*c*d)*(e*x+d)^(7/2)/
b^2/x/(c*x+b)^2-1/2*d*(e*x+d)^(9/2)/b/x^2/(c*x+b)^2-3/4*d^(7/2)*(33*b^2*e^
2-44*b*c*d*e+16*c^2*d^2)*arctanh((e*x+d)^(1/2)/d^(1/2))/b^5+3/4*(-b*e+c*d)
^(7/2)*(5*b^2*e^2+12*b*c*d*e+16*c^2*d^2)*arctanh(c^(1/2)*(e*x+d)^(1/2)/(-b
*e+c*d)^(1/2))/b^5/c^(7/2)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^{11/2}}{(bx+cx^2)^3}\,dx = \frac{\frac{b\sqrt{d+ex}\left(15b^6e^5x^2+24c^6d^5x^3+12bc^5d^4x^2(3d-5ex)+b^5ce^4x^2(-14d+25ex)+2b^4c^2e^3x^2\left(-7d^2-12dex+4e^2x^2\right)+b^2c^4\right.}{c^3x^2(b+cx)^2}}{}$$

input
```
Integrate[(d + e*x)^(11/2)/(b*x + c*x^2)^3,x]
```

output
```
((b*Sqrt[d + e*x]*(15*b^6*e^5*x^2 + 24*c^6*d^5*x^3 + 12*b*c^5*d^4*x^2*(3*d
 - 5*e*x) + b^5*c*e^4*x^2*(-14*d + 25*e*x) + 2*b^4*c^2*e^3*x^2*(-7*d^2 - 1
2*d*e*x + 4*e^2*x^2) + b^2*c^4*d^3*x*(8*d^2 - 91*d*e*x + 36*e^2*x^2) + b^3
*c^3*d^2*(-2*d^3 - 21*d^2*e*x + 56*d*e^2*x^2 + 6*e^3*x^3)))/(c^3*x^2*(b +
c*x)^2) - (3*(-(c*d) + b*e)^(7/2)*(16*c^2*d^2 + 12*b*c*d*e + 5*b^2*e^2)*Ar
cTan[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[-(c*d) + b*e]])/c^(7/2) - 3*d^(7/2)*(16*
c^2*d^2 - 44*b*c*d*e + 33*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*b^5)
```

Rubi [A] (verified)

Time = 1.50 (sec) , antiderivative size = 391, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {1164, 27, 1233, 27, 1196, 1196, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{11/2}}{(bx+cx^2)^3}dx$$

$$\Big\downarrow \text{1164}$$

$$-\frac{\int \frac{3(d+ex)^{7/2}(d(4cd-5be)-e(2cd-be)x)}{2(cx^2+bx)^2}dx}{2b^2}-\frac{(d+ex)^{9/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\int \frac{(d+ex)^{7/2}(d(4cd-5be)-e(2cd-be)x)}{(cx^2+bx)^2}dx}{4b^2}-\frac{(d+ex)^{9/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$\Big\downarrow$ 1233

$$-\frac{3\left(\frac{\int -\frac{(d+ex)^{3/2}\left(cd^2\left(16c^2d^2-44bced+33b^2e^2\right)-e(2cd-be)\left(12c^2d^2-12bced-5b^2e^2\right)x\right)}{2\left(cx^2+bx\right)}dx}{b^2c}-\frac{(d+ex)^{5/2}\left(x(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)+bcd^2(\cdots\right.}{b^2c\left(bx+cx^2\right)}\right.}{4b^2}\cdots$$

$$\frac{(d+ex)^{9/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$\Big\downarrow$ 27

$$-\frac{3\left(-\frac{\int\frac{(d+ex)^{3/2}\left(cd^2\left(16c^2d^2-44bced+33b^2e^2\right)-e(2cd-be)\left(12c^2d^2-12bced-5b^2e^2\right)x\right)}{cx^2+bx}dx}{2b^2c}-\frac{(d+ex)^{5/2}\left(x(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)+bcd^2\cdots\right.}{b^2c\left(bx+cx^2\right)}\right.}{4b^2}\cdots$$

$$\frac{(d+ex)^{9/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$\Big\downarrow$ 1196

$$-\frac{3\left(-\frac{\frac{\int\frac{\sqrt{d+ex}\left(c^2d^3\left(16c^2d^2-44bced+33b^2e^2\right)-e\left(c^2d^2-bced-b^2e^2\right)\left(8c^2d^2-8bced+5b^2e^2\right)x\right)}{cx^2+bx}dx}{c}-\frac{2e(d+ex)^{3/2}(2cd-be)\left(-5b^2e^2-12bcde+12c^2d^2\right)}{3c}}{2b^2c}-\cdots\right.}{4b^2}\cdots$$

$$\frac{(d+ex)^{9/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$\Big\downarrow$ 1196

$$-\frac{3\left(-\frac{\frac{\frac{\int\frac{c^3\left(16c^2d^2-44bced+33b^2e^2\right)d^4+e(2cd-be)\left(4c^4d^4-8bc^3ed^3+2b^2c^2e^2d^2+2b^3ce^3d+5b^4e^4\right)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{c}-\frac{2e\sqrt{d+ex}\left(-b^2e^2-bcde+c^2d^2\right)\left(5b^2e^2-8bcde+8c\cdots\right.}{c}}{c}}{2b^2c}\cdots\right.}{4b^2}\cdots$$

$$\frac{(d+ex)^{9/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$\Big\downarrow$ 1197

$$-\frac{3\left(-\frac{\frac{\frac{2\int \frac{e\left(d(cd-be)\left(c^2d^2-bced-b^2e^2\right)\left(8c^2d^2-8bced+5b^2e^2\right)+(2cd-be)\left(4c^4d^4-8bc^3ed^3+2b^2c^2e^2d^2+2b^3ce^3d+5b^4e^4\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}-\frac{2e\sqrt{d+ex}\left(-\cdots\right.}{\cdots}}{c}}{2b^2c}\cdots\right.}{\cdots}-\frac{(d+ex)^{9/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$\Big\downarrow$ 27

$$-\frac{3\left(-\frac{\frac{\frac{2e\int \frac{d(cd-be)\left(c^2d^2-bced-b^2e^2\right)\left(8c^2d^2-8bced+5b^2e^2\right)+(2cd-be)\left(4c^4d^4-8bc^3ed^3+2b^2c^2e^2d^2+2b^3ce^3d+5b^4e^4\right)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}-\frac{2e\sqrt{d+ex}\left(-b^2e\cdots\right.}{\cdots}}{c}}{2b^2c}\cdots\right.}{\cdots}-\frac{(d+ex)^{9/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$\Big\downarrow$ 1480

$$-\frac{3\left(-\frac{\frac{\frac{2e\left(\frac{c^4d^4\left(33b^2e^2-44bcde+16c^2d^2\right)\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{(cd-be)^4\left(5b^2e^2+12bcde+16c^2d^2\right)\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{c}-\frac{2e\sqrt{d+ex}\left(-b^2e^2-\cdots\right.}{\cdots}}{c}}{2b^2c}\cdots\right.}{\cdots}-\frac{(d+ex)^{9/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$\Big\downarrow$ 221

$$-\frac{3\left(-\frac{\frac{\frac{2e\left(\frac{(cd-be)^{7/2}\left(5b^2e^2+12bcde+16c^2d^2\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{c}e}-\frac{c^3d^{7/2}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(33b^2e^2-44bcde+16c^2d^2\right)}{be}\right)}{c}-\frac{2e\sqrt{d+ex}\left(-b^2e^2-\cdots\right.}{\cdots}}{c}}{2b^2c}\cdots\right.}{\cdots}-\frac{(d+ex)^{9/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

input
```
Int[(d + e*x)^(11/2)/(b*x + c*x^2)^3,x]
```

output
```
-1/2*((d + e*x)^(9/2)*(b*d + (2*c*d - b*e)*x))/(b^2*(b*x + c*x^2)^2) - (3*
(-(((d + e*x)^(5/2)*(b*c*d^2*(4*c*d - 5*b*e) + (2*c*d - b*e)*(4*c^2*d^2 -
4*b*c*d*e - b^2*e^2)*x))/(b^2*c*(b*x + c*x^2))) - ((-2*e*(2*c*d - b*e)*(12
*c^2*d^2 - 12*b*c*d*e - 5*b^2*e^2)*(d + e*x)^(3/2))/(3*c) + ((-2*e*(c^2*d^
2 - b*c*d*e - b^2*e^2)*(8*c^2*d^2 - 8*b*c*d*e + 5*b^2*e^2)*Sqrt[d + e*x])/
c + (2*e*(-((c^3*d^(7/2)*(16*c^2*d^2 - 44*b*c*d*e + 33*b^2*e^2)*ArcTanh[Sq
rt[d + e*x]/Sqrt[d]])/(b*e)) + ((c*d - b*e)^(7/2)*(16*c^2*d^2 + 12*b*c*d*e
 + 5*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*Sqrt[c]
*e)))/c)/c)/(2*b^2*c)))/(4*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.89

method	result
pseudoelliptic	$-\frac{\frac{3x^2(cx+b)^2\sqrt{d}\left(5b^2e^2+12bcde+16c^2d^2\right)(be-cd)^4\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{2}+\left(\frac{3\left(33b^2e^2-44bcde+16c^2d^2\right)x^2(cx+b)^2c^3d^4\,\mathrm{arctanl}}{2}\right.}{}$
risch	$-\frac{d^4\sqrt{ex+d}\,(21bex-12cdx+2bd)}{4b^4x^2}+\frac{e\left(\frac{8b^4e^4\sqrt{ex+d}}{c^3}-\frac{3d^{\frac{7}{2}}\left(33b^2e^2-44bcde+16c^2d^2\right)\mathrm{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be}-\frac{8\left(\frac{\left(-\frac{9}{8}b^6ce^6+\right.}{}\right.}{}\right.}{}$
derivativedivides	$2e^5\left(\frac{\sqrt{ex+d}}{c^3}-\frac{\frac{\left(-\frac{9}{8}b^6ce^6+3b^5de^5c^2-\frac{3}{4}b^4c^3d^2e^4-\frac{9}{2}b^3c^4d^3e^3+\frac{39}{8}b^2d^4e^2c^5-\frac{3}{2}bd^5ec^6\right)(ex+d)^{\frac{3}{2}}-\frac{be\left(7b^6e^6-23b^5de^5c-\right.}{}}{((ex+d)c+be-cd)^2}}{}\right.$
default	$2e^5\left(\frac{\sqrt{ex+d}}{c^3}-\frac{\frac{\left(-\frac{9}{8}b^6ce^6+3b^5de^5c^2-\frac{3}{4}b^4c^3d^2e^4-\frac{9}{2}b^3c^4d^3e^3+\frac{39}{8}b^2d^4e^2c^5-\frac{3}{2}bd^5ec^6\right)(ex+d)^{\frac{3}{2}}-\frac{be\left(7b^6e^6-23b^5de^5c-\right.}{}}{((ex+d)c+be-cd)^2}}{}\right.$

input
```
int((e*x+d)^(11/2)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d^(1/2)/(c*(b*e-c*d))^(1/2)*(3/2*x^2*(c*x+b)^2*d^(1/2)*(5*b^2*e^2+12*
b*c*d*e+16*c^2*d^2)*(b*e-c*d)^4*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2)
)+(3/2*(33*b^2*e^2-44*b*c*d*e+16*c^2*d^2)*x^2*(c*x+b)^2*c^3*d^4*arctanh((e
*x+d)^(1/2)/d^(1/2))+(-12*c^6*d^5*x^3-18*x^2*(-5/3*e*x+d)*b*d^4*c^5-4*x*b^
2*d^3*(9/2*e^2*x^2-91/8*d*e*x+d^2)*c^4+d^2*b^3*(-28*d*e^2*x^2+21/2*d^2*e*x
+d^3-3*e^3*x^3)*c^3+7*e^3*x^2*(-2/7*e*x+d)*(2*e*x+d)*b^4*c^2+7*e^4*(-25/14
*e*x+d)*x^2*b^5*c-15/2*b^6*x^2*e^5)*d^(1/2)*(e*x+d)^(1/2)*b)*(c*(b*e-c*d))
^(1/2))/c^3/b^5/x^2/(c*x+b)^2
```

Fricas [A] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 2702, normalized size of antiderivative = 7.15

$$\int \frac{(d+ex)^{11/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(11/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
[-1/8*(3*((16*c^7*d^5 - 36*b*c^6*d^4*e + 17*b^2*c^5*d^3*e^2 + 5*b^3*c^4*d^
2*e^3 + 3*b^4*c^3*d*e^4 - 5*b^5*c^2*e^5)*x^4 + 2*(16*b*c^6*d^5 - 36*b^2*c^
5*d^4*e + 17*b^3*c^4*d^3*e^2 + 5*b^4*c^3*d^2*e^3 + 3*b^5*c^2*d*e^4 - 5*b^6
*c*e^5)*x^3 + (16*b^2*c^5*d^5 - 36*b^3*c^4*d^4*e + 17*b^4*c^3*d^3*e^2 + 5*
b^5*c^2*d^2*e^3 + 3*b^6*c*d*e^4 - 5*b^7*e^5)*x^2)*sqrt((c*d - b*e)/c)*log(
(c*e*x + 2*c*d - b*e - 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b)) -
 3*((16*c^7*d^5 - 44*b*c^6*d^4*e + 33*b^2*c^5*d^3*e^2)*x^4 + 2*(16*b*c^6*d
^5 - 44*b^2*c^5*d^4*e + 33*b^3*c^4*d^3*e^2)*x^3 + (16*b^2*c^5*d^5 - 44*b^3
*c^4*d^4*e + 33*b^4*c^3*d^3*e^2)*x^2)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*s
qrt(d) + 2*d)/x) - 2*(8*b^5*c^2*e^5*x^4 - 2*b^4*c^3*d^5 + (24*b*c^6*d^5 -
60*b^2*c^5*d^4*e + 36*b^3*c^4*d^3*e^2 + 6*b^4*c^3*d^2*e^3 - 24*b^5*c^2*d*e
^4 + 25*b^6*c*e^5)*x^3 + (36*b^2*c^5*d^5 - 91*b^3*c^4*d^4*e + 56*b^4*c^3*d
^3*e^2 - 14*b^5*c^2*d^2*e^3 - 14*b^6*c*d*e^4 + 15*b^7*e^5)*x^2 + (8*b^3*c^
4*d^5 - 21*b^4*c^3*d^4*e)*x)*sqrt(e*x + d))/(b^5*c^5*x^4 + 2*b^6*c^4*x^3 +
 b^7*c^3*x^2), 1/8*(6*((16*c^7*d^5 - 36*b*c^6*d^4*e + 17*b^2*c^5*d^3*e^2 +
 5*b^3*c^4*d^2*e^3 + 3*b^4*c^3*d*e^4 - 5*b^5*c^2*e^5)*x^4 + 2*(16*b*c^6*d^
5 - 36*b^2*c^5*d^4*e + 17*b^3*c^4*d^3*e^2 + 5*b^4*c^3*d^2*e^3 + 3*b^5*c^2*
d*e^4 - 5*b^6*c*e^5)*x^3 + (16*b^2*c^5*d^5 - 36*b^3*c^4*d^4*e + 17*b^4*c^3
*d^3*e^2 + 5*b^5*c^2*d^2*e^3 + 3*b^6*c*d*e^4 - 5*b^7*e^5)*x^2)*sqrt(-(c*d
- b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + 3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{11/2}}{(bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(11/2)/(c*x**2+b*x)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{11/2}}{(bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(11/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(338) = 676$.

Time = 0.19 (sec) , antiderivative size = 749, normalized size of antiderivative = 1.98

$$\int \frac{(d+ex)^{11/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(11/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
2*sqrt(e*x + d)*e^5/c^3 + 3/4*(16*c^2*d^6 - 44*b*c*d^5*e + 33*b^2*d^4*e^2)
*arctan(sqrt(e*x + d)/sqrt(-d))/(b^5*sqrt(-d)) - 3/4*(16*c^6*d^6 - 52*b*c^
5*d^5*e + 53*b^2*c^4*d^4*e^2 - 12*b^3*c^3*d^3*e^3 - 2*b^4*c^2*d^2*e^4 - 8*
b^5*c*d*e^5 + 5*b^6*e^6)*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c*e))/(sqr
t(-c^2*d + b*c*e)*b^5*c^3) + 1/4*(24*(e*x + d)^(7/2)*c^6*d^5*e - 72*(e*x +
 d)^(5/2)*c^6*d^6*e + 72*(e*x + d)^(3/2)*c^6*d^7*e - 24*sqrt(e*x + d)*c^6*
d^8*e - 60*(e*x + d)^(7/2)*b*c^5*d^4*e^2 + 216*(e*x + d)^(5/2)*b*c^5*d^5*e
^2 - 252*(e*x + d)^(3/2)*b*c^5*d^6*e^2 + 96*sqrt(e*x + d)*b*c^5*d^7*e^2 +
36*(e*x + d)^(7/2)*b^2*c^4*d^3*e^3 - 199*(e*x + d)^(5/2)*b^2*c^4*d^4*e^3 +
 298*(e*x + d)^(3/2)*b^2*c^4*d^5*e^3 - 135*sqrt(e*x + d)*b^2*c^4*d^6*e^3 +
 6*(e*x + d)^(7/2)*b^3*c^3*d^2*e^4 + 38*(e*x + d)^(5/2)*b^3*c^3*d^3*e^4 -
115*(e*x + d)^(3/2)*b^3*c^3*d^4*e^4 + 69*sqrt(e*x + d)*b^3*c^3*d^5*e^4 - 2
4*(e*x + d)^(7/2)*b^4*c^2*d*e^5 + 58*(e*x + d)^(5/2)*b^4*c^2*d^2*e^5 - 44*
(e*x + d)^(3/2)*b^4*c^2*d^3*e^5 + 10*sqrt(e*x + d)*b^4*c^2*d^4*e^5 + 9*(e*
x + d)^(7/2)*b^5*c*e^6 - 41*(e*x + d)^(5/2)*b^5*c*d*e^6 + 55*(e*x + d)^(3/
2)*b^5*c*d^2*e^6 - 23*sqrt(e*x + d)*b^5*c*d^3*e^6 + 7*(e*x + d)^(5/2)*b^6*
e^7 - 14*(e*x + d)^(3/2)*b^6*d*e^7 + 7*sqrt(e*x + d)*b^6*d^2*e^7)/(((e*x +
 d)^2*c - 2*(e*x + d)*c*d + c*d^2 + (e*x + d)*b*e - b*d*e)^2*b^4*c^3)
```

Mupad [B] (verification not implemented)

Time = 6.34 (sec) , antiderivative size = 4450, normalized size of antiderivative = 11.77

$$\int \frac{(d+ex)^{11/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(11/2)/(b*x + c*x^2)^3,x)
```

output
```
(((d + e*x)^(5/2)*(7*b^6*e^7 - 72*c^6*d^6*e + 216*b*c^5*d^5*e^2 - 199*b^2*
c^4*d^4*e^3 + 38*b^3*c^3*d^3*e^4 + 58*b^4*c^2*d^2*e^5 - 41*b^5*c*d*e^6))/(
4*b^4) + ((d + e*x)^(1/2)*(7*b^6*d^2*e^7 - 24*c^6*d^8*e + 96*b*c^5*d^7*e^2
 - 23*b^5*c*d^3*e^6 - 135*b^2*c^4*d^6*e^3 + 69*b^3*c^3*d^5*e^4 + 10*b^4*c^
2*d^4*e^5))/(4*b^4) + (3*(d + e*x)^(7/2)*(3*b^5*c*e^6 + 8*c^6*d^5*e - 20*b
*c^5*d^4*e^2 - 8*b^4*c^2*d*e^5 + 12*b^2*c^4*d^3*e^3 + 2*b^3*c^3*d^2*e^4))/
(4*b^4) - ((d + e*x)^(3/2)*(14*b^6*d*e^7 - 72*c^6*d^7*e + 252*b*c^5*d^6*e^
2 - 55*b^5*c*d^2*e^6 - 298*b^2*c^4*d^5*e^3 + 115*b^3*c^3*d^4*e^4 + 44*b^4*
c^2*d^3*e^5))/(4*b^4))/(c^5*(d + e*x)^4 - (d + e*x)*(4*c^5*d^3 + 2*b^2*c^3
*d*e^2 - 6*b*c^4*d^2*e) - (4*c^5*d - 2*b*c^4*e)*(d + e*x)^3 + c^5*d^4 + (d
 + e*x)^2*(6*c^5*d^2 + b^2*c^3*e^2 - 6*b*c^4*d*e) + b^2*c^3*d^2*e^2 - 2*b*
c^4*d^3*e) + (2*e^5*(d + e*x)^(1/2))/c^3 + (atan((((((d + e*x)^(1/2)*(225*
b^12*e^14 + 4608*c^12*d^12*e^2 - 27648*b*c^11*d^11*e^3 + 66528*b^2*c^10*d^
10*e^4 - 79200*b^3*c^9*d^9*e^5 + 45738*b^4*c^8*d^8*e^6 - 11880*b^5*c^7*d^7
*e^7 + 8316*b^6*c^6*d^6*e^8 - 11880*b^7*c^5*d^5*e^9 + 6534*b^8*c^4*d^4*e^1
0 - 792*b^9*c^3*d^3*e^11 + 396*b^10*c^2*d^2*e^12 - 720*b^11*c*d*e^13))/(8*
b^8*c^5) - (3*((15*b^15*c^4*d*e^8 + 24*b^10*c^9*d^6*e^3 - 72*b^11*c^8*d^5*
e^4 + 63*b^12*c^7*d^4*e^5 - 6*b^13*c^6*d^3*e^6 - 24*b^14*c^5*d^2*e^7)/(b^1
2*c^5) - (3*(64*b^11*c^7*e^3 - 128*b^10*c^8*d*e^2)*(d^7)^(1/2)*(d + e*x)^(
1/2)*(33*b^2*e^2 + 16*c^2*d^2 - 44*b*c*d*e))/(64*b^13*c^5))*(d^7)^(1/2)...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1744, normalized size of antiderivative = 4.61

$$\int \frac{(d+ex)^{11/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(11/2)/(c*x^2+b*x)^3,x)
```

output

```
( - 30*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e -
c*d)))*b**7*e**5*x**2 + 18*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/
(sqrt(c)*sqrt(b*e - c*d)))*b**6*c*d*e**4*x**2 - 60*sqrt(c)*sqrt(b*e - c*d)
*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**6*c*e**5*x**3 + 30*s
qrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b
**5*c**2*d**2*e**3*x**2 + 36*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c
)/(sqrt(c)*sqrt(b*e - c*d)))*b**5*c**2*d*e**4*x**3 - 30*sqrt(c)*sqrt(b*e -
 c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**5*c**2*e**5*x**
4 + 102*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e -
 c*d)))*b**4*c**3*d**3*e**2*x**2 + 60*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d
 + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**4*c**3*d**2*e**3*x**3 + 18*sqrt(c
)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**4*c
**3*d*e**4*x**4 - 216*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt
(c)*sqrt(b*e - c*d)))*b**3*c**4*d**4*e*x**2 + 204*sqrt(c)*sqrt(b*e - c*d)*
atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*c**4*d**3*e**2*x**3
 + 30*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c
*d)))*b**3*c**4*d**2*e**3*x**4 + 96*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d +
 e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c**5*d**5*x**2 - 432*sqrt(c)*sqrt
(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c**5*d*
*4*e*x**3 + 102*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)...
```

3.122 $\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^3}\,dx$

Optimal result 936
Mathematica [A] (verified) 937
Rubi [A] (verified) 937
Maple [A] (verified) 941
Fricas [B] (verification not implemented) 943
Sympy [F(-1)] 944
Maxima [F(-2)] 944
Giac [B] (verification not implemented) 944
Mupad [B] (verification not implemented) 945
Reduce [B] (verification not implemented) 946

Optimal result

Integrand size = 21, antiderivative size = 312

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^3}\,dx = \frac{3(cd-be)(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)\sqrt{d+ex}}{4b^4c^2(b+cx)} + \frac{(cd-be)\left(12c^2d^2-17bcde+2b^2e^2\right)(d+ex)^{3/2}}{4b^3c(b+cx)^2} + \frac{d(8cd-11be)(d+ex)^{5/2}}{4b^2x(b+cx)^2} - \frac{d(d+ex)^{7/2}}{2bx^2(b+cx)^2} - \frac{3d^{5/2}\left(16c^2d^2-36bcde+21b^2e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4b^5} + \frac{3(cd-be)^{5/2}\left(16c^2d^2+4bcde+b^2e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{4b^5c^{5/2}}$$

output

```
3/4*(-b*e+c*d)*(-b*e+2*c*d)*(-b^2*e^2-4*b*c*d*e+4*c^2*d^2)*(e*x+d)^(1/2)/b
^4/c^2/(c*x+b)+1/4*(-b*e+c*d)*(2*b^2*e^2-17*b*c*d*e+12*c^2*d^2)*(e*x+d)^(3
/2)/b^3/c/(c*x+b)^2+1/4*d*(-11*b*e+8*c*d)*(e*x+d)^(5/2)/b^2/x/(c*x+b)^2-1/
2*d*(e*x+d)^(7/2)/b/x^2/(c*x+b)^2-3/4*d^(5/2)*(21*b^2*e^2-36*b*c*d*e+16*c^
2*d^2)*arctanh((e*x+d)^(1/2)/d^(1/2))/b^5+3/4*(-b*e+c*d)^(5/2)*(b^2*e^2+4*
b*c*d*e+16*c^2*d^2)*arctanh(c^(1/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b^5/c^
(5/2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^3}\,dx = \frac{\frac{b\sqrt{d+ex}(-3b^5e^4x^2+24c^5d^4x^3+12bc^4d^3x^2(3d-4ex)-5b^4ce^3x^2(d+ex)+b^2c^3d^2x(8d^2-73dex+21e^2x^2)+b^3c^2d(-2d^3-\cdots}{c^2x^2(b+cx)^2}}{\cdots}$$

input
```
Integrate[(d + e*x)^(9/2)/(b*x + c*x^2)^3,x]
```

output
```
((b*Sqrt[d + e*x]*(-3*b^5*e^4*x^2 + 24*c^5*d^4*x^3 + 12*b*c^4*d^3*x^2*(3*d
 - 4*e*x) - 5*b^4*c*e^3*x^2*(d + e*x) + b^2*c^3*d^2*x*(8*d^2 - 73*d*e*x +
21*e^2*x^2) + b^3*c^2*d*(-2*d^3 - 17*d^2*e*x + 33*d*e^2*x^2 + 3*e^3*x^3)))
/(c^2*x^2*(b + c*x)^2) + (3*(-(c*d) + b*e)^(5/2)*(16*c^2*d^2 + 4*b*c*d*e +
 b^2*e^2)*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[-(c*d) + b*e]])/c^(5/2) - 3*
d^(5/2)*(16*c^2*d^2 - 36*b*c*d*e + 21*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[
d]])/(4*b^5)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1164, 27, 1233, 27, 1196, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \ 1164$$

$$-\frac{\int \frac{(d+ex)^{5/2}(d(12cd-13be)-e(2cd-be)x)}{2(cx^2+bx)^2}dx}{2b^2}-\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{(d+ex)^{5/2}(d(12cd-13be)-e(2cd-be)x)}{(cx^2+bx)^2}dx}{4b^2}-\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \; 1233$$

$$-\frac{\frac{\int -\frac{3\sqrt{d+ex}\left(cd^2\left(16c^2d^2-36bced+21b^2e^2\right)-e(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)x\right)}{2\left(cx^2+bx\right)}dx}{b^2c}-\frac{(d+ex)^{3/2}\left(x(2cd-be)\left(-b^2e^2-12bcde+12c^2d^2\right)+bcd^2(12cd-\right.}{b^2c(bx+cx^2)}}{4b^2}$$

$$\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{3\int \frac{\sqrt{d+ex}\left(cd^2\left(16c^2d^2-36bced+21b^2e^2\right)-e(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)x\right)}{cx^2+bx}dx}{2b^2c}-\frac{(d+ex)^{3/2}\left(x(2cd-be)\left(-b^2e^2-12bcde+12c^2d^2\right)+bcd^2(12cd-\right.}{b^2c(bx+cx^2)}}{4b^2}$$

$$\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \; 1196$$

$$-\frac{-\frac{3\left(\frac{\int \frac{c^2\left(16c^2d^2-36bced+21b^2e^2\right)d^3+e\left(8c^4d^4-16bc^3ed^3+7b^2c^2e^2d^2+b^3ce^3d+b^4e^4\right)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{c}-\frac{2e\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)}{c}\right)}{2b^2c}-\frac{(d+ex)^{3/\cdots}}{\cdots}}{4b^2}$$

$$\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \; 1197$$

$$-\frac{-\frac{3\left(\frac{2\int \frac{e\left(d(cd-be)(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)+\left(8c^4d^4-16bc^3ed^3+7b^2c^2e^2d^2+b^3ce^3d+b^4e^4\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}-\frac{2e\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4\cdots\right.}{c}\right.}{2b^2c}}{4b^2}$$

$$\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{3\left(\frac{2e\int \frac{d(cd-be)(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)+\left(8c^4d^4-16bc^3ed^3+7b^2c^2e^2d^2+b^3ce^3d+b^4e^4\right)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{c}-\frac{2e\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2\cdots\right.}{c}\cdots\right.}{2b^2c}\cdots}{4b^2}-\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$\big\downarrow$ 1480

$$-\frac{-\frac{3\left(\frac{2e\left(\frac{c^3d^3\left(21b^2e^2-36bcde+16c^2d^2\right)\int\frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{(cd-be)^3\left(b^2e^2+4bcde+16c^2d^2\right)\int\frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{c}-\frac{2e\sqrt{d+ex}(2cd-be)\left(-b\cdots\right.}{c}\cdots\right.}{2b^2c}\cdots}{4b^2}-\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$\big\downarrow$ 221

$$-\frac{-\frac{3\left(\frac{2e\left(\frac{(cd-be)^{5/2}\left(b^2e^2+4bcde+16c^2d^2\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{c}e}-\frac{c^2d^{5/2}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(21b^2e^2-36bcde+16c^2d^2\right)}{be}\right)}{c}-\frac{2e\sqrt{d+ex}(2cd-be)\left(-b\cdots\right.}{c}\cdots\right.}{2b^2c}\cdots}{4b^2}-\frac{(d+ex)^{7/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

input

```
Int[(d + e*x)^(9/2)/(b*x + c*x^2)^3,x]
```

output

```
-1/2*((d + e*x)^(7/2)*(b*d + (2*c*d - b*e)*x))/(b^2*(b*x + c*x^2)^2) - (-(
((d + e*x)^(3/2)*(b*c*d^2*(12*c*d - 13*b*e) + (2*c*d - b*e)*(12*c^2*d^2 -
12*b*c*d*e - b^2*e^2)*x))/(b^2*c*(b*x + c*x^2))) - (3*((-2*e*(2*c*d - b*e)
*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*Sqrt[d + e*x])/c + (2*e*(-((c^2*d^(5/2)
*(16*c^2*d^2 - 36*b*c*d*e + 21*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b
*e)) + ((c*d - b*e)^(5/2)*(16*c^2*d^2 + 4*b*c*d*e + b^2*e^2)*ArcTanh[(Sqrt
[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*Sqrt[c]*e)))/c))/(2*b^2*c))/(4*b^2
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1196
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Simp[1/c   Int
[(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] &
& GtQ[m, 0]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.88

method	result
derivativedivides	$2e^5\left(\frac{(be-cd)^3\left(\frac{-\frac{be(5be+12cd)(ex+d)^{\frac{3}{2}}}{8c}-\frac{3be\left(b^2e^2+3bcde-4c^2d^2\right)\sqrt{ex+d}}{8c^2}}{((ex+d)c+be-cd)^2}+\frac{3\left(b^2e^2+4bcde+16c^2d^2\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{8c^2\sqrt{c(be-cd)}}\right.}{b^5e^5}\right.$
default	$2e^5\left(\frac{(be-cd)^3\left(\frac{-\frac{be(5be+12cd)(ex+d)^{\frac{3}{2}}}{8c}-\frac{3be\left(b^2e^2+3bcde-4c^2d^2\right)\sqrt{ex+d}}{8c^2}}{((ex+d)c+be-cd)^2}+\frac{3\left(b^2e^2+4bcde+16c^2d^2\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{8c^2\sqrt{c(be-cd)}}\right.}{b^5e^5}\right.$
pseudoelliptic	$-\frac{3\left(-\frac{(be-cd)^3x^2(cx+b)^2\sqrt{d}\left(b^2e^2+4bcde+16c^2d^2\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{2}+\frac{\sqrt{c(be-cd)}\left(\frac{3x^2(cx+b)^2c^2\left(21b^2e^2-36bcde+16c^2\right.}{2}\right.}{}\right.}{}$
risch	$-\frac{d^3\sqrt{ex+d}\,(17bex-12cdx+2bd)}{4b^4x^2}+\frac{e\left(-\frac{3d^{\frac{5}{2}}\left(21b^2e^2-36bcde+16c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be}+\frac{8\left(-\frac{be\left(5b^4e^4-3d\,e^3b^3c-21\right.}{}\right.}{}\right.}{}$

input
```
int((e*x+d)^(9/2)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
2*e^5*((b*e-c*d)^3/b^5/e^5*((-1/8*b*e*(5*b*e+12*c*d)/c*(e*x+d)^(3/2)-3/8*b
/c^2*e*(b^2*e^2+3*b*c*d*e-4*c^2*d^2)*(e*x+d)^(1/2))/((e*x+d)*c+b*e-c*d)^2+
3/8*(b^2*e^2+4*b*c*d*e+16*c^2*d^2)/c^2/(c*(b*e-c*d))^(1/2)*arctan(c*(e*x+d
)^(1/2)/(c*(b*e-c*d))^(1/2)))-d^3/b^5/e^5*(((17/8*b^2*e^2-3/2*b*c*d*e)*(e*
x+d)^(3/2)+(-15/8*d*e^2*b^2+3/2*d^2*e*b*c)*(e*x+d)^(1/2))/e^2/x^2+3/8*(21*
b^2*e^2-36*b*c*d*e+16*c^2*d^2)/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(276) = 552$.

Time = 0.96 (sec) , antiderivative size = 2367, normalized size of antiderivative = 7.59

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(9/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
[1/8*(3*((16*c^6*d^4 - 28*b*c^5*d^3*e + 9*b^2*c^4*d^2*e^2 + 2*b^3*c^3*d*e^
3 + b^4*c^2*e^4)*x^4 + 2*(16*b*c^5*d^4 - 28*b^2*c^4*d^3*e + 9*b^3*c^3*d^2*
e^2 + 2*b^4*c^2*d*e^3 + b^5*c*e^4)*x^3 + (16*b^2*c^4*d^4 - 28*b^3*c^3*d^3*
e + 9*b^4*c^2*d^2*e^2 + 2*b^5*c*d*e^3 + b^6*e^4)*x^2)*sqrt((c*d - b*e)/c)*
log((c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*sqrt((c*d - b*e)/c))/(c*x + b
)) + 3*((16*c^6*d^4 - 36*b*c^5*d^3*e + 21*b^2*c^4*d^2*e^2)*x^4 + 2*(16*b*c
^5*d^4 - 36*b^2*c^4*d^3*e + 21*b^3*c^3*d^2*e^2)*x^3 + (16*b^2*c^4*d^4 - 36
*b^3*c^3*d^3*e + 21*b^4*c^2*d^2*e^2)*x^2)*sqrt(d)*log((e*x - 2*sqrt(e*x +
d)*sqrt(d) + 2*d)/x) - 2*(2*b^4*c^2*d^4 - (24*b*c^5*d^4 - 48*b^2*c^4*d^3*e
 + 21*b^3*c^3*d^2*e^2 + 3*b^4*c^2*d*e^3 - 5*b^5*c*e^4)*x^3 - (36*b^2*c^4*d
^4 - 73*b^3*c^3*d^3*e + 33*b^4*c^2*d^2*e^2 - 5*b^5*c*d*e^3 - 3*b^6*e^4)*x^
2 - (8*b^3*c^3*d^4 - 17*b^4*c^2*d^3*e)*x)*sqrt(e*x + d))/(b^5*c^4*x^4 + 2*
b^6*c^3*x^3 + b^7*c^2*x^2), 1/8*(6*((16*c^6*d^4 - 28*b*c^5*d^3*e + 9*b^2*c
^4*d^2*e^2 + 2*b^3*c^3*d*e^3 + b^4*c^2*e^4)*x^4 + 2*(16*b*c^5*d^4 - 28*b^2
*c^4*d^3*e + 9*b^3*c^3*d^2*e^2 + 2*b^4*c^2*d*e^3 + b^5*c*e^4)*x^3 + (16*b^
2*c^4*d^4 - 28*b^3*c^3*d^3*e + 9*b^4*c^2*d^2*e^2 + 2*b^5*c*d*e^3 + b^6*e^4
)*x^2)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(
c*d - b*e)) + 3*((16*c^6*d^4 - 36*b*c^5*d^3*e + 21*b^2*c^4*d^2*e^2)*x^4 +
2*(16*b*c^5*d^4 - 36*b^2*c^4*d^3*e + 21*b^3*c^3*d^2*e^2)*x^3 + (16*b^2*c^4
*d^4 - 36*b^3*c^3*d^3*e + 21*b^4*c^2*d^2*e^2)*x^2)*sqrt(d)*log((e*x - 2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(9/2)/(c*x**2+b*x)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(9/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. 2(276) = 552.

Time = 0.15 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.98

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^3}\,dx = \frac{3\,(16\,c^2d^5 - 36\,bcd^4e + 21\,b^2d^3e^2)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{4\,b^5\sqrt{-d}} - \frac{3\,(16\,c^5d^5 - 44\,bc^4d^4e + 37\,b^2c^3d^3e^2 - 7\,b^3c^2d^2e^3 - b^4cde^4 - b^5e^5)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{4\,\sqrt{-c^2d+bce}b^5c^2} + \frac{24\,(ex+d)^{\frac{7}{2}}c^5d^4e - 72\,(ex+d)^{\frac{5}{2}}c^5d^5e + 72\,(ex+d)^{\frac{3}{2}}c^5d^6e - 24\,\sqrt{ex+d}c^5d^7e - 48\,(ex+d)^{\frac{7}{2}}bc^4d^3e^2 + \cdots}{\cdots}$$

input
```
integrate((e*x+d)^(9/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
3/4*(16*c^2*d^5 - 36*b*c*d^4*e + 21*b^2*d^3*e^2)*arctan(sqrt(e*x + d)/sqrt
(-d))/(b^5*sqrt(-d)) - 3/4*(16*c^5*d^5 - 44*b*c^4*d^4*e + 37*b^2*c^3*d^3*e
^2 - 7*b^3*c^2*d^2*e^3 - b^4*c*d*e^4 - b^5*e^5)*arctan(sqrt(e*x + d)*c/sqr
t(-c^2*d + b*c*e))/(sqrt(-c^2*d + b*c*e)*b^5*c^2) + 1/4*(24*(e*x + d)^(7/2
)*c^5*d^4*e - 72*(e*x + d)^(5/2)*c^5*d^5*e + 72*(e*x + d)^(3/2)*c^5*d^6*e
- 24*sqrt(e*x + d)*c^5*d^7*e - 48*(e*x + d)^(7/2)*b*c^4*d^3*e^2 + 180*(e*x
 + d)^(5/2)*b*c^4*d^4*e^2 - 216*(e*x + d)^(3/2)*b*c^4*d^5*e^2 + 84*sqrt(e*
x + d)*b*c^4*d^6*e^2 + 21*(e*x + d)^(7/2)*b^2*c^3*d^2*e^3 - 136*(e*x + d)^
(5/2)*b^2*c^3*d^3*e^3 + 217*(e*x + d)^(3/2)*b^2*c^3*d^4*e^3 - 102*sqrt(e*x
 + d)*b^2*c^3*d^5*e^3 + 3*(e*x + d)^(7/2)*b^3*c^2*d*e^4 + 24*(e*x + d)^(5/
2)*b^3*c^2*d^2*e^4 - 74*(e*x + d)^(3/2)*b^3*c^2*d^3*e^4 + 45*sqrt(e*x + d)
*b^3*c^2*d^4*e^4 - 5*(e*x + d)^(7/2)*b^4*c*e^5 + 10*(e*x + d)^(5/2)*b^4*c*
d*e^5 - 5*(e*x + d)^(3/2)*b^4*c*d^2*e^5 - 3*(e*x + d)^(5/2)*b^5*e^6 + 6*(e
*x + d)^(3/2)*b^5*d*e^6 - 3*sqrt(e*x + d)*b^5*d^2*e^6)/(((e*x + d)^2*c - 2
*(e*x + d)*c*d + c*d^2 + (e*x + d)*b*e - b*d*e)^2*b^4*c^2)
```

Mupad [B] (verification not implemented)

Time = 6.51 (sec) , antiderivative size = 3946, normalized size of antiderivative = 12.65

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(9/2)/(b*x + c*x^2)^3,x)
```

output
```
(((d + e*x)^(3/2)*(6*b^5*d*e^6 + 72*c^5*d^6*e - 216*b*c^4*d^5*e^2 - 5*b^4*
c*d^2*e^5 + 217*b^2*c^3*d^4*e^3 - 74*b^3*c^2*d^3*e^4))/(4*b^4*c^2) - (3*(d
 + e*x)^(1/2)*(8*c^5*d^7*e + b^5*d^2*e^6 - 28*b*c^4*d^6*e^2 + 34*b^2*c^3*d
^5*e^3 - 15*b^3*c^2*d^4*e^4))/(4*b^4*c^2) + (e*(d + e*x)^(7/2)*(24*c^4*d^4
 - 5*b^4*e^4 + 21*b^2*c^2*d^2*e^2 - 48*b*c^3*d^3*e + 3*b^3*c*d*e^3))/(4*b^
4*c) + ((b*e - 2*c*d)*(d + e*x)^(5/2)*(36*c^4*d^4*e - 3*b^4*e^5 - 72*b*c^3
*d^3*e^2 + 32*b^2*c^2*d^2*e^3 + 4*b^3*c*d*e^4))/(4*b^4*c^2))/(c^2*(d + e*x
)^4 - (d + e*x)*(4*c^2*d^3 + 2*b^2*d*e^2 - 6*b*c*d^2*e) - (4*c^2*d - 2*b*c
*e)*(d + e*x)^3 + (d + e*x)^2*(b^2*e^2 + 6*c^2*d^2 - 6*b*c*d*e) + c^2*d^4
+ b^2*d^2*e^2 - 2*b*c*d^3*e) + (atan(((((3*(d^5)^(1/2)*((3*b^14*c^3*d*e^7
- 24*b^10*c^7*d^5*e^3 + 60*b^11*c^6*d^4*e^4 - 42*b^12*c^5*d^3*e^5 + 3*b^13
*c^4*d^2*e^6)/(b^12*c^3) - (3*(64*b^11*c^5*e^3 - 128*b^10*c^6*d*e^2)*(d^5)
^(1/2)*(d + e*x)^(1/2)*(21*b^2*e^2 + 16*c^2*d^2 - 36*b*c*d*e))/(64*b^13*c^
3))*(21*b^2*e^2 + 16*c^2*d^2 - 36*b*c*d*e))/(8*b^5) - ((d + e*x)^(1/2)*(9*
b^10*e^12 + 4608*c^10*d^10*e^2 - 23040*b*c^9*d^9*e^3 + 45792*b^2*c^8*d^8*e
^4 - 44928*b^3*c^7*d^7*e^5 + 21546*b^4*c^6*d^6*e^6 - 4158*b^5*c^5*d^5*e^7
+ 567*b^6*c^4*d^4*e^8 - 540*b^7*c^3*d^3*e^9 + 135*b^8*c^2*d^2*e^10 + 18*b^
9*c*d*e^11))/(8*b^8*c^3))*(d^5)^(1/2)*(21*b^2*e^2 + 16*c^2*d^2 - 36*b*c*d*
e)*3i)/(8*b^5) - (((3*(d^5)^(1/2)*((3*b^14*c^3*d*e^7 - 24*b^10*c^7*d^5*e^3
 + 60*b^11*c^6*d^4*e^4 - 42*b^12*c^5*d^3*e^5 + 3*b^13*c^4*d^2*e^6)/(b^1...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1522, normalized size of antiderivative = 4.88

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(9/2)/(c*x^2+b*x)^3,x)
```

output

```
(6*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)
))*b**6*e**4*x**2 + 12*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqr
t(c)*sqrt(b*e - c*d)))*b**5*c*d*e**3*x**2 + 12*sqrt(c)*sqrt(b*e - c*d)*ata
n((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**5*c*e**4*x**3 + 54*sqrt(
c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**4*
c**2*d**2*e**2*x**2 + 24*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(s
qrt(c)*sqrt(b*e - c*d)))*b**4*c**2*d*e**3*x**3 + 6*sqrt(c)*sqrt(b*e - c*d)
*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**4*c**2*e**4*x**4 - 1
68*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)
))*b**3*c**3*d**3*e*x**2 + 108*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)
*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*c**3*d**2*e**2*x**3 + 12*sqrt(c)*sqrt(
b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*c**3*d*e
**3*x**4 + 96*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt
(b*e - c*d)))*b**2*c**4*d**4*x**2 - 336*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt
(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c**4*d**3*e*x**3 + 54*sqrt(c)
*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c*
*4*d**2*e**2*x**4 + 192*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sq
rt(c)*sqrt(b*e - c*d)))*b*c**5*d**4*x**3 - 168*sqrt(c)*sqrt(b*e - c*d)*ata
n((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**5*d**3*e*x**4 + 96*sqr
t(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*...
```

3.123 $\int \frac{(d+ex)^{7/2}}{\left(bx+cx^2\right)^3}\,dx$

Optimal result 948
Mathematica [A] (verified) 949
Rubi [A] (verified) 949
Maple [A] (verified) 952
Fricas [A] (verification not implemented) 953
Sympy [F(-1)] 954
Maxima [F(-2)] 955
Giac [A] (verification not implemented) 955
Mupad [B] (verification not implemented) 956
Reduce [B] (verification not implemented) 957

Optimal result

Integrand size = 21, antiderivative size = 305

$$\int \frac{(d+ex)^{7/2}}{\left(bx+cx^2\right)^3}\,dx = \frac{(cd-be)\left(12c^2d^2-15bcde+2b^2e^2\right)\sqrt{d+ex}}{4b^3c(b+cx)^2} + \frac{(2cd-be)\left(12c^2d^2-12bcde-b^2e^2\right)\sqrt{d+ex}}{4b^4c(b+cx)} + \frac{d(8cd-9be)(d+ex)^{3/2}}{4b^2x(b+cx)^2} - \frac{d(d+ex)^{5/2}}{2bx^2(b+cx)^2} - \frac{d^{3/2}\left(48c^2d^2-84bcde+35b^2e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4b^5} + \frac{(cd-be)^{3/2}\left(48c^2d^2-12bcde-b^2e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{4b^5c^{3/2}}$$

output
```
1/4*(-b*e+c*d)*(2*b^2*e^2-15*b*c*d*e+12*c^2*d^2)*(e*x+d)^(1/2)/b^3/c/(c*x+
b)^2+1/4*(-b*e+2*c*d)*(-b^2*e^2-12*b*c*d*e+12*c^2*d^2)*(e*x+d)^(1/2)/b^4/c
/(c*x+b)+1/4*d*(-9*b*e+8*c*d)*(e*x+d)^(3/2)/b^2/x/(c*x+b)^2-1/2*d*(e*x+d)^
(5/2)/b/x^2/(c*x+b)^2-1/4*d^(3/2)*(35*b^2*e^2-84*b*c*d*e+48*c^2*d^2)*arcta
nh((e*x+d)^(1/2)/d^(1/2))/b^5+1/4*(-b*e+c*d)^(3/2)*(-b^2*e^2-12*b*c*d*e+48
*c^2*d^2)*arctanh(c^(1/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b^5/c^(3/2)
```

Mathematica [A] (verified)

Time = 1.54 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.82

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^3}\,dx =$$

$$-\frac{-\frac{b\sqrt{d+ex}\left(-b^4e^3x^2+24c^4d^3x^3+36bc^3d^2x^2(d-ex)+b^2c^2dx\left(8d^2-55dex+10e^2x^2\right)+b^3c\left(-2d^3-13d^2ex+16de^2x^2+e^3x^3\right)\right)}{cx^2(b+cx)^2}+\frac{(-cd+be)^{3/2}(48}{}}{4b^5}$$

input
```
Integrate[(d + e*x)^(7/2)/(b*x + c*x^2)^3,x]
```

output
```
-1/4*(-((b*Sqrt[d + e*x]*(-(b^4*e^3*x^2) + 24*c^4*d^3*x^3 + 36*b*c^3*d^2*x
^2*(d - e*x) + b^2*c^2*d*x*(8*d^2 - 55*d*e*x + 10*e^2*x^2) + b^3*c*(-2*d^3
 - 13*d^2*e*x + 16*d*e^2*x^2 + e^3*x^3)))/(c*x^2*(b + c*x)^2)) + ((-(c*d)
+ b*e)^(3/2)*(48*c^2*d^2 - 12*b*c*d*e - b^2*e^2)*ArcTan[(Sqrt[c]*Sqrt[d +
e*x])/Sqrt[-(c*d) + b*e]])/c^(3/2) + d^(3/2)*(48*c^2*d^2 - 84*b*c*d*e + 35
*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/b^5
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.87, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1164, 27, 1233, 27, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{ 1164}$$

$$-\frac{\int \frac{(d+ex)^{3/2}(d(12cd-11be)+e(2cd-be)x)}{2(cx^2+bx)^2}dx}{2b^2}-\frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{(d+ex)^{3/2}(d(12cd-11be)+e(2cd-be)x)}{(cx^2+bx)^2}dx}{4b^2}-\frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \text{1233}$$

$$-\frac{\frac{\int -\frac{c\left(48c^2d^2-84bced+35b^2e^2\right)d^2+e(2cd-be)\left(12c^2d^2-12bced-b^2e^2\right)x}{2\sqrt{d+ex}\left(cx^2+bx\right)}dx}{b^2c}-\frac{\sqrt{d+ex}\left(x(2cd-be)\left(b^2e^2-12bcde+12c^2d^2\right)+bcd^2(12cd-11be)\right)}{b^2c(bx+cx^2)}}{4b^2}-\frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{c\left(48c^2d^2-84bced+35b^2e^2\right)d^2+e(2cd-be)\left(12c^2d^2-12bced-b^2e^2\right)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{2b^2c}-\frac{\sqrt{d+ex}\left(x(2cd-be)\left(b^2e^2-12bcde+12c^2d^2\right)+bcd^2(12cd-11be)\right)}{b^2c(bx+cx^2)}}{4b^2}-\frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \text{1197}$$

$$-\frac{-\frac{\int \frac{e\left(d(cd-be)\left(24c^2d^2-24bced+b^2e^2\right)+(2cd-be)\left(12c^2d^2-12bced-b^2e^2\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2c}-\frac{\sqrt{d+ex}\left(x(2cd-be)\left(b^2e^2-12bcde+12c^2d^2\right)+bcd^2(12\right.}{b^2c(bx+cx^2)}}{4b^2}-\frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{e\int \frac{d(cd-be)\left(24c^2d^2-24bced+b^2e^2\right)+(2cd-be)\left(12c^2d^2-12bced-b^2e^2\right)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2c}-\frac{\sqrt{d+ex}\left(x(2cd-be)\left(b^2e^2-12bcde+12c^2d^2\right)+bcd^2(12c\right.}{b^2c(bx+cx^2)}}{4b^2}-\frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{-\frac{e\left(\frac{c^2d^2\left(35b^2e^2-84bcde+48c^2d^2\right)\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{(cd-be)^2\left(-b^2e^2-12bcde+48c^2d^2\right)\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{b^2c}-\sqrt{d+ex}(x(2cd-be)(}{4b^2}-\frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow 221$$

$$-\frac{-\frac{e\left(\frac{(cd-be)^{3/2}\left(-b^2e^2-12bcde+48c^2d^2\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{c}e}-\frac{cd^{3/2}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(35b^2e^2-84bcde+48c^2d^2\right)}{be}\right)}{b^2c}-\frac{\sqrt{d+ex}(x(2cd-be)(b\ldots}{\ldots}}{4b^2}$$

$$\frac{(d+ex)^{5/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

input
```
Int[(d + e*x)^(7/2)/(b*x + c*x^2)^3,x]
```

output
```
-1/2*((d + e*x)^(5/2)*(b*d + (2*c*d - b*e)*x))/(b^2*(b*x + c*x^2)^2) - (-(
(Sqrt[d + e*x]*(b*c*d^2*(12*c*d - 11*b*e) + (2*c*d - b*e)*(12*c^2*d^2 - 12
*b*c*d*e + b^2*e^2)*x))/(b^2*c*(b*x + c*x^2))) - (e*(-((c*d^(3/2)*(48*c^2*
d^2 - 84*b*c*d*e + 35*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*e)) + ((
c*d - b*e)^(3/2)*(48*c^2*d^2 - 12*b*c*d*e - b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt
[d + e*x])/Sqrt[c*d - b*e]])/(b*Sqrt[c]*e)))/(b^2*c))/(4*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.90

method	result
derivativedivides	$2e^5\left(-\frac{d^2\left(\frac{\left(\frac{13}{8}b^2e^2-\frac{3}{2}bcde\right)(ex+d)^{\frac{3}{2}}+\left(-\frac{11}{8}d\,e^2b^2+\frac{3}{2}d^2ebc\right)\sqrt{ex+d}}{e^2x^2}+\frac{\left(35b^2e^2-84bcde+48c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8\sqrt{d}}\right)}{b^5e^5}+$
default	$2e^5\left(-\frac{d^2\left(\frac{\left(\frac{13}{8}b^2e^2-\frac{3}{2}bcde\right)(ex+d)^{\frac{3}{2}}+\left(-\frac{11}{8}d\,e^2b^2+\frac{3}{2}d^2ebc\right)\sqrt{ex+d}}{e^2x^2}+\frac{\left(35b^2e^2-84bcde+48c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8\sqrt{d}}\right)}{b^5e^5}+$
pseudoelliptic	$-\frac{-\frac{\left(b^2e^2+12bcde-48c^2d^2\right)x^2(cx+b)^2\sqrt{d}\,(be-cd)^2\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{2}+\left(\frac{x^2(cx+b)^2c\left(35b^2e^2-84bcde+48c^2d^2\right)d^2\operatorname{arctanh}(}{2}\right.}{}$
risch	$-\frac{d^2\sqrt{ex+d}\,(13bex-12cdx+2bd)}{4b^4x^2}-\frac{e\left(\frac{d^{\frac{3}{2}}\left(35b^2e^2-84bcde+48c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{eb}+\frac{8\left(\left(-\frac{1}{8}b^4e^4-\frac{5}{4}d\,e^3b^3c+\frac{23}{8}d^2e\right.\right.}{}\right.}{}$

input
```
int((e*x+d)^(7/2)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
2*e^5*(-d^2/b^5/e^5*(((13/8*b^2*e^2-3/2*b*c*d*e)*(e*x+d)^(3/2)+(-11/8*d*e^
2*b^2+3/2*d^2*e*b*c)*(e*x+d)^(1/2))/e^2/x^2+1/8*(35*b^2*e^2-84*b*c*d*e+48*
c^2*d^2)/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2)))+(b*e-c*d)^2/b^5/e^5*(((1/
8*b^2*e^2+3/2*b*c*d*e)*(e*x+d)^(3/2)-1/8*b*e*(b^2*e^2-13*b*c*d*e+12*c^2*d^
2)/c*(e*x+d)^(1/2))/((e*x+d)*c+b*e-c*d)^2+1/8*(b^2*e^2+12*b*c*d*e-48*c^2*d
^2)/c/(c*(b*e-c*d))^(1/2)*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 2021, normalized size of antiderivative = 6.63

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
[1/8*(((48*c^5*d^3 - 60*b*c^4*d^2*e + 11*b^2*c^3*d*e^2 + b^3*c^2*e^3)*x^4
+ 2*(48*b*c^4*d^3 - 60*b^2*c^3*d^2*e + 11*b^3*c^2*d*e^2 + b^4*c*e^3)*x^3 +
 (48*b^2*c^3*d^3 - 60*b^3*c^2*d^2*e + 11*b^4*c*d*e^2 + b^5*e^3)*x^2)*sqrt(
(c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*sqrt((c*d - b*
e)/c))/(c*x + b)) + ((48*c^5*d^3 - 84*b*c^4*d^2*e + 35*b^2*c^3*d*e^2)*x^4
+ 2*(48*b*c^4*d^3 - 84*b^2*c^3*d^2*e + 35*b^3*c^2*d*e^2)*x^3 + (48*b^2*c^3
*d^3 - 84*b^3*c^2*d^2*e + 35*b^4*c*d*e^2)*x^2)*sqrt(d)*log((e*x - 2*sqrt(e
*x + d)*sqrt(d) + 2*d)/x) - 2*(2*b^4*c*d^3 - (24*b*c^4*d^3 - 36*b^2*c^3*d^
2*e + 10*b^3*c^2*d*e^2 + b^4*c*e^3)*x^3 - (36*b^2*c^3*d^3 - 55*b^3*c^2*d^2
*e + 16*b^4*c*d*e^2 - b^5*e^3)*x^2 - (8*b^3*c^2*d^3 - 13*b^4*c*d^2*e)*x)*s
qrt(e*x + d))/(b^5*c^3*x^4 + 2*b^6*c^2*x^3 + b^7*c*x^2), 1/8*(2*((48*c^5*d
^3 - 60*b*c^4*d^2*e + 11*b^2*c^3*d*e^2 + b^3*c^2*e^3)*x^4 + 2*(48*b*c^4*d^
3 - 60*b^2*c^3*d^2*e + 11*b^3*c^2*d*e^2 + b^4*c*e^3)*x^3 + (48*b^2*c^3*d^3
 - 60*b^3*c^2*d^2*e + 11*b^4*c*d*e^2 + b^5*e^3)*x^2)*sqrt(-(c*d - b*e)/c)*
arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*d - b*e)) + ((48*c^5*d^3 -
 84*b*c^4*d^2*e + 35*b^2*c^3*d*e^2)*x^4 + 2*(48*b*c^4*d^3 - 84*b^2*c^3*d^2
*e + 35*b^3*c^2*d*e^2)*x^3 + (48*b^2*c^3*d^3 - 84*b^3*c^2*d^2*e + 35*b^4*c
*d*e^2)*x^2)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(2*b
^4*c*d^3 - (24*b*c^4*d^3 - 36*b^2*c^3*d^2*e + 10*b^3*c^2*d*e^2 + b^4*c*e^3
)*x^3 - (36*b^2*c^3*d^3 - 55*b^3*c^2*d^2*e + 16*b^4*c*d*e^2 - b^5*e^3)*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)/(c*x**2+b*x)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.76

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^3}\,dx = \frac{(48\,c^2d^4-84\,bcd^3e+35\,b^2d^2e^2)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{4\,b^5\sqrt{-d}} - \frac{(48\,c^4d^4-108\,bc^3d^3e+71\,b^2c^2d^2e^2-10\,b^3cde^3-b^4e^4)\arctan\left(\frac{\sqrt{ex+dc}}{\sqrt{-c^2d+bce}}\right)}{4\sqrt{-c^2d+bce}b^5c} + \frac{24\,(ex+d)^{\frac{7}{2}}c^4d^3e-72\,(ex+d)^{\frac{5}{2}}c^4d^4e+72\,(ex+d)^{\frac{3}{2}}c^4d^5e-24\sqrt{ex+d}c^4d^6e-36\,(ex+d)^{\frac{7}{2}}bc^3d^2e^2+}{}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
1/4*(48*c^2*d^4 - 84*b*c*d^3*e + 35*b^2*d^2*e^2)*arctan(sqrt(e*x + d)/sqrt
(-d))/(b^5*sqrt(-d)) - 1/4*(48*c^4*d^4 - 108*b*c^3*d^3*e + 71*b^2*c^2*d^2*
e^2 - 10*b^3*c*d*e^3 - b^4*e^4)*arctan(sqrt(e*x + d)*c/sqrt(-c^2*d + b*c*e
))/(sqrt(-c^2*d + b*c*e)*b^5*c) + 1/4*(24*(e*x + d)^(7/2)*c^4*d^3*e - 72*(
e*x + d)^(5/2)*c^4*d^4*e + 72*(e*x + d)^(3/2)*c^4*d^5*e - 24*sqrt(e*x + d)
*c^4*d^6*e - 36*(e*x + d)^(7/2)*b*c^3*d^2*e^2 + 144*(e*x + d)^(5/2)*b*c^3*
d^3*e^2 - 180*(e*x + d)^(3/2)*b*c^3*d^4*e^2 + 72*sqrt(e*x + d)*b*c^3*d^5*e
^2 + 10*(e*x + d)^(7/2)*b^2*c^2*d*e^3 - 85*(e*x + d)^(5/2)*b^2*c^2*d^2*e^3
 + 148*(e*x + d)^(3/2)*b^2*c^2*d^3*e^3 - 73*sqrt(e*x + d)*b^2*c^2*d^4*e^3
+ (e*x + d)^(7/2)*b^3*c*e^4 + 13*(e*x + d)^(5/2)*b^3*c*d*e^4 - 42*(e*x + d
)^(3/2)*b^3*c*d^2*e^4 + 26*sqrt(e*x + d)*b^3*c*d^3*e^4 - (e*x + d)^(5/2)*b
^4*e^5 + 2*(e*x + d)^(3/2)*b^4*d*e^5 - sqrt(e*x + d)*b^4*d^2*e^5)/(((e*x +
 d)^2*c - 2*(e*x + d)*c*d + c*d^2 + (e*x + d)*b*e - b*d*e)^2*b^4*c)
```

Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 1792, normalized size of antiderivative = 5.88

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(7/2)/(b*x + c*x^2)^3,x)
```

output
```
- (((d + e*x)^(1/2)*(24*c^4*d^6*e + b^4*d^2*e^5 - 72*b*c^3*d^5*e^2 - 26*b^
3*c*d^3*e^4 + 73*b^2*c^2*d^4*e^3))/(4*b^4*c) - (e*(d + e*x)^(7/2)*(b^3*e^3
 + 24*c^3*d^3 - 36*b*c^2*d^2*e + 10*b^2*c*d*e^2))/(4*b^4) - ((d + e*x)^(3/
2)*(b^4*d*e^5 + 36*c^4*d^5*e - 90*b*c^3*d^4*e^2 - 21*b^3*c*d^2*e^4 + 74*b^
2*c^2*d^3*e^3))/(2*b^4*c) + (e*(d + e*x)^(5/2)*(b^4*e^4 + 72*c^4*d^4 + 85*
b^2*c^2*d^2*e^2 - 144*b*c^3*d^3*e - 13*b^3*c*d*e^3))/(4*b^4*c))/(c^2*(d +
e*x)^4 - (d + e*x)*(4*c^2*d^3 + 2*b^2*d*e^2 - 6*b*c*d^2*e) - (4*c^2*d - 2*
b*c*e)*(d + e*x)^3 + (d + e*x)^2*(b^2*e^2 + 6*c^2*d^2 - 6*b*c*d*e) + c^2*d
^4 + b^2*d^2*e^2 - 2*b*c*d^3*e) - (atanh((35*e^12*(d^3)^(1/2)*(d + e*x)^(1
/2))/(32*((35*d^2*e^12)/32 + (77*c*d^3*e^11)/(4*b) - (1551*c^2*d^4*e^10)/(
16*b^2) + (5223*c^3*d^5*e^9)/(32*b^3) - (945*c^4*d^6*e^8)/(8*b^4) + (63*c^
5*d^7*e^7)/(2*b^5))) + (77*d*e^11*(d^3)^(1/2)*(d + e*x)^(1/2))/(4*((77*d^3
*e^11)/4 + (35*b*d^2*e^12)/(32*c) - (1551*c*d^4*e^10)/(16*b) + (5223*c^2*d
^5*e^9)/(32*b^2) - (945*c^3*d^6*e^8)/(8*b^3) + (63*c^4*d^7*e^7)/(2*b^4)))
+ (5223*c^2*d^3*e^9*(d^3)^(1/2)*(d + e*x)^(1/2))/(32*((77*b^2*d^3*e^11)/4
+ (5223*c^2*d^5*e^9)/32 - (945*c^3*d^6*e^8)/(8*b) + (35*b^3*d^2*e^12)/(32*
c) + (63*c^4*d^7*e^7)/(2*b^2) - (1551*b*c*d^4*e^10)/16)) - (945*c^3*d^4*e^
8*(d^3)^(1/2)*(d + e*x)^(1/2))/(8*((77*b^3*d^3*e^11)/4 - (945*c^3*d^6*e^8)
/8 + (5223*b*c^2*d^5*e^9)/32 - (1551*b^2*c*d^4*e^10)/16 + (63*c^4*d^7*e^7)
/(2*b) + (35*b^4*d^2*e^12)/(32*c))) + (63*c^4*d^5*e^7*(d^3)^(1/2)*(d + ...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1308, normalized size of antiderivative = 4.29

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(7/2)/(c*x^2+b*x)^3,x)
```

output

```
(2*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)
))*b**5*e**3*x**2 + 22*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqr
t(c)*sqrt(b*e - c*d)))*b**4*c*d*e**2*x**2 + 4*sqrt(c)*sqrt(b*e - c*d)*atan
((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**4*c*e**3*x**3 - 120*sqrt(
c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*
c**2*d**2*e*x**2 + 44*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt
(c)*sqrt(b*e - c*d)))*b**3*c**2*d*e**2*x**3 + 2*sqrt(c)*sqrt(b*e - c*d)*at
an((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*c**2*e**3*x**4 + 96*s
qrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b
**2*c**3*d**3*x**2 - 240*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(s
qrt(c)*sqrt(b*e - c*d)))*b**2*c**3*d**2*e*x**3 + 22*sqrt(c)*sqrt(b*e - c*d
)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c**3*d*e**2*x**4
+ 192*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c
*d)))*b*c**4*d**3*x**3 - 120*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c
)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**4*d**2*e*x**4 + 96*sqrt(c)*sqrt(b*e - c*
d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*c**5*d**3*x**4 - 2*sq
rt(d + e*x)*b**5*c*e**3*x**2 - 4*sqrt(d + e*x)*b**4*c**2*d**3 - 26*sqrt(d
+ e*x)*b**4*c**2*d**2*e*x + 32*sqrt(d + e*x)*b**4*c**2*d*e**2*x**2 + 2*sqr
t(d + e*x)*b**4*c**2*e**3*x**3 + 16*sqrt(d + e*x)*b**3*c**3*d**3*x - 110*s
qrt(d + e*x)*b**3*c**3*d**2*e*x**2 + 20*sqrt(d + e*x)*b**3*c**3*d*e**2*...
```

3.124 $\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^3}\,dx$

Optimal result 959
Mathematica [A] (verified) 960
Rubi [A] (verified) 960
Maple [A] (verified) 963
Fricas [A] (verification not implemented) 965
Sympy [F(-1)] 965
Maxima [F(-2)] 966
Giac [A] (verification not implemented) 966
Mupad [B] (verification not implemented) 967
Reduce [B] (verification not implemented) 968

Optimal result

Integrand size = 21, antiderivative size = 280

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^3}\,dx = \frac{(12c^2d^2-13bcde+2b^2e^2)\sqrt{d+ex}}{4b^3(b+cx)^2} + \frac{d(8cd-7be)\sqrt{d+ex}}{4b^2x(b+cx)^2} + \frac{3(8c^2d^2-8bcde+b^2e^2)\sqrt{d+ex}}{4b^4(b+cx)} - \frac{d(d+ex)^{3/2}}{2bx^2(b+cx)^2} - \frac{3\sqrt{d}(16c^2d^2-20bcde+5b^2e^2)\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4b^5} + \frac{3\sqrt{cd-be}(16c^2d^2-12bcde+b^2e^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{4b^5\sqrt{c}}$$

output
```
1/4*(2*b^2*e^2-13*b*c*d*e+12*c^2*d^2)*(e*x+d)^(1/2)/b^3/(c*x+b)^2+1/4*d*(-
7*b*e+8*c*d)*(e*x+d)^(1/2)/b^2/x/(c*x+b)^2+3/4*(b^2*e^2-8*b*c*d*e+8*c^2*d^
2)*(e*x+d)^(1/2)/b^4/(c*x+b)-1/2*d*(e*x+d)^(3/2)/b/x^2/(c*x+b)^2-3/4*d^(1/
2)*(5*b^2*e^2-20*b*c*d*e+16*c^2*d^2)*arctanh((e*x+d)^(1/2)/d^(1/2))/b^5+3/
4*(-b*e+c*d)^(1/2)*(b^2*e^2-12*b*c*d*e+16*c^2*d^2)*arctanh(c^(1/2)*(e*x+d)
^(1/2)/(-b*e+c*d)^(1/2))/b^5/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.79

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^3}\,dx = \frac{\frac{b\sqrt{d+ex}\left(24c^3d^2x^3+12bc^2dx^2(3d-2ex)+b^2cx\left(8d^2-37dex+3e^2x^2\right)+b^3\left(-2d^2-9dex+5e^2x^2\right)\right)}{x^2(b+cx)^2}+\frac{3\sqrt{-cd+be}\left(16c^2d^2\right.}{}}{4b^5}$$

input
```
Integrate[(d + e*x)^(5/2)/(b*x + c*x^2)^3,x]
```

output
```
((b*Sqrt[d + e*x]*(24*c^3*d^2*x^3 + 12*b*c^2*d*x^2*(3*d - 2*e*x) + b^2*c*x
*(8*d^2 - 37*d*e*x + 3*e^2*x^2) + b^3*(-2*d^2 - 9*d*e*x + 5*e^2*x^2)))/(x^
2*(b + c*x)^2) + (3*Sqrt[-(c*d) + b*e]*(16*c^2*d^2 - 12*b*c*d*e + b^2*e^2)
*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[-(c*d) + b*e]])/Sqrt[c] - 3*Sqrt[d]*(
16*c^2*d^2 - 20*b*c*d*e + 5*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(4*b^
5)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1164, 27, 1234, 27, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^3}dx$$

$$\Big\downarrow \; 1164$$

$$-\frac{\int \frac{3\sqrt{d+ex}(d(4cd-3be)+e(2cd-be)x)}{2(cx^2+bx)^2}dx}{2b^2}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{3\int \frac{\sqrt{d+ex}(d(4cd-3be)+e(2cd-be)x)}{(cx^2+bx)^2}dx}{4b^2}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \text{1234}$$

$$-\frac{3\left(-\frac{\int \frac{d\left(16c^2d^2-20bced+5b^2e^2\right)+e\left(8c^2d^2-8bced+b^2e^2\right)x}{2\sqrt{d+ex}\left(cx^2+bx\right)}dx}{b^2}-\frac{\sqrt{d+ex}\left(x\left(b^2e^2-8bcde+8c^2d^2\right)+bd(4cd-3be)\right)}{b^2\left(bx+cx^2\right)}\right)}{4b^2}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(-\frac{\int \frac{d\left(16c^2d^2-20bced+5b^2e^2\right)+e\left(8c^2d^2-8bced+b^2e^2\right)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{2b^2}-\frac{\sqrt{d+ex}\left(x\left(b^2e^2-8bcde+8c^2d^2\right)+bd(4cd-3be)\right)}{b^2\left(bx+cx^2\right)}\right)}{4b^2}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \text{1197}$$

$$-\frac{3\left(-\frac{\int \frac{e\left(4d(cd-be)(2cd-be)+\left(8c^2d^2-8bced+b^2e^2\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2}-\frac{\sqrt{d+ex}\left(x\left(b^2e^2-8bcde+8c^2d^2\right)+bd(4cd-3be)\right)}{b^2\left(bx+cx^2\right)}\right)}{4b^2}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{3\left(-\frac{e\int \frac{4d(cd-be)(2cd-be)+\left(8c^2d^2-8bced+b^2e^2\right)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2}-\frac{\sqrt{d+ex}\left(x\left(b^2e^2-8bcde+8c^2d^2\right)+bd(4cd-3be)\right)}{b^2\left(bx+cx^2\right)}\right)}{4b^2}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \text{1480}$$

$$-\frac{3\left(-\frac{e\left(\frac{cd\left(5b^2e^2-20bcde+16c^2d^2\right)\int\frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{(cd-be)\left(b^2e^2-12bcde+16c^2d^2\right)\int\frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{b^2}-\frac{\sqrt{d+ex}\left(x\left(b^2e^2-8bcd\cdots\right.\right.}{b^2(b\cdots}\right.}{4b^2}-\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$\downarrow$ 221

$$-\frac{3\left(-\frac{e\left(\frac{\sqrt{cd-be}\left(b^2e^2-12bcde+16c^2d^2\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{b\sqrt{c}e}-\frac{\sqrt{d}\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(5b^2e^2-20bcde+16c^2d^2\right)}{be}\right)}{b^2}-\frac{\sqrt{d+ex}\left(x\left(b^2e^2-8bcde+8\right.\right.}{b^2(bx+}\right.}{4b^2}$$

$$\frac{(d+ex)^{3/2}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

input
```
Int[(d + e*x)^(5/2)/(b*x + c*x^2)^3,x]
```

output
```
-1/2*((d + e*x)^(3/2)*(b*d + (2*c*d - b*e)*x))/(b^2*(b*x + c*x^2)^2) - (3*
(-((Sqrt[d + e*x]*(b*d*(4*c*d - 3*b*e) + (8*c^2*d^2 - 8*b*c*d*e + b^2*e^2)
*x))/(b^2*(b*x + c*x^2))) - (e*(-((Sqrt[d]*(16*c^2*d^2 - 20*b*c*d*e + 5*b^
2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*e)) + (Sqrt[c*d - b*e]*(16*c^2*d
^2 - 12*b*c*d*e + b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]
])/(b*Sqrt[c]*e)))/b^2))/(4*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1234
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p +
 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g
*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*
(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1
] && GtQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$-\frac{12\left(\left(-\frac{b^3e^3\sqrt{d}}{16}+c\,d^{\frac{3}{2}}\left(c^2d^2-\frac{7}{4}bcde+\frac{13}{16}b^2e^2\right)\right)x^2(cx+b)^2\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)+\frac{\left(\frac{15x^2(cx+b)^2d\left(b^2e^2-4bcde+\frac{16}{5}c^2d\cdots\right.}{2}\cdots\right.}{\cdots}\right.}{\sqrt{d}\cdots}$
derivativedivides	$2e^5\left(-\frac{d\left(\frac{\left(\frac{9}{8}b^2e^2-\frac{3}{2}bcde\right)(ex+d)^{\frac{3}{2}}+\left(-\frac{7}{8}d\,e^2b^2+\frac{3}{2}d^2ebc\right)\sqrt{ex+d}}{e^2x^2}+\frac{3\left(5b^2e^2-20bcde+16c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8\sqrt{d}}\right)}{b^5e^5}+\frac{(b\cdots}{\cdots}\right.$
default	$2e^5\left(-\frac{d\left(\frac{\left(\frac{9}{8}b^2e^2-\frac{3}{2}bcde\right)(ex+d)^{\frac{3}{2}}+\left(-\frac{7}{8}d\,e^2b^2+\frac{3}{2}d^2ebc\right)\sqrt{ex+d}}{e^2x^2}+\frac{3\left(5b^2e^2-20bcde+16c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8\sqrt{d}}\right)}{b^5e^5}+\frac{(b\cdots}{\cdots}\right.$
risch	$-\frac{d\sqrt{ex+d}\,(9bex-12cdx+2bd)}{4b^4x^2}-\frac{e\left(\frac{3\sqrt{d}\left(5b^2e^2-20bcde+16c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{eb}+\frac{8\left(\left(-\frac{3}{8}b^3c\,e^3+\frac{15}{8}d\,e^2b^2c^2-\frac{3}{2}b\,c^3\cdots\right.\right.}{\cdots}\right.}{\cdots}$

input
```
int((e*x+d)^(5/2)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-12/d^(1/2)*((-1/16*b^3*e^3*d^(1/2)+c*d^(3/2)*(c^2*d^2-7/4*b*c*d*e+13/16*b
^2*e^2))*x^2*(c*x+b)^2*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2))+1/24*(1
5/2*x^2*(c*x+b)^2*d*(b^2*e^2-4*b*c*d*e+16/5*c^2*d^2)*arctanh((e*x+d)^(1/2)
/d^(1/2))+(-5/2*e^2*(3/5*c*x+b)*x^2*b^2*d^(1/2)+(12*(b*c^2*e-c^3*d)*x^3+(-
18*b*c^2*d+37/2*c*e*b^2)*x^2+(-4*c*d*b^2+9/2*e*b^3)*x+b^3*d)*d^(3/2))*(e*x
+d)^(1/2)*b)*(c*(b*e-c*d))^(1/2))/(c*(b*e-c*d))^(1/2)/b^5/x^2/(c*x+b)^2
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1655, normalized size of antiderivative = 5.91

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
[1/8*(3*((16*c^4*d^2 - 12*b*c^3*d*e + b^2*c^2*e^2)*x^4 + 2*(16*b*c^3*d^2 -
 12*b^2*c^2*d*e + b^3*c*e^2)*x^3 + (16*b^2*c^2*d^2 - 12*b^3*c*d*e + b^4*e^
2)*x^2)*sqrt((c*d - b*e)/c)*log((c*e*x + 2*c*d - b*e + 2*sqrt(e*x + d)*c*s
qrt((c*d - b*e)/c))/(c*x + b)) + 3*((16*c^4*d^2 - 20*b*c^3*d*e + 5*b^2*c^2
*e^2)*x^4 + 2*(16*b*c^3*d^2 - 20*b^2*c^2*d*e + 5*b^3*c*e^2)*x^3 + (16*b^2*
c^2*d^2 - 20*b^3*c*d*e + 5*b^4*e^2)*x^2)*sqrt(d)*log((e*x - 2*sqrt(e*x + d
)*sqrt(d) + 2*d)/x) - 2*(2*b^4*d^2 - 3*(8*b*c^3*d^2 - 8*b^2*c^2*d*e + b^3*
c*e^2)*x^3 - (36*b^2*c^2*d^2 - 37*b^3*c*d*e + 5*b^4*e^2)*x^2 - (8*b^3*c*d^
2 - 9*b^4*d*e)*x)*sqrt(e*x + d))/(b^5*c^2*x^4 + 2*b^6*c*x^3 + b^7*x^2), 1/
8*(6*((16*c^4*d^2 - 12*b*c^3*d*e + b^2*c^2*e^2)*x^4 + 2*(16*b*c^3*d^2 - 12
*b^2*c^2*d*e + b^3*c*e^2)*x^3 + (16*b^2*c^2*d^2 - 12*b^3*c*d*e + b^4*e^2)*
x^2)*sqrt(-(c*d - b*e)/c)*arctan(-sqrt(e*x + d)*c*sqrt(-(c*d - b*e)/c)/(c*
d - b*e)) + 3*((16*c^4*d^2 - 20*b*c^3*d*e + 5*b^2*c^2*e^2)*x^4 + 2*(16*b*c
^3*d^2 - 20*b^2*c^2*d*e + 5*b^3*c*e^2)*x^3 + (16*b^2*c^2*d^2 - 20*b^3*c*d*
e + 5*b^4*e^2)*x^2)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) -
 2*(2*b^4*d^2 - 3*(8*b*c^3*d^2 - 8*b^2*c^2*d*e + b^3*c*e^2)*x^3 - (36*b^2*
c^2*d^2 - 37*b^3*c*d*e + 5*b^4*e^2)*x^2 - (8*b^3*c*d^2 - 9*b^4*d*e)*x)*sqr
t(e*x + d))/(b^5*c^2*x^4 + 2*b^6*c*x^3 + b^7*x^2), 1/8*(6*((16*c^4*d^2 - 2
0*b*c^3*d*e + 5*b^2*c^2*e^2)*x^4 + 2*(16*b*c^3*d^2 - 20*b^2*c^2*d*e + 5*b^
3*c*e^2)*x^3 + (16*b^2*c^2*d^2 - 20*b^3*c*d*e + 5*b^4*e^2)*x^2)*sqrt(-d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(5/2)/(c*x**2+b*x)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^3}\,dx = -\frac{3\,(16\,c^3d^3 - 28\,bc^2d^2e + 13\,b^2cde^2 - b^3e^3)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{4\,\sqrt{-c^2d+bce}b^5}$$
$$+ \frac{3\,(16\,c^2d^3 - 20\,bcd^2e + 5\,b^2de^2)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{4\,b^5\sqrt{-d}}$$
$$+ \frac{24\,(ex+d)^{\frac{7}{2}}c^3d^2e - 72\,(ex+d)^{\frac{5}{2}}c^3d^3e + 72\,(ex+d)^{\frac{3}{2}}c^3d^4e - 24\,\sqrt{ex+d}c^3d^5e - 24\,(ex+d)^{\frac{7}{2}}bc^2de^2 + 1}{}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
-3/4*(16*c^3*d^3 - 28*b*c^2*d^2*e + 13*b^2*c*d*e^2 - b^3*e^3)*arctan(sqrt(
e*x + d)*c/sqrt(-c^2*d + b*c*e))/(sqrt(-c^2*d + b*c*e)*b^5) + 3/4*(16*c^2*
d^3 - 20*b*c*d^2*e + 5*b^2*d*e^2)*arctan(sqrt(e*x + d)/sqrt(-d))/(b^5*sqrt
(-d)) + 1/4*(24*(e*x + d)^(7/2)*c^3*d^2*e - 72*(e*x + d)^(5/2)*c^3*d^3*e +
 72*(e*x + d)^(3/2)*c^3*d^4*e - 24*sqrt(e*x + d)*c^3*d^5*e - 24*(e*x + d)^
(7/2)*b*c^2*d*e^2 + 108*(e*x + d)^(5/2)*b*c^2*d^2*e^2 - 144*(e*x + d)^(3/2
)*b*c^2*d^3*e^2 + 60*sqrt(e*x + d)*b*c^2*d^4*e^2 + 3*(e*x + d)^(7/2)*b^2*c
*e^3 - 46*(e*x + d)^(5/2)*b^2*c*d*e^3 + 91*(e*x + d)^(3/2)*b^2*c*d^2*e^3 -
 48*sqrt(e*x + d)*b^2*c*d^3*e^3 + 5*(e*x + d)^(5/2)*b^3*e^4 - 19*(e*x + d)
^(3/2)*b^3*d*e^4 + 12*sqrt(e*x + d)*b^3*d^2*e^4)/(((e*x + d)^2*c - 2*(e*x
+ d)*c*d + c*d^2 + (e*x + d)*b*e - b*d*e)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 5.48 (sec) , antiderivative size = 910, normalized size of antiderivative = 3.25

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(5/2)/(b*x + c*x^2)^3,x)
```

output
```
(3*atanh((81*c^2*d^2*e^8*(c^2*d - b*c*e)^(1/2)*(d + e*x)^(1/2))/(8*((189*c
^3*d^3*e^8)/8 - (351*b*c^2*d^2*e^9)/32 - (27*c^4*d^4*e^7)/(2*b) + (27*b^2*
c*d*e^10)/32)) + (27*c^3*d^3*e^7*(c^2*d - b*c*e)^(1/2)*(d + e*x)^(1/2))/(2
*((27*c^4*d^4*e^7)/2 - (189*b*c^3*d^3*e^8)/8 + (351*b^2*c^2*d^2*e^9)/32 -
(27*b^3*c*d*e^10)/32)) + (27*c*d*e^9*(c^2*d - b*c*e)^(1/2)*(d + e*x)^(1/2)
)/(32*((351*c^2*d^2*e^9)/32 - (27*b*c*d*e^10)/32 - (189*c^3*d^3*e^8)/(8*b)
 + (27*c^4*d^4*e^7)/(2*b^2))))*(-c*(b*e - c*d))^(1/2)*(b^2*e^2 + 16*c^2*d^
2 - 12*b*c*d*e))/(4*b^5*c) - (3*d^(1/2)*atanh((135*c*d^(1/2)*e^10*(d + e*x
)^(1/2))/(32*((135*c*d*e^10)/32 - (675*c^2*d^2*e^9)/(32*b) + (243*c^3*d^3*
e^8)/(8*b^2) - (27*c^4*d^4*e^7)/(2*b^3))) + (675*c^2*d^(3/2)*e^9*(d + e*x)
^(1/2))/(32*((675*c^2*d^2*e^9)/32 - (135*b*c*d*e^10)/32 - (243*c^3*d^3*e^8
)/(8*b) + (27*c^4*d^4*e^7)/(2*b^2))) + (243*c^3*d^(5/2)*e^8*(d + e*x)^(1/2
))/(8*((243*c^3*d^3*e^8)/8 - (675*b*c^2*d^2*e^9)/32 - (27*c^4*d^4*e^7)/(2*
b) + (135*b^2*c*d*e^10)/32)) + (27*c^4*d^(7/2)*e^7*(d + e*x)^(1/2))/(2*((2
7*c^4*d^4*e^7)/2 - (243*b*c^3*d^3*e^8)/8 + (675*b^2*c^2*d^2*e^9)/32 - (135
*b^3*c*d*e^10)/32)))*(5*b^2*e^2 + 16*c^2*d^2 - 20*b*c*d*e))/(4*b^5) - (((d
 + e*x)^(3/2)*(19*b^3*d*e^4 - 72*c^3*d^4*e + 144*b*c^2*d^3*e^2 - 91*b^2*c*
d^2*e^3))/(4*b^4) + (3*(d + e*x)^(1/2)*(2*c^3*d^5*e - b^3*d^2*e^4 - 5*b*c^
2*d^4*e^2 + 4*b^2*c*d^3*e^3))/b^4 - ((b*e - 2*c*d)*(d + e*x)^(5/2)*(5*b^2*
e^3 + 36*c^2*d^2*e - 36*b*c*d*e^2))/(4*b^4) - (3*c*e*(d + e*x)^(7/2)*(b...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1078, normalized size of antiderivative = 3.85

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(5/2)/(c*x^2+b*x)^3,x)
```

output

```
(6*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)
))*b**4*e**2*x**2 - 72*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqr
t(c)*sqrt(b*e - c*d)))*b**3*c*d*e*x**2 + 12*sqrt(c)*sqrt(b*e - c*d)*atan((
sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*c*e**2*x**3 + 96*sqrt(c)*
sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c**
2*d**2*x**2 - 144*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*
sqrt(b*e - c*d)))*b**2*c**2*d*e*x**3 + 6*sqrt(c)*sqrt(b*e - c*d)*atan((sqr
t(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c**2*e**2*x**4 + 192*sqrt(c)
*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**3*
d**2*x**3 - 72*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqr
t(b*e - c*d)))*b*c**3*d*e*x**4 + 96*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d +
 e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*c**4*d**2*x**4 - 4*sqrt(d + e*x)*b**4*
c*d**2 - 18*sqrt(d + e*x)*b**4*c*d*e*x + 10*sqrt(d + e*x)*b**4*c*e**2*x**2
 + 16*sqrt(d + e*x)*b**3*c**2*d**2*x - 74*sqrt(d + e*x)*b**3*c**2*d*e*x**2
 + 6*sqrt(d + e*x)*b**3*c**2*e**2*x**3 + 72*sqrt(d + e*x)*b**2*c**3*d**2*x
**2 - 48*sqrt(d + e*x)*b**2*c**3*d*e*x**3 + 48*sqrt(d + e*x)*b*c**4*d**2*x
**3 + 15*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*b**4*c*e**2*x**2 - 60*sqrt(d
)*log(sqrt(d + e*x) - sqrt(d))*b**3*c**2*d*e*x**2 + 30*sqrt(d)*log(sqrt(d
+ e*x) - sqrt(d))*b**3*c**2*e**2*x**3 + 48*sqrt(d)*log(sqrt(d + e*x) - sqr
t(d))*b**2*c**3*d**2*x**2 - 120*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*b*...
```

3.125 $\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^3} dx$

Optimal result 970
Mathematica [A] (verified) 971
Rubi [A] (verified) 971
Maple [A] (verified) 974
Fricas [A] (verification not implemented) 975
Sympy [F(-1)] 976
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Giac [A] (verification not implemented) 977
Mupad [B] (verification not implemented) 978
Reduce [B] (verification not implemented) 979

Optimal result

Integrand size = 21, antiderivative size = 252

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^3} dx = \frac{c(12cd-7be)\sqrt{d+ex}}{4b^3(b+cx)^2} - \frac{d\sqrt{d+ex}}{2bx^2(b+cx)^2} + \frac{(8cd-5be)\sqrt{d+ex}}{4b^2x(b+cx)^2} + \frac{3c(2cd-be)\sqrt{d+ex}}{b^4(b+cx)} - \frac{3(16c^2d^2-12bcde+b^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4b^5\sqrt{d}} + \frac{3\sqrt{c}(16c^2d^2-20bcde+5b^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{4b^5\sqrt{cd-be}}$$

output
```
1/4*c*(-7*b*e+12*c*d)*(e*x+d)^(1/2)/b^3/(c*x+b)^2-1/2*d*(e*x+d)^(1/2)/b/x^
2/(c*x+b)^2+1/4*(-5*b*e+8*c*d)*(e*x+d)^(1/2)/b^2/x/(c*x+b)^2+3*c*(-b*e+2*c
*d)*(e*x+d)^(1/2)/b^4/(c*x+b)-3/4*(b^2*e^2-12*b*c*d*e+16*c^2*d^2)*arctanh(
(e*x+d)^(1/2)/d^(1/2))/b^5/d^(1/2)+3/4*c^(1/2)*(5*b^2*e^2-20*b*c*d*e+16*c^
2*d^2)*arctanh(c^(1/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b^5/(-b*e+c*d)^(1/2
)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^3}\,dx = \frac{\frac{b\sqrt{d+ex}(24c^3dx^3+b^2cx(8d-19ex)-12bc^2x^2(-3d+ex)-b^3(2d+5ex))}{x^2(b+cx)^2} - \frac{3\sqrt{c}(16c^2d^2-20bcde+5b^2e^2)\arctan\left(\frac{\sqrt{c}\sqrt{d}}{\sqrt{-cd}}\right.}{\sqrt{-cd+be}}}{4b^5}$$

input
```
Integrate[(d + e*x)^(3/2)/(b*x + c*x^2)^3,x]
```

output
```
((b*Sqrt[d + e*x]*(24*c^3*d*x^3 + b^2*c*x*(8*d - 19*e*x) - 12*b*c^2*x^2*(-
3*d + e*x) - b^3*(2*d + 5*e*x)))/(x^2*(b + c*x)^2) - (3*Sqrt[c]*(16*c^2*d^
2 - 20*b*c*d*e + 5*b^2*e^2)*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[-(c*d) + b
*e]])/Sqrt[-(c*d) + b*e] - (3*(16*c^2*d^2 - 12*b*c*d*e + b^2*e^2)*ArcTanh[
Sqrt[d + e*x]/Sqrt[d]])/Sqrt[d])/(4*b^5)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1164, 27, 1235, 27, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^3}dx$$

$$\Big\downarrow \text{ 1164}$$

$$-\frac{\int \frac{d(12cd-7be)+5e(2cd-be)x}{2\sqrt{d+ex}(cx^2+bx)^2}dx}{2b^2}-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{2b^2(bx+cx^2)^2}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\int \frac{d(12cd-7be)+5e(2cd-be)x}{\sqrt{d+ex}(cx^2+bx)^2}dx}{4b^2}-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{2b^2(bx+cx^2)^2}$$

$$\Big\downarrow \text{ 1235}$$

$$-\frac{-\frac{\int \frac{3d(cd-be)\left(16c^2d^2-12bced+b^2e^2+4ce(2cd-be)x\right)}{2\sqrt{d+ex}\left(cx^2+bx\right)}dx}{b^2d(cd-be)}-\frac{\sqrt{d+ex}(12cx(2cd-be)(cd-be)+b(12cd-7be)(cd-be))}{b^2(bx+cx^2)(cd-be)}}{4b^2}-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{3\int \frac{16c^2d^2-12bced+b^2e^2+4ce(2cd-be)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{2b^2}-\frac{\sqrt{d+ex}(12cx(2cd-be)(cd-be)+b(12cd-7be)(cd-be))}{b^2(bx+cx^2)(cd-be)}}{4b^2}-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \; 1197$$

$$-\frac{-\frac{3\int \frac{e\left(8c^2d^2-8bced+b^2e^2+4c(2cd-be)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2}-\frac{\sqrt{d+ex}(12cx(2cd-be)(cd-be)+b(12cd-7be)(cd-be))}{b^2(bx+cx^2)(cd-be)}}{4b^2}-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{3e\int \frac{8c^2d^2-8bced+b^2e^2+4c(2cd-be)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2}-\frac{\sqrt{d+ex}(12cx(2cd-be)(cd-be)+b(12cd-7be)(cd-be))}{b^2(bx+cx^2)(cd-be)}}{4b^2}-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \; 1480$$

$$-\frac{-\frac{3e\left(\frac{c\left(b^2e^2-12bcde+16c^2d^2\right)\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{c\left(5b^2e^2-20bcde+16c^2d^2\right)\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{b^2}-\frac{\sqrt{d+ex}(12cx(2cd-be)(cd-be)+}{b^2(bx+cx^2)(cd}}{4b^2}-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{-\frac{3e\left(\frac{\sqrt{c}\left(5b^2e^2-20bcde+16c^2d^2\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{be\sqrt{cd-be}}-\frac{\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)\left(b^2e^2-12bcde+16c^2d^2\right)}{b\sqrt{de}}\right)}{b^2}-\frac{\sqrt{d+ex}(12cx(2cd-be)(cd-be)+b(1}{b^2(bx+cx^2)(cd-be}}{4b^2}-\frac{\sqrt{d+ex}(x(2cd-be)+bd)}{2b^2\left(bx+cx^2\right)^2}$$

input
```
Int[(d + e*x)^(3/2)/(b*x + c*x^2)^3,x]
```

output
```
-1/2*(Sqrt[d + e*x]*(b*d + (2*c*d - b*e)*x))/(b^2*(b*x + c*x^2)^2) - (-((S
qrt[d + e*x]*(b*(12*c*d - 7*b*e)*(c*d - b*e) + 12*c*(c*d - b*e)*(2*c*d - b
*e)*x))/(b^2*(c*d - b*e)*(b*x + c*x^2))) - (3*e*(-(((16*c^2*d^2 - 12*b*c*d
*e + b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*Sqrt[d]*e)) + (Sqrt[c]*(1
6*c^2*d^2 - 20*b*c*d*e + 5*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c
*d - b*e]])/(b*e*Sqrt[c*d - b*e])))/b^2)/(4*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{12\left(x^2(cx+b)^2 c\left(\frac{5b^2e^2\sqrt{d}}{16}+c\left(cd-\frac{5be}{4}\right)d^{\frac{3}{2}}\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)+\frac{\left(\frac{3x^2(cx+b)^2\left(b^2e^2-12bcde+16c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex\cdots}}{\sqrt{\cdots}}\right)}{2}\cdots\right.}{\cdots}\right.}{\sqrt{d}\,\sqrt{c(be-cd)}\,b^5x^2(cx+b)^2\cdots}$
risch	$-\frac{\sqrt{ex+d}\,(5bex-12cdx+2bd)}{4b^4x^2}-\frac{e\left(-\frac{\left(-3b^2e^2+36bcde-48c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be\sqrt{d}}+8c\left(\frac{\left(\frac{7}{8}e^2b^2c-\frac{3}{2}c^2deb\right)(ex+d)^{\frac{3}{2}}+\frac{3b\cdots}{\cdots}}{((ex+d)c-\cdots}\cdots\right.\right.}{4b^4\cdots}$
derivativedivides	$2e^5\left(-\frac{\frac{\frac{be(5be-12cd)(ex+d)^{\frac{3}{2}}}{8}+\left(\frac{3}{2}d^2ebc-\frac{3}{8}d\,e^2b^2\right)\sqrt{ex+d}}{e^2x^2}+\frac{3\left(b^2e^2-12bcde+16c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8\sqrt{d}}}{b^5e^5}-\frac{c\left(\frac{\left(\frac{7}{8}e^2b^2c-\cdots\right.}{\cdots}\right.}{\cdots}\right.$
default	$2e^5\left(-\frac{\frac{\frac{be(5be-12cd)(ex+d)^{\frac{3}{2}}}{8}+\left(\frac{3}{2}d^2ebc-\frac{3}{8}d\,e^2b^2\right)\sqrt{ex+d}}{e^2x^2}+\frac{3\left(b^2e^2-12bcde+16c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8\sqrt{d}}}{b^5e^5}-\frac{c\left(\frac{\left(\frac{7}{8}e^2b^2c-\cdots\right.}{\cdots}\right.}{\cdots}\right.$

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-12/d^(1/2)*(x^2*(c*x+b)^2*c*(5/16*b^2*e^2*d^(1/2)+c*(c*d-5/4*b*e)*d^(3/2)
)*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2))+1/24*(3/2*x^2*(c*x+b)^2*(b^2
*e^2-12*b*c*d*e+16*c^2*d^2)*arctanh((e*x+d)^(1/2)/d^(1/2))+(5/2*(3*c*x+b)*
e*x*b*(4/5*c*x+b)*d^(1/2)+(-6*c^2*x^2-6*b*c*x+b^2)*d^(3/2)*(2*c*x+b))*(e*x
+d)^(1/2)*b)*(c*(b*e-c*d))^(1/2))/(c*(b*e-c*d))^(1/2)/b^5/x^2/(c*x+b)^2
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 1628, normalized size of antiderivative = 6.46

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
[1/8*(3*((16*c^4*d^3 - 20*b*c^3*d^2*e + 5*b^2*c^2*d*e^2)*x^4 + 2*(16*b*c^3
*d^3 - 20*b^2*c^2*d^2*e + 5*b^3*c*d*e^2)*x^3 + (16*b^2*c^2*d^3 - 20*b^3*c*
d^2*e + 5*b^4*d*e^2)*x^2)*sqrt(c/(c*d - b*e))*log((c*e*x + 2*c*d - b*e + 2
*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)) + 3*((16*c^4*d^
2 - 12*b*c^3*d*e + b^2*c^2*e^2)*x^4 + 2*(16*b*c^3*d^2 - 12*b^2*c^2*d*e + b
^3*c*e^2)*x^3 + (16*b^2*c^2*d^2 - 12*b^3*c*d*e + b^4*e^2)*x^2)*sqrt(d)*log
((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(2*b^4*d^2 - 12*(2*b*c^3*d^2
 - b^2*c^2*d*e)*x^3 - (36*b^2*c^2*d^2 - 19*b^3*c*d*e)*x^2 - (8*b^3*c*d^2 -
 5*b^4*d*e)*x)*sqrt(e*x + d))/(b^5*c^2*d*x^4 + 2*b^6*c*d*x^3 + b^7*d*x^2),
 -1/8*(6*((16*c^4*d^3 - 20*b*c^3*d^2*e + 5*b^2*c^2*d*e^2)*x^4 + 2*(16*b*c^
3*d^3 - 20*b^2*c^2*d^2*e + 5*b^3*c*d*e^2)*x^3 + (16*b^2*c^2*d^3 - 20*b^3*c
*d^2*e + 5*b^4*d*e^2)*x^2)*sqrt(-c/(c*d - b*e))*arctan(sqrt(e*x + d)*sqrt(
-c/(c*d - b*e))) - 3*((16*c^4*d^2 - 12*b*c^3*d*e + b^2*c^2*e^2)*x^4 + 2*(1
6*b*c^3*d^2 - 12*b^2*c^2*d*e + b^3*c*e^2)*x^3 + (16*b^2*c^2*d^2 - 12*b^3*c
*d*e + b^4*e^2)*x^2)*sqrt(d)*log((e*x - 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x)
+ 2*(2*b^4*d^2 - 12*(2*b*c^3*d^2 - b^2*c^2*d*e)*x^3 - (36*b^2*c^2*d^2 - 19
*b^3*c*d*e)*x^2 - (8*b^3*c*d^2 - 5*b^4*d*e)*x)*sqrt(e*x + d))/(b^5*c^2*d*x
^4 + 2*b^6*c*d*x^3 + b^7*d*x^2), 1/8*(6*((16*c^4*d^2 - 12*b*c^3*d*e + b^2*
c^2*e^2)*x^4 + 2*(16*b*c^3*d^2 - 12*b^2*c^2*d*e + b^3*c*e^2)*x^3 + (16*b^2
*c^2*d^2 - 12*b^3*c*d*e + b^4*e^2)*x^2)*sqrt(-d)*arctan(sqrt(-d)/sqrt(e...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(3/2)/(c*x**2+b*x)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.49

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^3}\,dx = -\frac{3\,(16\,c^3d^2-20\,bc^2de+5\,b^2ce^2)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{4\sqrt{-c^2d+bce}b^5} + \frac{3\,(16\,c^2d^2-12\,bcde+b^2e^2)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{4\,b^5\sqrt{-d}} + \frac{24\,(ex+d)^{\frac{7}{2}}c^3de-72\,(ex+d)^{\frac{5}{2}}c^3d^2e+72\,(ex+d)^{\frac{3}{2}}c^3d^3e-24\,\sqrt{ex+d}c^3d^4e-12\,(ex+d)^{\frac{7}{2}}bc^2e^2+72\,(}{4}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
-3/4*(16*c^3*d^2 - 20*b*c^2*d*e + 5*b^2*c*e^2)*arctan(sqrt(e*x + d)*c/sqrt
(-c^2*d + b*c*e))/(sqrt(-c^2*d + b*c*e)*b^5) + 3/4*(16*c^2*d^2 - 12*b*c*d*
e + b^2*e^2)*arctan(sqrt(e*x + d)/sqrt(-d))/(b^5*sqrt(-d)) + 1/4*(24*(e*x
+ d)^(7/2)*c^3*d*e - 72*(e*x + d)^(5/2)*c^3*d^2*e + 72*(e*x + d)^(3/2)*c^3
*d^3*e - 24*sqrt(e*x + d)*c^3*d^4*e - 12*(e*x + d)^(7/2)*b*c^2*e^2 + 72*(e
*x + d)^(5/2)*b*c^2*d*e^2 - 108*(e*x + d)^(3/2)*b*c^2*d^2*e^2 + 48*sqrt(e*
x + d)*b*c^2*d^3*e^2 - 19*(e*x + d)^(5/2)*b^2*c*e^3 + 46*(e*x + d)^(3/2)*b
^2*c*d*e^3 - 27*sqrt(e*x + d)*b^2*c*d^2*e^3 - 5*(e*x + d)^(3/2)*b^3*e^4 +
3*sqrt(e*x + d)*b^3*d*e^4)/(((e*x + d)^2*c - 2*(e*x + d)*c*d + c*d^2 + (e*
x + d)*b*e - b*d*e)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 5.73 (sec) , antiderivative size = 1880, normalized size of antiderivative = 7.46

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(3/2)/(b*x + c*x^2)^3,x)
```

output
```
((3*(d + e*x)^(1/2)*(b^3*d*e^4 - 8*c^3*d^4*e + 16*b*c^2*d^3*e^2 - 9*b^2*c*
d^2*e^3))/(4*b^4) - ((d + e*x)^(3/2)*(5*b^3*e^4 - 72*c^3*d^3*e + 108*b*c^2
*d^2*e^2 - 46*b^2*c*d*e^3))/(4*b^4) - (e*(d + e*x)^(5/2)*(72*c^3*d^2 + 19*
b^2*c*e^2 - 72*b*c^2*d*e))/(4*b^4) + (3*c*e*(2*c^2*d - b*c*e)*(d + e*x)^(7
/2))/b^4)/(c^2*(d + e*x)^4 - (d + e*x)*(4*c^2*d^3 + 2*b^2*d*e^2 - 6*b*c*d^
2*e) - (4*c^2*d - 2*b*c*e)*(d + e*x)^3 + (d + e*x)^2*(b^2*e^2 + 6*c^2*d^2
- 6*b*c*d*e) + c^2*d^4 + b^2*d^2*e^2 - 2*b*c*d^3*e) - (3*atanh((27*c^2*e^9
*(d + e*x)^(1/2))/(32*d^(3/2)*((27*c^2*e^9)/(32*d) - (81*c^3*e^8)/(8*b) +
(27*c^4*d*e^7)/(2*b^2))) - (81*c^3*e^8*(d + e*x)^(1/2))/(8*d^(1/2)*((27*b*
c^2*e^9)/(32*d) - (81*c^3*e^8)/8 + (27*c^4*d*e^7)/(2*b))) + (27*c^4*d^(1/2
)*e^7*(d + e*x)^(1/2))/(2*((27*c^4*d*e^7)/2 - (81*b*c^3*e^8)/8 + (27*b^2*c
^2*e^9)/(32*d))))*(b^2*e^2 + 16*c^2*d^2 - 12*b*c*d*e))/(4*b^5*d^(1/2)) + (
atan((((((d + e*x)^(1/2)*(117*b^4*c^3*e^6 + 2304*c^7*d^4*e^2 - 4608*b*c^6*
d^3*e^3 - 1008*b^3*c^4*d*e^5 + 3312*b^2*c^5*d^2*e^4))/(4*b^8) - (3*(-c*(b*
e - c*d))^(1/2)*((3*b^12*c^2*e^5 - 24*b^11*c^3*d*e^4 + 24*b^10*c^4*d^2*e^3
)/b^12 - (3*(32*b^11*c^2*e^3 - 64*b^10*c^3*d*e^2)*(-c*(b*e - c*d))^(1/2)*(
d + e*x)^(1/2)*(5*b^2*e^2 + 16*c^2*d^2 - 20*b*c*d*e))/(32*b^8*(b^6*e - b^5
*c*d)))*(5*b^2*e^2 + 16*c^2*d^2 - 20*b*c*d*e))/(8*(b^6*e - b^5*c*d)))*(-c*
(b*e - c*d))^(1/2)*(5*b^2*e^2 + 16*c^2*d^2 - 20*b*c*d*e)*3i)/(8*(b^6*e - b
^5*c*d)) + ((((d + e*x)^(1/2)*(117*b^4*c^3*e^6 + 2304*c^7*d^4*e^2 - 460...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1327, normalized size of antiderivative = 5.27

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x)^3,x)
```

output
```
( - 30*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e -
c*d)))*b**4*d*e**2*x**2 + 120*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*
c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*c*d**2*e*x**2 - 60*sqrt(c)*sqrt(b*e - c
*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*c*d*e**2*x**3 -
 96*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d
)))*b**2*c**2*d**3*x**2 + 240*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*
c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c**2*d**2*e*x**3 - 30*sqrt(c)*sqrt(b*e
- c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c**2*d*e**2*
x**4 - 192*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*
e - c*d)))*b*c**3*d**3*x**3 + 120*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e
*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**3*d**2*e*x**4 - 96*sqrt(c)*sqrt(b*e
 - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*c**4*d**3*x**4 -
 4*sqrt(d + e*x)*b**5*d**2*e - 10*sqrt(d + e*x)*b**5*d*e**2*x + 4*sqrt(d +
 e*x)*b**4*c*d**3 + 26*sqrt(d + e*x)*b**4*c*d**2*e*x - 38*sqrt(d + e*x)*b*
*4*c*d*e**2*x**2 - 16*sqrt(d + e*x)*b**3*c**2*d**3*x + 110*sqrt(d + e*x)*b
**3*c**2*d**2*e*x**2 - 24*sqrt(d + e*x)*b**3*c**2*d*e**2*x**3 - 72*sqrt(d
+ e*x)*b**2*c**3*d**3*x**2 + 72*sqrt(d + e*x)*b**2*c**3*d**2*e*x**3 - 48*s
qrt(d + e*x)*b*c**4*d**3*x**3 + 3*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*b**
5*e**3*x**2 - 39*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*b**4*c*d*e**2*x**2 +
 6*sqrt(d)*log(sqrt(d + e*x) - sqrt(d))*b**4*c*e**3*x**3 + 84*sqrt(d)*l...
```

3.126 $\int \frac{\sqrt{d+ex}}{(bx+cx^2)^3} \, dx$

Optimal result 981
Mathematica [A] (verified) 982
Rubi [A] (verified) 982
Maple [A] (verified) 985
Fricas [B] (verification not implemented) 987
Sympy [F] 988
Maxima [F(-2)] 988
Giac [A] (verification not implemented) 988
Mupad [B] (verification not implemented) 989
Reduce [B] (verification not implemented) 990

Optimal result

Integrand size = 21, antiderivative size = 286

$$\begin{aligned}\int \frac{\sqrt{d+ex}}{(bx+cx^2)^3} \, dx &= \frac{c(12cd-be)\sqrt{d+ex}}{4b^3d(b+cx)^2} - \frac{\sqrt{d+ex}}{2bx^2(b+cx)^2} \\ &+ \frac{(8cd-be)\sqrt{d+ex}}{4b^2dx(b+cx)^2} + \frac{c(24c^2d^2-24bcde+b^2e^2)\sqrt{d+ex}}{4b^4d(cd-be)(b+cx)} \\ &- \frac{(48c^2d^2-12bcde-b^2e^2)\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4b^5d^{3/2}} \\ &+ \frac{c^{3/2}(48c^2d^2-84bcde+35b^2e^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{4b^5(cd-be)^{3/2}}\end{aligned}$$

output
```
1/4*c*(-b*e+12*c*d)*(e*x+d)^(1/2)/b^3/d/(c*x+b)^2-1/2*(e*x+d)^(1/2)/b/x^2/
(c*x+b)^2+1/4*(-b*e+8*c*d)*(e*x+d)^(1/2)/b^2/d/x/(c*x+b)^2+1/4*c*(b^2*e^2-
24*b*c*d*e+24*c^2*d^2)*(e*x+d)^(1/2)/b^4/d/(-b*e+c*d)/(c*x+b)-1/4*(-b^2*e^
2-12*b*c*d*e+48*c^2*d^2)*arctanh((e*x+d)^(1/2)/d^(1/2))/b^5/d^(3/2)+1/4*c^
(3/2)*(35*b^2*e^2-84*b*c*d*e+48*c^2*d^2)*arctanh(c^(1/2)*(e*x+d)^(1/2)/(-b
*e+c*d)^(1/2))/b^5/(-b*e+c*d)^(3/2)
```

Mathematica [A] (verified)

Time = 1.50 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^3}\,dx$$
$$= \frac{\frac{b\sqrt{d+ex}(24c^4d^2x^3+12bc^3dx^2(3d-2ex)+b^4e(2d+ex)+b^2c^2x(8d^2-37dex+e^2x^2)+b^3c(-2d^2-9dex+2e^2x^2))}{d(cd-be)x^2(b+cx)^2} + \frac{c^{3/2}(48c^2d^2-84bcde+35b^2e^2}{(-cd+be)^3}}{4b^5}$$

input
```
Integrate[Sqrt[d + e*x]/(b*x + c*x^2)^3,x]
```

output
```
((b*Sqrt[d + e*x]*(24*c^4*d^2*x^3 + 12*b*c^3*d*x^2*(3*d - 2*e*x) + b^4*e*(
2*d + e*x) + b^2*c^2*x*(8*d^2 - 37*d*e*x + e^2*x^2) + b^3*c*(-2*d^2 - 9*d*
e*x + 2*e^2*x^2)))/(d*(c*d - b*e)*x^2*(b + c*x)^2) + (c^(3/2)*(48*c^2*d^2
- 84*b*c*d*e + 35*b^2*e^2)*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[-(c*d) + b*
e]])/(-(c*d) + b*e)^(3/2) + ((-48*c^2*d^2 + 12*b*c*d*e + b^2*e^2)*ArcTanh[
Sqrt[d + e*x]/Sqrt[d]])/d^(3/2))/(4*b^5)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.98, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1163, 27, 1235, 27, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{1163}$$

$$\frac{\int -\frac{12cd-be+10cex}{2\sqrt{d+ex}(cx^2+bx)^2}dx}{2b^2} - \frac{(b+2cx)\sqrt{d+ex}}{2b^2(bx+cx^2)^2}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{12cd-be+10cex}{\sqrt{d+ex}\left(cx^2+bx\right)^2}dx}{4b^2}-\frac{(b+2cx)\sqrt{d+ex}}{2b^2\left(bx+cx^2\right)^2}$$

$\Big\downarrow$ 1235

$$-\frac{-\frac{\int \frac{(cd-be)\left(48c^2d^2-12bced-b^2e^2\right)+ce\left(24c^2d^2-24bced+b^2e^2\right)x}{2\sqrt{d+ex}\left(cx^2+bx\right)}dx}{b^2d(cd-be)}-\frac{\sqrt{d+ex}\left(cx\left(b^2e^2-24bcde+24c^2d^2\right)+b(cd-be)(12cd-be)\right)}{b^2d\left(bx+cx^2\right)(cd-be)}}{4b^2}-\frac{(b+2cx)\sqrt{d+ex}}{2b^2\left(bx+cx^2\right)^2}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{\int \frac{(cd-be)\left(48c^2d^2-12bced-b^2e^2\right)+ce\left(24c^2d^2-24bced+b^2e^2\right)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{2b^2d(cd-be)}-\frac{\sqrt{d+ex}\left(cx\left(b^2e^2-24bcde+24c^2d^2\right)+b(cd-be)(12cd-be)\right)}{b^2d\left(bx+cx^2\right)(cd-be)}}{4b^2}-\frac{(b+2cx)\sqrt{d+ex}}{2b^2\left(bx+cx^2\right)^2}$$

$\Big\downarrow$ 1197

$$-\frac{-\frac{\int \frac{e\left((2cd-be)\left(12c^2d^2-12bced-b^2e^2\right)+c\left(24c^2d^2-24bced+b^2e^2\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2d(cd-be)}-\frac{\sqrt{d+ex}\left(cx\left(b^2e^2-24bcde+24c^2d^2\right)+b(cd-be)(12cd-be)\right)}{b^2d\left(bx+cx^2\right)(cd-be)}}{4b^2}-\frac{(b+2cx)\sqrt{d+ex}}{2b^2\left(bx+cx^2\right)^2}$$

$\Big\downarrow$ 27

$$-\frac{-\frac{e\int \frac{(2cd-be)\left(12c^2d^2-12bced-b^2e^2\right)+c\left(24c^2d^2-24bced+b^2e^2\right)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2d(cd-be)}-\frac{\sqrt{d+ex}\left(cx\left(b^2e^2-24bcde+24c^2d^2\right)+b(cd-be)(12cd-be)\right)}{b^2d\left(bx+cx^2\right)(cd-be)}}{4b^2}-\frac{(b+2cx)\sqrt{d+ex}}{2b^2\left(bx+cx^2\right)^2}$$

$\Big\downarrow$ 1480

$$-\frac{-\frac{e\left(\frac{c(cd-be)\left(-b^2e^2-12bcde+48c^2d^2\right)\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{c^2d\left(35b^2e^2-84bcde+48c^2d^2\right)\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{b^2d(cd-be)}-\frac{\sqrt{d+ex}\left(cx\left(b^2e^2-24b\cdots\right.\right.}{b^2d\cdots}}{4b^2}$$

$$\frac{(b+2cx)\sqrt{d+ex}}{2b^2\left(bx+cx^2\right)^2}$$

$\downarrow$ 221

$$-\frac{-\frac{e\left(\frac{c^{3/2}d\left(35b^2e^2-84bcde+48c^2d^2\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{be\sqrt{cd-be}}-\frac{\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(cd-be)\left(-b^2e^2-12bcde+48c^2d^2\right)}{b\sqrt{de}}\right)}{b^2d(cd-be)}-\frac{\sqrt{d+ex}\left(cx\left(b^2e^2-24bcd\right.\right.}{b^2d(b}}{4b^2}$$
$$\frac{(b+2cx)\sqrt{d+ex}}{2b^2\left(bx+cx^2\right)^2}$$

input
```
Int[Sqrt[d + e*x]/(b*x + c*x^2)^3,x]
```

output
```
-1/2*((b + 2*c*x)*Sqrt[d + e*x])/(b^2*(b*x + c*x^2)^2) - (-((Sqrt[d + e*x]
*(b*(c*d - b*e)*(12*c*d - b*e) + c*(24*c^2*d^2 - 24*b*c*d*e + b^2*e^2)*x))
/(b^2*d*(c*d - b*e)*(b*x + c*x^2))) - (e*(-((((c*d - b*e)*(48*c^2*d^2 - 12*
b*c*d*e - b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*Sqrt[d]*e)) + (c^(3/
2)*d*(48*c^2*d^2 - 84*b*c*d*e + 35*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x]
)/Sqrt[c*d - b*e]])/(b*e*Sqrt[c*d - b*e])))/(b^2*d*(c*d - b*e)))/(4*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1163
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] - Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1
)*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p +
1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 0] && (LtQ[
m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c, d,
e, m, p, x]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\sqrt{ex+d}\,(bex-12cdx+2bd)}{4d\,b^4x^2}-\frac{e\left(-\frac{\left(b^2e^2+12bcde-48c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be\sqrt{d}}-\frac{8c^2d\left(\frac{\frac{cbe(11be-12cd)(ex+d)^{\frac{3}{2}}}{8be-8cd}+\frac{be(13b}{}}{((ex+d)c+be-cd)^2}\right.}{}\right.}{4b^4d}$
derivativedivides	$2e^5\left(-\frac{\frac{\frac{be(be-12cd)(ex+d)^{\frac{3}{2}}}{8d}+\left(\frac{1}{8}b^2e^2+\frac{3}{2}bcde\right)\sqrt{ex+d}}{e^2x^2}-\frac{\left(b^2e^2+12bcde-48c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8d^{\frac{3}{2}}}}{b^5e^5}+\frac{c^2\left(\frac{cbe(11be-12c}{8be-8}\right.}{}\right.$
default	$2e^5\left(-\frac{\frac{\frac{be(be-12cd)(ex+d)^{\frac{3}{2}}}{8d}+\left(\frac{1}{8}b^2e^2+\frac{3}{2}bcde\right)\sqrt{ex+d}}{e^2x^2}-\frac{\left(b^2e^2+12bcde-48c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8d^{\frac{3}{2}}}}{b^5e^5}+\frac{c^2\left(\frac{cbe(11be-12c}{8be-8}\right.}{}\right.$
pseudoelliptic	$\frac{12x^2(cx+b)^2\left(c^2d^2-\frac{7}{4}bcde+\frac{35}{48}b^2e^2\right)c^2d^{\frac{5}{2}}\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)+\frac{\sqrt{c(be-cd)}\left(\frac{d\,x^2(cx+b)^2(be-cd)\left(b^2e^2+12bcde-48c^2d^2\right)}{2}\right.}{}}{\sqrt{c(be-cd)}\,d}$

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(e*x+d)^(1/2)*(b*e*x-12*c*d*x+2*b*d)/d/b^4/x^2-1/4/b^4/d*e*(-1/b/e*(b
^2*e^2+12*b*c*d*e-48*c^2*d^2)/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2))-8*c^2
*d/b/e*((1/8*c*b*e*(11*b*e-12*c*d)/(b*e-c*d)*(e*x+d)^(3/2)+1/8*b*e*(13*b*e
-12*c*d)*(e*x+d)^(1/2))/((e*x+d)*c+b*e-c*d)^2+1/8*(35*b^2*e^2-84*b*c*d*e+4
8*c^2*d^2)/(b*e-c*d)/(c*(b*e-c*d))^(1/2)*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*
d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 536 vs. $2(250) = 500$.

Time = 0.29 (sec) , antiderivative size = 2220, normalized size of antiderivative = 7.76

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
[-1/8*(((48*c^5*d^4 - 84*b*c^4*d^3*e + 35*b^2*c^3*d^2*e^2)*x^4 + 2*(48*b*c
^4*d^4 - 84*b^2*c^3*d^3*e + 35*b^3*c^2*d^2*e^2)*x^3 + (48*b^2*c^3*d^4 - 84
*b^3*c^2*d^3*e + 35*b^4*c*d^2*e^2)*x^2)*sqrt(c/(c*d - b*e))*log((c*e*x + 2
*c*d - b*e - 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)) +
 ((48*c^5*d^3 - 60*b*c^4*d^2*e + 11*b^2*c^3*d*e^2 + b^3*c^2*e^3)*x^4 + 2*(
48*b*c^4*d^3 - 60*b^2*c^3*d^2*e + 11*b^3*c^2*d*e^2 + b^4*c*e^3)*x^3 + (48*
b^2*c^3*d^3 - 60*b^3*c^2*d^2*e + 11*b^4*c*d*e^2 + b^5*e^3)*x^2)*sqrt(d)*lo
g((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(2*b^4*c*d^3 - 2*b^5*d^2*e
- (24*b*c^4*d^3 - 24*b^2*c^3*d^2*e + b^3*c^2*d*e^2)*x^3 - (36*b^2*c^3*d^3
- 37*b^3*c^2*d^2*e + 2*b^4*c*d*e^2)*x^2 - (8*b^3*c^2*d^3 - 9*b^4*c*d^2*e +
 b^5*d*e^2)*x)*sqrt(e*x + d))/((b^5*c^3*d^3 - b^6*c^2*d^2*e)*x^4 + 2*(b^6*
c^2*d^3 - b^7*c*d^2*e)*x^3 + (b^7*c*d^3 - b^8*d^2*e)*x^2), -1/8*(2*((48*c^
5*d^4 - 84*b*c^4*d^3*e + 35*b^2*c^3*d^2*e^2)*x^4 + 2*(48*b*c^4*d^4 - 84*b^
2*c^3*d^3*e + 35*b^3*c^2*d^2*e^2)*x^3 + (48*b^2*c^3*d^4 - 84*b^3*c^2*d^3*e
 + 35*b^4*c*d^2*e^2)*x^2)*sqrt(-c/(c*d - b*e))*arctan(sqrt(e*x + d)*sqrt(-
c/(c*d - b*e))) + ((48*c^5*d^3 - 60*b*c^4*d^2*e + 11*b^2*c^3*d*e^2 + b^3*c
^2*e^3)*x^4 + 2*(48*b*c^4*d^3 - 60*b^2*c^3*d^2*e + 11*b^3*c^2*d*e^2 + b^4*
c*e^3)*x^3 + (48*b^2*c^3*d^3 - 60*b^3*c^2*d^2*e + 11*b^4*c*d*e^2 + b^5*e^3
)*x^2)*sqrt(d)*log((e*x + 2*sqrt(e*x + d)*sqrt(d) + 2*d)/x) + 2*(2*b^4*c*d
^3 - 2*b^5*d^2*e - (24*b*c^4*d^3 - 24*b^2*c^3*d^2*e + b^3*c^2*d*e^2)*x^...
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^3}\,dx = \int \frac{\sqrt{d+ex}}{x^3(b+cx)^3}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(c*x**2+b*x)**3,x)
```

output
```
Integral(sqrt(d + e*x)/(x**3*(b + c*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 489, normalized size of antiderivative = 1.71

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^3}\,dx = -\frac{(48\,c^4d^2 - 84\,bc^3de + 35\,b^2c^2e^2)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{4\,(b^5cd - b^6e)\sqrt{-c^2d+bce}} + \frac{24\,(ex+d)^{\frac{7}{2}}c^4d^2e - 72\,(ex+d)^{\frac{5}{2}}c^4d^3e + 72\,(ex+d)^{\frac{3}{2}}c^4d^4e - 24\,\sqrt{ex+d}c^4d^5e - 24\,(ex+d)^{\frac{7}{2}}bc^3de^2 \cdots}{\cdots} + \frac{(48\,c^2d^2 - 12\,bcde - b^2e^2)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{4\,b^5\sqrt{-d}d}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
-1/4*(48*c^4*d^2 - 84*b*c^3*d*e + 35*b^2*c^2*e^2)*arctan(sqrt(e*x + d)*c/s
qrt(-c^2*d + b*c*e))/((b^5*c*d - b^6*e)*sqrt(-c^2*d + b*c*e)) + 1/4*(24*(e
*x + d)^(7/2)*c^4*d^2*e - 72*(e*x + d)^(5/2)*c^4*d^3*e + 72*(e*x + d)^(3/2
)*c^4*d^4*e - 24*sqrt(e*x + d)*c^4*d^5*e - 24*(e*x + d)^(7/2)*b*c^3*d*e^2
+ 108*(e*x + d)^(5/2)*b*c^3*d^2*e^2 - 144*(e*x + d)^(3/2)*b*c^3*d^3*e^2 +
60*sqrt(e*x + d)*b*c^3*d^4*e^2 + (e*x + d)^(7/2)*b^2*c^2*e^3 - 40*(e*x + d
)^(5/2)*b^2*c^2*d*e^3 + 85*(e*x + d)^(3/2)*b^2*c^2*d^2*e^3 - 46*sqrt(e*x +
 d)*b^2*c^2*d^3*e^3 + 2*(e*x + d)^(5/2)*b^3*c*e^4 - 13*(e*x + d)^(3/2)*b^3
*c*d*e^4 + 9*sqrt(e*x + d)*b^3*c*d^2*e^4 + (e*x + d)^(3/2)*b^4*e^5 + sqrt(
e*x + d)*b^4*d*e^5)/((b^4*c*d^2 - b^5*d*e)*((e*x + d)^2*c - 2*(e*x + d)*c*
d + c*d^2 + (e*x + d)*b*e - b*d*e)^2) + 1/4*(48*c^2*d^2 - 12*b*c*d*e - b^2
*e^2)*arctan(sqrt(e*x + d)/sqrt(-d))/(b^5*sqrt(-d)*d)
```

Mupad [B] (verification not implemented)

Time = 6.92 (sec) , antiderivative size = 4815, normalized size of antiderivative = 16.84

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((d + e*x)^(1/2)/(b*x + c*x^2)^3,x)
```

output
```
(((d + e*x)^(3/2)*(b^4*e^5 + 72*c^4*d^4*e - 144*b*c^3*d^3*e^2 + 85*b^2*c^2
*d^2*e^3 - 13*b^3*c*d*e^4))/(4*b^4*(c*d^2 - b*d*e)) - ((d + e*x)^(1/2)*(b^
3*e^4 + 24*c^3*d^3*e - 36*b*c^2*d^2*e^2 + 10*b^2*c*d*e^3))/(4*b^4) + ((b*e
 - 2*c*d)*(d + e*x)^(5/2)*(b^2*c*e^3 + 18*c^3*d^2*e - 18*b*c^2*d*e^2))/(2*
b^4*(c*d^2 - b*d*e)) + (c*e*(d + e*x)^(7/2)*(24*c^3*d^2 + b^2*c*e^2 - 24*b
*c^2*d*e))/(4*b^4*(c*d^2 - b*d*e)))/(c^2*(d + e*x)^4 - (d + e*x)*(4*c^2*d^
3 + 2*b^2*d*e^2 - 6*b*c*d^2*e) - (4*c^2*d - 2*b*c*e)*(d + e*x)^3 + (d + e*
x)^2*(b^2*e^2 + 6*c^2*d^2 - 6*b*c*d*e) + c^2*d^4 + b^2*d^2*e^2 - 2*b*c*d^3
*e) - (atan((((((d + e*x)^(1/2)*(b^6*c^3*e^8 + 4608*c^9*d^6*e^2 - 13824*b*
c^8*d^5*e^3 + 22*b^5*c^4*d*e^7 + 15072*b^2*c^7*d^4*e^4 - 7104*b^3*c^6*d^3*
e^5 + 1226*b^4*c^5*d^2*e^6))/(8*(b^8*c^2*d^4 + b^10*d^2*e^2 - 2*b^9*c*d^3*
e)) + (((b^14*c^2*d*e^7 - 24*b^10*c^6*d^5*e^3 + 60*b^11*c^5*d^4*e^4 - 46*b
^12*c^4*d^3*e^5 + 9*b^13*c^3*d^2*e^6)/(b^12*c^2*d^4 + b^14*d^2*e^2 - 2*b^1
3*c*d^3*e) - ((d + e*x)^(1/2)*(b^2*e^2 - 48*c^2*d^2 + 12*b*c*d*e)*(128*b^1
0*c^5*d^5*e^2 - 320*b^11*c^4*d^4*e^3 + 256*b^12*c^3*d^3*e^4 - 64*b^13*c^2*
d^2*e^5))/(64*b^5*(d^3)^(1/2)*(b^8*c^2*d^4 + b^10*d^2*e^2 - 2*b^9*c*d^3*e)
))*(b^2*e^2 - 48*c^2*d^2 + 12*b*c*d*e))/(8*b^5*(d^3)^(1/2)))*(b^2*e^2 - 48
*c^2*d^2 + 12*b*c*d*e)*1i)/(8*b^5*(d^3)^(1/2)) + ((((d + e*x)^(1/2)*(b^6*c
^3*e^8 + 4608*c^9*d^6*e^2 - 13824*b*c^8*d^5*e^3 + 22*b^5*c^4*d*e^7 + 15072
*b^2*c^7*d^4*e^4 - 7104*b^3*c^6*d^3*e^5 + 1226*b^4*c^5*d^2*e^6))/(8*(b^...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1647, normalized size of antiderivative = 5.76

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x)^3,x)
```

output

```
(70*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d
)))*b**4*c*d**2*e**2*x**2 - 168*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x
)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*c**2*d**3*e*x**2 + 140*sqrt(c)*sqrt(b
*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*c**2*d**2
*e**2*x**3 + 96*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sq
rt(b*e - c*d)))*b**2*c**3*d**4*x**2 - 336*sqrt(c)*sqrt(b*e - c*d)*atan((sq
rt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c**3*d**3*e*x**3 + 70*sqrt(
c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*
c**3*d**2*e**2*x**4 + 192*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(
sqrt(c)*sqrt(b*e - c*d)))*b*c**4*d**4*x**3 - 168*sqrt(c)*sqrt(b*e - c*d)*a
tan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**4*d**3*e*x**4 + 96*s
qrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*c
**5*d**4*x**4 - 4*sqrt(d + e*x)*b**6*d**2*e**2 - 2*sqrt(d + e*x)*b**6*d*e*
*3*x + 8*sqrt(d + e*x)*b**5*c*d**3*e + 20*sqrt(d + e*x)*b**5*c*d**2*e**2*x
 - 4*sqrt(d + e*x)*b**5*c*d*e**3*x**2 - 4*sqrt(d + e*x)*b**4*c**2*d**4 - 3
4*sqrt(d + e*x)*b**4*c**2*d**3*e*x + 78*sqrt(d + e*x)*b**4*c**2*d**2*e**2*
x**2 - 2*sqrt(d + e*x)*b**4*c**2*d*e**3*x**3 + 16*sqrt(d + e*x)*b**3*c**3*
d**4*x - 146*sqrt(d + e*x)*b**3*c**3*d**3*e*x**2 + 50*sqrt(d + e*x)*b**3*c
**3*d**2*e**2*x**3 + 72*sqrt(d + e*x)*b**2*c**4*d**4*x**2 - 96*sqrt(d + e*
x)*b**2*c**4*d**3*e*x**3 + 48*sqrt(d + e*x)*b*c**5*d**4*x**3 - sqrt(d)*...
```

3.127 $\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^3} dx$

Optimal result 992
Mathematica [A] (verified) 993
Rubi [A] (verified) 993
Maple [A] (verified) 997
Fricas [B] (verification not implemented) 998
Sympy [F] 999
Maxima [F(-2)] 999
Giac [B] (verification not implemented) 999
Mupad [B] (verification not implemented) 1000
Reduce [B] (verification not implemented) 1001

Optimal result

Integrand size = 21, antiderivative size = 322

$$\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^3} dx = \frac{c(12c^2d^2-7bcde-3b^2e^2)\sqrt{d+ex}}{4b^3d^2(cd-be)(b+cx)^2} - \frac{\sqrt{d+ex}}{2bdx^2(b+cx)^2} + \frac{(8cd+3be)\sqrt{d+ex}}{4b^2d^2x(b+cx)^2} + \frac{3c(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)\sqrt{d+ex}}{4b^4d^2(cd-be)^2(b+cx)} - \frac{3(16c^2d^2+4bcde+b^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4b^5d^{5/2}} + \frac{3c^{5/2}(16c^2d^2-36bcde+21b^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{4b^5(cd-be)^{5/2}}$$

output
```
1/4*c*(-3*b^2*e^2-7*b*c*d*e+12*c^2*d^2)*(e*x+d)^(1/2)/b^3/d^2/(-b*e+c*d)/(
c*x+b)^2-1/2*(e*x+d)^(1/2)/b/d/x^2/(c*x+b)^2+1/4*(3*b*e+8*c*d)*(e*x+d)^(1/
2)/b^2/d^2/x/(c*x+b)^2+3/4*c*(-b*e+2*c*d)*(-b^2*e^2-4*b*c*d*e+4*c^2*d^2)*(
e*x+d)^(1/2)/b^4/d^2/(-b*e+c*d)^2/(c*x+b)-3/4*(b^2*e^2+4*b*c*d*e+16*c^2*d^
2)*arctanh((e*x+d)^(1/2)/d^(1/2))/b^5/d^(5/2)+3/4*c^(5/2)*(21*b^2*e^2-36*b
*c*d*e+16*c^2*d^2)*arctanh(c^(1/2)*(e*x+d)^(1/2)/(-b*e+c*d)^(1/2))/b^5/(-b
*e+c*d)^(5/2)
```

Mathematica [A] (verified)

Time = 2.29 (sec) , antiderivative size = 292, normalized size of antiderivative = 0.91

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^3}\,dx$$
$$= \frac{\frac{b\sqrt{d+ex}\left(24c^5d^3x^3+36bc^4d^2x^2(d-ex)+b^5e^2(-2d+3ex)+2b^4ce\left(2d^2+dex+3e^2x^2\right)+b^2c^3dx\left(8d^2-55dex+6e^2x^2\right)+b^3c^2\left(-2d^3-13d^2ex+10de^2x\right.\right.}{d^2(cd-be)^2x^2(b+cx)^2}}{4b^5}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(b*x + c*x^2)^3),x]
```

output
```
((b*Sqrt[d + e*x]*(24*c^5*d^3*x^3 + 36*b*c^4*d^2*x^2*(d - e*x) + b^5*e^2*(
-2*d + 3*e*x) + 2*b^4*c*e*(2*d^2 + d*e*x + 3*e^2*x^2) + b^2*c^3*d*x*(8*d^2
 - 55*d*e*x + 6*e^2*x^2) + b^3*c^2*(-2*d^3 - 13*d^2*e*x + 10*d*e^2*x^2 + 3
*e^3*x^3)))/(d^2*(c*d - b*e)^2*x^2*(b + c*x)^2) - (3*c^(5/2)*(16*c^2*d^2 -
 36*b*c*d*e + 21*b^2*e^2)*ArcTan[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[-(c*d) + b*e
]])/(-(c*d) + b*e)^(5/2) - (3*(16*c^2*d^2 + 4*b*c*d*e + b^2*e^2)*ArcTanh[S
qrt[d + e*x]/Sqrt[d]])/d^(5/2))/(4*b^5)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1165, 27, 1235, 27, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^3\sqrt{d+ex}}\,dx$$
$$\Big\downarrow \; 1165$$
$$-\frac{\int \frac{12c^2d^2-7bced-3b^2e^2+5ce(2cd-be)x}{2\sqrt{d+ex}(cx^2+bx)^2}dx}{2b^2d(cd-be)}-\frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{2b^2d\,(bx+cx^2)^2\,(cd-be)}$$
$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{12c^2d^2-7bced-3b^2e^2+5ce(2cd-be)x}{\sqrt{d+ex}(cx^2+bx)^2}dx}{4b^2d(cd-be)}-\frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{2b^2d\left(bx+cx^2\right)^2(cd-be)}$$

$\downarrow$ 1235

$$-\frac{-\frac{\int \frac{3\left(\left(16c^2d^2+4bced+b^2e^2\right)(cd-be)^2+ce(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)x\right)}{2\sqrt{d+ex}\left(cx^2+bx\right)}dx}{b^2d(cd-be)}-\frac{\sqrt{d+ex}\left(3cx(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)+b(cd-be)\left(-3b^2e^2-\cdots\right.\right.}{b^2d(bx+cx^2)(cd-be)}}{4b^2d(cd-be)}\cdots$$
$$\frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{2b^2d\left(bx+cx^2\right)^2(cd-be)}$$

$\downarrow$ 27

$$-\frac{-\frac{3\int \frac{\left(16c^2d^2+4bced+b^2e^2\right)(cd-be)^2+ce(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{2b^2d(cd-be)}-\frac{\sqrt{d+ex}\left(3cx(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)+b(cd-be)\left(-3b^2e^2-\cdots\right.\right.}{b^2d(bx+cx^2)(cd-be)}}{4b^2d(cd-be)}\cdots$$
$$\frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{2b^2d\left(bx+cx^2\right)^2(cd-be)}$$

$\downarrow$ 1197

$$-\frac{-\frac{3\int \frac{e\left(8c^4d^4-16bc^3ed^3+7b^2c^2e^2d^2+b^3ce^3d+b^4e^4+c(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2d(cd-be)}-\frac{\sqrt{d+ex}\left(3cx(2cd-be)\left(-b^2e^2-4bcde+4c^2d\cdots\right.\right.}{b^2d(bx+cx^2\cdots}}{4b^2d(cd-be)}\cdots$$
$$\frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{2b^2d\left(bx+cx^2\right)^2(cd-be)}$$

$\downarrow$ 27

$$-\frac{-\frac{3e\int \frac{8c^4d^4-16bc^3ed^3+7b^2c^2e^2d^2+b^3ce^3d+b^4e^4+c(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{b^2d(cd-be)}-\frac{\sqrt{d+ex}\left(3cx(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)\cdots\right.}{b^2d(bx+cx^2)(\cdots}}{4b^2d(cd-be)}\cdots$$
$$\frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{2b^2d\left(bx+cx^2\right)^2(cd-be)}$$

$\downarrow$ 1480

$$-\frac{-\frac{3e\left(\frac{c(cd-be)^2\left(b^2e^2+4bcde+16c^2d^2\right)\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{c^3d^2\left(21b^2e^2-36bcde+16c^2d^2\right)\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{b^2d(cd-be)}-\frac{\sqrt{d+ex}(3cx(2cd-be\ldots}{\ldots}}{4b^2d(cd-be)}-\frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{2b^2d\left(bx+cx^2\right)^2(cd-be)}$$

$$\Big\downarrow \; 221$$

$$-\frac{-\frac{3e\left(\frac{c^{5/2}d^2\left(21b^2e^2-36bcde+16c^2d^2\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{be\sqrt{cd-be}}-\frac{\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(cd-be)^2\left(b^2e^2+4bcde+16c^2d^2\right)}{b\sqrt{d}e}\right)}{b^2d(cd-be)}-\frac{\sqrt{d+ex}(3cx(2cd-be)(\ldots}{\ldots}}{4b^2d(cd-be)}-\frac{\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{2b^2d\left(bx+cx^2\right)^2(cd-be)}$$

input

```
Int[1/(Sqrt[d + e*x]*(b*x + c*x^2)^3),x]
```

output

```
-1/2*(Sqrt[d + e*x]*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e
)*(b*x + c*x^2)^2) - (-((Sqrt[d + e*x]*(b*(c*d - b*e)*(12*c^2*d^2 - 7*b*c*
d*e - 3*b^2*e^2) + 3*c*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*x))
/(b^2*d*(c*d - b*e)*(b*x + c*x^2))) - (3*e*(-(((c*d - b*e)^2*(16*c^2*d^2 +
 4*b*c*d*e + b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/(b*Sqrt[d]*e)) + (c^
(5/2)*d^2*(16*c^2*d^2 - 36*b*c*d*e + 21*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[d +
 e*x])/Sqrt[c*d - b*e]])/(b*e*Sqrt[c*d - b*e])))/(b^2*d*(c*d - b*e)))/(4*b
^2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\sqrt{ex+d}\,(-3bex-12cdx+2bd)}{4d^2b^4x^2}+\frac{e\left(-\frac{(3b^2e^2+12bcde+48c^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be\sqrt{d}}-\frac{8c^3d^2\left(\frac{\frac{3cbe(5be-4cd)(ex+d)^{\frac{3}{2}}}{8(b^2e^2-2bcde+c^2d^2)}+\frac{(1\cdots}{}}{((ex+d)c+be-\cdots}\cdots\right.}{}\cdots\right.}{4b^4d^2\cdots}$
derivativedivides	$2e^5\left(-\frac{c^3\left(\frac{\frac{3cbe(5be-4cd)(ex+d)^{\frac{3}{2}}}{8(b^2e^2-2bcde+c^2d^2)}+\frac{(17be-12cd)be\sqrt{ex+d}}{8be-8cd}}{((ex+d)c+be-cd)^2}+\frac{3(21b^2e^2-36bcde+16c^2d^2)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{8(b^2e^2-2bcde+c^2d^2)\sqrt{c(be-cd)}}\right)}{b^5e^5}-\frac{-\frac{3be\cdots}{}}{}\cdots\right.$
default	$2e^5\left(-\frac{c^3\left(\frac{\frac{3cbe(5be-4cd)(ex+d)^{\frac{3}{2}}}{8(b^2e^2-2bcde+c^2d^2)}+\frac{(17be-12cd)be\sqrt{ex+d}}{8be-8cd}}{((ex+d)c+be-cd)^2}+\frac{3(21b^2e^2-36bcde+16c^2d^2)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)}{8(b^2e^2-2bcde+c^2d^2)\sqrt{c(be-cd)}}\right)}{b^5e^5}-\frac{-\frac{3be\cdots}{}}{}\cdots\right.$
pseudoelliptic	$\frac{12d^{\frac{9}{2}}x^2(cx+b)^2c^3(-be+cd)\left(c^2d^2-\frac{9}{4}bcde+\frac{21}{16}b^2e^2\right)\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)+\frac{\sqrt{c(be-cd)}\left(-\frac{3d^2x^2(cx+b)^2(b^2e^2+4bcde+16\cdots}{}\cdots\right.}{}}{\cdots}$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(e*x+d)^(1/2)*(-3*b*e*x-12*c*d*x+2*b*d)/d^2/b^4/x^2+1/4/b^4/d^2*e*(-(
3*b^2*e^2+12*b*c*d*e+48*c^2*d^2)/b/e/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2)
)-8*c^3*d^2/b/e*((3/8*c*b*e*(5*b*e-4*c*d)/(b^2*e^2-2*b*c*d*e+c^2*d^2)*(e*x
+d)^(3/2)+1/8*(17*b*e-12*c*d)*b*e/(b*e-c*d)*(e*x+d)^(1/2))/((e*x+d)*c+b*e-
c*d)^2+3/8*(21*b^2*e^2-36*b*c*d*e+16*c^2*d^2)/(b^2*e^2-2*b*c*d*e+c^2*d^2)/
(c*(b*e-c*d))^(1/2)*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(286) = 572$.

Time = 0.84 (sec) , antiderivative size = 2784, normalized size of antiderivative = 8.65

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
[1/8*(3*((16*c^6*d^5 - 36*b*c^5*d^4*e + 21*b^2*c^4*d^3*e^2)*x^4 + 2*(16*b*
c^5*d^5 - 36*b^2*c^4*d^4*e + 21*b^3*c^3*d^3*e^2)*x^3 + (16*b^2*c^4*d^5 - 3
6*b^3*c^3*d^4*e + 21*b^4*c^2*d^3*e^2)*x^2)*sqrt(c/(c*d - b*e))*log((c*e*x
+ 2*c*d - b*e + 2*(c*d - b*e)*sqrt(e*x + d)*sqrt(c/(c*d - b*e)))/(c*x + b)
) + 3*((16*c^6*d^4 - 28*b*c^5*d^3*e + 9*b^2*c^4*d^2*e^2 + 2*b^3*c^3*d*e^3
+ b^4*c^2*e^4)*x^4 + 2*(16*b*c^5*d^4 - 28*b^2*c^4*d^3*e + 9*b^3*c^3*d^2*e^
2 + 2*b^4*c^2*d*e^3 + b^5*c*e^4)*x^3 + (16*b^2*c^4*d^4 - 28*b^3*c^3*d^3*e
+ 9*b^4*c^2*d^2*e^2 + 2*b^5*c*d*e^3 + b^6*e^4)*x^2)*sqrt(d)*log((e*x - 2*s
qrt(e*x + d)*sqrt(d) + 2*d)/x) - 2*(2*b^4*c^2*d^4 - 4*b^5*c*d^3*e + 2*b^6*
d^2*e^2 - 3*(8*b*c^5*d^4 - 12*b^2*c^4*d^3*e + 2*b^3*c^3*d^2*e^2 + b^4*c^2*
d*e^3)*x^3 - (36*b^2*c^4*d^4 - 55*b^3*c^3*d^3*e + 10*b^4*c^2*d^2*e^2 + 6*b
^5*c*d*e^3)*x^2 - (8*b^3*c^3*d^4 - 13*b^4*c^2*d^3*e + 2*b^5*c*d^2*e^2 + 3*
b^6*d*e^3)*x)*sqrt(e*x + d))/((b^5*c^4*d^5 - 2*b^6*c^3*d^4*e + b^7*c^2*d^3
*e^2)*x^4 + 2*(b^6*c^3*d^5 - 2*b^7*c^2*d^4*e + b^8*c*d^3*e^2)*x^3 + (b^7*c
^2*d^5 - 2*b^8*c*d^4*e + b^9*d^3*e^2)*x^2), -1/8*(6*((16*c^6*d^5 - 36*b*c^
5*d^4*e + 21*b^2*c^4*d^3*e^2)*x^4 + 2*(16*b*c^5*d^5 - 36*b^2*c^4*d^4*e + 2
1*b^3*c^3*d^3*e^2)*x^3 + (16*b^2*c^4*d^5 - 36*b^3*c^3*d^4*e + 21*b^4*c^2*d
^3*e^2)*x^2)*sqrt(-c/(c*d - b*e))*arctan(sqrt(e*x + d)*sqrt(-c/(c*d - b*e)
)) - 3*((16*c^6*d^4 - 28*b*c^5*d^3*e + 9*b^2*c^4*d^2*e^2 + 2*b^3*c^3*d*e^3
 + b^4*c^2*e^4)*x^4 + 2*(16*b*c^5*d^4 - 28*b^2*c^4*d^3*e + 9*b^3*c^3*d^...
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^3}\,dx = \int \frac{1}{x^3\,(b+cx)^3\,\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x)**3,x)
```

output
```
Integral(1/(x**3*(b + c*x)**3*sqrt(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(286) = 572.

Time = 0.16 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.87

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^3}\,dx = -\frac{3\,(16\,c^5d^2-36\,bc^4de+21\,b^2c^3e^2)\arctan\left(\frac{\sqrt{ex+d}c}{\sqrt{-c^2d+bce}}\right)}{4\,(b^5c^2d^2-2\,b^6cde+b^7e^2)\sqrt{-c^2d+bce}} + \frac{24\,(ex+d)^{\frac{7}{2}}c^5d^3e-72\,(ex+d)^{\frac{5}{2}}c^5d^4e+72\,(ex+d)^{\frac{3}{2}}c^5d^5e-24\,\sqrt{ex+d}c^5d^6e-36\,(ex+d)^{\frac{7}{2}}bc^4d^2e^2}{} + \frac{3\,(16\,c^2d^2+4\,bcde+b^2e^2)\arctan\left(\frac{\sqrt{ex+d}}{\sqrt{-d}}\right)}{4\,b^5\sqrt{-d}d^2}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
-3/4*(16*c^5*d^2 - 36*b*c^4*d*e + 21*b^2*c^3*e^2)*arctan(sqrt(e*x + d)*c/s
qrt(-c^2*d + b*c*e))/((b^5*c^2*d^2 - 2*b^6*c*d*e + b^7*e^2)*sqrt(-c^2*d +
b*c*e)) + 1/4*(24*(e*x + d)^(7/2)*c^5*d^3*e - 72*(e*x + d)^(5/2)*c^5*d^4*e
 + 72*(e*x + d)^(3/2)*c^5*d^5*e - 24*sqrt(e*x + d)*c^5*d^6*e - 36*(e*x + d
)^(7/2)*b*c^4*d^2*e^2 + 144*(e*x + d)^(5/2)*b*c^4*d^3*e^2 - 180*(e*x + d)^
(3/2)*b*c^4*d^4*e^2 + 72*sqrt(e*x + d)*b*c^4*d^5*e^2 + 6*(e*x + d)^(7/2)*b
^2*c^3*d*e^3 - 73*(e*x + d)^(5/2)*b^2*c^3*d^2*e^3 + 136*(e*x + d)^(3/2)*b^
2*c^3*d^3*e^3 - 69*sqrt(e*x + d)*b^2*c^3*d^4*e^3 + 3*(e*x + d)^(7/2)*b^3*c
^2*e^4 + (e*x + d)^(5/2)*b^3*c^2*d*e^4 - 24*(e*x + d)^(3/2)*b^3*c^2*d^2*e^
4 + 18*sqrt(e*x + d)*b^3*c^2*d^3*e^4 + 6*(e*x + d)^(5/2)*b^4*c*e^5 - 10*(e
*x + d)^(3/2)*b^4*c*d*e^5 + 8*sqrt(e*x + d)*b^4*c*d^2*e^5 + 3*(e*x + d)^(3
/2)*b^5*e^6 - 5*sqrt(e*x + d)*b^5*d*e^6)/((b^4*c^2*d^4 - 2*b^5*c*d^3*e + b
^6*d^2*e^2)*((e*x + d)^2*c - 2*(e*x + d)*c*d + c*d^2 + (e*x + d)*b*e - b*d
*e)^2) + 3/4*(16*c^2*d^2 + 4*b*c*d*e + b^2*e^2)*arctan(sqrt(e*x + d)/sqrt(
-d))/(b^5*sqrt(-d)*d^2)
```

Mupad [B] (verification not implemented)

Time = 7.21 (sec) , antiderivative size = 6715, normalized size of antiderivative = 20.85

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((b*x + c*x^2)^3*(d + e*x)^(1/2)),x)
```

output
```
((e*(d + e*x)^(3/2)*(3*b^5*e^5 + 72*c^5*d^5 + 136*b^2*c^3*d^3*e^2 - 24*b^3
*c^2*d^2*e^3 - 180*b*c^4*d^4*e - 10*b^4*c*d*e^4))/(4*b^4*(c*d^2 - b*d*e)^2
) - ((d + e*x)^(1/2)*(24*c^4*d^4*e - 5*b^4*e^5 - 48*b*c^3*d^3*e^2 + 21*b^2
*c^2*d^2*e^3 + 3*b^3*c*d*e^4))/(4*b^4*(c*d^2 - b*d*e)) + (e*(d + e*x)^(5/2
)*(6*b^4*c*e^4 - 72*c^5*d^4 + b^3*c^2*d*e^3 - 73*b^2*c^3*d^2*e^2 + 144*b*c
^4*d^3*e))/(4*b^4*(c*d^2 - b*d*e)^2) + (3*c*e*(d + e*x)^(7/2)*(8*c^4*d^3 +
 b^3*c*e^3 + 2*b^2*c^2*d*e^2 - 12*b*c^3*d^2*e))/(4*b^4*(c*d^2 - b*d*e)^2))
/(c^2*(d + e*x)^4 - (d + e*x)*(4*c^2*d^3 + 2*b^2*d*e^2 - 6*b*c*d^2*e) - (4
*c^2*d - 2*b*c*e)*(d + e*x)^3 + (d + e*x)^2*(b^2*e^2 + 6*c^2*d^2 - 6*b*c*d
*e) + c^2*d^4 + b^2*d^2*e^2 - 2*b*c*d^3*e) - (atan((((-c^5*(b*e - c*d)^5)^
(1/2)*(((d + e*x)^(1/2)*(9*b^8*c^3*e^10 + 4608*c^11*d^8*e^2 - 18432*b*c^10
*d^7*e^3 + 36*b^7*c^4*d*e^9 + 27360*b^2*c^9*d^6*e^4 - 17568*b^3*c^8*d^5*e^
5 + 3978*b^4*c^7*d^4*e^6 - 180*b^5*c^6*d^3*e^7 + 198*b^6*c^5*d^2*e^8))/(8*
(b^8*c^4*d^8 + b^12*d^4*e^4 - 4*b^9*c^3*d^7*e - 4*b^11*c*d^5*e^3 + 6*b^10*
c^2*d^6*e^2)) + (3*(-c^5*(b*e - c*d)^5)^(1/2)*((24*b^10*c^8*d^8*e^3 - 96*b
^11*c^7*d^7*e^4 + 141*b^12*c^6*d^6*e^5 - 87*b^13*c^5*d^5*e^6 + 18*b^14*c^4
*d^4*e^7 - 3*b^15*c^3*d^3*e^8 + 3*b^16*c^2*d^2*e^9)/(b^12*c^4*d^8 + b^16*d
^4*e^4 - 4*b^13*c^3*d^7*e - 4*b^15*c*d^5*e^3 + 6*b^14*c^2*d^6*e^2) - (3*(-
c^5*(b*e - c*d)^5)^(1/2)*(d + e*x)^(1/2)*(21*b^2*e^2 + 16*c^2*d^2 - 36*b*c
*d*e)*(128*b^10*c^7*d^9*e^2 - 576*b^11*c^6*d^8*e^3 + 1024*b^12*c^5*d^7*...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1966, normalized size of antiderivative = 6.11

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x)^3,x)
```

output
```
( - 126*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e -
 c*d)))*b**4*c**2*d**3*e**2*x**2 + 216*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(
d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*c**3*d**4*e*x**2 - 252*sqrt(c)
*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*c*
*3*d**3*e**2*x**3 - 96*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqr
t(c)*sqrt(b*e - c*d)))*b**2*c**4*d**5*x**2 + 432*sqrt(c)*sqrt(b*e - c*d)*a
tan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c**4*d**4*e*x**3 - 1
26*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)
))*b**2*c**4*d**3*e**2*x**4 - 192*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e
*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**5*d**5*x**3 + 216*sqrt(c)*sqrt(b*e
- c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**5*d**4*e*x**
4 - 96*sqrt(c)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e -
c*d)))*c**6*d**5*x**4 - 4*sqrt(d + e*x)*b**7*d**2*e**3 + 6*sqrt(d + e*x)*b
**7*d*e**4*x + 12*sqrt(d + e*x)*b**6*c*d**3*e**2 - 2*sqrt(d + e*x)*b**6*c*
d**2*e**3*x + 12*sqrt(d + e*x)*b**6*c*d*e**4*x**2 - 12*sqrt(d + e*x)*b**5*
c**2*d**4*e - 30*sqrt(d + e*x)*b**5*c**2*d**3*e**2*x + 8*sqrt(d + e*x)*b**
5*c**2*d**2*e**3*x**2 + 6*sqrt(d + e*x)*b**5*c**2*d*e**4*x**3 + 4*sqrt(d +
 e*x)*b**4*c**3*d**5 + 42*sqrt(d + e*x)*b**4*c**3*d**4*e*x - 130*sqrt(d +
e*x)*b**4*c**3*d**3*e**2*x**2 + 6*sqrt(d + e*x)*b**4*c**3*d**2*e**3*x**3 -
 16*sqrt(d + e*x)*b**3*c**4*d**5*x + 182*sqrt(d + e*x)*b**3*c**4*d**4*e...
```

3.128 $\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)^3}\,dx$

Optimal result . . . 1003
Mathematica [A] (verified) . . . 1004
Rubi [A] (verified) . . . 1004
Maple [A] (verified) . . . 1008
Fricas [B] (verification not implemented) . . . 1009
Sympy [F] . . . 1010
Maxima [F(-2)] . . . 1010
Giac [B] (verification not implemented) . . . 1011
Mupad [B] (verification not implemented) . . . 1011
Reduce [B] (verification not implemented) . . . 1012

Optimal result

Integrand size = 21, antiderivative size = 398

$$\begin{aligned}\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)^3}\,dx &= \frac{3e(c^2d^2-bcde-b^2e^2)(8c^2d^2-8bcde+5b^2e^2)}{4b^4d^3(cd-be)^3\sqrt{d+ex}} \\ &+ \frac{c(12c^2d^2-5bcde-5b^2e^2)}{4b^3d^2(cd-be)(b+cx)^2\sqrt{d+ex}} - \frac{1}{2bdx^2(b+cx)^2\sqrt{d+ex}} \\ &+ \frac{8cd+5be}{4b^2d^2x(b+cx)^2\sqrt{d+ex}} + \frac{c(2cd-be)(12c^2d^2-12bcde-5b^2e^2)}{4b^4d^2(cd-be)^2(b+cx)\sqrt{d+ex}} \\ &- \frac{3(16c^2d^2+12bcde+5b^2e^2)\,\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)}{4b^5d^{7/2}} \\ &+ \frac{3c^{7/2}(16c^2d^2-44bcde+33b^2e^2)\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{4b^5(cd-be)^{7/2}}\end{aligned}$$

output
```
3/4*e*(-b^2*e^2-b*c*d*e+c^2*d^2)*(5*b^2*e^2-8*b*c*d*e+8*c^2*d^2)/b^4/d^3/(
-b*e+c*d)^3/(e*x+d)^(1/2)+1/4*c*(-5*b^2*e^2-5*b*c*d*e+12*c^2*d^2)/b^3/d^2/
(-b*e+c*d)/(c*x+b)^2/(e*x+d)^(1/2)-1/2/b/d/x^2/(c*x+b)^2/(e*x+d)^(1/2)+1/4
*(5*b*e+8*c*d)/b^2/d^2/x/(c*x+b)^2/(e*x+d)^(1/2)+1/4*c*(-b*e+2*c*d)*(-5*b^
2*e^2-12*b*c*d*e+12*c^2*d^2)/b^4/d^2/(-b*e+c*d)^2/(c*x+b)/(e*x+d)^(1/2)-3/
4*(5*b^2*e^2+12*b*c*d*e+16*c^2*d^2)*arctanh((e*x+d)^(1/2)/d^(1/2))/b^5/d^(
7/2)+3/4*c^(7/2)*(33*b^2*e^2-44*b*c*d*e+16*c^2*d^2)*arctanh(c^(1/2)*(e*x+d
)^(1/2)/(-b*e+c*d)^(1/2))/b^5/(-b*e+c*d)^(7/2)
```

Mathematica [A] (verified)

Time = 1.66 (sec) , antiderivative size = 408, normalized size of antiderivative = 1.03

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^3}\,dx = \frac{\frac{b\left(24c^6d^4x^3(d+ex)+b^6e^3\left(2d^2-5dex-15e^2x^2\right)-12bc^5d^3x^2\left(-3d^2+dex+4e^2x^2\right)+b^2c^4d^2x\left(8d^3-65d^2e\right.\right.}{}}{}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(b*x + c*x^2)^3),x]
```

output
```
((b*(24*c^6*d^4*x^3*(d + e*x) + b^6*e^3*(2*d^2 - 5*d*e*x - 15*e^2*x^2) - 1
2*b*c^5*d^3*x^2*(-3*d^2 + d*e*x + 4*e^2*x^2) + b^2*c^4*d^2*x*(8*d^3 - 65*d
^2*e*x - 58*d*e^2*x^2 + 15*e^3*x^3) - b^5*c*e^2*(6*d^3 - 7*d^2*e*x + d*e^2
*x^2 + 30*e^3*x^3) + b^4*c^2*e*(6*d^4 + 9*d^3*e*x + 23*d^2*e^2*x^2 + 13*d*
e^3*x^3 - 15*e^4*x^4) + b^3*c^3*d*(-2*d^4 - 19*d^3*e*x + 7*d^2*e^2*x^2 + 3
3*d*e^3*x^3 + 9*e^4*x^4)))/(d^3*(c*d - b*e)^3*x^2*(b + c*x)^2*Sqrt[d + e*x
]) + (3*c^(7/2)*(16*c^2*d^2 - 44*b*c*d*e + 33*b^2*e^2)*ArcTan[(Sqrt[c]*Sqr
t[d + e*x])/Sqrt[-(c*d) + b*e]])/(-(c*d) + b*e)^(7/2) - (3*(16*c^2*d^2 + 1
2*b*c*d*e + 5*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sqrt[d]])/d^(7/2))/(4*b^5)
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 436, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1165, 27, 1235, 27, 1198, 1197, 27, 1480, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(bx+cx^2\right)^3(d+ex)^{3/2}}dx$$

$$\Big\downarrow \textbf{1165}$$

$$-\frac{\int \frac{12c^2d^2-5bced-5b^2e^2+7ce(2cd-be)x}{2(d+ex)^{3/2}\left(cx^2+bx\right)^2}dx}{2b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{2b^2d\left(bx+cx^2\right)^2\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \textbf{27}$$

$$-\frac{\int \frac{12c^2d^2-5bced-5b^2e^2+7ce(2cd-be)x}{(d+ex)^{3/2}(cx^2+bx)^2}dx}{4b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{2b^2d\left(bx+cx^2\right)^2\sqrt{d+ex}(cd-be)}$$

$\downarrow$ 1235

$$-\frac{-\frac{\int \frac{3\left(\left(16c^2d^2+12bced+5b^2e^2\right)(cd-be)^2+ce(2cd-be)\left(12c^2d^2-12bced-5b^2e^2\right)x\right)}{2(d+ex)^{3/2}\left(cx^2+bx\right)}dx}{b^2d(cd-be)}-\frac{b\left(5b^3e^3-17bc^2d^2e+12c^3d^3\right)+cx(2cd-be)\left(-5b^2e^2-12bcde-\cdots\right.}{b^2d\left(bx+cx^2\right)\sqrt{d+ex}(cd-be)}}{4b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{2b^2d\left(bx+cx^2\right)^2\sqrt{d+ex}(cd-be)}$$

$\downarrow$ 27

$$-\frac{-\frac{3\int \frac{\left(16c^2d^2+12bced+5b^2e^2\right)(cd-be)^2+ce(2cd-be)\left(12c^2d^2-12bced-5b^2e^2\right)x}{(d+ex)^{3/2}\left(cx^2+bx\right)}dx}{2b^2d(cd-be)}-\frac{b\left(5b^3e^3-17bc^2d^2e+12c^3d^3\right)+cx(2cd-be)\left(-5b^2e^2-12bcde+1\cdots\right.}{b^2d\left(bx+cx^2\right)\sqrt{d+ex}(cd-be)}}{4b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{2b^2d\left(bx+cx^2\right)^2\sqrt{d+ex}(cd-be)}$$

$\downarrow$ 1198

$$-\frac{-\frac{3\left(\frac{\int \frac{\left(16c^2d^2+12bced+5b^2e^2\right)(cd-be)^3+ce\left(c^2d^2-bced-b^2e^2\right)\left(8c^2d^2-8bced+5b^2e^2\right)x}{\sqrt{d+ex}\left(cx^2+bx\right)}dx}{d(cd-be)}+\frac{2e\left(-b^2e^2-bcde+c^2d^2\right)\left(5b^2e^2-8bcde+8c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}\right)}{2b^2d(cd-be)}-\frac{b\left(5b^3\cdots\right.}{\cdots}}{4b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{2b^2d\left(bx+cx^2\right)^2\sqrt{d+ex}(cd-be)}$$

$\downarrow$ 1197

$$-\frac{-\frac{3\left(\frac{2\int \frac{e\left((2cd-be)\left(4c^4d^4-8bc^3ed^3+2b^2c^2e^2d^2+2b^3ce^3d+5b^4e^4\right)+c\left(c^2d^2-bced-b^2e^2\right)\left(8c^2d^2-8bced+5b^2e^2\right)(d+ex)\right)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{d(cd-be)}+\frac{2e\left(-b^2e^2-bcde+c^2d^2\cdots\right.}{d\sqrt{d+e\cdots}}\cdots\right.}{2b^2d(cd-be)}\cdots}{4b^2d(cd-be)}-\frac{cx(2cd-be)+b(cd-be)}{2b^2d\left(bx+cx^2\right)^2\sqrt{d+ex}(cd-be)}$$

$\downarrow$ 27

$$-\frac{-\frac{3\left(\frac{2e\int \frac{(2cd-be)\left(4c^4d^4-8bc^3ed^3+2b^2c^2e^2d^2+2b^3ce^3d+5b^4e^4\right)+c\left(c^2d^2-bced-b^2e^2\right)\left(8c^2d^2-8bced+5b^2e^2\right)(d+ex)}{c(d+ex)^2-(2cd-be)(d+ex)+d(cd-be)}d\sqrt{d+ex}}{d(cd-be)}+\frac{2e\left(-b^2e^2-bcde+c^2d^2\right)(}{d\sqrt{d+ex}(}\right.}{2b^2d(cd-be)}}{4b^2d(cd-be)}$$

$$\frac{cx(2cd-be)+b(cd-be)}{2b^2d\left(bx+cx^2\right)^2\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \; 1480$$

$$-\frac{-\frac{3\left(\frac{2e\left(\frac{c(cd-be)^3\left(5b^2e^2+12bcde+16c^2d^2\right)\int \frac{1}{c(d+ex)-cd}d\sqrt{d+ex}}{be}-\frac{c^4d^3\left(33b^2e^2-44bcde+16c^2d^2\right)\int \frac{1}{-cd+be+c(d+ex)}d\sqrt{d+ex}}{be}\right)}{d(cd-be)}+\frac{2e\left(-b^2e^2-bcde+c^2\right.}{d\sqrt{d}}\right.}{2b^2d(cd-be)}}{4b^2d(cd-be)}$$

$$\frac{cx(2cd-be)+b(cd-be)}{2b^2d\left(bx+cx^2\right)^2\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \; 221$$

$$-\frac{-\frac{3\left(\frac{2e\left(\frac{c^{7/2}d^3\left(33b^2e^2-44bcde+16c^2d^2\right)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{d+ex}}{\sqrt{cd-be}}\right)}{be\sqrt{cd-be}}-\frac{\text{arctanh}\left(\frac{\sqrt{d+ex}}{\sqrt{d}}\right)(cd-be)^3\left(5b^2e^2+12bcde+16c^2d^2\right)}{b\sqrt{d}e}\right)}{d(cd-be)}+\frac{2e\left(-b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}}\right.}{2b^2d(cd-be)}}{4b^2d(cd-be)}$$

$$\frac{cx(2cd-be)+b(cd-be)}{2b^2d\left(bx+cx^2\right)^2\sqrt{d+ex}(cd-be)}$$

input
```
Int[1/((d + e*x)^(3/2)*(b*x + c*x^2)^3),x]
```

output
```
-1/2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x)/(b^2*d*(c*d - b*e)*Sqrt[d + e*x]*
(b*x + c*x^2)^2) - (-((b*(12*c^3*d^3 - 17*b*c^2*d^2*e + 5*b^3*e^3) + c*(2*
c*d - b*e)*(12*c^2*d^2 - 12*b*c*d*e - 5*b^2*e^2)*x)/(b^2*d*(c*d - b*e)*Sqr
t[d + e*x]*(b*x + c*x^2))) - (3*((2*e*(c^2*d^2 - b*c*d*e - b^2*e^2)*(8*c^2
*d^2 - 8*b*c*d*e + 5*b^2*e^2))/(d*(c*d - b*e)*Sqrt[d + e*x]) + (2*e*(-(((c
*d - b*e)^3*(16*c^2*d^2 + 12*b*c*d*e + 5*b^2*e^2)*ArcTanh[Sqrt[d + e*x]/Sq
rt[d]])/(b*Sqrt[d]*e)) + (c^(7/2)*d^3*(16*c^2*d^2 - 44*b*c*d*e + 33*b^2*e^
2)*ArcTanh[(Sqrt[c]*Sqrt[d + e*x])/Sqrt[c*d - b*e]])/(b*e*Sqrt[c*d - b*e])
))/(d*(c*d - b*e))))/(2*b^2*d*(c*d - b*e)))/(4*b^2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1197
```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Simp[2   Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 -
b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fr
eeQ[{a, b, c, d, e, f, g}, x]
```

rule 1198
```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[(e*f - d*g)*((d + e*x)^(m + 1)/((m + 1)*(c
*d^2 - b*d*e + a*e^2))), x] + Simp[1/(c*d^2 - b*d*e + a*e^2)   Int[(d + e*x
)^(m + 1)*(Simp[c*d*f - f*b*e + a*e*g - c*(e*f - d*g)*x, x]/(a + b*x + c*x^
2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && FractionQ[m] && LtQ[m, -1
]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1480
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(e/2 + (2*c*d - b*e)/(2*q))   Int[1/(
b/2 - q/2 + c*x^2), x], x] + Simp[(e/2 - (2*c*d - b*e)/(2*q))   Int[1/(b/2
+ q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
 && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.70

method	result
risch	$-\frac{\sqrt{ex+d}\,(-7bex-12cdx+2bd)}{4d^3b^4x^2}+\frac{e\left(-\frac{(15b^2e^2+36bcde+48c^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{be\sqrt{d}}+\frac{8b^4e^4}{(be-cd)^3\sqrt{ex+d}}+\frac{8c^4d^3\left(\frac{\left(\frac{19}{8}e^2b^2 \cdots\right.}{\cdots}\right.}{\cdots}\right.}{\cdots}$
derivativedivides	$2e^5\left(-\frac{\frac{-\frac{be(7be+12cd)(ex+d)^{\frac{3}{2}}}{8}+\left(\frac{9}{8}d\,e^2b^2+\frac{3}{2}d^2ebc\right)\sqrt{ex+d}}{e^2x^2}+\frac{3(5b^2e^2+12bcde+16c^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8\sqrt{d}}}{d^3b^5e^5}+\frac{1}{d^3(be-cd)\cdots}\right.$
default	$2e^5\left(-\frac{\frac{-\frac{be(7be+12cd)(ex+d)^{\frac{3}{2}}}{8}+\left(\frac{9}{8}d\,e^2b^2+\frac{3}{2}d^2ebc\right)\sqrt{ex+d}}{e^2x^2}+\frac{3(5b^2e^2+12bcde+16c^2d^2)\operatorname{arctanh}\left(\frac{\sqrt{ex+d}}{\sqrt{d}}\right)}{8\sqrt{d}}}{d^3b^5e^5}+\frac{1}{d^3(be-cd)\cdots}\right.$
pseudoelliptic	$\frac{24x^2(cx+b)^2\left(\frac{33b^2e^2d^{\frac{7}{2}}}{16}+c\,d^{\frac{9}{2}}\left(cd-\frac{11be}{4}\right)\right)c^4\sqrt{ex+d}\,\arctan\left(\frac{c\sqrt{ex+d}}{\sqrt{c(be-cd)}}\right)+\sqrt{c(be-cd)}\left(-\frac{15\left(b^2e^2+\frac{12}{5}bcde+\frac{16}{5}c^2d^2 \cdots\right.}{\cdots}\right.}{\cdots}$

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(e*x+d)^(1/2)*(-7*b*e*x-12*c*d*x+2*b*d)/d^3/b^4/x^2+1/4/b^4/d^3*e*(-(
15*b^2*e^2+36*b*c*d*e+48*c^2*d^2)/b/e/d^(1/2)*arctanh((e*x+d)^(1/2)/d^(1/2
))+8*b^4*e^4/(b*e-c*d)^3/(e*x+d)^(1/2)+8*c^4*d^3/(b*e-c*d)^3/b/e*(((19/8*e
^2*b^2*c-3/2*c^2*d*e*b)*(e*x+d)^(3/2)+3/8*b*e*(7*b^2*e^2-11*b*c*d*e+4*c^2*
d^2)*(e*x+d)^(1/2))/((e*x+d)*c+b*e-c*d)^2+3/8*(33*b^2*e^2-44*b*c*d*e+16*c^
2*d^2)/(c*(b*e-c*d))^(1/2)*arctan(c*(e*x+d)^(1/2)/(c*(b*e-c*d))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1167 vs. $2(358) = 716$.

Time = 2.75 (sec) , antiderivative size = 4744, normalized size of antiderivative = 11.92

$$\int \frac{1}{(d+ex)^{3/2}\,(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^3}\,dx=\int \frac{1}{x^3\left(b+cx\right)^3\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x)**3,x)
```

output
```
Integral(1/(x**3*(b + c*x)**3*(d + e*x)**(3/2)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^3}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. 2(358) = 716.

Time = 0.15 (sec) , antiderivative size = 773, normalized size of antiderivative = 1.94

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
-2*e^5/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)*sqrt(e*x
 + d)) - 3/4*(16*c^6*d^2 - 44*b*c^5*d*e + 33*b^2*c^4*e^2)*arctan(sqrt(e*x
+ d)*c/sqrt(-c^2*d + b*c*e))/((b^5*c^3*d^3 - 3*b^6*c^2*d^2*e + 3*b^7*c*d*e
^2 - b^8*e^3)*sqrt(-c^2*d + b*c*e)) + 1/4*(24*(e*x + d)^(7/2)*c^6*d^4*e -
72*(e*x + d)^(5/2)*c^6*d^5*e + 72*(e*x + d)^(3/2)*c^6*d^6*e - 24*sqrt(e*x
+ d)*c^6*d^7*e - 48*(e*x + d)^(7/2)*b*c^5*d^3*e^2 + 180*(e*x + d)^(5/2)*b*
c^5*d^4*e^2 - 216*(e*x + d)^(3/2)*b*c^5*d^5*e^2 + 84*sqrt(e*x + d)*b*c^5*d
^6*e^2 + 15*(e*x + d)^(7/2)*b^2*c^4*d^2*e^3 - 118*(e*x + d)^(5/2)*b^2*c^4*
d^3*e^3 + 199*(e*x + d)^(3/2)*b^2*c^4*d^4*e^3 - 96*sqrt(e*x + d)*b^2*c^4*d
^5*e^3 + 9*(e*x + d)^(7/2)*b^3*c^3*d*e^4 - 3*(e*x + d)^(5/2)*b^3*c^3*d^2*e
^4 - 38*(e*x + d)^(3/2)*b^3*c^3*d^3*e^4 + 30*sqrt(e*x + d)*b^3*c^3*d^4*e^4
 - 7*(e*x + d)^(7/2)*b^4*c^2*e^5 + 41*(e*x + d)^(5/2)*b^4*c^2*d*e^5 - 58*(
e*x + d)^(3/2)*b^4*c^2*d^2*e^5 + 30*sqrt(e*x + d)*b^4*c^2*d^3*e^5 - 14*(e*
x + d)^(5/2)*b^5*c*e^6 + 41*(e*x + d)^(3/2)*b^5*c*d*e^6 - 33*sqrt(e*x + d)
*b^5*c*d^2*e^6 - 7*(e*x + d)^(3/2)*b^6*e^7 + 9*sqrt(e*x + d)*b^6*d*e^7)/((
b^4*c^3*d^6 - 3*b^5*c^2*d^5*e + 3*b^6*c*d^4*e^2 - b^7*d^3*e^3)*((e*x + d)^
2*c - 2*(e*x + d)*c*d + c*d^2 + (e*x + d)*b*e - b*d*e)^2) + 3/4*(16*c^2*d^
2 + 12*b*c*d*e + 5*b^2*e^2)*arctan(sqrt(e*x + d)/sqrt(-d))/(b^5*sqrt(-d)*d
^3)
```

Mupad [B] (verification not implemented)

Time = 7.93 (sec) , antiderivative size = 9635, normalized size of antiderivative = 24.21

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/((b*x + c*x^2)^3*(d + e*x)^(3/2)),x)
```

output
```
- ((2*e^5)/(c*d^2 - b*d*e) + (e*(d + e*x)^2*(15*b^6*e^6 - 72*c^6*d^6 - 199
*b^2*c^4*d^4*e^2 + 38*b^3*c^3*d^3*e^3 + 106*b^4*c^2*d^2*e^4 + 216*b*c^5*d^
5*e - 89*b^5*c*d*e^5))/(4*b^4*(c*d^2 - b*d*e)^3) + (e*(d + e*x)*(25*b^5*e^
5 + 24*c^5*d^5 + 36*b^2*c^3*d^3*e^2 + 6*b^3*c^2*d^2*e^3 - 60*b*c^4*d^4*e -
 56*b^4*c*d*e^4))/(4*b^4*(c*d^2 - b*d*e)^2) - (3*e*(d + e*x)^4*(8*c^6*d^4
- 5*b^4*c^2*e^4 + 3*b^3*c^3*d*e^3 + 5*b^2*c^4*d^2*e^2 - 16*b*c^5*d^3*e))/(
4*b^4*(c*d^2 - b*d*e)^3) + (e*(d + e*x)^3*(72*c^6*d^5 + 30*b^5*c*e^5 - 73*
b^4*c^2*d*e^4 + 118*b^2*c^4*d^3*e^2 + 3*b^3*c^3*d^2*e^3 - 180*b*c^5*d^4*e)
)/(4*b^4*(c*d^2 - b*d*e)^3))/(c^2*(d + e*x)^(9/2) - (4*c^2*d - 2*b*c*e)*(d
 + e*x)^(7/2) - (d + e*x)^(3/2)*(4*c^2*d^3 + 2*b^2*d*e^2 - 6*b*c*d^2*e) +
(d + e*x)^(5/2)*(b^2*e^2 + 6*c^2*d^2 - 6*b*c*d*e) + (d + e*x)^(1/2)*(c^2*d
^4 + b^2*d^2*e^2 - 2*b*c*d^3*e)) - (atan((((-c^7*(b*e - c*d)^7)^(1/2)*((d
+ e*x)^(1/2)*(589824*b^12*c^22*d^28*e^2 - 8257536*b^13*c^21*d^27*e^3 + 533
42208*b^14*c^20*d^26*e^4 - 210382848*b^15*c^19*d^25*e^5 + 564860160*b^16*c
^18*d^24*e^6 - 1089838080*b^17*c^17*d^23*e^7 + 1555380864*b^18*c^16*d^22*e
^8 - 1667850624*b^19*c^15*d^21*e^9 + 1358257536*b^20*c^14*d^20*e^10 - 8556
42240*b^21*c^13*d^19*e^11 + 438185088*b^22*c^12*d^18*e^12 - 201386880*b^23
*c^11*d^17*e^13 + 90100224*b^24*c^10*d^16*e^14 - 37986048*b^25*c^9*d^15*e^
15 + 15108480*b^26*c^8*d^14*e^16 - 6844032*b^27*c^7*d^13*e^17 + 3399552*b^
28*c^6*d^12*e^18 - 1300608*b^29*c^5*d^11*e^19 + 293760*b^30*c^4*d^10*e^...
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 2580, normalized size of antiderivative = 6.48

$$\int \frac{1}{(d+ex)^{3/2}\,(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+b*x)^3,x)
```

output
```
(198*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)
*sqrt(b*e - c*d)))*b**4*c**3*d**4*e**2*x**2 - 264*sqrt(c)*sqrt(d + e*x)*sq
rt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*c**4*
d**5*e*x**2 + 396*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*atan((sqrt(d + e*x
)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**3*c**4*d**4*e**2*x**3 + 96*sqrt(c)*sqrt
(d + e*x)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d))
)*b**2*c**5*d**6*x**2 - 528*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*atan((sq
rt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b**2*c**5*d**5*e*x**3 + 198*sqrt
(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e
 - c*d)))*b**2*c**5*d**4*e**2*x**4 + 192*sqrt(c)*sqrt(d + e*x)*sqrt(b*e -
c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*b*c**6*d**6*x**3 -
264*sqrt(c)*sqrt(d + e*x)*sqrt(b*e - c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*
sqrt(b*e - c*d)))*b*c**6*d**5*e*x**4 + 96*sqrt(c)*sqrt(d + e*x)*sqrt(b*e -
 c*d)*atan((sqrt(d + e*x)*c)/(sqrt(c)*sqrt(b*e - c*d)))*c**7*d**6*x**4 + 1
5*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b**8*e**6*x**2 - 24*s
qrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b**7*c*d*e**5*x**2 + 30*
sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b**7*c*e**6*x**3 - 6*sq
rt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b**6*c**2*d**2*e**4*x**2
- 48*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b**6*c**2*d*e**5*x
**3 + 15*sqrt(d)*sqrt(d + e*x)*log(sqrt(d + e*x) - sqrt(d))*b**6*c**2*e...
```

3.129 $\int (d+ex)^3\sqrt{bx+cx^2}\,dx$

Optimal result . . . 1014
Mathematica [A] (verified) . . . 1015
Rubi [A] (verified) . . . 1015
Maple [A] (verified) . . . 1018
Fricas [A] (verification not implemented) . . . 1019
Sympy [A] (verification not implemented) . . . 1020
Maxima [A] (verification not implemented) . . . 1021
Giac [A] (verification not implemented) . . . 1022
Mupad [B] (verification not implemented) . . . 1023
Reduce [B] (verification not implemented) . . . 1024

Optimal result

Integrand size = 21, antiderivative size = 275

$$\int (d+ex)^3\sqrt{bx+cx^2}\,dx = \frac{b(2cd-be)\left(16c^2d^2-16bcde+7b^2e^2\right)\sqrt{bx+cx^2}}{128c^4} + \frac{(2cd-be)\left(16c^2d^2-16bcde+7b^2e^2\right)x\sqrt{bx+cx^2}}{64c^3} + \frac{e(48c^2d^2-30bcde+7b^2e^2)\left(bx+cx^2\right)^{3/2}}{48c^3} + \frac{e^2(30cd-7be)x(bx+cx^2)^{3/2}}{40c^2} + \frac{e^3x^2(bx+cx^2)^{3/2}}{5c} - \frac{b^2(2cd-be)\left(16c^2d^2-16bcde+7b^2e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{128c^{9/2}}$$

output
```
1/128*b*(-b*e+2*c*d)*(7*b^2*e^2-16*b*c*d*e+16*c^2*d^2)*(c*x^2+b*x)^(1/2)/c
^4+1/64*(-b*e+2*c*d)*(7*b^2*e^2-16*b*c*d*e+16*c^2*d^2)*x*(c*x^2+b*x)^(1/2)
/c^3+1/48*e*(7*b^2*e^2-30*b*c*d*e+48*c^2*d^2)*(c*x^2+b*x)^(3/2)/c^3+1/40*e
^2*(-7*b*e+30*c*d)*x*(c*x^2+b*x)^(3/2)/c^2+1/5*e^3*x^2*(c*x^2+b*x)^(3/2)/c
-1/128*b^2*(-b*e+2*c*d)*(7*b^2*e^2-16*b*c*d*e+16*c^2*d^2)*arctanh(c^(1/2)*
x/(c*x^2+b*x)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 1.35 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.02

$$\int (d+ex)^3\sqrt{bx+cx^2}\,dx$$
$$=\frac{\sqrt{x(b+cx)}(480bc^3d^3-720b^2c^2d^2e+450b^3cde^2-105b^4e^3+960c^4d^3x+480bc^3d^2ex-300b^2c^2de^2x+7}{1920c^4}$$
$$+\frac{b^2(-32c^3d^3+48bc^2d^2e-30b^2cde^2+7b^3e^3)\sqrt{x(b+cx)}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{x}}{-\sqrt{b}+\sqrt{b+cx}}\right)}{64c^{9/2}\sqrt{x}\sqrt{b+cx}}$$

input
```
Integrate[(d + e*x)^3*Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[x*(b + c*x)]*(480*b*c^3*d^3 - 720*b^2*c^2*d^2*e + 450*b^3*c*d*e^2 -
105*b^4*e^3 + 960*c^4*d^3*x + 480*b*c^3*d^2*e*x - 300*b^2*c^2*d*e^2*x + 70
*b^3*c*e^3*x + 1920*c^4*d^2*e*x^2 + 240*b*c^3*d*e^2*x^2 - 56*b^2*c^2*e^3*x
^2 + 1440*c^4*d*e^2*x^3 + 48*b*c^3*e^3*x^3 + 384*c^4*e^3*x^4))/(1920*c^4)
+ (b^2*(-32*c^3*d^3 + 48*b*c^2*d^2*e - 30*b^2*c*d*e^2 + 7*b^3*e^3)*Sqrt[x*
(b + c*x)]*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])])/(64*c^(9
/2)*Sqrt[x]*Sqrt[b + c*x])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.71, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1166, 27, 1225, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{bx+cx^2}(d+ex)^3\,dx$$
$$\Big\downarrow \text{1166}$$
$$\frac{\int \frac{1}{2}(d+ex)(d(10cd-3be)+7e(2cd-be)x)\sqrt{cx^2+bx}dx}{5c}+\frac{e\left(bx+cx^2\right)^{3/2}(d+ex)^2}{5c}$$
$$\Big\downarrow \text{27}$$

$$\frac{\int (d+ex)(d(10cd-3be)+7e(2cd-be)x)\sqrt{cx^2+bx}dx}{10c}+\frac{e\left(bx+cx^2\right)^{3/2}(d+ex)^2}{5c}$$

$$\Big\downarrow \text{1225}$$

$$\frac{\frac{5(2cd-be)\left(7b^2e^2-16bcde+16c^2d^2\right)\int \sqrt{cx^2+bx}dx}{16c^2}+\frac{e\left(bx+cx^2\right)^{3/2}\left(35b^2e^2+42cex(2cd-be)-150bcde+192c^2d^2\right)}{24c^2}}{10c}+\frac{e\left(bx+cx^2\right)^{3/2}(d+ex)^2}{5c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{5(2cd-be)\left(7b^2e^2-16bcde+16c^2d^2\right)\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c^2}+\frac{e\left(bx+cx^2\right)^{3/2}\left(35b^2e^2+42cex(2cd-be)-150bcde+192c^2d^2\right)}{24c^2}}{10c}+\frac{e\left(bx+cx^2\right)^{3/2}(d+ex)^2}{5c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\frac{5(2cd-be)\left(7b^2e^2-16bcde+16c^2d^2\right)\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c^2}+\frac{e\left(bx+cx^2\right)^{3/2}\left(35b^2e^2+42cex(2cd-be)-150bcde+192c^2\right.}{24c^2}}{10c}$$

$$\frac{e\left(bx+cx^2\right)^{3/2}(d+ex)^2}{5c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{5\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)(2cd-be)\left(7b^2e^2-16bcde+16c^2d^2\right)}{16c^2}+\frac{e\left(bx+cx^2\right)^{3/2}\left(35b^2e^2+42cex(2cd-be)-150bcde+192c^2\right.}{24c^2}}{10c}$$

$$\frac{e\left(bx+cx^2\right)^{3/2}(d+ex)^2}{5c}$$

input

```
Int[(d + e*x)^3*Sqrt[b*x + c*x^2],x]
```

output
```
(e*(d + e*x)^2*(b*x + c*x^2)^(3/2))/(5*c) + ((e*(192*c^2*d^2 - 150*b*c*d*e
 + 35*b^2*e^2 + 42*c*e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(3/2))/(24*c^2) + (5
*(2*c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e + 7*b^2*e^2)*(((b + 2*c*x)*Sqrt[b*
x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2
))))/(16*c^2))/(10*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{\frac{7(be-2cd)\left(b^2e^2-\frac{16}{7}bcde+\frac{16}{7}c^2d^2\right)b^2\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{128}-\frac{7\left(-\frac{32\left(\frac{1}{10}e^3x^3+\frac{1}{2}de^2x^2+d^2ex+d^3\right)bc^{\frac{7}{2}}}{7}-\frac{64\left(\frac{2}{5}e^3x^3+\frac{3}{2}de^2x^2+2d^2\right.}{7}\right.}{}}{c^{\frac{9}{2}}}$
risch	$-\frac{\left(-384c^4e^3x^4-48bc^3e^3x^3-1440c^4de^2x^3+56b^2c^2e^3x^2-240bc^3de^2x^2-1920c^4d^2ex^2-70b^3ce^3x+300b^2c^2de^2x-480bc\right.}{1920c^4\sqrt{x(cx+b)}}$
default	$d^3\left(\frac{(2cx+b)\sqrt{cx^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx}\right)}{8c^{\frac{3}{2}}}\right)+e^3\left(\frac{x^2\left(cx^2+bx\right)^{\frac{3}{2}}}{5c}-\frac{7b\left(\frac{x\left(cx^2+bx\right)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{\left(cx^2+bx\right)^{\frac{3}{2}}}{3c}\right.}{}\right)}{}\right.$

input
```
int((e*x+d)^3*(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
7/128/c^(9/2)*((b*e-2*c*d)*(b^2*e^2-16/7*b*c*d*e+16/7*c^2*d^2)*b^2*arctanh
((x*(c*x+b))^(1/2)/x/c^(1/2))-(-32/7*(1/10*e^3*x^3+1/2*d*e^2*x^2+d^2*e*x+d
^3)*b*c^(7/2)-64/7*(2/5*e^3*x^3+3/2*d*e^2*x^2+2*d^2*e*x+d^3)*x*c^(9/2)+e*(
(8/15*e^2*x^2+20/7*d*e*x+48/7*d^2)*c^(5/2)+e*b*((-2/3*e*x-30/7*d)*c^(3/2)+
b*e*c^(1/2)))*b^2)*(x*(c*x+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 498, normalized size of antiderivative = 1.81

$$\int (d+ex)^3\sqrt{bx+cx^2}\,dx$$

$$= \left[-\frac{15\,(32\,b^2c^3d^3 - 48\,b^3c^2d^2e + 30\,b^4cde^2 - 7\,b^5e^3)\sqrt{c}\log\left(2\,cx + b + 2\,\sqrt{cx^2+bx}\sqrt{c}\right) - 2\,(384\,c^5e^3x^4 + \right.$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/3840*(15*(32*b^2*c^3*d^3 - 48*b^3*c^2*d^2*e + 30*b^4*c*d*e^2 - 7*b^5*e
^3)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(384*c^5*e^3*
x^4 + 480*b*c^4*d^3 - 720*b^2*c^3*d^2*e + 450*b^3*c^2*d*e^2 - 105*b^4*c*e^
3 + 48*(30*c^5*d*e^2 + b*c^4*e^3)*x^3 + 8*(240*c^5*d^2*e + 30*b*c^4*d*e^2
- 7*b^2*c^3*e^3)*x^2 + 10*(96*c^5*d^3 + 48*b*c^4*d^2*e - 30*b^2*c^3*d*e^2
+ 7*b^3*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/c^5, 1/1920*(15*(32*b^2*c^3*d^3 - 4
8*b^3*c^2*d^2*e + 30*b^4*c*d*e^2 - 7*b^5*e^3)*sqrt(-c)*arctan(sqrt(c*x^2 +
 b*x)*sqrt(-c)/(c*x + b)) + (384*c^5*e^3*x^4 + 480*b*c^4*d^3 - 720*b^2*c^3
*d^2*e + 450*b^3*c^2*d*e^2 - 105*b^4*c*e^3 + 48*(30*c^5*d*e^2 + b*c^4*e^3)
*x^3 + 8*(240*c^5*d^2*e + 30*b*c^4*d*e^2 - 7*b^2*c^3*e^3)*x^2 + 10*(96*c^5
*d^3 + 48*b*c^4*d^2*e - 30*b^2*c^3*d*e^2 + 7*b^3*c^2*e^3)*x)*sqrt(c*x^2 +
b*x))/c^5]
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.58

$$\int (d+ex)^3\sqrt{bx+cx^2}\,dx$$

$$= \begin{cases} -\frac{b\left(bd^3-\frac{3b\left(3bd^2e-\frac{5b\left(3bde^2-\frac{7b\left(\frac{be^3}{10}+3cde^2\right)}{8c}+3cd^2e\right)}{6c}+cd^3\right)}{4c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{2c}+\sqrt{bx+cx^2}\left(\right. \\ \frac{2\left(\frac{d^3(bx)^{\frac{3}{2}}}{3}+\frac{3d^2e(bx)^{\frac{5}{2}}}{5b}+\frac{3de^2(bx)^{\frac{7}{2}}}{7b^2}+\frac{e^3(bx)^{\frac{9}{2}}}{9b^3}\right)}{b} \\ 0 \end{cases}$$

input
```
integrate((e*x+d)**3*(c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((-b*(b*d**3 - 3*b*(3*b*d**2*e - 5*b*(3*b*d*e**2 - 7*b*(b*e**3/10
 + 3*c*d*e**2)/(8*c) + 3*c*d**2*e)/(6*c) + c*d**3)/(4*c))*Piecewise((log(b
 + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c
) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(2*c) + sqrt(b*x
+ c*x**2)*(e**3*x**4/5 + x**3*(b*e**3/10 + 3*c*d*e**2)/(4*c) + x**2*(3*b*d
*e**2 - 7*b*(b*e**3/10 + 3*c*d*e**2)/(8*c) + 3*c*d**2*e)/(3*c) + x*(3*b*d*
*2*e - 5*b*(3*b*d*e**2 - 7*b*(b*e**3/10 + 3*c*d*e**2)/(8*c) + 3*c*d**2*e)/
(6*c) + c*d**3)/(2*c) + (b*d**3 - 3*b*(3*b*d**2*e - 5*b*(3*b*d*e**2 - 7*b*
(b*e**3/10 + 3*c*d*e**2)/(8*c) + 3*c*d**2*e)/(6*c) + c*d**3)/(4*c))/c), Ne
(c, 0)), (2*(d**3*(b*x)**(3/2)/3 + 3*d**2*e*(b*x)**(5/2)/(5*b) + 3*d*e**2*
(b*x)**(7/2)/(7*b**2) + e**3*(b*x)**(9/2)/(9*b**3))/b, Ne(b, 0)), (0, True
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 439, normalized size of antiderivative = 1.60

$$\int (d+ex)^3\sqrt{bx+cx^2}\,dx = \frac{(cx^2+bx)^{\frac{3}{2}}e^3x^2}{5\,c} + \frac{1}{2}\sqrt{cx^2+bx}d^3x - \frac{3\sqrt{cx^2+bx}bd^2ex}{4\,c} + \frac{15\sqrt{cx^2+bx}b^2de^2x}{32\,c^2} + \frac{3\,(cx^2+bx)^{\frac{3}{2}}de^2x}{4\,c} - \frac{7\sqrt{cx^2+bx}b^3e^3x}{64\,c^3} - \frac{7\,(cx^2+bx)^{\frac{3}{2}}be^3x}{40\,c^2} - \frac{b^2d^3\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,c^{\frac{3}{2}}} + \frac{3\,b^3d^2e\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{16\,c^{\frac{5}{2}}} - \frac{15\,b^4de^2\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{128\,c^{\frac{7}{2}}} + \frac{7\,b^5e^3\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{256\,c^{\frac{9}{2}}} + \frac{\sqrt{cx^2+bx}bd^3}{4\,c} - \frac{3\sqrt{cx^2+bx}b^2d^2e}{8\,c^2} + \frac{(cx^2+bx)^{\frac{3}{2}}d^2e}{c} + \frac{15\sqrt{cx^2+bx}b^3de^2}{64\,c^3} - \frac{5\,(cx^2+bx)^{\frac{3}{2}}bde^2}{8\,c^2} - \frac{7\sqrt{cx^2+bx}b^4e^3}{128\,c^4} + \frac{7\,(cx^2+bx)^{\frac{3}{2}}b^2e^3}{48\,c^3}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(c*x^2 + b*x)^(3/2)*e^3*x^2/c + 1/2*sqrt(c*x^2 + b*x)*d^3*x - 3/4*sqrt
(c*x^2 + b*x)*b*d^2*e*x/c + 15/32*sqrt(c*x^2 + b*x)*b^2*d*e^2*x/c^2 + 3/4*
(c*x^2 + b*x)^(3/2)*d*e^2*x/c - 7/64*sqrt(c*x^2 + b*x)*b^3*e^3*x/c^3 - 7/4
0*(c*x^2 + b*x)^(3/2)*b*e^3*x/c^2 - 1/8*b^2*d^3*log(2*c*x + b + 2*sqrt(c*x
^2 + b*x)*sqrt(c))/c^(3/2) + 3/16*b^3*d^2*e*log(2*c*x + b + 2*sqrt(c*x^2 +
 b*x)*sqrt(c))/c^(5/2) - 15/128*b^4*d*e^2*log(2*c*x + b + 2*sqrt(c*x^2 + b
*x)*sqrt(c))/c^(7/2) + 7/256*b^5*e^3*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*s
qrt(c))/c^(9/2) + 1/4*sqrt(c*x^2 + b*x)*b*d^3/c - 3/8*sqrt(c*x^2 + b*x)*b^
2*d^2*e/c^2 + (c*x^2 + b*x)^(3/2)*d^2*e/c + 15/64*sqrt(c*x^2 + b*x)*b^3*d*
e^2/c^3 - 5/8*(c*x^2 + b*x)^(3/2)*b*d*e^2/c^2 - 7/128*sqrt(c*x^2 + b*x)*b^
4*e^3/c^4 + 7/48*(c*x^2 + b*x)^(3/2)*b^2*e^3/c^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 255, normalized size of antiderivative = 0.93

$$\int (d+ex)^3\sqrt{bx+cx^2}\,dx$$
$$=\frac{1}{1920}\sqrt{cx^2+bx}\left(2\left(4\left(6\left(8\,e^3x+\frac{30\,c^4de^2+bc^3e^3}{c^4}\right)x+\frac{240\,c^4d^2e+30\,bc^3de^2-7\,b^2c^2e^3}{c^4}\right)x+\frac{5\,(96\,c^4\right.\right.\right.$$
$$+\frac{(32\,b^2c^3d^3-48\,b^3c^2d^2e+30\,b^4cde^2-7\,b^5e^3)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{256\,c^{\frac{9}{2}}}$$

input

```
integrate((e*x+d)^3*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output

```
1/1920*sqrt(c*x^2 + b*x)*(2*(4*(6*(8*e^3*x + (30*c^4*d*e^2 + b*c^3*e^3)/c^
4)*x + (240*c^4*d^2*e + 30*b*c^3*d*e^2 - 7*b^2*c^2*e^3)/c^4)*x + 5*(96*c^4
*d^3 + 48*b*c^3*d^2*e - 30*b^2*c^2*d*e^2 + 7*b^3*c*e^3)/c^4)*x + 15*(32*b*
c^3*d^3 - 48*b^2*c^2*d^2*e + 30*b^3*c*d*e^2 - 7*b^4*e^3)/c^4) + 1/256*(32*
b^2*c^3*d^3 - 48*b^3*c^2*d^2*e + 30*b^4*c*d*e^2 - 7*b^5*e^3)*log(abs(2*(sq
rt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(9/2)
```

Mupad [B] (verification not implemented)

Time = 6.26 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.31

$$\int (d+ex)^3\sqrt{bx+cx^2}\,dx$$
$$= d^3\sqrt{c\,x^2+b\,x}\,\left(\frac{x}{2}+\frac{b}{4\,c}\right) - \frac{7\,b\,e^3\left(\frac{x\,\left(c\,x^2+b\,x\right)^{3/2}}{4\,c}-\frac{5\,b\,\left(\frac{b^3\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^2+b\,x}\,\left(-3\,b^2+2\,b\,c\,x+8\,c^2\,x^2\right)}{24\,c^2}\right)}{8\,c}\right)}{10\,c}$$
$$+ \frac{e^3\,x^2\,\left(c\,x^2+b\,x\right)^{3/2}}{5\,c} - \frac{b^2\,d^3\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}} + \frac{3\,d\,e^2\,x\,\left(c\,x^2+b\,x\right)^{3/2}}{4\,c}$$
$$- \frac{15\,b\,d\,e^2\left(\frac{b^3\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^2+b\,x}\,\left(-3\,b^2+2\,b\,c\,x+8\,c^2\,x^2\right)}{24\,c^2}\right)}{8\,c}$$
$$+ \frac{3\,b^3\,d^2\,e\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x}\right)}{16\,c^{5/2}} + \frac{d^2\,e\,\sqrt{c\,x^2+b\,x}\,\left(-3\,b^2+2\,b\,c\,x+8\,c^2\,x^2\right)}{8\,c^2}$$

input
```
int((b*x + c*x^2)^(1/2)*(d + e*x)^3,x)
```

output
```
d^3*(b*x + c*x^2)^(1/2)*(x/2 + b/(4*c)) - (7*b*e^3*((x*(b*x + c*x^2)^(3/2)
)/(4*c) - (5*b*((b^3*log((b + 2*c*x)/c^(1/2) + 2*(b*x + c*x^2)^(1/2)))/(16
*c^(5/2)) + ((b*x + c*x^2)^(1/2)*(8*c^2*x^2 - 3*b^2 + 2*b*c*x))/(24*c^2)))
/(8*c)))/(10*c) + (e^3*x^2*(b*x + c*x^2)^(3/2))/(5*c) - (b^2*d^3*log((b/2
+ c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/(8*c^(3/2)) + (3*d*e^2*x*(b*x + c*x
^2)^(3/2))/(4*c) - (15*b*d*e^2*((b^3*log((b + 2*c*x)/c^(1/2) + 2*(b*x + c*
x^2)^(1/2)))/(16*c^(5/2)) + ((b*x + c*x^2)^(1/2)*(8*c^2*x^2 - 3*b^2 + 2*b*
c*x))/(24*c^2)))/(8*c) + (3*b^3*d^2*e*log((b + 2*c*x)/c^(1/2) + 2*(b*x + c
*x^2)^(1/2)))/(16*c^(5/2)) + (d^2*e*(b*x + c*x^2)^(1/2)*(8*c^2*x^2 - 3*b^2
 + 2*b*c*x))/(8*c^2)
```

Reduce [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.45

$$\int (d+ex)^3\sqrt{bx+cx^2}\,dx$$

$$= \frac{-105\sqrt{x}\,\sqrt{cx+b}\,b^4c\,e^3+450\sqrt{x}\,\sqrt{cx+b}\,b^3c^2d\,e^2+70\sqrt{x}\,\sqrt{cx+b}\,b^3c^2e^3x-720\sqrt{x}\,\sqrt{cx+b}\,b^2c^3d^2e-}{}$$

input
```
int((e*x+d)^3*(c*x^2+b*x)^(1/2),x)
```

output
```
( - 105*sqrt(x)*sqrt(b + c*x)*b**4*c*e**3 + 450*sqrt(x)*sqrt(b + c*x)*b**3
*c**2*d*e**2 + 70*sqrt(x)*sqrt(b + c*x)*b**3*c**2*e**3*x - 720*sqrt(x)*sqr
t(b + c*x)*b**2*c**3*d**2*e - 300*sqrt(x)*sqrt(b + c*x)*b**2*c**3*d*e**2*x
 - 56*sqrt(x)*sqrt(b + c*x)*b**2*c**3*e**3*x**2 + 480*sqrt(x)*sqrt(b + c*x
)*b*c**4*d**3 + 480*sqrt(x)*sqrt(b + c*x)*b*c**4*d**2*e*x + 240*sqrt(x)*sq
rt(b + c*x)*b*c**4*d*e**2*x**2 + 48*sqrt(x)*sqrt(b + c*x)*b*c**4*e**3*x**3
 + 960*sqrt(x)*sqrt(b + c*x)*c**5*d**3*x + 1920*sqrt(x)*sqrt(b + c*x)*c**5
*d**2*e*x**2 + 1440*sqrt(x)*sqrt(b + c*x)*c**5*d*e**2*x**3 + 384*sqrt(x)*s
qrt(b + c*x)*c**5*e**3*x**4 + 105*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqr
t(c))/sqrt(b))*b**5*e**3 - 450*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c
))/sqrt(b))*b**4*c*d*e**2 + 720*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(
c))/sqrt(b))*b**3*c**2*d**2*e - 480*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*s
qrt(c))/sqrt(b))*b**2*c**3*d**3)/(1920*c**5)
```

3.130 $\int (d+ex)^2\sqrt{bx+cx^2}\,dx$

Optimal result 1025
Mathematica [A] (verified) 1026
Rubi [A] (verified) 1026
Maple [A] (verified) 1029
Fricas [A] (verification not implemented) 1029
Sympy [A] (verification not implemented) 1030
Maxima [A] (verification not implemented) 1031
Giac [A] (verification not implemented) 1032
Mupad [B] (verification not implemented) 1032
Reduce [B] (verification not implemented) 1033

Optimal result

Integrand size = 21, antiderivative size = 195

$$\begin{aligned}\int (d+ex)^2\sqrt{bx+cx^2}\,dx &= \frac{b(16c^2d^2-16bcde+5b^2e^2)\sqrt{bx+cx^2}}{64c^3} \\ &\quad+\frac{1}{32}\left(16d^2-\frac{be(16cd-5be)}{c^2}\right)x\sqrt{bx+cx^2} \\ &\quad+\frac{e(16cd-5be)\left(bx+cx^2\right)^{3/2}}{24c^2}+\frac{e^2x\left(bx+cx^2\right)^{3/2}}{4c} \\ &\quad-\frac{b^2(16c^2d^2-16bcde+5b^2e^2)\,\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{7/2}}\end{aligned}$$

output
```
1/64*b*(5*b^2*e^2-16*b*c*d*e+16*c^2*d^2)*(c*x^2+b*x)^(1/2)/c^3+1/32*(16*d^
2-b*e*(-5*b*e+16*c*d)/c^2)*x*(c*x^2+b*x)^(1/2)+1/24*e*(-5*b*e+16*c*d)*(c*x
^2+b*x)^(3/2)/c^2+1/4*e^2*x*(c*x^2+b*x)^(3/2)/c-1/64*b^2*(5*b^2*e^2-16*b*c
*d*e+16*c^2*d^2)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.89

$$\int (d+ex)^2\sqrt{bx+cx^2}\,dx$$
$$=\frac{\sqrt{x(b+cx)}\left(\sqrt{c}(15b^3e^2-2b^2ce(24d+5ex)+8bc^2(6d^2+4dex+e^2x^2)+16c^3x(6d^2+8dex+3e^2x^2))\right.}{192c^{7/2}}$$

input
```
Integrate[(d + e*x)^2*Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[c]*(15*b^3*e^2 - 2*b^2*c*e*(24*d + 5*e*x) + 8*b*c
^2*(6*d^2 + 4*d*e*x + e^2*x^2) + 16*c^3*x*(6*d^2 + 8*d*e*x + 3*e^2*x^2)) +
 (6*b^2*(16*c^2*d^2 - 16*b*c*d*e + 5*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[x])/(S
qrt[b] - Sqrt[b + c*x])])/(Sqrt[x]*Sqrt[b + c*x])))/(192*c^(7/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.79, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1166, 27, 1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{bx+cx^2}(d+ex)^2\,dx$$
$$\Big\downarrow \text{1166}$$
$$\frac{\int \frac{1}{2}(d(8cd-3be)+5e(2cd-be)x)\sqrt{cx^2+bx}dx}{4c}+\frac{e\left(bx+cx^2\right)^{3/2}(d+ex)}{4c}$$
$$\Big\downarrow \text{27}$$
$$\frac{\int (d(8cd-3be)+5e(2cd-be)x)\sqrt{cx^2+bx}dx}{8c}+\frac{e\left(bx+cx^2\right)^{3/2}(d+ex)}{4c}$$
$$\Big\downarrow \text{1160}$$

$$\frac{\frac{(5b^2e^2-16bcde+16c^2d^2)\int\sqrt{cx^2+bx}dx}{2c}+\frac{5e(bx+cx^2)^{3/2}(2cd-be)}{3c}}{8c}+\frac{e(bx+cx^2)^{3/2}(d+ex)}{4c}$$

$$\Big\downarrow \; 1087$$

$$\frac{\frac{(5b^2e^2-16bcde+16c^2d^2)\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{2c}+\frac{5e(bx+cx^2)^{3/2}(2cd-be)}{3c}}{8c}+\frac{e(bx+cx^2)^{3/2}(d+ex)}{4c}$$

$$\Big\downarrow \; 1091$$

$$\frac{\frac{(5b^2e^2-16bcde+16c^2d^2)\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{2c}+\frac{5e(bx+cx^2)^{3/2}(2cd-be)}{3c}}{8c}+\frac{e(bx+cx^2)^{3/2}(d+ex)}{4c}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)(5b^2e^2-16bcde+16c^2d^2)}{2c}+\frac{5e(bx+cx^2)^{3/2}(2cd-be)}{3c}}{8c}+\frac{e(bx+cx^2)^{3/2}(d+ex)}{4c}$$

input
```
Int[(d + e*x)^2*Sqrt[b*x + c*x^2],x]
```

output
```
(e*(d + e*x)*(b*x + c*x^2)^(3/2))/(4*c) + ((5*e*(2*c*d - b*e)*(b*x + c*x^2
)^(3/2))/(3*c) + ((16*c^2*d^2 - 16*b*c*d*e + 5*b^2*e^2)*(((b + 2*c*x)*Sqrt
[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(
3/2))))/(2*c))/(8*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{5\left(b^{2}\left(b^{2}e^{2}-\frac{16}{5}bcde+\frac{16}{5}c^{2}d^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)-\sqrt{x(cx+b)}\left(\frac{16\left(\frac{1}{6}e^{2}x^{2}+\frac{2}{3}dex+d^{2}\right)bc^{\frac{5}{2}}}{5}+\frac{32x\left(\frac{1}{2}e^{2}x^{2}+\frac{4}{3}dex+d^{2}\right)c^{\frac{7}{2}}}{5}\right.\right.}{64c^{\frac{7}{2}}}$
risch	$\frac{\left(48c^{3}e^{2}x^{3}+8bc^{2}e^{2}x^{2}+128c^{3}dex^{2}-10b^{2}ce^{2}x+32bc^{2}dex+96c^{3}d^{2}x+15e^{2}b^{3}-48b^{2}cde+48bc^{2}d^{2}\right)x(cx+b)}{192c^{3}\sqrt{x(cx+b)}}-\frac{b^{2}\left(5b^{2}e^{2}-1\right.}{}$
default	$d^{2}\left(\frac{(2cx+b)\sqrt{cx^{2}+bx}}{4c}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^{2}+bx}\right)}{8c^{\frac{3}{2}}}\right)+e^{2}\left(\frac{x\left(cx^{2}+bx\right)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{\left(cx^{2}+bx\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{cx^{2}+}}{4c}\right.}{}\right.}{8c}\right.$

input
```
int((e*x+d)^2*(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/64/c^(7/2)*(b^2*(b^2*e^2-16/5*b*c*d*e+16/5*c^2*d^2)*arctanh((x*(c*x+b))
^(1/2)/x/c^(1/2))-(x*(c*x+b))^(1/2)*(16/5*(1/6*e^2*x^2+2/3*d*e*x+d^2)*b*c^
(5/2)+32/5*x*(1/2*e^2*x^2+4/3*d*e*x+d^2)*c^(7/2)+e*((-2/3*e*x-16/5*d)*c^(3
/2)+b*e*c^(1/2))*b^2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.73

$$\int (d+ex)^2\sqrt{bx+cx^2}\,dx$$

$$=\left[\frac{3\left(16b^{2}c^{2}d^{2}-16b^{3}cde+5b^{4}e^{2}\right)\sqrt{c}\log\left(2cx+b-2\sqrt{cx^{2}+bx}\sqrt{c}\right)+2\left(48c^{4}e^{2}x^{3}+48bc^{3}d^{2}-48b^{2}c^{2}\right.}{384c^{4}}\right.$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/384*(3*(16*b^2*c^2*d^2 - 16*b^3*c*d*e + 5*b^4*e^2)*sqrt(c)*log(2*c*x +
b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(48*c^4*e^2*x^3 + 48*b*c^3*d^2 - 48*b
^2*c^2*d*e + 15*b^3*c*e^2 + 8*(16*c^4*d*e + b*c^3*e^2)*x^2 + 2*(48*c^4*d^2
 + 16*b*c^3*d*e - 5*b^2*c^2*e^2)*x)*sqrt(c*x^2 + b*x))/c^4, 1/192*(3*(16*b
^2*c^2*d^2 - 16*b^3*c*d*e + 5*b^4*e^2)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*s
qrt(-c)/(c*x + b)) + (48*c^4*e^2*x^3 + 48*b*c^3*d^2 - 48*b^2*c^2*d*e + 15*
b^3*c*e^2 + 8*(16*c^4*d*e + b*c^3*e^2)*x^2 + 2*(48*c^4*d^2 + 16*b*c^3*d*e
- 5*b^2*c^2*e^2)*x)*sqrt(c*x^2 + b*x))/c^4]
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.46

$$\int (d+ex)^2\sqrt{bx+cx^2}\,dx$$

$$= \begin{cases} -\frac{b\left(bd^2-\frac{3b\left(2bde-\frac{5b\left(\frac{be^2}{8}+2cde\right)}{6c}+cd^2\right)}{4c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{2c}+\sqrt{bx+cx^2}\left(\frac{e^2x^3}{4}+\frac{x^2\left(\frac{be^2}{8}+2cc\right.}{3c}\right. \\ \frac{2\left(\frac{d^2(bx)^{\frac{3}{2}}}{3}+\frac{2de(bx)^{\frac{5}{2}}}{5b}+\frac{e^2(bx)^{\frac{7}{2}}}{7b^2}\right)}{b} \\ 0 \end{cases}$$

input
```
integrate((e*x+d)**2*(c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((-b*(b*d**2 - 3*b*(2*b*d*e - 5*b*(b*e**2/8 + 2*c*d*e)/(6*c) + c*
d**2)/(4*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt
(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)*
*2), True))/(2*c) + sqrt(b*x + c*x**2)*(e**2*x**3/4 + x**2*(b*e**2/8 + 2*c
*d*e)/(3*c) + x*(2*b*d*e - 5*b*(b*e**2/8 + 2*c*d*e)/(6*c) + c*d**2)/(2*c)
+ (b*d**2 - 3*b*(2*b*d*e - 5*b*(b*e**2/8 + 2*c*d*e)/(6*c) + c*d**2)/(4*c))
/c), Ne(c, 0)), (2*(d**2*(b*x)**(3/2)/3 + 2*d*e*(b*x)**(5/2)/(5*b) + e**2*
(b*x)**(7/2)/(7*b**2))/b, Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.45

$$\begin{aligned}\int (d+ex)^2\sqrt{bx+cx^2}\,dx =& \frac{1}{2}\sqrt{cx^2+bx}d^2x - \frac{\sqrt{cx^2+bx}bdex}{2\,c} + \frac{5\sqrt{cx^2+bx}b^2e^2x}{32\,c^2} \\ &+ \frac{(cx^2+bx)^{\frac{3}{2}}e^2x}{4\,c} - \frac{b^2d^2\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,c^{\frac{3}{2}}} \\ &+ \frac{b^3de\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,c^{\frac{5}{2}}} \\ &- \frac{5\,b^4e^2\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{128\,c^{\frac{7}{2}}} \\ &+ \frac{\sqrt{cx^2+bx}bd^2}{4\,c} - \frac{\sqrt{cx^2+bx}b^2de}{4\,c^2} + \frac{2\,(cx^2+bx)^{\frac{3}{2}}de}{3\,c} \\ &+ \frac{5\sqrt{cx^2+bx}b^3e^2}{64\,c^3} - \frac{5\,(cx^2+bx)^{\frac{3}{2}}be^2}{24\,c^2}\end{aligned}$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^2 + b*x)*d^2*x - 1/2*sqrt(c*x^2 + b*x)*b*d*e*x/c + 5/32*sqrt(
c*x^2 + b*x)*b^2*e^2*x/c^2 + 1/4*(c*x^2 + b*x)^(3/2)*e^2*x/c - 1/8*b^2*d^2
*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(3/2) + 1/8*b^3*d*e*log(2*
c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(5/2) - 5/128*b^4*e^2*log(2*c*x +
 b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(7/2) + 1/4*sqrt(c*x^2 + b*x)*b*d^2/c
- 1/4*sqrt(c*x^2 + b*x)*b^2*d*e/c^2 + 2/3*(c*x^2 + b*x)^(3/2)*d*e/c + 5/64
*sqrt(c*x^2 + b*x)*b^3*e^2/c^3 - 5/24*(c*x^2 + b*x)^(3/2)*b*e^2/c^2
```

Giac [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.88

$$\int (d+ex)^2\sqrt{bx+cx^2}\,dx$$
$$= \frac{1}{192}\sqrt{cx^2+bx}\left(2\left(4\left(6e^2x+\frac{16c^3de+bc^2e^2}{c^3}\right)x+\frac{48c^3d^2+16bc^2de-5b^2ce^2}{c^3}\right)x+\frac{3(16bc^2d^2-16b}{c^3}\right.$$
$$+\frac{(16b^2c^2d^2-16b^3cde+5b^4e^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{128c^{\frac{7}{2}}}$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x)*(2*(4*(6*e^2*x + (16*c^3*d*e + b*c^2*e^2)/c^3)*x +
 (48*c^3*d^2 + 16*b*c^2*d*e - 5*b^2*c*e^2)/c^3)*x + 3*(16*b*c^2*d^2 - 16*b
^2*c*d*e + 5*b^3*e^2)/c^3) + 1/128*(16*b^2*c^2*d^2 - 16*b^3*c*d*e + 5*b^4*
e^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 5.90 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.18

$$\int (d+ex)^2\sqrt{bx+cx^2}\,dx$$
$$= d^2\sqrt{c\,x^2+b\,x}\left(\frac{x}{2}+\frac{b}{4\,c}\right)$$
$$-\frac{5\,b\,e^2\left(\frac{b^3\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^2+b\,x}\,\left(-3\,b^2+2\,b\,c\,x+8\,c^2\,x^2\right)}{24\,c^2}\right)}{8\,c}$$
$$-\frac{b^2\,d^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}+\frac{e^2\,x\,\left(c\,x^2+b\,x\right)^{3/2}}{4\,c}$$
$$+\frac{b^3\,d\,e\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x}\right)}{8\,c^{5/2}}+\frac{d\,e\,\sqrt{c\,x^2+b\,x}\,\left(-3\,b^2+2\,b\,c\,x+8\,c^2\,x^2\right)}{12\,c^2}$$

input
```
int((b*x + c*x^2)^(1/2)*(d + e*x)^2,x)
```

output
```
d^2*(b*x + c*x^2)^(1/2)*(x/2 + b/(4*c)) - (5*b*e^2*((b^3*log((b + 2*c*x)/c
^(1/2) + 2*(b*x + c*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x + c*x^2)^(1/2)*(8*c^
2*x^2 - 3*b^2 + 2*b*c*x))/(24*c^2)))/(8*c) - (b^2*d^2*log((b/2 + c*x)/c^(1
/2) + (b*x + c*x^2)^(1/2)))/(8*c^(3/2)) + (e^2*x*(b*x + c*x^2)^(3/2))/(4*c
) + (b^3*d*e*log((b + 2*c*x)/c^(1/2) + 2*(b*x + c*x^2)^(1/2)))/(8*c^(5/2))
 + (d*e*(b*x + c*x^2)^(1/2)*(8*c^2*x^2 - 3*b^2 + 2*b*c*x))/(12*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.31

$$\int (d+ex)^2\sqrt{bx+cx^2}\,dx$$

$$= \frac{15\sqrt{x}\,\sqrt{cx+b}\,b^3c\,e^2 - 48\sqrt{x}\,\sqrt{cx+b}\,b^2c^2de - 10\sqrt{x}\,\sqrt{cx+b}\,b^2c^2e^2x + 48\sqrt{x}\,\sqrt{cx+b}\,b\,c^3d^2 + 32\sqrt{x}\,\sqrt{}}{}$$

input
```
int((e*x+d)^2*(c*x^2+b*x)^(1/2),x)
```

output
```
(15*sqrt(x)*sqrt(b + c*x)*b**3*c*e**2 - 48*sqrt(x)*sqrt(b + c*x)*b**2*c**2
*d*e - 10*sqrt(x)*sqrt(b + c*x)*b**2*c**2*e**2*x + 48*sqrt(x)*sqrt(b + c*x
)*b*c**3*d**2 + 32*sqrt(x)*sqrt(b + c*x)*b*c**3*d*e*x + 8*sqrt(x)*sqrt(b +
 c*x)*b*c**3*e**2*x**2 + 96*sqrt(x)*sqrt(b + c*x)*c**4*d**2*x + 128*sqrt(x
)*sqrt(b + c*x)*c**4*d*e*x**2 + 48*sqrt(x)*sqrt(b + c*x)*c**4*e**2*x**3 -
15*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**4*e**2 + 48*s
qrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**3*c*d*e - 48*sqrt
(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**2*c**2*d**2)/(192*c*
*4)
```

3.131 $\int (d+ex)\sqrt{bx+cx^2}\,dx$

Optimal result 1034
Mathematica [A] (verified) 1034
Rubi [A] (verified) 1035
Maple [A] (verified) 1037
Fricas [A] (verification not implemented) 1037
Sympy [A] (verification not implemented) 1038
Maxima [A] (verification not implemented) 1039
Giac [A] (verification not implemented) 1039
Mupad [B] (verification not implemented) 1040
Reduce [B] (verification not implemented) 1040

Optimal result

Integrand size = 19, antiderivative size = 124

$$\int (d+ex)\sqrt{bx+cx^2}\,dx=\frac{b(2cd-be)\sqrt{bx+cx^2}}{8c^2}+\frac{(2cd-be)x\sqrt{bx+cx^2}}{4c}+\frac{e(bx+cx^2)^{3/2}}{3c}-\frac{b^2(2cd-be)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{5/2}}$$

output
```
1/8*b*(-b*e+2*c*d)*(c*x^2+b*x)^(1/2)/c^2+1/4*(-b*e+2*c*d)*x*(c*x^2+b*x)^(1
/2)/c+1/3*e*(c*x^2+b*x)^(3/2)/c-1/8*b^2*(-b*e+2*c*d)*arctanh(c^(1/2)*x/(c*
x^2+b*x)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int (d+ex)\sqrt{bx+cx^2}\,dx=\frac{\sqrt{x(b+cx)}\left(\sqrt{c}\sqrt{x}(-3b^2e+2bc(3d+ex)+4c^2x(3d+2ex))+\frac{6b^2(-2cd+be)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{x}}{-\sqrt{b}+\sqrt{b+cx}}\right)}{\sqrt{b+cx}}\right)}{24c^{5/2}\sqrt{x}}$$

input
```
Integrate[(d + e*x)*Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[c]*Sqrt[x]*(-3*b^2*e + 2*b*c*(3*d + e*x) + 4*c^2*
x*(3*d + 2*e*x)) + (6*b^2*(-2*c*d + b*e)*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[
b] + Sqrt[b + c*x])])/Sqrt[b + c*x]))/(24*c^(5/2)*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{bx+cx^2}(d+ex)\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{(2cd-be)\int \sqrt{cx^2+bx}dx}{2c}+\frac{e\left(bx+cx^2\right)^{3/2}}{3c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(2cd-be)\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{2c}+\frac{e\left(bx+cx^2\right)^{3/2}}{3c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(2cd-be)\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{2c}+\frac{e\left(bx+cx^2\right)^{3/2}}{3c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)(2cd-be)}{2c}+\frac{e\left(bx+cx^2\right)^{3/2}}{3c}$$

input
```
Int[(d + e*x)*Sqrt[b*x + c*x^2],x]
```

output
```
(e*(b*x + c*x^2)^(3/2))/(3*c) + ((2*c*d - b*e)*(((b + 2*c*x)*Sqrt[b*x + c*
x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))/(
2*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$\frac{(e\,b^{3}-2cd\,b^{2})\,\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)-\sqrt{x(cx+b)}\,\left(-2\left(\frac{ex}{3}+d\right)b\,c^{\frac{3}{2}}+\left(-\frac{8}{3}e\,x^{2}-4dx\right)c^{\frac{5}{2}}+\sqrt{c}\,b^{2}e\right)}{8c^{\frac{5}{2}}}$
risch	$-\frac{(-8c^{2}e\,x^{2}-2bcex-12c^{2}dx+3e\,b^{2}-6dbc)x(cx+b)}{24c^{2}\sqrt{x(cx+b)}}+\frac{b^{2}(be-2cd)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{16c^{\frac{5}{2}}}$
default	$d\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx}}{4c}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{8c^{\frac{3}{2}}}\right)+e\left(\frac{(c\,x^{2}+bx)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx}}{4c}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{8c^{\frac{3}{2}}}\right.}{2c}\right.$

input
```
int((e*x+d)*(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/8*((b^3*e-2*b^2*c*d)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))-(x*(c*x+b))^(1
/2)*(-2*(1/3*e*x+d)*b*c^(3/2)+(-8/3*e*x^2-4*d*x)*c^(5/2)+c^(1/2)*b^2*e))/c
^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.66

$$\int (d+ex)\sqrt{bx+cx^2}\,dx$$

$$=\left[-\frac{3\,(2\,b^2cd-b^3e)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(8\,c^3ex^2+6\,bc^2d-3\,b^2ce+2\,(6\,c^3d+bc^2e)x)}{48\,c^3}\right.$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(2*b^2*c*d - b^3*e)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*
sqrt(c)) - 2*(8*c^3*e*x^2 + 6*b*c^2*d - 3*b^2*c*e + 2*(6*c^3*d + b*c^2*e)*
x)*sqrt(c*x^2 + b*x))/c^3, 1/24*(3*(2*b^2*c*d - b^3*e)*sqrt(-c)*arctan(sqr
t(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (8*c^3*e*x^2 + 6*b*c^2*d - 3*b^2*c*e
+ 2*(6*c^3*d + b*c^2*e)*x)*sqrt(c*x^2 + b*x))/c^3]
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.35

$$\int (d+ex)\sqrt{bx+cx^2}\,dx$$
$$= \begin{cases} -\frac{b\left(bd-\frac{3b\left(\frac{be}{6}+cd\right)}{4c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{2c}+\sqrt{bx+cx^2}\left(\frac{ex^2}{3}+\frac{x\left(\frac{be}{6}+cd\right)}{2c}+\frac{bd-\frac{3b\left(\frac{be}{6}+cd\right)}{4c}}{c}\right) \\ \frac{2\left(\frac{d(bx)^{\frac{3}{2}}}{3}+\frac{e(bx)^{\frac{5}{2}}}{5b}\right)}{b} \\ 0 \end{cases}$$

input
```
integrate((e*x+d)*(c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((-b*(b*d - 3*b*(b*e/6 + c*d)/(4*c))*Piecewise((log(b + 2*sqrt(c)
*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b
/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(2*c) + sqrt(b*x + c*x**2)*(e
*x**2/3 + x*(b*e/6 + c*d)/(2*c) + (b*d - 3*b*(b*e/6 + c*d)/(4*c))/c), Ne(c
, 0)), (2*(d*(b*x)**(3/2)/3 + e*(b*x)**(5/2)/(5*b))/b, Ne(b, 0)), (0, True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.24

$$\int (d+ex)\sqrt{bx+cx^2}\,dx = \frac{1}{2}\sqrt{cx^2+bx}dx - \frac{\sqrt{cx^2+bx}bex}{4\,c} - \frac{b^2d\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,c^{\frac{3}{2}}} + \frac{b^3e\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{16\,c^{\frac{5}{2}}} + \frac{\sqrt{cx^2+bx}bd}{4\,c} - \frac{\sqrt{cx^2+bx}b^2e}{8\,c^2} + \frac{(cx^2+bx)^{\frac{3}{2}}e}{3\,c}$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^2 + b*x)*d*x - 1/4*sqrt(c*x^2 + b*x)*b*e*x/c - 1/8*b^2*d*log(
2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(3/2) + 1/16*b^3*e*log(2*c*x +
b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(5/2) + 1/4*sqrt(c*x^2 + b*x)*b*d/c - 1
/8*sqrt(c*x^2 + b*x)*b^2*e/c^2 + 1/3*(c*x^2 + b*x)^(3/2)*e/c
```

Giac [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.82

$$\int (d+ex)\sqrt{bx+cx^2}\,dx = \frac{1}{24}\sqrt{cx^2+bx}\left(2\left(4\,ex+\frac{6\,c^2d+bce}{c^2}\right)x+\frac{3\,(2\,bcd-b^2e)}{c^2}\right) + \frac{(2\,b^2cd-b^3e)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{5}{2}}}$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x)*(2*(4*e*x + (6*c^2*d + b*c*e)/c^2)*x + 3*(2*b*c*d -
 b^2*e)/c^2) + 1/16*(2*b^2*c*d - b^3*e)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2
+ b*x))*sqrt(c) + b))/c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\int (d+ex)\sqrt{bx+cx^2}\,dx = d\sqrt{c\,x^2+b\,x}\left(\frac{x}{2}+\frac{b}{4\,c}\right)-\frac{b^2\,d\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}+\frac{b^3\,e\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x}\right)}{16\,c^{5/2}}+\frac{e\,\sqrt{c\,x^2+b\,x}\,\left(-3\,b^2+2\,b\,c\,x+8\,c^2\,x^2\right)}{24\,c^2}$$

input
```
int((b*x + c*x^2)^(1/2)*(d + e*x),x)
```

output
```
d*(b*x + c*x^2)^(1/2)*(x/2 + b/(4*c)) - (b^2*d*log((b/2 + c*x)/c^(1/2) + (
b*x + c*x^2)^(1/2)))/(8*c^(3/2)) + (b^3*e*log((b + 2*c*x)/c^(1/2) + 2*(b*x
 + c*x^2)^(1/2)))/(16*c^(5/2)) + (e*(b*x + c*x^2)^(1/2)*(8*c^2*x^2 - 3*b^2
 + 2*b*c*x))/(24*c^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.10

$$\int (d+ex)\sqrt{bx+cx^2}\,dx = \frac{-3\sqrt{x}\,\sqrt{cx+b}\,b^2ce+6\sqrt{x}\,\sqrt{cx+b}\,b\,c^2d+2\sqrt{x}\,\sqrt{cx+b}\,b\,c^2ex+12\sqrt{x}\,\sqrt{cx+b}\,c^3dx+8\sqrt{x}\,\sqrt{cx+b}\,c^3}{24c^3}$$

input
```
int((e*x+d)*(c*x^2+b*x)^(1/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(b + c*x)*b**2*c*e + 6*sqrt(x)*sqrt(b + c*x)*b*c**2*d +
2*sqrt(x)*sqrt(b + c*x)*b*c**2*e*x + 12*sqrt(x)*sqrt(b + c*x)*c**3*d*x + 8
*sqrt(x)*sqrt(b + c*x)*c**3*e*x**2 + 3*sqrt(c)*log((sqrt(b + c*x) + sqrt(x
)*sqrt(c))/sqrt(b))*b**3*e - 6*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c
))/sqrt(b))*b**2*c*d)/(24*c**3)
```

3.132 $\int \sqrt{bx+cx^2}\,dx$

Optimal result . . . 1041
Mathematica [A] (verified) . . . 1041
Rubi [A] (verified) . . . 1042
Maple [A] (verified) . . . 1043
Fricas [A] (verification not implemented) . . . 1043
Sympy [A] (verification not implemented) . . . 1044
Maxima [A] (verification not implemented) . . . 1045
Giac [A] (verification not implemented) . . . 1045
Mupad [B] (verification not implemented) . . . 1045
Reduce [B] (verification not implemented) . . . 1046

Optimal result

Integrand size = 13, antiderivative size = 73

$$\int \sqrt{bx+cx^2}\,dx=\frac{b\sqrt{bx+cx^2}}{4c}+\frac{1}{2}x\sqrt{bx+cx^2}-\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}$$

output
```
1/4*b*(c*x^2+b*x)^(1/2)/c+1/2*x*(c*x^2+b*x)^(1/2)-1/4*b^2*arctanh(c^(1/2)*
x/(c*x^2+b*x)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.14

$$\int \sqrt{bx+cx^2}\,dx=\frac{\sqrt{x(b+cx)}\left(\sqrt{c}(b+2cx)+\frac{2b^2\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}-\sqrt{b+cx}}\right)}{\sqrt{x}\sqrt{b+cx}}\right)}{4c^{3/2}}$$

input
```
Integrate[Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[c]*(b + 2*c*x) + (2*b^2*ArcTanh[(Sqrt[c]*Sqrt[x])
/(Sqrt[b] - Sqrt[b + c*x])])/(Sqrt[x]*Sqrt[b + c*x])))/(4*c^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{bx+cx^2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^2+bx}}dx}{8c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}$$

$$\Big\downarrow \text{219}$$

$$\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}$$

input
```
Int[Sqrt[b*x + c*x^2],x]
```

output
```
((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x
+ c*x^2]])/(4*c^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{(2cx+b)\sqrt{c\,x^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{8c^{\frac{3}{2}}}$	56
pseudoelliptic	$\frac{2c^{\frac{3}{2}}\sqrt{x(cx+b)}\,x+b\sqrt{c}\,\sqrt{x(cx+b)}-\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)b^2}{4c^{\frac{3}{2}}}$	58
risch	$\frac{(2cx+b)x(cx+b)}{4c\sqrt{x(cx+b)}}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{8c^{\frac{3}{2}}}$	60

input
```
int((c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*c*x+b)/c*(c*x^2+b*x)^(1/2)-1/8*b^2/c^(3/2)*ln((1/2*b+c*x)/c^(1/2)+(
c*x^2+b*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.67

$$\int \sqrt{bx+cx^2}\,dx$$

$$=\left[\frac{b^2\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(2\,c^2x+bc\right)\sqrt{cx^2+bx}}{8\,c^2},\frac{b^2\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)+\left(2\,c^2x\right.}{4\,c^2}\right.$$

input
```
integrate((c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*(b^2*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(2*c^2*
x + b*c)*sqrt(c*x^2 + b*x))/c^2, 1/4*(b^2*sqrt(-c)*arctan(sqrt(c*x^2 + b*x
)*sqrt(-c)/(c*x + b)) + (2*c^2*x + b*c)*sqrt(c*x^2 + b*x))/c^2]
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.40

$$\int \sqrt{bx+cx^2}\,dx = \begin{cases} -\frac{b^2\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{8c} + \left(\frac{b}{4c}+\frac{x}{2}\right)\sqrt{bx+cx^2} & \text{for } c\neq 0 \\ \frac{2(bx)^{\frac{3}{2}}}{3b} & \text{for } b\neq 0 \\ 0 & \text{otherwise} \end{cases}$$

input
```
integrate((c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((-b**2*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/
sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) +
 x)**2), True))/(8*c) + (b/(4*c) + x/2)*sqrt(b*x + c*x**2), Ne(c, 0)), (2*
(b*x)**(3/2)/(3*b), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \sqrt{bx+cx^2}\,dx = \frac{1}{2}\sqrt{cx^2+bx}x - \frac{b^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^2+bx}b}{4\,c}$$

input
```
integrate((c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^2 + b*x)*x - 1/8*b^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt
(c))/c^(3/2) + 1/4*sqrt(c*x^2 + b*x)*b/c
```

Giac [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.81

$$\int \sqrt{bx+cx^2}\,dx = \frac{1}{4}\sqrt{cx^2+bx}\left(2\,x+\frac{b}{c}\right) + \frac{b^2\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{3}{2}}}$$

input
```
integrate((c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x)*(2*x + b/c) + 1/8*b^2*log(abs(2*(sqrt(c)*x - sqrt(c*
x^2 + b*x))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int \sqrt{bx+cx^2}\,dx = \sqrt{c\,x^2+b\,x}\,\left(\frac{x}{2}+\frac{b}{4\,c}\right) - \frac{b^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}$$

input
```
int((b*x + c*x^2)^(1/2),x)
```

output
```
(b*x + c*x^2)^(1/2)*(x/2 + b/(4*c)) - (b^2*log((b/2 + c*x)/c^(1/2) + (b*x
+ c*x^2)^(1/2)))/(8*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \sqrt{bx+cx^2}\,dx=\frac{\sqrt{x}\,\sqrt{cx+b}\,bc+2\sqrt{x}\,\sqrt{cx+b}\,c^2x-\sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^2}{4c^2}$$

input
```
int((c*x^2+b*x)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(b + c*x)*b*c + 2*sqrt(x)*sqrt(b + c*x)*c**2*x - sqrt(c)*log(
(sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**2)/(4*c**2)
```

3.133 $\int \frac{\sqrt{bx+cx^2}}{d+ex} \, dx$

Optimal result 1047
Mathematica [C] (verified) 1047
Rubi [A] (verified) 1048
Maple [A] (verified) 1050
Fricas [A] (verification not implemented) 1051
Sympy [F] 1051
Maxima [F(-2)] 1052
Giac [F(-2)] 1052
Mupad [F(-1)] 1052
Reduce [B] (verification not implemented) 1053

Optimal result

Integrand size = 21, antiderivative size = 113

$$\int \frac{\sqrt{bx+cx^2}}{d+ex} \, dx = \frac{\sqrt{bx+cx^2}}{e} - \frac{(2cd-be)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}e^2} + \frac{2\sqrt{d}\sqrt{cd-be}\text{arctanh}\left(\frac{\sqrt{cd-bex}}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{e^2}$$

output
```
(c*x^2+b*x)^(1/2)/e-(-b*e+2*c*d)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(1
/2)/e^2+2*d^(1/2)*(-b*e+c*d)^(1/2)*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(c*x
^2+b*x)^(1/2))/e^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.43 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.56

$$\int \frac{\sqrt{bx+cx^2}}{d+ex} \, dx$$
$$= \frac{\sqrt{x}\sqrt{b+cx}\left(c\sqrt{d}e\sqrt{x}\sqrt{b+cx}+2\left(cd-be-i\sqrt{b}\sqrt{e}\sqrt{cd-be}\right)\sqrt{-cd+2be-2i\sqrt{b}\sqrt{e}\sqrt{cd-be}}\,\text{arctan}\right.}{}$$

input
```
Integrate[Sqrt[b*x + c*x^2]/(d + e*x),x]
```

output
```
(Sqrt[x]*Sqrt[b + c*x]*(c*Sqrt[d]*e*Sqrt[x]*Sqrt[b + c*x] + 2*(c*d - b*e -
 I*Sqrt[b]*Sqrt[e]*Sqrt[c*d - b*e])*Sqrt[-(c*d) + 2*b*e - (2*I)*Sqrt[b]*Sq
rt[e]*Sqrt[c*d - b*e]]*ArcTan[(Sqrt[-(c*d) + 2*b*e - (2*I)*Sqrt[b]*Sqrt[e]
*Sqrt[c*d - b*e]]*Sqrt[x])/(Sqrt[d]*(Sqrt[b] - Sqrt[b + c*x]))] + 2*(c*d -
 b*e + I*Sqrt[b]*Sqrt[e]*Sqrt[c*d - b*e])*Sqrt[-(c*d) + 2*b*e + (2*I)*Sqrt
[b]*Sqrt[e]*Sqrt[c*d - b*e]]*ArcTan[(Sqrt[-(c*d) + 2*b*e + (2*I)*Sqrt[b]*S
qrt[e]*Sqrt[c*d - b*e]]*Sqrt[x])/(Sqrt[d]*(Sqrt[b] - Sqrt[b + c*x]))] + 2*
Sqrt[c]*Sqrt[d]*(2*c*d - b*e)*ArcTanh[(Sqrt[c]*Sqrt[x])/(Sqrt[b] - Sqrt[b
+ c*x])]))/(c*Sqrt[d]*e^2*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1162, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+cx^2}}{d+ex}\,dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{\sqrt{bx+cx^2}}{e}-\frac{\int \frac{bd+(2cd-be)x}{(d+ex)\sqrt{cx^2+bx}}dx}{2e}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\sqrt{bx+cx^2}}{e}-\frac{\frac{(2cd-be)\int \frac{1}{\sqrt{cx^2+bx}}dx}{e}-\frac{2d(cd-be)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{2e}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\sqrt{bx+cx^2}}{e}-\frac{\frac{2(2cd-be)\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{e}-\frac{2d(cd-be)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{bx+cx^2}}{e}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)}{\sqrt{c}e}-\frac{2d(cd-be)\int\frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{2e}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\sqrt{bx+cx^2}}{e}-\frac{\frac{4d(cd-be)\int\frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{e}+\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)}{\sqrt{c}e}}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{bx+cx^2}}{e}-\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)}{\sqrt{c}e}-\frac{2\sqrt{d}\sqrt{cd-be}\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{e}}{2e}$$

input
```
Int[Sqrt[b*x + c*x^2]/(d + e*x),x]
```

output
```
Sqrt[b*x + c*x^2]/e - ((2*(2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x
^2]])/(Sqrt[c]*e) - (2*Sqrt[d]*Sqrt[c*d - b*e]*ArcTanh[(b*d + (2*c*d - b*e
)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/e)/(2*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$-\frac{-e\sqrt{x(cx+b)}-\frac{(be-2cd)\,\mathrm{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{\sqrt{c}}-\frac{2(be-cd)d\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)}{\sqrt{d(be-cd)}}}{e^2}$
risch	$\frac{x(cx+b)}{e\sqrt{x(cx+b)}}+\frac{\frac{(be-2cd)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{e\sqrt{c}}+\frac{2d(be-cd)\ln\left(\frac{-\frac{2d(be-cd)}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{-\frac{d(be-cd)}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be\cdots}{}}}{x+\frac{d}{e}}\cdots\right)}{e^2\sqrt{-\frac{d(be-cd)}{e^2}}}\cdots}{2e}$
default	$\frac{\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}}+\frac{(be-2cd)\ln\left(\frac{\frac{be-2cd}{2e}+c\left(x+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}}\right)}{2e\sqrt{c}}+\frac{d(be-\cdots}{\cdots}}{e}$

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/e^2*(-e*(x*(c*x+b))^(1/2)-(b*e-2*c*d)/c^(1/2)*arctanh((x*(c*x+b))^(1/2)
/x/c^(1/2))-2*(b*e-c*d)*d/(d*(b*e-c*d))^(1/2)*arctan((x*(c*x+b))^(1/2)/x*d
/(d*(b*e-c*d))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 482, normalized size of antiderivative = 4.27

$$\int \frac{\sqrt{bx+cx^2}}{d+ex}\,dx$$
$$= \left[\frac{2\sqrt{cx^2+bx}ce-(2cd-be)\sqrt{c}\log\left(2cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)+2\sqrt{cd^2-bdec}\log\left(\frac{bd+(2cd-be)x+2\sqrt{cd}}{ex+d}\right.}{2ce^2}\right.$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/2*(2*sqrt(c*x^2 + b*x)*c*e - (2*c*d - b*e)*sqrt(c)*log(2*c*x + b + 2*sq
rt(c*x^2 + b*x)*sqrt(c)) + 2*sqrt(c*d^2 - b*d*e)*c*log((b*d + (2*c*d - b*e
)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)))/(c*e^2), 1/2*(2
*sqrt(c*x^2 + b*x)*c*e - 4*sqrt(-c*d^2 + b*d*e)*c*arctan(sqrt(-c*d^2 + b*d
*e)*sqrt(c*x^2 + b*x)/(c*d*x + b*d)) - (2*c*d - b*e)*sqrt(c)*log(2*c*x + b
 + 2*sqrt(c*x^2 + b*x)*sqrt(c)))/(c*e^2), (sqrt(c*x^2 + b*x)*c*e + (2*c*d
- b*e)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + sqrt(c*d^2
- b*d*e)*c*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 +
 b*x))/(e*x + d)))/(c*e^2), (sqrt(c*x^2 + b*x)*c*e - 2*sqrt(-c*d^2 + b*d*e
)*c*arctan(sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/(c*d*x + b*d)) + (2*c*d
- b*e)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)))/(c*e^2)]
```

Sympy [F]

$$\int \frac{\sqrt{bx+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{x(b+cx)}}{d+ex}\,dx$$

input
```
integrate((c*x**2+b*x)**(1/2)/(e*x+d),x)
```

output
```
Integral(sqrt(x*(b + c*x))/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{bx+cx^2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{bx+cx^2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx+cx^2}}{d+ex}\,dx = \int \frac{\sqrt{c\,x^2+b\,x}}{d+e\,x}\,dx$$

input
```
int((b*x + c*x^2)^(1/2)/(d + e*x),x)
```

output
```
int((b*x + c*x^2)^(1/2)/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.49

$$\int \frac{\sqrt{bx+cx^2}}{d+ex}\,dx$$
$$= \frac{2\sqrt{d}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{be-cd}-\sqrt{e}\,\sqrt{cx+b}-\sqrt{x}\,\sqrt{e}\,\sqrt{c}}{\sqrt{d}\,\sqrt{c}}\right)c+2\sqrt{d}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{be-cd}+\sqrt{e}\,\sqrt{cx+b}+\sqrt{x}\,\sqrt{e}\,\sqrt{c}}{\sqrt{d}\,\sqrt{c}}\right)c+\sqrt{x}}{c\,e^2}$$

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d),x)
```

output
```
(2*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) -
 sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*c + 2*sqrt(d)*sqrt(b*e - c*d)
*atan((sqrt(b*e - c*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/
(sqrt(d)*sqrt(c)))*c + sqrt(x)*sqrt(b + c*x)*c*e + sqrt(c)*log((sqrt(b + c
*x) + sqrt(x)*sqrt(c))/sqrt(b))*b*e - 2*sqrt(c)*log((sqrt(b + c*x) + sqrt(
x)*sqrt(c))/sqrt(b))*c*d)/(c*e**2)
```

3.134 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^2}\,dx$

Optimal result 1054
Mathematica [A] (verified) 1054
Rubi [A] (verified) 1055
Maple [A] (verified) 1057
Fricas [A] (verification not implemented) 1058
Sympy [F] 1058
Maxima [F(-2)] 1059
Giac [F(-2)] 1059
Mupad [F(-1)] 1059
Reduce [B] (verification not implemented) 1060

Optimal result

Integrand size = 21, antiderivative size = 121

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^2}\,dx = -\frac{\sqrt{bx+cx^2}}{e(d+ex)}+\frac{2\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{e^2}-\frac{(2cd-be)\text{arctanh}\left(\frac{\sqrt{cd-be}x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{\sqrt{d}e^2\sqrt{cd-be}}$$

output
```
-(c*x^2+b*x)^(1/2)/e/(e*x+d)+2*c^(1/2)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2)
)/e^2-(-b*e+2*c*d)*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(1/2))/d
^(1/2)/e^2/(-b*e+c*d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^2}\,dx = \frac{\sqrt{x(b+cx)}\left(-\frac{e}{d+ex}+\frac{(2cd-be)\arctan\left(\frac{-e\sqrt{x}\sqrt{b+cx}+\sqrt{c}(d+ex)}{\sqrt{d}\sqrt{-cd+be}}\right)}{\sqrt{d}\sqrt{-cd+be}\sqrt{x}\sqrt{b+cx}}-\frac{2\sqrt{c}\log\left(-\sqrt{c}\sqrt{x}+\sqrt{b+cx}\right)}{\sqrt{x}\sqrt{b+cx}}\right)}{e^2}$$

input
```
Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^2,x]
```

output
```
(Sqrt[x*(b + c*x)]*(-(e/(d + e*x)) + ((2*c*d - b*e)*ArcTan[(-(e*Sqrt[x]*Sq
rt[b + c*x]) + Sqrt[c]*(d + e*x))/(Sqrt[d]*Sqrt[-(c*d) + b*e])])/(Sqrt[d]*
Sqrt[-(c*d) + b*e]*Sqrt[x]*Sqrt[b + c*x]) - (2*Sqrt[c]*Log[-(Sqrt[c]*Sqrt[
x]) + Sqrt[b + c*x]])/(Sqrt[x]*Sqrt[b + c*x])))/e^2
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.21, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1161, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^2} \, dx$$

$$\Big\downarrow \text{1161}$$

$$\frac{\int \frac{b+2cx}{(d+ex)\sqrt{cx^2+bx}}dx}{2e} - \frac{\sqrt{bx+cx^2}}{e(d+ex)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{2c\int \frac{1}{\sqrt{cx^2+bx}}dx}{e} - \frac{(2cd-be)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{2e} - \frac{\sqrt{bx+cx^2}}{e(d+ex)}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\frac{4c\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{e} - \frac{(2cd-be)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{2e} - \frac{\sqrt{bx+cx^2}}{e(d+ex)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{e} - \frac{(2cd-be)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{2e} - \frac{\sqrt{bx+cx^2}}{e(d+ex)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{2(2cd-be)\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{e}+\frac{4\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{e}}{2e}-\frac{\sqrt{bx+cx^2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{4\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{e}-\frac{(2cd-be)\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{\sqrt{d}e\sqrt{cd-be}}}{2e}-\frac{\sqrt{bx+cx^2}}{e(d+ex)}$$

input

```
Int[Sqrt[b*x + c*x^2]/(d + e*x)^2,x]
```

output

```
-(Sqrt[b*x + c*x^2]/(e*(d + e*x))) + ((4*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[
b*x + c*x^2]])/e - ((2*c*d - b*e)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[
d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(Sqrt[d]*e*Sqrt[c*d - b*e]))/(2*e)
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.99

method	result
pseudoelliptic	$-\frac{(ex+d)(be-2cd)\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)-2\left(\sqrt{c}\,(ex+d)\,\mathrm{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)-\frac{e\sqrt{x(cx+b)}}{2}\right)\sqrt{d(be-cd)}}{\sqrt{d(be-cd)}\,e^2(ex+d)}$
default	$\frac{\frac{e^2\left(c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}\right)^{\frac{3}{2}}}{d(be-cd)\left(x+\frac{d}{e}\right)}-\frac{(be-2cd)e\left(\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}}+\frac{(be-2cd)\ln\left(\frac{\frac{be-2cd}{2e}+c(x}{\sqrt{c}}\right.}{}\right.}{}}{}$

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-((e*x+d)*(b*e-2*c*d)*arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))^(1/2))-2*
(c^(1/2)*(e*x+d)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))-1/2*e*(x*(c*x+b))^(1
/2))*(d*(b*e-c*d))^(1/2))/(d*(b*e-c*d))^(1/2)/e^2/(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 839, normalized size of antiderivative = 6.93

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[1/2*(2*(c*d^3 - b*d^2*e + (c*d^2*e - b*d*e^2)*x)*sqrt(c)*log(2*c*x + b +
2*sqrt(c*x^2 + b*x)*sqrt(c)) - (2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)*sqr
t(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c
*x^2 + b*x))/(e*x + d)) - 2*(c*d^2*e - b*d*e^2)*sqrt(c*x^2 + b*x))/(c*d^3*
e^2 - b*d^2*e^3 + (c*d^2*e^3 - b*d*e^4)*x), ((2*c*d^2 - b*d*e + (2*c*d*e -
 b*e^2)*x)*sqrt(-c*d^2 + b*d*e)*arctan(sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b
*x)/(c*d*x + b*d)) + (c*d^3 - b*d^2*e + (c*d^2*e - b*d*e^2)*x)*sqrt(c)*log
(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - (c*d^2*e - b*d*e^2)*sqrt(c*x^2
 + b*x))/(c*d^3*e^2 - b*d^2*e^3 + (c*d^2*e^3 - b*d*e^4)*x), -1/2*(4*(c*d^3
 - b*d^2*e + (c*d^2*e - b*d*e^2)*x)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt
(-c)/(c*x + b)) + (2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)*sqrt(c*d^2 - b*d
*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/
(e*x + d)) + 2*(c*d^2*e - b*d*e^2)*sqrt(c*x^2 + b*x))/(c*d^3*e^2 - b*d^2*e
^3 + (c*d^2*e^3 - b*d*e^4)*x), ((2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)*sq
rt(-c*d^2 + b*d*e)*arctan(sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/(c*d*x +
b*d)) - 2*(c*d^3 - b*d^2*e + (c*d^2*e - b*d*e^2)*x)*sqrt(-c)*arctan(sqrt(c
*x^2 + b*x)*sqrt(-c)/(c*x + b)) - (c*d^2*e - b*d*e^2)*sqrt(c*x^2 + b*x))/(
c*d^3*e^2 - b*d^2*e^3 + (c*d^2*e^3 - b*d*e^4)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{x\,(b+cx)}}{(d+ex)^2}\,dx$$

input
```
integrate((c*x**2+b*x)**(1/2)/(e*x+d)**2,x)
```

output
```
Integral(sqrt(x*(b + c*x))/(d + e*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^2}\,dx = \int \frac{\sqrt{c\,x^2+b\,x}}{{(d+e\,x)}^2}\,dx$$

input
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^2,x)
```

output
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 603, normalized size of antiderivative = 4.98

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^2}\,dx$$
$$= \frac{-\sqrt{d}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{be-cd}-\sqrt{e}\,\sqrt{cx+b}-\sqrt{x}\,\sqrt{e}\,\sqrt{c}}{\sqrt{d}\,\sqrt{c}}\right)bde-\sqrt{d}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{be-cd}-\sqrt{e}\,\sqrt{cx+b}-\sqrt{x}\,\sqrt{e}\,\sqrt{c}}{\sqrt{d}\,\sqrt{c}}\right)b\,e^2x}{}$$

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^2,x)
```

output
```
( - sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x)
- sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b*d*e - sqrt(d)*sqrt(b*e - c
*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c
))/(sqrt(d)*sqrt(c)))*b*e**2*x + 2*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e
- c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)
))*c*d**2 + 2*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt
(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*c*d*e*x - sqrt(d)*
sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sq
rt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b*d*e - sqrt(d)*sqrt(b*e - c*d)*atan((sq
rt(b*e - c*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*
sqrt(c)))*b*e**2*x + 2*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) + sqr
t(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*c*d**2 +
2*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) + sqrt(e)*sqrt(b + c*x) +
sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*c*d*e*x - sqrt(x)*sqrt(b + c*x
)*b*d*e**2 + sqrt(x)*sqrt(b + c*x)*c*d**2*e + 2*sqrt(c)*log((sqrt(b + c*x)
 + sqrt(x)*sqrt(c))/sqrt(b))*b*d**2*e + 2*sqrt(c)*log((sqrt(b + c*x) + sqr
t(x)*sqrt(c))/sqrt(b))*b*d*e**2*x - 2*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)
*sqrt(c))/sqrt(b))*c*d**3 - 2*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c)
)/sqrt(b))*c*d**2*e*x)/(d*e**2*(b*d*e + b*e**2*x - c*d**2 - c*d*e*x))
```

3.135 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^3} \, dx$

Optimal result 1061
Mathematica [A] (verified) 1061
Rubi [A] (verified) 1062
Maple [A] (verified) 1063
Fricas [B] (verification not implemented) 1064
Sympy [F] 1064
Maxima [F(-2)] 1065
Giac [B] (verification not implemented) 1065
Mupad [F(-1)] 1066
Reduce [B] (verification not implemented) 1066

Optimal result

Integrand size = 21, antiderivative size = 133

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^3} \, dx = -\frac{b\sqrt{bx+cx^2}}{4d(cd-be)(d+ex)} + \frac{(bx+cx^2)^{3/2}}{2(cd-be)x(d+ex)^2} - \frac{b^2 \operatorname{arctanh}\left(\frac{\sqrt{cd-be}x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{4d^{3/2}(cd-be)^{3/2}}$$

output

```
-1/4*b*(c*x^2+b*x)^(1/2)/d/(-b*e+c*d)/(e*x+d)+1/2*(c*x^2+b*x)^(3/2)/(-b*e+
c*d)/x/(e*x+d)^2-1/4*b^2*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(1
/2))/d^(3/2)/(-b*e+c*d)^(3/2)
```

Mathematica [A] (verified)

Time = 10.17 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^3} \, dx = \frac{\sqrt{x(b+cx)}\left(\frac{\sqrt{d}(2cdx+b(d-ex))}{(cd-be)(d+ex)^2} - \frac{b^2 \operatorname{arctanh}\left(\frac{\sqrt{cd-be}\sqrt{x}}{\sqrt{d}\sqrt{b+cx}}\right)}{(cd-be)^{3/2}\sqrt{x}\sqrt{b+cx}}\right)}{4d^{3/2}}$$

input

```
Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^3,x]
```

output
```
(Sqrt[x*(b + c*x)]*((Sqrt[d]*(2*c*d*x + b*(d - e*x)))/((c*d - b*e)*(d + e*
x)^2) - (b^2*ArcTanh[(Sqrt[c*d - b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])])/(
(c*d - b*e)^(3/2)*Sqrt[x]*Sqrt[b + c*x])))/(4*d^(3/2))
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^3} \, dx$$

$$\Big\downarrow \text{1152}$$

$$\frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)} - \frac{b^2 \int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{8d(cd-be)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{b^2 \int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{4d(cd-be)} + \frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)} - \frac{b^2 \text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{8d^{3/2}(cd-be)^{3/2}}$$

input
```
Int[Sqrt[b*x + c*x^2]/(d + e*x)^3,x]
```

output
```
((b*d + (2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(4*d*(c*d - b*e)*(d + e*x)^2)
- (b^2*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x
 + c*x^2])])/(8*d^(3/2)*(c*d - b*e)^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.71

method	result	size
pseudoelliptic	$-\frac{b^{2}\left(\frac{\sqrt{x(cx+b)}\,(-bex+2cdx+bd)}{b^{2}(ex+d)^{2}}+\frac{\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)}{\sqrt{d(be-cd)}}\right)}{4(be-cd)d}$	94
default	Expression too large to display	986

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*b^2/(b*e-c*d)/d*((x*(c*x+b))^(1/2)*(-b*e*x+2*c*d*x+b*d)/b^2/(e*x+d)^2
+1/(d*(b*e-c*d))^(1/2)*arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(113) = 226$.

Time = 0.10 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.56

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^3}\,dx$$
$$= \left[-\frac{(b^2e^2x^2+2\,b^2dex+b^2d^2)\sqrt{cd^2-bde}\log\left(\frac{bd+(2\,cd-be)x+2\,\sqrt{cd^2-bde}\sqrt{cx^2+bx}}{ex+d}\right)-2\,(bcd^3-b^2d^2e+(2\,c^2d^3}{8\,(c^2d^6-2\,bcd^5e+b^2d^4e^2+(c^2d^4e^2-2\,bcd^3e^3+b^2d^2e^4)x^2+2\,(c^2d^5e-2\,bcd^4e^2}\right.$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
[-1/8*((b^2*e^2*x^2 + 2*b^2*d*e*x + b^2*d^2)*sqrt(c*d^2 - b*d*e)*log((b*d
+ (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) -
2*(b*c*d^3 - b^2*d^2*e + (2*c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*x)*sqrt(c*x
^2 + b*x))/(c^2*d^6 - 2*b*c*d^5*e + b^2*d^4*e^2 + (c^2*d^4*e^2 - 2*b*c*d^3
*e^3 + b^2*d^2*e^4)*x^2 + 2*(c^2*d^5*e - 2*b*c*d^4*e^2 + b^2*d^3*e^3)*x),
1/4*((b^2*e^2*x^2 + 2*b^2*d*e*x + b^2*d^2)*sqrt(-c*d^2 + b*d*e)*arctan(sqr
t(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/(c*d*x + b*d)) + (b*c*d^3 - b^2*d^2*e
+ (2*c^2*d^3 - 3*b*c*d^2*e + b^2*d*e^2)*x)*sqrt(c*x^2 + b*x))/(c^2*d^6 - 2
*b*c*d^5*e + b^2*d^4*e^2 + (c^2*d^4*e^2 - 2*b*c*d^3*e^3 + b^2*d^2*e^4)*x^2
 + 2*(c^2*d^5*e - 2*b*c*d^4*e^2 + b^2*d^3*e^3)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^3}\,dx = \int \frac{\sqrt{x\,(b+cx)}}{(d+ex)^3}\,dx$$

input
```
integrate((c*x**2+b*x)**(1/2)/(e*x+d)**3,x)
```

output
```
Integral(sqrt(x*(b + c*x))/(d + e*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(113) = 226.

Time = 0.16 (sec) , antiderivative size = 407, normalized size of antiderivative = 3.06

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^3}\,dx = -\frac{b^2 \arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde}}\right)}{4\left(cd^2-bde\right)\sqrt{-cd^2+bde}} + \frac{8\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3c^2d^2e - 8\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3bcde^2 + \left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3b^2e^3 + 8\left(\sqrt{c}x-\sqrt{\right.}}{4\left(cd^2e^2\right.}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^3,x, algorithm="giac")
```

output
```
-1/4*b^2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d
^2 + b*d*e))/((c*d^2 - b*d*e)*sqrt(-c*d^2 + b*d*e)) + 1/4*(8*(sqrt(c)*x -
sqrt(c*x^2 + b*x))^3*c^2*d^2*e - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b*c*d
*e^2 + (sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^2*e^3 + 8*(sqrt(c)*x - sqrt(c*x
^2 + b*x))^2*c^(5/2)*d^3 - 5*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^2*sqrt(c)
*d*e^2 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b*c^2*d^3 - 4*(sqrt(c)*x - sqrt
(c*x^2 + b*x))*b^2*c*d^2*e - (sqrt(c)*x - sqrt(c*x^2 + b*x))*b^3*d*e^2 + 2
*b^2*c^(3/2)*d^3 - b^3*sqrt(c)*d^2*e)/((c*d^2*e^2 - b*d*e^3)*((sqrt(c)*x -
 sqrt(c*x^2 + b*x))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c)*d + b*
d)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^3}\,dx=\int \frac{\sqrt{c\,x^2+b\,x}}{{\left(d+e\,x\right)}^3}\,dx$$

input
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^3,x)
```

output
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 1056, normalized size of antiderivative = 7.94

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^3}\,dx=\text{Too large to display}$$

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^3,x)
```

output
```
( - 2*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x
) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**3*d**2*e - 4*sqrt(d)*sq
rt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt
(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**3*d*e**2*x - 2*sqrt(d)*sqrt(b*e - c*d)*
atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(
sqrt(d)*sqrt(c)))*b**3*e**3*x**2 + 4*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*
e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(
c)))*b**2*c*d**3 + 8*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(
e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**2*c*d**2
*e*x + 4*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b +
c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**2*c*d*e**2*x**2 - 2*
sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) + sqrt(e)*sqrt(b + c*x) + sq
rt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**3*d**2*e - 4*sqrt(d)*sqrt(b*e
 - c*d)*atan((sqrt(b*e - c*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sq
rt(c))/(sqrt(d)*sqrt(c)))*b**3*d*e**2*x - 2*sqrt(d)*sqrt(b*e - c*d)*atan((
sqrt(b*e - c*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d
)*sqrt(c)))*b**3*e**3*x**2 + 4*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*
d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b
**2*c*d**3 + 8*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) + sqrt(e)*sqr
t(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**2*c*d**2*e*...
```

3.136 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^4}\,dx$

Optimal result 1068
Mathematica [A] (verified) 1069
Rubi [A] (verified) 1069
Maple [A] (verified) 1071
Fricas [B] (verification not implemented) 1072
Sympy [F] 1073
Maxima [F(-2)] 1073
Giac [B] (verification not implemented) 1073
Mupad [F(-1)] 1074
Reduce [F] 1075

Optimal result

Integrand size = 21, antiderivative size = 206

$$\begin{aligned}\int \frac{\sqrt{bx+cx^2}}{(d+ex)^4}\,dx = &-\frac{\sqrt{bx+cx^2}}{3e(d+ex)^3}+\frac{(2cd-be)\sqrt{bx+cx^2}}{12de(cd-be)(d+ex)^2}\\ &+\frac{(4c^2d^2-4bcde+3b^2e^2)\sqrt{bx+cx^2}}{24d^2e(cd-be)^2(d+ex)}\\ &-\frac{b^2(2cd-be)\operatorname{arctanh}\left(\frac{\sqrt{cd-be}x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{8d^{5/2}(cd-be)^{5/2}}\end{aligned}$$

output

```
-1/3*(c*x^2+b*x)^(1/2)/e/(e*x+d)^3+1/12*(-b*e+2*c*d)*(c*x^2+b*x)^(1/2)/d/e
/(-b*e+c*d)/(e*x+d)^2+1/24*(3*b^2*e^2-4*b*c*d*e+4*c^2*d^2)*(c*x^2+b*x)^(1/
2)/d^2/e/(-b*e+c*d)^2/(e*x+d)-1/8*b^2*(-b*e+2*c*d)*arctanh((-b*e+c*d)^(1/2
)*x/d^(1/2)/(c*x^2+b*x)^(1/2))/d^(5/2)/(-b*e+c*d)^(5/2)
```

Mathematica [A] (verified)

Time = 10.23 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^4}\,dx = \frac{\sqrt{x(b+cx)}\left(\frac{ex^{3/2}(b+cx)}{(d+ex)^3}-\frac{3(2cd-be)\left(\sqrt{d}\sqrt{cd-be}\sqrt{x}\sqrt{b+cx}(2cdx+b(d-ex))-b^2(d+ex)^2\operatorname{arctanh}\left(\frac{\sqrt{cd-be}\sqrt{x}}{\sqrt{d}\sqrt{b+cx}}\right)\right)}{8d^{3/2}(cd-be)^{3/2}\sqrt{b+cx}(d+ex)^2}\right)}{3d(-cd+be)\sqrt{x}}$$

input
```
Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^4,x]
```

output
```
(Sqrt[x*(b + c*x)]*((e*x^(3/2)*(b + c*x))/(d + e*x)^3 - (3*(2*c*d - b*e)*(
Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[x]*Sqrt[b + c*x]*(2*c*d*x + b*(d - e*x)) - b^
2*(d + e*x)^2*ArcTanh[(Sqrt[c*d - b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])]))
/(8*d^(3/2)*(c*d - b*e)^(3/2)*Sqrt[b + c*x]*(d + e*x)^2)))/(3*d*(-(c*d) +
b*e)*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1157, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^4}\,dx$$

$$\Big\downarrow \text{ 1157}$$

$$\frac{(2cd-be)\int \frac{\sqrt{cx^2+bx}}{(d+ex)^3}dx}{2d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}}{3d(d+ex)^3(cd-be)}$$

$$\Big\downarrow \text{ 1152}$$

$$\frac{(2cd-be)\left(\frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)}-\frac{b^2\int\frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{8d(cd-be)}\right)}{2d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}}{3d(d+ex)^3(cd-be)}$$

$$\Big\downarrow \; 1154$$

$$\frac{(2cd-be)\left(\frac{b^2\int\frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{4d(cd-be)}+\frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)}\right)}{2d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}}{3d(d+ex)^3(cd-be)}$$

$$\Big\downarrow \; 219$$

$$\frac{(2cd-be)\left(\frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)}-\frac{b^2\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{8d^{3/2}(cd-be)^{3/2}}\right)}{2d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}}{3d(d+ex)^3(cd-be)}$$

input

```
Int[Sqrt[b*x + c*x^2]/(d + e*x)^4,x]
```

output

```
-1/3*(e*(b*x + c*x^2)^(3/2))/(d*(c*d - b*e)*(d + e*x)^3) + ((2*c*d - b*e)*
(((b*d + (2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(4*d*(c*d - b*e)*(d + e*x)^2)
 - (b^2*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*
x + c*x^2]])/(8*d^(3/2)*(c*d - b*e)^(3/2))))/(2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{b^{2}(ex+d)^{3}(be-2cd)\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)+\sqrt{x(cx+b)}\,\sqrt{d(be-cd)}\left(\left(-4c^{2}x-2bc\right)d^{3}+e\left(-\frac{4}{3}c^{2}x^{2}+\frac{14}{3}cbx+b^{2}\right)d^{2}-\frac{8e^{2}(}{}\right.}{8\sqrt{d(be-cd)}\,(ex+d)^{3}(be-cd)^{2}d^{2}}$
default	Expression too large to display

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(b^2*(e*x+d)^3*(b*e-2*c*d)*arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))
^(1/2))+(x*(c*x+b))^(1/2)*(d*(b*e-c*d))^(1/2)*((-4*c^2*x-2*b*c)*d^3+e*(-4/
3*c^2*x^2+14/3*c*b*x+b^2)*d^2-8/3*e^2*(-1/2*c*x+b)*x*b*d-b^2*e^3*x^2))/(d*
(b*e-c*d))^(1/2)/(e*x+d)^3/(b*e-c*d)^2/d^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(182) = 364$.

Time = 0.10 (sec) , antiderivative size = 964, normalized size of antiderivative = 4.68

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
[-1/48*(3*(2*b^2*c*d^4 - b^3*d^3*e + (2*b^2*c*d*e^3 - b^3*e^4)*x^3 + 3*(2*
b^2*c*d^2*e^2 - b^3*d*e^3)*x^2 + 3*(2*b^2*c*d^3*e - b^3*d^2*e^2)*x)*sqrt(c
*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^
2 + b*x))/(e*x + d)) - 2*(6*b*c^2*d^5 - 9*b^2*c*d^4*e + 3*b^3*d^3*e^2 + (4
*c^3*d^4*e - 8*b*c^2*d^3*e^2 + 7*b^2*c*d^2*e^3 - 3*b^3*d*e^4)*x^2 + 2*(6*c
^3*d^5 - 13*b*c^2*d^4*e + 11*b^2*c*d^3*e^2 - 4*b^3*d^2*e^3)*x)*sqrt(c*x^2
+ b*x))/(c^3*d^9 - 3*b*c^2*d^8*e + 3*b^2*c*d^7*e^2 - b^3*d^6*e^3 + (c^3*d^
6*e^3 - 3*b*c^2*d^5*e^4 + 3*b^2*c*d^4*e^5 - b^3*d^3*e^6)*x^3 + 3*(c^3*d^7*
e^2 - 3*b*c^2*d^6*e^3 + 3*b^2*c*d^5*e^4 - b^3*d^4*e^5)*x^2 + 3*(c^3*d^8*e
- 3*b*c^2*d^7*e^2 + 3*b^2*c*d^6*e^3 - b^3*d^5*e^4)*x), 1/24*(3*(2*b^2*c*d^
4 - b^3*d^3*e + (2*b^2*c*d*e^3 - b^3*e^4)*x^3 + 3*(2*b^2*c*d^2*e^2 - b^3*d
*e^3)*x^2 + 3*(2*b^2*c*d^3*e - b^3*d^2*e^2)*x)*sqrt(-c*d^2 + b*d*e)*arctan
(sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/(c*d*x + b*d)) + (6*b*c^2*d^5 - 9*
b^2*c*d^4*e + 3*b^3*d^3*e^2 + (4*c^3*d^4*e - 8*b*c^2*d^3*e^2 + 7*b^2*c*d^2
*e^3 - 3*b^3*d*e^4)*x^2 + 2*(6*c^3*d^5 - 13*b*c^2*d^4*e + 11*b^2*c*d^3*e^2
 - 4*b^3*d^2*e^3)*x)*sqrt(c*x^2 + b*x))/(c^3*d^9 - 3*b*c^2*d^8*e + 3*b^2*c
*d^7*e^2 - b^3*d^6*e^3 + (c^3*d^6*e^3 - 3*b*c^2*d^5*e^4 + 3*b^2*c*d^4*e^5
- b^3*d^3*e^6)*x^3 + 3*(c^3*d^7*e^2 - 3*b*c^2*d^6*e^3 + 3*b^2*c*d^5*e^4 -
b^3*d^4*e^5)*x^2 + 3*(c^3*d^8*e - 3*b*c^2*d^7*e^2 + 3*b^2*c*d^6*e^3 - b^3*
d^5*e^4)*x)]
```

Sympy [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^4}\,dx = \int \frac{\sqrt{x\,(b+cx)}}{(d+ex)^4}\,dx$$

input
```
integrate((c*x**2+b*x)**(1/2)/(e*x+d)**4,x)
```

output
```
Integral(sqrt(x*(b + c*x))/(d + e*x)**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(182) = 364$.

Time = 1.60 (sec) , antiderivative size = 831, normalized size of antiderivative = 4.03

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
1/8*(2*b^2*c*d - b^3*e)*arctan(((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c
)*d)/sqrt(-c*d^2 + b*d*e))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*sqrt(-c*
d^2 + b*d*e)) + 1/24*(6*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*b^2*c*d*e^4 - 3*
(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*b^3*e^5 + 48*(sqrt(c)*x - sqrt(c*x^2 + b
*x))^4*c^(7/2)*d^4*e - 96*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*b*c^(5/2)*d^3*
e^2 + 78*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*b^2*c^(3/2)*d^2*e^3 - 15*(sqrt(
c)*x - sqrt(c*x^2 + b*x))^4*b^3*sqrt(c)*d*e^4 + 32*(sqrt(c)*x - sqrt(c*x^2
 + b*x))^3*c^4*d^5 + 16*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b*c^3*d^4*e - 84
*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^2*c^2*d^3*e^2 + 74*(sqrt(c)*x - sqrt(
c*x^2 + b*x))^3*b^3*c*d^2*e^3 - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^4*d*
e^4 + 48*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b*c^(7/2)*d^5 - 36*(sqrt(c)*x -
 sqrt(c*x^2 + b*x))^2*b^2*c^(5/2)*d^4*e - 6*(sqrt(c)*x - sqrt(c*x^2 + b*x)
)^2*b^3*c^(3/2)*d^3*e^2 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^4*sqrt(c)
*d^2*e^3 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^2*c^3*d^5 - 24*(sqrt(c)*x
- sqrt(c*x^2 + b*x))*b^3*c^2*d^4*e + 12*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^
4*c*d^3*e^2 + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^5*d^2*e^3 + 4*b^3*c^(5/2
)*d^5 - 4*b^4*c^(3/2)*d^4*e + 3*b^5*sqrt(c)*d^3*e^2)/((c^2*d^4*e^2 - 2*b*c
*d^3*e^3 + b^2*d^2*e^4)*((sqrt(c)*x - sqrt(c*x^2 + b*x))^2*e + 2*(sqrt(c)*
x - sqrt(c*x^2 + b*x))*sqrt(c)*d + b*d)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^4}\,dx=\int \frac{\sqrt{c\,x^2+b\,x}}{{\left(d+e\,x\right)}^4}\,dx$$

input
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^4,x)
```

output
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^4, x)
```

Reduce [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^4}\,dx = \int \frac{\sqrt{c\,x^2+bx}}{(ex+d)^4}dx$$

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^4,x)
```

output
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^4,x)
```

3.137 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^5}\,dx$

Optimal result 1076
Mathematica [A] (verified) 1077
Rubi [A] (verified) 1077
Maple [A] (verified) 1080
Fricas [B] (verification not implemented) 1081
Sympy [F] 1082
Maxima [F(-2)] 1082
Giac [B] (verification not implemented) 1082
Mupad [F(-1)] 1083
Reduce [F] 1084

Optimal result

Integrand size = 21, antiderivative size = 292

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^5}\,dx = -\frac{\sqrt{bx+cx^2}}{4e(d+ex)^4}+\frac{(2cd-be)\sqrt{bx+cx^2}}{24de(cd-be)(d+ex)^3}+\frac{(8c^2d^2-8bcde+5b^2e^2)\sqrt{bx+cx^2}}{96d^2e(cd-be)^2(d+ex)^2}+\frac{(2cd-be)(8c^2d^2-8bcde+15b^2e^2)\sqrt{bx+cx^2}}{192d^3e(cd-be)^3(d+ex)}-\frac{b^2(16c^2d^2-16bcde+5b^2e^2)\,\mathrm{arctanh}\left(\frac{\sqrt{cd-bex}}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{64d^{7/2}(cd-be)^{7/2}}$$

output
```
-1/4*(c*x^2+b*x)^(1/2)/e/(e*x+d)^4+1/24*(-b*e+2*c*d)*(c*x^2+b*x)^(1/2)/d/e
/(-b*e+c*d)/(e*x+d)^3+1/96*(5*b^2*e^2-8*b*c*d*e+8*c^2*d^2)*(c*x^2+b*x)^(1/
2)/d^2/e/(-b*e+c*d)^2/(e*x+d)^2+1/192*(-b*e+2*c*d)*(15*b^2*e^2-8*b*c*d*e+8
*c^2*d^2)*(c*x^2+b*x)^(1/2)/d^3/e/(-b*e+c*d)^3/(e*x+d)-1/64*b^2*(5*b^2*e^2
-16*b*c*d*e+16*c^2*d^2)*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(1/
2))/d^(7/2)/(-b*e+c*d)^(7/2)
```

Mathematica [A] (verified)

Time = 10.61 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^5}\,dx$$
$$= \frac{\sqrt{x(b+cx)}\left(48ex^{3/2}(b+cx)+\frac{40e(2cd-be)x^{3/2}(b+cx)(d+ex)}{d(cd-be)}+\frac{3(16c^2d^2-16bcde+5b^2e^2)(d+ex)^2\left(\sqrt{d}\sqrt{cd-be}\sqrt{x}\sqrt{b+cx}(-b\right.}{d^{5/2}(cd-be)^{5/2}\sqrt{}}\right.}{192d(-cd+be)\sqrt{x}(d+ex)^4}$$

input
```
Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^5,x]
```

output
```
(Sqrt[x*(b + c*x)]*(48*e*x^(3/2)*(b + c*x) + (40*e*(2*c*d - b*e)*x^(3/2)*(
b + c*x)*(d + e*x))/(d*(c*d - b*e)) + (3*(16*c^2*d^2 - 16*b*c*d*e + 5*b^2*
e^2)*(d + e*x)^2*(Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[x]*Sqrt[b + c*x]*(-(b*d) -
2*c*d*x + b*e*x) + b^2*(d + e*x)^2*ArcTanh[(Sqrt[c*d - b*e]*Sqrt[x])/(Sqrt
[d]*Sqrt[b + c*x])]))/(d^(5/2)*(c*d - b*e)^(5/2)*Sqrt[b + c*x])))/(192*d*(
-(c*d) + b*e)*Sqrt[x]*(d + e*x)^4)
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 271, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1167, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^5}\,dx$$
$$\Big\downarrow \; 1167$$
$$-\frac{\int -\frac{(8cd-5be-2cex)\sqrt{cx^2+bx}}{2(d+ex)^4}dx}{4d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}}{4d(d+ex)^4(cd-be)}$$
$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(8cd-5be-2cex)\sqrt{cx^2+bx}}{(d+ex)^4}dx}{8d(cd-be)} - \frac{e(bx+cx^2)^{3/2}}{4d(d+ex)^4(cd-be)}$$

$$\Big\downarrow \text{1228}$$

$$\frac{\frac{(5b^2e^2-16bcde+16c^2d^2)\int \frac{\sqrt{cx^2+bx}}{(d+ex)^3}dx}{2d(cd-be)} - \frac{5e(bx+cx^2)^{3/2}(2cd-be)}{3d(d+ex)^3(cd-be)}}{8d(cd-be)} - \frac{e(bx+cx^2)^{3/2}}{4d(d+ex)^4(cd-be)}$$

$$\Big\downarrow \text{1152}$$

$$\frac{\frac{(5b^2e^2-16bcde+16c^2d^2)\left(\frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)} - \frac{b^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{8d(cd-be)}\right)}{2d(cd-be)} - \frac{5e(bx+cx^2)^{3/2}(2cd-be)}{3d(d+ex)^3(cd-be)}}{8d(cd-be)} - \frac{e(bx+cx^2)^{3/2}}{4d(d+ex)^4(cd-be)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\frac{(5b^2e^2-16bcde+16c^2d^2)\left(\frac{b^2\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{4d(cd-be)} + \frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)}\right)}{2d(cd-be)} - \frac{5e(bx+cx^2)^{3/2}(2cd-be)}{3d(d+ex)^3(cd-be)}}{8d(cd-be)} - \frac{e(bx+cx^2)^{3/2}}{4d(d+ex)^4(cd-be)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(5b^2e^2-16bcde+16c^2d^2)\left(\frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)} - \frac{b^2\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{8d^{3/2}(cd-be)^{3/2}}\right)}{2d(cd-be)} - \frac{5e(bx+cx^2)^{3/2}(2cd-be)}{3d(d+ex)^3(cd-be)}}{8d(cd-be)} - \frac{e(bx+cx^2)^{3/2}}{4d(d+ex)^4(cd-be)}$$

input
```
Int[Sqrt[b*x + c*x^2]/(d + e*x)^5,x]
```

output
```
-1/4*(e*(b*x + c*x^2)^(3/2))/(d*(c*d - b*e)*(d + e*x)^4) + ((-5*e*(2*c*d -
 b*e)*(b*x + c*x^2)^(3/2))/(3*d*(c*d - b*e)*(d + e*x)^3) + ((16*c^2*d^2 -
16*b*c*d*e + 5*b^2*e^2)*(((b*d + (2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(4*d*
(c*d - b*e)*(d + e*x)^2) - (b^2*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]
*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(8*d^(3/2)*(c*d - b*e)^(3/2))))/(2*d
*(c*d - b*e)))/(8*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.83

method	result
pseudoelliptic	$-\frac{5\left(\left(ex+d\right)^{4}\left(b^{2}e^{2}-\frac{16}{5}bcde+\frac{16}{5}c^{2}d^{2}\right)b^{2}\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)+\sqrt{x(cx+b)}\,\sqrt{d(be-cd)}\,\left(\frac{16c^{2}(2cx+b)d^{5}}{5}-\frac{16e(4cx+b)c\left(-\frac{c}{3}\right.}{5}\right.\right.}{64\sqrt{d(be-cd)}\,(ex}$
default	Expression too large to display

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-5/64*((e*x+d)^4*(b^2*e^2-16/5*b*c*d*e+16/5*c^2*d^2)*b^2*arctan((x*(c*x+b)
)^(1/2)/x*d/(d*(b*e-c*d))^(1/2))+(x*(c*x+b))^(1/2)*(d*(b*e-c*d))^(1/2)*(16
/5*c^2*(2*c*x+b)*d^5-16/5*e*(4*c*x+b)*c*(-1/3*c*x+b)*d^4+e^2*(16/15*c^3*x^
3-104/15*b*c^2*x^2+66/5*b^2*c*x+b^3)*d^3-73/15*(24/73*c^2*x^2-140/73*c*b*x
+b^2)*e^3*x*b*d^2-11/3*e^4*x^2*(-38/55*c*x+b)*b^2*d-b^3*e^5*x^3))/(d*(b*e-
c*d))^(1/2)/(e*x+d)^4/(b*e-c*d)^3/d^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 795 vs. $2(264) = 528$.

Time = 0.12 (sec) , antiderivative size = 1606, normalized size of antiderivative = 5.50

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^5}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^5,x, algorithm="fricas")
```

output
```
[-1/384*(3*(16*b^2*c^2*d^6 - 16*b^3*c*d^5*e + 5*b^4*d^4*e^2 + (16*b^2*c^2*
d^2*e^4 - 16*b^3*c*d*e^5 + 5*b^4*e^6)*x^4 + 4*(16*b^2*c^2*d^3*e^3 - 16*b^3
*c*d^2*e^4 + 5*b^4*d*e^5)*x^3 + 6*(16*b^2*c^2*d^4*e^2 - 16*b^3*c*d^3*e^3 +
 5*b^4*d^2*e^4)*x^2 + 4*(16*b^2*c^2*d^5*e - 16*b^3*c*d^4*e^2 + 5*b^4*d^3*e
^3)*x)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d
*e)*sqrt(c*x^2 + b*x))/(e*x + d)) - 2*(48*b*c^3*d^7 - 96*b^2*c^2*d^6*e + 6
3*b^3*c*d^5*e^2 - 15*b^4*d^4*e^3 + (16*c^4*d^5*e^2 - 40*b*c^3*d^4*e^3 + 62
*b^2*c^2*d^3*e^4 - 53*b^3*c*d^2*e^5 + 15*b^4*d*e^6)*x^3 + (64*c^4*d^6*e -
168*b*c^3*d^5*e^2 + 244*b^2*c^2*d^4*e^3 - 195*b^3*c*d^3*e^4 + 55*b^4*d^2*e
^5)*x^2 + (96*c^4*d^7 - 272*b*c^3*d^6*e + 374*b^2*c^2*d^5*e^2 - 271*b^3*c*
d^4*e^3 + 73*b^4*d^3*e^4)*x)*sqrt(c*x^2 + b*x))/(c^4*d^12 - 4*b*c^3*d^11*e
 + 6*b^2*c^2*d^10*e^2 - 4*b^3*c*d^9*e^3 + b^4*d^8*e^4 + (c^4*d^8*e^4 - 4*b
*c^3*d^7*e^5 + 6*b^2*c^2*d^6*e^6 - 4*b^3*c*d^5*e^7 + b^4*d^4*e^8)*x^4 + 4*
(c^4*d^9*e^3 - 4*b*c^3*d^8*e^4 + 6*b^2*c^2*d^7*e^5 - 4*b^3*c*d^6*e^6 + b^4
*d^5*e^7)*x^3 + 6*(c^4*d^10*e^2 - 4*b*c^3*d^9*e^3 + 6*b^2*c^2*d^8*e^4 - 4*
b^3*c*d^7*e^5 + b^4*d^6*e^6)*x^2 + 4*(c^4*d^11*e - 4*b*c^3*d^10*e^2 + 6*b^
2*c^2*d^9*e^3 - 4*b^3*c*d^8*e^4 + b^4*d^7*e^5)*x), 1/192*(3*(16*b^2*c^2*d^
6 - 16*b^3*c*d^5*e + 5*b^4*d^4*e^2 + (16*b^2*c^2*d^2*e^4 - 16*b^3*c*d*e^5
+ 5*b^4*e^6)*x^4 + 4*(16*b^2*c^2*d^3*e^3 - 16*b^3*c*d^2*e^4 + 5*b^4*d*e^5)
*x^3 + 6*(16*b^2*c^2*d^4*e^2 - 16*b^3*c*d^3*e^3 + 5*b^4*d^2*e^4)*x^2 + ...
```

Sympy [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^5}\,dx = \int \frac{\sqrt{x\,(b+cx)}}{(d+ex)^5}\,dx$$

input
```
integrate((c*x**2+b*x)**(1/2)/(e*x+d)**5,x)
```

output
```
Integral(sqrt(x*(b + c*x))/(d + e*x)**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1219 vs. 2(264) = 528.

Time = 0.34 (sec) , antiderivative size = 1219, normalized size of antiderivative = 4.17

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^5}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^5,x, algorithm="giac")
```

output
```
-1/384*((48*b^2*c^2*d^2*e^3*log(abs(2*c*d*e - b*e^2 - 2*sqrt(c*d^2 - b*d*e
)*sqrt(c)*abs(e))) - 48*b^3*c*d*e^4*log(abs(2*c*d*e - b*e^2 - 2*sqrt(c*d^2
 - b*d*e)*sqrt(c)*abs(e))) + 15*b^4*e^5*log(abs(2*c*d*e - b*e^2 - 2*sqrt(c
*d^2 - b*d*e)*sqrt(c)*abs(e))) + 32*sqrt(c*d^2 - b*d*e)*c^(7/2)*d^3*abs(e)
 - 48*sqrt(c*d^2 - b*d*e)*b*c^(5/2)*d^2*e*abs(e) + 76*sqrt(c*d^2 - b*d*e)*
b^2*c^(3/2)*d*e^2*abs(e) - 30*sqrt(c*d^2 - b*d*e)*b^3*sqrt(c)*e^3*abs(e))*
sgn(1/(e*x + d))*sgn(e)/(sqrt(c*d^2 - b*d*e)*c^3*d^6*e^4*abs(e) - 3*sqrt(c
*d^2 - b*d*e)*b*c^2*d^5*e^5*abs(e) + 3*sqrt(c*d^2 - b*d*e)*b^2*c*d^4*e^6*a
bs(e) - sqrt(c*d^2 - b*d*e)*b^3*d^3*e^7*abs(e)) - 2*sqrt(c - 2*c*d/(e*x +
d) + c*d^2/(e*x + d)^2 + b*e/(e*x + d) - b*d*e/(e*x + d)^2)*((16*c^3*d^3*e
^4*sgn(1/(e*x + d))*sgn(e) - 24*b*c^2*d^2*e^5*sgn(1/(e*x + d))*sgn(e) + 38
*b^2*c*d*e^6*sgn(1/(e*x + d))*sgn(e) - 15*b^3*e^7*sgn(1/(e*x + d))*sgn(e))
/(c^3*d^6*e^8 - 3*b*c^2*d^5*e^9 + 3*b^2*c*d^4*e^10 - b^3*d^3*e^11) + 2*((8
*c^3*d^4*e^5*sgn(1/(e*x + d))*sgn(e) - 16*b*c^2*d^3*e^6*sgn(1/(e*x + d))*s
gn(e) + 13*b^2*c*d^2*e^7*sgn(1/(e*x + d))*sgn(e) - 5*b^3*d*e^8*sgn(1/(e*x
+ d))*sgn(e))/(c^3*d^6*e^8 - 3*b*c^2*d^5*e^9 + 3*b^2*c*d^4*e^10 - b^3*d^3*
e^11) + 4*((2*c^3*d^5*e^6*sgn(1/(e*x + d))*sgn(e) - 5*b*c^2*d^4*e^7*sgn(1/
(e*x + d))*sgn(e) + 4*b^2*c*d^3*e^8*sgn(1/(e*x + d))*sgn(e) - b^3*d^2*e^9*
sgn(1/(e*x + d))*sgn(e))/(c^3*d^6*e^8 - 3*b*c^2*d^5*e^9 + 3*b^2*c*d^4*e^10
 - b^3*d^3*e^11) - 6*(c^3*d^6*e^7*sgn(1/(e*x + d))*sgn(e) - 3*b*c^2*d^5...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^5}\,dx = \int \frac{\sqrt{c\,x^2+b\,x}}{{(d+e\,x)}^5}\,dx$$

input
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^5,x)
```

output
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^5, x)
```

Reduce [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^5}\,dx = \int \frac{\sqrt{c\,x^2+bx}}{(ex+d)^5}dx$$

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^5,x)
```

output
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^5,x)
```

3.138 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^6}\,dx$

Optimal result 1085
Mathematica [A] (verified) 1086
Rubi [A] (verified) 1086
Maple [A] (verified) 1090
Fricas [B] (verification not implemented) 1090
Sympy [F] 1091
Maxima [F(-2)] 1092
Giac [B] (verification not implemented) 1092
Mupad [F(-1)] 1093
Reduce [F] 1094

Optimal result

Integrand size = 21, antiderivative size = 392

$$
\begin{aligned}
\int \frac{\sqrt{bx+cx^2}}{(d+ex)^6}\,dx
&= -\frac{\sqrt{bx+cx^2}}{5e(d+ex)^5} + \frac{(2cd-be)\sqrt{bx+cx^2}}{40de(cd-be)(d+ex)^4} + \frac{(12c^2d^2-12bcde+7b^2e^2)\sqrt{bx+cx^2}}{240d^2e(cd-be)^2(d+ex)^3} \\
&\quad + \frac{(2cd-be)(24c^2d^2-24bcde+35b^2e^2)\sqrt{bx+cx^2}}{960d^3e(cd-be)^3(d+ex)^2} \\
&\quad + \frac{(96c^4d^4-192bc^3d^3e+476b^2c^2d^2e^2-380b^3cde^3+105b^4e^4)\sqrt{bx+cx^2}}{1920d^4e(cd-be)^4(d+ex)} \\
&\quad - \frac{b^2(2cd-be)(16c^2d^2-16bcde+7b^2e^2)\,\text{arctanh}\left(\frac{\sqrt{cd-be}\,x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{128d^{9/2}(cd-be)^{9/2}}
\end{aligned}
$$

output
```
-1/5*(c*x^2+b*x)^(1/2)/e/(e*x+d)^5+1/40*(-b*e+2*c*d)*(c*x^2+b*x)^(1/2)/d/e
/(-b*e+c*d)/(e*x+d)^4+1/240*(7*b^2*e^2-12*b*c*d*e+12*c^2*d^2)*(c*x^2+b*x)^
(1/2)/d^2/e/(-b*e+c*d)^2/(e*x+d)^3+1/960*(-b*e+2*c*d)*(35*b^2*e^2-24*b*c*d
*e+24*c^2*d^2)*(c*x^2+b*x)^(1/2)/d^3/e/(-b*e+c*d)^3/(e*x+d)^2+1/1920*(105*
b^4*e^4-380*b^3*c*d*e^3+476*b^2*c^2*d^2*e^2-192*b*c^3*d^3*e+96*c^4*d^4)*(c
*x^2+b*x)^(1/2)/d^4/e/(-b*e+c*d)^4/(e*x+d)-1/128*b^2*(-b*e+2*c*d)*(7*b^2*e
^2-16*b*c*d*e+16*c^2*d^2)*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(
1/2))/d^(9/2)/(-b*e+c*d)^(9/2)
```

Mathematica [A] (verified)

Time = 10.87 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^6}\,dx = \frac{\sqrt{x(b+cx)}\left(384ex^{3/2}(b+cx)+\frac{336e(2cd-be)x^{3/2}(b+cx)(d+ex)}{d(cd-be)}+\frac{8e(108c^2d^2-108bcde+35b^2e^2)x^{3/2}(b+cx)(d+ex)^2}{d^2(cd-be)^2}+\frac{15(2\cdots}{\cdots}\right.}{1920d(-cd+be)\sqrt{x}(d\cdots}$$

input
```
Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^6,x]
```

output
```
(Sqrt[x*(b + c*x)]*(384*e*x^(3/2)*(b + c*x) + (336*e*(2*c*d - b*e)*x^(3/2)
*(b + c*x)*(d + e*x))/(d*(c*d - b*e)) + (8*e*(108*c^2*d^2 - 108*b*c*d*e +
35*b^2*e^2)*x^(3/2)*(b + c*x)*(d + e*x)^2)/(d^2*(c*d - b*e)^2) + (15*(2*c*
d - b*e)*(16*c^2*d^2 - 16*b*c*d*e + 7*b^2*e^2)*(d + e*x)^3*(Sqrt[d]*Sqrt[c
*d - b*e]*Sqrt[x]*Sqrt[b + c*x]*(-(b*d) - 2*c*d*x + b*e*x) + b^2*(d + e*x)
^2*ArcTanh[(Sqrt[c*d - b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])]))/(d^(7/2)*(
c*d - b*e)^(7/2)*Sqrt[b + c*x])))/(1920*d*(-(c*d) + b*e)*Sqrt[x]*(d + e*x)
^5)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 359, normalized size of antiderivative = 0.92, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1167, 27, 1237, 27, 1228, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^6}\,dx$$

$$\Big\downarrow \text{1167}$$

$$-\frac{\int -\frac{(10cd-7be-4cex)\sqrt{cx^2+bx}}{2(d+ex)^5}dx}{5d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}}{5d(d+ex)^5(cd-be)}$$

$$\downarrow 27$$

$$\frac{\int \frac{(10cd-7be-4cex)\sqrt{cx^2+bx}}{(d+ex)^5}dx}{10d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}}{5d(d+ex)^5(cd-be)}$$

$$\downarrow 1237$$

$$\frac{-\frac{\int -\frac{\left(80c^2d^2-94bced+35b^2e^2-14ce(2cd-be)x\right)\sqrt{cx^2+bx}}{2(d+ex)^4}dx}{4d(cd-be)}-\frac{7e\left(bx+cx^2\right)^{3/2}(2cd-be)}{4d(d+ex)^4(cd-be)}}{10d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}}{5d(d+ex)^5(cd-be)}$$

$$\downarrow 27$$

$$\frac{\frac{\int \frac{\left(80c^2d^2-94bced+35b^2e^2-14ce(2cd-be)x\right)\sqrt{cx^2+bx}}{(d+ex)^4}dx}{8d(cd-be)}-\frac{7e\left(bx+cx^2\right)^{3/2}(2cd-be)}{4d(d+ex)^4(cd-be)}}{10d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}}{5d(d+ex)^5(cd-be)}$$

$$\downarrow 1228$$

$$\frac{\frac{\frac{5(2cd-be)\left(7b^2e^2-16bcde+16c^2d^2\right)\int\frac{\sqrt{cx^2+bx}}{(d+ex)^3}dx}{2d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}\left(35b^2e^2-108bcde+108c^2d^2\right)}{3d(d+ex)^3(cd-be)}}{8d(cd-be)}-\frac{7e\left(bx+cx^2\right)^{3/2}(2cd-be)}{4d(d+ex)^4(cd-be)}}{10d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}}{5d(d+ex)^5(cd-be)}$$

$$\downarrow 1152$$

$$\frac{\frac{\frac{5(2cd-be)\left(7b^2e^2-16bcde+16c^2d^2\right)\left(\frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)}-\frac{b^2\int\frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{8d(cd-be)}\right)}{2d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}\left(35b^2e^2-108bcde+108c^2d^2\right)}{3d(d+ex)^3(cd-be)}}{8d(cd-be)}-\frac{7e\left(bx+cx^2\right)^{3/2}(2cd-be)}{4d(d+ex)^4(cd-be)}}{10d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}}{5d(d+ex)^5(cd-be)}$$

$$\downarrow 1154$$

$$\frac{\frac{\frac{5(2cd-be)\left(7b^2e^2-16bcde+16c^2d^2\right)\left(\frac{b^2\int\frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{4d(cd-be)}+\frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)}\right)}{2d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}\left(35b^2e^2-108bcde+108c^2d^2\right)}{3d(d+ex)^3(cd-be)}}{8d(cd-be)}-\frac{7e\left(bx+cx^2\right)^{3/2}(2cd-be)}{4d(d+ex)^4(cd-be)}}{10d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}}{5d(d+ex)^5(cd-be)}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{5(2cd-be)\left(7b^2e^2-16bcde+16c^2d^2\right)\left(\frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)}-\frac{b^2\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{8d^{3/2}(cd-be)^{3/2}}\right)}{2d(cd-be)}-\frac{e\left(bx+cx^2\right)^{3/2}\left(35b^2e^2-108bcde+108c^2d^2\right)}{3d(d+ex)^3(cd-be)}}{8d(cd-be)}-}{10d(cd-be)}$$

$$\frac{e\left(bx+cx^2\right)^{3/2}}{5d(d+ex)^5(cd-be)}$$

input
```
Int[Sqrt[b*x + c*x^2]/(d + e*x)^6,x]
```

output
```
-1/5*(e*(b*x + c*x^2)^(3/2))/(d*(c*d - b*e)*(d + e*x)^5) + ((-7*e*(2*c*d -
 b*e)*(b*x + c*x^2)^(3/2))/(4*d*(c*d - b*e)*(d + e*x)^4) + (-1/3*(e*(108*c
^2*d^2 - 108*b*c*d*e + 35*b^2*e^2)*(b*x + c*x^2)^(3/2))/(d*(c*d - b*e)*(d
+ e*x)^3) + (5*(2*c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e + 7*b^2*e^2)*(((b*d
+ (2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(4*d*(c*d - b*e)*(d + e*x)^2) - (b^2
*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x
^2])])/(8*d^(3/2)*(c*d - b*e)^(3/2))))/(2*d*(c*d - b*e)))/(8*d*(c*d - b*e)
))/(10*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 357, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$-\frac{7\left(\left(\left(e^{3}\left(-\frac{158}{21} d^{3} e x-\frac{14}{3} d\, e^{3} x^{3}-\frac{128}{15} d^{2} e^{2} x^{2}-e^{4} x^{4}+d^{4}\right) b^{4}-\frac{30 e^{2}\left(-\frac{38}{45} e^{4} x^{4}-\frac{889}{225} d\, e^{3} x^{3}-\frac{1631}{225} d^{2} e^{2} x^{2}-\frac{59}{9} d^{3} e x+d^{4}\right) c d\, b^{3}}{7}+\frac{48}{\cdots}\right.\right.\right.}{\cdots}$
default	Expression too large to display

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^6,x,method=_RETURNVERBOSE)
```

output
```
-7/128/(d*(b*e-c*d))^(1/2)*((e^3*(-158/21*d^3*e*x-14/3*d*e^3*x^3-128/15*d^
2*e^2*x^2-e^4*x^4+d^4)*b^4-30/7*e^2*(-38/45*e^4*x^4-889/225*d*e^3*x^3-1631
/225*d^2*e^2*x^2-59/9*d^3*e*x+d^4)*c*d*b^3+48/7*(-119/180*e^4*x^4-559/180*
d*e^3*x^3-1049/180*d^2*e^2*x^2-67/12*d^3*e*x+d^4)*e*c^2*d^2*b^2-32/7*(-2/5
*e^4*x^4-21/10*d*e^3*x^3-9/2*d^2*e^2*x^2-5*d^3*e*x+d^4)*c^3*d^3*b-64/7*(1/
10*e^3*x^3+1/2*d*e^2*x^2+d^2*e*x+d^3)*x*c^4*d^4)*(d*(b*e-c*d))^(1/2)*(x*(c
*x+b))^(1/2)+(e*x+d)^5*(b*e-2*c*d)*(b^2*e^2-16/7*b*c*d*e+16/7*c^2*d^2)*b^2
*arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))^(1/2)))/(e*x+d)^5/(b*e-c*d)^4/
d^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1214 vs. 2(360) = 720.

Time = 0.15 (sec) , antiderivative size = 2444, normalized size of antiderivative = 6.23

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^6}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^6,x, algorithm="fricas")
```

output
```
[-1/3840*(15*(32*b^2*c^3*d^8 - 48*b^3*c^2*d^7*e + 30*b^4*c*d^6*e^2 - 7*b^5
*d^5*e^3 + (32*b^2*c^3*d^3*e^5 - 48*b^3*c^2*d^2*e^6 + 30*b^4*c*d*e^7 - 7*b
^5*e^8)*x^5 + 5*(32*b^2*c^3*d^4*e^4 - 48*b^3*c^2*d^3*e^5 + 30*b^4*c*d^2*e^
6 - 7*b^5*d*e^7)*x^4 + 10*(32*b^2*c^3*d^5*e^3 - 48*b^3*c^2*d^4*e^4 + 30*b^
4*c*d^3*e^5 - 7*b^5*d^2*e^6)*x^3 + 10*(32*b^2*c^3*d^6*e^2 - 48*b^3*c^2*d^5
*e^3 + 30*b^4*c*d^4*e^4 - 7*b^5*d^3*e^5)*x^2 + 5*(32*b^2*c^3*d^7*e - 48*b^
3*c^2*d^6*e^2 + 30*b^4*c*d^5*e^3 - 7*b^5*d^4*e^4)*x)*sqrt(c*d^2 - b*d*e)*l
og((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x
+ d)) - 2*(480*b*c^4*d^9 - 1200*b^2*c^3*d^8*e + 1170*b^3*c^2*d^7*e^2 - 555
*b^4*c*d^6*e^3 + 105*b^5*d^5*e^4 + (96*c^5*d^6*e^3 - 288*b*c^4*d^5*e^4 + 6
68*b^2*c^3*d^4*e^5 - 856*b^3*c^2*d^3*e^6 + 485*b^4*c*d^2*e^7 - 105*b^5*d*e
^8)*x^4 + 2*(240*c^5*d^7*e^2 - 744*b*c^4*d^6*e^3 + 1622*b^2*c^3*d^5*e^4 -
2007*b^3*c^2*d^4*e^5 + 1134*b^4*c*d^3*e^6 - 245*b^5*d^2*e^7)*x^3 + 2*(480*
c^5*d^8*e - 1560*b*c^4*d^7*e^2 + 3178*b^2*c^3*d^6*e^3 - 3729*b^3*c^2*d^5*e
^4 + 2079*b^4*c*d^4*e^5 - 448*b^5*d^3*e^6)*x^2 + 10*(96*c^5*d^9 - 336*b*c^
4*d^8*e + 642*b^2*c^3*d^7*e^2 - 697*b^3*c^2*d^6*e^3 + 374*b^4*c*d^5*e^4 -
79*b^5*d^4*e^5)*x)*sqrt(c*x^2 + b*x))/(c^5*d^15 - 5*b*c^4*d^14*e + 10*b^2*
c^3*d^13*e^2 - 10*b^3*c^2*d^12*e^3 + 5*b^4*c*d^11*e^4 - b^5*d^10*e^5 + (c^
5*d^10*e^5 - 5*b*c^4*d^9*e^6 + 10*b^2*c^3*d^8*e^7 - 10*b^3*c^2*d^7*e^8 + 5
*b^4*c*d^6*e^9 - b^5*d^5*e^10)*x^5 + 5*(c^5*d^11*e^4 - 5*b*c^4*d^10*e^5...
```

Sympy [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^6}\,dx=\int \frac{\sqrt{x\,(b+cx)}}{(d+ex)^6}\,dx$$

input
```
integrate((c*x**2+b*x)**(1/2)/(e*x+d)**6,x)
```

output
```
Integral(sqrt(x*(b + c*x))/(d + e*x)**6, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sqrt{bx + cx^2}}{(d + ex)^6}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^6,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2144 vs. $2(360) = 720$.

Time = 0.18 (sec) , antiderivative size = 2144, normalized size of antiderivative = 5.47

$$\int \frac{\sqrt{bx + cx^2}}{(d + ex)^6}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^6,x, algorithm="giac")
```

output
```
1/128*(32*b^2*c^3*d^3 - 48*b^3*c^2*d^2*e + 30*b^4*c*d*e^2 - 7*b^5*e^3)*arc
tan(((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e))/
((c^4*d^8 - 4*b*c^3*d^7*e + 6*b^2*c^2*d^6*e^2 - 4*b^3*c*d^5*e^3 + b^4*d^4*
e^4)*sqrt(-c*d^2 + b*d*e)) + 1/1920*(480*(sqrt(c)*x - sqrt(c*x^2 + b*x))^9
*b^2*c^3*d^3*e^6 - 720*(sqrt(c)*x - sqrt(c*x^2 + b*x))^9*b^3*c^2*d^2*e^7 +
 450*(sqrt(c)*x - sqrt(c*x^2 + b*x))^9*b^4*c*d*e^8 - 105*(sqrt(c)*x - sqrt
(c*x^2 + b*x))^9*b^5*e^9 + 4320*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*b^2*c^(7
/2)*d^4*e^5 - 6480*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*b^3*c^(5/2)*d^3*e^6 +
 4050*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*b^4*c^(3/2)*d^2*e^7 - 945*(sqrt(c)
*x - sqrt(c*x^2 + b*x))^8*b^5*sqrt(c)*d*e^8 + 15040*(sqrt(c)*x - sqrt(c*x^
2 + b*x))^7*b^2*c^4*d^5*e^4 - 20320*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*b^3*
c^3*d^4*e^5 + 10740*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*b^4*c^2*d^3*e^6 - 11
90*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*b^5*c*d^2*e^7 - 490*(sqrt(c)*x - sqrt
(c*x^2 + b*x))^7*b^6*d*e^8 + 7680*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*c^(13/
2)*d^8*e - 30720*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*b*c^(11/2)*d^7*e^2 + 70
720*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*b^2*c^(9/2)*d^6*e^3 - 52000*(sqrt(c)
*x - sqrt(c*x^2 + b*x))^6*b^3*c^(7/2)*d^5*e^4 + 7260*(sqrt(c)*x - sqrt(c*x
^2 + b*x))^6*b^4*c^(5/2)*d^4*e^5 + 9310*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*
b^5*c^(3/2)*d^3*e^6 - 3430*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*b^6*sqrt(c)*d
^2*e^7 + 3072*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*c^7*d^9 + 9216*(sqrt(c)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^6}\,dx = \int \frac{\sqrt{c\,x^2+b\,x}}{(d+e\,x)^6}\,dx$$

input
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^6,x)
```

output
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^6, x)
```

Reduce [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^6}\,dx = \int \frac{\sqrt{c\,x^2+bx}}{\left(ex+d\right)^6}dx$$

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^6,x)
```

output
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^6,x)
```

3.139 $\int (d+ex)^3 \left(bx+cx^2\right)^{3/2} dx$

Optimal result . . . 1095
Mathematica [A] (verified) . . . 1096
Rubi [A] (verified) . . . 1097
Maple [A] (verified) . . . 1100
Fricas [A] (verification not implemented) . . . 1102
Sympy [B] (verification not implemented) . . . 1103
Maxima [A] (verification not implemented) . . . 1104
Giac [A] (verification not implemented) . . . 1105
Mupad [F(-1)] . . . 1106
Reduce [B] (verification not implemented) . . . 1106

Optimal result

Integrand size = 21, antiderivative size = 391

$$\begin{aligned}\int (d+ex)^3 \left(bx+cx^2\right)^{3/2} dx = & -\frac{3b^3(2cd-be)\left(8c^2d^2-8bcde+3b^2e^2\right)\sqrt{bx+cx^2}}{1024c^5} \\ & + \frac{b^2(2cd-be)\left(8c^2d^2-8bcde+3b^2e^2\right)x\sqrt{bx+cx^2}}{512c^4} \\ & + \frac{3b(2cd-be)\left(8c^2d^2-8bcde+3b^2e^2\right)x^2\sqrt{bx+cx^2}}{128c^3} \\ & + \frac{(2cd-be)\left(8c^2d^2-8bcde+3b^2e^2\right)x^3\sqrt{bx+cx^2}}{64c^2} \\ & + \frac{e\left(24c^2d^2-14bcde+3b^2e^2\right)\left(bx+cx^2\right)^{5/2}}{40c^3} + \frac{e^2(14cd-3be)x\left(bx+cx^2\right)^{5/2}}{28c^2} \\ & + \frac{e^3x^2\left(bx+cx^2\right)^{5/2}}{7c} + \frac{3b^4(2cd-be)\left(8c^2d^2-8bcde+3b^2e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{1024c^{11/2}}\end{aligned}$$

output
```
-3/1024*b^3*(-b*e+2*c*d)*(3*b^2*e^2-8*b*c*d*e+8*c^2*d^2)*(c*x^2+b*x)^(1/2)
/c^5+1/512*b^2*(-b*e+2*c*d)*(3*b^2*e^2-8*b*c*d*e+8*c^2*d^2)*x*(c*x^2+b*x)^
(1/2)/c^4+3/128*b*(-b*e+2*c*d)*(3*b^2*e^2-8*b*c*d*e+8*c^2*d^2)*x^2*(c*x^2+
b*x)^(1/2)/c^3+1/64*(-b*e+2*c*d)*(3*b^2*e^2-8*b*c*d*e+8*c^2*d^2)*x^3*(c*x^
2+b*x)^(1/2)/c^2+1/40*e*(3*b^2*e^2-14*b*c*d*e+24*c^2*d^2)*(c*x^2+b*x)^(5/2
)/c^3+1/28*e^2*(-3*b*e+14*c*d)*x*(c*x^2+b*x)^(5/2)/c^2+1/7*e^3*x^2*(c*x^2+
b*x)^(5/2)/c+3/1024*b^4*(-b*e+2*c*d)*(3*b^2*e^2-8*b*c*d*e+8*c^2*d^2)*arcta
nh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 1.92 (sec) , antiderivative size = 370, normalized size of antiderivative = 0.95

$$\int (d+ex)^3 \left(bx+cx^2\right)^{3/2} dx = \frac{\sqrt{x}\sqrt{b+cx}\left(\sqrt{c}\sqrt{x}\sqrt{b+cx}(315b^6e^3-210b^5ce^2(7d+ex)+28b^4c^2e(90d^2+35dex+6e^2x^2)\right.}{}$$

input
```
Integrate[(d + e*x)^3*(b*x + c*x^2)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[b + c*x]*(Sqrt[c]*Sqrt[x]*Sqrt[b + c*x]*(315*b^6*e^3 - 210*b
^5*c*e^2*(7*d + e*x) + 28*b^4*c^2*e*(90*d^2 + 35*d*e*x + 6*e^2*x^2) + 32*b
^2*c^4*x*(35*d^3 + 42*d^2*e*x + 21*d*e^2*x^2 + 4*e^3*x^3) - 16*b^3*c^3*(10
5*d^3 + 105*d^2*e*x + 49*d*e^2*x^2 + 9*e^3*x^3) + 256*c^6*x^3*(35*d^3 + 84
*d^2*e*x + 70*d*e^2*x^2 + 20*e^3*x^3) + 128*b*c^5*x^2*(105*d^3 + 231*d^2*e
*x + 182*d*e^2*x^2 + 50*e^3*x^3)) + 630*b^5*e*(8*c^2*d^2 + b^2*e^2)*ArcTan
h[(Sqrt[c]*Sqrt[x])/(Sqrt[b] - Sqrt[b + c*x])] + 420*b^4*c*d*(8*c^2*d^2 +
7*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])]))/(35840*
c^(11/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.59, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1166, 27, 1225, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{3/2} (d+ex)^3 \, dx$$

$$\Big\downarrow \ 1166$$

$$\frac{\int \frac{1}{2}(d+ex)(d(14cd-5be)+9e(2cd-be)x)\left(cx^2+bx\right)^{3/2}dx}{7c}+\frac{e\left(bx+cx^2\right)^{5/2}(d+ex)^2}{7c}$$

$$\Big\downarrow \ 27$$

$$\frac{\int (d+ex)(d(14cd-5be)+9e(2cd-be)x)\left(cx^2+bx\right)^{3/2}dx}{14c}+\frac{e\left(bx+cx^2\right)^{5/2}(d+ex)^2}{7c}$$

$$\Big\downarrow \ 1225$$

$$\frac{\frac{7(2cd-be)\left(3b^2e^2-8bcde+8c^2d^2\right)\int \left(cx^2+bx\right)^{3/2}dx}{8c^2}+\frac{e\left(bx+cx^2\right)^{5/2}\left(21b^2e^2+30cex(2cd-be)-98bcde+128c^2d^2\right)}{20c^2}}{14c}+\frac{e\left(bx+cx^2\right)^{5/2}(d+ex)^2}{7c}$$

$$\Big\downarrow \ 1087$$

$$\frac{\frac{7(2cd-be)\left(3b^2e^2-8bcde+8c^2d^2\right)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int \sqrt{cx^2+bx}dx}{16c}\right)}{8c^2}+\frac{e\left(bx+cx^2\right)^{5/2}\left(21b^2e^2+30cex(2cd-be)-98bcde+128c^2d^2\right)}{20c^2}}{14c}+\frac{e\left(bx+cx^2\right)^{5/2}(d+ex)^2}{7c}$$

$$\Big\downarrow \ 1087$$

$$\frac{\frac{7(2cd-be)(3b^2e^2-8bcde+8c^2d^2)\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)}{8c^2}+\frac{e(bx+cx^2)^{5/2}(21b^2e^2+30cex(2cd-be)}{20c^2}}{14c}$$

$$\frac{e(bx+cx^2)^{5/2}(d+ex)^2}{7c}$$

$\Big\downarrow$ 1091

$$\frac{\frac{7(2cd-be)(3b^2e^2-8bcde+8c^2d^2)\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)}{8c^2}+\frac{e(bx+cx^2)^{5/2}(21b^2e^2+30cex}{20}}{14c}$$

$$\frac{e(bx+cx^2)^{5/2}(d+ex)^2}{7c}$$

$\Big\downarrow$ 219

$$\frac{\frac{7\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)(2cd-be)(3b^2e^2-8bcde+8c^2d^2)}{8c^2}+\frac{e(bx+cx^2)^{5/2}(21b^2e^2+30cex(}{20c}}{14c}$$

$$\frac{e(bx+cx^2)^{5/2}(d+ex)^2}{7c}$$

input
```
Int[(d + e*x)^3*(b*x + c*x^2)^(3/2),x]
```

output
```
(e*(d + e*x)^2*(b*x + c*x^2)^(5/2))/(7*c) + ((e*(128*c^2*d^2 - 98*b*c*d*e
 + 21*b^2*e^2 + 30*c*e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(5/2))/(20*c^2) + (7*
(2*c*d - b*e)*(8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2)*(((b + 2*c*x)*(b*x + c*x
^2)^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*Ar
cTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))/(16*c)))/(8*c^2))/(14*
c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{9\left(\left(b^2e^2-\frac{8}{3}bcde+\frac{8}{3}c^2d^2\right)(be-2cd)b^4\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)-\left(\frac{128x^2b\left(\frac{10}{21}e^3x^3+\frac{26}{15}de^2x^2+\frac{11}{5}d^2ex+d^3\right)c^{\frac{11}{2}}}{3}+\frac{256\left(\frac{4}{7}e^3x^3+\cdots\right.}{\cdots}\right.\right.}{\cdots}$
risch	$\frac{(5120c^6e^3x^6+6400bc^5e^3x^5+17920c^6de^2x^5+128b^2c^4e^3x^4+23296bc^5de^2x^4+21504c^6d^2ex^4-144b^3c^3e^3x^3+672b^2c^4de^2x\cdots}{\cdots}$
default	$d^3\left(\frac{(2cx+b)(cx^2+bx)^{\frac{3}{2}}}{8c}-\frac{3b^2\left(\frac{(2cx+b)\sqrt{cx^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+e^3\left(\frac{x^2(cx^2+bx)^{\frac{5}{2}}}{7c}-\frac{9b\left(\frac{x(\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.$

input
```
int((e*x+d)^3*(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-9/1024/c^(11/2)*((b^2*e^2-8/3*b*c*d*e+8/3*c^2*d^2)*(b*e-2*c*d)*b^4*arctan
h((x*(c*x+b))^(1/2)/x/c^(1/2))-(128/3*x^2*b*(10/21*e^3*x^3+26/15*d*e^2*x^2
+11/5*d^2*e*x+d^3)*c^(11/2)+256/9*(4/7*e^3*x^3+2*d*e^2*x^2+12/5*d^2*e*x+d^
3)*x^3*c^(13/2)+b^2*(-16/3*(3/35*e^3*x^3+7/15*d*e^2*x^2+d^2*e*x+d^3)*b*c^(
7/2)+32/9*x*(4/35*e^3*x^3+3/5*d*e^2*x^2+6/5*d^2*e*x+d^3)*c^(9/2)+e*((8/15*
e^2*x^2+28/9*d*e*x+8*d^2)*c^(5/2)+((-2/3*e*x-14/3*d)*c^(3/2)+b*e*c^(1/2))*
e*b)*b^2))*(x*(c*x+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 707, normalized size of antiderivative = 1.81

$$\int (d+ex)^3 \left(bx+cx^2\right)^{3/2} dx = \left[-\frac{105 \left(16 b^4c^3d^3-24 b^5c^2d^2e+14 b^6cde^2-3 b^7e^3\right)\sqrt{c}\log\left(2 cx+b-2 \sqrt{cx^2+bx}\sqrt{c}\right)-2 \cdots}{\cdots}, -\frac{105 \left(16 b^4c^3d^3-24 b^5c^2d^2e+14 b^6cde^2-3 b^7e^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-\left(5120 c^7e^3x^6-1680 b^3c^4d^3 \cdots\right.}{\cdots}\right]$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/71680*(105*(16*b^4*c^3*d^3 - 24*b^5*c^2*d^2*e + 14*b^6*c*d*e^2 - 3*b^7
*e^3)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(5120*c^7*e
^3*x^6 - 1680*b^3*c^4*d^3 + 2520*b^4*c^3*d^2*e - 1470*b^5*c^2*d*e^2 + 315*
b^6*c*e^3 + 1280*(14*c^7*d*e^2 + 5*b*c^6*e^3)*x^5 + 128*(168*c^7*d^2*e + 1
82*b*c^6*d*e^2 + b^2*c^5*e^3)*x^4 + 16*(560*c^7*d^3 + 1848*b*c^6*d^2*e + 4
2*b^2*c^5*d*e^2 - 9*b^3*c^4*e^3)*x^3 + 56*(240*b*c^6*d^3 + 24*b^2*c^5*d^2*
e - 14*b^3*c^4*d*e^2 + 3*b^4*c^3*e^3)*x^2 + 70*(16*b^2*c^5*d^3 - 24*b^3*c^
4*d^2*e + 14*b^4*c^3*d*e^2 - 3*b^5*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/c^6, -1/
35840*(105*(16*b^4*c^3*d^3 - 24*b^5*c^2*d^2*e + 14*b^6*c*d*e^2 - 3*b^7*e^3
)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) - (5120*c^7*e^3*x^
6 - 1680*b^3*c^4*d^3 + 2520*b^4*c^3*d^2*e - 1470*b^5*c^2*d*e^2 + 315*b^6*c
*e^3 + 1280*(14*c^7*d*e^2 + 5*b*c^6*e^3)*x^5 + 128*(168*c^7*d^2*e + 182*b*
c^6*d*e^2 + b^2*c^5*e^3)*x^4 + 16*(560*c^7*d^3 + 1848*b*c^6*d^2*e + 42*b^2
*c^5*d*e^2 - 9*b^3*c^4*e^3)*x^3 + 56*(240*b*c^6*d^3 + 24*b^2*c^5*d^2*e - 1
4*b^3*c^4*d*e^2 + 3*b^4*c^3*e^3)*x^2 + 70*(16*b^2*c^5*d^3 - 24*b^3*c^4*d^2
*e + 14*b^4*c^3*d*e^2 - 3*b^5*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/c^6]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 864 vs. 2(388) = 776.

Time = 0.49 (sec) , antiderivative size = 864, normalized size of antiderivative = 2.21

$$\int (d+ex)^3 \left(bx+cx^2\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(c*x**2+b*x)**(3/2),x)
```

output
```
Piecewise((3*b**2*(b**2*d**3 - 5*b*(3*b**2*d**2*e + 2*b*c*d**3 - 7*b*(3*b*
*2*d*e**2 + 6*b*c*d**2*e - 9*b*(b**2*e**3 + 6*b*c*d*e**2 - 11*b*(15*b*c*e*
*3/14 + 3*c**2*d*e**2)/(12*c) + 3*c**2*d**2*e)/(10*c) + c**2*d**3)/(8*c))/
(6*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), N
e(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), T
rue))/(8*c**2) + sqrt(b*x + c*x**2)*(-3*b*(b**2*d**3 - 5*b*(3*b**2*d**2*e
+ 2*b*c*d**3 - 7*b*(3*b**2*d*e**2 + 6*b*c*d**2*e - 9*b*(b**2*e**3 + 6*b*c*
d*e**2 - 11*b*(15*b*c*e**3/14 + 3*c**2*d*e**2)/(12*c) + 3*c**2*d**2*e)/(10
*c) + c**2*d**3)/(8*c))/(6*c))/(4*c**2) + c*e**3*x**6/7 + x**5*(15*b*c*e**
3/14 + 3*c**2*d*e**2)/(6*c) + x**4*(b**2*e**3 + 6*b*c*d*e**2 - 11*b*(15*b*
c*e**3/14 + 3*c**2*d*e**2)/(12*c) + 3*c**2*d**2*e)/(5*c) + x**3*(3*b**2*d*
e**2 + 6*b*c*d**2*e - 9*b*(b**2*e**3 + 6*b*c*d*e**2 - 11*b*(15*b*c*e**3/14
 + 3*c**2*d*e**2)/(12*c) + 3*c**2*d**2*e)/(10*c) + c**2*d**3)/(4*c) + x**2
*(3*b**2*d**2*e + 2*b*c*d**3 - 7*b*(3*b**2*d*e**2 + 6*b*c*d**2*e - 9*b*(b*
*2*e**3 + 6*b*c*d*e**2 - 11*b*(15*b*c*e**3/14 + 3*c**2*d*e**2)/(12*c) + 3*
c**2*d**2*e)/(10*c) + c**2*d**3)/(8*c))/(3*c) + x*(b**2*d**3 - 5*b*(3*b**2
*d**2*e + 2*b*c*d**3 - 7*b*(3*b**2*d*e**2 + 6*b*c*d**2*e - 9*b*(b**2*e**3
+ 6*b*c*d*e**2 - 11*b*(15*b*c*e**3/14 + 3*c**2*d*e**2)/(12*c) + 3*c**2*d**
2*e)/(10*c) + c**2*d**3)/(8*c))/(6*c))/(2*c)), Ne(c, 0)), (2*(d**3*(b*x)**
(5/2)/5 + 3*d**2*e*(b*x)**(7/2)/(7*b) + d*e**2*(b*x)**(9/2)/(3*b**2) + ...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.60

$$\int (d+ex)^3 \left(bx+cx^2\right)^{3/2} \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
1/7*(c*x^2 + b*x)^(5/2)*e^3*x^2/c + 1/4*(c*x^2 + b*x)^(3/2)*d^3*x - 3/32*s
qrt(c*x^2 + b*x)*b^2*d^3*x/c + 9/64*sqrt(c*x^2 + b*x)*b^3*d^2*e*x/c^2 - 3/
8*(c*x^2 + b*x)^(3/2)*b*d^2*e*x/c - 21/256*sqrt(c*x^2 + b*x)*b^4*d*e^2*x/c
^3 + 7/32*(c*x^2 + b*x)^(3/2)*b^2*d*e^2*x/c^2 + 1/2*(c*x^2 + b*x)^(5/2)*d*
e^2*x/c + 9/512*sqrt(c*x^2 + b*x)*b^5*e^3*x/c^4 - 3/64*(c*x^2 + b*x)^(3/2)
*b^3*e^3*x/c^3 - 3/28*(c*x^2 + b*x)^(5/2)*b*e^3*x/c^2 + 3/128*b^4*d^3*log(
2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(5/2) - 9/256*b^5*d^2*e*log(2*c
*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(7/2) + 21/1024*b^6*d*e^2*log(2*c*
x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(9/2) - 9/2048*b^7*e^3*log(2*c*x +
b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(11/2) - 3/64*sqrt(c*x^2 + b*x)*b^3*d^3
/c^2 + 1/8*(c*x^2 + b*x)^(3/2)*b*d^3/c + 9/128*sqrt(c*x^2 + b*x)*b^4*d^2*e
/c^3 - 3/16*(c*x^2 + b*x)^(3/2)*b^2*d^2*e/c^2 + 3/5*(c*x^2 + b*x)^(5/2)*d^
2*e/c - 21/512*sqrt(c*x^2 + b*x)*b^5*d*e^2/c^4 + 7/64*(c*x^2 + b*x)^(3/2)*
b^3*d*e^2/c^3 - 7/20*(c*x^2 + b*x)^(5/2)*b*d*e^2/c^2 + 9/1024*sqrt(c*x^2 +
 b*x)*b^6*e^3/c^5 - 3/128*(c*x^2 + b*x)^(3/2)*b^4*e^3/c^4 + 3/40*(c*x^2 +
b*x)^(5/2)*b^2*e^3/c^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.95

$$\int (d+ex)^3 \left(bx+cx^2\right)^{3/2} dx = \frac{1}{35840}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\left(10\left(4\,ce^3x+\frac{14\,c^7de^2+5\,bc^6e^3}{c^6}\right)x+\frac{168\,c^7d^2e+182\,bc^6de}{c^6}\right.\right.\right.\right.\right.$$
$$-\frac{3\left(16\,b^4c^3d^3-24\,b^5c^2d^2e+14\,b^6cde^2-3\,b^7e^3\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{2048\,c^{\frac{11}{2}}}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
1/35840*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(10*(4*c*e^3*x + (14*c^7*d*e^2 + 5*b
*c^6*e^3)/c^6)*x + (168*c^7*d^2*e + 182*b*c^6*d*e^2 + b^2*c^5*e^3)/c^6)*x
+ (560*c^7*d^3 + 1848*b*c^6*d^2*e + 42*b^2*c^5*d*e^2 - 9*b^3*c^4*e^3)/c^6)
*x + 7*(240*b*c^6*d^3 + 24*b^2*c^5*d^2*e - 14*b^3*c^4*d*e^2 + 3*b^4*c^3*e^
3)/c^6)*x + 35*(16*b^2*c^5*d^3 - 24*b^3*c^4*d^2*e + 14*b^4*c^3*d*e^2 - 3*b
^5*c^2*e^3)/c^6)*x - 105*(16*b^3*c^4*d^3 - 24*b^4*c^3*d^2*e + 14*b^5*c^2*d
*e^2 - 3*b^6*c*e^3)/c^6) - 3/2048*(16*b^4*c^3*d^3 - 24*b^5*c^2*d^2*e + 14*
b^6*c*d*e^2 - 3*b^7*e^3)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c)
 + b))/c^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 \left(bx+cx^2\right)^{3/2} dx = \int \left(c\,x^2+b\,x\right)^{3/2} (d+e\,x)^3\,dx$$

input
```
int((b*x + c*x^2)^(3/2)*(d + e*x)^3,x)
```

output
```
int((b*x + c*x^2)^(3/2)*(d + e*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 26.31 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.48

$$\int (d+ex)^3 \left(bx+cx^2\right)^{3/2} dx = \frac{-315\sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\sqrt{c}}{\sqrt{b}}\right) b^7e^3 + 315\sqrt{x}\sqrt{cx+b}\,b^6ce^3 - 1680\sqrt{x}\sqrt{cx+b}\,b^3c^4d^3 + 8960\sqrt{}$$

input
```
int((e*x+d)^3*(c*x^2+b*x)^(3/2),x)
```

output
```
(315*sqrt(x)*sqrt(b + c*x)*b**6*c*e**3 - 1470*sqrt(x)*sqrt(b + c*x)*b**5*c
**2*d*e**2 - 210*sqrt(x)*sqrt(b + c*x)*b**5*c**2*e**3*x + 2520*sqrt(x)*sqr
t(b + c*x)*b**4*c**3*d**2*e + 980*sqrt(x)*sqrt(b + c*x)*b**4*c**3*d*e**2*x
 + 168*sqrt(x)*sqrt(b + c*x)*b**4*c**3*e**3*x**2 - 1680*sqrt(x)*sqrt(b + c
*x)*b**3*c**4*d**3 - 1680*sqrt(x)*sqrt(b + c*x)*b**3*c**4*d**2*e*x - 784*s
qrt(x)*sqrt(b + c*x)*b**3*c**4*d*e**2*x**2 - 144*sqrt(x)*sqrt(b + c*x)*b**
3*c**4*e**3*x**3 + 1120*sqrt(x)*sqrt(b + c*x)*b**2*c**5*d**3*x + 1344*sqrt
(x)*sqrt(b + c*x)*b**2*c**5*d**2*e*x**2 + 672*sqrt(x)*sqrt(b + c*x)*b**2*c
**5*d*e**2*x**3 + 128*sqrt(x)*sqrt(b + c*x)*b**2*c**5*e**3*x**4 + 13440*sq
rt(x)*sqrt(b + c*x)*b*c**6*d**3*x**2 + 29568*sqrt(x)*sqrt(b + c*x)*b*c**6*
d**2*e*x**3 + 23296*sqrt(x)*sqrt(b + c*x)*b*c**6*d*e**2*x**4 + 6400*sqrt(x
)*sqrt(b + c*x)*b*c**6*e**3*x**5 + 8960*sqrt(x)*sqrt(b + c*x)*c**7*d**3*x*
*3 + 21504*sqrt(x)*sqrt(b + c*x)*c**7*d**2*e*x**4 + 17920*sqrt(x)*sqrt(b +
 c*x)*c**7*d*e**2*x**5 + 5120*sqrt(x)*sqrt(b + c*x)*c**7*e**3*x**6 - 315*s
qrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**7*e**3 + 1470*sqr
t(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**6*c*d*e**2 - 2520*s
qrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**5*c**2*d**2*e + 1
680*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**4*c**3*d**3)
/(35840*c**6)
```

3.140 $\int (d+ex)^2 \left(bx+cx^2\right)^{3/2} \, dx$

Optimal result 1108
Mathematica [A] (verified) 1109
Rubi [A] (verified) 1109
Maple [A] (verified) 1112
Fricas [A] (verification not implemented) 1114
Sympy [A] (verification not implemented) 1115
Maxima [A] (verification not implemented) 1116
Giac [A] (verification not implemented) 1117
Mupad [F(-1)] 1118
Reduce [B] (verification not implemented) 1118

Optimal result

Integrand size = 21, antiderivative size = 297

$$\int (d+ex)^2 \left(bx+cx^2\right)^{3/2} \, dx = -\frac{b^3(24c^2d^2-24bcde+7b^2e^2)\sqrt{bx+cx^2}}{512c^4} + \frac{b^2(24c^2d^2-24bcde+7b^2e^2)\,x\sqrt{bx+cx^2}}{768c^3} + \frac{b(24c^2d^2-24bcde+7b^2e^2)\,x^2\sqrt{bx+cx^2}}{64c^2} + \frac{(24c^2d^2-24bcde+7b^2e^2)\,x^3\sqrt{bx+cx^2}}{96c} + \frac{e(24cd-7be)\left(bx+cx^2\right)^{5/2}}{60c^2} + \frac{e^2x(bx+cx^2)^{5/2}}{6c} + \frac{b^4(24c^2d^2-24bcde+7b^2e^2)\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{512c^{9/2}}$$

output

```
-1/512*b^3*(7*b^2*e^2-24*b*c*d*e+24*c^2*d^2)*(c*x^2+b*x)^(1/2)/c^4+1/768*b
^2*(7*b^2*e^2-24*b*c*d*e+24*c^2*d^2)*x*(c*x^2+b*x)^(1/2)/c^3+1/64*b*(7*b^2
*e^2-24*b*c*d*e+24*c^2*d^2)*x^2*(c*x^2+b*x)^(1/2)/c^2+1/96*(7*b^2*e^2-24*b
*c*d*e+24*c^2*d^2)*x^3*(c*x^2+b*x)^(1/2)/c+1/60*e*(-7*b*e+24*c*d)*(c*x^2+b
*x)^(5/2)/c^2+1/6*e^2*x*(c*x^2+b*x)^(5/2)/c+1/512*b^4*(7*b^2*e^2-24*b*c*d*
e+24*c^2*d^2)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.94

$$\int (d+ex)^2 \left(bx+cx^2\right)^{3/2} dx = \frac{\sqrt{x}\sqrt{b+cx}\left(\sqrt{c}\sqrt{x}\sqrt{b+cx}(-105b^5e^2+10b^4ce(36d+7ex)+48b^2c^3x(5d^2+4dex+e^2x^2)\right.}{}$$

input
```
Integrate[(d + e*x)^2*(b*x + c*x^2)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[b + c*x]*(Sqrt[c]*Sqrt[x]*Sqrt[b + c*x]*(-105*b^5*e^2 + 10*b
^4*c*e*(36*d + 7*e*x) + 48*b^2*c^3*x*(5*d^2 + 4*d*e*x + e^2*x^2) - 8*b^3*c
^2*(45*d^2 + 30*d*e*x + 7*e^2*x^2) + 128*c^5*x^3*(15*d^2 + 24*d*e*x + 10*e
^2*x^2) + 64*b*c^4*x^2*(45*d^2 + 66*d*e*x + 26*e^2*x^2)) + 720*b^5*c*d*e*A
rcTanh[(Sqrt[c]*Sqrt[x])/(Sqrt[b] - Sqrt[b + c*x])] + 30*b^4*(24*c^2*d^2 +
 7*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])]))/(7680*
c^(9/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.65, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1166, 27, 1160, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{3/2} (d+ex)^2 dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{\int \frac{1}{2}(d(12cd-5be)+7e(2cd-be)x)\left(cx^2+bx\right)^{3/2}dx}{6c}+\frac{e\left(bx+cx^2\right)^{5/2}(d+ex)}{6c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (d(12cd-5be)+7e(2cd-be)x)\left(cx^2+bx\right)^{3/2}dx}{12c}+\frac{e\left(bx+cx^2\right)^{5/2}(d+ex)}{6c}$$

$\downarrow$ 1160

$$\frac{\frac{(7b^2e^2-24bcde+24c^2d^2)\int \left(cx^2+bx\right)^{3/2}dx}{2c}+\frac{7e\left(bx+cx^2\right)^{5/2}(2cd-be)}{5c}}{12c}+\frac{e\left(bx+cx^2\right)^{5/2}(d+ex)}{6c}$$

$\downarrow$ 1087

$$\frac{\frac{(7b^2e^2-24bcde+24c^2d^2)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int \sqrt{cx^2+bx}dx}{16c}\right)}{2c}+\frac{7e\left(bx+cx^2\right)^{5/2}(2cd-be)}{5c}}{12c}+\frac{e\left(bx+cx^2\right)^{5/2}(d+ex)}{6c}$$

$\downarrow$ 1087

$$\frac{\frac{(7b^2e^2-24bcde+24c^2d^2)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)}{2c}+\frac{7e\left(bx+cx^2\right)^{5/2}(2cd-be)}{5c}}{12c}+\frac{e\left(bx+cx^2\right)^{5/2}(d+ex)}{6c}$$

$\downarrow$ 1091

$$\frac{\frac{(7b^2e^2-24bcde+24c^2d^2)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)}{2c}+\frac{7e\left(bx+cx^2\right)^{5/2}(2cd-be)}{5c}}{12c}+\frac{e\left(bx+cx^2\right)^{5/2}(d+ex)}{6c}$$

$\downarrow$ 219

$$\frac{\frac{\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)\left(7b^2e^2-24bcde+24c^2d^2\right)}{2c}+\frac{7e\left(bx+cx^2\right)^{5/2}(2cd-be)}{5c}}{12c}+\frac{e\left(bx+cx^2\right)^{5/2}(d+ex)}{6c}$$

input
```
Int[(d + e*x)^2*(b*x + c*x^2)^(3/2),x]
```

output
```
(e*(d + e*x)*(b*x + c*x^2)^(5/2))/(6*c) + ((7*e*(2*c*d - b*e)*(b*x + c*x^2
)^(5/2))/(5*c) + ((24*c^2*d^2 - 24*b*c*d*e + 7*b^2*e^2)*(((b + 2*c*x)*(b*x
 + c*x^2)^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (
b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))/(16*c)))/(2*c))/
(12*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{7b^4\left(b^2e^2-\frac{24}{7}bcde+\frac{24}{7}c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{512}-\frac{7\sqrt{x(cx+b)}\left(-\frac{128x^3\left(\frac{2}{3}e^2x^2+\frac{8}{5}dex+d^2\right)c^{\frac{11}{2}}}{7}+b\left(\frac{24\left(\frac{7}{45}e^2x^2+\frac{2}{3}dex+d^2\right)b^2c^{\frac{5}{2}}}{7}\right.\right.}{c^{\frac{9}{2}}}$
risch	$-\frac{(-1280c^5e^2x^5-1664b\,c^4e^2x^4-3072c^5de\,x^4-48b^2c^3e^2x^3-4224b\,c^4de\,x^3-1920c^5d^2x^3+56b^3c^2e^2x^2-192b^2c^3de\,x^2-288}{7680c^4\sqrt{x(cx+b)}}$
default	$d^2\left(\frac{(2cx+b)\left(c\,x^2+bx\right)^{\frac{3}{2}}}{8c}-\frac{3b^2\left(\frac{(2cx+b)\sqrt{c\,x^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+e^2\left(\frac{x\left(c\,x^2+bx\right)^{\frac{5}{2}}}{6c}-\frac{7b\left(\frac{(c\,x}{}\right.}{}\right.$

input
```
int((e*x+d)^2*(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
7/512/c^(9/2)*(b^4*(b^2*e^2-24/7*b*c*d*e+24/7*c^2*d^2)*arctanh((x*(c*x+b))
^(1/2)/x/c^(1/2))-(x*(c*x+b))^(1/2)*(-128/7*x^3*(2/3*e^2*x^2+8/5*d*e*x+d^2
)*c^(11/2)+b*(24/7*(7/45*e^2*x^2+2/3*d*e*x+d^2)*b^2*c^(5/2)-16/7*(1/5*e^2*
x^2+4/5*d*e*x+d^2)*x*b*c^(7/2)-192/7*x^2*(26/45*e^2*x^2+22/15*d*e*x+d^2)*c
^(9/2)+e*b^3*((-2/3*e*x-24/7*d)*c^(3/2)+b*e*c^(1/2)))))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 491, normalized size of antiderivative = 1.65

$$\int (d+ex)^2 \left(bx+cx^2\right)^{3/2} dx = \left[\frac{15 \left(24 b^4 c^2 d^2 - 24 b^5 cde + 7 b^6 e^2\right)\sqrt{c} \log\left(2 cx + b + 2 \sqrt{cx^2+bx}\sqrt{c}\right) + 2 \left(1280 c^6 e^2 x^5 - 36\right.}{} \right.$$

$$-\frac{15 \left(24 b^4 c^2 d^2 - 24 b^5 cde + 7 b^6 e^2\right)\sqrt{-c} \arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right) - \left(1280 c^6 e^2 x^5 - 360 b^3 c^3 d^2 + 360 b^4 c^2 de - \right.}{}$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[1/15360*(15*(24*b^4*c^2*d^2 - 24*b^5*c*d*e + 7*b^6*e^2)*sqrt(c)*log(2*c*x
 + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(1280*c^6*e^2*x^5 - 360*b^3*c^3*d^
2 + 360*b^4*c^2*d*e - 105*b^5*c*e^2 + 128*(24*c^6*d*e + 13*b*c^5*e^2)*x^4
+ 48*(40*c^6*d^2 + 88*b*c^5*d*e + b^2*c^4*e^2)*x^3 + 8*(360*b*c^5*d^2 + 24
*b^2*c^4*d*e - 7*b^3*c^3*e^2)*x^2 + 10*(24*b^2*c^4*d^2 - 24*b^3*c^3*d*e +
7*b^4*c^2*e^2)*x)*sqrt(c*x^2 + b*x))/c^5, -1/7680*(15*(24*b^4*c^2*d^2 - 24
*b^5*c*d*e + 7*b^6*e^2)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x +
b)) - (1280*c^6*e^2*x^5 - 360*b^3*c^3*d^2 + 360*b^4*c^2*d*e - 105*b^5*c*e^
2 + 128*(24*c^6*d*e + 13*b*c^5*e^2)*x^4 + 48*(40*c^6*d^2 + 88*b*c^5*d*e +
b^2*c^4*e^2)*x^3 + 8*(360*b*c^5*d^2 + 24*b^2*c^4*d*e - 7*b^3*c^3*e^2)*x^2
+ 10*(24*b^2*c^4*d^2 - 24*b^3*c^3*d*e + 7*b^4*c^2*e^2)*x)*sqrt(c*x^2 + b*x
))/c^5]
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.92

$$\int (d+ex)^2 \left(bx+cx^2\right)^{3/2} dx = \begin{cases} \frac{3b^2\left(b^2d^2-\frac{5b\left(2b^2de+2bcd^2-\frac{7b\left(b^2e^2+4bcde-\frac{9b\left(\frac{13bce^2}{12}+2c^2de\right)}{10c}+c^2d^2\right)}{8c}\right)}{6c}\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c} \neq \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwi} \end{cases}\right.}{8c^2} \\ \frac{2\left(\frac{d^2(bx)^{\frac{5}{2}}}{5}+\frac{2de(bx)^{\frac{7}{2}}}{7b}+\frac{e^2(bx)^{\frac{9}{2}}}{9b^2}\right)}{b} \\ 0 \end{cases}$$

input

```
integrate((e*x+d)**2*(c*x**2+b*x)**(3/2),x)
```

output
```
Piecewise((3*b**2*(b**2*d**2 - 5*b*(2*b**2*d*e + 2*b*c*d**2 - 7*b*(b**2*e*
*2 + 4*b*c*d*e - 9*b*(13*b*c*e**2/12 + 2*c**2*d*e)/(10*c) + c**2*d**2)/(8*
c))/(6*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c
), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2
), True))/(8*c**2) + sqrt(b*x + c*x**2)*(-3*b*(b**2*d**2 - 5*b*(2*b**2*d*e
 + 2*b*c*d**2 - 7*b*(b**2*e**2 + 4*b*c*d*e - 9*b*(13*b*c*e**2/12 + 2*c**2*
d*e)/(10*c) + c**2*d**2)/(8*c))/(6*c))/(4*c**2) + c*e**2*x**5/6 + x**4*(13
*b*c*e**2/12 + 2*c**2*d*e)/(5*c) + x**3*(b**2*e**2 + 4*b*c*d*e - 9*b*(13*b
*c*e**2/12 + 2*c**2*d*e)/(10*c) + c**2*d**2)/(4*c) + x**2*(2*b**2*d*e + 2*
b*c*d**2 - 7*b*(b**2*e**2 + 4*b*c*d*e - 9*b*(13*b*c*e**2/12 + 2*c**2*d*e)/
(10*c) + c**2*d**2)/(8*c))/(3*c) + x*(b**2*d**2 - 5*b*(2*b**2*d*e + 2*b*c*
d**2 - 7*b*(b**2*e**2 + 4*b*c*d*e - 9*b*(13*b*c*e**2/12 + 2*c**2*d*e)/(10*
c) + c**2*d**2)/(8*c))/(6*c))/(2*c)), Ne(c, 0)), (2*(d**2*(b*x)**(5/2)/5 +
 2*d*e*(b*x)**(7/2)/(7*b) + e**2*(b*x)**(9/2)/(9*b**2))/b, Ne(b, 0)), (0,
True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.40

$$\int (d+ex)^2 \left(bx+cx^2\right)^{3/2} \, dx = \frac{1}{4}\left(cx^2+bx\right)^{\frac{3}{2}} d^2 x - \frac{3\sqrt{cx^2+bx}b^2d^2x}{32\,c} + \frac{3\sqrt{cx^2+bx}b^3dex}{32\,c^2} - \frac{\left(cx^2+bx\right)^{\frac{3}{2}}bdex}{4\,c} - \frac{7\sqrt{cx^2+bx}b^4e^2x}{256\,c^3} + \frac{7\left(cx^2+bx\right)^{\frac{3}{2}}b^2e^2x}{96\,c^2} + \frac{\left(cx^2+bx\right)^{\frac{5}{2}}e^2x}{6\,c} + \frac{3\,b^4d^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{128\,c^{\frac{5}{2}}} - \frac{3\,b^5de\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{128\,c^{\frac{7}{2}}} + \frac{7\,b^6e^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{1024\,c^{\frac{9}{2}}} - \frac{3\sqrt{cx^2+bx}b^3d^2}{64\,c^2} + \frac{\left(cx^2+bx\right)^{\frac{3}{2}}bd^2}{8\,c} + \frac{3\sqrt{cx^2+bx}b^4de}{64\,c^3} - \frac{\left(cx^2+bx\right)^{\frac{3}{2}}b^2de}{8\,c^2} + \frac{2\left(cx^2+bx\right)^{\frac{5}{2}}de}{5\,c} - \frac{7\sqrt{cx^2+bx}b^5e^2}{512\,c^4} + \frac{7\left(cx^2+bx\right)^{\frac{3}{2}}b^3e^2}{192\,c^3} - \frac{7\left(cx^2+bx\right)^{\frac{5}{2}}be^2}{60\,c^2}$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(c*x^2 + b*x)^(3/2)*d^2*x - 3/32*sqrt(c*x^2 + b*x)*b^2*d^2*x/c + 3/32*
sqrt(c*x^2 + b*x)*b^3*d*e*x/c^2 - 1/4*(c*x^2 + b*x)^(3/2)*b*d*e*x/c - 7/25
6*sqrt(c*x^2 + b*x)*b^4*e^2*x/c^3 + 7/96*(c*x^2 + b*x)^(3/2)*b^2*e^2*x/c^2
 + 1/6*(c*x^2 + b*x)^(5/2)*e^2*x/c + 3/128*b^4*d^2*log(2*c*x + b + 2*sqrt(
c*x^2 + b*x)*sqrt(c))/c^(5/2) - 3/128*b^5*d*e*log(2*c*x + b + 2*sqrt(c*x^2
 + b*x)*sqrt(c))/c^(7/2) + 7/1024*b^6*e^2*log(2*c*x + b + 2*sqrt(c*x^2 + b
*x)*sqrt(c))/c^(9/2) - 3/64*sqrt(c*x^2 + b*x)*b^3*d^2/c^2 + 1/8*(c*x^2 + b
*x)^(3/2)*b*d^2/c + 3/64*sqrt(c*x^2 + b*x)*b^4*d*e/c^3 - 1/8*(c*x^2 + b*x)
^(3/2)*b^2*d*e/c^2 + 2/5*(c*x^2 + b*x)^(5/2)*d*e/c - 7/512*sqrt(c*x^2 + b*
x)*b^5*e^2/c^4 + 7/192*(c*x^2 + b*x)^(3/2)*b^3*e^2/c^3 - 7/60*(c*x^2 + b*x
)^(5/2)*b*e^2/c^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.88

$$\int (d+ex)^2 \left(bx+cx^2\right)^{3/2} dx = \frac{1}{7680}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\left(10\,ce^2x+\frac{24\,c^6de+13\,bc^5e^2}{c^5}\right)x+\frac{3\left(40\,c^6d^2+88\,bc^5de+b^2\right.}{c^5}\right.\right.\right.\right. - \frac{\left(24\,b^4c^2d^2-24\,b^5cde+7\,b^6e^2\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{1024\,c^{\frac{9}{2}}}$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(10*c*e^2*x + (24*c^6*d*e + 13*b*c^5*
e^2)/c^5)*x + 3*(40*c^6*d^2 + 88*b*c^5*d*e + b^2*c^4*e^2)/c^5)*x + (360*b*
c^5*d^2 + 24*b^2*c^4*d*e - 7*b^3*c^3*e^2)/c^5)*x + 5*(24*b^2*c^4*d^2 - 24*
b^3*c^3*d*e + 7*b^4*c^2*e^2)/c^5)*x - 15*(24*b^3*c^3*d^2 - 24*b^4*c^2*d*e
 + 7*b^5*c*e^2)/c^5) - 1/1024*(24*b^4*c^2*d^2 - 24*b^5*c*d*e + 7*b^6*e^2)*l
og(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(bx+cx^2\right)^{3/2} dx = \int \left(c\,x^2+b\,x\right)^{3/2} (d+e\,x)^2 \,dx$$

input
```
int((b*x + c*x^2)^(3/2)*(d + e*x)^2,x)
```

output
```
int((b*x + c*x^2)^(3/2)*(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.30

$$\int (d+ex)^2 \left(bx +cx^2\right)^{3/2} dx = \frac{-105\sqrt{x}\,\sqrt{cx+b}\,b^5c\,e^2+360\sqrt{x}\,\sqrt{cx+b}\,b^4c^2de+70\sqrt{x}\,\sqrt{cx+b}\,b^4c^2e^2x-360\sqrt{x}\,\sqrt{cx+b}}{}$$

input
```
int((e*x+d)^2*(c*x^2+b*x)^(3/2),x)
```

output
```
( - 105*sqrt(x)*sqrt(b + c*x)*b**5*c*e**2 + 360*sqrt(x)*sqrt(b + c*x)*b**4
*c**2*d*e + 70*sqrt(x)*sqrt(b + c*x)*b**4*c**2*e**2*x - 360*sqrt(x)*sqrt(b
 + c*x)*b**3*c**3*d**2 - 240*sqrt(x)*sqrt(b + c*x)*b**3*c**3*d*e*x - 56*sq
rt(x)*sqrt(b + c*x)*b**3*c**3*e**2*x**2 + 240*sqrt(x)*sqrt(b + c*x)*b**2*c
**4*d**2*x + 192*sqrt(x)*sqrt(b + c*x)*b**2*c**4*d*e*x**2 + 48*sqrt(x)*sqr
t(b + c*x)*b**2*c**4*e**2*x**3 + 2880*sqrt(x)*sqrt(b + c*x)*b*c**5*d**2*x*
*2 + 4224*sqrt(x)*sqrt(b + c*x)*b*c**5*d*e*x**3 + 1664*sqrt(x)*sqrt(b + c*
x)*b*c**5*e**2*x**4 + 1920*sqrt(x)*sqrt(b + c*x)*c**6*d**2*x**3 + 3072*sqr
t(x)*sqrt(b + c*x)*c**6*d*e*x**4 + 1280*sqrt(x)*sqrt(b + c*x)*c**6*e**2*x*
*5 + 105*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**6*e**2
- 360*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**5*c*d*e +
360*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**4*c**2*d**2)
/(7680*c**5)
```

3.141 $\int (d+ex) \left(bx+cx^2\right)^{3/2} dx$

Optimal result 1119
Mathematica [A] (verified) 1120
Rubi [A] (verified) 1120
Maple [A] (verified) 1122
Fricas [A] (verification not implemented) 1123
Sympy [A] (verification not implemented) 1124
Maxima [A] (verification not implemented) 1125
Giac [A] (verification not implemented) 1125
Mupad [B] (verification not implemented) 1126
Reduce [B] (verification not implemented) 1127

Optimal result

Integrand size = 19, antiderivative size = 191

$$\int (d+ex) \left(bx+cx^2\right)^{3/2} dx = -\frac{3b^3(2cd-be)\sqrt{bx+cx^2}}{128c^3} + \frac{b^2(2cd-be)x\sqrt{bx+cx^2}}{64c^2} + \frac{3b(2cd-be)x^2\sqrt{bx+cx^2}}{16c} + \frac{1}{8}(2cd-be)x^3\sqrt{bx+cx^2} + \frac{e\left(bx+cx^2\right)^{5/2}}{5c} + \frac{3b^4(2cd-be)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{128c^{7/2}}$$

output

```
-3/128*b^3*(-b*e+2*c*d)*(c*x^2+b*x)^(1/2)/c^3+1/64*b^2*(-b*e+2*c*d)*x*(c*x
^2+b*x)^(1/2)/c^2+3/16*b*(-b*e+2*c*d)*x^2*(c*x^2+b*x)^(1/2)/c+1/8*(-b*e+2*
c*d)*x^3*(c*x^2+b*x)^(1/2)+1/5*e*(c*x^2+b*x)^(5/2)/c+3/128*b^4*(-b*e+2*c*d
)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int (d+ex)\left(bx+cx^2\right)^{3/2}\,dx=\frac{(x(b+cx))^{3/2}\left(\frac{\sqrt{c}\sqrt{x}(15b^4e-10b^3c(3d+ex)+4b^2c^2x(5d+2ex)+32c^4x^3(5d+4ex)+16bc^3x^2(15d+11ex))}{b+cx}+\frac{30b^4(-2cd}{}\right.}{640c^{7/2}x^{3/2}}$$

input
```
Integrate[(d + e*x)*(b*x + c*x^2)^(3/2),x]
```

output
```
((x*(b + c*x))^(3/2)*((Sqrt[c]*Sqrt[x]*(15*b^4*e - 10*b^3*c*(3*d + e*x) +
4*b^2*c^2*x*(5*d + 2*e*x) + 32*c^4*x^3*(5*d + 4*e*x) + 16*b*c^3*x^2*(15*d
+ 11*e*x)))/(b + c*x) + (30*b^4*(-2*c*d + b*e)*ArcTanh[(Sqrt[c]*Sqrt[x])/(
Sqrt[b] - Sqrt[b + c*x])])/(b + c*x)^(3/2)))/(640*c^(7/2)*x^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {1160, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{3/2}(d+ex)\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{(2cd-be)\int \left(cx^2+bx\right)^{3/2}dx}{2c}+\frac{e\left(bx+cx^2\right)^{5/2}}{5c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(2cd-be)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int \sqrt{cx^2+bx}dx}{16c}\right)}{2c}+\frac{e\left(bx+cx^2\right)^{5/2}}{5c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(2cd-be)\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)}{2c}+\frac{e(bx+cx^2)^{5/2}}{5c}$$

$\Big\downarrow$ 1091

$$\frac{(2cd-be)\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)}{2c}+\frac{e(bx+cx^2)^{5/2}}{5c}$$

$\Big\downarrow$ 219

$$\frac{\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)(2cd-be)}{2c}+\frac{e(bx+cx^2)^{5/2}}{5c}$$

input
```
Int[(d + e*x)*(b*x + c*x^2)^(3/2),x]
```

output
```
(e*(b*x + c*x^2)^(5/2))/(5*c) + ((2*c*d - b*e)*(((b + 2*c*x)*(b*x + c*x^2)
^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTa
nh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))/(16*c)))/(2*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{3\left((b^5e-2b^4cd)\,\mathrm{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)-\left(-2\left(\frac{ex}{3}+d\right)b^3c^{\frac{3}{2}}+\frac{4\left(\frac{2ex}{5}+d\right)x\,b^2c^{\frac{5}{2}}}{3}+16x^2\left(\frac{11ex}{15}+d\right)b\,c^{\frac{7}{2}}+\frac{32x^3\left(\frac{4ex}{5}+d\right)c^{\frac{9}{2}}}{3}+\sqrt{\right.}\right.\right.}{128c^{\frac{7}{2}}}$
risch	$\frac{(128c^4e\,x^4+176b\,c^3e\,x^3+160c^4d\,x^3+8b^2c^2e\,x^2+240b\,c^3d\,x^2-10b^3cex+20b^2c^2dx+15b^4e-30b^3cd)x(cx+b)}{640c^3\sqrt{x(cx+b)}}-\frac{3b^4(be-2c}{}$
default	$d\left(\frac{(2cx+b)(c\,x^2+bx)^{\frac{3}{2}}}{8c}-\frac{3b^2\left(\frac{(2cx+b)\sqrt{c\,x^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+e\left(\frac{(c\,x^2+bx)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx+b)(}{8}\right.}{}\right.$

input
```
int((e*x+d)*(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3/128/c^(7/2)*((b^5*e-2*b^4*c*d)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))-(-2
*(1/3*e*x+d)*b^3*c^(3/2)+4/3*(2/5*e*x+d)*x*b^2*c^(5/2)+16*x^2*(11/15*e*x+d
)*b*c^(7/2)+32/3*x^3*(4/5*e*x+d)*c^(9/2)+c^(1/2)*b^4*e)*(x*(c*x+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.59

$$\int (d+ex)\left(bx+cx^2\right)^{3/2}\,dx=\left[-\frac{15\,(2\,b^4cd-b^5e)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(128\,c^5ex^4-30\,b^3c^2d+15\,b^4ce}{1280\,c}\right.$$

$$-\frac{15\,(2\,b^4cd-b^5e)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(128\,c^5ex^4-30\,b^3c^2d+15\,b^4ce+16\,(10\,c^5d+11\,bc^4e)x^3+}{640\,c^4}$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/1280*(15*(2*b^4*c*d - b^5*e)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*
x)*sqrt(c)) - 2*(128*c^5*e*x^4 - 30*b^3*c^2*d + 15*b^4*c*e + 16*(10*c^5*d
+ 11*b*c^4*e)*x^3 + 8*(30*b*c^4*d + b^2*c^3*e)*x^2 + 10*(2*b^2*c^3*d - b^3
*c^2*e)*x)*sqrt(c*x^2 + b*x))/c^4, -1/640*(15*(2*b^4*c*d - b^5*e)*sqrt(-c)
*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) - (128*c^5*e*x^4 - 30*b^3*c^
2*d + 15*b^4*c*e + 16*(10*c^5*d + 11*b*c^4*e)*x^3 + 8*(30*b*c^4*d + b^2*c^
3*e)*x^2 + 10*(2*b^2*c^3*d - b^3*c^2*e)*x)*sqrt(c*x^2 + b*x))/c^4]
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.71

$$\int (d+ex)\left(bx+cx^2\right)^{3/2}\,dx=\begin{cases}\frac{3b^2\left(b^2d-\frac{5b\left(b^2e+2bcd-\frac{7b\left(\frac{11bce}{10}+c^2d\right)}{8c}\right)}{6c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for}\ \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{8c^2}+\sqrt{bx+cx^2}\left(-\frac{3b\cdots}{}\right. \\ \frac{2\left(\frac{d(bx)^{\frac{5}{2}}}{5}+\frac{e(bx)^{\frac{7}{2}}}{7b}\right)}{b} \\ 0\end{cases}$$

input
```
integrate((e*x+d)*(c*x**2+b*x)**(3/2),x)
```

output
```
Piecewise((3*b**2*(b**2*d - 5*b*(b**2*e + 2*b*c*d - 7*b*(11*b*c*e/10 + c**
2*d)/(8*c))/(6*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x
)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c)
 + x)**2), True))/(8*c**2) + sqrt(b*x + c*x**2)*(-3*b*(b**2*d - 5*b*(b**2*
e + 2*b*c*d - 7*b*(11*b*c*e/10 + c**2*d)/(8*c))/(6*c))/(4*c**2) + c*e*x**4
/5 + x**3*(11*b*c*e/10 + c**2*d)/(4*c) + x**2*(b**2*e + 2*b*c*d - 7*b*(11*
b*c*e/10 + c**2*d)/(8*c))/(3*c) + x*(b**2*d - 5*b*(b**2*e + 2*b*c*d - 7*b*
(11*b*c*e/10 + c**2*d)/(8*c))/(6*c))/(2*c)), Ne(c, 0)), (2*(d*(b*x)**(5/2)
/5 + e*(b*x)**(7/2)/(7*b))/b, Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.24

$$\int (d+ex)\left(bx+cx^2\right)^{3/2}\,dx = \frac{1}{4}\left(cx^2+bx\right)^{\frac{3}{2}}dx - \frac{3\sqrt{cx^2+bx}b^2dx}{32\,c} + \frac{3\sqrt{cx^2+bx}b^3ex}{64\,c^2} - \frac{\left(cx^2+bx\right)^{\frac{3}{2}}bex}{8\,c} + \frac{3\,b^4d\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{128\,c^{\frac{5}{2}}} - \frac{3\,b^5e\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{256\,c^{\frac{7}{2}}} - \frac{3\sqrt{cx^2+bx}b^3d}{64\,c^2} + \frac{\left(cx^2+bx\right)^{\frac{3}{2}}bd}{8\,c} + \frac{3\sqrt{cx^2+bx}b^4e}{128\,c^3} - \frac{\left(cx^2+bx\right)^{\frac{3}{2}}b^2e}{16\,c^2} + \frac{\left(cx^2+bx\right)^{\frac{5}{2}}e}{5\,c}$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(c*x^2 + b*x)^(3/2)*d*x - 3/32*sqrt(c*x^2 + b*x)*b^2*d*x/c + 3/64*sqrt
(c*x^2 + b*x)*b^3*e*x/c^2 - 1/8*(c*x^2 + b*x)^(3/2)*b*e*x/c + 3/128*b^4*d*
log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(5/2) - 3/256*b^5*e*log(2*c
*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(7/2) - 3/64*sqrt(c*x^2 + b*x)*b^3
*d/c^2 + 1/8*(c*x^2 + b*x)^(3/2)*b*d/c + 3/128*sqrt(c*x^2 + b*x)*b^4*e/c^3
 - 1/16*(c*x^2 + b*x)^(3/2)*b^2*e/c^2 + 1/5*(c*x^2 + b*x)^(5/2)*e/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.85

$$\int (d+ex)\left(bx+cx^2\right)^{3/2}\,dx = \frac{1}{640}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\,cex+\frac{10\,c^5d+11\,bc^4e}{c^4}\right)x+\frac{30\,bc^4d+b^2c^3e}{c^4}\right)x+\frac{5\left(2\,b^2c^3d-b\right.}{c^4}\right.\right. - \frac{3\left(2\,b^4cd-b^5e\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{256\,c^{\frac{7}{2}}}$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
1/640*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*c*e*x + (10*c^5*d + 11*b*c^4*e)/c^4)*x
 + (30*b*c^4*d + b^2*c^3*e)/c^4)*x + 5*(2*b^2*c^3*d - b^3*c^2*e)/c^4)*x -
15*(2*b^3*c^2*d - b^4*c*e)/c^4) - 3/256*(2*b^4*c*d - b^5*e)*log(abs(2*(sqr
t(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.09

$$\int (d+ex)\left(bx+cx^2\right)^{3/2}\,dx=\frac{e\,{\left(c\,x^2+b\,x\right)}^{5/2}}{5\,c}-\frac{3\,b^2\,d\,\left(\sqrt{c\,x^2+b\,x}\,\left(\frac{x}{2}+\frac{b}{4\,c}\right)-\frac{b^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}\right)}{16\,c}+\frac{d\,{\left(c\,x^2+b\,x\right)}^{3/2}\,\left(\frac{b}{2}+c\,x\right)}{4\,c}-\frac{b\,e\,\left(\frac{x\,{\left(c\,x^2+b\,x\right)}^{3/2}}{4}+\frac{b\,{\left(c\,x^2+b\,x\right)}^{3/2}}{8\,c}-\frac{3\,b^2\,\left(\frac{\sqrt{c\,x^2+b\,x}\,\left(b+2\,c\,x\right)}{4\,c}-\frac{b^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}\right)}{16\,c}\right)}{2\,c}$$

input
```
int((b*x + c*x^2)^(3/2)*(d + e*x),x)
```

output
```
(e*(b*x + c*x^2)^(5/2))/(5*c) - (3*b^2*d*((b*x + c*x^2)^(1/2)*(x/2 + b/(4*
c)) - (b^2*log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/(8*c^(3/2))))/(
16*c) + (d*(b*x + c*x^2)^(3/2)*(b/2 + c*x))/(4*c) - (b*e*((x*(b*x + c*x^2)
^(3/2))/4 + (b*(b*x + c*x^2)^(3/2))/(8*c) - (3*b^2*(((b*x + c*x^2)^(1/2)*(
b + 2*c*x))/(4*c) - (b^2*log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/(
8*c^(3/2))))/(16*c)))/(2*c)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.14

$$\int (d+ex)\left(bx+cx^2\right)^{3/2}\,dx = \frac{15\sqrt{x}\,\sqrt{cx+b}\,b^4ce-30\sqrt{x}\,\sqrt{cx+b}\,b^3c^2d-10\sqrt{x}\,\sqrt{cx+b}\,b^3c^2ex+20\sqrt{x}\,\sqrt{cx+b}\,b^2c^3dx -}{}$$

input
```
int((e*x+d)*(c*x^2+b*x)^(3/2),x)
```

output
```
(15*sqrt(x)*sqrt(b + c*x)*b**4*c*e - 30*sqrt(x)*sqrt(b + c*x)*b**3*c**2*d
- 10*sqrt(x)*sqrt(b + c*x)*b**3*c**2*e*x + 20*sqrt(x)*sqrt(b + c*x)*b**2*c
**3*d*x + 8*sqrt(x)*sqrt(b + c*x)*b**2*c**3*e*x**2 + 240*sqrt(x)*sqrt(b +
c*x)*b*c**4*d*x**2 + 176*sqrt(x)*sqrt(b + c*x)*b*c**4*e*x**3 + 160*sqrt(x)
*sqrt(b + c*x)*c**5*d*x**3 + 128*sqrt(x)*sqrt(b + c*x)*c**5*e*x**4 - 15*sq
rt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**5*e + 30*sqrt(c)*l
og((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**4*c*d)/(640*c**4)
```

3.142 $\int (bx+cx^2)^{3/2}\,dx$

Optimal result 1128
Mathematica [A] (verified) 1128
Rubi [A] (verified) 1129
Maple [A] (verified) 1130
Fricas [A] (verification not implemented) 1131
Sympy [A] (verification not implemented) 1132
Maxima [A] (verification not implemented) 1133
Giac [A] (verification not implemented) 1133
Mupad [B] (verification not implemented) 1134
Reduce [B] (verification not implemented) 1134

Optimal result

Integrand size = 13, antiderivative size = 123

$$\int (bx+cx^2)^{3/2}\,dx=-\frac{3b^3\sqrt{bx+cx^2}}{64c^2}+\frac{b^2x\sqrt{bx+cx^2}}{32c}+\frac{3}{8}bx^2\sqrt{bx+cx^2}+\frac{1}{4}cx^3\sqrt{bx+cx^2}+\frac{3b^4\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{5/2}}$$

output
```
-3/64*b^3*(c*x^2+b*x)^(1/2)/c^2+1/32*b^2*x*(c*x^2+b*x)^(1/2)/c+3/8*b*x^2*(
c*x^2+b*x)^(1/2)+1/4*c*x^3*(c*x^2+b*x)^(1/2)+3/64*b^4*arctanh(c^(1/2)*x/(c
*x^2+b*x)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.87

$$\int (bx+cx^2)^{3/2}\,dx=\frac{\sqrt{x(b+cx)}\left(\sqrt{c}(-3b^3+2b^2cx+24bc^2x^2+16c^3x^3)+\frac{6b^4\text{arctanh}\left(\frac{\sqrt{c}\sqrt{x}}{-\sqrt{b}+\sqrt{b+cx}}\right)}{\sqrt{x}\sqrt{b+cx}}\right)}{64c^{5/2}}$$

input
```
Integrate[(b*x + c*x^2)^(3/2),x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[c]*(-3*b^3 + 2*b^2*c*x + 24*b*c^2*x^2 + 16*c^3*x^
3) + (6*b^4*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])])/(Sqrt[x
]*Sqrt[b + c*x])))/(64*c^(5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.79, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+cx^2)^{3/2}\,dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\int \sqrt{cx^2+bx}dx}{16c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}$$

$$\Big\downarrow \text{219}$$

$$\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}$$

input
```
Int[(b*x + c*x^2)^(3/2),x]
```

output
```
((b + 2*c*x)*(b*x + c*x^2)^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x)*Sqrt[b*x +
c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))
/(16*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\frac{\frac{3 \operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right) b^{4}}{64}-\frac{3\left(\sqrt{c}\, b^{3}-\frac{2c^{\frac{3}{2}} b^{2} x}{3}-8c^{\frac{5}{2}} b\, x^{2}-\frac{16c^{\frac{7}{2}} x^{3}}{3}\right)\sqrt{x(cx+b)}}{64}}{c^{\frac{5}{2}}}$	73
risch	$-\frac{\left(-16c^{3}x^{3}-24b\,c^{2}x^{2}-2b^{2}cx+3b^{3}\right)x(cx+b)}{64c^{2}\sqrt{x(cx+b)}}+\frac{3b^{4}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{128c^{\frac{5}{2}}}$	84
default	$\frac{(2cx+b)\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{8c}-\frac{3b^{2}\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx}}{4c}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}$	87

input
```
int((c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/64/c^(5/2)*(arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))*b^4-(c^(1/2)*b^3-2/3*c^
(3/2)*b^2*x-8*c^(5/2)*b*x^2-16/3*c^(7/2)*x^3)*(x*(c*x+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.39

$$\int (bx+cx^2)^{3/2}\,dx = \left[\frac{3\,b^4\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,(16\,c^4x^3+24\,bc^3x^2+2\,b^2c^2x-3\,b^3c)\sqrt{cx^2+bx}}{128\,c^3}, -\frac{3\,b^4\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(16\,c^4x^3+24\,bc^3x^2+2\,b^2c^2x-3\,b^3c)\sqrt{cx^2+bx}}{64\,c^3}\right]$$

input
```
integrate((c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[1/128*(3*b^4*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(16
*c^4*x^3 + 24*b*c^3*x^2 + 2*b^2*c^2*x - 3*b^3*c)*sqrt(c*x^2 + b*x))/c^3, -
1/64*(3*b^4*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) - (16*c^
4*x^3 + 24*b*c^3*x^2 + 2*b^2*c^2*x - 3*b^3*c)*sqrt(c*x^2 + b*x))/c^3]
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.09

$$\int (bx+cx^2)^{3/2}\,dx = b\left(\begin{cases} \frac{b^3\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{16c^2} + \sqrt{bx+cx^2}\left(-\frac{b^2}{8c^2}+\frac{bx}{12c}+\frac{x^2}{3}\right) & \text{for } c\neq 0 \\ \frac{2(bx)^{\frac{5}{2}}}{5b^2} & \text{for } b\neq 0 \\ 0 & \text{otherwise} \end{cases}\right) + c\left(\begin{cases} -\frac{5b^4\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{128c^3} + \sqrt{bx+cx^2}\cdot\left(\frac{5b^3}{64c^3}-\frac{5b^2x}{96c^2}+\frac{bx^2}{24c}+\frac{x^3}{4}\right) & \text{for } c\neq 0 \\ \frac{2(bx)^{\frac{7}{2}}}{7b^3} & \text{for } b\neq 0 \\ 0 & \text{otherwise} \end{cases}\right)$$

input
```
integrate((c*x**2+b*x)**(3/2),x)
```

output
```
b*Piecewise((b**3*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)
/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c)
+ x)**2), True))/(16*c**2) + sqrt(b*x + c*x**2)*(-b**2/(8*c**2) + b*x/(12*
c) + x**2/3), Ne(c, 0)), (2*(b*x)**(5/2)/(5*b**2), Ne(b, 0)), (0, True)) +
 c*Piecewise((-5*b**4*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*
c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2
*c) + x)**2), True))/(128*c**3) + sqrt(b*x + c*x**2)*(5*b**3/(64*c**3) - 5
*b**2*x/(96*c**2) + b*x**2/(24*c) + x**3/4), Ne(c, 0)), (2*(b*x)**(7/2)/(7
*b**3), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.83

$$\int \left(bx+cx^2\right)^{3/2}\,dx = \frac{1}{4}\left(cx^2+bx\right)^{\frac{3}{2}}x - \frac{3\sqrt{cx^2+bx}b^2x}{32\,c} + \frac{3\,b^4\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{128\,c^{\frac{5}{2}}} - \frac{3\,\sqrt{cx^2+bx}b^3}{64\,c^2} + \frac{\left(cx^2+bx\right)^{\frac{3}{2}}b}{8\,c}$$

input
```
integrate((c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(c*x^2 + b*x)^(3/2)*x - 3/32*sqrt(c*x^2 + b*x)*b^2*x/c + 3/128*b^4*log
(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(5/2) - 3/64*sqrt(c*x^2 + b*x)
*b^3/c^2 + 1/8*(c*x^2 + b*x)^(3/2)*b/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.66

$$\int \left(bx+cx^2\right)^{3/2}\,dx = -\frac{3\,b^4\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{5}{2}}} + \frac{1}{64}\sqrt{cx^2+bx}\left(2\left(4\,(2\,cx+3\,b)x+\frac{b^2}{c}\right)x-\frac{3\,b^3}{c^2}\right)$$

input
```
integrate((c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
-3/128*b^4*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(5/2)
 + 1/64*sqrt(c*x^2 + b*x)*(2*(4*(2*c*x + 3*b)*x + b^2/c)*x - 3*b^3/c^2)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

$$\int \left(bx+cx^2\right)^{3/2}\,dx=\frac{\left(c\,x^2+b\,x\right)^{3/2}\,\left(\frac{b}{2}+c\,x\right)}{4\,c}-\frac{3\,b^2\,\left(\sqrt{c\,x^2+b\,x}\,\left(\frac{x}{2}+\frac{b}{4\,c}\right)-\frac{b^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}\right)}{16\,c}$$

input
```
int((b*x + c*x^2)^(3/2),x)
```

output
```
((b*x + c*x^2)^(3/2)*(b/2 + c*x))/(4*c) - (3*b^2*((b*x + c*x^2)^(1/2)*(x/2
 + b/(4*c)) - (b^2*log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/(8*c^(3
/2))))/(16*c)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \left(bx+cx^2\right)^{3/2}\,dx=\frac{-3\sqrt{x}\,\sqrt{cx+b}\,b^3c+2\sqrt{x}\,\sqrt{cx+b}\,b^2c^2x+24\sqrt{x}\,\sqrt{cx+b}\,b\,c^3x^2+16\sqrt{x}\,\sqrt{cx+b}\,c^4x^3+3\sqrt{}}{64c^3}$$

input
```
int((c*x^2+b*x)^(3/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(b + c*x)*b**3*c + 2*sqrt(x)*sqrt(b + c*x)*b**2*c**2*x +
 24*sqrt(x)*sqrt(b + c*x)*b*c**3*x**2 + 16*sqrt(x)*sqrt(b + c*x)*c**4*x**3
 + 3*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**4)/(64*c**3
)
```

3.143 $\int \frac{(bx+cx^2)^{3/2}}{d+ex} \, dx$

Optimal result 1135
Mathematica [C] (verified) 1136
Rubi [A] (verified) 1136
Maple [A] (verified) 1140
Fricas [A] (verification not implemented) 1140
Sympy [F] 1141
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Giac [F(-2)] 1142
Mupad [F(-1)] 1142
Reduce [B] (verification not implemented) 1143

Optimal result

Integrand size = 21, antiderivative size = 214

$$\int \frac{(bx+cx^2)^{3/2}}{d+ex} \, dx = -\frac{\left(10bd-\frac{8cd^2}{e}-\frac{b^2e}{c}\right)\sqrt{bx+cx^2}}{8e^2} - \frac{(2cd-be)x\sqrt{bx+cx^2}}{4e^2} + \frac{(bx+cx^2)^{3/2}}{3e} - \frac{(2cd-be)\left(8c^2d^2-8bcde-b^2e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{3/2}e^4} + \frac{2d^{3/2}(cd-be)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{cd-bex}}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{e^4}$$

output

```
-1/8*(10*b*d-8*c*d^2/e-b^2*e/c)*(c*x^2+b*x)^(1/2)/e^2-1/4*(-b*e+2*c*d)*x*(
c*x^2+b*x)^(1/2)/e^2+1/3*(c*x^2+b*x)^(3/2)/e-1/8*(-b*e+2*c*d)*(-b^2*e^2-8*
b*c*d*e+8*c^2*d^2)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(3/2)/e^4+2*d^(3
/2)*(-b*e+c*d)^(3/2)*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(1/2))
/e^4
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.85 (sec) , antiderivative size = 500, normalized size of antiderivative = 2.34

$$\int \frac{(bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{(x(b+cx))^{3/2}\left(\sqrt{c}e\sqrt{x}\sqrt{b+cx}(3b^2e^2+2bce(-15d+7ex)+4c^2(6d^2-3dex+2e^2x\right.}{}$$

input
```
Integrate[(b*x + c*x^2)^(3/2)/(d + e*x),x]
```

output
```
((x*(b + c*x))^(3/2)*(Sqrt[c]*e*Sqrt[x]*Sqrt[b + c*x]*(3*b^2*e^2 + 2*b*c*e
*(-15*d + 7*e*x) + 4*c^2*(6*d^2 - 3*d*e*x + 2*e^2*x^2)) + 48*Sqrt[c]*Sqrt[
d]*(c*d - b*e)*(c*d - b*e - I*Sqrt[b]*Sqrt[e]*Sqrt[c*d - b*e])*Sqrt[-(c*d)
 + 2*b*e - (2*I)*Sqrt[b]*Sqrt[e]*Sqrt[c*d - b*e]]*ArcTan[(Sqrt[-(c*d) + 2*
b*e - (2*I)*Sqrt[b]*Sqrt[e]*Sqrt[c*d - b*e]]*Sqrt[x])/(Sqrt[d]*(Sqrt[b] -
Sqrt[b + c*x]))] + 48*Sqrt[c]*Sqrt[d]*(c*d - b*e)*(c*d - b*e + I*Sqrt[b]*S
qrt[e]*Sqrt[c*d - b*e])*Sqrt[-(c*d) + 2*b*e + (2*I)*Sqrt[b]*Sqrt[e]*Sqrt[c
*d - b*e]]*ArcTan[(Sqrt[-(c*d) + 2*b*e + (2*I)*Sqrt[b]*Sqrt[e]*Sqrt[c*d -
b*e]]*Sqrt[x])/(Sqrt[d]*(Sqrt[b] - Sqrt[b + c*x]))] + 6*(16*c^3*d^3 - 24*b
*c^2*d^2*e + 6*b^2*c*d*e^2 + b^3*e^3)*ArcTanh[(Sqrt[c]*Sqrt[x])/(Sqrt[b] -
 Sqrt[b + c*x])]))/(24*c^(3/2)*e^4*x^(3/2)*(b + c*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1162, 1231, 27, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^{3/2}}{d+ex}\,dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{\left(bx+cx^2\right)^{3/2}}{3e}-\frac{\int \frac{(bd+(2cd-be)x)\sqrt{cx^2+bx}}{d+ex}dx}{2e}$$

$$\Big\downarrow \text{1231}$$

$$\frac{\left(bx+cx^2\right)^{3/2}}{3e}-\frac{-\frac{\int -\frac{bd\left(8c^2d^2-10bced+b^2e^2\right)+(2cd-be)\left(8c^2d^2-8bced-b^2e^2\right)x}{2(d+ex)\sqrt{cx^2+bx}}dx}{4ce^2}-\frac{\sqrt{bx+cx^2}\left(b^2e^2-2cex(2cd-be)-10bcde+8c^2d^2\right)}{4ce^2}}{2e}$$

$$\Big\downarrow \text{27}$$

$$\frac{\left(bx+cx^2\right)^{3/2}}{3e}-\frac{\frac{\int \frac{bd\left(8c^2d^2-10bced+b^2e^2\right)+(2cd-be)\left(8c^2d^2-8bced-b^2e^2\right)x}{(d+ex)\sqrt{cx^2+bx}}dx}{8ce^2}-\frac{\sqrt{bx+cx^2}\left(b^2e^2-2cex(2cd-be)-10bcde+8c^2d^2\right)}{4ce^2}}{2e}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\left(bx+cx^2\right)^{3/2}}{3e}-\frac{\frac{\frac{(2cd-be)\left(-b^2e^2-8bcde+8c^2d^2\right)\int \frac{1}{\sqrt{cx^2+bx}}dx}{e}-\frac{16cd^2(cd-be)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{8ce^2}-\frac{\sqrt{bx+cx^2}\left(b^2e^2-2cex(2cd-be)-10bcde+8c^2d^2\right)}{4ce^2}}{2e}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\left(bx+cx^2\right)^{3/2}}{3e}-\frac{\frac{\frac{2(2cd-be)\left(-b^2e^2-8bcde+8c^2d^2\right)\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{e}-\frac{16cd^2(cd-be)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{8ce^2}-\frac{\sqrt{bx+cx^2}\left(b^2e^2-2cex(2cd-be)-10bcde+8c^2d^2\right)}{4ce^2}}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(bx+cx^2\right)^{3/2}}{3e}-\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)\left(-b^2e^2-8bcde+8c^2d^2\right)}{\sqrt{c}e}-\frac{16cd^2(cd-be)^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{8ce^2}-\frac{\sqrt{bx+cx^2}\left(b^2e^2-2cex(2cd-be)-10bcde+8c^2d^2\right)}{4ce^2}}{2e}$$

$$\Big\downarrow \text{1154}$$

$$\frac{\left(bx+cx^2\right)^{3/2}}{3e}-\frac{\frac{\frac{32cd^2(cd-be)^2\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{e}+\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)\left(-b^2e^2-8bcde+8c^2d^2\right)}{\sqrt{c}e}}{8ce^2}-\frac{\sqrt{bx+cx^2}\left(b^2e^2-2cex\right.}{}}{2e}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(bx+cx^2\right)^{3/2}}{3e}-\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)\left(-b^2e^2-8bcde+8c^2d^2\right)}{\sqrt{c}e}-\frac{16cd^{3/2}(cd-be)^{3/2}\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{e}}{8ce^2}-\frac{\sqrt{bx+cx^2}\left(b^2e^2-2cex(2cd-be)-\right.}{4ce^2}}{2e}$$

input
```
Int[(b*x + c*x^2)^(3/2)/(d + e*x),x]
```

output
```
(b*x + c*x^2)^(3/2)/(3*e) - (-1/4*((8*c^2*d^2 - 10*b*c*d*e + b^2*e^2 - 2*c
*e*(2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(c*e^2) + ((2*(2*c*d - b*e)*(8*c^2*
d^2 - 8*b*c*d*e - b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(Sqrt[c
]*e) - (16*c*d^(3/2)*(c*d - b*e)^(3/2)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*
Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/e)/(8*c*e^2))/(2*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$-\frac{-\frac{e\sqrt{x(cx+b)}\left(8c^2e^2x^2+14e^2xbc-12c^2dex+3b^2e^2-30bcde+24c^2d^2\right)}{24c}+\frac{\left(b^3e^3+6d\,e^2b^2c-24d^2eb\,c^2+16d^3c^3\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{8c^{\frac{3}{2}}}}{e^4}$
risch	$\frac{(8c^2e^2x^2+14e^2xbc-12c^2dex+3b^2e^2-30bcde+24c^2d^2)x(cx+b)}{24c\,e^3\sqrt{x(cx+b)}}-\frac{\frac{\left(b^3e^3+6d\,e^2b^2c-24d^2eb\,c^2+16d^3c^3\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+b}\right)}{e\sqrt{c}}}{}$
default	$\frac{\left(c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}\right)^{\frac{3}{2}}}{3}+\frac{(be-2cd)\left(\frac{\left(2c\left(x+\frac{d}{e}\right)+\frac{be-2cd}{e}\right)\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}}}{4c}+\frac{\left(-\frac{4cd}{}\right.}{}\right.}{}$

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/e^4*(-1/24*e*(x*(c*x+b))^(1/2)*(8*c^2*e^2*x^2+14*b*c*e^2*x-12*c^2*d*e*x
+3*b^2*e^2-30*b*c*d*e+24*c^2*d^2)/c+1/8*(b^3*e^3+6*b^2*c*d*e^2-24*b*c^2*d^
2*e+16*c^3*d^3)/c^(3/2)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))+2*(b*e-c*d)^2
*d^2/(d*(b*e-c*d))^(1/2)*arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))^(1/2))
)
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 896, normalized size of antiderivative = 4.19

$$\int \frac{(bx+cx^2)^{3/2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[1/48*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*b^2*c*d*e^2 + b^3*e^3)*sqrt(c)*l
og(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 48*(c^3*d^2 - b*c^2*d*e)*sqr
t(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x - 2*sqrt(c*d^2 - b*d*e)*sqrt(c
*x^2 + b*x))/(e*x + d)) + 2*(8*c^3*e^3*x^2 + 24*c^3*d^2*e - 30*b*c^2*d*e^2
 + 3*b^2*c*e^3 - 2*(6*c^3*d*e^2 - 7*b*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/(c^2*
e^4), -1/48*(96*(c^3*d^2 - b*c^2*d*e)*sqrt(-c*d^2 + b*d*e)*arctan(sqrt(-c*
d^2 + b*d*e)*sqrt(c*x^2 + b*x)/(c*d*x + b*d)) - 3*(16*c^3*d^3 - 24*b*c^2*d
^2*e + 6*b^2*c*d*e^2 + b^3*e^3)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x
)*sqrt(c)) - 2*(8*c^3*e^3*x^2 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + 3*b^2*c*e^
3 - 2*(6*c^3*d*e^2 - 7*b*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/(c^2*e^4), 1/24*(3
*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*b^2*c*d*e^2 + b^3*e^3)*sqrt(-c)*arctan(s
qrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) - 24*(c^3*d^2 - b*c^2*d*e)*sqrt(c*d^2
 - b*d*e)*log((b*d + (2*c*d - b*e)*x - 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 +
b*x))/(e*x + d)) + (8*c^3*e^3*x^2 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + 3*b^2*
c*e^3 - 2*(6*c^3*d*e^2 - 7*b*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/(c^2*e^4), -1/
24*(48*(c^3*d^2 - b*c^2*d*e)*sqrt(-c*d^2 + b*d*e)*arctan(sqrt(-c*d^2 + b*d
*e)*sqrt(c*x^2 + b*x)/(c*d*x + b*d)) - 3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*
b^2*c*d*e^2 + b^3*e^3)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b
)) - (8*c^3*e^3*x^2 + 24*c^3*d^2*e - 30*b*c^2*d*e^2 + 3*b^2*c*e^3 - 2*(6*c
^3*d*e^2 - 7*b*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/(c^2*e^4)]
```

Sympy [F]

$$\int \frac{(bx + cx^2)^{3/2}}{d + ex}\, dx = \int \frac{(x(b + cx))^{\frac{3}{2}}}{d + ex}\, dx$$

input
```
integrate((c*x**2+b*x)**(3/2)/(e*x+d),x)
```

output
```
Integral((x*(b + c*x))**(3/2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(bx + cx^2)^{3/2}}{d + ex} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(bx + cx^2)^{3/2}}{d + ex} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx + cx^2)^{3/2}}{d + ex} \, dx = \int \frac{(c\,x^2 + b\,x)^{3/2}}{d + e\,x} \, dx$$

input
```
int((b*x + c*x^2)^(3/2)/(d + e*x),x)
```

output
```
int((b*x + c*x^2)^(3/2)/(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.13

$$\int \frac{(bx+cx^2)^{3/2}}{d+ex}\,dx = \frac{-48\sqrt{d}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{be-cd}-\sqrt{e}\,\sqrt{cx+b}-\sqrt{x}\,\sqrt{e}\,\sqrt{c}}{\sqrt{d}\,\sqrt{c}}\right)b\,c^2de+48\sqrt{d}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{be-}}{}\right.}{}$$

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d),x)
```

output
```
( - 48*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*
x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b*c**2*d*e + 48*sqrt(d)*s
qrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqr
t(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*c**3*d**2 - 48*sqrt(d)*sqrt(b*e - c*d)*at
an((sqrt(b*e - c*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sq
rt(d)*sqrt(c)))*b*c**2*d*e + 48*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c
*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*
c**3*d**2 + 3*sqrt(x)*sqrt(b + c*x)*b**2*c*e**3 - 30*sqrt(x)*sqrt(b + c*x)
*b*c**2*d*e**2 + 14*sqrt(x)*sqrt(b + c*x)*b*c**2*e**3*x + 24*sqrt(x)*sqrt(
b + c*x)*c**3*d**2*e - 12*sqrt(x)*sqrt(b + c*x)*c**3*d*e**2*x + 8*sqrt(x)*
sqrt(b + c*x)*c**3*e**3*x**2 - 3*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt
(c))/sqrt(b))*b**3*e**3 - 18*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))
/sqrt(b))*b**2*c*d*e**2 + 72*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))
/sqrt(b))*b*c**2*d**2*e - 48*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))
/sqrt(b))*c**3*d**3)/(24*c**2*e**4)
```

3.144 $\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^2} dx$

Optimal result 1144
Mathematica [A] (verified) 1145
Rubi [A] (verified) 1145
Maple [A] (verified) 1149
Fricas [A] (verification not implemented) 1149
Sympy [F] 1150
Maxima [F(-2)] 1151
Giac [F(-1)] 1151
Mupad [F(-1)] 1151
Reduce [B] (verification not implemented) 1152

Optimal result

Integrand size = 21, antiderivative size = 196

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^2} dx = -\frac{3(4cd-3be)\sqrt{bx+cx^2}}{4e^3} + \frac{3cx\sqrt{bx+cx^2}}{2e^2} - \frac{(bx+cx^2)^{3/2}}{e(d+ex)} + \frac{3(8c^2d^2-8bcde+b^2e^2)\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4\sqrt{c}e^4} - \frac{3\sqrt{d}\sqrt{cd-be}(2cd-be)\text{arctanh}\left(\frac{\sqrt{cd-bex}}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{e^4}$$

output
```
-3/4*(-3*b*e+4*c*d)*(c*x^2+b*x)^(1/2)/e^3+3/2*c*x*(c*x^2+b*x)^(1/2)/e^2-(c
*x^2+b*x)^(3/2)/e/(e*x+d)+3/4*(b^2*e^2-8*b*c*d*e+8*c^2*d^2)*arctanh(c^(1/2
)*x/(c*x^2+b*x)^(1/2))/c^(1/2)/e^4-3*d^(1/2)*(-b*e+c*d)^(1/2)*(-b*e+2*c*d)
*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(1/2))/e^4
```

Mathematica [A] (verified)

Time = 10.57 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.05

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^2}\,dx = \frac{\sqrt{x(b+cx)}\left(\frac{e\sqrt{x}(be(9d+5ex)-2c(6d^2+3dex-e^2x^2))}{d+ex}+\frac{3(8c^2d^2-8bcde+b^2e^2)\text{arcsinh}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{b}\sqrt{c}\sqrt{1+\frac{cx}{b}}}-\frac{12\sqrt{}}{}\right.}{4e^4\sqrt{x}}$$

input
```
Integrate[(b*x + c*x^2)^(3/2)/(d + e*x)^2,x]
```

output
```
(Sqrt[x*(b + c*x)]*((e*Sqrt[x]*(b*e*(9*d + 5*e*x) - 2*c*(6*d^2 + 3*d*e*x -
 e^2*x^2)))/(d + e*x) + (3*(8*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*ArcSinh[(Sqrt
[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[b]*Sqrt[c]*Sqrt[1 + (c*x)/b]) - (12*Sqrt[d]*S
qrt[c*d - b*e]*(2*c*d - b*e)*ArcTanh[(Sqrt[c*d - b*e]*Sqrt[x])/(Sqrt[d]*Sq
rt[b + c*x])])/Sqrt[b + c*x]))/(4*e^4*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1161, 1231, 25, 27, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^2}dx$$

$$\Big\downarrow \; 1161$$

$$\frac{3\int \frac{(b+2cx)\sqrt{cx^2+bx}}{d+ex}dx}{2e}-\frac{(bx+cx^2)^{3/2}}{e(d+ex)}$$

$$\Big\downarrow \; 1231$$

$$\frac{3\left(-\frac{\int -\frac{c\left(bd(4cd-3be)+\left(8c^2d^2-8bced+b^2e^2\right)x\right)}{(d+ex)\sqrt{cx^2+bx}}dx}{4ce^2}-\frac{\sqrt{bx+cx^2}(-3be+4cd-2cex)}{2e^2}\right)}{2e}-\frac{(bx+cx^2)^{3/2}}{e(d+ex)}$$

$$\Big\downarrow \; 25$$

$$\frac{3\left(\frac{\int \frac{c\left(bd(4cd-3be)+\left(8c^2d^2-8bced+b^2e^2\right)x\right)}{(d+ex)\sqrt{cx^2+bx}}dx}{4ce^2}-\frac{\sqrt{bx+cx^2}(-3be+4cd-2cex)}{2e^2}\right)}{2e}-\frac{\left(bx+cx^2\right)^{3/2}}{e(d+ex)}$$

$$\Big\downarrow \; 27$$

$$\frac{3\left(\frac{\int \frac{bd(4cd-3be)+\left(8c^2d^2-8bced+b^2e^2\right)x}{(d+ex)\sqrt{cx^2+bx}}dx}{4e^2}-\frac{\sqrt{bx+cx^2}(-3be+4cd-2cex)}{2e^2}\right)}{2e}-\frac{\left(bx+cx^2\right)^{3/2}}{e(d+ex)}$$

$$\Big\downarrow \; 1269$$

$$\frac{3\left(\frac{\frac{\left(b^2e^2-8bcde+8c^2d^2\right)\int \frac{1}{\sqrt{cx^2+bx}}dx}{e}-\frac{4d(cd-be)(2cd-be)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{4e^2}-\frac{\sqrt{bx+cx^2}(-3be+4cd-2cex)}{2e^2}\right)}{2e}-\frac{\left(bx+cx^2\right)^{3/2}}{e(d+ex)}$$

$$\Big\downarrow \; 1091$$

$$\frac{3\left(\frac{\frac{2\left(b^2e^2-8bcde+8c^2d^2\right)\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{e}-\frac{4d(cd-be)(2cd-be)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{4e^2}-\frac{\sqrt{bx+cx^2}(-3be+4cd-2cex)}{2e^2}\right)}{2e}-\frac{\left(bx+cx^2\right)^{3/2}}{e(d+ex)}$$

$$\Big\downarrow \; 219$$

$$\frac{3\left(\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)\left(b^2e^2-8bcde+8c^2d^2\right)}{\sqrt{c}e}-\frac{4d(cd-be)(2cd-be)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{4e^2}-\frac{\sqrt{bx+cx^2}(-3be+4cd-2cex)}{2e^2}\right)}{2e}-\frac{\left(bx+cx^2\right)^{3/2}}{e(d+ex)}$$

$$\Big\downarrow \; 1154$$

$$\frac{3\left(\frac{\frac{8d(cd-be)(2cd-be)\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{e}+\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)\left(b^2e^2-8bcde+8c^2d^2\right)}{\sqrt{c}e}}{4e^2}-\frac{\sqrt{bx+cx^2}(-3be+4cd-2cex)}{2e^2}\right)}{2e}-\frac{\left(bx+cx^2\right)^{3/2}}{e(d+ex)}$$

$$\Big\downarrow \; 219$$

$$\frac{3\left(\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)\left(b^2e^2-8bcde+8c^2d^2\right)}{\sqrt{c}e}-\frac{4\sqrt{d}\sqrt{cd-be}(2cd-be)\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{e}}{4e^2}-\frac{\sqrt{bx+cx^2}(-3be+4cd-2cex)}{2e^2}\right)}{2e}-\frac{\left(bx+cx^2\right)^{3/2}}{e(d+ex)}$$

input
```
Int[(b*x + c*x^2)^(3/2)/(d + e*x)^2,x]
```

output
```
-((b*x + c*x^2)^(3/2)/(e*(d + e*x))) + (3*(-1/2*((4*c*d - 3*b*e - 2*c*e*x)
*Sqrt[b*x + c*x^2])/e^2 + ((2*(8*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*ArcTanh[(S
qrt[c]*x)/Sqrt[b*x + c*x^2]])/(Sqrt[c]*e) - (4*Sqrt[d]*Sqrt[c*d - b*e]*(2*
c*d - b*e)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt
[b*x + c*x^2])])/e)/(4*e^2)))/(2*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$\frac{6(ex+d)(-be+cd)\sqrt{c}\left(cd-\frac{be}{2}\right)d\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)-3\sqrt{d(be-cd)}\left(-\frac{\left(b^2e^2-8bcde+8c^2d^2\right)(ex+d)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{4}+\right.}{\sqrt{c}\,e^4(ex+d)\sqrt{d(be-cd)}}$
risch	$\frac{(2cex+5be-8cd)x(cx+b)}{4e^3\sqrt{x(cx+b)}}+\frac{\frac{3\left(b^2e^2-8bcde+8c^2d^2\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{e\sqrt{c}}+\frac{16d\left(b^2e^2-3bcde+2c^2d^2\right)\ln\left(-\frac{2d(be-cd)}{e^2}+\frac{(be-2\ldots}{\ldots}\right.}{\ldots}}{\ldots}$
default	Expression too large to display

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
6/c^(1/2)/(d*(b*e-c*d))^(1/2)*((e*x+d)*(-b*e+c*d)*c^(1/2)*(c*d-1/2*b*e)*d*
arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))^(1/2))-1/2*(d*(b*e-c*d))^(1/2)*
(-1/4*(b^2*e^2-8*b*c*d*e+8*c^2*d^2)*(e*x+d)*arctanh((x*(c*x+b))^(1/2)/x/c^
(1/2))+e*c^(1/2)*(x*(c*x+b))^(1/2)*(c*d^2-3/4*e*(-2/3*c*x+b)*d-5/12*e^2*x*
(2/5*c*x+b))))/e^4/(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 1008, normalized size of antiderivative = 5.14

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[1/8*(3*(8*c^2*d^3 - 8*b*c*d^2*e + b^2*d*e^2 + (8*c^2*d^2*e - 8*b*c*d*e^2
+ b^2*e^3)*x)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 12*(2
*c^2*d^2 - b*c*d*e + (2*c^2*d*e - b*c*e^2)*x)*sqrt(c*d^2 - b*d*e)*log((b*d
 + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) +
 2*(2*c^2*e^3*x^2 - 12*c^2*d^2*e + 9*b*c*d*e^2 - (6*c^2*d*e^2 - 5*b*c*e^3)
*x)*sqrt(c*x^2 + b*x))/(c*e^5*x + c*d*e^4), 1/8*(24*(2*c^2*d^2 - b*c*d*e +
 (2*c^2*d*e - b*c*e^2)*x)*sqrt(-c*d^2 + b*d*e)*arctan(sqrt(-c*d^2 + b*d*e)
*sqrt(c*x^2 + b*x)/(c*d*x + b*d)) + 3*(8*c^2*d^3 - 8*b*c*d^2*e + b^2*d*e^2
 + (8*c^2*d^2*e - 8*b*c*d*e^2 + b^2*e^3)*x)*sqrt(c)*log(2*c*x + b + 2*sqrt
(c*x^2 + b*x)*sqrt(c)) + 2*(2*c^2*e^3*x^2 - 12*c^2*d^2*e + 9*b*c*d*e^2 - (
6*c^2*d*e^2 - 5*b*c*e^3)*x)*sqrt(c*x^2 + b*x))/(c*e^5*x + c*d*e^4), -1/4*(
3*(8*c^2*d^3 - 8*b*c*d^2*e + b^2*d*e^2 + (8*c^2*d^2*e - 8*b*c*d*e^2 + b^2*
e^3)*x)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + 6*(2*c^2*d
^2 - b*c*d*e + (2*c^2*d*e - b*c*e^2)*x)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*
c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) - (2*c^
2*e^3*x^2 - 12*c^2*d^2*e + 9*b*c*d*e^2 - (6*c^2*d*e^2 - 5*b*c*e^3)*x)*sqrt
(c*x^2 + b*x))/(c*e^5*x + c*d*e^4), 1/4*(12*(2*c^2*d^2 - b*c*d*e + (2*c^2*
d*e - b*c*e^2)*x)*sqrt(-c*d^2 + b*d*e)*arctan(sqrt(-c*d^2 + b*d*e)*sqrt(c*
x^2 + b*x)/(c*d*x + b*d)) - 3*(8*c^2*d^3 - 8*b*c*d^2*e + b^2*d*e^2 + (8*c^
2*d^2*e - 8*b*c*d*e^2 + b^2*e^3)*x)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*s...
```

Sympy [F]

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^2}\,dx = \int \frac{(x(b+cx))^{\frac{3}{2}}}{(d+ex)^2}\,dx$$

input
```
integrate((c*x**2+b*x)**(3/2)/(e*x+d)**2,x)
```

output
```
Integral((x*(b + c*x))**(3/2)/(d + e*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^2}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^2}\,dx = \int \frac{\left(c\,x^2+b\,x\right)^{3/2}}{\left(d+e\,x\right)^2}\,dx$$

input
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^2,x)
```

output
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 719, normalized size of antiderivative = 3.67

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^2,x)
```

output
```
(12*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x)
- sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b*c*d*e + 12*sqrt(d)*sqrt(b*
e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*s
qrt(c))/(sqrt(d)*sqrt(c)))*b*c*e**2*x - 24*sqrt(d)*sqrt(b*e - c*d)*atan((s
qrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)
*sqrt(c)))*c**2*d**2 - 24*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) -
sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*c**2*d
*e*x + 12*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) + sqrt(e)*sqrt(b +
 c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b*c*d*e + 12*sqrt(d)*s
qrt(b*e - c*d)*atan((sqrt(b*e - c*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqr
t(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b*c*e**2*x - 24*sqrt(d)*sqrt(b*e - c*d)*a
tan((sqrt(b*e - c*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(s
qrt(d)*sqrt(c)))*c**2*d**2 - 24*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c
*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*
c**2*d*e*x + 9*sqrt(x)*sqrt(b + c*x)*b*c*d*e**2 + 5*sqrt(x)*sqrt(b + c*x)*
b*c*e**3*x - 12*sqrt(x)*sqrt(b + c*x)*c**2*d**2*e - 6*sqrt(x)*sqrt(b + c*x
)*c**2*d*e**2*x + 2*sqrt(x)*sqrt(b + c*x)*c**2*e**3*x**2 + 3*sqrt(c)*log((
sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**2*d*e**2 + 3*sqrt(c)*log((sqr
t(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**2*e**3*x - 24*sqrt(c)*log((sqrt(
b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b*c*d**2*e - 24*sqrt(c)*log((sqrt(...
```

3.145 $\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^3}\,dx$

Optimal result 1153
Mathematica [A] (verified) 1154
Rubi [A] (verified) 1154
Maple [A] (verified) 1157
Fricas [B] (verification not implemented) 1158
Sympy [F] 1159
Maxima [F(-2)] 1160
Giac [B] (verification not implemented) 1160
Mupad [F(-1)] 1161
Reduce [B] (verification not implemented) 1161

Optimal result

Integrand size = 21, antiderivative size = 219

$$\begin{aligned}\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^3}\,dx &= \frac{3(4cd-be)\sqrt{bx+cx^2}}{4de^3}-\frac{3(2cd-be)x\sqrt{bx+cx^2}}{4de^2(d+ex)} \\ &\quad -\frac{(bx+cx^2)^{3/2}}{2e(d+ex)^2}-\frac{3\sqrt{c}(2cd-be)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{e^4} \\ &\quad +\frac{3(8c^2d^2-8bcde+b^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{cd-be}x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{4\sqrt{d}e^4\sqrt{cd-be}}\end{aligned}$$

output
```
3/4*(-b*e+4*c*d)*(c*x^2+b*x)^(1/2)/d/e^3-3/4*(-b*e+2*c*d)*x*(c*x^2+b*x)^(1
/2)/d/e^2/(e*x+d)-1/2*(c*x^2+b*x)^(3/2)/e/(e*x+d)^2-3*c^(1/2)*(-b*e+2*c*d)
*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/e^4+3/4*(b^2*e^2-8*b*c*d*e+8*c^2*d^2
)*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(1/2))/d^(1/2)/e^4/(-b*e+
c*d)^(1/2)
```

Mathematica [A] (verified)

Time = 11.04 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.08

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^3}\,dx = \frac{\sqrt{x(b+cx)}\left(\frac{e(-cd+be)\sqrt{x}(-be(3d+5ex)+2c(6d^2+9dex+2e^2x^2))}{(d+ex)^2} + \frac{12\sqrt{c}(2c^2d^2-3bcde+b^2e^2)\operatorname{arcsinh}(}{\sqrt{b}\sqrt{1+\frac{cx}{b}}}\right.}{4e^4(-cd+be)\sqrt{x}}$$

input
```
Integrate[(b*x + c*x^2)^(3/2)/(d + e*x)^3,x]
```

output
```
(Sqrt[x*(b + c*x)]*((e*(-(c*d) + b*e)*Sqrt[x]*(-(b*e*(3*d + 5*e*x)) + 2*c*
(6*d^2 + 9*d*e*x + 2*e^2*x^2)))/(d + e*x)^2 + (12*Sqrt[c]*(2*c^2*d^2 - 3*b
*c*d*e + b^2*e^2)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[b]*Sqrt[1 + (c
*x)/b]) - (3*Sqrt[c*d - b*e]*(8*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*ArcTanh[(Sq
rt[c*d - b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])])/(Sqrt[d]*Sqrt[b + c*x]))
/(4*e^4*(-(c*d) + b*e)*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1161, 1230, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^3}dx$$

$$\Big\downarrow \ 1161$$

$$\frac{3\int \frac{(b+2cx)\sqrt{cx^2+bx}}{(d+ex)^2}dx}{4e} - \frac{(bx+cx^2)^{3/2}}{2e(d+ex)^2}$$

$$\Big\downarrow \ 1230$$

$$\frac{3\left(\frac{\sqrt{bx+cx^2}(-be+4cd+2cex)}{e^2(d+ex)} - \frac{\int \frac{b(4cd-be)+4c(2cd-be)x}{(d+ex)\sqrt{cx^2+bx}}dx}{2e^2}\right)}{4e} - \frac{(bx+cx^2)^{3/2}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{1269}$$

$$\frac{3\left(\frac{\sqrt{bx+cx^2}(-be+4cd+2cex)}{e^2(d+ex)}-\frac{\frac{4c(2cd-be)\int\frac{1}{\sqrt{cx^2+bx}}dx}{e}-\frac{\left(b^2e^2-8bcde+8c^2d^2\right)\int\frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{2e^2}\right)}{4e}-\frac{\left(bx+cx^2\right)^{3/2}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{3\left(\frac{\sqrt{bx+cx^2}(-be+4cd+2cex)}{e^2(d+ex)}-\frac{\frac{8c(2cd-be)\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{e}-\frac{\left(b^2e^2-8bcde+8c^2d^2\right)\int\frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{2e^2}\right)}{4e}-\frac{\left(bx+cx^2\right)^{3/2}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\left(\frac{\sqrt{bx+cx^2}(-be+4cd+2cex)}{e^2(d+ex)}-\frac{\frac{8\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)}{e}-\frac{\left(b^2e^2-8bcde+8c^2d^2\right)\int\frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{2e^2}\right)}{4e}-\frac{\left(bx+cx^2\right)^{3/2}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{3\left(\frac{\sqrt{bx+cx^2}(-be+4cd+2cex)}{e^2(d+ex)}-\frac{\frac{2\left(b^2e^2-8bcde+8c^2d^2\right)\int\frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{e}+\frac{8\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)}{e}}{2e^2}\right)}{4e}-\frac{\left(bx+cx^2\right)^{3/2}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\left(\frac{\sqrt{bx+cx^2}(-be+4cd+2cex)}{e^2(d+ex)}-\frac{\frac{8\sqrt{c}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)}{e}-\frac{(b^2e^2-8bcde+8c^2d^2)\operatorname{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{\sqrt{de}\sqrt{cd-be}}}{2e^2}\right)}{4e}-\frac{(bx+cx^2)^{3/2}}{2e(d+ex)^2}$$

input
```
Int[(b*x + c*x^2)^(3/2)/(d + e*x)^3,x]
```

output
```
-1/2*(b*x + c*x^2)^(3/2)/(e*(d + e*x)^2) + (3*(((4*c*d - b*e + 2*c*e*x)*Sq
rt[b*x + c*x^2])/(e^2*(d + e*x)) - ((8*Sqrt[c]*(2*c*d - b*e)*ArcTanh[(Sqrt
[c]*x)/Sqrt[b*x + c*x^2]])/e - ((8*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*ArcTanh[
(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(S
qrt[d]*e*Sqrt[c*d - b*e]))/(2*e^2)))/(4*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.01

method	result
pseudoelliptic	$-\frac{2\left(3(ex+d)^2x\sqrt{c}\left(c^2d^2-bcde+\frac{1}{8}b^2e^2\right)b\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)+\sqrt{d(be-cd)}\left(-\frac{3bcx(ex+d)^2(be-2cd)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{2}\right.\right.}{\sqrt{c}\,\sqrt{d(be-cd)}\,b\,e^4x(ex}$
risch	$\frac{x(cx+b)c}{e^3\sqrt{x(cx+b)}}+\frac{-\frac{2\left(b^2e^2-6bcde+6c^2d^2\right)\ln\left(\frac{-\frac{2d(be-cd)}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{-\frac{d(be-cd)}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(b}{}}}{x+\frac{d}{e}}\right.}{e^2\sqrt{-\frac{d(be-cd)}{e^2}}}}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/c^(1/2)*(3*(e*x+d)^2*x*c^(1/2)*(c^2*d^2-b*c*d*e+1/8*b^2*e^2)*b*arctan((
x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))^(1/2))+(d*(b*e-c*d))^(1/2)*(-3/2*b*c*x*
(e*x+d)^2*(b*e-2*c*d)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))+e*c^(1/2)*((-c*
d^2+3/8*b*d*e)*(x*(c*x+b))^(3/2)+5/8*(-4/5*c*(-2*c*x+b)*d^2-21/5*b*c*d*e*x
+b*e^2*x*(-4/5*c*x+b))*x*(x*(c*x+b))^(1/2))))/(d*(b*e-c*d))^(1/2)/b/e^4/x/
(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(189) = 378$.

Time = 0.14 (sec) , antiderivative size = 1745, normalized size of antiderivative = 7.97

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
[-1/8*(12*(2*c^2*d^5 - 3*b*c*d^4*e + b^2*d^3*e^2 + (2*c^2*d^3*e^2 - 3*b*c*
d^2*e^3 + b^2*d*e^4)*x^2 + 2*(2*c^2*d^4*e - 3*b*c*d^3*e^2 + b^2*d^2*e^3)*x
)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 3*(8*c^2*d^4 - 8*
b*c*d^3*e + b^2*d^2*e^2 + (8*c^2*d^2*e^2 - 8*b*c*d*e^3 + b^2*e^4)*x^2 + 2*
(8*c^2*d^3*e - 8*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqrt(c*d^2 - b*d*e)*log((b*d
+ (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) -
2*(12*c^2*d^4*e - 15*b*c*d^3*e^2 + 3*b^2*d^2*e^3 + 4*(c^2*d^2*e^3 - b*c*d*
e^4)*x^2 + (18*c^2*d^3*e^2 - 23*b*c*d^2*e^3 + 5*b^2*d*e^4)*x)*sqrt(c*x^2 +
 b*x))/(c*d^4*e^4 - b*d^3*e^5 + (c*d^2*e^6 - b*d*e^7)*x^2 + 2*(c*d^3*e^5 -
 b*d^2*e^6)*x), -1/4*(3*(8*c^2*d^4 - 8*b*c*d^3*e + b^2*d^2*e^2 + (8*c^2*d^
2*e^2 - 8*b*c*d*e^3 + b^2*e^4)*x^2 + 2*(8*c^2*d^3*e - 8*b*c*d^2*e^2 + b^2*
d*e^3)*x)*sqrt(-c*d^2 + b*d*e)*arctan(sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*
x)/(c*d*x + b*d)) + 6*(2*c^2*d^5 - 3*b*c*d^4*e + b^2*d^3*e^2 + (2*c^2*d^3*
e^2 - 3*b*c*d^2*e^3 + b^2*d*e^4)*x^2 + 2*(2*c^2*d^4*e - 3*b*c*d^3*e^2 + b^
2*d^2*e^3)*x)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - (12*c
^2*d^4*e - 15*b*c*d^3*e^2 + 3*b^2*d^2*e^3 + 4*(c^2*d^2*e^3 - b*c*d*e^4)*x^
2 + (18*c^2*d^3*e^2 - 23*b*c*d^2*e^3 + 5*b^2*d*e^4)*x)*sqrt(c*x^2 + b*x))/
(c*d^4*e^4 - b*d^3*e^5 + (c*d^2*e^6 - b*d*e^7)*x^2 + 2*(c*d^3*e^5 - b*d^2*
e^6)*x), 1/8*(24*(2*c^2*d^5 - 3*b*c*d^4*e + b^2*d^3*e^2 + (2*c^2*d^3*e^2 -
 3*b*c*d^2*e^3 + b^2*d*e^4)*x^2 + 2*(2*c^2*d^4*e - 3*b*c*d^3*e^2 + b^2*...
```

Sympy [F]

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^3}\,dx=\int \frac{(x(b+cx))^{\frac{3}{2}}}{(d+ex)^3}\,dx$$

input
```
integrate((c*x**2+b*x)**(3/2)/(e*x+d)**3,x)
```

output
```
Integral((x*(b + c*x))**(3/2)/(d + e*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(189) = 378$.

Time = 0.16 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.27

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^3}\,dx = \frac{\sqrt{cx^2+bx}c}{e^3} + \frac{3\,(8\,c^2d^2-8\,bcde+b^2e^2)\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde}}\right)}{4\,\sqrt{-cd^2+bde}e^4} + \frac{3\,(2\,c^2d-bce)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{2\,\sqrt{c}e^4} + \frac{24\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3c^2d^2e-24\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3bcde^2+5\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3b^2e^3+40\left(\sqrt{c}x-\right.}{}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^3,x, algorithm="giac")
```

output
```
sqrt(c*x^2 + b*x)*c/e^3 + 3/4*(8*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*arctan(-((
sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e))/(sqrt(
-c*d^2 + b*d*e)*e^4) + 3/2*(2*c^2*d - b*c*e)*log(abs(2*(sqrt(c)*x - sqrt(c
*x^2 + b*x))*sqrt(c) + b))/(sqrt(c)*e^4) + 1/4*(24*(sqrt(c)*x - sqrt(c*x^2
 + b*x))^3*c^2*d^2*e - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b*c*d*e^2 + 5*
(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^2*e^3 + 40*(sqrt(c)*x - sqrt(c*x^2 + b
*x))^2*c^(5/2)*d^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b*c^(3/2)*d^2*e
- (sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^2*sqrt(c)*d*e^2 + 40*(sqrt(c)*x - sq
rt(c*x^2 + b*x))*b*c^2*d^3 - 28*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^2*c*d^2*
e + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^3*d*e^2 + 10*b^2*c^(3/2)*d^3 - 5*b
^3*sqrt(c)*d^2*e)/(((sqrt(c)*x - sqrt(c*x^2 + b*x))^2*e + 2*(sqrt(c)*x - s
qrt(c*x^2 + b*x))*sqrt(c)*d + b*d)^2*e^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^3}\,dx = \int \frac{\left(c\,x^2+b\,x\right)^{3/2}}{\left(d+e\,x\right)^3}\,dx$$

input
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^3,x)
```

output
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.89 (sec) , antiderivative size = 2312, normalized size of antiderivative = 10.56

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^3,x)
```

output
```
( - 12*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*
x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**3*d**2*e**3 - 24*sqrt(
d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)
*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**3*d*e**4*x - 12*sqrt(d)*sqrt(b*e -
 c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt
(c))/(sqrt(d)*sqrt(c)))*b**3*e**5*x**2 + 120*sqrt(d)*sqrt(b*e - c*d)*atan(
(sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(
d)*sqrt(c)))*b**2*c*d**3*e**2 + 240*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e
 - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c
)))*b**2*c*d**2*e**3*x + 120*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d)
 - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**
2*c*d*e**4*x**2 - 288*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt
(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b*c**2*d**
4*e - 576*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b +
 c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b*c**2*d**3*e**2*x - 2
88*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) -
 sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b*c**2*d**2*e**3*x**2 + 192*s
qrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqr
t(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*c**3*d**5 + 384*sqrt(d)*sqrt(b*e
- c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*...
```

3.146 $\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^4} \, dx$

Optimal result . . . 1163
Mathematica [A] (verified) . . . 1164
Rubi [A] (verified) . . . 1164
Maple [A] (verified) . . . 1168
Fricas [B] (verification not implemented) . . . 1168
Sympy [F] . . . 1169
Maxima [F(-2)] . . . 1170
Giac [B] (verification not implemented) . . . 1170
Mupad [F(-1)] . . . 1171
Reduce [F] . . . 1172

Optimal result

Integrand size = 21, antiderivative size = 249

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^4} \, dx = -\frac{(2cd-be)x\sqrt{bx+cx^2}}{4de^2(d+ex)^2} - \frac{(8c^2d^2-be(6cd+be))\sqrt{bx+cx^2}}{8de^3(cd-be)(d+ex)} - \frac{(bx+cx^2)^{3/2}}{3e(d+ex)^3} + \frac{2c^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{e^4} - \frac{(2cd-be)(8c^2d^2-8bcde-b^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{cd-be}\,x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{8d^{3/2}e^4(cd-be)^{3/2}}$$

output
```
-1/4*(-b*e+2*c*d)*x*(c*x^2+b*x)^(1/2)/d/e^2/(e*x+d)^2-1/8*(8*c^2*d^2-b*e*(
b*e+6*c*d))*(c*x^2+b*x)^(1/2)/d/e^3/(-b*e+c*d)/(e*x+d)-1/3*(c*x^2+b*x)^(3/
2)/e/(e*x+d)^3+2*c^(3/2)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/e^4-1/8*(-b*
e+2*c*d)*(-b^2*e^2-8*b*c*d*e+8*c^2*d^2)*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)
/(c*x^2+b*x)^(1/2))/d^(3/2)/e^4/(-b*e+c*d)^(3/2)
```

Mathematica [A] (verified)

Time = 11.31 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.08

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^4} dx = \frac{\sqrt{x(b + cx)}\left(-\frac{e\sqrt{x}(b^2e^2(-3d^2-8dex+3e^2x^2)+4c^2d^2(6d^2+15dex+11e^2x^2)-2bcde(9d^2+23dex+22e^2x^2))}{d(cd-be)(d+ex)^3}\right.}{24e^4\sqrt{x}}$$

input
```
Integrate[(b*x + c*x^2)^(3/2)/(d + e*x)^4,x]
```

output
```
(Sqrt[x*(b + c*x)]*(-((e*Sqrt[x]*(b^2*e^2*(-3*d^2 - 8*d*e*x + 3*e^2*x^2) +
 4*c^2*d^2*(6*d^2 + 15*d*e*x + 11*e^2*x^2) - 2*b*c*d*e*(9*d^2 + 23*d*e*x +
 22*e^2*x^2)))/(d*(c*d - b*e)*(d + e*x)^3)) + (48*c^(3/2)*ArcSinh[(Sqrt[c]
*Sqrt[x])/Sqrt[b]])/(Sqrt[b]*Sqrt[1 + (c*x)/b]) - (3*(16*c^3*d^3 - 24*b*c^
2*d^2*e + 6*b^2*c*d*e^2 + b^3*e^3)*ArcTanh[(Sqrt[c*d - b*e]*Sqrt[x])/(Sqrt
[d]*Sqrt[b + c*x])])/(d^(3/2)*(c*d - b*e)^(3/2)*Sqrt[b + c*x])))/(24*e^4*S
qrt[x])
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.17, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1161, 1229, 27, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^4} dx$$

$$\Big\downarrow \text{1161}$$

$$\frac{\int \frac{(b+2cx)\sqrt{cx^2+bx}}{(d+ex)^3}dx}{2e} - \frac{(bx + cx^2)^{3/2}}{3e(d + ex)^3}$$

$$\Big\downarrow \text{1229}$$

$$\frac{-\frac{\int -\frac{16d(cd-be)xc^2+b\left(8c^2d^2-be(6cd+be)\right)}{2(d+ex)\sqrt{cx^2+bx}}dx}{4de^2(cd-be)}-\frac{\sqrt{bx+cx^2}\left(ex\left(b^2e^2-12bcde+12c^2d^2\right)+d\left(-b^2e^2-6bcde+8c^2d^2\right)\right)}{4de^2(d+ex)^2(cd-be)}}{2e}-\frac{\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{16d(cd-be)xc^2+b\left(8c^2d^2-be(6cd+be)\right)}{(d+ex)\sqrt{cx^2+bx}}dx}{8de^2(cd-be)}-\frac{\sqrt{bx+cx^2}\left(ex\left(b^2e^2-12bcde+12c^2d^2\right)+d\left(-b^2e^2-6bcde+8c^2d^2\right)\right)}{4de^2(d+ex)^2(cd-be)}}{2e}-\frac{\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^3}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{\frac{16c^2d(cd-be)\int \frac{1}{\sqrt{cx^2+bx}}dx}{e}-\frac{(2cd-be)\left(-b^2e^2-8bcde+8c^2d^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{8de^2(cd-be)}-\frac{\sqrt{bx+cx^2}\left(ex\left(b^2e^2-12bcde+12c^2d^2\right)+d\left(-b^2e^2-6bcde+8c^2d\right.\right.}{4de^2(d+ex)^2(cd-be)}}{2e}$$

$$\frac{\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^3}$$

$$\Big\downarrow \; 1091$$

$$\frac{\frac{\frac{32c^2d(cd-be)\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{e}-\frac{(2cd-be)\left(-b^2e^2-8bcde+8c^2d^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{8de^2(cd-be)}-\frac{\sqrt{bx+cx^2}\left(ex\left(b^2e^2-12bcde+12c^2d^2\right)+d\left(-b^2e^2-6b\right.\right.}{4de^2(d+ex)^2(cd-be)}}{2e}$$

$$\frac{\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^3}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{32c^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(cd-be)}{e}-\frac{(2cd-be)\left(-b^2e^2-8bcde+8c^2d^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{8de^2(cd-be)}-\frac{\sqrt{bx+cx^2}\left(ex\left(b^2e^2-12bcde+12c^2d^2\right)+d\left(-b^2e^2-6\right.\right.}{4de^2(d+ex)^2(cd-be)}}{2e}$$

$$\frac{\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^3}$$

$$\Big\downarrow \; 1154$$

$$\frac{\frac{\frac{2(2cd-be)\left(-b^2e^2-8bcde+8c^2d^2\right)\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{e}+\frac{32c^{3/2}d\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(cd-be)}{e}}{8de^2(cd-be)}-\frac{\sqrt{bx+cx^2}\left(ex\left(b^2e^2-\right.\right.}{}}{2e}$$

$$\frac{\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{32c^{3/2}d\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(cd-be)}{e}-\frac{(2cd-be)\left(-b^2e^2-8bcde+8c^2d^2\right)\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{\sqrt{d}e\sqrt{cd-be}}}{8de^2(cd-be)}-\frac{\sqrt{bx+cx^2}\left(ex\left(b^2e^2-12bcde+12c^2d^2\right)\right.}{4de^2(d+ex)^2(c}}{2e}$$

$$\frac{\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^3}$$

input
```
Int[(b*x + c*x^2)^(3/2)/(d + e*x)^4,x]
```

output
```
-1/3*(b*x + c*x^2)^(3/2)/(e*(d + e*x)^3) + (-1/4*((d*(8*c^2*d^2 - 6*b*c*d*
e - b^2*e^2) + e*(12*c^2*d^2 - 12*b*c*d*e + b^2*e^2)*x)*Sqrt[b*x + c*x^2])
/(d*e^2*(c*d - b*e)*(d + e*x)^2) + ((32*c^(3/2)*d*(c*d - b*e)*ArcTanh[(Sqr
t[c]*x)/Sqrt[b*x + c*x^2]])/e - ((2*c*d - b*e)*(8*c^2*d^2 - 8*b*c*d*e - b^
2*e^2)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x
 + c*x^2])])/(Sqrt[d]*e*Sqrt[c*d - b*e]))/(8*d*e^2*(c*d - b*e)))/(2*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{(ex+d)^3(be-2cd)(b^2e^2+8bcde-8c^2d^2)\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)+\sqrt{d(be-cd)}\left(-16d(ex+d)^3\left(eb\,c^{\frac{3}{2}}-d\,c^{\frac{5}{2}}\right)\operatorname{arctanh}\left(\frac{\sqrt{x}}{}\right.\right.}{8\sqrt{d(be-cd)}\,e^4(ex+}$
default	Expression too large to display

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/8/(d*(b*e-c*d))^(1/2)*((e*x+d)^3*(b*e-2*c*d)*(b^2*e^2+8*b*c*d*e-8*c^2*d
^2)*arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))^(1/2))+(d*(b*e-c*d))^(1/2)*
(-16*d*(e*x+d)^3*(e*b*c^(3/2)-d*c^(5/2))*arctanh((x*(c*x+b))^(1/2)/x/c^(1/
2))+e*(-8*c^2*d^4+6*e*x*(-10/3*c*x+b)*c*d^3+e^2*(-44/3*c^2*x^2+b^2+46/3*c*b*
x)*d^2+8/3*e^3*(11/2*c*x+b)*x*b*d-x^2*b^2*e^4)*(x*(c*x+b))^(1/2)))/e^4/(e*
x+d)^3/(b*e-c*d)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 704 vs. 2(219) = 438.

Time = 1.01 (sec) , antiderivative size = 2837, normalized size of antiderivative = 11.39

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
[1/48*(48*(c^3*d^7 - 2*b*c^2*d^6*e + b^2*c*d^5*e^2 + (c^3*d^4*e^3 - 2*b*c^
2*d^3*e^4 + b^2*c*d^2*e^5)*x^3 + 3*(c^3*d^5*e^2 - 2*b*c^2*d^4*e^3 + b^2*c*
d^3*e^4)*x^2 + 3*(c^3*d^6*e - 2*b*c^2*d^5*e^2 + b^2*c*d^4*e^3)*x)*sqrt(c)*
log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 3*(16*c^3*d^6 - 24*b*c^2*d^
5*e + 6*b^2*c*d^4*e^2 + b^3*d^3*e^3 + (16*c^3*d^3*e^3 - 24*b*c^2*d^2*e^4 +
 6*b^2*c*d*e^5 + b^3*e^6)*x^3 + 3*(16*c^3*d^4*e^2 - 24*b*c^2*d^3*e^3 + 6*b
^2*c*d^2*e^4 + b^3*d*e^5)*x^2 + 3*(16*c^3*d^5*e - 24*b*c^2*d^4*e^2 + 6*b^2
*c*d^3*e^3 + b^3*d^2*e^4)*x)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*
x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) - 2*(24*c^3*d^6*e
- 42*b*c^2*d^5*e^2 + 15*b^2*c*d^4*e^3 + 3*b^3*d^3*e^4 + (44*c^3*d^4*e^3 -
88*b*c^2*d^3*e^4 + 47*b^2*c*d^2*e^5 - 3*b^3*d*e^6)*x^2 + 2*(30*c^3*d^5*e^2
 - 53*b*c^2*d^4*e^3 + 19*b^2*c*d^3*e^4 + 4*b^3*d^2*e^5)*x)*sqrt(c*x^2 + b*
x))/(c^2*d^7*e^4 - 2*b*c*d^6*e^5 + b^2*d^5*e^6 + (c^2*d^4*e^7 - 2*b*c*d^3*
e^8 + b^2*d^2*e^9)*x^3 + 3*(c^2*d^5*e^6 - 2*b*c*d^4*e^7 + b^2*d^3*e^8)*x^2
 + 3*(c^2*d^6*e^5 - 2*b*c*d^5*e^6 + b^2*d^4*e^7)*x), 1/24*(3*(16*c^3*d^6 -
 24*b*c^2*d^5*e + 6*b^2*c*d^4*e^2 + b^3*d^3*e^3 + (16*c^3*d^3*e^3 - 24*b*c
^2*d^2*e^4 + 6*b^2*c*d*e^5 + b^3*e^6)*x^3 + 3*(16*c^3*d^4*e^2 - 24*b*c^2*d
^3*e^3 + 6*b^2*c*d^2*e^4 + b^3*d*e^5)*x^2 + 3*(16*c^3*d^5*e - 24*b*c^2*d^4
*e^2 + 6*b^2*c*d^3*e^3 + b^3*d^2*e^4)*x)*sqrt(-c*d^2 + b*d*e)*arctan(sqrt(
-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/(c*d*x + b*d)) + 24*(c^3*d^7 - 2*b*c^...
```

Sympy [F]

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^4}\,dx = \int \frac{(x(b + cx))^{\frac{3}{2}}}{(d + ex)^4}\,dx$$

input
```
integrate((c*x**2+b*x)**(3/2)/(e*x+d)**4,x)
```

output
```
Integral((x*(b + c*x))**(3/2)/(d + e*x)**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^4} dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 929 vs. 2(219) = 438.

Time = 0.18 (sec) , antiderivative size = 929, normalized size of antiderivative = 3.73

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^4} dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
1/8*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*b^2*c*d*e^2 + b^3*e^3)*arctan(((sqrt(
c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e))/((c*d^2*e^4
 - b*d*e^5)*sqrt(-c*d^2 + b*d*e)) - c^(3/2)*log(abs(2*(sqrt(c)*x - sqrt(c*
x^2 + b*x))*sqrt(c) + b))/e^4 - 1/24*(144*(sqrt(c)*x - sqrt(c*x^2 + b*x))^
5*c^3*d^3*e^2 - 216*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*b*c^2*d^2*e^3 + 78*(
sqrt(c)*x - sqrt(c*x^2 + b*x))^5*b^2*c*d*e^4 - 3*(sqrt(c)*x - sqrt(c*x^2 +
 b*x))^5*b^3*e^5 + 432*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*c^(7/2)*d^4*e - 5
04*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*b*c^(5/2)*d^3*e^2 + 54*(sqrt(c)*x - s
qrt(c*x^2 + b*x))^4*b^2*c^(3/2)*d^2*e^3 + 33*(sqrt(c)*x - sqrt(c*x^2 + b*x
))^4*b^3*sqrt(c)*d*e^4 + 352*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*c^4*d^5 - 1
6*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b*c^3*d^4*e - 420*(sqrt(c)*x - sqrt(c*
x^2 + b*x))^3*b^2*c^2*d^3*e^2 + 106*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^3*
c*d^2*e^3 + 8*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^4*d*e^4 + 528*(sqrt(c)*x
 - sqrt(c*x^2 + b*x))^2*b*c^(7/2)*d^5 - 516*(sqrt(c)*x - sqrt(c*x^2 + b*x)
)^2*b^2*c^(5/2)*d^4*e - 6*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^3*c^(3/2)*d^
3*e^2 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^4*sqrt(c)*d^2*e^3 + 264*(sq
rt(c)*x - sqrt(c*x^2 + b*x))*b^2*c^3*d^5 - 288*(sqrt(c)*x - sqrt(c*x^2 + b
*x))*b^3*c^2*d^4*e + 36*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^4*c*d^3*e^2 + 3*
(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^5*d^2*e^3 + 44*b^3*c^(5/2)*d^5 - 44*b^4*
c^(3/2)*d^4*e + 3*b^5*sqrt(c)*d^3*e^2)/((c*d^2*e^4 - b*d*e^5)*((sqrt(c)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^4}\,dx = \int \frac{(c\,x^2+b\,x)^{3/2}}{(d+e\,x)^4}\,dx$$

input
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^4,x)
```

output
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^4, x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^4}\, dx = \int \frac{(c\,x^2 + bx)^{\frac{3}{2}}}{(ex + d)^4} dx$$

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^4,x)
```

output
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^4,x)
```

3.147 $\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^5}\,dx$

Optimal result 1173
Mathematica [A] (verified) 1173
Rubi [A] (verified) 1174
Maple [A] (verified) 1176
Fricas [B] (verification not implemented) 1176
Sympy [F] 1177
Maxima [F(-2)] 1178
Giac [B] (verification not implemented) 1178
Mupad [F(-1)] 1179
Reduce [F] 1180

Optimal result

Integrand size = 21, antiderivative size = 216

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^5}\,dx = \frac{3b^3\sqrt{bx+cx^2}}{64d^2(cd-be)^2(d+ex)} + \frac{b^2(bx+cx^2)^{3/2}}{32d(cd-be)^2x(d+ex)^2} + \frac{(bx+cx^2)^{5/2}}{4(cd-be)x(d+ex)^4} - \frac{b(bx+cx^2)^{5/2}}{8(cd-be)^2x^2(d+ex)^3} + \frac{3b^4\operatorname{arctanh}\left(\frac{\sqrt{cd-be}x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{64d^{5/2}(cd-be)^{5/2}}$$

output
```
3/64*b^3*(c*x^2+b*x)^(1/2)/d^2/(-b*e+c*d)^2/(e*x+d)+1/32*b^2*(c*x^2+b*x)^(
3/2)/d/(-b*e+c*d)^2/x/(e*x+d)^2+1/4*(c*x^2+b*x)^(5/2)/(-b*e+c*d)/x/(e*x+d)
^4-1/8*b*(c*x^2+b*x)^(5/2)/(-b*e+c*d)^2/x^2/(e*x+d)^3+3/64*b^4*arctanh((-b
*e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(1/2))/d^(5/2)/(-b*e+c*d)^(5/2)
```

Mathematica [A] (verified)

Time = 10.33 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.96

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^5}\,dx = \frac{(x(b+cx))^{3/2}\left(-\frac{2(b+cx)}{(d+ex)^4} + \frac{b(b+cx)}{(cd-be)x(d+ex)^3} + \frac{b^2}{4d(-cd+be)x(d+ex)^2} + \frac{\frac{3b^3}{dx(b+cx)(d+ex)} + \frac{3b^4\operatorname{arcta}}{d^{3/2}\sqrt{cd-}}}{-8cd^2+8bd}\right.}{8(-cd+be)}$$

input
```
Integrate[(b*x + c*x^2)^(3/2)/(d + e*x)^5,x]
```

output
```
((x*(b + c*x))^(3/2)*((-2*(b + c*x))/(d + e*x)^4 + (b*(b + c*x))/((c*d - b
*e)*x*(d + e*x)^3) + b^2/(4*d*(-(c*d) + b*e)*x*(d + e*x)^2) + ((3*b^3)/(d*
x*(b + c*x)*(d + e*x)) + (3*b^4*ArcTanh[(Sqrt[c*d - b*e]*Sqrt[x])/(Sqrt[d]
*Sqrt[b + c*x])])/(d^(3/2)*Sqrt[c*d - b*e]*x^(3/2)*(b + c*x)^(3/2)))/(-8*c
*d^2 + 8*b*d*e)))/(8*(-(c*d) + b*e))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1152, 1152, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^5}dx$$

$$\Big\downarrow \text{1152}$$

$$\frac{(bx+cx^2)^{3/2}(x(2cd-be)+bd)}{8d(d+ex)^4(cd-be)}-\frac{3b^2\int \frac{\sqrt{cx^2+bx}}{(d+ex)^3}dx}{16d(cd-be)}$$

$$\Big\downarrow \text{1152}$$

$$\frac{(bx+cx^2)^{3/2}(x(2cd-be)+bd)}{8d(d+ex)^4(cd-be)}-\frac{3b^2\left(\frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)}-\frac{b^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{8d(cd-be)}\right)}{16d(cd-be)}$$

$$\Big\downarrow \text{1154}$$

$$\frac{(bx+cx^2)^{3/2}(x(2cd-be)+bd)}{8d(d+ex)^4(cd-be)}-\frac{3b^2\left(\frac{b^2\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{4d(cd-be)}+\frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)}\right)}{16d(cd-be)}$$

$$\Big\downarrow \text{219}$$

$$\frac{(bx+cx^2)^{3/2}(x(2cd-be)+bd)}{8d(d+ex)^4(cd-be)} - \frac{3b^2\left(\frac{\sqrt{bx+cx^2}(x(2cd-be)+bd)}{4d(d+ex)^2(cd-be)} - \frac{b^2\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{8d^{3/2}(cd-be)^{3/2}}\right)}{16d(cd-be)}$$

input
```
Int[(b*x + c*x^2)^(3/2)/(d + e*x)^5,x]
```

output
```
((b*d + (2*c*d - b*e)*x)*(b*x + c*x^2)^(3/2))/(8*d*(c*d - b*e)*(d + e*x)^4
) - (3*b^2*(((b*d + (2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(4*d*(c*d - b*e)*(
d + e*x)^2) - (b^2*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b
*e]*Sqrt[b*x + c*x^2])])/(8*d^(3/2)*(c*d - b*e)^(3/2))))/(16*d*(c*d - b*e)
)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1152
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2)))   Int[(d + e*x)^(m + 2)*(a + b*x +
 c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
 && GtQ[p, 0]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$-\frac{3(2cdx+b(-ex+d))\left(\left(e^2x^2+\frac{14}{3}dex+d^2\right)b^2-\frac{8dcx(-ex+d)b}{3}-\frac{8d^2c^2x^2}{3}\right)\sqrt{d(be-cd)}\,\sqrt{x(cx+b)}+3b^4(ex+d)^4\arctan\left(\cdots\right.}{64\sqrt{d(be-cd)}\,(ex+d)^4d^2(be-cd)^2}$
default	Expression too large to display

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/64/(d*(b*e-c*d))^(1/2)*(3*(2*c*d*x+b*(-e*x+d))*((e^2*x^2+14/3*d*e*x+d^2
)*b^2-8/3*d*c*x*(-e*x+d)*b-8/3*d^2*c^2*x^2)*(d*(b*e-c*d))^(1/2)*(x*(c*x+b)
)^(1/2)+3*b^4*(e*x+d)^4*arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))^(1/2)))
/(e*x+d)^4/d^2/(b*e-c*d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 525 vs. 2(188) = 376.

Time = 0.11 (sec) , antiderivative size = 1066, normalized size of antiderivative = 4.94

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^5}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^5,x, algorithm="fricas")
```

output
```
[1/128*(3*(b^4*e^4*x^4 + 4*b^4*d*e^3*x^3 + 6*b^4*d^2*e^2*x^2 + 4*b^4*d^3*e
*x + b^4*d^4)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^
2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) - 2*(3*b^3*c*d^5 - 3*b^4*d^4*e -
(16*c^4*d^5 - 40*b*c^3*d^4*e + 26*b^2*c^2*d^3*e^2 + b^3*c*d^2*e^3 - 3*b^4*
d*e^4)*x^3 - (24*b*c^3*d^5 - 68*b^2*c^2*d^4*e + 55*b^3*c*d^3*e^2 - 11*b^4*
d^2*e^3)*x^2 - (2*b^2*c^2*d^5 - 13*b^3*c*d^4*e + 11*b^4*d^3*e^2)*x)*sqrt(c
*x^2 + b*x))/(c^3*d^10 - 3*b*c^2*d^9*e + 3*b^2*c*d^8*e^2 - b^3*d^7*e^3 + (
c^3*d^6*e^4 - 3*b*c^2*d^5*e^5 + 3*b^2*c*d^4*e^6 - b^3*d^3*e^7)*x^4 + 4*(c^
3*d^7*e^3 - 3*b*c^2*d^6*e^4 + 3*b^2*c*d^5*e^5 - b^3*d^4*e^6)*x^3 + 6*(c^3*
d^8*e^2 - 3*b*c^2*d^7*e^3 + 3*b^2*c*d^6*e^4 - b^3*d^5*e^5)*x^2 + 4*(c^3*d^
9*e - 3*b*c^2*d^8*e^2 + 3*b^2*c*d^7*e^3 - b^3*d^6*e^4)*x), -1/64*(3*(b^4*e
^4*x^4 + 4*b^4*d*e^3*x^3 + 6*b^4*d^2*e^2*x^2 + 4*b^4*d^3*e*x + b^4*d^4)*sq
rt(-c*d^2 + b*d*e)*arctan(sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/(c*d*x +
b*d)) + (3*b^3*c*d^5 - 3*b^4*d^4*e - (16*c^4*d^5 - 40*b*c^3*d^4*e + 26*b^2
*c^2*d^3*e^2 + b^3*c*d^2*e^3 - 3*b^4*d*e^4)*x^3 - (24*b*c^3*d^5 - 68*b^2*c
^2*d^4*e + 55*b^3*c*d^3*e^2 - 11*b^4*d^2*e^3)*x^2 - (2*b^2*c^2*d^5 - 13*b^
3*c*d^4*e + 11*b^4*d^3*e^2)*x)*sqrt(c*x^2 + b*x))/(c^3*d^10 - 3*b*c^2*d^9*
e + 3*b^2*c*d^8*e^2 - b^3*d^7*e^3 + (c^3*d^6*e^4 - 3*b*c^2*d^5*e^5 + 3*b^2
*c*d^4*e^6 - b^3*d^3*e^7)*x^4 + 4*(c^3*d^7*e^3 - 3*b*c^2*d^6*e^4 + 3*b^2*c
*d^5*e^5 - b^3*d^4*e^6)*x^3 + 6*(c^3*d^8*e^2 - 3*b*c^2*d^7*e^3 + 3*b^2*...
```

Sympy [F]

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^5}\,dx = \int \frac{(x(b+cx))^{\frac{3}{2}}}{(d+ex)^5}\,dx$$

input
```
integrate((c*x**2+b*x)**(3/2)/(e*x+d)**5,x)
```

output
```
Integral((x*(b + c*x))**(3/2)/(d + e*x)**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1136 vs. 2(188) = 376.

Time = 0.29 (sec) , antiderivative size = 1136, normalized size of antiderivative = 5.26

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^5}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^5,x, algorithm="giac")
```

output
```
-1/128*(3*b^4*log(abs(2*c*d*e - b*e^2 - 2*sqrt(c*d^2 - b*d*e)*(sqrt(c - 2*
c*d/(e*x + d) + c*d^2/(e*x + d)^2 + b*e/(e*x + d) - b*d*e/(e*x + d)^2) + s
qrt(c*d^2*e^2 - b*d*e^3)/((e*x + d)*e))*abs(e)))*sgn(1/(e*x + d))*sgn(e)/(
(c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3)*sqrt(c*d^2 - b*d*e)*abs(e)) - (3
*b^4*e^5*log(abs(2*c*d*e - b*e^2 - 2*sqrt(c*d^2 - b*d*e)*sqrt(c)*abs(e)))
- 32*sqrt(c*d^2 - b*d*e)*c^(7/2)*d^3*abs(e) + 48*sqrt(c*d^2 - b*d*e)*b*c^(
5/2)*d^2*e*abs(e) - 4*sqrt(c*d^2 - b*d*e)*b^2*c^(3/2)*d*e^2*abs(e) - 6*sqr
t(c*d^2 - b*d*e)*b^3*sqrt(c)*e^3*abs(e))*sgn(1/(e*x + d))*sgn(e)/(sqrt(c*d
^2 - b*d*e)*c^2*d^4*e^6*abs(e) - 2*sqrt(c*d^2 - b*d*e)*b*c*d^3*e^7*abs(e)
+ sqrt(c*d^2 - b*d*e)*b^2*d^2*e^8*abs(e)) - 2*sqrt(c - 2*c*d/(e*x + d) + c
*d^2/(e*x + d)^2 + b*e/(e*x + d) - b*d*e/(e*x + d)^2)*((16*c^4*d^5*e^17*sg
n(1/(e*x + d))*sgn(e) - 40*b*c^3*d^4*e^18*sgn(1/(e*x + d))*sgn(e) + 26*b^2
*c^2*d^3*e^19*sgn(1/(e*x + d))*sgn(e) + b^3*c*d^2*e^20*sgn(1/(e*x + d))*sg
n(e) - 3*b^4*d*e^21*sgn(1/(e*x + d))*sgn(e))/(c^3*d^6*e^23 - 3*b*c^2*d^5*e
^24 + 3*b^2*c*d^4*e^25 - b^3*d^3*e^26) - 2*((24*c^4*d^6*e^18*sgn(1/(e*x +
d))*sgn(e) - 72*b*c^3*d^5*e^19*sgn(1/(e*x + d))*sgn(e) + 73*b^2*c^2*d^4*e^
20*sgn(1/(e*x + d))*sgn(e) - 26*b^3*c*d^3*e^21*sgn(1/(e*x + d))*sgn(e) + b
^4*d^2*e^22*sgn(1/(e*x + d))*sgn(e))/(c^3*d^6*e^23 - 3*b*c^2*d^5*e^24 + 3*
b^2*c*d^4*e^25 - b^3*d^3*e^26) - 4*(3*(2*c^4*d^7*e^19*sgn(1/(e*x + d))*sgn
(e) - 7*b*c^3*d^6*e^20*sgn(1/(e*x + d))*sgn(e) + 9*b^2*c^2*d^5*e^21*sgn...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^5}\,dx=\int \frac{(c\,x^2+b\,x)^{3/2}}{(d+e\,x)^5}\,dx$$

input
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^5,x)
```

output
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^5, x)
```

Reduce [F]

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^5}\,dx = \int \frac{(c\,x^2+bx)^{\frac{3}{2}}}{(ex+d)^5}dx$$

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^5,x)
```

output
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^5,x)
```

3.148 $\int (d+ex)^3 \left(bx+cx^2\right)^{5/2} \, dx$

Optimal result 1181
Mathematica [A] (verified) 1182
Rubi [A] (verified) 1183
Maple [A] (verified) 1186
Fricas [A] (verification not implemented) 1188
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Optimal result

Integrand size = 21, antiderivative size = 507

$$\begin{aligned}\int (d+ex)^3 \left(bx+cx^2\right)^{5/2} \, dx &= \frac{5b^5(2cd-be)\left(32c^2d^2-32bcde+11b^2e^2\right)\sqrt{bx+cx^2}}{32768c^6} \\ &- \frac{5b^4(2cd-be)\left(32c^2d^2-32bcde+11b^2e^2\right)x\sqrt{bx+cx^2}}{49152c^5} \\ &+ \frac{b^3(2cd-be)\left(32c^2d^2-32bcde+11b^2e^2\right)x^2\sqrt{bx+cx^2}}{12288c^4} \\ &+ \frac{9b^2(2cd-be)\left(32c^2d^2-32bcde+11b^2e^2\right)x^3\sqrt{bx+cx^2}}{2048c^3} \\ &+ \frac{5b(2cd-be)\left(32c^2d^2-32bcde+11b^2e^2\right)x^4\sqrt{bx+cx^2}}{768c^2} \\ &+ \frac{(2cd-be)\left(32c^2d^2-32bcde+11b^2e^2\right)x^5\sqrt{bx+cx^2}}{384c} \\ &+ \frac{e\left(96c^2d^2-54bcde+11b^2e^2\right)\left(bx+cx^2\right)^{7/2}}{224c^3} \\ &+ \frac{e^2(54cd-11be)x\left(bx+cx^2\right)^{7/2}}{144c^2} + \frac{e^3x^2\left(bx+cx^2\right)^{7/2}}{9c} \\ &- \frac{5b^6(2cd-be)\left(32c^2d^2-32bcde+11b^2e^2\right)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{32768c^{13/2}}\end{aligned}$$

output
```
5/32768*b^5*(-b*e+2*c*d)*(11*b^2*e^2-32*b*c*d*e+32*c^2*d^2)*(c*x^2+b*x)^(1
/2)/c^6-5/49152*b^4*(-b*e+2*c*d)*(11*b^2*e^2-32*b*c*d*e+32*c^2*d^2)*x*(c*x
^2+b*x)^(1/2)/c^5+1/12288*b^3*(-b*e+2*c*d)*(11*b^2*e^2-32*b*c*d*e+32*c^2*d
^2)*x^2*(c*x^2+b*x)^(1/2)/c^4+9/2048*b^2*(-b*e+2*c*d)*(11*b^2*e^2-32*b*c*d
*e+32*c^2*d^2)*x^3*(c*x^2+b*x)^(1/2)/c^3+5/768*b*(-b*e+2*c*d)*(11*b^2*e^2-
32*b*c*d*e+32*c^2*d^2)*x^4*(c*x^2+b*x)^(1/2)/c^2+1/384*(-b*e+2*c*d)*(11*b^
2*e^2-32*b*c*d*e+32*c^2*d^2)*x^5*(c*x^2+b*x)^(1/2)/c+1/224*e*(11*b^2*e^2-5
4*b*c*d*e+96*c^2*d^2)*(c*x^2+b*x)^(7/2)/c^3+1/144*e^2*(-11*b*e+54*c*d)*x*(
c*x^2+b*x)^(7/2)/c^2+1/9*e^3*x^2*(c*x^2+b*x)^(7/2)/c-5/32768*b^6*(-b*e+2*c
*d)*(11*b^2*e^2-32*b*c*d*e+32*c^2*d^2)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2)
)/c^(13/2)
```

Mathematica [A] (verified)

Time = 2.86 (sec) , antiderivative size = 454, normalized size of antiderivative = 0.90

$$\int (d+ex)^3 \left(bx+cx^2\right)^{5/2}\,dx = \frac{\sqrt{x}\sqrt{b+cx}\left(\sqrt{c}\sqrt{x}\sqrt{b+cx}(-3465b^8e^3+210b^7ce^2(81d+11ex)-84b^6c^2e(360d^2+135dex\right.}{}$$

input
```
Integrate[(d + e*x)^3*(b*x + c*x^2)^(5/2),x]
```

output
```
(Sqrt[x]*Sqrt[b + c*x]*(Sqrt[c]*Sqrt[x]*Sqrt[b + c*x]*(-3465*b^8*e^3 + 210
*b^7*c*e^2*(81*d + 11*e*x) - 84*b^6*c^2*e*(360*d^2 + 135*d*e*x + 22*e^2*x^
2) + 256*b^3*c^5*x^2*(42*d^3 + 54*d^2*e*x + 27*d*e^2*x^2 + 5*e^3*x^3) + 14
4*b^5*c^3*(140*d^3 + 140*d^2*e*x + 63*d*e^2*x^2 + 11*e^3*x^3) - 32*b^4*c^4
*x*(420*d^3 + 504*d^2*e*x + 243*d*e^2*x^2 + 44*e^3*x^3) + 4096*c^8*x^5*(84
*d^3 + 216*d^2*e*x + 189*d*e^2*x^2 + 56*e^3*x^3) + 1536*b^2*c^6*x^3*(378*d
^3 + 888*d^2*e*x + 729*d*e^2*x^2 + 206*e^3*x^3) + 2048*b*c^7*x^4*(420*d^3
+ 1044*d^2*e*x + 891*d*e^2*x^2 + 259*e^3*x^3)) + 1260*b^6*c*d*(32*c^2*d^2
+ 27*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[x])/(Sqrt[b] - Sqrt[b + c*x])] + 630*b
^7*e*(96*c^2*d^2 + 11*b^2*e^2)*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[
b + c*x])]))/(2064384*c^(13/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 268, normalized size of antiderivative = 0.53, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {1166, 27, 1225, 1087, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{5/2}(d+ex)^3dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{\int \frac{1}{2}(d+ex)(d(18cd-7be)+11e(2cd-be)x)\left(cx^2+bx\right)^{5/2}dx}{9c}+\frac{e\left(bx+cx^2\right)^{7/2}(d+ex)^2}{9c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (d+ex)(d(18cd-7be)+11e(2cd-be)x)\left(cx^2+bx\right)^{5/2}dx}{18c}+\frac{e\left(bx+cx^2\right)^{7/2}(d+ex)^2}{9c}$$

$$\Big\downarrow \text{1225}$$

$$\frac{\frac{9(2cd-be)\left(11b^2e^2-32bcde+32c^2d^2\right)\int \left(cx^2+bx\right)^{5/2}dx}{32c^2}+\frac{e\left(bx+cx^2\right)^{7/2}\left(99b^2e^2+154cex(2cd-be)-486bcde+640c^2d^2\right)}{112c^2}}{18c}+$$
$$\frac{e\left(bx+cx^2\right)^{7/2}(d+ex)^2}{9c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{9(2cd-be)\left(11b^2e^2-32bcde+32c^2d^2\right)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\int \left(cx^2+bx\right)^{3/2}dx}{24c}\right)}{32c^2}+\frac{e\left(bx+cx^2\right)^{7/2}\left(99b^2e^2+154cex(2cd-be)-486bcde+640c^2\right.}{112c^2}}{18c}$$
$$\frac{e\left(bx+cx^2\right)^{7/2}(d+ex)^2}{9c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{9(2cd-be)\left(11b^2e^2-32bcde+32c^2d^2\right)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int\sqrt{cx^2+bx}dx}{16c}\right)}{24c}\right)}{32c^2}+\frac{e\left(bx+cx^2\right)^{7/2}\left(99b^2e^2+154cex\cdots\right.}{112\cdots}}{18c}$$

$$\frac{e\left(bx+cx^2\right)^{7/2}(d+ex)^2}{9c}$$

$\big\downarrow$ 1087

$$\frac{\frac{9(2cd-be)\left(11b^2e^2-32bcde+32c^2d^2\right)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)}{24c}\right)}{32c^2}+\frac{e\left(bx+\cdots\right.}{\cdots}}{18c}$$

$$\frac{e\left(bx+cx^2\right)^{7/2}(d+ex)^2}{9c}$$

$\big\downarrow$ 1091

$$\frac{\frac{9(2cd-be)\left(11b^2e^2-32bcde+32c^2d^2\right)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)}{24c}\right)}{32c^2}\cdots}{18c}$$

$$\frac{e\left(bx+cx^2\right)^{7/2}(d+ex)^2}{9c}$$

$\big\downarrow$ 219

$$\frac{\frac{9\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)}{24c}\right)(2cd-be)\left(11b^2e^2-32bcde+32c^2d^2\right)}{32c^2} -}{18c}$$

$$\frac{e\left(bx+cx^2\right)^{7/2}(d+ex)^2}{9c}$$

input
```
Int[(d + e*x)^3*(b*x + c*x^2)^(5/2),x]
```

output
```
(e*(d + e*x)^2*(b*x + c*x^2)^(7/2))/(9*c) + ((e*(640*c^2*d^2 - 486*b*c*d*e
 + 99*b^2*e^2 + 154*c*e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(7/2))/(112*c^2) +
(9*(2*c*d - b*e)*(32*c^2*d^2 - 32*b*c*d*e + 11*b^2*e^2)*(((b + 2*c*x)*(b*x
 + c*x^2)^(5/2))/(12*c) - (5*b^2*(((b + 2*c*x)*(b*x + c*x^2)^(3/2))/(8*c)
- (3*b^2*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)
/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))/(16*c)))/(24*c)))/(32*c^2))/(18*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 480, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{(-229376c^8e^3x^8-530432b\,c^7e^3x^7-774144c^8d\,e^2x^7-316416b^2c^6e^3x^6-1824768b\,c^7d\,e^2x^6-884736c^8d^2e\,x^6-1280b^3c^5e^3x^5-111}{}$
default	$d^3\left(\frac{(2cx+b)(c\,x^2+bx)^{\frac{5}{2}}}{12c}-\frac{5b^2\left(\frac{(2cx+b)\left(c\,x^2+bx\right)^{\frac{3}{2}}}{8c}-\frac{3b^2\left(\frac{(2cx+b)\sqrt{c\,x^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{24c}\right)+e^3\left(\frac{x^2(c\,x^2}{9}\right.$

input
```
int((e*x+d)^3*(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2064384/c^6*(-229376*c^8*e^3*x^8-530432*b*c^7*e^3*x^7-774144*c^8*d*e^2*
x^7-316416*b^2*c^6*e^3*x^6-1824768*b*c^7*d*e^2*x^6-884736*c^8*d^2*e*x^6-12
80*b^3*c^5*e^3*x^5-1119744*b^2*c^6*d*e^2*x^5-2138112*b*c^7*d^2*e*x^5-34406
4*c^8*d^3*x^5+1408*b^4*c^4*e^3*x^4-6912*b^3*c^5*d*e^2*x^4-1363968*b^2*c^6*
d^2*e*x^4-860160*b*c^7*d^3*x^4-1584*b^5*c^3*e^3*x^3+7776*b^4*c^4*d*e^2*x^3
-13824*b^3*c^5*d^2*e*x^3-580608*b^2*c^6*d^3*x^3+1848*b^6*c^2*e^3*x^2-9072*
b^5*c^3*d*e^2*x^2+16128*b^4*c^4*d^2*e*x^2-10752*b^3*c^5*d^3*x^2-2310*b^7*c
*e^3*x+11340*b^6*c^2*d*e^2*x-20160*b^5*c^3*d^2*e*x+13440*b^4*c^4*d^3*x+346
5*b^8*e^3-17010*b^7*c*d*e^2+30240*b^6*c^2*d^2*e-20160*b^5*c^3*d^3)*x*(c*x+
b)/(x*(c*x+b))^(1/2)+5/65536*b^6*(11*b^3*e^3-54*b^2*c*d*e^2+96*b*c^2*d^2*e
-64*c^3*d^3)/c^(13/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 916, normalized size of antiderivative = 1.81

$$\int (d+ex)^3 \left(bx+cx^2\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/4128768*(315*(64*b^6*c^3*d^3 - 96*b^7*c^2*d^2*e + 54*b^8*c*d*e^2 - 11*
b^9*e^3)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(229376*
c^9*e^3*x^8 + 20160*b^5*c^4*d^3 - 30240*b^6*c^3*d^2*e + 17010*b^7*c^2*d*e^
2 - 3465*b^8*c*e^3 + 14336*(54*c^9*d*e^2 + 37*b*c^8*e^3)*x^7 + 3072*(288*c
^9*d^2*e + 594*b*c^8*d*e^2 + 103*b^2*c^7*e^3)*x^6 + 256*(1344*c^9*d^3 + 83
52*b*c^8*d^2*e + 4374*b^2*c^7*d*e^2 + 5*b^3*c^6*e^3)*x^5 + 128*(6720*b*c^8
*d^3 + 10656*b^2*c^7*d^2*e + 54*b^3*c^6*d*e^2 - 11*b^4*c^5*e^3)*x^4 + 144*
(4032*b^2*c^7*d^3 + 96*b^3*c^6*d^2*e - 54*b^4*c^5*d*e^2 + 11*b^5*c^4*e^3)*
x^3 + 168*(64*b^3*c^6*d^3 - 96*b^4*c^5*d^2*e + 54*b^5*c^4*d*e^2 - 11*b^6*c
^3*e^3)*x^2 - 210*(64*b^4*c^5*d^3 - 96*b^5*c^4*d^2*e + 54*b^6*c^3*d*e^2 -
11*b^7*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/c^7, 1/2064384*(315*(64*b^6*c^3*d^3
- 96*b^7*c^2*d^2*e + 54*b^8*c*d*e^2 - 11*b^9*e^3)*sqrt(-c)*arctan(sqrt(c*x
^2 + b*x)*sqrt(-c)/(c*x + b)) + (229376*c^9*e^3*x^8 + 20160*b^5*c^4*d^3 -
30240*b^6*c^3*d^2*e + 17010*b^7*c^2*d*e^2 - 3465*b^8*c*e^3 + 14336*(54*c^9
*d*e^2 + 37*b*c^8*e^3)*x^7 + 3072*(288*c^9*d^2*e + 594*b*c^8*d*e^2 + 103*b
^2*c^7*e^3)*x^6 + 256*(1344*c^9*d^3 + 8352*b*c^8*d^2*e + 4374*b^2*c^7*d*e^
2 + 5*b^3*c^6*e^3)*x^5 + 128*(6720*b*c^8*d^3 + 10656*b^2*c^7*d^2*e + 54*b^
3*c^6*d*e^2 - 11*b^4*c^5*e^3)*x^4 + 144*(4032*b^2*c^7*d^3 + 96*b^3*c^6*d^2
*e - 54*b^4*c^5*d*e^2 + 11*b^5*c^4*e^3)*x^3 + 168*(64*b^3*c^6*d^3 - 96*b^4
*c^5*d^2*e + 54*b^5*c^4*d*e^2 - 11*b^6*c^3*e^3)*x^2 - 210*(64*b^4*c^5*d...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1472 vs. $2(505) = 1010$.

Time = 0.60 (sec) , antiderivative size = 1472, normalized size of antiderivative = 2.90

$$\int (d+ex)^3 \left(bx+cx^2\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**3*(c*x**2+b*x)**(5/2),x)
```

output
```
Piecewise((-5*b**3*(b**3*d**3 - 7*b*(3*b**3*d**2*e + 3*b**2*c*d**3 - 9*b*(
3*b**3*d*e**2 + 9*b**2*c*d**2*e + 3*b*c**2*d**3 - 11*b*(b**3*e**3 + 9*b**2
*c*d*e**2 + 9*b*c**2*d**2*e - 13*b*(3*b**2*c*e**3 + 9*b*c**2*d*e**2 - 15*b
*(37*b*c**2*e**3/18 + 3*c**3*d*e**2)/(16*c) + 3*c**3*d**2*e)/(14*c) + c**3
*d**3)/(12*c))/(10*c))/(8*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x*
*2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt
(c*(b/(2*c) + x)**2), True))/(16*c**3) + sqrt(b*x + c*x**2)*(5*b**2*(b**3*
d**3 - 7*b*(3*b**3*d**2*e + 3*b**2*c*d**3 - 9*b*(3*b**3*d*e**2 + 9*b**2*c*
d**2*e + 3*b*c**2*d**3 - 11*b*(b**3*e**3 + 9*b**2*c*d*e**2 + 9*b*c**2*d**2
*e - 13*b*(3*b**2*c*e**3 + 9*b*c**2*d*e**2 - 15*b*(37*b*c**2*e**3/18 + 3*c
**3*d*e**2)/(16*c) + 3*c**3*d**2*e)/(14*c) + c**3*d**3)/(12*c))/(10*c))/(8
*c))/(8*c**3) - 5*b*x*(b**3*d**3 - 7*b*(3*b**3*d**2*e + 3*b**2*c*d**3 - 9*
b*(3*b**3*d*e**2 + 9*b**2*c*d**2*e + 3*b*c**2*d**3 - 11*b*(b**3*e**3 + 9*b
**2*c*d*e**2 + 9*b*c**2*d**2*e - 13*b*(3*b**2*c*e**3 + 9*b*c**2*d*e**2 - 1
5*b*(37*b*c**2*e**3/18 + 3*c**3*d*e**2)/(16*c) + 3*c**3*d**2*e)/(14*c) + c
**3*d**3)/(12*c))/(10*c))/(8*c))/(12*c**2) + c**2*e**3*x**8/9 + x**7*(37*b
*c**2*e**3/18 + 3*c**3*d*e**2)/(8*c) + x**6*(3*b**2*c*e**3 + 9*b*c**2*d*e*
*2 - 15*b*(37*b*c**2*e**3/18 + 3*c**3*d*e**2)/(16*c) + 3*c**3*d**2*e)/(7*c
) + x**5*(b**3*e**3 + 9*b**2*c*d*e**2 + 9*b*c**2*d**2*e - 13*b*(3*b**2*c*e
**3 + 9*b*c**2*d*e**2 - 15*b*(37*b*c**2*e**3/18 + 3*c**3*d*e**2)/(16*c)...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 808, normalized size of antiderivative = 1.59

$$\int (d+ex)^3 \left(bx+cx^2\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
1/9*(c*x^2 + b*x)^(7/2)*e^3*x^2/c + 1/6*(c*x^2 + b*x)^(5/2)*d^3*x + 5/256*
sqrt(c*x^2 + b*x)*b^4*d^3*x/c^2 - 5/96*(c*x^2 + b*x)^(3/2)*b^2*d^3*x/c - 1
5/512*sqrt(c*x^2 + b*x)*b^5*d^2*e*x/c^3 + 5/64*(c*x^2 + b*x)^(3/2)*b^3*d^2
*e*x/c^2 - 1/4*(c*x^2 + b*x)^(5/2)*b*d^2*e*x/c + 135/8192*sqrt(c*x^2 + b*x
)*b^6*d*e^2*x/c^4 - 45/1024*(c*x^2 + b*x)^(3/2)*b^4*d*e^2*x/c^3 + 9/64*(c*
x^2 + b*x)^(5/2)*b^2*d*e^2*x/c^2 + 3/8*(c*x^2 + b*x)^(7/2)*d*e^2*x/c - 55/
16384*sqrt(c*x^2 + b*x)*b^7*e^3*x/c^5 + 55/6144*(c*x^2 + b*x)^(3/2)*b^5*e^
3*x/c^4 - 11/384*(c*x^2 + b*x)^(5/2)*b^3*e^3*x/c^3 - 11/144*(c*x^2 + b*x)^
(7/2)*b*e^3*x/c^2 - 5/1024*b^6*d^3*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqr
t(c))/c^(7/2) + 15/2048*b^7*d^2*e*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt
(c))/c^(9/2) - 135/32768*b^8*d*e^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqr
t(c))/c^(11/2) + 55/65536*b^9*e^3*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt
(c))/c^(13/2) + 5/512*sqrt(c*x^2 + b*x)*b^5*d^3/c^3 - 5/192*(c*x^2 + b*x)^
(3/2)*b^3*d^3/c^2 + 1/12*(c*x^2 + b*x)^(5/2)*b*d^3/c - 15/1024*sqrt(c*x^2
+ b*x)*b^6*d^2*e/c^4 + 5/128*(c*x^2 + b*x)^(3/2)*b^4*d^2*e/c^3 - 1/8*(c*x^
2 + b*x)^(5/2)*b^2*d^2*e/c^2 + 3/7*(c*x^2 + b*x)^(7/2)*d^2*e/c + 135/16384
*sqrt(c*x^2 + b*x)*b^7*d*e^2/c^5 - 45/2048*(c*x^2 + b*x)^(3/2)*b^5*d*e^2/c
^4 + 9/128*(c*x^2 + b*x)^(5/2)*b^3*d*e^2/c^3 - 27/112*(c*x^2 + b*x)^(7/2)*
b*d*e^2/c^2 - 55/32768*sqrt(c*x^2 + b*x)*b^8*e^3/c^6 + 55/12288*(c*x^2 + b
*x)^(3/2)*b^6*e^3/c^5 - 11/768*(c*x^2 + b*x)^(5/2)*b^4*e^3/c^4 + 11/224...
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 489, normalized size of antiderivative = 0.96

$$\int (d+ex)^3 \left(bx+cx^2\right)^{5/2} dx = \frac{1}{2064384} \sqrt{cx^2+bx} \left(2 \left(4 \left(2 \left(8 \left(2 \left(4 \left(14 \left(16 c^2 e^3 x + \frac{54 c^{10} d e^2 + 37 b c^9 e^3}{c^8}\right) x + \frac{3 (288 c^1}{\ldots}\right.\right.\right.\right.\right.\right.\right. + \frac{5 (64 b^6 c^3 d^3 - 96 b^7 c^2 d^2 e + 54 b^8 c d e^2 - 11 b^9 e^3) \log \left(\left|2 \left(\sqrt{c} x - \sqrt{cx^2+bx}\right) \sqrt{c} + b\right|\right)}{65536 c^{\frac{13}{2}}}$$

input
```
integrate((e*x+d)^3*(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
1/2064384*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(2*(4*(14*(16*c^2*e^3*x + (54*c^10
*d*e^2 + 37*b*c^9*e^3)/c^8)*x + 3*(288*c^10*d^2*e + 594*b*c^9*d*e^2 + 103*
b^2*c^8*e^3)/c^8)*x + (1344*c^10*d^3 + 8352*b*c^9*d^2*e + 4374*b^2*c^8*d*e
^2 + 5*b^3*c^7*e^3)/c^8)*x + (6720*b*c^9*d^3 + 10656*b^2*c^8*d^2*e + 54*b^
3*c^7*d*e^2 - 11*b^4*c^6*e^3)/c^8)*x + 9*(4032*b^2*c^8*d^3 + 96*b^3*c^7*d^
2*e - 54*b^4*c^6*d*e^2 + 11*b^5*c^5*e^3)/c^8)*x + 21*(64*b^3*c^7*d^3 - 96*
b^4*c^6*d^2*e + 54*b^5*c^5*d*e^2 - 11*b^6*c^4*e^3)/c^8)*x - 105*(64*b^4*c^
6*d^3 - 96*b^5*c^5*d^2*e + 54*b^6*c^4*d*e^2 - 11*b^7*c^3*e^3)/c^8)*x + 315
*(64*b^5*c^5*d^3 - 96*b^6*c^4*d^2*e + 54*b^7*c^3*d*e^2 - 11*b^8*c^2*e^3)/c
^8) + 5/65536*(64*b^6*c^3*d^3 - 96*b^7*c^2*d^2*e + 54*b^8*c*d*e^2 - 11*b^9
*e^3)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(13/2)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^3 (bx+cx^2)^{5/2} \, dx = \int (c\,x^2+b\,x)^{5/2} (d+e\,x)^3 \, dx$$

input
```
int((b*x + c*x^2)^(5/2)*(d + e*x)^3,x)
```

output
```
int((b*x + c*x^2)^(5/2)*(d + e*x)^3, x)
```

Reduce [F]

$$\int (d+ex)^3 (bx+cx^2)^{5/2} \, dx = \int (ex+d)^3 (c\,x^2+bx)^{\frac{5}{2}} \, dx$$

input
```
int((e*x+d)^3*(c*x^2+b*x)^(5/2),x)
```

output
```
int((e*x+d)^3*(c*x^2+b*x)^(5/2),x)
```

3.149 $\int (d+ex)^2 \left(bx+cx^2\right)^{5/2} \, dx$

Optimal result . . . 1193
Mathematica [A] (verified) . . . 1194
Rubi [A] (verified) . . . 1195
Maple [A] (verified) . . . 1198
Fricas [A] (verification not implemented) . . . 1200
Sympy [B] (verification not implemented) . . . 1201
Maxima [A] (verification not implemented) . . . 1203
Giac [A] (verification not implemented) . . . 1204
Mupad [F(-1)] . . . 1204
Reduce [B] (verification not implemented) . . . 1205

Optimal result

Integrand size = 21, antiderivative size = 392

$$
\begin{aligned}
\int (d+ex)^2 \left(bx+cx^2\right)^{5/2} \, dx &= \frac{5b^5(32c^2d^2-32bcde+9b^2e^2)\sqrt{bx+cx^2}}{16384c^5} \\
&- \frac{5b^4(32c^2d^2-32bcde+9b^2e^2)\,x\sqrt{bx+cx^2}}{24576c^4} \\
&+ \frac{b^3(32c^2d^2-32bcde+9b^2e^2)\,x^2\sqrt{bx+cx^2}}{6144c^3} \\
&+ \frac{9b^2(32c^2d^2-32bcde+9b^2e^2)\,x^3\sqrt{bx+cx^2}}{1024c^2} \\
&+ \frac{5b(32c^2d^2-32bcde+9b^2e^2)\,x^4\sqrt{bx+cx^2}}{384c} \\
&+ \frac{1}{192}(32c^2d^2-32bcde+9b^2e^2)\,x^5\sqrt{bx+cx^2} + \frac{e(32cd-9be)\left(bx+cx^2\right)^{7/2}}{112c^2} \\
&+ \frac{e^2x(bx+cx^2)^{7/2}}{8c} - \frac{5b^6(32c^2d^2-32bcde+9b^2e^2)\,\mathrm{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{16384c^{11/2}}
\end{aligned}
$$

output
```
5/16384*b^5*(9*b^2*e^2-32*b*c*d*e+32*c^2*d^2)*(c*x^2+b*x)^(1/2)/c^5-5/2457
6*b^4*(9*b^2*e^2-32*b*c*d*e+32*c^2*d^2)*x*(c*x^2+b*x)^(1/2)/c^4+1/6144*b^3
*(9*b^2*e^2-32*b*c*d*e+32*c^2*d^2)*x^2*(c*x^2+b*x)^(1/2)/c^3+9/1024*b^2*(9
*b^2*e^2-32*b*c*d*e+32*c^2*d^2)*x^3*(c*x^2+b*x)^(1/2)/c^2+5/384*b*(9*b^2*e
^2-32*b*c*d*e+32*c^2*d^2)*x^4*(c*x^2+b*x)^(1/2)/c+1/192*(9*b^2*e^2-32*b*c*
d*e+32*c^2*d^2)*x^5*(c*x^2+b*x)^(1/2)+1/112*e*(-9*b*e+32*c*d)*(c*x^2+b*x)^
(7/2)/c^2+1/8*e^2*x*(c*x^2+b*x)^(7/2)/c-5/16384*b^6*(9*b^2*e^2-32*b*c*d*e+
32*c^2*d^2)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 1.95 (sec) , antiderivative size = 341, normalized size of antiderivative = 0.87

$$\int (d+ex)^2 \left(bx+cx^2\right)^{5/2} dx = \frac{\sqrt{x}\sqrt{b+cx}\left(\sqrt{c}\sqrt{x}\sqrt{b+cx}(945b^7e^2 - 210b^6ce(16d+3ex) + 128b^3c^4x^2(14d^2+12dex+3e^2\right.}{}$$

input
```
Integrate[(d + e*x)^2*(b*x + c*x^2)^(5/2),x]
```

output
```
(Sqrt[x]*Sqrt[b + c*x]*(Sqrt[c]*Sqrt[x]*Sqrt[b + c*x]*(945*b^7*e^2 - 210*b
^6*c*e*(16*d + 3*e*x) + 128*b^3*c^4*x^2*(14*d^2 + 12*d*e*x + 3*e^2*x^2) +
56*b^5*c^2*(60*d^2 + 40*d*e*x + 9*e^2*x^2) + 2048*c^7*x^5*(28*d^2 + 48*d*e
*x + 21*e^2*x^2) - 16*b^4*c^3*x*(140*d^2 + 112*d*e*x + 27*e^2*x^2) + 1024*
b*c^6*x^4*(140*d^2 + 232*d*e*x + 99*e^2*x^2) + 256*b^2*c^5*x^3*(378*d^2 +
592*d*e*x + 243*e^2*x^2)) + 210*b^6*(32*c^2*d^2 + 9*b^2*e^2)*ArcTanh[(Sqrt
[c]*Sqrt[x])/(Sqrt[b] - Sqrt[b + c*x])] + 6720*b^7*c*d*e*ArcTanh[(Sqrt[c]*
Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])]))/(344064*c^(11/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.58, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1166, 27, 1160, 1087, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{5/2} (d+ex)^2 \, dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{\int \frac{1}{2}(d(16cd-7be)+9e(2cd-be)x) \left(cx^2+bx\right)^{5/2}dx}{8c}+\frac{e\left(bx+cx^2\right)^{7/2} (d+ex)}{8c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int (d(16cd-7be)+9e(2cd-be)x) \left(cx^2+bx\right)^{5/2}dx}{16c}+\frac{e\left(bx+cx^2\right)^{7/2} (d+ex)}{8c}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{\left(9b^2e^2-32bcde+32c^2d^2\right)\int \left(cx^2+bx\right)^{5/2}dx}{2c}+\frac{9e\left(bx+cx^2\right)^{7/2}(2cd-be)}{7c}}{16c}+\frac{e\left(bx+cx^2\right)^{7/2} (d+ex)}{8c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{\left(9b^2e^2-32bcde+32c^2d^2\right)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\int \left(cx^2+bx\right)^{3/2}dx}{24c}\right)}{2c}+\frac{9e\left(bx+cx^2\right)^{7/2}(2cd-be)}{7c}}{16c}+\frac{e\left(bx+cx^2\right)^{7/2} (d+ex)}{8c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{\left(9b^2e^2-32bcde+32c^2d^2\right)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int \sqrt{cx^2+bx}dx}{16c}\right)}{24c}\right)}{2c}+\frac{9e\left(bx+cx^2\right)^{7/2}(2cd-be)}{7c}}{16c}+\frac{e\left(bx+cx^2\right)^{7/2} (d+ex)}{8c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{\frac{\left(9b^2e^2-32bcde+32c^2d^2\right)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)}{24c}\right)}{2c}+\frac{9e\left(bx+cx^2\right)^{7/2}(2c}{7c}}{16c}$$

$$\frac{e\left(bx+cx^2\right)^{7/2}(d+ex)}{8c}$$

$\Big\downarrow$ 1091

$$\frac{\frac{\left(9b^2e^2-32bcde+32c^2d^2\right)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)}{24c}\right)}{2c}+\frac{9e(bx+c}{}}{16c}$$

$$\frac{e\left(bx+cx^2\right)^{7/2}(d+ex)}{8c}$$

$\Big\downarrow$ 219

$$\frac{\frac{\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)}{24c}\right)\left(9b^2e^2-32bcde+32c^2d^2\right)}{2c}+\frac{9e(bx+cx}{}}{16c}$$

$$\frac{e\left(bx+cx^2\right)^{7/2}(d+ex)}{8c}$$

input
```
Int[(d + e*x)^2*(b*x + c*x^2)^(5/2),x]
```

output
```
(e*(d + e*x)*(b*x + c*x^2)^(7/2))/(8*c) + ((9*e*(2*c*d - b*e)*(b*x + c*x^2
)^(7/2))/(7*c) + ((32*c^2*d^2 - 32*b*c*d*e + 9*b^2*e^2)*(((b + 2*c*x)*(b*x
 + c*x^2)^(5/2))/(12*c) - (5*b^2*(((b + 2*c*x)*(b*x + c*x^2)^(3/2))/(8*c)
- (3*b^2*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)
/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))/(16*c)))/(24*c)))/(2*c))/(16*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.84

method	result
risch	$\frac{(43008c^7e^2x^7+101376b\,c^6e^2x^6+98304c^7de\,x^6+62208b^2c^5e^2x^5+237568b\,c^6de\,x^5+57344c^7d^2x^5+384b^3c^4e^2x^4+151552b^2c^5de\,x^4+}{}$
default	$d^2\left(\frac{(2cx+b)(c\,x^2+bx)^{\frac{5}{2}}}{12c}-\frac{5b^2\left(\frac{(2cx+b)\left(c\,x^2+bx\right)^{\frac{3}{2}}}{8c}-\frac{3b^2\left(\frac{(2cx+b)\sqrt{c\,x^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{24c}\right)+e^2\left(\frac{x(c\,x^2+}{8c}\right.$

input
```
int((e*x+d)^2*(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/344064/c^5*(43008*c^7*e^2*x^7+101376*b*c^6*e^2*x^6+98304*c^7*d*e*x^6+622
08*b^2*c^5*e^2*x^5+237568*b*c^6*d*e*x^5+57344*c^7*d^2*x^5+384*b^3*c^4*e^2*
x^4+151552*b^2*c^5*d*e*x^4+143360*b*c^6*d^2*x^4-432*b^4*c^3*e^2*x^3+1536*b
^3*c^4*d*e*x^3+96768*b^2*c^5*d^2*x^3+504*b^5*c^2*e^2*x^2-1792*b^4*c^3*d*e*
x^2+1792*b^3*c^4*d^2*x^2-630*b^6*c*e^2*x+2240*b^5*c^2*d*e*x-2240*b^4*c^3*d
^2*x+945*b^7*e^2-3360*b^6*c*d*e+3360*b^5*c^2*d^2)*x*(c*x+b)/(x*(c*x+b))^(1
/2)-5/32768*b^6*(9*b^2*e^2-32*b*c*d*e+32*c^2*d^2)/c^(11/2)*ln((1/2*b+c*x)/
c^(1/2)+(c*x^2+b*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.64

$$\int (d+ex)^2 \left(bx+cx^2\right)^{5/2} dx = \left[\frac{105 \left(32 b^6 c^2 d^2 - 32 b^7 cde + 9 b^8 e^2\right) \sqrt{c} \log\left(2 cx + b - 2 \sqrt{cx^2+bx} \sqrt{c}\right) + 2 \left(43008 c^8 e^2 x^7 + \right.}{}\right.$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/688128*(105*(32*b^6*c^2*d^2 - 32*b^7*c*d*e + 9*b^8*e^2)*sqrt(c)*log(2*c
*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(43008*c^8*e^2*x^7 + 3360*b^5*c^
3*d^2 - 3360*b^6*c^2*d*e + 945*b^7*c*e^2 + 3072*(32*c^8*d*e + 33*b*c^7*e^2
)*x^6 + 256*(224*c^8*d^2 + 928*b*c^7*d*e + 243*b^2*c^6*e^2)*x^5 + 128*(112
0*b*c^7*d^2 + 1184*b^2*c^6*d*e + 3*b^3*c^5*e^2)*x^4 + 48*(2016*b^2*c^6*d^2
 + 32*b^3*c^5*d*e - 9*b^4*c^4*e^2)*x^3 + 56*(32*b^3*c^5*d^2 - 32*b^4*c^4*d
*e + 9*b^5*c^3*e^2)*x^2 - 70*(32*b^4*c^4*d^2 - 32*b^5*c^3*d*e + 9*b^6*c^2*
e^2)*x)*sqrt(c*x^2 + b*x))/c^6, 1/344064*(105*(32*b^6*c^2*d^2 - 32*b^7*c*d
*e + 9*b^8*e^2)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (4
3008*c^8*e^2*x^7 + 3360*b^5*c^3*d^2 - 3360*b^6*c^2*d*e + 945*b^7*c*e^2 + 3
072*(32*c^8*d*e + 33*b*c^7*e^2)*x^6 + 256*(224*c^8*d^2 + 928*b*c^7*d*e + 2
43*b^2*c^6*e^2)*x^5 + 128*(1120*b*c^7*d^2 + 1184*b^2*c^6*d*e + 3*b^3*c^5*e
^2)*x^4 + 48*(2016*b^2*c^6*d^2 + 32*b^3*c^5*d*e - 9*b^4*c^4*e^2)*x^3 + 56*
(32*b^3*c^5*d^2 - 32*b^4*c^4*d*e + 9*b^5*c^3*e^2)*x^2 - 70*(32*b^4*c^4*d^2
 - 32*b^5*c^3*d*e + 9*b^6*c^2*e^2)*x)*sqrt(c*x^2 + b*x))/c^6]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 986 vs. $2(393) = 786$.

Time = 0.56 (sec) , antiderivative size = 986, normalized size of antiderivative = 2.52

$$\int (d+ex)^2 \left(bx+cx^2\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**2*(c*x**2+b*x)**(5/2),x)
```

output

```
Piecewise((-5*b**3*(b**3*d**2 - 7*b*(2*b**3*d*e + 3*b**2*c*d**2 - 9*b*(b**
3*e**2 + 6*b**2*c*d*e + 3*b*c**2*d**2 - 11*b*(3*b**2*c*e**2 + 6*b*c**2*d*e
 - 13*b*(33*b*c**2*e**2/16 + 2*c**3*d*e)/(14*c) + c**3*d**2)/(12*c))/(10*c
))/(8*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c)
, Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2)
, True))/(16*c**3) + sqrt(b*x + c*x**2)*(5*b**2*(b**3*d**2 - 7*b*(2*b**3*d
*e + 3*b**2*c*d**2 - 9*b*(b**3*e**2 + 6*b**2*c*d*e + 3*b*c**2*d**2 - 11*b*
(3*b**2*c*e**2 + 6*b*c**2*d*e - 13*b*(33*b*c**2*e**2/16 + 2*c**3*d*e)/(14*
c) + c**3*d**2)/(12*c))/(10*c))/(8*c))/(8*c**3) - 5*b*x*(b**3*d**2 - 7*b*(
2*b**3*d*e + 3*b**2*c*d**2 - 9*b*(b**3*e**2 + 6*b**2*c*d*e + 3*b*c**2*d**2
 - 11*b*(3*b**2*c*e**2 + 6*b*c**2*d*e - 13*b*(33*b*c**2*e**2/16 + 2*c**3*d
*e)/(14*c) + c**3*d**2)/(12*c))/(10*c))/(8*c))/(12*c**2) + c**2*e**2*x**7/
8 + x**6*(33*b*c**2*e**2/16 + 2*c**3*d*e)/(7*c) + x**5*(3*b**2*c*e**2 + 6*
b*c**2*d*e - 13*b*(33*b*c**2*e**2/16 + 2*c**3*d*e)/(14*c) + c**3*d**2)/(6*
c) + x**4*(b**3*e**2 + 6*b**2*c*d*e + 3*b*c**2*d**2 - 11*b*(3*b**2*c*e**2
+ 6*b*c**2*d*e - 13*b*(33*b*c**2*e**2/16 + 2*c**3*d*e)/(14*c) + c**3*d**2)
/(12*c))/(5*c) + x**3*(2*b**3*d*e + 3*b**2*c*d**2 - 9*b*(b**3*e**2 + 6*b**
2*c*d*e + 3*b*c**2*d**2 - 11*b*(3*b**2*c*e**2 + 6*b*c**2*d*e - 13*b*(33*b*
c**2*e**2/16 + 2*c**3*d*e)/(14*c) + c**3*d**2)/(12*c))/(10*c))/(4*c) + x**
2*(b**3*d**2 - 7*b*(2*b**3*d*e + 3*b**2*c*d**2 - 9*b*(b**3*e**2 + 6*b**...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.40

$$\int (d+ex)^2 \left(bx+cx^2\right)^{5/2} dx = \frac{1}{6} \left(cx^2+bx\right)^{\frac{5}{2}} d^2 x + \frac{5 \sqrt{cx^2+bx} b^4 d^2 x}{256 c^2} - \frac{5 \left(cx^2+bx\right)^{\frac{3}{2}} b^2 d^2 x}{96 c} - \frac{5 \sqrt{cx^2+bx} b^5 dex}{256 c^3} + \frac{5 \left(cx^2+bx\right)^{\frac{3}{2}} b^3 dex}{96 c^2} - \frac{\left(cx^2+bx\right)^{\frac{5}{2}} bdex}{6 c} + \frac{45 \sqrt{cx^2+bx} b^6 e^2 x}{8192 c^4} - \frac{15 \left(cx^2+bx\right)^{\frac{3}{2}} b^4 e^2 x}{1024 c^3} + \frac{3 \left(cx^2+bx\right)^{\frac{5}{2}} b^2 e^2 x}{64 c^2} + \frac{\left(cx^2+bx\right)^{\frac{7}{2}} e^2 x}{8 c} - \frac{5 b^6 d^2 \log\left(2 cx+b+2 \sqrt{cx^2+bx} \sqrt{c}\right)}{1024 c^{\frac{7}{2}}} + \frac{5 b^7 de \log\left(2 cx+b+2 \sqrt{cx^2+bx} \sqrt{c}\right)}{1024 c^{\frac{9}{2}}} - \frac{45 b^8 e^2 \log\left(2 cx+b+2 \sqrt{cx^2+bx} \sqrt{c}\right)}{32768 c^{\frac{11}{2}}} + \frac{5 \sqrt{cx^2+bx} b^5 d^2}{512 c^3} - \frac{5 \left(cx^2+bx\right)^{\frac{3}{2}} b^3 d^2}{192 c^2} + \frac{\left(cx^2+bx\right)^{\frac{5}{2}} bd^2}{12 c} - \frac{5 \sqrt{cx^2+bx} b^6 de}{512 c^4} + \frac{5 \left(cx^2+bx\right)^{\frac{3}{2}} b^4 de}{192 c^3} - \frac{\left(cx^2+bx\right)^{\frac{5}{2}} b^2 de}{12 c^2} + \frac{2 \left(cx^2+bx\right)^{\frac{7}{2}} de}{7 c} + \frac{45 \sqrt{cx^2+bx} b^7 e^2}{16384 c^5} - \frac{15 \left(cx^2+bx\right)^{\frac{3}{2}} b^5 e^2}{2048 c^4} + \frac{3 \left(cx^2+bx\right)^{\frac{5}{2}} b^3 e^2}{128 c^3} - \frac{9 \left(cx^2+bx\right)^{\frac{7}{2}} be^2}{112 c^2}$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(c*x^2 + b*x)^(5/2)*d^2*x + 5/256*sqrt(c*x^2 + b*x)*b^4*d^2*x/c^2 - 5/
96*(c*x^2 + b*x)^(3/2)*b^2*d^2*x/c - 5/256*sqrt(c*x^2 + b*x)*b^5*d*e*x/c^3
 + 5/96*(c*x^2 + b*x)^(3/2)*b^3*d*e*x/c^2 - 1/6*(c*x^2 + b*x)^(5/2)*b*d*e*
x/c + 45/8192*sqrt(c*x^2 + b*x)*b^6*e^2*x/c^4 - 15/1024*(c*x^2 + b*x)^(3/2
)*b^4*e^2*x/c^3 + 3/64*(c*x^2 + b*x)^(5/2)*b^2*e^2*x/c^2 + 1/8*(c*x^2 + b*
x)^(7/2)*e^2*x/c - 5/1024*b^6*d^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt
(c))/c^(7/2) + 5/1024*b^7*d*e*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))
/c^(9/2) - 45/32768*b^8*e^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c
^(11/2) + 5/512*sqrt(c*x^2 + b*x)*b^5*d^2/c^3 - 5/192*(c*x^2 + b*x)^(3/2)*
b^3*d^2/c^2 + 1/12*(c*x^2 + b*x)^(5/2)*b*d^2/c - 5/512*sqrt(c*x^2 + b*x)*b
^6*d*e/c^4 + 5/192*(c*x^2 + b*x)^(3/2)*b^4*d*e/c^3 - 1/12*(c*x^2 + b*x)^(5
/2)*b^2*d*e/c^2 + 2/7*(c*x^2 + b*x)^(7/2)*d*e/c + 45/16384*sqrt(c*x^2 + b*
x)*b^7*e^2/c^5 - 15/2048*(c*x^2 + b*x)^(3/2)*b^5*e^2/c^4 + 3/128*(c*x^2 +
b*x)^(5/2)*b^3*e^2/c^3 - 9/112*(c*x^2 + b*x)^(7/2)*b*e^2/c^2
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 349, normalized size of antiderivative = 0.89

$$\int (d+ex)^2 (bx+cx^2)^{5/2}\, dx = \frac{1}{344064}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\left(2\left(12\left(14\,c^2e^2x+\frac{32\,c^9de+33\,bc^8e^2}{c^7}\right)x+\frac{224\,c^9d^2+928}{}\right.\right.\right.\right.\right.\right. + \frac{5\,(32\,b^6c^2d^2-32\,b^7cde+9\,b^8e^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{32768\,c^{\frac{11}{2}}}$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
1/344064*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(2*(12*(14*c^2*e^2*x + (32*c^9*d*e
 + 33*b*c^8*e^2)/c^7)*x + (224*c^9*d^2 + 928*b*c^8*d*e + 243*b^2*c^7*e^2)/c
^7)*x + (1120*b*c^8*d^2 + 1184*b^2*c^7*d*e + 3*b^3*c^6*e^2)/c^7)*x + 3*(20
16*b^2*c^7*d^2 + 32*b^3*c^6*d*e - 9*b^4*c^5*e^2)/c^7)*x + 7*(32*b^3*c^6*d^
2 - 32*b^4*c^5*d*e + 9*b^5*c^4*e^2)/c^7)*x - 35*(32*b^4*c^5*d^2 - 32*b^5*c
^4*d*e + 9*b^6*c^3*e^2)/c^7)*x + 105*(32*b^5*c^4*d^2 - 32*b^6*c^3*d*e + 9*
b^7*c^2*e^2)/c^7) + 5/32768*(32*b^6*c^2*d^2 - 32*b^7*c*d*e + 9*b^8*e^2)*lo
g(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(bx+cx^2\right)^{5/2}\, dx = \int \left(c\,x^2+b\,x\right)^{5/2}\,(d+e\,x)^2\,dx$$

input
```
int((b*x + c*x^2)^(5/2)*(d + e*x)^2,x)
```

output
```
int((b*x + c*x^2)^(5/2)*(d + e*x)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.32

$$\int (d+ex)^2 \left(bx+cx^2\right)^{5/2} dx = \frac{945\sqrt{x}\,\sqrt{cx+b}\,b^7c\,e^2 - 3360\sqrt{x}\,\sqrt{cx+b}\,b^6c^2de - 630\sqrt{x}\,\sqrt{cx+b}\,b^6c^2e^2x + 3360\sqrt{x}\,\sqrt{cx+b}}{}$$

input
```
int((e*x+d)^2*(c*x^2+b*x)^(5/2),x)
```

output
```
(945*sqrt(x)*sqrt(b + c*x)*b**7*c*e**2 - 3360*sqrt(x)*sqrt(b + c*x)*b**6*c
**2*d*e - 630*sqrt(x)*sqrt(b + c*x)*b**6*c**2*e**2*x + 3360*sqrt(x)*sqrt(b
 + c*x)*b**5*c**3*d**2 + 2240*sqrt(x)*sqrt(b + c*x)*b**5*c**3*d*e*x + 504*
sqrt(x)*sqrt(b + c*x)*b**5*c**3*e**2*x**2 - 2240*sqrt(x)*sqrt(b + c*x)*b**
4*c**4*d**2*x - 1792*sqrt(x)*sqrt(b + c*x)*b**4*c**4*d*e*x**2 - 432*sqrt(x
)*sqrt(b + c*x)*b**4*c**4*e**2*x**3 + 1792*sqrt(x)*sqrt(b + c*x)*b**3*c**5
*d**2*x**2 + 1536*sqrt(x)*sqrt(b + c*x)*b**3*c**5*d*e*x**3 + 384*sqrt(x)*s
qrt(b + c*x)*b**3*c**5*e**2*x**4 + 96768*sqrt(x)*sqrt(b + c*x)*b**2*c**6*d
**2*x**3 + 151552*sqrt(x)*sqrt(b + c*x)*b**2*c**6*d*e*x**4 + 62208*sqrt(x)
*sqrt(b + c*x)*b**2*c**6*e**2*x**5 + 143360*sqrt(x)*sqrt(b + c*x)*b*c**7*d
**2*x**4 + 237568*sqrt(x)*sqrt(b + c*x)*b*c**7*d*e*x**5 + 101376*sqrt(x)*s
qrt(b + c*x)*b*c**7*e**2*x**6 + 57344*sqrt(x)*sqrt(b + c*x)*c**8*d**2*x**5
 + 98304*sqrt(x)*sqrt(b + c*x)*c**8*d*e*x**6 + 43008*sqrt(x)*sqrt(b + c*x)
*c**8*e**2*x**7 - 945*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b
))*b**8*e**2 + 3360*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))
*b**7*c*d*e - 3360*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*
b**6*c**2*d**2)/(344064*c**6)
```

3.150 $\int (d+ex) \left(bx+cx^2\right)^{5/2} \, dx$

Optimal result 1206
Mathematica [A] (verified) 1207
Rubi [A] (verified) 1207
Maple [A] (verified) 1210
Fricas [A] (verification not implemented) 1211
Sympy [B] (verification not implemented) 1212
Maxima [A] (verification not implemented) 1213
Giac [A] (verification not implemented) 1214
Mupad [F(-1)] 1215
Reduce [B] (verification not implemented) 1215

Optimal result

Integrand size = 19, antiderivative size = 259

$$\int (d+ex) \left(bx+cx^2\right)^{5/2} \, dx = \frac{5b^5(2cd-be)\sqrt{bx+cx^2}}{1024c^4} - \frac{5b^4(2cd-be)x\sqrt{bx+cx^2}}{1536c^3} + \frac{b^3(2cd-be)x^2\sqrt{bx+cx^2}}{384c^2} + \frac{9b^2(2cd-be)x^3\sqrt{bx+cx^2}}{64c} + \frac{5}{24}b(2cd-be)x^4\sqrt{bx+cx^2} + \frac{1}{12}c(2cd-be)x^5\sqrt{bx+cx^2} + \frac{e\left(bx+cx^2\right)^{7/2}}{7c} - \frac{5b^6(2cd-be)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{1024c^{9/2}}$$

output

```
5/1024*b^5*(-b*e+2*c*d)*(c*x^2+b*x)^(1/2)/c^4-5/1536*b^4*(-b*e+2*c*d)*x*(c
*x^2+b*x)^(1/2)/c^3+1/384*b^3*(-b*e+2*c*d)*x^2*(c*x^2+b*x)^(1/2)/c^2+9/64*
b^2*(-b*e+2*c*d)*x^3*(c*x^2+b*x)^(1/2)/c+5/24*b*(-b*e+2*c*d)*x^4*(c*x^2+b*
x)^(1/2)+1/12*c*(-b*e+2*c*d)*x^5*(c*x^2+b*x)^(1/2)+1/7*e*(c*x^2+b*x)^(7/2)
/c-5/1024*b^6*(-b*e+2*c*d)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.92

$$\int (d+ex)\left(bx+cx^2\right)^{5/2}\,dx=\frac{\sqrt{x}\sqrt{b+cx}\left(\sqrt{c}\sqrt{x}\sqrt{b+cx}(-105b^6e+70b^5c(3d+ex)-28b^4c^2x(5d+2ex)+16b^3c^3x^2(7d\right.}{}$$

input
```
Integrate[(d + e*x)*(b*x + c*x^2)^(5/2),x]
```

output
```
(Sqrt[x]*Sqrt[b + c*x]*(Sqrt[c]*Sqrt[x]*Sqrt[b + c*x]*(-105*b^6*e + 70*b^5
*c*(3*d + e*x) - 28*b^4*c^2*x*(5*d + 2*e*x) + 16*b^3*c^3*x^2*(7*d + 3*e*x)
 + 512*c^6*x^5*(7*d + 6*e*x) + 256*b*c^5*x^4*(35*d + 29*e*x) + 32*b^2*c^4*
x^3*(189*d + 148*e*x)) + 420*b^6*c*d*ArcTanh[(Sqrt[c]*Sqrt[x])/(Sqrt[b] -
Sqrt[b + c*x])] + 210*b^7*e*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b +
 c*x])]))/(21504*c^(9/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.66, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1160, 1087, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{5/2}(d+ex)\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{(2cd-be)\int \left(cx^2+bx\right)^{5/2}dx}{2c}+\frac{e\left(bx+cx^2\right)^{7/2}}{7c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(2cd-be)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\int \left(cx^2+bx\right)^{3/2}dx}{24c}\right)}{2c}+\frac{e\left(bx+cx^2\right)^{7/2}}{7c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(2cd-be)\left(\frac{(b+2cx)(bx+cx^2)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int\sqrt{cx^2+bx}dx}{16c}\right)}{24c}\right)}{2c}+\frac{e\left(bx+cx^2\right)^{7/2}}{7c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(2cd-be)\left(\frac{(b+2cx)(bx+cx^2)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)}{24c}\right)}{2c}+\frac{e\left(bx+cx^2\right)^{7/2}}{7c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(2cd-be)\left(\frac{(b+2cx)(bx+cx^2)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)}{24c}\right)}{2c}+\frac{e\left(bx+cx^2\right)^{7/2}}{7c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)}{24c}\right)(2cd-be)}{2c}+\frac{e\left(bx+cx^2\right)^{7/2}}{7c}$$

input
```
Int[(d + e*x)*(b*x + c*x^2)^(5/2),x]
```

output
```
(e*(b*x + c*x^2)^(7/2))/(7*c) + ((2*c*d - b*e)*(((b + 2*c*x)*(b*x + c*x^2)
^(5/2))/(12*c) - (5*b^2*(((b + 2*c*x)*(b*x + c*x^2)^(3/2))/(8*c) - (3*b^2*
(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x
 + c*x^2]])/(4*c^(3/2))))/(16*c)))/(24*c)))/(2*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\left(-3072c^{6}e\,x^{6}-7424b\,c^{5}e\,x^{5}-3584c^{6}d\,x^{5}-4736b^{2}c^{4}e\,x^{4}-8960b\,c^{5}d\,x^{4}-48b^{3}c^{3}e\,x^{3}-6048b^{2}c^{4}d\,x^{3}+56b^{4}c^{2}e\,x^{2}-112b^{3}c^{3}d\,x^{2}-70}{21504c^{4}\sqrt{x(cx+b)}}$
default	$d\left(\frac{(2cx+b)\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{12c}-\frac{5b^{2}\left(\frac{(2cx+b)\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{8c}-\frac{3b^{2}\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx}}{4c}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{24c}\right)+e\left(\frac{(c\,x^{2}+bx)}{7c}\right.$

input
```
int((e*x+d)*(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21504/c^4*(-3072*c^6*e*x^6-7424*b*c^5*e*x^5-3584*c^6*d*x^5-4736*b^2*c^4
*e*x^4-8960*b*c^5*d*x^4-48*b^3*c^3*e*x^3-6048*b^2*c^4*d*x^3+56*b^4*c^2*e*x
^2-112*b^3*c^3*d*x^2-70*b^5*c*e*x+140*b^4*c^2*d*x+105*b^6*e-210*b^5*c*d)*x
*(c*x+b)/(x*(c*x+b))^(1/2)+5/2048*b^6*(b*e-2*c*d)/c^(9/2)*ln((1/2*b+c*x)/c
^(1/2)+(c*x^2+b*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.54

$$\int (d+ex)\left(bx+cx^2\right)^{5/2}\,dx = \left[-\frac{105\,(2\,b^6cd-b^7e)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(3072\,c^7ex^6+210\,b^5c^2d-105\,b\right.$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/43008*(105*(2*b^6*c*d - b^7*e)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 +
b*x)*sqrt(c)) - 2*(3072*c^7*e*x^6 + 210*b^5*c^2*d - 105*b^6*c*e + 256*(14*
c^7*d + 29*b*c^6*e)*x^5 + 128*(70*b*c^6*d + 37*b^2*c^5*e)*x^4 + 48*(126*b^
2*c^5*d + b^3*c^4*e)*x^3 + 56*(2*b^3*c^4*d - b^4*c^3*e)*x^2 - 70*(2*b^4*c^
3*d - b^5*c^2*e)*x)*sqrt(c*x^2 + b*x))/c^5, 1/21504*(105*(2*b^6*c*d - b^7*
e)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (3072*c^7*e*x^6
 + 210*b^5*c^2*d - 105*b^6*c*e + 256*(14*c^7*d + 29*b*c^6*e)*x^5 + 128*(70
*b*c^6*d + 37*b^2*c^5*e)*x^4 + 48*(126*b^2*c^5*d + b^3*c^4*e)*x^3 + 56*(2*
b^3*c^4*d - b^4*c^3*e)*x^2 - 70*(2*b^4*c^3*d - b^5*c^2*e)*x)*sqrt(c*x^2 +
b*x))/c^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. 2(240) = 480.

Time = 0.52 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.20

$$\int (d+ex)\left(bx+cx^2\right)^{5/2}\,dx = \begin{cases} -\frac{5b^3\left(b^3d-\frac{7b\left(b^3e+3b^2cd-\frac{9b\left(3b^2ce+3bc^2d-\frac{11b\left(\frac{29bc^2e}{14}+c^3d\right)}{12c}\right)}{10c}\right)}{8c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{16c^3} \\ \frac{2\left(\frac{d(bx)^{\frac{7}{2}}}{7}+\frac{e(bx)^{\frac{9}{2}}}{9b}\right)}{b} \\ 0 \end{cases}$$

input
```
integrate((e*x+d)*(c*x**2+b*x)**(5/2),x)
```

output
```
Piecewise((-5*b**3*(b**3*d - 7*b*(b**3*e + 3*b**2*c*d - 9*b*(3*b**2*c*e +
3*b*c**2*d - 11*b*(29*b*c**2*e/14 + c**3*d)/(12*c))/(10*c))/(8*c))*Piecewi
se((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)),
 ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(16*c**3
) + sqrt(b*x + c*x**2)*(5*b**2*(b**3*d - 7*b*(b**3*e + 3*b**2*c*d - 9*b*(3
*b**2*c*e + 3*b*c**2*d - 11*b*(29*b*c**2*e/14 + c**3*d)/(12*c))/(10*c))/(8
*c))/(8*c**3) - 5*b*x*(b**3*d - 7*b*(b**3*e + 3*b**2*c*d - 9*b*(3*b**2*c*e
 + 3*b*c**2*d - 11*b*(29*b*c**2*e/14 + c**3*d)/(12*c))/(10*c))/(8*c))/(12*
c**2) + c**2*e*x**6/7 + x**5*(29*b*c**2*e/14 + c**3*d)/(6*c) + x**4*(3*b**
2*c*e + 3*b*c**2*d - 11*b*(29*b*c**2*e/14 + c**3*d)/(12*c))/(5*c) + x**3*(
b**3*e + 3*b**2*c*d - 9*b*(3*b**2*c*e + 3*b*c**2*d - 11*b*(29*b*c**2*e/14
+ c**3*d)/(12*c))/(10*c))/(4*c) + x**2*(b**3*d - 7*b*(b**3*e + 3*b**2*c*d
- 9*b*(3*b**2*c*e + 3*b*c**2*d - 11*b*(29*b*c**2*e/14 + c**3*d)/(12*c))/(1
0*c))/(8*c))/(3*c)), Ne(c, 0)), (2*(d*(b*x)**(7/2)/7 + e*(b*x)**(9/2)/(9*b
))/b, Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.23

$$\int (d+ex)\left(bx+cx^2\right)^{5/2}\,dx = \frac{1}{6}\left(cx^2+bx\right)^{\frac{5}{2}}dx + \frac{5\sqrt{cx^2+bx}b^4dx}{256\,c^2} - \frac{5\left(cx^2+bx\right)^{\frac{3}{2}}b^2dx}{96\,c} - \frac{5\sqrt{cx^2+bx}b^5ex}{512\,c^3} + \frac{5\left(cx^2+bx\right)^{\frac{3}{2}}b^3ex}{192\,c^2} - \frac{\left(cx^2+bx\right)^{\frac{5}{2}}bex}{12\,c} - \frac{5\,b^6d\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{1024\,c^{\frac{7}{2}}} + \frac{5\,b^7e\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{2048\,c^{\frac{9}{2}}} + \frac{5\sqrt{cx^2+bx}b^5d}{512\,c^3} - \frac{5\left(cx^2+bx\right)^{\frac{3}{2}}b^3d}{192\,c^2} + \frac{\left(cx^2+bx\right)^{\frac{5}{2}}bd}{12\,c} - \frac{5\sqrt{cx^2+bx}b^6e}{1024\,c^4} + \frac{5\left(cx^2+bx\right)^{\frac{3}{2}}b^4e}{384\,c^3} - \frac{\left(cx^2+bx\right)^{\frac{5}{2}}b^2e}{24\,c^2} + \frac{\left(cx^2+bx\right)^{\frac{7}{2}}e}{7\,c}$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(c*x^2 + b*x)^(5/2)*d*x + 5/256*sqrt(c*x^2 + b*x)*b^4*d*x/c^2 - 5/96*(
c*x^2 + b*x)^(3/2)*b^2*d*x/c - 5/512*sqrt(c*x^2 + b*x)*b^5*e*x/c^3 + 5/192
*(c*x^2 + b*x)^(3/2)*b^3*e*x/c^2 - 1/12*(c*x^2 + b*x)^(5/2)*b*e*x/c - 5/10
24*b^6*d*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(7/2) + 5/2048*b^7
*e*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(9/2) + 5/512*sqrt(c*x^2
 + b*x)*b^5*d/c^3 - 5/192*(c*x^2 + b*x)^(3/2)*b^3*d/c^2 + 1/12*(c*x^2 + b*
x)^(5/2)*b*d/c - 5/1024*sqrt(c*x^2 + b*x)*b^6*e/c^4 + 5/384*(c*x^2 + b*x)^
(3/2)*b^4*e/c^3 - 1/24*(c*x^2 + b*x)^(5/2)*b^2*e/c^2 + 1/7*(c*x^2 + b*x)^(
7/2)*e/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.86

$$\int (d+ex)\left(bx+cx^2\right)^{5/2}\,dx=\frac{1}{21504}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\left(2\left(12\,c^2ex+\frac{14\,c^8d+29\,bc^7e}{c^6}\right)x+\frac{70\,bc^7d+37\,b^2c^6e}{c^6}\right)x+\right.\right.\right.\right.\right.$$
$$+\frac{5\left(2\,b^6cd-b^7e\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{2048\,c^{\frac{9}{2}}}$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
1/21504*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(2*(12*c^2*e*x + (14*c^8*d + 29*b*c^
7*e)/c^6)*x + (70*b*c^7*d + 37*b^2*c^6*e)/c^6)*x + 3*(126*b^2*c^6*d + b^3*
c^5*e)/c^6)*x + 7*(2*b^3*c^5*d - b^4*c^4*e)/c^6)*x - 35*(2*b^4*c^4*d - b^5
*c^3*e)/c^6)*x + 105*(2*b^5*c^3*d - b^6*c^2*e)/c^6) + 5/2048*(2*b^6*c*d -
b^7*e)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(bx+cx^2\right)^{5/2}\,dx=\int \left(c\,x^2+b\,x\right)^{5/2}\,\left(d+e\,x\right)\,dx$$

input
```
int((b*x + c*x^2)^(5/2)*(d + e*x),x)
```

output
```
int((b*x + c*x^2)^(5/2)*(d + e*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.15

$$\int (d+ex)\left(bx+cx^2\right)^{5/2}\,dx=\frac{-105\sqrt{x}\,\sqrt{cx+b}\,b^6ce+210\sqrt{x}\,\sqrt{cx+b}\,b^5c^2d+70\sqrt{x}\,\sqrt{cx+b}\,b^5c^2ex-140\sqrt{x}\,\sqrt{cx+b}\,b^4c}{}$$

input
```
int((e*x+d)*(c*x^2+b*x)^(5/2),x)
```

output
```
( - 105*sqrt(x)*sqrt(b + c*x)*b**6*c*e + 210*sqrt(x)*sqrt(b + c*x)*b**5*c*
*2*d + 70*sqrt(x)*sqrt(b + c*x)*b**5*c**2*e*x - 140*sqrt(x)*sqrt(b + c*x)*
b**4*c**3*d*x - 56*sqrt(x)*sqrt(b + c*x)*b**4*c**3*e*x**2 + 112*sqrt(x)*sq
rt(b + c*x)*b**3*c**4*d*x**2 + 48*sqrt(x)*sqrt(b + c*x)*b**3*c**4*e*x**3 +
 6048*sqrt(x)*sqrt(b + c*x)*b**2*c**5*d*x**3 + 4736*sqrt(x)*sqrt(b + c*x)*
b**2*c**5*e*x**4 + 8960*sqrt(x)*sqrt(b + c*x)*b*c**6*d*x**4 + 7424*sqrt(x)
*sqrt(b + c*x)*b*c**6*e*x**5 + 3584*sqrt(x)*sqrt(b + c*x)*c**7*d*x**5 + 30
72*sqrt(x)*sqrt(b + c*x)*c**7*e*x**6 + 105*sqrt(c)*log((sqrt(b + c*x) + sq
rt(x)*sqrt(c))/sqrt(b))*b**7*e - 210*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*
sqrt(c))/sqrt(b))*b**6*c*d)/(21504*c**5)
```

3.151 $\int (bx+cx^2)^{5/2}\,dx$

Optimal result . . . 1216
Mathematica [A] (verified) . . . 1216
Rubi [A] (verified) . . . 1217
Maple [A] (verified) . . . 1219
Fricas [A] (verification not implemented) . . . 1219
Sympy [A] (verification not implemented) . . . 1220
Maxima [A] (verification not implemented) . . . 1221
Giac [A] (verification not implemented) . . . 1221
Mupad [B] (verification not implemented) . . . 1222
Reduce [B] (verification not implemented) . . . 1222

Optimal result

Integrand size = 13, antiderivative size = 175

$$\int (bx+cx^2)^{5/2}\,dx=\frac{5b^5\sqrt{bx+cx^2}}{512c^3}-\frac{5b^4x\sqrt{bx+cx^2}}{768c^2}+\frac{b^3x^2\sqrt{bx+cx^2}}{192c}+\frac{9}{32}b^2x^3\sqrt{bx+cx^2}+\frac{5}{12}bcx^4\sqrt{bx+cx^2}+\frac{1}{6}c^2x^5\sqrt{bx+cx^2}-\frac{5b^6\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{512c^{7/2}}$$

output
```
5/512*b^5*(c*x^2+b*x)^(1/2)/c^3-5/768*b^4*x*(c*x^2+b*x)^(1/2)/c^2+1/192*b^
3*x^2*(c*x^2+b*x)^(1/2)/c+9/32*b^2*x^3*(c*x^2+b*x)^(1/2)+5/12*b*c*x^4*(c*x
^2+b*x)^(1/2)+1/6*c^2*x^5*(c*x^2+b*x)^(1/2)-5/512*b^6*arctanh(c^(1/2)*x/(c
*x^2+b*x)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.74

$$\int (bx+cx^2)^{5/2}\,dx=\frac{\sqrt{x(b+cx)}\left(\sqrt{c}(15b^5-10b^4cx+8b^3c^2x^2+432b^2c^3x^3+640bc^4x^4+256c^5x^5)+\frac{30b^6\operatorname{arctanh}\ldots}{\sqrt{x}\sqrt{\ldots}}\right.}{1536c^{7/2}}$$

input
```
Integrate[(b*x + c*x^2)^(5/2),x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[c]*(15*b^5 - 10*b^4*c*x + 8*b^3*c^2*x^2 + 432*b^2
*c^3*x^3 + 640*b*c^4*x^4 + 256*c^5*x^5) + (30*b^6*ArcTanh[(Sqrt[c]*Sqrt[x]
)/(Sqrt[b] - Sqrt[b + c*x])])/(Sqrt[x]*Sqrt[b + c*x])))/(1536*c^(7/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {1087, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{5/2} dx$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\int \left(cx^2+bx\right)^{3/2}dx}{24c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int \sqrt{cx^2+bx}dx}{16c}\right)}{24c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)}{24c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)}{24c}$$

$$\Big\downarrow 219$$

$$\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)}{24c}$$

input
```
Int[(b*x + c*x^2)^(5/2),x]
```

output
```
((b + 2*c*x)*(b*x + c*x^2)^(5/2))/(12*c) - (5*b^2*(((b + 2*c*x)*(b*x + c*x
^2)^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*Ar
cTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))/(16*c)))/(24*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.61

method	result	size
risch	$\frac{(256c^5x^5+640b\,x^4c^4+432b^2c^3x^3+8c^2x^2b^3-10b^4cx+15b^5)x(cx+b)}{1536c^3\sqrt{x(cx+b)}}-\frac{5b^6\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{1024c^{\frac{7}{2}}}$	106
default	$\frac{(2cx+b)\left(c\,x^2+bx\right)^{\frac{5}{2}}}{12c}-\frac{5b^2\left(\frac{(2cx+b)\left(c\,x^2+bx\right)^{\frac{3}{2}}}{8c}-\frac{3b^2\left(\frac{(2cx+b)\sqrt{c\,x^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{24c}$	118

input
```
int((c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/1536*(256*c^5*x^5+640*b*c^4*x^4+432*b^2*c^3*x^3+8*b^3*c^2*x^2-10*b^4*c*x
+15*b^5)*x*(c*x+b)/c^3/(x*(c*x+b))^(1/2)-5/1024*b^6/c^(7/2)*ln((1/2*b+c*x)
/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.22

$$\int (bx+cx^2)^{5/2}\,dx=\left[\frac{15\,b^6\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,(256\,c^6x^5+640\,bc^5x^4+432\,b^2c^4x^3+8\,b^3c^3x^2}{3072\,c^4}\right.$$

input
```
integrate((c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/3072*(15*b^6*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 2*(
256*c^6*x^5 + 640*b*c^5*x^4 + 432*b^2*c^4*x^3 + 8*b^3*c^3*x^2 - 10*b^4*c^2
*x + 15*b^5*c)*sqrt(c*x^2 + b*x))/c^4, 1/1536*(15*b^6*sqrt(-c)*arctan(sqrt
(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (256*c^6*x^5 + 640*b*c^5*x^4 + 432*b^2
*c^4*x^3 + 8*b^3*c^3*x^2 - 10*b^4*c^2*x + 15*b^5*c)*sqrt(c*x^2 + b*x))/c^4
]
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.62

$$\int \left(bx + cx^2\right)^{5/2} dx = \text{Too large to display}$$

input
```
integrate((c*x**2+b*x)**(5/2),x)
```

output
```
b**2*Piecewise((-5*b**4*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) +
2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/
(2*c) + x)**2), True))/(128*c**3) + sqrt(b*x + c*x**2)*(5*b**3/(64*c**3) -
 5*b**2*x/(96*c**2) + b*x**2/(24*c) + x**3/4), Ne(c, 0)), (2*(b*x)**(7/2)/
(7*b**3), Ne(b, 0)), (0, True)) + 2*b*c*Piecewise((7*b**5*Piecewise((log(b
 + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c
) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(256*c**4) + sqrt
(b*x + c*x**2)*(-7*b**4/(128*c**4) + 7*b**3*x/(192*c**3) - 7*b**2*x**2/(24
0*c**2) + b*x**3/(40*c) + x**4/5), Ne(c, 0)), (2*(b*x)**(9/2)/(9*b**4), Ne
(b, 0)), (0, True)) + c**2*Piecewise((-21*b**6*Piecewise((log(b + 2*sqrt(c
)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(
b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(1024*c**5) + sqrt(b*x + c*x
**2)*(21*b**5/(512*c**5) - 7*b**4*x/(256*c**4) + 7*b**3*x**2/(320*c**3) -
3*b**2*x**3/(160*c**2) + b*x**4/(60*c) + x**5/6), Ne(c, 0)), (2*(b*x)**(11
/2)/(11*b**5), Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.81

$$\int \left(bx+cx^2\right)^{5/2}\,dx=\frac{1}{6}\left(cx^2+bx\right)^{\frac{5}{2}}x+\frac{5\sqrt{cx^2+bx}b^4x}{256\,c^2}-\frac{5\left(cx^2+bx\right)^{\frac{3}{2}}b^2x}{96\,c}-\frac{5\,b^6\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{1024\,c^{\frac{7}{2}}}+\frac{5\sqrt{cx^2+bx}b^5}{512\,c^3}-\frac{5\left(cx^2+bx\right)^{\frac{3}{2}}b^3}{192\,c^2}+\frac{\left(cx^2+bx\right)^{\frac{5}{2}}b}{12\,c}$$

input
```
integrate((c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(c*x^2 + b*x)^(5/2)*x + 5/256*sqrt(c*x^2 + b*x)*b^4*x/c^2 - 5/96*(c*x^
2 + b*x)^(3/2)*b^2*x/c - 5/1024*b^6*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sq
rt(c))/c^(7/2) + 5/512*sqrt(c*x^2 + b*x)*b^5/c^3 - 5/192*(c*x^2 + b*x)^(3/
2)*b^3/c^2 + 1/12*(c*x^2 + b*x)^(5/2)*b/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.60

$$\int \left(bx+cx^2\right)^{5/2}\,dx=\frac{5\,b^6\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{1024\,c^{\frac{7}{2}}}+\frac{1}{1536}\sqrt{cx^2+bx}\left(\frac{15\,b^5}{c^3}-2\left(\frac{5\,b^4}{c^2}-4\left(\frac{b^3}{c}+2\left(27\,b^2+8\left(2\,c^2x+5\,bc\right)x\right)x\right)x\right)x\right)$$

input
```
integrate((c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
5/1024*b^6*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(7/2)
 + 1/1536*sqrt(c*x^2 + b*x)*(15*b^5/c^3 - 2*(5*b^4/c^2 - 4*(b^3/c + 2*(27*
b^2 + 8*(2*c^2*x + 5*b*c)*x)*x)*x)*x)
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.68

$$\int \left(bx+cx^2\right)^{5/2}\,dx=\frac{\left(c\,x^2+b\,x\right)^{5/2}\,\left(\frac{b}{2}+c\,x\right)}{6\,c}-\frac{5\,b^2\,\left(\frac{\left(c\,x^2+b\,x\right)^{3/2}\,\left(\frac{b}{2}+c\,x\right)}{4\,c}-\frac{3\,b^2\,\left(\sqrt{c\,x^2+b\,x}\,\left(\frac{x}{2}+\frac{b}{4\,c}\right)-\frac{b^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}\right)}{16\,c}\right)}{24\,c}$$

input
```
int((b*x + c*x^2)^(5/2),x)
```

output
```
((b*x + c*x^2)^(5/2)*(b/2 + c*x))/(6*c) - (5*b^2*(((b*x + c*x^2)^(3/2)*(b/
2 + c*x))/(4*c) - (3*b^2*((b*x + c*x^2)^(1/2)*(x/2 + b/(4*c)) - (b^2*log((
b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/(8*c^(3/2))))/(16*c)))/(24*c)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.76

$$\int \left(bx+cx^2\right)^{5/2}\,dx=\frac{15\sqrt{x}\,\sqrt{cx+b}\,b^5c-10\sqrt{x}\,\sqrt{cx+b}\,b^4c^2x+8\sqrt{x}\,\sqrt{cx+b}\,b^3c^3x^2+432\sqrt{x}\,\sqrt{cx+b}\,b^2c^4x^3+}{1536c^4}$$

input
```
int((c*x^2+b*x)^(5/2),x)
```

output
```
(15*sqrt(x)*sqrt(b + c*x)*b**5*c - 10*sqrt(x)*sqrt(b + c*x)*b**4*c**2*x +
8*sqrt(x)*sqrt(b + c*x)*b**3*c**3*x**2 + 432*sqrt(x)*sqrt(b + c*x)*b**2*c*
*4*x**3 + 640*sqrt(x)*sqrt(b + c*x)*b*c**5*x**4 + 256*sqrt(x)*sqrt(b + c*x
)*c**6*x**5 - 15*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b*
*6)/(1536*c**4)
```

3.152 $\int \frac{(bx+cx^2)^{5/2}}{d+ex}\,dx$

Optimal result 1223
Mathematica [C] (verified) 1224
Rubi [A] (verified) 1224
Maple [A] (verified) 1228
Fricas [A] (verification not implemented) 1229
Sympy [F] 1230
Maxima [F(-2)] 1231
Giac [F(-2)] 1231
Mupad [F(-1)] 1231
Reduce [F] 1232

Optimal result

Integrand size = 21, antiderivative size = 380

$$\int \frac{(bx+cx^2)^{5/2}}{d+ex}\,dx = \frac{(128c^4d^4-288bc^3d^3e+176b^2c^2d^2e^2-10b^3cde^3-3b^4e^4)\sqrt{bx+cx^2}}{128c^2e^5} - \frac{(96c^3d^3-208bc^2d^2e+118b^2cde^2-3b^3e^3)\,x\sqrt{bx+cx^2}}{192ce^4} + \frac{(16c^2d^2-22bcde+3b^2e^2)\,x^2\sqrt{bx+cx^2}}{48e^3} - \frac{(2cd-be)x(bx+cx^2)^{3/2}}{8e^2} + \frac{(bx+cx^2)^{5/2}}{5e} - \frac{(2cd-be)\,(128c^4d^4-256bc^3d^3e+112b^2c^2d^2e^2+16b^3cde^3+3b^4e^4)\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{128c^{5/2}e^6} + \frac{2d^{5/2}(cd-be)^{5/2}\text{arctanh}\left(\frac{\sqrt{cd-be}\,x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{e^6}$$

output
```
1/128*(-3*b^4*e^4-10*b^3*c*d*e^3+176*b^2*c^2*d^2*e^2-288*b*c^3*d^3*e+128*c
^4*d^4)*(c*x^2+b*x)^(1/2)/c^2/e^5-1/192*(-3*b^3*e^3+118*b^2*c*d*e^2-208*b*
c^2*d^2*e+96*c^3*d^3)*x*(c*x^2+b*x)^(1/2)/c/e^4+1/48*(3*b^2*e^2-22*b*c*d*e
+16*c^2*d^2)*x^2*(c*x^2+b*x)^(1/2)/e^3-1/8*(-b*e+2*c*d)*x*(c*x^2+b*x)^(3/2
)/e^2+1/5*(c*x^2+b*x)^(5/2)/e-1/128*(-b*e+2*c*d)*(3*b^4*e^4+16*b^3*c*d*e^3
+112*b^2*c^2*d^2*e^2-256*b*c^3*d^3*e+128*c^4*d^4)*arctanh(c^(1/2)*x/(c*x^2
+b*x)^(1/2))/c^(5/2)/e^6+2*d^(5/2)*(-b*e+c*d)^(5/2)*arctanh((-b*e+c*d)^(1/
2)*x/d^(1/2)/(c*x^2+b*x)^(1/2))/e^6
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.96 (sec) , antiderivative size = 625, normalized size of antiderivative = 1.64

$$\int \frac{(bx+cx^2)^{5/2}}{d+ex}\,dx = \frac{(x(b+cx))^{5/2}\left(\sqrt{c}e\sqrt{x}\sqrt{b+cx}(-45b^4e^4+30b^3ce^3(-5d+ex)+4b^2c^2e^2(660d^2-2\right.}{}$$

input
```
Integrate[(b*x + c*x^2)^(5/2)/(d + e*x),x]
```

output
```
((x*(b + c*x))^(5/2)*(Sqrt[c]*e*Sqrt[x]*Sqrt[b + c*x]*(-45*b^4*e^4 + 30*b^
3*c*e^3*(-5*d + e*x) + 4*b^2*c^2*e^2*(660*d^2 - 295*d*e*x + 186*e^2*x^2) +
 16*b*c^3*e*(-270*d^3 + 130*d^2*e*x - 85*d*e^2*x^2 + 63*e^3*x^3) + 32*c^4*
(60*d^4 - 30*d^3*e*x + 20*d^2*e^2*x^2 - 15*d*e^3*x^3 + 12*e^4*x^4)) - 3840
*c^(3/2)*d^(3/2)*(c*d - b*e)^2*(c*d - b*e - I*Sqrt[b]*Sqrt[e]*Sqrt[c*d - b
*e])*Sqrt[-(c*d) + 2*b*e - (2*I)*Sqrt[b]*Sqrt[e]*Sqrt[c*d - b*e]]*ArcTan[(
Sqrt[-(c*d) + 2*b*e - (2*I)*Sqrt[b]*Sqrt[e]*Sqrt[c*d - b*e]]*Sqrt[x])/(Sqr
t[d]*(-Sqrt[b] + Sqrt[b + c*x]))] - 3840*c^(3/2)*d^(3/2)*(c*d - b*e)^2*(c*
d - b*e + I*Sqrt[b]*Sqrt[e]*Sqrt[c*d - b*e])*Sqrt[-(c*d) + 2*b*e + (2*I)*S
qrt[b]*Sqrt[e]*Sqrt[c*d - b*e]]*ArcTan[(Sqrt[-(c*d) + 2*b*e + (2*I)*Sqrt[b
]*Sqrt[e]*Sqrt[c*d - b*e]]*Sqrt[x])/(Sqrt[d]*(-Sqrt[b] + Sqrt[b + c*x]))]
+ 30*(256*c^5*d^5 - 640*b*c^4*d^4*e + 480*b^2*c^3*d^3*e^2 - 80*b^3*c^2*d^2
*e^3 - 10*b^4*c*d*e^4 - 3*b^5*e^5)*ArcTanh[(Sqrt[c]*Sqrt[x])/(Sqrt[b] - Sq
rt[b + c*x])]))/(1920*c^(5/2)*e^6*x^(5/2)*(b + c*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 1.02, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {1162, 1231, 27, 1231, 27, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^{5/2}}{d+ex}\,dx$$

$\Big\downarrow$ 1162

$$\frac{\left(bx+cx^2\right)^{5/2}}{5e}-\frac{\int \frac{(bd+(2cd-be)x)\left(cx^2+bx\right)^{3/2}}{d+ex}dx}{2e}$$

$\Big\downarrow$ 1231

$$\frac{\left(bx+cx^2\right)^{5/2}}{5e}-\frac{-\frac{\int -\frac{\left(bd\left(16c^2d^2-22bced+3b^2e^2\right)+(2cd-be)\left(16c^2d^2-16bced-3b^2e^2\right)x\right)\sqrt{cx^2+bx}}{2(d+ex)}dx}{8ce^2}-\frac{\left(bx+cx^2\right)^{3/2}\left(3b^2e^2-6cex(2cd-be)-22bcde+16c^2d^2\right)}{24ce^2}}{2e}$$

$\Big\downarrow$ 27

$$\frac{\left(bx+cx^2\right)^{5/2}}{5e}-\frac{\frac{\int \frac{\left(bd\left(16c^2d^2-22bced+3b^2e^2\right)+(2cd-be)\left(16c^2d^2-16bced-3b^2e^2\right)x\right)\sqrt{cx^2+bx}}{d+ex}dx}{16ce^2}-\frac{\left(bx+cx^2\right)^{3/2}\left(3b^2e^2-6cex(2cd-be)-22bcde+16c^2d^2\right)}{24ce^2}}{2e}$$

$\Big\downarrow$ 1231

$$\frac{\left(bx+cx^2\right)^{5/2}}{5e}-\frac{\frac{-\frac{\int -\frac{bd\left(128c^4d^4-288bc^3ed^3+176b^2c^2e^2d^2-10b^3ce^3d-3b^4e^4\right)+(2cd-be)\left(128c^4d^4-256bc^3ed^3+112b^2c^2e^2d^2+16b^3ce^3d+3b^4e^4\right)x}{2(d+ex)\sqrt{cx^2+bx}}dx}{4ce^2}-\frac{\sqrt{bx+cx^2}\left(-3b^4e^4-10\ldots\right.}{\ldots}}{16ce^2}}{2e}$$

$\Big\downarrow$ 27

$$\frac{\left(bx+cx^2\right)^{5/2}}{5e}-\frac{\frac{\frac{\int \frac{bd\left(128c^4d^4-288bc^3ed^3+176b^2c^2e^2d^2-10b^3ce^3d-3b^4e^4\right)+(2cd-be)\left(128c^4d^4-256bc^3ed^3+112b^2c^2e^2d^2+16b^3ce^3d+3b^4e^4\right)x}{(d+ex)\sqrt{cx^2+bx}}dx}{8ce^2}-\frac{\sqrt{bx+cx^2}\left(-3b^4e^4-10b^3c\ldots\right.}{\ldots}}{16ce^2}}{2e}$$

$\Big\downarrow$ 1269

$$\frac{\left(bx+cx^2\right)^{5/2}}{5e}-\frac{\frac{\frac{\frac{(2cd-be)\left(3b^4e^4+16b^3cde^3+112b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)\int \frac{1}{\sqrt{cx^2+bx}}dx}{e}-\frac{256c^2d^3(cd-be)^3\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{8ce^2}-\frac{\sqrt{bx+cx^2}\left(-3b^4e^4-10b^3cde^3-2ce\ldots\right.}{\ldots}}{16ce^2}}{2e}$$

$\Big\downarrow$ 1091

$$\frac{\left(bx+cx^2\right)^{5/2}}{5e}-\frac{\frac{\frac{\frac{2(2cd-be)\left(3b^4e^4+16b^3cde^3+112b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{e}-\frac{256c^2d^3(cd-be)^3\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{8ce^2}-\frac{\sqrt{bx+cx^2}\left(-3b^4e^4-10\ldots\right.}{\ldots}}{16ce^2}\ldots}{2e}$$

$\Big\downarrow$ 219

$$\frac{\left(bx+cx^2\right)^{5/2}}{5e}-\frac{\frac{\frac{\frac{2\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)\left(3b^4e^4+16b^3cde^3+112b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)}{\sqrt{c}e}-\frac{256c^2d^3(cd-be)^3\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{8ce^2}-\frac{\sqrt{bx+cx^2}\left(-3b^4e^4-10\ldots\right.}{\ldots}}{16ce^2}\ldots}{2e}$$

$\Big\downarrow$ 1154

$$\frac{\left(bx+cx^2\right)^{5/2}}{5e}-\frac{\frac{\frac{\frac{512c^2d^3(cd-be)^3\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{e}+\frac{2\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)\left(3b^4e^4+16b^3cde^3+112b^2c^2d^2e^2-256bc^3d^3e+128c\ldots\right.}{\sqrt{c}e}}{8ce^2}\ldots}{16ce^2}\ldots}{\ldots}$$

$\Big\downarrow$ 219

$$\frac{\left(bx+cx^2\right)^{5/2}}{5e}-\frac{\frac{\frac{\frac{2\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)\left(3b^4e^4+16b^3cde^3+112b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)}{\sqrt{c}e}-\frac{256c^2d^{5/2}(cd-be)^{5/2}\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{e}}{8ce^2}-\frac{\sqrt{bx\ldots}}{\ldots}}{16ce^2}\ldots}{\ldots}$$

input

```
Int[(b*x + c*x^2)^(5/2)/(d + e*x),x]
```

output
```
(b*x + c*x^2)^(5/2)/(5*e) - (-1/24*((16*c^2*d^2 - 22*b*c*d*e + 3*b^2*e^2 -
 6*c*e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(3/2))/(c*e^2) + (-1/4*((128*c^4*d^4
 - 288*b*c^3*d^3*e + 176*b^2*c^2*d^2*e^2 - 10*b^3*c*d*e^3 - 3*b^4*e^4 - 2*
c*e*(2*c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e - 3*b^2*e^2)*x)*Sqrt[b*x + c*x^
2])/(c*e^2) + ((2*(2*c*d - b*e)*(128*c^4*d^4 - 256*b*c^3*d^3*e + 112*b^2*c
^2*d^2*e^2 + 16*b^3*c*d*e^3 + 3*b^4*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*
x^2]])/(Sqrt[c]*e) - (256*c^2*d^(5/2)*(c*d - b*e)^(5/2)*ArcTanh[(b*d + (2*
c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/e)/(8*c*e^2)
)/(16*c*e^2))/(2*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.25

method	result
risch	$-\frac{\left(-384c^4e^4x^4-1008b\,c^3e^4x^3+480c^4d\,e^3x^3-744b^2c^2e^4x^2+1360b\,c^3d\,e^3x^2-640c^4d^2e^2x^2-30x\,b^3c\,e^4+1180b^2c^2d\,e^3x-2080b\,c^3d^2\right.}{1920c^2e^5\sqrt{x(cx+b)}}$
default	$\frac{\left(c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}\right)^{\frac{5}{2}}}{5}+\frac{(be-2cd)\left(\frac{\left(2c\left(x+\frac{d}{e}\right)+\frac{be-2cd}{e}\right)\left(c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}\right)^{\frac{3}{2}}}{8c}+\frac{3\left(-\frac{4cd(be-\right.}{e^2}}{}\right.}{}$

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/1920/c^2*(-384*c^4*e^4*x^4-1008*b*c^3*e^4*x^3+480*c^4*d*e^3*x^3-744*b^2
*c^2*e^4*x^2+1360*b*c^3*d*e^3*x^2-640*c^4*d^2*e^2*x^2-30*b^3*c*e^4*x+1180*
b^2*c^2*d*e^3*x-2080*b*c^3*d^2*e^2*x+960*c^4*d^3*e*x+45*b^4*e^4+150*b^3*c*
d*e^3-2640*b^2*c^2*d^2*e^2+4320*b*c^3*d^3*e-1920*c^4*d^4)*x*(c*x+b)/e^5/(x
*(c*x+b))^(1/2)+1/256/e^5/c^2*((3*b^5*e^5+10*b^4*c*d*e^4+80*b^3*c^2*d^2*e^
3-480*b^2*c^3*d^3*e^2+640*b*c^4*d^4*e-256*c^5*d^5)/e*ln((1/2*b+c*x)/c^(1/2
)+(c*x^2+b*x)^(1/2))/c^(1/2)+256*d^3*(b^3*e^3-3*b^2*c*d*e^2+3*b*c^2*d^2*e-
c^3*d^3)*c^2/e^2/(-d*(b*e-c*d)/e^2)^(1/2)*ln((-2*d*(b*e-c*d)/e^2+(b*e-2*c*
d)/e*(x+d/e)+2*(-d*(b*e-c*d)/e^2)^(1/2)*(c*(x+d/e)^2+(b*e-2*c*d)/e*(x+d/e)
-d*(b*e-c*d)/e^2)^(1/2))/(x+d/e)))
```

Fricas [A] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 1544, normalized size of antiderivative = 4.06

$$\int \frac{(bx+cx^2)^{5/2}}{d+ex}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d),x, algorithm="fricas")
```

output
```
[-1/3840*(15*(256*c^5*d^5 - 640*b*c^4*d^4*e + 480*b^2*c^3*d^3*e^2 - 80*b^3
*c^2*d^2*e^3 - 10*b^4*c*d*e^4 - 3*b^5*e^5)*sqrt(c)*log(2*c*x + b + 2*sqrt(
c*x^2 + b*x)*sqrt(c)) - 3840*(c^5*d^4 - 2*b*c^4*d^3*e + b^2*c^3*d^2*e^2)*s
qrt(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt
(c*x^2 + b*x))/(e*x + d)) - 2*(384*c^5*e^5*x^4 + 1920*c^5*d^4*e - 4320*b*c
^4*d^3*e^2 + 2640*b^2*c^3*d^2*e^3 - 150*b^3*c^2*d*e^4 - 45*b^4*c*e^5 - 48*
(10*c^5*d*e^4 - 21*b*c^4*e^5)*x^3 + 8*(80*c^5*d^2*e^3 - 170*b*c^4*d*e^4 +
93*b^2*c^3*e^5)*x^2 - 10*(96*c^5*d^3*e^2 - 208*b*c^4*d^2*e^3 + 118*b^2*c^3
*d*e^4 - 3*b^3*c^2*e^5)*x)*sqrt(c*x^2 + b*x))/(c^3*e^6), -1/3840*(7680*(c^
5*d^4 - 2*b*c^4*d^3*e + b^2*c^3*d^2*e^2)*sqrt(-c*d^2 + b*d*e)*arctan(sqrt(
-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/(c*d*x + b*d)) + 15*(256*c^5*d^5 - 640*b
*c^4*d^4*e + 480*b^2*c^3*d^3*e^2 - 80*b^3*c^2*d^2*e^3 - 10*b^4*c*d*e^4 - 3
*b^5*e^5)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(384*c^
5*e^5*x^4 + 1920*c^5*d^4*e - 4320*b*c^4*d^3*e^2 + 2640*b^2*c^3*d^2*e^3 - 1
50*b^3*c^2*d*e^4 - 45*b^4*c*e^5 - 48*(10*c^5*d*e^4 - 21*b*c^4*e^5)*x^3 + 8
*(80*c^5*d^2*e^3 - 170*b*c^4*d*e^4 + 93*b^2*c^3*e^5)*x^2 - 10*(96*c^5*d^3*
e^2 - 208*b*c^4*d^2*e^3 + 118*b^2*c^3*d*e^4 - 3*b^3*c^2*e^5)*x)*sqrt(c*x^2
 + b*x))/(c^3*e^6), 1/1920*(15*(256*c^5*d^5 - 640*b*c^4*d^4*e + 480*b^2*c^
3*d^3*e^2 - 80*b^3*c^2*d^2*e^3 - 10*b^4*c*d*e^4 - 3*b^5*e^5)*sqrt(-c)*arct
an(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + 1920*(c^5*d^4 - 2*b*c^4*d^3*...
```

Sympy [F]

$$\int \frac{(bx + cx^2)^{5/2}}{d + ex}\,dx = \int \frac{(x(b + cx))^{\frac{5}{2}}}{d + ex}\,dx$$

input
```
integrate((c*x**2+b*x)**(5/2)/(e*x+d),x)
```

output
```
Integral((x*(b + c*x))**(5/2)/(d + e*x), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(bx+cx^2)^{5/2}}{d+ex}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(bx+cx^2)^{5/2}}{d+ex}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{5/2}}{d+ex}\,dx = \int \frac{(c\,x^2+b\,x)^{5/2}}{d+e\,x}\,dx$$

input
```
int((b*x + c*x^2)^(5/2)/(d + e*x),x)
```

output
```
int((b*x + c*x^2)^(5/2)/(d + e*x), x)
```

Reduce [F]

$$\int \frac{(bx+cx^2)^{5/2}}{d+ex}\,dx = \int \frac{(c\,x^2+bx)^{\frac{5}{2}}}{ex+d}dx$$

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d),x)
```

output
```
int((c*x^2+b*x)^(5/2)/(e*x+d),x)
```

3.153 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^2}\,dx$

Optimal result 1233
Mathematica [A] (verified) 1234
Rubi [A] (verified) 1234
Maple [A] (verified) 1239
Fricas [A] (verification not implemented) 1239
Sympy [F] 1240
Maxima [F(-2)] 1241
Giac [F(-1)] 1241
Mupad [F(-1)] 1241
Reduce [F] 1242

Optimal result

Integrand size = 21, antiderivative size = 333

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^2}\,dx = -\frac{5(64c^3d^3-112bc^2d^2e+48b^2cde^2-b^3e^3)\sqrt{bx+cx^2}}{64ce^5} + \frac{5(48c^2d^2-80bcde+31b^2e^2)\,x\sqrt{bx+cx^2}}{96e^4} - \frac{5c(8cd-7be)x^2\sqrt{bx+cx^2}}{24e^3} + \frac{5cx(bx+cx^2)^{3/2}}{4e^2} - \frac{(bx+cx^2)^{5/2}}{e(d+ex)} + \frac{5(128c^4d^4-256bc^3d^3e+144b^2c^2d^2e^2-16b^3cde^3-b^4e^4)\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{3/2}e^6} - \frac{5d^{3/2}(cd-be)^{3/2}(2cd-be)\text{arctanh}\left(\frac{\sqrt{cd-be}\,x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{e^6}$$

output

```
-5/64*(-b^3*e^3+48*b^2*c*d*e^2-112*b*c^2*d^2*e+64*c^3*d^3)*(c*x^2+b*x)^(1/
2)/c/e^5+5/96*(31*b^2*e^2-80*b*c*d*e+48*c^2*d^2)*x*(c*x^2+b*x)^(1/2)/e^4-5
/24*c*(-7*b*e+8*c*d)*x^2*(c*x^2+b*x)^(1/2)/e^3+5/4*c*x*(c*x^2+b*x)^(3/2)/e
^2-(c*x^2+b*x)^(5/2)/e/(e*x+d)+5/64*(-b^4*e^4-16*b^3*c*d*e^3+144*b^2*c^2*d
^2*e^2-256*b*c^3*d^3*e+128*c^4*d^4)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c
^(3/2)/e^6-5*d^(3/2)*(-b*e+c*d)^(3/2)*(-b*e+2*c*d)*arctanh((-b*e+c*d)^(1/2
)*x/d^(1/2)/(c*x^2+b*x)^(1/2))/e^6
```

Mathematica [A] (verified)

Time = 10.96 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.04

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^2}\,dx = \frac{\sqrt{x(b+cx)}\left(\frac{\sqrt{c}e\sqrt{x}(15b^3e^3(d+ex)+2b^2ce^2(-360d^2-205dex+59e^2x^2)+8bc^2e(210d^3+110d^2ex-35de^2x^2+1}{d+ex}\right.}{}$$

input
```
Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^2,x]
```

output
```
(Sqrt[x*(b + c*x)]*((Sqrt[c]*e*Sqrt[x]*(15*b^3*e^3*(d + e*x) + 2*b^2*c*e^2
*(-360*d^2 - 205*d*e*x + 59*e^2*x^2) + 8*b*c^2*e*(210*d^3 + 110*d^2*e*x -
35*d*e^2*x^2 + 17*e^3*x^3) - 16*c^3*(60*d^4 + 30*d^3*e*x - 10*d^2*e^2*x^2
+ 5*d*e^3*x^3 - 3*e^4*x^4)))/(d + e*x) - (15*(-128*c^4*d^4 + 256*b*c^3*d^3
*e - 144*b^2*c^2*d^2*e^2 + 16*b^3*c*d*e^3 + b^4*e^4)*ArcSinh[(Sqrt[c]*Sqrt
[x])/Sqrt[b]])/(Sqrt[b]*Sqrt[1 + (c*x)/b]) - (960*c^(3/2)*d^(3/2)*Sqrt[c*d
 - b*e]*(2*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*ArcTanh[(Sqrt[c*d - b*e]*Sqrt[x]
)/(Sqrt[d]*Sqrt[b + c*x])])/Sqrt[b + c*x]))/(192*c^(3/2)*e^6*Sqrt[x])
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {1161, 1231, 25, 27, 1231, 27, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^2}dx$$

$$\Big\downarrow \text{ 1161}$$

$$\frac{5\int \frac{(b+2cx)(cx^2+bx)^{3/2}}{d+ex}dx}{2e} - \frac{(bx+cx^2)^{5/2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{5\left(-\frac{\int -\frac{c\left(bd(8cd-7be)+\left(16c^2d^2-16bced+b^2e^2\right)x\right)\sqrt{cx^2+bx}}{d+ex}dx}{8ce^2}-\frac{\left(bx+cx^2\right)^{3/2}(-7be+8cd-6cex)}{12e^2}\right)}{2e}-\frac{\left(bx+cx^2\right)^{5/2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{5\left(\frac{\int \frac{c\left(bd(8cd-7be)+\left(16c^2d^2-16bced+b^2e^2\right)x\right)\sqrt{cx^2+bx}}{d+ex}dx}{8ce^2}-\frac{\left(bx+cx^2\right)^{3/2}(-7be+8cd-6cex)}{12e^2}\right)}{2e}-\frac{\left(bx+cx^2\right)^{5/2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5\left(\frac{\int \frac{\left(bd(8cd-7be)+\left(16c^2d^2-16bced+b^2e^2\right)x\right)\sqrt{cx^2+bx}}{d+ex}dx}{8e^2}-\frac{\left(bx+cx^2\right)^{3/2}(-7be+8cd-6cex)}{12e^2}\right)}{2e}-\frac{\left(bx+cx^2\right)^{5/2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{5\left(\frac{-\frac{\int -\frac{bd\left(64c^3d^3-112bc^2ed^2+48b^2ce^2d-b^3e^3\right)+\left(128c^4d^4-256bc^3ed^3+144b^2c^2e^2d^2-16b^3ce^3d-b^4e^4\right)x}{2(d+ex)\sqrt{cx^2+bx}}dx}{4ce^2}-\frac{\sqrt{bx+cx^2}\left(-b^3e^3-2cex\left(b^2e^2-16bcde+16c^2\right.\right.}{4ce^2}}{8e^2}\right.}{2e}$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{5\left(\frac{\frac{\int \frac{bd\left(64c^3d^3-112bc^2ed^2+48b^2ce^2d-b^3e^3\right)+\left(128c^4d^4-256bc^3ed^3+144b^2c^2e^2d^2-16b^3ce^3d-b^4e^4\right)x}{(d+ex)\sqrt{cx^2+bx}}dx}{8ce^2}-\frac{\sqrt{bx+cx^2}\left(-b^3e^3-2cex\left(b^2e^2-16bcde+16c^2d^2\right)\right.}{4ce^2}}{8e^2}\right.}{2e}$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{e(d+ex)}$$

$$\Big\downarrow \text{ 1269}$$

$$5\left(\frac{\frac{\frac{\left(-b^4e^4-16b^3cde^3+144b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)\int\frac{1}{\sqrt{cx^2+bx}}dx}{e}-\frac{64cd^2(cd-be)^2(2cd-be)\int\frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{8ce^2}-\frac{\sqrt{bx+cx^2}\left(-b^3e^3-2cex\left(b^2e^2-\cdots\right.\right.}{\cdots}}{8e^2}\cdots\right.$$

$$\cdots 2e$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{e(d+ex)}$$

$\downarrow$ 1091

$$5\left(\frac{\frac{\frac{2\left(-b^4e^4-16b^3cde^3+144b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{e}-\frac{64cd^2(cd-be)^2(2cd-be)\int\frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{8ce^2}-\frac{\sqrt{bx+cx^2}\left(-b^3e^3-\cdots\right.}{\cdots}}{8e^2}\cdots\right.$$

$$\cdots 2e$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{e(d+ex)}$$

$\downarrow$ 219

$$5\left(\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)\left(-b^4e^4-16b^3cde^3+144b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)}{\sqrt{c}e}-\frac{64cd^2(cd-be)^2(2cd-be)\int\frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{8ce^2}-\frac{\sqrt{bx+cx^2}\left(-b^3e^3-2\cdots\right.}{\cdots}}{8e^2}\cdots\right.$$

$$\cdots 2e$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{e(d+ex)}$$

$\downarrow$ 1154

$$5\left(\frac{\frac{\frac{128cd^2(2cd-be)(cd-be)^2\int\frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{e}+\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)\left(-b^4e^4-16b^3cde^3+144b^2c^2d^2e^2-256bc^3d^3e+\cdots\right.}{\sqrt{c}e}}{8ce^2}\cdots}{8e^2}\cdots\right.$$

$$\cdots 2e$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{e(d+ex)}$$

$\downarrow$ 219

$$5\left(\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)\left(-b^4e^4-16b^3cde^3+144b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)}{\sqrt{c}e}-\frac{64cd^{3/2}(cd-be)^{3/2}(2cd-be)\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{e}}{8ce^2}-}{8e^2}\right.$$

$$2e$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{e(d+ex)}$$

input
```
Int[(b*x + c*x^2)^(5/2)/(d + e*x)^2,x]
```

output
```
-((b*x + c*x^2)^(5/2)/(e*(d + e*x))) + (5*(-1/12*((8*c*d - 7*b*e - 6*c*e*x
)*(b*x + c*x^2)^(3/2))/e^2 + (-1/4*((64*c^3*d^3 - 112*b*c^2*d^2*e + 48*b^2
*c*d*e^2 - b^3*e^3 - 2*c*e*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*x)*Sqrt[b*x
 + c*x^2])/(c*e^2) + ((2*(128*c^4*d^4 - 256*b*c^3*d^3*e + 144*b^2*c^2*d^2*
e^2 - 16*b^3*c*d*e^3 - b^4*e^4)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(S
qrt[c]*e) - (64*c*d^(3/2)*(c*d - b*e)^(3/2)*(2*c*d - b*e)*ArcTanh[(b*d + (
2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/e)/(8*c*e^
2))/(8*e^2)))/(2*e)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.98

method	result
pseudoelliptic	$-\frac{5\left(64d^2(ex+d)\left(b^3e^3c^{\frac{3}{2}}-4b^2c^{\frac{5}{2}}d\,e^2+5b\,d^2e\,c^{\frac{7}{2}}-2c^{\frac{9}{2}}d^3\right)\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)+\left((b^4e^4+16d\,e^3b^3c-144d^2e^2b^2c^2+256\right.}{}$
risch	$\frac{(48c^3e^3x^3+136e^3x^2b\,c^2-128c^3d\,e^2x^2+118x\,b^2c\,e^3-416b\,c^2d\,e^2x+288c^3d^2ex+15b^3e^3-528d\,e^2b^2c+1296d^2eb\,c^2-768d^3c^3}{192c\,e^5\sqrt{x(cx+b)}}$
default	Expression too large to display

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-5/64/c^(3/2)/(d*(b*e-c*d))^(1/2)*(64*d^2*(e*x+d)*(b^3*e^3*c^(3/2)-4*b^2*c
^(5/2)*d*e^2+5*b*d^2*e*c^(7/2)-2*c^(9/2)*d^3)*arctan((x*(c*x+b))^(1/2)/x*d
/(d*(b*e-c*d))^(1/2))+((b^4*e^4+16*b^3*c*d*e^3-144*b^2*c^2*d^2*e^2+256*b*c
^3*d^3*e-128*c^4*d^4)*(e*x+d)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))-e*((16/
5*e^4*x^4-16/3*d*e^3*x^3+32/3*d^2*e^2*x^2-32*d^3*e*x-64*d^4)*c^(7/2)+e*((1
36/15*e^3*x^3-56/3*d*e^2*x^2+176/3*d^2*e*x+112*d^3)*c^(5/2)+e*((118/15*e^2
*x^2-82/3*d*e*x-48*d^2)*c^(3/2)+b*e*c^(1/2)*(e*x+d))*b)*b)*(x*(c*x+b))^(1/
2))*(d*(b*e-c*d))^(1/2))/e^6/(e*x+d)
```

Fricas [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 1868, normalized size of antiderivative = 5.61

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^2}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^2,x, algorithm="fricas")
```

output
```
[-1/384*(15*(128*c^4*d^5 - 256*b*c^3*d^4*e + 144*b^2*c^2*d^3*e^2 - 16*b^3*
c*d^2*e^3 - b^4*d*e^4 + (128*c^4*d^4*e - 256*b*c^3*d^3*e^2 + 144*b^2*c^2*d
^2*e^3 - 16*b^3*c*d*e^4 - b^4*e^5)*x)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2
 + b*x)*sqrt(c)) - 960*(2*c^4*d^4 - 3*b*c^3*d^3*e + b^2*c^2*d^2*e^2 + (2*c
^4*d^3*e - 3*b*c^3*d^2*e^2 + b^2*c^2*d*e^3)*x)*sqrt(c*d^2 - b*d*e)*log((b*
d + (2*c*d - b*e)*x - 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d))
- 2*(48*c^4*e^5*x^4 - 960*c^4*d^4*e + 1680*b*c^3*d^3*e^2 - 720*b^2*c^2*d^2
*e^3 + 15*b^3*c*d*e^4 - 8*(10*c^4*d*e^4 - 17*b*c^3*e^5)*x^3 + 2*(80*c^4*d^
2*e^3 - 140*b*c^3*d*e^4 + 59*b^2*c^2*e^5)*x^2 - 5*(96*c^4*d^3*e^2 - 176*b*
c^3*d^2*e^3 + 82*b^2*c^2*d*e^4 - 3*b^3*c*e^5)*x)*sqrt(c*x^2 + b*x))/(c^2*e
^7*x + c^2*d*e^6), 1/384*(1920*(2*c^4*d^4 - 3*b*c^3*d^3*e + b^2*c^2*d^2*e^
2 + (2*c^4*d^3*e - 3*b*c^3*d^2*e^2 + b^2*c^2*d*e^3)*x)*sqrt(-c*d^2 + b*d*e
)*arctan(sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/(c*d*x + b*d)) - 15*(128*c
^4*d^5 - 256*b*c^3*d^4*e + 144*b^2*c^2*d^3*e^2 - 16*b^3*c*d^2*e^3 - b^4*d*
e^4 + (128*c^4*d^4*e - 256*b*c^3*d^3*e^2 + 144*b^2*c^2*d^2*e^3 - 16*b^3*c*
d*e^4 - b^4*e^5)*x)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) +
 2*(48*c^4*e^5*x^4 - 960*c^4*d^4*e + 1680*b*c^3*d^3*e^2 - 720*b^2*c^2*d^2*
e^3 + 15*b^3*c*d*e^4 - 8*(10*c^4*d*e^4 - 17*b*c^3*e^5)*x^3 + 2*(80*c^4*d^2
*e^3 - 140*b*c^3*d*e^4 + 59*b^2*c^2*e^5)*x^2 - 5*(96*c^4*d^3*e^2 - 176*b*c
^3*d^2*e^3 + 82*b^2*c^2*d*e^4 - 3*b^3*c*e^5)*x)*sqrt(c*x^2 + b*x))/(c^2...
```

Sympy [F]

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^2}\,dx = \int \frac{(x(b + cx))^{\frac{5}{2}}}{(d + ex)^2}\,dx$$

input
```
integrate((c*x**2+b*x)**(5/2)/(e*x+d)**2,x)
```

output
```
Integral((x*(b + c*x))**(5/2)/(d + e*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [F(-1)]

Timed out.

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^2} \, dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^2,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^2} \, dx = \int \frac{(c\,x^2 + b\,x)^{5/2}}{(d + e\,x)^2} \, dx$$

input
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^2,x)
```

output
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^2} \, dx = \int \frac{(c\,x^2 + bx)^{\frac{5}{2}}}{(ex + d)^2} dx$$

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^2,x)
```

output
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^2,x)
```

3.154 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^3} \, dx$

Optimal result 1243
Mathematica [A] (verified) 1244
Rubi [A] (verified) 1244
Maple [A] (verified) 1248
Fricas [A] (verification not implemented) 1249
Sympy [F] 1250
Maxima [F(-2)] 1251
Giac [B] (verification not implemented) 1251
Mupad [F(-1)] 1252
Reduce [F] 1253

Optimal result

Integrand size = 21, antiderivative size = 333

$$
\begin{aligned}
\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^3} \, dx = & \frac{5(16c^2d^2-20bcde+5b^2e^2)\sqrt{bx+cx^2}}{8e^5} \\
& - \frac{5(12c^2d^2-14bcde+3b^2e^2)\,x\sqrt{bx+cx^2}}{12de^4} + \frac{5c(8cd-3be)x^2\sqrt{bx+cx^2}}{12de^3} \\
& - \frac{5(2cd-be)x(bx+cx^2)^{3/2}}{4de^2(d+ex)} - \frac{(bx+cx^2)^{5/2}}{2e(d+ex)^2} \\
& - \frac{5(2cd-be)(16c^2d^2-16bcde+b^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8\sqrt{c}e^6} \\
& + \frac{5\sqrt{d}(4cd-3be)\sqrt{cd-be}(4cd-be)\operatorname{arctanh}\left(\frac{\sqrt{cd-be}x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{4e^6}
\end{aligned}
$$

output
```
5/8*(5*b^2*e^2-20*b*c*d*e+16*c^2*d^2)*(c*x^2+b*x)^(1/2)/e^5-5/12*(3*b^2*e^
2-14*b*c*d*e+12*c^2*d^2)*x*(c*x^2+b*x)^(1/2)/d/e^4+5/12*c*(-3*b*e+8*c*d)*x
^2*(c*x^2+b*x)^(1/2)/d/e^3-5/4*(-b*e+2*c*d)*x*(c*x^2+b*x)^(3/2)/d/e^2/(e*x
+d)-1/2*(c*x^2+b*x)^(5/2)/e/(e*x+d)^2-5/8*(-b*e+2*c*d)*(b^2*e^2-16*b*c*d*e
+16*c^2*d^2)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(1/2)/e^6+5/4*d^(1/2)*
(-3*b*e+4*c*d)*(-b*e+c*d)^(1/2)*(-b*e+4*c*d)*arctanh((-b*e+c*d)^(1/2)*x/d^
(1/2)/(c*x^2+b*x)^(1/2))/e^6
```

Mathematica [A] (verified)

Time = 11.09 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.92

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^3} dx = \frac{\sqrt{x(b+cx)}\left(\frac{e\sqrt{x}(3b^2e^2(25d^2+40dex+11e^2x^2)-2bce(150d^3+230d^2ex+55de^2x^2-13e^3x^3)+4c^2(60d^4+90d^3e}{(d+ex)^2}\right.}{}$$

input
```
Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^3,x]
```

output
```
(Sqrt[x*(b + c*x)]*((e*Sqrt[x]*(3*b^2*e^2*(25*d^2 + 40*d*e*x + 11*e^2*x^2)
 - 2*b*c*e*(150*d^3 + 230*d^2*e*x + 55*d*e^2*x^2 - 13*e^3*x^3) + 4*c^2*(60
*d^4 + 90*d^3*e*x + 20*d^2*e^2*x^2 - 5*d*e^3*x^3 + 2*e^4*x^4)))/(d + e*x)^
2 + (15*(-32*c^3*d^3 + 48*b*c^2*d^2*e - 18*b^2*c*d*e^2 + b^3*e^3)*ArcSinh[
(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[b]*Sqrt[c]*Sqrt[1 + (c*x)/b]) + (30*Sqrt
[d]*Sqrt[c*d - b*e]*(16*c^2*d^2 - 16*b*c*d*e + 3*b^2*e^2)*ArcTanh[(Sqrt[c*
d - b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])])/Sqrt[b + c*x]))/(24*e^6*Sqrt[x
])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.90, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1161, 1230, 1231, 27, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^3} dx$$

$$\Big\downarrow \text{ 1161}$$

$$\frac{5\int \frac{(b+2cx)(cx^2+bx)^{3/2}}{(d+ex)^2}dx}{4e} - \frac{(bx + cx^2)^{5/2}}{2e(d + ex)^2}$$

$$\Big\downarrow \text{ 1230}$$

$$\frac{5\left(\frac{\left(bx+cx^2\right)^{3/2}(-3be+8cd+2cex)}{3e^2(d+ex)}-\frac{\int \frac{(b(8cd-3be)+8c(2cd-be)x)\sqrt{cx^2+bx}}{d+ex}dx}{2e^2}\right)}{4e}-\frac{\left(bx+cx^2\right)^{5/2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 1231$$

$$\frac{5\left(\frac{\left(bx+cx^2\right)^{3/2}(-3be+8cd+2cex)}{3e^2(d+ex)}-\frac{-\frac{\int -\frac{2c\left(bd\left(16c^2d^2-20bced+5b^2e^2\right)+(2cd-be)\left(16c^2d^2-16bced+b^2e^2\right)x\right)}{(d+ex)\sqrt{cx^2+bx}}dx}{4ce^2}-\frac{\sqrt{bx+cx^2}\left(5b^2e^2-4cex(2cd-be)-\cdots\right.}{e^2}\cdots}{2e^2}\cdots\right.}{4e}$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 27$$

$$\frac{5\left(\frac{\left(bx+cx^2\right)^{3/2}(-3be+8cd+2cex)}{3e^2(d+ex)}-\frac{\frac{\int \frac{bd\left(16c^2d^2-20bced+5b^2e^2\right)+(2cd-be)\left(16c^2d^2-16bced+b^2e^2\right)x}{(d+ex)\sqrt{cx^2+bx}}dx}{2e^2}-\frac{\sqrt{bx+cx^2}\left(5b^2e^2-4cex(2cd-be)-20bcde+1\cdots\right.}{e^2}\cdots}{2e^2}\cdots\right.}{4e}$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 1269$$

$$\frac{5\left(\frac{\left(bx+cx^2\right)^{3/2}(-3be+8cd+2cex)}{3e^2(d+ex)}-\frac{\frac{\frac{(2cd-be)\left(b^2e^2-16bcde+16c^2d^2\right)\int \frac{1}{\sqrt{cx^2+bx}}dx}{e}-\frac{2d(4cd-3be)(cd-be)(4cd-be)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{2e^2}-\frac{\sqrt{bx+cx^2}\cdots}{\cdots}}{2e^2}\cdots\right.}{4e}$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{2e(d+ex)^2}$$

$$\Big\downarrow \; 1091$$

$$\frac{5\left(\frac{\left(bx+cx^2\right)^{3/2}(-3be+8cd+2cex)}{3e^2(d+ex)}-\frac{\frac{\frac{2(2cd-be)\left(b^2e^2-16bcde+16c^2d^2\right)\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{e}-\frac{2d(4cd-3be)(cd-be)(4cd-be)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}\cdots}{2e^2}\cdots}{2e^2}\cdots\right.}{4e}$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{5\left(\frac{(bx+cx^2)^{3/2}(-3be+8cd+2cex)}{3e^2(d+ex)}-\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)\left(b^2e^2-16bcde+16c^2d^2\right)}{\sqrt{c}e}-\frac{2d(4cd-3be)(cd-be)(4cd-be)\int\frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{e}}{2e^2}\cdots}{2e^2}\cdots\right)}{4e}\cdots$$

$$\frac{(bx+cx^2)^{5/2}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{1154}$$

$$\frac{5\left(\frac{(bx+cx^2)^{3/2}(-3be+8cd+2cex)}{3e^2(d+ex)}-\frac{\frac{\frac{4d(4cd-3be)(cd-be)(4cd-be)\int\frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{e}+\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2c\cdots}{\cdots}}{2e^2}\cdots}{2e^2}\cdots\right)}{4e}\cdots$$

$$\frac{(bx+cx^2)^{5/2}}{2e(d+ex)^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{5\left(\frac{(bx+cx^2)^{3/2}(-3be+8cd+2cex)}{3e^2(d+ex)}-\frac{\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)\left(b^2e^2-16bcde+16c^2d^2\right)}{\sqrt{c}e}-\frac{2\sqrt{d}(4cd-3be)\sqrt{cd-be}(4cd-be)\text{arctanh}\left(\frac{x(2c\cdots}{2\sqrt{d}\sqrt{bx\cdots}}\cdots\right)}{e}}{2e^2}\cdots}{2e^2}\cdots\right)}{4e}\cdots$$

$$\frac{(bx+cx^2)^{5/2}}{2e(d+ex)^2}$$

input
```
Int[(b*x + c*x^2)^(5/2)/(d + e*x)^3,x]
```

output
```
-1/2*(b*x + c*x^2)^(5/2)/(e*(d + e*x)^2) + (5*(((8*c*d - 3*b*e + 2*c*e*x)*
(b*x + c*x^2)^(3/2))/(3*e^2*(d + e*x)) - (-(((16*c^2*d^2 - 20*b*c*d*e + 5*
b^2*e^2 - 4*c*e*(2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/e^2) + ((2*(2*c*d - b*
e)*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^
2]])/(Sqrt[c]*e) - (2*Sqrt[d]*(4*c*d - 3*b*e)*Sqrt[c*d - b*e]*(4*c*d - b*e
)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*
x^2])])/e)/(2*e^2))/(2*e^2)))/(4*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.00

method	result
pseudoelliptic	$-\frac{4\left(5(-be+cd)(ex+d)^2\left(cd-\frac{be}{4}\right)\left(cd-\frac{3be}{4}\right)x\sqrt{c}\,bd\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)+\sqrt{d(be-cd)}\left(-\frac{5bx(ex+d)^2(be-2cd)\left(b^2e^2-16b\right.}{}\right.\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-4*(5*(-b*e+c*d)*(e*x+d)^2*(c*d-1/4*b*e)*(c*d-3/4*b*e)*x*c^(1/2)*b*d*arcta
n((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))^(1/2))+(d*(b*e-c*d))^(1/2)*(-5/32*b*
x*(e*x+d)^2*(b*e-2*c*d)*(b^2*e^2-16*b*c*d*e+16*c^2*d^2)*arctanh((x*(c*x+b)
)^(1/2)/x/c^(1/2))+e*((23/16*b*c*d^3*e-c^2*d^4-7/16*d^2*e^2*b^2)*(x*(c*x+b
))^(3/2)-11/32*(48/11*(-2/3*c*x+b)*c^2*d^4-54/11*e*c*(-83/27*c*x+b)*b*d^3+
b*e^2*(80/33*c^2*x^2-502/33*c*b*x+b^2)*d^2+40/11*e^3*(-1/6*c^2*x^2-11/12*c
*b*x+b^2)*x*b*d+b*e^4*x^2*(8/33*c^2*x^2+26/33*c*b*x+b^2))*x*(x*(c*x+b))^(1
/2))*c^(1/2)))/c^(1/2)/(d*(b*e-c*d))^(1/2)/b/e^6/x/(e*x+d)^2
```

Fricas [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 2028, normalized size of antiderivative = 6.09

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^3,x, algorithm="fricas")
```

output
```
[-1/48*(15*(32*c^3*d^5 - 48*b*c^2*d^4*e + 18*b^2*c*d^3*e^2 - b^3*d^2*e^3 +
 (32*c^3*d^3*e^2 - 48*b*c^2*d^2*e^3 + 18*b^2*c*d*e^4 - b^3*e^5)*x^2 + 2*(3
2*c^3*d^4*e - 48*b*c^2*d^3*e^2 + 18*b^2*c*d^2*e^3 - b^3*d*e^4)*x)*sqrt(c)*
log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 30*(16*c^3*d^4 - 16*b*c^2*d
^3*e + 3*b^2*c*d^2*e^2 + (16*c^3*d^2*e^2 - 16*b*c^2*d*e^3 + 3*b^2*c*e^4)*x
^2 + 2*(16*c^3*d^3*e - 16*b*c^2*d^2*e^2 + 3*b^2*c*d*e^3)*x)*sqrt(c*d^2 - b
*d*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x)
)/(e*x + d)) - 2*(8*c^3*e^5*x^4 + 240*c^3*d^4*e - 300*b*c^2*d^3*e^2 + 75*b
^2*c*d^2*e^3 - 2*(10*c^3*d*e^4 - 13*b*c^2*e^5)*x^3 + (80*c^3*d^2*e^3 - 110
*b*c^2*d*e^4 + 33*b^2*c*e^5)*x^2 + 20*(18*c^3*d^3*e^2 - 23*b*c^2*d^2*e^3 +
 6*b^2*c*d*e^4)*x)*sqrt(c*x^2 + b*x))/(c*e^8*x^2 + 2*c*d*e^7*x + c*d^2*e^6
), -1/48*(60*(16*c^3*d^4 - 16*b*c^2*d^3*e + 3*b^2*c*d^2*e^2 + (16*c^3*d^2*
e^2 - 16*b*c^2*d*e^3 + 3*b^2*c*e^4)*x^2 + 2*(16*c^3*d^3*e - 16*b*c^2*d^2*e
^2 + 3*b^2*c*d*e^3)*x)*sqrt(-c*d^2 + b*d*e)*arctan(sqrt(-c*d^2 + b*d*e)*sq
rt(c*x^2 + b*x)/(c*d*x + b*d)) + 15*(32*c^3*d^5 - 48*b*c^2*d^4*e + 18*b^2*
c*d^3*e^2 - b^3*d^2*e^3 + (32*c^3*d^3*e^2 - 48*b*c^2*d^2*e^3 + 18*b^2*c*d*
e^4 - b^3*e^5)*x^2 + 2*(32*c^3*d^4*e - 48*b*c^2*d^3*e^2 + 18*b^2*c*d^2*e^3
 - b^3*d*e^4)*x)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*
(8*c^3*e^5*x^4 + 240*c^3*d^4*e - 300*b*c^2*d^3*e^2 + 75*b^2*c*d^2*e^3 - 2*
(10*c^3*d*e^4 - 13*b*c^2*e^5)*x^3 + (80*c^3*d^2*e^3 - 110*b*c^2*d*e^4 +...
```

Sympy [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^3}\,dx=\int \frac{(x(b+cx))^{\frac{5}{2}}}{(d+ex)^3}\,dx$$

input
```
integrate((c*x**2+b*x)**(5/2)/(e*x+d)**3,x)
```

output
```
Integral((x*(b + c*x))**(5/2)/(d + e*x)**3, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 743 vs. $2(293) = 586$.

Time = 0.16 (sec) , antiderivative size = 743, normalized size of antiderivative = 2.23

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^3}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^3,x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x)*(2*x*(4*c^2*x/e^3 - (18*c^4*d*e^14 - 13*b*c^3*e^15)
/(c^2*e^18)) + 3*(48*c^4*d^2*e^13 - 54*b*c^3*d*e^14 + 11*b^2*c^2*e^15)/(c^
2*e^18)) + 5/4*(16*c^3*d^4 - 32*b*c^2*d^3*e + 19*b^2*c*d^2*e^2 - 3*b^3*d*e
^3)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^2 +
b*d*e))/(sqrt(-c*d^2 + b*d*e)*e^6) + 5/16*(32*c^3*d^3 - 48*b*c^2*d^2*e + 1
8*b^2*c*d*e^2 - b^3*e^3)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c)
 + b))/(sqrt(c)*e^6) + 1/4*(40*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*c^3*d^4*e
 - 80*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b*c^2*d^3*e^2 + 49*(sqrt(c)*x - sq
rt(c*x^2 + b*x))^3*b^2*c*d^2*e^3 - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^3
*d*e^4 + 72*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*c^(7/2)*d^5 - 120*(sqrt(c)*x
 - sqrt(c*x^2 + b*x))^2*b*c^(5/2)*d^4*e + 51*(sqrt(c)*x - sqrt(c*x^2 + b*x
))^2*b^2*c^(3/2)*d^3*e^2 - 3*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^3*sqrt(c)
*d^2*e^3 + 72*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b*c^3*d^5 - 124*(sqrt(c)*x -
 sqrt(c*x^2 + b*x))*b^2*c^2*d^4*e + 59*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^3
*c*d^3*e^2 - 7*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^4*d^2*e^3 + 18*b^2*c^(5/2
)*d^5 - 27*b^3*c^(3/2)*d^4*e + 9*b^4*sqrt(c)*d^3*e^2)/(((sqrt(c)*x - sqrt(
c*x^2 + b*x))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c)*d + b*d)^2*e
^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^3}\,dx=\int \frac{{\left(c\,x^2+b\,x\right)}^{5/2}}{{\left(d+e\,x\right)}^3}\,dx$$

input
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^3,x)
```

output
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^3, x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^3} \, dx = \int \frac{(c\,x^2 + bx)^{\frac{5}{2}}}{(ex + d)^3} dx$$

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^3,x)
```

output
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^3,x)
```

3.155 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^4} dx$

Optimal result . . . 1254
Mathematica [A] (verified) . . . 1255
Rubi [A] (verified) . . . 1255
Maple [A] (verified) . . . 1259
Fricas [B] (verification not implemented) . . . 1260
Sympy [F] . . . 1260
Maxima [F(-2)] . . . 1261
Giac [B] (verification not implemented) . . . 1261
Mupad [F(-1)] . . . 1262
Reduce [F] . . . 1263

Optimal result

Integrand size = 21, antiderivative size = 353

$$\begin{aligned}\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^4} dx = & -\frac{5(16c^2d^2-12bcde+b^2e^2)\sqrt{bx+cx^2}}{8de^5} \\ & + \frac{5(24c^2d^2-16bcde+b^2e^2)x\sqrt{bx+cx^2}}{24d^2e^4} \\ & - \frac{5(16c^2d^2-14bcde+b^2e^2)x^2\sqrt{bx+cx^2}}{24d^2e^3(d+ex)} - \frac{5(2cd-be)x(bx+cx^2)^{3/2}}{12de^2(d+ex)^2} \\ & - \frac{(bx+cx^2)^{5/2}}{3e(d+ex)^3} + \frac{5\sqrt{c}(4cd-3be)(4cd-be)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4e^6} \\ & - \frac{5(2cd-be)(16c^2d^2-16bcde+b^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{cd-be}x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{8\sqrt{d}e^6\sqrt{cd-be}}\end{aligned}$$

output

```
-5/8*(b^2*e^2-12*b*c*d*e+16*c^2*d^2)*(c*x^2+b*x)^(1/2)/d/e^5+5/24*(b^2*e^2
-16*b*c*d*e+24*c^2*d^2)*x*(c*x^2+b*x)^(1/2)/d^2/e^4-5/24*(b^2*e^2-14*b*c*d
*e+16*c^2*d^2)*x^2*(c*x^2+b*x)^(1/2)/d^2/e^3/(e*x+d)-5/12*(-b*e+2*c*d)*x*(
c*x^2+b*x)^(3/2)/d/e^2/(e*x+d)^2-1/3*(c*x^2+b*x)^(5/2)/e/(e*x+d)^3+5/4*c^(
1/2)*(-3*b*e+4*c*d)*(-b*e+4*c*d)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/e^6-
5/8*(-b*e+2*c*d)*(b^2*e^2-16*b*c*d*e+16*c^2*d^2)*arctanh((-b*e+c*d)^(1/2)*
x/d^(1/2)/(c*x^2+b*x)^(1/2))/d^(1/2)/e^6/(-b*e+c*d)^(1/2)
```

Mathematica [A] (verified)

Time = 11.58 (sec) , antiderivative size = 340, normalized size of antiderivative = 0.96

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^4} \, dx = \frac{\sqrt{x(b + cx)} \left(\frac{e(cd-be)\sqrt{x}(b^2e^2(15d^2+40dex+33e^2x^2)-2bce(90d^3+230d^2ex+175de^2x^2+27e^3x^3)+4c^2(60d^4 \cdots}{(d+ex)^3} \cdots \right.}{\cdots}$$

input
```
Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^4,x]
```

output
```
(Sqrt[x*(b + c*x)]*((e*(c*d - b*e)*Sqrt[x]*(b^2*e^2*(15*d^2 + 40*d*e*x + 3
3*e^2*x^2) - 2*b*c*e*(90*d^3 + 230*d^2*e*x + 175*d*e^2*x^2 + 27*e^3*x^3) +
 4*c^2*(60*d^4 + 150*d^3*e*x + 110*d^2*e^2*x^2 + 15*d*e^3*x^3 - 3*e^4*x^4)
))/(d + e*x)^3 + (30*Sqrt[c]*(-16*c^3*d^3 + 32*b*c^2*d^2*e - 19*b^2*c*d*e^
2 + 3*b^3*e^3)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[b]*Sqrt[1 + (c*x)
/b]) + (15*Sqrt[c*d - b*e]*(32*c^3*d^3 - 48*b*c^2*d^2*e + 18*b^2*c*d*e^2 -
 b^3*e^3)*ArcTanh[(Sqrt[c*d - b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c*x])])/(Sqr
t[d]*Sqrt[b + c*x])))/(24*e^6*(-(c*d) + b*e)*Sqrt[x])
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 305, normalized size of antiderivative = 0.86, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1161, 1230, 27, 1230, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^4} \, dx$$

$$\Big\downarrow \text{ 1161}$$

$$\frac{5 \int \frac{(b+2cx)(cx^2+bx)^{3/2}}{(d+ex)^3} dx}{6e} - \frac{(bx + cx^2)^{5/2}}{3e(d + ex)^3}$$

$$\Big\downarrow \text{ 1230}$$

$$\frac{5\left(\frac{(bx+cx^2)^{3/2}(-be+4cd+2cex)}{2e^2(d+ex)^2}-\frac{3\int \frac{2(b(4cd-be)+4c(2cd-be)x)\sqrt{cx^2+bx}}{(d+ex)^2}dx}{8e^2}\right)}{6e}-\frac{(bx+cx^2)^{5/2}}{3e(d+ex)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\left(\frac{(bx+cx^2)^{3/2}(-be+4cd+2cex)}{2e^2(d+ex)^2}-\frac{3\int \frac{(b(4cd-be)+4c(2cd-be)x)\sqrt{cx^2+bx}}{(d+ex)^2}dx}{4e^2}\right)}{6e}-\frac{(bx+cx^2)^{5/2}}{3e(d+ex)^3}$$

$$\Big\downarrow \text{1230}$$

$$\frac{5\left(\frac{(bx+cx^2)^{3/2}(-be+4cd+2cex)}{2e^2(d+ex)^2}-\frac{3\left(\frac{\sqrt{bx+cx^2}\left(b^2e^2+4cex(2cd-be)-12bcde+16c^2d^2\right)}{e^2(d+ex)}-\frac{\int \frac{b\left(16c^2d^2-12bced+b^2e^2\right)+2c(4cd-3be)(4cd-be)x}{(d+ex)\sqrt{cx^2+bx}}dx}{2e^2}\right)}{4e^2}\right)}{6e}$$

$$\frac{(bx+cx^2)^{5/2}}{3e(d+ex)^3}$$

$$\Big\downarrow \text{1269}$$

$$\frac{5\left(\frac{(bx+cx^2)^{3/2}(-be+4cd+2cex)}{2e^2(d+ex)^2}-\frac{3\left(\frac{\sqrt{bx+cx^2}\left(b^2e^2+4cex(2cd-be)-12bcde+16c^2d^2\right)}{e^2(d+ex)}-\frac{\frac{2c(4cd-3be)(4cd-be)\int \frac{1}{\sqrt{cx^2+bx}}dx}{e}-\frac{(2cd-be)\left(b^2e^2-16bc\right.}{}}{2e^2}\right.}{4e^2}\right.}{6e}$$

$$\frac{(bx+cx^2)^{5/2}}{3e(d+ex)^3}$$

$$\Big\downarrow \text{1091}$$

$$\frac{5\left(\frac{(bx+cx^2)^{3/2}(-be+4cd+2cex)}{2e^2(d+ex)^2}-\frac{3\left(\frac{\sqrt{bx+cx^2}\left(b^2e^2+4cex(2cd-be)-12bcde+16c^2d^2\right)}{e^2(d+ex)}-\frac{4c(4cd-3be)(4cd-be)\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{e}-\frac{(2cd-be)\left(\right.}{2e^2}\right.}{4e^2}\right.}{6e}$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{3e(d+ex)^3}$$

$\big\downarrow$ 219

$$\frac{5\left(\frac{(bx+cx^2)^{3/2}(-be+4cd+2cex)}{2e^2(d+ex)^2}-\frac{3\left(\frac{\sqrt{bx+cx^2}\left(b^2e^2+4cex(2cd-be)-12bcde+16c^2d^2\right)}{e^2(d+ex)}-\frac{4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(4cd-3be)(4cd-be)}{e}-\frac{(2cd-be)\left(\right.}{2e^2}\right.}{4e^2}\right.}{6e}$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{3e(d+ex)^3}$$

$\big\downarrow$ 1154

$$\frac{5\left(\frac{(bx+cx^2)^{3/2}(-be+4cd+2cex)}{2e^2(d+ex)^2}-\frac{3\left(\frac{\sqrt{bx+cx^2}\left(b^2e^2+4cex(2cd-be)-12bcde+16c^2d^2\right)}{e^2(d+ex)}-\frac{2(2cd-be)\left(b^2e^2-16bcde+16c^2d^2\right)\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-}{cx^2+b}}}{e}\right.}{4e^2}\right.}{6e}$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{3e(d+ex)^3}$$

$\big\downarrow$ 219

$$\frac{5\left(\frac{(bx+cx^2)^{3/2}(-be+4cd+2cex)}{2e^2(d+ex)^2}-\frac{3\left(\frac{\sqrt{bx+cx^2}\left(b^2e^2+4cex(2cd-be)-12bcde+16c^2d^2\right)}{e^2(d+ex)}-\frac{\frac{4\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(4cd-3be)(4cd-be)}{e}-\frac{(2cd-be)(\cdots}{2\cdots}}{\cdots}\right.}{4e^2}\right.}{6e}$$

$$\frac{(bx+cx^2)^{5/2}}{3e(d+ex)^3}$$

input
```
Int[(b*x + c*x^2)^(5/2)/(d + e*x)^4,x]
```

output
```
-1/3*(b*x + c*x^2)^(5/2)/(e*(d + e*x)^3) + (5*(((4*c*d - b*e + 2*c*e*x)*(b
*x + c*x^2)^(3/2))/(2*e^2*(d + e*x)^2) - (3*(((16*c^2*d^2 - 12*b*c*d*e + b
^2*e^2 + 4*c*e*(2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(e^2*(d + e*x)) - ((4*S
qrt[c]*(4*c*d - 3*b*e)*(4*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]
])/e - ((2*c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*ArcTanh[(b*d + (
2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(Sqrt[d]*e
*Sqrt[c*d - b*e]))/(2*e^2)))/(4*e^2)))/(6*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$-\frac{5\left((ex+d)^{3}\left(b^{3}e^{3}\sqrt{c}-18b^{2}d\,e^{2}c^{\frac{3}{2}}+48b\,d^{2}e\,c^{\frac{5}{2}}-32c^{\frac{7}{2}}d^{3}\right)\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)+\left(-6(ex+d)^{3}(be-4cd)c\left(be-\frac{4cd}{3}\right)\text{arct}\right.\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^4,x,method=_RETURNVERBOSE)
```

output
```
-5/8/(d*(b*e-c*d))^(1/2)/c^(1/2)*((e*x+d)^3*(b^3*e^3*c^(1/2)-18*b^2*d*e^2*
c^(3/2)+48*b*d^2*e*c^(5/2)-32*c^(7/2)*d^3)*arctan((x*(c*x+b))^(1/2)/x*d/(d
*(b*e-c*d))^(1/2))+(-6*(e*x+d)^3*(b*e-4*c*d)*c*(b*e-4/3*c*d)*arctanh((x*(c
*x+b))^(1/2)/x/c^(1/2))+((-4/5*e^4*x^4+16*d^4+88/3*d^2*e^2*x^2+40*d^3*e*x+
4*d*e^3*x^3)*c^(5/2)+e*((-70/3*d*e^2*x^2-92/3*d^2*e*x-18/5*e^3*x^3-12*d^3)
*c^(3/2)+b*e*c^(1/2)*(8/3*d*e*x+11/5*e^2*x^2+d^2))*b)*e*(x*(c*x+b))^(1/2))
*(d*(b*e-c*d))^(1/2))/e^6/(e*x+d)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 769 vs. 2(313) = 626.

Time = 0.28 (sec) , antiderivative size = 3097, normalized size of antiderivative = 8.77

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}}{(d+ex)^4}\,dx$$

input
```
integrate((c*x**2+b*x)**(5/2)/(e*x+d)**4,x)
```

output
```
Integral((x*(b + c*x))**(5/2)/(d + e*x)**4, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(313) = 626$.

Time = 0.25 (sec) , antiderivative size = 977, normalized size of antiderivative = 2.77

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^4}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^4,x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x)*(2*c^2*x/e^4 - (16*c^3*d*e^10 - 9*b*c^2*e^11)/(c*e^1
5)) - 5/8*(32*c^3*d^3 - 48*b*c^2*d^2*e + 18*b^2*c*d*e^2 - b^3*e^3)*arctan(
-((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e))/(sq
rt(-c*d^2 + b*d*e)*e^6) - 5/8*(16*c^3*d^2 - 16*b*c^2*d*e + 3*b^2*c*e^2)*lo
g(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/(sqrt(c)*e^6) - 1/24
*(480*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*c^3*d^3*e^2 - 720*(sqrt(c)*x - sqr
t(c*x^2 + b*x))^5*b*c^2*d^2*e^3 + 306*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*b^
2*c*d*e^4 - 33*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*b^3*e^5 + 1680*(sqrt(c)*x
 - sqrt(c*x^2 + b*x))^4*c^(7/2)*d^4*e - 2160*(sqrt(c)*x - sqrt(c*x^2 + b*x
))^4*b*c^(5/2)*d^3*e^2 + 666*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*b^2*c^(3/2)
*d^2*e^3 - 21*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*b^3*sqrt(c)*d*e^4 + 1504*(
sqrt(c)*x - sqrt(c*x^2 + b*x))^3*c^4*d^5 - 400*(sqrt(c)*x - sqrt(c*x^2 + b
*x))^3*b*c^3*d^4*e - 1308*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^2*c^2*d^3*e^
2 + 574*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^3*c*d^2*e^3 - 40*(sqrt(c)*x -
sqrt(c*x^2 + b*x))^3*b^4*d*e^4 + 2256*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b*
c^(7/2)*d^5 - 2412*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^2*c^(5/2)*d^4*e + 4
62*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^3*c^(3/2)*d^3*e^2 + 24*(sqrt(c)*x -
 sqrt(c*x^2 + b*x))^2*b^4*sqrt(c)*d^2*e^3 + 1128*(sqrt(c)*x - sqrt(c*x^2 +
 b*x))*b^2*c^3*d^5 - 1272*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^3*c^2*d^4*e +
324*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^4*c*d^3*e^2 - 15*(sqrt(c)*x - sqr...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{(c\,x^2+b\,x)^{5/2}}{(d+e\,x)^4}\,dx$$

input
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^4,x)
```

output
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^4, x)
```

Reduce [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^4}\,dx = \int \frac{(c\,x^2+bx)^{\frac{5}{2}}}{(ex+d)^4}dx$$

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^4,x)
```

output
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^4,x)
```

3.156 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^5}\,dx$

Optimal result 1264
Mathematica [A] (verified) 1265
Rubi [A] (verified) 1266
Maple [A] (verified) 1270
Fricas [B] (verification not implemented) 1271
Sympy [F] 1271
Maxima [F(-2)] 1272
Giac [F(-1)] 1272
Mupad [F(-1)] 1272
Reduce [F] 1273

Optimal result

Integrand size = 21, antiderivative size = 413

$$
\begin{aligned}
\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^5}\,dx &= \frac{5(64c^3d^3-80bc^2d^2e+16b^2cde^2+b^3e^3)\sqrt{bx+cx^2}}{64d^2e^5(cd-be)} \\
&+ \frac{5\left(12bc-\frac{16c^2d}{e}+\frac{b^2e}{d}\right)x^2\sqrt{bx+cx^2}}{96de^2(d+ex)^2} \\
&- \frac{5(4cd-be)\left(24c^2d^2-22bcde-b^2e^2\right)x\sqrt{bx+cx^2}}{192d^2e^4(cd-be)(d+ex)} \\
&- \frac{5(2cd-be)x(bx+cx^2)^{3/2}}{24de^2(d+ex)^3} - \frac{(bx+cx^2)^{5/2}}{4e(d+ex)^4} - \frac{5c^{3/2}(2cd-be)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{e^6} \\
&+ \frac{5(128c^4d^4-256bc^3d^3e+144b^2c^2d^2e^2-16b^3cde^3-b^4e^4)\operatorname{arctanh}\left(\frac{\sqrt{cd-be}x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{64d^{3/2}e^6(cd-be)^{3/2}}
\end{aligned}
$$

output
```
5/64*(b^3*e^3+16*b^2*c*d*e^2-80*b*c^2*d^2*e+64*c^3*d^3)*(c*x^2+b*x)^(1/2)/
d^2/e^5/(-b*e+c*d)+5/96*(12*b*c-16*c^2*d/e+b^2*e/d)*x^2*(c*x^2+b*x)^(1/2)/
d/e^2/(e*x+d)^2-5/192*(-b*e+4*c*d)*(-b^2*e^2-22*b*c*d*e+24*c^2*d^2)*x*(c*x
^2+b*x)^(1/2)/d^2/e^4/(-b*e+c*d)/(e*x+d)-5/24*(-b*e+2*c*d)*x*(c*x^2+b*x)^(
3/2)/d/e^2/(e*x+d)^3-1/4*(c*x^2+b*x)^(5/2)/e/(e*x+d)^4-5*c^(3/2)*(-b*e+2*c
*d)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/e^6+5/64*(-b^4*e^4-16*b^3*c*d*e^3
+144*b^2*c^2*d^2*e^2-256*b*c^3*d^3*e+128*c^4*d^4)*arctanh((-b*e+c*d)^(1/2)
*x/d^(1/2)/(c*x^2+b*x)^(1/2))/d^(3/2)/e^6/(-b*e+c*d)^(3/2)
```

Mathematica [A] (verified)

Time = 12.13 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.94

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^5}\,dx = \frac{\sqrt{x(b+cx)}\left(\frac{e\sqrt{x}(b^3e^3(15d^3+55d^2ex+73de^2x^2-15e^3x^3)+2b^2cde^2(120d^3+435d^2ex+566de^2x^2+323e^3x^3)+1}{}\right.}{}$$

input
```
Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^5,x]
```

output
```
(Sqrt[x*(b + c*x)]*((e*Sqrt[x]*(b^3*e^3*(15*d^3 + 55*d^2*e*x + 73*d*e^2*x^
2 - 15*e^3*x^3) + 2*b^2*c*d*e^2*(120*d^3 + 435*d^2*e*x + 566*d*e^2*x^2 + 3
23*e^3*x^3) + 16*c^3*d^2*(60*d^4 + 210*d^3*e*x + 260*d^2*e^2*x^2 + 125*d*e
^3*x^3 + 12*e^4*x^4) - 8*b*c^2*d*e*(150*d^4 + 530*d^3*e*x + 665*d^2*e^2*x^
2 + 327*d*e^3*x^3 + 24*e^4*x^4)))/(d*(c*d - b*e)*(d + e*x)^4) + (960*c^(3/
2)*(-2*c*d + b*e)*ArcSinh[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[b]*Sqrt[1 + (c
*x)/b]) + (15*(128*c^4*d^4 - 256*b*c^3*d^3*e + 144*b^2*c^2*d^2*e^2 - 16*b^
3*c*d*e^3 - b^4*e^4)*ArcTanh[(Sqrt[c*d - b*e]*Sqrt[x])/(Sqrt[d]*Sqrt[b + c
*x])])/(d^(3/2)*(c*d - b*e)^(3/2)*Sqrt[b + c*x])))/(192*e^6*Sqrt[x])
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 416, normalized size of antiderivative = 1.01, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {1161, 1229, 27, 1230, 1269, 1091, 219, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^5}dx$$

$$\Big\downarrow \text{1161}$$

$$\frac{5\int \frac{(b+2cx)(cx^2+bx)^{3/2}}{(d+ex)^4}dx}{8e}-\frac{(bx+cx^2)^{5/2}}{4e(d+ex)^4}$$

$$\Big\downarrow \text{1229}$$

$$\frac{5\left(-\frac{\int -\frac{\left(b\left(16c^2d^2-12bced-b^2e^2\right)+2c\left(16c^2d^2-16bced+b^2e^2\right)x\right)\sqrt{cx^2+bx}}{2(d+ex)^2}dx}{4de^2(cd-be)}-\frac{(bx+cx^2)^{3/2}(3ex(b^2e^2-8bcde+8c^2d^2)+d(-b^2e^2-12bcde+16c^2d}{12de^2(d+ex)^3(cd-be)}\right.}{8e}$$

$$\frac{(bx+cx^2)^{5/2}}{4e(d+ex)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\left(\frac{\int \frac{\left(b\left(16c^2d^2-12bced-b^2e^2\right)+2c\left(16c^2d^2-16bced+b^2e^2\right)x\right)\sqrt{cx^2+bx}}{(d+ex)^2}dx}{8de^2(cd-be)}-\frac{(bx+cx^2)^{3/2}(3ex(b^2e^2-8bcde+8c^2d^2)+d(-b^2e^2-12bcde+16c^2d^2))}{12de^2(d+ex)^3(cd-be)}\right)}{8e}$$

$$\frac{(bx+cx^2)^{5/2}}{4e(d+ex)^4}$$

$$\Big\downarrow \text{1230}$$

$$\frac{5\left(\frac{\frac{\sqrt{bx+cx^2}\left(b^3e^3+2cex\left(b^2e^2-16bcde+16c^2d^2\right)+16b^2cde^2-80bc^2d^2e+64c^3d^3\right)}{e^2(d+ex)}-\frac{\int \frac{64d(cd-be)(2cd-be)xc^2+b\left(64c^3d^3-80bc^2ed^2+16b^2ce^2d+b^3e^3\right)}{(d+ex)\sqrt{cx^2+bx}}dx}{2e^2}}{8de^2(cd-be)}-\right.}{8e}$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{4e(d+ex)^4}$$

$\big\downarrow$ 1269

$$\frac{5\left(\frac{\frac{\sqrt{bx+cx^2}\left(b^3e^3+2cex\left(b^2e^2-16bcde+16c^2d^2\right)+16b^2cde^2-80bc^2d^2e+64c^3d^3\right)}{e^2(d+ex)}-\frac{\frac{64c^2d(cd-be)(2cd-be)\int\frac{1}{\sqrt{cx^2+bx}}dx}{e}-\frac{\left(-b^4e^4-16b^3cde^3+144b^2c^2d^2e^2\right.}{}}{2e^2}}{8de^2(cd-be)}\right.}{8e}$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{4e(d+ex)^4}$$

$\big\downarrow$ 1091

$$\frac{5\left(\frac{\frac{\sqrt{bx+cx^2}\left(b^3e^3+2cex\left(b^2e^2-16bcde+16c^2d^2\right)+16b^2cde^2-80bc^2d^2e+64c^3d^3\right)}{e^2(d+ex)}-\frac{\frac{128c^2d(cd-be)(2cd-be)\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{e}-\frac{\left(-b^4e^4-16b^3cde^3+1\right.}{}}{2e^2}}{8de^2(cd-be)}\right.}{8e}$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{4e(d+ex)^4}$$

$\big\downarrow$ 219

$$\frac{5\left(\frac{\frac{\sqrt{bx+cx^2}\left(b^3e^3+2cex\left(b^2e^2-16bcde+16c^2d^2\right)+16b^2cde^2-80bc^2d^2e+64c^3d^3\right)}{e^2(d+ex)}-\frac{\frac{128c^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(cd-be)(2cd-be)}{e}-\frac{\left(-b^4e^4-16b^3cde^3+\right.}{}}{2e^2}}{8de^2(cd-be)}\right.}{8e}$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{4e(d+ex)^4}$$

$\big\downarrow$ 1154

$$5\left(\frac{\frac{\sqrt{bx+cx^2}\left(b^3e^3+2cex\left(b^2e^2-16bcde+16c^2d^2\right)+16b^2cde^2-80bc^2d^2e+64c^3d^3\right)}{e^2(d+ex)}-\frac{2\left(-b^4e^4-16b^3cde^3+144b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)\int\frac{}{4d(cd-be)-\cdots}}{e}}{8de^2(cd-be)}\right.$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{4e(d+ex)^4}$$

$$\Big\downarrow \; 219$$

$$5\left(\frac{\frac{\sqrt{bx+cx^2}\left(b^3e^3+2cex\left(b^2e^2-16bcde+16c^2d^2\right)+16b^2cde^2-80bc^2d^2e+64c^3d^3\right)}{e^2(d+ex)}-\frac{128c^{3/2}d\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(cd-be)(2cd-be)}{e}-\frac{\left(-b^4e^4-16b^3cde^3+\cdots\right.}{2e\cdots}}{8de^2(cd-be)}\right.$$

$$8e\cdots$$

$$\frac{\left(bx+cx^2\right)^{5/2}}{4e(d+ex)^4}$$

input
```
Int[(b*x + c*x^2)^(5/2)/(d + e*x)^5,x]
```

output
```
-1/4*(b*x + c*x^2)^(5/2)/(e*(d + e*x)^4) + (5*(-1/12*((d*(16*c^2*d^2 - 12*
b*c*d*e - b^2*e^2) + 3*e*(8*c^2*d^2 - 8*b*c*d*e + b^2*e^2)*x)*(b*x + c*x^2
)^(3/2))/(d*e^2*(c*d - b*e)*(d + e*x)^3) + (((64*c^3*d^3 - 80*b*c^2*d^2*e
+ 16*b^2*c*d*e^2 + b^3*e^3 + 2*c*e*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*x)*
Sqrt[b*x + c*x^2])/(e^2*(d + e*x)) - ((128*c^(3/2)*d*(c*d - b*e)*(2*c*d -
b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/e - ((128*c^4*d^4 - 256*b*c^3
*d^3*e + 144*b^2*c^2*d^2*e^2 - 16*b^3*c*d*e^3 - b^4*e^4)*ArcTanh[(b*d + (2
*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(Sqrt[d]*e*
Sqrt[c*d - b*e]))/(2*e^2))/(8*d*e^2*(c*d - b*e))))/(8*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.91

method	result
pseudoelliptic	$-\frac{5\left((ex+d)^4\left(b^4e^4\sqrt{c}+16b^3d\,e^3c^{\frac{3}{2}}-144b^2c^{\frac{5}{2}}d^2e^2+256b\,c^{\frac{7}{2}}d^3e-128c^{\frac{9}{2}}d^4\right)\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)+\sqrt{d(be-cd)}\left(-64c^2d\right.}{}$
risch	Expression too large to display
default	Expression too large to display

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^5,x,method=_RETURNVERBOSE)
```

output
```
-5/64/c^(1/2)*((e*x+d)^4*(b^4*e^4*c^(1/2)+16*b^3*d*e^3*c^(3/2)-144*b^2*c^(
5/2)*d^2*e^2+256*b*c^(7/2)*d^3*e-128*c^(9/2)*d^4)*arctan((x*(c*x+b))^(1/2)
/x*d/(d*(b*e-c*d))^(1/2))+(d*(b*e-c*d))^(1/2)*(-64*c^2*d*(e*x+d)^4*(b*e-c*
d)*(b*e-2*c*d)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))+e*(x*(c*x+b))^(1/2)*(-
80*e*(4/25*e^4*x^4+109/50*d*e^3*x^3+133/30*d^2*e^2*x^2+53/15*d^3*e*x+d^4)*
b*d*c^(5/2)+64*(1/5*e^4*x^4+25/12*d*e^3*x^3+13/3*d^2*e^2*x^2+7/2*d^3*e*x+d
^4)*d^2*c^(7/2)+e^2*((646/15*d*e^3*x^3+1132/15*d^2*e^2*x^2+58*d^3*e*x+16*d
^4)*c^(3/2)+b*e*c^(1/2)*(11/3*d^2*e*x+73/15*d*e^2*x^2-e^3*x^3+d^3))*b^2)))
/(d*(b*e-c*d))^(1/2)/(e*x+d)^4/e^6/(b*e-c*d)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1133 vs. $2(375) = 750$.

Time = 0.77 (sec) , antiderivative size = 4553, normalized size of antiderivative = 11.02

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^5}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^5,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^5}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}}{(d+ex)^5}\,dx$$

input
```
integrate((c*x**2+b*x)**(5/2)/(e*x+d)**5,x)
```

output
```
Integral((x*(b + c*x))**(5/2)/(d + e*x)**5, x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^5}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^5,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [F(-1)]

Timed out.

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^5}\,dx = \text{Timed out}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^5,x, algorithm="giac")
```

output
```
Timed out
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^5}\,dx = \int \frac{(c\,x^2 + b\,x)^{5/2}}{(d + e\,x)^5}\,dx$$

input
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^5,x)
```

output
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^5, x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^5}\, dx = \int \frac{(c\,x^2 + bx)^{\frac{5}{2}}}{(ex + d)^5} dx$$

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^5,x)
```

output
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^5,x)
```

3.157 $\int \frac{(d+ex)^3}{\sqrt{bx+cx^2}}\,dx$

Optimal result 1274
Mathematica [A] (verified) 1275
Rubi [A] (verified) 1275
Maple [A] (verified) 1277
Fricas [A] (verification not implemented) 1278
Sympy [A] (verification not implemented) 1279
Maxima [A] (verification not implemented) 1280
Giac [A] (verification not implemented) 1280
Mupad [F(-1)] 1281
Reduce [B] (verification not implemented) 1281

Optimal result

Integrand size = 21, antiderivative size = 166

$$\int \frac{(d+ex)^3}{\sqrt{bx+cx^2}}\,dx = \frac{e(24c^2d^2-18bcde+5b^2e^2)\sqrt{bx+cx^2}}{8c^3} + \frac{e^2(18cd-5be)x\sqrt{bx+cx^2}}{12c^2} + \frac{e^3x^2\sqrt{bx+cx^2}}{3c} + \frac{(2cd-be)\left(8c^2d^2-8bcde+5b^2e^2\right)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{7/2}}$$

output
```
1/8*e*(5*b^2*e^2-18*b*c*d*e+24*c^2*d^2)*(c*x^2+b*x)^(1/2)/c^3+1/12*e^2*(-5
*b*e+18*c*d)*x*(c*x^2+b*x)^(1/2)/c^2+1/3*e^3*x^2*(c*x^2+b*x)^(1/2)/c+1/8*(
-b*e+2*c*d)*(5*b^2*e^2-8*b*c*d*e+8*c^2*d^2)*arctanh(c^(1/2)*x/(c*x^2+b*x)^
(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^3}{\sqrt{bx+cx^2}}\,dx = \frac{\sqrt{c}ex(b+cx)\left(15b^2e^2-2bce(27d+5ex)+4c^2(18d^2+9dex+2e^2x^2)\right)+3(-16c^3d^3+24bc^2d^2e-18b^2c}{24c^{7/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[(d + e*x)^3/Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[c]*e*x*(b + c*x)*(15*b^2*e^2 - 2*b*c*e*(27*d + 5*e*x) + 4*c^2*(18*d^
2 + 9*d*e*x + 2*e^2*x^2)) + 3*(-16*c^3*d^3 + 24*b*c^2*d^2*e - 18*b^2*c*d*e
^2 + 5*b^3*e^3)*Sqrt[x]*Sqrt[b + c*x]*Log[-(Sqrt[c]*Sqrt[x]) + Sqrt[b + c*
x]])/(24*c^(7/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1166, 27, 1225, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{\sqrt{bx+cx^2}}\,dx$$

$$\Big\downarrow \; 1166$$

$$\frac{\int \frac{(d+ex)(d(6cd-be)+5e(2cd-be)x)}{2\sqrt{cx^2+bx}}dx}{3c}+\frac{e\sqrt{bx+cx^2}(d+ex)^2}{3c}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{(d+ex)(d(6cd-be)+5e(2cd-be)x)}{\sqrt{cx^2+bx}}dx}{6c}+\frac{e\sqrt{bx+cx^2}(d+ex)^2}{3c}$$

$$\Big\downarrow \; 1225$$

$$\frac{\frac{3(2cd-be)\left(5b^2e^2-8bcde+8c^2d^2\right)\int \frac{1}{\sqrt{cx^2+bx}}dx}{8c^2}+\frac{e\sqrt{bx+cx^2}\left(15b^2e^2+10cex(2cd-be)-54bcde+64c^2d^2\right)}{4c^2}}{6c}+\frac{e\sqrt{bx+cx^2}(d+ex)^2}{3c}$$

$\Big\downarrow$ 1091

$$\frac{\frac{3(2cd-be)\left(5b^2e^2-8bcde+8c^2d^2\right)\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c^2}+\frac{e\sqrt{bx+cx^2}\left(15b^2e^2+10cex(2cd-be)-54bcde+64c^2d^2\right)}{4c^2}}{6c}+\frac{e\sqrt{bx+cx^2}(d+ex)^2}{3c}$$

$\Big\downarrow$ 219

$$\frac{\frac{3\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)\left(5b^2e^2-8bcde+8c^2d^2\right)}{4c^{5/2}}+\frac{e\sqrt{bx+cx^2}\left(15b^2e^2+10cex(2cd-be)-54bcde+64c^2d^2\right)}{4c^2}}{6c}+\frac{e\sqrt{bx+cx^2}(d+ex)^2}{3c}$$

input
```
Int[(d + e*x)^3/Sqrt[b*x + c*x^2],x]
```

output
```
(e*(d + e*x)^2*Sqrt[b*x + c*x^2])/(3*c) + ((e*(64*c^2*d^2 - 54*b*c*d*e + 1
5*b^2*e^2 + 10*c*e*(2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(4*c^2) + (3*(2*c*d
 - b*e)*(8*c^2*d^2 - 8*b*c*d*e + 5*b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x +
 c*x^2]])/(4*c^(5/2)))/(6*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{5\left((be-2cd)\left(b^2e^2-\frac{8}{5}bcde+\frac{8}{5}c^2d^2\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)-e\sqrt{x(cx+b)}\left(\left(\frac{8}{15}e^2x^2+\frac{12}{5}dex+\frac{24}{5}d^2\right)c^{\frac{5}{2}}+e\left(\left(-\frac{2ex}{3}-\frac{18d}{5}\right)c\right.\right.\right.}{8c^{\frac{7}{2}}}$
risch	$\frac{\left(8c^2e^2x^2-10e^2xbc+36c^2dex+15b^2e^2-54bcde+72c^2d^2\right)ex(cx+b)}{24c^3\sqrt{x(cx+b)}}-\frac{\left(5b^3e^3-18d\,e^2b^2c+24d^2eb\,c^2-16d^3c^3\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\right.}{16c^{\frac{7}{2}}}$
default	$\frac{d^3\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{\sqrt{c}}+e^3\left(\frac{x^2\sqrt{c\,x^2+bx}}{3c}-\frac{5b\left(\frac{x\sqrt{c\,x^2+bx}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^2+bx}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{2c^{\frac{3}{2}}}\right)}{4c}\right)}{6c}\right)+3$

input
```
int((e*x+d)^3/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/8/c^(7/2)*((b*e-2*c*d)*(b^2*e^2-8/5*b*c*d*e+8/5*c^2*d^2)*arctanh((x*(c*
x+b))^(1/2)/x/c^(1/2))-e*(x*(c*x+b))^(1/2)*((8/15*e^2*x^2+12/5*d*e*x+24/5*
d^2)*c^(5/2)+e*((-2/3*e*x-18/5*d)*c^(3/2)+b*e*c^(1/2))*b))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.79

$$\int \frac{(d+ex)^3}{\sqrt{bx+cx^2}}\,dx$$

$$=\left[-\frac{3\left(16\,c^3d^3-24\,bc^2d^2e+18\,b^2cde^2-5\,b^3e^3\right)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\left(8\,c^3e^3x^2+72\,c^3d\right.}{48\,c^4}\right.$$

$$-\frac{3\left(16\,c^3d^3-24\,bc^2d^2e+18\,b^2cde^2-5\,b^3e^3\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-\left(8\,c^3e^3x^2+72\,c^3d^2e-54\,bc^2d\right.}{24\,c^4}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 18*b^2*c*d*e^2 - 5*b^3*e^3)*sqrt(
c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(8*c^3*e^3*x^2 + 72*c^
3*d^2*e - 54*b*c^2*d*e^2 + 15*b^2*c*e^3 + 2*(18*c^3*d*e^2 - 5*b*c^2*e^3)*x
)*sqrt(c*x^2 + b*x))/c^4, -1/24*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 18*b^2*c
*d*e^2 - 5*b^3*e^3)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b))
- (8*c^3*e^3*x^2 + 72*c^3*d^2*e - 54*b*c^2*d*e^2 + 15*b^2*c*e^3 + 2*(18*c^
3*d*e^2 - 5*b*c^2*e^3)*x)*sqrt(c*x^2 + b*x))/c^4]
```

Sympy [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.55

$$\int \frac{(d+ex)^3}{\sqrt{bx+cx^2}}\,dx = \begin{cases} \sqrt{bx+cx^2}\left(\frac{e^3x^2}{3c} + \frac{x\left(-\frac{5be^3}{6c}+3de^2\right)}{2c} + \frac{-\frac{3b\left(-\frac{5be^3}{6c}+3de^2\right)}{4c}+3d^2e}{c}\right) + \left(-\frac{b\left(-\frac{3b\left(-\frac{5be^3}{6c}+3de^2\right)}{4c}+3d^2e\right)}{2c} + d^3\right)\left(\left(\begin{cases} \frac{\log(\ldots}{\ldots} \\ \frac{\left(\frac{b}{2c}\ldots\right.}{\ldots} \end{cases}\right.\right. \\ \frac{2\left(d^3\sqrt{bx}+\frac{d^2e(bx)^{\frac{3}{2}}}{b}+\frac{3de^2(bx)^{\frac{5}{2}}}{5b^2}+\frac{e^3(bx)^{\frac{7}{2}}}{7b^3}\right)}{b} \\ \tilde{\infty}\left(\begin{cases} d^3x & \text{for } e=0 \\ \frac{(d+ex)^4}{4e} & \text{otherwise} \end{cases}\right) \end{cases}$$

input
```
integrate((e*x+d)**3/(c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((sqrt(b*x + c*x**2)*(e**3*x**2/(3*c) + x*(-5*b*e**3/(6*c) + 3*d*
e**2)/(2*c) + (-3*b*(-5*b*e**3/(6*c) + 3*d*e**2)/(4*c) + 3*d**2*e)/c) + (-
b*(-3*b*(-5*b*e**3/(6*c) + 3*d*e**2)/(4*c) + 3*d**2*e)/(2*c) + d**3)*Piece
wise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)
), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c,
 0)), (2*(d**3*sqrt(b*x) + d**2*e*(b*x)**(3/2)/b + 3*d*e**2*(b*x)**(5/2)/(
5*b**2) + e**3*(b*x)**(7/2)/(7*b**3))/b, Ne(b, 0)), (zoo*Piecewise((d**3*x
, Eq(e, 0)), ((d + e*x)**4/(4*e), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.57

$$\int \frac{(d+ex)^3}{\sqrt{bx+cx^2}}\,dx = \frac{\sqrt{cx^2+bx}e^3x^2}{3\,c} + \frac{3\,\sqrt{cx^2+bx}de^2x}{2\,c} - \frac{5\,\sqrt{cx^2+bx}be^3x}{12\,c^2} + \frac{d^3\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{\sqrt{c}} - \frac{3\,bd^2e\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{2\,c^{\frac{3}{2}}} + \frac{9\,b^2de^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,c^{\frac{5}{2}}} - \frac{5\,b^3e^3\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{16\,c^{\frac{7}{2}}} + \frac{3\,\sqrt{cx^2+bx}d^2e}{c} - \frac{9\,\sqrt{cx^2+bx}bde^2}{4\,c^2} + \frac{5\,\sqrt{cx^2+bx}b^2e^3}{8\,c^3}$$

input

```
integrate((e*x+d)^3/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output

```
1/3*sqrt(c*x^2 + b*x)*e^3*x^2/c + 3/2*sqrt(c*x^2 + b*x)*d*e^2*x/c - 5/12*s
qrt(c*x^2 + b*x)*b*e^3*x/c^2 + d^3*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqr
t(c))/sqrt(c) - 3/2*b*d^2*e*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c
^(3/2) + 9/8*b^2*d*e^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(5/2
) - 5/16*b^3*e^3*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(7/2) + 3*
sqrt(c*x^2 + b*x)*d^2*e/c - 9/4*sqrt(c*x^2 + b*x)*b*d*e^2/c^2 + 5/8*sqrt(c
*x^2 + b*x)*b^2*e^3/c^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^3}{\sqrt{bx+cx^2}}\,dx = \frac{1}{24}\,\sqrt{cx^2+bx}\left(2\left(\frac{4\,e^3x}{c}+\frac{18\,c^2de^2-5\,bce^3}{c^3}\right)x+\frac{3\,(24\,c^2d^2e-18\,bcde^2+5\,b^2e^3)}{c^3}\right) - \frac{(16\,c^3d^3-24\,bc^2d^2e+18\,b^2cde^2-5\,b^3e^3)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{7}{2}}}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x)*(2*(4*e^3*x/c + (18*c^2*d*e^2 - 5*b*c*e^3)/c^3)*x +
 3*(24*c^2*d^2*e - 18*b*c*d*e^2 + 5*b^2*e^3)/c^3) - 1/16*(16*c^3*d^3 - 24*
b*c^2*d^2*e + 18*b^2*c*d*e^2 - 5*b^3*e^3)*log(abs(2*(sqrt(c)*x - sqrt(c*x^
2 + b*x))*sqrt(c) + b))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{\sqrt{bx+cx^2}}\,dx=\int \frac{(d+e\,x)^3}{\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((d + e*x)^3/(b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x)^3/(b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.38

$$\int \frac{(d+ex)^3}{\sqrt{bx+cx^2}}\,dx = \frac{15\sqrt{x}\,\sqrt{cx+b}\,b^2ce^3-54\sqrt{x}\,\sqrt{cx+b}\,b\,c^2d\,e^2-10\sqrt{x}\,\sqrt{cx+b}\,b\,c^2e^3x+72\sqrt{x}\,\sqrt{cx+b}\,c^3d^2e+36\sqrt{x}\,\sqrt{}}{}$$

input
```
int((e*x+d)^3/(c*x^2+b*x)^(1/2),x)
```

output
```
(15*sqrt(x)*sqrt(b + c*x)*b**2*c*e**3 - 54*sqrt(x)*sqrt(b + c*x)*b*c**2*d*
e**2 - 10*sqrt(x)*sqrt(b + c*x)*b*c**2*e**3*x + 72*sqrt(x)*sqrt(b + c*x)*c
**3*d**2*e + 36*sqrt(x)*sqrt(b + c*x)*c**3*d*e**2*x + 8*sqrt(x)*sqrt(b + c
*x)*c**3*e**3*x**2 - 15*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt
(b))*b**3*e**3 + 54*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))
*b**2*c*d*e**2 - 72*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))
*b*c**2*d**2*e + 48*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))
*c**3*d**3)/(24*c**4)
```

3.158 $\int \frac{(d+ex)^2}{\sqrt{bx+cx^2}} dx$

Optimal result 1283
Mathematica [A] (verified) 1283
Rubi [A] (verified) 1284
Maple [A] (verified) 1286
Fricas [A] (verification not implemented) 1286
Sympy [A] (verification not implemented) 1287
Maxima [A] (verification not implemented) 1288
Giac [A] (verification not implemented) 1288
Mupad [F(-1)] 1289
Reduce [B] (verification not implemented) 1289

Optimal result

Integrand size = 21, antiderivative size = 108

$$\int \frac{(d+ex)^2}{\sqrt{bx+cx^2}} dx = \frac{e(8cd-3be)\sqrt{bx+cx^2}}{4c^2} + \frac{e^2x\sqrt{bx+cx^2}}{2c} + \frac{(8c^2d^2-8bcde+3b^2e^2)\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{5/2}}$$

output
```
1/4*e*(-3*b*e+8*c*d)*(c*x^2+b*x)^(1/2)/c^2+1/2*e^2*x*(c*x^2+b*x)^(1/2)/c+1
/4*(3*b^2*e^2-8*b*c*d*e+8*c^2*d^2)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^
(5/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^2}{\sqrt{bx+cx^2}} dx = \frac{\sqrt{c}ex(b+cx)(8cd-3be+2cex) + (-8c^2d^2+8bcde-3b^2e^2)\sqrt{x}\sqrt{b+cx}\log\left(-\sqrt{c}\sqrt{x}+\sqrt{b+cx}\right)}{4c^{5/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[(d + e*x)^2/Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[c]*e*x*(b + c*x)*(8*c*d - 3*b*e + 2*c*e*x) + (-8*c^2*d^2 + 8*b*c*d*e
 - 3*b^2*e^2)*Sqrt[x]*Sqrt[b + c*x]*Log[-(Sqrt[c]*Sqrt[x]) + Sqrt[b + c*x]
])/(4*c^(5/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.05, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {1166, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{\sqrt{bx+cx^2}}\,dx$$

$$\Big\downarrow \; 1166$$

$$\frac{\int \frac{d(4cd-be)+3e(2cd-be)x}{2\sqrt{cx^2+bx}}dx}{2c}+\frac{e\sqrt{bx+cx^2}(d+ex)}{2c}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{d(4cd-be)+3e(2cd-be)x}{\sqrt{cx^2+bx}}dx}{4c}+\frac{e\sqrt{bx+cx^2}(d+ex)}{2c}$$

$$\Big\downarrow \; 1160$$

$$\frac{\frac{(3b^2e^2-8bcde+8c^2d^2)\int \frac{1}{\sqrt{cx^2+bx}}dx}{2c}+\frac{3e\sqrt{bx+cx^2}(2cd-be)}{c}}{4c}+\frac{e\sqrt{bx+cx^2}(d+ex)}{2c}$$

$$\Big\downarrow \; 1091$$

$$\frac{\frac{(3b^2e^2-8bcde+8c^2d^2)\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{c}+\frac{3e\sqrt{bx+cx^2}(2cd-be)}{c}}{4c}+\frac{e\sqrt{bx+cx^2}(d+ex)}{2c}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(3b^2e^2-8bcde+8c^2d^2)}{c^{3/2}}+\frac{3e\sqrt{bx+cx^2}(2cd-be)}{c}}{4c}+\frac{e\sqrt{bx+cx^2}(d+ex)}{2c}$$

input
```
Int[(d + e*x)^2/Sqrt[b*x + c*x^2],x]
```

output
```
(e*(d + e*x)*Sqrt[b*x + c*x^2])/(2*c) + ((3*e*(2*c*d - b*e)*Sqrt[b*x + c*x
^2])/c + ((8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x
 + c*x^2]])/c^(3/2))/(4*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.73

method	result
pseudoelliptic	$\frac{\frac{3\left(b^{2} e^{2}-\frac{8}{3} b c d e+\frac{8}{3} c^{2} d^{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{x(c x+b)}}{x \sqrt{c}}\right)}{4}-\frac{3 e\left(\frac{2(-e x-4 d) c^{\frac{3}{2}}}{3}+b e \sqrt{c}\right) \sqrt{x(c x+b)}}{4}}{c^{\frac{5}{2}}}$
risch	$-\frac{(-2 c e x+3 b e-8 c d) e x(c x+b)}{4 c^{2} \sqrt{x(c x+b)}}+\frac{\left(3 b^{2} e^{2}-8 b c d e+8 c^{2} d^{2}\right) \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x}\right)}{8 c^{\frac{5}{2}}}$
default	$\frac{d^{2} \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x}\right)}{\sqrt{c}}+e^{2}\left(\frac{x \sqrt{c\,x^{2}+b x}}{2 c}-\frac{3 b\left(\frac{\sqrt{c\,x^{2}+b x}}{c}-\frac{b \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x}\right)}{2 c^{\frac{3}{2}}}\right)}{4 c}\right)+2 d e\left(\frac{\sqrt{c\,x^{2}+b x}}{c}-\right.$

input
```
int((e*x+d)^2/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
3/4/c^(5/2)*((b^2*e^2-8/3*b*c*d*e+8/3*c^2*d^2)*arctanh((x*(c*x+b))^(1/2)/x
/c^(1/2))-e*(2/3*(-e*x-4*d)*c^(3/2)+b*e*c^(1/2))*(x*(c*x+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.77

$$\int \frac{(d+ex)^2}{\sqrt{bx+cx^2}}\,dx$$
$$= \left[\frac{(8\,c^2d^2-8\,bcde+3\,b^2e^2)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,(2\,c^2e^2x+8\,c^2de-3\,bce^2)\sqrt{cx^2+bx}}{8\,c^3},\right.$$
$$\left.-\frac{(8\,c^2d^2-8\,bcde+3\,b^2e^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(2\,c^2e^2x+8\,c^2de-3\,bce^2)\sqrt{cx^2+bx}}{4\,c^3}\right]$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2)*sqrt(c)*log(2*c*x + b + 2*sqrt(c
*x^2 + b*x)*sqrt(c)) + 2*(2*c^2*e^2*x + 8*c^2*d*e - 3*b*c*e^2)*sqrt(c*x^2
+ b*x))/c^3, -1/4*((8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2)*sqrt(-c)*arctan(sqr
t(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) - (2*c^2*e^2*x + 8*c^2*d*e - 3*b*c*e^2)
*sqrt(c*x^2 + b*x))/c^3]
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.71

$$\int \frac{(d+ex)^2}{\sqrt{bx+cx^2}}\,dx = \begin{cases} \sqrt{bx+cx^2}\left(\frac{e^2x}{2c}+\frac{-\frac{3be^2}{4c}+2de}{c}\right)+\left(-\frac{b\left(-\frac{3be^2}{4c}+2de\right)}{2c}+d^2\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right) & \text{for } c \\ \frac{2\left(d^2\sqrt{bx}+\frac{2de(bx)^{\frac{3}{2}}}{3b}+\frac{e^2(bx)^{\frac{5}{2}}}{5b^2}\right)}{b} & \text{for } b \\ \tilde{\infty}\left(\begin{cases} d^2x & \text{for } e=0 \\ \frac{(d+ex)^3}{3e} & \text{otherwise}\end{cases}\right) & \text{other} \end{cases}$$

input
```
integrate((e*x+d)**2/(c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((sqrt(b*x + c*x**2)*(e**2*x/(2*c) + (-3*b*e**2/(4*c) + 2*d*e)/c)
 + (-b*(-3*b*e**2/(4*c) + 2*d*e)/(2*c) + d**2)*Piecewise((log(b + 2*sqrt(c
)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(
b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), (2*(d**2*sqrt(b*
x) + 2*d*e*(b*x)**(3/2)/(3*b) + e**2*(b*x)**(5/2)/(5*b**2))/b, Ne(b, 0)),
(zoo*Piecewise((d**2*x, Eq(e, 0)), ((d + e*x)**3/(3*e), True)), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.43

$$\int \frac{(d+ex)^2}{\sqrt{bx+cx^2}}\,dx = \frac{\sqrt{cx^2+bx}e^2x}{2\,c} + \frac{d^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{\sqrt{c}} - \frac{bde\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{c^{\frac{3}{2}}} + \frac{3\,b^2e^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,c^{\frac{5}{2}}} + \frac{2\,\sqrt{cx^2+bx}de}{c} - \frac{3\,\sqrt{cx^2+bx}be^2}{4\,c^2}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^2 + b*x)*e^2*x/c + d^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sq
rt(c))/sqrt(c) - b*d*e*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(3/2
) + 3/8*b^2*e^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(5/2) + 2*s
qrt(c*x^2 + b*x)*d*e/c - 3/4*sqrt(c*x^2 + b*x)*b*e^2/c^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^2}{\sqrt{bx+cx^2}}\,dx = \frac{1}{4}\,\sqrt{cx^2+bx}\left(\frac{2\,e^2x}{c} + \frac{8\,cde-3\,be^2}{c^2}\right) - \frac{\left(8\,c^2d^2-8\,bcde+3\,b^2e^2\right)\log\left(\left|2\,\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{5}{2}}}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x)*(2*e^2*x/c + (8*c*d*e - 3*b*e^2)/c^2) - 1/8*(8*c^2*d
^2 - 8*b*c*d*e + 3*b^2*e^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt
(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^2}{\sqrt{bx+cx^2}}\,dx = \int \frac{(d+e\,x)^2}{\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((d + e*x)^2/(b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x)^2/(b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.25

$$\int \frac{(d+ex)^2}{\sqrt{bx+cx^2}}\,dx = \frac{-3\sqrt{x}\,\sqrt{cx+b}\,bc\,e^2+8\sqrt{x}\,\sqrt{cx+b}\,c^2de+2\sqrt{x}\,\sqrt{cx+b}\,c^2e^2x+3\sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^2e^2-8\sqrt{c}\log\left(\right.}{4c^3}$$

input
```
int((e*x+d)^2/(c*x^2+b*x)^(1/2),x)
```

output
```
( - 3*sqrt(x)*sqrt(b + c*x)*b*c*e**2 + 8*sqrt(x)*sqrt(b + c*x)*c**2*d*e +
2*sqrt(x)*sqrt(b + c*x)*c**2*e**2*x + 3*sqrt(c)*log((sqrt(b + c*x) + sqrt(
x)*sqrt(c))/sqrt(b))*b**2*e**2 - 8*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sq
rt(c))/sqrt(b))*b*c*d*e + 8*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/
sqrt(b))*c**2*d**2)/(4*c**3)
```

3.159 $\int \frac{d+ex}{\sqrt{bx+cx^2}}\,dx$

Optimal result 1290
Mathematica [A] (verified) 1290
Rubi [A] (verified) 1291
Maple [A] (verified) 1292
Fricas [A] (verification not implemented) 1292
Sympy [B] (verification not implemented) 1293
Maxima [A] (verification not implemented) 1294
Giac [A] (verification not implemented) 1294
Mupad [B] (verification not implemented) 1294
Reduce [B] (verification not implemented) 1295

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{d+ex}{\sqrt{bx+cx^2}}\,dx = \frac{e\sqrt{bx+cx^2}}{c}+\frac{(2cd-be)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}$$

output
```
e*(c*x^2+b*x)^(1/2)/c+(-b*e+2*c*d)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^
(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.65

$$\int \frac{d+ex}{\sqrt{bx+cx^2}}\,dx = \frac{\sqrt{x}\left(\sqrt{c}e\sqrt{x}(b+cx)+2(2cd-be)\sqrt{b+cx}\,\text{arctanh}\left(\frac{\sqrt{c}\sqrt{x}}{-\sqrt{b}+\sqrt{b+cx}}\right)\right)}{c^{3/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[(d + e*x)/Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[x]*(Sqrt[c]*e*Sqrt[x]*(b + c*x) + 2*(2*c*d - b*e)*Sqrt[b + c*x]*ArcT
anh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])]))/(c^(3/2)*Sqrt[x*(b + c
*x)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{(2cd-be)\int \frac{1}{\sqrt{cx^2+bx}}dx}{2c}+\frac{e\sqrt{bx+cx^2}}{c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(2cd-be)\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{c}+\frac{e\sqrt{bx+cx^2}}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)}{c^{3/2}}+\frac{e\sqrt{bx+cx^2}}{c}$$

input
```
Int[(d + e*x)/Sqrt[b*x + c*x^2],x]
```

output
```
(e*Sqrt[b*x + c*x^2])/c + ((2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*
x^2]])/c^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{e\sqrt{x(cx+b)}}{c}-\frac{(be-2cd)\,\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{c^{\frac{3}{2}}}$	46
risch	$\frac{ex(cx+b)}{c\sqrt{x(cx+b)}}-\frac{(be-2cd)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{2c^{\frac{3}{2}}}$	59
default	$\frac{d\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{\sqrt{c}}+e\left(\frac{\sqrt{c\,x^2+bx}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{2c^{\frac{3}{2}}}\right)$	79

input
```
int((e*x+d)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
e*(x*(c*x+b))^(1/2)/c-(b*e-2*c*d)/c^(3/2)*arctanh((x*(c*x+b))^(1/2)/x/c^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.16

$$\int \frac{d+ex}{\sqrt{bx+cx^2}}\,dx$$
$$=\left[\frac{2\sqrt{cx^2+bx}ce-(2\,cd-be)\sqrt{c}\log\left(2\,cx+b-2\sqrt{cx^2+bx}\sqrt{c}\right)}{2\,c^2},\frac{\sqrt{cx^2+bx}ce-(2\,cd-be)\sqrt{-c}\,\text{arctan}}{c^2}\right.$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*sqrt(c*x^2 + b*x)*c*e - (2*c*d - b*e)*sqrt(c)*log(2*c*x + b - 2*sq
rt(c*x^2 + b*x)*sqrt(c)))/c^2, (sqrt(c*x^2 + b*x)*c*e - (2*c*d - b*e)*sqrt
(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)))/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(48) = 96$.

Time = 0.34 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.22

$$\int \frac{d+ex}{\sqrt{bx+cx^2}}\,dx = \begin{cases} \left(-\frac{be}{2c}+d\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for}\ \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise} \end{cases}\right)+\frac{e\sqrt{bx+cx^2}}{c} & \text{for}\ c\neq 0 \\ \frac{2d\sqrt{bx}+\frac{2e(bx)^{\frac{3}{2}}}{3b}}{b} & \text{for}\ b\neq 0 \\ \tilde{\infty}\left(dx+\frac{ex^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((e*x+d)/(c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise(((-b*e/(2*c) + d)*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2
) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c
*(b/(2*c) + x)**2), True)) + e*sqrt(b*x + c*x**2)/c, Ne(c, 0)), ((2*d*sqrt
(b*x) + 2*e*(b*x)**(3/2)/(3*b))/b, Ne(b, 0)), (zoo*(d*x + e*x**2/2), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.36

$$\int \frac{d+ex}{\sqrt{bx+cx^2}}\,dx = \frac{d\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{\sqrt{c}} - \frac{be\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{2\,c^{\frac{3}{2}}} + \frac{\sqrt{cx^2+bx}e}{c}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
d*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/sqrt(c) - 1/2*b*e*log(2*c*x
 + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(3/2) + sqrt(c*x^2 + b*x)*e/c
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.07

$$\int \frac{d+ex}{\sqrt{bx+cx^2}}\,dx = \frac{\sqrt{cx^2+bx}e}{c} - \frac{(2\,cd-be)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{3}{2}}}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
sqrt(c*x^2 + b*x)*e/c - 1/2*(2*c*d - b*e)*log(abs(2*(sqrt(c)*x - sqrt(c*x^
2 + b*x))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{d+ex}{\sqrt{bx+cx^2}}\,dx = \frac{e\,\sqrt{c\,x^2+b\,x}}{c} + \frac{d\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{\sqrt{c}} - \frac{b\,e\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{2\,c^{3/2}}$$

input
```
int((d + e*x)/(b*x + c*x^2)^(1/2),x)
```

output
```
(e*(b*x + c*x^2)^(1/2))/c + (d*log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/
2)))/c^(1/2) - (b*e*log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/(2*c^(
3/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.16

$$\int \frac{d+ex}{\sqrt{bx+cx^2}}\,dx = \frac{\sqrt{x}\,\sqrt{cx+b}\,ce-\sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)be+2\sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)cd}{c^2}$$

input
```
int((e*x+d)/(c*x^2+b*x)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(b + c*x)*c*e - sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))
/sqrt(b))*b*e + 2*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*c
*d)/c**2
```

3.160 $\int \frac{1}{\sqrt{bx+cx^2}}\,dx$

Optimal result 1296
Mathematica [A] (verified) 1296
Rubi [A] (verified) 1297
Maple [A] (verified) 1298
Fricas [A] (verification not implemented) 1298
Sympy [B] (verification not implemented) 1299
Maxima [A] (verification not implemented) 1299
Giac [B] (verification not implemented) 1300
Mupad [B] (verification not implemented) 1300
Reduce [B] (verification not implemented) 1300

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = \frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}$$

output
```
2*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.96

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = -\frac{2\sqrt{x}\sqrt{b+cx}\log\left(-\sqrt{c}\sqrt{x}+\sqrt{b+cx}\right)}{\sqrt{c}\sqrt{x(b+cx)}}$$

input
```
Integrate[1/Sqrt[b*x + c*x^2],x]
```

output
```
(-2*Sqrt[x]*Sqrt[b + c*x]*Log[-(Sqrt[c]*Sqrt[x]) + Sqrt[b + c*x]])/(Sqrt[c
]*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx$$
$$\Big\downarrow \text{ 1091}$$
$$2\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}$$
$$\Big\downarrow \text{ 219}$$
$$\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}$$

input
```
Int[1/Sqrt[b*x + c*x^2],x]
```

output
```
(2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.82

method	result	size
pseudoelliptic	$\frac{2\,\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{\sqrt{c}}$	23
default	$\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{\sqrt{c}}$	29

input
```
int(1/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/c^(1/2)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.25

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = \left[\frac{\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{\sqrt{c}}, -\frac{2\,\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)}{c}\right]$$

input
```
integrate(1/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/sqrt(c), -2*sqrt(-c)*arctan(
sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b))/c]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(26) = 52$.

Time = 0.29 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.68

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = \begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } c \neq 0 \wedge \frac{b^2}{c} \neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{for } c \neq 0 \\ \frac{2\sqrt{bx}}{b} & \text{for } b \neq 0 \\ \tilde{\infty}x & \text{otherwise} \end{cases}$$

input
```
integrate(1/(c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(c, 0)
 & Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2
), Ne(c, 0)), (2*sqrt(b*x)/b, Ne(b, 0)), (zoo*x, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = \frac{\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{\sqrt{c}}$$

input
```
integrate(1/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/sqrt(c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(22) = 44$.

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = \frac{1}{4}\sqrt{cx^2+bx}\left(2\,x+\frac{b}{c}\right)+\frac{b^2\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{3}{2}}}$$

input
```
integrate(1/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x)*(2*x + b/c) + 1/8*b^2*log(abs(2*(sqrt(c)*x - sqrt(c*
x^2 + b*x))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = \frac{\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{\sqrt{c}}$$

input
```
int(1/(b*x + c*x^2)^(1/2),x)
```

output
```
log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2))/c^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{bx+cx^2}}\,dx = \frac{2\sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)}{c}$$

input
```
int(1/(c*x^2+b*x)^(1/2),x)
```

output
```
(2*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b)))/c
```

3.161 $\int \frac{1}{(d+ex)\sqrt{bx+cx^2}} \, dx$

Optimal result . . . 1301
Mathematica [A] (verified) . . . 1301
Rubi [A] (verified) . . . 1302
Maple [A] (verified) . . . 1303
Fricas [A] (verification not implemented) . . . 1303
Sympy [F] . . . 1304
Maxima [F(-2)] . . . 1304
Giac [A] (verification not implemented) . . . 1304
Mupad [F(-1)] . . . 1305
Reduce [B] (verification not implemented) . . . 1305

Optimal result

Integrand size = 21, antiderivative size = 52

$$\int \frac{1}{(d+ex)\sqrt{bx+cx^2}} \, dx = \frac{2\text{arctanh}\left(\frac{\sqrt{cd-be}x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{\sqrt{d}\sqrt{cd-be}}$$

output

```
2*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(1/2))/d^(1/2)/(-b*e+c*d)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.77

$$\int \frac{1}{(d+ex)\sqrt{bx+cx^2}} \, dx = -\frac{2\sqrt{x}\sqrt{b+cx}\arctan\left(\frac{-e\sqrt{x}\sqrt{b+cx}+\sqrt{c}(d+ex)}{\sqrt{d}\sqrt{-cd+be}}\right)}{\sqrt{d}\sqrt{-cd+be}\sqrt{x(b+cx)}}$$

input

```
Integrate[1/((d + e*x)*Sqrt[b*x + c*x^2]),x]
```

output

```
(-2*Sqrt[x]*Sqrt[b + c*x]*ArcTan[(-(e*Sqrt[x]*Sqrt[b + c*x]) + Sqrt[c]*(d
+ e*x))/(Sqrt[d]*Sqrt[-(c*d) + b*e])])/(Sqrt[d]*Sqrt[-(c*d) + b*e]*Sqrt[x*
(b + c*x)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.31, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx+cx^2}(d+ex)}\,dx$$

$$\Big\downarrow \text{1154}$$

$$-2\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{\sqrt{d}\sqrt{cd-be}}$$

input
```
Int[1/((d + e*x)*Sqrt[b*x + c*x^2]),x]
```

output
```
ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^
2])]/(Sqrt[d]*Sqrt[c*d - b*e])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

method	result	size
pseudoelliptic	$-\frac{2\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)}{\sqrt{d(be-cd)}}$	42
default	$-\frac{\ln\left(\frac{-\frac{2d(be-cd)}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{-\frac{d(be-cd)}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}}}{x+\frac{d}{e}}\right)}{e\sqrt{-\frac{d(be-cd)}{e^2}}}$	132

input
```
int(1/(e*x+d)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(d*(b*e-c*d))^(1/2)*arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.48

$$\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx = \left[\frac{\log\left(\frac{bd+(2\,cd-be)x+2\,\sqrt{cd^2-bde}\sqrt{cx^2+bx}}{ex+d}\right)}{\sqrt{cd^2-bde}}, -\frac{2\,\sqrt{-cd^2+bde}\arctan\left(\frac{\sqrt{-cd^2+bde}\sqrt{cx^2+bx}}{cdx+bd}\right)}{cd^2-bde}\right]$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*
x + d))/sqrt(c*d^2 - b*d*e), -2*sqrt(-c*d^2 + b*d*e)*arctan(sqrt(-c*d^2 +
b*d*e)*sqrt(c*x^2 + b*x)/(c*d*x + b*d))/(c*d^2 - b*d*e)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{x\,(b+cx)}\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x*(b + c*x))*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12

$$\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx = \frac{2\arctan\left(-\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde}}\right)}{\sqrt{-cd^2+bde}}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*
d*e))/sqrt(-c*d^2 + b*d*e)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/((b*x + c*x^2)^(1/2)*(d + e*x)),x)
```

output
```
int(1/((b*x + c*x^2)^(1/2)*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.94

$$\int \frac{1}{(d+ex)\sqrt{bx+cx^2}}\,dx = -\frac{2\sqrt{d}\,\sqrt{be-cd}\,\left(atan\left(\frac{\sqrt{be-cd}-\sqrt{e}\,\sqrt{cx+b}-\sqrt{x}\,\sqrt{e}\,\sqrt{c}}{\sqrt{d}\,\sqrt{c}}\right)+atan\left(\frac{\sqrt{be-cd}+\sqrt{e}\,\sqrt{cx+b}+\sqrt{x}\,\sqrt{e}\,\sqrt{c}}{\sqrt{d}\,\sqrt{c}}\right)\right)}{d\,(be-cd)}$$

input
```
int(1/(e*x+d)/(c*x^2+b*x)^(1/2),x)
```

output
```
( - 2*sqrt(d)*sqrt(b*e - c*d)*(atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*
x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c))) + atan((sqrt(b*e - c*d) +
 sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))))/(d*
(b*e - c*d))
```

3.162 $\int \frac{1}{(d+ex)^2\sqrt{bx+cx^2}}\,dx$

Optimal result . . . 1306
Mathematica [A] (verified) . . . 1306
Rubi [A] (verified) . . . 1307
Maple [A] (verified) . . . 1308
Fricas [A] (verification not implemented) . . . 1309
Sympy [F] . . . 1309
Maxima [F(-2)] . . . 1310
Giac [F(-2)] . . . 1310
Mupad [F(-1)] . . . 1310
Reduce [B] (verification not implemented) . . . 1311

Optimal result

Integrand size = 21, antiderivative size = 97

$$\int \frac{1}{(d+ex)^2\sqrt{bx+cx^2}}\,dx=-\frac{e\sqrt{bx+cx^2}}{d(cd-be)(d+ex)}+\frac{(2cd-be)\,\mathrm{arctanh}\left(\frac{\sqrt{cd-be}\,x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{d^{3/2}(cd-be)^{3/2}}$$

output
```
-e*(c*x^2+b*x)^(1/2)/d/(-b*e+c*d)/(e*x+d)+(-b*e+2*c*d)*arctanh((-b*e+c*d)^
(1/2)*x/d^(1/2)/(c*x^2+b*x)^(1/2))/d^(3/2)/(-b*e+c*d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.41

$$\int \frac{1}{(d+ex)^2\sqrt{bx+cx^2}}\,dx=\frac{\sqrt{x}\left(-\frac{\sqrt{d}e\sqrt{x}(b+cx)}{(cd-be)(d+ex)}+\frac{(2cd-be)\sqrt{b+cx}\arctan\left(\frac{-e\sqrt{x}\sqrt{b+cx}+\sqrt{c}(d+ex)}{\sqrt{d}\sqrt{-cd+be}}\right)}{(-cd+be)^{3/2}}\right)}{d^{3/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[1/((d + e*x)^2*Sqrt[b*x + c*x^2]),x]
```

output
```
(Sqrt[x]*(-((Sqrt[d]*e*Sqrt[x]*(b + c*x))/((c*d - b*e)*(d + e*x))) + ((2*c
*d - b*e)*Sqrt[b + c*x]*ArcTan[(-(e*Sqrt[x]*Sqrt[b + c*x]) + Sqrt[c]*(d +
e*x))/(Sqrt[d]*Sqrt[-(c*d) + b*e])])/(-(c*d) + b*e)^(3/2)))/(d^(3/2)*Sqrt[
x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1157, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx+cx^2}(d+ex)^2}\,dx$$

$$\Big\downarrow \text{1157}$$

$$\frac{(2cd-be)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{2d(cd-be)}-\frac{e\sqrt{bx+cx^2}}{d(d+ex)(cd-be)}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{(2cd-be)\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{d(cd-be)}-\frac{e\sqrt{bx+cx^2}}{d(d+ex)(cd-be)}$$

$$\Big\downarrow \text{219}$$

$$\frac{(2cd-be)\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{2d^{3/2}(cd-be)^{3/2}}-\frac{e\sqrt{bx+cx^2}}{d(d+ex)(cd-be)}$$

input
```
Int[1/((d + e*x)^2*Sqrt[b*x + c*x^2]),x]
```

output
```
-((e*Sqrt[b*x + c*x^2])/(d*(c*d - b*e)*(d + e*x))) + ((2*c*d - b*e)*ArcTan
h[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/
(2*d^(3/2)*(c*d - b*e)^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1157
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2))
  Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{\frac{e\sqrt{x(cx+b)}}{ex+d}-\frac{(be-2cd)\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)}{\sqrt{d(be-cd)}}}{(be-cd)d}$
default	$\frac{\frac{e^2\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}}}{d(be-cd)\left(x+\frac{d}{e}\right)}-\frac{(be-2cd)e\ln\left(\frac{-\frac{2d(be-cd)}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{-\frac{d(be-cd)}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)(\cdots}{e}\cdots}}{x+\frac{d}{e}}\cdots\right)}{2d(be-cd)\sqrt{-\frac{d(be-cd)}{e^2}}}\cdots}{e^2}$

input
```
int(1/(e*x+d)^2/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(b*e-c*d)/d*(e*(x*(c*x+b))^(1/2)/(e*x+d)-(b*e-2*c*d)/(d*(b*e-c*d))^(1/2)
*arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.55

$$\int \frac{1}{(d+ex)^2\sqrt{bx+cx^2}}\,dx$$
$$= \left[\frac{(2\,cd^2-bde+(2\,cde-be^2)x)\sqrt{cd^2-bde}\log\left(\frac{bd+(2\,cd-be)x+2\,\sqrt{cd^2-bde}\sqrt{cx^2+bx}}{ex+d}\right)-2\,(cd^2e-bde^2)\sqrt{cx^2+}}{2\,(c^2d^5-2\,bcd^4e+b^2d^3e^2+(c^2d^4e-2\,bcd^3e^2+b^2d^2e^3)x)}\right.$$
$$\left.-\frac{(2\,cd^2-bde+(2\,cde-be^2)x)\sqrt{-cd^2+bde}\arctan\left(\frac{\sqrt{-cd^2+bde}\sqrt{cx^2+bx}}{cdx+bd}\right)+(cd^2e-bde^2)\sqrt{cx^2+bx}}{c^2d^5-2\,bcd^4e+b^2d^3e^2+(c^2d^4e-2\,bcd^3e^2+b^2d^2e^3)x}\right]$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*((2*c*d^2 - b*d*e + (2*c*d*e - b*e^2)*x)*sqrt(c*d^2 - b*d*e)*log((b*d
 + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) -
 2*(c*d^2*e - b*d*e^2)*sqrt(c*x^2 + b*x))/(c^2*d^5 - 2*b*c*d^4*e + b^2*d^3
*e^2 + (c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^3)*x), -((2*c*d^2 - b*d*e +
(2*c*d*e - b*e^2)*x)*sqrt(-c*d^2 + b*d*e)*arctan(sqrt(-c*d^2 + b*d*e)*sqrt
(c*x^2 + b*x)/(c*d*x + b*d)) + (c*d^2*e - b*d*e^2)*sqrt(c*x^2 + b*x))/(c^2
*d^5 - 2*b*c*d^4*e + b^2*d^3*e^2 + (c^2*d^4*e - 2*b*c*d^3*e^2 + b^2*d^2*e^
3)*x)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{x\,(b+cx)}\,(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x*(b + c*x))*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2\sqrt{bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2\sqrt{bx+cx^2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x}\,{\left(d+e\,x\right)}^2}\,dx$$

input
```
int(1/((b*x + c*x^2)^(1/2)*(d + e*x)^2),x)
```

output
```
int(1/((b*x + c*x^2)^(1/2)*(d + e*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 518, normalized size of antiderivative = 5.34

$$\int \frac{1}{(d+ex)^2\sqrt{bx+cx^2}}\,dx$$
$$= \frac{-\sqrt{d}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{be-cd}-\sqrt{e}\,\sqrt{cx+b}-\sqrt{x}\,\sqrt{e}\,\sqrt{c}}{\sqrt{d}\,\sqrt{c}}\right)bde-\sqrt{d}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{be-cd}-\sqrt{e}\,\sqrt{cx+b}-\sqrt{x}\,\sqrt{e}\,\sqrt{c}}{\sqrt{d}\,\sqrt{c}}\right)b\,e^2x}{}$$

input

```
int(1/(e*x+d)^2/(c*x^2+b*x)^(1/2),x)
```

output

```
( - sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x)
- sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b*d*e - sqrt(d)*sqrt(b*e - c
*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c
))/(sqrt(d)*sqrt(c)))*b*e**2*x + 2*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e
- c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)
))*c*d**2 + 2*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt
(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*c*d*e*x - sqrt(d)*
sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sq
rt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b*d*e - sqrt(d)*sqrt(b*e - c*d)*atan((sq
rt(b*e - c*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*
sqrt(c)))*b*e**2*x + 2*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) + sqr
t(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*c*d**2 +
2*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) + sqrt(e)*sqrt(b + c*x) +
sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*c*d*e*x + sqrt(x)*sqrt(b + c*x
)*b*d*e**2 - sqrt(x)*sqrt(b + c*x)*c*d**2*e)/(d**2*(b**2*d*e**2 + b**2*e**
3*x - 2*b*c*d**2*e - 2*b*c*d*e**2*x + c**2*d**3 + c**2*d**2*e*x))
```

3.163 $\int \frac{1}{(d+ex)^3\sqrt{bx+cx^2}}\,dx$

Optimal result 1312
Mathematica [A] (verified) 1312
Rubi [A] (verified) 1313
Maple [A] (verified) 1315
Fricas [B] (verification not implemented) 1316
Sympy [F] 1317
Maxima [F(-2)] 1317
Giac [B] (verification not implemented) 1317
Mupad [F(-1)] 1318
Reduce [B] (verification not implemented) 1318

Optimal result

Integrand size = 21, antiderivative size = 163

$$\int \frac{1}{(d+ex)^3\sqrt{bx+cx^2}}\,dx = -\frac{e\sqrt{bx+cx^2}}{2d(cd-be)(d+ex)^2} - \frac{3e(2cd-be)\sqrt{bx+cx^2}}{4d^2(cd-be)^2(d+ex)} + \frac{(8c^2d^2-8bcde+3b^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{cd-be}\,x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{4d^{5/2}(cd-be)^{5/2}}$$

output
```
-1/2*e*(c*x^2+b*x)^(1/2)/d/(-b*e+c*d)/(e*x+d)^2-3/4*e*(-b*e+2*c*d)*(c*x^2+
b*x)^(1/2)/d^2/(-b*e+c*d)^2/(e*x+d)+1/4*(3*b^2*e^2-8*b*c*d*e+8*c^2*d^2)*ar
ctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(1/2))/d^(5/2)/(-b*e+c*d)^(5/
2)
```

Mathematica [A] (verified)

Time = 0.85 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.10

$$\int \frac{1}{(d+ex)^3\sqrt{bx+cx^2}}\,dx = \frac{\sqrt{x}\left(-\frac{\sqrt{d}e\sqrt{x}(b+cx)(2cd(4d+3ex)-be(5d+3ex))}{(cd-be)^2(d+ex)^2} - \frac{(8c^2d^2-8bcde+3b^2e^2)\sqrt{b+cx}\arctan\left(\frac{-e\sqrt{x}\sqrt{b+cx}+\sqrt{c}(d+ex)}{\sqrt{d}\sqrt{-cd+be}}\right)}{(-cd+be)^{5/2}}\right)}{4d^{5/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[1/((d + e*x)^3*Sqrt[b*x + c*x^2]),x]
```

output
```
(Sqrt[x]*(-((Sqrt[d]*e*Sqrt[x]*(b + c*x)*(2*c*d*(4*d + 3*e*x) - b*e*(5*d +
 3*e*x)))/((c*d - b*e)^2*(d + e*x)^2)) - ((8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e
^2)*Sqrt[b + c*x]*ArcTan[(-(e*Sqrt[x]*Sqrt[b + c*x]) + Sqrt[c]*(d + e*x))/
(Sqrt[d]*Sqrt[-(c*d) + b*e])])/(-(c*d) + b*e)^(5/2)))/(4*d^(5/2)*Sqrt[x*(b
 + c*x)])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.20, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1167, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx+cx^2}(d+ex)^3}dx$$

$$\Big\downarrow \text{1167}$$

$$-\frac{\int -\frac{4cd-3be-2cex}{2(d+ex)^2\sqrt{cx^2+bx}}dx}{2d(cd-be)}-\frac{e\sqrt{bx+cx^2}}{2d(d+ex)^2(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{4cd-3be-2cex}{(d+ex)^2\sqrt{cx^2+bx}}dx}{4d(cd-be)}-\frac{e\sqrt{bx+cx^2}}{2d(d+ex)^2(cd-be)}$$

$$\Big\downarrow \text{1228}$$

$$\frac{\frac{(3b^2e^2-8bcde+8c^2d^2)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{2d(cd-be)}-\frac{3e\sqrt{bx+cx^2}(2cd-be)}{d(d+ex)(cd-be)}}{4d(cd-be)}-\frac{e\sqrt{bx+cx^2}}{2d(d+ex)^2(cd-be)}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{-\frac{\left(3b^2e^2-8bcde+8c^2d^2\right)\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{d(cd-be)}-\frac{3e\sqrt{bx+cx^2}(2cd-be)}{d(d+ex)(cd-be)}}{4d(cd-be)}-\frac{e\sqrt{bx+cx^2}}{2d(d+ex)^2(cd-be)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\left(3b^2e^2-8bcde+8c^2d^2\right)\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{2d^{3/2}(cd-be)^{3/2}}-\frac{3e\sqrt{bx+cx^2}(2cd-be)}{d(d+ex)(cd-be)}}{4d(cd-be)}-\frac{e\sqrt{bx+cx^2}}{2d(d+ex)^2(cd-be)}$$

input
```
Int[1/((d + e*x)^3*Sqrt[b*x + c*x^2]),x]
```

output
```
-1/2*(e*Sqrt[b*x + c*x^2])/(d*(c*d - b*e)*(d + e*x)^2) + ((-3*e*(2*c*d - b
*e)*Sqrt[b*x + c*x^2])/(d*(c*d - b*e)*(d + e*x)) + ((8*c^2*d^2 - 8*b*c*d*e
 + 3*b^2*e^2)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*S
qrt[b*x + c*x^2])])/(2*d^(3/2)*(c*d - b*e)^(3/2)))/(4*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$\frac{-\frac{3(ex+d)^2\left(b^2e^2-\frac{8}{3}bcde+\frac{8}{3}c^2d^2\right)\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)}{4}+\frac{5e\sqrt{x(cx+b)}\left(-\frac{8cd^2}{5}+e\left(-\frac{6cx}{5}+b\right)d+\frac{3xbe^2}{5}\right)\sqrt{d(be-cd)}}{4}}{(be-cd)^2\sqrt{d(be-cd)}\,d^2(ex+d)^2}$
default	$\frac{e^2\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}}}{2d(be-cd)\left(x+\frac{d}{e}\right)^2}+\frac{3(be-2cd)e\left(\frac{e^2\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}}}{d(be-cd)\left(x+\frac{d}{e}\right)}-\frac{(be-2cd)e\ln\left(\frac{-\frac{2d(be-cd}{e^2}\cdots}{\cdots}\right.}{\cdots}\right.}{4d(be-\cdots}$

input
```
int(1/(e*x+d)^3/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
5/4/(b*e-c*d)^2/(d*(b*e-c*d))^(1/2)*(-3/5*(e*x+d)^2*(b^2*e^2-8/3*b*c*d*e+8
/3*c^2*d^2)*arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))^(1/2))+e*(x*(c*x+b)
)^(1/2)*(-8/5*c*d^2+e*(-6/5*c*x+b)*d+3/5*x*b*e^2)*(d*(b*e-c*d))^(1/2))/d^2
/(e*x+d)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(143) = 286$.

Time = 0.10 (sec) , antiderivative size = 738, normalized size of antiderivative = 4.53

$$\int \frac{1}{(d+ex)^3\sqrt{bx+cx^2}}\,dx$$

$$= \left[\frac{(8\,c^2d^4 - 8\,bcd^3e + 3\,b^2d^2e^2 + (8\,c^2d^2e^2 - 8\,bcde^3 + 3\,b^2e^4)x^2 + 2\,(8\,c^2d^3e - 8\,bcd^2e^2 + 3\,b^2de^3)x)\sqrt{cd^2}}{8\,(c^3d^8 - 3\,bc^2d^7e + 3\,b^2cd^6e^2 - b^3d^5e^3 + (c^3d^6e^2 - 3\,bc^2}\right.$$

$$-\frac{(8\,c^2d^4 - 8\,bcd^3e + 3\,b^2d^2e^2 + (8\,c^2d^2e^2 - 8\,bcde^3 + 3\,b^2e^4)x^2 + 2\,(8\,c^2d^3e - 8\,bcd^2e^2 + 3\,b^2de^3)x)\sqrt{-c}}{4\,(c^3d^8 - 3\,bc^2d^7e + 3\,b^2cd^6e^2 - b^3d^5e^3 + (c^3d^6e^2 - 3\,bc^2d^5}$$

input
```
integrate(1/(e*x+d)^3/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/8*((8*c^2*d^4 - 8*b*c*d^3*e + 3*b^2*d^2*e^2 + (8*c^2*d^2*e^2 - 8*b*c*d*
e^3 + 3*b^2*e^4)*x^2 + 2*(8*c^2*d^3*e - 8*b*c*d^2*e^2 + 3*b^2*d*e^3)*x)*sq
rt(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(
c*x^2 + b*x))/(e*x + d)) - 2*(8*c^2*d^4*e - 13*b*c*d^3*e^2 + 5*b^2*d^2*e^3
 + 3*(2*c^2*d^3*e^2 - 3*b*c*d^2*e^3 + b^2*d*e^4)*x)*sqrt(c*x^2 + b*x))/(c^
3*d^8 - 3*b*c^2*d^7*e + 3*b^2*c*d^6*e^2 - b^3*d^5*e^3 + (c^3*d^6*e^2 - 3*b
*c^2*d^5*e^3 + 3*b^2*c*d^4*e^4 - b^3*d^3*e^5)*x^2 + 2*(c^3*d^7*e - 3*b*c^2
*d^6*e^2 + 3*b^2*c*d^5*e^3 - b^3*d^4*e^4)*x), -1/4*((8*c^2*d^4 - 8*b*c*d^3
*e + 3*b^2*d^2*e^2 + (8*c^2*d^2*e^2 - 8*b*c*d*e^3 + 3*b^2*e^4)*x^2 + 2*(8*
c^2*d^3*e - 8*b*c*d^2*e^2 + 3*b^2*d*e^3)*x)*sqrt(-c*d^2 + b*d*e)*arctan(sq
rt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/(c*d*x + b*d)) + (8*c^2*d^4*e - 13*b*
c*d^3*e^2 + 5*b^2*d^2*e^3 + 3*(2*c^2*d^3*e^2 - 3*b*c*d^2*e^3 + b^2*d*e^4)*
x)*sqrt(c*x^2 + b*x))/(c^3*d^8 - 3*b*c^2*d^7*e + 3*b^2*c*d^6*e^2 - b^3*d^5
*e^3 + (c^3*d^6*e^2 - 3*b*c^2*d^5*e^3 + 3*b^2*c*d^4*e^4 - b^3*d^3*e^5)*x^2
 + 2*(c^3*d^7*e - 3*b*c^2*d^6*e^2 + 3*b^2*c*d^5*e^3 - b^3*d^4*e^4)*x)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{x\,(b+cx)}\,(d+ex)^3}\,dx$$

input
```
integrate(1/(e*x+d)**3/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x*(b + c*x))*(d + e*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^3\sqrt{bx+cx^2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^3/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. 2(143) = 286.

Time = 0.19 (sec) , antiderivative size = 483, normalized size of antiderivative = 2.96

$$\int \frac{1}{(d+ex)^3\sqrt{bx+cx^2}}\,dx = -\frac{(8\,c^2d^2-8\,bcde+3\,b^2e^2)\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde}}\right)}{4\,(c^2d^4-2\,bcd^3e+b^2d^2e^2)\sqrt{-cd^2+bde}}$$
$$-\frac{8\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3c^2d^2e-8\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3bcde^2+3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3b^2e^3+24\left(\sqrt{c}x-\right.}{}$$

input
```
integrate(1/(e*x+d)^3/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
-1/4*(8*c^2*d^2 - 8*b*c*d*e + 3*b^2*e^2)*arctan(((sqrt(c)*x - sqrt(c*x^2 +
 b*x))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e))/((c^2*d^4 - 2*b*c*d^3*e + b^2*
d^2*e^2)*sqrt(-c*d^2 + b*d*e)) - 1/4*(8*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*
c^2*d^2*e - 8*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b*c*d*e^2 + 3*(sqrt(c)*x -
 sqrt(c*x^2 + b*x))^3*b^2*e^3 + 24*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*c^(5/
2)*d^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b*c^(3/2)*d^2*e + 9*(sqrt(c)
*x - sqrt(c*x^2 + b*x))^2*b^2*sqrt(c)*d*e^2 + 24*(sqrt(c)*x - sqrt(c*x^2 +
 b*x))*b*c^2*d^3 - 20*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^2*c*d^2*e + 5*(sqr
t(c)*x - sqrt(c*x^2 + b*x))*b^3*d*e^2 + 6*b^2*c^(3/2)*d^3 - 3*b^3*sqrt(c)*
d^2*e)/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*((sqrt(c)*x - sqrt(c*x^2 + b
*x))^2*e + 2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c)*d + b*d)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x}\,{(d+e\,x)}^3}\,dx$$

input
```
int(1/((b*x + c*x^2)^(1/2)*(d + e*x)^3),x)
```

output
```
int(1/((b*x + c*x^2)^(1/2)*(d + e*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.86 (sec) , antiderivative size = 1950, normalized size of antiderivative = 11.96

$$\int \frac{1}{(d+ex)^3\sqrt{bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^3/(c*x^2+b*x)^(1/2),x)
```

output
```
( - 6*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x
) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**3*d**2*e**3 - 12*sqrt(d
)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*
sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**3*d*e**4*x - 6*sqrt(d)*sqrt(b*e - c
*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c
))/(sqrt(d)*sqrt(c)))*b**3*e**5*x**2 + 28*sqrt(d)*sqrt(b*e - c*d)*atan((sq
rt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*
sqrt(c)))*b**2*c*d**3*e**2 + 56*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c
*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*
b**2*c*d**2*e**3*x + 28*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sq
rt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**2*c*d
*e**4*x**2 - 48*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sq
rt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b*c**2*d**4*e -
96*sqrt(d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) -
 sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b*c**2*d**3*e**2*x - 48*sqrt(
d)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)
*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b*c**2*d**2*e**3*x**2 + 32*sqrt(d)*sq
rt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt
(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*c**3*d**5 + 64*sqrt(d)*sqrt(b*e - c*d)*ata
n((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(...
```

3.164 $\int \frac{(d+ex)^3}{(bx+cx^2)^{3/2}}\,dx$

Optimal result 1320
Mathematica [A] (verified) 1320
Rubi [A] (verified) 1321
Maple [A] (verified) 1323
Fricas [A] (verification not implemented) 1324
Sympy [F] 1324
Maxima [A] (verification not implemented) 1325
Giac [A] (verification not implemented) 1325
Mupad [F(-1)] 1326
Reduce [B] (verification not implemented) 1326

Optimal result

Integrand size = 21, antiderivative size = 134

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{3/2}}\,dx = -\frac{2d^3}{b\sqrt{bx+cx^2}} - \frac{2(2cd-be)\left(c^2d^2-bcde+b^2e^2\right)x}{b^2c^2\sqrt{bx+cx^2}} + \frac{e^3\sqrt{bx+cx^2}}{c^2} + \frac{3e^2(2cd-be)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{5/2}}$$

output
```
-2*d^3/b/(c*x^2+b*x)^(1/2)-2*(-b*e+2*c*d)*(b^2*e^2-b*c*d*e+c^2*d^2)*x/b^2/
c^2/(c*x^2+b*x)^(1/2)+e^3*(c*x^2+b*x)^(1/2)/c^2+3*e^2*(-b*e+2*c*d)*arctanh
(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.06

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{3/2}}\,dx = \frac{x\left(\frac{\sqrt{c}(b+cx)\left(-4c^3d^3x+3b^3e^3x-2bc^2d^2(d-3ex)+b^2ce^2x(-6d+ex)\right)}{b^2} + 6e^2(2cd-be)\sqrt{x}(b+cx)^{3/2}\text{ar}\right.}{c^{5/2}(x(b+cx))^{3/2}}$$

input
```
Integrate[(d + e*x)^3/(b*x + c*x^2)^(3/2),x]
```

output
```
(x*((Sqrt[c]*(b + c*x)*(-4*c^3*d^3*x + 3*b^3*e^3*x - 2*b*c^2*d^2*(d - 3*e*
x) + b^2*c*e^2*x*(-6*d + e*x)))/b^2 + 6*e^2*(2*c*d - b*e)*Sqrt[x]*(b + c*x
)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])]))/(c^(5/2)*(
x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1164, 27, 1225, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{1164}$$

$$-\frac{2\int -\frac{2e(d+ex)(bd+(2cd-be)x)}{\sqrt{cx^2+bx}}dx}{b^2}-\frac{2(d+ex)^2(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{4e\int \frac{(d+ex)(bd+(2cd-be)x)}{\sqrt{cx^2+bx}}dx}{b^2}-\frac{2(d+ex)^2(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1225}$$

$$\frac{4e\left(\frac{3b^2e(2cd-be)\int \frac{1}{\sqrt{cx^2+bx}}dx}{8c^2}+\frac{\sqrt{bx+cx^2}(3b^2e^2+2cex(2cd-be)-6bcde+8c^2d^2)}{4c^2}\right)}{b^2}-\frac{2(d+ex)^2(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1091}$$

$$\frac{4e\left(\frac{3b^2e(2cd-be)\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c^2}+\frac{\sqrt{bx+cx^2}(3b^2e^2+2cex(2cd-be)-6bcde+8c^2d^2)}{4c^2}\right)}{b^2}-\frac{2(d+ex)^2(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{4e\left(\frac{3b^2e\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)(2cd-be)}{4c^{5/2}}+\frac{\sqrt{bx+cx^2}(3b^2e^2+2cex(2cd-be)-6bcde+8c^2d^2)}{4c^2}\right)}{b^2}-\frac{2(d+ex)^2(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

input

```
Int[(d + e*x)^3/(b*x + c*x^2)^(3/2),x]
```

output

```
(-2*(d + e*x)^2*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2]) + (4*e*((
(8*c^2*d^2 - 6*b*c*d*e + 3*b^2*e^2 + 2*c*e*(2*c*d - b*e)*x)*Sqrt[b*x + c*x
^2])/(4*c^2) + (3*b^2*e*(2*c*d - b*e)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2
]])/(4*c^(5/2))))/b^2
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091

```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1164

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.86

method	result
pseudoelliptic	$-\frac{3\left(b^{2}\sqrt{x(cx+b)}\,e^{2}(be-2cd)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)+2e^{2}\left(-\frac{ex}{6}+d\right)x\,b^{2}c^{\frac{3}{2}}+\frac{2d^{2}b(-3ex+d)c^{\frac{5}{2}}}{3}-\left(b^{3}e^{3}\sqrt{c}-\frac{4c^{\frac{7}{2}}d^{3}}{3}\right)x\right)}{\sqrt{x(cx+b)}\,c^{\frac{5}{2}}b^{2}}$
risch	$\frac{(cx+b)\left(e^{3}x\,b^{2}-2c^{2}d^{3}\right)}{b^{2}\sqrt{x(cx+b)}\,c^{2}}-\frac{\frac{2\left(-2b^{3}e^{3}+6d\,e^{2}b^{2}c-6d^{2}eb\,c^{2}+2d^{3}c^{3}\right)\sqrt{c\left(\frac{b}{c}+x\right)^{2}-\left(\frac{b}{c}+x\right)b}}{cb\left(\frac{b}{c}+x\right)}+\frac{3e^{3}b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{\sqrt{c}}-6bd\,e^{2}\sqrt{}}{2c^{2}b}$
default	$-\frac{2d^{3}(2cx+b)}{b^{2}\sqrt{c\,x^{2}+bx}}+e^{3}\left(\frac{x^{2}}{c\sqrt{c\,x^{2}+bx}}-\frac{3b\left(-\frac{x}{c\sqrt{c\,x^{2}+bx}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^{2}+bx}}+\frac{2cx+b}{bc\sqrt{c\,x^{2}+bx}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{c^{\frac{3}{2}}}\right)}{2c}\right)+$

input
```
int((e*x+d)^3/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-3/(x*(c*x+b))^(1/2)/c^(5/2)*(b^2*(x*(c*x+b))^(1/2)*e^2*(b*e-2*c*d)*arctan
h((x*(c*x+b))^(1/2)/x/c^(1/2))+2*e^2*(-1/6*e*x+d)*x*b^2*c^(3/2)+2/3*d^2*b*
(-3*e*x+d)*c^(5/2)-(b^3*e^3*c^(1/2)-4/3*c^(7/2)*d^3)*x)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.71

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{3/2}}\,dx = \left[-\frac{3\left((2\,b^2c^2de^2-b^3ce^3)x^2+(2\,b^3cde^2-b^4e^3)x\right)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{2\,(b^2c^4x^2 \cdots}\right.$$

$$-\frac{3\left((2\,b^2c^2de^2-b^3ce^3)x^2+(2\,b^3cde^2-b^4e^3)x\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(b^2c^2e^3x^2-2\,bc^3d^3-(4\,c^4d^3 \cdots}{b^2c^4x^2+b^3c^3x}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(3*((2*b^2*c^2*d*e^2 - b^3*c*e^3)*x^2 + (2*b^3*c*d*e^2 - b^4*e^3)*x)
*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(b^2*c^2*e^3*x^2
 - 2*b*c^3*d^3 - (4*c^4*d^3 - 6*b*c^3*d^2*e + 6*b^2*c^2*d*e^2 - 3*b^3*c*e^
3)*x)*sqrt(c*x^2 + b*x))/(b^2*c^4*x^2 + b^3*c^3*x), -(3*((2*b^2*c^2*d*e^2
- b^3*c*e^3)*x^2 + (2*b^3*c*d*e^2 - b^4*e^3)*x)*sqrt(-c)*arctan(sqrt(c*x^2
 + b*x)*sqrt(-c)/(c*x + b)) - (b^2*c^2*e^3*x^2 - 2*b*c^3*d^3 - (4*c^4*d^3
- 6*b*c^3*d^2*e + 6*b^2*c^2*d*e^2 - 3*b^3*c*e^3)*x)*sqrt(c*x^2 + b*x))/(b^
2*c^4*x^2 + b^3*c^3*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(d+ex)^3}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((d + e*x)**3/(x*(b + c*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.41

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{3/2}}\,dx = \frac{e^3x^2}{\sqrt{cx^2+bx}c} - \frac{4\,cd^3x}{\sqrt{cx^2+bx}b^2} + \frac{6\,d^2ex}{\sqrt{cx^2+bx}b} - \frac{6\,de^2x}{\sqrt{cx^2+bx}c} + \frac{3\,be^3x}{\sqrt{cx^2+bx}c^2} + \frac{3\,de^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{c^{\frac{3}{2}}} - \frac{3\,be^3\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{2\,c^{\frac{5}{2}}} - \frac{2\,d^3}{\sqrt{cx^2+bx}b}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
e^3*x^2/(sqrt(c*x^2 + b*x)*c) - 4*c*d^3*x/(sqrt(c*x^2 + b*x)*b^2) + 6*d^2*
e*x/(sqrt(c*x^2 + b*x)*b) - 6*d*e^2*x/(sqrt(c*x^2 + b*x)*c) + 3*b*e^3*x/(s
qrt(c*x^2 + b*x)*c^2) + 3*d*e^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c
))/c^(3/2) - 3/2*b*e^3*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(5/2
) - 2*d^3/(sqrt(c*x^2 + b*x)*b)
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.95

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{3/2}}\,dx = -\frac{\frac{2\,d^3}{b} - \left(\frac{e^3x}{c} - \frac{4\,c^3d^3-6\,bc^2d^2e+6\,b^2cde^2-3\,b^3e^3}{b^2c^2}\right)x}{\sqrt{cx^2+bx}} - \frac{3\,(2\,cde^2-be^3)\log\left(\left|2\,\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{5}{2}}}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
-(2*d^3/b - (e^3*x/c - (4*c^3*d^3 - 6*b*c^2*d^2*e + 6*b^2*c*d*e^2 - 3*b^3*
e^3)/(b^2*c^2))*x)/sqrt(c*x^2 + b*x) - 3/2*(2*c*d*e^2 - b*e^3)*log(abs(2*(
sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(d+e\,x)^3}{(c\,x^2+b\,x)^{3/2}}\,dx$$

input
```
int((d + e*x)^3/(b*x + c*x^2)^(3/2),x)
```

output
```
int((d + e*x)^3/(b*x + c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.79

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{3/2}}\,dx = \frac{-3\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^3e^3x+6\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^2cd\,e^2x+}{}$$

input
```
int((e*x+d)^3/(c*x^2+b*x)^(3/2),x)
```

output
```
( - 3*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))
*b**3*e**3*x + 6*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c
))/sqrt(b))*b**2*c*d*e**2*x + 2*sqrt(c)*sqrt(b + c*x)*b**3*e**3*x - 6*sqrt
(c)*sqrt(b + c*x)*b**2*c*d*e**2*x + 6*sqrt(c)*sqrt(b + c*x)*b*c**2*d**2*e*
x - 4*sqrt(c)*sqrt(b + c*x)*c**3*d**3*x + 3*sqrt(x)*b**3*c*e**3*x - 6*sqrt
(x)*b**2*c**2*d*e**2*x + sqrt(x)*b**2*c**2*e**3*x**2 - 2*sqrt(x)*b*c**3*d*
*3 + 6*sqrt(x)*b*c**3*d**2*e*x - 4*sqrt(x)*c**4*d**3*x)/(sqrt(b + c*x)*b**
2*c**3*x)
```

3.165 $\int \frac{(d+ex)^2}{(bx+cx^2)^{3/2}}\,dx$

Optimal result 1327
Mathematica [A] (verified) 1327
Rubi [A] (verified) 1328
Maple [A] (verified) 1330
Fricas [A] (verification not implemented) 1330
Sympy [F] 1331
Maxima [A] (verification not implemented) 1331
Giac [A] (verification not implemented) 1332
Mupad [B] (verification not implemented) 1332
Reduce [B] (verification not implemented) 1333

Optimal result

Integrand size = 21, antiderivative size = 91

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{3/2}}\,dx=-\frac{2d^2}{b\sqrt{bx+cx^2}}-\frac{2\left(\frac{e^2}{c}+\frac{2d(cd-be)}{b^2}\right)x}{\sqrt{bx+cx^2}}+\frac{2e^2\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}$$

output
```
-2*d^2/b/(c*x^2+b*x)^(1/2)-2*(e^2/c+2*d*(-b*e+c*d)/b^2)*x/(c*x^2+b*x)^(1/2
)+2*e^2*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.10

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{3/2}}\,dx=$$
$$-\frac{2\left(\sqrt{c}(2c^2d^2x+b^2e^2x+bcd(d-2ex))+b^2e^2\sqrt{x}\sqrt{b+cx}\log\left(-\sqrt{c}\sqrt{x}+\sqrt{b+cx}\right)\right)}{b^2c^{3/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[(d + e*x)^2/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(Sqrt[c]*(2*c^2*d^2*x + b^2*e^2*x + b*c*d*(d - 2*e*x)) + b^2*e^2*Sqrt[
x]*Sqrt[b + c*x]*Log[-(Sqrt[c]*Sqrt[x]) + Sqrt[b + c*x]]))/(b^2*c^(3/2)*Sq
rt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {1164, 25, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{1164}$$

$$-\frac{2\int -\frac{e(bd+(2cd-be)x)}{\sqrt{cx^2+bx}}dx}{b^2}-\frac{2(d+ex)(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \frac{e(bd+(2cd-be)x)}{\sqrt{cx^2+bx}}dx}{b^2}-\frac{2(d+ex)(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2e\int \frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}dx}{b^2}-\frac{2(d+ex)(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{2e\left(\frac{b^2e\int \frac{1}{\sqrt{cx^2+bx}}dx}{2c}+\frac{\sqrt{bx+cx^2}(2cd-be)}{c}\right)}{b^2}-\frac{2(d+ex)(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1091}$$

$$\frac{2e\left(\frac{b^2e\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{c}+\frac{\sqrt{bx+cx^2}(2cd-be)}{c}\right)}{b^2}-\frac{2(d+ex)(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\downarrow \text{219}$$

$$\frac{2e\left(\frac{b^2e\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}+\frac{\sqrt{bx+cx^2}(2cd-be)}{c}\right)}{b^2}-\frac{2(d+ex)(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

input
```
Int[(d + e*x)^2/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(d + e*x)*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2]) + (2*e*(((2
*c*d - b*e)*Sqrt[b*x + c*x^2])/c + (b^2*e*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c
*x^2]])/c^(3/2)))/b^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{-2bd(-2ex+d)c^{\frac{3}{2}}-4c^{\frac{5}{2}}d^{2}x+2e^{2}\left(\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)\sqrt{x(cx+b)}-\sqrt{c}\,x\right)b^{2}}{c^{\frac{3}{2}}\sqrt{x(cx+b)}\,b^{2}}$
risch	$-\frac{2d^{2}(cx+b)}{b^{2}\sqrt{x(cx+b)}}+\frac{\frac{b\,e^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{c^{\frac{3}{2}}}-\frac{2\left(b^{2}e^{2}-2bcde+c^{2}d^{2}\right)\sqrt{c\left(\frac{b}{c}+x\right)^{2}-\left(\frac{b}{c}+x\right)b}}{c^{2}b\left(\frac{b}{c}+x\right)}}{b}$
default	$-\frac{2d^{2}(2cx+b)}{b^{2}\sqrt{c\,x^{2}+bx}}+e^{2}\left(-\frac{x}{c\sqrt{c\,x^{2}+bx}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^{2}+bx}}+\frac{2cx+b}{bc\sqrt{c\,x^{2}+bx}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{c^{\frac{3}{2}}}\right)+2de\Big(-\frac{}{c\sqrt{}}$

input
```
int((e*x+d)^2/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/c^(3/2)/(x*(c*x+b))^(1/2)*(-b*d*(-2*e*x+d)*c^(3/2)-2*c^(5/2)*d^2*x+e^2*(
arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))*(x*(c*x+b))^(1/2)-c^(1/2)*x)*b^2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.67

$$\int\frac{(d+ex)^{2}}{(bx+cx^{2})^{3/2}}\,dx=\left[\frac{(b^{2}ce^{2}x^{2}+b^{3}e^{2}x)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^{2}+bx}\sqrt{c}\right)-2\left(bc^{2}d^{2}+(2\,c^{3}d^{2}-2\,bc^{2}de\right.}{b^{2}c^{3}x^{2}+b^{3}c^{2}x}\right.$$

$$\left.-\frac{2\left((b^{2}ce^{2}x^{2}+b^{3}e^{2}x)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{2}+bx}\sqrt{-c}}{cx+b}\right)+(bc^{2}d^{2}+(2\,c^{3}d^{2}-2\,bc^{2}de+b^{2}ce^{2})x)\sqrt{cx^{2}+bx}\right)}{b^{2}c^{3}x^{2}+b^{3}c^{2}x}\right]$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[((b^2*c*e^2*x^2 + b^3*e^2*x)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*
sqrt(c)) - 2*(b*c^2*d^2 + (2*c^3*d^2 - 2*b*c^2*d*e + b^2*c*e^2)*x)*sqrt(c*
x^2 + b*x))/(b^2*c^3*x^2 + b^3*c^2*x), -2*((b^2*c*e^2*x^2 + b^3*e^2*x)*sqr
t(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (b*c^2*d^2 + (2*c^3*d
^2 - 2*b*c^2*d*e + b^2*c*e^2)*x)*sqrt(c*x^2 + b*x))/(b^2*c^3*x^2 + b^3*c^2
*x)]
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(d+ex)^2}{(x(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((d + e*x)**2/(x*(b + c*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.21

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{3/2}}\,dx = -\frac{4\,cd^2x}{\sqrt{cx^2+bx}b^2} + \frac{4\,dex}{\sqrt{cx^2+bx}b} - \frac{2\,e^2x}{\sqrt{cx^2+bx}c} + \frac{e^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{c^{\frac{3}{2}}} - \frac{2\,d^2}{\sqrt{cx^2+bx}b}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
-4*c*d^2*x/(sqrt(c*x^2 + b*x)*b^2) + 4*d*e*x/(sqrt(c*x^2 + b*x)*b) - 2*e^2
*x/(sqrt(c*x^2 + b*x)*c) + e^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)
)/c^(3/2) - 2*d^2/(sqrt(c*x^2 + b*x)*b)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.97

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{3/2}}\,dx = -\frac{e^2\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{c^{\frac{3}{2}}} - \frac{2\left(\frac{d^2}{b}+\frac{(2c^2d^2-2bcde+b^2e^2)x}{b^2c}\right)}{\sqrt{cx^2+bx}}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
-e^2*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(3/2) - 2*(
d^2/b + (2*c^2*d^2 - 2*b*c*d*e + b^2*e^2)*x/(b^2*c))/sqrt(c*x^2 + b*x)
```

Mupad [B] (verification not implemented)

Time = 5.46 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{3/2}}\,dx = \frac{e^2\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{c^{3/2}} - \frac{d^2\,(2\,b+4\,c\,x)}{b^2\,\sqrt{c\,x^2+b\,x}} - \frac{2\,e^2\,x}{c\,\sqrt{c\,x^2+b\,x}} + \frac{4\,d\,e\,x}{b\,\sqrt{x\,(b+c\,x)}}$$

input
```
int((d + e*x)^2/(b*x + c*x^2)^(3/2),x)
```

output
```
(e^2*log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/c^(3/2) - (d^2*(2*b +
 4*c*x))/(b^2*(b*x + c*x^2)^(1/2)) - (2*e^2*x)/(c*(b*x + c*x^2)^(1/2)) + (
4*d*e*x)/(b*(x*(b + c*x))^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.63

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{3/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^2e^2x-2\sqrt{c}\,\sqrt{cx+b}\,b^2e^2x+4\sqrt{c}\,\sqrt{cx+b}\,bcdex-4}{\sqrt{cx+b}\,b^2c}$$

input
```
int((e*x+d)^2/(c*x^2+b*x)^(3/2),x)
```

output
```
(2*(sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b
**2*e**2*x - sqrt(c)*sqrt(b + c*x)*b**2*e**2*x + 2*sqrt(c)*sqrt(b + c*x)*b
*c*d*e*x - 2*sqrt(c)*sqrt(b + c*x)*c**2*d**2*x - sqrt(x)*b**2*c*e**2*x - s
qrt(x)*b*c**2*d**2 + 2*sqrt(x)*b*c**2*d*e*x - 2*sqrt(x)*c**3*d**2*x))/(sqr
t(b + c*x)*b**2*c**2*x)
```

3.166 $\int \frac{d+ex}{(bx+cx^2)^{3/2}}\,dx$

Optimal result 1334
Mathematica [A] (verified) 1334
Rubi [A] (verified) 1335
Maple [A] (verified) 1336
Fricas [A] (verification not implemented) 1336
Sympy [F] 1337
Maxima [A] (verification not implemented) 1337
Giac [A] (verification not implemented) 1337
Mupad [B] (verification not implemented) 1338
Reduce [B] (verification not implemented) 1338

Optimal result

Integrand size = 19, antiderivative size = 48

$$\int \frac{d+ex}{(bx+cx^2)^{3/2}}\,dx=-\frac{2d}{b\sqrt{bx+cx^2}}-\frac{2(2cd-be)x}{b^2\sqrt{bx+cx^2}}$$

output
```
-2*d/b/(c*x^2+b*x)^(1/2)-2*(-b*e+2*c*d)*x/b^2/(c*x^2+b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.62

$$\int \frac{d+ex}{(bx+cx^2)^{3/2}}\,dx=\frac{2(-bd-2cdx+bex)}{b^2\sqrt{x(b+cx)}}$$

input
```
Integrate[(d + e*x)/(b*x + c*x^2)^(3/2),x]
```

output
```
(2*(-(b*d) - 2*c*d*x + b*e*x))/(b^2*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(bx+cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1158}$$

$$-\frac{2(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

input
```
Int[(d + e*x)/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2])
```

Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.62

method	result	size
pseudoelliptic	$\frac{(2ex-2d)b-4cdx}{\sqrt{x(cx+b)}\, b^{2}}$	30
gosper	$-\frac{2x(cx+b)(-bex+2cdx+bd)}{b^{2}(c\,x^{2}+bx)^{\frac{3}{2}}}$	37
orering	$-\frac{2x(cx+b)(-bex+2cdx+bd)}{b^{2}(c\,x^{2}+bx)^{\frac{3}{2}}}$	37
trager	$-\frac{2(-bex+2cdx+bd)\sqrt{c\,x^{2}+bx}}{(cx+b)b^{2}x}$	41
risch	$-\frac{2d(cx+b)}{b^{2}\sqrt{x(cx+b)}}+\frac{2(be-cd)x}{\sqrt{x(cx+b)}\, b^{2}}$	45
default	$-\frac{2d(2cx+b)}{b^{2}\sqrt{c\,x^{2}+bx}}+e\left(-\frac{1}{c\sqrt{c\,x^{2}+bx}}+\frac{2cx+b}{bc\sqrt{c\,x^{2}+bx}}\right)$	68

input
```
int((e*x+d)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((2*e*x-2*d)*b-4*c*d*x)/(x*(c*x+b))^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.92

$$\int \frac{d+ex}{(bx+cx^2)^{3/2}}\,dx=-\frac{2\,\sqrt{cx^2+bx}(bd+(2\,cd-be)x)}{b^2cx^2+b^3x}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(c*x^2 + b*x)*(b*d + (2*c*d - b*e)*x)/(b^2*c*x^2 + b^3*x)
```

Sympy [F]

$$\int \frac{d+ex}{(bx+cx^2)^{3/2}}\,dx = \int \frac{d+ex}{(x(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((d + e*x)/(x*(b + c*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.15

$$\int \frac{d+ex}{(bx+cx^2)^{3/2}}\,dx = -\frac{4\,cdx}{\sqrt{cx^2+bx}b^2} + \frac{2\,ex}{\sqrt{cx^2+bx}b} - \frac{2\,d}{\sqrt{cx^2+bx}b}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
-4*c*d*x/(sqrt(c*x^2 + b*x)*b^2) + 2*e*x/(sqrt(c*x^2 + b*x)*b) - 2*d/(sqrt
(c*x^2 + b*x)*b)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.69

$$\int \frac{d+ex}{(bx+cx^2)^{3/2}}\,dx = -\frac{2\left(\frac{d}{b}+\frac{(2\,cd-be)x}{b^2}\right)}{\sqrt{cx^2+bx}}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
-2*(d/b + (2*c*d - b*e)*x/b^2)/sqrt(c*x^2 + b*x)
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.65

$$\int \frac{d+ex}{(bx+cx^2)^{3/2}}\,dx = -\frac{2\,b\,d-2\,b\,e\,x+4\,c\,d\,x}{b^2\,\sqrt{c\,x^2+b\,x}}$$

input
```
int((d + e*x)/(b*x + c*x^2)^(3/2),x)
```

output
```
-(2*b*d - 2*b*e*x + 4*c*d*x)/(b^2*(b*x + c*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.42

$$\int \frac{d+ex}{(bx+cx^2)^{3/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{cx+b}\,bex-4\sqrt{c}\,\sqrt{cx+b}\,cdx-2\sqrt{x}\,bcd+2\sqrt{x}\,bcex-4\sqrt{x}\,c^2dx}{\sqrt{cx+b}\,b^2cx}$$

input
```
int((e*x+d)/(c*x^2+b*x)^(3/2),x)
```

output
```
(2*(sqrt(c)*sqrt(b + c*x)*b*e*x - 2*sqrt(c)*sqrt(b + c*x)*c*d*x - sqrt(x)*
b*c*d + sqrt(x)*b*c*e*x - 2*sqrt(x)*c**2*d*x))/(sqrt(b + c*x)*b**2*c*x)
```

3.167 $\int \frac{1}{(bx+cx^2)^{3/2}} \, dx$

Optimal result 1339
Mathematica [A] (verified) 1339
Rubi [A] (verified) 1340
Maple [A] (verified) 1341
Fricas [A] (verification not implemented) 1341
Sympy [F] 1342
Maxima [A] (verification not implemented) 1342
Giac [A] (verification not implemented) 1342
Mupad [B] (verification not implemented) 1343
Reduce [B] (verification not implemented) 1343

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{1}{(bx+cx^2)^{3/2}} \, dx = \frac{2}{b\sqrt{bx+cx^2}} - \frac{4\sqrt{bx+cx^2}}{b^2x}$$

output
```
2/b/(c*x^2+b*x)^(1/2)-4*(c*x^2+b*x)^(1/2)/b^2/x
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.55

$$\int \frac{1}{(bx+cx^2)^{3/2}} \, dx = -\frac{2(b+2cx)}{b^2\sqrt{x(b+cx)}}$$

input
```
Integrate[(b*x + c*x^2)^(-3/2),x]
```

output
```
(-2*(b + 2*c*x))/(b^2*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.077, Rules used = {1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 1088$$

$$-\frac{2(b+2cx)}{b^2\sqrt{bx+cx^2}}$$

input
```
Int[(b*x + c*x^2)^(-3/2),x]
```

output
```
(-2*(b + 2*c*x))/(b^2*Sqrt[b*x + c*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.52

method	result	size
pseudoelliptic	$-\frac{2(2cx+b)}{b^{2}\sqrt{x(cx+b)}}$	21
default	$-\frac{2(2cx+b)}{b^{2}\sqrt{c\,x^{2}+bx}}$	23
gosper	$-\frac{2x(2cx+b)(cx+b)}{b^{2}(c\,x^{2}+bx)^{\frac{3}{2}}}$	29
orering	$-\frac{2x(2cx+b)(cx+b)}{b^{2}(c\,x^{2}+bx)^{\frac{3}{2}}}$	29
trager	$-\frac{2(2cx+b)\sqrt{c\,x^{2}+bx}}{(cx+b)b^{2}x}$	33
risch	$-\frac{2(cx+b)}{b^{2}\sqrt{x(cx+b)}}-\frac{2cx}{\sqrt{x(cx+b)}\,b^{2}}$	37

input
```
int(1/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(2*c*x+b)/b^2/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx = -\frac{2\sqrt{cx^2+bx}(2\,cx+b)}{b^2cx^2+b^3x}$$

input
```
integrate(1/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(c*x^2 + b*x)*(2*c*x + b)/(b^2*c*x^2 + b^3*x)
```

Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((b*x + c*x**2)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx = -\frac{4\,cx}{\sqrt{cx^2+bx}b^2} - \frac{2}{\sqrt{cx^2+bx}b}$$

input
```
integrate(1/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
-4*c*x/(sqrt(c*x^2 + b*x)*b^2) - 2/(sqrt(c*x^2 + b*x)*b)
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx = -\frac{2\left(\frac{2\,cx}{b^2}+\frac{1}{b}\right)}{\sqrt{cx^2+bx}}$$

input
```
integrate(1/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
-2*(2*c*x/b^2 + 1/b)/sqrt(c*x^2 + b*x)
```

Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.60

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx = -\frac{2\,b+4\,c\,x}{b^2\,\sqrt{c\,x^2+b\,x}}$$

input
```
int(1/(b*x + c*x^2)^(3/2),x)
```

output
```
-(2*b + 4*c*x)/(b^2*(b*x + c*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

$$\int \frac{1}{(bx+cx^2)^{3/2}}\,dx = \frac{-4\sqrt{c}\,\sqrt{cx+b}\,x-2\sqrt{x}\,b-4\sqrt{x}\,cx}{\sqrt{cx+b}\,b^2x}$$

input
```
int(1/(c*x^2+b*x)^(3/2),x)
```

output
```
(2*( - 2*sqrt(c)*sqrt(b + c*x)*x - sqrt(x)*b - 2*sqrt(x)*c*x))/(sqrt(b + c
*x)*b**2*x)
```

3.168 $\int \frac{1}{(d+ex)(bx+cx^2)^{3/2}}\,dx$

Optimal result 1344
Mathematica [A] (verified) 1344
Rubi [A] (verified) 1345
Maple [A] (verified) 1347
Fricas [B] (verification not implemented) 1347
Sympy [F] 1348
Maxima [F(-2)] 1348
Giac [A] (verification not implemented) 1349
Mupad [F(-1)] 1349
Reduce [B] (verification not implemented) 1350

Optimal result

Integrand size = 21, antiderivative size = 119

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2}{bd\sqrt{bx+cx^2}} - \frac{2c(2cd-be)x}{b^2d(cd-be)\sqrt{bx+cx^2}} + \frac{2e^2\operatorname{arctanh}\left(\frac{\sqrt{cd-be}\,x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{d^{3/2}(cd-be)^{3/2}}$$

output
```
-2/b/d/(c*x^2+b*x)^(1/2)-2*c*(-b*e+2*c*d)*x/b^2/d/(-b*e+c*d)/(c*x^2+b*x)^(
1/2)+2*e^2*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(1/2))/d^(3/2)/(
-b*e+c*d)^(3/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.21

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{3/2}}\,dx = \frac{2\left(\sqrt{d}\sqrt{-cd+be}\left(-b^2e+2c^2dx+bc(d-ex)\right)+b^2e^2\sqrt{x}\sqrt{b+cx}\arctan\left(\frac{-\cdots}{}\right.\right.}{b^2d^{3/2}(-cd+be)^{3/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[1/((d + e*x)*(b*x + c*x^2)^(3/2)),x]
```

output
```
(2*(Sqrt[d]*Sqrt[-(c*d) + b*e]*(-(b^2*e) + 2*c^2*d*x + b*c*(d - e*x)) + b^
2*e^2*Sqrt[x]*Sqrt[b + c*x]*ArcTan[(-(e*Sqrt[x]*Sqrt[b + c*x]) + Sqrt[c]*(
d + e*x))/(Sqrt[d]*Sqrt[-(c*d) + b*e])]))/(b^2*d^(3/2)*(-(c*d) + b*e)^(3/2
)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {1165, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{3/2}(d+ex)}dx$$

$$\Big\downarrow \text{1165}$$

$$-\frac{2\int -\frac{b^2e^2}{2(d+ex)\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$\frac{e^2\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{2e^2\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}$$

$$\Big\downarrow \text{219}$$

$$\frac{e^2\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{d^{3/2}(cd-be)^{3/2}}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}$$

input
```
Int[1/((d + e*x)*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*Sqrt[b*x + c*x
^2]) + (e^2*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqr
t[b*x + c*x^2]])])/(d^(3/2)*(c*d - b*e)^(3/2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$-\frac{2\sqrt{x(cx+b)}}{b^2dx}+\frac{2c^2x}{b^2(be-cd)\sqrt{x(cx+b)}}+\frac{2e^2\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)}{\sqrt{d(be-cd)}\,d(be-cd)}$
risch	$-\frac{2(cx+b)}{b^2d\sqrt{x(cx+b)}}+\frac{e\ln\left(\frac{-\frac{2d(be-cd)}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{-\frac{d(be-cd)}{e^2}}\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}}}{x+\frac{d}{e}}\right)}{d(be-cd)\sqrt{-\frac{d(be-cd)}{e^2}}}+\frac{2c\sqrt{c}}{b^2}$
default	$-\frac{e^2}{d(be-cd)\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}}}+\frac{(be-2cd)e\left(2c\left(x+\frac{d}{e}\right)+\frac{be-2cd}{e}\right)}{d(be-cd)\left(-\frac{4cd(be-cd)}{e^2}-\frac{(be-2cd)^2}{e^2}\right)\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-}{e^2}}}$ e

input
```
int(1/(e*x+d)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b^2/d*(x*(c*x+b))^(1/2)/x+2/b^2*c^2/(b*e-c*d)/(x*(c*x+b))^(1/2)*x+2*e^2
/(d*(b*e-c*d))^(1/2)*arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))^(1/2))/d/(
b*e-c*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(105) = 210$.

Time = 0.09 (sec) , antiderivative size = 448, normalized size of antiderivative = 3.76

$$\int \frac{1}{(d+ex)\,(bx+cx^2)^{3/2}}\,dx=\left[-\frac{(b^2ce^2x^2+b^3e^2x)\sqrt{cd^2-bde}\log\left(\frac{bd+(2\,cd-be)x-2\,\sqrt{cd^2-bde}\sqrt{cx^2+bx}}{ex+d}\right)+2\,(bc}{(b^2c^3d^4-2\,b^3c^2d^3e+b^4cd^2e^2)x^2+}\right.$$

$$-\frac{2\left((b^2ce^2x^2+b^3e^2x)\sqrt{-cd^2+bde}\arctan\left(\frac{\sqrt{-cd^2+bde}\sqrt{cx^2+bx}}{cdx+bd}\right)+(bc^2d^3-2\,b^2cd^2e+b^3de^2+(2\,c^3d^3-3\,bc\right.}{(b^2c^3d^4-2\,b^3c^2d^3e+b^4cd^2e^2)x^2+(b^3c^2d^4-2\,b^4cd^3e+b^5d^2e^2)x}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-((b^2*c*e^2*x^2 + b^3*e^2*x)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e
)*x - 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) + 2*(b*c^2*d^3 -
 2*b^2*c*d^2*e + b^3*d*e^2 + (2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2)*x)*
sqrt(c*x^2 + b*x))/((b^2*c^3*d^4 - 2*b^3*c^2*d^3*e + b^4*c*d^2*e^2)*x^2 +
(b^3*c^2*d^4 - 2*b^4*c*d^3*e + b^5*d^2*e^2)*x), -2*((b^2*c*e^2*x^2 + b^3*e
^2*x)*sqrt(-c*d^2 + b*d*e)*arctan(sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/(
c*d*x + b*d)) + (b*c^2*d^3 - 2*b^2*c*d^2*e + b^3*d*e^2 + (2*c^3*d^3 - 3*b*
c^2*d^2*e + b^2*c*d*e^2)*x)*sqrt(c*x^2 + b*x))/((b^2*c^3*d^4 - 2*b^3*c^2*d
^3*e + b^4*c*d^2*e^2)*x^2 + (b^3*c^2*d^4 - 2*b^4*c*d^3*e + b^5*d^2*e^2)*x)
]
```

Sympy [F]

$$\int \frac{1}{(d+ex)\,(bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(x\,(b+cx))^{\frac{3}{2}}\,(d+ex)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(1/((x*(b + c*x))**(3/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\,(bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.34

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2\,e^2\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde}}\right)}{(cd^2-bde)\sqrt{-cd^2+bde}} - \frac{2\left(\frac{(2\,c^2d^2-bcde)x}{b^2cd^3-b^3d^2e}+\frac{bcd^2-b^2de}{b^2cd^3-b^3d^2e}\right)}{\sqrt{cx^2+bx}}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
-2*e^2*arctan(((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^2
+ b*d*e))/((c*d^2 - b*d*e)*sqrt(-c*d^2 + b*d*e)) - 2*((2*c^2*d^2 - b*c*d*e
)*x/(b^2*c*d^3 - b^3*d^2*e) + (b*c*d^2 - b^2*d*e)/(b^2*c*d^3 - b^3*d^2*e))
/sqrt(c*x^2 + b*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(c\,x^2+b\,x\right)^{3/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/((b*x + c*x^2)^(3/2)*(d + e*x)),x)
```

output
```
int(1/((b*x + c*x^2)^(3/2)*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.44

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{3/2}}\,dx = \frac{2\sqrt{d}\,\sqrt{cx+b}\,\sqrt{be-cd}\,atan\left(\frac{\sqrt{be-cd}-\sqrt{e}\,\sqrt{cx+b}-\sqrt{x}\,\sqrt{e}\,\sqrt{c}}{\sqrt{d}\,\sqrt{c}}\right)b^2e^2x+2\sqrt{d}\,\sqrt{cx+}$$

input
```
int(1/(e*x+d)/(c*x^2+b*x)^(3/2),x)
```

output
```
(2*(sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*
sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**2*e**2*x +
sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) + sqrt(e)*sqrt
(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**2*e**2*x - sqrt
(c)*sqrt(b + c*x)*b**2*d*e**2*x + 3*sqrt(c)*sqrt(b + c*x)*b*c*d**2*e*x - 2
*sqrt(c)*sqrt(b + c*x)*c**2*d**3*x - sqrt(x)*b**3*d*e**2 + 2*sqrt(x)*b**2*
c*d**2*e - sqrt(x)*b**2*c*d*e**2*x - sqrt(x)*b*c**2*d**3 + 3*sqrt(x)*b*c**
2*d**2*e*x - 2*sqrt(x)*c**3*d**3*x))/(sqrt(b + c*x)*b**2*d**2*x*(b**2*e**2
 - 2*b*c*d*e + c**2*d**2))
```

3.169 $\int \frac{1}{(d+ex)^2 (bx+cx^2)^{3/2}} \, dx$

Optimal result 1351
Mathematica [A] (verified) 1352
Rubi [A] (verified) 1352
Maple [A] (verified) 1354
Fricas [B] (verification not implemented) 1355
Sympy [F] 1356
Maxima [F(-2)] 1357
Giac [B] (verification not implemented) 1357
Mupad [F(-1)] 1358
Reduce [B] (verification not implemented) 1359

Optimal result

Integrand size = 21, antiderivative size = 197

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^{3/2}} \, dx = -\frac{2cd-3be}{bd^2(cd-be)\sqrt{bx+cx^2}} - \frac{c(4c^2d^2-4bcde+3b^2e^2)\,x}{b^2d^2(cd-be)^2\sqrt{bx+cx^2}} - \frac{e}{d(cd-be)(d+ex)\sqrt{bx+cx^2}} + \frac{3e^2(2cd-be)\operatorname{arctanh}\left(\frac{\sqrt{cd-be}\,x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{d^{5/2}(cd-be)^{5/2}}$$

output

```
-(-3*b*e+2*c*d)/b/d^2/(-b*e+c*d)/(c*x^2+b*x)^(1/2)-c*(3*b^2*e^2-4*b*c*d*e+
4*c^2*d^2)*x/b^2/d^2/(-b*e+c*d)^2/(c*x^2+b*x)^(1/2)-e/d/(-b*e+c*d)/(e*x+d)
/(c*x^2+b*x)^(1/2)+3*e^2*(-b*e+2*c*d)*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(
c*x^2+b*x)^(1/2))/d^(5/2)/(-b*e+c*d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^{3/2}}\,dx = \frac{x\left(-\frac{\sqrt{d}(b+cx)(4c^3d^2x(d+ex)+b^3e^2(2d+3ex)+2bc^2d(d^2-dex-2e^2x^2)+b^2ce(-4d^2-2dex+3e^2x^2))}{b^2(cd-be)^2(d+ex)}\right.}{d^{5/2}(x(b+cx))^{3/2}}$$

input
```
Integrate[1/((d + e*x)^2*(b*x + c*x^2)^(3/2)),x]
```

output
```
(x*(-((Sqrt[d]*(b + c*x)*(4*c^3*d^2*x*(d + e*x) + b^3*e^2*(2*d + 3*e*x) +
2*b*c^2*d*(d^2 - d*e*x - 2*e^2*x^2) + b^2*c*e*(-4*d^2 - 2*d*e*x + 3*e^2*x^
2)))/(b^2*(c*d - b*e)^2*(d + e*x))) - (3*e^2*(2*c*d - b*e)*Sqrt[x]*(b + c*
x)^(3/2)*ArcTan[(-(e*Sqrt[x]*Sqrt[b + c*x]) + Sqrt[c]*(d + e*x))/(Sqrt[d]*
Sqrt[-(c*d) + b*e])])/(-(c*d) + b*e)^(5/2)))/(d^(5/2)*(x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {1165, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{3/2}(d+ex)^2}dx$$

$$\Big\downarrow \; 1165$$

$$-\frac{2\int \frac{e(b(2cd-3be)+2c(2cd-be)x)}{2(d+ex)^2\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)(cd-be)}$$

$$\Big\downarrow \; 27$$

$$-\frac{e\int \frac{b(2cd-3be)+2c(2cd-be)x}{(d+ex)^2\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)(cd-be)}$$

$$\Big\downarrow \; 1228$$

$$-\frac{e\left(\frac{\sqrt{bx+cx^2}(3b^2e^2-4bcde+4c^2d^2)}{d(d+ex)(cd-be)}-\frac{3b^2e(2cd-be)\int\frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{2d(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)(cd-be)}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{e\left(\frac{3b^2e(2cd-be)\int\frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{d(cd-be)}+\frac{\sqrt{bx+cx^2}(3b^2e^2-4bcde+4c^2d^2)}{d(d+ex)(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)(cd-be)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{e\left(\frac{\sqrt{bx+cx^2}(3b^2e^2-4bcde+4c^2d^2)}{d(d+ex)(cd-be)}-\frac{3b^2e(2cd-be)\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{2d^{3/2}(cd-be)^{3/2}}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)(cd-be)}$$

input
```
Int[1/((d + e*x)^2*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*(d + e*x)*Sqrt
[b*x + c*x^2]) - (e*(((4*c^2*d^2 - 4*b*c*d*e + 3*b^2*e^2)*Sqrt[b*x + c*x^2
])/(d*(c*d - b*e)*(d + e*x)) - (3*b^2*e*(2*c*d - b*e)*ArcTanh[(b*d + (2*c*
d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(2*d^(3/2)*(c*
d - b*e)^(3/2))))/(b^2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.95

method	result
pseudoelliptic	$-\frac{2\left(-\frac{3\sqrt{x(cx+b)}\,b^2e^2(ex+d)(be-2cd)\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)}{2}+\left(c^2(2cx+b)d^3-2ec\left(-c^2x^2+\frac{1}{2}cbx+b^2\right)d^2+b\,e^2(cx+b)(-2cx+\right.\right.}{\sqrt{x(cx+b)}\,\sqrt{d(be-cd)}\,d^2(be-cd)^2(ex+d)b^2}$
risch	$-\frac{2(cx+b)}{b^2d^2\sqrt{x(cx+b)}}-\frac{2c^2\sqrt{c\left(\frac{b}{c}+x\right)^2-\left(\frac{b}{c}+x\right)b}}{b^2(be-cd)^2\left(\frac{b}{c}+x\right)}-\frac{e^2\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}}}{d^2(be-cd)^2\left(x+\frac{d}{e}\right)}+\frac{3b\,e^2\ln\left(\frac{-\frac{2d(be-cd)}{e^2}}{}\right.}{}$
default	$\frac{e^2}{d(be-cd)\left(x+\frac{d}{e}\right)\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}}}+\frac{3(be-2cd)e\left(-\frac{e^2}{d(be-cd)\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}}}+\right.}{}$

input
```
int(1/(e*x+d)^2/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-3/2*(x*(c*x+b))^(1/2)*b^2*e^2*(e*x+d)*(b*e-2*c*d)*arctan((x*(c*x+b))^
(1/2)/x*d/(d*(b*e-c*d))^(1/2))+(c^2*(2*c*x+b)*d^3-2*e*c*(-c^2*x^2+1/2*c*b*
x+b^2)*d^2+b*e^2*(c*x+b)*(-2*c*x+b)*d+3/2*b^2*e^3*x*(c*x+b))*(d*(b*e-c*d))
^(1/2))/(x*(c*x+b))^(1/2)/(d*(b*e-c*d))^(1/2)/d^2/(b*e-c*d)^2/(e*x+d)/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(181) = 362$.

Time = 0.10 (sec) , antiderivative size = 910, normalized size of antiderivative = 4.62

$$\int \frac{1}{(d+ex)^2\left(bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(3*((2*b^2*c^2*d*e^3 - b^3*c*e^4)*x^3 + (2*b^2*c^2*d^2*e^2 + b^3*c*d
*e^3 - b^4*e^4)*x^2 + (2*b^3*c*d^2*e^2 - b^4*d*e^3)*x)*sqrt(c*d^2 - b*d*e)
*log((b*d + (2*c*d - b*e)*x - 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*
x + d)) + 2*(2*b*c^3*d^5 - 6*b^2*c^2*d^4*e + 6*b^3*c*d^3*e^2 - 2*b^4*d^2*e
^3 + (4*c^4*d^4*e - 8*b*c^3*d^3*e^2 + 7*b^2*c^2*d^2*e^3 - 3*b^3*c*d*e^4)*x
^2 + (4*c^4*d^5 - 6*b*c^3*d^4*e + 5*b^3*c*d^2*e^3 - 3*b^4*d*e^4)*x)*sqrt(c
*x^2 + b*x))/((b^2*c^4*d^6*e - 3*b^3*c^3*d^5*e^2 + 3*b^4*c^2*d^4*e^3 - b^5
*c*d^3*e^4)*x^3 + (b^2*c^4*d^7 - 2*b^3*c^3*d^6*e + 2*b^5*c*d^4*e^3 - b^6*d
^3*e^4)*x^2 + (b^3*c^3*d^7 - 3*b^4*c^2*d^6*e + 3*b^5*c*d^5*e^2 - b^6*d^4*e
^3)*x), -(3*((2*b^2*c^2*d*e^3 - b^3*c*e^4)*x^3 + (2*b^2*c^2*d^2*e^2 + b^3*
c*d*e^3 - b^4*e^4)*x^2 + (2*b^3*c*d^2*e^2 - b^4*d*e^3)*x)*sqrt(-c*d^2 + b*
d*e)*arctan(sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/(c*d*x + b*d)) + (2*b*c
^3*d^5 - 6*b^2*c^2*d^4*e + 6*b^3*c*d^3*e^2 - 2*b^4*d^2*e^3 + (4*c^4*d^4*e
- 8*b*c^3*d^3*e^2 + 7*b^2*c^2*d^2*e^3 - 3*b^3*c*d*e^4)*x^2 + (4*c^4*d^5 -
6*b*c^3*d^4*e + 5*b^3*c*d^2*e^3 - 3*b^4*d*e^4)*x)*sqrt(c*x^2 + b*x))/((b^2
*c^4*d^6*e - 3*b^3*c^3*d^5*e^2 + 3*b^4*c^2*d^4*e^3 - b^5*c*d^3*e^4)*x^3 +
(b^2*c^4*d^7 - 2*b^3*c^3*d^6*e + 2*b^5*c*d^4*e^3 - b^6*d^3*e^4)*x^2 + (b^3
*c^3*d^7 - 3*b^4*c^2*d^6*e + 3*b^5*c*d^5*e^2 - b^6*d^4*e^3)*x)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(x(b+cx))^{\frac{3}{2}}(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(1/((x*(b + c*x))**(3/2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^{3/2}}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 821 vs. 2(181) = 362.

Time = 0.49 (sec) , antiderivative size = 821, normalized size of antiderivative = 4.17

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
1/2*((6*b^2*c*d*e^5*log(abs(2*c*d*e - b*e^2 - 2*sqrt(c*d^2 - b*d*e)*sqrt(c
)*abs(e))) - 3*b^3*e^6*log(abs(2*c*d*e - b*e^2 - 2*sqrt(c*d^2 - b*d*e)*sqr
t(c)*abs(e))) + 8*sqrt(c*d^2 - b*d*e)*c^(5/2)*d^2*e^2*abs(e) - 8*sqrt(c*d^
2 - b*d*e)*b*c^(3/2)*d*e^3*abs(e) + 6*sqrt(c*d^2 - b*d*e)*b^2*sqrt(c)*e^4*
abs(e))*sgn(1/(e*x + d))*sgn(e)/(sqrt(c*d^2 - b*d*e)*b^2*c^2*d^4*abs(e) -
2*sqrt(c*d^2 - b*d*e)*b^3*c*d^3*e*abs(e) + sqrt(c*d^2 - b*d*e)*b^4*d^2*e^2
*abs(e)) - 2*((4*c^3*d^2*e^7 - 4*b*c^2*d*e^8 + 3*b^2*c*e^9)/(b^2*c^2*d^4*e
^5*sgn(1/(e*x + d))*sgn(e) - 2*b^3*c*d^3*e^6*sgn(1/(e*x + d))*sgn(e) + b^4
*d^2*e^7*sgn(1/(e*x + d))*sgn(e)) - ((4*c^3*d^3*e^8 - 6*b*c^2*d^2*e^9 + 8*
b^2*c*d*e^10 - 3*b^3*e^11)/(b^2*c^2*d^4*e^5*sgn(1/(e*x + d))*sgn(e) - 2*b^
3*c*d^3*e^6*sgn(1/(e*x + d))*sgn(e) + b^4*d^2*e^7*sgn(1/(e*x + d))*sgn(e))
 - (b^2*c*d^2*e^11 - b^3*d*e^12)/((b^2*c^2*d^4*e^5*sgn(1/(e*x + d))*sgn(e)
 - 2*b^3*c*d^3*e^6*sgn(1/(e*x + d))*sgn(e) + b^4*d^2*e^7*sgn(1/(e*x + d))*
sgn(e))*(e*x + d)*e))/((e*x + d)*e))/sqrt(c - 2*c*d/(e*x + d) + c*d^2/(e*x
 + d)^2 + b*e/(e*x + d) - b*d*e/(e*x + d)^2) - 3*(2*c*d*e^5 - b*e^6)*log(a
bs(2*c*d*e - b*e^2 - 2*sqrt(c*d^2 - b*d*e)*(sqrt(c - 2*c*d/(e*x + d) + c*d
^2/(e*x + d)^2 + b*e/(e*x + d) - b*d*e/(e*x + d)^2) + sqrt(c*d^2*e^2 - b*d
*e^3)/((e*x + d)*e))*abs(e)))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*sqrt(
c*d^2 - b*d*e)*abs(e)*sgn(1/(e*x + d))*sgn(e)))/e^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^{3/2}} \, dx = \int \frac{1}{(c\,x^2+b\,x)^{3/2}\,(d+e\,x)^2} \, dx$$

input
```
int(1/((b*x + c*x^2)^(3/2)*(d + e*x)^2),x)
```

output
```
int(1/((b*x + c*x^2)^(3/2)*(d + e*x)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 919, normalized size of antiderivative = 4.66

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^2/(c*x^2+b*x)^(3/2),x)
```

output
```
(3*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*s
qrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**3*d*e**3*x +
 3*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*s
qrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**3*e**4*x**2
- 6*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*
sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**2*c*d**2*e*
*2*x - 6*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqr
t(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**2*c*d*
e**3*x**2 + 3*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d)
+ sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**3
*d*e**3*x + 3*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d)
+ sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**3
*e**4*x**2 - 6*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d)
 + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**
2*c*d**2*e**2*x - 6*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e -
 c*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c))
)*b**2*c*d*e**3*x**2 - 4*sqrt(c)*sqrt(b + c*x)*b**2*c*d**3*e**2*x - 4*sqrt
(c)*sqrt(b + c*x)*b**2*c*d**2*e**3*x**2 + 8*sqrt(c)*sqrt(b + c*x)*b*c**2*d
**4*e*x + 8*sqrt(c)*sqrt(b + c*x)*b*c**2*d**3*e**2*x**2 - 4*sqrt(c)*sqrt(b
 + c*x)*c**3*d**5*x - 4*sqrt(c)*sqrt(b + c*x)*c**3*d**4*e*x**2 - 2*sqrt...
```

3.170 $\int \frac{1}{(d+ex)^3 (bx+cx^2)^{3/2}} \, dx$

Optimal result . . . 1360
Mathematica [A] (verified) . . . 1361
Rubi [A] (verified) . . . 1361
Maple [A] (verified) . . . 1364
Fricas [B] (verification not implemented) . . . 1365
Sympy [F] . . . 1366
Maxima [F(-2)] . . . 1367
Giac [B] (verification not implemented) . . . 1367
Mupad [F(-1)] . . . 1368
Reduce [B] (verification not implemented) . . . 1368

Optimal result

Integrand size = 21, antiderivative size = 289

$$\int \frac{1}{(d+ex)^3 (bx+cx^2)^{3/2}} \, dx = -\frac{8c^2d^2-28bcde+15b^2e^2}{4bd^3(cd-be)^2\sqrt{bx+cx^2}} - \frac{c(2cd-be)(8c^2d^2-8bcde+15b^2e^2)x}{4b^2d^3(cd-be)^3\sqrt{bx+cx^2}} - \frac{e}{2d(cd-be)(d+ex)^2\sqrt{bx+cx^2}} - \frac{5e(2cd-be)}{4d^2(cd-be)^2(d+ex)\sqrt{bx+cx^2}} + \frac{3e^2(16c^2d^2-16bcde+5b^2e^2)\operatorname{arctanh}\left(\frac{\sqrt{cd-be}x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{4d^{7/2}(cd-be)^{7/2}}$$

output
```
-1/4*(15*b^2*e^2-28*b*c*d*e+8*c^2*d^2)/b/d^3/(-b*e+c*d)^2/(c*x^2+b*x)^(1/2
)-1/4*c*(-b*e+2*c*d)*(15*b^2*e^2-8*b*c*d*e+8*c^2*d^2)*x/b^2/d^3/(-b*e+c*d)
^3/(c*x^2+b*x)^(1/2)-1/2*e/d/(-b*e+c*d)/(e*x+d)^2/(c*x^2+b*x)^(1/2)-5/4*e*
(-b*e+2*c*d)/d^2/(-b*e+c*d)^2/(e*x+d)/(c*x^2+b*x)^(1/2)+3/4*e^2*(5*b^2*e^2
-16*b*c*d*e+16*c^2*d^2)*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(1/
2))/d^(7/2)/(-b*e+c*d)^(7/2)
```

Mathematica [A] (verified)

Time = 10.62 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.00

$$\int \frac{1}{(d+ex)^3\,(bx+cx^2)^{3/2}}\,dx =$$
$$-\frac{2b^2d^{5/2}e(cd-be)^{5/2}+(d+ex)\left(5b^2d^{3/2}e(cd-be)^{3/2}(2cd-be)+(d+ex)\left(b\sqrt{d}(cd-be)^{3/2}\,(8c^2d^2-28\right.\right.}{4}$$

input
```
Integrate[1/((d + e*x)^3*(b*x + c*x^2)^(3/2)),x]
```

output
```
-1/4*(2*b^2*d^(5/2)*e*(c*d - b*e)^(5/2) + (d + e*x)*(5*b^2*d^(3/2)*e*(c*d
- b*e)^(3/2)*(2*c*d - b*e) + (d + e*x)*(b*Sqrt[d]*(c*d - b*e)^(3/2)*(8*c^2
*d^2 - 28*b*c*d*e + 15*b^2*e^2) + c*Sqrt[d]*Sqrt[c*d - b*e]*(16*c^3*d^3 -
24*b*c^2*d^2*e + 38*b^2*c*d*e^2 - 15*b^3*e^3)*x - 3*b^2*e^2*(16*c^2*d^2 -
16*b*c*d*e + 5*b^2*e^2)*Sqrt[x]*Sqrt[b + c*x]*ArcTanh[(Sqrt[c*d - b*e]*Sqr
t[x])/(Sqrt[d]*Sqrt[b + c*x])])))/(b^2*d^(7/2)*(c*d - b*e)^(7/2)*Sqrt[x*(b
 + c*x)]*(d + e*x)^2)
```

Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1165, 27, 1237, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{3/2}\,(d+ex)^3}\,dx$$

$$\Big\downarrow \; 1165$$

$$-\frac{2\int \frac{e(b(4cd-5be)+4c(2cd-be)x)}{2(d+ex)^3\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^2(cd-be)}$$

$$\Big\downarrow \; 27$$

$$-\frac{e\int \frac{b(4cd-5be)+4c(2cd-be)x}{(d+ex)^3\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^2(cd-be)}$$

$$\Big\downarrow \text{1237}$$

$$-\frac{e\left(\frac{\sqrt{bx+cx^2}(5b^2e^2-8bcde+8c^2d^2)}{2d(d+ex)^2(cd-be)}-\frac{\int -\frac{b\left(8c^2d^2-28bced+15b^2e^2\right)+2c\left(8c^2d^2-8bced+5b^2e^2\right)x}{2(d+ex)^2\sqrt{cx^2+bx}}dx}{2d(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^2(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e\left(\frac{\int \frac{b\left(8c^2d^2-28bced+15b^2e^2\right)+2c\left(8c^2d^2-8bced+5b^2e^2\right)x}{(d+ex)^2\sqrt{cx^2+bx}}dx}{4d(cd-be)}+\frac{\sqrt{bx+cx^2}(5b^2e^2-8bcde+8c^2d^2)}{2d(d+ex)^2(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^2(cd-be)}$$

$$\Big\downarrow \text{1228}$$

$$-\frac{e\left(\frac{\frac{\sqrt{bx+cx^2}(2cd-be)\left(15b^2e^2-8bcde+8c^2d^2\right)}{d(d+ex)(cd-be)}-\frac{3b^2e\left(5b^2e^2-16bcde+16c^2d^2\right)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{2d(cd-be)}}{4d(cd-be)}+\frac{\sqrt{bx+cx^2}(5b^2e^2-8bcde+8c^2d^2)}{2d(d+ex)^2(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^2(cd-be)}$$

$$\Big\downarrow \text{1154}$$

$$-\frac{e\left(\frac{\frac{3b^2e\left(5b^2e^2-16bcde+16c^2d^2\right)\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{d(cd-be)}+\frac{\sqrt{bx+cx^2}(2cd-be)\left(15b^2e^2-8bcde+8c^2d^2\right)}{d(d+ex)(cd-be)}}{4d(cd-be)}+\frac{\sqrt{bx+cx^2}(5b^2e^2 \ldots}{2d(d+ex \ldots}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^2(cd-be)}$$

$$\Big\downarrow \text{219}$$

$$-\frac{e\left(\frac{\frac{\sqrt{bx+cx^2}(2cd-be)\left(15b^2e^2-8bcde+8c^2d^2\right)}{d(d+ex)(cd-be)}-\frac{3b^2e\left(5b^2e^2-16bcde+16c^2d^2\right)\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{2d^{3/2}(cd-be)^{3/2}}}{4d(cd-be)}+\frac{\sqrt{bx+cx^2}\left(5b^2e^2-8bcde+8c^2d^2\right.}{2d(d+ex)^2(cd-be)}\right.}{b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^2(cd-be)}$$

input
```
Int[1/((d + e*x)^3*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*(d + e*x)^2*Sq
rt[b*x + c*x^2]) - (e*(((8*c^2*d^2 - 8*b*c*d*e + 5*b^2*e^2)*Sqrt[b*x + c*x
^2])/(2*d*(c*d - b*e)*(d + e*x)^2) + (((2*c*d - b*e)*(8*c^2*d^2 - 8*b*c*d*
e + 15*b^2*e^2)*Sqrt[b*x + c*x^2])/(d*(c*d - b*e)*(d + e*x)) - (3*b^2*e*(1
6*c^2*d^2 - 16*b*c*d*e + 5*b^2*e^2)*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqr
t[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])]])/(2*d^(3/2)*(c*d - b*e)^(3/2)))/(
4*d*(c*d - b*e))))/(b^2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.94

method	result
pseudoelliptic	$-\frac{2\left(-\frac{3e^{2}b^{2}\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)(ex+d)^{2}\left(5b^{2}e^{2}-16bcde+16c^{2}d^{2}\right)\sqrt{x(cx+b)}}{8}+\left(e^{3}\left(\frac{25}{8}dex+\frac{15}{8}e^{2}x^{2}+d^{2}\right)b^{4}-3\left(-\frac{5}{8}e^{3}x^{3}+\frac{13}{24}d\right.\right.\right.}{\sqrt{x(cx+b)}\,\sqrt{d(be-}}$
risch	$-\frac{2(cx+b)}{b^{2}d^{3}\sqrt{x(cx+b)}}-\frac{e\sqrt{c\left(x+\frac{d}{e}\right)^{2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^{2}}}}{2d^{2}(be-cd)^{2}\left(x+\frac{d}{e}\right)^{2}}-\frac{7b\,e^{3}\sqrt{c\left(x+\frac{d}{e}\right)^{2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^{2}}}}{4d^{3}(be-cd)^{3}\left(x+\frac{d}{e}\right)}+\frac{7e^{2}}{}$
default	Expression too large to display

input
```
int(1/(e*x+d)^3/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(x*(c*x+b))^(1/2)*(-3/8*e^2*b^2*arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*
d))^(1/2))*(e*x+d)^2*(5*b^2*e^2-16*b*c*d*e+16*c^2*d^2)*(x*(c*x+b))^(1/2)+(
e^3*(25/8*d*e*x+15/8*e^2*x^2+d^2)*b^4-3*(-5/8*e^3*x^3+13/24*d*e^2*x^2+7/3*
d^2*e*x+d^3)*e^2*c*b^3+3*e*(-19/12*e^3*x^3-5/3*d*e^2*x^2+d^2*e*x+d^3)*c^2*
d*b^2-d^2*b*(-3*e*x+d)*(e*x+d)^2*c^3-2*x*d^3*(e*x+d)^2*c^4)*(d*(b*e-c*d))^
(1/2))/(d*(b*e-c*d))^(1/2)/d^3/(b*e-c*d)^3/(e*x+d)^2/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(261) = 522$.

Time = 0.17 (sec) , antiderivative size = 1652, normalized size of antiderivative = 5.72

$$\int \frac{1}{(d+ex)^3 (bx+cx^2)^{3/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^3/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*((16*b^2*c^3*d^2*e^4 - 16*b^3*c^2*d*e^5 + 5*b^4*c*e^6)*x^4 + (32*
b^2*c^3*d^3*e^3 - 16*b^3*c^2*d^2*e^4 - 6*b^4*c*d*e^5 + 5*b^5*e^6)*x^3 + (1
6*b^2*c^3*d^4*e^2 + 16*b^3*c^2*d^3*e^3 - 27*b^4*c*d^2*e^4 + 10*b^5*d*e^5)*
x^2 + (16*b^3*c^2*d^4*e^2 - 16*b^4*c*d^3*e^3 + 5*b^5*d^2*e^4)*x)*sqrt(c*d^
2 - b*d*e)*log((b*d + (2*c*d - b*e)*x - 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 +
 b*x))/(e*x + d)) + 2*(8*b*c^4*d^7 - 32*b^2*c^3*d^6*e + 48*b^3*c^2*d^5*e^2
 - 32*b^4*c*d^4*e^3 + 8*b^5*d^3*e^4 + (16*c^5*d^5*e^2 - 40*b*c^4*d^4*e^3 +
 62*b^2*c^3*d^3*e^4 - 53*b^3*c^2*d^2*e^5 + 15*b^4*c*d*e^6)*x^3 + (32*c^5*d
^6*e - 72*b*c^4*d^5*e^2 + 80*b^2*c^3*d^4*e^3 - 27*b^3*c^2*d^3*e^4 - 28*b^4
*c*d^2*e^5 + 15*b^5*d*e^6)*x^2 + (16*c^5*d^7 - 24*b*c^4*d^6*e - 16*b^2*c^3
*d^5*e^2 + 80*b^3*c^2*d^4*e^3 - 81*b^4*c*d^3*e^4 + 25*b^5*d^2*e^5)*x)*sqrt
(c*x^2 + b*x))/((b^2*c^5*d^8*e^2 - 4*b^3*c^4*d^7*e^3 + 6*b^4*c^3*d^6*e^4 -
 4*b^5*c^2*d^5*e^5 + b^6*c*d^4*e^6)*x^4 + (2*b^2*c^5*d^9*e - 7*b^3*c^4*d^8
*e^2 + 8*b^4*c^3*d^7*e^3 - 2*b^5*c^2*d^6*e^4 - 2*b^6*c*d^5*e^5 + b^7*d^4*e
^6)*x^3 + (b^2*c^5*d^10 - 2*b^3*c^4*d^9*e - 2*b^4*c^3*d^8*e^2 + 8*b^5*c^2*
d^7*e^3 - 7*b^6*c*d^6*e^4 + 2*b^7*d^5*e^5)*x^2 + (b^3*c^4*d^10 - 4*b^4*c^3
*d^9*e + 6*b^5*c^2*d^8*e^2 - 4*b^6*c*d^7*e^3 + b^7*d^6*e^4)*x), -1/4*(3*((
16*b^2*c^3*d^2*e^4 - 16*b^3*c^2*d*e^5 + 5*b^4*c*e^6)*x^4 + (32*b^2*c^3*d^3
*e^3 - 16*b^3*c^2*d^2*e^4 - 6*b^4*c*d*e^5 + 5*b^5*e^6)*x^3 + (16*b^2*c^3*d
^4*e^2 + 16*b^3*c^2*d^3*e^3 - 27*b^4*c*d^2*e^4 + 10*b^5*d*e^5)*x^2 + (1...
```

Sympy [F]

$$\int \frac{1}{(d+ex)^3 (bx+cx^2)^{3/2}} \, dx = \int \frac{1}{(x(b+cx))^{\frac{3}{2}} (d+ex)^3} \, dx$$

input
```
integrate(1/(e*x+d)**3/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(1/((x*(b + c*x))**(3/2)*(d + e*x)**3), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^3 (bx+cx^2)^{3/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^3/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 738 vs. $2(261) = 522$.

Time = 0.34 (sec) , antiderivative size = 738, normalized size of antiderivative = 2.55

$$\int \frac{1}{(d+ex)^3 (bx+cx^2)^{3/2}}\,dx = -\frac{2\left(\frac{(2c^4d^6-3bc^3d^5e+3b^2c^2d^4e^2-b^3cd^3e^3)x}{b^2c^3d^9-3b^3c^2d^8e+3b^4cd^7e^2-b^5d^6e^3}+\frac{bc^3d^6-3b^2c^2d^5e+3b^3cd^4e^2-b^4d^3e^3}{b^2c^3d^9-3b^3c^2d^8e+3b^4cd^7e^2-b^5d^6e^3}\right)}{\sqrt{cx^2+bx}} + \frac{3(16c^2d^2e^2-16bcde^3+5b^2e^4)\arctan\left(-\frac{(\sqrt{c}x-\sqrt{cx^2+bx})e+\sqrt{c}d}{\sqrt{-cd^2+bde}}\right)}{4(c^3d^6-3bc^2d^5e+3b^2cd^4e^2-b^3d^3e^3)\sqrt{-cd^2+bde}} - \frac{24(\sqrt{c}x-\sqrt{cx^2+bx})^3c^2d^2e^3-24(\sqrt{c}x-\sqrt{cx^2+bx})^3bcde^4+7(\sqrt{c}x-\sqrt{cx^2+bx})^3b^2e^5+56(\sqrt{c}x\cdots}{\cdots}$$

input
```
integrate(1/(e*x+d)^3/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
-2*((2*c^4*d^6 - 3*b*c^3*d^5*e + 3*b^2*c^2*d^4*e^2 - b^3*c*d^3*e^3)*x/(b^2
*c^3*d^9 - 3*b^3*c^2*d^8*e + 3*b^4*c*d^7*e^2 - b^5*d^6*e^3) + (b*c^3*d^6 -
 3*b^2*c^2*d^5*e + 3*b^3*c*d^4*e^2 - b^4*d^3*e^3)/(b^2*c^3*d^9 - 3*b^3*c^2
*d^8*e + 3*b^4*c*d^7*e^2 - b^5*d^6*e^3))/sqrt(c*x^2 + b*x) + 3/4*(16*c^2*d
^2*e^2 - 16*b*c*d*e^3 + 5*b^2*e^4)*arctan(-((sqrt(c)*x - sqrt(c*x^2 + b*x)
)*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e))/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c
*d^4*e^2 - b^3*d^3*e^3)*sqrt(-c*d^2 + b*d*e)) - 1/4*(24*(sqrt(c)*x - sqrt(
c*x^2 + b*x))^3*c^2*d^2*e^3 - 24*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b*c*d*e
^4 + 7*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*b^2*e^5 + 56*(sqrt(c)*x - sqrt(c*
x^2 + b*x))^2*c^(5/2)*d^3*e^2 - 48*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b*c^(
3/2)*d^2*e^3 + 13*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*b^2*sqrt(c)*d*e^4 + 56
*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b*c^2*d^3*e^2 - 44*(sqrt(c)*x - sqrt(c*x^
2 + b*x))*b^2*c*d^2*e^3 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x))*b^3*d*e^4 + 14
*b^2*c^(3/2)*d^3*e^2 - 7*b^3*sqrt(c)*d^2*e^3)/((c^3*d^6 - 3*b*c^2*d^5*e +
3*b^2*c*d^4*e^2 - b^3*d^3*e^3)*((sqrt(c)*x - sqrt(c*x^2 + b*x))^2*e + 2*(s
qrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c)*d + b*d)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^3\left(bx+cx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(c\,x^2+b\,x\right)^{3/2}\,\left(d+e\,x\right)^3}\,dx$$

input
```
int(1/((b*x + c*x^2)^(3/2)*(d + e*x)^3),x)
```

output
```
int(1/((b*x + c*x^2)^(3/2)*(d + e*x)^3), x)
```

Reduce [B] (verification not implemented)

Time = 101.65 (sec) , antiderivative size = 2708, normalized size of antiderivative = 9.37

$$\int \frac{1}{(d+ex)^3\left(bx+cx^2\right)^{3/2}}\,dx=\text{Too large to display}$$

input
```
int(1/(e*x+d)^3/(c*x^2+b*x)^(3/2),x)
```

output

```
(30*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*
sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**5*d**2*e**5
*x + 60*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt
(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**5*d*e**
6*x**2 + 30*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) -
sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**5*e
**7*x**3 - 156*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d)
 - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**
4*c*d**3*e**4*x - 312*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e
 - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c
)))*b**4*c*d**2*e**5*x**2 - 156*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan
((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt
(d)*sqrt(c)))*b**4*c*d*e**6*x**3 + 288*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*
d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c)
)/(sqrt(d)*sqrt(c)))*b**3*c**2*d**4*e**3*x + 576*sqrt(d)*sqrt(b + c*x)*sqr
t(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(
e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**3*c**2*d**3*e**4*x**2 + 288*sqrt(d)*sqrt
(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*sqrt(b + c*x) -
sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**3*c**2*d**2*e**5*x**3 - 192
*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(e)*...
```

3.171 $\int \frac{(d+ex)^4}{(bx+cx^2)^{5/2}}\,dx$

Optimal result 1370
Mathematica [A] (verified) 1371
Rubi [A] (verified) 1371
Maple [A] (verified) 1374
Fricas [A] (verification not implemented) 1376
Sympy [F] 1377
Maxima [B] (verification not implemented) 1377
Giac [A] (verification not implemented) 1378
Mupad [F(-1)] 1379
Reduce [B] (verification not implemented) 1379

Optimal result

Integrand size = 21, antiderivative size = 234

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{5/2}}\,dx = -\frac{2d^4}{3b(bx+cx^2)^{3/2}} + \frac{2(2c^4d^4-4bc^3d^3e+6b^2c^2d^2e^2-4b^3cde^3+b^4e^4)\,x^2}{3b^3c^2(bx+cx^2)^{3/2}} + \frac{4d^3(cd-2be)}{b^3\sqrt{bx+cx^2}} + \frac{8(4c^4d^4-8bc^3d^3e+3b^2c^2d^2e^2+b^3cde^3-b^4e^4)\,x}{3b^4c^2\sqrt{bx+cx^2}} + \frac{2e^4\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{5/2}}$$

output
```
-2/3*d^4/b/(c*x^2+b*x)^(3/2)+2/3*(b^4*e^4-4*b^3*c*d*e^3+6*b^2*c^2*d^2*e^2-
4*b*c^3*d^3*e+2*c^4*d^4)*x^2/b^3/c^2/(c*x^2+b*x)^(3/2)+4*d^3*(-2*b*e+c*d)/
b^3/(c*x^2+b*x)^(1/2)+8/3*(-b^4*e^4+b^3*c*d*e^3+3*b^2*c^2*d^2*e^2-8*b*c^3*
d^3*e+4*c^4*d^4)*x/b^4/c^2/(c*x^2+b*x)^(1/2)+2*e^4*arctanh(c^(1/2)*x/(c*x^
2+b*x)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{5/2}}\,dx =$$
$$-\frac{2\left(\sqrt{c}(3b^5e^4x^2-16c^5d^4x^3+4b^4ce^4x^3+8bc^4d^3x^2(-3d+4ex)-6b^2c^3d^2x(d^2-8dex+2e^2x^2)+b^3c^2d(d^3\right.}{3b^4c^{5/2}(x(b+cx))^{3/2}}$$

input
```
Integrate[(d + e*x)^4/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(Sqrt[c]*(3*b^5*e^4*x^2 - 16*c^5*d^4*x^3 + 4*b^4*c*e^4*x^3 + 8*b*c^4*d
^3*x^2*(-3*d + 4*e*x) - 6*b^2*c^3*d^2*x*(d^2 - 8*d*e*x + 2*e^2*x^2) + b^3*
c^2*d*(d^3 + 12*d^2*e*x - 18*d*e^2*x^2 - 4*e^3*x^3)) + 3*b^4*e^4*x^(3/2)*(
b + c*x)^(3/2)*Log[-(Sqrt[c]*Sqrt[x]) + Sqrt[b + c*x]]))/(3*b^4*c^(5/2)*(x
*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1164, 1233, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1164}$$

$$-\frac{2\int \frac{(d+ex)^2(d(4cd-5be)-e(2cd-be)x)}{(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(d+ex)^3(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1233}$$

$$-\frac{2\left(\frac{2\int \frac{e\left(bd\left(8c^2d^2-12bced+b^2e^2\right)+(2cd-be)\left(8c^2d^2-8bced-3b^2e^2\right)x\right)}{2\sqrt{cx^2+bx}}dx}{b^2c}-\frac{2(d+ex)\left(x(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)+bcd^2(4cd-5be)\right)}{b^2c\sqrt{bx+cx^2}}\right)}{3b^2}-\frac{2(d+ex)^3(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{2\left(\frac{e\int \frac{bd\left(8c^2d^2-12bced+b^2e^2\right)+(2cd-be)\left(8c^2d^2-8bced-3b^2e^2\right)x}{\sqrt{cx^2+bx}}dx}{b^2c}-\frac{2(d+ex)\left(x(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)+bcd^2(4cd-5be)\right)}{b^2c\sqrt{bx+cx^2}}\right)}{3b^2}-\frac{2(d+ex)^3(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 1160$$

$$-\frac{2\left(\frac{e\left(\frac{\sqrt{bx+cx^2}(2cd-be)\left(-3b^2e^2-8bcde+8c^2d^2\right)}{c}-\frac{3b^4e^3\int \frac{1}{\sqrt{cx^2+bx}}dx}{2c}\right)}{b^2c}-\frac{2(d+ex)\left(x(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)+bcd^2(4cd-5be)\right)}{b^2c\sqrt{bx+cx^2}}\right)}{3b^2}-\frac{2(d+ex)^3(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 1091$$

$$-\frac{2\left(\frac{e\left(\frac{\sqrt{bx+cx^2}(2cd-be)\left(-3b^2e^2-8bcde+8c^2d^2\right)}{c}-\frac{3b^4e^3\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{c}\right)}{b^2c}-\frac{2(d+ex)\left(x(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)+bcd^2(4cd-5be)\right)}{b^2c\sqrt{bx+cx^2}}\right)}{3b^2}-\frac{2(d+ex)^3(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$-\frac{2\left(\frac{e\left(\frac{\sqrt{bx+cx^2}(2cd-be)(-3b^2e^2-8bcde+8c^2d^2)}{c}-\frac{3b^4e^3\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}\right)}{b^2c}-\frac{2(d+ex)(x(2cd-be)(-b^2e^2-4bcde+4c^2d^2)+bcd^2(4cd-5b\cdots}{b^2c\sqrt{bx+cx^2}}\right.}{3b^2}$$

$$\frac{2(d+ex)^3(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

input
```
Int[(d + e*x)^4/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^3*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) - (2*
((-2*(d + e*x)*(b*c*d^2*(4*c*d - 5*b*e) + (2*c*d - b*e)*(4*c^2*d^2 - 4*b*c
*d*e - b^2*e^2)*x))/(b^2*c*Sqrt[b*x + c*x^2]) + (e*(((2*c*d - b*e)*(8*c^2*
d^2 - 8*b*c*d*e - 3*b^2*e^2)*Sqrt[b*x + c*x^2])/c - (3*b^4*e^3*ArcTanh[(Sq
rt[c]*x)/Sqrt[b*x + c*x^2]])/c^(3/2)))/(b^2*c)))/(3*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{-\frac{2b^3d\left(-4e^3x^3-18d\,e^2x^2+12d^2ex+d^3\right)c^{\frac{5}{2}}}{3}+2x\left(2b^2d^2\left(2e^2x^2-8dex+d^2\right)c^{\frac{7}{2}}+8x\left(-\frac{4ex}{3}+d\right)b\,d^3c^{\frac{9}{2}}+\frac{16c^{\frac{11}{2}}d^4x^2}{3}+e^4\left(-\frac{4c^{\frac{3}{2}}x}{3}\right.\right.}{c^{\frac{5}{2}}x(cx+b)\sqrt{x(cx+b)}\,b^4}$
risch	$-\frac{2d^3(cx+b)(12bex-8cdx+bd)}{3b^4x\sqrt{x(cx+b)}}+\frac{\frac{e^4b^3\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{c^{\frac{5}{2}}}-\frac{4\left(b^4e^4-2d\,e^3b^3c+2d^3eb\,c^3-d^4c^4\right)\sqrt{c\left(\frac{b}{c}+x\right)^2-\left(\frac{b}{c}+x\right)b}}{c^3b\left(\frac{b}{c}+x\right)}+\frac{b(}{}}{}$
default	$d^4\left(-\frac{2(2cx+b)}{3b^2(c\,x^2+bx)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{c\,x^2+bx}}\right)+e^4\left(-\frac{x^3}{3c(c\,x^2+bx)^{\frac{3}{2}}}-\frac{b\left(-\frac{x^2}{c\left(c\,x^2+bx\right)^{\frac{3}{2}}}+\frac{b\left(-\frac{x}{2c\left(c\,x^2+bx\right)^{\frac{3}{2}}}-\frac{b\left(-\frac{}{3c}\right.}{}\right.}{}\right.}{}\right.$

input
```
int((e*x+d)^4/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/c^(5/2)/(x*(c*x+b))^(1/2)*(-1/3*b^3*d*(-4*e^3*x^3-18*d*e^2*x^2+12*d^2*e*
x+d^3)*c^(5/2)+x*(2*b^2*d^2*(2*e^2*x^2-8*d*e*x+d^2)*c^(7/2)+8*x*(-4/3*e*x+
d)*b*d^3*c^(9/2)+16/3*c^(11/2)*d^4*x^2+e^4*(-4/3*c^(3/2)*x^2-b*x*c^(1/2)+(
x*(c*x+b))^(1/2)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))*(c*x+b))*b^4))/x/(c*
x+b)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 526, normalized size of antiderivative = 2.25

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{5/2}}\,dx = \left[\frac{3\,(b^4c^2e^4x^4 + 2\,b^5ce^4x^3 + b^6e^4x^2)\sqrt{c}\log\left(2\,cx + b + 2\,\sqrt{cx^2+bx}\sqrt{c}\right) - 2\,(b^3c^3d^4 - \cdots}{\cdots}\right.$$

$$-\frac{2\left(3\,(b^4c^2e^4x^4 + 2\,b^5ce^4x^3 + b^6e^4x^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right) + (b^3c^3d^4 - 4\,(4\,c^6d^4 - 8\,bc^5d^3e + 3\,b^2c^4d^2 \cdots\right.}{3\,(b^4c^5x^4 + \cdots}$$

input
```
integrate((e*x+d)^4/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(b^4*c^2*e^4*x^4 + 2*b^5*c*e^4*x^3 + b^6*e^4*x^2)*sqrt(c)*log(2*c*
x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(b^3*c^3*d^4 - 4*(4*c^6*d^4 - 8*b
*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 + b^3*c^3*d*e^3 - b^4*c^2*e^4)*x^3 - 3*(8*b
*c^5*d^4 - 16*b^2*c^4*d^3*e + 6*b^3*c^3*d^2*e^2 - b^5*c*e^4)*x^2 - 6*(b^2*
c^4*d^4 - 2*b^3*c^3*d^3*e)*x)*sqrt(c*x^2 + b*x))/(b^4*c^5*x^4 + 2*b^5*c^4*
x^3 + b^6*c^3*x^2), -2/3*(3*(b^4*c^2*e^4*x^4 + 2*b^5*c*e^4*x^3 + b^6*e^4*x
^2)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (b^3*c^3*d^4 -
 4*(4*c^6*d^4 - 8*b*c^5*d^3*e + 3*b^2*c^4*d^2*e^2 + b^3*c^3*d*e^3 - b^4*c^
2*e^4)*x^3 - 3*(8*b*c^5*d^4 - 16*b^2*c^4*d^3*e + 6*b^3*c^3*d^2*e^2 - b^5*c
*e^4)*x^2 - 6*(b^2*c^4*d^4 - 2*b^3*c^3*d^3*e)*x)*sqrt(c*x^2 + b*x))/(b^4*c
^5*x^4 + 2*b^5*c^4*x^3 + b^6*c^3*x^2)]
```

Sympy [F]

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(d+ex)^4}{(x(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**4/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral((d + e*x)**4/(x*(b + c*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(214) = 428.

Time = 0.04 (sec) , antiderivative size = 458, normalized size of antiderivative = 1.96

$$\begin{aligned}\int \frac{(d+ex)^4}{(bx+cx^2)^{5/2}}\,dx =& -\frac{1}{3}\,e^4x\left(\frac{3\,x^2}{(cx^2+bx)^{\frac{3}{2}}c}+\frac{bx}{(cx^2+bx)^{\frac{3}{2}}c^2}-\frac{2\,x}{\sqrt{cx^2+bx}bc}-\frac{1}{\sqrt{cx^2+bx}c^2}\right)\\&-\frac{4\,de^3x^2}{(cx^2+bx)^{\frac{3}{2}}c}-\frac{4\,cd^4x}{3\,(cx^2+bx)^{\frac{3}{2}}b^2}+\frac{32\,c^2d^4x}{3\,\sqrt{cx^2+bx}b^4}\\&+\frac{8\,d^3ex}{3\,(cx^2+bx)^{\frac{3}{2}}b}-\frac{64\,cd^3ex}{3\,\sqrt{cx^2+bx}b^3}+\frac{8\,d^2e^2x}{\sqrt{cx^2+bx}b^2}-\frac{4\,d^2e^2x}{(cx^2+bx)^{\frac{3}{2}}c}\\&-\frac{4\,bde^3x}{3\,(cx^2+bx)^{\frac{3}{2}}c^2}+\frac{8\,de^3x}{3\,\sqrt{cx^2+bx}bc}-\frac{4\,e^4x}{3\,\sqrt{cx^2+bx}c^2}\\&+\frac{e^4\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{c^{\frac{5}{2}}}-\frac{2\,d^4}{3\,(cx^2+bx)^{\frac{3}{2}}b}+\frac{16\,cd^4}{3\,\sqrt{cx^2+bx}b^3}\\&-\frac{32\,d^3e}{3\,\sqrt{cx^2+bx}b^2}+\frac{4\,d^2e^2}{\sqrt{cx^2+bx}bc}+\frac{4\,de^3}{3\,\sqrt{cx^2+bx}c^2}-\frac{2\,\sqrt{cx^2+bx}e^4}{3\,bc^2}\end{aligned}$$

input
```
integrate((e*x+d)^4/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*e^4*x*(3*x^2/((c*x^2 + b*x)^(3/2)*c) + b*x/((c*x^2 + b*x)^(3/2)*c^2)
- 2*x/(sqrt(c*x^2 + b*x)*b*c) - 1/(sqrt(c*x^2 + b*x)*c^2)) - 4*d*e^3*x^2/(
(c*x^2 + b*x)^(3/2)*c) - 4/3*c*d^4*x/((c*x^2 + b*x)^(3/2)*b^2) + 32/3*c^2*
d^4*x/(sqrt(c*x^2 + b*x)*b^4) + 8/3*d^3*e*x/((c*x^2 + b*x)^(3/2)*b) - 64/3
*c*d^3*e*x/(sqrt(c*x^2 + b*x)*b^3) + 8*d^2*e^2*x/(sqrt(c*x^2 + b*x)*b^2) -
 4*d^2*e^2*x/((c*x^2 + b*x)^(3/2)*c) - 4/3*b*d*e^3*x/((c*x^2 + b*x)^(3/2)*
c^2) + 8/3*d*e^3*x/(sqrt(c*x^2 + b*x)*b*c) - 4/3*e^4*x/(sqrt(c*x^2 + b*x)*
c^2) + e^4*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(5/2) - 2/3*d^4/
((c*x^2 + b*x)^(3/2)*b) + 16/3*c*d^4/(sqrt(c*x^2 + b*x)*b^3) - 32/3*d^3*e/
(sqrt(c*x^2 + b*x)*b^2) + 4*d^2*e^2/(sqrt(c*x^2 + b*x)*b*c) + 4/3*d*e^3/(s
qrt(c*x^2 + b*x)*c^2) - 2/3*sqrt(c*x^2 + b*x)*e^4/(b*c^2)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{5/2}}\,dx = -\frac{e^4\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{c^{\frac{5}{2}}} - \frac{2\left(\frac{d^4}{b}-\left(x\left(\frac{4\left(4c^5d^4-8bc^4d^3e+3b^2c^3d^2e^2+b^3c^2de^3-b^4ce^4\right)x}{b^4c^2}+\frac{3\left(8bc^4d^4-16b^2c^3d^3e+6b^3c^2d^2e^2-b^5e^4\right)}{b^4c^2}\right)+\frac{6\left(b^2c^3d^4-2b^3c^2d^3e\right)}{b^4c^2}\right.\right.}{3\left(cx^2+bx\right)^{\frac{3}{2}}}$$

input
```
integrate((e*x+d)^4/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
-e^4*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(5/2) - 2/3
*(d^4/b - (x*(4*(4*c^5*d^4 - 8*b*c^4*d^3*e + 3*b^2*c^3*d^2*e^2 + b^3*c^2*d
*e^3 - b^4*c*e^4)*x/(b^4*c^2) + 3*(8*b*c^4*d^4 - 16*b^2*c^3*d^3*e + 6*b^3*
c^2*d^2*e^2 - b^5*e^4)/(b^4*c^2)) + 6*(b^2*c^3*d^4 - 2*b^3*c^2*d^3*e)/(b^4
*c^2))*x)/(c*x^2 + b*x)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(d+e\,x)^4}{(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((d + e*x)^4/(b*x + c*x^2)^(5/2),x)
```

output
```
int((d + e*x)^4/(b*x + c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.15

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{5/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^5e^4x^2+2\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^4c\,e^4x^3-8}{}$$

input
```
int((e*x+d)^4/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*(3*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))
*b**5*e**4*x**2 + 3*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqr
t(c))/sqrt(b))*b**4*c*e**4*x**3 - 4*sqrt(c)*sqrt(b + c*x)*b**5*e**4*x**2 +
 20*sqrt(c)*sqrt(b + c*x)*b**4*c*d*e**3*x**2 - 4*sqrt(c)*sqrt(b + c*x)*b**
4*c*e**4*x**3 - 36*sqrt(c)*sqrt(b + c*x)*b**3*c**2*d**2*e**2*x**2 + 20*sqr
t(c)*sqrt(b + c*x)*b**3*c**2*d*e**3*x**3 + 32*sqrt(c)*sqrt(b + c*x)*b**2*c
**3*d**3*e*x**2 - 36*sqrt(c)*sqrt(b + c*x)*b**2*c**3*d**2*e**2*x**3 - 16*s
qrt(c)*sqrt(b + c*x)*b*c**4*d**4*x**2 + 32*sqrt(c)*sqrt(b + c*x)*b*c**4*d*
*3*e*x**3 - 16*sqrt(c)*sqrt(b + c*x)*c**5*d**4*x**3 - 3*sqrt(x)*b**5*c*e**
4*x**2 - 4*sqrt(x)*b**4*c**2*e**4*x**3 - sqrt(x)*b**3*c**3*d**4 - 12*sqrt(
x)*b**3*c**3*d**3*e*x + 18*sqrt(x)*b**3*c**3*d**2*e**2*x**2 + 4*sqrt(x)*b*
*3*c**3*d*e**3*x**3 + 6*sqrt(x)*b**2*c**4*d**4*x - 48*sqrt(x)*b**2*c**4*d*
*3*e*x**2 + 12*sqrt(x)*b**2*c**4*d**2*e**2*x**3 + 24*sqrt(x)*b*c**5*d**4*x
**2 - 32*sqrt(x)*b*c**5*d**3*e*x**3 + 16*sqrt(x)*c**6*d**4*x**3))/(3*sqrt(
b + c*x)*b**4*c**3*x**2*(b + c*x))
```

3.172 $\int \frac{(d+ex)^3}{\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal result 1380
Mathematica [A] (verified) 1380
Rubi [A] (verified) 1381
Maple [A] (verified) 1382
Fricas [A] (verification not implemented) 1384
Sympy [F] 1384
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Reduce [B] (verification not implemented) 1386

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{(d+ex)^3}{\left(bx+cx^2\right)^{5/2}}\,dx = \frac{16(cd-be)^3x^2}{3b^3c\left(bx+cx^2\right)^{3/2}} + \frac{4(cd-be)x(d+ex)^2}{b^2\left(bx+cx^2\right)^{3/2}} - \frac{2(d+ex)^3}{3b\left(bx+cx^2\right)^{3/2}} + \frac{16(cd-be)^2(2cd+be)x}{3b^4c\sqrt{bx+cx^2}}$$

output
```
16/3*(-b*e+c*d)^3*x^2/b^3/c/(c*x^2+b*x)^(3/2)+4*(-b*e+c*d)*x*(e*x+d)^2/b^2
/(c*x^2+b*x)^(3/2)-2/3*(e*x+d)^3/b/(c*x^2+b*x)^(3/2)+16/3*(-b*e+c*d)^2*(b*
e+2*c*d)*x/b^4/c/(c*x^2+b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.75

$$\int \frac{(d+ex)^3}{\left(bx+cx^2\right)^{5/2}}\,dx = \frac{2(16c^3d^3x^3 + 24bc^2d^2x^2(d-ex) + 6b^2cdx(d^2-6dex+e^2x^2) + b^3(-d^3-9d^2ex+9e}{3b^4(x(b+cx))^{3/2}}$$

input
```
Integrate[(d + e*x)^3/(b*x + c*x^2)^(5/2),x]
```

output
```
(2*(16*c^3*d^3*x^3 + 24*b*c^2*d^2*x^2*(d - e*x) + 6*b^2*c*d*x*(d^2 - 6*d*e
*x + e^2*x^2) + b^3*(-d^3 - 9*d^2*e*x + 9*d*e^2*x^2 + e^3*x^3)))/(3*b^4*(x
*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1153, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1153}$$

$$-\frac{8d(cd-be)\int \frac{d+ex}{(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(d+ex)^2(x(2cd-be)+bd)}{3b^2(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1158}$$

$$\frac{16d(cd-be)(x(2cd-be)+bd)}{3b^4\sqrt{bx+cx^2}}-\frac{2(d+ex)^2(x(2cd-be)+bd)}{3b^2(bx+cx^2)^{3/2}}$$

input
```
Int[(d + e*x)^3/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^2*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (16
*d*(c*d - b*e)*(b*d + (2*c*d - b*e)*x))/(3*b^4*Sqrt[b*x + c*x^2])
```

Defintions of rubi rules used

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.59

method	result
pseudoelliptic	$-\frac{2\left(\left(e^{2} x^{2}+10 d e x+d^{2}\right) b^{2}-8 d c x(-e x+d) b-8 d^{2} c^{2} x^{2}\right)(2 c d x+b(-e x+d))}{3 \sqrt{x(c x+b)}\, x(c x+b) b^{4}}$
risch	$-\frac{2 d^{2}(c x+b)(9 b e x-8 c d x+b d)}{3 b^{4} x \sqrt{x(c x+b)}}+\frac{2 x(b e x+8 c d x+9 b d)\left(b^{2} e^{2}-2 b c d e+c^{2} d^{2}\right)}{3 \sqrt{x(c x+b)}\,(c x+b) b^{4}}$
gosper	$-\frac{2 x(c x+b)\left(-b^{3} e^{3} x^{3}-6 b^{2} c d\, e^{2} x^{3}+24 b\, c^{2} d^{2} e\, x^{3}-16 d^{3} c^{3} x^{3}-9 b^{3} d\, e^{2} x^{2}+36 b^{2} c\, d^{2} e\, x^{2}-24 b\, c^{2} d^{3} x^{2}+9 b^{3} d^{2} e x-6 b^{2} c\, d^{3} x+b \cdots\right.}{3 b^{4}\left(c\, x^{2}+b x\right)^{\frac{5}{2}}}$
orering	$-\frac{2 x(c x+b)\left(-b^{3} e^{3} x^{3}-6 b^{2} c d\, e^{2} x^{3}+24 b\, c^{2} d^{2} e\, x^{3}-16 d^{3} c^{3} x^{3}-9 b^{3} d\, e^{2} x^{2}+36 b^{2} c\, d^{2} e\, x^{2}-24 b\, c^{2} d^{3} x^{2}+9 b^{3} d^{2} e x-6 b^{2} c\, d^{3} x+b \cdots\right.}{3 b^{4}\left(c\, x^{2}+b x\right)^{\frac{5}{2}}}$
trager	$-\frac{2\left(-b^{3} e^{3} x^{3}-6 b^{2} c d\, e^{2} x^{3}+24 b\, c^{2} d^{2} e\, x^{3}-16 d^{3} c^{3} x^{3}-9 b^{3} d\, e^{2} x^{2}+36 b^{2} c\, d^{2} e\, x^{2}-24 b\, c^{2} d^{3} x^{2}+9 b^{3} d^{2} e x-6 b^{2} c\, d^{3} x+b^{3} d^{3}\right) \sqrt{c \cdots}}{3 b^{4} x^{2}(c x+b)^{2}}$
default	$d^{3}\left(-\frac{2(2 c x+b)}{3 b^{2}\left(c\, x^{2}+b x\right)^{\frac{3}{2}}}+\frac{16 c(2 c x+b)}{3 b^{4} \sqrt{c\, x^{2}+b x}}\right)+e^{3}\left(-\frac{x^{2}}{c\left(c\, x^{2}+b x\right)^{\frac{3}{2}}}+\frac{b\left(-\frac{x}{2 c\left(c\, x^{2}+b x\right)^{\frac{3}{2}}}-\frac{b\left(-\frac{1}{3 c\left(c\, x^{2}+b x\right)^{\frac{3}{2}}}-\frac{b\left(-\cdots\right)}{\cdots}\right)}{\cdots}\right)}{2 c} \cdots\right.$

input

```
int((e*x+d)^3/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
-2/3/(x*(c*x+b))^(1/2)*((e^2*x^2+10*d*e*x+d^2)*b^2-8*d*c*x*(-e*x+d)*b-8*d^
2*c^2*x^2)*(2*c*d*x+b*(-e*x+d))/x/(c*x+b)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{5/2}}\,dx = -\frac{2\,(b^3d^3 - (16\,c^3d^3 - 24\,bc^2d^2e + 6\,b^2cde^2 + b^3e^3)x^3 - 3\,(8\,bc^2d^3 - 12\,b^2cd^2e + 3\,b^3de^2)x^2 - 3\,(2\,b^2cd^3 - 3}{3\,(b^4c^2x^4 + 2\,b^5cx^3 + b^6x^2)}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(b^3*d^3 - (16*c^3*d^3 - 24*b*c^2*d^2*e + 6*b^2*c*d*e^2 + b^3*e^3)*x^
3 - 3*(8*b*c^2*d^3 - 12*b^2*c*d^2*e + 3*b^3*d*e^2)*x^2 - 3*(2*b^2*c*d^3 -
3*b^3*d^2*e)*x)*sqrt(c*x^2 + b*x)/(b^4*c^2*x^4 + 2*b^5*c*x^3 + b^6*x^2)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(d+ex)^3}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral((d + e*x)**3/(x*(b + c*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(126) = 252.

Time = 0.03 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.12

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{5/2}}\,dx = -\frac{e^3x^2}{(cx^2+bx)^{\frac{3}{2}}c} - \frac{4\,cd^3x}{3\,(cx^2+bx)^{\frac{3}{2}}b^2} + \frac{32\,c^2d^3x}{3\,\sqrt{cx^2+bx}b^4} + \frac{2\,d^2ex}{(cx^2+bx)^{\frac{3}{2}}b} - \frac{16\,cd^2ex}{\sqrt{cx^2+bx}b^3} + \frac{4\,de^2x}{\sqrt{cx^2+bx}b^2} - \frac{2\,de^2x}{(cx^2+bx)^{\frac{3}{2}}c} - \frac{be^3x}{3\,(cx^2+bx)^{\frac{3}{2}}c^2} + \frac{2\,e^3x}{3\,\sqrt{cx^2+bx}bc} - \frac{2\,d^3}{3\,(cx^2+bx)^{\frac{3}{2}}b} + \frac{16\,cd^3}{3\,\sqrt{cx^2+bx}b^3} - \frac{8\,d^2e}{\sqrt{cx^2+bx}b^2} + \frac{2\,de^2}{\sqrt{cx^2+bx}bc} + \frac{e^3}{3\,\sqrt{cx^2+bx}c^2}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
-e^3*x^2/((c*x^2 + b*x)^(3/2)*c) - 4/3*c*d^3*x/((c*x^2 + b*x)^(3/2)*b^2) +
 32/3*c^2*d^3*x/(sqrt(c*x^2 + b*x)*b^4) + 2*d^2*e*x/((c*x^2 + b*x)^(3/2)*b
) - 16*c*d^2*e*x/(sqrt(c*x^2 + b*x)*b^3) + 4*d*e^2*x/(sqrt(c*x^2 + b*x)*b^
2) - 2*d*e^2*x/((c*x^2 + b*x)^(3/2)*c) - 1/3*b*e^3*x/((c*x^2 + b*x)^(3/2)*
c^2) + 2/3*e^3*x/(sqrt(c*x^2 + b*x)*b*c) - 2/3*d^3/((c*x^2 + b*x)^(3/2)*b)
 + 16/3*c*d^3/(sqrt(c*x^2 + b*x)*b^3) - 8*d^2*e/(sqrt(c*x^2 + b*x)*b^2) +
2*d*e^2/(sqrt(c*x^2 + b*x)*b*c) + 1/3*e^3/(sqrt(c*x^2 + b*x)*c^2)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{5/2}}\,dx = -\frac{2\left(\frac{d^3}{b} - \left(x\left(\frac{(16\,c^3d^3-24\,bc^2d^2e+6\,b^2cde^2+b^3e^3)x}{b^4} + \frac{3\,(8\,bc^2d^3-12\,b^2cd^2e+3\,b^3de^2)}{b^4}\right) + \frac{3\,(2\,b^2cd^3-3\,b^3d^2e)}{b^4}\right)x\right)}{3\,(cx^2+bx)^{\frac{3}{2}}}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(d^3/b - (x*((16*c^3*d^3 - 24*b*c^2*d^2*e + 6*b^2*c*d*e^2 + b^3*e^3)*
x/b^4 + 3*(8*b*c^2*d^3 - 12*b^2*c*d^2*e + 3*b^3*d*e^2)/b^4) + 3*(2*b^2*c*d
^3 - 3*b^3*d^2*e)/b^4)*x)/(c*x^2 + b*x)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{5/2}}\,dx = \frac{2\,(-b^3\,d^3-9\,b^3\,d^2\,e\,x+9\,b^3\,d\,e^2\,x^2+b^3\,e^3\,x^3+6\,b^2\,c\,d^3\,x-36\,b^2\,c\,d^2\,e\,x^2+6\,b^2\,c\,d\,e^2}{3\,b^4\,(c\,x^2+b\,x)^{3/2}}$$

input
```
int((d + e*x)^3/(b*x + c*x^2)^(5/2),x)
```

output
```
(2*(b^3*e^3*x^3 - b^3*d^3 + 16*c^3*d^3*x^3 + 24*b*c^2*d^3*x^2 + 9*b^3*d*e^
2*x^2 + 6*b^2*c*d^3*x - 9*b^3*d^2*e*x - 36*b^2*c*d^2*e*x^2 - 24*b*c^2*d^2*
e*x^3 + 6*b^2*c*d*e^2*x^3))/(3*b^4*(b*x + c*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.45

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{5/2}}\,dx = \frac{\frac{10\sqrt{c}\,\sqrt{cx+b}\,b^4e^3x^2}{3}-12\sqrt{c}\,\sqrt{cx+b}\,b^3cd\,e^2x^2+\frac{10\sqrt{c}\,\sqrt{cx+b}\,b^3c\,e^3x^3}{3}+16\sqrt{c}\,\sqrt{cx+b}\,b^2c^2d^2}{}$$

input
```
int((e*x+d)^3/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*(5*sqrt(c)*sqrt(b + c*x)*b**4*e**3*x**2 - 18*sqrt(c)*sqrt(b + c*x)*b**3
*c*d*e**2*x**2 + 5*sqrt(c)*sqrt(b + c*x)*b**3*c*e**3*x**3 + 24*sqrt(c)*sqr
t(b + c*x)*b**2*c**2*d**2*e*x**2 - 18*sqrt(c)*sqrt(b + c*x)*b**2*c**2*d*e*
*2*x**3 - 16*sqrt(c)*sqrt(b + c*x)*b*c**3*d**3*x**2 + 24*sqrt(c)*sqrt(b +
c*x)*b*c**3*d**2*e*x**3 - 16*sqrt(c)*sqrt(b + c*x)*c**4*d**3*x**3 - sqrt(x
)*b**3*c**2*d**3 - 9*sqrt(x)*b**3*c**2*d**2*e*x + 9*sqrt(x)*b**3*c**2*d*e*
*2*x**2 + sqrt(x)*b**3*c**2*e**3*x**3 + 6*sqrt(x)*b**2*c**3*d**3*x - 36*sq
rt(x)*b**2*c**3*d**2*e*x**2 + 6*sqrt(x)*b**2*c**3*d*e**2*x**3 + 24*sqrt(x)
*b*c**4*d**3*x**2 - 24*sqrt(x)*b*c**4*d**2*e*x**3 + 16*sqrt(x)*c**5*d**3*x
**3))/(3*sqrt(b + c*x)*b**4*c**2*x**2*(b + c*x))
```

3.173 $\int \frac{(d+ex)^2}{(bx+cx^2)^{5/2}} \, dx$

Optimal result 1387
Mathematica [A] (verified) 1387
Rubi [A] (verified) 1388
Maple [A] (verified) 1389
Fricas [A] (verification not implemented) 1390
Sympy [F] 1390
Maxima [A] (verification not implemented) 1391
Giac [A] (verification not implemented) 1391
Mupad [B] (verification not implemented) 1392
Reduce [B] (verification not implemented) 1392

Optimal result

Integrand size = 21, antiderivative size = 169

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{5/2}} \, dx = \frac{8(cd-be)^2(2cd-be)x^2}{3b^3cd\,(bx+cx^2)^{3/2}} + \frac{2(2cd-be)x(d+ex)^2}{b^2d\,(bx+cx^2)^{3/2}} - \frac{2(d+ex)^3}{3bd\,(bx+cx^2)^{3/2}} + \frac{8(cd-be)(2cd-be)(2cd+be)x}{3b^4cd\sqrt{bx+cx^2}}$$

output
```
8/3*(-b*e+c*d)^2*(-b*e+2*c*d)*x^2/b^3/c/d/(c*x^2+b*x)^(3/2)+2*(-b*e+2*c*d)
*x*(e*x+d)^2/b^2/d/(c*x^2+b*x)^(3/2)-2/3*(e*x+d)^3/b/d/(c*x^2+b*x)^(3/2)+8
/3*(-b*e+c*d)*(-b*e+2*c*d)*(b*e+2*c*d)*x/b^4/c/d/(c*x^2+b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.56

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{5/2}} \, dx = \frac{32c^3d^2x^3 + 16bc^2dx^2(3d-2ex) - 2b^3(d^2+6dex-3e^2x^2) + 4b^2cx(3d^2-12dex+e^{2}}{3b^4(x(b+cx))^{3/2}}$$

input
```
Integrate[(d + e*x)^2/(b*x + c*x^2)^(5/2),x]
```

output
```
(32*c^3*d^2*x^3 + 16*b*c^2*d*x^2*(3*d - 2*e*x) - 2*b^3*(d^2 + 6*d*e*x - 3*
e^2*x^2) + 4*b^2*c*x*(3*d^2 - 12*d*e*x + e^2*x^2))/(3*b^4*(x*(b + c*x))^(3
/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1156, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1156}$$

$$-\frac{4(2cd-be)\int \frac{d+ex}{(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(b+2cx)(d+ex)^2}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1158}$$

$$\frac{8(2cd-be)(x(2cd-be)+bd)}{3b^4\sqrt{bx+cx^2}}-\frac{2(b+2cx)(d+ex)^2}{3b^2\left(bx+cx^2\right)^{3/2}}$$

input
```
Int[(d + e*x)^2/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(b + 2*c*x)*(d + e*x)^2)/(3*b^2*(b*x + c*x^2)^(3/2)) + (8*(2*c*d - b*e
)*(b*d + (2*c*d - b*e)*x))/(3*b^4*Sqrt[b*x + c*x^2])
```

Defintions of rubi rules used

rule 1156
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] + Simp[m*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c)))   Int[
(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.56

method	result
risch	$-\frac{2d(cx+b)(6bex-8cdx+bd)}{3b^4x\sqrt{x(cx+b)}}+\frac{2x(2bcex-8c^2dx+3eb^2-9dbc)(be-cd)}{3\sqrt{x(cx+b)}\,(cx+b)b^4}$
pseudoelliptic	$-\frac{2\left(\left(-3e^2x^2+6dex+d^2\right)b^3-6\left(\frac{1}{3}e^2x^2-4dex+d^2\right)xcb^2-24x^2bd\left(-\frac{2ex}{3}+d\right)c^2-16c^3d^2x^3\right)}{3\sqrt{x(cx+b)}\,x(cx+b)b^4}$
gosper	$-\frac{2x(cx+b)\left(-2b^2ce^2x^3+16bc^2dex^3-16c^3d^2x^3-3b^3e^2x^2+24b^2cdex^2-24bc^2d^2x^2+6b^3dex-6b^2cd^2x+b^3d^2\right)}{3b^4(cx^2+bx)^{\frac{5}{2}}}$
orering	$-\frac{2x(cx+b)\left(-2b^2ce^2x^3+16bc^2dex^3-16c^3d^2x^3-3b^3e^2x^2+24b^2cdex^2-24bc^2d^2x^2+6b^3dex-6b^2cd^2x+b^3d^2\right)}{3b^4(cx^2+bx)^{\frac{5}{2}}}$
trager	$-\frac{2\left(-2b^2ce^2x^3+16bc^2dex^3-16c^3d^2x^3-3b^3e^2x^2+24b^2cdex^2-24bc^2d^2x^2+6b^3dex-6b^2cd^2x+b^3d^2\right)\sqrt{cx^2+bx}}{3b^4x^2(cx+b)^2}$
default	$d^2\left(-\frac{2(2cx+b)}{3b^2(cx^2+bx)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{cx^2+bx}}\right)+e^2\left(-\frac{x}{2c(cx^2+bx)^{\frac{3}{2}}}-\frac{b\left(-\frac{1}{3c\left(cx^2+bx\right)^{\frac{3}{2}}}-\frac{b\left(-\frac{2(2cx+b)}{3b^2\left(cx^2+bx\right)^{\frac{3}{2}}}+\frac{16c}{3b^4\sqrt{}}\cdots\right.}{2c}\cdots\right.}{4c}\cdots\right.$

input
```
int((e*x+d)^2/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*d*(c*x+b)*(6*b*e*x-8*c*d*x+b*d)/b^4/x/(x*(c*x+b))^(1/2)+2/3*x*(2*b*c*
e*x-8*c^2*d*x+3*b^2*e-9*b*c*d)*(b*e-c*d)/(x*(c*x+b))^(1/2)/(c*x+b)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{5/2}}\,dx = -\frac{2\left(b^3d^2-2\left(8c^3d^2-8bc^2de+b^2ce^2\right)x^3-3\left(8bc^2d^2-8b^2cde+b^3e^2\right)x^2-6\left(b^2cd^2-b^3de\right)x\right)\sqrt{cx^2+bx}}{3\left(b^4c^2x^4+2b^5cx^3+b^6x^2\right)}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(b^3*d^2 - 2*(8*c^3*d^2 - 8*b*c^2*d*e + b^2*c*e^2)*x^3 - 3*(8*b*c^2*d
^2 - 8*b^2*c*d*e + b^3*e^2)*x^2 - 6*(b^2*c*d^2 - b^3*d*e)*x)*sqrt(c*x^2 +
b*x)/(b^4*c^2*x^4 + 2*b^5*c*x^3 + b^6*x^2)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(d+ex)^2}{(x(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral((d + e*x)**2/(x*(b + c*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.20

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{5/2}}\,dx = -\frac{4\,cd^2x}{3\,(cx^2+bx)^{\frac{3}{2}}b^2} + \frac{32\,c^2d^2x}{3\,\sqrt{cx^2+bx}b^4} + \frac{4\,dex}{3\,(cx^2+bx)^{\frac{3}{2}}b} - \frac{32\,cdex}{3\,\sqrt{cx^2+bx}b^3} + \frac{4\,e^2x}{3\,\sqrt{cx^2+bx}b^2} - \frac{2\,e^2x}{3\,(cx^2+bx)^{\frac{3}{2}}c} - \frac{2\,d^2}{3\,(cx^2+bx)^{\frac{3}{2}}b} + \frac{16\,cd^2}{3\,\sqrt{cx^2+bx}b^3} - \frac{16\,de}{3\,\sqrt{cx^2+bx}b^2} + \frac{2\,e^2}{3\,\sqrt{cx^2+bx}bc}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
-4/3*c*d^2*x/((c*x^2 + b*x)^(3/2)*b^2) + 32/3*c^2*d^2*x/(sqrt(c*x^2 + b*x)
*b^4) + 4/3*d*e*x/((c*x^2 + b*x)^(3/2)*b) - 32/3*c*d*e*x/(sqrt(c*x^2 + b*x
)*b^3) + 4/3*e^2*x/(sqrt(c*x^2 + b*x)*b^2) - 2/3*e^2*x/((c*x^2 + b*x)^(3/2
)*c) - 2/3*d^2/((c*x^2 + b*x)^(3/2)*b) + 16/3*c*d^2/(sqrt(c*x^2 + b*x)*b^3
) - 16/3*d*e/(sqrt(c*x^2 + b*x)*b^2) + 2/3*e^2/(sqrt(c*x^2 + b*x)*b*c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.65

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{5/2}}\,dx = \frac{2\left(\left(x\left(\frac{2\,(8\,c^3d^2-8\,bc^2de+b^2ce^2)x}{b^4}+\frac{3\,(8\,bc^2d^2-8\,b^2cde+b^3e^2)}{b^4}\right)+\frac{6\,(b^2cd^2-b^3de)}{b^4}\right)x-\frac{d^2}{b}\right)}{3\,(cx^2+bx)^{\frac{3}{2}}}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
2/3*((x*(2*(8*c^3*d^2 - 8*b*c^2*d*e + b^2*c*e^2)*x/b^4 + 3*(8*b*c^2*d^2 -
8*b^2*c*d*e + b^3*e^2)/b^4) + 6*(b^2*c*d^2 - b^3*d*e)/b^4)*x - d^2/b)/(c*x
^2 + b*x)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.66

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{5/2}}\,dx = \frac{2\left(-b^3\,d^2-6\,b^3\,d\,e\,x+3\,b^3\,e^2\,x^2+6\,b^2\,c\,d^2\,x-24\,b^2\,c\,d\,e\,x^2+2\,b^2\,c\,e^2\,x^3+24\,b\,c^2\,d^2\right.}{3\,b^4\,\left(c\,x^2+b\,x\right)^{3/2}}$$

input
```
int((d + e*x)^2/(b*x + c*x^2)^(5/2),x)
```

output
```
(2*(3*b^3*e^2*x^2 - b^3*d^2 + 16*c^3*d^2*x^3 + 24*b*c^2*d^2*x^2 + 2*b^2*c*
e^2*x^3 - 6*b^3*d*e*x + 6*b^2*c*d^2*x - 24*b^2*c*d*e*x^2 - 16*b*c^2*d*e*x^
3))/(3*b^4*(b*x + c*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.56

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{5/2}}\,dx = \frac{-4\sqrt{c}\,\sqrt{cx+b}\,b^3e^2x^2+\frac{32\sqrt{c}\,\sqrt{cx+b}\,b^2cde\,x^2}{3}-4\sqrt{c}\,\sqrt{cx+b}\,b^2c\,e^2x^3-\frac{32\sqrt{c}\,\sqrt{cx+b}\,b\,c^2d^2x^2}{3}}{}$$

input
```
int((e*x+d)^2/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*( - 6*sqrt(c)*sqrt(b + c*x)*b**3*e**2*x**2 + 16*sqrt(c)*sqrt(b + c*x)*b
**2*c*d*e*x**2 - 6*sqrt(c)*sqrt(b + c*x)*b**2*c*e**2*x**3 - 16*sqrt(c)*sqr
t(b + c*x)*b*c**2*d**2*x**2 + 16*sqrt(c)*sqrt(b + c*x)*b*c**2*d*e*x**3 - 1
6*sqrt(c)*sqrt(b + c*x)*c**3*d**2*x**3 - sqrt(x)*b**3*c*d**2 - 6*sqrt(x)*b
**3*c*d*e*x + 3*sqrt(x)*b**3*c*e**2*x**2 + 6*sqrt(x)*b**2*c**2*d**2*x - 24
*sqrt(x)*b**2*c**2*d*e*x**2 + 2*sqrt(x)*b**2*c**2*e**2*x**3 + 24*sqrt(x)*b
*c**3*d**2*x**2 - 16*sqrt(x)*b*c**3*d*e*x**3 + 16*sqrt(x)*c**4*d**2*x**3))
/(3*sqrt(b + c*x)*b**4*c*x**2*(b + c*x))
```

3.174 $\int \frac{d+ex}{(bx+cx^2)^{5/2}}\,dx$

Optimal result 1393
Mathematica [A] (verified) 1393
Rubi [A] (verified) 1394
Maple [A] (verified) 1395
Fricas [A] (verification not implemented) 1396
Sympy [F] 1396
Maxima [A] (verification not implemented) 1396
Giac [A] (verification not implemented) 1397
Mupad [B] (verification not implemented) 1397
Reduce [B] (verification not implemented) 1398

Optimal result

Integrand size = 19, antiderivative size = 113

$$\int \frac{d+ex}{(bx+cx^2)^{5/2}}\,dx = -\frac{2d}{3b\,(bx+cx^2)^{3/2}} - \frac{2(2cd-be)x}{3b^2\,(bx+cx^2)^{3/2}} - \frac{8(2cd-be)}{3b^3\sqrt{bx+cx^2}} + \frac{16(2cd-be)\sqrt{bx+cx^2}}{3b^4x}$$

output
```
-2/3*d/b/(c*x^2+b*x)^(3/2)-2/3*(-b*e+2*c*d)*x/b^2/(c*x^2+b*x)^(3/2)-8/3*(-
b*e+2*c*d)/b^3/(c*x^2+b*x)^(1/2)+16/3*(-b*e+2*c*d)*(c*x^2+b*x)^(1/2)/b^4/x
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.59

$$\int \frac{d+ex}{(bx+cx^2)^{5/2}}\,dx = -\frac{2(-16c^3dx^3-6b^2cx(d-2ex)+8bc^2x^2(-3d+ex)+b^3(d+3ex))}{3b^4(x(b+cx))^{3/2}}$$

input
```
Integrate[(d + e*x)/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(-16*c^3*d*x^3 - 6*b^2*c*x*(d - 2*e*x) + 8*b*c^2*x^2*(-3*d + e*x) + b^
3*(d + 3*e*x)))/(3*b^4*(x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.63, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1159, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1159}$$

$$-\frac{4(2cd-be)\int \frac{1}{(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{8(b+2cx)(2cd-be)}{3b^4\sqrt{bx+cx^2}}-\frac{2(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

input
```
Int[(d + e*x)/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (8*(2*c*d - b*e
)*(b + 2*c*x))/(3*b^4*Sqrt[b*x + c*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{2\left(\left(3 e x+d\right) b^{3}-6 c x \left(-2 e x+d\right) b^{2}-24 x^{2} \left(-\frac{e x}{3}+d\right) b \,c^{2}-16 c^{3} d \,x^{3}\right)}{3 \sqrt{x \left(c x+b\right)}\, x \left(c x+b\right) b^{4}}$	73
gosper	$-\frac{2 x \left(c x+b\right) \left(8 b \,c^{2} x^{3} e-16 c^{3} d \,x^{3}+12 b^{2} c e \,x^{2}-24 b \,c^{2} d \,x^{2}+3 b^{3} e x-6 b^{2} c x d+b^{3} d\right)}{3 b^{4} \left(c \,x^{2}+b x\right)^{\frac{5}{2}}}$	83
orering	$-\frac{2 x \left(c x+b\right) \left(8 b \,c^{2} x^{3} e-16 c^{3} d \,x^{3}+12 b^{2} c e \,x^{2}-24 b \,c^{2} d \,x^{2}+3 b^{3} e x-6 b^{2} c x d+b^{3} d\right)}{3 b^{4} \left(c \,x^{2}+b x\right)^{\frac{5}{2}}}$	83
risch	$-\frac{2 \left(c x+b\right) \left(3 b e x-8 c d x+b d\right)}{3 b^{4} x \sqrt{x \left(c x+b\right)}}-\frac{2 c \left(5 b c e x-8 c^{2} d x+6 e \,b^{2}-9 d b c\right) x}{3 \sqrt{x \left(c x+b\right)}\, \left(c x+b\right) b^{4}}$	86
trager	$-\frac{2 \left(8 b \,c^{2} x^{3} e-16 c^{3} d \,x^{3}+12 b^{2} c e \,x^{2}-24 b \,c^{2} d \,x^{2}+3 b^{3} e x-6 b^{2} c x d+b^{3} d\right) \sqrt{c \,x^{2}+b x}}{3 b^{4} x^{2} \left(c x+b\right)^{2}}$	87
default	$d \left(-\frac{2 \left(2 c x+b\right)}{3 b^{2} \left(c \,x^{2}+b x\right)^{\frac{3}{2}}}+\frac{16 c \left(2 c x+b\right)}{3 b^{4} \sqrt{c \,x^{2}+b x}}\right)+e \left(-\frac{1}{3 c \left(c \,x^{2}+b x\right)^{\frac{3}{2}}}-\frac{b \left(-\frac{2 \left(2 c x+b\right)}{3 b^{2} \left(c \,x^{2}+b x\right)^{\frac{3}{2}}}+\frac{16 c \left(2 c x+b\right)}{3 b^{4} \sqrt{c \,x^{2}+b x}}\right)}{2 c}\right)$	121

input
```
int((e*x+d)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(x*(c*x+b))^(1/2)*((3*e*x+d)*b^3-6*c*x*(-2*e*x+d)*b^2-24*x^2*(-1/3*e*
x+d)*b*c^2-16*c^3*d*x^3)/x/(c*x+b)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{d+ex}{(bx+cx^2)^{5/2}}\,dx = -\frac{2\,(b^3d-8\,(2\,c^3d-bc^2e)x^3-12\,(2\,bc^2d-b^2ce)x^2-3\,(2\,b^2cd-b^3e)x)\sqrt{cx^2+bx}}{3\,(b^4c^2x^4+2\,b^5cx^3+b^6x^2)}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(b^3*d - 8*(2*c^3*d - b*c^2*e)*x^3 - 12*(2*b*c^2*d - b^2*c*e)*x^2 - 3
*(2*b^2*c*d - b^3*e)*x)*sqrt(c*x^2 + b*x)/(b^4*c^2*x^4 + 2*b^5*c*x^3 + b^6
*x^2)
```

Sympy [F]

$$\int \frac{d+ex}{(bx+cx^2)^{5/2}}\,dx = \int \frac{d+ex}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral((d + e*x)/(x*(b + c*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.15

$$\int \frac{d+ex}{(bx+cx^2)^{5/2}}\,dx = -\frac{4\,cdx}{3\,(cx^2+bx)^{\frac{3}{2}}b^2}+\frac{32\,c^2dx}{3\,\sqrt{cx^2+bx}b^4}+\frac{2\,ex}{3\,(cx^2+bx)^{\frac{3}{2}}b} - \frac{16\,cex}{3\,\sqrt{cx^2+bx}b^3}-\frac{2\,d}{3\,(cx^2+bx)^{\frac{3}{2}}b}+\frac{16\,cd}{3\,\sqrt{cx^2+bx}b^3}-\frac{8\,e}{3\,\sqrt{cx^2+bx}b^2}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
-4/3*c*d*x/((c*x^2 + b*x)^(3/2)*b^2) + 32/3*c^2*d*x/(sqrt(c*x^2 + b*x)*b^4
) + 2/3*e*x/((c*x^2 + b*x)^(3/2)*b) - 16/3*c*e*x/(sqrt(c*x^2 + b*x)*b^3) -
 2/3*d/((c*x^2 + b*x)^(3/2)*b) + 16/3*c*d/(sqrt(c*x^2 + b*x)*b^3) - 8/3*e/
(sqrt(c*x^2 + b*x)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.76

$$\int \frac{d+ex}{(bx+cx^2)^{5/2}}\,dx = \frac{2\left(\left(4\,x\left(\frac{2\,(2\,c^3d-bc^2e)x}{b^4}+\frac{3\,(2\,bc^2d-b^2ce)}{b^4}\right)+\frac{3\,(2\,b^2cd-b^3e)}{b^4}\right)x-\frac{d}{b}\right)}{3\,(cx^2+bx)^{\frac{3}{2}}}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
2/3*((4*x*(2*(2*c^3*d - b*c^2*e)*x/b^4 + 3*(2*b*c^2*d - b^2*c*e)/b^4) + 3*
(2*b^2*c*d - b^3*e)/b^4)*x - d/b)/(c*x^2 + b*x)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.38 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.67

$$\int \frac{d+ex}{(bx+cx^2)^{5/2}}\,dx = -\frac{2\,(3\,e\,b^3\,x+d\,b^3+12\,e\,b^2\,c\,x^2-6\,d\,b^2\,c\,x+8\,e\,b\,c^2\,x^3-24\,d\,b\,c^2\,x^2-16\,d\,c^3\,x^3)}{3\,b^4\,(c\,x^2+b\,x)^{3/2}}$$

input
```
int((d + e*x)/(b*x + c*x^2)^(5/2),x)
```

output
```
-(2*(b^3*d - 16*c^3*d*x^3 + 3*b^3*e*x - 6*b^2*c*d*x - 24*b*c^2*d*x^2 + 12*
b^2*c*e*x^2 + 8*b*c^2*e*x^3))/(3*b^4*(b*x + c*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.45

$$\int \frac{d+ex}{(bx+cx^2)^{5/2}}\,dx = \frac{\frac{16\sqrt{c}\,\sqrt{cx+b}\,b^2e\,x^2}{3} - \frac{32\sqrt{c}\,\sqrt{cx+b}\,bcd\,x^2}{3} + \frac{16\sqrt{c}\,\sqrt{cx+b}\,bce\,x^3}{3} - \frac{32\sqrt{c}\,\sqrt{cx+b}\,c^2d\,x^3}{3} - \frac{2\sqrt{x}\,b^3d}{3} - 2\sqrt{x}}{\sqrt{cx+b}\,b^4x^2\,(c}$$

input
```
int((e*x+d)/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*(8*sqrt(c)*sqrt(b + c*x)*b**2*e*x**2 - 16*sqrt(c)*sqrt(b + c*x)*b*c*d*x
**2 + 8*sqrt(c)*sqrt(b + c*x)*b*c*e*x**3 - 16*sqrt(c)*sqrt(b + c*x)*c**2*d
*x**3 - sqrt(x)*b**3*d - 3*sqrt(x)*b**3*e*x + 6*sqrt(x)*b**2*c*d*x - 12*sq
rt(x)*b**2*c*e*x**2 + 24*sqrt(x)*b*c**2*d*x**2 - 8*sqrt(x)*b*c**2*e*x**3 +
 16*sqrt(x)*c**3*d*x**3))/(3*sqrt(b + c*x)*b**4*x**2*(b + c*x))
```

3.175 $\int \frac{1}{(bx+cx^2)^{5/2}}\,dx$

Optimal result 1399
Mathematica [A] (verified) 1399
Rubi [A] (verified) 1400
Maple [A] (verified) 1401
Fricas [A] (verification not implemented) 1401
Sympy [F] 1402
Maxima [A] (verification not implemented) 1402
Giac [A] (verification not implemented) 1402
Mupad [B] (verification not implemented) 1403
Reduce [B] (verification not implemented) 1403

Optimal result

Integrand size = 13, antiderivative size = 89

$$\int \frac{1}{(bx+cx^2)^{5/2}}\,dx = \frac{2}{3b\,(bx+cx^2)^{3/2}} + \frac{4}{b^2x\sqrt{bx+cx^2}} - \frac{16\sqrt{bx+cx^2}}{3b^3x^2} + \frac{32c\sqrt{bx+cx^2}}{3b^4x}$$

output
```
2/3/b/(c*x^2+b*x)^(3/2)+4/b^2/x/(c*x^2+b*x)^(1/2)-16/3*(c*x^2+b*x)^(1/2)/b
^3/x^2+32/3*c*(c*x^2+b*x)^(1/2)/b^4/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.54

$$\int \frac{1}{(bx+cx^2)^{5/2}}\,dx = \frac{-2b^3+12b^2cx+48bc^2x^2+32c^3x^3}{3b^4(x(b+cx))^{3/2}}$$

input
```
Integrate[(b*x + c*x^2)^(-5/2),x]
```

output
```
(-2*b^3 + 12*b^2*c*x + 48*b*c^2*x^2 + 32*c^3*x^3)/(3*b^4*(x*(b + c*x))^(3/
2))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{8c\int \frac{1}{(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{16c(b+2cx)}{3b^4\sqrt{bx+cx^2}}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}$$

input
```
Int[(b*x + c*x^2)^(-5/2),x]
```

output
```
(-2*(b + 2*c*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (16*c*(b + 2*c*x))/(3*b^4*S
qrt[b*x + c*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.53

method	result	size
default	$-\frac{2(2cx+b)}{3b^{2}(c\,x^{2}+bx)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^{4}\sqrt{c\,x^{2}+bx}}$	47
pseudoelliptic	$-\frac{2(2cx+b)\left(-8c^{2}x^{2}-8cbx+b^{2}\right)}{3\sqrt{x(cx+b)}\,x(cx+b)b^{4}}$	48
gosper	$-\frac{2x(cx+b)\left(-16c^{3}x^{3}-24b\,c^{2}x^{2}-6b^{2}cx+b^{3}\right)}{3b^{4}(c\,x^{2}+bx)^{\frac{5}{2}}}$	51
orering	$-\frac{2x(cx+b)\left(-16c^{3}x^{3}-24b\,c^{2}x^{2}-6b^{2}cx+b^{3}\right)}{3b^{4}(c\,x^{2}+bx)^{\frac{5}{2}}}$	51
trager	$-\frac{2\left(-16c^{3}x^{3}-24b\,c^{2}x^{2}-6b^{2}cx+b^{3}\right)\sqrt{c\,x^{2}+bx}}{3b^{4}x^{2}(cx+b)^{2}}$	55
risch	$-\frac{2(cx+b)(-8cx+b)}{3b^{4}x\sqrt{x(cx+b)}}+\frac{2c^{2}(8cx+9b)x}{3\sqrt{x(cx+b)}\,(cx+b)b^{4}}$	63

input
```
int(1/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(2*c*x+b)/b^2/(c*x^2+b*x)^(3/2)+16/3*c/b^4*(2*c*x+b)/(c*x^2+b*x)^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \frac{1}{(bx+cx^2)^{5/2}}\,dx = \frac{2\left(16\,c^3x^3+24\,bc^2x^2+6\,b^2cx-b^3\right)\sqrt{cx^2+bx}}{3\left(b^4c^2x^4+2\,b^5cx^3+b^6x^2\right)}$$

input
```
integrate(1/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(16*c^3*x^3 + 24*b*c^2*x^2 + 6*b^2*c*x - b^3)*sqrt(c*x^2 + b*x)/(b^4*c
^2*x^4 + 2*b^5*c*x^3 + b^6*x^2)
```

Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{5/2}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral((b*x + c*x**2)**(-5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.81

$$\int \frac{1}{(bx+cx^2)^{5/2}}\,dx = -\frac{4\,cx}{3\,(cx^2+bx)^{\frac{3}{2}}b^2} + \frac{32\,c^2x}{3\,\sqrt{cx^2+bx}b^4} - \frac{2}{3\,(cx^2+bx)^{\frac{3}{2}}b} + \frac{16\,c}{3\,\sqrt{cx^2+bx}b^3}$$

input
```
integrate(1/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
-4/3*c*x/((c*x^2 + b*x)^(3/2)*b^2) + 32/3*c^2*x/(sqrt(c*x^2 + b*x)*b^4) -
2/3/((c*x^2 + b*x)^(3/2)*b) + 16/3*c/(sqrt(c*x^2 + b*x)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.56

$$\int \frac{1}{(bx+cx^2)^{5/2}}\,dx = \frac{2\left(2\left(4\,x\left(\frac{2\,c^3x}{b^4}+\frac{3\,c^2}{b^3}\right)+\frac{3\,c}{b^2}\right)x-\frac{1}{b}\right)}{3\,(cx^2+bx)^{\frac{3}{2}}}$$

input
```
integrate(1/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
2/3*(2*(4*x*(2*c^3*x/b^4 + 3*c^2/b^3) + 3*c/b^2)*x - 1/b)/(c*x^2 + b*x)^(3
/2)
```

Mupad [B] (verification not implemented)

Time = 5.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.48

$$\int \frac{1}{(bx+cx^2)^{5/2}}\,dx = \frac{(2\,b+4\,c\,x)\,\left(-b^2+8\,b\,c\,x+8\,c^2\,x^2\right)}{3\,b^4\,\left(c\,x^2+b\,x\right)^{3/2}}$$

input
```
int(1/(b*x + c*x^2)^(5/2),x)
```

output
```
((2*b + 4*c*x)*(8*c^2*x^2 - b^2 + 8*b*c*x))/(3*b^4*(b*x + c*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int \frac{1}{(bx+cx^2)^{5/2}}\,dx = \frac{-\frac{32\sqrt{c}\,\sqrt{cx+b}\,bc\,x^2}{3}-\frac{32\sqrt{c}\,\sqrt{cx+b}\,c^2x^3}{3}-\frac{2\sqrt{x}\,b^3}{3}+4\sqrt{x}\,b^2cx+16\sqrt{x}\,b\,c^2x^2+\frac{32\sqrt{x}\,c^3x^3}{3}}{\sqrt{cx+b}\,b^4x^2\,(cx+b)}$$

input
```
int(1/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*( - 16*sqrt(c)*sqrt(b + c*x)*b*c*x**2 - 16*sqrt(c)*sqrt(b + c*x)*c**2*x
**3 - sqrt(x)*b**3 + 6*sqrt(x)*b**2*c*x + 24*sqrt(x)*b*c**2*x**2 + 16*sqrt
(x)*c**3*x**3))/(3*sqrt(b + c*x)*b**4*x**2*(b + c*x))
```

3.176 $\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal result 1404
Mathematica [A] (verified) 1405
Rubi [A] (verified) 1405
Maple [A] (verified) 1408
Fricas [B] (verification not implemented) 1408
Sympy [F] 1409
Maxima [F(-2)] 1410
Giac [B] (verification not implemented) 1410
Mupad [F(-1)] 1411
Reduce [B] (verification not implemented) 1411

Optimal result

Integrand size = 21, antiderivative size = 236

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2}{3bd\left(bx+cx^2\right)^{3/2}} + \frac{2(2cd+be)x}{b^2d^2\left(bx+cx^2\right)^{3/2}} + \frac{2c(8c^2d^2-4bcde-3b^2e^2)\,x^2}{3b^3d^2(cd-be)\left(bx+cx^2\right)^{3/2}} + \frac{2c(2cd-be)\left(8c^2d^2-8bcde-3b^2e^2\right)x}{3b^4d^2(cd-be)^2\sqrt{bx+cx^2}} + \frac{2e^4\operatorname{arctanh}\left(\frac{\sqrt{cd-be}\,x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{d^{5/2}(cd-be)^{5/2}}$$

output
```
-2/3/b/d/(c*x^2+b*x)^(3/2)+2*(b*e+2*c*d)*x/b^2/d^2/(c*x^2+b*x)^(3/2)+2/3*c
*(-3*b^2*e^2-4*b*c*d*e+8*c^2*d^2)*x^2/b^3/d^2/(-b*e+c*d)/(c*x^2+b*x)^(3/2)
+2/3*c*(-b*e+2*c*d)*(-3*b^2*e^2-8*b*c*d*e+8*c^2*d^2)*x/b^4/d^2/(-b*e+c*d)^
2/(c*x^2+b*x)^(1/2)+2*e^4*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(
1/2))/d^(5/2)/(-b*e+c*d)^(5/2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.07

$$\int \frac{1}{(d+ex)(bx+cx^2)^{5/2}}\,dx = \frac{2\sqrt{d}\sqrt{-cd+be}(16c^5d^3x^3+24bc^4d^2x^2(d-ex)+b^5e^2(-d+3ex)+2b^2c^3dx}{}$$

input
```
Integrate[1/((d + e*x)*(b*x + c*x^2)^(5/2)),x]
```

output
```
(2*Sqrt[d]*Sqrt[-(c*d) + b*e]*(16*c^5*d^3*x^3 + 24*b*c^4*d^2*x^2*(d - e*x)
 + b^5*e^2*(-d + 3*e*x) + 2*b^2*c^3*d*x*(3*d^2 - 18*d*e*x + e^2*x^2) + 2*b
^4*c*e*(d^2 + 3*e^2*x^2) - b^3*c^2*(d^3 + 9*d^2*e*x - 3*d*e^2*x^2 - 3*e^3*
x^3)) - 6*b^4*e^4*x^(3/2)*(b + c*x)^(3/2)*ArcTan[(-(e*Sqrt[x]*Sqrt[b + c*x
]) + Sqrt[c]*(d + e*x))/(Sqrt[d]*Sqrt[-(c*d) + b*e])])/(3*b^4*d^(5/2)*(-(c
*d) + b*e)^(5/2)*(x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1165, 27, 1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{5/2}(d+ex)}\,dx$$

$$\Big\downarrow \text{1165}$$

$$-\frac{2\int \frac{8c^2d^2-4bced-3b^2e^2+4ce(2cd-be)x}{2(d+ex)(cx^2+bx)^{3/2}}dx}{3b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d(bx+cx^2)^{3/2}(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{8c^2d^2-4bced-3b^2e^2+4ce(2cd-be)x}{(d+ex)(cx^2+bx)^{3/2}}dx}{3b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d(bx+cx^2)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 1235$$

$$-\frac{-\frac{2\int \frac{3b^4e^4}{2(d+ex)\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2\left(cx(2cd-be)\left(-3b^2e^2-8bcde+8c^2d^2\right)+b(cd-be)\left(-3b^2e^2-4bcde+8c^2d^2\right)\right)}{b^2d\sqrt{bx+cx^2}(cd-be)}}{3b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 27$$

$$-\frac{-\frac{3b^2e^4\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{d(cd-be)}-\frac{2\left(cx(2cd-be)\left(-3b^2e^2-8bcde+8c^2d^2\right)+b(cd-be)\left(-3b^2e^2-4bcde+8c^2d^2\right)\right)}{b^2d\sqrt{bx+cx^2}(cd-be)}}{3b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 1154$$

$$-\frac{\frac{6b^2e^4\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{d(cd-be)}-\frac{2\left(cx(2cd-be)\left(-3b^2e^2-8bcde+8c^2d^2\right)+b(cd-be)\left(-3b^2e^2-4bcde+8c^2d^2\right)\right)}{b^2d\sqrt{bx+cx^2}(cd-be)}}{3b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 219$$

$$-\frac{-\frac{3b^2e^4\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{d^{3/2}(cd-be)^{3/2}}-\frac{2\left(cx(2cd-be)\left(-3b^2e^2-8bcde+8c^2d^2\right)+b(cd-be)\left(-3b^2e^2-4bcde+8c^2d^2\right)\right)}{b^2d\sqrt{bx+cx^2}(cd-be)}}{3b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(cd-be)}$$

input
```
Int[1/((d + e*x)*(b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(3*b^2*d*(c*d - b*e)*(b*x + c*x^2
)^(3/2)) - ((-2*(b*(c*d - b*e)*(8*c^2*d^2 - 4*b*c*d*e - 3*b^2*e^2) + c*(2*
c*d - b*e)*(8*c^2*d^2 - 8*b*c*d*e - 3*b^2*e^2)*x))/(b^2*d*(c*d - b*e)*Sqrt
[b*x + c*x^2]) - (3*b^2*e^4*ArcTanh[(b*d + (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqr
t[c*d - b*e]*Sqrt[b*x + c*x^2])])/(d^(3/2)*(c*d - b*e)^(3/2)))/(3*b^2*d*(c
*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.92

method	result
pseudoelliptic	$-\frac{2\left(3b^4e^4x\sqrt{x(cx+b)}\,(cx+b)\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)+\sqrt{d(be-cd)}\,\left(c^2(2cx+b)\left(-8c^2x^2-8cbx+b^2\right)d^3-2ecb\left(-12c^3x^3-18\right.\right.\right.}{3\sqrt{x(cx+b)}\,\sqrt{d(be-cd)}\,x\,d^2(be-cd)^2(cx+b)b^4}$
risch	$-\frac{2(cx+b)(-3bex-8cdx+bd)}{3b^4d^2\sqrt{x(cx+b)}\,x}+\frac{-\frac{e^3b^3\ln\left(\frac{-\frac{2d(be-cd)}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{-\frac{d(be-cd)}{e^2}}\,\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(b}{}}}{x+\frac{d}{e}}\right.}{(be-cd)^2\sqrt{-\frac{d(be-cd)}{e^2}}}}{}$
default	$\frac{-\frac{e^2}{3d(be-cd)\left(c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}\right)^{\frac{3}{2}}}+\frac{(be-2cd)e\left(\frac{\frac{4c\left(x+\frac{d}{e}\right)}{3}+\frac{2(be-2cd)}{3e}}{\left(-\frac{4cd(be-cd)}{e^2}-\frac{(be-2cd)^2}{e^2}\right)\left(c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}\right.}\right.}{}}{}$

input
```
int(1/(e*x+d)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(x*(c*x+b))^(1/2)/(d*(b*e-c*d))^(1/2)*(3*b^4*e^4*x*(x*(c*x+b))^(1/2)*
(c*x+b)*arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))^(1/2))+(d*(b*e-c*d))^(1
/2)*(c^2*(2*c*x+b)*(-8*c^2*x^2-8*b*c*x+b^2)*d^3-2*e*c*b*(-12*c^3*x^3-18*b*
c^2*x^2-9/2*b^2*c*x+b^3)*d^2+b^2*e^2*(-2*c*x+b)*(c*x+b)^2*d-3*b^3*e^3*x*(c
*x+b)^2))/x/d^2/(b*e-c*d)^2/(c*x+b)/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(212) = 424$.

Time = 0.15 (sec) , antiderivative size = 1012, normalized size of antiderivative = 4.29

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(b^4*c^2*e^4*x^4 + 2*b^5*c*e^4*x^3 + b^6*e^4*x^2)*sqrt(c*d^2 - b*d
*e)*log((b*d + (2*c*d - b*e)*x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/
(e*x + d)) - 2*(b^3*c^3*d^5 - 3*b^4*c^2*d^4*e + 3*b^5*c*d^3*e^2 - b^6*d^2*
e^3 - (16*c^6*d^5 - 40*b*c^5*d^4*e + 26*b^2*c^4*d^3*e^2 + b^3*c^3*d^2*e^3
- 3*b^4*c^2*d*e^4)*x^3 - 3*(8*b*c^5*d^5 - 20*b^2*c^4*d^4*e + 13*b^3*c^3*d^
3*e^2 + b^4*c^2*d^2*e^3 - 2*b^5*c*d*e^4)*x^2 - 3*(2*b^2*c^4*d^5 - 5*b^3*c^
3*d^4*e + 3*b^4*c^2*d^3*e^2 + b^5*c*d^2*e^3 - b^6*d*e^4)*x)*sqrt(c*x^2 + b
*x))/((b^4*c^5*d^6 - 3*b^5*c^4*d^5*e + 3*b^6*c^3*d^4*e^2 - b^7*c^2*d^3*e^3
)*x^4 + 2*(b^5*c^4*d^6 - 3*b^6*c^3*d^5*e + 3*b^7*c^2*d^4*e^2 - b^8*c*d^3*e
^3)*x^3 + (b^6*c^3*d^6 - 3*b^7*c^2*d^5*e + 3*b^8*c*d^4*e^2 - b^9*d^3*e^3)*
x^2), -2/3*(3*(b^4*c^2*e^4*x^4 + 2*b^5*c*e^4*x^3 + b^6*e^4*x^2)*sqrt(-c*d^
2 + b*d*e)*arctan(sqrt(-c*d^2 + b*d*e)*sqrt(c*x^2 + b*x)/(c*d*x + b*d)) +
(b^3*c^3*d^5 - 3*b^4*c^2*d^4*e + 3*b^5*c*d^3*e^2 - b^6*d^2*e^3 - (16*c^6*d
^5 - 40*b*c^5*d^4*e + 26*b^2*c^4*d^3*e^2 + b^3*c^3*d^2*e^3 - 3*b^4*c^2*d*e
^4)*x^3 - 3*(8*b*c^5*d^5 - 20*b^2*c^4*d^4*e + 13*b^3*c^3*d^3*e^2 + b^4*c^2
*d^2*e^3 - 2*b^5*c*d*e^4)*x^2 - 3*(2*b^2*c^4*d^5 - 5*b^3*c^3*d^4*e + 3*b^4
*c^2*d^3*e^2 + b^5*c*d^2*e^3 - b^6*d*e^4)*x)*sqrt(c*x^2 + b*x))/((b^4*c^5*
d^6 - 3*b^5*c^4*d^5*e + 3*b^6*c^3*d^4*e^2 - b^7*c^2*d^3*e^3)*x^4 + 2*(b^5*
c^4*d^6 - 3*b^6*c^3*d^5*e + 3*b^7*c^2*d^4*e^2 - b^8*c*d^3*e^3)*x^3 + (b^6*
c^3*d^6 - 3*b^7*c^2*d^5*e + 3*b^8*c*d^4*e^2 - b^9*d^3*e^3)*x^2)]
```

Sympy [F]

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(x\left(b+cx\right)\right)^{\frac{5}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(1/((x*(b + c*x))**(5/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{5/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(212) = 424$.

Time = 0.13 (sec) , antiderivative size = 645, normalized size of antiderivative = 2.73

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2\,e^4\arctan\left(\frac{\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde}}\right)}{\left(c^2d^4-2\,bcd^3e+b^2d^2e^2\right)\sqrt{-cd^2+bde}} + \frac{2\left(\left(\left(\frac{\left(16\,c^7d^{10}-56\,bc^6d^9e+66\,b^2c^5d^8e^2-25\,b^3c^4d^7e^3-4\,b^4c^3d^6e^4+3\,b^5c^2d^5e^5\right)x}{b^4c^4d^{11}-4\,b^5c^3d^{10}e+6\,b^6c^2d^9e^2-4\,b^7cd^8e^3+b^8d^7e^4}+\frac{3\left(8\,bc^6d^{10}-28\,b^2c^5d^9e+33\,b^3c^4d^8e^2-12\,b^4c^3d^7e^3-3\,b^5\cdots\right.}{b^4c^4d^{11}-4\,b^5c^3d^{10}e+6\,b^6c^2d^9e^2-4\,b^7cd^8e^3+\cdots}\right.\right.\right.}{3\cdots}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
-2*e^4*arctan(((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^2
+ b*d*e))/((c^2*d^4 - 2*b*c*d^3*e + b^2*d^2*e^2)*sqrt(-c*d^2 + b*d*e)) + 2
/3*((((16*c^7*d^10 - 56*b*c^6*d^9*e + 66*b^2*c^5*d^8*e^2 - 25*b^3*c^4*d^7*
e^3 - 4*b^4*c^3*d^6*e^4 + 3*b^5*c^2*d^5*e^5)*x/(b^4*c^4*d^11 - 4*b^5*c^3*d
^10*e + 6*b^6*c^2*d^9*e^2 - 4*b^7*c*d^8*e^3 + b^8*d^7*e^4) + 3*(8*b*c^6*d^
10 - 28*b^2*c^5*d^9*e + 33*b^3*c^4*d^8*e^2 - 12*b^4*c^3*d^7*e^3 - 3*b^5*c^
2*d^6*e^4 + 2*b^6*c*d^5*e^5)/(b^4*c^4*d^11 - 4*b^5*c^3*d^10*e + 6*b^6*c^2*
d^9*e^2 - 4*b^7*c*d^8*e^3 + b^8*d^7*e^4))*x + 3*(2*b^2*c^5*d^10 - 7*b^3*c^
4*d^9*e + 8*b^4*c^3*d^8*e^2 - 2*b^5*c^2*d^7*e^3 - 2*b^6*c*d^6*e^4 + b^7*d^
5*e^5)/(b^4*c^4*d^11 - 4*b^5*c^3*d^10*e + 6*b^6*c^2*d^9*e^2 - 4*b^7*c*d^8*
e^3 + b^8*d^7*e^4))*x - (b^3*c^4*d^10 - 4*b^4*c^3*d^9*e + 6*b^5*c^2*d^8*e^
2 - 4*b^6*c*d^7*e^3 + b^7*d^6*e^4)/(b^4*c^4*d^11 - 4*b^5*c^3*d^10*e + 6*b^
6*c^2*d^9*e^2 - 4*b^7*c*d^8*e^3 + b^8*d^7*e^4))/(c*x^2 + b*x)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{5/2}}\,dx=\int \frac{1}{{\left(c\,x^2+b\,x\right)}^{5/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/((b*x + c*x^2)^(5/2)*(d + e*x)),x)
```

output
```
int(1/((b*x + c*x^2)^(5/2)*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 880, normalized size of antiderivative = 3.73

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
int(1/(e*x+d)/(c*x^2+b*x)^(5/2),x)
```

output

```
(2*( - 3*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqr
t(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**5*e**4
*x**2 - 3*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sq
rt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**4*c*e
**4*x**3 - 3*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) +
 sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**5*
e**4*x**2 - 3*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d)
+ sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**4
*c*e**4*x**3 - sqrt(c)*sqrt(b + c*x)*b**5*d*e**4*x**2 - 5*sqrt(c)*sqrt(b +
 c*x)*b**4*c*d**2*e**3*x**2 - sqrt(c)*sqrt(b + c*x)*b**4*c*d*e**4*x**3 + 3
0*sqrt(c)*sqrt(b + c*x)*b**3*c**2*d**3*e**2*x**2 - 5*sqrt(c)*sqrt(b + c*x)
*b**3*c**2*d**2*e**3*x**3 - 40*sqrt(c)*sqrt(b + c*x)*b**2*c**3*d**4*e*x**2
 + 30*sqrt(c)*sqrt(b + c*x)*b**2*c**3*d**3*e**2*x**3 + 16*sqrt(c)*sqrt(b +
 c*x)*b*c**4*d**5*x**2 - 40*sqrt(c)*sqrt(b + c*x)*b*c**4*d**4*e*x**3 + 16*
sqrt(c)*sqrt(b + c*x)*c**5*d**5*x**3 - sqrt(x)*b**6*d**2*e**3 + 3*sqrt(x)*
b**6*d*e**4*x + 3*sqrt(x)*b**5*c*d**3*e**2 - 3*sqrt(x)*b**5*c*d**2*e**3*x
+ 6*sqrt(x)*b**5*c*d*e**4*x**2 - 3*sqrt(x)*b**4*c**2*d**4*e - 9*sqrt(x)*b*
*4*c**2*d**3*e**2*x - 3*sqrt(x)*b**4*c**2*d**2*e**3*x**2 + 3*sqrt(x)*b**4*
c**2*d*e**4*x**3 + sqrt(x)*b**3*c**3*d**5 + 15*sqrt(x)*b**3*c**3*d**4*e*x
- 39*sqrt(x)*b**3*c**3*d**3*e**2*x**2 - sqrt(x)*b**3*c**3*d**2*e**3*x**...
```

3.177 $\int \frac{1}{(d+ex)^2(bx+cx^2)^{5/2}}\,dx$

Optimal result . . . 1413
Mathematica [A] (verified) . . . 1414
Rubi [A] (verified) . . . 1414
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Sympy [F] . . . 1419
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Giac [B] (verification not implemented) . . . 1420
Mupad [F(-1)] . . . 1421
Reduce [F] . . . 1422

Optimal result

Integrand size = 21, antiderivative size = 348

$$\int \frac{1}{(d+ex)^2(bx+cx^2)^{5/2}}\,dx = -\frac{2cd-5be}{3bd^2(cd-be)(bx+cx^2)^{3/2}} + \frac{\left(\frac{4c^2}{b^2}-\frac{5e^2}{d^2}\right)x}{d(cd-be)(bx+cx^2)^{3/2}} + \frac{c(16c^3d^3-16bc^2d^2e-10b^2cde^2+15b^3e^3)x^2}{3b^3d^3(cd-be)^2(bx+cx^2)^{3/2}} - \frac{e}{d(cd-be)(d+ex)(bx+cx^2)^{3/2}} + \frac{c(32c^4d^4-64bc^3d^3e+12b^2c^2d^2e^2+20b^3cde^3-15b^4e^4)x}{3b^4d^3(cd-be)^3\sqrt{bx+cx^2}} + \frac{5e^4(2cd-be)\text{arctanh}\left(\frac{\sqrt{cd-be}\,x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{d^{7/2}(cd-be)^{7/2}}$$

output
```
-1/3*(-5*b*e+2*c*d)/b/d^2/(-b*e+c*d)/(c*x^2+b*x)^(3/2)+(4*c^2/b^2-5*e^2/d^
2)*x/d/(-b*e+c*d)/(c*x^2+b*x)^(3/2)+1/3*c*(15*b^3*e^3-10*b^2*c*d*e^2-16*b*
c^2*d^2*e+16*c^3*d^3)*x^2/b^3/d^3/(-b*e+c*d)^2/(c*x^2+b*x)^(3/2)-e/d/(-b*e
+c*d)/(e*x+d)/(c*x^2+b*x)^(3/2)+1/3*c*(-15*b^4*e^4+20*b^3*c*d*e^3+12*b^2*c
^2*d^2*e^2-64*b*c^3*d^3*e+32*c^4*d^4)*x/b^4/d^3/(-b*e+c*d)^3/(c*x^2+b*x)^(
1/2)+5*e^4*(-b*e+2*c*d)*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(1/
2))/d^(7/2)/(-b*e+c*d)^(7/2)
```

Mathematica [A] (verified)

Time = 2.04 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex)^2 \left(bx+cx^2\right)^{5/2}}\,dx = \frac{x\left(\frac{\sqrt{d}(b+cx)\left(-32c^6d^4x^3(d+ex)+16bc^5d^3x^2\left(-3d^2+dex+4e^2x^2\right)+b^6e^3\left(-2d^2+10dex+15e^2x^2\right)-12\right.}{}\right.}{}$$

input
```
Integrate[1/((d + e*x)^2*(b*x + c*x^2)^(5/2)),x]
```

output
```
(x*((Sqrt[d]*(b + c*x)*(-32*c^6*d^4*x^3*(d + e*x) + 16*b*c^5*d^3*x^2*(-3*d
^2 + d*e*x + 4*e^2*x^2) + b^6*e^3*(-2*d^2 + 10*d*e*x + 15*e^2*x^2) - 12*b^
2*c^4*d^2*x*(d^3 - 7*d^2*e*x - 7*d*e^2*x^2 + e^3*x^3) + 6*b^5*c*e^2*(d^3 -
 3*d^2*e*x + 5*e^3*x^3) + 2*b^3*c^3*d*(d^4 + 13*d^3*e*x + 3*d^2*e^2*x^2 -
19*d*e^3*x^3 - 10*e^4*x^4) - 3*b^4*c^2*e*(2*d^4 + 2*d^3*e*x + 14*d^2*e^2*x
^2 + 10*d*e^3*x^3 - 5*e^4*x^4)))/(b^4*(-(c*d) + b*e)^3*(d + e*x)) + (15*e^
4*(2*c*d - b*e)*x^(3/2)*(b + c*x)^(5/2)*ArcTan[(-(e*Sqrt[x]*Sqrt[b + c*x])
 + Sqrt[c]*(d + e*x))/(Sqrt[d]*Sqrt[-(c*d) + b*e])])/(-(c*d) + b*e)^(7/2))
)/(3*d^(7/2)*(x*(b + c*x))^(5/2))
```

Rubi [A] (verified)

Time = 1.11 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1165, 27, 1235, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(bx+cx^2\right)^{5/2}(d+ex)^2}dx$$

$$\Big\downarrow \text{1165}$$

$$-\frac{2\int \frac{8c^2d^2-2bced-5b^2e^2+6ce(2cd-be)x}{2(d+ex)^2\left(cx^2+bx\right)^{3/2}}dx}{3b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(d+ex)(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{8c^2d^2-2bced-5b^2e^2+6ce(2cd-be)x}{(d+ex)^2(cx^2+bx)^{3/2}}dx}{3b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(d+ex)(cd-be)}$$

$\downarrow$ 1235

$$-\frac{-\frac{2\int \frac{e\left(b\left(16c^3d^3-16bc^2ed^2-10b^2ce^2d+15b^3e^3\right)+2c(2cd-be)\left(8c^2d^2-8bced-5b^2e^2\right)x\right)}{2(d+ex)^2\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2\left(cx(2cd-be)\left(-5b^2e^2-8bcde+8c^2d^2\right)+b(cd-be)\left(-5b\cdots\right.\right.}{b^2d\sqrt{bx+cx^2}(d+ex)(cd-be)}}{3b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(d+ex)(cd-be)}$$

$\downarrow$ 27

$$-\frac{-\frac{e\int \frac{b\left(16c^3d^3-16bc^2ed^2-10b^2ce^2d+15b^3e^3\right)+2c(2cd-be)\left(8c^2d^2-8bced-5b^2e^2\right)x}{(d+ex)^2\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2\left(cx(2cd-be)\left(-5b^2e^2-8bcde+8c^2d^2\right)+b(cd-be)\left(-5b^2e^2\cdots\right.\right.}{b^2d\sqrt{bx+cx^2}(d+ex)(cd-be)}}{3b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(d+ex)(cd-be)}$$

$\downarrow$ 1228

$$-\frac{-\frac{e\left(\frac{15b^4e^3(2cd-be)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{2d(cd-be)}+\frac{\sqrt{bx+cx^2}\left(-15b^4e^4+20b^3cde^3+12b^2c^2d^2e^2-64bc^3d^3e+32c^4d^4\right)}{d(d+ex)(cd-be)}\right)}{b^2d(cd-be)}-\frac{2\left(cx(2cd-be)\left(-5b^2e^2-8bcde+8c\cdots\right.\right.}{b^2d\sqrt{bx+c\cdots}}}{3b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(d+ex)(cd-be)}$$

$\downarrow$ 1154

$$-\frac{-\frac{e\left(\frac{\sqrt{bx+cx^2}\left(-15b^4e^4+20b^3cde^3+12b^2c^2d^2e^2-64bc^3d^3e+32c^4d^4\right)}{d(d+ex)(cd-be)}-\frac{15b^4e^3(2cd-be)\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{d(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx\cdots}{}}{3b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(d+ex)(cd-be)}$$

$\downarrow$ 219

$$-\frac{-\frac{e\left(\frac{15b^4e^3(2cd-be)\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{2d^{3/2}(cd-be)^{3/2}}+\frac{\sqrt{bx+cx^2}\left(-15b^4e^4+20b^3cde^3+12b^2c^2d^2e^2-64bc^3d^3e+32c^4d^4\right)}{d(d+ex)(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)(-5b^2\cdots}{\cdots}}{3b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(d+ex)(cd-be)}$$

input
```
Int[1/((d + e*x)^2*(b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(3*b^2*d*(c*d - b*e)*(d + e*x)*(b
*x + c*x^2)^(3/2)) - ((-2*(b*(c*d - b*e)*(8*c^2*d^2 - 2*b*c*d*e - 5*b^2*e^
2) + c*(2*c*d - b*e)*(8*c^2*d^2 - 8*b*c*d*e - 5*b^2*e^2)*x))/(b^2*d*(c*d -
 b*e)*(d + e*x)*Sqrt[b*x + c*x^2]) - (e*(((32*c^4*d^4 - 64*b*c^3*d^3*e + 1
2*b^2*c^2*d^2*e^2 + 20*b^3*c*d*e^3 - 15*b^4*e^4)*Sqrt[b*x + c*x^2])/(d*(c*
d - b*e)*(d + e*x)) + (15*b^4*e^3*(2*c*d - b*e)*ArcTanh[(b*d + (2*c*d - b*
e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(2*d^(3/2)*(c*d - b*
e)^(3/2))))/(b^2*d*(c*d - b*e)))/(3*b^2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.96

method	result
pseudoelliptic	$-\frac{2\left(\frac{15b^4e^4x\sqrt{x(cx+b)}\,(ex+d)(cx+b)(be-2cd)\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)}{2}+\left(\left(16c^6x^3+24b\,c^5x^2+6b^2c^4x-b^3c^3\right)d^5+3e\left(\frac{16}{3}c^4x^4-\frac{8}{3}\right.\right.\right.}{3}$
	$\frac{d\,e^2b^3\left(\frac{e^2\sqrt{c\left(x+\frac{d}{e}\right)^2+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}-\frac{d(be-cd)}{e^2}}}{d(be-cd)\left(x+\frac{d}{e}\right)}-\frac{(be-2cd)e\ln\left(\frac{-\frac{2d(be-cd)}{e^2}+\frac{(be-2cd)\left(x\right.}{e}}{}\right.}{}\right)}{(be-cd)^2}$
risch	$-\frac{2(cx+b)(-6bex-8cdx+bd)}{3b^4d^3\sqrt{x(cx+b)}\,x}+$
default	Expression too large to display

input
```
int(1/(e*x+d)^2/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(x*(c*x+b))^(1/2)/(d*(b*e-c*d))^(1/2)*(15/2*b^4*e^4*x*(x*(c*x+b))^(1/
2)*(e*x+d)*(c*x+b)*(b*e-2*c*d)*arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c*d))^
(1/2))+((16*c^6*x^3+24*b*c^5*x^2+6*b^2*c^4*x-b^3*c^3)*d^5+3*e*(16/3*c^4*x^
4-8/3*b*c^3*x^3-14*b^2*c^2*x^2-13/3*b^3*c*x+b^4)*c^2*d^4-3*(32/3*c^4*x^4+1
4*b*c^3*x^3+b^2*c^2*x^2-b^3*c*x+b^4)*e^2*c*b*d^3+b^2*e^3*(6*c*x+b)*(c*x+b)
^3*d^2-5*b^3*e^4*x*(-2*c*x+b)*(c*x+b)^2*d-15/2*b^4*e^5*x^2*(c*x+b)^2)*(d*(
b*e-c*d))^(1/2))/x/d^3/(b*e-c*d)^3/(c*x+b)/(e*x+d)/b^4
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 884 vs. 2(322) = 644.

Time = 0.25 (sec) , antiderivative size = 1784, normalized size of antiderivative = 5.13

$$\int \frac{1}{(d+ex)^2\,(bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/6*(15*((2*b^4*c^3*d*e^5 - b^5*c^2*e^6)*x^5 + (2*b^4*c^3*d^2*e^4 + 3*b^5
*c^2*d*e^5 - 2*b^6*c*e^6)*x^4 + (4*b^5*c^2*d^2*e^4 - b^7*e^6)*x^3 + (2*b^6
*c*d^2*e^4 - b^7*d*e^5)*x^2)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*
x + 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) - 2*(2*b^3*c^4*d^7
 - 8*b^4*c^3*d^6*e + 12*b^5*c^2*d^5*e^2 - 8*b^6*c*d^4*e^3 + 2*b^7*d^3*e^4
- (32*c^7*d^6*e - 96*b*c^6*d^5*e^2 + 76*b^2*c^5*d^4*e^3 + 8*b^3*c^4*d^3*e^
4 - 35*b^4*c^3*d^2*e^5 + 15*b^5*c^2*d*e^6)*x^4 - 2*(16*c^7*d^7 - 24*b*c^6*
d^6*e - 34*b^2*c^5*d^5*e^2 + 61*b^3*c^4*d^4*e^3 - 4*b^4*c^3*d^3*e^4 - 30*b
^5*c^2*d^2*e^5 + 15*b^6*c*d*e^6)*x^3 - 3*(16*b*c^6*d^7 - 44*b^2*c^5*d^6*e
+ 26*b^3*c^4*d^5*e^2 + 16*b^4*c^3*d^4*e^3 - 14*b^5*c^2*d^3*e^4 - 5*b^6*c*d
^2*e^5 + 5*b^7*d*e^6)*x^2 - 2*(6*b^2*c^5*d^7 - 19*b^3*c^4*d^6*e + 16*b^4*c
^3*d^5*e^2 + 6*b^5*c^2*d^4*e^3 - 14*b^6*c*d^3*e^4 + 5*b^7*d^2*e^5)*x)*sqrt
(c*x^2 + b*x))/((b^4*c^6*d^8*e - 4*b^5*c^5*d^7*e^2 + 6*b^6*c^4*d^6*e^3 - 4
*b^7*c^3*d^5*e^4 + b^8*c^2*d^4*e^5)*x^5 + (b^4*c^6*d^9 - 2*b^5*c^5*d^8*e -
 2*b^6*c^4*d^7*e^2 + 8*b^7*c^3*d^6*e^3 - 7*b^8*c^2*d^5*e^4 + 2*b^9*c*d^4*e
^5)*x^4 + (2*b^5*c^5*d^9 - 7*b^6*c^4*d^8*e + 8*b^7*c^3*d^7*e^2 - 2*b^8*c^2
*d^6*e^3 - 2*b^9*c*d^5*e^4 + b^10*d^4*e^5)*x^3 + (b^6*c^4*d^9 - 4*b^7*c^3*
d^8*e + 6*b^8*c^2*d^7*e^2 - 4*b^9*c*d^6*e^3 + b^10*d^5*e^4)*x^2), -1/3*(15
*((2*b^4*c^3*d*e^5 - b^5*c^2*e^6)*x^5 + (2*b^4*c^3*d^2*e^4 + 3*b^5*c^2*d*e
^5 - 2*b^6*c*e^6)*x^4 + (4*b^5*c^2*d^2*e^4 - b^7*e^6)*x^3 + (2*b^6*c*d^...
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2\,(bx+cx^2)^{5/2}}\,dx = \int \frac{1}{(x\,(b+cx))^{\frac{5}{2}}\,(d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(1/((x*(b + c*x))**(5/2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^{5/2}} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1457 vs. 2(322) = 644.

Time = 0.39 (sec) , antiderivative size = 1457, normalized size of antiderivative = 4.19

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^{5/2}} \, dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
1/6*((30*b^4*c^(3/2)*d*e^7*log(abs(2*c*d*e - b*e^2 - 2*sqrt(c*d^2 - b*d*e)
*sqrt(c)*abs(e))) - 15*b^5*sqrt(c)*e^8*log(abs(2*c*d*e - b*e^2 - 2*sqrt(c*
d^2 - b*d*e)*sqrt(c)*abs(e))) - 64*sqrt(c*d^2 - b*d*e)*c^5*d^4*e^2*abs(e)
+ 128*sqrt(c*d^2 - b*d*e)*b*c^4*d^3*e^3*abs(e) - 24*sqrt(c*d^2 - b*d*e)*b^
2*c^3*d^2*e^4*abs(e) - 40*sqrt(c*d^2 - b*d*e)*b^3*c^2*d*e^5*abs(e) + 30*sq
rt(c*d^2 - b*d*e)*b^4*c*e^6*abs(e))*sgn(1/(e*x + d))*sgn(e)/(sqrt(c*d^2 -
b*d*e)*b^4*c^(7/2)*d^6*abs(e) - 3*sqrt(c*d^2 - b*d*e)*b^5*c^(5/2)*d^5*e*ab
s(e) + 3*sqrt(c*d^2 - b*d*e)*b^6*c^(3/2)*d^4*e^2*abs(e) - sqrt(c*d^2 - b*d
*e)*b^7*sqrt(c)*d^3*e^3*abs(e)) + 2*((32*c^6*d^4*e^13 - 64*b*c^5*d^3*e^14
+ 12*b^2*c^4*d^2*e^15 + 20*b^3*c^3*d*e^16 - 15*b^4*c^2*e^17)/(b^4*c^3*d^6*
e^11*sgn(1/(e*x + d))*sgn(e) - 3*b^5*c^2*d^5*e^12*sgn(1/(e*x + d))*sgn(e)
+ 3*b^6*c*d^4*e^13*sgn(1/(e*x + d))*sgn(e) - b^7*d^3*e^14*sgn(1/(e*x + d))
*sgn(e)) - (6*(16*c^6*d^5*e^14 - 40*b*c^5*d^4*e^15 + 22*b^2*c^4*d^3*e^16 +
 7*b^3*c^3*d^2*e^17 - 15*b^4*c^2*d*e^18 + 5*b^5*c*e^19)/(b^4*c^3*d^6*e^11*
sgn(1/(e*x + d))*sgn(e) - 3*b^5*c^2*d^5*e^12*sgn(1/(e*x + d))*sgn(e) + 3*b
^6*c*d^4*e^13*sgn(1/(e*x + d))*sgn(e) - b^7*d^3*e^14*sgn(1/(e*x + d))*sgn(
e)) - (3*(32*c^6*d^6*e^15 - 96*b*c^5*d^5*e^16 + 80*b^2*c^4*d^4*e^17 - 46*b
^4*c^2*d^2*e^19 + 30*b^5*c*d*e^20 - 5*b^6*e^21)/(b^4*c^3*d^6*e^11*sgn(1/(e
*x + d))*sgn(e) - 3*b^5*c^2*d^5*e^12*sgn(1/(e*x + d))*sgn(e) + 3*b^6*c*d^4
*e^13*sgn(1/(e*x + d))*sgn(e) - b^7*d^3*e^14*sgn(1/(e*x + d))*sgn(e)) -...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\,(bx+cx^2)^{5/2}}\,dx = \int \frac{1}{\left(c\,x^2+b\,x\right)^{5/2}\,\left(d+e\,x\right)^2}\,dx$$

input
```
int(1/((b*x + c*x^2)^(5/2)*(d + e*x)^2),x)
```

output
```
int(1/((b*x + c*x^2)^(5/2)*(d + e*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^{5/2}} \, dx = \int \frac{1}{(ex+d)^2 (c\,x^2+bx)^{\frac{5}{2}}} dx$$

input
```
int(1/(e*x+d)^2/(c*x^2+b*x)^(5/2),x)
```

output
```
int(1/(e*x+d)^2/(c*x^2+b*x)^(5/2),x)
```

3.178 $\int \frac{(d+ex)^4}{(bx+cx^2)^{7/2}} \, dx$

Optimal result 1423
Mathematica [A] (verified) 1424
Rubi [A] (verified) 1424
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Reduce [B] (verification not implemented) 1431

Optimal result

Integrand size = 21, antiderivative size = 269

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{7/2}} \, dx = -\frac{32(cd-be)^2(2cd-be)x^3(d+ex)^2}{5b^4d\,(bx+cx^2)^{5/2}} - \frac{16(cd-be)(2cd-be)x^2(d+ex)^3}{3b^3d\,(bx+cx^2)^{5/2}} + \frac{2(2cd-be)x(d+ex)^4}{3b^2d\,(bx+cx^2)^{5/2}} - \frac{2(d+ex)^5}{5bd\,(bx+cx^2)^{5/2}} - \frac{128(cd-be)^3(2cd-be)x^2}{15b^5c\,(bx+cx^2)^{3/2}} - \frac{128(cd-be)^2(2cd-be)(2cd+be)x}{15b^6c\sqrt{bx+cx^2}}$$

output
```
-32/5*(-b*e+c*d)^2*(-b*e+2*c*d)*x^3*(e*x+d)^2/b^4/d/(c*x^2+b*x)^(5/2)-16/3
*(-b*e+c*d)*(-b*e+2*c*d)*x^2*(e*x+d)^3/b^3/d/(c*x^2+b*x)^(5/2)+2/3*(-b*e+2
*c*d)*x*(e*x+d)^4/b^2/d/(c*x^2+b*x)^(5/2)-2/5*(e*x+d)^5/b/d/(c*x^2+b*x)^(5
/2)-128/15*(-b*e+c*d)^3*(-b*e+2*c*d)*x^2/b^5/c/(c*x^2+b*x)^(3/2)-128/15*(-
b*e+c*d)^2*(-b*e+2*c*d)*(b*e+2*c*d)*x/b^6/c/(c*x^2+b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{7/2}}\,dx = \frac{-512c^5d^4x^5+256bc^4d^3x^4(-5d+4ex)-64b^2c^3d^2x^3(15d^2-40dex+9e^2x^2)+32b^3c}{}$$

input
```
Integrate[(d + e*x)^4/(b*x + c*x^2)^(7/2),x]
```

output
```
(-512*c^5*d^4*x^5 + 256*b*c^4*d^3*x^4*(-5*d + 4*e*x) - 64*b^2*c^3*d^2*x^3*
(15*d^2 - 40*d*e*x + 9*e^2*x^2) + 32*b^3*c^2*d*x^2*(-5*d^3 + 60*d^2*e*x -
45*d*e^2*x^2 + 2*e^3*x^3) - 2*b^5*(3*d^4 + 20*d^3*e*x + 90*d^2*e^2*x^2 - 6
0*d*e^3*x^3 - 5*e^4*x^4) + 4*b^4*c*x*(5*d^4 + 80*d^3*e*x - 270*d^2*e^2*x^2
 + 40*d*e^3*x^3 + e^4*x^4))/(15*b^6*(x*(b + c*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.51, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1156, 1153, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{1156}$$

$$-\frac{8(2cd-be)\int \frac{(d+ex)^3}{(cx^2+bx)^{5/2}}dx}{5b^2}-\frac{2(b+2cx)(d+ex)^4}{5b^2\left(bx+cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{1153}$$

$$-\frac{8(2cd-be)\left(-\frac{8d(cd-be)\int \frac{d+ex}{\left(cx^2+bx\right)^{3/2}}dx}{3b^2}-\frac{2(d+ex)^2(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}\right)}{5b^2}-\frac{2(b+2cx)(d+ex)^4}{5b^2\left(bx+cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{1158}$$

$$-\frac{2(b+2cx)(d+ex)^4}{5b^2(bx+cx^2)^{5/2}}-\frac{8(2cd-be)\left(\frac{16d(cd-be)(x(2cd-be)+bd)}{3b^4\sqrt{bx+cx^2}}-\frac{2(d+ex)^2(x(2cd-be)+bd)}{3b^2(bx+cx^2)^{3/2}}\right)}{5b^2}$$

input
```
Int[(d + e*x)^4/(b*x + c*x^2)^(7/2),x]
```

output
```
(-2*(b + 2*c*x)*(d + e*x)^4)/(5*b^2*(b*x + c*x^2)^(5/2)) - (8*(2*c*d - b*e
)*((-2*(d + e*x)^2*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) +
(16*d*(c*d - b*e)*(b*d + (2*c*d - b*e)*x))/(3*b^4*Sqrt[b*x + c*x^2])))/(5*
b^2)
```

Defintions of rubi rules used

rule 1153
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m - 2)*(a + b*x +
c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[m + 2*p + 2, 0]
&& LtQ[p, -1]
```

rule 1156
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] + Simp[m*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*c)))   Int[
(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e
, m, p}, x] && EqQ[m + 2*p + 3, 0] && LtQ[p, -1]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{2d^2(cx+b)\left(90b^2e^2x^2-220bcde\,x^2+128d^2c^2x^2+20b^2dex-19xbc\,d^2+3b^2d^2\right)}{15b^6x^2\sqrt{x(cx+b)}}+\frac{2x\left(2b^2c\,e^2x^2+36b\,c^2de\,x^2-128c^3d^2x^2-\right.}{}$
pseudoelliptic	$\frac{\left(10e^4x^4+120d\,e^3x^3-180d^2e^2x^2-40d^3ex-6d^4\right)b^5+20x\left(\frac{1}{5}e^4x^4+8d\,e^3x^3-54d^2e^2x^2+16d^3ex+d^4\right)c\,b^4-160\left(-\frac{2}{5}e^3x^3+9d\,e\right.}{15\sqrt{x(cx+b)}\,x^2(cx+b)^2}$
gosper	$-\frac{2x(cx+b)\left(-2b^4c\,e^4x^5-32b^3c^2d\,e^3x^5+288b^2c^3d^2e^2x^5-512b\,c^4d^3e\,x^5+256c^5d^4x^5-5b^5e^4x^4-80b^4cd\,e^3x^4+720b^3c^2d^2e^2\right.}{}$
orering	$-\frac{2x(cx+b)\left(-2b^4c\,e^4x^5-32b^3c^2d\,e^3x^5+288b^2c^3d^2e^2x^5-512b\,c^4d^3e\,x^5+256c^5d^4x^5-5b^5e^4x^4-80b^4cd\,e^3x^4+720b^3c^2d^2e^2\right.}{}$
trager	$-\frac{2\left(-2b^4c\,e^4x^5-32b^3c^2d\,e^3x^5+288b^2c^3d^2e^2x^5-512b\,c^4d^3e\,x^5+256c^5d^4x^5-5b^5e^4x^4-80b^4cd\,e^3x^4+720b^3c^2d^2e^2x^4-128\right.}{}$
default	$d^4\left(-\frac{2(2cx+b)}{5b^2(c\,x^2+bx)^{\frac{5}{2}}}-\frac{16c\left(-\frac{2(2cx+b)}{3b^2\left(c\,x^2+bx\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{c\,x^2+bx}}\right)}{5b^2}\right)+e^4\left(-\frac{x^3}{2c(c\,x^2+bx)^{\frac{5}{2}}}+\frac{b\left(-\frac{x^2}{3c\left(c\,x^2+bx\right)^{\frac{5}{2}}}\right.}{}\right.$

input
```
int((e*x+d)^4/(c*x^2+b*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*d^2*(c*x+b)*(90*b^2*e^2*x^2-220*b*c*d*e*x^2+128*c^2*d^2*x^2+20*b^2*d
*e*x-19*b*c*d^2*x+3*b^2*d^2)/b^6/x^2/(x*(c*x+b))^(1/2)+2/15*x*(2*b^2*c*e^2
*x^2+36*b*c^2*d*e*x^2-128*c^3*d^2*x^2+5*b^3*e^2*x+90*b^2*c*d*e*x-275*b*c^2
*d^2*x+60*b^3*d*e-150*b^2*c*d^2)*(b^2*e^2-2*b*c*d*e+c^2*d^2)/(x*(c*x+b))^(
1/2)/(c^2*x^2+2*b*c*x+b^2)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{7/2}}\,dx =$$
$$-\frac{2\,(3\,b^5d^4+2\,(128\,c^5d^4-256\,bc^4d^3e+144\,b^2c^3d^2e^2-16\,b^3c^2de^3-b^4ce^4)x^5+5\,(128\,bc^4d^4-256\,b^2c^3d^3e-\cdots}{\cdots}$$

input
```
integrate((e*x+d)^4/(c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(3*b^5*d^4 + 2*(128*c^5*d^4 - 256*b*c^4*d^3*e + 144*b^2*c^3*d^2*e^2
- 16*b^3*c^2*d*e^3 - b^4*c*e^4)*x^5 + 5*(128*b*c^4*d^4 - 256*b^2*c^3*d^3*e
 + 144*b^3*c^2*d^2*e^2 - 16*b^4*c*d*e^3 - b^5*e^4)*x^4 + 60*(8*b^2*c^3*d^4
 - 16*b^3*c^2*d^3*e + 9*b^4*c*d^2*e^2 - b^5*d*e^3)*x^3 + 10*(8*b^3*c^2*d^4
 - 16*b^4*c*d^3*e + 9*b^5*d^2*e^2)*x^2 - 10*(b^4*c*d^4 - 2*b^5*d^3*e)*x)*s
qrt(c*x^2 + b*x)/(b^6*c^3*x^6 + 3*b^7*c^2*x^5 + 3*b^8*c*x^4 + b^9*x^3)
```

Sympy [F]

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{7/2}}\,dx = \int \frac{(d+ex)^4}{(x\,(b+cx))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**4/(c*x**2+b*x)**(7/2),x)
```

output
```
Integral((d + e*x)**4/(x*(b + c*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(245) = 490$.

Time = 0.04 (sec) , antiderivative size = 640, normalized size of antiderivative = 2.38

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^4/(c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

output
```
-1/2*e^4*x^3/((c*x^2 + b*x)^(5/2)*c) - 4/3*d*e^3*x^2/((c*x^2 + b*x)^(5/2)*
c) - 1/12*b*e^4*x^2/((c*x^2 + b*x)^(5/2)*c^2) - 4/5*c*d^4*x/((c*x^2 + b*x)
^(5/2)*b^2) + 64/15*c^2*d^4*x/((c*x^2 + b*x)^(3/2)*b^4) - 512/15*c^3*d^4*x
/(sqrt(c*x^2 + b*x)*b^6) + 8/5*d^3*e*x/((c*x^2 + b*x)^(5/2)*b) - 128/15*c*
d^3*e*x/((c*x^2 + b*x)^(3/2)*b^3) + 1024/15*c^2*d^3*e*x/(sqrt(c*x^2 + b*x)
*b^5) + 24/5*d^2*e^2*x/((c*x^2 + b*x)^(3/2)*b^2) - 12/5*d^2*e^2*x/((c*x^2
+ b*x)^(5/2)*c) - 192/5*c*d^2*e^2*x/(sqrt(c*x^2 + b*x)*b^4) + 64/15*d*e^3*
x/(sqrt(c*x^2 + b*x)*b^3) + 4/15*b*d*e^3*x/((c*x^2 + b*x)^(5/2)*c^2) - 8/1
5*d*e^3*x/((c*x^2 + b*x)^(3/2)*b*c) + 1/60*b^2*e^4*x/((c*x^2 + b*x)^(5/2)*
c^3) - 1/30*e^4*x/((c*x^2 + b*x)^(3/2)*c^2) + 4/15*e^4*x/(sqrt(c*x^2 + b*x
)*b^2*c) - 2/5*d^4/((c*x^2 + b*x)^(5/2)*b) + 32/15*c*d^4/((c*x^2 + b*x)^(3
/2)*b^3) - 256/15*c^2*d^4/(sqrt(c*x^2 + b*x)*b^5) - 64/15*d^3*e/((c*x^2 +
b*x)^(3/2)*b^2) + 512/15*c*d^3*e/(sqrt(c*x^2 + b*x)*b^4) - 96/5*d^2*e^2/(s
qrt(c*x^2 + b*x)*b^3) + 12/5*d^2*e^2/((c*x^2 + b*x)^(3/2)*b*c) - 4/15*d*e^
3/((c*x^2 + b*x)^(3/2)*c^2) + 32/15*d*e^3/(sqrt(c*x^2 + b*x)*b^2*c) - 1/60
*b*e^4/((c*x^2 + b*x)^(3/2)*c^3) + 2/15*e^4/(sqrt(c*x^2 + b*x)*b*c^2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.98

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{7/2}}\,dx =$$
$$-\frac{2\left(\frac{3\,d^4}{b}+\left(\left(\left(x\left(\frac{2\,(128\,c^5d^4-256\,bc^4d^3e+144\,b^2c^3d^2e^2-16\,b^3c^2de^3-b^4ce^4)x}{b^6}+\frac{5\,(128\,bc^4d^4-256\,b^2c^3d^3e+144\,b^3c^2d^2e^2-16\,b^4cde^3-b^5}{b^6}\right.\right.\right.\right.\right.}{15\,(cx^2+bx}$$

input
```
integrate((e*x+d)^4/(c*x^2+b*x)^(7/2),x, algorithm="giac")
```

output
```
-2/15*(3*d^4/b + (((x*(2*(128*c^5*d^4 - 256*b*c^4*d^3*e + 144*b^2*c^3*d^2*
e^2 - 16*b^3*c^2*d*e^3 - b^4*c*e^4)*x/b^6 + 5*(128*b*c^4*d^4 - 256*b^2*c^3
*d^3*e + 144*b^3*c^2*d^2*e^2 - 16*b^4*c*d*e^3 - b^5*e^4)/b^6) + 60*(8*b^2*
c^3*d^4 - 16*b^3*c^2*d^3*e + 9*b^4*c*d^2*e^2 - b^5*d*e^3)/b^6)*x + 10*(8*b
^3*c^2*d^4 - 16*b^4*c*d^3*e + 9*b^5*d^2*e^2)/b^6)*x - 10*(b^4*c*d^4 - 2*b^
5*d^3*e)/b^6)*x)/(c*x^2 + b*x)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 5.33 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{7/2}}\,dx =$$
$$-\frac{2\,(3\,b^5\,d^4+20\,b^5\,d^3\,e\,x+90\,b^5\,d^2\,e^2\,x^2-60\,b^5\,d\,e^3\,x^3-5\,b^5\,e^4\,x^4-10\,b^4\,c\,d^4\,x-160\,b^4\,c\,d^3\,e\,x^2+540\,b^4\,c}{}$$

input
```
int((d + e*x)^4/(b*x + c*x^2)^(7/2),x)
```

output
```
-(2*(3*b^5*d^4 - 5*b^5*e^4*x^4 + 256*c^5*d^4*x^5 + 640*b*c^4*d^4*x^4 - 2*b
^4*c*e^4*x^5 - 60*b^5*d*e^3*x^3 + 80*b^3*c^2*d^4*x^2 + 480*b^2*c^3*d^4*x^3
 + 90*b^5*d^2*e^2*x^2 - 10*b^4*c*d^4*x + 20*b^5*d^3*e*x + 720*b^3*c^2*d^2*
e^2*x^4 + 288*b^2*c^3*d^2*e^2*x^5 - 160*b^4*c*d^3*e*x^2 - 80*b^4*c*d*e^3*x
^4 - 512*b*c^4*d^3*e*x^5 - 960*b^3*c^2*d^3*e*x^3 + 540*b^4*c*d^2*e^2*x^3 -
 1280*b^2*c^3*d^3*e*x^4 - 32*b^3*c^2*d*e^3*x^5))/(15*b^6*(b*x + c*x^2)^(5/
2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 695, normalized size of antiderivative = 2.58

$$\int \frac{(d+ex)^4}{(bx+cx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^4/(c*x^2+b*x)^(7/2),x)
```

output
```
(2*( - 14*sqrt(c)*sqrt(b + c*x)*b**6*e**4*x**3 - 32*sqrt(c)*sqrt(b + c*x)*
b**5*c*d*e**3*x**3 - 28*sqrt(c)*sqrt(b + c*x)*b**5*c*e**4*x**4 + 288*sqrt(
c)*sqrt(b + c*x)*b**4*c**2*d**2*e**2*x**3 - 64*sqrt(c)*sqrt(b + c*x)*b**4*
c**2*d*e**3*x**4 - 14*sqrt(c)*sqrt(b + c*x)*b**4*c**2*e**4*x**5 - 512*sqrt
(c)*sqrt(b + c*x)*b**3*c**3*d**3*e*x**3 + 576*sqrt(c)*sqrt(b + c*x)*b**3*c
**3*d**2*e**2*x**4 - 32*sqrt(c)*sqrt(b + c*x)*b**3*c**3*d*e**3*x**5 + 256*
sqrt(c)*sqrt(b + c*x)*b**2*c**4*d**4*x**3 - 1024*sqrt(c)*sqrt(b + c*x)*b**
2*c**4*d**3*e*x**4 + 288*sqrt(c)*sqrt(b + c*x)*b**2*c**4*d**2*e**2*x**5 +
512*sqrt(c)*sqrt(b + c*x)*b*c**5*d**4*x**4 - 512*sqrt(c)*sqrt(b + c*x)*b*c
**5*d**3*e*x**5 + 256*sqrt(c)*sqrt(b + c*x)*c**6*d**4*x**5 - 3*sqrt(x)*b**
5*c**2*d**4 - 20*sqrt(x)*b**5*c**2*d**3*e*x - 90*sqrt(x)*b**5*c**2*d**2*e*
*2*x**2 + 60*sqrt(x)*b**5*c**2*d*e**3*x**3 + 5*sqrt(x)*b**5*c**2*e**4*x**4
 + 10*sqrt(x)*b**4*c**3*d**4*x + 160*sqrt(x)*b**4*c**3*d**3*e*x**2 - 540*s
qrt(x)*b**4*c**3*d**2*e**2*x**3 + 80*sqrt(x)*b**4*c**3*d*e**3*x**4 + 2*sqr
t(x)*b**4*c**3*e**4*x**5 - 80*sqrt(x)*b**3*c**4*d**4*x**2 + 960*sqrt(x)*b*
*3*c**4*d**3*e*x**3 - 720*sqrt(x)*b**3*c**4*d**2*e**2*x**4 + 32*sqrt(x)*b*
*3*c**4*d*e**3*x**5 - 480*sqrt(x)*b**2*c**5*d**4*x**3 + 1280*sqrt(x)*b**2*
c**5*d**3*e*x**4 - 288*sqrt(x)*b**2*c**5*d**2*e**2*x**5 - 640*sqrt(x)*b*c*
*6*d**4*x**4 + 512*sqrt(x)*b*c**6*d**3*e*x**5 - 256*sqrt(x)*c**7*d**4*x**5
))/(15*sqrt(b + c*x)*b**6*c**2*x**3*(b**2 + 2*b*c*x + c**2*x**2))
```

3.179 $\int \frac{(d+ex)^3}{(bx+cx^2)^{7/2}} dx$

Optimal result 1432
Mathematica [A] (verified) 1433
Rubi [A] (verified) 1433
Maple [A] (verified) 1435
Fricas [A] (verification not implemented) 1437
Sympy [F] 1437
Maxima [B] (verification not implemented) 1438
Giac [A] (verification not implemented) 1439
Mupad [B] (verification not implemented) 1439
Reduce [B] (verification not implemented) 1440

Optimal result

Integrand size = 21, antiderivative size = 228

$$\begin{aligned}\int \frac{(d+ex)^3}{(bx+cx^2)^{7/2}} dx = &-\frac{2d^3}{5b(bx+cx^2)^{5/2}} + \frac{2d^2(2cd-3be)x}{3b^2(bx+cx^2)^{5/2}} \\ &- \frac{2d(4cd-3be)^2x^2}{3b^3(bx+cx^2)^{5/2}} + \frac{2(b^3e^3-2cd(4cd-3be)^2)x^3}{5b^4(bx+cx^2)^{5/2}} \\ &+ \frac{8(b^3e^3-2cd(4cd-3be)^2)x^2}{15b^5(bx+cx^2)^{3/2}} + \frac{16(b^3e^3-2cd(4cd-3be)^2)x}{15b^6\sqrt{bx+cx^2}}\end{aligned}$$

output
```
-2/5*d^3/b/(c*x^2+b*x)^(5/2)+2/3*d^2*(-3*b*e+2*c*d)*x/b^2/(c*x^2+b*x)^(5/2
)-2/3*d*(-3*b*e+4*c*d)^2*x^2/b^3/(c*x^2+b*x)^(5/2)+2/5*(b^3*e^3-2*c*d*(-3*
b*e+4*c*d)^2)*x^3/b^4/(c*x^2+b*x)^(5/2)+8/15*(b^3*e^3-2*c*d*(-3*b*e+4*c*d)
^2)*x^2/b^5/(c*x^2+b*x)^(3/2)+16/15*(b^3*e^3-2*c*d*(-3*b*e+4*c*d)^2)*x/b^6
/(c*x^2+b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{7/2}}\,dx = -\frac{2(256c^5d^3x^5+128bc^4d^2x^4(5d-3ex)+48b^2c^3dx^3(10d^2-20dex+3e^2x^2)+3b^5(d^3+5d^2ex+15de^2x^2-}{15b^6(x(b+c}$$

input
```
Integrate[(d + e*x)^3/(b*x + c*x^2)^(7/2),x]
```

output
```
(-2*(256*c^5*d^3*x^5 + 128*b*c^4*d^2*x^4*(5*d - 3*e*x) + 48*b^2*c^3*d*x^3*
(10*d^2 - 20*d*e*x + 3*e^2*x^2) + 3*b^5*(d^3 + 5*d^2*e*x + 15*d*e^2*x^2 -
5*e^3*x^3) - 8*b^3*c^2*x^2*(-10*d^3 + 90*d^2*e*x - 45*d*e^2*x^2 + e^3*x^3)
 - 10*b^4*c*x*(d^3 + 12*d^2*e*x - 27*d*e^2*x^2 + 2*e^3*x^3)))/(15*b^6*(x*(
b + c*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.64, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {1163, 25, 1227, 1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{7/2}}\,dx$$

$$\Big\downarrow \; 1163$$

$$\frac{2\int -\frac{(d+ex)^2(8cd-3be+2cex)}{(cx^2+bx)^{5/2}}dx}{5b^2}-\frac{2(b+2cx)(d+ex)^3}{5b^2\left(bx+cx^2\right)^{5/2}}$$

$$\Big\downarrow \; 25$$

$$-\frac{2\int \frac{(d+ex)^2(8cd-3be+2cex)}{(cx^2+bx)^{5/2}}dx}{5b^2}-\frac{2(b+2cx)(d+ex)^3}{5b^2\left(bx+cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{1227}$$

$$-\frac{2\left(-\frac{4(4cd-3be)(4cd-be)\int\frac{d+ex}{\left(cx^2+bx\right)^{3/2}}dx}{3b^2}-\frac{2(d+ex)^2(8cx(2cd-be)+b(8cd-3be))}{3b^2\left(bx+cx^2\right)^{3/2}}\right)}{5b^2}-\frac{2(b+2cx)(d+ex)^3}{5b^2\left(bx+cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{1158}$$

$$-\frac{2(b+2cx)(d+ex)^3}{5b^2\left(bx+cx^2\right)^{5/2}}-\frac{2\left(\frac{8(4cd-3be)(4cd-be)(x(2cd-be)+bd)}{3b^4\sqrt{bx+cx^2}}-\frac{2(d+ex)^2(8cx(2cd-be)+b(8cd-3be))}{3b^2\left(bx+cx^2\right)^{3/2}}\right)}{5b^2}$$

input
```
Int[(d + e*x)^3/(b*x + c*x^2)^(7/2),x]
```

output
```
(-2*(b + 2*c*x)*(d + e*x)^3)/(5*b^2*(b*x + c*x^2)^(5/2)) - (2*((-2*(d + e*
x)^2*(b*(8*c*d - 3*b*e) + 8*c*(2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)
) + (8*(4*c*d - 3*b*e)*(4*c*d - b*e)*(b*d + (2*c*d - b*e)*x))/(3*b^4*Sqrt[
b*x + c*x^2])))/(5*b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1163
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] - Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1
)*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p +
1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 0] && (LtQ[
m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c, d,
e, m, p, x]
```

rule 1227
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(b*f - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[m*((b*(
e*f + d*g) - 2*(c*d*f + a*e*g))/((p + 1)*(b^2 - 4*a*c)))   Int[(d + e*x)^(m
 - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x]
&& EqQ[Simplify[m + 2*p + 3], 0] && LtQ[p, -1]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.84

method	result
pseudoelliptic	$\frac{(30e^3x^3-90d\,e^2x^2-30d^2ex-6d^3)b^5+20cx(2e^3x^3-27d\,e^2x^2+12d^2ex+d^3)b^4-160x^2(-\frac{1}{10}e^3x^3+\frac{9}{2}d\,e^2x^2-9d^2ex+d^3)c^2b}{15\sqrt{x(cx+b)}\,x^2(cx+b)^2b^6}$
risch	$-\frac{2d(cx+b)(45b^2e^2x^2-165bcde\,x^2+128d^2c^2x^2+15b^2dex-19xbc\,d^2+3b^2d^2)}{15b^6x^2\sqrt{x(cx+b)}}+\frac{2x(8b^2c^2e^2x^2-91b\,c^3de\,x^2+128c^4d^2x^2+}{}$
gosper	$-\frac{2x(cx+b)(-8b^3c^2e^3x^5+144b^2c^3d\,e^2x^5-384b\,c^4d^2e\,x^5+256c^5d^3x^5-20b^4c\,e^3x^4+360b^3c^2d\,e^2x^4-960b^2c^3d^2e\,x^4+640b\,c}{}$
orering	$-\frac{2x(cx+b)(-8b^3c^2e^3x^5+144b^2c^3d\,e^2x^5-384b\,c^4d^2e\,x^5+256c^5d^3x^5-20b^4c\,e^3x^4+360b^3c^2d\,e^2x^4-960b^2c^3d^2e\,x^4+640b\,c}{}$
trager	$-\frac{2(-8b^3c^2e^3x^5+144b^2c^3d\,e^2x^5-384b\,c^4d^2e\,x^5+256c^5d^3x^5-20b^4c\,e^3x^4+360b^3c^2d\,e^2x^4-960b^2c^3d^2e\,x^4+640b\,c^4d^3x^4-}{}$
default	$d^3\left(-\frac{2(2cx+b)}{5b^2(c\,x^2+bx)^{\frac{5}{2}}}-\frac{16c\left(-\frac{2(2cx+b)}{3b^2\left(c\,x^2+bx\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{c\,x^2+bx}}\right)}{5b^2}\right)+e^3\left(-\frac{x^2}{3c(c\,x^2+bx)^{\frac{5}{2}}}-\frac{b\left(-\frac{x}{4c\left(c\,x^2+bx\right)^{\frac{5}{2}}}\right.}{}\right.$

input
```
int((e*x+d)^3/(c*x^2+b*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*((30*e^3*x^3-90*d*e^2*x^2-30*d^2*e*x-6*d^3)*b^5+20*c*x*(2*e^3*x^3-27*
d*e^2*x^2+12*d^2*e*x+d^3)*b^4-160*x^2*(-1/10*e^3*x^3+9/2*d*e^2*x^2-9*d^2*e
*x+d^3)*c^2*b^3-960*x^3*c^3*(3/10*e^2*x^2-2*d*e*x+d^2)*d*b^2-1280*x^4*c^4*
(-3/5*e*x+d)*d^2*b-512*c^5*d^3*x^5)/(x*(c*x+b))^(1/2)/x^2/(c*x+b)^2/b^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.15

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{7/2}}\,dx =$$
$$-\frac{2\,(3\,b^5d^3+8\,(32\,c^5d^3-48\,bc^4d^2e+18\,b^2c^3de^2-b^3c^2e^3)x^5+20\,(32\,bc^4d^3-48\,b^2c^3d^2e+18\,b^3c^2de^2-b^4c}{15\,(b^6c}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(3*b^5*d^3 + 8*(32*c^5*d^3 - 48*b*c^4*d^2*e + 18*b^2*c^3*d*e^2 - b^3
*c^2*e^3)*x^5 + 20*(32*b*c^4*d^3 - 48*b^2*c^3*d^2*e + 18*b^3*c^2*d*e^2 - b
^4*c*e^3)*x^4 + 15*(32*b^2*c^3*d^3 - 48*b^3*c^2*d^2*e + 18*b^4*c*d*e^2 - b
^5*e^3)*x^3 + 5*(16*b^3*c^2*d^3 - 24*b^4*c*d^2*e + 9*b^5*d*e^2)*x^2 - 5*(2
*b^4*c*d^3 - 3*b^5*d^2*e)*x)*sqrt(c*x^2 + b*x)/(b^6*c^3*x^6 + 3*b^7*c^2*x^
5 + 3*b^8*c*x^4 + b^9*x^3)
```

Sympy [F]

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{7/2}}\,dx = \int \frac{(d+ex)^3}{(x\,(b+cx))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**3/(c*x**2+b*x)**(7/2),x)
```

output
```
Integral((d + e*x)**3/(x*(b + c*x))**(7/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 471 vs. $2(204) = 408$.

Time = 0.04 (sec) , antiderivative size = 471, normalized size of antiderivative = 2.07

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{7/2}}\,dx = -\frac{e^3x^2}{3\,(cx^2+bx)^{\frac{5}{2}}c} - \frac{4\,cd^3x}{5\,(cx^2+bx)^{\frac{5}{2}}b^2} + \frac{64\,c^2d^3x}{15\,(cx^2+bx)^{\frac{3}{2}}b^4} - \frac{512\,c^3d^3x}{15\,\sqrt{cx^2+bx}b^6} + \frac{6\,d^2ex}{5\,(cx^2+bx)^{\frac{5}{2}}b} - \frac{32\,cd^2ex}{5\,(cx^2+bx)^{\frac{3}{2}}b^3} + \frac{256\,c^2d^2ex}{5\,\sqrt{cx^2+bx}b^5} + \frac{12\,de^2x}{5\,(cx^2+bx)^{\frac{3}{2}}b^2} - \frac{6\,de^2x}{5\,(cx^2+bx)^{\frac{5}{2}}c} - \frac{96\,cde^2x}{5\,\sqrt{cx^2+bx}b^4} + \frac{16\,e^3x}{15\,\sqrt{cx^2+bx}b^3} + \frac{be^3x}{15\,(cx^2+bx)^{\frac{5}{2}}c^2} - \frac{2\,e^3x}{15\,(cx^2+bx)^{\frac{3}{2}}bc} - \frac{2\,d^3}{5\,(cx^2+bx)^{\frac{5}{2}}b} + \frac{32\,cd^3}{15\,(cx^2+bx)^{\frac{3}{2}}b^3} - \frac{256\,c^2d^3}{15\,\sqrt{cx^2+bx}b^5} - \frac{16\,d^2e}{5\,(cx^2+bx)^{\frac{3}{2}}b^2} + \frac{128\,cd^2e}{5\,\sqrt{cx^2+bx}b^4} - \frac{48\,de^2}{5\,\sqrt{cx^2+bx}b^3} + \frac{6\,de^2}{5\,(cx^2+bx)^{\frac{3}{2}}bc} - \frac{e^3}{15\,(cx^2+bx)^{\frac{3}{2}}c^2} + \frac{8\,e^3}{15\,\sqrt{cx^2+bx}b^2c}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

output
```
-1/3*e^3*x^2/((c*x^2 + b*x)^(5/2)*c) - 4/5*c*d^3*x/((c*x^2 + b*x)^(5/2)*b^
2) + 64/15*c^2*d^3*x/((c*x^2 + b*x)^(3/2)*b^4) - 512/15*c^3*d^3*x/(sqrt(c*
x^2 + b*x)*b^6) + 6/5*d^2*e*x/((c*x^2 + b*x)^(5/2)*b) - 32/5*c*d^2*e*x/((c
*x^2 + b*x)^(3/2)*b^3) + 256/5*c^2*d^2*e*x/(sqrt(c*x^2 + b*x)*b^5) + 12/5*
d*e^2*x/((c*x^2 + b*x)^(3/2)*b^2) - 6/5*d*e^2*x/((c*x^2 + b*x)^(5/2)*c) -
96/5*c*d*e^2*x/(sqrt(c*x^2 + b*x)*b^4) + 16/15*e^3*x/(sqrt(c*x^2 + b*x)*b^
3) + 1/15*b*e^3*x/((c*x^2 + b*x)^(5/2)*c^2) - 2/15*e^3*x/((c*x^2 + b*x)^(3
/2)*b*c) - 2/5*d^3/((c*x^2 + b*x)^(5/2)*b) + 32/15*c*d^3/((c*x^2 + b*x)^(3
/2)*b^3) - 256/15*c^2*d^3/(sqrt(c*x^2 + b*x)*b^5) - 16/5*d^2*e/((c*x^2 + b
*x)^(3/2)*b^2) + 128/5*c*d^2*e/(sqrt(c*x^2 + b*x)*b^4) - 48/5*d*e^2/(sqrt(
c*x^2 + b*x)*b^3) + 6/5*d*e^2/((c*x^2 + b*x)^(3/2)*b*c) - 1/15*e^3/((c*x^2
 + b*x)^(3/2)*c^2) + 8/15*e^3/(sqrt(c*x^2 + b*x)*b^2*c)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.04

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{7/2}}\,dx = -\frac{2\left(\frac{3d^3}{b}+\left(\left(\left(4x\left(\frac{2(32c^5d^3-48bc^4d^2e+18b^2c^3de^2-b^3c^2e^3)x}{b^6}+\frac{5(32bc^4d^3-48b^2c^3d^2e+18b^3c^2de^2-b^4ce^3)}{b^6}\right)+\frac{15(32b^2c^3d^3-48b}{}\right.\right.\right.\right.}{15\,(cx^2+bx)^{\frac{5}{2}}}$$

input
```
integrate((e*x+d)^3/(c*x^2+b*x)^(7/2),x, algorithm="giac")
```

output
```
-2/15*(3*d^3/b + (((4*x*(2*(32*c^5*d^3 - 48*b*c^4*d^2*e + 18*b^2*c^3*d*e^2
 - b^3*c^2*e^3)*x/b^6 + 5*(32*b*c^4*d^3 - 48*b^2*c^3*d^2*e + 18*b^3*c^2*d*
e^2 - b^4*c*e^3)/b^6) + 15*(32*b^2*c^3*d^3 - 48*b^3*c^2*d^2*e + 18*b^4*c*d
*e^2 - b^5*e^3)/b^6)*x + 5*(16*b^3*c^2*d^3 - 24*b^4*c*d^2*e + 9*b^5*d*e^2)
/b^6)*x - 5*(2*b^4*c*d^3 - 3*b^5*d^2*e)/b^6)*x)/(c*x^2 + b*x)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 5.22 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{7/2}}\,dx = -\frac{2\,(3\,b^5\,d^3+15\,b^5\,d^2\,e\,x+45\,b^5\,d\,e^2\,x^2-15\,b^5\,e^3\,x^3-10\,b^4\,c\,d^3\,x-120\,b^4\,c\,d^2\,e\,x^2+270\,b^4\,c\,d\,e^2\,x^3-20\,b^4}{}$$

input
```
int((d + e*x)^3/(b*x + c*x^2)^(7/2),x)
```

output
```
-(2*(3*b^5*d^3 - 15*b^5*e^3*x^3 + 256*c^5*d^3*x^5 + 640*b*c^4*d^3*x^4 - 20
*b^4*c*e^3*x^4 + 45*b^5*d*e^2*x^2 + 80*b^3*c^2*d^3*x^2 + 480*b^2*c^3*d^3*x
^3 - 8*b^3*c^2*e^3*x^5 - 10*b^4*c*d^3*x + 15*b^5*d^2*e*x - 120*b^4*c*d^2*e
*x^2 + 270*b^4*c*d*e^2*x^3 - 384*b*c^4*d^2*e*x^5 - 720*b^3*c^2*d^2*e*x^3 -
 960*b^2*c^3*d^2*e*x^4 + 360*b^3*c^2*d*e^2*x^4 + 144*b^2*c^3*d*e^2*x^5))/(
15*b^6*(b*x + c*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 569, normalized size of antiderivative = 2.50

$$\int \frac{(d+ex)^3}{(bx+cx^2)^{7/2}}\,dx = \frac{-\frac{2\sqrt{x}\,b^5 c\,d^3}{5} - \frac{512\sqrt{x}\,c^6 d^3 x^5}{15} + \frac{96\sqrt{c}\,\sqrt{cx+b}\,b^4 cd\,e^2 x^3}{5} - \frac{256\sqrt{c}\,\sqrt{cx+b}\,b^3 c^2 d^2 e\,x^3}{5} + \frac{192\sqrt{c}\,\sqrt{cx+b}\,b^3 c^2 d\,e}{5}}{}$$

input
```
int((e*x+d)^3/(c*x^2+b*x)^(7/2),x)
```

output
```
(2*( - 8*sqrt(c)*sqrt(b + c*x)*b**5*e**3*x**3 + 144*sqrt(c)*sqrt(b + c*x)*
b**4*c*d*e**2*x**3 - 16*sqrt(c)*sqrt(b + c*x)*b**4*c*e**3*x**4 - 384*sqrt(
c)*sqrt(b + c*x)*b**3*c**2*d**2*e*x**3 + 288*sqrt(c)*sqrt(b + c*x)*b**3*c*
*2*d*e**2*x**4 - 8*sqrt(c)*sqrt(b + c*x)*b**3*c**2*e**3*x**5 + 256*sqrt(c)
*sqrt(b + c*x)*b**2*c**3*d**3*x**3 - 768*sqrt(c)*sqrt(b + c*x)*b**2*c**3*d
**2*e*x**4 + 144*sqrt(c)*sqrt(b + c*x)*b**2*c**3*d*e**2*x**5 + 512*sqrt(c)
*sqrt(b + c*x)*b*c**4*d**3*x**4 - 384*sqrt(c)*sqrt(b + c*x)*b*c**4*d**2*e*
x**5 + 256*sqrt(c)*sqrt(b + c*x)*c**5*d**3*x**5 - 3*sqrt(x)*b**5*c*d**3 -
15*sqrt(x)*b**5*c*d**2*e*x - 45*sqrt(x)*b**5*c*d*e**2*x**2 + 15*sqrt(x)*b*
*5*c*e**3*x**3 + 10*sqrt(x)*b**4*c**2*d**3*x + 120*sqrt(x)*b**4*c**2*d**2*
e*x**2 - 270*sqrt(x)*b**4*c**2*d*e**2*x**3 + 20*sqrt(x)*b**4*c**2*e**3*x**
4 - 80*sqrt(x)*b**3*c**3*d**3*x**2 + 720*sqrt(x)*b**3*c**3*d**2*e*x**3 - 3
60*sqrt(x)*b**3*c**3*d*e**2*x**4 + 8*sqrt(x)*b**3*c**3*e**3*x**5 - 480*sqr
t(x)*b**2*c**4*d**3*x**3 + 960*sqrt(x)*b**2*c**4*d**2*e*x**4 - 144*sqrt(x)
*b**2*c**4*d*e**2*x**5 - 640*sqrt(x)*b*c**5*d**3*x**4 + 384*sqrt(x)*b*c**5
*d**2*e*x**5 - 256*sqrt(x)*c**6*d**3*x**5))/(15*sqrt(b + c*x)*b**6*c*x**3*
(b**2 + 2*b*c*x + c**2*x**2))
```

3.180 $\int \frac{(d+ex)^2}{\left(bx+cx^2\right)^{7/2}}\,dx$

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Optimal result

Integrand size = 21, antiderivative size = 213

$$\int \frac{(d+ex)^2}{\left(bx+cx^2\right)^{7/2}}\,dx = -\frac{2d^2}{5b\left(bx+cx^2\right)^{5/2}} + \frac{4d(cd-be)x}{3b^2\left(bx+cx^2\right)^{5/2}} + \frac{2(4cd-3be)(4cd-be)x^2}{15b^3\left(bx+cx^2\right)^{5/2}} + \frac{4(4cd-3be)(4cd-be)x}{15b^4\left(bx+cx^2\right)^{3/2}} + \frac{16(4cd-3be)(4cd-be)}{15b^5\sqrt{bx+cx^2}} - \frac{32(4cd-3be)(4cd-be)\sqrt{bx+cx^2}}{15b^6x}$$

output

```
-2/5*d^2/b/(c*x^2+b*x)^(5/2)+4/3*d*(-b*e+c*d)*x/b^2/(c*x^2+b*x)^(5/2)+2/15
*(-3*b*e+4*c*d)*(-b*e+4*c*d)*x^2/b^3/(c*x^2+b*x)^(5/2)+4/15*(-3*b*e+4*c*d)
*(-b*e+4*c*d)*x/b^4/(c*x^2+b*x)^(3/2)+16/15*(-3*b*e+4*c*d)*(-b*e+4*c*d)/b^
5/(c*x^2+b*x)^(1/2)-32/15*(-3*b*e+4*c*d)*(-b*e+4*c*d)*(c*x^2+b*x)^(1/2)/b^
6/x
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.73

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{7/2}}\,dx = -\frac{2(256c^5d^2x^5+128bc^4dx^4(5d-2ex)-10b^4cx(d^2+8dex-9e^2x^2)+16b^2c^3x^3(30d^2-40dex+3e^2x^2)+}{15b^6(x(b+cx))^{5/2}}$$

input
```
Integrate[(d + e*x)^2/(b*x + c*x^2)^(7/2),x]
```

output
```
(-2*(256*c^5*d^2*x^5 + 128*b*c^4*d*x^4*(5*d - 2*e*x) - 10*b^4*c*x*(d^2 + 8
*d*e*x - 9*e^2*x^2) + 16*b^2*c^3*x^3*(30*d^2 - 40*d*e*x + 3*e^2*x^2) + 40*
b^3*c^2*x^2*(2*d^2 - 12*d*e*x + 3*e^2*x^2) + b^5*(3*d^2 + 10*d*e*x + 15*e^
2*x^2)))/(15*b^6*(x*(b + c*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.69, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1164, 1159, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{1164}$$

$$-\frac{2\int \frac{d(8cd-5be)+3e(2cd-be)x}{(cx^2+bx)^{5/2}}dx}{5b^2}-\frac{2(d+ex)(x(2cd-be)+bd)}{5b^2\left(bx+cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{1159}$$

$$-\frac{2\left(-\frac{4(4cd-3be)(4cd-be)\int \frac{1}{\left(cx^2+bx\right)^{3/2}}dx}{3b^2}-\frac{2(x(4cd-3be)(4cd-be)+bd(8cd-5be))}{3b^2(bx+cx^2)^{3/2}}\right)}{5b^2}-\frac{2(d+ex)(x(2cd-be)+bd)}{5b^2\left(bx+cx^2\right)^{5/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{2(d+ex)(x(2cd-be)+bd)}{5b^2\left(bx+cx^2\right)^{5/2}}-\frac{2\left(\frac{8(b+2cx)(4cd-3be)(4cd-be)}{3b^4\sqrt{bx+cx^2}}-\frac{2(x(4cd-3be)(4cd-be)+bd(8cd-5be))}{3b^2(bx+cx^2)^{3/2}}\right)}{5b^2}$$

input
```
Int[(d + e*x)^2/(b*x + c*x^2)^(7/2),x]
```

output
```
(-2*(d + e*x)*(b*d + (2*c*d - b*e)*x))/(5*b^2*(b*x + c*x^2)^(5/2)) - (2*((
-2*(b*d*(8*c*d - 5*b*e) + (4*c*d - 3*b*e)*(4*c*d - b*e)*x))/(3*b^2*(b*x +
c*x^2)^(3/2)) + (8*(4*c*d - 3*b*e)*(4*c*d - b*e)*(b + 2*c*x))/(3*b^4*Sqrt[
b*x + c*x^2])))/(5*b^2)
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.71

method	result
pseudoelliptic	$\frac{(-30e^2x^2-20dex-6d^2)b^5+20cx(9ex+d)(-ex+d)b^4-160\left(\frac{3}{2}e^2x^2-6dex+d^2\right)x^2c^2b^3-960x^3c^3\left(\frac{1}{10}e^2x^2-\frac{4}{3}dex+d^2\right)b^2-12}{15\sqrt{x(cx+b)}\,x^2(cx+b)^2b^6}$
gosper	$-\frac{2x(cx+b)\left(48b^2c^3e^2x^5-256b\,c^4de\,x^5+256c^5d^2x^5+120b^3c^2e^2x^4-640b^2c^3de\,x^4+640b\,c^4d^2x^4+90b^4c\,e^2x^3-480b^3c^2de\,x^3\right.}{15b^6(c\,x^2+bx)^{\frac{7}{2}}}$
orering	$-\frac{2x(cx+b)\left(48b^2c^3e^2x^5-256b\,c^4de\,x^5+256c^5d^2x^5+120b^3c^2e^2x^4-640b^2c^3de\,x^4+640b\,c^4d^2x^4+90b^4c\,e^2x^3-480b^3c^2de\,x^3\right.}{15b^6(c\,x^2+bx)^{\frac{7}{2}}}$
trager	$-\frac{2\left(48b^2c^3e^2x^5-256b\,c^4de\,x^5+256c^5d^2x^5+120b^3c^2e^2x^4-640b^2c^3de\,x^4+640b\,c^4d^2x^4+90b^4c\,e^2x^3-480b^3c^2de\,x^3+480b^2c\right.}{15b^6(cx+b)^3x^3}$
risch	$-\frac{2(cx+b)\left(15b^2e^2x^2-110bcde\,x^2+128d^2c^2x^2+10b^2dex-19xbc\,d^2+3b^2d^2\right)}{15b^6x^2\sqrt{x(cx+b)}}-\frac{2c\left(33b^2c^2e^2x^2-146b\,c^3de\,x^2+128c^4d^2x^2\right.}{15}$
default	$d^2\left(-\frac{2(2cx+b)}{5b^2(c\,x^2+bx)^{\frac{5}{2}}}-\frac{16c\left(-\frac{2(2cx+b)}{3b^2\left(c\,x^2+bx\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{c\,x^2+bx}}\right)}{5b^2}\right)+e^2\left(-\frac{x}{4c(c\,x^2+bx)^{\frac{5}{2}}}-\frac{3b\left(-\frac{1}{5c\left(c\,x^2+bx\right)^{\frac{5}{2}}}\right.}{}\right.$

input
```
int((e*x+d)^2/(c*x^2+b*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*((-30*e^2*x^2-20*d*e*x-6*d^2)*b^5+20*c*x*(9*e*x+d)*(-e*x+d)*b^4-160*(
3/2*e^2*x^2-6*d*e*x+d^2)*x^2*c^2*b^3-960*x^3*c^3*(1/10*e^2*x^2-4/3*d*e*x+d
^2)*b^2-1280*(-2/5*e*x+d)*x^4*c^4*d*b-512*c^5*d^2*x^5)/(x*(c*x+b))^(1/2)/x
^2/(c*x+b)^2/b^6
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.03

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{7/2}}\,dx =$$

$$-\frac{2\,(3\,b^5d^2+16\,(16\,c^5d^2-16\,bc^4de+3\,b^2c^3e^2)x^5+40\,(16\,bc^4d^2-16\,b^2c^3de+3\,b^3c^2e^2)x^4+30\,(16\,b^2c^3d^2-}{15\,(b^6c^3x^6+3\,b^7c^2x^5+3}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(3*b^5*d^2 + 16*(16*c^5*d^2 - 16*b*c^4*d*e + 3*b^2*c^3*e^2)*x^5 + 40
*(16*b*c^4*d^2 - 16*b^2*c^3*d*e + 3*b^3*c^2*e^2)*x^4 + 30*(16*b^2*c^3*d^2
- 16*b^3*c^2*d*e + 3*b^4*c*e^2)*x^3 + 5*(16*b^3*c^2*d^2 - 16*b^4*c*d*e + 3
*b^5*e^2)*x^2 - 10*(b^4*c*d^2 - b^5*d*e)*x)*sqrt(c*x^2 + b*x)/(b^6*c^3*x^6
 + 3*b^7*c^2*x^5 + 3*b^8*c*x^4 + b^9*x^3)
```

Sympy [F]

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{7/2}}\,dx = \int \frac{(d+ex)^2}{(x\,(b+cx))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)**2/(c*x**2+b*x)**(7/2),x)
```

output
```
Integral((d + e*x)**2/(x*(b + c*x))**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.54

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{7/2}}\,dx = -\frac{4\,cd^2x}{5\,(cx^2+bx)^{\frac{5}{2}}b^2} + \frac{64\,c^2d^2x}{15\,(cx^2+bx)^{\frac{3}{2}}b^4} - \frac{512\,c^3d^2x}{15\,\sqrt{cx^2+bx}b^6} + \frac{4\,dex}{5\,(cx^2+bx)^{\frac{5}{2}}b} - \frac{64\,cdex}{15\,(cx^2+bx)^{\frac{3}{2}}b^3} + \frac{512\,c^2dex}{15\,\sqrt{cx^2+bx}b^5} + \frac{4\,e^2x}{5\,(cx^2+bx)^{\frac{3}{2}}b^2} - \frac{2\,e^2x}{5\,(cx^2+bx)^{\frac{5}{2}}c} - \frac{32\,ce^2x}{5\,\sqrt{cx^2+bx}b^4} - \frac{2\,d^2}{5\,(cx^2+bx)^{\frac{5}{2}}b} + \frac{32\,cd^2}{15\,(cx^2+bx)^{\frac{3}{2}}b^3} - \frac{256\,c^2d^2}{15\,\sqrt{cx^2+bx}b^5} - \frac{32\,de}{15\,(cx^2+bx)^{\frac{3}{2}}b^2} + \frac{256\,cde}{15\,\sqrt{cx^2+bx}b^4} - \frac{16\,e^2}{5\,\sqrt{cx^2+bx}b^3} + \frac{2\,e^2}{5\,(cx^2+bx)^{\frac{3}{2}}bc}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

output
```
-4/5*c*d^2*x/((c*x^2 + b*x)^(5/2)*b^2) + 64/15*c^2*d^2*x/((c*x^2 + b*x)^(3
/2)*b^4) - 512/15*c^3*d^2*x/(sqrt(c*x^2 + b*x)*b^6) + 4/5*d*e*x/((c*x^2 +
b*x)^(5/2)*b) - 64/15*c*d*e*x/((c*x^2 + b*x)^(3/2)*b^3) + 512/15*c^2*d*e*x
/(sqrt(c*x^2 + b*x)*b^5) + 4/5*e^2*x/((c*x^2 + b*x)^(3/2)*b^2) - 2/5*e^2*x
/((c*x^2 + b*x)^(5/2)*c) - 32/5*c*e^2*x/(sqrt(c*x^2 + b*x)*b^4) - 2/5*d^2/
((c*x^2 + b*x)^(5/2)*b) + 32/15*c*d^2/((c*x^2 + b*x)^(3/2)*b^3) - 256/15*c
^2*d^2/(sqrt(c*x^2 + b*x)*b^5) - 32/15*d*e/((c*x^2 + b*x)^(3/2)*b^2) + 256
/15*c*d*e/(sqrt(c*x^2 + b*x)*b^4) - 16/5*e^2/(sqrt(c*x^2 + b*x)*b^3) + 2/5
*e^2/((c*x^2 + b*x)^(3/2)*b*c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{7/2}}\,dx = -\frac{2\left(\left(\left(2\left(4\,x\left(\frac{2\,(16\,c^5d^2-16\,bc^4de+3\,b^2c^3e^2)x}{b^6} + \frac{5\,(16\,bc^4d^2-16\,b^2c^3de+3\,b^3c^2e^2)}{b^6}\right) + \frac{15\,(16\,b^2c^3d^2-16\,b^3c^2de+3\,b^4ce^2)}{b^6}\right)x + \frac{5\,(16\cdots}{}\right.\right.\right.}{15\,(cx^2+bx)^{\frac{5}{2}}}$$

input
```
integrate((e*x+d)^2/(c*x^2+b*x)^(7/2),x, algorithm="giac")
```

output
```
-2/15*((((2*(4*x*(2*(16*c^5*d^2 - 16*b*c^4*d*e + 3*b^2*c^3*e^2)*x/b^6 + 5*(
16*b*c^4*d^2 - 16*b^2*c^3*d*e + 3*b^3*c^2*e^2)/b^6) + 15*(16*b^2*c^3*d^2 -
 16*b^3*c^2*d*e + 3*b^4*c*e^2)/b^6)*x + 5*(16*b^3*c^2*d^2 - 16*b^4*c*d*e +
 3*b^5*e^2)/b^6)*x - 10*(b^4*c*d^2 - b^5*d*e)/b^6)*x + 3*d^2/b)/(c*x^2 + b
*x)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.91

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{7/2}}\,dx = -\frac{2\,(3\,b^5\,d^2+10\,b^5\,d\,e\,x+15\,b^5\,e^2\,x^2-10\,b^4\,c\,d^2\,x-80\,b^4\,c\,d\,e\,x^2+90\,b^4\,c\,e^2\,x^3+80\,b^3\,c^2\,d^2\,x^2-480\,b^3\,c^2\,d}{15\,b}$$

input
```
int((d + e*x)^2/(b*x + c*x^2)^(7/2),x)
```

output
```
-(2*(3*b^5*d^2 + 15*b^5*e^2*x^2 + 256*c^5*d^2*x^5 + 640*b*c^4*d^2*x^4 + 90
*b^4*c*e^2*x^3 + 10*b^5*d*e*x + 80*b^3*c^2*d^2*x^2 + 480*b^2*c^3*d^2*x^3 +
 120*b^3*c^2*e^2*x^4 + 48*b^2*c^3*e^2*x^5 - 10*b^4*c*d^2*x - 80*b^4*c*d*e*
x^2 - 256*b*c^4*d*e*x^5 - 480*b^3*c^2*d*e*x^3 - 640*b^2*c^3*d*e*x^4))/(15*
b^6*(b*x + c*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 422, normalized size of antiderivative = 1.98

$$\int \frac{(d+ex)^2}{(bx+cx^2)^{7/2}}\,dx = \frac{\frac{32\sqrt{c}\,\sqrt{cx+b}\,b^4e^2x^3}{5}-\frac{512\sqrt{c}\,\sqrt{cx+b}\,b^3cde\,x^3}{15}+\frac{64\sqrt{c}\,\sqrt{cx+b}\,b^3c\,e^2x^4}{5}+\frac{512\sqrt{c}\,\sqrt{cx+b}\,b^2c^2d^2x^3}{15}-\frac{1024\sqrt{c}}{}}{}$$

input
```
int((e*x+d)^2/(c*x^2+b*x)^(7/2),x)
```

output
```
(2*(48*sqrt(c)*sqrt(b + c*x)*b**4*e**2*x**3 - 256*sqrt(c)*sqrt(b + c*x)*b*
*3*c*d*e*x**3 + 96*sqrt(c)*sqrt(b + c*x)*b**3*c*e**2*x**4 + 256*sqrt(c)*sq
rt(b + c*x)*b**2*c**2*d**2*x**3 - 512*sqrt(c)*sqrt(b + c*x)*b**2*c**2*d*e*
x**4 + 48*sqrt(c)*sqrt(b + c*x)*b**2*c**2*e**2*x**5 + 512*sqrt(c)*sqrt(b +
 c*x)*b*c**3*d**2*x**4 - 256*sqrt(c)*sqrt(b + c*x)*b*c**3*d*e*x**5 + 256*s
qrt(c)*sqrt(b + c*x)*c**4*d**2*x**5 - 3*sqrt(x)*b**5*d**2 - 10*sqrt(x)*b**
5*d*e*x - 15*sqrt(x)*b**5*e**2*x**2 + 10*sqrt(x)*b**4*c*d**2*x + 80*sqrt(x
)*b**4*c*d*e*x**2 - 90*sqrt(x)*b**4*c*e**2*x**3 - 80*sqrt(x)*b**3*c**2*d**
2*x**2 + 480*sqrt(x)*b**3*c**2*d*e*x**3 - 120*sqrt(x)*b**3*c**2*e**2*x**4
- 480*sqrt(x)*b**2*c**3*d**2*x**3 + 640*sqrt(x)*b**2*c**3*d*e*x**4 - 48*sq
rt(x)*b**2*c**3*e**2*x**5 - 640*sqrt(x)*b*c**4*d**2*x**4 + 256*sqrt(x)*b*c
**4*d*e*x**5 - 256*sqrt(x)*c**5*d**2*x**5))/(15*sqrt(b + c*x)*b**6*x**3*(b
**2 + 2*b*c*x + c**2*x**2))
```

3.181 $\int \frac{d+ex}{(bx+cx^2)^{7/2}}\,dx$

Optimal result 1449
Mathematica [A] (verified) 1449
Rubi [A] (verified) 1450
Maple [A] (verified) 1451
Fricas [A] (verification not implemented) 1452
Sympy [F] 1453
Maxima [A] (verification not implemented) 1453
Giac [A] (verification not implemented) 1454
Mupad [B] (verification not implemented) 1454
Reduce [B] (verification not implemented) 1455

Optimal result

Integrand size = 19, antiderivative size = 178

$$\int \frac{d+ex}{(bx+cx^2)^{7/2}}\,dx = -\frac{2d}{5b\,(bx+cx^2)^{5/2}} - \frac{2(2cd-be)x}{5b^2\,(bx+cx^2)^{5/2}} - \frac{16(2cd-be)}{15b^3\,(bx+cx^2)^{3/2}} - \frac{32(2cd-be)}{5b^4x\sqrt{bx+cx^2}} + \frac{128(2cd-be)\sqrt{bx+cx^2}}{15b^5x^2} - \frac{256c(2cd-be)\sqrt{bx+cx^2}}{15b^6x}$$

output
```
-2/5*d/b/(c*x^2+b*x)^(5/2)-2/5*(-b*e+2*c*d)*x/b^2/(c*x^2+b*x)^(5/2)-16/15*
(-b*e+2*c*d)/b^3/(c*x^2+b*x)^(3/2)-32/5*(-b*e+2*c*d)/b^4/x/(c*x^2+b*x)^(1/
2)+128/15*(-b*e+2*c*d)*(c*x^2+b*x)^(1/2)/b^5/x^2-256/15*c*(-b*e+2*c*d)*(c*
x^2+b*x)^(1/2)/b^6/x
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.59

$$\int \frac{d+ex}{(bx+cx^2)^{7/2}}\,dx = -\frac{2(256c^5dx^5+80b^3c^2x^2(d-3ex)+160b^2c^3x^3(3d-2ex)-128bc^4x^4(-5d+ex)-10b^4cx(d+4ex)+b^5(}{15b^6(x(b+cx))^{5/2}}$$

input
```
Integrate[(d + e*x)/(b*x + c*x^2)^(7/2),x]
```

output
```
(-2*(256*c^5*d*x^5 + 80*b^3*c^2*x^2*(d - 3*e*x) + 160*b^2*c^3*x^3*(3*d - 2
*e*x) - 128*b*c^4*x^4*(-5*d + e*x) - 10*b^4*c*x*(d + 4*e*x) + b^5*(3*d + 5
*e*x)))/(15*b^6*(x*(b + c*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.60, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1159, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(bx+cx^2)^{7/2}}\,dx$$

$$\Big\downarrow \; 1159$$

$$-\frac{8(2cd-be)\int \frac{1}{(cx^2+bx)^{5/2}}dx}{5b^2}-\frac{2(x(2cd-be)+bd)}{5b^2(bx+cx^2)^{5/2}}$$

$$\Big\downarrow \; 1089$$

$$-\frac{8(2cd-be)\left(-\frac{8c\int \frac{1}{(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}\right)}{5b^2}-\frac{2(x(2cd-be)+bd)}{5b^2(bx+cx^2)^{5/2}}$$

$$\Big\downarrow \; 1088$$

$$-\frac{2(x(2cd-be)+bd)}{5b^2(bx+cx^2)^{5/2}}-\frac{8\left(\frac{16c(b+2cx)}{3b^4\sqrt{bx+cx^2}}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}\right)(2cd-be)}{5b^2}$$

input
```
Int[(d + e*x)/(b*x + c*x^2)^(7/2),x]
```

output
```
(-2*(b*d + (2*c*d - b*e)*x))/(5*b^2*(b*x + c*x^2)^(5/2)) - (8*(2*c*d - b*e
)*((-2*(b + 2*c*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (16*c*(b + 2*c*x))/(3*b^
4*Sqrt[b*x + c*x^2])))/(5*b^2)
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$\frac{(-10ex-6d)b^5+20cx(4ex+d)b^4-160c^2x^2(-3ex+d)b^3-960x^3c^3\left(-\frac{2ex}{3}+d\right)b^2-1280\left(-\frac{ex}{5}+d\right)x^4c^4b-512c^5d\,x^5}{15\sqrt{x(cx+b)}\,x^2(cx+b)^2b^6}$
gosper	$-\frac{2x(cx+b)\left(-128b\,c^4e\,x^5+256c^5d\,x^5-320b^2c^3e\,x^4+640b\,c^4d\,x^4-240b^3c^2e\,x^3+480c^3b^2d\,x^3-40b^4ce\,x^2+80b^3c^2d\,x^2+5b^5\right.}{15b^6(c\,x^2+bx)^{\frac{7}{2}}}$
orering	$-\frac{2x(cx+b)\left(-128b\,c^4e\,x^5+256c^5d\,x^5-320b^2c^3e\,x^4+640b\,c^4d\,x^4-240b^3c^2e\,x^3+480c^3b^2d\,x^3-40b^4ce\,x^2+80b^3c^2d\,x^2+5b^5\right.}{15b^6(c\,x^2+bx)^{\frac{7}{2}}}$
trager	$-\frac{2\left(-128b\,c^4e\,x^5+256c^5d\,x^5-320b^2c^3e\,x^4+640b\,c^4d\,x^4-240b^3c^2e\,x^3+480c^3b^2d\,x^3-40b^4ce\,x^2+80b^3c^2d\,x^2+5b^5ex-10b^4\right.}{15b^6(cx+b)^3x^3}$
risch	$-\frac{2(cx+b)(-55bce\,x^2+128c^2d\,x^2+5b^2ex-19bcdx+3b^2d)}{15b^6x^2\sqrt{x(cx+b)}}+\frac{2c^2\left(73be\,x^2c^2-128c^3d\,x^2+160x\,b^2ce-275b\,c^2dx+90e\,b^3-1\right.}{15\sqrt{x(cx+b)}\,(c^2x^2+2cbx+b^2)b^6}$
default	$d\left(-\frac{2(2cx+b)}{5b^2(c\,x^2+bx)^{\frac{5}{2}}}-\frac{16c\left(-\frac{2(2cx+b)}{3b^2\left(c\,x^2+bx\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{c\,x^2+bx}}\right)}{5b^2}\right)+e\left(-\frac{1}{5c(c\,x^2+bx)^{\frac{5}{2}}}-\frac{b\left(-\frac{2(2cx+b)}{5b^2\left(c\,x^2+bx\right)^{\frac{5}{2}}}-\right.}{}\right.$

input
```
int((e*x+d)/(c*x^2+b*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*((-10*e*x-6*d)*b^5+20*c*x*(4*e*x+d)*b^4-160*c^2*x^2*(-3*e*x+d)*b^3-96
0*x^3*c^3*(-2/3*e*x+d)*b^2-1280*(-1/5*e*x+d)*x^4*c^4*b-512*c^5*d*x^5)/(x*(
c*x+b))^(1/2)/x^2/(c*x+b)^2/b^6
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.92

$$\int \frac{d+ex}{(bx+cx^2)^{7/2}}\,dx =$$

$$-\frac{2\left(3\,b^5d+128\,(2\,c^5d-bc^4e)x^5+320\,(2\,bc^4d-b^2c^3e)x^4+240\,(2\,b^2c^3d-b^3c^2e)x^3+40\,(2\,b^3c^2d-b^4ce)x^2\right.}{15\,(b^6c^3x^6+3\,b^7c^2x^5+3\,b^8cx^4+b^9x^3)}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(3*b^5*d + 128*(2*c^5*d - b*c^4*e)*x^5 + 320*(2*b*c^4*d - b^2*c^3*e)
*x^4 + 240*(2*b^2*c^3*d - b^3*c^2*e)*x^3 + 40*(2*b^3*c^2*d - b^4*c*e)*x^2
- 5*(2*b^4*c*d - b^5*e)*x)*sqrt(c*x^2 + b*x)/(b^6*c^3*x^6 + 3*b^7*c^2*x^5
+ 3*b^8*c*x^4 + b^9*x^3)
```

Sympy [F]

$$\int \frac{d+ex}{(bx+cx^2)^{7/2}}\,dx = \int \frac{d+ex}{(x\,(b+cx))^{\frac{7}{2}}}\,dx$$

input
```
integrate((e*x+d)/(c*x**2+b*x)**(7/2),x)
```

output
```
Integral((d + e*x)/(x*(b + c*x))**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 210, normalized size of antiderivative = 1.18

$$\begin{aligned}\int \frac{d+ex}{(bx+cx^2)^{7/2}}\,dx = &-\frac{4\,cdx}{5\,(cx^2+bx)^{\frac{5}{2}}b^2}+\frac{64\,c^2dx}{15\,(cx^2+bx)^{\frac{3}{2}}b^4}-\frac{512\,c^3dx}{15\,\sqrt{cx^2+bx}b^6}\\&+\frac{2\,ex}{5\,(cx^2+bx)^{\frac{5}{2}}b}-\frac{32\,cex}{15\,(cx^2+bx)^{\frac{3}{2}}b^3}+\frac{256\,c^2ex}{15\,\sqrt{cx^2+bx}b^5}-\frac{2\,d}{5\,(cx^2+bx)^{\frac{5}{2}}b}\\&+\frac{32\,cd}{15\,(cx^2+bx)^{\frac{3}{2}}b^3}-\frac{256\,c^2d}{15\,\sqrt{cx^2+bx}b^5}-\frac{16\,e}{15\,(cx^2+bx)^{\frac{3}{2}}b^2}+\frac{128\,ce}{15\,\sqrt{cx^2+bx}b^4}\end{aligned}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

output
```
-4/5*c*d*x/((c*x^2 + b*x)^(5/2)*b^2) + 64/15*c^2*d*x/((c*x^2 + b*x)^(3/2)*
b^4) - 512/15*c^3*d*x/(sqrt(c*x^2 + b*x)*b^6) + 2/5*e*x/((c*x^2 + b*x)^(5/
2)*b) - 32/15*c*e*x/((c*x^2 + b*x)^(3/2)*b^3) + 256/15*c^2*e*x/(sqrt(c*x^2
 + b*x)*b^5) - 2/5*d/((c*x^2 + b*x)^(5/2)*b) + 32/15*c*d/((c*x^2 + b*x)^(3
/2)*b^3) - 256/15*c^2*d/(sqrt(c*x^2 + b*x)*b^5) - 16/15*e/((c*x^2 + b*x)^(
3/2)*b^2) + 128/15*c*e/(sqrt(c*x^2 + b*x)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.80

$$\int \frac{d+ex}{(bx+cx^2)^{7/2}}\,dx =$$
$$-\frac{2\left(\left(8\left(2\left(4x\left(\frac{2(2c^5d-bc^4e)x}{b^6}+\frac{5(2bc^4d-b^2c^3e)}{b^6}\right)+\frac{15(2b^2c^3d-b^3c^2e)}{b^6}\right)x+\frac{5(2b^3c^2d-b^4ce)}{b^6}\right)x-\frac{5(2b^4cd-b^5e)}{b^6}\right)x+\frac{3}{b}\right.}{15(cx^2+bx)^{\frac{5}{2}}}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^(7/2),x, algorithm="giac")
```

output
```
-2/15*((8*(2*(4*x*(2*(2*c^5*d - b*c^4*e)*x/b^6 + 5*(2*b*c^4*d - b^2*c^3*e)
/b^6) + 15*(2*b^2*c^3*d - b^3*c^2*e)/b^6)*x + 5*(2*b^3*c^2*d - b^4*c*e)/b^
6)*x - 5*(2*b^4*c*d - b^5*e)/b^6)*x + 3*d/b)/(c*x^2 + b*x)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 5.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.70

$$\int \frac{d+ex}{(bx+cx^2)^{7/2}}\,dx =$$
$$-\frac{2\left(5\,e\,b^5\,x+3\,d\,b^5-40\,e\,b^4\,c\,x^2-10\,d\,b^4\,c\,x-240\,e\,b^3\,c^2\,x^3+80\,d\,b^3\,c^2\,x^2-320\,e\,b^2\,c^3\,x^4+480\,d\,b^2\,c^3\,x^3\right.}{15\,b^6\,\left(c\,x^2+b\,x\right)^{5/2}}$$

input
```
int((d + e*x)/(b*x + c*x^2)^(7/2),x)
```

output
```
-(2*(3*b^5*d + 256*c^5*d*x^5 + 5*b^5*e*x + 80*b^3*c^2*d*x^2 + 480*b^2*c^3*
d*x^3 - 240*b^3*c^2*e*x^3 - 320*b^2*c^3*e*x^4 - 10*b^4*c*d*x + 640*b*c^4*d
*x^4 - 40*b^4*c*e*x^2 - 128*b*c^4*e*x^5))/(15*b^6*(b*x + c*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.55

$$\int \frac{d+ex}{(bx+cx^2)^{7/2}}\,dx = \frac{-\frac{256\sqrt{c}\,\sqrt{cx+b}\,b^3ce\,x^3}{15}+\frac{512\sqrt{c}\,\sqrt{cx+b}\,b^2c^2d\,x^3}{15}-\frac{512\sqrt{c}\,\sqrt{cx+b}\,b^2c^2e\,x^4}{15}+\frac{1024\sqrt{c}\,\sqrt{cx+b}\,b\,c^3d\,x^4}{15}-256}{}$$

input
```
int((e*x+d)/(c*x^2+b*x)^(7/2),x)
```

output
```
(2*( - 128*sqrt(c)*sqrt(b + c*x)*b**3*c*e*x**3 + 256*sqrt(c)*sqrt(b + c*x)
*b**2*c**2*d*x**3 - 256*sqrt(c)*sqrt(b + c*x)*b**2*c**2*e*x**4 + 512*sqrt(
c)*sqrt(b + c*x)*b*c**3*d*x**4 - 128*sqrt(c)*sqrt(b + c*x)*b*c**3*e*x**5 +
 256*sqrt(c)*sqrt(b + c*x)*c**4*d*x**5 - 3*sqrt(x)*b**5*d - 5*sqrt(x)*b**5
*e*x + 10*sqrt(x)*b**4*c*d*x + 40*sqrt(x)*b**4*c*e*x**2 - 80*sqrt(x)*b**3*
c**2*d*x**2 + 240*sqrt(x)*b**3*c**2*e*x**3 - 480*sqrt(x)*b**2*c**3*d*x**3
+ 320*sqrt(x)*b**2*c**3*e*x**4 - 640*sqrt(x)*b*c**4*d*x**4 + 128*sqrt(x)*b
*c**4*e*x**5 - 256*sqrt(x)*c**5*d*x**5))/(15*sqrt(b + c*x)*b**6*x**3*(b**2
 + 2*b*c*x + c**2*x**2))
```

3.182 $\int \frac{1}{(bx+cx^2)^{7/2}} dx$

Optimal result 1456
Mathematica [A] (verified) 1456
Rubi [A] (verified) 1457
Maple [A] (verified) 1458
Fricas [A] (verification not implemented) 1459
Sympy [F] 1459
Maxima [A] (verification not implemented) 1459
Giac [A] (verification not implemented) 1460
Mupad [B] (verification not implemented) 1460
Reduce [B] (verification not implemented) 1461

Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{1}{(bx+cx^2)^{7/2}} dx = \frac{2}{5b(bx+cx^2)^{5/2}} + \frac{4}{3b^2x(bx+cx^2)^{3/2}} + \frac{32}{3b^3x^2\sqrt{bx+cx^2}} - \frac{64\sqrt{bx+cx^2}}{5b^4x^3} + \frac{256c\sqrt{bx+cx^2}}{15b^5x^2} - \frac{512c^2\sqrt{bx+cx^2}}{15b^6x}$$

output
```
2/5/b/(c*x^2+b*x)^(5/2)+4/3/b^2/x/(c*x^2+b*x)^(3/2)+32/3/b^3/x^2/(c*x^2+b*
x)^(1/2)-64/5*(c*x^2+b*x)^(1/2)/b^4/x^3+256/15*c*(c*x^2+b*x)^(1/2)/b^5/x^2
-512/15*c^2*(c*x^2+b*x)^(1/2)/b^6/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.50

$$\int \frac{1}{(bx+cx^2)^{7/2}} dx = -\frac{2(3b^5 - 10b^4cx + 80b^3c^2x^2 + 480b^2c^3x^3 + 640bc^4x^4 + 256c^5x^5)}{15b^6(x(b+cx))^{5/2}}$$

input
```
Integrate[(b*x + c*x^2)^(-7/2),x]
```

output
```
(-2*(3*b^5 - 10*b^4*c*x + 80*b^3*c^2*x^2 + 480*b^2*c^3*x^3 + 640*b*c^4*x^4
 + 256*c^5*x^5))/(15*b^6*(x*(b + c*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.64, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1089, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx$$

$$\Big\downarrow \text{1089}$$

$$-\frac{16c\int \frac{1}{(cx^2+bx)^{5/2}}dx}{5b^2}-\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}}$$

$$\Big\downarrow \text{1089}$$

$$-\frac{16c\left(-\frac{8c\int \frac{1}{(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}\right)}{5b^2}-\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{2(b+2cx)}{5b^2(bx+cx^2)^{5/2}}-\frac{16c\left(\frac{16c(b+2cx)}{3b^4\sqrt{bx+cx^2}}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}\right)}{5b^2}$$

input
```
Int[(b*x + c*x^2)^(-7/2),x]
```

output
```
(-2*(b + 2*c*x))/(5*b^2*(b*x + c*x^2)^(5/2)) - (16*c*((-2*(b + 2*c*x))/(3*
b^2*(b*x + c*x^2)^(3/2)) + (16*c*(b + 2*c*x))/(3*b^4*Sqrt[b*x + c*x^2])))/
(5*b^2)
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.54

method	result	size
gosper	$-\frac{2x(cx+b)\left(256c^{5}x^{5}+640b\,x^{4}c^{4}+480b^{2}c^{3}x^{3}+80c^{2}x^{2}b^{3}-10b^{4}cx+3b^{5}\right)}{15b^{6}\left(c\,x^{2}+bx\right)^{\frac{7}{2}}}$	75
orering	$-\frac{2x(cx+b)\left(256c^{5}x^{5}+640b\,x^{4}c^{4}+480b^{2}c^{3}x^{3}+80c^{2}x^{2}b^{3}-10b^{4}cx+3b^{5}\right)}{15b^{6}\left(c\,x^{2}+bx\right)^{\frac{7}{2}}}$	75
default	$-\frac{2(2cx+b)}{5b^{2}\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}-\frac{16c\left(-\frac{2(2cx+b)}{3b^{2}\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^{4}\sqrt{c\,x^{2}+bx}}\right)}{5b^{2}}$	76
pseudoelliptic	$\frac{-\frac{512}{15}c^{5}x^{5}-\frac{256}{3}b\,x^{4}c^{4}-64b^{2}c^{3}x^{3}-\frac{32}{3}c^{2}x^{2}b^{3}+\frac{4}{3}b^{4}cx-\frac{2}{5}b^{5}}{x^{2}(cx+b)^{2}\sqrt{x(cx+b)}\,b^{6}}$	77
trager	$-\frac{2\left(256c^{5}x^{5}+640b\,x^{4}c^{4}+480b^{2}c^{3}x^{3}+80c^{2}x^{2}b^{3}-10b^{4}cx+3b^{5}\right)\sqrt{c\,x^{2}+bx}}{15b^{6}(cx+b)^{3}x^{3}}$	79
risch	$-\frac{2(cx+b)\left(128c^{2}x^{2}-19cbx+3b^{2}\right)}{15b^{6}x^{2}\sqrt{x(cx+b)}}-\frac{2c^{3}\left(128c^{2}x^{2}+275cbx+150b^{2}\right)x}{15\sqrt{x(cx+b)}\,\left(c^{2}x^{2}+2cbx+b^{2}\right)b^{6}}$	98

input
```
int(1/(c*x^2+b*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*x*(c*x+b)*(256*c^5*x^5+640*b*c^4*x^4+480*b^2*c^3*x^3+80*b^3*c^2*x^2-
10*b^4*c*x+3*b^5)/b^6/(c*x^2+b*x)^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.75

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = -\frac{2\,(256\,c^5x^5+640\,bc^4x^4+480\,b^2c^3x^3+80\,b^3c^2x^2-10\,b^4cx+3\,b^5)\sqrt{cx^2+bx}}{15\,(b^6c^3x^6+3\,b^7c^2x^5+3\,b^8cx^4+b^9x^3)}$$

input
```
integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*(256*c^5*x^5 + 640*b*c^4*x^4 + 480*b^2*c^3*x^3 + 80*b^3*c^2*x^2 - 10
*b^4*c*x + 3*b^5)*sqrt(c*x^2 + b*x)/(b^6*c^3*x^6 + 3*b^7*c^2*x^5 + 3*b^8*c
*x^4 + b^9*x^3)
```

Sympy [F]

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = \int \frac{1}{(bx+cx^2)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(c*x**2+b*x)**(7/2),x)
```

output
```
Integral((b*x + c*x**2)**(-7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.79

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = -\frac{4\,cx}{5\,(cx^2+bx)^{\frac{5}{2}}b^2}+\frac{64\,c^2x}{15\,(cx^2+bx)^{\frac{3}{2}}b^4} -\frac{512\,c^3x}{15\,\sqrt{cx^2+bx}b^6}-\frac{2}{5\,(cx^2+bx)^{\frac{5}{2}}b}+\frac{32\,c}{15\,(cx^2+bx)^{\frac{3}{2}}b^3}-\frac{256\,c^2}{15\,\sqrt{cx^2+bx}b^5}$$

input
```
integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

output
```
-4/5*c*x/((c*x^2 + b*x)^(5/2)*b^2) + 64/15*c^2*x/((c*x^2 + b*x)^(3/2)*b^4)
 - 512/15*c^3*x/(sqrt(c*x^2 + b*x)*b^6) - 2/5/((c*x^2 + b*x)^(5/2)*b) + 32
/15*c/((c*x^2 + b*x)^(3/2)*b^3) - 256/15*c^2/(sqrt(c*x^2 + b*x)*b^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.53

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = -\frac{2\left(2\left(8\left(2\left(4\,x\left(\frac{2\,c^5x}{b^6}+\frac{5\,c^4}{b^5}\right)+\frac{15\,c^3}{b^4}\right)x+\frac{5\,c^2}{b^3}\right)x-\frac{5\,c}{b^2}\right)x+\frac{3}{b}\right)}{15\,(cx^2+bx)^{\frac{5}{2}}}$$

input
```
integrate(1/(c*x^2+b*x)^(7/2),x, algorithm="giac")
```

output
```
-2/15*(2*(8*(2*(4*x*(2*c^5*x/b^6 + 5*c^4/b^5) + 15*c^3/b^4)*x + 5*c^2/b^3)
*x - 5*c/b^2)*x + 3/b)/(c*x^2 + b*x)^(5/2)
```

Mupad [B] (verification not implemented)

Time = 5.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.69

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx =$$
$$-\frac{6\,b^5+256\,b\,c^2\,\left(c\,x^2+b\,x\right)^2+512\,c^3\,x\,\left(c\,x^2+b\,x\right)^2-32\,b^3\,c\,\left(c\,x^2+b\,x\right)+12\,b^4\,c\,x-64\,b^2\,c^2\,x\,\left(c\,x^2+b\,x\right)}{15\,b^6\,\left(c\,x^2+b\,x\right)^{5/2}}$$

input
```
int(1/(b*x + c*x^2)^(7/2),x)
```

output
```
-(6*b^5 + 256*b*c^2*(b*x + c*x^2)^2 + 512*c^3*x*(b*x + c*x^2)^2 - 32*b^3*c
*(b*x + c*x^2) + 12*b^4*c*x - 64*b^2*c^2*x*(b*x + c*x^2))/(15*b^6*(b*x + c
*x^2)^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.07

$$\int \frac{1}{(bx+cx^2)^{7/2}}\,dx = \frac{\frac{512\sqrt{c}\,\sqrt{cx+b}\,b^2c^2x^3}{15} + \frac{1024\sqrt{c}\,\sqrt{cx+b}\,b\,c^3x^4}{15} + \frac{512\sqrt{c}\,\sqrt{cx+b}\,c^4x^5}{15} - \frac{2\sqrt{x}\,b^5}{5} + \frac{4\sqrt{x}\,b^4cx}{3} - \frac{32\sqrt{x}\,b^3c^2x^2}{3}}{\sqrt{cx+b}\,b^6x^3\left(c^2x^2+2bcx+b^2\right)}$$

input
```
int(1/(c*x^2+b*x)^(7/2),x)
```

output
```
(2*(256*sqrt(c)*sqrt(b + c*x)*b**2*c**2*x**3 + 512*sqrt(c)*sqrt(b + c*x)*b
*c**3*x**4 + 256*sqrt(c)*sqrt(b + c*x)*c**4*x**5 - 3*sqrt(x)*b**5 + 10*sqr
t(x)*b**4*c*x - 80*sqrt(x)*b**3*c**2*x**2 - 480*sqrt(x)*b**2*c**3*x**3 - 6
40*sqrt(x)*b*c**4*x**4 - 256*sqrt(x)*c**5*x**5))/(15*sqrt(b + c*x)*b**6*x*
*3*(b**2 + 2*b*c*x + c**2*x**2))
```

3.183 $\int \frac{1}{(d+ex)(bx+cx^2)^{7/2}}\,dx$

Optimal result 1462
Mathematica [A] (verified) 1463
Rubi [A] (verified) 1463
Maple [A] (verified) 1466
Fricas [B] (verification not implemented) 1467
Sympy [F] 1468
Maxima [F(-2)] 1469
Giac [B] (verification not implemented) 1469
Mupad [F(-1)] 1470
Reduce [B] (verification not implemented) 1471

Optimal result

Integrand size = 21, antiderivative size = 417

$$\int \frac{1}{(d+ex)(bx+cx^2)^{7/2}}\,dx = -\frac{2}{5bd(bx+cx^2)^{5/2}}+\frac{2(2cd+be)x}{3b^2d^2(bx+cx^2)^{5/2}} -\frac{2(16c^2d^2+8bcde+3b^2e^2)x^2}{3b^3d^3(bx+cx^2)^{5/2}}-\frac{2c(32c^3d^3-16bc^2d^2e-10b^2cde^2-5b^3e^3)x^3}{5b^4d^3(cd-be)(bx+cx^2)^{5/2}} -\frac{2c(128c^4d^4-192bc^3d^3e+24b^2c^2d^2e^2+20b^3cde^3+15b^4e^4)x^2}{15b^5d^3(cd-be)^2(bx+cx^2)^{3/2}} -\frac{2c(2cd-be)(128c^4d^4-256bc^3d^3e+88b^2c^2d^2e^2+40b^3cde^3+15b^4e^4)x}{15b^6d^3(cd-be)^3\sqrt{bx+cx^2}} +\frac{2e^6\operatorname{arctanh}\left(\frac{\sqrt{cd-be}\,x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{d^{7/2}(cd-be)^{7/2}}$$

output
```
-2/5/b/d/(c*x^2+b*x)^(5/2)+2/3*(b*e+2*c*d)*x/b^2/d^2/(c*x^2+b*x)^(5/2)-2/3
*(3*b^2*e^2+8*b*c*d*e+16*c^2*d^2)*x^2/b^3/d^3/(c*x^2+b*x)^(5/2)-2/5*c*(-5*
b^3*e^3-10*b^2*c*d*e^2-16*b*c^2*d^2*e+32*c^3*d^3)*x^3/b^4/d^3/(-b*e+c*d)/(
c*x^2+b*x)^(5/2)-2/15*c*(15*b^4*e^4+20*b^3*c*d*e^3+24*b^2*c^2*d^2*e^2-192*
b*c^3*d^3*e+128*c^4*d^4)*x^2/b^5/d^3/(-b*e+c*d)^2/(c*x^2+b*x)^(3/2)-2/15*c
*(-b*e+2*c*d)*(15*b^4*e^4+40*b^3*c*d*e^3+88*b^2*c^2*d^2*e^2-256*b*c^3*d^3*
e+128*c^4*d^4)*x/b^6/d^3/(-b*e+c*d)^3/(c*x^2+b*x)^(1/2)+2*e^6*arctanh((-b*
e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(1/2))/d^(7/2)/(-b*e+c*d)^(7/2)
```

Mathematica [A] (verified)

Time = 2.14 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.08

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{7/2}}\,dx = \frac{2x\left(-\frac{\sqrt{d}(b+cx)\left(-256c^8d^5x^5-640bc^7d^4x^4(d-ex)+b^8e^3\left(3d^2-5dex+15e^2x^2\right)-16b^2c^6d^3x^3\left(30d^2-10\right.\right.}{}\right.}{}$$

input
```
Integrate[1/((d + e*x)*(b*x + c*x^2)^(7/2)),x]
```

output
```
(2*x*(-((Sqrt[d]*(b + c*x)*(-256*c^8*d^5*x^5 - 640*b*c^7*d^4*x^4*(d - e*x)
 + b^8*e^3*(3*d^2 - 5*d*e*x + 15*e^2*x^2) - 16*b^2*c^6*d^3*x^3*(30*d^2 - 1
00*d*e*x + 27*e^2*x^2) + 8*b^3*c^5*d^2*x^2*(-10*d^3 + 150*d^2*e*x - 135*d*
e^2*x^2 + e^3*x^3) + b^7*c*e^2*(-9*d^3 + 5*d^2*e*x - 5*d*e^2*x^2 + 45*e^3*
x^3) + 10*b^4*c^4*d*x*(d^4 + 20*d^3*e*x - 81*d^2*e^2*x^2 + 2*d*e^3*x^3 + e
^4*x^4) + b^6*c^2*e*(9*d^4 + 15*d^3*e*x + 5*d^2*e^2*x^2 + 15*d*e^3*x^3 + 4
5*e^4*x^4) + b^5*c^3*(-3*d^5 - 25*d^4*e*x - 135*d^3*e^2*x^2 + 15*d^2*e^3*x
^3 + 25*d*e^4*x^4 + 15*e^5*x^5)))/(b^6*(-(c*d) + b*e)^3)) + (15*e^6*x^(5/2
)*(b + c*x)^(7/2)*ArcTan[(-(e*Sqrt[x]*Sqrt[b + c*x]) + Sqrt[c]*(d + e*x))/
(Sqrt[d]*Sqrt[-(c*d) + b*e])])/(-(c*d) + b*e)^(7/2)))/(15*d^(7/2)*(x*(b +
c*x))^(7/2))
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {1165, 27, 1235, 27, 1235, 27, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(bx+cx^2\right)^{7/2}(d+ex)}dx$$

$$\Big\downarrow \; 1165$$

$$-\frac{2\int \frac{16c^2d^2-8bced-5b^2e^2+8ce(2cd-be)x}{2(d+ex)\left(cx^2+bx\right)^{5/2}}dx}{5b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{16c^2d^2-8bced-5b^2e^2+8ce(2cd-be)x}{(d+ex)(cx^2+bx)^{5/2}}dx}{5b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 1235$$

$$-\frac{-\frac{2\int \frac{128c^4d^4-192bc^3ed^3+24b^2c^2e^2d^2+20b^3ce^3d+15b^4e^4+4ce(4cd-5be)(2cd-be)(4cd+be)x}{2(d+ex)\left(cx^2+bx\right)^{3/2}}dx}{3b^2d(cd-be)}-\frac{2\left(b(cd-be)\left(-5b^2e^2-8bcde+16c^2d^2\right)+cx(4cd-5b\right.}{3b^2d(bx+cx^2)^{3/2}(cd-be)}}{5b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\int \frac{128c^4d^4-192bc^3ed^3+24b^2c^2e^2d^2+20b^3ce^3d+15b^4e^4+4ce(4cd-5be)(2cd-be)(4cd+be)x}{(d+ex)\left(cx^2+bx\right)^{3/2}}dx}{3b^2d(cd-be)}-\frac{2\left(b(cd-be)\left(-5b^2e^2-8bcde+16c^2d^2\right)+cx(4cd-5be\right.}{3b^2d(bx+cx^2)^{3/2}(cd-be)}}{5b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 1235$$

$$-\frac{-\frac{-\frac{2\int -\frac{15b^6e^6}{2(d+ex)\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2\left(cx(2cd-be)\left(15b^4e^4+40b^3cde^3+88b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)+b(cd-be)\left(15b^4e^4+20b^3cde^3+24b^2c^2d^2e^2-192bc^3d^3\right.\right.}{b^2d\sqrt{bx+cx^2}(cd-be)}}{3b^2d(cd-be)}}{5b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 27$$

$$-\frac{-\frac{\frac{15b^4e^6\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{d(cd-be)}-\frac{2\left(cx(2cd-be)\left(15b^4e^4+40b^3cde^3+88b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)+b(cd-be)\left(15b^4e^4+20b^3cde^3+24b^2c^2d^2e^2-192bc^3d\right.\right.}{b^2d\sqrt{bx+cx^2}(cd-be)}}{3b^2d(cd-be)}}{5b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 1154$$

$$-\frac{-\frac{-\frac{30b^4e^6\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{d(cd-be)}-\frac{2\left(cx(2cd-be)\left(15b^4e^4+40b^3cde^3+88b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)+b(cd-be)\left(15b^4e^4\cdots\right)\right)}{b^2d\sqrt{bx+cx^2}(cd-be)}}{3b^2d(cd-be)}}{5b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(cd-be)}$$

$$\Big\downarrow \ 219$$

$$-\frac{-\frac{-\frac{15b^4e^6\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd-be}}\right)}{d^{3/2}(cd-be)^{3/2}}-\frac{2\left(cx(2cd-be)\left(15b^4e^4+40b^3cde^3+88b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)+b(cd-be)\left(15b^4e^4+20b^3cde^3+24b^2c^2\cdots\right)\right)}{b^2d\sqrt{bx+cx^2}(cd-be)}}{3b^2d(cd-be)}}{5b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(cd-be)}$$

input
```
Int[1/((d + e*x)*(b*x + c*x^2)^(7/2)),x]
```

output
```
(-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(5*b^2*d*(c*d - b*e)*(b*x + c*x^2
)^(5/2)) - ((-2*(b*(c*d - b*e)*(16*c^2*d^2 - 8*b*c*d*e - 5*b^2*e^2) + c*(4
*c*d - 5*b*e)*(2*c*d - b*e)*(4*c*d + b*e)*x))/(3*b^2*d*(c*d - b*e)*(b*x +
c*x^2)^(3/2)) - ((-2*(b*(c*d - b*e)*(128*c^4*d^4 - 192*b*c^3*d^3*e + 24*b^
2*c^2*d^2*e^2 + 20*b^3*c*d*e^3 + 15*b^4*e^4) + c*(2*c*d - b*e)*(128*c^4*d^
4 - 256*b*c^3*d^3*e + 88*b^2*c^2*d^2*e^2 + 40*b^3*c*d*e^3 + 15*b^4*e^4)*x)
)/(b^2*d*(c*d - b*e)*Sqrt[b*x + c*x^2]) + (15*b^4*e^6*ArcTanh[(b*d + (2*c*
d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(d^(3/2)*(c*d
- b*e)^(3/2)))/(3*b^2*d*(c*d - b*e)))/(5*b^2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 369, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{2b^6e^6x^2\sqrt{x(cx+b)}\,(cx+b)^2\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)-\frac{2\sqrt{d(be-cd)}\left(-c^3(2cx+b)\left(\frac{128}{3}c^4x^4+\frac{256}{3}bc^3x^3+\frac{112}{3}b^2c^2x^2-\frac{16}{3}b^3cx+b^4\right)c\right.}{}}{}$
risch	$-\frac{2(cx+b)(15b^2e^2x^2+55bcde\,x^2+128d^2c^2x^2-5b^2dex-19xbc\,d^2+3b^2d^2)}{15b^6d^3\sqrt{x(cx+b)}\,x^2}-\frac{-\frac{b^5e^5\ln\left(\frac{-\frac{2d(be-cd)}{e^2}+\frac{(be-2cd)\left(x+\frac{d}{e}\right)}{e}+2\sqrt{}}{}\right)}{}}{(be-cc}$
default	Expression too large to display

input
```
int(1/(e*x+d)/(c*x^2+b*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b^6*e^6*x^2*(x*(c*x+b))^(1/2)*(c*x+b)^2*arctan((x*(c*x+b))^(1/2)/x*d/(d
*(b*e-c*d))^(1/2))-1/5*(d*(b*e-c*d))^(1/2)*(-c^3*(2*c*x+b)*(128/3*c^4*x^4+
256/3*b*c^3*x^3+112/3*b^2*c^2*x^2-16/3*b^3*c*x+b^4)*d^5+3*e*(640/9*c^5*x^5
+1600/9*b*x^4*c^4+400/3*b^2*c^3*x^3+200/9*c^2*x^2*b^3-25/9*b^4*c*x+b^5)*c^
2*b*d^4-3*e^2*(48*c^5*x^5+120*b*x^4*c^4+90*b^2*c^3*x^3+15*c^2*x^2*b^3-5/3*
b^4*c*x+b^5)*c*b^2*d^3+(8/3*c^2*x^2-4/3*c*b*x+b^2)*e^3*(c*x+b)^3*b^3*d^2-5
/3*b^4*e^4*x*(-2*c*x+b)*(c*x+b)^3*d+5*b^5*e^5*x^2*(c*x+b)^3))/(x*(c*x+b))^
(1/2)/(d*(b*e-c*d))^(1/2)/x^2/d^3/(b*e-c*d)^3/(c*x+b)^2/b^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 883 vs. 2(383) = 766.

Time = 0.19 (sec) , antiderivative size = 1782, normalized size of antiderivative = 4.27

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

output
```
[-1/15*(15*(b^6*c^3*e^6*x^6 + 3*b^7*c^2*e^6*x^5 + 3*b^8*c*e^6*x^4 + b^9*e^
6*x^3)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*e)*x - 2*sqrt(c*d^2 - b*d
*e)*sqrt(c*x^2 + b*x))/(e*x + d)) + 2*(3*b^5*c^4*d^7 - 12*b^6*c^3*d^6*e +
18*b^7*c^2*d^5*e^2 - 12*b^8*c*d^4*e^3 + 3*b^9*d^3*e^4 + (256*c^9*d^7 - 896
*b*c^8*d^6*e + 1072*b^2*c^7*d^5*e^2 - 440*b^3*c^6*d^4*e^3 - 2*b^4*c^5*d^3*
e^4 - 5*b^5*c^4*d^2*e^5 + 15*b^6*c^3*d*e^6)*x^5 + 5*(128*b*c^8*d^7 - 448*b
^2*c^7*d^6*e + 536*b^3*c^6*d^5*e^2 - 220*b^4*c^5*d^4*e^3 - b^5*c^4*d^3*e^4
 - 4*b^6*c^3*d^2*e^5 + 9*b^7*c^2*d*e^6)*x^4 + 15*(32*b^2*c^7*d^7 - 112*b^3
*c^6*d^6*e + 134*b^4*c^5*d^5*e^2 - 55*b^5*c^4*d^4*e^3 - 2*b^7*c^2*d^2*e^5
+ 3*b^8*c*d*e^6)*x^3 + 5*(16*b^3*c^6*d^7 - 56*b^4*c^5*d^6*e + 67*b^5*c^4*d
^5*e^2 - 28*b^6*c^3*d^4*e^3 + 2*b^7*c^2*d^3*e^4 - 4*b^8*c*d^2*e^5 + 3*b^9*
d*e^6)*x^2 - 5*(2*b^4*c^5*d^7 - 7*b^5*c^4*d^6*e + 8*b^6*c^3*d^5*e^2 - 2*b^
7*c^2*d^4*e^3 - 2*b^8*c*d^3*e^4 + b^9*d^2*e^5)*x)*sqrt(c*x^2 + b*x))/((b^6
*c^7*d^8 - 4*b^7*c^6*d^7*e + 6*b^8*c^5*d^6*e^2 - 4*b^9*c^4*d^5*e^3 + b^10*
c^3*d^4*e^4)*x^6 + 3*(b^7*c^6*d^8 - 4*b^8*c^5*d^7*e + 6*b^9*c^4*d^6*e^2 -
4*b^10*c^3*d^5*e^3 + b^11*c^2*d^4*e^4)*x^5 + 3*(b^8*c^5*d^8 - 4*b^9*c^4*d^
7*e + 6*b^10*c^3*d^6*e^2 - 4*b^11*c^2*d^5*e^3 + b^12*c*d^4*e^4)*x^4 + (b^9
*c^4*d^8 - 4*b^10*c^3*d^7*e + 6*b^11*c^2*d^6*e^2 - 4*b^12*c*d^5*e^3 + b^13
*d^4*e^4)*x^3), -2/15*(15*(b^6*c^3*e^6*x^6 + 3*b^7*c^2*e^6*x^5 + 3*b^8*c*e
^6*x^4 + b^9*e^6*x^3)*sqrt(-c*d^2 + b*d*e)*arctan(sqrt(-c*d^2 + b*d*e)*...
```

Sympy [F]

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{7/2}}\,dx = \int \frac{1}{\left(x\left(b+cx\right)\right)^{\frac{7}{2}}\left(d+ex\right)}\,dx$$

input
```
integrate(1/(e*x+d)/(c*x**2+b*x)**(7/2),x)
```

output
```
Integral(1/((x*(b + c*x))**(7/2)*(d + e*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1851 vs. 2(383) = 766.

Time = 0.18 (sec) , antiderivative size = 1851, normalized size of antiderivative = 4.44

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)/(c*x^2+b*x)^(7/2),x, algorithm="giac")
```

output
```
-2*e^6*arctan(((sqrt(c)*x - sqrt(c*x^2 + b*x))*e + sqrt(c)*d)/sqrt(-c*d^2
+ b*d*e))/((c^3*d^6 - 3*b*c^2*d^5*e + 3*b^2*c*d^4*e^2 - b^3*d^3*e^3)*sqrt(
-c*d^2 + b*d*e)) - 2/15*((((((256*c^14*d^23 - 2176*b*c^13*d^22*e + 8112*b^
2*c^12*d^21*e^2 - 17320*b^3*c^11*d^20*e^3 + 23158*b^4*c^10*d^19*e^4 - 1985
1*b^5*c^9*d^18*e^5 + 10676*b^6*c^8*d^17*e^6 - 3377*b^7*c^7*d^16*e^7 + 630*
b^8*c^6*d^15*e^8 - 173*b^9*c^5*d^14*e^9 + 80*b^10*c^4*d^13*e^10 - 15*b^11*
c^3*d^12*e^11)*x/(b^6*c^9*d^24 - 9*b^7*c^8*d^23*e + 36*b^8*c^7*d^22*e^2 -
84*b^9*c^6*d^21*e^3 + 126*b^10*c^5*d^20*e^4 - 126*b^11*c^4*d^19*e^5 + 84*b
^12*c^3*d^18*e^6 - 36*b^13*c^2*d^17*e^7 + 9*b^14*c*d^16*e^8 - b^15*d^15*e^
9) + 5*(128*b*c^13*d^23 - 1088*b^2*c^12*d^22*e + 4056*b^3*c^11*d^21*e^2 -
8660*b^4*c^10*d^20*e^3 + 11579*b^5*c^9*d^19*e^4 - 9927*b^6*c^8*d^18*e^5 +
5347*b^7*c^7*d^17*e^6 - 1711*b^8*c^6*d^16*e^7 + 345*b^9*c^5*d^15*e^8 - 109
*b^10*c^4*d^14*e^9 + 49*b^11*c^3*d^13*e^10 - 9*b^12*c^2*d^12*e^11)/(b^6*c^
9*d^24 - 9*b^7*c^8*d^23*e + 36*b^8*c^7*d^22*e^2 - 84*b^9*c^6*d^21*e^3 + 12
6*b^10*c^5*d^20*e^4 - 126*b^11*c^4*d^19*e^5 + 84*b^12*c^3*d^18*e^6 - 36*b^
13*c^2*d^17*e^7 + 9*b^14*c*d^16*e^8 - b^15*d^15*e^9))*x + 15*(32*b^2*c^12*
d^23 - 272*b^3*c^11*d^22*e + 1014*b^4*c^10*d^21*e^2 - 2165*b^5*c^9*d^20*e^
3 + 2895*b^6*c^8*d^19*e^4 - 2484*b^7*c^7*d^18*e^5 + 1345*b^8*c^6*d^17*e^6
- 444*b^9*c^5*d^16*e^7 + 105*b^10*c^4*d^15*e^8 - 40*b^11*c^3*d^14*e^9 + 17
*b^12*c^2*d^13*e^10 - 3*b^13*c*d^12*e^11)/(b^6*c^9*d^24 - 9*b^7*c^8*d^2...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{7/2}}\,dx=\int \frac{1}{\left(c\,x^2+b\,x\right)^{7/2}\,\left(d+e\,x\right)}\,dx$$

input
```
int(1/((b*x + c*x^2)^(7/2)*(d + e*x)),x)
```

output
```
int(1/((b*x + c*x^2)^(7/2)*(d + e*x)), x)
```

Reduce [B] (verification not implemented)

Time = 0.70 (sec) , antiderivative size = 1726, normalized size of antiderivative = 4.14

$$\int \frac{1}{(d+ex)\left(bx+cx^2\right)^{7/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)/(c*x^2+b*x)^(7/2),x)
```

output
```
(2*(15*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqrt(
e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**8*e**6*x
**3 + 30*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) - sqr
t(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**7*c*e*
*6*x**4 + 15*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e - c*d) -
 sqrt(e)*sqrt(b + c*x) - sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c)))*b**6*
c**2*e**6*x**5 + 15*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e -
 c*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c))
)*b**8*e**6*x**3 + 30*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt(b*e
 - c*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sqrt(c
)))*b**7*c*e**6*x**4 + 15*sqrt(d)*sqrt(b + c*x)*sqrt(b*e - c*d)*atan((sqrt
(b*e - c*d) + sqrt(e)*sqrt(b + c*x) + sqrt(x)*sqrt(e)*sqrt(c))/(sqrt(d)*sq
rt(c)))*b**6*c**2*e**6*x**5 + 9*sqrt(c)*sqrt(b + c*x)*b**8*d*e**6*x**3 + 1
3*sqrt(c)*sqrt(b + c*x)*b**7*c*d**2*e**5*x**3 + 18*sqrt(c)*sqrt(b + c*x)*b
**7*c*d*e**6*x**4 - 14*sqrt(c)*sqrt(b + c*x)*b**6*c**2*d**3*e**4*x**3 + 26
*sqrt(c)*sqrt(b + c*x)*b**6*c**2*d**2*e**5*x**4 + 9*sqrt(c)*sqrt(b + c*x)*
b**6*c**2*d*e**6*x**5 - 440*sqrt(c)*sqrt(b + c*x)*b**5*c**3*d**4*e**3*x**3
 - 28*sqrt(c)*sqrt(b + c*x)*b**5*c**3*d**3*e**4*x**4 + 13*sqrt(c)*sqrt(b +
 c*x)*b**5*c**3*d**2*e**5*x**5 + 1072*sqrt(c)*sqrt(b + c*x)*b**4*c**4*d**5
*e**2*x**3 - 880*sqrt(c)*sqrt(b + c*x)*b**4*c**4*d**4*e**3*x**4 - 14*sq...
```

3.184 $\int \frac{1}{(d+ex)^2(bx+cx^2)^{7/2}}\,dx$

Optimal result 1472
Mathematica [A] (verified) 1473
Rubi [A] (verified) 1474
Maple [A] (verified) 1478
Fricas [B] (verification not implemented) 1479
Sympy [F] 1480
Maxima [F(-2)] 1480
Giac [B] (verification not implemented) 1480
Mupad [F(-1)] 1481
Reduce [F] 1482

Optimal result

Integrand size = 21, antiderivative size = 548

$$\int \frac{1}{(d+ex)^2(bx+cx^2)^{7/2}}\,dx = -\frac{2cd-7be}{5bd^2(cd-be)(bx+cx^2)^{5/2}} + \frac{\left(\frac{4c^2}{b^2}-\frac{7e^2}{d^2}\right)x}{3d(cd-be)(bx+cx^2)^{5/2}} - \frac{(32c^3d^3-14b^2cde^2-21b^3e^3)x^2}{3b^3d^4(cd-be)(bx+cx^2)^{5/2}} - \frac{c(64c^4d^4-64bc^3d^3e-28b^2c^2d^2e^2+35b^4e^4)x^3}{5b^4d^4(cd-be)^2(bx+cx^2)^{5/2}} - \frac{e}{d(cd-be)(d+ex)(bx+cx^2)^{5/2}} - \frac{c(256c^5d^5-512bc^4d^4e+144b^2c^3d^3e^2+112b^3c^2d^2e^3+70b^4cde^4-105b^5e^5)x^2}{15b^5d^4(cd-be)^3(bx+cx^2)^{3/2}} - \frac{c(512c^6d^6-1536bc^5d^5e+1312b^2c^4d^4e^2-64b^3c^3d^3e^3-84b^4c^2d^2e^4-140b^5cde^5+105b^6e^6)x}{15b^6d^4(cd-be)^4\sqrt{bx+cx^2}} + \frac{7e^6(2cd-be)\operatorname{arctanh}\left(\frac{\sqrt{cd-be}\,x}{\sqrt{d}\sqrt{bx+cx^2}}\right)}{d^{9/2}(cd-be)^{9/2}}$$

output
```
-1/5*(-7*b*e+2*c*d)/b/d^2/(-b*e+c*d)/(c*x^2+b*x)^(5/2)+1/3*(4*c^2/b^2-7*e^
2/d^2)*x/d/(-b*e+c*d)/(c*x^2+b*x)^(5/2)-1/3*(-21*b^3*e^3-14*b^2*c*d*e^2+32
*c^3*d^3)*x^2/b^3/d^4/(-b*e+c*d)/(c*x^2+b*x)^(5/2)-1/5*c*(35*b^4*e^4-28*b^
2*c^2*d^2*e^2-64*b*c^3*d^3*e+64*c^4*d^4)*x^3/b^4/d^4/(-b*e+c*d)^2/(c*x^2+b
*x)^(5/2)-e/d/(-b*e+c*d)/(e*x+d)/(c*x^2+b*x)^(5/2)-1/15*c*(-105*b^5*e^5+70
*b^4*c*d*e^4+112*b^3*c^2*d^2*e^3+144*b^2*c^3*d^3*e^2-512*b*c^4*d^4*e+256*c
^5*d^5)*x^2/b^5/d^4/(-b*e+c*d)^3/(c*x^2+b*x)^(3/2)-1/15*c*(105*b^6*e^6-140
*b^5*c*d*e^5-84*b^4*c^2*d^2*e^4-64*b^3*c^3*d^3*e^3+1312*b^2*c^4*d^4*e^2-15
36*b*c^5*d^5*e+512*c^6*d^6)*x/b^6/d^4/(-b*e+c*d)^4/(c*x^2+b*x)^(1/2)+7*e^6
*(-b*e+2*c*d)*arctanh((-b*e+c*d)^(1/2)*x/d^(1/2)/(c*x^2+b*x)^(1/2))/d^(9/2
)/(-b*e+c*d)^(9/2)
```

Mathematica [A] (verified)

Time = 5.02 (sec) , antiderivative size = 644, normalized size of antiderivative = 1.18

$$\int \frac{1}{(d+ex)^2\left(bx+cx^2\right)^{7/2}}\,dx = \frac{x\Bigg(-\frac{\sqrt{d}(b+cx)\left(512c^9d^6x^5(d+ex)-256bc^8d^5x^4\left(-5d^2+dex+6e^2x^2\right)+32b^2c^7d^4x^3\left(30d^3-90d^2ex-79\right.\right.}{}}{}$$

input
```
Integrate[1/((d + e*x)^2*(b*x + c*x^2)^(7/2)),x]
```

output
```
(x*(-((Sqrt[d]*(b + c*x)*(512*c^9*d^6*x^5*(d + e*x) - 256*b*c^8*d^5*x^4*(-
5*d^2 + d*e*x + 6*e^2*x^2) + 32*b^2*c^7*d^4*x^3*(30*d^3 - 90*d^2*e*x - 79*
d*e^2*x^2 + 41*e^3*x^3) + b^9*e^4*(6*d^3 - 14*d^2*e*x + 70*d*e^2*x^2 + 105
*e^3*x^3) + 16*b^3*c^6*d^3*x^2*(10*d^4 - 170*d^3*e*x + 25*d^2*e^2*x^2 + 20
1*d*e^3*x^3 - 4*e^4*x^4) + b^8*c*e^3*(-24*d^4 + 36*d^3*e*x - 140*d^2*e^2*x
^2 + 70*d*e^3*x^3 + 315*e^4*x^4) - 4*b^4*c^5*d^2*x*(5*d^5 + 125*d^4*e*x -
495*d^3*e^2*x^2 - 575*d^2*e^3*x^3 + 61*d*e^4*x^4 + 21*e^5*x^5) + b^7*c^2*e
^2*(36*d^5 - 4*d^4*e*x + 20*d^3*e^2*x^2 - 420*d^2*e^3*x^3 - 210*d*e^4*x^4
+ 315*e^5*x^5) - b^6*c^3*e*(24*d^6 + 64*d^5*e*x + 80*d^4*e^2*x^2 + 160*d^3
*e^3*x^3 + 560*d^2*e^4*x^4 + 350*d*e^5*x^5 - 105*e^6*x^6) + 2*b^5*c^4*d*(3
*d^6 + 33*d^5*e*x + 235*d^4*e^2*x^2 + 145*d^3*e^3*x^3 - 165*d^2*e^4*x^4 -
175*d*e^5*x^5 - 70*e^6*x^6)))/(b^6*(c*d - b*e)^4*(d + e*x))) - (105*e^6*(2
*c*d - b*e)*x^(5/2)*(b + c*x)^(7/2)*ArcTan[(-(e*Sqrt[x]*Sqrt[b + c*x]) + S
qrt[c]*(d + e*x))/(Sqrt[d]*Sqrt[-(c*d) + b*e])])/(-(c*d) + b*e)^(9/2)))/(1
5*d^(9/2)*(x*(b + c*x))^(7/2))
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 591, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {1165, 27, 1235, 27, 1235, 27, 1228, 1154, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{7/2}(d+ex)^2}dx$$

$$\Big\downarrow \text{1165}$$

$$-\frac{2\int \frac{(8cd-7be)(2cd+be)+10ce(2cd-be)x}{2(d+ex)^2(cx^2+bx)^{5/2}}dx}{5b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(d+ex)(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(8cd-7be)(2cd+be)+10ce(2cd-be)x}{(d+ex)^2(cx^2+bx)^{5/2}}dx}{5b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(d+ex)(cd-be)}$$

$$\Big\downarrow \text{1235}$$

$$-\frac{-\frac{2\int \frac{128c^4d^4-160bc^3ed^3-24b^2c^2e^2d^2+14b^3ce^3d+35b^4e^4+6ce(2cd-be)\left(16c^2d^2-16bced-7b^2e^2\right)x}{2(d+ex)^2\left(cx^2+bx\right)^{3/2}}dx}{3b^2d(cd-be)}-\frac{2\left(cx(2cd-be)\left(-7b^2e^2-16bcde+16c^2d^2\right)+b \cdots}{3b^2d(bx+cx^2)^{3/2}(d+ex \cdots}}{5b^2d(cd-be)}-$$
$$\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(d+ex)(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{\int \frac{128c^4d^4-160bc^3ed^3-24b^2c^2e^2d^2+14b^3ce^3d+35b^4e^4+6ce(2cd-be)\left(16c^2d^2-16bced-7b^2e^2\right)x}{(d+ex)^2\left(cx^2+bx\right)^{3/2}}dx}{3b^2d(cd-be)}-\frac{2\left(cx(2cd-be)\left(-7b^2e^2-16bcde+16c^2d^2\right)+b(8 \cdots}{3b^2d(bx+cx^2)^{3/2}(d+ex)(\cdots}}{5b^2d(cd-be)}-$$
$$\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(d+ex)(cd-be)}$$

$$\Big\downarrow \text{1235}$$

$$-\frac{-\frac{-\frac{2\int \frac{e\left(b\left(256c^5d^5-512bc^4ed^4+144b^2c^3e^2d^3+112b^3c^2e^3d^2+70b^4ce^4d-105b^5e^5\right)+2c(2cd-be)\left(128c^4d^4-256bc^3ed^3+72b^2c^2e^2d^2+56b^3ce^3d+35b^4e^4\right)x\right)}{2(d+ex)^2\sqrt{cx^2+bx}}d}{b^2d(cd-be)}}{3b^2d(cd-be}}{}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(d+ex)(cd-be)}$$

$\big\downarrow$ 27

$$-\frac{-\frac{-\frac{e\int \frac{b\left(256c^5d^5-512bc^4ed^4+144b^2c^3e^2d^3+112b^3c^2e^3d^2+70b^4ce^4d-105b^5e^5\right)+2c(2cd-be)\left(128c^4d^4-256bc^3ed^3+72b^2c^2e^2d^2+56b^3ce^3d+35b^4e^4\right)x}{(d+ex)^2\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-}{3b^2d(cd-be)}}{}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(d+ex)(cd-be)}$$

$\big\downarrow$ 1228

$$-\frac{-\frac{-\frac{e\left(\frac{\sqrt{bx+cx^2}\left(105b^6e^6-140b^5cde^5-84b^4c^2d^2e^4-64b^3c^3d^3e^3+1312b^2c^4d^4e^2-1536bc^5d^5e+512c^6d^6\right)}{d(d+ex)(cd-be)}-\frac{105b^6e^5(2cd-be)\int \frac{1}{(d+ex)\sqrt{cx^2+bx}}dx}{2d(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx}{}}{3b^2d(cd-be)}}{}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(d+ex)(cd-be)}$$

$\big\downarrow$ 1154

$$-\frac{-\frac{-\frac{e\left(\frac{105b^6e^5(2cd-be)\int \frac{1}{4d(cd-be)-\frac{(bd+(2cd-be)x)^2}{cx^2+bx}}d\left(-\frac{bd+(2cd-be)x}{\sqrt{cx^2+bx}}\right)}{d(cd-be)}+\frac{\sqrt{bx+cx^2}\left(105b^6e^6-140b^5cde^5-84b^4c^2d^2e^4-64b^3c^3d^3e^3+1312b^2c^4d^4e^2-\right.}{d(d+ex)(cd-be)}\right.}{b^2d(cd-be)}}{3b}}{}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(d+ex)(cd-be)}$$

$\big\downarrow$ 219

$$-\frac{-\frac{-\frac{e\left(\frac{\sqrt{bx+cx^2}\left(105b^6e^6-140b^5cde^5-84b^4c^2d^2e^4-64b^3c^3d^3e^3+1312b^2c^4d^4e^2-1536bc^5d^5e+512c^6d^6\right)}{d(d+ex)(cd-be)}-\frac{105b^6e^5(2cd-be)\text{arctanh}\left(\frac{x(2cd-be)+bd}{2\sqrt{d}\sqrt{bx+cx^2}\sqrt{cd}}\right)}{2d^{3/2}(cd-be)^{3/2}}\right)}{b^2d(cd-be)}}{3b^2d(cd-}}{}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{5b^2d\left(bx+cx^2\right)^{5/2}(d+ex)(cd-be)}$$

input
```
Int[1/((d + e*x)^2*(b*x + c*x^2)^(7/2)),x]
```

output
```
(-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(5*b^2*d*(c*d - b*e)*(d + e*x)*(b
*x + c*x^2)^(5/2)) - ((-2*(b*(8*c*d - 7*b*e)*(c*d - b*e)*(2*c*d + b*e) + c
*(2*c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e - 7*b^2*e^2)*x))/(3*b^2*d*(c*d - b
*e)*(d + e*x)*(b*x + c*x^2)^(3/2)) - ((-2*(b*(c*d - b*e)*(128*c^4*d^4 - 16
0*b*c^3*d^3*e - 24*b^2*c^2*d^2*e^2 + 14*b^3*c*d*e^3 + 35*b^4*e^4) + c*(2*c
*d - b*e)*(128*c^4*d^4 - 256*b*c^3*d^3*e + 72*b^2*c^2*d^2*e^2 + 56*b^3*c*d
*e^3 + 35*b^4*e^4)*x))/(b^2*d*(c*d - b*e)*(d + e*x)*Sqrt[b*x + c*x^2]) - (
e*(((512*c^6*d^6 - 1536*b*c^5*d^5*e + 1312*b^2*c^4*d^4*e^2 - 64*b^3*c^3*d^
3*e^3 - 84*b^4*c^2*d^2*e^4 - 140*b^5*c*d*e^5 + 105*b^6*e^6)*Sqrt[b*x + c*x
^2])/(d*(c*d - b*e)*(d + e*x)) - (105*b^6*e^5*(2*c*d - b*e)*ArcTanh[(b*d +
 (2*c*d - b*e)*x)/(2*Sqrt[d]*Sqrt[c*d - b*e]*Sqrt[b*x + c*x^2])])/(2*d^(3/
2)*(c*d - b*e)^(3/2))))/(b^2*d*(c*d - b*e)))/(3*b^2*d*(c*d - b*e)))/(5*b^2
*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1154
```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Simp[-2   Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (
2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c
, d, e}, x]
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1228
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Simp[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^
(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x
] && EqQ[Simplify[m + 2*p + 3], 0]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 509, normalized size of antiderivative = 0.93

method	result
pseudoelliptic	$-\frac{2\left(-\frac{35b^6e^6x^2\sqrt{x(cx+b)}\,(ex+d)(cx+b)^2(be-2cd)\arctan\left(\frac{\sqrt{x(cx+b)}\,d}{x\sqrt{d(be-cd)}}\right)}{2}+\sqrt{d(be-cd)}\,\left(c^4(2cx+b)\left(\frac{128}{3}c^4x^4+\frac{256}{3}b\,c^3x^3+\frac{112}{3}\right.\right.\right.}{}$
risch	$-\frac{2(cx+b)\left(45b^2e^2x^2+110bcde\,x^2+128d^2c^2x^2-10b^2dex-19xbc\,d^2+3b^2d^2\right)}{15b^6d^4\sqrt{x(cx+b)}\,x^2}-\frac{b^2c^2d^4\left(\frac{2\sqrt{c\left(\frac{b}{c}+x\right)^2-\left(\frac{b}{c}+x\right)b}}{5b\left(\frac{b}{c}+x\right)^3}+\frac{4c\left(\frac{2\sqrt{c\left(\frac{b}{c}\right.}}{3}\right.}{}\right.}{(be-}$
default	Expression too large to display

input
```
int(1/(e*x+d)^2/(c*x^2+b*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(x*(c*x+b))^(1/2)/(d*(b*e-c*d))^(1/2)*(-35/2*b^6*e^6*x^2*(x*(c*x+b))^
(1/2)*(e*x+d)*(c*x+b)^2*(b*e-2*c*d)*arctan((x*(c*x+b))^(1/2)/x*d/(d*(b*e-c
*d))^(1/2))+(d*(b*e-c*d))^(1/2)*(c^4*(2*c*x+b)*(128/3*c^4*x^4+256/3*b*c^3*
x^3+112/3*b^2*c^2*x^2-16/3*b^3*c*x+b^4)*d^7-4*e*c^3*(-64/3*x^6*c^6+32/3*x^
5*b*c^5+120*x^4*b^2*c^4+340/3*b^3*c^3*x^3+125/6*c^2*x^2*b^4-11/4*x*c*b^5+b
^6)*d^6+6*e^2*c^2*(-128/3*x^6*c^6-632/9*x^5*b*c^5+100/9*x^4*b^2*c^4+55*b^3
*c^3*x^3+235/18*c^2*x^2*b^4-16/9*x*c*b^5+b^6)*b*d^5-4*e^3*(-164/3*x^6*c^6-
134*x^5*b*c^5-575/6*x^4*b^2*c^4-145/12*b^3*c^3*x^3+10/3*c^2*x^2*b^4+1/6*x*
c*b^5+b^6)*c*b^2*d^4+e^4*(-32/3*c^2*x^2+2*c*b*x+b^2)*(c*x+b)^4*b^3*d^3-7/3
*b^4*e^5*x*(6*c*x+b)*(c*x+b)^4*d^2+35/3*b^5*e^6*x^2*(-2*c*x+b)*(c*x+b)^3*d
+35/2*b^6*e^7*x^3*(c*x+b)^3))/x^2/d^4/(b*e-c*d)^4/(c*x+b)^2/(e*x+d)/b^6
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1408 vs. $2(512) = 1024$.

Time = 1.03 (sec) , antiderivative size = 2832, normalized size of antiderivative = 5.17

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^{7/2}} dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^(7/2),x, algorithm="fricas")
```

output
```
[-1/30*(105*((2*b^6*c^4*d*e^7 - b^7*c^3*e^8)*x^7 + (2*b^6*c^4*d^2*e^6 + 5*
b^7*c^3*d*e^7 - 3*b^8*c^2*e^8)*x^6 + 3*(2*b^7*c^3*d^2*e^6 + b^8*c^2*d*e^7
- b^9*c*e^8)*x^5 + (6*b^8*c^2*d^2*e^6 - b^9*c*d*e^7 - b^10*e^8)*x^4 + (2*b
^9*c*d^2*e^6 - b^10*d*e^7)*x^3)*sqrt(c*d^2 - b*d*e)*log((b*d + (2*c*d - b*
e)*x - 2*sqrt(c*d^2 - b*d*e)*sqrt(c*x^2 + b*x))/(e*x + d)) + 2*(6*b^5*c^5*
d^9 - 30*b^6*c^4*d^8*e + 60*b^7*c^3*d^7*e^2 - 60*b^8*c^2*d^6*e^3 + 30*b^9*
c*d^5*e^4 - 6*b^10*d^4*e^5 + (512*c^10*d^8*e - 2048*b*c^9*d^7*e^2 + 2848*b
^2*c^8*d^6*e^3 - 1376*b^3*c^7*d^5*e^4 - 20*b^4*c^6*d^4*e^5 - 56*b^5*c^5*d^
3*e^6 + 245*b^6*c^4*d^2*e^7 - 105*b^7*c^3*d*e^8)*x^6 + (512*c^10*d^9 - 768
*b*c^9*d^8*e - 2272*b^2*c^8*d^7*e^2 + 5744*b^3*c^7*d^6*e^3 - 3460*b^4*c^6*
d^5*e^4 - 106*b^5*c^5*d^4*e^5 + 665*b^7*c^3*d^2*e^7 - 315*b^8*c^2*d*e^8)*x
^5 + 5*(256*b*c^9*d^9 - 832*b^2*c^8*d^8*e + 656*b^3*c^7*d^7*e^2 + 380*b^4*
c^6*d^6*e^3 - 526*b^5*c^5*d^5*e^4 - 46*b^6*c^4*d^4*e^5 + 70*b^7*c^3*d^3*e^
6 + 105*b^8*c^2*d^2*e^7 - 63*b^9*c*d*e^8)*x^4 + 5*(192*b^2*c^8*d^9 - 736*b
^3*c^7*d^8*e + 940*b^4*c^6*d^7*e^2 - 338*b^5*c^5*d^6*e^3 - 90*b^6*c^4*d^5*
e^4 - 52*b^7*c^3*d^4*e^5 + 98*b^8*c^2*d^3*e^6 + 7*b^9*c*d^2*e^7 - 21*b^10*
d*e^8)*x^3 + 10*(16*b^3*c^7*d^9 - 66*b^4*c^6*d^8*e + 97*b^5*c^5*d^7*e^2 -
55*b^6*c^4*d^6*e^3 + 10*b^7*c^3*d^5*e^4 - 16*b^8*c^2*d^4*e^5 + 21*b^9*c*d^
3*e^6 - 7*b^10*d^2*e^7)*x^2 - 2*(10*b^4*c^6*d^9 - 43*b^5*c^5*d^8*e + 65*b^
6*c^4*d^7*e^2 - 30*b^7*c^3*d^6*e^3 - 20*b^8*c^2*d^5*e^4 + 25*b^9*c*d^4*...
```

Sympy [F]

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^{7/2}}\,dx = \int \frac{1}{(x(b+cx))^{\frac{7}{2}} (d+ex)^2}\,dx$$

input
```
integrate(1/(e*x+d)**2/(c*x**2+b*x)**(7/2),x)
```

output
```
Integral(1/((x*(b + c*x))**(7/2)*(d + e*x)**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^{7/2}}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^(7/2),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(b*e-c*d>0)', see `assume?` for m
ore detail
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2349 vs. $2(512) = 1024$.

Time = 0.52 (sec) , antiderivative size = 2349, normalized size of antiderivative = 4.29

$$\int \frac{1}{(d+ex)^2 (bx+cx^2)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^2/(c*x^2+b*x)^(7/2),x, algorithm="giac")
```

output
```
1/30*((210*b^6*c^(3/2)*d*e^9*log(abs(2*c*d*e - b*e^2 - 2*sqrt(c*d^2 - b*d*
e)*sqrt(c)*abs(e))) - 105*b^7*sqrt(c)*e^10*log(abs(2*c*d*e - b*e^2 - 2*sqr
t(c*d^2 - b*d*e)*sqrt(c)*abs(e))) + 1024*sqrt(c*d^2 - b*d*e)*c^7*d^6*e^2*a
bs(e) - 3072*sqrt(c*d^2 - b*d*e)*b*c^6*d^5*e^3*abs(e) + 2624*sqrt(c*d^2 -
b*d*e)*b^2*c^5*d^4*e^4*abs(e) - 128*sqrt(c*d^2 - b*d*e)*b^3*c^4*d^3*e^5*ab
s(e) - 168*sqrt(c*d^2 - b*d*e)*b^4*c^3*d^2*e^6*abs(e) - 280*sqrt(c*d^2 - b
*d*e)*b^5*c^2*d*e^7*abs(e) + 210*sqrt(c*d^2 - b*d*e)*b^6*c*e^8*abs(e))*sgn
(1/(e*x + d))*sgn(e)/(sqrt(c*d^2 - b*d*e)*b^6*c^(9/2)*d^8*abs(e) - 4*sqrt(
c*d^2 - b*d*e)*b^7*c^(7/2)*d^7*e*abs(e) + 6*sqrt(c*d^2 - b*d*e)*b^8*c^(5/2
)*d^6*e^2*abs(e) - 4*sqrt(c*d^2 - b*d*e)*b^9*c^(3/2)*d^5*e^3*abs(e) + sqrt
(c*d^2 - b*d*e)*b^10*sqrt(c)*d^4*e^4*abs(e)) - 2*((512*c^9*d^6*e^19 - 1536
*b*c^8*d^5*e^20 + 1312*b^2*c^7*d^4*e^21 - 64*b^3*c^6*d^3*e^22 - 84*b^4*c^5
*d^2*e^23 - 140*b^5*c^4*d*e^24 + 105*b^6*c^3*e^25)/(b^6*c^4*d^8*e^17*sgn(1
/(e*x + d))*sgn(e) - 4*b^7*c^3*d^7*e^18*sgn(1/(e*x + d))*sgn(e) + 6*b^8*c^
2*d^6*e^19*sgn(1/(e*x + d))*sgn(e) - 4*b^9*c*d^5*e^20*sgn(1/(e*x + d))*sgn
(e) + b^10*d^4*e^21*sgn(1/(e*x + d))*sgn(e)) - (5*(512*c^9*d^7*e^20 - 1792
*b*c^8*d^6*e^21 + 2080*b^2*c^7*d^5*e^22 - 720*b^3*c^6*d^4*e^23 - 52*b^4*c^
5*d^3*e^24 - 98*b^5*c^4*d^2*e^25 + 196*b^6*c^3*d*e^26 - 63*b^7*c^2*e^27)/(
b^6*c^4*d^8*e^17*sgn(1/(e*x + d))*sgn(e) - 4*b^7*c^3*d^7*e^18*sgn(1/(e*x +
 d))*sgn(e) + 6*b^8*c^2*d^6*e^19*sgn(1/(e*x + d))*sgn(e) - 4*b^9*c*d^5*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^2\,(bx+cx^2)^{7/2}}\,dx = \int \frac{1}{(c\,x^2+b\,x)^{7/2}\,(d+e\,x)^2}\,dx$$

input
```
int(1/((b*x + c*x^2)^(7/2)*(d + e*x)^2),x)
```

output
```
int(1/((b*x + c*x^2)^(7/2)*(d + e*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^2\,(bx+cx^2)^{7/2}}\,dx = \int \frac{1}{(ex+d)^2\,(c\,x^2+bx)^{\frac{7}{2}}}dx$$

input
```
int(1/(e*x+d)^2/(c*x^2+b*x)^(7/2),x)
```

output
```
int(1/(e*x+d)^2/(c*x^2+b*x)^(7/2),x)
```

3.185 $\int (d+ex)^{3/2}\sqrt{bx+cx^2}\,dx$

Optimal result 1483
Mathematica [C] (verified) 1484
Rubi [A] (verified) 1485
Maple [A] (verified) 1489
Fricas [A] (verification not implemented) 1490
Sympy [F] 1490
Maxima [F] 1491
Giac [F] 1491
Mupad [F(-1)] 1491
Reduce [F] 1492

Optimal result

Integrand size = 23, antiderivative size = 428

$$\begin{aligned}\int (d+ex)^{3/2}\sqrt{bx+cx^2}\,dx &= -\frac{2(2cd-be)\left(3c^2d^2-3bcde+8b^2e^2\right)x\sqrt{d+ex}}{105c^2e^2\sqrt{bx+cx^2}}\\
&+\frac{2\left(9bd+\frac{3cd^2}{e}-\frac{4b^2e}{c}\right)\sqrt{d+ex}\sqrt{bx+cx^2}}{105c}\\
&+\frac{2(3cd+be)x\sqrt{d+ex}\sqrt{bx+cx^2}}{35c}+\frac{2}{7}x(d+ex)^{3/2}\sqrt{bx+cx^2}\\
&+\frac{2\sqrt{b}(2cd-be)\left(3c^2d^2-3bcde+8b^2e^2\right)\sqrt{x}\sqrt{d+ex}E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{105c^{5/2}e^2\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}\\
&-\frac{2b^{3/2}\left(3c^2d^2+9bcde-4b^2e^2\right)\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{105c^{5/2}e\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}\end{aligned}$$

output
```
-2/105*(-b*e+2*c*d)*(8*b^2*e^2-3*b*c*d*e+3*c^2*d^2)*x*(e*x+d)^(1/2)/c^2/e^
2/(c*x^2+b*x)^(1/2)+2/105*(9*b*d+3*c*d^2/e-4*b^2*e/c)*(e*x+d)^(1/2)*(c*x^2
+b*x)^(1/2)/c+2/35*(b*e+3*c*d)*x*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/c+2/7*x*(
e*x+d)^(3/2)*(c*x^2+b*x)^(1/2)+2/105*b^(1/2)*(-b*e+2*c*d)*(8*b^2*e^2-3*b*c
*d*e+3*c^2*d^2)*x^(1/2)*(e*x+d)^(1/2)*EllipticE(c^(1/2)*x^(1/2)/b^(1/2)/(1
+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/c^(5/2)/e^2/(b*(e*x+d)/d/(c*x+b))^(1/2)/(
c*x^2+b*x)^(1/2)-2/105*b^(3/2)*(-4*b^2*e^2+9*b*c*d*e+3*c^2*d^2)*x^(1/2)*(e
*x+d)^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e/c/d)^(1
/2))/c^(5/2)/e/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.00 (sec) , antiderivative size = 372, normalized size of antiderivative = 0.87

$$\int (d+ex)^{3/2}\sqrt{bx+cx^2}\,dx = \frac{2\left(bex(b+cx)(d+ex)\left(-4b^2e^2+3bce(3d+ex)+3c^2(d^2+8dex+5e^2x^2)\right)-\sqrt{\frac{b}{c}}\right.}{}$$

input
```
Integrate[(d + e*x)^(3/2)*Sqrt[b*x + c*x^2],x]
```

output
```
(2*(b*e*x*(b + c*x)*(d + e*x)*(-4*b^2*e^2 + 3*b*c*e*(3*d + e*x) + 3*c^2*(d
^2 + 8*d*e*x + 5*e^2*x^2)) - Sqrt[b/c]*(Sqrt[b/c]*(6*c^3*d^3 - 9*b*c^2*d^2
*e + 19*b^2*c*d*e^2 - 8*b^3*e^3)*(b + c*x)*(d + e*x) + I*b*e*(6*c^3*d^3 -
9*b*c^2*d^2*e + 19*b^2*c*d*e^2 - 8*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(
e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e
*(3*c^3*d^3 - 18*b*c^2*d^2*e + 23*b^2*c*d*e^2 - 8*b^3*e^3)*Sqrt[1 + b/(c*x
)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)
/(b*e)])))/(105*b*c^2*e^2*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 377, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1166, 27, 1231, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{bx+cx^2}(d+ex)^{3/2} \, dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{2\int \frac{(d(7cd-3be)+4e(2cd-be)x)\sqrt{cx^2+bx}}{2\sqrt{d+ex}}dx}{7c}+\frac{2e\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d(7cd-3be)+4e(2cd-be)x)\sqrt{cx^2+bx}}{\sqrt{d+ex}}dx}{7c}+\frac{2e\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$$\Big\downarrow \text{1231}$$

$$\frac{\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-4b^2e^2+12cex(2cd-be)+9bcde+3c^2d^2\right)}{15ce}-\frac{2\int \frac{e\left(bd\left(3c^2d^2+9bced-4b^2e^2\right)+(2cd-be)\left(3c^2d^2-3bced+8b^2e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{15ce^2}}{7c}+\frac{2e\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-4b^2e^2+12cex(2cd-be)+9bcde+3c^2d^2\right)}{15ce}-\frac{\int \frac{bd\left(3c^2d^2+9bced-4b^2e^2\right)+(2cd-be)\left(3c^2d^2-3bced+8b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{15ce}}{7c}+\frac{2e\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-4b^2e^2+12cex(2cd-be)+9bcde+3c^2d^2\right)}{15ce}-\frac{\frac{(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{2d(cd-be)\left(2b^2e^2-3bcde+3c^2d^2\right)\int\frac{}{\sqrt{c}}}{e}}{15ce}}{7c}$$

$$\frac{2e\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$\Big\downarrow$ 1169

$$\frac{\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-4b^2e^2+12cex(2cd-be)+9bcde+3c^2d^2\right)}{15ce}-\frac{\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\left(2b^2e^2-\right.}{e}}{15ce}}{7c}$$

$$\frac{2e\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$\Big\downarrow$ 122

$$\frac{\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-4b^2e^2+12cex(2cd-be)+9bcde+3c^2d^2\right)}{15ce}-\frac{\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be}{}}{15ce}}{7c}$$

$$\frac{2e\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$\Big\downarrow$ 120

$$\frac{\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-4b^2e^2+12cex(2cd-be)+9bcde+3c^2d^2\right)}{15ce}-\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}}{}}{15ce}}{7c}$$

$$\frac{2e\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$\Big\downarrow$ 127

$$\frac{\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-4b^2e^2+12cex(2cd-be)+9bcde+3c^2d^2\right)}{15ce}-\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}}{}}{15ce}}{7c}$$

$$\frac{2e\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

$\Big\downarrow$ 126

$$\frac{\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-4b^2e^2+12cex(2cd-be)+9bcde+3c^2d^2\right)}{15ce}-\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4\sqrt{-b}\cdots}{\cdots}}{15ce}}{7c}$$

$$\frac{2e\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7c}$$

input
```
Int[(d + e*x)^(3/2)*Sqrt[b*x + c*x^2],x]
```

output
```
(2*e*Sqrt[d + e*x]*(b*x + c*x^2)^(3/2))/(7*c) + ((2*Sqrt[d + e*x]*(3*c^2*d
^2 + 9*b*c*d*e - 4*b^2*e^2 + 12*c*e*(2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(1
5*c*e) - ((2*Sqrt[-b]*(2*c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 8*b^2*e^2)*Sq
rt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/S
qrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) -
(4*Sqrt[-b]*d*(c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[x]*Sqrt
[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b
]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]))/(15*c*e))/(
7*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.55

method	result
elliptic	$\frac{\sqrt{x(cx+b)}\,\sqrt{(cx+b)x(ex+d)}\left(\frac{2e x^2\sqrt{ce x^3+be x^2+cd x^2+bdx}}{7}+\frac{2\left(b e^2+2dec-\frac{2e(3be+3cd)}{7}\right)x\sqrt{ce x^3+be x^2+cd x^2+bdx}}{5ce}+\frac{2\left(\frac{9bde}{7}+c d^2-\right.}{}\right.}{}$
default	$-\frac{2\sqrt{ex+d}\,\sqrt{x(cx+b)}\left(-15c^4e^5x^5+8\sqrt{\frac{ex+d}{d}}\sqrt{\frac{e(cx+b)}{be-cd}}\sqrt{-\frac{ex}{d}}\operatorname{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^4de^4-23\sqrt{\frac{ex+d}{d}}\sqrt{\frac{e(cx+b)}{be-cd}}\sqrt{-\right.}{}$

input
```
int((e*x+d)^(3/2)*(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)*x*(e*x+d))^(1/2)/x/(c*x+b)*(2/7
*e*x^2*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2/5*(b*e^2+2*d*e*c-2/7*e*(3*b
*e+3*c*d))/c/e*x*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2/3*(9/7*b*d*e+c*d^
2-2/5*(b*e^2+2*d*e*c-2/7*e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(c*e*x^3+
b*e*x^2+c*d*x^2+b*d*x)^(1/2)-2/3*(9/7*b*d*e+c*d^2-2/5*(b*e^2+2*d*e*c-2/7*e
*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e^2*b*d^2*((x+d/e)/d*e)^(1/2)*((b/c+x
)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*E
llipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(b*d^2-3/5*(b*e^2+
2*d*e*c-2/7*e*(3*b*e+3*c*d))/c/e*b*d-2/3*(9/7*b*d*e+c*d^2-2/5*(b*e^2+2*d*e
*c-2/7*e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(b*e+c*d))*d/e*((x+d/e)/d*e
)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2
+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^
(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 466, normalized size of antiderivative = 1.09

$$\int (d+ex)^{3/2}\sqrt{bx+cx^2}\,dx = \frac{2\left((6c^4d^4-12bc^3d^3e-17b^2c^2d^2e^2+23b^3cde^3-8b^4e^4)\sqrt{ce}\,\text{weierstrassPInverse}\left(\right.\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2/315*((6*c^4*d^4 - 12*b*c^3*d^3*e - 17*b^2*c^2*d^2*e^2 + 23*b^3*c*d*e^3 -
 8*b^4*e^4)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2
)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)
/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(6*c^4*d^3*e - 9*b*c^3*d^
2*e^2 + 19*b^2*c^2*d*e^3 - 8*b^3*c*e^4)*sqrt(c*e)*weierstrassZeta(4/3*(c^2
*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*
b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c
*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^
2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(15*c^4*e^
4*x^2 + 3*c^4*d^2*e^2 + 9*b*c^3*d*e^3 - 4*b^2*c^2*e^4 + 3*(8*c^4*d*e^3 + b
*c^3*e^4)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/(c^4*e^3)
```

Sympy [F]

$$\int (d+ex)^{3/2}\sqrt{bx+cx^2}\,dx = \int \sqrt{x(b+cx)}(d+ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(sqrt(x*(b + c*x))*(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int (d+ex)^{3/2}\sqrt{bx+cx^2}\,dx = \int \sqrt{cx^2+bx}(ex+d)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x)*(e*x + d)^(3/2), x)
```

Giac [F]

$$\int (d+ex)^{3/2}\sqrt{bx+cx^2}\,dx = \int \sqrt{cx^2+bx}(ex+d)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x)*(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{3/2}\sqrt{bx+cx^2}\,dx = \int \sqrt{c\,x^2+b\,x}\,(d+e\,x)^{3/2}\,dx$$

input
```
int((b*x + c*x^2)^(1/2)*(d + e*x)^(3/2),x)
```

output
```
int((b*x + c*x^2)^(1/2)*(d + e*x)^(3/2), x)
```

Reduce [F]

$$\int (d+ex)^{3/2}\sqrt{bx+cx^2}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(3/2)*(c*x^2+b*x)^(1/2),x)
```

output
```
( - 6*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*d*e + 4*sqrt(x)*sqrt(d + e*
x)*sqrt(b + c*x)*b**2*e**2*x + 22*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c*
d**2 + 36*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c*d*e*x + 20*sqrt(x)*sqrt(
d + e*x)*sqrt(b + c*x)*b*c*e**2*x**2 + 32*sqrt(x)*sqrt(d + e*x)*sqrt(b + c
*x)*c**2*d**2*x + 20*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**2*d*e*x**2 - 8
*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c
*d**2 + 2*b*c*d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*b**4
*e**4 + 11*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e*
*2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**
2),x)*b**3*c*d*e**3 + 10*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2
*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*x**2 + c**2*d**2*x
+ c**2*d*e*x**2),x)*b**2*c**2*d**2*e**2 - 3*int((sqrt(x)*sqrt(d + e*x)*sqr
t(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*
x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*b*c**3*d**3*e + 6*int((sqrt(x)*sqrt
(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*
x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*c**4*d**4 + 3*int((sqr
t(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b**2*d*e*x + b**2*e**2*x**2 + b*c*d**2*
x + 2*b*c*d*e*x**2 + b*c*e**2*x**3 + c**2*d**2*x**2 + c**2*d*e*x**3),x)*b*
*4*d**2*e**2 - 8*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b**2*d*e*x + b
**2*e**2*x**2 + b*c*d**2*x + 2*b*c*d*e*x**2 + b*c*e**2*x**3 + c**2*d**2...
```

3.186 $\int \sqrt{d+ex}\sqrt{bx+cx^2}\,dx$

Optimal result 1493
Mathematica [C] (verified) 1494
Rubi [A] (verified) 1494
Maple [A] (verified) 1498
Fricas [A] (verification not implemented) 1499
Sympy [F] 1499
Maxima [F] 1500
Giac [F] 1500
Mupad [F(-1)] 1500
Reduce [F] 1501

Optimal result

Integrand size = 23, antiderivative size = 333

$$\begin{aligned}\int \sqrt{d+ex}\sqrt{bx+cx^2}\,dx &= \frac{4(3bcde-(cd+be)^2)\,x\sqrt{d+ex}}{15ce^2\sqrt{bx+cx^2}} \\ &+ \frac{2(cd+be)\sqrt{d+ex}\sqrt{bx+cx^2}}{15ce} + \frac{2}{5}x\sqrt{d+ex}\sqrt{bx+cx^2} \\ &- \frac{4\sqrt{b}(3bcde-(cd+be)^2)\,\sqrt{x}\sqrt{d+ex}E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{15c^{3/2}e^2\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} \\ &- \frac{2b^{3/2}(cd+be)\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{15c^{3/2}e\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}\end{aligned}$$

output
```
4/15*(3*b*c*d*e-(b*e+c*d)^2)*x*(e*x+d)^(1/2)/c/e^2/(c*x^2+b*x)^(1/2)+2/15*
(b*e+c*d)*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/c/e+2/5*x*(e*x+d)^(1/2)*(c*x^2+b
*x)^(1/2)-4/15*b^(1/2)*(3*b*c*d*e-(b*e+c*d)^2)*x^(1/2)*(e*x+d)^(1/2)*Ellip
ticE(c^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/c^(3/2)/e^
2/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)-2/15*b^(3/2)*(b*e+c*d)*x^(
1/2)*(e*x+d)^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e/
c/d)^(1/2))/c^(3/2)/e/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.18 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.88

$$\int \sqrt{d+ex}\sqrt{bx+cx^2}\,dx$$
$$= \frac{2\left(bex(b+cx)(d+ex)(be+c(d+3ex))+\sqrt{\frac{b}{c}}\left(-2\sqrt{\frac{b}{c}}(c^2d^2-bcde+b^2e^2)(b+cx)(d+ex)-2ibe(c^2d\right.\right.}{}$$

input
```
Integrate[Sqrt[d + e*x]*Sqrt[b*x + c*x^2],x]
```

output
```
(2*(b*e*x*(b + c*x)*(d + e*x)*(b*e + c*(d + 3*e*x)) + Sqrt[b/c]*(-2*Sqrt[b
/c]*(c^2*d^2 - b*c*d*e + b^2*e^2)*(b + c*x)*(d + e*x) - (2*I)*b*e*(c^2*d^2
 - b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*Elliptic
E[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + I*b*e*(c^2*d^2 - 3*b*c*d*e
+ 2*b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSi
nh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(15*b*c*e^2*Sqrt[x*(b + c*x)]*Sqrt[
d + e*x])
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1162, 1236, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{bx+cx^2}\sqrt{d+ex}\,dx$$
$$\Big\downarrow \text{ 1162}$$
$$\frac{2\sqrt{bx+cx^2}(d+ex)^{3/2}}{5e}-\frac{\int \frac{\sqrt{d+ex}(bd+(2cd-be)x)}{\sqrt{cx^2+bx}}dx}{5e}$$

$$\Big\downarrow \; 1236$$

$$\frac{2\sqrt{bx+cx^2}(d+ex)^{3/2}}{5e}-\frac{\frac{2\int \frac{bd(cd+be)+2\left(c^2d^2-bced+b^2e^2\right)x}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}}{5e}$$

$$\Big\downarrow \; 27$$

$$\frac{2\sqrt{bx+cx^2}(d+ex)^{3/2}}{5e}-\frac{\frac{\int \frac{bd(cd+be)+2\left(c^2d^2-bced+b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}}{5e}$$

$$\Big\downarrow \; 1269$$

$$\frac{2\sqrt{bx+cx^2}(d+ex)^{3/2}}{5e}-\frac{\frac{\frac{2\left(b^2e^2-bcde+c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{d(cd-be)(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}}{3c}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}}{5e}$$

$$\Big\downarrow \; 1169$$

$$\frac{2\sqrt{bx+cx^2}(d+ex)^{3/2}}{5e}-\frac{\frac{\frac{2\sqrt{x}\sqrt{b+cx}\left(b^2e^2-bcde+c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{3c}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}}{5e}$$

$$\Big\downarrow \; 122$$

$$\frac{2\sqrt{bx+cx^2}(d+ex)^{3/2}}{5e}-\frac{\frac{\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{3c}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}}{5e}$$

$$\Big\downarrow \; 120$$

$$\frac{2\sqrt{bx+cx^2}(d+ex)^{3/2}}{5e}-\frac{\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{3c}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}}{5e}$$

$$\Big\downarrow \; 127$$

$$\frac{2\sqrt{bx+cx^2}(d+ex)^{3/2}}{5e}-$$
$$\frac{\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{d+ex}}}{3c}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(\cdots}{3c}}{5e}$$

$$\Big\downarrow \ 126$$

$$\frac{2\sqrt{bx+cx^2}(d+ex)^{3/2}}{5e}-$$
$$\frac{\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}}{3c}+2\sqrt{bx+c\cdots}}{5e}$$

input
```
Int[Sqrt[d + e*x]*Sqrt[b*x + c*x^2],x]
```

output
```
(2*(d + e*x)^(3/2)*Sqrt[b*x + c*x^2])/(5*e) - ((2*(2*c*d - b*e)*Sqrt[d + e
*x]*Sqrt[b*x + c*x^2])/(3*c) + ((4*Sqrt[-b]*(c^2*d^2 - b*c*d*e + b^2*e^2)*
Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])
/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2])
- (2*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1
 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(S
qrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]))/(3*c))/(5*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.33

method	result
elliptic	$\frac{\sqrt{x(cx+b)}\,\sqrt{(cx+b)x(ex+d)}\left(\frac{2x\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{5}+\frac{2\left(\frac{be}{5}+\frac{cd}{5}\right)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3ce}-\frac{2\left(\frac{be}{5}+\frac{cd}{5}\right)b\,d^2\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}\sqrt{\frac{\frac{b}{c}+x}{-\frac{d}{e}+\frac{b}{c}}}}{3c\,e^2\sqrt{ce\,x}}\right.}{}$
default	$\frac{2\sqrt{ex+d}\,\sqrt{x(cx+b)}\left(2\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\operatorname{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^3d\,e^3-3\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\operatorname{EllipticF}\left(\sqrt{\right.\right.}{}$

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)*x*(e*x+d))^(1/2)/x/(c*x+b)*(2/5
*x*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2/3*(1/5*b*e+1/5*c*d)/c/e*(c*e*x^
3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)-2/3*(1/5*b*e+1/5*c*d)/c/e^2*b*d^2*((x+d/e)/
d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*
x^2+b*d*x)^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*
(2/5*b*d-2/3*(1/5*b*e+1/5*c*d)/c/e*(b*e+c*d))*d/e*((x+d/e)/d*e)^(1/2)*((b/
c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2
)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*E
llipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 397, normalized size of antiderivative = 1.19

$$\int \sqrt{d+ex}\sqrt{bx+cx^2}\,dx = \frac{2\left((2\,c^3d^3-3\,bc^2d^2e-3\,b^2cde^2+2\,b^3e^3)\sqrt{ce}\text{weierstrassPInverse}\left(\frac{4\,(c^2d^2-bcde+b^2e^2)}{3\,c^2e^2},-\frac{4\,(2\,c^3d^3-3\,bc^2d^2e-3\,b^2c\right.\right.}{}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2/45*((2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)*sqrt(c*e)*we
ierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^
3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x
 + c*d + b*e)/(c*e)) + 6*(c^3*d^2*e - b*c^2*d*e^2 + b^2*c*e^3)*sqrt(c*e)*w
eierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d
^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInv
erse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c
^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)
/(c*e))) + 3*(3*c^3*e^3*x + c^3*d*e^2 + b*c^2*e^3)*sqrt(c*x^2 + b*x)*sqrt(
e*x + d))/(c^3*e^3)
```

Sympy [F]

$$\int \sqrt{d+ex}\sqrt{bx+cx^2}\,dx = \int \sqrt{x\,(b+cx)}\sqrt{d+ex}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(sqrt(x*(b + c*x))*sqrt(d + e*x), x)
```

Maxima [F]

$$\int \sqrt{d+ex}\sqrt{bx+cx^2}\,dx = \int \sqrt{cx^2+bx}\sqrt{ex+d}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x)*sqrt(e*x + d), x)
```

Giac [F]

$$\int \sqrt{d+ex}\sqrt{bx+cx^2}\,dx = \int \sqrt{cx^2+bx}\sqrt{ex+d}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x)*sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\sqrt{bx+cx^2}\,dx = \int \sqrt{c\,x^2+b\,x}\,\sqrt{d+e\,x}\,dx$$

input
```
int((b*x + c*x^2)^(1/2)*(d + e*x)^(1/2),x)
```

output
```
int((b*x + c*x^2)^(1/2)*(d + e*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+ex}\sqrt{bx+cx^2}\,dx$$
$$= \frac{2\sqrt{x}\,\sqrt{ex+d}\,\sqrt{cx+b}\,bd+2\sqrt{x}\,\sqrt{ex+d}\,\sqrt{cx+b}\,bex+2\sqrt{x}\,\sqrt{ex+d}\,\sqrt{cx+b}\,cdx+\left(\int \frac{\sqrt{x}}{bc\,e^2x^2+c^2de\,x^2+b^2}\right.}{}$$

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*d + 2*sqrt(x)*sqrt(d + e*x)*sqrt(
b + c*x)*b*e*x + 2*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c*d*x + int((sqrt(x
)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*
c*d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*b**3*e**3 + int(
(sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2
 + 2*b*c*d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*c**3*d**3
 - int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b**2*d*e*x + b**2*e**2*x**2
+ b*c*d**2*x + 2*b*c*d*e*x**2 + b*c*e**2*x**3 + c**2*d**2*x**2 + c**2*d*e*
x**3),x)*b**3*d**2*e - int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b**2*d*e
*x + b**2*e**2*x**2 + b*c*d**2*x + 2*b*c*d*e*x**2 + b*c*e**2*x**3 + c**2*d
**2*x**2 + c**2*d*e*x**3),x)*b**2*c*d**3)/(5*(b*e + c*d))
```

3.187 $\int \frac{\sqrt{bx+cx^2}}{\sqrt{d+ex}}\,dx$

Optimal result 1502
Mathematica [C] (verified) 1503
Rubi [A] (verified) 1503
Maple [A] (verified) 1506
Fricas [A] (verification not implemented) 1507
Sympy [F] 1508
Maxima [F] 1508
Giac [F] 1508
Mupad [F(-1)] 1509
Reduce [F] 1509

Optimal result

Integrand size = 23, antiderivative size = 268

$$
\begin{aligned}
\int \frac{\sqrt{bx+cx^2}}{\sqrt{d+ex}}\,dx = & -\frac{2(2cd-be)x\sqrt{d+ex}}{3e^2\sqrt{bx+cx^2}}+\frac{2\sqrt{d+ex}\sqrt{bx+cx^2}}{3e} \\
& +\frac{2\sqrt{b}(2cd-be)\sqrt{x}\sqrt{d+ex}E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{3\sqrt{c}e^2\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} \\
& -\frac{2b^{3/2}\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{3\sqrt{c}e\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}
\end{aligned}
$$

output
```
-2/3*(-b*e+2*c*d)*x*(e*x+d)^(1/2)/e^2/(c*x^2+b*x)^(1/2)+2/3*(e*x+d)^(1/2)*
(c*x^2+b*x)^(1/2)/e+2/3*b^(1/2)*(-b*e+2*c*d)*x^(1/2)*(e*x+d)^(1/2)*Ellipti
cE(c^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/c^(1/2)/e^2/
(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)-2/3*b^(3/2)*x^(1/2)*(e*x+d)^
(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e/c/d)^(1/2))/c
^(1/2)/e/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.24 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{bx+cx^2}}{\sqrt{d+ex}}\,dx = \frac{2\left((b+cx)(d+ex)(-2cd+be+cex)+i\sqrt{\frac{b}{c}}ce(-2cd+be)\sqrt{1+\frac{b}{cx}}\sqrt{1+\frac{d}{ex}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right)|\frac{cd}{be}\right)\right.}{3ce^2\sqrt{x(b+cx)}\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[b*x + c*x^2]/Sqrt[d + e*x],x]
```

output
```
(2*((b + c*x)*(d + e*x)*(-2*c*d + b*e + c*e*x) + I*Sqrt[b/c]*c*e*(-2*c*d +
 b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt
[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*Sqrt[b/c]*c*e*(-(c*d) + b*e)*Sqrt[1 + b/(
c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c
*d)/(b*e)]))/(3*c*e^2*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 250, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1162, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+cx^2}}{\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}}{3e}-\frac{\int \frac{bd+(2cd-be)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3e}$$

$$\Big\downarrow \text{1269}$$

$$\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}}{3e}-\frac{\frac{(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{2d(cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}}{3e}$$

$$\Big\downarrow \text{ 1169}$$

$$\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}}{3e}-\frac{\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{3e}$$

$$\Big\downarrow \text{ 122}$$

$$\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}}{3e}-\frac{\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{3e}$$

$$\Big\downarrow \text{ 120}$$

$$\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}}{3e}-\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{3e}$$

$$\Big\downarrow \text{ 127}$$

$$\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}}{3e}-\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{d+ex}}}{3e}$$

$$\Big\downarrow \text{ 126}$$

$$\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}}{3e}-\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}}{3e}$$

input
```
Int[Sqrt[b*x + c*x^2]/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(3*e) - ((2*Sqrt[-b]*(2*c*d - b*e)*Sqr
t[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sq
rt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (
4*Sqrt[-b]*d*(c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*Ellip
ticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d +
 e*x]*Sqrt[b*x + c*x^2]))/(3*e)
```

Defintions of rubi rules used

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.41

method	result
elliptic	$\frac{\sqrt{x(cx+b)}\,\sqrt{(cx+b)x(ex+d)}\left(\frac{2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3e}-\frac{2d^2b\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}\,\sqrt{\frac{\frac{b}{c}+x}{-\frac{d}{e}+\frac{b}{c}}}\,\sqrt{-\frac{ex}{d}}\;\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}},\sqrt{-\frac{d}{e\left(-\frac{d}{e}+\frac{b}{c}\right)}}\right)}{3e^2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}+\cdots\right.}{\sqrt{ex+d}\cdots}$
default	$-\frac{2\sqrt{x(cx+b)}\,\sqrt{ex+d}\left(\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\;\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^2d\,e^2-d^2b\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\;\mathrm{EllipticF}\left(\cdots\right.\right.}{\cdots}$

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)*x*(e*x+d))^(1/2)/x/(c*x+b)*(2/3
/e*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)-2/3*d^2/e^2*b*((x+d/e)/d*e)^(1/2)
*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)
^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(b-2/3/e*(
b*e+c*d))*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2
)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e
)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(
-d/e+b/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.31

$$\int \frac{\sqrt{bx+cx^2}}{\sqrt{d+ex}}\,dx = \frac{2\left(3\sqrt{cx^2+bx}\sqrt{ex+d}c^2e^2+(2\,c^2d^2-2\,bcde-b^2e^2)\sqrt{ce}\text{weierstrassPInverse}\left(\frac{4\,(c^2d^2-bcde+b^2e^2)}{3\,c^2e^2},-\frac{4\,(2\,c^3d}{}\right.\right.}{}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(3*sqrt(c*x^2 + b*x)*sqrt(e*x + d)*c^2*e^2 + (2*c^2*d^2 - 2*b*c*d*e -
b^2*e^2)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(
c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c
^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(2*c^2*d*e - b*c*e^2)*sqrt(c
*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*
c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstras
sPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 -
3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d +
 b*e)/(c*e))))/(c^2*e^3)
```

Sympy [F]

$$\int \frac{\sqrt{bx + cx^2}}{\sqrt{d + ex}}\, dx = \int \frac{\sqrt{x\,(b + cx)}}{\sqrt{d + ex}}\, dx$$

input
```
integrate((c*x**2+b*x)**(1/2)/(e*x+d)**(1/2),x)
```

output
```
Integral(sqrt(x*(b + c*x))/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{\sqrt{bx + cx^2}}{\sqrt{d + ex}}\, dx = \int \frac{\sqrt{cx^2 + bx}}{\sqrt{ex + d}}\, dx$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x)/sqrt(e*x + d), x)
```

Giac [F]

$$\int \frac{\sqrt{bx + cx^2}}{\sqrt{d + ex}}\, dx = \int \frac{\sqrt{cx^2 + bx}}{\sqrt{ex + d}}\, dx$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x)/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx+cx^2}}{\sqrt{d+ex}}\,dx=\int \frac{\sqrt{c\,x^2+b\,x}}{\sqrt{d+e\,x}}\,dx$$

input
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^(1/2),x)
```

output
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{bx+cx^2}}{\sqrt{d+ex}}\,dx = \frac{2\sqrt{x}\,\sqrt{ex+d}\,\sqrt{cx+b}\,b-\left(\int \frac{\sqrt{x}\,\sqrt{ex+d}\,\sqrt{cx+b}\,x}{bc\,e^2x^2+c^2de\,x^2+b^2e^2x+2bcdex+c^2d^2x+b^2de+bc\,d^2}dx\right)b^2c\,e^2+\left(\int \frac{\sqrt{x}\,\sqrt{ex\cdots}}{bc\,e^2x^2+c^2de\,x^2+b^2e^2x+\cdots}\right.}{\cdots}$$

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b - int((sqrt(x)*sqrt(d + e*x)*sqrt
(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*x
**2 + c**2*d**2*x + c**2*d*e*x**2),x)*b**2*c*e**2 + int((sqrt(x)*sqrt(d +
e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*x + b
*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*b*c**2*d*e + 2*int((sqrt(x)
*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c
*d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*c**3*d**2 - int((
sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b**2*d*e*x + b**2*e**2*x**2 + b*c*d*
*2*x + 2*b*c*d*e*x**2 + b*c*e**2*x**3 + c**2*d**2*x**2 + c**2*d*e*x**3),x)
*b**3*d*e - int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b**2*d*e*x + b**2*e
**2*x**2 + b*c*d**2*x + 2*b*c*d*e*x**2 + b*c*e**2*x**3 + c**2*d**2*x**2 +
c**2*d*e*x**3),x)*b**2*c*d**2)/(2*(b*e + c*d))
```

3.188 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{3/2}}\,dx$

Optimal result 1510
Mathematica [C] (verified) 1511
Rubi [A] (verified) 1511
Maple [A] (verified) 1514
Fricas [A] (verification not implemented) 1515
Sympy [F] 1516
Maxima [F] 1516
Giac [F] 1516
Mupad [F(-1)] 1517
Reduce [F] 1517

Optimal result

Integrand size = 23, antiderivative size = 208

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{bx+cx^2}}{e\sqrt{d+ex}} - \frac{4\sqrt{d}\sqrt{bx+cx^2}\,E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{e^{3/2}\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}} + \frac{2\sqrt{d}\sqrt{bx+cx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{e^{3/2}\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}$$

output
```
2*(c*x^2+b*x)^(1/2)/e/(e*x+d)^(1/2)-4*d^(1/2)*(c*x^2+b*x)^(1/2)*EllipticE(
e^(1/2)*x^(1/2)/d^(1/2)/(1+e*x/d)^(1/2),(1-c*d/b/e)^(1/2))/e^(3/2)/x^(1/2)
/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)+2*d^(1/2)*(c*x^2+b*x)^(1/2)*Inv
erseJacobiAM(arctan(e^(1/2)*x^(1/2)/d^(1/2)),(1-c*d/b/e)^(1/2))/e^(3/2)/x^
(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.70 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.94

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{3/2}}\,dx = \frac{2\left(\sqrt{\frac{b}{c}}(b+cx)(2d+ex)+2ibe\sqrt{1+\frac{b}{cx}}\sqrt{1+\frac{d}{ex}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right)|\frac{cd}{be}\right)-ibe\sqrt{1
}{\sqrt{\frac{b}{c}}e^2\sqrt{x(b+cx)}\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^(3/2),x]
```

output
```
(2*(Sqrt[b/c]*(b + c*x)*(2*d + e*x) + (2*I)*b*e*Sqrt[1 + b/(c*x)]*Sqrt[1 +
 d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I
*b*e*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[
b/c]/Sqrt[x]], (c*d)/(b*e)]))/(Sqrt[b/c]*e^2*Sqrt[x*(b + c*x)]*Sqrt[d + e*
x])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {1161, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{1161}$$

$$\frac{\int \frac{b+2cx}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}-\frac{2\sqrt{bx+cx^2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{2c\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}}{e}-\frac{2\sqrt{bx+cx^2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{1169}$$

$$\frac{\frac{2c\sqrt{x}\sqrt{b+cx}\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{e}-\frac{2\sqrt{bx+cx^2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{122}$$

$$\frac{\frac{2c\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{e}-\frac{2\sqrt{bx+cx^2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{120}$$

$$\frac{\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{e}-\frac{2\sqrt{bx+cx^2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{127}$$

$$\frac{\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{d+ex}}}{e}-\frac{2\sqrt{bx+cx^2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{126}$$

$$\frac{\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}}{e}-\frac{2\sqrt{bx+cx^2}}{e\sqrt{d+ex}}$$

input
```
Int[Sqrt[b*x + c*x^2]/(d + e*x)^(3/2),x]
```

output
```
(-2*Sqrt[b*x + c*x^2])/(e*Sqrt[d + e*x]) + ((4*Sqrt[-b]*Sqrt[c]*Sqrt[x]*Sq
rt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]]
, (b*e)/(c*d)])/(e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[-b]*(2*c
*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sq
rt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x
+ c*x^2]))/e
```

Defintions of rubi rules used

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.33

method	result
default	$-\frac{2\sqrt{x(cx+b)}\,\sqrt{ex+d}\left(\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\operatorname{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)bde-2\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\operatorname{EllipticE}\left(\sqrt{\frac{e}{}}\right.\right.}{x(ce\,x^2+bex+cdx+bd)e^3}$
elliptic	$\frac{\sqrt{x(cx+b)}\,\sqrt{(cx+b)x(ex+d)}\left(-\frac{2\left(ce\,x^2+bex\right)}{e^2\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2+bex\right)}}+\frac{2bd\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}\,\sqrt{\frac{\frac{b}{c}+x}{-\frac{d}{e}+\frac{b}{c}}}\,\sqrt{-\frac{ex}{d}}\,\operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}},\sqrt{-\frac{d}{e\left(-\frac{d}{e}+\frac{b}{c}\right)}}\right)}{e^2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}+\frac{4cd}{}\right.}{\sqrt{ex+d}\,x(cx+}$

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(x*(c*x+b))^(1/2)*(e*x+d)^(1/2)*(((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d)
)^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))
*b*d*e-2*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*Elli
pticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b*d*e+2*((e*x+d)/d)^(1/2)*
(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d
*c/(b*e-c*d))^(1/2))*c*d^2+x^2*c*e^2+x*b*e^2)/x/(c*e*x^2+b*e*x+c*d*x+b*d)/
e^3
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.72

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{3/2}}\,dx =$$
$$-\frac{2\left(3\sqrt{cx^2+bx}\sqrt{ex+d}ce^2+(2\,cd^2-bde+(2\,cde-be^2)x)\sqrt{ce}\text{weierstrassPInverse}\left(\frac{4\,(c^2d^2-bcde+b^2e^2)}{3\,c^2e^2},-\right.\right.}{}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(c*x^2 + b*x)*sqrt(e*x + d)*c*e^2 + (2*c*d^2 - b*d*e + (2*c*d*
e - b*e^2)*x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e
^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^
3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 6*(c*e^2*x + c*d*e)*sqrt(
c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2
*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstra
ssPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 -
 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d
+ b*e)/(c*e))))/(c*e^4*x + c*d*e^3)
```

Sympy [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{x\,(b+cx)}}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x)**(1/2)/(e*x+d)**(3/2),x)
```

output
```
Integral(sqrt(x*(b + c*x))/(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{cx^2+bx}}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x)/(e*x + d)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{cx^2+bx}}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x)/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{3/2}}\,dx = \int \frac{\sqrt{c\,x^2+b\,x}}{{\left(d+e\,x\right)}^{3/2}}\,dx$$

input
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^(3/2),x)
```

output
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{3/2}}\,dx = \frac{2\sqrt{x}\,\sqrt{ex+d}\,\sqrt{cx+b}\,b-\left(\int \frac{\sqrt{ex+d}\,\sqrt{cx+b}}{\sqrt{x}\,b\,d^2+2\sqrt{x}\,bdex+\sqrt{x}\,b\,e^2x^2+\sqrt{x}\,c\,d^2x+2\sqrt{x}\,cde\,x^2+\sqrt{x}\,c\,e^2x^3}dx\right)b^2d}{}$$

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^(3/2),x)
```

output
```
(2*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b - int((sqrt(d + e*x)*sqrt(b + c*x
))/(sqrt(x)*b*d**2 + 2*sqrt(x)*b*d*e*x + sqrt(x)*b*e**2*x**2 + sqrt(x)*c*d
**2*x + 2*sqrt(x)*c*d*e*x**2 + sqrt(x)*c*e**2*x**3),x)*b**2*d**2 - int((sq
rt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b*d**2 + 2*sqrt(x)*b*d*e*x + sqrt(x)*b
*e**2*x**2 + sqrt(x)*c*d**2*x + 2*sqrt(x)*c*d*e*x**2 + sqrt(x)*c*e**2*x**3
),x)*b**2*d*e*x - int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b*d**2 + 2*
b*d*e*x + b*e**2*x**2 + c*d**2*x + 2*c*d*e*x**2 + c*e**2*x**3),x)*b*c*d*e
- int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b*d**2 + 2*b*d*e*x + b*e**2
*x**2 + c*d**2*x + 2*c*d*e*x**2 + c*e**2*x**3),x)*b*c*e**2*x + 2*int((sqrt
(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b*d**2 + 2*b*d*e*x + b*e**2*x**2 + c*d
**2*x + 2*c*d*e*x**2 + c*e**2*x**3),x)*c**2*d**2 + 2*int((sqrt(x)*sqrt(d +
 e*x)*sqrt(b + c*x)*x)/(b*d**2 + 2*b*d*e*x + b*e**2*x**2 + c*d**2*x + 2*c*
d*e*x**2 + c*e**2*x**3),x)*c**2*d*e*x)/(2*c*d*(d + e*x))
```

3.189 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{5/2}}\,dx$

Optimal result 1518
Mathematica [C] (verified) 1519
Rubi [A] (verified) 1519
Maple [B] (verified) 1523
Fricas [B] (verification not implemented) 1524
Sympy [F] 1524
Maxima [F] 1525
Giac [F] 1525
Mupad [F(-1)] 1525
Reduce [F] 1526

Optimal result

Integrand size = 23, antiderivative size = 244

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{5/2}}\,dx = -\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}} + \frac{2(2cd-be)\sqrt{bx+cx^2}\,E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{3\sqrt{d}e^{3/2}(cd-be)\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}} - \frac{2c\sqrt{d}\sqrt{bx+cx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{3e^{3/2}(cd-be)\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}$$

output
```
-2/3*(c*x^2+b*x)^(1/2)/e/(e*x+d)^(3/2)+2/3*(-b*e+2*c*d)*(c*x^2+b*x)^(1/2)*
EllipticE(e^(1/2)*x^(1/2)/d^(1/2)/(1+e*x/d)^(1/2),(1-c*d/b/e)^(1/2))/d^(1/
2)/e^(3/2)/(-b*e+c*d)/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)-2/
3*c*d^(1/2)*(c*x^2+b*x)^(1/2)*InverseJacobiAM(arctan(e^(1/2)*x^(1/2)/d^(1/
2)),(1-c*d/b/e)^(1/2))/e^(3/2)/(-b*e+c*d)/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1
/2)/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.64 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{5/2}}\,dx =$$

$$-\frac{2\left(ex(b+cx)\left(be^2x-cd(d+2ex)\right)+(d+ex)\left((2cd-be)(b+cx)(d+ex)-i\sqrt{\frac{b}{c}}ce(-2cd+be)\sqrt{1+\right.\right.}{3de^2(cd-be}$$

input
```
Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^(5/2),x]
```

output
```
(-2*(e*x*(b + c*x)*(b*e^2*x - c*d*(d + 2*e*x)) + (d + e*x)*((2*c*d - b*e)*
(b + c*x)*(d + e*x) - I*Sqrt[b/c]*c*e*(-2*c*d + b*e)*Sqrt[1 + b/(c*x)]*Sqr
t[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)
] + I*Sqrt[b/c]*c*e*(-(c*d) + b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(
3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(3*d*e^2*(c*d
 - b*e)*Sqrt[x*(b + c*x)]*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1161, 1237, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{ 1161}$$

$$\frac{\int \frac{b+2cx}{(d+ex)^{3/2}\sqrt{cx^2+bx}}dx}{3e}-\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{ 1237}$$

$$\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}-\frac{2\int \frac{c(bd+(2cd-be)x)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{d(cd-be)}}{3e}-\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}-\frac{c\int \frac{bd+(2cd-be)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{d(cd-be)}}{3e}-\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \ 1269$$

$$\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{2d(cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}\right)}{d(cd-be)}}{3e}-\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \ 1169$$

$$\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{d(cd-be)}}{3e}-\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \ 122$$

$$\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{d(cd-be)}}{3e}-\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \ 120$$

$$\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{d(cd-be)}}{3e}-\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \ 127$$

$$\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\int\frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{d(cd-be)}}{3e}-\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{ 126}$$

$$\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{d(cd-be)}}{3e}-\frac{2\sqrt{bx+cx^2}}{3e(d+ex)^{3/2}}$$

input
```
Int[Sqrt[b*x + c*x^2]/(d + e*x)^(5/2),x]
```

output
```
(-2*Sqrt[b*x + c*x^2])/(3*e*(d + e*x)^(3/2)) + ((2*(2*c*d - b*e)*Sqrt[b*x
+ c*x^2])/(d*(c*d - b*e)*Sqrt[d + e*x]) - (c*((2*Sqrt[-b]*(2*c*d - b*e)*Sq
rt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/S
qrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) -
(4*Sqrt[-b]*d*(c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*Elli
pticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d
+ e*x]*Sqrt[b*x + c*x^2])))/(d*(c*d - b*e)))/(3*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 494 vs. $2(211) = 422$.

Time = 0.75 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.03

method	result
elliptic	$\frac{\sqrt{x(cx+b)}\,\sqrt{(cx+b)x(ex+d)}\left(-\frac{2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3e^3\left(x+\frac{d}{e}\right)^2}+\frac{2\left(ce\,x^2+bex\right)(be-2cd)}{3e^2d(be-cd)\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2+bex\right)}}+\frac{2\left(\frac{2c}{3e^2}+\frac{be-2cd}{3e^2d}-\frac{b(be-2cd)}{3ed(be-cd)}\right)d\sqrt{\frac{(x+}{}}}{e}\right.}{}$
default	$\frac{2\left(\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\ \text{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^2d\,e^3x-\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\ \text{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)bc\right.}{}$

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)*x*(e*x+d))^(1/2)/x/(c*x+b)*(-2/
3/e^3*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(x+d/e)^2+2/3*(c*e*x^2+b*e*x)/
e^2/d/(b*e-c*d)*(b*e-2*c*d)/((x+d/e)*(c*e*x^2+b*e*x))^(1/2)+2*(2/3*c/e^2+1
/3/e^2*(b*e-2*c*d)/d-1/3*b/e/d/(b*e-c*d)*(b*e-2*c*d))*d/e*((x+d/e)/d*e)^(1
/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d
*x)^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-2/3*c/e^2
*(b*e-2*c*d)/(b*e-c*d)*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*
x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x
+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2
),(-d/e/(-d/e+b/c))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(211) = 422.

Time = 0.15 (sec) , antiderivative size = 557, normalized size of antiderivative = 2.28

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{5/2}}\,dx = \frac{2\left((2\,c^2d^4-2\,bcd^3e-b^2d^2e^2+(2\,c^2d^2e^2-2\,bcde^3-b^2e^4)x^2+2\,(2\,c^2d^3e-2\,bcd^2e^2-\right.}{}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
2/9*((2*c^2*d^4 - 2*b*c*d^3*e - b^2*d^2*e^2 + (2*c^2*d^2*e^2 - 2*b*c*d*e^3
 - b^2*e^4)*x^2 + 2*(2*c^2*d^3*e - 2*b*c*d^2*e^2 - b^2*d*e^3)*x)*sqrt(c*e)
*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2
*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*
e*x + c*d + b*e)/(c*e)) + 3*(2*c^2*d^3*e - b*c*d^2*e^2 + (2*c^2*d*e^3 - b*
c*e^4)*x^2 + 2*(2*c^2*d^2*e^2 - b*c*d*e^3)*x)*sqrt(c*e)*weierstrassZeta(4/
3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*
e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2
 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*
c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(c^2
*d^2*e^2 + (2*c^2*d*e^3 - b*c*e^4)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/(c^
2*d^4*e^3 - b*c*d^3*e^4 + (c^2*d^2*e^5 - b*c*d*e^6)*x^2 + 2*(c^2*d^3*e^4 -
 b*c*d^2*e^5)*x)
```

Sympy [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{5/2}}\,dx = \int \frac{\sqrt{x\,(b+cx)}}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x)**(1/2)/(e*x+d)**(5/2),x)
```

output
```
Integral(sqrt(x*(b + c*x))/(d + e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{5/2}}\,dx = \int \frac{\sqrt{cx^2+bx}}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x)/(e*x + d)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{5/2}}\,dx = \int \frac{\sqrt{cx^2+bx}}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x)/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{5/2}}\,dx = \int \frac{\sqrt{c\,x^2+b\,x}}{(d+e\,x)^{5/2}}\,dx$$

input
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^(5/2),x)
```

output
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^(5/2),x)
```

output
```
( - 2*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b + int((sqrt(d + e*x)*sqrt(b +
c*x))/(sqrt(x)*b**2*d**3*e + 3*sqrt(x)*b**2*d**2*e**2*x + 3*sqrt(x)*b**2*d
*e**3*x**2 + sqrt(x)*b**2*e**4*x**3 - sqrt(x)*b*c*d**4 - 2*sqrt(x)*b*c*d**
3*e*x + 2*sqrt(x)*b*c*d*e**3*x**3 + sqrt(x)*b*c*e**4*x**4 - sqrt(x)*c**2*d
**4*x - 3*sqrt(x)*c**2*d**3*e*x**2 - 3*sqrt(x)*c**2*d**2*e**2*x**3 - sqrt(
x)*c**2*d*e**3*x**4),x)*b**3*d**3*e + 2*int((sqrt(d + e*x)*sqrt(b + c*x))/
(sqrt(x)*b**2*d**3*e + 3*sqrt(x)*b**2*d**2*e**2*x + 3*sqrt(x)*b**2*d*e**3*
x**2 + sqrt(x)*b**2*e**4*x**3 - sqrt(x)*b*c*d**4 - 2*sqrt(x)*b*c*d**3*e*x
+ 2*sqrt(x)*b*c*d*e**3*x**3 + sqrt(x)*b*c*e**4*x**4 - sqrt(x)*c**2*d**4*x
- 3*sqrt(x)*c**2*d**3*e*x**2 - 3*sqrt(x)*c**2*d**2*e**2*x**3 - sqrt(x)*c**
2*d*e**3*x**4),x)*b**3*d**2*e**2*x + int((sqrt(d + e*x)*sqrt(b + c*x))/(sq
rt(x)*b**2*d**3*e + 3*sqrt(x)*b**2*d**2*e**2*x + 3*sqrt(x)*b**2*d*e**3*x**
2 + sqrt(x)*b**2*e**4*x**3 - sqrt(x)*b*c*d**4 - 2*sqrt(x)*b*c*d**3*e*x + 2
*sqrt(x)*b*c*d*e**3*x**3 + sqrt(x)*b*c*e**4*x**4 - sqrt(x)*c**2*d**4*x - 3
*sqrt(x)*c**2*d**3*e*x**2 - 3*sqrt(x)*c**2*d**2*e**2*x**3 - sqrt(x)*c**2*d
*e**3*x**4),x)*b**3*d*e**3*x**2 - int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(
x)*b**2*d**3*e + 3*sqrt(x)*b**2*d**2*e**2*x + 3*sqrt(x)*b**2*d*e**3*x**2 +
 sqrt(x)*b**2*e**4*x**3 - sqrt(x)*b*c*d**4 - 2*sqrt(x)*b*c*d**3*e*x + 2*sq
rt(x)*b*c*d*e**3*x**3 + sqrt(x)*b*c*e**4*x**4 - sqrt(x)*c**2*d**4*x - 3*sq
rt(x)*c**2*d**3*e*x**2 - 3*sqrt(x)*c**2*d**2*e**2*x**3 - sqrt(x)*c**2*d...
```

3.190 $\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{7/2}}\,dx$

Optimal result 1527
Mathematica [C] (verified) 1528
Rubi [A] (verified) 1528
Maple [B] (verified) 1532
Fricas [B] (verification not implemented) 1533
Sympy [F] 1534
Maxima [F] 1535
Giac [F] 1535
Mupad [F(-1)] 1535
Reduce [F] 1536

Optimal result

Integrand size = 23, antiderivative size = 314

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{7/2}}\,dx = -\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}}+\frac{2(2cd-be)\sqrt{bx+cx^2}}{15de(cd-be)(d+ex)^{3/2}} + \frac{4(c^2d^2-bcde+b^2e^2)\sqrt{bx+cx^2}\,E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{15d^{3/2}e^{3/2}(cd-be)^2\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}} - \frac{2c(cd+be)\sqrt{bx+cx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{15\sqrt{d}e^{3/2}(cd-be)^2\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}$$

output
```
-2/5*(c*x^2+b*x)^(1/2)/e/(e*x+d)^(5/2)+2/15*(-b*e+2*c*d)*(c*x^2+b*x)^(1/2)
/d/e/(-b*e+c*d)/(e*x+d)^(3/2)+4/15*(b^2*e^2-b*c*d*e+c^2*d^2)*(c*x^2+b*x)^(
1/2)*EllipticE(e^(1/2)*x^(1/2)/d^(1/2)/(1+e*x/d)^(1/2),(1-c*d/b/e)^(1/2))/
d^(3/2)/e^(3/2)/(-b*e+c*d)^2/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(
1/2)-2/15*c*(b*e+c*d)*(c*x^2+b*x)^(1/2)*InverseJacobiAM(arctan(e^(1/2)*x^(
1/2)/d^(1/2)),(1-c*d/b/e)^(1/2))/d^(1/2)/e^(3/2)/(-b*e+c*d)^2/x^(1/2)/(d*(
c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.80 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.15

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{7/2}}\,dx =$$

$$-\frac{2\left(bex(b+cx)\left(-b^2e^3x(5d+2ex)-c^2d^2(d^2+6dex+2e^2x^2)+bcde(-d^2+7dex+2e^2x^2)\right)+\sqrt{\frac{b}{c}}c(d+\right.}{}$$

input
```
Integrate[Sqrt[b*x + c*x^2]/(d + e*x)^(7/2),x]
```

output
```
(-2*(b*e*x*(b + c*x)*(-(b^2*e^3*x*(5*d + 2*e*x)) - c^2*d^2*(d^2 + 6*d*e*x
+ 2*e^2*x^2) + b*c*d*e*(-d^2 + 7*d*e*x + 2*e^2*x^2)) + Sqrt[b/c]*c*(d + e*
x)^2*(2*Sqrt[b/c]*(c^2*d^2 - b*c*d*e + b^2*e^2)*(b + c*x)*(d + e*x) + (2*I
)*b*e*(c^2*d^2 - b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^
(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(c^2*d^
2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*Ell
ipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(15*b*d^2*e^2*(c*d -
b*e)^2*Sqrt[x*(b + c*x)]*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 412, normalized size of antiderivative = 1.31, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {1161, 1237, 27, 1237, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{7/2}}\,dx$$

$$\Big\downarrow \; 1161$$

$$\frac{\int \frac{b+2cx}{(d+ex)^{5/2}\sqrt{cx^2+bx}}dx}{5e}-\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \ 1237$$

$$\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}-\frac{2\int -\frac{b(cd-2be)+c(2cd-be)x}{2(d+ex)^{3/2}\sqrt{cx^2+bx}}dx}{3d(cd-be)}}{5e}-\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{b(cd-2be)+c(2cd-be)x}{(d+ex)^{3/2}\sqrt{cx^2+bx}}dx}{3d(cd-be)}+\frac{2\sqrt{bx+cx^2}(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{5e}-\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \ 1237$$

$$\frac{\frac{\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{2\int \frac{c\left(bd(cd+be)+2\left(c^2d^2-bced+b^2e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{d(cd-be)}}{3d(cd-be)}+\frac{2\sqrt{bx+cx^2}(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{5e}-\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\int \frac{bd(cd+be)+2\left(c^2d^2-bced+b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{d(cd-be)}}{3d(cd-be)}+\frac{2\sqrt{bx+cx^2}(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{5e}-\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \ 1269$$

$$\frac{\frac{\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\left(b^2e^2-bcde+c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{d(cd-be)(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}\right)}{d(cd-be)}}{3d(cd-be)}+\frac{2\sqrt{bx+cx^2}(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{5e}-\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \ 1169$$

$$\frac{\frac{\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\sqrt{x}\sqrt{b+cx}\left(b^2e^2-bcde+c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{d(cd-be)}}{3d(cd-be)}+\frac{2\sqrt{bx+cx^2}(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{5e}-\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}}$$

$\big\downarrow$ 122

$$\frac{\frac{\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{d(cd-be)}}{3d(cd-be)}+\frac{2\sqrt{bx\cdots}}{3d(d+\cdots}}{5e}$$

$$\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}}$$

$\big\downarrow$ 120

$$\frac{\frac{\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\cdots}{d(cd-be)}}{3d(cd-be)}\cdots}{5e}$$

$$\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}}$$

$\big\downarrow$ 127

$$\frac{\frac{\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\cdots}}\cdots}{e\sqrt{bx+cx^2}\sqrt{d+ex}}\cdots}{d(cd-be)}}{3d(cd-be)}\cdots}{5e}$$

$$\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}}$$

$\big\downarrow$ 126

$$\frac{\frac{\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\,\text{EllipticF}\cdots}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}\cdots}{d(cd-be)}}{3d(cd-be)}\cdots}{5e}$$

$$\frac{2\sqrt{bx+cx^2}}{5e(d+ex)^{5/2}}$$

input
```
Int[Sqrt[b*x + c*x^2]/(d + e*x)^(7/2),x]
```

output
```
(-2*Sqrt[b*x + c*x^2])/(5*e*(d + e*x)^(5/2)) + ((2*(2*c*d - b*e)*Sqrt[b*x
+ c*x^2])/(3*d*(c*d - b*e)*(d + e*x)^(3/2)) + ((4*(c^2*d^2 - b*c*d*e + b^2
*e^2)*Sqrt[b*x + c*x^2])/(d*(c*d - b*e)*Sqrt[d + e*x]) - (c*((4*Sqrt[-b]*(
c^2*d^2 - b*c*d*e + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*Ellip
ticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 +
 (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*Sqr
t[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x]
)/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])))/(
d*(c*d - b*e)))/(3*d*(c*d - b*e)))/(5*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(275) = 550$.

Time = 1.04 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.04

method	result
elliptic	$\frac{\sqrt{x(cx+b)}\,\sqrt{(cx+b)x(ex+d)}\left(-\frac{2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{5e^4\left(x+\frac{d}{e}\right)^3}+\frac{2(be-2cd)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{15e^3d(be-cd)\left(x+\frac{d}{e}\right)^2}+\frac{4\left(ce\,x^2+bex\right)\left(b^2e^2-bcde+c^2d^2\right)}{15e^2d^2(be-cd)^2\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2+bex\right)}}\right.}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)*x*(e*x+d))^(1/2)/x/(c*x+b)*(-2/
5/e^4*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(x+d/e)^3+2/15*(b*e-2*c*d)/e^3
/d/(b*e-c*d)*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(x+d/e)^2+4/15*(c*e*x^2
+b*e*x)/e^2/d^2/(b*e-c*d)^2*(b^2*e^2-b*c*d*e+c^2*d^2)/((x+d/e)*(c*e*x^2+b*
e*x))^(1/2)+2*(1/15*c*(b*e-2*c*d)/e^2/d/(b*e-c*d)+2/15/e^2/(b*e-c*d)*(b^2*
e^2-b*c*d*e+c^2*d^2)/d^2-2/15*b/e/d^2/(b*e-c*d)^2*(b^2*e^2-b*c*d*e+c^2*d^2
))*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*
x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e
+b/c))^(1/2))-4/15*c/e^2*(b^2*e^2-b*c*d*e+c^2*d^2)/d/(b*e-c*d)^2*((x+d/e)/
d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*
x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c
))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 868 vs. $2(275) = 550$.

Time = 0.15 (sec) , antiderivative size = 868, normalized size of antiderivative = 2.76

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
2/45*((2*c^3*d^6 - 3*b*c^2*d^5*e - 3*b^2*c*d^4*e^2 + 2*b^3*d^3*e^3 + (2*c^
3*d^3*e^3 - 3*b*c^2*d^2*e^4 - 3*b^2*c*d*e^5 + 2*b^3*e^6)*x^3 + 3*(2*c^3*d^
4*e^2 - 3*b*c^2*d^3*e^3 - 3*b^2*c*d^2*e^4 + 2*b^3*d*e^5)*x^2 + 3*(2*c^3*d^
5*e - 3*b*c^2*d^4*e^2 - 3*b^2*c*d^3*e^3 + 2*b^3*d^2*e^4)*x)*sqrt(c*e)*weie
rstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*
d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x +
 c*d + b*e)/(c*e)) + 6*(c^3*d^5*e - b*c^2*d^4*e^2 + b^2*c*d^3*e^3 + (c^3*d
^2*e^4 - b*c^2*d*e^5 + b^2*c*e^6)*x^3 + 3*(c^3*d^3*e^3 - b*c^2*d^2*e^4 + b
^2*c*d*e^5)*x^2 + 3*(c^3*d^4*e^2 - b*c^2*d^3*e^3 + b^2*c*d^2*e^4)*x)*sqrt(
c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2
*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstra
ssPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 -
 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d
+ b*e)/(c*e))) + 3*(c^3*d^4*e^2 + b*c^2*d^3*e^3 + 2*(c^3*d^2*e^4 - b*c^2*d
*e^5 + b^2*c*e^6)*x^2 + (6*c^3*d^3*e^3 - 7*b*c^2*d^2*e^4 + 5*b^2*c*d*e^5)*
x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/(c^3*d^7*e^3 - 2*b*c^2*d^6*e^4 + b^2*c
*d^5*e^5 + (c^3*d^4*e^6 - 2*b*c^2*d^3*e^7 + b^2*c*d^2*e^8)*x^3 + 3*(c^3*d^
5*e^5 - 2*b*c^2*d^4*e^6 + b^2*c*d^3*e^7)*x^2 + 3*(c^3*d^6*e^4 - 2*b*c^2*d^
5*e^5 + b^2*c*d^4*e^6)*x)
```

Sympy [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{7/2}}\,dx = \int \frac{\sqrt{x\,(b+cx)}}{(d+ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x)**(1/2)/(e*x+d)**(7/2),x)
```

output
```
Integral(sqrt(x*(b + c*x))/(d + e*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{7/2}}\,dx=\int \frac{\sqrt{cx^2+bx}}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x)/(e*x + d)^(7/2), x)
```

Giac [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{7/2}}\,dx=\int \frac{\sqrt{cx^2+bx}}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(1/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x)/(e*x + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{7/2}}\,dx=\int \frac{\sqrt{c\,x^2+b\,x}}{{\left(d+e\,x\right)}^{7/2}}\,dx$$

input
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^(7/2),x)
```

output
```
int((b*x + c*x^2)^(1/2)/(d + e*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{bx+cx^2}}{(d+ex)^{7/2}}\,dx = \text{too large to display}$$

input
```
int((c*x^2+b*x)^(1/2)/(e*x+d)^(7/2),x)
```

output
```
( - 2*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b + 2*int((sqrt(d + e*x)*sqrt(b
+ c*x))/(2*sqrt(x)*b**2*d**4*e + 8*sqrt(x)*b**2*d**3*e**2*x + 12*sqrt(x)*b
**2*d**2*e**3*x**2 + 8*sqrt(x)*b**2*d*e**4*x**3 + 2*sqrt(x)*b**2*e**5*x**4
 - sqrt(x)*b*c*d**5 - 2*sqrt(x)*b*c*d**4*e*x + 2*sqrt(x)*b*c*d**3*e**2*x**
2 + 8*sqrt(x)*b*c*d**2*e**3*x**3 + 7*sqrt(x)*b*c*d*e**4*x**4 + 2*sqrt(x)*b
*c*e**5*x**5 - sqrt(x)*c**2*d**5*x - 4*sqrt(x)*c**2*d**4*e*x**2 - 6*sqrt(x
)*c**2*d**3*e**2*x**3 - 4*sqrt(x)*c**2*d**2*e**3*x**4 - sqrt(x)*c**2*d*e**
4*x**5),x)*b**3*d**4*e + 6*int((sqrt(d + e*x)*sqrt(b + c*x))/(2*sqrt(x)*b*
*2*d**4*e + 8*sqrt(x)*b**2*d**3*e**2*x + 12*sqrt(x)*b**2*d**2*e**3*x**2 +
8*sqrt(x)*b**2*d*e**4*x**3 + 2*sqrt(x)*b**2*e**5*x**4 - sqrt(x)*b*c*d**5 -
 2*sqrt(x)*b*c*d**4*e*x + 2*sqrt(x)*b*c*d**3*e**2*x**2 + 8*sqrt(x)*b*c*d**
2*e**3*x**3 + 7*sqrt(x)*b*c*d*e**4*x**4 + 2*sqrt(x)*b*c*e**5*x**5 - sqrt(x
)*c**2*d**5*x - 4*sqrt(x)*c**2*d**4*e*x**2 - 6*sqrt(x)*c**2*d**3*e**2*x**3
 - 4*sqrt(x)*c**2*d**2*e**3*x**4 - sqrt(x)*c**2*d*e**4*x**5),x)*b**3*d**3*
e**2*x + 6*int((sqrt(d + e*x)*sqrt(b + c*x))/(2*sqrt(x)*b**2*d**4*e + 8*sq
rt(x)*b**2*d**3*e**2*x + 12*sqrt(x)*b**2*d**2*e**3*x**2 + 8*sqrt(x)*b**2*d
*e**4*x**3 + 2*sqrt(x)*b**2*e**5*x**4 - sqrt(x)*b*c*d**5 - 2*sqrt(x)*b*c*d
**4*e*x + 2*sqrt(x)*b*c*d**3*e**2*x**2 + 8*sqrt(x)*b*c*d**2*e**3*x**3 + 7*
sqrt(x)*b*c*d*e**4*x**4 + 2*sqrt(x)*b*c*e**5*x**5 - sqrt(x)*c**2*d**5*x -
4*sqrt(x)*c**2*d**4*e*x**2 - 6*sqrt(x)*c**2*d**3*e**2*x**3 - 4*sqrt(x)*...
```

3.191 $\int (d+ex)^{3/2} \left(bx+cx^2\right)^{3/2} dx$

Optimal result 1537
Mathematica [C] (verified) 1538
Rubi [A] (verified) 1539
Maple [B] (verified) 1544
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Maxima [F] 1546
Giac [F] 1546
Mupad [F(-1)] 1546
Reduce [F] 1547

Optimal result

Integrand size = 23, antiderivative size = 620

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^{3/2} dx = -\frac{16(cd-2be)(2cd-be)(cd+be)\left(c^2d^2-bcde+b^2e^2\right)x\sqrt{d+ex}}{1155c^3e^4\sqrt{bx+cx^2}} + \frac{2\left(8c^4d^4-19bc^3d^3e+6b^2c^2d^2e^2-19b^3cde^3+8b^4e^4\right)\sqrt{d+ex}\sqrt{bx+cx^2}}{1155c^3e^3} - \frac{4(cd-3be)(3cd-be)(cd+be)x\sqrt{d+ex}\sqrt{bx+cx^2}}{1155c^2e^2} + \frac{2}{231}\left(13bd+\frac{cd^2}{e}-\frac{6b^2e}{c}\right)x^2\sqrt{d+ex}\sqrt{bx+cx^2} + \frac{2(cd+be)x\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}}{33c} + \frac{2}{11}x(d+ex)^{3/2}\left(bx+cx^2\right)^{3/2} + \frac{16\sqrt{b}(cd-2be)(2cd-be)(cd+be)\left(c^2d^2-bcde+b^2e^2\right)\sqrt{x}\sqrt{d+ex}E\left(\text{arct}\right.}{1155c^{7/2}e^4\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$

output
```
-16/1155*(-2*b*e+c*d)*(-b*e+2*c*d)*(b*e+c*d)*(b^2*e^2-b*c*d*e+c^2*d^2)*x*(
e*x+d)^(1/2)/c^3/e^4/(c*x^2+b*x)^(1/2)+2/1155*(8*b^4*e^4-19*b^3*c*d*e^3+6*
b^2*c^2*d^2*e^2-19*b*c^3*d^3*e+8*c^4*d^4)*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/
c^3/e^3-4/1155*(-3*b*e+c*d)*(-b*e+3*c*d)*(b*e+c*d)*x*(e*x+d)^(1/2)*(c*x^2+
b*x)^(1/2)/c^2/e^2+2/231*(13*b*d+c*d^2/e-6*b^2*e/c)*x^2*(e*x+d)^(1/2)*(c*x
^2+b*x)^(1/2)+2/33*(b*e+c*d)*x*(e*x+d)^(1/2)*(c*x^2+b*x)^(3/2)/c+2/11*x*(e
*x+d)^(3/2)*(c*x^2+b*x)^(3/2)+16/1155*b^(1/2)*(-2*b*e+c*d)*(-b*e+2*c*d)*(b
*e+c*d)*(b^2*e^2-b*c*d*e+c^2*d^2)*x^(1/2)*(e*x+d)^(1/2)*EllipticE(c^(1/2)*
x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/c^(7/2)/e^4/(b*(e*x+d)/
d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)-2/1155*b^(3/2)*(8*b^4*e^4-19*b^3*c*d*e^
3+6*b^2*c^2*d^2*e^2-19*b*c^3*d^3*e+8*c^4*d^4)*x^(1/2)*(e*x+d)^(1/2)*Invers
eJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e/c/d)^(1/2))/c^(7/2)/e^3/(
b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.67 (sec) , antiderivative size = 559, normalized size of antiderivative = 0.90

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^{3/2} dx = \frac{2(x(b+cx))^{3/2} \left(bex(b+cx)(d+ex) \left(8b^4e^4 - b^3ce^3(19d+6ex) + b^2c^2e^2(6d^2+14dex+5e^2 \cdots\right.\right.}{\cdots}$$

input
```
Integrate[(d + e*x)^(3/2)*(b*x + c*x^2)^(3/2),x]
```

output
```
(2*(x*(b + c*x))^(3/2)*(b*e*x*(b + c*x)*(d + e*x)*(8*b^4*e^4 - b^3*c*e^3*(
19*d + 6*e*x) + b^2*c^2*e^2*(6*d^2 + 14*d*e*x + 5*e^2*x^2) + b*c^3*e*(-19*
d^3 + 14*d^2*e*x + 205*d*e^2*x^2 + 140*e^3*x^3) + c^4*(8*d^4 - 6*d^3*e*x +
 5*d^2*e^2*x^2 + 140*d*e^3*x^3 + 105*e^4*x^4)) + Sqrt[b/c]*(-8*Sqrt[b/c]*(
2*c^5*d^5 - 5*b*c^4*d^4*e + 2*b^2*c^3*d^3*e^2 + 2*b^3*c^2*d^2*e^3 - 5*b^4*
c*d*e^4 + 2*b^5*e^5)*(b + c*x)*(d + e*x) - (8*I)*b*e*(2*c^5*d^5 - 5*b*c^4*
d^4*e + 2*b^2*c^3*d^3*e^2 + 2*b^3*c^2*d^2*e^3 - 5*b^4*c*d*e^4 + 2*b^5*e^5)
*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]
/Sqrt[x]], (c*d)/(b*e)] + I*b*e*(8*c^5*d^5 - 21*b*c^4*d^4*e + 10*b^2*c^3*d
^3*e^2 + 35*b^3*c^2*d^2*e^3 - 48*b^4*c*d*e^4 + 16*b^5*e^5)*Sqrt[1 + b/(c*x
)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)
/(b*e)]))/(1155*b*c^3*e^4*x^2*(b + c*x)^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 547, normalized size of antiderivative = 0.88, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {1166, 27, 1231, 27, 1231, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{3/2}(d+ex)^{3/2}\,dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{2\int \frac{(d(11cd-5be)+6e(2cd-be)x)\left(cx^2+bx\right)^{3/2}}{2\sqrt{d+ex}}dx}{11c}+\frac{2e\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(d(11cd-5be)+6e(2cd-be)x)\left(cx^2+bx\right)^{3/2}}{\sqrt{d+ex}}dx}{11c}+\frac{2e\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$$\Big\downarrow \text{1231}$$

$$\frac{\frac{2(bx+cx^2)^{3/2}\sqrt{d+ex}\left(-6b^2e^2+14cex(2cd-be)+13bcde+c^2d^2\right)}{21ce}-\frac{2\int\frac{3e\left(bd\left(c^2d^2+13bced-6b^2e^2\right)+(2cd-be)\left(c^2d^2-bced+8b^2e^2\right)x\right)\sqrt{cx^2+bx}}{2\sqrt{d+ex}}dx}{21ce^2}}{11c}+\frac{2e\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\big\downarrow$ 27

$$\frac{\frac{2(bx+cx^2)^{3/2}\sqrt{d+ex}\left(-6b^2e^2+14cex(2cd-be)+13bcde+c^2d^2\right)}{21ce}-\frac{\int\frac{\left(bd\left(c^2d^2+13bced-6b^2e^2\right)+(2cd-be)\left(c^2d^2-bced+8b^2e^2\right)x\right)\sqrt{cx^2+bx}}{\sqrt{d+ex}}dx}{7ce}}{11c}+\frac{2e\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\big\downarrow$ 1231

$$\frac{\frac{2(bx+cx^2)^{3/2}\sqrt{d+ex}\left(-6b^2e^2+14cex(2cd-be)+13bcde+c^2d^2\right)}{21ce}-\frac{-\frac{2\int-\frac{bd\left(8c^4d^4-19bc^3ed^3+6b^2c^2e^2d^2-19b^3ce^3d+8b^4e^4\right)+8(cd-2be)(2cd-be)(cd+}{2\sqrt{d+ex}\sqrt{cx^2+bx}}}{15ce^2}}{}}{11c}$$
$$\frac{2e\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\big\downarrow$ 27

$$\frac{\frac{2(bx+cx^2)^{3/2}\sqrt{d+ex}\left(-6b^2e^2+14cex(2cd-be)+13bcde+c^2d^2\right)}{21ce}-\frac{\frac{\int\frac{bd\left(8c^4d^4-19bc^3ed^3+6b^2c^2e^2d^2-19b^3ce^3d+8b^4e^4\right)+8(cd-2be)(2cd-be)(cd+be)(}{\sqrt{d+ex}\sqrt{cx^2+bx}}}{15ce^2}}{}}{11c}$$
$$\frac{2e\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\big\downarrow$ 1269

$$\frac{2(bx+cx^2)^{3/2}\sqrt{d+ex}\left(-6b^2e^2+14cex(2cd-be)+13bcde+c^2d^2\right)}{21ce}-\frac{\frac{8(cd-2be)(2cd-be)(be+cd)\left(b^2e^2-bcde+c^2d^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{d(cd-be)\left(-8b^4e^4+\right.}{15ce^2}}{}$$
$$\frac{2e\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\big\downarrow$ 1169

$$\frac{\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}\left(-6b^2e^2+14cex(2cd-be)+13bcde+c^2d^2\right)}{21ce}-\frac{\frac{\frac{8\sqrt{x}\sqrt{b+cx}(cd-2be)(2cd-be)(be+cd)\left(b^2e^2-bcde+c^2d^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{d\sqrt{x}\sqrt{b+c\cdots}}{\cdots}}{15\cdots}}{\cdots}}{\cdots}$$

$$\frac{2e\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\Big\downarrow$ 122

$$\frac{\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}\left(-6b^2e^2+14cex(2cd-be)+13bcde+c^2d^2\right)}{21ce}-\frac{\frac{\frac{8\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(cd-2be)(2cd-be)(be+cd)\left(b^2e^2-bcde+c^2d^2\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2e\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\Big\downarrow$ 120

$$\frac{\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}\left(-6b^2e^2+14cex(2cd-be)+13bcde+c^2d^2\right)}{21ce}-\frac{\frac{\frac{16\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(cd-2be)(2cd-be)(be+cd)\left(b^2e^2-bcde+c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c\cdots}}{\sqrt{\cdots}}\cdots\right.\right.}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2e\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\Big\downarrow$ 127

$$\frac{\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}\left(-6b^2e^2+14cex(2cd-be)+13bcde+c^2d^2\right)}{21ce}-\frac{\frac{\frac{16\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(cd-2be)(2cd-be)(be+cd)\left(b^2e^2-bcde+c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c\cdots}}{\sqrt{\cdots}}\cdots\right.\right.}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2e\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

$\Big\downarrow$ 126

$$\frac{\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}\left(-6b^2e^2+14cex(2cd-be)+13bcde+c^2d^2\right)}{21ce}-\frac{\frac{\frac{16\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(cd-2be)(2cd-be)(be+cd)\left(b^2e^2-bcde+c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c\cdots}}{\sqrt{\cdots}}\cdots\right.\right.}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}\cdots}{\cdots}}{\cdots}}{\cdots}$$

$$\frac{2e\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11c}$$

input
```
Int[(d + e*x)^(3/2)*(b*x + c*x^2)^(3/2),x]
```

output
```
(2*e*Sqrt[d + e*x]*(b*x + c*x^2)^(5/2))/(11*c) + ((2*Sqrt[d + e*x]*(c^2*d^
2 + 13*b*c*d*e - 6*b^2*e^2 + 14*c*e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(3/2))/
(21*c*e) - ((-2*Sqrt[d + e*x]*(8*c^4*d^4 - 19*b*c^3*d^3*e + 6*b^2*c^2*d^2*
e^2 - 19*b^3*c*d*e^3 + 8*b^4*e^4 - 3*c*e*(2*c*d - b*e)*(c^2*d^2 - b*c*d*e
+ 8*b^2*e^2)*x)*Sqrt[b*x + c*x^2])/(15*c*e^2) + ((16*Sqrt[-b]*(c*d - 2*b*e
)*(2*c*d - b*e)*(c*d + b*e)*(c^2*d^2 - b*c*d*e + b^2*e^2)*Sqrt[x]*Sqrt[1 +
 (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e
)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[-b]*d*
(c*d - b*e)*(16*c^4*d^4 - 32*b*c^3*d^3*e + 3*b^2*c^2*d^2*e^2 + 13*b^3*c*d*
e^3 - 8*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[Arc
Sin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sq
rt[b*x + c*x^2]))/(15*c*e^2))/(7*c*e))/(11*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1358 vs. 2(557) = 1114.

Time = 1.62 (sec) , antiderivative size = 1359, normalized size of antiderivative = 2.19

method	result	size
default	Expression too large to display	1359
elliptic	Expression too large to display	1623

input
```
int((e*x+d)^(3/2)*(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/1155*(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*(105*c^6*e^7*x^7+16*((e*x+d)/d)^(1/
2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),
(-d*c/(b*e-c*d))^(1/2))*b^6*d*e^6-16*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d
))^(1/2)*(-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2)
)*b^6*d*e^6-48*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2
)*EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^5*c*d^2*e^5+35*((e
*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+
d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^4*c^2*d^3*e^4+10*((e*x+d)/d)^(1/2)*(
e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(-d*
c/(b*e-c*d))^(1/2))*b^3*c^3*d^4*e^3-21*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c
*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/
2))*b^2*c^4*d^5*e^2+8*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/
d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b*c^5*d^6*e+5
6*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticE((
(e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^5*c*d^2*e^5-56*((e*x+d)/d)^(1/2
)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(
-d*c/(b*e-c*d))^(1/2))*b^4*c^2*d^3*e^4+56*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*
e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^
(1/2))*b^2*c^4*d^5*e^2-56*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-
e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b*c^5*...
```

Fricas [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.03

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^{3/2} \, dx = \frac{2\left(\left(16\,c^6d^6-48\,bc^5d^5e+33\,b^2c^4d^4e^2+14\,b^3c^3d^3e^3+33\,b^4c^2d^2e^4-48\,b^5cde^5+16\,b^6e^6\right)\sqrt{ce}\right.}{}$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
2/3465*((16*c^6*d^6 - 48*b*c^5*d^5*e + 33*b^2*c^4*d^4*e^2 + 14*b^3*c^3*d^3
*e^3 + 33*b^4*c^2*d^2*e^4 - 48*b^5*c*d*e^5 + 16*b^6*e^6)*sqrt(c*e)*weierst
rassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3
 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*
d + b*e)/(c*e)) + 24*(2*c^6*d^5*e - 5*b*c^5*d^4*e^2 + 2*b^2*c^4*d^3*e^3 +
2*b^3*c^3*d^2*e^4 - 5*b^4*c^2*d*e^5 + 2*b^5*c*e^6)*sqrt(c*e)*weierstrassZe
ta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2
*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^
2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3
*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3
*(105*c^6*e^6*x^4 + 8*c^6*d^4*e^2 - 19*b*c^5*d^3*e^3 + 6*b^2*c^4*d^2*e^4 -
 19*b^3*c^3*d*e^5 + 8*b^4*c^2*e^6 + 140*(c^6*d*e^5 + b*c^5*e^6)*x^3 + 5*(c
^6*d^2*e^4 + 41*b*c^5*d*e^5 + b^2*c^4*e^6)*x^2 - 2*(3*c^6*d^3*e^3 - 7*b*c^
5*d^2*e^4 - 7*b^2*c^4*d*e^5 + 3*b^3*c^3*e^6)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x
 + d))/(c^5*e^5)
```

Sympy [F]

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^{3/2} \, dx = \int \left(x(b+cx)\right)^{\frac{3}{2}} (d+ex)^{\frac{3}{2}} \, dx$$

input
```
integrate((e*x+d)**(3/2)*(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^{3/2} dx = \int \left(cx^2+bx\right)^{\frac{3}{2}} (ex+d)^{\frac{3}{2}} dx$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(3/2)*(e*x + d)^(3/2), x)
```

Giac [F]

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^{3/2} dx = \int \left(cx^2+bx\right)^{\frac{3}{2}} (ex+d)^{\frac{3}{2}} dx$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(3/2)*(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^{3/2} dx = \int \left(c\,x^2+b\,x\right)^{3/2} (d+e\,x)^{3/2} dx$$

input
```
int((b*x + c*x^2)^(3/2)*(d + e*x)^(3/2),x)
```

output
```
int((b*x + c*x^2)^(3/2)*(d + e*x)^(3/2), x)
```

Reduce [F]

$$\int (d+ex)^{3/2} (bx +cx^2)^{3/2} dx = \frac{18\sqrt{x}\,\sqrt{ex+d}\,\sqrt{cx+b}\,b^3 d\,e^2 - 12\sqrt{x}\,\sqrt{ex+d}\,\sqrt{cx+b}\,b^3 e^3 x - 60\sqrt{x}\,\sqrt{ex+d}\,\sqrt{cx+b}\,b^2 c}{}$$

input
```
int((e*x+d)^(3/2)*(c*x^2+b*x)^(3/2),x)
```

output
```
(18*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**3*d*e**2 - 12*sqrt(x)*sqrt(d +
e*x)*sqrt(b + c*x)*b**3*e**3*x - 60*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*
*2*c*d**2*e + 28*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c*d*e**2*x + 10*
sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c*e**3*x**2 + 18*sqrt(x)*sqrt(d +
 e*x)*sqrt(b + c*x)*b*c**2*d**3 + 28*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b
*c**2*d**2*e*x + 410*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**2*d*e**2*x**
2 + 280*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**2*e**3*x**3 - 12*sqrt(x)*
sqrt(d + e*x)*sqrt(b + c*x)*c**3*d**3*x + 10*sqrt(x)*sqrt(d + e*x)*sqrt(b
+ c*x)*c**3*d**2*e*x**2 + 280*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**3*d*e
**2*x**3 + 210*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**3*e**3*x**4 + 24*int
((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b*d + b*e*x + c*d*x + c*e*x**2),
x)*b**4*e**4 - 84*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b*d + b*e*x
 + c*d*x + c*e*x**2),x)*b**3*c*d*e**3 + 108*int((sqrt(x)*sqrt(d + e*x)*sqr
t(b + c*x)*x)/(b*d + b*e*x + c*d*x + c*e*x**2),x)*b**2*c**2*d**2*e**2 - 84
*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b*d + b*e*x + c*d*x + c*e*x*
*2),x)*b*c**3*d**3*e + 24*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b*d
 + b*e*x + c*d*x + c*e*x**2),x)*c**4*d**4 - 9*int((sqrt(x)*sqrt(d + e*x)*s
qrt(b + c*x))/(b*d*x + b*e*x**2 + c*d*x**2 + c*e*x**3),x)*b**4*d**2*e**2 +
 30*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b*d*x + b*e*x**2 + c*d*x**2
 + c*e*x**3),x)*b**3*c*d**3*e - 9*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c...
```

3.192 $\int \sqrt{d+ex}(bx+cx^2)^{3/2}\,dx$

Optimal result 1548
Mathematica [C] (verified) 1549
Rubi [A] (verified) 1550
Maple [A] (verified) 1555
Fricas [A] (verification not implemented) 1556
Sympy [F] 1557
Maxima [F] 1557
Giac [F] 1558
Mupad [F(-1)] 1558
Reduce [F] 1558

Optimal result

Integrand size = 23, antiderivative size = 552

$$\begin{aligned}
&\int \sqrt{d+ex}(bx+cx^2)^{3/2}\,dx = \\
&-\frac{2(16c^4d^4-32bc^3d^3e+9b^2c^2d^2e^2+7b^3cde^3-8b^4e^4)\,x\sqrt{d+ex}}{315c^2e^4\sqrt{bx+cx^2}} \\
&+\frac{2(8c^3d^3-15bc^2d^2e+3b^2cde^2-4b^3e^3)\sqrt{d+ex}\sqrt{bx+cx^2}}{315c^2e^3} \\
&+\frac{2\left(11bd-\frac{6cd^2}{e}+\frac{3b^2e}{c}\right)x\sqrt{d+ex}\sqrt{bx+cx^2}}{315e} \\
&+\frac{2(cd+3be)x^2\sqrt{d+ex}\sqrt{bx+cx^2}}{63e}+\frac{2}{9}x\sqrt{d+ex}(bx+cx^2)^{3/2} \\
&+\frac{2\sqrt{b}(16c^4d^4-32bc^3d^3e+9b^2c^2d^2e^2+7b^3cde^3-8b^4e^4)\sqrt{x}\sqrt{d+ex}\,E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{315c^{5/2}e^4\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} \\
&-\frac{2b^{3/2}(8c^3d^3-15bc^2d^2e+3b^2cde^2-4b^3e^3)\sqrt{x}\sqrt{d+ex}\,\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{315c^{5/2}e^3\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}
\end{aligned}$$

output
```
-2/315*(-8*b^4*e^4+7*b^3*c*d*e^3+9*b^2*c^2*d^2*e^2-32*b*c^3*d^3*e+16*c^4*d
^4)*x*(e*x+d)^(1/2)/c^2/e^4/(c*x^2+b*x)^(1/2)+2/315*(-4*b^3*e^3+3*b^2*c*d*
e^2-15*b*c^2*d^2*e+8*c^3*d^3)*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/c^2/e^3+2/31
5*(11*b*d-6*c*d^2/e+3*b^2*e/c)*x*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/e+2/63*(3
*b*e+c*d)*x^2*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/e+2/9*x*(e*x+d)^(1/2)*(c*x^2
+b*x)^(3/2)+2/315*b^(1/2)*(-8*b^4*e^4+7*b^3*c*d*e^3+9*b^2*c^2*d^2*e^2-32*b
*c^3*d^3*e+16*c^4*d^4)*x^(1/2)*(e*x+d)^(1/2)*EllipticE(c^(1/2)*x^(1/2)/b^(
1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/c^(5/2)/e^4/(b*(e*x+d)/d/(c*x+b))^
(1/2)/(c*x^2+b*x)^(1/2)-2/315*b^(3/2)*(-4*b^3*e^3+3*b^2*c*d*e^2-15*b*c^2*d
^2*e+8*c^3*d^3)*x^(1/2)*(e*x+d)^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/
2)/b^(1/2)),(1-b*e/c/d)^(1/2))/c^(5/2)/e^3/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*
x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.24 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.84

$$\int \sqrt{d+ex}(bx+cx^2)^{3/2}\,dx = \frac{2(x(b+cx))^{3/2}\Big(bex(b+cx)(d+ex)\left(-4b^3e^3+3b^2ce^2(d+ex)+bc^2e(-15d^2+11dex+50\right.}{}$$

input
```
Integrate[Sqrt[d + e*x]*(b*x + c*x^2)^(3/2),x]
```

output
```
(2*(x*(b + c*x))^(3/2)*(b*e*x*(b + c*x)*(d + e*x)*(-4*b^3*e^3 + 3*b^2*c*e^
2*(d + e*x) + b*c^2*e*(-15*d^2 + 11*d*e*x + 50*e^2*x^2) + c^3*(8*d^3 - 6*d
^2*e*x + 5*d*e^2*x^2 + 35*e^3*x^3)) - Sqrt[b/c]*(Sqrt[b/c]*(16*c^4*d^4 - 3
2*b*c^3*d^3*e + 9*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*(b + c*x)*(
d + e*x) + I*b*e*(16*c^4*d^4 - 32*b*c^3*d^3*e + 9*b^2*c^2*d^2*e^2 + 7*b^3*
c*d*e^3 - 8*b^4*e^4)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE
[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(8*c^4*d^4 - 17*b*c^3*
d^3*e + 6*b^2*c^2*d^2*e^2 + 11*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[1 + b/(c*x)]*
Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b
*e)])))/(315*b*c^2*e^4*x^2*(b + c*x)^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 477, normalized size of antiderivative = 0.86, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {1162, 1236, 27, 1231, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{3/2}\sqrt{d+ex}\,dx$$

$$\Big\downarrow \ 1162$$

$$\frac{2\left(bx+cx^2\right)^{3/2}(d+ex)^{3/2}}{9e}-\frac{\int \sqrt{d+ex}(bd+(2cd-be)x)\sqrt{cx^2+bx}dx}{3e}$$

$$\Big\downarrow \ 1236$$

$$\frac{2\left(bx+cx^2\right)^{3/2}(d+ex)^{3/2}}{9e}-\frac{\frac{2\int \frac{\left(bd(cd+3be)+2\left(c^2d^2-bced+2b^2e^2\right)x\right)\sqrt{cx^2+bx}}{2\sqrt{d+ex}}dx}{7c}+\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}(2cd-be)}{7c}}{3e}$$

$$\Big\downarrow \ 27$$

$$\frac{2\left(bx+cx^2\right)^{3/2}(d+ex)^{3/2}}{9e}-\frac{\frac{\int \frac{\left(bd(cd+3be)+2\left(c^2d^2-bced+2b^2e^2\right)x\right)\sqrt{cx^2+bx}}{\sqrt{d+ex}}dx}{7c}+\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}(2cd-be)}{7c}}{3e}$$

$$\Big\downarrow \ 1231$$

$$\frac{2\left(bx+cx^2\right)^{3/2}(d+ex)^{3/2}}{9e}-\frac{-\frac{-\frac{2\int -\frac{bd\left(8c^3d^3-15bc^2ed^2+3b^2ce^2d-4b^3e^3\right)+\left(16c^4d^4-32bc^3ed^3+9b^2c^2e^2d^2+7b^3ce^3d-8b^4e^4\right)x}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{15ce^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-4b^3e^3-6cex\left(2b^2e^2-bcde+c^2d^2\right)\cdots\right.}{15ce^2}}{7c}\cdots}{3e}$$

$$\Big\downarrow \ 27$$

$$\frac{2\left(bx+cx^2\right)^{3/2}(d+ex)^{3/2}}{9e}-\frac{\frac{\frac{\int \frac{bd\left(8c^3d^3-15bc^2ed^2+3b^2ce^2d-4b^3e^3\right)+\left(16c^4d^4-32bc^3ed^3+9b^2c^2e^2d^2+7b^3ce^3d-8b^4e^4\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{15ce^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-4b^3e^3-6cex\left(2b^2e^2-bcde+c^2d^2\right)+3b^2\ldots\right.}{15ce^2}}{7c}}{3e}$$

$\downarrow$ 1269

$$\frac{2\left(bx+cx^2\right)^{3/2}(d+ex)^{3/2}}{9e}-\frac{\frac{\frac{\frac{\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)\int\frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{4d(cd-be)(2cd-be)\left(-b^2e^2-2bcde+2c^2d^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}}{15ce^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-\ldots\right.}{\ldots}}{7c}}{3e}$$

$\downarrow$ 1169

$$\frac{2\left(bx+cx^2\right)^{3/2}(d+ex)^{3/2}}{9e}-\frac{\frac{\frac{\frac{\sqrt{x}\sqrt{b+cx}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)\int\frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{4d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\left(-b^2e^2-2bcde+2c^2d^2\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{15ce^2}-\ldots}{7c}}{3e}$$

$\downarrow$ 122

$$\frac{2\left(bx+cx^2\right)^{3/2}(d+ex)^{3/2}}{9e}-\frac{\frac{\frac{\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\left(-b^2e^2-2bcde+2c^2d^2\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+\ldots}}}{e\sqrt{bx+cx^2}}}{15ce^2}}{7c}}{3e}$$

$\downarrow$ 120

$$\frac{2\left(bx+cx^2\right)^{3/2}(d+ex)^{3/2}}{9e}-\frac{\frac{\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\left(-b^2e^2-2bcde+2c^2d^2\right)\int\ldots}{e\sqrt{bx+cx^2}}}{15ce^2}}{7c}}{3e}$$

$\downarrow$ 127

$$\frac{2\left(bx+cx^2\right)^{3/2}(d+ex)^{3/2}}{9e}-$$

$$\frac{\frac{\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\left(-b^2e^2-2bcde+2c\right.}{e\sqrt{bx+cx^2}\sqrt{d+ex}}}{15ce^2}}{7c}}{3e}$$

↓ 126

$$\frac{2\left(bx+cx^2\right)^{3/2}(d+ex)^{3/2}}{9e}-$$

$$\frac{\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\left(-b^2e^2-2bcd\right.}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+}}}{15ce^2}}{7c}$$

input
```
Int[Sqrt[d + e*x]*(b*x + c*x^2)^(3/2),x]
```

output
```
(2*(d + e*x)^(3/2)*(b*x + c*x^2)^(3/2))/(9*e) - ((2*(2*c*d - b*e)*Sqrt[d +
 e*x]*(b*x + c*x^2)^(3/2))/(7*c) + ((-2*Sqrt[d + e*x]*(8*c^3*d^3 - 15*b*c^
2*d^2*e + 3*b^2*c*d*e^2 - 4*b^3*e^3 - 6*c*e*(c^2*d^2 - b*c*d*e + 2*b^2*e^2
)*x)*Sqrt[b*x + c*x^2])/(15*c*e^2) + ((2*Sqrt[-b]*(16*c^4*d^4 - 32*b*c^3*d
^3*e + 9*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*
x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c
*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (8*Sqrt[-b]*d*(c*d
 - b*e)*(2*c*d - b*e)*(2*c^2*d^2 - 2*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (
c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b
*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]))/(15*c*e^2))/(7*c)
)/(3*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 942, normalized size of antiderivative = 1.71

method	result
elliptic	$$\frac{\sqrt{x(cx+b)}\,\sqrt{(cx+b)x(ex+d)}\left(\frac{2c\,x^{3}\sqrt{ce\,x^{3}+be\,x^{2}+cd\,x^{2}+bdx}}{9}+\frac{2\left(2bce+c^{2}d-\frac{2c(4be+4cd)}{9}\right)x^{2}\sqrt{ce\,x^{3}+be\,x^{2}+cd\,x^{2}+bdx}}{7ce}+\frac{2\left(e\,b^{2}+\frac{11dbc}{9}\right.}{}\right.}{}$$
default	Expression too large to display

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)*x*(e*x+d))^(1/2)/x/(c*x+b)*(2/9
*c*x^3*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2/7*(2*b*c*e+c^2*d-2/9*c*(4*b
*e+4*c*d))/c/e*x^2*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2/5*(e*b^2+11/9*d
*b*c-2/7*(2*b*c*e+c^2*d-2/9*c*(4*b*e+4*c*d))/c/e*(3*b*e+3*c*d))/c/e*x*(c*e
*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2/3*(b^2*d-5/7*(2*b*c*e+c^2*d-2/9*c*(4*b
*e+4*c*d))/c/e*b*d-2/5*(e*b^2+11/9*d*b*c-2/7*(2*b*c*e+c^2*d-2/9*c*(4*b*e+4
*c*d))/c/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+
b*d*x)^(1/2)-2/3*(b^2*d-5/7*(2*b*c*e+c^2*d-2/9*c*(4*b*e+4*c*d))/c/e*b*d-2/
5*(e*b^2+11/9*d*b*c-2/7*(2*b*c*e+c^2*d-2/9*c*(4*b*e+4*c*d))/c/e*(3*b*e+3*c
*d))/c/e*(2*b*e+2*c*d))/c/e^2*b*d^2*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c
))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((
x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(-3/5*(e*b^2+11/9*d*b*c-2/7*(
2*b*c*e+c^2*d-2/9*c*(4*b*e+4*c*d))/c/e*(3*b*e+3*c*d))/c/e*b*d-2/3*(b^2*d-5
/7*(2*b*c*e+c^2*d-2/9*c*(4*b*e+4*c*d))/c/e*b*d-2/5*(e*b^2+11/9*d*b*c-2/7*(
2*b*c*e+c^2*d-2/9*c*(4*b*e+4*c*d))/c/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c
/e*(b*e+c*d))*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^
(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)
/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d
/e/(-d/e+b/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 546, normalized size of antiderivative = 0.99

$$\int \sqrt{d+ex}(bx+cx^2)^{3/2}\,dx = \frac{2\left((16\,c^5d^5-40\,bc^4d^4e+22\,b^2c^3d^3e^2+7\,b^3c^2d^2e^3+11\,b^4cde^4-8\,b^5e^5)\sqrt{ce}\text{weierstrassPInve}\right.}{}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
2/945*((16*c^5*d^5 - 40*b*c^4*d^4*e + 22*b^2*c^3*d^3*e^2 + 7*b^3*c^2*d^2*e
^3 + 11*b^4*c*d*e^4 - 8*b^5*e^5)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^
2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2
*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(16*
c^5*d^4*e - 32*b*c^4*d^3*e^2 + 9*b^2*c^3*d^2*e^3 + 7*b^3*c^2*d*e^4 - 8*b^4
*c*e^5)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e
^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^
3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27
*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3
*c*e*x + c*d + b*e)/(c*e))) + 3*(35*c^5*e^5*x^3 + 8*c^5*d^3*e^2 - 15*b*c^4
*d^2*e^3 + 3*b^2*c^3*d*e^4 - 4*b^3*c^2*e^5 + 5*(c^5*d*e^4 + 10*b*c^4*e^5)*
x^2 - (6*c^5*d^2*e^3 - 11*b*c^4*d*e^4 - 3*b^2*c^3*e^5)*x)*sqrt(c*x^2 + b*x
)*sqrt(e*x + d))/(c^4*e^5)
```

Sympy [F]

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^{3/2}\,dx=\int (x(b+cx))^{\frac{3}{2}}\sqrt{d+ex}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((x*(b + c*x))**(3/2)*sqrt(d + e*x), x)
```

Maxima [F]

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^{3/2}\,dx=\int \left(cx^2+bx\right)^{\frac{3}{2}}\sqrt{ex+d}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(3/2)*sqrt(e*x + d), x)
```

Giac [F]

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^{3/2}\,dx=\int \left(cx^2+bx\right)^{\frac{3}{2}}\sqrt{ex+d}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(3/2)*sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^{3/2}\,dx=\int \left(c\,x^2+b\,x\right)^{3/2}\,\sqrt{d+e\,x}\,dx$$

input
```
int((b*x + c*x^2)^(3/2)*(d + e*x)^(1/2),x)
```

output
```
int((b*x + c*x^2)^(3/2)*(d + e*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x)^(3/2),x)
```

output
```
( - 18*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**3*d*e**2 + 12*sqrt(x)*sqrt(d
 + e*x)*sqrt(b + c*x)*b**3*e**3*x - 66*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)
*b**2*c*d**2*e + 56*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c*d*e**2*x +
200*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c*e**3*x**2 + 36*sqrt(x)*sqrt
(d + e*x)*sqrt(b + c*x)*b*c**2*d**3 + 20*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*
x)*b*c**2*d**2*e*x + 220*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**2*d*e**2
*x**2 + 140*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**2*e**3*x**3 - 24*sqrt
(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**3*d**3*x + 20*sqrt(x)*sqrt(d + e*x)*sqr
t(b + c*x)*c**3*d**2*e*x**2 + 140*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**3
*d*e**2*x**3 - 24*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e +
b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*
d*e*x**2),x)*b**5*e**5 - 3*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b*
*2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*x**2 + c**2*d**2*
x + c**2*d*e*x**2),x)*b**4*c*d*e**4 + 48*int((sqrt(x)*sqrt(d + e*x)*sqrt(b
 + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*x**
2 + c**2*d**2*x + c**2*d*e*x**2),x)*b**3*c**2*d**2*e**3 - 69*int((sqrt(x)*
sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*
d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*b**2*c**3*d**3*e**
2 - 48*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x
 + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2...
```

3.193 $\int \frac{(bx+cx^2)^{3/2}}{\sqrt{d+ex}} \, dx$

Optimal result 1560
Mathematica [C] (verified) 1561
Rubi [A] (verified) 1562
Maple [A] (verified) 1566
Fricas [A] (verification not implemented) 1567
Sympy [F] 1567
Maxima [F] 1568
Giac [F] 1568
Mupad [F(-1)] 1568
Reduce [F] 1569

Optimal result

Integrand size = 23, antiderivative size = 436

$$\begin{aligned}\int \frac{(bx+cx^2)^{3/2}}{\sqrt{d+ex}} \, dx = & -\frac{4(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)x\sqrt{d+ex}}{35ce^4\sqrt{bx+cx^2}} \\ & + \frac{2\left(8c^2d^2-11bcde+b^2e^2\right)\sqrt{d+ex}\sqrt{bx+cx^2}}{35ce^3} \\ & - \frac{4(3cd-4be)x\sqrt{d+ex}\sqrt{bx+cx^2}}{35e^2} + \frac{2cx^2\sqrt{d+ex}\sqrt{bx+cx^2}}{7e} \\ & + \frac{4\sqrt{b}(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)\sqrt{x}\sqrt{d+ex}\,E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{35c^{3/2}e^4\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} \\ & - \frac{2b^{3/2}\left(8c^2d^2-11bcde+b^2e^2\right)\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{35c^{3/2}e^3\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}\end{aligned}$$

output
```
-4/35*(-b*e+2*c*d)*(-b^2*e^2-4*b*c*d*e+4*c^2*d^2)*x*(e*x+d)^(1/2)/c/e^4/(c
*x^2+b*x)^(1/2)+2/35*(b^2*e^2-11*b*c*d*e+8*c^2*d^2)*(e*x+d)^(1/2)*(c*x^2+b
*x)^(1/2)/c/e^3-4/35*(-4*b*e+3*c*d)*x*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/e^2+
2/7*c*x^2*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/e+4/35*b^(1/2)*(-b*e+2*c*d)*(-b^
2*e^2-4*b*c*d*e+4*c^2*d^2)*x^(1/2)*(e*x+d)^(1/2)*EllipticE(c^(1/2)*x^(1/2)
/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/c^(3/2)/e^4/(b*(e*x+d)/d/(c*x+
b))^(1/2)/(c*x^2+b*x)^(1/2)-2/35*b^(3/2)*(b^2*e^2-11*b*c*d*e+8*c^2*d^2)*x^
(1/2)*(e*x+d)^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e
/c/d)^(1/2))/c^(3/2)/e^3/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.61 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.87

$$\int \frac{(bx+cx^2)^{3/2}}{\sqrt{d+ex}}\,dx = \frac{2(x(b+cx))^{3/2}\Big(bex(b+cx)(d+ex)\left(b^2e^2+bce(-11d+8ex)+c^2(8d^2-6dex+5e\right.}{}$$

input
```
Integrate[(b*x + c*x^2)^(3/2)/Sqrt[d + e*x],x]
```

output
```
(2*(x*(b + c*x))^(3/2)*(b*e*x*(b + c*x)*(d + e*x)*(b^2*e^2 + b*c*e*(-11*d
+ 8*e*x) + c^2*(8*d^2 - 6*d*e*x + 5*e^2*x^2)) + Sqrt[b/c]*(-2*Sqrt[b/c]*(8
*c^3*d^3 - 12*b*c^2*d^2*e + 2*b^2*c*d*e^2 + b^3*e^3)*(b + c*x)*(d + e*x) -
 (2*I)*b*e*(8*c^3*d^3 - 12*b*c^2*d^2*e + 2*b^2*c*d*e^2 + b^3*e^3)*Sqrt[1 +
 b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]]
, (c*d)/(b*e)] + I*b*e*(8*c^3*d^3 - 13*b*c^2*d^2*e + 3*b^2*c*d*e^2 + 2*b^3
*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt
[b/c]/Sqrt[x]], (c*d)/(b*e)]))/(35*b*c*e^4*x^2*(b + c*x)^2*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {1162, 1231, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx+cx^2\right)^{3/2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 1162}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7e}-\frac{3\int \frac{(bd+(2cd-be)x)\sqrt{cx^2+bx}}{\sqrt{d+ex}}dx}{7e}$$

$$\Big\downarrow \text{ 1231}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7e}-\frac{3\left(-\frac{2\int -\frac{bd\left(8c^2d^2-11bced+b^2e^2\right)+2(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)x}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{15ce^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(b^2e^2-3cex(2cd-be)-11bcde+8c^2d^2\right)}{15ce^2}\right)}{7e}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7e}-\frac{3\left(\frac{\int \frac{bd\left(8c^2d^2-11bced+b^2e^2\right)+2(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{15ce^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(b^2e^2-3cex(2cd-be)-11bcde+8c^2d^2\right)}{15ce^2}\right)}{7e}$$

$$\Big\downarrow \text{ 1269}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7e}-\frac{3\left(\frac{\frac{2(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{d(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}}{15ce^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(b^2e^2-3cex(2cd-\right.}{15ce^2}\right.}{7e}$$

$$\Big\downarrow \text{ 1169}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7e}-$$

$$\frac{3\left(\frac{\frac{2\sqrt{x}\sqrt{b+cx}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{15ce^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\ldots}{}\right.}{7e}$$

$\downarrow$ 122

$$\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7e}-$$

$$\frac{3\left(\frac{\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{15ce^2}-\frac{2\sqrt{bx+cx^2}\ldots}{}\right.}{7e}$$

$\downarrow$ 120

$$\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7e}-$$

$$\frac{3\left(\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{15ce^2}-\ldots\right.}{7e}$$

$\downarrow$ 127

$$\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7e}-$$

$$\frac{3\left(\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int\frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\ldots}}}{e\sqrt{bx+cx^2}\sqrt{d+ex}}}{15ce^2}\ldots\right.}{7e}$$

$\downarrow$ 126

$$\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}}{7e}-$$

$$\frac{3\left(\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\text{EllipticF}\left(\ldots\right.}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}}{15ce^2}\ldots\right.}{7e}$$

input
```
Int[(b*x + c*x^2)^(3/2)/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(b*x + c*x^2)^(3/2))/(7*e) - (3*((-2*Sqrt[d + e*x]*(8*c^2
*d^2 - 11*b*c*d*e + b^2*e^2 - 3*c*e*(2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(1
5*c*e^2) + ((4*Sqrt[-b]*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*Sq
rt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/S
qrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) -
(2*Sqrt[-b]*d*(c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt
[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b
]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]))/(15*c*e^2))
)/(7*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 650, normalized size of antiderivative = 1.49

method	result
elliptic	$\frac{\sqrt{x(cx+b)}\,\sqrt{(cx+b)x(ex+d)}\left(\frac{2c\,x^{2}\sqrt{ce\,x^{3}+be\,x^{2}+cd\,x^{2}+bdx}}{7e}+\frac{2\left(2bc-\frac{2c(3be+3cd)}{7e}\right)x\sqrt{ce\,x^{3}+be\,x^{2}+cd\,x^{2}+bdx}}{5ce}+\frac{2\left(b^{2}-\frac{5cbd}{7e}-\frac{2\left(2bc-\right.}{}\right.}{}\right.}{}$
default	$\frac{2\sqrt{x(cx+b)}\,\sqrt{ex+d}\left(5c^{4}e^{5}x^{5}+2\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\text{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^{4}d\,e^{4}+3\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\text{Elli}\right.}{}$

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)*x*(e*x+d))^(1/2)/x/(c*x+b)*(2/7
*c/e*x^2*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2/5*(2*b*c-2/7*c/e*(3*b*e+3
*c*d))/c/e*x*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2/3*(b^2-5/7*c/e*b*d-2/
5*(2*b*c-2/7*c/e*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(c*e*x^3+b*e*x^2+c*
d*x^2+b*d*x)^(1/2)-2/3*(b^2-5/7*c/e*b*d-2/5*(2*b*c-2/7*c/e*(3*b*e+3*c*d))/
c/e*(2*b*e+2*c*d))/c/e^2*b*d^2*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1
/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e
)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(-3/5*(2*b*c-2/7*c/e*(3*b*e+3*c*d)
)/c/e*b*d-2/3*(b^2-5/7*c/e*b*d-2/5*(2*b*c-2/7*c/e*(3*b*e+3*c*d))/c/e*(2*b*
e+2*c*d))/c/e*(b*e+c*d))*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2
)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*Ellipti
cE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e
)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.07

$$\int \frac{(bx+cx^2)^{3/2}}{\sqrt{d+ex}}\,dx = \frac{2\left((16\,c^4d^4 - 32\,bc^3d^3e + 13\,b^2c^2d^2e^2 + 3\,b^3cde^3 + 2\,b^4e^4)\sqrt{ce}\,\text{weierstrassPInverse}\left(\frac{4}{}\right.\right.}{}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/105*((16*c^4*d^4 - 32*b*c^3*d^3*e + 13*b^2*c^2*d^2*e^2 + 3*b^3*c*d*e^3 +
 2*b^4*e^4)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2
)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)
/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 6*(8*c^4*d^3*e - 12*b*c^3*d
^2*e^2 + 2*b^2*c^2*d*e^3 + b^3*c*e^4)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d
^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^
2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d
*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2
+ 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(5*c^4*e^4*x
^2 + 8*c^4*d^2*e^2 - 11*b*c^3*d*e^3 + b^2*c^2*e^4 - 2*(3*c^4*d*e^3 - 4*b*c
^3*e^4)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/(c^3*e^5)
```

Sympy [F]

$$\int \frac{(bx+cx^2)^{3/2}}{\sqrt{d+ex}}\,dx = \int \frac{(x(b+cx))^{\frac{3}{2}}}{\sqrt{d+ex}}\,dx$$

input
```
integrate((c*x**2+b*x)**(3/2)/(e*x+d)**(1/2),x)
```

output
```
Integral((x*(b + c*x))**(3/2)/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{(bx+cx^2)^{3/2}}{\sqrt{d+ex}}\,dx = \int \frac{(cx^2+bx)^{\frac{3}{2}}}{\sqrt{ex+d}}\,dx$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(3/2)/sqrt(e*x + d), x)
```

Giac [F]

$$\int \frac{(bx+cx^2)^{3/2}}{\sqrt{d+ex}}\,dx = \int \frac{(cx^2+bx)^{\frac{3}{2}}}{\sqrt{ex+d}}\,dx$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(3/2)/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{3/2}}{\sqrt{d+ex}}\,dx = \int \frac{(c\,x^2+b\,x)^{3/2}}{\sqrt{d+e\,x}}\,dx$$

input
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^(1/2),x)
```

output
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^{3/2}}{\sqrt{d + ex}} \, dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^(1/2),x)
```

output
```
( - 24*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*d*e + 16*sqrt(x)*sqrt(d +
e*x)*sqrt(b + c*x)*b**2*e**2*x + 18*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*
c*d**2 + 4*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c*d*e*x + 10*sqrt(x)*sqrt
(d + e*x)*sqrt(b + c*x)*b*c*e**2*x**2 - 12*sqrt(x)*sqrt(d + e*x)*sqrt(b +
c*x)*c**2*d**2*x + 10*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**2*d*e*x**2 +
3*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*
c*d**2 + 2*b*c*d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*b**
4*e**4 + 9*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e*
*2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**
2),x)*b**3*c*d*e**3 - 30*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2
*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*x**2 + c**2*d**2*x
+ c**2*d*e*x**2),x)*b**2*c**2*d**2*e**2 - 12*int((sqrt(x)*sqrt(d + e*x)*sq
rt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2
*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*b*c**3*d**3*e + 24*int((sqrt(x)*sq
rt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*
e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*c**4*d**4 + 12*int((
sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b**2*d*e*x + b**2*e**2*x**2 + b*c*d*
*2*x + 2*b*c*d*e*x**2 + b*c*e**2*x**3 + c**2*d**2*x**2 + c**2*d*e*x**3),x)
*b**4*d**2*e**2 + 3*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b**2*d*e*x
+ b**2*e**2*x**2 + b*c*d**2*x + 2*b*c*d*e*x**2 + b*c*e**2*x**3 + c**2*d...
```

3.194 $\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{3/2}}\,dx$

Optimal result 1570
Mathematica [C] (verified) 1571
Rubi [A] (verified) 1571
Maple [B] (verified) 1575
Fricas [A] (verification not implemented) 1576
Sympy [F] 1577
Maxima [F] 1577
Giac [F] 1578
Mupad [F(-1)] 1578
Reduce [F] 1578

Optimal result

Integrand size = 23, antiderivative size = 370

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2(16c^2d^2-16bcde+b^2e^2)\,x\sqrt{d+ex}}{5e^4\sqrt{bx+cx^2}} - \frac{2(8cd-7be)\sqrt{d+ex}\sqrt{bx+cx^2}}{5e^3} + \frac{12cx\sqrt{d+ex}\sqrt{bx+cx^2}}{5e^2} - \frac{2(bx+cx^2)^{3/2}}{e\sqrt{d+ex}} - \frac{2\sqrt{b}(16c^2d^2-16bcde+b^2e^2)\sqrt{x}\sqrt{d+ex}\,E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{5\sqrt{c}e^4\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} + \frac{2b^{3/2}(8cd-7be)\sqrt{x}\sqrt{d+ex}\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{5\sqrt{c}e^3\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$

output

```
2/5*(b^2*e^2-16*b*c*d*e+16*c^2*d^2)*x*(e*x+d)^(1/2)/e^4/(c*x^2+b*x)^(1/2)-
2/5*(-7*b*e+8*c*d)*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/e^3+12/5*c*x*(e*x+d)^(1
/2)*(c*x^2+b*x)^(1/2)/e^2-2*(c*x^2+b*x)^(3/2)/e/(e*x+d)^(1/2)-2/5*b^(1/2)*
(b^2*e^2-16*b*c*d*e+16*c^2*d^2)*x^(1/2)*(e*x+d)^(1/2)*EllipticE(c^(1/2)*x^
(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/c^(1/2)/e^4/(b*(e*x+d)/d/
(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)+2/5*b^(3/2)*(-7*b*e+8*c*d)*x^(1/2)*(e*x+d
)^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e/c/d)^(1/2))
/c^(1/2)/e^3/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 19.27 (sec) , antiderivative size = 306, normalized size of antiderivative = 0.83

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{3/2}}\,dx = \frac{2\Big((b+cx)\left(b^2e^2(d+ex)+bce(-16d^2-9dex+2e^2x^2)+c^2(16d^3+8d^2ex-2de^2x^2\right.}{}$$

input
```
Integrate[(b*x + c*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

output
```
(2*((b + c*x)*(b^2*e^2*(d + e*x) + b*c*e*(-16*d^2 - 9*d*e*x + 2*e^2*x^2) +
 c^2*(16*d^3 + 8*d^2*e*x - 2*d*e^2*x^2 + e^3*x^3)) + I*Sqrt[b/c]*c*e*(16*c
^2*d^2 - 16*b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)
*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*Sqrt[b/c]*c*e*(8
*c^2*d^2 - 9*b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2
)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)]))/(5*c*e^4*Sqrt[x*(
b + c*x)]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1161, 1231, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1161}$$

$$\frac{3\int \frac{(b+2cx)\sqrt{cx^2+bx}}{\sqrt{d+ex}}dx}{e} - \frac{2(bx+cx^2)^{3/2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \text{1231}$$

$$\frac{3\left(-\frac{2\int -\frac{c\left(bd(8cd-7be)+\left(16c^2d^2-16bced+b^2e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{15ce^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(-7be+8cd-6cex)}{15e^2}\right)}{e}-\frac{2\left(bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{\int \frac{bd(8cd-7be)+\left(16c^2d^2-16bced+b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{15e^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(-7be+8cd-6cex)}{15e^2}\right)}{e}-\frac{2\left(bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}}$$

$\Big\downarrow$ 1269

$$\frac{3\left(\frac{\frac{\left(b^2e^2-16bcde+16c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{8d(cd-be)(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}}{15e^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(-7be+8cd-6cex)}{15e^2}\right)}{e}-\frac{2\left(bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}}$$

$\Big\downarrow$ 1169

$$\frac{3\left(\frac{\frac{\sqrt{x}\sqrt{b+cx}\left(b^2e^2-16bcde+16c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{8d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{15e^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(-7be+8cd-6cex)}{15e^2}\right)}{e}-\frac{2\left(bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}}$$

$\Big\downarrow$ 122

$$\frac{3\left(\frac{\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{15e^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(-7be+8cd-6\ldots}{15e^2}\right.}{e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}}$$

$\Big\downarrow$ 120

$$\frac{3\left(\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{15e^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(-}{15e^2}\right.}{e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}}$$

$\downarrow$ 127

$$\frac{3\left(\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{d+ex}}}{15e^2}-\frac{2\sqrt{bx+cx^2}}{}\right.}{e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}}$$

$\downarrow$ 126

$$\frac{3\left(\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{16\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}}{15e^2}-\right.}{e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{e\sqrt{d+ex}}$$

input
```
Int[(b*x + c*x^2)^(3/2)/(d + e*x)^(3/2),x]
```

output
```
(-2*(b*x + c*x^2)^(3/2))/(e*Sqrt[d + e*x]) + (3*((-2*Sqrt[d + e*x]*(8*c*d
 - 7*b*e - 6*c*e*x)*Sqrt[b*x + c*x^2])/(15*e^2) + ((2*Sqrt[-b]*(16*c^2*d^2
 - 16*b*c*d*e + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[
ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x
)/d]*Sqrt[b*x + c*x^2]) - (16*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]
*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sq
rt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]))/(15*e^
2)))/e
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. $2(321) = 642$.

Time = 1.50 (sec) , antiderivative size = 663, normalized size of antiderivative = 1.79

method	result
elliptic	$\sqrt{x(cx+b)}\,\sqrt{(cx+b)x(ex+d)}\left(\frac{2\left(ce\,x^2+bex\right)d(be-cd)}{e^4\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2+bex\right)}}+\frac{2cx\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{5e^2}+\frac{2\left(\frac{c(2be-cd)}{e^2}-\frac{2c(2be+2cd)}{5e^2}\right)\sqrt{ce\,x^3+be\,x^2+cd\,x}}{3ce}\right.$
default	$-\frac{2\sqrt{x(cx+b)}\,\sqrt{ex+d}\left(\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^3d\,e^3-9\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\mathrm{EllipticF}\left(\right.}{}$

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)*x*(e*x+d))^(1/2)/x/(c*x+b)*(2*(
c*e*x^2+b*e*x)*d*(b*e-c*d)/e^4/((x+d/e)*(c*e*x^2+b*e*x))^(1/2)+2/5*c/e^2*x
*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2/3*(c/e^2*(2*b*e-c*d)-2/5*c/e^2*(2
*b*e+2*c*d))/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2*(-d*(b^2*e^2-2*b*
c*d*e+c^2*d^2)/e^4+d*(b*e-c*d)^2/e^4-b/e^3*d*(b*e-c*d)-1/3*(c/e^2*(2*b*e-c
*d)-2/5*c/e^2*(2*b*e+2*c*d))/c/e*b*d)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d
/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*Ellipt
icF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(1/e^3*(b^2*e^2-2*b*c*d
*e+c^2*d^2)-d*(b*e-c*d)/e^3*c-3/5*c/e^2*b*d-2/3*(c/e^2*(2*b*e-c*d)-2/5*c/e
^2*(2*b*e+2*c*d))/c/e*(b*e+c*d))*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/
c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)
*EllipticE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+
d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.39

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{3/2}}\,dx =$$
$$-\frac{2\left(\left(16\,c^3d^4-24\,bc^2d^3e+6\,b^2cd^2e^2+b^3de^3+\left(16\,c^3d^3e-24\,bc^2d^2e^2+6\,b^2cde^3+b^3e^4\right)x\right)\sqrt{ce}\,\mathrm{weierstrassI}\right.}{}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*((16*c^3*d^4 - 24*b*c^2*d^3*e + 6*b^2*c*d^2*e^2 + b^3*d*e^3 + (16*c^
3*d^3*e - 24*b*c^2*d^2*e^2 + 6*b^2*c*d*e^3 + b^3*e^4)*x)*sqrt(c*e)*weierst
rassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3
 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*
d + b*e)/(c*e)) + 3*(16*c^3*d^3*e - 16*b*c^2*d^2*e^2 + b^2*c*d*e^3 + (16*c
^3*d^2*e^2 - 16*b*c^2*d*e^3 + b^2*c*e^4)*x)*sqrt(c*e)*weierstrassZeta(4/3*
(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e
- 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 -
 b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*
d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) - 3*(c^3*e
^4*x^2 - 8*c^3*d^2*e^2 + 7*b*c^2*d*e^3 - 2*(c^3*d*e^3 - b*c^2*e^4)*x)*sqrt
(c*x^2 + b*x)*sqrt(e*x + d))/(c^2*e^6*x + c^2*d*e^5)
```

Sympy [F]

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{3/2}}\,dx = \int \frac{(x(b + cx))^{\frac{3}{2}}}{(d + ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x)**(3/2)/(e*x+d)**(3/2),x)
```

output
```
Integral((x*(b + c*x))**(3/2)/(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{3/2}}\,dx = \int \frac{(cx^2 + bx)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(3/2), x)
```

Giac [F]

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{3/2}}\, dx = \int \frac{(cx^2 + bx)^{\frac{3}{2}}}{(ex + d)^{\frac{3}{2}}}\, dx$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{3/2}}\, dx = \int \frac{(c\,x^2 + b\,x)^{3/2}}{(d + e\,x)^{3/2}}\, dx$$

input
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^(3/2),x)
```

output
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{3/2}}\, dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^(3/2),x)
```

output

```
( - 6*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*e + 6*sqrt(x)*sqrt(d + e*x)
*sqrt(b + c*x)*b*c*d + 4*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c*e*x - 4*s
qrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**2*d*x + 2*sqrt(x)*sqrt(d + e*x)*sqrt
(b + c*x)*c**2*e*x**2 + 3*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b*d**
2 + 2*sqrt(x)*b*d*e*x + sqrt(x)*b*e**2*x**2 + sqrt(x)*c*d**2*x + 2*sqrt(x)
*c*d*e*x**2 + sqrt(x)*c*e**2*x**3),x)*b**3*d**2*e + 3*int((sqrt(d + e*x)*s
qrt(b + c*x))/(sqrt(x)*b*d**2 + 2*sqrt(x)*b*d*e*x + sqrt(x)*b*e**2*x**2 +
sqrt(x)*c*d**2*x + 2*sqrt(x)*c*d*e*x**2 + sqrt(x)*c*e**2*x**3),x)*b**3*d*e
**2*x - 3*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b*d**2 + 2*sqrt(x)*b*
d*e*x + sqrt(x)*b*e**2*x**2 + sqrt(x)*c*d**2*x + 2*sqrt(x)*c*d*e*x**2 + sq
rt(x)*c*e**2*x**3),x)*b**2*c*d**3 - 3*int((sqrt(d + e*x)*sqrt(b + c*x))/(s
qrt(x)*b*d**2 + 2*sqrt(x)*b*d*e*x + sqrt(x)*b*e**2*x**2 + sqrt(x)*c*d**2*x
 + 2*sqrt(x)*c*d*e*x**2 + sqrt(x)*c*e**2*x**3),x)*b**2*c*d**2*e*x + 4*int(
(sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b*d**2 + 2*b*d*e*x + b*e**2*x**2
+ c*d**2*x + 2*c*d*e*x**2 + c*e**2*x**3),x)*b**2*c*d*e**2 + 4*int((sqrt(x)
*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b*d**2 + 2*b*d*e*x + b*e**2*x**2 + c*d**2
*x + 2*c*d*e*x**2 + c*e**2*x**3),x)*b**2*c*e**3*x - 12*int((sqrt(x)*sqrt(d
 + e*x)*sqrt(b + c*x)*x)/(b*d**2 + 2*b*d*e*x + b*e**2*x**2 + c*d**2*x + 2*
c*d*e*x**2 + c*e**2*x**3),x)*b*c**2*d**2*e - 12*int((sqrt(x)*sqrt(d + e*x)
*sqrt(b + c*x)*x)/(b*d**2 + 2*b*d*e*x + b*e**2*x**2 + c*d**2*x + 2*c*d*...
```

3.195 $\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{5/2}}\,dx$

Optimal result 1580
Mathematica [C] (verified) 1581
Rubi [A] (verified) 1581
Maple [B] (verified) 1585
Fricas [B] (verification not implemented) 1586
Sympy [F] 1587
Maxima [F] 1587
Giac [F] 1587
Mupad [F(-1)] 1588
Reduce [F] 1588

Optimal result

Integrand size = 23, antiderivative size = 301

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{5/2}}\,dx = -\frac{2(8cd-5be)\sqrt{bx+cx^2}}{3e^3\sqrt{d+ex}} + \frac{4cx\sqrt{bx+cx^2}}{3e^2\sqrt{d+ex}} - \frac{2(bx+cx^2)^{3/2}}{3e(d+ex)^{3/2}} + \frac{16\sqrt{d}(2cd-be)\sqrt{bx+cx^2}\,E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{3e^{7/2}\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}} - \frac{2\sqrt{d}(8cd-3be)\sqrt{bx+cx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{3e^{7/2}\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}$$

output

```
-2/3*(-5*b*e+8*c*d)*(c*x^2+b*x)^(1/2)/e^3/(e*x+d)^(1/2)+4/3*c*x*(c*x^2+b*x
)^(1/2)/e^2/(e*x+d)^(1/2)-2/3*(c*x^2+b*x)^(3/2)/e/(e*x+d)^(3/2)+16/3*d^(1/
2)*(-b*e+2*c*d)*(c*x^2+b*x)^(1/2)*EllipticE(e^(1/2)*x^(1/2)/d^(1/2)/(1+e*x
/d)^(1/2),(1-c*d/b/e)^(1/2))/e^(7/2)/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(
e*x+d)^(1/2)-2/3*d^(1/2)*(-3*b*e+8*c*d)*(c*x^2+b*x)^(1/2)*InverseJacobiAM(
arctan(e^(1/2)*x^(1/2)/d^(1/2)),(1-c*d/b/e)^(1/2))/e^(7/2)/x^(1/2)/(d*(c*x
+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.30 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.93

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{5/2}}\,dx = \frac{2(x(b+cx))^{3/2}\left(8(-2cd+be)(b+cx)(d+ex)+\frac{ex(b+cx)(-be(3d+4ex)+c(8d^2+10dex+e^2x^2)}{d+ex}\right.}{}$$

input
```
Integrate[(b*x + c*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

output
```
(2*(x*(b + c*x))^(3/2)*(8*(-2*c*d + b*e)*(b + c*x)*(d + e*x) + (e*x*(b + c
*x)*(-(b*e*(3*d + 4*e*x)) + c*(8*d^2 + 10*d*e*x + e^2*x^2)))/(d + e*x) + (
8*I)*Sqrt[b/c]*c*e*(-2*c*d + b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3
/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*Sqrt[b/c]*c*e
*(-8*c*d + 5*b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*
ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)]))/(3*e^4*x^2*(b + c*x)^2*Sqrt[d +
 e*x])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1161, 1230, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \ 1161$$

$$\frac{\int \frac{(b+2cx)\sqrt{cx^2+bx}}{(d+ex)^{3/2}}dx}{e} - \frac{2(bx+cx^2)^{3/2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \ 1230$$

$$\frac{\frac{2\sqrt{bx+cx^2}(-3be+8cd+2cex)}{3e^2\sqrt{d+ex}}-\frac{2\int\frac{b(8cd-3be)+8c(2cd-be)x}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3e^2}}{e}-\frac{2\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{2\sqrt{bx+cx^2}(-3be+8cd+2cex)}{3e^2\sqrt{d+ex}}-\frac{\int\frac{b(8cd-3be)+8c(2cd-be)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3e^2}}{e}-\frac{2\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{2\sqrt{bx+cx^2}(-3be+8cd+2cex)}{3e^2\sqrt{d+ex}}-\frac{\frac{8c(2cd-be)\int\frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{(4cd-3be)(4cd-be)\int\frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}}{3e^2}}{e}-\frac{2\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \; 1169$$

$$\frac{\frac{2\sqrt{bx+cx^2}(-3be+8cd+2cex)}{3e^2\sqrt{d+ex}}-\frac{\frac{8c\sqrt{x}\sqrt{b+cx}(2cd-be)\int\frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{\sqrt{x}\sqrt{b+cx}(4cd-3be)(4cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{3e^2}}{e}-\frac{2\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \; 122$$

$$\frac{\frac{2\sqrt{bx+cx^2}(-3be+8cd+2cex)}{3e^2\sqrt{d+ex}}-\frac{\frac{8c\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{\sqrt{x}\sqrt{b+cx}(4cd-3be)(4cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{3e^2}}{e}-\frac{2\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \; 120$$

$$\frac{\frac{2\sqrt{bx+cx^2}(-3be+8cd+2cex)}{3e^2\sqrt{d+ex}}-\frac{\frac{16\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{\sqrt{x}\sqrt{b+cx}(4cd-3be)(4cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{3e^2}}{e}-\frac{2\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \; 127$$

$$\frac{\frac{2\sqrt{bx+cx^2}(-3be+8cd+2cex)}{3e^2\sqrt{d+ex}}-\frac{\frac{16\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(4cd-3be)(4cd-be)\int\frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+\dots}}}{e\sqrt{bx+cx^2}\sqrt{d+ex}}}{3e^2}}{e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^{3/2}}$$

$\Big\downarrow$ 126

$$\frac{\frac{2\sqrt{bx+cx^2}(-3be+8cd+2cex)}{3e^2\sqrt{d+ex}}-\frac{\frac{16\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(4cd-3be)(4cd-be)\,\mathrm{EllipticF}\left(\mathrm{arc}\dots\right.}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}}{3e^2}}{e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{3e(d+ex)^{3/2}}$$

input
```
Int[(b*x + c*x^2)^(3/2)/(d + e*x)^(5/2),x]
```

output
```
(-2*(b*x + c*x^2)^(3/2))/(3*e*(d + e*x)^(3/2)) + ((2*(8*c*d - 3*b*e + 2*c*
e*x)*Sqrt[b*x + c*x^2])/(3*e^2*Sqrt[d + e*x]) - ((16*Sqrt[-b]*Sqrt[c]*(2*c
*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c
]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]
) - (2*Sqrt[-b]*(4*c*d - 3*b*e)*(4*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sq
rt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)]
)/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]))/(3*e^2))/e
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(256) = 512$.

Time = 2.46 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.93

method	result
elliptic	$\frac{\sqrt{x(cx+b)}\,\sqrt{(cx+b)x(ex+d)}\left(\frac{2d(be-cd)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3e^5\left(x+\frac{d}{e}\right)^2}-\frac{8\left(ce\,x^2+bex\right)(be-2cd)}{3e^4\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2+bex\right)}}+\frac{2c\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3e^3}+\frac{2\left(\frac{b^2e^2-4\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{2\sqrt{x(cx+b)}\left(5\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\text{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^2d\,e^3x-8\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\text{EllipticF}\left(\sqrt{\frac{ex+\cdots}{d}}\right.\right.}{\cdots}$

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)*x*(e*x+d))^(1/2)/x/(c*x+b)*(2/3
*d*(b*e-c*d)/e^5*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(x+d/e)^2-8/3*(c*e*
x^2+b*e*x)*(b*e-2*c*d)/e^4/((x+d/e)*(c*e*x^2+b*e*x))^(1/2)+2/3*c/e^3*(c*e*
x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2*((b^2*e^2-4*b*c*d*e+3*c^2*d^2)/e^4+1/3*
d*(b*e-c*d)/e^4*c-4/3*(b*e-2*c*d)/e^4*(b*e-c*d)+4/3*b/e^3*(b*e-2*c*d)-1/3*
c/e^3*b*d)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/
2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d
/e/(-d/e+b/c))^(1/2))+2*(2*c/e^3*(b*e-c*d)+4/3*(b*e-2*c*d)/e^3*c-2/3*c/e^3
*(b*e+c*d))*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1
/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d
*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e
/(-d/e+b/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. $2(256) = 512$.

Time = 0.17 (sec) , antiderivative size = 531, normalized size of antiderivative = 1.76

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{5/2}}\,dx = \frac{2\left((16\,c^2d^4 - 16\,bcd^3e + b^2d^2e^2 + (16\,c^2d^2e^2 - 16\,bcde^3 + b^2e^4)x^2 + 2\,(16\,c^2d^3e - 16\right.}{}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
2/9*((16*c^2*d^4 - 16*b*c*d^3*e + b^2*d^2*e^2 + (16*c^2*d^2*e^2 - 16*b*c*d
*e^3 + b^2*e^4)*x^2 + 2*(16*c^2*d^3*e - 16*b*c*d^2*e^2 + b^2*d*e^3)*x)*sqr
t(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4
/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3
*(3*c*e*x + c*d + b*e)/(c*e)) + 24*(2*c^2*d^3*e - b*c*d^2*e^2 + (2*c^2*d*e
^3 - b*c*e^4)*x^2 + 2*(2*c^2*d^2*e^2 - b*c*d*e^3)*x)*sqrt(c*e)*weierstrass
Zeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c
^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(
c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e -
 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) +
 3*(c^2*e^4*x^2 + 8*c^2*d^2*e^2 - 3*b*c*d*e^3 + 2*(5*c^2*d*e^3 - 2*b*c*e^4
)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/(c*e^7*x^2 + 2*c*d*e^6*x + c*d^2*e^5
)
```

Sympy [F]

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{5/2}}\, dx = \int \frac{(x(b + cx))^{\frac{3}{2}}}{(d + ex)^{\frac{5}{2}}}\, dx$$

input
```
integrate((c*x**2+b*x)**(3/2)/(e*x+d)**(5/2),x)
```

output
```
Integral((x*(b + c*x))**(3/2)/(d + e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{5/2}}\, dx = \int \frac{(cx^2 + bx)^{\frac{3}{2}}}{(ex + d)^{\frac{5}{2}}}\, dx$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(5/2), x)
```

Giac [F]

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{5/2}}\, dx = \int \frac{(cx^2 + bx)^{\frac{3}{2}}}{(ex + d)^{\frac{5}{2}}}\, dx$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{5/2}}\,dx=\int \frac{{\left(c\,x^2+b\,x\right)}^{3/2}}{{\left(d+e\,x\right)}^{5/2}}\,dx$$

input
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^(5/2),x)
```

output
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{5/2}}\,dx=\text{too large to display}$$

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^(5/2),x)
```

output
```
(12*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*d*e + 8*sqrt(x)*sqrt(d + e*x)
*sqrt(b + c*x)*b**2*e**2*x - 18*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c*d*
*2 - 20*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c*d*e*x + 2*sqrt(x)*sqrt(d +
 e*x)*sqrt(b + c*x)*b*c*e**2*x**2 + 12*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)
*c**2*d**2*x - 2*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**2*d*e*x**2 - 6*int
((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**2*d**3*e + 3*sqrt(x)*b**2*d**2*
e**2*x + 3*sqrt(x)*b**2*d*e**3*x**2 + sqrt(x)*b**2*e**4*x**3 - sqrt(x)*b*c
*d**4 - 2*sqrt(x)*b*c*d**3*e*x + 2*sqrt(x)*b*c*d*e**3*x**3 + sqrt(x)*b*c*e
**4*x**4 - sqrt(x)*c**2*d**4*x - 3*sqrt(x)*c**2*d**3*e*x**2 - 3*sqrt(x)*c*
*2*d**2*e**2*x**3 - sqrt(x)*c**2*d*e**3*x**4),x)*b**4*d**4*e**2 - 12*int((
sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**2*d**3*e + 3*sqrt(x)*b**2*d**2*e*
*2*x + 3*sqrt(x)*b**2*d*e**3*x**2 + sqrt(x)*b**2*e**4*x**3 - sqrt(x)*b*c*d
**4 - 2*sqrt(x)*b*c*d**3*e*x + 2*sqrt(x)*b*c*d*e**3*x**3 + sqrt(x)*b*c*e**
4*x**4 - sqrt(x)*c**2*d**4*x - 3*sqrt(x)*c**2*d**3*e*x**2 - 3*sqrt(x)*c**2
*d**2*e**2*x**3 - sqrt(x)*c**2*d*e**3*x**4),x)*b**4*d**3*e**3*x - 6*int((s
qrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**2*d**3*e + 3*sqrt(x)*b**2*d**2*e**
2*x + 3*sqrt(x)*b**2*d*e**3*x**2 + sqrt(x)*b**2*e**4*x**3 - sqrt(x)*b*c*d*
*4 - 2*sqrt(x)*b*c*d**3*e*x + 2*sqrt(x)*b*c*d*e**3*x**3 + sqrt(x)*b*c*e**4
*x**4 - sqrt(x)*c**2*d**4*x - 3*sqrt(x)*c**2*d**3*e*x**2 - 3*sqrt(x)*c**2*
d**2*e**2*x**3 - sqrt(x)*c**2*d*e**3*x**4),x)*b**4*d**2*e**4*x**2 + 15*...
```

3.196 $\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{7/2}}\,dx$

Optimal result . . . 1590
Mathematica [C] (verified) . . . 1591
Rubi [A] (verified) . . . 1591
Maple [B] (verified) . . . 1596
Fricas [B] (verification not implemented) . . . 1597
Sympy [F] . . . 1597
Maxima [F] . . . 1598
Giac [F] . . . 1598
Mupad [F(-1)] . . . 1598
Reduce [F] . . . 1599

Optimal result

Integrand size = 23, antiderivative size = 335

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{7/2}}\,dx = \frac{2(8cd-be)\sqrt{bx+cx^2}}{5e^3(d+ex)^{3/2}}+\frac{12cx\sqrt{bx+cx^2}}{5e^2(d+ex)^{3/2}}-\frac{2(bx+cx^2)^{3/2}}{5e(d+ex)^{5/2}}-\frac{2(16c^2d^2-16bcde+b^2e^2)\sqrt{bx+cx^2}E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{5\sqrt{d}e^{7/2}(cd-be)\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}+\frac{2c\sqrt{d}(8cd-7be)\sqrt{bx+cx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{5e^{7/2}(cd-be)\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}$$

output
```
2/5*(-b*e+8*c*d)*(c*x^2+b*x)^(1/2)/e^3/(e*x+d)^(3/2)+12/5*c*x*(c*x^2+b*x)^
(1/2)/e^2/(e*x+d)^(3/2)-2/5*(c*x^2+b*x)^(3/2)/e/(e*x+d)^(5/2)-2/5*(b^2*e^2
-16*b*c*d*e+16*c^2*d^2)*(c*x^2+b*x)^(1/2)*EllipticE(e^(1/2)*x^(1/2)/d^(1/2
)/(1+e*x/d)^(1/2),(1-c*d/b/e)^(1/2))/d^(1/2)/e^(7/2)/(-b*e+c*d)/x^(1/2)/(d
*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)+2/5*c*d^(1/2)*(-7*b*e+8*c*d)*(c*x^
2+b*x)^(1/2)*InverseJacobiAM(arctan(e^(1/2)*x^(1/2)/d^(1/2)),(1-c*d/b/e)^(
1/2))/e^(7/2)/(-b*e+c*d)/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.53 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.10

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{7/2}}\, dx =$$

$$-\frac{2(x(b+cx))^{3/2}\left(bex(b+cx)\left(b^2e^4x^2 - bcde(7d^2 + 16dex + 11e^2x^2) + c^2d^2(8d^2 + 18dex + 11e^2x^2)\right) - \sqrt{\ }\right.}{}$$

input
```
Integrate[(b*x + c*x^2)^(3/2)/(d + e*x)^(7/2),x]
```

output
```
(-2*(x*(b + c*x))^(3/2)*(b*e*x*(b + c*x)*(b^2*e^4*x^2 - b*c*d*e*(7*d^2 + 1
6*d*e*x + 11*e^2*x^2) + c^2*d^2*(8*d^2 + 18*d*e*x + 11*e^2*x^2)) - Sqrt[b/
c]*c*(d + e*x)^2*(Sqrt[b/c]*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*(b + c*x)*
(d + e*x) + I*b*e*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sq
rt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e
)] - I*b*e*(8*c^2*d^2 - 9*b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/
(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(5*
b*d*e^4*(c*d - b*e)*x^2*(b + c*x)^2*(d + e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {1161, 1229, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{7/2}}\, dx$$

$$\Big\downarrow \; 1161$$

$$\frac{3\int \frac{(b+2cx)\sqrt{cx^2+bx}}{(d+ex)^{5/2}}dx}{5e}-\frac{2\left(bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}}$$

$\Big\downarrow$ 1229

$$\frac{3\left(-\frac{2\int -\frac{c\left(bd(8cd-7be)+\left(16c^2d^2-16bced+b^2e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3de^2(cd-be)}-\frac{2\sqrt{bx+cx^2}\left(ex\left(b^2e^2-10bcde+10c^2d^2\right)+cd^2(8cd-7be)\right)}{3de^2(d+ex)^{3/2}(cd-be)}\right)}{5e}-\frac{2\left(bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}}$$

$\Big\downarrow$ 27

$$\frac{3\left(\frac{c\int \frac{bd(8cd-7be)+\left(16c^2d^2-16bced+b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3de^2(cd-be)}-\frac{2\sqrt{bx+cx^2}\left(ex\left(b^2e^2-10bcde+10c^2d^2\right)+cd^2(8cd-7be)\right)}{3de^2(d+ex)^{3/2}(cd-be)}\right)}{5e}-\frac{2\left(bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}}$$

$\Big\downarrow$ 1269

$$\frac{3\left(\frac{c\left(\frac{\left(b^2e^2-16bcde+16c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{8d(cd-be)(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}\right)}{3de^2(cd-be)}-\frac{2\sqrt{bx+cx^2}\left(ex\left(b^2e^2-10bcde+10c^2d^2\right)+cd^2(8cd-7be)\right)}{3de^2(d+ex)^{3/2}(cd-be)}\cdots\right)}{5e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}}$$

$\Big\downarrow$ 1169

$$\frac{3\left(\frac{c\left(\frac{\sqrt{x}\sqrt{b+cx}\left(b^2e^2-16bcde+16c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{8d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{3de^2(cd-be)}-\frac{2\sqrt{bx+cx^2}\left(ex\left(b^2e^2-10bcde+10c^2d\cdots\right.\right.}{3de^2(d+ex)^{3/2}(cd-\cdots}\right)}{5e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}}$$

$\Big\downarrow$ 122

$$\frac{3\left(\frac{c\left(\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{3de^2(cd-be)}-\frac{2\sqrt{bx+cx^2}\left(ex\left(b^2e^2-10bcd\right.\right.}{3de^2(d+ex)}\right.}{5e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 120$$

$$\frac{3\left(\frac{c\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{3de^2(cd-be)}-\frac{2\sqrt{bx+cx^2}\left(ex\left(\right.\right.}{}\right.}{5e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 127$$

$$\frac{3\left(\frac{c\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{3de^2(cd-be)}-\frac{2\sqrt{bx}}{}\right.}{5e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \; 126$$

$$\frac{3\left(\frac{c\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{16\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\text{ EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}\right.}{3de^2(cd-be)}\right.}{5e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{5e(d+ex)^{5/2}}$$

input
```
Int[(b*x + c*x^2)^(3/2)/(d + e*x)^(7/2),x]
```

output
```
(-2*(b*x + c*x^2)^(3/2))/(5*e*(d + e*x)^(5/2)) + (3*((-2*(c*d^2*(8*c*d - 7
*b*e) + e*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)*x)*Sqrt[b*x + c*x^2])/(3*d*e
^2*(c*d - b*e)*(d + e*x)^(3/2)) + (c*((2*Sqrt[-b]*(16*c^2*d^2 - 16*b*c*d*e
 + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt
[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*
x + c*x^2]) - (16*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c
*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*
e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])))/(3*d*e^2*(c*d - b
*e))))/(5*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1161

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1169

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1229

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 645 vs. $2(290) = 580$.

Time = 3.54 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.93

method	result
elliptic	$\frac{\sqrt{x(cx+b)}\,\sqrt{(cx+b)x(ex+d)}\left(\frac{2d(be-cd)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{5e^6\left(x+\frac{d}{e}\right)^3}-\frac{4(be-2cd)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{5e^5\left(x+\frac{d}{e}\right)^2}+\frac{2\left(ce\,x^2+bex\right)\left(b^2e^2-11bcde+1\right.}{5e^4d(be-cd)\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2\right.}}\right.}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)*x*(e*x+d))^(1/2)/x/(c*x+b)*(2/5
*d*(b*e-c*d)/e^6*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(x+d/e)^3-4/5*(b*e-
2*c*d)/e^5*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(x+d/e)^2+2/5*(c*e*x^2+b*
e*x)/e^4/d/(b*e-c*d)*(b^2*e^2-11*b*c*d*e+11*c^2*d^2)/((x+d/e)*(c*e*x^2+b*e
*x))^(1/2)+2*(c*(2*b*e-3*c*d)/e^4-2/5*c*(b*e-2*c*d)/e^4+1/5/e^4*(b^2*e^2-1
1*b*c*d*e+11*c^2*d^2)/d-1/5*b/e^3/d/(b*e-c*d)*(b^2*e^2-11*b*c*d*e+11*c^2*d
^2))*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*
e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d
/e+b/c))^(1/2))+2*(c^2/e^3-1/5/e^3*c*(b^2*e^2-11*b*c*d*e+11*c^2*d^2)/d/(b*
e-c*d))*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/
(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e)^
(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d
/e+b/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(290) = 580$.

Time = 0.28 (sec) , antiderivative size = 807, normalized size of antiderivative = 2.41

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
-2/15*((16*c^3*d^6 - 24*b*c^2*d^5*e + 6*b^2*c*d^4*e^2 + b^3*d^3*e^3 + (16*
c^3*d^3*e^3 - 24*b*c^2*d^2*e^4 + 6*b^2*c*d*e^5 + b^3*e^6)*x^3 + 3*(16*c^3*
d^4*e^2 - 24*b*c^2*d^3*e^3 + 6*b^2*c*d^2*e^4 + b^3*d*e^5)*x^2 + 3*(16*c^3*
d^5*e - 24*b*c^2*d^4*e^2 + 6*b^2*c*d^3*e^3 + b^3*d^2*e^4)*x)*sqrt(c*e)*wei
erstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3
*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x
+ c*d + b*e)/(c*e)) + 3*(16*c^3*d^5*e - 16*b*c^2*d^4*e^2 + b^2*c*d^3*e^3 +
 (16*c^3*d^2*e^4 - 16*b*c^2*d*e^5 + b^2*c*e^6)*x^3 + 3*(16*c^3*d^3*e^3 - 1
6*b*c^2*d^2*e^4 + b^2*c*d*e^5)*x^2 + 3*(16*c^3*d^4*e^2 - 16*b*c^2*d^3*e^3
+ b^2*c*d^2*e^4)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2
*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*
e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2
*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*
e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(8*c^3*d^4*e^2 - 7*b*c^2*d^3*e
^3 + (11*c^3*d^2*e^4 - 11*b*c^2*d*e^5 + b^2*c*e^6)*x^2 + 2*(9*c^3*d^3*e^3
- 8*b*c^2*d^2*e^4)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/(c^2*d^5*e^5 - b*c*
d^4*e^6 + (c^2*d^2*e^8 - b*c*d*e^9)*x^3 + 3*(c^2*d^3*e^7 - b*c*d^2*e^8)*x^
2 + 3*(c^2*d^4*e^6 - b*c*d^3*e^7)*x)
```

Sympy [F]

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{7/2}}\,dx = \int \frac{(x(b + cx))^{\frac{3}{2}}}{(d + ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x)**(3/2)/(e*x+d)**(7/2),x)
```

output
```
Integral((x*(b + c*x))**(3/2)/(d + e*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{7/2}}\, dx = \int \frac{(cx^2 + bx)^{\frac{3}{2}}}{(ex + d)^{\frac{7}{2}}}\, dx$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(7/2), x)
```

Giac [F]

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{7/2}}\, dx = \int \frac{(cx^2 + bx)^{\frac{3}{2}}}{(ex + d)^{\frac{7}{2}}}\, dx$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{7/2}}\, dx = \int \frac{(c\,x^2 + b\,x)^{3/2}}{(d + e\,x)^{7/2}}\, dx$$

input
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^(7/2),x)
```

output
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{7/2}}\, dx = \text{too large to display}$$

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^(7/2),x)
```

output
```
( - 6*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*d*e - 8*sqrt(x)*sqrt(d + e*
x)*sqrt(b + c*x)*b**2*e**2*x + 18*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c*
d**2 + 28*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c*d*e*x + 4*sqrt(x)*sqrt(d
 + e*x)*sqrt(b + c*x)*b*c*e**2*x**2 - 12*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*
x)*c**2*d**2*x - 2*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**2*d*e*x**2 + 6*i
nt((sqrt(d + e*x)*sqrt(b + c*x))/(2*sqrt(x)*b**2*d**4*e + 8*sqrt(x)*b**2*d
**3*e**2*x + 12*sqrt(x)*b**2*d**2*e**3*x**2 + 8*sqrt(x)*b**2*d*e**4*x**3 +
 2*sqrt(x)*b**2*e**5*x**4 - sqrt(x)*b*c*d**5 - 2*sqrt(x)*b*c*d**4*e*x + 2*
sqrt(x)*b*c*d**3*e**2*x**2 + 8*sqrt(x)*b*c*d**2*e**3*x**3 + 7*sqrt(x)*b*c*
d*e**4*x**4 + 2*sqrt(x)*b*c*e**5*x**5 - sqrt(x)*c**2*d**5*x - 4*sqrt(x)*c*
*2*d**4*e*x**2 - 6*sqrt(x)*c**2*d**3*e**2*x**3 - 4*sqrt(x)*c**2*d**2*e**3*
x**4 - sqrt(x)*c**2*d*e**4*x**5),x)*b**4*d**5*e**2 + 18*int((sqrt(d + e*x)
*sqrt(b + c*x))/(2*sqrt(x)*b**2*d**4*e + 8*sqrt(x)*b**2*d**3*e**2*x + 12*s
qrt(x)*b**2*d**2*e**3*x**2 + 8*sqrt(x)*b**2*d*e**4*x**3 + 2*sqrt(x)*b**2*e
**5*x**4 - sqrt(x)*b*c*d**5 - 2*sqrt(x)*b*c*d**4*e*x + 2*sqrt(x)*b*c*d**3*
e**2*x**2 + 8*sqrt(x)*b*c*d**2*e**3*x**3 + 7*sqrt(x)*b*c*d*e**4*x**4 + 2*s
qrt(x)*b*c*e**5*x**5 - sqrt(x)*c**2*d**5*x - 4*sqrt(x)*c**2*d**4*e*x**2 -
6*sqrt(x)*c**2*d**3*e**2*x**3 - 4*sqrt(x)*c**2*d**2*e**3*x**4 - sqrt(x)*c*
*2*d*e**4*x**5),x)*b**4*d**4*e**3*x + 18*int((sqrt(d + e*x)*sqrt(b + c*x))
/(2*sqrt(x)*b**2*d**4*e + 8*sqrt(x)*b**2*d**3*e**2*x + 12*sqrt(x)*b**2*...
```

3.197 $\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{9/2}}\,dx$

Optimal result 1600
Mathematica [C] (verified) 1601
Rubi [A] (verified) 1602
Maple [B] (verified) 1607
Fricas [B] (verification not implemented) 1608
Sympy [F] 1609
Maxima [F] 1610
Giac [F] 1610
Mupad [F(-1)] 1610
Reduce [F] 1611

Optimal result

Integrand size = 23, antiderivative size = 422

$$\begin{aligned}\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{9/2}}\,dx &= -\frac{6(8cd+be)\sqrt{bx+cx^2}}{35e^3(d+ex)^{5/2}}-\frac{12cx\sqrt{bx+cx^2}}{7e^2(d+ex)^{5/2}}\\
&+\frac{2(16c^2d^2-16bcde-b^2e^2)\sqrt{bx+cx^2}}{35de^3(cd-be)(d+ex)^{3/2}}-\frac{2(bx+cx^2)^{3/2}}{7e(d+ex)^{7/2}}\\
&+\frac{4(2cd-be)(4c^2d^2-4bcde-b^2e^2)\sqrt{bx+cx^2}\,E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{35d^{3/2}e^{7/2}(cd-be)^2\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}\\
&-\frac{2c(8c^2d^2-11bcde+b^2e^2)\sqrt{bx+cx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{35\sqrt{d}e^{7/2}(cd-be)^2\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}\end{aligned}$$

output
```
-6/35*(b*e+8*c*d)*(c*x^2+b*x)^(1/2)/e^3/(e*x+d)^(5/2)-12/7*c*x*(c*x^2+b*x)
^(1/2)/e^2/(e*x+d)^(5/2)+2/35*(-b^2*e^2-16*b*c*d*e+16*c^2*d^2)*(c*x^2+b*x)
^(1/2)/d/e^3/(-b*e+c*d)/(e*x+d)^(3/2)-2/7*(c*x^2+b*x)^(3/2)/e/(e*x+d)^(7/2
)+4/35*(-b*e+2*c*d)*(-b^2*e^2-4*b*c*d*e+4*c^2*d^2)*(c*x^2+b*x)^(1/2)*Ellip
ticE(e^(1/2)*x^(1/2)/d^(1/2)/(1+e*x/d)^(1/2),(1-c*d/b/e)^(1/2))/d^(3/2)/e^
(7/2)/(-b*e+c*d)^2/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)-2/35*
c*(b^2*e^2-11*b*c*d*e+8*c^2*d^2)*(c*x^2+b*x)^(1/2)*InverseJacobiAM(arctan(
e^(1/2)*x^(1/2)/d^(1/2)),(1-c*d/b/e)^(1/2))/d^(1/2)/e^(7/2)/(-b*e+c*d)^2/x
^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.91 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.14

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{9/2}}\,dx =$$
$$-\frac{2(x(b+cx))^{3/2}\left(bex(b+cx)\left(5d^3(cd-be)^3-8d^2(cd-be)^2(2cd-be)(d+ex)+d(cd-be)\left(19c^2d^2-1\right.\right.\right.}{}$$

input
```
Integrate[(b*x + c*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

output
```
(-2*(x*(b + c*x))^(3/2)*(b*e*x*(b + c*x)*(5*d^3*(c*d - b*e)^3 - 8*d^2*(c*d
 - b*e)^2*(2*c*d - b*e)*(d + e*x) + d*(c*d - b*e)*(19*c^2*d^2 - 19*b*c*d*e
 + b^2*e^2)*(d + e*x)^2 - 2*(8*c^3*d^3 - 12*b*c^2*d^2*e + 2*b^2*c*d*e^2 +
b^3*e^3)*(d + e*x)^3) + Sqrt[b/c]*c*(d + e*x)^3*(2*Sqrt[b/c]*(8*c^3*d^3 -
12*b*c^2*d^2*e + 2*b^2*c*d*e^2 + b^3*e^3)*(b + c*x)*(d + e*x) + (2*I)*b*e*
(8*c^3*d^3 - 12*b*c^2*d^2*e + 2*b^2*c*d*e^2 + b^3*e^3)*Sqrt[1 + b/(c*x)]*S
qrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*
e)] - I*b*e*(8*c^3*d^3 - 13*b*c^2*d^2*e + 3*b^2*c*d*e^2 + 2*b^3*e^3)*Sqrt[
1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[
x]], (c*d)/(b*e)])))/(35*b*d^2*e^4*(c*d - b*e)^2*x^2*(b + c*x)^2*(d + e*x)
^(7/2))
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 496, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {1161, 1229, 27, 1237, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{9/2}}dx$$

$$\Big\downarrow \ 1161$$

$$\frac{3\int \frac{(b+2cx)\sqrt{cx^2+bx}}{(d+ex)^{7/2}}dx}{7e}-\frac{2(bx+cx^2)^{3/2}}{7e(d+ex)^{7/2}}$$

$$\Big\downarrow \ 1229$$

$$\frac{3\left(-\frac{2\int -\frac{b(8c^2d^2-5bced-2b^2e^2)+c(16c^2d^2-16bced-b^2e^2)x}{2(d+ex)^{3/2}\sqrt{cx^2+bx}}dx}{15de^2(cd-be)}-\frac{2\sqrt{bx+cx^2}(ex(b^2e^2-14bcde+14c^2d^2)+d(-2b^2e^2-5bcde+8c^2d^2))}{15de^2(d+ex)^{5/2}(cd-be)}\right)}{7e}-\frac{2(bx+cx^2)^{3/2}}{7e(d+ex)^{7/2}}$$

$$\Big\downarrow \ 27$$

$$\frac{3\left(\frac{\int \frac{b(8c^2d^2-5bced-2b^2e^2)+c(16c^2d^2-16bced-b^2e^2)x}{(d+ex)^{3/2}\sqrt{cx^2+bx}}dx}{15de^2(cd-be)}-\frac{2\sqrt{bx+cx^2}(ex(b^2e^2-14bcde+14c^2d^2)+d(-2b^2e^2-5bcde+8c^2d^2))}{15de^2(d+ex)^{5/2}(cd-be)}\right)}{7e}-\frac{2(bx+cx^2)^{3/2}}{7e(d+ex)^{7/2}}$$

$$\Big\downarrow \ 1237$$

$$\frac{3\left(\frac{\frac{4\sqrt{bx+cx^2}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{2\int\frac{c\left(bd\left(8c^2d^2-11bced+b^2e^2\right)+2(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{d(cd-be)}}{15de^2(cd-be)}-\frac{2\sqrt{bx+cx^2}\left(ex\left(b^2e^2-14bcde-\right.\right.}{15de^2(d}\right.}{7e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}}$$

$$\Big\downarrow\ 27$$

$$\frac{3\left(\frac{\frac{4\sqrt{bx+cx^2}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\int\frac{bd\left(8c^2d^2-11bced+b^2e^2\right)+2(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{d(cd-be)}}{15de^2(cd-be)}-\frac{2\sqrt{bx+cx^2}\left(ex\left(b^2e^2-14bcde+14\right.\right.}{15de^2(d+e}\right.}{7e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}}$$

$$\Big\downarrow\ 1269$$

$$\frac{3\left(\frac{\frac{4\sqrt{bx+cx^2}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{d(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{cx^2+b}}}{e}\right.}{d(cd-be)}}{15de^2(cd-be)}\right.}{7e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}}$$

$$\Big\downarrow\ 1169$$

$$\frac{3\left(\frac{\frac{4\sqrt{bx+cx^2}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\sqrt{x}\sqrt{b+cx}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right.}{e\sqrt{bx+cx^2}}\right.}{d(cd-be)}}{15de^2(cd-be)}\right.}{7e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}}$$

$$\Big\downarrow\ 122$$

$$\frac{3\left(\frac{\frac{4\sqrt{bx+cx^2}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\left(-b^2e^2-16bcd\cdots\right.}{e\sqrt{bx\cdots}}\right)}{d(cd-be)}}{15de^2(cd-be)}\right.}{7e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}}$$

$\downarrow$ 120

$$\frac{3\left(\frac{\frac{4\sqrt{bx+cx^2}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\left(\cdots\right.}{\cdots}\right)}{d(cd-be)}}{15de^2(cd-be)}\right.}{7e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}}$$

$\downarrow$ 127

$$\frac{3\left(\frac{\frac{4\sqrt{bx+cx^2}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(\cdots}{\cdots}\right)}{d(cd-be)}}{15de^2(cd-be)}\right.}{7e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}}$$

$\downarrow$ 126

$$\frac{3\left(\frac{\frac{4\sqrt{bx+cx^2}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\cdots}}{\cdots}\right)}{d(cd-be)}}{15de^2(cd-be)}\cdots\right)}{7e}$$

$$\frac{2\left(bx+cx^2\right)^{3/2}}{7e(d+ex)^{7/2}}$$

input
```
Int[(b*x + c*x^2)^(3/2)/(d + e*x)^(9/2),x]
```

output
```
(-2*(b*x + c*x^2)^(3/2))/(7*e*(d + e*x)^(7/2)) + (3*((-2*(d*(8*c^2*d^2 - 5
*b*c*d*e - 2*b^2*e^2) + e*(14*c^2*d^2 - 14*b*c*d*e + b^2*e^2)*x)*Sqrt[b*x
+ c*x^2])/(15*d*e^2*(c*d - b*e)*(d + e*x)^(5/2)) + ((4*(2*c*d - b*e)*(4*c^
2*d^2 - 4*b*c*d*e - b^2*e^2)*Sqrt[b*x + c*x^2])/(d*(c*d - b*e)*Sqrt[d + e*
x]) - (c*((4*Sqrt[-b]*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*Sqrt
[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqr
t[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2
*Sqrt[-b]*d*(c*d - b*e)*(16*c^2*d^2 - 16*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1
 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]]
, (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])))/(d*(c*d - b*
e)))/(15*d*e^2*(c*d - b*e))))/(7*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(371) = 742$.

Time = 5.11 (sec) , antiderivative size = 792, normalized size of antiderivative = 1.88

method	result
elliptic	$\frac{\sqrt{x(cx+b)}\,\sqrt{(cx+b)x(ex+d)}\left(\frac{2d(be-cd)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{7e^7\left(x+\frac{d}{e}\right)^4}-\frac{16(be-2cd)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{35e^6\left(x+\frac{d}{e}\right)^3}+\frac{2\left(b^2e^2-19bcde+19c^2d^2\right)\sqrt{ce}}{35d(be-cd)e^5(}\right.}{}$
default	Expression too large to display

input

```
int((c*x^2+b*x)^(3/2)/(e*x+d)^(9/2),x,method=_RETURNVERBOSE)
```

output

```
1/(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)*x*(e*x+d))^(1/2)/x/(c*x+b)*(2/7
*d*(b*e-c*d)/e^7*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(x+d/e)^4-16/35*(b*
e-2*c*d)/e^6*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(x+d/e)^3+2/35*(b^2*e^2
-19*b*c*d*e+19*c^2*d^2)/d/(b*e-c*d)/e^5*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1
/2)/(x+d/e)^2+4/35*(c*e*x^2+b*e*x)/e^4/d^2/(b*e-c*d)^2*(b^3*e^3+2*b^2*c*d*
e^2-12*b*c^2*d^2*e+8*c^3*d^3)/((x+d/e)*(c*e*x^2+b*e*x))^(1/2)+2*(c^2/e^4+1
/35*c*(b^2*e^2-19*b*c*d*e+19*c^2*d^2)/e^4/d/(b*e-c*d)+2/35/e^4/(b*e-c*d)*(
b^3*e^3+2*b^2*c*d*e^2-12*b*c^2*d^2*e+8*c^3*d^3)/d^2-2/35*b/e^3/d^2/(b*e-c*
d)^2*(b^3*e^3+2*b^2*c*d*e^2-12*b*c^2*d^2*e+8*c^3*d^3))*d/e*((x+d/e)/d*e)^(
1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*
d*x)^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-4/35/e^4
*c*(b^3*e^3+2*b^2*c*d*e^2-12*b*c^2*d^2*e+8*c^3*d^3)/d/(b*e-c*d)^2*((x+d/e)
/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d
*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/
c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1230 vs. $2(371) = 742$.

Time = 0.17 (sec) , antiderivative size = 1230, normalized size of antiderivative = 2.91

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

output
```
2/105*((16*c^4*d^8 - 32*b*c^3*d^7*e + 13*b^2*c^2*d^6*e^2 + 3*b^3*c*d^5*e^3
 + 2*b^4*d^4*e^4 + (16*c^4*d^4*e^4 - 32*b*c^3*d^3*e^5 + 13*b^2*c^2*d^2*e^6
 + 3*b^3*c*d*e^7 + 2*b^4*e^8)*x^4 + 4*(16*c^4*d^5*e^3 - 32*b*c^3*d^4*e^4 +
 13*b^2*c^2*d^3*e^5 + 3*b^3*c*d^2*e^6 + 2*b^4*d*e^7)*x^3 + 6*(16*c^4*d^6*e
^2 - 32*b*c^3*d^5*e^3 + 13*b^2*c^2*d^4*e^4 + 3*b^3*c*d^3*e^5 + 2*b^4*d^2*e
^6)*x^2 + 4*(16*c^4*d^7*e - 32*b*c^3*d^6*e^2 + 13*b^2*c^2*d^5*e^3 + 3*b^3*
c*d^4*e^4 + 2*b^4*d^3*e^5)*x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 -
 b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*
d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 6*(8*c^4*
d^7*e - 12*b*c^3*d^6*e^2 + 2*b^2*c^2*d^5*e^3 + b^3*c*d^4*e^4 + (8*c^4*d^3*
e^5 - 12*b*c^3*d^2*e^6 + 2*b^2*c^2*d*e^7 + b^3*c*e^8)*x^4 + 4*(8*c^4*d^4*e
^4 - 12*b*c^3*d^3*e^5 + 2*b^2*c^2*d^2*e^6 + b^3*c*d*e^7)*x^3 + 6*(8*c^4*d^
5*e^3 - 12*b*c^3*d^4*e^4 + 2*b^2*c^2*d^3*e^5 + b^3*c*d^2*e^6)*x^2 + 4*(8*c
^4*d^6*e^2 - 12*b*c^3*d^5*e^3 + 2*b^2*c^2*d^4*e^4 + b^3*c*d^3*e^5)*x)*sqrt
(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(
2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstr
assPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3
- 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d
 + b*e)/(c*e))) + 3*(8*c^4*d^6*e^2 - 11*b*c^3*d^5*e^3 + b^2*c^2*d^4*e^4 +
2*(8*c^4*d^3*e^5 - 12*b*c^3*d^2*e^6 + 2*b^2*c^2*d*e^7 + b^3*c*e^8)*x^3 ...
```

Sympy [F]

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{9/2}}\, dx = \int \frac{(x(b + cx))^{\frac{3}{2}}}{(d + ex)^{\frac{9}{2}}}\, dx$$

input
```
integrate((c*x**2+b*x)**(3/2)/(e*x+d)**(9/2),x)
```

output
```
Integral((x*(b + c*x))**(3/2)/(d + e*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{9/2}}\,dx = \int \frac{(cx^2+bx)^{\frac{3}{2}}}{(ex+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(9/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(9/2), x)
```

Giac [F]

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{9/2}}\,dx = \int \frac{(cx^2+bx)^{\frac{3}{2}}}{(ex+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(3/2)/(e*x+d)^(9/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(3/2)/(e*x + d)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{3/2}}{(d+ex)^{9/2}}\,dx = \int \frac{(c\,x^2+b\,x)^{3/2}}{(d+e\,x)^{9/2}}\,dx$$

input
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^(9/2),x)
```

output
```
int((b*x + c*x^2)^(3/2)/(d + e*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^{3/2}}{(d + ex)^{9/2}} \, dx = \text{too large to display}$$

input
```
int((c*x^2+b*x)^(3/2)/(e*x+d)^(9/2),x)
```

output
```
( - 6*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c*d**2 - 12*sqrt(x)*sqrt(d + e
*x)*sqrt(b + c*x)*b*c*d*e*x - 6*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c*e*
*2*x**2 + 4*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**2*d**2*x + 2*sqrt(x)*sq
rt(d + e*x)*sqrt(b + c*x)*c**2*d*e*x**2 + 9*int((sqrt(d + e*x)*sqrt(b + c*
x))/(3*sqrt(x)*b**2*d**5*e + 15*sqrt(x)*b**2*d**4*e**2*x + 30*sqrt(x)*b**2
*d**3*e**3*x**2 + 30*sqrt(x)*b**2*d**2*e**4*x**3 + 15*sqrt(x)*b**2*d*e**5*
x**4 + 3*sqrt(x)*b**2*e**6*x**5 - sqrt(x)*b*c*d**6 - 2*sqrt(x)*b*c*d**5*e*
x + 5*sqrt(x)*b*c*d**4*e**2*x**2 + 20*sqrt(x)*b*c*d**3*e**3*x**3 + 25*sqrt
(x)*b*c*d**2*e**4*x**4 + 14*sqrt(x)*b*c*d*e**5*x**5 + 3*sqrt(x)*b*c*e**6*x
**6 - sqrt(x)*c**2*d**6*x - 5*sqrt(x)*c**2*d**5*e*x**2 - 10*sqrt(x)*c**2*d
**4*e**2*x**3 - 10*sqrt(x)*c**2*d**3*e**3*x**4 - 5*sqrt(x)*c**2*d**2*e**4*
x**5 - sqrt(x)*c**2*d*e**5*x**6),x)*b**3*c*d**7*e + 36*int((sqrt(d + e*x)*
sqrt(b + c*x))/(3*sqrt(x)*b**2*d**5*e + 15*sqrt(x)*b**2*d**4*e**2*x + 30*s
qrt(x)*b**2*d**3*e**3*x**2 + 30*sqrt(x)*b**2*d**2*e**4*x**3 + 15*sqrt(x)*b
**2*d*e**5*x**4 + 3*sqrt(x)*b**2*e**6*x**5 - sqrt(x)*b*c*d**6 - 2*sqrt(x)*
b*c*d**5*e*x + 5*sqrt(x)*b*c*d**4*e**2*x**2 + 20*sqrt(x)*b*c*d**3*e**3*x**
3 + 25*sqrt(x)*b*c*d**2*e**4*x**4 + 14*sqrt(x)*b*c*d*e**5*x**5 + 3*sqrt(x)
*b*c*e**6*x**6 - sqrt(x)*c**2*d**6*x - 5*sqrt(x)*c**2*d**5*e*x**2 - 10*sqr
t(x)*c**2*d**4*e**2*x**3 - 10*sqrt(x)*c**2*d**3*e**3*x**4 - 5*sqrt(x)*c**2
*d**2*e**4*x**5 - sqrt(x)*c**2*d*e**5*x**6),x)*b**3*c*d**6*e**2*x + 54*...
```

3.198 $\int \sqrt{d+ex}(bx+cx^2)^{5/2}\,dx$

Optimal result 1612
Mathematica [C] (verified) 1613
Rubi [A] (verified) 1614
Maple [B] (verified) 1620
Fricas [A] (verification not implemented) 1621
Sympy [F] 1622
Maxima [F] 1622
Giac [F] 1623
Mupad [F(-1)] 1623
Reduce [F] 1623

Optimal result

Integrand size = 23, antiderivative size = 817

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^{5/2}\,dx =$$
$$-\frac{4(128c^6d^6-384bc^5d^5e+343b^2c^4d^4e^2-46b^3c^3d^3e^3-21b^4c^2d^2e^4-20b^5cde^5+24b^6e^6)\,x\sqrt{d+ex}}{9009c^3e^6\sqrt{bx+cx^2}}$$
$$+\frac{2(128c^5d^5-368bc^4d^4e+303b^2c^3d^3e^2-22b^3c^2d^2e^3-17b^4cde^4+24b^5e^5)\sqrt{d+ex}\sqrt{bx+cx^2}}{9009c^3e^5}$$
$$-\frac{4(48c^4d^4-136bc^3d^3e+109b^2c^2d^2e^2-6b^3cde^3+9b^4e^4)\,x\sqrt{d+ex}\sqrt{bx+cx^2}}{9009c^2e^4}$$
$$+\frac{2(80c^3d^3-225bc^2d^2e+178b^2cde^2+15b^3e^3)\,x^2\sqrt{d+ex}\sqrt{bx+cx^2}}{9009ce^3}$$
$$-\frac{2(2cd-5be)(5cd+3be)x^3\sqrt{d+ex}\sqrt{bx+cx^2}}{1287e^2}$$
$$+\frac{2(cd+5be)x^2\sqrt{d+ex}(bx+cx^2)^{3/2}}{143e}+\frac{2}{13}x\sqrt{d+ex}\left(bx+cx^2\right)^{5/2}$$
$$+\frac{4\sqrt{b}(128c^6d^6-384bc^5d^5e+343b^2c^4d^4e^2-46b^3c^3d^3e^3-21b^4c^2d^2e^4-20b^5cde^5+24b^6e^6)\sqrt{x}\sqrt{d+ex}\,E\Big(}{9009c^{7/2}e^6\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$
$$-\frac{2b^{3/2}(128c^5d^5-368bc^4d^4e+303b^2c^3d^3e^2-22b^3c^2d^2e^3-17b^4cde^4+24b^5e^5)\sqrt{x}\sqrt{d+ex}\,\mathrm{EllipticF}\Big(\mathrm{arct}}{9009c^{7/2}e^5\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$

output
```
-4/9009*(24*b^6*e^6-20*b^5*c*d*e^5-21*b^4*c^2*d^2*e^4-46*b^3*c^3*d^3*e^3+3
43*b^2*c^4*d^4*e^2-384*b*c^5*d^5*e+128*c^6*d^6)*x*(e*x+d)^(1/2)/c^3/e^6/(c
*x^2+b*x)^(1/2)+2/9009*(24*b^5*e^5-17*b^4*c*d*e^4-22*b^3*c^2*d^2*e^3+303*b
^2*c^3*d^3*e^2-368*b*c^4*d^4*e+128*c^5*d^5)*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2
)/c^3/e^5-4/9009*(9*b^4*e^4-6*b^3*c*d*e^3+109*b^2*c^2*d^2*e^2-136*b*c^3*d^
3*e+48*c^4*d^4)*x*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/c^2/e^4+2/9009*(15*b^3*e
^3+178*b^2*c*d*e^2-225*b*c^2*d^2*e+80*c^3*d^3)*x^2*(e*x+d)^(1/2)*(c*x^2+b*
x)^(1/2)/c/e^3-2/1287*(-5*b*e+2*c*d)*(3*b*e+5*c*d)*x^3*(e*x+d)^(1/2)*(c*x^
2+b*x)^(1/2)/e^2+2/143*(5*b*e+c*d)*x^2*(e*x+d)^(1/2)*(c*x^2+b*x)^(3/2)/e+2
/13*x*(e*x+d)^(1/2)*(c*x^2+b*x)^(5/2)+4/9009*b^(1/2)*(24*b^6*e^6-20*b^5*c*
d*e^5-21*b^4*c^2*d^2*e^4-46*b^3*c^3*d^3*e^3+343*b^2*c^4*d^4*e^2-384*b*c^5*
d^5*e+128*c^6*d^6)*x^(1/2)*(e*x+d)^(1/2)*EllipticE(c^(1/2)*x^(1/2)/b^(1/2)
/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/c^(7/2)/e^6/(b*(e*x+d)/d/(c*x+b))^(1/2
)/(c*x^2+b*x)^(1/2)-2/9009*b^(3/2)*(24*b^5*e^5-17*b^4*c*d*e^4-22*b^3*c^2*d
^2*e^3+303*b^2*c^3*d^3*e^2-368*b*c^4*d^4*e+128*c^5*d^5)*x^(1/2)*(e*x+d)^(1
/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e/c/d)^(1/2))/c^(
7/2)/e^5/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 20.73 (sec) , antiderivative size = 663, normalized size of antiderivative = 0.81

$$\int \sqrt{d+ex}(bx+cx^2)^{5/2}\,dx = \frac{2(x(b+cx))^{5/2}\left(bex(b+cx)(d+ex)\left(24b^5e^5-b^4ce^4(17d+18ex)+b^3c^2e^3(-22d^2+12dex\right.\right.}{}$$

input
```
Integrate[Sqrt[d + e*x]*(b*x + c*x^2)^(5/2),x]
```

output
```
(2*(x*(b + c*x))^(5/2)*(b*e*x*(b + c*x)*(d + e*x)*(24*b^5*e^5 - b^4*c*e^4*
(17*d + 18*e*x) + b^3*c^2*e^3*(-22*d^2 + 12*d*e*x + 15*e^2*x^2) + b^2*c^3*
e^2*(303*d^3 - 218*d^2*e*x + 178*d*e^2*x^2 + 1113*e^3*x^3) + b*c^4*e*(-368
*d^4 + 272*d^3*e*x - 225*d^2*e^2*x^2 + 196*d*e^3*x^3 + 1701*e^4*x^4) + c^5
*(128*d^5 - 96*d^4*e*x + 80*d^3*e^2*x^2 - 70*d^2*e^3*x^3 + 63*d*e^4*x^4 +
693*e^5*x^5)) + Sqrt[b/c]*(-2*Sqrt[b/c]*(128*c^6*d^6 - 384*b*c^5*d^5*e + 3
43*b^2*c^4*d^4*e^2 - 46*b^3*c^3*d^3*e^3 - 21*b^4*c^2*d^2*e^4 - 20*b^5*c*d*
e^5 + 24*b^6*e^6)*(b + c*x)*(d + e*x) - (2*I)*b*e*(128*c^6*d^6 - 384*b*c^5
*d^5*e + 343*b^2*c^4*d^4*e^2 - 46*b^3*c^3*d^3*e^3 - 21*b^4*c^2*d^2*e^4 - 2
0*b^5*c*d*e^5 + 24*b^6*e^6)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*El
lipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + I*b*e*(128*c^6*d^6 -
400*b*c^5*d^5*e + 383*b^2*c^4*d^4*e^2 - 70*b^3*c^3*d^3*e^3 - 25*b^4*c^2*d^
2*e^4 - 64*b^5*c*d*e^5 + 48*b^6*e^6)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x
^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(9009*b*c^3
*e^6*x^3*(b + c*x)^3*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 697, normalized size of antiderivative = 0.85, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.565$, Rules used = {1162, 1236, 27, 1231, 27, 1231, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{5/2}\sqrt{d+ex}\,dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}(d+ex)^{3/2}}{13e}-\frac{5\int \sqrt{d+ex}(bd+(2cd-be)x)\left(cx^2+bx\right)^{3/2}dx}{13e}$$

$$\Big\downarrow \text{1236}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}(d+ex)^{3/2}}{13e}-\frac{5\left(\frac{2\int \frac{\left(bd(cd+5be)+2\left(c^2d^2-bced+3b^2e^2\right)x\right)\left(cx^2+bx\right)^{3/2}}{2\sqrt{d+ex}}dx}{11c}+\frac{2\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}(2cd-be)}{11c}\right)}{13e}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(bx+cx^2\right)^{5/2}(d+ex)^{3/2}}{13e}-\frac{5\left(\frac{\int \frac{\left(bd(cd+5be)+2\left(c^2d^2-bced+3b^2e^2\right)x\right)\left(cx^2+bx\right)^{3/2}}{\sqrt{d+ex}}dx}{11c}+\frac{2\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}(2cd-be)}{11c}\right)}{13e}$$

$$\Big\downarrow \; 1231$$

$$\frac{2\left(bx+cx^2\right)^{5/2}(d+ex)^{3/2}}{13e}-\frac{5\left(-\frac{-\frac{2\int -\frac{\left(bd\left(16c^3d^3-31bc^2ed^2+9b^2ce^2d-18b^3e^3\right)+\left(32c^4d^4-64bc^3ed^3+21b^2c^2e^2d^2+11b^3ce^3d-24b^4e^4\right)x\right)\sqrt{cx^2+bx}}{2\sqrt{d+ex}}dx}{21ce^2}-\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}\left(-18b^3e^3 \ldots}{\ldots}}{11c} \ldots}{13e}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(bx+cx^2\right)^{5/2}(d+ex)^{3/2}}{13e}-\frac{5\left(\frac{\frac{\int \frac{\left(bd\left(16c^3d^3-31bc^2ed^2+9b^2ce^2d-18b^3e^3\right)+\left(32c^4d^4-64bc^3ed^3+21b^2c^2e^2d^2+11b^3ce^3d-24b^4e^4\right)x\right)\sqrt{cx^2+bx}}{\sqrt{d+ex}}dx}{21ce^2}-\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}\left(-18b^3e^3-14c \ldots}{\ldots}}{11c} \ldots}{13e}$$

$$\Big\downarrow \; 1231$$

$$\frac{2\left(bx+cx^2\right)^{5/2}(d+ex)^{3/2}}{13e}-\frac{5\left(\frac{-\frac{2\int -\frac{bd\left(128c^5d^5-368bc^4ed^4+303b^2c^3e^2d^3-22b^3c^2e^3d^2-17b^4ce^4d+24b^5e^5\right)+2\left(128c^6d^6-384bc^5ed^5+343b^2c^4e^2d^4-46b^3c^3e^3d^3-21b^4c^2e^4d^2-20b^5ce \ldots}{2\sqrt{d+ex}\sqrt{cx^2+bx}}}{15ce^2} \ldots}{\ldots} \ldots}{\ldots}$$

$$\Big\downarrow \; 27$$

$$\frac{2\left(bx+cx^2\right)^{5/2}(d+ex)^{3/2}}{13e}-$$

$$5\left(\frac{\frac{\int \frac{bd\left(128c^5d^5-368bc^4ed^4+303b^2c^3e^2d^3-22b^3c^2e^3d^2-17b^4ce^4d+24b^5e^5\right)+2\left(128c^6d^6-384bc^5ed^5+343b^2c^4e^2d^4-46b^3c^3e^3d^3-21b^4c^2e^4d^2-20b^5ce^5d+2\right.}{\sqrt{d+ex}\sqrt{cx^2+bx}}}{15ce^2}}{}\right.$$

$\downarrow$ 1269

$$\frac{2\left(bx+cx^2\right)^{5/2}(d+ex)^{3/2}}{13e}-$$

$$5\left(\frac{\frac{\frac{2\left(24b^6e^6-20b^5cde^5-21b^4c^2d^2e^4-46b^3c^3d^3e^3+343b^2c^4d^4e^2-384bc^5d^5e+128c^6d^6\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{d(cd-be)(2cd-be)\left(24b^4e^4+49b^3cde^3+79b^2c^2d^2e^2-\right.}{e}}{15ce^2}}{}\right.$$

$\downarrow$ 1169

$$\frac{2\left(bx+cx^2\right)^{5/2}(d+ex)^{3/2}}{13e}-$$

$$5\left(\frac{\frac{\frac{2\sqrt{x}\sqrt{b+cx}\left(24b^6e^6-20b^5cde^5-21b^4c^2d^2e^4-46b^3c^3d^3e^3+343b^2c^4d^4e^2-384bc^5d^5e+128c^6d^6\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\left(24b^4e^4+49b\right.}{}}{15ce^2}}{}\right.$$

$\downarrow$ 122

$$\frac{2\left(bx+cx^2\right)^{5/2}(d+ex)^{3/2}}{13e}-$$

$$5\left(\frac{\frac{\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(24b^6e^6-20b^5cde^5-21b^4c^2d^2e^4-46b^3c^3d^3e^3+343b^2c^4d^4e^2-384bc^5d^5e+128c^6d^6\right)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\left(24\right.}{}}{15ce^2}}{}\right.$$

$\downarrow$ 120

$$\frac{2\left(bx+cx^2\right)^{5/2}(d+ex)^{3/2}}{13e}-$$

$$5\left(\frac{\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(24b^6e^6-20b^5cde^5-21b^4c^2d^2e^4-46b^3c^3d^3e^3+343b^2c^4d^4e^2-384bc^5d^5e+128c^6d^6\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\cdots}{\cdots}}{15ce^2}}{\cdots}\right.$$

$\Big\downarrow$ 127

$$\frac{2\left(bx+cx^2\right)^{5/2}(d+ex)^{3/2}}{13e}-$$

$$5\left(\frac{\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(24b^6e^6-20b^5cde^5-21b^4c^2d^2e^4-46b^3c^3d^3e^3+343b^2c^4d^4e^2-384bc^5d^5e+128c^6d^6\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+\cdots}}{\cdots}}{15ce^2}}{\cdots}\right.$$

$\Big\downarrow$ 126

$$\frac{2\left(bx+cx^2\right)^{5/2}(d+ex)^{3/2}}{13e}-$$

$$5\left(\frac{\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(24b^6e^6-20b^5cde^5-21b^4c^2d^2e^4-46b^3c^3d^3e^3+343b^2c^4d^4e^2-384bc^5d^5e+128c^6d^6\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\cdots}{\cdots}}{15ce^2}}{\cdots}\right.$$

input
```
Int[Sqrt[d + e*x]*(b*x + c*x^2)^(5/2),x]
```

output
```
(2*(d + e*x)^(3/2)*(b*x + c*x^2)^(5/2))/(13*e) - (5*((2*(2*c*d - b*e)*Sqrt
[d + e*x]*(b*x + c*x^2)^(5/2))/(11*c) + ((-2*Sqrt[d + e*x]*(16*c^3*d^3 - 3
1*b*c^2*d^2*e + 9*b^2*c*d*e^2 - 18*b^3*e^3 - 14*c*e*(c^2*d^2 - b*c*d*e + 3
*b^2*e^2)*x)*(b*x + c*x^2)^(3/2))/(63*c*e^2) + ((-2*Sqrt[d + e*x]*(128*c^5
*d^5 - 368*b*c^4*d^4*e + 303*b^2*c^3*d^3*e^2 - 22*b^3*c^2*d^2*e^3 - 17*b^4
*c*d*e^4 + 24*b^5*e^5 - 3*c*e*(32*c^4*d^4 - 64*b*c^3*d^3*e + 21*b^2*c^2*d^
2*e^2 + 11*b^3*c*d*e^3 - 24*b^4*e^4)*x)*Sqrt[b*x + c*x^2])/(15*c*e^2) + ((
4*Sqrt[-b]*(128*c^6*d^6 - 384*b*c^5*d^5*e + 343*b^2*c^4*d^4*e^2 - 46*b^3*c
^3*d^3*e^3 - 21*b^4*c^2*d^2*e^4 - 20*b^5*c*d*e^5 + 24*b^6*e^6)*Sqrt[x]*Sqr
t[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]],
 (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[-
b]*d*(c*d - b*e)*(2*c*d - b*e)*(128*c^4*d^4 - 256*b*c^3*d^3*e + 79*b^2*c^2
*d^2*e^2 + 49*b^3*c*d*e^3 + 24*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 +
 (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqr
t[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]))/(15*c*e^2))/(21*c*e^2))/(11*c)))/
(13*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1727 vs. $2(748) = 1496$.

Time = 1.96 (sec) , antiderivative size = 1728, normalized size of antiderivative = 2.12

method	result	size
default	Expression too large to display	1728
elliptic	Expression too large to display	2334

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/9009*(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*(48*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b
*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))
^(1/2))*b^7*d*e^7-48*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d
)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^7*d*e^7+756*
c^7*d*e^7*x^7-36*b*c^6*d^2*e^6*x^5+1318*b^3*c^4*d*e^7*x^4-69*b^2*c^5*d^2*e
^6*x^4+57*b*c^6*d^3*e^5*x^4-8*b^4*c^3*d*e^7*x^3-50*b^3*c^4*d^2*e^6*x^3+132
*b^2*c^5*d^3*e^5*x^3-112*b*c^6*d^4*e^4*x^3-11*b^5*c^2*d*e^7*x^2-27*b^4*c^3
*d^2*e^6*x^2+63*b^3*c^4*d^3*e^5*x^2+207*b^2*c^5*d^4*e^4*x^2-336*b*c^6*d^5*
e^3*x^2+24*b^6*c*d*e^7*x-17*b^5*c^2*d^2*e^6*x-22*b^4*c^3*d^3*e^5*x+303*b^3
*c^4*d^4*e^4*x-368*b^2*c^5*d^5*e^3*x+128*b*c^6*d^6*e^2*x-3*b^4*c^3*e^8*x^4
-16*c^7*d^4*e^4*x^4+6*b^5*c^2*e^8*x^3+32*c^7*d^5*e^3*x^3+24*b^6*c*e^8*x^2+
128*c^7*d^6*e^2*x^2+2394*b*c^6*e^8*x^7+2814*b^2*c^5*e^8*x^6-7*c^7*d^2*e^6*
x^6+1128*b^3*c^4*e^8*x^5+10*c^7*d^3*e^5*x^5+693*c^7*e^8*x^8+256*((e*x+d)/d
)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(
1/2),(-d*c/(b*e-c*d))^(1/2))*c^7*d^8-64*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-
c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1
/2))*b^6*c*d^2*e^6-25*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/
d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^5*c^2*d^3*e
^5-70*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*Ellipti
cF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^4*c^3*d^4*e^4+383*((e*x+...
```

Fricas [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 747, normalized size of antiderivative = 0.91

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/27027*((256*c^7*d^7 - 896*b*c^6*d^6*e + 1022*b^2*c^5*d^5*e^2 - 315*b^3*c
^4*d^4*e^3 - 68*b^4*c^3*d^3*e^4 - 31*b^5*c^2*d^2*e^5 - 64*b^6*c*d*e^6 + 48
*b^7*e^7)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/
(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(
c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 6*(128*c^7*d^6*e - 384*b*c^6*
d^5*e^2 + 343*b^2*c^5*d^4*e^3 - 46*b^3*c^4*d^3*e^4 - 21*b^4*c^3*d^2*e^5 -
20*b^5*c^2*d*e^6 + 24*b^6*c*e^7)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 -
b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d
*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e +
b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b
^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(693*c^7*e^7*x^5
+ 128*c^7*d^5*e^2 - 368*b*c^6*d^4*e^3 + 303*b^2*c^5*d^3*e^4 - 22*b^3*c^4*d
^2*e^5 - 17*b^4*c^3*d*e^6 + 24*b^5*c^2*e^7 + 63*(c^7*d*e^6 + 27*b*c^6*e^7)
*x^4 - 7*(10*c^7*d^2*e^5 - 28*b*c^6*d*e^6 - 159*b^2*c^5*e^7)*x^3 + (80*c^7
*d^3*e^4 - 225*b*c^6*d^2*e^5 + 178*b^2*c^5*d*e^6 + 15*b^3*c^4*e^7)*x^2 - 2
*(48*c^7*d^4*e^3 - 136*b*c^6*d^3*e^4 + 109*b^2*c^5*d^2*e^5 - 6*b^3*c^4*d*e
^6 + 9*b^4*c^3*e^7)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/(c^5*e^7)
```

Sympy [F]

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^{5/2}\,dx=\int (x(b+cx))^{\frac{5}{2}}\sqrt{d+ex}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(c*x**2+b*x)**(5/2),x)
```

output
```
Integral((x*(b + c*x))**(5/2)*sqrt(d + e*x), x)
```

Maxima [F]

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^{5/2}\,dx=\int \left(cx^2+bx\right)^{\frac{5}{2}}\sqrt{ex+d}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(5/2)*sqrt(e*x + d), x)
```

Giac [F]

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^{5/2}\,dx=\int \left(cx^2+bx\right)^{\frac{5}{2}}\sqrt{ex+d}\,dx$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(5/2)*sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^{5/2}\,dx=\int {\left(c\,x^2+b\,x\right)}^{5/2}\,\sqrt{d+e\,x}\,dx$$

input
```
int((b*x + c*x^2)^(5/2)*(d + e*x)^(1/2),x)
```

output
```
int((b*x + c*x^2)^(5/2)*(d + e*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^{5/2}\,dx=\text{too large to display}$$

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x)^(5/2),x)
```

output
```
(54*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**5*d*e**4 - 36*sqrt(x)*sqrt(d +
e*x)*sqrt(b + c*x)*b**5*e**5*x - 36*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*
*4*c*d**2*e**3 - 12*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**4*c*d*e**4*x +
30*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**4*c*e**5*x**2 + 654*sqrt(x)*sqrt
(d + e*x)*sqrt(b + c*x)*b**3*c**2*d**3*e**2 - 412*sqrt(x)*sqrt(d + e*x)*sq
rt(b + c*x)*b**3*c**2*d**2*e**3*x + 386*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x
)*b**3*c**2*d*e**4*x**2 + 2226*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**3*c*
*2*e**5*x**3 - 816*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c**3*d**4*e +
108*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c**3*d**3*e**2*x - 94*sqrt(x)
*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c**3*d**2*e**3*x**2 + 2618*sqrt(x)*sqrt(
d + e*x)*sqrt(b + c*x)*b**2*c**3*d*e**4*x**3 + 3402*sqrt(x)*sqrt(d + e*x)*
sqrt(b + c*x)*b**2*c**3*e**5*x**4 + 288*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x
)*b*c**4*d**5 + 352*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**4*d**4*e*x -
290*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**4*d**3*e**2*x**2 + 252*sqrt(x
)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**4*d**2*e**3*x**3 + 3528*sqrt(x)*sqrt(d
+ e*x)*sqrt(b + c*x)*b*c**4*d*e**4*x**4 + 1386*sqrt(x)*sqrt(d + e*x)*sqrt(
b + c*x)*b*c**4*e**5*x**5 - 192*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**5*d
**5*x + 160*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**5*d**4*e*x**2 - 140*sqr
t(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**5*d**3*e**2*x**3 + 126*sqrt(x)*sqrt(d
+ e*x)*sqrt(b + c*x)*c**5*d**2*e**3*x**4 + 1386*sqrt(x)*sqrt(d + e*x)*s...
```

3.199 $\int \frac{(bx+cx^2)^{5/2}}{\sqrt{d+ex}}\,dx$

Optimal result . . . 1625
Mathematica [C] (verified) . . . 1626
Rubi [A] (verified) . . . 1627
Maple [B] (verified) . . . 1632
Fricas [A] (verification not implemented) . . . 1633
Sympy [F] . . . 1633
Maxima [F] . . . 1634
Giac [F] . . . 1634
Mupad [F(-1)] . . . 1634
Reduce [F] . . . 1635

Optimal result

Integrand size = 23, antiderivative size = 678

$$\int \frac{(bx+cx^2)^{5/2}}{\sqrt{d+ex}}\,dx = -\frac{2(2cd-be)\left(128c^4d^4-256bc^3d^3e+99b^2c^2d^2e^2+29b^3cde^3+8b^4e^4\right)x\sqrt{d+ex}}{693c^2e^6\sqrt{bx+cx^2}}$$
$$+\frac{2\left(128c^4d^4-304bc^3d^3e+195b^2c^2d^2e^2-7b^3cde^3-4b^4e^4\right)\sqrt{d+ex}\sqrt{bx+cx^2}}{693c^2e^5}$$
$$-\frac{2\left(96c^3d^3-224bc^2d^2e+139b^2cde^2-3b^3e^3\right)x\sqrt{d+ex}\sqrt{bx+cx^2}}{693ce^4}$$
$$+\frac{2\left(80c^2d^2-185bcde+113b^2e^2\right)x^2\sqrt{d+ex}\sqrt{bx+cx^2}}{693e^3}$$
$$-\frac{4c(5cd-7be)x^3\sqrt{d+ex}\sqrt{bx+cx^2}}{99e^2}+\frac{2cx^2\sqrt{d+ex}(bx+cx^2)^{3/2}}{11e}$$
$$+\frac{2\sqrt{b}(2cd-be)\left(128c^4d^4-256bc^3d^3e+99b^2c^2d^2e^2+29b^3cde^3+8b^4e^4\right)\sqrt{x}\sqrt{d+ex}E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\right.}{693c^{5/2}e^6\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$
$$-\frac{2b^{3/2}\left(128c^4d^4-304bc^3d^3e+195b^2c^2d^2e^2-7b^3cde^3-4b^4e^4\right)\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\right.}{693c^{5/2}e^5\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$

output
```
-2/693*(-b*e+2*c*d)*(8*b^4*e^4+29*b^3*c*d*e^3+99*b^2*c^2*d^2*e^2-256*b*c^3
*d^3*e+128*c^4*d^4)*x*(e*x+d)^(1/2)/c^2/e^6/(c*x^2+b*x)^(1/2)+2/693*(-4*b^
4*e^4-7*b^3*c*d*e^3+195*b^2*c^2*d^2*e^2-304*b*c^3*d^3*e+128*c^4*d^4)*(e*x+
d)^(1/2)*(c*x^2+b*x)^(1/2)/c^2/e^5-2/693*(-3*b^3*e^3+139*b^2*c*d*e^2-224*b
*c^2*d^2*e+96*c^3*d^3)*x*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/c/e^4+2/693*(113*
b^2*e^2-185*b*c*d*e+80*c^2*d^2)*x^2*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/e^3-4/
99*c*(-7*b*e+5*c*d)*x^3*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/e^2+2/11*c*x^2*(e*
x+d)^(1/2)*(c*x^2+b*x)^(3/2)/e+2/693*b^(1/2)*(-b*e+2*c*d)*(8*b^4*e^4+29*b^
3*c*d*e^3+99*b^2*c^2*d^2*e^2-256*b*c^3*d^3*e+128*c^4*d^4)*x^(1/2)*(e*x+d)^
(1/2)*EllipticE(c^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))
/c^(5/2)/e^6/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)-2/693*b^(3/2)*(
-4*b^4*e^4-7*b^3*c*d*e^3+195*b^2*c^2*d^2*e^2-304*b*c^3*d^3*e+128*c^4*d^4)*
x^(1/2)*(e*x+d)^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b
*e/c/d)^(1/2))/c^(5/2)/e^5/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 21.09 (sec) , antiderivative size = 557, normalized size of antiderivative = 0.82

$$\int \frac{(bx+cx^2)^{5/2}}{\sqrt{d+ex}}\,dx = \frac{2(x(b+cx))^{5/2}\left(bex(b+cx)(d+ex)\left(-4b^4e^4+b^3ce^3(-7d+3ex)+b^2c^2e^2(195d^2-\right.\right.}{}$$

input
```
Integrate[(b*x + c*x^2)^(5/2)/Sqrt[d + e*x],x]
```

output
```
(2*(x*(b + c*x))^(5/2)*(b*e*x*(b + c*x)*(d + e*x)*(-4*b^4*e^4 + b^3*c*e^3*
(-7*d + 3*e*x) + b^2*c^2*e^2*(195*d^2 - 139*d*e*x + 113*e^2*x^2) + b*c^3*e
*(-304*d^3 + 224*d^2*e*x - 185*d*e^2*x^2 + 161*e^3*x^3) + c^4*(128*d^4 - 9
6*d^3*e*x + 80*d^2*e^2*x^2 - 70*d*e^3*x^3 + 63*e^4*x^4)) + Sqrt[b/c]*(Sqrt
[b/c]*(-256*c^5*d^5 + 640*b*c^4*d^4*e - 454*b^2*c^3*d^3*e^2 + 41*b^3*c^2*d
^2*e^3 + 13*b^4*c*d*e^4 + 8*b^5*e^5)*(b + c*x)*(d + e*x) - I*b*e*(256*c^5*
d^5 - 640*b*c^4*d^4*e + 454*b^2*c^3*d^3*e^2 - 41*b^3*c^2*d^2*e^3 - 13*b^4*
c*d*e^4 - 8*b^5*e^5)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE
[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + I*b*e*(128*c^5*d^5 - 336*b*c
^4*d^4*e + 259*b^2*c^3*d^3*e^2 - 34*b^3*c^2*d^2*e^3 - 9*b^4*c*d*e^4 - 8*b^
5*e^5)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqr
t[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(693*b*c^2*e^6*x^3*(b + c*x)^3*Sqrt[d + e
*x])
```

Rubi [A] (verified)

Time = 1.46 (sec) , antiderivative size = 563, normalized size of antiderivative = 0.83, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {1162, 1231, 27, 1231, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx+cx^2\right)^{5/2}}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1162}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11e}-\frac{5\int \frac{(bd+(2cd-be)x)\left(cx^2+bx\right)^{3/2}}{\sqrt{d+ex}}dx}{11e}$$

$$\Big\downarrow \text{1231}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11e}-\frac{5\left(-\frac{2\int -\frac{\left(bd\left(16c^2d^2-23bced+3b^2e^2\right)+4(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)x\right)\sqrt{cx^2+bx}}{2\sqrt{d+ex}}dx}{21ce^2}-\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}\left(3b^2e^2-7cex(2cd-be)-23bcde+16\right.}{63ce^2}\right.}{11e}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11e}-\frac{5\Bigg(\frac{\int \frac{\left(bd\left(16c^2d^2-23bced+3b^2e^2\right)+4(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)x\right)\sqrt{cx^2+bx}}{\sqrt{d+ex}}dx}{21ce^2}-\frac{2\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}\left(3b^2e^2-7cex(2cd-be)-23bcde+16c^2d^2\right)}{63ce^2}}{11e}$$

$\downarrow$ 1231

$$\frac{2\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11e}-\frac{5\Bigg(\frac{-\frac{2\int -\frac{bd\left(128c^4d^4-304bc^3ed^3+195b^2c^2e^2d^2-7b^3ce^3d-4b^4e^4\right)+(2cd-be)\left(128c^4d^4-256bc^3ed^3+99b^2c^2e^2d^2+29b^3ce^3d+8b^4e^4\right)x}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{15ce^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}}{}}{21ce^2}}{}$$

$\downarrow$ 27

$$\frac{2\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11e}-\frac{5\Bigg(\frac{\frac{\int \frac{bd\left(128c^4d^4-304bc^3ed^3+195b^2c^2e^2d^2-7b^3ce^3d-4b^4e^4\right)+(2cd-be)\left(128c^4d^4-256bc^3ed^3+99b^2c^2e^2d^2+29b^3ce^3d+8b^4e^4\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{15ce^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(-4b}{}}{21ce^2}}{}$$

$\downarrow$ 1269

$$\frac{2\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11e}-\frac{5\Bigg(\frac{\frac{\frac{(2cd-be)\left(8b^4e^4+29b^3cde^3+99b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{2d(cd-be)\left(2b^4e^4+5b^3cde^3+123b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)\int \frac{}{\sqrt{d+e}}}{e}}{15ce^2}}{21ce^2}}{}$$

$\downarrow$ 1169

$$\frac{2\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11e}-\frac{5\Bigg(\frac{\frac{\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\left(8b^4e^4+29b^3cde^3+99b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\left(2b^4e^4+5b^3cde^3+123b^2c^2d^2e^2-256bc^3d^3}{e\sqrt{bx+cx^2}}}{15ce^2}}{21ce}}{}$$

$\downarrow$ 122

$$\frac{2\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11e}-$$

$$5\left(\frac{\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^4e^4+29b^3cde^3+99b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\left(2b^4e^4+5b^3cde^3+123b^2c^2d^2e^2-\right.}{e\sqrt{bx+cx^2}}}{15ce^2}\right.$$

$\downarrow$ 120

$$\frac{2\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11e}-$$

$$5\left(\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^4e^4+29b^3cde^3+99b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\left(2b^4e^4+5b^3cde^3+1\right.}{}}{15ce^2}\right.$$

$\downarrow$ 127

$$\frac{2\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11e}-$$

$$5\left(\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^4e^4+29b^3cde^3+99b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(2b^4e^4+5b\right.}{}}{15ce^2}\right.$$

$\downarrow$ 126

$$\frac{2\left(bx+cx^2\right)^{5/2}\sqrt{d+ex}}{11e}-$$

$$5\left(\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^4e^4+29b^3cde^3+99b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(2b^4e\right.}{}}{15ce^2}\right.$$

input
```
Int[(b*x + c*x^2)^(5/2)/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(b*x + c*x^2)^(5/2))/(11*e) - (5*((-2*Sqrt[d + e*x]*(16*c
^2*d^2 - 23*b*c*d*e + 3*b^2*e^2 - 7*c*e*(2*c*d - b*e)*x)*(b*x + c*x^2)^(3/
2))/(63*c*e^2) + ((-2*Sqrt[d + e*x]*(128*c^4*d^4 - 304*b*c^3*d^3*e + 195*b
^2*c^2*d^2*e^2 - 7*b^3*c*d*e^3 - 4*b^4*e^4 - 12*c*e*(2*c*d - b*e)*(4*c^2*d
^2 - 4*b*c*d*e - b^2*e^2)*x)*Sqrt[b*x + c*x^2])/(15*c*e^2) + ((2*Sqrt[-b]*
(2*c*d - b*e)*(128*c^4*d^4 - 256*b*c^3*d^3*e + 99*b^2*c^2*d^2*e^2 + 29*b^3
*c*d*e^3 + 8*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[Ar
cSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/
d]*Sqrt[b*x + c*x^2]) - (4*Sqrt[-b]*d*(c*d - b*e)*(128*c^4*d^4 - 256*b*c^3
*d^3*e + 123*b^2*c^2*d^2*e^2 + 5*b^3*c*d*e^3 + 2*b^4*e^4)*Sqrt[x]*Sqrt[1 +
 (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]],
(b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]))/(15*c*e^2))/(21
*c*e^2)))/(11*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1162
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p/(e*(m + 2*p + 1))   Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d -
b*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x
] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &&
!ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1441 vs. $2(615) = 1230$.

Time = 2.97 (sec) , antiderivative size = 1442, normalized size of antiderivative = 2.13

method	result	size
default	Expression too large to display	1442
elliptic	Expression too large to display	1442

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/693*(x*(c*x+b))^(1/2)*(e*x+d)^(1/2)*(63*c^6*e^7*x^7-8*((e*x+d)/d)^(1/2)*
(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(-d
*c/(b*e-c*d))^(1/2))*b^6*d*e^6+8*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(
1/2)*(-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^
6*d*e^6-9*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*Ell
ipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^5*c*d^2*e^5-34*((e*x+d)
/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+d)/d)
^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^4*c^2*d^3*e^4+259*((e*x+d)/d)^(1/2)*(e*(c
*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b
*e-c*d))^(1/2))*b^3*c^3*d^4*e^3-336*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d)
)^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))
*b^2*c^4*d^5*e^2+128*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d
)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b*c^5*d^6*e+5*
((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticE(((e
*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^5*c*d^2*e^5+28*((e*x+d)/d)^(1/2)*
(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d
*c/(b*e-c*d))^(1/2))*b^4*c^2*d^3*e^4+1094*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*
e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^
(1/2))*b^2*c^4*d^5*e^2-896*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(
-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b*c^5...
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 640, normalized size of antiderivative = 0.94

$$\int \frac{(bx+cx^2)^{5/2}}{\sqrt{d+ex}}\,dx = \frac{2\left((256\,c^6d^6-768\,bc^5d^5e+726\,b^2c^4d^4e^2-172\,b^3c^3d^3e^3-33\,b^4c^2d^2e^4-9\,b^5cde^5-8\right.}{}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
2/2079*((256*c^6*d^6 - 768*b*c^5*d^5*e + 726*b^2*c^4*d^4*e^2 - 172*b^3*c^3
*d^3*e^3 - 33*b^4*c^2*d^2*e^4 - 9*b^5*c*d*e^5 - 8*b^6*e^6)*sqrt(c*e)*weier
strassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d
^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x +
c*d + b*e)/(c*e)) + 3*(256*c^6*d^5*e - 640*b*c^5*d^4*e^2 + 454*b^2*c^4*d^3
*e^3 - 41*b^3*c^3*d^2*e^4 - 13*b^4*c^2*d*e^5 - 8*b^5*c*e^6)*sqrt(c*e)*weie
rstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3
- 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInvers
e(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*
d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c
*e))) + 3*(63*c^6*e^6*x^4 + 128*c^6*d^4*e^2 - 304*b*c^5*d^3*e^3 + 195*b^2*
c^4*d^2*e^4 - 7*b^3*c^3*d*e^5 - 4*b^4*c^2*e^6 - 7*(10*c^6*d*e^5 - 23*b*c^5
*e^6)*x^3 + (80*c^6*d^2*e^4 - 185*b*c^5*d*e^5 + 113*b^2*c^4*e^6)*x^2 - (96
*c^6*d^3*e^3 - 224*b*c^5*d^2*e^4 + 139*b^2*c^4*d*e^5 - 3*b^3*c^3*e^6)*x)*s
qrt(c*x^2 + b*x)*sqrt(e*x + d))/(c^4*e^7)
```

Sympy [F]

$$\int \frac{(bx+cx^2)^{5/2}}{\sqrt{d+ex}}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}}{\sqrt{d+ex}}\,dx$$

input
```
integrate((c*x**2+b*x)**(5/2)/(e*x+d)**(1/2),x)
```

output
```
Integral((x*(b + c*x))**(5/2)/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{(bx+cx^2)^{5/2}}{\sqrt{d+ex}}\,dx = \int \frac{(cx^2+bx)^{\frac{5}{2}}}{\sqrt{ex+d}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(5/2)/sqrt(e*x + d), x)
```

Giac [F]

$$\int \frac{(bx+cx^2)^{5/2}}{\sqrt{d+ex}}\,dx = \int \frac{(cx^2+bx)^{\frac{5}{2}}}{\sqrt{ex+d}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(5/2)/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{5/2}}{\sqrt{d+ex}}\,dx = \int \frac{(c\,x^2+b\,x)^{5/2}}{\sqrt{d+e\,x}}\,dx$$

input
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^(1/2),x)
```

output
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^{5/2}}{\sqrt{d + ex}}\,dx = \text{too large to display}$$

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^(1/2),x)
```

output
```
( - 18*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**4*d*e**3 + 12*sqrt(x)*sqrt(d
 + e*x)*sqrt(b + c*x)*b**4*e**4*x + 834*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x
)*b**3*c*d**2*e**2 - 544*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**3*c*d*e**3
*x + 452*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**3*c*e**4*x**2 - 1344*sqrt(
x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c**2*d**3*e + 340*sqrt(x)*sqrt(d + e*x
)*sqrt(b + c*x)*b**2*c**2*d**2*e**2*x - 288*sqrt(x)*sqrt(d + e*x)*sqrt(b +
 c*x)*b**2*c**2*d*e**3*x**2 + 644*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2
*c**2*e**4*x**3 + 576*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**3*d**4 + 51
2*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**3*d**3*e*x - 420*sqrt(x)*sqrt(d
 + e*x)*sqrt(b + c*x)*b*c**3*d**2*e**2*x**2 + 364*sqrt(x)*sqrt(d + e*x)*sq
rt(b + c*x)*b*c**3*d*e**3*x**3 + 252*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b
*c**3*e**4*x**4 - 384*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**4*d**4*x + 32
0*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**4*d**3*e*x**2 - 280*sqrt(x)*sqrt(
d + e*x)*sqrt(b + c*x)*c**4*d**2*e**2*x**3 + 252*sqrt(x)*sqrt(d + e*x)*sqr
t(b + c*x)*c**4*d*e**3*x**4 - 24*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*
x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*x**2 + c**2
*d**2*x + c**2*d*e*x**2),x)*b**6*e**6 - 63*int((sqrt(x)*sqrt(d + e*x)*sqrt
(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*x
**2 + c**2*d**2*x + c**2*d*e*x**2),x)*b**5*c*d*e**5 - 162*int((sqrt(x)*sqr
t(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*...
```

3.200 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{3/2}}\,dx$

Optimal result . . . 1636
Mathematica [C] (verified) . . . 1637
Rubi [A] (verified) . . . 1638
Maple [B] (verified) . . . 1643
Fricas [A] (verification not implemented) . . . 1644
Sympy [F] . . . 1644
Maxima [F] . . . 1645
Giac [F] . . . 1645
Mupad [F(-1)] . . . 1645
Reduce [F] . . . 1646

Optimal result

Integrand size = 23, antiderivative size = 580

$$\begin{aligned}\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{3/2}}\,dx &= \frac{4(128c^4d^4-256bc^3d^3e+135b^2c^2d^2e^2-7b^3cde^3-b^4e^4)\,x\sqrt{d+ex}}{63ce^6\sqrt{bx+cx^2}} \\
&- \frac{2(128c^3d^3-240bc^2d^2e+111b^2cde^2-b^3e^3)\,\sqrt{d+ex}\sqrt{bx+cx^2}}{63ce^5} \\
&+ \frac{4(12cd-13be)(4cd-3be)x\sqrt{d+ex}\sqrt{bx+cx^2}}{63e^4} \\
&- \frac{10c(16cd-15be)x^2\sqrt{d+ex}\sqrt{bx+cx^2}}{63e^3} + \frac{20cx\sqrt{d+ex}(bx+cx^2)^{3/2}}{9e^2} - \frac{2(bx+cx^2)^{5/2}}{e\sqrt{d+ex}} \\
&- \frac{4\sqrt{b}(128c^4d^4-256bc^3d^3e+135b^2c^2d^2e^2-7b^3cde^3-b^4e^4)\,\sqrt{x}\sqrt{d+ex}E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{63c^{3/2}e^6\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} \\
&+ \frac{2b^{3/2}(128c^3d^3-240bc^2d^2e+111b^2cde^2-b^3e^3)\,\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{63c^{3/2}e^5\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}\end{aligned}$$

output
```
4/63*(-b^4*e^4-7*b^3*c*d*e^3+135*b^2*c^2*d^2*e^2-256*b*c^3*d^3*e+128*c^4*d
^4)*x*(e*x+d)^(1/2)/c/e^6/(c*x^2+b*x)^(1/2)-2/63*(-b^3*e^3+111*b^2*c*d*e^2
-240*b*c^2*d^2*e+128*c^3*d^3)*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/c/e^5+4/63*(
-13*b*e+12*c*d)*(-3*b*e+4*c*d)*x*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/e^4-10/63
*c*(-15*b*e+16*c*d)*x^2*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/e^3+20/9*c*x*(e*x+
d)^(1/2)*(c*x^2+b*x)^(3/2)/e^2-2*(c*x^2+b*x)^(5/2)/e/(e*x+d)^(1/2)-4/63*b^
(1/2)*(-b^4*e^4-7*b^3*c*d*e^3+135*b^2*c^2*d^2*e^2-256*b*c^3*d^3*e+128*c^4*
d^4)*x^(1/2)*(e*x+d)^(1/2)*EllipticE(c^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/b)^(1/
2),(1-b*e/c/d)^(1/2))/c^(3/2)/e^6/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^
(1/2)+2/63*b^(3/2)*(-b^3*e^3+111*b^2*c*d*e^2-240*b*c^2*d^2*e+128*c^3*d^3)*
x^(1/2)*(e*x+d)^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b
*e/c/d)^(1/2))/c^(3/2)/e^5/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.89 (sec) , antiderivative size = 498, normalized size of antiderivative = 0.86

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{3/2}}\,dx = \frac{2(x(b+cx))^{5/2}\left(\frac{2(128c^4d^4-256bc^3d^3e+135b^2c^2d^2e^2-7b^3cde^3-b^4e^4)(b+cx)(d+ex)}{c\sqrt{x}} - e\sqrt{x}(b+cx)\,(\cdots\right.}{\cdots}$$

input
```
Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^(3/2),x]
```

output
```
(2*(x*(b + c*x))^(5/2)*((2*(128*c^4*d^4 - 256*b*c^3*d^3*e + 135*b^2*c^2*d^
2*e^2 - 7*b^3*c*d*e^3 - b^4*e^4)*(b + c*x)*(d + e*x))/(c*Sqrt[x]) - e*Sqrt
[x]*(b + c*x)*(-(b^3*e^3*(d + e*x)) + 3*b^2*c*e^2*(37*d^2 + 11*d*e*x - 5*e
^2*x^2) - b*c^2*e*(240*d^3 + 64*d^2*e*x - 31*d*e^2*x^2 + 19*e^3*x^3) + c^3
*(128*d^4 + 32*d^3*e*x - 16*d^2*e^2*x^2 + 10*d*e^3*x^3 - 7*e^4*x^4)) - (2*
I)*Sqrt[b/c]*e*(-128*c^4*d^4 + 256*b*c^3*d^3*e - 135*b^2*c^2*d^2*e^2 + 7*b
^3*c*d*e^3 + b^4*e^4)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x*EllipticE[I*Ar
cSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + I*Sqrt[b/c]*e*(-128*c^4*d^4 + 272
*b*c^3*d^3*e - 159*b^2*c^2*d^2*e^2 + 13*b^3*c*d*e^3 + 2*b^4*e^4)*Sqrt[1 +
b/(c*x)]*Sqrt[1 + d/(e*x)]*x*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)
/(b*e)]))/(63*c*e^6*x^(5/2)*(b + c*x)^3*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 478, normalized size of antiderivative = 0.82, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {1161, 1231, 27, 1231, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{3/2}}dx$$

$$\Big\downarrow \ 1161$$

$$\frac{5\int \frac{(b+2cx)(cx^2+bx)^{3/2}}{\sqrt{d+ex}}dx}{e}-\frac{2(bx+cx^2)^{5/2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \ 1231$$

$$\frac{5\left(-\frac{2\int -\frac{c\left(bd(16cd-15be)+\left(32c^2d^2-32bced+b^2e^2\right)x\right)\sqrt{cx^2+bx}}{2\sqrt{d+ex}}dx}{21ce^2}-\frac{2(bx+cx^2)^{3/2}\sqrt{d+ex}(-15be+16cd-14cex)}{63e^2}\right)}{e}-\frac{2(bx+cx^2)^{5/2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \ 27$$

$$\frac{5\left(\frac{\int \frac{\left(bd(16cd-15be)+\left(32c^2d^2-32bced+b^2e^2\right)x\right)\sqrt{cx^2+bx}}{\sqrt{d+ex}}dx}{21e^2}-\frac{2(bx+cx^2)^{3/2}\sqrt{d+ex}(-15be+16cd-14cex)}{63e^2}\right)}{e}-\frac{2(bx+cx^2)^{5/2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \ 1231$$

$$\frac{5\left(\frac{-\frac{2\int -\frac{bd\left(128c^3d^3-240bc^2ed^2+111b^2ce^2d-b^3e^3\right)+2\left(128c^4d^4-256bc^3ed^3+135b^2c^2e^2d^2-7b^3ce^3d-b^4e^4\right)x}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{15ce^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-b^3e^3-3cex\left(b^2e^2-3\cdots\right.\right.}{\cdots}}{21e^2}\cdots\right)}{e}-\frac{2(bx+cx^2)^{5/2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \; 27$$

$$5\left(\frac{\frac{\int \frac{bd\left(128c^3d^3-240bc^2ed^2+111b^2ce^2d-b^3e^3\right)+2\left(128c^4d^4-256bc^3ed^3+135b^2c^2e^2d^2-7b^3ce^3d-b^4e^4\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{15ce^2}}{21e^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(-b^3e^3-3cex\left(b^2e^2-32bcde\cdots\right.\right.}{15c\cdots}\cdots\right.$$

$$\cdots e$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \; 1269$$

$$5\left(\frac{\frac{\frac{2\left(-b^4e^4-7b^3cde^3+135b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{d(cd-be)(2cd-be)\left(-b^2e^2-128bcde+128c^2d^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}}{15ce^2}}{21e^2}-\frac{2\sqrt{bx+c\cdots}}{\cdots}\cdots\right.$$

$$\cdots e$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \; 1169$$

$$5\left(\frac{\frac{\frac{2\sqrt{x}\sqrt{b+cx}\left(-b^4e^4-7b^3cde^3+135b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\left(-b^2e^2-128bcde+128c^2d^2\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\cdots}\cdots}{e\sqrt{bx+cx^2}}}{15ce^2}}{21e^2}\cdots\right.$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \; 122$$

$$5\left(\frac{\frac{\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-b^4e^4-7b^3cde^3+135b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\left(-b^2e^2-128bcde+128c^2d^2\right)\int \frac{\cdots}{\sqrt{\cdots}}\cdots}{e\sqrt{bx+cx^2}}}{15ce^2}}{21e^2}\cdots\right.$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{e\sqrt{d+ex}}$$

$$\Big\downarrow \; 120$$

$$\frac{5\left(\frac{\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-b^4e^4-7b^3cde^3+135b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\left(-b^2e^2-128bcde+1\cdots\right.}{e\sqrt{bx+cx^2}\cdots}}{15ce^2}\cdots}{21e^2}\cdots\right.}{\cdots}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{e\sqrt{d+ex}}$$

$\Big\downarrow$ 127

$$\frac{5\left(\frac{\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-b^4e^4-7b^3cde^3+135b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\left(-b^2e^2-1\cdots\right.}{e\sqrt{bx+cx^2}\sqrt{\cdots}}}{15ce^2}\cdots}{21e^2}\cdots\right.}{\cdots}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{e\sqrt{d+ex}}$$

$\Big\downarrow$ 126

$$\frac{5\left(\frac{\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-b^4e^4-7b^3cde^3+135b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\left(-b^2\cdots\right.}{\sqrt{c}e\sqrt{b\cdots}}}{15ce^2}\cdots}{21e^2}\cdots\right.}{\cdots}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{e\sqrt{d+ex}}$$

input
```
Int[(b*x + c*x^2)^(5/2)/(d + e*x)^(3/2),x]
```

output
```
(-2*(b*x + c*x^2)^(5/2))/(e*Sqrt[d + e*x]) + (5*((-2*Sqrt[d + e*x]*(16*c*d
 - 15*b*e - 14*c*e*x)*(b*x + c*x^2)^(3/2))/(63*e^2) + ((-2*Sqrt[d + e*x]*(
128*c^3*d^3 - 240*b*c^2*d^2*e + 111*b^2*c*d*e^2 - b^3*e^3 - 3*c*e*(32*c^2*
d^2 - 32*b*c*d*e + b^2*e^2)*x)*Sqrt[b*x + c*x^2])/(15*c*e^2) + ((4*Sqrt[-b
]*(128*c^4*d^4 - 256*b*c^3*d^3*e + 135*b^2*c^2*d^2*e^2 - 7*b^3*c*d*e^3 - b
^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*
Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x +
c*x^2]) - (2*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*(128*c^2*d^2 - 128*b*c*d
*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin
[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[
b*x + c*x^2]))/(15*c*e^2))/(21*e^2)))/e
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1161

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1169

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1231

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1170 vs. 2(519) = 1038.

Time = 3.03 (sec) , antiderivative size = 1171, normalized size of antiderivative = 2.02

method	result	size
default	Expression too large to display	1171
elliptic	Expression too large to display	1532

input

```
int((c*x^2+b*x)^(5/2)/(e*x+d)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
2/63*(x*(c*x+b))^(1/2)*(e*x+d)^(1/2)*(13*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e
-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(
1/2))*b^4*c*d^2*e^4-159*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*
x/d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^3*c^2*d^3
*e^3+272*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*Elli
pticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^2*c^3*d^4*e^2-128*((e*x+
d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+d)/
d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b*c^4*d^5*e-12*((e*x+d)/d)^(1/2)*(e*(c*x+
b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-
c*d))^(1/2))*b^4*c*d^2*e^4+284*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/
2)*(-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^3*
c^2*d^3*e^3-782*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/
2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^2*c^3*d^4*e^2+768
*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticE(((
e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b*c^4*d^5*e+2*((e*x+d)/d)^(1/2)*(e
*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(-d*c
/(b*e-c*d))^(1/2))*b^5*d*e^5-2*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/
2)*(-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^5*
d*e^5+240*b^2*c^3*d^3*e^3*x-256*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1
/2)*(-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*...
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 750, normalized size of antiderivative = 1.29

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
-2/189*((256*c^5*d^6 - 640*b*c^4*d^5*e + 478*b^2*c^3*d^4*e^2 - 77*b^3*c^2*
d^3*e^3 - 13*b^4*c*d^2*e^4 - 2*b^5*d*e^5 + (256*c^5*d^5*e - 640*b*c^4*d^4*
e^2 + 478*b^2*c^3*d^3*e^3 - 77*b^3*c^2*d^2*e^4 - 13*b^4*c*d*e^5 - 2*b^5*e^
6)*x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2
*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*
e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 6*(128*c^5*d^5*e - 256*b*c^4*d^4*
e^2 + 135*b^2*c^3*d^3*e^3 - 7*b^3*c^2*d^2*e^4 - b^4*c*d*e^5 + (128*c^5*d^4
*e^2 - 256*b*c^4*d^3*e^3 + 135*b^2*c^3*d^2*e^4 - 7*b^3*c^2*d*e^5 - b^4*c*e
^6)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^
2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3
), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*
(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*
c*e*x + c*d + b*e)/(c*e))) - 3*(7*c^5*e^6*x^4 - 128*c^5*d^4*e^2 + 240*b*c^
4*d^3*e^3 - 111*b^2*c^3*d^2*e^4 + b^3*c^2*d*e^5 - (10*c^5*d*e^5 - 19*b*c^4
*e^6)*x^3 + (16*c^5*d^2*e^4 - 31*b*c^4*d*e^5 + 15*b^2*c^3*e^6)*x^2 - (32*c
^5*d^3*e^3 - 64*b*c^4*d^2*e^4 + 33*b^2*c^3*d*e^5 - b^3*c^2*e^6)*x)*sqrt(c*
x^2 + b*x)*sqrt(e*x + d))/(c^3*e^8*x + c^3*d*e^7)
```

Sympy [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{3/2}}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x)**(5/2)/(e*x+d)**(3/2),x)
```

output
```
Integral((x*(b + c*x))**(5/2)/(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{3/2}}\,dx = \int \frac{(cx^2+bx)^{\frac{5}{2}}}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(3/2), x)
```

Giac [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{3/2}}\,dx = \int \frac{(cx^2+bx)^{\frac{5}{2}}}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{3/2}}\,dx = \int \frac{(c\,x^2+b\,x)^{5/2}}{(d+e\,x)^{3/2}}\,dx$$

input
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^(3/2),x)
```

output
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^(3/2),x)
```

output
```
( - 6*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**4*e**3 + 198*sqrt(x)*sqrt(d +
 e*x)*sqrt(b + c*x)*b**3*c*d*e**2 + 4*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*
b**3*c*e**3*x - 384*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c**2*d**2*e -
 132*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c**2*d*e**2*x + 60*sqrt(x)*s
qrt(d + e*x)*sqrt(b + c*x)*b**2*c**2*e**3*x**2 + 192*sqrt(x)*sqrt(d + e*x)
*sqrt(b + c*x)*b*c**3*d**3 + 256*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**
3*d**2*e*x - 124*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**3*d*e**2*x**2 +
76*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**3*e**3*x**3 - 128*sqrt(x)*sqrt
(d + e*x)*sqrt(b + c*x)*c**4*d**3*x + 64*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*
x)*c**4*d**2*e*x**2 - 40*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**4*d*e**2*x
**3 + 28*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**4*e**3*x**4 + 3*int((sqrt(
d + e*x)*sqrt(b + c*x))/(sqrt(x)*b*d**2 + 2*sqrt(x)*b*d*e*x + sqrt(x)*b*e*
*2*x**2 + sqrt(x)*c*d**2*x + 2*sqrt(x)*c*d*e*x**2 + sqrt(x)*c*e**2*x**3),x
)*b**5*d**2*e**3 + 3*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b*d**2 + 2
*sqrt(x)*b*d*e*x + sqrt(x)*b*e**2*x**2 + sqrt(x)*c*d**2*x + 2*sqrt(x)*c*d*
e*x**2 + sqrt(x)*c*e**2*x**3),x)*b**5*d*e**4*x - 99*int((sqrt(d + e*x)*sqr
t(b + c*x))/(sqrt(x)*b*d**2 + 2*sqrt(x)*b*d*e*x + sqrt(x)*b*e**2*x**2 + sq
rt(x)*c*d**2*x + 2*sqrt(x)*c*d*e*x**2 + sqrt(x)*c*e**2*x**3),x)*b**4*c*d**
3*e**2 - 99*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b*d**2 + 2*sqrt(x)*
b*d*e*x + sqrt(x)*b*e**2*x**2 + sqrt(x)*c*d**2*x + 2*sqrt(x)*c*d*e*x**2...
```

3.201 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{5/2}}\,dx$

Optimal result . . . 1647
Mathematica [C] (verified) . . . 1648
Rubi [A] (verified) . . . 1649
Maple [B] (verified) . . . 1654
Fricas [B] (verification not implemented) . . . 1655
Sympy [F] . . . 1656
Maxima [F] . . . 1657
Giac [F] . . . 1657
Mupad [F(-1)] . . . 1657
Reduce [F] . . . 1658

Optimal result

Integrand size = 23, antiderivative size = 455

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{5/2}}\,dx = -\frac{2(cd-be)\left(128c^2d^2-80bcde+3b^2e^2\right)\sqrt{bx+cx^2}}{21ce^5\sqrt{d+ex}} + \frac{32(cd-be)(2cd-be)x\sqrt{bx+cx^2}}{21e^4\sqrt{d+ex}} - \frac{2c(16cd-13be)x^2\sqrt{bx+cx^2}}{21e^3\sqrt{d+ex}} + \frac{20cx(bx+cx^2)^{3/2}}{21e^2\sqrt{d+ex}} - \frac{2(bx+cx^2)^{5/2}}{3e(d+ex)^{3/2}} + \frac{2\sqrt{d}(2cd-be)\left(128c^2d^2-128bcde+3b^2e^2\right)\sqrt{bx+cx^2}\,E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{21ce^{11/2}\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}} - \frac{2d^{3/2}\left(128c^2d^2-176bcde+51b^2e^2\right)\sqrt{bx+cx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{21e^{11/2}\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}$$

output
```
-2/21*(-b*e+c*d)*(3*b^2*e^2-80*b*c*d*e+128*c^2*d^2)*(c*x^2+b*x)^(1/2)/c/e^
5/(e*x+d)^(1/2)+32/21*(-b*e+c*d)*(-b*e+2*c*d)*x*(c*x^2+b*x)^(1/2)/e^4/(e*x
+d)^(1/2)-2/21*c*(-13*b*e+16*c*d)*x^2*(c*x^2+b*x)^(1/2)/e^3/(e*x+d)^(1/2)+
20/21*c*x*(c*x^2+b*x)^(3/2)/e^2/(e*x+d)^(1/2)-2/3*(c*x^2+b*x)^(5/2)/e/(e*x
+d)^(3/2)+2/21*d^(1/2)*(-b*e+2*c*d)*(3*b^2*e^2-128*b*c*d*e+128*c^2*d^2)*(c
*x^2+b*x)^(1/2)*EllipticE(e^(1/2)*x^(1/2)/d^(1/2)/(1+e*x/d)^(1/2),(1-c*d/b
/e)^(1/2))/c/e^(11/2)/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)-2/
21*d^(3/2)*(51*b^2*e^2-176*b*c*d*e+128*c^2*d^2)*(c*x^2+b*x)^(1/2)*InverseJ
acobiAM(arctan(e^(1/2)*x^(1/2)/d^(1/2)),(1-c*d/b/e)^(1/2))/e^(11/2)/x^(1/2
)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 23.72 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.97

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \frac{2(x(b+cx))^{5/2}\left(-\frac{(256c^3d^3-384bc^2d^2e+134b^2cde^2-3b^3e^3)(b+cx)(d+ex)}{c\sqrt{x}}+\frac{e\sqrt{x}(b+cx)(b^2e^2(51d^2+67d}{}\right.}{}$$

input
```
Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

output
```
(2*(x*(b + c*x))^(5/2)*(-(((256*c^3*d^3 - 384*b*c^2*d^2*e + 134*b^2*c*d*e^
2 - 3*b^3*e^3)*(b + c*x)*(d + e*x))/(c*Sqrt[x])) + (e*Sqrt[x]*(b + c*x)*(b
^2*e^2*(51*d^2 + 67*d*e*x + 9*e^2*x^2) + b*c*e*(-176*d^3 - 224*d^2*e*x - 2
5*d*e^2*x^2 + 9*e^3*x^3) + c^2*(128*d^4 + 160*d^3*e*x + 16*d^2*e^2*x^2 - 6
*d*e^3*x^3 + 3*e^4*x^4)))/(d + e*x) + I*Sqrt[b/c]*e*(-256*c^3*d^3 + 384*b*
c^2*d^2*e - 134*b^2*c*d*e^2 + 3*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x
)]*x*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + I*Sqrt[b/c]*e*
(128*c^3*d^3 - 208*b*c^2*d^2*e + 83*b^2*c*d*e^2 - 3*b^3*e^3)*Sqrt[1 + b/(c
*x)]*Sqrt[1 + d/(e*x)]*x*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*
e)]))/(21*e^6*x^(5/2)*(b + c*x)^3*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 421, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {1161, 1230, 27, 1231, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1161}$$

$$\frac{5\int \frac{(b+2cx)(cx^2+bx)^{3/2}}{(d+ex)^{3/2}}dx}{3e}-\frac{2(bx+cx^2)^{5/2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1230}$$

$$\frac{5\left(\frac{2(bx+cx^2)^{3/2}(-7be+16cd+2cex)}{7e^2\sqrt{d+ex}}-\frac{6\int \frac{(b(16cd-7be)+16c(2cd-be)x)\sqrt{cx^2+bx}}{2\sqrt{d+ex}}dx}{7e^2}\right)}{3e}-\frac{2(bx+cx^2)^{5/2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\left(\frac{2(bx+cx^2)^{3/2}(-7be+16cd+2cex)}{7e^2\sqrt{d+ex}}-\frac{3\int \frac{(b(16cd-7be)+16c(2cd-be)x)\sqrt{cx^2+bx}}{\sqrt{d+ex}}dx}{7e^2}\right)}{3e}-\frac{2(bx+cx^2)^{5/2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{1231}$$

$$5\left(\frac{2(bx+cx^2)^{3/2}(-7be+16cd+2cex)}{7e^2\sqrt{d+ex}}-\frac{3\left(-\frac{2\int -\frac{c\left(bd\left(128c^2d^2-176bced+51b^2e^2\right)+(2cd-be)\left(128c^2d^2-128bced+3b^2e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{15ce^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(51}{}\right.}{7e^2}\right.$$

$$3e$$

$$\frac{2(bx+cx^2)^{5/2}}{3e(d+ex)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\left(\frac{2\left(bx+cx^2\right)^{3/2}(-7be+16cd+2cex)}{7e^2\sqrt{d+ex}}-\frac{3\left(\frac{\int \frac{bd\left(128c^2d^2-176bced+51b^2e^2\right)+(2cd-be)\left(128c^2d^2-128bced+3b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{15e^2}-\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}\left(51b^2e^2-48\cdots\right.}{1\cdots}\cdots\right.}{7e^2}\cdots\right.}{3e}\cdots$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}}$$

$\Big\downarrow$ 1269

$$\frac{5\left(\frac{2\left(bx+cx^2\right)^{3/2}(-7be+16cd+2cex)}{7e^2\sqrt{d+ex}}-\frac{3\left(\frac{\frac{(2cd-be)\left(3b^2e^2-128bcde+128c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{2d(cd-be)\left(27b^2e^2-128bcde+128c^2d^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2\cdots}}\cdots}{e}}{15e^2}\cdots\right.}{7e^2}\cdots\right.}{3e}\cdots$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}}$$

$\Big\downarrow$ 1169

$$\frac{5\left(\frac{2\left(bx+cx^2\right)^{3/2}(-7be+16cd+2cex)}{7e^2\sqrt{d+ex}}-\frac{3\left(\frac{\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\left(3b^2e^2-128bcde+128c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\left(27b^2e^2-128bcde+128\cdots\right.}{e\sqrt{bx+cx^2}}}{15e^2}\cdots\right.}{7e^2}\cdots\right.}{3e}\cdots$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}}$$

$\Big\downarrow$ 122

$$\frac{5\left(\frac{2\left(bx+cx^2\right)^{3/2}(-7be+16cd+2cex)}{7e^2\sqrt{d+ex}}-\frac{3\left(\frac{\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(3b^2e^2-128bcde+128c^2d^2\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\left(27b^2e^2-128b\ldots\right.}{e\sqrt{b\ldots}}}{15e^2}\ldots\right)}{7e^2}\right)}{3e}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}}$$

$\Big\downarrow$ 120

$$\frac{5\left(\frac{2\left(bx+cx^2\right)^{3/2}(-7be+16cd+2cex)}{7e^2\sqrt{d+ex}}-\frac{3\left(\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(3b^2e^2-128bcde+128c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\ldots}{\ldots}}{15e^2}\ldots\right)}{\ldots}\right)}{3e}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}}$$

$\Big\downarrow$ 127

$$\frac{5\left(\frac{2\left(bx+cx^2\right)^{3/2}(-7be+16cd+2cex)}{7e^2\sqrt{d+ex}}-\frac{3\left(\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(3b^2e^2-128bcde+128c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+\ldots}}{\ldots}}{15e^2}\ldots\right)}{\ldots}\right)}{3e}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}}$$

$\Big\downarrow$ 126

$$\frac{5\left(\frac{2\left(bx+cx^2\right)^{3/2}(-7be+16cd+2cex)}{7e^2\sqrt{d+ex}}-\frac{3\left(\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(3b^2e^2-128bcde+128c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\cdots}}{\cdots}}{15e^2}\cdots\right)}{\cdots}\right)}{3e}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{3e(d+ex)^{3/2}}$$

input
```
Int[(b*x + c*x^2)^(5/2)/(d + e*x)^(5/2),x]
```

output
```
(-2*(b*x + c*x^2)^(5/2))/(3*e*(d + e*x)^(3/2)) + (5*((2*(16*c*d - 7*b*e +
2*c*e*x)*(b*x + c*x^2)^(3/2))/(7*e^2*Sqrt[d + e*x]) - (3*((-2*Sqrt[d + e*x
]*(128*c^2*d^2 - 176*b*c*d*e + 51*b^2*e^2 - 48*c*e*(2*c*d - b*e)*x)*Sqrt[b
*x + c*x^2])/(15*e^2) + ((2*Sqrt[-b]*(2*c*d - b*e)*(128*c^2*d^2 - 128*b*c*
d*e + 3*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[
(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sq
rt[b*x + c*x^2]) - (4*Sqrt[-b]*d*(c*d - b*e)*(128*c^2*d^2 - 128*b*c*d*e +
27*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(
Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*
x + c*x^2]))/(15*e^2)))/(7*e^2)))/(3*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1231
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Simp[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*
a*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*
c*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c
^2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x
] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !R
ationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Integer
Q[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(398) = 796$.

Time = 3.10 (sec) , antiderivative size = 1135, normalized size of antiderivative = 2.49

method	result	size
elliptic	Expression too large to display	1135
default	Expression too large to display	1688

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)*x*(e*x+d))^(1/2)/x/(c*x+b)*(-2/
3*d^2*(b^2*e^2-2*b*c*d*e+c^2*d^2)/e^7*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2
)/(x+d/e)^2+14/3*(c*e*x^2+b*e*x)*d*(b^2*e^2-3*b*c*d*e+2*c^2*d^2)/e^6/((x+d
/e)*(c*e*x^2+b*e*x))^(1/2)+2/7*c^2/e^3*x^2*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)
^(1/2)+2/5*(c^2/e^3*(3*b*e-2*c*d)-2/7*c^2/e^3*(3*b*e+3*c*d))/c/e*x*(c*e*x^
3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2/3*(3*c/e^4*(b^2*e^2-2*b*c*d*e+c^2*d^2)-5/
7*c^2/e^3*b*d-2/5*(c^2/e^3*(3*b*e-2*c*d)-2/7*c^2/e^3*(3*b*e+3*c*d))/c/e*(2
*b*e+2*c*d))/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2*(-d*(2*b^3*e^3-9*
b^2*c*d*e^2+12*b*c^2*d^2*e-5*c^3*d^3)/e^6-1/3*d^2*(b^2*e^2-2*b*c*d*e+c^2*d
^2)/e^6*c+7/3*d*(b^2*e^2-3*b*c*d*e+2*c^2*d^2)/e^6*(b*e-c*d)-7/3*b/e^5*d*(b
^2*e^2-3*b*c*d*e+2*c^2*d^2)-1/3*(3*c/e^4*(b^2*e^2-2*b*c*d*e+c^2*d^2)-5/7*c
^2/e^3*b*d-2/5*(c^2/e^3*(3*b*e-2*c*d)-2/7*c^2/e^3*(3*b*e+3*c*d))/c/e*(2*b*
e+2*c*d))/c/e*b*d)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*
x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e)/d*e)^(
1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(1/e^5*(b^3*e^3-6*b^2*c*d*e^2+9*b*c^2*d^2*
e-4*c^3*d^3)-7/3*d*(b^2*e^2-3*b*c*d*e+2*c^2*d^2)/e^5*c-3/5*(c^2/e^3*(3*b*e
-2*c*d)-2/7*c^2/e^3*(3*b*e+3*c*d))/c/e*b*d-2/3*(3*c/e^4*(b^2*e^2-2*b*c*d*e
+c^2*d^2)-5/7*c^2/e^3*b*d-2/5*(c^2/e^3*(3*b*e-2*c*d)-2/7*c^2/e^3*(3*b*e+3*
c*d))/c/e*(2*b*e+2*c*d))/c/e*(b*e+c*d))*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(
-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(398) = 796$.

Time = 0.19 (sec) , antiderivative size = 806, normalized size of antiderivative = 1.77

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(5/2),x, algorithm="fricas")
```

output
```
2/63*((256*c^4*d^6 - 512*b*c^3*d^5*e + 278*b^2*c^2*d^4*e^2 - 22*b^3*c*d^3*
e^3 - 3*b^4*d^2*e^4 + (256*c^4*d^4*e^2 - 512*b*c^3*d^3*e^3 + 278*b^2*c^2*d
^2*e^4 - 22*b^3*c*d*e^5 - 3*b^4*e^6)*x^2 + 2*(256*c^4*d^5*e - 512*b*c^3*d^
4*e^2 + 278*b^2*c^2*d^3*e^3 - 22*b^3*c*d^2*e^4 - 3*b^4*d*e^5)*x)*sqrt(c*e)
*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2
*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*
e*x + c*d + b*e)/(c*e)) + 3*(256*c^4*d^5*e - 384*b*c^3*d^4*e^2 + 134*b^2*c
^2*d^3*e^3 - 3*b^3*c*d^2*e^4 + (256*c^4*d^3*e^3 - 384*b*c^3*d^2*e^4 + 134*
b^2*c^2*d*e^5 - 3*b^3*c*e^6)*x^2 + 2*(256*c^4*d^4*e^2 - 384*b*c^3*d^3*e^3
+ 134*b^2*c^2*d^2*e^4 - 3*b^3*c*d*e^5)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c
^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e -
3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b
*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*
e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(3*c^4*e
^6*x^4 + 128*c^4*d^4*e^2 - 176*b*c^3*d^3*e^3 + 51*b^2*c^2*d^2*e^4 - 3*(2*c
^4*d*e^5 - 3*b*c^3*e^6)*x^3 + (16*c^4*d^2*e^4 - 25*b*c^3*d*e^5 + 9*b^2*c^2
*e^6)*x^2 + (160*c^4*d^3*e^3 - 224*b*c^3*d^2*e^4 + 67*b^2*c^2*d*e^5)*x)*sq
rt(c*x^2 + b*x)*sqrt(e*x + d))/(c^2*e^9*x^2 + 2*c^2*d*e^8*x + c^2*d^2*e^7)
```

Sympy [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}}{(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x)**(5/2)/(e*x+d)**(5/2),x)
```

output
```
Integral((x*(b + c*x))**(5/2)/(d + e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(cx^2+bx)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(5/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(5/2), x)
```

Giac [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(cx^2+bx)^{\frac{5}{2}}}{(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(5/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{5/2}}\,dx = \int \frac{(c\,x^2+b\,x)^{5/2}}{(d+e\,x)^{5/2}}\,dx$$

input
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^(5/2),x)
```

output
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^{5/2}} dx = \text{too large to display}$$

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^(5/2),x)
```

output
```
(18*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**3*d*e**2 + 12*sqrt(x)*sqrt(d +
e*x)*sqrt(b + c*x)*b**3*e**3*x - 384*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b
**2*c*d**2*e - 268*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c*d*e**2*x + 3
6*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c*e**3*x**2 + 576*sqrt(x)*sqrt(
d + e*x)*sqrt(b + c*x)*b*c**2*d**3 + 640*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*
x)*b*c**2*d**2*e*x - 100*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**2*d*e**2
*x**2 + 36*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**2*e**3*x**3 - 384*sqrt
(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**3*d**3*x + 64*sqrt(x)*sqrt(d + e*x)*sqr
t(b + c*x)*c**3*d**2*e*x**2 - 24*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**3*
d*e**2*x**3 + 12*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**3*e**3*x**4 - 9*in
t((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b*d**3 + 3*sqrt(x)*b*d**2*e*x + 3
*sqrt(x)*b*d*e**2*x**2 + sqrt(x)*b*e**3*x**3 + sqrt(x)*c*d**3*x + 3*sqrt(x
)*c*d**2*e*x**2 + 3*sqrt(x)*c*d*e**2*x**3 + sqrt(x)*c*e**3*x**4),x)*b**4*d
**4*e**2 - 18*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b*d**3 + 3*sqrt(x
)*b*d**2*e*x + 3*sqrt(x)*b*d*e**2*x**2 + sqrt(x)*b*e**3*x**3 + sqrt(x)*c*d
**3*x + 3*sqrt(x)*c*d**2*e*x**2 + 3*sqrt(x)*c*d*e**2*x**3 + sqrt(x)*c*e**3
*x**4),x)*b**4*d**3*e**3*x - 9*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*
b*d**3 + 3*sqrt(x)*b*d**2*e*x + 3*sqrt(x)*b*d*e**2*x**2 + sqrt(x)*b*e**3*x
**3 + sqrt(x)*c*d**3*x + 3*sqrt(x)*c*d**2*e*x**2 + 3*sqrt(x)*c*d*e**2*x**3
 + sqrt(x)*c*e**3*x**4),x)*b**4*d**2*e**4*x**2 + 192*int((sqrt(d + e*x)...
```

3.202 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{7/2}}\,dx$

Optimal result 1659
Mathematica [C] (verified) 1660
Rubi [A] (verified) 1661
Maple [B] (verified) 1665
Fricas [B] (verification not implemented) 1666
Sympy [F] 1667
Maxima [F] 1668
Giac [F] 1668
Mupad [F(-1)] 1668
Reduce [F] 1669

Optimal result

Integrand size = 23, antiderivative size = 438

$$\begin{aligned}
\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{7/2}}\,dx &= -\frac{2(32c^2d^2-32bcde+5b^2e^2)\,x\sqrt{bx+cx^2}}{15e^4(d+ex)^{3/2}} \\
&- \frac{2c(16cd-11be)x^2\sqrt{bx+cx^2}}{15e^3(d+ex)^{3/2}} + \frac{2(128c^2d^2-144bcde+31b^2e^2)\sqrt{bx+cx^2}}{15e^5\sqrt{d+ex}} \\
&+ \frac{4cx(bx+cx^2)^{3/2}}{5e^2(d+ex)^{3/2}} - \frac{2(bx+cx^2)^{5/2}}{5e(d+ex)^{5/2}} \\
&- \frac{4\sqrt{d}(128c^2d^2-128bcde+23b^2e^2)\sqrt{bx+cx^2}\,E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{15e^{11/2}\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}} \\
&+ \frac{2\sqrt{d}(128c^2d^2-112bcde+15b^2e^2)\sqrt{bx+cx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{15e^{11/2}\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}
\end{aligned}$$

output
```
-2/15*(5*b^2*e^2-32*b*c*d*e+32*c^2*d^2)*x*(c*x^2+b*x)^(1/2)/e^4/(e*x+d)^(3
/2)-2/15*c*(-11*b*e+16*c*d)*x^2*(c*x^2+b*x)^(1/2)/e^3/(e*x+d)^(3/2)+2/15*(
31*b^2*e^2-144*b*c*d*e+128*c^2*d^2)*(c*x^2+b*x)^(1/2)/e^5/(e*x+d)^(1/2)+4/
5*c*x*(c*x^2+b*x)^(3/2)/e^2/(e*x+d)^(3/2)-2/5*(c*x^2+b*x)^(5/2)/e/(e*x+d)^
(5/2)-4/15*d^(1/2)*(23*b^2*e^2-128*b*c*d*e+128*c^2*d^2)*(c*x^2+b*x)^(1/2)*
EllipticE(e^(1/2)*x^(1/2)/d^(1/2)/(1+e*x/d)^(1/2),(1-c*d/b/e)^(1/2))/e^(11
/2)/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)+2/15*d^(1/2)*(15*b^2
*e^2-112*b*c*d*e+128*c^2*d^2)*(c*x^2+b*x)^(1/2)*InverseJacobiAM(arctan(e^(
1/2)*x^(1/2)/d^(1/2)),(1-c*d/b/e)^(1/2))/e^(11/2)/x^(1/2)/(d*(c*x+b)/b/(e*
x+d))^(1/2)/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 22.87 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.92

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{7/2}}\,dx = \frac{2(x(b+cx))^{5/2}\left(\frac{2(128c^2d^2-128bcde+23b^2e^2)(b+cx)(d+ex)}{\sqrt{x}} - \frac{e\sqrt{x}(b+cx)(b^2e^2(15d^2+35dex+23e^2x^2)-\cdots}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^(7/2),x]
```

output
```
(2*(x*(b + c*x))^(5/2)*((2*(128*c^2*d^2 - 128*b*c*d*e + 23*b^2*e^2)*(b + c
*x)*(d + e*x))/Sqrt[x] - (e*Sqrt[x]*(b + c*x)*(b^2*e^2*(15*d^2 + 35*d*e*x
+ 23*e^2*x^2) - b*c*e*(112*d^3 + 256*d^2*e*x + 161*d*e^2*x^2 + 11*e^3*x^3)
 + c^2*(128*d^4 + 288*d^3*e*x + 176*d^2*e^2*x^2 + 10*d*e^3*x^3 - 3*e^4*x^4
)))/(d + e*x)^2 + (2*I)*Sqrt[b/c]*c*e*(128*c^2*d^2 - 128*b*c*d*e + 23*b^2*
e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x*EllipticE[I*ArcSinh[Sqrt[b/c]/S
qrt[x]], (c*d)/(b*e)] - I*Sqrt[b/c]*c*e*(128*c^2*d^2 - 144*b*c*d*e + 31*b^
2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x*EllipticF[I*ArcSinh[Sqrt[b/c]
/Sqrt[x]], (c*d)/(b*e)]))/(15*e^6*x^(5/2)*(b + c*x)^3*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.93, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {1161, 1230, 27, 1230, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(bx+cx^2\right)^{5/2}}{(d+ex)^{7/2}}dx$$

$$\Big\downarrow \text{1161}$$

$$\frac{\int \frac{(b+2cx)\left(cx^2+bx\right)^{3/2}}{(d+ex)^{5/2}}dx}{e}-\frac{2\left(bx+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1230}$$

$$\frac{\frac{2\left(bx+cx^2\right)^{3/2}(-5be+16cd+6cex)}{15e^2(d+ex)^{3/2}}-\frac{2\int \frac{(b(16cd-5be)+16c(2cd-be)x)\sqrt{cx^2+bx}}{2(d+ex)^{3/2}}dx}{5e^2}}{e}-\frac{2\left(bx+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2\left(bx+cx^2\right)^{3/2}(-5be+16cd+6cex)}{15e^2(d+ex)^{3/2}}-\frac{\int \frac{(b(16cd-5be)+16c(2cd-be)x)\sqrt{cx^2+bx}}{(d+ex)^{3/2}}dx}{5e^2}}{e}-\frac{2\left(bx+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{1230}$$

$$\frac{\frac{2\left(bx+cx^2\right)^{3/2}(-5be+16cd+6cex)}{15e^2(d+ex)^{3/2}}-\frac{\frac{2\sqrt{bx+cx^2}\left(15b^2e^2+16cex(2cd-be)-112bcde+128c^2d^2\right)}{3e^2\sqrt{d+ex}}-\frac{2\int \frac{b\left(128c^2d^2-112bced+15b^2e^2\right)+2c\left(128c^2d^2-128bced+ \cdots\right.}{2\sqrt{d+ex}\sqrt{cx^2+bx}}}{3e^2}}{5e^2}}{e}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{2(bx+cx^2)^{3/2}(-5be+16cd+6cex)}{15e^2(d+ex)^{3/2}}-\frac{\frac{2\sqrt{bx+cx^2}\left(15b^2e^2+16cex(2cd-be)-112bcde+128c^2d^2\right)}{3e^2\sqrt{d+ex}}-\frac{\int\frac{b\left(128c^2d^2-112bced+15b^2e^2\right)+2c\left(128c^2d^2-128bced+2\cdots\right.}{\sqrt{d+ex}\sqrt{cx^2+bx}}}{3e^2}}{5e^2}}{e}$$

$$\frac{2(bx+cx^2)^{5/2}}{5e(d+ex)^{5/2}}$$

$\downarrow$ 1269

$$\frac{\frac{2(bx+cx^2)^{3/2}(-5be+16cd+6cex)}{15e^2(d+ex)^{3/2}}-\frac{\frac{2\sqrt{bx+cx^2}\left(15b^2e^2+16cex(2cd-be)-112bcde+128c^2d^2\right)}{3e^2\sqrt{d+ex}}-\frac{\frac{2c\left(23b^2e^2-128bcde+128c^2d^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{(2cd-be)\cdots}{\cdots}}{3e^2}}{5e^2}}{e}$$

$$\frac{2(bx+cx^2)^{5/2}}{5e(d+ex)^{5/2}}$$

$\downarrow$ 1169

$$\frac{\frac{2(bx+cx^2)^{3/2}(-5be+16cd+6cex)}{15e^2(d+ex)^{3/2}}-\frac{\frac{2\sqrt{bx+cx^2}\left(15b^2e^2+16cex(2cd-be)-112bcde+128c^2d^2\right)}{3e^2\sqrt{d+ex}}-\frac{\frac{2c\sqrt{x}\sqrt{b+cx}\left(23b^2e^2-128bcde+128c^2d^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\cdots}{\cdots}}{5e^2}}{e}$$

$$\frac{2(bx+cx^2)^{5/2}}{5e(d+ex)^{5/2}}$$

$\downarrow$ 122

$$\frac{\frac{2(bx+cx^2)^{3/2}(-5be+16cd+6cex)}{15e^2(d+ex)^{3/2}}-\frac{\frac{2\sqrt{bx+cx^2}\left(15b^2e^2+16cex(2cd-be)-112bcde+128c^2d^2\right)}{3e^2\sqrt{d+ex}}-\frac{\frac{2c\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(23b^2e^2-128bcde+128c^2d^2\right)\int\frac{\sqrt{\frac{ex}{d}\cdots}}{\sqrt{x}\sqrt{\cdots}}}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}\cdots}{\cdots}}{5e^2}}{e}$$

$$\frac{2(bx+cx^2)^{5/2}}{5e(d+ex)^{5/2}}$$

$\downarrow$ 120

$$\frac{\frac{2(bx+cx^2)^{3/2}(-5be+16cd+6cex)}{15e^2(d+ex)^{3/2}}-\frac{\frac{2\sqrt{bx+cx^2}\left(15b^2e^2+16cex(2cd-be)-112bcde+128c^2d^2\right)}{3e^2\sqrt{d+ex}}-\frac{\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(23b^2e^2-128bcde+128c^2d^2\right)E\cdots}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}\cdots}{\cdots}}{5e^2}}{e}$$

$$\frac{2(bx+cx^2)^{5/2}}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{127}$$

$$\frac{\frac{2\left(bx+cx^2\right)^{3/2}(-5be+16cd+6cex)}{15e^2(d+ex)^{3/2}}-\frac{\frac{2\sqrt{bx+cx^2}\left(15b^2e^2+16cex(2cd-be)-112bcde+128c^2d^2\right)}{3e^2\sqrt{d+ex}}-\frac{\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(23b^2e^2-128bcde+128c^2d^2\right)E}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}}{}}{5e^2}}{e}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}}$$

$$\Big\downarrow \text{126}$$

$$\frac{\frac{2\left(bx+cx^2\right)^{3/2}(-5be+16cd+6cex)}{15e^2(d+ex)^{3/2}}-\frac{\frac{2\sqrt{bx+cx^2}\left(15b^2e^2+16cex(2cd-be)-112bcde+128c^2d^2\right)}{3e^2\sqrt{d+ex}}-\frac{\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(23b^2e^2-128bcde+128c^2d^2\right)E}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}}{}}{5}}{e}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{5e(d+ex)^{5/2}}$$

input
```
Int[(b*x + c*x^2)^(5/2)/(d + e*x)^(7/2),x]
```

output
```
(-2*(b*x + c*x^2)^(5/2))/(5*e*(d + e*x)^(5/2)) + ((2*(16*c*d - 5*b*e + 6*c
*e*x)*(b*x + c*x^2)^(3/2))/(15*e^2*(d + e*x)^(3/2)) - ((2*(128*c^2*d^2 - 1
12*b*c*d*e + 15*b^2*e^2 + 16*c*e*(2*c*d - b*e)*x)*Sqrt[b*x + c*x^2])/(3*e^
2*Sqrt[d + e*x]) - ((4*Sqrt[-b]*Sqrt[c]*(128*c^2*d^2 - 128*b*c*d*e + 23*b^
2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*S
qrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) -
 (2*Sqrt[-b]*(2*c*d - b*e)*(128*c^2*d^2 - 128*b*c*d*e + 15*b^2*e^2)*Sqrt[x
]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/S
qrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]))/(3*e^
2))/(5*e^2))/e
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 911 vs. $2(381) = 762$.

Time = 4.21 (sec) , antiderivative size = 912, normalized size of antiderivative = 2.08

method	result
elliptic	$\frac{\sqrt{x(cx+b)}\,\sqrt{(cx+b)x(ex+d)}\left(-\frac{2d^2\left(b^2e^2-2bcde+c^2d^2\right)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{5e^8\left(x+\frac{d}{e}\right)^3}+\frac{22d\left(b^2e^2-3bcde+2c^2d^2\right)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{15e^7\left(x+\frac{d}{e}\right)^2}-\right.}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)*x*(e*x+d))^(1/2)/x/(c*x+b)*(-2/
5*d^2*(b^2*e^2-2*b*c*d*e+c^2*d^2)/e^8*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2
)/(x+d/e)^3+22/15*d*(b^2*e^2-3*b*c*d*e+2*c^2*d^2)/e^7*(c*e*x^3+b*e*x^2+c*d
*x^2+b*d*x)^(1/2)/(x+d/e)^2-2/15*(c*e*x^2+b*e*x)*(23*b^2*e^2-128*b*c*d*e+1
28*c^2*d^2)/e^6/((x+d/e)*(c*e*x^2+b*e*x))^(1/2)+2/5*c^2/e^4*x*(c*e*x^3+b*e
*x^2+c*d*x^2+b*d*x)^(1/2)+2/3*(3*c^2/e^4*(b*e-c*d)-2/5*c^2/e^4*(2*b*e+2*c*
d))/c/e*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2*((b^3*e^3-9*b^2*c*d*e^2+18
*b*c^2*d^2*e-10*c^3*d^3)/e^6+11/15*d*(b^2*e^2-3*b*c*d*e+2*c^2*d^2)*c/e^6-1
/15*(23*b^2*e^2-128*b*c*d*e+128*c^2*d^2)/e^6*(b*e-c*d)+1/15*b/e^5*(23*b^2*
e^2-128*b*c*d*e+128*c^2*d^2)-1/3*(3*c^2/e^4*(b*e-c*d)-2/5*c^2/e^4*(2*b*e+2
*c*d))/c/e*b*d)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d
)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2
),(-d/e/(-d/e+b/c))^(1/2))+2*(3*c/e^5*(b^2*e^2-3*b*c*d*e+2*c^2*d^2)+1/15*(
23*b^2*e^2-128*b*c*d*e+128*c^2*d^2)/e^5*c-3/5*c^2/e^4*b*d-2/3*(3*c^2/e^4*(
b*e-c*d)-2/5*c^2/e^4*(2*b*e+2*c*d))/c/e*(b*e+c*d))*d/e*((x+d/e)/d*e)^(1/2)
*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)
^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-
b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 813 vs. $2(381) = 762$.

Time = 0.22 (sec) , antiderivative size = 813, normalized size of antiderivative = 1.86

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^{7/2}} \, dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(7/2),x, algorithm="fricas")
```

output
```
-2/45*((256*c^3*d^6 - 384*b*c^2*d^5*e + 126*b^2*c*d^4*e^2 + b^3*d^3*e^3 +
(256*c^3*d^3*e^3 - 384*b*c^2*d^2*e^4 + 126*b^2*c*d*e^5 + b^3*e^6)*x^3 + 3*
(256*c^3*d^4*e^2 - 384*b*c^2*d^3*e^3 + 126*b^2*c*d^2*e^4 + b^3*d*e^5)*x^2
+ 3*(256*c^3*d^5*e - 384*b*c^2*d^4*e^2 + 126*b^2*c*d^3*e^3 + b^3*d^2*e^4)*
x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^
2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3
), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 6*(128*c^3*d^5*e - 128*b*c^2*d^4*e^2
 + 23*b^2*c*d^3*e^3 + (128*c^3*d^2*e^4 - 128*b*c^2*d*e^5 + 23*b^2*c*e^6)*x
^3 + 3*(128*c^3*d^3*e^3 - 128*b*c^2*d^2*e^4 + 23*b^2*c*d*e^5)*x^2 + 3*(128
*c^3*d^4*e^2 - 128*b*c^2*d^3*e^3 + 23*b^2*c*d^2*e^4)*x)*sqrt(c*e)*weierstr
assZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*
b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/
3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*
e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))
) - 3*(3*c^3*e^6*x^4 - 128*c^3*d^4*e^2 + 112*b*c^2*d^3*e^3 - 15*b^2*c*d^2*
e^4 - (10*c^3*d*e^5 - 11*b*c^2*e^6)*x^3 - (176*c^3*d^2*e^4 - 161*b*c^2*d*e
^5 + 23*b^2*c*e^6)*x^2 - (288*c^3*d^3*e^3 - 256*b*c^2*d^2*e^4 + 35*b^2*c*d
*e^5)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/(c*e^10*x^3 + 3*c*d*e^9*x^2 + 3*
c*d^2*e^8*x + c*d^3*e^7)
```

Sympy [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{7/2}}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}}{(d+ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x)**(5/2)/(e*x+d)**(7/2),x)
```

output
```
Integral((x*(b + c*x))**(5/2)/(d + e*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{7/2}}\,dx=\int \frac{(cx^2+bx)^{\frac{5}{2}}}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(7/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(7/2), x)
```

Giac [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{7/2}}\,dx=\int \frac{(cx^2+bx)^{\frac{5}{2}}}{(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(7/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{7/2}}\,dx=\int \frac{(c\,x^2+b\,x)^{5/2}}{(d+e\,x)^{7/2}}\,dx$$

input
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^(7/2),x)
```

output
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^{5/2}}{(d + ex)^{7/2}}\, dx = \text{too large to display}$$

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^(7/2),x)
```

output
```
( - 90*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**4*d*e**3 - 120*sqrt(x)*sqrt(
d + e*x)*sqrt(b + c*x)*b**4*e**4*x + 1290*sqrt(x)*sqrt(d + e*x)*sqrt(b + c
*x)*b**3*c*d**2*e**2 + 1780*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**3*c*d*e
**3*x + 184*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**3*c*e**4*x**2 - 3840*sq
rt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c**2*d**3*e - 5980*sqrt(x)*sqrt(d +
 e*x)*sqrt(b + c*x)*b**2*c**2*d**2*e**2*x - 852*sqrt(x)*sqrt(d + e*x)*sqrt
(b + c*x)*b**2*c**2*d*e**3*x**2 + 88*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b
**2*c**2*e**4*x**3 + 2880*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**3*d**4
+ 6400*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**3*d**3*e*x + 1020*sqrt(x)*
sqrt(d + e*x)*sqrt(b + c*x)*b*c**3*d**2*e**2*x**2 - 124*sqrt(x)*sqrt(d + e
*x)*sqrt(b + c*x)*b*c**3*d*e**3*x**3 + 24*sqrt(x)*sqrt(d + e*x)*sqrt(b + c
*x)*b*c**3*e**4*x**4 - 1920*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**4*d**4*
x - 320*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**4*d**3*e*x**2 + 40*sqrt(x)*
sqrt(d + e*x)*sqrt(b + c*x)*c**4*d**2*e**2*x**3 - 12*sqrt(x)*sqrt(d + e*x)
*sqrt(b + c*x)*c**4*d*e**3*x**4 + 90*int((sqrt(d + e*x)*sqrt(b + c*x))/(2*
sqrt(x)*b**2*d**4*e + 8*sqrt(x)*b**2*d**3*e**2*x + 12*sqrt(x)*b**2*d**2*e*
*3*x**2 + 8*sqrt(x)*b**2*d*e**4*x**3 + 2*sqrt(x)*b**2*e**5*x**4 - sqrt(x)*
b*c*d**5 - 2*sqrt(x)*b*c*d**4*e*x + 2*sqrt(x)*b*c*d**3*e**2*x**2 + 8*sqrt(
x)*b*c*d**2*e**3*x**3 + 7*sqrt(x)*b*c*d*e**4*x**4 + 2*sqrt(x)*b*c*e**5*x**
5 - sqrt(x)*c**2*d**5*x - 4*sqrt(x)*c**2*d**4*e*x**2 - 6*sqrt(x)*c**2*d...
```

3.203 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{9/2}}\,dx$

Optimal result 1670
Mathematica [C] (verified) 1671
Rubi [A] (verified) 1672
Maple [B] (verified) 1678
Fricas [B] (verification not implemented) 1679
Sympy [F] 1680
Maxima [F] 1680
Giac [F] 1680
Mupad [F(-1)] 1681
Reduce [F] 1681

Optimal result

Integrand size = 23, antiderivative size = 468

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{9/2}}\,dx = -\frac{2(96c^2d^2-64bcde+3b^2e^2)\,x\sqrt{bx+cx^2}}{21e^4(d+ex)^{5/2}} - \frac{10c(16cd-9be)x^2\sqrt{bx+cx^2}}{21e^3(d+ex)^{5/2}} - \frac{2(128c^2d^2-80bcde+3b^2e^2)\sqrt{bx+cx^2}}{21e^5(d+ex)^{3/2}} + \frac{20cx(bx+cx^2)^{3/2}}{21e^2(d+ex)^{5/2}} - \frac{2(bx+cx^2)^{5/2}}{7e(d+ex)^{7/2}} + \frac{2(2cd-be)(128c^2d^2-128bcde+3b^2e^2)\sqrt{bx+cx^2}\,E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{21\sqrt{d}e^{11/2}(cd-be)\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}} - \frac{2c\sqrt{d}(128c^2d^2-176bcde+51b^2e^2)\sqrt{bx+cx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{21e^{11/2}(cd-be)\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}$$

output
```
-2/21*(3*b^2*e^2-64*b*c*d*e+96*c^2*d^2)*x*(c*x^2+b*x)^(1/2)/e^4/(e*x+d)^(5
/2)-10/21*c*(-9*b*e+16*c*d)*x^2*(c*x^2+b*x)^(1/2)/e^3/(e*x+d)^(5/2)-2/21*(
3*b^2*e^2-80*b*c*d*e+128*c^2*d^2)*(c*x^2+b*x)^(1/2)/e^5/(e*x+d)^(3/2)+20/2
1*c*x*(c*x^2+b*x)^(3/2)/e^2/(e*x+d)^(5/2)-2/7*(c*x^2+b*x)^(5/2)/e/(e*x+d)^
(7/2)+2/21*(-b*e+2*c*d)*(3*b^2*e^2-128*b*c*d*e+128*c^2*d^2)*(c*x^2+b*x)^(1
/2)*EllipticE(e^(1/2)*x^(1/2)/d^(1/2)/(1+e*x/d)^(1/2),(1-c*d/b/e)^(1/2))/d
^(1/2)/e^(11/2)/(-b*e+c*d)/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/
2)-2/21*c*d^(1/2)*(51*b^2*e^2-176*b*c*d*e+128*c^2*d^2)*(c*x^2+b*x)^(1/2)*I
nverseJacobiAM(arctan(e^(1/2)*x^(1/2)/d^(1/2)),(1-c*d/b/e)^(1/2))/e^(11/2)
/(-b*e+c*d)/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.97 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.07

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{9/2}}\,dx =$$

$$-\frac{2(x(b+cx))^{5/2}\left(bex(b+cx)\left(3b^3e^6x^3-b^2cde^2(51d^3+169d^2ex+194de^2x^2+85e^3x^3)-c^3d^2(128d^4+4\right.\right.}{}$$

input
```
Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^(9/2),x]
```

output
```
(-2*(x*(b + c*x))^(5/2)*(b*e*x*(b + c*x)*(3*b^3*e^6*x^3 - b^2*c*d*e^2*(51*
d^3 + 169*d^2*e*x + 194*d*e^2*x^2 + 85*e^3*x^3) - c^3*d^2*(128*d^4 + 416*d
^3*e*x + 464*d^2*e^2*x^2 + 186*d*e^3*x^3 + 7*e^4*x^4) + b*c^2*d*e*(176*d^4
 + 576*d^3*e*x + 649*d^2*e^2*x^2 + 265*d*e^3*x^3 + 7*e^4*x^4)) + Sqrt[b/c]
*c*(d + e*x)^3*(Sqrt[b/c]*(256*c^3*d^3 - 384*b*c^2*d^2*e + 134*b^2*c*d*e^2
 - 3*b^3*e^3)*(b + c*x)*(d + e*x) + I*b*e*(256*c^3*d^3 - 384*b*c^2*d^2*e +
 134*b^2*c*d*e^2 - 3*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*
EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(128*c^3*d^3
- 208*b*c^2*d^2*e + 83*b^2*c*d*e^2 - 3*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 +
 d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/
(21*b*d*e^6*(c*d - b*e)*x^3*(b + c*x)^3*(d + e*x)^(7/2))
```

Rubi [A] (verified)

Time = 1.24 (sec) , antiderivative size = 490, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {1161, 1229, 27, 1230, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{9/2}}dx$$

$$\Big\downarrow \text{1161}$$

$$\frac{5\int \frac{(b+2cx)(cx^2+bx)^{3/2}}{(d+ex)^{7/2}}dx}{7e}-\frac{2(bx+cx^2)^{5/2}}{7e(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1229}$$

$$\frac{5\left(-\frac{2\int -\frac{c\left(bd(16cd-13be)+\left(32c^2d^2-32bced+3b^2e^2\right)x\right)\sqrt{cx^2+bx}}{2(d+ex)^{3/2}}dx}{5de^2(cd-be)}-\frac{2(bx+cx^2)^{3/2}\left(ex\left(3b^2e^2-22bcde+22c^2d^2\right)+cd^2(16cd-13be)\right)}{15de^2(d+ex)^{5/2}(cd-be)}\right)}{7e}-\frac{2(bx+cx^2)^{5/2}}{7e(d+ex)^{7/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\left(\frac{c\int \frac{\left(bd(16cd-13be)+\left(32c^2d^2-32bced+3b^2e^2\right)x\right)\sqrt{cx^2+bx}}{(d+ex)^{3/2}}dx}{5de^2(cd-be)}-\frac{2(bx+cx^2)^{3/2}\left(ex\left(3b^2e^2-22bcde+22c^2d^2\right)+cd^2(16cd-13be)\right)}{15de^2(d+ex)^{5/2}(cd-be)}\right)}{7e}-\frac{2(bx+cx^2)^{5/2}}{7e(d+ex)^{7/2}}$$

$$\Big\downarrow \text{1230}$$

$$\frac{5\left(\frac{c\left(\frac{2\sqrt{bx+cx^2}\left(ex\left(3b^2e^2-32bcde+32c^2d^2\right)+d\left(51b^2e^2-176bcde+128c^2d^2\right)\right)}{3e^2\sqrt{d+ex}}-\frac{2\int\frac{bd\left(128c^2d^2-176bced+51b^2e^2\right)+(2cd-be)\left(128c^2d^2-128bced+3b^2e^2\right)x}{2\sqrt{d+ex}\sqrt{cx^2+bx}}d}{3e^2}\right.}{5de^2(cd-be)}\right.}{7e}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{7e(d+ex)^{7/2}}$$

$\Big\downarrow$ 27

$$\frac{5\left(\frac{c\left(\frac{2\sqrt{bx+cx^2}\left(ex\left(3b^2e^2-32bcde+32c^2d^2\right)+d\left(51b^2e^2-176bcde+128c^2d^2\right)\right)}{3e^2\sqrt{d+ex}}-\frac{\int\frac{bd\left(128c^2d^2-176bced+51b^2e^2\right)+(2cd-be)\left(128c^2d^2-128bced+3b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3e^2}\right.}{5de^2(cd-be)}\right.}{7e}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{7e(d+ex)^{7/2}}$$

$\Big\downarrow$ 1269

$$\frac{5\left(\frac{c\left(\frac{2\sqrt{bx+cx^2}\left(ex\left(3b^2e^2-32bcde+32c^2d^2\right)+d\left(51b^2e^2-176bcde+128c^2d^2\right)\right)}{3e^2\sqrt{d+ex}}-\frac{\frac{(2cd-be)\left(3b^2e^2-128bcde+128c^2d^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{2d(cd-be)\left(27b^2e^2-1\right.}{}}{3e^2}\right.}{5de^2(cd-be)}\right.}{7e}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{7e(d+ex)^{7/2}}$$

$\Big\downarrow$ 1169

$$5\left(\frac{c\left(\frac{2\sqrt{bx+cx^2}\left(ex\left(3b^2e^2-32bcde+32c^2d^2\right)+d\left(51b^2e^2-176bcde+128c^2d^2\right)\right)}{3e^2\sqrt{d+ex}}-\frac{\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\left(3b^2e^2-128bcde+128c^2d^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{2d\sqrt{x}\sqrt{b+c\cdots}}{\cdots}}{3e^2}\right.}{5de^2(cd-be)}\right.$$

$$7e$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{7e(d+ex)^{7/2}}$$

$\downarrow$ 122

$$5\left(\frac{c\left(\frac{2\sqrt{bx+cx^2}\left(ex\left(3b^2e^2-32bcde+32c^2d^2\right)+d\left(51b^2e^2-176bcde+128c^2d^2\right)\right)}{3e^2\sqrt{d+ex}}-\frac{\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(3b^2e^2-128bcde+128c^2d^2\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\cdots}{\cdots}}{3e^2}\right.}{5de^2(cd-be)}\right.$$

$$7e$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{7e(d+ex)^{7/2}}$$

$\downarrow$ 120

$$5\left(\frac{c\left(\frac{2\sqrt{bx+cx^2}\left(ex\left(3b^2e^2-32bcde+32c^2d^2\right)+d\left(51b^2e^2-176bcde+128c^2d^2\right)\right)}{3e^2\sqrt{d+ex}}-\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(3b^2e^2-128bcde+128c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\cdots}{\sqrt{-\cdots}}\right.\right.}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}\right.}{5de^2(cd-be)}\right.$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{7e(d+ex)^{7/2}}$$

$\downarrow$ 127

$$5\left(\frac{c\left(\frac{2\sqrt{bx+cx^2}\left(ex\left(3b^2e^2-32bcde+32c^2d^2\right)+d\left(51b^2e^2-176bcde+128c^2d^2\right)\right)}{3e^2\sqrt{d+ex}}-\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(3b^2e^2-128bcde+128c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}}{\sqrt{-}}\right.\right.}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}}{5de^2(cd-be)}\right.$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{7e(d+ex)^{7/2}}$$

$$\Big\downarrow \; 126$$

$$5\left(\frac{c\left(\frac{2\sqrt{bx+cx^2}\left(ex\left(3b^2e^2-32bcde+32c^2d^2\right)+d\left(51b^2e^2-176bcde+128c^2d^2\right)\right)}{3e^2\sqrt{d+ex}}-\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(3b^2e^2-128bcde+128c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}}{\sqrt{-}}\right.\right.}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}}{5de^2(cd-be)}\right.$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{7e(d+ex)^{7/2}}$$

input
```
Int[(b*x + c*x^2)^(5/2)/(d + e*x)^(9/2),x]
```

output
```
(-2*(b*x + c*x^2)^(5/2))/(7*e*(d + e*x)^(7/2)) + (5*((-2*(c*d^2*(16*c*d -
13*b*e) + e*(22*c^2*d^2 - 22*b*c*d*e + 3*b^2*e^2)*x)*(b*x + c*x^2)^(3/2))/
(15*d*e^2*(c*d - b*e)*(d + e*x)^(5/2)) + (c*((2*(d*(128*c^2*d^2 - 176*b*c*
d*e + 51*b^2*e^2) + e*(32*c^2*d^2 - 32*b*c*d*e + 3*b^2*e^2)*x)*Sqrt[b*x +
c*x^2])/(3*e^2*Sqrt[d + e*x]) - ((2*Sqrt[-b]*(2*c*d - b*e)*(128*c^2*d^2 -
128*b*c*d*e + 3*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE
[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*
x)/d]*Sqrt[b*x + c*x^2]) - (4*Sqrt[-b]*d*(c*d - b*e)*(128*c^2*d^2 - 128*b*
c*d*e + 27*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[
ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]
*Sqrt[b*x + c*x^2]))/(3*e^2)))/(5*d*e^2*(c*d - b*e))))/(7*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1230
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Simp[p/(e^2*(m + 1)*(m + 2*p + 2))   Int[(d + e*x)^(m + 1)*(a
+ b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m
+ 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
 x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && GtQ[p, 0] && (LtQ[m, -
1] || EqQ[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ
[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 893 vs. 2(411) = 822.

Time = 7.61 (sec) , antiderivative size = 894, normalized size of antiderivative = 1.91

method	result
elliptic	$\frac{\sqrt{x(cx+b)}\,\sqrt{(cx+b)x(ex+d)}\left(-\frac{2d^2\left(b^2e^2-2bcde+c^2d^2\right)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{7e^9\left(x+\frac{d}{e}\right)^4}+\frac{6d\left(b^2e^2-3bcde+2c^2d^2\right)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{7e^8\left(x+\frac{d}{e}\right)^3}-\right.}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)*x*(e*x+d))^(1/2)/x/(c*x+b)*(-2/
7*d^2*(b^2*e^2-2*b*c*d*e+c^2*d^2)/e^9*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2
)/(x+d/e)^4+6/7*d*(b^2*e^2-3*b*c*d*e+2*c^2*d^2)/e^8*(c*e*x^3+b*e*x^2+c*d*x
^2+b*d*x)^(1/2)/(x+d/e)^3-2/21*(9*b^2*e^2-52*b*c*d*e+52*c^2*d^2)/e^7*(c*e*
x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(x+d/e)^2+2/21*(c*e*x^2+b*e*x)/e^6/d/(b*e
-c*d)*(3*b^3*e^3-85*b^2*c*d*e^2+237*b*c^2*d^2*e-158*c^3*d^3)/((x+d/e)*(c*e
*x^2+b*e*x))^(1/2)+2/3*c^2/e^5*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2*(c*
(3*b^2*e^2-12*b*c*d*e+10*c^2*d^2)/e^6-1/21*c*(9*b^2*e^2-52*b*c*d*e+52*c^2*
d^2)/e^6+1/21/e^6*(3*b^3*e^3-85*b^2*c*d*e^2+237*b*c^2*d^2*e-158*c^3*d^3)/d
-1/21*b/e^5/d/(b*e-c*d)*(3*b^3*e^3-85*b^2*c*d*e^2+237*b*c^2*d^2*e-158*c^3*
d^3)-1/3*c^2/e^5*b*d)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(
-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e)/d*e
)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(1/e^5*c^2*(3*b*e-4*c*d)-1/21/e^5*c*(3*
b^3*e^3-85*b^2*c*d*e^2+237*b*c^2*d^2*e-158*c^3*d^3)/d/(b*e-c*d)-2/3*c^2/e^
5*(b*e+c*d))*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(
1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/
d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/
e/(-d/e+b/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1184 vs. 2(411) = 822.

Time = 0.34 (sec) , antiderivative size = 1184, normalized size of antiderivative = 2.53

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(9/2),x, algorithm="fricas")
```

output
```
2/63*((256*c^4*d^8 - 512*b*c^3*d^7*e + 278*b^2*c^2*d^6*e^2 - 22*b^3*c*d^5*
e^3 - 3*b^4*d^4*e^4 + (256*c^4*d^4*e^4 - 512*b*c^3*d^3*e^5 + 278*b^2*c^2*d
^2*e^6 - 22*b^3*c*d*e^7 - 3*b^4*e^8)*x^4 + 4*(256*c^4*d^5*e^3 - 512*b*c^3*
d^4*e^4 + 278*b^2*c^2*d^3*e^5 - 22*b^3*c*d^2*e^6 - 3*b^4*d*e^7)*x^3 + 6*(2
56*c^4*d^6*e^2 - 512*b*c^3*d^5*e^3 + 278*b^2*c^2*d^4*e^4 - 22*b^3*c*d^3*e^
5 - 3*b^4*d^2*e^6)*x^2 + 4*(256*c^4*d^7*e - 512*b*c^3*d^6*e^2 + 278*b^2*c^
2*d^5*e^3 - 22*b^3*c*d^4*e^4 - 3*b^4*d^3*e^5)*x)*sqrt(c*e)*weierstrassPInv
erse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c
^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)
/(c*e)) + 3*(256*c^4*d^7*e - 384*b*c^3*d^6*e^2 + 134*b^2*c^2*d^5*e^3 - 3*b
^3*c*d^4*e^4 + (256*c^4*d^3*e^5 - 384*b*c^3*d^2*e^6 + 134*b^2*c^2*d*e^7 -
3*b^3*c*e^8)*x^4 + 4*(256*c^4*d^4*e^4 - 384*b*c^3*d^3*e^5 + 134*b^2*c^2*d^
2*e^6 - 3*b^3*c*d*e^7)*x^3 + 6*(256*c^4*d^5*e^3 - 384*b*c^3*d^4*e^4 + 134*
b^2*c^2*d^3*e^5 - 3*b^3*c*d^2*e^6)*x^2 + 4*(256*c^4*d^6*e^2 - 384*b*c^3*d^
5*e^3 + 134*b^2*c^2*d^4*e^4 - 3*b^3*c*d^3*e^5)*x)*sqrt(c*e)*weierstrassZet
a(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*
d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2
*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*
b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*
(128*c^4*d^6*e^2 - 176*b*c^3*d^5*e^3 + 51*b^2*c^2*d^4*e^4 + 7*(c^4*d^2*...
```

Sympy [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{9/2}}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}}{(d+ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x)**(5/2)/(e*x+d)**(9/2),x)
```

output
```
Integral((x*(b + c*x))**(5/2)/(d + e*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{9/2}}\,dx = \int \frac{(cx^2+bx)^{\frac{5}{2}}}{(ex+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(9/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(9/2), x)
```

Giac [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{9/2}}\,dx = \int \frac{(cx^2+bx)^{\frac{5}{2}}}{(ex+d)^{\frac{9}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(9/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{9/2}}\,dx = \int \frac{(c\,x^2+b\,x)^{5/2}}{(d+e\,x)^{9/2}}\,dx$$

input
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^(9/2),x)
```

output
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{9/2}}\,dx = \text{too large to display}$$

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^(9/2),x)
```

output

```
(30*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**4*d*e**3 + 60*sqrt(x)*sqrt(d +
e*x)*sqrt(b + c*x)*b**4*e**4*x - 270*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b
**3*c*d**2*e**2 - 560*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**3*c*d*e**3*x
- 108*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**3*c*e**4*x**2 + 960*sqrt(x)*s
qrt(d + e*x)*sqrt(b + c*x)*b**2*c**2*d**3*e + 2100*sqrt(x)*sqrt(d + e*x)*s
qrt(b + c*x)*b**2*c**2*d**2*e**2*x + 816*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*
x)*b**2*c**2*d*e**3*x**2 + 84*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c**
2*e**4*x**3 - 960*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**3*d**4 - 2560*s
qrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**3*d**3*e*x - 1220*sqrt(x)*sqrt(d +
 e*x)*sqrt(b + c*x)*b*c**3*d**2*e**2*x**2 - 148*sqrt(x)*sqrt(d + e*x)*sqrt
(b + c*x)*b*c**3*d*e**3*x**3 + 12*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c*
*3*e**4*x**4 + 640*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**4*d**4*x + 320*s
qrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**4*d**3*e*x**2 + 40*sqrt(x)*sqrt(d +
e*x)*sqrt(b + c*x)*c**4*d**2*e**2*x**3 - 4*sqrt(x)*sqrt(d + e*x)*sqrt(b +
c*x)*c**4*d*e**3*x**4 - 45*int((sqrt(d + e*x)*sqrt(b + c*x))/(3*sqrt(x)*b*
*2*d**5*e + 15*sqrt(x)*b**2*d**4*e**2*x + 30*sqrt(x)*b**2*d**3*e**3*x**2 +
 30*sqrt(x)*b**2*d**2*e**4*x**3 + 15*sqrt(x)*b**2*d*e**5*x**4 + 3*sqrt(x)*
b**2*e**6*x**5 - sqrt(x)*b*c*d**6 - 2*sqrt(x)*b*c*d**5*e*x + 5*sqrt(x)*b*c
*d**4*e**2*x**2 + 20*sqrt(x)*b*c*d**3*e**3*x**3 + 25*sqrt(x)*b*c*d**2*e**4
*x**4 + 14*sqrt(x)*b*c*d*e**5*x**5 + 3*sqrt(x)*b*c*e**6*x**6 - sqrt(x)*...
```

3.204 $\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{11/2}}\,dx$

Optimal result . . . 1683
Mathematica [C] (verified) . . . 1684
Rubi [A] (verified) . . . 1685
Maple [A] (verified) . . . 1692
Fricas [B] (verification not implemented) . . . 1693
Sympy [F] . . . 1694
Maxima [F] . . . 1694
Giac [F] . . . 1694
Mupad [F(-1)] . . . 1695
Reduce [F] . . . 1695

Optimal result

Integrand size = 23, antiderivative size = 575

$$
\begin{aligned}
\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{11/2}}\,dx &= \frac{10(96c^2d^2-32bcde-b^2e^2)\,x\sqrt{bx+cx^2}}{63e^4(d+ex)^{7/2}} \\
&+ \frac{10c(16cd-7be)x^2\sqrt{bx+cx^2}}{9e^3(d+ex)^{7/2}} + \frac{2(128c^2d^2-48bcde-b^2e^2)\sqrt{bx+cx^2}}{21e^5(d+ex)^{5/2}} \\
&- \frac{2(2cd-be)\,(128c^2d^2-128bcde-b^2e^2)\sqrt{bx+cx^2}}{63de^5(cd-be)(d+ex)^{3/2}} \\
&+ \frac{20cx(bx+cx^2)^{3/2}}{9e^2(d+ex)^{7/2}} - \frac{2(bx+cx^2)^{5/2}}{9e(d+ex)^{9/2}} \\
&- \frac{4(128c^4d^4-256bc^3d^3e+135b^2c^2d^2e^2-7b^3cde^3-b^4e^4)\sqrt{bx+cx^2}\,E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{63d^{3/2}e^{11/2}(cd-be)^2\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}} \\
&+ \frac{2c(128c^3d^3-240bc^2d^2e+111b^2cde^2-b^3e^3)\sqrt{bx+cx^2}\,\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{63\sqrt{d}e^{11/2}(cd-be)^2\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}
\end{aligned}
$$

output
```
10/63*(-b^2*e^2-32*b*c*d*e+96*c^2*d^2)*x*(c*x^2+b*x)^(1/2)/e^4/(e*x+d)^(7/
2)+10/9*c*(-7*b*e+16*c*d)*x^2*(c*x^2+b*x)^(1/2)/e^3/(e*x+d)^(7/2)+2/21*(-b
^2*e^2-48*b*c*d*e+128*c^2*d^2)*(c*x^2+b*x)^(1/2)/e^5/(e*x+d)^(5/2)-2/63*(-
b*e+2*c*d)*(-b^2*e^2-128*b*c*d*e+128*c^2*d^2)*(c*x^2+b*x)^(1/2)/d/e^5/(-b*
e+c*d)/(e*x+d)^(3/2)+20/9*c*x*(c*x^2+b*x)^(3/2)/e^2/(e*x+d)^(7/2)-2/9*(c*x
^2+b*x)^(5/2)/e/(e*x+d)^(9/2)-4/63*(-b^4*e^4-7*b^3*c*d*e^3+135*b^2*c^2*d^2
*e^2-256*b*c^3*d^3*e+128*c^4*d^4)*(c*x^2+b*x)^(1/2)*EllipticE(e^(1/2)*x^(1
/2)/d^(1/2)/(1+e*x/d)^(1/2),(1-c*d/b/e)^(1/2))/d^(3/2)/e^(11/2)/(-b*e+c*d)
^2/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)+2/63*c*(-b^3*e^3+111*
b^2*c*d*e^2-240*b*c^2*d^2*e+128*c^3*d^3)*(c*x^2+b*x)^(1/2)*InverseJacobiAM
(arctan(e^(1/2)*x^(1/2)/d^(1/2)),(1-c*d/b/e)^(1/2))/d^(1/2)/e^(11/2)/(-b*e
+c*d)^2/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.75 (sec) , antiderivative size = 610, normalized size of antiderivative = 1.06

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{11/2}}\,dx =$$

$$-\frac{2(x(b+cx))^{5/2}\left(bex(b+cx)\left(7d^4(cd-be)^4-19d^3(cd-be)^2\left(2c^2d^2-3bcde+b^2e^2\right)(d+ex)+d^2(cd-\right.\right.}{}$$

input
```
Integrate[(b*x + c*x^2)^(5/2)/(d + e*x)^(11/2),x]
```

output
```
(-2*(x*(b + c*x))^(5/2)*(b*e*x*(b + c*x)*(7*d^4*(c*d - b*e)^4 - 19*d^3*(c*
d - b*e)^2*(2*c^2*d^2 - 3*b*c*d*e + b^2*e^2)*(d + e*x) + d^2*(c*d - b*e)^2
*(88*c^2*d^2 - 88*b*c*d*e + 15*b^2*e^2)*(d + e*x)^2 - d*(c*d - b*e)*(122*c
^3*d^3 - 183*b*c^2*d^2*e + 63*b^2*c*d*e^2 - b^3*e^3)*(d + e*x)^3 + (193*c^
4*d^4 - 386*b*c^3*d^3*e + 207*b^2*c^2*d^2*e^2 - 14*b^3*c*d*e^3 - 2*b^4*e^4
)*(d + e*x)^4) - Sqrt[b/c]*c*(d + e*x)^4*(-2*Sqrt[b/c]*(-128*c^4*d^4 + 256
*b*c^3*d^3*e - 135*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 + b^4*e^4)*(b + c*x)*(d
 + e*x) + (2*I)*b*e*(128*c^4*d^4 - 256*b*c^3*d^3*e + 135*b^2*c^2*d^2*e^2 -
 7*b^3*c*d*e^3 - b^4*e^4)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*Elli
pticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(128*c^4*d^4 - 27
2*b*c^3*d^3*e + 159*b^2*c^2*d^2*e^2 - 13*b^3*c*d*e^3 - 2*b^4*e^4)*Sqrt[1 +
 b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]]
, (c*d)/(b*e)])))/(63*b*d^2*e^6*(c*d - b*e)^2*x^3*(b + c*x)^3*(d + e*x)^(9
/2))
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 600, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {1161, 1229, 27, 1229, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{11/2}}\,dx$$

$$\Big\downarrow \text{1161}$$

$$\frac{5\int \frac{(b+2cx)(cx^2+bx)^{3/2}}{(d+ex)^{9/2}}dx}{9e}-\frac{2(bx+cx^2)^{5/2}}{9e(d+ex)^{9/2}}$$

$$\Big\downarrow \text{1229}$$

$$\frac{5\left(-\frac{6\int -\frac{\left(b\left(16c^2d^2-11bced-2b^2e^2\right)+c\left(32c^2d^2-32bced+b^2e^2\right)x\right)\sqrt{cx^2+bx}}{2(d+ex)^{5/2}}dx}{35de^2(cd-be)}-\frac{2(bx+cx^2)^{3/2}\left(ex\left(3b^2e^2-26bcde+26c^2d^2\right)+d\left(-2b^2e^2-11bcde+\right.\right.}{35de^2(d+ex)^{7/2}(cd-be)}\right.}{9e}$$

$$\frac{2(bx+cx^2)^{5/2}}{9e(d+ex)^{9/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{5\left(\frac{3\int \frac{\left(b\left(16c^2d^2-11bced-2b^2e^2\right)+c\left(32c^2d^2-32bced+b^2e^2\right)x\right)\sqrt{cx^2+bx}}{(d+ex)^{5/2}}dx}{35de^2(cd-be)}-\frac{2\left(bx+cx^2\right)^{3/2}\left(ex\left(3b^2e^2-26bcde+26c^2d^2\right)+d\left(-2b^2e^2-11bcde+16c\right.\right.}{35de^2(d+ex)^{7/2}(cd-be)}\right.}{9e}$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{9e(d+ex)^{9/2}}$$

$$\Big\downarrow \; 1229$$

$$5\left(\frac{3\left(-\frac{2\int -\frac{c\left(bd\left(128c^3d^3-240bc^2ed^2+111b^2ce^2d-b^3e^3\right)+2\left(128c^4d^4-256bc^3ed^3+135b^2c^2e^2d^2-7b^3ce^3d-b^4e^4\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3de^2(cd-be)}-\frac{2\sqrt{bx+cx^2}\left(cd^2\left(-b^3e^3+111b^2cd\right.\right.}{}\right.}{35de^2(cd-be)}\right.$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{9e(d+ex)^{9/2}}$$

$$\Big\downarrow \; 27$$

$$5\left(\frac{3\left(\frac{c\int \frac{bd\left(128c^3d^3-240bc^2ed^2+111b^2ce^2d-b^3e^3\right)+2\left(128c^4d^4-256bc^3ed^3+135b^2c^2e^2d^2-7b^3ce^3d-b^4e^4\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3de^2(cd-be)}-\frac{2\sqrt{bx+cx^2}\left(cd^2\left(-b^3e^3+111b^2cde^2-240\right.\right.}{}\right.}{35de^2(cd-be)}\right.$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{9e(d+ex)^{9/2}}$$

$$\Big\downarrow \; 1269$$

$$5\left(\frac{3\left(\frac{c\left(\frac{2\left(-b^4e^4-7b^3cde^3+135b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)\int\frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{d(cd-be)(2cd-be)\left(-b^2e^2-128bcde+128c^2d^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}\right)}{3de^2(cd-be)}-\cdots}{35de^2(cd-be)}\cdots\right.$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{9e(d+ex)^{9/2}}$$

$$\Big\downarrow \text{ 1169}$$

$$5\left(\frac{3\left(\frac{c\left(\frac{2\sqrt{x}\sqrt{b+cx}\left(-b^4e^4-7b^3cde^3+135b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)\int\frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\left(-b^2e^2-128bcde+128c^2d^2\right)\int\frac{\cdots}{\sqrt{x}\cdots}}{e\sqrt{bx+cx^2}}\cdots\right.}{3de^2(cd-be)}\cdots}{35de^2(cd-b\cdots}\cdots\right.$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{9e(d+ex)^{9/2}}$$

$$\Big\downarrow \text{ 122}$$

$$5\left(\frac{3\left(\frac{c\left(\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-b^4e^4-7b^3cde^3+135b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\left(-b^2e^2-128bcde+128c^2d^2\cdots\right.}{e\sqrt{bx+cx^2}\cdots}\right)}{3de^2(cd-be)}\cdots}{35de^2(c\cdots}\cdots\right.$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{9e(d+ex)^{9/2}}$$

$\Big\downarrow$ 120

$$5\left(\frac{3\left(\frac{c\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-b^4e^4-7b^3cde^3+135b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\left(-b^2e^2-128b\cdots\right.}{e\sqrt{bx+c\cdots}}\right)}{3de^2(cd-be)}\cdots}{35\cdots}\cdots\right.$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{9e(d+ex)^{9/2}}$$

$\Big\downarrow$ 127

$$5\left(\frac{3\left(\frac{c\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-b^4e^4-7b^3cde^3+135b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\left(-b^2\cdots\right.}{e\sqrt{bx\cdots}}\right.}{3de^2(cd-be)}\cdots}{\cdots}\right.$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{9e(d+ex)^{9/2}}$$

$$\Big\downarrow \; 126$$

$$5\left(\frac{3\left(\frac{c\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-b^4e^4-7b^3cde^3+135b^2c^2d^2e^2-256bc^3d^3e+128c^4d^4\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be\cdots}{\cdots}\right.}{3de^2(cd-be)}\cdots}{\cdots}\right.$$

$$\frac{2\left(bx+cx^2\right)^{5/2}}{9e(d+ex)^{9/2}}$$

input
```
Int[(b*x + c*x^2)^(5/2)/(d + e*x)^(11/2),x]
```

output
```
(-2*(b*x + c*x^2)^(5/2))/(9*e*(d + e*x)^(9/2)) + (5*((-2*(d*(16*c^2*d^2 -
11*b*c*d*e - 2*b^2*e^2) + e*(26*c^2*d^2 - 26*b*c*d*e + 3*b^2*e^2)*x)*(b*x
+ c*x^2)^(3/2))/(35*d*e^2*(c*d - b*e)*(d + e*x)^(7/2)) + (3*((-2*(c*d^2*(1
28*c^3*d^3 - 240*b*c^2*d^2*e + 111*b^2*c*d*e^2 - b^3*e^3) + e*(160*c^4*d^4
 - 320*b*c^3*d^3*e + 171*b^2*c^2*d^2*e^2 - 11*b^3*c*d*e^3 - 2*b^4*e^4)*x)*
Sqrt[b*x + c*x^2])/(3*d*e^2*(c*d - b*e)*(d + e*x)^(3/2)) + (c*((4*Sqrt[-b]
*(128*c^4*d^4 - 256*b*c^3*d^3*e + 135*b^2*c^2*d^2*e^2 - 7*b^3*c*d*e^3 - b^
4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*S
qrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c
*x^2]) - (2*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*(128*c^2*d^2 - 128*b*c*d*
e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[
(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b
*x + c*x^2])))/(3*d*e^2*(c*d - b*e))))/(35*d*e^2*(c*d - b*e))))/(9*e)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1161
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Si
mp[p/(e*(m + 1))   Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||
 LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a, b,
 c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1229
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Simp[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m +
p + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c
*(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(
m - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g
}, x] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3,
0]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 9.71 (sec) , antiderivative size = 995, normalized size of antiderivative = 1.73

method	result
elliptic	$\frac{\sqrt{x(cx+b)}\,\sqrt{(cx+b)x(ex+d)}\left(-\frac{2\left(b^2e^2-2bcde+c^2d^2\right)d^2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{9e^{10}\left(x+\frac{d}{e}\right)^5}+\frac{38d\left(b^2e^2-3bcde+2c^2d^2\right)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{63e^9\left(x+\frac{d}{e}\right)^4}-\right.}{}$
default	Expression too large to display

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((c*x+b)*x*(e*x+d))^(1/2)/x/(c*x+b)*(-2/
9*(b^2*e^2-2*b*c*d*e+c^2*d^2)*d^2/e^10*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/
2)/(x+d/e)^5+38/63*d*(b^2*e^2-3*b*c*d*e+2*c^2*d^2)/e^9*(c*e*x^3+b*e*x^2+c*
d*x^2+b*d*x)^(1/2)/(x+d/e)^4-2/63*(15*b^2*e^2-88*b*c*d*e+88*c^2*d^2)/e^8*(
c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(x+d/e)^3+2/63*(b^3*e^3-63*b^2*c*d*e^
2+183*b*c^2*d^2*e-122*c^3*d^3)/e^7/d/(b*e-c*d)*(c*e*x^3+b*e*x^2+c*d*x^2+b*
d*x)^(1/2)/(x+d/e)^2+2/63*(c*e*x^2+b*e*x)/e^6/d^2/(b*e-c*d)^2*(2*b^4*e^4+1
4*b^3*c*d*e^3-207*b^2*c^2*d^2*e^2+386*b*c^3*d^3*e-193*c^4*d^4)/((x+d/e)*(c
*e*x^2+b*e*x))^(1/2)+2*(c^2*(3*b*e-5*c*d)/e^6+1/63*c*(b^3*e^3-63*b^2*c*d*e
^2+183*b*c^2*d^2*e-122*c^3*d^3)/e^6/d/(b*e-c*d)+1/63/e^6/(b*e-c*d)*(2*b^4*
e^4+14*b^3*c*d*e^3-207*b^2*c^2*d^2*e^2+386*b*c^3*d^3*e-193*c^4*d^4)/d^2-1/
63*b/e^5/d^2/(b*e-c*d)^2*(2*b^4*e^4+14*b^3*c*d*e^3-207*b^2*c^2*d^2*e^2+386
*b*c^3*d^3*e-193*c^4*d^4))*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1
/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e
)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(c^3/e^5-1/63/e^5*c*(2*b^4*e^4+14*
b^3*c*d*e^3-207*b^2*c^2*d^2*e^2+386*b*c^3*d^3*e-193*c^4*d^4)/d^2/(b*e-c*d)
^2)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e
*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e)^(1/2
),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b
/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1675 vs. $2(512) = 1024$.

Time = 0.33 (sec) , antiderivative size = 1675, normalized size of antiderivative = 2.91

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{11/2}}\,dx = \text{Too large to display}$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(11/2),x, algorithm="fricas")
```

output
```
-2/189*((256*c^5*d^10 - 640*b*c^4*d^9*e + 478*b^2*c^3*d^8*e^2 - 77*b^3*c^2
*d^7*e^3 - 13*b^4*c*d^6*e^4 - 2*b^5*d^5*e^5 + (256*c^5*d^5*e^5 - 640*b*c^4
*d^4*e^6 + 478*b^2*c^3*d^3*e^7 - 77*b^3*c^2*d^2*e^8 - 13*b^4*c*d*e^9 - 2*b
^5*e^10)*x^5 + 5*(256*c^5*d^6*e^4 - 640*b*c^4*d^5*e^5 + 478*b^2*c^3*d^4*e^
6 - 77*b^3*c^2*d^3*e^7 - 13*b^4*c*d^2*e^8 - 2*b^5*d*e^9)*x^4 + 10*(256*c^5
*d^7*e^3 - 640*b*c^4*d^6*e^4 + 478*b^2*c^3*d^5*e^5 - 77*b^3*c^2*d^4*e^6 -
13*b^4*c*d^3*e^7 - 2*b^5*d^2*e^8)*x^3 + 10*(256*c^5*d^8*e^2 - 640*b*c^4*d^
7*e^3 + 478*b^2*c^3*d^6*e^4 - 77*b^3*c^2*d^5*e^5 - 13*b^4*c*d^4*e^6 - 2*b^
5*d^3*e^7)*x^2 + 5*(256*c^5*d^9*e - 640*b*c^4*d^8*e^2 + 478*b^2*c^3*d^7*e^
3 - 77*b^3*c^2*d^6*e^4 - 13*b^4*c*d^5*e^5 - 2*b^5*d^4*e^6)*x)*sqrt(c*e)*we
ierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^
3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x
 + c*d + b*e)/(c*e)) + 6*(128*c^5*d^9*e - 256*b*c^4*d^8*e^2 + 135*b^2*c^3*
d^7*e^3 - 7*b^3*c^2*d^6*e^4 - b^4*c*d^5*e^5 + (128*c^5*d^4*e^6 - 256*b*c^4
*d^3*e^7 + 135*b^2*c^3*d^2*e^8 - 7*b^3*c^2*d*e^9 - b^4*c*e^10)*x^5 + 5*(12
8*c^5*d^5*e^5 - 256*b*c^4*d^4*e^6 + 135*b^2*c^3*d^3*e^7 - 7*b^3*c^2*d^2*e^
8 - b^4*c*d*e^9)*x^4 + 10*(128*c^5*d^6*e^4 - 256*b*c^4*d^5*e^5 + 135*b^2*c
^3*d^4*e^6 - 7*b^3*c^2*d^3*e^7 - b^4*c*d^2*e^8)*x^3 + 10*(128*c^5*d^7*e^3
- 256*b*c^4*d^6*e^4 + 135*b^2*c^3*d^5*e^5 - 7*b^3*c^2*d^4*e^6 - b^4*c*d^3*
e^7)*x^2 + 5*(128*c^5*d^8*e^2 - 256*b*c^4*d^7*e^3 + 135*b^2*c^3*d^6*e^4...
```

Sympy [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{11/2}}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}}{(d+ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x)**(5/2)/(e*x+d)**(11/2),x)
```

output
```
Integral((x*(b + c*x))**(5/2)/(d + e*x)**(11/2), x)
```

Maxima [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{11/2}}\,dx = \int \frac{(cx^2+bx)^{\frac{5}{2}}}{(ex+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(11/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(11/2), x)
```

Giac [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{11/2}}\,dx = \int \frac{(cx^2+bx)^{\frac{5}{2}}}{(ex+d)^{\frac{11}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^(5/2)/(e*x+d)^(11/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(5/2)/(e*x + d)^(11/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{11/2}}\,dx=\int \frac{{\left(c\,x^2+b\,x\right)}^{5/2}}{{\left(d+e\,x\right)}^{11/2}}\,dx$$

input
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^(11/2),x)
```

output
```
int((b*x + c*x^2)^(5/2)/(d + e*x)^(11/2), x)
```

Reduce [F]

$$\int \frac{(bx+cx^2)^{5/2}}{(d+ex)^{11/2}}\,dx=\text{too large to display}$$

input
```
int((c*x^2+b*x)^(5/2)/(e*x+d)^(11/2),x)
```

output
```
( - 18*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**4*d*e**3 - 48*sqrt(x)*sqrt(d
 + e*x)*sqrt(b + c*x)*b**4*e**4*x + 114*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x
)*b**3*c*d**2*e**2 + 316*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**3*c*d*e**3
*x + 48*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**3*c*e**4*x**2 - 384*sqrt(x)
*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c**2*d**3*e - 1100*sqrt(x)*sqrt(d + e*x)
*sqrt(b + c*x)*b**2*c**2*d**2*e**2*x - 572*sqrt(x)*sqrt(d + e*x)*sqrt(b +
c*x)*b**2*c**2*d*e**3*x**2 - 144*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*
c**2*e**4*x**3 + 576*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**3*d**4 + 179
2*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c**3*d**3*e*x + 1420*sqrt(x)*sqrt(
d + e*x)*sqrt(b + c*x)*b*c**3*d**2*e**2*x**2 + 516*sqrt(x)*sqrt(d + e*x)*s
qrt(b + c*x)*b*c**3*d*e**3*x**3 + 48*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b
*c**3*e**4*x**4 - 384*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**4*d**4*x - 32
0*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**4*d**3*e*x**2 - 120*sqrt(x)*sqrt(
d + e*x)*sqrt(b + c*x)*c**4*d**2*e**2*x**3 - 12*sqrt(x)*sqrt(d + e*x)*sqrt
(b + c*x)*c**4*d*e**3*x**4 + 36*int((sqrt(d + e*x)*sqrt(b + c*x))/(4*sqrt(
x)*b**2*d**6*e + 24*sqrt(x)*b**2*d**5*e**2*x + 60*sqrt(x)*b**2*d**4*e**3*x
**2 + 80*sqrt(x)*b**2*d**3*e**4*x**3 + 60*sqrt(x)*b**2*d**2*e**5*x**4 + 24
*sqrt(x)*b**2*d*e**6*x**5 + 4*sqrt(x)*b**2*e**7*x**6 - sqrt(x)*b*c*d**7 -
2*sqrt(x)*b*c*d**6*e*x + 9*sqrt(x)*b*c*d**5*e**2*x**2 + 40*sqrt(x)*b*c*d**
4*e**3*x**3 + 65*sqrt(x)*b*c*d**3*e**4*x**4 + 54*sqrt(x)*b*c*d**2*e**5*...
```

3.205 $\int \frac{(d+ex)^{7/2}}{\sqrt{bx+cx^2}}\,dx$

Optimal result . . . 1697
Mathematica [C] (verified) . . . 1698
Rubi [A] (verified) . . . 1699
Maple [A] (verified) . . . 1703
Fricas [A] (verification not implemented) . . . 1704
Sympy [F] . . . 1705
Maxima [F] . . . 1705
Giac [F] . . . 1705
Mupad [F(-1)] . . . 1706
Reduce [F] . . . 1706

Optimal result

Integrand size = 23, antiderivative size = 433

$$\int \frac{(d+ex)^{7/2}}{\sqrt{bx+cx^2}}\,dx = \frac{16(2cd-be)\left(11c^2d^2-11bcde+6b^2e^2\right)x\sqrt{d+ex}}{105c^3\sqrt{bx+cx^2}} + \frac{2e(71c^2d^2-71bcde+24b^2e^2)\sqrt{d+ex}\sqrt{bx+cx^2}}{105c^3} + \frac{12e(2cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}}{35c^2} + \frac{2e(d+ex)^{5/2}\sqrt{bx+cx^2}}{7c} - \frac{16\sqrt{b}(2cd-be)\left(11c^2d^2-11bcde+6b^2e^2\right)\sqrt{x}\sqrt{d+ex}\,E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{105c^{7/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} + \frac{2\sqrt{b}(7cd-3be)\left(15c^2d^2-11bcde+8b^2e^2\right)\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{105c^{7/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$

output
```
16/105*(-b*e+2*c*d)*(6*b^2*e^2-11*b*c*d*e+11*c^2*d^2)*x*(e*x+d)^(1/2)/c^3/
(c*x^2+b*x)^(1/2)+2/105*e*(24*b^2*e^2-71*b*c*d*e+71*c^2*d^2)*(e*x+d)^(1/2)
*(c*x^2+b*x)^(1/2)/c^3+12/35*e*(-b*e+2*c*d)*(e*x+d)^(3/2)*(c*x^2+b*x)^(1/2
)/c^2+2/7*e*(e*x+d)^(5/2)*(c*x^2+b*x)^(1/2)/c-16/105*b^(1/2)*(-b*e+2*c*d)*
(6*b^2*e^2-11*b*c*d*e+11*c^2*d^2)*x^(1/2)*(e*x+d)^(1/2)*EllipticE(c^(1/2)*
x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/c^(7/2)/(b*(e*x+d)/d/(c
*x+b))^(1/2)/(c*x^2+b*x)^(1/2)+2/105*b^(1/2)*(-3*b*e+7*c*d)*(8*b^2*e^2-11*
b*c*d*e+15*c^2*d^2)*x^(1/2)*(e*x+d)^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x
^(1/2)/b^(1/2)),(1-b*e/c/d)^(1/2))/c^(7/2)/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*
x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 19.12 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^{7/2}}{\sqrt{bx+cx^2}}\,dx = \frac{2\sqrt{x}\left(\frac{8(22c^3d^3-33bc^2d^2e+23b^2cde^2-6b^3e^3)(b+cx)(d+ex)}{c\sqrt{x}}+e\sqrt{x}(b+cx)(d+ex)\left(24b^2e^2-bce(8\right.\right.}{}$$

input
```
Integrate[(d + e*x)^(7/2)/Sqrt[b*x + c*x^2],x]
```

output
```
(2*Sqrt[x]*((8*(22*c^3*d^3 - 33*b*c^2*d^2*e + 23*b^2*c*d*e^2 - 6*b^3*e^3)*
(b + c*x)*(d + e*x))/(c*Sqrt[x]) + e*Sqrt[x]*(b + c*x)*(d + e*x)*(24*b^2*e
^2 - b*c*e*(89*d + 18*e*x) + c^2*(122*d^2 + 66*d*e*x + 15*e^2*x^2)) + (8*I
)*Sqrt[b/c]*e*(22*c^3*d^3 - 33*b*c^2*d^2*e + 23*b^2*c*d*e^2 - 6*b^3*e^3)*S
qrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]
], (c*d)/(b*e)] + (I*Sqrt[b/c]*(105*c^4*d^4 - 298*b*c^3*d^3*e + 353*b^2*c^
2*d^2*e^2 - 208*b^3*c*d*e^3 + 48*b^4*e^4)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*
x)]*x*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])/b))/(105*c^3*S
qrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 399, normalized size of antiderivative = 0.92, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {1166, 27, 1236, 27, 1236, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \ 1166$$

$$\frac{2\int \frac{(d+ex)^{3/2}(d(7cd-be)+6e(2cd-be)x)}{2\sqrt{cx^2+bx}}dx}{7c}+\frac{2e\sqrt{bx+cx^2}(d+ex)^{5/2}}{7c}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{(d+ex)^{3/2}(d(7cd-be)+6e(2cd-be)x)}{\sqrt{cx^2+bx}}dx}{7c}+\frac{2e\sqrt{bx+cx^2}(d+ex)^{5/2}}{7c}$$

$$\Big\downarrow \ 1236$$

$$\frac{\frac{2\int \frac{\sqrt{d+ex}\left(d\left(35c^2d^2-17bced+6b^2e^2\right)+e\left(71c^2d^2-71bced+24b^2e^2\right)x\right)}{2\sqrt{cx^2+bx}}dx}{5c}+\frac{12e\sqrt{bx+cx^2}(d+ex)^{3/2}(2cd-be)}{5c}}{7c}+\frac{2e\sqrt{bx+cx^2}(d+ex)^{5/2}}{7c}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\int \frac{\sqrt{d+ex}\left(d\left(35c^2d^2-17bced+6b^2e^2\right)+e\left(71c^2d^2-71bced+24b^2e^2\right)x\right)}{\sqrt{cx^2+bx}}dx}{5c}+\frac{12e\sqrt{bx+cx^2}(d+ex)^{3/2}(2cd-be)}{5c}}{7c}+\frac{2e\sqrt{bx+cx^2}(d+ex)^{5/2}}{7c}$$

$$\Big\downarrow \ 1236$$

$$\frac{\frac{\frac{2\int \frac{d(7cd-3be)\left(15c^2d^2-11bced+8b^2e^2\right)+8e(2cd-be)\left(11c^2d^2-11bced+6b^2e^2\right)x}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{2e\sqrt{bx+cx^2}\sqrt{d+ex}\left(24b^2e^2-71bcde+71c^2d^2\right)}{3c}}{5c}+\frac{12e\sqrt{bx+cx^2}(d+ex)}{5c}}{7c}+\frac{2e\sqrt{bx+cx^2}(d+ex)^{5/2}}{7c}$$

$$\downarrow 27$$

$$\frac{\frac{\frac{\int \frac{d(7cd-3be)\left(15c^2d^2-11bced+8b^2e^2\right)+8e(2cd-be)\left(11c^2d^2-11bced+6b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{2e\sqrt{bx+cx^2}\sqrt{d+ex}\left(24b^2e^2-71bcde+71c^2d^2\right)}{3c}}{5c}+\frac{12e\sqrt{bx+cx^2}(d+ex)^{3/}}{5c}}{7c}$$

$$\frac{2e\sqrt{bx+cx^2}(d+ex)^{5/2}}{7c}$$

$$\downarrow 1269$$

$$\frac{\frac{8(2cd-be)\left(6b^2e^2-11bcde+11c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx-d(cd-be)\left(24b^2e^2-71bcde+71c^2d^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{2e\sqrt{bx+cx^2}\sqrt{d+ex}\left(24b^2e^2-71bcde+71c^2d^2\right)}{3c}}{5c}$$
$$7c$$

$$\frac{2e\sqrt{bx+cx^2}(d+ex)^{5/2}}{7c}$$

$$\downarrow 1169$$

$$\frac{\frac{\frac{8\sqrt{x}\sqrt{b+cx}(2cd-be)\left(6b^2e^2-11bcde+11c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{\sqrt{bx+cx^2}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\left(24b^2e^2-71bcde+71c^2d^2\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}}{3c}+\frac{2e\sqrt{bx+cx^2}\sqrt{d+ex}\left(24b\right.}{3c}}{5c}$$
$$7c$$

$$\frac{2e\sqrt{bx+cx^2}(d+ex)^{5/2}}{7c}$$

$$\downarrow 122$$

$$\frac{\frac{\frac{8\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(6b^2e^2-11bcde+11c^2d^2\right)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\left(24b^2e^2-71bcde+71c^2d^2\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}}{3c}+2e\sqrt{bx+cx^2}\sqrt{d}}{5c}$$
$$7c$$

$$\frac{2e\sqrt{bx+cx^2}(d+ex)^{5/2}}{7c}$$

$$\downarrow 120$$

$$\frac{\frac{\frac{16\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(6b^2e^2-11bcde+11c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\left(24b^2e^2-71bcde+71c^2d^2\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}}{3c}+2}{5c}$$
$$7c$$

$$\frac{2e\sqrt{bx+cx^2}(d+ex)^{5/2}}{7c}$$

$$\downarrow 127$$

$$\frac{\frac{\frac{16\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(6b^2e^2-11bcde+11c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(24b^2e^2-71bcde+71c^2d^2\right)\int\frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}}}}{\sqrt{bx+cx^2}\sqrt{d+ex}}}{3c}}{5c}}{7c}$$

$$\frac{2e\sqrt{bx+cx^2}(d+ex)^{5/2}}{7c}$$

$$\Big\downarrow \text{126}$$

$$\frac{\frac{\frac{16\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(6b^2e^2-11bcde+11c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(24b^2e^2-71bcde+71c^2d^2\right)\text{EllipticF}\left(\text{a}\right.}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{d+ex}}}{3c}}{5c}}{7c}$$

$$\frac{2e\sqrt{bx+cx^2}(d+ex)^{5/2}}{7c}$$

input
```
Int[(d + e*x)^(7/2)/Sqrt[b*x + c*x^2],x]
```

output
```
(2*e*(d + e*x)^(5/2)*Sqrt[b*x + c*x^2])/(7*c) + ((12*e*(2*c*d - b*e)*(d +
e*x)^(3/2)*Sqrt[b*x + c*x^2])/(5*c) + ((2*e*(71*c^2*d^2 - 71*b*c*d*e + 24*
b^2*e^2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(3*c) + ((16*Sqrt[-b]*(2*c*d - b
*e)*(11*c^2*d^2 - 11*b*c*d*e + 6*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d
 + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[
c]*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[-b]*d*(c*d - b*e)*(71*c^
2*d^2 - 71*b*c*d*e + 24*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/
d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*Sq
rt[d + e*x]*Sqrt[b*x + c*x^2]))/(3*c))/(5*c))/(7*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 697, normalized size of antiderivative = 1.61

method	result
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(\frac{2e^3x^2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{7c}+\frac{2\left(4d\,e^3-\frac{2e^3(3be+3cd)}{7c}\right)x\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{5ce}+\frac{2\left(6d^2e^2-\frac{5e^3bd}{7c}-\frac{2\left(4d\,e^3-\frac{2}{}\right.}{}\right.}{}\right.}{}$
default	$\frac{2\sqrt{ex+d}\,\sqrt{x(cx+b)}\left(15c^4e^5x^5+48\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\text{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^4d\,e^4-208\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\right.}{}$

input
```
int((e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(2/7/c*e^3*x^2*(
c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2/5*(4*d*e^3-2/7/c*e^3*(3*b*e+3*c*d))
/c/e*x*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2/3*(6*d^2*e^2-5/7/c*e^3*b*d-
2/5*(4*d*e^3-2/7/c*e^3*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(c*e*x^3+b*e*
x^2+c*d*x^2+b*d*x)^(1/2)+2*(d^4-1/3*(6*d^2*e^2-5/7/c*e^3*b*d-2/5*(4*d*e^3-
2/7/c*e^3*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*b*d)*d/e*((x+d/e)/d*e)^(1/
2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*
x)^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(4*d^3*e
-3/5*(4*d*e^3-2/7/c*e^3*(3*b*e+3*c*d))/c/e*b*d-2/3*(6*d^2*e^2-5/7/c*e^3*b*
d-2/5*(4*d*e^3-2/7/c*e^3*(3*b*e+3*c*d))/c/e*(2*b*e+2*c*d))/c/e*(b*e+c*d))*
d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3
+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-
d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))
^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 467, normalized size of antiderivative = 1.08

$$\int \frac{(d+ex)^{7/2}}{\sqrt{bx+cx^2}}\,dx = \frac{2\left((139\,c^4d^4-278\,bc^3d^3e+347\,b^2c^2d^2e^2-208\,b^3cde^3+48\,b^4e^4)\sqrt{ce}\text{weierstrassPInver}\right.}{}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2/315*((139*c^4*d^4 - 278*b*c^3*d^3*e + 347*b^2*c^2*d^2*e^2 - 208*b^3*c*d*
e^3 + 48*b^4*e^4)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b
^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^
3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) - 24*(22*c^4*d^3*e - 33
*b*c^3*d^2*e^2 + 23*b^2*c^2*d*e^3 - 6*b^3*c*e^4)*sqrt(c*e)*weierstrassZeta
(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d
^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*
d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b
^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(
15*c^4*e^4*x^2 + 122*c^4*d^2*e^2 - 89*b*c^3*d*e^3 + 24*b^2*c^2*e^4 + 6*(11
*c^4*d*e^3 - 3*b*c^3*e^4)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/(c^5*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{7/2}}{\sqrt{bx+cx^2}}\,dx = \int \frac{(d+ex)^{\frac{7}{2}}}{\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate((e*x+d)**(7/2)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral((d + e*x)**(7/2)/sqrt(x*(b + c*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{7/2}}{\sqrt{bx+cx^2}}\,dx = \int \frac{(ex+d)^{\frac{7}{2}}}{\sqrt{cx^2+bx}}\,dx$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(7/2)/sqrt(c*x^2 + b*x), x)
```

Giac [F]

$$\int \frac{(d+ex)^{7/2}}{\sqrt{bx+cx^2}}\,dx = \int \frac{(ex+d)^{\frac{7}{2}}}{\sqrt{cx^2+bx}}\,dx$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(7/2)/sqrt(c*x^2 + b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{\sqrt{bx+cx^2}}\,dx = \int \frac{(d+e\,x)^{7/2}}{\sqrt{c\,x^2+b\,x}}\,dx$$

input

```
int((d + e*x)^(7/2)/(b*x + c*x^2)^(1/2),x)
```

output

```
int((d + e*x)^(7/2)/(b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{7/2}}{\sqrt{bx+cx^2}}\,dx = \text{Too large to display}$$

input

```
int((e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x)
```

output

```
(18*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b**2*d*e**3 - 12*sqrt(x)*sqrt(d +
e*x)*sqrt(b + c*x)*b**2*e**4*x - 66*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*
c*d**2*e**2 + 32*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c*d*e**3*x + 10*sqr
t(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*c*e**4*x**2 + 140*sqrt(x)*sqrt(d + e*x)
*sqrt(b + c*x)*c**2*d**3*e + 44*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**2*d
**2*e**2*x + 10*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c**2*d*e**3*x**2 + 24*
int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*
d**2 + 2*b*c*d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*b**4*
e**6 - 68*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**
2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2
),x)*b**3*c*d*e**5 + 40*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*
d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*x**2 + c**2*d**2*x +
 c**2*d*e*x**2),x)*b**2*c**2*d**2*e**4 + 44*int((sqrt(x)*sqrt(d + e*x)*sqr
t(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*
x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*b*c**3*d**3*e**3 - 88*int((sqrt(x)*
sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*
d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*c**4*d**4*e**2 - 9
*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b**2*d*e*x + b**2*e**2*x**2 +
b*c*d**2*x + 2*b*c*d*e*x**2 + b*c*e**2*x**3 + c**2*d**2*x**2 + c**2*d*e*x*
*3),x)*b**4*d**2*e**4 + 24*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b...
```

3.206 $\int \frac{(d+ex)^{5/2}}{\sqrt{bx+cx^2}}\,dx$

Optimal result . . . 1708
Mathematica [C] (verified) . . . 1709
Rubi [A] (verified) . . . 1709
Maple [A] (verified) . . . 1713
Fricas [A] (verification not implemented) . . . 1714
Sympy [F] . . . 1715
Maxima [F] . . . 1715
Giac [F] . . . 1715
Mupad [F(-1)] . . . 1716
Reduce [F] . . . 1716

Optimal result

Integrand size = 23, antiderivative size = 353

$$\int \frac{(d+ex)^{5/2}}{\sqrt{bx+cx^2}}\,dx = \frac{2(23c^2d^2-23bcde+8b^2e^2)\,x\sqrt{d+ex}}{15c^2\sqrt{bx+cx^2}} + \frac{8e(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{15c^2} + \frac{2e(d+ex)^{3/2}\sqrt{bx+cx^2}}{5c} - \frac{2\sqrt{b}(23c^2d^2-23bcde+8b^2e^2)\,\sqrt{x}\sqrt{d+ex}\,E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{15c^{5/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} + \frac{2\sqrt{b}(15c^2d^2-11bcde+4b^2e^2)\,\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{15c^{5/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$

output
```
2/15*(8*b^2*e^2-23*b*c*d*e+23*c^2*d^2)*x*(e*x+d)^(1/2)/c^2/(c*x^2+b*x)^(1/
2)+8/15*e*(-b*e+2*c*d)*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/c^2+2/5*e*(e*x+d)^(
3/2)*(c*x^2+b*x)^(1/2)/c-2/15*b^(1/2)*(8*b^2*e^2-23*b*c*d*e+23*c^2*d^2)*x^
(1/2)*(e*x+d)^(1/2)*EllipticE(c^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b
*e/c/d)^(1/2))/c^(5/2)/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)+2/15*
b^(1/2)*(4*b^2*e^2-11*b*c*d*e+15*c^2*d^2)*x^(1/2)*(e*x+d)^(1/2)*InverseJac
obiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e/c/d)^(1/2))/c^(5/2)/(b*(e*x+d
)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 17.72 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^{5/2}}{\sqrt{bx+cx^2}}\,dx = \frac{2\sqrt{x}\left(\frac{(23c^2d^2-23bcde+8b^2e^2)(b+cx)(d+ex)}{c\sqrt{x}}+e\sqrt{x}(b+cx)(d+ex)(11cd-4be+3cex)+i\sqrt{}\right.}{}$$

input
```
Integrate[(d + e*x)^(5/2)/Sqrt[b*x + c*x^2],x]
```

output
```
(2*Sqrt[x]*(((23*c^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*(b + c*x)*(d + e*x))/(c
*Sqrt[x]) + e*Sqrt[x]*(b + c*x)*(d + e*x)*(11*c*d - 4*b*e + 3*c*e*x) + I*S
qrt[b/c]*e*(23*c^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1
+ d/(e*x)]*x*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - (I*Sqr
t[b/c]*(-15*c^3*d^3 + 34*b*c^2*d^2*e - 27*b^2*c*d*e^2 + 8*b^3*e^3)*Sqrt[1
+ b/(c*x)]*Sqrt[1 + d/(e*x)]*x*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*
d)/(b*e)])/b))/(15*c^2*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.89, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1166, 27, 1236, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{\sqrt{bx+cx^2}}\,dx$$

$$\Big\downarrow \; 1166$$

$$\frac{2\int \frac{\sqrt{d+ex}(d(5cd-be)+4e(2cd-be)x)}{2\sqrt{cx^2+bx}}dx}{5c}+\frac{2e\sqrt{bx+cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{\sqrt{d+ex}(d(5cd-be)+4e(2cd-be)x)}{\sqrt{cx^2+bx}}dx}{5c}+\frac{2e\sqrt{bx+cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \text{1236}$$

$$\frac{\frac{2\int \frac{d\left(15c^2d^2-11bced+4b^2e^2\right)+e\left(23c^2d^2-23bced+8b^2e^2\right)x}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{8e\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}}{5c}+\frac{2e\sqrt{bx+cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{d\left(15c^2d^2-11bced+4b^2e^2\right)+e\left(23c^2d^2-23bced+8b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{8e\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}}{5c}+\frac{2e\sqrt{bx+cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \text{1269}$$

$$\frac{\frac{\left(8b^2e^2-23bcde+23c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx-4d(cd-be)(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{8e\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}}{5c}+\frac{2e\sqrt{bx+cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \text{1169}$$

$$\frac{\frac{\frac{\sqrt{x}\sqrt{b+cx}\left(8b^2e^2-23bcde+23c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{\sqrt{bx+cx^2}}-\frac{4d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}}{3c}+\frac{8e\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}}{5c}+\frac{2e\sqrt{bx+cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \text{122}$$

$$\frac{\frac{\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(8b^2e^2-23bcde+23c^2d^2\right)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}}{3c}+\frac{8e\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}}{5c}+\frac{2e\sqrt{bx+cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \text{120}$$

$$\frac{\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(8b^2e^2-23bcde+23c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}}{3c}+\frac{8e\sqrt{bx+cx^2}\sqrt{d+ex}(2cd}{3c}}{5c}$$

$$\frac{2e\sqrt{bx+cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \text{127}$$

$$\frac{\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(8b^2e^2-23bcde+23c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{\sqrt{bx+cx^2}\sqrt{d+ex}}}{3c}+\frac{8e\sqrt{bx+cx^2}\sqrt{}}{}}{5c}$$

$$\frac{2e\sqrt{bx+cx^2}(d+ex)^{3/2}}{5c}$$

$$\Big\downarrow \text{126}$$

$$\frac{\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(8b^2e^2-23bcde+23c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\text{ EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{d+ex}}}{3c}+\frac{8e}{}}{5c}$$

$$\frac{2e\sqrt{bx+cx^2}(d+ex)^{3/2}}{5c}$$

input
```
Int[(d + e*x)^(5/2)/Sqrt[b*x + c*x^2],x]
```

output
```
(2*e*(d + e*x)^(3/2)*Sqrt[b*x + c*x^2])/(5*c) + ((8*e*(2*c*d - b*e)*Sqrt[d
 + e*x]*Sqrt[b*x + c*x^2])/(3*c) + ((2*Sqrt[-b]*(23*c^2*d^2 - 23*b*c*d*e +
 8*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt
[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*Sqrt[1 + (e*x)/d]*Sqrt[b*x
+ c*x^2]) - (8*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)
/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/
(c*d)])/(Sqrt[c]*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]))/(3*c))/(5*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.43

method	result
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(\frac{2e^2x\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{5c}+\frac{2\left(3d\,e^2-\frac{2e^2(2be+2cd)}{5c}\right)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3ce}+\frac{2\left(d^3-\frac{\left(3d\,e^2-\frac{2e^2(2be+2cd)}{5c}\right)b}{3ce}\right.}{}\right.}{}$
default	$-\frac{2\sqrt{ex+d}\,\sqrt{x(cx+b)}\left(8\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\operatorname{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^3d\,e^3-27\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\operatorname{EllipticF}\left(\right.\right.}{}$

input
```
int((e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(2/5*e^2/c*x*(c*
e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2/3*(3*d*e^2-2/5*e^2/c*(2*b*e+2*c*d))/c
/e*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2*(d^3-1/3*(3*d*e^2-2/5*e^2/c*(2*
b*e+2*c*d))/c/e*b*d)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-
e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e)/d*e)
^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(3*d^2*e-3/5*e^2/c*b*d-2/3*(3*d*e^2-2/5*
e^2/c*(2*b*e+2*c*d))/c/e*(b*e+c*d))*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e
+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b
/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF((
(x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 401, normalized size of antiderivative = 1.14

$$\int \frac{(d+ex)^{5/2}}{\sqrt{bx+cx^2}}\,dx = \frac{2\left(\left(22\,c^3d^3-33\,bc^2d^2e+27\,b^2cde^2-8\,b^3e^3\right)\sqrt{ce}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(c^2d^2-bcde+b^2e^2\right)}{3\,c^2e^2},\right.\right.}{}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2/45*((22*c^3*d^3 - 33*b*c^2*d^2*e + 27*b^2*c*d*e^2 - 8*b^3*e^3)*sqrt(c*e)
*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2
*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*
e*x + c*d + b*e)/(c*e)) - 3*(23*c^3*d^2*e - 23*b*c^2*d*e^2 + 8*b^2*c*e^3)*
sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/
27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weie
rstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*
d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x +
 c*d + b*e)/(c*e))) + 3*(3*c^3*e^3*x + 11*c^3*d*e^2 - 4*b*c^2*e^3)*sqrt(c*
x^2 + b*x)*sqrt(e*x + d))/(c^4*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{5/2}}{\sqrt{bx+cx^2}}\,dx = \int \frac{(d+ex)^{\frac{5}{2}}}{\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate((e*x+d)**(5/2)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral((d + e*x)**(5/2)/sqrt(x*(b + c*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{\sqrt{bx+cx^2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{\sqrt{cx^2+bx}}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/sqrt(c*x^2 + b*x), x)
```

Giac [F]

$$\int \frac{(d+ex)^{5/2}}{\sqrt{bx+cx^2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{\sqrt{cx^2+bx}}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(5/2)/sqrt(c*x^2 + b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{\sqrt{bx+cx^2}}\,dx = \int \frac{(d+e\,x)^{5/2}}{\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((d + e*x)^(5/2)/(b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x)^(5/2)/(b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{5/2}}{\sqrt{bx+cx^2}}\,dx = \frac{-6\sqrt{x}\,\sqrt{ex+d}\,\sqrt{cx+b}\,bd\,e^2+4\sqrt{x}\,\sqrt{ex+d}\,\sqrt{cx+b}\,b\,e^3x+30\sqrt{x}\,\sqrt{ex+d}\,\sqrt{cx+b}}{}$$

input
```
int((e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x)
```

output

```
( - 6*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*b*d*e**2 + 4*sqrt(x)*sqrt(d + e*
x)*sqrt(b + c*x)*b*e**3*x + 30*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c*d**2*
e + 4*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*c*d*e**2*x - 8*int((sqrt(x)*sqrt
(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*
x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*b**3*e**5 + 15*int((sq
rt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 +
2*b*c*d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*b**2*c*d*e**
4 - 23*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x
 + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x
)*c**3*d**3*e**2 + 3*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b**2*d*e*x
 + b**2*e**2*x**2 + b*c*d**2*x + 2*b*c*d*e*x**2 + b*c*e**2*x**3 + c**2*d**
2*x**2 + c**2*d*e*x**3),x)*b**3*d**2*e**3 - 2*int((sqrt(x)*sqrt(d + e*x)*s
qrt(b + c*x))/(b**2*d*e*x + b**2*e**2*x**2 + b*c*d**2*x + 2*b*c*d*e*x**2 +
 b*c*e**2*x**3 + c**2*d**2*x**2 + c**2*d*e*x**3),x)*b**2*c*d**3*e**2 + 5*i
nt((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b**2*d*e*x + b**2*e**2*x**2 + b*
c*d**2*x + 2*b*c*d*e*x**2 + b*c*e**2*x**3 + c**2*d**2*x**2 + c**2*d*e*x**3
),x)*b*c**2*d**4*e + 10*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b**2*d*
e*x + b**2*e**2*x**2 + b*c*d**2*x + 2*b*c*d*e*x**2 + b*c*e**2*x**3 + c**2*
d**2*x**2 + c**2*d*e*x**3),x)*c**3*d**5)/(10*c*(b*e + c*d))
```

3.207 $\int \frac{(d+ex)^{3/2}}{\sqrt{bx+cx^2}}\,dx$

Optimal result 1718
Mathematica [C] (verified) 1719
Rubi [A] (verified) 1719
Maple [A] (verified) 1722
Fricas [A] (verification not implemented) 1723
Sympy [F] 1723
Maxima [F] 1724
Giac [F] 1724
Mupad [F(-1)] 1724
Reduce [F] 1725

Optimal result

Integrand size = 23, antiderivative size = 272

$$\int \frac{(d+ex)^{3/2}}{\sqrt{bx+cx^2}}\,dx = \frac{4(2cd-be)x\sqrt{d+ex}}{3c\sqrt{bx+cx^2}} + \frac{2e\sqrt{d+ex}\sqrt{bx+cx^2}}{3c} - \frac{4\sqrt{b}(2cd-be)\sqrt{x}\sqrt{d+ex}E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{3c^{3/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} + \frac{2\sqrt{b}(3cd-be)\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{3c^{3/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$

output
```
4/3*(-b*e+2*c*d)*x*(e*x+d)^(1/2)/c/(c*x^2+b*x)^(1/2)+2/3*e*(e*x+d)^(1/2)*(
c*x^2+b*x)^(1/2)/c-4/3*b^(1/2)*(-b*e+2*c*d)*x^(1/2)*(e*x+d)^(1/2)*Elliptic
E(c^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/c^(3/2)/(b*(e
*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)+2/3*b^(1/2)*(-b*e+3*c*d)*x^(1/2)*
(e*x+d)^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e/c/d)^
(1/2))/c^(3/2)/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.90

$$\int \frac{(d+ex)^{3/2}}{\sqrt{bx+cx^2}}\,dx = \frac{-2b(b+cx)(d+ex)(2be-c(4d+ex)) - 4ib\sqrt{\frac{b}{c}}ce(-2cd+be)\sqrt{1+\frac{b}{cx}}\sqrt{1+\frac{d}{ex}}x^{3/2}E}{3}$$

input
```
Integrate[(d + e*x)^(3/2)/Sqrt[b*x + c*x^2],x]
```

output
```
(-2*b*(b + c*x)*(d + e*x)*(2*b*e - c*(4*d + e*x)) - (4*I)*b*Sqrt[b/c]*c*e*
(-2*c*d + b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*Arc
Sinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + (2*I)*Sqrt[b/c]*c*(3*c^2*d^2 - 5*b
*c*d*e + 2*b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[
I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])/(3*b*c^2*Sqrt[x*(b + c*x)]*Sqr
t[d + e*x])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1166, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{ 1166}$$

$$\frac{2\int \frac{d(3cd-be)+2e(2cd-be)x}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{2e\sqrt{bx+cx^2}\sqrt{d+ex}}{3c}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{d(3cd-be)+2e(2cd-be)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{2e\sqrt{bx+cx^2}\sqrt{d+ex}}{3c}$$

$$\Big\downarrow \text{1269}$$

$$\frac{2(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx-d(cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{2e\sqrt{bx+cx^2}\sqrt{d+ex}}{3c}$$

$$\Big\downarrow \text{1169}$$

$$\frac{\frac{2\sqrt{x}\sqrt{b+cx}(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{\sqrt{bx+cx^2}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}}{3c}+\frac{2e\sqrt{bx+cx^2}\sqrt{d+ex}}{3c}$$

$$\Big\downarrow \text{122}$$

$$\frac{\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}}{3c}+\frac{2e\sqrt{bx+cx^2}\sqrt{d+ex}}{3c}$$

$$\Big\downarrow \text{120}$$

$$\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}}{3c}+\frac{2e\sqrt{bx+cx^2}\sqrt{d+ex}}{3c}$$

$$\Big\downarrow \text{127}$$

$$\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{\sqrt{bx+cx^2}\sqrt{d+ex}}}{3c}+\frac{2e\sqrt{bx+cx^2}\sqrt{d+ex}}{3c}$$

$$\Big\downarrow \text{126}$$

$$\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{d+ex}}}{3c}+\frac{2e\sqrt{bx+cx^2}\sqrt{d+ex}}{3c}$$

input
```
Int[(d + e*x)^(3/2)/Sqrt[b*x + c*x^2],x]
```

output
```
(2*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(3*c) + ((4*Sqrt[-b]*(2*c*d - b*e)*S
qrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/
Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (
2*Sqrt[-b]*d*(c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*Ellip
ticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*Sqrt[d + e
*x]*Sqrt[b*x + c*x^2]))/(3*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.40

method	result
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(\frac{2e\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3c}+\frac{2\left(d^2-\frac{ebd}{3c}\right)d\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}\sqrt{\frac{\frac{b}{c}+x}{-\frac{d}{e}+\frac{b}{c}}}\sqrt{-\frac{ex}{d}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}},\sqrt{-\frac{d}{e\left(-\frac{d}{e}+\frac{b}{c}\right)}}\right)}{e\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}+\frac{2\left(2 \cdots}{\cdots}\right.}{\sqrt{x(cx+\cdots}}$
default	$\frac{2\sqrt{ex+d}\,\sqrt{x(cx+b)}\left(2\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\text{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^2d\,e^2-5d^2b\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\text{EllipticF}\left(\cdots\right.\right.}{\cdots}$

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(2/3*e/c*(c*e*x^
3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2*(d^2-1/3*e/c*b*d)*d/e*((x+d/e)/d*e)^(1/2)
*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)
^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(2*d*e-2/3
*e/c*(b*e+c*d))*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d
)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/
e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(
-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.29

$$\int \frac{(d+ex)^{3/2}}{\sqrt{bx+cx^2}}\,dx = \frac{2\left(3\sqrt{cx^2+bx}\sqrt{ex+d}c^2e^2+(5c^2d^2-5bcde+2b^2e^2)\sqrt{ce}\text{weierstrassPInverse}\left(\frac{4(c^2d\cdots}{\cdots}\right.\cdots\right)}{\cdots}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2/9*(3*sqrt(c*x^2 + b*x)*sqrt(e*x + d)*c^2*e^2 + (5*c^2*d^2 - 5*b*c*d*e +
2*b^2*e^2)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)
/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/
(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) - 6*(2*c^2*d*e - b*c*e^2)*sqrt
(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(
2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstr
assPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3
- 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d
 + b*e)/(c*e))))/(c^3*e)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{bx+cx^2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{\sqrt{x(b+cx)}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral((d + e*x)**(3/2)/sqrt(x*(b + c*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{bx+cx^2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+bx}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/sqrt(c*x^2 + b*x), x)
```

Giac [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{bx+cx^2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{\sqrt{cx^2+bx}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(3/2)/sqrt(c*x^2 + b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{\sqrt{bx+cx^2}}\,dx = \int \frac{(d+e\,x)^{3/2}}{\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((d + e*x)^(3/2)/(b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x)^(3/2)/(b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{3/2}}{\sqrt{bx+cx^2}}\,dx = \frac{2\sqrt{x}\,\sqrt{ex+d}\,\sqrt{cx+b}\,de+\left(\int \frac{\sqrt{x}\,\sqrt{ex+d}\,\sqrt{cx+b}\,x}{bc\,e^2x^2+c^2de\,x^2+b^2e^2x+2bcdex+c^2d^2x+b^2de+bc\,d^2}dx\right)b^2e^4-\left(\int \right.}{}$$

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x)
```

output
```
(2*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x)*d*e + int((sqrt(x)*sqrt(d + e*x)*sq
rt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*x + b*c*e**2
*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*b**2*e**4 - int((sqrt(x)*sqrt(d +
e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c*d*e*x + b
*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*b*c*d*e**3 - 2*int((sqrt(x)
*sqrt(d + e*x)*sqrt(b + c*x)*x)/(b**2*d*e + b**2*e**2*x + b*c*d**2 + 2*b*c
*d*e*x + b*c*e**2*x**2 + c**2*d**2*x + c**2*d*e*x**2),x)*c**2*d**2*e**2 +
int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b**2*d*e*x + b**2*e**2*x**2 + b
*c*d**2*x + 2*b*c*d*e*x**2 + b*c*e**2*x**3 + c**2*d**2*x**2 + c**2*d*e*x**
3),x)*b*c*d**3*e + int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b**2*d*e*x +
 b**2*e**2*x**2 + b*c*d**2*x + 2*b*c*d*e*x**2 + b*c*e**2*x**3 + c**2*d**2*
x**2 + c**2*d*e*x**3),x)*c**2*d**4)/(b*e + c*d)
```

3.208 $\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx$

Optimal result 1726
Mathematica [A] (verified) 1727
Rubi [A] (verified) 1727
Maple [A] (verified) 1729
Fricas [A] (verification not implemented) 1729
Sympy [F] 1730
Maxima [F] 1730
Giac [F] 1731
Mupad [F(-1)] 1731
Reduce [F] 1731

Optimal result

Integrand size = 23, antiderivative size = 206

$$\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx = \frac{2x\sqrt{d+ex}}{\sqrt{bx+cx^2}} - \frac{2\sqrt{b}\sqrt{x}\sqrt{d+ex}\,E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{\sqrt{c}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} + \frac{2\sqrt{b}\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{\sqrt{c}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$

output
```
2*x*(e*x+d)^(1/2)/(c*x^2+b*x)^(1/2)-2*b^(1/2)*x^(1/2)*(e*x+d)^(1/2)*Ellipt
icE(c^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/c^(1/2)/(b*
(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)+2*b^(1/2)*x^(1/2)*(e*x+d)^(1/2)
*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e/c/d)^(1/2))/c^(1/2
)/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 4.88 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx = -\frac{2\left(c+\frac{b}{x}\right)\sqrt{x}\sqrt{d+ex}\left(-\sqrt{x}+\frac{d\sqrt{1+\frac{d}{ex}}E\left(\arcsin\left(\frac{\sqrt{-\frac{d}{e}}}{\sqrt{x}}\right)|\frac{be}{cd}\right)}{\sqrt{-\frac{d}{e}}\sqrt{1+\frac{b}{cx}}\left(e+\frac{d}{x}\right)}\right)}{c\sqrt{x(b+cx)}}$$

input
```
Integrate[Sqrt[d + e*x]/Sqrt[b*x + c*x^2],x]
```

output
```
(-2*(c + b/x)*Sqrt[x]*Sqrt[d + e*x]*(-Sqrt[x] + (d*Sqrt[1 + d/(e*x)]*Ellip
ticE[ArcSin[Sqrt[-(d/e)]/Sqrt[x]], (b*e)/(c*d)])/(Sqrt[-(d/e)]*Sqrt[1 + b/
(c*x)]*(e + d/x))))/(c*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.46, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {1169, 122, 120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{ 1169}$$

$$\frac{\sqrt{x}\sqrt{b+cx}\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{ 122}$$

$$\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}$$

$$\Big\downarrow \text{ 120}$$

$$\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}$$

input
```
Int[Sqrt[d + e*x]/Sqrt[b*x + c*x^2],x]
```

output
```
(2*Sqrt[-b]*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt
[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*Sqrt[1 + (e*x)/d]*Sqrt[b*x
+ c*x^2])
```

Defintions of rubi rules used

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.04

method	result
default	$-\frac{2\sqrt{ex+d}\,\sqrt{x(cx+b)}\,\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,d\left(b\,\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)e-\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)cd-\mathrm{Elliptic}\right.}{cex(ce\,x^2+bex+cdx+bd)}$
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(\frac{2d^2\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}\,\sqrt{\frac{\frac{b}{c}+x}{-\frac{d}{e}+\frac{b}{c}}}\,\sqrt{-\frac{ex}{d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}},\sqrt{-\frac{d}{e\left(-\frac{d}{e}+\frac{b}{c}\right)}}\right)}{e\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}+\frac{2d\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}\,\sqrt{\frac{\frac{b}{c}+x}{-\frac{d}{e}+\frac{b}{c}}}\,\sqrt{-\frac{ex}{d}}\left(\left(-\frac{d}{e}+\frac{b}{c}\right)\mathrm{E}\right.}{}\right.}{\sqrt{x(cx+b)}\,\sqrt{ex+d}}$

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))
^(1/2)*(-e*x/d)^(1/2)*d*(b*EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1
/2))*e-EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*c*d-EllipticE((
(e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b*e+EllipticE(((e*x+d)/d)^(1/2),(
-d*c/(b*e-c*d))^(1/2))*c*d)/c/e/x/(c*e*x^2+b*e*x+c*d*x+b*d)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.45

$$\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx =$$
$$-\frac{2\left(3\sqrt{ce}\,ce\,\mathrm{weierstrassZeta}\left(\frac{4\,(c^2d^2-bcde+b^2e^2)}{3\,c^2e^2},-\frac{4\,(2\,c^3d^3-3\,bc^2d^2e-3\,b^2cde^2+2\,b^3e^3)}{27\,c^3e^3},\mathrm{weierstrassPInverse}\left(\frac{4\,(c^2d\right.\right.\right.}{}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(c*e)*c*e*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c
^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^
3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -
4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/
3*(3*c*e*x + c*d + b*e)/(c*e))) - (2*c*d - b*e)*sqrt(c*e)*weierstrassPInve
rse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^
2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/
(c*e)))/(c^2*e)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/sqrt(x*(b + c*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/sqrt(c*x^2 + b*x), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx=\int \frac{\sqrt{ex+d}}{\sqrt{cx^2+bx}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)/sqrt(c*x^2 + b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx=\int \frac{\sqrt{d+e\,x}}{\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((d + e*x)^(1/2)/(b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x)^(1/2)/(b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{bx+cx^2}}\,dx=\int \frac{\sqrt{ex+d}\,\sqrt{cx+b}}{\sqrt{x}\,b+\sqrt{x}\,cx}dx$$

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x)
```

output
```
int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b + sqrt(x)*c*x),x)
```

3.209 $\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx$

Optimal result . . . 1732
Mathematica [A] (verified) . . . 1732
Rubi [A] (verified) . . . 1733
Maple [A] (verified) . . . 1734
Fricas [A] (verification not implemented) . . . 1735
Sympy [F] . . . 1735
Maxima [F] . . . 1735
Giac [F] . . . 1736
Mupad [F(-1)] . . . 1736
Reduce [F] . . . 1736

Optimal result

Integrand size = 23, antiderivative size = 93

$$\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx=\frac{2\sqrt{b}\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{\sqrt{c}d\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$

output
```
2*b^(1/2)*x^(1/2)*(e*x+d)^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(
1/2)),(1-b*e/c/d)^(1/2))/c^(1/2)/d/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)
^(1/2)
```

Mathematica [A] (verified)

Time = 3.71 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx=-\frac{2\sqrt{\frac{c+\frac{b}{x}}{c}}\sqrt{\frac{e+\frac{d}{x}}{e}}x^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{-\frac{b}{c}}}{\sqrt{x}}\right),\frac{cd}{be}\right)}{\sqrt{-\frac{b}{c}}\sqrt{x(b+cx)}\sqrt{d+ex}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*Sqrt[(c + b/x)/c]*Sqrt[(e + d/x)/e]*x^(3/2)*EllipticF[ArcSin[Sqrt[-(b/
c)]/Sqrt[x]], (c*d)/(b*e)])/(Sqrt[-(b/c)]*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {1169, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx+cx^2}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1169}$$

$$\frac{\sqrt{x}\sqrt{b+cx}\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{127}$$

$$\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}\int \frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{\sqrt{bx+cx^2}\sqrt{d+ex}}$$

$$\Big\downarrow \text{126}$$

$$\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{d+ex}}$$

input
```
Int[1/(Sqrt[d + e*x]*Sqrt[b*x + c*x^2]),x]
```

output
```
(2*Sqrt[-b]*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(
Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*Sqrt[d + e*x]*Sqrt[b*x
+ c*x^2])
```

Defintions of rubi rules used

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

Maple [A] (verified)

Time = 1.49 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.22

method	result	size
default	$\frac{2\,\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}}, \sqrt{-\frac{dc}{be-cd}}\right)\sqrt{-\frac{ex}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{\frac{ex+d}{d}}\,d\sqrt{ex+d}\,\sqrt{x(cx+b)}}{(ce\,x^{2}+bex+cdx+bd)ex}$	113
elliptic	$\frac{2\sqrt{(cx+b)x(ex+d)}\,d\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}\,\sqrt{\frac{\frac{b}{c}+x}{-\frac{d}{e}+\frac{b}{c}}}\,\sqrt{-\frac{ex}{d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}, \sqrt{-\frac{d}{e\left(-\frac{d}{e}+\frac{b}{c}\right)}}\right)}{\sqrt{x(cx+b)}\,\sqrt{ex+d}\,e\sqrt{ce\,x^{3}+be\,x^{2}+cd\,x^{2}+bdx}}$	146

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*(-e*x/d)^(1/2)*(e*(c
*x+b)/(b*e-c*d))^(1/2)*((e*x+d)/d)^(1/2)*d*(e*x+d)^(1/2)*(x*(c*x+b))^(1/2)
/(c*e*x^2+b*e*x+c*d*x+b*d)/e/x
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.16

$$\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx = \frac{2\,\sqrt{ce}\text{weierstrassPInverse}\left(\frac{4\,(c^2d^2-bcde+b^2e^2)}{3\,c^2e^2},-\frac{4\,(2\,c^3d^3-3\,bc^2d^2e-3\,b^2cde^2+2\,b^3e^3)}{27\,c^3e^3},\frac{3\,cex+cd+be}{3\,ce}\right)}{ce}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2
), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3)
, 1/3*(3*c*e*x + c*d + b*e)/(c*e))/(c*e)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{x\,(b+cx)}\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x*(b + c*x))*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx}\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx}\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x}\,\sqrt{d+e\,x}}\,dx$$

input
```
int(1/((b*x + c*x^2)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
int(1/((b*x + c*x^2)^(1/2)*(d + e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{bx+cx^2}}\,dx = \int \frac{\sqrt{x}\,\sqrt{ex+d}\,\sqrt{cx+b}}{ce\,x^3+be\,x^2+cd\,x^2+bdx}dx$$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(1/2),x)
```

output
```
int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b*d*x + b*e*x**2 + c*d*x**2 + c
*e*x**3),x)
```

3.210 $\int \frac{1}{(d+ex)^{3/2}\sqrt{bx+cx^2}}\,dx$

Optimal result . . . 1737
Mathematica [A] (verified) . . . 1738
Rubi [A] (verified) . . . 1738
Maple [A] (verified) . . . 1740
Fricas [B] (verification not implemented) . . . 1741
Sympy [F] . . . 1742
Maxima [F] . . . 1742
Giac [F] . . . 1742
Mupad [F(-1)] . . . 1743
Reduce [F] . . . 1743

Optimal result

Integrand size = 23, antiderivative size = 205

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{bx+cx^2}}\,dx = -\frac{2\sqrt{e}\sqrt{bx+cx^2}E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{\sqrt{d}(cd-be)\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}} + \frac{2c\sqrt{d}\sqrt{bx+cx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{b\sqrt{e}(cd-be)\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}$$

output

```
-2*e^(1/2)*(c*x^2+b*x)^(1/2)*EllipticE(e^(1/2)*x^(1/2)/d^(1/2)/(1+e*x/d)^(
1/2),(1-c*d/b/e)^(1/2))/d^(1/2)/(-b*e+c*d)/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(
1/2)/(e*x+d)^(1/2)+2*c*d^(1/2)*(c*x^2+b*x)^(1/2)*InverseJacobiAM(arctan(e^
(1/2)*x^(1/2)/d^(1/2)),(1-c*d/b/e)^(1/2))/b/e^(1/2)/(-b*e+c*d)/x^(1/2)/(d*
(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 4.58 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.62

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{bx+cx^2}}\,dx = \frac{2\sqrt{x(b+cx)}\left(d\sqrt{1+\frac{b}{cx}}+\sqrt{-\frac{d}{e}}e\sqrt{1+\frac{d}{ex}}\sqrt{x}E\left(\arcsin\left(\frac{\sqrt{-\frac{d}{e}}}{\sqrt{x}}\right)|\frac{be}{cd}\right)\right)}{d(cd-be)\sqrt{1+\frac{b}{cx}}x\sqrt{d+ex}}$$

input
```
Integrate[1/((d + e*x)^(3/2)*Sqrt[b*x + c*x^2]),x]
```

output
```
(2*Sqrt[x*(b + c*x)]*(d*Sqrt[1 + b/(c*x)] + Sqrt[-(d/e)]*e*Sqrt[1 + d/(e*x
)]*Sqrt[x]*EllipticE[ArcSin[Sqrt[-(d/e)]/Sqrt[x]], (b*e)/(c*d)]))/(d*(c*d
- b*e)*Sqrt[1 + b/(c*x)]*x*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1167, 27, 1169, 122, 120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx+cx^2}(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \; 1167$$

$$-\frac{2\int -\frac{c\sqrt{d+ex}}{2\sqrt{cx^2+bx}}dx}{d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{d\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \; 27$$

$$\frac{c\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{d\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \; 1169$$

$$\frac{c\sqrt{x}\sqrt{b+cx}\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{d\sqrt{bx+cx^2}(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{d\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{122}$$

$$\frac{c\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{d\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{d\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{120}$$

$$\frac{2\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{d\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{d\sqrt{d+ex}(cd-be)}$$

input
```
Int[1/((d + e*x)^(3/2)*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*e*Sqrt[b*x + c*x^2])/(d*(c*d - b*e)*Sqrt[d + e*x]) + (2*Sqrt[-b]*Sqrt[
c]*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[
x])/Sqrt[-b]], (b*e)/(c*d)])/(d*(c*d - b*e)*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c
*x^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

Maple [A] (verified)

Time = 2.44 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.76

method	result
default	$\frac{2\left(\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\ \operatorname{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)bde-\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\ \operatorname{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)c\,d^2-\cdots}{de\cdots}$
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(\frac{2ce\,x^2+2bex}{(be-cd)d\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2+bex\right)}}+\frac{2\left(\frac{1}{d}-\frac{be}{d(be-cd)}\right)d\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}\,\sqrt{\frac{\frac{b}{c}+x}{-\frac{d}{e}+\frac{b}{c}}}\,\sqrt{-\frac{ex}{d}}\ \operatorname{EllipticF}\left(\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}},\sqrt{-\frac{d}{e\left(-\frac{d}{e}+\frac{b}{c}\right)}}\cdots\right.}{e\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}\cdots\right)}{\sqrt{x(cx+b)}\,\sqrt{\cdots}}$

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(
((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b*d*e-((e*x+d)/d)^(1/2)*(e*(c*x+
b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(-d*c/(b*e-
c*d))^(1/2))*c*d^2-((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^
(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b*d*e+((e*x+d)/d
)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(
1/2),(-d*c/(b*e-c*d))^(1/2))*c*d^2+x^2*c*e^2+x*b*e^2)*(x*(c*x+b))^(1/2)*(e
*x+d)^(1/2)/d/e/(b*e-c*d)/x/(c*e*x^2+b*e*x+c*d*x+b*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(182) = 364$.

Time = 0.13 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.88

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{bx+cx^2}}\,dx = -\frac{2\left(3\sqrt{cx^2+bx}\sqrt{ex+d}ce^2-(2cd^2-bde+(2cde-be^2)x)\sqrt{ce}\,\text{weierstrassPInverse}\left(\frac{4(c^2d^2-bcde+b^2e^2)}{3c^2e^2},-\right.\right.}{}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(3*sqrt(c*x^2 + b*x)*sqrt(e*x + d)*c*e^2 - (2*c*d^2 - b*d*e + (2*c*d*
e - b*e^2)*x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e
^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^
3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*(c*e^2*x + c*d*e)*sqrt(
c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2
*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstra
ssPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 -
 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d
+ b*e)/(c*e))))/(c^2*d^3*e - b*c*d^2*e^2 + (c^2*d^2*e^2 - b*c*d*e^3)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{x\,(b+cx)}\,(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x*(b + c*x))*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x)*(e*x + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x)*(e*x + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x}\,(d+e\,x)^{3/2}}\,dx$$

input
```
int(1/((b*x + c*x^2)^(1/2)*(d + e*x)^(3/2)),x)
```

output
```
int(1/((b*x + c*x^2)^(1/2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^{3/2}\sqrt{bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}\,\sqrt{cx+b}}{\sqrt{x}\,b\,d^2+2\sqrt{x}\,bdex+\sqrt{x}\,b\,e^2x^2+\sqrt{x}\,c\,d^2x+2\sqrt{x}\,cde\,x^2+\sqrt{x}\,c\,e^2x^3}\,dx$$

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(1/2),x)
```

output
```
int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b*d**2 + 2*sqrt(x)*b*d*e*x + sq
rt(x)*b*e**2*x**2 + sqrt(x)*c*d**2*x + 2*sqrt(x)*c*d*e*x**2 + sqrt(x)*c*e*
*2*x**3),x)
```

3.211 $\int \frac{1}{(d+ex)^{5/2}\sqrt{bx+cx^2}}\,dx$

Optimal result . . . 1744
Mathematica [C] (verified) . . . 1745
Rubi [A] (verified) . . . 1745
Maple [B] (verified) . . . 1749
Fricas [B] (verification not implemented) . . . 1750
Sympy [F] . . . 1751
Maxima [F] . . . 1751
Giac [F] . . . 1751
Mupad [F(-1)] . . . 1752
Reduce [F] . . . 1752

Optimal result

Integrand size = 23, antiderivative size = 267

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{bx+cx^2}}\,dx = -\frac{2e\sqrt{bx+cx^2}}{3d(cd-be)(d+ex)^{3/2}} - \frac{4\sqrt{e}(2cd-be)\sqrt{bx+cx^2}E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{3d^{3/2}(cd-be)^2\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}} + \frac{2c(3cd-be)\sqrt{bx+cx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{3b\sqrt{d}\sqrt{e}(cd-be)^2\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}$$

output
```
-2/3*e*(c*x^2+b*x)^(1/2)/d/(-b*e+c*d)/(e*x+d)^(3/2)-4/3*e^(1/2)*(-b*e+2*c*
d)*(c*x^2+b*x)^(1/2)*EllipticE(e^(1/2)*x^(1/2)/d^(1/2)/(1+e*x/d)^(1/2),(1-
c*d/b/e)^(1/2))/d^(3/2)/(-b*e+c*d)^2/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(
e*x+d)^(1/2)+2/3*c*(-b*e+3*c*d)*(c*x^2+b*x)^(1/2)*InverseJacobiAM(arctan(e
^(1/2)*x^(1/2)/d^(1/2)),(1-c*d/b/e)^(1/2))/b/d^(1/2)/e^(1/2)/(-b*e+c*d)^2/
x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 14.12 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.09

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{bx+cx^2}}\,dx =$$

$$-\frac{2\left(-bex(b+cx)(be(3d+2ex)-cd(5d+4ex))-\sqrt{\frac{b}{c}}c(d+ex)\left(-2\sqrt{\frac{b}{c}}(-2cd+be)(b+cx)(d+ex)+2\right.\right.}{}$$

input
```
Integrate[1/((d + e*x)^(5/2)*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*(-(b*e*x*(b + c*x)*(b*e*(3*d + 2*e*x) - c*d*(5*d + 4*e*x))) - Sqrt[b/c
]*c*(d + e*x)*(-2*Sqrt[b/c]*(-2*c*d + b*e)*(b + c*x)*(d + e*x) + (2*I)*b*e
*(2*c*d - b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*Arc
Sinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + I*(3*c^2*d^2 - 5*b*c*d*e + 2*b^2*e
^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b
/c]/Sqrt[x]], (c*d)/(b*e)])))/(3*b*d^2*(c*d - b*e)^2*Sqrt[x*(b + c*x)]*(d
+ e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 328, normalized size of antiderivative = 1.23, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1167, 27, 1237, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx+cx^2}(d+ex)^{5/2}}dx$$

$$\Big\downarrow \text{1167}$$

$$-\frac{2\int -\frac{3cd-2be-cex}{2(d+ex)^{3/2}\sqrt{cx^2+bx}}dx}{3d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{3d(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 27$$

$$\frac{\int \frac{3cd-2be-cex}{(d+ex)^{3/2}\sqrt{cx^2+bx}}dx}{3d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{3d(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 1237$$

$$\frac{-\frac{2\int -\frac{c(d(3cd-be)+2e(2cd-be)x)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{d(cd-be)}-\frac{4e\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}}{3d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{3d(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{c\int \frac{d(3cd-be)+2e(2cd-be)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{d(cd-be)}-\frac{4e\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}}{3d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{3d(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 1269$$

$$\frac{\frac{c\left(2(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx-d(cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx\right)}{d(cd-be)}-\frac{4e\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}}{3d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{3d(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 1169$$

$$\frac{\frac{c\left(\frac{2\sqrt{x}\sqrt{b+cx}(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{\sqrt{bx+cx^2}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}\right)}{d(cd-be)}-\frac{4e\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}}{3d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{3d(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 122$$

$$\frac{\frac{c\left(\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}\right)}{d(cd-be)}-\frac{4e\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}}{3d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{3d(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 120$$

$$\frac{\frac{c\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}\right)}{d(cd-be)}-\frac{4e\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}}{3d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{3d(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \; 127$$

$$\frac{\frac{c\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\int\frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{d(cd-be)}-\frac{4e\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}}{3d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{3d(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \; 126$$

$$\frac{\frac{c\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{d(cd-be)}-\frac{4e\sqrt{bx+cx^2}(2cd-be)}{d\sqrt{d+ex}(cd-be)}}{3d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{3d(d+ex)^{3/2}(cd-be)}$$

input
```
Int[1/((d + e*x)^(5/2)*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*e*Sqrt[b*x + c*x^2])/(3*d*(c*d - b*e)*(d + e*x)^(3/2)) + ((-4*e*(2*c*d
 - b*e)*Sqrt[b*x + c*x^2])/(d*(c*d - b*e)*Sqrt[d + e*x]) + (c*((4*Sqrt[-b]
*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(S
qrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*Sqrt[1 + (e*x)/d]*Sqrt[b
*x + c*x^2]) - (2*Sqrt[-b]*d*(c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1
+ (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sq
rt[c]*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])))/(d*(c*d - b*e)))/(3*d*(c*d - b*e)
)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(234) = 468$.

Time = 3.48 (sec) , antiderivative size = 510, normalized size of antiderivative = 1.91

method	result
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(\frac{2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3ed(be-cd)\left(x+\frac{d}{e}\right)^2}+\frac{4\left(ce\,x^2+bex\right)(be-2cd)}{3d^2(be-cd)^2\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2+bex\right)}}+\frac{2\left(\frac{c}{3d(be-cd)}+\frac{-\frac{4cd}{3}+\frac{2be}{3}}{d^2(be-cd)}-\frac{2be(be-2cd)}{3d^2(be-cd)^2}\right)d\sqrt{\frac{(x+\ldots}{d}}\ldots}{e\sqrt{ce\,x^3\ldots}}\ldots\right.}{\ldots}$
default	Expression too large to display

input
```
int(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(2/3/e/d/(b*e-c*
d)*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(x+d/e)^2+4/3*(c*e*x^2+b*e*x)/d^2
/(b*e-c*d)^2*(b*e-2*c*d)/((x+d/e)*(c*e*x^2+b*e*x))^(1/2)+2*(1/3*c/d/(b*e-c
*d)+2/3*(b*e-2*c*d)/(b*e-c*d)/d^2-2/3*b*e/d^2/(b*e-c*d)^2*(b*e-2*c*d))*d/e
*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*
e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))
^(1/2))-4/3*(b*e-2*c*d)*c/d/(b*e-c*d)^2*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e
+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b
/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF((
(x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(234) = 468$.

Time = 0.13 (sec) , antiderivative size = 606, normalized size of antiderivative = 2.27

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{bx+cx^2}}\,dx = \frac{2\left((5\,c^2d^4 - 5\,bcd^3e + 2\,b^2d^2e^2 + (5\,c^2d^2e^2 - 5\,bcde^3 + 2\,b^2e^4)x^2 + 2\,(5\,c^2d^3e\right.}{}$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2/9*((5*c^2*d^4 - 5*b*c*d^3*e + 2*b^2*d^2*e^2 + (5*c^2*d^2*e^2 - 5*b*c*d*e
^3 + 2*b^2*e^4)*x^2 + 2*(5*c^2*d^3*e - 5*b*c*d^2*e^2 + 2*b^2*d*e^3)*x)*sqr
t(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4
/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3
*(3*c*e*x + c*d + b*e)/(c*e)) - 6*(2*c^2*d^3*e - b*c*d^2*e^2 + (2*c^2*d*e^
3 - b*c*e^4)*x^2 + 2*(2*c^2*d^2*e^2 - b*c*d*e^3)*x)*sqrt(c*e)*weierstrassZ
eta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^
2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c
^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e -
3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) -
3*(5*c^2*d^2*e^2 - 3*b*c*d*e^3 + 2*(2*c^2*d*e^3 - b*c*e^4)*x)*sqrt(c*x^2 +
 b*x)*sqrt(e*x + d))/(c^3*d^6*e - 2*b*c^2*d^5*e^2 + b^2*c*d^4*e^3 + (c^3*d
^4*e^3 - 2*b*c^2*d^3*e^4 + b^2*c*d^2*e^5)*x^2 + 2*(c^3*d^5*e^2 - 2*b*c^2*d
^4*e^3 + b^2*c*d^3*e^4)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{x\,(b+cx)}\,(d+ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(5/2)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x*(b + c*x))*(d + e*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx}(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x)*(e*x + d)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx}(ex+d)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x)*(e*x + d)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x}\,{(d+e\,x)}^{5/2}}\,dx$$

input
```
int(1/((b*x + c*x^2)^(1/2)*(d + e*x)^(5/2)),x)
```

output
```
int(1/((b*x + c*x^2)^(1/2)*(d + e*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^{5/2}\sqrt{bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}\,\sqrt{cx+b}}{\sqrt{x}\,b\,d^3+3\sqrt{x}\,b\,d^2ex+3\sqrt{x}\,bd\,e^2x^2+\sqrt{x}\,b\,e^3x^3+\sqrt{x}\,c\,d^3x+3\sqrt{x}\,c\,d^2e$$

input
```
int(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(1/2),x)
```

output
```
int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b*d**3 + 3*sqrt(x)*b*d**2*e*x +
 3*sqrt(x)*b*d*e**2*x**2 + sqrt(x)*b*e**3*x**3 + sqrt(x)*c*d**3*x + 3*sqrt
(x)*c*d**2*e*x**2 + 3*sqrt(x)*c*d*e**2*x**3 + sqrt(x)*c*e**3*x**4),x)
```

3.212 $\int \frac{1}{(d+ex)^{7/2}\sqrt{bx+cx^2}}\,dx$

Optimal result 1753
Mathematica [C] (verified) 1754
Rubi [A] (verified) 1754
Maple [B] (verified) 1759
Fricas [B] (verification not implemented) 1760
Sympy [F] 1761
Maxima [F] 1761
Giac [F] 1761
Mupad [F(-1)] 1762
Reduce [F] 1762

Optimal result

Integrand size = 23, antiderivative size = 344

$$\int \frac{1}{(d+ex)^{7/2}\sqrt{bx+cx^2}}\,dx = -\frac{2e\sqrt{bx+cx^2}}{5d(cd-be)(d+ex)^{5/2}} - \frac{8e(2cd-be)\sqrt{bx+cx^2}}{15d^2(cd-be)^2(d+ex)^{3/2}} - \frac{2\sqrt{e}(23c^2d^2-23bcde+8b^2e^2)\sqrt{bx+cx^2}\,E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{15d^{5/2}(cd-be)^3\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}} + \frac{2c(15c^2d^2-11bcde+4b^2e^2)\sqrt{bx+cx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{15bd^{3/2}\sqrt{e}(cd-be)^3\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}$$

output
```
-2/5*e*(c*x^2+b*x)^(1/2)/d/(-b*e+c*d)/(e*x+d)^(5/2)-8/15*e*(-b*e+2*c*d)*(c
*x^2+b*x)^(1/2)/d^2/(-b*e+c*d)^2/(e*x+d)^(3/2)-2/15*e^(1/2)*(8*b^2*e^2-23*
b*c*d*e+23*c^2*d^2)*(c*x^2+b*x)^(1/2)*EllipticE(e^(1/2)*x^(1/2)/d^(1/2)/(1
+e*x/d)^(1/2),(1-c*d/b/e)^(1/2))/d^(5/2)/(-b*e+c*d)^3/x^(1/2)/(d*(c*x+b)/b
/(e*x+d))^(1/2)/(e*x+d)^(1/2)+2/15*c*(4*b^2*e^2-11*b*c*d*e+15*c^2*d^2)*(c*
x^2+b*x)^(1/2)*InverseJacobiAM(arctan(e^(1/2)*x^(1/2)/d^(1/2)),(1-c*d/b/e)
^(1/2))/b/d^(3/2)/e^(1/2)/(-b*e+c*d)^3/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)
/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.16 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.11

$$\int \frac{1}{(d+ex)^{7/2}\sqrt{bx+cx^2}}\,dx =$$
$$-\frac{2\Big(bex(b+cx)\left(3d^2(cd-be)^2+4d(cd-be)(2cd-be)(d+ex)+\left(23c^2d^2-23bcde+8b^2e^2\right)(d+ex)^2\right)-$$

input
```
Integrate[1/((d + e*x)^(7/2)*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*(b*e*x*(b + c*x)*(3*d^2*(c*d - b*e)^2 + 4*d*(c*d - b*e)*(2*c*d - b*e)*
(d + e*x) + (23*c^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*(d + e*x)^2) - Sqrt[b/c]
*c*(d + e*x)^2*(Sqrt[b/c]*(23*c^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*(b + c*x)*
(d + e*x) + I*b*e*(23*c^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*Sqrt[1 + b/(c*x)]*
Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b
*e)] + I*(15*c^3*d^3 - 34*b*c^2*d^2*e + 27*b^2*c*d*e^2 - 8*b^3*e^3)*Sqrt[1
 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x
]], (c*d)/(b*e)])))/(15*b*d^3*(c*d - b*e)^3*Sqrt[x*(b + c*x)]*(d + e*x)^(5
/2))
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.26, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {1167, 27, 1237, 27, 1237, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{bx+cx^2}(d+ex)^{7/2}}\,dx$$

$$\Big\downarrow \text{1167}$$

$$-\frac{2\int -\frac{5cd-4be-3cex}{2(d+ex)^{5/2}\sqrt{cx^2+bx}}dx}{5d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 27$$

$$\frac{\int \frac{5cd-4be-3cex}{(d+ex)^{5/2}\sqrt{cx^2+bx}}dx}{5d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 1237$$

$$\frac{-\frac{2\int -\frac{15c^2d^2-19bced+8b^2e^2-4ce(2cd-be)x}{2(d+ex)^{3/2}\sqrt{cx^2+bx}}dx}{3d(cd-be)}-\frac{8e\sqrt{bx+cx^2}(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{5d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{15c^2d^2-19bced+8b^2e^2-4ce(2cd-be)x}{(d+ex)^{3/2}\sqrt{cx^2+bx}}dx}{3d(cd-be)}-\frac{8e\sqrt{bx+cx^2}(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{5d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 1237$$

$$\frac{\frac{-\frac{2\int -\frac{c\left(d\left(15c^2d^2-11bced+4b^2e^2\right)+e\left(23c^2d^2-23bced+8b^2e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{d(cd-be)}-\frac{2e\sqrt{bx+cx^2}\left(8b^2e^2-23bcde+23c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{3d(cd-be)}-\frac{8e\sqrt{bx+cx^2}(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{5d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{c\int \frac{d\left(15c^2d^2-11bced+4b^2e^2\right)+e\left(23c^2d^2-23bced+8b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{d(cd-be)}-\frac{2e\sqrt{bx+cx^2}\left(8b^2e^2-23bcde+23c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{3d(cd-be)}-\frac{8e\sqrt{bx+cx^2}(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{5d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 1269$$

$$\frac{\frac{\frac{c\left(\left(8b^2e^2-23bcde+23c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx-4d(cd-be)(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx\right)}{d(cd-be)}-\frac{2e\sqrt{bx+cx^2}\left(8b^2e^2-23bcde+23c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{3d(cd-be)}-\frac{8e\sqrt{bx+cx^2}(2cd-be)}{3d(d+ex)^{3/2}(cd-be)}}{5d(cd-be)}-\frac{2e\sqrt{bx+cx^2}}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 1169$$

$$\frac{\frac{\frac{c\left(\frac{\sqrt{x}\sqrt{b+cx}\left(8b^2e^2-23bcde+23c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{\sqrt{bx+cx^2}}-\frac{4d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}\right)}{d(cd-be)}-\frac{2e\sqrt{bx+cx^2}\left(8b^2e^2-23bcde+23c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}}{3d(cd-be)}-\frac{8e\cdots}{3d(\cdots}}{5d(cd-be)}$$

$$\frac{2e\sqrt{bx+cx^2}}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 122$$

$$\frac{\frac{c\left(\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(8b^2e^2-23bcde+23c^2d^2\right)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}\right)}{d(cd-be)}-\frac{2e\sqrt{bx+cx^2}\left(8b^2e^2-23bcde+23c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}\cdots}{5d(cd-be)}$$

$$\frac{2e\sqrt{bx+cx^2}}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 120$$

$$\frac{\frac{c\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(8b^2e^2-23bcde+23c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{\sqrt{bx+cx^2}}\right)}{d(cd-be)}-\frac{2e\sqrt{bx+cx^2}\left(8b^2e^2-23\cdots\right.}{d\sqrt{d+ex}(cd-\cdots}}{5d(cd-be)}$$

$$\frac{2e\sqrt{bx+cx^2}}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 127$$

$$\frac{\frac{c\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(8b^2e^2-23bcde+23c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{d(cd-be)}-\frac{2e\sqrt{bx+cx^2}\cdots}{d\cdots}}{5d(cd-be)}$$

$$\frac{2e\sqrt{bx+cx^2}}{5d(d+ex)^{5/2}(cd-be)}$$

$$\Big\downarrow \; 126$$

$$\frac{\frac{\frac{c\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(8b^2e^2-23bcde+23c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{d(cd-be)}-}{3d(cd-be)}}{5d(cd-be)}$$

$$\frac{2e\sqrt{bx+cx^2}}{5d(d+ex)^{5/2}(cd-be)}$$

input
```
Int[1/((d + e*x)^(7/2)*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*e*Sqrt[b*x + c*x^2])/(5*d*(c*d - b*e)*(d + e*x)^(5/2)) + ((-8*e*(2*c*d
 - b*e)*Sqrt[b*x + c*x^2])/(3*d*(c*d - b*e)*(d + e*x)^(3/2)) + ((-2*e*(23*
c^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*Sqrt[b*x + c*x^2])/(d*(c*d - b*e)*Sqrt[d
 + e*x]) + (c*((2*Sqrt[-b]*(23*c^2*d^2 - 23*b*c*d*e + 8*b^2*e^2)*Sqrt[x]*S
qrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]
], (b*e)/(c*d)])/(Sqrt[c]*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (8*Sqrt[-
b]*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]
*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*Sqrt
[d + e*x]*Sqrt[b*x + c*x^2])))/(d*(c*d - b*e)))/(3*d*(c*d - b*e)))/(5*d*(c
*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1167
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)*(c*d^2 - b*d*e + a*e^2))   Int[
(d + e*x)^(m + 1)*Simp[c*d*(m + 1) - b*e*(m + p + 2) - c*e*(m + 2*p + 3)*x,
 x]*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[m
, -1] && ((LtQ[m, -1] && IntQuadraticQ[a, b, c, d, e, m, p, x]) || (SumSimp
lerQ[m, 1] && IntegerQ[p]) || ILtQ[Simplify[m + 2*p + 3], 0])
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 637 vs. $2(305) = 610$.

Time = 5.57 (sec) , antiderivative size = 638, normalized size of antiderivative = 1.85

method	result
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(\frac{2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{5(be-cd)d\,e^2\left(x+\frac{d}{e}\right)^3}+\frac{8(be-2cd)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{15e\,d^2(be-cd)^2\left(x+\frac{d}{e}\right)^2}+\frac{2\left(ce\,x^2+bex\right)\left(8b^2e^2-23bcde+23c^2d^2\right)}{15d^3(be-cd)^3\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2+bex\right)}}+\frac{2\left(\frac{4c(be}{15d^2(b}\right.}{}\right.}{}$
default	Expression too large to display

input
```
int(1/(e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(2/5/(b*e-c*d)/d
/e^2*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(x+d/e)^3+8/15*(b*e-2*c*d)/e/d^
2/(b*e-c*d)^2*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(x+d/e)^2+2/15*(c*e*x^
2+b*e*x)/d^3/(b*e-c*d)^3*(8*b^2*e^2-23*b*c*d*e+23*c^2*d^2)/((x+d/e)*(c*e*x
^2+b*e*x))^(1/2)+2*(4/15*c*(b*e-2*c*d)/d^2/(b*e-c*d)^2+1/15/(b*e-c*d)^2*(8
*b^2*e^2-23*b*c*d*e+23*c^2*d^2)/d^3-1/15*b*e/d^3/(b*e-c*d)^3*(8*b^2*e^2-23
*b*c*d*e+23*c^2*d^2))*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(
-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e)/d*e
)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-2/15*c*(8*b^2*e^2-23*b*c*d*e+23*c^2*d^2)/
d^2/(b*e-c*d)^3*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1
/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d
*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e
/(-d/e+b/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 950 vs. $2(305) = 610$.

Time = 0.14 (sec) , antiderivative size = 950, normalized size of antiderivative = 2.76

$$\int \frac{1}{(d+ex)^{7/2}\sqrt{bx+cx^2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2/45*((22*c^3*d^6 - 33*b*c^2*d^5*e + 27*b^2*c*d^4*e^2 - 8*b^3*d^3*e^3 + (2
2*c^3*d^3*e^3 - 33*b*c^2*d^2*e^4 + 27*b^2*c*d*e^5 - 8*b^3*e^6)*x^3 + 3*(22
*c^3*d^4*e^2 - 33*b*c^2*d^3*e^3 + 27*b^2*c*d^2*e^4 - 8*b^3*d*e^5)*x^2 + 3*
(22*c^3*d^5*e - 33*b*c^2*d^4*e^2 + 27*b^2*c*d^3*e^3 - 8*b^3*d^2*e^4)*x)*sq
rt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -
4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/
3*(3*c*e*x + c*d + b*e)/(c*e)) - 3*(23*c^3*d^5*e - 23*b*c^2*d^4*e^2 + 8*b^
2*c*d^3*e^3 + (23*c^3*d^2*e^4 - 23*b*c^2*d*e^5 + 8*b^2*c*e^6)*x^3 + 3*(23*
c^3*d^3*e^3 - 23*b*c^2*d^2*e^4 + 8*b^2*c*d*e^5)*x^2 + 3*(23*c^3*d^4*e^2 -
23*b*c^2*d^3*e^3 + 8*b^2*c*d^2*e^4)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*
d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b
^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*
d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2
 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) - 3*(34*c^3*d^4
*e^2 - 41*b*c^2*d^3*e^3 + 15*b^2*c*d^2*e^4 + (23*c^3*d^2*e^4 - 23*b*c^2*d*
e^5 + 8*b^2*c*e^6)*x^2 + 2*(27*c^3*d^3*e^3 - 29*b*c^2*d^2*e^4 + 10*b^2*c*d
*e^5)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/(c^4*d^9*e - 3*b*c^3*d^8*e^2 + 3
*b^2*c^2*d^7*e^3 - b^3*c*d^6*e^4 + (c^4*d^6*e^4 - 3*b*c^3*d^5*e^5 + 3*b^2*
c^2*d^4*e^6 - b^3*c*d^3*e^7)*x^3 + 3*(c^4*d^7*e^3 - 3*b*c^3*d^6*e^4 + 3*b^
2*c^2*d^5*e^5 - b^3*c*d^4*e^6)*x^2 + 3*(c^4*d^8*e^2 - 3*b*c^3*d^7*e^3 +...
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{7/2}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{x\,(b+cx)}\,(d+ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(7/2)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(x*(b + c*x))*(d + e*x)**(7/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{7/2}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx}(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x)*(e*x + d)^(7/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^{7/2}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{cx^2+bx}(ex+d)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(c*x^2 + b*x)*(e*x + d)^(7/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{7/2}\sqrt{bx+cx^2}}\,dx = \int \frac{1}{\sqrt{c\,x^2+b\,x}\,{\left(d+e\,x\right)}^{7/2}}\,dx$$

input
```
int(1/((b*x + c*x^2)^(1/2)*(d + e*x)^(7/2)),x)
```

output
```
int(1/((b*x + c*x^2)^(1/2)*(d + e*x)^(7/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^{7/2}\sqrt{bx+cx^2}}\,dx = \int \frac{\sqrt{ex+d}\,\sqrt{cx}}{\sqrt{x}\,b\,d^4+4\sqrt{x}\,b\,d^3ex+6\sqrt{x}\,b\,d^2e^2x^2+4\sqrt{x}\,bd\,e^3x^3+\sqrt{x}\,b\,e^4x^4+\sqrt{x}\,c}$$

input
```
int(1/(e*x+d)^(7/2)/(c*x^2+b*x)^(1/2),x)
```

output
```
int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b*d**4 + 4*sqrt(x)*b*d**3*e*x +
 6*sqrt(x)*b*d**2*e**2*x**2 + 4*sqrt(x)*b*d*e**3*x**3 + sqrt(x)*b*e**4*x**
4 + sqrt(x)*c*d**4*x + 4*sqrt(x)*c*d**3*e*x**2 + 6*sqrt(x)*c*d**2*e**2*x**
3 + 4*sqrt(x)*c*d*e**3*x**4 + sqrt(x)*c*e**4*x**5),x)
```

3.213 $\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{3/2}}\,dx$

Optimal result 1763
Mathematica [C] (verified) 1764
Rubi [A] (verified) 1765
Maple [A] (verified) 1770
Fricas [A] (verification not implemented) 1771
Sympy [F] 1771
Maxima [F] 1772
Giac [F] 1772
Mupad [F(-1)] 1772
Reduce [F] 1773

Optimal result

Integrand size = 23, antiderivative size = 443

$$
\begin{aligned}
\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{3/2}}\,dx &= \frac{2(2cd-be)\left(3c^2d^2-3bcde+8b^2e^2\right)x\sqrt{d+ex}}{3b^2c^2\sqrt{bx+cx^2}} \\
&+ \frac{2(cd-be)(d+ex)^{5/2}}{bc\sqrt{bx+cx^2}} + \frac{4e(3c^2d^2-3bcde+2b^2e^2)\sqrt{d+ex}\sqrt{bx+cx^2}}{3b^2c^2} \\
&- \frac{2d(2cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}}{b^2cx} \\
&- \frac{2(2cd-be)\left(3c^2d^2-3bcde+8b^2e^2\right)\sqrt{x}\sqrt{d+ex}E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{3b^{3/2}c^{5/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} \\
&+ \frac{2e(3c^2d^2+9bcde-4b^2e^2)\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{3\sqrt{b}c^{5/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}
\end{aligned}
$$

output
```
2/3*(-b*e+2*c*d)*(8*b^2*e^2-3*b*c*d*e+3*c^2*d^2)*x*(e*x+d)^(1/2)/b^2/c^2/(
c*x^2+b*x)^(1/2)+2*(-b*e+c*d)*(e*x+d)^(5/2)/b/c/(c*x^2+b*x)^(1/2)+4/3*e*(2
*b^2*e^2-3*b*c*d*e+3*c^2*d^2)*(e*x+d)^(1/2)*(c*x^2+b*x)^(1/2)/b^2/c^2-2*d*
(-b*e+2*c*d)*(e*x+d)^(3/2)*(c*x^2+b*x)^(1/2)/b^2/c/x-2/3*(-b*e+2*c*d)*(8*b
^2*e^2-3*b*c*d*e+3*c^2*d^2)*x^(1/2)*(e*x+d)^(1/2)*EllipticE(c^(1/2)*x^(1/2
)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/b^(3/2)/c^(5/2)/(b*(e*x+d)/d/
(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)+2/3*e*(-4*b^2*e^2+9*b*c*d*e+3*c^2*d^2)*x^
(1/2)*(e*x+d)^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e
/c/d)^(1/2))/b^(1/2)/c^(5/2)/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.89 (sec) , antiderivative size = 360, normalized size of antiderivative = 0.81

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{3/2}}\,dx =$$

$$-\frac{2\left(b(d+ex)\left(3(cd-be)^3x+3c^2d^3(b+cx)-b^2e^3x(b+cx)\right)-\sqrt{\frac{b}{c}}\left(\sqrt{\frac{b}{c}}(6c^3d^3-9bc^2d^2e+19b^2cde^2-8\right.\right.}{}$$

input
```
Integrate[(d + e*x)^(7/2)/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(b*(d + e*x)*(3*(c*d - b*e)^3*x + 3*c^2*d^3*(b + c*x) - b^2*e^3*x*(b +
 c*x)) - Sqrt[b/c]*(Sqrt[b/c]*(6*c^3*d^3 - 9*b*c^2*d^2*e + 19*b^2*c*d*e^2
- 8*b^3*e^3)*(b + c*x)*(d + e*x) + I*b*e*(6*c^3*d^3 - 9*b*c^2*d^2*e + 19*b
^2*c*d*e^2 - 8*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*Ellipt
icE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(3*c^3*d^3 - 18*b*c
^2*d^2*e + 23*b^2*c*d*e^2 - 8*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]
*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(3*b^3*c^
2*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.14 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.93, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {1164, 27, 1236, 27, 1236, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{1164}$$

$$-\frac{2\int -\frac{5e(d+ex)^{3/2}(bd+(2cd-be)x)}{2\sqrt{cx^2+bx}}dx}{b^2}-\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5e\int \frac{(d+ex)^{3/2}(bd+(2cd-be)x)}{\sqrt{cx^2+bx}}dx}{b^2}-\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1236}$$

$$\frac{5e\left(\frac{2\int \frac{\sqrt{d+ex}\left(bd(3cd+be)+2\left(3c^2d^2-3bced+2b^2e^2\right)x\right)}{2\sqrt{cx^2+bx}}dx}{5c}+\frac{2\sqrt{bx+cx^2}(d+ex)^{3/2}(2cd-be)}{5c}\right)}{b^2}-\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{5e\left(\frac{\int \frac{\sqrt{d+ex}\left(bd(3cd+be)+2\left(3c^2d^2-3bced+2b^2e^2\right)x\right)}{\sqrt{cx^2+bx}}dx}{5c}+\frac{2\sqrt{bx+cx^2}(d+ex)^{3/2}(2cd-be)}{5c}\right)}{b^2}-\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1236}$$

$$\frac{5e\left(\frac{\frac{2\int \frac{bd\left(3c^2d^2+9bced-4b^2e^2\right)+(2cd-be)\left(3c^2d^2-3bced+8b^2e^2\right)x}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{4\sqrt{bx+cx^2}\sqrt{d+ex}\left(2b^2e^2-3bcde+3c^2d^2\right)}{3c}}{5c}+\frac{2\sqrt{bx+cx^2}(d+ex)^{3/2}(2cd-be)}{5c}\right)}{b^2}$$

$$\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{5e\left(\frac{\frac{\int \frac{bd\left(3c^2d^2+9bced-4b^2e^2\right)+(2cd-be)\left(3c^2d^2-3bced+8b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{4\sqrt{bx+cx^2}\sqrt{d+ex}\left(2b^2e^2-3bcde+3c^2d^2\right)}{3c}}{5c}+\frac{2\sqrt{bx+cx^2}(d+ex)^{3/2}(2cd-be)}{5c}\right)}{b^2} -$$

$$\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \; 1269$$

$$\frac{5e\left(\frac{\frac{\frac{(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{2d(cd-be)\left(2b^2e^2-3bcde+3c^2d^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}}{3c}+\frac{4\sqrt{bx+cx^2}\sqrt{d+ex}\left(2b^2e^2-3bcde+3c^2d^2\right)}{3c}}{5c}+\right.}{b^2}$$

$$\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \; 1169$$

$$\frac{5e\left(\frac{\frac{\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\left(2b^2e^2-3bcde+3c^2d^2\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{3c}+\frac{4\sqrt{bx+cx^2}\sqrt{d+ex}\left(2b^2e\right.}{3c}}{5c}\right.}{b^2}$$

$$\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \; 122$$

$$\frac{5e\left(\frac{\frac{\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\left(2b^2e^2-3bcde+3c^2d^2\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{3c}+\frac{4\sqrt{bx+cx^2}\sqrt{d\cdots}}{\cdots}}{5c}\cdots\right)}{b^2}$$

$$\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$\downarrow$ 120

$$\frac{5e\left(\frac{\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\left(2b^2e^2-3bcde+3c^2d^2\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{3c}+\cdots}{5c}\cdots\right)}{b^2}$$

$$\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$\downarrow$ 127

$$\frac{5e\left(\frac{\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(2b^2e^2-3bcde+3c^2d^2\right)\int\frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\cdots}}\cdots}{e\sqrt{bx+cx^2}\sqrt{d+ex}}}{3c}\cdots}{5c}\cdots\right)}{b^2}$$

$$\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$\downarrow$ 126

$$\frac{5e\left(\frac{\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(2b^2e^2-3bcde+3c^2d^2\right)\text{EllipticF}\left(\text{ar}\cdots\right.}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}}{3c}\cdots}{5c}\cdots\right)}{b^2}$$

$$\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

input
```
Int[(d + e*x)^(7/2)/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(d + e*x)^(5/2)*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2]) + (5*
e*((2*(2*c*d - b*e)*(d + e*x)^(3/2)*Sqrt[b*x + c*x^2])/(5*c) + ((4*(3*c^2*
d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(3*c) + ((2*
Sqrt[-b]*(2*c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 8*b^2*e^2)*Sqrt[x]*Sqrt[1
+ (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*
e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (4*Sqrt[-b]*d
*(c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]
*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*
d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]))/(3*c))/(5*c)))/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127

```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1164

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169

```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1236

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 2.51 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.58

method	result
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(\frac{2\left(ce\,x^2+cdx\right)\left(b^3e^3-3d\,e^2b^2c+3d^2eb\,c^2-d^3c^3\right)}{c^3b^2\sqrt{\left(\frac{b}{c}+x\right)\left(ce\,x^2+cdx\right)}}-\frac{2\left(ce\,x^2+bex+cdx+bd\right)d^3}{b^2\sqrt{x\left(ce\,x^2+bex+cdx+bd\right)}}+\frac{2e^3\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3c^2}+\frac{2\left(\frac{e^2\left(b\right.}{}\right.}{}\right.}{}$
default	$\frac{2\left(8\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^4d\,e^4-23\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)\right.}{}$

input
```
int((e*x+d)^(7/2)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(2*(c*e*x^2+c*d*
x)*(b^3*e^3-3*b^2*c*d*e^2+3*b*c^2*d^2*e-c^3*d^3)/c^3/b^2/((b/c+x)*(c*e*x^2
+c*d*x))^(1/2)-2*(c*e*x^2+b*e*x+c*d*x+b*d)*d^3/b^2/(x*(c*e*x^2+b*e*x+c*d*x
+b*d))^(1/2)+2/3*e^3/c^2*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)+2*(e^2*(b^2
*e^2-4*b*c*d*e+6*c^2*d^2)/c^3-(b^3*e^3-3*b^2*c*d*e^2+3*b*c^2*d^2*e-c^3*d^3
)/c^3*(b*e-c*d)/b^2-1/c^2*d*(b^3*e^3-3*b^2*c*d*e^2+3*b*c^2*d^2*e-c^3*d^3)/
b^2-1/3*e^3/c^2*b*d)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-
e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e)/d*e)
^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(-1/c^2*e^3*(b*e-4*c*d)-(b^3*e^3-3*b^2*c
*d*e^2+3*b*c^2*d^2*e-c^3*d^3)/c^2*e/b^2+c*d^3*e/b^2-2/3*e^3/c^2*(b*e+c*d))
*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^
3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e)^(1/2),(
-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c)
)^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.37

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{3/2}}\,dx =$$
$$-\frac{2\left(((6\,c^5d^4-12\,bc^4d^3e-17\,b^2c^3d^2e^2+23\,b^3c^2de^3-8\,b^4ce^4)x^2+(6\,bc^4d^4-12\,b^2c^3d^3e-17\,b^3c^2d^2e^2+23\right.}{}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/9*(((6*c^5*d^4 - 12*b*c^4*d^3*e - 17*b^2*c^3*d^2*e^2 + 23*b^3*c^2*d*e^3
 - 8*b^4*c*e^4)*x^2 + (6*b*c^4*d^4 - 12*b^2*c^3*d^3*e - 17*b^3*c^2*d^2*e^2
 + 23*b^4*c*d*e^3 - 8*b^5*e^4)*x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d
^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^
2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*((6
*c^5*d^3*e - 9*b*c^4*d^2*e^2 + 19*b^2*c^3*d*e^3 - 8*b^3*c^2*e^4)*x^2 + (6*
b*c^4*d^3*e - 9*b^2*c^3*d^2*e^2 + 19*b^3*c^2*d*e^3 - 8*b^4*c*e^4)*x)*sqrt(
c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2
*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstra
ssPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 -
 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d
+ b*e)/(c*e))) - 3*(b^2*c^3*e^4*x^2 - 3*b*c^4*d^3*e - (6*c^5*d^3*e - 9*b*c
^4*d^2*e^2 + 9*b^2*c^3*d*e^3 - 4*b^3*c^2*e^4)*x)*sqrt(c*x^2 + b*x)*sqrt(e*
x + d))/(b^2*c^5*e*x^2 + b^3*c^4*e*x)
```

Sympy [F]

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{7}{2}}}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(7/2)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((d + e*x)**(7/2)/(x*(b + c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{7}{2}}}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(7/2)/(c*x^2 + b*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{7}{2}}}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(7/2)/(c*x^2 + b*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(d+e\,x)^{7/2}}{(c\,x^2+b\,x)^{3/2}}\,dx$$

input
```
int((d + e*x)^(7/2)/(b*x + c*x^2)^(3/2),x)
```

output
```
int((d + e*x)^(7/2)/(b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{3/2}}\,dx = \text{too large to display}$$

input
```
int((e*x+d)^(7/2)/(c*x^2+b*x)^(3/2),x)
```

output
```
( - 8*sqrt(d + e*x)*sqrt(b + c*x)*b**2*e**3*x + 20*sqrt(d + e*x)*sqrt(b +
c*x)*b*c*d*e**2*x + 2*sqrt(d + e*x)*sqrt(b + c*x)*b*c*e**3*x**2 - 6*sqrt(d
 + e*x)*sqrt(b + c*x)*c**2*d**3 + 2*sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*
x)*x)/(sqrt(x)*b**2*d + sqrt(x)*b**2*e*x + 2*sqrt(x)*b*c*d*x + 2*sqrt(x)*b
*c*e*x**2 + sqrt(x)*c**2*d*x**2 + sqrt(x)*c**2*e*x**3),x)*b**4*e**4 - 5*sq
rt(x)*int((sqrt(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**2*d + sqrt(x)*b**2*e
*x + 2*sqrt(x)*b*c*d*x + 2*sqrt(x)*b*c*e*x**2 + sqrt(x)*c**2*d*x**2 + sqrt
(x)*c**2*e*x**3),x)*b**3*c*d*e**3 + 2*sqrt(x)*int((sqrt(d + e*x)*sqrt(b +
c*x)*x)/(sqrt(x)*b**2*d + sqrt(x)*b**2*e*x + 2*sqrt(x)*b*c*d*x + 2*sqrt(x)
*b*c*e*x**2 + sqrt(x)*c**2*d*x**2 + sqrt(x)*c**2*e*x**3),x)*b**3*c*e**4*x
+ 6*sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**2*d + sqrt(x)*
b**2*e*x + 2*sqrt(x)*b*c*d*x + 2*sqrt(x)*b*c*e*x**2 + sqrt(x)*c**2*d*x**2
+ sqrt(x)*c**2*e*x**3),x)*b**2*c**2*d**2*e**2 - 5*sqrt(x)*int((sqrt(d + e*
x)*sqrt(b + c*x)*x)/(sqrt(x)*b**2*d + sqrt(x)*b**2*e*x + 2*sqrt(x)*b*c*d*x
 + 2*sqrt(x)*b*c*e*x**2 + sqrt(x)*c**2*d*x**2 + sqrt(x)*c**2*e*x**3),x)*b*
*2*c**2*d*e**3*x - 3*sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*
b**2*d + sqrt(x)*b**2*e*x + 2*sqrt(x)*b*c*d*x + 2*sqrt(x)*b*c*e*x**2 + sqr
t(x)*c**2*d*x**2 + sqrt(x)*c**2*e*x**3),x)*b*c**3*d**3*e + 6*sqrt(x)*int((
sqrt(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**2*d + sqrt(x)*b**2*e*x + 2*sqrt
(x)*b*c*d*x + 2*sqrt(x)*b*c*e*x**2 + sqrt(x)*c**2*d*x**2 + sqrt(x)*c**2...
```

3.214 $\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{3/2}}\,dx$

Optimal result 1774
Mathematica [C] (verified) 1775
Rubi [A] (verified) 1775
Maple [A] (verified) 1779
Fricas [A] (verification not implemented) 1780
Sympy [F] 1781
Maxima [F] 1781
Giac [F] 1781
Mupad [F(-1)] 1782
Reduce [F] 1782

Optimal result

Integrand size = 23, antiderivative size = 343

$$
\begin{aligned}
\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{3/2}}\,dx &= \frac{4(c^2d^2-bcde+b^2e^2)\,x\sqrt{d+ex}}{b^2c\sqrt{bx+cx^2}} \\
&+ \frac{2(cd-be)(d+ex)^{3/2}}{bc\sqrt{bx+cx^2}} - \frac{2d(2cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}}{b^2cx} \\
&- \frac{4(c^2d^2-bcde+b^2e^2)\sqrt{x}\sqrt{d+ex}\,E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{b^{3/2}c^{3/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} \\
&+ \frac{2e(cd+be)\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{\sqrt{b}c^{3/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}
\end{aligned}
$$

output

```
4*(b^2*e^2-b*c*d*e+c^2*d^2)*x*(e*x+d)^(1/2)/b^2/c/(c*x^2+b*x)^(1/2)+2*(-b*
e+c*d)*(e*x+d)^(3/2)/b/c/(c*x^2+b*x)^(1/2)-2*d*(-b*e+2*c*d)*(e*x+d)^(1/2)*
(c*x^2+b*x)^(1/2)/b^2/c/x-4*(b^2*e^2-b*c*d*e+c^2*d^2)*x^(1/2)*(e*x+d)^(1/2
)*EllipticE(c^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/b^(
3/2)/c^(3/2)/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)+2*e*(b*e+c*d)*x
^(1/2)*(e*x+d)^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*
e/c/d)^(1/2))/b^(1/2)/c^(3/2)/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2
)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.69 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.76

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{3/2}}\,dx = \frac{2b(d+ex)\left(c^2d^2+2b^2e^2+bce(-2d+ex)\right)+4i\sqrt{\frac{b}{c}}ce(c^2d^2-bcde+b^2e^2)\sqrt{1+\frac{b}{cx}}\sqrt{}}{}$$

input
```
Integrate[(d + e*x)^(5/2)/(b*x + c*x^2)^(3/2),x]
```

output
```
(2*b*(d + e*x)*(c^2*d^2 + 2*b^2*e^2 + b*c*e*(-2*d + e*x)) + (4*I)*Sqrt[b/c
]*c*e*(c^2*d^2 - b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^
(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - (2*I)*Sqrt[b/
c]*c*e*(c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x
)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])/(b^2*c^2*
Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.96, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1164, 27, 1236, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1164$$

$$-\frac{2\int -\frac{3e\sqrt{d+ex}(bd+(2cd-be)x)}{2\sqrt{cx^2+bx}}dx}{b^2}-\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{3e\int \frac{\sqrt{d+ex}(bd+(2cd-be)x)}{\sqrt{cx^2+bx}}dx}{b^2}-\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1236}$$

$$\frac{3e\left(\frac{2\int \frac{bd(cd+be)+2\left(c^2d^2-bced+b^2e^2\right)x}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}\right)}{b^2}-\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{3e\left(\frac{\int \frac{bd(cd+be)+2\left(c^2d^2-bced+b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}\right)}{b^2}-\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1269}$$

$$\frac{3e\left(\frac{\frac{2\left(b^2e^2-bcde+c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{d(cd-be)(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}}{3c}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}\right)}{b^2}-\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1169}$$

$$\frac{3e\left(\frac{\frac{2\sqrt{x}\sqrt{b+cx}\left(b^2e^2-bcde+c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{3c}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}\right)}{b^2}-\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{122}$$

$$\frac{3e\left(\frac{\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{3c}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-be)}{3c}\right)}{b^2}-\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{120}$$

$$\frac{3e\left(\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}}{3c}+\frac{2\sqrt{bx+cx^2}\sqrt{d+ex}(2cd-}{3c}\right.}{b^2}$$

$$\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{127}$$

$$\frac{3e\left(\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{d+ex}}}{3c}+\frac{2\sqrt{bx+cx^2}\sqrt{d}}{3}\right.}{b^2}$$

$$\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{126}$$

$$\frac{3e\left(\frac{\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}}{3c}+2\sqrt{}\right.}{b^2}$$

$$\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

input

```
Int[(d + e*x)^(5/2)/(b*x + c*x^2)^(3/2),x]
```

output

```
(-2*(d + e*x)^(3/2)*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2]) + (3*
e*((2*(2*c*d - b*e)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(3*c) + ((4*Sqrt[-b]*
(c^2*d^2 - b*c*d*e + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*Elli
pticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1
+ (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*Sq
rt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x
])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]))/(
3*c)))/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 2.20 (sec) , antiderivative size = 557, normalized size of antiderivative = 1.62

method	result
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(-\frac{2\left(ce\,x^{2}+cdx\right)\left(b^{2}e^{2}-2bcde+c^{2}d^{2}\right)}{b^{2}c^{2}\sqrt{\left(\frac{b}{c}+x\right)\left(ce\,x^{2}+cdx\right)}}-\frac{2\left(ce\,x^{2}+bex+cdx+bd\right)d^{2}}{b^{2}\sqrt{x\left(ce\,x^{2}+bex+cdx+bd\right)}}+\frac{2\left(-\frac{e^{2}(be-3cd)}{c^{2}}+\frac{\left(b^{2}e^{2}-2bcde+c^{2}d^{2}\right)(be-cd)}{c^{2}b^{2}}+\cdots\right.}{\cdots}\right.}{\cdots}$
default	$-\frac{2\left(2\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\;\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^{3}d\,e^{3}-3\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\;\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right.\cdots}{\cdots}$

input
```
int((e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(-2*(c*e*x^2+c*d
*x)*(b^2*e^2-2*b*c*d*e+c^2*d^2)/b^2/c^2/((b/c+x)*(c*e*x^2+c*d*x))^(1/2)-2*
(c*e*x^2+b*e*x+c*d*x+b*d)*d^2/b^2/(x*(c*e*x^2+b*e*x+c*d*x+b*d))^(1/2)+2*(-
e^2*(b*e-3*c*d)/c^2+(b^2*e^2-2*b*c*d*e+c^2*d^2)/c^2*(b*e-c*d)/b^2+1/c*d*(b
^2*e^2-2*b*c*d*e+c^2*d^2)/b^2)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c)
)^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x
+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(e^3/c+(b^2*e^2-2*b*c*d*e+c^2*
d^2)/c*e/b^2+c*d^2*e/b^2)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/
2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*Ellipt
icE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*
e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 519, normalized size of antiderivative = 1.51

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{3/2}}\,dx =$$
$$-\frac{2\left(((2\,c^4d^3-3\,bc^3d^2e-3\,b^2c^2de^2+2\,b^3ce^3)x^2+(2\,bc^3d^3-3\,b^2c^2d^2e-3\,b^3cde^2+2\,b^4e^3)x)\sqrt{ce}\text{weierstra}\right.}{}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(((2*c^4*d^3 - 3*b*c^3*d^2*e - 3*b^2*c^2*d*e^2 + 2*b^3*c*e^3)*x^2 + (
2*b*c^3*d^3 - 3*b^2*c^2*d^2*e - 3*b^3*c*d*e^2 + 2*b^4*e^3)*x)*sqrt(c*e)*we
ierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^
3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x
 + c*d + b*e)/(c*e)) + 6*((c^4*d^2*e - b*c^3*d*e^2 + b^2*c^2*e^3)*x^2 + (b
*c^3*d^2*e - b^2*c^2*d*e^2 + b^3*c*e^3)*x)*sqrt(c*e)*weierstrassZeta(4/3*(
c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e -
 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 -
b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d
*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(b*c^3*
d^2*e + (2*c^4*d^2*e - 2*b*c^3*d*e^2 + b^2*c^2*e^3)*x)*sqrt(c*x^2 + b*x)*s
qrt(e*x + d))/(b^2*c^4*e*x^2 + b^3*c^3*e*x)
```

Sympy [F]

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{5}{2}}}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(5/2)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((d + e*x)**(5/2)/(x*(b + c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/(c*x^2 + b*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(5/2)/(c*x^2 + b*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(d+e\,x)^{5/2}}{(c\,x^2+b\,x)^{3/2}}\,dx$$

input
```
int((d + e*x)^(5/2)/(b*x + c*x^2)^(3/2),x)
```

output
```
int((d + e*x)^(5/2)/(b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int((e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x)
```

output
```
(4*sqrt(d + e*x)*sqrt(b + c*x)*b*e**2*x - 4*sqrt(d + e*x)*sqrt(b + c*x)*c*
d**2 - sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**2*d + sqrt(
x)*b**2*e*x + 2*sqrt(x)*b*c*d*x + 2*sqrt(x)*b*c*e*x**2 + sqrt(x)*c**2*d*x*
*2 + sqrt(x)*c**2*e*x**3),x)*b**3*e**3 + 3*sqrt(x)*int((sqrt(d + e*x)*sqrt
(b + c*x)*x)/(sqrt(x)*b**2*d + sqrt(x)*b**2*e*x + 2*sqrt(x)*b*c*d*x + 2*sq
rt(x)*b*c*e*x**2 + sqrt(x)*c**2*d*x**2 + sqrt(x)*c**2*e*x**3),x)*b**2*c*d*
e**2 - sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**2*d + sqrt(
x)*b**2*e*x + 2*sqrt(x)*b*c*d*x + 2*sqrt(x)*b*c*e*x**2 + sqrt(x)*c**2*d*x*
*2 + sqrt(x)*c**2*e*x**3),x)*b**2*c*e**3*x - 2*sqrt(x)*int((sqrt(d + e*x)*
sqrt(b + c*x)*x)/(sqrt(x)*b**2*d + sqrt(x)*b**2*e*x + 2*sqrt(x)*b*c*d*x +
2*sqrt(x)*b*c*e*x**2 + sqrt(x)*c**2*d*x**2 + sqrt(x)*c**2*e*x**3),x)*b*c**
2*d**2*e + 3*sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**2*d +
 sqrt(x)*b**2*e*x + 2*sqrt(x)*b*c*d*x + 2*sqrt(x)*b*c*e*x**2 + sqrt(x)*c**
2*d*x**2 + sqrt(x)*c**2*e*x**3),x)*b*c**2*d*e**2*x - 2*sqrt(x)*int((sqrt(d
 + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**2*d + sqrt(x)*b**2*e*x + 2*sqrt(x)*b*
c*d*x + 2*sqrt(x)*b*c*e*x**2 + sqrt(x)*c**2*d*x**2 + sqrt(x)*c**2*e*x**3),
x)*c**3*d**2*e*x - 2*sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b*
*2*d + sqrt(x)*b**2*e*x + 2*sqrt(x)*b*c*d*x + 2*sqrt(x)*b*c*e*x**2 + sqrt(
x)*c**2*d*x**2 + sqrt(x)*c**2*e*x**3),x)*b**3*d*e**2 + 6*sqrt(x)*int((sqrt
(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**2*d + sqrt(x)*b**2*e*x + 2*sqrt(x)...
```

3.215 $\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{3/2}}\,dx$

Optimal result 1784
Mathematica [C] (verified) 1785
Rubi [A] (verified) 1785
Maple [A] (verified) 1789
Fricas [A] (verification not implemented) 1789
Sympy [F] 1790
Maxima [F] 1790
Giac [F] 1791
Mupad [F(-1)] 1791
Reduce [F] 1791

Optimal result

Integrand size = 23, antiderivative size = 268

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{3/2}}\,dx = \frac{2(cd-be)\sqrt{d+ex}}{bc\sqrt{bx+cx^2}} - \frac{2(2cd-be)\sqrt{d+ex}}{bc\sqrt{bx+cx^2}} - \frac{2(2cd-be)\sqrt{x}\sqrt{d+ex}E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{b^{3/2}\sqrt{c}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} + \frac{2e\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{\sqrt{b}\sqrt{c}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$

output
```
2*(-b*e+c*d)*(e*x+d)^(1/2)/b/c/(c*x^2+b*x)^(1/2)-2*(-b*e+2*c*d)*(e*x+d)^(1
/2)/b/c/(c*x^2+b*x)^(1/2)-2*(-b*e+2*c*d)*x^(1/2)*(e*x+d)^(1/2)*EllipticE(c
^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/b^(3/2)/c^(1/2)/
(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)+2*e*x^(1/2)*(e*x+d)^(1/2)*In
verseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e/c/d)^(1/2))/b^(1/2)/c
^(1/2)/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 9.79 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.78

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{3/2}}\,dx = \frac{-2i\sqrt{\frac{b}{c}}ce(-2cd+be)\sqrt{1+\frac{b}{cx}}\sqrt{1+\frac{d}{ex}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right)|\frac{cd}{be}\right)+2(cd-be)\left(b(\cdots\right.}{b^2c\sqrt{x(b+cx)}\sqrt{\cdots}}$$

input
```
Integrate[(d + e*x)^(3/2)/(b*x + c*x^2)^(3/2),x]
```

output
```
((-2*I)*Sqrt[b/c]*c*e*(-2*c*d + b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x
^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + 2*(c*d - b*e
)*(b*(d + e*x) - I*Sqrt[b/c]*c*e*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/
2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)]))/(b^2*c*Sqrt[x*(b
 + c*x)]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1164, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1164}$$

$$-\frac{2\int -\frac{e(bd+(2cd-be)x)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2}-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{e\int \frac{bd+(2cd-be)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2}-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1269}$$

$$\frac{e\left(\frac{(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{2d(cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}\right)}{b^2}-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1169}$$

$$\frac{e\left(\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2}-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{122}$$

$$\frac{e\left(\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2}-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{120}$$

$$\frac{e\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2}-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{127}$$

$$\frac{e\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{b^2}-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{126}$$

$$\frac{e\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{b^2}-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{b^2\sqrt{bx+cx^2}}$$

input
```
Int[(d + e*x)^(3/2)/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(b*d + (2*c*d - b*e)*x))/(b^2*Sqrt[b*x + c*x^2]) + (e*((
2*Sqrt[-b]*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE
[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*
x)/d]*Sqrt[b*x + c*x^2]) - (4*Sqrt[-b]*d*(c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x
)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)
/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])))/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 2.18 (sec) , antiderivative size = 454, normalized size of antiderivative = 1.69

method	result
default	$\frac{2\left(\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\ \mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^2 d\,e^2-d^2 b\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\ \mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)\cdots\right.}{}$
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(\frac{2\left(ce\,x^2+cdx\right)(be-cd)}{b^2c\sqrt{\left(\frac{b}{c}+x\right)\left(ce\,x^2+cdx\right)}}-\frac{2\left(ce\,x^2+bex+cdx+bd\right)d}{b^2\sqrt{x\left(ce\,x^2+bex+cdx+bd\right)}}+\frac{2\left(\frac{e^2}{c}-\frac{(be-cd)^2}{c\,b^2}-\frac{d(be-cd)}{b^2}\right)d\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}\sqrt{\frac{\frac{b}{c}+x}{-\frac{d}{e}+\frac{b}{c}}}\sqrt{-\frac{ex}{d}}\cdots}{e\sqrt{ce\,x^3+be\,x^2+cd\,x^2+b\cdots}}\cdots\right.}{}$

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(
((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^2*d*e^2-d^2*b*((e*x+d)/d)^(1/2
)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF(((e*x+d)/d)^(1/2),(
-d*c/(b*e-c*d))^(1/2))*e*c-((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(
-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b^2*d*e^
2+3*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticE
(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b*c*d^2*e-2*((e*x+d)/d)^(1/2)*(
e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*
c/(b*e-c*d))^(1/2))*c^2*d^3+e^3*x^2*c*b-2*d*e^2*c^2*x^2-2*d^2*e*c^2*x-d^2*
e*b*c)/x*(x*(c*x+b))^(1/2)/(c*x+b)/e/b^2/c/(e*x+d)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.69

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{3/2}}\,dx =$$

$$-\frac{2\left(\left((2\,c^3d^2-2\,bc^2de-b^2ce^2)x^2+(2\,bc^2d^2-2\,b^2cde-b^3e^2)x\right)\sqrt{ce}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(c^2d^2-bcde+b^2e^2)}{3\,c^2e^2},\cdots\right.\right.}{\cdots}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(((2*c^3*d^2 - 2*b*c^2*d*e - b^2*c*e^2)*x^2 + (2*b*c^2*d^2 - 2*b^2*c*
d*e - b^3*e^2)*x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b
^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^
3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*((2*c^3*d*e - b*c^2
*e^2)*x^2 + (2*b*c^2*d*e - b^2*c*e^2)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^
2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3
*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*
c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e
^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(b*c^2*d*
e + (2*c^3*d*e - b*c^2*e^2)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/(b^2*c^3*e
*x^2 + b^3*c^2*e*x)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{(x(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((d + e*x)**(3/2)/(x*(b + c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(c*x^2 + b*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(3/2)/(c*x^2 + b*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(d+e\,x)^{3/2}}{(c\,x^2+b\,x)^{3/2}}\,dx$$

input
```
int((d + e*x)^(3/2)/(b*x + c*x^2)^(3/2),x)
```

output
```
int((d + e*x)^(3/2)/(b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{3/2}}\,dx = \frac{-4\sqrt{ex+d}\,\sqrt{cx+b}\,bd+2\sqrt{ex+d}\,\sqrt{cx+b}\,bex-2\sqrt{ex+d}\,\sqrt{cx+b}\,cdx+3\sqrt{x}\,\Big(\int}{}$$

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x)
```

output

```
( - 4*sqrt(d + e*x)*sqrt(b + c*x)*b*d + 2*sqrt(d + e*x)*sqrt(b + c*x)*b*e*
x - 2*sqrt(d + e*x)*sqrt(b + c*x)*c*d*x + 3*sqrt(x)*int((sqrt(d + e*x)*sqr
t(b + c*x))/(sqrt(x)*b**2*d + sqrt(x)*b**2*e*x + 2*sqrt(x)*b*c*d*x + 2*sqr
t(x)*b*c*e*x**2 + sqrt(x)*c**2*d*x**2 + sqrt(x)*c**2*e*x**3),x)*b**3*d*e -
 3*sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**2*d + sqrt(x)*b**
2*e*x + 2*sqrt(x)*b*c*d*x + 2*sqrt(x)*b*c*e*x**2 + sqrt(x)*c**2*d*x**2 + s
qrt(x)*c**2*e*x**3),x)*b**2*c*d**2 + 3*sqrt(x)*int((sqrt(d + e*x)*sqrt(b +
 c*x))/(sqrt(x)*b**2*d + sqrt(x)*b**2*e*x + 2*sqrt(x)*b*c*d*x + 2*sqrt(x)*
b*c*e*x**2 + sqrt(x)*c**2*d*x**2 + sqrt(x)*c**2*e*x**3),x)*b**2*c*d*e*x -
3*sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**2*d + sqrt(x)*b**2
*e*x + 2*sqrt(x)*b*c*d*x + 2*sqrt(x)*b*c*e*x**2 + sqrt(x)*c**2*d*x**2 + sq
rt(x)*c**2*e*x**3),x)*b*c**2*d**2*x - sqrt(x)*int((sqrt(x)*sqrt(d + e*x)*s
qrt(b + c*x)*x)/(b**2*d + b**2*e*x + 2*b*c*d*x + 2*b*c*e*x**2 + c**2*d*x**
2 + c**2*e*x**3),x)*b**2*c*e**2 + sqrt(x)*int((sqrt(x)*sqrt(d + e*x)*sqrt(
b + c*x)*x)/(b**2*d + b**2*e*x + 2*b*c*d*x + 2*b*c*e*x**2 + c**2*d*x**2 +
c**2*e*x**3),x)*b*c**2*d*e - sqrt(x)*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c
*x)*x)/(b**2*d + b**2*e*x + 2*b*c*d*x + 2*b*c*e*x**2 + c**2*d*x**2 + c**2*
e*x**3),x)*b*c**2*e**2*x + sqrt(x)*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x
)*x)/(b**2*d + b**2*e*x + 2*b*c*d*x + 2*b*c*e*x**2 + c**2*d*x**2 + c**2*e*
x**3),x)*c**3*d*e*x)/(2*sqrt(x)*b**2*(b + c*x))
```

3.216 $\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{3/2}}\,dx$

Optimal result . . . 1793
Mathematica [C] (verified) . . . 1794
Rubi [A] (verified) . . . 1794
Maple [A] (verified) . . . 1797
Fricas [B] (verification not implemented) . . . 1798
Sympy [F] . . . 1799
Maxima [F] . . . 1799
Giac [F] . . . 1799
Mupad [F(-1)] . . . 1800
Reduce [F] . . . 1800

Optimal result

Integrand size = 23, antiderivative size = 212

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{b\sqrt{bx+cx^2}} - \frac{4\sqrt{c}\sqrt{x}\sqrt{d+ex}\,E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{b^{3/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} + \frac{2e\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{\sqrt{b}\sqrt{c}d\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$

output
```
-2*(e*x+d)^(1/2)/b/(c*x^2+b*x)^(1/2)-4*c^(1/2)*x^(1/2)*(e*x+d)^(1/2)*Ellip
ticE(c^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/b^(3/2)/(b
*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)+2*e*x^(1/2)*(e*x+d)^(1/2)*Inve
rseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e/c/d)^(1/2))/b^(1/2)/c^(
1/2)/d/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.61 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{3/2}}\,dx = \frac{2\sqrt{\frac{b}{c}}(d+ex)+4ie\sqrt{1+\frac{b}{cx}}\sqrt{1+\frac{d}{ex}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{b}{c}}}{\sqrt{x}}\right)|\frac{cd}{be}\right)-2ie\sqrt{1+\frac{b}{cx}}\sqrt{1-}}{b\sqrt{\frac{b}{c}}\sqrt{x(b+cx)}\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[d + e*x]/(b*x + c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[b/c]*(d + e*x) + (4*I)*e*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/
2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - (2*I)*e*Sqrt[1 +
 b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]]
, (c*d)/(b*e)])/(b*Sqrt[b/c]*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1163, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1163$$

$$\frac{2\int \frac{e(b+2cx)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2}-\frac{2(b+2cx)\sqrt{d+ex}}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \; 27$$

$$\frac{e\int \frac{b+2cx}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2}-\frac{2(b+2cx)\sqrt{d+ex}}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1269}$$

$$\frac{e\left(\frac{2c\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}\right)}{b^2}-\frac{2(b+2cx)\sqrt{d+ex}}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1169}$$

$$\frac{e\left(\frac{2c\sqrt{x}\sqrt{b+cx}\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2}-\frac{2(b+2cx)\sqrt{d+ex}}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{122}$$

$$\frac{e\left(\frac{2c\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2}-\frac{2(b+2cx)\sqrt{d+ex}}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{120}$$

$$\frac{e\left(\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2}-\frac{2(b+2cx)\sqrt{d+ex}}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{127}$$

$$\frac{e\left(\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{b^2}-\frac{2(b+2cx)\sqrt{d+ex}}{b^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{126}$$

$$\frac{e\left(\frac{4\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(2cd-be)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{ce}\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{b^2}-\frac{2(b+2cx)\sqrt{d+ex}}{b^2\sqrt{bx+cx^2}}$$

input
```
Int[Sqrt[d + e*x]/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(b + 2*c*x)*Sqrt[d + e*x])/(b^2*Sqrt[b*x + c*x^2]) + (e*((4*Sqrt[-b]*S
qrt[c]*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*S
qrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) -
 (2*Sqrt[-b]*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*Ell
ipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d
 + e*x]*Sqrt[b*x + c*x^2])))/b^2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1163
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] - Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1
)*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p +
1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 0] && (LtQ[
m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c, d,
e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 1.99 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.31

method	result
default	$-\frac{2\left(\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\ \text{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)bde-2\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\ \text{EllipticE}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)bd}{x(cx+b)e\,b^2\sqrt{ex+d}}$
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(-\frac{2\left(ce\,x^2+cdx\right)}{b^2\sqrt{\left(\frac{b}{c}+x\right)\left(ce\,x^2+cdx\right)}}-\frac{2\left(ce\,x^2+bex+cdx+bd\right)}{b^2\sqrt{x\left(ce\,x^2+bex+cdx+bd\right)}}+\frac{2\left(\frac{be-cd}{b^2}+\frac{cd}{b^2}\right)d\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}\,\sqrt{\frac{\frac{b}{c}+x}{-\frac{d}{e}+\frac{b}{c}}}\,\sqrt{-\frac{ex}{d}}\ \text{EllipticF}\left(\sqrt{\frac{(}{}}\right.}{e\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}\right.}{\sqrt{}}$

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticF
(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*b*d*e-2*((e*x+d)/d)^(1/2)*(e*(c
*x+b)/(b*e-c*d))^(1/2)*(-e*x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b
*e-c*d))^(1/2))*b*d*e+2*((e*x+d)/d)^(1/2)*(e*(c*x+b)/(b*e-c*d))^(1/2)*(-e*
x/d)^(1/2)*EllipticE(((e*x+d)/d)^(1/2),(-d*c/(b*e-c*d))^(1/2))*c*d^2+2*x^2
*c*e^2+x*b*e^2+2*c*d*x*e+b*d*e)/x*(x*(c*x+b))^(1/2)/(c*x+b)/e/b^2/(e*x+d)^
(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(185) = 370$.

Time = 0.12 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.79

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{3/2}}\,dx =$$

$$-\frac{2\left(((2c^2d-bce)x^2+(2bcd-b^2e)x)\sqrt{ce}\,\text{weierstrassPInverse}\left(\frac{4(c^2d^2-bcde+b^2e^2)}{3c^2e^2},-\frac{4(2c^3d^3-3bc^2d^2e-3b^2cde^2+2}{27c^3e^3}\right.\right.}{}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(((2*c^2*d - b*c*e)*x^2 + (2*b*c*d - b^2*e)*x)*sqrt(c*e)*weierstrassP
Inverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*
b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b
*e)/(c*e)) + 6*(c^2*e*x^2 + b*c*e*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^
2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2
*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*
e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 +
 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(2*c^2*e*x +
b*c*e)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/(b^2*c^2*e*x^2 + b^3*c*e*x)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{\sqrt{d+ex}}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(sqrt(d + e*x)/(x*(b + c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(c*x^2 + b*x)^(3/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)/(c*x^2 + b*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{\sqrt{d+e\,x}}{(c\,x^2+b\,x)^{3/2}}\,dx$$

input
```
int((d + e*x)^(1/2)/(b*x + c*x^2)^(3/2),x)
```

output
```
int((d + e*x)^(1/2)/(b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{3/2}}\,dx = \int \frac{\sqrt{ex+d}\,\sqrt{cx+b}}{\sqrt{x}\,b^2x+2\sqrt{x}\,bc\,x^2+\sqrt{x}\,c^2x^3}dx$$

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x)
```

output
```
int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**2*x + 2*sqrt(x)*b*c*x**2 + s
qrt(x)*c**2*x**3),x)
```

3.217 $\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal result . . . 1801
Mathematica [C] (verified) . . . 1802
Rubi [A] (verified) . . . 1802
Maple [B] (verified) . . . 1806
Fricas [B] (verification not implemented) . . . 1807
Sympy [F] . . . 1807
Maxima [F] . . . 1808
Giac [F] . . . 1808
Mupad [F(-1)] . . . 1808
Reduce [F] . . . 1809

Optimal result

Integrand size = 23, antiderivative size = 247

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^{3/2}}\,dx = -\frac{2\sqrt{d+ex}}{bd\sqrt{bx+cx^2}} - \frac{2\sqrt{c}(2cd-be)\sqrt{x}\sqrt{d+ex}\,E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{b^{3/2}d(cd-be)\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} + \frac{2\sqrt{c}e\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{\sqrt{b}d(cd-be)\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$

output

```
-2*(e*x+d)^(1/2)/b/d/(c*x^2+b*x)^(1/2)-2*c^(1/2)*(-b*e+2*c*d)*x^(1/2)*(e*x
+d)^(1/2)*EllipticE(c^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1
/2))/b^(3/2)/d/(-b*e+c*d)/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)+2*
c^(1/2)*e*x^(1/2)*(e*x+d)^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(
1/2)),(1-b*e/c/d)^(1/2))/b^(1/2)/d/(-b*e+c*d)/(b*(e*x+d)/d/(c*x+b))^(1/2)/
(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 6.59 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.89

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^{3/2}}\,dx = \frac{-2bcd(d+ex)+2i\sqrt{\frac{b}{c}}ce(-2cd+be)\sqrt{1+\frac{b}{cx}}\sqrt{1+\frac{d}{ex}}x^{3/2}E\left(i\text{arcsinh}\left(\frac{\sqrt{\ }}{\sqrt{\ }}\right.\right.}{b^2d(-cd+be}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*b*c*d*(d + e*x) + (2*I)*Sqrt[b/c]*c*e*(-2*c*d + b*e)*Sqrt[1 + b/(c*x)]
*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(
b*e)] - (2*I)*Sqrt[b/c]*c*e*(-(c*d) + b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e
*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])/(b^2*d*
(-(c*d) + b*e)*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {1165, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{3/2}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 1165}$$

$$-\frac{2\int -\frac{ce(bd+(2cd-be)x)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{ce\int \frac{bd+(2cd-be)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}$$

$$\Big\downarrow \text{1269}$$

$$\frac{ce\left(\frac{(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{2d(cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}\right)}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}$$

$$\Big\downarrow \text{1169}$$

$$\frac{ce\left(\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}$$

$$\Big\downarrow \text{122}$$

$$\frac{ce\left(\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}$$

$$\Big\downarrow \text{120}$$

$$\frac{ce\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}$$

$$\Big\downarrow \text{127}$$

$$\frac{ce\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\int \frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}$$

$$\Big\downarrow \text{126}$$

$$\frac{ce\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}$$

input
```
Int[1/(Sqrt[d + e*x]*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*Sqrt[d + e*x]*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*
Sqrt[b*x + c*x^2]) + (c*e*((2*Sqrt[-b]*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x
)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*
d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (4*Sqrt[-b]*d*(c*d
- b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[
c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c
*x^2])))/(b^2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 470 vs. $2(220) = 440$.

Time = 2.68 (sec) , antiderivative size = 471, normalized size of antiderivative = 1.91

method	result
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(\frac{2(ce\,x^2+cdx)c}{(be-cd)b^2\sqrt{\left(\frac{b}{c}+x\right)(ce\,x^2+cdx)}}-\frac{2(ce\,x^2+bex+cdx+bd)}{b^2d\sqrt{x(ce\,x^2+bex+cdx+bd)}}+\frac{2\left(-\frac{c}{b^2}-\frac{c^2d}{(be-cd)b^2}\right)d\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}\sqrt{\frac{\frac{b}{c}+x}{-\frac{d}{e}+\frac{b}{c}}}\sqrt{-\frac{ex}{d}}\,\mathrm{E}}{e\sqrt{ce\,x^3+be\,x^2+cd\,x^2}}\right.}{}$
default	$-\frac{2\left(\sqrt{\frac{ex+d}{d}}\sqrt{\frac{e(cx+b)}{be-cd}}\sqrt{-\frac{ex}{d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^2de^2-d^2b\sqrt{\frac{ex+d}{d}}\sqrt{\frac{e(cx+b)}{be-cd}}\sqrt{-\frac{ex}{d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right.\right.}{}$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(2*(c*e*x^2+c*d*
x)/(b*e-c*d)*c/b^2/((b/c+x)*(c*e*x^2+c*d*x))^(1/2)-2*(c*e*x^2+b*e*x+c*d*x+
b*d)/b^2/d/(x*(c*e*x^2+b*e*x+c*d*x+b*d))^(1/2)+2*(-c/b^2-c^2*d/(b*e-c*d)/b
^2)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e
*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/
e+b/c))^(1/2))+2*(-1/(b*e-c*d)*c^2*e/b^2+c*e/b^2/d)*d/e*((x+d/e)/d*e)^(1/2
)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x
)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))
-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(220) = 440$.

Time = 0.10 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.00

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^{3/2}}\,dx = -\frac{2\left(((2\,c^3d^2-2\,bc^2de-b^2ce^2)x^2+(2\,bc^2d^2-2\,b^2cde-b^3e^2)x)\sqrt{ce}\text{weierstrassPInverse}\left(\frac{4\,(c^2d^2-bcde+b^2e^2)}{3\,c^2e^2},\right.\right.}{}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(((2*c^3*d^2 - 2*b*c^2*d*e - b^2*c*e^2)*x^2 + (2*b*c^2*d^2 - 2*b^2*c*
d*e - b^3*e^2)*x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b
^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^
3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*((2*c^3*d*e - b*c^2
*e^2)*x^2 + (2*b*c^2*d*e - b^2*c*e^2)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^
2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3
*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*
c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e
^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(b*c^2*d*
e - b^2*c*e^2 + (2*c^3*d*e - b*c^2*e^2)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d)
)/((b^2*c^3*d^2*e - b^3*c^2*d*e^2)*x^2 + (b^3*c^2*d^2*e - b^4*c*d*e^2)*x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(x\,(b+cx))^{\frac{3}{2}}\,\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(1/((x*(b + c*x))**(3/2)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{3}{2}}\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x)^(3/2)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{3}{2}}\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^2 + b*x)^(3/2)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^{3/2}}\,dx = \int \frac{1}{{\left(c\,x^2+b\,x\right)}^{3/2}\,\sqrt{d+e\,x}}\,dx$$

input
```
int(1/((b*x + c*x^2)^(3/2)*(d + e*x)^(1/2)),x)
```

output
```
int(1/((b*x + c*x^2)^(3/2)*(d + e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^{3/2}}\,dx = \frac{-2\sqrt{ex+d}\,\sqrt{cx+b}-2\sqrt{x}\left(\int \frac{\sqrt{ex+d}\,\sqrt{cx+b}}{\sqrt{x}\,b^2d+\sqrt{x}\,b^2ex+2\sqrt{x}\,bcdx+2\sqrt{x}\,bce\,x^2+\sqrt{x}\,c^2d\,x^2+\sqrt{x}\,c}\right.}{}$$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(3/2),x)
```

output
```
( - 2*sqrt(d + e*x)*sqrt(b + c*x) - 2*sqrt(x)*int((sqrt(d + e*x)*sqrt(b +
c*x))/(sqrt(x)*b**2*d + sqrt(x)*b**2*e*x + 2*sqrt(x)*b*c*d*x + 2*sqrt(x)*b
*c*e*x**2 + sqrt(x)*c**2*d*x**2 + sqrt(x)*c**2*e*x**3),x)*b*c*d - 2*sqrt(x
)*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**2*d + sqrt(x)*b**2*e*x + 2
*sqrt(x)*b*c*d*x + 2*sqrt(x)*b*c*e*x**2 + sqrt(x)*c**2*d*x**2 + sqrt(x)*c*
*2*e*x**3),x)*c**2*d*x - sqrt(x)*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))
/(b**2*d + b**2*e*x + 2*b*c*d*x + 2*b*c*e*x**2 + c**2*d*x**2 + c**2*e*x**3
),x)*b*c*e - sqrt(x)*int((sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x))/(b**2*d + b
**2*e*x + 2*b*c*d*x + 2*b*c*e*x**2 + c**2*d*x**2 + c**2*e*x**3),x)*c**2*e*
x)/(sqrt(x)*b*d*(b + c*x))
```

3.218 $\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)^{3/2}}\,dx$

Optimal result . . . 1810
Mathematica [C] (verified) . . . 1811
Rubi [A] (verified) . . . 1811
Maple [B] (verified) . . . 1816
Fricas [B] (verification not implemented) . . . 1817
Sympy [F] . . . 1817
Maxima [F] . . . 1818
Giac [F] . . . 1818
Mupad [F(-1)] . . . 1818
Reduce [F] . . . 1819

Optimal result

Integrand size = 23, antiderivative size = 326

$$\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)^{3/2}}\,dx = \frac{2c}{b(cd-be)\sqrt{d+ex}\sqrt{bx+cx^2}} - \frac{2(2cd-be)\sqrt{bx+cx^2}}{b^2d(cd-be)x\sqrt{d+ex}} - \frac{4\sqrt{e}(c^2d^2-bcde+b^2e^2)\sqrt{bx+cx^2}E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{b^2d^{3/2}(cd-be)^2\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}} + \frac{2c\sqrt{e}(cd+be)\sqrt{bx+cx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{b^2\sqrt{d}(cd-be)^2\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}$$

output
```
2*c/b/(-b*e+c*d)/(e*x+d)^(1/2)/(c*x^2+b*x)^(1/2)-2*(-b*e+2*c*d)*(c*x^2+b*x
)^(1/2)/b^2/d/(-b*e+c*d)/x/(e*x+d)^(1/2)-4*e^(1/2)*(b^2*e^2-b*c*d*e+c^2*d^
2)*(c*x^2+b*x)^(1/2)*EllipticE(e^(1/2)*x^(1/2)/d^(1/2)/(1+e*x/d)^(1/2),(1-
c*d/b/e)^(1/2))/b^2/d^(3/2)/(-b*e+c*d)^2/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/
2)/(e*x+d)^(1/2)+2*c*e^(1/2)*(b*e+c*d)*(c*x^2+b*x)^(1/2)*InverseJacobiAM(a
rctan(e^(1/2)*x^(1/2)/d^(1/2)),(1-c*d/b/e)^(1/2))/b^2/d^(1/2)/(-b*e+c*d)^2
/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 11.59 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.82

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^{3/2}}\,dx = \frac{2bd(b^2e^2+bce^2x+c^2d(d+ex))+4i\sqrt{\frac{b}{c}}ce(c^2d^2-bcde+b^2e^2)\sqrt{1+\frac{b}{cx}}}{}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(b*x + c*x^2)^(3/2)),x]
```

output
```
(2*b*d*(b^2*e^2 + b*c*e^2*x + c^2*d*(d + e*x)) + (4*I)*Sqrt[b/c]*c*e*(c^2*
d^2 - b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*Ellip
ticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - (2*I)*Sqrt[b/c]*c*e*(c^2
*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*
EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])/(b^2*d^2*(c*d - b*e)
^2*Sqrt[x*(b + c*x)]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.03 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1165, 27, 1237, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(bx+cx^2\right)^{3/2}(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1165}$$

$$-\frac{2\int \frac{e(b(cd-2be)+c(2cd-be)x)}{2(d+ex)^{3/2}\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{e\int \frac{b(cd-2be)+c(2cd-be)x}{(d+ex)^{3/2}\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}$$

$\Big\downarrow$ 1237

$$-\frac{e\left(\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{2\int \frac{c\left(bd(cd+be)+2\left(c^2d^2-bced+b^2e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{d(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}$$

$\Big\downarrow$ 27

$$-\frac{e\left(\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\int \frac{bd(cd+be)+2\left(c^2d^2-bced+b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{d(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}$$

$\Big\downarrow$ 1269

$$-\frac{e\left(\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\left(b^2e^2-bcde+c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{d(cd-be)(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}\right)}{d(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}$$

$\Big\downarrow$ 1169

$$-\frac{e\left(\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\sqrt{x}\sqrt{b+cx}\left(b^2e^2-bcde+c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{d(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}$$

$\Big\downarrow$ 122

$$-\frac{e\left(\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{d(cd-be)}\cdots\right)}{b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}$$

$\Big\downarrow$ 120

$$-\frac{e\left(\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\cdots}}{e\sqrt{bx+cx^2}}\cdots\right)}{d(cd-be)}\cdots\right)}{b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}$$

$\Big\downarrow$ 127

$$-\frac{e\left(\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\int\cdots}{e\sqrt{bx+cx^2}\sqrt{d+ex}\cdots}\cdots\right)}{d(cd-be)}\cdots\right)}{b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}$$

$\Big\downarrow$ 126

$$-\frac{e\left(\frac{4\sqrt{bx+cx^2}\left(b^2e^2-bcde+c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-bcde+c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-\cdots}{\sqrt{c}e\sqrt{bx+cx^2}\cdots}\cdots\right)}{d(cd-be)}\cdots\right)}{b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}$$

input
```
Int[1/((d + e*x)^(3/2)*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*Sqrt[d + e*x]*
Sqrt[b*x + c*x^2]) - (e*((4*(c^2*d^2 - b*c*d*e + b^2*e^2)*Sqrt[b*x + c*x^2
])/(d*(c*d - b*e)*Sqrt[d + e*x]) - (c*((4*Sqrt[-b]*(c^2*d^2 - b*c*d*e + b^
2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*S
qrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c
*x^2]) - (2*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]
*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*
d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])))/(d*(c*d - b*e))))/(b^2*
d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 648 vs. $2(295) = 590$.

Time = 3.71 (sec) , antiderivative size = 649, normalized size of antiderivative = 1.99

method	result
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(-\frac{2cex\left(\frac{\left(b^2e^2+c^2d^2\right)x}{b^2\left(b^2e^2-2bcde+c^2d^2\right)d^2}+\frac{(be+cd)\left(b^2e^2-bcde+c^2d^2\right)}{\left(b^2e^2-2bcde+c^2d^2\right)d^2b^2ec}\right)}{\sqrt{\left(\frac{(be+cd)x}{ce}+x^2+\frac{db}{ce}\right)cex}}-\frac{2\left(ce\,x^2+bex+cdx+bd\right)}{b^2d^2\sqrt{x\left(ce\,x^2+bex+cdx+bd\right)}}+\frac{2\left(-\frac{be+cd}{b^2d^2}+\frac{(be+\cdots}{\left(b^2\cdots\right.}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$
default	$-\frac{2\left(2\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\operatorname{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^3d\,e^3-3\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\operatorname{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right.\cdots\right.}{\cdots}$

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(-2*c*e*x*((b^2*
e^2+c^2*d^2)/b^2/(b^2*e^2-2*b*c*d*e+c^2*d^2)/d^2*x+(b*e+c*d)*(b^2*e^2-b*c*
d*e+c^2*d^2)/(b^2*e^2-2*b*c*d*e+c^2*d^2)/d^2/b^2/e/c)/(((b*e+c*d)/c/e*x+x^
2+1/c*d*b/e)*c*e*x)^(1/2)-2*(c*e*x^2+b*e*x+c*d*x+b*d)/b^2/d^2/(x*(c*e*x^2+
b*e*x+c*d*x+b*d))^(1/2)+2*(-(b*e+c*d)/b^2/d^2+(b*e+c*d)*(b^2*e^2-b*c*d*e+c
^2*d^2)/(b^2*e^2-2*b*c*d*e+c^2*d^2)/d^2/b^2)*d/e*((x+d/e)/d*e)^(1/2)*((b/c
+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)
*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*((b^2*e^2+c^2*d^
2)*c*e/b^2/(b^2*e^2-2*b*c*d*e+c^2*d^2)/d^2+c*e/b^2/d^2)*d/e*((x+d/e)/d*e)^
(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b
*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1
/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 797 vs. $2(295) = 590$.

Time = 0.10 (sec) , antiderivative size = 797, normalized size of antiderivative = 2.44

$$\int \frac{1}{(d+ex)^{3/2}\,(bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(((2*c^4*d^3*e - 3*b*c^3*d^2*e^2 - 3*b^2*c^2*d*e^3 + 2*b^3*c*e^4)*x^3
 + (2*c^4*d^4 - b*c^3*d^3*e - 6*b^2*c^2*d^2*e^2 - b^3*c*d*e^3 + 2*b^4*e^4)
*x^2 + (2*b*c^3*d^4 - 3*b^2*c^2*d^3*e - 3*b^3*c*d^2*e^2 + 2*b^4*d*e^3)*x)*
sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2),
 -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3),
1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 6*((c^4*d^2*e^2 - b*c^3*d*e^3 + b^2*c^2
*e^4)*x^3 + (c^4*d^3*e + b^3*c*e^4)*x^2 + (b*c^3*d^3*e - b^2*c^2*d^2*e^2 +
 b^3*c*d*e^3)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^
2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3
)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^
2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3
), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(b*c^3*d^3*e - 2*b^2*c^2*d^2*e^2
+ b^3*c*d*e^3 + 2*(c^4*d^2*e^2 - b*c^3*d*e^3 + b^2*c^2*e^4)*x^2 + (2*c^4*d
^3*e - b*c^3*d^2*e^2 - b^2*c^2*d*e^3 + 2*b^3*c*e^4)*x)*sqrt(c*x^2 + b*x)*s
qrt(e*x + d))/((b^2*c^4*d^4*e^2 - 2*b^3*c^3*d^3*e^3 + b^4*c^2*d^2*e^4)*x^3
 + (b^2*c^4*d^5*e - b^3*c^3*d^4*e^2 - b^4*c^2*d^3*e^3 + b^5*c*d^2*e^4)*x^2
 + (b^3*c^3*d^5*e - 2*b^4*c^2*d^4*e^2 + b^5*c*d^3*e^3)*x)
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\,(bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(x\,(b+cx))^{\frac{3}{2}}\,(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(1/((x*(b + c*x))**(3/2)*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{3}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x)^(3/2)*(e*x + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{3}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^2 + b*x)^(3/2)*(e*x + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)^{3/2}}\,dx = \int \frac{1}{(c\,x^2+b\,x)^{3/2}\,(d+e\,x)^{3/2}}\,dx$$

input
```
int(1/((b*x + c*x^2)^(3/2)*(d + e*x)^(3/2)),x)
```

output
```
int(1/((b*x + c*x^2)^(3/2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^{3/2}\,(bx+cx^2)^{3/2}}\,dx = \text{Too large to display}$$

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(3/2),x)
```

output
```
( - 2*sqrt(x)*sqrt(d + e*x)*sqrt(b + c*x) - 2*int((sqrt(d + e*x)*sqrt(b +
c*x))/(sqrt(x)*b**2*d**2 + 2*sqrt(x)*b**2*d*e*x + sqrt(x)*b**2*e**2*x**2 +
 2*sqrt(x)*b*c*d**2*x + 4*sqrt(x)*b*c*d*e*x**2 + 2*sqrt(x)*b*c*e**2*x**3 +
 sqrt(x)*c**2*d**2*x**2 + 2*sqrt(x)*c**2*d*e*x**3 + sqrt(x)*c**2*e**2*x**4
),x)*b**2*d*e*x - 2*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**2*d**2 +
 2*sqrt(x)*b**2*d*e*x + sqrt(x)*b**2*e**2*x**2 + 2*sqrt(x)*b*c*d**2*x + 4*
sqrt(x)*b*c*d*e*x**2 + 2*sqrt(x)*b*c*e**2*x**3 + sqrt(x)*c**2*d**2*x**2 +
2*sqrt(x)*c**2*d*e*x**3 + sqrt(x)*c**2*e**2*x**4),x)*b**2*e**2*x**2 - 2*in
t((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**2*d**2 + 2*sqrt(x)*b**2*d*e*x
+ sqrt(x)*b**2*e**2*x**2 + 2*sqrt(x)*b*c*d**2*x + 4*sqrt(x)*b*c*d*e*x**2 +
 2*sqrt(x)*b*c*e**2*x**3 + sqrt(x)*c**2*d**2*x**2 + 2*sqrt(x)*c**2*d*e*x**
3 + sqrt(x)*c**2*e**2*x**4),x)*b*c*d**2*x - 4*int((sqrt(d + e*x)*sqrt(b +
c*x))/(sqrt(x)*b**2*d**2 + 2*sqrt(x)*b**2*d*e*x + sqrt(x)*b**2*e**2*x**2 +
 2*sqrt(x)*b*c*d**2*x + 4*sqrt(x)*b*c*d*e*x**2 + 2*sqrt(x)*b*c*e**2*x**3 +
 sqrt(x)*c**2*d**2*x**2 + 2*sqrt(x)*c**2*d*e*x**3 + sqrt(x)*c**2*e**2*x**4
),x)*b*c*d*e*x**2 - 2*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**2*d**2
 + 2*sqrt(x)*b**2*d*e*x + sqrt(x)*b**2*e**2*x**2 + 2*sqrt(x)*b*c*d**2*x +
4*sqrt(x)*b*c*d*e*x**2 + 2*sqrt(x)*b*c*e**2*x**3 + sqrt(x)*c**2*d**2*x**2
+ 2*sqrt(x)*c**2*d*e*x**3 + sqrt(x)*c**2*e**2*x**4),x)*b*c*e**2*x**3 - 2*i
nt((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**2*d**2 + 2*sqrt(x)*b**2*d*...
```

3.219 $\int \frac{1}{(d+ex)^{5/2}(bx+cx^2)^{3/2}}\,dx$

Optimal result . . . 1820
Mathematica [C] (verified) . . . 1821
Rubi [A] (verified) . . . 1821
Maple [A] (verified) . . . 1827
Fricas [B] (verification not implemented) . . . 1828
Sympy [F] . . . 1829
Maxima [F] . . . 1829
Giac [F] . . . 1829
Mupad [F(-1)] . . . 1830
Reduce [F] . . . 1830

Optimal result

Integrand size = 23, antiderivative size = 427

$$\int \frac{1}{(d+ex)^{5/2}(bx+cx^2)^{3/2}}\,dx = \frac{2c}{b(cd-be)(d+ex)^{3/2}\sqrt{bx+cx^2}} + \frac{2e(3cd+be)\sqrt{bx+cx^2}}{3bd(cd-be)^2x(d+ex)^{3/2}} - \frac{4(3c^2d^2-3bcde+2b^2e^2)\sqrt{bx+cx^2}}{3b^2d^2(cd-be)^2x\sqrt{d+ex}} - \frac{2\sqrt{e}(2cd-be)(3c^2d^2-3bcde+8b^2e^2)\sqrt{bx+cx^2}E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{3b^2d^{5/2}(cd-be)^3\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}} + \frac{2c\sqrt{e}(3c^2d^2+9bcde-4b^2e^2)\sqrt{bx+cx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{3b^2d^{3/2}(cd-be)^3\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}$$

output
```
2*c/b/(-b*e+c*d)/(e*x+d)^(3/2)/(c*x^2+b*x)^(1/2)+2/3*e*(b*e+3*c*d)*(c*x^2+
b*x)^(1/2)/b/d/(-b*e+c*d)^2/x/(e*x+d)^(3/2)-4/3*(2*b^2*e^2-3*b*c*d*e+3*c^2
*d^2)*(c*x^2+b*x)^(1/2)/b^2/d^2/(-b*e+c*d)^2/x/(e*x+d)^(1/2)-2/3*e^(1/2)*(
-b*e+2*c*d)*(8*b^2*e^2-3*b*c*d*e+3*c^2*d^2)*(c*x^2+b*x)^(1/2)*EllipticE(e^
(1/2)*x^(1/2)/d^(1/2)/(1+e*x/d)^(1/2),(1-c*d/b/e)^(1/2))/b^2/d^(5/2)/(-b*e
+c*d)^3/x^(1/2)/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)+2/3*c*e^(1/2)*(-
4*b^2*e^2+9*b*c*d*e+3*c^2*d^2)*(c*x^2+b*x)^(1/2)*InverseJacobiAM(arctan(e^
(1/2)*x^(1/2)/d^(1/2)),(1-c*d/b/e)^(1/2))/b^2/d^(3/2)/(-b*e+c*d)^3/x^(1/2)
/(d*(c*x+b)/b/(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.00 (sec) , antiderivative size = 420, normalized size of antiderivative = 0.98

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx =$$

$$-\frac{2\Big(b(b^2de^3(cd-be)x(b+cx)-5b^2e^3(-2cd+be)x(b+cx)(d+ex)+3c^4d^3x(d+ex)^2+3(cd-be)^3(b+}{}$$

input
```
Integrate[1/((d + e*x)^(5/2)*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(b*(b^2*d*e^3*(c*d - b*e)*x*(b + c*x) - 5*b^2*e^3*(-2*c*d + b*e)*x*(b
+ c*x)*(d + e*x) + 3*c^4*d^3*x*(d + e*x)^2 + 3*(c*d - b*e)^3*(b + c*x)*(d
+ e*x)^2) - Sqrt[b/c]*c*(d + e*x)*(Sqrt[b/c]*(6*c^3*d^3 - 9*b*c^2*d^2*e +
19*b^2*c*d*e^2 - 8*b^3*e^3)*(b + c*x)*(d + e*x) + I*b*e*(6*c^3*d^3 - 9*b*c
^2*d^2*e + 19*b^2*c*d*e^2 - 8*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]
*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(3*c
^3*d^3 - 18*b*c^2*d^2*e + 23*b^2*c*d*e^2 - 8*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sq
rt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e
)])))/(3*b^3*d^3*(c*d - b*e)^3*Sqrt[x*(b + c*x)]*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 508, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {1165, 27, 1237, 27, 1237, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(bx+cx^2\right)^{3/2}(d+ex)^{5/2}}\,dx$$

$$\Big\downarrow \; 1165$$

$$-\frac{2\int \frac{e(b(3cd-4be)+3c(2cd-be)x)}{2(d+ex)^{5/2}\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 27$$

$$-\frac{e\int \frac{b(3cd-4be)+3c(2cd-be)x}{(d+ex)^{5/2}\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 1237$$

$$-\frac{e\left(\frac{4\sqrt{bx+cx^2}(2b^2e^2-3bcde+3c^2d^2)}{3d(d+ex)^{3/2}(cd-be)}-\frac{2\int -\frac{b\left(3c^2d^2-15bced+8b^2e^2\right)+2c\left(3c^2d^2-3bced+2b^2e^2\right)x}{2(d+ex)^{3/2}\sqrt{cx^2+bx}}dx}{3d(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 27$$

$$-\frac{e\left(\frac{\int \frac{b\left(3c^2d^2-15bced+8b^2e^2\right)+2c\left(3c^2d^2-3bced+2b^2e^2\right)x}{(d+ex)^{3/2}\sqrt{cx^2+bx}}dx}{3d(cd-be)}+\frac{4\sqrt{bx+cx^2}(2b^2e^2-3bcde+3c^2d^2)}{3d(d+ex)^{3/2}(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 1237$$

$$-\frac{e\left(\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{2\int \frac{c\left(bd\left(3c^2d^2+9bced-4b^2e^2\right)+(2cd-be)\left(3c^2d^2-3bced+8b^2e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{d(cd-be)}}{3d(cd-be)}+\frac{4\sqrt{bx+cx^2}(2b^2e^2-3bcde+3}{3d(d+ex)^{3/2}(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 27$$

$$-\frac{e\left(\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\int \frac{bd\left(3c^2d^2+9bced-4b^2e^2\right)+(2cd-be)\left(3c^2d^2-3bced+8b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{d(cd-be)}}{3d(cd-be)}+\frac{4\sqrt{bx+cx^2}(2b^2e^2-3bcde+3c^2}{3d(d+ex)^{3/2}(cd-be)}\right)}{b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 1269$$

$$-\frac{e\left(\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{2d(cd-be)\left(2b^2e^2-3bcde+3c^2d^2\right)\int\frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}\cdots\right.}{d(cd-be)}}{3d(cd-be)}\cdots\right.}{b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 1169$$

$$-\frac{e\left(\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{\sqrt{x}\sqrt{b+cx}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)\int\frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\left(2b^2e^2-3bcde+3c^2d^2\right)\cdots}{e\sqrt{bx+cx^2}}\right.}{d(cd-be)}}{3d(cd-be)}\cdots\right.}{b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 122$$

$$-\frac{e\left(\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)\left(2b^2e^2-3bcd\cdots\right.}{e\sqrt{bx\cdots}}\right.}{d(cd-be)}}{3d(cd-be)}\cdots\right.}{b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 120$$

$$-\frac{e\left(\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{b+cx}(cd-be)}{\cdots}\right.}{d(cd-be)}}{3d(cd-be)}\right.}{b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^{3/2}(cd-be)}$$

$\downarrow$ 127

$$-\frac{e\left(\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+\cdots}}{\cdots}\right.}{d(cd-be)}}{3d(cd-be)}\right.}{b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^{3/2}(cd-be)}$$

$\downarrow$ 126

$$-\frac{e\left(\frac{\frac{2\sqrt{bx+cx^2}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(8b^2e^2-3bcde+3c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\cdots}}{\cdots}\right.}{d(cd-be)}}{3d(cd-be)}\right.}{b^2d(cd-be)}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{b^2d\sqrt{bx+cx^2}(d+ex)^{3/2}(cd-be)}$$

input
```
Int[1/((d + e*x)^(5/2)*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*e)*(d + e*x)^(3/2
)*Sqrt[b*x + c*x^2]) - (e*((4*(3*c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[b*x
 + c*x^2])/(3*d*(c*d - b*e)*(d + e*x)^(3/2)) + ((2*(2*c*d - b*e)*(3*c^2*d^
2 - 3*b*c*d*e + 8*b^2*e^2)*Sqrt[b*x + c*x^2])/(d*(c*d - b*e)*Sqrt[d + e*x]
) - (c*((2*Sqrt[-b]*(2*c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 8*b^2*e^2)*Sqrt
[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqr
t[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (4
*Sqrt[-b]*d*(c*d - b*e)*(3*c^2*d^2 - 3*b*c*d*e + 2*b^2*e^2)*Sqrt[x]*Sqrt[1
 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]]
, (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])))/(d*(c*d - b*
e)))/(3*d*(c*d - b*e))))/(b^2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 4.92 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.62

method	result
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(\frac{2\left(ce\,x^2+cdx\right)c^3}{(be-cd)^3b^2\sqrt{\left(\frac{b}{c}+x\right)\left(ce\,x^2+cdx\right)}}-\frac{2e\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3d^2(be-cd)^2\left(x+\frac{d}{e}\right)^2}-\frac{10\left(ce\,x^2+bex\right)e^2(be-2cd)}{3d^3(be-cd)^3\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2+bex\right)}}-\frac{2\left(ce\,x^2+bex\right.}{b^2d^3\sqrt{x\left(ce\,x^2+\right.}}\right.}{}$
default	Expression too large to display

input
```
int(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(2*(c*e*x^2+c*d*
x)/(b*e-c*d)^3*c^3/b^2/((b/c+x)*(c*e*x^2+c*d*x))^(1/2)-2/3*e/d^2/(b*e-c*d)
^2*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(x+d/e)^2-10/3*(c*e*x^2+b*e*x)*e^
2/d^3/(b*e-c*d)^3*(b*e-2*c*d)/((x+d/e)*(c*e*x^2+b*e*x))^(1/2)-2*(c*e*x^2+b
*e*x+c*d*x+b*d)/b^2/d^3/(x*(c*e*x^2+b*e*x+c*d*x+b*d))^(1/2)+2*(-c^3/(b*e-c
*d)^2/b^2-c^4*d/(b*e-c*d)^3/b^2-1/3*c*e^2/d^2/(b*e-c*d)^2-5/3*e^2/(b*e-c*d
)^2*(b*e-2*c*d)/d^3+5/3*b*e^3/d^3/(b*e-c*d)^3*(b*e-2*c*d))*d/e*((x+d/e)/d*
e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^
2+b*d*x)^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(-
c^4*e/(b*e-c*d)^3/b^2+5/3*(b*e-2*c*d)*c*e^3/d^3/(b*e-c*d)^3+c*e/b^2/d^3)*d
/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+
b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-d
/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^
(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1277 vs. $2(384) = 768$.

Time = 0.15 (sec) , antiderivative size = 1277, normalized size of antiderivative = 2.99

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/9*(((6*c^5*d^4*e^2 - 12*b*c^4*d^3*e^3 - 17*b^2*c^3*d^2*e^4 + 23*b^3*c^2
*d*e^5 - 8*b^4*c*e^6)*x^4 + (12*c^5*d^5*e - 18*b*c^4*d^4*e^2 - 46*b^2*c^3*
d^3*e^3 + 29*b^3*c^2*d^2*e^4 + 7*b^4*c*d*e^5 - 8*b^5*e^6)*x^3 + (6*c^5*d^6
 - 41*b^2*c^3*d^4*e^2 - 11*b^3*c^2*d^3*e^3 + 38*b^4*c*d^2*e^4 - 16*b^5*d*e
^5)*x^2 + (6*b*c^4*d^6 - 12*b^2*c^3*d^5*e - 17*b^3*c^2*d^4*e^2 + 23*b^4*c*
d^3*e^3 - 8*b^5*d^2*e^4)*x)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b
*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*
e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 3*((6*c^5*d
^3*e^3 - 9*b*c^4*d^2*e^4 + 19*b^2*c^3*d*e^5 - 8*b^3*c^2*e^6)*x^4 + (12*c^5
*d^4*e^2 - 12*b*c^4*d^3*e^3 + 29*b^2*c^3*d^2*e^4 + 3*b^3*c^2*d*e^5 - 8*b^4
*c*e^6)*x^3 + (6*c^5*d^5*e + 3*b*c^4*d^4*e^2 + b^2*c^3*d^3*e^3 + 30*b^3*c^
2*d^2*e^4 - 16*b^4*c*d*e^5)*x^2 + (6*b*c^4*d^5*e - 9*b^2*c^3*d^4*e^2 + 19*
b^3*c^2*d^3*e^3 - 8*b^4*c*d^2*e^4)*x)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d
^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^
2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d
*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2
+ 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) + 3*(3*b*c^4*d^5
*e - 9*b^2*c^3*d^4*e^2 + 9*b^3*c^2*d^3*e^3 - 3*b^4*c*d^2*e^4 + (6*c^5*d^3*
e^3 - 9*b*c^4*d^2*e^4 + 19*b^2*c^3*d*e^5 - 8*b^3*c^2*e^6)*x^3 + (12*c^5*d^
4*e^2 - 15*b*c^4*d^3*e^3 + 20*b^2*c^3*d^2*e^4 + 7*b^3*c^2*d*e^5 - 8*b^4...
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(x\left(b+cx\right)\right)^{\frac{3}{2}}\left(d+ex\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(5/2)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(1/((x*(b + c*x))**(3/2)*(d + e*x)**(5/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(cx^2+bx\right)^{\frac{3}{2}}\left(ex+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x)^(3/2)*(e*x + d)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx = \int \frac{1}{\left(cx^2+bx\right)^{\frac{3}{2}}\left(ex+d\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^2 + b*x)^(3/2)*(e*x + d)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx=\int \frac{1}{\left(c\,x^2+b\,x\right)^{3/2}\,\left(d+e\,x\right)^{5/2}}\,dx$$

input
```
int(1/((b*x + c*x^2)^(3/2)*(d + e*x)^(5/2)),x)
```

output
```
int(1/((b*x + c*x^2)^(3/2)*(d + e*x)^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx=\text{too large to display}$$

input
```
int(1/(e*x+d)^(5/2)/(c*x^2+b*x)^(3/2),x)
```

output

```
( - 2*sqrt(d + e*x)*sqrt(b + c*x) - 4*sqrt(x)*int((sqrt(d + e*x)*sqrt(b +
c*x))/(sqrt(x)*b**2*d**3 + 3*sqrt(x)*b**2*d**2*e*x + 3*sqrt(x)*b**2*d*e**2
*x**2 + sqrt(x)*b**2*e**3*x**3 + 2*sqrt(x)*b*c*d**3*x + 6*sqrt(x)*b*c*d**2
*e*x**2 + 6*sqrt(x)*b*c*d*e**2*x**3 + 2*sqrt(x)*b*c*e**3*x**4 + sqrt(x)*c*
*2*d**3*x**2 + 3*sqrt(x)*c**2*d**2*e*x**3 + 3*sqrt(x)*c**2*d*e**2*x**4 + s
qrt(x)*c**2*e**3*x**5),x)*b**2*d**2*e - 8*sqrt(x)*int((sqrt(d + e*x)*sqrt(
b + c*x))/(sqrt(x)*b**2*d**3 + 3*sqrt(x)*b**2*d**2*e*x + 3*sqrt(x)*b**2*d*
e**2*x**2 + sqrt(x)*b**2*e**3*x**3 + 2*sqrt(x)*b*c*d**3*x + 6*sqrt(x)*b*c*
d**2*e*x**2 + 6*sqrt(x)*b*c*d*e**2*x**3 + 2*sqrt(x)*b*c*e**3*x**4 + sqrt(x
)*c**2*d**3*x**2 + 3*sqrt(x)*c**2*d**2*e*x**3 + 3*sqrt(x)*c**2*d*e**2*x**4
 + sqrt(x)*c**2*e**3*x**5),x)*b**2*d*e**2*x - 4*sqrt(x)*int((sqrt(d + e*x)
*sqrt(b + c*x))/(sqrt(x)*b**2*d**3 + 3*sqrt(x)*b**2*d**2*e*x + 3*sqrt(x)*b
**2*d*e**2*x**2 + sqrt(x)*b**2*e**3*x**3 + 2*sqrt(x)*b*c*d**3*x + 6*sqrt(x
)*b*c*d**2*e*x**2 + 6*sqrt(x)*b*c*d*e**2*x**3 + 2*sqrt(x)*b*c*e**3*x**4 +
sqrt(x)*c**2*d**3*x**2 + 3*sqrt(x)*c**2*d**2*e*x**3 + 3*sqrt(x)*c**2*d*e**
2*x**4 + sqrt(x)*c**2*e**3*x**5),x)*b**2*e**3*x**2 - 2*sqrt(x)*int((sqrt(d
 + e*x)*sqrt(b + c*x))/(sqrt(x)*b**2*d**3 + 3*sqrt(x)*b**2*d**2*e*x + 3*sq
rt(x)*b**2*d*e**2*x**2 + sqrt(x)*b**2*e**3*x**3 + 2*sqrt(x)*b*c*d**3*x + 6
*sqrt(x)*b*c*d**2*e*x**2 + 6*sqrt(x)*b*c*d*e**2*x**3 + 2*sqrt(x)*b*c*e**3*
x**4 + sqrt(x)*c**2*d**3*x**2 + 3*sqrt(x)*c**2*d**2*e*x**3 + 3*sqrt(x)*...
```

3.220 $\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^{5/2}}\,dx$

Optimal result . . . 1832
Mathematica [C] (verified) . . . 1833
Rubi [A] (verified) . . . 1834
Maple [B] (verified) . . . 1839
Fricas [B] (verification not implemented) . . . 1840
Sympy [F(-1)] . . . 1841
Maxima [F] . . . 1842
Giac [F] . . . 1842
Mupad [F(-1)] . . . 1842
Reduce [F] . . . 1843

Optimal result

Integrand size = 23, antiderivative size = 485

$$\begin{aligned}\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^{5/2}}\,dx &= \frac{2(cd-be)(d+ex)^{7/2}}{3bc\,(bx+cx^2)^{3/2}}\\
&- \frac{2e(8c^3d^3-15bc^2d^2e+3b^2cde^2-4b^3e^3)\,x\sqrt{d+ex}}{3b^3c^2\sqrt{bx+cx^2}}\\
&+ \frac{2d(8c^2d^2-13bcde+b^2e^2)\,(d+ex)^{3/2}}{3b^3c\sqrt{bx+cx^2}} - \frac{2d(2cd-be)(d+ex)^{5/2}}{3b^2cx\sqrt{bx+cx^2}}\\
&+ \frac{2(16c^4d^4-32bc^3d^3e+9b^2c^2d^2e^2+7b^3cde^3-8b^4e^4)\,\sqrt{x}\sqrt{d+ex}E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{3b^{7/2}c^{5/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}\\
&- \frac{2e(8c^3d^3-15bc^2d^2e+3b^2cde^2-4b^3e^3)\,\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{3b^{5/2}c^{5/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}\end{aligned}$$

output
```
2/3*(-b*e+c*d)*(e*x+d)^(7/2)/b/c/(c*x^2+b*x)^(3/2)-2/3*e*(-4*b^3*e^3+3*b^2
*c*d*e^2-15*b*c^2*d^2*e+8*c^3*d^3)*x*(e*x+d)^(1/2)/b^3/c^2/(c*x^2+b*x)^(1/
2)+2/3*d*(b^2*e^2-13*b*c*d*e+8*c^2*d^2)*(e*x+d)^(3/2)/b^3/c/(c*x^2+b*x)^(1
/2)-2/3*d*(-b*e+2*c*d)*(e*x+d)^(5/2)/b^2/c/x/(c*x^2+b*x)^(1/2)+2/3*(-8*b^4
*e^4+7*b^3*c*d*e^3+9*b^2*c^2*d^2*e^2-32*b*c^3*d^3*e+16*c^4*d^4)*x^(1/2)*(e
*x+d)^(1/2)*EllipticE(c^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^
(1/2))/b^(7/2)/c^(5/2)/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)-2/3*e
*(-4*b^3*e^3+3*b^2*c*d*e^2-15*b*c^2*d^2*e+8*c^3*d^3)*x^(1/2)*(e*x+d)^(1/2)
*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e/c/d)^(1/2))/b^(5/2
)/c^(5/2)/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 24.21 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.93

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^{5/2}}\,dx = \frac{2\Big(b(d+ex)\,(b(cd-be)^4x^2+(cd-be)^3(8cd+5be)x^2(b+cx)-bc^2d^4(b+cx)^2+c^2}{}$$

input
```
Integrate[(d + e*x)^(9/2)/(b*x + c*x^2)^(5/2),x]
```

output
```
(2*(b*(d + e*x)*(b*(c*d - b*e)^4*x^2 + (c*d - b*e)^3*(8*c*d + 5*b*e)*x^2*(
b + c*x) - b*c^2*d^4*(b + c*x)^2 + c^2*d^3*(8*c*d - 13*b*e)*x*(b + c*x)^2)
 - Sqrt[b/c]*x*(b + c*x)*(Sqrt[b/c]*(16*c^4*d^4 - 32*b*c^3*d^3*e + 9*b^2*c
^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*(b + c*x)*(d + e*x) + I*b*e*(16*c^
4*d^4 - 32*b*c^3*d^3*e + 9*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*Sq
rt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sq
rt[x]], (c*d)/(b*e)] - I*b*e*(8*c^4*d^4 - 17*b*c^3*d^3*e + 6*b^2*c^2*d^2*e
^2 + 11*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/
2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(3*b^5*c^2*(x*(
b + c*x))^(3/2)*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {1164, 27, 1233, 27, 1236, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{1164}$$

$$-\frac{2\int \frac{(d+ex)^{5/2}(d(8cd-11be)-3e(2cd-be)x)}{2(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(d+ex)^{7/2}(x(2cd-be)+bd)}{3b^2(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{(d+ex)^{5/2}(d(8cd-11be)-3e(2cd-be)x)}{(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(d+ex)^{7/2}(x(2cd-be)+bd)}{3b^2(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1233}$$

$$-\frac{\frac{2\int \frac{3e\sqrt{d+ex}\left(bd\left(8c^2d^2-13bced+b^2e^2\right)+4\left(4c^3d^3-6bc^2ed^2+b^3e^3\right)x\right)}{2\sqrt{cx^2+bx}}dx}{b^2c}-\frac{2(d+ex)^{3/2}\left(x(2cd-be)\left(-3b^2e^2-8bcde+8c^2d^2\right)+bcd^2(8cd-11be)\right)}{b^2c\sqrt{bx+cx^2}}}{3b^2}-\frac{2(d+ex)^{7/2}(x(2cd-be)+bd)}{3b^2(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{3e\int \frac{\sqrt{d+ex}\left(bd\left(8c^2d^2-13bced+b^2e^2\right)+4\left(4c^3d^3-6bc^2ed^2+b^3e^3\right)x\right)}{\sqrt{cx^2+bx}}dx}{b^2c}-\frac{2(d+ex)^{3/2}\left(x(2cd-be)\left(-3b^2e^2-8bcde+8c^2d^2\right)+bcd^2(8cd-11be)\right)}{b^2c\sqrt{bx+cx^2}}}{3b^2}-\frac{2(d+ex)^{7/2}(x(2cd-be)+bd)}{3b^2(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1236}$$

$$-\frac{\frac{3e\left(\frac{2\int \frac{bd\left(8c^3d^3-15bc^2ed^2+3b^2ce^2d-4b^3e^3\right)+\left(16c^4d^4-32bc^3ed^3+9b^2c^2e^2d^2+7b^3ce^3d-8b^4e^4\right)x}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{8\sqrt{bx+cx^2}\sqrt{d+ex}\left(b^3e^3-6bc^2d^2e+4c^3d^3\right)}{3c}\right)}{b^2c}-2\cdots}{3b^2}\cdots$$

$$\frac{2(d+ex)^{7/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$\downarrow$ 27

$$-\frac{\frac{3e\left(\frac{\int \frac{bd\left(8c^3d^3-15bc^2ed^2+3b^2ce^2d-4b^3e^3\right)+\left(16c^4d^4-32bc^3ed^3+9b^2c^2e^2d^2+7b^3ce^3d-8b^4e^4\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{3c}+\frac{8\sqrt{bx+cx^2}\sqrt{d+ex}\left(b^3e^3-6bc^2d^2e+4c^3d^3\right)}{3c}\right)}{b^2c}-\frac{2(\cdots}{\cdots}}{3b^2}\cdots$$

$$\frac{2(d+ex)^{7/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$\downarrow$ 1269

$$-\frac{\frac{3e\left(\frac{\frac{\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{4d(cd-be)(2cd-be)\left(-b^2e^2-2bcde+2c^2d^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}}{3c}+\frac{8\sqrt{bx+cx^2}\sqrt{d\cdots}}{\cdots}\right)}{b^2c}\cdots}{3b^2}\cdots$$

$$\frac{2(d+ex)^{7/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$\downarrow$ 1169

$$-\frac{\frac{3e\left(\frac{\frac{\sqrt{x}\sqrt{b+cx}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{4d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\left(-b^2e^2-2bcde+2c^2d^2\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+e\cdots}}\cdots}{e\sqrt{bx+cx^2}}}{3c}\cdots\right)}{b^2c}\cdots}{3b^2}\cdots$$

$$\frac{2(d+ex)^{7/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$\downarrow$ 122

$$-\frac{3e\left(\frac{\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\left(-b^2e^2-2bcde+2c^2d^2\right)\int\frac{}{\sqrt{x}\sqrt{b-}}}{e\sqrt{bx+cx^2}}}{3c}\right.}{b^2c}$$

$$3b^2$$

$$\frac{2(d+ex)^{7/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{120}$$

$$-\frac{3e\left(\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\left(-b^2e^2-2bcde+2c^2\right.}{e\sqrt{bx+cx^2}}}{3c}\right.}{b^2c}$$

$$3b^2$$

$$\frac{2(d+ex)^{7/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{127}$$

$$-\frac{3e\left(\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{4d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\left(-b^2e^2-2\right.}{e\sqrt{bx+cx^2}\sqrt{d-}}}{3c}\right.}{b^2c}$$

$$\frac{2(d+ex)^{7/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \textbf{126}$$

$$-\frac{3e\left(\frac{\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\left(-b^2e\right.}{\sqrt{c}e\sqrt{bx+}}}{3c}\right.}{b^2c}$$

$$\frac{2(d+ex)^{7/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

input
```
Int[(d + e*x)^(9/2)/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(7/2)*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) -
 ((-2*(d + e*x)^(3/2)*(b*c*d^2*(8*c*d - 11*b*e) + (2*c*d - b*e)*(8*c^2*d^2
 - 8*b*c*d*e - 3*b^2*e^2)*x))/(b^2*c*Sqrt[b*x + c*x^2]) + (3*e*((8*(4*c^3*
d^3 - 6*b*c^2*d^2*e + b^3*e^3)*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])/(3*c) + ((
2*Sqrt[-b]*(16*c^4*d^4 - 32*b*c^3*d^3*e + 9*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e^
3 - 8*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(S
qrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt
[b*x + c*x^2]) - (8*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*(2*c^2*d^2 - 2*b*
c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[Arc
Sin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sq
rt[b*x + c*x^2]))/(3*c)))/(b^2*c))/(3*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1236
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Simp[1/(c*(m + 2*p + 2))   Int[(d + e*x)^(m - 1
)*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m
*(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[
{a, b, c, d, e, f, g, p}, x] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 894 vs. $2(434) = 868$.

Time = 4.14 (sec) , antiderivative size = 895, normalized size of antiderivative = 1.85

method	result
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(\frac{2\left(b^4e^4-4d\,e^3b^3c+6d^2e^2b^2c^2-4d^3eb\,c^3+d^4c^4\right)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3b^3c^4\left(\frac{b}{c}+x\right)^2}-\frac{2\left(ce\,x^2+cdx\right)\left(5b^4e^4-7d\,e^3b^3c-9d^2e^2b^2c^2+\right.}{3b^4c^3\sqrt{\left(\frac{b}{c}+x\right)\left(ce\,x^2+cdx\right.}}\right.}{}$
default	Expression too large to display

input
```
int((e*x+d)^(9/2)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(2/3*(b^4*e^4-4*
b^3*c*d*e^3+6*b^2*c^2*d^2*e^2-4*b*c^3*d^3*e+c^4*d^4)/b^3/c^4*(c*e*x^3+b*e*
x^2+c*d*x^2+b*d*x)^(1/2)/(b/c+x)^2-2/3*(c*e*x^2+c*d*x)*(5*b^4*e^4-7*b^3*c*
d*e^3-9*b^2*c^2*d^2*e^2+19*b*c^3*d^3*e-8*c^4*d^4)/b^4/c^3/((b/c+x)*(c*e*x^
2+c*d*x))^(1/2)-2/3*d^4/b^3*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/x^2-2/3*
(c*e*x^2+b*e*x+c*d*x+b*d)*d^3/b^4*(13*b*e-8*c*d)/(x*(c*e*x^2+b*e*x+c*d*x+b
*d))^(1/2)+2*(-e^4*(2*b*e-5*c*d)/c^3+1/3*(b^4*e^4-4*b^3*c*d*e^3+6*b^2*c^2*
d^2*e^2-4*b*c^3*d^3*e+c^4*d^4)/c^3*e/b^3+1/3*(5*b^4*e^4-7*b^3*c*d*e^3-9*b^
2*c^2*d^2*e^2+19*b*c^3*d^3*e-8*c^4*d^4)/c^3*(b*e-c*d)/b^4+1/3/c^2*d*(5*b^4
*e^4-7*b^3*c*d*e^3-9*b^2*c^2*d^2*e^2+19*b*c^3*d^3*e-8*c^4*d^4)/b^4-1/3*d^4
/b^3*c*e)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2
)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d/
e/(-d/e+b/c))^(1/2))+2*(e^5/c^2+1/3*(5*b^4*e^4-7*b^3*c*d*e^3-9*b^2*c^2*d^2
*e^2+19*b*c^3*d^3*e-8*c^4*d^4)/c^2*e/b^4+1/3*c*d^3*e*(13*b*e-8*c*d)/b^4)*d
/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+
b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-d
/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^
(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(434) = 868$.

Time = 0.16 (sec) , antiderivative size = 933, normalized size of antiderivative = 1.92

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(9/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/9*(((16*c^7*d^5 - 40*b*c^6*d^4*e + 22*b^2*c^5*d^3*e^2 + 7*b^3*c^4*d^2*e^
3 + 11*b^4*c^3*d*e^4 - 8*b^5*c^2*e^5)*x^4 + 2*(16*b*c^6*d^5 - 40*b^2*c^5*d
^4*e + 22*b^3*c^4*d^3*e^2 + 7*b^4*c^3*d^2*e^3 + 11*b^5*c^2*d*e^4 - 8*b^6*c
*e^5)*x^3 + (16*b^2*c^5*d^5 - 40*b^3*c^4*d^4*e + 22*b^4*c^3*d^3*e^2 + 7*b^
5*c^2*d^2*e^3 + 11*b^6*c*d*e^4 - 8*b^7*e^5)*x^2)*sqrt(c*e)*weierstrassPInv
erse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c
^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)
/(c*e)) + 3*((16*c^7*d^4*e - 32*b*c^6*d^3*e^2 + 9*b^2*c^5*d^2*e^3 + 7*b^3*
c^4*d*e^4 - 8*b^4*c^3*e^5)*x^4 + 2*(16*b*c^6*d^4*e - 32*b^2*c^5*d^3*e^2 +
9*b^3*c^4*d^2*e^3 + 7*b^4*c^3*d*e^4 - 8*b^5*c^2*e^5)*x^3 + (16*b^2*c^5*d^4
*e - 32*b^3*c^4*d^3*e^2 + 9*b^4*c^3*d^2*e^3 + 7*b^5*c^2*d*e^4 - 8*b^6*c*e^
5)*x^2)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e
^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^
3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27
*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3
*c*e*x + c*d + b*e)/(c*e))) - 3*(b^3*c^4*d^4*e - (16*c^7*d^4*e - 32*b*c^6*
d^3*e^2 + 9*b^2*c^5*d^2*e^3 + 7*b^3*c^4*d*e^4 - 5*b^4*c^3*e^5)*x^3 - (24*b
*c^6*d^4*e - 49*b^2*c^5*d^3*e^2 + 15*b^3*c^4*d^2*e^3 + 3*b^4*c^3*d*e^4 - 4
*b^5*c^2*e^5)*x^2 - (6*b^2*c^5*d^4*e - 13*b^3*c^4*d^3*e^2)*x)*sqrt(c*x^2 +
 b*x)*sqrt(e*x + d))/(b^4*c^6*e*x^4 + 2*b^5*c^5*e*x^3 + b^6*c^4*e*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(9/2)/(c*x**2+b*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{9}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(9/2)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(9/2)/(c*x^2 + b*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{9}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(9/2)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(9/2)/(c*x^2 + b*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(d+e\,x)^{9/2}}{(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((d + e*x)^(9/2)/(b*x + c*x^2)^(5/2),x)
```

output
```
int((d + e*x)^(9/2)/(b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{9/2}}{(bx+cx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((e*x+d)^(9/2)/(c*x^2+b*x)^(5/2),x)
```

output
```
(16*sqrt(d + e*x)*sqrt(b + c*x)*b**3*e**4*x - 30*sqrt(d + e*x)*sqrt(b + c*
x)*b**2*c*d*e**3*x + 24*sqrt(d + e*x)*sqrt(b + c*x)*b**2*c*e**4*x**2 - 30*
sqrt(d + e*x)*sqrt(b + c*x)*b*c**2*d*e**3*x**2 + 6*sqrt(d + e*x)*sqrt(b +
c*x)*b*c**2*e**4*x**3 - 2*sqrt(d + e*x)*sqrt(b + c*x)*c**3*d**4 + 8*sqrt(x
)*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**4*d*e*x + sqrt(x)*b**4*e**
2*x**2 + 3*sqrt(x)*b**3*c*d**2*x + 6*sqrt(x)*b**3*c*d*e*x**2 + 3*sqrt(x)*b
**3*c*e**2*x**3 + 9*sqrt(x)*b**2*c**2*d**2*x**2 + 12*sqrt(x)*b**2*c**2*d*e
*x**3 + 3*sqrt(x)*b**2*c**2*e**2*x**4 + 9*sqrt(x)*b*c**3*d**2*x**3 + 10*sq
rt(x)*b*c**3*d*e*x**4 + sqrt(x)*b*c**3*e**2*x**5 + 3*sqrt(x)*c**4*d**2*x**
4 + 3*sqrt(x)*c**4*d*e*x**5),x)*b**7*d*e**5*x + 9*sqrt(x)*int((sqrt(d + e*
x)*sqrt(b + c*x))/(sqrt(x)*b**4*d*e*x + sqrt(x)*b**4*e**2*x**2 + 3*sqrt(x)
*b**3*c*d**2*x + 6*sqrt(x)*b**3*c*d*e*x**2 + 3*sqrt(x)*b**3*c*e**2*x**3 +
9*sqrt(x)*b**2*c**2*d**2*x**2 + 12*sqrt(x)*b**2*c**2*d*e*x**3 + 3*sqrt(x)*
b**2*c**2*e**2*x**4 + 9*sqrt(x)*b*c**3*d**2*x**3 + 10*sqrt(x)*b*c**3*d*e*x
**4 + sqrt(x)*b*c**3*e**2*x**5 + 3*sqrt(x)*c**4*d**2*x**4 + 3*sqrt(x)*c**4
*d*e*x**5),x)*b**6*c*d**2*e**4*x + 16*sqrt(x)*int((sqrt(d + e*x)*sqrt(b +
c*x))/(sqrt(x)*b**4*d*e*x + sqrt(x)*b**4*e**2*x**2 + 3*sqrt(x)*b**3*c*d**2
*x + 6*sqrt(x)*b**3*c*d*e*x**2 + 3*sqrt(x)*b**3*c*e**2*x**3 + 9*sqrt(x)*b*
*2*c**2*d**2*x**2 + 12*sqrt(x)*b**2*c**2*d*e*x**3 + 3*sqrt(x)*b**2*c**2*e*
*2*x**4 + 9*sqrt(x)*b*c**3*d**2*x**3 + 10*sqrt(x)*b*c**3*d*e*x**4 + sqr...
```

3.221 $\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{5/2}}\,dx$

Optimal result . . . 1844
Mathematica [C] (verified) . . . 1845
Rubi [A] (verified) . . . 1845
Maple [B] (verified) . . . 1850
Fricas [B] (verification not implemented) . . . 1851
Sympy [F(-1)] . . . 1851
Maxima [F] . . . 1852
Giac [F] . . . 1852
Mupad [F(-1)] . . . 1852
Reduce [F] . . . 1853

Optimal result

Integrand size = 23, antiderivative size = 380

$$\begin{aligned}\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{5/2}}\,dx &= \frac{2(cd-be)(d+ex)^{5/2}}{3bc\,(bx+cx^2)^{3/2}} \\ &+ \frac{2d(8c^2d^2-11bcde+b^2e^2)\sqrt{d+ex}}{3b^3c\sqrt{bx+cx^2}} - \frac{2d(2cd-be)(d+ex)^{3/2}}{3b^2cx\sqrt{bx+cx^2}} \\ &+ \frac{4(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)\sqrt{x}\sqrt{d+ex}\,E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{3b^{7/2}c^{3/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} \\ &- \frac{2e(8c^2d^2-11bcde+b^2e^2)\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{3b^{5/2}c^{3/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}\end{aligned}$$

output
```
2/3*(-b*e+c*d)*(e*x+d)^(5/2)/b/c/(c*x^2+b*x)^(3/2)+2/3*d*(b^2*e^2-11*b*c*d
*e+8*c^2*d^2)*(e*x+d)^(1/2)/b^3/c/(c*x^2+b*x)^(1/2)-2/3*d*(-b*e+2*c*d)*(e*
x+d)^(3/2)/b^2/c/x/(c*x^2+b*x)^(1/2)+4/3*(-b*e+2*c*d)*(-b^2*e^2-4*b*c*d*e+
4*c^2*d^2)*x^(1/2)*(e*x+d)^(1/2)*EllipticE(c^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/
b)^(1/2),(1-b*e/c/d)^(1/2))/b^(7/2)/c^(3/2)/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c
*x^2+b*x)^(1/2)-2/3*e*(b^2*e^2-11*b*c*d*e+8*c^2*d^2)*x^(1/2)*(e*x+d)^(1/2)
*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e/c/d)^(1/2))/b^(5/2
)/c^(3/2)/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.80 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.07

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{5/2}}\,dx = \frac{2\Big(b(d+ex)\left(b(cd-be)^3x^2+2(cd-be)^2(4cd+be)x^2(b+cx)-bcd^3(b+cx)^2+2cd\right.}{}$$

input
```
Integrate[(d + e*x)^(7/2)/(b*x + c*x^2)^(5/2),x]
```

output
```
(2*(b*(d + e*x)*(b*(c*d - b*e)^3*x^2 + 2*(c*d - b*e)^2*(4*c*d + b*e)*x^2*(
b + c*x) - b*c*d^3*(b + c*x)^2 + 2*c*d^2*(4*c*d - 5*b*e)*x*(b + c*x)^2) -
Sqrt[b/c]*x*(b + c*x)*(2*Sqrt[b/c]*(8*c^3*d^3 - 12*b*c^2*d^2*e + 2*b^2*c*d
*e^2 + b^3*e^3)*(b + c*x)*(d + e*x) + (2*I)*b*e*(8*c^3*d^3 - 12*b*c^2*d^2*
e + 2*b^2*c*d*e^2 + b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*E
llipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(8*c^3*d^3 - 1
3*b*c^2*d^2*e + 3*b^2*c*d*e^2 + 2*b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e
*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(3*b^
5*c*(x*(b + c*x))^(3/2)*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1164, 27, 1233, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \; 1164$$

$$-\frac{2\int \frac{(d+ex)^{3/2}(d(8cd-9be)-e(2cd-be)x)}{2(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{3b^2(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\int \frac{(d+ex)^{3/2}(d(8cd-9be)-e(2cd-be)x)}{(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 1233$$

$$-\frac{\frac{2\int \frac{e\left(bd\left(8c^2d^2-11bced+b^2e^2\right)+2(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2c}-\frac{2\sqrt{d+ex}\left(x(2cd-be)\left(-b^2e^2-8bcde+8c^2d^2\right)+bcd^2(8cd-9be)\right)}{b^2c\sqrt{bx+cx^2}}}{3b^2}-\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{e\int \frac{bd\left(8c^2d^2-11bced+b^2e^2\right)+2(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2c}-\frac{2\sqrt{d+ex}\left(x(2cd-be)\left(-b^2e^2-8bcde+8c^2d^2\right)+bcd^2(8cd-9be)\right)}{b^2c\sqrt{bx+cx^2}}}{3b^2}-\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 1269$$

$$-\frac{\frac{e\left(\frac{2(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{d(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}\right)}{b^2c}-\frac{2\sqrt{d+ex}\left(x(2cd-be)\left(-b^2e^2-8bc\cdots\right.\right.}{b^2c\sqrt{bx+\cdots}}}{3b^2}-\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 1169$$

$$-\frac{\frac{e\left(\frac{2\sqrt{x}\sqrt{b+cx}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2c}-\frac{2\sqrt{d+ex}\left(x(2c\cdots\right.}{\cdots}}{3b^2}-\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 122$$

$$-\frac{e\left(\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2c} - 2\sqrt{d+}$$

$$3b^2$$

$$\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 120$$

$$-\frac{e\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2c}$$

$$3b^2$$

$$\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 127$$

$$-\frac{e\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int\frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{}}}{e\sqrt{bx+cx^2}\sqrt{d+ex}}}{b^2c}$$

$$3b^2$$

$$\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \; 126$$

$$-\frac{e\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\text{EllipticF}}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}}{b^2c}$$

$$3b^2$$

$$\frac{2(d+ex)^{5/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

input
```
Int[(d + e*x)^(7/2)/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(5/2)*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) -
 ((-2*Sqrt[d + e*x]*(b*c*d^2*(8*c*d - 9*b*e) + (2*c*d - b*e)*(8*c^2*d^2 -
8*b*c*d*e - b^2*e^2)*x))/(b^2*c*Sqrt[b*x + c*x^2]) + (e*((4*Sqrt[-b]*(2*c*
d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[
d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt
[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[-b]*d*(c*d - b*e)*(16
*c^2*d^2 - 16*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/
d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*
Sqrt[d + e*x]*Sqrt[b*x + c*x^2])))/(b^2*c))/(3*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1233
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Simp[1/(c*(
p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Sim
p[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f
*(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(
m + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*
p + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1] &&
GtQ[m, 1] && ((EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) |
|  !ILtQ[m + 2*p + 3, 0])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(335) = 670$.

Time = 4.37 (sec) , antiderivative size = 792, normalized size of antiderivative = 2.08

method	result
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(-\frac{2\left(b^3e^3-3d\,e^2b^2c+3d^2eb\,c^2-d^3c^3\right)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3b^3c^3\left(\frac{b}{c}+x\right)^2}+\frac{4\left(ce\,x^2+cdx\right)\left(b^3e^3+2d\,e^2b^2c-7d^2eb\,c^2+4d^3c^3\right)}{3b^4c^2\sqrt{\left(\frac{b}{c}+x\right)\left(ce\,x^2+cdx\right)}}-\frac{2d^3}{}\right.}{}$
default	Expression too large to display

input
```
int((e*x+d)^(7/2)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(-2/3*(b^3*e^3-3
*b^2*c*d*e^2+3*b*c^2*d^2*e-c^3*d^3)/b^3/c^3*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x
)^(1/2)/(b/c+x)^2+4/3*(c*e*x^2+c*d*x)*(b^3*e^3+2*b^2*c*d*e^2-7*b*c^2*d^2*e
+4*c^3*d^3)/b^4/c^2/((b/c+x)*(c*e*x^2+c*d*x))^(1/2)-2/3*d^3/b^3*(c*e*x^3+b
*e*x^2+c*d*x^2+b*d*x)^(1/2)/x^2-4/3*(c*e*x^2+b*e*x+c*d*x+b*d)*d^2/b^4*(5*b
*e-4*c*d)/(x*(c*e*x^2+b*e*x+c*d*x+b*d))^(1/2)+2*(e^4/c^2-1/3*(b^3*e^3-3*b^
2*c*d*e^2+3*b*c^2*d^2*e-c^3*d^3)/c^2*e/b^3-2/3*(b^3*e^3+2*b^2*c*d*e^2-7*b*
c^2*d^2*e+4*c^3*d^3)/c^2*(b*e-c*d)/b^4-2/3/c*d*(b^3*e^3+2*b^2*c*d*e^2-7*b*
c^2*d^2*e+4*c^3*d^3)/b^4-1/3*d^3/b^3*c*e)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)
/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*El
lipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(-2/3*(b^3*e^3+2*b^
2*c*d*e^2-7*b*c^2*d^2*e+4*c^3*d^3)/c*e/b^4+2/3*c*d^2*e*(5*b*e-4*c*d)/b^4)*
d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3
+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-
d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))
^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(335) = 670$.

Time = 0.10 (sec) , antiderivative size = 816, normalized size of antiderivative = 2.15

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/9*(((16*c^6*d^4 - 32*b*c^5*d^3*e + 13*b^2*c^4*d^2*e^2 + 3*b^3*c^3*d*e^3
+ 2*b^4*c^2*e^4)*x^4 + 2*(16*b*c^5*d^4 - 32*b^2*c^4*d^3*e + 13*b^3*c^3*d^2
*e^2 + 3*b^4*c^2*d*e^3 + 2*b^5*c*e^4)*x^3 + (16*b^2*c^4*d^4 - 32*b^3*c^3*d
^3*e + 13*b^4*c^2*d^2*e^2 + 3*b^5*c*d*e^3 + 2*b^6*e^4)*x^2)*sqrt(c*e)*weie
rstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*
d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x +
 c*d + b*e)/(c*e)) + 6*((8*c^6*d^3*e - 12*b*c^5*d^2*e^2 + 2*b^2*c^4*d*e^3
+ b^3*c^3*e^4)*x^4 + 2*(8*b*c^5*d^3*e - 12*b^2*c^4*d^2*e^2 + 2*b^3*c^3*d*e
^3 + b^4*c^2*e^4)*x^3 + (8*b^2*c^4*d^3*e - 12*b^3*c^3*d^2*e^2 + 2*b^4*c^2*
d*e^3 + b^5*c*e^4)*x^2)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e +
 b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*
b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/
(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(
c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) - 3*(b^3*c^3*d^3*e - 2*(8*c^6*
d^3*e - 12*b*c^5*d^2*e^2 + 2*b^2*c^4*d*e^3 + b^3*c^3*e^4)*x^3 - (24*b*c^5*
d^3*e - 37*b^2*c^4*d^2*e^2 + 7*b^3*c^3*d*e^3 + b^4*c^2*e^4)*x^2 - 2*(3*b^2
*c^4*d^3*e - 5*b^3*c^3*d^2*e^2)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/(b^4*c
^5*e*x^4 + 2*b^5*c^4*e*x^3 + b^6*c^3*e*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x+d)**(7/2)/(c*x**2+b*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{7}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(7/2)/(c*x^2 + b*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{7}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(7/2)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(7/2)/(c*x^2 + b*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(d+e\,x)^{7/2}}{(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((d + e*x)^(7/2)/(b*x + c*x^2)^(5/2),x)
```

output
```
int((d + e*x)^(7/2)/(b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{7/2}}{(bx+cx^2)^{5/2}}\,dx = \text{too large to display}$$

input

```
int((e*x+d)^(7/2)/(c*x^2+b*x)^(5/2),x)
```

output

```
( - 24*sqrt(d + e*x)*sqrt(b + c*x)*b**2*e**3*x**2 - 8*sqrt(d + e*x)*sqrt(b
 + c*x)*b*c*d**3 - 80*sqrt(d + e*x)*sqrt(b + c*x)*b*c*d**2*e*x + 48*sqrt(d
 + e*x)*sqrt(b + c*x)*c**2*d**3*x + 9*sqrt(x)*int((sqrt(d + e*x)*sqrt(b +
c*x)*x)/(sqrt(x)*b**3*d + sqrt(x)*b**3*e*x + 3*sqrt(x)*b**2*c*d*x + 3*sqrt
(x)*b**2*c*e*x**2 + 3*sqrt(x)*b*c**2*d*x**2 + 3*sqrt(x)*b*c**2*e*x**3 + sq
rt(x)*c**3*d*x**3 + sqrt(x)*c**3*e*x**4),x)*b**5*e**4*x + 54*sqrt(x)*int((
sqrt(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**3*d + sqrt(x)*b**3*e*x + 3*sqrt
(x)*b**2*c*d*x + 3*sqrt(x)*b**2*c*e*x**2 + 3*sqrt(x)*b*c**2*d*x**2 + 3*sqr
t(x)*b*c**2*e*x**3 + sqrt(x)*c**3*d*x**3 + sqrt(x)*c**3*e*x**4),x)*b**4*c*
d*e**3*x + 18*sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**3*d
+ sqrt(x)*b**3*e*x + 3*sqrt(x)*b**2*c*d*x + 3*sqrt(x)*b**2*c*e*x**2 + 3*sq
rt(x)*b*c**2*d*x**2 + 3*sqrt(x)*b*c**2*e*x**3 + sqrt(x)*c**3*d*x**3 + sqrt
(x)*c**3*e*x**4),x)*b**4*c*e**4*x**2 - 135*sqrt(x)*int((sqrt(d + e*x)*sqrt
(b + c*x)*x)/(sqrt(x)*b**3*d + sqrt(x)*b**3*e*x + 3*sqrt(x)*b**2*c*d*x + 3
*sqrt(x)*b**2*c*e*x**2 + 3*sqrt(x)*b*c**2*d*x**2 + 3*sqrt(x)*b*c**2*e*x**3
 + sqrt(x)*c**3*d*x**3 + sqrt(x)*c**3*e*x**4),x)*b**3*c**2*d**2*e**2*x + 1
08*sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**3*d + sqrt(x)*b
**3*e*x + 3*sqrt(x)*b**2*c*d*x + 3*sqrt(x)*b**2*c*e*x**2 + 3*sqrt(x)*b*c**
2*d*x**2 + 3*sqrt(x)*b*c**2*e*x**3 + sqrt(x)*c**3*d*x**3 + sqrt(x)*c**3*e*
x**4),x)*b**3*c**2*d*e**3*x**2 + 9*sqrt(x)*int((sqrt(d + e*x)*sqrt(b + ...
```

3.222 $\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{5/2}}\,dx$

Optimal result 1854
Mathematica [C] (verified) 1855
Rubi [A] (verified) 1855
Maple [B] (verified) 1860
Fricas [B] (verification not implemented) 1861
Sympy [F] 1861
Maxima [F] 1862
Giac [F] 1862
Mupad [F(-1)] 1862
Reduce [F] 1863

Optimal result

Integrand size = 23, antiderivative size = 351

$$\begin{aligned}\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{5/2}}\,dx &= \frac{2(cd-be)(d+ex)^{3/2}}{3bc\,(bx+cx^2)^{3/2}} \\ &+ \frac{2(cd-be)(8cd-be)\sqrt{d+ex}}{3b^3c\sqrt{bx+cx^2}} - \frac{2d(2cd-be)\sqrt{d+ex}}{3b^2cx\sqrt{bx+cx^2}} \\ &+ \frac{2(16c^2d^2-16bcde+b^2e^2)\,\sqrt{x}\sqrt{d+ex}\,E\!\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{3b^{7/2}\sqrt{c}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} \\ &- \frac{2e(8cd-7be)\sqrt{x}\sqrt{d+ex}\,\operatorname{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{3b^{5/2}\sqrt{c}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}\end{aligned}$$

output

```
2/3*(-b*e+c*d)*(e*x+d)^(3/2)/b/c/(c*x^2+b*x)^(3/2)+2/3*(-b*e+c*d)*(-b*e+8*
c*d)*(e*x+d)^(1/2)/b^3/c/(c*x^2+b*x)^(1/2)-2/3*d*(-b*e+2*c*d)*(e*x+d)^(1/2
)/b^2/c/x/(c*x^2+b*x)^(1/2)+2/3*(b^2*e^2-16*b*c*d*e+16*c^2*d^2)*x^(1/2)*(e
*x+d)^(1/2)*EllipticE(c^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^
(1/2))/b^(7/2)/c^(1/2)/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)-2/3*e
*(-7*b*e+8*c*d)*x^(1/2)*(e*x+d)^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/
2)/b^(1/2)),(1-b*e/c/d)^(1/2))/b^(5/2)/c^(1/2)/(b*(e*x+d)/d/(c*x+b))^(1/2)
/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 12.04 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.01

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{5/2}}\,dx = \frac{2\Big(b(d+ex)\left(16c^3d^2x^3+8bc^2dx^2(3d-2ex)-b^3(d^2+7dex-2e^2x^2)+b^2cx(6d^2-\right.}{}$$

input
```
Integrate[(d + e*x)^(5/2)/(b*x + c*x^2)^(5/2),x]
```

output
```
(2*(b*(d + e*x)*(16*c^3*d^2*x^3 + 8*b*c^2*d*x^2*(3*d - 2*e*x) - b^3*(d^2 +
 7*d*e*x - 2*e^2*x^2) + b^2*c*x*(6*d^2 - 25*d*e*x + e^2*x^2)) - Sqrt[b/c]*
x*(b + c*x)*(Sqrt[b/c]*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*(b + c*x)*(d +
e*x) + I*b*e*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1
+ d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] -
I*b*e*(8*c^2*d^2 - 9*b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)
]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(3*b^5*(
x*(b + c*x))^(3/2)*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.02, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1164, 27, 1234, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 1164$$

$$-\frac{2\int \frac{\sqrt{d+ex}(d(8cd-7be)+e(2cd-be)x)}{2(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{\sqrt{d+ex}(d(8cd-7be)+e(2cd-be)x)}{(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 1234$$

$$-\frac{-\frac{2\int -\frac{e\left(bd(8cd-7be)+\left(16c^2d^2-16bced+b^2e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2}-\frac{2\sqrt{d+ex}\left(x\left(b^2e^2-16bcde+16c^2d^2\right)+bd(8cd-7be)\right)}{b^2\sqrt{bx+cx^2}}}{3b^2}-\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{e\int \frac{bd(8cd-7be)+\left(16c^2d^2-16bced+b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2}-\frac{2\sqrt{d+ex}\left(x\left(b^2e^2-16bcde+16c^2d^2\right)+bd(8cd-7be)\right)}{b^2\sqrt{bx+cx^2}}}{3b^2}-\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 1269$$

$$-\frac{\frac{e\left(\frac{\left(b^2e^2-16bcde+16c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{8d(cd-be)(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}\right)}{b^2}-\frac{2\sqrt{d+ex}\left(x\left(b^2e^2-16bcde+16c^2d^2\right)+bd(8cd-7be)\right)}{b^2\sqrt{bx+cx^2}}}{3b^2}-\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 1169$$

$$-\frac{\frac{e\left(\frac{\sqrt{x}\sqrt{b+cx}\left(b^2e^2-16bcde+16c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{8d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2}-\frac{2\sqrt{d+ex}\left(x\left(b^2e^2-16bcde+16c^2d^2\right)+b\cdots\right.}{b^2\sqrt{bx+cx^2}}}{3b^2}-\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 122$$

$$-\frac{e\left(\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2}-\frac{2\sqrt{d+ex}\left(x\left(b^2e^2-16bcde+16\ldots\right.\right.}{b^2\sqrt{bx+c\ldots}}}{3b^2}$$

$$\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$\big\downarrow$ 120

$$-\frac{e\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2}-\frac{2\sqrt{d+ex}\left(x\left(b^2e^2\ldots\right.\right.}{}}{3b^2}$$

$$\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$\big\downarrow$ 127

$$-\frac{e\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{b^2}-\frac{2\sqrt{d+e\ldots}}{}}{3b^2}$$

$$\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$\big\downarrow$ 126

$$-\frac{e\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{16\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{b^2}}{3b^2}$$

$$\frac{2(d+ex)^{3/2}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

input
```
Int[(d + e*x)^(5/2)/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(d + e*x)^(3/2)*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) -
 ((-2*Sqrt[d + e*x]*(b*d*(8*c*d - 7*b*e) + (16*c^2*d^2 - 16*b*c*d*e + b^2*
e^2)*x))/(b^2*Sqrt[b*x + c*x^2]) + (e*((2*Sqrt[-b]*(16*c^2*d^2 - 16*b*c*d*
e + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE[ArcSin[(Sqr
t[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b
*x + c*x^2]) - (16*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (
c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b
*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])))/b^2)/(3*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1234
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p +
 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g
*(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*
(m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && LtQ[p, -1
] && GtQ[m, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(306) = 612$.

Time = 4.10 (sec) , antiderivative size = 659, normalized size of antiderivative = 1.88

method	result
elliptic	$\sqrt{(cx+b)x(ex+d)}\left(\frac{2\left(b^{2}e^{2}-2bcde+c^{2}d^{2}\right)\sqrt{ce\,x^{3}+be\,x^{2}+cd\,x^{2}+bdx}}{3b^{3}c^{2}\left(\frac{b}{c}+x\right)^{2}}+\frac{2\left(ce\,x^{2}+cdx\right)(be-8cd)(be-cd)}{3b^{4}c\sqrt{\left(\frac{b}{c}+x\right)\left(ce\,x^{2}+cdx\right)}}-\frac{2d^{2}\sqrt{ce\,x^{3}+be\,x^{2}+cd\,x^{2}+bdx}}{3b^{3}x^{2}}-\frac{2(ce\ldots$
default	Expression too large to display

input
```
int((e*x+d)^(5/2)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(2/3*(b^2*e^2-2*
b*c*d*e+c^2*d^2)/b^3/c^2*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(b/c+x)^2+2
/3*(c*e*x^2+c*d*x)*(b*e-8*c*d)*(b*e-c*d)/b^4/c/((b/c+x)*(c*e*x^2+c*d*x))^(
1/2)-2/3*d^2/b^3*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/x^2-2/3*(c*e*x^2+b*
e*x+c*d*x+b*d)*d/b^4*(7*b*e-8*c*d)/(x*(c*e*x^2+b*e*x+c*d*x+b*d))^(1/2)+2*(
1/3*(b^2*e^2-2*b*c*d*e+c^2*d^2)/c*e/b^3-1/3*(b*e-8*c*d)*(b*e-c*d)^2/c/b^4-
1/3*d*(b*e-8*c*d)*(b*e-c*d)/b^4-1/3*d^2/b^3*c*e)*d/e*((x+d/e)/d*e)^(1/2)*(
(b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(
1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(-1/3*(b*e-8
*c*d)*(b*e-c*d)*e/b^4+1/3*d*e*c*(7*b*e-8*c*d)/b^4)*d/e*((x+d/e)/d*e)^(1/2)
*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)
^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-
b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 700 vs. $2(306) = 612$.

Time = 0.12 (sec) , antiderivative size = 700, normalized size of antiderivative = 1.99

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{5/2}}\,dx = \frac{2\left(((16\,c^5d^3-24\,bc^4d^2e+6\,b^2c^3de^2+b^3c^2e^3)x^4+2\,(16\,bc^4d^3-24\,b^2c^3d^2e+6\,b^3c^2de\right.}{}$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/9*(((16*c^5*d^3 - 24*b*c^4*d^2*e + 6*b^2*c^3*d*e^2 + b^3*c^2*e^3)*x^4 +
2*(16*b*c^4*d^3 - 24*b^2*c^3*d^2*e + 6*b^3*c^2*d*e^2 + b^4*c*e^3)*x^3 + (1
6*b^2*c^3*d^3 - 24*b^3*c^2*d^2*e + 6*b^4*c*d*e^2 + b^5*e^3)*x^2)*sqrt(c*e)
*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2
*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*
e*x + c*d + b*e)/(c*e)) + 3*((16*c^5*d^2*e - 16*b*c^4*d*e^2 + b^2*c^3*e^3)
*x^4 + 2*(16*b*c^4*d^2*e - 16*b^2*c^3*d*e^2 + b^3*c^2*e^3)*x^3 + (16*b^2*c
^3*d^2*e - 16*b^3*c^2*d*e^2 + b^4*c*e^3)*x^2)*sqrt(c*e)*weierstrassZeta(4/
3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*
e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2
 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*
c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) - 3*(b^3
*c^2*d^2*e - (16*c^5*d^2*e - 16*b*c^4*d*e^2 + b^2*c^3*e^3)*x^3 - (24*b*c^4
*d^2*e - 25*b^2*c^3*d*e^2 + 2*b^3*c^2*e^3)*x^2 - (6*b^2*c^3*d^2*e - 7*b^3*
c^2*d*e^2)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/(b^4*c^4*e*x^4 + 2*b^5*c^3*
e*x^3 + b^6*c^2*e*x^2)
```

Sympy [F]

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(d+ex)^{\frac{5}{2}}}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**(5/2)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral((d + e*x)**(5/2)/(x*(b + c*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(5/2)/(c*x^2 + b*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{5}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(5/2)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(5/2)/(c*x^2 + b*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(d+e\,x)^{5/2}}{(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((d + e*x)^(5/2)/(b*x + c*x^2)^(5/2),x)
```

output
```
int((d + e*x)^(5/2)/(b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{5/2}}{(bx+cx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((e*x+d)^(5/2)/(c*x^2+b*x)^(5/2),x)
```

output
```
( - 2*sqrt(d + e*x)*sqrt(b + c*x)*b*e**2*x - 2*sqrt(d + e*x)*sqrt(b + c*x)
*c*d**2 - sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**4*d*e*x +
sqrt(x)*b**4*e**2*x**2 + 3*sqrt(x)*b**3*c*d**2*x + 6*sqrt(x)*b**3*c*d*e*x*
*2 + 3*sqrt(x)*b**3*c*e**2*x**3 + 9*sqrt(x)*b**2*c**2*d**2*x**2 + 12*sqrt(
x)*b**2*c**2*d*e*x**3 + 3*sqrt(x)*b**2*c**2*e**2*x**4 + 9*sqrt(x)*b*c**3*d
**2*x**3 + 10*sqrt(x)*b*c**3*d*e*x**4 + sqrt(x)*b*c**3*e**2*x**5 + 3*sqrt(
x)*c**4*d**2*x**4 + 3*sqrt(x)*c**4*d*e*x**5),x)*b**5*d*e**3*x + 4*sqrt(x)*
int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**4*d*e*x + sqrt(x)*b**4*e**2*
x**2 + 3*sqrt(x)*b**3*c*d**2*x + 6*sqrt(x)*b**3*c*d*e*x**2 + 3*sqrt(x)*b**
3*c*e**2*x**3 + 9*sqrt(x)*b**2*c**2*d**2*x**2 + 12*sqrt(x)*b**2*c**2*d*e*x
**3 + 3*sqrt(x)*b**2*c**2*e**2*x**4 + 9*sqrt(x)*b*c**3*d**2*x**3 + 10*sqrt
(x)*b*c**3*d*e*x**4 + sqrt(x)*b*c**3*e**2*x**5 + 3*sqrt(x)*c**4*d**2*x**4
+ 3*sqrt(x)*c**4*d*e*x**5),x)*b**4*c*d**2*e**2*x - 2*sqrt(x)*int((sqrt(d +
 e*x)*sqrt(b + c*x))/(sqrt(x)*b**4*d*e*x + sqrt(x)*b**4*e**2*x**2 + 3*sqrt
(x)*b**3*c*d**2*x + 6*sqrt(x)*b**3*c*d*e*x**2 + 3*sqrt(x)*b**3*c*e**2*x**3
 + 9*sqrt(x)*b**2*c**2*d**2*x**2 + 12*sqrt(x)*b**2*c**2*d*e*x**3 + 3*sqrt(
x)*b**2*c**2*e**2*x**4 + 9*sqrt(x)*b*c**3*d**2*x**3 + 10*sqrt(x)*b*c**3*d*
e*x**4 + sqrt(x)*b*c**3*e**2*x**5 + 3*sqrt(x)*c**4*d**2*x**4 + 3*sqrt(x)*c
**4*d*e*x**5),x)*b**4*c*d*e**3*x**2 + 15*sqrt(x)*int((sqrt(d + e*x)*sqrt(b
 + c*x))/(sqrt(x)*b**4*d*e*x + sqrt(x)*b**4*e**2*x**2 + 3*sqrt(x)*b**3*...
```

3.223 $\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{5/2}}\,dx$

Optimal result . . . 1864
Mathematica [C] (verified) . . . 1865
Rubi [A] (verified) . . . 1865
Maple [B] (verified) . . . 1870
Fricas [B] (verification not implemented) . . . 1871
Sympy [F] . . . 1871
Maxima [F] . . . 1872
Giac [F] . . . 1872
Mupad [F(-1)] . . . 1872
Reduce [F] . . . 1873

Optimal result

Integrand size = 23, antiderivative size = 329

$$\begin{aligned}\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{5/2}}\,dx &= \frac{2(cd-be)\sqrt{d+ex}}{3bc\,(bx+cx^2)^{3/2}}+\frac{2(8cd-5be)\sqrt{d+ex}}{3b^3\sqrt{bx+cx^2}}\\ &-\frac{2(2cd-be)\sqrt{d+ex}}{3b^2cx\sqrt{bx+cx^2}}+\frac{16\sqrt{c}(2cd-be)\sqrt{x}\sqrt{d+ex}E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{3b^{7/2}\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}\\ &-\frac{2e(8cd-3be)\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{3b^{5/2}\sqrt{c}d\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}\end{aligned}$$

output
```
2/3*(-b*e+c*d)*(e*x+d)^(1/2)/b/c/(c*x^2+b*x)^(3/2)+2/3*(-5*b*e+8*c*d)*(e*x
+d)^(1/2)/b^3/(c*x^2+b*x)^(1/2)-2/3*(-b*e+2*c*d)*(e*x+d)^(1/2)/b^2/c/x/(c*
x^2+b*x)^(1/2)+16/3*c^(1/2)*(-b*e+2*c*d)*x^(1/2)*(e*x+d)^(1/2)*EllipticE(c
^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/b^(7/2)/(b*(e*x+
d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)-2/3*e*(-3*b*e+8*c*d)*x^(1/2)*(e*x+d)
^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e/c/d)^(1/2))/
b^(5/2)/c^(1/2)/d/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 18.79 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.88

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{5/2}}\,dx = \frac{2\left(-8(2cd-be)x(b+cx)(d+ex)+\frac{(d+ex)\left(16c^3dx^3+b^2cx(6d-13ex)-8bc^2x^2(-3d+ex)-b^3(d+4ex\right.}{b+cx}\right.}{}$$

input
```
Integrate[(d + e*x)^(3/2)/(b*x + c*x^2)^(5/2),x]
```

output
```
(2*(-8*(2*c*d - b*e)*x*(b + c*x)*(d + e*x) + ((d + e*x)*(16*c^3*d*x^3 + b^
2*c*x*(6*d - 13*e*x) - 8*b*c^2*x^2*(-3*d + e*x) - b^3*(d + 4*e*x)))/(b + c
*x) + (8*I)*Sqrt[b/c]*c*e*(-2*c*d + b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x
)]*x^(5/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] + I*Sqrt[b
/c]*c*e*(8*c*d - 5*b*e)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(5/2)*Ellipt
icF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)]))/(3*b^4*x*Sqrt[x*(b + c*x)
]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1164, 27, 1235, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1164}$$

$$-\frac{2\int \frac{d(8cd-5be)+3e(2cd-be)x}{2\sqrt{d+ex}(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{3b^2(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{d(8cd-5be)+3e(2cd-be)x}{\sqrt{d+ex}(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{3b^2(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 1235}$$

$$-\frac{-\frac{2\int -\frac{de(b(8cd-3be)(cd-be)+8c(2cd-be)x(cd-be))}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}(8cx(2cd-be)(cd-be)+b(8cd-5be)(cd-be))}{b^2\sqrt{bx+cx^2}(cd-be)}}{3b^2}-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{3b^2(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$-\frac{\frac{e\int \frac{b(8cd-3be)(cd-be)+8c(2cd-be)x(cd-be)}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2(cd-be)}-\frac{2\sqrt{d+ex}(8cx(2cd-be)(cd-be)+b(8cd-5be)(cd-be))}{b^2\sqrt{bx+cx^2}(cd-be)}}{3b^2}-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{3b^2(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 1269}$$

$$-\frac{\frac{e\left(\frac{8c(cd-be)(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{(4cd-3be)(cd-be)(4cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}\right)}{b^2(cd-be)}-\frac{2\sqrt{d+ex}(8cx(2cd-be)(cd-be)+b(8cd-5be)(cd-be))}{b^2\sqrt{bx+cx^2}(cd-be)}}{3b^2}-\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{3b^2(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 1169}$$

$$-\frac{\frac{e\left(\frac{8c\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{\sqrt{x}\sqrt{b+cx}(4cd-3be)(cd-be)(4cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2(cd-be)}-\frac{2\sqrt{d+ex}(8cx(2cd-be)(cd-be)+b(8}{b^2\sqrt{bx+cx^2}(cd-be}}{3b^2}$$
$$\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{3b^2(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 122}$$

$$-\frac{\frac{e\left(\frac{8c\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(cd-be)(2cd-be)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{\sqrt{x}\sqrt{b+cx}(4cd-3be)(cd-be)(4cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2(cd-be)}-\frac{2\sqrt{d+ex}(8cx(2cd-be)(cd-}{b^2\sqrt{bx+cx}}}{3b^2}$$

$$\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 120$$

$$-\frac{\frac{e\left(\frac{16\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(cd-be)(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{\sqrt{x}\sqrt{b+cx}(4cd-3be)(cd-be)(4cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2(cd-be)}-\frac{2\sqrt{d+ex}(8cx}{}}{3b^2}$$

$$\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 127$$

$$-\frac{\frac{e\left(\frac{16\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(cd-be)(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(4cd-3be)(cd-be)(4cd-be)\int\frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{b^2(cd-be)}-2\sqrt{}}{3b^2}$$

$$\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 126$$

$$-\frac{\frac{e\left(\frac{16\sqrt{-b}\sqrt{c}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(cd-be)(2cd-be)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(4cd-3be)(cd-be)(4cd-be)\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right.}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}}{b^2(cd-be)}}{3b^2}$$

$$\frac{2\sqrt{d+ex}(x(2cd-be)+bd)}{3b^2\left(bx+cx^2\right)^{3/2}}$$

input

```
Int[(d + e*x)^(3/2)/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*Sqrt[d + e*x]*(b*d + (2*c*d - b*e)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) - (
(-2*Sqrt[d + e*x]*(b*(8*c*d - 5*b*e)*(c*d - b*e) + 8*c*(c*d - b*e)*(2*c*d
- b*e)*x))/(b^2*(c*d - b*e)*Sqrt[b*x + c*x^2]) + (e*((16*Sqrt[-b]*Sqrt[c]*
(c*d - b*e)*(2*c*d - b*e)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*Elliptic
E[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(e*Sqrt[1 + (e*x)/d]*S
qrt[b*x + c*x^2]) - (2*Sqrt[-b]*(4*c*d - 3*b*e)*(c*d - b*e)*(4*c*d - b*e)*
Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt
[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2]))
)/(b^2*(c*d - b*e)))/(3*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1164
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] + Simp[1/((p + 1)*(b^2 - 4*a*
c))   Int[(d + e*x)^(m - 2)*Simp[e*(2*a*e*(m - 1) + b*d*(2*p - m + 4)) - 2*
c*d^2*(2*p + 3) + e*(b*e - 2*d*c)*(m + 2*p + 2)*x, x]*(a + b*x + c*x^2)^(p
+ 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 1] && Int
QuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(284) = 568$.

Time = 4.02 (sec) , antiderivative size = 570, normalized size of antiderivative = 1.73

method	result
elliptic	$\frac{\sqrt{(cx+b)x(ex+d)}\left(-\frac{2(be-cd)\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3b^3c\left(\frac{b}{c}+x\right)^2}-\frac{8\left(ce\,x^2+cdx\right)(be-2cd)}{3b^4\sqrt{\left(\frac{b}{c}+x\right)\left(ce\,x^2+cdx\right)}}-\frac{2d\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3b^3x^2}-\frac{8\left(ce\,x^2+bex+cdx+bd\right.}{3b^4\sqrt{x\left(ce\,x^2+bex+\right.}}\right.}{}$
default	$-\frac{2\left(5x^2\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\frac{dc}{be-cd}}\right)b^2cd\,e^2-8x^2\sqrt{\frac{ex+d}{d}}\,\sqrt{\frac{e(cx+b)}{be-cd}}\,\sqrt{-\frac{ex}{d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{-\right.\right.}{}$

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(-2/3*(b*e-c*d)/
b^3/c*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(b/c+x)^2-8/3*(c*e*x^2+c*d*x)/
b^4*(b*e-2*c*d)/((b/c+x)*(c*e*x^2+c*d*x))^(1/2)-2/3*d/b^3*(c*e*x^3+b*e*x^2
+c*d*x^2+b*d*x)^(1/2)/x^2-8/3*(c*e*x^2+b*e*x+c*d*x+b*d)/b^4*(b*e-2*c*d)/(x
*(c*e*x^2+b*e*x+c*d*x+b*d))^(1/2)+2*(-1/3*e*(b*e-c*d)/b^3+4/3*(b*e-c*d)*(b
*e-2*c*d)/b^4+4/3*c*d/b^4*(b*e-2*c*d)-1/3/b^3*c*d*e)*d/e*((x+d/e)/d*e)^(1/
2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*
x)^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+16/3*c*(b*
e-2*c*d)/b^4*d*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/
2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c)*EllipticE(((x+d/e)/d*
e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/
(-d/e+b/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(284) = 568$.

Time = 0.10 (sec) , antiderivative size = 581, normalized size of antiderivative = 1.77

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{5/2}}\,dx = \frac{2\left(((16\,c^4d^2-16\,bc^3de+b^2c^2e^2)x^4+2\,(16\,bc^3d^2-16\,b^2c^2de+b^3ce^2)x^3+(16\,b^2c^2d^2\right.}{}$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/9*(((16*c^4*d^2 - 16*b*c^3*d*e + b^2*c^2*e^2)*x^4 + 2*(16*b*c^3*d^2 - 16
*b^2*c^2*d*e + b^3*c*e^2)*x^3 + (16*b^2*c^2*d^2 - 16*b^3*c*d*e + b^4*e^2)*
x^2)*sqrt(c*e)*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*
e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e
^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e)) + 24*((2*c^4*d*e - b*c^3*e^2)*x^4 +
2*(2*b*c^3*d*e - b^2*c^2*e^2)*x^3 + (2*b^2*c^2*d*e - b^3*c*e^2)*x^2)*sqrt(
c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2
*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstra
ssPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 -
 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d
+ b*e)/(c*e))) - 3*(b^3*c*d*e - 8*(2*c^4*d*e - b*c^3*e^2)*x^3 - (24*b*c^3*
d*e - 13*b^2*c^2*e^2)*x^2 - 2*(3*b^2*c^2*d*e - 2*b^3*c*e^2)*x)*sqrt(c*x^2
+ b*x)*sqrt(e*x + d))/(b^4*c^3*e*x^4 + 2*b^5*c^2*e*x^3 + b^6*c*e*x^2)
```

Sympy [F]

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(d+ex)^{\frac{3}{2}}}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**(3/2)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral((d + e*x)**(3/2)/(x*(b + c*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)/(c*x^2 + b*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(ex+d)^{\frac{3}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^(3/2)/(c*x^2 + b*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(d+e\,x)^{3/2}}{(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((d + e*x)^(3/2)/(b*x + c*x^2)^(5/2),x)
```

output
```
int((d + e*x)^(3/2)/(b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^{3/2}}{(bx+cx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int((e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x)
```

output
```
( - 2*sqrt(d + e*x)*sqrt(b + c*x)*d + 4*sqrt(x)*int((sqrt(d + e*x)*sqrt(b
+ c*x))/(sqrt(x)*b**4*d*e*x + sqrt(x)*b**4*e**2*x**2 + 3*sqrt(x)*b**3*c*d*
*2*x + 6*sqrt(x)*b**3*c*d*e*x**2 + 3*sqrt(x)*b**3*c*e**2*x**3 + 9*sqrt(x)*
b**2*c**2*d**2*x**2 + 12*sqrt(x)*b**2*c**2*d*e*x**3 + 3*sqrt(x)*b**2*c**2*
e**2*x**4 + 9*sqrt(x)*b*c**3*d**2*x**3 + 10*sqrt(x)*b*c**3*d*e*x**4 + sqrt
(x)*b*c**3*e**2*x**5 + 3*sqrt(x)*c**4*d**2*x**4 + 3*sqrt(x)*c**4*d*e*x**5)
,x)*b**4*d*e**2*x + 6*sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b
**4*d*e*x + sqrt(x)*b**4*e**2*x**2 + 3*sqrt(x)*b**3*c*d**2*x + 6*sqrt(x)*b
**3*c*d*e*x**2 + 3*sqrt(x)*b**3*c*e**2*x**3 + 9*sqrt(x)*b**2*c**2*d**2*x**
2 + 12*sqrt(x)*b**2*c**2*d*e*x**3 + 3*sqrt(x)*b**2*c**2*e**2*x**4 + 9*sqrt
(x)*b*c**3*d**2*x**3 + 10*sqrt(x)*b*c**3*d*e*x**4 + sqrt(x)*b*c**3*e**2*x*
*5 + 3*sqrt(x)*c**4*d**2*x**4 + 3*sqrt(x)*c**4*d*e*x**5),x)*b**3*c*d**2*e*
x + 8*sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**4*d*e*x + sqrt
(x)*b**4*e**2*x**2 + 3*sqrt(x)*b**3*c*d**2*x + 6*sqrt(x)*b**3*c*d*e*x**2 +
 3*sqrt(x)*b**3*c*e**2*x**3 + 9*sqrt(x)*b**2*c**2*d**2*x**2 + 12*sqrt(x)*b
**2*c**2*d*e*x**3 + 3*sqrt(x)*b**2*c**2*e**2*x**4 + 9*sqrt(x)*b*c**3*d**2*
x**3 + 10*sqrt(x)*b*c**3*d*e*x**4 + sqrt(x)*b*c**3*e**2*x**5 + 3*sqrt(x)*c
**4*d**2*x**4 + 3*sqrt(x)*c**4*d*e*x**5),x)*b**3*c*d*e**2*x**2 - 18*sqrt(x
)*int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**4*d*e*x + sqrt(x)*b**4*e**
2*x**2 + 3*sqrt(x)*b**3*c*d**2*x + 6*sqrt(x)*b**3*c*d*e*x**2 + 3*sqrt(x...
```

3.224 $\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{5/2}} dx$

Optimal result 1874
Mathematica [C] (verified) 1875
Rubi [A] (verified) 1875
Maple [A] (verified) 1880
Fricas [B] (verification not implemented) 1880
Sympy [F] 1881
Maxima [F] 1882
Giac [F] 1882
Mupad [F(-1)] 1882
Reduce [F] 1883

Optimal result

Integrand size = 23, antiderivative size = 345

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{5/2}} dx = \frac{2\sqrt{d+ex}}{3b(bx+cx^2)^{3/2}} + \frac{2(8cd-be)\sqrt{d+ex}}{3b^3d\sqrt{bx+cx^2}} - \frac{4\sqrt{d+ex}}{3b^2x\sqrt{bx+cx^2}} + \frac{2\sqrt{c}(16c^2d^2-16bcde+b^2e^2)\sqrt{x}\sqrt{d+ex}E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{3b^{7/2}d(cd-be)\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}} - \frac{2\sqrt{c}e(8cd-7be)\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right), 1-\frac{be}{cd}\right)}{3b^{5/2}d(cd-be)\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$

output
```
2/3*(e*x+d)^(1/2)/b/(c*x^2+b*x)^(3/2)+2/3*(-b*e+8*c*d)*(e*x+d)^(1/2)/b^3/d
/(c*x^2+b*x)^(1/2)-4/3*(e*x+d)^(1/2)/b^2/x/(c*x^2+b*x)^(1/2)+2/3*c^(1/2)*(
b^2*e^2-16*b*c*d*e+16*c^2*d^2)*x^(1/2)*(e*x+d)^(1/2)*EllipticE(c^(1/2)*x^(
1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/b^(7/2)/d/(-b*e+c*d)/(b*(e
*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)-2/3*c^(1/2)*e*(-7*b*e+8*c*d)*x^(1
/2)*(e*x+d)^(1/2)*InverseJacobiAM(arctan(c^(1/2)*x^(1/2)/b^(1/2)),(1-b*e/c
/d)^(1/2))/b^(5/2)/d/(-b*e+c*d)/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1
/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 8.83 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.09

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{5/2}}\,dx = \frac{2\Big(b(d+ex)\left(bc^2d(cd-be)x^2+c^2d(8cd-7be)x^2(b+cx)+bd(-cd+be)(b+cx)^2\right.\right.}{}$$

input
```
Integrate[Sqrt[d + e*x]/(b*x + c*x^2)^(5/2),x]
```

output
```
(2*(b*(d + e*x)*(b*c^2*d*(c*d - b*e)*x^2 + c^2*d*(8*c*d - 7*b*e)*x^2*(b +
c*x) + b*d*(-(c*d) + b*e)*(b + c*x)^2 + (c*d - b*e)*(8*c*d - b*e)*x*(b + c
*x)^2) - Sqrt[b/c]*c*x*(b + c*x)*(Sqrt[b/c]*(16*c^2*d^2 - 16*b*c*d*e + b^2
*e^2)*(b + c*x)*(d + e*x) + I*b*e*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)*Sqrt
[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt
[x]], (c*d)/(b*e)] - I*b*e*(8*c^2*d^2 - 9*b*c*d*e + b^2*e^2)*Sqrt[1 + b/(c
*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*
d)/(b*e)])))/(3*b^5*d*(c*d - b*e)*(x*(b + c*x))^(3/2)*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1163, 27, 1235, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 1163$$

$$\frac{2\int -\frac{8cd-be+6cex}{2\sqrt{d+ex}(cx^2+bx)^{3/2}}dx}{3b^2} - \frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{8cd-be+6cex}{\sqrt{d+ex}(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$\Big\downarrow$ 1235

$$-\frac{-\frac{2\int -\frac{ce\left(bd(8cd-7be)+\left(16c^2d^2-16bced+b^2e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(b^2e^2-16bcde+16c^2d^2)+b(cd-be)(8cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}}{3b^2}-\frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$\Big\downarrow$ 27

$$-\frac{\frac{ce\int \frac{bd(8cd-7be)+\left(16c^2d^2-16bced+b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(b^2e^2-16bcde+16c^2d^2)+b(cd-be)(8cd-be))}{b^2d\sqrt{bx+cx^2}(cd-be)}}{3b^2}-\frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$\Big\downarrow$ 1269

$$-\frac{\frac{ce\left(\frac{\left(b^2e^2-16bcde+16c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{8d(cd-be)(2cd-be)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}\right)}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(b^2e^2-16bcde+16c^2d^2)+b(cd-be)(8cd-be}{b^2d\sqrt{bx+cx^2}(cd-be)}}{3b^2}$$

$$\frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$\Big\downarrow$ 1169

$$-\frac{\frac{ce\left(\frac{\sqrt{x}\sqrt{b+cx}\left(b^2e^2-16bcde+16c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{8d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(b^2e^2-16bcde+16c^2d^2)}{b^2d\sqrt{bx+cx^2}(cd-}}{3b^2}$$

$$\frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$\Big\downarrow$ 122

$$-\frac{\frac{ce\left(\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(b^2e^2-16bcde+}{b^2d\sqrt{bx+c}}}{3b^2}$$

$$\frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$\big\downarrow$ 120

$$-\frac{\frac{ce\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8d\sqrt{x}\sqrt{b+cx}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(b^2}{}}{3b^2}$$

$$\frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$\big\downarrow$ 127

$$-\frac{\frac{ce\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{8d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\int\frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{b^2d(cd-be)}-\frac{2\sqrt{d+}}{}}{3b^2}$$

$$\frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$\big\downarrow$ 126

$$-\frac{\frac{ce\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(b^2e^2-16bcde+16c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{16\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)(2cd-be)\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right),\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}\right)}{b^2d(cd-be)}}{3b^2}$$

$$\frac{2(b+2cx)\sqrt{d+ex}}{3b^2\left(bx+cx^2\right)^{3/2}}$$

input

```
Int[Sqrt[d + e*x]/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(b + 2*c*x)*Sqrt[d + e*x])/(3*b^2*(b*x + c*x^2)^(3/2)) - ((-2*Sqrt[d +
 e*x]*(b*(c*d - b*e)*(8*c*d - b*e) + c*(16*c^2*d^2 - 16*b*c*d*e + b^2*e^2)
*x))/(b^2*d*(c*d - b*e)*Sqrt[b*x + c*x^2]) + (c*e*((2*Sqrt[-b]*(16*c^2*d^2
 - 16*b*c*d*e + b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*x]*EllipticE
[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[1 + (e*
x)/d]*Sqrt[b*x + c*x^2]) - (16*Sqrt[-b]*d*(c*d - b*e)*(2*c*d - b*e)*Sqrt[x
]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/S
qrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])))/(b^2
*d*(c*d - b*e)))/(3*b^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1163
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^m*(b + 2*c*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*
(b^2 - 4*a*c))), x] - Simp[1/((p + 1)*(b^2 - 4*a*c))   Int[(d + e*x)^(m - 1
)*(b*e*m + 2*c*d*(2*p + 3) + 2*c*e*(m + 2*p + 3)*x)*(a + b*x + c*x^2)^(p +
1), x], x] /; FreeQ[{a, b, c, d, e}, x] && LtQ[p, -1] && GtQ[m, 0] && (LtQ[
m, 1] || (ILtQ[m + 2*p + 3, 0] && NeQ[m, 2])) && IntQuadraticQ[a, b, c, d,
e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 3.66 (sec) , antiderivative size = 596, normalized size of antiderivative = 1.73

method	result
elliptic	$\sqrt{(cx+b)x(ex+d)}\left(\frac{2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3b^3\left(\frac{b}{c}+x\right)^2}+\frac{2\left(ce\,x^2+cdx\right)c(7be-8cd)}{3b^4(be-cd)\sqrt{\left(\frac{b}{c}+x\right)\left(ce\,x^2+cdx\right)}}-\frac{2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3b^3x^2}-\frac{2\left(ce\,x^2+bex+cdx+bd\right)(b}{3d\,b^4\sqrt{x\left(ce\,x^2+bex+cd\right.}}\right.$
default	Expression too large to display

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(2/3/b^3*(c*e*x^
3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(b/c+x)^2+2/3*(c*e*x^2+c*d*x)/b^4/(b*e-c*d)
*c*(7*b*e-8*c*d)/((b/c+x)*(c*e*x^2+c*d*x))^(1/2)-2/3/b^3*(c*e*x^3+b*e*x^2+
c*d*x^2+b*d*x)^(1/2)/x^2-2/3*(c*e*x^2+b*e*x+c*d*x+b*d)/d/b^4*(b*e-8*c*d)/(
x*(c*e*x^2+b*e*x+c*d*x+b*d))^(1/2)+2*(-1/3*c*(7*b*e-8*c*d)/b^4-1/3*c^2*d/b
^4/(b*e-c*d)*(7*b*e-8*c*d))*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(
1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/
e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(-1/3*e*c^2*(7*b*e-8*c*d)/b^4/(b*
e-c*d)+1/3*c*e*(b*e-8*c*d)/d/b^4)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b
/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b/c
)*EllipticE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF(((x
+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 767 vs. 2(300) = 600.

Time = 0.11 (sec) , antiderivative size = 767, normalized size of antiderivative = 2.22

$$\int \frac{\sqrt{d+ex}}{\left(bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/9*(((16*c^5*d^3 - 24*b*c^4*d^2*e + 6*b^2*c^3*d*e^2 + b^3*c^2*e^3)*x^4 +
2*(16*b*c^4*d^3 - 24*b^2*c^3*d^2*e + 6*b^3*c^2*d*e^2 + b^4*c*e^3)*x^3 + (1
6*b^2*c^3*d^3 - 24*b^3*c^2*d^2*e + 6*b^4*c*d*e^2 + b^5*e^3)*x^2)*sqrt(c*e)
*weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2
*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*
e*x + c*d + b*e)/(c*e)) + 3*((16*c^5*d^2*e - 16*b*c^4*d*e^2 + b^2*c^3*e^3)
*x^4 + 2*(16*b*c^4*d^2*e - 16*b^2*c^3*d*e^2 + b^3*c^2*e^3)*x^3 + (16*b^2*c
^3*d^2*e - 16*b^3*c^2*d*e^2 + b^4*c*e^3)*x^2)*sqrt(c*e)*weierstrassZeta(4/
3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*
e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2
 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*
c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) - 3*(b^3
*c^2*d^2*e - b^4*c*d*e^2 - (16*c^5*d^2*e - 16*b*c^4*d*e^2 + b^2*c^3*e^3)*x
^3 - (24*b*c^4*d^2*e - 25*b^2*c^3*d*e^2 + 2*b^3*c^2*e^3)*x^2 - (6*b^2*c^3*
d^2*e - 7*b^3*c^2*d*e^2 + b^4*c*e^3)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/(
(b^4*c^4*d^2*e - b^5*c^3*d*e^2)*x^4 + 2*(b^5*c^3*d^2*e - b^6*c^2*d*e^2)*x^
3 + (b^6*c^2*d^2*e - b^7*c*d*e^2)*x^2)
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{\sqrt{d+ex}}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(sqrt(d + e*x)/(x*(b + c*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/(c*x^2 + b*x)^(5/2), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{\sqrt{ex+d}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)/(c*x^2 + b*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{\sqrt{d+e\,x}}{(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((d + e*x)^(1/2)/(b*x + c*x^2)^(5/2),x)
```

output
```
int((d + e*x)^(1/2)/(b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}}{(bx+cx^2)^{5/2}}\,dx = \int \frac{\sqrt{ex+d}\,\sqrt{cx+b}}{\sqrt{x}\,b^3x^2+3\sqrt{x}\,b^2c\,x^3+3\sqrt{x}\,b\,c^2x^4+\sqrt{x}\,c^3x^5}dx$$

input
```
int((e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x)
```

output
```
int((sqrt(d + e*x)*sqrt(b + c*x))/(sqrt(x)*b**3*x**2 + 3*sqrt(x)*b**2*c*x*
*3 + 3*sqrt(x)*b*c**2*x**4 + sqrt(x)*c**3*x**5),x)
```

3.225 $\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal result . . . 1884
Mathematica [C] (verified) . . . 1885
Rubi [A] (verified) . . . 1885
Maple [A] (verified) . . . 1890
Fricas [B] (verification not implemented) . . . 1891
Sympy [F] . . . 1892
Maxima [F] . . . 1892
Giac [F] . . . 1892
Mupad [F(-1)] . . . 1893
Reduce [F] . . . 1893

Optimal result

Integrand size = 23, antiderivative size = 425

$$\int \frac{1}{\sqrt{d+ex}\left(bx+cx^2\right)^{5/2}}\,dx=\frac{2c\sqrt{d+ex}}{3b(cd-be)\left(bx+cx^2\right)^{3/2}}+\frac{2(8c^2d^2-5bcde-2b^2e^2)\sqrt{d+ex}}{3b^3d^2(cd-be)\sqrt{bx+cx^2}}-\frac{2(2cd-be)\sqrt{d+ex}}{3b^2d(cd-be)x\sqrt{bx+cx^2}}+\frac{4\sqrt{c}(2cd-be)\left(4c^2d^2-4bcde-b^2e^2\right)\sqrt{x}\sqrt{d+ex}E\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)|1-\frac{be}{cd}\right)}{3b^{7/2}d^2(cd-be)^2\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}-\frac{2\sqrt{c}e(8c^2d^2-11bcde+b^2e^2)\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right),1-\frac{be}{cd}\right)}{3b^{5/2}d^2(cd-be)^2\sqrt{\frac{b(d+ex)}{d(b+cx)}}\sqrt{bx+cx^2}}$$

output
```
2/3*c*(e*x+d)^(1/2)/b/(-b*e+c*d)/(c*x^2+b*x)^(3/2)+2/3*(-2*b^2*e^2-5*b*c*d
*e+8*c^2*d^2)*(e*x+d)^(1/2)/b^3/d^2/(-b*e+c*d)/(c*x^2+b*x)^(1/2)-2/3*(-b*e
+2*c*d)*(e*x+d)^(1/2)/b^2/d/(-b*e+c*d)/x/(c*x^2+b*x)^(1/2)+4/3*c^(1/2)*(-b
*e+2*c*d)*(-b^2*e^2-4*b*c*d*e+4*c^2*d^2)*x^(1/2)*(e*x+d)^(1/2)*EllipticE(c
^(1/2)*x^(1/2)/b^(1/2)/(1+c*x/b)^(1/2),(1-b*e/c/d)^(1/2))/b^(7/2)/d^2/(-b*
e+c*d)^2/(b*(e*x+d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)-2/3*c^(1/2)*e*(b^2*
e^2-11*b*c*d*e+8*c^2*d^2)*x^(1/2)*(e*x+d)^(1/2)*InverseJacobiAM(arctan(c^(
1/2)*x^(1/2)/b^(1/2)),(1-b*e/c/d)^(1/2))/b^(5/2)/d^2/(-b*e+c*d)^2/(b*(e*x+
d)/d/(c*x+b))^(1/2)/(c*x^2+b*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 10.14 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.01

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^{5/2}}\,dx = \frac{2\Big(b(d+ex)\,(bc^3d^2(cd-be)x^2+2c^3d^2(4cd-5be)x^2(b+cx)-bd(cd-be)^2}{}$$

input
```
Integrate[1/(Sqrt[d + e*x]*(b*x + c*x^2)^(5/2)),x]
```

output
```
(2*(b*(d + e*x)*(b*c^3*d^2*(c*d - b*e)*x^2 + 2*c^3*d^2*(4*c*d - 5*b*e)*x^2
*(b + c*x) - b*d*(c*d - b*e)^2*(b + c*x)^2 + 2*(c*d - b*e)^2*(4*c*d + b*e)
*x*(b + c*x)^2) - Sqrt[b/c]*c*x*(b + c*x)*(2*Sqrt[b/c]*(8*c^3*d^3 - 12*b*c
^2*d^2*e + 2*b^2*c*d*e^2 + b^3*e^3)*(b + c*x)*(d + e*x) + (2*I)*b*e*(8*c^3
*d^3 - 12*b*c^2*d^2*e + 2*b^2*c*d*e^2 + b^3*e^3)*Sqrt[1 + b/(c*x)]*Sqrt[1
+ d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)] -
I*b*e*(8*c^3*d^3 - 13*b*c^2*d^2*e + 3*b^2*c*d*e^2 + 2*b^3*e^3)*Sqrt[1 + b/
(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (
c*d)/(b*e)])))/(3*b^5*d^2*(c*d - b*e)^2*(x*(b + c*x))^(3/2)*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.21 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {1165, 27, 1235, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{5/2}\sqrt{d+ex}}\,dx$$

$$\Big\downarrow \; 1165$$

$$-\frac{2\int \frac{8c^2d^2-5bced-2b^2e^2+3ce(2cd-be)x}{2\sqrt{d+ex}(cx^2+bx)^{3/2}}dx}{3b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{3b^2d\,(bx+cx^2)^{3/2}\,(cd-be)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\int \frac{8c^2d^2-5bced-2b^2e^2+3ce(2cd-be)x}{\sqrt{d+ex}(cx^2+bx)^{3/2}}dx}{3b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 1235$$

$$-\frac{-\frac{2\int -\frac{ce\left(bd\left(8c^2d^2-11bced+b^2e^2\right)+2(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)x\right)}{2\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}\left(2cx(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)+b(cd-be)\left(-2b^2e^2-5\right.\right.}{b^2d\sqrt{bx+cx^2}(cd-be)}}{3b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 27$$

$$-\frac{\frac{ce\int \frac{bd\left(8c^2d^2-11bced+b^2e^2\right)+2(2cd-be)\left(4c^2d^2-4bced-b^2e^2\right)x}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}\left(2cx(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)+b(cd-be)\left(-2b^2e^2-5bcde+8\right.\right.}{b^2d\sqrt{bx+cx^2}(cd-be)}}{3b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 1269$$

$$-\frac{\frac{ce\left(\frac{2(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{d(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int \frac{1}{\sqrt{d+ex}\sqrt{cx^2+bx}}dx}{e}\right)}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}\left(2cx(2cd-be)\left(-b^2e^2-\right.\right.}{b^2\cdots}}{3b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 1169$$

$$-\frac{\frac{ce\left(\frac{2\sqrt{x}\sqrt{b+cx}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)\int \frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2d(cd-be)}-\frac{2\sqrt{d+ex}\left(2cx\cdots\right.}{\cdots}}{3b^2d(cd-be)}-\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 122$$

$$-\frac{\frac{ce\left(\frac{2\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+1}}dx}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\right)}{b^2d(cd-be)}-\frac{2\sqrt{d\cdots}}{\cdots}}{3b^2d(cd-be)}$$

$$\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 120$$

$$-\frac{\frac{ce\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{b+cx}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int \frac{1}{\sqrt{x}\sqrt{b+cx}\sqrt{d+ex}}dx}{e\sqrt{bx+cx^2}}\cdots\right)}{b^2d(cd-be)}\cdots}{3b^2d(cd-be)}$$

$$\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 127$$

$$-\frac{\frac{ce\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\int \frac{1}{\sqrt{x}\sqrt{\frac{cx}{b}+1}\cdots}\cdots}{e\sqrt{bx+cx^2}\sqrt{d+ex}}\cdots\right)}{b^2d(cd-be)}\cdots}{3b^2d(cd-be)}$$

$$\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(cd-be)}$$

$$\Big\downarrow \ 126$$

$$-\frac{\frac{ce\left(\frac{4\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}(2cd-be)\left(-b^2e^2-4bcde+4c^2d^2\right)E\left(\arcsin\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{-b}}\right)|\frac{be}{cd}\right)}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}-\frac{2\sqrt{-b}d\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{\frac{ex}{d}+1}(cd-be)\left(-b^2e^2-16bcde+16c^2d^2\right)\text{EllipticF}\cdots}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{d+ex}}\cdots\right)}{b^2d(cd-be)}\cdots}{3b^2d(cd-be)}$$

$$\frac{2\sqrt{d+ex}(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}(cd-be)}$$

input
```
Int[1/(Sqrt[d + e*x]*(b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*Sqrt[d + e*x]*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(3*b^2*d*(c*d - b*e
)*(b*x + c*x^2)^(3/2)) - ((-2*Sqrt[d + e*x]*(b*(c*d - b*e)*(8*c^2*d^2 - 5*
b*c*d*e - 2*b^2*e^2) + 2*c*(2*c*d - b*e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)
*x))/(b^2*d*(c*d - b*e)*Sqrt[b*x + c*x^2]) + (c*e*((4*Sqrt[-b]*(2*c*d - b*
e)*(4*c^2*d^2 - 4*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[d + e*
x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*
Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (2*Sqrt[-b]*d*(c*d - b*e)*(16*c^2*d
^2 - 16*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[1 + (e*x)/d]*Ell
ipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt[c]*e*Sqrt[d
 + e*x]*Sqrt[b*x + c*x^2])))/(b^2*d*(c*d - b*e)))/(3*b^2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 4.63 (sec) , antiderivative size = 653, normalized size of antiderivative = 1.54

method	result
elliptic	$\sqrt{(cx+b)x(ex+d)}\left(-\frac{2c\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3b^3(be-cd)\left(\frac{b}{c}+x\right)^2}-\frac{4\left(ce\,x^2+cdx\right)c^2(5be-4cd)}{3b^4(be-cd)^2\sqrt{\left(\frac{b}{c}+x\right)\left(ce\,x^2+cdx\right)}}-\frac{2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3b^3d\,x^2}+\frac{4\left(ce\,x^2+bex+cdx+b\right.}{3d^2b^4\sqrt{x\left(ce\,x^2+bex\right.}}\right.$
default	Expression too large to display

input

```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(-2/3/b^3/(b*e-c
*d)*c*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(b/c+x)^2-4/3*(c*e*x^2+c*d*x)/
b^4/(b*e-c*d)^2*c^2*(5*b*e-4*c*d)/((b/c+x)*(c*e*x^2+c*d*x))^(1/2)-2/3/b^3/
d*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/x^2+4/3*(c*e*x^2+b*e*x+c*d*x+b*d)/
d^2/b^4*(b*e+4*c*d)/(x*(c*e*x^2+b*e*x+c*d*x+b*d))^(1/2)+2*(-1/3*e*c^2/(b*e
-c*d)/b^3+2/3*c^2/(b*e-c*d)*(5*b*e-4*c*d)/b^4+2/3*c^3*d/b^4/(b*e-c*d)^2*(5
*b*e-4*c*d)-1/3/d/b^3*c*e)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1
/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e
)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))+2*(2/3*c^3*e*(5*b*e-4*c*d)/(b*e-c*d)
^2/b^4-2/3*c*e*(b*e+4*c*d)/d^2/b^4)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e
+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-d/e+b
/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*EllipticF((
(x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(380) = 760$.

Time = 0.11 (sec) , antiderivative size = 953, normalized size of antiderivative = 2.24

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/9*(((16*c^6*d^4 - 32*b*c^5*d^3*e + 13*b^2*c^4*d^2*e^2 + 3*b^3*c^3*d*e^3
+ 2*b^4*c^2*e^4)*x^4 + 2*(16*b*c^5*d^4 - 32*b^2*c^4*d^3*e + 13*b^3*c^3*d^2
*e^2 + 3*b^4*c^2*d*e^3 + 2*b^5*c*e^4)*x^3 + (16*b^2*c^4*d^4 - 32*b^3*c^3*d
^3*e + 13*b^4*c^2*d^2*e^2 + 3*b^5*c*d*e^3 + 2*b^6*e^4)*x^2)*sqrt(c*e)*weie
rstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*
d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x +
 c*d + b*e)/(c*e)) + 6*((8*c^6*d^3*e - 12*b*c^5*d^2*e^2 + 2*b^2*c^4*d*e^3
+ b^3*c^3*e^4)*x^4 + 2*(8*b*c^5*d^3*e - 12*b^2*c^4*d^2*e^2 + 2*b^3*c^3*d*e
^3 + b^4*c^2*e^4)*x^3 + (8*b^2*c^4*d^3*e - 12*b^3*c^3*d^2*e^2 + 2*b^4*c^2*
d*e^3 + b^5*c*e^4)*x^2)*sqrt(c*e)*weierstrassZeta(4/3*(c^2*d^2 - b*c*d*e +
 b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*
b^3*e^3)/(c^3*e^3), weierstrassPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/
(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(
c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/(c*e))) - 3*(b^3*c^3*d^3*e - 2*b^4*c^2
*d^2*e^2 + b^5*c*d*e^3 - 2*(8*c^6*d^3*e - 12*b*c^5*d^2*e^2 + 2*b^2*c^4*d*e
^3 + b^3*c^3*e^4)*x^3 - (24*b*c^5*d^3*e - 37*b^2*c^4*d^2*e^2 + 7*b^3*c^3*d
*e^3 + 4*b^4*c^2*e^4)*x^2 - 2*(3*b^2*c^4*d^3*e - 5*b^3*c^3*d^2*e^2 + b^4*c
^2*d*e^3 + b^5*c*e^4)*x)*sqrt(c*x^2 + b*x)*sqrt(e*x + d))/((b^4*c^5*d^4*e
- 2*b^5*c^4*d^3*e^2 + b^6*c^3*d^2*e^3)*x^4 + 2*(b^5*c^4*d^4*e - 2*b^6*c^3*
d^3*e^2 + b^7*c^2*d^2*e^3)*x^3 + (b^6*c^3*d^4*e - 2*b^7*c^2*d^3*e^2 + b...
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^{5/2}}\,dx = \int \frac{1}{(x\,(b+cx))^{\frac{5}{2}}\,\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(1/((x*(b + c*x))**(5/2)*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^{5/2}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{5}{2}}\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x)^(5/2)*sqrt(e*x + d)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^{5/2}}\,dx = \int \frac{1}{(cx^2+bx)^{\frac{5}{2}}\sqrt{ex+d}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^2 + b*x)^(5/2)*sqrt(e*x + d)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^{5/2}}\,dx = \int \frac{1}{(c\,x^2+b\,x)^{5/2}\,\sqrt{d+e\,x}}\,dx$$

input
```
int(1/((b*x + c*x^2)^(5/2)*(d + e*x)^(1/2)),x)
```

output
```
int(1/((b*x + c*x^2)^(5/2)*(d + e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex}\,(bx+cx^2)^{5/2}}\,dx = \text{too large to display}$$

input
```
int(1/(e*x+d)^(1/2)/(c*x^2+b*x)^(5/2),x)
```

output

```
( - 2*sqrt(d + e*x)*sqrt(b + c*x)*b**2*d*e + 4*sqrt(d + e*x)*sqrt(b + c*x)
*b**2*e**2*x + 2*sqrt(d + e*x)*sqrt(b + c*x)*b*c*d**2 + 8*sqrt(d + e*x)*sq
rt(b + c*x)*b*c*d*e*x - 12*sqrt(d + e*x)*sqrt(b + c*x)*c**2*d**2*x + 6*sqr
t(x)*int((sqrt(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**3*d + sqrt(x)*b**3*e*
x + 3*sqrt(x)*b**2*c*d*x + 3*sqrt(x)*b**2*c*e*x**2 + 3*sqrt(x)*b*c**2*d*x*
*2 + 3*sqrt(x)*b*c**2*e*x**3 + sqrt(x)*c**3*d*x**3 + sqrt(x)*c**3*e*x**4),
x)*b**4*c*e**3*x + 12*sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)
*b**3*d + sqrt(x)*b**3*e*x + 3*sqrt(x)*b**2*c*d*x + 3*sqrt(x)*b**2*c*e*x**
2 + 3*sqrt(x)*b*c**2*d*x**2 + 3*sqrt(x)*b*c**2*e*x**3 + sqrt(x)*c**3*d*x**
3 + sqrt(x)*c**3*e*x**4),x)*b**3*c**2*d*e**2*x + 12*sqrt(x)*int((sqrt(d +
e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**3*d + sqrt(x)*b**3*e*x + 3*sqrt(x)*b**2*
c*d*x + 3*sqrt(x)*b**2*c*e*x**2 + 3*sqrt(x)*b*c**2*d*x**2 + 3*sqrt(x)*b*c*
*2*e*x**3 + sqrt(x)*c**3*d*x**3 + sqrt(x)*c**3*e*x**4),x)*b**3*c**2*e**3*x
**2 - 18*sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**3*d + sqr
t(x)*b**3*e*x + 3*sqrt(x)*b**2*c*d*x + 3*sqrt(x)*b**2*c*e*x**2 + 3*sqrt(x)
*b*c**2*d*x**2 + 3*sqrt(x)*b*c**2*e*x**3 + sqrt(x)*c**3*d*x**3 + sqrt(x)*c
**3*e*x**4),x)*b**2*c**3*d**2*e*x + 24*sqrt(x)*int((sqrt(d + e*x)*sqrt(b +
 c*x)*x)/(sqrt(x)*b**3*d + sqrt(x)*b**3*e*x + 3*sqrt(x)*b**2*c*d*x + 3*sqr
t(x)*b**2*c*e*x**2 + 3*sqrt(x)*b*c**2*d*x**2 + 3*sqrt(x)*b*c**2*e*x**3 + s
qrt(x)*c**3*d*x**3 + sqrt(x)*c**3*e*x**4),x)*b**2*c**3*d*e**2*x**2 + 6*...
```

3.226 $\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)^{5/2}} \, dx$

Optimal result . . . 1895
Mathematica [C] (verified) . . . 1896
Rubi [A] (verified) . . . 1897
Maple [A] (verified) . . . 1902
Fricas [B] (verification not implemented) . . . 1903
Sympy [F] . . . 1904
Maxima [F] . . . 1905
Giac [F] . . . 1905
Mupad [F(-1)] . . . 1905
Reduce [F] . . . 1906

Optimal result

Integrand size = 23, antiderivative size = 527

$$
\begin{aligned}
\int \frac{1}{(d+ex)^{3/2}(bx+cx^2)^{5/2}} \, dx &= \frac{2c}{3b(cd-be)\sqrt{d+ex}\,(bx+cx^2)^{3/2}} \\
&+ \frac{4c(3cd-5be)}{3b^2(cd-be)^2x\sqrt{d+ex}\sqrt{bx+cx^2}} - \frac{2\left(\frac{8c^2d}{b}-13ce+\frac{be^2}{d}\right)\sqrt{bx+cx^2}}{3b^2(cd-be)^2x^2\sqrt{d+ex}} \\
&+ \frac{8(4c^3d^3-6bc^2d^2e+b^3e^3)\sqrt{bx+cx^2}}{3b^4d^2(cd-be)^2x\sqrt{d+ex}} \\
&+ \frac{2\sqrt{e}(16c^4d^4-32bc^3d^3e+9b^2c^2d^2e^2+7b^3cde^3-8b^4e^4)\sqrt{bx+cx^2}\,E\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right)|1-\frac{cd}{be}\right)}{3b^4d^{5/2}(cd-be)^3\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}} \\
&- \frac{2c\sqrt{e}(8c^3d^3-15bc^2d^2e+3b^2cde^2-4b^3e^3)\sqrt{bx+cx^2}\,\text{EllipticF}\left(\arctan\left(\frac{\sqrt{e}\sqrt{x}}{\sqrt{d}}\right),1-\frac{cd}{be}\right)}{3b^4d^{3/2}(cd-be)^3\sqrt{x}\sqrt{\frac{d(b+cx)}{b(d+ex)}}\sqrt{d+ex}}
\end{aligned}
$$

output
```
2/3*c/b/(-b*e+c*d)/(e*x+d)^(1/2)/(c*x^2+b*x)^(3/2)+4/3*c*(-5*b*e+3*c*d)/b^
2/(-b*e+c*d)^2/x/(e*x+d)^(1/2)/(c*x^2+b*x)^(1/2)-2/3*(8*c^2*d/b-13*c*e+b*e
^2/d)*(c*x^2+b*x)^(1/2)/b^2/(-b*e+c*d)^2/x^2/(e*x+d)^(1/2)+8/3*(b^3*e^3-6*
b*c^2*d^2*e+4*c^3*d^3)*(c*x^2+b*x)^(1/2)/b^4/d^2/(-b*e+c*d)^2/x/(e*x+d)^(1
/2)+2/3*e^(1/2)*(-8*b^4*e^4+7*b^3*c*d*e^3+9*b^2*c^2*d^2*e^2-32*b*c^3*d^3*e
+16*c^4*d^4)*(c*x^2+b*x)^(1/2)*EllipticE(e^(1/2)*x^(1/2)/d^(1/2)/(1+e*x/d)
^(1/2),(1-c*d/b/e)^(1/2))/b^4/d^(5/2)/(-b*e+c*d)^3/x^(1/2)/(d*(c*x+b)/b/(e
*x+d))^(1/2)/(e*x+d)^(1/2)-2/3*c*e^(1/2)*(-4*b^3*e^3+3*b^2*c*d*e^2-15*b*c^
2*d^2*e+8*c^3*d^3)*(c*x^2+b*x)^(1/2)*InverseJacobiAM(arctan(e^(1/2)*x^(1/2
)/d^(1/2)),(1-c*d/b/e)^(1/2))/b^4/d^(3/2)/(-b*e+c*d)^3/x^(1/2)/(d*(c*x+b)/
b/(e*x+d))^(1/2)/(e*x+d)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 13.25 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.96

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx =$$

$$-\frac{2\Big(b(3b^4e^5x^2(b+cx)^2+bc^4d^3(-cd+be)x^2(d+ex)-c^4d^3(8cd-13be)x^2(b+cx)(d+ex)+bd(cd-be)}{}$$

input
```
Integrate[1/((d + e*x)^(3/2)*(b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*(b*(3*b^4*e^5*x^2*(b + c*x)^2 + b*c^4*d^3*(-(c*d) + b*e)*x^2*(d + e*x)
 - c^4*d^3*(8*c*d - 13*b*e)*x^2*(b + c*x)*(d + e*x) + b*d*(c*d - b*e)^3*(b
 + c*x)^2*(d + e*x) - (c*d - b*e)^3*(8*c*d + 5*b*e)*x*(b + c*x)^2*(d + e*x
)) + Sqrt[b/c]*c*x*(b + c*x)*(Sqrt[b/c]*(16*c^4*d^4 - 32*b*c^3*d^3*e + 9*b
^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*(b + c*x)*(d + e*x) + I*b*e*(1
6*c^4*d^4 - 32*b*c^3*d^3*e + 9*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4
)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticE[I*ArcSinh[Sqrt[b/c
]/Sqrt[x]], (c*d)/(b*e)] - I*b*e*(8*c^4*d^4 - 17*b*c^3*d^3*e + 6*b^2*c^2*d
^2*e^2 + 11*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[1 + b/(c*x)]*Sqrt[1 + d/(e*x)]*x
^(3/2)*EllipticF[I*ArcSinh[Sqrt[b/c]/Sqrt[x]], (c*d)/(b*e)])))/(3*b^5*d^3*
(c*d - b*e)^3*(x*(b + c*x))^(3/2)*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 602, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {1165, 27, 1235, 27, 1237, 27, 1269, 1169, 122, 120, 127, 126}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(bx+cx^2)^{5/2}(d+ex)^{3/2}}dx$$

$$\Big\downarrow \text{1165}$$

$$-\frac{2\int \frac{8c^2d^2-3bced-4b^2e^2+5ce(2cd-be)x}{2(d+ex)^{3/2}(cx^2+bx)^{3/2}}dx}{3b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d(bx+cx^2)^{3/2}\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{8c^2d^2-3bced-4b^2e^2+5ce(2cd-be)x}{(d+ex)^{3/2}(cx^2+bx)^{3/2}}dx}{3b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d(bx+cx^2)^{3/2}\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{1235}$$

$$-\frac{-\frac{2\int \frac{e\left(b\left(8c^3d^3-9bc^2ed^2-3b^2ce^2d+8b^3e^3\right)+4c\left(4c^3d^3-6bc^2ed^2+b^3e^3\right)x\right)}{2(d+ex)^{3/2}\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2(4cx(b^3e^3-6bc^2d^2e+4c^3d^3)+b(cd-be)(-4b^2e^2-3bcde+8c^2d^2)}{b^2d\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}}{3b^2d(cd-be)}$$
$$\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d(bx+cx^2)^{3/2}\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{e\int \frac{b\left(8c^3d^3-9bc^2ed^2-3b^2ce^2d+8b^3e^3\right)+4c\left(4c^3d^3-6bc^2ed^2+b^3e^3\right)x}{(d+ex)^{3/2}\sqrt{cx^2+bx}}dx}{b^2d(cd-be)}-\frac{2(4cx(b^3e^3-6bc^2d^2e+4c^3d^3)+b(cd-be)(-4b^2e^2-3bcde+8c^2d^2))}{b^2d\sqrt{bx+cx^2}\sqrt{d+ex}(cd-be)}}{3b^2d(cd-be)}-$$
$$\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d(bx+cx^2)^{3/2}\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{1237}$$

$$-\frac{-\frac{e\left(\frac{2\sqrt{bx+cx^2}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)}{d\sqrt{d+ex}(cd-be)}-\frac{2\int\frac{c\left(bd\left(8c^3d^3-15bc^2ed^2+3b^2ce^2d-4b^3e^3\right)+\left(16c^4d^4-32bc^3ed^3+9b^2c^2e^2d^2+7b^3ce\cdots\right.\right.}{2\sqrt{d+ex}\sqrt{cx^2+bx}}\cdots}{d(cd-be)}\right.\cdots}{b^2d(cd-be)}\cdots}{3b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}(cd-be)}$$

$\big\downarrow$ 27

$$-\frac{-\frac{e\left(\frac{2\sqrt{bx+cx^2}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\int\frac{bd\left(8c^3d^3-15bc^2ed^2+3b^2ce^2d-4b^3e^3\right)+\left(16c^4d^4-32bc^3ed^3+9b^2c^2e^2d^2+7b^3ce^3d\cdots\right.}{\sqrt{d+ex}\sqrt{cx^2+bx}}\cdots}{d(cd-be)}\right.\cdots}{b^2d(cd-be)}\cdots}{3b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}(cd-be)}$$

$\big\downarrow$ 1269

$$-\frac{-\frac{e\left(\frac{2\sqrt{bx+cx^2}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)\int\frac{\sqrt{d+ex}}{\sqrt{cx^2+bx}}dx}{e}-\frac{4d(cd-be)(2c\cdots}{\cdots}\right.}{d(cd-be)}\right.\cdots}{b^2d(cd-be)}\cdots}{3b^2d(cd-be)}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}(cd-be)}$$

$\big\downarrow$ 1169

$$-\frac{-\frac{e\left(\frac{2\sqrt{bx+cx^2}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{\sqrt{x}\sqrt{b+cx}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)\int\frac{\sqrt{d+ex}}{\sqrt{x}\sqrt{b+cx}}dx}{e\sqrt{bx+cx^2}}-\frac{4d\cdots}{\cdots}\right.}{d(cd-be)}\right.\cdots}{b^2d(cd-be)}\cdots}{3b^2d(c\cdots}-\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}(cd-be)}$$

$\big\downarrow$ 122

$$-\frac{-\frac{e\left(\frac{2\sqrt{bx+cx^2}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)\int\frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{x}\sqrt{\frac{cx}{b}+\cdots}}}{e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}\cdots\right.}{d(cd\cdots}\right.}{b^2d(cd-be)}\cdots}{3b^2c\cdots}-$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{ 120}$$

$$-\frac{-\frac{e\left(\frac{2\sqrt{bx+cx^2}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)E\left(\text{arc}\cdots\right.}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}\cdots\right.}{\cdots}\right.}{b^2d(cd-be)}\cdots}{\cdots}-$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{ 127}$$

$$-\frac{-\frac{e\left(\frac{2\sqrt{bx+cx^2}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)E\left(\text{arc}\cdots\right.}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}\cdots\right.}{\cdots}\right.}{b^2d(cd-be)}\cdots}{\cdots}-$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}(cd-be)}$$

$$\Big\downarrow \text{ 126}$$

$$-\frac{-\frac{e\left(\frac{2\sqrt{bx+cx^2}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)}{d\sqrt{d+ex}(cd-be)}-\frac{c\left(\frac{2\sqrt{-b}\sqrt{x}\sqrt{\frac{cx}{b}+1}\sqrt{d+ex}\left(-8b^4e^4+7b^3cde^3+9b^2c^2d^2e^2-32bc^3d^3e+16c^4d^4\right)E\left(\text{arc}\right.}{\sqrt{c}e\sqrt{bx+cx^2}\sqrt{\frac{ex}{d}+1}}\right.}{}\right)}{b^2d(cd-be)}}{}$$

$$\frac{2(cx(2cd-be)+b(cd-be))}{3b^2d\left(bx+cx^2\right)^{3/2}\sqrt{d+ex}(cd-be)}$$

input
```
Int[1/((d + e*x)^(3/2)*(b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(3*b^2*d*(c*d - b*e)*Sqrt[d + e*x
]*(b*x + c*x^2)^(3/2)) - ((-2*(b*(c*d - b*e)*(8*c^2*d^2 - 3*b*c*d*e - 4*b^
2*e^2) + 4*c*(4*c^3*d^3 - 6*b*c^2*d^2*e + b^3*e^3)*x))/(b^2*d*(c*d - b*e)*
Sqrt[d + e*x]*Sqrt[b*x + c*x^2]) - (e*((2*(16*c^4*d^4 - 32*b*c^3*d^3*e + 9
*b^2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[b*x + c*x^2])/(d*(c*d -
 b*e)*Sqrt[d + e*x]) - (c*((2*Sqrt[-b]*(16*c^4*d^4 - 32*b*c^3*d^3*e + 9*b^
2*c^2*d^2*e^2 + 7*b^3*c*d*e^3 - 8*b^4*e^4)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqrt[
d + e*x]*EllipticE[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])/(Sqrt
[c]*e*Sqrt[1 + (e*x)/d]*Sqrt[b*x + c*x^2]) - (8*Sqrt[-b]*d*(c*d - b*e)*(2*
c*d - b*e)*(2*c^2*d^2 - 2*b*c*d*e - b^2*e^2)*Sqrt[x]*Sqrt[1 + (c*x)/b]*Sqr
t[1 + (e*x)/d]*EllipticF[ArcSin[(Sqrt[c]*Sqrt[x])/Sqrt[-b]], (b*e)/(c*d)])
/(Sqrt[c]*e*Sqrt[d + e*x]*Sqrt[b*x + c*x^2])))/(d*(c*d - b*e))))/(b^2*d*(c
*d - b*e)))/(3*b^2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 126
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (PosQ[-b/d] || NegQ[-b/f])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1169
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Simp[Sqrt[x]*(Sqrt[b + c*x]/Sqrt[b*x + c*x^2])   Int[(d + e*x)^m/(Sqrt[x]*
Sqrt[b + c*x]), x], x] /; FreeQ[{b, c, d, e}, x] && NeQ[c*d - b*e, 0] && Eq
Q[m^2, 1/4]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 1237
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Simp[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[
(c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && LtQ[m, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1269
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + b*x +
 c*x^2)^p, x], x] + Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + b*x + c*x^2)^
p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

Maple [A] (verified)

Time = 5.43 (sec) , antiderivative size = 762, normalized size of antiderivative = 1.45

method	result
elliptic	$\sqrt{(cx+b)x(ex+d)}\left(\frac{2c^2\sqrt{ce\,x^3+be\,x^2+cd\,x^2+bdx}}{3b^3(be-cd)^2\left(\frac{b}{c}+x\right)^2}+\frac{2\left(ce\,x^2+cdx\right)c^3(13be-8cd)}{3b^4(be-cd)^3\sqrt{\left(\frac{b}{c}+x\right)\left(ce\,x^2+cdx\right)}}+\frac{2\left(ce\,x^2+bex\right)e^4}{d^3(be-cd)^3\sqrt{\left(x+\frac{d}{e}\right)\left(ce\,x^2+bex\right)}}-\frac{2\sqrt{ce\,x^3+be\,x^2}}{3b^3d^2}\right.$
default	Expression too large to display

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((c*x+b)*x*(e*x+d))^(1/2)/(x*(c*x+b))^(1/2)/(e*x+d)^(1/2)*(2/3/b^3/(b*e-c*
d)^2*c^2*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/(b/c+x)^2+2/3*(c*e*x^2+c*d*
x)/b^4/(b*e-c*d)^3*c^3*(13*b*e-8*c*d)/((b/c+x)*(c*e*x^2+c*d*x))^(1/2)+2*(c
*e*x^2+b*e*x)*e^4/d^3/(b*e-c*d)^3/((x+d/e)*(c*e*x^2+b*e*x))^(1/2)-2/3/b^3/
d^2*(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)/x^2+2/3*(c*e*x^2+b*e*x+c*d*x+b*d
)/d^3/b^4*(5*b*e+8*c*d)/(x*(c*e*x^2+b*e*x+c*d*x+b*d))^(1/2)+2*(1/3*c^3*e/(
b*e-c*d)^2/b^3-1/3*c^3/(b*e-c*d)^2*(13*b*e-8*c*d)/b^4-1/3*c^4*d/b^4/(b*e-c
*d)^3*(13*b*e-8*c*d)+e^4/(b*e-c*d)^2/d^3-b*e^5/d^3/(b*e-c*d)^3-1/3/b^3/d^2
*c*e)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c
*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*EllipticF(((x+d/e)/d*e)^(1/2),(-d/e/(-
d/e+b/c))^(1/2))+2*(-1/3*c^4*e*(13*b*e-8*c*d)/(b*e-c*d)^3/b^4-c*e^5/d^3/(b
*e-c*d)^3-1/3*c*e*(5*b*e+8*c*d)/b^4/d^3)*d/e*((x+d/e)/d*e)^(1/2)*((b/c+x)/
(-d/e+b/c))^(1/2)*(-e*x/d)^(1/2)/(c*e*x^3+b*e*x^2+c*d*x^2+b*d*x)^(1/2)*((-
d/e+b/c)*EllipticE(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))-b/c*Ellipt
icF(((x+d/e)/d*e)^(1/2),(-d/e/(-d/e+b/c))^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1510 vs. $2(476) = 952$.

Time = 0.17 (sec) , antiderivative size = 1510, normalized size of antiderivative = 2.87

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/9*(((16*c^7*d^5*e - 40*b*c^6*d^4*e^2 + 22*b^2*c^5*d^3*e^3 + 7*b^3*c^4*d^
2*e^4 + 11*b^4*c^3*d*e^5 - 8*b^5*c^2*e^6)*x^5 + (16*c^7*d^6 - 8*b*c^6*d^5*
e - 58*b^2*c^5*d^4*e^2 + 51*b^3*c^4*d^3*e^3 + 25*b^4*c^3*d^2*e^4 + 14*b^5*
c^2*d*e^5 - 16*b^6*c*e^6)*x^4 + (32*b*c^6*d^6 - 64*b^2*c^5*d^5*e + 4*b^3*c
^4*d^4*e^2 + 36*b^4*c^3*d^3*e^3 + 29*b^5*c^2*d^2*e^4 - 5*b^6*c*d*e^5 - 8*b
^7*e^6)*x^3 + (16*b^2*c^5*d^6 - 40*b^3*c^4*d^5*e + 22*b^4*c^3*d^4*e^2 + 7*
b^5*c^2*d^3*e^3 + 11*b^6*c*d^2*e^4 - 8*b^7*d*e^5)*x^2)*sqrt(c*e)*weierstra
ssPInverse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 -
 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d
+ b*e)/(c*e)) + 3*((16*c^7*d^4*e^2 - 32*b*c^6*d^3*e^3 + 9*b^2*c^5*d^2*e^4
+ 7*b^3*c^4*d*e^5 - 8*b^4*c^3*e^6)*x^5 + (16*c^7*d^5*e - 55*b^2*c^5*d^3*e^
3 + 25*b^3*c^4*d^2*e^4 + 6*b^4*c^3*d*e^5 - 16*b^5*c^2*e^6)*x^4 + (32*b*c^6
*d^5*e - 48*b^2*c^5*d^4*e^2 - 14*b^3*c^4*d^3*e^3 + 23*b^4*c^3*d^2*e^4 - 9*
b^5*c^2*d*e^5 - 8*b^6*c*e^6)*x^3 + (16*b^2*c^5*d^5*e - 32*b^3*c^4*d^4*e^2
+ 9*b^4*c^3*d^3*e^3 + 7*b^5*c^2*d^2*e^4 - 8*b^6*c*d*e^5)*x^2)*sqrt(c*e)*we
ierstrassZeta(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^
3 - 3*b*c^2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), weierstrassPInve
rse(4/3*(c^2*d^2 - b*c*d*e + b^2*e^2)/(c^2*e^2), -4/27*(2*c^3*d^3 - 3*b*c^
2*d^2*e - 3*b^2*c*d*e^2 + 2*b^3*e^3)/(c^3*e^3), 1/3*(3*c*e*x + c*d + b*e)/
(c*e))) - 3*(b^3*c^4*d^5*e - 3*b^4*c^3*d^4*e^2 + 3*b^5*c^2*d^3*e^3 - b^...
```

Sympy [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx=\int \frac{1}{\left(x\left(b+cx\right)\right)^{\frac{5}{2}}\left(d+ex\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)**(3/2)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(1/((x*(b + c*x))**(5/2)*(d + e*x)**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(cx^2+bx\right)^{\frac{5}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/((c*x^2 + b*x)^(5/2)*(e*x + d)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(cx^2+bx\right)^{\frac{5}{2}}(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(1/((c*x^2 + b*x)^(5/2)*(e*x + d)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx = \int \frac{1}{\left(c\,x^2+b\,x\right)^{5/2}\,\left(d+e\,x\right)^{3/2}}\,dx$$

input
```
int(1/((b*x + c*x^2)^(5/2)*(d + e*x)^(3/2)),x)
```

output
```
int(1/((b*x + c*x^2)^(5/2)*(d + e*x)^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{(d+ex)^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx = \text{too large to display}$$

input
```
int(1/(e*x+d)^(3/2)/(c*x^2+b*x)^(5/2),x)
```

output
```
( - 2*sqrt(d + e*x)*sqrt(b + c*x)*b*d + 8*sqrt(d + e*x)*sqrt(b + c*x)*b*e*
x + 12*sqrt(d + e*x)*sqrt(b + c*x)*c*d*x + 20*sqrt(x)*int((sqrt(d + e*x)*s
qrt(b + c*x)*x)/(sqrt(x)*b**3*d**2 + 2*sqrt(x)*b**3*d*e*x + sqrt(x)*b**3*e
**2*x**2 + 3*sqrt(x)*b**2*c*d**2*x + 6*sqrt(x)*b**2*c*d*e*x**2 + 3*sqrt(x)
*b**2*c*e**2*x**3 + 3*sqrt(x)*b*c**2*d**2*x**2 + 6*sqrt(x)*b*c**2*d*e*x**3
 + 3*sqrt(x)*b*c**2*e**2*x**4 + sqrt(x)*c**3*d**2*x**3 + 2*sqrt(x)*c**3*d*
e*x**4 + sqrt(x)*c**3*e**2*x**5),x)*b**3*c*d*e**2*x + 20*sqrt(x)*int((sqrt
(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**3*d**2 + 2*sqrt(x)*b**3*d*e*x + sqr
t(x)*b**3*e**2*x**2 + 3*sqrt(x)*b**2*c*d**2*x + 6*sqrt(x)*b**2*c*d*e*x**2
+ 3*sqrt(x)*b**2*c*e**2*x**3 + 3*sqrt(x)*b*c**2*d**2*x**2 + 6*sqrt(x)*b*c*
*2*d*e*x**3 + 3*sqrt(x)*b*c**2*e**2*x**4 + sqrt(x)*c**3*d**2*x**3 + 2*sqrt
(x)*c**3*d*e*x**4 + sqrt(x)*c**3*e**2*x**5),x)*b**3*c*e**3*x**2 + 30*sqrt(
x)*int((sqrt(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**3*d**2 + 2*sqrt(x)*b**3
*d*e*x + sqrt(x)*b**3*e**2*x**2 + 3*sqrt(x)*b**2*c*d**2*x + 6*sqrt(x)*b**2
*c*d*e*x**2 + 3*sqrt(x)*b**2*c*e**2*x**3 + 3*sqrt(x)*b*c**2*d**2*x**2 + 6*
sqrt(x)*b*c**2*d*e*x**3 + 3*sqrt(x)*b*c**2*e**2*x**4 + sqrt(x)*c**3*d**2*x
**3 + 2*sqrt(x)*c**3*d*e*x**4 + sqrt(x)*c**3*e**2*x**5),x)*b**2*c**2*d**2*
e*x + 70*sqrt(x)*int((sqrt(d + e*x)*sqrt(b + c*x)*x)/(sqrt(x)*b**3*d**2 +
2*sqrt(x)*b**3*d*e*x + sqrt(x)*b**3*e**2*x**2 + 3*sqrt(x)*b**2*c*d**2*x +
6*sqrt(x)*b**2*c*d*e*x**2 + 3*sqrt(x)*b**2*c*e**2*x**3 + 3*sqrt(x)*b*c*...
```

3.227 $\int \frac{\sqrt{d+ex}}{\sqrt{2x-3x^2}}\,dx$

Optimal result . . . 1907
Mathematica [B] (verified) . . . 1907
Rubi [A] (verified) . . . 1908
Maple [B] (verified) . . . 1910
Fricas [B] (verification not implemented) . . . 1910
Sympy [F] . . . 1911
Maxima [F] . . . 1911
Giac [F] . . . 1912
Mupad [F(-1)] . . . 1912
Reduce [F] . . . 1912

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{\sqrt{d+ex}}{\sqrt{2x-3x^2}}\,dx=\frac{2\sqrt{d+ex}E\left(\arcsin\left(\sqrt{\frac{3}{2}}\sqrt{x}\right)|-\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{1+\frac{ex}{d}}}$$

output
```
2/3*(e*x+d)^(1/2)*EllipticE(1/2*6^(1/2)*x^(1/2),1/3*(-6*e/d)^(1/2))*3^(1/2
)/(1+e*x/d)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 117 vs. 2(51) = 102.

Time = 4.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.29

$$\int \frac{\sqrt{d+ex}}{\sqrt{2x-3x^2}}\,dx = \frac{2\sqrt{-\frac{d}{e}}(-2+3x)(d+ex)-2d\sqrt{9-\frac{6}{x}}\sqrt{1+\frac{d}{ex}}x^{3/2}E\left(\arcsin\left(\frac{\sqrt{-\frac{d}{e}}}{\sqrt{x}}\right)|-\frac{2e}{3d}\right)}{3\sqrt{-\frac{d}{e}}\sqrt{-x(-2+3x)}\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[d + e*x]/Sqrt[2*x - 3*x^2],x]
```

output
```
(2*Sqrt[-(d/e)]*(-2 + 3*x)*(d + e*x) - 2*d*Sqrt[9 - 6/x]*Sqrt[1 + d/(e*x)]
*x^(3/2)*EllipticE[ArcSin[Sqrt[-(d/e)]/Sqrt[x]], (-2*e)/(3*d)])/(3*Sqrt[-(
d/e)]*Sqrt[-(x*(-2 + 3*x))]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1168, 27, 27, 122, 27, 120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{\sqrt{2x-3x^2}}dx$$

$$\Big\downarrow \text{1168}$$

$$\int \frac{\sqrt{d+ex}}{\sqrt{2}\sqrt{1-\frac{3x}{2}}\sqrt{x}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{2}\sqrt{d+ex}}{\sqrt{2-3x}\sqrt{x}}dx}{\sqrt{2}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{\sqrt{d+ex}}{\sqrt{2-3x}\sqrt{x}}dx$$

$$\Big\downarrow \text{122}$$

$$\frac{\sqrt{d+ex}\int \frac{\sqrt{2}\sqrt{\frac{ex}{d}+1}}{\sqrt{2-3x}\sqrt{x}}dx}{\sqrt{2}\sqrt{\frac{ex}{d}+1}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{d+ex}\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{2-3x}\sqrt{x}}dx}{\sqrt{\frac{ex}{d}+1}}$$

$$\Big\downarrow \text{120}$$

$$\frac{2\sqrt{d+ex}E\left(\arcsin\left(\sqrt{\frac{3}{2}}\sqrt{x}\right)|-\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{\frac{ex}{d}+1}}$$

input
```
Int[Sqrt[d + e*x]/Sqrt[2*x - 3*x^2],x]
```

output
```
(2*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[3/2]*Sqrt[x]], (-2*e)/(3*d)])/(Sqrt
[3]*Sqrt[1 + (e*x)/d])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1168
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c/b)*x]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0] && RationalQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(41) = 82$.

Time = 0.88 (sec) , antiderivative size = 215, normalized size of antiderivative = 4.22

method	result
default	$-\frac{2\sqrt{ex+d}\,\sqrt{-x(-2+3x)}\,d\sqrt{\frac{ex+d}{d}}\,\sqrt{-\frac{(-2+3x)e}{3d+2e}}\,\sqrt{-\frac{ex}{d}}\left(3d\,\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{3}\,\sqrt{\frac{d}{3d+2e}}\right)+2\,\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{3}\,\sqrt{\frac{d}{3d+2e}}\right.\right.}{3ex(3e\,x^2+3dx-2ex-2d)}$
elliptic	$\frac{\sqrt{-x(-2+3x)(ex+d)}\left(\frac{2d^2\sqrt{\frac{(x+\frac{d}{e})e}{d}}\,\sqrt{\frac{x-\frac{2}{3}}{-\frac{d}{e}-\frac{2}{3}}}\,\sqrt{-\frac{ex}{d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{(x+\frac{d}{e})e}{d}},\sqrt{-\frac{d}{e(-\frac{d}{e}-\frac{2}{3})}}\right)}{e\sqrt{-3x^3e-3d\,x^2+2e\,x^2+2dx}}+\frac{2d\sqrt{\frac{(x+\frac{d}{e})e}{d}}\,\sqrt{\frac{x-\frac{2}{3}}{-\frac{d}{e}-\frac{2}{3}}}\,\sqrt{-\frac{ex}{d}}\left(\left(-\frac{d}{e}-\right.\right.}{}\right.}{\sqrt{-x(-2+3x)}\,\sqrt{ex+d}}$

input
```
int((e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(e*x+d)^(1/2)*(-x*(-2+3*x))^(1/2)*d*((e*x+d)/d)^(1/2)*(-(-2+3*x)*e/(3
*d+2*e))^(1/2)*(-e*x/d)^(1/2)*(3*d*EllipticF(((e*x+d)/d)^(1/2),3^(1/2)*(d/
(3*d+2*e))^(1/2))+2*EllipticF(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d+2*e))^(1/2
))*e-3*EllipticE(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d+2*e))^(1/2))*d-2*Ellipt
icE(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d+2*e))^(1/2))*e)/e/x/(3*e*x^2+3*d*x-2
*e*x-2*d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(44) = 88$.

Time = 0.11 (sec) , antiderivative size = 211, normalized size of antiderivative = 4.14

$$\int \frac{\sqrt{d+ex}}{\sqrt{2x-3x^2}}\,dx =$$
$$-\frac{2\left(2\sqrt{3}(3d+e)\sqrt{-e}\,\mathrm{weierstrassPInverse}\left(\frac{4\left(9d^2+6de+4e^2\right)}{27e^2},-\frac{8\left(27d^3+27d^2e-18de^2-8e^3\right)}{729e^3},\frac{9ex+3d-2e}{9e}\right)-9\sqrt{\cdots}\right.}{}$$

input
```
integrate((e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/27*(2*sqrt(3)*(3*d + e)*sqrt(-e)*weierstrassPInverse(4/27*(9*d^2 + 6*d*
e + 4*e^2)/e^2, -8/729*(27*d^3 + 27*d^2*e - 18*d*e^2 - 8*e^3)/e^3, 1/9*(9*
e*x + 3*d - 2*e)/e) - 9*sqrt(3)*sqrt(-e)*e*weierstrassZeta(4/27*(9*d^2 + 6
*d*e + 4*e^2)/e^2, -8/729*(27*d^3 + 27*d^2*e - 18*d*e^2 - 8*e^3)/e^3, weie
rstrassPInverse(4/27*(9*d^2 + 6*d*e + 4*e^2)/e^2, -8/729*(27*d^3 + 27*d^2*
e - 18*d*e^2 - 8*e^3)/e^3, 1/9*(9*e*x + 3*d - 2*e)/e)))/e
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{2x-3x^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{-x\,(3x-2)}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(-3*x**2+2*x)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/sqrt(-x*(3*x - 2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{2x-3x^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{-3\,x^2+2\,x}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/sqrt(-3*x^2 + 2*x), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{2x-3x^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{-3\,x^2+2\,x}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)/sqrt(-3*x^2 + 2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{2x-3x^2}}\,dx = \int \frac{\sqrt{d+e\,x}}{\sqrt{2\,x-3\,x^2}}\,dx$$

input
```
int((d + e*x)^(1/2)/(2*x - 3*x^2)^(1/2),x)
```

output
```
int((d + e*x)^(1/2)/(2*x - 3*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{2x-3x^2}}\,dx = -\left(\int \frac{\sqrt{ex+d}\,\sqrt{-3x+2}}{3\sqrt{x}\,x-2\sqrt{x}}dx\right)$$

input
```
int((e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x)
```

output
```
- int((sqrt(d + e*x)*sqrt( - 3*x + 2))/(3*sqrt(x)*x - 2*sqrt(x)),x)
```

3.228 $\int \frac{1}{\sqrt{d+ex}\sqrt{2x-3x^2}}\,dx$

Optimal result . . . 1913
Mathematica [A] (verified) . . . 1913
Rubi [A] (verified) . . . 1914
Maple [B] (verified) . . . 1915
Fricas [A] (verification not implemented) . . . 1916
Sympy [F] . . . 1916
Maxima [F] . . . 1917
Giac [F] . . . 1917
Mupad [F(-1)] . . . 1917
Reduce [F] . . . 1918

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \frac{1}{\sqrt{d+ex}\sqrt{2x-3x^2}}\,dx = \frac{2\sqrt{1+\frac{ex}{d}}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}\sqrt{x}\right),-\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{d+ex}}$$

output
```
2/3*(1+e*x/d)^(1/2)*EllipticF(1/2*6^(1/2)*x^(1/2),1/3*(-6*e/d)^(1/2))*3^(1
/2)/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 3.56 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{1}{\sqrt{d+ex}\sqrt{2x-3x^2}}\,dx = -\frac{\sqrt{6-\frac{4}{x}}\sqrt{1+\frac{d}{ex}}\,x^{3/2}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right),-\frac{3d}{2e}\right)}{\sqrt{-x(-2+3x)}\sqrt{d+ex}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*Sqrt[2*x - 3*x^2]),x]
```

output
```
-((Sqrt[6 - 4/x]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[ArcSin[Sqrt[2/3]/Sqrt
[x]], (-3*d)/(2*e)])/(Sqrt[-(x*(-2 + 3*x))]*Sqrt[d + e*x]))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1168, 27, 27, 127, 27, 125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{2x-3x^2}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 1168}$$

$$\int \frac{1}{\sqrt{2}\sqrt{1-\frac{3x}{2}}\sqrt{x}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{2}}{\sqrt{2-3x}\sqrt{x}\sqrt{d+ex}}dx}{\sqrt{2}}$$

$$\Big\downarrow \text{ 27}$$

$$\int \frac{1}{\sqrt{2-3x}\sqrt{x}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{ 127}$$

$$\frac{\sqrt{\frac{ex}{d}+1}\int \frac{\sqrt{2}}{\sqrt{2-3x}\sqrt{x}\sqrt{\frac{ex}{d}+1}}dx}{\sqrt{2}\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\sqrt{\frac{ex}{d}+1}\int \frac{1}{\sqrt{2-3x}\sqrt{x}\sqrt{\frac{ex}{d}+1}}dx}{\sqrt{d+ex}}$$

$$\Big\downarrow \text{ 125}$$

$$\frac{2\sqrt{\frac{ex}{d}+1}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}\sqrt{x}\right),-\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{d+ex}}$$

input
```
Int[1/(Sqrt[d + e*x]*Sqrt[2*x - 3*x^2]),x]
```

output
```
(2*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[Sqrt[3/2]*Sqrt[x]], (-2*e)/(3*d)])/(
Sqrt[3]*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 125
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (GtQ[-b/d, 0] || LtQ[-b/f, 0])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1168
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c/b)*x]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0] && RationalQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(41) = 82$.

Time = 1.24 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.25

method	result	size
default	$-\frac{2\,\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{3}\,\sqrt{\frac{d}{3d+2e}}\right)\sqrt{-\frac{ex}{d}}\,\sqrt{-\frac{(-2+3x)e}{3d+2e}}\,\sqrt{\frac{ex+d}{d}}\,d\sqrt{ex+d}\,\sqrt{-x(-2+3x)}}{ex(3e\,x^{2}+3dx-2ex-2d)}$	115
elliptic	$\frac{2\sqrt{-x(-2+3x)(ex+d)}\,d\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}\,\sqrt{\frac{x-\frac{2}{3}}{-\frac{d}{e}-\frac{2}{3}}}\,\sqrt{-\frac{ex}{d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}},\sqrt{-\frac{d}{e\left(-\frac{d}{e}-\frac{2}{3}\right)}}\right)}{\sqrt{-x(-2+3x)}\,\sqrt{ex+d}\,e\sqrt{-3x^{3}e-3d\,x^{2}+2e\,x^{2}+2dx}}$	136

input
```
int(1/(e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*EllipticF(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d+2*e))^(1/2))*(-e*x/d)^(1/2)
*(-(-2+3*x)*e/(3*d+2*e))^(1/2)*((e*x+d)/d)^(1/2)*d*(e*x+d)^(1/2)*(-x*(-2+3
*x))^(1/2)/e/x/(3*e*x^2+3*d*x-2*e*x-2*d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int \frac{1}{\sqrt{d+ex}\sqrt{2x-3x^2}}\,dx = -\frac{2\sqrt{3}\sqrt{-e}\,\text{weierstrassPInverse}\left(\frac{4\,(9\,d^2+6\,de+4\,e^2)}{27\,e^2}, -\frac{8\,(27\,d^3+27\,d^2e-18\,de^2-8\,e^3)}{729\,e^3}, \frac{9\,ex+3\,d-2\,e}{9\,e}\right)}{3\,e}$$

input
```
integrate(1/(e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(3)*sqrt(-e)*weierstrassPInverse(4/27*(9*d^2 + 6*d*e + 4*e^2)/e^2
, -8/729*(27*d^3 + 27*d^2*e - 18*d*e^2 - 8*e^3)/e^3, 1/9*(9*e*x + 3*d - 2*
e)/e)/e
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{2x-3x^2}}\,dx = \int \frac{1}{\sqrt{-x\,(3x-2)}\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(-3*x**2+2*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(-x*(3*x - 2))*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{2x-3x^2}}\,dx = \int \frac{1}{\sqrt{ex+d}\sqrt{-3\,x^2+2\,x}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(e*x + d)*sqrt(-3*x^2 + 2*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{2x-3x^2}}\,dx = \int \frac{1}{\sqrt{ex+d}\sqrt{-3\,x^2+2\,x}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*x + d)*sqrt(-3*x^2 + 2*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{2x-3x^2}}\,dx = \int \frac{1}{\sqrt{2\,x-3\,x^2}\,\sqrt{d+e\,x}}\,dx$$

input
```
int(1/((2*x - 3*x^2)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
int(1/((2*x - 3*x^2)^(1/2)*(d + e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{2x-3x^2}}\,dx = -\left(\int \frac{\sqrt{x}\,\sqrt{ex+d}\,\sqrt{-3x+2}}{3e\,x^3+3d\,x^2-2e\,x^2-2dx}dx\right)$$

input
```
int(1/(e*x+d)^(1/2)/(-3*x^2+2*x)^(1/2),x)
```

output
```
- int((sqrt(x)*sqrt(d + e*x)*sqrt( - 3*x + 2))/(3*d*x**2 - 2*d*x + 3*e*x*
*3 - 2*e*x**2),x)
```

3.229 $\int \frac{\sqrt{d+ex}}{\sqrt{-2x-3x^2}}\,dx$

Optimal result . . . 1919
Mathematica [A] (verified) . . . 1920
Rubi [A] (verified) . . . 1920
Maple [A] (verified) . . . 1922
Fricas [A] (verification not implemented) . . . 1923
Sympy [F] . . . 1923
Maxima [F] . . . 1924
Giac [F] . . . 1924
Mupad [F(-1)] . . . 1924
Reduce [F] . . . 1925

Optimal result

Integrand size = 23, antiderivative size = 184

$$\int \frac{\sqrt{d+ex}}{\sqrt{-2x-3x^2}}\,dx=\frac{2x\sqrt{d+ex}}{\sqrt{-2x-3x^2}}-\frac{2\sqrt{x}\sqrt{d+ex}E\left(\arctan\left(\sqrt{\frac{3}{2}}\sqrt{x}\right)|1-\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{\frac{d+ex}{d(2+3x)}}\sqrt{-2x-3x^2}}+\frac{2\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{3}{2}}\sqrt{x}\right),1-\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{\frac{d+ex}{d(2+3x)}}\sqrt{-2x-3x^2}}$$

output
```
2*x*(e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2)-2/3*x^(1/2)*(e*x+d)^(1/2)*EllipticE(6
^(1/2)*x^(1/2)/(4+6*x)^(1/2),1/3*(9-6*e/d)^(1/2))*3^(1/2)/((e*x+d)/d/(2+3*
x))^(1/2)/(-3*x^2-2*x)^(1/2)+2/3*x^(1/2)*(e*x+d)^(1/2)*InverseJacobiAM(arc
tan(1/2*6^(1/2)*x^(1/2)),1/3*(9-6*e/d)^(1/2))*3^(1/2)/((e*x+d)/d/(2+3*x))^
(1/2)/(-3*x^2-2*x)^(1/2)
```

Mathematica [A] (verified)

Time = 4.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{d+ex}}{\sqrt{-2x-3x^2}}\,dx = \frac{2\sqrt{-\frac{d}{e}}(2+3x)(d+ex)-2d\sqrt{9+\frac{6}{x}}\sqrt{1+\frac{d}{ex}}x^{3/2}E\left(\arcsin\left(\frac{\sqrt{-\frac{d}{e}}}{\sqrt{x}}\right)|\frac{2e}{3d}\right)}{3\sqrt{-\frac{d}{e}}\sqrt{-x(2+3x)}\sqrt{d+ex}}$$

input
```
Integrate[Sqrt[d + e*x]/Sqrt[-2*x - 3*x^2],x]
```

output
```
(2*Sqrt[-(d/e)]*(2 + 3*x)*(d + e*x) - 2*d*Sqrt[9 + 6/x]*Sqrt[1 + d/(e*x)]*
x^(3/2)*EllipticE[ArcSin[Sqrt[-(d/e)]/Sqrt[x]], (2*e)/(3*d)])/(3*Sqrt[-(d/
e)]*Sqrt[-(x*(2 + 3*x))]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.29, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {1168, 27, 27, 122, 27, 120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{d+ex}}{\sqrt{-3x^2-2x}}\,dx$$

$$\Big\downarrow \text{ 1168}$$

$$\int \frac{\sqrt{d+ex}}{\sqrt{2}\sqrt{-x}\sqrt{\frac{3x}{2}+1}}dx$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{\sqrt{2}\sqrt{d+ex}}{\sqrt{-x}\sqrt{3x+2}}dx}{\sqrt{2}}$$

$$\Big\downarrow \; 27$$

$$\int \frac{\sqrt{d+ex}}{\sqrt{-x}\sqrt{3x+2}}dx$$

$$\Big\downarrow \; 122$$

$$\frac{\sqrt{d+ex}\int \frac{\sqrt{2}\sqrt{\frac{ex}{d}+1}}{\sqrt{-x}\sqrt{3x+2}}dx}{\sqrt{2}\sqrt{\frac{ex}{d}+1}}$$

$$\Big\downarrow \; 27$$

$$\frac{\sqrt{d+ex}\int \frac{\sqrt{\frac{ex}{d}+1}}{\sqrt{-x}\sqrt{3x+2}}dx}{\sqrt{\frac{ex}{d}+1}}$$

$$\Big\downarrow \; 120$$

$$-\frac{2\sqrt{d+ex}E\left(\arcsin\left(\sqrt{\frac{3}{2}}\sqrt{-x}\right)|\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{\frac{ex}{d}+1}}$$

input
```
Int[Sqrt[d + e*x]/Sqrt[-2*x - 3*x^2],x]
```

output
```
(-2*Sqrt[d + e*x]*EllipticE[ArcSin[Sqrt[3/2]*Sqrt[-x]], (2*e)/(3*d)])/(Sqr
t[3]*Sqrt[1 + (e*x)/d])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 122
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[Sqrt[e + f*x]*(Sqrt[1 + d*(x/c)]/(Sqrt[c + d*x]*Sqrt[1 + f*(x/e)])
)   Int[Sqrt[1 + f*(x/e)]/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]), x], x] /; FreeQ[{b
, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1168
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c/b)*x]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0] && RationalQ[b]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.17

method	result
default	$-\frac{2\sqrt{ex+d}\,\sqrt{-x(3x+2)}\,d\sqrt{\frac{ex+d}{d}}\,\sqrt{-\frac{(3x+2)e}{3d-2e}}\,\sqrt{-\frac{ex}{d}}\left(3d\,\text{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{3}\,\sqrt{\frac{d}{3d-2e}}\right)-2\,\text{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{3}\,\sqrt{\frac{d}{3d-2e}}\right)e\cdots\right.}{3ex(3e\,x^2+3dx+2ex+2d)}$
elliptic	$\frac{\sqrt{-x(3x+2)(ex+d)}\left(\frac{2d^2\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}\,\sqrt{\frac{x+\frac{2}{3}}{-\frac{d}{e}+\frac{2}{3}}}\,\sqrt{-\frac{ex}{d}}\,\text{EllipticF}\left(\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}},\sqrt{-\frac{d}{e\left(-\frac{d}{e}+\frac{2}{3}\right)}}\right)}{e\sqrt{-3x^3e-3d\,x^2-2e\,x^2-2dx}}+\frac{2d\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}\,\sqrt{\frac{x+\frac{2}{3}}{-\frac{d}{e}+\frac{2}{3}}}\,\sqrt{-\frac{ex}{d}}\left(\left(-\frac{d}{e}+\frac{2}{3}\cdots\right.\right.}{\cdots}\right.}{\sqrt{-x(3x+2)}\,\sqrt{ex+d}}$

input
```
int((e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(e*x+d)^(1/2)*(-x*(3*x+2))^(1/2)*d*((e*x+d)/d)^(1/2)*(-(3*x+2)*e/(3*d
-2*e))^(1/2)*(-e*x/d)^(1/2)*(3*d*EllipticF(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3
*d-2*e))^(1/2))-2*EllipticF(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d-2*e))^(1/2))
*e-3*EllipticE(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d-2*e))^(1/2))*d+2*Elliptic
E(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d-2*e))^(1/2))*e)/e/x/(3*e*x^2+3*d*x+2*e
*x+2*d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.16

$$\int \frac{\sqrt{d+ex}}{\sqrt{-2x-3x^2}}\,dx = -\frac{2\left(2\sqrt{3}(3d-e)\sqrt{-e}\,\text{weierstrassPInverse}\left(\frac{4\,(9\,d^2-6\,de+4\,e^2)}{27\,e^2}, -\frac{8\,(27\,d^3-27\,d^2e-18\,de^2+8\,e^3)}{729\,e^3}, \frac{9\,ex+3\,d+2\,e}{9\,e}\right)-9\sqrt{}\right.}{}$$

input
```
integrate((e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/27*(2*sqrt(3)*(3*d - e)*sqrt(-e)*weierstrassPInverse(4/27*(9*d^2 - 6*d*
e + 4*e^2)/e^2, -8/729*(27*d^3 - 27*d^2*e - 18*d*e^2 + 8*e^3)/e^3, 1/9*(9*
e*x + 3*d + 2*e)/e) - 9*sqrt(3)*sqrt(-e)*e*weierstrassZeta(4/27*(9*d^2 - 6
*d*e + 4*e^2)/e^2, -8/729*(27*d^3 - 27*d^2*e - 18*d*e^2 + 8*e^3)/e^3, weie
rstrassPInverse(4/27*(9*d^2 - 6*d*e + 4*e^2)/e^2, -8/729*(27*d^3 - 27*d^2*
e - 18*d*e^2 + 8*e^3)/e^3, 1/9*(9*e*x + 3*d + 2*e)/e)))/e
```

Sympy [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{-2x-3x^2}}\,dx = \int \frac{\sqrt{d+ex}}{\sqrt{-x\,(3x+2)}}\,dx$$

input
```
integrate((e*x+d)**(1/2)/(-3*x**2-2*x)**(1/2),x)
```

output
```
Integral(sqrt(d + e*x)/sqrt(-x*(3*x + 2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{-2x-3x^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{-3\,x^2-2\,x}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)/sqrt(-3*x^2 - 2*x), x)
```

Giac [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{-2x-3x^2}}\,dx = \int \frac{\sqrt{ex+d}}{\sqrt{-3\,x^2-2\,x}}\,dx$$

input
```
integrate((e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)/sqrt(-3*x^2 - 2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{d+ex}}{\sqrt{-2x-3x^2}}\,dx = \int \frac{\sqrt{d+e\,x}}{\sqrt{-3\,x^2-2\,x}}\,dx$$

input
```
int((d + e*x)^(1/2)/(- 2*x - 3*x^2)^(1/2),x)
```

output
```
int((d + e*x)^(1/2)/(- 2*x - 3*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{d+ex}}{\sqrt{-2x-3x^2}}\,dx = -\left(\int \frac{\sqrt{ex+d}\,\sqrt{-3x-2}}{3\sqrt{x}\,x+2\sqrt{x}}dx\right)$$

input
```
int((e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x)
```

output
```
- int((sqrt(d + e*x)*sqrt( - 3*x - 2))/(3*sqrt(x)*x + 2*sqrt(x)),x)
```

3.230 $\int \frac{1}{\sqrt{d+ex}\sqrt{-2x-3x^2}}\,dx$

Optimal result . . . 1926
Mathematica [C] (verified) . . . 1926
Rubi [A] (verified) . . . 1927
Maple [A] (verified) . . . 1929
Fricas [A] (verification not implemented) . . . 1929
Sympy [F] . . . 1930
Maxima [F] . . . 1930
Giac [F] . . . 1930
Mupad [F(-1)] . . . 1931
Reduce [F] . . . 1931

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{1}{\sqrt{d+ex}\sqrt{-2x-3x^2}}\,dx=\frac{2\sqrt{x}\sqrt{d+ex}\,\text{EllipticF}\left(\arctan\left(\sqrt{\frac{3}{2}}\sqrt{x}\right),1-\frac{2e}{3d}\right)}{\sqrt{3}d\sqrt{\frac{d+ex}{d(2+3x)}}\sqrt{-2x-3x^2}}$$

output
```
2/3*x^(1/2)*(e*x+d)^(1/2)*InverseJacobiAM(arctan(1/2*6^(1/2)*x^(1/2)),1/3*
(9-6*e/d)^(1/2))*3^(1/2)/d/((e*x+d)/d/(2+3*x))^(1/2)/(-3*x^2-2*x)^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.46 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{d+ex}\sqrt{-2x-3x^2}}\,dx=\frac{i\sqrt{6+\frac{4}{x}}\sqrt{1+\frac{d}{ex}}x^{3/2}\,\text{EllipticF}\left(i\text{arcsinh}\left(\frac{\sqrt{\frac{2}{3}}}{\sqrt{x}}\right),\frac{3d}{2e}\right)}{\sqrt{-x(2+3x)}\sqrt{d+ex}}$$

input
```
Integrate[1/(Sqrt[d + e*x]*Sqrt[-2*x - 3*x^2]),x]
```

output
```
(I*Sqrt[6 + 4/x]*Sqrt[1 + d/(e*x)]*x^(3/2)*EllipticF[I*ArcSinh[Sqrt[2/3]/S
qrt[x]], (3*d)/(2*e)])/(Sqrt[-(x*(2 + 3*x))]*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {1168, 27, 27, 127, 27, 125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-3x^2-2x}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{1168}$$

$$\int \frac{1}{\sqrt{2}\sqrt{-x}\sqrt{\frac{3x}{2}+1}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{\sqrt{2}}{\sqrt{-x}\sqrt{3x+2}\sqrt{d+ex}}dx}{\sqrt{2}}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{\sqrt{-x}\sqrt{3x+2}\sqrt{d+ex}}dx$$

$$\Big\downarrow \text{127}$$

$$\frac{\sqrt{\frac{ex}{d}+1}\int \frac{\sqrt{2}}{\sqrt{-x}\sqrt{3x+2}\sqrt{\frac{ex}{d}+1}}dx}{\sqrt{2}\sqrt{d+ex}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\frac{ex}{d}+1}\int \frac{1}{\sqrt{-x}\sqrt{3x+2}\sqrt{\frac{ex}{d}+1}}dx}{\sqrt{d+ex}}$$

$$\Big\downarrow \text{125}$$

$$-\frac{2\sqrt{\frac{ex}{d}+1}\,\text{EllipticF}\left(\arcsin\left(\sqrt{\frac{3}{2}}\sqrt{-x}\right),\frac{2e}{3d}\right)}{\sqrt{3}\sqrt{d+ex}}$$

input
```
Int[1/(Sqrt[d + e*x]*Sqrt[-2*x - 3*x^2]),x]
```

output
```
(-2*Sqrt[1 + (e*x)/d]*EllipticF[ArcSin[Sqrt[3/2]*Sqrt[-x]], (2*e)/(3*d)])/
(Sqrt[3]*Sqrt[d + e*x])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 125
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (GtQ[-b/d, 0] || LtQ[-b/f, 0])
```

rule 127
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[Sqrt[1 + d*(x/c)]*(Sqrt[1 + f*(x/e)]/(Sqrt[c + d*x]*Sqrt[e + f*x
]))   Int[1/(Sqrt[b*x]*Sqrt[1 + d*(x/c)]*Sqrt[1 + f*(x/e)]), x], x] /; Free
Q[{b, c, d, e, f}, x] &&  !(GtQ[c, 0] && GtQ[e, 0])
```

rule 1168
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c/b)*x]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0] && RationalQ[b]
```

Maple [A] (verified)

Time = 1.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.40

method	result	size
default	$-\frac{2\,\mathrm{EllipticF}\left(\sqrt{\frac{ex+d}{d}},\sqrt{3}\,\sqrt{\frac{d}{3d-2e}}\right)\sqrt{-\frac{ex}{d}}\,\sqrt{-\frac{(3x+2)e}{3d-2e}}\,\sqrt{\frac{ex+d}{d}}\,d\sqrt{ex+d}\,\sqrt{-x(3x+2)}}{ex\left(3e\,x^{2}+3dx+2ex+2d\right)}$	115
elliptic	$\frac{2\sqrt{-x(3x+2)(ex+d)}\,d\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}}\,\sqrt{\frac{x+\frac{2}{3}}{-\frac{d}{e}+\frac{2}{3}}}\,\sqrt{-\frac{ex}{d}}\,\mathrm{EllipticF}\left(\sqrt{\frac{\left(x+\frac{d}{e}\right)e}{d}},\sqrt{-\frac{d}{e\left(-\frac{d}{e}+\frac{2}{3}\right)}}\right)}{\sqrt{-x(3x+2)}\,\sqrt{ex+d}\,e\sqrt{-3x^{3}e-3d\,x^{2}-2e\,x^{2}-2dx}}$	136

input
```
int(1/(e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*EllipticF(((e*x+d)/d)^(1/2),3^(1/2)*(d/(3*d-2*e))^(1/2))*(-e*x/d)^(1/2)
*(-(3*x+2)*e/(3*d-2*e))^(1/2)*((e*x+d)/d)^(1/2)*d*(e*x+d)^(1/2)*(-x*(3*x+2
))^(1/2)/e/x/(3*e*x^2+3*d*x+2*e*x+2*d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.95

$$\int \frac{1}{\sqrt{d+ex}\sqrt{-2x-3x^2}}\,dx = -\frac{2\,\sqrt{3}\sqrt{-e}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(9\,d^2-6\,de+4\,e^2)}{27\,e^2},-\frac{8\,(27\,d^3-27\,d^2e-18\,de^2+8\,e^3)}{729\,e^3},\frac{9\,ex+3\,d+2\,e}{9\,e}\right)}{3\,e}$$

input
```
integrate(1/(e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(3)*sqrt(-e)*weierstrassPInverse(4/27*(9*d^2 - 6*d*e + 4*e^2)/e^2
, -8/729*(27*d^3 - 27*d^2*e - 18*d*e^2 + 8*e^3)/e^3, 1/9*(9*e*x + 3*d + 2*
e)/e)/e
```

Sympy [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{-2x-3x^2}}\,dx = \int \frac{1}{\sqrt{-x\,(3x+2)}\sqrt{d+ex}}\,dx$$

input
```
integrate(1/(e*x+d)**(1/2)/(-3*x**2-2*x)**(1/2),x)
```

output
```
Integral(1/(sqrt(-x*(3*x + 2))*sqrt(d + e*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{-2x-3x^2}}\,dx = \int \frac{1}{\sqrt{ex+d}\sqrt{-3\,x^2-2\,x}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(e*x + d)*sqrt(-3*x^2 - 2*x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{-2x-3x^2}}\,dx = \int \frac{1}{\sqrt{ex+d}\sqrt{-3\,x^2-2\,x}}\,dx$$

input
```
integrate(1/(e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(e*x + d)*sqrt(-3*x^2 - 2*x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{d+ex}\sqrt{-2x-3x^2}}\,dx = \int \frac{1}{\sqrt{-3\,x^2-2\,x}\,\sqrt{d+e\,x}}\,dx$$

input
```
int(1/((- 2*x - 3*x^2)^(1/2)*(d + e*x)^(1/2)),x)
```

output
```
int(1/((- 2*x - 3*x^2)^(1/2)*(d + e*x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{d+ex}\sqrt{-2x-3x^2}}\,dx = -\left(\int \frac{\sqrt{x}\,\sqrt{ex+d}\,\sqrt{-3x-2}}{3e\,x^3+3d\,x^2+2e\,x^2+2dx}dx\right)$$

input
```
int(1/(e*x+d)^(1/2)/(-3*x^2-2*x)^(1/2),x)
```

output
```
- int((sqrt(x)*sqrt(d + e*x)*sqrt( - 3*x - 2))/(3*d*x**2 + 2*d*x + 3*e*x*
*3 + 2*e*x**2),x)
```

3.231 $\int \frac{\sqrt{1+x}}{\sqrt{x-x^2}} \, dx$

Optimal result . . . 1932
Mathematica [C] (verified) . . . 1932
Rubi [A] (verified) . . . 1933
Maple [B] (verified) . . . 1934
Fricas [B] (verification not implemented) . . . 1934
Sympy [F] . . . 1935
Maxima [F] . . . 1935
Giac [F] . . . 1935
Mupad [F(-1)] . . . 1936
Reduce [F] . . . 1936

Optimal result

Integrand size = 19, antiderivative size = 10

$$\int \frac{\sqrt{1+x}}{\sqrt{x-x^2}} \, dx = 2E\left(\arcsin\left(\sqrt{x}\right)|-1\right)$$

output
```
2*EllipticE(x^(1/2),I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.53 (sec) , antiderivative size = 64, normalized size of antiderivative = 6.40

$$\int \frac{\sqrt{1+x}}{\sqrt{x-x^2}} \, dx = \frac{2x\sqrt{1-x^2}\left(3\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^2\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^2\right)\right)}{3\sqrt{-((-1+x)x)}\sqrt{1+x}}$$

input
```
Integrate[Sqrt[1 + x]/Sqrt[x - x^2],x]
```

output
```
(2*x*Sqrt[1 - x^2]*(3*Hypergeometric2F1[1/4, 1/2, 5/4, x^2] + x*Hypergeome
tric2F1[1/2, 3/4, 7/4, x^2]))/(3*Sqrt[-((-1 + x)*x)]*Sqrt[1 + x])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1168, 120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x+1}}{\sqrt{x-x^2}}dx$$

$$\Big\downarrow \text{1168}$$

$$\int \frac{\sqrt{x+1}}{\sqrt{1-x}\sqrt{x}}dx$$

$$\Big\downarrow \text{120}$$

$$2E\left(\arcsin\left(\sqrt{x}\right)|-1\right)$$

input
```
Int[Sqrt[1 + x]/Sqrt[x - x^2],x]
```

output
```
2*EllipticE[ArcSin[Sqrt[x]], -1]
```

Defintions of rubi rules used

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] && !LtQ[-b/d, 0]
```

rule 1168
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c/b)*x]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0] && RationalQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(8) = 16$.

Time = 0.68 (sec) , antiderivative size = 56, normalized size of antiderivative = 5.60

method	result
default	$-\frac{2\left(\operatorname{EllipticF}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)-\operatorname{EllipticE}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)\right)\sqrt{-x}\,\sqrt{-2x+2}\,\sqrt{-x(x-1)}}{x(x-1)}$
elliptic	$\frac{\sqrt{-x(x^2-1)}\left(\frac{\sqrt{x+1}\,\sqrt{-2x+2}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)}{\sqrt{-x^3+x}}+\frac{\sqrt{x+1}\,\sqrt{-2x+2}\,\sqrt{-x}\,\left(-2\operatorname{EllipticE}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)+\operatorname{EllipticF}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)\right)}{\sqrt{-x^3+x}}\right)}{\sqrt{x+1}\,\sqrt{-x(x-1)}}$

input
```
int((x+1)^(1/2)/(-x^2+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(EllipticF((x+1)^(1/2),1/2*2^(1/2))-EllipticE((x+1)^(1/2),1/2*2^(1/2)))
*(-x)^(1/2)*(-2*x+2)^(1/2)*(-x*(x-1))^(1/2)/x/(x-1)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 16 vs. $2(7) = 14$.

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\sqrt{1+x}}{\sqrt{x-x^2}}\,dx = -2i\,\text{weierstrassPInverse}(4,0,x) + 2i\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,x))$$

input
```
integrate((1+x)^(1/2)/(-x^2+x)^(1/2),x, algorithm="fricas")
```

output
```
-2*I*weierstrassPInverse(4, 0, x) + 2*I*weierstrassZeta(4, 0, weierstrassP
Inverse(4, 0, x))
```

Sympy [F]

$$\int \frac{\sqrt{1+x}}{\sqrt{x-x^2}}\,dx = \int \frac{\sqrt{x+1}}{\sqrt{-x\,(x-1)}}\,dx$$

input
```
integrate((1+x)**(1/2)/(-x**2+x)**(1/2),x)
```

output
```
Integral(sqrt(x + 1)/sqrt(-x*(x - 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1+x}}{\sqrt{x-x^2}}\,dx = \int \frac{\sqrt{x+1}}{\sqrt{-x^2+x}}\,dx$$

input
```
integrate((1+x)^(1/2)/(-x^2+x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(x + 1)/sqrt(-x^2 + x), x)
```

Giac [F]

$$\int \frac{\sqrt{1+x}}{\sqrt{x-x^2}}\,dx = \int \frac{\sqrt{x+1}}{\sqrt{-x^2+x}}\,dx$$

input
```
integrate((1+x)^(1/2)/(-x^2+x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(x + 1)/sqrt(-x^2 + x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1+x}}{\sqrt{x-x^2}}\,dx=\int \frac{\sqrt{x+1}}{\sqrt{x-x^2}}\,dx$$

input
```
int((x + 1)^(1/2)/(x - x^2)^(1/2),x)
```

output
```
int((x + 1)^(1/2)/(x - x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{1+x}}{\sqrt{x-x^2}}\,dx=-\left(\int \frac{\sqrt{x}\,\sqrt{x+1}\,\sqrt{1-x}}{x^2-x}dx\right)$$

input
```
int((1+x)^(1/2)/(-x^2+x)^(1/2),x)
```

output
```
- int((sqrt(x)*sqrt(x + 1)*sqrt( - x + 1))/(x**2 - x),x)
```

3.232 $\int \frac{1}{\sqrt{1+x}\sqrt{x-x^2}}\,dx$

Optimal result 1937
Mathematica [C] (verified) 1937
Rubi [A] (verified) 1938
Maple [B] (verified) 1939
Fricas [A] (verification not implemented) 1939
Sympy [F] 1940
Maxima [F] 1940
Giac [F] 1940
Mupad [F(-1)] 1941
Reduce [F] 1941

Optimal result

Integrand size = 19, antiderivative size = 10

$$\int \frac{1}{\sqrt{1+x}\sqrt{x-x^2}}\,dx = 2\,\text{EllipticF}\left(\arcsin\left(\sqrt{x}\right),-1\right)$$

output
```
2*EllipticF(x^(1/2),I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.44 (sec) , antiderivative size = 44, normalized size of antiderivative = 4.40

$$\int \frac{1}{\sqrt{1+x}\sqrt{x-x^2}}\,dx = \frac{2x\sqrt{1-x^2}\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^2\right)}{\sqrt{-((-1+x)x)}\sqrt{1+x}}$$

input
```
Integrate[1/(Sqrt[1 + x]*Sqrt[x - x^2]),x]
```

output
```
(2*x*Sqrt[1 - x^2]*Hypergeometric2F1[1/4, 1/2, 5/4, x^2])/(Sqrt[-((-1 + x)
*x)]*Sqrt[1 + x])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1168, 125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{x+1}\sqrt{x-x^2}} \, dx$$
$$\Big\downarrow \text{ 1168}$$
$$\int \frac{1}{\sqrt{1-x}\sqrt{x}\sqrt{x+1}}dx$$
$$\Big\downarrow \text{ 125}$$
$$2 \operatorname{EllipticF}\left(\arcsin\left(\sqrt{x}\right),-1\right)$$

input
```
Int[1/(Sqrt[1 + x]*Sqrt[x - x^2]),x]
```

output
```
2*EllipticF[ArcSin[Sqrt[x]], -1]
```

Defintions of rubi rules used

rule 125
```
Int[1/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]*Sqrt[(e_) + (f_.)*(x_)]), x
_] :> Simp[(2/(b*Sqrt[e]))*Rt[-b/d, 2]*EllipticF[ArcSin[Sqrt[b*x]/(Sqrt[c]*
Rt[-b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] &
& GtQ[e, 0] && (GtQ[-b/d, 0] || LtQ[-b/f, 0])
```

rule 1168
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c/b)*x]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0] && RationalQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. 2(8) = 16.

Time = 0.69 (sec) , antiderivative size = 42, normalized size of antiderivative = 4.20

method	result	size
default	$-\frac{\mathrm{EllipticF}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)\sqrt{-x}\,\sqrt{-2x+2}\,\sqrt{-x(x-1)}}{x(x-1)}$	42
elliptic	$\frac{\sqrt{-x(x^2-1)}\,\sqrt{-2x+2}\,\sqrt{-x}\,\mathrm{EllipticF}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)}{\sqrt{-x(x-1)}\,\sqrt{-x^3+x}}$	52

input
```
int(1/(x+1)^(1/2)/(-x^2+x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-EllipticF((x+1)^(1/2),1/2*2^(1/2))*(-x)^(1/2)*(-2*x+2)^(1/2)*(-x*(x-1))^(
1/2)/x/(x-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.60

$$\int \frac{1}{\sqrt{1+x}\sqrt{x-x^2}}\,dx = -2i\,\mathrm{weierstrassPInverse}(4,0,x)$$

input
```
integrate(1/(1+x)^(1/2)/(-x^2+x)^(1/2),x, algorithm="fricas")
```

output
```
-2*I*weierstrassPInverse(4, 0, x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{x-x^2}}\,dx = \int \frac{1}{\sqrt{-x\,(x-1)}\sqrt{x+1}}\,dx$$

input
```
integrate(1/(1+x)**(1/2)/(-x**2+x)**(1/2),x)
```

output
```
Integral(1/(sqrt(-x*(x - 1))*sqrt(x + 1)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{x-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+x}\sqrt{x+1}}\,dx$$

input
```
integrate(1/(1+x)^(1/2)/(-x^2+x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(-x^2 + x)*sqrt(x + 1)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{x-x^2}}\,dx = \int \frac{1}{\sqrt{-x^2+x}\sqrt{x+1}}\,dx$$

input
```
integrate(1/(1+x)^(1/2)/(-x^2+x)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(-x^2 + x)*sqrt(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{1+x}\sqrt{x-x^2}}\,dx = \int \frac{1}{\sqrt{x-x^2}\,\sqrt{x+1}}\,dx$$

input
```
int(1/((x - x^2)^(1/2)*(x + 1)^(1/2)),x)
```

output
```
int(1/((x - x^2)^(1/2)*(x + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1+x}\sqrt{x-x^2}}\,dx = -\left(\int \frac{\sqrt{x}\,\sqrt{x+1}\,\sqrt{1-x}}{x^3-x}dx\right)$$

input
```
int(1/(1+x)^(1/2)/(-x^2+x)^(1/2),x)
```

output
```
- int((sqrt(x)*sqrt(x + 1)*sqrt( - x + 1))/(x**3 - x),x)
```

3.233 $\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{1+x}} \, dx$

Optimal result . . . 1942
Mathematica [C] (verified) . . . 1942
Rubi [A] (verified) . . . 1943
Maple [A] (verified) . . . 1944
Fricas [A] (verification not implemented) . . . 1944
Sympy [F] . . . 1945
Maxima [F] . . . 1945
Giac [F] . . . 1945
Mupad [F(-1)] . . . 1946
Reduce [F] . . . 1946

Optimal result

Integrand size = 24, antiderivative size = 12

$$\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{1+x}} \, dx = -2E\left(\arcsin\left(\sqrt{-x}\right)|-1\right)$$

output
```
-2*EllipticE((-x)^(1/2),I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.55 (sec) , antiderivative size = 66, normalized size of antiderivative = 5.50

$$\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{1+x}} \, dx = -\frac{2x\sqrt{1-x^2}\left(-3\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^2\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^2\right)\right)}{3\sqrt{1-x}\sqrt{-x(1+x)}}$$

input
```
Integrate[Sqrt[1 - x]/(Sqrt[-x]*Sqrt[1 + x]),x]
```

output
```
(-2*x*Sqrt[1 - x^2]*(-3*Hypergeometric2F1[1/4, 1/2, 5/4, x^2] + x*Hypergeo
metric2F1[1/2, 3/4, 7/4, x^2]))/(3*Sqrt[1 - x]*Sqrt[-(x*(1 + x))])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = $\{120\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{x+1}}\,dx$$
$$\Big\downarrow \; 120$$
$$-2E\left(\arcsin\left(\sqrt{-x}\right)|-1\right)$$

input
```
Int[Sqrt[1 - x]/(Sqrt[-x]*Sqrt[1 + x]),x]
```

output
```
-2*EllipticE[ArcSin[Sqrt[-x]], -1]
```

Defintions of rubi rules used

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.42

method	result	si
default	$2\sqrt{2}\ \mathrm{EllipticE}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)$	1
elliptic	$\frac{\sqrt{x(x^2-1)}\left(\frac{\sqrt{x+1}\sqrt{-2x+2}\sqrt{-x}\ \mathrm{EllipticF}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-x}}-\frac{\sqrt{x+1}\sqrt{-2x+2}\sqrt{-x}\left(-2\ \mathrm{EllipticE}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)+\mathrm{EllipticF}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)\right)}{\sqrt{x^3-x}}\right)}{\sqrt{-x}\sqrt{1-x}\sqrt{x+1}}$	1

input
```
int((1-x)^(1/2)/(-x)^(1/2)/(x+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*2^(1/2)*EllipticE((x+1)^(1/2),1/2*2^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{1+x}}\,dx = 2\,\mathrm{weierstrassPInverse}(4,0,x) + 2\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,x))$$

input
```
integrate((1-x)^(1/2)/(-x)^(1/2)/(1+x)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassPInverse(4, 0, x) + 2*weierstrassZeta(4, 0, weierstrassPInver
se(4, 0, x))
```

Sympy [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{1+x}}\,dx = \int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{x+1}}\,dx$$

input
```
integrate((1-x)**(1/2)/(-x)**(1/2)/(1+x)**(1/2),x)
```

output
```
Integral(sqrt(1 - x)/(sqrt(-x)*sqrt(x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{1+x}}\,dx = \int \frac{\sqrt{-x+1}}{\sqrt{-x}\sqrt{x+1}}\,dx$$

input
```
integrate((1-x)^(1/2)/(-x)^(1/2)/(1+x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x + 1)/(sqrt(-x)*sqrt(x + 1)), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{1+x}}\,dx = \int \frac{\sqrt{-x+1}}{\sqrt{-x}\sqrt{x+1}}\,dx$$

input
```
integrate((1-x)^(1/2)/(-x)^(1/2)/(1+x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x + 1)/(sqrt(-x)*sqrt(x + 1)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{1+x}}\,dx = \int \frac{\sqrt{1-x}}{\sqrt{-x}\,\sqrt{x+1}}\,dx$$

input
```
int((1 - x)^(1/2)/((-x)^(1/2)*(x + 1)^(1/2)),x)
```

output
```
int((1 - x)^(1/2)/((-x)^(1/2)*(x + 1)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{1+x}}\,dx = -\left(\int \frac{\sqrt{x}\,\sqrt{x+1}\,\sqrt{1-x}}{x^2+x}dx\right)i$$

input
```
int((1-x)^(1/2)/(-x)^(1/2)/(1+x)^(1/2),x)
```

output
```
- int((sqrt(x)*sqrt(x + 1)*sqrt( - x + 1))/(x**2 + x),x)*i
```

3.234 $\int \frac{1-x}{\sqrt{-x}\sqrt{1-x^2}}\,dx$

Optimal result . . . 1947
Mathematica [C] (verified) . . . 1947
Rubi [B] (verified) . . . 1948
Maple [B] (verified) . . . 1950
Fricas [A] (verification not implemented) . . . 1951
Sympy [B] (verification not implemented) . . . 1951
Maxima [F] . . . 1952
Giac [F] . . . 1952
Mupad [B] (verification not implemented) . . . 1952
Reduce [F] . . . 1953

Optimal result

Integrand size = 24, antiderivative size = 12

$$\int \frac{1-x}{\sqrt{-x}\sqrt{1-x^2}}\,dx = -2E\left(\arcsin\left(\sqrt{-x}\right)\middle|-1\right)$$

output
```
-2*EllipticE((-x)^(1/2),I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 10.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.50

$$\int \frac{1-x}{\sqrt{-x}\sqrt{1-x^2}}\,dx = \frac{2}{3}\sqrt{-x}\left(-3\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^2\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^2\right)\right)$$

input
```
Integrate[(1 - x)/(Sqrt[-x]*Sqrt[1 - x^2]),x]
```

output
```
(2*Sqrt[-x]*(-3*Hypergeometric2F1[1/4, 1/2, 5/4, x^2] + x*Hypergeometric2F
1[1/2, 3/4, 7/4, x^2]))/3
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 35 vs. $2(12) = 24$.

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {556, 555, 1388, 326, 284, 327, 762}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1-x}{\sqrt{-x}\sqrt{1-x^2}}\,dx$$

$$\Big\downarrow \text{556}$$

$$\frac{\sqrt{x}\int \frac{1-x}{\sqrt{x}\sqrt{1-x^2}}dx}{\sqrt{-x}}$$

$$\Big\downarrow \text{555}$$

$$\frac{2\sqrt{x}\int \frac{1-x}{\sqrt{1-x^2}}d\sqrt{x}}{\sqrt{-x}}$$

$$\Big\downarrow \text{1388}$$

$$\frac{2\sqrt{x}\int \frac{\sqrt{1-x}}{\sqrt{x+1}}d\sqrt{x}}{\sqrt{-x}}$$

$$\Big\downarrow \text{326}$$

$$\frac{2\sqrt{x}\left(2\int \frac{1}{\sqrt{1-x}\sqrt{x+1}}d\sqrt{x}-\int \frac{\sqrt{x+1}}{\sqrt{1-x}}d\sqrt{x}\right)}{\sqrt{-x}}$$

$$\Big\downarrow \text{284}$$

$$\frac{2\sqrt{x}\left(2\int \frac{1}{\sqrt{1-x^2}}d\sqrt{x}-\int \frac{\sqrt{x+1}}{\sqrt{1-x}}d\sqrt{x}\right)}{\sqrt{-x}}$$

$$\Big\downarrow \text{327}$$

$$\frac{2\sqrt{x}\left(2\int\frac{1}{\sqrt{1-x^2}}d\sqrt{x}-E\left(\arcsin\left(\sqrt{x}\right)|-1\right)\right)}{\sqrt{-x}}$$

$$\Big\downarrow \; 762$$

$$\frac{2\sqrt{x}\left(2\,\text{EllipticF}\left(\arcsin\left(\sqrt{x}\right),-1\right)-E\left(\arcsin\left(\sqrt{x}\right)|-1\right)\right)}{\sqrt{-x}}$$

input
```
Int[(1 - x)/(Sqrt[-x]*Sqrt[1 - x^2]),x]
```

output
```
(2*Sqrt[x]*(-EllipticE[ArcSin[Sqrt[x]], -1] + 2*EllipticF[ArcSin[Sqrt[x]],
 -1]))/Sqrt[-x]
```

Defintions of rubi rules used

rule 284
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(p_.), x_Symbol] :> I
nt[(a*c + b*d*x^4)^p, x] /; FreeQ[{a, b, c, d, p}, x] && EqQ[b*c + a*d, 0]
&& (IntegerQ[p] || (GtQ[a, 0] && GtQ[c, 0]))
```

rule 326
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
b/d   Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Simp[(b*c - a*d)/d   In
t[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] &&
PosQ[d/c] && NegQ[b/a]
```

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 555
```
Int[((f_) + (g_.)*(x_))/(Sqrt[x_]*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :>
Simp[2   Subst[Int[(f + g*x^2)/Sqrt[a + c*x^4], x], x, Sqrt[x]], x] /; Free
Q[{a, c, f, g}, x]
```

rule 556
```
Int[((c_) + (d_.)*(x_))/(Sqrt[(e_)*(x_)]*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symb
ol] :> Simp[Sqrt[x]/Sqrt[e*x]   Int[(c + d*x)/(Sqrt[x]*Sqrt[a + b*x^2]), x]
, x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(10) = 20$.

Time = 0.60 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.92

method	result	si
default	$\frac{2\sqrt{x+1}\,\sqrt{-2x+2}\,\operatorname{EllipticE}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)}{\sqrt{-x^{2}+1}}$	3
meijerg	$\frac{2x\,\operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}\right], \left[\frac{5}{4}\right], x^{2}\right)}{\sqrt{-x}} - \frac{2x^{2}\,\operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], x^{2}\right)}{3\sqrt{-x}}$	3
elliptic	$\frac{\sqrt{x(x^{2}-1)}\left(\frac{\sqrt{x+1}\,\sqrt{-2x+2}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)}{\sqrt{x^{3}-x}} - \frac{\sqrt{x+1}\,\sqrt{-2x+2}\,\sqrt{-x}\,\left(-2\operatorname{EllipticE}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)+\operatorname{EllipticF}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)\right)}{\sqrt{x^{3}-x}}\right)}{\sqrt{-x}\,\sqrt{-x^{2}+1}}$	1

input
```
int((1-x)/(-x)^(1/2)/(-x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(x+1)^(1/2)*(-2*x+2)^(1/2)/(-x^2+1)^(1/2)*EllipticE((x+1)^(1/2),1/2*2^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{1-x}{\sqrt{-x}\sqrt{1-x^2}}\,dx = 2\,\text{weierstrassPInverse}(4,0,x) + 2\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,x))$$

input
```
integrate((1-x)/(-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassPInverse(4, 0, x) + 2*weierstrassZeta(4, 0, weierstrassPInver
se(4, 0, x))
```

Sympy [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(10) = 20.

Time = 4.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 5.83

$$\int \frac{1-x}{\sqrt{-x}\sqrt{1-x^2}}\,dx = \frac{ix^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\middle|x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{7}{4}\right)} - \frac{i\sqrt{x}\,\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\middle|x^2e^{2i\pi}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate((1-x)/(-x)**(1/2)/(-x**2+1)**(1/2),x)
```

output
```
I*x**(3/2)*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**2*exp_polar(2*I*pi))/(2
*gamma(7/4)) - I*sqrt(x)*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**2*exp_pol
ar(2*I*pi))/(2*gamma(5/4))
```

Maxima [F]

$$\int \frac{1-x}{\sqrt{-x}\sqrt{1-x^2}}\,dx = \int -\frac{x-1}{\sqrt{-x^2+1}\sqrt{-x}}\,dx$$

input
```
integrate((1-x)/(-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="maxima")
```

output
```
-integrate((x - 1)/(sqrt(-x^2 + 1)*sqrt(-x)), x)
```

Giac [F]

$$\int \frac{1-x}{\sqrt{-x}\sqrt{1-x^2}}\,dx = \int -\frac{x-1}{\sqrt{-x^2+1}\sqrt{-x}}\,dx$$

input
```
integrate((1-x)/(-x)^(1/2)/(-x^2+1)^(1/2),x, algorithm="giac")
```

output
```
integrate(-(x - 1)/(sqrt(-x^2 + 1)*sqrt(-x)), x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.08

$$\int \frac{1-x}{\sqrt{-x}\sqrt{1-x^2}}\,dx = 2\,\mathrm{E}\left(\mathrm{asin}\left(\sqrt{\frac{x}{2}+\frac{1}{2}}\right)\middle|\,2\right) + 2\,\mathrm{F}\left(\mathrm{asin}\left(\sqrt{\frac{x}{2}+\frac{1}{2}}\right)\middle|\,2\right)$$

input
```
int(-(x - 1)/((-x)^(1/2)*(1 - x^2)^(1/2)),x)
```

output
```
2*ellipticE(asin((x/2 + 1/2)^(1/2)), 2) + 2*ellipticF(asin((x/2 + 1/2)^(1/
2)), 2)
```

Reduce [F]

$$\int \frac{1-x}{\sqrt{-x}\sqrt{1-x^2}}\,dx = -\left(\int \frac{\sqrt{x}\,\sqrt{-x^2+1}}{x^2+x}dx\right)i$$

input
```
int((1-x)/(-x)^(1/2)/(-x^2+1)^(1/2),x)
```

output
```
- int((sqrt(x)*sqrt( - x**2 + 1))/(x**2 + x),x)*i
```

3.235 $\int \frac{\sqrt{1-x}}{\sqrt{-x-x^2}} \, dx$

Optimal result 1954
Mathematica [C] (verified) 1954
Rubi [A] (verified) 1955
Maple [B] (verified) 1956
Fricas [A] (verification not implemented) 1956
Sympy [F] 1957
Maxima [F] 1957
Giac [F] 1957
Mupad [F(-1)] 1958
Reduce [F] 1958

Optimal result

Integrand size = 23, antiderivative size = 12

$$\int \frac{\sqrt{1-x}}{\sqrt{-x-x^2}} \, dx = -2E\left(\arcsin\left(\sqrt{-x}\right)|-1\right)$$

output
```
-2*EllipticE((-x)^(1/2),I)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.00 (sec) , antiderivative size = 66, normalized size of antiderivative = 5.50

$$\int \frac{\sqrt{1-x}}{\sqrt{-x-x^2}} \, dx = -\frac{2x\sqrt{1-x^2}\left(-3\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},x^2\right)+x\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},x^2\right)\right)}{3\sqrt{1-x}\sqrt{-x(1+x)}}$$

input
```
Integrate[Sqrt[1 - x]/Sqrt[-x - x^2],x]
```

output
```
(-2*x*Sqrt[1 - x^2]*(-3*Hypergeometric2F1[1/4, 1/2, 5/4, x^2] + x*Hypergeo
metric2F1[1/2, 3/4, 7/4, x^2]))/(3*Sqrt[1 - x]*Sqrt[-(x*(1 + x))])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1168, 120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}}{\sqrt{-x^2-x}}dx$$

$$\Big\downarrow \text{1168}$$

$$\int \frac{\sqrt{1-x}}{\sqrt{-x}\sqrt{x+1}}dx$$

$$\Big\downarrow \text{120}$$

$$-2E\left(\arcsin\left(\sqrt{-x}\right)|-1\right)$$

input
```
Int[Sqrt[1 - x]/Sqrt[-x - x^2],x]
```

output
```
-2*EllipticE[ArcSin[Sqrt[-x]], -1]
```

Defintions of rubi rules used

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

rule 1168
```
Int[((d_.) + (e_.)*(x_))^(m_)/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :>
 Int[(d + e*x)^m/(Sqrt[b*x]*Sqrt[1 + (c/b)*x]), x] /; FreeQ[{b, c, d, e}, x
] && NeQ[c*d - b*e, 0] && EqQ[m^2, 1/4] && LtQ[c, 0] && RationalQ[b]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(10) = 20.

Time = 0.66 (sec) , antiderivative size = 50, normalized size of antiderivative = 4.17

method	result	si
default	$\frac{2(1-x)\sqrt{-(x+1)x}\,\sqrt{x+1}\,\sqrt{2}\,\sqrt{-x}\,\operatorname{EllipticE}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)}{x(x^2-1)}$	50
elliptic	$\frac{\sqrt{x(x^2-1)}\left(\frac{\sqrt{x+1}\,\sqrt{-2x+2}\,\sqrt{-x}\,\operatorname{EllipticF}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)}{\sqrt{x^3-x}}-\frac{\sqrt{x+1}\,\sqrt{-2x+2}\,\sqrt{-x}\,\left(-2\operatorname{EllipticE}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)+\operatorname{EllipticF}\left(\sqrt{x+1}, \frac{\sqrt{2}}{2}\right)\right)}{\sqrt{x^3-x}}\right)}{\sqrt{1-x}\,\sqrt{-(x+1)x}}$	1

input
```
int((1-x)^(1/2)/(-x^2-x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(1-x)*(-(x+1)*x)^(1/2)*(x+1)^(1/2)*2^(1/2)*(-x)^(1/2)*EllipticE((x+1)^(1
/2),1/2*2^(1/2))/x/(x^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.33

$$\int \frac{\sqrt{1-x}}{\sqrt{-x-x^2}}\,dx = 2\,\text{weierstrassPInverse}(4,0,x) + 2\,\text{weierstrassZeta}(4,0,\text{weierstrassPInverse}(4,0,x))$$

input
```
integrate((1-x)^(1/2)/(-x^2-x)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassPInverse(4, 0, x) + 2*weierstrassZeta(4, 0, weierstrassPInver
se(4, 0, x))
```

Sympy [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{-x-x^2}}\,dx = \int \frac{\sqrt{1-x}}{\sqrt{-x\,(x+1)}}\,dx$$

input
```
integrate((1-x)**(1/2)/(-x**2-x)**(1/2),x)
```

output
```
Integral(sqrt(1 - x)/sqrt(-x*(x + 1)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{-x-x^2}}\,dx = \int \frac{\sqrt{-x+1}}{\sqrt{-x^2-x}}\,dx$$

input
```
integrate((1-x)^(1/2)/(-x^2-x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x + 1)/sqrt(-x^2 - x), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{-x-x^2}}\,dx = \int \frac{\sqrt{-x+1}}{\sqrt{-x^2-x}}\,dx$$

input
```
integrate((1-x)^(1/2)/(-x^2-x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x + 1)/sqrt(-x^2 - x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}}{\sqrt{-x-x^2}}\,dx = \int \frac{\sqrt{1-x}}{\sqrt{-x^2-x}}\,dx$$

input
```
int((1 - x)^(1/2)/(- x - x^2)^(1/2),x)
```

output
```
int((1 - x)^(1/2)/(- x - x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{-x-x^2}}\,dx = -\left(\int \frac{\sqrt{x}\,\sqrt{x+1}\,\sqrt{1-x}}{x^2+x}dx\right)i$$

input
```
int((1-x)^(1/2)/(-x^2-x)^(1/2),x)
```

output
```
- int((sqrt(x)*sqrt(x + 1)*sqrt( - x + 1))/(x**2 + x),x)*i
```

3.236 $\int \frac{\sqrt{1-x}}{\sqrt{2-x}\sqrt{x}} \, dx$

Optimal result 1959
Mathematica [C] (verified) 1959
Rubi [A] (verified) 1960
Maple [C] (verified) 1960
Fricas [A] (verification not implemented) 1961
Sympy [C] (verification not implemented) 1961
Maxima [F] 1962
Giac [F] 1962
Mupad [F(-1)] 1963
Reduce [F] 1963

Optimal result

Integrand size = 24, antiderivative size = 16

$$\int \frac{\sqrt{1-x}}{\sqrt{2-x}\sqrt{x}} \, dx = 2E\left(\arcsin\left(\frac{\sqrt{x}}{\sqrt{2}}\right)\middle|2\right)$$

output
```
2*EllipticE(1/2*2^(1/2)*x^(1/2),2^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 1.81 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{1-x}}{\sqrt{2-x}\sqrt{x}} \, dx = -\frac{2}{3}(1-x)^{3/2} \, {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(-1+x)^2\right)$$

input
```
Integrate[Sqrt[1 - x]/(Sqrt[2 - x]*Sqrt[x]),x]
```

output
```
(-2*(1 - x)^(3/2)*HypergeometricPFQ[{1/2, 3/4}, {7/4}, (-1 + x)^2])/3
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.042$, Rules used = {120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}}{\sqrt{2-x}\sqrt{x}}\,dx$$
$$\Big\downarrow \; 120$$
$$2E\left(\arcsin\left(\frac{\sqrt{x}}{\sqrt{2}}\right)\middle|2\right)$$

input
```
Int[Sqrt[1 - x]/(Sqrt[2 - x]*Sqrt[x]),x]
```

output
```
2*EllipticE[ArcSin[Sqrt[x]/Sqrt[2]], 2]
```

Defintions of rubi rules used

rule 120
```
Int[Sqrt[(e_) + (f_.)*(x_)]/(Sqrt[(b_.)*(x_)]*Sqrt[(c_) + (d_.)*(x_)]), x_]
 :> Simp[2*(Sqrt[e]/b)*Rt[-b/d, 2]*EllipticE[ArcSin[Sqrt[b*x]/(Sqrt[c]*Rt[-
b/d, 2])], c*(f/(d*e))], x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[c, 0] && Gt
Q[e, 0] &&  !LtQ[-b/d, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.62

method	result	size
default	$2\,\mathrm{EllipticF}\left(\sqrt{1-x}, i\right)-2\,\mathrm{EllipticE}\left(\sqrt{1-x}, i\right)$	26
elliptic	$\frac{\sqrt{(x-1)(x-2)x}\left(-\frac{2\sqrt{1-x}\,\sqrt{2-x}\,\sqrt{x}\,\mathrm{EllipticF}\left(\sqrt{1-x},i\right)}{\sqrt{x^{3}-3x^{2}+2x}}+\frac{2\sqrt{1-x}\,\sqrt{2-x}\,\sqrt{x}\,\left(-\mathrm{EllipticE}\left(\sqrt{1-x},i\right)+2\,\mathrm{EllipticF}\left(\sqrt{1-x},i\right)\right)}{\sqrt{x^{3}-3x^{2}+2x}}\right)}{\sqrt{1-x}\,\sqrt{2-x}\,\sqrt{x}}$	131

input
```
int((1-x)^(1/2)/(2-x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*EllipticF((1-x)^(1/2),I)-2*EllipticE((1-x)^(1/2),I)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{1-x}}{\sqrt{2-x}\sqrt{x}}\,dx = 2\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,x-1))$$

input
```
integrate((1-x)^(1/2)/(2-x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, x - 1))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 101.82 (sec) , antiderivative size = 76, normalized size of antiderivative = 4.75

$$\int \frac{\sqrt{1-x}}{\sqrt{2-x}\sqrt{x}}\,dx = \frac{G_{6,6}^{6,2}\left(\begin{matrix} 0, \frac{1}{2} & \frac{1}{4}, \frac{1}{4}, \frac{3}{4}, 1 \\ -\frac{1}{4}, 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 0 & \end{matrix} \middle| {\frac{1}{(x-1)^{2}}} \right)}{4\pi^{\frac{3}{2}}} - \frac{G_{6,6}^{3,5}\left(\begin{matrix} -\frac{3}{4}, -\frac{1}{2}, -\frac{1}{4}, 0, \frac{1}{4} & 1 \\ -\frac{1}{2}, 0, 0 & -\frac{3}{4}, -\frac{1}{4}, -\frac{1}{4} \end{matrix} \middle| {\frac{e^{-2i\pi}}{(x-1)^{2}}} \right)}{4\pi^{\frac{3}{2}}}$$

input
```
integrate((1-x)**(1/2)/(2-x)**(1/2)/x**(1/2),x)
```

output
```
meijerg((((0, 1/2), (1/4, 1/4, 3/4, 1)), ((-1/4, 0, 1/4, 1/2, 3/4, 0), ()),
 (x - 1)**(-2))/(4*pi**(3/2)) - meijerg((((-3/4, -1/2, -1/4, 0, 1/4), (1,))
, ((-1/2, 0, 0), (-3/4, -1/4, -1/4)), exp_polar(-2*I*pi)/(x - 1)**2)/(4*pi
**(3/2))
```

Maxima [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{2-x}\sqrt{x}}\,dx = \int \frac{\sqrt{-x+1}}{\sqrt{x}\sqrt{-x+2}}\,dx$$

input
```
integrate((1-x)^(1/2)/(2-x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x + 1)/(sqrt(x)*sqrt(-x + 2)), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{2-x}\sqrt{x}}\,dx = \int \frac{\sqrt{-x+1}}{\sqrt{x}\sqrt{-x+2}}\,dx$$

input
```
integrate((1-x)^(1/2)/(2-x)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x + 1)/(sqrt(x)*sqrt(-x + 2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}}{\sqrt{2-x}\sqrt{x}}\,dx = \int \frac{\sqrt{1-x}}{\sqrt{x}\,\sqrt{2-x}}\,dx$$

input
```
int((1 - x)^(1/2)/(x^(1/2)*(2 - x)^(1/2)),x)
```

output
```
int((1 - x)^(1/2)/(x^(1/2)*(2 - x)^(1/2)), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{2-x}\sqrt{x}}\,dx = -\left(\int \frac{\sqrt{1-x}\,\sqrt{-x+2}}{\sqrt{x}\,x-2\sqrt{x}}dx\right)$$

input
```
int((1-x)^(1/2)/(2-x)^(1/2)/x^(1/2),x)
```

output
```
- int((sqrt( - x + 1)*sqrt( - x + 2))/(sqrt(x)*x - 2*sqrt(x)),x)
```

3.237 $\int \frac{\sqrt{1-x}}{\sqrt{2x-x^2}} \, dx$

Optimal result . . . 1964
Mathematica [C] (verified) . . . 1964
Rubi [A] (verified) . . . 1965
Maple [C] (verified) . . . 1966
Fricas [A] (verification not implemented) . . . 1967
Sympy [F] . . . 1967
Maxima [F] . . . 1968
Giac [F] . . . 1968
Mupad [F(-1)] . . . 1968
Reduce [F] . . . 1969

Optimal result

Integrand size = 23, antiderivative size = 16

$$\int \frac{\sqrt{1-x}}{\sqrt{2x-x^2}} \, dx = 2E\left(\arcsin\left(\frac{\sqrt{x}}{\sqrt{2}}\right)\middle|2\right)$$

output
```
2*EllipticE(1/2*2^(1/2)*x^(1/2),2^(1/2))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.01 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.88

$$\int \frac{\sqrt{1-x}}{\sqrt{2x-x^2}} \, dx = -\frac{2}{3}(1-x)^{3/2} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};(-1+x)^2\right)$$

input
```
Integrate[Sqrt[1 - x]/Sqrt[2*x - x^2],x]
```

output
```
(-2*(1 - x)^(3/2)*HypergeometricPFQ[{1/2, 3/4}, {7/4}, (-1 + x)^2])/3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {1114, 836, 762, 1388, 327}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{1-x}}{\sqrt{2x-x^2}}dx$$

$$\Big\downarrow \text{1114}$$

$$-2\int \frac{1-x}{\sqrt{1-(1-x)^2}}d\sqrt{1-x}$$

$$\Big\downarrow \text{836}$$

$$-2\left(\int \frac{2-x}{\sqrt{1-(1-x)^2}}d\sqrt{1-x}-\int \frac{1}{\sqrt{1-(1-x)^2}}d\sqrt{1-x}\right)$$

$$\Big\downarrow \text{762}$$

$$-2\left(\int \frac{2-x}{\sqrt{1-(1-x)^2}}d\sqrt{1-x}-\text{EllipticF}\left(\arcsin\left(\sqrt{1-x}\right),-1\right)\right)$$

$$\Big\downarrow \text{1388}$$

$$-2\left(\int \frac{\sqrt{2-x}}{\sqrt{x}}d\sqrt{1-x}-\text{EllipticF}\left(\arcsin\left(\sqrt{1-x}\right),-1\right)\right)$$

$$\Big\downarrow \text{327}$$

$$-2\left(E\left(\arcsin\left(\sqrt{1-x}\right)|-1\right)-\text{EllipticF}\left(\arcsin\left(\sqrt{1-x}\right),-1\right)\right)$$

input
```
Int[Sqrt[1 - x]/Sqrt[2*x - x^2],x]
```

output
```
-2*(EllipticE[ArcSin[Sqrt[1 - x]], -1] - EllipticF[ArcSin[Sqrt[1 - x]], -1
])
```

Defintions of rubi rules used

rule 327
```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

rule 762
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[(1/(Sqrt[a]*Rt[-b/a, 4])
)*EllipticF[ArcSin[Rt[-b/a, 4]*x], -1], x] /; FreeQ[{a, b}, x] && NegQ[b/a]
 && GtQ[a, 0]
```

rule 836
```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[-b/a, 2]},
Simp[-q^(-1)   Int[1/Sqrt[a + b*x^4], x], x] + Simp[1/q   Int[(1 + q*x^2)/S
qrt[a + b*x^4], x], x]] /; FreeQ[{a, b}, x] && NegQ[b/a]
```

rule 1114
```
Int[Sqrt[(d_) + (e_.)*(x_)]/Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symb
ol] :> Simp[(4/e)*Sqrt[-c/(b^2 - 4*a*c)]   Subst[Int[x^2/Sqrt[Simp[1 - b^2*
(x^4/(d^2*(b^2 - 4*a*c))), x]], x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c
, d, e}, x] && EqQ[2*c*d - b*e, 0] && LtQ[c/(b^2 - 4*a*c), 0]
```

rule 1388
```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.))^(p_.)*((d_) + (e_.)*(x_)^(n_))^(q_.),
x_Symbol] :> Int[u*(d + e*x^n)^(p + q)*(a/d + (c/e)*x^n)^p, x] /; FreeQ[{a,
 c, d, e, n, p, q}, x] && EqQ[n2, 2*n] && EqQ[c*d^2 + a*e^2, 0] && (Integer
Q[p] || (GtQ[a, 0] && GtQ[d, 0]))
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.75

method	result	size
default	$\frac{2\left(\operatorname{EllipticF}\left(\sqrt{1-x}, i\right)-\operatorname{EllipticE}\left(\sqrt{1-x}, i\right)\right) \sqrt{-x(x-2)}}{\sqrt{2-x}\,\sqrt{x}}$	44
elliptic	$\frac{\sqrt{(x-1)(x-2)x}\left(-\frac{2\sqrt{1-x}\,\sqrt{2-x}\,\sqrt{x}\,\operatorname{EllipticF}\left(\sqrt{1-x}, i\right)}{\sqrt{x^3-3x^2+2x}}+\frac{2\sqrt{1-x}\,\sqrt{2-x}\,\sqrt{x}\,\left(-\operatorname{EllipticE}\left(\sqrt{1-x}, i\right)+2\operatorname{EllipticF}\left(\sqrt{1-x}, i\right)\right)}{\sqrt{x^3-3x^2+2x}}\right)}{\sqrt{1-x}\,\sqrt{-x(x-2)}}$	129

input
```
int((1-x)^(1/2)/(-x^2+2*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(EllipticF((1-x)^(1/2),I)-EllipticE((1-x)^(1/2),I))/(2-x)^(1/2)/x^(1/2)*
(-x*(x-2))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{1-x}}{\sqrt{2x-x^2}}\,dx = 2\,\mathrm{weierstrassZeta}(4,0,\mathrm{weierstrassPInverse}(4,0,x-1))$$

input
```
integrate((1-x)^(1/2)/(-x^2+2*x)^(1/2),x, algorithm="fricas")
```

output
```
2*weierstrassZeta(4, 0, weierstrassPInverse(4, 0, x - 1))
```

Sympy [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{2x-x^2}}\,dx = \int \frac{\sqrt{1-x}}{\sqrt{-x\,(x-2)}}\,dx$$

input
```
integrate((1-x)**(1/2)/(-x**2+2*x)**(1/2),x)
```

output
```
Integral(sqrt(1 - x)/sqrt(-x*(x - 2)), x)
```

Maxima [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{2x-x^2}}\,dx = \int \frac{\sqrt{-x+1}}{\sqrt{-x^2+2\,x}}\,dx$$

input
```
integrate((1-x)^(1/2)/(-x^2+2*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-x + 1)/sqrt(-x^2 + 2*x), x)
```

Giac [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{2x-x^2}}\,dx = \int \frac{\sqrt{-x+1}}{\sqrt{-x^2+2\,x}}\,dx$$

input
```
integrate((1-x)^(1/2)/(-x^2+2*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(-x + 1)/sqrt(-x^2 + 2*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{1-x}}{\sqrt{2x-x^2}}\,dx = \int \frac{\sqrt{1-x}}{\sqrt{2\,x-x^2}}\,dx$$

input
```
int((1 - x)^(1/2)/(2*x - x^2)^(1/2),x)
```

output
```
int((1 - x)^(1/2)/(2*x - x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\sqrt{1-x}}{\sqrt{2x-x^2}}\,dx = -\left(\int \frac{\sqrt{1-x}\,\sqrt{-x+2}}{\sqrt{x}\,x-2\sqrt{x}}dx\right)$$

input
```
int((1-x)^(1/2)/(-x^2+2*x)^(1/2),x)
```

output
```
- int((sqrt( - x + 1)*sqrt( - x + 2))/(sqrt(x)*x - 2*sqrt(x)),x)
```

3.238 $\int (d+ex)^m \left(bx+cx^2\right)^3 \, dx$

Optimal result . . . 1970
Mathematica [A] (verified) . . . 1971
Rubi [A] (verified) . . . 1971
Maple [B] (verified) . . . 1972
Fricas [B] (verification not implemented) . . . 1973
Sympy [B] (verification not implemented) . . . 1974
Maxima [B] (verification not implemented) . . . 1975
Giac [B] (verification not implemented) . . . 1976
Mupad [B] (verification not implemented) . . . 1977
Reduce [B] (verification not implemented) . . . 1978

Optimal result

Integrand size = 19, antiderivative size = 267

$$
\begin{aligned}
\int (d+ex)^m \left(bx+cx^2\right)^3 \, dx = {} & \frac{d^3(cd-be)^3(d+ex)^{1+m}}{e^7(1+m)} \\
& - \frac{3d^2(cd-be)^2(2cd-be)(d+ex)^{2+m}}{e^7(2+m)} \\
& + \frac{3d(cd-be)\left(5c^2d^2-5bcde+b^2e^2\right)(d+ex)^{3+m}}{e^7(3+m)} \\
& - \frac{(2cd-be)\left(10c^2d^2-10bcde+b^2e^2\right)(d+ex)^{4+m}}{e^7(4+m)} \\
& + \frac{3c(5c^2d^2-5bcde+b^2e^2)(d+ex)^{5+m}}{e^7(5+m)} \\
& - \frac{3c^2(2cd-be)(d+ex)^{6+m}}{e^7(6+m)} + \frac{c^3(d+ex)^{7+m}}{e^7(7+m)}
\end{aligned}
$$

output

```
d^3*(-b*e+c*d)^3*(e*x+d)^(1+m)/e^7/(1+m)-3*d^2*(-b*e+c*d)^2*(-b*e+2*c*d)*(
e*x+d)^(2+m)/e^7/(2+m)+3*d*(-b*e+c*d)*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d
)^(3+m)/e^7/(3+m)-(-b*e+2*c*d)*(b^2*e^2-10*b*c*d*e+10*c^2*d^2)*(e*x+d)^(4+
m)/e^7/(4+m)+3*c*(b^2*e^2-5*b*c*d*e+5*c^2*d^2)*(e*x+d)^(5+m)/e^7/(5+m)-3*c
^2*(-b*e+2*c*d)*(e*x+d)^(6+m)/e^7/(6+m)+c^3*(e*x+d)^(7+m)/e^7/(7+m)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.88

$$\int (d+ex)^m \left(bx+cx^2\right)^3 \, dx$$
$$= \frac{(d+ex)^{1+m} \left(\frac{d^3(cd-be)^3}{1+m} - \frac{3d^2(cd-be)^2(2cd-be)(d+ex)}{2+m} + \frac{3d(cd-be)(5c^2d^2-5bcde+b^2e^2)(d+ex)^2}{3+m} - \frac{(2cd-be)(10c^2d^2-10bcde+b}{4+m}\right.}{e^7}$$

input
```
Integrate[(d + e*x)^m*(b*x + c*x^2)^3,x]
```

output
```
((d + e*x)^(1 + m)*((d^3*(c*d - b*e)^3)/(1 + m) - (3*d^2*(c*d - b*e)^2*(2*
c*d - b*e)*(d + e*x))/(2 + m) + (3*d*(c*d - b*e)*(5*c^2*d^2 - 5*b*c*d*e +
b^2*e^2)*(d + e*x)^2)/(3 + m) - ((2*c*d - b*e)*(10*c^2*d^2 - 10*b*c*d*e +
b^2*e^2)*(d + e*x)^3)/(4 + m) + (3*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d
+ e*x)^4)/(5 + m) - (3*c^2*(2*c*d - b*e)*(d + e*x)^5)/(6 + m) + (c^3*(d +
e*x)^6)/(7 + m)))/e^7
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^3 (d+ex)^m \, dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)(d+ex)^{m+2}}{e^6} + \frac{(2cd-be)\left(-b^2e^2+10bcde-10c^2d^2\right)(d+ex)^{m+3}}{e^6} + \frac{3c}{}\right.$$

$$\Big\downarrow \text{2009}$$

$$
\frac{3d(cd-be)\left(b^2e^2-5bcde+5c^2d^2\right)(d+ex)^{m+3}}{e^7(m+3)}-\frac{(2cd-be)\left(b^2e^2-10bcde+10c^2d^2\right)(d+ex)^{m+4}}{e^7(m+4)}+\frac{3c\left(b^2e^2-5bcde+5c^2d^2\right)(d+ex)^{m+5}}{e^7(m+5)}-\frac{3c^2(2cd-be)(d+ex)^{m+6}}{e^7(m+6)}+\frac{d^3(cd-be)^3(d+ex)^{m+1}}{e^7(m+1)}-\frac{3d^2(cd-be)^2(2cd-be)(d+ex)^{m+2}}{e^7(m+2)}+\frac{c^3(d+ex)^{m+7}}{e^7(m+7)}
$$

input
```
Int[(d + e*x)^m*(b*x + c*x^2)^3,x]
```

output
```
(d^3*(c*d - b*e)^3*(d + e*x)^(1 + m))/(e^7*(1 + m)) - (3*d^2*(c*d - b*e)^2
*(2*c*d - b*e)*(d + e*x)^(2 + m))/(e^7*(2 + m)) + (3*d*(c*d - b*e)*(5*c^2*
d^2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^(3 + m))/(e^7*(3 + m)) - ((2*c*d - b*
e)*(10*c^2*d^2 - 10*b*c*d*e + b^2*e^2)*(d + e*x)^(4 + m))/(e^7*(4 + m)) +
(3*c*(5*c^2*d^2 - 5*b*c*d*e + b^2*e^2)*(d + e*x)^(5 + m))/(e^7*(5 + m)) -
(3*c^2*(2*c*d - b*e)*(d + e*x)^(6 + m))/(e^7*(6 + m)) + (c^3*(d + e*x)^(7
+ m))/(e^7*(7 + m))
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(267) = 534$.

Time = 0.81 (sec) , antiderivative size = 904, normalized size of antiderivative = 3.39

method	result
norman	$\frac{c^3x^7e^{m\ln(ex+d)}}{7+m}+\frac{(b^3e^3m^3+3b^2cd\,e^2m^3+18b^3e^3m^2+39b^2cd\,e^2m^2-15b\,c^2d^2e\,m^2+107b^3e^3m+126b^2cd\,e^2m-105b\,c^2d^2em}{e^3(m^4+22m^3+179m^2+638m+840)}$
gosper	Expression too large to display
orering	Expression too large to display
risch	Expression too large to display
parallelrisch	Expression too large to display

input
```
int((e*x+d)^m*(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
c^3/(7+m)*x^7*exp(m*ln(e*x+d))+(b^3*e^3*m^3+3*b^2*c*d*e^2*m^3+18*b^3*e^3*m
^2+39*b^2*c*d*e^2*m^2-15*b*c^2*d^2*e*m^2+107*b^3*e^3*m+126*b^2*c*d*e^2*m-1
05*b*c^2*d^2*e*m+30*c^3*d^3*m+210*b^3*e^3)/e^3/(m^4+22*m^3+179*m^2+638*m+8
40)*x^4*exp(m*ln(e*x+d))+c^2*(3*b*e*m+c*d*m+21*b*e)/e/(m^2+13*m+42)*x^6*ex
p(m*ln(e*x+d))+m*d*(b^3*e^3*m^3+18*b^3*e^3*m^2-12*b^2*c*d*e^2*m^2+107*b^3*
e^3*m-156*b^2*c*d*e^2*m+60*b*c^2*d^2*e*m+210*b^3*e^3-504*b^2*c*d*e^2+420*b
*c^2*d^2*e-120*c^3*d^3)/e^4/(m^5+25*m^4+245*m^3+1175*m^2+2754*m+2520)*x^3*
exp(m*ln(e*x+d))-6*d^4*(b^3*e^3*m^3+18*b^3*e^3*m^2-12*b^2*c*d*e^2*m^2+107*
b^3*e^3*m-156*b^2*c*d*e^2*m+60*b*c^2*d^2*e*m+210*b^3*e^3-504*b^2*c*d*e^2+4
20*b*c^2*d^2*e-120*c^3*d^3)/e^7/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+1313
2*m^2+13068*m+5040)*exp(m*ln(e*x+d))+3*(b^2*e^2*m^2+b*c*d*e*m^2+13*b^2*e^2
*m+7*b*c*d*e*m-2*c^2*d^2*m+42*b^2*e^2)/e^2*c/(m^3+18*m^2+107*m+210)*x^5*ex
p(m*ln(e*x+d))+6/e^6*m*d^3*(b^3*e^3*m^3+18*b^3*e^3*m^2-12*b^2*c*d*e^2*m^2+
107*b^3*e^3*m-156*b^2*c*d*e^2*m+60*b*c^2*d^2*e*m+210*b^3*e^3-504*b^2*c*d*e
^2+420*b*c^2*d^2*e-120*c^3*d^3)/(m^7+28*m^6+322*m^5+1960*m^4+6769*m^3+1313
2*m^2+13068*m+5040)*x*exp(m*ln(e*x+d))-3*(b^3*e^3*m^3+18*b^3*e^3*m^2-12*b^
2*c*d*e^2*m^2+107*b^3*e^3*m-156*b^2*c*d*e^2*m+60*b*c^2*d^2*e*m+210*b^3*e^3
-504*b^2*c*d*e^2+420*b*c^2*d^2*e-120*c^3*d^3)*d^2/e^5*m/(m^6+27*m^5+295*m^
4+1665*m^3+5104*m^2+8028*m+5040)*x^2*exp(m*ln(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1449 vs. $2(267) = 534$.

Time = 0.11 (sec) , antiderivative size = 1449, normalized size of antiderivative = 5.43

$$\int (d+ex)^m \left(bx+cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
-(6*b^3*d^4*e^3*m^3 - 720*c^3*d^7 + 2520*b*c^2*d^6*e - 3024*b^2*c*d^5*e^2
+ 1260*b^3*d^4*e^3 - (c^3*e^7*m^6 + 21*c^3*e^7*m^5 + 175*c^3*e^7*m^4 + 735
*c^3*e^7*m^3 + 1624*c^3*e^7*m^2 + 1764*c^3*e^7*m + 720*c^3*e^7)*x^7 - (252
0*b*c^2*e^7 + (c^3*d*e^6 + 3*b*c^2*e^7)*m^6 + 3*(5*c^3*d*e^6 + 22*b*c^2*e^
7)*m^5 + 5*(17*c^3*d*e^6 + 114*b*c^2*e^7)*m^4 + 15*(15*c^3*d*e^6 + 164*b*c
^2*e^7)*m^3 + (274*c^3*d*e^6 + 5547*b*c^2*e^7)*m^2 + 6*(20*c^3*d*e^6 + 101
9*b*c^2*e^7)*m)*x^6 - 3*(1008*b^2*c*e^7 + (b*c^2*d*e^6 + b^2*c*e^7)*m^6 -
(2*c^3*d^2*e^5 - 17*b*c^2*d*e^6 - 23*b^2*c*e^7)*m^5 - (20*c^3*d^2*e^5 - 10
5*b*c^2*d*e^6 - 207*b^2*c*e^7)*m^4 - 5*(14*c^3*d^2*e^5 - 59*b*c^2*d*e^6 -
185*b^2*c*e^7)*m^3 - 2*(50*c^3*d^2*e^5 - 187*b*c^2*d*e^6 - 1072*b^2*c*e^7)
*m^2 - 12*(4*c^3*d^2*e^5 - 14*b*c^2*d*e^6 - 201*b^2*c*e^7)*m)*x^5 - (1260*
b^3*e^7 + (3*b^2*c*d*e^6 + b^3*e^7)*m^6 - 3*(5*b*c^2*d^2*e^5 - 19*b^2*c*d*
e^6 - 8*b^3*e^7)*m^5 + (30*c^3*d^3*e^4 - 195*b*c^2*d^2*e^5 + 393*b^2*c*d*e
^6 + 226*b^3*e^7)*m^4 + 3*(60*c^3*d^3*e^4 - 265*b*c^2*d^2*e^5 + 401*b^2*c*
d*e^6 + 352*b^3*e^7)*m^3 + 5*(66*c^3*d^3*e^4 - 249*b*c^2*d^2*e^5 + 324*b^2
*c*d*e^6 + 509*b^3*e^7)*m^2 + 18*(10*c^3*d^3*e^4 - 35*b*c^2*d^2*e^5 + 42*b
^2*c*d*e^6 + 164*b^3*e^7)*m)*x^4 - (b^3*d*e^6*m^6 - 3*(4*b^2*c*d^2*e^5 - 7
*b^3*d*e^6)*m^5 + (60*b*c^2*d^3*e^4 - 192*b^2*c*d^2*e^5 + 163*b^3*d*e^6)*m
^4 - 3*(40*c^3*d^4*e^3 - 200*b*c^2*d^3*e^4 + 332*b^2*c*d^2*e^5 - 189*b^3*d
*e^6)*m^3 - 4*(90*c^3*d^4*e^3 - 345*b*c^2*d^3*e^4 + 456*b^2*c*d^2*e^5 -...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21005 vs. $2(250) = 500$.

Time = 4.14 (sec) , antiderivative size = 21005, normalized size of antiderivative = 78.67

$$\int (d+ex)^m \left(bx+cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x)**3,x)
```

output
```
Piecewise((d**m*(b**3*x**4/4 + 3*b**2*c*x**5/5 + b*c**2*x**6/2 + c**3*x**7
/7), Eq(e, 0)), (-b**3*d**3*e**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**
4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**
5 + 60*e**13*x**6) - 6*b**3*d**2*e**4*x/(60*d**6*e**7 + 360*d**5*e**8*x +
900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e*
*12*x**5 + 60*e**13*x**6) - 15*b**3*d*e**5*x**2/(60*d**6*e**7 + 360*d**5*e
**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 +
360*d*e**12*x**5 + 60*e**13*x**6) - 20*b**3*e**6*x**3/(60*d**6*e**7 + 360*
d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x
**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 6*b**2*c*d**4*e**2/(60*d**6*e**7
 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*
e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 36*b**2*c*d**3*e**3*x/(60
*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3*e**10*x**3 +
 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 90*b**2*c*d**2*
e**4*x**2/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2 + 1200*d**3
*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13*x**6) - 12
0*b**2*c*d*e**5*x**3/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e**9*x**2
+ 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5 + 60*e**13
*x**6) - 90*b**2*c*e**6*x**4/(60*d**6*e**7 + 360*d**5*e**8*x + 900*d**4*e*
*9*x**2 + 1200*d**3*e**10*x**3 + 900*d**2*e**11*x**4 + 360*d*e**12*x**5...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(267) = 534$.

Time = 0.05 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.51

$$\int (d+ex)^m \left(bx+cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
((m^3 + 6*m^2 + 11*m + 6)*e^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2
 + m)*d^2*e^2*x^2 + 6*d^3*e*m*x - 6*d^4)*(e*x + d)^m*b^3/((m^4 + 10*m^3 +
35*m^2 + 50*m + 24)*e^4) + 3*((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5
+ (m^4 + 6*m^3 + 11*m^2 + 6*m)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x
^3 + 12*(m^2 + m)*d^3*e^2*x^2 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*b^2*c/(
(m^5 + 15*m^4 + 85*m^3 + 225*m^2 + 274*m + 120)*e^5) + 3*((m^5 + 15*m^4 +
85*m^3 + 225*m^2 + 274*m + 120)*e^6*x^6 + (m^5 + 10*m^4 + 35*m^3 + 50*m^2
+ 24*m)*d*e^5*x^5 - 5*(m^4 + 6*m^3 + 11*m^2 + 6*m)*d^2*e^4*x^4 + 20*(m^3 +
 3*m^2 + 2*m)*d^3*e^3*x^3 - 60*(m^2 + m)*d^4*e^2*x^2 + 120*d^5*e*m*x - 120
*d^6)*(e*x + d)^m*b*c^2/((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 17
64*m + 720)*e^6) + ((m^6 + 21*m^5 + 175*m^4 + 735*m^3 + 1624*m^2 + 1764*m
+ 720)*e^7*x^7 + (m^6 + 15*m^5 + 85*m^4 + 225*m^3 + 274*m^2 + 120*m)*d*e^6
*x^6 - 6*(m^5 + 10*m^4 + 35*m^3 + 50*m^2 + 24*m)*d^2*e^5*x^5 + 30*(m^4 + 6
*m^3 + 11*m^2 + 6*m)*d^3*e^4*x^4 - 120*(m^3 + 3*m^2 + 2*m)*d^4*e^3*x^3 + 3
60*(m^2 + m)*d^5*e^2*x^2 - 720*d^6*e*m*x + 720*d^7)*(e*x + d)^m*c^3/((m^7
+ 28*m^6 + 322*m^5 + 1960*m^4 + 6769*m^3 + 13132*m^2 + 13068*m + 5040)*e^7
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2539 vs. $2(267) = 534$.

Time = 0.15 (sec) , antiderivative size = 2539, normalized size of antiderivative = 9.51

$$\int (d+ex)^m \left(bx+cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
((e*x + d)^m*c^3*e^7*m^6*x^7 + (e*x + d)^m*c^3*d*e^6*m^6*x^6 + 3*(e*x + d)
^m*b*c^2*e^7*m^6*x^6 + 21*(e*x + d)^m*c^3*e^7*m^5*x^7 + 3*(e*x + d)^m*b*c^
2*d*e^6*m^6*x^5 + 3*(e*x + d)^m*b^2*c*e^7*m^6*x^5 + 15*(e*x + d)^m*c^3*d*e
^6*m^5*x^6 + 66*(e*x + d)^m*b*c^2*e^7*m^5*x^6 + 175*(e*x + d)^m*c^3*e^7*m^
4*x^7 + 3*(e*x + d)^m*b^2*c*d*e^6*m^6*x^4 + (e*x + d)^m*b^3*e^7*m^6*x^4 -
6*(e*x + d)^m*c^3*d^2*e^5*m^5*x^5 + 51*(e*x + d)^m*b*c^2*d*e^6*m^5*x^5 + 6
9*(e*x + d)^m*b^2*c*e^7*m^5*x^5 + 85*(e*x + d)^m*c^3*d*e^6*m^4*x^6 + 570*(
e*x + d)^m*b*c^2*e^7*m^4*x^6 + 735*(e*x + d)^m*c^3*e^7*m^3*x^7 + (e*x + d)
^m*b^3*d*e^6*m^6*x^3 - 15*(e*x + d)^m*b*c^2*d^2*e^5*m^5*x^4 + 57*(e*x + d)
^m*b^2*c*d*e^6*m^5*x^4 + 24*(e*x + d)^m*b^3*e^7*m^5*x^4 - 60*(e*x + d)^m*c
^3*d^2*e^5*m^4*x^5 + 315*(e*x + d)^m*b*c^2*d*e^6*m^4*x^5 + 621*(e*x + d)^m
*b^2*c*e^7*m^4*x^5 + 225*(e*x + d)^m*c^3*d*e^6*m^3*x^6 + 2460*(e*x + d)^m*
b*c^2*e^7*m^3*x^6 + 1624*(e*x + d)^m*c^3*e^7*m^2*x^7 - 12*(e*x + d)^m*b^2*
c*d^2*e^5*m^5*x^3 + 21*(e*x + d)^m*b^3*d*e^6*m^5*x^3 + 30*(e*x + d)^m*c^3*
d^3*e^4*m^4*x^4 - 195*(e*x + d)^m*b*c^2*d^2*e^5*m^4*x^4 + 393*(e*x + d)^m*
b^2*c*d*e^6*m^4*x^4 + 226*(e*x + d)^m*b^3*e^7*m^4*x^4 - 210*(e*x + d)^m*c^
3*d^2*e^5*m^3*x^5 + 885*(e*x + d)^m*b*c^2*d*e^6*m^3*x^5 + 2775*(e*x + d)^m
*b^2*c*e^7*m^3*x^5 + 274*(e*x + d)^m*c^3*d*e^6*m^2*x^6 + 5547*(e*x + d)^m*
b*c^2*e^7*m^2*x^6 + 1764*(e*x + d)^m*c^3*e^7*m*x^7 - 3*(e*x + d)^m*b^3*d^2
*e^5*m^5*x^2 + 60*(e*x + d)^m*b*c^2*d^3*e^4*m^4*x^3 - 192*(e*x + d)^m*b...
```

Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 1085, normalized size of antiderivative = 4.06

$$\int (d+ex)^m \left(bx+cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
int((b*x + c*x^2)^3*(d + e*x)^m,x)
```

output
```
(c^3*x^7*(d + e*x)^m*(1764*m + 1624*m^2 + 735*m^3 + 175*m^4 + 21*m^5 + m^6
 + 720))/(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m
^7 + 5040) - (6*d^4*(d + e*x)^m*(210*b^3*e^3 - 120*c^3*d^3 + 107*b^3*e^3*m
 + 18*b^3*e^3*m^2 + b^3*e^3*m^3 + 420*b*c^2*d^2*e - 504*b^2*c*d*e^2 + 60*b
*c^2*d^2*e*m - 156*b^2*c*d*e^2*m - 12*b^2*c*d*e^2*m^2))/(e^7*(13068*m + 13
132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (x^4*(d
+ e*x)^m*(11*m + 6*m^2 + m^3 + 6)*(210*b^3*e^3 + 107*b^3*e^3*m + 30*c^3*d^
3*m + 18*b^3*e^3*m^2 + b^3*e^3*m^3 - 105*b*c^2*d^2*e*m + 126*b^2*c*d*e^2*m
 - 15*b*c^2*d^2*e*m^2 + 39*b^2*c*d*e^2*m^2 + 3*b^2*c*d*e^2*m^3))/(e^3*(130
68*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) +
 (3*c*x^5*(d + e*x)^m*(50*m + 35*m^2 + 10*m^3 + m^4 + 24)*(42*b^2*e^2 + 13
*b^2*e^2*m - 2*c^2*d^2*m + b^2*e^2*m^2 + 7*b*c*d*e*m + b*c*d*e*m^2))/(e^2*
(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040
)) + (6*d^3*m*x*(d + e*x)^m*(210*b^3*e^3 - 120*c^3*d^3 + 107*b^3*e^3*m + 1
8*b^3*e^3*m^2 + b^3*e^3*m^3 + 420*b*c^2*d^2*e - 504*b^2*c*d*e^2 + 60*b*c^2
*d^2*e*m - 156*b^2*c*d*e^2*m - 12*b^2*c*d*e^2*m^2))/(e^6*(13068*m + 13132*
m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^6 + m^7 + 5040)) + (c^2*x^6*(d
+ e*x)^m*(21*b*e + 3*b*e*m + c*d*m)*(274*m + 225*m^2 + 85*m^3 + 15*m^4 + m
^5 + 120))/(e*(13068*m + 13132*m^2 + 6769*m^3 + 1960*m^4 + 322*m^5 + 28*m^
6 + m^7 + 5040)) + (d*m*x^3*(d + e*x)^m*(3*m + m^2 + 2)*(210*b^3*e^3 - ...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1739, normalized size of antiderivative = 6.51

$$\int (d+ex)^m \left(bx+cx^2\right)^3 \, dx = \text{Too large to display}$$

input
```
int((e*x+d)^m*(c*x^2+b*x)^3,x)
```

output

```
((d + e*x)**m*( - 6*b**3*d**4*e**3*m**3 - 108*b**3*d**4*e**3*m**2 - 642*b*
*3*d**4*e**3*m - 1260*b**3*d**4*e**3 + 6*b**3*d**3*e**4*m**4*x + 108*b**3*
d**3*e**4*m**3*x + 642*b**3*d**3*e**4*m**2*x + 1260*b**3*d**3*e**4*m*x - 3
*b**3*d**2*e**5*m**5*x**2 - 57*b**3*d**2*e**5*m**4*x**2 - 375*b**3*d**2*e*
*5*m**3*x**2 - 951*b**3*d**2*e**5*m**2*x**2 - 630*b**3*d**2*e**5*m*x**2 +
b**3*d*e**6*m**6*x**3 + 21*b**3*d*e**6*m**5*x**3 + 163*b**3*d*e**6*m**4*x*
*3 + 567*b**3*d*e**6*m**3*x**3 + 844*b**3*d*e**6*m**2*x**3 + 420*b**3*d*e*
*6*m*x**3 + b**3*e**7*m**6*x**4 + 24*b**3*e**7*m**5*x**4 + 226*b**3*e**7*m
**4*x**4 + 1056*b**3*e**7*m**3*x**4 + 2545*b**3*e**7*m**2*x**4 + 2952*b**3
*e**7*m*x**4 + 1260*b**3*e**7*x**4 + 72*b**2*c*d**5*e**2*m**2 + 936*b**2*c
*d**5*e**2*m + 3024*b**2*c*d**5*e**2 - 72*b**2*c*d**4*e**3*m**3*x - 936*b*
*2*c*d**4*e**3*m**2*x - 3024*b**2*c*d**4*e**3*m*x + 36*b**2*c*d**3*e**4*m*
*4*x**2 + 504*b**2*c*d**3*e**4*m**3*x**2 + 1980*b**2*c*d**3*e**4*m**2*x**2
 + 1512*b**2*c*d**3*e**4*m*x**2 - 12*b**2*c*d**2*e**5*m**5*x**3 - 192*b**2
*c*d**2*e**5*m**4*x**3 - 996*b**2*c*d**2*e**5*m**3*x**3 - 1824*b**2*c*d**2
*e**5*m**2*x**3 - 1008*b**2*c*d**2*e**5*m*x**3 + 3*b**2*c*d*e**6*m**6*x**4
 + 57*b**2*c*d*e**6*m**5*x**4 + 393*b**2*c*d*e**6*m**4*x**4 + 1203*b**2*c*
d*e**6*m**3*x**4 + 1620*b**2*c*d*e**6*m**2*x**4 + 756*b**2*c*d*e**6*m*x**4
 + 3*b**2*c*e**7*m**6*x**5 + 69*b**2*c*e**7*m**5*x**5 + 621*b**2*c*e**7*m*
*4*x**5 + 2775*b**2*c*e**7*m**3*x**5 + 6432*b**2*c*e**7*m**2*x**5 + 723...
```

3.239 $\int (d+ex)^m \left(bx+cx^2\right)^2 \, dx$

Optimal result 1980
Mathematica [A] (verified) 1981
Rubi [A] (verified) 1981
Maple [B] (verified) 1982
Fricas [B] (verification not implemented) 1983
Sympy [B] (verification not implemented) 1984
Maxima [A] (verification not implemented) 1985
Giac [B] (verification not implemented) 1985
Mupad [B] (verification not implemented) 1986
Reduce [B] (verification not implemented) 1987

Optimal result

Integrand size = 19, antiderivative size = 159

$$\begin{aligned}\int (d+ex)^m \left(bx+cx^2\right)^2 \, dx = {} & \frac{d^2(cd-be)^2(d+ex)^{1+m}}{e^5(1+m)} \\ & - \frac{2d(cd-be)(2cd-be)(d+ex)^{2+m}}{e^5(2+m)} \\ & + \frac{\left(6c^2d^2-6bcde+b^2e^2\right)(d+ex)^{3+m}}{e^5(3+m)} \\ & - \frac{2c(2cd-be)(d+ex)^{4+m}}{e^5(4+m)} + \frac{c^2(d+ex)^{5+m}}{e^5(5+m)}\end{aligned}$$

output
```
d^2*(-b*e+c*d)^2*(e*x+d)^(1+m)/e^5/(1+m)-2*d*(-b*e+c*d)*(-b*e+2*c*d)*(e*x+
d)^(2+m)/e^5/(2+m)+(b^2*e^2-6*b*c*d*e+6*c^2*d^2)*(e*x+d)^(3+m)/e^5/(3+m)-2
*c*(-b*e+2*c*d)*(e*x+d)^(4+m)/e^5/(4+m)+c^2*(e*x+d)^(5+m)/e^5/(5+m)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.87

$$\int (d+ex)^m \left(bx+cx^2\right)^2 \, dx = \frac{(d+ex)^{1+m} \left(\frac{d^2(cd-be)^2}{1+m} - \frac{2d(cd-be)(2cd-be)(d+ex)}{2+m} + \frac{(6c^2d^2-6bcde+b^2e^2)(d+ex)^2}{3+m} - \frac{2c(2cd-be)(d+ex)^3}{4+m} + \frac{c^2(d+ex)^4}{5+m}\right)}{e^5}$$

input
```
Integrate[(d + e*x)^m*(b*x + c*x^2)^2,x]
```

output
```
((d + e*x)^(1 + m)*((d^2*(c*d - b*e)^2)/(1 + m) - (2*d*(c*d - b*e)*(2*c*d
- b*e)*(d + e*x))/(2 + m) + ((6*c^2*d^2 - 6*b*c*d*e + b^2*e^2)*(d + e*x)^2
)/(3 + m) - (2*c*(2*c*d - b*e)*(d + e*x)^3)/(4 + m) + (c^2*(d + e*x)^4)/(5
 + m)))/e^5
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^2 (d+ex)^m \, dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(\frac{\left(b^2e^2-6bcde+6c^2d^2\right)(d+ex)^{m+2}}{e^4} + \frac{d^2(cd-be)^2(d+ex)^m}{e^4} + \frac{2d(cd-be)(be-2cd)(d+ex)^{m+1}}{e^4} - 2c(2c$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\left(b^2e^2-6bcde+6c^2d^2\right)(d+ex)^{m+3}}{e^5(m+3)} + \frac{d^2(cd-be)^2(d+ex)^{m+1}}{e^5(m+1)} - \frac{2d(cd-be)(2cd-be)(d+ex)^{m+2}}{e^5(m+2)} - \frac{2c(2cd-be)(d+ex)^{m+4}}{e^5(m+4)} + \frac{c^2(d+ex)^{m+5}}{e^5(m+5)}$$

input
```
Int[(d + e*x)^m*(b*x + c*x^2)^2,x]
```

output
```
(d^2*(c*d - b*e)^2*(d + e*x)^(1 + m))/(e^5*(1 + m)) - (2*d*(c*d - b*e)*(2*
c*d - b*e)*(d + e*x)^(2 + m))/(e^5*(2 + m)) + ((6*c^2*d^2 - 6*b*c*d*e + b^
2*e^2)*(d + e*x)^(3 + m))/(e^5*(3 + m)) - (2*c*(2*c*d - b*e)*(d + e*x)^(4
+ m))/(e^5*(4 + m)) + (c^2*(d + e*x)^(5 + m))/(e^5*(5 + m))
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(159) = 318$.

Time = 0.74 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.64

method	result
norman	$\frac{c^2x^5\mathrm{e}^{m\ln(ex+d)}}{5+m}+\frac{(b^2e^2m^2+2bcde\,m^2+9b^2e^2m+10bcdem-4c^2d^2m+20b^2e^2)x^3\mathrm{e}^{m\ln(ex+d)}}{e^2(m^3+12m^2+47m+60)}+\frac{(2bem+cdm+10be)c\,x^4\mathrm{e}^{}}{e(m^2+9m+20)}$
gosper	$\frac{(ex+d)^{1+m}(c^2e^4m^4x^4+2bc\,e^4m^4x^3+10c^2e^4m^3x^4+b^2e^4m^4x^2+22bc\,e^4m^3x^3-4c^2d\,e^3m^3x^3+35c^2e^4m^2x^4+12b^2e^4m^3x^2-6}{}$
orering	$\frac{(c^2e^4m^4x^4+2bc\,e^4m^4x^3+10c^2e^4m^3x^4+b^2e^4m^4x^2+22bc\,e^4m^3x^3-4c^2d\,e^3m^3x^3+35c^2e^4m^2x^4+12b^2e^4m^3x^2-6bcd\,e^3m^3x^2}{}$
risch	$\frac{(c^2e^5m^4x^5+2bc\,e^5m^4x^4+c^2d\,e^4m^4x^4+10c^2e^5m^3x^5+b^2e^5m^4x^3+2bcd\,e^4m^4x^3+22bc\,e^5m^3x^4+6c^2d\,e^4m^3x^4+35c^2e^5m^2x^5-}{}$
parallelrisch	Expression too large to display

input
```
int((e*x+d)^m*(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
c^2/(5+m)*x^5*exp(m*ln(e*x+d))+(b^2*e^2*m^2+2*b*c*d*e*m^2+9*b^2*e^2*m+10*b
*c*d*e*m-4*c^2*d^2*m+20*b^2*e^2)/e^2/(m^3+12*m^2+47*m+60)*x^3*exp(m*ln(e*x
+d))+(2*b*e*m+c*d*m+10*b*e)/e*c/(m^2+9*m+20)*x^4*exp(m*ln(e*x+d))+(b^2*e^2
*m^2+9*b^2*e^2*m-6*b*c*d*e*m+20*b^2*e^2-30*b*c*d*e+12*c^2*d^2)*d/e^3*m/(m^
4+14*m^3+71*m^2+154*m+120)*x^2*exp(m*ln(e*x+d))+2*d^3*(b^2*e^2*m^2+9*b^2*e
^2*m-6*b*c*d*e*m+20*b^2*e^2-30*b*c*d*e+12*c^2*d^2)/e^5/(m^5+15*m^4+85*m^3+
225*m^2+274*m+120)*exp(m*ln(e*x+d))-2/e^4*m*d^2*(b^2*e^2*m^2+9*b^2*e^2*m-6
*b*c*d*e*m+20*b^2*e^2-30*b*c*d*e+12*c^2*d^2)/(m^5+15*m^4+85*m^3+225*m^2+27
4*m+120)*x*exp(m*ln(e*x+d))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(159) = 318$.

Time = 0.10 (sec) , antiderivative size = 584, normalized size of antiderivative = 3.67

$$\int (d+ex)^m (bx+cx^2)^2 \, dx = \frac{(2 b^2 d^3 e^2 m^2 + 24 c^2 d^5 - 60 bcd^4 e + 40 b^2 d^3 e^2 + (c^2 e^5 m^4 + 10 c^2 e^5 m^3 + 35 c^2 e^5 m^2 + 50 c^2 e^5 m + 24 c^2 e^5) x}{}$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
(2*b^2*d^3*e^2*m^2 + 24*c^2*d^5 - 60*b*c*d^4*e + 40*b^2*d^3*e^2 + (c^2*e^5
*m^4 + 10*c^2*e^5*m^3 + 35*c^2*e^5*m^2 + 50*c^2*e^5*m + 24*c^2*e^5)*x^5 +
(60*b*c*e^5 + (c^2*d*e^4 + 2*b*c*e^5)*m^4 + 2*(3*c^2*d*e^4 + 11*b*c*e^5)*m
^3 + (11*c^2*d*e^4 + 82*b*c*e^5)*m^2 + 2*(3*c^2*d*e^4 + 61*b*c*e^5)*m)*x^4
 + (40*b^2*e^5 + (2*b*c*d*e^4 + b^2*e^5)*m^4 - 4*(c^2*d^2*e^3 - 4*b*c*d*e^
4 - 3*b^2*e^5)*m^3 - (12*c^2*d^2*e^3 - 34*b*c*d*e^4 - 49*b^2*e^5)*m^2 - 2*
(4*c^2*d^2*e^3 - 10*b*c*d*e^4 - 39*b^2*e^5)*m)*x^3 + (b^2*d*e^4*m^4 - 2*(3
*b*c*d^2*e^3 - 5*b^2*d*e^4)*m^3 + (12*c^2*d^3*e^2 - 36*b*c*d^2*e^3 + 29*b^
2*d*e^4)*m^2 + 2*(6*c^2*d^3*e^2 - 15*b*c*d^2*e^3 + 10*b^2*d*e^4)*m)*x^2 -
6*(2*b*c*d^4*e - 3*b^2*d^3*e^2)*m - 2*(b^2*d^2*e^3*m^3 - 3*(2*b*c*d^3*e^2
- 3*b^2*d^2*e^3)*m^2 + 2*(6*c^2*d^4*e - 15*b*c*d^3*e^2 + 10*b^2*d^2*e^3)*m
)*x)*(e*x + d)^m/(e^5*m^5 + 15*e^5*m^4 + 85*e^5*m^3 + 225*e^5*m^2 + 274*e^
5*m + 120*e^5)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6418 vs. $2(144) = 288$.

Time = 1.39 (sec) , antiderivative size = 6418, normalized size of antiderivative = 40.36

$$\int (d+ex)^m \left(bx+cx^2\right)^2 \, dx = \text{Too large to display}$$

input

```
integrate((e*x+d)**m*(c*x**2+b*x)**2,x)
```

output

```
Piecewise((d**m*(b**2*x**3/3 + b*c*x**4/2 + c**2*x**5/5), Eq(e, 0)), (-b**
2*d**2*e**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8
*x**3 + 12*e**9*x**4) - 4*b**2*d*e**3*x/(12*d**4*e**5 + 48*d**3*e**6*x + 7
2*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 6*b**2*e**4*x**2/(12*d
**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x
**4) - 6*b*c*d**3*e/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 4
8*d*e**8*x**3 + 12*e**9*x**4) - 24*b*c*d**2*e**2*x/(12*d**4*e**5 + 48*d**3
*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) - 36*b*c*d*e*
*3*x**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**
3 + 12*e**9*x**4) - 24*b*c*e**4*x**3/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d
**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 12*c**2*d**4*log(d/e + x)
/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*
e**9*x**4) + 25*c**2*d**4/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x*
*2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 48*c**2*d**3*e*x*log(d/e + x)/(12*d*
*4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x*
*4) + 88*c**2*d**3*e*x/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2
+ 48*d*e**8*x**3 + 12*e**9*x**4) + 72*c**2*d**2*e**2*x**2*log(d/e + x)/(12
*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*e**7*x**2 + 48*d*e**8*x**3 + 12*e**9
*x**4) + 108*c**2*d**2*e**2*x**2/(12*d**4*e**5 + 48*d**3*e**6*x + 72*d**2*
e**7*x**2 + 48*d*e**8*x**3 + 12*e**9*x**4) + 48*c**2*d*e**3*x**3*log(d/...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 318, normalized size of antiderivative = 2.00

$$\int (d+ex)^m \left(bx+cx^2\right)^2 \, dx$$
$$= \frac{((m^2+3\,m+2)e^3x^3+(m^2+m)de^2x^2-2\,d^2emx+2\,d^3)(ex+d)^m b^2}{(m^3+6\,m^2+11\,m+6)e^3}$$
$$+ \frac{2\,((m^3+6\,m^2+11\,m+6)e^4x^4+(m^3+3\,m^2+2\,m)de^3x^3-3\,(m^2+m)d^2e^2x^2+6\,d^3emx-6\,d^4)(ex}{(m^4+10\,m^3+35\,m^2+50\,m+24)e^4}$$
$$+ \frac{((m^4+10\,m^3+35\,m^2+50\,m+24)e^5x^5+(m^4+6\,m^3+11\,m^2+6\,m)de^4x^4-4\,(m^3+3\,m^2+2\,m)d}{(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+12}$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
((m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x
 + d)^m*b^2/((m^3 + 6*m^2 + 11*m + 6)*e^3) + 2*((m^3 + 6*m^2 + 11*m + 6)*e
^4*x^4 + (m^3 + 3*m^2 + 2*m)*d*e^3*x^3 - 3*(m^2 + m)*d^2*e^2*x^2 + 6*d^3*e
*m*x - 6*d^4)*(e*x + d)^m*b*c/((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^4) +
((m^4 + 10*m^3 + 35*m^2 + 50*m + 24)*e^5*x^5 + (m^4 + 6*m^3 + 11*m^2 + 6*m
)*d*e^4*x^4 - 4*(m^3 + 3*m^2 + 2*m)*d^2*e^3*x^3 + 12*(m^2 + m)*d^3*e^2*x^2
 - 24*d^4*e*m*x + 24*d^5)*(e*x + d)^m*c^2/((m^5 + 15*m^4 + 85*m^3 + 225*m^
2 + 274*m + 120)*e^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. 2(159) = 318.

Time = 0.13 (sec) , antiderivative size = 1001, normalized size of antiderivative = 6.30

$$\int (d+ex)^m \left(bx+cx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
((e*x + d)^m*c^2*e^5*m^4*x^5 + (e*x + d)^m*c^2*d*e^4*m^4*x^4 + 2*(e*x + d)
^m*b*c*e^5*m^4*x^4 + 10*(e*x + d)^m*c^2*e^5*m^3*x^5 + 2*(e*x + d)^m*b*c*d*
e^4*m^4*x^3 + (e*x + d)^m*b^2*e^5*m^4*x^3 + 6*(e*x + d)^m*c^2*d*e^4*m^3*x^
4 + 22*(e*x + d)^m*b*c*e^5*m^3*x^4 + 35*(e*x + d)^m*c^2*e^5*m^2*x^5 + (e*x
 + d)^m*b^2*d*e^4*m^4*x^2 - 4*(e*x + d)^m*c^2*d^2*e^3*m^3*x^3 + 16*(e*x +
d)^m*b*c*d*e^4*m^3*x^3 + 12*(e*x + d)^m*b^2*e^5*m^3*x^3 + 11*(e*x + d)^m*c
^2*d*e^4*m^2*x^4 + 82*(e*x + d)^m*b*c*e^5*m^2*x^4 + 50*(e*x + d)^m*c^2*e^5
*m*x^5 - 6*(e*x + d)^m*b*c*d^2*e^3*m^3*x^2 + 10*(e*x + d)^m*b^2*d*e^4*m^3*
x^2 - 12*(e*x + d)^m*c^2*d^2*e^3*m^2*x^3 + 34*(e*x + d)^m*b*c*d*e^4*m^2*x^
3 + 49*(e*x + d)^m*b^2*e^5*m^2*x^3 + 6*(e*x + d)^m*c^2*d*e^4*m*x^4 + 122*(
e*x + d)^m*b*c*e^5*m*x^4 + 24*(e*x + d)^m*c^2*e^5*x^5 - 2*(e*x + d)^m*b^2*
d^2*e^3*m^3*x + 12*(e*x + d)^m*c^2*d^3*e^2*m^2*x^2 - 36*(e*x + d)^m*b*c*d^
2*e^3*m^2*x^2 + 29*(e*x + d)^m*b^2*d*e^4*m^2*x^2 - 8*(e*x + d)^m*c^2*d^2*e
^3*m*x^3 + 20*(e*x + d)^m*b*c*d*e^4*m*x^3 + 78*(e*x + d)^m*b^2*e^5*m*x^3 +
 60*(e*x + d)^m*b*c*e^5*x^4 + 12*(e*x + d)^m*b*c*d^3*e^2*m^2*x - 18*(e*x +
 d)^m*b^2*d^2*e^3*m^2*x + 12*(e*x + d)^m*c^2*d^3*e^2*m*x^2 - 30*(e*x + d)^
m*b*c*d^2*e^3*m*x^2 + 20*(e*x + d)^m*b^2*d*e^4*m*x^2 + 40*(e*x + d)^m*b^2*
e^5*x^3 + 2*(e*x + d)^m*b^2*d^3*e^2*m^2 - 24*(e*x + d)^m*c^2*d^4*e*m*x + 6
0*(e*x + d)^m*b*c*d^3*e^2*m*x - 40*(e*x + d)^m*b^2*d^2*e^3*m*x - 12*(e*x +
 d)^m*b*c*d^4*e*m + 18*(e*x + d)^m*b^2*d^3*e^2*m + 24*(e*x + d)^m*c^2*d...
```

Mupad [B] (verification not implemented)

Time = 5.58 (sec) , antiderivative size = 464, normalized size of antiderivative = 2.92

$$\int (d+ex)^m \left(bx+cx^2\right)^2 \, dx = (d+e\,x)^m \left(\frac{c^2\,x^5\,\left(m^4+10\,m^3+35\,m^2+50\,m+24\right)}{m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120} + \frac{2\,d^3\,\left(b^2\,e^2\,m^2+9\,b^2\,e^2\,m+20\,b^2\,e^2-6\,b\,c\,d\,e\,m-30\,b\,c\,d\,e+12\,c^2\,d^2\right)}{e^5\,\left(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120\right)} + \frac{x^3\,\left(m^2+3\,m+2\right)\,\left(b^2\,e^2\,m^2+9\,b^2\,e^2\,m+20\,b^2\,e^2+2\,b\,c\,d\,e\,m^2+10\,b\,c\,d\,e\,m-4\,c^2\,d^2\,m\right)}{e^2\,\left(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120\right)} + \frac{c\,x^4\,\left(10\,b\,e+2\,b\,e\,m+c\,d\,m\right)\,\left(m^3+6\,m^2+11\,m+6\right)}{e\,\left(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120\right)} - \frac{2\,d^2\,m\,x\,\left(b^2\,e^2\,m^2+9\,b^2\,e^2\,m+20\,b^2\,e^2-6\,b\,c\,d\,e\,m-30\,b\,c\,d\,e+12\,c^2\,d^2\right)}{e^4\,\left(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120\right)} + \frac{d\,m\,x^2\,\left(m+1\right)\,\left(b^2\,e^2\,m^2+9\,b^2\,e^2\,m+20\,b^2\,e^2-6\,b\,c\,d\,e\,m-30\,b\,c\,d\,e+12\,c^2\,d^2\right)}{e^3\,\left(m^5+15\,m^4+85\,m^3+225\,m^2+274\,m+120\right)}\right)$$

input
```
int((b*x + c*x^2)^2*(d + e*x)^m,x)
```

output
```
(d + e*x)^m*((c^2*x^5*(50*m + 35*m^2 + 10*m^3 + m^4 + 24))/(274*m + 225*m^
2 + 85*m^3 + 15*m^4 + m^5 + 120) + (2*d^3*(20*b^2*e^2 + 12*c^2*d^2 + 9*b^2
*e^2*m + b^2*e^2*m^2 - 30*b*c*d*e - 6*b*c*d*e*m))/(e^5*(274*m + 225*m^2 +
85*m^3 + 15*m^4 + m^5 + 120)) + (x^3*(3*m + m^2 + 2)*(20*b^2*e^2 + 9*b^2*e
^2*m - 4*c^2*d^2*m + b^2*e^2*m^2 + 10*b*c*d*e*m + 2*b*c*d*e*m^2))/(e^2*(27
4*m + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 120)) + (c*x^4*(10*b*e + 2*b*e*m +
 c*d*m)*(11*m + 6*m^2 + m^3 + 6))/(e*(274*m + 225*m^2 + 85*m^3 + 15*m^4 +
m^5 + 120)) - (2*d^2*m*x*(20*b^2*e^2 + 12*c^2*d^2 + 9*b^2*e^2*m + b^2*e^2*
m^2 - 30*b*c*d*e - 6*b*c*d*e*m))/(e^4*(274*m + 225*m^2 + 85*m^3 + 15*m^4 +
 m^5 + 120)) + (d*m*x^2*(m + 1)*(20*b^2*e^2 + 12*c^2*d^2 + 9*b^2*e^2*m + b
^2*e^2*m^2 - 30*b*c*d*e - 6*b*c*d*e*m))/(e^3*(274*m + 225*m^2 + 85*m^3 + 1
5*m^4 + m^5 + 120)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 662, normalized size of antiderivative = 4.16

$$\int (d+ex)^m \left(bx+cx^2\right)^2 \, dx = \frac{(ex+d)^m \left(c^2e^5m^4x^5+2bc\,e^5m^4x^4+c^2d\,e^4m^4x^4+10c^2e^5m^3x^5+b^2e^5m^4x^3+2bcd\,e^4m^4x^3+22bc\,e^5m^3x^4 \cdots\right)}{\cdots}$$

input
```
int((e*x+d)^m*(c*x^2+b*x)^2,x)
```

output
```
((d + e*x)**m*(2*b**2*d**3*e**2*m**2 + 18*b**2*d**3*e**2*m + 40*b**2*d**3*
e**2 - 2*b**2*d**2*e**3*m**3*x - 18*b**2*d**2*e**3*m**2*x - 40*b**2*d**2*e
**3*m*x + b**2*d*e**4*m**4*x**2 + 10*b**2*d*e**4*m**3*x**2 + 29*b**2*d*e**
4*m**2*x**2 + 20*b**2*d*e**4*m*x**2 + b**2*e**5*m**4*x**3 + 12*b**2*e**5*m
**3*x**3 + 49*b**2*e**5*m**2*x**3 + 78*b**2*e**5*m*x**3 + 40*b**2*e**5*x**
3 - 12*b*c*d**4*e*m - 60*b*c*d**4*e + 12*b*c*d**3*e**2*m**2*x + 60*b*c*d**
3*e**2*m*x - 6*b*c*d**2*e**3*m**3*x**2 - 36*b*c*d**2*e**3*m**2*x**2 - 30*b
*c*d**2*e**3*m*x**2 + 2*b*c*d*e**4*m**4*x**3 + 16*b*c*d*e**4*m**3*x**3 + 3
4*b*c*d*e**4*m**2*x**3 + 20*b*c*d*e**4*m*x**3 + 2*b*c*e**5*m**4*x**4 + 22*
b*c*e**5*m**3*x**4 + 82*b*c*e**5*m**2*x**4 + 122*b*c*e**5*m*x**4 + 60*b*c*
e**5*x**4 + 24*c**2*d**5 - 24*c**2*d**4*e*m*x + 12*c**2*d**3*e**2*m**2*x**
2 + 12*c**2*d**3*e**2*m*x**2 - 4*c**2*d**2*e**3*m**3*x**3 - 12*c**2*d**2*e
**3*m**2*x**3 - 8*c**2*d**2*e**3*m*x**3 + c**2*d*e**4*m**4*x**4 + 6*c**2*d
*e**4*m**3*x**4 + 11*c**2*d*e**4*m**2*x**4 + 6*c**2*d*e**4*m*x**4 + c**2*e
**5*m**4*x**5 + 10*c**2*e**5*m**3*x**5 + 35*c**2*e**5*m**2*x**5 + 50*c**2*
e**5*m*x**5 + 24*c**2*e**5*x**5))/(e**5*(m**5 + 15*m**4 + 85*m**3 + 225*m*
*2 + 274*m + 120))
```

3.240 $\int (d+ex)^m (bx+cx^2) \, dx$

Optimal result 1989
Mathematica [A] (verified) 1989
Rubi [A] (verified) 1990
Maple [A] (verified) 1991
Fricas [B] (verification not implemented) 1991
Sympy [B] (verification not implemented) 1992
Maxima [A] (verification not implemented) 1993
Giac [B] (verification not implemented) 1993
Mupad [B] (verification not implemented) 1994
Reduce [B] (verification not implemented) 1994

Optimal result

Integrand size = 17, antiderivative size = 75

$$\int (d+ex)^m (bx+cx^2) \, dx = \frac{d(cd-be)(d+ex)^{1+m}}{e^3(1+m)} - \frac{(2cd-be)(d+ex)^{2+m}}{e^3(2+m)} + \frac{c(d+ex)^{3+m}}{e^3(3+m)}$$

output
```
d*(-b*e+c*d)*(e*x+d)^(1+m)/e^3/(1+m)-(-b*e+2*c*d)*(e*x+d)^(2+m)/e^3/(2+m)+
c*(e*x+d)^(3+m)/e^3/(3+m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int (d+ex)^m (bx+cx^2) \, dx = \frac{d(cd-be)(d+ex)^{1+m}}{e^3(1+m)} - \frac{(2cd-be)(d+ex)^{2+m}}{e^3(2+m)} + \frac{c(d+ex)^{3+m}}{e^3(3+m)}$$

input
```
Integrate[(d + e*x)^m*(b*x + c*x^2),x]
```

output
```
(d*(c*d - b*e)*(d + e*x)^(1 + m))/(e^3*(1 + m)) - ((2*c*d - b*e)*(d + e*x)
^(2 + m))/(e^3*(2 + m)) + (c*(d + e*x)^(3 + m))/(e^3*(3 + m))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)(d+ex)^m\,dx$$

$$\Big\downarrow \ 1140$$

$$\int \left(\frac{d(cd-be)(d+ex)^m}{e^2}+\frac{(be-2cd)(d+ex)^{m+1}}{e^2}+\frac{c(d+ex)^{m+2}}{e^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{d(cd-be)(d+ex)^{m+1}}{e^3(m+1)}-\frac{(2cd-be)(d+ex)^{m+2}}{e^3(m+2)}+\frac{c(d+ex)^{m+3}}{e^3(m+3)}$$

input
```
Int[(d + e*x)^m*(b*x + c*x^2),x]
```

output
```
(d*(c*d - b*e)*(d + e*x)^(1 + m))/(e^3*(1 + m)) - ((2*c*d - b*e)*(d + e*x)
^(2 + m))/(e^3*(2 + m)) + (c*(d + e*x)^(3 + m))/(e^3*(3 + m))
```


Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.55

method	result
gosper	$-\frac{(ex+d)^{1+m}\left(-c\,e^{2}m^{2}x^{2}-b\,e^{2}m^{2}x-3c\,e^{2}m\,x^{2}-4b\,e^{2}mx+2cdemx-2x^{2}c\,e^{2}+bdem-3xb\,e^{2}+2cdxe+3bde-2c\,d^{2}\right)}{e^{3}(m^{3}+6m^{2}+11m+6)}$
orering	$-\frac{\left(-c\,e^{2}m^{2}x^{2}-b\,e^{2}m^{2}x-3c\,e^{2}m\,x^{2}-4b\,e^{2}mx+2cdemx-2x^{2}c\,e^{2}+bdem-3xb\,e^{2}+2cdxe+3bde-2c\,d^{2}\right)(ex+d)(ex+d)^{m}(c\,x^{2}}{e^{3}(m^{3}+6m^{2}+11m+6)x(cx+b)}$
norman	$\frac{c\,x^{3}\mathrm{e}^{m\ln(ex+d)}}{3+m}+\frac{(bem+cdm+3be)x^{2}\mathrm{e}^{m\ln(ex+d)}}{e(m^{2}+5m+6)}+\frac{md(bem+3be-2cd)x\,\mathrm{e}^{m\ln(ex+d)}}{e^{2}(m^{3}+6m^{2}+11m+6)}-\frac{d^{2}(bem+3be-2cd)\mathrm{e}^{m\ln(ex+d)}}{e^{3}(m^{3}+6m^{2}+11m+6)}$
risch	$-\frac{\left(-c\,e^{3}m^{2}x^{3}-b\,e^{3}m^{2}x^{2}-cd\,e^{2}m^{2}x^{2}-3c\,e^{3}m\,x^{3}-bd\,e^{2}m^{2}x-4b\,e^{3}m\,x^{2}-cd\,e^{2}m\,x^{2}-2c\,e^{3}x^{3}-3bd\,e^{2}mx-3b\,e^{3}x^{2}+2c\,d^{2}em\right.}{(2+m)(3+m)(1+m)e^{3}}$
parallelrisch	$\frac{x^{3}(ex+d)^{m}c\,e^{3}m^{2}+3x^{3}(ex+d)^{m}c\,e^{3}m+x^{2}(ex+d)^{m}b\,e^{3}m^{2}+x^{2}(ex+d)^{m}cd\,e^{2}m^{2}+2x^{3}(ex+d)^{m}c\,e^{3}+4x^{2}(ex+d)^{m}b\,e^{3}m+x^{}}{}$

input
```
int((e*x+d)^m*(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-1/e^3*(e*x+d)^(1+m)/(m^3+6*m^2+11*m+6)*(-c*e^2*m^2*x^2-b*e^2*m^2*x-3*c*e^
2*m*x^2-4*b*e^2*m*x+2*c*d*e*m*x-2*c*e^2*x^2+b*d*e*m-3*b*e^2*x+2*c*d*e*x+3*
b*d*e-2*c*d^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(75) = 150$.

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.13

$$\int (d+ex)^m\left(bx+cx^2\right)\,dx = -\frac{(bd^2em-2\,cd^3+3\,bd^2e-(ce^3m^2+3\,ce^3m+2\,ce^3)x^3-(3\,be^3+(cde^2+be^3)m^2+(cde^2+4\,be^3)m)x}{e^3m^3+6\,e^3m^2+11\,e^3m+6\,e^3}$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x),x, algorithm="fricas")
```

output
```
-(b*d^2*e*m - 2*c*d^3 + 3*b*d^2*e - (c*e^3*m^2 + 3*c*e^3*m + 2*c*e^3)*x^3
- (3*b*e^3 + (c*d*e^2 + b*e^3)*m^2 + (c*d*e^2 + 4*b*e^3)*m)*x^2 - (b*d*e^2
*m^2 - (2*c*d^2*e - 3*b*d*e^2)*m)*x)*(e*x + d)^m/(e^3*m^3 + 6*e^3*m^2 + 11
*e^3*m + 6*e^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1095 vs. $2(63) = 126$.

Time = 0.50 (sec) , antiderivative size = 1095, normalized size of antiderivative = 14.60

$$\int (d+ex)^m \left(bx+cx^2\right) \, dx = \text{Too large to display}$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x),x)
```

output
```
Piecewise((d**m*(b*x**2/2 + c*x**3/3), Eq(e, 0)), (-b*d*e/(2*d**2*e**3 + 4
*d*e**4*x + 2*e**5*x**2) - 2*b*e**2*x/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x
**2) + 2*c*d**2*log(d/e + x)/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 3*
c*d**2/(2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 4*c*d*e*x*log(d/e + x)/(
2*d**2*e**3 + 4*d*e**4*x + 2*e**5*x**2) + 4*c*d*e*x/(2*d**2*e**3 + 4*d*e**
4*x + 2*e**5*x**2) + 2*c*e**2*x**2*log(d/e + x)/(2*d**2*e**3 + 4*d*e**4*x
+ 2*e**5*x**2), Eq(m, -3)), (b*d*e*log(d/e + x)/(d*e**3 + e**4*x) + b*d*e/
(d*e**3 + e**4*x) + b*e**2*x*log(d/e + x)/(d*e**3 + e**4*x) - 2*c*d**2*log
(d/e + x)/(d*e**3 + e**4*x) - 2*c*d**2/(d*e**3 + e**4*x) - 2*c*d*e*x*log(d
/e + x)/(d*e**3 + e**4*x) + c*e**2*x**2/(d*e**3 + e**4*x), Eq(m, -2)), (-b
*d*log(d/e + x)/e**2 + b*x/e + c*d**2*log(d/e + x)/e**3 - c*d*x/e**2 + c*x
**2/(2*e), Eq(m, -1)), (-b*d**2*e*m*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2
+ 11*e**3*m + 6*e**3) - 3*b*d**2*e*(d + e*x)**m/(e**3*m**3 + 6*e**3*m**2 +
 11*e**3*m + 6*e**3) + b*d*e**2*m**2*x*(d + e*x)**m/(e**3*m**3 + 6*e**3*m*
*2 + 11*e**3*m + 6*e**3) + 3*b*d*e**2*m*x*(d + e*x)**m/(e**3*m**3 + 6*e**3
*m**2 + 11*e**3*m + 6*e**3) + b*e**3*m**2*x**2*(d + e*x)**m/(e**3*m**3 + 6
*e**3*m**2 + 11*e**3*m + 6*e**3) + 4*b*e**3*m*x**2*(d + e*x)**m/(e**3*m**3
 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 3*b*e**3*x**2*(d + e*x)**m/(e**3*m*
*3 + 6*e**3*m**2 + 11*e**3*m + 6*e**3) + 2*c*d**3*(d + e*x)**m/(e**3*m**3
+ 6*e**3*m**2 + 11*e**3*m + 6*e**3) - 2*c*d**2*e*m*x*(d + e*x)**m/(e**3...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.51

$$\int (d+ex)^m \left(bx+cx^2\right) dx = \frac{(e^2(m+1)x^2 + demx - d^2)(ex+d)^m b}{(m^2+3m+2)e^2} + \frac{((m^2+3m+2)e^3x^3 + (m^2+m)de^2x^2 - 2d^2emx + 2d^3)(ex+d)^m c}{(m^3+6m^2+11m+6)e^3}$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x),x, algorithm="maxima")
```

output
```
(e^2*(m + 1)*x^2 + d*e*m*x - d^2)*(e*x + d)^m*b/((m^2 + 3*m + 2)*e^2) + ((
m^2 + 3*m + 2)*e^3*x^3 + (m^2 + m)*d*e^2*x^2 - 2*d^2*e*m*x + 2*d^3)*(e*x +
 d)^m*c/((m^3 + 6*m^2 + 11*m + 6)*e^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. 2(75) = 150.

Time = 0.20 (sec) , antiderivative size = 260, normalized size of antiderivative = 3.47

$$\int (d+ex)^m \left(bx+cx^2\right) dx = \frac{(ex+d)^m ce^3m^2x^3 + (ex+d)^m cde^2m^2x^2 + (ex+d)^m be^3m^2x^2 + 3(ex+d)^m ce^3mx^3 + (ex+d)^m bde^2m}{}$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x),x, algorithm="giac")
```

output
```
((e*x + d)^m*c*e^3*m^2*x^3 + (e*x + d)^m*c*d*e^2*m^2*x^2 + (e*x + d)^m*b*e
^3*m^2*x^2 + 3*(e*x + d)^m*c*e^3*m*x^3 + (e*x + d)^m*b*d*e^2*m^2*x + (e*x
+ d)^m*c*d*e^2*m*x^2 + 4*(e*x + d)^m*b*e^3*m*x^2 + 2*(e*x + d)^m*c*e^3*x^3
 - 2*(e*x + d)^m*c*d^2*e*m*x + 3*(e*x + d)^m*b*d*e^2*m*x + 3*(e*x + d)^m*b
*e^3*x^2 - (e*x + d)^m*b*d^2*e*m + 2*(e*x + d)^m*c*d^3 - 3*(e*x + d)^m*b*d
^2*e)/(e^3*m^3 + 6*e^3*m^2 + 11*e^3*m + 6*e^3)
```

Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.95

$$\int (d+ex)^m \left(bx+cx^2\right)\,dx = (d+e\,x)^m\left(\frac{c\,x^3\,\left(m^2+3\,m+2\right)}{m^3+6\,m^2+11\,m+6} - \frac{d^2\,\left(3\,b\,e-2\,c\,d+b\,e\,m\right)}{e^3\,\left(m^3+6\,m^2+11\,m+6\right)} + \frac{x^2\,\left(m+1\right)\,\left(3\,b\,e+b\,e\,m+c\,d\,m\right)}{e\,\left(m^3+6\,m^2+11\,m+6\right)} + \frac{d\,m\,x\,\left(3\,b\,e-2\,c\,d+b\,e\,m\right)}{e^2\,\left(m^3+6\,m^2+11\,m+6\right)}\right)$$

input
```
int((b*x + c*x^2)*(d + e*x)^m,x)
```

output
```
(d + e*x)^m*((c*x^3*(3*m + m^2 + 2))/(11*m + 6*m^2 + m^3 + 6) - (d^2*(3*b*
e - 2*c*d + b*e*m))/(e^3*(11*m + 6*m^2 + m^3 + 6)) + (x^2*(m + 1)*(3*b*e +
 b*e*m + c*d*m))/(e*(11*m + 6*m^2 + m^3 + 6)) + (d*m*x*(3*b*e - 2*c*d + b*
e*m))/(e^2*(11*m + 6*m^2 + m^3 + 6)))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.11

$$\int (d+ex)^m \left(bx+cx^2\right)\,dx = \frac{(ex+d)^m \left(c\,e^3m^2x^3+b\,e^3m^2x^2+cd\,e^2m^2x^2+3c\,e^3m\,x^3+bd\,e^2m^2x+4b\,e^3m\,x^2+cd\,e^2m\,x^2+2c\,e^3x^3\right.}{e^3\left(m^3+6m^2+11m+6\right)}$$

input
```
int((e*x+d)^m*(c*x^2+b*x),x)
```

output
```
((d + e*x)**m*( - b*d**2*e*m - 3*b*d**2*e + b*d*e**2*m**2*x + 3*b*d*e**2*m
*x + b*e**3*m**2*x**2 + 4*b*e**3*m*x**2 + 3*b*e**3*x**2 + 2*c*d**3 - 2*c*d
**2*e*m*x + c*d*e**2*m**2*x**2 + c*d*e**2*m*x**2 + c*e**3*m**2*x**3 + 3*c*
e**3*m*x**3 + 2*c*e**3*x**3))/(e**3*(m**3 + 6*m**2 + 11*m + 6))
```

3.241 $\int \frac{(d+ex)^m}{bx+cx^2}\,dx$

Optimal result . . . 1995
Mathematica [A] (verified) . . . 1995
Rubi [A] (verified) . . . 1996
Maple [F] . . . 1997
Fricas [F] . . . 1997
Sympy [F] . . . 1998
Maxima [F] . . . 1998
Giac [F] . . . 1998
Mupad [F(-1)] . . . 1999
Reduce [F] . . . 1999

Optimal result

Integrand size = 19, antiderivative size = 93

$$\int \frac{(d+ex)^m}{bx+cx^2}\,dx=\frac{c(d+ex)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{c(d+ex)}{cd-be}\right)}{b(cd-be)(1+m)}-\frac{(d+ex)^{1+m}\text{Hypergeometric2F1}\left(1,1+m,2+m,1+\frac{ex}{d}\right)}{bd(1+m)}$$

output
```
c*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],c*(e*x+d)/(-b*e+c*d))/b/(-b*e+c*d
)/(1+m)-(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],1+e*x/d)/b/d/(1+m)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^m}{bx+cx^2}\,dx=-\frac{(d+ex)^{1+m}\left(cd\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{c(d+ex)}{cd-be}\right)+(-cd+be)\,\text{Hypergeometric2F1}\left(1,\right.\right.}{bd(-cd+be)(1+m)}$$

input
```
Integrate[(d + e*x)^m/(b*x + c*x^2),x]
```

output
```
-(((d + e*x)^(1 + m)*(c*d*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(d + e*x))
/(c*d - b*e)] + (-(c*d) + b*e)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + (e*x
)/d]))/(b*d*(-(c*d) + b*e)*(1 + m)))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1150, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{bx+cx^2} \, dx$$

$$\Big\downarrow \text{1150}$$

$$\int \left(\frac{(d+ex)^m}{bx}-\frac{c(d+ex)^m}{b(b+cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{c(d+ex)}{cd-be}\right)}{b(m+1)(cd-be)}-\frac{(d+ex)^{m+1}\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{ex}{d}+1\right)}{bd(m+1)}$$

input
```
Int[(d + e*x)^m/(b*x + c*x^2),x]
```

output
```
(c*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(d + e*x))/(c*d
 - b*e)])/(b*(c*d - b*e)*(1 + m)) - ((d + e*x)^(1 + m)*Hypergeometric2F1[1
, 1 + m, 2 + m, 1 + (e*x)/d])/(b*d*(1 + m))
```

Defintions of rubi rules used

rule 1150
```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && !IntegerQ[2*m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex+d)^m}{c\,x^2+bx}dx$$

input
```
int((e*x+d)^m/(c*x^2+b*x),x)
```

output
```
int((e*x+d)^m/(c*x^2+b*x),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{bx+cx^2}\,dx=\int \frac{(ex+d)^m}{cx^2+bx}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m/(c*x^2 + b*x), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m}{bx+cx^2}\,dx=\int \frac{(d+ex)^m}{x\,(b+cx)}\,dx$$

input
```
integrate((e*x+d)**m/(c*x**2+b*x),x)
```

output
```
Integral((d + e*x)**m/(x*(b + c*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m}{bx+cx^2}\,dx=\int \frac{(ex+d)^m}{cx^2+bx}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/(c*x^2 + b*x), x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{bx+cx^2}\,dx=\int \frac{(ex+d)^m}{cx^2+bx}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/(c*x^2 + b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{bx+cx^2}\,dx=\int \frac{(d+e\,x)^m}{c\,x^2+b\,x}\,dx$$

input
```
int((d + e*x)^m/(b*x + c*x^2),x)
```

output
```
int((d + e*x)^m/(b*x + c*x^2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{bx+cx^2}\,dx=\int \frac{(ex+d)^m}{c\,x^2+bx}dx$$

input
```
int((e*x+d)^m/(c*x^2+b*x),x)
```

output
```
int((d + e*x)**m/(b*x + c*x**2),x)
```

3.242 $\int \frac{(d+ex)^m}{(bx+cx^2)^2} dx$

Optimal result 2000
Mathematica [A] (verified) 2001
Rubi [A] (verified) 2001
Maple [F] 2003
Fricas [F] 2003
Sympy [F] 2003
Maxima [F] 2004
Giac [F] 2004
Mupad [F(-1)] 2004
Reduce [F] 2005

Optimal result

Integrand size = 19, antiderivative size = 190

$$\int \frac{(d+ex)^m}{(bx+cx^2)^2} dx = -\frac{c(2cd-be)(d+ex)^{1+m}}{b^2d(cd-be)(b+cx)} - \frac{(d+ex)^{1+m}}{bdx(b+cx)} - \frac{c^2(2cd-be(2-m))(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\frac{c(d+ex)}{cd-be}\right)}{b^3(cd-be)^2(1+m)} + \frac{(2cd-bem)(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,1+\frac{ex}{d}\right)}{b^3d^2(1+m)}$$

output
```
-c*(-b*e+2*c*d)*(e*x+d)^(1+m)/b^2/d/(-b*e+c*d)/(c*x+b)-(e*x+d)^(1+m)/b/d/x
/(c*x+b)-c^2*(2*c*d-b*e*(2-m))*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],c*(e
*x+d)/(-b*e+c*d))/b^3/(-b*e+c*d)^2/(1+m)+(-b*e*m+2*c*d)*(e*x+d)^(1+m)*hype
rgeom([1, 1+m],[2+m],1+e*x/d)/b^3/d^2/(1+m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.92

$$\int \frac{(d+ex)^m}{(bx+cx^2)^2}\,dx = -\frac{(d+ex)^{1+m}\left(b^2d(cd-be)^2(1+m)+bcd(-2cd+be)(-cd+be)(1+m)x+x(b+cx)\left(c^2d^2(2cd+be\right.\right.}{}$$

input
```
Integrate[(d + e*x)^m/(b*x + c*x^2)^2,x]
```

output
```
-(((d + e*x)^(1 + m)*(b^2*d*(c*d - b*e)^2*(1 + m) + b*c*d*(-2*c*d + b*e)*(
-(c*d) + b*e)*(1 + m)*x + x*(b + c*x)*(c^2*d^2*(2*c*d + b*e*(-2 + m))*Hype
rgeometric2F1[1, 1 + m, 2 + m, (c*(d + e*x))/(c*d - b*e)] - (c*d - b*e)^2*
(2*c*d - b*e*m)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + (e*x)/d])))/(b^3*d^
2*(c*d - b*e)^2*(1 + m)*x*(b + c*x)))
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {1165, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{(bx+cx^2)^2}\,dx$$

$$\Big\downarrow \; 1165$$

$$-\frac{\int \frac{(d+ex)^m((cd-be)(2cd-bem)-ce(2cd-be)mx)}{cx^2+bx}dx}{b^2d(cd-be)}-\frac{(d+ex)^{m+1}(cx(2cd-be)+b(cd-be))}{b^2d\,(bx+cx^2)\,(cd-be)}$$

$$\Big\downarrow \; 1200$$

$$-\frac{\int \left(\frac{(be-cd)(bem-2cd)(d+ex)^m}{bx}+\frac{c^2d(be(2-m)-2cd)(d+ex)^m}{b(b+cx)}\right)dx}{b^2d(cd-be)}-\frac{(d+ex)^{m+1}(cx(2cd-be)+b(cd-be))}{b^2d\,(bx+cx^2)\,(cd-be)}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{c^2d(d+ex)^{m+1}(2cd-be(2-m))\,\text{Hypergeometric2F1}\left(1,m+1,m+2,\frac{c(d+ex)}{cd-be}\right)}{b(m+1)(cd-be)}-\frac{(cd-be)(d+ex)^{m+1}(2cd-bem)\,\text{Hypergeometric2F1}(1,m+1,}{bd(m+1)}}{b^2d(cd-be)}$$
$$\frac{(d+ex)^{m+1}(cx(2cd-be)+b(cd-be))}{b^2d\,(bx+cx^2)\,(cd-be)}$$

input
```
Int[(d + e*x)^m/(b*x + c*x^2)^2,x]
```

output
```
-(((d + e*x)^(1 + m)*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d - b*
e)*(b*x + c*x^2))) - ((c^2*d*(2*c*d - b*e*(2 - m))*(d + e*x)^(1 + m)*Hyper
geometric2F1[1, 1 + m, 2 + m, (c*(d + e*x))/(c*d - b*e)])/(b*(c*d - b*e)*(
1 + m)) - ((c*d - b*e)*(2*c*d - b*e*m)*(d + e*x)^(1 + m)*Hypergeometric2F1
[1, 1 + m, 2 + m, 1 + (e*x)/d])/(b*d*(1 + m)))/(b^2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex+d)^m}{(c\,x^2+bx)^2}dx$$

input
```
int((e*x+d)^m/(c*x^2+b*x)^2,x)
```

output
```
int((e*x+d)^m/(c*x^2+b*x)^2,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^2}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx)^2}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
integral((e*x + d)^m/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^2}\,dx = \int \frac{(d+ex)^m}{x^2\,(b+cx)^2}\,dx$$

input
```
integrate((e*x+d)**m/(c*x**2+b*x)**2,x)
```

output
```
Integral((d + e*x)**m/(x**2*(b + c*x)**2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^2}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx)^2}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/(c*x^2 + b*x)^2, x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^2}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx)^2}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/(c*x^2 + b*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{(bx+cx^2)^2}\,dx = \int \frac{(d+e\,x)^m}{(c\,x^2+b\,x)^2}\,dx$$

input
```
int((d + e*x)^m/(b*x + c*x^2)^2,x)
```

output
```
int((d + e*x)^m/(b*x + c*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^2}\,dx = \int \frac{(ex+d)^m}{c^2x^4+2bc\,x^3+b^2x^2}dx$$

input
```
int((e*x+d)^m/(c*x^2+b*x)^2,x)
```

output
```
int((d + e*x)**m/(b**2*x**2 + 2*b*c*x**3 + c**2*x**4),x)
```

3.243 $\int \frac{(d+ex)^m}{(bx+cx^2)^3} \, dx$

Optimal result 2006
Mathematica [A] (verified) 2007
Rubi [A] (verified) 2007
Maple [F] 2010
Fricas [F] 2010
Sympy [F] 2010
Maxima [F] 2011
Giac [F] 2011
Mupad [F(-1)] 2011
Reduce [F] 2012

Optimal result

Integrand size = 19, antiderivative size = 378

$$\int \frac{(d+ex)^m}{(bx+cx^2)^3} \, dx = \frac{c(6c^2d^2-b^2e^2(1-m)-bcde(4+m))\,(d+ex)^{1+m}}{2b^3d^2(cd-be)(b+cx)^2} - \frac{(d+ex)^{1+m}}{2bdx^2(b+cx)^2} + \frac{(4cd+be(1-m))(d+ex)^{1+m}}{2b^2d^2x(b+cx)^2} + \frac{c(2cd-be)\,(6c^2d^2-6bcde-b^2e^2(1-m))\,(d+ex)^{1+m}}{2b^4d^2(cd-be)^2(b+cx)} + \frac{c^3(12c^2d^2-6bcde(4-m)+b^2e^2(12-7m+m^2))\,(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,\right.}{2b^5(cd-be)^3(1+m)} - \frac{(12c^2d^2-6bcdem-b^2e^2(1-m)m)\,(d+ex)^{1+m}\,\text{Hypergeometric2F1}\left(1,1+m,2+m,1+\frac{ex}{d}\right)}{2b^5d^3(1+m)}$$

output
```
1/2*c*(6*c^2*d^2-b^2*e^2*(1-m)-b*c*d*e*(4+m))*(e*x+d)^(1+m)/b^3/d^2/(-b*e+
c*d)/(c*x+b)^2-1/2*(e*x+d)^(1+m)/b/d/x^2/(c*x+b)^2+1/2*(4*c*d+b*e*(1-m))*(
e*x+d)^(1+m)/b^2/d^2/x/(c*x+b)^2+1/2*c*(-b*e+2*c*d)*(6*c^2*d^2-6*b*c*d*e-b
^2*e^2*(1-m))*(e*x+d)^(1+m)/b^4/d^2/(-b*e+c*d)^2/(c*x+b)+1/2*c^3*(12*c^2*d
^2-6*b*c*d*e*(4-m)+b^2*e^2*(m^2-7*m+12))*(e*x+d)^(1+m)*hypergeom([1, 1+m],
[2+m],c*(e*x+d)/(-b*e+c*d))/b^5/(-b*e+c*d)^3/(1+m)-1/2*(12*c^2*d^2-6*b*c*d
*e*m-b^2*e^2*(1-m)*m)*(e*x+d)^(1+m)*hypergeom([1, 1+m],[2+m],1+e*x/d)/b^5/
d^3/(1+m)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.89

$$\int \frac{(d+ex)^m}{(bx+cx^2)^3}\,dx =$$

$$-\frac{(d+ex)^{1+m}\left(2b^4d^2(cd-be)^3(1+m)-2b^3d(cd-be)^3(4cd-be(-1+m))(1+m)x+x^2\left(-2b^2cd(cd\right.\right.}{}$$

input
```
Integrate[(d + e*x)^m/(b*x + c*x^2)^3,x]
```

output
```
-1/4*((d + e*x)^(1 + m)*(2*b^4*d^2*(c*d - b*e)^3*(1 + m) - 2*b^3*d*(c*d -
b*e)^3*(4*c*d - b*e*(-1 + m))*(1 + m)*x + x^2*(-2*b^2*c*d*(c*d - b*e)^2*(1
 + m)*(6*c^2*d^2 + b^2*e^2*(-1 + m) - b*c*d*e*(4 + m)) - (b + c*x)*(-2*b*c
*d*(2*c*d - b*e)*(-(c*d) + b*e)*(6*c^2*d^2 - 6*b*c*d*e + b^2*e^2*(-1 + m))
*(1 + m) + (b + c*x)*(2*c^3*d^3*(12*c^2*d^2 + 6*b*c*d*e*(-4 + m) + b^2*e^2
*(12 - 7*m + m^2))*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(d + e*x))/(c*d -
 b*e)] - 2*(c*d - b*e)^3*(12*c^2*d^2 - 6*b*c*d*e*m + b^2*e^2*(-1 + m)*m)*H
ypergeometric2F1[1, 1 + m, 2 + m, 1 + (e*x)/d])))))/(b^5*d^3*(c*d - b*e)^3
*(1 + m)*x^2*(b + c*x)^2)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1165, 1235, 1200, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \; 1165$$

$$-\frac{\int \frac{(d+ex)^m(6c^2d^2-bce(m+4)d-b^2e^2(1-m)+ce(2cd-be)(2-m)x)}{(cx^2+bx)^2}dx}{2b^2d(cd-be)}-\frac{(d+ex)^{m+1}(cx(2cd-be)+b(cd-be))}{2b^2d\left(bx+cx^2\right)^2(cd-be)}$$

$\Big\downarrow$ 1235

$$-\frac{-\frac{\int \frac{(d+ex)^m\left((cd-be)^2\left(12c^2d^2-6bcemd-b^2e^2(1-m)m\right)-ce(2cd-be)\left(6c^2d^2-6bced-b^2e^2(1-m)\right)mx\right)}{cx^2+bx}dx}{b^2d(cd-be)}-\frac{(d+ex)^{m+1}(cx(2cd-be)(-b^2e^2(1-m)-}{}}{2b^2d(cd-be)}-\frac{(d+ex)^{m+1}(cx(2cd-be)+b(cd-be))}{2b^2d\left(bx+cx^2\right)^2(cd-be)}$$

$\Big\downarrow$ 1200

$$-\frac{-\frac{\int \left(\frac{(cd-be)^2\left(12c^2d^2-6bcemd-b^2e^2(1-m)m\right)(d+ex)^m}{bx}+\frac{c^3d^2\left(-12c^2d^2+6bce(4-m)d-b^2e^2\left(m^2-7m+12\right)\right)(d+ex)^m}{b(b+cx)}\right)dx}{b^2d(cd-be)}-\frac{(d+ex)^{m+1}(cx(2cd-b}{}}{2b^2d(cd-be)}-\frac{(d+ex)^{m+1}(cx(2cd-be)+b(cd-be))}{2b^2d\left(bx+cx^2\right)^2(cd-be)}$$

$\Big\downarrow$ 2009

$$-\frac{-\frac{(d+ex)^{m+1}\left(cx(2cd-be)\left(-b^2e^2(1-m)-6bcde+6c^2d^2\right)+b(cd-be)\left(-b^2e^2(1-m)-bcde(m+4)+6c^2d^2\right)\right)}{b^2d(bx+cx^2)(cd-be)}-\frac{c^3d^2(d+ex)^{m+1}\left(b^2e^2\left(m^2-7m+1\right.\right.}{}}{2b}-\frac{(d+ex)^{m+1}(cx(2cd-be)+b(cd-be))}{2b^2d\left(bx+cx^2\right)^2(cd-be)}$$

input
```
Int[(d + e*x)^m/(b*x + c*x^2)^3,x]
```

output
```
-1/2*((d + e*x)^(1 + m)*(b*(c*d - b*e) + c*(2*c*d - b*e)*x))/(b^2*d*(c*d -
 b*e)*(b*x + c*x^2)^2) - (-(((d + e*x)^(1 + m)*(b*(c*d - b*e)*(6*c^2*d^2 -
 b^2*e^2*(1 - m) - b*c*d*e*(4 + m)) + c*(2*c*d - b*e)*(6*c^2*d^2 - 6*b*c*d
*e - b^2*e^2*(1 - m))*x))/(b^2*d*(c*d - b*e)*(b*x + c*x^2))) - ((c^3*d^2*(
12*c^2*d^2 - 6*b*c*d*e*(4 - m) + b^2*e^2*(12 - 7*m + m^2))*(d + e*x)^(1 +
m)*Hypergeometric2F1[1, 1 + m, 2 + m, (c*(d + e*x))/(c*d - b*e)])/(b*(c*d
- b*e)*(1 + m)) - ((c*d - b*e)^2*(12*c^2*d^2 - 6*b*c*d*e*m - b^2*e^2*(1 -
m)*m)*(d + e*x)^(1 + m)*Hypergeometric2F1[1, 1 + m, 2 + m, 1 + (e*x)/d])/(
b*d*(1 + m)))/(b^2*d*(c*d - b*e)))/(2*b^2*d*(c*d - b*e))
```

Defintions of rubi rules used

rule 1165
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d
+ e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p
+ 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
 b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && LtQ[p, -1]
 && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

rule 1200
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*
x)^n/(a + b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && In
tegersQ[n]
```

rule 1235
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Simp[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))   Int[(d + e*x)^m
*(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
 2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*
m + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) -
f*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
)
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \frac{(ex+d)^m}{(c\,x^2+bx)^3}dx$$

input
```
int((e*x+d)^m/(c*x^2+b*x)^3,x)
```

output
```
int((e*x+d)^m/(c*x^2+b*x)^3,x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^3}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx)^3}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
integral((e*x + d)^m/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^4 + b^3*x^3), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^3}\,dx = \int \frac{(d+ex)^m}{x^3\,(b+cx)^3}\,dx$$

input
```
integrate((e*x+d)**m/(c*x**2+b*x)**3,x)
```

output
```
Integral((d + e*x)**m/(x**3*(b + c*x)**3), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^3}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx)^3}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/(c*x^2 + b*x)^3, x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^3}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx)^3}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/(c*x^2 + b*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{(bx+cx^2)^3}\,dx = \int \frac{(d+e\,x)^m}{(c\,x^2+b\,x)^3}\,dx$$

input
```
int((d + e*x)^m/(b*x + c*x^2)^3,x)
```

output
```
int((d + e*x)^m/(b*x + c*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^3}\,dx = \int \frac{(ex+d)^m}{c^3x^6+3b\,c^2x^5+3b^2c\,x^4+b^3x^3}dx$$

input
```
int((e*x+d)^m/(c*x^2+b*x)^3,x)
```

output
```
int((d + e*x)**m/(b**3*x**3 + 3*b**2*c*x**4 + 3*b*c**2*x**5 + c**3*x**6),x
)
```

3.244 $\int (d+ex)^m \left(bx+cx^2\right)^{3/2} dx$

Optimal result 2013
Mathematica [A] (verified) 2013
Rubi [A] (verified) 2014
Maple [F] 2015
Fricas [F] 2015
Sympy [F] 2016
Maxima [F] 2016
Giac [F] 2016
Mupad [F(-1)] 2017
Reduce [F] 2017

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int (d+ex)^m \left(bx+cx^2\right)^{3/2} dx = \frac{2bx^2(d+ex)^m \left(1+\frac{ex}{d}\right)^{-m} \sqrt{bx+cx^2}\,\text{AppellF1}\left(\frac{5}{2},-\frac{3}{2},-m,\frac{7}{2},-\frac{cx}{b},-\frac{ex}{d}\right)}{5\sqrt{1+\frac{cx}{b}}}$$

output
```
2/5*b*x^2*(e*x+d)^m*(c*x^2+b*x)^(1/2)*AppellF1(5/2,-3/2,-m,7/2,-c*x/b,-e*x
/d)/(1+c*x/b)^(1/2)/((1+e*x/d)^m)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.41

$$\int (d+ex)^m \left(bx+cx^2\right)^{3/2} dx = \frac{2x^2\sqrt{x(b+cx)}(d+ex)^m \left(1+\frac{ex}{d}\right)^{-m} \left(7b\,\text{AppellF1}\left(\frac{5}{2},-\frac{1}{2},-m,\frac{7}{2},-\frac{cx}{b},-\frac{ex}{d}\right)+5cx\,\text{AppellF}\right.}{35\sqrt{1+\frac{cx}{b}}}$$

input
```
Integrate[(d + e*x)^m*(b*x + c*x^2)^(3/2),x]
```

output
```
(2*x^2*Sqrt[x*(b + c*x)]*(d + e*x)^m*(7*b*AppellF1[5/2, -1/2, -m, 7/2, -((
c*x)/b), -((e*x)/d)] + 5*c*x*AppellF1[7/2, -1/2, -m, 9/2, -((c*x)/b), -((e
*x)/d)]))/(35*Sqrt[1 + (c*x)/b]*(1 + (e*x)/d)^m)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.33, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^{3/2}(d+ex)^m\,dx$$

$$\Big\downarrow \text{1179}$$

$$\frac{\left(bx+cx^2\right)^{3/2}\int (d+ex)^m\left(1-\frac{d+ex}{d}\right)^{3/2}\left(1-\frac{c(d+ex)}{cd-be}\right)^{3/2}d(d+ex)}{e\left(-\frac{ex}{d}\right)^{3/2}\left(1-\frac{c(d+ex)}{cd-be}\right)^{3/2}}$$

$$\Big\downarrow \text{150}$$

$$\frac{\left(bx+cx^2\right)^{3/2}(d+ex)^{m+1}\text{AppellF1}\left(m+1,-\frac{3}{2},-\frac{3}{2},m+2,\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)\left(-\frac{ex}{d}\right)^{3/2}\left(1-\frac{c(d+ex)}{cd-be}\right)^{3/2}}$$

input
```
Int[(d + e*x)^m*(b*x + c*x^2)^(3/2),x]
```

output
```
((d + e*x)^(1 + m)*(b*x + c*x^2)^(3/2)*AppellF1[1 + m, -3/2, -3/2, 2 + m,
(d + e*x)/d, (c*(d + e*x))/(c*d - b*e)])/(e*(1 + m)*(-((e*x)/d))^(3/2)*(1
- (c*(d + e*x))/(c*d - b*e))^(3/2))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int (ex+d)^m \left(c\,x^2+bx\right)^{\frac{3}{2}} dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x)^(3/2),x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x)^(3/2),x)
```

Fricas [F]

$$\int (d+ex)^m \left(bx+cx^2\right)^{3/2} dx = \int \left(cx^2+bx\right)^{\frac{3}{2}} (ex+d)^m \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^(3/2)*(e*x + d)^m, x)
```

Sympy [F]

$$\int (d+ex)^m \left(bx+cx^2\right)^{3/2} dx = \int (x(b+cx))^{\frac{3}{2}} (d+ex)^m dx$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(d + e*x)**m, x)
```

Maxima [F]

$$\int (d+ex)^m \left(bx+cx^2\right)^{3/2} dx = \int \left(cx^2+bx\right)^{\frac{3}{2}} (ex+d)^m dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(3/2)*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m \left(bx+cx^2\right)^{3/2} dx = \int \left(cx^2+bx\right)^{\frac{3}{2}} (ex+d)^m dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^(3/2)*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \left(bx+cx^2\right)^{3/2}\,dx=\int \left(c\,x^2+b\,x\right)^{3/2}\,(d+e\,x)^m\,dx$$

input
```
int((b*x + c*x^2)^(3/2)*(d + e*x)^m,x)
```

output
```
int((b*x + c*x^2)^(3/2)*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m \left(bx+cx^2\right)^{3/2}\,dx=\int (ex+d)^m \left(c\,x^2+bx\right)^{\frac{3}{2}}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x)^(3/2),x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x)^(3/2),x)
```

3.245 $\int (d+ex)^m \sqrt{bx+cx^2}\,dx$

Optimal result 2018
Mathematica [A] (verified) 2018
Rubi [A] (verified) 2019
Maple [F] 2020
Fricas [F] 2020
Sympy [F] 2021
Maxima [F] 2021
Giac [F] 2021
Mupad [F(-1)] 2022
Reduce [F] 2022

Optimal result

Integrand size = 21, antiderivative size = 76

$$\int (d+ex)^m \sqrt{bx+cx^2}\,dx = \frac{2x(d+ex)^m \left(1+\frac{ex}{d}\right)^{-m} \sqrt{bx+cx^2}\,\text{AppellF1}\left(\frac{3}{2},-\frac{1}{2},-m,\frac{5}{2},-\frac{cx}{b},-\frac{ex}{d}\right)}{3\sqrt{1+\frac{cx}{b}}}$$

output
```
2/3*x*(e*x+d)^m*(c*x^2+b*x)^(1/2)*AppellF1(3/2,-1/2,-m,5/2,-c*x/b,-e*x/d)/
(1+c*x/b)^(1/2)/((1+e*x/d)^m)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int (d+ex)^m \sqrt{bx+cx^2}\,dx = \frac{2x\sqrt{x(b+cx)}(d+ex)^m \left(\frac{d+ex}{d}\right)^{-m} \text{AppellF1}\left(\frac{3}{2},-\frac{1}{2},-m,\frac{5}{2},-\frac{cx}{b},-\frac{ex}{d}\right)}{3\sqrt{\frac{b+cx}{b}}}$$

input
```
Integrate[(d + e*x)^m*Sqrt[b*x + c*x^2],x]
```

output
```
(2*x*Sqrt[x*(b + c*x)]*(d + e*x)^m*AppellF1[3/2, -1/2, -m, 5/2, -((c*x)/b)
, -((e*x)/d)])/(3*Sqrt[(b + c*x)/b]*((d + e*x)/d)^m)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.38, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{bx+cx^2}(d+ex)^m\,dx$$

$$\Big\downarrow \text{ 1179}$$

$$\frac{\sqrt{bx+cx^2}\int (d+ex)^m\sqrt{1-\frac{d+ex}{d}}\sqrt{1-\frac{c(d+ex)}{cd-be}}d(d+ex)}{e\sqrt{-\frac{ex}{d}}\sqrt{1-\frac{c(d+ex)}{cd-be}}}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{\sqrt{bx+cx^2}(d+ex)^{m+1}\operatorname{AppellF1}\left(m+1,-\frac{1}{2},-\frac{1}{2},m+2,\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)\sqrt{-\frac{ex}{d}}\sqrt{1-\frac{c(d+ex)}{cd-be}}}$$

input
```
Int[(d + e*x)^m*Sqrt[b*x + c*x^2],x]
```

output
```
((d + e*x)^(1 + m)*Sqrt[b*x + c*x^2]*AppellF1[1 + m, -1/2, -1/2, 2 + m, (d
 + e*x)/d, (c*(d + e*x))/(c*d - b*e)])/(e*(1 + m)*Sqrt[-((e*x)/d)]*Sqrt[1
- (c*(d + e*x))/(c*d - b*e)])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int (ex+d)^m \sqrt{c\,x^2+bx}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x)^(1/2),x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x)^(1/2),x)
```

Fricas [F]

$$\int (d+ex)^m\sqrt{bx+cx^2}\,dx = \int \sqrt{cx^2+bx}(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x)*(e*x + d)^m, x)
```

Sympy [F]

$$\int (d+ex)^m\sqrt{bx+cx^2}\,dx=\int \sqrt{x\,(b+cx)}(d+ex)^m\,dx$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(sqrt(x*(b + c*x))*(d + e*x)**m, x)
```

Maxima [F]

$$\int (d+ex)^m\sqrt{bx+cx^2}\,dx=\int \sqrt{cx^2+bx}(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x)*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m\sqrt{bx+cx^2}\,dx=\int \sqrt{cx^2+bx}(ex+d)^m\,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(c*x^2 + b*x)*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m\sqrt{bx+cx^2}\,dx=\int \sqrt{c\,x^2+b\,x}\,(d+e\,x)^m\,dx$$

input
```
int((b*x + c*x^2)^(1/2)*(d + e*x)^m,x)
```

output
```
int((b*x + c*x^2)^(1/2)*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m\sqrt{bx+cx^2}\,dx=\int \sqrt{x}\,(ex+d)^m\,\sqrt{cx+b}dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x)^(1/2),x)
```

output
```
int(sqrt(x)*(d + e*x)**m*sqrt(b + c*x),x)
```

3.246 $\int \frac{(d+ex)^m}{\sqrt{bx+cx^2}}\,dx$

Optimal result . . . 2023
Mathematica [A] (verified) . . . 2023
Rubi [A] (verified) . . . 2024
Maple [F] . . . 2025
Fricas [F] . . . 2025
Sympy [F] . . . 2026
Maxima [F] . . . 2026
Giac [F] . . . 2026
Mupad [F(-1)] . . . 2027
Reduce [F] . . . 2027

Optimal result

Integrand size = 21, antiderivative size = 74

$$\int \frac{(d+ex)^m}{\sqrt{bx+cx^2}}\,dx=\frac{2x\sqrt{1+\frac{cx}{b}}(d+ex)^m\left(1+\frac{ex}{d}\right)^{-m}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},-\frac{cx}{b},-\frac{ex}{d}\right)}{\sqrt{bx+cx^2}}$$

output
```
2*x*(1+c*x/b)^(1/2)*(e*x+d)^m*AppellF1(1/2,1/2,-m,3/2,-c*x/b,-e*x/d)/((1+e
*x/d)^m)/(c*x^2+b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{(d+ex)^m}{\sqrt{bx+cx^2}}\,dx=\frac{2x\sqrt{\frac{b+cx}{b}}(d+ex)^m\left(\frac{d+ex}{d}\right)^{-m}\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2},-\frac{cx}{b},-\frac{ex}{d}\right)}{\sqrt{x(b+cx)}}$$

input
```
Integrate[(d + e*x)^m/Sqrt[b*x + c*x^2],x]
```

output
```
(2*x*Sqrt[(b + c*x)/b]*(d + e*x)^m*AppellF1[1/2, 1/2, -m, 3/2, -((c*x)/b),
 -((e*x)/d)])/(Sqrt[x*(b + c*x)]*((d + e*x)/d)^m)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.42, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{ 1179}$$

$$\frac{\sqrt{-\frac{ex}{d}}\sqrt{1-\frac{c(d+ex)}{cd-be}}\int \frac{(d+ex)^m}{\sqrt{1-\frac{d+ex}{d}}\sqrt{1-\frac{c(d+ex)}{cd-be}}}d(d+ex)}{e\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{\sqrt{-\frac{ex}{d}}(d+ex)^{m+1}\sqrt{1-\frac{c(d+ex)}{cd-be}}\text{AppellF1}\left(m+1,\frac{1}{2},\frac{1}{2},m+2,\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)\sqrt{bx+cx^2}}$$

input
```
Int[(d + e*x)^m/Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[-((e*x)/d)]*(d + e*x)^(1 + m)*Sqrt[1 - (c*(d + e*x))/(c*d - b*e)]*Ap
pellF1[1 + m, 1/2, 1/2, 2 + m, (d + e*x)/d, (c*(d + e*x))/(c*d - b*e)])/(e
*(1 + m)*Sqrt[b*x + c*x^2])
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int \frac{(ex+d)^m}{\sqrt{c\,x^2+bx}}dx$$

input
```
int((e*x+d)^m/(c*x^2+b*x)^(1/2),x)
```

output
```
int((e*x+d)^m/(c*x^2+b*x)^(1/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{\sqrt{bx+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+bx}}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
integral((e*x + d)^m/sqrt(c*x^2 + b*x), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m}{\sqrt{bx+cx^2}}\,dx = \int \frac{(d+ex)^m}{\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate((e*x+d)**m/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral((d + e*x)**m/sqrt(x*(b + c*x)), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m}{\sqrt{bx+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+bx}}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/sqrt(c*x^2 + b*x), x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{\sqrt{bx+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{cx^2+bx}}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/sqrt(c*x^2 + b*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{\sqrt{bx+cx^2}}\,dx = \int \frac{(d+e\,x)^m}{\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((d + e*x)^m/(b*x + c*x^2)^(1/2),x)
```

output
```
int((d + e*x)^m/(b*x + c*x^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{\sqrt{bx+cx^2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{x}\,\sqrt{cx+b}}dx$$

input
```
int((e*x+d)^m/(c*x^2+b*x)^(1/2),x)
```

output
```
int((d + e*x)**m/(sqrt(x)*sqrt(b + c*x)),x)
```

3.247 $\int \frac{(d+ex)^m}{(bx+cx^2)^{3/2}}\,dx$

Optimal result 2028
Mathematica [A] (verified) 2028
Rubi [A] (verified) 2029
Maple [F] 2030
Fricas [F] 2030
Sympy [F] 2031
Maxima [F] 2031
Giac [F] 2031
Mupad [F(-1)] 2032
Reduce [F] 2032

Optimal result

Integrand size = 21, antiderivative size = 76

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{3/2}}\,dx=-\frac{2\sqrt{1+\frac{cx}{b}}(d+ex)^m\left(1+\frac{ex}{d}\right)^{-m}\operatorname{AppellF1}\left(-\frac{1}{2},\frac{3}{2},-m,\frac{1}{2},-\frac{cx}{b},-\frac{ex}{d}\right)}{b\sqrt{bx+cx^2}}$$

output
```
-2*(1+c*x/b)^(1/2)*(e*x+d)^m*AppellF1(-1/2,3/2,-m,1/2,-c*x/b,-e*x/d)/b/((1
+e*x/d)^m)/(c*x^2+b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.82

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{3/2}}\,dx =$$
$$-\frac{2\sqrt{x(b+cx)}(d+ex)^m\left(1+\frac{ex}{d}\right)^{-m}\left(b\operatorname{AppellF1}\left(-\frac{1}{2},-\frac{1}{2},-m,\frac{1}{2},-\frac{cx}{b},-\frac{ex}{d}\right)+cx\left(\operatorname{AppellF1}\left(\frac{1}{2},\frac{1}{2},-m,\frac{3}{2}\right.\right.\right.}{b^3x\sqrt{1+\frac{cx}{b}}}$$

input
```
Integrate[(d + e*x)^m/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[x*(b + c*x)]*(d + e*x)^m*(b*AppellF1[-1/2, -1/2, -m, 1/2, -((c*x)
/b), -((e*x)/d)] + c*x*(AppellF1[1/2, 1/2, -m, 3/2, -((c*x)/b), -((e*x)/d)
] + AppellF1[1/2, 3/2, -m, 3/2, -((c*x)/b), -((e*x)/d)])))/(b^3*x*Sqrt[1 +
 (c*x)/b]*(1 + (e*x)/d)^m)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.38, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{ 1179}$$

$$\frac{\left(-\frac{ex}{d}\right)^{3/2}\left(1-\frac{c(d+ex)}{cd-be}\right)^{3/2}\int \frac{(d+ex)^m}{\left(1-\frac{d+ex}{d}\right)^{3/2}\left(1-\frac{c(d+ex)}{cd-be}\right)^{3/2}}d(d+ex)}{e\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{\left(-\frac{ex}{d}\right)^{3/2}(d+ex)^{m+1}\left(1-\frac{c(d+ex)}{cd-be}\right)^{3/2}\text{AppellF1}\left(m+1,\frac{3}{2},\frac{3}{2},m+2,\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)\left(bx+cx^2\right)^{3/2}}$$

input
```
Int[(d + e*x)^m/(b*x + c*x^2)^(3/2),x]
```

output
```
((-((e*x)/d))^(3/2)*(d + e*x)^(1 + m)*(1 - (c*(d + e*x))/(c*d - b*e))^(3/2
)*AppellF1[1 + m, 3/2, 3/2, 2 + m, (d + e*x)/d, (c*(d + e*x))/(c*d - b*e)]
)/(e*(1 + m)*(b*x + c*x^2)^(3/2))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int \frac{(ex+d)^m}{(c\,x^2+bx)^{\frac{3}{2}}}dx$$

input
```
int((e*x+d)^m/(c*x^2+b*x)^(3/2),x)
```

output
```
int((e*x+d)^m/(c*x^2+b*x)^(3/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x)*(e*x + d)^m/(c^2*x^4 + 2*b*c*x^3 + b^2*x^2), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(d+ex)^m}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)**m/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((d + e*x)**m/(x*(b + c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/(c*x^2 + b*x)^(3/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/(c*x^2 + b*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(d+e\,x)^m}{(c\,x^2+b\,x)^{3/2}}\,dx$$

input
```
int((d + e*x)^m/(b*x + c*x^2)^(3/2),x)
```

output
```
int((d + e*x)^m/(b*x + c*x^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{x}\,\sqrt{cx+b}\,bx+\sqrt{x}\,\sqrt{cx+b}\,c\,x^2}dx$$

input
```
int((e*x+d)^m/(c*x^2+b*x)^(3/2),x)
```

output
```
int((d + e*x)**m/(sqrt(x)*sqrt(b + c*x)*b*x + sqrt(x)*sqrt(b + c*x)*c*x**2
),x)
```

3.248 $\int \frac{(d+ex)^m}{(bx+cx^2)^{5/2}}\,dx$

Optimal result . . . 2033
Mathematica [A] (verified) . . . 2033
Rubi [A] (verified) . . . 2034
Maple [F] . . . 2035
Fricas [F] . . . 2035
Sympy [F] . . . 2036
Maxima [F] . . . 2036
Giac [F] . . . 2036
Mupad [F(-1)] . . . 2037
Reduce [F] . . . 2037

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{5/2}}\,dx = -\frac{2\sqrt{1+\frac{cx}{b}}(d+ex)^m\left(1+\frac{ex}{d}\right)^{-m}\text{AppellF1}\left(-\frac{3}{2},\frac{5}{2},-m,-\frac{1}{2},-\frac{cx}{b},-\frac{ex}{d}\right)}{3b^2x\sqrt{bx+cx^2}}$$

output
```
-2/3*(1+c*x/b)^(1/2)*(e*x+d)^m*AppellF1(-3/2,5/2,-m,-1/2,-c*x/b,-e*x/d)/b^
2/x/((1+e*x/d)^m)/(c*x^2+b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 2.02 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.94

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{5/2}}\,dx = -\frac{2x\left(\frac{b+cx}{b}\right)^{5/2}(d+ex)^m\left(\frac{d+ex}{d}\right)^{-m}\text{AppellF1}\left(-\frac{3}{2},\frac{5}{2},-m,-\frac{1}{2},-\frac{cx}{b},-\frac{ex}{d}\right)}{3(x(b+cx))^{5/2}}$$

input
```
Integrate[(d + e*x)^m/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*x*((b + c*x)/b)^(5/2)*(d + e*x)^m*AppellF1[-3/2, 5/2, -m, -1/2, -((c*x
)/b), -((e*x)/d)])/(3*(x*(b + c*x))^(5/2)*((d + e*x)/d)^m)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.30, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \ 1179$$

$$\frac{\left(-\frac{ex}{d}\right)^{5/2}\left(1-\frac{c(d+ex)}{cd-be}\right)^{5/2}\int \frac{(d+ex)^m}{\left(1-\frac{d+ex}{d}\right)^{5/2}\left(1-\frac{c(d+ex)}{cd-be}\right)^{5/2}}d(d+ex)}{e\,(bx+cx^2)^{5/2}}$$

$$\Big\downarrow \ 150$$

$$\frac{\left(-\frac{ex}{d}\right)^{5/2}(d+ex)^{m+1}\left(1-\frac{c(d+ex)}{cd-be}\right)^{5/2}\text{AppellF1}\left(m+1,\frac{5}{2},\frac{5}{2},m+2,\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)\,(bx+cx^2)^{5/2}}$$

input
```
Int[(d + e*x)^m/(b*x + c*x^2)^(5/2),x]
```

output
```
((-((e*x)/d))^(5/2)*(d + e*x)^(1 + m)*(1 - (c*(d + e*x))/(c*d - b*e))^(5/2
)*AppellF1[1 + m, 5/2, 5/2, 2 + m, (d + e*x)/d, (c*(d + e*x))/(c*d - b*e)]
)/(e*(1 + m)*(b*x + c*x^2)^(5/2))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int \frac{(ex+d)^m}{(c\,x^2+bx)^{\frac{5}{2}}}dx$$

input
```
int((e*x+d)^m/(c*x^2+b*x)^(5/2),x)
```

output
```
int((e*x+d)^m/(c*x^2+b*x)^(5/2),x)
```

Fricas [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
integral(sqrt(c*x^2 + b*x)*(e*x + d)^m/(c^3*x^6 + 3*b*c^2*x^5 + 3*b^2*c*x^
4 + b^3*x^3), x)
```

Sympy [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(d+ex)^m}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)**m/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral((d + e*x)**m/(x*(b + c*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((e*x + d)^m/(c*x^2 + b*x)^(5/2), x)
```

Giac [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(ex+d)^m}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x+d)^m/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((e*x + d)^m/(c*x^2 + b*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(d+e\,x)^m}{(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((d + e*x)^m/(b*x + c*x^2)^(5/2),x)
```

output
```
int((d + e*x)^m/(b*x + c*x^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{(d+ex)^m}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(ex+d)^m}{\sqrt{x}\,\sqrt{cx+b}\,b^2x^2+2\sqrt{x}\,\sqrt{cx+b}\,bc\,x^3+\sqrt{x}\,\sqrt{cx+b}\,c^2x^4}dx$$

input
```
int((e*x+d)^m/(c*x^2+b*x)^(5/2),x)
```

output
```
int((d + e*x)**m/(sqrt(x)*sqrt(b + c*x)*b**2*x**2 + 2*sqrt(x)*sqrt(b + c*x
)*b*c*x**3 + sqrt(x)*sqrt(b + c*x)*c**2*x**4),x)
```

3.249 $\int (d+ex)^2 (bx+cx^2)^p \, dx$

Optimal result 2038
Mathematica [A] (verified) 2039
Rubi [A] (verified) 2039
Maple [F] 2041
Fricas [F] 2041
Sympy [F] 2042
Maxima [F] 2042
Giac [F] 2042
Mupad [F(-1)] 2043
Reduce [F] 2043

Optimal result

Integrand size = 19, antiderivative size = 165

$$\int (d+ex)^2 (bx+cx^2)^p \, dx$$
$$= -\frac{e(be(2+p)-2cd(3+2p))(bx+cx^2)^{1+p}}{2c^2(1+p)(3+2p)} + \frac{e^2x(bx+cx^2)^{1+p}}{c(3+2p)}$$
$$+ \frac{(b^2e^2(2+p)+2c^2d^2(3+2p)-2bcde(3+2p))(bx+cx^2)^{1+p} \text{Hypergeometric2F1}(1,2(1+p),2+p,-}{2bc^2(1+p)(3+2p)}$$

output
```
-1/2*e*(b*e*(2+p)-2*c*d*(3+2*p))*(c*x^2+b*x)^(p+1)/c^2/(p+1)/(3+2*p)+e^2*x
*(c*x^2+b*x)^(p+1)/c/(3+2*p)+1/2*(b^2*e^2*(2+p)+2*c^2*d^2*(3+2*p)-2*b*c*d*
e*(3+2*p))*(c*x^2+b*x)^(p+1)*hypergeom([1, 2*p+2],[2+p],-c*x/b)/b/c^2/(p+1
)/(3+2*p)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.83

$$\int (d+ex)^2 \left(bx+cx^2\right)^p \, dx$$
$$= \frac{x(x(b+cx))^p \left(1+\frac{cx}{b}\right)^{-p} \left(-e(b+cx) \left(1+\frac{cx}{b}\right)^p (be(2+p)-2c(d(3+2p)+e(1+p)x))+(b^2e^2(2+p)\right.}{2c^2(1+p)(3+2p)}$$

input
```
Integrate[(d + e*x)^2*(b*x + c*x^2)^p,x]
```

output
```
(x*(x*(b + c*x))^p*(-(e*(b + c*x)*(1 + (c*x)/b)^p*(b*e*(2 + p) - 2*c*(d*(3
 + 2*p) + e*(1 + p)*x))) + (b^2*e^2*(2 + p) + 2*c^2*d^2*(3 + 2*p) - 2*b*c*
d*e*(3 + 2*p))*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x)/b)]))/(2*c^2*(1
 + p)*(3 + 2*p)*(1 + (c*x)/b)^p)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.07, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1166, 25, 1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^2 \left(bx+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{1166}$$

$$\frac{\int -\left((d(be(p+1)-cd(2p+3))-e(2cd-be)(p+2)x) \left(cx^2+bx\right)^p\right)dx}{c(2p+3)}+\frac{e(d+ex)\left(bx+cx^2\right)^{p+1}}{c(2p+3)}$$

$$\Big\downarrow \text{25}$$

$$\frac{e(d+ex)\left(bx+cx^2\right)^{p+1}}{c(2p+3)}-\frac{\int (d(be(p+1)-cd(2p+3))-e(2cd-be)(p+2)x) \left(cx^2+bx\right)^p dx}{c(2p+3)}$$

$$\Big\downarrow \ 1160$$

$$\frac{e(d+ex)\left(bx+cx^2\right)^{p+1}}{c(2p+3)}-\frac{-\frac{\left(b^2e^2(p+2)-2bcde(2p+3)+2c^2d^2(2p+3)\right)\int \left(cx^2+bx\right)^p dx}{2c}-\frac{e(p+2)(2cd-be)\left(bx+cx^2\right)^{p+1}}{2c(p+1)}}{c(2p+3)}$$

$$\Big\downarrow \ 1096$$

$$\frac{e(d+ex)\left(bx+cx^2\right)^{p+1}}{c(2p+3)}-\frac{\frac{\left(-\frac{cx}{b}\right)^{-p-1}\left(bx+cx^2\right)^{p+1}\left(b^2e^2(p+2)-2bcde(2p+3)+2c^2d^2(2p+3)\right)\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+cx}{b}\right)}{2bc(p+1)}-\frac{e(p+2)(2cd-be)\left(bx+cx^2\right)^{p+1}}{2c(p+1)}}{c(2p+3)}$$

input
```
Int[(d + e*x)^2*(b*x + c*x^2)^p,x]
```

output
```
(e*(d + e*x)*(b*x + c*x^2)^(1 + p))/(c*(3 + 2*p)) - (-1/2*(e*(2*c*d - b*e)
*(2 + p)*(b*x + c*x^2)^(1 + p))/(c*(1 + p)) + ((b^2*e^2*(2 + p) + 2*c^2*d^
2*(3 + 2*p) - 2*b*c*d*e*(3 + 2*p))*(-((c*x)/b))^(-1 - p)*(b*x + c*x^2)^(1
+ p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + c*x)/b])/(2*b*c*(1 + p)))/(c
*(3 + 2*p))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1166
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[1/(c*(m + 2*p + 1))   Int[(d + e*x)^(m - 2)*Simp[c*d^2*(m
+ 2*p + 1) - e*(a*e*(m - 1) + b*d*(p + 1)) + e*(2*c*d - b*e)*(m + p)*x, x]*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && If[Ration
alQ[m], GtQ[m, 1], SumSimplerQ[m, -2]] && NeQ[m + 2*p + 1, 0] && IntQuadrat
icQ[a, b, c, d, e, m, p, x]
```

Maple [F]

$$\int (ex+d)^2 \left(c\,x^2+bx\right)^p dx$$

input
```
int((e*x+d)^2*(c*x^2+b*x)^p,x)
```

output
```
int((e*x+d)^2*(c*x^2+b*x)^p,x)
```

Fricas [F]

$$\int (d+ex)^2 \left(bx+cx^2\right)^p \, dx = \int (ex+d)^2 \left(cx^2+bx\right)^p dx$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^p,x, algorithm="fricas")
```

output
```
integral((e^2*x^2 + 2*d*e*x + d^2)*(c*x^2 + b*x)^p, x)
```

Sympy [F]

$$\int (d+ex)^2\left(bx+cx^2\right)^p\,dx=\int (x(b+cx))^p(d+ex)^2\,dx$$

input
```
integrate((e*x+d)**2*(c*x**2+b*x)**p,x)
```

output
```
Integral((x*(b + c*x))**p*(d + e*x)**2, x)
```

Maxima [F]

$$\int (d+ex)^2\left(bx+cx^2\right)^p\,dx=\int (ex+d)^2\left(cx^2+bx\right)^p\,dx$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^2*(c*x^2 + b*x)^p, x)
```

Giac [F]

$$\int (d+ex)^2\left(bx+cx^2\right)^p\,dx=\int (ex+d)^2\left(cx^2+bx\right)^p\,dx$$

input
```
integrate((e*x+d)^2*(c*x^2+b*x)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^2*(c*x^2 + b*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^2 \left(bx+cx^2\right)^p \, dx = \int \left(c\,x^2+b\,x\right)^p\,(d+e\,x)^2 \,dx$$

input
```
int((b*x + c*x^2)^p*(d + e*x)^2,x)
```

output
```
int((b*x + c*x^2)^p*(d + e*x)^2, x)
```

Reduce [F]

$$\int (d+ex)^2 \left(bx+cx^2\right)^p \, dx = \text{Too large to display}$$

input
```
int((e*x+d)^2*(c*x^2+b*x)^p,x)
```

output
```
((b*x + c*x**2)**p*b**3*e**2*p**2 + 3*(b*x + c*x**2)**p*b**3*e**2*p + 2*(b
*x + c*x**2)**p*b**3*e**2 - 4*(b*x + c*x**2)**p*b**2*c*d*e*p**2 - 10*(b*x
+ c*x**2)**p*b**2*c*d*e*p - 6*(b*x + c*x**2)**p*b**2*c*d*e - 2*(b*x + c*x*
*2)**p*b**2*c*e**2*p**2*x - 4*(b*x + c*x**2)**p*b**2*c*e**2*p*x + 4*(b*x +
 c*x**2)**p*b*c**2*d**2*p**2 + 10*(b*x + c*x**2)**p*b*c**2*d**2*p + 6*(b*x
 + c*x**2)**p*b*c**2*d**2 + 8*(b*x + c*x**2)**p*b*c**2*d*e*p**2*x + 12*(b*
x + c*x**2)**p*b*c**2*d*e*p*x + 4*(b*x + c*x**2)**p*b*c**2*e**2*p**2*x**2
+ 2*(b*x + c*x**2)**p*b*c**2*e**2*p*x**2 + 8*(b*x + c*x**2)**p*c**3*d**2*p
**2*x + 20*(b*x + c*x**2)**p*c**3*d**2*p*x + 12*(b*x + c*x**2)**p*c**3*d**
2*x + 16*(b*x + c*x**2)**p*c**3*d*e*p**2*x**2 + 32*(b*x + c*x**2)**p*c**3*
d*e*p*x**2 + 12*(b*x + c*x**2)**p*c**3*d*e*x**2 + 8*(b*x + c*x**2)**p*c**3
*e**2*p**2*x**3 + 12*(b*x + c*x**2)**p*c**3*e**2*p*x**3 + 4*(b*x + c*x**2)
**p*c**3*e**2*x**3 - 4*int((b*x + c*x**2)**p/(4*b*p**2*x + 8*b*p*x + 3*b*x
 + 4*c*p**2*x**2 + 8*c*p*x**2 + 3*c*x**2),x)*b**4*e**2*p**5 - 20*int((b*x
+ c*x**2)**p/(4*b*p**2*x + 8*b*p*x + 3*b*x + 4*c*p**2*x**2 + 8*c*p*x**2 +
3*c*x**2),x)*b**4*e**2*p**4 - 35*int((b*x + c*x**2)**p/(4*b*p**2*x + 8*b*p
*x + 3*b*x + 4*c*p**2*x**2 + 8*c*p*x**2 + 3*c*x**2),x)*b**4*e**2*p**3 - 25
*int((b*x + c*x**2)**p/(4*b*p**2*x + 8*b*p*x + 3*b*x + 4*c*p**2*x**2 + 8*c
*p*x**2 + 3*c*x**2),x)*b**4*e**2*p**2 - 6*int((b*x + c*x**2)**p/(4*b*p**2*
x + 8*b*p*x + 3*b*x + 4*c*p**2*x**2 + 8*c*p*x**2 + 3*c*x**2),x)*b**4*e*...
```

3.250 $\int (d+ex) \left(bx+cx^2\right)^p \, dx$

Optimal result 2045
Mathematica [A] (verified) 2045
Rubi [A] (verified) 2046
Maple [F] 2047
Fricas [F] 2047
Sympy [F] 2048
Maxima [F] 2048
Giac [F] 2048
Mupad [F(-1)] 2049
Reduce [F] 2049

Optimal result

Integrand size = 17, antiderivative size = 81

$$\int (d+ex) \left(bx+cx^2\right)^p \, dx = \frac{e\left(bx+cx^2\right)^{1+p}}{2c(1+p)} + \frac{(2cd-be)\left(bx+cx^2\right)^{1+p} \text{Hypergeometric2F1}\left(1,2(1+p),2+p,-\frac{cx}{b}\right)}{2bc(1+p)}$$

output
```
1/2*e*(c*x^2+b*x)^(p+1)/c/(p+1)+1/2*(-b*e+2*c*d)*(c*x^2+b*x)^(p+1)*hyperge
om([1, 2*p+2],[2+p],-c*x/b)/b/c/(p+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

$$\int (d+ex) \left(bx+cx^2\right)^p \, dx = \frac{x(x(b+cx))^p \left(1+\frac{cx}{b}\right)^{-p} \left(e(b+cx)\left(1+\frac{cx}{b}\right)^p + (2cd-be)\,\text{Hypergeometric2F1}\left(-p,1+p,2+p,-\frac{cx}{b}\right)\right)}{2c(1+p)}$$

input
```
Integrate[(d + e*x)*(b*x + c*x^2)^p,x]
```

output
```
(x*(x*(b + c*x))^p*(e*(b + c*x)*(1 + (c*x)/b)^p + (2*c*d - b*e)*Hypergeome
tric2F1[-p, 1 + p, 2 + p, -((c*x)/b)]))/(2*c*(1 + p)*(1 + (c*x)/b)^p)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.19, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1160, 1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)\left(bx+cx^2\right)^p\,dx$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{(2cd-be)\int \left(cx^2+bx\right)^p dx}{2c}+\frac{e\left(bx+cx^2\right)^{p+1}}{2c(p+1)}$$

$$\Big\downarrow \text{ 1096}$$

$$\frac{e\left(bx+cx^2\right)^{p+1}}{2c(p+1)}-\frac{(2cd-be)\left(-\frac{cx}{b}\right)^{-p-1}\left(bx+cx^2\right)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+cx}{b}\right)}{2bc(p+1)}$$

input
```
Int[(d + e*x)*(b*x + c*x^2)^p,x]
```

output
```
(e*(b*x + c*x^2)^(1 + p))/(2*c*(1 + p)) - ((2*c*d - b*e)*(-((c*x)/b))^(-1
- p)*(b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p, 2 + p, (b + c*x)/b
])/(2*b*c*(1 + p))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [F]

$$\int (ex+d) \left(c\,x^{2}+bx\right)^{p}dx$$

input
```
int((e*x+d)*(c*x^2+b*x)^p,x)
```

output
```
int((e*x+d)*(c*x^2+b*x)^p,x)
```

Fricas [F]

$$\int (d+ex) \left(bx+cx^2\right)^p \, dx = \int (ex+d)(cx^2+bx)^p \, dx$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

output
```
integral((e*x + d)*(c*x^2 + b*x)^p, x)
```

Sympy [F]

$$\int (d+ex)\left(bx+cx^2\right)^p\,dx = \int (x(b+cx))^p\,(d+ex)\,dx$$

input
```
integrate((e*x+d)*(c*x**2+b*x)**p,x)
```

output
```
Integral((x*(b + c*x))**p*(d + e*x), x)
```

Maxima [F]

$$\int (d+ex)\left(bx+cx^2\right)^p\,dx = \int (ex+d)\left(cx^2+bx\right)^p\,dx$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)*(c*x^2 + b*x)^p, x)
```

Giac [F]

$$\int (d+ex)\left(bx+cx^2\right)^p\,dx = \int (ex+d)\left(cx^2+bx\right)^p\,dx$$

input
```
integrate((e*x+d)*(c*x^2+b*x)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)*(c*x^2 + b*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)\left(bx+cx^2\right)^p\,dx=\int \left(c\,x^2+b\,x\right)^p\,(d+e\,x)\,dx$$

input
```
int((b*x + c*x^2)^p*(d + e*x),x)
```

output
```
int((b*x + c*x^2)^p*(d + e*x), x)
```

Reduce [F]

$$\int (d+ex)\left(bx+cx^2\right)^p\,dx$$
$$=\frac{-(c\,x^2+bx)^p\,b^2ep-(c\,x^2+bx)^p\,b^2e+2(c\,x^2+bx)^p\,bcdp+2(c\,x^2+bx)^p\,bcd+2(c\,x^2+bx)^p\,bcepx+4(}{}$$

input
```
int((e*x+d)*(c*x^2+b*x)^p,x)
```

output
```
( - (b*x + c*x**2)**p*b**2*e*p - (b*x + c*x**2)**p*b**2*e + 2*(b*x + c*x**
2)**p*b*c*d*p + 2*(b*x + c*x**2)**p*b*c*d + 2*(b*x + c*x**2)**p*b*c*e*p*x
+ 4*(b*x + c*x**2)**p*c**2*d*p*x + 4*(b*x + c*x**2)**p*c**2*d*x + 4*(b*x +
 c*x**2)**p*c**2*e*p*x**2 + 2*(b*x + c*x**2)**p*c**2*e*x**2 + 2*int((b*x +
 c*x**2)**p/(2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**3*e*p**3 + 3*int((
b*x + c*x**2)**p/(2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**3*e*p**2 + in
t((b*x + c*x**2)**p/(2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**3*e*p - 4*
int((b*x + c*x**2)**p/(2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**2*c*d*p*
*3 - 6*int((b*x + c*x**2)**p/(2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**2
*c*d*p**2 - 2*int((b*x + c*x**2)**p/(2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),
x)*b**2*c*d*p)/(4*c**2*(2*p**2 + 3*p + 1))
```

3.251 $\int (bx + cx^2)^p \, dx$

Optimal result 2050
Mathematica [A] (verified) 2050
Rubi [A] (verified) 2051
Maple [F] 2051
Fricas [F] 2052
Sympy [F] 2052
Maxima [F] 2052
Giac [F] 2053
Mupad [B] (verification not implemented) 2053
Reduce [F] 2053

Optimal result

Integrand size = 11, antiderivative size = 39

$$\int (bx + cx^2)^p \, dx = \frac{(bx + cx^2)^{1+p} \text{Hypergeometric2F1}\left(1, 2(1+p), 2+p, -\frac{cx}{b}\right)}{b(1+p)}$$

output
```
(c*x^2+b*x)^(p+1)*hypergeom([1, 2*p+2],[2+p],-c*x/b)/b/(p+1)
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.15

$$\int (bx+cx^2)^p \, dx = \frac{x(x(b+cx))^p \left(1+\frac{cx}{b}\right)^{-p} \text{Hypergeometric2F1}\left(-p, 1+p, 2+p, -\frac{cx}{b}\right)}{1+p}$$

input
```
Integrate[(b*x + c*x^2)^p,x]
```

output
```
(x*(x*(b + c*x))^p*Hypergeometric2F1[-p, 1 + p, 2 + p, -((c*x)/b)])/((1 +
p)*(1 + (c*x)/b)^p)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.41, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1096}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (bx+cx^2)^p\,dx$$

$$\Big\downarrow \text{1096}$$

$$-\frac{\left(-\frac{cx}{b}\right)^{-p-1}(bx+cx^2)^{p+1}\text{Hypergeometric2F1}\left(-p,p+1,p+2,\frac{b+cx}{b}\right)}{b(p+1)}$$

input
```
Int[(b*x + c*x^2)^p,x]
```

output
```
-(((-((c*x)/b))^(-1 - p)*(b*x + c*x^2)^(1 + p)*Hypergeometric2F1[-p, 1 + p
, 2 + p, (b + c*x)/b])/(b*(1 + p)))
```

Defintions of rubi rules used

rule 1096
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Simp[(-(a + b*x + c*x^2)^(p + 1)/(q*(p + 1)*((q - b - 2*c*x)
/(2*q))^(p + 1)))*Hypergeometric2F1[-p, p + 1, p + 2, (b + q + 2*c*x)/(2*q)
], x]] /; FreeQ[{a, b, c, p}, x] &&  !IntegerQ[4*p] &&  !IntegerQ[3*p]
```

Maple [F]

$$\int (c\,x^2+bx)^p\,dx$$

input
```
int((c*x^2+b*x)^p,x)
```

output
```
int((c*x^2+b*x)^p,x)
```

Fricas [F]

$$\int (bx + cx^2)^p \, dx = \int (cx^2 + bx)^p \, dx$$

input
```
integrate((c*x^2+b*x)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^p, x)
```

Sympy [F]

$$\int (bx + cx^2)^p \, dx = \int (bx + cx^2)^p \, dx$$

input
```
integrate((c*x**2+b*x)**p,x)
```

output
```
Integral((b*x + c*x**2)**p, x)
```

Maxima [F]

$$\int (bx + cx^2)^p \, dx = \int (cx^2 + bx)^p \, dx$$

input
```
integrate((c*x^2+b*x)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^p, x)
```

Giac [F]

$$\int (bx + cx^2)^p \, dx = \int (cx^2 + bx)^p \, dx$$

input
```
integrate((c*x^2+b*x)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^p, x)
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.23

$$\int (bx + cx^2)^p \, dx = \frac{x\,(c\,x^2 + b\,x)^p \, {}_2\mathrm{F}_1\left(-p, p+1;\; p+2;\; -\frac{c\,x}{b}\right)}{\left(\frac{c\,x}{b} + 1\right)^p \, (p+1)}$$

input
```
int((b*x + c*x^2)^p,x)
```

output
```
(x*(b*x + c*x^2)^p*hypergeom([-p, p + 1], p + 2, -(c*x)/b))/(((c*x)/b + 1)
^p*(p + 1))
```

Reduce [F]

$$\int (bx + cx^2)^p \, dx = \frac{(c\,x^2 + bx)^p\, b + 2(c\,x^2 + bx)^p\, cx - 2\left(\int \frac{(c\,x^2+bx)^p}{2cp\,x^2+2bpx+c\,x^2+bx} dx\right) b^2 p^2 - \left(\int \frac{(c\,x^2+bx)^p}{2cp\,x^2+2bpx+c\,x^2+bx} dx\right) b^2 p}{2c\,(2p+1)}$$

input
```
int((c*x^2+b*x)^p,x)
```

output
```
((b*x + c*x**2)**p*b + 2*(b*x + c*x**2)**p*c*x - 2*int((b*x + c*x**2)**p/(
2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**2*p**2 - int((b*x + c*x**2)**p/
(2*b*p*x + b*x + 2*c*p*x**2 + c*x**2),x)*b**2*p)/(2*c*(2*p + 1))
```

3.252 $\int \frac{(bx+cx^2)^p}{d+ex}\,dx$

Optimal result 2054
Mathematica [A] (verified) 2054
Rubi [A] (verified) 2055
Maple [F] 2056
Fricas [F] 2056
Sympy [F] 2056
Maxima [F] 2057
Giac [F] 2057
Mupad [F(-1)] 2057
Reduce [F] 2058

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{(bx+cx^2)^p}{d+ex}\,dx = \frac{x\left(1+\frac{cx}{b}\right)^{-p}(bx+cx^2)^p \operatorname{AppellF1}\left(1+p,-p,1,2+p,-\frac{cx}{b},-\frac{ex}{d}\right)}{d(1+p)}$$

output
```
x*(c*x^2+b*x)^p*AppellF1(p+1,-p,1,2+p,-c*x/b,-e*x/d)/d/(p+1)/((1+c*x/b)^p)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{(bx+cx^2)^p}{d+ex}\,dx = \frac{x\left(\frac{b+cx}{b}\right)^{-p}(x(b+cx))^p \operatorname{AppellF1}\left(1+p,-p,1,2+p,-\frac{cx}{b},-\frac{ex}{d}\right)}{d(1+p)}$$

input
```
Integrate[(b*x + c*x^2)^p/(d + e*x),x]
```

output
```
(x*(x*(b + c*x))^p*AppellF1[1 + p, -p, 1, 2 + p, -((c*x)/b), -((e*x)/d)])/
(d*(1 + p)*((b + c*x)/b)^p)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.67, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^p}{d+ex}dx$$

$$\Big\downarrow \ 1178$$

$$-\frac{(bx+cx^2)^p \left(\frac{1}{d+ex}\right)^{2p} \left(\frac{ex}{d+ex}\right)^{-p} \left(\frac{e(b+cx)}{c(d+ex)}\right)^{-p} \int \left(\frac{1}{d+ex}\right)^{-2p-1} \left(1-\frac{d}{d+ex}\right)^p \left(1-\frac{d-\frac{be}{c}}{d+ex}\right)^p d\frac{1}{d+ex}}{e}$$

$$\Big\downarrow \ 150$$

$$\frac{(bx+cx^2)^p \left(\frac{ex}{d+ex}\right)^{-p} \left(\frac{e(b+cx)}{c(d+ex)}\right)^{-p} \text{AppellF1}\left(-2p,-p,-p,1-2p,\frac{d}{d+ex},\frac{d-\frac{be}{c}}{d+ex}\right)}{2ep}$$

input
```
Int[(b*x + c*x^2)^p/(d + e*x),x]
```

output
```
((b*x + c*x^2)^p*AppellF1[-2*p, -p, -p, 1 - 2*p, d/(d + e*x), (d - (b*e)/c
)/(d + e*x)])/(2*e*p*((e*x)/(d + e*x))^p*((e*(b + c*x))/(c*(d + e*x)))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

Maple [F]

$$\int \frac{(c\,x^{2}+bx)^{p}}{ex+d}dx$$

input
```
int((c*x^2+b*x)^p/(e*x+d),x)
```

output
```
int((c*x^2+b*x)^p/(e*x+d),x)
```

Fricas [F]

$$\int \frac{(bx+cx^2)^p}{d+ex}\,dx = \int \frac{(cx^2+bx)^p}{ex+d}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(e*x+d),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^p/(e*x + d), x)
```

Sympy [F]

$$\int \frac{(bx+cx^2)^p}{d+ex}\,dx = \int \frac{(x(b+cx))^p}{d+ex}\,dx$$

input
```
integrate((c*x**2+b*x)**p/(e*x+d),x)
```

output
```
Integral((x*(b + c*x))**p/(d + e*x), x)
```

Maxima [F]

$$\int \frac{(bx+cx^2)^p}{d+ex}\,dx = \int \frac{(cx^2+bx)^p}{ex+d}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(e*x+d),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^p/(e*x + d), x)
```

Giac [F]

$$\int \frac{(bx+cx^2)^p}{d+ex}\,dx = \int \frac{(cx^2+bx)^p}{ex+d}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(e*x+d),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^p/(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx+cx^2)^p}{d+ex}\,dx = \int \frac{(c\,x^2+b\,x)^p}{d+e\,x}\,dx$$

input
```
int((b*x + c*x^2)^p/(d + e*x),x)
```

output
```
int((b*x + c*x^2)^p/(d + e*x), x)
```

Reduce [F]

$$\int \frac{(bx+cx^2)^p}{d+ex}\,dx$$
$$= \frac{(c\,x^2+bx)^p\,b-\left(\int \frac{(c\,x^2+bx)^p}{bc\,e^2x^3+2c^2de\,x^3+b^2e^2x^2+3bcde\,x^2+2c^2d^2x^2+b^2dex+2bc\,d^2x}dx\right)b^3dep-2\left(\int \frac{(c}{bc\,e^2x^3+2c^2de\,x^3+b^2e^2x^2+}\right.}{}$$

input
```
int((c*x^2+b*x)^p/(e*x+d),x)
```

output
```
((b*x + c*x**2)**p*b - int((b*x + c*x**2)**p/(b**2*d*e*x + b**2*e**2*x**2
+ 2*b*c*d**2*x + 3*b*c*d*e*x**2 + b*c*e**2*x**3 + 2*c**2*d**2*x**2 + 2*c**
2*d*e*x**3),x)*b**3*d*e*p - 2*int((b*x + c*x**2)**p/(b**2*d*e*x + b**2*e**
2*x**2 + 2*b*c*d**2*x + 3*b*c*d*e*x**2 + b*c*e**2*x**3 + 2*c**2*d**2*x**2
+ 2*c**2*d*e*x**3),x)*b**2*c*d**2*p - int(((b*x + c*x**2)**p*x)/(b**2*d*e
+ b**2*e**2*x + 2*b*c*d**2 + 3*b*c*d*e*x + b*c*e**2*x**2 + 2*c**2*d**2*x +
 2*c**2*d*e*x**2),x)*b**2*c*e**2*p + 4*int(((b*x + c*x**2)**p*x)/(b**2*d*e
 + b**2*e**2*x + 2*b*c*d**2 + 3*b*c*d*e*x + b*c*e**2*x**2 + 2*c**2*d**2*x
+ 2*c**2*d*e*x**2),x)*c**3*d**2*p)/(p*(b*e + 2*c*d))
```

3.253 $\int \frac{(bx+cx^2)^p}{(d+ex)^2}\,dx$

Optimal result 2059
Mathematica [A] (verified) 2059
Rubi [A] (verified) 2060
Maple [F] 2061
Fricas [F] 2061
Sympy [F] 2062
Maxima [F] 2062
Giac [F] 2062
Mupad [F(-1)] 2063
Reduce [F] 2063

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \frac{(bx+cx^2)^p}{(d+ex)^2}\,dx = \frac{x\left(1+\frac{cx}{b}\right)^{-p}(bx+cx^2)^p \operatorname{AppellF1}\left(1+p,-p,2,2+p,-\frac{cx}{b},-\frac{ex}{d}\right)}{d^2(1+p)}$$

output
```
x*(c*x^2+b*x)^p*AppellF1(p+1,-p,2,2+p,-c*x/b,-e*x/d)/d^2/(p+1)/((1+c*x/b)^
p)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{(bx+cx^2)^p}{(d+ex)^2}\,dx = \frac{x\left(\frac{b+cx}{b}\right)^{-p}(x(b+cx))^p \operatorname{AppellF1}\left(1+p,-p,2,2+p,-\frac{cx}{b},-\frac{ex}{d}\right)}{d^2(1+p)}$$

input
```
Integrate[(b*x + c*x^2)^p/(d + e*x)^2,x]
```

output
```
(x*(x*(b + c*x))^p*AppellF1[1 + p, -p, 2, 2 + p, -((c*x)/b), -((e*x)/d)])/
(d^2*(1 + p)*((b + c*x)/b)^p)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.86, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1178, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^p}{(d+ex)^2}\,dx$$

$$\Big\downarrow \text{1178}$$

$$-\frac{(bx+cx^2)^p\left(\frac{1}{d+ex}\right)^{2p}\left(\frac{ex}{d+ex}\right)^{-p}\left(\frac{e(b+cx)}{c(d+ex)}\right)^{-p}\int\left(\frac{1}{d+ex}\right)^{-2p}\left(1-\frac{d}{d+ex}\right)^p\left(1-\frac{d-\frac{be}{c}}{d+ex}\right)^p d\frac{1}{d+ex}}{e}$$

$$\Big\downarrow \text{150}$$

$$-\frac{(bx+cx^2)^p\left(\frac{ex}{d+ex}\right)^{-p}\left(\frac{e(b+cx)}{c(d+ex)}\right)^{-p}\text{AppellF1}\left(1-2p,-p,-p,2-2p,\frac{d}{d+ex},\frac{d-\frac{be}{c}}{d+ex}\right)}{e(1-2p)(d+ex)}$$

input
```
Int[(b*x + c*x^2)^p/(d + e*x)^2,x]
```

output
```
-(((b*x + c*x^2)^p*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, d/(d + e*x), (d - (b
*e)/c)/(d + e*x)])/(e*(1 - 2*p)*((e*x)/(d + e*x))^p*((e*(b + c*x))/(c*(d +
 e*x)))^p*(d + e*x)))
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] && !In
tegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1178
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(-(1/(d + e*x))^(2*p))*((a +
b*x + c*x^2)^p/(e*(e*((b - q + 2*c*x)/(2*c*(d + e*x))))^p*(e*((b + q + 2*c*
x)/(2*c*(d + e*x))))^p))   Subst[Int[x^(-m - 2*(p + 1))*Simp[1 - (d - e*((b
 - q)/(2*c)))*x, x]^p*Simp[1 - (d - e*((b + q)/(2*c)))*x, x]^p, x], x, 1/(d
 + e*x)], x]] /; FreeQ[{a, b, c, d, e, p}, x] && ILtQ[m, 0]
```

Maple [F]

$$\int \frac{(c\,x^{2}+bx)^{p}}{(ex+d)^{2}}dx$$

input
```
int((c*x^2+b*x)^p/(e*x+d)^2,x)
```

output
```
int((c*x^2+b*x)^p/(e*x+d)^2,x)
```

Fricas [F]

$$\int \frac{(bx+cx^2)^p}{(d+ex)^2}\,dx=\int \frac{(cx^2+bx)^p}{(ex+d)^2}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{(bx+cx^2)^p}{(d+ex)^2}\,dx = \int \frac{(x(b+cx))^p}{(d+ex)^2}\,dx$$

input
```
integrate((c*x**2+b*x)**p/(e*x+d)**2,x)
```

output
```
Integral((x*(b + c*x))**p/(d + e*x)**2, x)
```

Maxima [F]

$$\int \frac{(bx+cx^2)^p}{(d+ex)^2}\,dx = \int \frac{(cx^2+bx)^p}{(ex+d)^2}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(e*x+d)^2,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^p/(e*x + d)^2, x)
```

Giac [F]

$$\int \frac{(bx+cx^2)^p}{(d+ex)^2}\,dx = \int \frac{(cx^2+bx)^p}{(ex+d)^2}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^p/(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx + cx^2)^p}{(d + ex)^2} dx = \int \frac{(c x^2 + b x)^p}{(d + e x)^2} dx$$

input
```
int((b*x + c*x^2)^p/(d + e*x)^2,x)
```

output
```
int((b*x + c*x^2)^p/(d + e*x)^2, x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^p}{(d + ex)^2} dx = \text{too large to display}$$

input
```
int((c*x^2+b*x)^p/(e*x+d)^2,x)
```

output
```
((b*x + c*x**2)**p*b - int((b*x + c*x**2)**p/(b**2*d**2*e*p*x - b**2*d**2*
e*x + 2*b**2*d*e**2*p*x**2 - 2*b**2*d*e**2*x**2 + b**2*e**3*p*x**3 - b**2*
e**3*x**3 + 2*b*c*d**3*p*x + 5*b*c*d**2*e*p*x**2 - b*c*d**2*e*x**2 + 4*b*c
*d*e**2*p*x**3 - 2*b*c*d*e**2*x**3 + b*c*e**3*p*x**4 - b*c*e**3*x**4 + 2*c
**2*d**3*p*x**2 + 4*c**2*d**2*e*p*x**3 + 2*c**2*d*e**2*p*x**4),x)*b**3*d**
2*e*p**2 + int((b*x + c*x**2)**p/(b**2*d**2*e*p*x - b**2*d**2*e*x + 2*b**2
*d*e**2*p*x**2 - 2*b**2*d*e**2*x**2 + b**2*e**3*p*x**3 - b**2*e**3*x**3 +
2*b*c*d**3*p*x + 5*b*c*d**2*e*p*x**2 - b*c*d**2*e*x**2 + 4*b*c*d*e**2*p*x*
*3 - 2*b*c*d*e**2*x**3 + b*c*e**3*p*x**4 - b*c*e**3*x**4 + 2*c**2*d**3*p*x
**2 + 4*c**2*d**2*e*p*x**3 + 2*c**2*d*e**2*p*x**4),x)*b**3*d**2*e*p - int(
(b*x + c*x**2)**p/(b**2*d**2*e*p*x - b**2*d**2*e*x + 2*b**2*d*e**2*p*x**2
- 2*b**2*d*e**2*x**2 + b**2*e**3*p*x**3 - b**2*e**3*x**3 + 2*b*c*d**3*p*x
+ 5*b*c*d**2*e*p*x**2 - b*c*d**2*e*x**2 + 4*b*c*d*e**2*p*x**3 - 2*b*c*d*e*
*2*x**3 + b*c*e**3*p*x**4 - b*c*e**3*x**4 + 2*c**2*d**3*p*x**2 + 4*c**2*d*
*2*e*p*x**3 + 2*c**2*d*e**2*p*x**4),x)*b**3*d*e**2*p**2*x + int((b*x + c*x
**2)**p/(b**2*d**2*e*p*x - b**2*d**2*e*x + 2*b**2*d*e**2*p*x**2 - 2*b**2*d
*e**2*x**2 + b**2*e**3*p*x**3 - b**2*e**3*x**3 + 2*b*c*d**3*p*x + 5*b*c*d*
*2*e*p*x**2 - b*c*d**2*e*x**2 + 4*b*c*d*e**2*p*x**3 - 2*b*c*d*e**2*x**3 +
b*c*e**3*p*x**4 - b*c*e**3*x**4 + 2*c**2*d**3*p*x**2 + 4*c**2*d**2*e*p*x**
3 + 2*c**2*d*e**2*p*x**4),x)*b**3*d*e**2*p*x - 2*int((b*x + c*x**2)**p/...
```

3.254 $\int (d+ex)^{3/2} (bx+cx^2)^p \, dx$

Optimal result 2065
Mathematica [A] (verified) 2065
Rubi [A] (verified) 2066
Maple [F] 2067
Fricas [F] 2067
Sympy [F] 2068
Maxima [F] 2068
Giac [F] 2068
Mupad [F(-1)] 2069
Reduce [F] 2069

Optimal result

Integrand size = 21, antiderivative size = 79

$$\int (d+ex)^{3/2} (bx+cx^2)^p \, dx = \frac{dx\left(1+\frac{cx}{b}\right)^{-p} \sqrt{d+ex}(bx+cx^2)^p \operatorname{AppellF1}\left(1+p,-p,-\frac{3}{2},2+p,-\frac{cx}{b},-\frac{ex}{d}\right)}{(1+p)\sqrt{1+\frac{ex}{d}}}$$

output
```
d*x*(e*x+d)^(1/2)*(c*x^2+b*x)^p*AppellF1(p+1,-p,-3/2,2+p,-c*x/b,-e*x/d)/(p
+1)/((1+c*x/b)^p)/(1+e*x/d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.52

$$\int (d+ex)^{3/2} (bx+cx^2)^p \, dx = \frac{x(x(b+cx))^p \left(1+\frac{cx}{b}\right)^{-p} \sqrt{d+ex}\left(d(2+p)\operatorname{AppellF1}\left(1+p,-p,-\frac{1}{2},2+p,-\frac{cx}{b},-\frac{ex}{d}\right)+e(1\right.}{(1+p)(2+p)\sqrt{1+\frac{ex}{d}}}$$

input
```
Integrate[(d + e*x)^(3/2)*(b*x + c*x^2)^p,x]
```

output
```
(x*(x*(b + c*x))^p*Sqrt[d + e*x]*(d*(2 + p)*AppellF1[1 + p, -p, -1/2, 2 +
p, -((c*x)/b), -((e*x)/d)] + e*(1 + p)*x*AppellF1[2 + p, -p, -1/2, 3 + p,
-((c*x)/b), -((e*x)/d)]))/((1 + p)*(2 + p)*(1 + (c*x)/b)^p*Sqrt[1 + (e*x)/
d])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{ 1179}$$

$$\frac{\left(bx+cx^2\right)^p \left(-\frac{ex}{d}\right)^{-p} \left(1-\frac{c(d+ex)}{cd-be}\right)^{-p} \int (d+ex)^{3/2} \left(1-\frac{d+ex}{d}\right)^p \left(1-\frac{c(d+ex)}{cd-be}\right)^p d(d+ex)}{e}$$

$$\Big\downarrow \text{ 150}$$

$$\frac{2(d+ex)^{5/2} \left(bx+cx^2\right)^p \left(-\frac{ex}{d}\right)^{-p} \left(1-\frac{c(d+ex)}{cd-be}\right)^{-p} \text{AppellF1}\left(\frac{5}{2},-p,-p,\frac{7}{2},\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{5e}$$

input
```
Int[(d + e*x)^(3/2)*(b*x + c*x^2)^p,x]
```

output
```
(2*(d + e*x)^(5/2)*(b*x + c*x^2)^p*AppellF1[5/2, -p, -p, 7/2, (d + e*x)/d,
 (c*(d + e*x))/(c*d - b*e)])/(5*e*(-((e*x)/d))^p*(1 - (c*(d + e*x))/(c*d -
 b*e))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int (ex+d)^{\frac{3}{2}} \left(c\,x^{2}+bx\right)^{p} dx$$

input
```
int((e*x+d)^(3/2)*(c*x^2+b*x)^p,x)
```

output
```
int((e*x+d)^(3/2)*(c*x^2+b*x)^p,x)
```

Fricas [F]

$$\int (d+ex)^{3/2} \left(bx+cx^2\right)^p \, dx = \int (ex+d)^{\frac{3}{2}} \left(cx^2+bx\right)^p dx$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

output
```
integral((e*x + d)^(3/2)*(c*x^2 + b*x)^p, x)
```

Sympy [F]

$$\int (d+ex)^{3/2}\left(bx+cx^2\right)^p\,dx=\int (x(b+cx))^p\,(d+ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x+d)**(3/2)*(c*x**2+b*x)**p,x)
```

output
```
Integral((x*(b + c*x))**p*(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int (d+ex)^{3/2}\left(bx+cx^2\right)^p\,dx=\int (ex+d)^{\frac{3}{2}}\left(cx^2+bx\right)^p\,dx$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

output
```
integrate((e*x + d)^(3/2)*(c*x^2 + b*x)^p, x)
```

Giac [F]

$$\int (d+ex)^{3/2}\left(bx+cx^2\right)^p\,dx=\int (ex+d)^{\frac{3}{2}}\left(cx^2+bx\right)^p\,dx$$

input
```
integrate((e*x+d)^(3/2)*(c*x^2+b*x)^p,x, algorithm="giac")
```

output
```
integrate((e*x + d)^(3/2)*(c*x^2 + b*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^{3/2}\left(bx+cx^2\right)^p\,dx=\int \left(c\,x^2+b\,x\right)^p\,\left(d+e\,x\right)^{3/2}\,dx$$

input
```
int((b*x + c*x^2)^p*(d + e*x)^(3/2),x)
```

output
```
int((b*x + c*x^2)^p*(d + e*x)^(3/2), x)
```

Reduce [F]

$$\int (d+ex)^{3/2}\left(bx+cx^2\right)^p\,dx=\text{too large to display}$$

input
```
int((e*x+d)^(3/2)*(c*x^2+b*x)^p,x)
```

output
```
(2*( - 4*sqrt(d + e*x)*(b*x + c*x**2)**p*b**2*d*e*p**2 - 4*sqrt(d + e*x)*(
b*x + c*x**2)**p*b**2*d*e*p + 4*sqrt(d + e*x)*(b*x + c*x**2)**p*b**2*e**2*
p**2*x + 2*sqrt(d + e*x)*(b*x + c*x**2)**p*b**2*e**2*p*x + 8*sqrt(d + e*x)
*(b*x + c*x**2)**p*b*c*d**2*p**2 + 12*sqrt(d + e*x)*(b*x + c*x**2)**p*b*c*
d**2*p + 3*sqrt(d + e*x)*(b*x + c*x**2)**p*b*c*d**2 + 16*sqrt(d + e*x)*(b*
x + c*x**2)**p*b*c*d*e*p**2*x + 16*sqrt(d + e*x)*(b*x + c*x**2)**p*b*c*d*e
*p*x + 6*sqrt(d + e*x)*(b*x + c*x**2)**p*b*c*d*e*x + 8*sqrt(d + e*x)*(b*x
+ c*x**2)**p*b*c*e**2*p**2*x**2 + 10*sqrt(d + e*x)*(b*x + c*x**2)**p*b*c*e
**2*p*x**2 + 3*sqrt(d + e*x)*(b*x + c*x**2)**p*b*c*e**2*x**2 + 16*sqrt(d +
 e*x)*(b*x + c*x**2)**p*c**2*d**2*p**2*x + 24*sqrt(d + e*x)*(b*x + c*x**2)
**p*c**2*d**2*p*x + 16*sqrt(d + e*x)*(b*x + c*x**2)**p*c**2*d*e*p**2*x**2
+ 12*sqrt(d + e*x)*(b*x + c*x**2)**p*c**2*d*e*p*x**2 - 128*int((sqrt(d + e
*x)*(b*x + c*x**2)**p*x)/(32*b**2*d*e*p**3 + 80*b**2*d*e*p**2 + 62*b**2*d*
e*p + 15*b**2*d*e + 32*b**2*e**2*p**3*x + 80*b**2*e**2*p**2*x + 62*b**2*e*
*2*p*x + 15*b**2*e**2*x + 64*b*c*d**2*p**3 + 128*b*c*d**2*p**2 + 60*b*c*d*
*2*p + 96*b*c*d*e*p**3*x + 208*b*c*d*e*p**2*x + 122*b*c*d*e*p*x + 15*b*c*d
*e*x + 32*b*c*e**2*p**3*x**2 + 80*b*c*e**2*p**2*x**2 + 62*b*c*e**2*p*x**2
+ 15*b*c*e**2*x**2 + 64*c**2*d**2*p**3*x + 128*c**2*d**2*p**2*x + 60*c**2*
d**2*p*x + 64*c**2*d*e*p**3*x**2 + 128*c**2*d*e*p**2*x**2 + 60*c**2*d*e*p*
x**2),x)*b**4*e**4*p**6 - 576*int((sqrt(d + e*x)*(b*x + c*x**2)**p*x)/(...
```

3.255 $\int \sqrt{d+ex}(bx+cx^2)^p\,dx$

Optimal result . . . 2071
Mathematica [A] (verified) . . . 2071
Rubi [A] (verified) . . . 2072
Maple [F] . . . 2073
Fricas [F] . . . 2073
Sympy [F] . . . 2074
Maxima [F] . . . 2074
Giac [F] . . . 2074
Mupad [F(-1)] . . . 2075
Reduce [F] . . . 2075

Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \sqrt{d+ex}(bx+cx^2)^p\,dx = \frac{x\left(1+\frac{cx}{b}\right)^{-p}\sqrt{d+ex}(bx+cx^2)^p\,\text{AppellF1}\left(1+p,-p,-\frac{1}{2},2+p,-\frac{cx}{b},-\frac{ex}{d}\right)}{(1+p)\sqrt{1+\frac{ex}{d}}}$$

output
```
x*(e*x+d)^(1/2)*(c*x^2+b*x)^p*AppellF1(p+1,-p,-1/2,2+p,-c*x/b,-e*x/d)/(p+1
)/((1+c*x/b)^p)/(1+e*x/d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \sqrt{d+ex}(bx+cx^2)^p\,dx = \frac{x\left(\frac{b+cx}{b}\right)^{-p}(x(b+cx))^p\sqrt{d+ex}\,\text{AppellF1}\left(1+p,-p,-\frac{1}{2},2+p,-\frac{cx}{b},-\frac{ex}{d}\right)}{(1+p)\sqrt{\frac{d+ex}{d}}}$$

input
```
Integrate[Sqrt[d + e*x]*(b*x + c*x^2)^p,x]
```

output
```
(x*(x*(b + c*x))^p*Sqrt[d + e*x]*AppellF1[1 + p, -p, -1/2, 2 + p, -((c*x)/
b), -((e*x)/d)])/((1 + p)*((b + c*x)/b)^p*Sqrt[(d + e*x)/d])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.29, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{d+ex}(bx+cx^2)^p\,dx$$

$$\Big\downarrow \text{1179}$$

$$\frac{(bx+cx^2)^p\left(-\frac{ex}{d}\right)^{-p}\left(1-\frac{c(d+ex)}{cd-be}\right)^{-p}\int \sqrt{d+ex}\left(1-\frac{d+ex}{d}\right)^p\left(1-\frac{c(d+ex)}{cd-be}\right)^p d(d+ex)}{e}$$

$$\Big\downarrow \text{150}$$

$$\frac{2(d+ex)^{3/2}(bx+cx^2)^p\left(-\frac{ex}{d}\right)^{-p}\left(1-\frac{c(d+ex)}{cd-be}\right)^{-p}\text{AppellF1}\left(\frac{3}{2},-p,-p,\frac{5}{2},\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{3e}$$

input
```
Int[Sqrt[d + e*x]*(b*x + c*x^2)^p,x]
```

output
```
(2*(d + e*x)^(3/2)*(b*x + c*x^2)^p*AppellF1[3/2, -p, -p, 5/2, (d + e*x)/d,
 (c*(d + e*x))/(c*d - b*e)])/(3*e*(-((e*x)/d))^p*(1 - (c*(d + e*x))/(c*d -
 b*e))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int \sqrt{ex+d}\,\left(c\,x^{2}+bx\right)^{p}dx$$

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x)^p,x)
```

output
```
int((e*x+d)^(1/2)*(c*x^2+b*x)^p,x)
```

Fricas [F]

$$\int \sqrt{d+ex}(bx+cx^2)^p\,dx = \int \sqrt{ex+d}(cx^2+bx)^p\,dx$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(c*x^2 + b*x)^p, x)
```

Sympy [F]

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^p\,dx=\int (x(b+cx))^p\sqrt{d+ex}\,dx$$

input
```
integrate((e*x+d)**(1/2)*(c*x**2+b*x)**p,x)
```

output
```
Integral((x*(b + c*x))**p*sqrt(d + e*x), x)
```

Maxima [F]

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^p\,dx=\int \sqrt{ex+d}\left(cx^2+bx\right)^p\,dx$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

output
```
integrate(sqrt(e*x + d)*(c*x^2 + b*x)^p, x)
```

Giac [F]

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^p\,dx=\int \sqrt{ex+d}\left(cx^2+bx\right)^p\,dx$$

input
```
integrate((e*x+d)^(1/2)*(c*x^2+b*x)^p,x, algorithm="giac")
```

output
```
integrate(sqrt(e*x + d)*(c*x^2 + b*x)^p, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^p\,dx=\int \left(c\,x^2+b\,x\right)^p\,\sqrt{d+e\,x}\,dx$$

input
```
int((b*x + c*x^2)^p*(d + e*x)^(1/2),x)
```

output
```
int((b*x + c*x^2)^p*(d + e*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{d+ex}\left(bx+cx^2\right)^p\,dx=\text{too large to display}$$

input
```
int((e*x+d)^(1/2)*(c*x^2+b*x)^p,x)
```

output

```
(2*(2*sqrt(d + e*x)*(b*x + c*x**2)**p*b*d*p + sqrt(d + e*x)*(b*x + c*x**2)
**p*b*d + 2*sqrt(d + e*x)*(b*x + c*x**2)**p*b*e*p*x + sqrt(d + e*x)*(b*x +
 c*x**2)**p*b*e*x + 4*sqrt(d + e*x)*(b*x + c*x**2)**p*c*d*p*x + 16*int((sq
rt(d + e*x)*(b*x + c*x**2)**p*x)/(8*b**2*d*e*p**2 + 10*b**2*d*e*p + 3*b**2
*d*e + 8*b**2*e**2*p**2*x + 10*b**2*e**2*p*x + 3*b**2*e**2*x + 16*b*c*d**2
*p**2 + 12*b*c*d**2*p + 24*b*c*d*e*p**2*x + 22*b*c*d*e*p*x + 3*b*c*d*e*x +
 8*b*c*e**2*p**2*x**2 + 10*b*c*e**2*p*x**2 + 3*b*c*e**2*x**2 + 16*c**2*d**
2*p**2*x + 12*c**2*d**2*p*x + 16*c**2*d*e*p**2*x**2 + 12*c**2*d*e*p*x**2),
x)*b**3*e**3*p**4 + 28*int((sqrt(d + e*x)*(b*x + c*x**2)**p*x)/(8*b**2*d*e
*p**2 + 10*b**2*d*e*p + 3*b**2*d*e + 8*b**2*e**2*p**2*x + 10*b**2*e**2*p*x
 + 3*b**2*e**2*x + 16*b*c*d**2*p**2 + 12*b*c*d**2*p + 24*b*c*d*e*p**2*x +
22*b*c*d*e*p*x + 3*b*c*d*e*x + 8*b*c*e**2*p**2*x**2 + 10*b*c*e**2*p*x**2 +
 3*b*c*e**2*x**2 + 16*c**2*d**2*p**2*x + 12*c**2*d**2*p*x + 16*c**2*d*e*p*
*2*x**2 + 12*c**2*d*e*p*x**2),x)*b**3*e**3*p**3 + 16*int((sqrt(d + e*x)*(b
*x + c*x**2)**p*x)/(8*b**2*d*e*p**2 + 10*b**2*d*e*p + 3*b**2*d*e + 8*b**2*
e**2*p**2*x + 10*b**2*e**2*p*x + 3*b**2*e**2*x + 16*b*c*d**2*p**2 + 12*b*c
*d**2*p + 24*b*c*d*e*p**2*x + 22*b*c*d*e*p*x + 3*b*c*d*e*x + 8*b*c*e**2*p*
*2*x**2 + 10*b*c*e**2*p*x**2 + 3*b*c*e**2*x**2 + 16*c**2*d**2*p**2*x + 12*
c**2*d**2*p*x + 16*c**2*d*e*p**2*x**2 + 12*c**2*d*e*p*x**2),x)*b**3*e**3*p
**2 + 3*int((sqrt(d + e*x)*(b*x + c*x**2)**p*x)/(8*b**2*d*e*p**2 + 10*b...
```

3.256 $\int \frac{(bx+cx^2)^p}{\sqrt{d+ex}}\,dx$

Optimal result . . . 2077
Mathematica [A] (verified) . . . 2077
Rubi [A] (verified) . . . 2078
Maple [F] . . . 2079
Fricas [F] . . . 2079
Sympy [F] . . . 2080
Maxima [F] . . . 2080
Giac [F] . . . 2080
Mupad [F(-1)] . . . 2081
Reduce [F] . . . 2081

Optimal result

Integrand size = 21, antiderivative size = 78

$$\int \frac{(bx+cx^2)^p}{\sqrt{d+ex}}\,dx = \frac{x\left(1+\frac{cx}{b}\right)^{-p}\sqrt{1+\frac{ex}{d}}(bx+cx^2)^p \operatorname{AppellF1}\left(1+p,-p,\frac{1}{2},2+p,-\frac{cx}{b},-\frac{ex}{d}\right)}{(1+p)\sqrt{d+ex}}$$

output
```
x*(1+e*x/d)^(1/2)*(c*x^2+b*x)^p*AppellF1(p+1,-p,1/2,2+p,-c*x/b,-e*x/d)/(p+
1)/((1+c*x/b)^p)/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00

$$\int \frac{(bx+cx^2)^p}{\sqrt{d+ex}}\,dx = \frac{x\left(\frac{b+cx}{b}\right)^{-p}(x(b+cx))^p\sqrt{\frac{d+ex}{d}} \operatorname{AppellF1}\left(1+p,-p,\frac{1}{2},2+p,-\frac{cx}{b},-\frac{ex}{d}\right)}{(1+p)\sqrt{d+ex}}$$

input
```
Integrate[(b*x + c*x^2)^p/Sqrt[d + e*x],x]
```

output
```
(x*(x*(b + c*x))^p*Sqrt[(d + e*x)/d]*AppellF1[1 + p, -p, 1/2, 2 + p, -((c*
x)/b), -((e*x)/d)])/((1 + p)*((b + c*x)/b)^p*Sqrt[d + e*x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.27, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^p}{\sqrt{d+ex}}dx$$

$$\Big\downarrow \ 1179$$

$$\frac{(bx+cx^2)^p\left(-\frac{ex}{d}\right)^{-p}\left(1-\frac{c(d+ex)}{cd-be}\right)^{-p}\int \frac{\left(1-\frac{d+ex}{d}\right)^p\left(1-\frac{c(d+ex)}{cd-be}\right)^p}{\sqrt{d+ex}}d(d+ex)}{e}$$

$$\Big\downarrow \ 150$$

$$\frac{2\sqrt{d+ex}(bx+cx^2)^p\left(-\frac{ex}{d}\right)^{-p}\left(1-\frac{c(d+ex)}{cd-be}\right)^{-p}\text{AppellF1}\left(\frac{1}{2},-p,-p,\frac{3}{2},\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e}$$

input
```
Int[(b*x + c*x^2)^p/Sqrt[d + e*x],x]
```

output
```
(2*Sqrt[d + e*x]*(b*x + c*x^2)^p*AppellF1[1/2, -p, -p, 3/2, (d + e*x)/d, (
c*(d + e*x))/(c*d - b*e)])/(e*(-((e*x)/d))^p*(1 - (c*(d + e*x))/(c*d - b*e
))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int \frac{(c\,x^2+bx)^p}{\sqrt{ex+d}}dx$$

input
```
int((c*x^2+b*x)^p/(e*x+d)^(1/2),x)
```

output
```
int((c*x^2+b*x)^p/(e*x+d)^(1/2),x)
```

Fricas [F]

$$\int \frac{(bx+cx^2)^p}{\sqrt{d+ex}}\,dx=\int \frac{(cx^2+bx)^p}{\sqrt{ex+d}}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(e*x+d)^(1/2),x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^p/sqrt(e*x + d), x)
```

Sympy [F]

$$\int \frac{(bx+cx^2)^p}{\sqrt{d+ex}}\,dx = \int \frac{(x(b+cx))^p}{\sqrt{d+ex}}\,dx$$

input
```
integrate((c*x**2+b*x)**p/(e*x+d)**(1/2),x)
```

output
```
Integral((x*(b + c*x))**p/sqrt(d + e*x), x)
```

Maxima [F]

$$\int \frac{(bx+cx^2)^p}{\sqrt{d+ex}}\,dx = \int \frac{(cx^2+bx)^p}{\sqrt{ex+d}}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(e*x+d)^(1/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^p/sqrt(e*x + d), x)
```

Giac [F]

$$\int \frac{(bx+cx^2)^p}{\sqrt{d+ex}}\,dx = \int \frac{(cx^2+bx)^p}{\sqrt{ex+d}}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(e*x+d)^(1/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^p/sqrt(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx + cx^2)^p}{\sqrt{d + ex}}\, dx = \int \frac{(c\,x^2 + b\,x)^p}{\sqrt{d + e\,x}}\, dx$$

input
```
int((b*x + c*x^2)^p/(d + e*x)^(1/2),x)
```

output
```
int((b*x + c*x^2)^p/(d + e*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^p}{\sqrt{d + ex}}\, dx = \text{Too large to display}$$

input
```
int((c*x^2+b*x)^p/(e*x+d)^(1/2),x)
```

output

```
(2*(sqrt(d + e*x)*(b*x + c*x**2)**p*b - 2*int((sqrt(d + e*x)*(b*x + c*x**2
)**p*x)/(2*b**2*d*e*p + b**2*d*e + 2*b**2*e**2*p*x + b**2*e**2*x + 4*b*c*d
**2*p + 6*b*c*d*e*p*x + b*c*d*e*x + 2*b*c*e**2*p*x**2 + b*c*e**2*x**2 + 4*
c**2*d**2*p*x + 4*c**2*d*e*p*x**2),x)*b**2*c*e**2*p**2 - int((sqrt(d + e*x
)*(b*x + c*x**2)**p*x)/(2*b**2*d*e*p + b**2*d*e + 2*b**2*e**2*p*x + b**2*e
**2*x + 4*b*c*d**2*p + 6*b*c*d*e*p*x + b*c*d*e*x + 2*b*c*e**2*p*x**2 + b*c
*e**2*x**2 + 4*c**2*d**2*p*x + 4*c**2*d*e*p*x**2),x)*b**2*c*e**2*p + 2*int
((sqrt(d + e*x)*(b*x + c*x**2)**p*x)/(2*b**2*d*e*p + b**2*d*e + 2*b**2*e**
2*p*x + b**2*e**2*x + 4*b*c*d**2*p + 6*b*c*d*e*p*x + b*c*d*e*x + 2*b*c*e**
2*p*x**2 + b*c*e**2*x**2 + 4*c**2*d**2*p*x + 4*c**2*d*e*p*x**2),x)*b*c**2*
d*e*p + 8*int((sqrt(d + e*x)*(b*x + c*x**2)**p*x)/(2*b**2*d*e*p + b**2*d*e
 + 2*b**2*e**2*p*x + b**2*e**2*x + 4*b*c*d**2*p + 6*b*c*d*e*p*x + b*c*d*e*
x + 2*b*c*e**2*p*x**2 + b*c*e**2*x**2 + 4*c**2*d**2*p*x + 4*c**2*d*e*p*x**
2),x)*c**3*d**2*p**2 - 2*int((sqrt(d + e*x)*(b*x + c*x**2)**p)/(2*b**2*d*e
*p*x + b**2*d*e*x + 2*b**2*e**2*p*x**2 + b**2*e**2*x**2 + 4*b*c*d**2*p*x +
 6*b*c*d*e*p*x**2 + b*c*d*e*x**2 + 2*b*c*e**2*p*x**3 + b*c*e**2*x**3 + 4*c
**2*d**2*p*x**2 + 4*c**2*d*e*p*x**3),x)*b**3*d*e*p**2 - int((sqrt(d + e*x)
*(b*x + c*x**2)**p)/(2*b**2*d*e*p*x + b**2*d*e*x + 2*b**2*e**2*p*x**2 + b*
*2*e**2*x**2 + 4*b*c*d**2*p*x + 6*b*c*d*e*p*x**2 + b*c*d*e*x**2 + 2*b*c*e*
*2*p*x**3 + b*c*e**2*x**3 + 4*c**2*d**2*p*x**2 + 4*c**2*d*e*p*x**3),x)*...
```

3.257 $\int \frac{(bx+cx^2)^p}{(d+ex)^{3/2}}\,dx$

Optimal result 2083
Mathematica [A] (verified) 2083
Rubi [A] (verified) 2084
Maple [F] 2085
Fricas [F] 2085
Sympy [F] 2086
Maxima [F] 2086
Giac [F] 2086
Mupad [F(-1)] 2087
Reduce [F] 2087

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \frac{(bx+cx^2)^p}{(d+ex)^{3/2}}\,dx = \frac{x\left(1+\frac{cx}{b}\right)^{-p}\sqrt{1+\frac{ex}{d}}\,(bx+cx^2)^p\,\text{AppellF1}\left(1+p,-p,\frac{3}{2},2+p,-\frac{cx}{b},-\frac{ex}{d}\right)}{d(1+p)\sqrt{d+ex}}$$

output
```
x*(1+e*x/d)^(1/2)*(c*x^2+b*x)^p*AppellF1(p+1,-p,3/2,2+p,-c*x/b,-e*x/d)/d/(
p+1)/((1+c*x/b)^p)/(e*x+d)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{(bx+cx^2)^p}{(d+ex)^{3/2}}\,dx = \frac{x\left(\frac{b+cx}{b}\right)^{-p}(x(b+cx))^p\left(\frac{d+ex}{d}\right)^{3/2}\text{AppellF1}\left(1+p,-p,\frac{3}{2},2+p,-\frac{cx}{b},-\frac{ex}{d}\right)}{(1+p)(d+ex)^{3/2}}$$

input
```
Integrate[(b*x + c*x^2)^p/(d + e*x)^(3/2),x]
```

output
```
(x*(x*(b + c*x))^p*((d + e*x)/d)^(3/2)*AppellF1[1 + p, -p, 3/2, 2 + p, -((
c*x)/b), -((e*x)/d)])/((1 + p)*((b + c*x)/b)^p*(d + e*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.22, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(bx+cx^2)^p}{(d+ex)^{3/2}}\,dx$$

$$\Big\downarrow \text{1179}$$

$$\frac{(bx+cx^2)^p \left(-\frac{ex}{d}\right)^{-p} \left(1-\frac{c(d+ex)}{cd-be}\right)^{-p} \int \frac{\left(1-\frac{d+ex}{d}\right)^p \left(1-\frac{c(d+ex)}{cd-be}\right)^p}{(d+ex)^{3/2}}d(d+ex)}{e}$$

$$\Big\downarrow \text{150}$$

$$-\frac{2(bx+cx^2)^p \left(-\frac{ex}{d}\right)^{-p} \left(1-\frac{c(d+ex)}{cd-be}\right)^{-p} \text{AppellF1}\left(-\frac{1}{2},-p,-p,\frac{1}{2},\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e\sqrt{d+ex}}$$

input
```
Int[(b*x + c*x^2)^p/(d + e*x)^(3/2),x]
```

output
```
(-2*(b*x + c*x^2)^p*AppellF1[-1/2, -p, -p, 1/2, (d + e*x)/d, (c*(d + e*x))
/(c*d - b*e)])/(e*(-((e*x)/d))^p*Sqrt[d + e*x]*(1 - (c*(d + e*x))/(c*d - b
*e))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int \frac{(c\,x^{2}+bx)^{p}}{(ex+d)^{\frac{3}{2}}}dx$$

input
```
int((c*x^2+b*x)^p/(e*x+d)^(3/2),x)
```

output
```
int((c*x^2+b*x)^p/(e*x+d)^(3/2),x)
```

Fricas [F]

$$\int \frac{(bx+cx^2)^p}{(d+ex)^{3/2}}\,dx = \int \frac{(cx^2+bx)^p}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(e*x+d)^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(e*x + d)*(c*x^2 + b*x)^p/(e^2*x^2 + 2*d*e*x + d^2), x)
```

Sympy [F]

$$\int \frac{(bx+cx^2)^p}{(d+ex)^{3/2}}\,dx = \int \frac{(x(b+cx))^p}{(d+ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x**2+b*x)**p/(e*x+d)**(3/2),x)
```

output
```
Integral((x*(b + c*x))**p/(d + e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(bx+cx^2)^p}{(d+ex)^{3/2}}\,dx = \int \frac{(cx^2+bx)^p}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(e*x+d)^(3/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^p/(e*x + d)^(3/2), x)
```

Giac [F]

$$\int \frac{(bx+cx^2)^p}{(d+ex)^{3/2}}\,dx = \int \frac{(cx^2+bx)^p}{(ex+d)^{\frac{3}{2}}}\,dx$$

input
```
integrate((c*x^2+b*x)^p/(e*x+d)^(3/2),x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^p/(e*x + d)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(bx + cx^2)^p}{(d + ex)^{3/2}}\, dx = \int \frac{(c\,x^2 + b\,x)^p}{(d + e\,x)^{3/2}}\, dx$$

input
```
int((b*x + c*x^2)^p/(d + e*x)^(3/2),x)
```

output
```
int((b*x + c*x^2)^p/(d + e*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{(bx + cx^2)^p}{(d + ex)^{3/2}}\, dx = \text{too large to display}$$

input
```
int((c*x^2+b*x)^p/(e*x+d)^(3/2),x)
```

output

```
(2*(sqrt(d + e*x)*(b*x + c*x**2)**p*b - 2*int((sqrt(d + e*x)*(b*x + c*x**2
)**p*x)/(2*b**2*d**2*e*p - b**2*d**2*e + 4*b**2*d*e**2*p*x - 2*b**2*d*e**2
*x + 2*b**2*e**3*p*x**2 - b**2*e**3*x**2 + 4*b*c*d**3*p + 10*b*c*d**2*e*p*
x - b*c*d**2*e*x + 8*b*c*d*e**2*p*x**2 - 2*b*c*d*e**2*x**2 + 2*b*c*e**3*p*
x**3 - b*c*e**3*x**3 + 4*c**2*d**3*p*x + 8*c**2*d**2*e*p*x**2 + 4*c**2*d*e
**2*p*x**3),x)*b**2*c*d*e**2*p**2 + int((sqrt(d + e*x)*(b*x + c*x**2)**p*x
)/(2*b**2*d**2*e*p - b**2*d**2*e + 4*b**2*d*e**2*p*x - 2*b**2*d*e**2*x + 2
*b**2*e**3*p*x**2 - b**2*e**3*x**2 + 4*b*c*d**3*p + 10*b*c*d**2*e*p*x - b*
c*d**2*e*x + 8*b*c*d*e**2*p*x**2 - 2*b*c*d*e**2*x**2 + 2*b*c*e**3*p*x**3 -
 b*c*e**3*x**3 + 4*c**2*d**3*p*x + 8*c**2*d**2*e*p*x**2 + 4*c**2*d*e**2*p*
x**3),x)*b**2*c*d*e**2*p - 2*int((sqrt(d + e*x)*(b*x + c*x**2)**p*x)/(2*b*
*2*d**2*e*p - b**2*d**2*e + 4*b**2*d*e**2*p*x - 2*b**2*d*e**2*x + 2*b**2*e
**3*p*x**2 - b**2*e**3*x**2 + 4*b*c*d**3*p + 10*b*c*d**2*e*p*x - b*c*d**2*
e*x + 8*b*c*d*e**2*p*x**2 - 2*b*c*d*e**2*x**2 + 2*b*c*e**3*p*x**3 - b*c*e*
*3*x**3 + 4*c**2*d**3*p*x + 8*c**2*d**2*e*p*x**2 + 4*c**2*d*e**2*p*x**3),x
)*b**2*c*e**3*p**2*x + int((sqrt(d + e*x)*(b*x + c*x**2)**p*x)/(2*b**2*d**
2*e*p - b**2*d**2*e + 4*b**2*d*e**2*p*x - 2*b**2*d*e**2*x + 2*b**2*e**3*p*
x**2 - b**2*e**3*x**2 + 4*b*c*d**3*p + 10*b*c*d**2*e*p*x - b*c*d**2*e*x +
8*b*c*d*e**2*p*x**2 - 2*b*c*d*e**2*x**2 + 2*b*c*e**3*p*x**3 - b*c*e**3*x**
3 + 4*c**2*d**3*p*x + 8*c**2*d**2*e*p*x**2 + 4*c**2*d*e**2*p*x**3),x)*b...
```

3.258 $\int (3+ex)^m (2x+cx^2)^p \, dx$

Optimal result . . . 2089
Mathematica [A] (verified) . . . 2089
Rubi [A] (verified) . . . 2090
Maple [F] . . . 2091
Fricas [F] . . . 2091
Sympy [F] . . . 2092
Maxima [F] . . . 2092
Giac [F] . . . 2092
Mupad [F(-1)] . . . 2093
Reduce [F] . . . 2093

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int (3+ex)^m (2x+cx^2)^p \, dx = \frac{2^p 3^m x (2+cx)^{-p} (2x+cx^2)^p \operatorname{AppellF1}\left(1+p,-p,-m,2+p,-\frac{cx}{2},-\frac{ex}{3}\right)}{1+p}$$

output
```
2^p*3^m*x*(c*x^2+2*x)^p*AppellF1(p+1,-p,-m,2+p,-1/2*c*x,-1/3*e*x)/(p+1)/((
c*x+2)^p)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int (3+ex)^m (2x+cx^2)^p \, dx = \frac{2^p 3^m x (2+cx)^{-p} (x(2+cx))^p \operatorname{AppellF1}\left(1+p,-p,-m,2+p,-\frac{cx}{2},-\frac{ex}{3}\right)}{1+p}$$

input
```
Integrate[(3 + e*x)^m*(2*x + c*x^2)^p,x]
```

output
```
(2^p*3^m*x*(x*(2 + c*x))^p*AppellF1[1 + p, -p, -m, 2 + p, -1/2*(c*x), -1/3
*(e*x)])/((1 + p)*(2 + c*x)^p)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.74, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (cx^2+2x)^p (ex+3)^m\,dx$$

$$\Big\downarrow \text{1179}$$

$$\frac{3^p (cx^2+2x)^p (-ex)^{-p} \left(1-\frac{c(ex+3)}{3c-2e}\right)^{-p} \int (ex+3)^m \left(\frac{1}{3}(-ex-3)+1\right)^p \left(1-\frac{c(ex+3)}{3c-2e}\right)^p d(ex+3)}{e}$$

$$\Big\downarrow \text{150}$$

$$\frac{3^p (cx^2+2x)^p (ex+3)^{m+1}(-ex)^{-p} \left(1-\frac{c(ex+3)}{3c-2e}\right)^{-p} \text{AppellF1}\left(m+1,-p,-p,m+2,\frac{1}{3}(ex+3),\frac{c(ex+3)}{3c-2e}\right)}{e(m+1)}$$

input
```
Int[(3 + e*x)^m*(2*x + c*x^2)^p,x]
```

output
```
(3^p*(3 + e*x)^(1 + m)*(2*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (3 +
 e*x)/3, (c*(3 + e*x))/(3*c - 2*e)])/(e*(1 + m)*(-(e*x))^p*(1 - (c*(3 + e*
x))/(3*c - 2*e))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c)))
^p))   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
 - e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int (ex+3)^{m} \left(c\,x^{2}+2x\right)^{p}dx$$

input
```
int((e*x+3)^m*(c*x^2+2*x)^p,x)
```

output
```
int((e*x+3)^m*(c*x^2+2*x)^p,x)
```

Fricas [F]

$$\int (3+ex)^m \left(2x+cx^2\right)^p\,dx=\int \left(cx^{2}+2\,x\right)^{p}(ex+3)^{m}\,dx$$

input
```
integrate((e*x+3)^m*(c*x^2+2*x)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + 2*x)^p*(e*x + 3)^m, x)
```

Sympy [F]

$$\int (3+ex)^m \left(2x+cx^2\right)^p \, dx = \int (x(cx+2))^p (ex+3)^m \, dx$$

input
```
integrate((e*x+3)**m*(c*x**2+2*x)**p,x)
```

output
```
Integral((x*(c*x + 2))**p*(e*x + 3)**m, x)
```

Maxima [F]

$$\int (3+ex)^m \left(2x+cx^2\right)^p \, dx = \int \left(cx^2+2\,x\right)^p (ex+3)^m \, dx$$

input
```
integrate((e*x+3)^m*(c*x^2+2*x)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + 2*x)^p*(e*x + 3)^m, x)
```

Giac [F]

$$\int (3+ex)^m \left(2x+cx^2\right)^p \, dx = \int \left(cx^2+2\,x\right)^p (ex+3)^m \, dx$$

input
```
integrate((e*x+3)^m*(c*x^2+2*x)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + 2*x)^p*(e*x + 3)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (3+ex)^m \left(2x+cx^2\right)^p \, dx = \int (e\,x+3)^m \,\left(c\,x^2+2\,x\right)^p \,dx$$

input
```
int((e*x + 3)^m*(2*x + c*x^2)^p,x)
```

output
```
int((e*x + 3)^m*(2*x + c*x^2)^p, x)
```

Reduce [F]

$$\int (3+ex)^m \left(2x+cx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((e*x+3)^m*(c*x^2+2*x)^p,x)
```

output

```
(3*(e*x + 3)**m*(c*x**2 + 2*x)**p*c*p*x + (e*x + 3)**m*(c*x**2 + 2*x)**p*e
*m*x + (e*x + 3)**m*(c*x**2 + 2*x)**p*e*p*x + 3*(e*x + 3)**m*(c*x**2 + 2*x
)**p*m + 3*(e*x + 3)**m*(c*x**2 + 2*x)**p*p + 27*int(((e*x + 3)**m*(c*x**2
 + 2*x)**p*x)/(3*c**2*e*m*p*x**2 + 6*c**2*e*p**2*x**2 + 3*c**2*e*p*x**2 +
9*c**2*m*p*x + 18*c**2*p**2*x + 9*c**2*p*x + c*e**2*m**2*x**2 + 3*c*e**2*m
*p*x**2 + c*e**2*m*x**2 + 2*c*e**2*p**2*x**2 + c*e**2*p*x**2 + 3*c*e*m**2*
x + 15*c*e*m*p*x + 3*c*e*m*x + 18*c*e*p**2*x + 9*c*e*p*x + 18*c*m*p + 36*c
*p**2 + 18*c*p + 2*e**2*m**2*x + 6*e**2*m*p*x + 2*e**2*m*x + 4*e**2*p**2*x
 + 2*e**2*p*x + 6*e*m**2 + 18*e*m*p + 6*e*m + 12*e*p**2 + 6*e*p),x)*c**3*m
**2*p**2 + 54*int(((e*x + 3)**m*(c*x**2 + 2*x)**p*x)/(3*c**2*e*m*p*x**2 +
6*c**2*e*p**2*x**2 + 3*c**2*e*p*x**2 + 9*c**2*m*p*x + 18*c**2*p**2*x + 9*c
**2*p*x + c*e**2*m**2*x**2 + 3*c*e**2*m*p*x**2 + c*e**2*m*x**2 + 2*c*e**2*
p**2*x**2 + c*e**2*p*x**2 + 3*c*e*m**2*x + 15*c*e*m*p*x + 3*c*e*m*x + 18*c
*e*p**2*x + 9*c*e*p*x + 18*c*m*p + 36*c*p**2 + 18*c*p + 2*e**2*m**2*x + 6*
e**2*m*p*x + 2*e**2*m*x + 4*e**2*p**2*x + 2*e**2*p*x + 6*e*m**2 + 18*e*m*p
 + 6*e*m + 12*e*p**2 + 6*e*p),x)*c**3*m*p**3 + 27*int(((e*x + 3)**m*(c*x**
2 + 2*x)**p*x)/(3*c**2*e*m*p*x**2 + 6*c**2*e*p**2*x**2 + 3*c**2*e*p*x**2 +
 9*c**2*m*p*x + 18*c**2*p**2*x + 9*c**2*p*x + c*e**2*m**2*x**2 + 3*c*e**2*
m*p*x**2 + c*e**2*m*x**2 + 2*c*e**2*p**2*x**2 + c*e**2*p*x**2 + 3*c*e*m**2
*x + 15*c*e*m*p*x + 3*c*e*m*x + 18*c*e*p**2*x + 9*c*e*p*x + 18*c*m*p + ...
```

3.259 $\int (3+ex)^m \left(bx+cx^2\right)^p\,dx$

Optimal result 2095
Mathematica [A] (verified) 2095
Rubi [A] (verified) 2096
Maple [F] 2097
Fricas [F] 2097
Sympy [F] 2098
Maxima [F] 2098
Giac [F] 2098
Mupad [F(-1)] 2099
Reduce [F] 2099

Optimal result

Integrand size = 19, antiderivative size = 59

$$\int (3+ex)^m \left(bx+cx^2\right)^p\,dx = \frac{3^m x \left(1+\frac{cx}{b}\right)^{-p} \left(bx+cx^2\right)^p \operatorname{AppellF1}\left(1+p,-p,-m,2+p,-\frac{cx}{b},-\frac{ex}{3}\right)}{1+p}$$

output
```
3^m*x*(c*x^2+b*x)^p*AppellF1(p+1,-m,-p,2+p,-1/3*e*x,-c*x/b)/(p+1)/((1+c*x/
b)^p)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

$$\int (3+ex)^m \left(bx+cx^2\right)^p\,dx = \frac{3^m x \left(\frac{b+cx}{b}\right)^{-p} (x(b+cx))^p \operatorname{AppellF1}\left(1+p,-p,-m,2+p,-\frac{cx}{b},-\frac{ex}{3}\right)}{1+p}$$

input
```
Integrate[(3 + e*x)^m*(b*x + c*x^2)^p,x]
```

output
```
(3^m*x*(x*(b + c*x))^p*AppellF1[1 + p, -p, -m, 2 + p, -((c*x)/b), -1/3*(e*
x)])/((1 + p)*((b + c*x)/b)^p)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.75, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex+3)^m \left(bx+cx^2\right)^p \, dx$$

$$\Big\downarrow \text{1179}$$

$$\frac{3^p(-ex)^{-p} \left(bx+cx^2\right)^p \left(1-\frac{c(ex+3)}{3c-be}\right)^{-p} \int (ex+3)^m \left(\frac{1}{3}(-ex-3)+1\right)^p \left(1-\frac{c(ex+3)}{3c-be}\right)^p d(ex+3)}{e}$$

$$\Big\downarrow \text{150}$$

$$\frac{3^p(ex+3)^{m+1}(-ex)^{-p} \left(bx+cx^2\right)^p \left(1-\frac{c(ex+3)}{3c-be}\right)^{-p} \text{AppellF1}\left(m+1,-p,-p,m+2,\frac{1}{3}(ex+3),\frac{c(ex+3)}{3c-be}\right)}{e(m+1)}$$

input
```
Int[(3 + e*x)^m*(b*x + c*x^2)^p,x]
```

output
```
(3^p*(3 + e*x)^(1 + m)*(b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (3 +
 e*x)/3, (c*(3 + e*x))/(3*c - b*e)])/(e*(1 + m)*(-(e*x))^p*(1 - (c*(3 + e*
x))/(3*c - b*e))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int (ex+3)^{m} \left(c\,x^{2}+bx\right)^{p} dx$$

input
```
int((e*x+3)^m*(c*x^2+b*x)^p,x)
```

output
```
int((e*x+3)^m*(c*x^2+b*x)^p,x)
```

Fricas [F]

$$\int (3+ex)^m \left(bx+cx^2\right)^p \, dx = \int \left(cx^2+bx\right)^p (ex+3)^m \, dx$$

input
```
integrate((e*x+3)^m*(c*x^2+b*x)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^p*(e*x + 3)^m, x)
```

Sympy [F]

$$\int (3+ex)^m \left(bx+cx^2\right)^p \, dx = \int (x(b+cx))^p \, (ex+3)^m \, dx$$

input
```
integrate((e*x+3)**m*(c*x**2+b*x)**p,x)
```

output
```
Integral((x*(b + c*x))**p*(e*x + 3)**m, x)
```

Maxima [F]

$$\int (3+ex)^m \left(bx+cx^2\right)^p \, dx = \int (cx^2+bx)^p (ex+3)^m \, dx$$

input
```
integrate((e*x+3)^m*(c*x^2+b*x)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^p*(e*x + 3)^m, x)
```

Giac [F]

$$\int (3+ex)^m \left(bx+cx^2\right)^p \, dx = \int (cx^2+bx)^p (ex+3)^m \, dx$$

input
```
integrate((e*x+3)^m*(c*x^2+b*x)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^p*(e*x + 3)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (3+ex)^m \left(bx+cx^2\right)^p \, dx = \int \left(c\,x^2+b\,x\right)^p \,(e\,x+3)^m \, dx$$

input
```
int((b*x + c*x^2)^p*(e*x + 3)^m,x)
```

output
```
int((b*x + c*x^2)^p*(e*x + 3)^m, x)
```

Reduce [F]

$$\int (3+ex)^m \left(bx+cx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((e*x+3)^m*(c*x^2+b*x)^p,x)
```

output

```
((e*x + 3)**m*(b*x + c*x**2)**p*b*e*m*x + (e*x + 3)**m*(b*x + c*x**2)**p*b
*e*p*x + 3*(e*x + 3)**m*(b*x + c*x**2)**p*b*m + 3*(e*x + 3)**m*(b*x + c*x*
*2)**p*b*p + 6*(e*x + 3)**m*(b*x + c*x**2)**p*c*p*x + int(((e*x + 3)**m*(b
*x + c*x**2)**p*x)/(b**2*e**2*m**2*x + 3*b**2*e**2*m*p*x + b**2*e**2*m*x +
 2*b**2*e**2*p**2*x + b**2*e**2*p*x + 3*b**2*e*m**2 + 9*b**2*e*m*p + 3*b**
2*e*m + 6*b**2*e*p**2 + 3*b**2*e*p + b*c*e**2*m**2*x**2 + 3*b*c*e**2*m*p*x
**2 + b*c*e**2*m*x**2 + 2*b*c*e**2*p**2*x**2 + b*c*e**2*p*x**2 + 3*b*c*e*m
**2*x + 15*b*c*e*m*p*x + 3*b*c*e*m*x + 18*b*c*e*p**2*x + 9*b*c*e*p*x + 18*
b*c*m*p + 36*b*c*p**2 + 18*b*c*p + 6*c**2*e*m*p*x**2 + 12*c**2*e*p**2*x**2
 + 6*c**2*e*p*x**2 + 18*c**2*m*p*x + 36*c**2*p**2*x + 18*c**2*p*x),x)*b**3
*e**3*m**3*p + 4*int(((e*x + 3)**m*(b*x + c*x**2)**p*x)/(b**2*e**2*m**2*x
+ 3*b**2*e**2*m*p*x + b**2*e**2*m*x + 2*b**2*e**2*p**2*x + b**2*e**2*p*x +
 3*b**2*e*m**2 + 9*b**2*e*m*p + 3*b**2*e*m + 6*b**2*e*p**2 + 3*b**2*e*p +
b*c*e**2*m**2*x**2 + 3*b*c*e**2*m*p*x**2 + b*c*e**2*m*x**2 + 2*b*c*e**2*p*
*2*x**2 + b*c*e**2*p*x**2 + 3*b*c*e*m**2*x + 15*b*c*e*m*p*x + 3*b*c*e*m*x
+ 18*b*c*e*p**2*x + 9*b*c*e*p*x + 18*b*c*m*p + 36*b*c*p**2 + 18*b*c*p + 6*
c**2*e*m*p*x**2 + 12*c**2*e*p**2*x**2 + 6*c**2*e*p*x**2 + 18*c**2*m*p*x +
36*c**2*p**2*x + 18*c**2*p*x),x)*b**3*e**3*m**2*p**2 + int(((e*x + 3)**m*(
b*x + c*x**2)**p*x)/(b**2*e**2*m**2*x + 3*b**2*e**2*m*p*x + b**2*e**2*m*x
+ 2*b**2*e**2*p**2*x + b**2*e**2*p*x + 3*b**2*e*m**2 + 9*b**2*e*m*p + 3...
```

3.260 $\int (d+ex)^m \left(2x+cx^2\right)^p \, dx$

Optimal result . . . 2101
Mathematica [A] (verified) . . . 2101
Rubi [A] (verified) . . . 2102
Maple [F] . . . 2103
Fricas [F] . . . 2103
Sympy [F] . . . 2104
Maxima [F] . . . 2104
Giac [F] . . . 2104
Mupad [F(-1)] . . . 2105
Reduce [F] . . . 2105

Optimal result

Integrand size = 19, antiderivative size = 75

$$\int (d+ex)^m \left(2x+cx^2\right)^p \, dx = \frac{2^p x (2+cx)^{-p} (d+ex)^m \left(1+\frac{ex}{d}\right)^{-m} \left(2x+cx^2\right)^p \operatorname{AppellF1}\left(1+p,-p,-m,2+p,-\frac{cx}{2},-\frac{ex}{d}\right)}{1+p}$$

output
```
2^p*x*(e*x+d)^m*(c*x^2+2*x)^p*AppellF1(p+1,-p,-m,2+p,-1/2*c*x,-e*x/d)/(p+1
)/((c*x+2)^p)/((1+e*x/d)^m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int (d+ex)^m \left(2x+cx^2\right)^p \, dx = \frac{2^p x (2+cx)^{-p} (x(2+cx))^p (d+ex)^m \left(\frac{d+ex}{d}\right)^{-m} \operatorname{AppellF1}\left(1+p,-p,-m,2+p,-\frac{cx}{2},-\frac{ex}{d}\right)}{1+p}$$

input
```
Integrate[(d + e*x)^m*(2*x + c*x^2)^p,x]
```

output
```
(2^p*x*(x*(2 + c*x))^p*(d + e*x)^m*AppellF1[1 + p, -p, -m, 2 + p, -1/2*(c*
x), -((e*x)/d)])/((1 + p)*(2 + c*x)^p*((d + e*x)/d)^m)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.35, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(cx^2+2x\right)^p (d+ex)^m \, dx$$

$$\Big\downarrow \text{1179}$$

$$\frac{\left(cx^2+2x\right)^p \left(-\frac{ex}{d}\right)^{-p} \left(1-\frac{c(d+ex)}{cd-2e}\right)^{-p} \int (d+ex)^m \left(1-\frac{d+ex}{d}\right)^p \left(1-\frac{c(d+ex)}{cd-2e}\right)^p d(d+ex)}{e}$$

$$\Big\downarrow \text{150}$$

$$\frac{\left(cx^2+2x\right)^p (d+ex)^{m+1} \left(-\frac{ex}{d}\right)^{-p} \left(1-\frac{c(d+ex)}{cd-2e}\right)^{-p} \text{AppellF1}\left(m+1,-p,-p,m+2,\frac{d+ex}{d},\frac{c(d+ex)}{cd-2e}\right)}{e(m+1)}$$

input
```
Int[(d + e*x)^m*(2*x + c*x^2)^p,x]
```

output
```
((d + e*x)^(1 + m)*(2*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (d + e*x
)/d, (c*(d + e*x))/(c*d - 2*e)])/(e*(1 + m)*(-((e*x)/d))^p*(1 - (c*(d + e*
x))/(c*d - 2*e))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int (ex+d)^m \left(c\,x^2+2x\right)^p dx$$

input
```
int((e*x+d)^m*(c*x^2+2*x)^p,x)
```

output
```
int((e*x+d)^m*(c*x^2+2*x)^p,x)
```

Fricas [F]

$$\int (d+ex)^m \left(2x+cx^2\right)^p \, dx = \int \left(cx^2+2\,x\right)^p (ex+d)^m \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+2*x)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + 2*x)^p*(e*x + d)^m, x)
```

Sympy [F]

$$\int (d+ex)^m \left(2x+cx^2\right)^p \,dx = \int (x(cx+2))^p (d+ex)^m \,dx$$

input
```
integrate((e*x+d)**m*(c*x**2+2*x)**p,x)
```

output
```
Integral((x*(c*x + 2))**p*(d + e*x)**m, x)
```

Maxima [F]

$$\int (d+ex)^m \left(2x+cx^2\right)^p \,dx = \int \left(cx^2+2\,x\right)^p (ex+d)^m \,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+2*x)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + 2*x)^p*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m \left(2x+cx^2\right)^p \,dx = \int \left(cx^2+2\,x\right)^p (ex+d)^m \,dx$$

input
```
integrate((e*x+d)^m*(c*x^2+2*x)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + 2*x)^p*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \left(2x+cx^2\right)^p \, dx = \int \left(c\,x^2+2\,x\right)^p\,\left(d+e\,x\right)^m \, dx$$

input
```
int((2*x + c*x^2)^p*(d + e*x)^m,x)
```

output
```
int((2*x + c*x^2)^p*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m \left(2x+cx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((e*x+d)^m*(c*x^2+2*x)^p,x)
```

output

```
((d + e*x)**m*(c*x**2 + 2*x)**p*c*d*p*x + (d + e*x)**m*(c*x**2 + 2*x)**p*d
*m + (d + e*x)**m*(c*x**2 + 2*x)**p*d*p + (d + e*x)**m*(c*x**2 + 2*x)**p*e
*m*x + (d + e*x)**m*(c*x**2 + 2*x)**p*e*p*x + int(((d + e*x)**m*(c*x**2 +
2*x)**p*x)/(c**2*d**2*m*p*x + 2*c**2*d**2*p**2*x + c**2*d**2*p*x + c**2*d*
e*m*p*x**2 + 2*c**2*d*e*p**2*x**2 + c**2*d*e*p*x**2 + 2*c*d**2*m*p + 4*c*d
**2*p**2 + 2*c*d**2*p + c*d*e*m**2*x + 5*c*d*e*m*p*x + c*d*e*m*x + 6*c*d*e
*p**2*x + 3*c*d*e*p*x + c*e**2*m**2*x**2 + 3*c*e**2*m*p*x**2 + c*e**2*m*x*
*2 + 2*c*e**2*p**2*x**2 + c*e**2*p*x**2 + 2*d*e*m**2 + 6*d*e*m*p + 2*d*e*m
 + 4*d*e*p**2 + 2*d*e*p + 2*e**2*m**2*x + 6*e**2*m*p*x + 2*e**2*m*x + 4*e*
*2*p**2*x + 2*e**2*p*x),x)*c**3*d**3*m**2*p**2 + 2*int(((d + e*x)**m*(c*x*
*2 + 2*x)**p*x)/(c**2*d**2*m*p*x + 2*c**2*d**2*p**2*x + c**2*d**2*p*x + c*
*2*d*e*m*p*x**2 + 2*c**2*d*e*p**2*x**2 + c**2*d*e*p*x**2 + 2*c*d**2*m*p +
4*c*d**2*p**2 + 2*c*d**2*p + c*d*e*m**2*x + 5*c*d*e*m*p*x + c*d*e*m*x + 6*
c*d*e*p**2*x + 3*c*d*e*p*x + c*e**2*m**2*x**2 + 3*c*e**2*m*p*x**2 + c*e**2
*m*x**2 + 2*c*e**2*p**2*x**2 + c*e**2*p*x**2 + 2*d*e*m**2 + 6*d*e*m*p + 2*
d*e*m + 4*d*e*p**2 + 2*d*e*p + 2*e**2*m**2*x + 6*e**2*m*p*x + 2*e**2*m*x +
 4*e**2*p**2*x + 2*e**2*p*x),x)*c**3*d**3*m*p**3 + int(((d + e*x)**m*(c*x*
*2 + 2*x)**p*x)/(c**2*d**2*m*p*x + 2*c**2*d**2*p**2*x + c**2*d**2*p*x + c*
*2*d*e*m*p*x**2 + 2*c**2*d*e*p**2*x**2 + c**2*d*e*p*x**2 + 2*c*d**2*m*p +
4*c*d**2*p**2 + 2*c*d**2*p + c*d*e*m**2*x + 5*c*d*e*m*p*x + c*d*e*m*x +...
```

3.261 $\int (d+ex)^m (bx+cx^2)^p \, dx$

Optimal result 2107
Mathematica [A] (verified) 2107
Rubi [A] (verified) 2108
Maple [F] 2109
Fricas [F] 2109
Sympy [F] 2110
Maxima [F] 2110
Giac [F] 2110
Mupad [F(-1)] 2111
Reduce [F] 2111

Optimal result

Integrand size = 19, antiderivative size = 76

$$\int (d+ex)^m (bx+cx^2)^p \, dx = \frac{x\left(1+\frac{cx}{b}\right)^{-p}(d+ex)^m\left(1+\frac{ex}{d}\right)^{-m}(bx+cx^2)^p \operatorname{AppellF1}\left(1+p,-p,-m,2+p,-\frac{cx}{b},-\frac{ex}{d}\right)}{1+p}$$

output
```
x*(e*x+d)^m*(c*x^2+b*x)^p*AppellF1(p+1,-p,-m,2+p,-c*x/b,-e*x/d)/(p+1)/((1+
c*x/b)^p)/((1+e*x/d)^m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int (d+ex)^m (bx+cx^2)^p \, dx = \frac{x\left(\frac{b+cx}{b}\right)^{-p}(x(b+cx))^p(d+ex)^m\left(\frac{d+ex}{d}\right)^{-m} \operatorname{AppellF1}\left(1+p,-p,-m,2+p,-\frac{cx}{b},-\frac{ex}{d}\right)}{1+p}$$

input
```
Integrate[(d + e*x)^m*(b*x + c*x^2)^p,x]
```

output
```
(x*(x*(b + c*x))^p*(d + e*x)^m*AppellF1[1 + p, -p, -m, 2 + p, -((c*x)/b),
-((e*x)/d)])/((1 + p)*((b + c*x)/b)^p*((d + e*x)/d)^m)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.36, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1179, 150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(bx+cx^2\right)^p (d+ex)^m\,dx$$

$$\Big\downarrow \text{1179}$$

$$\frac{\left(bx+cx^2\right)^p \left(-\frac{ex}{d}\right)^{-p} \left(1-\frac{c(d+ex)}{cd-be}\right)^{-p} \int (d+ex)^m \left(1-\frac{d+ex}{d}\right)^p \left(1-\frac{c(d+ex)}{cd-be}\right)^p d(d+ex)}{e}$$

$$\Big\downarrow \text{150}$$

$$\frac{\left(bx+cx^2\right)^p (d+ex)^{m+1} \left(-\frac{ex}{d}\right)^{-p} \left(1-\frac{c(d+ex)}{cd-be}\right)^{-p} \text{AppellF1}\left(m+1,-p,-p,m+2,\frac{d+ex}{d},\frac{c(d+ex)}{cd-be}\right)}{e(m+1)}$$

input
```
Int[(d + e*x)^m*(b*x + c*x^2)^p,x]
```

output
```
((d + e*x)^(1 + m)*(b*x + c*x^2)^p*AppellF1[1 + m, -p, -p, 2 + m, (d + e*x
)/d, (c*(d + e*x))/(c*d - b*e)])/(e*(1 + m)*(-((e*x)/d))^p*(1 - (c*(d + e*
x))/(c*d - b*e))^p)
```

Defintions of rubi rules used

rule 150
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p, m + 2
, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &&  !In
tegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

rule 1179
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[(a + b*x + c*x^2)^p/(e*(1 - (
d + e*x)/(d - e*((b - q)/(2*c))))^p*(1 - (d + e*x)/(d - e*((b + q)/(2*c))))
^p)   Subst[Int[x^m*Simp[1 - x/(d - e*((b - q)/(2*c))), x]^p*Simp[1 - x/(d
- e*((b + q)/(2*c))), x]^p, x], x, d + e*x], x]] /; FreeQ[{a, b, c, d, e, m
, p}, x]
```

Maple [F]

$$\int (ex+d)^{m} \left(c\,x^{2}+bx\right)^{p} dx$$

input
```
int((e*x+d)^m*(c*x^2+b*x)^p,x)
```

output
```
int((e*x+d)^m*(c*x^2+b*x)^p,x)
```

Fricas [F]

$$\int (d+ex)^m \left(bx+cx^2\right)^p \, dx = \int \left(cx^2+bx\right)^p (ex+d)^m \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x)^p,x, algorithm="fricas")
```

output
```
integral((c*x^2 + b*x)^p*(e*x + d)^m, x)
```

Sympy [F]

$$\int (d+ex)^m \left(bx+cx^2\right)^p \, dx = \int (x(b+cx))^p (d+ex)^m \, dx$$

input
```
integrate((e*x+d)**m*(c*x**2+b*x)**p,x)
```

output
```
Integral((x*(b + c*x))**p*(d + e*x)**m, x)
```

Maxima [F]

$$\int (d+ex)^m \left(bx+cx^2\right)^p \, dx = \int \left(cx^2+bx\right)^p (ex+d)^m \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x)^p,x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^p*(e*x + d)^m, x)
```

Giac [F]

$$\int (d+ex)^m \left(bx+cx^2\right)^p \, dx = \int \left(cx^2+bx\right)^p (ex+d)^m \, dx$$

input
```
integrate((e*x+d)^m*(c*x^2+b*x)^p,x, algorithm="giac")
```

output
```
integrate((c*x^2 + b*x)^p*(e*x + d)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (d+ex)^m \left(bx+cx^2\right)^p \, dx = \int \left(c\,x^2+b\,x\right)^p\,(d+e\,x)^m \,dx$$

input
```
int((b*x + c*x^2)^p*(d + e*x)^m,x)
```

output
```
int((b*x + c*x^2)^p*(d + e*x)^m, x)
```

Reduce [F]

$$\int (d+ex)^m \left(bx+cx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((e*x+d)^m*(c*x^2+b*x)^p,x)
```

output
```
((d + e*x)**m*(b*x + c*x**2)**p*b*d*m + (d + e*x)**m*(b*x + c*x**2)**p*b*d
*p + (d + e*x)**m*(b*x + c*x**2)**p*b*e*m*x + (d + e*x)**m*(b*x + c*x**2)*
*p*b*e*p*x + 2*(d + e*x)**m*(b*x + c*x**2)**p*c*d*p*x + int(((d + e*x)**m*
(b*x + c*x**2)**p*x)/(b**2*d*e*m**2 + 3*b**2*d*e*m*p + b**2*d*e*m + 2*b**2
*d*e*p**2 + b**2*d*e*p + b**2*e**2*m**2*x + 3*b**2*e**2*m*p*x + b**2*e**2*
m*x + 2*b**2*e**2*p**2*x + b**2*e**2*p*x + 2*b*c*d**2*m*p + 4*b*c*d**2*p**
2 + 2*b*c*d**2*p + b*c*d*e*m**2*x + 5*b*c*d*e*m*p*x + b*c*d*e*m*x + 6*b*c*
d*e*p**2*x + 3*b*c*d*e*p*x + b*c*e**2*m**2*x**2 + 3*b*c*e**2*m*p*x**2 + b*
c*e**2*m*x**2 + 2*b*c*e**2*p**2*x**2 + b*c*e**2*p*x**2 + 2*c**2*d**2*m*p*x
 + 4*c**2*d**2*p**2*x + 2*c**2*d**2*p*x + 2*c**2*d*e*m*p*x**2 + 4*c**2*d*e
*p**2*x**2 + 2*c**2*d*e*p*x**2),x)*b**3*e**3*m**3*p + 4*int(((d + e*x)**m*
(b*x + c*x**2)**p*x)/(b**2*d*e*m**2 + 3*b**2*d*e*m*p + b**2*d*e*m + 2*b**2
*d*e*p**2 + b**2*d*e*p + b**2*e**2*m**2*x + 3*b**2*e**2*m*p*x + b**2*e**2*
m*x + 2*b**2*e**2*p**2*x + b**2*e**2*p*x + 2*b*c*d**2*m*p + 4*b*c*d**2*p**
2 + 2*b*c*d**2*p + b*c*d*e*m**2*x + 5*b*c*d*e*m*p*x + b*c*d*e*m*x + 6*b*c*
d*e*p**2*x + 3*b*c*d*e*p*x + b*c*e**2*m**2*x**2 + 3*b*c*e**2*m*p*x**2 + b*
c*e**2*m*x**2 + 2*b*c*e**2*p**2*x**2 + b*c*e**2*p*x**2 + 2*c**2*d**2*m*p*x
 + 4*c**2*d**2*p**2*x + 2*c**2*d**2*p*x + 2*c**2*d*e*m*p*x**2 + 4*c**2*d*e
*p**2*x**2 + 2*c**2*d*e*p*x**2),x)*b**3*e**3*m**2*p**2 + int(((d + e*x)**m
*(b*x + c*x**2)**p*x)/(b**2*d*e*m**2 + 3*b**2*d*e*m*p + b**2*d*e*m + 2*...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 2113
4.2 Links to plain text integration problems used in this report for each CAS . 2131

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expn
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).__name__ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file