Computer Algebra Independent Integration Tests

Summer 2024

1-Algebraic-functions/1.1-Binomial/1.1.6-Improper-linear-binomial/82-1.1.6.4

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**376**]. This is test number [82].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (376)	0.00 (0)
Mathematica	100.00 (376)	0.00 (0)
Maple	94.95 (357)	5.05 (19)
Fricas	94.95 (357)	5.05 (19)
Reduce	94.95 (357)	5.05 (19)
Giac	92.29 (347)	7.71 (29)
Maxima	60.90 (229)	39.10 (147)
Mupad	53.72 (202)	46.28 (174)
Sympy	33.51 (126)	66.49 (250)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

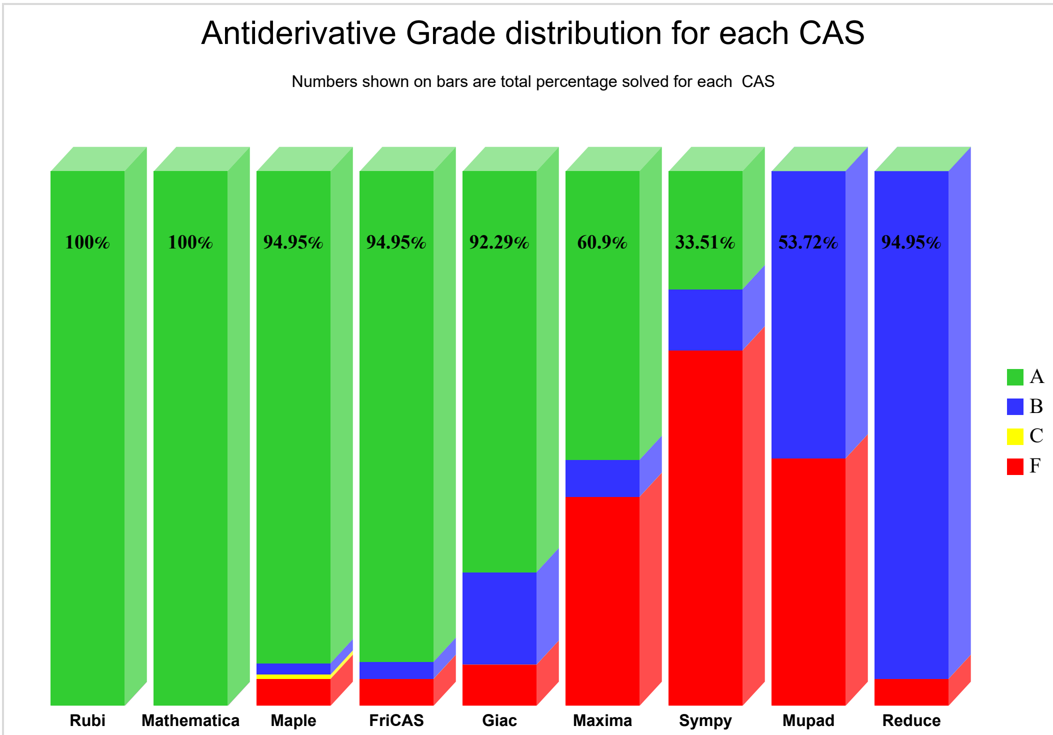
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

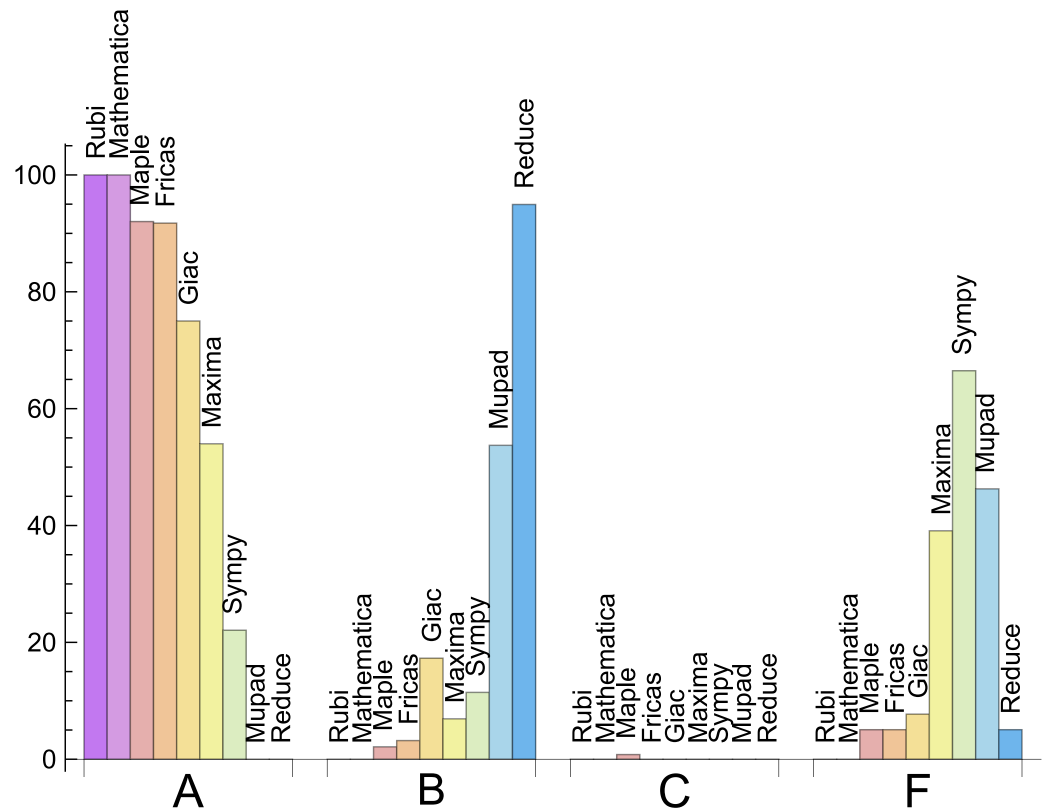
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.000	0.000	0.000	0.000
Mathematica	100.000	0.000	0.000	0.000
Maple	92.021	2.128	0.798	5.053
Fricas	91.755	3.191	0.000	5.053
Giac	75.000	17.287	0.000	7.713
Maxima	53.989	6.915	0.000	39.096
Sympy	22.074	11.436	0.000	66.489
Mupad	0.000	53.723	0.000	46.277
Reduce	0.000	94.947	0.000	5.053

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	19	68.42	0.00	31.58
Maple	19	100.00	0.00	0.00
Reduce	19	100.00	0.00	0.00
Giac	29	93.10	6.90	0.00
Maxima	147	100.00	0.00	0.00
Mupad	174	0.00	100.00	0.00
Sympy	250	85.60	14.40	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.04
Fricas	0.10
Giac	0.19
Mathematica	0.20
Reduce	0.21
Rubi	0.46
Maple	0.75
Mupad	4.93
Sympy	6.85

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Maple	93.27	0.83	78.00	0.76
Mathematica	93.86	0.83	86.00	0.84
Mupad	113.07	1.15	80.50	0.93
Rubi	116.93	1.00	109.00	1.00
Maxima	127.44	1.21	97.00	1.07
Reduce	146.83	1.25	133.00	1.12
Fricas	180.43	1.49	152.00	1.45
Giac	183.71	1.52	125.00	1.05
Sympy	482.60	4.50	93.50	1.31

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

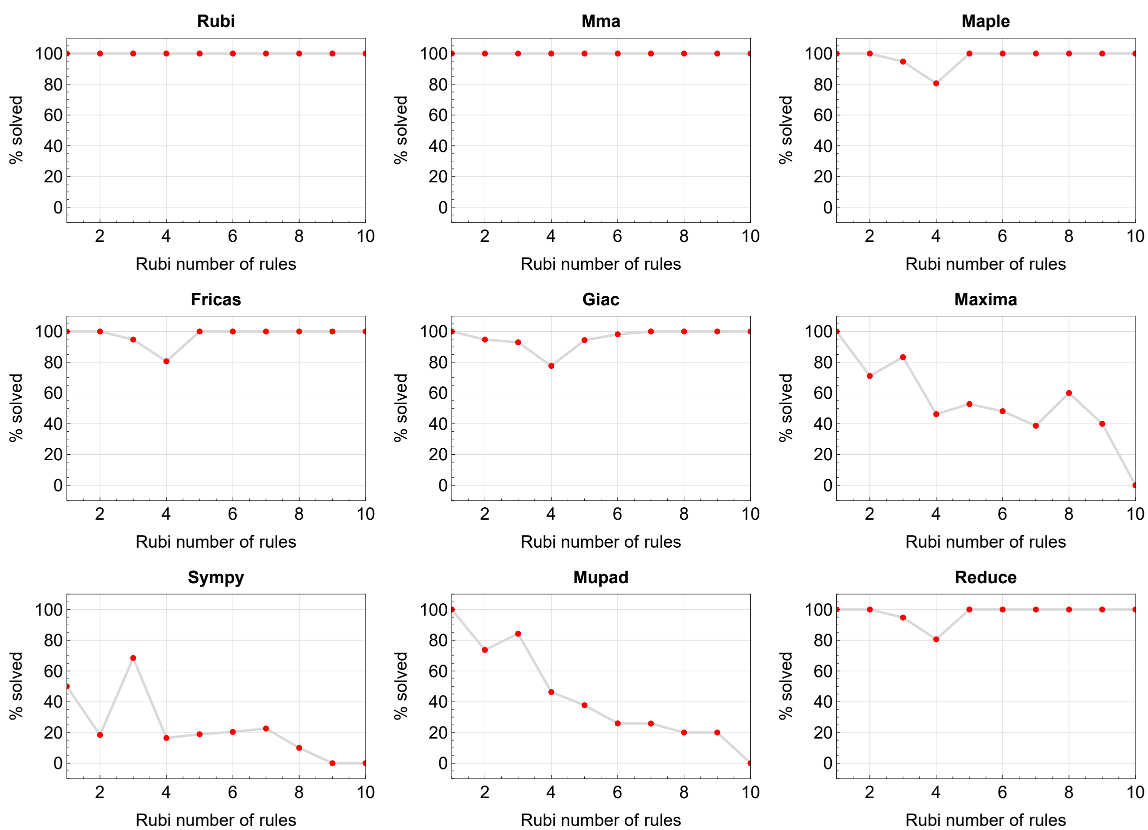


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

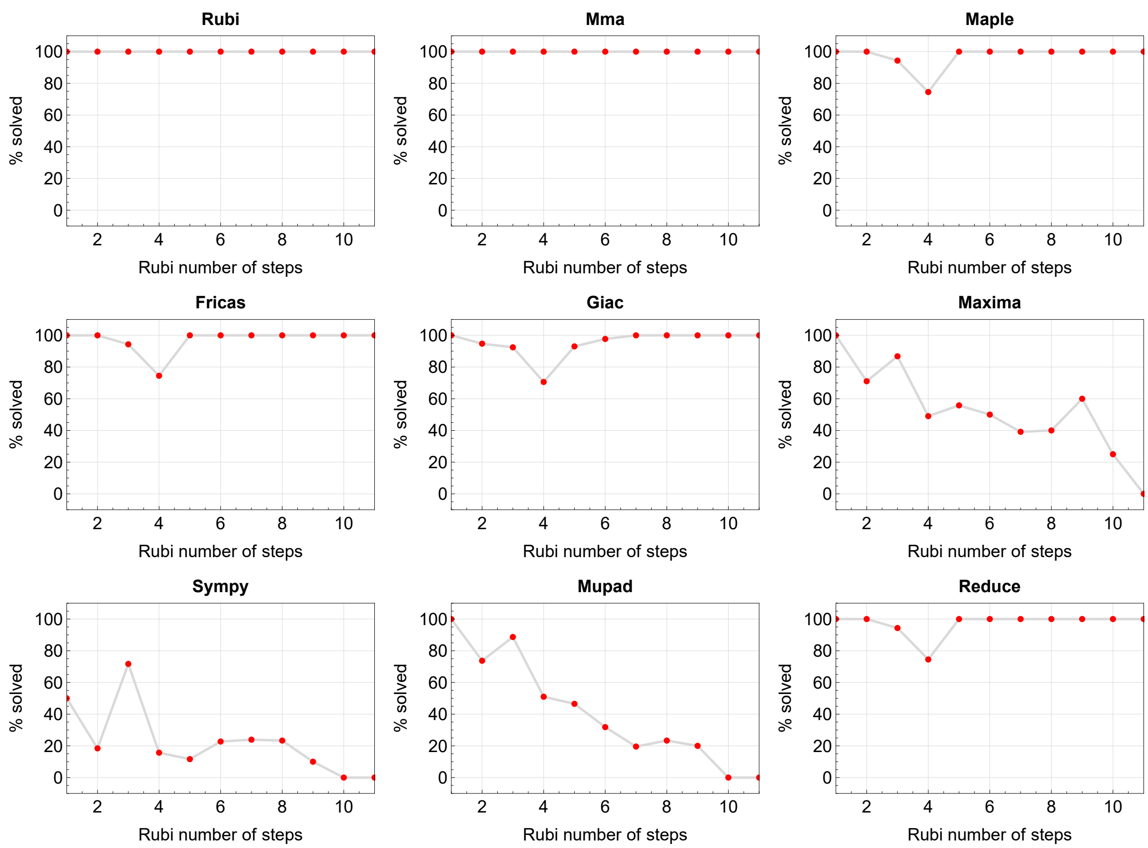


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

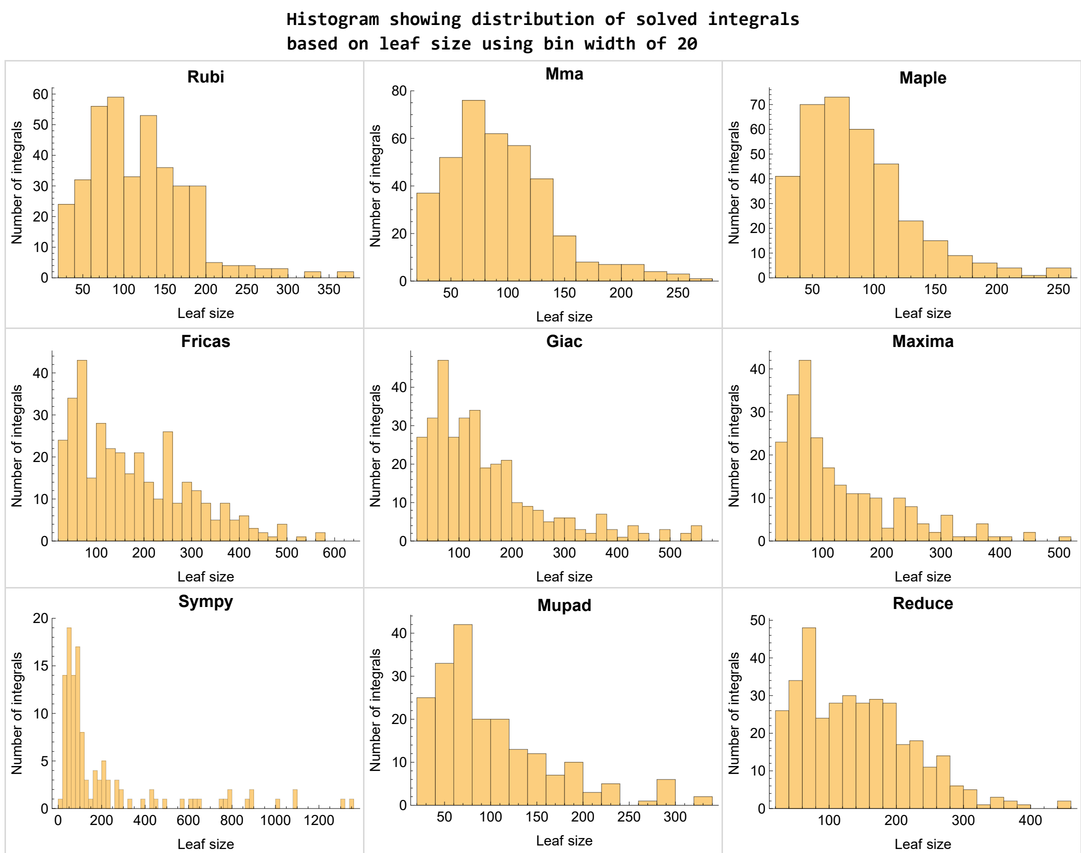


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

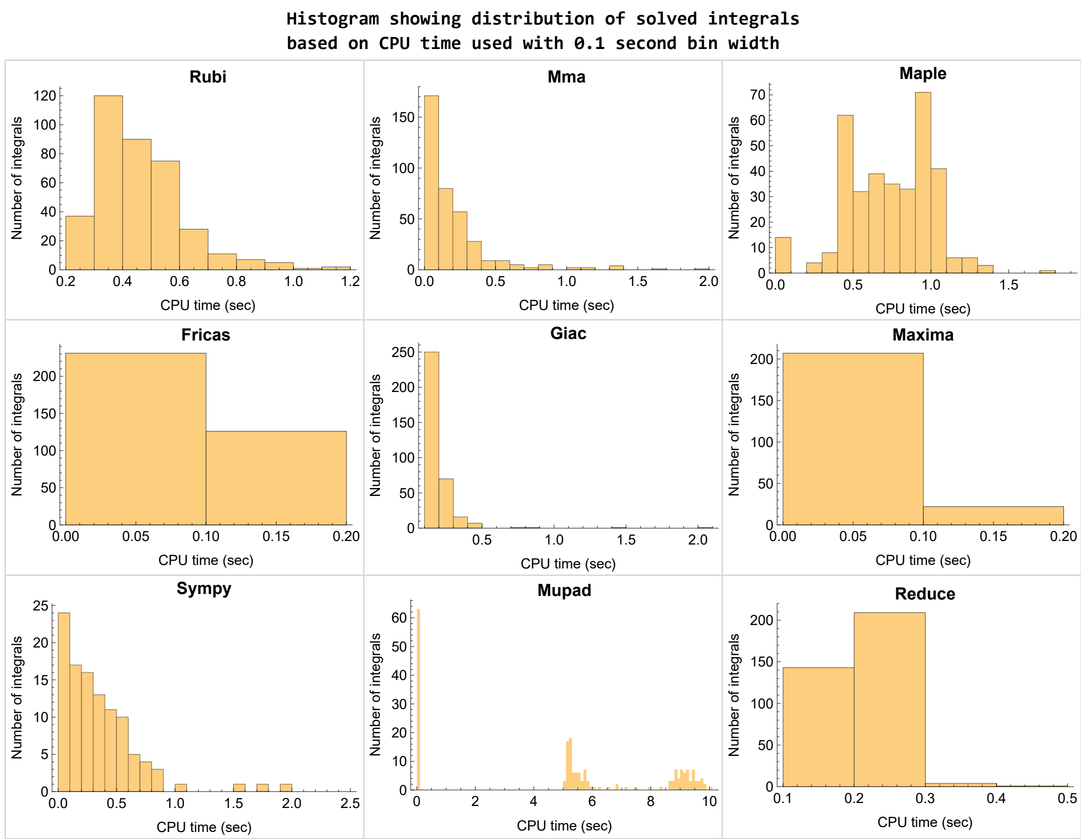


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

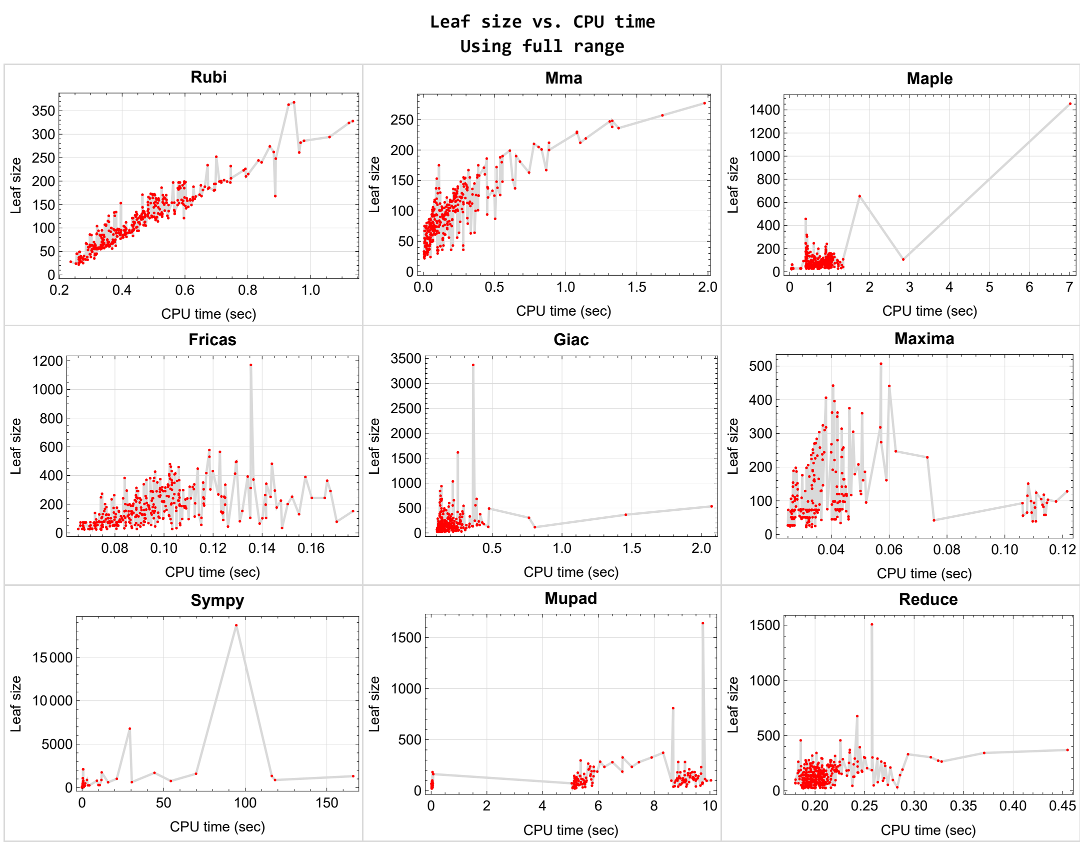


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple $\{4, 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

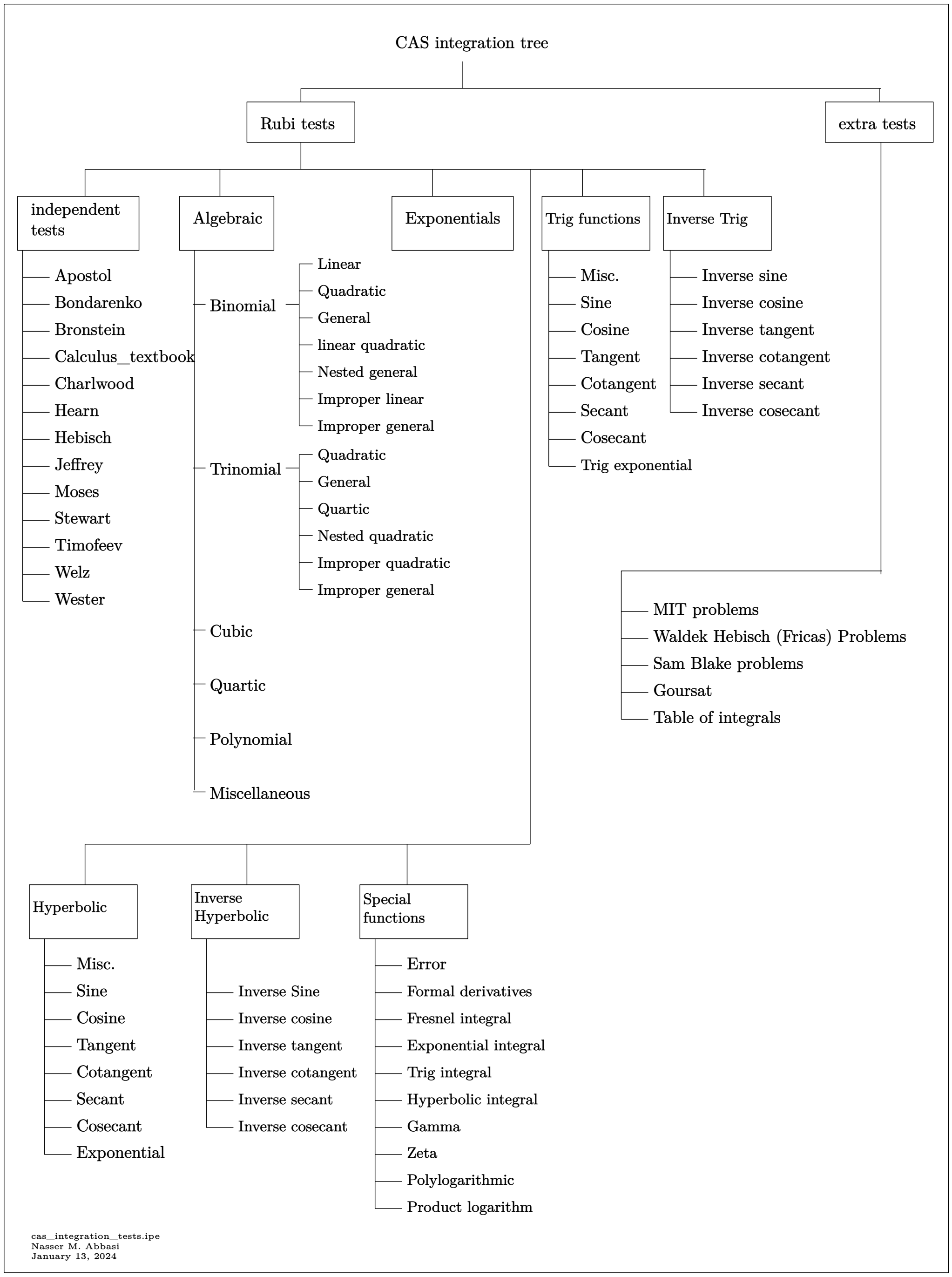
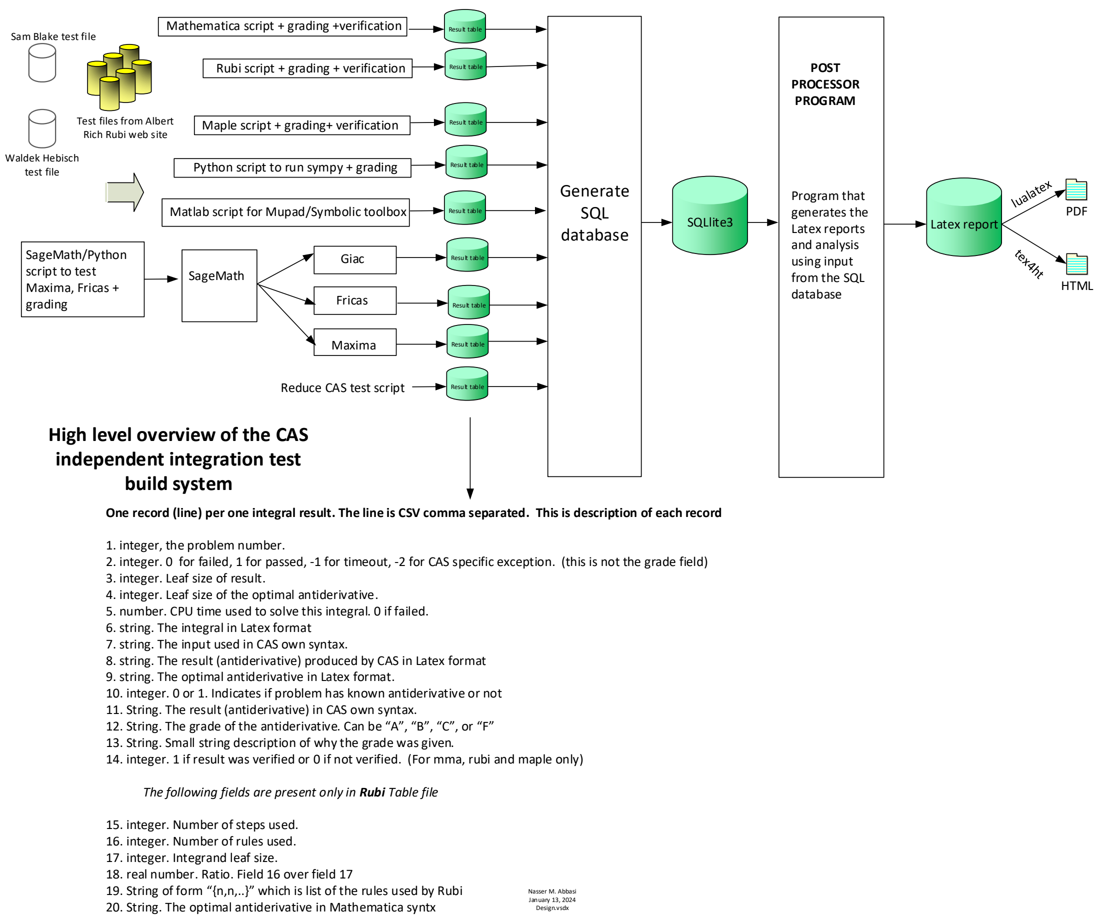


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 343, 344, 345, 346, 347, 350, 351, 352, 353, 354, 355, 357, 358 }

B grade { 26, 342, 348, 349, 356, 364, 365, 366 }

C grade { 249, 261, 262 }

F normal fail { 232, 233, 234, 235, 359, 360, 361, 362, 363, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 231, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 358 }

B grade { 26, 56, 57, 137, 197, 229, 230, 356, 357, 364, 365, 366 }

C grade { }

F normal fail { 232, 233, 234, 235, 359, 360, 361, 362, 363, 367, 368, 369, 376 }

F(-1) timedout fail { }

F(-2) exception fail { 370, 371, 372, 373, 374, 375 }

Maxima

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 130, 131, 132, 133, 134, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 182, 183, 204, 205, 206, 207, 229, 230, 231, 236, 237, 238, 239, 240, 246, 247, 248, 249, 250, 251, 258, 259, 260, 261, 262, 263, 264, 265, 273, 274, 275, 276, 282, 283, 284, 285, 291, 292, 293, 294, 356, 357, 358, 364, 365, 366 }

B grade { 26, 109, 122, 123, 124, 125, 126, 135, 136, 137, 138, 139, 140, 141, 162, 163, 164, 165, 184, 185, 186, 193, 194, 195, 196, 197 }

C grade { }

F normal fail { 176, 177, 178, 179, 180, 181, 187, 188, 189, 190, 191, 192, 198, 199, 200, 201, 202, 203, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 232, 233, 234, 235, 241, 242, 243, 244, 245, 252, 253, 254, 255, 256, 257, 266, 267, 268, 269, 270, 271, 272, 277, 278, 279, 280, 281, 286, 287, 288, 289, 290, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 359, 360, 361, 362, 363, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 114, 115, 116, 117, 118, 119, 120, 127, 128, 129, 130, 131, 132, 133, 134, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 162, 167, 171, 172, 173, 174, 176, 177, 178, 179, 180, 181, 182, 183, 184, 187, 188, 189, 190, 191, 192, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 237, 241, 242, 243, 244, 245, 252, 253, 254, 255, 256, 257, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 347, 348, 349, 350, 351 }

B grade { 26, 108, 109, 110, 111, 112, 113, 121, 122, 123, 124, 125, 126, 135, 136, 137, 138, 139, 140, 141, 163, 164, 165, 175, 185, 186, 193, 194, 195, 196, 197, 229, 230, 231, 236, 238, 239, 240, 246, 247, 248, 249, 250, 251, 258, 259, 260, 261, 262, 263, 264, 265, 310, 335, 344, 345, 346, 352, 355, 356, 357, 358, 364, 365, 366 }

C grade { }

F normal fail { 158, 159, 160, 161, 166, 168, 169, 170, 232, 233, 234, 235, 359, 360, 361, 362, 363, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

F(-1) timedout fail { 353, 354 }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 109, 110, 111, 112, 113, 116, 122, 123, 124, 125, 126, 137, 138, 139, 140, 141, 146, 147, 148, 149, 150, 151, 152, 156, 157, 158, 159, 160, 161, 165, 166, 167, 168, 169, 170, 229, 230, 231, 236, 237, 238, 239, 246, 247, 248, 249, 250, 251, 258, 259, 260, 261, 262, 263, 264, 265, 273, 274, 275, 276, 282, 283, 284, 285, 291, 292, 293, 294, 307, 308, 309, 318, 319, 320, 329, 330, 331, 337, 338, 339, 343, 344, 345, 346, 350, 351, 352, 353, 354, 355, 356, 357, 358, 364, 365, 366 }

C grade { }

F normal fail { }

F(-1) timedout fail { 107, 108, 114, 115, 117, 118, 119, 120, 121, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 142, 143, 144, 145, 153, 154, 155, 162, 163, 164, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 232, 233, 234, 235, 240, 241, 242, 243, 244, 245, 252, 253, 254, 255, 256, 257, 266, 267, 268, 269, 270, 271, 272, 277, 278, 279, 280, 281, 286, 287, 288, 289, 290, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 310, 311, 312, 313, 314, 315, 316, 317, 321, 322, 323, 324, 325, 326, 327, 328, 332, 333, 334, 335, 336, 340, 341, 342, 347, 348, 349, 359, 360, 361, 362, 363, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 28, 29, 30, 31, 33, 34, 35, 36, 37, 38, 39, 43, 44, 45, 46, 47, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 103, 104, 105, 114, 115, 116, 117, 130, 131, 142, 143, 144, 145 }

B grade { 26, 32, 40, 41, 42, 48, 49, 50, 56, 57, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 99, 100, 101, 102, 127, 128, 129, 146, 229, 230, 231, 236, 356, 357, 358, 364, 365, 366 }

C grade { }

F normal fail { 106, 107, 108, 109, 110, 111, 112, 113, 118, 119, 120, 121, 122, 123, 124, 125, 126, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 194, 195, 196, 197, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 210, 211, 214, 215, 216, 217, 218, 219, 220, 223, 224, 225, 226, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 311, 312, 313, 314, 315, 321, 322, 323, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 351, 352, 353, 354, 355, 359, 360, 361, 362, 363, 367, 368, 372, 373, 374, 375 }

F(-1) timedout fail { 96, 97, 98, 181, 192, 193, 202, 203, 212, 213, 221, 222, 227, 228, 309, 316, 317, 318, 319, 320, 324, 325, 326, 327, 328, 329, 330, 331, 347, 348, 349, 350, 369, 370, 371, 376 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 364, 365, 366 }

C grade { }

F normal fail { 232, 233, 234, 235, 359, 360, 361, 362, 363, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85	0.85
time (sec)	N/A	0.276	0.005	0.087	0.025	0.067	0.021	0.139	0.208	0.025

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	27	27	29	29	28	28
N.S.	1	1.00	1.00	0.85	0.82	0.82	0.88	0.88	0.85	0.85
time (sec)	N/A	0.283	0.005	0.085	0.026	0.089	0.016	0.133	0.197	0.021

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	28	27	27	29	29	28	28
N.S.	1	1.00	0.88	0.85	0.82	0.82	0.88	0.88	0.85	0.85
time (sec)	N/A	0.283	0.005	0.086	0.027	0.065	0.017	0.117	0.216	0.021

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	25	24	24	26	26	26	25
N.S.	1	1.00	1.00	0.89	0.86	0.86	0.93	0.93	0.93	0.89
time (sec)	N/A	0.271	0.008	0.040	0.026	0.068	0.017	0.121	0.189	0.023

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	22	22	22	22	22	22	22
N.S.	1	1.00	1.00	0.92	0.92	0.92	0.92	0.92	0.92	0.92
time (sec)	N/A	0.253	0.006	0.046	0.033	0.074	0.042	0.114	0.188	5.152

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	22	23	22	26	19	23	28	22
N.S.	1	1.00	1.00	1.05	1.00	1.18	0.86	1.05	1.27	1.00
time (sec)	N/A	0.262	0.008	0.049	0.031	0.070	0.064	0.105	0.190	0.024

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	28	26	25	29	27	26	30	25
N.S.	1	1.00	1.04	0.96	0.93	1.07	1.00	0.96	1.11	0.93
time (sec)	N/A	0.267	0.009	0.042	0.026	0.071	0.111	0.220	0.197	0.019

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	28	28	27	27	31	27	28	27
N.S.	1	1.00	0.90	0.90	0.87	0.87	1.00	0.87	0.90	0.87
time (sec)	N/A	0.266	0.008	0.038	0.027	0.070	0.139	0.190	0.189	0.022

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	29	28	27	27	31	27	28	28
N.S.	1	1.00	0.88	0.85	0.82	0.82	0.94	0.82	0.85	0.85
time (sec)	N/A	0.265	0.007	0.040	0.025	0.069	0.166	0.110	0.187	0.022

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	54	53	51	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	0.98	0.96	0.93	0.93
time (sec)	N/A	0.332	0.010	0.678	0.026	0.066	0.023	0.126	0.204	0.034

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	52	51	51	54	53	51	51
N.S.	1	1.00	1.00	0.95	0.93	0.93	0.98	0.96	0.93	0.93
time (sec)	N/A	0.320	0.008	0.681	0.030	0.080	0.022	0.126	0.220	0.027

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	52	51	51	54	53	51	51
N.S.	1	1.00	0.89	0.95	0.93	0.93	0.98	0.96	0.93	0.93
time (sec)	N/A	0.323	0.009	0.606	0.031	0.072	0.046	0.132	0.190	0.028

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	49	52	51	51	54	53	51	51
N.S.	1	1.00	0.89	0.95	0.93	0.93	0.98	0.96	0.93	0.93
time (sec)	N/A	0.317	0.009	0.668	0.034	0.084	0.022	0.183	0.205	0.027

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	47	49	48	48	49	49	49	47
N.S.	1	1.00	1.24	1.29	1.26	1.26	1.29	1.29	1.29	1.24
time (sec)	N/A	0.301	0.008	0.580	0.027	0.070	0.023	0.153	0.259	0.026

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	43	46	46	46	46	46	45	45
N.S.	1	1.00	1.08	1.15	1.15	1.15	1.15	1.15	1.12	1.12
time (sec)	N/A	0.279	0.014	0.579	0.026	0.078	0.060	0.226	0.238	0.023

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	43	44	46	52	42	46	54	46
N.S.	1	1.00	0.98	1.00	1.05	1.18	0.95	1.05	1.23	1.05
time (sec)	N/A	0.297	0.023	0.598	0.030	0.072	0.092	0.138	0.273	0.027

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	44	43	46	53	46	47	55	46
N.S.	1	1.00	1.00	0.98	1.05	1.20	1.05	1.07	1.25	1.05
time (sec)	N/A	0.299	0.028	0.634	0.027	0.070	0.164	0.123	0.196	0.035

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	47	46	50	53	54	51	53	48
N.S.	1	1.00	0.96	0.94	1.02	1.08	1.10	1.04	1.08	0.98
time (sec)	N/A	0.305	0.024	0.642	0.033	0.072	0.270	0.121	0.194	5.167

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	50	48	51	51	56	51	51	49
N.S.	1	1.00	1.11	1.07	1.13	1.13	1.24	1.13	1.13	1.09
time (sec)	N/A	0.257	0.012	0.645	0.029	0.067	0.380	0.144	0.191	0.023

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	53	48	51	51	56	51	51	51
N.S.	1	1.00	0.96	0.87	0.93	0.93	1.02	0.93	0.93	0.93
time (sec)	N/A	0.310	0.013	0.651	0.026	0.067	0.421	0.253	0.186	0.021

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	75	73	73	82	77	74	69
N.S.	1	1.00	1.00	1.00	0.97	0.97	1.09	1.03	0.99	0.92
time (sec)	N/A	0.378	0.012	0.596	0.033	0.066	0.023	0.109	0.208	0.022

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	75	73	73	82	77	74	69
N.S.	1	1.00	1.00	1.00	0.97	0.97	1.09	1.03	0.99	0.92
time (sec)	N/A	0.353	0.011	0.652	0.030	0.073	0.024	0.107	0.200	0.018

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	75	73	73	80	77	74	69
N.S.	1	1.00	1.00	1.00	0.97	0.97	1.07	1.03	0.99	0.92
time (sec)	N/A	0.371	0.010	0.654	0.033	0.067	0.025	0.107	0.188	0.017

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	75	73	73	82	77	74	69
N.S.	1	1.00	1.00	1.00	0.97	0.97	1.09	1.03	0.99	0.92
time (sec)	N/A	0.353	0.008	0.596	0.034	0.068	0.026	0.157	0.200	0.018

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	67	76	73	73	80	76	74	68
N.S.	1	1.00	1.08	1.23	1.18	1.18	1.29	1.23	1.19	1.10
time (sec)	N/A	0.338	0.014	0.621	0.025	0.084	0.026	0.243	0.191	0.018

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	67	73	69	69	73	72	72	65
N.S.	1	1.00	1.76	1.92	1.82	1.82	1.92	1.89	1.89	1.71
time (sec)	N/A	0.289	0.008	0.599	0.032	0.072	0.025	0.125	0.188	0.017

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	54	53	63	70	68	68	73	70	68	63
N.S.	1	0.98	1.17	1.30	1.26	1.26	1.35	1.30	1.26	1.17
time (sec)	N/A	0.291	0.023	0.664	0.026	0.086	0.071	0.131	0.203	0.020

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	67	69	69	75	70	71	78	65
N.S.	1	1.00	1.03	1.06	1.06	1.15	1.08	1.09	1.20	1.00
time (sec)	N/A	0.338	0.032	0.612	0.035	0.074	0.098	0.137	0.194	5.125

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	71	63	69	74	68	69	77	70
N.S.	1	1.00	1.09	0.97	1.06	1.14	1.05	1.06	1.18	1.08
time (sec)	N/A	0.338	0.022	0.616	0.032	0.081	0.180	0.121	0.215	5.249

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	73	61	69	75	73	70	78	70
N.S.	1	1.00	1.14	0.95	1.08	1.17	1.14	1.09	1.22	1.09
time (sec)	N/A	0.333	0.040	0.611	0.026	0.070	0.336	0.109	0.194	0.038

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	59	58	71	64	72	75	80	73	76	71
N.S.	1	0.98	1.20	1.08	1.22	1.27	1.36	1.24	1.29	1.20
time (sec)	N/A	0.310	0.033	0.720	0.033	0.086	0.510	0.141	0.197	0.046

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	72	66	73	73	82	75	74	71
N.S.	1	1.00	1.60	1.47	1.62	1.62	1.82	1.67	1.64	1.58
time (sec)	N/A	0.256	0.018	0.658	0.035	0.068	0.672	0.189	0.210	5.132

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	74	66	73	73	82	75	74	73
N.S.	1	1.00	0.99	0.88	0.97	0.97	1.09	1.00	0.99	0.97
time (sec)	N/A	0.341	0.014	0.601	0.031	0.067	0.822	0.109	0.195	0.026

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	75	66	73	73	82	75	74	74
N.S.	1	1.00	1.00	0.88	0.97	0.97	1.09	1.00	0.99	0.99
time (sec)	N/A	0.341	0.015	0.599	0.032	0.068	1.019	0.124	0.210	0.026

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	80	82	93	94	85	95	99	94
N.S.	1	1.00	0.92	0.94	1.07	1.08	0.98	1.09	1.14	1.08
time (sec)	N/A	0.412	0.023	0.757	0.034	0.072	0.122	0.143	0.213	0.030

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	61	63	69	71	61	70	75	72
N.S.	1	1.00	0.92	0.95	1.05	1.08	0.92	1.06	1.14	1.09
time (sec)	N/A	0.365	0.018	0.754	0.029	0.071	0.109	0.116	0.193	5.110

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	43	46	47	37	46	50	46
N.S.	1	1.00	0.91	0.96	1.02	1.04	0.82	1.02	1.11	1.02
time (sec)	N/A	0.314	0.013	0.684	0.027	0.072	0.096	0.184	0.209	5.184

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	26	25	24	20	26	28	26
N.S.	1	1.00	1.00	1.04	1.00	0.96	0.80	1.04	1.12	1.04
time (sec)	N/A	0.276	0.007	0.734	0.034	0.072	0.073	0.160	0.201	0.028

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	35	29	30	30	29	41	32	32	28
N.S.	1	1.17	0.97	1.00	1.00	0.97	1.37	1.07	1.07	0.93
time (sec)	N/A	0.311	0.007	0.698	0.033	0.081	0.216	0.117	0.191	5.070

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	42	44	43	41	95	51	46	33
N.S.	1	1.00	0.98	1.02	1.00	0.95	2.21	1.19	1.07	0.77
time (sec)	N/A	0.316	0.014	0.767	0.037	0.075	0.175	0.115	0.200	0.051

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	58	62	63	68	131	75	78	73
N.S.	1	1.00	0.94	1.00	1.02	1.10	2.11	1.21	1.26	1.18
time (sec)	N/A	0.348	0.025	0.702	0.034	0.089	0.218	0.112	0.208	5.043

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	81	82	89	94	165	99	104	97
N.S.	1	1.00	0.94	0.95	1.03	1.09	1.92	1.15	1.21	1.13
time (sec)	N/A	0.372	0.045	0.766	0.036	0.076	0.266	0.107	0.193	5.128

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	87	92	98	139	92	102	136	115
N.S.	1	1.00	0.97	1.02	1.09	1.54	1.02	1.13	1.51	1.28
time (sec)	N/A	0.420	0.049	0.821	0.032	0.072	0.212	0.123	0.199	5.212

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	66	67	75	111	68	77	109	77
N.S.	1	1.00	0.96	0.97	1.09	1.61	0.99	1.12	1.58	1.12
time (sec)	N/A	0.369	0.037	0.755	0.030	0.080	0.198	0.114	0.190	0.042

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	41	47	50	69	44	48	79	56
N.S.	1	1.00	0.91	1.04	1.11	1.53	0.98	1.07	1.76	1.24
time (sec)	N/A	0.319	0.022	0.706	0.032	0.077	0.143	0.174	0.211	5.297

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	31	32	35	39	27	33	49	31
N.S.	1	1.00	0.97	1.00	1.09	1.22	0.84	1.03	1.53	0.97
time (sec)	N/A	0.290	0.009	0.768	0.032	0.076	0.092	0.145	0.191	0.029

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	38	41	43	61	32	47	53	40
N.S.	1	1.00	0.90	0.98	1.02	1.45	0.76	1.12	1.26	0.95
time (sec)	N/A	0.313	0.023	0.712	0.032	0.081	0.135	0.128	0.190	5.092

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	81	56	63	69	111	128	74	128	57
N.S.	1	1.25	0.86	0.97	1.06	1.71	1.97	1.14	1.97	0.88
time (sec)	N/A	0.414	0.030	0.754	0.027	0.075	0.248	0.114	0.240	5.177

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	85	85	100	151	184	107	166	105
N.S.	1	1.00	1.00	1.00	1.18	1.78	2.16	1.26	1.95	1.24
time (sec)	N/A	0.410	0.051	0.709	0.033	0.084	0.286	0.115	0.243	5.167

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	106	107	129	180	219	134	192	132
N.S.	1	1.00	0.94	0.95	1.14	1.59	1.94	1.19	1.70	1.17
time (sec)	N/A	0.473	0.068	0.793	0.031	0.077	0.323	0.114	0.288	5.213

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	86	92	106	167	107	99	175	108
N.S.	1	1.00	0.91	0.98	1.13	1.78	1.14	1.05	1.86	1.15
time (sec)	N/A	0.448	0.040	0.767	0.028	0.076	0.385	0.142	0.224	5.206

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	75	71	83	131	83	70	148	87
N.S.	1	1.00	1.06	1.00	1.17	1.85	1.17	0.99	2.08	1.23
time (sec)	N/A	0.374	0.021	0.746	0.028	0.075	0.280	0.117	0.191	0.047

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	54	51	63	79	63	52	92	63
N.S.	1	1.00	0.98	0.93	1.15	1.44	1.15	0.95	1.67	1.15
time (sec)	N/A	0.331	0.014	0.762	0.030	0.076	0.182	0.129	0.208	5.264

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	26	25	38	38	39	24	37	39
N.S.	1	1.00	0.93	0.89	1.36	1.36	1.39	0.86	1.32	1.39
time (sec)	N/A	0.236	0.011	0.701	0.032	0.067	0.134	0.134	0.188	0.019

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	53	55	68	109	63	59	121	62
N.S.	1	1.00	0.93	0.96	1.19	1.91	1.11	1.04	2.12	1.09
time (sec)	N/A	0.334	0.043	0.700	0.032	0.080	0.220	0.182	0.207	0.040

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	81	83	104	195	168	103	223	84
N.S.	1	1.00	0.92	0.94	1.18	2.22	1.91	1.17	2.53	0.95
time (sec)	N/A	0.413	0.035	0.789	0.028	0.085	0.304	0.120	0.186	5.191

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	128	102	106	136	234	219	127	263	132
N.S.	1	1.16	0.93	0.96	1.24	2.13	1.99	1.15	2.39	1.20
time (sec)	N/A	0.527	0.063	2.839	0.033	0.079	0.361	0.115	0.183	5.213

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	129	132	165	263	262	159	291	163
N.S.	1	1.00	0.92	0.94	1.18	1.88	1.87	1.14	2.08	1.16
time (sec)	N/A	0.520	0.101	0.556	0.036	0.080	0.392	0.113	0.207	0.077

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	31	28	27	32	46	29	30	27
N.S.	1	1.00	0.79	0.72	0.69	0.82	1.18	0.74	0.77	0.69
time (sec)	N/A	0.270	0.026	0.270	0.027	0.072	0.185	0.111	0.193	0.028

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	28	27	32	37	29	30	27
N.S.	1	1.00	0.85	0.72	0.69	0.82	0.95	0.74	0.77	0.69
time (sec)	N/A	0.269	0.029	0.283	0.026	0.070	0.529	0.109	0.196	0.026

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	33	28	27	30	46	29	28	27
N.S.	1	1.00	0.85	0.72	0.69	0.77	1.18	0.74	0.72	0.69
time (sec)	N/A	0.267	0.023	0.266	0.025	0.074	0.104	0.112	0.205	5.166

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	27	27	27	44	29	27	27
N.S.	1	1.00	0.84	0.73	0.73	0.73	1.19	0.78	0.73	0.73
time (sec)	N/A	0.269	0.026	0.269	0.026	0.070	0.114	0.136	0.199	5.203

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	31	28	27	26	41	29	28	27
N.S.	1	1.00	0.89	0.80	0.77	0.74	1.17	0.83	0.80	0.77
time (sec)	N/A	0.275	0.024	0.076	0.032	0.074	0.197	0.138	0.193	0.026

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	28	27	27	27	41	27	32	27
N.S.	1	1.00	0.80	0.77	0.77	0.77	1.17	0.77	0.91	0.77
time (sec)	N/A	0.267	0.029	0.082	0.026	0.103	0.249	0.143	0.193	5.128

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	31	28	27	27	46	27	32	28
N.S.	1	1.00	0.84	0.76	0.73	0.73	1.24	0.73	0.86	0.76
time (sec)	N/A	0.262	0.029	0.076	0.031	0.077	0.381	0.128	0.195	0.020

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	55	52	51	56	80	53	53	51
N.S.	1	1.00	0.87	0.83	0.81	0.89	1.27	0.84	0.84	0.81
time (sec)	N/A	0.305	0.046	0.415	0.028	0.075	0.294	0.114	0.195	0.035

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	55	52	51	56	66	53	53	51
N.S.	1	1.00	0.87	0.83	0.81	0.89	1.05	0.84	0.84	0.81
time (sec)	N/A	0.315	0.033	0.420	0.031	0.080	0.690	0.110	0.209	0.028

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	55	52	51	56	80	53	53	51
N.S.	1	1.00	0.87	0.83	0.81	0.89	1.27	0.84	0.84	0.81
time (sec)	N/A	0.301	0.036	0.402	0.027	0.080	0.203	0.120	0.194	0.028

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	54	52	51	54	80	53	51	51
N.S.	1	1.00	0.86	0.83	0.81	0.86	1.27	0.84	0.81	0.81
time (sec)	N/A	0.302	0.034	0.409	0.032	0.107	0.200	0.133	0.205	0.030

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	54	51	51	51	78	53	50	51
N.S.	1	1.00	0.89	0.84	0.84	0.84	1.28	0.87	0.82	0.84
time (sec)	N/A	0.298	0.032	0.732	0.032	0.075	0.239	0.126	0.192	0.028

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	52	51	51	75	53	52	51
N.S.	1	1.00	0.93	0.88	0.86	0.86	1.27	0.90	0.88	0.86
time (sec)	N/A	0.292	0.038	0.756	0.031	0.079	0.384	0.110	0.196	0.029

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	51	51	50	73	51	54	51
N.S.	1	1.00	0.92	0.86	0.86	0.85	1.24	0.86	0.92	0.86
time (sec)	N/A	0.294	0.048	0.679	0.035	0.081	0.414	0.112	0.218	0.030

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	76	73	78	114	77	76	69
N.S.	1	1.00	0.91	0.89	0.86	0.92	1.34	0.91	0.89	0.81
time (sec)	N/A	0.340	0.054	0.714	0.029	0.075	0.477	0.112	0.202	0.023

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	76	73	78	95	77	76	69
N.S.	1	1.00	0.91	0.89	0.86	0.92	1.12	0.91	0.89	0.81
time (sec)	N/A	0.336	0.047	0.737	0.035	0.082	0.807	0.114	0.193	0.020

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	76	73	78	114	77	76	69
N.S.	1	1.00	0.91	0.89	0.86	0.92	1.34	0.91	0.89	0.81
time (sec)	N/A	0.329	0.046	0.707	0.035	0.074	0.296	0.108	0.199	0.020

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	76	73	78	114	77	76	69
N.S.	1	1.00	0.91	0.89	0.86	0.92	1.34	0.91	0.89	0.81
time (sec)	N/A	0.342	0.050	0.697	0.032	0.076	0.317	0.115	0.202	0.020

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	77	76	73	76	114	77	74	69
N.S.	1	1.00	0.91	0.89	0.86	0.89	1.34	0.91	0.87	0.81
time (sec)	N/A	0.330	0.074	0.700	0.027	0.075	0.384	0.149	0.203	0.019

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	76	75	73	73	110	77	73	69
N.S.	1	1.00	0.92	0.90	0.88	0.88	1.33	0.93	0.88	0.83
time (sec)	N/A	0.332	0.052	0.691	0.036	0.072	0.435	0.163	0.202	0.019

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	75	76	73	73	105	77	75	69
N.S.	1	1.00	0.95	0.96	0.92	0.92	1.33	0.97	0.95	0.87
time (sec)	N/A	0.335	0.067	0.697	0.026	0.075	0.626	0.125	0.206	0.022

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	74	75	73	73	105	75	78	70
N.S.	1	1.00	0.91	0.93	0.90	0.90	1.30	0.93	0.96	0.86
time (sec)	N/A	0.343	0.071	0.724	0.033	0.081	0.734	0.116	0.263	0.034

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	105	101	100	105	229	294	115	127	125
N.S.	1	0.93	0.89	0.88	0.93	2.03	2.60	1.02	1.12	1.11
time (sec)	N/A	0.349	0.128	0.799	0.113	0.088	10.788	0.138	0.239	5.144

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	86	81	76	82	180	260	91	98	101
N.S.	1	0.96	0.90	0.84	0.91	2.00	2.89	1.01	1.09	1.12
time (sec)	N/A	0.297	0.121	0.865	0.113	0.095	4.445	0.135	0.279	0.042

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	67	63	53	58	129	221	64	73	76
N.S.	1	0.97	0.91	0.77	0.84	1.87	3.20	0.93	1.06	1.10
time (sec)	N/A	0.285	0.075	0.789	0.114	0.092	1.730	0.152	0.209	0.046

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	39	102	180	39	58	37
N.S.	1	1.00	1.00	0.82	0.80	2.08	3.67	0.80	1.18	0.76
time (sec)	N/A	0.264	0.049	0.802	0.110	0.086	0.658	0.130	0.200	0.037

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	40	39	112	178	39	64	50
N.S.	1	1.00	1.00	0.82	0.80	2.29	3.63	0.80	1.31	1.02
time (sec)	N/A	0.265	0.059	0.871	0.111	0.088	0.840	0.241	0.212	5.154

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	66	64	54	56	141	218	55	80	54
N.S.	1	0.96	0.93	0.78	0.81	2.04	3.16	0.80	1.16	0.78
time (sec)	N/A	0.287	0.109	0.799	0.106	0.098	1.548	0.262	0.205	5.202

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	85	83	76	80	192	262	80	108	71
N.S.	1	0.94	0.92	0.84	0.89	2.13	2.91	0.89	1.20	0.79
time (sec)	N/A	0.302	0.112	0.808	0.110	0.093	3.636	0.216	0.193	0.061

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	104	107	95	103	241	299	104	133	90
N.S.	1	0.92	0.95	0.84	0.91	2.13	2.65	0.92	1.18	0.80
time (sec)	N/A	0.320	0.162	0.818	0.115	0.104	9.984	0.207	0.200	5.189

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	126	110	99	115	290	877	122	183	146
N.S.	1	1.09	0.95	0.85	0.99	2.50	7.56	1.05	1.58	1.26
time (sec)	N/A	0.357	0.168	0.826	0.108	0.097	118.255	0.186	0.201	5.133

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	107	88	77	88	231	762	95	153	107
N.S.	1	1.14	0.94	0.82	0.94	2.46	8.11	1.01	1.63	1.14
time (sec)	N/A	0.345	0.138	0.811	0.113	0.091	54.383	0.139	0.212	5.266

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	88	67	62	65	198	634	65	133	62
N.S.	1	1.19	0.91	0.84	0.88	2.68	8.57	0.88	1.80	0.84
time (sec)	N/A	0.320	0.099	0.815	0.109	0.096	30.475	0.104	0.205	5.232

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	63	57	58	177	615	60	119	51
N.S.	1	1.00	0.98	0.89	0.91	2.77	9.61	0.94	1.86	0.80
time (sec)	N/A	0.297	0.094	0.805	0.114	0.094	15.924	0.164	0.192	0.059

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	87	67	64	65	215	794	60	138	65
N.S.	1	1.21	0.93	0.89	0.90	2.99	11.03	0.83	1.92	0.90
time (sec)	N/A	0.319	0.091	0.822	0.108	0.100	9.060	0.182	0.210	5.262

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	107	92	77	93	257	882	85	165	81
N.S.	1	1.11	0.96	0.80	0.97	2.68	9.19	0.89	1.72	0.84
time (sec)	N/A	0.349	0.147	0.777	0.106	0.098	11.835	0.117	0.188	5.149

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	126	115	101	118	316	1017	110	197	103
N.S.	1	1.07	0.97	0.86	1.00	2.68	8.62	0.93	1.67	0.87
time (sec)	N/A	0.377	0.188	0.836	0.113	0.101	21.255	0.111	0.204	5.239

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	153	129	120	151	408	0	146	278	183
N.S.	1	1.01	0.85	0.79	0.99	2.68	0.00	0.96	1.83	1.20
time (sec)	N/A	0.396	0.212	0.836	0.108	0.104	0.000	0.115	0.194	0.050

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	134	110	98	124	349	0	119	248	143
N.S.	1	1.03	0.85	0.75	0.95	2.68	0.00	0.92	1.91	1.10
time (sec)	N/A	0.381	0.182	0.842	0.111	0.104	0.000	0.123	0.195	5.346

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	115	94	83	99	319	0	87	229	96
N.S.	1	1.07	0.88	0.78	0.93	2.98	0.00	0.81	2.14	0.90
time (sec)	N/A	0.355	0.162	0.819	0.109	0.099	0.000	0.187	0.207	5.287

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	95	86	79	94	291	1316	82	213	84
N.S.	1	0.97	0.88	0.81	0.96	2.97	13.43	0.84	2.17	0.86
time (sec)	N/A	0.336	0.156	0.826	0.110	0.095	166.174	0.124	0.201	5.371

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	94	86	80	94	291	1345	82	213	84
N.S.	1	0.94	0.86	0.80	0.94	2.91	13.45	0.82	2.13	0.84
time (sec)	N/A	0.325	0.147	0.807	0.115	0.100	116.264	0.129	0.207	5.524

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	113	96	84	98	331	1598	86	237	116
N.S.	1	1.07	0.91	0.79	0.92	3.12	15.08	0.81	2.24	1.09
time (sec)	N/A	0.358	0.172	0.845	0.118	0.102	69.836	0.133	0.216	5.392

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	133	117	98	128	375	1703	108	260	114
N.S.	1	1.03	0.91	0.76	0.99	2.91	13.20	0.84	2.02	0.88
time (sec)	N/A	0.375	0.191	0.846	0.122	0.098	44.361	0.126	0.198	5.430

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	160	201	116	242	303	238	158	214	215
N.S.	1	0.81	1.02	0.59	1.23	1.54	1.21	0.80	1.09	1.09
time (sec)	N/A	0.541	0.831	0.936	0.034	0.119	0.432	0.143	0.219	5.674

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	112	137	102	198	254	206	130	175	165
N.S.	1	0.70	0.86	0.64	1.24	1.59	1.29	0.81	1.09	1.03
time (sec)	N/A	0.413	0.645	0.886	0.028	0.102	0.532	0.335	0.209	5.488

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	97	122	82	154	205	167	100	136	127
N.S.	1	0.80	1.01	0.68	1.27	1.69	1.38	0.83	1.12	1.05
time (sec)	N/A	0.373	0.454	0.892	0.027	0.103	0.412	0.207	0.202	5.556

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	82	87	61	109	154	0	75	98	101
N.S.	1	0.89	0.95	0.66	1.18	1.67	0.00	0.82	1.07	1.10
time (sec)	N/A	0.384	0.141	0.867	0.030	0.105	0.000	0.118	0.207	5.477

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	77	82	57	89	139	0	80	100	0
N.S.	1	1.10	1.17	0.81	1.27	1.99	0.00	1.14	1.43	0.00
time (sec)	N/A	0.395	0.259	0.898	0.040	0.095	0.000	0.132	0.198	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	74	91	61	85	142	0	140	94	0
N.S.	1	1.01	1.25	0.84	1.16	1.95	0.00	1.92	1.29	0.00
time (sec)	N/A	0.376	0.139	0.893	0.035	0.124	0.000	0.121	0.198	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	36	37	100	55	0	191	107	100
N.S.	1	1.00	0.63	0.65	1.75	0.96	0.00	3.35	1.88	1.75
time (sec)	N/A	0.337	0.133	0.868	0.027	0.095	0.000	0.127	0.216	5.574

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	89	56	54	146	80	0	251	148	146
N.S.	1	0.99	0.62	0.60	1.62	0.89	0.00	2.79	1.64	1.62
time (sec)	N/A	0.389	0.180	0.898	0.037	0.092	0.000	0.141	0.200	5.710

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	121	78	70	192	105	0	311	187	192
N.S.	1	0.97	0.62	0.56	1.54	0.84	0.00	2.49	1.50	1.54
time (sec)	N/A	0.452	0.189	0.941	0.040	0.115	0.000	0.129	0.257	5.849

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	153	100	88	238	129	0	371	226	238
N.S.	1	0.96	0.62	0.55	1.49	0.81	0.00	2.32	1.41	1.49
time (sec)	N/A	0.581	0.213	0.934	0.037	0.087	0.000	0.271	0.271	5.708

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	185	123	105	284	153	0	431	265	284
N.S.	1	0.95	0.63	0.54	1.46	0.78	0.00	2.21	1.36	1.46
time (sec)	N/A	0.598	0.255	0.996	0.035	0.096	0.000	0.287	0.328	6.063

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	197	236	148	324	400	444	220	292	0
N.S.	1	0.75	0.89	0.56	1.23	1.52	1.68	0.83	1.11	0.00
time (sec)	N/A	0.599	1.372	0.965	0.037	0.102	0.480	0.270	0.263	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	149	219	133	280	351	386	192	253	0
N.S.	1	0.66	0.96	0.59	1.23	1.55	1.70	0.85	1.11	0.00
time (sec)	N/A	0.483	1.141	0.987	0.042	0.116	0.566	0.254	0.234	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	134	167	116	236	298	326	160	214	208
N.S.	1	0.72	0.90	0.62	1.27	1.60	1.75	0.86	1.15	1.12
time (sec)	N/A	0.445	0.863	0.915	0.033	0.108	0.437	0.180	0.214	5.648

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	124	181	99	189	257	490	136	175	0
N.S.	1	0.78	1.14	0.62	1.19	1.62	3.08	0.86	1.10	0.00
time (sec)	N/A	0.464	0.679	0.891	0.027	0.101	1.957	0.186	0.220	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	118	110	82	147	206	0	107	136	0
N.S.	1	0.98	0.91	0.68	1.21	1.70	0.00	0.88	1.12	0.00
time (sec)	N/A	0.461	0.243	0.911	0.028	0.113	0.000	0.225	0.197	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	103	101	73	129	187	0	107	137	0
N.S.	1	0.98	0.96	0.70	1.23	1.78	0.00	1.02	1.30	0.00
time (sec)	N/A	0.435	0.384	0.888	0.036	0.108	0.000	0.253	0.194	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	106	111	78	146	171	0	170	125	0
N.S.	1	1.07	1.12	0.79	1.47	1.73	0.00	1.72	1.26	0.00
time (sec)	N/A	0.472	0.282	0.928	0.033	0.109	0.000	0.285	0.196	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	97	100	78	158	189	0	259	136	0
N.S.	1	0.99	1.02	0.80	1.61	1.93	0.00	2.64	1.39	0.00
time (sec)	N/A	0.436	0.242	0.876	0.034	0.094	0.000	0.271	0.205	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	36	39	176	78	0	311	148	142
N.S.	1	1.00	0.63	0.68	3.09	1.37	0.00	5.46	2.60	2.49
time (sec)	N/A	0.343	0.195	0.919	0.034	0.170	0.000	0.185	0.203	5.825

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	89	56	56	222	104	0	371	187	188
N.S.	1	0.99	0.62	0.62	2.47	1.16	0.00	4.12	2.08	2.09
time (sec)	N/A	0.395	0.238	0.937	0.037	0.142	0.000	0.127	0.205	5.961

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	121	79	73	268	130	0	431	226	234
N.S.	1	0.97	0.63	0.58	2.14	1.04	0.00	3.45	1.81	1.87
time (sec)	N/A	0.460	0.312	1.012	0.040	0.155	0.000	0.149	0.216	6.213

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	153	100	90	314	153	0	491	265	280
N.S.	1	0.96	0.62	0.56	1.96	0.96	0.00	3.07	1.66	1.75
time (sec)	N/A	0.532	0.367	1.033	0.044	0.132	0.000	0.157	0.243	6.502

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	185	122	106	360	177	0	551	304	326
N.S.	1	0.95	0.63	0.54	1.85	0.91	0.00	2.83	1.56	1.67
time (sec)	N/A	0.586	0.165	1.072	0.051	0.146	0.000	0.217	0.250	6.869

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	332	234	277	241	406	497	741	280	370	0
N.S.	1	0.70	0.83	0.73	1.22	1.50	2.23	0.84	1.11	0.00
time (sec)	N/A	0.672	1.976	0.927	0.038	0.130	0.583	0.243	0.455	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	186	257	217	362	448	656	251	331	0
N.S.	1	0.63	0.87	0.74	1.23	1.52	2.22	0.85	1.12	0.00
time (sec)	N/A	0.526	1.680	0.896	0.040	0.114	0.643	0.159	0.294	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	171	238	193	318	393	570	219	292	0
N.S.	1	0.68	0.94	0.77	1.26	1.56	2.26	0.87	1.16	0.00
time (sec)	N/A	0.465	1.327	0.901	0.038	0.134	0.517	0.141	0.240	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	161	212	169	271	352	886	196	253	0
N.S.	1	0.71	0.93	0.74	1.19	1.55	3.90	0.86	1.11	0.00
time (sec)	N/A	0.510	1.103	0.901	0.035	0.097	2.985	0.125	0.204	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	161	200	116	226	305	796	168	214	0
N.S.	1	0.84	1.04	0.60	1.18	1.59	4.15	0.88	1.11	0.00
time (sec)	N/A	0.513	0.883	0.948	0.034	0.112	0.785	0.129	0.201	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	144	129	97	187	254	0	139	175	0
N.S.	1	0.94	0.84	0.63	1.22	1.66	0.00	0.91	1.14	0.00
time (sec)	N/A	0.501	0.271	0.937	0.028	0.123	0.000	0.161	0.222	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	140	126	92	172	241	0	139	180	0
N.S.	1	1.04	0.93	0.68	1.27	1.79	0.00	1.03	1.33	0.00
time (sec)	N/A	0.483	0.540	0.971	0.032	0.111	0.000	0.179	0.206	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	135	137	93	191	225	0	200	175	0
N.S.	1	0.96	0.98	0.66	1.36	1.61	0.00	1.43	1.25	0.00
time (sec)	N/A	0.567	0.453	0.973	0.034	0.106	0.000	0.214	0.196	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	132	136	97	244	225	0	293	179	0
N.S.	1	1.02	1.05	0.75	1.88	1.73	0.00	2.25	1.38	0.00
time (sec)	N/A	0.502	0.344	0.984	0.040	0.095	0.000	0.196	0.203	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	120	113	95	258	239	0	379	179	0
N.S.	1	0.96	0.90	0.76	2.06	1.91	0.00	3.03	1.43	0.00
time (sec)	N/A	0.464	0.309	1.006	0.036	0.116	0.000	0.149	0.201	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	36	39	258	102	0	431	187	188
N.S.	1	1.00	0.63	0.68	4.53	1.79	0.00	7.56	3.28	3.30
time (sec)	N/A	0.329	0.281	1.009	0.044	0.096	0.000	0.135	0.221	6.858

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	89	63	56	304	127	0	491	226	234
N.S.	1	0.99	0.70	0.62	3.38	1.41	0.00	5.46	2.51	2.60
time (sec)	N/A	0.382	0.333	1.065	0.036	0.092	0.000	0.136	0.217	7.199

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	121	87	73	350	153	0	551	265	280
N.S.	1	0.97	0.70	0.58	2.80	1.22	0.00	4.41	2.12	2.24
time (sec)	N/A	0.432	0.504	1.126	0.042	0.095	0.000	0.128	0.267	7.453

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	153	107	90	396	177	0	611	304	326
N.S.	1	0.96	0.67	0.56	2.48	1.11	0.00	3.82	1.90	2.04
time (sec)	N/A	0.502	0.130	1.207	0.041	0.096	0.000	0.124	0.317	7.909

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	185	130	107	442	202	0	671	343	372
N.S.	1	0.95	0.67	0.55	2.27	1.04	0.00	3.44	1.76	1.91
time (sec)	N/A	0.579	0.119	1.335	0.041	0.092	0.000	0.130	0.371	8.321

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	182	163	116	252	304	228	163	214	0
N.S.	1	0.92	0.83	0.59	1.28	1.54	1.16	0.83	1.09	0.00
time (sec)	N/A	0.579	0.743	1.018	0.034	0.116	0.503	0.146	0.222	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	150	188	99	206	257	204	135	175	0
N.S.	1	0.93	1.16	0.61	1.27	1.59	1.26	0.83	1.08	0.00
time (sec)	N/A	0.534	0.540	0.970	0.032	0.093	0.531	0.188	0.184	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	118	125	82	160	208	180	107	136	0
N.S.	1	0.93	0.98	0.65	1.26	1.64	1.42	0.84	1.07	0.00
time (sec)	N/A	0.447	0.499	0.961	0.034	0.104	0.479	0.202	0.216	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	75	108	64	115	159	151	81	99	0
N.S.	1	0.82	1.17	0.70	1.25	1.73	1.64	0.88	1.08	0.00
time (sec)	N/A	0.338	0.331	1.033	0.027	0.091	0.556	0.186	0.199	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	86	46	75	116	122	58	65	77
N.S.	1	1.00	1.56	0.84	1.36	2.11	2.22	1.05	1.18	1.40
time (sec)	N/A	0.304	0.204	0.927	0.029	0.085	0.366	0.176	0.200	5.787

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	74	49	49	117	0	59	56	50
N.S.	1	1.00	1.42	0.94	0.94	2.25	0.00	1.13	1.08	0.96
time (sec)	N/A	0.303	0.096	0.947	0.032	0.102	0.000	0.148	0.201	5.769

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	32	62	34	0	76	66	33
N.S.	1	1.00	0.61	0.56	1.09	0.60	0.00	1.33	1.16	0.58
time (sec)	N/A	0.311	0.105	0.881	0.026	0.087	0.000	0.133	0.201	5.435

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	89	54	49	106	57	0	133	107	56
N.S.	1	0.99	0.60	0.54	1.18	0.63	0.00	1.48	1.19	0.62
time (sec)	N/A	0.381	0.136	1.003	0.026	0.081	0.000	0.361	0.212	5.294

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	121	79	66	152	82	0	191	148	113
N.S.	1	0.97	0.63	0.53	1.22	0.66	0.00	1.53	1.18	0.90
time (sec)	N/A	0.435	0.169	0.952	0.028	0.078	0.000	0.174	0.209	5.269

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	153	100	83	198	106	0	251	187	146
N.S.	1	0.96	0.62	0.52	1.24	0.66	0.00	1.57	1.17	0.91
time (sec)	N/A	0.491	0.183	0.993	0.031	0.084	0.000	0.162	0.220	5.340

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	185	123	100	244	130	0	311	226	177
N.S.	1	0.95	0.63	0.51	1.25	0.67	0.00	1.59	1.16	0.91
time (sec)	N/A	0.577	0.217	1.007	0.038	0.080	0.000	0.267	0.215	5.260

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	168	148	107	212	314	0	164	181	0
N.S.	1	1.08	0.95	0.69	1.36	2.01	0.00	1.05	1.16	0.00
time (sec)	N/A	0.888	0.556	1.039	0.034	0.089	0.000	0.282	0.206	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	125	87	163	263	0	136	150	0
N.S.	1	1.00	1.03	0.72	1.35	2.17	0.00	1.12	1.24	0.00
time (sec)	N/A	0.597	0.387	0.987	0.034	0.089	0.000	0.246	0.198	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	86	103	67	115	203	0	107	125	0
N.S.	1	1.04	1.24	0.81	1.39	2.45	0.00	1.29	1.51	0.00
time (sec)	N/A	0.407	0.271	1.000	0.028	0.087	0.000	0.237	0.206	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	77	63	65	165	0	80	86	64
N.S.	1	1.00	1.28	1.05	1.08	2.75	0.00	1.33	1.43	1.07
time (sec)	N/A	0.336	0.115	0.948	0.035	0.094	0.000	0.158	0.200	5.420

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	33	30	30	55	44	0	33	70	31
N.S.	1	0.70	0.64	0.64	1.17	0.94	0.00	0.70	1.49	0.66
time (sec)	N/A	0.262	0.100	0.967	0.029	0.090	0.000	0.169	0.205	5.205

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	60	52	49	96	68	0	0	94	62
N.S.	1	0.70	0.60	0.57	1.12	0.79	0.00	0.00	1.09	0.72
time (sec)	N/A	0.331	0.137	0.945	0.027	0.094	0.000	0.000	0.198	5.352

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	92	75	66	142	93	0	0	124	87
N.S.	1	0.75	0.61	0.54	1.16	0.76	0.00	0.00	1.02	0.71
time (sec)	N/A	0.395	0.200	0.960	0.027	0.082	0.000	0.000	0.213	5.561

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	124	98	83	188	117	0	0	153	161
N.S.	1	0.79	0.62	0.53	1.20	0.75	0.00	0.00	0.97	1.03
time (sec)	N/A	0.444	0.225	0.984	0.034	0.084	0.000	0.000	0.204	5.725

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	156	123	102	234	142	0	0	180	191
N.S.	1	0.81	0.64	0.53	1.22	0.74	0.00	0.00	0.94	0.99
time (sec)	N/A	0.506	0.233	0.997	0.035	0.080	0.000	0.000	0.207	5.755

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	149	180	120	362	381	0	246	286	0
N.S.	1	0.96	1.16	0.77	2.34	2.46	0.00	1.59	1.85	0.00
time (sec)	N/A	0.621	0.551	1.079	0.042	0.094	0.000	0.159	0.228	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	123	125	103	310	322	0	217	225	0
N.S.	1	1.07	1.09	0.90	2.70	2.80	0.00	1.89	1.96	0.00
time (sec)	N/A	0.511	0.389	1.031	0.038	0.110	0.000	0.145	0.196	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	88	93	88	221	240	0	188	141	0
N.S.	1	1.05	1.11	1.05	2.63	2.86	0.00	2.24	1.68	0.00
time (sec)	N/A	0.411	0.174	1.050	0.041	0.110	0.000	0.136	0.189	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	35	37	134	51	0	119	106	37
N.S.	1	1.00	0.52	0.55	2.00	0.76	0.00	1.78	1.58	0.55
time (sec)	N/A	0.370	0.098	0.967	0.028	0.081	0.000	0.188	0.184	5.527

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	70	55	53	111	77	0	0	141	63
N.S.	1	0.86	0.68	0.65	1.37	0.95	0.00	0.00	1.74	0.78
time (sec)	N/A	0.335	0.137	0.962	0.027	0.082	0.000	0.000	0.193	5.514

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	70	72	73	130	101	0	82	163	76
N.S.	1	0.64	0.65	0.66	1.18	0.92	0.00	0.75	1.48	0.69
time (sec)	N/A	0.310	0.167	1.033	0.037	0.083	0.000	0.239	0.203	5.487

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	95	98	92	176	128	0	0	195	111
N.S.	1	0.63	0.65	0.61	1.17	0.85	0.00	0.00	1.29	0.74
time (sec)	N/A	0.388	0.278	1.018	0.030	0.095	0.000	0.000	0.200	5.744

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	127	123	109	224	153	0	0	224	235
N.S.	1	0.67	0.65	0.58	1.19	0.81	0.00	0.00	1.19	1.24
time (sec)	N/A	0.441	0.264	1.002	0.033	0.082	0.000	0.000	0.195	5.831

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	159	145	126	270	177	0	0	251	266
N.S.	1	0.71	0.65	0.56	1.21	0.79	0.00	0.00	1.12	1.19
time (sec)	N/A	0.517	0.295	1.028	0.034	0.091	0.000	0.000	0.270	5.635

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	197	113	129	142	150	0	137	137	0
N.S.	1	0.96	0.55	0.63	0.69	0.73	0.00	0.67	0.67	0.00
time (sec)	N/A	0.584	0.092	0.908	0.044	0.100	0.000	0.136	0.232	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	163	94	105	120	126	0	113	114	0
N.S.	1	0.96	0.56	0.62	0.71	0.75	0.00	0.67	0.67	0.00
time (sec)	N/A	0.516	0.062	0.966	0.042	0.076	0.000	0.116	0.270	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	129	72	81	98	102	0	89	91	0
N.S.	1	0.98	0.55	0.61	0.74	0.77	0.00	0.67	0.69	0.00
time (sec)	N/A	0.482	0.053	0.913	0.044	0.078	0.000	0.113	0.213	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	56	57	75	78	0	65	68	0
N.S.	1	1.00	0.59	0.60	0.79	0.82	0.00	0.68	0.72	0.00
time (sec)	N/A	0.397	0.045	0.925	0.039	0.079	0.000	0.306	0.180	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	61	37	37	45	53	0	100	44	0
N.S.	1	1.02	0.62	0.62	0.75	0.88	0.00	1.67	0.73	0.00
time (sec)	N/A	0.336	0.030	0.927	0.046	0.081	0.000	0.314	0.195	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	82	80	79	0	151	0	55	72	0
N.S.	1	1.01	0.99	0.98	0.00	1.86	0.00	0.68	0.89	0.00
time (sec)	N/A	0.385	0.070	1.184	0.000	0.086	0.000	0.210	0.180	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	93	80	78	0	160	0	63	104	0
N.S.	1	1.12	0.96	0.94	0.00	1.93	0.00	0.76	1.25	0.00
time (sec)	N/A	0.408	0.097	0.966	0.000	0.092	0.000	0.124	0.203	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	97	94	83	0	190	0	110	132	0
N.S.	1	0.96	0.93	0.82	0.00	1.88	0.00	1.09	1.31	0.00
time (sec)	N/A	0.413	0.148	0.967	0.000	0.087	0.000	0.114	0.185	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	127	116	108	0	241	0	114	170	0
N.S.	1	0.92	0.84	0.78	0.00	1.75	0.00	0.83	1.23	0.00
time (sec)	N/A	0.474	0.230	0.963	0.000	0.090	0.000	0.806	0.197	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	161	140	133	0	290	0	176	205	0
N.S.	1	0.92	0.80	0.76	0.00	1.66	0.00	1.01	1.17	0.00
time (sec)	N/A	0.530	0.242	0.980	0.000	0.095	0.000	0.123	0.192	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	195	159	157	0	337	0	181	240	0
N.S.	1	0.92	0.75	0.74	0.00	1.59	0.00	0.85	1.13	0.00
time (sec)	N/A	0.598	0.380	0.984	0.000	0.102	0.000	0.167	0.203	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	197	110	131	318	174	0	299	160	0
N.S.	1	0.96	0.53	0.64	1.54	0.84	0.00	1.45	0.78	0.00
time (sec)	N/A	0.580	0.099	0.946	0.057	0.089	0.000	0.221	0.183	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	163	94	107	274	150	0	251	137	0
N.S.	1	0.96	0.56	0.63	1.62	0.89	0.00	1.49	0.81	0.00
time (sec)	N/A	0.537	0.091	0.959	0.057	0.087	0.000	0.214	0.205	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	129	75	83	229	127	0	203	114	0
N.S.	1	0.98	0.57	0.63	1.73	0.96	0.00	1.54	0.86	0.00
time (sec)	N/A	0.462	0.084	0.974	0.073	0.077	0.000	0.194	0.195	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	56	59	182	102	0	155	91	0
N.S.	1	1.00	0.59	0.62	1.92	1.07	0.00	1.63	0.96	0.00
time (sec)	N/A	0.396	0.051	0.977	0.044	0.077	0.000	0.156	0.196	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	61	37	39	129	76	0	106	67	0
N.S.	1	1.02	0.62	0.65	2.15	1.27	0.00	1.77	1.12	0.00
time (sec)	N/A	0.348	0.044	0.948	0.050	0.077	0.000	0.143	0.214	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	107	100	113	0	198	0	72	105	0
N.S.	1	1.02	0.95	1.08	0.00	1.89	0.00	0.69	1.00	0.00
time (sec)	N/A	0.437	0.117	0.980	0.000	0.106	0.000	0.110	0.197	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	119	94	102	0	187	0	102	126	0
N.S.	1	1.04	0.82	0.89	0.00	1.64	0.00	0.89	1.11	0.00
time (sec)	N/A	0.465	0.132	1.013	0.000	0.107	0.000	0.109	0.198	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	121	101	95	0	209	0	119	148	0
N.S.	1	1.01	0.84	0.79	0.00	1.74	0.00	0.99	1.23	0.00
time (sec)	N/A	0.464	0.177	0.987	0.000	0.093	0.000	0.233	0.208	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	125	118	108	0	242	0	128	170	0
N.S.	1	0.93	0.88	0.81	0.00	1.81	0.00	0.96	1.27	0.00
time (sec)	N/A	0.457	0.242	0.995	0.000	0.093	0.000	0.152	0.199	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	155	140	133	0	291	0	176	205	0
N.S.	1	0.90	0.81	0.77	0.00	1.68	0.00	1.02	1.18	0.00
time (sec)	N/A	0.529	0.307	0.981	0.000	0.100	0.000	0.135	0.203	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	189	158	156	0	338	0	168	240	0
N.S.	1	0.90	0.76	0.75	0.00	1.62	0.00	0.80	1.15	0.00
time (sec)	N/A	0.583	0.393	1.106	0.000	0.112	0.000	0.124	0.189	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	197	120	131	507	199	0	489	183	0
N.S.	1	0.96	0.58	0.64	2.46	0.97	0.00	2.37	0.89	0.00
time (sec)	N/A	0.562	0.094	0.978	0.057	0.086	0.000	0.478	0.183	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	163	101	107	441	174	0	417	160	0
N.S.	1	0.96	0.60	0.63	2.61	1.03	0.00	2.47	0.95	0.00
time (sec)	N/A	0.508	0.074	0.999	0.060	0.079	0.000	0.130	0.206	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	129	82	83	375	150	0	345	137	0
N.S.	1	0.98	0.62	0.63	2.84	1.14	0.00	2.61	1.04	0.00
time (sec)	N/A	0.447	0.066	0.963	0.046	0.087	0.000	0.123	0.209	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	63	59	305	125	0	273	114	0
N.S.	1	1.00	0.66	0.62	3.21	1.32	0.00	2.87	1.20	0.00
time (sec)	N/A	0.382	0.053	1.018	0.048	0.091	0.000	0.142	0.189	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	61	45	39	230	100	0	200	90	0
N.S.	1	1.02	0.75	0.65	3.83	1.67	0.00	3.33	1.50	0.00
time (sec)	N/A	0.348	0.047	0.967	0.044	0.092	0.000	0.117	0.198	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	132	114	151	0	248	0	88	144	0
N.S.	1	1.01	0.87	1.15	0.00	1.89	0.00	0.67	1.10	0.00
time (sec)	N/A	0.485	0.150	0.998	0.000	0.089	0.000	0.115	0.190	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	144	118	129	0	241	0	137	163	0
N.S.	1	0.99	0.81	0.88	0.00	1.65	0.00	0.94	1.12	0.00
time (sec)	N/A	0.515	0.188	1.055	0.000	0.095	0.000	0.117	0.205	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	147	122	118	0	241	0	155	175	0
N.S.	1	0.95	0.79	0.76	0.00	1.55	0.00	1.00	1.13	0.00
time (sec)	N/A	0.507	0.272	0.993	0.000	0.089	0.000	0.146	0.206	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	149	127	119	0	261	0	134	187	0
N.S.	1	0.96	0.81	0.76	0.00	1.67	0.00	0.86	1.20	0.00
time (sec)	N/A	0.505	0.231	0.984	0.000	0.099	0.000	0.181	0.191	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	153	140	132	0	292	0	177	205	0
N.S.	1	0.90	0.82	0.78	0.00	1.72	0.00	1.04	1.21	0.00
time (sec)	N/A	0.513	0.285	1.014	0.000	0.089	0.000	0.181	0.252	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	183	160	157	0	339	0	181	240	0
N.S.	1	0.89	0.78	0.76	0.00	1.65	0.00	0.88	1.17	0.00
time (sec)	N/A	0.580	0.432	1.010	0.000	0.092	0.000	0.174	0.239	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	163	94	100	120	103	0	116	91	0
N.S.	1	0.98	0.56	0.60	0.72	0.62	0.00	0.69	0.54	0.00
time (sec)	N/A	0.488	0.066	0.975	0.045	0.077	0.000	0.144	0.269	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	129	75	76	98	79	0	89	68	0
N.S.	1	0.99	0.58	0.58	0.75	0.61	0.00	0.68	0.52	0.00
time (sec)	N/A	0.447	0.065	0.990	0.041	0.079	0.000	0.120	0.200	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	95	55	52	75	55	0	62	45	0
N.S.	1	1.02	0.59	0.56	0.81	0.59	0.00	0.67	0.48	0.00
time (sec)	N/A	0.377	0.041	0.957	0.041	0.073	0.000	0.221	0.205	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	61	36	31	45	32	0	34	25	0
N.S.	1	1.05	0.62	0.53	0.78	0.55	0.00	0.59	0.43	0.00
time (sec)	N/A	0.310	0.033	0.995	0.040	0.084	0.000	0.134	0.211	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	68	57	0	134	0	36	52	0
N.S.	1	1.00	1.17	0.98	0.00	2.31	0.00	0.62	0.90	0.00
time (sec)	N/A	0.317	0.046	1.016	0.000	0.087	0.000	0.161	0.207	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	73	68	0	147	0	57	92	0
N.S.	1	1.00	1.11	1.03	0.00	2.23	0.00	0.86	1.39	0.00
time (sec)	N/A	0.342	0.084	0.977	0.000	0.082	0.000	0.144	0.188	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	99	92	85	0	191	0	111	133	0
N.S.	1	0.94	0.88	0.81	0.00	1.82	0.00	1.06	1.27	0.00
time (sec)	N/A	0.412	0.134	1.013	0.000	0.116	0.000	0.119	0.194	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	133	115	109	0	244	0	127	170	0
N.S.	1	0.94	0.81	0.77	0.00	1.72	0.00	0.89	1.20	0.00
time (sec)	N/A	0.466	0.225	0.993	0.000	0.089	0.000	0.130	0.193	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	170	93	107	0	116	0	134	93	0
N.S.	1	1.04	0.57	0.66	0.00	0.71	0.00	0.82	0.57	0.00
time (sec)	N/A	0.540	0.086	1.017	0.000	0.082	0.000	0.140	0.197	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	136	74	83	0	92	0	102	70	0
N.S.	1	1.06	0.58	0.65	0.00	0.72	0.00	0.80	0.55	0.00
time (sec)	N/A	0.467	0.080	1.032	0.000	0.080	0.000	0.113	0.201	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	102	54	58	0	67	0	69	46	0
N.S.	1	1.12	0.59	0.64	0.00	0.74	0.00	0.76	0.51	0.00
time (sec)	N/A	0.409	0.043	0.976	0.000	0.078	0.000	0.114	0.192	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	70	34	38	0	45	0	34	27	0
N.S.	1	1.25	0.61	0.68	0.00	0.80	0.00	0.61	0.48	0.00
time (sec)	N/A	0.354	0.030	1.043	0.000	0.096	0.000	0.113	0.198	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	69	63	0	195	0	49	71	0
N.S.	1	1.00	1.01	0.93	0.00	2.87	0.00	0.72	1.04	0.00
time (sec)	N/A	0.342	0.075	0.986	0.000	0.085	0.000	0.110	0.192	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	81	94	0	251	0	87	133	0
N.S.	1	1.00	0.84	0.97	0.00	2.59	0.00	0.90	1.37	0.00
time (sec)	N/A	0.391	0.120	1.055	0.000	0.098	0.000	0.191	0.191	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	131	105	109	0	307	0	125	168	0
N.S.	1	0.96	0.77	0.80	0.00	2.24	0.00	0.91	1.23	0.00
time (sec)	N/A	0.472	0.189	1.027	0.000	0.103	0.000	0.116	0.205	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	165	132	133	0	362	0	165	195	0
N.S.	1	0.94	0.75	0.76	0.00	2.06	0.00	0.94	1.11	0.00
time (sec)	N/A	0.533	0.237	1.039	0.000	0.096	0.000	0.123	0.191	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	199	153	157	0	409	0	197	218	0
N.S.	1	0.93	0.72	0.74	0.00	1.92	0.00	0.92	1.02	0.00
time (sec)	N/A	0.598	0.288	1.034	0.000	0.102	0.000	0.138	0.199	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	170	93	107	0	127	0	125	100	0
N.S.	1	1.03	0.56	0.65	0.00	0.77	0.00	0.76	0.61	0.00
time (sec)	N/A	0.530	0.071	1.017	0.000	0.086	0.000	0.119	0.196	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	135	70	82	0	102	0	92	76	0
N.S.	1	1.05	0.55	0.64	0.00	0.80	0.00	0.72	0.59	0.00
time (sec)	N/A	0.470	0.061	1.045	0.000	0.092	0.000	0.120	0.194	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	104	53	59	0	79	0	55	54	0
N.S.	1	1.14	0.58	0.65	0.00	0.87	0.00	0.60	0.59	0.00
time (sec)	N/A	0.405	0.048	1.012	0.000	0.082	0.000	0.124	0.209	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	73	36	38	0	56	0	28	35	0
N.S.	1	1.26	0.62	0.66	0.00	0.97	0.00	0.48	0.60	0.00
time (sec)	N/A	0.339	0.039	0.997	0.000	0.081	0.000	0.280	0.193	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	98	83	101	0	265	0	61	144	0
N.S.	1	1.04	0.88	1.07	0.00	2.82	0.00	0.65	1.53	0.00
time (sec)	N/A	0.386	0.093	1.079	0.000	0.106	0.000	0.169	0.192	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	144	105	112	0	370	0	90	275	0
N.S.	1	1.11	0.81	0.86	0.00	2.85	0.00	0.69	2.12	0.00
time (sec)	N/A	0.465	0.148	1.025	0.000	0.093	0.000	0.122	0.216	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	164	129	127	0	427	0	149	316	0
N.S.	1	0.95	0.75	0.74	0.00	2.48	0.00	0.87	1.84	0.00
time (sec)	N/A	0.498	0.236	1.118	0.000	0.096	0.000	0.121	0.220	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	198	154	151	0	480	0	200	343	0
N.S.	1	0.93	0.72	0.71	0.00	2.25	0.00	0.94	1.61	0.00
time (sec)	N/A	0.602	0.295	1.071	0.000	0.102	0.000	0.122	0.201	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	125	90	122	161	383	2111	683	457	282
N.S.	1	1.03	0.74	1.01	1.33	3.17	17.45	5.64	3.78	2.33
time (sec)	N/A	0.473	0.138	1.058	0.051	0.084	0.576	0.138	0.185	8.863

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	95	74	92	115	219	1081	388	249	181
N.S.	1	1.04	0.81	1.01	1.26	2.41	11.88	4.26	2.74	1.99
time (sec)	N/A	0.407	0.107	0.987	0.045	0.089	0.411	0.180	0.197	8.834

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	64	60	61	69	96	425	173	101	99
N.S.	1	1.07	1.00	1.02	1.15	1.60	7.08	2.88	1.68	1.65
time (sec)	N/A	0.350	0.057	0.057	0.043	0.103	0.291	0.130	0.187	8.617

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	50	62	50	0	0	0	0	0	59	0
N.S.	1	1.24	1.00	0.00	0.00	0.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.314	0.067	0.000	0.000	0.000	0.000	0.000	0.192	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	92	67	0	0	0	0	0	809	0
N.S.	1	1.16	0.85	0.00	0.00	0.00	0.00	0.00	10.24	0.00
time (sec)	N/A	0.364	0.074	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	100	74	0	0	0	0	0	1674	0
N.S.	1	1.18	0.87	0.00	0.00	0.00	0.00	0.00	19.69	0.00
time (sec)	N/A	0.366	0.073	0.000	0.000	0.000	0.000	0.000	0.243	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	96	108	91	0	0	0	0	0	0	0
N.S.	1	1.12	0.95	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.457	0.075	0.000	0.000	0.000	0.000	0.000	0.272	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	24	31	42	33	61	61	32	45
N.S.	1	1.00	0.92	1.19	1.62	1.27	2.35	2.35	1.23	1.73
time (sec)	N/A	0.262	0.050	1.288	0.076	0.105	0.783	0.211	0.283	8.736

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	158	94	57	120	128	0	286	117	121
N.S.	1	0.95	0.56	0.34	0.72	0.77	0.00	1.71	0.70	0.72
time (sec)	N/A	0.608	0.066	1.300	0.050	0.077	0.000	0.110	0.206	8.822

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	126	72	41	98	104	0	237	93	101
N.S.	1	0.96	0.55	0.31	0.75	0.79	0.00	1.81	0.71	0.77
time (sec)	N/A	0.494	0.067	1.335	0.041	0.098	0.000	0.114	0.189	8.775

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	136	56	27	75	79	0	188	69	81
N.S.	1	1.45	0.60	0.29	0.80	0.84	0.00	2.00	0.73	0.86
time (sec)	N/A	0.482	0.040	1.208	0.043	0.073	0.000	0.114	0.201	8.740

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	61	37	41	45	55	0	130	44	0
N.S.	1	1.02	0.62	0.68	0.75	0.92	0.00	2.17	0.73	0.00
time (sec)	N/A	0.360	0.025	1.115	0.042	0.088	0.000	0.115	0.187	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	80	74	49	0	154	0	111	71	0
N.S.	1	0.99	0.91	0.60	0.00	1.90	0.00	1.37	0.88	0.00
time (sec)	N/A	0.404	0.078	1.217	0.000	0.084	0.000	0.123	0.190	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	91	75	64	0	163	0	71	101	0
N.S.	1	1.10	0.90	0.77	0.00	1.96	0.00	0.86	1.22	0.00
time (sec)	N/A	0.431	0.139	1.215	0.000	0.089	0.000	0.219	0.180	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	94	94	82	0	190	0	120	131	0
N.S.	1	0.94	0.94	0.82	0.00	1.90	0.00	1.20	1.31	0.00
time (sec)	N/A	0.434	0.155	1.261	0.000	0.106	0.000	0.199	0.184	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	124	116	100	0	241	0	127	170	0
N.S.	1	0.91	0.85	0.74	0.00	1.77	0.00	0.93	1.25	0.00
time (sec)	N/A	0.527	0.258	1.190	0.000	0.083	0.000	0.244	0.189	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	159	137	118	0	292	0	196	206	0
N.S.	1	0.91	0.78	0.67	0.00	1.67	0.00	1.12	1.18	0.00
time (sec)	N/A	0.625	0.242	0.614	0.000	0.089	0.000	0.268	0.195	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	226	137	58	186	201	0	600	189	171
N.S.	1	0.94	0.57	0.24	0.78	0.84	0.00	2.50	0.79	0.71
time (sec)	N/A	0.792	0.111	0.619	0.042	0.076	0.000	0.192	0.186	9.299

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	194	115	41	164	176	0	528	165	152
N.S.	1	0.95	0.56	0.20	0.80	0.86	0.00	2.58	0.80	0.74
time (sec)	N/A	0.697	0.078	0.605	0.044	0.081	0.000	0.188	0.191	9.212

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	252	99	27	142	152	0	456	141	132
N.S.	1	1.51	0.59	0.16	0.85	0.91	0.00	2.73	0.84	0.79
time (sec)	N/A	0.700	0.065	0.578	0.042	0.076	0.000	0.157	0.197	9.285

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	129	80	64	120	129	0	384	117	122
N.S.	1	0.98	0.61	0.49	0.92	0.98	0.00	2.93	0.89	0.93
time (sec)	N/A	0.520	0.058	0.473	0.043	0.074	0.000	0.123	0.187	9.035

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	95	61	61	97	103	0	309	93	101
N.S.	1	1.01	0.65	0.65	1.03	1.10	0.00	3.29	0.99	1.07
time (sec)	N/A	0.433	0.049	0.541	0.043	0.075	0.000	0.140	0.199	9.013

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	61	42	41	72	78	0	226	68	42
N.S.	1	1.02	0.70	0.68	1.20	1.30	0.00	3.77	1.13	0.70
time (sec)	N/A	0.358	0.039	0.465	0.042	0.079	0.000	0.130	0.183	9.496

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	93	82	0	201	0	133	105	0
N.S.	1	1.00	0.89	0.78	0.00	1.91	0.00	1.27	1.00	0.00
time (sec)	N/A	0.475	0.096	0.493	0.000	0.084	0.000	0.124	0.192	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	117	88	100	0	190	0	114	123	0
N.S.	1	1.03	0.77	0.88	0.00	1.67	0.00	1.00	1.08	0.00
time (sec)	N/A	0.506	0.138	0.536	0.000	0.087	0.000	0.336	0.201	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	119	101	94	0	212	0	133	146	0
N.S.	1	0.99	0.84	0.78	0.00	1.77	0.00	1.11	1.22	0.00
time (sec)	N/A	0.508	0.164	0.574	0.000	0.090	0.000	0.189	0.187	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	122	119	108	0	242	0	142	170	0
N.S.	1	0.92	0.89	0.81	0.00	1.82	0.00	1.07	1.28	0.00
time (sec)	N/A	0.505	0.216	0.609	0.000	0.096	0.000	0.190	0.194	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	153	136	132	0	295	0	196	206	0
N.S.	1	0.88	0.79	0.76	0.00	1.71	0.00	1.13	1.19	0.00
time (sec)	N/A	0.592	0.304	0.617	0.000	0.085	0.000	0.183	0.202	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	186	155	156	0	334	0	189	242	0
N.S.	1	0.90	0.75	0.76	0.00	1.62	0.00	0.92	1.17	0.00
time (sec)	N/A	0.677	0.327	0.682	0.000	0.103	0.000	0.189	0.195	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	314	294	175	58	252	273	0	1034	261	232
N.S.	1	0.94	0.56	0.18	0.80	0.87	0.00	3.29	0.83	0.74
time (sec)	N/A	1.060	0.109	0.639	0.044	0.075	0.000	0.218	0.191	9.685

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	262	153	41	230	248	0	938	237	211
N.S.	1	0.94	0.55	0.15	0.82	0.89	0.00	3.36	0.85	0.76
time (sec)	N/A	0.883	0.094	0.601	0.044	0.074	0.000	0.136	0.213	9.259

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	368	137	27	208	225	0	842	213	192
N.S.	1	1.53	0.57	0.11	0.87	0.94	0.00	3.51	0.89	0.80
time (sec)	N/A	0.947	0.090	0.597	0.049	0.148	0.000	0.126	0.198	9.087

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	197	118	82	186	201	0	746	189	172
N.S.	1	0.96	0.58	0.40	0.91	0.98	0.00	3.64	0.92	0.84
time (sec)	N/A	0.716	0.080	0.494	0.052	0.150	0.000	0.130	0.196	9.212

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	163	99	86	164	176	0	650	165	153
N.S.	1	0.98	0.59	0.51	0.98	1.05	0.00	3.89	0.99	0.92
time (sec)	N/A	0.606	0.067	0.550	0.044	0.108	0.000	0.136	0.191	9.095

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	129	80	85	142	152	0	554	141	141
N.S.	1	0.98	0.61	0.65	1.08	1.16	0.00	4.23	1.08	1.08
time (sec)	N/A	0.523	0.059	0.516	0.045	0.177	0.000	0.380	0.186	8.895

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	95	61	61	119	126	0	456	117	120
N.S.	1	1.01	0.65	0.65	1.27	1.34	0.00	4.85	1.24	1.28
time (sec)	N/A	0.436	0.051	0.566	0.045	0.114	0.000	0.170	0.201	8.866

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	61	42	41	94	102	0	344	92	123
N.S.	1	1.02	0.70	0.68	1.57	1.70	0.00	5.73	1.53	2.05
time (sec)	N/A	0.350	0.044	0.529	0.042	0.140	0.000	0.124	0.201	9.563

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	130	112	96	0	251	0	151	145	0
N.S.	1	0.99	0.85	0.73	0.00	1.92	0.00	1.15	1.11	0.00
time (sec)	N/A	0.567	0.117	0.579	0.000	0.143	0.000	0.122	0.209	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	142	108	128	0	244	0	153	165	0
N.S.	1	0.97	0.74	0.88	0.00	1.67	0.00	1.05	1.13	0.00
time (sec)	N/A	0.604	0.226	0.615	0.000	0.160	0.000	0.160	0.244	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	145	115	117	0	244	0	173	173	0
N.S.	1	0.94	0.74	0.75	0.00	1.57	0.00	1.12	1.12	0.00
time (sec)	N/A	0.611	0.196	0.644	0.000	0.166	0.000	0.159	0.248	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	147	125	118	0	264	0	152	186	0
N.S.	1	0.94	0.80	0.76	0.00	1.69	0.00	0.97	1.19	0.00
time (sec)	N/A	0.601	0.218	0.718	0.000	0.141	0.000	0.154	0.234	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	150	138	132	0	292	0	195	206	0
N.S.	1	0.89	0.82	0.78	0.00	1.73	0.00	1.15	1.22	0.00
time (sec)	N/A	0.596	0.310	0.754	0.000	0.168	0.000	0.161	0.204	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	181	155	156	0	343	0	205	242	0
N.S.	1	0.88	0.75	0.76	0.00	1.67	0.00	1.00	1.17	0.00
time (sec)	N/A	0.695	0.379	0.891	0.000	0.142	0.000	0.165	0.192	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	215	175	180	0	390	0	268	278	0
N.S.	1	0.88	0.72	0.74	0.00	1.60	0.00	1.10	1.14	0.00
time (sec)	N/A	0.801	0.386	0.903	0.000	0.158	0.000	0.198	0.216	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	158	94	92	120	105	0	143	93	104
N.S.	1	0.96	0.57	0.56	0.73	0.64	0.00	0.87	0.56	0.63
time (sec)	N/A	0.594	0.059	0.648	0.044	0.135	0.000	0.121	0.191	9.072

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	124	75	75	98	81	0	120	69	84
N.S.	1	0.96	0.58	0.58	0.76	0.63	0.00	0.93	0.53	0.65
time (sec)	N/A	0.502	0.048	0.645	0.041	0.131	0.000	0.146	0.209	9.112

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	90	55	56	75	56	0	93	45	62
N.S.	1	0.98	0.60	0.61	0.82	0.61	0.00	1.01	0.49	0.67
time (sec)	N/A	0.410	0.037	0.605	0.044	0.115	0.000	0.121	0.204	8.916

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	36	36	45	34	0	65	24	40
N.S.	1	1.00	0.62	0.62	0.78	0.59	0.00	1.12	0.41	0.69
time (sec)	N/A	0.324	0.031	0.503	0.041	0.148	0.000	0.111	0.200	8.691

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	56	66	27	0	137	0	77	51	0
N.S.	1	0.97	1.14	0.47	0.00	2.36	0.00	1.33	0.88	0.00
time (sec)	N/A	0.368	0.041	0.415	0.000	0.124	0.000	0.118	0.199	0.000

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	62	68	38	0	148	0	60	90	0
N.S.	1	0.97	1.06	0.59	0.00	2.31	0.00	0.94	1.41	0.00
time (sec)	N/A	0.360	0.082	0.448	0.000	0.123	0.000	0.138	0.200	0.000

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	97	93	43	0	197	0	115	132	0
N.S.	1	0.92	0.89	0.41	0.00	1.88	0.00	1.10	1.26	0.00
time (sec)	N/A	0.438	0.142	0.463	0.000	0.125	0.000	0.146	0.205	0.000

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	131	116	64	0	246	0	131	170	0
N.S.	1	0.92	0.82	0.45	0.00	1.73	0.00	0.92	1.20	0.00
time (sec)	N/A	0.526	0.205	0.506	0.000	0.125	0.000	0.155	0.205	0.000

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	165	135	82	0	295	0	180	206	0
N.S.	1	0.92	0.75	0.46	0.00	1.65	0.00	1.01	1.15	0.00
time (sec)	N/A	0.599	0.225	0.498	0.000	0.145	0.000	0.154	0.189	0.000

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	167	91	109	97	114	0	166	95	110
N.S.	1	1.05	0.57	0.69	0.61	0.72	0.00	1.04	0.60	0.69
time (sec)	N/A	0.634	0.062	0.635	0.045	0.113	0.000	0.119	0.204	9.135

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	133	72	85	74	90	0	134	71	86
N.S.	1	1.06	0.58	0.68	0.59	0.72	0.00	1.07	0.57	0.69
time (sec)	N/A	0.526	0.075	0.543	0.042	0.124	0.000	0.132	0.196	9.075

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	101	51	61	52	65	0	99	46	61
N.S.	1	1.15	0.58	0.69	0.59	0.74	0.00	1.12	0.52	0.69
time (sec)	N/A	0.445	0.049	0.523	0.042	0.139	0.000	0.235	0.196	8.951

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	69	32	40	34	44	0	60	26	41
N.S.	1	1.28	0.59	0.74	0.63	0.81	0.00	1.11	0.48	0.76
time (sec)	N/A	0.368	0.032	0.575	0.043	0.126	0.000	0.307	0.210	9.154

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	64	67	58	0	197	0	106	72	0
N.S.	1	0.97	1.02	0.88	0.00	2.98	0.00	1.61	1.09	0.00
time (sec)	N/A	0.358	0.066	0.451	0.000	0.141	0.000	0.293	0.204	0.000

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	90	77	41	0	252	0	97	132	0
N.S.	1	0.96	0.82	0.44	0.00	2.68	0.00	1.03	1.40	0.00
time (sec)	N/A	0.413	0.150	0.553	0.000	0.152	0.000	0.237	0.204	0.000

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	191	105	27	0	313	0	137	168	0
N.S.	1	1.41	0.78	0.20	0.00	2.32	0.00	1.01	1.24	0.00
time (sec)	N/A	0.592	0.177	0.542	0.000	0.135	0.000	0.356	0.208	0.000

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	160	129	46	0	364	0	177	196	0
N.S.	1	0.92	0.74	0.26	0.00	2.09	0.00	1.02	1.13	0.00
time (sec)	N/A	0.589	0.220	0.469	0.000	0.167	0.000	0.440	0.196	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	194	148	65	0	413	0	209	220	0
N.S.	1	0.92	0.70	0.31	0.00	1.96	0.00	0.99	1.04	0.00
time (sec)	N/A	0.693	0.263	0.481	0.000	0.129	0.000	0.429	0.202	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	166	92	109	117	125	0	157	102	110
N.S.	1	1.03	0.57	0.68	0.73	0.78	0.00	0.98	0.63	0.68
time (sec)	N/A	0.625	0.075	0.705	0.047	0.089	0.000	0.315	0.205	9.412

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	131	70	85	95	99	0	121	77	85
N.S.	1	1.05	0.56	0.68	0.76	0.79	0.00	0.97	0.62	0.68
time (sec)	N/A	0.515	0.067	1.007	0.052	0.100	0.000	0.114	0.208	9.251

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	102	53	61	74	77	0	81	54	62
N.S.	1	1.16	0.60	0.69	0.84	0.88	0.00	0.92	0.61	0.70
time (sec)	N/A	0.448	0.050	0.895	0.043	0.097	0.000	0.112	0.188	9.179

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	71	36	40	53	55	0	51	34	41
N.S.	1	1.27	0.64	0.71	0.95	0.98	0.00	0.91	0.61	0.73
time (sec)	N/A	0.372	0.040	0.622	0.046	0.092	0.000	0.258	0.189	8.860

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	94	83	79	0	268	0	120	145	0
N.S.	1	1.02	0.90	0.86	0.00	2.91	0.00	1.30	1.58	0.00
time (sec)	N/A	0.452	0.094	0.524	0.000	0.122	0.000	0.215	0.187	0.000

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	136	104	75	0	372	0	103	277	0
N.S.	1	1.06	0.81	0.59	0.00	2.91	0.00	0.80	2.16	0.00
time (sec)	N/A	0.548	0.133	0.520	0.000	0.136	0.000	0.294	0.190	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	171	122	58	0	431	0	161	319	0
N.S.	1	1.01	0.72	0.34	0.00	2.54	0.00	0.95	1.88	0.00
time (sec)	N/A	0.646	0.225	0.510	0.000	0.103	0.000	0.227	0.197	0.000

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	188	144	45	0	482	0	208	347	0
N.S.	1	0.89	0.68	0.21	0.00	2.28	0.00	0.99	1.64	0.00
time (sec)	N/A	0.663	0.281	0.494	0.000	0.144	0.000	0.245	0.235	0.000

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	363	167	26	0	531	0	235	371	0
N.S.	1	1.46	0.67	0.10	0.00	2.14	0.00	0.95	1.50	0.00
time (sec)	N/A	0.930	0.317	0.476	0.000	0.119	0.000	0.202	0.235	0.000

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	261	186	65	0	578	0	267	395	0
N.S.	1	0.92	0.65	0.23	0.00	2.03	0.00	0.94	1.39	0.00
time (sec)	N/A	0.964	0.445	0.523	0.000	0.118	0.000	0.212	0.245	0.000

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	248	210	199	0	378	0	303	220	0
N.S.	1	0.96	0.81	0.77	0.00	1.47	0.00	1.17	0.85	0.00
time (sec)	N/A	0.889	0.778	0.715	0.000	0.102	0.000	0.183	0.243	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	206	151	171	0	313	0	269	179	0
N.S.	1	0.96	0.71	0.80	0.00	1.46	0.00	1.26	0.84	0.00
time (sec)	N/A	0.747	0.628	0.517	0.000	0.098	0.000	0.197	0.202	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	164	172	144	0	270	0	235	142	0
N.S.	1	0.95	1.00	0.84	0.00	1.57	0.00	1.37	0.83	0.00
time (sec)	N/A	0.625	0.514	0.570	0.000	0.104	0.000	0.169	0.200	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	124	110	127	0	221	0	184	103	0
N.S.	1	0.95	0.84	0.97	0.00	1.69	0.00	1.40	0.79	0.00
time (sec)	N/A	0.497	0.173	0.411	0.000	0.100	0.000	0.159	0.196	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	121	105	117	0	199	0	124	103	0
N.S.	1	1.11	0.96	1.07	0.00	1.83	0.00	1.14	0.94	0.00
time (sec)	N/A	0.547	0.211	0.405	0.000	0.103	0.000	0.240	0.201	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	114	98	119	0	204	0	122	96	0
N.S.	1	1.07	0.92	1.11	0.00	1.91	0.00	1.14	0.90	0.00
time (sec)	N/A	0.514	0.157	0.408	0.000	0.116	0.000	0.472	0.221	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	43	45	0	66	0	98	111	74
N.S.	1	1.00	0.61	0.64	0.00	0.94	0.00	1.40	1.59	1.06
time (sec)	N/A	0.374	0.141	0.399	0.000	0.106	0.000	0.158	0.205	9.002

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	70	67	0	91	0	142	153	99
N.S.	1	1.00	0.62	0.60	0.00	0.81	0.00	1.27	1.37	0.88
time (sec)	N/A	0.472	0.168	0.445	0.000	0.086	0.000	0.160	0.191	9.143

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	154	85	91	0	115	0	182	193	122
N.S.	1	0.99	0.54	0.58	0.00	0.74	0.00	1.17	1.24	0.78
time (sec)	N/A	0.540	0.203	0.396	0.000	0.090	0.000	0.188	0.209	8.944

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	328	248	243	0	459	0	655	299	0
N.S.	1	0.96	0.72	0.71	0.00	1.34	0.00	1.91	0.87	0.00
time (sec)	N/A	1.134	1.327	0.401	0.000	0.106	0.000	0.225	0.250	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	286	228	217	0	414	0	369	262	0
N.S.	1	0.95	0.76	0.72	0.00	1.38	0.00	1.23	0.87	0.00
time (sec)	N/A	0.979	1.079	0.430	0.000	0.105	0.000	0.191	0.225	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	244	205	198	0	367	0	329	222	0
N.S.	1	0.94	0.79	0.76	0.00	1.41	0.00	1.27	0.85	0.00
time (sec)	N/A	0.834	0.810	0.411	0.000	0.103	0.000	0.203	0.197	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	202	190	175	0	320	0	286	182	0
N.S.	1	0.94	0.88	0.81	0.00	1.48	0.00	1.32	0.84	0.00
time (sec)	N/A	0.724	0.652	0.417	0.000	0.112	0.000	0.190	0.216	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	162	136	150	0	273	0	230	142	0
N.S.	1	0.94	0.79	0.87	0.00	1.59	0.00	1.34	0.83	0.00
time (sec)	N/A	0.631	0.211	0.431	0.000	0.106	0.000	0.404	0.196	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	159	126	133	0	247	0	158	143	0
N.S.	1	1.04	0.82	0.87	0.00	1.61	0.00	1.03	0.93	0.00
time (sec)	N/A	0.617	0.371	0.500	0.000	0.116	0.000	0.410	0.205	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	161	121	134	0	233	0	187	127	0
N.S.	1	1.10	0.82	0.91	0.00	1.59	0.00	1.27	0.86	0.00
time (sec)	N/A	0.617	0.267	0.447	0.000	0.109	0.000	0.269	0.209	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	149	125	145	0	252	0	153	139	0
N.S.	1	1.06	0.89	1.03	0.00	1.79	0.00	1.09	0.99	0.00
time (sec)	N/A	0.576	0.227	0.426	0.000	0.125	0.000	0.390	0.209	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	43	45	0	89	0	105	153	99
N.S.	1	1.00	0.61	0.64	0.00	1.27	0.00	1.50	2.19	1.41
time (sec)	N/A	0.377	0.208	0.471	0.000	0.083	0.000	0.245	0.208	9.490

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	64	67	0	115	0	145	193	122
N.S.	1	1.00	0.57	0.60	0.00	1.03	0.00	1.29	1.72	1.09
time (sec)	N/A	0.455	0.251	0.435	0.000	0.089	0.000	0.211	0.222	9.375

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	154	88	91	0	140	0	189	233	146
N.S.	1	0.99	0.56	0.58	0.00	0.90	0.00	1.21	1.49	0.94
time (sec)	N/A	0.558	0.309	0.472	0.000	0.088	0.000	0.225	0.220	9.294

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	324	247	246	0	462	0	421	302	0
N.S.	1	0.94	0.71	0.71	0.00	1.34	0.00	1.22	0.87	0.00
time (sec)	N/A	1.122	1.310	0.600	0.000	0.103	0.000	0.252	0.258	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	302	282	230	223	0	417	0	375	262	0
N.S.	1	0.93	0.76	0.74	0.00	1.38	0.00	1.24	0.87	0.00
time (sec)	N/A	0.968	1.079	0.444	0.000	0.117	0.000	0.212	0.216	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	240	212	199	0	368	0	331	222	0
N.S.	1	0.93	0.82	0.77	0.00	1.43	0.00	1.28	0.86	0.00
time (sec)	N/A	0.844	0.883	0.446	0.000	0.106	0.000	0.339	0.223	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	200	155	174	0	321	0	271	182	0
N.S.	1	0.93	0.72	0.81	0.00	1.50	0.00	1.27	0.85	0.00
time (sec)	N/A	0.715	0.282	0.418	0.000	0.102	0.000	0.205	0.222	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	197	148	159	0	301	0	193	187	0
N.S.	1	1.02	0.76	0.82	0.00	1.55	0.00	0.99	0.96	0.00
time (sec)	N/A	0.745	0.514	0.410	0.000	0.099	0.000	0.263	0.219	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	199	139	159	0	287	0	235	182	0
N.S.	1	1.01	0.71	0.81	0.00	1.46	0.00	1.19	0.92	0.00
time (sec)	N/A	0.710	0.451	0.431	0.000	0.095	0.000	0.274	0.209	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	199	145	160	0	287	0	245	182	0
N.S.	1	1.06	0.78	0.86	0.00	1.53	0.00	1.31	0.97	0.00
time (sec)	N/A	0.734	0.359	0.414	0.000	0.098	0.000	0.375	0.211	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	184	143	169	0	304	0	183	183	0
N.S.	1	1.04	0.81	0.95	0.00	1.72	0.00	1.03	1.03	0.00
time (sec)	N/A	0.674	0.302	0.418	0.000	0.098	0.000	0.338	0.232	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	43	45	0	113	0	105	193	121
N.S.	1	1.00	0.61	0.64	0.00	1.61	0.00	1.50	2.76	1.73
time (sec)	N/A	0.357	0.280	0.409	0.000	0.098	0.000	0.277	0.221	9.438

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	64	67	0	138	0	149	233	144
N.S.	1	1.00	0.57	0.60	0.00	1.23	0.00	1.33	2.08	1.29
time (sec)	N/A	0.452	0.360	0.419	0.000	0.090	0.000	0.192	0.225	9.587

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	154	94	91	0	163	0	189	273	168
N.S.	1	0.99	0.60	0.58	0.00	1.04	0.00	1.21	1.75	1.08
time (sec)	N/A	0.566	0.449	0.412	0.000	0.093	0.000	0.254	0.324	9.680

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	210	199	174	0	350	0	256	182	0
N.S.	1	0.96	0.91	0.79	0.00	1.60	0.00	1.17	0.83	0.00
time (sec)	N/A	0.794	0.608	0.405	0.000	0.103	0.000	0.139	0.247	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	168	138	150	0	297	0	218	142	0
N.S.	1	0.96	0.79	0.86	0.00	1.70	0.00	1.25	0.81	0.00
time (sec)	N/A	0.653	0.521	0.411	0.000	0.106	0.000	0.272	0.286	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	126	112	128	0	226	0	172	102	0
N.S.	1	0.96	0.85	0.98	0.00	1.73	0.00	1.31	0.78	0.00
time (sec)	N/A	0.544	0.322	0.398	0.000	0.106	0.000	0.134	0.216	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	100	108	0	185	0	135	66	0
N.S.	1	1.00	1.20	1.30	0.00	2.23	0.00	1.63	0.80	0.00
time (sec)	N/A	0.435	0.210	0.401	0.000	0.109	0.000	0.139	0.211	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	80	84	94	0	177	0	85	60	0
N.S.	1	1.04	1.09	1.22	0.00	2.30	0.00	1.10	0.78	0.00
time (sec)	N/A	0.401	0.112	0.394	0.000	0.105	0.000	0.214	0.207	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	42	44	0	45	0	78	69	51
N.S.	1	1.00	0.60	0.63	0.00	0.64	0.00	1.11	0.99	0.73
time (sec)	N/A	0.358	0.120	0.401	0.000	0.087	0.000	0.245	0.201	9.661

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	63	67	0	68	0	132	111	77
N.S.	1	1.00	0.56	0.60	0.00	0.61	0.00	1.18	0.99	0.69
time (sec)	N/A	0.440	0.141	0.424	0.000	0.091	0.000	0.189	0.207	9.740

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	154	83	91	0	92	0	172	153	102
N.S.	1	0.99	0.53	0.58	0.00	0.59	0.00	1.10	0.98	0.65
time (sec)	N/A	0.542	0.182	0.403	0.000	0.083	0.000	0.163	0.227	10.038

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	180	171	214	0	362	0	230	157	0
N.S.	1	1.05	1.00	1.25	0.00	2.12	0.00	1.35	0.92	0.00
time (sec)	N/A	0.692	0.426	0.428	0.000	0.105	0.000	0.173	0.217	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	138	112	199	0	301	0	178	127	0
N.S.	1	1.12	0.91	1.62	0.00	2.45	0.00	1.45	1.03	0.00
time (sec)	N/A	0.566	0.343	0.418	0.000	0.103	0.000	0.151	0.203	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	90	90	174	0	246	0	126	88	0
N.S.	1	1.03	1.03	2.00	0.00	2.83	0.00	1.45	1.01	0.00
time (sec)	N/A	0.425	0.119	0.398	0.000	0.106	0.000	0.173	0.191	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	40	44	0	55	0	111	71	68
N.S.	1	1.00	0.61	0.67	0.00	0.83	0.00	1.68	1.08	1.03
time (sec)	N/A	0.356	0.100	0.408	0.000	0.090	0.000	0.190	0.211	9.729

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	110	61	66	0	76	0	304	95	98
N.S.	1	0.99	0.55	0.59	0.00	0.68	0.00	2.74	0.86	0.88
time (sec)	N/A	0.439	0.151	0.415	0.000	0.092	0.000	0.763	0.209	9.881

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	149	82	91	0	103	0	323	129	115
N.S.	1	0.99	0.54	0.60	0.00	0.68	0.00	2.14	0.85	0.76
time (sec)	N/A	0.548	0.171	0.415	0.000	0.100	0.000	0.284	0.204	9.823

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	191	101	115	0	131	0	375	159	148
N.S.	1	0.99	0.53	0.60	0.00	0.68	0.00	1.95	0.83	0.77
time (sec)	N/A	0.651	0.229	0.430	0.000	0.091	0.000	0.414	0.224	9.488

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	223	190	319	0	494	0	290	296	0
N.S.	1	1.04	0.89	1.49	0.00	2.31	0.00	1.36	1.38	0.00
time (sec)	N/A	0.787	0.557	0.425	0.000	0.129	0.000	0.181	0.207	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	181	133	304	0	431	0	236	230	0
N.S.	1	1.11	0.82	1.87	0.00	2.64	0.00	1.45	1.41	0.00
time (sec)	N/A	0.657	0.486	0.431	0.000	0.120	0.000	0.230	0.212	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	130	108	247	0	336	0	151	146	0
N.S.	1	1.08	0.90	2.06	0.00	2.80	0.00	1.26	1.22	0.00
time (sec)	N/A	0.528	0.200	0.397	0.000	0.125	0.000	0.151	0.211	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	42	44	0	68	0	73	110	80
N.S.	1	1.00	0.49	0.52	0.00	0.80	0.00	0.86	1.29	0.94
time (sec)	N/A	0.394	0.120	0.401	0.000	0.103	0.000	0.141	0.204	9.731

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	123	61	67	0	97	0	171	146	109
N.S.	1	1.13	0.56	0.61	0.00	0.89	0.00	1.57	1.34	1.00
time (sec)	N/A	0.495	0.154	0.425	0.000	0.107	0.000	0.391	0.212	9.512

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	159	77	90	0	123	0	365	169	134
N.S.	1	1.05	0.51	0.59	0.00	0.81	0.00	2.40	1.11	0.88
time (sec)	N/A	0.577	0.170	0.437	0.000	0.102	0.000	1.457	0.209	9.393

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	188	102	115	0	141	0	0	200	152
N.S.	1	0.97	0.53	0.59	0.00	0.73	0.00	0.00	1.03	0.78
time (sec)	N/A	0.627	0.239	0.427	0.000	0.096	0.000	0.000	0.217	9.497

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F(-1)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	232	121	139	0	160	0	0	229	183
N.S.	1	0.97	0.51	0.58	0.00	0.67	0.00	0.00	0.96	0.77
time (sec)	N/A	0.746	0.295	0.447	0.000	0.101	0.000	0.000	0.218	9.452

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	274	134	163	0	187	0	533	260	183
N.S.	1	0.98	0.48	0.58	0.00	0.67	0.00	1.90	0.93	0.65
time (sec)	N/A	0.870	0.309	0.474	0.000	0.101	0.000	2.072	0.219	9.367

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	125	90	457	161	383	2111	683	457	282
N.S.	1	1.03	0.74	3.78	1.33	3.17	17.45	5.64	3.78	2.33
time (sec)	N/A	0.482	0.142	0.399	0.059	0.096	0.704	0.389	0.226	9.106

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	95	74	92	115	219	1081	388	249	181
N.S.	1	1.04	0.81	1.01	1.26	2.41	11.88	4.26	2.74	1.99
time (sec)	N/A	0.423	0.109	0.342	0.044	0.099	0.465	0.291	0.197	8.939

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	64	60	61	69	96	425	173	101	99
N.S.	1	1.07	1.00	1.02	1.15	1.60	7.08	2.88	1.68	1.65
time (sec)	N/A	0.353	0.065	0.062	0.045	0.110	0.327	0.203	0.207	8.836

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	65	52	0	0	0	0	0	117	0
N.S.	1	0.98	0.79	0.00	0.00	0.00	0.00	0.00	1.77	0.00
time (sec)	N/A	0.313	0.067	0.000	0.000	0.000	0.000	0.000	0.207	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	94	69	0	0	0	0	0	821	0
N.S.	1	1.11	0.81	0.00	0.00	0.00	0.00	0.00	9.66	0.00
time (sec)	N/A	0.358	0.066	0.000	0.000	0.000	0.000	0.000	0.203	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	100	74	0	0	0	0	0	1674	0
N.S.	1	1.18	0.87	0.00	0.00	0.00	0.00	0.00	19.69	0.00
time (sec)	N/A	0.365	0.081	0.000	0.000	0.000	0.000	0.000	0.209	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	111	123	101	0	0	0	0	0	0	0
N.S.	1	1.11	0.91	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.472	0.112	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	106	121	115	0	0	0	0	0	1514	0
N.S.	1	1.14	1.08	0.00	0.00	0.00	0.00	0.00	14.28	0.00
time (sec)	N/A	0.487	0.082	0.000	0.000	0.000	0.000	0.000	0.232	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	137	115	1454	247	1171	18686	3372	1508	1641
N.S.	1	0.94	0.79	9.96	1.69	8.02	127.99	23.10	10.33	11.24
time (sec)	N/A	0.536	0.198	7.014	0.062	0.135	94.522	0.364	0.257	9.747

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	106	96	653	179	565	6790	1616	677	809
N.S.	1	0.95	0.86	5.88	1.61	5.09	61.17	14.56	6.10	7.29
time (sec)	N/A	0.468	0.155	1.748	0.048	0.123	29.238	0.254	0.243	8.680

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	62	74	205	99	190	1756	544	211	297
N.S.	1	0.98	1.17	3.25	1.57	3.02	27.87	8.63	3.35	4.71
time (sec)	N/A	0.380	0.085	0.409	0.043	0.094	11.991	0.295	0.276	5.362

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	82	85	61	0	0	0	0	0	421	0
N.S.	1	1.04	0.74	0.00	0.00	0.00	0.00	0.00	5.13	0.00
time (sec)	N/A	0.402	0.083	0.000	0.000	0.000	0.000	0.000	0.204	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	94	106	86	0	0	0	0	0	0	0
N.S.	1	1.13	0.91	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.431	0.080	0.000	0.000	0.000	0.000	0.000	0.206	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	99	113	93	0	0	0	0	0	0	0
N.S.	1	1.14	0.94	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.436	0.074	0.000	0.000	0.000	0.000	0.000	0.208	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	172	127	0	0	0	0	0	28	0
N.S.	1	1.17	0.86	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.503	0.345	0.000	0.000	0.000	0.000	0.000	200.037	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	147	172	127	0	0	0	0	0	86	0
N.S.	1	1.17	0.86	0.00	0.00	0.00	0.00	0.00	0.59	0.00
time (sec)	N/A	0.496	0.279	0.000	0.000	0.000	0.000	0.000	34.695	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	141	166	121	0	0	0	0	0	42	0
N.S.	1	1.18	0.86	0.00	0.00	0.00	0.00	0.00	0.30	0.00
time (sec)	N/A	0.484	0.221	0.000	0.000	0.000	0.000	0.000	1.723	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	170	106	0	0	0	0	0	52	0
N.S.	1	1.17	0.73	0.00	0.00	0.00	0.00	0.00	0.36	0.00
time (sec)	N/A	0.504	0.216	0.000	0.000	0.000	0.000	0.000	0.304	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	170	114	0	0	0	0	0	82	0
N.S.	1	1.17	0.79	0.00	0.00	0.00	0.00	0.00	0.57	0.00
time (sec)	N/A	0.498	0.281	0.000	0.000	0.000	0.000	0.000	0.373	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	173	115	0	0	0	0	0	126	0
N.S.	1	1.18	0.79	0.00	0.00	0.00	0.00	0.00	0.86	0.00
time (sec)	N/A	0.491	0.327	0.000	0.000	0.000	0.000	0.000	0.290	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	155	103	0	0	0	0	0	0	0
N.S.	1	1.23	0.82	0.00	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.482	0.077	0.000	0.000	0.000	0.000	0.000	0.288	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [153] had the largest ratio of [.4090909999999999982]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	18	0.167
2	A	3	3	1.00	16	0.188
3	A	2	2	1.00	15	0.133
4	A	3	3	1.00	18	0.167
5	A	3	3	1.00	18	0.167
6	A	3	3	1.00	18	0.167
7	A	3	3	1.00	18	0.167
8	A	3	3	1.00	18	0.167
9	A	3	3	1.00	18	0.167
10	A	3	3	1.00	20	0.150
11	A	3	3	1.00	18	0.167
12	A	2	2	1.00	17	0.118
13	A	3	3	1.00	20	0.150
14	A	3	3	1.00	20	0.150
15	A	4	4	1.00	20	0.200
16	A	3	3	1.00	20	0.150
17	A	3	3	1.00	20	0.150
18	A	3	3	1.00	20	0.150
19	A	3	3	1.00	20	0.150
20	A	3	3	1.00	20	0.150
21	A	3	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	3	3	1.00	18	0.167
23	A	2	2	1.00	17	0.118
24	A	3	3	1.00	20	0.150
25	A	3	3	1.00	20	0.150
26	A	3	3	1.00	20	0.150
27	A	4	4	0.98	20	0.200
28	A	3	3	1.00	20	0.150
29	A	3	3	1.00	20	0.150
30	A	3	3	1.00	20	0.150
31	A	4	4	0.98	20	0.200
32	A	3	3	1.00	20	0.150
33	A	3	3	1.00	20	0.150
34	A	3	3	1.00	20	0.150
35	A	3	3	1.00	20	0.150
36	A	3	3	1.00	20	0.150
37	A	3	3	1.00	20	0.150
38	A	3	3	1.00	18	0.167
39	A	2	2	1.17	17	0.118
40	A	3	3	1.00	20	0.150
41	A	3	3	1.00	20	0.150
42	A	3	3	1.00	20	0.150
43	A	3	3	1.00	20	0.150
44	A	3	3	1.00	20	0.150
45	A	3	3	1.00	20	0.150
46	A	3	3	1.00	20	0.150
47	A	3	3	1.00	18	0.167
48	A	2	2	1.25	17	0.118
49	A	3	3	1.00	20	0.150
50	A	3	3	1.00	20	0.150
51	A	3	3	1.00	20	0.150
52	A	3	3	1.00	20	0.150
53	A	3	3	1.00	20	0.150

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	20	0.100
55	A	3	3	1.00	20	0.150
56	A	3	3	1.00	18	0.167
57	A	2	2	1.16	17	0.118
58	A	3	3	1.00	20	0.150
59	A	3	3	1.00	20	0.150
60	A	3	3	1.00	20	0.150
61	A	3	3	1.00	20	0.150
62	A	3	3	1.00	20	0.150
63	A	3	3	1.00	20	0.150
64	A	3	3	1.00	20	0.150
65	A	3	3	1.00	20	0.150
66	A	3	3	1.00	22	0.136
67	A	3	3	1.00	22	0.136
68	A	3	3	1.00	22	0.136
69	A	3	3	1.00	22	0.136
70	A	3	3	1.00	22	0.136
71	A	3	3	1.00	22	0.136
72	A	3	3	1.00	22	0.136
73	A	3	3	1.00	22	0.136
74	A	3	3	1.00	22	0.136
75	A	3	3	1.00	22	0.136
76	A	3	3	1.00	22	0.136
77	A	3	3	1.00	22	0.136
78	A	3	3	1.00	22	0.136
79	A	3	3	1.00	22	0.136
80	A	3	3	1.00	22	0.136
81	A	8	7	0.93	22	0.318
82	A	7	6	0.96	22	0.273
83	A	6	5	0.97	22	0.227
84	A	5	4	1.00	22	0.182
85	A	5	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	6	5	0.96	22	0.227
87	A	7	6	0.94	22	0.273
88	A	8	7	0.92	22	0.318
89	A	8	7	1.09	22	0.318
90	A	7	6	1.14	22	0.273
91	A	6	5	1.19	22	0.227
92	A	5	4	1.00	22	0.182
93	A	6	5	1.21	22	0.227
94	A	7	6	1.11	22	0.273
95	A	8	7	1.07	22	0.318
96	A	9	8	1.01	22	0.364
97	A	8	7	1.03	22	0.318
98	A	7	6	1.07	22	0.273
99	A	6	5	0.97	22	0.227
100	A	6	5	0.94	22	0.227
101	A	7	6	1.07	22	0.273
102	A	8	7	1.03	22	0.318
103	A	7	6	0.81	22	0.273
104	A	5	4	0.70	20	0.200
105	A	5	4	0.80	19	0.211
106	A	5	4	0.89	22	0.182
107	A	5	4	1.10	22	0.182
108	A	7	6	1.01	22	0.273
109	A	2	2	1.00	22	0.091
110	A	3	3	0.99	22	0.136
111	A	4	4	0.97	22	0.182
112	A	5	5	0.96	22	0.227
113	A	6	6	0.95	22	0.273
114	A	8	7	0.75	22	0.318
115	A	6	5	0.66	20	0.250
116	A	6	5	0.72	19	0.263
117	A	6	5	0.78	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	6	5	0.98	22	0.227
119	A	6	5	0.98	22	0.227
120	A	8	7	1.07	22	0.318
121	A	8	7	0.99	22	0.318
122	A	2	2	1.00	22	0.091
123	A	3	3	0.99	22	0.136
124	A	4	4	0.97	22	0.182
125	A	5	5	0.96	22	0.227
126	A	6	6	0.95	22	0.273
127	A	9	8	0.70	22	0.364
128	A	7	6	0.63	20	0.300
129	A	7	6	0.68	19	0.316
130	A	7	6	0.71	22	0.273
131	A	7	6	0.84	22	0.273
132	A	7	6	0.94	22	0.273
133	A	7	6	1.04	22	0.273
134	A	9	8	0.96	22	0.364
135	A	9	8	1.02	22	0.364
136	A	9	8	0.96	22	0.364
137	A	2	2	1.00	22	0.091
138	A	3	3	0.99	22	0.136
139	A	4	4	0.97	22	0.182
140	A	5	5	0.96	22	0.227
141	A	6	6	0.95	22	0.273
142	A	8	7	0.92	22	0.318
143	A	7	6	0.93	22	0.273
144	A	6	5	0.93	22	0.227
145	A	4	3	0.82	20	0.150
146	A	4	3	1.00	19	0.158
147	A	4	3	1.00	22	0.136
148	A	2	2	1.00	22	0.091
149	A	3	3	0.99	22	0.136

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	4	4	0.97	22	0.182
151	A	5	5	0.96	22	0.227
152	A	6	6	0.95	22	0.273
153	A	10	9	1.08	22	0.409
154	A	7	6	1.00	22	0.273
155	A	6	5	1.04	22	0.227
156	A	5	4	1.00	20	0.200
157	A	1	1	0.70	19	0.053
158	A	2	2	0.70	22	0.091
159	A	3	3	0.75	22	0.136
160	A	4	4	0.79	22	0.182
161	A	5	5	0.81	22	0.227
162	A	8	7	0.96	22	0.318
163	A	7	6	1.07	22	0.273
164	A	5	4	1.05	22	0.182
165	A	3	3	1.00	22	0.136
166	A	2	2	0.86	20	0.100
167	A	2	2	0.64	19	0.105
168	A	3	3	0.63	22	0.136
169	A	4	4	0.67	22	0.182
170	A	5	5	0.71	22	0.227
171	A	6	6	0.96	24	0.250
172	A	5	5	0.96	24	0.208
173	A	4	4	0.98	24	0.167
174	A	3	3	1.00	24	0.125
175	A	2	2	1.02	24	0.083
176	A	5	4	1.01	24	0.167
177	A	5	4	1.12	24	0.167
178	A	5	4	0.96	24	0.167
179	A	6	5	0.92	24	0.208
180	A	7	6	0.92	24	0.250
181	A	8	7	0.92	24	0.292

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	6	6	0.96	24	0.250
183	A	5	5	0.96	24	0.208
184	A	4	4	0.98	24	0.167
185	A	3	3	1.00	24	0.125
186	A	2	2	1.02	24	0.083
187	A	6	5	1.02	24	0.208
188	A	6	5	1.04	24	0.208
189	A	6	5	1.01	24	0.208
190	A	6	5	0.93	24	0.208
191	A	7	6	0.90	24	0.250
192	A	8	7	0.90	24	0.292
193	A	6	6	0.96	24	0.250
194	A	5	5	0.96	24	0.208
195	A	4	4	0.98	24	0.167
196	A	3	3	1.00	24	0.125
197	A	2	2	1.02	24	0.083
198	A	7	6	1.01	24	0.250
199	A	7	6	0.99	24	0.250
200	A	7	6	0.95	24	0.250
201	A	7	6	0.96	24	0.250
202	A	7	6	0.90	24	0.250
203	A	8	7	0.89	24	0.292
204	A	5	5	0.98	24	0.208
205	A	4	4	0.99	24	0.167
206	A	3	3	1.02	24	0.125
207	A	2	2	1.05	24	0.083
208	A	4	3	1.00	24	0.125
209	A	4	3	1.00	24	0.125
210	A	5	4	0.94	24	0.167
211	A	6	5	0.94	24	0.208
212	A	5	5	1.04	24	0.208
213	A	4	4	1.06	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	3	3	1.12	24	0.125
215	A	2	2	1.25	24	0.083
216	A	4	3	1.00	24	0.125
217	A	5	4	1.00	24	0.167
218	A	6	5	0.96	24	0.208
219	A	7	6	0.94	24	0.250
220	A	8	7	0.93	24	0.292
221	A	5	5	1.03	24	0.208
222	A	4	4	1.05	24	0.167
223	A	3	3	1.14	24	0.125
224	A	2	2	1.26	24	0.083
225	A	5	4	1.04	24	0.167
226	A	6	5	1.11	24	0.208
227	A	7	6	0.95	24	0.250
228	A	8	7	0.93	24	0.292
229	A	3	3	1.03	22	0.136
230	A	3	3	1.04	22	0.136
231	A	3	3	1.07	20	0.150
232	A	3	3	1.24	22	0.136
233	A	3	3	1.16	22	0.136
234	A	3	3	1.18	22	0.136
235	A	4	4	1.12	22	0.182
236	A	1	1	1.00	27	0.037
237	A	5	5	0.95	24	0.208
238	A	4	4	0.96	22	0.182
239	A	2	2	1.45	21	0.095
240	A	2	2	1.02	24	0.083
241	A	5	4	0.99	24	0.167
242	A	5	4	1.10	24	0.167
243	A	5	4	0.94	24	0.167
244	A	6	5	0.91	24	0.208
245	A	7	6	0.91	24	0.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	7	7	0.94	24	0.292
247	A	6	6	0.95	22	0.273
248	A	2	2	1.51	21	0.095
249	A	4	4	0.98	24	0.167
250	A	3	3	1.01	24	0.125
251	A	2	2	1.02	24	0.083
252	A	6	5	1.00	24	0.208
253	A	6	5	1.03	24	0.208
254	A	6	5	0.99	24	0.208
255	A	6	5	0.92	24	0.208
256	A	7	6	0.88	24	0.250
257	A	8	7	0.90	24	0.292
258	A	9	9	0.94	24	0.375
259	A	8	8	0.94	22	0.364
260	A	2	2	1.53	21	0.095
261	A	6	6	0.96	24	0.250
262	A	5	5	0.98	24	0.208
263	A	4	4	0.98	24	0.167
264	A	3	3	1.01	24	0.125
265	A	2	2	1.02	24	0.083
266	A	7	6	0.99	24	0.250
267	A	7	6	0.97	24	0.250
268	A	7	6	0.94	24	0.250
269	A	7	6	0.94	24	0.250
270	A	7	6	0.89	24	0.250
271	A	8	7	0.88	24	0.292
272	A	9	8	0.88	24	0.333
273	A	5	5	0.96	24	0.208
274	A	4	4	0.96	24	0.167
275	A	3	3	0.98	24	0.125
276	A	2	2	1.00	22	0.091
277	A	2	2	0.97	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
278	A	4	3	0.97	24	0.125
279	A	5	4	0.92	24	0.167
280	A	6	5	0.92	24	0.208
281	A	7	6	0.92	24	0.250
282	A	5	5	1.05	24	0.208
283	A	4	4	1.06	24	0.167
284	A	3	3	1.15	24	0.125
285	A	2	2	1.28	24	0.083
286	A	4	3	0.97	24	0.125
287	A	5	4	0.96	22	0.182
288	A	2	2	1.41	21	0.095
289	A	7	6	0.92	24	0.250
290	A	8	7	0.92	24	0.292
291	A	5	5	1.03	24	0.208
292	A	4	4	1.05	24	0.167
293	A	3	3	1.16	24	0.125
294	A	2	2	1.27	24	0.083
295	A	5	4	1.02	24	0.167
296	A	6	5	1.06	24	0.208
297	A	7	6	1.01	24	0.250
298	A	8	7	0.89	22	0.318
299	A	2	2	1.46	21	0.095
300	A	10	9	0.92	24	0.375
301	A	9	8	0.96	28	0.286
302	A	8	7	0.96	28	0.250
303	A	7	6	0.95	28	0.214
304	A	6	5	0.95	28	0.179
305	A	6	5	1.11	28	0.179
306	A	6	5	1.07	28	0.179
307	A	2	2	1.00	28	0.071
308	A	3	3	1.00	28	0.107
309	A	4	4	0.99	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
310	A	11	10	0.96	28	0.357
311	A	10	9	0.95	28	0.321
312	A	9	8	0.94	28	0.286
313	A	8	7	0.94	28	0.250
314	A	7	6	0.94	28	0.214
315	A	7	6	1.04	28	0.214
316	A	7	6	1.10	28	0.214
317	A	7	6	1.06	28	0.214
318	A	2	2	1.00	28	0.071
319	A	3	3	1.00	28	0.107
320	A	4	4	0.99	28	0.143
321	A	11	10	0.94	28	0.357
322	A	10	9	0.93	28	0.321
323	A	9	8	0.93	28	0.286
324	A	8	7	0.93	28	0.250
325	A	8	7	1.02	28	0.250
326	A	8	7	1.01	28	0.250
327	A	8	7	1.06	28	0.250
328	A	8	7	1.04	28	0.250
329	A	2	2	1.00	28	0.071
330	A	3	3	1.00	28	0.107
331	A	4	4	0.99	28	0.143
332	A	8	7	0.96	28	0.250
333	A	7	6	0.96	28	0.214
334	A	6	5	0.96	28	0.179
335	A	5	4	1.00	28	0.143
336	A	5	4	1.04	28	0.143
337	A	2	2	1.00	28	0.071
338	A	3	3	1.00	28	0.107
339	A	4	4	0.99	28	0.143
340	A	7	6	1.05	28	0.214
341	A	6	5	1.12	28	0.179

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
342	A	5	4	1.03	28	0.143
343	A	2	2	1.00	28	0.071
344	A	3	3	0.99	28	0.107
345	A	4	4	0.99	28	0.143
346	A	5	5	0.99	28	0.179
347	A	8	7	1.04	28	0.250
348	A	7	6	1.11	28	0.214
349	A	6	5	1.08	28	0.179
350	A	2	2	1.00	28	0.071
351	A	3	3	1.13	28	0.107
352	A	4	4	1.05	28	0.143
353	A	5	5	0.97	28	0.179
354	A	6	6	0.97	28	0.214
355	A	7	7	0.98	28	0.250
356	A	3	3	1.03	24	0.125
357	A	3	3	1.04	24	0.125
358	A	3	3	1.07	22	0.136
359	A	3	3	0.98	24	0.125
360	A	3	3	1.11	24	0.125
361	A	3	3	1.18	24	0.125
362	A	4	4	1.11	24	0.167
363	A	4	4	1.14	29	0.138
364	A	4	4	0.94	26	0.154
365	A	4	4	0.95	26	0.154
366	A	4	4	0.98	24	0.167
367	A	4	4	1.04	26	0.154
368	A	4	4	1.13	26	0.154
369	A	4	4	1.14	26	0.154
370	A	4	4	1.17	28	0.143
371	A	4	4	1.17	28	0.143
372	A	4	4	1.18	28	0.143
373	A	4	4	1.17	28	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
374	A	4	4	1.17	28	0.143
375	A	4	4	1.18	28	0.143
376	A	4	4	1.23	26	0.154

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int x^2(A+Bx)\left(bx+cx^2\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int x^2(A+Bx)\left(bx+cx^2\right)\,dx=\frac{1}{4}Abx^4+\frac{1}{5}(bB+Ac)x^5+\frac{1}{6}Bcx^6$$

output
```
1/4*A*b*x^4+1/5*(A*c+B*b)*x^5+1/6*B*c*x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx)\left(bx+cx^2\right)\,dx=\frac{1}{4}Abx^4+\frac{1}{5}(bB+Ac)x^5+\frac{1}{6}Bcx^6$$

input
```
Integrate[x^2*(A + B*x)*(b*x + c*x^2),x]
```

output
```
(A*b*x^4)/4 + ((b*B + A*c)*x^5)/5 + (B*c*x^6)/6
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(A+Bx)\left(bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{9}$$

$$\int x^3(A+Bx)(b+cx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(x^4(Ac+bB)+Abx^3+Bcx^5\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}x^5(Ac+bB)+\frac{1}{4}Abx^4+\frac{1}{6}Bcx^6$$

input
```
Int[x^2*(A + B*x)*(b*x + c*x^2),x]
```

output
```
(A*b*x^4)/4 + ((b*B + A*c)*x^5)/5 + (B*c*x^6)/6
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{x^{4}\left(10 B c \,x^{2}+12 A c x +12 B b x +15 A b \right)}{60}$	28
default	$\frac{A b \,x^{4}}{4}+\frac{\left(A c +B b \right) x^{5}}{5}+\frac{B c \,x^{6}}{6}$	28
norman	$\frac{B c \,x^{6}}{6}+\left(\frac{A c}{5}+\frac{B b}{5}\right) x^{5}+\frac{A b \,x^{4}}{4}$	29
risch	$\frac{1}{4} A b \,x^{4}+\frac{1}{5} x^{5} A c +\frac{1}{5} b B \,x^{5}+\frac{1}{6} B c \,x^{6}$	30
parallelrisch	$\frac{1}{4} A b \,x^{4}+\frac{1}{5} x^{5} A c +\frac{1}{5} b B \,x^{5}+\frac{1}{6} B c \,x^{6}$	30
orering	$\frac{x^{3}\left(10 B c \,x^{2}+12 A c x +12 B b x +15 A b \right)\left(c \,x^{2}+b x \right)}{60 c x +60 b}$	44

input
```
int(x^2*(B*x+A)*(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
1/60*x^4*(10*B*c*x^2+12*A*c*x+12*B*b*x+15*A*b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(A+Bx)\left(bx+cx^2\right)\,dx=\frac{1}{6}\,Bcx^6+\frac{1}{4}\,Abx^4+\frac{1}{5}\,(Bb+Ac)x^5$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x),x, algorithm="fricas")
```

output
```
1/6*B*c*x^6 + 1/4*A*b*x^4 + 1/5*(B*b + A*c)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(A+Bx)\left(bx+cx^2\right)\,dx=\frac{Abx^4}{4}+\frac{Bcx^6}{6}+x^5\left(\frac{Ac}{5}+\frac{Bb}{5}\right)$$

input
```
integrate(x**2*(B*x+A)*(c*x**2+b*x),x)
```

output
```
A*b*x**4/4 + B*c*x**6/6 + x**5*(A*c/5 + B*b/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x^2(A+Bx)\left(bx+cx^2\right)\,dx=\frac{1}{6}\,Bcx^6+\frac{1}{4}\,Abx^4+\frac{1}{5}\,(Bb+Ac)x^5$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x),x, algorithm="maxima")
```

output
```
1/6*B*c*x^6 + 1/4*A*b*x^4 + 1/5*(B*b + A*c)*x^5
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x^2(A+Bx)\left(bx+cx^2\right)\,dx=\frac{1}{6}\,Bcx^6+\frac{1}{5}\,Bbx^5+\frac{1}{5}\,Acx^5+\frac{1}{4}\,Abx^4$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x),x, algorithm="giac")
```

output
```
1/6*B*c*x^6 + 1/5*B*b*x^5 + 1/5*A*c*x^5 + 1/4*A*b*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^2(A+Bx)\left(bx+cx^2\right)\,dx=\frac{B\,c\,x^6}{6}+\left(\frac{A\,c}{5}+\frac{B\,b}{5}\right)\,x^5+\frac{A\,b\,x^4}{4}$$

input
```
int(x^2*(b*x + c*x^2)*(A + B*x),x)
```

output
```
x^5*((A*c)/5 + (B*b)/5) + (A*b*x^4)/4 + (B*c*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x^2(A+Bx)\left(bx+cx^2\right)\,dx=\frac{x^4(10bc\,x^2+12acx+12b^2x+15ab)}{60}$$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x),x)
```

output
```
(x**4*(15*a*b + 12*a*c*x + 12*b**2*x + 10*b*c*x**2))/60
```

3.2 $\int x(A+Bx)\left(bx+cx^2\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 33

$$\int x(A+Bx)\left(bx+cx^2\right)\,dx=\frac{1}{3}Abx^3+\frac{1}{4}(bB+Ac)x^4+\frac{1}{5}Bcx^5$$

output
```
1/3*A*b*x^3+1/4*(A*c+B*b)*x^4+1/5*B*c*x^5
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x(A+Bx)\left(bx+cx^2\right)\,dx=\frac{1}{3}Abx^3+\frac{1}{4}(bB+Ac)x^4+\frac{1}{5}Bcx^5$$

input
```
Integrate[x*(A + B*x)*(b*x + c*x^2),x]
```

output
```
(A*b*x^3)/3 + ((b*B + A*c)*x^4)/4 + (B*c*x^5)/5
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(A+Bx)\left(bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^2(A+Bx)(b+cx)dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(x^3(Ac+bB)+Abx^2+Bcx^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}x^4(Ac+bB)+\frac{1}{3}Abx^3+\frac{1}{5}Bcx^5$$

input
```
Int[x*(A + B*x)*(b*x + c*x^2),x]
```

output
```
(A*b*x^3)/3 + ((b*B + A*c)*x^4)/4 + (B*c*x^5)/5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{x^{3}\left(12 B c \,x^{2}+15 A c x +15 B b x +20 A b \right)}{60}$	28
default	$\frac{A b \,x^{3}}{3}+\frac{\left(A c +B b \right) x^{4}}{4}+\frac{B c \,x^{5}}{5}$	28
norman	$\frac{B c \,x^{5}}{5}+\left(\frac{A c}{4}+\frac{B b}{4}\right) x^{4}+\frac{A b \,x^{3}}{3}$	29
risch	$\frac{1}{3} A b \,x^{3}+\frac{1}{4} x^{4} A c +\frac{1}{4} x^{4} B b +\frac{1}{5} B c \,x^{5}$	30
parallelrisch	$\frac{1}{3} A b \,x^{3}+\frac{1}{4} x^{4} A c +\frac{1}{4} x^{4} B b +\frac{1}{5} B c \,x^{5}$	30
orering	$\frac{x^{2}\left(12 B c \,x^{2}+15 A c x +15 B b x +20 A b \right) \left(c \,x^{2}+b x \right)}{60 c x +60 b}$	44

input
```
int(x*(B*x+A)*(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
1/60*x^3*(12*B*c*x^2+15*A*c*x+15*B*b*x+20*A*b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x(A+Bx)\left(bx+cx^2\right)\,dx=\frac{1}{5}\,Bcx^5+\frac{1}{3}\,Abx^3+\frac{1}{4}\,(Bb+Ac)x^4$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x),x, algorithm="fricas")
```

output
```
1/5*B*c*x^5 + 1/3*A*b*x^3 + 1/4*(B*b + A*c)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(A+Bx)\left(bx+cx^2\right)\,dx=\frac{Abx^3}{3}+\frac{Bcx^5}{5}+x^4\left(\frac{Ac}{4}+\frac{Bb}{4}\right)$$

input
```
integrate(x*(B*x+A)*(c*x**2+b*x),x)
```

output
```
A*b*x**3/3 + B*c*x**5/5 + x**4*(A*c/4 + B*b/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int x(A+Bx)\left(bx+cx^2\right)\,dx=\frac{1}{5}\,Bcx^5+\frac{1}{3}\,Abx^3+\frac{1}{4}\,(Bb+Ac)x^4$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x),x, algorithm="maxima")
```

output
```
1/5*B*c*x^5 + 1/3*A*b*x^3 + 1/4*(B*b + A*c)*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int x(A+Bx)\left(bx+cx^2\right)\,dx=\frac{1}{5}Bcx^5+\frac{1}{4}Bbx^4+\frac{1}{4}Acx^4+\frac{1}{3}Abx^3$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x),x, algorithm="giac")
```

output
```
1/5*B*c*x^5 + 1/4*B*b*x^4 + 1/4*A*c*x^4 + 1/3*A*b*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x(A+Bx)\left(bx+cx^2\right)\,dx=\frac{B\,c\,x^5}{5}+\left(\frac{A\,c}{4}+\frac{B\,b}{4}\right)\,x^4+\frac{A\,b\,x^3}{3}$$

input
```
int(x*(b*x + c*x^2)*(A + B*x),x)
```

output
```
x^4*((A*c)/4 + (B*b)/4) + (A*b*x^3)/3 + (B*c*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int x(A+Bx)\left(bx+cx^2\right)\,dx=\frac{x^3(12bc\,x^2+15acx+15b^2x+20ab)}{60}$$

input
```
int(x*(B*x+A)*(c*x^2+b*x),x)
```

output
```
(x**3*(20*a*b + 15*a*c*x + 15*b**2*x + 12*b*c*x**2))/60
```

3.3 $\int (A+Bx)\left(bx+cx^2\right) dx$

Optimal result

Integrand size = 15, antiderivative size = 33

$$\int (A+Bx)\left(bx+cx^2\right) dx=\frac{1}{2}Abx^2+\frac{1}{3}(bB+Ac)x^3+\frac{1}{4}Bcx^4$$

output
```
1/2*A*b*x^2+1/3*(A*c+B*b)*x^3+1/4*B*c*x^4
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (A+Bx)\left(bx+cx^2\right) dx=\frac{1}{12}x^2(Bx(4b+3cx)+A(6b+4cx))$$

input
```
Integrate[(A + B*x)*(b*x + c*x^2),x]
```

output
```
(x^2*(B*x*(4*b + 3*c*x) + A*(6*b + 4*c*x)))/12
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = $\{1140, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)\left(bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(x^2(Ac+bB)+Abx+Bcx^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{3}x^3(Ac+bB)+\frac{1}{2}Abx^2+\frac{1}{4}Bcx^4$$

input
```
Int[(A + B*x)*(b*x + c*x^2),x]
```

output
```
(A*b*x^2)/2 + ((b*B + A*c)*x^3)/3 + (B*c*x^4)/4
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
gosper	$\frac{x^{2}\left(3 B c \,x^{2}+4 A c x +4 B b x +6 A b \right)}{12}$	28
default	$\frac{A b \,x^{2}}{2}+\frac{\left(A c +B b \right) x^{3}}{3}+\frac{B c \,x^{4}}{4}$	28
norman	$\frac{B c \,x^{4}}{4}+\left(\frac{A c}{3}+\frac{B b}{3}\right) x^{3}+\frac{A b \,x^{2}}{2}$	29
risch	$\frac{1}{2} A b \,x^{2}+\frac{1}{3} x^{3} A c +\frac{1}{3} B b \,x^{3}+\frac{1}{4} B c \,x^{4}$	30
parallelrisch	$\frac{1}{2} A b \,x^{2}+\frac{1}{3} x^{3} A c +\frac{1}{3} B b \,x^{3}+\frac{1}{4} B c \,x^{4}$	30
orering	$\frac{x \left(3 B c \,x^{2}+4 A c x +4 B b x +6 A b \right) \left(c \,x^{2}+b x \right)}{12 c x +12 b}$	42

input
```
int((B*x+A)*(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
1/12*x^2*(3*B*c*x^2+4*A*c*x+4*B*b*x+6*A*b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int (A+Bx)\left(bx+cx^2\right)\,dx=\frac{1}{4}\,Bcx^4+\frac{1}{2}\,Abx^2+\frac{1}{3}\,(Bb+Ac)x^3$$

input
```
integrate((B*x+A)*(c*x^2+b*x),x, algorithm="fricas")
```

output
```
1/4*B*c*x^4 + 1/2*A*b*x^2 + 1/3*(B*b + A*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (A+Bx)\left(bx+cx^2\right)\,dx=\frac{Abx^2}{2}+\frac{Bcx^4}{4}+x^3\left(\frac{Ac}{3}+\frac{Bb}{3}\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x),x)
```

output
```
A*b*x**2/2 + B*c*x**4/4 + x**3*(A*c/3 + B*b/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int (A+Bx)\left(bx+cx^2\right)\,dx=\frac{1}{4}\,Bcx^4+\frac{1}{2}\,Abx^2+\frac{1}{3}\,(Bb+Ac)x^3$$

input
```
integrate((B*x+A)*(c*x^2+b*x),x, algorithm="maxima")
```

output
```
1/4*B*c*x^4 + 1/2*A*b*x^2 + 1/3*(B*b + A*c)*x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int (A+Bx)\left(bx+cx^2\right)\,dx=\frac{1}{4}\,Bcx^4+\frac{1}{3}\,Bbx^3+\frac{1}{3}\,Acx^3+\frac{1}{2}\,Abx^2$$

input
```
integrate((B*x+A)*(c*x^2+b*x),x, algorithm="giac")
```

output
```
1/4*B*c*x^4 + 1/3*B*b*x^3 + 1/3*A*c*x^3 + 1/2*A*b*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int (A+Bx)\left(bx+cx^2\right)\,dx=\frac{B\,c\,x^4}{4}+\left(\frac{A\,c}{3}+\frac{B\,b}{3}\right)\,x^3+\frac{A\,b\,x^2}{2}$$

input
```
int((b*x + c*x^2)*(A + B*x),x)
```

output
```
x^3*((A*c)/3 + (B*b)/3) + (A*b*x^2)/2 + (B*c*x^4)/4
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int (A+Bx)\left(bx+cx^2\right)\,dx=\frac{x^2(3bc\,x^2+4acx+4b^2x+6ab)}{12}$$

input
```
int((B*x+A)*(c*x^2+b*x),x)
```

output
```
(x**2*(6*a*b + 4*a*c*x + 4*b**2*x + 3*b*c*x**2))/12
```

3.4 $\int \frac{(A+Bx)(bx+cx^2)}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 28

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x}\,dx = Abx+\frac{1}{2}(bB+Ac)x^2+\frac{1}{3}Bcx^3$$

output
```
A*b*x+1/2*(A*c+B*b)*x^2+1/3*B*c*x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x}\,dx = Abx+\frac{1}{2}(bB+Ac)x^2+\frac{1}{3}Bcx^3$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2))/x,x]
```

output
```
A*b*x + ((b*B + A*c)*x^2)/2 + (B*c*x^3)/3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x}dx$$

$$\Big\downarrow \; 9$$

$$\int (A+Bx)(b+cx)dx$$

$$\Big\downarrow \; 49$$

$$\int \left(x(Ac+bB)+Ab+Bcx^2\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}x^2(Ac+bB)+Abx+\frac{1}{3}Bcx^3$$

input
```
Int[((A + B*x)*(b*x + c*x^2))/x,x]
```

output
```
A*b*x + ((b*B + A*c)*x^2)/2 + (B*c*x^3)/3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
default	$Abx+\frac{(Ac+Bb)x^2}{2}+\frac{Bc\,x^3}{3}$	25
gosper	$\frac{x(2Bc\,x^2+3Acx+3Bbx+6Ab)}{6}$	26
norman	$\frac{Bc\,x^3}{3}+\left(\frac{Ac}{2}+\frac{Bb}{2}\right)x^2+Abx$	26
risch	$Abx+\frac{1}{2}Ac\,x^2+\frac{1}{2}Bb\,x^2+\frac{1}{3}Bc\,x^3$	27
parallelrisch	$Abx+\frac{1}{2}Ac\,x^2+\frac{1}{2}Bb\,x^2+\frac{1}{3}Bc\,x^3$	27
orering	$\frac{(2Bc\,x^2+3Acx+3Bbx+6Ab)(c\,x^2+bx)}{6cx+6b}$	41

input
```
int((B*x+A)*(c*x^2+b*x)/x,x,method=_RETURNVERBOSE)
```

output
```
A*b*x+1/2*(A*c+B*b)*x^2+1/3*B*c*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x}\,dx=\frac{1}{3}\,Bcx^3+Abx+\frac{1}{2}\,(Bb+Ac)x^2$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x,x, algorithm="fricas")
```

output
```
1/3*B*c*x^3 + A*b*x + 1/2*(B*b + A*c)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)(bx+cx^2)}{x}\,dx = Abx + \frac{Bcx^3}{3} + x^2\left(\frac{Ac}{2} + \frac{Bb}{2}\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x)/x,x)
```

output
```
A*b*x + B*c*x**3/3 + x**2*(A*c/2 + B*b/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)(bx+cx^2)}{x}\,dx = \frac{1}{3}\,Bcx^3 + Abx + \frac{1}{2}\,(Bb+Ac)x^2$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x,x, algorithm="maxima")
```

output
```
1/3*B*c*x^3 + A*b*x + 1/2*(B*b + A*c)*x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)(bx+cx^2)}{x}\,dx = \frac{1}{3}\,Bcx^3 + \frac{1}{2}\,Bbx^2 + \frac{1}{2}\,Acx^2 + Abx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x,x, algorithm="giac")
```

output
```
1/3*B*c*x^3 + 1/2*B*b*x^2 + 1/2*A*c*x^2 + A*b*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x}\,dx=\frac{B\,c\,x^3}{3}+\left(\frac{A\,c}{2}+\frac{B\,b}{2}\right)\,x^2+A\,b\,x$$

input
```
int(((b*x + c*x^2)*(A + B*x))/x,x)
```

output
```
x^2*((A*c)/2 + (B*b)/2) + A*b*x + (B*c*x^3)/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x}\,dx=\frac{x(2bc\,x^2+3acx+3b^2x+6ab)}{6}$$

input
```
int((B*x+A)*(c*x^2+b*x)/x,x)
```

output
```
(x*(6*a*b + 3*a*c*x + 3*b**2*x + 2*b*c*x**2))/6
```

3.5 $\int \frac{(A+Bx)(bx+cx^2)}{x^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 24

$$\int \frac{(A+Bx)(bx+cx^2)}{x^2} dx = (bB+Ac)x + \frac{1}{2}Bcx^2 + Ab\log(x)$$

output
```
(A*c+B*b)*x+1/2*B*c*x^2+A*b*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)(bx+cx^2)}{x^2} dx = (bB+Ac)x + \frac{1}{2}Bcx^2 + Ab\log(x)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2))/x^2,x]
```

output
```
(b*B + A*c)*x + (B*c*x^2)/2 + A*b*Log[x]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^2}dx$$
$$\Big\downarrow \ 9$$
$$\int \frac{(A+Bx)(b+cx)}{x}dx$$
$$\Big\downarrow \ 85$$
$$\int \left(\frac{Ab}{x}+Ac+bB+Bcx\right)dx$$
$$\Big\downarrow \ 2009$$
$$x(Ac+bB)+Ab\log(x)+\frac{1}{2}Bcx^2$$

input
```
Int[((A + B*x)*(b*x + c*x^2))/x^2,x]
```

output
```
(b*B + A*c)*x + (B*c*x^2)/2 + A*b*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{Bc\,x^{2}}{2}+Acx+Bbx+Ab\ln\left(x\right)$	22
risch	$\frac{Bc\,x^{2}}{2}+Acx+Bbx+Ab\ln\left(x\right)$	22
parallelrisch	$\frac{Bc\,x^{2}}{2}+Acx+Bbx+Ab\ln\left(x\right)$	22
norman	$\frac{(Ac+Bb)x^{2}+\frac{Bc\,x^{3}}{2}}{x}+Ab\ln\left(x\right)$	30

input
```
int((B*x+A)*(c*x^2+b*x)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*B*c*x^2+A*c*x+B*b*x+A*b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int\frac{(A+Bx)\left(bx+cx^{2}\right)}{x^{2}}\,dx=\frac{1}{2}\,Bcx^{2}+Ab\log\left(x\right)+(Bb+Ac)x$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^2,x, algorithm="fricas")
```

output
```
1/2*B*c*x^2 + A*b*log(x) + (B*b + A*c)*x
```

Sympy [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)(bx+cx^2)}{x^2}\,dx = Ab\log(x) + \frac{Bcx^2}{2} + x(Ac+Bb)$$

input
```
integrate((B*x+A)*(c*x**2+b*x)/x**2,x)
```

output
```
A*b*log(x) + B*c*x**2/2 + x*(A*c + B*b)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)(bx+cx^2)}{x^2}\,dx = \frac{1}{2}Bcx^2 + Ab\log(x) + (Bb+Ac)x$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^2,x, algorithm="maxima")
```

output
```
1/2*B*c*x^2 + A*b*log(x) + (B*b + A*c)*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)(bx+cx^2)}{x^2}\,dx = \frac{1}{2}Bcx^2 + Bbx + Acx + Ab\log(|x|)$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^2,x, algorithm="giac")
```

output
```
1/2*B*c*x^2 + B*b*x + A*c*x + A*b*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)(bx+cx^2)}{x^2}\,dx = x\,(A\,c+B\,b)+\frac{B\,c\,x^2}{2}+A\,b\,\ln\left(x\right)$$

input
```
int(((b*x + c*x^2)*(A + B*x))/x^2,x)
```

output
```
x*(A*c + B*b) + (B*c*x^2)/2 + A*b*log(x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)(bx+cx^2)}{x^2}\,dx = \log(x)\,ab+acx+b^2x+\frac{bc\,x^2}{2}$$

input
```
int((B*x+A)*(c*x^2+b*x)/x^2,x)
```

output
```
(2*log(x)*a*b + 2*a*c*x + 2*b**2*x + b*c*x**2)/2
```

3.6 $\int \frac{(A+Bx)(bx+cx^2)}{x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 22

$$\int \frac{(A+Bx)(bx+cx^2)}{x^3} dx = -\frac{Ab}{x} + Bcx + (bB + Ac)\log(x)$$

output
```
-A*b/x+B*c*x+(A*c+B*b)*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)(bx+cx^2)}{x^3} dx = -\frac{Ab}{x} + Bcx + (bB + Ac)\log(x)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2))/x^3,x]
```

output
```
-((A*b)/x) + B*c*x + (b*B + A*c)*Log[x]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^3}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{(A+Bx)(b+cx)}{x^2}dx$$

$$\Big\downarrow \ 85$$

$$\int \left(\frac{Ac+bB}{x}+\frac{Ab}{x^2}+Bc\right)dx$$

$$\Big\downarrow \ 2009$$

$$\log(x)(Ac+bB)-\frac{Ab}{x}+Bcx$$

input
```
Int[((A + B*x)*(b*x + c*x^2))/x^3,x]
```

output
```
-((A*b)/x) + B*c*x + (b*B + A*c)*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

method	result	size
default	$-\frac{Ab}{x}+Bcx+(Ac+Bb)\ln(x)$	23
risch	$-\frac{Ab}{x}+Bcx+A\ln(x)\,c+B\ln(x)\,b$	23
norman	$\frac{Bc\,x^{3}-Abx}{x^{2}}+(Ac+Bb)\ln(x)$	28
parallelrisch	$\frac{A\ln(x)xc+B\ln(x)xb+Bc\,x^{2}-Ab}{x}$	28

input
```
int((B*x+A)*(c*x^2+b*x)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-A*b/x+B*c*x+(A*c+B*b)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18

$$\int\frac{(A+Bx)\,(bx+cx^2)}{x^3}\,dx=\frac{Bcx^2+(Bb+Ac)x\log(x)-Ab}{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^3,x, algorithm="fricas")
```

output
```
(B*c*x^2 + (B*b + A*c)*x*log(x) - A*b)/x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^3}\,dx = -\frac{Ab}{x} + Bcx + (Ac+Bb)\log{(x)}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)/x**3,x)
```

output
```
-A*b/x + B*c*x + (A*c + B*b)*log(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^3}\,dx = Bcx + (Bb+Ac)\log{(x)} - \frac{Ab}{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^3,x, algorithm="maxima")
```

output
```
B*c*x + (B*b + A*c)*log(x) - A*b/x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^3}\,dx = Bcx + (Bb+Ac)\log{(|x|)} - \frac{Ab}{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^3,x, algorithm="giac")
```

output
```
B*c*x + (B*b + A*c)*log(abs(x)) - A*b/x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)(bx+cx^2)}{x^3}\,dx = \ln(x)\ (A\,c+B\,b)+B\,c\,x-\frac{A\,b}{x}$$

input
```
int(((b*x + c*x^2)*(A + B*x))/x^3,x)
```

output
```
log(x)*(A*c + B*b) + B*c*x - (A*b)/x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)(bx+cx^2)}{x^3}\,dx = \frac{\log(x)\,acx+\log(x)\,b^2x-ab+bc\,x^2}{x}$$

input
```
int((B*x+A)*(c*x^2+b*x)/x^3,x)
```

output
```
(log(x)*a*c*x + log(x)*b**2*x - a*b + b*c*x**2)/x
```

3.7 $\int \frac{(A+Bx)(bx+cx^2)}{x^4} dx$

Optimal result

Integrand size = 18, antiderivative size = 27

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^4} dx = -\frac{Ab}{2x^2} - \frac{bB+Ac}{x} + Bc\log(x)$$

output
```
-1/2*A*b/x^2-(A*c+B*b)/x+B*c*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^4} dx = -\frac{Ab}{2x^2} + \frac{-bB-Ac}{x} + Bc\log(x)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2))/x^4,x]
```

output
```
-1/2*(A*b)/x^2 + (-(b*B) - A*c)/x + B*c*Log[x]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^4}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx)(b+cx)}{x^3}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{Ac+bB}{x^2}+\frac{Ab}{x^3}+\frac{Bc}{x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{Ac+bB}{x}-\frac{Ab}{2x^2}+Bc\log(x)$$

input
```
Int[((A + B*x)*(b*x + c*x^2))/x^4,x]
```

output
```
-1/2*(A*b)/x^2 - (b*B + A*c)/x + B*c*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{Ab}{2x^2}-\frac{Ac+Bb}{x}+Bc\ln(x)$	26
risch	$\frac{(-Ac-Bb)x-\frac{Ab}{2}}{x^2}+Bc\ln(x)$	27
parallelrisch	$-\frac{-2Bc\ln(x)x^2+2Acx+2Bbx+Ab}{2x^2}$	29
norman	$\frac{(-Ac-Bb)x^2-\frac{Abx}{2}}{x^3}+Bc\ln(x)$	30

input
```
int((B*x+A)*(c*x^2+b*x)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A*b/x^2-(A*c+B*b)/x+B*c*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.07

$$\int\frac{(A+Bx)(bx+cx^2)}{x^4}\,dx=\frac{2\,Bcx^2\log(x)-Ab-2\,(Bb+Ac)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^4,x, algorithm="fricas")
```

output
```
1/2*(2*B*c*x^2*log(x) - A*b - 2*(B*b + A*c)*x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^4}\,dx = Bc\log{(x)} + \frac{-Ab+x(-2Ac-2Bb)}{2x^2}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)/x**4,x)
```

output
```
B*c*log(x) + (-A*b + x*(-2*A*c - 2*B*b))/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^4}\,dx = Bc\log{(x)} - \frac{Ab+2\,(Bb+Ac)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^4,x, algorithm="maxima")
```

output
```
B*c*log(x) - 1/2*(A*b + 2*(B*b + A*c)*x)/x^2
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^4}\,dx = Bc\log{(|x|)} - \frac{Ab+2\,(Bb+Ac)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^4,x, algorithm="giac")
```

output
```
B*c*log(abs(x)) - 1/2*(A*b + 2*(B*b + A*c)*x)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^4}\,dx = B\,c\,\ln\left(x\right)-\frac{\frac{A\,b}{2}+x\,\left(A\,c+B\,b\right)}{x^2}$$

input
```
int(((b*x + c*x^2)*(A + B*x))/x^4,x)
```

output
```
B*c*log(x) - ((A*b)/2 + x*(A*c + B*b))/x^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^4}\,dx = \frac{2\log(x)\,bc\,x^2-ab-2acx-2b^2x}{2x^2}$$

input
```
int((B*x+A)*(c*x^2+b*x)/x^4,x)
```

output
```
(2*log(x)*b*c*x**2 - a*b - 2*a*c*x - 2*b**2*x)/(2*x**2)
```

3.8 $\int \frac{(A+Bx)(bx+cx^2)}{x^5} dx$

Optimal result

Integrand size = 18, antiderivative size = 31

$$\int \frac{(A+Bx)(bx+cx^2)}{x^5} dx = -\frac{Ab}{3x^3} - \frac{bB+Ac}{2x^2} - \frac{Bc}{x}$$

output
```
-1/3*A*b/x^3-1/2*(A*c+B*b)/x^2-B*c/x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)(bx+cx^2)}{x^5} dx = -\frac{3Bx(b+2cx)+A(2b+3cx)}{6x^3}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2))/x^5,x]
```

output
```
-1/6*(3*B*x*(b + 2*c*x) + A*(2*b + 3*c*x))/x^3
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^5} \, dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{(A+Bx)(b+cx)}{x^4}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{Ac+bB}{x^3}+\frac{Ab}{x^4}+\frac{Bc}{x^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ac+bB}{2x^2}-\frac{Ab}{3x^3}-\frac{Bc}{x}$$

input
```
Int[((A + B*x)*(b*x + c*x^2))/x^5,x]
```

output
```
-1/3*(A*b)/x^3 - (b*B + A*c)/(2*x^2) - (B*c)/x
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

method	result	size
gosper	$-\frac{6Bc\,x^{2}+3Acx+3Bbx+2Ab}{6x^{3}}$	28
default	$-\frac{Ab}{3x^{3}}-\frac{Ac+Bb}{2x^{2}}-\frac{Bc}{x}$	28
risch	$\frac{-Bc\,x^{2}+\left(-\frac{Ac}{2}-\frac{Bb}{2}\right)x-\frac{Ab}{3}}{x^{3}}$	28
parallelrisch	$-\frac{6Bc\,x^{2}+3Acx+3Bbx+2Ab}{6x^{3}}$	28
norman	$\frac{\left(-\frac{Ac}{2}-\frac{Bb}{2}\right)x^{2}-\frac{Abx}{3}-Bc\,x^{3}}{x^{4}}$	31
orering	$-\frac{\left(6Bc\,x^{2}+3Acx+3Bbx+2Ab\right)\left(c\,x^{2}+bx\right)}{6x^{4}\left(cx+b\right)}$	44

input
```
int((B*x+A)*(c*x^2+b*x)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/6/x^3*(6*B*c*x^2+3*A*c*x+3*B*b*x+2*A*b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^5}\,dx = -\frac{6\,Bcx^2+2\,Ab+3\,(Bb+Ac)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^5,x, algorithm="fricas")
```

output
```
-1/6*(6*B*c*x^2 + 2*A*b + 3*(B*b + A*c)*x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^5}\,dx = \frac{-2Ab-6Bcx^2+x(-3Ac-3Bb)}{6x^3}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)/x**5,x)
```

output
```
(-2*A*b - 6*B*c*x**2 + x*(-3*A*c - 3*B*b))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^5}\,dx = -\frac{6\,Bcx^2+2\,Ab+3\,(Bb+Ac)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^5,x, algorithm="maxima")
```

output
```
-1/6*(6*B*c*x^2 + 2*A*b + 3*(B*b + A*c)*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^5}\,dx = -\frac{6\,Bcx^2+3\,Bbx+3\,Acx+2\,Ab}{6\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^5,x, algorithm="giac")
```

output
```
-1/6*(6*B*c*x^2 + 3*B*b*x + 3*A*c*x + 2*A*b)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^5}\,dx = -\frac{B\,c\,x^2+\left(\frac{A\,c}{2}+\frac{B\,b}{2}\right)\,x+\frac{A\,b}{3}}{x^3}$$

input
```
int(((b*x + c*x^2)*(A + B*x))/x^5,x)
```

output
```
-((A*b)/3 + x*((A*c)/2 + (B*b)/2) + B*c*x^2)/x^3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^5}\,dx = \frac{-6bc\,x^2-3acx-3b^2x-2ab}{6x^3}$$

input
```
int((B*x+A)*(c*x^2+b*x)/x^5,x)
```

output
```
( - 2*a*b - 3*a*c*x - 3*b**2*x - 6*b*c*x**2)/(6*x**3)
```

3.9 $\int \frac{(A+Bx)(bx+cx^2)}{x^6} dx$

Optimal result

Integrand size = 18, antiderivative size = 33

$$\int \frac{(A+Bx)(bx+cx^2)}{x^6} dx = -\frac{Ab}{4x^4} - \frac{bB+Ac}{3x^3} - \frac{Bc}{2x^2}$$

output
```
-1/4*A*b/x^4-1/3*(A*c+B*b)/x^3-1/2*B*c/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)(bx+cx^2)}{x^6} dx = -\frac{3Ab+4bBx+4Acx+6Bcx^2}{12x^4}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2))/x^6,x]
```

output
```
-1/12*(3*A*b + 4*b*B*x + 4*A*c*x + 6*B*c*x^2)/x^4
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^6}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx)(b+cx)}{x^5}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{Ac+bB}{x^4}+\frac{Ab}{x^5}+\frac{Bc}{x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{Ac+bB}{3x^3}-\frac{Ab}{4x^4}-\frac{Bc}{2x^2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2))/x^6,x]
```

output
```
-1/4*(A*b)/x^4 - (b*B + A*c)/(3*x^3) - (B*c)/(2*x^2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p, x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
gosper	$-\frac{6Bc\,x^2+4Acx+4Bbx+3Ab}{12x^4}$	28
default	$-\frac{Ab}{4x^4}-\frac{Ac+Bb}{3x^3}-\frac{Bc}{2x^2}$	28
risch	$\frac{-\frac{Bc\,x^2}{2}+\left(-\frac{Ac}{3}-\frac{Bb}{3}\right)x-\frac{Ab}{4}}{x^4}$	28
parallelrisch	$-\frac{6Bc\,x^2+4Acx+4Bbx+3Ab}{12x^4}$	28
norman	$\frac{\left(-\frac{Ac}{3}-\frac{Bb}{3}\right)x^2-\frac{Abx}{4}-\frac{Bc\,x^3}{2}}{x^5}$	31
orering	$-\frac{(6Bc\,x^2+4Acx+4Bbx+3Ab)\,(c\,x^2+bx)}{12x^5(cx+b)}$	44

input
```
int((B*x+A)*(c*x^2+b*x)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/12/x^4*(6*B*c*x^2+4*A*c*x+4*B*b*x+3*A*b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)(bx+cx^2)}{x^6}\,dx = -\frac{6\,Bcx^2+3\,Ab+4\,(Bb+Ac)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^6,x, algorithm="fricas")
```

output
```
-1/12*(6*B*c*x^2 + 3*A*b + 4*(B*b + A*c)*x)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)(bx+cx^2)}{x^6}\,dx = \frac{-3Ab-6Bcx^2+x(-4Ac-4Bb)}{12x^4}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)/x**6,x)
```

output
```
(-3*A*b - 6*B*c*x**2 + x*(-4*A*c - 4*B*b))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)(bx+cx^2)}{x^6}\,dx = -\frac{6\,Bcx^2+3\,Ab+4\,(Bb+Ac)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^6,x, algorithm="maxima")
```

output
```
-1/12*(6*B*c*x^2 + 3*A*b + 4*(B*b + A*c)*x)/x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)(bx+cx^2)}{x^6}\,dx = -\frac{6\,Bcx^2+4\,Bbx+4\,Acx+3\,Ab}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^6,x, algorithm="giac")
```

output
```
-1/12*(6*B*c*x^2 + 4*B*b*x + 4*A*c*x + 3*A*b)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)(bx+cx^2)}{x^6}\,dx = -\frac{\frac{B\,c\,x^2}{2}+\left(\frac{A\,c}{3}+\frac{B\,b}{3}\right)\,x+\frac{A\,b}{4}}{x^4}$$

input
```
int(((b*x + c*x^2)*(A + B*x))/x^6,x)
```

output
```
-((A*b)/4 + x*((A*c)/3 + (B*b)/3) + (B*c*x^2)/2)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)(bx+cx^2)}{x^6}\,dx = \frac{-6bc\,x^2-4acx-4b^2x-3ab}{12x^4}$$

input
```
int((B*x+A)*(c*x^2+b*x)/x^6,x)
```

output
```
( - 3*a*b - 4*a*c*x - 4*b**2*x - 6*b*c*x**2)/(12*x**4)
```

3.10 $\int x^2(A+Bx)\left(bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int x^2(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{1}{5}Ab^2x^5+\frac{1}{6}b(bB+2Ac)x^6+\frac{1}{7}c(2bB+Ac)x^7+\frac{1}{8}Bc^2x^8$$

output
```
1/5*A*b^2*x^5+1/6*b*(2*A*c+B*b)*x^6+1/7*c*(A*c+2*B*b)*x^7+1/8*B*c^2*x^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{1}{5}Ab^2x^5+\frac{1}{6}b(bB+2Ac)x^6+\frac{1}{7}c(2bB+Ac)x^7+\frac{1}{8}Bc^2x^8$$

input
```
Integrate[x^2*(A + B*x)*(b*x + c*x^2)^2,x]
```

output
```
(A*b^2*x^5)/5 + (b*(b*B + 2*A*c)*x^6)/6 + (c*(2*b*B + A*c)*x^7)/7 + (B*c^2
*x^8)/8
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(A+Bx)\left(bx+cx^2\right)^2\,dx$$
$$\Big\downarrow \text{ 9}$$
$$\int x^4(A+Bx)(b+cx)^2dx$$
$$\Big\downarrow \text{ 85}$$
$$\int \left(Ab^2x^4+cx^6(Ac+2bB)+bx^5(2Ac+bB)+Bc^2x^7\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{5}Ab^2x^5+\frac{1}{7}cx^7(Ac+2bB)+\frac{1}{6}bx^6(2Ac+bB)+\frac{1}{8}Bc^2x^8$$

input
```
Int[x^2*(A + B*x)*(b*x + c*x^2)^2,x]
```

output
```
(A*b^2*x^5)/5 + (b*(b*B + 2*A*c)*x^6)/6 + (c*(2*b*B + A*c)*x^7)/7 + (B*c^2
*x^8)/8
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{x^{5} \left(105 B \,c^{2} x^{3}+120 A \,c^{2} x^{2}+240 x^{2} B b c +280 A b c x +140 x B \,b^{2}+168 b^{2} A \right)}{840}$	52
default	$\frac{B \,c^{2} x^{8}}{8}+\frac{\left(A \,c^{2}+2 B b c \right) x^{7}}{7}+\frac{\left(2 A b c +B \,b^{2}\right) x^{6}}{6}+\frac{A \,b^{2} x^{5}}{5}$	52
norman	$\frac{B \,c^{2} x^{8}}{8}+\left(\frac{1}{7} A \,c^{2}+\frac{2}{7} B b c \right) x^{7}+\left(\frac{1}{3} A b c +\frac{1}{6} B \,b^{2}\right) x^{6}+\frac{A \,b^{2} x^{5}}{5}$	52
risch	$\frac{1}{8} B \,c^{2} x^{8}+\frac{1}{7} x^{7} A \,c^{2}+\frac{2}{7} x^{7} B b c +\frac{1}{3} x^{6} A b c +\frac{1}{6} x^{6} B \,b^{2}+\frac{1}{5} A \,b^{2} x^{5}$	54
parallelrisch	$\frac{1}{8} B \,c^{2} x^{8}+\frac{1}{7} x^{7} A \,c^{2}+\frac{2}{7} x^{7} B b c +\frac{1}{3} x^{6} A b c +\frac{1}{6} x^{6} B \,b^{2}+\frac{1}{5} A \,b^{2} x^{5}$	54
orering	$\frac{x^{3} \left(105 B \,c^{2} x^{3}+120 A \,c^{2} x^{2}+240 x^{2} B b c +280 A b c x +140 x B \,b^{2}+168 b^{2} A \right) \left(c \,x^{2}+b x \right)^{2}}{840 \left(c x +b \right)^{2}}$	70

input
```
int(x^2*(B*x+A)*(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/840*x^5*(105*B*c^2*x^3+120*A*c^2*x^2+240*B*b*c*x^2+280*A*b*c*x+140*B*b^2
*x+168*A*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{1}{8}\,Bc^2x^8+\frac{1}{5}\,Ab^2x^5+\frac{1}{7}\,\left(2\,Bbc+Ac^2\right)x^7+\frac{1}{6}\,\left(Bb^2+2\,Abc\right)x^6$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
1/8*B*c^2*x^8 + 1/5*A*b^2*x^5 + 1/7*(2*B*b*c + A*c^2)*x^7 + 1/6*(B*b^2 + 2
*A*b*c)*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int x^2(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{Ab^2x^5}{5}+\frac{Bc^2x^8}{8}+x^7\left(\frac{Ac^2}{7}+\frac{2Bbc}{7}\right)+x^6\left(\frac{Abc}{3}+\frac{Bb^2}{6}\right)$$

input
```
integrate(x**2*(B*x+A)*(c*x**2+b*x)**2,x)
```

output
```
A*b**2*x**5/5 + B*c**2*x**8/8 + x**7*(A*c**2/7 + 2*B*b*c/7) + x**6*(A*b*c/
3 + B*b**2/6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{1}{8}\,Bc^2x^8+\frac{1}{5}\,Ab^2x^5+\frac{1}{7}\,\left(2\,Bbc+Ac^2\right)x^7+\frac{1}{6}\,\left(Bb^2+2\,Abc\right)x^6$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
1/8*B*c^2*x^8 + 1/5*A*b^2*x^5 + 1/7*(2*B*b*c + A*c^2)*x^7 + 1/6*(B*b^2 + 2
*A*b*c)*x^6
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x^2(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{1}{8}Bc^2x^8+\frac{2}{7}Bbcx^7+\frac{1}{7}Ac^2x^7+\frac{1}{6}Bb^2x^6+\frac{1}{3}Abcx^6+\frac{1}{5}Ab^2x^5$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
1/8*B*c^2*x^8 + 2/7*B*b*c*x^7 + 1/7*A*c^2*x^7 + 1/6*B*b^2*x^6 + 1/3*A*b*c*
x^6 + 1/5*A*b^2*x^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(A+Bx)\left(bx+cx^2\right)^2\,dx=x^6\left(\frac{B\,b^2}{6}+\frac{A\,c\,b}{3}\right)+x^7\left(\frac{A\,c^2}{7}+\frac{2\,B\,b\,c}{7}\right)+\frac{A\,b^2\,x^5}{5}+\frac{B\,c^2\,x^8}{8}$$

input
```
int(x^2*(b*x + c*x^2)^2*(A + B*x),x)
```

output
```
x^6*((B*b^2)/6 + (A*b*c)/3) + x^7*((A*c^2)/7 + (2*B*b*c)/7) + (A*b^2*x^5)/
5 + (B*c^2*x^8)/8
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x^2(A+Bx)\left(bx+cx^2\right)^2\,dx$$
$$=\frac{x^5(105b\,c^2x^3+120a\,c^2x^2+240b^2c\,x^2+280abcx+140b^3x+168a\,b^2)}{840}$$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x)^2,x)
```

output
```
(x**5*(168*a*b**2 + 280*a*b*c*x + 120*a*c**2*x**2 + 140*b**3*x + 240*b**2*
c*x**2 + 105*b*c**2*x**3))/840
```

3.11 $\int x(A+Bx)\left(bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 55

$$\int x(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{1}{4}Ab^2x^4+\frac{1}{5}b(bB+2Ac)x^5+\frac{1}{6}c(2bB+Ac)x^6+\frac{1}{7}Bc^2x^7$$

output
```
1/4*A*b^2*x^4+1/5*b*(2*A*c+B*b)*x^5+1/6*c*(A*c+2*B*b)*x^6+1/7*B*c^2*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{1}{4}Ab^2x^4+\frac{1}{5}b(bB+2Ac)x^5+\frac{1}{6}c(2bB+Ac)x^6+\frac{1}{7}Bc^2x^7$$

input
```
Integrate[x*(A + B*x)*(b*x + c*x^2)^2,x]
```

output
```
(A*b^2*x^4)/4 + (b*(b*B + 2*A*c)*x^5)/5 + (c*(2*b*B + A*c)*x^6)/6 + (B*c^2
*x^7)/7
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(A+Bx)\left(bx+cx^2\right)^2\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^3(A+Bx)(b+cx)^2dx$$

$$\Big\downarrow \; 85$$

$$\int \left(Ab^2x^3+cx^5(Ac+2bB)+bx^4(2Ac+bB)+Bc^2x^6\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{4}Ab^2x^4+\frac{1}{6}cx^6(Ac+2bB)+\frac{1}{5}bx^5(2Ac+bB)+\frac{1}{7}Bc^2x^7$$

input
```
Int[x*(A + B*x)*(b*x + c*x^2)^2,x]
```

output
```
(A*b^2*x^4)/4 + (b*(b*B + 2*A*c)*x^5)/5 + (c*(2*b*B + A*c)*x^6)/6 + (B*c^2
*x^7)/7
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{x^{4}\left(60B\,c^{2}x^{3}+70A\,c^{2}x^{2}+140x^{2}Bbc+168Abcx+84xB\,b^{2}+105b^{2}A\right)}{420}$	52
default	$\frac{B\,c^{2}x^{7}}{7}+\frac{\left(A\,c^{2}+2Bbc\right)x^{6}}{6}+\frac{\left(2Abc+B\,b^{2}\right)x^{5}}{5}+\frac{b^{2}A\,x^{4}}{4}$	52
norman	$\frac{B\,c^{2}x^{7}}{7}+\left(\frac{1}{6}A\,c^{2}+\frac{1}{3}Bbc\right)x^{6}+\left(\frac{2}{5}Abc+\frac{1}{5}B\,b^{2}\right)x^{5}+\frac{b^{2}A\,x^{4}}{4}$	52
risch	$\frac{1}{7}B\,c^{2}x^{7}+\frac{1}{6}x^{6}A\,c^{2}+\frac{1}{3}x^{6}Bbc+\frac{2}{5}x^{5}Abc+\frac{1}{5}B\,b^{2}x^{5}+\frac{1}{4}b^{2}A\,x^{4}$	54
parallelrisch	$\frac{1}{7}B\,c^{2}x^{7}+\frac{1}{6}x^{6}A\,c^{2}+\frac{1}{3}x^{6}Bbc+\frac{2}{5}x^{5}Abc+\frac{1}{5}B\,b^{2}x^{5}+\frac{1}{4}b^{2}A\,x^{4}$	54
orering	$\frac{x^{2}\left(60B\,c^{2}x^{3}+70A\,c^{2}x^{2}+140x^{2}Bbc+168Abcx+84xB\,b^{2}+105b^{2}A\right)\left(c\,x^{2}+bx\right)^{2}}{420\left(cx+b\right)^{2}}$	70

input
```
int(x*(B*x+A)*(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/420*x^4*(60*B*c^2*x^3+70*A*c^2*x^2+140*B*b*c*x^2+168*A*b*c*x+84*B*b^2*x+
105*A*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{1}{7}\,Bc^2x^7+\frac{1}{4}\,Ab^2x^4+\frac{1}{6}\,\left(2\,Bbc+Ac^2\right)x^6+\frac{1}{5}\,\left(Bb^2+2\,Abc\right)x^5$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
1/7*B*c^2*x^7 + 1/4*A*b^2*x^4 + 1/6*(2*B*b*c + A*c^2)*x^6 + 1/5*(B*b^2 + 2
*A*b*c)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int x(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{Ab^2x^4}{4}+\frac{Bc^2x^7}{7}+x^6\left(\frac{Ac^2}{6}+\frac{Bbc}{3}\right)+x^5\cdot\left(\frac{2Abc}{5}+\frac{Bb^2}{5}\right)$$

input
```
integrate(x*(B*x+A)*(c*x**2+b*x)**2,x)
```

output
```
A*b**2*x**4/4 + B*c**2*x**7/7 + x**6*(A*c**2/6 + B*b*c/3) + x**5*(2*A*b*c/
5 + B*b**2/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{1}{7}\,Bc^2x^7+\frac{1}{4}\,Ab^2x^4+\frac{1}{6}\,\left(2\,Bbc+Ac^2\right)x^6+\frac{1}{5}\,\left(Bb^2+2\,Abc\right)x^5$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
1/7*B*c^2*x^7 + 1/4*A*b^2*x^4 + 1/6*(2*B*b*c + A*c^2)*x^6 + 1/5*(B*b^2 + 2
*A*b*c)*x^5
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int x(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{1}{7}\,Bc^2x^7+\frac{1}{3}\,Bbcx^6+\frac{1}{6}\,Ac^2x^6+\frac{1}{5}\,Bb^2x^5+\frac{2}{5}\,Abcx^5+\frac{1}{4}\,Ab^2x^4$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
1/7*B*c^2*x^7 + 1/3*B*b*c*x^6 + 1/6*A*c^2*x^6 + 1/5*B*b^2*x^5 + 2/5*A*b*c*
x^5 + 1/4*A*b^2*x^4
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(A+Bx)\left(bx+cx^2\right)^2\,dx=x^5\left(\frac{B\,b^2}{5}+\frac{2\,A\,c\,b}{5}\right)+x^6\left(\frac{A\,c^2}{6}+\frac{B\,b\,c}{3}\right)+\frac{A\,b^2\,x^4}{4}+\frac{B\,c^2\,x^7}{7}$$

input
```
int(x*(b*x + c*x^2)^2*(A + B*x),x)
```

output
```
x^5*((B*b^2)/5 + (2*A*b*c)/5) + x^6*((A*c^2)/6 + (B*b*c)/3) + (A*b^2*x^4)/
4 + (B*c^2*x^7)/7
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int x(A+Bx)\left(bx+cx^2\right)^2\,dx = \frac{x^4(60b\,c^2x^3+70a\,c^2x^2+140b^2c\,x^2+168abcx+84b^3x+105a\,b^2)}{420}$$

input
```
int(x*(B*x+A)*(c*x^2+b*x)^2,x)
```

output
```
(x**4*(105*a*b**2 + 168*a*b*c*x + 70*a*c**2*x**2 + 84*b**3*x + 140*b**2*c*
x**2 + 60*b*c**2*x**3))/420
```

3.12 $\int (A+Bx)\left(bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int (A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{1}{3}Ab^2x^3+\frac{1}{4}b(bB+2Ac)x^4+\frac{1}{5}c(2bB+Ac)x^5+\frac{1}{6}Bc^2x^6$$

output
```
1/3*A*b^2*x^3+1/4*b*(2*A*c+B*b)*x^4+1/5*c*(A*c+2*B*b)*x^5+1/6*B*c^2*x^6
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int (A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{1}{60}x^3\left(20Ab^2+15b(bB+2Ac)x+12c(2bB+Ac)x^2+10Bc^2x^3\right)$$

input
```
Integrate[(A + B*x)*(b*x + c*x^2)^2,x]
```

output
```
(x^3*(20*A*b^2 + 15*b*(b*B + 2*A*c)*x + 12*c*(2*b*B + A*c)*x^2 + 10*B*c^2*
x^3))/60
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)\left(bx+cx^2\right)^2 dx$$

$$\Big\downarrow \text{ 1140}$$

$$\int \left(Ab^2x^2+cx^4(Ac+2bB)+bx^3(2Ac+bB)+Bc^2x^5\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}Ab^2x^3+\frac{1}{5}cx^5(Ac+2bB)+\frac{1}{4}bx^4(2Ac+bB)+\frac{1}{6}Bc^2x^6$$

input
```
Int[(A + B*x)*(b*x + c*x^2)^2,x]
```

output
```
(A*b^2*x^3)/3 + (b*(b*B + 2*A*c)*x^4)/4 + (c*(2*b*B + A*c)*x^5)/5 + (B*c^2
*x^6)/6
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{x^{3}\left(10B\,c^{2}x^{3}+12A\,c^{2}x^{2}+24x^{2}Bbc+30Abcx+15xB\,b^{2}+20b^{2}A\right)}{60}$	52
default	$\frac{B\,c^{2}x^{6}}{6}+\frac{\left(A\,c^{2}+2Bbc\right)x^{5}}{5}+\frac{\left(2Abc+B\,b^{2}\right)x^{4}}{4}+\frac{A\,x^{3}b^{2}}{3}$	52
norman	$\frac{B\,c^{2}x^{6}}{6}+\left(\frac{1}{5}A\,c^{2}+\frac{2}{5}Bbc\right)x^{5}+\left(\frac{1}{2}Abc+\frac{1}{4}B\,b^{2}\right)x^{4}+\frac{A\,x^{3}b^{2}}{3}$	52
risch	$\frac{1}{6}B\,c^{2}x^{6}+\frac{1}{5}x^{5}A\,c^{2}+\frac{2}{5}x^{5}Bbc+\frac{1}{2}x^{4}Abc+\frac{1}{4}x^{4}B\,b^{2}+\frac{1}{3}A\,x^{3}b^{2}$	54
parallelrisch	$\frac{1}{6}B\,c^{2}x^{6}+\frac{1}{5}x^{5}A\,c^{2}+\frac{2}{5}x^{5}Bbc+\frac{1}{2}x^{4}Abc+\frac{1}{4}x^{4}B\,b^{2}+\frac{1}{3}A\,x^{3}b^{2}$	54
orering	$\frac{x\left(10B\,c^{2}x^{3}+12A\,c^{2}x^{2}+24x^{2}Bbc+30Abcx+15xB\,b^{2}+20b^{2}A\right)\left(c\,x^{2}+bx\right)^{2}}{60(cx+b)^{2}}$	68

input
```
int((B*x+A)*(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*x^3*(10*B*c^2*x^3+12*A*c^2*x^2+24*B*b*c*x^2+30*A*b*c*x+15*B*b^2*x+20*
A*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{1}{6}\,Bc^2x^6+\frac{1}{3}\,Ab^2x^3+\frac{1}{5}\,(2\,Bbc+Ac^2)x^5+\frac{1}{4}\,(Bb^2+2\,Abc)x^4$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
1/6*B*c^2*x^6 + 1/3*A*b^2*x^3 + 1/5*(2*B*b*c + A*c^2)*x^5 + 1/4*(B*b^2 + 2
*A*b*c)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int (A+Bx) \left(bx+cx^2\right)^2 \, dx = \frac{Ab^2x^3}{3} + \frac{Bc^2x^6}{6} + x^5\left(\frac{Ac^2}{5} + \frac{2Bbc}{5}\right) + x^4\left(\frac{Abc}{2} + \frac{Bb^2}{4}\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**2,x)
```

output
```
A*b**2*x**3/3 + B*c**2*x**6/6 + x**5*(A*c**2/5 + 2*B*b*c/5) + x**4*(A*b*c/
2 + B*b**2/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (A+Bx) \left(bx+cx^2\right)^2 \, dx = \frac{1}{6} Bc^2x^6 + \frac{1}{3} Ab^2x^3 + \frac{1}{5} \left(2 Bbc + Ac^2\right)x^5 + \frac{1}{4} \left(Bb^2 + 2 Abc\right)x^4$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
1/6*B*c^2*x^6 + 1/3*A*b^2*x^3 + 1/5*(2*B*b*c + A*c^2)*x^5 + 1/4*(B*b^2 + 2
*A*b*c)*x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int (A+Bx) \left(bx+cx^2\right)^2 \, dx = \frac{1}{6} Bc^2x^6 + \frac{2}{5} Bbcx^5 + \frac{1}{5} Ac^2x^5 + \frac{1}{4} Bb^2x^4 + \frac{1}{2} Abcx^4 + \frac{1}{3} Ab^2x^3$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
1/6*B*c^2*x^6 + 2/5*B*b*c*x^5 + 1/5*A*c^2*x^5 + 1/4*B*b^2*x^4 + 1/2*A*b*c*
x^4 + 1/3*A*b^2*x^3
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (A+Bx) \left(bx+cx^2\right)^2 \, dx = x^4 \left(\frac{B\,b^2}{4}+\frac{A\,c\,b}{2}\right)+x^5 \left(\frac{A\,c^2}{5}+\frac{2\,B\,b\,c}{5}\right)+\frac{A\,b^2\,x^3}{3}+\frac{B\,c^2\,x^6}{6}$$

input
```
int((b*x + c*x^2)^2*(A + B*x),x)
```

output
```
x^4*((B*b^2)/4 + (A*b*c)/2) + x^5*((A*c^2)/5 + (2*B*b*c)/5) + (A*b^2*x^3)/
3 + (B*c^2*x^6)/6
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (A+Bx) \left(bx+cx^2\right)^2 \, dx = \frac{x^3(10b\,c^2x^3+12a\,c^2x^2+24b^2c\,x^2+30abcx+15b^3x+20a\,b^2)}{60}$$

input
```
int((B*x+A)*(c*x^2+b*x)^2,x)
```

output
```
(x**3*(20*a*b**2 + 30*a*b*c*x + 12*a*c**2*x**2 + 15*b**3*x + 24*b**2*c*x**
2 + 10*b*c**2*x**3))/60
```

3.13 $\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x} \, dx = \frac{1}{2}Ab^2x^2 + \frac{1}{3}b(bB+2Ac)x^3 + \frac{1}{4}c(2bB+Ac)x^4 + \frac{1}{5}Bc^2x^5$$

output
```
1/2*A*b^2*x^2+1/3*b*(2*A*c+B*b)*x^3+1/4*c*(A*c+2*B*b)*x^4+1/5*B*c^2*x^5
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x} \, dx = \frac{1}{60}x^2\left(30Ab^2 + 20b(bB+2Ac)x + 15c(2bB+Ac)x^2 + 12Bc^2x^3\right)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^2)/x,x]
```

output
```
(x^2*(30*A*b^2 + 20*b*(b*B + 2*A*c)*x + 15*c*(2*b*B + A*c)*x^2 + 12*B*c^2*
x^3))/60
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x(A+Bx)(b+cx)^2dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(Ab^2x+cx^3(Ac+2bB)+bx^2(2Ac+bB)+Bc^2x^4\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}Ab^2x^2+\frac{1}{4}cx^4(Ac+2bB)+\frac{1}{3}bx^3(2Ac+bB)+\frac{1}{5}Bc^2x^5$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^2)/x,x]
```

output
```
(A*b^2*x^2)/2 + (b*(b*B + 2*A*c)*x^3)/3 + (c*(2*b*B + A*c)*x^4)/4 + (B*c^2
*x^5)/5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.67 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result	size
gosper	$\frac{x^{2}\left(12B\,c^{2}x^{3}+15A\,c^{2}x^{2}+30x^{2}Bbc+40Abcx+20xB\,b^{2}+30b^{2}A\right)}{60}$	52
default	$\frac{B\,c^{2}x^{5}}{5}+\frac{\left(A\,c^{2}+2Bbc\right)x^{4}}{4}+\frac{\left(2Abc+B\,b^{2}\right)x^{3}}{3}+\frac{x^{2}b^{2}A}{2}$	52
norman	$\frac{B\,c^{2}x^{5}}{5}+\left(\frac{1}{4}A\,c^{2}+\frac{1}{2}Bbc\right)x^{4}+\left(\frac{2}{3}Abc+\frac{1}{3}B\,b^{2}\right)x^{3}+\frac{x^{2}b^{2}A}{2}$	52
risch	$\frac{1}{5}B\,c^{2}x^{5}+\frac{1}{4}x^{4}A\,c^{2}+\frac{1}{2}x^{4}Bbc+\frac{2}{3}x^{3}Abc+\frac{1}{3}x^{3}B\,b^{2}+\frac{1}{2}x^{2}b^{2}A$	54
parallelrisch	$\frac{1}{5}B\,c^{2}x^{5}+\frac{1}{4}x^{4}A\,c^{2}+\frac{1}{2}x^{4}Bbc+\frac{2}{3}x^{3}Abc+\frac{1}{3}x^{3}B\,b^{2}+\frac{1}{2}x^{2}b^{2}A$	54
orering	$\frac{\left(12B\,c^{2}x^{3}+15A\,c^{2}x^{2}+30x^{2}Bbc+40Abcx+20xB\,b^{2}+30b^{2}A\right)\left(c\,x^{2}+bx\right)^{2}}{60\left(cx+b\right)^{2}}$	67

input
```
int((B*x+A)*(c*x^2+b*x)^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/60*x^2*(12*B*c^2*x^3+15*A*c^2*x^2+30*B*b*c*x^2+40*A*b*c*x+20*B*b^2*x+30*
A*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x}\,dx = \frac{1}{5}\,Bc^2x^5 + \frac{1}{2}\,Ab^2x^2 + \frac{1}{4}\,\left(2\,Bbc + Ac^2\right)x^4 + \frac{1}{3}\,\left(Bb^2 + 2\,Abc\right)x^3$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x,x, algorithm="fricas")
```

output
```
1/5*B*c^2*x^5 + 1/2*A*b^2*x^2 + 1/4*(2*B*b*c + A*c^2)*x^4 + 1/3*(B*b^2 + 2
*A*b*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x}\,dx = \frac{Ab^2x^2}{2} + \frac{Bc^2x^5}{5} + x^4\left(\frac{Ac^2}{4} + \frac{Bbc}{2}\right) + x^3\cdot\left(\frac{2Abc}{3} + \frac{Bb^2}{3}\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**2/x,x)
```

output
```
A*b**2*x**2/2 + B*c**2*x**5/5 + x**4*(A*c**2/4 + B*b*c/2) + x**3*(2*A*b*c/
3 + B*b**2/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x}\,dx = \frac{1}{5}\,Bc^2x^5 + \frac{1}{2}\,Ab^2x^2 + \frac{1}{4}\,\left(2\,Bbc + Ac^2\right)x^4 + \frac{1}{3}\,\left(Bb^2 + 2\,Abc\right)x^3$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x,x, algorithm="maxima")
```

output
```
1/5*B*c^2*x^5 + 1/2*A*b^2*x^2 + 1/4*(2*B*b*c + A*c^2)*x^4 + 1/3*(B*b^2 + 2
*A*b*c)*x^3
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)(bx+cx^2)^2}{x}\,dx = \frac{1}{5}Bc^2x^5+\frac{1}{2}Bbcx^4+\frac{1}{4}Ac^2x^4+\frac{1}{3}Bb^2x^3+\frac{2}{3}Abcx^3+\frac{1}{2}Ab^2x^2$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x,x, algorithm="giac")
```

output
```
1/5*B*c^2*x^5 + 1/2*B*b*c*x^4 + 1/4*A*c^2*x^4 + 1/3*B*b^2*x^3 + 2/3*A*b*c*
x^3 + 1/2*A*b^2*x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)(bx+cx^2)^2}{x}\,dx = x^3\left(\frac{B\,b^2}{3}+\frac{2\,A\,c\,b}{3}\right)+x^4\left(\frac{A\,c^2}{4}+\frac{B\,b\,c}{2}\right)+\frac{A\,b^2\,x^2}{2}+\frac{B\,c^2\,x^5}{5}$$

input
```
int(((b*x + c*x^2)^2*(A + B*x))/x,x)
```

output
```
x^3*((B*b^2)/3 + (2*A*b*c)/3) + x^4*((A*c^2)/4 + (B*b*c)/2) + (A*b^2*x^2)/
2 + (B*c^2*x^5)/5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)(bx+cx^2)^2}{x}\,dx$$
$$= \frac{x^2(12b\,c^2x^3+15a\,c^2x^2+30b^2c\,x^2+40abcx+20b^3x+30a\,b^2)}{60}$$

input
```
int((B*x+A)*(c*x^2+b*x)^2/x,x)
```

output
```
(x**2*(30*a*b**2 + 40*a*b*c*x + 15*a*c**2*x**2 + 20*b**3*x + 30*b**2*c*x**
2 + 12*b*c**2*x**3))/60
```

3.14 $\int \frac{(A+Bx)(bx+cx^2)^2}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{(A+Bx)(bx+cx^2)^2}{x^2}\,dx=-\frac{(bB-Ac)(b+cx)^3}{3c^2}+\frac{B(b+cx)^4}{4c^2}$$

output
```
-1/3*(-A*c+B*b)*(c*x+b)^3/c^2+1/4*B*(c*x+b)^4/c^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx)(bx+cx^2)^2}{x^2}\,dx=\frac{1}{12}x\left(12Ab^2+6b(bB+2Ac)x+4c(2bB+Ac)x^2+3Bc^2x^3\right)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^2)/x^2,x]
```

output
```
(x*(12*A*b^2 + 6*b*(b*B + 2*A*c)*x + 4*c*(2*b*B + A*c)*x^2 + 3*B*c^2*x^3))
/12
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^2}dx$$

$$\Big\downarrow \ 9$$

$$\int (A+Bx)(b+cx)^2dx$$

$$\Big\downarrow \ 49$$

$$\int \left(\frac{(b+cx)^2(Ac-bB)}{c}+\frac{B(b+cx)^3}{c}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{B(b+cx)^4}{4c^2}-\frac{(b+cx)^3(bB-Ac)}{3c^2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^2)/x^2,x]
```

output
```
-1/3*((b*B - A*c)*(b + c*x)^3)/c^2 + (B*(b + c*x)^4)/(4*c^2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

method	result	size
default	$\frac{B\,c^{2} x^{4}}{4}+\frac{\left(A\,c^{2}+2 B b c \right) x^{3}}{3}+\frac{\left(2 A b c +B\,b^{2}\right) x^{2}}{2}+x\,b^{2} A$	49
gosper	$\frac{x \left(3 B\,c^{2} x^{3}+4 A\,c^{2} x^{2}+8 x^{2} B b c +12 A b c x +6 x B\,b^{2}+12 b^{2} A \right)}{12}$	50
risch	$\frac{1}{4} B\,c^{2} x^{4}+\frac{1}{3} x^{3} A\,c^{2}+\frac{2}{3} x^{3} B b c +x^{2} A b c +\frac{1}{2} x^{2} B\,b^{2}+x\,b^{2} A$	50
parallelrisch	$\frac{1}{4} B\,c^{2} x^{4}+\frac{1}{3} x^{3} A\,c^{2}+\frac{2}{3} x^{3} B b c +x^{2} A b c +\frac{1}{2} x^{2} B\,b^{2}+x\,b^{2} A$	50
norman	$\frac{\left(\frac{1}{3} A\,c^{2}+\frac{2}{3} B b c \right) x^{4}+\left(A b c +\frac{1}{2} B\,b^{2}\right) x^{3}+x^{2} b^{2} A +\frac{B\,c^{2} x^{5}}{4}}{x}$	54
orering	$\frac{\left(3 B\,c^{2} x^{3}+4 A\,c^{2} x^{2}+8 x^{2} B b c +12 A b c x +6 x B\,b^{2}+12 b^{2} A \right) \left(c\,x^{2}+b x \right)^{2}}{12 x \left(c x +b \right)^{2}}$	70

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*B*c^2*x^4+1/3*(A*c^2+2*B*b*c)*x^3+1/2*(2*A*b*c+B*b^2)*x^2+x*b^2*A
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^2}\,dx=\frac{1}{4}\,Bc^2x^4+Ab^2x+\frac{1}{3}\,\left(2\,Bbc+Ac^2\right)x^3+\frac{1}{2}\,\left(Bb^2+2\,Abc\right)x^2$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^2,x, algorithm="fricas")
```

output
```
1/4*B*c^2*x^4 + A*b^2*x + 1/3*(2*B*b*c + A*c^2)*x^3 + 1/2*(B*b^2 + 2*A*b*c
)*x^2
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^2}\,dx = Ab^2x + \frac{Bc^2x^4}{4} + x^3\left(\frac{Ac^2}{3} + \frac{2Bbc}{3}\right) + x^2\left(Abc + \frac{Bb^2}{2}\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**2/x**2,x)
```

output
```
A*b**2*x + B*c**2*x**4/4 + x**3*(A*c**2/3 + 2*B*b*c/3) + x**2*(A*b*c + B*b
**2/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.26

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^2}\,dx = \frac{1}{4}\,Bc^2x^4 + Ab^2x + \frac{1}{3}\,\left(2\,Bbc + Ac^2\right)x^3 + \frac{1}{2}\,\left(Bb^2 + 2\,Abc\right)x^2$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^2,x, algorithm="maxima")
```

output
```
1/4*B*c^2*x^4 + A*b^2*x + 1/3*(2*B*b*c + A*c^2)*x^3 + 1/2*(B*b^2 + 2*A*b*c
)*x^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^2}\,dx = \frac{1}{4}\,Bc^2x^4 + \frac{2}{3}\,Bbcx^3 + \frac{1}{3}\,Ac^2x^3 + \frac{1}{2}\,Bb^2x^2 + Abcx^2 + Ab^2x$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^2,x, algorithm="giac")
```

output
```
1/4*B*c^2*x^4 + 2/3*B*b*c*x^3 + 1/3*A*c^2*x^3 + 1/2*B*b^2*x^2 + A*b*c*x^2
+ A*b^2*x
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^2}\,dx = x^2\left(\frac{B\,b^2}{2}+A\,c\,b\right)+x^3\left(\frac{A\,c^2}{3}+\frac{2\,B\,b\,c}{3}\right)+\frac{B\,c^2\,x^4}{4}+A\,b^2\,x$$

input
```
int(((b*x + c*x^2)^2*(A + B*x))/x^2,x)
```

output
```
x^2*((B*b^2)/2 + A*b*c) + x^3*((A*c^2)/3 + (2*B*b*c)/3) + (B*c^2*x^4)/4 +
A*b^2*x
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.29

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^2}\,dx = \frac{x(3b\,c^2x^3+4a\,c^2x^2+8b^2c\,x^2+12abcx+6b^3x+12a\,b^2)}{12}$$

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^2,x)
```

output
```
(x*(12*a*b**2 + 12*a*b*c*x + 4*a*c**2*x**2 + 6*b**3*x + 8*b**2*c*x**2 + 3*
b*c**2*x**3))/12
```

3.15 $\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 40

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^3} \, dx = 2Abcx + \frac{1}{2}Ac^2x^2 + \frac{B(b+cx)^3}{3c} + Ab^2\log(x)$$

output
```
2*A*b*c*x+1/2*A*c^2*x^2+1/3*B*(c*x+b)^3/c+A*b^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^3} \, dx = b^2Bx + bcx(2A+Bx) + \frac{1}{6}c^2x^2(3A+2Bx) + Ab^2\log(x)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^2)/x^3,x]
```

output
```
b^2*B*x + b*c*x*(2*A + B*x) + (c^2*x^2*(3*A + 2*B*x))/6 + A*b^2*Log[x]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {9, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx)(b+cx)^2}{x}dx$$

$$\Big\downarrow \text{ 90}$$

$$A\int \frac{(b+cx)^2}{x}dx+\frac{B(b+cx)^3}{3c}$$

$$\Big\downarrow \text{ 49}$$

$$A\int \left(\frac{b^2}{x}+2cb+c^2x\right)dx+\frac{B(b+cx)^3}{3c}$$

$$\Big\downarrow \text{ 2009}$$

$$A\left(b^2\log(x)+2bcx+\frac{c^2x^2}{2}\right)+\frac{B(b+cx)^3}{3c}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^2)/x^3,x]
```

output
```
(B*(b + c*x)^3)/(3*c) + A*(2*b*c*x + (c^2*x^2)/2 + b^2*Log[x])
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{B\,c^{2} x^{3}}{3}+\frac{A\,c^{2} x^{2}}{2}+x^{2} B b c+2 A b c x+x B\,b^{2}+A\,b^{2} \ln \left(x \right)$	46
risch	$\frac{B\,c^{2} x^{3}}{3}+\frac{A\,c^{2} x^{2}}{2}+x^{2} B b c+2 A b c x+x B\,b^{2}+A\,b^{2} \ln \left(x \right)$	46
parallelrisch	$\frac{B\,c^{2} x^{3}}{3}+\frac{A\,c^{2} x^{2}}{2}+x^{2} B b c+2 A b c x+x B\,b^{2}+A\,b^{2} \ln \left(x \right)$	46
norman	$\frac{\left(\frac{1}{2} A\,c^{2}+B b c \right) x^{4}+\left(2 A b c +B\,b^{2}\right) x^{3}+\frac{B\,c^{2} x^{5}}{3}}{x^{2}}+A\,b^{2} \ln \left(x \right)$	53

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*B*c^2*x^3+1/2*A*c^2*x^2+x^2*B*b*c+2*A*b*c*x+x*B*b^2+A*b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^3}\,dx=\frac{1}{3}\,Bc^2x^3+Ab^2\log\left(x\right)+\frac{1}{2}\,\left(2\,Bbc+Ac^2\right)x^2+\left(Bb^2+2\,Abc\right)x$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^3,x, algorithm="fricas")
```

output
```
1/3*B*c^2*x^3 + A*b^2*log(x) + 1/2*(2*B*b*c + A*c^2)*x^2 + (B*b^2 + 2*A*b*
c)*x
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^3}\,dx=Ab^2\log\left(x\right)+\frac{Bc^2x^3}{3}+x^2\left(\frac{Ac^2}{2}+Bbc\right)+x\left(2Abc+Bb^2\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**2/x**3,x)
```

output
```
A*b**2*log(x) + B*c**2*x**3/3 + x**2*(A*c**2/2 + B*b*c) + x*(2*A*b*c + B*b
**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^3}\,dx=\frac{1}{3}\,Bc^2x^3+Ab^2\log\left(x\right)+\frac{1}{2}\,\left(2\,Bbc+Ac^2\right)x^2+\left(Bb^2+2\,Abc\right)x$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^3,x, algorithm="maxima")
```

output
```
1/3*B*c^2*x^3 + A*b^2*log(x) + 1/2*(2*B*b*c + A*c^2)*x^2 + (B*b^2 + 2*A*b*
c)*x
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^3}\,dx = \frac{1}{3}\,Bc^2x^3+Bbcx^2+\frac{1}{2}\,Ac^2x^2+Bb^2x+2\,Abcx+Ab^2\log\left(|x|\right)$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^3,x, algorithm="giac")
```

output
```
1/3*B*c^2*x^3 + B*b*c*x^2 + 1/2*A*c^2*x^2 + B*b^2*x + 2*A*b*c*x + A*b^2*lo
g(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^3}\,dx = x^2\left(\frac{A\,c^2}{2}+B\,b\,c\right)+x\,\left(B\,b^2+2\,A\,c\,b\right)+\frac{B\,c^2\,x^3}{3}+A\,b^2\,\ln\left(x\right)$$

input
```
int(((b*x + c*x^2)^2*(A + B*x))/x^3,x)
```

output
```
x^2*((A*c^2)/2 + B*b*c) + x*(B*b^2 + 2*A*b*c) + (B*c^2*x^3)/3 + A*b^2*log(
x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^3}\,dx = \log(x)\,a\,b^2+2abcx+\frac{a\,c^2x^2}{2}+b^3x+b^2c\,x^2+\frac{b\,c^2x^3}{3}$$

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^3,x)
```

output
```
(6*log(x)*a*b**2 + 12*a*b*c*x + 3*a*c**2*x**2 + 6*b**3*x + 6*b**2*c*x**2 +
 2*b*c**2*x**3)/6
```

3.16 $\int \frac{(A+Bx)(bx+cx^2)^2}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^4}\,dx=-\frac{Ab^2}{x}+c(2bB+Ac)x+\frac{1}{2}Bc^2x^2+b(bB+2Ac)\log(x)$$

output
```
-A*b^2/x+c*(A*c+2*B*b)*x+1/2*B*c^2*x^2+b*(2*A*c+B*b)*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^4}\,dx=\frac{1}{2}Bcx(4b+cx)+A\left(-\frac{b^2}{x}+c^2x\right)+b(bB+2Ac)\log(x)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^2)/x^4,x]
```

output
```
(B*c*x*(4*b + c*x))/2 + A*(-(b^2/x) + c^2*x) + b*(b*B + 2*A*c)*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^4}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx)(b+cx)^2}{x^2}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{Ab^2}{x^2}+\frac{b(2Ac+bB)}{x}+c(Ac+2bB)+Bc^2x\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{Ab^2}{x}+cx(Ac+2bB)+b\log(x)(2Ac+bB)+\frac{1}{2}Bc^2x^2$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^2)/x^4,x]
```

output
```
-((A*b^2)/x) + c*(2*b*B + A*c)*x + (B*c^2*x^2)/2 + b*(b*B + 2*A*c)*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{B\,c^{2} x^{2}}{2}+A\,c^{2} x+2 B b c x+b \left(2 A c+B b\right) \ln \left(x\right)-\frac{A\,b^{2}}{x}$	44
risch	$\frac{B\,c^{2} x^{2}}{2}+A\,c^{2} x+2 B b c x-\frac{A\,b^{2}}{x}+2 A \ln \left(x\right) b c+B \ln \left(x\right) b^{2}$	46
norman	$\frac{\left(A\,c^{2}+2 B b c\right) x^{4}+\frac{B\,c^{2} x^{5}}{2}-x^{2} b^{2} A}{x^{3}}+\left(2 A b c+B\,b^{2}\right) \ln \left(x\right)$	54
parallelrisch	$\frac{B\,c^{2} x^{3}+4 A \ln \left(x\right) x b c+2 A\,c^{2} x^{2}+2 B \ln \left(x\right) x\,b^{2}+4 x^{2} B b c-2 b^{2} A}{2 x}$	55

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/2*B*c^2*x^2+A*c^2*x+2*B*b*c*x+b*(2*A*c+B*b)*ln(x)-A*b^2/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^4}\,dx = \frac{Bc^2x^3-2\,Ab^2+2\,(2\,Bbc+Ac^2)x^2+2\,(Bb^2+2\,Abc)x\log{(x)}}{2\,x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^4,x, algorithm="fricas")
```

output
```
1/2*(B*c^2*x^3 - 2*A*b^2 + 2*(2*B*b*c + A*c^2)*x^2 + 2*(B*b^2 + 2*A*b*c)*x
*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)(bx+cx^2)^2}{x^4}\,dx = -\frac{Ab^2}{x} + \frac{Bc^2x^2}{2} + b(2Ac+Bb)\log(x) + x\left(Ac^2+2Bbc\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**2/x**4,x)
```

output
```
-A*b**2/x + B*c**2*x**2/2 + b*(2*A*c + B*b)*log(x) + x*(A*c**2 + 2*B*b*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)(bx+cx^2)^2}{x^4}\,dx = \frac{1}{2}\,Bc^2x^2 - \frac{Ab^2}{x} + \left(2\,Bbc+Ac^2\right)x + \left(Bb^2+2\,Abc\right)\log(x)$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^4,x, algorithm="maxima")
```

output
```
1/2*B*c^2*x^2 - A*b^2/x + (2*B*b*c + A*c^2)*x + (B*b^2 + 2*A*b*c)*log(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)(bx+cx^2)^2}{x^4}\,dx = \frac{1}{2}\,Bc^2x^2 + 2\,Bbcx + Ac^2x - \frac{Ab^2}{x} + \left(Bb^2+2\,Abc\right)\log(|x|)$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^4,x, algorithm="giac")
```

output
```
1/2*B*c^2*x^2 + 2*B*b*c*x + A*c^2*x - A*b^2/x + (B*b^2 + 2*A*b*c)*log(abs(
x))
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^4}\,dx = x\left(A\,c^2+2\,B\,b\,c\right)+\ln\left(x\right)\,\left(B\,b^2+2\,A\,c\,b\right)-\frac{A\,b^2}{x}+\frac{B\,c^2\,x^2}{2}$$

input
```
int(((b*x + c*x^2)^2*(A + B*x))/x^4,x)
```

output
```
x*(A*c^2 + 2*B*b*c) + log(x)*(B*b^2 + 2*A*b*c) - (A*b^2)/x + (B*c^2*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^4}\,dx = \frac{4\log(x)\,abcx+2\log(x)\,b^3x-2a\,b^2+2a\,c^2x^2+4b^2c\,x^2+b\,c^2x^3}{2x}$$

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^4,x)
```

output
```
(4*log(x)*a*b*c*x + 2*log(x)*b**3*x - 2*a*b**2 + 2*a*c**2*x**2 + 4*b**2*c*
x**2 + b*c**2*x**3)/(2*x)
```

3.17 $\int \frac{(A+Bx)(bx+cx^2)^2}{x^5}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 44

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^5}\,dx=-\frac{Ab^2}{2x^2}-\frac{b(bB+2Ac)}{x}+Bc^2x+c(2bB+Ac)\log(x)$$

output
```
-1/2*A*b^2/x^2-b*(2*A*c+B*b)/x+B*c^2*x+c*(A*c+2*B*b)*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^5}\,dx=-\frac{Ab^2}{2x^2}-\frac{b(bB+2Ac)}{x}+Bc^2x+c(2bB+Ac)\log(x)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^2)/x^5,x]
```

output
```
-1/2*(A*b^2)/x^2 - (b*(b*B + 2*A*c))/x + B*c^2*x + c*(2*b*B + A*c)*Log[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^5}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(A+Bx)(b+cx)^2}{x^3}dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{Ab^2}{x^3}+\frac{b(2Ac+bB)}{x^2}+\frac{c(Ac+2bB)}{x}+Bc^2\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{Ab^2}{2x^2}-\frac{b(2Ac+bB)}{x}+c\log(x)(Ac+2bB)+Bc^2x$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^2)/x^5,x]
```

output
```
-1/2*(A*b^2)/x^2 - (b*(b*B + 2*A*c))/x + B*c^2*x + c*(2*b*B + A*c)*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)    Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.63 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{A\,b^{2}}{2x^{2}}-\frac{b(2Ac+Bb)}{x}+c^{2}xB+c(Ac+2Bb)\ln(x)$	43
risch	$c^{2}xB+\frac{(-2Abc-B\,b^{2})x-\frac{b^{2}A}{2}}{x^{2}}+A\ln(x)\,c^{2}+2B\ln(x)\,bc$	47
norman	$\frac{(-2Abc-B\,b^{2})x^{3}+B\,c^{2}x^{5}-\frac{x^{2}b^{2}A}{2}}{x^{4}}+(A\,c^{2}+2Bbc)\ln(x)$	54
parallelrisch	$\frac{2A\ln(x)x^{2}c^{2}+4B\ln(x)x^{2}bc+2B\,c^{2}x^{3}-4Abcx-2xB\,b^{2}-b^{2}A}{2x^{2}}$	56

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A*b^2/x^2-b*(2*A*c+B*b)/x+c^2*x*B+c*(A*c+2*B*b)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.20

$$\int\frac{(A+Bx)\,(bx+cx^2)^2}{x^5}\,dx=\frac{2\,Bc^2x^3+2\,(2\,Bbc+Ac^2)x^2\log(x)-Ab^2-2\,(Bb^2+2\,Abc)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^5,x, algorithm="fricas")
```

output
```
1/2*(2*B*c^2*x^3 + 2*(2*B*b*c + A*c^2)*x^2*log(x) - A*b^2 - 2*(B*b^2 + 2*A
*b*c)*x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^5}\,dx = Bc^2x + c(Ac+2Bb)\log{(x)} + \frac{-Ab^2 + x(-4Abc-2Bb^2)}{2x^2}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**2/x**5,x)
```

output
```
B*c**2*x + c*(A*c + 2*B*b)*log(x) + (-A*b**2 + x*(-4*A*b*c - 2*B*b**2))/(2
*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^5}\,dx = Bc^2x + \left(2\,Bbc + Ac^2\right)\log\left(x\right) - \frac{Ab^2 + 2\,(Bb^2 + 2\,Abc)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^5,x, algorithm="maxima")
```

output
```
B*c^2*x + (2*B*b*c + A*c^2)*log(x) - 1/2*(A*b^2 + 2*(B*b^2 + 2*A*b*c)*x)/x
^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^5}\,dx = Bc^2x + \left(2\,Bbc + Ac^2\right)\log\left(|x|\right) - \frac{Ab^2 + 2\,(Bb^2 + 2\,Abc)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^5,x, algorithm="giac")
```

output
```
B*c^2*x + (2*B*b*c + A*c^2)*log(abs(x)) - 1/2*(A*b^2 + 2*(B*b^2 + 2*A*b*c)
*x)/x^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^5}\,dx = \ln\left(x\right)\,\left(A\,c^2 + 2\,B\,b\,c\right) - \frac{\frac{A\,b^2}{2} + x\,\left(B\,b^2 + 2\,A\,c\,b\right)}{x^2} + B\,c^2\,x$$

input
```
int(((b*x + c*x^2)^2*(A + B*x))/x^5,x)
```

output
```
log(x)*(A*c^2 + 2*B*b*c) - ((A*b^2)/2 + x*(B*b^2 + 2*A*b*c))/x^2 + B*c^2*x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^5}\,dx = \frac{2\log(x)\,a\,c^2x^2 + 4\log(x)\,b^2c\,x^2 - a\,b^2 - 4abcx - 2b^3x + 2b\,c^2x^3}{2x^2}$$

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^5,x)
```

output
```
(2*log(x)*a*c**2*x**2 + 4*log(x)*b**2*c*x**2 - a*b**2 - 4*a*b*c*x - 2*b**3
*x + 2*b*c**2*x**3)/(2*x**2)
```

3.18 $\int \frac{(A+Bx)(bx+cx^2)^2}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 49

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^6}\,dx = -\frac{Ab^2}{3x^3}-\frac{b(bB+2Ac)}{2x^2}-\frac{c(2bB+Ac)}{x}+Bc^2\log(x)$$

output
```
-1/3*A*b^2/x^3-1/2*b*(2*A*c+B*b)/x^2-c*(A*c+2*B*b)/x+B*c^2*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^6}\,dx = -\frac{3bBx(b+4cx)+2A(b^2+3bcx+3c^2x^2)}{6x^3}+Bc^2\log(x)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^2)/x^6,x]
```

output
```
-1/6*(3*b*B*x*(b + 4*c*x) + 2*A*(b^2 + 3*b*c*x + 3*c^2*x^2))/x^3 + B*c^2*L
og[x]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^6}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{(A+Bx)(b+cx)^2}{x^4}dx$$

$$\Big\downarrow \ 85$$

$$\int \left(\frac{Ab^2}{x^4}+\frac{b(2Ac+bB)}{x^3}+\frac{c(Ac+2bB)}{x^2}+\frac{Bc^2}{x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{Ab^2}{3x^3}-\frac{b(2Ac+bB)}{2x^2}-\frac{c(Ac+2bB)}{x}+Bc^2\log(x)$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^2)/x^6,x]
```

output
```
-1/3*(A*b^2)/x^3 - (b*(b*B + 2*A*c))/(2*x^2) - (c*(2*b*B + A*c))/x + B*c^2
*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

method	result	size
default	$-\frac{A\,b^{2}}{3x^{3}}-\frac{b(2Ac+Bb)}{2x^{2}}-\frac{c(Ac+2Bb)}{x}+B\,c^{2}\ln\left(x\right)$	46
risch	$\frac{\left(-A\,c^{2}-2Bbc\right)x^{2}+\left(-Abc-\frac{1}{2}B\,b^{2}\right)x-\frac{b^{2}A}{3}}{x^{3}}+B\,c^{2}\ln\left(x\right)$	50
parallelrisch	$-\frac{-6B\,c^{2}\ln(x)x^{3}+6A\,c^{2}x^{2}+12x^{2}Bbc+6Abcx+3xB\,b^{2}+2b^{2}A}{6x^{3}}$	54
norman	$\frac{\left(-Abc-\frac{1}{2}B\,b^{2}\right)x^{3}+\left(-A\,c^{2}-2Bbc\right)x^{4}-\frac{x^{2}b^{2}A}{3}}{x^{5}}+B\,c^{2}\ln\left(x\right)$	55

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^6,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A*b^2/x^3-1/2*b*(2*A*c+B*b)/x^2-c*(A*c+2*B*b)/x+B*c^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int\frac{(A+Bx)\left(bx+cx^{2}\right)^{2}}{x^{6}}\,dx=\frac{6\,Bc^{2}x^{3}\log\left(x\right)-2\,Ab^{2}-6\,(2\,Bbc+Ac^{2})x^{2}-3\,(Bb^{2}+2\,Abc)x}{6\,x^{3}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^6,x, algorithm="fricas")
```

output
```
1/6*(6*B*c^2*x^3*log(x) - 2*A*b^2 - 6*(2*B*b*c + A*c^2)*x^2 - 3*(B*b^2 + 2
*A*b*c)*x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)(bx+cx^2)^2}{x^6}\,dx = Bc^2\log(x) + \frac{-2Ab^2+x^2(-6Ac^2-12Bbc)+x(-6Abc-3Bb^2)}{6x^3}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**2/x**6,x)
```

output
```
B*c**2*log(x) + (-2*A*b**2 + x**2*(-6*A*c**2 - 12*B*b*c) + x*(-6*A*b*c - 3
*B*b**2))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)(bx+cx^2)^2}{x^6}\,dx = Bc^2\log(x) - \frac{2\,Ab^2+6\,(2\,Bbc+Ac^2)x^2+3\,(Bb^2+2\,Abc)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^6,x, algorithm="maxima")
```

output
```
B*c^2*log(x) - 1/6*(2*A*b^2 + 6*(2*B*b*c + A*c^2)*x^2 + 3*(B*b^2 + 2*A*b*c
)*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^6}\,dx$$
$$= Bc^2\log\left(|x|\right) - \frac{2\,Ab^2 + 6\,(2\,Bbc + Ac^2)x^2 + 3\,(Bb^2 + 2\,Abc)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^6,x, algorithm="giac")
```

output
```
B*c^2*log(abs(x)) - 1/6*(2*A*b^2 + 6*(2*B*b*c + A*c^2)*x^2 + 3*(B*b^2 + 2*
A*b*c)*x)/x^3
```

Mupad [B] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^6}\,dx = B\,c^2\,\ln\left(x\right) - \frac{x^2\,\left(A\,c^2 + 2\,B\,b\,c\right) + \frac{A\,b^2}{3} + x\,\left(\frac{B\,b^2}{2} + A\,c\,b\right)}{x^3}$$

input
```
int(((b*x + c*x^2)^2*(A + B*x))/x^6,x)
```

output
```
B*c^2*log(x) - (x^2*(A*c^2 + 2*B*b*c) + (A*b^2)/3 + x*((B*b^2)/2 + A*b*c))
/x^3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^6}\,dx$$
$$= \frac{6\log(x)\,b\,c^2x^3 - 2a\,b^2 - 6abcx - 6a\,c^2x^2 - 3b^3x - 12b^2c\,x^2}{6x^3}$$

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^6,x)
```

output
```
(6*log(x)*b*c**2*x**3 - 2*a*b**2 - 6*a*b*c*x - 6*a*c**2*x**2 - 3*b**3*x -
12*b**2*c*x**2)/(6*x**3)
```

3.19 $\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^7} dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^7} dx = -\frac{A(b+cx)^3}{4bx^4} - \frac{(4bB-Ac)(b+cx)^3}{12b^2x^3}$$

output
```
-1/4*A*(c*x+b)^3/b/x^4-1/12*(-A*c+4*B*b)*(c*x+b)^3/b^2/x^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^7} dx = -\frac{4Bx(b^2+3bcx+3c^2x^2)+A(3b^2+8bcx+6c^2x^2)}{12x^4}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^2)/x^7,x]
```

output
```
-1/12*(4*B*x*(b^2 + 3*b*c*x + 3*c^2*x^2) + A*(3*b^2 + 8*b*c*x + 6*c^2*x^2)
)/x^4
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^7}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(A+Bx)(b+cx)^2}{x^5}dx$$

$$\Big\downarrow \; 87$$

$$\frac{(4bB-Ac)\int \frac{(b+cx)^2}{x^4}dx}{4b}-\frac{A(b+cx)^3}{4bx^4}$$

$$\Big\downarrow \; 48$$

$$-\frac{(b+cx)^3(4bB-Ac)}{12b^2x^3}-\frac{A(b+cx)^3}{4bx^4}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^2)/x^7,x]
```

output
```
-1/4*(A*(b + c*x)^3)/(b*x^4) - ((4*b*B - A*c)*(b + c*x)^3)/(12*b^2*x^3)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (warning: unable to verify)

Time = 0.64 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{b(2Ac+Bb)}{3x^{3}}-\frac{c(Ac+2Bb)}{2x^{2}}-\frac{b^{2}A}{4x^{4}}-\frac{B\,c^{2}}{x}$	48
risch	$\frac{-B\,c^{2}x^{3}+\left(-\frac{1}{2}A\,c^{2}-Bbc\right)x^{2}+\left(-\frac{2}{3}Abc-\frac{1}{3}B\,b^{2}\right)x-\frac{b^{2}A}{4}}{x^{4}}$	51
gosper	$-\frac{12B\,c^{2}x^{3}+6A\,c^{2}x^{2}+12x^{2}Bbc+8Abcx+4xB\,b^{2}+3b^{2}A}{12x^{4}}$	52
parallelrisch	$-\frac{12B\,c^{2}x^{3}+6A\,c^{2}x^{2}+12x^{2}Bbc+8Abcx+4xB\,b^{2}+3b^{2}A}{12x^{4}}$	52
norman	$\frac{\left(-\frac{1}{2}A\,c^{2}-Bbc\right)x^{4}+\left(-\frac{2}{3}Abc-\frac{1}{3}B\,b^{2}\right)x^{3}-B\,c^{2}x^{5}-\frac{x^{2}b^{2}A}{4}}{x^{6}}$	56
orering	$-\frac{\left(12B\,c^{2}x^{3}+6A\,c^{2}x^{2}+12x^{2}Bbc+8Abcx+4xB\,b^{2}+3b^{2}A\right)\left(c\,x^{2}+bx\right)^{2}}{12x^{6}\left(cx+b\right)^{2}}$	70

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b*(2*A*c+B*b)/x^3-1/2*c*(A*c+2*B*b)/x^2-1/4*b^2*A/x^4-B*c^2/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^7}\,dx = -\frac{12\,Bc^2x^3+3\,Ab^2+6\,(2\,Bbc+Ac^2)x^2+4\,(Bb^2+2\,Abc)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^7,x, algorithm="fricas")
```

output
```
-1/12*(12*B*c^2*x^3 + 3*A*b^2 + 6*(2*B*b*c + A*c^2)*x^2 + 4*(B*b^2 + 2*A*b
*c)*x)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^7}\,dx = \frac{-3Ab^2-12Bc^2x^3+x^2(-6Ac^2-12Bbc)+x(-8Abc-4Bb^2)}{12x^4}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**2/x**7,x)
```

output
```
(-3*A*b**2 - 12*B*c**2*x**3 + x**2*(-6*A*c**2 - 12*B*b*c) + x*(-8*A*b*c -
4*B*b**2))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^7}\,dx = -\frac{12\,Bc^2x^3+3\,Ab^2+6\,(2\,Bbc+Ac^2)x^2+4\,(Bb^2+2\,Abc)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^7,x, algorithm="maxima")
```

output
```
-1/12*(12*B*c^2*x^3 + 3*A*b^2 + 6*(2*B*b*c + A*c^2)*x^2 + 4*(B*b^2 + 2*A*b
*c)*x)/x^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^7}\,dx = -\frac{12\,Bc^2x^3+12\,Bbcx^2+6\,Ac^2x^2+4\,Bb^2x+8\,Abcx+3\,Ab^2}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^7,x, algorithm="giac")
```

output
```
-1/12*(12*B*c^2*x^3 + 12*B*b*c*x^2 + 6*A*c^2*x^2 + 4*B*b^2*x + 8*A*b*c*x +
 3*A*b^2)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\,(bx+cx^2)^2}{x^7}\,dx = -\frac{x^2\left(\frac{A\,c^2}{2}+B\,b\,c\right)+\frac{A\,b^2}{4}+x\left(\frac{B\,b^2}{3}+\frac{2\,A\,c\,b}{3}\right)+B\,c^2\,x^3}{x^4}$$

input
```
int(((b*x + c*x^2)^2*(A + B*x))/x^7,x)
```

output
```
-(x^2*((A*c^2)/2 + B*b*c) + (A*b^2)/4 + x*((B*b^2)/3 + (2*A*b*c)/3) + B*c^
2*x^3)/x^4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx)\,(bx+cx^2)^2}{x^7}\,dx = \frac{-12b\,c^2x^3-6a\,c^2x^2-12b^2c\,x^2-8abcx-4b^3x-3a\,b^2}{12x^4}$$

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^7,x)
```

output
```
( - 3*a*b**2 - 8*a*b*c*x - 6*a*c**2*x**2 - 4*b**3*x - 12*b**2*c*x**2 - 12*
b*c**2*x**3)/(12*x**4)
```

3.20 $\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^8}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^8}\,dx=-\frac{Ab^2}{5x^5}-\frac{b(bB+2Ac)}{4x^4}-\frac{c(2bB+Ac)}{3x^3}-\frac{Bc^2}{2x^2}$$

output
```
-1/5*A*b^2/x^5-1/4*b*(2*A*c+B*b)/x^4-1/3*c*(A*c+2*B*b)/x^3-1/2*B*c^2/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^8}\,dx=-\frac{5Bx(3b^2+8bcx+6c^2x^2)+2A(6b^2+15bcx+10c^2x^2)}{60x^5}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^2)/x^8,x]
```

output
```
-1/60*(5*B*x*(3*b^2 + 8*b*c*x + 6*c^2*x^2) + 2*A*(6*b^2 + 15*b*c*x + 10*c^
2*x^2))/x^5
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^8}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx)(b+cx)^2}{x^6}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{Ab^2}{x^6}+\frac{b(2Ac+bB)}{x^5}+\frac{c(Ac+2bB)}{x^4}+\frac{Bc^2}{x^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{Ab^2}{5x^5}-\frac{b(2Ac+bB)}{4x^4}-\frac{c(Ac+2bB)}{3x^3}-\frac{Bc^2}{2x^2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^2)/x^8,x]
```

output
```
-1/5*(A*b^2)/x^5 - (b*(b*B + 2*A*c))/(4*x^4) - (c*(2*b*B + A*c))/(3*x^3) -
 (B*c^2)/(2*x^2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.65 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.87

method	result	size
default	$-\frac{A\,b^{2}}{5x^{5}}-\frac{b(2Ac+Bb)}{4x^{4}}-\frac{c(Ac+2Bb)}{3x^{3}}-\frac{B\,c^{2}}{2x^{2}}$	48
risch	$\frac{-\frac{B\,c^{2}x^{3}}{2}+\left(-\frac{1}{3}A\,c^{2}-\frac{2}{3}Bbc\right)x^{2}+\left(-\frac{1}{2}Abc-\frac{1}{4}B\,b^{2}\right)x-\frac{b^{2}A}{5}}{x^{5}}$	51
gosper	$-\frac{30B\,c^{2}x^{3}+20A\,c^{2}x^{2}+40x^{2}Bbc+30Abcx+15xB\,b^{2}+12b^{2}A}{60x^{5}}$	52
parallelrisch	$-\frac{30B\,c^{2}x^{3}+20A\,c^{2}x^{2}+40x^{2}Bbc+30Abcx+15xB\,b^{2}+12b^{2}A}{60x^{5}}$	52
norman	$\frac{\left(-\frac{1}{3}A\,c^{2}-\frac{2}{3}Bbc\right)x^{4}+\left(-\frac{1}{2}Abc-\frac{1}{4}B\,b^{2}\right)x^{3}-\frac{B\,c^{2}x^{5}}{2}-\frac{x^{2}b^{2}A}{5}}{x^{7}}$	56
orering	$-\frac{\left(30B\,c^{2}x^{3}+20A\,c^{2}x^{2}+40x^{2}Bbc+30Abcx+15xB\,b^{2}+12b^{2}A\right)\left(c\,x^{2}+bx\right)^{2}}{60x^{7}\left(cx+b\right)^{2}}$	70

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/5*A*b^2/x^5-1/4*b*(2*A*c+B*b)/x^4-1/3*c*(A*c+2*B*b)/x^3-1/2*B*c^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^8}\,dx = -\frac{30\,Bc^2x^3+12\,Ab^2+20\,(2\,Bbc+Ac^2)x^2+15\,(Bb^2+2\,Abc)x}{60\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^8,x, algorithm="fricas")
```

output
```
-1/60*(30*B*c^2*x^3 + 12*A*b^2 + 20*(2*B*b*c + A*c^2)*x^2 + 15*(B*b^2 + 2*
A*b*c)*x)/x^5
```

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^8}\,dx = \frac{-12Ab^2-30Bc^2x^3+x^2(-20Ac^2-40Bbc)+x(-30Abc-15Bb^2)}{60x^5}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**2/x**8,x)
```

output
```
(-12*A*b**2 - 30*B*c**2*x**3 + x**2*(-20*A*c**2 - 40*B*b*c) + x*(-30*A*b*c
 - 15*B*b**2))/(60*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^8}\,dx = -\frac{30\,Bc^2x^3+12\,Ab^2+20\,(2\,Bbc+Ac^2)x^2+15\,(Bb^2+2\,Abc)x}{60\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^8,x, algorithm="maxima")
```

output
```
-1/60*(30*B*c^2*x^3 + 12*A*b^2 + 20*(2*B*b*c + A*c^2)*x^2 + 15*(B*b^2 + 2*
A*b*c)*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^8}\,dx = -\frac{30\,Bc^2x^3+40\,Bbcx^2+20\,Ac^2x^2+15\,Bb^2x+30\,Abcx+12\,Ab^2}{60\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^8,x, algorithm="giac")
```

output
```
-1/60*(30*B*c^2*x^3 + 40*B*b*c*x^2 + 20*A*c^2*x^2 + 15*B*b^2*x + 30*A*b*c*
x + 12*A*b^2)/x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^8}\,dx = -\frac{x^2\left(\frac{A\,c^2}{3}+\frac{2\,B\,b\,c}{3}\right)+\frac{A\,b^2}{5}+x\left(\frac{B\,b^2}{4}+\frac{A\,c\,b}{2}\right)+\frac{B\,c^2\,x^3}{2}}{x^5}$$

input
```
int(((b*x + c*x^2)^2*(A + B*x))/x^8,x)
```

output
```
-(x^2*((A*c^2)/3 + (2*B*b*c)/3) + (A*b^2)/5 + x*((B*b^2)/4 + (A*b*c)/2) +
(B*c^2*x^3)/2)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^8}\,dx = \frac{-30b\,c^2x^3-20a\,c^2x^2-40b^2c\,x^2-30abcx-15b^3x-12a\,b^2}{60x^5}$$

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^8,x)
```

output
```
( - 12*a*b**2 - 30*a*b*c*x - 20*a*c**2*x**2 - 15*b**3*x - 40*b**2*c*x**2 -
 30*b*c**2*x**3)/(60*x**5)
```

3.21 $\int x^2(A+Bx)\left(bx+cx^2\right)^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 75

$$\int x^2(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{1}{6}Ab^3x^6+\frac{1}{7}b^2(bB+3Ac)x^7+\frac{3}{8}bc(bB+Ac)x^8+\frac{1}{9}c^2(3bB+Ac)x^9+\frac{1}{10}Bc^3x^{10}$$

output
```
1/6*A*b^3*x^6+1/7*b^2*(3*A*c+B*b)*x^7+3/8*b*c*(A*c+B*b)*x^8+1/9*c^2*(A*c+3
*B*b)*x^9+1/10*B*c^3*x^10
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x^2(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{1}{6}Ab^3x^6+\frac{1}{7}b^2(bB+3Ac)x^7+\frac{3}{8}bc(bB+Ac)x^8+\frac{1}{9}c^2(3bB+Ac)x^9+\frac{1}{10}Bc^3x^{10}$$

input
```
Integrate[x^2*(A + B*x)*(b*x + c*x^2)^3,x]
```

output
```
(A*b^3*x^6)/6 + (b^2*(b*B + 3*A*c)*x^7)/7 + (3*b*c*(b*B + A*c)*x^8)/8 + (c
^2*(3*b*B + A*c)*x^9)/9 + (B*c^3*x^10)/10
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(A+Bx)\left(bx+cx^2\right)^3\,dx$$

$$\Big\downarrow \ 9$$

$$\int x^5(A+Bx)(b+cx)^3dx$$

$$\Big\downarrow \ 85$$

$$\int \left(Ab^3x^5+b^2x^6(3Ac+bB)+c^2x^8(Ac+3bB)+3bcx^7(Ac+bB)+Bc^3x^9\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{6}Ab^3x^6+\frac{1}{7}b^2x^7(3Ac+bB)+\frac{1}{9}c^2x^9(Ac+3bB)+\frac{3}{8}bcx^8(Ac+bB)+\frac{1}{10}Bc^3x^{10}$$

input
```
Int[x^2*(A + B*x)*(b*x + c*x^2)^3,x]
```

output

```
(A*b^3*x^6)/6 + (b^2*(b*B + 3*A*c)*x^7)/7 + (3*b*c*(b*B + A*c)*x^8)/8 + (c
^2*(3*b*B + A*c)*x^9)/9 + (B*c^3*x^10)/10
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

method	result
norman	$\frac{B\,c^3x^{10}}{10}+\left(\frac{1}{9}A\,c^3+\frac{1}{3}Bb\,c^2\right)x^9+\left(\frac{3}{8}Ab\,c^2+\frac{3}{8}B\,b^2c\right)x^8+\left(\frac{3}{7}A\,b^2c+\frac{1}{7}B\,b^3\right)x^7+\frac{A\,b^3x^6}{6}$
gosper	$\frac{x^6\left(252B\,c^3x^4+280A\,c^3x^3+840x^3Bb\,c^2+945Ab\,c^2x^2+945x^2B\,b^2c+1080A\,b^2cx+360xB\,b^3+420A\,b^3\right)}{2520}$
default	$\frac{B\,c^3x^{10}}{10}+\frac{\left(A\,c^3+3Bb\,c^2\right)x^9}{9}+\frac{\left(3Ab\,c^2+3B\,b^2c\right)x^8}{8}+\frac{\left(3A\,b^2c+B\,b^3\right)x^7}{7}+\frac{A\,b^3x^6}{6}$
risch	$\frac{1}{10}B\,c^3x^{10}+\frac{1}{9}x^9A\,c^3+\frac{1}{3}x^9Bb\,c^2+\frac{3}{8}x^8Ab\,c^2+\frac{3}{8}x^8B\,b^2c+\frac{3}{7}x^7A\,b^2c+\frac{1}{7}B\,b^3x^7+\frac{1}{6}A\,b^3x^6$
parallelrisch	$\frac{1}{10}B\,c^3x^{10}+\frac{1}{9}x^9A\,c^3+\frac{1}{3}x^9Bb\,c^2+\frac{3}{8}x^8Ab\,c^2+\frac{3}{8}x^8B\,b^2c+\frac{3}{7}x^7A\,b^2c+\frac{1}{7}B\,b^3x^7+\frac{1}{6}A\,b^3x^6$
orering	$\frac{x^3\left(252B\,c^3x^4+280A\,c^3x^3+840x^3Bb\,c^2+945Ab\,c^2x^2+945x^2B\,b^2c+1080A\,b^2cx+360xB\,b^3+420A\,b^3\right)\left(c\,x^2+bx\right)^3}{2520\left(cx+b\right)^3}$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/10*B*c^3*x^10+(1/9*A*c^3+1/3*B*b*c^2)*x^9+(3/8*A*b*c^2+3/8*B*b^2*c)*x^8+
(3/7*A*b^2*c+1/7*B*b^3)*x^7+1/6*A*b^3*x^6
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int x^2(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{1}{10}Bc^3x^{10}+\frac{1}{6}Ab^3x^6+\frac{1}{9}\left(3\,Bbc^2+Ac^3\right)x^9+\frac{3}{8}\left(Bb^2c+Abc^2\right)x^8+\frac{1}{7}\left(Bb^3+3\,Ab^2c\right)x^7$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/10*B*c^3*x^10 + 1/6*A*b^3*x^6 + 1/9*(3*B*b*c^2 + A*c^3)*x^9 + 3/8*(B*b^2
*c + A*b*c^2)*x^8 + 1/7*(B*b^3 + 3*A*b^2*c)*x^7
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int x^2(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{Ab^3x^6}{6}+\frac{Bc^3x^{10}}{10}+x^9\left(\frac{Ac^3}{9}+\frac{Bbc^2}{3}\right)+x^8\cdot\left(\frac{3Abc^2}{8}+\frac{3Bb^2c}{8}\right)+x^7\cdot\left(\frac{3Ab^2c}{7}+\frac{Bb^3}{7}\right)$$

input
```
integrate(x**2*(B*x+A)*(c*x**2+b*x)**3,x)
```

output
```
A*b**3*x**6/6 + B*c**3*x**10/10 + x**9*(A*c**3/9 + B*b*c**2/3) + x**8*(3*A
*b*c**2/8 + 3*B*b**2*c/8) + x**7*(3*A*b**2*c/7 + B*b**3/7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int x^2(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{1}{10}\,Bc^3x^{10}+\frac{1}{6}\,Ab^3x^6+\frac{1}{9}\left(3\,Bbc^2+Ac^3\right)x^9+\frac{3}{8}\left(Bb^2c+Abc^2\right)x^8+\frac{1}{7}\left(Bb^3+3\,Ab^2c\right)x^7$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/10*B*c^3*x^10 + 1/6*A*b^3*x^6 + 1/9*(3*B*b*c^2 + A*c^3)*x^9 + 3/8*(B*b^2
*c + A*b*c^2)*x^8 + 1/7*(B*b^3 + 3*A*b^2*c)*x^7
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int x^2(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{1}{10}\,Bc^3x^{10}+\frac{1}{3}\,Bbc^2x^9+\frac{1}{9}\,Ac^3x^9+\frac{3}{8}\,Bb^2cx^8+\frac{3}{8}\,Abc^2x^8+\frac{1}{7}\,Bb^3x^7+\frac{3}{7}\,Ab^2cx^7+\frac{1}{6}\,Ab^3x^6$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
1/10*B*c^3*x^10 + 1/3*B*b*c^2*x^9 + 1/9*A*c^3*x^9 + 3/8*B*b^2*c*x^8 + 3/8*
A*b*c^2*x^8 + 1/7*B*b^3*x^7 + 3/7*A*b^2*c*x^7 + 1/6*A*b^3*x^6
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int x^2(A+Bx)\left(bx+cx^2\right)^3\,dx=x^7\left(\frac{B\,b^3}{7}+\frac{3\,A\,c\,b^2}{7}\right)+x^9\left(\frac{A\,c^3}{9}+\frac{B\,b\,c^2}{3}\right)+\frac{A\,b^3\,x^6}{6}+\frac{B\,c^3\,x^{10}}{10}+\frac{3\,b\,c\,x^8\,\left(A\,c+B\,b\right)}{8}$$

input
```
int(x^2*(b*x + c*x^2)^3*(A + B*x),x)
```

output
```
x^7*((B*b^3)/7 + (3*A*b^2*c)/7) + x^9*((A*c^3)/9 + (B*b*c^2)/3) + (A*b^3*x
^6)/6 + (B*c^3*x^10)/10 + (3*b*c*x^8*(A*c + B*b))/8
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int x^2(A+Bx)\left(bx+cx^2\right)^3\,dx$$
$$=\frac{x^6(252b\,c^3x^4+280a\,c^3x^3+840b^2c^2x^3+945ab\,c^2x^2+945b^3c\,x^2+1080a\,b^2cx+360b^4x+420a\,b^3)}{2520}$$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x)^3,x)
```

output
```
(x**6*(420*a*b**3 + 1080*a*b**2*c*x + 945*a*b*c**2*x**2 + 280*a*c**3*x**3
+ 360*b**4*x + 945*b**3*c*x**2 + 840*b**2*c**2*x**3 + 252*b*c**3*x**4))/25
20
```

3.22 $\int x(A+Bx)\left(bx+cx^2\right)^3\,dx$

Optimal result

Integrand size = 18, antiderivative size = 75

$$\int x(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{1}{5}Ab^3x^5+\frac{1}{6}b^2(bB+3Ac)x^6+\frac{3}{7}bc(bB+Ac)x^7+\frac{1}{8}c^2(3bB+Ac)x^8+\frac{1}{9}Bc^3x^9$$

output
```
1/5*A*b^3*x^5+1/6*b^2*(3*A*c+B*b)*x^6+3/7*b*c*(A*c+B*b)*x^7+1/8*c^2*(A*c+3
*B*b)*x^8+1/9*B*c^3*x^9
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int x(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{1}{5}Ab^3x^5+\frac{1}{6}b^2(bB+3Ac)x^6+\frac{3}{7}bc(bB+Ac)x^7+\frac{1}{8}c^2(3bB+Ac)x^8+\frac{1}{9}Bc^3x^9$$

input
```
Integrate[x*(A + B*x)*(b*x + c*x^2)^3,x]
```

output
```
(A*b^3*x^5)/5 + (b^2*(b*B + 3*A*c)*x^6)/6 + (3*b*c*(b*B + A*c)*x^7)/7 + (c
^2*(3*b*B + A*c)*x^8)/8 + (B*c^3*x^9)/9
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(A+Bx)\left(bx+cx^2\right)^3\,dx$$

$$\Big\downarrow \text{9}$$

$$\int x^4(A+Bx)(b+cx)^3dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(Ab^3x^4+b^2x^5(3Ac+bB)+c^2x^7(Ac+3bB)+3bcx^6(Ac+bB)+Bc^3x^8\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{5}Ab^3x^5+\frac{1}{6}b^2x^6(3Ac+bB)+\frac{1}{8}c^2x^8(Ac+3bB)+\frac{3}{7}bcx^7(Ac+bB)+\frac{1}{9}Bc^3x^9$$

input
```
Int[x*(A + B*x)*(b*x + c*x^2)^3,x]
```

output

```
(A*b^3*x^5)/5 + (b^2*(b*B + 3*A*c)*x^6)/6 + (3*b*c*(b*B + A*c)*x^7)/7 + (c
^2*(3*b*B + A*c)*x^8)/8 + (B*c^3*x^9)/9
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

method	result	s
norman	$\frac{B\,c^{3}x^{9}}{9}+\left(\frac{1}{8}A\,c^{3}+\frac{3}{8}Bb\,c^{2}\right)x^{8}+\left(\frac{3}{7}Ab\,c^{2}+\frac{3}{7}B\,b^{2}c\right)x^{7}+\left(\frac{1}{2}A\,b^{2}c+\frac{1}{6}B\,b^{3}\right)x^{6}+\frac{A\,b^{3}x^{5}}{5}$	
gosper	$\frac{x^{5}\left(280B\,c^{3}x^{4}+315A\,c^{3}x^{3}+945x^{3}Bb\,c^{2}+1080Ab\,c^{2}x^{2}+1080x^{2}B\,b^{2}c+1260A\,b^{2}cx+420xB\,b^{3}+504A\,b^{3}\right)}{2520}$	
default	$\frac{B\,c^{3}x^{9}}{9}+\frac{\left(A\,c^{3}+3Bb\,c^{2}\right)x^{8}}{8}+\frac{\left(3Ab\,c^{2}+3B\,b^{2}c\right)x^{7}}{7}+\frac{\left(3A\,b^{2}c+B\,b^{3}\right)x^{6}}{6}+\frac{A\,b^{3}x^{5}}{5}$	
risch	$\frac{1}{9}B\,c^{3}x^{9}+\frac{1}{8}x^{8}A\,c^{3}+\frac{3}{8}x^{8}Bb\,c^{2}+\frac{3}{7}x^{7}Ab\,c^{2}+\frac{3}{7}x^{7}B\,b^{2}c+\frac{1}{2}x^{6}A\,b^{2}c+\frac{1}{6}B\,b^{3}x^{6}+\frac{1}{5}A\,b^{3}x^{5}$	
parallelrisch	$\frac{1}{9}B\,c^{3}x^{9}+\frac{1}{8}x^{8}A\,c^{3}+\frac{3}{8}x^{8}Bb\,c^{2}+\frac{3}{7}x^{7}Ab\,c^{2}+\frac{3}{7}x^{7}B\,b^{2}c+\frac{1}{2}x^{6}A\,b^{2}c+\frac{1}{6}B\,b^{3}x^{6}+\frac{1}{5}A\,b^{3}x^{5}$	
orering	$\frac{x^{2}\left(280B\,c^{3}x^{4}+315A\,c^{3}x^{3}+945x^{3}Bb\,c^{2}+1080Ab\,c^{2}x^{2}+1080x^{2}B\,b^{2}c+1260A\,b^{2}cx+420xB\,b^{3}+504A\,b^{3}\right)\left(c\,x^{2}+bx\right)^{3}}{2520\left(cx+b\right)^{3}}$	

input
```
int(x*(B*x+A)*(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/9*B*c^3*x^9+(1/8*A*c^3+3/8*B*b*c^2)*x^8+(3/7*A*b*c^2+3/7*B*b^2*c)*x^7+(1
/2*A*b^2*c+1/6*B*b^3)*x^6+1/5*A*b^3*x^5
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int x(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{1}{9}\,Bc^3x^9+\frac{1}{5}\,Ab^3x^5+\frac{1}{8}\,\left(3\,Bbc^2+Ac^3\right)x^8+\frac{3}{7}\,\left(Bb^2c+Abc^2\right)x^7+\frac{1}{6}\,\left(Bb^3+3\,Ab^2c\right)x^6$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/9*B*c^3*x^9 + 1/5*A*b^3*x^5 + 1/8*(3*B*b*c^2 + A*c^3)*x^8 + 3/7*(B*b^2*c
 + A*b*c^2)*x^7 + 1/6*(B*b^3 + 3*A*b^2*c)*x^6
```

Sympy [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int x(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{Ab^3x^5}{5}+\frac{Bc^3x^9}{9}+x^8\left(\frac{Ac^3}{8}+\frac{3Bbc^2}{8}\right)+x^7\cdot\left(\frac{3Abc^2}{7}+\frac{3Bb^2c}{7}\right)+x^6\left(\frac{Ab^2c}{2}+\frac{Bb^3}{6}\right)$$

input
```
integrate(x*(B*x+A)*(c*x**2+b*x)**3,x)
```

output
```
A*b**3*x**5/5 + B*c**3*x**9/9 + x**8*(A*c**3/8 + 3*B*b*c**2/8) + x**7*(3*A
*b*c**2/7 + 3*B*b**2*c/7) + x**6*(A*b**2*c/2 + B*b**3/6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int x(A+Bx)\left(bx+cx^2\right)^3\,dx = \frac{1}{9}\,Bc^3x^9+\frac{1}{5}\,Ab^3x^5+\frac{1}{8}\,\left(3\,Bbc^2+Ac^3\right)x^8+\frac{3}{7}\,\left(Bb^2c+Abc^2\right)x^7+\frac{1}{6}\,\left(Bb^3+3\,Ab^2c\right)x^6$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/9*B*c^3*x^9 + 1/5*A*b^3*x^5 + 1/8*(3*B*b*c^2 + A*c^3)*x^8 + 3/7*(B*b^2*c
 + A*b*c^2)*x^7 + 1/6*(B*b^3 + 3*A*b^2*c)*x^6
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int x(A+Bx)\left(bx+cx^2\right)^3\,dx = \frac{1}{9}\,Bc^3x^9+\frac{3}{8}\,Bbc^2x^8+\frac{1}{8}\,Ac^3x^8+\frac{3}{7}\,Bb^2cx^7+\frac{3}{7}\,Abc^2x^7+\frac{1}{6}\,Bb^3x^6+\frac{1}{2}\,Ab^2cx^6+\frac{1}{5}\,Ab^3x^5$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
1/9*B*c^3*x^9 + 3/8*B*b*c^2*x^8 + 1/8*A*c^3*x^8 + 3/7*B*b^2*c*x^7 + 3/7*A*
b*c^2*x^7 + 1/6*B*b^3*x^6 + 1/2*A*b^2*c*x^6 + 1/5*A*b^3*x^5
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int x(A+Bx)\left(bx+cx^2\right)^3\,dx = x^6\,\left(\frac{B\,b^3}{6}+\frac{A\,c\,b^2}{2}\right)+x^8\,\left(\frac{A\,c^3}{8}+\frac{3\,B\,b\,c^2}{8}\right)+\frac{A\,b^3\,x^5}{5}+\frac{B\,c^3\,x^9}{9}+\frac{3\,b\,c\,x^7\,\left(A\,c+B\,b\right)}{7}$$

input
```
int(x*(b*x + c*x^2)^3*(A + B*x),x)
```

output
```
x^6*((B*b^3)/6 + (A*b^2*c)/2) + x^8*((A*c^3)/8 + (3*B*b*c^2)/8) + (A*b^3*x
^5)/5 + (B*c^3*x^9)/9 + (3*b*c*x^7*(A*c + B*b))/7
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int x(A+Bx)\left(bx+cx^2\right)^3\,dx$$
$$=\frac{x^5(280b\,c^3x^4+315a\,c^3x^3+945b^2c^2x^3+1080ab\,c^2x^2+1080b^3c\,x^2+1260a\,b^2cx+420b^4x+504a\,b^3)}{2520}$$

input
```
int(x*(B*x+A)*(c*x^2+b*x)^3,x)
```

output
```
(x**5*(504*a*b**3 + 1260*a*b**2*c*x + 1080*a*b*c**2*x**2 + 315*a*c**3*x**3
 + 420*b**4*x + 1080*b**3*c*x**2 + 945*b**2*c**2*x**3 + 280*b*c**3*x**4))/
2520
```

3.23 $\int (A+Bx)\left(bx+cx^2\right)^3 dx$

Optimal result

Integrand size = 17, antiderivative size = 75

$$\int (A+Bx)\left(bx+cx^2\right)^3 dx=\frac{1}{4}Ab^3x^4+\frac{1}{5}b^2(bB+3Ac)x^5+\frac{1}{2}bc(bB+Ac)x^6+\frac{1}{7}c^2(3bB+Ac)x^7+\frac{1}{8}Bc^3x^8$$

output
```
1/4*A*b^3*x^4+1/5*b^2*(3*A*c+B*b)*x^5+1/2*b*c*(A*c+B*b)*x^6+1/7*c^2*(A*c+3
*B*b)*x^7+1/8*B*c^3*x^8
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int (A+Bx)\left(bx+cx^2\right)^3 dx=\frac{1}{4}Ab^3x^4+\frac{1}{5}b^2(bB+3Ac)x^5+\frac{1}{2}bc(bB+Ac)x^6+\frac{1}{7}c^2(3bB+Ac)x^7+\frac{1}{8}Bc^3x^8$$

input
```
Integrate[(A + B*x)*(b*x + c*x^2)^3,x]
```

output
```
(A*b^3*x^4)/4 + (b^2*(b*B + 3*A*c)*x^5)/5 + (b*c*(b*B + A*c)*x^6)/2 + (c^2
*(3*b*B + A*c)*x^7)/7 + (B*c^3*x^8)/8
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)\left(bx+cx^2\right)^3\,dx$$

$$\Big\downarrow \text{1140}$$

$$\int \left(Ab^3x^3+b^2x^4(3Ac+bB)+c^2x^6(Ac+3bB)+3bcx^5(Ac+bB)+Bc^3x^7\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}Ab^3x^4+\frac{1}{5}b^2x^5(3Ac+bB)+\frac{1}{7}c^2x^7(Ac+3bB)+\frac{1}{2}bcx^6(Ac+bB)+\frac{1}{8}Bc^3x^8$$

input
```
Int[(A + B*x)*(b*x + c*x^2)^3,x]
```

output
```
(A*b^3*x^4)/4 + (b^2*(b*B + 3*A*c)*x^5)/5 + (b*c*(b*B + A*c)*x^6)/2 + (c^2
*(3*b*B + A*c)*x^7)/7 + (B*c^3*x^8)/8
```

Defintions of rubi rules used

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

method	result
norman	$\frac{B\,c^{3} x^{8}}{8}+\left(\frac{1}{7} A\,c^{3}+\frac{3}{7} B b\,c^{2}\right) x^{7}+\left(\frac{1}{2} A b\,c^{2}+\frac{1}{2} B\,b^{2} c\right) x^{6}+\left(\frac{3}{5} A\,b^{2} c+\frac{1}{5} B\,b^{3}\right) x^{5}+\frac{A\,b^{3} x^{4}}{4}$
gosper	$\frac{x^{4} \left(35 B\,c^{3} x^{4}+40 A\,c^{3} x^{3}+120 x^{3} B b\,c^{2}+140 A b\,c^{2} x^{2}+140 x^{2} B\,b^{2} c+168 A\,b^{2} c x+56 x B\,b^{3}+70 A\,b^{3}\right)}{280}$
default	$\frac{B\,c^{3} x^{8}}{8}+\frac{\left(A\,c^{3}+3 B b\,c^{2}\right) x^{7}}{7}+\frac{\left(3 A b\,c^{2}+3 B\,b^{2} c\right) x^{6}}{6}+\frac{\left(3 A\,b^{2} c+B\,b^{3}\right) x^{5}}{5}+\frac{A\,b^{3} x^{4}}{4}$
risch	$\frac{1}{8} B\,c^{3} x^{8}+\frac{1}{7} x^{7} A\,c^{3}+\frac{3}{7} x^{7} B b\,c^{2}+\frac{1}{2} x^{6} A b\,c^{2}+\frac{1}{2} x^{6} B\,b^{2} c+\frac{3}{5} x^{5} A\,b^{2} c+\frac{1}{5} B\,b^{3} x^{5}+\frac{1}{4} A\,b^{3} x^{4}$
parallelrisch	$\frac{1}{8} B\,c^{3} x^{8}+\frac{1}{7} x^{7} A\,c^{3}+\frac{3}{7} x^{7} B b\,c^{2}+\frac{1}{2} x^{6} A b\,c^{2}+\frac{1}{2} x^{6} B\,b^{2} c+\frac{3}{5} x^{5} A\,b^{2} c+\frac{1}{5} B\,b^{3} x^{5}+\frac{1}{4} A\,b^{3} x^{4}$
orering	$\frac{x \left(35 B\,c^{3} x^{4}+40 A\,c^{3} x^{3}+120 x^{3} B b\,c^{2}+140 A b\,c^{2} x^{2}+140 x^{2} B\,b^{2} c+168 A\,b^{2} c x+56 x B\,b^{3}+70 A\,b^{3}\right) \left(c\,x^{2}+b x \right)^{3}}{280 \left(c x +b \right)^{3}}$

input
```
int((B*x+A)*(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*B*c^3*x^8+(1/7*A*c^3+3/7*B*b*c^2)*x^7+(1/2*A*b*c^2+1/2*B*b^2*c)*x^6+(3
/5*A*b^2*c+1/5*B*b^3)*x^5+1/4*A*b^3*x^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int (A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{1}{8}\,Bc^3x^8+\frac{1}{4}\,Ab^3x^4+\frac{1}{7}\,\left(3\,Bbc^2+Ac^3\right)x^7+\frac{1}{2}\,\left(Bb^2c+Abc^2\right)x^6+\frac{1}{5}\,\left(Bb^3+3\,Ab^2c\right)x^5$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/8*B*c^3*x^8 + 1/4*A*b^3*x^4 + 1/7*(3*B*b*c^2 + A*c^3)*x^7 + 1/2*(B*b^2*c
 + A*b*c^2)*x^6 + 1/5*(B*b^3 + 3*A*b^2*c)*x^5
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.07

$$\int (A+Bx)\left(bx+cx^2\right)^3\,dx = \frac{Ab^3x^4}{4}+\frac{Bc^3x^8}{8}+x^7\left(\frac{Ac^3}{7}+\frac{3Bbc^2}{7}\right)+x^6\left(\frac{Abc^2}{2}+\frac{Bb^2c}{2}\right)+x^5\cdot\left(\frac{3Ab^2c}{5}+\frac{Bb^3}{5}\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3,x)
```

output
```
A*b**3*x**4/4 + B*c**3*x**8/8 + x**7*(A*c**3/7 + 3*B*b*c**2/7) + x**6*(A*b
*c**2/2 + B*b**2*c/2) + x**5*(3*A*b**2*c/5 + B*b**3/5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int (A+Bx)\left(bx+cx^2\right)^3\,dx = \frac{1}{8}\,Bc^3x^8+\frac{1}{4}\,Ab^3x^4+\frac{1}{7}\left(3\,Bbc^2+Ac^3\right)x^7+\frac{1}{2}\left(Bb^2c+Abc^2\right)x^6+\frac{1}{5}\left(Bb^3+3\,Ab^2c\right)x^5$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/8*B*c^3*x^8 + 1/4*A*b^3*x^4 + 1/7*(3*B*b*c^2 + A*c^3)*x^7 + 1/2*(B*b^2*c
 + A*b*c^2)*x^6 + 1/5*(B*b^3 + 3*A*b^2*c)*x^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int (A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{1}{8}Bc^3x^8+\frac{3}{7}Bbc^2x^7+\frac{1}{7}Ac^3x^7+\frac{1}{2}Bb^2cx^6+\frac{1}{2}Abc^2x^6+\frac{1}{5}Bb^3x^5+\frac{3}{5}Ab^2cx^5+\frac{1}{4}Ab^3x^4$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
1/8*B*c^3*x^8 + 3/7*B*b*c^2*x^7 + 1/7*A*c^3*x^7 + 1/2*B*b^2*c*x^6 + 1/2*A*
b*c^2*x^6 + 1/5*B*b^3*x^5 + 3/5*A*b^2*c*x^5 + 1/4*A*b^3*x^4
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int (A+Bx)\left(bx+cx^2\right)^3\,dx=x^5\left(\frac{B\,b^3}{5}+\frac{3\,A\,c\,b^2}{5}\right)+x^7\left(\frac{A\,c^3}{7}+\frac{3\,B\,b\,c^2}{7}\right)+\frac{A\,b^3\,x^4}{4}+\frac{B\,c^3\,x^8}{8}+\frac{b\,c\,x^6\,(A\,c+B\,b)}{2}$$

input
```
int((b*x + c*x^2)^3*(A + B*x),x)
```

output
```
x^5*((B*b^3)/5 + (3*A*b^2*c)/5) + x^7*((A*c^3)/7 + (3*B*b*c^2)/7) + (A*b^3
*x^4)/4 + (B*c^3*x^8)/8 + (b*c*x^6*(A*c + B*b))/2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int (A+Bx)\left(bx+cx^2\right)^3\,dx = \frac{x^4(35b\,c^3x^4+40a\,c^3x^3+120b^2c^2x^3+140ab\,c^2x^2+140b^3c\,x^2+168a\,b^2cx+56b^4x+70a\,b^3)}{280}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3,x)
```

output
```
(x**4*(70*a*b**3 + 168*a*b**2*c*x + 140*a*b*c**2*x**2 + 40*a*c**3*x**3 + 5
6*b**4*x + 140*b**3*c*x**2 + 120*b**2*c**2*x**3 + 35*b*c**3*x**4))/280
```

3.24 $\int \frac{(A+Bx)(bx+cx^2)^3}{x} dx$

Optimal result

Integrand size = 20, antiderivative size = 75

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x} dx = \frac{1}{3}Ab^3x^3 + \frac{1}{4}b^2(bB+3Ac)x^4 + \frac{3}{5}bc(bB+Ac)x^5 + \frac{1}{6}c^2(3bB+Ac)x^6 + \frac{1}{7}Bc^3x^7$$

output
```
1/3*A*b^3*x^3+1/4*b^2*(3*A*c+B*b)*x^4+3/5*b*c*(A*c+B*b)*x^5+1/6*c^2*(A*c+3
*B*b)*x^6+1/7*B*c^3*x^7
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x} dx = \frac{1}{3}Ab^3x^3 + \frac{1}{4}b^2(bB+3Ac)x^4 + \frac{3}{5}bc(bB+Ac)x^5 + \frac{1}{6}c^2(3bB+Ac)x^6 + \frac{1}{7}Bc^3x^7$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x,x]
```

output
```
(A*b^3*x^3)/3 + (b^2*(b*B + 3*A*c)*x^4)/4 + (3*b*c*(b*B + A*c)*x^5)/5 + (c
^2*(3*b*B + A*c)*x^6)/6 + (B*c^3*x^7)/7
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^2(A+Bx)(b+cx)^3dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(Ab^3x^2+b^2x^3(3Ac+bB)+c^2x^5(Ac+3bB)+3bcx^4(Ac+bB)+Bc^3x^6\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{3}Ab^3x^3+\frac{1}{4}b^2x^4(3Ac+bB)+\frac{1}{6}c^2x^6(Ac+3bB)+\frac{3}{5}bcx^5(Ac+bB)+\frac{1}{7}Bc^3x^7$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x,x]
```

output

```
(A*b^3*x^3)/3 + (b^2*(b*B + 3*A*c)*x^4)/4 + (3*b*c*(b*B + A*c)*x^5)/5 + (c
^2*(3*b*B + A*c)*x^6)/6 + (B*c^3*x^7)/7
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

method	result
norman	$\frac{B\,c^3 x^7}{7}+\left(\frac{1}{6}A\,c^3+\frac{1}{2}Bb\,c^2\right)x^6+\left(\frac{3}{5}Ab\,c^2+\frac{3}{5}B\,b^2c\right)x^5+\left(\frac{3}{4}A\,b^2c+\frac{1}{4}B\,b^3\right)x^4+\frac{A\,b^3x^3}{3}$
gosper	$\frac{x^3\left(60B\,c^3x^4+70A\,c^3x^3+210x^3Bb\,c^2+252Ab\,c^2x^2+252x^2B\,b^2c+315A\,b^2cx+105xB\,b^3+140A\,b^3\right)}{420}$
default	$\frac{B\,c^3x^7}{7}+\frac{\left(A\,c^3+3Bb\,c^2\right)x^6}{6}+\frac{\left(3Ab\,c^2+3B\,b^2c\right)x^5}{5}+\frac{\left(3A\,b^2c+B\,b^3\right)x^4}{4}+\frac{A\,b^3x^3}{3}$
risch	$\frac{1}{7}B\,c^3x^7+\frac{1}{6}x^6A\,c^3+\frac{1}{2}x^6Bb\,c^2+\frac{3}{5}x^5Ab\,c^2+\frac{3}{5}x^5B\,b^2c+\frac{3}{4}x^4A\,b^2c+\frac{1}{4}B\,b^3x^4+\frac{1}{3}A\,b^3x^3$
parallelrisch	$\frac{1}{7}B\,c^3x^7+\frac{1}{6}x^6A\,c^3+\frac{1}{2}x^6Bb\,c^2+\frac{3}{5}x^5Ab\,c^2+\frac{3}{5}x^5B\,b^2c+\frac{3}{4}x^4A\,b^2c+\frac{1}{4}B\,b^3x^4+\frac{1}{3}A\,b^3x^3$
orering	$\frac{\left(60B\,c^3x^4+70A\,c^3x^3+210x^3Bb\,c^2+252Ab\,c^2x^2+252x^2B\,b^2c+315A\,b^2cx+105xB\,b^3+140A\,b^3\right)\left(c\,x^2+bx\right)^3}{420(cx+b)^3}$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/7*B*c^3*x^7+(1/6*A*c^3+1/2*B*b*c^2)*x^6+(3/5*A*b*c^2+3/5*B*b^2*c)*x^5+(3
/4*A*b^2*c+1/4*B*b^3)*x^4+1/3*A*b^3*x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x}\,dx = \frac{1}{7}\,Bc^3x^7+\frac{1}{3}\,Ab^3x^3+\frac{1}{6}\,\left(3\,Bbc^2+Ac^3\right)x^6+\frac{3}{5}\,\left(Bb^2c+Abc^2\right)x^5+\frac{1}{4}\,\left(Bb^3+3\,Ab^2c\right)x^4$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x,x, algorithm="fricas")
```

output
```
1/7*B*c^3*x^7 + 1/3*A*b^3*x^3 + 1/6*(3*B*b*c^2 + A*c^3)*x^6 + 3/5*(B*b^2*c
 + A*b*c^2)*x^5 + 1/4*(B*b^3 + 3*A*b^2*c)*x^4
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x}\,dx = \frac{Ab^3x^3}{3}+\frac{Bc^3x^7}{7}+x^6\left(\frac{Ac^3}{6}+\frac{Bbc^2}{2}\right)+x^5\cdot\left(\frac{3Abc^2}{5}+\frac{3Bb^2c}{5}\right)+x^4\cdot\left(\frac{3Ab^2c}{4}+\frac{Bb^3}{4}\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x,x)
```

output
```
A*b**3*x**3/3 + B*c**3*x**7/7 + x**6*(A*c**3/6 + B*b*c**2/2) + x**5*(3*A*b
*c**2/5 + 3*B*b**2*c/5) + x**4*(3*A*b**2*c/4 + B*b**3/4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x}\,dx = \frac{1}{7}Bc^3x^7 + \frac{1}{3}Ab^3x^3 + \frac{1}{6}\left(3\,Bbc^2 + Ac^3\right)x^6 + \frac{3}{5}\left(Bb^2c + Abc^2\right)x^5 + \frac{1}{4}\left(Bb^3 + 3\,Ab^2c\right)x^4$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x,x, algorithm="maxima")
```

output
```
1/7*B*c^3*x^7 + 1/3*A*b^3*x^3 + 1/6*(3*B*b*c^2 + A*c^3)*x^6 + 3/5*(B*b^2*c
 + A*b*c^2)*x^5 + 1/4*(B*b^3 + 3*A*b^2*c)*x^4
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x}\,dx = \frac{1}{7}Bc^3x^7 + \frac{1}{2}Bbc^2x^6 + \frac{1}{6}Ac^3x^6 + \frac{3}{5}Bb^2cx^5 + \frac{3}{5}Abc^2x^5 + \frac{1}{4}Bb^3x^4 + \frac{3}{4}Ab^2cx^4 + \frac{1}{3}Ab^3x^3$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x,x, algorithm="giac")
```

output
```
1/7*B*c^3*x^7 + 1/2*B*b*c^2*x^6 + 1/6*A*c^3*x^6 + 3/5*B*b^2*c*x^5 + 3/5*A*
b*c^2*x^5 + 1/4*B*b^3*x^4 + 3/4*A*b^2*c*x^4 + 1/3*A*b^3*x^3
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x}\,dx = x^4\left(\frac{B\,b^3}{4}+\frac{3\,A\,c\,b^2}{4}\right)+x^6\left(\frac{A\,c^3}{6}+\frac{B\,b\,c^2}{2}\right)+\frac{A\,b^3\,x^3}{3}+\frac{B\,c^3\,x^7}{7}+\frac{3\,b\,c\,x^5\,(A\,c+B\,b)}{5}$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x,x)
```

output
```
x^4*((B*b^3)/4 + (3*A*b^2*c)/4) + x^6*((A*c^3)/6 + (B*b*c^2)/2) + (A*b^3*x
^3)/3 + (B*c^3*x^7)/7 + (3*b*c*x^5*(A*c + B*b))/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x}\,dx = \frac{x^3(60b\,c^3x^4+70a\,c^3x^3+210b^2c^2x^3+252ab\,c^2x^2+252b^3c\,x^2+315a\,b^2cx+105b^4x+140a\,b^3)}{420}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x,x)
```

output
```
(x**3*(140*a*b**3 + 315*a*b**2*c*x + 252*a*b*c**2*x**2 + 70*a*c**3*x**3 +
105*b**4*x + 252*b**3*c*x**2 + 210*b**2*c**2*x**3 + 60*b*c**3*x**4))/420
```

3.25 $\int \frac{(A+Bx)(bx+cx^2)^3}{x^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{x^2}\,dx = \frac{b(bB-Ac)(b+cx)^4}{4c^3} - \frac{(2bB-Ac)(b+cx)^5}{5c^3} + \frac{B(b+cx)^6}{6c^3}$$

output
```
1/4*b*(-A*c+B*b)*(c*x+b)^4/c^3-1/5*(-A*c+2*B*b)*(c*x+b)^5/c^3+1/6*B*(c*x+b
)^6/c^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{x^2}\,dx = \frac{1}{60}x^2\big(30Ab^3+20b^2(bB+3Ac)x+45bc(bB+Ac)x^2 + 12c^2(3bB+Ac)x^3+10Bc^3x^4\big)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x^2,x]
```

output
```
(x^2*(30*A*b^3 + 20*b^2*(b*B + 3*A*c)*x + 45*b*c*(b*B + A*c)*x^2 + 12*c^2*
(3*b*B + A*c)*x^3 + 10*B*c^3*x^4))/60
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^2}\,dx$$

$$\Big\downarrow \ 9$$

$$\int x(A+Bx)(b+cx)^3dx$$

$$\Big\downarrow \ 85$$

$$\int \left(\frac{(b+cx)^4(Ac-2bB)}{c^2}+\frac{b(b+cx)^3(bB-Ac)}{c^2}+\frac{B(b+cx)^5}{c^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{(b+cx)^5(2bB-Ac)}{5c^3}+\frac{b(b+cx)^4(bB-Ac)}{4c^3}+\frac{B(b+cx)^6}{6c^3}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x^2,x]
```

output
```
(b*(b*B - A*c)*(b + c*x)^4)/(4*c^3) - ((2*b*B - A*c)*(b + c*x)^5)/(5*c^3)
+ (B*(b + c*x)^6)/(6*c^3)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

method	result	si
gosper	$\frac{x^{2}\left(10B\,c^{3}x^{4}+12A\,c^{3}x^{3}+36x^{3}Bb\,c^{2}+45Ab\,c^{2}x^{2}+45x^{2}B\,b^{2}c+60A\,b^{2}cx+20xB\,b^{3}+30A\,b^{3}\right)}{60}$	76
default	$\frac{B\,c^{3}x^{6}}{6}+\frac{\left(A\,c^{3}+3Bb\,c^{2}\right)x^{5}}{5}+\frac{\left(3Ab\,c^{2}+3B\,b^{2}c\right)x^{4}}{4}+\frac{\left(3A\,b^{2}c+B\,b^{3}\right)x^{3}}{3}+\frac{A\,b^{3}x^{2}}{2}$	76
risch	$\frac{1}{6}B\,c^{3}x^{6}+\frac{1}{5}x^{5}A\,c^{3}+\frac{3}{5}x^{5}Bb\,c^{2}+\frac{3}{4}x^{4}Ab\,c^{2}+\frac{3}{4}x^{4}B\,b^{2}c+x^{3}A\,b^{2}c+\frac{1}{3}x^{3}B\,b^{3}+\frac{1}{2}A\,b^{3}x^{2}$	77
parallelrisch	$\frac{1}{6}B\,c^{3}x^{6}+\frac{1}{5}x^{5}A\,c^{3}+\frac{3}{5}x^{5}Bb\,c^{2}+\frac{3}{4}x^{4}Ab\,c^{2}+\frac{3}{4}x^{4}B\,b^{2}c+x^{3}A\,b^{2}c+\frac{1}{3}x^{3}B\,b^{3}+\frac{1}{2}A\,b^{3}x^{2}$	77
norman	$\frac{\left(\frac{1}{5}A\,c^{3}+\frac{3}{5}Bb\,c^{2}\right)x^{6}+\left(\frac{3}{4}Ab\,c^{2}+\frac{3}{4}B\,b^{2}c\right)x^{5}+\left(A\,b^{2}c+\frac{1}{3}B\,b^{3}\right)x^{4}+\frac{A\,b^{3}x^{3}}{2}+\frac{B\,c^{3}x^{7}}{6}}{x}$	78
orering	$\frac{\left(10B\,c^{3}x^{4}+12A\,c^{3}x^{3}+36x^{3}Bb\,c^{2}+45Ab\,c^{2}x^{2}+45x^{2}B\,b^{2}c+60A\,b^{2}cx+20xB\,b^{3}+30A\,b^{3}\right)\left(c\,x^{2}+bx\right)^{3}}{60x\left(cx+b\right)^{3}}$	94

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/60*x^2*(10*B*c^3*x^4+12*A*c^3*x^3+36*B*b*c^2*x^3+45*A*b*c^2*x^2+45*B*b^2
*c*x^2+60*A*b^2*c*x+20*B*b^3*x+30*A*b^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^2}\,dx = \frac{1}{6}\,Bc^3x^6 + \frac{1}{2}\,Ab^3x^2 + \frac{1}{5}\,\left(3\,Bbc^2 + Ac^3\right)x^5 + \frac{3}{4}\,\left(Bb^2c + Abc^2\right)x^4 + \frac{1}{3}\,\left(Bb^3 + 3\,Ab^2c\right)x^3$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^2,x, algorithm="fricas")
```

output
```
1/6*B*c^3*x^6 + 1/2*A*b^3*x^2 + 1/5*(3*B*b*c^2 + A*c^3)*x^5 + 3/4*(B*b^2*c
 + A*b*c^2)*x^4 + 1/3*(B*b^3 + 3*A*b^2*c)*x^3
```

Sympy [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.29

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^2}\,dx = \frac{Ab^3x^2}{2} + \frac{Bc^3x^6}{6} + x^5\left(\frac{Ac^3}{5} + \frac{3Bbc^2}{5}\right) + x^4 \cdot \left(\frac{3Abc^2}{4} + \frac{3Bb^2c}{4}\right) + x^3\left(Ab^2c + \frac{Bb^3}{3}\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**2,x)
```

output
```
A*b**3*x**2/2 + B*c**3*x**6/6 + x**5*(A*c**3/5 + 3*B*b*c**2/5) + x**4*(3*A
*b*c**2/4 + 3*B*b**2*c/4) + x**3*(A*b**2*c + B*b**3/3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^2}\,dx = \frac{1}{6}\,Bc^3x^6 + \frac{1}{2}\,Ab^3x^2 + \frac{1}{5}\,\left(3\,Bbc^2 + Ac^3\right)x^5 + \frac{3}{4}\,\left(Bb^2c + Abc^2\right)x^4 + \frac{1}{3}\,\left(Bb^3 + 3\,Ab^2c\right)x^3$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^2,x, algorithm="maxima")
```

output
```
1/6*B*c^3*x^6 + 1/2*A*b^3*x^2 + 1/5*(3*B*b*c^2 + A*c^3)*x^5 + 3/4*(B*b^2*c
 + A*b*c^2)*x^4 + 1/3*(B*b^3 + 3*A*b^2*c)*x^3
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^2}\,dx = \frac{1}{6}\,Bc^3x^6 + \frac{3}{5}\,Bbc^2x^5 + \frac{1}{5}\,Ac^3x^5 + \frac{3}{4}\,Bb^2cx^4 + \frac{3}{4}\,Abc^2x^4 + \frac{1}{3}\,Bb^3x^3 + Ab^2cx^3 + \frac{1}{2}\,Ab^3x^2$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^2,x, algorithm="giac")
```

output
```
1/6*B*c^3*x^6 + 3/5*B*b*c^2*x^5 + 1/5*A*c^3*x^5 + 3/4*B*b^2*c*x^4 + 3/4*A*
b*c^2*x^4 + 1/3*B*b^3*x^3 + A*b^2*c*x^3 + 1/2*A*b^3*x^2
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^2}\,dx = x^3\left(\frac{B\,b^3}{3}+A\,c\,b^2\right)+x^5\left(\frac{A\,c^3}{5}+\frac{3\,B\,b\,c^2}{5}\right) + \frac{A\,b^3\,x^2}{2}+\frac{B\,c^3\,x^6}{6}+\frac{3\,b\,c\,x^4\,(A\,c+B\,b)}{4}$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^2,x)
```

output
```
x^3*((B*b^3)/3 + A*b^2*c) + x^5*((A*c^3)/5 + (3*B*b*c^2)/5) + (A*b^3*x^2)/
2 + (B*c^3*x^6)/6 + (3*b*c*x^4*(A*c + B*b))/4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^2}\,dx = \frac{x^2(10b\,c^3x^4+12a\,c^3x^3+36b^2c^2x^3+45ab\,c^2x^2+45b^3c\,x^2+60a\,b^2cx+20b^4x+30a\,b^3)}{60}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^2,x)
```

output
```
(x**2*(30*a*b**3 + 60*a*b**2*c*x + 45*a*b*c**2*x**2 + 12*a*c**3*x**3 + 20*
b**4*x + 45*b**3*c*x**2 + 36*b**2*c**2*x**3 + 10*b*c**3*x**4))/60
```

3.26 $\int \frac{(A+Bx)(bx+cx^2)^3}{x^3} dx$

Optimal result

Integrand size = 20, antiderivative size = 38

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^3} dx = -\frac{(bB-Ac)(b+cx)^4}{4c^2}+\frac{B(b+cx)^5}{5c^2}$$

output
```
-1/4*(-A*c+B*b)*(c*x+b)^4/c^2+1/5*B*(c*x+b)^5/c^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.76

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^3} dx = Ab^3x+\frac{1}{2}b^2(bB+3Ac)x^2+bc(bB+Ac)x^3 + \frac{1}{4}c^2(3bB+Ac)x^4+\frac{1}{5}Bc^3x^5$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x^3,x]
```

output
```
A*b^3*x + (b^2*(b*B + 3*A*c)*x^2)/2 + b*c*(b*B + A*c)*x^3 + (c^2*(3*b*B +
A*c)*x^4)/4 + (B*c^3*x^5)/5
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int (A+Bx)(b+cx)^3dx$$

$$\Big\downarrow \text{ 49}$$

$$\int \left(\frac{(b+cx)^3(Ac-bB)}{c}+\frac{B(b+cx)^4}{c}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{B(b+cx)^5}{5c^2}-\frac{(b+cx)^4(bB-Ac)}{4c^2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x^3,x]
```

output
```
-1/4*((b*B - A*c)*(b + c*x)^4)/c^2 + (B*(b + c*x)^5)/(5*c^2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(34) = 68$.

Time = 0.60 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

method	result	size
default	$\frac{B\,c^3 x^5}{5}+\frac{(A\,c^3+3Bb\,c^2)x^4}{4}+\frac{(3Ab\,c^2+3B\,b^2c)x^3}{3}+\frac{(3A\,b^2c+B\,b^3)x^2}{2}+A\,b^3x$	73
risch	$\frac{1}{5}B\,c^3x^5+\frac{1}{4}x^4A\,c^3+\frac{3}{4}x^4Bb\,c^2+Ab\,c^2x^3+B\,b^2c\,x^3+\frac{3}{2}x^2A\,b^2c+\frac{1}{2}x^2B\,b^3+A\,b^3x$	73
parallelrisch	$\frac{1}{5}B\,c^3x^5+\frac{1}{4}x^4A\,c^3+\frac{3}{4}x^4Bb\,c^2+Ab\,c^2x^3+B\,b^2c\,x^3+\frac{3}{2}x^2A\,b^2c+\frac{1}{2}x^2B\,b^3+A\,b^3x$	73
gosper	$\frac{x(4B\,c^3x^4+5A\,c^3x^3+15x^3Bb\,c^2+20Ab\,c^2x^2+20x^2B\,b^2c+30A\,b^2cx+10xB\,b^3+20A\,b^3)}{20}$	74
norman	$\frac{(\frac{1}{4}A\,c^3+\frac{3}{4}Bb\,c^2)x^6+(\frac{3}{2}A\,b^2c+\frac{1}{2}B\,b^3)x^4+(Ab\,c^2+B\,b^2c)x^5+A\,b^3x^3+\frac{B\,c^3x^7}{5}}{x^2}$	76
orering	$\frac{(4B\,c^3x^4+5A\,c^3x^3+15x^3Bb\,c^2+20Ab\,c^2x^2+20x^2B\,b^2c+30A\,b^2cx+10xB\,b^3+20A\,b^3)\,(c\,x^2+bx)^3}{20x^2(cx+b)^3}$	94

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/5*B*c^3*x^5+1/4*(A*c^3+3*B*b*c^2)*x^4+1/3*(3*A*b*c^2+3*B*b^2*c)*x^3+1/2*
(3*A*b^2*c+B*b^3)*x^2+A*b^3*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^3}\,dx = \frac{1}{5}\,Bc^3x^5 + Ab^3x + \frac{1}{4}\,\left(3\,Bbc^2 + Ac^3\right)x^4 + \left(Bb^2c + Abc^2\right)x^3 + \frac{1}{2}\,\left(Bb^3 + 3\,Ab^2c\right)x^2$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^3,x, algorithm="fricas")
```

output
```
1/5*B*c^3*x^5 + A*b^3*x + 1/4*(3*B*b*c^2 + A*c^3)*x^4 + (B*b^2*c + A*b*c^2
)*x^3 + 1/2*(B*b^3 + 3*A*b^2*c)*x^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(32) = 64$.

Time = 0.02 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^3}\,dx = Ab^3x + \frac{Bc^3x^5}{5} + x^4\left(\frac{Ac^3}{4} + \frac{3Bbc^2}{4}\right) + x^3\left(Abc^2 + Bb^2c\right) + x^2\cdot\left(\frac{3Ab^2c}{2} + \frac{Bb^3}{2}\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**3,x)
```

output
```
A*b**3*x + B*c**3*x**5/5 + x**4*(A*c**3/4 + 3*B*b*c**2/4) + x**3*(A*b*c**2
 + B*b**2*c) + x**2*(3*A*b**2*c/2 + B*b**3/2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(34) = 68$.

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.82

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^3}\,dx = \frac{1}{5}\,Bc^3x^5 + Ab^3x + \frac{1}{4}\left(3\,Bbc^2 + Ac^3\right)x^4 + \left(Bb^2c + Abc^2\right)x^3 + \frac{1}{2}\left(Bb^3 + 3\,Ab^2c\right)x^2$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^3,x, algorithm="maxima")
```

output
```
1/5*B*c^3*x^5 + A*b^3*x + 1/4*(3*B*b*c^2 + A*c^3)*x^4 + (B*b^2*c + A*b*c^2
)*x^3 + 1/2*(B*b^3 + 3*A*b^2*c)*x^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(34) = 68$.

Time = 0.12 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^3}\,dx = \frac{1}{5}\,Bc^3x^5 + \frac{3}{4}\,Bbc^2x^4 + \frac{1}{4}\,Ac^3x^4 + Bb^2cx^3 + Abc^2x^3 + \frac{1}{2}\,Bb^3x^2 + \frac{3}{2}\,Ab^2cx^2 + Ab^3x$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^3,x, algorithm="giac")
```

output
```
1/5*B*c^3*x^5 + 3/4*B*b*c^2*x^4 + 1/4*A*c^3*x^4 + B*b^2*c*x^3 + A*b*c^2*x^
3 + 1/2*B*b^3*x^2 + 3/2*A*b^2*c*x^2 + A*b^3*x
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.71

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^3}\,dx = x^2\left(\frac{B\,b^3}{2}+\frac{3\,A\,c\,b^2}{2}\right)+x^4\left(\frac{A\,c^3}{4}+\frac{3\,B\,b\,c^2}{4}\right)+\frac{B\,c^3\,x^5}{5}+A\,b^3\,x+b\,c\,x^3\,(A\,c+B\,b)$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^3,x)
```

output
```
x^2*((B*b^3)/2 + (3*A*b^2*c)/2) + x^4*((A*c^3)/4 + (3*B*b*c^2)/4) + (B*c^3
*x^5)/5 + A*b^3*x + b*c*x^3*(A*c + B*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.89

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^3}\,dx = \frac{x(4b\,c^3x^4+5a\,c^3x^3+15b^2c^2x^3+20ab\,c^2x^2+20b^3c\,x^2+30a\,b^2cx+10b^4x+20a\,b^3)}{20}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^3,x)
```

output
```
(x*(20*a*b**3 + 30*a*b**2*c*x + 20*a*b*c**2*x**2 + 5*a*c**3*x**3 + 10*b**4
*x + 20*b**3*c*x**2 + 15*b**2*c**2*x**3 + 4*b*c**3*x**4))/20
```

3.27 $\int \frac{(A+Bx)(bx+cx^2)^3}{x^4}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 54

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^4}\,dx = 3Ab^2cx+\frac{3}{2}Abc^2x^2+\frac{1}{3}Ac^3x^3+\frac{B(b+cx)^4}{4c}+Ab^3\log(x)$$

output
```
3*A*b^2*c*x+3/2*A*b*c^2*x^2+1/3*A*c^3*x^3+1/4*B*(c*x+b)^4/c+A*b^3*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^4}\,dx = \frac{1}{12}x\left(12b^3B+18b^2c(2A+Bx)+6bc^2x(3A+2Bx)+c^3x^2(4A+3Bx)\right)+Ab^3\log(x)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x^4,x]
```

output
```
(x*(12*b^3*B + 18*b^2*c*(2*A + B*x) + 6*b*c^2*x*(3*A + 2*B*x) + c^3*x^2*(4
*A + 3*B*x)))/12 + A*b^3*Log[x]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 90, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^4}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx)(b+cx)^3}{x}dx$$

$$\Big\downarrow \text{ 90}$$

$$A\int \frac{(b+cx)^3}{x}dx+\frac{B(b+cx)^4}{4c}$$

$$\Big\downarrow \text{ 49}$$

$$A\int \left(\frac{b^3}{x}+3cb^2+3c^2xb+c^3x^2\right)dx+\frac{B(b+cx)^4}{4c}$$

$$\Big\downarrow \text{ 2009}$$

$$A\left(b^3\log(x)+3b^2cx+\frac{3}{2}bc^2x^2+\frac{c^3x^3}{3}\right)+\frac{B(b+cx)^4}{4c}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x^4,x]
```

output
```
(B*(b + c*x)^4)/(4*c) + A*(3*b^2*c*x + (3*b*c^2*x^2)/2 + (c^3*x^3)/3 + b^3
*Log[x])
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

method	result	size
default	$\frac{B\,c^{3} x^{4}}{4}+\frac{A\,c^{3} x^{3}}{3}+x^{3} B b\,c^{2}+\frac{3 A b\,c^{2} x^{2}}{2}+\frac{3 x^{2} B\,b^{2} c}{2}+3 A\,b^{2} c x+x B\,b^{3}+A\,b^{3} \ln \left(x \right)$	70
risch	$\frac{B\,c^{3} x^{4}}{4}+\frac{A\,c^{3} x^{3}}{3}+x^{3} B b\,c^{2}+\frac{3 A b\,c^{2} x^{2}}{2}+\frac{3 x^{2} B\,b^{2} c}{2}+3 A\,b^{2} c x+x B\,b^{3}+A\,b^{3} \ln \left(x \right)$	70
parallelrisch	$\frac{B\,c^{3} x^{4}}{4}+\frac{A\,c^{3} x^{3}}{3}+x^{3} B b\,c^{2}+\frac{3 A b\,c^{2} x^{2}}{2}+\frac{3 x^{2} B\,b^{2} c}{2}+3 A\,b^{2} c x+x B\,b^{3}+A\,b^{3} \ln \left(x \right)$	70
norman	$\frac{\left(\frac{1}{3} A\,c^{3}+B b\,c^{2}\right) x^{6}+\left(\frac{3}{2} A b\,c^{2}+\frac{3}{2} B\,b^{2} c \right) x^{5}+\left(3 A\,b^{2} c+B\,b^{3}\right) x^{4}+\frac{B\,c^{3} x^{7}}{4}}{x^{3}}+A\,b^{3} \ln \left(x \right)$	76

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*B*c^3*x^4+1/3*A*c^3*x^3+x^3*B*b*c^2+3/2*A*b*c^2*x^2+3/2*x^2*B*b^2*c+3*
A*b^2*c*x+x*B*b^3+A*b^3*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^4}\,dx = \frac{1}{4}\,Bc^3x^4 + Ab^3\log\left(x\right) + \frac{1}{3}\left(3\,Bbc^2 + Ac^3\right)x^3 + \frac{3}{2}\left(Bb^2c + Abc^2\right)x^2 + \left(Bb^3 + 3\,Ab^2c\right)x$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^4,x, algorithm="fricas")
```

output
```
1/4*B*c^3*x^4 + A*b^3*log(x) + 1/3*(3*B*b*c^2 + A*c^3)*x^3 + 3/2*(B*b^2*c
+ A*b*c^2)*x^2 + (B*b^3 + 3*A*b^2*c)*x
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.35

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^4}\,dx = Ab^3\log\left(x\right) + \frac{Bc^3x^4}{4} + x^3\left(\frac{Ac^3}{3} + Bbc^2\right) + x^2\cdot\left(\frac{3Abc^2}{2} + \frac{3Bb^2c}{2}\right) + x\left(3Ab^2c + Bb^3\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**4,x)
```

output
```
A*b**3*log(x) + B*c**3*x**4/4 + x**3*(A*c**3/3 + B*b*c**2) + x**2*(3*A*b*c
**2/2 + 3*B*b**2*c/2) + x*(3*A*b**2*c + B*b**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^4}\,dx = \frac{1}{4}\,Bc^3x^4 + Ab^3\log\left(x\right) + \frac{1}{3}\left(3\,Bbc^2 + Ac^3\right)x^3 + \frac{3}{2}\left(Bb^2c + Abc^2\right)x^2 + \left(Bb^3 + 3\,Ab^2c\right)x$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^4,x, algorithm="maxima")
```

output
```
1/4*B*c^3*x^4 + A*b^3*log(x) + 1/3*(3*B*b*c^2 + A*c^3)*x^3 + 3/2*(B*b^2*c
+ A*b*c^2)*x^2 + (B*b^3 + 3*A*b^2*c)*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^4}\,dx = \frac{1}{4}\,Bc^3x^4 + Bbc^2x^3 + \frac{1}{3}\,Ac^3x^3 + \frac{3}{2}\,Bb^2cx^2 + \frac{3}{2}\,Abc^2x^2 + Bb^3x + 3\,Ab^2cx + Ab^3\log\left(|x|\right)$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^4,x, algorithm="giac")
```

output
```
1/4*B*c^3*x^4 + B*b*c^2*x^3 + 1/3*A*c^3*x^3 + 3/2*B*b^2*c*x^2 + 3/2*A*b*c^
2*x^2 + B*b^3*x + 3*A*b^2*c*x + A*b^3*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^4}\,dx = x\left(B\,b^3+3\,A\,c\,b^2\right)+x^3\left(\frac{A\,c^3}{3}+B\,b\,c^2\right)+\frac{B\,c^3\,x^4}{4}+A\,b^3\,\ln\left(x\right)+\frac{3\,b\,c\,x^2\,\left(A\,c+B\,b\right)}{2}$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^4,x)
```

output
```
x*(B*b^3 + 3*A*b^2*c) + x^3*((A*c^3)/3 + B*b*c^2) + (B*c^3*x^4)/4 + A*b^3*
log(x) + (3*b*c*x^2*(A*c + B*b))/2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^4}\,dx = \log(x)\,a\,b^3+3a\,b^2cx+\frac{3ab\,c^2x^2}{2}+\frac{a\,c^3x^3}{3}+b^4x+\frac{3b^3c\,x^2}{2}+b^2c^2x^3+\frac{b\,c^3x^4}{4}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^4,x)
```

output
```
(12*log(x)*a*b**3 + 36*a*b**2*c*x + 18*a*b*c**2*x**2 + 4*a*c**3*x**3 + 12*
b**4*x + 18*b**3*c*x**2 + 12*b**2*c**2*x**3 + 3*b*c**3*x**4)/12
```

3.28 $\int \frac{(A+Bx)(bx+cx^2)^3}{x^5} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^5} \, dx = -\frac{Ab^3}{x} + 3bc(bB+Ac)x + \frac{1}{2}c^2(3bB+Ac)x^2 + \frac{1}{3}Bc^3x^3 + b^2(bB+3Ac)\log(x)$$

output
```
-A*b^3/x+3*b*c*(A*c+B*b)*x+1/2*c^2*(A*c+3*B*b)*x^2+1/3*B*c^3*x^3+b^2*(3*A*
c+B*b)*ln(x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^5} \, dx = -\frac{Ab^3}{x} + 3bc(bB+Ac)x + \frac{1}{2}c^2(3bB+Ac)x^2 + \frac{1}{3}Bc^3x^3 + (b^3B+3Ab^2c)\log(x)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x^5,x]
```

output
```
-((A*b^3)/x) + 3*b*c*(b*B + A*c)*x + (c^2*(3*b*B + A*c)*x^2)/2 + (B*c^3*x^
3)/3 + (b^3*B + 3*A*b^2*c)*Log[x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^5}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx)(b+cx)^3}{x^2}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{Ab^3}{x^2}+\frac{b^2(3Ac+bB)}{x}+c^2x(Ac+3bB)+3bc(Ac+bB)+Bc^3x^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{Ab^3}{x}+b^2\log(x)(3Ac+bB)+\frac{1}{2}c^2x^2(Ac+3bB)+3bcx(Ac+bB)+\frac{1}{3}Bc^3x^3$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x^5,x]
```

output
```
-((A*b^3)/x) + 3*b*c*(b*B + A*c)*x + (c^2*(3*b*B + A*c)*x^2)/2 + (B*c^3*x^
3)/3 + b^2*(b*B + 3*A*c)*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

method	result	size
default	$\frac{B\,c^3x^3}{3}+\frac{A\,c^3x^2}{2}+\frac{3Bb\,c^2x^2}{2}+3Ab\,c^2x+3B\,b^2cx+b^2(3Ac+Bb)\ln\left(x\right)-\frac{A\,b^3}{x}$	69
risch	$\frac{B\,c^3x^3}{3}+\frac{A\,c^3x^2}{2}+\frac{3Bb\,c^2x^2}{2}+3Ab\,c^2x+3B\,b^2cx-\frac{A\,b^3}{x}+3A\ln\left(x\right)b^2c+B\ln\left(x\right)b^3$	71
norman	$\frac{\left(\frac{1}{2}A\,c^3+\frac{3}{2}Bb\,c^2\right)x^6+\left(3Ab\,c^2+3B\,b^2c\right)x^5-A\,b^3x^3+\frac{B\,c^3x^7}{3}}{x^4}+\left(3A\,b^2c+B\,b^3\right)\ln\left(x\right)$	78
parallelrisch	$\frac{2B\,c^3x^4+3A\,c^3x^3+9x^3Bb\,c^2+18A\ln(x)x\,b^2c+18Ab\,c^2x^2+6B\ln(x)x\,b^3+18x^2B\,b^2c-6A\,b^3}{6x}$	80

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/3*B*c^3*x^3+1/2*A*c^3*x^2+3/2*B*b*c^2*x^2+3*A*b*c^2*x+3*B*b^2*c*x+b^2*(3
*A*c+B*b)*ln(x)-A*b^3/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^5}\,dx = \frac{2\,Bc^3x^4 - 6\,Ab^3 + 3\,(3\,Bbc^2 + Ac^3)x^3 + 18\,(Bb^2c + Abc^2)x^2 + 6\,(Bb^3 + 3\,Ab^2c)x\log(x)}{6\,x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^5,x, algorithm="fricas")
```

output
```
1/6*(2*B*c^3*x^4 - 6*A*b^3 + 3*(3*B*b*c^2 + A*c^3)*x^3 + 18*(B*b^2*c + A*b
*c^2)*x^2 + 6*(B*b^3 + 3*A*b^2*c)*x*log(x))/x
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^5}\,dx = -\frac{Ab^3}{x} + \frac{Bc^3x^3}{3} + b^2\cdot(3Ac+Bb)\log(x) + x^2\left(\frac{Ac^3}{2} + \frac{3Bbc^2}{2}\right) + x\left(3Abc^2 + 3Bb^2c\right)$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**5,x)
```

output
```
-A*b**3/x + B*c**3*x**3/3 + b**2*(3*A*c + B*b)*log(x) + x**2*(A*c**3/2 + 3
*B*b*c**2/2) + x*(3*A*b*c**2 + 3*B*b**2*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{x^5}\,dx = \frac{1}{3}\,Bc^3x^3 - \frac{Ab^3}{x} + \frac{1}{2}\,(3\,Bbc^2 + Ac^3)x^2 + 3\,(Bb^2c + Abc^2)x + (Bb^3 + 3\,Ab^2c)\log(x)$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^5,x, algorithm="maxima")
```

output
```
1/3*B*c^3*x^3 - A*b^3/x + 1/2*(3*B*b*c^2 + A*c^3)*x^2 + 3*(B*b^2*c + A*b*c
^2)*x + (B*b^3 + 3*A*b^2*c)*log(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{x^5}\,dx = \frac{1}{3}\,Bc^3x^3 + \frac{3}{2}\,Bbc^2x^2 + \frac{1}{2}\,Ac^3x^2 + 3\,Bb^2cx + 3\,Abc^2x - \frac{Ab^3}{x} + (Bb^3 + 3\,Ab^2c)\log(|x|)$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^5,x, algorithm="giac")
```

output
```
1/3*B*c^3*x^3 + 3/2*B*b*c^2*x^2 + 1/2*A*c^3*x^2 + 3*B*b^2*c*x + 3*A*b*c^2*
x - A*b^3/x + (B*b^3 + 3*A*b^2*c)*log(abs(x))
```

Mupad [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{x^5}\,dx = x^2\left(\frac{A\,c^3}{2} + \frac{3\,B\,b\,c^2}{2}\right) + \ln(x)\,\left(B\,b^3 + 3\,A\,c\,b^2\right) - \frac{A\,b^3}{x} + \frac{B\,c^3\,x^3}{3} + 3\,b\,c\,x\,(A\,c + B\,b)$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^5,x)
```

output
```
x^2*((A*c^3)/2 + (3*B*b*c^2)/2) + log(x)*(B*b^3 + 3*A*b^2*c) - (A*b^3)/x +
 (B*c^3*x^3)/3 + 3*b*c*x*(A*c + B*b)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{x^5}\,dx = \frac{18\log(x)\,a\,b^2cx+6\log(x)\,b^4x-6a\,b^3+18ab\,c^2x^2+3a\,c^3x^3+18b^3c\,x^2+9b^2c^2x^3+2b\,c^3x^4}{6x}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^5,x)
```

output
```
(18*log(x)*a*b**2*c*x + 6*log(x)*b**4*x - 6*a*b**3 + 18*a*b*c**2*x**2 + 3*
a*c**3*x**3 + 18*b**3*c*x**2 + 9*b**2*c**2*x**3 + 2*b*c**3*x**4)/(6*x)
```

3.29 $\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^6}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 65

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^6}\,dx = -\frac{Ab^3}{2x^2}-\frac{b^2(bB+3Ac)}{x}+c^2(3bB+Ac)x+\frac{1}{2}Bc^3x^2+3bc(bB+Ac)\log(x)$$

output
```
-1/2*A*b^3/x^2-b^2*(3*A*c+B*b)/x+c^2*(A*c+3*B*b)*x+1/2*B*c^3*x^2+3*b*c*(A*
c+B*b)*ln(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^6}\,dx = -\frac{Ab^3}{2x^2}+\frac{-b^3B-3Ab^2c}{x}+c^2(3bB+Ac)x+\frac{1}{2}Bc^3x^2+3\left(b^2Bc+Abc^2\right)\log(x)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x^6,x]
```

output
```
-1/2*(A*b^3)/x^2 + (-(b^3*B) - 3*A*b^2*c)/x + c^2*(3*b*B + A*c)*x + (B*c^3
*x^2)/2 + 3*(b^2*B*c + A*b*c^2)*Log[x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^6}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{(A+Bx)(b+cx)^3}{x^3}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{Ab^3}{x^3}+\frac{b^2(3Ac+bB)}{x^2}+c^2(Ac+3bB)+\frac{3bc(Ac+bB)}{x}+Bc^3x\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{Ab^3}{2x^2}-\frac{b^2(3Ac+bB)}{x}+c^2x(Ac+3bB)+3bc\log(x)(Ac+bB)+\frac{1}{2}Bc^3x^2$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x^6,x]
```

output
```
-1/2*(A*b^3)/x^2 - (b^2*(b*B + 3*A*c))/x + c^2*(3*b*B + A*c)*x + (B*c^3*x^
2)/2 + 3*b*c*(b*B + A*c)*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{B\,c^{3}x^{2}}{2}+A\,c^{3}x+3Bb\,c^{2}x-\frac{A\,b^{3}}{2x^{2}}+3bc(Ac+Bb)\ln\left(x\right)-\frac{b^{2}(3Ac+Bb)}{x}$	63
risch	$\frac{B\,c^{3}x^{2}}{2}+A\,c^{3}x+3Bb\,c^{2}x+\frac{(-3A\,b^{2}c-B\,b^{3})x-\frac{A\,b^{3}}{2}}{x^{2}}+3A\ln\left(x\right)b\,c^{2}+3B\ln\left(x\right)b^{2}c$	70
norman	$\frac{(A\,c^{3}+3Bb\,c^{2})x^{6}+(-3A\,b^{2}c-B\,b^{3})x^{4}-\frac{A\,b^{3}x^{3}}{2}+\frac{B\,c^{3}x^{7}}{2}}{x^{5}}+(3Ab\,c^{2}+3B\,b^{2}c)\ln\left(x\right)$	78
parallelrisch	$\frac{B\,c^{3}x^{4}+6A\ln(x)x^{2}b\,c^{2}+2A\,c^{3}x^{3}+6B\ln(x)x^{2}b^{2}c+6x^{3}Bb\,c^{2}-6A\,b^{2}cx-2xB\,b^{3}-A\,b^{3}}{2x^{2}}$	79

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^6,x,method=_RETURNVERBOSE)
```

output
```
1/2*B*c^3*x^2+A*c^3*x+3*B*b*c^2*x-1/2*A*b^3/x^2+3*b*c*(A*c+B*b)*ln(x)-b^2*
(3*A*c+B*b)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{x^6}\,dx = \frac{Bc^3x^4 - Ab^3 + 2\,(3\,Bbc^2 + Ac^3)x^3 + 6\,(Bb^2c + Abc^2)x^2\log{(x)} - 2\,(Bb^3 + 3\,Ab^2c)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^6,x, algorithm="fricas")
```

output
```
1/2*(B*c^3*x^4 - A*b^3 + 2*(3*B*b*c^2 + A*c^3)*x^3 + 6*(B*b^2*c + A*b*c^2)
**x^2*log(x) - 2*(B*b^3 + 3*A*b^2*c)*x)/x^2
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{x^6}\,dx = \frac{Bc^3x^2}{2} + 3bc(Ac+Bb)\log{(x)} + x\left(Ac^3 + 3Bbc^2\right) + \frac{-Ab^3 + x(-6Ab^2c - 2Bb^3)}{2x^2}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**6,x)
```

output
```
B*c**3*x**2/2 + 3*b*c*(A*c + B*b)*log(x) + x*(A*c**3 + 3*B*b*c**2) + (-A*b
**3 + x*(-6*A*b**2*c - 2*B*b**3))/(2*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^6}\,dx = \frac{1}{2}\,Bc^3x^2 + \left(3\,Bbc^2 + Ac^3\right)x + 3\left(Bb^2c + Abc^2\right)\log\left(x\right) - \frac{Ab^3 + 2\,(Bb^3 + 3\,Ab^2c)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^6,x, algorithm="maxima")
```

output
```
1/2*B*c^3*x^2 + (3*B*b*c^2 + A*c^3)*x + 3*(B*b^2*c + A*b*c^2)*log(x) - 1/2
*(A*b^3 + 2*(B*b^3 + 3*A*b^2*c)*x)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^6}\,dx = \frac{1}{2}\,Bc^3x^2 + 3\,Bbc^2x + Ac^3x + 3\left(Bb^2c + Abc^2\right)\log\left(|x|\right) - \frac{Ab^3 + 2\,(Bb^3 + 3\,Ab^2c)x}{2\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^6,x, algorithm="giac")
```

output
```
1/2*B*c^3*x^2 + 3*B*b*c^2*x + A*c^3*x + 3*(B*b^2*c + A*b*c^2)*log(abs(x))
- 1/2*(A*b^3 + 2*(B*b^3 + 3*A*b^2*c)*x)/x^2
```

Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^6}\,dx = \ln(x)\,\left(3\,B\,b^2\,c+3\,A\,b\,c^2\right)-\frac{x\,(B\,b^3+3\,A\,c\,b^2)+\frac{A\,b^3}{2}}{x^2}+x\,\left(A\,c^3+3\,B\,b\,c^2\right)+\frac{B\,c^3\,x^2}{2}$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^6,x)
```

output
```
log(x)*(3*A*b*c^2 + 3*B*b^2*c) - (x*(B*b^3 + 3*A*b^2*c) + (A*b^3)/2)/x^2 +
 x*(A*c^3 + 3*B*b*c^2) + (B*c^3*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^6}\,dx = \frac{6\log(x)\,ab\,c^2x^2+6\log(x)\,b^3c\,x^2-a\,b^3-6a\,b^2cx+2a\,c^3x^3-2b^4x+6b^2c^2x^3+b\,c^3x^4}{2x^2}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^6,x)
```

output
```
(6*log(x)*a*b*c**2*x**2 + 6*log(x)*b**3*c*x**2 - a*b**3 - 6*a*b**2*c*x + 2
*a*c**3*x**3 - 2*b**4*x + 6*b**2*c**2*x**3 + b*c**3*x**4)/(2*x**2)
```

3.30 $\int \frac{(A+Bx)(bx+cx^2)^3}{x^7} dx$

Optimal result

Integrand size = 20, antiderivative size = 64

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{x^7}\,dx = -\frac{Ab^3}{3x^3} - \frac{b^2(bB+3Ac)}{2x^2} - \frac{3bc(bB+Ac)}{x} + Bc^3x + c^2(3bB+Ac)\log(x)$$

output
```
-1/3*A*b^3/x^3-1/2*b^2*(3*A*c+B*b)/x^2-3*b*c*(A*c+B*b)/x+B*c^3*x+c^2*(A*c+
3*B*b)*ln(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{x^7}\,dx = -\frac{Ab^3}{3x^3} + \frac{-b^3B-3Ab^2c}{2x^2} - \frac{3(b^2Bc+Abc^2)}{x} + Bc^3x + \left(3bBc^2+Ac^3\right)\log(x)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x^7,x]
```

output
```
-1/3*(A*b^3)/x^3 + (-(b^3*B) - 3*A*b^2*c)/(2*x^2) - (3*(b^2*B*c + A*b*c^2)
)/x + B*c^3*x + (3*b*B*c^2 + A*c^3)*Log[x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^7}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx)(b+cx)^3}{x^4}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{Ab^3}{x^4}+\frac{b^2(3Ac+bB)}{x^3}+\frac{c^2(Ac+3bB)}{x}+\frac{3bc(Ac+bB)}{x^2}+Bc^3\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{Ab^3}{3x^3}-\frac{b^2(3Ac+bB)}{2x^2}+c^2\log(x)(Ac+3bB)-\frac{3bc(Ac+bB)}{x}+Bc^3x$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x^7,x]
```

output
```
-1/3*(A*b^3)/x^3 - (b^2*(b*B + 3*A*c))/(2*x^2) - (3*b*c*(b*B + A*c))/x + B
*c^3*x + c^2*(3*b*B + A*c)*Log[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.61 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{A\,b^{3}}{3x^{3}}-\frac{b^{2}(3Ac+Bb)}{2x^{2}}-\frac{3bc(Ac+Bb)}{x}+B\,c^{3}x+c^{2}(Ac+3Bb)\ln\left(x\right)$	61
risch	$B\,c^{3}x+\frac{(-3Ab\,c^{2}-3B\,b^{2}c)x^{2}+(-\frac{3}{2}A\,b^{2}c-\frac{1}{2}B\,b^{3})x-\frac{A\,b^{3}}{3}}{x^{3}}+A\ln\left(x\right)c^{3}+3B\ln\left(x\right)b\,c^{2}$	70
norman	$\frac{(-\frac{3}{2}A\,b^{2}c-\frac{1}{2}B\,b^{3})x^{4}+(-3Ab\,c^{2}-3B\,b^{2}c)x^{5}+B\,c^{3}x^{7}-\frac{A\,b^{3}x^{3}}{3}}{x^{6}}+\left(A\,c^{3}+3Bb\,c^{2}\right)\ln\left(x\right)$	77
parallelrisch	$\frac{6A\ln(x)x^{3}c^{3}+18B\ln(x)x^{3}b\,c^{2}+6B\,c^{3}x^{4}-18Ab\,c^{2}x^{2}-18x^{2}B\,b^{2}c-9A\,b^{2}cx-3xB\,b^{3}-2A\,b^{3}}{6x^{3}}$	80

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/3*A*b^3/x^3-1/2*b^2*(3*A*c+B*b)/x^2-3*b*c*(A*c+B*b)/x+B*c^3*x+c^2*(A*c+
3*B*b)*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.17

$$\int\frac{(A+Bx)\left(bx+cx^{2}\right)^{3}}{x^{7}}\,dx$$
$$=\frac{6\,Bc^{3}x^{4}+6\,(3\,Bbc^{2}+Ac^{3})x^{3}\log\left(x\right)-2\,Ab^{3}-18\,(Bb^{2}c+Abc^{2})x^{2}-3\,(Bb^{3}+3\,Ab^{2}c)x}{6\,x^{3}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^7,x, algorithm="fricas")
```

output
```
1/6*(6*B*c^3*x^4 + 6*(3*B*b*c^2 + A*c^3)*x^3*log(x) - 2*A*b^3 - 18*(B*b^2*
c + A*b*c^2)*x^2 - 3*(B*b^3 + 3*A*b^2*c)*x)/x^3
```

Sympy [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^7}\,dx = Bc^3x + c^2(Ac+3Bb)\log(x) + \frac{-2Ab^3 + x^2(-18Abc^2 - 18Bb^2c) + x(-9Ab^2c - 3Bb^3)}{6x^3}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**7,x)
```

output
```
B*c**3*x + c**2*(A*c + 3*B*b)*log(x) + (-2*A*b**3 + x**2*(-18*A*b*c**2 - 1
8*B*b**2*c) + x*(-9*A*b**2*c - 3*B*b**3))/(6*x**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.08

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^7}\,dx = Bc^3x + \left(3\,Bbc^2 + Ac^3\right)\log(x) - \frac{2\,Ab^3 + 18\,(Bb^2c + Abc^2)x^2 + 3\,(Bb^3 + 3\,Ab^2c)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^7,x, algorithm="maxima")
```

output
```
B*c^3*x + (3*B*b*c^2 + A*c^3)*log(x) - 1/6*(2*A*b^3 + 18*(B*b^2*c + A*b*c^
2)*x^2 + 3*(B*b^3 + 3*A*b^2*c)*x)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^7}\,dx = Bc^3x+\left(3\,Bbc^2+Ac^3\right)\log\left(|x|\right) - \frac{2\,Ab^3+18\,(Bb^2c+Abc^2)x^2+3\,(Bb^3+3\,Ab^2c)x}{6\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^7,x, algorithm="giac")
```

output
```
B*c^3*x + (3*B*b*c^2 + A*c^3)*log(abs(x)) - 1/6*(2*A*b^3 + 18*(B*b^2*c + A
*b*c^2)*x^2 + 3*(B*b^3 + 3*A*b^2*c)*x)/x^3
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^7}\,dx = \ln\left(x\right)\,\left(A\,c^3+3\,B\,b\,c^2\right) - \frac{x^2\,\left(3\,B\,b^2\,c+3\,A\,b\,c^2\right)+x\,\left(\frac{B\,b^3}{2}+\frac{3\,A\,c\,b^2}{2}\right)+\frac{A\,b^3}{3}}{x^3} + B\,c^3\,x$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^7,x)
```

output
```
log(x)*(A*c^3 + 3*B*b*c^2) - (x^2*(3*A*b*c^2 + 3*B*b^2*c) + x*((B*b^3)/2 +
 (3*A*b^2*c)/2) + (A*b^3)/3)/x^3 + B*c^3*x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{x^7}\,dx$$
$$= \frac{6\log(x)\,a\,c^3x^3+18\log(x)\,b^2c^2x^3-2a\,b^3-9a\,b^2cx-18ab\,c^2x^2-3b^4x-18b^3c\,x^2+6b\,c^3x^4}{6x^3}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^7,x)
```

output
```
(6*log(x)*a*c**3*x**3 + 18*log(x)*b**2*c**2*x**3 - 2*a*b**3 - 9*a*b**2*c*x
 - 18*a*b*c**2*x**2 - 3*b**4*x - 18*b**3*c*x**2 + 6*b*c**3*x**4)/(6*x**3)
```

3.31 $\int \frac{(A+Bx)(bx+cx^2)^3}{x^8} dx$

Optimal result

Integrand size = 20, antiderivative size = 59

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^8} dx = -\frac{b^3B}{3x^3} - \frac{3b^2Bc}{2x^2} - \frac{3bBc^2}{x} - \frac{A(b+cx)^4}{4bx^4} + Bc^3\log(x)$$

output
```
-1/3*b^3*B/x^3-3/2*b^2*B*c/x^2-3*b*B*c^2/x-1/4*A*(c*x+b)^4/b/x^4+B*c^3*ln(
x)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^8} dx$$
$$= -\frac{2bBx(2b^2+9bcx+18c^2x^2)+3A(b^3+4b^2cx+6bc^2x^2+4c^3x^3)}{12x^4} + Bc^3\log(x)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x^8,x]
```

output
```
-1/12*(2*b*B*x*(2*b^2 + 9*b*c*x + 18*c^2*x^2) + 3*A*(b^3 + 4*b^2*c*x + 6*b
*c^2*x^2 + 4*c^3*x^3))/x^4 + B*c^3*Log[x]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {9, 87, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^8}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{(A+Bx)(b+cx)^3}{x^5}dx$$

$$\Big\downarrow \ 87$$

$$B\int \frac{(b+cx)^3}{x^4}dx-\frac{A(b+cx)^4}{4bx^4}$$

$$\Big\downarrow \ 49$$

$$B\int \left(\frac{b^3}{x^4}+\frac{3cb^2}{x^3}+\frac{3c^2b}{x^2}+\frac{c^3}{x}\right)dx-\frac{A(b+cx)^4}{4bx^4}$$

$$\Big\downarrow \ 2009$$

$$B\left(-\frac{b^3}{3x^3}-\frac{3b^2c}{2x^2}-\frac{3bc^2}{x}+c^3\log(x)\right)-\frac{A(b+cx)^4}{4bx^4}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x^8,x]
```

output
```
-1/4*(A*(b + c*x)^4)/(b*x^4) + B*(-1/3*b^3/x^3 - (3*b^2*c)/(2*x^2) - (3*b*
c^2)/x + c^3*Log[x])
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.72 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

method	result	size
default	$-\frac{b^{2}(3Ac+Bb)}{3x^{3}}-\frac{3bc(Ac+Bb)}{2x^{2}}-\frac{A\,b^{3}}{4x^{4}}+B\,c^{3}\ln\left(x\right)-\frac{c^{2}(Ac+3Bb)}{x}$	64
risch	$\frac{\left(-A\,c^{3}-3Bb\,c^{2}\right)x^{3}+\left(-\frac{3}{2}Ab\,c^{2}-\frac{3}{2}B\,b^{2}c\right)x^{2}+\left(-A\,b^{2}c-\frac{1}{3}B\,b^{3}\right)x-\frac{A\,b^{3}}{4}}{x^{4}}+B\,c^{3}\ln\left(x\right)$	73
norman	$\frac{\left(-\frac{3}{2}Ab\,c^{2}-\frac{3}{2}B\,b^{2}c\right)x^{5}+\left(-A\,b^{2}c-\frac{1}{3}B\,b^{3}\right)x^{4}+\left(-A\,c^{3}-3Bb\,c^{2}\right)x^{6}-\frac{A\,b^{3}x^{3}}{4}}{x^{7}}+B\,c^{3}\ln\left(x\right)$	78
parallelrisch	$-\frac{-12B\,c^{3}\ln(x)x^{4}+12A\,c^{3}x^{3}+36x^{3}Bb\,c^{2}+18Ab\,c^{2}x^{2}+18x^{2}B\,b^{2}c+12A\,b^{2}cx+4xB\,b^{3}+3A\,b^{3}}{12x^{4}}$	78

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/3*b^2*(3*A*c+B*b)/x^3-3/2*b*c*(A*c+B*b)/x^2-1/4*A*b^3/x^4+B*c^3*ln(x)-c
^2*(A*c+3*B*b)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^8}\,dx = \frac{12\,Bc^3x^4\log\left(x\right)-3\,Ab^3-12\,(3\,Bbc^2+Ac^3)x^3-18\,(Bb^2c+Abc^2)x^2-4\,(Bb^3+3\,Ab^2c)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^8,x, algorithm="fricas")
```

output
```
1/12*(12*B*c^3*x^4*log(x) - 3*A*b^3 - 12*(3*B*b*c^2 + A*c^3)*x^3 - 18*(B*b
^2*c + A*b*c^2)*x^2 - 4*(B*b^3 + 3*A*b^2*c)*x)/x^4
```

Sympy [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^8}\,dx = Bc^3\log\left(x\right) + \frac{-3Ab^3+x^3(-12Ac^3-36Bbc^2)+x^2(-18Abc^2-18Bb^2c)+x(-12Ab^2c-4Bb^3)}{12x^4}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**8,x)
```

output
```
B*c**3*log(x) + (-3*A*b**3 + x**3*(-12*A*c**3 - 36*B*b*c**2) + x**2*(-18*A
*b*c**2 - 18*B*b**2*c) + x*(-12*A*b**2*c - 4*B*b**3))/(12*x**4)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^8}\,dx$$
$$= Bc^3\log(x) - \frac{3\,Ab^3+12\,(3\,Bbc^2+Ac^3)x^3+18\,(Bb^2c+Abc^2)x^2+4\,(Bb^3+3\,Ab^2c)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^8,x, algorithm="maxima")
```

output
```
B*c^3*log(x) - 1/12*(3*A*b^3 + 12*(3*B*b*c^2 + A*c^3)*x^3 + 18*(B*b^2*c +
A*b*c^2)*x^2 + 4*(B*b^3 + 3*A*b^2*c)*x)/x^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^8}\,dx$$
$$= Bc^3\log(|x|) - \frac{3\,Ab^3+12\,(3\,Bbc^2+Ac^3)x^3+18\,(Bb^2c+Abc^2)x^2+4\,(Bb^3+3\,Ab^2c)x}{12\,x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^8,x, algorithm="giac")
```

output
```
B*c^3*log(abs(x)) - 1/12*(3*A*b^3 + 12*(3*B*b*c^2 + A*c^3)*x^3 + 18*(B*b^2
*c + A*b*c^2)*x^2 + 4*(B*b^3 + 3*A*b^2*c)*x)/x^4
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^8}\,dx$$
$$= B\,c^3\,\ln(x) - \frac{x^2\left(\frac{3\,B\,b^2\,c}{2}+\frac{3\,A\,b\,c^2}{2}\right)+x\left(\frac{B\,b^3}{3}+A\,c\,b^2\right)+\frac{A\,b^3}{4}+x^3\left(A\,c^3+3\,B\,b\,c^2\right)}{x^4}$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^8,x)
```

output
```
B*c^3*log(x) - (x^2*((3*A*b*c^2)/2 + (3*B*b^2*c)/2) + x*((B*b^3)/3 + A*b^2
*c) + (A*b^3)/4 + x^3*(A*c^3 + 3*B*b*c^2))/x^4
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.29

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^8}\,dx$$
$$= \frac{12\log(x)\,b\,c^3x^4 - 3a\,b^3 - 12a\,b^2cx - 18ab\,c^2x^2 - 12a\,c^3x^3 - 4b^4x - 18b^3c\,x^2 - 36b^2c^2x^3}{12x^4}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^8,x)
```

output
```
(12*log(x)*b*c**3*x**4 - 3*a*b**3 - 12*a*b**2*c*x - 18*a*b*c**2*x**2 - 12*
a*c**3*x**3 - 4*b**4*x - 18*b**3*c*x**2 - 36*b**2*c**2*x**3)/(12*x**4)
```

3.32 $\int \frac{(A+Bx)(bx+cx^2)^3}{x^9} dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^9} dx = -\frac{A(b+cx)^4}{5bx^5} - \frac{(5bB-Ac)(b+cx)^4}{20b^2x^4}$$

output
```
-1/5*A*(c*x+b)^4/b/x^5-1/20*(-A*c+5*B*b)*(c*x+b)^4/b^2/x^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^9} dx$$
$$= -\frac{5Bx(b^3+4b^2cx+6bc^2x^2+4c^3x^3)+A(4b^3+15b^2cx+20bc^2x^2+10c^3x^3)}{20x^5}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x^9,x]
```

output
```
-1/20*(5*B*x*(b^3 + 4*b^2*c*x + 6*b*c^2*x^2 + 4*c^3*x^3) + A*(4*b^3 + 15*b
^2*c*x + 20*b*c^2*x^2 + 10*c^3*x^3))/x^5
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 87, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^9}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(A+Bx)(b+cx)^3}{x^6}dx$$

$$\Big\downarrow \; 87$$

$$\frac{(5bB-Ac)\int \frac{(b+cx)^3}{x^5}dx}{5b}-\frac{A(b+cx)^4}{5bx^5}$$

$$\Big\downarrow \; 48$$

$$-\frac{(b+cx)^4(5bB-Ac)}{20b^2x^4}-\frac{A(b+cx)^4}{5bx^5}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x^9,x]
```

output
```
-1/5*(A*(b + c*x)^4)/(b*x^5) - ((5*b*B - A*c)*(b + c*x)^4)/(20*b^2*x^4)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [A] (warning: unable to verify)

Time = 0.66 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

method	result	size
default	$-\frac{A\,b^{3}}{5x^{5}}-\frac{bc(Ac+Bb)}{x^{3}}-\frac{c^{2}(Ac+3Bb)}{2x^{2}}-\frac{b^{2}(3Ac+Bb)}{4x^{4}}-\frac{B\,c^{3}}{x}$	66
risch	$\frac{-B\,c^{3}x^{4}+\left(-\frac{1}{2}A\,c^{3}-\frac{3}{2}Bb\,c^{2}\right)x^{3}+\left(-Ab\,c^{2}-B\,b^{2}c\right)x^{2}+\left(-\frac{3}{4}A\,b^{2}c-\frac{1}{4}B\,b^{3}\right)x-\frac{A\,b^{3}}{5}}{x^{5}}$	74
gosper	$-\frac{20B\,c^{3}x^{4}+10A\,c^{3}x^{3}+30x^{3}Bb\,c^{2}+20Ab\,c^{2}x^{2}+20x^{2}B\,b^{2}c+15A\,b^{2}cx+5xB\,b^{3}+4A\,b^{3}}{20x^{5}}$	76
parallelrisch	$-\frac{20B\,c^{3}x^{4}+10A\,c^{3}x^{3}+30x^{3}Bb\,c^{2}+20Ab\,c^{2}x^{2}+20x^{2}B\,b^{2}c+15A\,b^{2}cx+5xB\,b^{3}+4A\,b^{3}}{20x^{5}}$	76
norman	$\frac{\left(-\frac{1}{2}A\,c^{3}-\frac{3}{2}Bb\,c^{2}\right)x^{6}+\left(-\frac{3}{4}A\,b^{2}c-\frac{1}{4}B\,b^{3}\right)x^{4}+\left(-Ab\,c^{2}-B\,b^{2}c\right)x^{5}-\frac{A\,b^{3}x^{3}}{5}-B\,c^{3}x^{7}}{x^{8}}$	79
orering	$-\frac{\left(20B\,c^{3}x^{4}+10A\,c^{3}x^{3}+30x^{3}Bb\,c^{2}+20Ab\,c^{2}x^{2}+20x^{2}B\,b^{2}c+15A\,b^{2}cx+5xB\,b^{3}+4A\,b^{3}\right)\left(c\,x^{2}+bx\right)^{3}}{20x^{8}\left(cx+b\right)^{3}}$	94

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/5*A*b^3/x^5-b*c*(A*c+B*b)/x^3-1/2*c^2*(A*c+3*B*b)/x^2-1/4*b^2*(3*A*c+B*
b)/x^4-B*c^3/x
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^9}\,dx = -\frac{20\,Bc^3x^4+4\,Ab^3+10\,(3\,Bbc^2+Ac^3)x^3+20\,(Bb^2c+Abc^2)x^2+5\,(Bb^3+3\,Ab^2c)x}{20\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^9,x, algorithm="fricas")
```

output
```
-1/20*(20*B*c^3*x^4 + 4*A*b^3 + 10*(3*B*b*c^2 + A*c^3)*x^3 + 20*(B*b^2*c +
 A*b*c^2)*x^2 + 5*(B*b^3 + 3*A*b^2*c)*x)/x^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(39) = 78.

Time = 0.67 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.82

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^9}\,dx = \frac{-4Ab^3-20Bc^3x^4+x^3(-10Ac^3-30Bbc^2)+x^2(-20Abc^2-20Bb^2c)+x(-15Ab^2c-5Bb^3)}{20x^5}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**9,x)
```

output
```
(-4*A*b**3 - 20*B*c**3*x**4 + x**3*(-10*A*c**3 - 30*B*b*c**2) + x**2*(-20*
A*b*c**2 - 20*B*b**2*c) + x*(-15*A*b**2*c - 5*B*b**3))/(20*x**5)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.62

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^9}\,dx = -\frac{20\,Bc^3x^4+4\,Ab^3+10\,(3\,Bbc^2+Ac^3)x^3+20\,(Bb^2c+Abc^2)x^2+5\,(Bb^3+3\,Ab^2c)x}{20\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^9,x, algorithm="maxima")
```

output
```
-1/20*(20*B*c^3*x^4 + 4*A*b^3 + 10*(3*B*b*c^2 + A*c^3)*x^3 + 20*(B*b^2*c +
 A*b*c^2)*x^2 + 5*(B*b^3 + 3*A*b^2*c)*x)/x^5
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.67

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^9}\,dx = -\frac{20\,Bc^3x^4+30\,Bbc^2x^3+10\,Ac^3x^3+20\,Bb^2cx^2+20\,Abc^2x^2+5\,Bb^3x+15\,Ab^2cx+4\,Ab^3}{20\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^9,x, algorithm="giac")
```

output
```
-1/20*(20*B*c^3*x^4 + 30*B*b*c^2*x^3 + 10*A*c^3*x^3 + 20*B*b^2*c*x^2 + 20*
A*b*c^2*x^2 + 5*B*b^3*x + 15*A*b^2*c*x + 4*A*b^3)/x^5
```

Mupad [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{x^9}\,dx = -\frac{x^2\,(B\,b^2\,c+A\,b\,c^2)+x\,\left(\frac{B\,b^3}{4}+\frac{3\,A\,c\,b^2}{4}\right)+\frac{A\,b^3}{5}+x^3\,\left(\frac{A\,c^3}{2}+\frac{3\,B\,b\,c^2}{2}\right)+B\,c^3\,x^4}{x^5}$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^9,x)
```

output
```
-(x^2*(A*b*c^2 + B*b^2*c) + x*((B*b^3)/4 + (3*A*b^2*c)/4) + (A*b^3)/5 + x^
3*((A*c^3)/2 + (3*B*b*c^2)/2) + B*c^3*x^4)/x^5
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.64

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{x^9}\,dx = \frac{-20b\,c^3x^4-10a\,c^3x^3-30b^2c^2x^3-20ab\,c^2x^2-20b^3c\,x^2-15a\,b^2cx-5b^4x-4a\,b^3}{20x^5}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^9,x)
```

output
```
( - 4*a*b**3 - 15*a*b**2*c*x - 20*a*b*c**2*x**2 - 10*a*c**3*x**3 - 5*b**4*
x - 20*b**3*c*x**2 - 30*b**2*c**2*x**3 - 20*b*c**3*x**4)/(20*x**5)
```

3.33 $\int \frac{(A+Bx)(bx+cx^2)^3}{x^{10}} dx$

Optimal result

Integrand size = 20, antiderivative size = 75

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^{10}} dx = -\frac{Ab^3}{6x^6} - \frac{b^2(bB+3Ac)}{5x^5} - \frac{3bc(bB+Ac)}{4x^4} - \frac{c^2(3bB+Ac)}{3x^3} - \frac{Bc^3}{2x^2}$$

output
```
-1/6*A*b^3/x^6-1/5*b^2*(3*A*c+B*b)/x^5-3/4*b*c*(A*c+B*b)/x^4-1/3*c^2*(A*c+
3*B*b)/x^3-1/2*B*c^3/x^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^{10}} dx = -\frac{3Bx(4b^3+15b^2cx+20bc^2x^2+10c^3x^3)+A(10b^3+36b^2cx+45bc^2x^2+20c^3x^3)}{60x^6}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x^10,x]
```

output
```
-1/60*(3*B*x*(4*b^3 + 15*b^2*c*x + 20*b*c^2*x^2 + 10*c^3*x^3) + A*(10*b^3
+ 36*b^2*c*x + 45*b*c^2*x^2 + 20*c^3*x^3))/x^6
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{10}}\,dx$$

$$\Big\downarrow 9$$

$$\int \frac{(A+Bx)(b+cx)^3}{x^7}dx$$

$$\Big\downarrow 85$$

$$\int \left(\frac{Ab^3}{x^7}+\frac{b^2(3Ac+bB)}{x^6}+\frac{c^2(Ac+3bB)}{x^4}+\frac{3bc(Ac+bB)}{x^5}+\frac{Bc^3}{x^3}\right)dx$$

$$\Big\downarrow 2009$$

$$-\frac{Ab^3}{6x^6}-\frac{b^2(3Ac+bB)}{5x^5}-\frac{c^2(Ac+3bB)}{3x^3}-\frac{3bc(Ac+bB)}{4x^4}-\frac{Bc^3}{2x^2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x^10,x]
```

output
```
-1/6*(A*b^3)/x^6 - (b^2*(b*B + 3*A*c))/(5*x^5) - (3*b*c*(b*B + A*c))/(4*x^
4) - (c^2*(3*b*B + A*c))/(3*x^3) - (B*c^3)/(2*x^2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{A\,b^{3}}{6x^{6}}-\frac{b^{2}(3Ac+Bb)}{5x^{5}}-\frac{3bc(Ac+Bb)}{4x^{4}}-\frac{c^{2}(Ac+3Bb)}{3x^{3}}-\frac{B\,c^{3}}{2x^{2}}$	66
risch	$\frac{-\frac{B\,c^{3}x^{4}}{2}+\left(-\frac{1}{3}A\,c^{3}-Bb\,c^{2}\right)x^{3}+\left(-\frac{3}{4}Ab\,c^{2}-\frac{3}{4}B\,b^{2}c\right)x^{2}+\left(-\frac{3}{5}A\,b^{2}c-\frac{1}{5}B\,b^{3}\right)x-\frac{A\,b^{3}}{6}}{x^{6}}$	74
gosper	$-\frac{30B\,c^{3}x^{4}+20A\,c^{3}x^{3}+60x^{3}Bb\,c^{2}+45Ab\,c^{2}x^{2}+45x^{2}B\,b^{2}c+36A\,b^{2}cx+12xB\,b^{3}+10A\,b^{3}}{60x^{6}}$	76
parallelrisch	$-\frac{30B\,c^{3}x^{4}+20A\,c^{3}x^{3}+60x^{3}Bb\,c^{2}+45Ab\,c^{2}x^{2}+45x^{2}B\,b^{2}c+36A\,b^{2}cx+12xB\,b^{3}+10A\,b^{3}}{60x^{6}}$	76
norman	$\frac{\left(-\frac{1}{3}A\,c^{3}-Bb\,c^{2}\right)x^{6}+\left(-\frac{3}{4}Ab\,c^{2}-\frac{3}{4}B\,b^{2}c\right)x^{5}+\left(-\frac{3}{5}A\,b^{2}c-\frac{1}{5}B\,b^{3}\right)x^{4}-\frac{A\,b^{3}x^{3}}{6}-\frac{B\,c^{3}x^{7}}{2}}{x^{9}}$	79
orering	$-\frac{\left(30B\,c^{3}x^{4}+20A\,c^{3}x^{3}+60x^{3}Bb\,c^{2}+45Ab\,c^{2}x^{2}+45x^{2}B\,b^{2}c+36A\,b^{2}cx+12xB\,b^{3}+10A\,b^{3}\right)\left(c\,x^{2}+bx\right)^{3}}{60x^{9}(cx+b)^{3}}$	94

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/6*A*b^3/x^6-1/5*b^2*(3*A*c+B*b)/x^5-3/4*b*c*(A*c+B*b)/x^4-1/3*c^2*(A*c+
3*B*b)/x^3-1/2*B*c^3/x^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{10}}\,dx = -\frac{30\,Bc^3x^4+10\,Ab^3+20\,(3\,Bbc^2+Ac^3)x^3+45\,(Bb^2c+Abc^2)x^2+12\,(Bb^3+3\,Ab^2c)x}{60\,x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^10,x, algorithm="fricas")
```

output
```
-1/60*(30*B*c^3*x^4 + 10*A*b^3 + 20*(3*B*b*c^2 + A*c^3)*x^3 + 45*(B*b^2*c
+ A*b*c^2)*x^2 + 12*(B*b^3 + 3*A*b^2*c)*x)/x^6
```

Sympy [A] (verification not implemented)

Time = 0.82 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{10}}\,dx = \frac{-10Ab^3-30Bc^3x^4+x^3(-20Ac^3-60Bbc^2)+x^2(-45Abc^2-45Bb^2c)+x(-36Ab^2c-12Bb^3)}{60x^6}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**10,x)
```

output
```
(-10*A*b**3 - 30*B*c**3*x**4 + x**3*(-20*A*c**3 - 60*B*b*c**2) + x**2*(-45
*A*b*c**2 - 45*B*b**2*c) + x*(-36*A*b**2*c - 12*B*b**3))/(60*x**6)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{10}}\,dx = -\frac{30\,Bc^3x^4+10\,Ab^3+20\,(3\,Bbc^2+Ac^3)x^3+45\,(Bb^2c+Abc^2)x^2+12\,(Bb^3+3\,Ab^2c)x}{60\,x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^10,x, algorithm="maxima")
```

output
```
-1/60*(30*B*c^3*x^4 + 10*A*b^3 + 20*(3*B*b*c^2 + A*c^3)*x^3 + 45*(B*b^2*c
+ A*b*c^2)*x^2 + 12*(B*b^3 + 3*A*b^2*c)*x)/x^6
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{10}}\,dx = -\frac{30\,Bc^3x^4+60\,Bbc^2x^3+20\,Ac^3x^3+45\,Bb^2cx^2+45\,Abc^2x^2+12\,Bb^3x+36\,Ab^2cx+10\,Ab^3}{60\,x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^10,x, algorithm="giac")
```

output
```
-1/60*(30*B*c^3*x^4 + 60*B*b*c^2*x^3 + 20*A*c^3*x^3 + 45*B*b^2*c*x^2 + 45*
A*b*c^2*x^2 + 12*B*b^3*x + 36*A*b^2*c*x + 10*A*b^3)/x^6
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{10}}\,dx = -\frac{x^2\left(\frac{3\,B\,b^2\,c}{4}+\frac{3\,A\,b\,c^2}{4}\right)+x\left(\frac{B\,b^3}{5}+\frac{3\,A\,c\,b^2}{5}\right)+\frac{A\,b^3}{6}+x^3\left(\frac{A\,c^3}{3}+B\,b\,c^2\right)+\frac{B\,c^3\,x^4}{2}}{x^6}$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^10,x)
```

output
```
-(x^2*((3*A*b*c^2)/4 + (3*B*b^2*c)/4) + x*((B*b^3)/5 + (3*A*b^2*c)/5) + (A
*b^3)/6 + x^3*((A*c^3)/3 + B*b*c^2) + (B*c^3*x^4)/2)/x^6
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{10}}\,dx = \frac{-30b\,c^3x^4-20a\,c^3x^3-60b^2c^2x^3-45ab\,c^2x^2-45b^3c\,x^2-36a\,b^2cx-12b^4x-10a\,b^3}{60x^6}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^10,x)
```

output
```
( - 10*a*b**3 - 36*a*b**2*c*x - 45*a*b*c**2*x**2 - 20*a*c**3*x**3 - 12*b**
4*x - 45*b**3*c*x**2 - 60*b**2*c**2*x**3 - 30*b*c**3*x**4)/(60*x**6)
```

3.34 $\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{11}} dx$

Optimal result

Integrand size = 20, antiderivative size = 75

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{11}} dx = -\frac{Ab^3}{7x^7} - \frac{b^2(bB+3Ac)}{6x^6} - \frac{3bc(bB+Ac)}{5x^5} - \frac{c^2(3bB+Ac)}{4x^4} - \frac{Bc^3}{3x^3}$$

output
```
-1/7*A*b^3/x^7-1/6*b^2*(3*A*c+B*b)/x^6-3/5*b*c*(A*c+B*b)/x^5-1/4*c^2*(A*c+
3*B*b)/x^4-1/3*B*c^3/x^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{11}} dx = -\frac{7Bx(10b^3+36b^2cx+45bc^2x^2+20c^3x^3)+3A(20b^3+70b^2cx+84bc^2x^2+35c^3x^3)}{420x^7}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x^11,x]
```

output
```
-1/420*(7*B*x*(10*b^3 + 36*b^2*c*x + 45*b*c^2*x^2 + 20*c^3*x^3) + 3*A*(20*
b^3 + 70*b^2*c*x + 84*b*c^2*x^2 + 35*c^3*x^3))/x^7
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{11}}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{(A+Bx)(b+cx)^3}{x^8}dx$$

$$\Big\downarrow \ 85$$

$$\int \left(\frac{Ab^3}{x^8}+\frac{b^2(3Ac+bB)}{x^7}+\frac{c^2(Ac+3bB)}{x^5}+\frac{3bc(Ac+bB)}{x^6}+\frac{Bc^3}{x^4}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{Ab^3}{7x^7}-\frac{b^2(3Ac+bB)}{6x^6}-\frac{c^2(Ac+3bB)}{4x^4}-\frac{3bc(Ac+bB)}{5x^5}-\frac{Bc^3}{3x^3}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x^11,x]
```

output
```
-1/7*(A*b^3)/x^7 - (b^2*(b*B + 3*A*c))/(6*x^6) - (3*b*c*(b*B + A*c))/(5*x^
5) - (c^2*(3*b*B + A*c))/(4*x^4) - (B*c^3)/(3*x^3)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

method	result	size
default	$-\frac{A\,b^{3}}{7x^{7}}-\frac{b^{2}(3Ac+Bb)}{6x^{6}}-\frac{3bc(Ac+Bb)}{5x^{5}}-\frac{c^{2}(Ac+3Bb)}{4x^{4}}-\frac{B\,c^{3}}{3x^{3}}$	66
risch	$\frac{-\frac{B\,c^{3}x^{4}}{3}+(-\frac{1}{4}A\,c^{3}-\frac{3}{4}Bb\,c^{2})x^{3}+(-\frac{3}{5}Ab\,c^{2}-\frac{3}{5}B\,b^{2}c)x^{2}+(-\frac{1}{2}A\,b^{2}c-\frac{1}{6}B\,b^{3})x-\frac{A\,b^{3}}{7}}{x^{7}}$	74
gosper	$-\frac{140B\,c^{3}x^{4}+105A\,c^{3}x^{3}+315x^{3}Bb\,c^{2}+252Ab\,c^{2}x^{2}+252x^{2}B\,b^{2}c+210A\,b^{2}cx+70xB\,b^{3}+60A\,b^{3}}{420x^{7}}$	76
parallelrisch	$-\frac{140B\,c^{3}x^{4}+105A\,c^{3}x^{3}+315x^{3}Bb\,c^{2}+252Ab\,c^{2}x^{2}+252x^{2}B\,b^{2}c+210A\,b^{2}cx+70xB\,b^{3}+60A\,b^{3}}{420x^{7}}$	76
norman	$\frac{(-\frac{1}{4}A\,c^{3}-\frac{3}{4}Bb\,c^{2})x^{6}+(-\frac{3}{5}Ab\,c^{2}-\frac{3}{5}B\,b^{2}c)x^{5}+(-\frac{1}{2}A\,b^{2}c-\frac{1}{6}B\,b^{3})x^{4}-\frac{A\,b^{3}x^{3}}{7}-\frac{B\,c^{3}x^{7}}{3}}{x^{10}}$	79
orering	$-\frac{(140B\,c^{3}x^{4}+105A\,c^{3}x^{3}+315x^{3}Bb\,c^{2}+252Ab\,c^{2}x^{2}+252x^{2}B\,b^{2}c+210A\,b^{2}cx+70xB\,b^{3}+60A\,b^{3})\,(c\,x^{2}+bx)^{3}}{420x^{10}(cx+b)^{3}}$	94

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/7*A*b^3/x^7-1/6*b^2*(3*A*c+B*b)/x^6-3/5*b*c*(A*c+B*b)/x^5-1/4*c^2*(A*c+
3*B*b)/x^4-1/3*B*c^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{11}}\,dx = -\frac{140\,Bc^3x^4+60\,Ab^3+105\,(3\,Bbc^2+Ac^3)x^3+252\,(Bb^2c+Abc^2)x^2+70\,(Bb^3+3\,Ab^2c)x}{420\,x^7}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^11,x, algorithm="fricas")
```

output
```
-1/420*(140*B*c^3*x^4 + 60*A*b^3 + 105*(3*B*b*c^2 + A*c^3)*x^3 + 252*(B*b^
2*c + A*b*c^2)*x^2 + 70*(B*b^3 + 3*A*b^2*c)*x)/x^7
```

Sympy [A] (verification not implemented)

Time = 1.02 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{11}}\,dx = \frac{-60Ab^3-140Bc^3x^4+x^3(-105Ac^3-315Bbc^2)+x^2(-252Abc^2-252Bb^2c)+x(-210Ab^2c-70Bb^3)}{420x^7}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**11,x)
```

output
```
(-60*A*b**3 - 140*B*c**3*x**4 + x**3*(-105*A*c**3 - 315*B*b*c**2) + x**2*(
-252*A*b*c**2 - 252*B*b**2*c) + x*(-210*A*b**2*c - 70*B*b**3))/(420*x**7)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{11}}\,dx = -\frac{140\,Bc^3x^4+60\,Ab^3+105\,(3\,Bbc^2+Ac^3)x^3+252\,(Bb^2c+Abc^2)x^2+70\,(Bb^3+3\,Ab^2c)x}{420\,x^7}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^11,x, algorithm="maxima")
```

output
```
-1/420*(140*B*c^3*x^4 + 60*A*b^3 + 105*(3*B*b*c^2 + A*c^3)*x^3 + 252*(B*b^
2*c + A*b*c^2)*x^2 + 70*(B*b^3 + 3*A*b^2*c)*x)/x^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{11}}\,dx = -\frac{140\,Bc^3x^4+315\,Bbc^2x^3+105\,Ac^3x^3+252\,Bb^2cx^2+252\,Abc^2x^2+70\,Bb^3x+210\,Ab^2cx+60\,Ab^3}{420\,x^7}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^11,x, algorithm="giac")
```

output
```
-1/420*(140*B*c^3*x^4 + 315*B*b*c^2*x^3 + 105*A*c^3*x^3 + 252*B*b^2*c*x^2
+ 252*A*b*c^2*x^2 + 70*B*b^3*x + 210*A*b^2*c*x + 60*A*b^3)/x^7
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{11}}\,dx = -\frac{x^2\left(\frac{3\,B\,b^2\,c}{5}+\frac{3\,A\,b\,c^2}{5}\right)+x\left(\frac{B\,b^3}{6}+\frac{A\,c\,b^2}{2}\right)+\frac{A\,b^3}{7}+x^3\left(\frac{A\,c^3}{4}+\frac{3\,B\,b\,c^2}{4}\right)+\frac{B\,c^3\,x^4}{3}}{x^7}$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^11,x)
```

output
```
-(x^2*((3*A*b*c^2)/5 + (3*B*b^2*c)/5) + x*((B*b^3)/6 + (A*b^2*c)/2) + (A*b
^3)/7 + x^3*((A*c^3)/4 + (3*B*b*c^2)/4) + (B*c^3*x^4)/3)/x^7
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{11}}\,dx = \frac{-140b\,c^3x^4-105a\,c^3x^3-315b^2c^2x^3-252ab\,c^2x^2-252b^3c\,x^2-210a\,b^2cx-70b^4x-60a\,b^3}{420x^7}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^11,x)
```

output
```
( - 60*a*b**3 - 210*a*b**2*c*x - 252*a*b*c**2*x**2 - 105*a*c**3*x**3 - 70*
b**4*x - 252*b**3*c*x**2 - 315*b**2*c**2*x**3 - 140*b*c**3*x**4)/(420*x**7
)
```

3.35 $\int \frac{x^4(d+ex)}{bx+cx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 87

$$\int \frac{x^4(d+ex)}{bx+cx^2}\,dx=\frac{b^2(cd-be)x}{c^4}-\frac{b(cd-be)x^2}{2c^3}+\frac{(cd-be)x^3}{3c^2}+\frac{ex^4}{4c}-\frac{b^3(cd-be)\log(b+cx)}{c^5}$$

output
```
b^2*(-b*e+c*d)*x/c^4-1/2*b*(-b*e+c*d)*x^2/c^3+1/3*(-b*e+c*d)*x^3/c^2+1/4*e
*x^4/c-b^3*(-b*e+c*d)*ln(c*x+b)/c^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{x^4(d+ex)}{bx+cx^2}\,dx=\frac{cx(-12b^3e+6b^2c(2d+ex)-2bc^2x(3d+2ex)+c^3x^2(4d+3ex))+12b^3(-cd+be)\log(b+cx)}{12c^5}$$

input
```
Integrate[(x^4*(d + e*x))/(b*x + c*x^2),x]
```

output
```
(c*x*(-12*b^3*e + 6*b^2*c*(2*d + e*x) - 2*b*c^2*x*(3*d + 2*e*x) + c^3*x^2*
(4*d + 3*e*x)) + 12*b^3*(-(c*d) + b*e)*Log[b + c*x])/(12*c^5)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex)}{bx+cx^2}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x^3(d+ex)}{b+cx}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{b^3(be-cd)}{c^4(b+cx)}-\frac{b^2(be-cd)}{c^4}+\frac{bx(be-cd)}{c^3}+\frac{x^2(cd-be)}{c^2}+\frac{ex^3}{c}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^3(cd-be)\log(b+cx)}{c^5}+\frac{b^2x(cd-be)}{c^4}-\frac{bx^2(cd-be)}{2c^3}+\frac{x^3(cd-be)}{3c^2}+\frac{ex^4}{4c}$$

input
```
Int[(x^4*(d + e*x))/(b*x + c*x^2),x]
```

output
```
(b^2*(c*d - b*e)*x)/c^4 - (b*(c*d - b*e)*x^2)/(2*c^3) + ((c*d - b*e)*x^3)/
(3*c^2) + (e*x^4)/(4*c) - (b^3*(c*d - b*e)*Log[b + c*x])/c^5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

method	result	size
norman	$-\frac{(be-cd)x^3}{3c^2}+\frac{e\,x^4}{4c}+\frac{b(be-cd)x^2}{2c^3}-\frac{b^2(be-cd)x}{c^4}+\frac{b^3(be-cd)\ln(cx+b)}{c^5}$	82
default	$-\frac{-\frac{1}{4}c^3x^4e+\frac{1}{3}b\,c^2x^3e-\frac{1}{3}c^3d\,x^3-\frac{1}{2}b^2ce\,x^2+\frac{1}{2}b\,c^2d\,x^2+b^3ex-b^2cxd}{c^4}+\frac{b^3(be-cd)\ln(cx+b)}{c^5}$	91
risch	$\frac{e\,x^4}{4c}-\frac{b\,x^3e}{3c^2}+\frac{x^3d}{3c}+\frac{b^2e\,x^2}{2c^3}-\frac{bd\,x^2}{2c^2}-\frac{b^3ex}{c^4}+\frac{b^2xd}{c^3}+\frac{b^4\ln(cx+b)e}{c^5}-\frac{b^3\ln(cx+b)d}{c^4}$	100
parallelrisch	$\frac{3e\,x^4c^4-4x^3b\,c^3e+4c^4d\,x^3+6x^2b^2c^2e-6b\,c^3d\,x^2+12\ln(cx+b)b^4e-12\ln(cx+b)b^3cd-12x\,b^3ce+12b^2c^2dx}{12c^5}$	100

input
```
int(x^4*(e*x+d)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-1/3/c^2*(b*e-c*d)*x^3+1/4*e*x^4/c+1/2*b/c^3*(b*e-c*d)*x^2-b^2*(b*e-c*d)/c
^4*x+b^3*(b*e-c*d)/c^5*ln(c*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int \frac{x^4(d+ex)}{bx+cx^2}\,dx = \frac{3\,c^4ex^4+4\,(c^4d-bc^3e)x^3-6\,(bc^3d-b^2c^2e)x^2+12\,(b^2c^2d-b^3ce)x-12\,(b^3cd-b^4e)\log{(cx+b)}}{12\,c^5}$$

input
```
integrate(x^4*(e*x+d)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
1/12*(3*c^4*e*x^4 + 4*(c^4*d - b*c^3*e)*x^3 - 6*(b*c^3*d - b^2*c^2*e)*x^2
+ 12*(b^2*c^2*d - b^3*c*e)*x - 12*(b^3*c*d - b^4*e)*log(c*x + b))/c^5
```

Sympy [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \frac{x^4(d+ex)}{bx+cx^2}\,dx = \frac{b^3(be-cd)\log{(b+cx)}}{c^5}+x^3\left(-\frac{be}{3c^2}+\frac{d}{3c}\right) + x^2\left(\frac{b^2e}{2c^3}-\frac{bd}{2c^2}\right)+x\left(-\frac{b^3e}{c^4}+\frac{b^2d}{c^3}\right)+\frac{ex^4}{4c}$$

input
```
integrate(x**4*(e*x+d)/(c*x**2+b*x),x)
```

output
```
b**3*(b*e - c*d)*log(b + c*x)/c**5 + x**3*(-b*e/(3*c**2) + d/(3*c)) + x**2
*(b**2*e/(2*c**3) - b*d/(2*c**2)) + x*(-b**3*e/c**4 + b**2*d/c**3) + e*x**
4/(4*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.07

$$\int \frac{x^4(d+ex)}{bx+cx^2}\,dx = \frac{3\,c^3ex^4 + 4\,(c^3d - bc^2e)x^3 - 6\,(bc^2d - b^2ce)x^2 + 12\,(b^2cd - b^3e)x}{12\,c^4} - \frac{(b^3cd - b^4e)\log(cx+b)}{c^5}$$

input
```
integrate(x^4*(e*x+d)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
1/12*(3*c^3*e*x^4 + 4*(c^3*d - b*c^2*e)*x^3 - 6*(b*c^2*d - b^2*c*e)*x^2 +
12*(b^2*c*d - b^3*e)*x)/c^4 - (b^3*c*d - b^4*e)*log(c*x + b)/c^5
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.09

$$\int \frac{x^4(d+ex)}{bx+cx^2}\,dx = \frac{3\,c^3ex^4 + 4\,c^3dx^3 - 4\,bc^2ex^3 - 6\,bc^2dx^2 + 6\,b^2cex^2 + 12\,b^2cdx - 12\,b^3ex}{12\,c^4} - \frac{(b^3cd - b^4e)\log(|cx+b|)}{c^5}$$

input
```
integrate(x^4*(e*x+d)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
1/12*(3*c^3*e*x^4 + 4*c^3*d*x^3 - 4*b*c^2*e*x^3 - 6*b*c^2*d*x^2 + 6*b^2*c*
e*x^2 + 12*b^2*c*d*x - 12*b^3*e*x)/c^4 - (b^3*c*d - b^4*e)*log(abs(c*x + b
))/c^5
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.08

$$\int \frac{x^4(d+ex)}{bx+cx^2}\,dx = x^3\left(\frac{d}{3\,c}-\frac{b\,e}{3\,c^2}\right)+\frac{\ln\left(b+c\,x\right)\,\left(b^4\,e-b^3\,c\,d\right)}{c^5} + \frac{e\,x^4}{4\,c}-\frac{b\,x^2\,\left(\frac{d}{c}-\frac{b\,e}{c^2}\right)}{2\,c}+\frac{b^2\,x\,\left(\frac{d}{c}-\frac{b\,e}{c^2}\right)}{c^2}$$

input
```
int((x^4*(d + e*x))/(b*x + c*x^2),x)
```

output
```
x^3*(d/(3*c) - (b*e)/(3*c^2)) + (log(b + c*x)*(b^4*e - b^3*c*d))/c^5 + (e*
x^4)/(4*c) - (b*x^2*(d/c - (b*e)/c^2))/(2*c) + (b^2*x*(d/c - (b*e)/c^2))/c
^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.14

$$\int \frac{x^4(d+ex)}{bx+cx^2}\,dx = \frac{12\log(cx+b)\,b^4e-12\log(cx+b)\,b^3cd-12b^3cex+12b^2c^2dx+6b^2c^2e\,x^2-6b\,c^3d\,x^2-4b\,c^3e\,x^3+4c^4d}{12c^5}$$

input
```
int(x^4*(e*x+d)/(c*x^2+b*x),x)
```

output
```
(12*log(b + c*x)*b**4*e - 12*log(b + c*x)*b**3*c*d - 12*b**3*c*e*x + 12*b*
*2*c**2*d*x + 6*b**2*c**2*e*x**2 - 6*b*c**3*d*x**2 - 4*b*c**3*e*x**3 + 4*c
**4*d*x**3 + 3*c**4*e*x**4)/(12*c**5)
```

3.36 $\int \frac{x^3(d+ex)}{bx+cx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 66

$$\int \frac{x^3(d+ex)}{bx+cx^2}\,dx=-\frac{b(cd-be)x}{c^3}+\frac{(cd-be)x^2}{2c^2}+\frac{ex^3}{3c}+\frac{b^2(cd-be)\log(b+cx)}{c^4}$$

output
```
-b*(-b*e+c*d)*x/c^3+1/2*(-b*e+c*d)*x^2/c^2+1/3*e*x^3/c+b^2*(-b*e+c*d)*ln(c
*x+b)/c^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{x^3(d+ex)}{bx+cx^2}\,dx=\frac{cx(6b^2e-3bc(2d+ex)+c^2x(3d+2ex))+6b^2(cd-be)\log(b+cx)}{6c^4}$$

input
```
Integrate[(x^3*(d + e*x))/(b*x + c*x^2),x]
```

output
```
(c*x*(6*b^2*e - 3*b*c*(2*d + e*x) + c^2*x*(3*d + 2*e*x)) + 6*b^2*(c*d - b*
e)*Log[b + c*x])/(6*c^4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)}{bx+cx^2}dx$$
$$\Big\downarrow 9$$
$$\int \frac{x^2(d+ex)}{b+cx}dx$$
$$\Big\downarrow 86$$
$$\int \left(-\frac{b^2(be-cd)}{c^3(b+cx)}+\frac{b(be-cd)}{c^3}+\frac{x(cd-be)}{c^2}+\frac{ex^2}{c}\right)dx$$
$$\Big\downarrow 2009$$
$$\frac{b^2(cd-be)\log(b+cx)}{c^4}-\frac{bx(cd-be)}{c^3}+\frac{x^2(cd-be)}{2c^2}+\frac{ex^3}{3c}$$

input
```
Int[(x^3*(d + e*x))/(b*x + c*x^2),x]
```

output
```
-((b*(c*d - b*e)*x)/c^3) + ((c*d - b*e)*x^2)/(2*c^2) + (e*x^3)/(3*c) + (b^
2*(c*d - b*e)*Log[b + c*x])/c^4
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

method	result	size
norman	$\frac{b(be-cd)x}{c^3}-\frac{(be-cd)x^2}{2c^2}+\frac{e\,x^3}{3c}-\frac{b^2(be-cd)\ln(cx+b)}{c^4}$	63
default	$\frac{\frac{1}{3}e\,x^3c^2-\frac{1}{2}bce\,x^2+\frac{1}{2}c^2d\,x^2+b^2ex-bcdx}{c^3}-\frac{b^2(be-cd)\ln(cx+b)}{c^4}$	67
risch	$\frac{e\,x^3}{3c}-\frac{be\,x^2}{2c^2}+\frac{x^2d}{2c}+\frac{b^2ex}{c^3}-\frac{dbx}{c^2}-\frac{b^3\ln(cx+b)e}{c^4}+\frac{b^2\ln(cx+b)d}{c^3}$	76
parallelrisch	$-\frac{-2e\,x^3c^3+3be\,x^2c^2-3c^3d\,x^2+6\ln(cx+b)b^3e-6\ln(cx+b)b^2cd-6x\,b^2ce+6b\,c^2dx}{6c^4}$	76

input
```
int(x^3*(e*x+d)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
b/c^3*(b*e-c*d)*x-1/2/c^2*(b*e-c*d)*x^2+1/3*e*x^3/c-b^2*(b*e-c*d)/c^4*ln(c
*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int \frac{x^3(d+ex)}{bx+cx^2}\,dx = \frac{2\,c^3ex^3+3\,(c^3d-bc^2e)x^2-6\,(bc^2d-b^2ce)x+6\,(b^2cd-b^3e)\log{(cx+b)}}{6\,c^4}$$

input
```
integrate(x^3*(e*x+d)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
1/6*(2*c^3*e*x^3 + 3*(c^3*d - b*c^2*e)*x^2 - 6*(b*c^2*d - b^2*c*e)*x + 6*(
b^2*c*d - b^3*e)*log(c*x + b))/c^4
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.92

$$\int \frac{x^3(d+ex)}{bx+cx^2}\,dx = -\frac{b^2(be-cd)\log(b+cx)}{c^4} + x^2\left(-\frac{be}{2c^2}+\frac{d}{2c}\right) + x\left(\frac{b^2e}{c^3}-\frac{bd}{c^2}\right) + \frac{ex^3}{3c}$$

input
```
integrate(x**3*(e*x+d)/(c*x**2+b*x),x)
```

output
```
-b**2*(b*e - c*d)*log(b + c*x)/c**4 + x**2*(-b*e/(2*c**2) + d/(2*c)) + x*(
b**2*e/c**3 - b*d/c**2) + e*x**3/(3*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.05

$$\int \frac{x^3(d+ex)}{bx+cx^2}\,dx = \frac{2\,c^2ex^3+3\,(c^2d-bce)x^2-6\,(bcd-b^2e)x}{6\,c^3} + \frac{(b^2cd-b^3e)\log(cx+b)}{c^4}$$

input
```
integrate(x^3*(e*x+d)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
1/6*(2*c^2*e*x^3 + 3*(c^2*d - b*c*e)*x^2 - 6*(b*c*d - b^2*e)*x)/c^3 + (b^2
*c*d - b^3*e)*log(c*x + b)/c^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{x^3(d+ex)}{bx+cx^2}\,dx = \frac{2\,c^2ex^3 + 3\,c^2dx^2 - 3\,bcex^2 - 6\,bcdx + 6\,b^2ex}{6\,c^3} + \frac{(b^2cd - b^3e)\log\left(|cx+b|\right)}{c^4}$$

input
```
integrate(x^3*(e*x+d)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
1/6*(2*c^2*e*x^3 + 3*c^2*d*x^2 - 3*b*c*e*x^2 - 6*b*c*d*x + 6*b^2*e*x)/c^3
+ (b^2*c*d - b^3*e)*log(abs(c*x + b))/c^4
```

Mupad [B] (verification not implemented)

Time = 5.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{x^3(d+ex)}{bx+cx^2}\,dx = x^2\left(\frac{d}{2\,c} - \frac{b\,e}{2\,c^2}\right) - \frac{\ln\left(b+c\,x\right)\,\left(b^3\,e - b^2\,c\,d\right)}{c^4} + \frac{e\,x^3}{3\,c} - \frac{b\,x\,\left(\frac{d}{c} - \frac{b\,e}{c^2}\right)}{c}$$

input
```
int((x^3*(d + e*x))/(b*x + c*x^2),x)
```

output
```
x^2*(d/(2*c) - (b*e)/(2*c^2)) - (log(b + c*x)*(b^3*e - b^2*c*d))/c^4 + (e*
x^3)/(3*c) - (b*x*(d/c - (b*e)/c^2))/c
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.14

$$\int \frac{x^3(d+ex)}{bx+cx^2}\,dx = \frac{-6\log(cx+b)\,b^3e + 6\log(cx+b)\,b^2cd + 6b^2cex - 6b\,c^2dx - 3b\,c^2e\,x^2 + 3c^3d\,x^2 + 2c^3e\,x^3}{6c^4}$$

input
```
int(x^3*(e*x+d)/(c*x^2+b*x),x)
```

output
```
( - 6*log(b + c*x)*b**3*e + 6*log(b + c*x)*b**2*c*d + 6*b**2*c*e*x - 6*b*c
**2*d*x - 3*b*c**2*e*x**2 + 3*c**3*d*x**2 + 2*c**3*e*x**3)/(6*c**4)
```

3.37 $\int \frac{x^2(d+ex)}{bx+cx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{x^2(d+ex)}{bx+cx^2}\,dx=\frac{(cd-be)x}{c^2}+\frac{ex^2}{2c}-\frac{b(cd-be)\log(b+cx)}{c^3}$$

output
```
(-b*e+c*d)*x/c^2+1/2*e*x^2/c-b*(-b*e+c*d)*ln(c*x+b)/c^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{x^2(d+ex)}{bx+cx^2}\,dx=\frac{cx(2cd-2be+cex)+2b(-cd+be)\log(b+cx)}{2c^3}$$

input
```
Integrate[(x^2*(d + e*x))/(b*x + c*x^2),x]
```

output
```
(c*x*(2*c*d - 2*b*e + c*e*x) + 2*b*(-(c*d) + b*e)*Log[b + c*x])/(2*c^3)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)}{bx+cx^2}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x(d+ex)}{b+cx}dx$$

$$\Big\downarrow \ 86$$

$$\int \left(\frac{b(be-cd)}{c^2(b+cx)}+\frac{cd-be}{c^2}+\frac{ex}{c}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{b(cd-be)\log(b+cx)}{c^3}+\frac{x(cd-be)}{c^2}+\frac{ex^2}{2c}$$

input
```
Int[(x^2*(d + e*x))/(b*x + c*x^2),x]
```

output
```
((c*d - b*e)*x)/c^2 + (e*x^2)/(2*c) - (b*(c*d - b*e)*Log[b + c*x])/c^3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

method	result	size
default	$-\frac{-\frac{1}{2}ce\,x^2+bex-cdx}{c^2}+\frac{b(be-cd)\ln(cx+b)}{c^3}$	43
norman	$-\frac{(be-cd)x}{c^2}+\frac{e\,x^2}{2c}+\frac{b(be-cd)\ln(cx+b)}{c^3}$	44
parallelrisch	$\frac{e\,x^2c^2+2\ln(cx+b)b^2e-2\ln(cx+b)bcd-2bcex+2c^2dx}{2c^3}$	51
risch	$\frac{e\,x^2}{2c}-\frac{bex}{c^2}+\frac{xd}{c}+\frac{b^2\ln(cx+b)e}{c^3}-\frac{b\ln(cx+b)d}{c^2}$	52

input
```
int(x^2*(e*x+d)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-1/c^2*(-1/2*c*e*x^2+b*e*x-c*d*x)+b/c^3*(b*e-c*d)*ln(c*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

$$\int \frac{x^2(d+ex)}{bx+cx^2}\,dx=\frac{c^2ex^2+2\left(c^2d-bce\right)x-2\left(bcd-b^2e\right)\log\left(cx+b\right)}{2\,c^3}$$

input
```
integrate(x^2*(e*x+d)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
1/2*(c^2*e*x^2 + 2*(c^2*d - b*c*e)*x - 2*(b*c*d - b^2*e)*log(c*x + b))/c^3
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{x^2(d+ex)}{bx+cx^2}\,dx = \frac{b(be-cd)\log{(b+cx)}}{c^3} + x\left(-\frac{be}{c^2}+\frac{d}{c}\right) + \frac{ex^2}{2c}$$

input
```
integrate(x**2*(e*x+d)/(c*x**2+b*x),x)
```

output
```
b*(b*e - c*d)*log(b + c*x)/c**3 + x*(-b*e/c**2 + d/c) + e*x**2/(2*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{x^2(d+ex)}{bx+cx^2}\,dx = \frac{cex^2+2\,(cd-be)x}{2\,c^2} - \frac{(bcd-b^2e)\log{(cx+b)}}{c^3}$$

input
```
integrate(x^2*(e*x+d)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
1/2*(c*e*x^2 + 2*(c*d - b*e)*x)/c^2 - (b*c*d - b^2*e)*log(c*x + b)/c^3
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{x^2(d+ex)}{bx+cx^2}\,dx = \frac{cex^2+2\,cdx-2\,bex}{2\,c^2} - \frac{(bcd-b^2e)\log{(|cx+b|)}}{c^3}$$

input
```
integrate(x^2*(e*x+d)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
1/2*(c*e*x^2 + 2*c*d*x - 2*b*e*x)/c^2 - (b*c*d - b^2*e)*log(abs(c*x + b))/
c^3
```

Mupad [B] (verification not implemented)

Time = 5.18 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int \frac{x^2(d+ex)}{bx+cx^2}\,dx = x\left(\frac{d}{c}-\frac{b\,e}{c^2}\right)+\frac{e\,x^2}{2\,c}+\frac{\ln(b+c\,x)\,\left(b^2\,e-b\,c\,d\right)}{c^3}$$

input
```
int((x^2*(d + e*x))/(b*x + c*x^2),x)
```

output
```
x*(d/c - (b*e)/c^2) + (e*x^2)/(2*c) + (log(b + c*x)*(b^2*e - b*c*d))/c^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \frac{x^2(d+ex)}{bx+cx^2}\,dx = \frac{2\log(cx+b)\,b^2e-2\log(cx+b)\,bcd-2bcex+2c^2dx+c^2e\,x^2}{2c^3}$$

input
```
int(x^2*(e*x+d)/(c*x^2+b*x),x)
```

output
```
(2*log(b + c*x)*b**2*e - 2*log(b + c*x)*b*c*d - 2*b*c*e*x + 2*c**2*d*x + c
**2*e*x**2)/(2*c**3)
```

3.38 $\int \frac{x(d+ex)}{bx+cx^2} dx$

Optimal result

Integrand size = 18, antiderivative size = 25

$$\int \frac{x(d+ex)}{bx+cx^2}\,dx = \frac{ex}{c} + \frac{(cd-be)\log(b+cx)}{c^2}$$

output
```
e*x/c+(-b*e+c*d)*ln(c*x+b)/c^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x(d+ex)}{bx+cx^2}\,dx = \frac{ex}{c} + \frac{(cd-be)\log(b+cx)}{c^2}$$

input
```
Integrate[(x*(d + e*x))/(b*x + c*x^2),x]
```

output
```
(e*x)/c + ((c*d - b*e)*Log[b + c*x])/c^2
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)}{bx+cx^2}\,dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{d+ex}{b+cx}dx$$
$$\Big\downarrow \; 49$$
$$\int \left(\frac{cd-be}{c(b+cx)}+\frac{e}{c}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{(cd-be)\log(b+cx)}{c^2}+\frac{ex}{c}$$

input
```
Int[(x*(d + e*x))/(b*x + c*x^2),x]
```

output
```
(e*x)/c + ((c*d - b*e)*Log[b + c*x])/c^2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{ex}{c}+\frac{(-be+cd)\ln(cx+b)}{c^2}$	26
norman	$\frac{ex}{c}-\frac{(be-cd)\ln(cx+b)}{c^2}$	27
parallelrisch	$-\frac{\ln(cx+b)be-\ln(cx+b)cd-cex}{c^2}$	31
risch	$\frac{ex}{c}-\frac{\ln(cx+b)be}{c^2}+\frac{\ln(cx+b)d}{c}$	32

input
```
int(x*(e*x+d)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
e/c*x+(-b*e+c*d)*ln(c*x+b)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{x(d+ex)}{bx+cx^2}\,dx=\frac{cex+(cd-be)\log(cx+b)}{c^2}$$

input
```
integrate(x*(e*x+d)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
(c*e*x + (c*d - b*e)*log(c*x + b))/c^2
```

Sympy [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

$$\int \frac{x(d+ex)}{bx+cx^2}\,dx = \frac{ex}{c} - \frac{(be-cd)\log(b+cx)}{c^2}$$

input
```
integrate(x*(e*x+d)/(c*x**2+b*x),x)
```

output
```
e*x/c - (b*e - c*d)*log(b + c*x)/c**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{x(d+ex)}{bx+cx^2}\,dx = \frac{ex}{c} + \frac{(cd-be)\log(cx+b)}{c^2}$$

input
```
integrate(x*(e*x+d)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
e*x/c + (c*d - b*e)*log(c*x + b)/c^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{x(d+ex)}{bx+cx^2}\,dx = \frac{ex}{c} + \frac{(cd-be)\log(|cx+b|)}{c^2}$$

input
```
integrate(x*(e*x+d)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
e*x/c + (c*d - b*e)*log(abs(c*x + b))/c^2
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04

$$\int \frac{x(d+ex)}{bx+cx^2}\,dx = \frac{e\,x}{c} - \frac{\ln\left(b+c\,x\right)\,\left(b\,e-c\,d\right)}{c^2}$$

input
```
int((x*(d + e*x))/(b*x + c*x^2),x)
```

output
```
(e*x)/c - (log(b + c*x)*(b*e - c*d))/c^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.12

$$\int \frac{x(d+ex)}{bx+cx^2}\,dx = \frac{-\log(cx+b)\,be+\log(cx+b)\,cd+cex}{c^2}$$

input
```
int(x*(e*x+d)/(c*x^2+b*x),x)
```

output
```
( - log(b + c*x)*b*e + log(b + c*x)*c*d + c*e*x)/c**2
```

3.39 $\int \frac{d+ex}{bx+cx^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 30

$$\int \frac{d+ex}{bx+cx^2} \, dx = \frac{d\log(x)}{b} - \frac{(cd-be)\log(b+cx)}{bc}$$

output
```
d*ln(x)/b-(-b*e+c*d)*ln(c*x+b)/b/c
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{d+ex}{bx+cx^2} \, dx = \frac{d\log(x)}{b} + \frac{(-cd+be)\log(b+cx)}{bc}$$

input
```
Integrate[(d + e*x)/(b*x + c*x^2),x]
```

output
```
(d*Log[x])/b + ((-(c*d) + b*e)*Log[b + c*x])/(b*c)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{bx+cx^2}dx$$

$$\Big\downarrow \text{1141}$$

$$c\int \left(\frac{d}{bcx}-\frac{cd-be}{bc(b+cx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c\left(\frac{d\log(x)}{bc}-\frac{(cd-be)\log(b+cx)}{bc^2}\right)$$

input
```
Int[(d + e*x)/(b*x + c*x^2),x]
```

output
```
c*((d*Log[x])/(b*c) - ((c*d - b*e)*Log[b + c*x])/(b*c^2))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{(be-cd)\ln(cx+b)}{bc}+\frac{d\ln(x)}{b}$	30
norman	$\frac{(be-cd)\ln(cx+b)}{bc}+\frac{d\ln(x)}{b}$	30
parallelrisch	$\frac{d\ln(x)c+\ln(cx+b)be-\ln(cx+b)cd}{bc}$	33
risch	$\frac{d\ln(x)}{b}+\frac{\ln(-cx-b)e}{c}-\frac{\ln(-cx-b)d}{b}$	38

input
```
int((e*x+d)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
(b*e-c*d)/b/c*ln(c*x+b)+d/b*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

$$\int \frac{d+ex}{bx+cx^2}\,dx=\frac{cd\log(x)-(cd-be)\log(cx+b)}{bc}$$

input
```
integrate((e*x+d)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
(c*d*log(x) - (c*d - b*e)*log(c*x + b))/(b*c)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int \frac{d+ex}{bx+cx^2}\,dx=\frac{d\log(x)}{b}+\frac{(be-cd)\log\left(x+\frac{-bd+\frac{b(be-cd)}{c}}{be-2cd}\right)}{bc}$$

input
```
integrate((e*x+d)/(c*x**2+b*x),x)
```

output
```
d*log(x)/b + (b*e - c*d)*log(x + (-b*d + b*(b*e - c*d)/c)/(b*e - 2*c*d))/(
b*c)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{bx+cx^2}\,dx = \frac{d\log{(x)}}{b} - \frac{(cd-be)\log{(cx+b)}}{bc}$$

input
```
integrate((e*x+d)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
d*log(x)/b - (c*d - b*e)*log(c*x + b)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{d+ex}{bx+cx^2}\,dx = \frac{d\log{(|x|)}}{b} - \frac{(cd-be)\log{(|cx+b|)}}{bc}$$

input
```
integrate((e*x+d)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
d*log(abs(x))/b - (c*d - b*e)*log(abs(c*x + b))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 5.07 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93

$$\int \frac{d+ex}{bx+cx^2}\,dx = \frac{d\,\ln{(x)}}{b} - \ln{(b+c\,x)}\,\left(\frac{d}{b}-\frac{e}{c}\right)$$

input
```
int((d + e*x)/(b*x + c*x^2),x)
```

output
```
(d*log(x))/b - log(b + c*x)*(d/b - e/c)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.07

$$\int \frac{d+ex}{bx+cx^2}\,dx = \frac{\log(cx+b)\,be-\log(cx+b)\,cd+\log(x)\,cd}{bc}$$

input
```
int((e*x+d)/(c*x^2+b*x),x)
```

output
```
(log(b + c*x)*b*e - log(b + c*x)*c*d + log(x)*c*d)/(b*c)
```

3.40 $\int \frac{d+ex}{x(bx+cx^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 43

$$\int \frac{d+ex}{x\,(bx+cx^2)}\,dx = -\frac{d}{bx} - \frac{(cd-be)\log(x)}{b^2} + \frac{(cd-be)\log(b+cx)}{b^2}$$

output
```
-d/b/x-(-b*e+c*d)*ln(x)/b^2+(-b*e+c*d)*ln(c*x+b)/b^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{d+ex}{x\,(bx+cx^2)}\,dx = -\frac{d}{bx} + \frac{(-cd+be)\log(x)}{b^2} + \frac{(cd-be)\log(b+cx)}{b^2}$$

input
```
Integrate[(d + e*x)/(x*(b*x + c*x^2)),x]
```

output
```
-(d/(b*x)) + ((-(c*d) + b*e)*Log[x])/b^2 + ((c*d - b*e)*Log[b + c*x])/b^2
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x(bx+cx^2)}dx$$
$$\Big\downarrow \; 9$$
$$\int \frac{d+ex}{x^2(b+cx)}dx$$
$$\Big\downarrow \; 86$$
$$\int \left(\frac{be-cd}{b^2x}-\frac{c(be-cd)}{b^2(b+cx)}+\frac{d}{bx^2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$-\frac{\log(x)(cd-be)}{b^2}+\frac{(cd-be)\log(b+cx)}{b^2}-\frac{d}{bx}$$

input
```
Int[(d + e*x)/(x*(b*x + c*x^2)),x]
```

output
```
-(d/(b*x)) - ((c*d - b*e)*Log[x])/b^2 + ((c*d - b*e)*Log[b + c*x])/b^2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02

method	result	size
default	$-\frac{(be-cd)\ln(cx+b)}{b^2}-\frac{d}{bx}+\frac{(be-cd)\ln(x)}{b^2}$	44
norman	$-\frac{(be-cd)\ln(cx+b)}{b^2}-\frac{d}{bx}+\frac{(be-cd)\ln(x)}{b^2}$	44
parallelrisch	$\frac{\ln(x)xbe-\ln(x)xcd-\ln(cx+b)xbe+\ln(cx+b)xcd-bd}{x\,b^2}$	47
risch	$-\frac{d}{bx}-\frac{\ln(cx+b)e}{b}+\frac{\ln(cx+b)cd}{b^2}+\frac{\ln(-x)e}{b}-\frac{\ln(-x)cd}{b^2}$	55

input
```
int((e*x+d)/x/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-(b*e-c*d)/b^2*ln(c*x+b)-d/b/x+(b*e-c*d)/b^2*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{d+ex}{x\,(bx+cx^2)}\,dx=\frac{(cd-be)x\log{(cx+b)}-(cd-be)x\log{(x)}-bd}{b^2x}$$

input
```
integrate((e*x+d)/x/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
((c*d - b*e)*x*log(c*x + b) - (c*d - b*e)*x*log(x) - b*d)/(b^2*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(34) = 68$.

Time = 0.18 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.21

$$\int \frac{d+ex}{x\,(bx+cx^2)}\,dx = -\frac{d}{bx} + \frac{(be-cd)\log\left(x+\frac{b^2e-bcd-b(be-cd)}{2bce-2c^2d}\right)}{b^2} - \frac{(be-cd)\log\left(x+\frac{b^2e-bcd+b(be-cd)}{2bce-2c^2d}\right)}{b^2}$$

input
```
integrate((e*x+d)/x/(c*x**2+b*x),x)
```

output
```
-d/(b*x) + (b*e - c*d)*log(x + (b**2*e - b*c*d - b*(b*e - c*d))/(2*b*c*e -
 2*c**2*d))/b**2 - (b*e - c*d)*log(x + (b**2*e - b*c*d + b*(b*e - c*d))/(2
*b*c*e - 2*c**2*d))/b**2
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{x\,(bx+cx^2)}\,dx = \frac{(cd-be)\log(cx+b)}{b^2} - \frac{(cd-be)\log(x)}{b^2} - \frac{d}{bx}$$

input
```
integrate((e*x+d)/x/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
(c*d - b*e)*log(c*x + b)/b^2 - (c*d - b*e)*log(x)/b^2 - d/(b*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{d+ex}{x\,(bx+cx^2)}\,dx = -\frac{(cd-be)\log{(|x|)}}{b^2} - \frac{d}{bx} + \frac{(c^2d-bce)\log{(|cx+b|)}}{b^2c}$$

input
```
integrate((e*x+d)/x/(c*x^2+b*x),x, algorithm="giac")
```

output
```
-(c*d - b*e)*log(abs(x))/b^2 - d/(b*x) + (c^2*d - b*c*e)*log(abs(c*x + b))
/(b^2*c)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.77

$$\int \frac{d+ex}{x\,(bx+cx^2)}\,dx = -\frac{d}{b\,x} - \frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)\,\left(b\,e-c\,d\right)}{b^2}$$

input
```
int((d + e*x)/(x*(b*x + c*x^2)),x)
```

output
```
- d/(b*x) - (2*atanh((2*c*x)/b + 1)*(b*e - c*d))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

$$\int \frac{d+ex}{x\,(bx+cx^2)}\,dx = \frac{-\log(cx+b)\,bex + \log(cx+b)\,cdx + \log(x)\,bex - \log(x)\,cdx - bd}{b^2x}$$

input
```
int((e*x+d)/x/(c*x^2+b*x),x)
```

output
```
( - log(b + c*x)*b*e*x + log(b + c*x)*c*d*x + log(x)*b*e*x - log(x)*c*d*x
- b*d)/(b**2*x)
```

3.41 $\int \frac{d+ex}{x^2(bx+cx^2)} dx$

Optimal result

Integrand size = 20, antiderivative size = 62

$$\int \frac{d+ex}{x^2(bx+cx^2)}\,dx=-\frac{d}{2bx^2}+\frac{cd-be}{b^2x}+\frac{c(cd-be)\log(x)}{b^3}-\frac{c(cd-be)\log(b+cx)}{b^3}$$

output
```
-1/2*d/b/x^2+(-b*e+c*d)/b^2/x+c*(-b*e+c*d)*ln(x)/b^3-c*(-b*e+c*d)*ln(c*x+b
)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.94

$$\int \frac{d+ex}{x^2(bx+cx^2)}\,dx=\frac{-\frac{b(bd-2cdx+2bex)}{x^2}+2c(cd-be)\log(x)+2c(-cd+be)\log(b+cx)}{2b^3}$$

input
```
Integrate[(d + e*x)/(x^2*(b*x + c*x^2)),x]
```

output
```
(-((b*(b*d - 2*c*d*x + 2*b*e*x))/x^2) + 2*c*(c*d - b*e)*Log[x] + 2*c*(-(c*
d) + b*e)*Log[b + c*x])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^2\left(bx+cx^2\right)}dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{d+ex}{x^3(b+cx)}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{c^2(be-cd)}{b^3(b+cx)}-\frac{c(be-cd)}{b^3x}+\frac{be-cd}{b^2x^2}+\frac{d}{bx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{c\log(x)(cd-be)}{b^3}-\frac{c(cd-be)\log(b+cx)}{b^3}+\frac{cd-be}{b^2x}-\frac{d}{2bx^2}$$

input
```
Int[(d + e*x)/(x^2*(b*x + c*x^2)),x]
```

output
```
-1/2*d/(b*x^2) + (c*d - b*e)/(b^2*x) + (c*(c*d - b*e)*Log[x])/b^3 - (c*(c*
d - b*e)*Log[b + c*x])/b^3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

method	result	size
default	$\frac{(be-cd)c\ln(cx+b)}{b^3}-\frac{d}{2b\,x^2}-\frac{be-cd}{b^2x}-\frac{(be-cd)c\ln(x)}{b^3}$	62
norman	$\frac{-\frac{d}{2b}-\frac{(be-cd)x}{b^2}}{x^2}+\frac{(be-cd)c\ln(cx+b)}{b^3}-\frac{(be-cd)c\ln(x)}{b^3}$	62
parallelrisch	$-\frac{2\ln(x)x^2bce-2\ln(x)x^2c^2d-2\ln(cx+b)x^2bce+2\ln(cx+b)x^2c^2d+2b^2ex-2bcdx+b^2d}{2x^2b^3}$	78
risch	$\frac{-\frac{d}{2b}-\frac{(be-cd)x}{b^2}}{x^2}+\frac{c\ln(-cx-b)e}{b^2}-\frac{c^2\ln(-cx-b)d}{b^3}-\frac{c\ln(x)e}{b^2}+\frac{c^2\ln(x)d}{b^3}$	79

input
```
int((e*x+d)/x^2/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
(b*e-c*d)/b^3*c*ln(c*x+b)-1/2*d/b/x^2-(b*e-c*d)/b^2/x-(b*e-c*d)/b^3*c*ln(x
)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.10

$$\int \frac{d+ex}{x^2(bx+cx^2)}\,dx$$
$$=-\frac{2\,(c^2d-bce)x^2\log{(cx+b)}-2\,(c^2d-bce)x^2\log{(x)}+b^2d-2\,(bcd-b^2e)x}{2\,b^3x^2}$$

input
```
integrate((e*x+d)/x^2/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
-1/2*(2*(c^2*d - b*c*e)*x^2*log(c*x + b) - 2*(c^2*d - b*c*e)*x^2*log(x) +
b^2*d - 2*(b*c*d - b^2*e)*x)/(b^3*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(53) = 106$.

Time = 0.22 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.11

$$\int \frac{d+ex}{x^2\left(bx+cx^2\right)}\,dx = \frac{-bd+x(-2be+2cd)}{2b^2x^2} - \frac{c(be-cd)\log\left(x+\frac{b^2ce-bc^2d-bc(be-cd)}{2bc^2e-2c^3d}\right)}{b^3} + \frac{c(be-cd)\log\left(x+\frac{b^2ce-bc^2d+bc(be-cd)}{2bc^2e-2c^3d}\right)}{b^3}$$

input
```
integrate((e*x+d)/x**2/(c*x**2+b*x),x)
```

output
```
(-b*d + x*(-2*b*e + 2*c*d))/(2*b**2*x**2) - c*(b*e - c*d)*log(x + (b**2*c*
e - b*c**2*d - b*c*(b*e - c*d))/(2*b*c**2*e - 2*c**3*d))/b**3 + c*(b*e - c
*d)*log(x + (b**2*c*e - b*c**2*d + b*c*(b*e - c*d))/(2*b*c**2*e - 2*c**3*d
))/b**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{d+ex}{x^2\left(bx+cx^2\right)}\,dx = -\frac{(c^2d-bce)\log(cx+b)}{b^3} + \frac{(c^2d-bce)\log(x)}{b^3} - \frac{bd-2(cd-be)x}{2\,b^2x^2}$$

input
```
integrate((e*x+d)/x^2/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
-(c^2*d - b*c*e)*log(c*x + b)/b^3 + (c^2*d - b*c*e)*log(x)/b^3 - 1/2*(b*d
- 2*(c*d - b*e)*x)/(b^2*x^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.21

$$\int \frac{d+ex}{x^2\,(bx+cx^2)}\,dx = \frac{(c^2d-bce)\log(|x|)}{b^3} - \frac{(c^3d-bc^2e)\log(|cx+b|)}{b^3c} - \frac{b^2d-2\,(bcd-b^2e)x}{2\,b^3x^2}$$

input
```
integrate((e*x+d)/x^2/(c*x^2+b*x),x, algorithm="giac")
```

output
```
(c^2*d - b*c*e)*log(abs(x))/b^3 - (c^3*d - b*c^2*e)*log(abs(c*x + b))/(b^3
*c) - 1/2*(b^2*d - 2*(b*c*d - b^2*e)*x)/(b^3*x^2)
```

Mupad [B] (verification not implemented)

Time = 5.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{d+ex}{x^2\,(bx+cx^2)}\,dx = -\frac{\frac{d}{2\,b}+\frac{x\,(b\,e-c\,d)}{b^2}}{x^2} - \frac{2\,c\,\mathrm{atanh}\left(\frac{c\,(b\,e-c\,d)\,(b+2\,c\,x)}{b\,(c^2\,d-b\,c\,e)}\right)\,(b\,e-c\,d)}{b^3}$$

input
```
int((d + e*x)/(x^2*(b*x + c*x^2)),x)
```

output
```
- (d/(2*b) + (x*(b*e - c*d))/b^2)/x^2 - (2*c*atanh((c*(b*e - c*d)*(b + 2*c
*x))/(b*(c^2*d - b*c*e)))*(b*e - c*d))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.26

$$\int \frac{d+ex}{x^2\,(bx+cx^2)}\,dx = \frac{2\log(cx+b)\,bce\,x^2-2\log(cx+b)\,c^2d\,x^2-2\log(x)\,bce\,x^2+2\log(x)\,c^2d\,x^2-b^2d-2b^2ex+2bcdx}{2b^3x^2}$$

input
```
int((e*x+d)/x^2/(c*x^2+b*x),x)
```

output
```
(2*log(b + c*x)*b*c*e*x**2 - 2*log(b + c*x)*c**2*d*x**2 - 2*log(x)*b*c*e*x
**2 + 2*log(x)*c**2*d*x**2 - b**2*d - 2*b**2*e*x + 2*b*c*d*x)/(2*b**3*x**2
)
```

3.42 $\int \frac{d+ex}{x^3(bx+cx^2)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 86

$$\int \frac{d+ex}{x^3(bx+cx^2)}\,dx = -\frac{d}{3bx^3}+\frac{cd-be}{2b^2x^2}-\frac{c(cd-be)}{b^3x} - \frac{c^2(cd-be)\log(x)}{b^4}+\frac{c^2(cd-be)\log(b+cx)}{b^4}$$

output
```
-1/3*d/b/x^3+1/2*(-b*e+c*d)/b^2/x^2-c*(-b*e+c*d)/b^3/x-c^2*(-b*e+c*d)*ln(x
)/b^4+c^2*(-b*e+c*d)*ln(c*x+b)/b^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{d+ex}{x^3(bx+cx^2)}\,dx = \frac{\frac{b(-6c^2dx^2+3bcx(d+2ex)-b^2(2d+3ex))}{x^3}+6c^2(-cd+be)\log(x)+6c^2(cd-be)\log(b+cx)}{6b^4}$$

input
```
Integrate[(d + e*x)/(x^3*(b*x + c*x^2)),x]
```

output
```
((b*(-6*c^2*d*x^2 + 3*b*c*x*(d + 2*e*x) - b^2*(2*d + 3*e*x)))/x^3 + 6*c^2*
(-(c*d) + b*e)*Log[x] + 6*c^2*(c*d - b*e)*Log[b + c*x])/(6*b^4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^3 (bx+cx^2)} \, dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{d+ex}{x^4(b+cx)}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(-\frac{c^3(be-cd)}{b^4(b+cx)}+\frac{c^2(be-cd)}{b^4x}-\frac{c(be-cd)}{b^3x^2}+\frac{be-cd}{b^2x^3}+\frac{d}{bx^4}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{c^2\log(x)(cd-be)}{b^4}+\frac{c^2(cd-be)\log(b+cx)}{b^4}-\frac{c(cd-be)}{b^3x}+\frac{cd-be}{2b^2x^2}-\frac{d}{3bx^3}$$

input
```
Int[(d + e*x)/(x^3*(b*x + c*x^2)),x]
```

output
```
-1/3*d/(b*x^3) + (c*d - b*e)/(2*b^2*x^2) - (c*(c*d - b*e))/(b^3*x) - (c^2*
(c*d - b*e)*Log[x])/b^4 + (c^2*(c*d - b*e)*Log[b + c*x])/b^4
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.95

method	result	size
default	$-\frac{(be-cd)c^2\ln(cx+b)}{b^4}-\frac{d}{3b\,x^3}-\frac{be-cd}{2b^2x^2}+\frac{(be-cd)c^2\ln(x)}{b^4}+\frac{(be-cd)c}{b^3x}$	82
norman	$\frac{\frac{(be-cd)c\,x^2}{b^3}-\frac{d}{3b}-\frac{(be-cd)x}{2b^2}}{x^3}+\frac{(be-cd)c^2\ln(x)}{b^4}-\frac{(be-cd)c^2\ln(cx+b)}{b^4}$	82
risch	$\frac{\frac{(be-cd)c\,x^2}{b^3}-\frac{d}{3b}-\frac{(be-cd)x}{2b^2}}{x^3}+\frac{c^2\ln(-x)e}{b^3}-\frac{c^3\ln(-x)d}{b^4}-\frac{c^2\ln(cx+b)e}{b^3}+\frac{c^3\ln(cx+b)d}{b^4}$	97
parallelrisch	$\frac{6\ln(x)x^3b\,c^2e-6\ln(x)x^3c^3d-6\ln(cx+b)x^3b\,c^2e+6\ln(cx+b)x^3c^3d+6b^2ce\,x^2-6b\,c^2d\,x^2-3b^3ex+3b^2cxd-2b^3d}{6b^4x^3}$	105

input
```
int((e*x+d)/x^3/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-(b*e-c*d)/b^4*c^2*ln(c*x+b)-1/3*d/b/x^3-1/2*(b*e-c*d)/b^2/x^2+(b*e-c*d)/b
^4*c^2*ln(x)+(b*e-c*d)/b^3*c/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

$$\int \frac{d+ex}{x^3(bx+cx^2)}\,dx = \frac{6\,(c^3d-bc^2e)x^3\log\left(cx+b\right)-6\,(c^3d-bc^2e)x^3\log\left(x\right)-2\,b^3d-6\,(bc^2d-b^2ce)x^2+3\,(b^2cd-b^3e)x}{6\,b^4x^3}$$

input
```
integrate((e*x+d)/x^3/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
1/6*(6*(c^3*d - b*c^2*e)*x^3*log(c*x + b) - 6*(c^3*d - b*c^2*e)*x^3*log(x)
 - 2*b^3*d - 6*(b*c^2*d - b^2*c*e)*x^2 + 3*(b^2*c*d - b^3*e)*x)/(b^4*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. 2(75) = 150.

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.92

$$\int \frac{d+ex}{x^3(bx+cx^2)}\,dx = \frac{-2b^2d+x^2\cdot(6bce-6c^2d)+x(-3b^2e+3bcd)}{6b^3x^3} + \frac{c^2(be-cd)\log\left(x+\frac{b^2c^2e-bc^3d-bc^2(be-cd)}{2bc^3e-2c^4d}\right)}{b^4} - \frac{c^2(be-cd)\log\left(x+\frac{b^2c^2e-bc^3d+bc^2(be-cd)}{2bc^3e-2c^4d}\right)}{b^4}$$

input
```
integrate((e*x+d)/x**3/(c*x**2+b*x),x)
```

output
```
(-2*b**2*d + x**2*(6*b*c*e - 6*c**2*d) + x*(-3*b**2*e + 3*b*c*d))/(6*b**3*
x**3) + c**2*(b*e - c*d)*log(x + (b**2*c**2*e - b*c**3*d - b*c**2*(b*e - c
*d))/(2*b*c**3*e - 2*c**4*d))/b**4 - c**2*(b*e - c*d)*log(x + (b**2*c**2*e
 - b*c**3*d + b*c**2*(b*e - c*d))/(2*b*c**3*e - 2*c**4*d))/b**4
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.03

$$\int \frac{d+ex}{x^3\left(bx+cx^2\right)}\,dx = \frac{\left(c^3d-bc^2e\right)\log\left(cx+b\right)}{b^4} - \frac{\left(c^3d-bc^2e\right)\log\left(x\right)}{b^4} - \frac{2\,b^2d+6\left(c^2d-bce\right)x^2-3\left(bcd-b^2e\right)x}{6\,b^3x^3}$$

input
```
integrate((e*x+d)/x^3/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
(c^3*d - b*c^2*e)*log(c*x + b)/b^4 - (c^3*d - b*c^2*e)*log(x)/b^4 - 1/6*(2
*b^2*d + 6*(c^2*d - b*c*e)*x^2 - 3*(b*c*d - b^2*e)*x)/(b^3*x^3)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{d+ex}{x^3\left(bx+cx^2\right)}\,dx = -\frac{\left(c^3d-bc^2e\right)\log\left(|x|\right)}{b^4} + \frac{\left(c^4d-bc^3e\right)\log\left(|cx+b|\right)}{b^4c} - \frac{2\,b^3d+6\left(bc^2d-b^2ce\right)x^2-3\left(b^2cd-b^3e\right)x}{6\,b^4x^3}$$

input
```
integrate((e*x+d)/x^3/(c*x^2+b*x),x, algorithm="giac")
```

output
```
-(c^3*d - b*c^2*e)*log(abs(x))/b^4 + (c^4*d - b*c^3*e)*log(abs(c*x + b))/(
b^4*c) - 1/6*(2*b^3*d + 6*(b*c^2*d - b^2*c*e)*x^2 - 3*(b^2*c*d - b^3*e)*x)
/(b^4*x^3)
```

Mupad [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13

$$\int \frac{d+ex}{x^3\left(bx+cx^2\right)}\,dx = \frac{2\,c^2\,\mathrm{atanh}\left(\frac{c^2\,(b\,e-c\,d)\,(b+2\,c\,x)}{b\,(c^3\,d-b\,c^2\,e)}\right)\,(b\,e-c\,d)}{b^4}-\frac{\frac{d}{3\,b}+\frac{x\,(b\,e-c\,d)}{2\,b^2}-\frac{c\,x^2\,(b\,e-c\,d)}{b^3}}{x^3}$$

input
```
int((d + e*x)/(x^3*(b*x + c*x^2)),x)
```

output
```
(2*c^2*atanh((c^2*(b*e - c*d)*(b + 2*c*x))/(b*(c^3*d - b*c^2*e)))*(b*e - c
*d))/b^4 - (d/(3*b) + (x*(b*e - c*d))/(2*b^2) - (c*x^2*(b*e - c*d))/b^3)/x
^3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.21

$$\int \frac{d+ex}{x^3\left(bx+cx^2\right)}\,dx = \frac{-6\log(cx+b)\,b\,c^2e\,x^3+6\log(cx+b)\,c^3d\,x^3+6\log(x)\,b\,c^2e\,x^3-6\log(x)\,c^3d\,x^3-2b^3d-3b^3ex+3b^2cd}{6b^4x^3}$$

input
```
int((e*x+d)/x^3/(c*x^2+b*x),x)
```

output
```
( - 6*log(b + c*x)*b*c**2*e*x**3 + 6*log(b + c*x)*c**3*d*x**3 + 6*log(x)*b
*c**2*e*x**3 - 6*log(x)*c**3*d*x**3 - 2*b**3*d - 3*b**3*e*x + 3*b**2*c*d*x
 + 6*b**2*c*e*x**2 - 6*b*c**2*d*x**2)/(6*b**4*x**3)
```

3.43 $\int \frac{x^5(d+ex)}{(bx+cx^2)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 90

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^2} \, dx = -\frac{b(2cd-3be)x}{c^4} + \frac{(cd-2be)x^2}{2c^3} + \frac{ex^3}{3c^2} + \frac{b^3(cd-be)}{c^5(b+cx)} + \frac{b^2(3cd-4be)\log(b+cx)}{c^5}$$

output
```
-b*(-3*b*e+2*c*d)*x/c^4+1/2*(-2*b*e+c*d)*x^2/c^3+1/3*e*x^3/c^2+b^3*(-b*e+c
*d)/c^5/(c*x+b)+b^2*(-4*b*e+3*c*d)*ln(c*x+b)/c^5
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.97

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^2} \, dx = \frac{6bc(-2cd+3be)x + 3c^2(cd-2be)x^2 + 2c^3ex^3 + \frac{6b^3(cd-be)}{b+cx} + 6b^2(3cd-4be)\log(b+cx)}{6c^5}$$

input
```
Integrate[(x^5*(d + e*x))/(b*x + c*x^2)^2,x]
```

output
```
(6*b*c*(-2*c*d + 3*b*e)*x + 3*c^2*(c*d - 2*b*e)*x^2 + 2*c^3*e*x^3 + (6*b^3
*(c*d - b*e))/(b + c*x) + 6*b^2*(3*c*d - 4*b*e)*Log[b + c*x])/(6*c^5)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^2}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{x^3(d+ex)}{(b+cx)^2}dx$$

$$\Big\downarrow \text{86}$$

$$\int \left(\frac{b^3(be-cd)}{c^4(b+cx)^2}-\frac{b^2(4be-3cd)}{c^4(b+cx)}+\frac{b(3be-2cd)}{c^4}+\frac{x(cd-2be)}{c^3}+\frac{ex^2}{c^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^3(cd-be)}{c^5(b+cx)}+\frac{b^2(3cd-4be)\log(b+cx)}{c^5}-\frac{bx(2cd-3be)}{c^4}+\frac{x^2(cd-2be)}{2c^3}+\frac{ex^3}{3c^2}$$

input
```
Int[(x^5*(d + e*x))/(b*x + c*x^2)^2,x]
```

output
```
-((b*(2*c*d - 3*b*e)*x)/c^4) + ((c*d - 2*b*e)*x^2)/(2*c^3) + (e*x^3)/(3*c^
2) + (b^3*(c*d - b*e))/(c^5*(b + c*x)) + (b^2*(3*c*d - 4*b*e)*Log[b + c*x]
)/c^5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

method	result
default	$\frac{\frac{1}{3}e\,x^{3}c^{2}-bce\,x^{2}+\frac{1}{2}c^{2}d\,x^{2}+3b^{2}ex-2bcdx}{c^{4}}-\frac{b^{2}\left(4be-3cd\right)\ln\left(cx+b\right)}{c^{5}}-\frac{b^{3}\left(be-cd\right)}{c^{5}\left(cx+b\right)}$
norman	$\frac{\frac{e\,x^{5}}{3c}-\frac{\left(4be-3cd\right)x^{4}}{6c^{2}}+\frac{b\left(4be-3cd\right)x^{3}}{2c^{3}}-\frac{b\left(4e\,b^{3}-3cd\,b^{2}\right)x}{c^{5}}}{x\left(cx+b\right)}-\frac{b^{2}\left(4be-3cd\right)\ln\left(cx+b\right)}{c^{5}}$
risch	$\frac{e\,x^{3}}{3c^{2}}-\frac{be\,x^{2}}{c^{3}}+\frac{d\,x^{2}}{2c^{2}}+\frac{3b^{2}ex}{c^{4}}-\frac{2bdx}{c^{3}}-\frac{4b^{3}\ln\left(cx+b\right)e}{c^{5}}+\frac{3b^{2}\ln\left(cx+b\right)d}{c^{4}}-\frac{b^{4}e}{c^{5}\left(cx+b\right)}+\frac{b^{3}d}{c^{4}\left(cx+b\right)}$
parallelrisch	$-\frac{-2e\,x^{4}c^{4}+4x^{3}b\,c^{3}e-3c^{4}d\,x^{3}+24\ln\left(cx+b\right)x\,b^{3}ce-18\ln\left(cx+b\right)x\,b^{2}c^{2}d-12x^{2}b^{2}c^{2}e+9b\,c^{3}d\,x^{2}+24\ln\left(cx+b\right)b^{4}e-18\ln(cx+b}{6c^{5}\left(cx+b\right)}$

input
```
int(x^5*(e*x+d)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
1/c^4*(1/3*e*x^3*c^2-b*c*e*x^2+1/2*c^2*d*x^2+3*b^2*e*x-2*b*c*d*x)-b^2/c^5*
(4*b*e-3*c*d)*ln(c*x+b)-b^3*(b*e-c*d)/c^5/(c*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.54

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^2}\,dx = \frac{2\,c^4ex^4+6\,b^3cd-6\,b^4e+(3\,c^4d-4\,bc^3e)x^3-3\,(3\,bc^3d-4\,b^2c^2e)x^2-6\,(2\,b^2c^2d-3\,b^3ce)x+6\,(3\,b^3cd-}{6\,(c^6x+bc^5)}$$

input
```
integrate(x^5*(e*x+d)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
1/6*(2*c^4*e*x^4 + 6*b^3*c*d - 6*b^4*e + (3*c^4*d - 4*b*c^3*e)*x^3 - 3*(3*
b*c^3*d - 4*b^2*c^2*e)*x^2 - 6*(2*b^2*c^2*d - 3*b^3*c*e)*x + 6*(3*b^3*c*d
- 4*b^4*e + (3*b^2*c^2*d - 4*b^3*c*e)*x)*log(c*x + b))/(c^6*x + b*c^5)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.02

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^2}\,dx = -\frac{b^2\cdot(4be-3cd)\log{(b+cx)}}{c^5} + x^2\left(-\frac{be}{c^3}+\frac{d}{2c^2}\right) + x\left(\frac{3b^2e}{c^4}-\frac{2bd}{c^3}\right) + \frac{-b^4e+b^3cd}{bc^5+c^6x} + \frac{ex^3}{3c^2}$$

input
```
integrate(x**5*(e*x+d)/(c*x**2+b*x)**2,x)
```

output
```
-b**2*(4*b*e - 3*c*d)*log(b + c*x)/c**5 + x**2*(-b*e/c**3 + d/(2*c**2)) +
x*(3*b**2*e/c**4 - 2*b*d/c**3) + (-b**4*e + b**3*c*d)/(b*c**5 + c**6*x) +
e*x**3/(3*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^2}\,dx = \frac{b^3cd-b^4e}{c^6x+bc^5} + \frac{2\,c^2ex^3+3\,(c^2d-2\,bce)x^2-6\,(2\,bcd-3\,b^2e)x}{6\,c^4} + \frac{(3\,b^2cd-4\,b^3e)\log{(cx+b)}}{c^5}$$

input
```
integrate(x^5*(e*x+d)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
(b^3*c*d - b^4*e)/(c^6*x + b*c^5) + 1/6*(2*c^2*e*x^3 + 3*(c^2*d - 2*b*c*e)
*x^2 - 6*(2*b*c*d - 3*b^2*e)*x)/c^4 + (3*b^2*c*d - 4*b^3*e)*log(c*x + b)/c
^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.13

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^2}\,dx = \frac{(3\,b^2cd-4\,b^3e)\log{(|cx+b|)}}{c^5} + \frac{2\,c^4ex^3+3\,c^4dx^2-6\,bc^3ex^2-12\,bc^3dx+18\,b^2c^2ex}{6\,c^6} + \frac{b^3cd-b^4e}{(cx+b)c^5}$$

input
```
integrate(x^5*(e*x+d)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
(3*b^2*c*d - 4*b^3*e)*log(abs(c*x + b))/c^5 + 1/6*(2*c^4*e*x^3 + 3*c^4*d*x
^2 - 6*b*c^3*e*x^2 - 12*b*c^3*d*x + 18*b^2*c^2*e*x)/c^6 + (b^3*c*d - b^4*e
)/((c*x + b)*c^5)
```

Mupad [B] (verification not implemented)

Time = 5.21 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.28

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^2}\,dx = x^2\left(\frac{d}{2\,c^2}-\frac{b\,e}{c^3}\right)-x\left(\frac{b^2\,e}{c^4}+\frac{2\,b\left(\frac{d}{c^2}-\frac{2\,b\,e}{c^3}\right)}{c}\right) - \frac{\ln(b+c\,x)\,\left(4\,b^3\,e-3\,b^2\,c\,d\right)}{c^5}+\frac{e\,x^3}{3\,c^2}-\frac{b^4\,e-b^3\,c\,d}{c\,\left(x\,c^5+b\,c^4\right)}$$

input
```
int((x^5*(d + e*x))/(b*x + c*x^2)^2,x)
```

output
```
x^2*(d/(2*c^2) - (b*e)/c^3) - x*((b^2*e)/c^4 + (2*b*(d/c^2 - (2*b*e)/c^3))
/c) - (log(b + c*x)*(4*b^3*e - 3*b^2*c*d))/c^5 + (e*x^3)/(3*c^2) - (b^4*e
- b^3*c*d)/(c*(b*c^4 + c^5*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.51

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^2}\,dx = \frac{-24\log(cx+b)\,b^4e+18\log(cx+b)\,b^3cd-24\log(cx+b)\,b^3cex+18\log(cx+b)\,b^2c^2dx+24b^3cex-18b}{6c^5\,(cx+b)}$$

input
```
int(x^5*(e*x+d)/(c*x^2+b*x)^2,x)
```

output
```
( - 24*log(b + c*x)*b**4*e + 18*log(b + c*x)*b**3*c*d - 24*log(b + c*x)*b*
*3*c*e*x + 18*log(b + c*x)*b**2*c**2*d*x + 24*b**3*c*e*x - 18*b**2*c**2*d*
x + 12*b**2*c**2*e*x**2 - 9*b*c**3*d*x**2 - 4*b*c**3*e*x**3 + 3*c**4*d*x**
3 + 2*c**4*e*x**4)/(6*c**5*(b + c*x))
```

3.44 $\int \frac{x^4(d+ex)}{(bx+cx^2)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 69

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^2} dx = \frac{(cd-2be)x}{c^3} + \frac{ex^2}{2c^2} - \frac{b^2(cd-be)}{c^4(b+cx)} - \frac{b(2cd-3be)\log(b+cx)}{c^4}$$

output
```
(-2*b*e+c*d)*x/c^3+1/2*e*x^2/c^2-b^2*(-b*e+c*d)/c^4/(c*x+b)-b*(-3*b*e+2*c*
d)*ln(c*x+b)/c^4
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^2} dx = \frac{2c(cd-2be)x + c^2ex^2 + \frac{2b^2(-cd+be)}{b+cx} + 2b(-2cd+3be)\log(b+cx)}{2c^4}$$

input
```
Integrate[(x^4*(d + e*x))/(b*x + c*x^2)^2,x]
```

output
```
(2*c*(c*d - 2*b*e)*x + c^2*e*x^2 + (2*b^2*(-(c*d) + b*e))/(b + c*x) + 2*b*
(-2*c*d + 3*b*e)*Log[b + c*x])/(2*c^4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^2}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^2(d+ex)}{(b+cx)^2}dx$$

$$\Big\downarrow \ 86$$

$$\int \left(-\frac{b^2(be-cd)}{c^3(b+cx)^2}+\frac{b(3be-2cd)}{c^3(b+cx)}+\frac{cd-2be}{c^3}+\frac{ex}{c^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{b^2(cd-be)}{c^4(b+cx)}-\frac{b(2cd-3be)\log(b+cx)}{c^4}+\frac{x(cd-2be)}{c^3}+\frac{ex^2}{2c^2}$$

input
```
Int[(x^4*(d + e*x))/(b*x + c*x^2)^2,x]
```

output
```
((c*d - 2*b*e)*x)/c^3 + (e*x^2)/(2*c^2) - (b^2*(c*d - b*e))/(c^4*(b + c*x)
) - (b*(2*c*d - 3*b*e)*Log[b + c*x])/c^4
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

method	result	size
default	$-\frac{-\frac{1}{2} c e \,x^{2}+2 b e x -c d x}{c^{3}}+\frac{b \left(3 b e -2 c d \right) \ln \left(c x +b \right)}{c^{4}}+\frac{b^{2} \left(b e -c d \right)}{c^{4} \left(c x +b \right)}$	67
norman	$\frac{\frac{b \left(3 e \,b^{2}-2 d b c \right) x}{c^{4}}-\frac{\left(3 b e -2 c d \right) x^{3}}{2 c^{2}}+\frac{e \,x^{4}}{2 c}}{x \left(c x +b \right)}+\frac{b \left(3 b e -2 c d \right) \ln \left(c x +b \right)}{c^{4}}$	78
risch	$\frac{e \,x^{2}}{2 c^{2}}-\frac{2 b e x}{c^{3}}+\frac{d x}{c^{2}}+\frac{b^{3} e}{c^{4} \left(c x +b \right)}-\frac{b^{2} d}{c^{3} \left(c x +b \right)}+\frac{3 b^{2} \ln \left(c x +b \right) e}{c^{4}}-\frac{2 b \ln \left(c x +b \right) d}{c^{3}}$	84
parallelrisch	$\frac{e \,x^{3} c^{3}+6 \ln \left(c x +b \right) x \,b^{2} c e -4 \ln \left(c x +b \right) x b \,c^{2} d -3 b e \,x^{2} c^{2}+2 c^{3} d \,x^{2}+6 \ln \left(c x +b \right) b^{3} e -4 \ln \left(c x +b \right) b^{2} c d +6 e \,b^{3}-4 c d \,b^{2}}{2 c^{4} \left(c x +b \right)}$	107

input
```
int(x^4*(e*x+d)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/c^3*(-1/2*c*e*x^2+2*b*e*x-c*d*x)+b/c^4*(3*b*e-2*c*d)*ln(c*x+b)+b^2*(b*e
-c*d)/c^4/(c*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.61

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^2}\,dx = \frac{c^3ex^3 - 2\,b^2cd + 2\,b^3e + (2\,c^3d - 3\,bc^2e)x^2 + 2\,(bc^2d - 2\,b^2ce)x - 2\,(2\,b^2cd - 3\,b^3e + (2\,bc^2d - 3\,b^2ce)x)\,\mathrm{l}}{2\,(c^5x + bc^4)}$$

input
```
integrate(x^4*(e*x+d)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
1/2*(c^3*e*x^3 - 2*b^2*c*d + 2*b^3*e + (2*c^3*d - 3*b*c^2*e)*x^2 + 2*(b*c^
2*d - 2*b^2*c*e)*x - 2*(2*b^2*c*d - 3*b^3*e + (2*b*c^2*d - 3*b^2*c*e)*x)*l
og(c*x + b))/(c^5*x + b*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.99

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^2}\,dx = \frac{b(3be-2cd)\log(b+cx)}{c^4} + x\left(-\frac{2be}{c^3}+\frac{d}{c^2}\right) + \frac{b^3e-b^2cd}{bc^4+c^5x} + \frac{ex^2}{2c^2}$$

input
```
integrate(x**4*(e*x+d)/(c*x**2+b*x)**2,x)
```

output
```
b*(3*b*e - 2*c*d)*log(b + c*x)/c**4 + x*(-2*b*e/c**3 + d/c**2) + (b**3*e -
 b**2*c*d)/(b*c**4 + c**5*x) + e*x**2/(2*c**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^2}\,dx = -\frac{b^2cd-b^3e}{c^5x+bc^4} + \frac{cex^2+2\,(cd-2\,be)x}{2\,c^3} - \frac{(2\,bcd-3\,b^2e)\log(cx+b)}{c^4}$$

input
```
integrate(x^4*(e*x+d)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-(b^2*c*d - b^3*e)/(c^5*x + b*c^4) + 1/2*(c*e*x^2 + 2*(c*d - 2*b*e)*x)/c^3
 - (2*b*c*d - 3*b^2*e)*log(c*x + b)/c^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^2}\,dx = -\frac{(2\,bcd-3\,b^2e)\log\left(|cx+b|\right)}{c^4}+\frac{c^2ex^2+2\,c^2dx-4\,bcex}{2\,c^4}-\frac{b^2cd-b^3e}{(cx+b)c^4}$$

input
```
integrate(x^4*(e*x+d)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
-(2*b*c*d - 3*b^2*e)*log(abs(c*x + b))/c^4 + 1/2*(c^2*e*x^2 + 2*c^2*d*x -
4*b*c*e*x)/c^4 - (b^2*c*d - b^3*e)/((c*x + b)*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.12

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^2}\,dx = x\left(\frac{d}{c^2}-\frac{2\,b\,e}{c^3}\right)+\frac{e\,x^2}{2\,c^2}+\frac{b^3\,e-b^2\,c\,d}{c\,\left(x\,c^4+b\,c^3\right)}+\frac{\ln\left(b+c\,x\right)\,\left(3\,b^2\,e-2\,b\,c\,d\right)}{c^4}$$

input
```
int((x^4*(d + e*x))/(b*x + c*x^2)^2,x)
```

output
```
x*(d/c^2 - (2*b*e)/c^3) + (e*x^2)/(2*c^2) + (b^3*e - b^2*c*d)/(c*(b*c^3 +
c^4*x)) + (log(b + c*x)*(3*b^2*e - 2*b*c*d))/c^4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.58

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^2}\,dx$$
$$= \frac{6\log(cx+b)\,b^3e-4\log(cx+b)\,b^2cd+6\log(cx+b)\,b^2cex-4\log(cx+b)\,b\,c^2dx-6b^2cex+4b\,c^2dx-3}{2c^4\,(cx+b)}$$

input
```
int(x^4*(e*x+d)/(c*x^2+b*x)^2,x)
```

output
```
(6*log(b + c*x)*b**3*e - 4*log(b + c*x)*b**2*c*d + 6*log(b + c*x)*b**2*c*e
*x - 4*log(b + c*x)*b*c**2*d*x - 6*b**2*c*e*x + 4*b*c**2*d*x - 3*b*c**2*e*
x**2 + 2*c**3*d*x**2 + c**3*e*x**3)/(2*c**4*(b + c*x))
```

3.45 $\int \frac{x^3(d+ex)}{(bx+cx^2)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 45

$$\int \frac{x^3(d+ex)}{(bx+cx^2)^2} \, dx = \frac{ex}{c^2} + \frac{b(cd-be)}{c^3(b+cx)} + \frac{(cd-2be)\log(b+cx)}{c^3}$$

output
```
e*x/c^2+b*(-b*e+c*d)/c^3/(c*x+b)+(-2*b*e+c*d)*ln(c*x+b)/c^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.91

$$\int \frac{x^3(d+ex)}{(bx+cx^2)^2} \, dx = \frac{cex + \frac{b(cd-be)}{b+cx} + (cd-2be)\log(b+cx)}{c^3}$$

input
```
Integrate[(x^3*(d + e*x))/(b*x + c*x^2)^2,x]
```

output
```
(c*e*x + (b*(c*d - b*e))/(b + c*x) + (c*d - 2*b*e)*Log[b + c*x])/c^3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)}{(bx+cx^2)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x(d+ex)}{(b+cx)^2}dx$$

$$\Big\downarrow \; 86$$

$$\int \left(\frac{cd-2be}{c^2(b+cx)}+\frac{b(be-cd)}{c^2(b+cx)^2}+\frac{e}{c^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{b(cd-be)}{c^3(b+cx)}+\frac{(cd-2be)\log(b+cx)}{c^3}+\frac{ex}{c^2}$$

input
```
Int[(x^3*(d + e*x))/(b*x + c*x^2)^2,x]
```

output
```
(e*x)/c^2 + (b*(c*d - b*e))/(c^3*(b + c*x)) + ((c*d - 2*b*e)*Log[b + c*x])
/c^3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.04

method	result	size
default	$\frac{ex}{c^2}+\frac{(-2be+cd)\ln(cx+b)}{c^3}-\frac{b(be-cd)}{c^3(cx+b)}$	47
norman	$\frac{\frac{e\,x^3}{c}-\frac{b(2be-cd)x}{c^3}}{x(cx+b)}-\frac{(2be-cd)\ln(cx+b)}{c^3}$	58
risch	$\frac{ex}{c^2}-\frac{b^2e}{c^3(cx+b)}+\frac{bd}{c^2(cx+b)}-\frac{2\ln(cx+b)be}{c^3}+\frac{\ln(cx+b)d}{c^2}$	61
parallelrisch	$-\frac{2\ln(cx+b)xbce-\ln(cx+b)x\,c^2d-e\,x^2c^2+2\ln(cx+b)b^2e-\ln(cx+b)bcd+2e\,b^2-dbc}{c^3(cx+b)}$	82

input
```
int(x^3*(e*x+d)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
e*x/c^2+(-2*b*e+c*d)*ln(c*x+b)/c^3-b/c^3*(b*e-c*d)/(c*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.53

$$\int\frac{x^3(d+ex)}{(bx+cx^2)^2}\,dx=\frac{c^2ex^2+bcex+bcd-b^2e+(bcd-2\,b^2e+(c^2d-2\,bce)x)\log(cx+b)}{c^4x+bc^3}$$

input
```
integrate(x^3*(e*x+d)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
(c^2*e*x^2 + b*c*e*x + b*c*d - b^2*e + (b*c*d - 2*b^2*e + (c^2*d - 2*b*c*e
)*x)*log(c*x + b))/(c^4*x + b*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

$$\int \frac{x^3(d+ex)}{(bx+cx^2)^2}\,dx = \frac{-b^2e+bcd}{bc^3+c^4x} + \frac{ex}{c^2} - \frac{(2be-cd)\log(b+cx)}{c^3}$$

input
```
integrate(x**3*(e*x+d)/(c*x**2+b*x)**2,x)
```

output
```
(-b**2*e + b*c*d)/(b*c**3 + c**4*x) + e*x/c**2 - (2*b*e - c*d)*log(b + c*x
)/c**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.11

$$\int \frac{x^3(d+ex)}{(bx+cx^2)^2}\,dx = \frac{bcd-b^2e}{c^4x+bc^3} + \frac{ex}{c^2} + \frac{(cd-2\,be)\log(cx+b)}{c^3}$$

input
```
integrate(x^3*(e*x+d)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
(b*c*d - b^2*e)/(c^4*x + b*c^3) + e*x/c^2 + (c*d - 2*b*e)*log(c*x + b)/c^3
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.07

$$\int \frac{x^3(d+ex)}{(bx+cx^2)^2}\,dx = \frac{ex}{c^2} + \frac{(cd-2\,be)\log(|cx+b|)}{c^3} + \frac{bcd-b^2e}{(cx+b)c^3}$$

input
```
integrate(x^3*(e*x+d)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
e*x/c^2 + (c*d - 2*b*e)*log(abs(c*x + b))/c^3 + (b*c*d - b^2*e)/((c*x + b)
*c^3)
```

Mupad [B] (verification not implemented)

Time = 5.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int \frac{x^3(d+ex)}{(bx+cx^2)^2}\,dx = \frac{e\,x}{c^2} - \frac{b^2\,e-b\,c\,d}{c\,(x\,c^3+b\,c^2)} - \frac{\ln(b+c\,x)\,(2\,b\,e-c\,d)}{c^3}$$

input
```
int((x^3*(d + e*x))/(b*x + c*x^2)^2,x)
```

output
```
(e*x)/c^2 - (b^2*e - b*c*d)/(c*(b*c^2 + c^3*x)) - (log(b + c*x)*(2*b*e - c
*d))/c^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.76

$$\int \frac{x^3(d+ex)}{(bx+cx^2)^2}\,dx = \frac{-2\log(cx+b)\,b^2e+\log(cx+b)\,bcd-2\log(cx+b)\,bcex+\log(cx+b)\,c^2dx+2bcex-c^2dx+c^2e\,x^2}{c^3\,(cx+b)}$$

input
```
int(x^3*(e*x+d)/(c*x^2+b*x)^2,x)
```

output
```
( - 2*log(b + c*x)*b**2*e + log(b + c*x)*b*c*d - 2*log(b + c*x)*b*c*e*x +
log(b + c*x)*c**2*d*x + 2*b*c*e*x - c**2*d*x + c**2*e*x**2)/(c**3*(b + c*x
))
```

3.46 $\int \frac{x^2(d+ex)}{\left(bx+cx^2\right)^2} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 32

$$\int \frac{x^2(d+ex)}{\left(bx+cx^2\right)^2} \, dx = -\frac{cd-be}{c^2(b+cx)}+\frac{e\log(b+cx)}{c^2}$$

output
```
-(-b*e+c*d)/c^2/(c*x+b)+e*ln(c*x+b)/c^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{x^2(d+ex)}{\left(bx+cx^2\right)^2} \, dx = \frac{-cd+be}{c^2(b+cx)}+\frac{e\log(b+cx)}{c^2}$$

input
```
Integrate[(x^2*(d + e*x))/(b*x + c*x^2)^2,x]
```

output
```
(-(c*d) + b*e)/(c^2*(b + c*x)) + (e*Log[b + c*x])/c^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^2}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{d+ex}{(b+cx)^2}dx$$
$$\Big\downarrow \text{ 49}$$
$$\int \left(\frac{cd-be}{c(b+cx)^2}+\frac{e}{c(b+cx)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{e\log(b+cx)}{c^2}-\frac{cd-be}{c^2(b+cx)}$$

input
```
Int[(x^2*(d + e*x))/(b*x + c*x^2)^2,x]
```

output
```
-((c*d - b*e)/(c^2*(b + c*x))) + (e*Log[b + c*x])/c^2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)    Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

method	result	size
norman	$\frac{be-cd}{c^2(cx+b)}+\frac{e\ln(cx+b)}{c^2}$	32
default	$-\frac{-be+cd}{c^2(cx+b)}+\frac{e\ln(cx+b)}{c^2}$	33
risch	$\frac{be}{c^2(cx+b)}-\frac{d}{c(cx+b)}+\frac{e\ln(cx+b)}{c^2}$	39
parallelrisch	$\frac{\ln(cx+b)xce+\ln(cx+b)be+be-cd}{c^2(cx+b)}$	39

input
```
int(x^2*(e*x+d)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(b*e-c*d)/c^2/(c*x+b)+e*ln(c*x+b)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^2}\,dx = -\frac{cd-be-(cex+be)\log(cx+b)}{c^3x+bc^2}$$

input
```
integrate(x^2*(e*x+d)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
-(c*d - b*e - (c*e*x + b*e)*log(c*x + b))/(c^3*x + b*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.84

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^2}\,dx = \frac{be-cd}{bc^2+c^3x} + \frac{e\log(b+cx)}{c^2}$$

input
```
integrate(x**2*(e*x+d)/(c*x**2+b*x)**2,x)
```

output
```
(b*e - c*d)/(b*c**2 + c**3*x) + e*log(b + c*x)/c**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^2}\,dx = -\frac{cd-be}{c^3x+bc^2} + \frac{e\log(cx+b)}{c^2}$$

input
```
integrate(x^2*(e*x+d)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-(c*d - b*e)/(c^3*x + b*c^2) + e*log(c*x + b)/c^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.03

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^2}\,dx = \frac{e\log(|cx+b|)}{c^2} - \frac{cd-be}{(cx+b)c^2}$$

input
```
integrate(x^2*(e*x+d)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
e*log(abs(c*x + b))/c^2 - (c*d - b*e)/((c*x + b)*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.97

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^2}\,dx = \frac{b\,e-c\,d}{c^2\,\left(b+c\,x\right)}+\frac{e\,\ln\left(b+c\,x\right)}{c^2}$$

input
```
int((x^2*(d + e*x))/(b*x + c*x^2)^2,x)
```

output
```
(b*e - c*d)/(c^2*(b + c*x)) + (e*log(b + c*x))/c^2
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.53

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^2}\,dx = \frac{\log(cx+b)\,b^2e+\log(cx+b)\,bcex-bcex+c^2dx}{b\,c^2\,(cx+b)}$$

input
```
int(x^2*(e*x+d)/(c*x^2+b*x)^2,x)
```

output
```
(log(b + c*x)*b**2*e + log(b + c*x)*b*c*e*x - b*c*e*x + c**2*d*x)/(b*c**2*
(b + c*x))
```

3.47 $\int \frac{x(d+ex)}{(bx+cx^2)^2} \, dx$

Optimal result

Integrand size = 18, antiderivative size = 42

$$\int \frac{x(d+ex)}{(bx+cx^2)^2} \, dx = \frac{cd-be}{bc(b+cx)} + \frac{d\log(x)}{b^2} - \frac{d\log(b+cx)}{b^2}$$

output
```
(-b*e+c*d)/b/c/(c*x+b)+d*ln(x)/b^2-d*ln(c*x+b)/b^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{x(d+ex)}{(bx+cx^2)^2} \, dx = \frac{\frac{b(cd-be)}{c(b+cx)} + d\log(x) - d\log(b+cx)}{b^2}$$

input
```
Integrate[(x*(d + e*x))/(b*x + c*x^2)^2,x]
```

output
```
((b*(c*d - b*e))/(c*(b + c*x)) + d*Log[x] - d*Log[b + c*x])/b^2
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)}{(bx+cx^2)^2}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \frac{d+ex}{x(b+cx)^2}dx$$
$$\Big\downarrow \text{ 86}$$
$$\int \left(-\frac{cd}{b^2(b+cx)}+\frac{d}{b^2x}+\frac{be-cd}{b(b+cx)^2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{d\log(b+cx)}{b^2}+\frac{d\log(x)}{b^2}+\frac{cd-be}{bc(b+cx)}$$

input
```
Int[(x*(d + e*x))/(b*x + c*x^2)^2,x]
```

output
```
(c*d - b*e)/(b*c*(b + c*x)) + (d*Log[x])/b^2 - (d*Log[b + c*x])/b^2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

method	result	size
norman	$\frac{(be-cd)x}{b^2(cx+b)}+\frac{d\ln(x)}{b^2}-\frac{d\ln(cx+b)}{b^2}$	41
default	$-\frac{be-cd}{bc(cx+b)}-\frac{d\ln(cx+b)}{b^2}+\frac{d\ln(x)}{b^2}$	44
risch	$-\frac{e}{(cx+b)c}+\frac{d}{b(cx+b)}+\frac{d\ln(-x)}{b^2}-\frac{d\ln(cx+b)}{b^2}$	48
parallelrisch	$\frac{\ln(x)xcd-\ln(cx+b)xcd+bd\ln(x)-\ln(cx+b)bd+bex-cdx}{b^2(cx+b)}$	54

input
```
int(x*(e*x+d)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(b*e-c*d)/b^2*x/(c*x+b)+d*ln(x)/b^2-d*ln(c*x+b)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int \frac{x(d+ex)}{(bx+cx^2)^2}\,dx = \frac{bcd-b^2e-(c^2dx+bcd)\log(cx+b)+(c^2dx+bcd)\log(x)}{b^2c^2x+b^3c}$$

input
```
integrate(x*(e*x+d)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
(b*c*d - b^2*e - (c^2*d*x + b*c*d)*log(c*x + b) + (c^2*d*x + b*c*d)*log(x)
)/(b^2*c^2*x + b^3*c)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.76

$$\int \frac{x(d+ex)}{(bx+cx^2)^2}\,dx = \frac{-be+cd}{b^2c+bc^2x} + \frac{d\left(\log(x)-\log\left(\frac{b}{c}+x\right)\right)}{b^2}$$

input
```
integrate(x*(e*x+d)/(c*x**2+b*x)**2,x)
```

output
```
(-b*e + c*d)/(b**2*c + b*c**2*x) + d*(log(x) - log(b/c + x))/b**2
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{x(d+ex)}{(bx+cx^2)^2}\,dx = \frac{cd-be}{bc^2x+b^2c} - \frac{d\log(cx+b)}{b^2} + \frac{d\log(x)}{b^2}$$

input
```
integrate(x*(e*x+d)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
(c*d - b*e)/(b*c^2*x + b^2*c) - d*log(c*x + b)/b^2 + d*log(x)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12

$$\int \frac{x(d+ex)}{(bx+cx^2)^2}\,dx = -\frac{d\log(|cx+b|)}{b^2} + \frac{d\log(|x|)}{b^2} + \frac{bcd-b^2e}{(cx+b)b^2c}$$

input
```
integrate(x*(e*x+d)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
-d*log(abs(c*x + b))/b^2 + d*log(abs(x))/b^2 + (b*c*d - b^2*e)/((c*x + b)*
b^2*c)
```

Mupad [B] (verification not implemented)

Time = 5.09 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{x(d+ex)}{(bx+cx^2)^2}\,dx = -\frac{2\,d\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b^2} - \frac{b\,e-c\,d}{b\,c\,(b+c\,x)}$$

input
```
int((x*(d + e*x))/(b*x + c*x^2)^2,x)
```

output
```
- (2*d*atanh((2*c*x)/b + 1))/b^2 - (b*e - c*d)/(b*c*(b + c*x))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{x(d+ex)}{(bx+cx^2)^2}\,dx = \frac{-\log(cx+b)\,bd-\log(cx+b)\,cdx+\log(x)\,bd+\log(x)\,cdx+bex-cdx}{b^2\,(cx+b)}$$

input
```
int(x*(e*x+d)/(c*x^2+b*x)^2,x)
```

output
```
( - log(b + c*x)*b*d - log(b + c*x)*c*d*x + log(x)*b*d + log(x)*c*d*x + b*
e*x - c*d*x)/(b**2*(b + c*x))
```

3.48 $\int \frac{d+ex}{\left(bx+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{d+ex}{\left(bx+cx^2\right)^2}\,dx=-\frac{d}{b^2x}-\frac{cd-be}{b^2(b+cx)}-\frac{(2cd-be)\log(x)}{b^3}+\frac{(2cd-be)\log(b+cx)}{b^3}$$

output
```
-d/b^2/x-(-b*e+c*d)/b^2/(c*x+b)-(-b*e+2*c*d)*ln(x)/b^3+(-b*e+2*c*d)*ln(c*x
+b)/b^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.86

$$\int \frac{d+ex}{\left(bx+cx^2\right)^2}\,dx=\frac{-\frac{bd}{x}+\frac{b(-cd+be)}{b+cx}+(-2cd+be)\log(x)+(2cd-be)\log(b+cx)}{b^3}$$

input
```
Integrate[(d + e*x)/(b*x + c*x^2)^2,x]
```

output
```
(-((b*d)/x) + (b*(-(c*d) + b*e))/(b + c*x) + (-2*c*d + b*e)*Log[x] + (2*c*
d - b*e)*Log[b + c*x])/b^3
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(bx+cx^2)^2}dx$$

$$\Big\downarrow \text{ 1141}$$

$$c^2\int \left(\frac{d}{b^2c^2x^2}-\frac{2cd-be}{b^3c^2x}+\frac{2cd-be}{b^3c(b+cx)}+\frac{cd-be}{b^2c(b+cx)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$c^2\left(-\frac{\log(x)(2cd-be)}{b^3c^2}+\frac{(2cd-be)\log(b+cx)}{b^3c^2}-\frac{cd-be}{b^2c^2(b+cx)}-\frac{d}{b^2c^2x}\right)$$

input
```
Int[(d + e*x)/(b*x + c*x^2)^2,x]
```

output
```
c^2*(-(d/(b^2*c^2*x)) - (c*d - b*e)/(b^2*c^2*(b + c*x)) - ((2*c*d - b*e)*L
og[x])/(b^3*c^2) + ((2*c*d - b*e)*Log[b + c*x])/(b^3*c^2))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.97

method	result
default	$-\frac{(be-2cd)\ln(cx+b)}{b^3}+\frac{be-cd}{b^2(cx+b)}-\frac{d}{b^2x}+\frac{(be-2cd)\ln(x)}{b^3}$
norman	$\frac{\frac{c(-be+2cd)x^2}{b^3}-\frac{d}{b}}{x(cx+b)}+\frac{(be-2cd)\ln(x)}{b^3}-\frac{(be-2cd)\ln(cx+b)}{b^3}$
risch	$\frac{\frac{(be-2cd)x}{b^2}-\frac{d}{b}}{x(cx+b)}-\frac{\ln(cx+b)e}{b^2}+\frac{2\ln(cx+b)cd}{b^3}+\frac{\ln(-x)e}{b^2}-\frac{2\ln(-x)cd}{b^3}$
parallelrisch	$\frac{\ln(x)x^2bc^2e-2\ln(x)x^2c^3d-\ln(cx+b)x^2bc^2e+2\ln(cx+b)x^2c^3d+\ln(x)xb^2ce-2\ln(x)xbc^2d-\ln(cx+b)xb^2ce+2\ln(cx+b)xbc}{b^3cx(cx+b)}$

input
```
int((e*x+d)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-(b*e-2*c*d)/b^3*ln(c*x+b)+(b*e-c*d)/b^2/(c*x+b)-d/b^2/x+(b*e-2*c*d)/b^3*l
n(x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.71

$$\int \frac{d+ex}{(bx+cx^2)^2}\,dx =$$
$$-\frac{b^2d+(2\,bcd-b^2e)x-((2\,c^2d-bce)x^2+(2\,bcd-b^2e)x)\log(cx+b)+((2\,c^2d-bce)x^2+(2\,bcd-b^2}{b^3cx^2+b^4x}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
-(b^2*d + (2*b*c*d - b^2*e)*x - ((2*c^2*d - b*c*e)*x^2 + (2*b*c*d - b^2*e)
*x)*log(c*x + b) + ((2*c^2*d - b*c*e)*x^2 + (2*b*c*d - b^2*e)*x)*log(x))/(
b^3*c*x^2 + b^4*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(54) = 108$.

Time = 0.25 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.97

$$\int \frac{d+ex}{(bx+cx^2)^2}\,dx = \frac{-bd+x(be-2cd)}{b^3x+b^2cx^2} + \frac{(be-2cd)\log\left(x+\frac{b^2e-2bcd-b(be-2cd)}{2bce-4c^2d}\right)}{b^3} - \frac{(be-2cd)\log\left(x+\frac{b^2e-2bcd+b(be-2cd)}{2bce-4c^2d}\right)}{b^3}$$

input
```
integrate((e*x+d)/(c*x**2+b*x)**2,x)
```

output
```
(-b*d + x*(b*e - 2*c*d))/(b**3*x + b**2*c*x**2) + (b*e - 2*c*d)*log(x + (b
**2*e - 2*b*c*d - b*(b*e - 2*c*d))/(2*b*c*e - 4*c**2*d))/b**3 - (b*e - 2*c
*d)*log(x + (b**2*e - 2*b*c*d + b*(b*e - 2*c*d))/(2*b*c*e - 4*c**2*d))/b**
3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.06

$$\int \frac{d+ex}{(bx+cx^2)^2}\,dx = -\frac{bd+(2\,cd-be)x}{b^2cx^2+b^3x} + \frac{(2\,cd-be)\log(cx+b)}{b^3} - \frac{(2\,cd-be)\log(x)}{b^3}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-(b*d + (2*c*d - b*e)*x)/(b^2*c*x^2 + b^3*x) + (2*c*d - b*e)*log(c*x + b)/
b^3 - (2*c*d - b*e)*log(x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.14

$$\int \frac{d+ex}{(bx+cx^2)^2}\,dx = -\frac{(2\,cd-be)\log(|x|)}{b^3} - \frac{2\,cdx-bex+bd}{(cx^2+bx)b^2} + \frac{(2\,c^2d-bce)\log(|cx+b|)}{b^3c}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
-(2*c*d - b*e)*log(abs(x))/b^3 - (2*c*d*x - b*e*x + b*d)/((c*x^2 + b*x)*b^
2) + (2*c^2*d - b*c*e)*log(abs(c*x + b))/(b^3*c)
```

Mupad [B] (verification not implemented)

Time = 5.18 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.88

$$\int \frac{d+ex}{(bx+cx^2)^2}\,dx = -\frac{\frac{d}{b}-\frac{x\,(b\,e-2\,c\,d)}{b^2}}{c\,x^2+b\,x} - \frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)\,(b\,e-2\,c\,d)}{b^3}$$

input
```
int((d + e*x)/(b*x + c*x^2)^2,x)
```

output
```
- (d/b - (x*(b*e - 2*c*d))/b^2)/(b*x + c*x^2) - (2*atanh((2*c*x)/b + 1)*(b
*e - 2*c*d))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.97

$$\int \frac{d+ex}{(bx+cx^2)^2}\,dx = \frac{-\log(cx+b)\,b^2ex+2\log(cx+b)\,bcdx-\log(cx+b)\,bce\,x^2+2\log(cx+b)\,c^2d\,x^2+\log(x)\,b^2ex-2\log(x}{b^3x\,(cx+b)}$$

input
```
int((e*x+d)/(c*x^2+b*x)^2,x)
```

output
```
( - log(b + c*x)*b**2*e*x + 2*log(b + c*x)*b*c*d*x - log(b + c*x)*b*c*e*x*
*2 + 2*log(b + c*x)*c**2*d*x**2 + log(x)*b**2*e*x - 2*log(x)*b*c*d*x + log
(x)*b*c*e*x**2 - 2*log(x)*c**2*d*x**2 - b**2*d - b*c*e*x**2 + 2*c**2*d*x**
2)/(b**3*x*(b + c*x))
```

3.49 $\int \frac{d+ex}{x\left(bx+cx^2\right)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 85

$$\int \frac{d+ex}{x\left(bx+cx^2\right)^2}\,dx=-\frac{d}{2b^2x^2}+\frac{2cd-be}{b^3x}+\frac{c(cd-be)}{b^3(b+cx)}+\frac{c(3cd-2be)\log(x)}{b^4}-\frac{c(3cd-2be)\log(b+cx)}{b^4}$$

output
```
-1/2*d/b^2/x^2+(-b*e+2*c*d)/b^3/x+c*(-b*e+c*d)/b^3/(c*x+b)+c*(-2*b*e+3*c*d
)*ln(x)/b^4-c*(-2*b*e+3*c*d)*ln(c*x+b)/b^4
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

$$\int \frac{d+ex}{x\left(bx+cx^2\right)^2}\,dx=\frac{-\frac{b\left(-6c^2dx^2+b^2(d+2ex)+bcx(-3d+4ex)\right)}{x^2(b+cx)}+2c(3cd-2be)\log(x)+2c(-3cd+2be)\log(b+cx)}{2b^4}$$

input
```
Integrate[(d + e*x)/(x*(b*x + c*x^2)^2),x]
```

output
```
(-((b*(-6*c^2*d*x^2 + b^2*(d + 2*e*x) + b*c*x*(-3*d + 4*e*x)))/(x^2*(b + c
*x))) + 2*c*(3*c*d - 2*b*e)*Log[x] + 2*c*(-3*c*d + 2*b*e)*Log[b + c*x])/(2
*b^4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x\left(bx+cx^2\right)^2}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{d+ex}{x^3(b+cx)^2}dx$$

$$\Big\downarrow \ 86$$

$$\int \left(\frac{c^2(2be-3cd)}{b^4(b+cx)}-\frac{c(2be-3cd)}{b^4x}+\frac{c^2(be-cd)}{b^3(b+cx)^2}+\frac{be-2cd}{b^3x^2}+\frac{d}{b^2x^3}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{c\log(x)(3cd-2be)}{b^4}-\frac{c(3cd-2be)\log(b+cx)}{b^4}+\frac{2cd-be}{b^3x}+\frac{c(cd-be)}{b^3(b+cx)}-\frac{d}{2b^2x^2}$$

input
```
Int[(d + e*x)/(x*(b*x + c*x^2)^2),x]
```

output
```
-1/2*d/(b^2*x^2) + (2*c*d - b*e)/(b^3*x) + (c*(c*d - b*e))/(b^3*(b + c*x))
 + (c*(3*c*d - 2*b*e)*Log[x])/b^4 - (c*(3*c*d - 2*b*e)*Log[b + c*x])/b^4
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00

method	result
default	$\frac{c(2be-3cd)\ln(cx+b)}{b^4}-\frac{(be-cd)c}{b^3(cx+b)}-\frac{d}{2b^2x^2}-\frac{be-2cd}{b^3x}-\frac{c(2be-3cd)\ln(x)}{b^4}$
norman	$\frac{\frac{c\left(2bce-3c^2d\right)x^3}{b^4}-\frac{d}{2b}-\frac{(2be-3cd)x}{2b^2}}{(cx+b)x^2}+\frac{c(2be-3cd)\ln(cx+b)}{b^4}-\frac{c(2be-3cd)\ln(x)}{b^4}$
risch	$\frac{-\frac{c(2be-3cd)x^2}{b^3}-\frac{(2be-3cd)x}{2b^2}-\frac{d}{2b}}{(cx+b)x^2}-\frac{2c\ln(x)e}{b^3}+\frac{3c^2\ln(x)d}{b^4}+\frac{2c\ln(-cx-b)e}{b^3}-\frac{3c^2\ln(-cx-b)d}{b^4}$
parallelrisch	$-\frac{4\ln(x)x^3bc^2e-6\ln(x)x^3c^3d-4\ln(cx+b)x^3bc^2e+6\ln(cx+b)x^3c^3d+4\ln(x)x^2b^2ce-6\ln(x)x^2bc^2d-4\ln(cx+b)x^2b^2ce+6\ln}{2b^4x^2(cx+b)}$

input
```
int((e*x+d)/x/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
c*(2*b*e-3*c*d)/b^4*ln(c*x+b)-(b*e-c*d)/b^3*c/(c*x+b)-1/2*d/b^2/x^2-(b*e-2
*c*d)/b^3/x-c*(2*b*e-3*c*d)/b^4*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.78

$$\int \frac{d+ex}{x\left(bx+cx^2\right)^2}\,dx = -\frac{b^3d-2\left(3\,bc^2d-2\,b^2ce\right)x^2-\left(3\,b^2cd-2\,b^3e\right)x+2\left(\left(3\,c^3d-2\,bc^2e\right)x^3+\left(3\,bc^2d-2\,b^2ce\right)x^2\right)\log\left(cx+\right.}{2\left(b^4cx^3+b^5x^2\right)}$$

input
```
integrate((e*x+d)/x/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
-1/2*(b^3*d - 2*(3*b*c^2*d - 2*b^2*c*e)*x^2 - (3*b^2*c*d - 2*b^3*e)*x + 2*
((3*c^3*d - 2*b*c^2*e)*x^3 + (3*b*c^2*d - 2*b^2*c*e)*x^2)*log(c*x + b) - 2
*((3*c^3*d - 2*b*c^2*e)*x^3 + (3*b*c^2*d - 2*b^2*c*e)*x^2)*log(x))/(b^4*c*
x^3 + b^5*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(80) = 160$.

Time = 0.29 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.16

$$\int \frac{d+ex}{x\left(bx+cx^2\right)^2}\,dx = \frac{-b^2d+x^2(-4bce+6c^2d)+x(-2b^2e+3bcd)}{2b^4x^2+2b^3cx^3} - \frac{c(2be-3cd)\log\left(x+\frac{2b^2ce-3bc^2d-bc(2be-3cd)}{4bc^2e-6c^3d}\right)}{b^4} + \frac{c(2be-3cd)\log\left(x+\frac{2b^2ce-3bc^2d+bc(2be-3cd)}{4bc^2e-6c^3d}\right)}{b^4}$$

input
```
integrate((e*x+d)/x/(c*x**2+b*x)**2,x)
```

output
```
(-b**2*d + x**2*(-4*b*c*e + 6*c**2*d) + x*(-2*b**2*e + 3*b*c*d))/(2*b**4*x
**2 + 2*b**3*c*x**3) - c*(2*b*e - 3*c*d)*log(x + (2*b**2*c*e - 3*b*c**2*d
- b*c*(2*b*e - 3*c*d))/(4*b*c**2*e - 6*c**3*d))/b**4 + c*(2*b*e - 3*c*d)*l
og(x + (2*b**2*c*e - 3*b*c**2*d + b*c*(2*b*e - 3*c*d))/(4*b*c**2*e - 6*c**
3*d))/b**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

$$\int \frac{d+ex}{x\left(bx+cx^2\right)^2}\,dx = -\frac{b^2d-2\,(3\,c^2d-2\,bce)x^2-(3\,bcd-2\,b^2e)x}{2\,(b^3cx^3+b^4x^2)} - \frac{(3\,c^2d-2\,bce)\log(cx+b)}{b^4} + \frac{(3\,c^2d-2\,bce)\log(x)}{b^4}$$

input
```
integrate((e*x+d)/x/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-1/2*(b^2*d - 2*(3*c^2*d - 2*b*c*e)*x^2 - (3*b*c*d - 2*b^2*e)*x)/(b^3*c*x^
3 + b^4*x^2) - (3*c^2*d - 2*b*c*e)*log(c*x + b)/b^4 + (3*c^2*d - 2*b*c*e)*
log(x)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.26

$$\int \frac{d+ex}{x\left(bx+cx^2\right)^2}\,dx = \frac{(3\,c^2d-2\,bce)\log(|x|)}{b^4} - \frac{(3\,c^3d-2\,bc^2e)\log(|cx+b|)}{b^4c} - \frac{b^3d-2\,(3\,bc^2d-2\,b^2ce)x^2-(3\,b^2cd-2\,b^3e)x}{2\,(cx+b)b^4x^2}$$

input
```
integrate((e*x+d)/x/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
(3*c^2*d - 2*b*c*e)*log(abs(x))/b^4 - (3*c^3*d - 2*b*c^2*e)*log(abs(c*x +
b))/(b^4*c) - 1/2*(b^3*d - 2*(3*b*c^2*d - 2*b^2*c*e)*x^2 - (3*b^2*c*d - 2*
b^3*e)*x)/((c*x + b)*b^4*x^2)
```

Mupad [B] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.24

$$\int \frac{d+ex}{x\,(bx+cx^2)^2}\,dx = -\frac{\frac{d}{2\,b}+\frac{x\,(2\,b\,e-3\,c\,d)}{2\,b^2}+\frac{c\,x^2\,(2\,b\,e-3\,c\,d)}{b^3}}{c\,x^3+b\,x^2} - \frac{2\,c\,\mathrm{atanh}\left(\frac{c\,(2\,b\,e-3\,c\,d)\,(b+2\,c\,x)}{b\,(3\,c^2\,d-2\,b\,c\,e)}\right)\,(2\,b\,e-3\,c\,d)}{b^4}$$

input
```
int((d + e*x)/(x*(b*x + c*x^2)^2),x)
```

output
```
- (d/(2*b) + (x*(2*b*e - 3*c*d))/(2*b^2) + (c*x^2*(2*b*e - 3*c*d))/b^3)/(b
*x^2 + c*x^3) - (2*c*atanh((c*(2*b*e - 3*c*d)*(b + 2*c*x))/(b*(3*c^2*d - 2
*b*c*e)))*(2*b*e - 3*c*d))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.95

$$\int \frac{d+ex}{x\,(bx+cx^2)^2}\,dx = \frac{4\log(cx+b)\,b^2ce\,x^2-6\log(cx+b)\,b\,c^2d\,x^2+4\log(cx+b)\,b\,c^2e\,x^3-6\log(cx+b)\,c^3d\,x^3-4\log(x)\,b^2ce}{2b^4x^2\,(c}$$

input
```
int((e*x+d)/x/(c*x^2+b*x)^2,x)
```

output
```
(4*log(b + c*x)*b**2*c*e*x**2 - 6*log(b + c*x)*b*c**2*d*x**2 + 4*log(b + c
*x)*b*c**2*e*x**3 - 6*log(b + c*x)*c**3*d*x**3 - 4*log(x)*b**2*c*e*x**2 +
6*log(x)*b*c**2*d*x**2 - 4*log(x)*b*c**2*e*x**3 + 6*log(x)*c**3*d*x**3 - b
**3*d - 2*b**3*e*x + 3*b**2*c*d*x + 4*b*c**2*e*x**3 - 6*c**3*d*x**3)/(2*b*
*4*x**2*(b + c*x))
```

3.50 $\int \frac{d+ex}{x^2(bx+cx^2)^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 113

$$\int \frac{d+ex}{x^2(bx+cx^2)^2} dx = -\frac{d}{3b^2x^3}+\frac{2cd-be}{2b^3x^2}-\frac{c(3cd-2be)}{b^4x}-\frac{c^2(cd-be)}{b^4(b+cx)} -\frac{c^2(4cd-3be)\log(x)}{b^5}+\frac{c^2(4cd-3be)\log(b+cx)}{b^5}$$

output
```
-1/3*d/b^2/x^3+1/2*(-b*e+2*c*d)/b^3/x^2-c*(-2*b*e+3*c*d)/b^4/x-c^2*(-b*e+c
*d)/b^4/(c*x+b)-c^2*(-3*b*e+4*c*d)*ln(x)/b^5+c^2*(-3*b*e+4*c*d)*ln(c*x+b)/
b^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.94

$$\int \frac{d+ex}{x^2(bx+cx^2)^2} dx = \frac{-\frac{2b^3d}{x^3}-\frac{3b^2(-2cd+be)}{x^2}+\frac{6bc(-3cd+2be)}{x}+\frac{6bc^2(-cd+be)}{b+cx}+6c^2(-4cd+3be)\log(x)+6c^2(4cd-3be)\log(b+cx)}{6b^5}$$

input
```
Integrate[(d + e*x)/(x^2*(b*x + c*x^2)^2),x]
```

output
```
((-2*b^3*d)/x^3 - (3*b^2*(-2*c*d + b*e))/x^2 + (6*b*c*(-3*c*d + 2*b*e))/x
+ (6*b*c^2*(-(c*d) + b*e))/(b + c*x) + 6*c^2*(-4*c*d + 3*b*e)*Log[x] + 6*c
^2*(4*c*d - 3*b*e)*Log[b + c*x])/(6*b^5)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x^2 (bx+cx^2)^2} \, dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{d+ex}{x^4(b+cx)^2}dx$$

$$\Big\downarrow \; 86$$

$$\int \left(-\frac{c^3(3be-4cd)}{b^5(b+cx)}+\frac{c^2(3be-4cd)}{b^5x}-\frac{c^3(be-cd)}{b^4(b+cx)^2}-\frac{c(2be-3cd)}{b^4x^2}+\frac{be-2cd}{b^3x^3}+\frac{d}{b^2x^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{c^2\log(x)(4cd-3be)}{b^5}+\frac{c^2(4cd-3be)\log(b+cx)}{b^5}-\frac{c^2(cd-be)}{b^4(b+cx)}-\frac{c(3cd-2be)}{b^4x}+\frac{2cd-be}{2b^3x^2}-\frac{d}{3b^2x^3}$$

input
```
Int[(d + e*x)/(x^2*(b*x + c*x^2)^2),x]
```

output
```
-1/3*d/(b^2*x^3) + (2*c*d - b*e)/(2*b^3*x^2) - (c*(3*c*d - 2*b*e))/(b^4*x)
 - (c^2*(c*d - b*e))/(b^4*(b + c*x)) - (c^2*(4*c*d - 3*b*e)*Log[x])/b^5 +
(c^2*(4*c*d - 3*b*e)*Log[b + c*x])/b^5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

method	result
default	$-\frac{c^{2}(3be-4cd)\ln(cx+b)}{b^{5}}+\frac{(be-cd)c^{2}}{b^{4}(cx+b)}-\frac{d}{3b^{2}x^{3}}-\frac{be-2cd}{2b^{3}x^{2}}+\frac{c^{2}(3be-4cd)\ln(x)}{b^{5}}+\frac{c(2be-3cd)}{b^{4}x}$
norman	$\frac{\frac{c\left(-3b\,c^{2}e+4d\,c^{3}\right)x^{4}}{b^{5}}-\frac{d}{3b}-\frac{(3be-4cd)x}{6b^{2}}+\frac{c(3be-4cd)x^{2}}{2b^{3}}}{(cx+b)x^{3}}+\frac{c^{2}(3be-4cd)\ln(x)}{b^{5}}-\frac{c^{2}(3be-4cd)\ln(cx+b)}{b^{5}}$
risch	$\frac{\frac{c^{2}(3be-4cd)x^{3}}{b^{4}}+\frac{c(3be-4cd)x^{2}}{2b^{3}}-\frac{(3be-4cd)x}{6b^{2}}-\frac{d}{3b}}{(cx+b)x^{3}}-\frac{3c^{2}\ln(cx+b)e}{b^{4}}+\frac{4c^{3}\ln(cx+b)d}{b^{5}}+\frac{3c^{2}\ln(-x)e}{b^{4}}-\frac{4c^{3}\ln(-x)d}{b^{5}}$
parallelrisch	$\frac{18\ln(x)x^{4}b\,c^{3}e-24\ln(x)x^{4}c^{4}d-18\ln(cx+b)x^{4}b\,c^{3}e+24\ln(cx+b)x^{4}c^{4}d+18\ln(x)x^{3}b^{2}c^{2}e-24\ln(x)x^{3}b\,c^{3}d-18\ln(cx+b)x^{3}b^{2}}{6b^{5}(cx+b)x^{3}}$

input
```
int((e*x+d)/x^2/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-c^2*(3*b*e-4*c*d)/b^5*ln(c*x+b)+(b*e-c*d)/b^4*c^2/(c*x+b)-1/3*d/b^2/x^3-1
/2*(b*e-2*c*d)/b^3/x^2+c^2*(3*b*e-4*c*d)/b^5*ln(x)+c*(2*b*e-3*c*d)/b^4/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.59

$$\int \frac{d+ex}{x^2\left(bx+cx^2\right)^2}\,dx = -\frac{2\,b^4d+6\,(4\,bc^3d-3\,b^2c^2e)x^3+3\,(4\,b^2c^2d-3\,b^3ce)x^2-(4\,b^3cd-3\,b^4e)x-6\,((4\,c^4d-3\,bc^3e)x^4+(4\,b}{6\,(b^5cx^4+b^6x^3)}$$

input
```
integrate((e*x+d)/x^2/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
-1/6*(2*b^4*d + 6*(4*b*c^3*d - 3*b^2*c^2*e)*x^3 + 3*(4*b^2*c^2*d - 3*b^3*c
*e)*x^2 - (4*b^3*c*d - 3*b^4*e)*x - 6*((4*c^4*d - 3*b*c^3*e)*x^4 + (4*b*c^
3*d - 3*b^2*c^2*e)*x^3)*log(c*x + b) + 6*((4*c^4*d - 3*b*c^3*e)*x^4 + (4*b
*c^3*d - 3*b^2*c^2*e)*x^3)*log(x))/(b^5*c*x^4 + b^6*x^3)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(104) = 208.

Time = 0.32 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.94

$$\int \frac{d+ex}{x^2\left(bx+cx^2\right)^2}\,dx = \frac{-2b^3d+x^3\cdot(18bc^2e-24c^3d)+x^2\cdot(9b^2ce-12bc^2d)+x(-3b^3e+4b^2cd)}{6b^5x^3+6b^4cx^4} + \frac{c^2\cdot(3be-4cd)\log\left(x+\frac{3b^2c^2e-4bc^3d-bc^2\cdot(3be-4cd)}{6bc^3e-8c^4d}\right)}{b^5} - \frac{c^2\cdot(3be-4cd)\log\left(x+\frac{3b^2c^2e-4bc^3d+bc^2\cdot(3be-4cd)}{6bc^3e-8c^4d}\right)}{b^5}$$

input
```
integrate((e*x+d)/x**2/(c*x**2+b*x)**2,x)
```

output
```
(-2*b**3*d + x**3*(18*b*c**2*e - 24*c**3*d) + x**2*(9*b**2*c*e - 12*b*c**2
*d) + x*(-3*b**3*e + 4*b**2*c*d))/(6*b**5*x**3 + 6*b**4*c*x**4) + c**2*(3*
b*e - 4*c*d)*log(x + (3*b**2*c**2*e - 4*b*c**3*d - b*c**2*(3*b*e - 4*c*d))
/(6*b*c**3*e - 8*c**4*d))/b**5 - c**2*(3*b*e - 4*c*d)*log(x + (3*b**2*c**2
*e - 4*b*c**3*d + b*c**2*(3*b*e - 4*c*d))/(6*b*c**3*e - 8*c**4*d))/b**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14

$$\int \frac{d+ex}{x^2\left(bx+cx^2\right)^2}\,dx = -\frac{2\,b^3d+6\,(4\,c^3d-3\,bc^2e)x^3+3\,(4\,bc^2d-3\,b^2ce)x^2-(4\,b^2cd-3\,b^3e)x}{6\,(b^4cx^4+b^5x^3)} + \frac{(4\,c^3d-3\,bc^2e)\log(cx+b)}{b^5} - \frac{(4\,c^3d-3\,bc^2e)\log(x)}{b^5}$$

input
```
integrate((e*x+d)/x^2/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-1/6*(2*b^3*d + 6*(4*c^3*d - 3*b*c^2*e)*x^3 + 3*(4*b*c^2*d - 3*b^2*c*e)*x^
2 - (4*b^2*c*d - 3*b^3*e)*x)/(b^4*c*x^4 + b^5*x^3) + (4*c^3*d - 3*b*c^2*e)
*log(c*x + b)/b^5 - (4*c^3*d - 3*b*c^2*e)*log(x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.19

$$\int \frac{d+ex}{x^2\left(bx+cx^2\right)^2}\,dx = -\frac{(4\,c^3d-3\,bc^2e)\log(|x|)}{b^5} + \frac{(4\,c^4d-3\,bc^3e)\log(|cx+b|)}{b^5c} - \frac{2\,b^4d+6\,(4\,bc^3d-3\,b^2c^2e)x^3+3\,(4\,b^2c^2d-3\,b^3ce)x^2-(4\,b^3cd-3\,b^4e)x}{6\,(cx+b)b^5x^3}$$

input
```
integrate((e*x+d)/x^2/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
-(4*c^3*d - 3*b*c^2*e)*log(abs(x))/b^5 + (4*c^4*d - 3*b*c^3*e)*log(abs(c*x
 + b))/(b^5*c) - 1/6*(2*b^4*d + 6*(4*b*c^3*d - 3*b^2*c^2*e)*x^3 + 3*(4*b^2
*c^2*d - 3*b^3*c*e)*x^2 - (4*b^3*c*d - 3*b^4*e)*x)/((c*x + b)*b^5*x^3)
```

Mupad [B] (verification not implemented)

Time = 5.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.17

$$\int \frac{d+ex}{x^2\left(bx+cx^2\right)^2}\,dx=\frac{2\,c^2\,\mathrm{atanh}\left(\frac{c^2\,\left(3\,b\,e-4\,c\,d\right)\,\left(b+2\,c\,x\right)}{b\,\left(4\,c^3\,d-3\,b\,c^2\,e\right)}\right)\,\left(3\,b\,e-4\,c\,d\right)}{b^5}-\frac{\frac{d}{3\,b}+\frac{x\,\left(3\,b\,e-4\,c\,d\right)}{6\,b^2}-\frac{c\,x^2\,\left(3\,b\,e-4\,c\,d\right)}{2\,b^3}-\frac{c^2\,x^3\,\left(3\,b\,e-4\,c\,d\right)}{b^4}}{c\,x^4+b\,x^3}$$

input
```
int((d + e*x)/(x^2*(b*x + c*x^2)^2),x)
```

output
```
(2*c^2*atanh((c^2*(3*b*e - 4*c*d)*(b + 2*c*x))/(b*(4*c^3*d - 3*b*c^2*e)))*
(3*b*e - 4*c*d))/b^5 - (d/(3*b) + (x*(3*b*e - 4*c*d))/(6*b^2) - (c*x^2*(3*
b*e - 4*c*d))/(2*b^3) - (c^2*x^3*(3*b*e - 4*c*d))/b^4)/(b*x^3 + c*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.70

$$\int \frac{d+ex}{x^2\left(bx+cx^2\right)^2}\,dx$$
$$=\frac{-18\log(cx+b)\,b^2c^2e\,x^3+24\log(cx+b)\,b\,c^3d\,x^3-18\log(cx+b)\,b\,c^3e\,x^4+24\log(cx+b)\,c^4d\,x^4+18\,\mathrm{lo}}{}$$

input
```
int((e*x+d)/x^2/(c*x^2+b*x)^2,x)
```

output
```
( - 18*log(b + c*x)*b**2*c**2*e*x**3 + 24*log(b + c*x)*b*c**3*d*x**3 - 18*
log(b + c*x)*b*c**3*e*x**4 + 24*log(b + c*x)*c**4*d*x**4 + 18*log(x)*b**2*
c**2*e*x**3 - 24*log(x)*b*c**3*d*x**3 + 18*log(x)*b*c**3*e*x**4 - 24*log(x
)*c**4*d*x**4 - 2*b**4*d - 3*b**4*e*x + 4*b**3*c*d*x + 9*b**3*c*e*x**2 - 1
2*b**2*c**2*d*x**2 - 18*b*c**3*e*x**4 + 24*c**4*d*x**4)/(6*b**5*x**3*(b +
c*x))
```

3.51 $\int \frac{x^6(d+ex)}{(bx+cx^2)^3} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 94

$$\int \frac{x^6(d+ex)}{(bx+cx^2)^3} \, dx = \frac{(cd-3be)x}{c^4} + \frac{ex^2}{2c^3} + \frac{b^3(cd-be)}{2c^5(b+cx)^2} - \frac{b^2(3cd-4be)}{c^5(b+cx)} - \frac{3b(cd-2be)\log(b+cx)}{c^5}$$

output
```
(-3*b*e+c*d)*x/c^4+1/2*e*x^2/c^3+1/2*b^3*(-b*e+c*d)/c^5/(c*x+b)^2-b^2*(-4*
b*e+3*c*d)/c^5/(c*x+b)-3*b*(-2*b*e+c*d)*ln(c*x+b)/c^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \frac{x^6(d+ex)}{(bx+cx^2)^3} \, dx = \frac{2c(cd-3be)x + c^2ex^2 + \frac{b^3(cd-be)}{(b+cx)^2} + \frac{2b^2(-3cd+4be)}{b+cx} + 6b(-cd+2be)\log(b+cx)}{2c^5}$$

input
```
Integrate[(x^6*(d + e*x))/(b*x + c*x^2)^3,x]
```

output
```
(2*c*(c*d - 3*b*e)*x + c^2*e*x^2 + (b^3*(c*d - b*e))/(b + c*x)^2 + (2*b^2*
(-3*c*d + 4*b*e))/(b + c*x) + 6*b*(-(c*d) + 2*b*e)*Log[b + c*x])/(2*c^5)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(d+ex)}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^3(d+ex)}{(b+cx)^3}dx$$

$$\Big\downarrow \; 86$$

$$\int \left(\frac{b^3(be-cd)}{c^4(b+cx)^3}-\frac{b^2(4be-3cd)}{c^4(b+cx)^2}+\frac{3b(2be-cd)}{c^4(b+cx)}+\frac{cd-3be}{c^4}+\frac{ex}{c^3}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{b^3(cd-be)}{2c^5(b+cx)^2}-\frac{b^2(3cd-4be)}{c^5(b+cx)}-\frac{3b(cd-2be)\log(b+cx)}{c^5}+\frac{x(cd-3be)}{c^4}+\frac{ex^2}{2c^3}$$

input
```
Int[(x^6*(d + e*x))/(b*x + c*x^2)^3,x]
```

output
```
((c*d - 3*b*e)*x)/c^4 + (e*x^2)/(2*c^3) + (b^3*(c*d - b*e))/(2*c^5*(b + c*
x)^2) - (b^2*(3*c*d - 4*b*e))/(c^5*(b + c*x)) - (3*b*(c*d - 2*b*e)*Log[b +
 c*x])/c^5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

method	result
default	$-\frac{-\frac{1}{2} c e\, x^{2}+3 b e x -c d x}{c^{4}}+\frac{3 b \left(2 b e -c d \right) \ln \left(c x +b \right)}{c^{5}}+\frac{b^{2} \left(4 b e -3 c d \right)}{c^{5} \left(c x +b \right)}-\frac{b^{3} \left(b e -c d \right)}{2 c^{5} \left(c x +b \right)^{2}}$
risch	$\frac{e\, x^{2}}{2 c^{3}}-\frac{3 b e x}{c^{4}}+\frac{d x}{c^{3}}+\frac{\left(4 e\, b^{3}-3 c d\, b^{2}\right) x +\frac{b^{3} \left(7 b e -5 c d \right)}{2 c}}{c^{4} \left(c x +b \right)^{2}}+\frac{6 b^{2} \ln \left(c x +b \right) e}{c^{5}}-\frac{3 b \ln \left(c x +b \right) d}{c^{4}}$
norman	$\frac{-\frac{\left(2 b e -c d \right) x^{5}}{c^{2}}+\frac{e\, x^{6}}{2 c}+\frac{2 b \left(6 e\, b^{2}-3 d b c \right) x^{3}}{c^{4}}+\frac{b^{2} \left(18 e\, b^{2}-9 d b c \right) x^{2}}{2 c^{5}}}{x^{2} \left(c x +b \right)^{2}}+\frac{3 b \left(2 b e -c d \right) \ln \left(c x +b \right)}{c^{5}}$
parallelrisch	$\frac{e\, x^{4} c^{4}+12 \ln \left(c x +b \right) x^{2} b^{2} c^{2} e -6 \ln \left(c x +b \right) x^{2} b\, c^{3} d -4 x^{3} b\, c^{3} e +2 c^{4} d\, x^{3}+24 \ln \left(c x +b \right) x\, b^{3} c e -12 \ln \left(c x +b \right) x\, b^{2} c^{2} d +12 \ln \left(c x +b \right) b^{4} e}{2 c^{5} \left(c x +b \right)^{2}}$

input
```
int(x^6*(e*x+d)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/c^4*(-1/2*c*e*x^2+3*b*e*x-c*d*x)+3*b/c^5*(2*b*e-c*d)*ln(c*x+b)+b^2/c^5*
(4*b*e-3*c*d)/(c*x+b)-1/2*b^3*(b*e-c*d)/c^5/(c*x+b)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.78

$$\int \frac{x^6(d+ex)}{(bx+cx^2)^3}\,dx$$
$$= \frac{c^4ex^4 - 5\,b^3cd + 7\,b^4e + 2\,(c^4d - 2\,bc^3e)x^3 + (4\,bc^3d - 11\,b^2c^2e)x^2 - 2\,(2\,b^2c^2d - b^3ce)x - 6\,(b^3cd - 2\,b^4}{2\,(c^7x^2 + 2\,bc^6x + b^2c^5)}$$

input
```
integrate(x^6*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/2*(c^4*e*x^4 - 5*b^3*c*d + 7*b^4*e + 2*(c^4*d - 2*b*c^3*e)*x^3 + (4*b*c^
3*d - 11*b^2*c^2*e)*x^2 - 2*(2*b^2*c^2*d - b^3*c*e)*x - 6*(b^3*c*d - 2*b^4
*e + (b*c^3*d - 2*b^2*c^2*e)*x^2 + 2*(b^2*c^2*d - 2*b^3*c*e)*x)*log(c*x +
b))/(c^7*x^2 + 2*b*c^6*x + b^2*c^5)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

$$\int \frac{x^6(d+ex)}{(bx+cx^2)^3}\,dx = \frac{3b(2be-cd)\log(b+cx)}{c^5} + x\left(-\frac{3be}{c^4} + \frac{d}{c^3}\right) + \frac{7b^4e - 5b^3cd + x(8b^3ce - 6b^2c^2d)}{2b^2c^5 + 4bc^6x + 2c^7x^2} + \frac{ex^2}{2c^3}$$

input
```
integrate(x**6*(e*x+d)/(c*x**2+b*x)**3,x)
```

output
```
3*b*(2*b*e - c*d)*log(b + c*x)/c**5 + x*(-3*b*e/c**4 + d/c**3) + (7*b**4*e
 - 5*b**3*c*d + x*(8*b**3*c*e - 6*b**2*c**2*d))/(2*b**2*c**5 + 4*b*c**6*x
+ 2*c**7*x**2) + e*x**2/(2*c**3)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \frac{x^6(d+ex)}{(bx+cx^2)^3}\,dx = -\frac{5\,b^3cd-7\,b^4e+2\,(3\,b^2c^2d-4\,b^3ce)x}{2\,(c^7x^2+2\,bc^6x+b^2c^5)} + \frac{cex^2+2\,(cd-3\,be)x}{2\,c^4} - \frac{3\,(bcd-2\,b^2e)\log{(cx+b)}}{c^5}$$

input
```
integrate(x^6*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(5*b^3*c*d - 7*b^4*e + 2*(3*b^2*c^2*d - 4*b^3*c*e)*x)/(c^7*x^2 + 2*b*
c^6*x + b^2*c^5) + 1/2*(c*e*x^2 + 2*(c*d - 3*b*e)*x)/c^4 - 3*(b*c*d - 2*b^
2*e)*log(c*x + b)/c^5
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.05

$$\int \frac{x^6(d+ex)}{(bx+cx^2)^3}\,dx = -\frac{3\,(bcd-2\,b^2e)\log{(|cx+b|)}}{c^5} + \frac{c^3ex^2+2\,c^3dx-6\,bc^2ex}{2\,c^6} - \frac{5\,b^3cd-7\,b^4e+2\,(3\,b^2c^2d-4\,b^3ce)x}{2\,(cx+b)^2c^5}$$

input
```
integrate(x^6*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
-3*(b*c*d - 2*b^2*e)*log(abs(c*x + b))/c^5 + 1/2*(c^3*e*x^2 + 2*c^3*d*x -
6*b*c^2*e*x)/c^6 - 1/2*(5*b^3*c*d - 7*b^4*e + 2*(3*b^2*c^2*d - 4*b^3*c*e)*
x)/((c*x + b)^2*c^5)
```

Mupad [B] (verification not implemented)

Time = 5.21 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

$$\int \frac{x^6(d+ex)}{(bx+cx^2)^3}\,dx = x\left(\frac{d}{c^3}-\frac{3\,b\,e}{c^4}\right)+\frac{x\,(4\,b^3\,e-3\,b^2\,c\,d)+\frac{7\,b^4\,e-5\,b^3\,c\,d}{2\,c}}{b^2\,c^4+2\,b\,c^5\,x+c^6\,x^2}+\frac{e\,x^2}{2\,c^3}+\frac{\ln(b+c\,x)\,(6\,b^2\,e-3\,b\,c\,d)}{c^5}$$

input
```
int((x^6*(d + e*x))/(b*x + c*x^2)^3,x)
```

output
```
x*(d/c^3 - (3*b*e)/c^4) + (x*(4*b^3*e - 3*b^2*c*d) + (7*b^4*e - 5*b^3*c*d)
/(2*c))/(b^2*c^4 + c^6*x^2 + 2*b*c^5*x) + (e*x^2)/(2*c^3) + (log(b + c*x)*
(6*b^2*e - 3*b*c*d))/c^5
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.86

$$\int \frac{x^6(d+ex)}{(bx+cx^2)^3}\,dx = \frac{12\log(cx+b)\,b^4e-6\log(cx+b)\,b^3cd+24\log(cx+b)\,b^3cex-12\log(cx+b)\,b^2c^2dx+12\log(cx+b)\,b^2 \cdots}{2c^5\,(c^2x^2+2b \cdots}$$

input
```
int(x^6*(e*x+d)/(c*x^2+b*x)^3,x)
```

output
```
(12*log(b + c*x)*b**4*e - 6*log(b + c*x)*b**3*c*d + 24*log(b + c*x)*b**3*c
*e*x - 12*log(b + c*x)*b**2*c**2*d*x + 12*log(b + c*x)*b**2*c**2*e*x**2 -
6*log(b + c*x)*b*c**3*d*x**2 + 6*b**4*e - 3*b**3*c*d - 12*b**2*c**2*e*x**2
 + 6*b*c**3*d*x**2 - 4*b*c**3*e*x**3 + 2*c**4*d*x**3 + c**4*e*x**4)/(2*c**
5*(b**2 + 2*b*c*x + c**2*x**2))
```

3.52 $\int \frac{x^5(d+ex)}{(bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 71

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^3}\,dx = \frac{ex}{c^3} - \frac{b^2(cd-be)}{2c^4(b+cx)^2} + \frac{b(2cd-3be)}{c^4(b+cx)} + \frac{(cd-3be)\log(b+cx)}{c^4}$$

output
```
e*x/c^3-1/2*b^2*(-b*e+c*d)/c^4/(c*x+b)^2+b*(-3*b*e+2*c*d)/c^4/(c*x+b)+(-3*
b*e+c*d)*ln(c*x+b)/c^4
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.06

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^3}\,dx = \frac{ex}{c^3} + \frac{-b^2cd+b^3e}{2c^4(b+cx)^2} + \frac{2bcd-3b^2e}{c^4(b+cx)} + \frac{(cd-3be)\log(b+cx)}{c^4}$$

input
```
Integrate[(x^5*(d + e*x))/(b*x + c*x^2)^3,x]
```

output
```
(e*x)/c^3 + (-(b^2*c*d) + b^3*e)/(2*c^4*(b + c*x)^2) + (2*b*c*d - 3*b^2*e)
/(c^4*(b + c*x)) + ((c*d - 3*b*e)*Log[b + c*x])/c^4
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \, 9$$

$$\int \frac{x^2(d+ex)}{(b+cx)^3}dx$$

$$\Big\downarrow \, 86$$

$$\int \left(-\frac{b^2(be-cd)}{c^3(b+cx)^3}+\frac{b(3be-2cd)}{c^3(b+cx)^2}+\frac{cd-3be}{c^3(b+cx)}+\frac{e}{c^3}\right)dx$$

$$\Big\downarrow \, 2009$$

$$-\frac{b^2(cd-be)}{2c^4(b+cx)^2}+\frac{b(2cd-3be)}{c^4(b+cx)}+\frac{(cd-3be)\log(b+cx)}{c^4}+\frac{ex}{c^3}$$

input
```
Int[(x^5*(d + e*x))/(b*x + c*x^2)^3,x]
```

output
```
(e*x)/c^3 - (b^2*(c*d - b*e))/(2*c^4*(b + c*x)^2) + (b*(2*c*d - 3*b*e))/(c
^4*(b + c*x)) + ((c*d - 3*b*e)*Log[b + c*x])/c^4
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

method	result
default	$\frac{ex}{c^3}+\frac{(-3be+cd)\ln(cx+b)}{c^4}-\frac{b(3be-2cd)}{c^4(cx+b)}+\frac{b^2(be-cd)}{2c^4(cx+b)^2}$
risch	$\frac{ex}{c^3}+\frac{(-3e\,b^2+2dbc)x-\frac{b^2(5be-3cd)}{2c}}{c^3(cx+b)^2}-\frac{3\ln(cx+b)be}{c^4}+\frac{\ln(cx+b)d}{c^3}$
norman	$\frac{\frac{e\,x^5}{c}-\frac{2b(3be-cd)x^3}{c^3}-\frac{b^2(9be-3cd)x^2}{2c^4}}{x^2(cx+b)^2}-\frac{(3be-cd)\ln(cx+b)}{c^4}$
parallelrisch	$-\frac{6\ln(cx+b)x^2b\,c^2e-2\ln(cx+b)x^2c^3d-2e\,x^3c^3+12\ln(cx+b)x\,b^2ce-4\ln(cx+b)xb\,c^2d+6\ln(cx+b)b^3e-2\ln(cx+b)b^2cd+12x}{2c^4(cx+b)^2}$

input
```
int(x^5*(e*x+d)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
e*x/c^3+(-3*b*e+c*d)*ln(c*x+b)/c^4-b/c^4*(3*b*e-2*c*d)/(c*x+b)+1/2*b^2*(b*
e-c*d)/c^4/(c*x+b)^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.85

$$\int\frac{x^5(d+ex)}{(bx+cx^2)^3}\,dx$$
$$=\frac{2\,c^3ex^3+4\,bc^2ex^2+3\,b^2cd-5\,b^3e+4\,(bc^2d-b^2ce)x+2\,(b^2cd-3\,b^3e+(c^3d-3\,bc^2e)x^2+2\,(bc^2d-3\,b}{2\,(c^6x^2+2\,bc^5x+b^2c^4)}$$

input
```
integrate(x^5*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/2*(2*c^3*e*x^3 + 4*b*c^2*e*x^2 + 3*b^2*c*d - 5*b^3*e + 4*(b*c^2*d - b^2*
c*e)*x + 2*(b^2*c*d - 3*b^3*e + (c^3*d - 3*b*c^2*e)*x^2 + 2*(b*c^2*d - 3*b
^2*c*e)*x)*log(c*x + b))/(c^6*x^2 + 2*b*c^5*x + b^2*c^4)
```

Sympy [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^3}\,dx = \frac{-5b^3e+3b^2cd+x(-6b^2ce+4bc^2d)}{2b^2c^4+4bc^5x+2c^6x^2} + \frac{ex}{c^3} - \frac{(3be-cd)\log(b+cx)}{c^4}$$

input
```
integrate(x**5*(e*x+d)/(c*x**2+b*x)**3,x)
```

output
```
(-5*b**3*e + 3*b**2*c*d + x*(-6*b**2*c*e + 4*b*c**2*d))/(2*b**2*c**4 + 4*b
*c**5*x + 2*c**6*x**2) + e*x/c**3 - (3*b*e - c*d)*log(b + c*x)/c**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^3}\,dx = \frac{3\,b^2cd-5\,b^3e+2\,(2\,bc^2d-3\,b^2ce)x}{2\,(c^6x^2+2\,bc^5x+b^2c^4)} + \frac{ex}{c^3} + \frac{(cd-3\,be)\log(cx+b)}{c^4}$$

input
```
integrate(x^5*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/2*(3*b^2*c*d - 5*b^3*e + 2*(2*b*c^2*d - 3*b^2*c*e)*x)/(c^6*x^2 + 2*b*c^5
*x + b^2*c^4) + e*x/c^3 + (c*d - 3*b*e)*log(c*x + b)/c^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^3}\,dx=\frac{ex}{c^3}+\frac{(cd-3\,be)\log\left(|cx+b|\right)}{c^4}+\frac{3\,b^2cd-5\,b^3e+2\,(2\,bc^2d-3\,b^2ce)x}{2\,(cx+b)^2c^4}$$

input
```
integrate(x^5*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
e*x/c^3 + (c*d - 3*b*e)*log(abs(c*x + b))/c^4 + 1/2*(3*b^2*c*d - 5*b^3*e +
 2*(2*b*c^2*d - 3*b^2*c*e)*x)/((c*x + b)^2*c^4)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.23

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^3}\,dx=\frac{e\,x}{c^3}-\frac{\ln\left(b+c\,x\right)\,\left(3\,b\,e-c\,d\right)}{c^4}-\frac{x\,\left(3\,b^2\,e-2\,b\,c\,d\right)+\frac{5\,b^3\,e-3\,b^2\,c\,d}{2\,c}}{b^2\,c^3+2\,b\,c^4\,x+c^5\,x^2}$$

input
```
int((x^5*(d + e*x))/(b*x + c*x^2)^3,x)
```

output
```
(e*x)/c^3 - (log(b + c*x)*(3*b*e - c*d))/c^4 - (x*(3*b^2*e - 2*b*c*d) + (5
*b^3*e - 3*b^2*c*d)/(2*c))/(b^2*c^3 + c^5*x^2 + 2*b*c^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.08

$$\int \frac{x^5(d+ex)}{(bx+cx^2)^3}\,dx$$
$$=\frac{-6\log(cx+b)\,b^3e+2\log(cx+b)\,b^2cd-12\log(cx+b)\,b^2cex+4\log(cx+b)\,b\,c^2dx-6\log(cx+b)\,b\,c^2e}{2c^4\left(c^2x^2+2bcx+b^2\right)}$$

input
```
int(x^5*(e*x+d)/(c*x^2+b*x)^3,x)
```

output
```
( - 6*log(b + c*x)*b**3*e + 2*log(b + c*x)*b**2*c*d - 12*log(b + c*x)*b**2
*c*e*x + 4*log(b + c*x)*b*c**2*d*x - 6*log(b + c*x)*b*c**2*e*x**2 + 2*log(
b + c*x)*c**3*d*x**2 - 3*b**3*e + b**2*c*d + 6*b*c**2*e*x**2 - 2*c**3*d*x*
*2 + 2*c**3*e*x**3)/(2*c**4*(b**2 + 2*b*c*x + c**2*x**2))
```

3.53 $\int \frac{x^4(d+ex)}{(bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 55

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^3}\,dx = \frac{b(cd-be)}{2c^3(b+cx)^2} - \frac{cd-2be}{c^3(b+cx)} + \frac{e\log(b+cx)}{c^3}$$

output
```
1/2*b*(-b*e+c*d)/c^3/(c*x+b)^2-(-2*b*e+c*d)/c^3/(c*x+b)+e*ln(c*x+b)/c^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^3}\,dx = \frac{3b^2e-2c^2dx-bc(d-4ex)+2e(b+cx)^2\log(b+cx)}{2c^3(b+cx)^2}$$

input
```
Integrate[(x^4*(d + e*x))/(b*x + c*x^2)^3,x]
```

output
```
(3*b^2*e - 2*c^2*d*x - b*c*(d - 4*e*x) + 2*e*(b + c*x)^2*Log[b + c*x])/(2*
c^3*(b + c*x)^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x(d+ex)}{(b+cx)^3}dx$$

$$\Big\downarrow \; 86$$

$$\int \left(\frac{cd-2be}{c^2(b+cx)^2}+\frac{b(be-cd)}{c^2(b+cx)^3}+\frac{e}{c^2(b+cx)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{cd-2be}{c^3(b+cx)}+\frac{b(cd-be)}{2c^3(b+cx)^2}+\frac{e\log(b+cx)}{c^3}$$

input
```
Int[(x^4*(d + e*x))/(b*x + c*x^2)^3,x]
```

output
```
(b*(c*d - b*e))/(2*c^3*(b + c*x)^2) - (c*d - 2*b*e)/(c^3*(b + c*x)) + (e*L
og[b + c*x])/c^3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

method	result	size
risch	$\frac{\frac{(2be-cd)x}{c^2}+\frac{b(3be-cd)}{2c^3}}{(cx+b)^2}+\frac{e\ln(cx+b)}{c^3}$	51
default	$\frac{e\ln(cx+b)}{c^3}-\frac{-2be+cd}{c^3(cx+b)}-\frac{b(be-cd)}{2c^3(cx+b)^2}$	54
norman	$\frac{\frac{(2be-cd)x^3}{c^2}+\frac{b(3be-cd)x^2}{2c^3}}{x^2(cx+b)^2}+\frac{e\ln(cx+b)}{c^3}$	59
parallelrisch	$\frac{2\ln(cx+b)x^2c^2e+4\ln(cx+b)xbce+2\ln(cx+b)b^2e+4bcex-2c^2dx+3e\,b^2-dbc}{2c^3(cx+b)^2}$	77

input
```
int(x^4*(e*x+d)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
(1/c^2*(2*b*e-c*d)*x+1/2*b*(3*b*e-c*d)/c^3)/(c*x+b)^2+e*ln(c*x+b)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^3}\,dx = -\frac{bcd-3\,b^2e+2\,(c^2d-2\,bce)x-2\,(c^2ex^2+2\,bcex+b^2e)\log(cx+b)}{2\,(c^5x^2+2\,bc^4x+b^2c^3)}$$

input
```
integrate(x^4*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(b*c*d - 3*b^2*e + 2*(c^2*d - 2*b*c*e)*x - 2*(c^2*e*x^2 + 2*b*c*e*x +
 b^2*e)*log(c*x + b))/(c^5*x^2 + 2*b*c^4*x + b^2*c^3)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^3}\,dx = \frac{3b^2e-bcd+x(4bce-2c^2d)}{2b^2c^3+4bc^4x+2c^5x^2}+\frac{e\log{(b+cx)}}{c^3}$$

input
```
integrate(x**4*(e*x+d)/(c*x**2+b*x)**3,x)
```

output
```
(3*b**2*e - b*c*d + x*(4*b*c*e - 2*c**2*d))/(2*b**2*c**3 + 4*b*c**4*x + 2*
c**5*x**2) + e*log(b + c*x)/c**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^3}\,dx = -\frac{bcd-3\,b^2e+2\,(c^2d-2\,bce)x}{2\,(c^5x^2+2\,bc^4x+b^2c^3)}+\frac{e\log{(cx+b)}}{c^3}$$

input
```
integrate(x^4*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(b*c*d - 3*b^2*e + 2*(c^2*d - 2*b*c*e)*x)/(c^5*x^2 + 2*b*c^4*x + b^2*
c^3) + e*log(c*x + b)/c^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^3}\,dx = \frac{e\log(|cx+b|)}{c^3} - \frac{2\,(cd-2\,be)x + \frac{bcd-3\,b^2e}{c}}{2\,(cx+b)^2c^2}$$

input
```
integrate(x^4*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
e*log(abs(c*x + b))/c^3 - 1/2*(2*(c*d - 2*b*e)*x + (b*c*d - 3*b^2*e)/c)/((
c*x + b)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^3}\,dx = \frac{\frac{3\,b^2\,e-b\,c\,d}{2\,c^3} + \frac{x\,(2\,b\,e-c\,d)}{c^2}}{b^2+2\,b\,c\,x+c^2\,x^2} + \frac{e\,\ln(b+c\,x)}{c^3}$$

input
```
int((x^4*(d + e*x))/(b*x + c*x^2)^3,x)
```

output
```
((3*b^2*e - b*c*d)/(2*c^3) + (x*(2*b*e - c*d))/c^2)/(b^2 + c^2*x^2 + 2*b*c
*x) + (e*log(b + c*x))/c^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.67

$$\int \frac{x^4(d+ex)}{(bx+cx^2)^3}\,dx = \frac{2\log(cx+b)\,b^3e + 4\log(cx+b)\,b^2cex + 2\log(cx+b)\,b\,c^2e\,x^2 + b^3e - 2b\,c^2e\,x^2 + c^3d\,x^2}{2b\,c^3\,(c^2x^2+2bcx+b^2)}$$

input
```
int(x^4*(e*x+d)/(c*x^2+b*x)^3,x)
```

output

```
(2*log(b + c*x)*b**3*e + 4*log(b + c*x)*b**2*c*e*x + 2*log(b + c*x)*b*c**2
*e*x**2 + b**3*e - 2*b*c**2*e*x**2 + c**3*d*x**2)/(2*b*c**3*(b**2 + 2*b*c*
x + c**2*x**2))
```

3.54 $\int \frac{x^3(d+ex)}{\left(bx+cx^2\right)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 28

$$\int \frac{x^3(d+ex)}{\left(bx+cx^2\right)^3}\,dx=-\frac{(d+ex)^2}{2(cd-be)(b+cx)^2}$$

output
```
-1/2*(e*x+d)^2/(-b*e+c*d)/(c*x+b)^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93

$$\int \frac{x^3(d+ex)}{\left(bx+cx^2\right)^3}\,dx=-\frac{be+c(d+2ex)}{2c^2(b+cx)^2}$$

input
```
Integrate[(x^3*(d + e*x))/(b*x + c*x^2)^3,x]
```

output
```
-1/2*(b*e + c*(d + 2*e*x))/(c^2*(b + c*x)^2)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {9, 48}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(d+ex)}{(bx+cx^2)^3}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{d+ex}{(b+cx)^3}dx$$

$$\Big\downarrow \; 48$$

$$-\frac{(d+ex)^2}{2(b+cx)^2(cd-be)}$$

input
```
Int[(x^3*(d + e*x))/(b*x + c*x^2)^3,x]
```

output
```
-1/2*(d + e*x)^2/((c*d - b*e)*(b + c*x)^2)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 48
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp
[(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] /; FreeQ[{
a, b, c, d, m, n}, x] && EqQ[m + n + 2, 0] && NeQ[m, -1]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
gosper	$-\frac{2cex+be+cd}{2(cx+b)^{2} c^{2}}$	25
parallelrisch	$\frac{-2cex-be-cd}{2c^{2} (cx+b)^{2}}$	27
risch	$\frac{-\frac{ex}{c}-\frac{be+cd}{2c^{2}}}{(cx+b)^{2}}$	29
default	$-\frac{e}{(cx+b)c^{2}}-\frac{-be+cd}{2c^{2} (cx+b)^{2}}$	35
orering	$-\frac{(2cex+be+cd)(cx+b)x^{3}}{2c^{2} (c\,x^{2}+bx)^{3}}$	37
norman	$\frac{-\frac{e\,x^{3}}{c}+\frac{(-be-cd)x^{2}}{2c^{2}}}{x^{2} (cx+b)^{2}}$	39

input
```
int(x^3*(e*x+d)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*c*e*x+b*e+c*d)/(c*x+b)^2/c^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{x^3(d+ex)}{(bx+cx^2)^3}\,dx = -\frac{2\,cex+cd+be}{2\,(c^4x^2+2\,bc^3x+b^2c^2)}$$

input
```
integrate(x^3*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*c*e*x + c*d + b*e)/(c^4*x^2 + 2*b*c^3*x + b^2*c^2)
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{x^3(d+ex)}{(bx+cx^2)^3}\,dx = \frac{-be-cd-2cex}{2b^2c^2+4bc^3x+2c^4x^2}$$

input
```
integrate(x**3*(e*x+d)/(c*x**2+b*x)**3,x)
```

output
```
(-b*e - c*d - 2*c*e*x)/(2*b**2*c**2 + 4*b*c**3*x + 2*c**4*x**2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.36

$$\int \frac{x^3(d+ex)}{(bx+cx^2)^3}\,dx = -\frac{2\,cex+cd+be}{2\,(c^4x^2+2\,bc^3x+b^2c^2)}$$

input
```
integrate(x^3*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*c*e*x + c*d + b*e)/(c^4*x^2 + 2*b*c^3*x + b^2*c^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.86

$$\int \frac{x^3(d+ex)}{(bx+cx^2)^3}\,dx = -\frac{2\,cex+cd+be}{2\,(cx+b)^2c^2}$$

input
```
integrate(x^3*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
-1/2*(2*c*e*x + c*d + b*e)/((c*x + b)^2*c^2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{x^3(d+ex)}{(bx+cx^2)^3}\,dx = -\frac{\frac{b\,e+c\,d}{2\,c^2}+\frac{e\,x}{c}}{b^2+2\,b\,c\,x+c^2\,x^2}$$

input
```
int((x^3*(d + e*x))/(b*x + c*x^2)^3,x)
```

output
```
-((b*e + c*d)/(2*c^2) + (e*x)/c)/(b^2 + c^2*x^2 + 2*b*c*x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.32

$$\int \frac{x^3(d+ex)}{(bx+cx^2)^3}\,dx = \frac{ce\,x^2-bd}{2bc\,(c^2x^2+2bcx+b^2)}$$

input
```
int(x^3*(e*x+d)/(c*x^2+b*x)^3,x)
```

output
```
( - b*d + c*e*x**2)/(2*b*c*(b**2 + 2*b*c*x + c**2*x**2))
```

3.55 $\int \frac{x^2(d+ex)}{(bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 57

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^3}\,dx=\frac{cd-be}{2bc(b+cx)^2}+\frac{d}{b^2(b+cx)}+\frac{d\log(x)}{b^3}-\frac{d\log(b+cx)}{b^3}$$

output
```
1/2*(-b*e+c*d)/b/c/(c*x+b)^2+d/b^2/(c*x+b)+d*ln(x)/b^3-d*ln(c*x+b)/b^3
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^3}\,dx=\frac{\frac{b(3bcd-b^2e+2c^2dx)}{c(b+cx)^2}+2d\log(x)-2d\log(b+cx)}{2b^3}$$

input
```
Integrate[(x^2*(d + e*x))/(b*x + c*x^2)^3,x]
```

output
```
((b*(3*b*c*d - b^2*e + 2*c^2*d*x))/(c*(b + c*x)^2) + 2*d*Log[x] - 2*d*Log[
b + c*x])/(2*b^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{d+ex}{x(b+cx)^3}dx$$

$$\Big\downarrow \text{ 86}$$

$$\int \left(-\frac{cd}{b^3(b+cx)}+\frac{d}{b^3x}-\frac{cd}{b^2(b+cx)^2}+\frac{be-cd}{b(b+cx)^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{d\log(b+cx)}{b^3}+\frac{d\log(x)}{b^3}+\frac{d}{b^2(b+cx)}+\frac{cd-be}{2bc(b+cx)^2}$$

input
```
Int[(x^2*(d + e*x))/(b*x + c*x^2)^3,x]
```

output
```
(c*d - b*e)/(2*b*c*(b + c*x)^2) + d/(b^2*(b + c*x)) + (d*Log[x])/b^3 - (d*
Log[b + c*x])/b^3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

method	result
risch	$\frac{\frac{cdx}{b^2}-\frac{be-3cd}{2bc}}{(cx+b)^2}+\frac{d\ln(-x)}{b^3}-\frac{d\ln(cx+b)}{b^3}$
default	$-\frac{be-cd}{2bc(cx+b)^2}-\frac{d\ln(cx+b)}{b^3}+\frac{d}{b^2(cx+b)}+\frac{d\ln(x)}{b^3}$
norman	$\frac{\frac{(be-2cd)x^3}{b^2}+\frac{c(be-3cd)x^4}{2b^3}}{x^2(cx+b)^2}+\frac{d\ln(x)}{b^3}-\frac{d\ln(cx+b)}{b^3}$
parallelrisch	$\frac{2\ln(x)x^2c^2d-2\ln(cx+b)x^2c^2d+4\ln(x)xbcd-4\ln(cx+b)xbcd+bce\,x^2-3c^2d\,x^2+2b^2d\ln(x)-2\ln(cx+b)b^2d+2b^2ex-4bcdx}{2b^3(cx+b)^2}$

input
```
int(x^2*(e*x+d)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
(c*d*x/b^2-1/2*(b*e-3*c*d)/b/c)/(c*x+b)^2+d/b^3*ln(-x)-d*ln(c*x+b)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.91

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^3}\,dx$$
$$=\frac{2\,bc^2dx+3\,b^2cd-b^3e-2\,(c^3dx^2+2\,bc^2dx+b^2cd)\log(cx+b)+2\,(c^3dx^2+2\,bc^2dx+b^2cd)\log(x)}{2\,(b^3c^3x^2+2\,b^4c^2x+b^5c)}$$

input
```
integrate(x^2*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
1/2*(2*b*c^2*d*x + 3*b^2*c*d - b^3*e - 2*(c^3*d*x^2 + 2*b*c^2*d*x + b^2*c*
d)*log(c*x + b) + 2*(c^3*d*x^2 + 2*b*c^2*d*x + b^2*c*d)*log(x))/(b^3*c^3*x
^2 + 2*b^4*c^2*x + b^5*c)
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^3}\,dx = \frac{-b^2e+3bcd+2c^2dx}{2b^4c+4b^3c^2x+2b^2c^3x^2}+\frac{d\left(\log{(x)}-\log{\left(\frac{b}{c}+x\right)}\right)}{b^3}$$

input
```
integrate(x**2*(e*x+d)/(c*x**2+b*x)**3,x)
```

output
```
(-b**2*e + 3*b*c*d + 2*c**2*d*x)/(2*b**4*c + 4*b**3*c**2*x + 2*b**2*c**3*x
**2) + d*(log(x) - log(b/c + x))/b**3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.19

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^3}\,dx = \frac{2\,c^2dx+3\,bcd-b^2e}{2\,(b^2c^3x^2+2\,b^3c^2x+b^4c)}-\frac{d\log{(cx+b)}}{b^3}+\frac{d\log{(x)}}{b^3}$$

input
```
integrate(x^2*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/2*(2*c^2*d*x + 3*b*c*d - b^2*e)/(b^2*c^3*x^2 + 2*b^3*c^2*x + b^4*c) - d*
log(c*x + b)/b^3 + d*log(x)/b^3
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^3}\,dx = -\frac{d\log(|cx+b|)}{b^3} + \frac{d\log(|x|)}{b^3} + \frac{2\,bc^2dx + 3\,b^2cd - b^3e}{2\,(cx+b)^2b^3c}$$

input
```
integrate(x^2*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
-d*log(abs(c*x + b))/b^3 + d*log(abs(x))/b^3 + 1/2*(2*b*c^2*d*x + 3*b^2*c*
d - b^3*e)/((c*x + b)^2*b^3*c)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^3}\,dx = -\frac{\frac{b\,e-3\,c\,d}{2\,b\,c} - \frac{c\,d\,x}{b^2}}{b^2+2\,b\,c\,x+c^2\,x^2} - \frac{2\,d\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)}{b^3}$$

input
```
int((x^2*(d + e*x))/(b*x + c*x^2)^3,x)
```

output
```
- ((b*e - 3*c*d)/(2*b*c) - (c*d*x)/b^2)/(b^2 + c^2*x^2 + 2*b*c*x) - (2*d*a
tanh((2*c*x)/b + 1))/b^3
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 121, normalized size of antiderivative = 2.12

$$\int \frac{x^2(d+ex)}{(bx+cx^2)^3}\,dx = \frac{-2\log(cx+b)\,b^2cd - 4\log(cx+b)\,b\,c^2dx - 2\log(cx+b)\,c^3d\,x^2 + 2\log(x)\,b^2cd + 4\log(x)\,b\,c^2dx + 2\log(}{2b^3c\,(c^2x^2+2bcx+b^2)}$$

input
```
int(x^2*(e*x+d)/(c*x^2+b*x)^3,x)
```

output
```
( - 2*log(b + c*x)*b**2*c*d - 4*log(b + c*x)*b*c**2*d*x - 2*log(b + c*x)*c
**3*d*x**2 + 2*log(x)*b**2*c*d + 4*log(x)*b*c**2*d*x + 2*log(x)*c**3*d*x**
2 - b**3*e + 2*b**2*c*d - c**3*d*x**2)/(2*b**3*c*(b**2 + 2*b*c*x + c**2*x*
*2))
```

3.56 $\int \frac{x(d+ex)}{(bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 88

$$\int \frac{x(d+ex)}{(bx+cx^2)^3}\,dx = -\frac{d}{b^3x} - \frac{cd-be}{2b^2(b+cx)^2} - \frac{2cd-be}{b^3(b+cx)} - \frac{(3cd-be)\log(x)}{b^4} + \frac{(3cd-be)\log(b+cx)}{b^4}$$

output
```
-d/b^3/x-1/2*(-b*e+c*d)/b^2/(c*x+b)^2-(-b*e+2*c*d)/b^3/(c*x+b)-(-b*e+3*c*d
)*ln(x)/b^4+(-b*e+3*c*d)*ln(c*x+b)/b^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \frac{x(d+ex)}{(bx+cx^2)^3}\,dx = \frac{-\frac{2bd}{x} + \frac{b^2(-cd+be)}{(b+cx)^2} + \frac{2b(-2cd+be)}{b+cx} + 2(-3cd+be)\log(x) + 2(3cd-be)\log(b+cx)}{2b^4}$$

input
```
Integrate[(x*(d + e*x))/(b*x + c*x^2)^3,x]
```

output
```
((-2*b*d)/x + (b^2*(-(c*d) + b*e))/(b + c*x)^2 + (2*b*(-2*c*d + b*e))/(b +
 c*x) + 2*(-3*c*d + b*e)*Log[x] + 2*(3*c*d - b*e)*Log[b + c*x])/(2*b^4)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(d+ex)}{(bx+cx^2)^3}dx$$
$$\Big\downarrow 9$$
$$\int \frac{d+ex}{x^2(b+cx)^3}dx$$
$$\Big\downarrow 86$$
$$\int \left(\frac{be-3cd}{b^4x}-\frac{c(be-3cd)}{b^4(b+cx)}-\frac{c(be-2cd)}{b^3(b+cx)^2}+\frac{d}{b^3x^2}-\frac{c(be-cd)}{b^2(b+cx)^3}\right)dx$$
$$\Big\downarrow 2009$$
$$-\frac{\log(x)(3cd-be)}{b^4}+\frac{(3cd-be)\log(b+cx)}{b^4}-\frac{2cd-be}{b^3(b+cx)}-\frac{d}{b^3x}-\frac{cd-be}{2b^2(b+cx)^2}$$

input
```
Int[(x*(d + e*x))/(b*x + c*x^2)^3,x]
```

output
```
-(d/(b^3*x)) - (c*d - b*e)/(2*b^2*(b + c*x)^2) - (2*c*d - b*e)/(b^3*(b + c
*x)) - ((3*c*d - b*e)*Log[x])/b^4 + ((3*c*d - b*e)*Log[b + c*x])/b^4
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.94

method	result
default	$-\frac{(be-3cd)\ln(cx+b)}{b^4}+\frac{be-2cd}{b^3(cx+b)}+\frac{be-cd}{2b^2(cx+b)^2}-\frac{d}{b^3x}+\frac{(be-3cd)\ln(x)}{b^4}$
norman	$\frac{-\frac{dx}{b}+\frac{2c(-be+3cd)x^3}{b^3}+\frac{c^2(-3be+9cd)x^4}{2b^4}}{x^2(cx+b)^2}+\frac{(be-3cd)\ln(x)}{b^4}-\frac{(be-3cd)\ln(cx+b)}{b^4}$
risch	$\frac{\frac{c(be-3cd)x^2}{b^3}+\frac{3(be-3cd)x}{2b^2}-\frac{d}{b}}{(cx+b)^2x}-\frac{\ln(cx+b)e}{b^3}+\frac{3\ln(cx+b)cd}{b^4}+\frac{\ln(-x)e}{b^3}-\frac{3\ln(-x)cd}{b^4}$
parallelrisch	$\frac{2\ln(x)x^3bc^2e-6\ln(x)x^3c^3d-2\ln(cx+b)x^3bc^2e+6\ln(cx+b)x^3c^3d+4\ln(x)x^2b^2ce-12\ln(x)x^2bc^2d-4\ln(cx+b)x^2b^2ce+12\ln}{2b^4x}$

input
```
int(x*(e*x+d)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-(b*e-3*c*d)/b^4*ln(c*x+b)+(b*e-2*c*d)/b^3/(c*x+b)+1/2*(b*e-c*d)/b^2/(c*x+
b)^2-d/b^3/x+(b*e-3*c*d)/b^4*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(86) = 172$.

Time = 0.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.22

$$\int \frac{x(d+ex)}{(bx+cx^2)^3}\,dx = -\frac{2\,b^3d + 2\,(3\,bc^2d - b^2ce)x^2 + 3\,(3\,b^2cd - b^3e)x - 2\,((3\,c^3d - bc^2e)x^3 + 2\,(3\,bc^2d - b^2ce)x^2 + (3\,b^2cd -}{2\,(b^4c^2x^3 + 2\,b^5cx^2 +}$$

input
```
integrate(x*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*b^3*d + 2*(3*b*c^2*d - b^2*c*e)*x^2 + 3*(3*b^2*c*d - b^3*e)*x - 2*
((3*c^3*d - b*c^2*e)*x^3 + 2*(3*b*c^2*d - b^2*c*e)*x^2 + (3*b^2*c*d - b^3*
e)*x)*log(c*x + b) + 2*((3*c^3*d - b*c^2*e)*x^3 + 2*(3*b*c^2*d - b^2*c*e)*
x^2 + (3*b^2*c*d - b^3*e)*x)*log(x))/(b^4*c^2*x^3 + 2*b^5*c*x^2 + b^6*x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(76) = 152$.

Time = 0.30 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.91

$$\begin{aligned}\int \frac{x(d+ex)}{(bx+cx^2)^3}\,dx &= \frac{-2b^2d + x^2\cdot(2bce - 6c^2d) + x(3b^2e - 9bcd)}{2b^5x + 4b^4cx^2 + 2b^3c^2x^3} \\ &\quad + \frac{(be-3cd)\log\left(x + \frac{b^2e-3bcd-b(be-3cd)}{2bce-6c^2d}\right)}{b^4} \\ &\quad - \frac{(be-3cd)\log\left(x + \frac{b^2e-3bcd+b(be-3cd)}{2bce-6c^2d}\right)}{b^4}\end{aligned}$$

input
```
integrate(x*(e*x+d)/(c*x**2+b*x)**3,x)
```

output
```
(-2*b**2*d + x**2*(2*b*c*e - 6*c**2*d) + x*(3*b**2*e - 9*b*c*d))/(2*b**5*x
 + 4*b**4*c*x**2 + 2*b**3*c**2*x**3) + (b*e - 3*c*d)*log(x + (b**2*e - 3*b
*c*d - b*(b*e - 3*c*d))/(2*b*c*e - 6*c**2*d))/b**4 - (b*e - 3*c*d)*log(x +
 (b**2*e - 3*b*c*d + b*(b*e - 3*c*d))/(2*b*c*e - 6*c**2*d))/b**4
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.18

$$\int \frac{x(d+ex)}{(bx+cx^2)^3}\,dx = -\frac{2\,b^2d+2\,(3\,c^2d-bce)x^2+3\,(3\,bcd-b^2e)x}{2\,(b^3c^2x^3+2\,b^4cx^2+b^5x)} + \frac{(3\,cd-be)\log(cx+b)}{b^4} - \frac{(3\,cd-be)\log(x)}{b^4}$$

input
```
integrate(x*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(2*b^2*d + 2*(3*c^2*d - b*c*e)*x^2 + 3*(3*b*c*d - b^2*e)*x)/(b^3*c^2*
x^3 + 2*b^4*c*x^2 + b^5*x) + (3*c*d - b*e)*log(c*x + b)/b^4 - (3*c*d - b*e
)*log(x)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.17

$$\int \frac{x(d+ex)}{(bx+cx^2)^3}\,dx = -\frac{(3\,cd-be)\log(|x|)}{b^4} + \frac{(3\,c^2d-bce)\log(|cx+b|)}{b^4c} - \frac{2\,b^3d+2\,(3\,bc^2d-b^2ce)x^2+3\,(3\,b^2cd-b^3e)x}{2\,(cx+b)^2b^4x}$$

input
```
integrate(x*(e*x+d)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
-(3*c*d - b*e)*log(abs(x))/b^4 + (3*c^2*d - b*c*e)*log(abs(c*x + b))/(b^4*
c) - 1/2*(2*b^3*d + 2*(3*b*c^2*d - b^2*c*e)*x^2 + 3*(3*b^2*c*d - b^3*e)*x)
/((c*x + b)^2*b^4*x)
```

Mupad [B] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.95

$$\int \frac{x(d+ex)}{(bx+cx^2)^3}\,dx = \frac{\frac{3\,x\,(b\,e-3\,c\,d)}{2\,b^2}-\frac{d}{b}+\frac{c\,x^2\,(b\,e-3\,c\,d)}{b^3}}{b^2\,x+2\,b\,c\,x^2+c^2\,x^3}-\frac{2\,\mathrm{atanh}\left(\frac{2\,c\,x}{b}+1\right)\,(b\,e-3\,c\,d)}{b^4}$$

input
```
int((x*(d + e*x))/(b*x + c*x^2)^3,x)
```

output
```
((3*x*(b*e - 3*c*d))/(2*b^2) - d/b + (c*x^2*(b*e - 3*c*d))/b^3)/(b^2*x + c
^2*x^3 + 2*b*c*x^2) - (2*atanh((2*c*x)/b + 1)*(b*e - 3*c*d))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.53

$$\int \frac{x(d+ex)}{(bx+cx^2)^3}\,dx$$
$$= \frac{-2\log(cx+b)\,b^3ex+6\log(cx+b)\,b^2cdx-4\log(cx+b)\,b^2ce\,x^2+12\log(cx+b)\,b\,c^2d\,x^2-2\log(cx+b)}{}$$

input
```
int(x*(e*x+d)/(c*x^2+b*x)^3,x)
```

output
```
( - 2*log(b + c*x)*b**3*e*x + 6*log(b + c*x)*b**2*c*d*x - 4*log(b + c*x)*b
**2*c*e*x**2 + 12*log(b + c*x)*b*c**2*d*x**2 - 2*log(b + c*x)*b*c**2*e*x**
3 + 6*log(b + c*x)*c**3*d*x**3 + 2*log(x)*b**3*e*x - 6*log(x)*b**2*c*d*x +
 4*log(x)*b**2*c*e*x**2 - 12*log(x)*b*c**2*d*x**2 + 2*log(x)*b*c**2*e*x**3
 - 6*log(x)*c**3*d*x**3 - 2*b**3*d + 2*b**3*e*x - 6*b**2*c*d*x - b*c**2*e*
x**3 + 3*c**3*d*x**3)/(2*b**4*x*(b**2 + 2*b*c*x + c**2*x**2))
```

3.57 $\int \frac{d+ex}{\left(bx+cx^2\right)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 110

$$\int \frac{d+ex}{\left(bx+cx^2\right)^3}\,dx = -\frac{d}{2b^3x^2}+\frac{3cd-be}{b^4x}+\frac{c(cd-be)}{2b^3(b+cx)^2}+\frac{c(3cd-2be)}{b^4(b+cx)}+\frac{3c(2cd-be)\log(x)}{b^5}-\frac{3c(2cd-be)\log(b+cx)}{b^5}$$

output
```
-1/2*d/b^3/x^2+(-b*e+3*c*d)/b^4/x+1/2*c*(-b*e+c*d)/b^3/(c*x+b)^2+c*(-2*b*e
+3*c*d)/b^4/(c*x+b)+3*c*(-b*e+2*c*d)*ln(x)/b^5-3*c*(-b*e+2*c*d)*ln(c*x+b)/
b^5
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int \frac{d+ex}{\left(bx+cx^2\right)^3}\,dx = \frac{-\frac{b\left(-12c^3dx^3+6bc^2x^2(-3d+ex)+b^3(d+2ex)+b^2cx(-4d+9ex)\right)}{x^2(b+cx)^2}+6c(2cd-be)\log(x)+6c(-2cd+be)\log(b+cx)}{2b^5}$$

input
```
Integrate[(d + e*x)/(b*x + c*x^2)^3,x]
```

output
```
(-((b*(-12*c^3*d*x^3 + 6*b*c^2*x^2*(-3*d + e*x) + b^3*(d + 2*e*x) + b^2*c*
x*(-4*d + 9*e*x)))/(x^2*(b + c*x)^2)) + 6*c*(2*c*d - b*e)*Log[x] + 6*c*(-2
*c*d + b*e)*Log[b + c*x])/(2*b^5)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {1141, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{1141}$$

$$c^3\int \left(\frac{d}{b^3c^3x^3}+\frac{3(2cd-be)}{b^5c^2x}-\frac{3(2cd-be)}{b^5c(b+cx)}-\frac{3cd-be}{b^4c^3x^2}-\frac{3cd-2be}{b^4c(b+cx)^2}-\frac{cd-be}{b^3c(b+cx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$c^3\left(\frac{3\log(x)(2cd-be)}{b^5c^2}-\frac{3(2cd-be)\log(b+cx)}{b^5c^2}+\frac{3cd-be}{b^4c^3x}+\frac{3cd-2be}{b^4c^2(b+cx)}-\frac{d}{2b^3c^3x^2}+\frac{cd-be}{2b^3c^2(b+cx)^2}\right)$$

input
```
Int[(d + e*x)/(b*x + c*x^2)^3,x]
```

output
```
c^3*(-1/2*d/(b^3*c^3*x^2) + (3*c*d - b*e)/(b^4*c^3*x) + (c*d - b*e)/(2*b^3
*c^2*(b + c*x)^2) + (3*c*d - 2*b*e)/(b^4*c^2*(b + c*x)) + (3*(2*c*d - b*e)
*Log[x])/(b^5*c^2) - (3*(2*c*d - b*e)*Log[b + c*x])/(b^5*c^2))
```

Defintions of rubi rules used

rule 1141
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[1/c^p   Int[ExpandIntegrand[
(d + e*x)^m*(b/2 - q/2 + c*x)^p*(b/2 + q/2 + c*x)^p, x], x], x] /; EqQ[p, -
1] ||  !FractionalPowerFactorQ[q]] /; FreeQ[{a, b, c, d, e}, x] && ILtQ[p,
0] && IntegerQ[m] && NiceSqrtQ[b^2 - 4*a*c]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96

method	result
default	$-\frac{c(2be-3cd)}{b^4(cx+b)}-\frac{(be-cd)c}{2b^3(cx+b)^2}+\frac{3c(be-2cd)\ln(cx+b)}{b^5}-\frac{d}{2b^3x^2}-\frac{be-3cd}{b^4x}-\frac{3c(be-2cd)\ln(x)}{b^5}$
norman	$\frac{-\frac{d}{2b}-\frac{(be-2cd)x}{b^2}+\frac{2c\left(3bce-6c^2d\right)x^3}{b^4}+\frac{c^2\left(9bce-18c^2d\right)x^4}{2b^5}}{x^2(cx+b)^2}-\frac{3c(be-2cd)\ln(x)}{b^5}+\frac{3c(be-2cd)\ln(cx+b)}{b^5}$
risch	$\frac{-\frac{3c^2(be-2cd)x^3}{b^4}-\frac{9c(be-2cd)x^2}{2b^3}-\frac{(be-2cd)x}{b^2}-\frac{d}{2b}}{x^2(cx+b)^2}-\frac{3c\ln(x)e}{b^4}+\frac{6c^2\ln(x)d}{b^5}+\frac{3c\ln(-cx-b)e}{b^4}-\frac{6c^2\ln(-cx-b)d}{b^5}$
parallelrisch	$-\frac{6\ln(x)x^4bc^3e-12\ln(x)x^4c^4d-6\ln(cx+b)x^4bc^3e+12\ln(cx+b)x^4c^4d+12\ln(x)x^3b^2c^2e-24\ln(x)x^3bc^3d-12\ln(cx+b)x^3b^2}{}$

input
```
int((e*x+d)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-c*(2*b*e-3*c*d)/b^4/(c*x+b)-1/2*(b*e-c*d)/b^3*c/(c*x+b)^2+3*c*(b*e-2*c*d)
/b^5*ln(c*x+b)-1/2*d/b^3/x^2-(b*e-3*c*d)/b^4/x-3*c*(b*e-2*c*d)/b^5*ln(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(106) = 212$.

Time = 0.08 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.13

$$\int \frac{d+ex}{(bx+cx^2)^3}\,dx = -\frac{b^4d - 6\,(2\,bc^3d - b^2c^2e)x^3 - 9\,(2\,b^2c^2d - b^3ce)x^2 - 2\,(2\,b^3cd - b^4e)x + 6\,((2\,c^4d - bc^3e)x^4 + 2\,(2\,bc^3d -}{2\,(b^5c}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
-1/2*(b^4*d - 6*(2*b*c^3*d - b^2*c^2*e)*x^3 - 9*(2*b^2*c^2*d - b^3*c*e)*x^
2 - 2*(2*b^3*c*d - b^4*e)*x + 6*((2*c^4*d - b*c^3*e)*x^4 + 2*(2*b*c^3*d -
b^2*c^2*e)*x^3 + (2*b^2*c^2*d - b^3*c*e)*x^2)*log(c*x + b) - 6*((2*c^4*d -
 b*c^3*e)*x^4 + 2*(2*b*c^3*d - b^2*c^2*e)*x^3 + (2*b^2*c^2*d - b^3*c*e)*x^
2)*log(x))/(b^5*c^2*x^4 + 2*b^6*c*x^3 + b^7*x^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(104) = 208$.

Time = 0.36 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.99

$$\begin{aligned}\int \frac{d+ex}{(bx+cx^2)^3}\,dx &= \frac{-b^3d + x^3(-6bc^2e + 12c^3d) + x^2(-9b^2ce + 18bc^2d) + x(-2b^3e + 4b^2cd)}{2b^6x^2 + 4b^5cx^3 + 2b^4c^2x^4} \\ &\quad - \frac{3c(be-2cd)\log\left(x + \frac{3b^2ce - 6bc^2d - 3bc(be-2cd)}{6bc^2e - 12c^3d}\right)}{b^5} \\ &\quad + \frac{3c(be-2cd)\log\left(x + \frac{3b^2ce - 6bc^2d + 3bc(be-2cd)}{6bc^2e - 12c^3d}\right)}{b^5}\end{aligned}$$

input
```
integrate((e*x+d)/(c*x**2+b*x)**3,x)
```

output
```
(-b**3*d + x**3*(-6*b*c**2*e + 12*c**3*d) + x**2*(-9*b**2*c*e + 18*b*c**2*
d) + x*(-2*b**3*e + 4*b**2*c*d))/(2*b**6*x**2 + 4*b**5*c*x**3 + 2*b**4*c**
2*x**4) - 3*c*(b*e - 2*c*d)*log(x + (3*b**2*c*e - 6*b*c**2*d - 3*b*c*(b*e
- 2*c*d))/(6*b*c**2*e - 12*c**3*d))/b**5 + 3*c*(b*e - 2*c*d)*log(x + (3*b*
*2*c*e - 6*b*c**2*d + 3*b*c*(b*e - 2*c*d))/(6*b*c**2*e - 12*c**3*d))/b**5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.24

$$\int \frac{d+ex}{(bx+cx^2)^3}\,dx = -\frac{b^3d-6\,(2\,c^3d-bc^2e)x^3-9\,(2\,bc^2d-b^2ce)x^2-2\,(2\,b^2cd-b^3e)x}{2\,(b^4c^2x^4+2\,b^5cx^3+b^6x^2)} - \frac{3\,(2\,c^2d-bce)\log(cx+b)}{b^5} + \frac{3\,(2\,c^2d-bce)\log(x)}{b^5}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/2*(b^3*d - 6*(2*c^3*d - b*c^2*e)*x^3 - 9*(2*b*c^2*d - b^2*c*e)*x^2 - 2*
(2*b^2*c*d - b^3*e)*x)/(b^4*c^2*x^4 + 2*b^5*c*x^3 + b^6*x^2) - 3*(2*c^2*d
- b*c*e)*log(c*x + b)/b^5 + 3*(2*c^2*d - b*c*e)*log(x)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

$$\int \frac{d+ex}{(bx+cx^2)^3}\,dx = \frac{3\,(2\,c^2d-bce)\log(|x|)}{b^5} - \frac{3\,(2\,c^3d-bc^2e)\log(|cx+b|)}{b^5c} + \frac{12\,c^3dx^3-6\,bc^2ex^3+18\,bc^2dx^2-9\,b^2cex^2+4\,b^2cdx-2\,b^3ex-b^3d}{2\,(cx^2+bx)^2b^4}$$

input
```
integrate((e*x+d)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
3*(2*c^2*d - b*c*e)*log(abs(x))/b^5 - 3*(2*c^3*d - b*c^2*e)*log(abs(c*x +
b))/(b^5*c) + 1/2*(12*c^3*d*x^3 - 6*b*c^2*e*x^3 + 18*b*c^2*d*x^2 - 9*b^2*c
*e*x^2 + 4*b^2*c*d*x - 2*b^3*e*x - b^3*d)/((c*x^2 + b*x)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 5.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.20

$$\int \frac{d+ex}{(bx+cx^2)^3}\,dx = -\frac{\frac{d}{2\,b}+\frac{x\,(b\,e-2\,c\,d)}{b^2}+\frac{9\,c\,x^2\,(b\,e-2\,c\,d)}{2\,b^3}+\frac{3\,c^2\,x^3\,(b\,e-2\,c\,d)}{b^4}}{b^2\,x^2+2\,b\,c\,x^3+c^2\,x^4} - \frac{6\,c\,\mathrm{atanh}\left(\frac{3\,c\,(b\,e-2\,c\,d)\,(b+2\,c\,x)}{b\,(6\,c^2\,d-3\,b\,c\,e)}\right)\,(b\,e-2\,c\,d)}{b^5}$$

input
```
int((d + e*x)/(b*x + c*x^2)^3,x)
```

output
```
- (d/(2*b) + (x*(b*e - 2*c*d))/b^2 + (9*c*x^2*(b*e - 2*c*d))/(2*b^3) + (3*
c^2*x^3*(b*e - 2*c*d))/b^4)/(b^2*x^2 + c^2*x^4 + 2*b*c*x^3) - (6*c*atanh((
3*c*(b*e - 2*c*d)*(b + 2*c*x))/(b*(6*c^2*d - 3*b*c*e)))*(b*e - 2*c*d))/b^5
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.39

$$\int \frac{d+ex}{(bx+cx^2)^3}\,dx = \frac{6\log(cx+b)\,b^3ce\,x^2-12\log(cx+b)\,b^2c^2d\,x^2+12\log(cx+b)\,b^2c^2e\,x^3-24\log(cx+b)\,b\,c^3d\,x^3+6\log(c}{}$$

input
```
int((e*x+d)/(c*x^2+b*x)^3,x)
```

output
```
(6*log(b + c*x)*b**3*c*e*x**2 - 12*log(b + c*x)*b**2*c**2*d*x**2 + 12*log(
b + c*x)*b**2*c**2*e*x**3 - 24*log(b + c*x)*b*c**3*d*x**3 + 6*log(b + c*x)
*b*c**3*e*x**4 - 12*log(b + c*x)*c**4*d*x**4 - 6*log(x)*b**3*c*e*x**2 + 12
*log(x)*b**2*c**2*d*x**2 - 12*log(x)*b**2*c**2*e*x**3 + 24*log(x)*b*c**3*d
*x**3 - 6*log(x)*b*c**3*e*x**4 + 12*log(x)*c**4*d*x**4 - b**4*d - 2*b**4*e
*x + 4*b**3*c*d*x - 6*b**3*c*e*x**2 + 12*b**2*c**2*d*x**2 + 3*b*c**3*e*x**
4 - 6*c**4*d*x**4)/(2*b**5*x**2*(b**2 + 2*b*c*x + c**2*x**2))
```

3.58 $\int \frac{d+ex}{x(bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 140

$$\int \frac{d+ex}{x\left(bx+cx^2\right)^3}\,dx=-\frac{d}{3b^3x^3}+\frac{3cd-be}{2b^4x^2}-\frac{3c(2cd-be)}{b^5x}-\frac{c^2(cd-be)}{2b^4(b+cx)^2}-\frac{c^2(4cd-3be)}{b^5(b+cx)}-\frac{2c^2(5cd-3be)\log(x)}{b^6}+\frac{2c^2(5cd-3be)\log(b+cx)}{b^6}$$

output
```
-1/3*d/b^3/x^3+1/2*(-b*e+3*c*d)/b^4/x^2-3*c*(-b*e+2*c*d)/b^5/x-1/2*c^2*(-b
*e+c*d)/b^4/(c*x+b)^2-c^2*(-3*b*e+4*c*d)/b^5/(c*x+b)-2*c^2*(-3*b*e+5*c*d)*
ln(x)/b^6+2*c^2*(-3*b*e+5*c*d)*ln(c*x+b)/b^6
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.92

$$\int \frac{d+ex}{x\left(bx+cx^2\right)^3}\,dx = \frac{\frac{b\left(-60c^4dx^4+18bc^3x^3(-5d+2ex)-b^4(2d+3ex)+b^3cx(5d+12ex)+2b^2c^2x^2(-10d+27ex)\right)}{x^3(b+cx)^2}+12c^2(-5cd+3be)\log(x)+12c^2(5cd}{6b^6}$$

input
```
Integrate[(d + e*x)/(x*(b*x + c*x^2)^3),x]
```

output
```
((b*(-60*c^4*d*x^4 + 18*b*c^3*x^3*(-5*d + 2*e*x) - b^4*(2*d + 3*e*x) + b^3
*c*x*(5*d + 12*e*x) + 2*b^2*c^2*x^2*(-10*d + 27*e*x)))/(x^3*(b + c*x)^2) +
 12*c^2*(-5*c*d + 3*b*e)*Log[x] + 12*c^2*(5*c*d - 3*b*e)*Log[b + c*x])/(6*
b^6)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{d+ex}{x\left(bx+cx^2\right)^3}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{d+ex}{x^4(b+cx)^3}dx$$

$$\Big\downarrow \; 86$$

$$\int \left(-\frac{2c^3(3be-5cd)}{b^6(b+cx)}+\frac{2c^2(3be-5cd)}{b^6x}-\frac{c^3(3be-4cd)}{b^5(b+cx)^2}-\frac{3c(be-2cd)}{b^5x^2}-\frac{c^3(be-cd)}{b^4(b+cx)^3}+\frac{be-3cd}{b^4x^3}+\frac{d}{b^3x^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2c^2\log(x)(5cd-3be)}{b^6}+\frac{2c^2(5cd-3be)\log(b+cx)}{b^6}-\frac{c^2(4cd-3be)}{b^5(b+cx)}-\frac{3c(2cd-be)}{b^5x}-\frac{c^2(cd-be)}{2b^4(b+cx)^2}+\frac{3cd-be}{2b^4x^2}-\frac{d}{3b^3x^3}$$

input
```
Int[(d + e*x)/(x*(b*x + c*x^2)^3),x]
```

output
```
-1/3*d/(b^3*x^3) + (3*c*d - b*e)/(2*b^4*x^2) - (3*c*(2*c*d - b*e))/(b^5*x)
 - (c^2*(c*d - b*e))/(2*b^4*(b + c*x)^2) - (c^2*(4*c*d - 3*b*e))/(b^5*(b +
 c*x)) - (2*c^2*(5*c*d - 3*b*e)*Log[x])/b^6 + (2*c^2*(5*c*d - 3*b*e)*Log[b
 + c*x])/b^6
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.94

method	result
default	$-\frac{2c^2(3be-5cd)\ln(cx+b)}{b^6}+\frac{c^2(3be-4cd)}{b^5(cx+b)}+\frac{(be-cd)c^2}{2b^4(cx+b)^2}-\frac{d}{3b^3x^3}-\frac{be-3cd}{2b^4x^2}+\frac{3c(be-2cd)}{b^5x}+\frac{2c^2(3be-5cd)\ln(x)}{b^6}$
norman	$\frac{\frac{\left(9b\,c^4e-15c^5d\right)x^3}{b^4c^2}-\frac{d}{3b}-\frac{(3be-5cd)x}{6b^2}+\frac{2c(3be-5cd)x^2}{3b^3}+\frac{2\left(3b\,c^4e-5c^5d\right)x^4}{b^5c}}{(cx+b)^2x^3}+\frac{2c^2(3be-5cd)\ln(x)}{b^6}-\frac{2c^2(3be-5cd)\ln(cx+b)}{b^6}$
risch	$\frac{\frac{2c^3(3be-5cd)x^4}{b^5}+\frac{3c^2(3be-5cd)x^3}{b^4}+\frac{2c(3be-5cd)x^2}{3b^3}-\frac{(3be-5cd)x}{6b^2}-\frac{d}{3b}}{(cx+b)^2x^3}+\frac{6c^2\ln(-x)e}{b^5}-\frac{10c^3\ln(-x)d}{b^6}-\frac{6c^2\ln(cx+b)e}{b^5}+$
parallelrisch	$36\ln(x)x^5b\,c^6e-60\ln(x)x^5c^7d-36\ln(cx+b)x^5b\,c^6e+60\ln(cx+b)x^5c^7d+72\ln(x)x^4b^2c^5e-120\ln(x)x^4b\,c^6d-72\ln(cx+b)x^4b$

input
```
int((e*x+d)/x/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*c^2*(3*b*e-5*c*d)/b^6*ln(c*x+b)+c^2*(3*b*e-4*c*d)/b^5/(c*x+b)+1/2*(b*e-
c*d)/b^4*c^2/(c*x+b)^2-1/3*d/b^3/x^3-1/2*(b*e-3*c*d)/b^4/x^2+3*c*(b*e-2*c*
d)/b^5/x+2*c^2*(3*b*e-5*c*d)/b^6*ln(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.88

$$\int \frac{d+ex}{x(bx+cx^2)^3}\,dx = -\frac{2\,b^5d+12\,(5\,bc^4d-3\,b^2c^3e)x^4+18\,(5\,b^2c^3d-3\,b^3c^2e)x^3+4\,(5\,b^3c^2d-3\,b^4ce)x^2-(5\,b^4cd-3\,b^5e)x-$$

input
```
integrate((e*x+d)/x/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
-1/6*(2*b^5*d + 12*(5*b*c^4*d - 3*b^2*c^3*e)*x^4 + 18*(5*b^2*c^3*d - 3*b^3
*c^2*e)*x^3 + 4*(5*b^3*c^2*d - 3*b^4*c*e)*x^2 - (5*b^4*c*d - 3*b^5*e)*x -
12*((5*c^5*d - 3*b*c^4*e)*x^5 + 2*(5*b*c^4*d - 3*b^2*c^3*e)*x^4 + (5*b^2*c
^3*d - 3*b^3*c^2*e)*x^3)*log(c*x + b) + 12*((5*c^5*d - 3*b*c^4*e)*x^5 + 2*
(5*b*c^4*d - 3*b^2*c^3*e)*x^4 + (5*b^2*c^3*d - 3*b^3*c^2*e)*x^3)*log(x))/(
b^6*c^2*x^5 + 2*b^7*c*x^4 + b^8*x^3)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.87

$$\int \frac{d+ex}{x(bx+cx^2)^3}\,dx = \frac{-2b^4d+x^4\cdot(36bc^3e-60c^4d)+x^3\cdot(54b^2c^2e-90bc^3d)+x^2\cdot(12b^3ce-20b^2c^2d)+x(-3b^4e+5b^3cd)}{6b^7x^3+12b^6cx^4+6b^5c^2x^5} + \frac{2c^2\cdot(3be-5cd)\log\left(x+\frac{6b^2c^2e-10bc^3d-2bc^2\cdot(3be-5cd)}{12bc^3e-20c^4d}\right)}{b^6} - \frac{2c^2\cdot(3be-5cd)\log\left(x+\frac{6b^2c^2e-10bc^3d+2bc^2\cdot(3be-5cd)}{12bc^3e-20c^4d}\right)}{b^6}$$

input
```
integrate((e*x+d)/x/(c*x**2+b*x)**3,x)
```

output
```
(-2*b**4*d + x**4*(36*b*c**3*e - 60*c**4*d) + x**3*(54*b**2*c**2*e - 90*b*
c**3*d) + x**2*(12*b**3*c*e - 20*b**2*c**2*d) + x*(-3*b**4*e + 5*b**3*c*d)
)/(6*b**7*x**3 + 12*b**6*c*x**4 + 6*b**5*c**2*x**5) + 2*c**2*(3*b*e - 5*c*
d)*log(x + (6*b**2*c**2*e - 10*b*c**3*d - 2*b*c**2*(3*b*e - 5*c*d))/(12*b*
c**3*e - 20*c**4*d))/b**6 - 2*c**2*(3*b*e - 5*c*d)*log(x + (6*b**2*c**2*e
- 10*b*c**3*d + 2*b*c**2*(3*b*e - 5*c*d))/(12*b*c**3*e - 20*c**4*d))/b**6
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.18

$$\int \frac{d+ex}{x\,(bx+cx^2)^3}\,dx = -\frac{2\,b^4d+12\,(5\,c^4d-3\,bc^3e)x^4+18\,(5\,bc^3d-3\,b^2c^2e)x^3+4\,(5\,b^2c^2d-3\,b^3ce)x^2-(5\,b^3cd-3\,b^4e)x}{6\,(b^5c^2x^5+2\,b^6cx^4+b^7x^3)} + \frac{2\,(5\,c^3d-3\,bc^2e)\log{(cx+b)}}{b^6} - \frac{2\,(5\,c^3d-3\,bc^2e)\log{(x)}}{b^6}$$

input
```
integrate((e*x+d)/x/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/6*(2*b^4*d + 12*(5*c^4*d - 3*b*c^3*e)*x^4 + 18*(5*b*c^3*d - 3*b^2*c^2*e
)*x^3 + 4*(5*b^2*c^2*d - 3*b^3*c*e)*x^2 - (5*b^3*c*d - 3*b^4*e)*x)/(b^5*c^
2*x^5 + 2*b^6*c*x^4 + b^7*x^3) + 2*(5*c^3*d - 3*b*c^2*e)*log(c*x + b)/b^6
- 2*(5*c^3*d - 3*b*c^2*e)*log(x)/b^6
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.14

$$\int \frac{d+ex}{x\,(bx+cx^2)^3}\,dx = -\frac{2\,(5\,c^3d-3\,bc^2e)\log{(|x|)}}{b^6} + \frac{2\,(5\,c^4d-3\,bc^3e)\log{(|cx+b|)}}{b^6c} - \frac{2\,b^5d+12\,(5\,bc^4d-3\,b^2c^3e)x^4+18\,(5\,b^2c^3d-3\,b^3c^2e)x^3+4\,(5\,b^3c^2d-3\,b^4ce)x^2-(5\,b^4cd-3\,b^5e)x}{6\,(cx+b)^2b^6x^3}$$

input
```
integrate((e*x+d)/x/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
-2*(5*c^3*d - 3*b*c^2*e)*log(abs(x))/b^6 + 2*(5*c^4*d - 3*b*c^3*e)*log(abs
(c*x + b))/(b^6*c) - 1/6*(2*b^5*d + 12*(5*b*c^4*d - 3*b^2*c^3*e)*x^4 + 18*
(5*b^2*c^3*d - 3*b^3*c^2*e)*x^3 + 4*(5*b^3*c^2*d - 3*b^4*c*e)*x^2 - (5*b^4
*c*d - 3*b^5*e)*x)/((c*x + b)^2*b^6*x^3)
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.16

$$\int \frac{d+ex}{x\left(bx+cx^2\right)^3}\,dx = \frac{\frac{2\,c\,x^2\,(3\,b\,e-5\,c\,d)}{3\,b^3}-\frac{x\,(3\,b\,e-5\,c\,d)}{6\,b^2}-\frac{d}{3\,b}+\frac{3\,c^2\,x^3\,(3\,b\,e-5\,c\,d)}{b^4}+\frac{2\,c^3\,x^4\,(3\,b\,e-5\,c\,d)}{b^5}}{b^2\,x^3+2\,b\,c\,x^4+c^2\,x^5} + \frac{4\,c^2\,\mathrm{atanh}\left(\frac{2\,c^2\,(3\,b\,e-5\,c\,d)\,(b+2\,c\,x)}{b\,(10\,c^3\,d-6\,b\,c^2\,e)}\right)\,(3\,b\,e-5\,c\,d)}{b^6}$$

input
```
int((d + e*x)/(x*(b*x + c*x^2)^3),x)
```

output
```
((2*c*x^2*(3*b*e - 5*c*d))/(3*b^3) - (x*(3*b*e - 5*c*d))/(6*b^2) - d/(3*b)
 + (3*c^2*x^3*(3*b*e - 5*c*d))/b^4 + (2*c^3*x^4*(3*b*e - 5*c*d))/b^5)/(b^2
*x^3 + c^2*x^5 + 2*b*c*x^4) + (4*c^2*atanh((2*c^2*(3*b*e - 5*c*d)*(b + 2*c
*x))/(b*(10*c^3*d - 6*b*c^2*e)))*(3*b*e - 5*c*d))/b^6
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.08

$$\int \frac{d+ex}{x\left(bx+cx^2\right)^3}\,dx = \frac{-36\log(cx+b)\,b^3c^2e\,x^3+60\log(cx+b)\,b^2c^3d\,x^3-72\log(cx+b)\,b^2c^3e\,x^4+120\log(cx+b)\,b\,c^4d\,x^4-3}{}$$

input
```
int((e*x+d)/x/(c*x^2+b*x)^3,x)
```

output

```
( - 36*log(b + c*x)*b**3*c**2*e*x**3 + 60*log(b + c*x)*b**2*c**3*d*x**3 -
72*log(b + c*x)*b**2*c**3*e*x**4 + 120*log(b + c*x)*b*c**4*d*x**4 - 36*log
(b + c*x)*b*c**4*e*x**5 + 60*log(b + c*x)*c**5*d*x**5 + 36*log(x)*b**3*c**
2*e*x**3 - 60*log(x)*b**2*c**3*d*x**3 + 72*log(x)*b**2*c**3*e*x**4 - 120*l
og(x)*b*c**4*d*x**4 + 36*log(x)*b*c**4*e*x**5 - 60*log(x)*c**5*d*x**5 - 2*
b**5*d - 3*b**5*e*x + 5*b**4*c*d*x + 12*b**4*c*e*x**2 - 20*b**3*c**2*d*x**
2 + 36*b**3*c**2*e*x**3 - 60*b**2*c**3*d*x**3 - 18*b*c**4*e*x**5 + 30*c**5
*d*x**5)/(6*b**6*x**3*(b**2 + 2*b*c*x + c**2*x**2))
```

3.59 $\int x^{3/2}(A+Bx)\left(bx+cx^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2}{7}Abx^{7/2}+\frac{2}{9}(bB+Ac)x^{9/2}+\frac{2}{11}Bcx^{11/2}$$

output
```
2/7*A*b*x^(7/2)+2/9*(A*c+B*b)*x^(9/2)+2/11*B*c*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.79

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2}{693}x^{7/2}\left(99Ab+77bBx+77Acx+63Bcx^2\right)$$

input
```
Integrate[x^(3/2)*(A + B*x)*(b*x + c*x^2),x]
```

output
```
(2*x^(7/2)*(99*A*b + 77*b*B*x + 77*A*c*x + 63*B*c*x^2))/693
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{9}$$

$$\int x^{5/2}(A+Bx)(b+cx)dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(x^{7/2}(Ac+bB)+Abx^{5/2}+Bcx^{9/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{9}x^{9/2}(Ac+bB)+\frac{2}{7}Abx^{7/2}+\frac{2}{11}Bcx^{11/2}$$

input
```
Int[x^(3/2)*(A + B*x)*(b*x + c*x^2),x]
```

output
```
(2*A*b*x^(7/2))/7 + (2*(b*B + A*c)*x^(9/2))/9 + (2*B*c*x^(11/2))/11
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}\left(63Bc\,x^{2}+77Acx+77Bbx+99Ab\right)}{693}$	28
derivativedivides	$\frac{2Ab\,x^{\frac{7}{2}}}{7}+\frac{2(Ac+Bb)x^{\frac{9}{2}}}{9}+\frac{2Bc\,x^{\frac{11}{2}}}{11}$	28
default	$\frac{2Ab\,x^{\frac{7}{2}}}{7}+\frac{2(Ac+Bb)x^{\frac{9}{2}}}{9}+\frac{2Bc\,x^{\frac{11}{2}}}{11}$	28
trager	$\frac{2x^{\frac{7}{2}}\left(63Bc\,x^{2}+77Acx+77Bbx+99Ab\right)}{693}$	28
risch	$\frac{2x^{\frac{7}{2}}\left(63Bc\,x^{2}+77Acx+77Bbx+99Ab\right)}{693}$	28
orering	$\frac{2(63Bc\,x^{2}+77Acx+77Bbx+99Ab)x^{\frac{5}{2}}\left(c\,x^{2}+bx\right)}{693(cx+b)}$	44

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
2/693*x^(7/2)*(63*B*c*x^2+77*A*c*x+77*B*b*x+99*A*b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2}{693}\left(63\,Bcx^5+99\,Abx^3+77\,(Bb+Ac)x^4\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x),x, algorithm="fricas")
```

output
```
2/693*(63*B*c*x^5 + 99*A*b*x^3 + 77*(B*b + A*c)*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2Abx^{\frac{7}{2}}}{7}+\frac{2Acx^{\frac{9}{2}}}{9}+\frac{2Bbx^{\frac{9}{2}}}{9}+\frac{2Bcx^{\frac{11}{2}}}{11}$$

input
```
integrate(x**(3/2)*(B*x+A)*(c*x**2+b*x),x)
```

output
```
2*A*b*x**(7/2)/7 + 2*A*c*x**(9/2)/9 + 2*B*b*x**(9/2)/9 + 2*B*c*x**(11/2)/1
1
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2}{11}\,Bcx^{\frac{11}{2}}+\frac{2}{7}\,Abx^{\frac{7}{2}}+\frac{2}{9}\,(Bb+Ac)x^{\frac{9}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x),x, algorithm="maxima")
```

output
```
2/11*B*c*x^(11/2) + 2/7*A*b*x^(7/2) + 2/9*(B*b + A*c)*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2}{11}\,Bcx^{\frac{11}{2}}+\frac{2}{9}\,Bbx^{\frac{9}{2}}+\frac{2}{9}\,Acx^{\frac{9}{2}}+\frac{2}{7}\,Abx^{\frac{7}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x),x, algorithm="giac")
```

output
```
2/11*B*c*x^(11/2) + 2/9*B*b*x^(9/2) + 2/9*A*c*x^(9/2) + 2/7*A*b*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2\,x^{7/2}\,\left(99\,A\,b+77\,A\,c\,x+77\,B\,b\,x+63\,B\,c\,x^2\right)}{693}$$

input
```
int(x^(3/2)*(b*x + c*x^2)*(A + B*x),x)
```

output
```
(2*x^(7/2)*(99*A*b + 77*A*c*x + 77*B*b*x + 63*B*c*x^2))/693
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2\sqrt{x}\,x^3(63bc\,x^2+77acx+77b^2x+99ab)}{693}$$

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x),x)
```

output
```
(2*sqrt(x)*x**3*(99*a*b + 77*a*c*x + 77*b**2*x + 63*b*c*x**2))/693
```

3.60 $\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2}{5}Abx^{5/2}+\frac{2}{7}(bB+Ac)x^{7/2}+\frac{2}{9}Bcx^{9/2}$$

output
```
2/5*A*b*x^(5/2)+2/7*(A*c+B*b)*x^(7/2)+2/9*B*c*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2}{315}x^{5/2}(9A(7b+5cx)+5Bx(9b+7cx))$$

input
```
Integrate[Sqrt[x]*(A + B*x)*(b*x + c*x^2),x]
```

output
```
(2*x^(5/2)*(9*A*(7*b + 5*c*x) + 5*B*x*(9*b + 7*c*x)))/315
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{3/2}(A+Bx)(b+cx)dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(x^{5/2}(Ac+bB)+Abx^{3/2}+Bcx^{7/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{7}x^{7/2}(Ac+bB)+\frac{2}{5}Abx^{5/2}+\frac{2}{9}Bcx^{9/2}$$

input
```
Int[Sqrt[x]*(A + B*x)*(b*x + c*x^2),x]
```

output
```
(2*A*b*x^(5/2))/5 + (2*(b*B + A*c)*x^(7/2))/7 + (2*B*c*x^(9/2))/9
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}\left(35Bc\,x^{2}+45Acx+45Bbx+63Ab\right)}{315}$	28
derivativedivides	$\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2(Ac+Bb)x^{\frac{7}{2}}}{7}+\frac{2Bc\,x^{\frac{9}{2}}}{9}$	28
default	$\frac{2Ab\,x^{\frac{5}{2}}}{5}+\frac{2(Ac+Bb)x^{\frac{7}{2}}}{7}+\frac{2Bc\,x^{\frac{9}{2}}}{9}$	28
trager	$\frac{2x^{\frac{5}{2}}\left(35Bc\,x^{2}+45Acx+45Bbx+63Ab\right)}{315}$	28
risch	$\frac{2x^{\frac{5}{2}}\left(35Bc\,x^{2}+45Acx+45Bbx+63Ab\right)}{315}$	28
orering	$\frac{2\left(35Bc\,x^{2}+45Acx+45Bbx+63Ab\right)x^{\frac{3}{2}}\left(c\,x^{2}+bx\right)}{315(cx+b)}$	44

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
2/315*x^(5/2)*(35*B*c*x^2+45*A*c*x+45*B*b*x+63*A*b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2}{315}\left(35\,Bcx^4+63\,Abx^2+45\,(Bb+Ac)x^3\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x),x, algorithm="fricas")
```

output
```
2/315*(35*B*c*x^4 + 63*A*b*x^2 + 45*(B*b + A*c)*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.95

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2Abx^{\frac{5}{2}}}{5}+\frac{2Bcx^{\frac{9}{2}}}{9}+\frac{2x^{\frac{7}{2}}(Ac+Bb)}{7}$$

input
```
integrate(x**(1/2)*(B*x+A)*(c*x**2+b*x),x)
```

output
```
2*A*b*x**(5/2)/5 + 2*B*c*x**(9/2)/9 + 2*x**(7/2)*(A*c + B*b)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2}{9}\,Bcx^{\frac{9}{2}}+\frac{2}{5}\,Abx^{\frac{5}{2}}+\frac{2}{7}\,(Bb+Ac)x^{\frac{7}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x),x, algorithm="maxima")
```

output
```
2/9*B*c*x^(9/2) + 2/5*A*b*x^(5/2) + 2/7*(B*b + A*c)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2}{9}\,Bcx^{\frac{9}{2}}+\frac{2}{7}\,Bbx^{\frac{7}{2}}+\frac{2}{7}\,Acx^{\frac{7}{2}}+\frac{2}{5}\,Abx^{\frac{5}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x),x, algorithm="giac")
```

output
```
2/9*B*c*x^(9/2) + 2/7*B*b*x^(7/2) + 2/7*A*c*x^(7/2) + 2/5*A*b*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2\,x^{5/2}\,\left(63\,A\,b+45\,A\,c\,x+45\,B\,b\,x+35\,B\,c\,x^2\right)}{315}$$

input
```
int(x^(1/2)*(b*x + c*x^2)*(A + B*x),x)
```

output
```
(2*x^(5/2)*(63*A*b + 45*A*c*x + 45*B*b*x + 35*B*c*x^2))/315
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)\,dx=\frac{2\sqrt{x}\,x^2(35bc\,x^2+45acx+45b^2x+63ab)}{315}$$

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x),x)
```

output
```
(2*sqrt(x)*x**2*(63*a*b + 45*a*c*x + 45*b**2*x + 35*b*c*x**2))/315
```

3.61 $\int \frac{(A+Bx)\left(bx+cx^2\right)}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 39

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{\sqrt{x}}\,dx=\frac{2}{3}Abx^{3/2}+\frac{2}{5}(bB+Ac)x^{5/2}+\frac{2}{7}Bcx^{7/2}$$

output
```
2/3*A*b*x^(3/2)+2/5*(A*c+B*b)*x^(5/2)+2/7*B*c*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{\sqrt{x}}\,dx=\frac{2}{105}x^{3/2}(7A(5b+3cx)+3Bx(7b+5cx))$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2))/Sqrt[x],x]
```

output
```
(2*x^(3/2)*(7*A*(5*b + 3*c*x) + 3*B*x*(7*b + 5*c*x)))/105
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{\sqrt{x}}dx$$
$$\Big\downarrow \text{ 9}$$
$$\int \sqrt{x}(A+Bx)(b+cx)dx$$
$$\Big\downarrow \text{ 85}$$
$$\int \left(x^{3/2}(Ac+bB)+Ab\sqrt{x}+Bcx^{5/2}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{2}{5}x^{5/2}(Ac+bB)+\frac{2}{3}Abx^{3/2}+\frac{2}{7}Bcx^{7/2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2))/Sqrt[x],x]
```

output
```
(2*A*b*x^(3/2))/3 + (2*(b*B + A*c)*x^(5/2))/5 + (2*B*c*x^(7/2))/7
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}\left(15Bc\,x^{2}+21Acx+21Bbx+35Ab\right)}{105}$	28
derivativedivides	$\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2(Ac+Bb)x^{\frac{5}{2}}}{5}+\frac{2Bc\,x^{\frac{7}{2}}}{7}$	28
default	$\frac{2Ab\,x^{\frac{3}{2}}}{3}+\frac{2(Ac+Bb)x^{\frac{5}{2}}}{5}+\frac{2Bc\,x^{\frac{7}{2}}}{7}$	28
trager	$\frac{2x^{\frac{3}{2}}\left(15Bc\,x^{2}+21Acx+21Bbx+35Ab\right)}{105}$	28
risch	$\frac{2x^{\frac{3}{2}}\left(15Bc\,x^{2}+21Acx+21Bbx+35Ab\right)}{105}$	28
orering	$\frac{2\sqrt{x}\,\left(15Bc\,x^{2}+21Acx+21Bbx+35Ab\right)\left(c\,x^{2}+bx\right)}{105(cx+b)}$	44

input
```
int((B*x+A)*(c*x^2+b*x)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105*x^(3/2)*(15*B*c*x^2+21*A*c*x+21*B*b*x+35*A*b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx)\,(bx+cx^2)}{\sqrt{x}}\,dx = \frac{2}{105}\left(15\,Bcx^3+35\,Abx+21\,(Bb+Ac)x^2\right)\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*B*c*x^3 + 35*A*b*x + 21*(B*b + A*c)*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\,(bx+cx^2)}{\sqrt{x}}\,dx = \frac{2Abx^{\frac{3}{2}}}{3}+\frac{2Acx^{\frac{5}{2}}}{5}+\frac{2Bbx^{\frac{5}{2}}}{5}+\frac{2Bcx^{\frac{7}{2}}}{7}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)/x**(1/2),x)
```

output
```
2*A*b*x**(3/2)/3 + 2*A*c*x**(5/2)/5 + 2*B*b*x**(5/2)/5 + 2*B*c*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{(A+Bx)\,(bx+cx^2)}{\sqrt{x}}\,dx = \frac{2}{7}\,Bcx^{\frac{7}{2}}+\frac{2}{3}\,Abx^{\frac{3}{2}}+\frac{2}{5}\,(Bb+Ac)x^{\frac{5}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^(1/2),x, algorithm="maxima")
```

output
```
2/7*B*c*x^(7/2) + 2/3*A*b*x^(3/2) + 2/5*(B*b + A*c)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.74

$$\int \frac{(A+Bx)(bx+cx^2)}{\sqrt{x}}\,dx = \frac{2}{7}Bcx^{\frac{7}{2}} + \frac{2}{5}Bbx^{\frac{5}{2}} + \frac{2}{5}Acx^{\frac{5}{2}} + \frac{2}{3}Abx^{\frac{3}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^(1/2),x, algorithm="giac")
```

output
```
2/7*B*c*x^(7/2) + 2/5*B*b*x^(5/2) + 2/5*A*c*x^(5/2) + 2/3*A*b*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.17 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.69

$$\int \frac{(A+Bx)(bx+cx^2)}{\sqrt{x}}\,dx = \frac{2\,x^{3/2}\,(35\,A\,b + 21\,A\,c\,x + 21\,B\,b\,x + 15\,B\,c\,x^2)}{105}$$

input
```
int(((b*x + c*x^2)*(A + B*x))/x^(1/2),x)
```

output
```
(2*x^(3/2)*(35*A*b + 21*A*c*x + 21*B*b*x + 15*B*c*x^2))/105
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.72

$$\int \frac{(A+Bx)(bx+cx^2)}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,x(15bc\,x^2 + 21acx + 21b^2x + 35ab)}{105}$$

input
```
int((B*x+A)*(c*x^2+b*x)/x^(1/2),x)
```

output
```
(2*sqrt(x)*x*(35*a*b + 21*a*c*x + 21*b**2*x + 15*b*c*x**2))/105
```

3.62 $\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^{3/2}} dx$

Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^{3/2}}\,dx=2Ab\sqrt{x}+\frac{2}{3}(bB+Ac)x^{3/2}+\frac{2}{5}Bcx^{5/2}$$

output
```
2*A*b*x^(1/2)+2/3*(A*c+B*b)*x^(3/2)+2/5*B*c*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^{3/2}}\,dx=\frac{2}{15}\sqrt{x}(5A(3b+cx)+Bx(5b+3cx))$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2))/x^(3/2),x]
```

output
```
(2*Sqrt[x]*(5*A*(3*b + c*x) + B*x*(5*b + 3*c*x)))/15
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx)(b+cx)}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\sqrt{x}(Ac+bB)+\frac{Ab}{\sqrt{x}}+Bcx^{3/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{3}x^{3/2}(Ac+bB)+2Ab\sqrt{x}+\frac{2}{5}Bcx^{5/2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2))/x^(3/2),x]
```

output
```
2*A*b*Sqrt[x] + (2*(b*B + A*c)*x^(3/2))/3 + (2*B*c*x^(5/2))/5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

method	result	size
trager	$\left(\frac{2}{5}Bc\,x^{2}+\frac{2}{3}Acx+\frac{2}{3}Bbx+2Ab\right)\sqrt{x}$	27
gosper	$\frac{2\sqrt{x}\,\left(3Bc\,x^{2}+5Acx+5Bbx+15Ab\right)}{15}$	28
derivativedivides	$2Ab\sqrt{x}+\frac{2(Ac+Bb)x^{\frac{3}{2}}}{3}+\frac{2Bc\,x^{\frac{5}{2}}}{5}$	28
default	$2Ab\sqrt{x}+\frac{2(Ac+Bb)x^{\frac{3}{2}}}{3}+\frac{2Bc\,x^{\frac{5}{2}}}{5}$	28
risch	$\frac{2\sqrt{x}\,\left(3Bc\,x^{2}+5Acx+5Bbx+15Ab\right)}{15}$	28
orering	$\frac{2\left(3Bc\,x^{2}+5Acx+5Bbx+15Ab\right)\left(c\,x^{2}+bx\right)}{15\sqrt{x}\,\left(cx+b\right)}$	44

input
```
int((B*x+A)*(c*x^2+b*x)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2/5*B*c*x^2+2/3*A*c*x+2/3*B*b*x+2*A*b)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{3/2}}\,dx = \frac{2}{15}\left(3\,Bcx^2+15\,Ab+5\,(Bb+Ac)x\right)\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*c*x^2 + 15*A*b + 5*(B*b + A*c)*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{3/2}}\,dx = 2Ab\sqrt{x}+\frac{2Acx^{\frac{3}{2}}}{3}+\frac{2Bbx^{\frac{3}{2}}}{3}+\frac{2Bcx^{\frac{5}{2}}}{5}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)/x**(3/2),x)
```

output
```
2*A*b*sqrt(x) + 2*A*c*x**(3/2)/3 + 2*B*b*x**(3/2)/3 + 2*B*c*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{3/2}}\,dx = \frac{2}{5}\,Bcx^{\frac{5}{2}}+2\,Ab\sqrt{x}+\frac{2}{3}\,(Bb+Ac)x^{\frac{3}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^(3/2),x, algorithm="maxima")
```

output
```
2/5*B*c*x^(5/2) + 2*A*b*sqrt(x) + 2/3*(B*b + A*c)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^{3/2}}\,dx = \frac{2}{5}\,Bcx^{\frac{5}{2}} + \frac{2}{3}\,Bbx^{\frac{3}{2}} + \frac{2}{3}\,Acx^{\frac{3}{2}} + 2\,Ab\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^(3/2),x, algorithm="giac")
```

output
```
2/5*B*c*x^(5/2) + 2/3*B*b*x^(3/2) + 2/3*A*c*x^(3/2) + 2*A*b*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^{3/2}}\,dx = \frac{2\,\sqrt{x}\,\left(15\,A\,b+5\,A\,c\,x+5\,B\,b\,x+3\,B\,c\,x^2\right)}{15}$$

input
```
int(((b*x + c*x^2)*(A + B*x))/x^(3/2),x)
```

output
```
(2*x^(1/2)*(15*A*b + 5*A*c*x + 5*B*b*x + 3*B*c*x^2))/15
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^{3/2}}\,dx = \frac{2\sqrt{x}\,\left(3bc\,x^2+5acx+5b^2x+15ab\right)}{15}$$

input
```
int((B*x+A)*(c*x^2+b*x)/x^(3/2),x)
```

output
```
(2*sqrt(x)*(15*a*b + 5*a*c*x + 5*b**2*x + 3*b*c*x**2))/15
```

3.63 $\int \frac{(A+Bx)(bx+cx^2)}{x^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^{5/2}}\,dx = -\frac{2Ab}{\sqrt{x}} + 2(bB+Ac)\sqrt{x} + \frac{2}{3}Bcx^{3/2}$$

output
```
-2*A*b/x^(1/2)+2*(A*c+B*b)*x^(1/2)+2/3*B*c*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^{5/2}}\,dx = -\frac{2(3Ab-3bBx-3Acx-Bcx^2)}{3\sqrt{x}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2))/x^(5/2),x]
```

output
```
(-2*(3*A*b - 3*b*B*x - 3*A*c*x - B*c*x^2))/(3*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^{5/2}} \, dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{(A+Bx)(b+cx)}{x^{3/2}}dx$$

$$\Big\downarrow \ 85$$

$$\int \left(\frac{Ac+bB}{\sqrt{x}}+\frac{Ab}{x^{3/2}}+Bc\sqrt{x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$2\sqrt{x}(Ac+bB)-\frac{2Ab}{\sqrt{x}}+\frac{2}{3}Bcx^{3/2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2))/x^(5/2),x]
```

output
```
(-2*A*b)/Sqrt[x] + 2*(b*B + A*c)*Sqrt[x] + (2*B*c*x^(3/2))/3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
gosper	$-\frac{2(-Bc\,x^{2}-3Acx-3Bbx+3Ab)}{3\sqrt{x}}$	28
trager	$-\frac{2(-Bc\,x^{2}-3Acx-3Bbx+3Ab)}{3\sqrt{x}}$	28
risch	$-\frac{2(-Bc\,x^{2}-3Acx-3Bbx+3Ab)}{3\sqrt{x}}$	28
derivativedivides	$\frac{2Bc\,x^{\frac{3}{2}}}{3}+2Ac\sqrt{x}+2Bb\sqrt{x}-\frac{2Ab}{\sqrt{x}}$	30
default	$\frac{2Bc\,x^{\frac{3}{2}}}{3}+2Ac\sqrt{x}+2Bb\sqrt{x}-\frac{2Ab}{\sqrt{x}}$	30
orering	$-\frac{2(-Bc\,x^{2}-3Acx-3Bbx+3Ab)\,(c\,x^{2}+bx)}{3x^{\frac{3}{2}}\,(cx+b)}$	44

input
```
int((B*x+A)*(c*x^2+b*x)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/x^(1/2)*(-B*c*x^2-3*A*c*x-3*B*b*x+3*A*b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^{5/2}}\,dx=\frac{2\left(Bcx^2-3\,Ab+3\,(Bb+Ac)x\right)}{3\,\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^(5/2),x, algorithm="fricas")
```

output
```
2/3*(B*c*x^2 - 3*A*b + 3*(B*b + A*c)*x)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^{5/2}}\,dx=-\frac{2Ab}{\sqrt{x}}+2Ac\sqrt{x}+2Bb\sqrt{x}+\frac{2Bcx^{\frac{3}{2}}}{3}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)/x**(5/2),x)
```

output
```
-2*A*b/sqrt(x) + 2*A*c*sqrt(x) + 2*B*b*sqrt(x) + 2*B*c*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^{5/2}}\,dx=\frac{2}{3}\,Bcx^{\frac{3}{2}}-\frac{2\,Ab}{\sqrt{x}}+2\,(Bb+Ac)\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^(5/2),x, algorithm="maxima")
```

output
```
2/3*B*c*x^(3/2) - 2*A*b/sqrt(x) + 2*(B*b + A*c)*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{5/2}}\,dx = \frac{2}{3}Bcx^{\frac{3}{2}} + 2Bb\sqrt{x} + 2Ac\sqrt{x} - \frac{2Ab}{\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^(5/2),x, algorithm="giac")
```

output
```
2/3*B*c*x^(3/2) + 2*B*b*sqrt(x) + 2*A*c*sqrt(x) - 2*A*b/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{5/2}}\,dx = \frac{6\,A\,c\,x - 6\,A\,b + 6\,B\,b\,x + 2\,B\,c\,x^2}{3\,\sqrt{x}}$$

input
```
int(((b*x + c*x^2)*(A + B*x))/x^(5/2),x)
```

output
```
(6*A*c*x - 6*A*b + 6*B*b*x + 2*B*c*x^2)/(3*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{5/2}}\,dx = \frac{\frac{2}{3}bc\,x^2 + 2acx + 2b^2x - 2ab}{\sqrt{x}}$$

input
```
int((B*x+A)*(c*x^2+b*x)/x^(5/2),x)
```

output
```
(2*( - 3*a*b + 3*a*c*x + 3*b**2*x + b*c*x**2))/(3*sqrt(x))
```

3.64 $\int \frac{(A+Bx)(bx+cx^2)}{x^{7/2}} \, dx$

Optimal result

Integrand size = 20, antiderivative size = 35

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{7/2}} \, dx = -\frac{2Ab}{3x^{3/2}} - \frac{2(bB+Ac)}{\sqrt{x}} + 2Bc\sqrt{x}$$

output
```
-2/3*A*b/x^(3/2)-2*(A*c+B*b)/x^(1/2)+2*B*c*x^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{7/2}} \, dx = -\frac{2(3Bx(b-cx)+A(b+3cx))}{3x^{3/2}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2))/x^(7/2),x]
```

output
```
(-2*(3*B*x*(b - c*x) + A*(b + 3*c*x)))/(3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int \frac{(A+Bx)(b+cx)}{x^{5/2}}dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(\frac{Ac+bB}{x^{3/2}}+\frac{Ab}{x^{5/2}}+\frac{Bc}{\sqrt{x}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2(Ac+bB)}{\sqrt{x}}-\frac{2Ab}{3x^{3/2}}+2Bc\sqrt{x}$$

input
```
Int[((A + B*x)*(b*x + c*x^2))/x^(7/2),x]
```

output
```
(-2*A*b)/(3*x^(3/2)) - (2*(b*B + A*c))/Sqrt[x] + 2*B*c*Sqrt[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

method	result	size
gosper	$-\frac{2\left(-3Bc\,x^{2}+3Acx+3Bbx+Ab\right)}{3x^{\frac{3}{2}}}$	27
trager	$-\frac{2\left(-3Bc\,x^{2}+3Acx+3Bbx+Ab\right)}{3x^{\frac{3}{2}}}$	27
risch	$-\frac{2\left(-3Bc\,x^{2}+3Acx+3Bbx+Ab\right)}{3x^{\frac{3}{2}}}$	27
derivativedivides	$-\frac{2Ab}{3x^{\frac{3}{2}}}-\frac{2(Ac+Bb)}{\sqrt{x}}+2Bc\sqrt{x}$	28
default	$-\frac{2Ab}{3x^{\frac{3}{2}}}-\frac{2(Ac+Bb)}{\sqrt{x}}+2Bc\sqrt{x}$	28
orering	$-\frac{2\left(-3Bc\,x^{2}+3Acx+3Bbx+Ab\right)\left(c\,x^{2}+bx\right)}{3x^{\frac{5}{2}}\left(cx+b\right)}$	43

input
```
int((B*x+A)*(c*x^2+b*x)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/x^(3/2)*(-3*B*c*x^2+3*A*c*x+3*B*b*x+A*b)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^{7/2}}\,dx = \frac{2\,(3\,Bcx^2 - Ab - 3\,(Bb+Ac)x)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^(7/2),x, algorithm="fricas")
```

output
```
2/3*(3*B*c*x^2 - A*b - 3*(B*b + A*c)*x)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^{7/2}}\,dx = -\frac{2Ab}{3x^{\frac{3}{2}}} - \frac{2Ac}{\sqrt{x}} - \frac{2Bb}{\sqrt{x}} + 2Bc\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)/x**(7/2),x)
```

output
```
-2*A*b/(3*x**(3/2)) - 2*A*c/sqrt(x) - 2*B*b/sqrt(x) + 2*B*c*sqrt(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^{7/2}}\,dx = 2\,Bc\sqrt{x} - \frac{2\,(Ab+3\,(Bb+Ac)x)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^(7/2),x, algorithm="maxima")
```

output
```
2*B*c*sqrt(x) - 2/3*(A*b + 3*(B*b + A*c)*x)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{7/2}}\,dx = 2\,Bc\sqrt{x} - \frac{2\,(3\,Bbx+3\,Acx+Ab)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^(7/2),x, algorithm="giac")
```

output
```
2*B*c*sqrt(x) - 2/3*(3*B*b*x + 3*A*c*x + A*b)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{7/2}}\,dx = -\frac{2\,A\,b+6\,A\,c\,x+6\,B\,b\,x-6\,B\,c\,x^2}{3\,x^{3/2}}$$

input
```
int(((b*x + c*x^2)*(A + B*x))/x^(7/2),x)
```

output
```
-(2*A*b + 6*A*c*x + 6*B*b*x - 6*B*c*x^2)/(3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{7/2}}\,dx = \frac{2bc\,x^2-2acx-2b^2x-\frac{2}{3}ab}{\sqrt{x}\,x}$$

input
```
int((B*x+A)*(c*x^2+b*x)/x^(7/2),x)
```

output
```
(2*( - a*b - 3*a*c*x - 3*b**2*x + 3*b*c*x**2))/(3*sqrt(x)*x)
```

3.65 $\int \frac{(A+Bx)(bx+cx^2)}{x^{9/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 37

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{9/2}}\,dx = -\frac{2Ab}{5x^{5/2}} - \frac{2(bB+Ac)}{3x^{3/2}} - \frac{2Bc}{\sqrt{x}}$$

output
```
-2/5*A*b/x^(5/2)-2/3*(A*c+B*b)/x^(3/2)-2*B*c/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{9/2}}\,dx = -\frac{2(3Ab+5bBx+5Acx+15Bcx^2)}{15x^{5/2}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2))/x^(9/2),x]
```

output
```
(-2*(3*A*b + 5*b*B*x + 5*A*c*x + 15*B*c*x^2))/(15*x^(5/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)}{x^{9/2}}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx)(b+cx)}{x^{7/2}}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{Ac+bB}{x^{5/2}}+\frac{Ab}{x^{7/2}}+\frac{Bc}{x^{3/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2(Ac+bB)}{3x^{3/2}}-\frac{2Ab}{5x^{5/2}}-\frac{2Bc}{\sqrt{x}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2))/x^(9/2),x]
```

output
```
(-2*A*b)/(5*x^(5/2)) - (2*(b*B + A*c))/(3*x^(3/2)) - (2*B*c)/Sqrt[x]
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

method	result	size
gosper	$-\frac{2(15Bc\,x^2+5Acx+5Bbx+3Ab)}{15x^{\frac{5}{2}}}$	28
derivativedivides	$-\frac{2Ab}{5x^{\frac{5}{2}}}-\frac{2(Ac+Bb)}{3x^{\frac{3}{2}}}-\frac{2Bc}{\sqrt{x}}$	28
default	$-\frac{2Ab}{5x^{\frac{5}{2}}}-\frac{2(Ac+Bb)}{3x^{\frac{3}{2}}}-\frac{2Bc}{\sqrt{x}}$	28
trager	$-\frac{2(15Bc\,x^2+5Acx+5Bbx+3Ab)}{15x^{\frac{5}{2}}}$	28
risch	$-\frac{2(15Bc\,x^2+5Acx+5Bbx+3Ab)}{15x^{\frac{5}{2}}}$	28
orering	$-\frac{2(15Bc\,x^2+5Acx+5Bbx+3Ab)\,(c\,x^2+bx)}{15x^{\frac{7}{2}}\,(cx+b)}$	44

input
```
int((B*x+A)*(c*x^2+b*x)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/x^(5/2)*(15*B*c*x^2+5*A*c*x+5*B*b*x+3*A*b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{9/2}}\,dx = -\frac{2\,(15\,Bcx^2+3\,Ab+5\,(Bb+Ac)x)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^(9/2),x, algorithm="fricas")
```

output
```
-2/15*(15*B*c*x^2 + 3*A*b + 5*(B*b + A*c)*x)/x^(5/2)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{9/2}}\,dx = -\frac{2Ab}{5x^{\frac{5}{2}}}-\frac{2Ac}{3x^{\frac{3}{2}}}-\frac{2Bb}{3x^{\frac{3}{2}}}-\frac{2Bc}{\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)/x**(9/2),x)
```

output
```
-2*A*b/(5*x**(5/2)) - 2*A*c/(3*x**(3/2)) - 2*B*b/(3*x**(3/2)) - 2*B*c/sqrt
(x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(A+Bx)(bx+cx^2)}{x^{9/2}}\,dx = -\frac{2\,(15\,Bcx^2+3\,Ab+5\,(Bb+Ac)x)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^(9/2),x, algorithm="maxima")
```

output
```
-2/15*(15*B*c*x^2 + 3*A*b + 5*(B*b + A*c)*x)/x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.73

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^{9/2}}\,dx = -\frac{2\,(15\,Bcx^2+5\,Bbx+5\,Acx+3\,Ab)}{15\,x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)/x^(9/2),x, algorithm="giac")
```

output
```
-2/15*(15*B*c*x^2 + 5*B*b*x + 5*A*c*x + 3*A*b)/x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^{9/2}}\,dx = -\frac{2\,B\,c\,x^2+\left(\frac{2\,A\,c}{3}+\frac{2\,B\,b}{3}\right)\,x+\frac{2\,A\,b}{5}}{x^{5/2}}$$

input
```
int(((b*x + c*x^2)*(A + B*x))/x^(9/2),x)
```

output
```
-((2*A*b)/5 + x*((2*A*c)/3 + (2*B*b)/3) + 2*B*c*x^2)/x^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\,(bx+cx^2)}{x^{9/2}}\,dx = \frac{-2bc\,x^2-\frac{2}{3}acx-\frac{2}{3}b^2x-\frac{2}{5}ab}{\sqrt{x}\,x^2}$$

input
```
int((B*x+A)*(c*x^2+b*x)/x^(9/2),x)
```

output
```
(2*( - 3*a*b - 5*a*c*x - 5*b**2*x - 15*b*c*x**2))/(15*sqrt(x)*x**2)
```

3.66 $\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{2}{9}Ab^2x^{9/2}+\frac{2}{11}b(bB+2Ac)x^{11/2}+\frac{2}{13}c(2bB+Ac)x^{13/2}+\frac{2}{15}Bc^2x^{15/2}$$

output
```
2/9*A*b^2*x^(9/2)+2/11*b*(2*A*c+B*b)*x^(11/2)+2/13*c*(A*c+2*B*b)*x^(13/2)+
2/15*B*c^2*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{2x^{9/2}(715Ab^2+585b^2Bx+1170Abcx+990bBcx^2+495Ac^2x^2+429Bc^2x^3)}{6435}$$

input
```
Integrate[x^(3/2)*(A + B*x)*(b*x + c*x^2)^2,x]
```

output
```
(2*x^(9/2)*(715*A*b^2 + 585*b^2*B*x + 1170*A*b*c*x + 990*b*B*c*x^2 + 495*A
*c^2*x^2 + 429*B*c^2*x^3))/6435
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^2\,dx$$

$$\Big\downarrow 9$$

$$\int x^{7/2}(A+Bx)(b+cx)^2dx$$

$$\Big\downarrow 85$$

$$\int \left(Ab^2x^{7/2}+cx^{11/2}(Ac+2bB)+bx^{9/2}(2Ac+bB)+Bc^2x^{13/2}\right)dx$$

$$\Big\downarrow 2009$$

$$\frac{2}{9}Ab^2x^{9/2}+\frac{2}{13}cx^{13/2}(Ac+2bB)+\frac{2}{11}bx^{11/2}(2Ac+bB)+\frac{2}{15}Bc^2x^{15/2}$$

input
```
Int[x^(3/2)*(A + B*x)*(b*x + c*x^2)^2,x]
```

output
```
(2*A*b^2*x^(9/2))/9 + (2*b*(b*B + 2*A*c)*x^(11/2))/11 + (2*c*(2*b*B + A*c)
*x^(13/2))/13 + (2*B*c^2*x^(15/2))/15
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2x^{\frac{9}{2}}\left(429B\,c^{2}x^{3}+495A\,c^{2}x^{2}+990x^{2}Bbc+1170Abcx+585xB\,b^{2}+715b^{2}A\right)}{6435}$	52
derivativedivides	$\frac{2B\,c^{2}x^{\frac{15}{2}}}{15}+\frac{2\left(A\,c^{2}+2Bbc\right)x^{\frac{13}{2}}}{13}+\frac{2\left(2Abc+B\,b^{2}\right)x^{\frac{11}{2}}}{11}+\frac{2A\,b^{2}x^{\frac{9}{2}}}{9}$	52
default	$\frac{2B\,c^{2}x^{\frac{15}{2}}}{15}+\frac{2\left(A\,c^{2}+2Bbc\right)x^{\frac{13}{2}}}{13}+\frac{2\left(2Abc+B\,b^{2}\right)x^{\frac{11}{2}}}{11}+\frac{2A\,b^{2}x^{\frac{9}{2}}}{9}$	52
trager	$\frac{2x^{\frac{9}{2}}\left(429B\,c^{2}x^{3}+495A\,c^{2}x^{2}+990x^{2}Bbc+1170Abcx+585xB\,b^{2}+715b^{2}A\right)}{6435}$	52
risch	$\frac{2x^{\frac{9}{2}}\left(429B\,c^{2}x^{3}+495A\,c^{2}x^{2}+990x^{2}Bbc+1170Abcx+585xB\,b^{2}+715b^{2}A\right)}{6435}$	52
orering	$\frac{2\left(429B\,c^{2}x^{3}+495A\,c^{2}x^{2}+990x^{2}Bbc+1170Abcx+585xB\,b^{2}+715b^{2}A\right)x^{\frac{5}{2}}\left(c\,x^{2}+bx\right)^{2}}{6435\left(cx+b\right)^{2}}$	70

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
2/6435*x^(9/2)*(429*B*c^2*x^3+495*A*c^2*x^2+990*B*b*c*x^2+1170*A*b*c*x+585
*B*b^2*x+715*A*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{2}{6435}\left(429\,Bc^2x^7+715\,Ab^2x^4+495\,(2\,Bbc+Ac^2)x^6+585\,(Bb^2+2\,Abc)x^5\right)\sqrt{x}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
2/6435*(429*B*c^2*x^7 + 715*A*b^2*x^4 + 495*(2*B*b*c + A*c^2)*x^6 + 585*(B
*b^2 + 2*A*b*c)*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{2Ab^2x^{\frac{9}{2}}}{9}+\frac{4Abcx^{\frac{11}{2}}}{11}+\frac{2Ac^2x^{\frac{13}{2}}}{13}+\frac{2Bb^2x^{\frac{11}{2}}}{11}+\frac{4Bbcx^{\frac{13}{2}}}{13}+\frac{2Bc^2x^{\frac{15}{2}}}{15}$$

input
```
integrate(x**(3/2)*(B*x+A)*(c*x**2+b*x)**2,x)
```

output
```
2*A*b**2*x**(9/2)/9 + 4*A*b*c*x**(11/2)/11 + 2*A*c**2*x**(13/2)/13 + 2*B*b
**2*x**(11/2)/11 + 4*B*b*c*x**(13/2)/13 + 2*B*c**2*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{2}{15}\,Bc^2x^{\frac{15}{2}}+\frac{2}{9}\,Ab^2x^{\frac{9}{2}}+\frac{2}{13}\,\left(2\,Bbc+Ac^2\right)x^{\frac{13}{2}}+\frac{2}{11}\,\left(Bb^2+2\,Abc\right)x^{\frac{11}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
2/15*B*c^2*x^(15/2) + 2/9*A*b^2*x^(9/2) + 2/13*(2*B*b*c + A*c^2)*x^(13/2)
+ 2/11*(B*b^2 + 2*A*b*c)*x^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{2}{15}\,Bc^2x^{\frac{15}{2}}+\frac{4}{13}\,Bbcx^{\frac{13}{2}}+\frac{2}{13}\,Ac^2x^{\frac{13}{2}}+\frac{2}{11}\,Bb^2x^{\frac{11}{2}}+\frac{4}{11}\,Abcx^{\frac{11}{2}}+\frac{2}{9}\,Ab^2x^{\frac{9}{2}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
2/15*B*c^2*x^(15/2) + 4/13*B*b*c*x^(13/2) + 2/13*A*c^2*x^(13/2) + 2/11*B*b
^2*x^(11/2) + 4/11*A*b*c*x^(11/2) + 2/9*A*b^2*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^2\,dx=x^{11/2}\,\left(\frac{2\,B\,b^2}{11}+\frac{4\,A\,c\,b}{11}\right)+x^{13/2}\,\left(\frac{2\,A\,c^2}{13}+\frac{4\,B\,b\,c}{13}\right)+\frac{2\,A\,b^2\,x^{9/2}}{9}+\frac{2\,B\,c^2\,x^{15/2}}{15}$$

input
```
int(x^(3/2)*(b*x + c*x^2)^2*(A + B*x),x)
```

output
```
x^(11/2)*((2*B*b^2)/11 + (4*A*b*c)/11) + x^(13/2)*((2*A*c^2)/13 + (4*B*b*c
)/13) + (2*A*b^2*x^(9/2))/9 + (2*B*c^2*x^(15/2))/15
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{2\sqrt{x}\,x^4(429b\,c^2x^3+495a\,c^2x^2+990b^2c\,x^2+1170abcx+585b^3x+715a\,b^2)}{6435}$$

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x)^2,x)
```

output
```
(2*sqrt(x)*x**4*(715*a*b**2 + 1170*a*b*c*x + 495*a*c**2*x**2 + 585*b**3*x
+ 990*b**2*c*x**2 + 429*b*c**2*x**3))/6435
```

3.67 $\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{2}{7}Ab^2x^{7/2}+\frac{2}{9}b(bB+2Ac)x^{9/2}+\frac{2}{11}c(2bB+Ac)x^{11/2}+\frac{2}{13}Bc^2x^{13/2}$$

output
```
2/7*A*b^2*x^(7/2)+2/9*b*(2*A*c+B*b)*x^(9/2)+2/11*c*(A*c+2*B*b)*x^(11/2)+2/
13*B*c^2*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{2x^{7/2}(13A(99b^2+154bcx+63c^2x^2)+7Bx(143b^2+234bcx+99c^2x^2))}{9009}$$

input
```
Integrate[Sqrt[x]*(A + B*x)*(b*x + c*x^2)^2,x]
```

output
```
(2*x^(7/2)*(13*A*(99*b^2 + 154*b*c*x + 63*c^2*x^2) + 7*B*x*(143*b^2 + 234*
b*c*x + 99*c^2*x^2)))/9009
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^2\,dx$$

$$\Big\downarrow \; 9$$

$$\int x^{5/2}(A+Bx)(b+cx)^2dx$$

$$\Big\downarrow \; 85$$

$$\int \left(Ab^2x^{5/2}+cx^{9/2}(Ac+2bB)+bx^{7/2}(2Ac+bB)+Bc^2x^{11/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{7}Ab^2x^{7/2}+\frac{2}{11}cx^{11/2}(Ac+2bB)+\frac{2}{9}bx^{9/2}(2Ac+bB)+\frac{2}{13}Bc^2x^{13/2}$$

input
```
Int[Sqrt[x]*(A + B*x)*(b*x + c*x^2)^2,x]
```

output
```
(2*A*b^2*x^(7/2))/7 + (2*b*(b*B + 2*A*c)*x^(9/2))/9 + (2*c*(2*b*B + A*c)*x
^(11/2))/11 + (2*B*c^2*x^(13/2))/13
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2x^{\frac{7}{2}}\left(693Bc^{2}x^{3}+819Ac^{2}x^{2}+1638x^{2}Bbc+2002Abcx+1001xBb^{2}+1287b^{2}A\right)}{9009}$	52
derivativedivides	$\frac{2Bc^{2}x^{\frac{13}{2}}}{13}+\frac{2(Ac^{2}+2Bbc)x^{\frac{11}{2}}}{11}+\frac{2(2Abc+Bb^{2})x^{\frac{9}{2}}}{9}+\frac{2Ab^{2}x^{\frac{7}{2}}}{7}$	52
default	$\frac{2Bc^{2}x^{\frac{13}{2}}}{13}+\frac{2(Ac^{2}+2Bbc)x^{\frac{11}{2}}}{11}+\frac{2(2Abc+Bb^{2})x^{\frac{9}{2}}}{9}+\frac{2Ab^{2}x^{\frac{7}{2}}}{7}$	52
trager	$\frac{2x^{\frac{7}{2}}\left(693Bc^{2}x^{3}+819Ac^{2}x^{2}+1638x^{2}Bbc+2002Abcx+1001xBb^{2}+1287b^{2}A\right)}{9009}$	52
risch	$\frac{2x^{\frac{7}{2}}\left(693Bc^{2}x^{3}+819Ac^{2}x^{2}+1638x^{2}Bbc+2002Abcx+1001xBb^{2}+1287b^{2}A\right)}{9009}$	52
orering	$\frac{2(693Bc^{2}x^{3}+819Ac^{2}x^{2}+1638x^{2}Bbc+2002Abcx+1001xBb^{2}+1287b^{2}A)x^{\frac{3}{2}}\left(cx^{2}+bx\right)^{2}}{9009(cx+b)^{2}}$	70

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
2/9009*x^(7/2)*(693*B*c^2*x^3+819*A*c^2*x^2+1638*B*b*c*x^2+2002*A*b*c*x+10
01*B*b^2*x+1287*A*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^2\,dx$$
$$=\frac{2}{9009}\left(693\,Bc^2x^6+1287\,Ab^2x^3+819\left(2\,Bbc+Ac^2\right)x^5+1001\left(Bb^2+2\,Abc\right)x^4\right)\sqrt{x}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
2/9009*(693*B*c^2*x^6 + 1287*A*b^2*x^3 + 819*(2*B*b*c + A*c^2)*x^5 + 1001*
(B*b^2 + 2*A*b*c)*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^2\,dx=\frac{2Ab^2x^{\frac{7}{2}}}{7}+\frac{2Bc^2x^{\frac{13}{2}}}{13}+\frac{2x^{\frac{11}{2}}\left(Ac^2+2Bbc\right)}{11}+\frac{2x^{\frac{9}{2}}\cdot\left(2Abc+Bb^2\right)}{9}$$

input
```
integrate(x**(1/2)*(B*x+A)*(c*x**2+b*x)**2,x)
```

output
```
2*A*b**2*x**(7/2)/7 + 2*B*c**2*x**(13/2)/13 + 2*x**(11/2)*(A*c**2 + 2*B*b*
c)/11 + 2*x**(9/2)*(2*A*b*c + B*b**2)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^2\,dx = \frac{2}{13}\,Bc^2x^{\frac{13}{2}}+\frac{2}{7}\,Ab^2x^{\frac{7}{2}}+\frac{2}{11}\,(2\,Bbc+Ac^2)x^{\frac{11}{2}}+\frac{2}{9}\,(Bb^2+2\,Abc)x^{\frac{9}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
2/13*B*c^2*x^(13/2) + 2/7*A*b^2*x^(7/2) + 2/11*(2*B*b*c + A*c^2)*x^(11/2)
+ 2/9*(B*b^2 + 2*A*b*c)*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^2\,dx = \frac{2}{13}\,Bc^2x^{\frac{13}{2}}+\frac{4}{11}\,Bbcx^{\frac{11}{2}}+\frac{2}{11}\,Ac^2x^{\frac{11}{2}}+\frac{2}{9}\,Bb^2x^{\frac{9}{2}}+\frac{4}{9}\,Abcx^{\frac{9}{2}}+\frac{2}{7}\,Ab^2x^{\frac{7}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
2/13*B*c^2*x^(13/2) + 4/11*B*b*c*x^(11/2) + 2/11*A*c^2*x^(11/2) + 2/9*B*b^
2*x^(9/2) + 4/9*A*b*c*x^(9/2) + 2/7*A*b^2*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^2\,dx = x^{9/2}\left(\frac{2\,B\,b^2}{9}+\frac{4\,A\,c\,b}{9}\right)+x^{11/2}\left(\frac{2\,A\,c^2}{11}+\frac{4\,B\,b\,c}{11}\right)+\frac{2\,A\,b^2\,x^{7/2}}{7}+\frac{2\,B\,c^2\,x^{13/2}}{13}$$

input
```
int(x^(1/2)*(b*x + c*x^2)^2*(A + B*x),x)
```

output
```
x^(9/2)*((2*B*b^2)/9 + (4*A*b*c)/9) + x^(11/2)*((2*A*c^2)/11 + (4*B*b*c)/1
1) + (2*A*b^2*x^(7/2))/7 + (2*B*c^2*x^(13/2))/13
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^2\,dx = \frac{2\sqrt{x}\,x^3(693b\,c^2x^3+819a\,c^2x^2+1638b^2c\,x^2+2002abcx+1001b^3x+1287a\,b^2)}{9009}$$

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x)^2,x)
```

output
```
(2*sqrt(x)*x**3*(1287*a*b**2 + 2002*a*b*c*x + 819*a*c**2*x**2 + 1001*b**3*
x + 1638*b**2*c*x**2 + 693*b*c**2*x**3))/9009
```

3.68 $\int \frac{(A+Bx)(bx+cx^2)^2}{\sqrt{x}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{(A+Bx)(bx+cx^2)^2}{\sqrt{x}} \, dx = \frac{2}{5}Ab^2x^{5/2} + \frac{2}{7}b(bB+2Ac)x^{7/2} + \frac{2}{9}c(2bB+Ac)x^{9/2} + \frac{2}{11}Bc^2x^{11/2}$$

output
```
2/5*A*b^2*x^(5/2)+2/7*b*(2*A*c+B*b)*x^(7/2)+2/9*c*(A*c+2*B*b)*x^(9/2)+2/11
*B*c^2*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)(bx+cx^2)^2}{\sqrt{x}} \, dx = \frac{2x^{5/2}(11A(63b^2+90bcx+35c^2x^2)+5Bx(99b^2+154bcx+63c^2x^2))}{3465}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^2)/Sqrt[x],x]
```

output
```
(2*x^(5/2)*(11*A*(63*b^2 + 90*b*c*x + 35*c^2*x^2) + 5*B*x*(99*b^2 + 154*b*
c*x + 63*c^2*x^2)))/3465
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{3/2}(A+Bx)(b+cx)^2dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(Ab^2x^{3/2}+cx^{7/2}(Ac+2bB)+bx^{5/2}(2Ac+bB)+Bc^2x^{9/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5}Ab^2x^{5/2}+\frac{2}{9}cx^{9/2}(Ac+2bB)+\frac{2}{7}bx^{7/2}(2Ac+bB)+\frac{2}{11}Bc^2x^{11/2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^2)/Sqrt[x],x]
```

output
```
(2*A*b^2*x^(5/2))/5 + (2*b*(b*B + 2*A*c)*x^(7/2))/7 + (2*c*(2*b*B + A*c)*x
^(9/2))/9 + (2*B*c^2*x^(11/2))/11
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2x^{\frac{5}{2}}\left(315B\,c^{2}x^{3}+385A\,c^{2}x^{2}+770x^{2}Bbc+990Abcx+495xB\,b^{2}+693b^{2}A\right)}{3465}$	52
derivativedivides	$\frac{2B\,c^{2}x^{\frac{11}{2}}}{11}+\frac{2\left(A\,c^{2}+2Bbc\right)x^{\frac{9}{2}}}{9}+\frac{2\left(2Abc+B\,b^{2}\right)x^{\frac{7}{2}}}{7}+\frac{2A\,b^{2}x^{\frac{5}{2}}}{5}$	52
default	$\frac{2B\,c^{2}x^{\frac{11}{2}}}{11}+\frac{2\left(A\,c^{2}+2Bbc\right)x^{\frac{9}{2}}}{9}+\frac{2\left(2Abc+B\,b^{2}\right)x^{\frac{7}{2}}}{7}+\frac{2A\,b^{2}x^{\frac{5}{2}}}{5}$	52
trager	$\frac{2x^{\frac{5}{2}}\left(315B\,c^{2}x^{3}+385A\,c^{2}x^{2}+770x^{2}Bbc+990Abcx+495xB\,b^{2}+693b^{2}A\right)}{3465}$	52
risch	$\frac{2x^{\frac{5}{2}}\left(315B\,c^{2}x^{3}+385A\,c^{2}x^{2}+770x^{2}Bbc+990Abcx+495xB\,b^{2}+693b^{2}A\right)}{3465}$	52
orering	$\frac{2\left(315B\,c^{2}x^{3}+385A\,c^{2}x^{2}+770x^{2}Bbc+990Abcx+495xB\,b^{2}+693b^{2}A\right)\sqrt{x}\,\left(c\,x^{2}+bx\right)^{2}}{3465\left(cx+b\right)^{2}}$	70

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3465*x^(5/2)*(315*B*c^2*x^3+385*A*c^2*x^2+770*B*b*c*x^2+990*A*b*c*x+495*
B*b^2*x+693*A*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{\sqrt{x}}\,dx = \frac{2}{3465}\left(315\,Bc^2x^5+693\,Ab^2x^2+385\left(2\,Bbc+Ac^2\right)x^4+495\left(Bb^2+2\,Abc\right)x^3\right)\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*B*c^2*x^5 + 693*A*b^2*x^2 + 385*(2*B*b*c + A*c^2)*x^4 + 495*(B
*b^2 + 2*A*b*c)*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{\sqrt{x}}\,dx = \frac{2Ab^2x^{\frac{5}{2}}}{5}+\frac{4Abcx^{\frac{7}{2}}}{7}+\frac{2Ac^2x^{\frac{9}{2}}}{9}+\frac{2Bb^2x^{\frac{7}{2}}}{7}+\frac{4Bbcx^{\frac{9}{2}}}{9}+\frac{2Bc^2x^{\frac{11}{2}}}{11}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**2/x**(1/2),x)
```

output
```
2*A*b**2*x**(5/2)/5 + 4*A*b*c*x**(7/2)/7 + 2*A*c**2*x**(9/2)/9 + 2*B*b**2*
x**(7/2)/7 + 4*B*b*c*x**(9/2)/9 + 2*B*c**2*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{\sqrt{x}}\,dx = \frac{2}{11}\,Bc^2x^{\frac{11}{2}} + \frac{2}{5}\,Ab^2x^{\frac{5}{2}} + \frac{2}{9}\left(2\,Bbc + Ac^2\right)x^{\frac{9}{2}} + \frac{2}{7}\left(Bb^2 + 2\,Abc\right)x^{\frac{7}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^(1/2),x, algorithm="maxima")
```

output
```
2/11*B*c^2*x^(11/2) + 2/5*A*b^2*x^(5/2) + 2/9*(2*B*b*c + A*c^2)*x^(9/2) +
2/7*(B*b^2 + 2*A*b*c)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{\sqrt{x}}\,dx = \frac{2}{11}\,Bc^2x^{\frac{11}{2}} + \frac{4}{9}\,Bbcx^{\frac{9}{2}} + \frac{2}{9}\,Ac^2x^{\frac{9}{2}} + \frac{2}{7}\,Bb^2x^{\frac{7}{2}} + \frac{4}{7}\,Abcx^{\frac{7}{2}} + \frac{2}{5}\,Ab^2x^{\frac{5}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^(1/2),x, algorithm="giac")
```

output
```
2/11*B*c^2*x^(11/2) + 4/9*B*b*c*x^(9/2) + 2/9*A*c^2*x^(9/2) + 2/7*B*b^2*x^
(7/2) + 4/7*A*b*c*x^(7/2) + 2/5*A*b^2*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\,(bx+cx^2)^2}{\sqrt{x}}\,dx = x^{7/2}\left(\frac{2\,B\,b^2}{7}+\frac{4\,A\,c\,b}{7}\right) + x^{9/2}\left(\frac{2\,A\,c^2}{9}+\frac{4\,B\,b\,c}{9}\right)+\frac{2\,A\,b^2\,x^{5/2}}{5}+\frac{2\,B\,c^2\,x^{11/2}}{11}$$

input
```
int(((b*x + c*x^2)^2*(A + B*x))/x^(1/2),x)
```

output
```
x^(7/2)*((2*B*b^2)/7 + (4*A*b*c)/7) + x^(9/2)*((2*A*c^2)/9 + (4*B*b*c)/9)
+ (2*A*b^2*x^(5/2))/5 + (2*B*c^2*x^(11/2))/11
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\,(bx+cx^2)^2}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,x^2(315b\,c^2x^3+385a\,c^2x^2+770b^2c\,x^2+990abcx+495b^3x+693a\,b^2)}{3465}$$

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^(1/2),x)
```

output
```
(2*sqrt(x)*x**2*(693*a*b**2 + 990*a*b*c*x + 385*a*c**2*x**2 + 495*b**3*x +
 770*b**2*c*x**2 + 315*b*c**2*x**3))/3465
```

3.69 $\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 63

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{3/2}}\,dx=\frac{2}{3}Ab^2x^{3/2}+\frac{2}{5}b(bB+2Ac)x^{5/2}+\frac{2}{7}c(2bB+Ac)x^{7/2}+\frac{2}{9}Bc^2x^{9/2}$$

output
```
2/3*A*b^2*x^(3/2)+2/5*b*(2*A*c+B*b)*x^(5/2)+2/7*c*(A*c+2*B*b)*x^(7/2)+2/9*
B*c^2*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{3/2}}\,dx=\frac{2}{315}x^{3/2}\left(3A\left(35b^2+42bcx+15c^2x^2\right)+Bx\left(63b^2+90bcx+35c^2x^2\right)\right)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^2)/x^(3/2),x]
```

output
```
(2*x^(3/2)*(3*A*(35*b^2 + 42*b*c*x + 15*c^2*x^2) + B*x*(63*b^2 + 90*b*c*x
+ 35*c^2*x^2)))/315
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{3/2}}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \sqrt{x}(A+Bx)(b+cx)^2dx$$

$$\Big\downarrow \; 85$$

$$\int \left(Ab^2\sqrt{x}+cx^{5/2}(Ac+2bB)+bx^{3/2}(2Ac+bB)+Bc^2x^{7/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{3}Ab^2x^{3/2}+\frac{2}{7}cx^{7/2}(Ac+2bB)+\frac{2}{5}bx^{5/2}(2Ac+bB)+\frac{2}{9}Bc^2x^{9/2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^2)/x^(3/2),x]
```

output
```
(2*A*b^2*x^(3/2))/3 + (2*b*(b*B + 2*A*c)*x^(5/2))/5 + (2*c*(2*b*B + A*c)*x
^(7/2))/7 + (2*B*c^2*x^(9/2))/9
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.83

method	result	size
gosper	$\frac{2x^{\frac{3}{2}}\left(35B\,c^{2}x^{3}+45A\,c^{2}x^{2}+90x^{2}Bbc+126Abcx+63xB\,b^{2}+105b^{2}A\right)}{315}$	52
derivativedivides	$\frac{2B\,c^{2}x^{\frac{9}{2}}}{9}+\frac{2\left(A\,c^{2}+2Bbc\right)x^{\frac{7}{2}}}{7}+\frac{2\left(2Abc+B\,b^{2}\right)x^{\frac{5}{2}}}{5}+\frac{2A\,b^{2}x^{\frac{3}{2}}}{3}$	52
default	$\frac{2B\,c^{2}x^{\frac{9}{2}}}{9}+\frac{2\left(A\,c^{2}+2Bbc\right)x^{\frac{7}{2}}}{7}+\frac{2\left(2Abc+B\,b^{2}\right)x^{\frac{5}{2}}}{5}+\frac{2A\,b^{2}x^{\frac{3}{2}}}{3}$	52
trager	$\frac{2x^{\frac{3}{2}}\left(35B\,c^{2}x^{3}+45A\,c^{2}x^{2}+90x^{2}Bbc+126Abcx+63xB\,b^{2}+105b^{2}A\right)}{315}$	52
risch	$\frac{2x^{\frac{3}{2}}\left(35B\,c^{2}x^{3}+45A\,c^{2}x^{2}+90x^{2}Bbc+126Abcx+63xB\,b^{2}+105b^{2}A\right)}{315}$	52
orering	$\frac{2\left(35B\,c^{2}x^{3}+45A\,c^{2}x^{2}+90x^{2}Bbc+126Abcx+63xB\,b^{2}+105b^{2}A\right)\left(c\,x^{2}+bx\right)^{2}}{315\sqrt{x}\,\left(cx+b\right)^{2}}$	70

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*x^(3/2)*(35*B*c^2*x^3+45*A*c^2*x^2+90*B*b*c*x^2+126*A*b*c*x+63*B*b^2
*x+105*A*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{3/2}}\,dx=\frac{2}{315}\left(35\,Bc^2x^4+105\,Ab^2x+45\left(2\,Bbc+Ac^2\right)x^3+63\left(Bb^2+2\,Abc\right)x^2\right)\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^(3/2),x, algorithm="fricas")
```

output
```
2/315*(35*B*c^2*x^4 + 105*A*b^2*x + 45*(2*B*b*c + A*c^2)*x^3 + 63*(B*b^2 +
 2*A*b*c)*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{3/2}}\,dx=\frac{2Ab^2x^{\frac{3}{2}}}{3}+\frac{4Abcx^{\frac{5}{2}}}{5}+\frac{2Ac^2x^{\frac{7}{2}}}{7}+\frac{2Bb^2x^{\frac{5}{2}}}{5}+\frac{4Bbcx^{\frac{7}{2}}}{7}+\frac{2Bc^2x^{\frac{9}{2}}}{9}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**2/x**(3/2),x)
```

output
```
2*A*b**2*x**(3/2)/3 + 4*A*b*c*x**(5/2)/5 + 2*A*c**2*x**(7/2)/7 + 2*B*b**2*
x**(5/2)/5 + 4*B*b*c*x**(7/2)/7 + 2*B*c**2*x**(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{3/2}}\,dx=\frac{2}{9}\,Bc^2x^{\frac{9}{2}}+\frac{2}{3}\,Ab^2x^{\frac{3}{2}}+\frac{2}{7}\left(2\,Bbc+Ac^2\right)x^{\frac{7}{2}}+\frac{2}{5}\left(Bb^2+2\,Abc\right)x^{\frac{5}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^(3/2),x, algorithm="maxima")
```

output
```
2/9*B*c^2*x^(9/2) + 2/3*A*b^2*x^(3/2) + 2/7*(2*B*b*c + A*c^2)*x^(7/2) + 2/
5*(B*b^2 + 2*A*b*c)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)(bx+cx^2)^2}{x^{3/2}}\,dx = \frac{2}{9}Bc^2x^{\frac{9}{2}} + \frac{4}{7}Bbcx^{\frac{7}{2}} + \frac{2}{7}Ac^2x^{\frac{7}{2}} + \frac{2}{5}Bb^2x^{\frac{5}{2}} + \frac{4}{5}Abcx^{\frac{5}{2}} + \frac{2}{3}Ab^2x^{\frac{3}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^(3/2),x, algorithm="giac")
```

output
```
2/9*B*c^2*x^(9/2) + 4/7*B*b*c*x^(7/2) + 2/7*A*c^2*x^(7/2) + 2/5*B*b^2*x^(5
/2) + 4/5*A*b*c*x^(5/2) + 2/3*A*b^2*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)(bx+cx^2)^2}{x^{3/2}}\,dx = x^{5/2}\left(\frac{2\,B\,b^2}{5} + \frac{4\,A\,c\,b}{5}\right) + x^{7/2}\left(\frac{2\,A\,c^2}{7} + \frac{4\,B\,b\,c}{7}\right) + \frac{2\,A\,b^2\,x^{3/2}}{3} + \frac{2\,B\,c^2\,x^{9/2}}{9}$$

input
```
int(((b*x + c*x^2)^2*(A + B*x))/x^(3/2),x)
```

output
```
x^(5/2)*((2*B*b^2)/5 + (4*A*b*c)/5) + x^(7/2)*((2*A*c^2)/7 + (4*B*b*c)/7)
+ (2*A*b^2*x^(3/2))/3 + (2*B*c^2*x^(9/2))/9
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\,(bx+cx^2)^2}{x^{3/2}}\,dx = \frac{2\sqrt{x}\,x(35b\,c^2x^3+45a\,c^2x^2+90b^2c\,x^2+126abcx+63b^3x+105a\,b^2)}{315}$$

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^(3/2),x)
```

output
```
(2*sqrt(x)*x*(105*a*b**2 + 126*a*b*c*x + 45*a*c**2*x**2 + 63*b**3*x + 90*b
**2*c*x**2 + 35*b*c**2*x**3))/315
```

3.70 $\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 61

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{5/2}}\,dx = 2Ab^2\sqrt{x} + \frac{2}{3}b(bB+2Ac)x^{3/2} + \frac{2}{5}c(2bB+Ac)x^{5/2} + \frac{2}{7}Bc^2x^{7/2}$$

output
```
2*A*b^2*x^(1/2)+2/3*b*(2*A*c+B*b)*x^(3/2)+2/5*c*(A*c+2*B*b)*x^(5/2)+2/7*B*
c^2*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{5/2}}\,dx = \frac{2}{105}\sqrt{x}\left(7A\left(15b^2+10bcx+3c^2x^2\right) + Bx\left(35b^2+42bcx+15c^2x^2\right)\right)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^2)/x^(5/2),x]
```

output
```
(2*Sqrt[x]*(7*A*(15*b^2 + 10*b*c*x + 3*c^2*x^2) + B*x*(35*b^2 + 42*b*c*x +
 15*c^2*x^2)))/105
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{5/2}}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx)(b+cx)^2}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{Ab^2}{\sqrt{x}}+cx^{3/2}(Ac+2bB)+b\sqrt{x}(2Ac+bB)+Bc^2x^{5/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$2Ab^2\sqrt{x}+\frac{2}{5}cx^{5/2}(Ac+2bB)+\frac{2}{3}bx^{3/2}(2Ac+bB)+\frac{2}{7}Bc^2x^{7/2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^2)/x^(5/2),x]
```

output
```
2*A*b^2*Sqrt[x] + (2*b*(b*B + 2*A*c)*x^(3/2))/3 + (2*c*(2*b*B + A*c)*x^(5/
2))/5 + (2*B*c^2*x^(7/2))/7
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

method	result	size
trager	$\left(\frac{2}{7}B\,c^{2}x^{3}+\frac{2}{5}A\,c^{2}x^{2}+\frac{4}{5}x^{2}Bbc+\frac{4}{3}Abcx+\frac{2}{3}xB\,b^{2}+2b^{2}A\right)\sqrt{x}$	51
gosper	$\frac{2\sqrt{x}\,(15B\,c^{2}x^{3}+21A\,c^{2}x^{2}+42x^{2}Bbc+70Abcx+35xB\,b^{2}+105b^{2}A)}{105}$	52
derivativedivides	$\frac{2B\,c^{2}x^{\frac{7}{2}}}{7}+\frac{2(A\,c^{2}+2Bbc)x^{\frac{5}{2}}}{5}+\frac{2(2Abc+B\,b^{2})x^{\frac{3}{2}}}{3}+2A\,b^{2}\sqrt{x}$	52
default	$\frac{2B\,c^{2}x^{\frac{7}{2}}}{7}+\frac{2(A\,c^{2}+2Bbc)x^{\frac{5}{2}}}{5}+\frac{2(2Abc+B\,b^{2})x^{\frac{3}{2}}}{3}+2A\,b^{2}\sqrt{x}$	52
risch	$\frac{2\sqrt{x}\,(15B\,c^{2}x^{3}+21A\,c^{2}x^{2}+42x^{2}Bbc+70Abcx+35xB\,b^{2}+105b^{2}A)}{105}$	52
orering	$\frac{2(15B\,c^{2}x^{3}+21A\,c^{2}x^{2}+42x^{2}Bbc+70Abcx+35xB\,b^{2}+105b^{2}A)\,(c\,x^{2}+bx)^{2}}{105x^{\frac{3}{2}}\,(cx+b)^{2}}$	70

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
(2/7*B*c^2*x^3+2/5*A*c^2*x^2+4/5*x^2*B*b*c+4/3*A*b*c*x+2/3*x*B*b^2+2*b^2*A
)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\,(bx+cx^2)^2}{x^{5/2}}\,dx = \frac{2}{105}\left(15\,Bc^2x^3+105\,Ab^2+21\left(2\,Bbc+Ac^2\right)x^2+35\left(Bb^2+2\,Abc\right)x\right)\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^(5/2),x, algorithm="fricas")
```

output
```
2/105*(15*B*c^2*x^3 + 105*A*b^2 + 21*(2*B*b*c + A*c^2)*x^2 + 35*(B*b^2 + 2
*A*b*c)*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

$$\int \frac{(A+Bx)\,(bx+cx^2)^2}{x^{5/2}}\,dx = 2Ab^2\sqrt{x}+\frac{4Abcx^{\frac{3}{2}}}{3} + \frac{2Ac^2x^{\frac{5}{2}}}{5}+\frac{2Bb^2x^{\frac{3}{2}}}{3}+\frac{4Bbcx^{\frac{5}{2}}}{5}+\frac{2Bc^2x^{\frac{7}{2}}}{7}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**2/x**(5/2),x)
```

output
```
2*A*b**2*sqrt(x) + 4*A*b*c*x**(3/2)/3 + 2*A*c**2*x**(5/2)/5 + 2*B*b**2*x**
(3/2)/3 + 4*B*b*c*x**(5/2)/5 + 2*B*c**2*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{5/2}}\,dx = \frac{2}{7}\,Bc^2x^{\frac{7}{2}} + 2\,Ab^2\sqrt{x} + \frac{2}{5}\left(2\,Bbc + Ac^2\right)x^{\frac{5}{2}} + \frac{2}{3}\left(Bb^2 + 2\,Abc\right)x^{\frac{3}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^(5/2),x, algorithm="maxima")
```

output
```
2/7*B*c^2*x^(7/2) + 2*A*b^2*sqrt(x) + 2/5*(2*B*b*c + A*c^2)*x^(5/2) + 2/3*
(B*b^2 + 2*A*b*c)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{5/2}}\,dx = \frac{2}{7}\,Bc^2x^{\frac{7}{2}} + \frac{4}{5}\,Bbcx^{\frac{5}{2}} + \frac{2}{5}\,Ac^2x^{\frac{5}{2}} + \frac{2}{3}\,Bb^2x^{\frac{3}{2}} + \frac{4}{3}\,Abcx^{\frac{3}{2}} + 2\,Ab^2\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^(5/2),x, algorithm="giac")
```

output
```
2/7*B*c^2*x^(7/2) + 4/5*B*b*c*x^(5/2) + 2/5*A*c^2*x^(5/2) + 2/3*B*b^2*x^(3
/2) + 4/3*A*b*c*x^(3/2) + 2*A*b^2*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{5/2}}\,dx = x^{3/2}\left(\frac{2\,B\,b^2}{3}+\frac{4\,A\,c\,b}{3}\right) + x^{5/2}\left(\frac{2\,A\,c^2}{5}+\frac{4\,B\,b\,c}{5}\right)+2\,A\,b^2\,\sqrt{x}+\frac{2\,B\,c^2\,x^{7/2}}{7}$$

input
```
int(((b*x + c*x^2)^2*(A + B*x))/x^(5/2),x)
```

output
```
x^(3/2)*((2*B*b^2)/3 + (4*A*b*c)/3) + x^(5/2)*((2*A*c^2)/5 + (4*B*b*c)/5)
+ 2*A*b^2*x^(1/2) + (2*B*c^2*x^(7/2))/7
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{5/2}}\,dx = \frac{2\sqrt{x}\,(15b\,c^2x^3+21a\,c^2x^2+42b^2c\,x^2+70abcx+35b^3x+105a\,b^2)}{105}$$

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^(5/2),x)
```

output
```
(2*sqrt(x)*(105*a*b**2 + 70*a*b*c*x + 21*a*c**2*x**2 + 35*b**3*x + 42*b**2
*c*x**2 + 15*b*c**2*x**3))/105
```

3.71 $\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 59

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{7/2}}\,dx=-\frac{2Ab^2}{\sqrt{x}}+2b(bB+2Ac)\sqrt{x}+\frac{2}{3}c(2bB+Ac)x^{3/2}+\frac{2}{5}Bc^2x^{5/2}$$

output
```
-2*A*b^2/x^(1/2)+2*b*(2*A*c+B*b)*x^(1/2)+2/3*c*(A*c+2*B*b)*x^(3/2)+2/5*B*c
^2*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{7/2}}\,dx=\frac{2(-15Ab^2+15b^2Bx+30Abcx+10bBcx^2+5Ac^2x^2+3Bc^2x^3)}{15\sqrt{x}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^2)/x^(7/2),x]
```

output
```
(2*(-15*A*b^2 + 15*b^2*B*x + 30*A*b*c*x + 10*b*B*c*x^2 + 5*A*c^2*x^2 + 3*B
*c^2*x^3))/(15*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx)(b+cx)^2}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{Ab^2}{x^{3/2}}+\frac{b(2Ac+bB)}{\sqrt{x}}+c\sqrt{x}(Ac+2bB)+Bc^2x^{3/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2Ab^2}{\sqrt{x}}+\frac{2}{3}cx^{3/2}(Ac+2bB)+2b\sqrt{x}(2Ac+bB)+\frac{2}{5}Bc^2x^{5/2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^2)/x^(7/2),x]
```

output
```
(-2*A*b^2)/Sqrt[x] + 2*b*(b*B + 2*A*c)*Sqrt[x] + (2*c*(2*b*B + A*c)*x^(3/2
))/3 + (2*B*c^2*x^(5/2))/5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

method	result	size
gosper	$-\frac{2\left(-3B\,c^{2}x^{3}-5A\,c^{2}x^{2}-10x^{2}Bbc-30Abcx-15xB\,b^{2}+15b^{2}A\right)}{15\sqrt{x}}$	52
trager	$-\frac{2\left(-3B\,c^{2}x^{3}-5A\,c^{2}x^{2}-10x^{2}Bbc-30Abcx-15xB\,b^{2}+15b^{2}A\right)}{15\sqrt{x}}$	52
risch	$-\frac{2\left(-3B\,c^{2}x^{3}-5A\,c^{2}x^{2}-10x^{2}Bbc-30Abcx-15xB\,b^{2}+15b^{2}A\right)}{15\sqrt{x}}$	52
derivativedivides	$\frac{2B\,c^{2}x^{\frac{5}{2}}}{5}+\frac{2A\,c^{2}x^{\frac{3}{2}}}{3}+\frac{4Bbc\,x^{\frac{3}{2}}}{3}+4Abc\sqrt{x}+2B\,b^{2}\sqrt{x}-\frac{2A\,b^{2}}{\sqrt{x}}$	54
default	$\frac{2B\,c^{2}x^{\frac{5}{2}}}{5}+\frac{2A\,c^{2}x^{\frac{3}{2}}}{3}+\frac{4Bbc\,x^{\frac{3}{2}}}{3}+4Abc\sqrt{x}+2B\,b^{2}\sqrt{x}-\frac{2A\,b^{2}}{\sqrt{x}}$	54
orering	$-\frac{2\left(-3B\,c^{2}x^{3}-5A\,c^{2}x^{2}-10x^{2}Bbc-30Abcx-15xB\,b^{2}+15b^{2}A\right)\left(c\,x^{2}+bx\right)^{2}}{15x^{\frac{5}{2}}\left(cx+b\right)^{2}}$	70

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/x^(1/2)*(-3*B*c^2*x^3-5*A*c^2*x^2-10*B*b*c*x^2-30*A*b*c*x-15*B*b^2*x
+15*A*b^2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{7/2}}\,dx = \frac{2\left(3\,Bc^2x^3-15\,Ab^2+5\left(2\,Bbc+Ac^2\right)x^2+15\left(Bb^2+2\,Abc\right)x\right)}{15\,\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^(7/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*c^2*x^3 - 15*A*b^2 + 5*(2*B*b*c + A*c^2)*x^2 + 15*(B*b^2 + 2*A*b
*c)*x)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{7/2}}\,dx = -\frac{2Ab^2}{\sqrt{x}}+4Abc\sqrt{x} + \frac{2Ac^2x^{\frac{3}{2}}}{3}+2Bb^2\sqrt{x}+\frac{4Bbcx^{\frac{3}{2}}}{3}+\frac{2Bc^2x^{\frac{5}{2}}}{5}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**2/x**(7/2),x)
```

output
```
-2*A*b**2/sqrt(x) + 4*A*b*c*sqrt(x) + 2*A*c**2*x**(3/2)/3 + 2*B*b**2*sqrt(
x) + 4*B*b*c*x**(3/2)/3 + 2*B*c**2*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{7/2}}\,dx = \frac{2}{5}\,Bc^2x^{\frac{5}{2}}-\frac{2\,Ab^2}{\sqrt{x}}+\frac{2}{3}\left(2\,Bbc+Ac^2\right)x^{\frac{3}{2}}+2\left(Bb^2+2\,Abc\right)\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^(7/2),x, algorithm="maxima")
```

output
```
2/5*B*c^2*x^(5/2) - 2*A*b^2/sqrt(x) + 2/3*(2*B*b*c + A*c^2)*x^(3/2) + 2*(B
*b^2 + 2*A*b*c)*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{7/2}}\,dx = \frac{2}{5}\,Bc^2x^{\frac{5}{2}} + \frac{4}{3}\,Bbcx^{\frac{3}{2}} + \frac{2}{3}\,Ac^2x^{\frac{3}{2}} + 2\,Bb^2\sqrt{x} + 4\,Abc\sqrt{x} - \frac{2\,Ab^2}{\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^(7/2),x, algorithm="giac")
```

output
```
2/5*B*c^2*x^(5/2) + 4/3*B*b*c*x^(3/2) + 2/3*A*c^2*x^(3/2) + 2*B*b^2*sqrt(x
) + 4*A*b*c*sqrt(x) - 2*A*b^2/sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{7/2}}\,dx = \sqrt{x}\left(2\,B\,b^2+4\,A\,c\,b\right) + x^{3/2}\left(\frac{2\,A\,c^2}{3}+\frac{4\,B\,b\,c}{3}\right) - \frac{2\,A\,b^2}{\sqrt{x}} + \frac{2\,B\,c^2\,x^{5/2}}{5}$$

input
```
int(((b*x + c*x^2)^2*(A + B*x))/x^(7/2),x)
```

output
```
x^(1/2)*(2*B*b^2 + 4*A*b*c) + x^(3/2)*((2*A*c^2)/3 + (4*B*b*c)/3) - (2*A*b
^2)/x^(1/2) + (2*B*c^2*x^(5/2))/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{7/2}}\,dx = \frac{\frac{2}{5}b\,c^2x^3+\frac{2}{3}a\,c^2x^2+\frac{4}{3}b^2c\,x^2+4abcx+2b^3x-2a\,b^2}{\sqrt{x}}$$

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^(7/2),x)
```

output
```
(2*( - 15*a*b**2 + 30*a*b*c*x + 5*a*c**2*x**2 + 15*b**3*x + 10*b**2*c*x**2
 + 3*b*c**2*x**3))/(15*sqrt(x))
```

3.72 $\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{9/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 59

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{9/2}} \, dx = -\frac{2Ab^2}{3x^{3/2}} - \frac{2b(bB+2Ac)}{\sqrt{x}} + 2c(2bB+Ac)\sqrt{x} + \frac{2}{3}Bc^2x^{3/2}$$

output
```
-2/3*A*b^2/x^(3/2)-2*b*(2*A*c+B*b)/x^(1/2)+2*c*(A*c+2*B*b)*x^(1/2)+2/3*B*c
^2*x^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{9/2}} \, dx = -\frac{2(Ab^2+3b^2Bx+6Abcx-6bBcx^2-3Ac^2x^2-Bc^2x^3)}{3x^{3/2}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^2)/x^(9/2),x]
```

output
```
(-2*(A*b^2 + 3*b^2*B*x + 6*A*b*c*x - 6*b*B*c*x^2 - 3*A*c^2*x^2 - B*c^2*x^3
))/(3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{9/2}}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx)(b+cx)^2}{x^{5/2}}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{Ab^2}{x^{5/2}}+\frac{b(2Ac+bB)}{x^{3/2}}+\frac{c(Ac+2bB)}{\sqrt{x}}+Bc^2\sqrt{x}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2Ab^2}{3x^{3/2}}-\frac{2b(2Ac+bB)}{\sqrt{x}}+2c\sqrt{x}(Ac+2bB)+\frac{2}{3}Bc^2x^{3/2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^2)/x^(9/2),x]
```

output
```
(-2*A*b^2)/(3*x^(3/2)) - (2*b*(b*B + 2*A*c))/Sqrt[x] + 2*c*(2*b*B + A*c)*S
qrt[x] + (2*B*c^2*x^(3/2))/3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

method	result	size
gosper	$-\frac{2\left(-B\,c^{2} x^{3}-3 A\,c^{2} x^{2}-6 x^{2} B b c+6 A b c x+3 x B\,b^{2}+b^{2} A\right)}{3 x^{\frac{3}{2}}}$	51
derivativedivides	$\frac{2 B\,c^{2} x^{\frac{3}{2}}}{3}+2 A\,c^{2} \sqrt{x}+4 B b c \sqrt{x}-\frac{2 A\,b^{2}}{3 x^{\frac{3}{2}}}-\frac{2 b \left(2 A c+B b\right)}{\sqrt{x}}$	51
default	$\frac{2 B\,c^{2} x^{\frac{3}{2}}}{3}+2 A\,c^{2} \sqrt{x}+4 B b c \sqrt{x}-\frac{2 A\,b^{2}}{3 x^{\frac{3}{2}}}-\frac{2 b \left(2 A c+B b\right)}{\sqrt{x}}$	51
trager	$-\frac{2\left(-B\,c^{2} x^{3}-3 A\,c^{2} x^{2}-6 x^{2} B b c+6 A b c x+3 x B\,b^{2}+b^{2} A\right)}{3 x^{\frac{3}{2}}}$	51
risch	$-\frac{2\left(-B\,c^{2} x^{3}-3 A\,c^{2} x^{2}-6 x^{2} B b c+6 A b c x+3 x B\,b^{2}+b^{2} A\right)}{3 x^{\frac{3}{2}}}$	51
orering	$-\frac{2\left(-B\,c^{2} x^{3}-3 A\,c^{2} x^{2}-6 x^{2} B b c+6 A b c x+3 x B\,b^{2}+b^{2} A\right) \left(c\,x^{2}+b x \right)^{2}}{3 x^{\frac{7}{2}} \left(c x +b \right)^{2}}$	69

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/x^(3/2)*(-B*c^2*x^3-3*A*c^2*x^2-6*B*b*c*x^2+6*A*b*c*x+3*B*b^2*x+A*b^2
)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{9/2}}\,dx=\frac{2\left(Bc^2x^3-Ab^2+3\left(2\,Bbc+Ac^2\right)x^2-3\left(Bb^2+2\,Abc\right)x\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^(9/2),x, algorithm="fricas")
```

output
```
2/3*(B*c^2*x^3 - A*b^2 + 3*(2*B*b*c + A*c^2)*x^2 - 3*(B*b^2 + 2*A*b*c)*x)/
x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{9/2}}\,dx=-\frac{2Ab^2}{3x^{\frac{3}{2}}}-\frac{4Abc}{\sqrt{x}}+2Ac^2\sqrt{x}-\frac{2Bb^2}{\sqrt{x}}+4Bbc\sqrt{x}+\frac{2Bc^2x^{\frac{3}{2}}}{3}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**2/x**(9/2),x)
```

output
```
-2*A*b**2/(3*x**(3/2)) - 4*A*b*c/sqrt(x) + 2*A*c**2*sqrt(x) - 2*B*b**2/sqr
t(x) + 4*B*b*c*sqrt(x) + 2*B*c**2*x**(3/2)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^2}{x^{9/2}}\,dx=\frac{2}{3}\,Bc^2x^{\frac{3}{2}}+2\left(2\,Bbc+Ac^2\right)\sqrt{x}-\frac{2\left(Ab^2+3\left(Bb^2+2\,Abc\right)x\right)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^(9/2),x, algorithm="maxima")
```

output
```
2/3*B*c^2*x^(3/2) + 2*(2*B*b*c + A*c^2)*sqrt(x) - 2/3*(A*b^2 + 3*(B*b^2 +
2*A*b*c)*x)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\,(bx+cx^2)^2}{x^{9/2}}\,dx = \frac{2}{3}\,Bc^2x^{\frac{3}{2}} + 4\,Bbc\sqrt{x} + 2\,Ac^2\sqrt{x} - \frac{2\,(3\,Bb^2x + 6\,Abcx + Ab^2)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^2/x^(9/2),x, algorithm="giac")
```

output
```
2/3*B*c^2*x^(3/2) + 4*B*b*c*sqrt(x) + 2*A*c^2*sqrt(x) - 2/3*(3*B*b^2*x + 6
*A*b*c*x + A*b^2)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\,(bx+cx^2)^2}{x^{9/2}}\,dx = -\frac{6\,B\,b^2\,x + 2\,A\,b^2 - 12\,B\,b\,c\,x^2 + 12\,A\,b\,c\,x - 2\,B\,c^2\,x^3 - 6\,A\,c^2\,x^2}{3\,x^{3/2}}$$

input
```
int(((b*x + c*x^2)^2*(A + B*x))/x^(9/2),x)
```

output
```
-(2*A*b^2 - 6*A*c^2*x^2 - 2*B*c^2*x^3 + 6*B*b^2*x - 12*B*b*c*x^2 + 12*A*b*
c*x)/(3*x^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\,(bx+cx^2)^2}{x^{9/2}}\,dx = \frac{\frac{2}{3}b\,c^2x^3 + 2a\,c^2x^2 + 4b^2c\,x^2 - 4abcx - 2b^3x - \frac{2}{3}a\,b^2}{\sqrt{x}\,x}$$

input
```
int((B*x+A)*(c*x^2+b*x)^2/x^(9/2),x)
```

output
```
(2*( - a*b**2 - 6*a*b*c*x + 3*a*c**2*x**2 - 3*b**3*x + 6*b**2*c*x**2 + b*c
**2*x**3))/(3*sqrt(x)*x)
```

3.73 $\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{2}{11}Ab^3x^{11/2}+\frac{2}{13}b^2(bB+3Ac)x^{13/2}+\frac{2}{5}bc(bB+Ac)x^{15/2}+\frac{2}{17}c^2(3bB+Ac)x^{17/2}+\frac{2}{19}Bc^3x^{19/2}$$

output
```
2/11*A*b^3*x^(11/2)+2/13*b^2*(3*A*c+B*b)*x^(13/2)+2/5*b*c*(A*c+B*b)*x^(15/
2)+2/17*c^2*(A*c+3*B*b)*x^(17/2)+2/19*B*c^3*x^(19/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{2x^{11/2}(19A(1105b^3+2805b^2cx+2431bc^2x^2+715c^3x^3)+11Bx(1615b^3+4199b^2cx+3705bc}{230945}$$

input
```
Integrate[x^(3/2)*(A + B*x)*(b*x + c*x^2)^3,x]
```

output
```
(2*x^(11/2)*(19*A*(1105*b^3 + 2805*b^2*c*x + 2431*b*c^2*x^2 + 715*c^3*x^3)
 + 11*B*x*(1615*b^3 + 4199*b^2*c*x + 3705*b*c^2*x^2 + 1105*c^3*x^3)))/2309
45
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^3\,dx$$

$$\Big\downarrow \text{9}$$

$$\int x^{9/2}(A+Bx)(b+cx)^3dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(Ab^3x^{9/2}+b^2x^{11/2}(3Ac+bB)+c^2x^{15/2}(Ac+3bB)+3bcx^{13/2}(Ac+bB)+Bc^3x^{17/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{11}Ab^3x^{11/2}+\frac{2}{13}b^2x^{13/2}(3Ac+bB)+\frac{2}{17}c^2x^{17/2}(Ac+3bB)+\frac{2}{5}bcx^{15/2}(Ac+bB)+\frac{2}{19}Bc^3x^{19/2}$$

input
```
Int[x^(3/2)*(A + B*x)*(b*x + c*x^2)^3,x]
```

output
```
(2*A*b^3*x^(11/2))/11 + (2*b^2*(b*B + 3*A*c)*x^(13/2))/13 + (2*b*c*(b*B +
A*c)*x^(15/2))/5 + (2*c^2*(3*b*B + A*c)*x^(17/2))/17 + (2*B*c^3*x^(19/2))/
19
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
gosper	$\frac{2x^{\frac{11}{2}}\left(12155Bc^3x^4+13585Ac^3x^3+40755x^3Bbc^2+46189Abc^2x^2+46189x^2Bb^2c+53295Ab^2cx+17765xBb^3+20995\right.}{230945}$
derivativedivides	$\frac{2Bc^3x^{\frac{19}{2}}}{19}+\frac{2(Ac^3+3Bbc^2)x^{\frac{17}{2}}}{17}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{15}{2}}}{15}+\frac{2(3Ab^2c+Bb^3)x^{\frac{13}{2}}}{13}+\frac{2Ab^3x^{\frac{11}{2}}}{11}$
default	$\frac{2Bc^3x^{\frac{19}{2}}}{19}+\frac{2(Ac^3+3Bbc^2)x^{\frac{17}{2}}}{17}+\frac{2(3Abc^2+3Bb^2c)x^{\frac{15}{2}}}{15}+\frac{2(3Ab^2c+Bb^3)x^{\frac{13}{2}}}{13}+\frac{2Ab^3x^{\frac{11}{2}}}{11}$
trager	$\frac{2x^{\frac{11}{2}}\left(12155Bc^3x^4+13585Ac^3x^3+40755x^3Bbc^2+46189Abc^2x^2+46189x^2Bb^2c+53295Ab^2cx+17765xBb^3+20995\right.}{230945}$
risch	$\frac{2x^{\frac{11}{2}}\left(12155Bc^3x^4+13585Ac^3x^3+40755x^3Bbc^2+46189Abc^2x^2+46189x^2Bb^2c+53295Ab^2cx+17765xBb^3+20995\right.}{230945}$
orering	$\frac{2(12155Bc^3x^4+13585Ac^3x^3+40755x^3Bbc^2+46189Abc^2x^2+46189x^2Bb^2c+53295Ab^2cx+17765xBb^3+20995Ab^3)}{230945(cx+b)^3}$

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
2/230945*x^(11/2)*(12155*B*c^3*x^4+13585*A*c^3*x^3+40755*B*b*c^2*x^3+46189
*A*b*c^2*x^2+46189*B*b^2*c*x^2+53295*A*b^2*c*x+17765*B*b^3*x+20995*A*b^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{2}{230945}\left(12155\,Bc^3x^9+20995\,Ab^3x^5+13585\left(3\,Bbc^2+Ac^3\right)x^8+46189\left(Bb^2c+Abc^2\right)x^7+\right.$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
2/230945*(12155*B*c^3*x^9 + 20995*A*b^3*x^5 + 13585*(3*B*b*c^2 + A*c^3)*x^
8 + 46189*(B*b^2*c + A*b*c^2)*x^7 + 17765*(B*b^3 + 3*A*b^2*c)*x^6)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{2Ab^3x^{\frac{11}{2}}}{11}+\frac{6Ab^2cx^{\frac{13}{2}}}{13}+\frac{2Abc^2x^{\frac{15}{2}}}{5}+\frac{2Ac^3x^{\frac{17}{2}}}{17}+\frac{2Bb^3x^{\frac{13}{2}}}{13}+\frac{2Bb^2cx^{\frac{15}{2}}}{5}+\frac{6Bbc^2x^{\frac{17}{2}}}{17}+\frac{2Bc^3x^{\frac{19}{2}}}{19}$$

input
```
integrate(x**(3/2)*(B*x+A)*(c*x**2+b*x)**3,x)
```

output
```
2*A*b**3*x**(11/2)/11 + 6*A*b**2*c*x**(13/2)/13 + 2*A*b*c**2*x**(15/2)/5 +
 2*A*c**3*x**(17/2)/17 + 2*B*b**3*x**(13/2)/13 + 2*B*b**2*c*x**(15/2)/5 +
6*B*b*c**2*x**(17/2)/17 + 2*B*c**3*x**(19/2)/19
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{2}{19}\,Bc^3x^{\frac{19}{2}}+\frac{2}{11}\,Ab^3x^{\frac{11}{2}}+\frac{2}{17}\left(3\,Bbc^2+Ac^3\right)x^{\frac{17}{2}}+\frac{2}{5}\left(Bb^2c+Abc^2\right)x^{\frac{15}{2}}+\frac{2}{13}\left(Bb^3+3\,Ab^2c\right)x^{\frac{13}{2}}$$

input

```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x)^3,x, algorithm="maxima")
```

output

```
2/19*B*c^3*x^(19/2) + 2/11*A*b^3*x^(11/2) + 2/17*(3*B*b*c^2 + A*c^3)*x^(17
/2) + 2/5*(B*b^2*c + A*b*c^2)*x^(15/2) + 2/13*(B*b^3 + 3*A*b^2*c)*x^(13/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{2}{19}\,Bc^3x^{\frac{19}{2}}+\frac{6}{17}\,Bbc^2x^{\frac{17}{2}}+\frac{2}{17}\,Ac^3x^{\frac{17}{2}}+\frac{2}{5}\,Bb^2cx^{\frac{15}{2}}+\frac{2}{5}\,Abc^2x^{\frac{15}{2}}+\frac{2}{13}\,Bb^3x^{\frac{13}{2}}+\frac{6}{13}\,Ab^2cx^{\frac{13}{2}}+\frac{2}{11}\,Ab^3x^{\frac{11}{2}}$$

input

```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x)^3,x, algorithm="giac")
```

output

```
2/19*B*c^3*x^(19/2) + 6/17*B*b*c^2*x^(17/2) + 2/17*A*c^3*x^(17/2) + 2/5*B*
b^2*c*x^(15/2) + 2/5*A*b*c^2*x^(15/2) + 2/13*B*b^3*x^(13/2) + 6/13*A*b^2*c
*x^(13/2) + 2/11*A*b^3*x^(11/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^3\,dx = x^{13/2}\left(\frac{2\,B\,b^3}{13}+\frac{6\,A\,c\,b^2}{13}\right) + x^{17/2}\left(\frac{2\,A\,c^3}{17}+\frac{6\,B\,b\,c^2}{17}\right)+\frac{2\,A\,b^3\,x^{11/2}}{11}+\frac{2\,B\,c^3\,x^{19/2}}{19}+\frac{2\,b\,c\,x^{15/2}\,(A\,c+B\,b)}{5}$$

input
```
int(x^(3/2)*(b*x + c*x^2)^3*(A + B*x),x)
```

output
```
x^(13/2)*((2*B*b^3)/13 + (6*A*b^2*c)/13) + x^(17/2)*((2*A*c^3)/17 + (6*B*b
*c^2)/17) + (2*A*b^3*x^(11/2))/11 + (2*B*c^3*x^(19/2))/19 + (2*b*c*x^(15/2
)*(A*c + B*b))/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^3\,dx = \frac{2\sqrt{x}\,x^5(12155b\,c^3x^4+13585a\,c^3x^3+40755b^2c^2x^3+46189ab\,c^2x^2+46189b^3c\,x^2+53295a\,b^2c}{230945}$$

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x)^3,x)
```

output
```
(2*sqrt(x)*x**5*(20995*a*b**3 + 53295*a*b**2*c*x + 46189*a*b*c**2*x**2 + 1
3585*a*c**3*x**3 + 17765*b**4*x + 46189*b**3*c*x**2 + 40755*b**2*c**2*x**3
 + 12155*b*c**3*x**4))/230945
```

3.74 $\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{2}{9}Ab^3x^{9/2}+\frac{2}{11}b^2(bB+3Ac)x^{11/2}+\frac{6}{13}bc(bB+Ac)x^{13/2}+\frac{2}{15}c^2(3bB+Ac)x^{15/2}+\frac{2}{17}Bc^3x^{17/2}$$

output
```
2/9*A*b^3*x^(9/2)+2/11*b^2*(3*A*c+B*b)*x^(11/2)+6/13*b*c*(A*c+B*b)*x^(13/2
)+2/15*c^2*(A*c+3*B*b)*x^(15/2)+2/17*B*c^3*x^(17/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^3\,dx =\frac{2x^{9/2}(17A(715b^3+1755b^2cx+1485bc^2x^2+429c^3x^3)+9Bx(1105b^3+2805b^2cx+2431bc^2x^2+715c^3x^3}{109395}$$

input
```
Integrate[Sqrt[x]*(A + B*x)*(b*x + c*x^2)^3,x]
```

output
```
(2*x^(9/2)*(17*A*(715*b^3 + 1755*b^2*c*x + 1485*b*c^2*x^2 + 429*c^3*x^3) +
 9*B*x*(1105*b^3 + 2805*b^2*c*x + 2431*b*c^2*x^2 + 715*c^3*x^3)))/109395
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^3\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{7/2}(A+Bx)(b+cx)^3dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(Ab^3x^{7/2}+b^2x^{9/2}(3Ac+bB)+c^2x^{13/2}(Ac+3bB)+3bcx^{11/2}(Ac+bB)+Bc^3x^{15/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{9}Ab^3x^{9/2}+\frac{2}{11}b^2x^{11/2}(3Ac+bB)+\frac{2}{15}c^2x^{15/2}(Ac+3bB)+\frac{6}{13}bcx^{13/2}(Ac+bB)+\frac{2}{17}Bc^3x^{17/2}$$

input
```
Int[Sqrt[x]*(A + B*x)*(b*x + c*x^2)^3,x]
```

output
```
(2*A*b^3*x^(9/2))/9 + (2*b^2*(b*B + 3*A*c)*x^(11/2))/11 + (6*b*c*(b*B + A*
c)*x^(13/2))/13 + (2*c^2*(3*b*B + A*c)*x^(15/2))/15 + (2*B*c^3*x^(17/2))/1
7
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.74 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
gosper	$\frac{2x^{\frac{9}{2}}\left(6435B\,c^{3}x^{4}+7293A\,c^{3}x^{3}+21879x^{3}Bb\,c^{2}+25245Ab\,c^{2}x^{2}+25245x^{2}B\,b^{2}c+29835A\,b^{2}cx+9945xB\,b^{3}+12155A\,b^{3}\right)}{109395}$
derivativedivides	$\frac{2B\,c^{3}x^{\frac{17}{2}}}{17}+\frac{2\left(A\,c^{3}+3Bb\,c^{2}\right)x^{\frac{15}{2}}}{15}+\frac{2\left(3Ab\,c^{2}+3B\,b^{2}c\right)x^{\frac{13}{2}}}{13}+\frac{2\left(3A\,b^{2}c+B\,b^{3}\right)x^{\frac{11}{2}}}{11}+\frac{2A\,b^{3}x^{\frac{9}{2}}}{9}$
default	$\frac{2B\,c^{3}x^{\frac{17}{2}}}{17}+\frac{2\left(A\,c^{3}+3Bb\,c^{2}\right)x^{\frac{15}{2}}}{15}+\frac{2\left(3Ab\,c^{2}+3B\,b^{2}c\right)x^{\frac{13}{2}}}{13}+\frac{2\left(3A\,b^{2}c+B\,b^{3}\right)x^{\frac{11}{2}}}{11}+\frac{2A\,b^{3}x^{\frac{9}{2}}}{9}$
trager	$\frac{2x^{\frac{9}{2}}\left(6435B\,c^{3}x^{4}+7293A\,c^{3}x^{3}+21879x^{3}Bb\,c^{2}+25245Ab\,c^{2}x^{2}+25245x^{2}B\,b^{2}c+29835A\,b^{2}cx+9945xB\,b^{3}+12155A\,b^{3}\right)}{109395}$
risch	$\frac{2x^{\frac{9}{2}}\left(6435B\,c^{3}x^{4}+7293A\,c^{3}x^{3}+21879x^{3}Bb\,c^{2}+25245Ab\,c^{2}x^{2}+25245x^{2}B\,b^{2}c+29835A\,b^{2}cx+9945xB\,b^{3}+12155A\,b^{3}\right)}{109395}$
orering	$\frac{2\left(6435B\,c^{3}x^{4}+7293A\,c^{3}x^{3}+21879x^{3}Bb\,c^{2}+25245Ab\,c^{2}x^{2}+25245x^{2}B\,b^{2}c+29835A\,b^{2}cx+9945xB\,b^{3}+12155A\,b^{3}\right)x^{\frac{3}{2}}}{109395(cx+b)^{3}}$

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
2/109395*x^(9/2)*(6435*B*c^3*x^4+7293*A*c^3*x^3+21879*B*b*c^2*x^3+25245*A*
b*c^2*x^2+25245*B*b^2*c*x^2+29835*A*b^2*c*x+9945*B*b^3*x+12155*A*b^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^3\,dx$$
$$=\frac{2}{109395}\left(6435\,Bc^3x^8+12155\,Ab^3x^4+7293\left(3\,Bbc^2+Ac^3\right)x^7+25245\left(Bb^2c+Abc^2\right)x^6+9945\left(Bb^3+3\right.\right.$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
2/109395*(6435*B*c^3*x^8 + 12155*A*b^3*x^4 + 7293*(3*B*b*c^2 + A*c^3)*x^7
+ 25245*(B*b^2*c + A*b*c^2)*x^6 + 9945*(B*b^3 + 3*A*b^2*c)*x^5)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^3\,dx=\frac{2Ab^3x^{\frac{9}{2}}}{9}+\frac{2Bc^3x^{\frac{17}{2}}}{17}+\frac{2x^{\frac{15}{2}}\left(Ac^3+3Bbc^2\right)}{15}+\frac{2x^{\frac{13}{2}}\cdot\left(3Abc^2+3Bb^2c\right)}{13}+\frac{2x^{\frac{11}{2}}\cdot\left(3Ab^2c+Bb^3\right)}{11}$$

input
```
integrate(x**(1/2)*(B*x+A)*(c*x**2+b*x)**3,x)
```

output
```
2*A*b**3*x**(9/2)/9 + 2*B*c**3*x**(17/2)/17 + 2*x**(15/2)*(A*c**3 + 3*B*b*
c**2)/15 + 2*x**(13/2)*(3*A*b*c**2 + 3*B*b**2*c)/13 + 2*x**(11/2)*(3*A*b**
2*c + B*b**3)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^3 dx = \frac{2}{17}Bc^3x^{\frac{17}{2}} + \frac{2}{9}Ab^3x^{\frac{9}{2}} + \frac{2}{15}\left(3Bbc^2+Ac^3\right)x^{\frac{15}{2}} + \frac{6}{13}\left(Bb^2c+Abc^2\right)x^{\frac{13}{2}} + \frac{2}{11}\left(Bb^3+3Ab^2c\right)x^{\frac{11}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
2/17*B*c^3*x^(17/2) + 2/9*A*b^3*x^(9/2) + 2/15*(3*B*b*c^2 + A*c^3)*x^(15/2
) + 6/13*(B*b^2*c + A*b*c^2)*x^(13/2) + 2/11*(B*b^3 + 3*A*b^2*c)*x^(11/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^3 dx = \frac{2}{17}Bc^3x^{\frac{17}{2}} + \frac{2}{5}Bbc^2x^{\frac{15}{2}} + \frac{2}{15}Ac^3x^{\frac{15}{2}} + \frac{6}{13}Bb^2cx^{\frac{13}{2}} + \frac{6}{13}Abc^2x^{\frac{13}{2}} + \frac{2}{11}Bb^3x^{\frac{11}{2}} + \frac{6}{11}Ab^2cx^{\frac{11}{2}} + \frac{2}{9}Ab^3x^{\frac{9}{2}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
2/17*B*c^3*x^(17/2) + 2/5*B*b*c^2*x^(15/2) + 2/15*A*c^3*x^(15/2) + 6/13*B*
b^2*c*x^(13/2) + 6/13*A*b*c^2*x^(13/2) + 2/11*B*b^3*x^(11/2) + 6/11*A*b^2*
c*x^(11/2) + 2/9*A*b^3*x^(9/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^3\,dx = x^{11/2}\left(\frac{2\,B\,b^3}{11}+\frac{6\,A\,c\,b^2}{11}\right)+x^{15/2}\left(\frac{2\,A\,c^3}{15}+\frac{2\,B\,b\,c^2}{5}\right)+\frac{2\,A\,b^3\,x^{9/2}}{9}+\frac{2\,B\,c^3\,x^{17/2}}{17}+\frac{6\,b\,c\,x^{13/2}\,(A\,c+B\,b)}{13}$$

input
```
int(x^(1/2)*(b*x + c*x^2)^3*(A + B*x),x)
```

output
```
x^(11/2)*((2*B*b^3)/11 + (6*A*b^2*c)/11) + x^(15/2)*((2*A*c^3)/15 + (2*B*b
*c^2)/5) + (2*A*b^3*x^(9/2))/9 + (2*B*c^3*x^(17/2))/17 + (6*b*c*x^(13/2)*(
A*c + B*b))/13
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^3\,dx = \frac{2\sqrt{x}\,x^4(6435b\,c^3x^4+7293a\,c^3x^3+21879b^2c^2x^3+25245ab\,c^2x^2+25245b^3c\,x^2+29835a\,b^2cx+9945b^4x}{109395}$$

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x)^3,x)
```

output
```
(2*sqrt(x)*x**4*(12155*a*b**3 + 29835*a*b**2*c*x + 25245*a*b*c**2*x**2 + 7
293*a*c**3*x**3 + 9945*b**4*x + 25245*b**3*c*x**2 + 21879*b**2*c**2*x**3 +
 6435*b*c**3*x**4))/109395
```

3.75 $\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{\sqrt{x}}\,dx = \frac{2}{7}Ab^3x^{7/2}+\frac{2}{9}b^2(bB+3Ac)x^{9/2}+\frac{6}{11}bc(bB+Ac)x^{11/2}+\frac{2}{13}c^2(3bB+Ac)x^{13/2}+\frac{2}{15}Bc^3x^{15/2}$$

output
```
2/7*A*b^3*x^(7/2)+2/9*b^2*(3*A*c+B*b)*x^(9/2)+6/11*b*c*(A*c+B*b)*x^(11/2)+
2/13*c^2*(A*c+3*B*b)*x^(13/2)+2/15*B*c^3*x^(15/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{\sqrt{x}}\,dx = \frac{2x^{7/2}(15A(429b^3+1001b^2cx+819bc^2x^2+231c^3x^3)+7Bx(715b^3+1755b^2cx+1485bc^2x^2+429c^3x^3))}{45045}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/Sqrt[x],x]
```

output
```
(2*x^(7/2)*(15*A*(429*b^3 + 1001*b^2*c*x + 819*b*c^2*x^2 + 231*c^3*x^3) +
7*B*x*(715*b^3 + 1755*b^2*c*x + 1485*b*c^2*x^2 + 429*c^3*x^3)))/45045
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{\sqrt{x}}\,dx$$

$$\Big\downarrow \text{9}$$

$$\int x^{5/2}(A+Bx)(b+cx)^3dx$$

$$\Big\downarrow \text{85}$$

$$\int \left(Ab^3x^{5/2}+b^2x^{7/2}(3Ac+bB)+c^2x^{11/2}(Ac+3bB)+3bcx^{9/2}(Ac+bB)+Bc^3x^{13/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{7}Ab^3x^{7/2}+\frac{2}{9}b^2x^{9/2}(3Ac+bB)+\frac{2}{13}c^2x^{13/2}(Ac+3bB)+\frac{6}{11}bcx^{11/2}(Ac+bB)+\frac{2}{15}Bc^3x^{15/2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/Sqrt[x],x]
```

output
```
(2*A*b^3*x^(7/2))/7 + (2*b^2*(b*B + 3*A*c)*x^(9/2))/9 + (6*b*c*(b*B + A*c)
*x^(11/2))/11 + (2*c^2*(3*b*B + A*c)*x^(13/2))/13 + (2*B*c^3*x^(15/2))/15
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
gosper	$\frac{2x^{\frac{7}{2}}\left(3003B\,c^{3}x^{4}+3465A\,c^{3}x^{3}+10395x^{3}Bb\,c^{2}+12285Ab\,c^{2}x^{2}+12285x^{2}B\,b^{2}c+15015A\,b^{2}cx+5005xB\,b^{3}+6435A\,b^{3}\right)}{45045}$
derivativedivides	$\frac{2B\,c^{3}x^{\frac{15}{2}}}{15}+\frac{2\left(A\,c^{3}+3Bb\,c^{2}\right)x^{\frac{13}{2}}}{13}+\frac{2\left(3Ab\,c^{2}+3B\,b^{2}c\right)x^{\frac{11}{2}}}{11}+\frac{2\left(3A\,b^{2}c+B\,b^{3}\right)x^{\frac{9}{2}}}{9}+\frac{2A\,b^{3}x^{\frac{7}{2}}}{7}$
default	$\frac{2B\,c^{3}x^{\frac{15}{2}}}{15}+\frac{2\left(A\,c^{3}+3Bb\,c^{2}\right)x^{\frac{13}{2}}}{13}+\frac{2\left(3Ab\,c^{2}+3B\,b^{2}c\right)x^{\frac{11}{2}}}{11}+\frac{2\left(3A\,b^{2}c+B\,b^{3}\right)x^{\frac{9}{2}}}{9}+\frac{2A\,b^{3}x^{\frac{7}{2}}}{7}$
trager	$\frac{2x^{\frac{7}{2}}\left(3003B\,c^{3}x^{4}+3465A\,c^{3}x^{3}+10395x^{3}Bb\,c^{2}+12285Ab\,c^{2}x^{2}+12285x^{2}B\,b^{2}c+15015A\,b^{2}cx+5005xB\,b^{3}+6435A\,b^{3}\right)}{45045}$
risch	$\frac{2x^{\frac{7}{2}}\left(3003B\,c^{3}x^{4}+3465A\,c^{3}x^{3}+10395x^{3}Bb\,c^{2}+12285Ab\,c^{2}x^{2}+12285x^{2}B\,b^{2}c+15015A\,b^{2}cx+5005xB\,b^{3}+6435A\,b^{3}\right)}{45045}$
orering	$\frac{2\left(3003B\,c^{3}x^{4}+3465A\,c^{3}x^{3}+10395x^{3}Bb\,c^{2}+12285Ab\,c^{2}x^{2}+12285x^{2}B\,b^{2}c+15015A\,b^{2}cx+5005xB\,b^{3}+6435A\,b^{3}\right)\sqrt{x}\,(}{45045(cx+b)^{3}}$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/45045*x^(7/2)*(3003*B*c^3*x^4+3465*A*c^3*x^3+10395*B*b*c^2*x^3+12285*A*b
*c^2*x^2+12285*B*b^2*c*x^2+15015*A*b^2*c*x+5005*B*b^3*x+6435*A*b^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)(bx+cx^2)^3}{\sqrt{x}}\,dx$$
$$=\frac{2}{45045}\left(3003\,Bc^3x^7+6435\,Ab^3x^3+3465\left(3\,Bbc^2+Ac^3\right)x^6+12285\left(Bb^2c+Abc^2\right)x^5+5005\left(Bb^3+3\,A\right.\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(1/2),x, algorithm="fricas")
```

output
```
2/45045*(3003*B*c^3*x^7 + 6435*A*b^3*x^3 + 3465*(3*B*b*c^2 + A*c^3)*x^6 +
12285*(B*b^2*c + A*b*c^2)*x^5 + 5005*(B*b^3 + 3*A*b^2*c)*x^4)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)(bx+cx^2)^3}{\sqrt{x}}\,dx=\frac{2Ab^3x^{\frac{7}{2}}}{7}+\frac{2Ab^2cx^{\frac{9}{2}}}{3}+\frac{6Abc^2x^{\frac{11}{2}}}{11}+\frac{2Ac^3x^{\frac{13}{2}}}{13}+\frac{2Bb^3x^{\frac{9}{2}}}{9}+\frac{6Bb^2cx^{\frac{11}{2}}}{11}+\frac{6Bbc^2x^{\frac{13}{2}}}{13}+\frac{2Bc^3x^{\frac{15}{2}}}{15}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**(1/2),x)
```

output
```
2*A*b**3*x**(7/2)/7 + 2*A*b**2*c*x**(9/2)/3 + 6*A*b*c**2*x**(11/2)/11 + 2*
A*c**3*x**(13/2)/13 + 2*B*b**3*x**(9/2)/9 + 6*B*b**2*c*x**(11/2)/11 + 6*B*
b*c**2*x**(13/2)/13 + 2*B*c**3*x**(15/2)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{\sqrt{x}}\,dx = \frac{2}{15}\,Bc^3x^{\frac{15}{2}} + \frac{2}{7}\,Ab^3x^{\frac{7}{2}} + \frac{2}{13}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{13}{2}} + \frac{6}{11}\left(Bb^2c + Abc^2\right)x^{\frac{11}{2}} + \frac{2}{9}\left(Bb^3 + 3\,Ab^2c\right)x^{\frac{9}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(1/2),x, algorithm="maxima")
```

output
```
2/15*B*c^3*x^(15/2) + 2/7*A*b^3*x^(7/2) + 2/13*(3*B*b*c^2 + A*c^3)*x^(13/2
) + 6/11*(B*b^2*c + A*b*c^2)*x^(11/2) + 2/9*(B*b^3 + 3*A*b^2*c)*x^(9/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{\sqrt{x}}\,dx = \frac{2}{15}\,Bc^3x^{\frac{15}{2}} + \frac{6}{13}\,Bbc^2x^{\frac{13}{2}} + \frac{2}{13}\,Ac^3x^{\frac{13}{2}} + \frac{6}{11}\,Bb^2cx^{\frac{11}{2}} + \frac{6}{11}\,Abc^2x^{\frac{11}{2}} + \frac{2}{9}\,Bb^3x^{\frac{9}{2}} + \frac{2}{3}\,Ab^2cx^{\frac{9}{2}} + \frac{2}{7}\,Ab^3x^{\frac{7}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(1/2),x, algorithm="giac")
```

output
```
2/15*B*c^3*x^(15/2) + 6/13*B*b*c^2*x^(13/2) + 2/13*A*c^3*x^(13/2) + 6/11*B
*b^2*c*x^(11/2) + 6/11*A*b*c^2*x^(11/2) + 2/9*B*b^3*x^(9/2) + 2/3*A*b^2*c*
x^(9/2) + 2/7*A*b^3*x^(7/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{\sqrt{x}}\,dx = x^{9/2}\left(\frac{2\,B\,b^3}{9}+\frac{2\,A\,c\,b^2}{3}\right)+x^{13/2}\left(\frac{2\,A\,c^3}{13}+\frac{6\,B\,b\,c^2}{13}\right) + \frac{2\,A\,b^3\,x^{7/2}}{7}+\frac{2\,B\,c^3\,x^{15/2}}{15}+\frac{6\,b\,c\,x^{11/2}\,(A\,c+B\,b)}{11}$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^(1/2),x)
```

output
```
x^(9/2)*((2*B*b^3)/9 + (2*A*b^2*c)/3) + x^(13/2)*((2*A*c^3)/13 + (6*B*b*c^
2)/13) + (2*A*b^3*x^(7/2))/7 + (2*B*c^3*x^(15/2))/15 + (6*b*c*x^(11/2)*(A*
c + B*b))/11
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{\sqrt{x}}\,dx = \frac{2\sqrt{x}\,x^3(3003b\,c^3x^4+3465a\,c^3x^3+10395b^2c^2x^3+12285ab\,c^2x^2+12285b^3c\,x^2+15015a\,b^2cx+5005b^4x}{45045}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^(1/2),x)
```

output
```
(2*sqrt(x)*x**3*(6435*a*b**3 + 15015*a*b**2*c*x + 12285*a*b*c**2*x**2 + 34
65*a*c**3*x**3 + 5005*b**4*x + 12285*b**3*c*x**2 + 10395*b**2*c**2*x**3 +
3003*b*c**3*x**4))/45045
```

3.76 $\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{3/2}}\,dx = \frac{2}{5}Ab^3x^{5/2} + \frac{2}{7}b^2(bB+3Ac)x^{7/2} + \frac{2}{3}bc(bB+Ac)x^{9/2} + \frac{2}{11}c^2(3bB+Ac)x^{11/2} + \frac{2}{13}Bc^3x^{13/2}$$

output
```
2/5*A*b^3*x^(5/2)+2/7*b^2*(3*A*c+B*b)*x^(7/2)+2/3*b*c*(A*c+B*b)*x^(9/2)+2/
11*c^2*(A*c+3*B*b)*x^(11/2)+2/13*B*c^3*x^(13/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{3/2}}\,dx = \frac{2x^{5/2}(13A(231b^3+495b^2cx+385bc^2x^2+105c^3x^3)+5Bx(429b^3+1001b^2c}{15015}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x^(3/2),x]
```

output
```
(2*x^(5/2)*(13*A*(231*b^3 + 495*b^2*c*x + 385*b*c^2*x^2 + 105*c^3*x^3) + 5
*B*x*(429*b^3 + 1001*b^2*c*x + 819*b*c^2*x^2 + 231*c^3*x^3)))/15015
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{3/2}}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int x^{3/2}(A+Bx)(b+cx)^3dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(Ab^3x^{3/2}+b^2x^{5/2}(3Ac+bB)+c^2x^{9/2}(Ac+3bB)+3bcx^{7/2}(Ac+bB)+Bc^3x^{11/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2}{5}Ab^3x^{5/2}+\frac{2}{7}b^2x^{7/2}(3Ac+bB)+\frac{2}{11}c^2x^{11/2}(Ac+3bB)+\frac{2}{3}bcx^{9/2}(Ac+bB)+\frac{2}{13}Bc^3x^{13/2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x^(3/2),x]
```

output
```
(2*A*b^3*x^(5/2))/5 + (2*b^2*(b*B + 3*A*c)*x^(7/2))/7 + (2*b*c*(b*B + A*c)
*x^(9/2))/3 + (2*c^2*(3*b*B + A*c)*x^(11/2))/11 + (2*B*c^3*x^(13/2))/13
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
gosper	$\frac{2x^{\frac{5}{2}}\left(1155B\,c^{3}x^{4}+1365A\,c^{3}x^{3}+4095x^{3}Bb\,c^{2}+5005Ab\,c^{2}x^{2}+5005x^{2}B\,b^{2}c+6435A\,b^{2}cx+2145xB\,b^{3}+3003A\,b^{3}\right)}{15015}$
derivativedivides	$\frac{2B\,c^{3}x^{\frac{13}{2}}}{13}+\frac{2\left(A\,c^{3}+3Bb\,c^{2}\right)x^{\frac{11}{2}}}{11}+\frac{2\left(3Ab\,c^{2}+3B\,b^{2}c\right)x^{\frac{9}{2}}}{9}+\frac{2\left(3A\,b^{2}c+B\,b^{3}\right)x^{\frac{7}{2}}}{7}+\frac{2A\,b^{3}x^{\frac{5}{2}}}{5}$
default	$\frac{2B\,c^{3}x^{\frac{13}{2}}}{13}+\frac{2\left(A\,c^{3}+3Bb\,c^{2}\right)x^{\frac{11}{2}}}{11}+\frac{2\left(3Ab\,c^{2}+3B\,b^{2}c\right)x^{\frac{9}{2}}}{9}+\frac{2\left(3A\,b^{2}c+B\,b^{3}\right)x^{\frac{7}{2}}}{7}+\frac{2A\,b^{3}x^{\frac{5}{2}}}{5}$
trager	$\frac{2x^{\frac{5}{2}}\left(1155B\,c^{3}x^{4}+1365A\,c^{3}x^{3}+4095x^{3}Bb\,c^{2}+5005Ab\,c^{2}x^{2}+5005x^{2}B\,b^{2}c+6435A\,b^{2}cx+2145xB\,b^{3}+3003A\,b^{3}\right)}{15015}$
risch	$\frac{2x^{\frac{5}{2}}\left(1155B\,c^{3}x^{4}+1365A\,c^{3}x^{3}+4095x^{3}Bb\,c^{2}+5005Ab\,c^{2}x^{2}+5005x^{2}B\,b^{2}c+6435A\,b^{2}cx+2145xB\,b^{3}+3003A\,b^{3}\right)}{15015}$
orering	$\frac{2\left(1155B\,c^{3}x^{4}+1365A\,c^{3}x^{3}+4095x^{3}Bb\,c^{2}+5005Ab\,c^{2}x^{2}+5005x^{2}B\,b^{2}c+6435A\,b^{2}cx+2145xB\,b^{3}+3003A\,b^{3}\right)\left(c\,x^{2}+bx\right.}{15015\sqrt{x}\,\left(cx+b\right)^{3}}$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15015*x^(5/2)*(1155*B*c^3*x^4+1365*A*c^3*x^3+4095*B*b*c^2*x^3+5005*A*b*c
^2*x^2+5005*B*b^2*c*x^2+6435*A*b^2*c*x+2145*B*b^3*x+3003*A*b^3)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{3/2}}\,dx = \frac{2}{15015}\left(1155\,Bc^3x^6 + 3003\,Ab^3x^2 + 1365\left(3\,Bbc^2 + Ac^3\right)x^5 + 5005\left(Bb^2c + \right.\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(3/2),x, algorithm="fricas")
```

output
```
2/15015*(1155*B*c^3*x^6 + 3003*A*b^3*x^2 + 1365*(3*B*b*c^2 + A*c^3)*x^5 +
5005*(B*b^2*c + A*b*c^2)*x^4 + 2145*(B*b^3 + 3*A*b^2*c)*x^3)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{3/2}}\,dx = \frac{2Ab^3x^{\frac{5}{2}}}{5} + \frac{6Ab^2cx^{\frac{7}{2}}}{7} + \frac{2Abc^2x^{\frac{9}{2}}}{3} + \frac{2Ac^3x^{\frac{11}{2}}}{11} + \frac{2Bb^3x^{\frac{7}{2}}}{7} + \frac{2Bb^2cx^{\frac{9}{2}}}{3} + \frac{6Bbc^2x^{\frac{11}{2}}}{11} + \frac{2Bc^3x^{\frac{13}{2}}}{13}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**(3/2),x)
```

output
```
2*A*b**3*x**(5/2)/5 + 6*A*b**2*c*x**(7/2)/7 + 2*A*b*c**2*x**(9/2)/3 + 2*A*
c**3*x**(11/2)/11 + 2*B*b**3*x**(7/2)/7 + 2*B*b**2*c*x**(9/2)/3 + 6*B*b*c*
*2*x**(11/2)/11 + 2*B*c**3*x**(13/2)/13
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{3/2}}\,dx = \frac{2}{13}\,Bc^3x^{\frac{13}{2}} + \frac{2}{5}\,Ab^3x^{\frac{5}{2}} + \frac{2}{11}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{11}{2}} + \frac{2}{3}\left(Bb^2c + Abc^2\right)x^{\frac{9}{2}} + \frac{2}{7}\left(Bb^3 + 3\,Ab^2c\right)x^{\frac{7}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(3/2),x, algorithm="maxima")
```

output
```
2/13*B*c^3*x^(13/2) + 2/5*A*b^3*x^(5/2) + 2/11*(3*B*b*c^2 + A*c^3)*x^(11/2
) + 2/3*(B*b^2*c + A*b*c^2)*x^(9/2) + 2/7*(B*b^3 + 3*A*b^2*c)*x^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{3/2}}\,dx = \frac{2}{13}\,Bc^3x^{\frac{13}{2}} + \frac{6}{11}\,Bbc^2x^{\frac{11}{2}} + \frac{2}{11}\,Ac^3x^{\frac{11}{2}} + \frac{2}{3}\,Bb^2cx^{\frac{9}{2}} + \frac{2}{3}\,Abc^2x^{\frac{9}{2}} + \frac{2}{7}\,Bb^3x^{\frac{7}{2}} + \frac{6}{7}\,Ab^2cx^{\frac{7}{2}} + \frac{2}{5}\,Ab^3x^{\frac{5}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(3/2),x, algorithm="giac")
```

output
```
2/13*B*c^3*x^(13/2) + 6/11*B*b*c^2*x^(11/2) + 2/11*A*c^3*x^(11/2) + 2/3*B*
b^2*c*x^(9/2) + 2/3*A*b*c^2*x^(9/2) + 2/7*B*b^3*x^(7/2) + 6/7*A*b^2*c*x^(7
/2) + 2/5*A*b^3*x^(5/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{3/2}}\,dx = x^{7/2}\left(\frac{2\,B\,b^3}{7}+\frac{6\,A\,c\,b^2}{7}\right) + x^{11/2}\left(\frac{2\,A\,c^3}{11}+\frac{6\,B\,b\,c^2}{11}\right) + \frac{2\,A\,b^3\,x^{5/2}}{5} + \frac{2\,B\,c^3\,x^{13/2}}{13} + \frac{2\,b\,c\,x^{9/2}\,(A\,c+B\,b)}{3}$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^(3/2),x)
```

output
```
x^(7/2)*((2*B*b^3)/7 + (6*A*b^2*c)/7) + x^(11/2)*((2*A*c^3)/11 + (6*B*b*c^
2)/11) + (2*A*b^3*x^(5/2))/5 + (2*B*c^3*x^(13/2))/13 + (2*b*c*x^(9/2)*(A*c
 + B*b))/3
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{x^{3/2}}\,dx = \frac{2\sqrt{x}\,x^2(1155b\,c^3x^4+1365a\,c^3x^3+4095b^2c^2x^3+5005ab\,c^2x^2+5005b^3c\,x^2+}{15015}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^(3/2),x)
```

output
```
(2*sqrt(x)*x**2*(3003*a*b**3 + 6435*a*b**2*c*x + 5005*a*b*c**2*x**2 + 1365
*a*c**3*x**3 + 2145*b**4*x + 5005*b**3*c*x**2 + 4095*b**2*c**2*x**3 + 1155
*b*c**3*x**4))/15015
```

3.77 $\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{5/2}}\,dx = \frac{2}{3}Ab^3x^{3/2} + \frac{2}{5}b^2(bB+3Ac)x^{5/2} + \frac{6}{7}bc(bB+Ac)x^{7/2} + \frac{2}{9}c^2(3bB+Ac)x^{9/2} + \frac{2}{11}Bc^3x^{11/2}$$

output
```
2/3*A*b^3*x^(3/2)+2/5*b^2*(3*A*c+B*b)*x^(5/2)+6/7*b*c*(A*c+B*b)*x^(7/2)+2/
9*c^2*(A*c+3*B*b)*x^(9/2)+2/11*B*c^3*x^(11/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{5/2}}\,dx = \frac{2x^{3/2}(11A(105b^3+189b^2cx+135bc^2x^2+35c^3x^3)+3Bx(231b^3+495b^2cx -}{3465}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x^(5/2),x]
```

output
```
(2*x^(3/2)*(11*A*(105*b^3 + 189*b^2*c*x + 135*b*c^2*x^2 + 35*c^3*x^3) + 3*
B*x*(231*b^3 + 495*b^2*c*x + 385*b*c^2*x^2 + 105*c^3*x^3)))/3465
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{5/2}}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \sqrt{x}(A+Bx)(b+cx)^3dx$$

$$\Big\downarrow \; 85$$

$$\int \left(Ab^3\sqrt{x}+b^2x^{3/2}(3Ac+bB)+c^2x^{7/2}(Ac+3bB)+3bcx^{5/2}(Ac+bB)+Bc^3x^{9/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2}{3}Ab^3x^{3/2}+\frac{2}{5}b^2x^{5/2}(3Ac+bB)+\frac{2}{9}c^2x^{9/2}(Ac+3bB)+\frac{6}{7}bcx^{7/2}(Ac+bB)+\frac{2}{11}Bc^3x^{11/2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x^(5/2),x]
```

output
```
(2*A*b^3*x^(3/2))/3 + (2*b^2*(b*B + 3*A*c)*x^(5/2))/5 + (6*b*c*(b*B + A*c)
*x^(7/2))/7 + (2*c^2*(3*b*B + A*c)*x^(9/2))/9 + (2*B*c^3*x^(11/2))/11
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

method	result
gosper	$\frac{2x^{\frac{3}{2}}\left(315B\,c^{3}x^{4}+385A\,c^{3}x^{3}+1155x^{3}Bb\,c^{2}+1485Ab\,c^{2}x^{2}+1485x^{2}B\,b^{2}c+2079A\,b^{2}cx+693xB\,b^{3}+1155A\,b^{3}\right)}{3465}$
derivativedivides	$\frac{2B\,c^{3}x^{\frac{11}{2}}}{11}+\frac{2\left(A\,c^{3}+3Bb\,c^{2}\right)x^{\frac{9}{2}}}{9}+\frac{2\left(3Ab\,c^{2}+3B\,b^{2}c\right)x^{\frac{7}{2}}}{7}+\frac{2\left(3A\,b^{2}c+B\,b^{3}\right)x^{\frac{5}{2}}}{5}+\frac{2A\,b^{3}x^{\frac{3}{2}}}{3}$
default	$\frac{2B\,c^{3}x^{\frac{11}{2}}}{11}+\frac{2\left(A\,c^{3}+3Bb\,c^{2}\right)x^{\frac{9}{2}}}{9}+\frac{2\left(3Ab\,c^{2}+3B\,b^{2}c\right)x^{\frac{7}{2}}}{7}+\frac{2\left(3A\,b^{2}c+B\,b^{3}\right)x^{\frac{5}{2}}}{5}+\frac{2A\,b^{3}x^{\frac{3}{2}}}{3}$
trager	$\frac{2x^{\frac{3}{2}}\left(315B\,c^{3}x^{4}+385A\,c^{3}x^{3}+1155x^{3}Bb\,c^{2}+1485Ab\,c^{2}x^{2}+1485x^{2}B\,b^{2}c+2079A\,b^{2}cx+693xB\,b^{3}+1155A\,b^{3}\right)}{3465}$
risch	$\frac{2x^{\frac{3}{2}}\left(315B\,c^{3}x^{4}+385A\,c^{3}x^{3}+1155x^{3}Bb\,c^{2}+1485Ab\,c^{2}x^{2}+1485x^{2}B\,b^{2}c+2079A\,b^{2}cx+693xB\,b^{3}+1155A\,b^{3}\right)}{3465}$
orering	$\frac{2\left(315B\,c^{3}x^{4}+385A\,c^{3}x^{3}+1155x^{3}Bb\,c^{2}+1485Ab\,c^{2}x^{2}+1485x^{2}B\,b^{2}c+2079A\,b^{2}cx+693xB\,b^{3}+1155A\,b^{3}\right)\left(c\,x^{2}+bx\right)^{3}}{3465x^{\frac{3}{2}}\left(cx+b\right)^{3}}$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3465*x^(3/2)*(315*B*c^3*x^4+385*A*c^3*x^3+1155*B*b*c^2*x^3+1485*A*b*c^2*
x^2+1485*B*b^2*c*x^2+2079*A*b^2*c*x+693*B*b^3*x+1155*A*b^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{5/2}}\,dx = \frac{2}{3465}\left(315\,Bc^3x^5+1155\,Ab^3x+385\left(3\,Bbc^2+Ac^3\right)x^4+1485\left(Bb^2c+Abc^2\right.\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(5/2),x, algorithm="fricas")
```

output
```
2/3465*(315*B*c^3*x^5 + 1155*A*b^3*x + 385*(3*B*b*c^2 + A*c^3)*x^4 + 1485*
(B*b^2*c + A*b*c^2)*x^3 + 693*(B*b^3 + 3*A*b^2*c)*x^2)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.34

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{5/2}}\,dx = \frac{2Ab^3x^{\frac{3}{2}}}{3}+\frac{6Ab^2cx^{\frac{5}{2}}}{5}+\frac{6Abc^2x^{\frac{7}{2}}}{7} + \frac{2Ac^3x^{\frac{9}{2}}}{9}+\frac{2Bb^3x^{\frac{5}{2}}}{5}+\frac{6Bb^2cx^{\frac{7}{2}}}{7}+\frac{2Bbc^2x^{\frac{9}{2}}}{3}+\frac{2Bc^3x^{\frac{11}{2}}}{11}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**(5/2),x)
```

output
```
2*A*b**3*x**(3/2)/3 + 6*A*b**2*c*x**(5/2)/5 + 6*A*b*c**2*x**(7/2)/7 + 2*A*
c**3*x**(9/2)/9 + 2*B*b**3*x**(5/2)/5 + 6*B*b**2*c*x**(7/2)/7 + 2*B*b*c**2
*x**(9/2)/3 + 2*B*c**3*x**(11/2)/11
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{5/2}}\,dx = \frac{2}{11}\,Bc^3x^{\frac{11}{2}}+\frac{2}{3}\,Ab^3x^{\frac{3}{2}} + \frac{2}{9}\left(3\,Bbc^2+Ac^3\right)x^{\frac{9}{2}}+\frac{6}{7}\left(Bb^2c+Abc^2\right)x^{\frac{7}{2}}+\frac{2}{5}\left(Bb^3+3\,Ab^2c\right)x^{\frac{5}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(5/2),x, algorithm="maxima")
```

output
```
2/11*B*c^3*x^(11/2) + 2/3*A*b^3*x^(3/2) + 2/9*(3*B*b*c^2 + A*c^3)*x^(9/2)
+ 6/7*(B*b^2*c + A*b*c^2)*x^(7/2) + 2/5*(B*b^3 + 3*A*b^2*c)*x^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{5/2}}\,dx = \frac{2}{11}\,Bc^3x^{\frac{11}{2}} + \frac{2}{3}\,Bbc^2x^{\frac{9}{2}} + \frac{2}{9}\,Ac^3x^{\frac{9}{2}} + \frac{6}{7}\,Bb^2cx^{\frac{7}{2}} + \frac{6}{7}\,Abc^2x^{\frac{7}{2}} + \frac{2}{5}\,Bb^3x^{\frac{5}{2}} + \frac{6}{5}\,Ab^2cx^{\frac{5}{2}} + \frac{2}{3}\,Ab^3x^{\frac{3}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(5/2),x, algorithm="giac")
```

output
```
2/11*B*c^3*x^(11/2) + 2/3*B*b*c^2*x^(9/2) + 2/9*A*c^3*x^(9/2) + 6/7*B*b^2*
c*x^(7/2) + 6/7*A*b*c^2*x^(7/2) + 2/5*B*b^3*x^(5/2) + 6/5*A*b^2*c*x^(5/2)
+ 2/3*A*b^3*x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{5/2}}\,dx = x^{5/2}\left(\frac{2\,B\,b^3}{5} + \frac{6\,A\,c\,b^2}{5}\right) + x^{9/2}\left(\frac{2\,A\,c^3}{9} + \frac{2\,B\,b\,c^2}{3}\right) + \frac{2\,A\,b^3\,x^{3/2}}{3} + \frac{2\,B\,c^3\,x^{11/2}}{11} + \frac{6\,b\,c\,x^{7/2}\,(A\,c+B\,b)}{7}$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^(5/2),x)
```

output
```
x^(5/2)*((2*B*b^3)/5 + (6*A*b^2*c)/5) + x^(9/2)*((2*A*c^3)/9 + (2*B*b*c^2)
/3) + (2*A*b^3*x^(3/2))/3 + (2*B*c^3*x^(11/2))/11 + (6*b*c*x^(7/2)*(A*c +
B*b))/7
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\,(bx+cx^2)^3}{x^{5/2}}\,dx = \frac{2\sqrt{x}\,x(315b\,c^3x^4+385a\,c^3x^3+1155b^2c^2x^3+1485ab\,c^2x^2+1485b^3c\,x^2+20}{3465}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^(5/2),x)
```

output
```
(2*sqrt(x)*x*(1155*a*b**3 + 2079*a*b**2*c*x + 1485*a*b*c**2*x**2 + 385*a*c
**3*x**3 + 693*b**4*x + 1485*b**3*c*x**2 + 1155*b**2*c**2*x**3 + 315*b*c**
3*x**4))/3465
```

3.78 $\int \frac{(A+Bx)(bx+cx^2)^3}{x^{7/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^{7/2}}\,dx = 2Ab^3\sqrt{x} + \frac{2}{3}b^2(bB+3Ac)x^{3/2} + \frac{6}{5}bc(bB+Ac)x^{5/2} + \frac{2}{7}c^2(3bB+Ac)x^{7/2} + \frac{2}{9}Bc^3x^{9/2}$$

output
```
2*A*b^3*x^(1/2)+2/3*b^2*(3*A*c+B*b)*x^(3/2)+6/5*b*c*(A*c+B*b)*x^(5/2)+2/7*
c^2*(A*c+3*B*b)*x^(7/2)+2/9*B*c^3*x^(9/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^{7/2}}\,dx = \frac{2}{315}\sqrt{x}\big(9A\big(35b^3+35b^2cx+21bc^2x^2+5c^3x^3\big) + Bx\big(105b^3+189b^2cx+135bc^2x^2+35c^3x^3\big)\big)$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x^(7/2),x]
```

output
```
(2*Sqrt[x]*(9*A*(35*b^3 + 35*b^2*c*x + 21*b*c^2*x^2 + 5*c^3*x^3) + B*x*(10
5*b^3 + 189*b^2*c*x + 135*b*c^2*x^2 + 35*c^3*x^3)))/315
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{7/2}}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(A+Bx)(b+cx)^3}{\sqrt{x}}dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{Ab^3}{\sqrt{x}}+b^2\sqrt{x}(3Ac+bB)+c^2x^{5/2}(Ac+3bB)+3bcx^{3/2}(Ac+bB)+Bc^3x^{7/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$2Ab^3\sqrt{x}+\frac{2}{3}b^2x^{3/2}(3Ac+bB)+\frac{2}{7}c^2x^{7/2}(Ac+3bB)+\frac{6}{5}bcx^{5/2}(Ac+bB)+\frac{2}{9}Bc^3x^{9/2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x^(7/2),x]
```

output
```
2*A*b^3*Sqrt[x] + (2*b^2*(b*B + 3*A*c)*x^(3/2))/3 + (6*b*c*(b*B + A*c)*x^(
5/2))/5 + (2*c^2*(3*b*B + A*c)*x^(7/2))/7 + (2*B*c^3*x^(9/2))/9
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

method	result
trager	$\left(\frac{2}{9}B\,c^{3}x^{4}+\frac{2}{7}A\,c^{3}x^{3}+\frac{6}{7}x^{3}Bb\,c^{2}+\frac{6}{5}Ab\,c^{2}x^{2}+\frac{6}{5}x^{2}B\,b^{2}c+2A\,b^{2}cx+\frac{2}{3}xB\,b^{3}+2A\,b^{3}\right)$
gosper	$\frac{2\sqrt{x}\,(35B\,c^{3}x^{4}+45A\,c^{3}x^{3}+135x^{3}Bb\,c^{2}+189Ab\,c^{2}x^{2}+189x^{2}B\,b^{2}c+315A\,b^{2}cx+105xB\,b^{3}+315A\,b^{3})}{315}$
derivativedivides	$\frac{2B\,c^{3}x^{\frac{9}{2}}}{9}+\frac{2(A\,c^{3}+3Bb\,c^{2})x^{\frac{7}{2}}}{7}+\frac{2(3Ab\,c^{2}+3B\,b^{2}c)x^{\frac{5}{2}}}{5}+\frac{2(3A\,b^{2}c+B\,b^{3})x^{\frac{3}{2}}}{3}+2A\,b^{3}\sqrt{x}$
default	$\frac{2B\,c^{3}x^{\frac{9}{2}}}{9}+\frac{2(A\,c^{3}+3Bb\,c^{2})x^{\frac{7}{2}}}{7}+\frac{2(3Ab\,c^{2}+3B\,b^{2}c)x^{\frac{5}{2}}}{5}+\frac{2(3A\,b^{2}c+B\,b^{3})x^{\frac{3}{2}}}{3}+2A\,b^{3}\sqrt{x}$
risch	$\frac{2\sqrt{x}\,(35B\,c^{3}x^{4}+45A\,c^{3}x^{3}+135x^{3}Bb\,c^{2}+189Ab\,c^{2}x^{2}+189x^{2}B\,b^{2}c+315A\,b^{2}cx+105xB\,b^{3}+315A\,b^{3})}{315}$
orering	$\frac{2(35B\,c^{3}x^{4}+45A\,c^{3}x^{3}+135x^{3}Bb\,c^{2}+189Ab\,c^{2}x^{2}+189x^{2}B\,b^{2}c+315A\,b^{2}cx+105xB\,b^{3}+315A\,b^{3})\,(c\,x^{2}+bx)^{3}}{315x^{\frac{5}{2}}\,(cx+b)^{3}}$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
(2/9*B*c^3*x^4+2/7*A*c^3*x^3+6/7*x^3*B*b*c^2+6/5*A*b*c^2*x^2+6/5*x^2*B*b^2
*c+2*A*b^2*c*x+2/3*x*B*b^3+2*A*b^3)*x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{7/2}}\,dx = \frac{2}{315}\left(35\,Bc^3x^4+315\,Ab^3+45\left(3\,Bbc^2+Ac^3\right)x^3+189\left(Bb^2c+Abc^2\right)x^2+\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(7/2),x, algorithm="fricas")
```

output
```
2/315*(35*B*c^3*x^4 + 315*A*b^3 + 45*(3*B*b*c^2 + A*c^3)*x^3 + 189*(B*b^2*
c + A*b*c^2)*x^2 + 105*(B*b^3 + 3*A*b^2*c)*x)*sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.33

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{7/2}}\,dx = 2Ab^3\sqrt{x}+2Ab^2cx^{\frac{3}{2}}+\frac{6Abc^2x^{\frac{5}{2}}}{5} + \frac{2Ac^3x^{\frac{7}{2}}}{7}+\frac{2Bb^3x^{\frac{3}{2}}}{3}+\frac{6Bb^2cx^{\frac{5}{2}}}{5}+\frac{6Bbc^2x^{\frac{7}{2}}}{7}+\frac{2Bc^3x^{\frac{9}{2}}}{9}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**(7/2),x)
```

output
```
2*A*b**3*sqrt(x) + 2*A*b**2*c*x**(3/2) + 6*A*b*c**2*x**(5/2)/5 + 2*A*c**3*
x**(7/2)/7 + 2*B*b**3*x**(3/2)/3 + 6*B*b**2*c*x**(5/2)/5 + 6*B*b*c**2*x**(
7/2)/7 + 2*B*c**3*x**(9/2)/9
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{7/2}}\,dx = \frac{2}{9}\,Bc^3x^{\frac{9}{2}} + 2\,Ab^3\sqrt{x} + \frac{2}{7}\left(3\,Bbc^2 + Ac^3\right)x^{\frac{7}{2}} + \frac{6}{5}\left(Bb^2c + Abc^2\right)x^{\frac{5}{2}} + \frac{2}{3}\left(Bb^3 + 3\,Ab^2c\right)x^{\frac{3}{2}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(7/2),x, algorithm="maxima")
```

output
```
2/9*B*c^3*x^(9/2) + 2*A*b^3*sqrt(x) + 2/7*(3*B*b*c^2 + A*c^3)*x^(7/2) + 6/
5*(B*b^2*c + A*b*c^2)*x^(5/2) + 2/3*(B*b^3 + 3*A*b^2*c)*x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{7/2}}\,dx = \frac{2}{9}\,Bc^3x^{\frac{9}{2}} + \frac{6}{7}\,Bbc^2x^{\frac{7}{2}} + \frac{2}{7}\,Ac^3x^{\frac{7}{2}} + \frac{6}{5}\,Bb^2cx^{\frac{5}{2}} + \frac{6}{5}\,Abc^2x^{\frac{5}{2}} + \frac{2}{3}\,Bb^3x^{\frac{3}{2}} + 2\,Ab^2cx^{\frac{3}{2}} + 2\,Ab^3\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(7/2),x, algorithm="giac")
```

output
```
2/9*B*c^3*x^(9/2) + 6/7*B*b*c^2*x^(7/2) + 2/7*A*c^3*x^(7/2) + 6/5*B*b^2*c*
x^(5/2) + 6/5*A*b*c^2*x^(5/2) + 2/3*B*b^3*x^(3/2) + 2*A*b^2*c*x^(3/2) + 2*
A*b^3*sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^{7/2}}\,dx = x^{3/2}\left(\frac{2\,B\,b^3}{3}+2\,A\,c\,b^2\right) + x^{7/2}\left(\frac{2\,A\,c^3}{7}+\frac{6\,B\,b\,c^2}{7}\right)+2\,A\,b^3\,\sqrt{x}+\frac{2\,B\,c^3\,x^{9/2}}{9}+\frac{6\,b\,c\,x^{5/2}\,(A\,c+B\,b)}{5}$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^(7/2),x)
```

output
```
x^(3/2)*((2*B*b^3)/3 + 2*A*b^2*c) + x^(7/2)*((2*A*c^3)/7 + (6*B*b*c^2)/7)
+ 2*A*b^3*x^(1/2) + (2*B*c^3*x^(9/2))/9 + (6*b*c*x^(5/2)*(A*c + B*b))/5
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^{7/2}}\,dx = \frac{2\sqrt{x}\,(35b\,c^3x^4+45a\,c^3x^3+135b^2c^2x^3+189ab\,c^2x^2+189b^3c\,x^2+315a\,b^2cx}{315}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^(7/2),x)
```

output
```
(2*sqrt(x)*(315*a*b**3 + 315*a*b**2*c*x + 189*a*b*c**2*x**2 + 45*a*c**3*x*
*3 + 105*b**4*x + 189*b**3*c*x**2 + 135*b**2*c**2*x**3 + 35*b*c**3*x**4))/
315
```

3.79 $\int \frac{(A+Bx)(bx+cx^2)^3}{x^{9/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{9/2}}\,dx = -\frac{2Ab^3}{\sqrt{x}} + 2b^2(bB+3Ac)\sqrt{x} + 2bc(bB+Ac)x^{3/2} + \frac{2}{5}c^2(3bB+Ac)x^{5/2} + \frac{2}{7}Bc^3x^{7/2}$$

output
```
-2*A*b^3/x^(1/2)+2*b^2*(3*A*c+B*b)*x^(1/2)+2*b*c*(A*c+B*b)*x^(3/2)+2/5*c^2
*(A*c+3*B*b)*x^(5/2)+2/7*B*c^3*x^(7/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{9/2}}\,dx = \frac{2(7A(-5b^3+15b^2cx+5bc^2x^2+c^3x^3)+Bx(35b^3+35b^2cx+21bc^2x^2+5c^3}{35\sqrt{x}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x^(9/2),x]
```

output
```
(2*(7*A*(-5*b^3 + 15*b^2*c*x + 5*b*c^2*x^2 + c^3*x^3) + B*x*(35*b^3 + 35*b
^2*c*x + 21*b*c^2*x^2 + 5*c^3*x^3)))/(35*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{9/2}}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{(A+Bx)(b+cx)^3}{x^{3/2}}dx$$

$$\Big\downarrow \; 85$$

$$\int \left(\frac{Ab^3}{x^{3/2}}+\frac{b^2(3Ac+bB)}{\sqrt{x}}+c^2x^{3/2}(Ac+3bB)+3bc\sqrt{x}(Ac+bB)+Bc^3x^{5/2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{2Ab^3}{\sqrt{x}}+2b^2\sqrt{x}(3Ac+bB)+\frac{2}{5}c^2x^{5/2}(Ac+3bB)+2bcx^{3/2}(Ac+bB)+\frac{2}{7}Bc^3x^{7/2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x^(9/2),x]
```

output
```
(-2*A*b^3)/Sqrt[x] + 2*b^2*(b*B + 3*A*c)*Sqrt[x] + 2*b*c*(b*B + A*c)*x^(3/
2) + (2*c^2*(3*b*B + A*c)*x^(5/2))/5 + (2*B*c^3*x^(7/2))/7
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.96

method	result
gosper	$-\frac{2\left(-5B\,c^{3}x^{4}-7A\,c^{3}x^{3}-21x^{3}Bb\,c^{2}-35Ab\,c^{2}x^{2}-35x^{2}B\,b^{2}c-105A\,b^{2}cx-35xB\,b^{3}+35A\,b^{3}\right)}{35\sqrt{x}}$
trager	$-\frac{2\left(-5B\,c^{3}x^{4}-7A\,c^{3}x^{3}-21x^{3}Bb\,c^{2}-35Ab\,c^{2}x^{2}-35x^{2}B\,b^{2}c-105A\,b^{2}cx-35xB\,b^{3}+35A\,b^{3}\right)}{35\sqrt{x}}$
risch	$-\frac{2\left(-5B\,c^{3}x^{4}-7A\,c^{3}x^{3}-21x^{3}Bb\,c^{2}-35Ab\,c^{2}x^{2}-35x^{2}B\,b^{2}c-105A\,b^{2}cx-35xB\,b^{3}+35A\,b^{3}\right)}{35\sqrt{x}}$
derivativedivides	$\frac{2B\,c^{3}x^{\frac{7}{2}}}{7}+\frac{2A\,c^{3}x^{\frac{5}{2}}}{5}+\frac{6Bb\,c^{2}x^{\frac{5}{2}}}{5}+2Ab\,c^{2}x^{\frac{3}{2}}+2B\,b^{2}c\,x^{\frac{3}{2}}+6A\,b^{2}c\sqrt{x}+2B\,b^{3}\sqrt{x}-\frac{2A\,b^{3}}{\sqrt{x}}$
default	$\frac{2B\,c^{3}x^{\frac{7}{2}}}{7}+\frac{2A\,c^{3}x^{\frac{5}{2}}}{5}+\frac{6Bb\,c^{2}x^{\frac{5}{2}}}{5}+2Ab\,c^{2}x^{\frac{3}{2}}+2B\,b^{2}c\,x^{\frac{3}{2}}+6A\,b^{2}c\sqrt{x}+2B\,b^{3}\sqrt{x}-\frac{2A\,b^{3}}{\sqrt{x}}$
orering	$-\frac{2\left(-5B\,c^{3}x^{4}-7A\,c^{3}x^{3}-21x^{3}Bb\,c^{2}-35Ab\,c^{2}x^{2}-35x^{2}B\,b^{2}c-105A\,b^{2}cx-35xB\,b^{3}+35A\,b^{3}\right)\left(c\,x^{2}+bx\right)^{3}}{35x^{\frac{7}{2}}\left(cx+b\right)^{3}}$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35/x^(1/2)*(-5*B*c^3*x^4-7*A*c^3*x^3-21*B*b*c^2*x^3-35*A*b*c^2*x^2-35*B
*b^2*c*x^2-105*A*b^2*c*x-35*B*b^3*x+35*A*b^3)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{9/2}}\,dx = \frac{2\left(5\,Bc^3x^4 - 35\,Ab^3 + 7\,(3\,Bbc^2 + Ac^3)x^3 + 35\,(Bb^2c + Abc^2)x^2 + 35\,(Bb^3 -\right.}{35\,\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(9/2),x, algorithm="fricas")
```

output
```
2/35*(5*B*c^3*x^4 - 35*A*b^3 + 7*(3*B*b*c^2 + A*c^3)*x^3 + 35*(B*b^2*c + A
*b*c^2)*x^2 + 35*(B*b^3 + 3*A*b^2*c)*x)/sqrt(x)
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.33

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{9/2}}\,dx = -\frac{2Ab^3}{\sqrt{x}} + 6Ab^2c\sqrt{x} + 2Abc^2x^{\frac{3}{2}} + \frac{2Ac^3x^{\frac{5}{2}}}{5} + 2Bb^3\sqrt{x} + 2Bb^2cx^{\frac{3}{2}} + \frac{6Bbc^2x^{\frac{5}{2}}}{5} + \frac{2Bc^3x^{\frac{7}{2}}}{7}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**(9/2),x)
```

output
```
-2*A*b**3/sqrt(x) + 6*A*b**2*c*sqrt(x) + 2*A*b*c**2*x**(3/2) + 2*A*c**3*x*
*(5/2)/5 + 2*B*b**3*sqrt(x) + 2*B*b**2*c*x**(3/2) + 6*B*b*c**2*x**(5/2)/5
+ 2*B*c**3*x**(7/2)/7
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.92

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{9/2}}\,dx = \frac{2}{7}\,Bc^3x^{\frac{7}{2}} - \frac{2\,Ab^3}{\sqrt{x}} + \frac{2}{5}\,(3\,Bbc^2 + Ac^3)x^{\frac{5}{2}} + 2\,\left(Bb^2c + Abc^2\right)x^{\frac{3}{2}} + 2\,\left(Bb^3 + 3\,Ab^2c\right)\sqrt{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(9/2),x, algorithm="maxima")
```

output
```
2/7*B*c^3*x^(7/2) - 2*A*b^3/sqrt(x) + 2/5*(3*B*b*c^2 + A*c^3)*x^(5/2) + 2*
(B*b^2*c + A*b*c^2)*x^(3/2) + 2*(B*b^3 + 3*A*b^2*c)*sqrt(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.97

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{9/2}}\,dx = \frac{2}{7}\,Bc^3x^{\frac{7}{2}} + \frac{6}{5}\,Bbc^2x^{\frac{5}{2}} + \frac{2}{5}\,Ac^3x^{\frac{5}{2}} + 2\,Bb^2cx^{\frac{3}{2}} + 2\,Abc^2x^{\frac{3}{2}} + 2\,Bb^3\sqrt{x} + 6\,Ab^2c\sqrt{x} - \frac{2\,Ab^3}{\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(9/2),x, algorithm="giac")
```

output
```
2/7*B*c^3*x^(7/2) + 6/5*B*b*c^2*x^(5/2) + 2/5*A*c^3*x^(5/2) + 2*B*b^2*c*x^
(3/2) + 2*A*b*c^2*x^(3/2) + 2*B*b^3*sqrt(x) + 6*A*b^2*c*sqrt(x) - 2*A*b^3/
sqrt(x)
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{9/2}}\,dx = \sqrt{x}\,\left(2\,B\,b^3+6\,A\,c\,b^2\right) + x^{5/2}\,\left(\frac{2\,A\,c^3}{5}+\frac{6\,B\,b\,c^2}{5}\right) - \frac{2\,A\,b^3}{\sqrt{x}} + \frac{2\,B\,c^3\,x^{7/2}}{7} + 2\,b\,c\,x^{3/2}\,\left(A\,c+B\,b\right)$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^(9/2),x)
```

output
```
x^(1/2)*(2*B*b^3 + 6*A*b^2*c) + x^(5/2)*((2*A*c^3)/5 + (6*B*b*c^2)/5) - (2
*A*b^3)/x^(1/2) + (2*B*c^3*x^(7/2))/7 + 2*b*c*x^(3/2)*(A*c + B*b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{9/2}}\,dx = \frac{\frac{2}{7}b\,c^3x^4+\frac{2}{5}a\,c^3x^3+\frac{6}{5}b^2c^2x^3+2ab\,c^2x^2+2b^3c\,x^2+6a\,b^2cx+2b^4x-2a\,b^3}{\sqrt{x}}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^(9/2),x)
```

output
```
(2*( - 35*a*b**3 + 105*a*b**2*c*x + 35*a*b*c**2*x**2 + 7*a*c**3*x**3 + 35*
b**4*x + 35*b**3*c*x**2 + 21*b**2*c**2*x**3 + 5*b*c**3*x**4))/(35*sqrt(x))
```

3.80 $\int \frac{(A+Bx)(bx+cx^2)^3}{x^{11/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 81

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^{11/2}}\,dx = -\frac{2Ab^3}{3x^{3/2}} - \frac{2b^2(bB+3Ac)}{\sqrt{x}} + 6bc(bB+Ac)\sqrt{x} + \frac{2}{3}c^2(3bB+Ac)x^{3/2} + \frac{2}{5}Bc^3x^{5/2}$$

output
```
-2/3*A*b^3/x^(3/2)-2*b^2*(3*A*c+B*b)/x^(1/2)+6*b*c*(A*c+B*b)*x^(1/2)+2/3*c
^2*(A*c+3*B*b)*x^(3/2)+2/5*B*c^3*x^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^{11/2}}\,dx = \frac{-10A(b^3+9b^2cx-9bc^2x^2-c^3x^3)+6Bx(-5b^3+15b^2cx+5bc^2x^2+c^3x^3)}{15x^{3/2}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^3)/x^(11/2),x]
```

output
```
(-10*A*(b^3 + 9*b^2*c*x - 9*b*c^2*x^2 - c^3*x^3) + 6*B*x*(-5*b^3 + 15*b^2*
c*x + 5*b*c^2*x^2 + c^3*x^3))/(15*x^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{11/2}}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{(A+Bx)(b+cx)^3}{x^{5/2}}dx$$

$$\Big\downarrow \text{ 85}$$

$$\int \left(\frac{Ab^3}{x^{5/2}}+\frac{b^2(3Ac+bB)}{x^{3/2}}+c^2\sqrt{x}(Ac+3bB)+\frac{3bc(Ac+bB)}{\sqrt{x}}+Bc^3x^{3/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{2Ab^3}{3x^{3/2}}-\frac{2b^2(3Ac+bB)}{\sqrt{x}}+\frac{2}{3}c^2x^{3/2}(Ac+3bB)+6bc\sqrt{x}(Ac+bB)+\frac{2}{5}Bc^3x^{5/2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^3)/x^(11/2),x]
```

output
```
(-2*A*b^3)/(3*x^(3/2)) - (2*b^2*(b*B + 3*A*c))/Sqrt[x] + 6*b*c*(b*B + A*c)
*Sqrt[x] + (2*c^2*(3*b*B + A*c)*x^(3/2))/3 + (2*B*c^3*x^(5/2))/5
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$\frac{2B\,c^{3}x^{\frac{5}{2}}}{5}+\frac{2A\,c^{3}x^{\frac{3}{2}}}{3}+2Bb\,c^{2}x^{\frac{3}{2}}+6Ab\,c^{2}\sqrt{x}+6B\,b^{2}c\sqrt{x}-\frac{2A\,b^{3}}{3x^{\frac{3}{2}}}-\frac{2b^{2}(3Ac+Bb)}{\sqrt{x}}$	75
default	$\frac{2B\,c^{3}x^{\frac{5}{2}}}{5}+\frac{2A\,c^{3}x^{\frac{3}{2}}}{3}+2Bb\,c^{2}x^{\frac{3}{2}}+6Ab\,c^{2}\sqrt{x}+6B\,b^{2}c\sqrt{x}-\frac{2A\,b^{3}}{3x^{\frac{3}{2}}}-\frac{2b^{2}(3Ac+Bb)}{\sqrt{x}}$	75
gosper	$-\frac{2(-3B\,c^{3}x^{4}-5A\,c^{3}x^{3}-15x^{3}Bb\,c^{2}-45Ab\,c^{2}x^{2}-45x^{2}B\,b^{2}c+45A\,b^{2}cx+15xB\,b^{3}+5A\,b^{3})}{15x^{\frac{3}{2}}}$	76
trager	$-\frac{2(-3B\,c^{3}x^{4}-5A\,c^{3}x^{3}-15x^{3}Bb\,c^{2}-45Ab\,c^{2}x^{2}-45x^{2}B\,b^{2}c+45A\,b^{2}cx+15xB\,b^{3}+5A\,b^{3})}{15x^{\frac{3}{2}}}$	76
risch	$-\frac{2(-3B\,c^{3}x^{4}-5A\,c^{3}x^{3}-15x^{3}Bb\,c^{2}-45Ab\,c^{2}x^{2}-45x^{2}B\,b^{2}c+45A\,b^{2}cx+15xB\,b^{3}+5A\,b^{3})}{15x^{\frac{3}{2}}}$	76
orering	$-\frac{2(-3B\,c^{3}x^{4}-5A\,c^{3}x^{3}-15x^{3}Bb\,c^{2}-45Ab\,c^{2}x^{2}-45x^{2}B\,b^{2}c+45A\,b^{2}cx+15xB\,b^{3}+5A\,b^{3})\,(c\,x^{2}+bx)^{3}}{15x^{\frac{9}{2}}\,(cx+b)^{3}}$	94

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*B*c^3*x^(5/2)+2/3*A*c^3*x^(3/2)+2*B*b*c^2*x^(3/2)+6*A*b*c^2*x^(1/2)+6*
B*b^2*c*x^(1/2)-2/3*A*b^3/x^(3/2)-2*b^2*(3*A*c+B*b)/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^{11/2}}\,dx = \frac{2\left(3\,Bc^3x^4 - 5\,Ab^3 + 5\,(3\,Bbc^2 + Ac^3)x^3 + 45\,(Bb^2c + Abc^2)x^2 - 15\,(Bb^3 + \right.}{15\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(11/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*c^3*x^4 - 5*A*b^3 + 5*(3*B*b*c^2 + A*c^3)*x^3 + 45*(B*b^2*c + A*
b*c^2)*x^2 - 15*(B*b^3 + 3*A*b^2*c)*x)/x^(3/2)
```

Sympy [A] (verification not implemented)

Time = 0.73 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^{11/2}}\,dx = -\frac{2Ab^3}{3x^{\frac{3}{2}}} - \frac{6Ab^2c}{\sqrt{x}} + 6Abc^2\sqrt{x} + \frac{2Ac^3x^{\frac{3}{2}}}{3} - \frac{2Bb^3}{\sqrt{x}} + 6Bb^2c\sqrt{x} + 2Bbc^2x^{\frac{3}{2}} + \frac{2Bc^3x^{\frac{5}{2}}}{5}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**3/x**(11/2),x)
```

output
```
-2*A*b**3/(3*x**(3/2)) - 6*A*b**2*c/sqrt(x) + 6*A*b*c**2*sqrt(x) + 2*A*c**
3*x**(3/2)/3 - 2*B*b**3/sqrt(x) + 6*B*b**2*c*sqrt(x) + 2*B*b*c**2*x**(3/2)
 + 2*B*c**3*x**(5/2)/5
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^{11/2}}\,dx = \frac{2}{5}\,Bc^3x^{\frac{5}{2}} + \frac{2}{3}\,(3\,Bbc^2 + Ac^3)x^{\frac{3}{2}} + 6\,(Bb^2c + Abc^2)\sqrt{x} - \frac{2\,(Ab^3 + 3\,(Bb^3 + 3\,Ab^2c)x)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(11/2),x, algorithm="maxima")
```

output
```
2/5*B*c^3*x^(5/2) + 2/3*(3*B*b*c^2 + A*c^3)*x^(3/2) + 6*(B*b^2*c + A*b*c^2
)*sqrt(x) - 2/3*(A*b^3 + 3*(B*b^3 + 3*A*b^2*c)*x)/x^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^{11/2}}\,dx = \frac{2}{5}\,Bc^3x^{\frac{5}{2}} + 2\,Bbc^2x^{\frac{3}{2}} + \frac{2}{3}\,Ac^3x^{\frac{3}{2}} + 6\,Bb^2c\sqrt{x} + 6\,Abc^2\sqrt{x} - \frac{2\,(3\,Bb^3x + 9\,Ab^2cx + Ab^3)}{3\,x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^3/x^(11/2),x, algorithm="giac")
```

output
```
2/5*B*c^3*x^(5/2) + 2*B*b*c^2*x^(3/2) + 2/3*A*c^3*x^(3/2) + 6*B*b^2*c*sqrt
(x) + 6*A*b*c^2*sqrt(x) - 2/3*(3*B*b^3*x + 9*A*b^2*c*x + A*b^3)/x^(3/2)
```

Mupad [B] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)(bx+cx^2)^3}{x^{11/2}}\,dx = x^{3/2}\left(\frac{2\,A\,c^3}{3} + 2\,B\,b\,c^2\right) - \frac{x\,(2\,B\,b^3 + 6\,A\,c\,b^2) + \frac{2\,A\,b^3}{3}}{x^{3/2}} + \frac{2\,B\,c^3\,x^{5/2}}{5} + 6\,b\,c\,\sqrt{x}\,(A\,c + B\,b)$$

input
```
int(((b*x + c*x^2)^3*(A + B*x))/x^(11/2),x)
```

output
```
x^(3/2)*((2*A*c^3)/3 + 2*B*b*c^2) - (x*(2*B*b^3 + 6*A*b^2*c) + (2*A*b^3)/3
)/x^(3/2) + (2*B*c^3*x^(5/2))/5 + 6*b*c*x^(1/2)*(A*c + B*b)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^3}{x^{11/2}}\,dx = \frac{\frac{2}{5}b\,c^3x^4+\frac{2}{3}a\,c^3x^3+2b^2c^2x^3+6ab\,c^2x^2+6b^3c\,x^2-6a\,b^2cx-2b^4x-\frac{2}{3}a\,b^3}{\sqrt{x}\,x}$$

input
```
int((B*x+A)*(c*x^2+b*x)^3/x^(11/2),x)
```

output
```
(2*( - 5*a*b**3 - 45*a*b**2*c*x + 45*a*b*c**2*x**2 + 5*a*c**3*x**3 - 15*b*
*4*x + 45*b**3*c*x**2 + 15*b**2*c**2*x**3 + 3*b*c**3*x**4))/(15*sqrt(x)*x)
```

3.81 $\int \frac{x^{7/2}(A+Bx)}{bx+cx^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{x^{7/2}(A+Bx)}{bx+cx^2} dx = -\frac{2b^2(bB-Ac)\sqrt{x}}{c^4} + \frac{2b(bB-Ac)x^{3/2}}{3c^3} - \frac{2(bB-Ac)x^{5/2}}{5c^2} + \frac{2Bx^{7/2}}{7c} + \frac{2b^{5/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{9/2}}$$

output
```
-2*b^2*(-A*c+B*b)*x^(1/2)/c^4+2/3*b*(-A*c+B*b)*x^(3/2)/c^3-2/5*(-A*c+B*b)*
x^(5/2)/c^2+2/7*B*x^(7/2)/c+2*b^(5/2)*(-A*c+B*b)*arctan(c^(1/2)*x^(1/2)/b^
(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{x^{7/2}(A+Bx)}{bx+cx^2} dx = \frac{2\sqrt{x}(-105b^3B+35b^2c(3A+Bx)-7bc^2x(5A+3Bx)+3c^3x^2(7A+5Bx))}{105c^4} + \frac{2b^{5/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{9/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/(b*x + c*x^2),x]
```

output
```
(2*Sqrt[x]*(-105*b^3*B + 35*b^2*c*(3*A + B*x) - 7*b*c^2*x*(5*A + 3*B*x) +
3*c^3*x^2*(7*A + 5*B*x)))/(105*c^4) + (2*b^(5/2)*(b*B - A*c)*ArcTan[(Sqrt[
c]*Sqrt[x])/Sqrt[b]])/c^(9/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {9, 90, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{bx+cx^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^{5/2}(A+Bx)}{b+cx}dx$$

$$\Big\downarrow \; 90$$

$$\frac{2Bx^{7/2}}{7c}-\frac{(bB-Ac)\int \frac{x^{5/2}}{b+cx}dx}{c}$$

$$\Big\downarrow \; 60$$

$$\frac{2Bx^{7/2}}{7c}-\frac{(bB-Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\int \frac{x^{3/2}}{b+cx}dx}{c}\right)}{c}$$

$$\Big\downarrow \; 60$$

$$\frac{2Bx^{7/2}}{7c}-\frac{(bB-Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{b\int \frac{\sqrt{x}}{b+cx}dx}{c}\right)}{c}\right)}{c}$$

$$\Big\downarrow \; 60$$

$$\frac{2Bx^{7/2}}{7c}-\frac{(bB-Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{b\int\frac{1}{\sqrt{x}(b+cx)}dx}{c}\right)}{c}\right)}{c}\right)}{c}$$

$$\Big\downarrow \; 73$$

$$\frac{2Bx^{7/2}}{7c}-\frac{(bB-Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{b+cx}d\sqrt{x}}{c}\right)}{c}\right)}{c}\right)}{c}$$

$$\Big\downarrow \; 218$$

$$\frac{2Bx^{7/2}}{7c}-\frac{(bB-Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{3/2}}\right)}{c}\right)}{c}\right)}{c}$$

input
```
Int[(x^(7/2)*(A + B*x))/(b*x + c*x^2),x]
```

output
```
(2*B*x^(7/2))/(7*c) - ((b*B - A*c)*((2*x^(5/2))/(5*c) - (b*((2*x^(3/2))/(3
*c) - (b*((2*Sqrt[x])/c - (2*Sqrt[b]*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/c^
(3/2)))/c))/c))/c
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

method	result
risch	$\frac{2(15Bc^3x^3+21Ac^3x^2-21Bbc^2x^2-35Abc^2x+35Bb^2cx+105Ab^2c-105Bb^3)\sqrt{x}}{105c^4}-\frac{2b^3(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{c^4\sqrt{bc}}$
derivativedivides	$\frac{\frac{2Bc^3x^{\frac{7}{2}}}{7}+\frac{2Ac^3x^{\frac{5}{2}}}{5}-\frac{2Bbc^2x^{\frac{5}{2}}}{5}-\frac{2Abc^2x^{\frac{3}{2}}}{3}+\frac{2Bb^2cx^{\frac{3}{2}}}{3}+2Ab^2c\sqrt{x}-2Bb^3\sqrt{x}}{c^4}-\frac{2b^3(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{c^4\sqrt{bc}}$
default	$\frac{\frac{2Bc^3x^{\frac{7}{2}}}{7}+\frac{2Ac^3x^{\frac{5}{2}}}{5}-\frac{2Bbc^2x^{\frac{5}{2}}}{5}-\frac{2Abc^2x^{\frac{3}{2}}}{3}+\frac{2Bb^2cx^{\frac{3}{2}}}{3}+2Ab^2c\sqrt{x}-2Bb^3\sqrt{x}}{c^4}-\frac{2b^3(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{c^4\sqrt{bc}}$

input
```
int(x^(7/2)*(B*x+A)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
2/105*(15*B*c^3*x^3+21*A*c^3*x^2-21*B*b*c^2*x^2-35*A*b*c^2*x+35*B*b^2*c*x+
105*A*b^2*c-105*B*b^3)*x^(1/2)/c^4-2*b^3*(A*c-B*b)/c^4/(b*c)^(1/2)*arctan(
c*x^(1/2)/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.03

$$\int \frac{x^{7/2}(A+Bx)}{bx+cx^2}\,dx = \left[-\frac{105\,(Bb^3-Ab^2c)\sqrt{-\frac{b}{c}}\log\left(\frac{cx-2\,c\sqrt{x}\sqrt{-\frac{b}{c}}-b}{cx+b}\right)-2\,(15\,Bc^3x^3-105\,Bb^3+105\,Ab^2}{105\,c^4}\right.$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
[-1/105*(105*(B*b^3 - A*b^2*c)*sqrt(-b/c)*log((c*x - 2*c*sqrt(x)*sqrt(-b/c
) - b)/(c*x + b)) - 2*(15*B*c^3*x^3 - 105*B*b^3 + 105*A*b^2*c - 21*(B*b*c^
2 - A*c^3)*x^2 + 35*(B*b^2*c - A*b*c^2)*x)*sqrt(x))/c^4, 2/105*(105*(B*b^3
 - A*b^2*c)*sqrt(b/c)*arctan(c*sqrt(x)*sqrt(b/c)/b) + (15*B*c^3*x^3 - 105*
B*b^3 + 105*A*b^2*c - 21*(B*b*c^2 - A*c^3)*x^2 + 35*(B*b^2*c - A*b*c^2)*x)
*sqrt(x))/c^4]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(109) = 218$.

Time = 10.79 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.60

$$\int \frac{x^{7/2}(A+Bx)}{bx+cx^2}\,dx = \begin{cases} \tilde{\infty}\left(\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{7}{2}}}{7}\right) \\ \frac{\frac{2Ax^{\frac{7}{2}}}{7}+\frac{2Bx^{\frac{9}{2}}}{9}}{b} \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{7}{2}}}{7}}{c} \\ -\frac{Ab^3\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{c^4\sqrt{-\frac{b}{c}}}+\frac{Ab^3\log\left(\sqrt{x}+\sqrt{-\frac{b}{c}}\right)}{c^4\sqrt{-\frac{b}{c}}}+\frac{2Ab^2\sqrt{x}}{c^3}-\frac{2Abx^{\frac{3}{2}}}{3c^2}+\frac{2Ax^{\frac{5}{2}}}{5c}+\frac{Bb^4\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{c^5\sqrt{-\frac{b}{c}}} \end{cases}$$

input
```
integrate(x**(7/2)*(B*x+A)/(c*x**2+b*x),x)
```

output
```
Piecewise((zoo*(2*A*x**(5/2)/5 + 2*B*x**(7/2)/7), Eq(b, 0) & Eq(c, 0)), ((
2*A*x**(7/2)/7 + 2*B*x**(9/2)/9)/b, Eq(c, 0)), ((2*A*x**(5/2)/5 + 2*B*x**(
7/2)/7)/c, Eq(b, 0)), (-A*b**3*log(sqrt(x) - sqrt(-b/c))/(c**4*sqrt(-b/c))
 + A*b**3*log(sqrt(x) + sqrt(-b/c))/(c**4*sqrt(-b/c)) + 2*A*b**2*sqrt(x)/c
**3 - 2*A*b*x**(3/2)/(3*c**2) + 2*A*x**(5/2)/(5*c) + B*b**4*log(sqrt(x) -
sqrt(-b/c))/(c**5*sqrt(-b/c)) - B*b**4*log(sqrt(x) + sqrt(-b/c))/(c**5*sqr
t(-b/c)) - 2*B*b**3*sqrt(x)/c**4 + 2*B*b**2*x**(3/2)/(3*c**3) - 2*B*b*x**(
5/2)/(5*c**2) + 2*B*x**(7/2)/(7*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.93

$$\int \frac{x^{7/2}(A+Bx)}{bx+cx^2}\,dx = \frac{2\,(Bb^4-Ab^3c)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}c^4} + \frac{2\left(15\,Bc^3x^{\frac{7}{2}}-21\,(Bbc^2-Ac^3)x^{\frac{5}{2}}+35\,(Bb^2c-Abc^2)x^{\frac{3}{2}}-105\,(Bb^3-Ab^2c)\sqrt{x}\right)}{105\,c^4}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
2*(B*b^4 - A*b^3*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*c^4) + 2/105*(1
5*B*c^3*x^(7/2) - 21*(B*b*c^2 - A*c^3)*x^(5/2) + 35*(B*b^2*c - A*b*c^2)*x^
(3/2) - 105*(B*b^3 - A*b^2*c)*sqrt(x))/c^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02

$$\int \frac{x^{7/2}(A+Bx)}{bx+cx^2}\,dx = \frac{2\,(Bb^4-Ab^3c)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}c^4} + \frac{2\left(15\,Bc^6x^{\frac{7}{2}}-21\,Bbc^5x^{\frac{5}{2}}+21\,Ac^6x^{\frac{5}{2}}+35\,Bb^2c^4x^{\frac{3}{2}}-35\,Abc^5x^{\frac{3}{2}}-105\,Bb^3c^3\sqrt{x}+105\,Ab^2c^4\sqrt{x}\right)}{105\,c^7}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
2*(B*b^4 - A*b^3*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*c^4) + 2/105*(1
5*B*c^6*x^(7/2) - 21*B*b*c^5*x^(5/2) + 21*A*c^6*x^(5/2) + 35*B*b^2*c^4*x^(
3/2) - 35*A*b*c^5*x^(3/2) - 105*B*b^3*c^3*sqrt(x) + 105*A*b^2*c^4*sqrt(x))
/c^7
```

Mupad [B] (verification not implemented)

Time = 5.14 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.11

$$\int \frac{x^{7/2}(A+Bx)}{bx+cx^2}\,dx = x^{5/2}\left(\frac{2\,A}{5\,c}-\frac{2\,B\,b}{5\,c^2}\right)+\frac{2\,B\,x^{7/2}}{7\,c}+\frac{b^2\,\sqrt{x}\,\left(\frac{2\,A}{c}-\frac{2\,B\,b}{c^2}\right)}{c^2} + \frac{2\,b^{5/2}\,\mathrm{atan}\left(\frac{b^{5/2}\,\sqrt{c}\,\sqrt{x}\,(A\,c-B\,b)}{B\,b^4-A\,b^3\,c}\right)\,(A\,c-B\,b)}{c^{9/2}}-\frac{b\,x^{3/2}\,\left(\frac{2\,A}{c}-\frac{2\,B\,b}{c^2}\right)}{3\,c}$$

input
```
int((x^(7/2)*(A + B*x))/(b*x + c*x^2),x)
```

output
```
x^(5/2)*((2*A)/(5*c) - (2*B*b)/(5*c^2)) + (2*B*x^(7/2))/(7*c) + (b^2*x^(1/
2)*((2*A)/c - (2*B*b)/c^2))/c^2 + (2*b^(5/2)*atan((b^(5/2)*c^(1/2)*x^(1/2)
*(A*c - B*b))/(B*b^4 - A*b^3*c))*(A*c - B*b))/c^(9/2) - (b*x^(3/2)*((2*A)/
c - (2*B*b)/c^2))/(3*c)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.12

$$\int \frac{x^{7/2}(A+Bx)}{bx+cx^2}\,dx = \frac{-2\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,b^2c+2\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)b^4+2\sqrt{x}\,a\,b^2c^2-\frac{2\sqrt{x}\,ab\,c^3x}{3}+\frac{2\sqrt{}}{}}{c^5}$$

input
```
int(x^(7/2)*(B*x+A)/(c*x^2+b*x),x)
```

output
```
(2*( - 105*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b**2*c +
105*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**4 + 105*sqrt(x)
*a*b**2*c**2 - 35*sqrt(x)*a*b*c**3*x + 21*sqrt(x)*a*c**4*x**2 - 105*sqrt(x
)*b**4*c + 35*sqrt(x)*b**3*c**2*x - 21*sqrt(x)*b**2*c**3*x**2 + 15*sqrt(x)
*b*c**4*x**3))/(105*c**5)
```

3.82 $\int \frac{x^{5/2}(A+Bx)}{bx+cx^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{x^{5/2}(A+Bx)}{bx+cx^2} \, dx = \frac{2b(bB-Ac)\sqrt{x}}{c^3} - \frac{2(bB-Ac)x^{3/2}}{3c^2} + \frac{2Bx^{5/2}}{5c} - \frac{2b^{3/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{7/2}}$$

output
```
2*b*(-A*c+B*b)*x^(1/2)/c^3-2/3*(-A*c+B*b)*x^(3/2)/c^2+2/5*B*x^(5/2)/c-2*b^
(3/2)*(-A*c+B*b)*arctan(c^(1/2)*x^(1/2)/b^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.90

$$\int \frac{x^{5/2}(A+Bx)}{bx+cx^2} \, dx = \frac{2\sqrt{x}(15b^2B - 5bc(3A+Bx) + c^2x(5A+3Bx))}{15c^3} - \frac{2b^{3/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{7/2}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(b*x + c*x^2),x]
```

output
```
(2*Sqrt[x]*(15*b^2*B - 5*b*c*(3*A + B*x) + c^2*x*(5*A + 3*B*x)))/(15*c^3)
- (2*b^(3/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/c^(7/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {9, 90, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{bx+cx^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^{3/2}(A+Bx)}{b+cx}dx$$

$$\Big\downarrow \text{ 90}$$

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\int \frac{x^{3/2}}{b+cx}dx}{c}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\left(\frac{2x^{3/2}}{3c}-\frac{b\int \frac{\sqrt{x}}{b+cx}dx}{c}\right)}{c}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{b\int \frac{1}{\sqrt{x}(b+cx)}dx}{c}\right)}{c}\right)}{c}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\int \frac{1}{b+cx}d\sqrt{x}}{c}\right)}{c}\right)}{c}$$

$$\Big\downarrow \text{218}$$

$$\frac{2Bx^{5/2}}{5c}-\frac{(bB-Ac)\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{3/2}}\right)}{c}\right)}{c}$$

input
```
Int[(x^(5/2)*(A + B*x))/(b*x + c*x^2),x]
```

output
```
(2*B*x^(5/2))/(5*c) - ((b*B - A*c)*((2*x^(3/2))/(3*c) - (b*((2*Sqrt[x])/c
- (2*Sqrt[b]*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/c^(3/2)))/c))/c
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

method	result	size
risch	$-\frac{2\left(-3B\,c^{2}x^{2}-5A\,c^{2}x+5Bbcx+15Abc-15B\,b^{2}\right)\sqrt{x}}{15c^{3}}+\frac{2b^{2}\left(Ac-Bb\right)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{c^{3}\sqrt{bc}}$	76
derivativedivides	$-\frac{2\left(-\frac{B\,c^{2}x^{\frac{5}{2}}}{5}-\frac{A\,c^{2}x^{\frac{3}{2}}}{3}+\frac{Bbc\,x^{\frac{3}{2}}}{3}+Abc\sqrt{x}-B\,b^{2}\sqrt{x}\right)}{c^{3}}+\frac{2b^{2}\left(Ac-Bb\right)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{c^{3}\sqrt{bc}}$	82
default	$-\frac{2\left(-\frac{B\,c^{2}x^{\frac{5}{2}}}{5}-\frac{A\,c^{2}x^{\frac{3}{2}}}{3}+\frac{Bbc\,x^{\frac{3}{2}}}{3}+Abc\sqrt{x}-B\,b^{2}\sqrt{x}\right)}{c^{3}}+\frac{2b^{2}\left(Ac-Bb\right)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{c^{3}\sqrt{bc}}$	82

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*B*c^2*x^2-5*A*c^2*x+5*B*b*c*x+15*A*b*c-15*B*b^2)*x^(1/2)/c^3+2*b
^2*(A*c-B*b)/c^3/(b*c)^(1/2)*arctan(c*x^(1/2)/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 2.00

$$\int \frac{x^{5/2}(A+Bx)}{bx+cx^2}\,dx = \left[-\frac{15\,(Bb^2-Abc)\sqrt{-\frac{b}{c}}\log\left(\frac{cx+2\,c\sqrt{x}\sqrt{-\frac{b}{c}}-b}{cx+b}\right)-2\,(3\,Bc^2x^2+15\,Bb^2-15\,Abc-5\,(}{15\,c^3}\right.$$

$$\left.-\frac{2\left(15\,(Bb^2-Abc)\sqrt{\frac{b}{c}}\arctan\left(\frac{c\sqrt{x}\sqrt{\frac{b}{c}}}{b}\right)-(3\,Bc^2x^2+15\,Bb^2-15\,Abc-5\,(Bbc-Ac^2)x)\sqrt{x}\right)}{15\,c^3}\right]$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
[-1/15*(15*(B*b^2 - A*b*c)*sqrt(-b/c)*log((c*x + 2*c*sqrt(x)*sqrt(-b/c) -
b)/(c*x + b)) - 2*(3*B*c^2*x^2 + 15*B*b^2 - 15*A*b*c - 5*(B*b*c - A*c^2)*x
)*sqrt(x))/c^3, -2/15*(15*(B*b^2 - A*b*c)*sqrt(b/c)*arctan(c*sqrt(x)*sqrt(
b/c)/b) - (3*B*c^2*x^2 + 15*B*b^2 - 15*A*b*c - 5*(B*b*c - A*c^2)*x)*sqrt(x
))/c^3]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(87) = 174$.

Time = 4.45 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.89

$$\int \frac{x^{5/2}(A+Bx)}{bx+cx^2}\,dx = \begin{cases} \tilde{\infty}\left(\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{5}{2}}}{5}\right) \\ \frac{\frac{2Ax^{\frac{5}{2}}}{5}+\frac{2Bx^{\frac{7}{2}}}{7}}{b} \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{5}{2}}}{5}}{c} \\ \frac{Ab^2\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{c^3\sqrt{-\frac{b}{c}}}-\frac{Ab^2\log\left(\sqrt{x}+\sqrt{-\frac{b}{c}}\right)}{c^3\sqrt{-\frac{b}{c}}}-\frac{2Ab\sqrt{x}}{c^2}+\frac{2Ax^{\frac{3}{2}}}{3c}-\frac{Bb^3\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{c^4\sqrt{-\frac{b}{c}}}+\frac{Bb^3\log}{c} \end{cases}$$

input
```
integrate(x**(5/2)*(B*x+A)/(c*x**2+b*x),x)
```

output
```
Piecewise((zoo*(2*A*x**(3/2)/3 + 2*B*x**(5/2)/5), Eq(b, 0) & Eq(c, 0)), ((
2*A*x**(5/2)/5 + 2*B*x**(7/2)/7)/b, Eq(c, 0)), ((2*A*x**(3/2)/3 + 2*B*x**(
5/2)/5)/c, Eq(b, 0)), (A*b**2*log(sqrt(x) - sqrt(-b/c))/(c**3*sqrt(-b/c))
- A*b**2*log(sqrt(x) + sqrt(-b/c))/(c**3*sqrt(-b/c)) - 2*A*b*sqrt(x)/c**2
+ 2*A*x**(3/2)/(3*c) - B*b**3*log(sqrt(x) - sqrt(-b/c))/(c**4*sqrt(-b/c))
+ B*b**3*log(sqrt(x) + sqrt(-b/c))/(c**4*sqrt(-b/c)) + 2*B*b**2*sqrt(x)/c*
*3 - 2*B*b*x**(3/2)/(3*c**2) + 2*B*x**(5/2)/(5*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.91

$$\int \frac{x^{5/2}(A+Bx)}{bx+cx^2}\,dx = -\frac{2\left(Bb^3-Ab^2c\right)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} + \frac{2\left(3\,Bc^2x^{\frac{5}{2}}-5\left(Bbc-Ac^2\right)x^{\frac{3}{2}}+15\left(Bb^2-Abc\right)\sqrt{x}\right)}{15\,c^3}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
-2*(B*b^3 - A*b^2*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*c^3) + 2/15*(3
*B*c^2*x^(5/2) - 5*(B*b*c - A*c^2)*x^(3/2) + 15*(B*b^2 - A*b*c)*sqrt(x))/c
^3
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.01

$$\int \frac{x^{5/2}(A+Bx)}{bx+cx^2}\,dx = -\frac{2\left(Bb^3-Ab^2c\right)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} + \frac{2\left(3\,Bc^4x^{\frac{5}{2}}-5\,Bbc^3x^{\frac{3}{2}}+5\,Ac^4x^{\frac{3}{2}}+15\,Bb^2c^2\sqrt{x}-15\,Abc^3\sqrt{x}\right)}{15\,c^5}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
-2*(B*b^3 - A*b^2*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*c^3) + 2/15*(3
*B*c^4*x^(5/2) - 5*B*b*c^3*x^(3/2) + 5*A*c^4*x^(3/2) + 15*B*b^2*c^2*sqrt(x
) - 15*A*b*c^3*sqrt(x))/c^5
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.12

$$\int \frac{x^{5/2}(A+Bx)}{bx+cx^2}\,dx = x^{3/2}\left(\frac{2\,A}{3\,c}-\frac{2\,B\,b}{3\,c^2}\right)+\frac{2\,B\,x^{5/2}}{5\,c} - \frac{2\,b^{3/2}\,\mathrm{atan}\left(\frac{b^{3/2}\,\sqrt{c}\,\sqrt{x}\,(A\,c-B\,b)}{B\,b^3-A\,b^2\,c}\right)\,(A\,c-B\,b)}{c^{7/2}}-\frac{b\,\sqrt{x}\,\left(\frac{2\,A}{c}-\frac{2\,B\,b}{c^2}\right)}{c}$$

input
```
int((x^(5/2)*(A + B*x))/(b*x + c*x^2),x)
```

output
```
x^(3/2)*((2*A)/(3*c) - (2*B*b)/(3*c^2)) + (2*B*x^(5/2))/(5*c) - (2*b^(3/2)
*atan((b^(3/2)*c^(1/2)*x^(1/2)*(A*c - B*b))/(B*b^3 - A*b^2*c))*(A*c - B*b)
)/c^(7/2) - (b*x^(1/2)*((2*A)/c - (2*B*b)/c^2))/c
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.09

$$\int \frac{x^{5/2}(A+Bx)}{bx+cx^2}\,dx = \frac{2\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)abc-2\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)b^3-2\sqrt{x}\,ab\,c^2+\frac{2\sqrt{x}\,a\,c^3x}{3}+2\sqrt{x}\,b^3}{c^4}$$

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x),x)
```

output
```
(2*(15*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b*c - 15*sqrt
(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**3 - 15*sqrt(x)*a*b*c**2
 + 5*sqrt(x)*a*c**3*x + 15*sqrt(x)*b**3*c - 5*sqrt(x)*b**2*c**2*x + 3*sqrt
(x)*b*c**3*x**2))/(15*c**4)
```

3.83 $\int \frac{x^{3/2}(A+Bx)}{bx+cx^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \frac{x^{3/2}(A+Bx)}{bx+cx^2} \, dx = -\frac{2(bB-Ac)\sqrt{x}}{c^2} + \frac{2Bx^{3/2}}{3c} + \frac{2\sqrt{b}(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{5/2}}$$

output
```
-2*(-A*c+B*b)*x^(1/2)/c^2+2/3*B*x^(3/2)/c+2*b^(1/2)*(-A*c+B*b)*arctan(c^(1
/2)*x^(1/2)/b^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{x^{3/2}(A+Bx)}{bx+cx^2} \, dx = \frac{2\sqrt{x}(-3bB+3Ac+Bcx)}{3c^2} + \frac{2\sqrt{b}(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{5/2}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(b*x + c*x^2),x]
```

output
```
(2*Sqrt[x]*(-3*b*B + 3*A*c + B*c*x))/(3*c^2) + (2*Sqrt[b]*(b*B - A*c)*ArcT
an[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/c^(5/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {9, 90, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{bx+cx^2}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{\sqrt{x}(A+Bx)}{b+cx}dx$$

$$\Big\downarrow \ 90$$

$$\frac{2Bx^{3/2}}{3c}-\frac{(bB-Ac)\int \frac{\sqrt{x}}{b+cx}dx}{c}$$

$$\Big\downarrow \ 60$$

$$\frac{2Bx^{3/2}}{3c}-\frac{(bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{b\int \frac{1}{\sqrt{x}(b+cx)}dx}{c}\right)}{c}$$

$$\Big\downarrow \ 73$$

$$\frac{2Bx^{3/2}}{3c}-\frac{(bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\int \frac{1}{b+cx}d\sqrt{x}}{c}\right)}{c}$$

$$\Big\downarrow \ 218$$

$$\frac{2Bx^{3/2}}{3c}-\frac{(bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{3/2}}\right)}{c}$$

input
```
Int[(x^(3/2)*(A + B*x))/(b*x + c*x^2),x]
```

output
```
(2*B*x^(3/2))/(3*c) - ((b*B - A*c)*((2*Sqrt[x])/c - (2*Sqrt[b]*ArcTan[(Sqr
t[c]*Sqrt[x])/Sqrt[b]])/c^(3/2)))/c
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.77

method	result	size
risch	$\frac{2(Bcx+3Ac-3Bb)\sqrt{x}}{3c^2}-\frac{2b(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{c^2\sqrt{bc}}$	53
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{3}{2}}}{3}+2Ac\sqrt{x}-2Bb\sqrt{x}}{c^2}-\frac{2b(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{c^2\sqrt{bc}}$	58
default	$\frac{\frac{2Bc\,x^{\frac{3}{2}}}{3}+2Ac\sqrt{x}-2Bb\sqrt{x}}{c^2}-\frac{2b(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{c^2\sqrt{bc}}$	58

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
2/3*(B*c*x+3*A*c-3*B*b)*x^(1/2)/c^2-2*b*(A*c-B*b)/c^2/(b*c)^(1/2)*arctan(c
*x^(1/2)/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.87

$$\int \frac{x^{3/2}(A+Bx)}{bx+cx^2}\,dx = \left[-\frac{3\,(Bb-Ac)\sqrt{-\frac{b}{c}}\log\left(\frac{cx-2\,c\sqrt{x}\sqrt{-\frac{b}{c}}-b}{cx+b}\right)-2\,(Bcx-3\,Bb+3\,Ac)\sqrt{x}}{3\,c^2},\ \frac{2\left(3\,(Bb-\right.}{}\right.$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
[-1/3*(3*(B*b - A*c)*sqrt(-b/c)*log((c*x - 2*c*sqrt(x)*sqrt(-b/c) - b)/(c*
x + b)) - 2*(B*c*x - 3*B*b + 3*A*c)*sqrt(x))/c^2, 2/3*(3*(B*b - A*c)*sqrt(
b/c)*arctan(c*sqrt(x)*sqrt(b/c)/b) + (B*c*x - 3*B*b + 3*A*c)*sqrt(x))/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(65) = 130$.

Time = 1.73 (sec) , antiderivative size = 221, normalized size of antiderivative = 3.20

$$\int \frac{x^{3/2}(A+Bx)}{bx+cx^2}\,dx = \begin{cases} \tilde{\infty}\left(2A\sqrt{x}+\frac{2Bx^{\frac{3}{2}}}{3}\right) \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{5}{2}}}{5}}{b} \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{3}{2}}}{3}}{c} \\ -\frac{Ab\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{c^2\sqrt{-\frac{b}{c}}}+\frac{Ab\log\left(\sqrt{x}+\sqrt{-\frac{b}{c}}\right)}{c^2\sqrt{-\frac{b}{c}}}+\frac{2A\sqrt{x}}{c}+\frac{Bb^2\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{c^3\sqrt{-\frac{b}{c}}}-\frac{Bb^2\log\left(\sqrt{x}+\sqrt{-\frac{b}{c}}\right)}{c^3\sqrt{-\frac{b}{c}}} \end{cases}$$

input
```
integrate(x**(3/2)*(B*x+A)/(c*x**2+b*x),x)
```

output
```
Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(3/2)/3), Eq(b, 0) & Eq(c, 0)), ((2*A
*x**(3/2)/3 + 2*B*x**(5/2)/5)/b, Eq(c, 0)), ((2*A*sqrt(x) + 2*B*x**(3/2)/3
)/c, Eq(b, 0)), (-A*b*log(sqrt(x) - sqrt(-b/c))/(c**2*sqrt(-b/c)) + A*b*lo
g(sqrt(x) + sqrt(-b/c))/(c**2*sqrt(-b/c)) + 2*A*sqrt(x)/c + B*b**2*log(sqr
t(x) - sqrt(-b/c))/(c**3*sqrt(-b/c)) - B*b**2*log(sqrt(x) + sqrt(-b/c))/(c
**3*sqrt(-b/c)) - 2*B*b*sqrt(x)/c**2 + 2*B*x**(3/2)/(3*c), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.84

$$\int \frac{x^{3/2}(A+Bx)}{bx+cx^2}\,dx = \frac{2\,(Bb^2-Abc)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}c^2}+\frac{2\left(Bcx^{\frac{3}{2}}-3\,(Bb-Ac)\sqrt{x}\right)}{3\,c^2}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
2*(B*b^2 - A*b*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*c^2) + 2/3*(B*c*x
^(3/2) - 3*(B*b - A*c)*sqrt(x))/c^2
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \frac{x^{3/2}(A+Bx)}{bx+cx^2}\,dx = \frac{2\,(Bb^2-Abc)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}c^2}+\frac{2\left(Bc^2x^{\frac{3}{2}}-3\,Bbc\sqrt{x}+3\,Ac^2\sqrt{x}\right)}{3\,c^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
2*(B*b^2 - A*b*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*c^2) + 2/3*(B*c^2
*x^(3/2) - 3*B*b*c*sqrt(x) + 3*A*c^2*sqrt(x))/c^3
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.10

$$\int \frac{x^{3/2}(A+Bx)}{bx+cx^2}\,dx = \sqrt{x}\left(\frac{2\,A}{c}-\frac{2\,B\,b}{c^2}\right)+\frac{2\,B\,x^{3/2}}{3\,c} + \frac{2\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c}\,\sqrt{x}\,(A\,c-B\,b)}{B\,b^2-A\,b\,c}\right)\,(A\,c-B\,b)}{c^{5/2}}$$

input
```
int((x^(3/2)*(A + B*x))/(b*x + c*x^2),x)
```

output
```
x^(1/2)*((2*A)/c - (2*B*b)/c^2) + (2*B*x^(3/2))/(3*c) + (2*b^(1/2)*atan((b
^(1/2)*c^(1/2)*x^(1/2)*(A*c - B*b))/(B*b^2 - A*b*c))*(A*c - B*b))/c^(5/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.06

$$\int \frac{x^{3/2}(A+Bx)}{bx+cx^2}\,dx = \frac{-2\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)ac+2\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)b^2+2\sqrt{x}\,a\,c^2-2\sqrt{x}\,b^2c+\frac{2\sqrt{x}\,b\,c^2x}{3}}{c^3}$$

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x),x)
```

output
```
(2*( - 3*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c + 3*sqrt(
c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**2 + 3*sqrt(x)*a*c**2 - 3
*sqrt(x)*b**2*c + sqrt(x)*b*c**2*x))/(3*c**3)
```

3.84 $\int \frac{\sqrt{x}(A+Bx)}{bx+cx^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 49

$$\int \frac{\sqrt{x}(A+Bx)}{bx+cx^2} dx = \frac{2B\sqrt{x}}{c} - \frac{2(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}$$

output
```
2*B*x^(1/2)/c-2*(-A*c+B*b)*arctan(c^(1/2)*x^(1/2)/b^(1/2))/b^(1/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{x}(A+Bx)}{bx+cx^2} dx = \frac{2B\sqrt{x}}{c} - \frac{2(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(b*x + c*x^2),x]
```

output
```
(2*B*Sqrt[x])/c - (2*(b*B - A*c)*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[
b]*c^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {9, 90, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{bx+cx^2} \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{A+Bx}{\sqrt{x}(b+cx)}dx$$

$$\Big\downarrow \text{ 90}$$

$$\frac{2B\sqrt{x}}{c}-\frac{(bB-Ac)\int \frac{1}{\sqrt{x}(b+cx)}dx}{c}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2B\sqrt{x}}{c}-\frac{2(bB-Ac)\int \frac{1}{b+cx}d\sqrt{x}}{c}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2B\sqrt{x}}{c}-\frac{2(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(b*x + c*x^2),x]
```

output
```
(2*B*Sqrt[x])/c - (2*(b*B - A*c)*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[
b]*c^(3/2))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{2B\sqrt{x}}{c}+\frac{2(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{c\sqrt{bc}}$	40
default	$\frac{2B\sqrt{x}}{c}+\frac{2(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{c\sqrt{bc}}$	40
risch	$\frac{2B\sqrt{x}}{c}+\frac{2(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{c\sqrt{bc}}$	40

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
2*B*x^(1/2)/c+2*(A*c-B*b)/c/(b*c)^(1/2)*arctan(c*x^(1/2)/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.08

$$\int \frac{\sqrt{x}(A+Bx)}{bx+cx^2}\,dx = \left[\frac{2\,Bbc\sqrt{x}+(Bb-Ac)\sqrt{-bc}\log\left(\frac{cx-b-2\,\sqrt{-bc}\sqrt{x}}{cx+b}\right)}{bc^2}, \frac{2\left(Bbc\sqrt{x}+(Bb-Ac)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}}{c\sqrt{x}}\right)\right)}{bc^2}\right]$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
[(2*B*b*c*sqrt(x) + (B*b - A*c)*sqrt(-b*c)*log((c*x - b - 2*sqrt(-b*c)*sqr
t(x))/(c*x + b)))/(b*c^2), 2*(B*b*c*sqrt(x) + (B*b - A*c)*sqrt(b*c)*arctan
(sqrt(b*c)/(c*sqrt(x))))/(b*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(46) = 92$.

Time = 0.66 (sec) , antiderivative size = 180, normalized size of antiderivative = 3.67

$$\int \frac{\sqrt{x}(A+Bx)}{bx+cx^2}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{\sqrt{x}}+2B\sqrt{x}\right) & \text{for } b=0 \wedge c=0 \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{3}{2}}}{3}}{b} & \text{for } c=0 \\ \frac{-\frac{2A}{\sqrt{x}}+2B\sqrt{x}}{c} & \text{for } b=0 \\ \frac{A\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{c\sqrt{-\frac{b}{c}}} - \frac{A\log\left(\sqrt{x}+\sqrt{-\frac{b}{c}}\right)}{c\sqrt{-\frac{b}{c}}} - \frac{Bb\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{c^2\sqrt{-\frac{b}{c}}} + \frac{Bb\log\left(\sqrt{x}+\sqrt{-\frac{b}{c}}\right)}{c^2\sqrt{-\frac{b}{c}}} + \frac{2B\sqrt{x}}{c} & \text{otherwise} \end{cases}$$

input
```
integrate(x**(1/2)*(B*x+A)/(c*x**2+b*x),x)
```

output
```
Piecewise((zoo*(-2*A/sqrt(x) + 2*B*sqrt(x)), Eq(b, 0) & Eq(c, 0)), ((2*A*s
qrt(x) + 2*B*x**(3/2)/3)/b, Eq(c, 0)), ((-2*A/sqrt(x) + 2*B*sqrt(x))/c, Eq
(b, 0)), (A*log(sqrt(x) - sqrt(-b/c))/(c*sqrt(-b/c)) - A*log(sqrt(x) + sqr
t(-b/c))/(c*sqrt(-b/c)) - B*b*log(sqrt(x) - sqrt(-b/c))/(c**2*sqrt(-b/c))
+ B*b*log(sqrt(x) + sqrt(-b/c))/(c**2*sqrt(-b/c)) + 2*B*sqrt(x)/c, True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{x}(A+Bx)}{bx+cx^2}\,dx = \frac{2\,B\sqrt{x}}{c} - \frac{2\,(Bb-Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}c}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
2*B*sqrt(x)/c - 2*(B*b - A*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{x}(A+Bx)}{bx+cx^2}\,dx = \frac{2\,B\sqrt{x}}{c} - \frac{2\,(Bb-Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}c}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
2*B*sqrt(x)/c - 2*(B*b - A*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*c)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.76

$$\int \frac{\sqrt{x}(A+Bx)}{bx+cx^2}\,dx = \frac{2\,B\,\sqrt{x}}{c} + \frac{2\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{b}}\right)\,(A\,c-B\,b)}{\sqrt{b}\,c^{3/2}}$$

input
```
int((x^(1/2)*(A + B*x))/(b*x + c*x^2),x)
```

output
```
(2*B*x^(1/2))/c + (2*atan((c^(1/2)*x^(1/2))/b^(1/2))*(A*c - B*b))/(b^(1/2)
*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.18

$$\int \frac{\sqrt{x}(A+Bx)}{bx+cx^2}\,dx = \frac{2\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)ac - 2\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)b^2 + 2\sqrt{x}\,b^2c}{b\,c^2}$$

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x),x)
```

output
```
(2*(sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c - sqrt(c)*sqrt
(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**2 + sqrt(x)*b**2*c))/(b*c**2)
```

3.85 $\int \frac{A+Bx}{\sqrt{x}(bx+cx^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 49

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)}\,dx=-\frac{2A}{b\sqrt{x}}+\frac{2(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

output
```
-2*A/b/x^(1/2)+2*(-A*c+B*b)*arctan(c^(1/2)*x^(1/2)/b^(1/2))/b^(3/2)/c^(1/2
)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)}\,dx=-\frac{2A}{b\sqrt{x}}+\frac{2(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(b*x + c*x^2)),x]
```

output
```
(-2*A)/(b*Sqrt[x]) + (2*(b*B - A*c)*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(b^
(3/2)*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {9, 87, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}\left(bx+cx^2\right)}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{A+Bx}{x^{3/2}(b+cx)}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(bB-Ac)\int \frac{1}{\sqrt{x}(b+cx)}dx}{b}-\frac{2A}{b\sqrt{x}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{2(bB-Ac)\int \frac{1}{b+cx}d\sqrt{x}}{b}-\frac{2A}{b\sqrt{x}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}-\frac{2A}{b\sqrt{x}}$$

input
```
Int[(A + B*x)/(Sqrt[x]*(b*x + c*x^2)),x]
```

output
```
(-2*A)/(b*Sqrt[x]) + (2*(b*B - A*c)*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(b^
(3/2)*Sqrt[c])
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$-\frac{2A}{b\sqrt{x}}+\frac{2(-Ac+Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{b\sqrt{bc}}$	40
default	$-\frac{2A}{b\sqrt{x}}+\frac{2(-Ac+Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{b\sqrt{bc}}$	40
risch	$-\frac{2A}{b\sqrt{x}}-\frac{2(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{b\sqrt{bc}}$	40

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-2*A/b/x^(1/2)+2*(-A*c+B*b)/b/(b*c)^(1/2)*arctan(c*x^(1/2)/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)}\,dx = \left[-\frac{2\,Abc\sqrt{x}-(Bb-Ac)\sqrt{-bc}x\log\left(\frac{cx-b+2\,\sqrt{-bc}\sqrt{x}}{cx+b}\right)}{b^2cx},\right.$$
$$\left.-\frac{2\left(Abc\sqrt{x}+(Bb-Ac)\sqrt{bc}x\arctan\left(\frac{\sqrt{bc}}{c\sqrt{x}}\right)\right)}{b^2cx}\right]$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
[-(2*A*b*c*sqrt(x) - (B*b - A*c)*sqrt(-b*c)*x*log((c*x - b + 2*sqrt(-b*c)*
sqrt(x))/(c*x + b)))/(b^2*c*x), -2*(A*b*c*sqrt(x) + (B*b - A*c)*sqrt(b*c)*
x*arctan(sqrt(b*c)/(c*sqrt(x))))/(b^2*c*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. 2(46) = 92.

Time = 0.84 (sec) , antiderivative size = 178, normalized size of antiderivative = 3.63

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}-\frac{2B}{\sqrt{x}}\right) & \text{for } b=0\wedge c=0 \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}-\frac{2B}{\sqrt{x}}}{c} & \text{for } b=0 \\ \frac{-\frac{2A}{\sqrt{x}}+2B\sqrt{x}}{b} & \text{for } c=0 \\ -\frac{A\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{b\sqrt{-\frac{b}{c}}}+\frac{A\log\left(\sqrt{x}+\sqrt{-\frac{b}{c}}\right)}{b\sqrt{-\frac{b}{c}}}-\frac{2A}{b\sqrt{x}}+\frac{B\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{c\sqrt{-\frac{b}{c}}}-\frac{B\log\left(\sqrt{x}+\sqrt{-\frac{b}{c}}\right)}{c\sqrt{-\frac{b}{c}}} & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)/x**(1/2)/(c*x**2+b*x),x)
```

output
```
Piecewise((zoo*(-2*A/(3*x**(3/2)) - 2*B/sqrt(x)), Eq(b, 0) & Eq(c, 0)), ((
-2*A/(3*x**(3/2)) - 2*B/sqrt(x))/c, Eq(b, 0)), ((-2*A/sqrt(x) + 2*B*sqrt(x
))/b, Eq(c, 0)), (-A*log(sqrt(x) - sqrt(-b/c))/(b*sqrt(-b/c)) + A*log(sqrt
(x) + sqrt(-b/c))/(b*sqrt(-b/c)) - 2*A/(b*sqrt(x)) + B*log(sqrt(x) - sqrt(
-b/c))/(c*sqrt(-b/c)) - B*log(sqrt(x) + sqrt(-b/c))/(c*sqrt(-b/c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)}\,dx=\frac{2\,(Bb-Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}b}-\frac{2\,A}{b\sqrt{x}}$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
2*(B*b - A*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b) - 2*A/(b*sqrt(x))
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)}\,dx=\frac{2\,(Bb-Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}b}-\frac{2\,A}{b\sqrt{x}}$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
2*(B*b - A*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b) - 2*A/(b*sqrt(x))
```

Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)}\,dx = \frac{2\,B\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{b}\,\sqrt{c}} - \frac{2\,A\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}} - \frac{2\,A}{b\,\sqrt{x}}$$

input
```
int((A + B*x)/(x^(1/2)*(b*x + c*x^2)),x)
```

output
```
(2*B*atan((c^(1/2)*x^(1/2))/b^(1/2)))/(b^(1/2)*c^(1/2)) - (2*A*c^(1/2)*ata
n((c^(1/2)*x^(1/2))/b^(1/2)))/b^(3/2) - (2*A)/(b*x^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)}\,dx = \frac{-2\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)ac + 2\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)b^2 - 2abc}{\sqrt{x}\,b^2c}$$

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x),x)
```

output
```
(2*( - sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c + s
qrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**2 - a*b*c))/
(sqrt(x)*b**2*c)
```

3.86 $\int \frac{A+Bx}{x^{3/2}(bx+cx^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 69

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)}\,dx=-\frac{2A}{3bx^{3/2}}-\frac{2(bB-Ac)}{b^2\sqrt{x}}-\frac{2\sqrt{c}(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}}$$

output
```
-2/3*A/b/x^(3/2)-2*(-A*c+B*b)/b^2/x^(1/2)-2*c^(1/2)*(-A*c+B*b)*arctan(c^(1
/2)*x^(1/2)/b^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)}\,dx=-\frac{2(Ab+3bBx-3Acx)}{3b^2x^{3/2}}-\frac{2\sqrt{c}(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(b*x + c*x^2)),x]
```

output
```
(-2*(A*b + 3*b*B*x - 3*A*c*x))/(3*b^2*x^(3/2)) - (2*Sqrt[c]*(b*B - A*c)*Ar
cTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/b^(5/2)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {9, 87, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx}{x^{5/2}(b+cx)}dx$$

$$\Big\downarrow \; 87$$

$$\frac{(bB-Ac)\int \frac{1}{x^{3/2}(b+cx)}dx}{b}-\frac{2A}{3bx^{3/2}}$$

$$\Big\downarrow \; 61$$

$$\frac{(bB-Ac)\left(-\frac{c\int \frac{1}{\sqrt{x}(b+cx)}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2A}{3bx^{3/2}}$$

$$\Big\downarrow \; 73$$

$$\frac{(bB-Ac)\left(-\frac{2c\int \frac{1}{b+cx}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2A}{3bx^{3/2}}$$

$$\Big\downarrow \; 218$$

$$\frac{(bB-Ac)\left(-\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2A}{3bx^{3/2}}$$

input
```
Int[(A + B*x)/(x^(3/2)*(b*x + c*x^2)),x]
```

output
```
(-2*A)/(3*b*x^(3/2)) + ((b*B - A*c)*(-2/(b*Sqrt[x]) - (2*Sqrt[c]*ArcTan[(S
qrt[c]*Sqrt[x])/Sqrt[b]])/b^(3/2)))/b
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

method	result	size
risch	$-\frac{2(-3Acx+3Bbx+Ab)}{3b^2x^{\frac{3}{2}}}+\frac{2c(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{b^2\sqrt{bc}}$	54
derivativedivides	$-\frac{2A}{3b\,x^{\frac{3}{2}}}-\frac{2(-Ac+Bb)}{b^2\sqrt{x}}+\frac{2c(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{b^2\sqrt{bc}}$	57
default	$-\frac{2A}{3b\,x^{\frac{3}{2}}}-\frac{2(-Ac+Bb)}{b^2\sqrt{x}}+\frac{2c(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{b^2\sqrt{bc}}$	57

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-3*A*c*x+3*B*b*x+A*b)/b^2/x^(3/2)+2*c*(A*c-B*b)/b^2/(b*c)^(1/2)*arct
an(c*x^(1/2)/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.04

$$\int \frac{A+Bx}{x^{3/2}\,(bx+cx^2)}\,dx = \left[-\frac{3\,(Bb-Ac)x^2\sqrt{-\frac{c}{b}}\log\left(\frac{cx+2\,b\sqrt{x}\sqrt{-\frac{c}{b}}-b}{cx+b}\right)+2\,(Ab+3\,(Bb-Ac)x)\sqrt{x}}{3\,b^2x^2},\right.$$
$$\left.-\frac{2\left(3\,(Bb-Ac)x^2\sqrt{\frac{c}{b}}\arctan\left(\sqrt{x}\sqrt{\frac{c}{b}}\right)+(Ab+3\,(Bb-Ac)x)\sqrt{x}\right)}{3\,b^2x^2}\right]$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
[-1/3*(3*(B*b - A*c)*x^2*sqrt(-c/b)*log((c*x + 2*b*sqrt(x)*sqrt(-c/b) - b)
/(c*x + b)) + 2*(A*b + 3*(B*b - A*c)*x)*sqrt(x))/(b^2*x^2), -2/3*(3*(B*b -
 A*c)*x^2*sqrt(c/b)*arctan(sqrt(x)*sqrt(c/b)) + (A*b + 3*(B*b - A*c)*x)*sq
rt(x))/(b^2*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(66) = 132$.

Time = 1.55 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.16

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}\right) \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{c} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}-\frac{2B}{\sqrt{x}}}{b} \\ -\frac{2A}{3bx^{\frac{3}{2}}}+\frac{Ac\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{b^2\sqrt{-\frac{b}{c}}}-\frac{Ac\log\left(\sqrt{x}+\sqrt{-\frac{b}{c}}\right)}{b^2\sqrt{-\frac{b}{c}}}+\frac{2Ac}{b^2\sqrt{x}}-\frac{B\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{b\sqrt{-\frac{b}{c}}}+\frac{B\log\left(\sqrt{x}\right.}{b\sqrt{-}} \end{cases}$$

input
```
integrate((B*x+A)/x**(3/2)/(c*x**2+b*x),x)
```

output
```
Piecewise((zoo*(-2*A/(5*x**(5/2)) - 2*B/(3*x**(3/2))), Eq(b, 0) & Eq(c, 0)
), ((-2*A/(5*x**(5/2)) - 2*B/(3*x**(3/2)))/c, Eq(b, 0)), ((-2*A/(3*x**(3/2
)) - 2*B/sqrt(x))/b, Eq(c, 0)), (-2*A/(3*b*x**(3/2)) + A*c*log(sqrt(x) - s
qrt(-b/c))/(b**2*sqrt(-b/c)) - A*c*log(sqrt(x) + sqrt(-b/c))/(b**2*sqrt(-b
/c)) + 2*A*c/(b**2*sqrt(x)) - B*log(sqrt(x) - sqrt(-b/c))/(b*sqrt(-b/c)) +
 B*log(sqrt(x) + sqrt(-b/c))/(b*sqrt(-b/c)) - 2*B/(b*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)}\,dx = -\frac{2\left(Bbc-Ac^2\right)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}b^2}-\frac{2\left(Ab+3\left(Bb-Ac\right)x\right)}{3\,b^2x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
-2*(B*b*c - A*c^2)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b^2) - 2/3*(A*b
+ 3*(B*b - A*c)*x)/(b^2*x^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{x^{3/2}\,(bx+cx^2)}\,dx = -\frac{2\,(Bbc-Ac^2)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}b^2} - \frac{2\,(3\,Bbx-3\,Acx+Ab)}{3\,b^2x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
-2*(B*b*c - A*c^2)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b^2) - 2/3*(3*B*
b*x - 3*A*c*x + A*b)/(b^2*x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 5.20 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.78

$$\int \frac{A+Bx}{x^{3/2}\,(bx+cx^2)}\,dx = \frac{2\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{b}}\right)\,(A\,c-B\,b)}{b^{5/2}} - \frac{\frac{2\,A}{3\,b}-\frac{2\,x\,(A\,c-B\,b)}{b^2}}{x^{3/2}}$$

input
```
int((A + B*x)/(x^(3/2)*(b*x + c*x^2)),x)
```

output
```
(2*c^(1/2)*atan((c^(1/2)*x^(1/2))/b^(1/2))*(A*c - B*b))/b^(5/2) - ((2*A)/(
3*b) - (2*x*(A*c - B*b))/b^2)/x^(3/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{x^{3/2}\,(bx+cx^2)}\,dx = \frac{2\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)acx - 2\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)b^2x - \frac{2a\,b^2}{3} + 2abcx - 2b^3x}{\sqrt{x}\,b^3x}$$

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x),x)
```

output
```
(2*(3*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c*x -
3*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**2*x - a*b
**2 + 3*a*b*c*x - 3*b**3*x))/(3*sqrt(x)*b**3*x)
```

3.87 $\int \frac{A+Bx}{x^{5/2}(bx+cx^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{A+Bx}{x^{5/2}\left(bx+cx^2\right)}\,dx = -\frac{2A}{5bx^{5/2}}-\frac{2(bB-Ac)}{3b^2x^{3/2}}+\frac{2c(bB-Ac)}{b^3\sqrt{x}}+\frac{2c^{3/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}$$

output
```
-2/5*A/b/x^(5/2)-2/3*(-A*c+B*b)/b^2/x^(3/2)+2*c*(-A*c+B*b)/b^3/x^(1/2)+2*c
^(3/2)*(-A*c+B*b)*arctan(c^(1/2)*x^(1/2)/b^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^{5/2}\left(bx+cx^2\right)}\,dx = -\frac{2(5bBx(b-3cx)+A(3b^2-5bcx+15c^2x^2))}{15b^3x^{5/2}}+\frac{2c^{3/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*(b*x + c*x^2)),x]
```

output
```
(-2*(5*b*B*x*(b - 3*c*x) + A*(3*b^2 - 5*b*c*x + 15*c^2*x^2)))/(15*b^3*x^(5
/2)) + (2*c^(3/2)*(b*B - A*c)*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {9, 87, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}\left(bx+cx^2\right)}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{A+Bx}{x^{7/2}(b+cx)}dx$$

$$\Big\downarrow \ 87$$

$$\frac{(bB-Ac)\int \frac{1}{x^{5/2}(b+cx)}dx}{b}-\frac{2A}{5bx^{5/2}}$$

$$\Big\downarrow \ 61$$

$$\frac{(bB-Ac)\left(-\frac{c\int \frac{1}{x^{3/2}(b+cx)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2A}{5bx^{5/2}}$$

$$\Big\downarrow \ 61$$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{c\int \frac{1}{\sqrt{x}(b+cx)}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2A}{5bx^{5/2}}$$

$$\Big\downarrow \ 73$$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2c\int \frac{1}{b+cx}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2A}{5bx^{5/2}}$$

$$\Big\downarrow \text{218}$$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2A}{5bx^{5/2}}$$

input
```
Int[(A + B*x)/(x^(5/2)*(b*x + c*x^2)),x]
```

output
```
(-2*A)/(5*b*x^(5/2)) + ((b*B - A*c)*(-2/(3*b*x^(3/2)) - (c*(-2/(b*Sqrt[x])
 - (2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/b^(3/2)))/b))/b
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

method	result	size
derivativedivides	$-\frac{2A}{5b\,x^{\frac{5}{2}}}-\frac{2(-Ac+Bb)}{3b^{2}x^{\frac{3}{2}}}-\frac{2(Ac-Bb)c}{b^{3}\sqrt{x}}-\frac{2c^{2}(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{b^{3}\sqrt{bc}}$	76
default	$-\frac{2A}{5b\,x^{\frac{5}{2}}}-\frac{2(-Ac+Bb)}{3b^{2}x^{\frac{3}{2}}}-\frac{2(Ac-Bb)c}{b^{3}\sqrt{x}}-\frac{2c^{2}(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{b^{3}\sqrt{bc}}$	76
risch	$-\frac{2\left(15A\,c^{2}x^{2}-15x^{2}Bbc-5Abcx+5xB\,b^{2}+3b^{2}A\right)}{15b^{3}x^{\frac{5}{2}}}-\frac{2c^{2}(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{b^{3}\sqrt{bc}}$	79

input
```
int((B*x+A)/x^(5/2)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
-2/5*A/b/x^(5/2)-2/3*(-A*c+B*b)/b^2/x^(3/2)-2*(A*c-B*b)/b^3*c/x^(1/2)-2*c^
2*(A*c-B*b)/b^3/(b*c)^(1/2)*arctan(c*x^(1/2)/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.13

$$\int \frac{A+Bx}{x^{5/2}\left(bx+cx^2\right)}\,dx=\left[-\frac{15\left(Bbc-Ac^2\right)x^3\sqrt{-\frac{c}{b}}\log\left(\frac{cx-2\,b\sqrt{x}\sqrt{-\frac{c}{b}}-b}{cx+b}\right)+2\left(3\,Ab^2-15\left(Bbc-Ac^2\right)x^2+\right.}{15\,b^3x^3}\right.$$

input
```
integrate((B*x+A)/x^(5/2)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
[-1/15*(15*(B*b*c - A*c^2)*x^3*sqrt(-c/b)*log((c*x - 2*b*sqrt(x)*sqrt(-c/b
) - b)/(c*x + b)) + 2*(3*A*b^2 - 15*(B*b*c - A*c^2)*x^2 + 5*(B*b^2 - A*b*c
)*x)*sqrt(x))/(b^3*x^3), 2/15*(15*(B*b*c - A*c^2)*x^3*sqrt(c/b)*arctan(sqr
t(x)*sqrt(c/b)) - (3*A*b^2 - 15*(B*b*c - A*c^2)*x^2 + 5*(B*b^2 - A*b*c)*x)
*sqrt(x))/(b^3*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(87) = 174$.

Time = 3.64 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.91

$$\int \frac{A+Bx}{x^{5/2}\,(bx+cx^2)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{5x^{\frac{5}{2}}}\right) \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{5x^{\frac{5}{2}}}}{c} \\ \frac{-\frac{2A}{5x^{\frac{5}{2}}}-\frac{2B}{3x^{\frac{3}{2}}}}{b} \\ -\frac{2A}{5bx^{\frac{5}{2}}}+\frac{2Ac}{3b^2x^{\frac{3}{2}}}-\frac{Ac^2\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{b^3\sqrt{-\frac{b}{c}}}+\frac{Ac^2\log\left(\sqrt{x}+\sqrt{-\frac{b}{c}}\right)}{b^3\sqrt{-\frac{b}{c}}}-\frac{2Ac^2}{b^3\sqrt{x}}-\frac{2B}{3bx^{\frac{3}{2}}}+\frac{Bc\log\left(\sqrt{x}-\right.}{b^2\sqrt{-}} \end{cases}$$

input
```
integrate((B*x+A)/x**(5/2)/(c*x**2+b*x),x)
```

output
```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(5*x**(5/2))), Eq(b, 0) & Eq(c, 0)
), ((-2*A/(7*x**(7/2)) - 2*B/(5*x**(5/2)))/c, Eq(b, 0)), ((-2*A/(5*x**(5/2
)) - 2*B/(3*x**(3/2)))/b, Eq(c, 0)), (-2*A/(5*b*x**(5/2)) + 2*A*c/(3*b**2*
x**(3/2)) - A*c**2*log(sqrt(x) - sqrt(-b/c))/(b**3*sqrt(-b/c)) + A*c**2*lo
g(sqrt(x) + sqrt(-b/c))/(b**3*sqrt(-b/c)) - 2*A*c**2/(b**3*sqrt(x)) - 2*B/
(3*b*x**(3/2)) + B*c*log(sqrt(x) - sqrt(-b/c))/(b**2*sqrt(-b/c)) - B*c*log
(sqrt(x) + sqrt(-b/c))/(b**2*sqrt(-b/c)) + 2*B*c/(b**2*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{x^{5/2}\,(bx+cx^2)}\,dx = \frac{2\,(Bbc^2-Ac^3)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}b^3} - \frac{2\,(3\,Ab^2-15\,(Bbc-Ac^2)x^2+5\,(Bb^2-Abc)x)}{15\,b^3x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)/x^(5/2)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
2*(B*b*c^2 - A*c^3)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b^3) - 2/15*(3*
A*b^2 - 15*(B*b*c - A*c^2)*x^2 + 5*(B*b^2 - A*b*c)*x)/(b^3*x^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{x^{5/2}\,(bx+cx^2)}\,dx = \frac{2\,(Bbc^2-Ac^3)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}b^3} + \frac{2\,(15\,Bbcx^2-15\,Ac^2x^2-5\,Bb^2x+5\,Abcx-3\,Ab^2)}{15\,b^3x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)/x^(5/2)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
2*(B*b*c^2 - A*c^3)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b^3) + 2/15*(15
*B*b*c*x^2 - 15*A*c^2*x^2 - 5*B*b^2*x + 5*A*b*c*x - 3*A*b^2)/(b^3*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx}{x^{5/2}\left(bx+cx^2\right)}\,dx=-\frac{\frac{2\,A}{5\,b}-\frac{2\,x\,(A\,c-B\,b)}{3\,b^2}+\frac{2\,c\,x^2\,(A\,c-B\,b)}{b^3}}{x^{5/2}}-\frac{2\,c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{b}}\right)\,(A\,c-B\,b)}{b^{7/2}}$$

input
```
int((A + B*x)/(x^(5/2)*(b*x + c*x^2)),x)
```

output
```
- ((2*A)/(5*b) - (2*x*(A*c - B*b))/(3*b^2) + (2*c*x^2*(A*c - B*b))/b^3)/x^
(5/2) - (2*c^(3/2)*atan((c^(1/2)*x^(1/2))/b^(1/2))*(A*c - B*b))/b^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx}{x^{5/2}\left(bx+cx^2\right)}\,dx=\frac{-2\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,c^2x^2+2\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)b^2c\,x^2-\frac{2a\,b^3}{5}+\frac{2a\,b^2cx}{3}}{\sqrt{x}\,b^4x^2}$$

input
```
int((B*x+A)/x^(5/2)/(c*x^2+b*x),x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c*
*2*x**2 + 15*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b
**2*c*x**2 - 3*a*b**3 + 5*a*b**2*c*x - 15*a*b*c**2*x**2 - 5*b**4*x + 15*b*
*3*c*x**2))/(15*sqrt(x)*b**4*x**2)
```

3.88 $\int \frac{A+Bx}{x^{7/2}(bx+cx^2)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 113

$$\int \frac{A+Bx}{x^{7/2}(bx+cx^2)}\,dx = -\frac{2A}{7bx^{7/2}} - \frac{2(bB-Ac)}{5b^2x^{5/2}} + \frac{2c(bB-Ac)}{3b^3x^{3/2}} - \frac{2c^2(bB-Ac)}{b^4\sqrt{x}} - \frac{2c^{5/2}(bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{9/2}}$$

output
```
-2/7*A/b/x^(7/2)-2/5*(-A*c+B*b)/b^2/x^(5/2)+2/3*c*(-A*c+B*b)/b^3/x^(3/2)-2
*c^2*(-A*c+B*b)/b^4/x^(1/2)-2*c^(5/2)*(-A*c+B*b)*arctan(c^(1/2)*x^(1/2)/b^
(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

$$\int \frac{A+Bx}{x^{7/2}(bx+cx^2)}\,dx = \frac{-14bBx(3b^2-5bcx+15c^2x^2)+A(-30b^3+42b^2cx-70bc^2x^2+210c^3x^3)}{105b^4x^{7/2}} + \frac{2c^{5/2}(-bB+Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{9/2}}$$

input
```
Integrate[(A + B*x)/(x^(7/2)*(b*x + c*x^2)),x]
```

output
```
(-14*b*B*x*(3*b^2 - 5*b*c*x + 15*c^2*x^2) + A*(-30*b^3 + 42*b^2*c*x - 70*b
*c^2*x^2 + 210*c^3*x^3))/(105*b^4*x^(7/2)) + (2*c^(5/2)*(-(b*B) + A*c)*Arc
Tan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/b^(9/2)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {9, 87, 61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{7/2}(bx+cx^2)}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx}{x^{9/2}(b+cx)}dx$$

$$\Big\downarrow \; 87$$

$$\frac{(bB-Ac)\int \frac{1}{x^{7/2}(b+cx)}dx}{b}-\frac{2A}{7bx^{7/2}}$$

$$\Big\downarrow \; 61$$

$$\frac{(bB-Ac)\left(-\frac{c\int \frac{1}{x^{5/2}(b+cx)}dx}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2A}{7bx^{7/2}}$$

$$\Big\downarrow \; 61$$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{c\int \frac{1}{x^{3/2}(b+cx)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2A}{7bx^{7/2}}$$

$$\Big\downarrow \; 61$$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{c\left(-\frac{c\int\frac{1}{\sqrt{x}(b+cx)}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2A}{7bx^{7/2}}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{c\left(-\frac{2c\int\frac{1}{b+cx}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2A}{7bx^{7/2}}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(bB-Ac)\left(-\frac{c\left(-\frac{c\left(-\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{b}-\frac{2A}{7bx^{7/2}}$$

input
```
Int[(A + B*x)/(x^(7/2)*(b*x + c*x^2)),x]
```

output
```
(-2*A)/(7*b*x^(7/2)) + ((b*B - A*c)*(-2/(5*b*x^(5/2)) - (c*(-2/(3*b*x^(3/2
)) - (c*(-2/(b*Sqrt[x]) - (2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/b^
(3/2)))/b))/b))/b
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.84

method	result
derivativedivides	$\frac{2c^3(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{b^4\sqrt{bc}}-\frac{2A}{7b\,x^{\frac{7}{2}}}-\frac{2(-Ac+Bb)}{5b^2x^{\frac{5}{2}}}-\frac{2(Ac-Bb)c}{3b^3x^{\frac{3}{2}}}+\frac{2c^2(Ac-Bb)}{b^4\sqrt{x}}$
default	$\frac{2c^3(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{b^4\sqrt{bc}}-\frac{2A}{7b\,x^{\frac{7}{2}}}-\frac{2(-Ac+Bb)}{5b^2x^{\frac{5}{2}}}-\frac{2(Ac-Bb)c}{3b^3x^{\frac{3}{2}}}+\frac{2c^2(Ac-Bb)}{b^4\sqrt{x}}$
risch	$-\frac{2(-105A\,c^3x^3+105x^3Bb\,c^2+35Ab\,c^2x^2-35x^2B\,b^2c-21A\,b^2cx+21xB\,b^3+15A\,b^3)}{105b^4x^{\frac{7}{2}}}+\frac{2c^3(Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{b^4\sqrt{bc}}$

input
```
int((B*x+A)/x^(7/2)/(c*x^2+b*x),x,method=_RETURNVERBOSE)
```

output
```
2*c^3*(A*c-B*b)/b^4/(b*c)^(1/2)*arctan(c*x^(1/2)/(b*c)^(1/2))-2/7*A/b/x^(7
/2)-2/5*(-A*c+B*b)/b^2/x^(5/2)-2/3*(A*c-B*b)/b^3*c/x^(3/2)+2*c^2*(A*c-B*b)
/b^4/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 241, normalized size of antiderivative = 2.13

$$\int \frac{A+Bx}{x^{7/2}\left(bx+cx^2\right)}\,dx=\left[-\frac{105\,(Bbc^2-Ac^3)x^4\sqrt{-\frac{c}{b}}\log\left(\frac{cx+2\,b\sqrt{x}\sqrt{-\frac{c}{b}}-b}{cx+b}\right)+2\,(15\,Ab^3+105\,(Bbc^2-Ac^3)}{105\,b^4x^4}\right.$$

$$-\frac{2\,(105\,(Bbc^2-Ac^3)x^4\sqrt{\frac{c}{b}}\arctan\left(\sqrt{x}\sqrt{\frac{c}{b}}\right)+(15\,Ab^3+105\,(Bbc^2-Ac^3)x^3-35\,(Bb^2c-Abc^2)x^2+21\,(}{105\,b^4x^4}$$

input
```
integrate((B*x+A)/x^(7/2)/(c*x^2+b*x),x, algorithm="fricas")
```

output
```
[-1/105*(105*(B*b*c^2 - A*c^3)*x^4*sqrt(-c/b)*log((c*x + 2*b*sqrt(x)*sqrt(
-c/b) - b)/(c*x + b)) + 2*(15*A*b^3 + 105*(B*b*c^2 - A*c^3)*x^3 - 35*(B*b^
2*c - A*b*c^2)*x^2 + 21*(B*b^3 - A*b^2*c)*x)*sqrt(x))/(b^4*x^4), -2/105*(1
05*(B*b*c^2 - A*c^3)*x^4*sqrt(c/b)*arctan(sqrt(x)*sqrt(c/b)) + (15*A*b^3 +
 105*(B*b*c^2 - A*c^3)*x^3 - 35*(B*b^2*c - A*b*c^2)*x^2 + 21*(B*b^3 - A*b^
2*c)*x)*sqrt(x))/(b^4*x^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(109) = 218$.

Time = 9.98 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.65

$$\int \frac{A+Bx}{x^{7/2}\left(bx+cx^2\right)}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{7x^{\frac{7}{2}}}\right) \\ \frac{-\frac{2A}{9x^{\frac{9}{2}}}-\frac{2B}{7x^{\frac{7}{2}}}}{c} \\ \frac{-\frac{2A}{7x^{\frac{7}{2}}}-\frac{2B}{5x^{\frac{5}{2}}}}{b} \\ -\frac{2A}{7bx^{\frac{7}{2}}}+\frac{2Ac}{5b^2x^{\frac{5}{2}}}-\frac{2Ac^2}{3b^3x^{\frac{3}{2}}}+\frac{Ac^3\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{b^4\sqrt{-\frac{b}{c}}}-\frac{Ac^3\log\left(\sqrt{x}+\sqrt{-\frac{b}{c}}\right)}{b^4\sqrt{-\frac{b}{c}}}+\frac{2Ac^3}{b^4\sqrt{x}}-\frac{2B}{5bx^{\frac{5}{2}}}+ \end{cases}$$

input
```
integrate((B*x+A)/x**(7/2)/(c*x**2+b*x),x)
```

output
```
Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(7*x**(7/2))), Eq(b, 0) & Eq(c, 0)
), ((-2*A/(9*x**(9/2)) - 2*B/(7*x**(7/2)))/c, Eq(b, 0)), ((-2*A/(7*x**(7/2
)) - 2*B/(5*x**(5/2)))/b, Eq(c, 0)), (-2*A/(7*b*x**(7/2)) + 2*A*c/(5*b**2*
x**(5/2)) - 2*A*c**2/(3*b**3*x**(3/2)) + A*c**3*log(sqrt(x) - sqrt(-b/c))/
(b**4*sqrt(-b/c)) - A*c**3*log(sqrt(x) + sqrt(-b/c))/(b**4*sqrt(-b/c)) + 2
*A*c**3/(b**4*sqrt(x)) - 2*B/(5*b*x**(5/2)) + 2*B*c/(3*b**2*x**(3/2)) - B*
c**2*log(sqrt(x) - sqrt(-b/c))/(b**3*sqrt(-b/c)) + B*c**2*log(sqrt(x) + sq
rt(-b/c))/(b**3*sqrt(-b/c)) - 2*B*c**2/(b**3*sqrt(x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{x^{7/2}\,(bx+cx^2)}\,dx = -\frac{2\,(Bbc^3-Ac^4)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}b^4} - \frac{2\,(15\,Ab^3+105\,(Bbc^2-Ac^3)x^3-35\,(Bb^2c-Abc^2)x^2+21\,(Bb^3-Ab^2c)x)}{105\,b^4x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)/x^(7/2)/(c*x^2+b*x),x, algorithm="maxima")
```

output
```
-2*(B*b*c^3 - A*c^4)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b^4) - 2/105*(
15*A*b^3 + 105*(B*b*c^2 - A*c^3)*x^3 - 35*(B*b^2*c - A*b*c^2)*x^2 + 21*(B*
b^3 - A*b^2*c)*x)/(b^4*x^(7/2))
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^{7/2}\,(bx+cx^2)}\,dx = -\frac{2\,(Bbc^3-Ac^4)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}b^4} - \frac{2\,(105\,Bbc^2x^3-105\,Ac^3x^3-35\,Bb^2cx^2+35\,Abc^2x^2+21\,Bb^3x-21\,Ab^2cx+15\,Ab^3)}{105\,b^4x^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)/x^(7/2)/(c*x^2+b*x),x, algorithm="giac")
```

output
```
-2*(B*b*c^3 - A*c^4)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b^4) - 2/105*(
105*B*b*c^2*x^3 - 105*A*c^3*x^3 - 35*B*b^2*c*x^2 + 35*A*b*c^2*x^2 + 21*B*b
^3*x - 21*A*b^2*c*x + 15*A*b^3)/(b^4*x^(7/2))
```

Mupad [B] (verification not implemented)

Time = 5.19 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.80

$$\int \frac{A+Bx}{x^{7/2}\,(bx+cx^2)}\,dx = \frac{2\,c^{5/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{b}}\right)\,(A\,c-B\,b)}{b^{9/2}} - \frac{\frac{2\,A}{7\,b}-\frac{2\,x\,(A\,c-B\,b)}{5\,b^2}-\frac{2\,c^2\,x^3\,(A\,c-B\,b)}{b^4}+\frac{2\,c\,x^2\,(A\,c-B\,b)}{3\,b^3}}{x^{7/2}}$$

input
```
int((A + B*x)/(x^(7/2)*(b*x + c*x^2)),x)
```

output
```
(2*c^(5/2)*atan((c^(1/2)*x^(1/2))/b^(1/2))*(A*c - B*b))/b^(9/2) - ((2*A)/(
7*b) - (2*x*(A*c - B*b))/(5*b^2) - (2*c^2*x^3*(A*c - B*b))/b^4 + (2*c*x^2*
(A*c - B*b))/(3*b^3))/x^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx}{x^{7/2}\,(bx+cx^2)}\,dx = \frac{2\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,c^3x^3-2\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)b^2c^2x^3-\frac{2a\,b^4}{7}+\frac{2a\,b^3cx}{5}-}{\sqrt{x}\,b^5x^3}$$

input
```
int((B*x+A)/x^(7/2)/(c*x^2+b*x),x)
```

output
```
(2*(105*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c**3
*x**3 - 105*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b*
*2*c**2*x**3 - 15*a*b**4 + 21*a*b**3*c*x - 35*a*b**2*c**2*x**2 + 105*a*b*c
**3*x**3 - 21*b**5*x + 35*b**4*c*x**2 - 105*b**3*c**2*x**3))/(105*sqrt(x)*
b**5*x**3)
```

3.89 $\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 116

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \frac{b(7bB-5Ac)\sqrt{x}}{c^4} - \frac{(7bB-5Ac)x^{3/2}}{3c^3} + \frac{2Bx^{5/2}}{5c^2} + \frac{(bB-Ac)x^{5/2}}{c^2(b+cx)} - \frac{b^{3/2}(7bB-5Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{9/2}}$$

output
```
b*(-5*A*c+7*B*b)*x^(1/2)/c^4-1/3*(-5*A*c+7*B*b)*x^(3/2)/c^3+2/5*B*x^(5/2)/
c^2+(-A*c+B*b)*x^(5/2)/c^2/(c*x+b)-b^(3/2)*(-5*A*c+7*B*b)*arctan(c^(1/2)*x
^(1/2)/b^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.95

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \frac{\sqrt{x}(105b^3B+2c^3x^2(5A+3Bx)-2bc^2x(25A+7Bx)+b^2(-75Ac+70Bcx))}{15c^4(b+cx)} - \frac{b^{3/2}(7bB-5Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{9/2}}$$

input
```
Integrate[(x^(9/2)*(A + B*x))/(b*x + c*x^2)^2,x]
```

output
```
(Sqrt[x]*(105*b^3*B + 2*c^3*x^2*(5*A + 3*B*x) - 2*b*c^2*x*(25*A + 7*B*x) +
 b^2*(-75*A*c + 70*B*c*x)))/(15*c^4*(b + c*x)) - (b^(3/2)*(7*b*B - 5*A*c)*
ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/c^(9/2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {9, 87, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^{5/2}(A+Bx)}{(b+cx)^2}dx$$

$$\Big\downarrow \; 87$$

$$\frac{(7bB-5Ac)\int \frac{x^{5/2}}{b+cx}dx}{2bc}-\frac{x^{7/2}(bB-Ac)}{bc(b+cx)}$$

$$\Big\downarrow \; 60$$

$$\frac{(7bB-5Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\int \frac{x^{3/2}}{b+cx}dx}{c}\right)}{2bc}-\frac{x^{7/2}(bB-Ac)}{bc(b+cx)}$$

$$\Big\downarrow \; 60$$

$$\frac{(7bB-5Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{b\int \frac{\sqrt{x}}{b+cx}dx}{c}\right)}{c}\right)}{2bc}-\frac{x^{7/2}(bB-Ac)}{bc(b+cx)}$$

$$\Big\downarrow \; 60$$

$$\frac{(7bB-5Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{b\int\frac{1}{\sqrt{x}(b+cx)}dx}{c}\right)}{c}\right)}{c}\right)}{2bc}-\frac{x^{7/2}(bB-Ac)}{bc(b+cx)}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(7bB-5Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{b+cx}d\sqrt{x}}{c}\right)}{c}\right)}{c}\right)}{2bc}-\frac{x^{7/2}(bB-Ac)}{bc(b+cx)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(7bB-5Ac)\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{3/2}}\right)}{c}\right)}{c}\right)}{2bc}-\frac{x^{7/2}(bB-Ac)}{bc(b+cx)}$$

input

```
Int[(x^(9/2)*(A + B*x))/(b*x + c*x^2)^2,x]
```

output

```
-(((b*B - A*c)*x^(7/2))/(b*c*(b + c*x))) + ((7*b*B - 5*A*c)*((2*x^(5/2))/(
5*c) - (b*((2*x^(3/2))/(3*c) - (b*((2*Sqrt[x])/c - (2*Sqrt[b]*ArcTan[(Sqrt
[c]*Sqrt[x])/Sqrt[b]])/c^(3/2)))/c))/c))/(2*b*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.85

method	result	si
risch	$-\frac{2(-3B\,c^2x^2-5A\,c^2x+10Bbcx+30Abc-45B\,b^2)\sqrt{x}}{15c^4}+\frac{b^2\left(\frac{2\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(5Ac-7Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}}\right)}{c^4}$	99
derivativedivides	$-\frac{2\left(-\frac{B\,c^2x^{\frac{5}{2}}}{5}-\frac{A\,c^2x^{\frac{3}{2}}}{3}+\frac{2Bbc\,x^{\frac{3}{2}}}{3}+2Abc\sqrt{x}-3B\,b^2\sqrt{x}\right)}{c^4}+\frac{2b^2\left(\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(5Ac-7Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^4}$	10
default	$-\frac{2\left(-\frac{B\,c^2x^{\frac{5}{2}}}{5}-\frac{A\,c^2x^{\frac{3}{2}}}{3}+\frac{2Bbc\,x^{\frac{3}{2}}}{3}+2Abc\sqrt{x}-3B\,b^2\sqrt{x}\right)}{c^4}+\frac{2b^2\left(\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(5Ac-7Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^4}$	10

input
```
int(x^(9/2)*(B*x+A)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*B*c^2*x^2-5*A*c^2*x+10*B*b*c*x+30*A*b*c-45*B*b^2)*x^(1/2)/c^4+b^
2/c^4*(2*(-1/2*A*c+1/2*B*b)*x^(1/2)/(c*x+b)+(5*A*c-7*B*b)/(b*c)^(1/2)*arct
an(c*x^(1/2)/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.50

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \left[-\frac{15\,(7\,Bb^3-5\,Ab^2c+(7\,Bb^2c-5\,Abc^2)x)\sqrt{-\frac{b}{c}}\log\left(\frac{cx+2\,c\sqrt{x}\sqrt{-\frac{b}{c}}-b}{cx+b}\right)-2\,(6\,Bc^3x}{30\,(c^5x+}\right.$$

$$-\frac{15\,(7\,Bb^3-5\,Ab^2c+(7\,Bb^2c-5\,Abc^2)x)\sqrt{\frac{b}{c}}\arctan\left(\frac{c\sqrt{x}\sqrt{\frac{b}{c}}}{b}\right)-(6\,Bc^3x^3+105\,Bb^3-75\,Ab^2c-2\,(7\,B}{15\,(c^5x+bc^4)}$$

input
```
integrate(x^(9/2)*(B*x+A)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
[-1/30*(15*(7*B*b^3 - 5*A*b^2*c + (7*B*b^2*c - 5*A*b*c^2)*x)*sqrt(-b/c)*lo
g((c*x + 2*c*sqrt(x)*sqrt(-b/c) - b)/(c*x + b)) - 2*(6*B*c^3*x^3 + 105*B*b
^3 - 75*A*b^2*c - 2*(7*B*b*c^2 - 5*A*c^3)*x^2 + 10*(7*B*b^2*c - 5*A*b*c^2)
*x)*sqrt(x))/(c^5*x + b*c^4), -1/15*(15*(7*B*b^3 - 5*A*b^2*c + (7*B*b^2*c
- 5*A*b*c^2)*x)*sqrt(b/c)*arctan(c*sqrt(x)*sqrt(b/c)/b) - (6*B*c^3*x^3 + 1
05*B*b^3 - 75*A*b^2*c - 2*(7*B*b*c^2 - 5*A*c^3)*x^2 + 10*(7*B*b^2*c - 5*A*
b*c^2)*x)*sqrt(x))/(c^5*x + b*c^4)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 877 vs. $2(112) = 224$.

Time = 118.26 (sec) , antiderivative size = 877, normalized size of antiderivative = 7.56

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(9/2)*(B*x+A)/(c*x**2+b*x)**2,x)
```

output
```
Piecewise((zoo*(2*A*x**(3/2)/3 + 2*B*x**(5/2)/5), Eq(b, 0) & Eq(c, 0)), ((
2*A*x**(7/2)/7 + 2*B*x**(9/2)/9)/b**2, Eq(c, 0)), ((2*A*x**(3/2)/3 + 2*B*x
**(5/2)/5)/c**2, Eq(b, 0)), (75*A*b**3*c*log(sqrt(x) - sqrt(-b/c))/(30*b*c
**5*sqrt(-b/c) + 30*c**6*x*sqrt(-b/c)) - 75*A*b**3*c*log(sqrt(x) + sqrt(-b
/c))/(30*b*c**5*sqrt(-b/c) + 30*c**6*x*sqrt(-b/c)) - 150*A*b**2*c**2*sqrt(
x)*sqrt(-b/c)/(30*b*c**5*sqrt(-b/c) + 30*c**6*x*sqrt(-b/c)) + 75*A*b**2*c*
*2*x*log(sqrt(x) - sqrt(-b/c))/(30*b*c**5*sqrt(-b/c) + 30*c**6*x*sqrt(-b/c
)) - 75*A*b**2*c**2*x*log(sqrt(x) + sqrt(-b/c))/(30*b*c**5*sqrt(-b/c) + 30
*c**6*x*sqrt(-b/c)) - 100*A*b*c**3*x**(3/2)*sqrt(-b/c)/(30*b*c**5*sqrt(-b/
c) + 30*c**6*x*sqrt(-b/c)) + 20*A*c**4*x**(5/2)*sqrt(-b/c)/(30*b*c**5*sqrt
(-b/c) + 30*c**6*x*sqrt(-b/c)) - 105*B*b**4*log(sqrt(x) - sqrt(-b/c))/(30*
b*c**5*sqrt(-b/c) + 30*c**6*x*sqrt(-b/c)) + 105*B*b**4*log(sqrt(x) + sqrt(
-b/c))/(30*b*c**5*sqrt(-b/c) + 30*c**6*x*sqrt(-b/c)) + 210*B*b**3*c*sqrt(x
)*sqrt(-b/c)/(30*b*c**5*sqrt(-b/c) + 30*c**6*x*sqrt(-b/c)) - 105*B*b**3*c*
x*log(sqrt(x) - sqrt(-b/c))/(30*b*c**5*sqrt(-b/c) + 30*c**6*x*sqrt(-b/c))
+ 105*B*b**3*c*x*log(sqrt(x) + sqrt(-b/c))/(30*b*c**5*sqrt(-b/c) + 30*c**6
*x*sqrt(-b/c)) + 140*B*b**2*c**2*x**(3/2)*sqrt(-b/c)/(30*b*c**5*sqrt(-b/c)
 + 30*c**6*x*sqrt(-b/c)) - 28*B*b*c**3*x**(5/2)*sqrt(-b/c)/(30*b*c**5*sqrt
(-b/c) + 30*c**6*x*sqrt(-b/c)) + 12*B*c**4*x**(7/2)*sqrt(-b/c)/(30*b*c**5*
sqrt(-b/c) + 30*c**6*x*sqrt(-b/c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.99

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \frac{(Bb^3-Ab^2c)\sqrt{x}}{c^5x+bc^4} - \frac{(7\,Bb^3-5\,Ab^2c)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}c^4} + \frac{2\left(3\,Bc^2x^{\frac{5}{2}}-5\,(2\,Bbc-Ac^2)x^{\frac{3}{2}}+15\,(3\,Bb^2-2\,Abc)\sqrt{x}\right)}{15\,c^4}$$

input
```
integrate(x^(9/2)*(B*x+A)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
(B*b^3 - A*b^2*c)*sqrt(x)/(c^5*x + b*c^4) - (7*B*b^3 - 5*A*b^2*c)*arctan(c
*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*c^4) + 2/15*(3*B*c^2*x^(5/2) - 5*(2*B*b*c -
 A*c^2)*x^(3/2) + 15*(3*B*b^2 - 2*A*b*c)*sqrt(x))/c^4
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.05

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^2}\,dx = -\frac{(7\,Bb^3-5\,Ab^2c)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}c^4}+\frac{Bb^3\sqrt{x}-Ab^2c\sqrt{x}}{(cx+b)c^4} + \frac{2\left(3\,Bc^8x^{\frac{5}{2}}-10\,Bbc^7x^{\frac{3}{2}}+5\,Ac^8x^{\frac{3}{2}}+45\,Bb^2c^6\sqrt{x}-30\,Abc^7\sqrt{x}\right)}{15\,c^{10}}$$

input
```
integrate(x^(9/2)*(B*x+A)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
-(7*B*b^3 - 5*A*b^2*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*c^4) + (B*b^
3*sqrt(x) - A*b^2*c*sqrt(x))/((c*x + b)*c^4) + 2/15*(3*B*c^8*x^(5/2) - 10*
B*b*c^7*x^(3/2) + 5*A*c^8*x^(3/2) + 45*B*b^2*c^6*sqrt(x) - 30*A*b*c^7*sqrt
(x))/c^10
```

Mupad [B] (verification not implemented)

Time = 5.13 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.26

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^2}\,dx = x^{3/2}\left(\frac{2\,A}{3\,c^2}-\frac{4\,B\,b}{3\,c^3}\right)-\sqrt{x}\left(\frac{2\,b\left(\frac{2\,A}{c^2}-\frac{4\,B\,b}{c^3}\right)}{c}+\frac{2\,B\,b^2}{c^4}\right) + \frac{2\,B\,x^{5/2}}{5\,c^2}+\frac{\sqrt{x}\,(B\,b^3-A\,b^2\,c)}{x\,c^5+b\,c^4}-\frac{b^{3/2}\,\mathrm{atan}\left(\frac{b^{3/2}\,\sqrt{c}\,\sqrt{x}\,(5\,A\,c-7\,B\,b)}{7\,B\,b^3-5\,A\,b^2\,c}\right)\,(5\,A\,c-7\,B\,b)}{c^{9/2}}$$

input
```
int((x^(9/2)*(A + B*x))/(b*x + c*x^2)^2,x)
```

output
```
x^(3/2)*((2*A)/(3*c^2) - (4*B*b)/(3*c^3)) - x^(1/2)*((2*b*((2*A)/c^2 - (4*
B*b)/c^3))/c + (2*B*b^2)/c^4) + (2*B*x^(5/2))/(5*c^2) + (x^(1/2)*(B*b^3 -
A*b^2*c))/(b*c^4 + c^5*x) - (b^(3/2)*atan((b^(3/2)*c^(1/2)*x^(1/2)*(5*A*c
- 7*B*b))/(7*B*b^3 - 5*A*b^2*c))*(5*A*c - 7*B*b))/c^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.58

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \frac{75\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,b^2c+75\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)ab\,c^2x-105\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right.}{}$$

input
```
int(x^(9/2)*(B*x+A)/(c*x^2+b*x)^2,x)
```

output
```
(75*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b**2*c + 75*sqrt
(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b*c**2*x - 105*sqrt(c)*s
qrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**4 - 105*sqrt(c)*sqrt(b)*atan
((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**3*c*x - 75*sqrt(x)*a*b**2*c**2 - 50*sqr
t(x)*a*b*c**3*x + 10*sqrt(x)*a*c**4*x**2 + 105*sqrt(x)*b**4*c + 70*sqrt(x)
*b**3*c**2*x - 14*sqrt(x)*b**2*c**3*x**2 + 6*sqrt(x)*b*c**4*x**3)/(15*c**5
*(b + c*x))
```

3.90 $\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^2}\,dx=-\frac{(5bB-3Ac)\sqrt{x}}{c^3}+\frac{2Bx^{3/2}}{3c^2}+\frac{(bB-Ac)x^{3/2}}{c^2(b+cx)}+\frac{\sqrt{b}(5bB-3Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{7/2}}$$

output
```
-(-3*A*c+5*B*b)*x^(1/2)/c^3+2/3*B*x^(3/2)/c^2+(-A*c+B*b)*x^(3/2)/c^2/(c*x+
b)+b^(1/2)*(-3*A*c+5*B*b)*arctan(c^(1/2)*x^(1/2)/b^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^2}\,dx=\frac{\sqrt{x}(-15b^2B+bc(9A-10Bx)+2c^2x(3A+Bx))}{3c^3(b+cx)}+\frac{\sqrt{b}(5bB-3Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{7/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/(b*x + c*x^2)^2,x]
```

output
```
(Sqrt[x]*(-15*b^2*B + b*c*(9*A - 10*B*x) + 2*c^2*x*(3*A + B*x)))/(3*c^3*(b
 + c*x)) + (Sqrt[b]*(5*b*B - 3*A*c)*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/c^(
7/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {9, 87, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^2}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^{3/2}(A+Bx)}{(b+cx)^2}dx$$

$$\Big\downarrow \ 87$$

$$\frac{(5bB-3Ac)\int \frac{x^{3/2}}{b+cx}dx}{2bc}-\frac{x^{5/2}(bB-Ac)}{bc(b+cx)}$$

$$\Big\downarrow \ 60$$

$$\frac{(5bB-3Ac)\left(\frac{2x^{3/2}}{3c}-\frac{b\int \frac{\sqrt{x}}{b+cx}dx}{c}\right)}{2bc}-\frac{x^{5/2}(bB-Ac)}{bc(b+cx)}$$

$$\Big\downarrow \ 60$$

$$\frac{(5bB-3Ac)\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{b\int \frac{1}{\sqrt{x}(b+cx)}dx}{c}\right)}{c}\right)}{2bc}-\frac{x^{5/2}(bB-Ac)}{bc(b+cx)}$$

$$\Big\downarrow \ 73$$

$$\frac{(5bB-3Ac)\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{b+cx}d\sqrt{x}}{c}\right)}{c}\right)}{2bc}-\frac{x^{5/2}(bB-Ac)}{bc(b+cx)}$$

$\big\downarrow$ 218

$$\frac{(5bB-3Ac)\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{3/2}}\right)}{c}\right)}{2bc}-\frac{x^{5/2}(bB-Ac)}{bc(b+cx)}$$

input
```
Int[(x^(7/2)*(A + B*x))/(b*x + c*x^2)^2,x]
```

output
```
-(((b*B - A*c)*x^(5/2))/(b*c*(b + c*x))) + ((5*b*B - 3*A*c)*((2*x^(3/2))/(
3*c) - (b*((2*Sqrt[x])/c - (2*Sqrt[b]*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/c
^(3/2)))/c))/(2*b*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

method	result	size
risch	$\frac{2(Bcx+3Ac-6Bb)\sqrt{x}}{3c^3}-\frac{b\left(\frac{2\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(3Ac-5Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}}\right)}{c^3}$	77
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{3}{2}}}{3}+2Ac\sqrt{x}-4Bb\sqrt{x}}{c^3}-\frac{2b\left(\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(3Ac-5Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^3}$	82
default	$\frac{\frac{2Bc\,x^{\frac{3}{2}}}{3}+2Ac\sqrt{x}-4Bb\sqrt{x}}{c^3}-\frac{2b\left(\frac{\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(3Ac-5Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{c^3}$	82

input
```
int(x^(7/2)*(B*x+A)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
2/3*(B*c*x+3*A*c-6*B*b)*x^(1/2)/c^3-b/c^3*(2*(-1/2*A*c+1/2*B*b)*x^(1/2)/(c
*x+b)+(3*A*c-5*B*b)/(b*c)^(1/2)*arctan(c*x^(1/2)/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.46

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \left[-\frac{3\,(5\,Bb^2-3\,Abc+(5\,Bbc-3\,Ac^2)x)\sqrt{-\frac{b}{c}}\log\left(\frac{cx-2\,c\sqrt{x}\sqrt{-\frac{b}{c}}-b}{cx+b}\right)-2\,(2\,Bc^2x^2-}{6\,(c^4x+bc^3)}\right.$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
[-1/6*(3*(5*B*b^2 - 3*A*b*c + (5*B*b*c - 3*A*c^2)*x)*sqrt(-b/c)*log((c*x -
 2*c*sqrt(x)*sqrt(-b/c) - b)/(c*x + b)) - 2*(2*B*c^2*x^2 - 15*B*b^2 + 9*A*
b*c - 2*(5*B*b*c - 3*A*c^2)*x)*sqrt(x))/(c^4*x + b*c^3), 1/3*(3*(5*B*b^2 -
 3*A*b*c + (5*B*b*c - 3*A*c^2)*x)*sqrt(b/c)*arctan(c*sqrt(x)*sqrt(b/c)/b)
+ (2*B*c^2*x^2 - 15*B*b^2 + 9*A*b*c - 2*(5*B*b*c - 3*A*c^2)*x)*sqrt(x))/(c
^4*x + b*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(88) = 176.

Time = 54.38 (sec) , antiderivative size = 762, normalized size of antiderivative = 8.11

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(7/2)*(B*x+A)/(c*x**2+b*x)**2,x)
```

output
```
Piecewise((zoo*(2*A*sqrt(x) + 2*B*x**(3/2)/3), Eq(b, 0) & Eq(c, 0)), ((2*A
*x**(5/2)/5 + 2*B*x**(7/2)/7)/b**2, Eq(c, 0)), ((2*A*sqrt(x) + 2*B*x**(3/2
)/3)/c**2, Eq(b, 0)), (-9*A*b**2*c*log(sqrt(x) - sqrt(-b/c))/(6*b*c**4*sqr
t(-b/c) + 6*c**5*x*sqrt(-b/c)) + 9*A*b**2*c*log(sqrt(x) + sqrt(-b/c))/(6*b
*c**4*sqrt(-b/c) + 6*c**5*x*sqrt(-b/c)) + 18*A*b*c**2*sqrt(x)*sqrt(-b/c)/(
6*b*c**4*sqrt(-b/c) + 6*c**5*x*sqrt(-b/c)) - 9*A*b*c**2*x*log(sqrt(x) - sq
rt(-b/c))/(6*b*c**4*sqrt(-b/c) + 6*c**5*x*sqrt(-b/c)) + 9*A*b*c**2*x*log(s
qrt(x) + sqrt(-b/c))/(6*b*c**4*sqrt(-b/c) + 6*c**5*x*sqrt(-b/c)) + 12*A*c*
*3*x**(3/2)*sqrt(-b/c)/(6*b*c**4*sqrt(-b/c) + 6*c**5*x*sqrt(-b/c)) + 15*B*
b**3*log(sqrt(x) - sqrt(-b/c))/(6*b*c**4*sqrt(-b/c) + 6*c**5*x*sqrt(-b/c))
 - 15*B*b**3*log(sqrt(x) + sqrt(-b/c))/(6*b*c**4*sqrt(-b/c) + 6*c**5*x*sqr
t(-b/c)) - 30*B*b**2*c*sqrt(x)*sqrt(-b/c)/(6*b*c**4*sqrt(-b/c) + 6*c**5*x*
sqrt(-b/c)) + 15*B*b**2*c*x*log(sqrt(x) - sqrt(-b/c))/(6*b*c**4*sqrt(-b/c)
 + 6*c**5*x*sqrt(-b/c)) - 15*B*b**2*c*x*log(sqrt(x) + sqrt(-b/c))/(6*b*c**
4*sqrt(-b/c) + 6*c**5*x*sqrt(-b/c)) - 20*B*b*c**2*x**(3/2)*sqrt(-b/c)/(6*b
*c**4*sqrt(-b/c) + 6*c**5*x*sqrt(-b/c)) + 4*B*c**3*x**(5/2)*sqrt(-b/c)/(6*
b*c**4*sqrt(-b/c) + 6*c**5*x*sqrt(-b/c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.94

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^2}\,dx = -\frac{(Bb^2-Abc)\sqrt{x}}{c^4x+bc^3} + \frac{(5\,Bb^2-3\,Abc)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} + \frac{2\left(Bcx^{\frac{3}{2}}-3\,(2\,Bb-Ac)\sqrt{x}\right)}{3\,c^3}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-(B*b^2 - A*b*c)*sqrt(x)/(c^4*x + b*c^3) + (5*B*b^2 - 3*A*b*c)*arctan(c*sq
rt(x)/sqrt(b*c))/(sqrt(b*c)*c^3) + 2/3*(B*c*x^(3/2) - 3*(2*B*b - A*c)*sqrt
(x))/c^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \frac{(5\,Bb^2-3\,Abc)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}c^3} - \frac{Bb^2\sqrt{x}-Abc\sqrt{x}}{(cx+b)c^3} + \frac{2\left(Bc^4x^{\frac{3}{2}}-6\,Bbc^3\sqrt{x}+3\,Ac^4\sqrt{x}\right)}{3\,c^6}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
(5*B*b^2 - 3*A*b*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*c^3) - (B*b^2*s
qrt(x) - A*b*c*sqrt(x))/((c*x + b)*c^3) + 2/3*(B*c^4*x^(3/2) - 6*B*b*c^3*s
qrt(x) + 3*A*c^4*sqrt(x))/c^6
```

Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.14

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \sqrt{x}\left(\frac{2\,A}{c^2}-\frac{4\,B\,b}{c^3}\right) - \frac{\sqrt{x}\,(B\,b^2-A\,b\,c)}{x\,c^4+b\,c^3} + \frac{2\,B\,x^{3/2}}{3\,c^2} + \frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c}\,\sqrt{x}\,(3\,A\,c-5\,B\,b)}{5\,B\,b^2-3\,A\,b\,c}\right)\,(3\,A\,c-5\,B\,b)}{c^{7/2}}$$

input
```
int((x^(7/2)*(A + B*x))/(b*x + c*x^2)^2,x)
```

output
```
x^(1/2)*((2*A)/c^2 - (4*B*b)/c^3) - (x^(1/2)*(B*b^2 - A*b*c))/(b*c^3 + c^4
*x) + (2*B*x^(3/2))/(3*c^2) + (b^(1/2)*atan((b^(1/2)*c^(1/2)*x^(1/2)*(3*A*
c - 5*B*b))/(5*B*b^2 - 3*A*b*c))*(3*A*c - 5*B*b))/c^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.63

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \frac{-9\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)abc-9\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,c^2x+15\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)b^3}{3}$$

input
```
int(x^(7/2)*(B*x+A)/(c*x^2+b*x)^2,x)
```

output
```
( - 9*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b*c - 9*sqrt(c
)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c**2*x + 15*sqrt(c)*sqrt(b
)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**3 + 15*sqrt(c)*sqrt(b)*atan((sqrt
(x)*c)/(sqrt(c)*sqrt(b)))*b**2*c*x + 9*sqrt(x)*a*b*c**2 + 6*sqrt(x)*a*c**3
*x - 15*sqrt(x)*b**3*c - 10*sqrt(x)*b**2*c**2*x + 2*sqrt(x)*b*c**3*x**2)/(
3*c**4*(b + c*x))
```

3.91 $\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 74

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^2} dx = \frac{2B\sqrt{x}}{c^2} + \frac{(bB-Ac)\sqrt{x}}{c^2(b+cx)} - \frac{(3bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{b}c^{5/2}}$$

output
```
2*B*x^(1/2)/c^2+(-A*c+B*b)*x^(1/2)/c^2/(c*x+b)-(-A*c+3*B*b)*arctan(c^(1/2)
*x^(1/2)/b^(1/2))/b^(1/2)/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^2} dx = \frac{\sqrt{x}(3bB-Ac+2Bcx)}{c^2(b+cx)} + \frac{(-3bB+Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{b}c^{5/2}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(b*x + c*x^2)^2,x]
```

output
```
(Sqrt[x]*(3*b*B - A*c + 2*B*c*x))/(c^2*(b + c*x)) + ((-3*b*B + A*c)*ArcTan
[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(Sqrt[b]*c^(5/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.19, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {9, 87, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^2}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{\sqrt{x}(A+Bx)}{(b+cx)^2}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(3bB-Ac)\int \frac{\sqrt{x}}{b+cx}dx}{2bc}-\frac{x^{3/2}(bB-Ac)}{bc(b+cx)}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(3bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{b\int \frac{1}{\sqrt{x}(b+cx)}dx}{c}\right)}{2bc}-\frac{x^{3/2}(bB-Ac)}{bc(b+cx)}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(3bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2b\int \frac{1}{b+cx}d\sqrt{x}}{c}\right)}{2bc}-\frac{x^{3/2}(bB-Ac)}{bc(b+cx)}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(3bB-Ac)\left(\frac{2\sqrt{x}}{c}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{3/2}}\right)}{2bc}-\frac{x^{3/2}(bB-Ac)}{bc(b+cx)}$$

input
```
Int[(x^(5/2)*(A + B*x))/(b*x + c*x^2)^2,x]
```

output
```
-(((b*B - A*c)*x^(3/2))/(b*c*(b + c*x))) + ((3*b*B - A*c)*((2*Sqrt[x])/c -
 (2*Sqrt[b]*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/c^(3/2)))/(2*b*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

method	result	size
risch	$\frac{2B\sqrt{x}}{c^2}+\frac{\frac{2\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(Ac-3Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}}}{c^2}$	62
derivativedivides	$\frac{2B\sqrt{x}}{c^2}+\frac{\frac{2\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(Ac-3Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}}}{c^2}$	63
default	$\frac{2B\sqrt{x}}{c^2}+\frac{\frac{2\left(-\frac{Ac}{2}+\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(Ac-3Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}}}{c^2}$	63

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
2*B*x^(1/2)/c^2+1/c^2*(2*(-1/2*A*c+1/2*B*b)*x^(1/2)/(c*x+b)+(A*c-3*B*b)/(b
*c)^(1/2)*arctan(c*x^(1/2)/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.68

$$\int \frac{x^{5/2}(A+Bx)}{\left(bx+cx^2\right)^2}\,dx=\left[\frac{\left(3\,Bb^2-Abc+\left(3\,Bbc-Ac^2\right)x\right)\sqrt{-bc}\log\left(\frac{cx-b-2\,\sqrt{-bc}\sqrt{x}}{cx+b}\right)+2\,(2\,Bbc^2x+3\,Bb^2c}{2\,(bc^4x+b^2c^3)}\right.$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
[1/2*((3*B*b^2 - A*b*c + (3*B*b*c - A*c^2)*x)*sqrt(-b*c)*log((c*x - b - 2*
sqrt(-b*c)*sqrt(x))/(c*x + b)) + 2*(2*B*b*c^2*x + 3*B*b^2*c - A*b*c^2)*sqr
t(x))/(b*c^4*x + b^2*c^3), ((3*B*b^2 - A*b*c + (3*B*b*c - A*c^2)*x)*sqrt(b
*c)*arctan(sqrt(b*c)/(c*sqrt(x))) + (2*B*b*c^2*x + 3*B*b^2*c - A*b*c^2)*sq
rt(x))/(b*c^4*x + b^2*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(66) = 132$.

Time = 30.48 (sec) , antiderivative size = 634, normalized size of antiderivative = 8.57

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{\sqrt{x}}+2B\sqrt{x}\right) \\ \frac{\frac{2Ax^{\frac{3}{2}}}{3}+\frac{2Bx^{\frac{5}{2}}}{5}}{b^2} \\ \frac{-\frac{2A}{\sqrt{x}}+2B\sqrt{x}}{c^2} \\ \frac{Abc\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{2bc^3\sqrt{-\frac{b}{c}}+2c^4x\sqrt{-\frac{b}{c}}} - \frac{Abc\log\left(\sqrt{x}+\sqrt{-\frac{b}{c}}\right)}{2bc^3\sqrt{-\frac{b}{c}}+2c^4x\sqrt{-\frac{b}{c}}} - \frac{2Ac^2\sqrt{x}\sqrt{-\frac{b}{c}}}{2bc^3\sqrt{-\frac{b}{c}}+2c^4x\sqrt{-\frac{b}{c}}} + \frac{Ac^2x\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{2bc^3\sqrt{-\frac{b}{c}}+2c^4x\sqrt{-\frac{b}{c}}} - \frac{Ac}{2b} \end{cases}$$

input
```
integrate(x**(5/2)*(B*x+A)/(c*x**2+b*x)**2,x)
```

output
```
Piecewise((zoo*(-2*A/sqrt(x) + 2*B*sqrt(x)), Eq(b, 0) & Eq(c, 0)), ((2*A*x
**(3/2)/3 + 2*B*x**(5/2)/5)/b**2, Eq(c, 0)), ((-2*A/sqrt(x) + 2*B*sqrt(x))
/c**2, Eq(b, 0)), (A*b*c*log(sqrt(x) - sqrt(-b/c))/(2*b*c**3*sqrt(-b/c) +
2*c**4*x*sqrt(-b/c)) - A*b*c*log(sqrt(x) + sqrt(-b/c))/(2*b*c**3*sqrt(-b/c
) + 2*c**4*x*sqrt(-b/c)) - 2*A*c**2*sqrt(x)*sqrt(-b/c)/(2*b*c**3*sqrt(-b/c
) + 2*c**4*x*sqrt(-b/c)) + A*c**2*x*log(sqrt(x) - sqrt(-b/c))/(2*b*c**3*sq
rt(-b/c) + 2*c**4*x*sqrt(-b/c)) - A*c**2*x*log(sqrt(x) + sqrt(-b/c))/(2*b*
c**3*sqrt(-b/c) + 2*c**4*x*sqrt(-b/c)) - 3*B*b**2*log(sqrt(x) - sqrt(-b/c)
)/(2*b*c**3*sqrt(-b/c) + 2*c**4*x*sqrt(-b/c)) + 3*B*b**2*log(sqrt(x) + sqr
t(-b/c))/(2*b*c**3*sqrt(-b/c) + 2*c**4*x*sqrt(-b/c)) + 6*B*b*c*sqrt(x)*sqr
t(-b/c)/(2*b*c**3*sqrt(-b/c) + 2*c**4*x*sqrt(-b/c)) - 3*B*b*c*x*log(sqrt(x
) - sqrt(-b/c))/(2*b*c**3*sqrt(-b/c) + 2*c**4*x*sqrt(-b/c)) + 3*B*b*c*x*lo
g(sqrt(x) + sqrt(-b/c))/(2*b*c**3*sqrt(-b/c) + 2*c**4*x*sqrt(-b/c)) + 4*B*
c**2*x**(3/2)*sqrt(-b/c)/(2*b*c**3*sqrt(-b/c) + 2*c**4*x*sqrt(-b/c)), True
))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \frac{(Bb-Ac)\sqrt{x}}{c^3x+bc^2} + \frac{2\,B\sqrt{x}}{c^2} - \frac{(3\,Bb-Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}c^2}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
(B*b - A*c)*sqrt(x)/(c^3*x + b*c^2) + 2*B*sqrt(x)/c^2 - (3*B*b - A*c)*arct
an(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*c^2)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.88

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \frac{2\,B\sqrt{x}}{c^2} - \frac{(3\,Bb-Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}c^2} + \frac{Bb\sqrt{x}-Ac\sqrt{x}}{(cx+b)c^2}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
2*B*sqrt(x)/c^2 - (3*B*b - A*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*c^2
) + (B*b*sqrt(x) - A*c*sqrt(x))/((c*x + b)*c^2)
```

Mupad [B] (verification not implemented)

Time = 5.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.84

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \frac{2\,B\,\sqrt{x}}{c^2} - \frac{\sqrt{x}\,(A\,c-B\,b)}{x\,c^3+b\,c^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{b}}\right)\,(A\,c-3\,B\,b)}{\sqrt{b}\,c^{5/2}}$$

input
```
int((x^(5/2)*(A + B*x))/(b*x + c*x^2)^2,x)
```

output
```
(2*B*x^(1/2))/c^2 - (x^(1/2)*(A*c - B*b))/(b*c^2 + c^3*x) + (atan((c^(1/2)
*x^(1/2))/b^(1/2))*(A*c - 3*B*b))/(b^(1/2)*c^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.80

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \frac{\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)abc+\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,c^2x-3\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)b^3-3\sqrt{}}{b\,c^3\,(cx+b)}$$

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x)^2,x)
```

output
```
(sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b*c + sqrt(c)*sqrt(
b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c**2*x - 3*sqrt(c)*sqrt(b)*atan((
sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**3 - 3*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(s
qrt(c)*sqrt(b)))*b**2*c*x - sqrt(x)*a*b*c**2 + 3*sqrt(x)*b**3*c + 2*sqrt(x
)*b**2*c**2*x)/(b*c**3*(b + c*x))
```

3.92 $\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 64

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^2}\,dx = -\frac{(bB-Ac)\sqrt{x}}{bc(b+cx)} + \frac{(bB+Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}c^{3/2}}$$

output
```
-(-A*c+B*b)*x^(1/2)/b/c/(c*x+b)+(A*c+B*b)*arctan(c^(1/2)*x^(1/2)/b^(1/2))/
b^(3/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \frac{(-bB+Ac)\sqrt{x}}{bc(b+cx)} + \frac{(bB+Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}c^{3/2}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(b*x + c*x^2)^2,x]
```

output
```
((-(b*B) + A*c)*Sqrt[x])/(b*c*(b + c*x)) + ((b*B + A*c)*ArcTan[(Sqrt[c]*Sq
rt[x])/Sqrt[b]])/(b^(3/2)*c^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {9, 87, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^2} \, dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{A+Bx}{\sqrt{x}(b+cx)^2}dx$$

$$\Big\downarrow \ 87$$

$$\frac{(Ac+bB)\int \frac{1}{\sqrt{x}(b+cx)}dx}{2bc} - \frac{\sqrt{x}(bB-Ac)}{bc(b+cx)}$$

$$\Big\downarrow \ 73$$

$$\frac{(Ac+bB)\int \frac{1}{b+cx}d\sqrt{x}}{bc} - \frac{\sqrt{x}(bB-Ac)}{bc(b+cx)}$$

$$\Big\downarrow \ 218$$

$$\frac{(Ac+bB)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}c^{3/2}} - \frac{\sqrt{x}(bB-Ac)}{bc(b+cx)}$$

input
```
Int[(x^(3/2)*(A + B*x))/(b*x + c*x^2)^2,x]
```

output
```
-(((b*B - A*c)*Sqrt[x])/(b*c*(b + c*x))) + ((b*B + A*c)*ArcTan[(Sqrt[c]*Sq
rt[x])/Sqrt[b]])/(b^(3/2)*c^(3/2))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p, x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] && !MonomialQ[Px, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) + d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p + 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || IntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/Rt[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{(Ac-Bb)\sqrt{x}}{bc(cx+b)}+\frac{(Ac+Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{bc\sqrt{bc}}$	57
default	$\frac{(Ac-Bb)\sqrt{x}}{bc(cx+b)}+\frac{(Ac+Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{bc\sqrt{bc}}$	57

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
(A*c-B*b)/b/c*x^(1/2)/(c*x+b)+(A*c+B*b)/b/c/(b*c)^(1/2)*arctan(c*x^(1/2)/(
b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.77

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \left[-\frac{(Bb^2+Abc+(Bbc+Ac^2)x)\sqrt{-bc}\log\left(\frac{cx-b-2\sqrt{-bc}\sqrt{x}}{cx+b}\right)+2\left(Bb^2c-Abc^2\right)\sqrt{x}}{2\left(b^2c^3x+b^3c^2\right)}, -\frac{(Bb^2+Abc+(Bbc+Ac^2)x)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}}{c\sqrt{x}}\right)+\left(Bb^2c-Abc^2\right)\sqrt{x}}{b^2c^3x+b^3c^2}\right]$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
[-1/2*((B*b^2 + A*b*c + (B*b*c + A*c^2)*x)*sqrt(-b*c)*log((c*x - b - 2*sqr
t(-b*c)*sqrt(x))/(c*x + b)) + 2*(B*b^2*c - A*b*c^2)*sqrt(x))/(b^2*c^3*x +
b^3*c^2), -((B*b^2 + A*b*c + (B*b*c + A*c^2)*x)*sqrt(b*c)*arctan(sqrt(b*c)
/(c*sqrt(x))) + (B*b^2*c - A*b*c^2)*sqrt(x))/(b^2*c^3*x + b^3*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(53) = 106$.

Time = 15.92 (sec) , antiderivative size = 615, normalized size of antiderivative = 9.61

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \begin{cases} \tilde{\infty}\left(-\frac{2A}{3x^{\frac{3}{2}}}-\frac{2B}{\sqrt{x}}\right) \\ \frac{2A\sqrt{x}+\frac{2Bx^{\frac{3}{2}}}{3}}{b^2} \\ \frac{-\frac{2A}{3x^{\frac{3}{2}}}-\frac{2B}{\sqrt{x}}}{c^2} \\ \frac{Abc\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{2b^2c^2\sqrt{-\frac{b}{c}}+2bc^3x\sqrt{-\frac{b}{c}}} - \frac{Abc\log\left(\sqrt{x}+\sqrt{-\frac{b}{c}}\right)}{2b^2c^2\sqrt{-\frac{b}{c}}+2bc^3x\sqrt{-\frac{b}{c}}} + \frac{2Ac^2\sqrt{x}\sqrt{-\frac{b}{c}}}{2b^2c^2\sqrt{-\frac{b}{c}}+2bc^3x\sqrt{-\frac{b}{c}}} + \frac{Ac^2x\log\left(\sqrt{x}-\sqrt{-\frac{b}{c}}\right)}{2b^2c^2\sqrt{-\frac{b}{c}}+2bc^3x\sqrt{-\frac{b}{c}}} \end{cases}$$

input
```
integrate(x**(3/2)*(B*x+A)/(c*x**2+b*x)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(3*x**(3/2)) - 2*B/sqrt(x)), Eq(b, 0) & Eq(c, 0)), ((
2*A*sqrt(x) + 2*B*x**(3/2)/3)/b**2, Eq(c, 0)), ((-2*A/(3*x**(3/2)) - 2*B/s
qrt(x))/c**2, Eq(b, 0)), (A*b*c*log(sqrt(x) - sqrt(-b/c))/(2*b**2*c**2*sqr
t(-b/c) + 2*b*c**3*x*sqrt(-b/c)) - A*b*c*log(sqrt(x) + sqrt(-b/c))/(2*b**2
*c**2*sqrt(-b/c) + 2*b*c**3*x*sqrt(-b/c)) + 2*A*c**2*sqrt(x)*sqrt(-b/c)/(2
*b**2*c**2*sqrt(-b/c) + 2*b*c**3*x*sqrt(-b/c)) + A*c**2*x*log(sqrt(x) - sq
rt(-b/c))/(2*b**2*c**2*sqrt(-b/c) + 2*b*c**3*x*sqrt(-b/c)) - A*c**2*x*log(
sqrt(x) + sqrt(-b/c))/(2*b**2*c**2*sqrt(-b/c) + 2*b*c**3*x*sqrt(-b/c)) + B
*b**2*log(sqrt(x) - sqrt(-b/c))/(2*b**2*c**2*sqrt(-b/c) + 2*b*c**3*x*sqrt(
-b/c)) - B*b**2*log(sqrt(x) + sqrt(-b/c))/(2*b**2*c**2*sqrt(-b/c) + 2*b*c*
*3*x*sqrt(-b/c)) - 2*B*b*c*sqrt(x)*sqrt(-b/c)/(2*b**2*c**2*sqrt(-b/c) + 2*
b*c**3*x*sqrt(-b/c)) + B*b*c*x*log(sqrt(x) - sqrt(-b/c))/(2*b**2*c**2*sqrt
(-b/c) + 2*b*c**3*x*sqrt(-b/c)) - B*b*c*x*log(sqrt(x) + sqrt(-b/c))/(2*b**
2*c**2*sqrt(-b/c) + 2*b*c**3*x*sqrt(-b/c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.91

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^2}\,dx = -\frac{(Bb-Ac)\sqrt{x}}{bc^2x+b^2c} + \frac{(Bb+Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}bc}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-(B*b - A*c)*sqrt(x)/(b*c^2*x + b^2*c) + (B*b + A*c)*arctan(c*sqrt(x)/sqrt
(b*c))/(sqrt(b*c)*b*c)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \frac{(Bb+Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}bc} - \frac{Bb\sqrt{x}-Ac\sqrt{x}}{(cx+b)bc}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
(B*b + A*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b*c) - (B*b*sqrt(x) - A
*c*sqrt(x))/((c*x + b)*b*c)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.80

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)\,(A\,c+B\,b)}{b^{3/2}\,c^{3/2}} + \frac{\sqrt{x}\,(A\,c-B\,b)}{b\,c\,(b+c\,x)}$$

input
```
int((x^(3/2)*(A + B*x))/(b*x + c*x^2)^2,x)
```

output
```
(atan((c^(1/2)*x^(1/2))/b^(1/2))*(A*c + B*b))/(b^(3/2)*c^(3/2)) + (x^(1/2)
*(A*c - B*b))/(b*c*(b + c*x))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.86

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^2}\,dx = \frac{\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)abc+\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,c^2x+\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)b^3+\sqrt{c}\,\sqrt{\ }}{b^2c^2\,(cx+b)}$$

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x)^2,x)
```

output
```
(sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b*c + sqrt(c)*sqrt(
b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c**2*x + sqrt(c)*sqrt(b)*atan((sq
rt(x)*c)/(sqrt(c)*sqrt(b)))*b**3 + sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(
c)*sqrt(b)))*b**2*c*x + sqrt(x)*a*b*c**2 - sqrt(x)*b**3*c)/(b**2*c**2*(b +
 c*x))
```

3.93 $\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 72

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^2} dx = -\frac{2A}{b^2\sqrt{x}} + \frac{(bB-Ac)\sqrt{x}}{b^2(b+cx)} + \frac{(bB-3Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}\sqrt{c}}$$

output
```
-2*A/b^2/x^(1/2)+(-A*c+B*b)*x^(1/2)/b^2/(c*x+b)+(-3*A*c+B*b)*arctan(c^(1/2
)*x^(1/2)/b^(1/2))/b^(5/2)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^2} dx = \frac{-2Ab+bBx-3Acx}{b^2\sqrt{x}(b+cx)} + \frac{(bB-3Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{5/2}\sqrt{c}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(b*x + c*x^2)^2,x]
```

output
```
(-2*A*b + b*B*x - 3*A*c*x)/(b^2*Sqrt[x]*(b + c*x)) + ((b*B - 3*A*c)*ArcTan
[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(b^(5/2)*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.21, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {9, 87, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^2}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx}{x^{3/2}(b+cx)^2}dx$$

$$\Big\downarrow \; 87$$

$$-\frac{(bB-3Ac)\int \frac{1}{x^{3/2}(b+cx)}dx}{2bc}-\frac{bB-Ac}{bc\sqrt{x}(b+cx)}$$

$$\Big\downarrow \; 61$$

$$-\frac{(bB-3Ac)\left(-\frac{c\int \frac{1}{\sqrt{x}(b+cx)}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{2bc}-\frac{bB-Ac}{bc\sqrt{x}(b+cx)}$$

$$\Big\downarrow \; 73$$

$$-\frac{(bB-3Ac)\left(-\frac{2c\int \frac{1}{b+cx}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{2bc}-\frac{bB-Ac}{bc\sqrt{x}(b+cx)}$$

$$\Big\downarrow \; 218$$

$$-\frac{(bB-3Ac)\left(-\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{2bc}-\frac{bB-Ac}{bc\sqrt{x}(b+cx)}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(b*x + c*x^2)^2,x]
```

output
```
-((b*B - A*c)/(b*c*Sqrt[x]*(b + c*x))) - ((b*B - 3*A*c)*(-2/(b*Sqrt[x]) -
(2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/b^(3/2)))/(2*b*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$-\frac{2\left(\frac{\left(\frac{Ac}{2}-\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(3Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^2}-\frac{2A}{b^2\sqrt{x}}$	64
default	$-\frac{2\left(\frac{\left(\frac{Ac}{2}-\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(3Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^2}-\frac{2A}{b^2\sqrt{x}}$	64
risch	$-\frac{2A}{b^2\sqrt{x}}-\frac{\frac{2\left(\frac{Ac}{2}-\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(3Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}}}{b^2}$	64

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/b^2*((1/2*A*c-1/2*B*b)*x^(1/2)/(c*x+b)+1/2*(3*A*c-B*b)/(b*c)^(1/2)*arct
an(c*x^(1/2)/(b*c)^(1/2)))-2*A/b^2/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.99

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^2}\,dx$$
$$=\left[\frac{((Bbc-3\,Ac^2)x^2+(Bb^2-3\,Abc)x)\sqrt{-bc}\log\left(\frac{cx-b+2\sqrt{-bc}\sqrt{x}}{cx+b}\right)-2\,(2\,Ab^2c-(Bb^2c-3\,Abc^2)x)\sqrt{x}}{2\,(b^3c^2x^2+b^4cx)},\right.$$
$$\left.-\frac{((Bbc-3\,Ac^2)x^2+(Bb^2-3\,Abc)x)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}}{c\sqrt{x}}\right)+(2\,Ab^2c-(Bb^2c-3\,Abc^2)x)\sqrt{x}}{b^3c^2x^2+b^4cx}\right]$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
[1/2*(((B*b*c - 3*A*c^2)*x^2 + (B*b^2 - 3*A*b*c)*x)*sqrt(-b*c)*log((c*x -
b + 2*sqrt(-b*c)*sqrt(x))/(c*x + b)) - 2*(2*A*b^2*c - (B*b^2*c - 3*A*b*c^2
)*x)*sqrt(x))/(b^3*c^2*x^2 + b^4*c*x), -(((B*b*c - 3*A*c^2)*x^2 + (B*b^2 -
 3*A*b*c)*x)*sqrt(b*c)*arctan(sqrt(b*c)/(c*sqrt(x))) + (2*A*b^2*c - (B*b^2
*c - 3*A*b*c^2)*x)*sqrt(x))/(b^3*c^2*x^2 + b^4*c*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(66) = 132$.

Time = 9.06 (sec) , antiderivative size = 794, normalized size of antiderivative = 11.03

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x**(1/2)*(B*x+A)/(c*x**2+b*x)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(5*x**(5/2)) - 2*B/(3*x**(3/2))), Eq(b, 0) & Eq(c, 0)
), ((-2*A/sqrt(x) + 2*B*sqrt(x))/b**2, Eq(c, 0)), ((-2*A/(5*x**(5/2)) - 2*
B/(3*x**(3/2)))/c**2, Eq(b, 0)), (-3*A*b*c*sqrt(x)*log(sqrt(x) - sqrt(-b/c
))/(2*b**3*c*sqrt(x)*sqrt(-b/c) + 2*b**2*c**2*x**(3/2)*sqrt(-b/c)) + 3*A*b
*c*sqrt(x)*log(sqrt(x) + sqrt(-b/c))/(2*b**3*c*sqrt(x)*sqrt(-b/c) + 2*b**2
*c**2*x**(3/2)*sqrt(-b/c)) - 4*A*b*c*sqrt(-b/c)/(2*b**3*c*sqrt(x)*sqrt(-b/
c) + 2*b**2*c**2*x**(3/2)*sqrt(-b/c)) - 3*A*c**2*x**(3/2)*log(sqrt(x) - sq
rt(-b/c))/(2*b**3*c*sqrt(x)*sqrt(-b/c) + 2*b**2*c**2*x**(3/2)*sqrt(-b/c))
+ 3*A*c**2*x**(3/2)*log(sqrt(x) + sqrt(-b/c))/(2*b**3*c*sqrt(x)*sqrt(-b/c)
 + 2*b**2*c**2*x**(3/2)*sqrt(-b/c)) - 6*A*c**2*x*sqrt(-b/c)/(2*b**3*c*sqrt
(x)*sqrt(-b/c) + 2*b**2*c**2*x**(3/2)*sqrt(-b/c)) + B*b**2*sqrt(x)*log(sqr
t(x) - sqrt(-b/c))/(2*b**3*c*sqrt(x)*sqrt(-b/c) + 2*b**2*c**2*x**(3/2)*sqr
t(-b/c)) - B*b**2*sqrt(x)*log(sqrt(x) + sqrt(-b/c))/(2*b**3*c*sqrt(x)*sqrt
(-b/c) + 2*b**2*c**2*x**(3/2)*sqrt(-b/c)) + B*b*c*x**(3/2)*log(sqrt(x) - s
qrt(-b/c))/(2*b**3*c*sqrt(x)*sqrt(-b/c) + 2*b**2*c**2*x**(3/2)*sqrt(-b/c))
 - B*b*c*x**(3/2)*log(sqrt(x) + sqrt(-b/c))/(2*b**3*c*sqrt(x)*sqrt(-b/c) +
 2*b**2*c**2*x**(3/2)*sqrt(-b/c)) + 2*B*b*c*x*sqrt(-b/c)/(2*b**3*c*sqrt(x)
*sqrt(-b/c) + 2*b**2*c**2*x**(3/2)*sqrt(-b/c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^2}\,dx = -\frac{2\,Ab-(Bb-3\,Ac)x}{b^2cx^{\frac{3}{2}}+b^3\sqrt{x}}+\frac{(Bb-3\,Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}b^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-(2*A*b - (B*b - 3*A*c)*x)/(b^2*c*x^(3/2) + b^3*sqrt(x)) + (B*b - 3*A*c)*a
rctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^2}\,dx = \frac{(Bb-3\,Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}b^2}+\frac{Bbx-3\,Acx-2\,Ab}{\left(cx^{\frac{3}{2}}+b\sqrt{x}\right)b^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
(B*b - 3*A*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b^2) + (B*b*x - 3*A*c
*x - 2*A*b)/((c*x^(3/2) + b*sqrt(x))*b^2)
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^2}\,dx = -\frac{\frac{2\,A}{b}+\frac{x\,(3\,A\,c-B\,b)}{b^2}}{b\,\sqrt{x}+c\,x^{3/2}}-\frac{\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{b}}\right)\,(3\,A\,c-B\,b)}{b^{5/2}\,\sqrt{c}}$$

input
```
int((x^(1/2)*(A + B*x))/(b*x + c*x^2)^2,x)
```

output
```
- ((2*A)/b + (x*(3*A*c - B*b))/b^2)/(b*x^(1/2) + c*x^(3/2)) - (atan((c^(1/
2)*x^(1/2))/b^(1/2))*(3*A*c - B*b))/(b^(5/2)*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.92

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^2}\,dx$$
$$= \frac{-3\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)abc - 3\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,c^2x + \sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)b^3 + \sqrt{x}\,\sqrt{c}\,\sqrt{b}}{\sqrt{x}\,b^3c\,(cx+b)}$$

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x)^2,x)
```

output
```
( - 3*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b*c -
3*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c**2*x + s
qrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**3 + sqrt(x)*
sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**2*c*x - 2*a*b**2*c
- 3*a*b*c**2*x + b**3*c*x)/(sqrt(x)*b**3*c*(b + c*x))
```

3.94 $\int \frac{A+Bx}{\sqrt{x}(bx+cx^2)^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 96

$$\int \frac{A+Bx}{\sqrt{x}(bx+cx^2)^2} \, dx = -\frac{2A}{3b^2x^{3/2}} - \frac{2(bB-2Ac)}{b^3\sqrt{x}} - \frac{c(bB-Ac)\sqrt{x}}{b^3(b+cx)} - \frac{\sqrt{c}(3bB-5Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}$$

output
```
-2/3*A/b^2/x^(3/2)-2*(-2*A*c+B*b)/b^3/x^(1/2)-c*(-A*c+B*b)*x^(1/2)/b^3/(c*
x+b)-c^(1/2)*(-5*A*c+3*B*b)*arctan(c^(1/2)*x^(1/2)/b^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx}{\sqrt{x}(bx+cx^2)^2} \, dx = \frac{-3bBx(2b+3cx)+A(-2b^2+10bcx+15c^2x^2)}{3b^3x^{3/2}(b+cx)} + \frac{\sqrt{c}(-3bB+5Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{7/2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(b*x + c*x^2)^2),x]
```

output
```
(-3*b*B*x*(2*b + 3*c*x) + A*(-2*b^2 + 10*b*c*x + 15*c^2*x^2))/(3*b^3*x^(3/
2)*(b + c*x)) + (Sqrt[c]*(-3*b*B + 5*A*c)*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]
])/b^(7/2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {9, 87, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^2}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{A+Bx}{x^{5/2}(b+cx)^2}dx$$

$$\Big\downarrow \; 87$$

$$-\frac{(3bB-5Ac)\int \frac{1}{x^{5/2}(b+cx)}dx}{2bc}-\frac{bB-Ac}{bcx^{3/2}(b+cx)}$$

$$\Big\downarrow \; 61$$

$$-\frac{(3bB-5Ac)\left(-\frac{c\int \frac{1}{x^{3/2}(b+cx)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{2bc}-\frac{bB-Ac}{bcx^{3/2}(b+cx)}$$

$$\Big\downarrow \; 61$$

$$-\frac{(3bB-5Ac)\left(-\frac{c\left(-\frac{c\int \frac{1}{\sqrt{x}(b+cx)}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{2bc}-\frac{bB-Ac}{bcx^{3/2}(b+cx)}$$

$$\Big\downarrow \; 73$$

$$-\frac{(3bB-5Ac)\left(-\frac{c\left(-\frac{2c\int \frac{1}{b+cx}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{2bc}-\frac{bB-Ac}{bcx^{3/2}(b+cx)}$$

$\downarrow$ 218

$$-\frac{(3bB-5Ac)\left(-\frac{c\left(-\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{2bc}-\frac{bB-Ac}{bcx^{3/2}(b+cx)}$$

input
```
Int[(A + B*x)/(Sqrt[x]*(b*x + c*x^2)^2),x]
```

output
```
-((b*B - A*c)/(b*c*x^(3/2)*(b + c*x))) - ((3*b*B - 5*A*c)*(-2/(3*b*x^(3/2)
) - (c*(-2/(b*Sqrt[x]) - (2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/b^(
3/2)))/b))/(2*b*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

method	result	size
risch	$-\frac{2(-6Acx+3Bbx+Ab)}{3b^3x^{\frac{3}{2}}}+\frac{c\left(\frac{2\left(\frac{Ac}{2}-\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(5Ac-3Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}}\right)}{b^3}$	77
derivativedivides	$\frac{2c\left(\frac{\left(\frac{Ac}{2}-\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(5Ac-3Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^3}-\frac{2A}{3b^2x^{\frac{3}{2}}}-\frac{2(-2Ac+Bb)}{b^3\sqrt{x}}$	81
default	$\frac{2c\left(\frac{\left(\frac{Ac}{2}-\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(5Ac-3Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^3}-\frac{2A}{3b^2x^{\frac{3}{2}}}-\frac{2(-2Ac+Bb)}{b^3\sqrt{x}}$	81

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-6*A*c*x+3*B*b*x+A*b)/b^3/x^(3/2)+1/b^3*c*(2*(1/2*A*c-1/2*B*b)*x^(1/
2)/(c*x+b)+(5*A*c-3*B*b)/(b*c)^(1/2)*arctan(c*x^(1/2)/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.68

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^2}\,dx$$
$$= \left[-\frac{3\left((3\,Bbc-5\,Ac^2)x^3+(3\,Bb^2-5\,Abc)x^2\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx+2\,b\sqrt{x}\sqrt{-\frac{c}{b}}-b}{cx+b}\right)+2\,(2\,Ab^2+3\,(3\,Bbc-5\,Ac^2)\ldots}{6\,(b^3cx^3+b^4x^2)}\right.$$
$$-\frac{3\left((3\,Bbc-5\,Ac^2)x^3+(3\,Bb^2-5\,Abc)x^2\right)\sqrt{\frac{c}{b}}\arctan\left(\sqrt{x}\sqrt{\frac{c}{b}}\right)+(2\,Ab^2+3\,(3\,Bbc-5\,Ac^2)x^2+2\,(3\ldots}{3\,(b^3cx^3+b^4x^2)}$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
[-1/6*(3*((3*B*b*c - 5*A*c^2)*x^3 + (3*B*b^2 - 5*A*b*c)*x^2)*sqrt(-c/b)*lo
g((c*x + 2*b*sqrt(x)*sqrt(-c/b) - b)/(c*x + b)) + 2*(2*A*b^2 + 3*(3*B*b*c
- 5*A*c^2)*x^2 + 2*(3*B*b^2 - 5*A*b*c)*x)*sqrt(x))/(b^3*c*x^3 + b^4*x^2),
-1/3*(3*((3*B*b*c - 5*A*c^2)*x^3 + (3*B*b^2 - 5*A*b*c)*x^2)*sqrt(c/b)*arct
an(sqrt(x)*sqrt(c/b)) + (2*A*b^2 + 3*(3*B*b*c - 5*A*c^2)*x^2 + 2*(3*B*b^2
- 5*A*b*c)*x)*sqrt(x))/(b^3*c*x^3 + b^4*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. 2(92) = 184.

Time = 11.83 (sec) , antiderivative size = 882, normalized size of antiderivative = 9.19

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(1/2)/(c*x**2+b*x)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(5*x**(5/2))), Eq(b, 0) & Eq(c, 0)
), ((-2*A/(3*x**(3/2)) - 2*B/sqrt(x))/b**2, Eq(c, 0)), ((-2*A/(7*x**(7/2))
 - 2*B/(5*x**(5/2)))/c**2, Eq(b, 0)), (-4*A*b**2*sqrt(-b/c)/(6*b**4*x**(3/
2)*sqrt(-b/c) + 6*b**3*c*x**(5/2)*sqrt(-b/c)) + 15*A*b*c*x**(3/2)*log(sqrt
(x) - sqrt(-b/c))/(6*b**4*x**(3/2)*sqrt(-b/c) + 6*b**3*c*x**(5/2)*sqrt(-b/
c)) - 15*A*b*c*x**(3/2)*log(sqrt(x) + sqrt(-b/c))/(6*b**4*x**(3/2)*sqrt(-b
/c) + 6*b**3*c*x**(5/2)*sqrt(-b/c)) + 20*A*b*c*x*sqrt(-b/c)/(6*b**4*x**(3/
2)*sqrt(-b/c) + 6*b**3*c*x**(5/2)*sqrt(-b/c)) + 15*A*c**2*x**(5/2)*log(sqr
t(x) - sqrt(-b/c))/(6*b**4*x**(3/2)*sqrt(-b/c) + 6*b**3*c*x**(5/2)*sqrt(-b
/c)) - 15*A*c**2*x**(5/2)*log(sqrt(x) + sqrt(-b/c))/(6*b**4*x**(3/2)*sqrt(
-b/c) + 6*b**3*c*x**(5/2)*sqrt(-b/c)) + 30*A*c**2*x**2*sqrt(-b/c)/(6*b**4*
x**(3/2)*sqrt(-b/c) + 6*b**3*c*x**(5/2)*sqrt(-b/c)) - 9*B*b**2*x**(3/2)*lo
g(sqrt(x) - sqrt(-b/c))/(6*b**4*x**(3/2)*sqrt(-b/c) + 6*b**3*c*x**(5/2)*sq
rt(-b/c)) + 9*B*b**2*x**(3/2)*log(sqrt(x) + sqrt(-b/c))/(6*b**4*x**(3/2)*s
qrt(-b/c) + 6*b**3*c*x**(5/2)*sqrt(-b/c)) - 12*B*b**2*x*sqrt(-b/c)/(6*b**4
*x**(3/2)*sqrt(-b/c) + 6*b**3*c*x**(5/2)*sqrt(-b/c)) - 9*B*b*c*x**(5/2)*lo
g(sqrt(x) - sqrt(-b/c))/(6*b**4*x**(3/2)*sqrt(-b/c) + 6*b**3*c*x**(5/2)*sq
rt(-b/c)) + 9*B*b*c*x**(5/2)*log(sqrt(x) + sqrt(-b/c))/(6*b**4*x**(3/2)*sq
rt(-b/c) + 6*b**3*c*x**(5/2)*sqrt(-b/c)) - 18*B*b*c*x**2*sqrt(-b/c)/(6*b**
4*x**(3/2)*sqrt(-b/c) + 6*b**3*c*x**(5/2)*sqrt(-b/c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{\sqrt{x}\left(bx+cx^2\right)^2}\,dx = -\frac{2\,Ab^2+3\,(3\,Bbc-5\,Ac^2)x^2+2\,(3\,Bb^2-5\,Abc)x}{3\left(b^3cx^{\frac{5}{2}}+b^4x^{\frac{3}{2}}\right)} - \frac{(3\,Bbc-5\,Ac^2)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}b^3}$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-1/3*(2*A*b^2 + 3*(3*B*b*c - 5*A*c^2)*x^2 + 2*(3*B*b^2 - 5*A*b*c)*x)/(b^3*
c*x^(5/2) + b^4*x^(3/2)) - (3*B*b*c - 5*A*c^2)*arctan(c*sqrt(x)/sqrt(b*c))
/(sqrt(b*c)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^2}\,dx = -\frac{(3\,Bbc-5\,Ac^2)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}b^3} - \frac{Bbc\sqrt{x}-Ac^2\sqrt{x}}{(cx+b)b^3} - \frac{2\,(3\,Bbx-6\,Acx+Ab)}{3\,b^3x^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
-(3*B*b*c - 5*A*c^2)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b^3) - (B*b*c*
sqrt(x) - A*c^2*sqrt(x))/((c*x + b)*b^3) - 2/3*(3*B*b*x - 6*A*c*x + A*b)/(
b^3*x^(3/2))
```

Mupad [B] (verification not implemented)

Time = 5.15 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^2}\,dx = \frac{\frac{2\,x\,(5\,A\,c-3\,B\,b)}{3\,b^2}-\frac{2\,A}{3\,b}+\frac{c\,x^2\,(5\,A\,c-3\,B\,b)}{b^3}}{b\,x^{3/2}+c\,x^{5/2}} + \frac{\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{b}}\right)\,(5\,A\,c-3\,B\,b)}{b^{7/2}}$$

input
```
int((A + B*x)/(x^(1/2)*(b*x + c*x^2)^2),x)
```

output
```
((2*x*(5*A*c - 3*B*b))/(3*b^2) - (2*A)/(3*b) + (c*x^2*(5*A*c - 3*B*b))/b^3
)/(b*x^(3/2) + c*x^(5/2)) + (c^(1/2)*atan((c^(1/2)*x^(1/2))/b^(1/2))*(5*A*
c - 3*B*b))/b^(7/2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.72

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^2}\,dx$$
$$= \frac{15\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)abcx+15\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,c^2x^2-9\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)b^3x-9\sqrt{x}}{3\sqrt{x}\,b^4x\,(cx+b)}$$

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x)^2,x)
```

output
```
(15*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b*c*x +
15*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c**2*x**2
 - 9*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**3*x -
9*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**2*c*x**2
- 2*a*b**3 + 10*a*b**2*c*x + 15*a*b*c**2*x**2 - 6*b**4*x - 9*b**3*c*x**2)/
(3*sqrt(x)*b**4*x*(b + c*x))
```

3.95 $\int \frac{A+Bx}{x^{3/2}(bx+cx^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 118

$$\int \frac{A+Bx}{x^{3/2}(bx+cx^2)^2} dx = -\frac{2A}{5b^2x^{5/2}} - \frac{2(bB-2Ac)}{3b^3x^{3/2}} + \frac{2c(2bB-3Ac)}{b^4\sqrt{x}} + \frac{c^2(bB-Ac)\sqrt{x}}{b^4(b+cx)} + \frac{c^{3/2}(5bB-7Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{9/2}}$$

output
```
-2/5*A/b^2/x^(5/2)-2/3*(-2*A*c+B*b)/b^3/x^(3/2)+2*c*(-3*A*c+2*B*b)/b^4/x^(
1/2)+c^2*(-A*c+B*b)*x^(1/2)/b^4/(c*x+b)+c^(3/2)*(-7*A*c+5*B*b)*arctan(c^(1
/2)*x^(1/2)/b^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{x^{3/2}(bx+cx^2)^2} dx = \frac{5bBx(-2b^2+10bcx+15c^2x^2)-A(6b^3-14b^2cx+70bc^2x^2+105c^3x^3)}{15b^4x^{5/2}(b+cx)} + \frac{c^{3/2}(5bB-7Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{9/2}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(b*x + c*x^2)^2),x]
```

output
```
(5*b*B*x*(-2*b^2 + 10*b*c*x + 15*c^2*x^2) - A*(6*b^3 - 14*b^2*c*x + 70*b*c
^2*x^2 + 105*c^3*x^3))/(15*b^4*x^(5/2)*(b + c*x)) + (c^(3/2)*(5*b*B - 7*A*
c)*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/b^(9/2)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {9, 87, 61, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^2}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{A+Bx}{x^{7/2}(b+cx)^2}dx$$

$$\Big\downarrow \ 87$$

$$-\frac{(5bB-7Ac)\int \frac{1}{x^{7/2}(b+cx)}dx}{2bc}-\frac{bB-Ac}{bcx^{5/2}(b+cx)}$$

$$\Big\downarrow \ 61$$

$$-\frac{(5bB-7Ac)\left(-\frac{c\int \frac{1}{x^{5/2}(b+cx)}dx}{b}-\frac{2}{5bx^{5/2}}\right)}{2bc}-\frac{bB-Ac}{bcx^{5/2}(b+cx)}$$

$$\Big\downarrow \ 61$$

$$-\frac{(5bB-7Ac)\left(-\frac{c\left(-\frac{c\int \frac{1}{x^{3/2}(b+cx)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{2bc}-\frac{bB-Ac}{bcx^{5/2}(b+cx)}$$

$$\Big\downarrow \text{61}$$

$$-\frac{(5bB-7Ac)\left(-\frac{c\left(-\frac{c\left(-\frac{c\int\frac{1}{\sqrt{x}(b+cx)}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{2bc}-\frac{bB-Ac}{bcx^{5/2}(b+cx)}$$

$$\Big\downarrow \text{73}$$

$$-\frac{(5bB-7Ac)\left(-\frac{c\left(-\frac{c\left(-\frac{2c\int\frac{1}{b+cx}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{2bc}-\frac{bB-Ac}{bcx^{5/2}(b+cx)}$$

$$\Big\downarrow \text{218}$$

$$-\frac{(5bB-7Ac)\left(-\frac{c\left(-\frac{c\left(-\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{b}-\frac{2}{5bx^{5/2}}\right)}{2bc}-\frac{bB-Ac}{bcx^{5/2}(b+cx)}$$

input

```
Int[(A + B*x)/(x^(3/2)*(b*x + c*x^2)^2),x]
```

output

```
-((b*B - A*c)/(b*c*x^(5/2)*(b + c*x))) - ((5*b*B - 7*A*c)*(-2/(5*b*x^(5/2)
) - (c*(-2/(3*b*x^(3/2)) - (c*(-2/(b*Sqrt[x]) - (2*Sqrt[c]*ArcTan[(Sqrt[c]
*Sqrt[x])/Sqrt[b]])/b^(3/2)))/b))/b))/(2*b*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$-\frac{2c^2\left(\frac{\left(\frac{Ac}{2}-\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(7Ac-5Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^4}-\frac{2A}{5b^2x^{\frac{5}{2}}}-\frac{2(-2Ac+Bb)}{3b^3x^{\frac{3}{2}}}-\frac{2c(3Ac-2Bb)}{b^4\sqrt{x}}$	101
default	$-\frac{2c^2\left(\frac{\left(\frac{Ac}{2}-\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(7Ac-5Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{2\sqrt{bc}}\right)}{b^4}-\frac{2A}{5b^2x^{\frac{5}{2}}}-\frac{2(-2Ac+Bb)}{3b^3x^{\frac{3}{2}}}-\frac{2c(3Ac-2Bb)}{b^4\sqrt{x}}$	101
risch	$-\frac{2(45A\,c^2x^2-30x^2Bbc-10Abcx+5xB\,b^2+3b^2A)}{15b^4x^{\frac{5}{2}}}-\frac{c^2\left(\frac{2\left(\frac{Ac}{2}-\frac{Bb}{2}\right)\sqrt{x}}{cx+b}+\frac{(7Ac-5Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}}\right)}{b^4}$	103

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/b^4*c^2*((1/2*A*c-1/2*B*b)*x^(1/2)/(c*x+b)+1/2*(7*A*c-5*B*b)/(b*c)^(1/2
)*arctan(c*x^(1/2)/(b*c)^(1/2)))-2/5*A/b^2/x^(5/2)-2/3*(-2*A*c+B*b)/b^3/x^
(3/2)-2*c*(3*A*c-2*B*b)/b^4/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.68

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^2}\,dx = \left[-\frac{15\left((5\,Bbc^2-7\,Ac^3)x^4+(5\,Bb^2c-7\,Abc^2)x^3\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx-2\,b\sqrt{x}\sqrt{-\frac{c}{b}}-b}{cx+b}\right)+2}{30\,(b^4}\right.$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x)^2,x, algorithm="fricas")
```

output
```
[-1/30*(15*((5*B*b*c^2 - 7*A*c^3)*x^4 + (5*B*b^2*c - 7*A*b*c^2)*x^3)*sqrt(
-c/b)*log((c*x - 2*b*sqrt(x)*sqrt(-c/b) - b)/(c*x + b)) + 2*(6*A*b^3 - 15*
(5*B*b*c^2 - 7*A*c^3)*x^3 - 10*(5*B*b^2*c - 7*A*b*c^2)*x^2 + 2*(5*B*b^3 -
7*A*b^2*c)*x)*sqrt(x))/(b^4*c*x^4 + b^5*x^3), 1/15*(15*((5*B*b*c^2 - 7*A*c
^3)*x^4 + (5*B*b^2*c - 7*A*b*c^2)*x^3)*sqrt(c/b)*arctan(sqrt(x)*sqrt(c/b))
 - (6*A*b^3 - 15*(5*B*b*c^2 - 7*A*c^3)*x^3 - 10*(5*B*b^2*c - 7*A*b*c^2)*x^
2 + 2*(5*B*b^3 - 7*A*b^2*c)*x)*sqrt(x))/(b^4*c*x^4 + b^5*x^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1017 vs. $2(117) = 234$.

Time = 21.25 (sec) , antiderivative size = 1017, normalized size of antiderivative = 8.62

$$\int \frac{A + Bx}{x^{3/2} (bx + cx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate((B*x+A)/x**(3/2)/(c*x**2+b*x)**2,x)
```

output
```
Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(7*x**(7/2))), Eq(b, 0) & Eq(c, 0)
), ((-2*A/(5*x**(5/2)) - 2*B/(3*x**(3/2)))/b**2, Eq(c, 0)), ((-2*A/(9*x**(
9/2)) - 2*B/(7*x**(7/2)))/c**2, Eq(b, 0)), (-12*A*b**3*sqrt(-b/c)/(30*b**5
*x**(5/2)*sqrt(-b/c) + 30*b**4*c*x**(7/2)*sqrt(-b/c)) + 28*A*b**2*c*x*sqrt
(-b/c)/(30*b**5*x**(5/2)*sqrt(-b/c) + 30*b**4*c*x**(7/2)*sqrt(-b/c)) - 105
*A*b*c**2*x**(5/2)*log(sqrt(x) - sqrt(-b/c))/(30*b**5*x**(5/2)*sqrt(-b/c)
+ 30*b**4*c*x**(7/2)*sqrt(-b/c)) + 105*A*b*c**2*x**(5/2)*log(sqrt(x) + sqr
t(-b/c))/(30*b**5*x**(5/2)*sqrt(-b/c) + 30*b**4*c*x**(7/2)*sqrt(-b/c)) - 1
40*A*b*c**2*x**2*sqrt(-b/c)/(30*b**5*x**(5/2)*sqrt(-b/c) + 30*b**4*c*x**(7
/2)*sqrt(-b/c)) - 105*A*c**3*x**(7/2)*log(sqrt(x) - sqrt(-b/c))/(30*b**5*x
**(5/2)*sqrt(-b/c) + 30*b**4*c*x**(7/2)*sqrt(-b/c)) + 105*A*c**3*x**(7/2)*
log(sqrt(x) + sqrt(-b/c))/(30*b**5*x**(5/2)*sqrt(-b/c) + 30*b**4*c*x**(7/2
)*sqrt(-b/c)) - 210*A*c**3*x**3*sqrt(-b/c)/(30*b**5*x**(5/2)*sqrt(-b/c) +
30*b**4*c*x**(7/2)*sqrt(-b/c)) - 20*B*b**3*x*sqrt(-b/c)/(30*b**5*x**(5/2)*
sqrt(-b/c) + 30*b**4*c*x**(7/2)*sqrt(-b/c)) + 75*B*b**2*c*x**(5/2)*log(sqr
t(x) - sqrt(-b/c))/(30*b**5*x**(5/2)*sqrt(-b/c) + 30*b**4*c*x**(7/2)*sqrt(
-b/c)) - 75*B*b**2*c*x**(5/2)*log(sqrt(x) + sqrt(-b/c))/(30*b**5*x**(5/2)*
sqrt(-b/c) + 30*b**4*c*x**(7/2)*sqrt(-b/c)) + 100*B*b**2*c*x**2*sqrt(-b/c)
/(30*b**5*x**(5/2)*sqrt(-b/c) + 30*b**4*c*x**(7/2)*sqrt(-b/c)) + 75*B*b*c*
*2*x**(7/2)*log(sqrt(x) - sqrt(-b/c))/(30*b**5*x**(5/2)*sqrt(-b/c) + 30...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.00

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^2}\,dx = -\frac{6\,Ab^3-15\,(5\,Bbc^2-7\,Ac^3)x^3-10\,(5\,Bb^2c-7\,Abc^2)x^2+2\,(5\,Bb^3-7\,Ab^2c)x}{15\left(b^4cx^{\frac{7}{2}}+b^5x^{\frac{5}{2}}\right)} + \frac{(5\,Bbc^2-7\,Ac^3)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}b^4}$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x)^2,x, algorithm="maxima")
```

output
```
-1/15*(6*A*b^3 - 15*(5*B*b*c^2 - 7*A*c^3)*x^3 - 10*(5*B*b^2*c - 7*A*b*c^2)
*x^2 + 2*(5*B*b^3 - 7*A*b^2*c)*x)/(b^4*c*x^(7/2) + b^5*x^(5/2)) + (5*B*b*c
^2 - 7*A*c^3)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.93

$$\int \frac{A+Bx}{x^{3/2}(bx+cx^2)^2}\,dx = \frac{(5\,Bbc^2-7\,Ac^3)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{\sqrt{bc}b^4} + \frac{Bbc^2\sqrt{x}-Ac^3\sqrt{x}}{(cx+b)b^4} + \frac{2\,(30\,Bbcx^2-45\,Ac^2x^2-5\,Bb^2x+10\,Abcx-3\,Ab^2)}{15\,b^4x^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x)^2,x, algorithm="giac")
```

output
```
(5*B*b*c^2 - 7*A*c^3)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b^4) + (B*b*c
^2*sqrt(x) - A*c^3*sqrt(x))/((c*x + b)*b^4) + 2/15*(30*B*b*c*x^2 - 45*A*c^
2*x^2 - 5*B*b^2*x + 10*A*b*c*x - 3*A*b^2)/(b^4*x^(5/2))
```

Mupad [B] (verification not implemented)

Time = 5.24 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{x^{3/2}(bx+cx^2)^2}\,dx = -\frac{\frac{2\,A}{5\,b}-\frac{2\,x\,(7\,A\,c-5\,B\,b)}{15\,b^2}+\frac{c^2\,x^3\,(7\,A\,c-5\,B\,b)}{b^4}+\frac{2\,c\,x^2\,(7\,A\,c-5\,B\,b)}{3\,b^3}}{b\,x^{5/2}+c\,x^{7/2}} - \frac{c^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{b}}\right)\,(7\,A\,c-5\,B\,b)}{b^{9/2}}$$

input
```
int((A + B*x)/(x^(3/2)*(b*x + c*x^2)^2),x)
```

output
```
- ((2*A)/(5*b) - (2*x*(7*A*c - 5*B*b))/(15*b^2) + (c^2*x^3*(7*A*c - 5*B*b)
)/b^4 + (2*c*x^2*(7*A*c - 5*B*b))/(3*b^3))/(b*x^(5/2) + c*x^(7/2)) - (c^(3
/2)*atan((c^(1/2)*x^(1/2))/b^(1/2))*(7*A*c - 5*B*b))/b^(9/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.67

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^2}\,dx = \frac{-105\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)ab\,c^2x^2-105\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,c^3x^3+75\sqrt{x}$$

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x)^2,x)
```

output
```
( - 105*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b*c*
*2*x**2 - 105*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*
a*c**3*x**3 + 75*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)
))*b**3*c*x**2 + 75*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt
(b)))*b**2*c**2*x**3 - 6*a*b**4 + 14*a*b**3*c*x - 70*a*b**2*c**2*x**2 - 10
5*a*b*c**3*x**3 - 10*b**5*x + 50*b**4*c*x**2 + 75*b**3*c**2*x**3)/(15*sqrt
(x)*b**5*x**2*(b + c*x))
```

3.96 $\int \frac{x^{13/2}(A+Bx)}{(bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 152

$$\int \frac{x^{13/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{7b(9bB-5Ac)\sqrt{x}}{4c^5} - \frac{7(9bB-5Ac)x^{3/2}}{12c^4} + \frac{2Bx^{5/2}}{5c^3} + \frac{(bB-Ac)x^{7/2}}{2c^2(b+cx)^2} + \frac{(11bB-7Ac)x^{5/2}}{4c^3(b+cx)} - \frac{7b^{3/2}(9bB-5Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{4c^{11/2}}$$

output
```
7/4*b*(-5*A*c+9*B*b)*x^(1/2)/c^5-7/12*(-5*A*c+9*B*b)*x^(3/2)/c^4+2/5*B*x^(
5/2)/c^3+1/2*(-A*c+B*b)*x^(7/2)/c^2/(c*x+b)^2+1/4*(-7*A*c+11*B*b)*x^(5/2)/
c^3/(c*x+b)-7/4*b^(3/2)*(-5*A*c+9*B*b)*arctan(c^(1/2)*x^(1/2)/b^(1/2))/c^(
11/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int \frac{x^{13/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{\sqrt{x}(945b^4B-525b^3c(A-3Bx)+8c^4x^3(5A+3Bx)-8bc^3x^2(35A+9Bx)+7b^2c}{60c^5(b+cx)^2} - \frac{7b^{3/2}(9bB-5Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{4c^{11/2}}$$

input
```
Integrate[(x^(13/2)*(A + B*x))/(b*x + c*x^2)^3,x]
```

output
```
(Sqrt[x]*(945*b^4*B - 525*b^3*c*(A - 3*B*x) + 8*c^4*x^3*(5*A + 3*B*x) - 8*
b*c^3*x^2*(35*A + 9*B*x) + 7*b^2*c^2*x*(-125*A + 72*B*x)))/(60*c^5*(b + c*
x)^2) - (7*b^(3/2)*(9*b*B - 5*A*c)*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(4*c
^(11/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {9, 87, 51, 60, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{13/2}(A+Bx)}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{x^{7/2}(A+Bx)}{(b+cx)^3}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(9bB-5Ac)\int \frac{x^{7/2}}{(b+cx)^2}dx}{4bc}-\frac{x^{9/2}(bB-Ac)}{2bc(b+cx)^2}$$

$$\Big\downarrow \text{ 51}$$

$$\frac{(9bB-5Ac)\left(\frac{7\int \frac{x^{5/2}}{b+cx}dx}{2c}-\frac{x^{7/2}}{c(b+cx)}\right)}{4bc}-\frac{x^{9/2}(bB-Ac)}{2bc(b+cx)^2}$$

$$\Big\downarrow \text{ 60}$$

$$\frac{(9bB-5Ac)\left(\frac{7\left(\frac{2x^{5/2}}{5c}-\frac{b\int\frac{x^{3/2}}{b+cx}dx}{c}\right)}{2c}-\frac{x^{7/2}}{c(b+cx)}\right)}{4bc}-\frac{x^{9/2}(bB-Ac)}{2bc(b+cx)^2}$$

$\Big\downarrow$ 60

$$\frac{(9bB-5Ac)\left(\frac{7\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{b\int\frac{\sqrt{x}}{b+cx}dx}{c}\right)}{c}\right)}{2c}-\frac{x^{7/2}}{c(b+cx)}\right)}{4bc}-\frac{x^{9/2}(bB-Ac)}{2bc(b+cx)^2}$$

$\Big\downarrow$ 60

$$\frac{(9bB-5Ac)\left(\frac{7\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{b\int\frac{1}{\sqrt{x}(b+cx)}dx}{c}\right)}{c}\right)}{c}\right)}{2c}-\frac{x^{7/2}}{c(b+cx)}\right)}{4bc}-\frac{x^{9/2}(bB-Ac)}{2bc(b+cx)^2}$$

$\Big\downarrow$ 73

$$\frac{(9bB-5Ac)\left(\frac{7\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{b+cx}d\sqrt{x}}{c}\right)}{c}\right)}{c}\right)}{2c}-\frac{x^{7/2}}{c(b+cx)}\right)}{4bc}-\frac{x^{9/2}(bB-Ac)}{2bc(b+cx)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{(9bB-5Ac)\left(\frac{7\left(\frac{2x^{5/2}}{5c}-\frac{b\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{3/2}}\right)}{c}\right)}{c}\right)}{2c}-\frac{x^{7/2}}{c(b+cx)}\right)}{4bc}-\frac{x^{9/2}(bB-Ac)}{2bc(b+cx)^2}$$

input
```
Int[(x^(13/2)*(A + B*x))/(b*x + c*x^2)^3,x]
```

output
```
-1/2*((b*B - A*c)*x^(9/2))/(b*c*(b + c*x)^2) + ((9*b*B - 5*A*c)*(-(x^(7/2)
/(c*(b + c*x))) + (7*((2*x^(5/2))/(5*c) - (b*((2*x^(3/2))/(3*c) - (b*((2*S
qrt[x])/c - (2*Sqrt[b]*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/c^(3/2)))/c))/c)
)/(2*c)))/(4*b*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{2(-3B\,c^2x^2-5A\,c^2x+15Bbcx+45Abc-90B\,b^2)\sqrt{x}}{15c^5}+\frac{b^2\left(\frac{2\left(-\frac{13}{8}A\,c^2+\frac{17}{8}Bbc\right)x^{\frac{3}{2}}-\frac{b(11Ac-15Bb)\sqrt{x}}{4}}{(cx+b)^2}+\frac{7(5Ac-9Bb)}{4}\right.}{c^5}$
derivativedivides	$-\frac{2\left(-\frac{B\,c^2x^{\frac{5}{2}}}{5}-\frac{A\,c^2x^{\frac{3}{2}}}{3}+Bbc\,x^{\frac{3}{2}}+3Abc\sqrt{x}-6B\,b^2\sqrt{x}\right)}{c^5}+\frac{2b^2\left(\frac{\left(-\frac{13}{8}A\,c^2+\frac{17}{8}Bbc\right)x^{\frac{3}{2}}-\frac{b(11Ac-15Bb)\sqrt{x}}{8}}{(cx+b)^2}+\frac{7(5Ac-9\right.}{c^5}$
default	$-\frac{2\left(-\frac{B\,c^2x^{\frac{5}{2}}}{5}-\frac{A\,c^2x^{\frac{3}{2}}}{3}+Bbc\,x^{\frac{3}{2}}+3Abc\sqrt{x}-6B\,b^2\sqrt{x}\right)}{c^5}+\frac{2b^2\left(\frac{\left(-\frac{13}{8}A\,c^2+\frac{17}{8}Bbc\right)x^{\frac{3}{2}}-\frac{b(11Ac-15Bb)\sqrt{x}}{8}}{(cx+b)^2}+\frac{7(5Ac-9\right.}{c^5}$

input
```
int(x^(13/2)*(B*x+A)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(-3*B*c^2*x^2-5*A*c^2*x+15*B*b*c*x+45*A*b*c-90*B*b^2)*x^(1/2)/c^5+b^
2/c^5*(2*((-13/8*A*c^2+17/8*B*b*c)*x^(3/2)-1/8*b*(11*A*c-15*B*b)*x^(1/2))/
(c*x+b)^2+7/4*(5*A*c-9*B*b)/(b*c)^(1/2)*arctan(c*x^(1/2)/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.68

$$\int \frac{x^{13/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \left[-\frac{105\,(9\,Bb^4-5\,Ab^3c+(9\,Bb^2c^2-5\,Abc^3)x^2+2\,(9\,Bb^3c-5\,Ab^2c^2)x)\sqrt{-\frac{b}{c}}\log\left(\right.}{}\right.$$

$$-\frac{105\,(9\,Bb^4-5\,Ab^3c+(9\,Bb^2c^2-5\,Abc^3)x^2+2\,(9\,Bb^3c-5\,Ab^2c^2)x)\sqrt{\frac{b}{c}}\arctan\left(\frac{c\sqrt{x}\sqrt{\frac{b}{c}}}{b}\right)-(24\,Bc^4x^4+}{60\,(c^7x^2+2\,b}$$

input
```
integrate(x^(13/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
[-1/120*(105*(9*B*b^4 - 5*A*b^3*c + (9*B*b^2*c^2 - 5*A*b*c^3)*x^2 + 2*(9*B
*b^3*c - 5*A*b^2*c^2)*x)*sqrt(-b/c)*log((c*x + 2*c*sqrt(x)*sqrt(-b/c) - b)
/(c*x + b)) - 2*(24*B*c^4*x^4 + 945*B*b^4 - 525*A*b^3*c - 8*(9*B*b*c^3 - 5
*A*c^4)*x^3 + 56*(9*B*b^2*c^2 - 5*A*b*c^3)*x^2 + 175*(9*B*b^3*c - 5*A*b^2*
c^2)*x)*sqrt(x))/(c^7*x^2 + 2*b*c^6*x + b^2*c^5), -1/60*(105*(9*B*b^4 - 5*
A*b^3*c + (9*B*b^2*c^2 - 5*A*b*c^3)*x^2 + 2*(9*B*b^3*c - 5*A*b^2*c^2)*x)*s
qrt(b/c)*arctan(c*sqrt(x)*sqrt(b/c)/b) - (24*B*c^4*x^4 + 945*B*b^4 - 525*A
*b^3*c - 8*(9*B*b*c^3 - 5*A*c^4)*x^3 + 56*(9*B*b^2*c^2 - 5*A*b*c^3)*x^2 +
175*(9*B*b^3*c - 5*A*b^2*c^2)*x)*sqrt(x))/(c^7*x^2 + 2*b*c^6*x + b^2*c^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{13/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(13/2)*(B*x+A)/(c*x**2+b*x)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.99

$$\begin{aligned}\int \frac{x^{13/2}(A+Bx)}{(bx+cx^2)^3}\,dx &= \frac{(17\,Bb^3c-13\,Ab^2c^2)x^{\frac{3}{2}}+(15\,Bb^4-11\,Ab^3c)\sqrt{x}}{4\,(c^7x^2+2\,bc^6x+b^2c^5)}\\&\quad-\frac{7\,(9\,Bb^3-5\,Ab^2c)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\,\sqrt{bc}c^5}\\&\quad+\frac{2\left(3\,Bc^2x^{\frac{5}{2}}-5\,(3\,Bbc-Ac^2)x^{\frac{3}{2}}+45\,(2\,Bb^2-Abc)\sqrt{x}\right)}{15\,c^5}\end{aligned}$$

input
```
integrate(x^(13/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/4*((17*B*b^3*c - 13*A*b^2*c^2)*x^(3/2) + (15*B*b^4 - 11*A*b^3*c)*sqrt(x)
)/(c^7*x^2 + 2*b*c^6*x + b^2*c^5) - 7/4*(9*B*b^3 - 5*A*b^2*c)*arctan(c*sqr
t(x)/sqrt(b*c))/(sqrt(b*c)*c^5) + 2/15*(3*B*c^2*x^(5/2) - 5*(3*B*b*c - A*c
^2)*x^(3/2) + 45*(2*B*b^2 - A*b*c)*sqrt(x))/c^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.96

$$\int \frac{x^{13/2}(A+Bx)}{(bx+cx^2)^3}\,dx = -\frac{7\,(9\,Bb^3-5\,Ab^2c)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\,\sqrt{bc}c^5} + \frac{17\,Bb^3cx^{\frac{3}{2}}-13\,Ab^2c^2x^{\frac{3}{2}}+15\,Bb^4\sqrt{x}-11\,Ab^3c\sqrt{x}}{4\,(cx+b)^2c^5} + \frac{2\left(3\,Bc^{12}x^{\frac{5}{2}}-15\,Bbc^{11}x^{\frac{3}{2}}+5\,Ac^{12}x^{\frac{3}{2}}+90\,Bb^2c^{10}\sqrt{x}-45\,Abc^{11}\sqrt{x}\right)}{15\,c^{15}}$$

input
```
integrate(x^(13/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
-7/4*(9*B*b^3 - 5*A*b^2*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*c^5) + 1
/4*(17*B*b^3*c*x^(3/2) - 13*A*b^2*c^2*x^(3/2) + 15*B*b^4*sqrt(x) - 11*A*b^
3*c*sqrt(x))/((c*x + b)^2*c^5) + 2/15*(3*B*c^12*x^(5/2) - 15*B*b*c^11*x^(3
/2) + 5*A*c^12*x^(3/2) + 90*B*b^2*c^10*sqrt(x) - 45*A*b*c^11*sqrt(x))/c^15
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.20

$$\int \frac{x^{13/2}(A+Bx)}{(bx+cx^2)^3}\,dx = x^{3/2}\left(\frac{2\,A}{3\,c^3}-\frac{2\,B\,b}{c^4}\right) - \frac{x^{3/2}\left(\frac{13\,A\,b^2\,c^2}{4}-\frac{17\,B\,b^3\,c}{4}\right)-\sqrt{x}\left(\frac{15\,B\,b^4}{4}-\frac{11\,A\,b^3\,c}{4}\right)}{b^2\,c^5+2\,b\,c^6\,x+c^7\,x^2} - \sqrt{x}\left(\frac{3\,b\left(\frac{2\,A}{c^3}-\frac{6\,B\,b}{c^4}\right)}{c}+\frac{6\,B\,b^2}{c^5}\right)+\frac{2\,B\,x^{5/2}}{5\,c^3}-\frac{7\,b^{3/2}\,\mathrm{atan}\left(\frac{b^{3/2}\,\sqrt{c}\,\sqrt{x}\,(5\,A\,c-9\,B\,b)}{9\,B\,b^3-5\,A\,b^2\,c}\right)\,(5\,A\,c-9\,B\,b)}{4\,c^{11/2}}$$

input
```
int((x^(13/2)*(A + B*x))/(b*x + c*x^2)^3,x)
```

output
```
x^(3/2)*((2*A)/(3*c^3) - (2*B*b)/c^4) - (x^(3/2)*((13*A*b^2*c^2)/4 - (17*B
*b^3*c)/4) - x^(1/2)*((15*B*b^4)/4 - (11*A*b^3*c)/4))/(b^2*c^5 + c^7*x^2 +
 2*b*c^6*x) - x^(1/2)*((3*b*((2*A)/c^3 - (6*B*b)/c^4))/c + (6*B*b^2)/c^5)
+ (2*B*x^(5/2))/(5*c^3) - (7*b^(3/2)*atan((b^(3/2)*c^(1/2)*x^(1/2)*(5*A*c
- 9*B*b))/(9*B*b^3 - 5*A*b^2*c))*(5*A*c - 9*B*b))/(4*c^(11/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.83

$$\int \frac{x^{13/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{525\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,b^3c + 1050\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,b^2c^2x + 525\sqrt{c}\,\sqrt{b}\,atan\Big(}{}$$

input
```
int(x^(13/2)*(B*x+A)/(c*x^2+b*x)^3,x)
```

output
```
(525*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b**3*c + 1050*s
qrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b**2*c**2*x + 525*sqr
t(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b*c**3*x**2 - 945*sqrt(
c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**5 - 1890*sqrt(c)*sqrt(b)
*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**4*c*x - 945*sqrt(c)*sqrt(b)*atan((
sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**3*c**2*x**2 - 525*sqrt(x)*a*b**3*c**2 - 8
75*sqrt(x)*a*b**2*c**3*x - 280*sqrt(x)*a*b*c**4*x**2 + 40*sqrt(x)*a*c**5*x
**3 + 945*sqrt(x)*b**5*c + 1575*sqrt(x)*b**4*c**2*x + 504*sqrt(x)*b**3*c**
3*x**2 - 72*sqrt(x)*b**2*c**4*x**3 + 24*sqrt(x)*b*c**5*x**4)/(60*c**6*(b**
2 + 2*b*c*x + c**2*x**2))
```

3.97 $\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^3} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 130

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^3} \, dx = -\frac{5(7bB-3Ac)\sqrt{x}}{4c^4} + \frac{2Bx^{3/2}}{3c^3} + \frac{(bB-Ac)x^{5/2}}{2c^2(b+cx)^2} + \frac{(9bB-5Ac)x^{3/2}}{4c^3(b+cx)} + \frac{5\sqrt{b}(7bB-3Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{4c^{9/2}}$$

output
```
-5/4*(-3*A*c+7*B*b)*x^(1/2)/c^4+2/3*B*x^(3/2)/c^3+1/2*(-A*c+B*b)*x^(5/2)/c
^2/(c*x+b)^2+1/4*(-5*A*c+9*B*b)*x^(3/2)/c^3/(c*x+b)+5/4*b^(1/2)*(-3*A*c+7*
B*b)*arctan(c^(1/2)*x^(1/2)/b^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.85

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^3} \, dx = \frac{\sqrt{x}(-105b^3B + bc^2x(75A-56Bx) + 5b^2c(9A-35Bx) + 8c^3x^2(3A+Bx))}{12c^4(b+cx)^2} + \frac{5\sqrt{b}(7bB-3Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{4c^{9/2}}$$

input
```
Integrate[(x^(11/2)*(A + B*x))/(b*x + c*x^2)^3,x]
```

output
```
(Sqrt[x]*(-105*b^3*B + b*c^2*x*(75*A - 56*B*x) + 5*b^2*c*(9*A - 35*B*x) +
8*c^3*x^2*(3*A + B*x)))/(12*c^4*(b + c*x)^2) + (5*Sqrt[b]*(7*b*B - 3*A*c)*
ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(4*c^(9/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {9, 87, 51, 60, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^3}dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{x^{5/2}(A+Bx)}{(b+cx)^3}dx$$

$$\Big\downarrow \; 87$$

$$\frac{(7bB-3Ac)\int \frac{x^{5/2}}{(b+cx)^2}dx}{4bc}-\frac{x^{7/2}(bB-Ac)}{2bc(b+cx)^2}$$

$$\Big\downarrow \; 51$$

$$\frac{(7bB-3Ac)\left(\frac{5\int \frac{x^{3/2}}{b+cx}dx}{2c}-\frac{x^{5/2}}{c(b+cx)}\right)}{4bc}-\frac{x^{7/2}(bB-Ac)}{2bc(b+cx)^2}$$

$$\Big\downarrow \; 60$$

$$\frac{(7bB-3Ac)\left(\frac{5\left(\frac{2x^{3/2}}{3c}-\frac{b\int \frac{\sqrt{x}}{b+cx}dx}{c}\right)}{2c}-\frac{x^{5/2}}{c(b+cx)}\right)}{4bc}-\frac{x^{7/2}(bB-Ac)}{2bc(b+cx)^2}$$

$$\downarrow \text{60}$$

$$\frac{(7bB-3Ac)\left(\frac{5\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{b\int\frac{1}{\sqrt{x}(b+cx)}dx}{c}\right)}{c}\right)}{2c}-\frac{x^{5/2}}{c(b+cx)}\right)}{4bc}-\frac{x^{7/2}(bB-Ac)}{2bc(b+cx)^2}$$

$$\downarrow \text{73}$$

$$\frac{(7bB-3Ac)\left(\frac{5\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{b+cx}d\sqrt{x}}{c}\right)}{c}\right)}{2c}-\frac{x^{5/2}}{c(b+cx)}\right)}{4bc}-\frac{x^{7/2}(bB-Ac)}{2bc(b+cx)^2}$$

$$\downarrow \text{218}$$

$$\frac{(7bB-3Ac)\left(\frac{5\left(\frac{2x^{3/2}}{3c}-\frac{b\left(\frac{2\sqrt{x}}{c}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{3/2}}\right)}{c}\right)}{2c}-\frac{x^{5/2}}{c(b+cx)}\right)}{4bc}-\frac{x^{7/2}(bB-Ac)}{2bc(b+cx)^2}$$

input
```
Int[(x^(11/2)*(A + B*x))/(b*x + c*x^2)^3,x]
```

output
```
-1/2*((b*B - A*c)*x^(7/2))/(b*c*(b + c*x)^2) + ((7*b*B - 3*A*c)*(-(x^(5/2)
/(c*(b + c*x))) + (5*((2*x^(3/2))/(3*c) - (b*((2*Sqrt[x])/c - (2*Sqrt[b]*A
rcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/c^(3/2)))/c))/(2*c)))/(4*b*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
 !MonomialQ[Px, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{2(Bcx+3Ac-9Bb)\sqrt{x}}{3c^4}-\frac{b\left(\frac{2\left(-\frac{9}{8}A\,c^2+\frac{13}{8}Bbc\right)x^{\frac{3}{2}}-\frac{b(7Ac-11Bb)\sqrt{x}}{4}}{(cx+b)^2}+\frac{5(3Ac-7Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\sqrt{bc}}\right)}{c^4}$	98
derivativedivides	$\frac{\frac{2Bc\,x^{\frac{3}{2}}}{3}+2Ac\sqrt{x}-6Bb\sqrt{x}}{c^4}-\frac{2b\left(\frac{\left(-\frac{9}{8}A\,c^2+\frac{13}{8}Bbc\right)x^{\frac{3}{2}}-\frac{b(7Ac-11Bb)\sqrt{x}}{8}}{(cx+b)^2}+\frac{5(3Ac-7Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{c^4}$	102
default	$\frac{\frac{2Bc\,x^{\frac{3}{2}}}{3}+2Ac\sqrt{x}-6Bb\sqrt{x}}{c^4}-\frac{2b\left(\frac{\left(-\frac{9}{8}A\,c^2+\frac{13}{8}Bbc\right)x^{\frac{3}{2}}-\frac{b(7Ac-11Bb)\sqrt{x}}{8}}{(cx+b)^2}+\frac{5(3Ac-7Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{c^4}$	102

input
```
int(x^(11/2)*(B*x+A)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
2/3*(B*c*x+3*A*c-9*B*b)*x^(1/2)/c^4-b/c^4*(2*((-9/8*A*c^2+13/8*B*b*c)*x^(3
/2)-1/8*b*(7*A*c-11*B*b)*x^(1/2))/(c*x+b)^2+5/4*(3*A*c-7*B*b)/(b*c)^(1/2)*
arctan(c*x^(1/2)/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 349, normalized size of antiderivative = 2.68

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \left[-\frac{15\,(7\,Bb^3-3\,Ab^2c+(7\,Bbc^2-3\,Ac^3)x^2+2\,(7\,Bb^2c-3\,Abc^2)x)\sqrt{-\frac{b}{c}}\log\left(\frac{cx-}{}\right.}{24}\right.$$

input
```
integrate(x^(11/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
[-1/24*(15*(7*B*b^3 - 3*A*b^2*c + (7*B*b*c^2 - 3*A*c^3)*x^2 + 2*(7*B*b^2*c
 - 3*A*b*c^2)*x)*sqrt(-b/c)*log((c*x - 2*c*sqrt(x)*sqrt(-b/c) - b)/(c*x +
b)) - 2*(8*B*c^3*x^3 - 105*B*b^3 + 45*A*b^2*c - 8*(7*B*b*c^2 - 3*A*c^3)*x^
2 - 25*(7*B*b^2*c - 3*A*b*c^2)*x)*sqrt(x))/(c^6*x^2 + 2*b*c^5*x + b^2*c^4)
, 1/12*(15*(7*B*b^3 - 3*A*b^2*c + (7*B*b*c^2 - 3*A*c^3)*x^2 + 2*(7*B*b^2*c
 - 3*A*b*c^2)*x)*sqrt(b/c)*arctan(c*sqrt(x)*sqrt(b/c)/b) + (8*B*c^3*x^3 -
105*B*b^3 + 45*A*b^2*c - 8*(7*B*b*c^2 - 3*A*c^3)*x^2 - 25*(7*B*b^2*c - 3*A
*b*c^2)*x)*sqrt(x))/(c^6*x^2 + 2*b*c^5*x + b^2*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)*(B*x+A)/(c*x**2+b*x)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^3}\,dx = -\frac{(13\,Bb^2c-9\,Abc^2)x^{\frac{3}{2}}+(11\,Bb^3-7\,Ab^2c)\sqrt{x}}{4\,(c^6x^2+2\,bc^5x+b^2c^4)} + \frac{5\,(7\,Bb^2-3\,Abc)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\,\sqrt{bc}c^4} + \frac{2\left(Bcx^{\frac{3}{2}}-3\,(3\,Bb-Ac)\sqrt{x}\right)}{3\,c^4}$$

input
```
integrate(x^(11/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/4*((13*B*b^2*c - 9*A*b*c^2)*x^(3/2) + (11*B*b^3 - 7*A*b^2*c)*sqrt(x))/(
c^6*x^2 + 2*b*c^5*x + b^2*c^4) + 5/4*(7*B*b^2 - 3*A*b*c)*arctan(c*sqrt(x)/
sqrt(b*c))/(sqrt(b*c)*c^4) + 2/3*(B*c*x^(3/2) - 3*(3*B*b - A*c)*sqrt(x))/c
^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.92

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{5\,(7\,Bb^2-3\,Abc)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\,\sqrt{bc}c^4} - \frac{13\,Bb^2cx^{\frac{3}{2}}-9\,Abc^2x^{\frac{3}{2}}+11\,Bb^3\sqrt{x}-7\,Ab^2c\sqrt{x}}{4\,(cx+b)^2c^4} + \frac{2\left(Bc^6x^{\frac{3}{2}}-9\,Bbc^5\sqrt{x}+3\,Ac^6\sqrt{x}\right)}{3\,c^9}$$

input
```
integrate(x^(11/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
5/4*(7*B*b^2 - 3*A*b*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*c^4) - 1/4*
(13*B*b^2*c*x^(3/2) - 9*A*b*c^2*x^(3/2) + 11*B*b^3*sqrt(x) - 7*A*b^2*c*sqr
t(x))/((c*x + b)^2*c^4) + 2/3*(B*c^6*x^(3/2) - 9*B*b*c^5*sqrt(x) + 3*A*c^6
*sqrt(x))/c^9
```

Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.10

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{x^{3/2}\left(\frac{9\,A\,b\,c^2}{4}-\frac{13\,B\,b^2\,c}{4}\right)-\sqrt{x}\left(\frac{11\,B\,b^3}{4}-\frac{7\,A\,b^2\,c}{4}\right)}{b^2\,c^4+2\,b\,c^5\,x+c^6\,x^2} + \sqrt{x}\left(\frac{2\,A}{c^3}-\frac{6\,B\,b}{c^4}\right) + \frac{2\,B\,x^{3/2}}{3\,c^3} + \frac{5\,\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{b}\,\sqrt{c}\,\sqrt{x}\,(3\,A\,c-7\,B\,b)}{7\,B\,b^2-3\,A\,b\,c}\right)\,(3\,A\,c-7\,B\,b)}{4\,c^{9/2}}$$

input
```
int((x^(11/2)*(A + B*x))/(b*x + c*x^2)^3,x)
```

output
```
(x^(3/2)*((9*A*b*c^2)/4 - (13*B*b^2*c)/4) - x^(1/2)*((11*B*b^3)/4 - (7*A*b
^2*c)/4))/(b^2*c^4 + c^6*x^2 + 2*b*c^5*x) + x^(1/2)*((2*A)/c^3 - (6*B*b)/c
^4) + (2*B*x^(3/2))/(3*c^3) + (5*b^(1/2)*atan((b^(1/2)*c^(1/2)*x^(1/2)*(3*
A*c - 7*B*b))/(7*B*b^2 - 3*A*b*c))*(3*A*c - 7*B*b))/(4*c^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.91

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{-45\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,b^2c - 90\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)ab\,c^2x - 45\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\sqrt{}}\right.}{}$$

input
```
int(x^(11/2)*(B*x+A)/(c*x^2+b*x)^3,x)
```

output
```
( - 45*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b**2*c - 90*s
qrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b*c**2*x - 45*sqrt(c)
*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c**3*x**2 + 105*sqrt(c)*sqr
t(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**4 + 210*sqrt(c)*sqrt(b)*atan((
sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**3*c*x + 105*sqrt(c)*sqrt(b)*atan((sqrt(x)
*c)/(sqrt(c)*sqrt(b)))*b**2*c**2*x**2 + 45*sqrt(x)*a*b**2*c**2 + 75*sqrt(x
)*a*b*c**3*x + 24*sqrt(x)*a*c**4*x**2 - 105*sqrt(x)*b**4*c - 175*sqrt(x)*b
**3*c**2*x - 56*sqrt(x)*b**2*c**3*x**2 + 8*sqrt(x)*b*c**4*x**3)/(12*c**5*(
b**2 + 2*b*c*x + c**2*x**2))
```

3.98 $\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 107

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{2B\sqrt{x}}{c^3}+\frac{(bB-Ac)x^{3/2}}{2c^2(b+cx)^2}+\frac{(7bB-3Ac)\sqrt{x}}{4c^3(b+cx)}-\frac{3(5bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{4\sqrt{b}c^{7/2}}$$

output
```
2*B*x^(1/2)/c^3+1/2*(-A*c+B*b)*x^(3/2)/c^2/(c*x+b)^2+1/4*(-3*A*c+7*B*b)*x^
(1/2)/c^3/(c*x+b)-3/4*(-A*c+5*B*b)*arctan(c^(1/2)*x^(1/2)/b^(1/2))/b^(1/2)
/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.88

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{\sqrt{x}(15b^2B-3Abc+25bBcx-5Ac^2x+8Bc^2x^2)}{4c^3(b+cx)^2}-\frac{3(5bB-Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{4\sqrt{b}c^{7/2}}$$

input
```
Integrate[(x^(9/2)*(A + B*x))/(b*x + c*x^2)^3,x]
```

output
```
(Sqrt[x]*(15*b^2*B - 3*A*b*c + 25*b*B*c*x - 5*A*c^2*x + 8*B*c^2*x^2))/(4*c
^3*(b + c*x)^2) - (3*(5*b*B - A*c)*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(4*S
qrt[b]*c^(7/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {9, 87, 51, 60, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{x^{3/2}(A+Bx)}{(b+cx)^3}dx$$

$$\Big\downarrow \ 87$$

$$\frac{(5bB-Ac)\int \frac{x^{3/2}}{(b+cx)^2}dx}{4bc}-\frac{x^{5/2}(bB-Ac)}{2bc(b+cx)^2}$$

$$\Big\downarrow \ 51$$

$$\frac{(5bB-Ac)\left(\frac{3\int \frac{\sqrt{x}}{b+cx}dx}{2c}-\frac{x^{3/2}}{c(b+cx)}\right)}{4bc}-\frac{x^{5/2}(bB-Ac)}{2bc(b+cx)^2}$$

$$\Big\downarrow \ 60$$

$$\frac{(5bB-Ac)\left(\frac{3\left(\frac{2\sqrt{x}}{c}-\frac{b\int \frac{1}{\sqrt{x}(b+cx)}dx}{c}\right)}{2c}-\frac{x^{3/2}}{c(b+cx)}\right)}{4bc}-\frac{x^{5/2}(bB-Ac)}{2bc(b+cx)^2}$$

$$\Big\downarrow \ 73$$

$$\frac{(5bB-Ac)\left(\frac{3\left(\frac{2\sqrt{x}}{c}-\frac{2b\int\frac{1}{b+cx}d\sqrt{x}}{c}\right)}{2c}-\frac{x^{3/2}}{c(b+cx)}\right)}{4bc}-\frac{x^{5/2}(bB-Ac)}{2bc(b+cx)^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{(5bB-Ac)\left(\frac{3\left(\frac{2\sqrt{x}}{c}-\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{c^{3/2}}\right)}{2c}-\frac{x^{3/2}}{c(b+cx)}\right)}{4bc}-\frac{x^{5/2}(bB-Ac)}{2bc(b+cx)^2}$$

input
```
Int[(x^(9/2)*(A + B*x))/(b*x + c*x^2)^3,x]
```

output
```
-1/2*((b*B - A*c)*x^(5/2))/(b*c*(b + c*x)^2) + ((5*b*B - A*c)*(-(x^(3/2)/(
c*(b + c*x))) + (3*((2*Sqrt[x])/c - (2*Sqrt[b]*ArcTan[(Sqrt[c]*Sqrt[x])/Sq
rt[b]])/c^(3/2)))/(2*c)))/(4*b*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{2B\sqrt{x}}{c^3}+\frac{\frac{2\left(\left(-\frac{5}{8}Ac^2+\frac{9}{8}Bbc\right)x^{\frac{3}{2}}-\frac{b(3Ac-7Bb)\sqrt{x}}{8}\right)}{(cx+b)^2}+\frac{3(Ac-5Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\sqrt{bc}}}{c^3}$	83
default	$\frac{2B\sqrt{x}}{c^3}+\frac{\frac{2\left(\left(-\frac{5}{8}Ac^2+\frac{9}{8}Bbc\right)x^{\frac{3}{2}}-\frac{b(3Ac-7Bb)\sqrt{x}}{8}\right)}{(cx+b)^2}+\frac{3(Ac-5Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\sqrt{bc}}}{c^3}$	83
risch	$\frac{2B\sqrt{x}}{c^3}+\frac{\frac{2\left(-\frac{5}{8}Ac^2+\frac{9}{8}Bbc\right)x^{\frac{3}{2}}-\frac{b(3Ac-7Bb)\sqrt{x}}{4}}{(cx+b)^2}+\frac{3(Ac-5Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\sqrt{bc}}}{c^3}$	83

input
```
int(x^(9/2)*(B*x+A)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
2*B*x^(1/2)/c^3+2/c^3*(((-5/8*A*c^2+9/8*B*b*c)*x^(3/2)-1/8*b*(3*A*c-7*B*b)
*x^(1/2))/(c*x+b)^2+3/8*(A*c-5*B*b)/(b*c)^(1/2)*arctan(c*x^(1/2)/(b*c)^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.98

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \left[\frac{3\,(5\,Bb^3 - Ab^2c + (5\,Bbc^2 - Ac^3)x^2 + 2\,(5\,Bb^2c - Abc^2)x)\sqrt{-bc}\log\left(\frac{cx-b-2\sqrt{-bc}\sqrt{}}{cx+b}\right.}{8\,(bc^6x^2 + 2\,b^2c^5x + b}\right.$$

input
```
integrate(x^(9/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
[1/8*(3*(5*B*b^3 - A*b^2*c + (5*B*b*c^2 - A*c^3)*x^2 + 2*(5*B*b^2*c - A*b*
c^2)*x)*sqrt(-b*c)*log((c*x - b - 2*sqrt(-b*c)*sqrt(x))/(c*x + b)) + 2*(8*
B*b*c^3*x^2 + 15*B*b^3*c - 3*A*b^2*c^2 + 5*(5*B*b^2*c^2 - A*b*c^3)*x)*sqrt
(x))/(b*c^6*x^2 + 2*b^2*c^5*x + b^3*c^4), 1/4*(3*(5*B*b^3 - A*b^2*c + (5*B
*b*c^2 - A*c^3)*x^2 + 2*(5*B*b^2*c - A*b*c^2)*x)*sqrt(b*c)*arctan(sqrt(b*c
)/(c*sqrt(x))) + (8*B*b*c^3*x^2 + 15*B*b^3*c - 3*A*b^2*c^2 + 5*(5*B*b^2*c^
2 - A*b*c^3)*x)*sqrt(x))/(b*c^6*x^2 + 2*b^2*c^5*x + b^3*c^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)*(B*x+A)/(c*x**2+b*x)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.93

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{(9\,Bbc-5\,Ac^2)x^{\frac{3}{2}}+(7\,Bb^2-3\,Abc)\sqrt{x}}{4\,(c^5x^2+2\,bc^4x+b^2c^3)} + \frac{2\,B\sqrt{x}}{c^3} - \frac{3\,(5\,Bb-Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\,\sqrt{bc}c^3}$$

input
```
integrate(x^(9/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/4*((9*B*b*c - 5*A*c^2)*x^(3/2) + (7*B*b^2 - 3*A*b*c)*sqrt(x))/(c^5*x^2 +
 2*b*c^4*x + b^2*c^3) + 2*B*sqrt(x)/c^3 - 3/4*(5*B*b - A*c)*arctan(c*sqrt(
x)/sqrt(b*c))/(sqrt(b*c)*c^3)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.81

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{2\,B\sqrt{x}}{c^3} - \frac{3\,(5\,Bb-Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\,\sqrt{bc}c^3} + \frac{9\,Bbcx^{\frac{3}{2}}-5\,Ac^2x^{\frac{3}{2}}+7\,Bb^2\sqrt{x}-3\,Abc\sqrt{x}}{4\,(cx+b)^2c^3}$$

input
```
integrate(x^(9/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
2*B*sqrt(x)/c^3 - 3/4*(5*B*b - A*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)
*c^3) + 1/4*(9*B*b*c*x^(3/2) - 5*A*c^2*x^(3/2) + 7*B*b^2*sqrt(x) - 3*A*b*c
*sqrt(x))/((c*x + b)^2*c^3)
```

Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^3}\,dx=\frac{\sqrt{x}\left(\frac{7\,B\,b^2}{4}-\frac{3\,A\,b\,c}{4}\right)-x^{3/2}\left(\frac{5\,A\,c^2}{4}-\frac{9\,B\,b\,c}{4}\right)}{b^2\,c^3+2\,b\,c^4\,x+c^5\,x^2}+\frac{2\,B\,\sqrt{x}}{c^3}+\frac{3\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{b}}\right)\,(A\,c-5\,B\,b)}{4\,\sqrt{b}\,c^{7/2}}$$

input
```
int((x^(9/2)*(A + B*x))/(b*x + c*x^2)^3,x)
```

output
```
(x^(1/2)*((7*B*b^2)/4 - (3*A*b*c)/4) - x^(3/2)*((5*A*c^2)/4 - (9*B*b*c)/4)
)/(b^2*c^3 + c^5*x^2 + 2*b*c^4*x) + (2*B*x^(1/2))/c^3 + (3*atan((c^(1/2)*x
^(1/2))/b^(1/2))*(A*c - 5*B*b))/(4*b^(1/2)*c^(7/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 229, normalized size of antiderivative = 2.14

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^3}\,dx=\frac{3\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,b^2c+6\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)ab\,c^2x+3\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,c^3}{}$$

input
```
int(x^(9/2)*(B*x+A)/(c*x^2+b*x)^3,x)
```

output
```
(3*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b**2*c + 6*sqrt(c
)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b*c**2*x + 3*sqrt(c)*sqrt(
b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c**3*x**2 - 15*sqrt(c)*sqrt(b)*at
an((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**4 - 30*sqrt(c)*sqrt(b)*atan((sqrt(x)*
c)/(sqrt(c)*sqrt(b)))*b**3*c*x - 15*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt
(c)*sqrt(b)))*b**2*c**2*x**2 - 3*sqrt(x)*a*b**2*c**2 - 5*sqrt(x)*a*b*c**3*
x + 15*sqrt(x)*b**4*c + 25*sqrt(x)*b**3*c**2*x + 8*sqrt(x)*b**2*c**3*x**2)
/(4*b*c**4*(b**2 + 2*b*c*x + c**2*x**2))
```

3.99 $\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^3}\,dx=\frac{(bB-Ac)\sqrt{x}}{2c^2(b+cx)^2}-\frac{(5bB-Ac)\sqrt{x}}{4bc^2(b+cx)}+\frac{(3bB+Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{4b^{3/2}c^{5/2}}$$

output
```
1/2*(-A*c+B*b)*x^(1/2)/c^2/(c*x+b)^2-1/4*(-A*c+5*B*b)*x^(1/2)/b/c^2/(c*x+b
)+1/4*(A*c+3*B*b)*arctan(c^(1/2)*x^(1/2)/b^(1/2))/b^(3/2)/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.88

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^3}\,dx=-\frac{\sqrt{x}(3b^2B+Abc+5bBcx-Ac^2x)}{4bc^2(b+cx)^2}+\frac{(3bB+Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{4b^{3/2}c^{5/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/(b*x + c*x^2)^3,x]
```

output
```
-1/4*(Sqrt[x]*(3*b^2*B + A*b*c + 5*b*B*c*x - A*c^2*x))/(b*c^2*(b + c*x)^2)
 + ((3*b*B + A*c)*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(4*b^(3/2)*c^(5/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {9, 87, 51, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^3}\,dx$$

$$\Big\downarrow \; 9$$

$$\int \frac{\sqrt{x}(A+Bx)}{(b+cx)^3}dx$$

$$\Big\downarrow \; 87$$

$$\frac{(Ac+3bB)\int \frac{\sqrt{x}}{(b+cx)^2}dx}{4bc}-\frac{x^{3/2}(bB-Ac)}{2bc(b+cx)^2}$$

$$\Big\downarrow \; 51$$

$$\frac{(Ac+3bB)\left(\frac{\int \frac{1}{\sqrt{x}(b+cx)}dx}{2c}-\frac{\sqrt{x}}{c(b+cx)}\right)}{4bc}-\frac{x^{3/2}(bB-Ac)}{2bc(b+cx)^2}$$

$$\Big\downarrow \; 73$$

$$\frac{(Ac+3bB)\left(\frac{\int \frac{1}{b+cx}d\sqrt{x}}{c}-\frac{\sqrt{x}}{c(b+cx)}\right)}{4bc}-\frac{x^{3/2}(bB-Ac)}{2bc(b+cx)^2}$$

$$\Big\downarrow \; 218$$

$$\frac{(Ac+3bB)\left(\frac{\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{\sqrt{b}c^{3/2}}-\frac{\sqrt{x}}{c(b+cx)}\right)}{4bc}-\frac{x^{3/2}(bB-Ac)}{2bc(b+cx)^2}$$

input
```
Int[(x^(7/2)*(A + B*x))/(b*x + c*x^2)^3,x]
```

output
```
-1/2*((b*B - A*c)*x^(3/2))/(b*c*(b + c*x)^2) + ((3*b*B + A*c)*(-(Sqrt[x]/(
c*(b + c*x))) + ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]]/(Sqrt[b]*c^(3/2))))/(4*b
*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.81

method	result	size
derivativedivides	$\frac{\frac{(Ac-5Bb)x^{\frac{3}{2}}}{4bc}-\frac{(Ac+3Bb)\sqrt{x}}{4c^2}}{(cx+b)^2}+\frac{(Ac+3Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4c^2b\sqrt{bc}}$	79
default	$\frac{\frac{(Ac-5Bb)x^{\frac{3}{2}}}{4bc}-\frac{(Ac+3Bb)\sqrt{x}}{4c^2}}{(cx+b)^2}+\frac{(Ac+3Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4c^2b\sqrt{bc}}$	79

input
```
int(x^(7/2)*(B*x+A)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/8*(A*c-5*B*b)/b/c*x^(3/2)-1/8*(A*c+3*B*b)/c^2*x^(1/2))/(c*x+b)^2+1/4*
(A*c+3*B*b)/c^2/b/(b*c)^(1/2)*arctan(c*x^(1/2)/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.97

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \left[-\frac{(3\,Bb^3+Ab^2c+(3\,Bbc^2+Ac^3)x^2+2\,(3\,Bb^2c+Abc^2)x)\sqrt{-bc}\log\left(\frac{cx-b-2\,\sqrt{-bc}\sqrt{}}{cx+b}\right.}{8\,(b^2c^5x^2+2\,b^3c^4x+b^4c^3)}\right.$$

$$-\frac{(3\,Bb^3+Ab^2c+(3\,Bbc^2+Ac^3)x^2+2\,(3\,Bb^2c+Abc^2)x)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}}{c\sqrt{x}}\right)+(3\,Bb^3c+Ab^2c^2+(5\,Bb^2c}{4\,(b^2c^5x^2+2\,b^3c^4x+b^4c^3)}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
[-1/8*((3*B*b^3 + A*b^2*c + (3*B*b*c^2 + A*c^3)*x^2 + 2*(3*B*b^2*c + A*b*c
^2)*x)*sqrt(-b*c)*log((c*x - b - 2*sqrt(-b*c)*sqrt(x))/(c*x + b)) + 2*(3*B
*b^3*c + A*b^2*c^2 + (5*B*b^2*c^2 - A*b*c^3)*x)*sqrt(x))/(b^2*c^5*x^2 + 2*
b^3*c^4*x + b^4*c^3), -1/4*((3*B*b^3 + A*b^2*c + (3*B*b*c^2 + A*c^3)*x^2 +
 2*(3*B*b^2*c + A*b*c^2)*x)*sqrt(b*c)*arctan(sqrt(b*c)/(c*sqrt(x))) + (3*B
*b^3*c + A*b^2*c^2 + (5*B*b^2*c^2 - A*b*c^3)*x)*sqrt(x))/(b^2*c^5*x^2 + 2*
b^3*c^4*x + b^4*c^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1316 vs. $2(90) = 180$.

Time = 166.17 (sec) , antiderivative size = 1316, normalized size of antiderivative = 13.43

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**(7/2)*(B*x+A)/(c*x**2+b*x)**3,x)
```

output
```
Piecewise((zoo*(-2*A/(3*x**(3/2)) - 2*B/sqrt(x)), Eq(b, 0) & Eq(c, 0)), ((
2*A*x**(3/2)/3 + 2*B*x**(5/2)/5)/b**3, Eq(c, 0)), ((-2*A/(3*x**(3/2)) - 2*
B/sqrt(x))/c**3, Eq(b, 0)), (A*b**2*c*log(sqrt(x) - sqrt(-b/c))/(8*b**3*c*
*3*sqrt(-b/c) + 16*b**2*c**4*x*sqrt(-b/c) + 8*b*c**5*x**2*sqrt(-b/c)) - A*
b**2*c*log(sqrt(x) + sqrt(-b/c))/(8*b**3*c**3*sqrt(-b/c) + 16*b**2*c**4*x*
sqrt(-b/c) + 8*b*c**5*x**2*sqrt(-b/c)) - 2*A*b*c**2*sqrt(x)*sqrt(-b/c)/(8*
b**3*c**3*sqrt(-b/c) + 16*b**2*c**4*x*sqrt(-b/c) + 8*b*c**5*x**2*sqrt(-b/c
)) + 2*A*b*c**2*x*log(sqrt(x) - sqrt(-b/c))/(8*b**3*c**3*sqrt(-b/c) + 16*b
**2*c**4*x*sqrt(-b/c) + 8*b*c**5*x**2*sqrt(-b/c)) - 2*A*b*c**2*x*log(sqrt(
x) + sqrt(-b/c))/(8*b**3*c**3*sqrt(-b/c) + 16*b**2*c**4*x*sqrt(-b/c) + 8*b
*c**5*x**2*sqrt(-b/c)) + 2*A*c**3*x**(3/2)*sqrt(-b/c)/(8*b**3*c**3*sqrt(-b
/c) + 16*b**2*c**4*x*sqrt(-b/c) + 8*b*c**5*x**2*sqrt(-b/c)) + A*c**3*x**2*
log(sqrt(x) - sqrt(-b/c))/(8*b**3*c**3*sqrt(-b/c) + 16*b**2*c**4*x*sqrt(-b
/c) + 8*b*c**5*x**2*sqrt(-b/c)) - A*c**3*x**2*log(sqrt(x) + sqrt(-b/c))/(8
*b**3*c**3*sqrt(-b/c) + 16*b**2*c**4*x*sqrt(-b/c) + 8*b*c**5*x**2*sqrt(-b/
c)) + 3*B*b**3*log(sqrt(x) - sqrt(-b/c))/(8*b**3*c**3*sqrt(-b/c) + 16*b**2
*c**4*x*sqrt(-b/c) + 8*b*c**5*x**2*sqrt(-b/c)) - 3*B*b**3*log(sqrt(x) + sq
rt(-b/c))/(8*b**3*c**3*sqrt(-b/c) + 16*b**2*c**4*x*sqrt(-b/c) + 8*b*c**5*x
**2*sqrt(-b/c)) - 6*B*b**2*c*sqrt(x)*sqrt(-b/c)/(8*b**3*c**3*sqrt(-b/c) +
16*b**2*c**4*x*sqrt(-b/c) + 8*b*c**5*x**2*sqrt(-b/c)) + 6*B*b**2*c*x*lo...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.96

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^3}\,dx = -\frac{(5\,Bbc-Ac^2)x^{\frac{3}{2}}+(3\,Bb^2+Abc)\sqrt{x}}{4\,(bc^4x^2+2\,b^2c^3x+b^3c^2)}+\frac{(3\,Bb+Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\,\sqrt{bc}bc^2}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/4*((5*B*b*c - A*c^2)*x^(3/2) + (3*B*b^2 + A*b*c)*sqrt(x))/(b*c^4*x^2 +
2*b^2*c^3*x + b^3*c^2) + 1/4*(3*B*b + A*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sq
rt(b*c)*b*c^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{(3\,Bb+Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\,\sqrt{bc}bc^2}-\frac{5\,Bbcx^{\frac{3}{2}}-Ac^2x^{\frac{3}{2}}+3\,Bb^2\sqrt{x}+Abc\sqrt{x}}{4\,(cx+b)^2bc^2}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
1/4*(3*B*b + A*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b*c^2) - 1/4*(5*B
*b*c*x^(3/2) - A*c^2*x^(3/2) + 3*B*b^2*sqrt(x) + A*b*c*sqrt(x))/((c*x + b)
^2*b*c^2)
```

Mupad [B] (verification not implemented)

Time = 5.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.86

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{b}}\right)\,\left(A\,c+3\,B\,b\right)}{4\,b^{3/2}\,c^{5/2}}-\frac{\frac{\sqrt{x}\,\left(A\,c+3\,B\,b\right)}{4\,c^2}-\frac{x^{3/2}\,\left(A\,c-5\,B\,b\right)}{4\,b\,c}}{b^2+2\,b\,c\,x+c^2\,x^2}$$

input
```
int((x^(7/2)*(A + B*x))/(b*x + c*x^2)^3,x)
```

output
```
(atan((c^(1/2)*x^(1/2))/b^(1/2))*(A*c + 3*B*b))/(4*b^(3/2)*c^(5/2)) - ((x^
(1/2)*(A*c + 3*B*b))/(4*c^2) - (x^(3/2)*(A*c - 5*B*b))/(4*b*c))/(b^2 + c^2
*x^2 + 2*b*c*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.17

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,b^2c+2\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)ab\,c^2x+\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,c^3x^2}{}$$

input
```
int(x^(7/2)*(B*x+A)/(c*x^2+b*x)^3,x)
```

output
```
(sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b**2*c + 2*sqrt(c)*
sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b*c**2*x + sqrt(c)*sqrt(b)*a
tan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c**3*x**2 + 3*sqrt(c)*sqrt(b)*atan((s
qrt(x)*c)/(sqrt(c)*sqrt(b)))*b**4 + 6*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sq
rt(c)*sqrt(b)))*b**3*c*x + 3*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqr
t(b)))*b**2*c**2*x**2 - sqrt(x)*a*b**2*c**2 + sqrt(x)*a*b*c**3*x - 3*sqrt(
x)*b**4*c - 5*sqrt(x)*b**3*c**2*x)/(4*b**2*c**3*(b**2 + 2*b*c*x + c**2*x**
2))
```

3.100 $\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 100

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^3}\,dx = -\frac{(bB-Ac)\sqrt{x}}{2bc(b+cx)^2}+\frac{(bB+3Ac)\sqrt{x}}{4b^2c(b+cx)}+\frac{(bB+3Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{4b^{5/2}c^{3/2}}$$

output
```
-1/2*(-A*c+B*b)*x^(1/2)/b/c/(c*x+b)^2+1/4*(3*A*c+B*b)*x^(1/2)/b^2/c/(c*x+b
)+1/4*(3*A*c+B*b)*arctan(c^(1/2)*x^(1/2)/b^(1/2))/b^(5/2)/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.86

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^3}\,dx = -\frac{\sqrt{x}(b^2B-5Abc-bBcx-3Ac^2x)}{4b^2c(b+cx)^2}+\frac{(bB+3Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{4b^{5/2}c^{3/2}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(b*x + c*x^2)^3,x]
```

output
```
-1/4*(Sqrt[x]*(b^2*B - 5*A*b*c - b*B*c*x - 3*A*c^2*x))/(b^2*c*(b + c*x)^2)
 + ((b*B + 3*A*c)*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(4*b^(5/2)*c^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {9, 87, 52, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{A+Bx}{\sqrt{x}(b+cx)^3}dx$$

$$\Big\downarrow \text{ 87}$$

$$\frac{(3Ac+bB)\int \frac{1}{\sqrt{x}(b+cx)^2}dx}{4bc}-\frac{\sqrt{x}(bB-Ac)}{2bc(b+cx)^2}$$

$$\Big\downarrow \text{ 52}$$

$$\frac{(3Ac+bB)\left(\frac{\int \frac{1}{\sqrt{x}(b+cx)}dx}{2b}+\frac{\sqrt{x}}{b(b+cx)}\right)}{4bc}-\frac{\sqrt{x}(bB-Ac)}{2bc(b+cx)^2}$$

$$\Big\downarrow \text{ 73}$$

$$\frac{(3Ac+bB)\left(\frac{\int \frac{1}{b+cx}d\sqrt{x}}{b}+\frac{\sqrt{x}}{b(b+cx)}\right)}{4bc}-\frac{\sqrt{x}(bB-Ac)}{2bc(b+cx)^2}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{(3Ac+bB)\left(\frac{\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}\sqrt{c}}+\frac{\sqrt{x}}{b(b+cx)}\right)}{4bc}-\frac{\sqrt{x}(bB-Ac)}{2bc(b+cx)^2}$$

input
```
Int[(x^(5/2)*(A + B*x))/(b*x + c*x^2)^3,x]
```

output
```
-1/2*((b*B - A*c)*Sqrt[x])/(b*c*(b + c*x)^2) + ((b*B + 3*A*c)*(Sqrt[x]/(b*
(b + c*x)) + ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]]/(b^(3/2)*Sqrt[c])))/(4*b*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$\frac{\frac{(3Ac+Bb)x^{\frac{3}{2}}}{4b^2}+\frac{(5Ac-Bb)\sqrt{x}}{4bc}}{(cx+b)^2}+\frac{(3Ac+Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4b^2c\sqrt{bc}}$	80
default	$\frac{\frac{(3Ac+Bb)x^{\frac{3}{2}}}{4b^2}+\frac{(5Ac-Bb)\sqrt{x}}{4bc}}{(cx+b)^2}+\frac{(3Ac+Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4b^2c\sqrt{bc}}$	80

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
2*(1/8*(3*A*c+B*b)/b^2*x^(3/2)+1/8*(5*A*c-B*b)/b/c*x^(1/2))/(c*x+b)^2+1/4*
(3*A*c+B*b)/b^2/c/(b*c)^(1/2)*arctan(c*x^(1/2)/(b*c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.91

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \left[-\frac{(Bb^3+3\,Ab^2c+(Bbc^2+3\,Ac^3)x^2+2\,(Bb^2c+3\,Abc^2)x)\sqrt{-bc}\log\left(\frac{cx-b-2\,\sqrt{-bc}\sqrt{}}{cx+b}\right.}{8\,(b^3c^4x^2+2\,b^4c^3x+b^5c^2)}\right.$$
$$-\frac{(Bb^3+3\,Ab^2c+(Bbc^2+3\,Ac^3)x^2+2\,(Bb^2c+3\,Abc^2)x)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}}{c\sqrt{x}}\right)+(Bb^3c-5\,Ab^2c^2-(Bb^2c^2}{4\,(b^3c^4x^2+2\,b^4c^3x+b^5c^2)}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
[-1/8*((B*b^3 + 3*A*b^2*c + (B*b*c^2 + 3*A*c^3)*x^2 + 2*(B*b^2*c + 3*A*b*c
^2)*x)*sqrt(-b*c)*log((c*x - b - 2*sqrt(-b*c)*sqrt(x))/(c*x + b)) + 2*(B*b
^3*c - 5*A*b^2*c^2 - (B*b^2*c^2 + 3*A*b*c^3)*x)*sqrt(x))/(b^3*c^4*x^2 + 2*
b^4*c^3*x + b^5*c^2), -1/4*((B*b^3 + 3*A*b^2*c + (B*b*c^2 + 3*A*c^3)*x^2 +
 2*(B*b^2*c + 3*A*b*c^2)*x)*sqrt(b*c)*arctan(sqrt(b*c)/(c*sqrt(x))) + (B*b
^3*c - 5*A*b^2*c^2 - (B*b^2*c^2 + 3*A*b*c^3)*x)*sqrt(x))/(b^3*c^4*x^2 + 2*
b^4*c^3*x + b^5*c^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1345 vs. 2(90) = 180.

Time = 116.26 (sec) , antiderivative size = 1345, normalized size of antiderivative = 13.45

$$\int \frac{x^{5/2}(A+Bx)}{\left(bx+cx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**(5/2)*(B*x+A)/(c*x**2+b*x)**3,x)
```

output
```
Piecewise((zoo*(-2*A/(5*x**(5/2)) - 2*B/(3*x**(3/2))), Eq(b, 0) & Eq(c, 0)
), ((2*A*sqrt(x) + 2*B*x**(3/2)/3)/b**3, Eq(c, 0)), ((-2*A/(5*x**(5/2)) -
2*B/(3*x**(3/2)))/c**3, Eq(b, 0)), (3*A*b**2*c*log(sqrt(x) - sqrt(-b/c))/(
8*b**4*c**2*sqrt(-b/c) + 16*b**3*c**3*x*sqrt(-b/c) + 8*b**2*c**4*x**2*sqrt
(-b/c)) - 3*A*b**2*c*log(sqrt(x) + sqrt(-b/c))/(8*b**4*c**2*sqrt(-b/c) + 1
6*b**3*c**3*x*sqrt(-b/c) + 8*b**2*c**4*x**2*sqrt(-b/c)) + 10*A*b*c**2*sqrt
(x)*sqrt(-b/c)/(8*b**4*c**2*sqrt(-b/c) + 16*b**3*c**3*x*sqrt(-b/c) + 8*b**
2*c**4*x**2*sqrt(-b/c)) + 6*A*b*c**2*x*log(sqrt(x) - sqrt(-b/c))/(8*b**4*c
**2*sqrt(-b/c) + 16*b**3*c**3*x*sqrt(-b/c) + 8*b**2*c**4*x**2*sqrt(-b/c))
- 6*A*b*c**2*x*log(sqrt(x) + sqrt(-b/c))/(8*b**4*c**2*sqrt(-b/c) + 16*b**3
*c**3*x*sqrt(-b/c) + 8*b**2*c**4*x**2*sqrt(-b/c)) + 6*A*c**3*x**(3/2)*sqrt
(-b/c)/(8*b**4*c**2*sqrt(-b/c) + 16*b**3*c**3*x*sqrt(-b/c) + 8*b**2*c**4*x
**2*sqrt(-b/c)) + 3*A*c**3*x**2*log(sqrt(x) - sqrt(-b/c))/(8*b**4*c**2*sqr
t(-b/c) + 16*b**3*c**3*x*sqrt(-b/c) + 8*b**2*c**4*x**2*sqrt(-b/c)) - 3*A*c
**3*x**2*log(sqrt(x) + sqrt(-b/c))/(8*b**4*c**2*sqrt(-b/c) + 16*b**3*c**3*
x*sqrt(-b/c) + 8*b**2*c**4*x**2*sqrt(-b/c)) + B*b**3*log(sqrt(x) - sqrt(-b
/c))/(8*b**4*c**2*sqrt(-b/c) + 16*b**3*c**3*x*sqrt(-b/c) + 8*b**2*c**4*x**
2*sqrt(-b/c)) - B*b**3*log(sqrt(x) + sqrt(-b/c))/(8*b**4*c**2*sqrt(-b/c) +
 16*b**3*c**3*x*sqrt(-b/c) + 8*b**2*c**4*x**2*sqrt(-b/c)) - 2*B*b**2*c*sqr
t(x)*sqrt(-b/c)/(8*b**4*c**2*sqrt(-b/c) + 16*b**3*c**3*x*sqrt(-b/c) + 8...
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{(Bbc+3\,Ac^2)x^{\frac{3}{2}}-(Bb^2-5\,Abc)\sqrt{x}}{4\,(b^2c^3x^2+2\,b^3c^2x+b^4c)} + \frac{(Bb+3\,Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\,\sqrt{bc}b^2c}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
1/4*((B*b*c + 3*A*c^2)*x^(3/2) - (B*b^2 - 5*A*b*c)*sqrt(x))/(b^2*c^3*x^2 +
 2*b^3*c^2*x + b^4*c) + 1/4*(B*b + 3*A*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqr
t(b*c)*b^2*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{(Bb+3\,Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\,\sqrt{bc}b^2c} + \frac{Bbcx^{\frac{3}{2}}+3\,Ac^2x^{\frac{3}{2}}-Bb^2\sqrt{x}+5\,Abc\sqrt{x}}{4\,(cx+b)^2b^2c}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
1/4*(B*b + 3*A*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b^2*c) + 1/4*(B*b
*c*x^(3/2) + 3*A*c^2*x^(3/2) - B*b^2*sqrt(x) + 5*A*b*c*sqrt(x))/((c*x + b)
^2*b^2*c)
```

Mupad [B] (verification not implemented)

Time = 5.52 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.84

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{\frac{x^{3/2}\,(3\,A\,c+B\,b)}{4\,b^2}+\frac{\sqrt{x}\,(5\,A\,c-B\,b)}{4\,b\,c}}{b^2+2\,b\,c\,x+c^2\,x^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{b}}\right)\,(3\,A\,c+B\,b)}{4\,b^{5/2}\,c^{3/2}}$$

input
```
int((x^(5/2)*(A + B*x))/(b*x + c*x^2)^3,x)
```

output
```
((x^(3/2)*(3*A*c + B*b))/(4*b^2) + (x^(1/2)*(5*A*c - B*b))/(4*b*c))/(b^2 +
 c^2*x^2 + 2*b*c*x) + (atan((c^(1/2)*x^(1/2))/b^(1/2))*(3*A*c + B*b))/(4*b
^(5/2)*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.13

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{3\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,b^2c+6\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)ab\,c^2x+3\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,c^3}{}$$

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x)^3,x)
```

output
```
(3*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b**2*c + 6*sqrt(c
)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b*c**2*x + 3*sqrt(c)*sqrt(
b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c**3*x**2 + sqrt(c)*sqrt(b)*atan(
(sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**4 + 2*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(
sqrt(c)*sqrt(b)))*b**3*c*x + sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqr
t(b)))*b**2*c**2*x**2 + 5*sqrt(x)*a*b**2*c**2 + 3*sqrt(x)*a*b*c**3*x - sqr
t(x)*b**4*c + sqrt(x)*b**3*c**2*x)/(4*b**3*c**2*(b**2 + 2*b*c*x + c**2*x**
2))
```

3.101 $\int \frac{x^{3/2}(A+Bx)}{\left(bx+cx^2\right)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 106

$$\int \frac{x^{3/2}(A+Bx)}{\left(bx+cx^2\right)^3}\,dx = -\frac{2A}{b^3\sqrt{x}}+\frac{(bB-Ac)\sqrt{x}}{2b^2(b+cx)^2} +\frac{(3bB-7Ac)\sqrt{x}}{4b^3(b+cx)}+\frac{3(bB-5Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{4b^{7/2}\sqrt{c}}$$

output
```
-2*A/b^3/x^(1/2)+1/2*(-A*c+B*b)*x^(1/2)/b^2/(c*x+b)^2+1/4*(-7*A*c+3*B*b)*x
^(1/2)/b^3/(c*x+b)+3/4*(-5*A*c+B*b)*arctan(c^(1/2)*x^(1/2)/b^(1/2))/b^(7/2
)/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.91

$$\int \frac{x^{3/2}(A+Bx)}{\left(bx+cx^2\right)^3}\,dx = \frac{-8Ab^2+5b^2Bx-25Abcx+3bBcx^2-15Ac^2x^2}{4b^3\sqrt{x}(b+cx)^2} +\frac{3(bB-5Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{4b^{7/2}\sqrt{c}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(b*x + c*x^2)^3,x]
```

output
```
(-8*A*b^2 + 5*b^2*B*x - 25*A*b*c*x + 3*b*B*c*x^2 - 15*A*c^2*x^2)/(4*b^3*Sq
rt[x]*(b + c*x)^2) + (3*(b*B - 5*A*c)*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(
4*b^(7/2)*Sqrt[c])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {9, 87, 52, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^3}dx$$

$$\Big\downarrow \text{ 9}$$

$$\int \frac{A+Bx}{x^{3/2}(b+cx)^3}dx$$

$$\Big\downarrow \text{ 87}$$

$$-\frac{(bB-5Ac)\int \frac{1}{x^{3/2}(b+cx)^2}dx}{4bc}-\frac{bB-Ac}{2bc\sqrt{x}(b+cx)^2}$$

$$\Big\downarrow \text{ 52}$$

$$-\frac{(bB-5Ac)\left(\frac{3\int \frac{1}{x^{3/2}(b+cx)}dx}{2b}+\frac{1}{b\sqrt{x}(b+cx)}\right)}{4bc}-\frac{bB-Ac}{2bc\sqrt{x}(b+cx)^2}$$

$$\Big\downarrow \text{ 61}$$

$$-\frac{(bB-5Ac)\left(\frac{3\left(-\frac{c\int \frac{1}{\sqrt{x}(b+cx)}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{2b}+\frac{1}{b\sqrt{x}(b+cx)}\right)}{4bc}-\frac{bB-Ac}{2bc\sqrt{x}(b+cx)^2}$$

$$\Big\downarrow \text{ 73}$$

$$-\frac{(bB-5Ac)\left(\frac{3\left(-\frac{2c\int\frac{1}{b+cx}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{2b}+\frac{1}{b\sqrt{x}(b+cx)}\right)}{4bc}-\frac{bB-Ac}{2bc\sqrt{x}(b+cx)^2}$$

$\big\downarrow$ 218

$$-\frac{(bB-5Ac)\left(\frac{3\left(-\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{2b}+\frac{1}{b\sqrt{x}(b+cx)}\right)}{4bc}-\frac{bB-Ac}{2bc\sqrt{x}(b+cx)^2}$$

input

```
Int[(x^(3/2)*(A + B*x))/(b*x + c*x^2)^3,x]
```

output

```
-1/2*(b*B - A*c)/(b*c*Sqrt[x]*(b + c*x)^2) - ((b*B - 5*A*c)*(1/(b*Sqrt[x]*
(b + c*x)) + (3*(-2/(b*Sqrt[x]) - (2*Sqrt[c]*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt
[b]])/b^(3/2)))/(2*b)))/(4*b*c)
```

Defintions of rubi rules used

rule 9

```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$-\frac{2A}{b^{3}\sqrt{x}}-\frac{2\left(\frac{\left(\frac{7}{8}A\,c^{2}-\frac{3}{8}Bbc\right)x^{\frac{3}{2}}+\frac{b(9Ac-5Bb)\sqrt{x}}{8}}{(cx+b)^{2}}+\frac{3(5Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^{3}}$	84
default	$-\frac{2A}{b^{3}\sqrt{x}}-\frac{2\left(\frac{\left(\frac{7}{8}A\,c^{2}-\frac{3}{8}Bbc\right)x^{\frac{3}{2}}+\frac{b(9Ac-5Bb)\sqrt{x}}{8}}{(cx+b)^{2}}+\frac{3(5Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^{3}}$	84
risch	$-\frac{2A}{b^{3}\sqrt{x}}-\frac{\frac{2\left(\frac{7}{8}A\,c^{2}-\frac{3}{8}Bbc\right)x^{\frac{3}{2}}+\frac{b(9Ac-5Bb)\sqrt{x}}{4}}{(cx+b)^{2}}+\frac{3(5Ac-Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\sqrt{bc}}}{b^{3}}$	85

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-2*A/b^3/x^(1/2)-2/b^3*(((7/8*A*c^2-3/8*B*b*c)*x^(3/2)+1/8*b*(9*A*c-5*B*b)
*x^(1/2))/(c*x+b)^2+3/8*(5*A*c-B*b)/(b*c)^(1/2)*arctan(c*x^(1/2)/(b*c)^(1/
2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 331, normalized size of antiderivative = 3.12

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \left[\frac{3\,((Bbc^2-5\,Ac^3)x^3+2\,(Bb^2c-5\,Abc^2)x^2+(Bb^3-5\,Ab^2c)x)\sqrt{-bc}\log\left(\frac{cx-b+2\sqrt{}}{cx+}\right.}{8\,(b^4c^3x^3+2\,b^5c^2x^2}\right.$$

$$-\frac{3\,((Bbc^2-5\,Ac^3)x^3+2\,(Bb^2c-5\,Abc^2)x^2+(Bb^3-5\,Ab^2c)x)\sqrt{bc}\arctan\left(\frac{\sqrt{bc}}{c\sqrt{x}}\right)+(8\,Ab^3c-3\,(Bb^2c^2-}{4\,(b^4c^3x^3+2\,b^5c^2x^2+b^6cx)}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
[1/8*(3*((B*b*c^2 - 5*A*c^3)*x^3 + 2*(B*b^2*c - 5*A*b*c^2)*x^2 + (B*b^3 -
5*A*b^2*c)*x)*sqrt(-b*c)*log((c*x - b + 2*sqrt(-b*c)*sqrt(x))/(c*x + b)) -
 2*(8*A*b^3*c - 3*(B*b^2*c^2 - 5*A*b*c^3)*x^2 - 5*(B*b^3*c - 5*A*b^2*c^2)*
x)*sqrt(x))/(b^4*c^3*x^3 + 2*b^5*c^2*x^2 + b^6*c*x), -1/4*(3*((B*b*c^2 - 5
*A*c^3)*x^3 + 2*(B*b^2*c - 5*A*b*c^2)*x^2 + (B*b^3 - 5*A*b^2*c)*x)*sqrt(b*
c)*arctan(sqrt(b*c)/(c*sqrt(x))) + (8*A*b^3*c - 3*(B*b^2*c^2 - 5*A*b*c^3)*
x^2 - 5*(B*b^3*c - 5*A*b^2*c^2)*x)*sqrt(x))/(b^4*c^3*x^3 + 2*b^5*c^2*x^2 +
 b^6*c*x)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1598 vs. $2(104) = 208$.

Time = 69.84 (sec) , antiderivative size = 1598, normalized size of antiderivative = 15.08

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x**(3/2)*(B*x+A)/(c*x**2+b*x)**3,x)
```

output
```
Piecewise((zoo*(-2*A/(7*x**(7/2)) - 2*B/(5*x**(5/2))), Eq(b, 0) & Eq(c, 0)
), ((-2*A/sqrt(x) + 2*B*sqrt(x))/b**3, Eq(c, 0)), ((-2*A/(7*x**(7/2)) - 2*
B/(5*x**(5/2)))/c**3, Eq(b, 0)), (-15*A*b**2*c*sqrt(x)*log(sqrt(x) - sqrt(
-b/c))/(8*b**5*c*sqrt(x)*sqrt(-b/c) + 16*b**4*c**2*x**(3/2)*sqrt(-b/c) + 8
*b**3*c**3*x**(5/2)*sqrt(-b/c)) + 15*A*b**2*c*sqrt(x)*log(sqrt(x) + sqrt(-
b/c))/(8*b**5*c*sqrt(x)*sqrt(-b/c) + 16*b**4*c**2*x**(3/2)*sqrt(-b/c) + 8*
b**3*c**3*x**(5/2)*sqrt(-b/c)) - 16*A*b**2*c*sqrt(-b/c)/(8*b**5*c*sqrt(x)*
sqrt(-b/c) + 16*b**4*c**2*x**(3/2)*sqrt(-b/c) + 8*b**3*c**3*x**(5/2)*sqrt(
-b/c)) - 30*A*b*c**2*x**(3/2)*log(sqrt(x) - sqrt(-b/c))/(8*b**5*c*sqrt(x)*
sqrt(-b/c) + 16*b**4*c**2*x**(3/2)*sqrt(-b/c) + 8*b**3*c**3*x**(5/2)*sqrt(
-b/c)) + 30*A*b*c**2*x**(3/2)*log(sqrt(x) + sqrt(-b/c))/(8*b**5*c*sqrt(x)*
sqrt(-b/c) + 16*b**4*c**2*x**(3/2)*sqrt(-b/c) + 8*b**3*c**3*x**(5/2)*sqrt(
-b/c)) - 50*A*b*c**2*x*sqrt(-b/c)/(8*b**5*c*sqrt(x)*sqrt(-b/c) + 16*b**4*c
**2*x**(3/2)*sqrt(-b/c) + 8*b**3*c**3*x**(5/2)*sqrt(-b/c)) - 15*A*c**3*x**
(5/2)*log(sqrt(x) - sqrt(-b/c))/(8*b**5*c*sqrt(x)*sqrt(-b/c) + 16*b**4*c**
2*x**(3/2)*sqrt(-b/c) + 8*b**3*c**3*x**(5/2)*sqrt(-b/c)) + 15*A*c**3*x**(5
/2)*log(sqrt(x) + sqrt(-b/c))/(8*b**5*c*sqrt(x)*sqrt(-b/c) + 16*b**4*c**2*
x**(3/2)*sqrt(-b/c) + 8*b**3*c**3*x**(5/2)*sqrt(-b/c)) - 30*A*c**3*x**2*sq
rt(-b/c)/(8*b**5*c*sqrt(x)*sqrt(-b/c) + 16*b**4*c**2*x**(3/2)*sqrt(-b/c) +
 8*b**3*c**3*x**(5/2)*sqrt(-b/c)) + 3*B*b**3*sqrt(x)*log(sqrt(x) - sqrt...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^3}\,dx = -\frac{8\,Ab^2-3\,(Bbc-5\,Ac^2)x^2-5\,(Bb^2-5\,Abc)x}{4\left(b^3c^2x^{\frac{5}{2}}+2\,b^4cx^{\frac{3}{2}}+b^5\sqrt{x}\right)} + \frac{3\,(Bb-5\,Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\,\sqrt{bc}b^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/4*(8*A*b^2 - 3*(B*b*c - 5*A*c^2)*x^2 - 5*(B*b^2 - 5*A*b*c)*x)/(b^3*c^2*
x^(5/2) + 2*b^4*c*x^(3/2) + b^5*sqrt(x)) + 3/4*(B*b - 5*A*c)*arctan(c*sqrt
(x)/sqrt(b*c))/(sqrt(b*c)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.81

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{3\,(Bb-5\,Ac)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\,\sqrt{bc}b^3} - \frac{2\,A}{b^3\sqrt{x}} + \frac{3\,Bbcx^{\frac{3}{2}}-7\,Ac^2x^{\frac{3}{2}}+5\,Bb^2\sqrt{x}-9\,Abc\sqrt{x}}{4\,(cx+b)^2b^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
3/4*(B*b - 5*A*c)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b^3) - 2*A/(b^3*s
qrt(x)) + 1/4*(3*B*b*c*x^(3/2) - 7*A*c^2*x^(3/2) + 5*B*b^2*sqrt(x) - 9*A*b
*c*sqrt(x))/((c*x + b)^2*b^3)
```

Mupad [B] (verification not implemented)

Time = 5.39 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.09

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^3}\,dx = -\frac{\frac{2A}{b}+\frac{5x(5Ac-Bb)}{4b^2}+\frac{3cx^2(5Ac-Bb)}{4b^3}}{b^2\sqrt{x}+c^2x^{5/2}+2bcx^{3/2}} - \frac{3\,\mathrm{atan}\left(\frac{3\sqrt{c}\sqrt{x}(5Ac-Bb)}{\sqrt{b}(15Ac-3Bb)}\right)(5Ac-Bb)}{4b^{7/2}\sqrt{c}}$$

input
```
int((x^(3/2)*(A + B*x))/(b*x + c*x^2)^3,x)
```

output
```
- ((2*A)/b + (5*x*(5*A*c - B*b))/(4*b^2) + (3*c*x^2*(5*A*c - B*b))/(4*b^3)
)/(b^2*x^(1/2) + c^2*x^(5/2) + 2*b*c*x^(3/2)) - (3*atan((3*c^(1/2)*x^(1/2)
*(5*A*c - B*b))/(b^(1/2)*(15*A*c - 3*B*b)))*(5*A*c - B*b))/(4*b^(7/2)*c^(1
/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.24

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{-15\sqrt{x}\sqrt{c}\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\sqrt{b}}\right)ab^2c-30\sqrt{x}\sqrt{c}\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\sqrt{b}}\right)abc^2x-15\sqrt{x}\sqrt{c}\sqrt{b}}{}$$

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x)^3,x)
```

output
```
( - 15*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b**2*
c - 30*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b*c**
2*x - 15*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*c**
3*x**2 + 3*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**
4 + 6*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**3*c*x
 + 3*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*b**2*c**2
*x**2 - 8*a*b**3*c - 25*a*b**2*c**2*x - 15*a*b*c**3*x**2 + 5*b**4*c*x + 3*
b**3*c**2*x**2)/(4*sqrt(x)*b**4*c*(b**2 + 2*b*c*x + c**2*x**2))
```

3.102 $\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 129

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^3} dx = -\frac{2A}{3b^3x^{3/2}} - \frac{2(bB-3Ac)}{b^4\sqrt{x}} - \frac{c(bB-Ac)\sqrt{x}}{2b^3(b+cx)^2} - \frac{c(7bB-11Ac)\sqrt{x}}{4b^4(b+cx)} - \frac{5\sqrt{c}(3bB-7Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{4b^{9/2}}$$

output
```
-2/3*A/b^3/x^(3/2)-2*(-3*A*c+B*b)/b^4/x^(1/2)-1/2*c*(-A*c+B*b)*x^(1/2)/b^3
/(c*x+b)^2-1/4*c*(-11*A*c+7*B*b)*x^(1/2)/b^4/(c*x+b)-5/4*c^(1/2)*(-7*A*c+3
*B*b)*arctan(c^(1/2)*x^(1/2)/b^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.91

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^3} dx = \frac{-3bBx(8b^2+25bcx+15c^2x^2)+A(-8b^3+56b^2cx+175bc^2x^2+105c^3x^3)}{12b^4x^{3/2}(b+cx)^2} + \frac{5\sqrt{c}(-3bB+7Ac)\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{4b^{9/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(b*x + c*x^2)^3,x]
```

output
```
(-3*b*B*x*(8*b^2 + 25*b*c*x + 15*c^2*x^2) + A*(-8*b^3 + 56*b^2*c*x + 175*b
*c^2*x^2 + 105*c^3*x^3))/(12*b^4*x^(3/2)*(b + c*x)^2) + (5*Sqrt[c]*(-3*b*B
 + 7*A*c)*ArcTan[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/(4*b^(9/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {9, 87, 52, 61, 61, 73, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^3}dx$$

$$\Big\downarrow \ 9$$

$$\int \frac{A+Bx}{x^{5/2}(b+cx)^3}dx$$

$$\Big\downarrow \ 87$$

$$-\frac{(3bB-7Ac)\int \frac{1}{x^{5/2}(b+cx)^2}dx}{4bc}-\frac{bB-Ac}{2bcx^{3/2}(b+cx)^2}$$

$$\Big\downarrow \ 52$$

$$-\frac{(3bB-7Ac)\left(\frac{5\int \frac{1}{x^{5/2}(b+cx)}dx}{2b}+\frac{1}{bx^{3/2}(b+cx)}\right)}{4bc}-\frac{bB-Ac}{2bcx^{3/2}(b+cx)^2}$$

$$\Big\downarrow \ 61$$

$$-\frac{(3bB-7Ac)\left(\frac{5\left(-\frac{c\int \frac{1}{x^{3/2}(b+cx)}dx}{b}-\frac{2}{3bx^{3/2}}\right)}{2b}+\frac{1}{bx^{3/2}(b+cx)}\right)}{4bc}-\frac{bB-Ac}{2bcx^{3/2}(b+cx)^2}$$

$$\Big\downarrow \; 61$$

$$-\frac{(3bB-7Ac)\left(\frac{5\left(-\frac{c\left(-\frac{c\int \frac{1}{\sqrt{x}(b+cx)}dx}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{2b}+\frac{1}{bx^{3/2}(b+cx)}\right)}{4bc}-\frac{bB-Ac}{2bcx^{3/2}(b+cx)^2}$$

$$\Big\downarrow \; 73$$

$$-\frac{(3bB-7Ac)\left(\frac{5\left(-\frac{c\left(-\frac{2c\int \frac{1}{b+cx}d\sqrt{x}}{b}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{2b}+\frac{1}{bx^{3/2}(b+cx)}\right)}{4bc}-\frac{bB-Ac}{2bcx^{3/2}(b+cx)^2}$$

$$\Big\downarrow \; 218$$

$$-\frac{(3bB-7Ac)\left(\frac{5\left(-\frac{c\left(-\frac{2\sqrt{c}\arctan\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}}\right)}{b^{3/2}}-\frac{2}{b\sqrt{x}}\right)}{b}-\frac{2}{3bx^{3/2}}\right)}{2b}+\frac{1}{bx^{3/2}(b+cx)}\right)}{4bc}-\frac{bB-Ac}{2bcx^{3/2}(b+cx)^2}$$

input

```
Int[(Sqrt[x]*(A + B*x))/(b*x + c*x^2)^3,x]
```

output

```
-1/2*(b*B - A*c)/(b*c*x^(3/2)*(b + c*x)^2) - ((3*b*B - 7*A*c)*(1/(b*x^(3/2
)*(b + c*x)) + (5*(-2/(3*b*x^(3/2)) - (c*(-2/(b*Sqrt[x]) - (2*Sqrt[c]*ArcT
an[(Sqrt[c]*Sqrt[x])/Sqrt[b]])/b^(3/2)))/b))/(2*b)))/(4*b*c)
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 52
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && ILtQ[m, -1] && FractionQ[n] && LtQ[n, 0]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.76

method	result	size
risch	$-\frac{2(-9Acx+3Bbx+Ab)}{3b^4x^{\frac{3}{2}}}+\frac{c\left(\frac{2\left(\frac{11}{8}Ac^2-\frac{7}{8}Bbc\right)x^{\frac{3}{2}}+\frac{b(13Ac-9Bb)\sqrt{x}}{4}}{(cx+b)^2}+\frac{5(7Ac-3Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\sqrt{bc}}\right)}{b^4}$	98
derivativedivides	$\frac{2c\left(\frac{\left(\frac{11}{8}Ac^2-\frac{7}{8}Bbc\right)x^{\frac{3}{2}}+\frac{b(13Ac-9Bb)\sqrt{x}}{8}}{(cx+b)^2}+\frac{5(7Ac-3Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^4}-\frac{2A}{3b^3x^{\frac{3}{2}}}-\frac{2(-3Ac+Bb)}{b^4\sqrt{x}}$	101
default	$\frac{2c\left(\frac{\left(\frac{11}{8}Ac^2-\frac{7}{8}Bbc\right)x^{\frac{3}{2}}+\frac{b(13Ac-9Bb)\sqrt{x}}{8}}{(cx+b)^2}+\frac{5(7Ac-3Bb)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{8\sqrt{bc}}\right)}{b^4}-\frac{2A}{3b^3x^{\frac{3}{2}}}-\frac{2(-3Ac+Bb)}{b^4\sqrt{x}}$	101

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x)^3,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-9*A*c*x+3*B*b*x+A*b)/b^4/x^(3/2)+1/b^4*c*(2*((11/8*A*c^2-7/8*B*b*c)
*x^(3/2)+1/8*b*(13*A*c-9*B*b)*x^(1/2))/(c*x+b)^2+5/4*(7*A*c-3*B*b)/(b*c)^(
1/2)*arctan(c*x^(1/2)/(b*c)^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.91

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^3}\,dx$$

$$= \Bigg[-\frac{15\left((3Bbc^2-7Ac^3)x^4+2(3Bb^2c-7Abc^2)x^3+(3Bb^3-7Ab^2c)x^2\right)\sqrt{-\frac{c}{b}}\log\left(\frac{cx+2b\sqrt{x}\sqrt{-\frac{c}{b}}-b}{cx+b}\right)+2}{24(b^4c^2x^4+2b^5cx^3+}$$

$$-\frac{15\left((3Bbc^2-7Ac^3)x^4+2(3Bb^2c-7Abc^2)x^3+(3Bb^3-7Ab^2c)x^2\right)\sqrt{\frac{c}{b}}\arctan\left(\sqrt{x}\sqrt{\frac{c}{b}}\right)+(8Ab^3+}{12(b^4c^2x^4+2b^5cx^3+b^6x^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="fricas")
```

output
```
[-1/24*(15*((3*B*b*c^2 - 7*A*c^3)*x^4 + 2*(3*B*b^2*c - 7*A*b*c^2)*x^3 + (3
*B*b^3 - 7*A*b^2*c)*x^2)*sqrt(-c/b)*log((c*x + 2*b*sqrt(x)*sqrt(-c/b) - b)
/(c*x + b)) + 2*(8*A*b^3 + 15*(3*B*b*c^2 - 7*A*c^3)*x^3 + 25*(3*B*b^2*c -
7*A*b*c^2)*x^2 + 8*(3*B*b^3 - 7*A*b^2*c)*x)*sqrt(x))/(b^4*c^2*x^4 + 2*b^5*
c*x^3 + b^6*x^2), -1/12*(15*((3*B*b*c^2 - 7*A*c^3)*x^4 + 2*(3*B*b^2*c - 7*
A*b*c^2)*x^3 + (3*B*b^3 - 7*A*b^2*c)*x^2)*sqrt(c/b)*arctan(sqrt(x)*sqrt(c/
b)) + (8*A*b^3 + 15*(3*B*b*c^2 - 7*A*c^3)*x^3 + 25*(3*B*b^2*c - 7*A*b*c^2)
*x^2 + 8*(3*B*b^3 - 7*A*b^2*c)*x)*sqrt(x))/(b^4*c^2*x^4 + 2*b^5*c*x^3 + b^
6*x^2)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1703 vs. $2(126) = 252$.

Time = 44.36 (sec) , antiderivative size = 1703, normalized size of antiderivative = 13.20

$$\int \frac{\sqrt{x}(A + Bx)}{(bx + cx^2)^3}\, dx = \text{Too large to display}$$

input
```
integrate(x**(1/2)*(B*x+A)/(c*x**2+b*x)**3,x)
```

output
```
Piecewise((zoo*(-2*A/(9*x**(9/2)) - 2*B/(7*x**(7/2))), Eq(b, 0) & Eq(c, 0)
), ((-2*A/(3*x**(3/2)) - 2*B/sqrt(x))/b**3, Eq(c, 0)), ((-2*A/(9*x**(9/2))
 - 2*B/(7*x**(7/2)))/c**3, Eq(b, 0)), (-16*A*b**3*sqrt(-b/c)/(24*b**6*x**(
3/2)*sqrt(-b/c) + 48*b**5*c*x**(5/2)*sqrt(-b/c) + 24*b**4*c**2*x**(7/2)*sq
rt(-b/c)) + 105*A*b**2*c*x**(3/2)*log(sqrt(x) - sqrt(-b/c))/(24*b**6*x**(3
/2)*sqrt(-b/c) + 48*b**5*c*x**(5/2)*sqrt(-b/c) + 24*b**4*c**2*x**(7/2)*sqr
t(-b/c)) - 105*A*b**2*c*x**(3/2)*log(sqrt(x) + sqrt(-b/c))/(24*b**6*x**(3/
2)*sqrt(-b/c) + 48*b**5*c*x**(5/2)*sqrt(-b/c) + 24*b**4*c**2*x**(7/2)*sqrt
(-b/c)) + 112*A*b**2*c*x*sqrt(-b/c)/(24*b**6*x**(3/2)*sqrt(-b/c) + 48*b**5
*c*x**(5/2)*sqrt(-b/c) + 24*b**4*c**2*x**(7/2)*sqrt(-b/c)) + 210*A*b*c**2*
x**(5/2)*log(sqrt(x) - sqrt(-b/c))/(24*b**6*x**(3/2)*sqrt(-b/c) + 48*b**5*
c*x**(5/2)*sqrt(-b/c) + 24*b**4*c**2*x**(7/2)*sqrt(-b/c)) - 210*A*b*c**2*x
**(5/2)*log(sqrt(x) + sqrt(-b/c))/(24*b**6*x**(3/2)*sqrt(-b/c) + 48*b**5*c
*x**(5/2)*sqrt(-b/c) + 24*b**4*c**2*x**(7/2)*sqrt(-b/c)) + 350*A*b*c**2*x*
*2*sqrt(-b/c)/(24*b**6*x**(3/2)*sqrt(-b/c) + 48*b**5*c*x**(5/2)*sqrt(-b/c)
 + 24*b**4*c**2*x**(7/2)*sqrt(-b/c)) + 105*A*c**3*x**(7/2)*log(sqrt(x) - s
qrt(-b/c))/(24*b**6*x**(3/2)*sqrt(-b/c) + 48*b**5*c*x**(5/2)*sqrt(-b/c) +
24*b**4*c**2*x**(7/2)*sqrt(-b/c)) - 105*A*c**3*x**(7/2)*log(sqrt(x) + sqrt
(-b/c))/(24*b**6*x**(3/2)*sqrt(-b/c) + 48*b**5*c*x**(5/2)*sqrt(-b/c) + 24*
b**4*c**2*x**(7/2)*sqrt(-b/c)) + 210*A*c**3*x**3*sqrt(-b/c)/(24*b**6*x*...
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.99

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^3}\,dx = -\frac{8\,Ab^3+15\,(3\,Bbc^2-7\,Ac^3)x^3+25\,(3\,Bb^2c-7\,Abc^2)x^2+8\,(3\,Bb^3-7\,Ab^2c)x}{12\left(b^4c^2x^{\frac{7}{2}}+2\,b^5cx^{\frac{5}{2}}+b^6x^{\frac{3}{2}}\right)} - \frac{5\,(3\,Bbc-7\,Ac^2)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\,\sqrt{bc}b^4}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="maxima")
```

output
```
-1/12*(8*A*b^3 + 15*(3*B*b*c^2 - 7*A*c^3)*x^3 + 25*(3*B*b^2*c - 7*A*b*c^2)
*x^2 + 8*(3*B*b^3 - 7*A*b^2*c)*x)/(b^4*c^2*x^(7/2) + 2*b^5*c*x^(5/2) + b^6
*x^(3/2)) - 5/4*(3*B*b*c - 7*A*c^2)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)
*b^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.84

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^3}\,dx = -\frac{5\,(3\,Bbc-7\,Ac^2)\arctan\left(\frac{c\sqrt{x}}{\sqrt{bc}}\right)}{4\,\sqrt{bc}b^4} - \frac{2\,(3\,Bbx-9\,Acx+Ab)}{3\,b^4x^{\frac{3}{2}}} - \frac{7\,Bbc^2x^{\frac{3}{2}}-11\,Ac^3x^{\frac{3}{2}}+9\,Bb^2c\sqrt{x}-13\,Abc^2\sqrt{x}}{4\,(cx+b)^2b^4}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x)^3,x, algorithm="giac")
```

output
```
-5/4*(3*B*b*c - 7*A*c^2)*arctan(c*sqrt(x)/sqrt(b*c))/(sqrt(b*c)*b^4) - 2/3
*(3*B*b*x - 9*A*c*x + A*b)/(b^4*x^(3/2)) - 1/4*(7*B*b*c^2*x^(3/2) - 11*A*c
^3*x^(3/2) + 9*B*b^2*c*sqrt(x) - 13*A*b*c^2*sqrt(x))/((c*x + b)^2*b^4)
```

Mupad [B] (verification not implemented)

Time = 5.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^3}\,dx = \frac{\frac{2\,x\,(7\,A\,c-3\,B\,b)}{3\,b^2}-\frac{2\,A}{3\,b}+\frac{5\,c^2\,x^3\,(7\,A\,c-3\,B\,b)}{4\,b^4}+\frac{25\,c\,x^2\,(7\,A\,c-3\,B\,b)}{12\,b^3}}{b^2\,x^{3/2}+c^2\,x^{7/2}+2\,b\,c\,x^{5/2}} + \frac{5\,\sqrt{c}\,\mathrm{atan}\left(\frac{\sqrt{c}\,\sqrt{x}}{\sqrt{b}}\right)\,(7\,A\,c-3\,B\,b)}{4\,b^{9/2}}$$

input
```
int((x^(1/2)*(A + B*x))/(b*x + c*x^2)^3,x)
```

output
```
((2*x*(7*A*c - 3*B*b))/(3*b^2) - (2*A)/(3*b) + (5*c^2*x^3*(7*A*c - 3*B*b))
/(4*b^4) + (25*c*x^2*(7*A*c - 3*B*b))/(12*b^3))/(b^2*x^(3/2) + c^2*x^(7/2)
 + 2*b*c*x^(5/2)) + (5*c^(1/2)*atan((c^(1/2)*x^(1/2))/b^(1/2))*(7*A*c - 3*
B*b))/(4*b^(9/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.02

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^3}\,dx$$
$$= \frac{105\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,b^2cx+210\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)ab\,c^2x^2+105\sqrt{x}\,\sqrt{c}\,\sqrt{b}\,atan\left(\frac{\sqrt{x}\,c}{\sqrt{c}\,\sqrt{b}}\right)a\,c^3}{}$$

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x)^3,x)
```

output
```
(105*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b**2*c*
x + 210*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*a*b*c*
*2*x**2 + 105*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))*
a*c**3*x**3 - 45*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)
))*b**4*x - 90*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b)))
*b**3*c*x**2 - 45*sqrt(x)*sqrt(c)*sqrt(b)*atan((sqrt(x)*c)/(sqrt(c)*sqrt(b
)))*b**2*c**2*x**3 - 8*a*b**4 + 56*a*b**3*c*x + 175*a*b**2*c**2*x**2 + 105
*a*b*c**3*x**3 - 24*b**5*x - 75*b**4*c*x**2 - 45*b**3*c**2*x**3)/(12*sqrt(
x)*b**5*x*(b**2 + 2*b*c*x + c**2*x**2))
```

3.103 $\int x^2(A+Bx)\sqrt{bx+cx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 197

$$\int x^2(A+Bx)\sqrt{bx+cx^2}\,dx = -\frac{b^3(7bB-10Ac)\sqrt{bx+cx^2}}{128c^4} + \frac{b^2(7bB-10Ac)x\sqrt{bx+cx^2}}{192c^3} - \frac{b(7bB-10Ac)x^2\sqrt{bx+cx^2}}{240c^2} - \frac{(7bB-10Ac)x^3\sqrt{bx+cx^2}}{40c} + \frac{Bx^2(bx+cx^2)^{3/2}}{5c} + \frac{b^4(7bB-10Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{128c^{9/2}}$$

output
```
-1/128*b^3*(-10*A*c+7*B*b)*(c*x^2+b*x)^(1/2)/c^4+1/192*b^2*(-10*A*c+7*B*b)
*x*(c*x^2+b*x)^(1/2)/c^3-1/240*b*(-10*A*c+7*B*b)*x^2*(c*x^2+b*x)^(1/2)/c^2
-1/40*(-10*A*c+7*B*b)*x^3*(c*x^2+b*x)^(1/2)/c+1/5*B*x^2*(c*x^2+b*x)^(3/2)/
c+1/128*b^4*(-10*A*c+7*B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.02

$$\int x^2(A+Bx)\sqrt{bx+cx^2}\,dx$$
$$=\frac{\sqrt{x}\sqrt{b+cx}\left(\sqrt{c}\sqrt{x}\sqrt{b+cx}(-105b^4B+16bc^3x^2(5A+3Bx)+96c^4x^3(5A+4Bx)+10b^3c(15A+7Bx)\right.}{1920c^{9/2}\sqrt{x(b+}}$$

input
```
Integrate[x^2*(A + B*x)*Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[x]*Sqrt[b + c*x]*(Sqrt[c]*Sqrt[x]*Sqrt[b + c*x]*(-105*b^4*B + 16*b*c
^3*x^2*(5*A + 3*B*x) + 96*c^4*x^3*(5*A + 4*B*x) + 10*b^3*c*(15*A + 7*B*x)
- 4*b^2*c^2*x*(25*A + 14*B*x)) + 300*A*b^4*c*ArcTanh[(Sqrt[c]*Sqrt[x])/(Sq
rt[b] - Sqrt[b + c*x])] + 210*b^5*B*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] +
Sqrt[b + c*x])]))/(1920*c^(9/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.81, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.273$, Rules used = {1221, 1134, 1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(A+Bx)\sqrt{bx+cx^2}\,dx$$
$$\Big\downarrow \text{ 1221}$$
$$\frac{Bx^2(bx+cx^2)^{3/2}}{5c}-\frac{(7bB-10Ac)\int x^2\sqrt{cx^2+bx}dx}{10c}$$
$$\Big\downarrow \text{ 1134}$$
$$\frac{Bx^2(bx+cx^2)^{3/2}}{5c}-\frac{(7bB-10Ac)\left(\frac{x(bx+cx^2)^{3/2}}{4c}-\frac{5b\int x\sqrt{cx^2+bx}dx}{8c}\right)}{10c}$$

$$\Big\downarrow \text{1160}$$

$$\frac{Bx^2\left(bx+cx^2\right)^{3/2}}{5c}-\frac{(7bB-10Ac)\left(\frac{x\left(bx+cx^2\right)^{3/2}}{4c}-\frac{5b\left(\frac{\left(bx+cx^2\right)^{3/2}}{3c}-\frac{b\int\sqrt{cx^2+bx}dx}{2c}\right)}{8c}\right)}{10c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{Bx^2\left(bx+cx^2\right)^{3/2}}{5c}-\frac{(7bB-10Ac)\left(\frac{x\left(bx+cx^2\right)^{3/2}}{4c}-\frac{5b\left(\frac{\left(bx+cx^2\right)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{2c}\right)}{8c}\right)}{10c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{Bx^2\left(bx+cx^2\right)^{3/2}}{5c}-\frac{(7bB-10Ac)\left(\frac{x\left(bx+cx^2\right)^{3/2}}{4c}-\frac{5b\left(\frac{\left(bx+cx^2\right)^{3/2}}{3c}-\frac{b\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{2c}\right)}{8c}\right)}{10c}$$

$$\Big\downarrow \text{219}$$

$$\frac{Bx^2(bx+cx^2)^{3/2}}{5c} - \frac{(7bB-10Ac)\left(\frac{x(bx+cx^2)^{3/2}}{4c} - \frac{5b\left(\frac{(bx+cx^2)^{3/2}}{3c} - \frac{b\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c} - \frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{2c}\right)}{8c}\right)}{10c}$$

input
```
Int[x^2*(A + B*x)*Sqrt[b*x + c*x^2],x]
```

output
```
(B*x^2*(b*x + c*x^2)^(3/2))/(5*c) - ((7*b*B - 10*A*c)*((x*(b*x + c*x^2)^(3
/2))/(4*c) - (5*b*((b*x + c*x^2)^(3/2)/(3*c) - (b*(((b + 2*c*x)*Sqrt[b*x +
 c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2)))
)/(2*c)))/(8*c)))/(10*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.59

method	result
pseudoelliptic	$-\frac{5\left(\frac{3\left(A\,b^{4}c-\frac{7}{10}b^{5}B\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{2}+\left(-\frac{3\left(\frac{7Bx}{15}+A\right)b^{3}c^{\frac{3}{2}}}{2}+b^{2}x\left(\frac{14Bx}{25}+A\right)c^{\frac{5}{2}}-\frac{4x^{2}\left(\frac{3Bx}{5}+A\right)b\,c^{\frac{7}{2}}}{5}-\frac{24x^{3}\left(\frac{4Bx}{5}+A\right)c}{5}\right.}{96c^{\frac{9}{2}}}$
risch	$\frac{(384B\,c^{4}x^{4}+480A\,c^{4}x^{3}+48Bb\,c^{3}x^{3}+80Ab\,c^{3}x^{2}-56B\,b^{2}c^{2}x^{2}-100A\,b^{2}c^{2}x+70B\,b^{3}cx+150A\,b^{3}c-105B\,b^{4})x(cx+b)}{1920c^{4}\sqrt{x(cx+b)}}-$
default	$A\left(\frac{x(c\,x^{2}+bx)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{(c\,x^{2}+bx)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx}}{4c}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{8c^{\frac{3}{2}}}\right)}{2c}\right)}{8c}\right)+B\left(\frac{x^{2}(c\,x^{2}+bx)^{\frac{3}{2}}}{5c}-\right.$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/96/c^(9/2)*(3/2*(A*b^4*c-7/10*b^5*B)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2
))+(-3/2*(7/15*B*x+A)*b^3*c^(3/2)+b^2*x*(14/25*B*x+A)*c^(5/2)-4/5*x^2*(3/5
*B*x+A)*b*c^(7/2)-24/5*x^3*(4/5*B*x+A)*c^(9/2)+21/20*B*c^(1/2)*b^4)*(x*(c*
x+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.54

$$\int x^2(A+Bx)\sqrt{bx+cx^2}\,dx$$
$$=\left[-\frac{15\,(7\,Bb^5-10\,Ab^4c)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(384\,Bc^5x^4-105\,Bb^4c+150\,Ab^3c^2+48}{3840\,c^5}\right.$$
$$-\frac{15\,(7\,Bb^5-10\,Ab^4c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(384\,Bc^5x^4-105\,Bb^4c+150\,Ab^3c^2+48\,(Bbc^4+10}{1920\,c^5}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/3840*(15*(7*B*b^5 - 10*A*b^4*c)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 +
 b*x)*sqrt(c)) - 2*(384*B*c^5*x^4 - 105*B*b^4*c + 150*A*b^3*c^2 + 48*(B*b*
c^4 + 10*A*c^5)*x^3 - 8*(7*B*b^2*c^3 - 10*A*b*c^4)*x^2 + 10*(7*B*b^3*c^2 -
 10*A*b^2*c^3)*x)*sqrt(c*x^2 + b*x))/c^5, -1/1920*(15*(7*B*b^5 - 10*A*b^4*
c)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) - (384*B*c^5*x^4
- 105*B*b^4*c + 150*A*b^3*c^2 + 48*(B*b*c^4 + 10*A*c^5)*x^3 - 8*(7*B*b^2*c
^3 - 10*A*b*c^4)*x^2 + 10*(7*B*b^3*c^2 - 10*A*b^2*c^3)*x)*sqrt(c*x^2 + b*x
))/c^5]
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.21

$$\int x^2(A+Bx)\sqrt{bx+cx^2}\,dx$$

$$= \begin{cases} -\frac{5b^3\left(Ab-\frac{7b\left(Ac+\frac{Bb}{10}\right)}{8c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{16c^3} + \sqrt{bx+cx^2}\left(\frac{Bx^4}{5}+\frac{5b^2\left(Ab-\frac{7b\left(Ac+\frac{Bb}{10}\right)}{8c}\right)}{8c^3} - \cdots \right. \\ \frac{2\left(\frac{A(bx)^{\frac{7}{2}}}{7}+\frac{B(bx)^{\frac{9}{2}}}{9b}\right)}{b^3} \\ 0 \end{cases}$$

input
```
integrate(x**2*(B*x+A)*(c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((-5*b**3*(A*b - 7*b*(A*c + B*b/10)/(8*c))*Piecewise((log(b + 2*s
qrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)
*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(16*c**3) + sqrt(b*x +
c*x**2)*(B*x**4/5 + 5*b**2*(A*b - 7*b*(A*c + B*b/10)/(8*c))/(8*c**3) - 5*b
*x*(A*b - 7*b*(A*c + B*b/10)/(8*c))/(12*c**2) + x**3*(A*c + B*b/10)/(4*c)
+ x**2*(A*b - 7*b*(A*c + B*b/10)/(8*c))/(3*c)), Ne(c, 0)), (2*(A*(b*x)**(7
/2)/7 + B*(b*x)**(9/2)/(9*b))/b**3, Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.23

$$\int x^2(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{(cx^2+bx)^{\frac{3}{2}}Bx^2}{5\,c} - \frac{7\,\sqrt{cx^2+bx}Bb^3x}{64\,c^3} - \frac{7\,(cx^2+bx)^{\frac{3}{2}}Bbx}{40\,c^2} + \frac{5\,\sqrt{cx^2+bx}Ab^2x}{32\,c^2} + \frac{(cx^2+bx)^{\frac{3}{2}}Ax}{4\,c} + \frac{7\,Bb^5\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{256\,c^{\frac{9}{2}}} - \frac{5\,Ab^4\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{128\,c^{\frac{7}{2}}} - \frac{7\,\sqrt{cx^2+bx}Bb^4}{128\,c^4} + \frac{7\,(cx^2+bx)^{\frac{3}{2}}Bb^2}{48\,c^3} + \frac{5\,\sqrt{cx^2+bx}Ab^3}{64\,c^3} - \frac{5\,(cx^2+bx)^{\frac{3}{2}}Ab}{24\,c^2}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
1/5*(c*x^2 + b*x)^(3/2)*B*x^2/c - 7/64*sqrt(c*x^2 + b*x)*B*b^3*x/c^3 - 7/4
0*(c*x^2 + b*x)^(3/2)*B*b*x/c^2 + 5/32*sqrt(c*x^2 + b*x)*A*b^2*x/c^2 + 1/4
*(c*x^2 + b*x)^(3/2)*A*x/c + 7/256*B*b^5*log(2*c*x + b + 2*sqrt(c*x^2 + b*
x)*sqrt(c))/c^(9/2) - 5/128*A*b^4*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt
(c))/c^(7/2) - 7/128*sqrt(c*x^2 + b*x)*B*b^4/c^4 + 7/48*(c*x^2 + b*x)^(3/2
)*B*b^2/c^3 + 5/64*sqrt(c*x^2 + b*x)*A*b^3/c^3 - 5/24*(c*x^2 + b*x)^(3/2)*
A*b/c^2
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.80

$$\int x^2(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{1}{1920}\,\sqrt{cx^2+bx}\left(2\left(4\left(6\left(8\,Bx+\frac{Bbc^3+10\,Ac^4}{c^4}\right)x-\frac{7\,Bb^2c^2-10\,Abc^3}{c^4}\right)x+\frac{5\,(7\,Bb^3c-10\,Ab^2c^2)}{c^4}\right.\right. - \frac{(7\,Bb^5-10\,Ab^4c)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{256\,c^{\frac{9}{2}}}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(c*x^2 + b*x)*(2*(4*(6*(8*B*x + (B*b*c^3 + 10*A*c^4)/c^4)*x - (
7*B*b^2*c^2 - 10*A*b*c^3)/c^4)*x + 5*(7*B*b^3*c - 10*A*b^2*c^2)/c^4)*x - 1
5*(7*B*b^4 - 10*A*b^3*c)/c^4) - 1/256*(7*B*b^5 - 10*A*b^4*c)*log(abs(2*(sq
rt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(9/2)
```

Mupad [B] (verification not implemented)

Time = 5.67 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.09

$$\int x^2(A+Bx)\sqrt{bx+cx^2}\,dx$$

$$= \frac{A\,x\,(c\,x^2+b\,x)^{3/2}}{4\,c} - \frac{5\,A\,b\left(\frac{b^3\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x}\right)}{16\,c^{5/2}} + \frac{\sqrt{c\,x^2+b\,x}\,\left(-3\,b^2+2\,b\,c\,x+8\,c^2\,x^2\right)}{24\,c^2}\right)}{8\,c} - \frac{7\,B\,b\left(\frac{x\,(c\,x^2+b\,x)^{3/2}}{4\,c} - \frac{5\,b\left(\frac{b^3\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x}\right)}{16\,c^{5/2}} + \frac{\sqrt{c\,x^2+b\,x}\,\left(-3\,b^2+2\,b\,c\,x+8\,c^2\,x^2\right)}{24\,c^2}\right)}{8\,c}\right)}{10\,c} + \frac{B\,x^2\,(c\,x^2+b\,x)^{3/2}}{5\,c}$$

input
```
int(x^2*(b*x + c*x^2)^(1/2)*(A + B*x),x)
```

output
```
(A*x*(b*x + c*x^2)^(3/2))/(4*c) - (5*A*b*((b^3*log((b + 2*c*x)/c^(1/2) + 2
*(b*x + c*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x + c*x^2)^(1/2)*(8*c^2*x^2 - 3*
b^2 + 2*b*c*x))/(24*c^2)))/(8*c) - (7*B*b*((x*(b*x + c*x^2)^(3/2))/(4*c) -
 (5*b*((b^3*log((b + 2*c*x)/c^(1/2) + 2*(b*x + c*x^2)^(1/2)))/(16*c^(5/2))
 + ((b*x + c*x^2)^(1/2)*(8*c^2*x^2 - 3*b^2 + 2*b*c*x))/(24*c^2)))/(8*c)))/
(10*c) + (B*x^2*(b*x + c*x^2)^(3/2))/(5*c)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.09

$$\int x^2(A+Bx)\sqrt{bx+cx^2}\,dx$$

$$= \frac{150\sqrt{x}\,\sqrt{cx+b}\,a\,b^3c^2 - 100\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^3x + 80\sqrt{x}\,\sqrt{cx+b}\,ab\,c^4x^2 + 480\sqrt{x}\,\sqrt{cx+b}\,a\,c^5x^3 - 105}{}$$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x)^(1/2),x)
```

output
```
(150*sqrt(x)*sqrt(b + c*x)*a*b**3*c**2 - 100*sqrt(x)*sqrt(b + c*x)*a*b**2*
c**3*x + 80*sqrt(x)*sqrt(b + c*x)*a*b*c**4*x**2 + 480*sqrt(x)*sqrt(b + c*x
)*a*c**5*x**3 - 105*sqrt(x)*sqrt(b + c*x)*b**5*c + 70*sqrt(x)*sqrt(b + c*x
)*b**4*c**2*x - 56*sqrt(x)*sqrt(b + c*x)*b**3*c**3*x**2 + 48*sqrt(x)*sqrt(
b + c*x)*b**2*c**4*x**3 + 384*sqrt(x)*sqrt(b + c*x)*b*c**5*x**4 - 150*sqrt
(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*b**4*c + 105*sqrt(c)*
log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**6)/(1920*c**5)
```

3.104 $\int x(A+Bx)\sqrt{bx+cx^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 160

$$\int x(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{b^2(5bB-8Ac)\sqrt{bx+cx^2}}{64c^3} - \frac{b(5bB-8Ac)x\sqrt{bx+cx^2}}{96c^2} - \frac{(5bB-8Ac)x^2\sqrt{bx+cx^2}}{24c} + \frac{Bx(bx+cx^2)^{3/2}}{4c} - \frac{b^3(5bB-8Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{7/2}}$$

output
```
1/64*b^2*(-8*A*c+5*B*b)*(c*x^2+b*x)^(1/2)/c^3-1/96*b*(-8*A*c+5*B*b)*x*(c*x
^2+b*x)^(1/2)/c^2-1/24*(-8*A*c+5*B*b)*x^2*(c*x^2+b*x)^(1/2)/c+1/4*B*x*(c*x
^2+b*x)^(3/2)/c-1/64*b^3*(-8*A*c+5*B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2
))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.64 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.86

$$\int x(A+Bx)\sqrt{bx+cx^2}\,dx$$
$$=\frac{\sqrt{x(b+cx)}\left(\sqrt{c}(15b^3B+8bc^2x(2A+Bx)+16c^3x^2(4A+3Bx)-2b^2c(12A+5Bx))+\frac{6b^3(5bB-8Ac)\text{arc}}{\sqrt{x}\sqrt{}}\right.}{192c^{7/2}}$$

input
```
Integrate[x*(A + B*x)*Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[c]*(15*b^3*B + 8*b*c^2*x*(2*A + B*x) + 16*c^3*x^2
*(4*A + 3*B*x) - 2*b^2*c*(12*A + 5*B*x)) + (6*b^3*(5*b*B - 8*A*c)*ArcTanh[
(Sqrt[c]*Sqrt[x])/(Sqrt[b] - Sqrt[b + c*x])])/(Sqrt[x]*Sqrt[b + c*x])))/(1
92*c^(7/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1225, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(A+Bx)\sqrt{bx+cx^2}\,dx$$

$$\Big\downarrow \text{1225}$$

$$\frac{b(5bB-8Ac)\int\sqrt{cx^2+bx}dx}{16c^2}-\frac{(bx+cx^2)^{3/2}(-8Ac+5bB-6Bcx)}{24c^2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{b(5bB-8Ac)\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c^2}-\frac{(bx+cx^2)^{3/2}(-8Ac+5bB-6Bcx)}{24c^2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{b(5bB-8Ac)\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c^2}-\frac{(bx+cx^2)^{3/2}(-8Ac+5bB-6Bcx)}{24c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{b(5bB-8Ac)\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c^2}-\frac{(bx+cx^2)^{3/2}(-8Ac+5bB-6Bcx)}{24c^2}$$

input
```
Int[x*(A + B*x)*Sqrt[b*x + c*x^2],x]
```

output
```
-1/24*((5*b*B - 8*A*c - 6*B*c*x)*(b*x + c*x^2)^(3/2))/c^2 + (b*(5*b*B - 8*
A*c)*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqr
t[b*x + c*x^2]])/(4*c^(3/2))))/(16*c^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.64

method	result
pseudoelliptic	$\frac{\frac{3\left(A\,b^{3}c-\frac{5}{8}B\,b^{4}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{2}+\left(-\frac{3\left(\frac{5Bx}{12}+A\right)b^{2}c^{\frac{3}{2}}}{2}+bx\left(\frac{Bx}{2}+A\right)c^{\frac{5}{2}}+\left(3B\,x^{3}+4A\,x^{2}\right)c^{\frac{7}{2}}+\frac{15B\sqrt{c}\,b^{3}}{16}\right)\sqrt{x(cx+b)}}{12c^{\frac{7}{2}}}$
risch	$-\frac{\left(-48B\,c^{3}x^{3}-64A\,c^{3}x^{2}-8Bb\,c^{2}x^{2}-16Ab\,c^{2}x+10B\,b^{2}cx+24A\,b^{2}c-15B\,b^{3}\right)x(cx+b)}{192c^{3}\sqrt{x(cx+b)}}+\frac{b^{3}(8Ac-5Bb)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x\ldots\right.}{128c^{\frac{7}{2}}}$
default	$A\left(\frac{\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{3c}-\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx}}{4c}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{8c^{\frac{3}{2}}}\right)}{2c}\right)+B\left(\frac{x\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{4c}-\frac{5b\left(\frac{\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{3c}-\ldots\right.}{\ldots}\right.$

input
```
int(x*(B*x+A)*(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/12/c^(7/2)*(3/2*(A*b^3*c-5/8*B*b^4)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))
+(-3/2*(5/12*B*x+A)*b^2*c^(3/2)+b*x*(1/2*B*x+A)*c^(5/2)+(3*B*x^3+4*A*x^2)*
c^(7/2)+15/16*B*c^(1/2)*b^3)*(x*(c*x+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.59

$$\int x(A+Bx)\sqrt{bx+cx^2}\,dx$$
$$=\left[-\frac{3\,(5\,Bb^4-8\,Ab^3c)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(48\,Bc^4x^3+15\,Bb^3c-24\,Ab^2c^2+8\,(Bbc^3}{384\,c^4}\right.$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(3*(5*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*
x)*sqrt(c)) - 2*(48*B*c^4*x^3 + 15*B*b^3*c - 24*A*b^2*c^2 + 8*(B*b*c^3 + 8
*A*c^4)*x^2 - 2*(5*B*b^2*c^2 - 8*A*b*c^3)*x)*sqrt(c*x^2 + b*x))/c^4, 1/192
*(3*(5*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x
+ b)) + (48*B*c^4*x^3 + 15*B*b^3*c - 24*A*b^2*c^2 + 8*(B*b*c^3 + 8*A*c^4)*
x^2 - 2*(5*B*b^2*c^2 - 8*A*b*c^3)*x)*sqrt(c*x^2 + b*x))/c^4]
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.29

$$\int x(A+Bx)\sqrt{bx+cx^2}\,dx$$
$$=\begin{cases}\frac{3b^2\left(Ab-\frac{5b\left(Ac+\frac{Bb}{8}\right)}{6c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for}\ \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{8c^2}+\sqrt{bx+cx^2}\left(\frac{Bx^3}{4}-\frac{3b\left(Ab-\frac{5b\left(Ac+\frac{Bb}{8}\right)}{6c}\right)}{4c^2}+\frac{x^2(A\right.\\ \frac{2\left(\frac{A(bx)^{\frac{5}{2}}}{5}+\frac{B(bx)^{\frac{7}{2}}}{7b}\right)}{b^2}\\ 0\end{cases}$$

input
```
integrate(x*(B*x+A)*(c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((3*b**2*(A*b - 5*b*(A*c + B*b/8)/(6*c))*Piecewise((log(b + 2*sqr
t(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*l
og(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(8*c**2) + sqrt(b*x + c*x
**2)*(B*x**3/4 - 3*b*(A*b - 5*b*(A*c + B*b/8)/(6*c))/(4*c**2) + x**2*(A*c
+ B*b/8)/(3*c) + x*(A*b - 5*b*(A*c + B*b/8)/(6*c))/(2*c)), Ne(c, 0)), (2*(
A*(b*x)**(5/2)/5 + B*(b*x)**(7/2)/(7*b))/b**2, Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.24

$$\int x(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{5\sqrt{cx^2+bx}Bb^2x}{32\,c^2} + \frac{(cx^2+bx)^{\frac{3}{2}}Bx}{4\,c} - \frac{\sqrt{cx^2+bx}Abx}{4\,c} - \frac{5\,Bb^4\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{128\,c^{\frac{7}{2}}} + \frac{Ab^3\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{16\,c^{\frac{5}{2}}} + \frac{5\sqrt{cx^2+bx}Bb^3}{64\,c^3} - \frac{5\,(cx^2+bx)^{\frac{3}{2}}Bb}{24\,c^2} - \frac{\sqrt{cx^2+bx}Ab^2}{8\,c^2} + \frac{(cx^2+bx)^{\frac{3}{2}}A}{3\,c}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
5/32*sqrt(c*x^2 + b*x)*B*b^2*x/c^2 + 1/4*(c*x^2 + b*x)^(3/2)*B*x/c - 1/4*s
qrt(c*x^2 + b*x)*A*b*x/c - 5/128*B*b^4*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)
*sqrt(c))/c^(7/2) + 1/16*A*b^3*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)
)/c^(5/2) + 5/64*sqrt(c*x^2 + b*x)*B*b^3/c^3 - 5/24*(c*x^2 + b*x)^(3/2)*B*
b/c^2 - 1/8*sqrt(c*x^2 + b*x)*A*b^2/c^2 + 1/3*(c*x^2 + b*x)^(3/2)*A/c
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.81

$$\int x(A+Bx)\sqrt{bx+cx^2}\,dx$$
$$=\frac{1}{192}\sqrt{cx^2+bx}\left(2\left(4\left(6\,Bx+\frac{Bbc^2+8\,Ac^3}{c^3}\right)x-\frac{5\,Bb^2c-8\,Abc^2}{c^3}\right)x+\frac{3\,(5\,Bb^3-8\,Ab^2c)}{c^3}\right)$$
$$+\frac{(5\,Bb^4-8\,Ab^3c)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{7}{2}}}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x)*(2*(4*(6*B*x + (B*b*c^2 + 8*A*c^3)/c^3)*x - (5*B*b
^2*c - 8*A*b*c^2)/c^3)*x + 3*(5*B*b^3 - 8*A*b^2*c)/c^3) + 1/128*(5*B*b^4 -
 8*A*b^3*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(7/2
)
```

Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.03

$$\int x(A+Bx)\sqrt{bx+cx^2}\,dx$$
$$=\frac{A\sqrt{c\,x^2+b\,x}\,(-3\,b^2+2\,b\,c\,x+8\,c^2\,x^2)}{24\,c^2}+\frac{B\,x\,(c\,x^2+b\,x)^{3/2}}{4\,c}$$
$$-\frac{5\,B\,b\left(\frac{b^3\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x}\right)}{16\,c^{5/2}}+\frac{\sqrt{c\,x^2+b\,x}\,(-3\,b^2+2\,b\,c\,x+8\,c^2\,x^2)}{24\,c^2}\right)}{8\,c}$$
$$+\frac{A\,b^3\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x}\right)}{16\,c^{5/2}}$$

input
```
int(x*(b*x + c*x^2)^(1/2)*(A + B*x),x)
```

output
```
(A*(b*x + c*x^2)^(1/2)*(8*c^2*x^2 - 3*b^2 + 2*b*c*x))/(24*c^2) + (B*x*(b*x
 + c*x^2)^(3/2))/(4*c) - (5*B*b*((b^3*log((b + 2*c*x)/c^(1/2) + 2*(b*x + c
*x^2)^(1/2)))/(16*c^(5/2)) + ((b*x + c*x^2)^(1/2)*(8*c^2*x^2 - 3*b^2 + 2*b
*c*x))/(24*c^2)))/(8*c) + (A*b^3*log((b + 2*c*x)/c^(1/2) + 2*(b*x + c*x^2)
^(1/2)))/(16*c^(5/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.09

$$\int x(A+Bx)\sqrt{bx+cx^2}\,dx$$

$$=\frac{-24\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^2+16\sqrt{x}\,\sqrt{cx+b}\,ab\,c^3x+64\sqrt{x}\,\sqrt{cx+b}\,a\,c^4x^2+15\sqrt{x}\,\sqrt{cx+b}\,b^4c-10\sqrt{x}\,\sqrt{cx}}{}$$

input
```
int(x*(B*x+A)*(c*x^2+b*x)^(1/2),x)
```

output
```
( - 24*sqrt(x)*sqrt(b + c*x)*a*b**2*c**2 + 16*sqrt(x)*sqrt(b + c*x)*a*b*c*
*3*x + 64*sqrt(x)*sqrt(b + c*x)*a*c**4*x**2 + 15*sqrt(x)*sqrt(b + c*x)*b**
4*c - 10*sqrt(x)*sqrt(b + c*x)*b**3*c**2*x + 8*sqrt(x)*sqrt(b + c*x)*b**2*
c**3*x**2 + 48*sqrt(x)*sqrt(b + c*x)*b*c**4*x**3 + 24*sqrt(c)*log((sqrt(b
+ c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*b**3*c - 15*sqrt(c)*log((sqrt(b + c*x
) + sqrt(x)*sqrt(c))/sqrt(b))*b**5)/(192*c**4)
```

3.105 $\int (A+Bx)\sqrt{bx+cx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 121

$$\int (A+Bx)\sqrt{bx+cx^2}\,dx = -\frac{b(bB-2Ac)\sqrt{bx+cx^2}}{8c^2}-\frac{(bB-2Ac)x\sqrt{bx+cx^2}}{4c}+\frac{B(bx+cx^2)^{3/2}}{3c}+\frac{b^2(bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{5/2}}$$

output
```
-1/8*b*(-2*A*c+B*b)*(c*x^2+b*x)^(1/2)/c^2-1/4*(-2*A*c+B*b)*x*(c*x^2+b*x)^(
1/2)/c+1/3*B*(c*x^2+b*x)^(3/2)/c+1/8*b^2*(-2*A*c+B*b)*arctanh(c^(1/2)*x/(c
*x^2+b*x)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.01

$$\int (A+Bx)\sqrt{bx+cx^2}\,dx = \frac{\sqrt{x(b+cx)}\left(\sqrt{c}\sqrt{x}(-3b^2B+2bc(3A+Bx)+4c^2x(3A+2Bx))+\frac{6b^2(bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}\sqrt{x}}{-\sqrt{b}+\sqrt{b+cx}}\right)}{\sqrt{b+cx}}\right)}{24c^{5/2}\sqrt{x}}$$

input
```
Integrate[(A + B*x)*Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[c]*Sqrt[x]*(-3*b^2*B + 2*b*c*(3*A + B*x) + 4*c^2*
x*(3*A + 2*B*x)) + (6*b^2*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b
] + Sqrt[b + c*x])])/Sqrt[b + c*x]))/(24*c^(5/2)*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.80, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {1160, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)\sqrt{bx+cx^2}\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{B\left(bx+cx^2\right)^{3/2}}{3c}-\frac{(bB-2Ac)\int \sqrt{cx^2+bx}dx}{2c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{B\left(bx+cx^2\right)^{3/2}}{3c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{2c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{B\left(bx+cx^2\right)^{3/2}}{3c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{2c}$$

$$\Big\downarrow \text{219}$$

$$\frac{B\left(bx+cx^2\right)^{3/2}}{3c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{2c}$$

input
```
Int[(A + B*x)*Sqrt[b*x + c*x^2],x]
```

output
```
(B*(b*x + c*x^2)^(3/2))/(3*c) - ((b*B - 2*A*c)*(((b + 2*c*x)*Sqrt[b*x + c*
x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))/(
2*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{\left(-\frac{1}{2} A\, b^{2} c+\frac{1}{4} B\, b^{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{x (c x+b)}}{x \sqrt{c}}\right)+\left(\frac{\left(\frac{B x}{3}+A\right) b\, c^{\frac{3}{2}}}{2}+x \left(\frac{2 B x}{3}+A\right) c^{\frac{5}{2}}-\frac{B \sqrt{c}\, b^{2}}{4}\right) \sqrt{x (c x+b)}}{2 c^{\frac{5}{2}}}$
risch	$\frac{\left(8 B\, c^{2} x^{2}+12 A\, c^{2} x+2 B b c x+6 A b c-3 B\, b^{2}\right) x (c x+b)}{24 c^{2} \sqrt{x (c x+b)}}-\frac{b^{2} (2 A c-B b) \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\, x^{2}+b x}\right)}{16 c^{\frac{5}{2}}}$
default	$A \left(\frac{(2 c x+b) \sqrt{c\, x^{2}+b x}}{4 c}-\frac{b^{2} \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\, x^{2}+b x}\right)}{8 c^{\frac{3}{2}}}\right)+B \left(\frac{\left(c\, x^{2}+b x\right)^{\frac{3}{2}}}{3 c}-\frac{b \left(\frac{(2 c x+b) \sqrt{c\, x^{2}+b x}}{4 c}-\frac{b^{2} \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{\right.}\right.}{8 c^{\frac{3}{2}}}\right.}{2 c}\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/c^(5/2)*((-1/2*A*b^2*c+1/4*B*b^3)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))
+(1/2*(1/3*B*x+A)*b*c^(3/2)+x*(2/3*B*x+A)*c^(5/2)-1/4*B*c^(1/2)*b^2)*(x*(c
*x+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.69

$$\int (A+Bx)\sqrt{bx+cx^2}\,dx$$
$$= \left[-\frac{3\,(Bb^3-2\,Ab^2c)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(8\,Bc^3x^2-3\,Bb^2c+6\,Abc^2+2\,(Bbc^2+6\,A}{48\,c^3}\right.$$
$$-\frac{3\,(Bb^3-2\,Ab^2c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(8\,Bc^3x^2-3\,Bb^2c+6\,Abc^2+2\,(Bbc^2+6\,Ac^3)x)\sqrt{cx^2}}{24\,c^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(B*b^3 - 2*A*b^2*c)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*
sqrt(c)) - 2*(8*B*c^3*x^2 - 3*B*b^2*c + 6*A*b*c^2 + 2*(B*b*c^2 + 6*A*c^3)*
x)*sqrt(c*x^2 + b*x))/c^3, -1/24*(3*(B*b^3 - 2*A*b^2*c)*sqrt(-c)*arctan(sq
rt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) - (8*B*c^3*x^2 - 3*B*b^2*c + 6*A*b*c^2
 + 2*(B*b*c^2 + 6*A*c^3)*x)*sqrt(c*x^2 + b*x))/c^3]
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.38

$$\int (A+Bx)\sqrt{bx+cx^2}\,dx$$
$$= \begin{cases} -\frac{b\left(Ab-\frac{3b\left(Ac+\frac{Bb}{6}\right)}{4c}\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{2c} + \sqrt{bx+cx^2}\left(\frac{Bx^2}{3}+\frac{x\left(Ac+\frac{Bb}{6}\right)}{2c}+\frac{Ab-\frac{3b\left(Ac+\frac{Bb}{6}\right)}{4c}}{c}\right. \\ \frac{2\left(\frac{A(bx)^{\frac{3}{2}}}{3}+\frac{B(bx)^{\frac{5}{2}}}{5b}\right)}{b} \\ 0 \end{cases}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((-b*(A*b - 3*b*(A*c + B*b/6)/(4*c))*Piecewise((log(b + 2*sqrt(c)
*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b
/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(2*c) + sqrt(b*x + c*x**2)*(B
*x**2/3 + x*(A*c + B*b/6)/(2*c) + (A*b - 3*b*(A*c + B*b/6)/(4*c))/c), Ne(c
, 0)), (2*(A*(b*x)**(3/2)/3 + B*(b*x)**(5/2)/(5*b))/b, Ne(b, 0)), (0, True
))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.27

$$\int (A+Bx)\sqrt{bx+cx^2}\,dx = \frac{1}{2}\sqrt{cx^2+bx}Ax - \frac{\sqrt{cx^2+bx}Bbx}{4\,c} + \frac{Bb^3\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{16\,c^{\frac{5}{2}}} - \frac{Ab^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,c^{\frac{3}{2}}} - \frac{\sqrt{cx^2+bx}Bb^2}{8\,c^2} + \frac{(cx^2+bx)^{\frac{3}{2}}B}{3\,c} + \frac{\sqrt{cx^2+bx}Ab}{4\,c}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^2 + b*x)*A*x - 1/4*sqrt(c*x^2 + b*x)*B*b*x/c + 1/16*B*b^3*log
(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(5/2) - 1/8*A*b^2*log(2*c*x +
b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(3/2) - 1/8*sqrt(c*x^2 + b*x)*B*b^2/c^2
 + 1/3*(c*x^2 + b*x)^(3/2)*B/c + 1/4*sqrt(c*x^2 + b*x)*A*b/c
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.83

$$\int (A+Bx)\sqrt{bx+cx^2}\,dx = \frac{1}{24}\sqrt{cx^2+bx}\left(2\left(4\,Bx+\frac{Bbc+6\,Ac^2}{c^2}\right)x - \frac{3\,(Bb^2-2\,Abc)}{c^2}\right) - \frac{(Bb^3-2\,Ab^2c)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x)*(2*(4*B*x + (B*b*c + 6*A*c^2)/c^2)*x - 3*(B*b^2 - 2
*A*b*c)/c^2) - 1/16*(B*b^3 - 2*A*b^2*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2
+ b*x))*sqrt(c) + b))/c^(5/2)
```

Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.05

$$\int (A+Bx)\sqrt{bx+cx^2}\,dx = A\sqrt{c\,x^2+b\,x}\left(\frac{x}{2}+\frac{b}{4\,c}\right)+\frac{B\,b^3\,\ln\left(\frac{b+2\,c\,x}{\sqrt{c}}+2\,\sqrt{c\,x^2+b\,x}\right)}{16\,c^{5/2}} + \frac{B\,\sqrt{c\,x^2+b\,x}\,\left(-3\,b^2+2\,b\,c\,x+8\,c^2\,x^2\right)}{24\,c^2} - \frac{A\,b^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}$$

input
```
int((b*x + c*x^2)^(1/2)*(A + B*x),x)
```

output
```
A*(b*x + c*x^2)^(1/2)*(x/2 + b/(4*c)) + (B*b^3*log((b + 2*c*x)/c^(1/2) + 2
*(b*x + c*x^2)^(1/2)))/(16*c^(5/2)) + (B*(b*x + c*x^2)^(1/2)*(8*c^2*x^2 -
3*b^2 + 2*b*c*x))/(24*c^2) - (A*b^2*log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2
)^(1/2)))/(8*c^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.12

$$\int (A+Bx)\sqrt{bx+cx^2}\,dx = \frac{6\sqrt{x}\,\sqrt{cx+b}\,ab\,c^2+12\sqrt{x}\,\sqrt{cx+b}\,a\,c^3x-3\sqrt{x}\,\sqrt{cx+b}\,b^3c+2\sqrt{x}\,\sqrt{cx+b}\,b^2c^2x+8\sqrt{x}\,\sqrt{cx+b}\,b\,c^3x}{24c^3}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2),x)
```

output
```
(6*sqrt(x)*sqrt(b + c*x)*a*b*c**2 + 12*sqrt(x)*sqrt(b + c*x)*a*c**3*x - 3*
sqrt(x)*sqrt(b + c*x)*b**3*c + 2*sqrt(x)*sqrt(b + c*x)*b**2*c**2*x + 8*sqr
t(x)*sqrt(b + c*x)*b*c**3*x**2 - 6*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sq
rt(c))/sqrt(b))*a*b**2*c + 3*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))
/sqrt(b))*b**4)/(24*c**3)
```

3.106 $\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 92

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x}\,dx = -\frac{(bB-4Ac)\sqrt{bx+cx^2}}{4c} + \frac{B(bx+cx^2)^{3/2}}{2cx} - \frac{b(bB-4Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}$$

output
```
-1/4*(-4*A*c+B*b)*(c*x^2+b*x)^(1/2)/c+1/2*B*(c*x^2+b*x)^(3/2)/c/x-1/4*b*(-
4*A*c+B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x}\,dx = \frac{\sqrt{x(b+cx)}\left(\sqrt{c}(bB+4Ac+2Bcx) + \frac{b(bB-4Ac)\log\left(-\sqrt{c}\sqrt{x}+\sqrt{b+cx}\right)}{\sqrt{x}\sqrt{b+cx}}\right)}{4c^{3/2}}$$

input
```
Integrate[((A + B*x)*Sqrt[b*x + c*x^2])/x,x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[c]*(b*B + 4*A*c + 2*B*c*x) + (b*(b*B - 4*A*c)*Log
[-(Sqrt[c]*Sqrt[x]) + Sqrt[b + c*x]])/(Sqrt[x]*Sqrt[b + c*x])))/(4*c^(3/2)
)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1221, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{B\left(bx+cx^2\right)^{3/2}}{2cx}-\frac{(bB-4Ac)\int \frac{\sqrt{cx^2+bx}}{x}dx}{4c}$$

$$\Big\downarrow \text{1131}$$

$$\frac{B\left(bx+cx^2\right)^{3/2}}{2cx}-\frac{(bB-4Ac)\left(\frac{1}{2}b\int \frac{1}{\sqrt{cx^2+bx}}dx+\sqrt{bx+cx^2}\right)}{4c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{B\left(bx+cx^2\right)^{3/2}}{2cx}-\frac{(bB-4Ac)\left(b\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}+\sqrt{bx+cx^2}\right)}{4c}$$

$$\Big\downarrow \text{219}$$

$$\frac{B\left(bx+cx^2\right)^{3/2}}{2cx}-\frac{(bB-4Ac)\left(\frac{b\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}+\sqrt{bx+cx^2}\right)}{4c}$$

input
```
Int[((A + B*x)*Sqrt[b*x + c*x^2])/x,x]
```

output
```
(B*(b*x + c*x^2)^(3/2))/(2*c*x) - ((b*B - 4*A*c)*(Sqrt[b*x + c*x^2] + (b*A
rcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c]))/(4*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.66

method	result	siz
pseudoelliptic	$\frac{b\left(Ac-\frac{Bb}{4}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)+\left(\left(\frac{Bx}{2}+A\right)c^{\frac{3}{2}}+\frac{B\sqrt{c}\,b}{4}\right)\sqrt{x(cx+b)}}{c^{\frac{3}{2}}}$	61
risch	$\frac{(2Bcx+4Ac+Bb)x(cx+b)}{4c\sqrt{x(cx+b)}}+\frac{b(4Ac-Bb)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{8c^{\frac{3}{2}}}$	74
default	$B\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx}}{4c}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{8c^{\frac{3}{2}}}\right)+A\left(\sqrt{c\,x^{2}+bx}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{2\sqrt{c}}\right)$	10

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
1/c^(3/2)*(b*(A*c-1/4*B*b)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))+((1/2*B*x+
A)*c^(3/2)+1/4*B*c^(1/2)*b)*(x*(c*x+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.67

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x}\,dx$$

$$= \left[-\frac{(Bb^2-4\,Abc)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\left(2\,Bc^2x+Bbc+4\,Ac^2\right)\sqrt{cx^2+bx}}{8\,c^2}, \frac{(Bb^2-4\,A}{}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x,x, algorithm="fricas")
```

output
```
[-1/8*((B*b^2 - 4*A*b*c)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(
c)) - 2*(2*B*c^2*x + B*b*c + 4*A*c^2)*sqrt(c*x^2 + b*x))/c^2, 1/4*((B*b^2
- 4*A*b*c)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (2*B*c^
2*x + B*b*c + 4*A*c^2)*sqrt(c*x^2 + b*x))/c^2]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x}\,dx = \int \frac{\sqrt{x(b+cx)}(A+Bx)}{x}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2)/x,x)
```

output
```
Integral(sqrt(x*(b + c*x))*(A + B*x)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x}\,dx = \frac{1}{2}\sqrt{cx^2+bx}Bx - \frac{Bb^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,c^{\frac{3}{2}}} + \frac{Ab\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{2\,\sqrt{c}} + \sqrt{cx^2+bx}A + \frac{\sqrt{cx^2+bx}Bb}{4\,c}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x,x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^2 + b*x)*B*x - 1/8*B*b^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*
sqrt(c))/c^(3/2) + 1/2*A*b*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/sq
rt(c) + sqrt(c*x^2 + b*x)*A + 1/4*sqrt(c*x^2 + b*x)*B*b/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x}\,dx = \frac{1}{4}\sqrt{cx^2+bx}\left(2\,Bx+\frac{Bb+4\,Ac}{c}\right) + \frac{(Bb^2-4\,Abc)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x,x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x)*(2*B*x + (B*b + 4*A*c)/c) + 1/8*(B*b^2 - 4*A*b*c)*lo
g(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.48 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x}\,dx = A\sqrt{c\,x^2+b\,x}+B\sqrt{c\,x^2+b\,x}\left(\frac{x}{2}+\frac{b}{4\,c}\right) - \frac{B\,b^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}} + \frac{A\,b\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{2\,\sqrt{c}}$$

input
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x,x)
```

output
```
A*(b*x + c*x^2)^(1/2) + B*(b*x + c*x^2)^(1/2)*(x/2 + b/(4*c)) - (B*b^2*log
((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/(8*c^(3/2)) + (A*b*log((b/2 +
 c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/(2*c^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x}\,dx$$
$$= \frac{4\sqrt{x}\,\sqrt{cx+b}\,a\,c^2+\sqrt{x}\,\sqrt{cx+b}\,b^2c+2\sqrt{x}\,\sqrt{cx+b}\,b\,c^2x+4\sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)abc-\sqrt{c}\log\left(\frac{\sqrt{cx+b}+}{\sqrt{b}}\right.}{4c^2}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x,x)
```

output
```
(4*sqrt(x)*sqrt(b + c*x)*a*c**2 + sqrt(x)*sqrt(b + c*x)*b**2*c + 2*sqrt(x)
*sqrt(b + c*x)*b*c**2*x + 4*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/
sqrt(b))*a*b*c - sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b*
*3)/(4*c**2)
```

3.107 $\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 70

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^2}\,dx = B\sqrt{bx+cx^2} - \frac{2A\sqrt{bx+cx^2}}{x} + \frac{(bB+2Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}$$

output
```
B*(c*x^2+b*x)^(1/2)-2*A*(c*x^2+b*x)^(1/2)/x+(2*A*c+B*b)*arctanh(c^(1/2)*x/
(c*x^2+b*x)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^2}\,dx = \frac{\sqrt{x(b+cx)}\left(-2A+Bx+\frac{2(bB+2Ac)\sqrt{x}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{x}}{-\sqrt{b}+\sqrt{b+cx}}\right)}{\sqrt{c}\sqrt{b+cx}}\right)}{x}$$

input
```
Integrate[((A + B*x)*Sqrt[b*x + c*x^2])/x^2,x]
```

output
```
(Sqrt[x*(b + c*x)]*(-2*A + B*x + (2*(b*B + 2*A*c)*Sqrt[x]*ArcTanh[(Sqrt[c]
*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])])/(Sqrt[c]*Sqrt[b + c*x])))/x
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1220, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^2}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(2Ac+bB)\int \frac{\sqrt{cx^2+bx}}{x}dx}{b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{bx^2}$$

$$\Big\downarrow \text{1131}$$

$$\frac{(2Ac+bB)\left(\frac{1}{2}b\int \frac{1}{\sqrt{cx^2+bx}}dx+\sqrt{bx+cx^2}\right)}{b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{bx^2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(2Ac+bB)\left(b\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}+\sqrt{bx+cx^2}\right)}{b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{bx^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{(2Ac+bB)\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}+\sqrt{bx+cx^2}\right)}{b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{bx^2}$$

input
```
Int[((A + B*x)*Sqrt[b*x + c*x^2])/x^2,x]
```

output
```
(-2*A*(b*x + c*x^2)^(3/2))/(b*x^2) + ((b*B + 2*A*c)*(Sqrt[b*x + c*x^2] + (
b*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c]))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$-\frac{2\left(-x\left(Ac+\frac{Bb}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)+\left(-\frac{Bx}{2}+A\right)\sqrt{c}\,\sqrt{x(cx+b)}\right)}{\sqrt{c}\,x}$
risch	$-\frac{(cx+b)(-Bx+2A)}{\sqrt{x(cx+b)}}+\frac{\left(Ac+\frac{Bb}{2}\right)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{\sqrt{c}}$
default	$A\left(-\frac{2(c\,x^2+bx)^{\frac{3}{2}}}{b\,x^2}+\frac{2c\left(\sqrt{c\,x^2+bx}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{2\sqrt{c}}\right)}{b}\right)+B\left(\sqrt{c\,x^2+bx}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+b\right.}{2\sqrt{c}}\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2/c^(1/2)*(-x*(A*c+1/2*B*b)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))+(-1/2*B*
x+A)*c^(1/2)*(x*(c*x+b))^(1/2))/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.99

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^2}\,dx = \left[\frac{(Bb+2\,Ac)\sqrt{c}x\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,(Bcx-2\,Ac)\sqrt{cx^2+bx}}{2\,cx}, -\frac{(Bb+2\,Ac)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(Bcx-2\,Ac)\sqrt{cx^2+bx}}{cx}\right]$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/2*((B*b + 2*A*c)*sqrt(c)*x*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))
 + 2*(B*c*x - 2*A*c)*sqrt(c*x^2 + b*x))/(c*x), -((B*b + 2*A*c)*sqrt(-c)*x*
arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) - (B*c*x - 2*A*c)*sqrt(c*x^2
+ b*x))/(c*x)]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^2}\,dx = \int \frac{\sqrt{x\,(b+cx)}(A+Bx)}{x^2}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(x*(b + c*x))*(A + B*x)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^2}\,dx = \frac{Bb\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{2\,\sqrt{c}} + A\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right) + \sqrt{cx^2+bx}B - \frac{2\,\sqrt{cx^2+bx}A}{x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
1/2*B*b*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/sqrt(c) + A*sqrt(c)*l
og(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + sqrt(c*x^2 + b*x)*B - 2*sqrt
(c*x^2 + b*x)*A/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^2}\,dx = \sqrt{cx^2+bx}B - \frac{(Bb+2\,Ac)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{2\,\sqrt{c}} + \frac{2\,Ab}{\sqrt{c}x-\sqrt{cx^2+bx}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^2,x, algorithm="giac")
```

output
```
sqrt(c*x^2 + b*x)*B - 1/2*(B*b + 2*A*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2
+ b*x))*sqrt(c) + b))/sqrt(c) + 2*A*b/(sqrt(c)*x - sqrt(c*x^2 + b*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^2}\,dx = \int \frac{\sqrt{c\,x^2+b\,x}\,(A+B\,x)}{x^2}\,dx$$

input
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^2,x)
```

output
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^2}\,dx = \frac{-8\sqrt{x}\,\sqrt{cx+b}\,ac+4\sqrt{x}\,\sqrt{cx+b}\,bcx+8\sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)acx+4\sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^2x-8\sqrt{c}}{4cx}$$

input

```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^2,x)
```

output

```
( - 8*sqrt(x)*sqrt(b + c*x)*a*c + 4*sqrt(x)*sqrt(b + c*x)*b*c*x + 8*sqrt(c
)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*c*x + 4*sqrt(c)*log((sq
rt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**2*x - 8*sqrt(c)*a*c*x - sqrt(c)
*b**2*x)/(4*c*x)
```

3.108 $\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 73

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^3}\,dx = -\frac{2B\sqrt{bx+cx^2}}{x} - \frac{2A(bx+cx^2)^{3/2}}{3bx^3} + 2B\sqrt{c}\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)$$

output
```
-2*B*(c*x^2+b*x)^(1/2)/x-2/3*A*(c*x^2+b*x)^(3/2)/b/x^3+2*B*c^(1/2)*arctanh
(c^(1/2)*x/(c*x^2+b*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^3}\,dx = -\frac{2\sqrt{x(b+cx)}(Ab+3bBx+Acx)}{3bx^2} - \frac{2B\sqrt{c}\sqrt{x(b+cx)}\log\left(-\sqrt{c}\sqrt{x}+\sqrt{b+cx}\right)}{\sqrt{x}\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*Sqrt[b*x + c*x^2])/x^3,x]
```

output
```
(-2*Sqrt[x*(b + c*x)]*(A*b + 3*b*B*x + A*c*x))/(3*b*x^2) - (2*B*Sqrt[c]*Sq
rt[x*(b + c*x)]*Log[-(Sqrt[c]*Sqrt[x]) + Sqrt[b + c*x]])/(Sqrt[x]*Sqrt[b +
 c*x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1220, 1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^3}dx$$

$$\Big\downarrow \text{1220}$$

$$B\int \frac{\sqrt{cx^2+bx}}{x^2}dx-\frac{2A\left(bx+cx^2\right)^{3/2}}{3bx^3}$$

$$\Big\downarrow \text{1125}$$

$$B\left(-\int -\frac{c}{\sqrt{cx^2+bx}}dx-\frac{2\sqrt{bx+cx^2}}{x}\right)-\frac{2A\left(bx+cx^2\right)^{3/2}}{3bx^3}$$

$$\Big\downarrow \text{25}$$

$$B\left(\int \frac{c}{\sqrt{cx^2+bx}}dx-\frac{2\sqrt{bx+cx^2}}{x}\right)-\frac{2A\left(bx+cx^2\right)^{3/2}}{3bx^3}$$

$$\Big\downarrow \text{27}$$

$$B\left(c\int \frac{1}{\sqrt{cx^2+bx}}dx-\frac{2\sqrt{bx+cx^2}}{x}\right)-\frac{2A\left(bx+cx^2\right)^{3/2}}{3bx^3}$$

$$\Big\downarrow \text{1091}$$

$$B\left(2c\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}-\frac{2\sqrt{bx+cx^2}}{x}\right)-\frac{2A\left(bx+cx^2\right)^{3/2}}{3bx^3}$$

$$\Big\downarrow \text{219}$$

$$B\left(2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)-\frac{2\sqrt{bx+cx^2}}{x}\right)-\frac{2A(bx+cx^2)^{3/2}}{3bx^3}$$

input
```
Int[((A + B*x)*Sqrt[b*x + c*x^2])/x^3,x]
```

output
```
(-2*A*(b*x + c*x^2)^(3/2))/(3*b*x^3) + B*((-2*Sqrt[b*x + c*x^2])/x + 2*Sqr
t[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{6Bb\sqrt{c}\,\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)x^{2}-2\sqrt{x(cx+b)}\,((3Bx+A)b+Acx)}{3b\,x^{2}}$	61
risch	$-\frac{2(cx+b)(Acx+3Bbx+Ab)}{3x\sqrt{x(cx+b)}\,b}+B\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)$	66
default	$-\frac{2A(c\,x^{2}+bx)^{\frac{3}{2}}}{3b\,x^{3}}+B\left(-\frac{2(c\,x^{2}+bx)^{\frac{3}{2}}}{b\,x^{2}}+\frac{2c\left(\sqrt{c\,x^{2}+bx}+\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{2\sqrt{c}}\right)}{b}\right)$	92

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(6*B*b*c^(1/2)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))*x^2-2*(x*(c*x+b))^
(1/2)*((3*B*x+A)*b+A*c*x))/b/x^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.95

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^3}\,dx = \left[\frac{3\,Bb\sqrt{c}x^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,\sqrt{cx^2+bx}(Ab+(3\,Bb+Ac)x)}{3\,bx^2},\ -\frac{2\left(3\,Bb\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)+\sqrt{cx^2+bx}(Ab+(3\,Bb+Ac)x)\right)}{3\,bx^2}\right]$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/3*(3*B*b*sqrt(c)*x^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*s
qrt(c*x^2 + b*x)*(A*b + (3*B*b + A*c)*x))/(b*x^2), -2/3*(3*B*b*sqrt(-c)*x^
2*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + sqrt(c*x^2 + b*x)*(A*b +
(3*B*b + A*c)*x))/(b*x^2)]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^3}\,dx = \int \frac{\sqrt{x\,(b+cx)}(A+Bx)}{x^3}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(x*(b + c*x))*(A + B*x)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^3}\,dx = \left(\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-\frac{2\,\sqrt{cx^2+bx}}{x}\right)B - \frac{2}{3}\,A\left(\frac{\sqrt{cx^2+bx}c}{bx}+\frac{\sqrt{cx^2+bx}}{x^2}\right)$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
(sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*sqrt(c*x^2 + b*x
)/x)*B - 2/3*A*(sqrt(c*x^2 + b*x)*c/(b*x) + sqrt(c*x^2 + b*x)/x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(61) = 122.

Time = 0.12 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.92

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^3}\,dx = -B\sqrt{c}\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right) + \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2Bb+3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2Ac+3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)Ab\sqrt{c}+Ab^2\right)}{3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^3,x, algorithm="giac")
```

output
```
-B*sqrt(c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b)) + 2/3*(
3*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*b + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x)
)^2*A*c + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x))*A*b*sqrt(c) + A*b^2)/(sqrt(c)*
x - sqrt(c*x^2 + b*x))^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^3}\,dx = \int \frac{\sqrt{c\,x^2+b\,x}\,(A+B\,x)}{x^3}\,dx$$

input
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^3,x)
```

output
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.29

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^3}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,ab}{3}-\frac{2\sqrt{x}\,\sqrt{cx+b}\,acx}{3}-2\sqrt{x}\,\sqrt{cx+b}\,b^2x+2\sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^2x^2-\frac{2\sqrt{c}\,ac\,x^2}{3}+\frac{2\sqrt{c}\,b^2x^2}{3}}{b\,x^2}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^3,x)
```

output
```
(2*( - sqrt(x)*sqrt(b + c*x)*a*b - sqrt(x)*sqrt(b + c*x)*a*c*x - 3*sqrt(x)
*sqrt(b + c*x)*b**2*x + 3*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sq
rt(b))*b**2*x**2 - sqrt(c)*a*c*x**2 + sqrt(c)*b**2*x**2))/(3*b*x**2)
```

3.109 $\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^4}\,dx = -\frac{2A(bx+cx^2)^{3/2}}{5bx^4} - \frac{2(5bB-2Ac)\,(bx+cx^2)^{3/2}}{15b^2x^3}$$

output
```
-2/5*A*(c*x^2+b*x)^(3/2)/b/x^4-2/15*(-2*A*c+5*B*b)*(c*x^2+b*x)^(3/2)/b^2/x
^3
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^4}\,dx = -\frac{2(x(b+cx))^{3/2}(3Ab+5bBx-2Acx)}{15b^2x^4}$$

input
```
Integrate[((A + B*x)*Sqrt[b*x + c*x^2])/x^4,x]
```

output
```
(-2*(x*(b + c*x))^(3/2)*(3*A*b + 5*b*B*x - 2*A*c*x))/(15*b^2*x^4)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1220, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^4}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(5bB-2Ac)\int \frac{\sqrt{cx^2+bx}}{x^3}dx}{5b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{5bx^4}$$

$$\Big\downarrow \text{1123}$$

$$-\frac{2\left(bx+cx^2\right)^{3/2}(5bB-2Ac)}{15b^2x^3}-\frac{2A\left(bx+cx^2\right)^{3/2}}{5bx^4}$$

input
```
Int[((A + B*x)*Sqrt[b*x + c*x^2])/x^4,x]
```

output
```
(-2*A*(b*x + c*x^2)^(3/2))/(5*b*x^4) - (2*(5*b*B - 2*A*c)*(b*x + c*x^2)^(3
/2))/(15*b^2*x^3)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.65

method	result	size
pseudoelliptic	$-\frac{2(cx+b)\sqrt{x(cx+b)}\left(\left(\frac{5Bx}{3}+A\right)b-\frac{2Acx}{3}\right)}{5x^{3}b^{2}}$	37
gosper	$-\frac{2(cx+b)(-2Acx+5Bbx+3Ab)\sqrt{c\,x^{2}+bx}}{15b^{2}x^{3}}$	40
orering	$-\frac{2(cx+b)(-2Acx+5Bbx+3Ab)\sqrt{c\,x^{2}+bx}}{15b^{2}x^{3}}$	40
trager	$-\frac{2\left(-2A\,c^{2}x^{2}+5x^{2}Bbc+Abcx+5xB\,b^{2}+3b^{2}A\right)\sqrt{c\,x^{2}+bx}}{15b^{2}x^{3}}$	56
risch	$-\frac{2(cx+b)\left(-2A\,c^{2}x^{2}+5x^{2}Bbc+Abcx+5xB\,b^{2}+3b^{2}A\right)}{15x^{2}\sqrt{x(cx+b)}\,b^{2}}$	59
default	$A\left(-\frac{2\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{5b\,x^{4}}+\frac{4c\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{15b^{2}x^{3}}\right)-\frac{2B\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{3b\,x^{3}}$	64

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/5*(c*x+b)*(x*(c*x+b))^(1/2)*((5/3*B*x+A)*b-2/3*A*c*x)/x^3/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^4}\,dx = -\frac{2\,(3\,Ab^2+(5\,Bbc-2\,Ac^2)x^2+(5\,Bb^2+Abc)x)\sqrt{cx^2+bx}}{15\,b^2x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
-2/15*(3*A*b^2 + (5*B*b*c - 2*A*c^2)*x^2 + (5*B*b^2 + A*b*c)*x)*sqrt(c*x^2
 + b*x)/(b^2*x^3)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^4}\,dx = \int \frac{\sqrt{x\,(b+cx)}(A+Bx)}{x^4}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(x*(b + c*x))*(A + B*x)/x**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(49) = 98.

Time = 0.03 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^4}\,dx = -\frac{2\,\sqrt{cx^2+bx}Bc}{3\,bx}+\frac{4\,\sqrt{cx^2+bx}Ac^2}{15\,b^2x} -\frac{2\,\sqrt{cx^2+bx}B}{3\,x^2}-\frac{2\,\sqrt{cx^2+bx}Ac}{15\,bx^2}-\frac{2\,\sqrt{cx^2+bx}A}{5\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
-2/3*sqrt(c*x^2 + b*x)*B*c/(b*x) + 4/15*sqrt(c*x^2 + b*x)*A*c^2/(b^2*x) -
2/3*sqrt(c*x^2 + b*x)*B/x^2 - 2/15*sqrt(c*x^2 + b*x)*A*c/(b*x^2) - 2/5*sqr
t(c*x^2 + b*x)*A/x^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(49) = 98$.

Time = 0.13 (sec) , antiderivative size = 191, normalized size of antiderivative = 3.35

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^4}\,dx = \frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4Bc+15\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3Bb\sqrt{c}+15\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3Ac^{\frac{3}{2}}+5\left(\sqrt{c}x\right.\right.}{15\left(\sqrt{c}x-\sqrt{cx^2\right.}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^4,x, algorithm="giac")
```

output
```
2/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*B*c + 15*(sqrt(c)*x - sqrt(c*x^
2 + b*x))^3*B*b*sqrt(c) + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*A*c^(3/2) +
 5*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*b^2 + 25*(sqrt(c)*x - sqrt(c*x^2 +
b*x))^2*A*b*c + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x))*A*b^2*sqrt(c) + 3*A*b^3
)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^5
```

Mupad [B] (verification not implemented)

Time = 5.57 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.75

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^4}\,dx = \frac{4\,A\,c^2\,\sqrt{c\,x^2+b\,x}}{15\,b^2\,x} - \frac{2\,B\,\sqrt{c\,x^2+b\,x}}{3\,x^2} - \frac{2\,A\,c\,\sqrt{c\,x^2+b\,x}}{15\,b\,x^2} - \frac{2\,B\,c\,\sqrt{c\,x^2+b\,x}}{3\,b\,x} - \frac{2\,A\,\sqrt{c\,x^2+b\,x}}{5\,x^3}$$

input
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^4,x)
```

output
```
(4*A*c^2*(b*x + c*x^2)^(1/2))/(15*b^2*x) - (2*B*(b*x + c*x^2)^(1/2))/(3*x^
2) - (2*A*c*(b*x + c*x^2)^(1/2))/(15*b*x^2) - (2*B*c*(b*x + c*x^2)^(1/2))/
(3*b*x) - (2*A*(b*x + c*x^2)^(1/2))/(5*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.88

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^4} \, dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^2}{5} - \frac{2\sqrt{x}\,\sqrt{cx+b}\,abcx}{15} + \frac{4\sqrt{x}\,\sqrt{cx+b}\,a\,c^2x^2}{15} - \frac{2\sqrt{x}\,\sqrt{cx+b}\,b^3x}{3} - \frac{2\sqrt{x}\,\sqrt{cx+b}\,b^2c\,x^2}{3} - \frac{4\sqrt{c}\,a\,c^2x^3}{15} - \frac{2\sqrt{c}\,b^2c\,x^3}{15}}{b^2x^3}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^4,x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(b + c*x)*a*b**2 - sqrt(x)*sqrt(b + c*x)*a*b*c*x + 2*
sqrt(x)*sqrt(b + c*x)*a*c**2*x**2 - 5*sqrt(x)*sqrt(b + c*x)*b**3*x - 5*sqr
t(x)*sqrt(b + c*x)*b**2*c*x**2 - 2*sqrt(c)*a*c**2*x**3 - sqrt(c)*b**2*c*x*
*3))/(15*b**2*x**3)
```

3.110 $\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^5}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^5}\,dx = -\frac{2A(bx+cx^2)^{3/2}}{7bx^5} - \frac{2(7bB-4Ac)\,(bx+cx^2)^{3/2}}{35b^2x^4} + \frac{4c(7bB-4Ac)\,(bx+cx^2)^{3/2}}{105b^3x^3}$$

output
```
-2/7*A*(c*x^2+b*x)^(3/2)/b/x^5-2/35*(-4*A*c+7*B*b)*(c*x^2+b*x)^(3/2)/b^2/x
^4+4/105*c*(-4*A*c+7*B*b)*(c*x^2+b*x)^(3/2)/b^3/x^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^5}\,dx = \frac{2(x(b+cx))^{3/2}\,(7bBx(-3b+2cx)+A(-15b^2+12bcx-8c^2x^2))}{105b^3x^5}$$

input
```
Integrate[((A + B*x)*Sqrt[b*x + c*x^2])/x^5,x]
```

output
```
(2*(x*(b + c*x))^(3/2)*(7*b*B*x*(-3*b + 2*c*x) + A*(-15*b^2 + 12*b*c*x - 8
*c^2*x^2)))/(105*b^3*x^5)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^5}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(7bB-4Ac)\int \frac{\sqrt{cx^2+bx}}{x^4}dx}{7b}-\frac{2A(bx+cx^2)^{3/2}}{7bx^5}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(7bB-4Ac)\left(-\frac{2c\int \frac{\sqrt{cx^2+bx}}{x^3}dx}{5b}-\frac{2(bx+cx^2)^{3/2}}{5bx^4}\right)}{7b}-\frac{2A(bx+cx^2)^{3/2}}{7bx^5}$$

$$\Big\downarrow \text{ 1123}$$

$$\frac{\left(\frac{4c(bx+cx^2)^{3/2}}{15b^2x^3}-\frac{2(bx+cx^2)^{3/2}}{5bx^4}\right)(7bB-4Ac)}{7b}-\frac{2A(bx+cx^2)^{3/2}}{7bx^5}$$

input
```
Int[((A + B*x)*Sqrt[b*x + c*x^2])/x^5,x]
```

output
```
(-2*A*(b*x + c*x^2)^(3/2))/(7*b*x^5) + ((7*b*B - 4*A*c)*((-2*(b*x + c*x^2)
^(3/2))/(5*b*x^4) + (4*c*(b*x + c*x^2)^(3/2))/(15*b^2*x^3)))/(7*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60

method	result	size
pseudoelliptic	$-\frac{2(cx+b)\sqrt{x(cx+b)}\left(\left(\frac{7Bx}{5}+A\right)b^{2}-\frac{4cx\left(\frac{7Bx}{6}+A\right)b}{5}+\frac{8A\,c^{2}x^{2}}{15}\right)}{7x^{4}b^{3}}$	54
gosper	$-\frac{2(cx+b)\left(8A\,c^{2}x^{2}-14x^{2}Bbc-12Abcx+21xB\,b^{2}+15b^{2}A\right)\sqrt{c\,x^{2}+bx}}{105b^{3}x^{4}}$	62
orering	$-\frac{2(cx+b)\left(8A\,c^{2}x^{2}-14x^{2}Bbc-12Abcx+21xB\,b^{2}+15b^{2}A\right)\sqrt{c\,x^{2}+bx}}{105b^{3}x^{4}}$	62
trager	$-\frac{2\left(8A\,c^{3}x^{3}-14x^{3}Bb\,c^{2}-4Ab\,c^{2}x^{2}+7x^{2}B\,b^{2}c+3A\,b^{2}cx+21xB\,b^{3}+15A\,b^{3}\right)\sqrt{c\,x^{2}+bx}}{105b^{3}x^{4}}$	81
risch	$-\frac{2(cx+b)\left(8A\,c^{3}x^{3}-14x^{3}Bb\,c^{2}-4Ab\,c^{2}x^{2}+7x^{2}B\,b^{2}c+3A\,b^{2}cx+21xB\,b^{3}+15A\,b^{3}\right)}{105x^{3}\sqrt{x(cx+b)}\,b^{3}}$	84
default	$A\left(-\frac{2\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{7b\,x^{5}}-\frac{4c\left(-\frac{2\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{5b\,x^{4}}+\frac{4c\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{15b^{2}x^{3}}\right)}{7b}\right)+B\left(-\frac{2\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{5b\,x^{4}}+\frac{4c\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{15b^{2}x^{3}}\right)$	112

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/7*(c*x+b)*(x*(c*x+b))^(1/2)*((7/5*B*x+A)*b^2-4/5*c*x*(7/6*B*x+A)*b+8/15
*A*c^2*x^2)/x^4/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^5}\,dx = -\frac{2\left(15\,Ab^3-2\left(7\,Bbc^2-4\,Ac^3\right)x^3+\left(7\,Bb^2c-4\,Abc^2\right)x^2+3\left(7\,Bb^3+Ab^2c\right)x\right)\sqrt{cx^2+bx}}{105\,b^3x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
-2/105*(15*A*b^3 - 2*(7*B*b*c^2 - 4*A*c^3)*x^3 + (7*B*b^2*c - 4*A*b*c^2)*x
^2 + 3*(7*B*b^3 + A*b^2*c)*x)*sqrt(c*x^2 + b*x)/(b^3*x^4)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^5}\,dx = \int \frac{\sqrt{x\,(b+cx)}(A+Bx)}{x^5}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(x*(b + c*x))*(A + B*x)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

$$\begin{aligned}\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^5}\,dx &= \frac{4\,\sqrt{cx^2+bx}Bc^2}{15\,b^2x} - \frac{16\,\sqrt{cx^2+bx}Ac^3}{105\,b^3x} \\ &\quad - \frac{2\,\sqrt{cx^2+bx}Bc}{15\,bx^2} + \frac{8\,\sqrt{cx^2+bx}Ac^2}{105\,b^2x^2} \\ &\quad - \frac{2\,\sqrt{cx^2+bx}B}{5\,x^3} - \frac{2\,\sqrt{cx^2+bx}Ac}{35\,bx^3} - \frac{2\,\sqrt{cx^2+bx}A}{7\,x^4}\end{aligned}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
4/15*sqrt(c*x^2 + b*x)*B*c^2/(b^2*x) - 16/105*sqrt(c*x^2 + b*x)*A*c^3/(b^3
*x) - 2/15*sqrt(c*x^2 + b*x)*B*c/(b*x^2) + 8/105*sqrt(c*x^2 + b*x)*A*c^2/(
b^2*x^2) - 2/5*sqrt(c*x^2 + b*x)*B/x^3 - 2/35*sqrt(c*x^2 + b*x)*A*c/(b*x^3
) - 2/7*sqrt(c*x^2 + b*x)*A/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(78) = 156$.

Time = 0.14 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.79

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^5}\,dx$$
$$= \frac{2\left(105\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5Bc^{\frac{3}{2}}+175\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4Bbc+140\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4Ac^2+105\left(\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^5,x, algorithm="giac")
```

output
```
2/105*(105*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*B*c^(3/2) + 175*(sqrt(c)*x -
sqrt(c*x^2 + b*x))^4*B*b*c + 140*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*A*c^2 +
 105*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*B*b^2*sqrt(c) + 315*(sqrt(c)*x - sq
rt(c*x^2 + b*x))^3*A*b*c^(3/2) + 21*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*b^
3 + 273*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*A*b^2*c + 105*(sqrt(c)*x - sqrt(
c*x^2 + b*x))*A*b^3*sqrt(c) + 15*A*b^4)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^7
```

Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.62

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^5}\,dx = \frac{8\,A\,c^2\,\sqrt{c\,x^2+b\,x}}{105\,b^2\,x^2} - \frac{2\,B\,\sqrt{c\,x^2+b\,x}}{5\,x^3} - \frac{2\,A\,c\,\sqrt{c\,x^2+b\,x}}{35\,b\,x^3} - \frac{2\,B\,c\,\sqrt{c\,x^2+b\,x}}{15\,b\,x^2} - \frac{2\,A\,\sqrt{c\,x^2+b\,x}}{7\,x^4} - \frac{16\,A\,c^3\,\sqrt{c\,x^2+b\,x}}{105\,b^3\,x} + \frac{4\,B\,c^2\,\sqrt{c\,x^2+b\,x}}{15\,b^2\,x}$$

input
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^5,x)
```

output
```
(8*A*c^2*(b*x + c*x^2)^(1/2))/(105*b^2*x^2) - (2*B*(b*x + c*x^2)^(1/2))/(5
*x^3) - (2*A*c*(b*x + c*x^2)^(1/2))/(35*b*x^3) - (2*B*c*(b*x + c*x^2)^(1/2
))/(15*b*x^2) - (2*A*(b*x + c*x^2)^(1/2))/(7*x^4) - (16*A*c^3*(b*x + c*x^2
)^(1/2))/(105*b^3*x) + (4*B*c^2*(b*x + c*x^2)^(1/2))/(15*b^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.64

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^5}\,dx$$
$$= \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^3}{7} - \frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^2cx}{35} + \frac{8\sqrt{x}\,\sqrt{cx+b}\,ab\,c^2x^2}{105} - \frac{16\sqrt{x}\,\sqrt{cx+b}\,a\,c^3x^3}{105} - \frac{2\sqrt{x}\,\sqrt{cx+b}\,b^4x}{5} - \frac{2\sqrt{x}\,\sqrt{cx+b}\,b^3c\,x^2}{15} + 4\sqrt{}}{b^3x^4}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^5,x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(b + c*x)*a*b**3 - 3*sqrt(x)*sqrt(b + c*x)*a*b**2*c*
x + 4*sqrt(x)*sqrt(b + c*x)*a*b*c**2*x**2 - 8*sqrt(x)*sqrt(b + c*x)*a*c**3
*x**3 - 21*sqrt(x)*sqrt(b + c*x)*b**4*x - 7*sqrt(x)*sqrt(b + c*x)*b**3*c*x
**2 + 14*sqrt(x)*sqrt(b + c*x)*b**2*c**2*x**3 + 8*sqrt(c)*a*c**3*x**4 - 14
*sqrt(c)*b**2*c**2*x**4))/(105*b**3*x**4)
```

3.111 $\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^6} dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^6}\,dx=-\frac{2A(bx+cx^2)^{3/2}}{9bx^6}-\frac{2(3bB-2Ac)\,(bx+cx^2)^{3/2}}{21b^2x^5}+\frac{8c(3bB-2Ac)\,(bx+cx^2)^{3/2}}{105b^3x^4}-\frac{16c^2(3bB-2Ac)\,(bx+cx^2)^{3/2}}{315b^4x^3}$$

output
```
-2/9*A*(c*x^2+b*x)^(3/2)/b/x^6-2/21*(-2*A*c+3*B*b)*(c*x^2+b*x)^(3/2)/b^2/x
^5+8/105*c*(-2*A*c+3*B*b)*(c*x^2+b*x)^(3/2)/b^3/x^4-16/315*c^2*(-2*A*c+3*B
*b)*(c*x^2+b*x)^(3/2)/b^4/x^3
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^6}\,dx=-\frac{2(x(b+cx))^{3/2}\,(3bBx(15b^2-12bcx+8c^2x^2)+A(35b^3-30b^2cx+24bc^2x^2-16c^3x^3))}{315b^4x^6}$$

input
```
Integrate[((A + B*x)*Sqrt[b*x + c*x^2])/x^6,x]
```

output
```
(-2*(x*(b + c*x))^(3/2)*(3*b*B*x*(15*b^2 - 12*b*c*x + 8*c^2*x^2) + A*(35*b
^3 - 30*b^2*c*x + 24*b*c^2*x^2 - 16*c^3*x^3)))/(315*b^4*x^6)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^6}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(3bB-2Ac)\int \frac{\sqrt{cx^2+bx}}{x^5}dx}{3b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{9bx^6}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(3bB-2Ac)\left(-\frac{4c\int \frac{\sqrt{cx^2+bx}}{x^4}dx}{7b}-\frac{2\left(bx+cx^2\right)^{3/2}}{7bx^5}\right)}{3b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{9bx^6}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(3bB-2Ac)\left(-\frac{4c\left(-\frac{2c\int \frac{\sqrt{cx^2+bx}}{x^3}dx}{5b}-\frac{2\left(bx+cx^2\right)^{3/2}}{5bx^4}\right)}{7b}-\frac{2\left(bx+cx^2\right)^{3/2}}{7bx^5}\right)}{3b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{9bx^6}$$

$$\Big\downarrow \text{ 1123}$$

$$\frac{\left(-\frac{4c\left(\frac{4c\left(bx+cx^2\right)^{3/2}}{15b^2x^3}-\frac{2\left(bx+cx^2\right)^{3/2}}{5bx^4}\right)}{7b}-\frac{2\left(bx+cx^2\right)^{3/2}}{7bx^5}\right)(3bB-2Ac)}{3b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{9bx^6}$$

input
```
Int[((A + B*x)*Sqrt[b*x + c*x^2])/x^6,x]
```

output
```
(-2*A*(b*x + c*x^2)^(3/2))/(9*b*x^6) + ((3*b*B - 2*A*c)*((-2*(b*x + c*x^2)
^(3/2))/(7*b*x^5) - (4*c*((-2*(b*x + c*x^2)^(3/2))/(5*b*x^4) + (4*c*(b*x +
 c*x^2)^(3/2))/(15*b^2*x^3)))/(7*b)))/(3*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

method	result
pseudoelliptic	$-\frac{2(cx+b)\sqrt{x(cx+b)}\left(\left(\frac{9Bx}{7}+A\right)b^{3}-\frac{6cx\left(\frac{6Bx}{5}+A\right)b^{2}}{7}+\frac{24c^{2}x^{2}(Bx+A)b}{35}-\frac{16A\,c^{3}x^{3}}{35}\right)}{9x^{5}b^{4}}$
gosper	$-\frac{2(cx+b)\left(-16A\,c^{3}x^{3}+24x^{3}Bb\,c^{2}+24Ab\,c^{2}x^{2}-36x^{2}B\,b^{2}c-30A\,b^{2}cx+45xB\,b^{3}+35A\,b^{3}\right)\sqrt{c\,x^{2}+bx}}{315b^{4}x^{5}}$
orering	$-\frac{2(cx+b)\left(-16A\,c^{3}x^{3}+24x^{3}Bb\,c^{2}+24Ab\,c^{2}x^{2}-36x^{2}B\,b^{2}c-30A\,b^{2}cx+45xB\,b^{3}+35A\,b^{3}\right)\sqrt{c\,x^{2}+bx}}{315b^{4}x^{5}}$
trager	$-\frac{2\left(-16A\,c^{4}x^{4}+24Bb\,c^{3}x^{4}+8Ab\,c^{3}x^{3}-12B\,b^{2}c^{2}x^{3}-6A\,b^{2}c^{2}x^{2}+9B\,b^{3}c\,x^{2}+5A\,b^{3}cx+45B\,b^{4}x+35A\,b^{4}\right)\sqrt{c\,x^{2}+bx}}{315b^{4}x^{5}}$
risch	$-\frac{2(cx+b)\left(-16A\,c^{4}x^{4}+24Bb\,c^{3}x^{4}+8Ab\,c^{3}x^{3}-12B\,b^{2}c^{2}x^{3}-6A\,b^{2}c^{2}x^{2}+9B\,b^{3}c\,x^{2}+5A\,b^{3}cx+45B\,b^{4}x+35A\,b^{4}\right)}{315x^{4}\sqrt{x(cx+b)}\,b^{4}}$
default	$A\left(-\frac{2\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{9b\,x^{6}}-\frac{2c\left(-\frac{2\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{7b\,x^{5}}-\frac{4c\left(-\frac{2\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{5b\,x^{4}}+\frac{4c\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{15b^{2}x^{3}}\right)}{7b}\right)}{3b}\right)+B\left(-\frac{2\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{7b\,x^{5}}-\frac{4c}{}\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/9*(c*x+b)*(x*(c*x+b))^(1/2)*((9/7*B*x+A)*b^3-6/7*c*x*(6/5*B*x+A)*b^2+24
/35*c^2*x^2*(B*x+A)*b-16/35*A*c^3*x^3)/x^5/b^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^6}\,dx =$$
$$-\frac{2\,(35\,Ab^4+8\,(3\,Bbc^3-2\,Ac^4)x^4-4\,(3\,Bb^2c^2-2\,Abc^3)x^3+3\,(3\,Bb^3c-2\,Ab^2c^2)x^2+5\,(9\,Bb^4+Ab^3c}{315\,b^4x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
-2/315*(35*A*b^4 + 8*(3*B*b*c^3 - 2*A*c^4)*x^4 - 4*(3*B*b^2*c^2 - 2*A*b*c^
3)*x^3 + 3*(3*B*b^3*c - 2*A*b^2*c^2)*x^2 + 5*(9*B*b^4 + A*b^3*c)*x)*sqrt(c
*x^2 + b*x)/(b^4*x^5)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^6}\,dx = \int \frac{\sqrt{x\,(b+cx)}(A+Bx)}{x^6}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2)/x**6,x)
```

output
```
Integral(sqrt(x*(b + c*x))*(A + B*x)/x**6, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.54

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^6}\,dx = -\frac{16\,\sqrt{cx^2+bx}Bc^3}{105\,b^3x} + \frac{32\,\sqrt{cx^2+bx}Ac^4}{315\,b^4x} + \frac{8\,\sqrt{cx^2+bx}Bc^2}{105\,b^2x^2} - \frac{16\,\sqrt{cx^2+bx}Ac^3}{315\,b^3x^2} - \frac{2\,\sqrt{cx^2+bx}Bc}{35\,bx^3} + \frac{4\,\sqrt{cx^2+bx}Ac^2}{105\,b^2x^3} - \frac{2\,\sqrt{cx^2+bx}B}{7\,x^4} - \frac{2\,\sqrt{cx^2+bx}Ac}{63\,bx^4} - \frac{2\,\sqrt{cx^2+bx}A}{9\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
-16/105*sqrt(c*x^2 + b*x)*B*c^3/(b^3*x) + 32/315*sqrt(c*x^2 + b*x)*A*c^4/(
b^4*x) + 8/105*sqrt(c*x^2 + b*x)*B*c^2/(b^2*x^2) - 16/315*sqrt(c*x^2 + b*x
)*A*c^3/(b^3*x^2) - 2/35*sqrt(c*x^2 + b*x)*B*c/(b*x^3) + 4/105*sqrt(c*x^2
+ b*x)*A*c^2/(b^2*x^3) - 2/7*sqrt(c*x^2 + b*x)*B/x^4 - 2/63*sqrt(c*x^2 + b
*x)*A*c/(b*x^4) - 2/9*sqrt(c*x^2 + b*x)*A/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(109) = 218$.

Time = 0.13 (sec) , antiderivative size = 311, normalized size of antiderivative = 2.49

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^6}\,dx$$
$$= \frac{2\left(420\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^6Bc^2+945\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5Bbc^{\frac{3}{2}}+630\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5Ac^{\frac{5}{2}}+819\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^6,x, algorithm="giac")
```

output
```
2/315*(420*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*B*c^2 + 945*(sqrt(c)*x - sqrt
(c*x^2 + b*x))^5*B*b*c^(3/2) + 630*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*A*c^(
5/2) + 819*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*B*b^2*c + 1764*(sqrt(c)*x - s
qrt(c*x^2 + b*x))^4*A*b*c^2 + 315*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*B*b^3*
sqrt(c) + 1995*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*A*b^2*c^(3/2) + 45*(sqrt(
c)*x - sqrt(c*x^2 + b*x))^2*B*b^4 + 1125*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2
*A*b^3*c + 315*(sqrt(c)*x - sqrt(c*x^2 + b*x))*A*b^4*sqrt(c) + 35*A*b^5)/(
sqrt(c)*x - sqrt(c*x^2 + b*x))^9
```

Mupad [B] (verification not implemented)

Time = 5.85 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.54

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^6}\,dx = \frac{4\,A\,c^2\,\sqrt{c\,x^2+b\,x}}{105\,b^2\,x^3}-\frac{2\,B\,\sqrt{c\,x^2+b\,x}}{7\,x^4}-\frac{2\,A\,c\,\sqrt{c\,x^2+b\,x}}{63\,b\,x^4}$$
$$-\frac{2\,B\,c\,\sqrt{c\,x^2+b\,x}}{35\,b\,x^3}-\frac{2\,A\,\sqrt{c\,x^2+b\,x}}{9\,x^5}$$
$$-\frac{16\,A\,c^3\,\sqrt{c\,x^2+b\,x}}{315\,b^3\,x^2}+\frac{32\,A\,c^4\,\sqrt{c\,x^2+b\,x}}{315\,b^4\,x}$$
$$+\frac{8\,B\,c^2\,\sqrt{c\,x^2+b\,x}}{105\,b^2\,x^2}-\frac{16\,B\,c^3\,\sqrt{c\,x^2+b\,x}}{105\,b^3\,x}$$

input
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^6,x)
```

output
```
(4*A*c^2*(b*x + c*x^2)^(1/2))/(105*b^2*x^3) - (2*B*(b*x + c*x^2)^(1/2))/(7
*x^4) - (2*A*c*(b*x + c*x^2)^(1/2))/(63*b*x^4) - (2*B*c*(b*x + c*x^2)^(1/2
))/(35*b*x^3) - (2*A*(b*x + c*x^2)^(1/2))/(9*x^5) - (16*A*c^3*(b*x + c*x^2
)^(1/2))/(315*b^3*x^2) + (32*A*c^4*(b*x + c*x^2)^(1/2))/(315*b^4*x) + (8*B
*c^2*(b*x + c*x^2)^(1/2))/(105*b^2*x^2) - (16*B*c^3*(b*x + c*x^2)^(1/2))/(
105*b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.50

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^6}\,dx$$
$$= \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^4}{9}-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^3cx}{63}+\frac{4\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^2x^2}{105}-\frac{16\sqrt{x}\,\sqrt{cx+b}\,ab\,c^3x^3}{315}+\frac{32\sqrt{x}\,\sqrt{cx+b}\,a\,c^4x^4}{315}-\frac{2\sqrt{x}\,\sqrt{cx+b}\,b^5x}{7}-}{b^4x^5}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^6,x)
```

output
```
(2*( - 35*sqrt(x)*sqrt(b + c*x)*a*b**4 - 5*sqrt(x)*sqrt(b + c*x)*a*b**3*c*
x + 6*sqrt(x)*sqrt(b + c*x)*a*b**2*c**2*x**2 - 8*sqrt(x)*sqrt(b + c*x)*a*b
*c**3*x**3 + 16*sqrt(x)*sqrt(b + c*x)*a*c**4*x**4 - 45*sqrt(x)*sqrt(b + c*
x)*b**5*x - 9*sqrt(x)*sqrt(b + c*x)*b**4*c*x**2 + 12*sqrt(x)*sqrt(b + c*x)
*b**3*c**2*x**3 - 24*sqrt(x)*sqrt(b + c*x)*b**2*c**3*x**4 - 16*sqrt(c)*a*c
**4*x**5 + 24*sqrt(c)*b**2*c**3*x**5))/(315*b**4*x**5)
```

3.112 $\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^7} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^7} \, dx = -\frac{2A(bx+cx^2)^{3/2}}{11bx^7} - \frac{2(11bB-8Ac)\left(bx+cx^2\right)^{3/2}}{99b^2x^6} + \frac{4c(11bB-8Ac)\left(bx+cx^2\right)^{3/2}}{231b^3x^5} - \frac{16c^2(11bB-8Ac)\left(bx+cx^2\right)^{3/2}}{1155b^4x^4} + \frac{32c^3(11bB-8Ac)\left(bx+cx^2\right)^{3/2}}{3465b^5x^3}$$

output

```
-2/11*A*(c*x^2+b*x)^(3/2)/b/x^7-2/99*(-8*A*c+11*B*b)*(c*x^2+b*x)^(3/2)/b^2
/x^6+4/231*c*(-8*A*c+11*B*b)*(c*x^2+b*x)^(3/2)/b^3/x^5-16/1155*c^2*(-8*A*c
+11*B*b)*(c*x^2+b*x)^(3/2)/b^4/x^4+32/3465*c^3*(-8*A*c+11*B*b)*(c*x^2+b*x)
^(3/2)/b^5/x^3
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^7}\,dx = \frac{2(x(b+cx))^{3/2}\left(11bBx(-35b^3+30b^2cx-24bc^2x^2+16c^3x^3)+A(-315b^4+280b^3cx-240b^2c^2x^2+192\right.}{3465b^5x^7}$$

input
```
Integrate[((A + B*x)*Sqrt[b*x + c*x^2])/x^7,x]
```

output
```
(2*(x*(b + c*x))^(3/2)*(11*b*B*x*(-35*b^3 + 30*b^2*c*x - 24*b*c^2*x^2 + 16
*c^3*x^3) + A*(-315*b^4 + 280*b^3*c*x - 240*b^2*c^2*x^2 + 192*b*c^3*x^3 -
128*c^4*x^4)))/(3465*b^5*x^7)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^7}\,dx$$

$$\Big\downarrow \; 1220$$

$$\frac{(11bB-8Ac)\int \frac{\sqrt{cx^2+bx}}{x^6}dx}{11b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{11bx^7}$$

$$\Big\downarrow \; 1129$$

$$\frac{(11bB-8Ac)\left(-\frac{2c\int \frac{\sqrt{cx^2+bx}}{x^5}dx}{3b}-\frac{2\left(bx+cx^2\right)^{3/2}}{9bx^6}\right)}{11b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{11bx^7}$$

$$\Big\downarrow \; 1129$$

$$\frac{(11bB-8Ac)\left(-\frac{2c\left(-\frac{4c\int \frac{\sqrt{cx^2+bx}}{x^4}dx}{7b}-\frac{2\left(bx+cx^2\right)^{3/2}}{7bx^5}\right)}{3b}-\frac{2\left(bx+cx^2\right)^{3/2}}{9bx^6}\right)}{11b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{11bx^7}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(11bB-8Ac)\left(-\frac{2c\left(-\frac{4c\left(-\frac{2c\int \frac{\sqrt{cx^2+bx}}{x^3}dx}{5b}-\frac{2\left(bx+cx^2\right)^{3/2}}{5bx^4}\right)}{7b}-\frac{2\left(bx+cx^2\right)^{3/2}}{7bx^5}\right)}{3b}-\frac{2\left(bx+cx^2\right)^{3/2}}{9bx^6}\right)}{11b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{11bx^7}$$

$$\Big\downarrow \text{1123}$$

$$\frac{\left(-\frac{2c\left(-\frac{4c\left(\frac{4c\left(bx+cx^2\right)^{3/2}}{15b^2x^3}-\frac{2\left(bx+cx^2\right)^{3/2}}{5bx^4}\right)}{7b}-\frac{2\left(bx+cx^2\right)^{3/2}}{7bx^5}\right)}{3b}-\frac{2\left(bx+cx^2\right)^{3/2}}{9bx^6}\right)(11bB-8Ac)}{11b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{11bx^7}$$

input
```
Int[((A + B*x)*Sqrt[b*x + c*x^2])/x^7,x]
```

output
```
(-2*A*(b*x + c*x^2)^(3/2))/(11*b*x^7) + ((11*b*B - 8*A*c)*((-2*(b*x + c*x^
2)^(3/2))/(9*b*x^6) - (2*c*((-2*(b*x + c*x^2)^(3/2))/(7*b*x^5) - (4*c*((-2
*(b*x + c*x^2)^(3/2))/(5*b*x^4) + (4*c*(b*x + c*x^2)^(3/2))/(15*b^2*x^3)))
/(7*b)))/(3*b)))/(11*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.55

method	result
pseudoelliptic	$-\frac{2(cx+b)\sqrt{x(cx+b)}\left(\left(\frac{11Bx}{9}+A\right)b^4-\frac{8cx\left(\frac{33Bx}{28}+A\right)b^3}{9}+\frac{16c^2x^2\left(\frac{11Bx}{10}+A\right)b^2}{21}-\frac{64\left(\frac{11Bx}{12}+A\right)c^3x^3b}{105}+\frac{128Ac^4x^4}{315}\right)}{11x^6b^5}$
gosper	$-\frac{2(cx+b)(128A\,c^4x^4-176Bb\,c^3x^4-192Ab\,c^3x^3+264B\,b^2c^2x^3+240A\,b^2c^2x^2-330B\,b^3c\,x^2-280A\,b^3cx+385B\,b^4x+315A}{3465x^6b^5}$
orering	$-\frac{2(cx+b)(128A\,c^4x^4-176Bb\,c^3x^4-192Ab\,c^3x^3+264B\,b^2c^2x^3+240A\,b^2c^2x^2-330B\,b^3c\,x^2-280A\,b^3cx+385B\,b^4x+315A}{3465x^6b^5}$
trager	$-\frac{2(128A\,c^5x^5-176Bb\,c^4x^5-64Ab\,c^4x^4+88B\,b^2c^3x^4+48A\,b^2c^3x^3-66B\,b^3c^2x^3-40A\,b^3c^2x^2+55B\,b^4c\,x^2+35A\,b^4cx+38}{3465x^6b^5}$
risch	$-\frac{2(cx+b)(128A\,c^5x^5-176Bb\,c^4x^5-64Ab\,c^4x^4+88B\,b^2c^3x^4+48A\,b^2c^3x^3-66B\,b^3c^2x^3-40A\,b^3c^2x^2+55B\,b^4c\,x^2+35A\,b^4}{3465x^5\sqrt{x(cx+b)}\,b^5}$
default	$A\left(-\frac{2(c\,x^2+bx)^{\frac{3}{2}}}{11b\,x^7}-\frac{8c\left(-\frac{2(c\,x^2+bx)^{\frac{3}{2}}}{9b\,x^6}-\frac{2c\left(-\frac{2(c\,x^2+bx)^{\frac{3}{2}}}{7b\,x^5}-\frac{4c\left(-\frac{2(c\,x^2+bx)^{\frac{3}{2}}}{5b\,x^4}+\frac{4c(c\,x^2+bx)^{\frac{3}{2}}}{15b^2x^3}\right)}{7b}\right)}{3b}\right)}{11b}\right)+B\Bigg(-$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-2/11*(c*x+b)*(x*(c*x+b))^(1/2)*((11/9*B*x+A)*b^4-8/9*c*x*(33/28*B*x+A)*b^
3+16/21*c^2*x^2*(11/10*B*x+A)*b^2-64/105*(11/12*B*x+A)*c^3*x^3*b+128/315*A
*c^4*x^4)/x^6/b^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^7}\,dx = -\frac{2\,(315\,Ab^5 - 16\,(11\,Bbc^4 - 8\,Ac^5)x^5 + 8\,(11\,Bb^2c^3 - 8\,Abc^4)x^4 - 6\,(11\,Bb^3c^2 - 8\,Ab^2c^3)x^3 + 5\,(11\,Bb}{3465\,b^5x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^7,x, algorithm="fricas")
```

output
```
-2/3465*(315*A*b^5 - 16*(11*B*b*c^4 - 8*A*c^5)*x^5 + 8*(11*B*b^2*c^3 - 8*A
*b*c^4)*x^4 - 6*(11*B*b^3*c^2 - 8*A*b^2*c^3)*x^3 + 5*(11*B*b^4*c - 8*A*b^3
*c^2)*x^2 + 35*(11*B*b^5 + A*b^4*c)*x)*sqrt(c*x^2 + b*x)/(b^5*x^6)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^7}\,dx = \int \frac{\sqrt{x\,(b+cx)}(A+Bx)}{x^7}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2)/x**7,x)
```

output
```
Integral(sqrt(x*(b + c*x))*(A + B*x)/x**7, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.49

$$\begin{aligned}\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^7}\,dx &= \frac{32\,\sqrt{cx^2+bx}Bc^4}{315\,b^4x} - \frac{256\,\sqrt{cx^2+bx}Ac^5}{3465\,b^5x} - \frac{16\,\sqrt{cx^2+bx}Bc^3}{315\,b^3x^2} \\ &\quad + \frac{128\,\sqrt{cx^2+bx}Ac^4}{3465\,b^4x^2} + \frac{4\,\sqrt{cx^2+bx}Bc^2}{105\,b^2x^3} \\ &\quad - \frac{32\,\sqrt{cx^2+bx}Ac^3}{1155\,b^3x^3} - \frac{2\,\sqrt{cx^2+bx}Bc}{63\,bx^4} + \frac{16\,\sqrt{cx^2+bx}Ac^2}{693\,b^2x^4} \\ &\quad - \frac{2\,\sqrt{cx^2+bx}B}{9\,x^5} - \frac{2\,\sqrt{cx^2+bx}Ac}{99\,bx^5} - \frac{2\,\sqrt{cx^2+bx}A}{11\,x^6}\end{aligned}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^7,x, algorithm="maxima")
```

output
```
32/315*sqrt(c*x^2 + b*x)*B*c^4/(b^4*x) - 256/3465*sqrt(c*x^2 + b*x)*A*c^5/
(b^5*x) - 16/315*sqrt(c*x^2 + b*x)*B*c^3/(b^3*x^2) + 128/3465*sqrt(c*x^2 +
 b*x)*A*c^4/(b^4*x^2) + 4/105*sqrt(c*x^2 + b*x)*B*c^2/(b^2*x^3) - 32/1155*
sqrt(c*x^2 + b*x)*A*c^3/(b^3*x^3) - 2/63*sqrt(c*x^2 + b*x)*B*c/(b*x^4) + 1
6/693*sqrt(c*x^2 + b*x)*A*c^2/(b^2*x^4) - 2/9*sqrt(c*x^2 + b*x)*B/x^5 - 2/
99*sqrt(c*x^2 + b*x)*A*c/(b*x^5) - 2/11*sqrt(c*x^2 + b*x)*A/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(140) = 280$.

Time = 0.27 (sec) , antiderivative size = 371, normalized size of antiderivative = 2.32

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^7}\,dx = \frac{2\left(6930\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^7Bc^{\frac{5}{2}}+19404\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^6Bbc^2+11088\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^6Ac^3\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^7,x, algorithm="giac")
```

output
```
2/3465*(6930*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*B*c^(5/2) + 19404*(sqrt(c)*
x - sqrt(c*x^2 + b*x))^6*B*b*c^2 + 11088*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6
*A*c^3 + 21945*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*B*b^2*c^(3/2) + 36960*(sq
rt(c)*x - sqrt(c*x^2 + b*x))^5*A*b*c^(5/2) + 12375*(sqrt(c)*x - sqrt(c*x^2
 + b*x))^4*B*b^3*c + 51480*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*A*b^2*c^2 + 3
465*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*B*b^4*sqrt(c) + 38115*(sqrt(c)*x - s
qrt(c*x^2 + b*x))^3*A*b^3*c^(3/2) + 385*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*
B*b^5 + 15785*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*A*b^4*c + 3465*(sqrt(c)*x
- sqrt(c*x^2 + b*x))*A*b^5*sqrt(c) + 315*A*b^6)/(sqrt(c)*x - sqrt(c*x^2 +
b*x))^11
```

Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.49

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^7}\,dx = \frac{16\,A\,c^2\,\sqrt{c\,x^2+b\,x}}{693\,b^2\,x^4} - \frac{2\,B\,\sqrt{c\,x^2+b\,x}}{9\,x^5} - \frac{2\,A\,c\,\sqrt{c\,x^2+b\,x}}{99\,b\,x^5} - \frac{2\,B\,c\,\sqrt{c\,x^2+b\,x}}{63\,b\,x^4} - \frac{2\,A\,\sqrt{c\,x^2+b\,x}}{11\,x^6} - \frac{32\,A\,c^3\,\sqrt{c\,x^2+b\,x}}{1155\,b^3\,x^3} + \frac{128\,A\,c^4\,\sqrt{c\,x^2+b\,x}}{3465\,b^4\,x^2} - \frac{256\,A\,c^5\,\sqrt{c\,x^2+b\,x}}{3465\,b^5\,x} + \frac{4\,B\,c^2\,\sqrt{c\,x^2+b\,x}}{105\,b^2\,x^3} - \frac{16\,B\,c^3\,\sqrt{c\,x^2+b\,x}}{315\,b^3\,x^2} + \frac{32\,B\,c^4\,\sqrt{c\,x^2+b\,x}}{315\,b^4\,x}$$

input
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^7,x)
```

output
```
(16*A*c^2*(b*x + c*x^2)^(1/2))/(693*b^2*x^4) - (2*B*(b*x + c*x^2)^(1/2))/(
9*x^5) - (2*A*c*(b*x + c*x^2)^(1/2))/(99*b*x^5) - (2*B*c*(b*x + c*x^2)^(1/
2))/(63*b*x^4) - (2*A*(b*x + c*x^2)^(1/2))/(11*x^6) - (32*A*c^3*(b*x + c*x
^2)^(1/2))/(1155*b^3*x^3) + (128*A*c^4*(b*x + c*x^2)^(1/2))/(3465*b^4*x^2)
 - (256*A*c^5*(b*x + c*x^2)^(1/2))/(3465*b^5*x) + (4*B*c^2*(b*x + c*x^2)^(
1/2))/(105*b^2*x^3) - (16*B*c^3*(b*x + c*x^2)^(1/2))/(315*b^3*x^2) + (32*B
*c^4*(b*x + c*x^2)^(1/2))/(315*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.41

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^7}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^5}{11} - \frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^4cx}{99} + \frac{16\sqrt{x}\,\sqrt{cx+b}\,a\,b^3c^2x^2}{693} - \frac{32\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^3x^3}{1155} + \frac{128\sqrt{x}\,\sqrt{cx+b}\,ab\,c^4x^4}{3465} - \frac{256\sqrt{x}\,\sqrt{cx+b}\,c}{3465}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^7,x)
```

output
```
(2*( - 315*sqrt(x)*sqrt(b + c*x)*a*b**5 - 35*sqrt(x)*sqrt(b + c*x)*a*b**4*
c*x + 40*sqrt(x)*sqrt(b + c*x)*a*b**3*c**2*x**2 - 48*sqrt(x)*sqrt(b + c*x)
*a*b**2*c**3*x**3 + 64*sqrt(x)*sqrt(b + c*x)*a*b*c**4*x**4 - 128*sqrt(x)*s
qrt(b + c*x)*a*c**5*x**5 - 385*sqrt(x)*sqrt(b + c*x)*b**6*x - 55*sqrt(x)*s
qrt(b + c*x)*b**5*c*x**2 + 66*sqrt(x)*sqrt(b + c*x)*b**4*c**2*x**3 - 88*sq
rt(x)*sqrt(b + c*x)*b**3*c**3*x**4 + 176*sqrt(x)*sqrt(b + c*x)*b**2*c**4*x
**5 + 128*sqrt(c)*a*c**5*x**6 - 176*sqrt(c)*b**2*c**4*x**6))/(3465*b**5*x*
*6)
```

3.113 $\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^8}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 195

$$\begin{aligned}\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^8}\,dx = &-\frac{2A(bx+cx^2)^{3/2}}{13bx^8}-\frac{2(13bB-10Ac)\,(bx+cx^2)^{3/2}}{143b^2x^7}\\&+\frac{16c(13bB-10Ac)\,(bx+cx^2)^{3/2}}{1287b^3x^6}\\&-\frac{32c^2(13bB-10Ac)\,(bx+cx^2)^{3/2}}{3003b^4x^5}\\&+\frac{128c^3(13bB-10Ac)\,(bx+cx^2)^{3/2}}{15015b^5x^4}\\&-\frac{256c^4(13bB-10Ac)\,(bx+cx^2)^{3/2}}{45045b^6x^3}\end{aligned}$$

output
```
-2/13*A*(c*x^2+b*x)^(3/2)/b/x^8-2/143*(-10*A*c+13*B*b)*(c*x^2+b*x)^(3/2)/b
^2/x^7+16/1287*c*(-10*A*c+13*B*b)*(c*x^2+b*x)^(3/2)/b^3/x^6-32/3003*c^2*(-
10*A*c+13*B*b)*(c*x^2+b*x)^(3/2)/b^4/x^5+128/15015*c^3*(-10*A*c+13*B*b)*(c
*x^2+b*x)^(3/2)/b^5/x^4-256/45045*c^4*(-10*A*c+13*B*b)*(c*x^2+b*x)^(3/2)/b
^6/x^3
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.63

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^8}\,dx = -\frac{2(x(b+cx))^{3/2}\left(13bBx(315b^4-280b^3cx+240b^2c^2x^2-192bc^3x^3+128c^4x^4)+5A(693b^5-630b^4cx+\right.}{45045b^6x^8}$$

input
```
Integrate[((A + B*x)*Sqrt[b*x + c*x^2])/x^8,x]
```

output
```
(-2*(x*(b + c*x))^(3/2)*(13*b*B*x*(315*b^4 - 280*b^3*c*x + 240*b^2*c^2*x^2
 - 192*b*c^3*x^3 + 128*c^4*x^4) + 5*A*(693*b^5 - 630*b^4*c*x + 560*b^3*c^2
*x^2 - 480*b^2*c^3*x^3 + 384*b*c^4*x^4 - 256*c^5*x^5)))/(45045*b^6*x^8)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1220, 1129, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^8}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(13bB-10Ac)\int \frac{\sqrt{cx^2+bx}}{x^7}dx}{13b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{13bx^8}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(13bB-10Ac)\left(-\frac{8c\int \frac{\sqrt{cx^2+bx}}{x^6}dx}{11b}-\frac{2\left(bx+cx^2\right)^{3/2}}{11bx^7}\right)}{13b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{13bx^8}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(13bB-10Ac)\left(-\frac{8c\left(-\frac{2c\int \frac{\sqrt{cx^2+bx}}{x^5}dx}{3b}-\frac{2(bx+cx^2)^{3/2}}{9bx^6}\right)}{11b}-\frac{2(bx+cx^2)^{3/2}}{11bx^7}\right)}{13b}-\frac{2A(bx+cx^2)^{3/2}}{13bx^8}$$

$\downarrow$ 1129

$$\frac{(13bB-10Ac)\left(-\frac{8c\left(-\frac{2c\left(-\frac{4c\int \frac{\sqrt{cx^2+bx}}{x^4}dx}{7b}-\frac{2(bx+cx^2)^{3/2}}{7bx^5}\right)}{3b}-\frac{2(bx+cx^2)^{3/2}}{9bx^6}\right)}{11b}-\frac{2(bx+cx^2)^{3/2}}{11bx^7}\right)}{13b}-\frac{2A(bx+cx^2)^{3/2}}{13bx^8}$$

$\downarrow$ 1129

$$\frac{(13bB-10Ac)\left(-\frac{8c\left(-\frac{2c\left(-\frac{4c\left(-\frac{2c\int \frac{\sqrt{cx^2+bx}}{x^3}dx}{5b}-\frac{2(bx+cx^2)^{3/2}}{5bx^4}\right)}{7b}-\frac{2(bx+cx^2)^{3/2}}{7bx^5}\right)}{3b}-\frac{2(bx+cx^2)^{3/2}}{9bx^6}\right)}{11b}-\frac{2(bx+cx^2)^{3/2}}{11bx^7}\right)}{13b}-\frac{2A(bx+cx^2)^{3/2}}{13bx^8}$$

$\downarrow$ 1123

$$\frac{\left(-\frac{8c\left(-\frac{2c\left(-\frac{4c\left(\frac{4c\left(bx+cx^2\right)^{3/2}}{15b^2x^3}-\frac{2\left(bx+cx^2\right)^{3/2}}{5bx^4}\right)}{7b}-\frac{2\left(bx+cx^2\right)^{3/2}}{7bx^5}\right)}{3b}-\frac{2\left(bx+cx^2\right)^{3/2}}{9bx^6}\right)}{11b}-\frac{2\left(bx+cx^2\right)^{3/2}}{11bx^7}\right)(13bB-10Ac)}{13b}-\frac{2A\left(bx+cx^2\right)^{3/2}}{13bx^8}$$

input
```
Int[((A + B*x)*Sqrt[b*x + c*x^2])/x^8,x]
```

output
```
(-2*A*(b*x + c*x^2)^(3/2))/(13*b*x^8) + ((13*b*B - 10*A*c)*((-2*(b*x + c*x
^2)^(3/2))/(11*b*x^7) - (8*c*((-2*(b*x + c*x^2)^(3/2))/(9*b*x^6) - (2*c*((
-2*(b*x + c*x^2)^(3/2))/(7*b*x^5) - (4*c*((-2*(b*x + c*x^2)^(3/2))/(5*b*x^
4) + (4*c*(b*x + c*x^2)^(3/2))/(15*b^2*x^3)))/(7*b)))/(3*b)))/(11*b)))/(13
*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.54

method	result
pseudoelliptic	$-\frac{2(cx+b)\sqrt{x(cx+b)}\left(\left(\frac{13Bx}{11}+A\right)b^5-\frac{10c\left(\frac{52Bx}{45}+A\right)x\,b^4}{11}+\frac{80\left(\frac{39Bx}{35}+A\right)c^2x^2b^3}{99}-\frac{160c^3x^3\left(\frac{26Bx}{25}+A\right)b^2}{231}+\frac{128c^4\left(\frac{13Bx}{15}+A\right)x\cdots}{231}\cdots\right)}{13x^7b^6}$
gosper	$-\frac{2(cx+b)\left(-1280A\,c^5x^5+1664Bb\,c^4x^5+1920Ab\,c^4x^4-2496B\,b^2c^3x^4-2400A\,b^2c^3x^3+3120B\,b^3c^2x^3+2800A\,b^3c^2x^2-364\cdots\right)}{45045b^6x^7}$
orering	$-\frac{2(cx+b)\left(-1280A\,c^5x^5+1664Bb\,c^4x^5+1920Ab\,c^4x^4-2496B\,b^2c^3x^4-2400A\,b^2c^3x^3+3120B\,b^3c^2x^3+2800A\,b^3c^2x^2-364\cdots\right)}{45045b^6x^7}$
trager	$-\frac{2\left(-1280A\,c^6x^6+1664Bb\,c^5x^6+640Ab\,c^5x^5-832B\,b^2c^4x^5-480A\,b^2c^4x^4+624B\,b^3c^3x^4+400A\,b^3c^3x^3-520B\,b^4c^2x^3-3\cdots\right)}{45045b^6x^7}$
risch	$-\frac{2(cx+b)\left(-1280A\,c^6x^6+1664Bb\,c^5x^6+640Ab\,c^5x^5-832B\,b^2c^4x^5-480A\,b^2c^4x^4+624B\,b^3c^3x^4+400A\,b^3c^3x^3-520B\,b^4c\cdots\right)}{45045x^6\sqrt{x(cx+b)}\,b^6}$
default	$A\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{3}{2}}}{13b\,x^8}-\frac{10c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{3}{2}}}{11b\,x^7}-\frac{8c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{3}{2}}}{9b\,x^6}-\frac{2c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{3}{2}}}{7b\,x^5}-\frac{4c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{3}{2}}}{5b\,x^4}+\frac{4c\left(c\,x^2+bx\right)^{\frac{3}{2}}}{15b^2x^3}\cdots\right)}{7b}\cdots\right)}{3b}\cdots\right)}{11b}\cdots\right)}{13b}\cdots\right)$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-2/13*(c*x+b)*(x*(c*x+b))^(1/2)*((13/11*B*x+A)*b^5-10/11*c*(52/45*B*x+A)*x
*b^4+80/99*(39/35*B*x+A)*c^2*x^2*b^3-160/231*c^3*x^3*(26/25*B*x+A)*b^2+128
/231*c^4*(13/15*B*x+A)*x^4*b-256/693*A*c^5*x^5)/x^7/b^6
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^8}\,dx =$$
$$-\frac{2\,(3465\,Ab^6+128\,(13\,Bbc^5-10\,Ac^6)x^6-64\,(13\,Bb^2c^4-10\,Abc^5)x^5+48\,(13\,Bb^3c^3-10\,Ab^2c^4)x^4-}{45045\,b^6x^7}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^8,x, algorithm="fricas")
```

output
```
-2/45045*(3465*A*b^6 + 128*(13*B*b*c^5 - 10*A*c^6)*x^6 - 64*(13*B*b^2*c^4
- 10*A*b*c^5)*x^5 + 48*(13*B*b^3*c^3 - 10*A*b^2*c^4)*x^4 - 40*(13*B*b^4*c^
2 - 10*A*b^3*c^3)*x^3 + 35*(13*B*b^5*c - 10*A*b^4*c^2)*x^2 + 315*(13*B*b^6
 + A*b^5*c)*x)*sqrt(c*x^2 + b*x)/(b^6*x^7)
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^8}\,dx = \int \frac{\sqrt{x\,(b+cx)}(A+Bx)}{x^8}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2)/x**8,x)
```

output
```
Integral(sqrt(x*(b + c*x))*(A + B*x)/x**8, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.46

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^8}\,dx = -\frac{256\,\sqrt{cx^2+bx}Bc^5}{3465\,b^5x} + \frac{512\,\sqrt{cx^2+bx}Ac^6}{9009\,b^6x} + \frac{128\,\sqrt{cx^2+bx}Bc^4}{3465\,b^4x^2} - \frac{256\,\sqrt{cx^2+bx}Ac^5}{9009\,b^5x^2} - \frac{32\,\sqrt{cx^2+bx}Bc^3}{1155\,b^3x^3} + \frac{64\,\sqrt{cx^2+bx}Ac^4}{3003\,b^4x^3} + \frac{16\,\sqrt{cx^2+bx}Bc^2}{693\,b^2x^4} - \frac{160\,\sqrt{cx^2+bx}Ac^3}{9009\,b^3x^4} - \frac{2\,\sqrt{cx^2+bx}Bc}{99\,bx^5} + \frac{20\,\sqrt{cx^2+bx}Ac^2}{1287\,b^2x^5} - \frac{2\,\sqrt{cx^2+bx}B}{11\,x^6} - \frac{2\,\sqrt{cx^2+bx}Ac}{143\,bx^6} - \frac{2\,\sqrt{cx^2+bx}A}{13\,x^7}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^8,x, algorithm="maxima")
```

output
```
-256/3465*sqrt(c*x^2 + b*x)*B*c^5/(b^5*x) + 512/9009*sqrt(c*x^2 + b*x)*A*c
^6/(b^6*x) + 128/3465*sqrt(c*x^2 + b*x)*B*c^4/(b^4*x^2) - 256/9009*sqrt(c*
x^2 + b*x)*A*c^5/(b^5*x^2) - 32/1155*sqrt(c*x^2 + b*x)*B*c^3/(b^3*x^3) + 6
4/3003*sqrt(c*x^2 + b*x)*A*c^4/(b^4*x^3) + 16/693*sqrt(c*x^2 + b*x)*B*c^2/
(b^2*x^4) - 160/9009*sqrt(c*x^2 + b*x)*A*c^3/(b^3*x^4) - 2/99*sqrt(c*x^2 +
 b*x)*B*c/(b*x^5) + 20/1287*sqrt(c*x^2 + b*x)*A*c^2/(b^2*x^5) - 2/11*sqrt(
c*x^2 + b*x)*B/x^6 - 2/143*sqrt(c*x^2 + b*x)*A*c/(b*x^6) - 2/13*sqrt(c*x^2
 + b*x)*A/x^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(171) = 342$.

Time = 0.29 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.21

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^8}\,dx = \frac{2\left(144144\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^8Bc^3+480480\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^7Bbc^{\frac{5}{2}}+240240\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^7\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^8,x, algorithm="giac")
```

output
```
2/45045*(144144*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*B*c^3 + 480480*(sqrt(c)*
x - sqrt(c*x^2 + b*x))^7*B*b*c^(5/2) + 240240*(sqrt(c)*x - sqrt(c*x^2 + b*
x))^7*A*c^(7/2) + 669240*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*B*b^2*c^2 + 926
640*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*A*b*c^3 + 495495*(sqrt(c)*x - sqrt(c
*x^2 + b*x))^5*B*b^3*c^(3/2) + 1531530*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*A
*b^2*c^(5/2) + 205205*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*B*b^4*c + 1401400*
(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*A*b^3*c^2 + 45045*(sqrt(c)*x - sqrt(c*x^
2 + b*x))^3*B*b^5*sqrt(c) + 765765*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*A*b^4
*c^(3/2) + 4095*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*b^6 + 249795*(sqrt(c)*
x - sqrt(c*x^2 + b*x))^2*A*b^5*c + 45045*(sqrt(c)*x - sqrt(c*x^2 + b*x))*A
*b^6*sqrt(c) + 3465*A*b^7)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^13
```

Mupad [B] (verification not implemented)

Time = 6.06 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.46

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^8}\,dx = \frac{20\,A\,c^2\,\sqrt{c\,x^2+b\,x}}{1287\,b^2\,x^5} - \frac{2\,B\,\sqrt{c\,x^2+b\,x}}{11\,x^6} - \frac{2\,A\,c\,\sqrt{c\,x^2+b\,x}}{143\,b\,x^6} - \frac{2\,B\,c\,\sqrt{c\,x^2+b\,x}}{99\,b\,x^5} - \frac{2\,A\,\sqrt{c\,x^2+b\,x}}{13\,x^7} - \frac{160\,A\,c^3\,\sqrt{c\,x^2+b\,x}}{9009\,b^3\,x^4} + \frac{64\,A\,c^4\,\sqrt{c\,x^2+b\,x}}{3003\,b^4\,x^3} - \frac{256\,A\,c^5\,\sqrt{c\,x^2+b\,x}}{9009\,b^5\,x^2} + \frac{512\,A\,c^6\,\sqrt{c\,x^2+b\,x}}{9009\,b^6\,x} + \frac{16\,B\,c^2\,\sqrt{c\,x^2+b\,x}}{693\,b^2\,x^4} - \frac{32\,B\,c^3\,\sqrt{c\,x^2+b\,x}}{1155\,b^3\,x^3} + \frac{128\,B\,c^4\,\sqrt{c\,x^2+b\,x}}{3465\,b^4\,x^2} - \frac{256\,B\,c^5\,\sqrt{c\,x^2+b\,x}}{3465\,b^5\,x}$$

input
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^8,x)
```

output
```
(20*A*c^2*(b*x + c*x^2)^(1/2))/(1287*b^2*x^5) - (2*B*(b*x + c*x^2)^(1/2))/
(11*x^6) - (2*A*c*(b*x + c*x^2)^(1/2))/(143*b*x^6) - (2*B*c*(b*x + c*x^2)^
(1/2))/(99*b*x^5) - (2*A*(b*x + c*x^2)^(1/2))/(13*x^7) - (160*A*c^3*(b*x +
 c*x^2)^(1/2))/(9009*b^3*x^4) + (64*A*c^4*(b*x + c*x^2)^(1/2))/(3003*b^4*x
^3) - (256*A*c^5*(b*x + c*x^2)^(1/2))/(9009*b^5*x^2) + (512*A*c^6*(b*x + c
*x^2)^(1/2))/(9009*b^6*x) + (16*B*c^2*(b*x + c*x^2)^(1/2))/(693*b^2*x^4) -
 (32*B*c^3*(b*x + c*x^2)^(1/2))/(1155*b^3*x^3) + (128*B*c^4*(b*x + c*x^2)^
(1/2))/(3465*b^4*x^2) - (256*B*c^5*(b*x + c*x^2)^(1/2))/(3465*b^5*x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^8}\,dx$$
$$= \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^6}{13} - \frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^5cx}{143} + \frac{20\sqrt{x}\,\sqrt{cx+b}\,a\,b^4c^2x^2}{1287} - \frac{160\sqrt{x}\,\sqrt{cx+b}\,a\,b^3c^3x^3}{9009} + \frac{64\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^4x^4}{3003} - \frac{256\sqrt{x}\,\sqrt{cx+b}}{9009}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^8,x)
```

output
```
(2*( - 3465*sqrt(x)*sqrt(b + c*x)*a*b**6 - 315*sqrt(x)*sqrt(b + c*x)*a*b**
5*c*x + 350*sqrt(x)*sqrt(b + c*x)*a*b**4*c**2*x**2 - 400*sqrt(x)*sqrt(b +
c*x)*a*b**3*c**3*x**3 + 480*sqrt(x)*sqrt(b + c*x)*a*b**2*c**4*x**4 - 640*s
qrt(x)*sqrt(b + c*x)*a*b*c**5*x**5 + 1280*sqrt(x)*sqrt(b + c*x)*a*c**6*x**
6 - 4095*sqrt(x)*sqrt(b + c*x)*b**7*x - 455*sqrt(x)*sqrt(b + c*x)*b**6*c*x
**2 + 520*sqrt(x)*sqrt(b + c*x)*b**5*c**2*x**3 - 624*sqrt(x)*sqrt(b + c*x)
*b**4*c**3*x**4 + 832*sqrt(x)*sqrt(b + c*x)*b**3*c**4*x**5 - 1664*sqrt(x)*
sqrt(b + c*x)*b**2*c**5*x**6 - 1280*sqrt(c)*a*c**6*x**7 + 1664*sqrt(c)*b**
2*c**5*x**7))/(45045*b**6*x**7)
```

3.114 $\int x^2(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 264

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{b^5(9bB-14Ac)\sqrt{bx+cx^2}}{1024c^5}-\frac{b^4(9bB-14Ac)x\sqrt{bx+cx^2}}{1536c^4}+\frac{b^3(9bB-14Ac)x^2\sqrt{bx+cx^2}}{1920c^3}-\frac{b^2(9bB-14Ac)x^3\sqrt{bx+cx^2}}{2240c^2}-\frac{13b(9bB-14Ac)x^4\sqrt{bx+cx^2}}{840c}-\frac{1}{84}(9bB-14Ac)x^5\sqrt{bx+cx^2}+\frac{Bx^2\left(bx+cx^2\right)^{5/2}}{7c}-\frac{b^6(9bB-14Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{1024c^{11/2}}$$

output
```
1/1024*b^5*(-14*A*c+9*B*b)*(c*x^2+b*x)^(1/2)/c^5-1/1536*b^4*(-14*A*c+9*B*b
)*x*(c*x^2+b*x)^(1/2)/c^4+1/1920*b^3*(-14*A*c+9*B*b)*x^2*(c*x^2+b*x)^(1/2)
/c^3-1/2240*b^2*(-14*A*c+9*B*b)*x^3*(c*x^2+b*x)^(1/2)/c^2-13/840*b*(-14*A*
c+9*B*b)*x^4*(c*x^2+b*x)^(1/2)/c-1/84*(-14*A*c+9*B*b)*x^5*(c*x^2+b*x)^(1/2
)+1/7*B*x^2*(c*x^2+b*x)^(5/2)/c-1/1024*b^6*(-14*A*c+9*B*b)*arctanh(c^(1/2)
*x/(c*x^2+b*x)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 1.37 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.89

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \frac{(x(b+cx))^{3/2}\left(945b^6B-1470Ab^5c-630b^5Bcx+980Ab^4c^2x+504b^4Bc^2x^2-784Ab^3c^3x^2\right.}{10} - \frac{b^6(9bB-14Ac)(x(b+cx))^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{x}}{-\sqrt{b}+\sqrt{b+cx}}\right)}{512c^{11/2}x^{3/2}(b+cx)^{3/2}}$$

input
```
Integrate[x^2*(A + B*x)*(b*x + c*x^2)^(3/2),x]
```

output
```
((x*(b + c*x))^(3/2)*(945*b^6*B - 1470*A*b^5*c - 630*b^5*B*c*x + 980*A*b^4
*c^2*x + 504*b^4*B*c^2*x^2 - 784*A*b^3*c^3*x^2 - 432*b^3*B*c^3*x^3 + 672*A
*b^2*c^4*x^3 + 384*b^2*B*c^4*x^4 + 23296*A*b*c^5*x^4 + 19200*b*B*c^5*x^5 +
 17920*A*c^6*x^5 + 15360*B*c^6*x^6))/(107520*c^5*x*(b + c*x)) - (b^6*(9*b*
B - 14*A*c)*(x*(b + c*x))^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt
[b + c*x])])/(512*c^(11/2)*x^(3/2)*(b + c*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.75, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1221, 1134, 1160, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{Bx^2\left(bx+cx^2\right)^{5/2}}{7c}-\frac{(9bB-14Ac)\int x^2\left(cx^2+bx\right)^{3/2}dx}{14c}$$

$$\Big\downarrow \text{1134}$$

$$\frac{Bx^2\left(bx+cx^2\right)^{5/2}}{7c}-\frac{(9bB-14Ac)\left(\frac{x\left(bx+cx^2\right)^{5/2}}{6c}-\frac{7b\int x\left(cx^2+bx\right)^{3/2}dx}{12c}\right)}{14c}$$

$\big\downarrow$ 1160

$$\frac{Bx^2\left(bx+cx^2\right)^{5/2}}{7c}-\frac{(9bB-14Ac)\left(\frac{x\left(bx+cx^2\right)^{5/2}}{6c}-\frac{7b\left(\frac{\left(bx+cx^2\right)^{5/2}}{5c}-\frac{b\int\left(cx^2+bx\right)^{3/2}dx}{2c}\right)}{12c}\right)}{14c}$$

$\big\downarrow$ 1087

$$\frac{Bx^2\left(bx+cx^2\right)^{5/2}}{7c}-\frac{(9bB-14Ac)\left(\frac{x\left(bx+cx^2\right)^{5/2}}{6c}-\frac{7b\left(\frac{\left(bx+cx^2\right)^{5/2}}{5c}-\frac{b\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int\sqrt{cx^2+bx}dx}{16c}\right)}{2c}\right)}{12c}\right)}{14c}$$

$\big\downarrow$ 1087

$$\frac{Bx^2\left(bx+cx^2\right)^{5/2}}{7c}-\frac{(9bB-14Ac)\left(\frac{x\left(bx+cx^2\right)^{5/2}}{6c}-\frac{7b\left(\frac{\left(bx+cx^2\right)^{5/2}}{5c}-\frac{b\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)}{2c}\right)}{12c}\right)}{14c}$$

$\big\downarrow$ 1091

$$\frac{Bx^2\left(bx+cx^2\right)^{5/2}}{7c}-\frac{(9bB-14Ac)\left(\frac{x\left(bx+cx^2\right)^{5/2}}{6c}-\frac{7b\left(\frac{\left(bx+cx^2\right)^{5/2}}{5c}-\frac{b\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)}{2c}\right)}{12c}\right)}{14c}$$

$$\downarrow$$

219

$$\frac{Bx^2(bx+cx^2)^{5/2}}{7c} - \frac{(9bB-14Ac)\left(\frac{x(bx+cx^2)^{5/2}}{6c} - \frac{7b\left(\frac{(bx+cx^2)^{5/2}}{5c} - \frac{b\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c} - \frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c} - \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)}{2c}\right)}{12c}\right)}{14c}$$

input
```
Int[x^2*(A + B*x)*(b*x + c*x^2)^(3/2),x]
```

output
```
(B*x^2*(b*x + c*x^2)^(5/2))/(7*c) - ((9*b*B - 14*A*c)*((x*(b*x + c*x^2)^(5
/2))/(6*c) - (7*b*((b*x + c*x^2)^(5/2)/(5*c) - (b*(((b + 2*c*x)*(b*x + c*x
^2)^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*Ar
cTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))/(16*c)))/(2*c)))/(12*c
)))/(14*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.56

method	result
pseudoelliptic	$-\frac{7\left(-\frac{15b^{6}\left(Ac-\frac{9Bb}{14}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{8}+\left(-\frac{208x^{4}b\left(\frac{75Bx}{91}+A\right)c^{\frac{11}{2}}}{7}-\frac{160x^{5}\left(\frac{6Bx}{7}+A\right)c^{\frac{13}{2}}}{7}+\left(\frac{15\left(\frac{3Bx}{7}+A\right)b^{3}c^{\frac{3}{2}}}{8}-\frac{5x\,b^{2}(\ldots}{}\right.\right.\right.}{960c^{\frac{11}{2}}}$
risch	$-\frac{(-15360Bc^{6}x^{6}-17920Ac^{6}x^{5}-19200Bbc^{5}x^{5}-23296Abc^{5}x^{4}-384Bb^{2}c^{4}x^{4}-672Ab^{2}c^{4}x^{3}+432Bb^{3}c^{3}x^{3}+784Ab^{3}c^{3}x^{2}\ldots}{107520c^{5}\sqrt{x(cx+b)}}$
default	$A\left(\frac{x(cx^{2}+bx)^{\frac{5}{2}}}{6c}-\frac{7b\left(\frac{(cx^{2}+bx)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx+b)(cx^{2}+bx)^{\frac{3}{2}}}{8c}-\frac{3b^{2}\left(\frac{(2cx+b)\sqrt{cx^{2}+bx}}{4c}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^{2}+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{2c}\right)}{12c}\right)\ldots$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-7/960*(-15/8*b^6*(A*c-9/14*B*b)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))+(-20
8/7*x^4*b*(75/91*B*x+A)*c^(11/2)-160/7*x^5*(6/7*B*x+A)*c^(13/2)+(15/8*(3/7
*B*x+A)*b^3*c^(3/2)-5/4*x*b^2*(18/35*B*x+A)*c^(5/2)+b*x^2*(27/49*B*x+A)*c^
(7/2)-6/7*(4/7*B*x+A)*x^3*c^(9/2)-135/112*B*c^(1/2)*b^4)*b^2)*(x*(c*x+b))^
(1/2))/c^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.52

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \left[-\frac{105\left(9\,Bb^7-14\,Ab^6c\right)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\left(15360\,Bc^7x^6+945\,Bb^6c-\right.}{}\right.$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/215040*(105*(9*B*b^7 - 14*A*b^6*c)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^
2 + b*x)*sqrt(c)) - 2*(15360*B*c^7*x^6 + 945*B*b^6*c - 1470*A*b^5*c^2 + 12
80*(15*B*b*c^6 + 14*A*c^7)*x^5 + 128*(3*B*b^2*c^5 + 182*A*b*c^6)*x^4 - 48*
(9*B*b^3*c^4 - 14*A*b^2*c^5)*x^3 + 56*(9*B*b^4*c^3 - 14*A*b^3*c^4)*x^2 - 7
0*(9*B*b^5*c^2 - 14*A*b^4*c^3)*x)*sqrt(c*x^2 + b*x))/c^6, 1/107520*(105*(9
*B*b^7 - 14*A*b^6*c)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b))
 + (15360*B*c^7*x^6 + 945*B*b^6*c - 1470*A*b^5*c^2 + 1280*(15*B*b*c^6 + 14
*A*c^7)*x^5 + 128*(3*B*b^2*c^5 + 182*A*b*c^6)*x^4 - 48*(9*B*b^3*c^4 - 14*A
*b^2*c^5)*x^3 + 56*(9*B*b^4*c^3 - 14*A*b^3*c^4)*x^2 - 70*(9*B*b^5*c^2 - 14
*A*b^4*c^3)*x)*sqrt(c*x^2 + b*x))/c^6]
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.68

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\begin{cases}\frac{35b^4\left(Ab^2-\frac{9b\left(2Abc+Bb^2-\frac{11b\left(Ac^2+\frac{15Bbc}{14}\right)}{12c}\right)}{10c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{128c^4}+\sqrt{bx+cx^2}\Bigg(\\ \frac{2\left(\frac{A(bx)^{\frac{9}{2}}}{9}+\frac{B(bx)^{\frac{11}{2}}}{11b}\right)}{b^3} \\ 0\end{cases}$$

input
```
integrate(x**2*(B*x+A)*(c*x**2+b*x)**(3/2),x)
```

output
```
Piecewise((35*b**4*(A*b**2 - 9*b*(2*A*b*c + B*b**2 - 11*b*(A*c**2 + 15*B*b
*c/14)/(12*c))/(10*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2
*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(
2*c) + x)**2), True))/(128*c**4) + sqrt(b*x + c*x**2)*(B*c*x**6/7 - 35*b**
3*(A*b**2 - 9*b*(2*A*b*c + B*b**2 - 11*b*(A*c**2 + 15*B*b*c/14)/(12*c))/(1
0*c))/(64*c**4) + 35*b**2*x*(A*b**2 - 9*b*(2*A*b*c + B*b**2 - 11*b*(A*c**2
 + 15*B*b*c/14)/(12*c))/(10*c))/(96*c**3) - 7*b*x**2*(A*b**2 - 9*b*(2*A*b*
c + B*b**2 - 11*b*(A*c**2 + 15*B*b*c/14)/(12*c))/(10*c))/(24*c**2) + x**5*
(A*c**2 + 15*B*b*c/14)/(6*c) + x**4*(2*A*b*c + B*b**2 - 11*b*(A*c**2 + 15*
B*b*c/14)/(12*c))/(5*c) + x**3*(A*b**2 - 9*b*(2*A*b*c + B*b**2 - 11*b*(A*c
**2 + 15*B*b*c/14)/(12*c))/(10*c))/(4*c)), Ne(c, 0)), (2*(A*(b*x)**(9/2)/9
 + B*(b*x)**(11/2)/(11*b))/b**3, Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.23

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \frac{(cx^2+bx)^{\frac{5}{2}}Bx^2}{7\,c} + \frac{9\sqrt{cx^2+bx}Bb^5x}{512\,c^4} - \frac{3\,(cx^2+bx)^{\frac{3}{2}}Bb^3x}{64\,c^3} - \frac{7\sqrt{cx^2+bx}Ab^4x}{256\,c^3} - \frac{3\,(cx^2+bx)^{\frac{5}{2}}Bbx}{28\,c^2} + \frac{7\,(cx^2+bx)^{\frac{3}{2}}Ab^2x}{96\,c^2} + \frac{(cx^2+bx)^{\frac{5}{2}}Ax}{6\,c} - \frac{9\,Bb^7\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{2048\,c^{\frac{11}{2}}} + \frac{7\,Ab^6\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{1024\,c^{\frac{9}{2}}} + \frac{9\sqrt{cx^2+bx}Bb^6}{1024\,c^5} - \frac{3\,(cx^2+bx)^{\frac{3}{2}}Bb^4}{128\,c^4} - \frac{7\sqrt{cx^2+bx}Ab^5}{512\,c^4} + \frac{3\,(cx^2+bx)^{\frac{5}{2}}Bb^2}{40\,c^3} + \frac{7\,(cx^2+bx)^{\frac{3}{2}}Ab^3}{192\,c^3} - \frac{7\,(cx^2+bx)^{\frac{5}{2}}Ab}{60\,c^2}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
1/7*(c*x^2 + b*x)^(5/2)*B*x^2/c + 9/512*sqrt(c*x^2 + b*x)*B*b^5*x/c^4 - 3/
64*(c*x^2 + b*x)^(3/2)*B*b^3*x/c^3 - 7/256*sqrt(c*x^2 + b*x)*A*b^4*x/c^3 -
 3/28*(c*x^2 + b*x)^(5/2)*B*b*x/c^2 + 7/96*(c*x^2 + b*x)^(3/2)*A*b^2*x/c^2
 + 1/6*(c*x^2 + b*x)^(5/2)*A*x/c - 9/2048*B*b^7*log(2*c*x + b + 2*sqrt(c*x
^2 + b*x)*sqrt(c))/c^(11/2) + 7/1024*A*b^6*log(2*c*x + b + 2*sqrt(c*x^2 +
b*x)*sqrt(c))/c^(9/2) + 9/1024*sqrt(c*x^2 + b*x)*B*b^6/c^5 - 3/128*(c*x^2
+ b*x)^(3/2)*B*b^4/c^4 - 7/512*sqrt(c*x^2 + b*x)*A*b^5/c^4 + 3/40*(c*x^2 +
 b*x)^(5/2)*B*b^2/c^3 + 7/192*(c*x^2 + b*x)^(3/2)*A*b^3/c^3 - 7/60*(c*x^2
+ b*x)^(5/2)*A*b/c^2
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.83

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \frac{1}{107520}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\left(10\left(12\,Bcx+\frac{15\,Bbc^6+14\,Ac^7}{c^6}\right)x+\frac{3\,Bb^2c^5+182\,Abc^6}{c^6}\right.\right.\right.\right.\right. + \frac{(9\,Bb^7-14\,Ab^6c)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{2048\,c^{\frac{11}{2}}}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
1/107520*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(10*(12*B*c*x + (15*B*b*c^6 + 14*A*
c^7)/c^6)*x + (3*B*b^2*c^5 + 182*A*b*c^6)/c^6)*x - 3*(9*B*b^3*c^4 - 14*A*b
^2*c^5)/c^6)*x + 7*(9*B*b^4*c^3 - 14*A*b^3*c^4)/c^6)*x - 35*(9*B*b^5*c^2 -
 14*A*b^4*c^3)/c^6)*x + 105*(9*B*b^6*c - 14*A*b^5*c^2)/c^6) + 1/2048*(9*B*
b^7 - 14*A*b^6*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/
c^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\int x^2\,\left(c\,x^2+b\,x\right)^{3/2}\,(A+B\,x)\,dx$$

input
```
int(x^2*(b*x + c*x^2)^(3/2)*(A + B*x),x)
```

output
```
int(x^2*(b*x + c*x^2)^(3/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.11

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{-1470\sqrt{x}\,\sqrt{cx+b}\,a\,b^5c^2+980\sqrt{x}\,\sqrt{cx+b}\,a\,b^4c^3x-784\sqrt{x}\,\sqrt{cx+b}\,a\,b^3c^4x^2+672\sqrt{x}\,\sqrt{cx}$$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x)^(3/2),x)
```

output
```
( - 1470*sqrt(x)*sqrt(b + c*x)*a*b**5*c**2 + 980*sqrt(x)*sqrt(b + c*x)*a*b
**4*c**3*x - 784*sqrt(x)*sqrt(b + c*x)*a*b**3*c**4*x**2 + 672*sqrt(x)*sqrt
(b + c*x)*a*b**2*c**5*x**3 + 23296*sqrt(x)*sqrt(b + c*x)*a*b*c**6*x**4 + 1
7920*sqrt(x)*sqrt(b + c*x)*a*c**7*x**5 + 945*sqrt(x)*sqrt(b + c*x)*b**7*c
- 630*sqrt(x)*sqrt(b + c*x)*b**6*c**2*x + 504*sqrt(x)*sqrt(b + c*x)*b**5*c
**3*x**2 - 432*sqrt(x)*sqrt(b + c*x)*b**4*c**4*x**3 + 384*sqrt(x)*sqrt(b +
 c*x)*b**3*c**5*x**4 + 19200*sqrt(x)*sqrt(b + c*x)*b**2*c**6*x**5 + 15360*
sqrt(x)*sqrt(b + c*x)*b*c**7*x**6 + 1470*sqrt(c)*log((sqrt(b + c*x) + sqrt
(x)*sqrt(c))/sqrt(b))*a*b**6*c - 945*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*
sqrt(c))/sqrt(b))*b**8)/(107520*c**6)
```

3.115 $\int x(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 227

$$\int x(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=-\frac{b^4(7bB-12Ac)\sqrt{bx+cx^2}}{512c^4}+\frac{b^3(7bB-12Ac)x\sqrt{bx+cx^2}}{768c^3}-\frac{b^2(7bB-12Ac)x^2\sqrt{bx+cx^2}}{960c^2}-\frac{11b(7bB-12Ac)x^3\sqrt{bx+cx^2}}{480c}-\frac{1}{60}(7bB-12Ac)x^4\sqrt{bx+cx^2}+\frac{Bx\left(bx+cx^2\right)^{5/2}}{6c}+\frac{b^5(7bB-12Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{512c^{9/2}}$$

output
```
-1/512*b^4*(-12*A*c+7*B*b)*(c*x^2+b*x)^(1/2)/c^4+1/768*b^3*(-12*A*c+7*B*b)
*x*(c*x^2+b*x)^(1/2)/c^3-1/960*b^2*(-12*A*c+7*B*b)*x^2*(c*x^2+b*x)^(1/2)/c
^2-11/480*b*(-12*A*c+7*B*b)*x^3*(c*x^2+b*x)^(1/2)/c-1/60*(-12*A*c+7*B*b)*x
^4*(c*x^2+b*x)^(1/2)+1/6*B*x*(c*x^2+b*x)^(5/2)/c+1/512*b^5*(-12*A*c+7*B*b)
*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.96

$$\int x(A+Bx)\left(bx+cx^2\right)^{3/2}dx = \frac{\sqrt{x}\sqrt{b+cx}\left(\sqrt{c}\sqrt{x}\sqrt{b+cx}(-105b^5B+48b^2c^3x^2(2A+Bx)+256c^5x^4(6A+5Bx)-8b^3c^2\right.}{}$$

input
```
Integrate[x*(A + B*x)*(b*x + c*x^2)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[b + c*x]*(Sqrt[c]*Sqrt[x]*Sqrt[b + c*x]*(-105*b^5*B + 48*b^2
*c^3*x^2*(2*A + B*x) + 256*c^5*x^4*(6*A + 5*B*x) - 8*b^3*c^2*x*(15*A + 7*B
*x) + 10*b^4*c*(18*A + 7*B*x) + 64*b*c^4*x^3*(33*A + 26*B*x)) + 360*A*b^5*
c*ArcTanh[(Sqrt[c]*Sqrt[x])/(Sqrt[b] - Sqrt[b + c*x])] + 210*b^6*B*ArcTanh
[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])]))/(7680*c^(9/2)*Sqrt[x*(b +
 c*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.66, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1225, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(A+Bx)\left(bx+cx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{1225}$$

$$\frac{b(7bB-12Ac)\int \left(cx^2+bx\right)^{3/2}dx}{24c^2}-\frac{\left(bx+cx^2\right)^{5/2}(-12Ac+7bB-10Bcx)}{60c^2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{b(7bB-12Ac)\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\int\sqrt{cx^2+bx}dx}{16c}\right)}{24c^2}-\frac{(bx+cx^2)^{5/2}(-12Ac+7bB-10Bcx)}{60c^2}$$

$$\Big\downarrow \text{ 1087}$$

$$\frac{b(7bB-12Ac)\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)}{24c^2}-\frac{(bx+cx^2)^{5/2}(-12Ac+7bB-10Bcx)}{60c^2}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{b(7bB-12Ac)\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)}{24c^2}-\frac{(bx+cx^2)^{5/2}(-12Ac+7bB-10Bcx)}{60c^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{b(7bB-12Ac)\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)}{24c^2}-\frac{(bx+cx^2)^{5/2}(-12Ac+7bB-10Bcx)}{60c^2}$$

input
```
Int[x*(A + B*x)*(b*x + c*x^2)^(3/2),x]
```

output
```
-1/60*((7*b*B - 12*A*c - 10*B*c*x)*(b*x + c*x^2)^(5/2))/c^2 + (b*(7*b*B -
12*A*c)*(((b + 2*c*x)*(b*x + c*x^2)^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x)*Sq
rt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c
^(3/2))))/(16*c)))/(24*c^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.59

method	result
pseudoelliptic	$-\frac{\left(\frac{3}{2}A\,b^5c-\frac{7}{8}B\,b^6\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)+\sqrt{x(cx+b)}\left(-\frac{64x^4\left(\frac{5Bx}{6}+A\right)c^{\frac{11}{2}}}{5}+\left(-\frac{3\left(\frac{7Bx}{18}+A\right)b^3c^{\frac{3}{2}}}{2}+b^2x\left(\frac{7Bx}{15}+A\right)c^{\frac{5}{2}}-\ldots\right.\right.}{64c^{\frac{9}{2}}}$
risch	$\frac{(1280B\,c^5x^5+1536A\,c^5x^4+1664Bb\,c^4x^4+2112Ab\,c^4x^3+48B\,b^2c^3x^3+96A\,b^2c^3x^2-56B\,b^3c^2x^2-120A\,b^3c^2x+70B\,b^4cx+\ldots}{7680c^4\sqrt{x(cx+b)}}$
default	$A\left(\frac{(c\,x^2+bx)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx+b)(c\,x^2+bx)^{\frac{3}{2}}}{8c}-\frac{3b^2\left(\frac{(2cx+b)\sqrt{c\,x^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{2c}\right)+B\left(\frac{x(c\,x^2+bx\ldots}{6c}\ldots\right.$

input
```
int(x*(B*x+A)*(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/64/c^(9/2)*((3/2*A*b^5*c-7/8*B*b^6)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2)
)+(x*(c*x+b))^(1/2)*(-64/5*x^4*(5/6*B*x+A)*c^(11/2)+(-3/2*(7/18*B*x+A)*b^3
*c^(3/2)+b^2*x*(7/15*B*x+A)*c^(5/2)-4/5*(1/2*B*x+A)*x^2*b*c^(7/2)-88/5*(26
/33*B*x+A)*x^3*c^(9/2)+7/8*B*c^(1/2)*b^4)*b))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 351, normalized size of antiderivative = 1.55

$$\int x(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \left[-\frac{15\,(7\,Bb^6-12\,Ab^5c)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(1280\,Bc^6x^5-105\,Bb^5c+1}{}\right.$$

$$-\frac{15\,(7\,Bb^6-12\,Ab^5c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(1280\,Bc^6x^5-105\,Bb^5c+180\,Ab^4c^2+128\,(13\,Bbc^5+}{768\mathrm{0}}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/15360*(15*(7*B*b^6 - 12*A*b^5*c)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2
+ b*x)*sqrt(c)) - 2*(1280*B*c^6*x^5 - 105*B*b^5*c + 180*A*b^4*c^2 + 128*(1
3*B*b*c^5 + 12*A*c^6)*x^4 + 48*(B*b^2*c^4 + 44*A*b*c^5)*x^3 - 8*(7*B*b^3*c
^3 - 12*A*b^2*c^4)*x^2 + 10*(7*B*b^4*c^2 - 12*A*b^3*c^3)*x)*sqrt(c*x^2 + b
*x))/c^5, -1/7680*(15*(7*B*b^6 - 12*A*b^5*c)*sqrt(-c)*arctan(sqrt(c*x^2 +
b*x)*sqrt(-c)/(c*x + b)) - (1280*B*c^6*x^5 - 105*B*b^5*c + 180*A*b^4*c^2 +
 128*(13*B*b*c^5 + 12*A*c^6)*x^4 + 48*(B*b^2*c^4 + 44*A*b*c^5)*x^3 - 8*(7*
B*b^3*c^3 - 12*A*b^2*c^4)*x^2 + 10*(7*B*b^4*c^2 - 12*A*b^3*c^3)*x)*sqrt(c*
x^2 + b*x))/c^5]
```

Sympy [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 386, normalized size of antiderivative = 1.70

$$\int x(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \begin{cases} -\frac{5b^3\left(Ab^2-\frac{7b\left(2Abc+Bb^2-\frac{9b\left(Ac^2+\frac{13Bbc}{12}\right)}{10c}\right)}{8c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{16c^3}+\sqrt{bx+cx^2}\left(\right. \\ \frac{2\left(\frac{A(bx)^{\frac{7}{2}}}{7}+\frac{B(bx)^{\frac{9}{2}}}{9b}\right)}{b^2} \\ 0 \end{cases}$$

input
```
integrate(x*(B*x+A)*(c*x**2+b*x)**(3/2),x)
```

output
```
Piecewise((-5*b**3*(A*b**2 - 7*b*(2*A*b*c + B*b**2 - 9*b*(A*c**2 + 13*B*b*
c/12)/(10*c))/(8*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c
*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*
c) + x)**2), True))/(16*c**3) + sqrt(b*x + c*x**2)*(B*c*x**5/6 + 5*b**2*(A
*b**2 - 7*b*(2*A*b*c + B*b**2 - 9*b*(A*c**2 + 13*B*b*c/12)/(10*c))/(8*c))/
(8*c**3) - 5*b*x*(A*b**2 - 7*b*(2*A*b*c + B*b**2 - 9*b*(A*c**2 + 13*B*b*c/
12)/(10*c))/(8*c))/(12*c**2) + x**4*(A*c**2 + 13*B*b*c/12)/(5*c) + x**3*(2
*A*b*c + B*b**2 - 9*b*(A*c**2 + 13*B*b*c/12)/(10*c))/(4*c) + x**2*(A*b**2
- 7*b*(2*A*b*c + B*b**2 - 9*b*(A*c**2 + 13*B*b*c/12)/(10*c))/(8*c))/(3*c))
, Ne(c, 0)), (2*(A*(b*x)**(7/2)/7 + B*(b*x)**(9/2)/(9*b))/b**2, Ne(b, 0)),
 (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.23

$$\int x(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = -\frac{7\sqrt{cx^2+bx}Bb^4x}{256\,c^3} + \frac{7\,(cx^2+bx)^{\frac{3}{2}}Bb^2x}{96\,c^2} + \frac{3\sqrt{cx^2+bx}Ab^3x}{64\,c^2} + \frac{(cx^2+bx)^{\frac{5}{2}}Bx}{6\,c} - \frac{(cx^2+bx)^{\frac{3}{2}}Abx}{8\,c} + \frac{7\,Bb^6\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{1024\,c^{\frac{9}{2}}} - \frac{3\,Ab^5\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{256\,c^{\frac{7}{2}}} - \frac{7\sqrt{cx^2+bx}Bb^5}{512\,c^4} + \frac{7\,(cx^2+bx)^{\frac{3}{2}}Bb^3}{192\,c^3} + \frac{3\sqrt{cx^2+bx}Ab^4}{128\,c^3} - \frac{7\,(cx^2+bx)^{\frac{5}{2}}Bb}{60\,c^2} - \frac{(cx^2+bx)^{\frac{3}{2}}Ab^2}{16\,c^2} + \frac{(cx^2+bx)^{\frac{5}{2}}A}{5\,c}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
-7/256*sqrt(c*x^2 + b*x)*B*b^4*x/c^3 + 7/96*(c*x^2 + b*x)^(3/2)*B*b^2*x/c^
2 + 3/64*sqrt(c*x^2 + b*x)*A*b^3*x/c^2 + 1/6*(c*x^2 + b*x)^(5/2)*B*x/c - 1
/8*(c*x^2 + b*x)^(3/2)*A*b*x/c + 7/1024*B*b^6*log(2*c*x + b + 2*sqrt(c*x^2
 + b*x)*sqrt(c))/c^(9/2) - 3/256*A*b^5*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)
*sqrt(c))/c^(7/2) - 7/512*sqrt(c*x^2 + b*x)*B*b^5/c^4 + 7/192*(c*x^2 + b*x
)^(3/2)*B*b^3/c^3 + 3/128*sqrt(c*x^2 + b*x)*A*b^4/c^3 - 7/60*(c*x^2 + b*x)
^(5/2)*B*b/c^2 - 1/16*(c*x^2 + b*x)^(3/2)*A*b^2/c^2 + 1/5*(c*x^2 + b*x)^(5
/2)*A/c
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.85

$$\int x(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \frac{1}{7680}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\left(10\,Bcx+\frac{13\,Bbc^5+12\,Ac^6}{c^5}\right)x+\frac{3\,(Bb^2c^4+44\,Abc^5)}{c^5}\right)x-\right.\right.\right.\right. - \frac{(7\,Bb^6-12\,Ab^5c)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{1024\,c^{\frac{9}{2}}}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(10*B*c*x + (13*B*b*c^5 + 12*A*c^6)/c
^5)*x + 3*(B*b^2*c^4 + 44*A*b*c^5)/c^5)*x - (7*B*b^3*c^3 - 12*A*b^2*c^4)/c
^5)*x + 5*(7*B*b^4*c^2 - 12*A*b^3*c^3)/c^5)*x - 15*(7*B*b^5*c - 12*A*b^4*c
^2)/c^5) - 1/1024*(7*B*b^6 - 12*A*b^5*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2
 + b*x))*sqrt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\int x\,\left(c\,x^2+b\,x\right)^{3/2}\,\left(A+B\,x\right)\,dx$$

input
```
int(x*(b*x + c*x^2)^(3/2)*(A + B*x),x)
```

output
```
int(x*(b*x + c*x^2)^(3/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.11

$$\int x(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{180\sqrt{x}\,\sqrt{cx+b}\,a\,b^4c^2-120\sqrt{x}\,\sqrt{cx+b}\,a\,b^3c^3x+96\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^4x^2+2112\sqrt{x}\,\sqrt{cx+}}{}$$

input
```
int(x*(B*x+A)*(c*x^2+b*x)^(3/2),x)
```

output

```
(180*sqrt(x)*sqrt(b + c*x)*a*b**4*c**2 - 120*sqrt(x)*sqrt(b + c*x)*a*b**3*
c**3*x + 96*sqrt(x)*sqrt(b + c*x)*a*b**2*c**4*x**2 + 2112*sqrt(x)*sqrt(b +
 c*x)*a*b*c**5*x**3 + 1536*sqrt(x)*sqrt(b + c*x)*a*c**6*x**4 - 105*sqrt(x)
*sqrt(b + c*x)*b**6*c + 70*sqrt(x)*sqrt(b + c*x)*b**5*c**2*x - 56*sqrt(x)*
sqrt(b + c*x)*b**4*c**3*x**2 + 48*sqrt(x)*sqrt(b + c*x)*b**3*c**4*x**3 + 1
664*sqrt(x)*sqrt(b + c*x)*b**2*c**5*x**4 + 1280*sqrt(x)*sqrt(b + c*x)*b*c*
*6*x**5 - 180*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*b**
5*c + 105*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**7)/(76
80*c**5)
```

3.116 $\int (A+Bx)\left(bx+cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 186

$$\int (A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{3b^3(bB-2Ac)\sqrt{bx+cx^2}}{128c^3}-\frac{b^2(bB-2Ac)x\sqrt{bx+cx^2}}{64c^2}-\frac{3b(bB-2Ac)x^2\sqrt{bx+cx^2}}{16c}-\frac{1}{8}(bB-2Ac)x^3\sqrt{bx+cx^2}+\frac{B\left(bx+cx^2\right)^{5/2}}{5c}-\frac{3b^4(bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{128c^{7/2}}$$

output

```
3/128*b^3*(-2*A*c+B*b)*(c*x^2+b*x)^(1/2)/c^3-1/64*b^2*(-2*A*c+B*b)*x*(c*x^
2+b*x)^(1/2)/c^2-3/16*b*(-2*A*c+B*b)*x^2*(c*x^2+b*x)^(1/2)/c-1/8*(-2*A*c+B
*b)*x^3*(c*x^2+b*x)^(1/2)+1/5*B*(c*x^2+b*x)^(5/2)/c-3/128*b^4*(-2*A*c+B*b)
*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.86 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.90

$$\int (A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{(x(b+cx))^{3/2}\left(\frac{\sqrt{c}\sqrt{x}(15b^4B-10b^3c(3A+Bx)+4b^2c^2x(5A+2Bx)+32c^4x^3(5A+4Bx)+16bc^3x^2(15A+11Bx))}{b+cx}+\frac{30b^4\cdots}{\cdots}\right.}{640c^{7/2}x^{3/2}}$$

input
```
Integrate[(A + B*x)*(b*x + c*x^2)^(3/2),x]
```

output
```
((x*(b + c*x))^(3/2)*((Sqrt[c]*Sqrt[x]*(15*b^4*B - 10*b^3*c*(3*A + B*x) +
4*b^2*c^2*x*(5*A + 2*B*x) + 32*c^4*x^3*(5*A + 4*B*x) + 16*b*c^3*x^2*(15*A
+ 11*B*x)))/(b + c*x) + (30*b^4*(b*B - 2*A*c)*ArcTanh[(Sqrt[c]*Sqrt[x])/(S
qrt[b] - Sqrt[b + c*x])])/(b + c*x)^(3/2)))/(640*c^(7/2)*x^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {1160, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)\left(bx+cx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{B\left(bx+cx^2\right)^{5/2}}{5c}-\frac{(bB-2Ac)\int \left(cx^2+bx\right)^{3/2}dx}{2c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{B\left(bx+cx^2\right)^{5/2}}{5c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int \sqrt{cx^2+bx}dx}{16c}\right)}{2c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{B\left(bx+cx^2\right)^{5/2}}{5c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)}{2c}$$

$$\Big\downarrow \ 1091$$

$$\frac{B\left(bx+cx^2\right)^{5/2}}{5c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)}{2c}$$

$$\Big\downarrow \ 219$$

$$\frac{B\left(bx+cx^2\right)^{5/2}}{5c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)}{2c}$$

input

```
Int[(A + B*x)*(b*x + c*x^2)^(3/2),x]
```

output

```
(B*(b*x + c*x^2)^(5/2))/(5*c) - ((b*B - 2*A*c)*(((b + 2*c*x)*(b*x + c*x^2)
^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTa
nh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))/(16*c)))/(2*c)
```

Defintions of rubi rules used

rule 219

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$\frac{\left(\frac{3}{2} A b^4 c-\frac{3}{4} b^5 B\right) \operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)+\left(-\frac{3\left(\frac{Bx}{3}+A\right) b^3 c^{\frac{3}{2}}}{2}+b^2 x\left(\frac{2Bx}{5}+A\right) c^{\frac{5}{2}}+12\left(\frac{11Bx}{15}+A\right) x^2 b c^{\frac{7}{2}}+8x^3\left(\frac{4Bx}{5}+A\right) c^{\frac{9}{2}} -}{32c^{\frac{7}{2}}}$
risch	$-\frac{\left(-128B c^4x^4-160A c^4x^3-176Bb c^3x^3-240Ab c^3x^2-8B b^2c^2x^2-20A b^2c^2x+10B b^3cx+30A b^3c-15B b^4\right)x(cx+b)}{640c^3\sqrt{x(cx+b)}}+$
default	$A\left(\frac{(2cx+b)(c x^2+bx)^{\frac{3}{2}}}{8c}-\frac{3b^2\left(\frac{(2cx+b)\sqrt{c x^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c x^2+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+B\left(\frac{(c x^2+bx)^{\frac{5}{2}}}{5c}-\frac{b\left(\frac{(2cx+b)}{}\right.}{}\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/32/c^(7/2)*((3/2*A*b^4*c-3/4*b^5*B)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))
+(-3/2*(1/3*B*x+A)*b^3*c^(3/2)+b^2*x*(2/5*B*x+A)*c^(5/2)+12*(11/15*B*x+A)*
x^2*b*c^(7/2)+8*x^3*(4/5*B*x+A)*c^(9/2)+3/4*B*c^(1/2)*b^4)*(x*(c*x+b))^(1/
2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.60

$$\int (A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \left[-\frac{15\left(Bb^5-2\,Ab^4c\right)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\left(128\,Bc^5x^4+15\,Bb^4c-30\,Ab^3\right.}{12}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/1280*(15*(B*b^5 - 2*A*b^4*c)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*
x)*sqrt(c)) - 2*(128*B*c^5*x^4 + 15*B*b^4*c - 30*A*b^3*c^2 + 16*(11*B*b*c^
4 + 10*A*c^5)*x^3 + 8*(B*b^2*c^3 + 30*A*b*c^4)*x^2 - 10*(B*b^3*c^2 - 2*A*b
^2*c^3)*x)*sqrt(c*x^2 + b*x))/c^4, 1/640*(15*(B*b^5 - 2*A*b^4*c)*sqrt(-c)*
arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (128*B*c^5*x^4 + 15*B*b^4*c
 - 30*A*b^3*c^2 + 16*(11*B*b*c^4 + 10*A*c^5)*x^3 + 8*(B*b^2*c^3 + 30*A*b*c
^4)*x^2 - 10*(B*b^3*c^2 - 2*A*b^2*c^3)*x)*sqrt(c*x^2 + b*x))/c^4]
```

Sympy [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.75

$$\int (A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \begin{cases} \frac{3b^2\left(Ab^2-\frac{5b\left(2Abc+Bb^2-\frac{7b\left(Ac^2+\frac{11Bbc}{10}\right)}{8c}\right)}{6c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{8c^2}+\sqrt{bx+cx^2}\left(\frac{B \cdots}{}\right. \\ \frac{2\left(\frac{A(bx)^{\frac{5}{2}}}{5}+\frac{B(bx)^{\frac{7}{2}}}{7b}\right)}{b} \\ 0 \end{cases}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2),x)
```

output
```
Piecewise((3*b**2*(A*b**2 - 5*b*(2*A*b*c + B*b**2 - 7*b*(A*c**2 + 11*B*b*c
/10)/(8*c))/(6*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x
)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c)
 + x)**2), True))/(8*c**2) + sqrt(b*x + c*x**2)*(B*c*x**4/5 - 3*b*(A*b**2
- 5*b*(2*A*b*c + B*b**2 - 7*b*(A*c**2 + 11*B*b*c/10)/(8*c))/(6*c))/(4*c**2
) + x**3*(A*c**2 + 11*B*b*c/10)/(4*c) + x**2*(2*A*b*c + B*b**2 - 7*b*(A*c*
*2 + 11*B*b*c/10)/(8*c))/(3*c) + x*(A*b**2 - 5*b*(2*A*b*c + B*b**2 - 7*b*(
A*c**2 + 11*B*b*c/10)/(8*c))/(6*c))/(2*c)), Ne(c, 0)), (2*(A*(b*x)**(5/2)/
5 + B*(b*x)**(7/2)/(7*b))/b, Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.27

$$\int (A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \frac{1}{4}\left(cx^2+bx\right)^{\frac{3}{2}}Ax + \frac{3\sqrt{cx^2+bx}Bb^3x}{64\,c^2} - \frac{\left(cx^2+bx\right)^{\frac{3}{2}}Bbx}{8\,c} - \frac{3\sqrt{cx^2+bx}Ab^2x}{32\,c} - \frac{3\,Bb^5\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{256\,c^{\frac{7}{2}}} + \frac{3\,Ab^4\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{128\,c^{\frac{5}{2}}} + \frac{3\sqrt{cx^2+bx}Bb^4}{128\,c^3} - \frac{\left(cx^2+bx\right)^{\frac{3}{2}}Bb^2}{16\,c^2} - \frac{3\sqrt{cx^2+bx}Ab^3}{64\,c^2} + \frac{\left(cx^2+bx\right)^{\frac{5}{2}}B}{5\,c} + \frac{\left(cx^2+bx\right)^{\frac{3}{2}}Ab}{8\,c}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
1/4*(c*x^2 + b*x)^(3/2)*A*x + 3/64*sqrt(c*x^2 + b*x)*B*b^3*x/c^2 - 1/8*(c*
x^2 + b*x)^(3/2)*B*b*x/c - 3/32*sqrt(c*x^2 + b*x)*A*b^2*x/c - 3/256*B*b^5*
log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(7/2) + 3/128*A*b^4*log(2*c
*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(5/2) + 3/128*sqrt(c*x^2 + b*x)*B*
b^4/c^3 - 1/16*(c*x^2 + b*x)^(3/2)*B*b^2/c^2 - 3/64*sqrt(c*x^2 + b*x)*A*b^
3/c^2 + 1/5*(c*x^2 + b*x)^(5/2)*B/c + 1/8*(c*x^2 + b*x)^(3/2)*A*b/c
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

$$\int (A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \frac{1}{640}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\,Bcx+\frac{11\,Bbc^4+10\,Ac^5}{c^4}\right)x+\frac{Bb^2c^3+30\,Abc^4}{c^4}\right)x-\frac{5\left(Bb^3c^2 \cdots\right.}{c \cdots}\right.\right. + \frac{3\left(Bb^5-2\,Ab^4c\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{256\,c^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
1/640*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*B*c*x + (11*B*b*c^4 + 10*A*c^5)/c^4)*x
 + (B*b^2*c^3 + 30*A*b*c^4)/c^4)*x - 5*(B*b^3*c^2 - 2*A*b^2*c^3)/c^4)*x +
15*(B*b^4*c - 2*A*b^3*c^2)/c^4) + 3/256*(B*b^5 - 2*A*b^4*c)*log(abs(2*(sqr
t(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(7/2)
```

Mupad [B] (verification not implemented)

Time = 5.65 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.12

$$\int (A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{B\left(c\,x^2+b\,x\right)^{5/2}}{5\,c}+\frac{A\left(c\,x^2+b\,x\right)^{3/2}\left(\frac{b}{2}+c\,x\right)}{4\,c}-\frac{B\,b\left(\frac{x\left(c\,x^2+b\,x\right)^{3/2}}{4}+\frac{b\left(c\,x^2+b\,x\right)^{3/2}}{8\,c}-\frac{3\,b^2\left(\frac{\sqrt{c\,x^2+b\,x}\,(b+2\,c\,x)}{4\,c}-\frac{b^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}\right)}{16\,c}\right)}{2\,c}-\frac{3\,A\,b^2\left(\sqrt{c\,x^2+b\,x}\left(\frac{x}{2}+\frac{b}{4\,c}\right)-\frac{b^2\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{8\,c^{3/2}}\right)}{16\,c}$$

input
```
int((b*x + c*x^2)^(3/2)*(A + B*x),x)
```

output
```
(B*(b*x + c*x^2)^(5/2))/(5*c) + (A*(b*x + c*x^2)^(3/2)*(b/2 + c*x))/(4*c)
- (B*b*((x*(b*x + c*x^2)^(3/2))/4 + (b*(b*x + c*x^2)^(3/2))/(8*c) - (3*b^2
*(((b*x + c*x^2)^(1/2)*(b + 2*c*x))/(4*c) - (b^2*log((b/2 + c*x)/c^(1/2) +
 (b*x + c*x^2)^(1/2)))/(8*c^(3/2))))/(16*c)))/(2*c) - (3*A*b^2*((b*x + c*x
^2)^(1/2)*(x/2 + b/(4*c)) - (b^2*log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(
1/2)))/(8*c^(3/2))))/(16*c)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.15

$$\int (A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \frac{-30\sqrt{x}\,\sqrt{cx+b}\,a\,b^3c^2+20\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^3x+240\sqrt{x}\,\sqrt{cx+b}\,ab\,c^4x^2+160\sqrt{x}\,\sqrt{cx+b}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2),x)
```

output
```
( - 30*sqrt(x)*sqrt(b + c*x)*a*b**3*c**2 + 20*sqrt(x)*sqrt(b + c*x)*a*b**2
*c**3*x + 240*sqrt(x)*sqrt(b + c*x)*a*b*c**4*x**2 + 160*sqrt(x)*sqrt(b + c
*x)*a*c**5*x**3 + 15*sqrt(x)*sqrt(b + c*x)*b**5*c - 10*sqrt(x)*sqrt(b + c*
x)*b**4*c**2*x + 8*sqrt(x)*sqrt(b + c*x)*b**3*c**3*x**2 + 176*sqrt(x)*sqrt
(b + c*x)*b**2*c**4*x**3 + 128*sqrt(x)*sqrt(b + c*x)*b*c**5*x**4 + 30*sqrt
(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*b**4*c - 15*sqrt(c)*l
og((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**6)/(640*c**4)
```

3.117 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 159

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x}\,dx = -\frac{b^2(3bB-8Ac)\sqrt{bx+cx^2}}{64c^2} - \frac{7b(3bB-8Ac)x\sqrt{bx+cx^2}}{96c} - \frac{1}{24}(3bB-8Ac)x^2\sqrt{bx+cx^2} + \frac{B\left(bx+cx^2\right)^{5/2}}{4cx} + \frac{b^3(3bB-8Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{5/2}}$$

output
```
-1/64*b^2*(-8*A*c+3*B*b)*(c*x^2+b*x)^(1/2)/c^2-7/96*b*(-8*A*c+3*B*b)*x*(c*
x^2+b*x)^(1/2)/c-1/24*(-8*A*c+3*B*b)*x^2*(c*x^2+b*x)^(1/2)+1/4*B*(c*x^2+b*
x)^(5/2)/c/x+1/64*b^3*(-8*A*c+3*B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/
c^(5/2)
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x}\,dx = \frac{\sqrt{x}\sqrt{b+cx}\left(\sqrt{c}\sqrt{x}\sqrt{b+cx}(-9b^3B+6b^2c(4A+Bx)+16c^3x^2(4A+3Bx\right.}{192}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x,x]
```

output
```
(Sqrt[x]*Sqrt[b + c*x]*(Sqrt[c]*Sqrt[x]*Sqrt[b + c*x]*(-9*b^3*B + 6*b^2*c*
(4*A + B*x) + 16*c^3*x^2*(4*A + 3*B*x) + 8*b*c^2*x*(14*A + 9*B*x)) + 48*A*
b^3*c*ArcTanh[(Sqrt[c]*Sqrt[x])/(Sqrt[b] - Sqrt[b + c*x])] + 18*b^4*B*ArcT
anh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])]))/(192*c^(5/2)*Sqrt[x*(b
 + c*x)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.78, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1221, 1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x}\,dx$$

$$\Big\downarrow \ 1221$$

$$\frac{B\left(bx+cx^2\right)^{5/2}}{4cx}-\frac{(3bB-8Ac)\int \frac{\left(cx^2+bx\right)^{3/2}}{x}dx}{8c}$$

$$\Big\downarrow \ 1131$$

$$\frac{B\left(bx+cx^2\right)^{5/2}}{4cx}-\frac{(3bB-8Ac)\left(\frac{1}{2}b\int \sqrt{cx^2+bx}dx+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)}{8c}$$

$$\Big\downarrow \ 1087$$

$$\frac{B\left(bx+cx^2\right)^{5/2}}{4cx}-\frac{(3bB-8Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)}{8c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{B\left(bx+cx^2\right)^{5/2}}{4cx}-\frac{(3bB-8Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)}{8c}$$

$$\Big\downarrow \text{219}$$

$$\frac{B\left(bx+cx^2\right)^{5/2}}{4cx}-\frac{(3bB-8Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)}{8c}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x,x]
```

output
```
(B*(b*x + c*x^2)^(5/2))/(4*c*x) - ((3*b*B - 8*A*c)*((b*x + c*x^2)^(3/2)/3
+ (b*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqr
t[b*x + c*x^2]])/(4*c^(3/2))))/2))/(8*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$\frac{\frac{7\left(-\frac{3}{14}A\,b^{3}c+\frac{9}{112}B\,b^{4}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{12}+\frac{7\sqrt{x(cx+b)}\left(\frac{3b^{2}\left(\frac{Bx}{4}+A\right)c^{\frac{3}{2}}}{14}+bx\left(\frac{9Bx}{14}+A\right)c^{\frac{5}{2}}+\frac{4x^{2}\left(\frac{3Bx}{4}+A\right)c^{\frac{7}{2}}}{7}-\frac{9B\sqrt{c}\,b^{3}}{112}\right)}{12}}{c^{\frac{5}{2}}}$
risch	$\frac{\left(48B\,c^{3}x^{3}+64A\,c^{3}x^{2}+72Bb\,c^{2}x^{2}+112Ab\,c^{2}x+6B\,b^{2}cx+24A\,b^{2}c-9B\,b^{3}\right)x(cx+b)}{192c^{2}\sqrt{x(cx+b)}}-\frac{b^{3}(8Ac-3Bb)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right.}{128c^{\frac{5}{2}}}$
default	$B\left(\frac{(2cx+b)\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{8c}-\frac{3b^{2}\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx}}{4c}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)+A\left(\frac{\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{3}+\frac{b\left(\frac{(2cx+b)}{\right.}}{}\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
7/12/c^(5/2)*((-3/14*A*b^3*c+9/112*B*b^4)*arctanh((x*(c*x+b))^(1/2)/x/c^(1
/2))+(x*(c*x+b))^(1/2)*(3/14*b^2*(1/4*B*x+A)*c^(3/2)+b*x*(9/14*B*x+A)*c^(5
/2)+4/7*x^2*(3/4*B*x+A)*c^(7/2)-9/112*B*c^(1/2)*b^3))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.62

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x}\,dx = \left[-\frac{3\left(3\,Bb^4-8\,Ab^3c\right)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\left(48\,Bc^4x^3-\right.}{}\right.$$

$$-\frac{3\left(3\,Bb^4-8\,Ab^3c\right)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-\left(48\,Bc^4x^3-9\,Bb^3c+24\,Ab^2c^2+8\left(9\,Bbc^3+8\,Ac^4\right)x^2+\right.}{192\,c^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x,x, algorithm="fricas")
```

output
```
[-1/384*(3*(3*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*
x)*sqrt(c)) - 2*(48*B*c^4*x^3 - 9*B*b^3*c + 24*A*b^2*c^2 + 8*(9*B*b*c^3 +
8*A*c^4)*x^2 + 2*(3*B*b^2*c^2 + 56*A*b*c^3)*x)*sqrt(c*x^2 + b*x))/c^3, -1/
192*(3*(3*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c
*x + b)) - (48*B*c^4*x^3 - 9*B*b^3*c + 24*A*b^2*c^2 + 8*(9*B*b*c^3 + 8*A*c
^4)*x^2 + 2*(3*B*b^2*c^2 + 56*A*b*c^3)*x)*sqrt(c*x^2 + b*x))/c^3]
```

Sympy [A] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 490, normalized size of antiderivative = 3.08

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x,x)
```

output
```
A*b*Piecewise((-b**2*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c
*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*
c) + x)**2), True))/(8*c) + (b/(4*c) + x/2)*sqrt(b*x + c*x**2), Ne(c, 0)),
 (2*(b*x)**(3/2)/(3*b), Ne(b, 0)), (0, True)) + A*c*Piecewise((b**3*Piecew
ise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0))
, ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(16*c**
2) + sqrt(b*x + c*x**2)*(-b**2/(8*c**2) + b*x/(12*c) + x**2/3), Ne(c, 0)),
 (2*(b*x)**(5/2)/(5*b**2), Ne(b, 0)), (0, True)) + B*b*Piecewise((b**3*Pie
cewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c,
0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(16*
c**2) + sqrt(b*x + c*x**2)*(-b**2/(8*c**2) + b*x/(12*c) + x**2/3), Ne(c, 0
)), (2*(b*x)**(5/2)/(5*b**2), Ne(b, 0)), (0, True)) + B*c*Piecewise((-5*b*
*4*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b*
*2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)
)/(128*c**3) + sqrt(b*x + c*x**2)*(5*b**3/(64*c**3) - 5*b**2*x/(96*c**2) +
 b*x**2/(24*c) + x**3/4), Ne(c, 0)), (2*(b*x)**(7/2)/(7*b**3), Ne(b, 0)),
(0, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.19

$$
\begin{aligned}
\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x}\,dx &= \frac{1}{4}\left(cx^2+bx\right)^{\frac{3}{2}}Bx + \frac{1}{4}\sqrt{cx^2+bx}Abx \\
&- \frac{3\sqrt{cx^2+bx}Bb^2x}{32\,c} + \frac{3\,Bb^4\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{128\,c^{\frac{5}{2}}} \\
&- \frac{Ab^3\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{16\,c^{\frac{3}{2}}} + \frac{1}{3}\left(cx^2+bx\right)^{\frac{3}{2}}A \\
&- \frac{3\sqrt{cx^2+bx}Bb^3}{64\,c^2} + \frac{\left(cx^2+bx\right)^{\frac{3}{2}}Bb}{8\,c} + \frac{\sqrt{cx^2+bx}Ab^2}{8\,c}
\end{aligned}
$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x,x, algorithm="maxima")
```

output
```
1/4*(c*x^2 + b*x)^(3/2)*B*x + 1/4*sqrt(c*x^2 + b*x)*A*b*x - 3/32*sqrt(c*x^
2 + b*x)*B*b^2*x/c + 3/128*B*b^4*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(
c))/c^(5/2) - 1/16*A*b^3*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(3
/2) + 1/3*(c*x^2 + b*x)^(3/2)*A - 3/64*sqrt(c*x^2 + b*x)*B*b^3/c^2 + 1/8*(
c*x^2 + b*x)^(3/2)*B*b/c + 1/8*sqrt(c*x^2 + b*x)*A*b^2/c
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x}\,dx = \frac{1}{192}\sqrt{cx^2+bx}\left(2\left(4\left(6\,Bcx+\frac{9\,Bbc^3+8\,Ac^4}{c^3}\right)x+\frac{3\,Bb^2c^2+56\,Abc^3}{c^3}\right)\right.$$
$$-\frac{\left(3\,Bb^4-8\,Ab^3c\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x,x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x)*(2*(4*(6*B*c*x + (9*B*b*c^3 + 8*A*c^4)/c^3)*x + (3
*B*b^2*c^2 + 56*A*b*c^3)/c^3)*x - 3*(3*B*b^3*c - 8*A*b^2*c^2)/c^3) - 1/128
*(3*B*b^4 - 8*A*b^3*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) +
 b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x}\,dx = \int \frac{{\left(c\,x^2+b\,x\right)}^{3/2}\,\left(A+B\,x\right)}{x}\,dx$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x,x)
```

output
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\,(bx+cx^2)^{3/2}}{x}\,dx = \frac{24\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^2+112\sqrt{x}\,\sqrt{cx+b}\,ab\,c^3x+64\sqrt{x}\,\sqrt{cx+b}\,a\,c^4x^2-9\sqrt{}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x,x)
```

output
```
(24*sqrt(x)*sqrt(b + c*x)*a*b**2*c**2 + 112*sqrt(x)*sqrt(b + c*x)*a*b*c**3
*x + 64*sqrt(x)*sqrt(b + c*x)*a*c**4*x**2 - 9*sqrt(x)*sqrt(b + c*x)*b**4*c
 + 6*sqrt(x)*sqrt(b + c*x)*b**3*c**2*x + 72*sqrt(x)*sqrt(b + c*x)*b**2*c**
3*x**2 + 48*sqrt(x)*sqrt(b + c*x)*b*c**4*x**3 - 24*sqrt(c)*log((sqrt(b + c
*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*b**3*c + 9*sqrt(c)*log((sqrt(b + c*x) +
sqrt(x)*sqrt(c))/sqrt(b))*b**5)/(192*c**3)
```

3.118 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^2} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^2} \, dx = -\frac{5b(bB-6Ac)\sqrt{bx+cx^2}}{24c} - \frac{1}{12}(bB-6Ac)x\sqrt{bx+cx^2} + \frac{B\left(bx+cx^2\right)^{5/2}}{3cx^2} - \frac{b^2(bB-6Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{3/2}}$$

output
```
-5/24*b*(-6*A*c+B*b)*(c*x^2+b*x)^(1/2)/c-1/12*(-6*A*c+B*b)*x*(c*x^2+b*x)^(
1/2)+1/3*B*(c*x^2+b*x)^(5/2)/c/x^2-1/8*b^2*(-6*A*c+B*b)*arctanh(c^(1/2)*x/
(c*x^2+b*x)^(1/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^2} \, dx = \frac{\sqrt{x(b+cx)}\left(\sqrt{c}(3b^2B+4c^2x(3A+2Bx)+2bc(15A+7Bx)) + \frac{3b^2(bB-6A}{}\right.}{24c^{3/2}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^2,x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[c]*(3*b^2*B + 4*c^2*x*(3*A + 2*B*x) + 2*b*c*(15*A
 + 7*B*x)) + (3*b^2*(b*B - 6*A*c)*Log[-(Sqrt[c]*Sqrt[x]) + Sqrt[b + c*x]])
/(Sqrt[x]*Sqrt[b + c*x])))/(24*c^(3/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1220, 1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^2}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(bB-6Ac)\int \frac{\left(cx^2+bx\right)^{3/2}}{x}dx}{b}+\frac{2A\left(bx+cx^2\right)^{5/2}}{bx^2}$$

$$\Big\downarrow \text{1131}$$

$$\frac{(bB-6Ac)\left(\frac{1}{2}b\int \sqrt{cx^2+bx}dx+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)}{b}+\frac{2A\left(bx+cx^2\right)^{5/2}}{bx^2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(bB-6Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)}{b}+\frac{2A\left(bx+cx^2\right)^{5/2}}{bx^2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(bB-6Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)}{b}+\frac{2A\left(bx+cx^2\right)^{5/2}}{bx^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{(bB-6Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)}{b}+\frac{2A\left(bx+cx^2\right)^{5/2}}{bx^2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^2,x]
```

output
```
(2*A*(b*x + c*x^2)^(5/2))/(b*x^2) + ((b*B - 6*A*c)*((b*x + c*x^2)^(3/2)/3
+ (b*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqr
t[b*x + c*x^2]])/(4*c^(3/2))))/2))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$\frac{\left(\frac{3}{2} A\,b^{2} c-\frac{1}{4} B\,b^{3}\right) \operatorname{arctanh}\left(\frac{\sqrt{x \left(c x+b\right)}}{x \sqrt{c}}\right)+\left(\frac{5 \left(\frac{7 B x}{15}+A\right) b\,c^{\frac{3}{2}}}{2}+x \left(\frac{2 B x}{3}+A\right) c^{\frac{5}{2}}+\frac{B \sqrt{c}\,b^{2}}{4}\right) \sqrt{x \left(c x+b\right)}}{2 c^{\frac{3}{2}}}$
risch	$\frac{\left(8 B\,c^{2} x^{2}+12 A\,c^{2} x+14 B b c x+30 A b c+3 B\,b^{2}\right) x \left(c x+b\right)}{24 c \sqrt{x \left(c x+b\right)}}+\frac{b^{2} \left(6 A c-B b\right) \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x}\right)}{16 c^{\frac{3}{2}}}$
default	$A \left(\frac{2 \left(c\,x^{2}+b x\right)^{\frac{5}{2}}}{b\,x^{2}}-\frac{6 c \left(\frac{\left(c\,x^{2}+b x\right)^{\frac{3}{2}}}{3}+\frac{b \left(\frac{\left(2 c x+b\right) \sqrt{c\,x^{2}+b x}}{4 c}-\frac{b^{2} \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c\,x^{2}+b x}\right)}{8 c^{\frac{3}{2}}}\right)}{2}\right)}{b}\right)+B \left(\frac{\left(c\,x^{2}+b x\right)^{\frac{3}{2}}}{3}+\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/c^(3/2)*((3/2*A*b^2*c-1/4*B*b^3)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))+
(5/2*(7/15*B*x+A)*b*c^(3/2)+x*(2/3*B*x+A)*c^(5/2)+1/4*B*c^(1/2)*b^2)*(x*(c
*x+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.70

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^2}\,dx = \left[-\frac{3\,(Bb^3-6\,Ab^2c)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(8\,Bc^3x^2+3\,B\right.}{48\,c^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
[-1/48*(3*(B*b^3 - 6*A*b^2*c)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*
sqrt(c)) - 2*(8*B*c^3*x^2 + 3*B*b^2*c + 30*A*b*c^2 + 2*(7*B*b*c^2 + 6*A*c^
3)*x)*sqrt(c*x^2 + b*x))/c^2, 1/24*(3*(B*b^3 - 6*A*b^2*c)*sqrt(-c)*arctan(
sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (8*B*c^3*x^2 + 3*B*b^2*c + 30*A*b*
c^2 + 2*(7*B*b*c^2 + 6*A*c^3)*x)*sqrt(c*x^2 + b*x))/c^2]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^2}\,dx = \int \frac{(x(b+cx))^{\frac{3}{2}}\,(A+Bx)}{x^2}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**2,x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^2}\,dx = \frac{1}{4}\,\sqrt{cx^2+bx}Bbx - \frac{Bb^3\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{16\,c^{\frac{3}{2}}} + \frac{3\,Ab^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,\sqrt{c}} + \frac{1}{3}\,\left(cx^2+bx\right)^{\frac{3}{2}}B + \frac{3}{4}\,\sqrt{cx^2+bx}Ab + \frac{\sqrt{cx^2+bx}Bb^2}{8\,c} + \frac{\left(cx^2+bx\right)^{\frac{3}{2}}A}{2\,x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
1/4*sqrt(c*x^2 + b*x)*B*b*x - 1/16*B*b^3*log(2*c*x + b + 2*sqrt(c*x^2 + b*
x)*sqrt(c))/c^(3/2) + 3/8*A*b^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c
))/sqrt(c) + 1/3*(c*x^2 + b*x)^(3/2)*B + 3/4*sqrt(c*x^2 + b*x)*A*b + 1/8*s
qrt(c*x^2 + b*x)*B*b^2/c + 1/2*(c*x^2 + b*x)^(3/2)*A/x
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^2}\,dx = \frac{1}{24}\sqrt{cx^2+bx}\left(2\left(4\,Bcx+\frac{7\,Bbc^2+6\,Ac^3}{c^2}\right)x+\frac{3\left(Bb^2c+10\,Abc^2\right)}{c^2}\right) + \frac{\left(Bb^3-6\,Ab^2c\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x)*(2*(4*B*c*x + (7*B*b*c^2 + 6*A*c^3)/c^2)*x + 3*(B*b
^2*c + 10*A*b*c^2)/c^2) + 1/16*(B*b^3 - 6*A*b^2*c)*log(abs(2*(sqrt(c)*x -
sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^2}\,dx = \int \frac{\left(c\,x^2+b\,x\right)^{3/2}\,\left(A+B\,x\right)}{x^2}\,dx$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^2,x)
```

output
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^2}\,dx = \frac{30\sqrt{x}\,\sqrt{cx+b}\,ab\,c^2+12\sqrt{x}\,\sqrt{cx+b}\,a\,c^3x+3\sqrt{x}\,\sqrt{cx+b}\,b^3c+14\sqrt{x}\,\sqrt{c}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^2,x)
```

output
```
(30*sqrt(x)*sqrt(b + c*x)*a*b*c**2 + 12*sqrt(x)*sqrt(b + c*x)*a*c**3*x + 3
*sqrt(x)*sqrt(b + c*x)*b**3*c + 14*sqrt(x)*sqrt(b + c*x)*b**2*c**2*x + 8*s
qrt(x)*sqrt(b + c*x)*b*c**3*x**2 + 18*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)
*sqrt(c))/sqrt(b))*a*b**2*c - 3*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(
c))/sqrt(b))*b**4)/(24*c**2)
```

3.119 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 105

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^3}\,dx=\frac{3}{4}(bB+4Ac)\sqrt{bx+cx^2}-\frac{2A\left(bx+cx^2\right)^{3/2}}{x^2}+\frac{B\left(bx+cx^2\right)^{3/2}}{2x}+\frac{3b(bB+4Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4\sqrt{c}}$$

output
```
3/4*(4*A*c+B*b)*(c*x^2+b*x)^(1/2)-2*A*(c*x^2+b*x)^(3/2)/x^2+1/2*B*(c*x^2+b
*x)^(3/2)/x+3/4*b*(4*A*c+B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^3}\,dx=\frac{\sqrt{x(b+cx)}\left(Bx(5b+2cx)+A(-8b+4cx)+\frac{6b(bB+4Ac)\sqrt{x}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{}}{-\sqrt{b+\sqrt{}}}\right.}{\sqrt{c}\sqrt{b+cx}}\right.}{4x}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^3,x]
```

output
```
(Sqrt[x*(b + c*x)]*(B*x*(5*b + 2*c*x) + A*(-8*b + 4*c*x) + (6*b*(b*B + 4*A
*c)*Sqrt[x]*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])])/(Sqrt[c
]*Sqrt[b + c*x])))/(4*x)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1220, 1131, 1131, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(4Ac+bB)\int \frac{\left(cx^2+bx\right)^{3/2}}{x^2}dx}{b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{bx^3}$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{(4Ac+bB)\left(\frac{3}{4}b\int \frac{\sqrt{cx^2+bx}}{x}dx+\frac{\left(bx+cx^2\right)^{3/2}}{2x}\right)}{b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{bx^3}$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{(4Ac+bB)\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{\sqrt{cx^2+bx}}dx+\sqrt{bx+cx^2}\right)+\frac{\left(bx+cx^2\right)^{3/2}}{2x}\right)}{b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{bx^3}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{(4Ac+bB)\left(\frac{3}{4}b\left(b\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}+\sqrt{bx+cx^2}\right)+\frac{\left(bx+cx^2\right)^{3/2}}{2x}\right)}{b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{bx^3}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(4Ac+bB)\left(\frac{3}{4}b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}+\sqrt{bx+cx^2}\right)+\frac{\left(bx+cx^2\right)^{3/2}}{2x}\right)}{b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{bx^3}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^3,x]
```

output
```
(-2*A*(b*x + c*x^2)^(5/2))/(b*x^3) + ((b*B + 4*A*c)*((b*x + c*x^2)^(3/2)/(
2*x) + (3*b*(Sqrt[b*x + c*x^2] + (b*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]]
)/Sqrt[c]))/4))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{3bx\left(Ac+\frac{Bb}{4}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)-2\left(-\frac{\left(\frac{Bx}{2}+A\right)x\,c^{\frac{3}{2}}}{2}+b\sqrt{c}\left(-\frac{5Bx}{8}+A\right)\right)\sqrt{x(cx+b)}}{\sqrt{c}\,x}$
risch	$-\frac{(cx+b)\left(-2Bc\,x^{2}-4Acx-5Bbx+8Ab\right)}{4\sqrt{x(cx+b)}}+\frac{3(4Ac+Bb)b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{8\sqrt{c}}$
default	$A\left(-\frac{2\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{b\,x^{3}}+\frac{4c\left(\frac{2\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{b\,x^{2}}-\frac{6c\left(\frac{\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{3}+\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx}}{4c}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{8c^{\frac{3}{2}}}\right)}{2}\right)}{b}\right)}{b}\right)+$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
3/c^(1/2)*(b*x*(A*c+1/4*B*b)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))-2/3*(-1/
2*(1/2*B*x+A)*x*c^(3/2)+b*c^(1/2)*(-5/8*B*x+A))*(x*(c*x+b))^(1/2))/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.78

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^3}\,dx = \left[\frac{3\left(Bb^2+4\,Abc\right)\sqrt{c}x\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\left(2\,Bc^2x^2-8\,Ab\right.}{8\,cx}\right.$$

$$\left.-\frac{3\left(Bb^2+4\,Abc\right)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-\left(2\,Bc^2x^2-8\,Abc+\left(5\,Bbc+4\,Ac^2\right)x\right)\sqrt{cx^2+bx}}{4\,cx}\right]$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
[1/8*(3*(B*b^2 + 4*A*b*c)*sqrt(c)*x*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sq
rt(c)) + 2*(2*B*c^2*x^2 - 8*A*b*c + (5*B*b*c + 4*A*c^2)*x)*sqrt(c*x^2 + b*
x))/(c*x), -1/4*(3*(B*b^2 + 4*A*b*c)*sqrt(-c)*x*arctan(sqrt(c*x^2 + b*x)*s
qrt(-c)/(c*x + b)) - (2*B*c^2*x^2 - 8*A*b*c + (5*B*b*c + 4*A*c^2)*x)*sqrt(
c*x^2 + b*x))/(c*x)]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^3}\,dx = \int \frac{\left(x(b+cx)\right)^{\frac{3}{2}}(A+Bx)}{x^3}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**3,x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^3}\,dx = \frac{3\,Bb^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,\sqrt{c}} + \frac{3}{2}\,Ab\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right) + \frac{3}{4}\,\sqrt{cx^2+bx}Bb + \frac{\left(cx^2+bx\right)^{\frac{3}{2}}B}{2\,x} - \frac{3\,\sqrt{cx^2+bx}Ab}{x} + \frac{\left(cx^2+bx\right)^{\frac{3}{2}}A}{x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
3/8*B*b^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/sqrt(c) + 3/2*A*b*s
qrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 3/4*sqrt(c*x^2 + b*x
)*B*b + 1/2*(c*x^2 + b*x)^(3/2)*B/x - 3*sqrt(c*x^2 + b*x)*A*b/x + (c*x^2 +
 b*x)^(3/2)*A/x^2
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^3}\,dx = \frac{2\,Ab^2}{\sqrt{c}x-\sqrt{cx^2+bx}} + \frac{1}{4}\left(2\,Bcx+\frac{5\,Bbc+4\,Ac^2}{c}\right)\sqrt{cx^2+bx} - \frac{3\left(Bb^2+4\,Abc\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{8\,\sqrt{c}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
2*A*b^2/(sqrt(c)*x - sqrt(c*x^2 + b*x)) + 1/4*(2*B*c*x + (5*B*b*c + 4*A*c^
2)/c)*sqrt(c*x^2 + b*x) - 3/8*(B*b^2 + 4*A*b*c)*log(abs(2*(sqrt(c)*x - sqr
t(c*x^2 + b*x))*sqrt(c) + b))/sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^3}\,dx = \int \frac{\left(c\,x^2+b\,x\right)^{3/2}\,(A+B\,x)}{x^3}\,dx$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^3,x)
```

output
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.30

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^3}\,dx = \frac{-8\sqrt{x}\,\sqrt{cx+b}\,abc+4\sqrt{x}\,\sqrt{cx+b}\,a\,c^2x+5\sqrt{x}\,\sqrt{cx+b}\,b^2cx+2\sqrt{x}\,\sqrt{cx}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^3,x)
```

output
```
( - 8*sqrt(x)*sqrt(b + c*x)*a*b*c + 4*sqrt(x)*sqrt(b + c*x)*a*c**2*x + 5*s
qrt(x)*sqrt(b + c*x)*b**2*c*x + 2*sqrt(x)*sqrt(b + c*x)*b*c**2*x**2 + 12*s
qrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*b*c*x + 3*sqrt(c)*
log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**3*x - 9*sqrt(c)*a*b*c*x
- sqrt(c)*b**3*x)/(4*c*x)
```

3.120 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 99

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^4}\,dx = Bc\sqrt{bx+cx^2} - \frac{2(bB+Ac)\sqrt{bx+cx^2}}{x} - \frac{2A\left(bx+cx^2\right)^{3/2}}{3x^3} + \sqrt{c}(3bB+2Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)$$

output
```
B*c*(c*x^2+b*x)^(1/2)-2*(A*c+B*b)*(c*x^2+b*x)^(1/2)/x-2/3*A*(c*x^2+b*x)^(3
/2)/x^3+c^(1/2)*(2*A*c+3*B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^4}\,dx = \frac{\sqrt{x(b+cx)}\left(\sqrt{b+cx}(3Bx(-2b+cx)-2A(b+4cx))+6\sqrt{c}(3bB+2Ac\right.}{3x^2\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^4,x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[b + c*x]*(3*B*x*(-2*b + c*x) - 2*A*(b + 4*c*x)) +
 6*Sqrt[c]*(3*b*B + 2*A*c)*x^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + S
qrt[b + c*x])]))/(3*x^2*Sqrt[b + c*x])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1220, 1125, 25, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^4}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(2Ac+3bB)\int \frac{\left(cx^2+bx\right)^{3/2}}{x^3}dx}{3b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{3bx^4}$$

$$\Big\downarrow \text{ 1125}$$

$$\frac{(2Ac+3bB)\left(-\int -\frac{c(2b+cx)}{\sqrt{cx^2+bx}}dx-\frac{2b\sqrt{bx+cx^2}}{x}\right)}{3b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{3bx^4}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{(2Ac+3bB)\left(\int \frac{c(2b+cx)}{\sqrt{cx^2+bx}}dx-\frac{2b\sqrt{bx+cx^2}}{x}\right)}{3b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{3bx^4}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{(2Ac+3bB)\left(c\int \frac{2b+cx}{\sqrt{cx^2+bx}}dx-\frac{2b\sqrt{bx+cx^2}}{x}\right)}{3b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{3bx^4}$$

$$\Big\downarrow \text{ 1160}$$

$$\frac{(2Ac+3bB)\left(c\left(\frac{3}{2}b\int \frac{1}{\sqrt{cx^2+bx}}dx+\sqrt{bx+cx^2}\right)-\frac{2b\sqrt{bx+cx^2}}{x}\right)}{3b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{3bx^4}$$

$$\Big\downarrow \text{ 1091}$$

$$\frac{(2Ac+3bB)\left(c\left(3b\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}+\sqrt{bx+cx^2}\right)-\frac{2b\sqrt{bx+cx^2}}{x}\right)}{3b}-\frac{2A(bx+cx^2)^{5/2}}{3bx^4}$$

$$\Big\downarrow \; 219$$

$$\frac{(2Ac+3bB)\left(c\left(\frac{3b\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}+\sqrt{bx+cx^2}\right)-\frac{2b\sqrt{bx+cx^2}}{x}\right)}{3b}-\frac{2A(bx+cx^2)^{5/2}}{3bx^4}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^4,x]
```

output
```
(-2*A*(b*x + c*x^2)^(5/2))/(3*b*x^4) + ((3*b*B + 2*A*c)*((-2*b*Sqrt[b*x +
c*x^2])/x + c*(Sqrt[b*x + c*x^2] + (3*b*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x
^2]])/Sqrt[c])))/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.79

method	result
pseudoelliptic	$\frac{2 c x^{2} \left(A c+\frac{3 B b}{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{x (c x+b)}}{x \sqrt{c}}\right)-\frac{2 \left(\left(-\frac{3}{2} B x^{2}+4 A x\right) c^{\frac{3}{2}}+b \sqrt{c}\, (3 B x+A)\right) \sqrt{x (c x+b)}}{3}}{\sqrt{c}\, x^{2}}$
risch	$-\frac{(c x+b) \left(-3 B c x^{2}+8 A c x+6 B b x+2 A b\right)}{3 x \sqrt{x (c x+b)}}+\frac{(2 A c+3 B b) \sqrt{c}\, \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c x^{2}+b x}\right)}{2}$
default	$A \left(-\frac{2 \left(c x^{2}+b x\right)^{\frac{5}{2}}}{3 b x^{4}}+\frac{2 c \left(-\frac{2 \left(c x^{2}+b x\right)^{\frac{5}{2}}}{b x^{3}}+\frac{4 c \left(\frac{2 \left(c x^{2}+b x\right)^{\frac{5}{2}}}{b x^{2}}-\frac{6 c \left(\frac{\left(c x^{2}+b x\right)^{\frac{3}{2}}}{3}+\frac{b \left(\frac{(2 c x+b) \sqrt{c x^{2}+b x}}{4 c}-\frac{b^{2} \ln\left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c \cdots}\right)}{8 c^{\frac{3}{2}}} \cdots\right)}{2} \cdots\right)}{b} \cdots\right)}{b} \cdots\right)}{3 b} \cdots\right)$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2*(c*x^2*(A*c+3/2*B*b)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))-1/3*((-3/2*B*x
^2+4*A*x)*c^(3/2)+b*c^(1/2)*(3*B*x+A))*(x*(c*x+b))^(1/2))/c^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.73

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^4}\,dx = \left[\frac{3\,(3\,Bb+2\,Ac)\sqrt{c}x^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,(3\,Bcx^2-2\,Ab\right.$$

$$\left.-\frac{3\,(3\,Bb+2\,Ac)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(3\,Bcx^2-2\,Ab-2\,(3\,Bb+4\,Ac)x)\sqrt{cx^2+bx}}{3\,x^2}\right]$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/6*(3*(3*B*b + 2*A*c)*sqrt(c)*x^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sq
rt(c)) + 2*(3*B*c*x^2 - 2*A*b - 2*(3*B*b + 4*A*c)*x)*sqrt(c*x^2 + b*x))/x^
2, -1/3*(3*(3*B*b + 2*A*c)*sqrt(-c)*x^2*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/
(c*x + b)) - (3*B*c*x^2 - 2*A*b - 2*(3*B*b + 4*A*c)*x)*sqrt(c*x^2 + b*x))/
x^2]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^4}\,dx = \int \frac{(x(b+cx))^{\frac{3}{2}}\,(A+Bx)}{x^4}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**4,x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.47

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^4}\,dx = \frac{3}{2}\,Bb\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right) + Ac^{\frac{3}{2}}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right) - \frac{3\,\sqrt{cx^2+bx}Bb}{x} - \frac{7\,\sqrt{cx^2+bx}Ac}{3\,x} + \frac{\left(cx^2+bx\right)^{\frac{3}{2}}B}{x^2} - \frac{\sqrt{cx^2+bx}Ab}{3\,x^2} - \frac{\left(cx^2+bx\right)^{\frac{3}{2}}A}{3\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
3/2*B*b*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + A*c^(3/2)*l
og(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 3*sqrt(c*x^2 + b*x)*B*b/x -
7/3*sqrt(c*x^2 + b*x)*A*c/x + (c*x^2 + b*x)^(3/2)*B/x^2 - 1/3*sqrt(c*x^2 +
 b*x)*A*b/x^2 - 1/3*(c*x^2 + b*x)^(3/2)*A/x^3
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.72

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^4}\,dx = \sqrt{cx^2+bx}Bc - \frac{\left(3\,Bbc+2\,Ac^2\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{2\,\sqrt{c}} + \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2Bb^2+6\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2Abc+3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)Ab^2\sqrt{c}+Ab^3\right)}{3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^4,x, algorithm="giac")
```

output
```
sqrt(c*x^2 + b*x)*B*c - 1/2*(3*B*b*c + 2*A*c^2)*log(abs(2*(sqrt(c)*x - sqr
t(c*x^2 + b*x))*sqrt(c) + b))/sqrt(c) + 2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b
*x))^2*B*b^2 + 6*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*A*b*c + 3*(sqrt(c)*x -
sqrt(c*x^2 + b*x))*A*b^2*sqrt(c) + A*b^3)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^
3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^4}\,dx = \int \frac{\left(c\,x^2+b\,x\right)^{3/2}\,(A+B\,x)}{x^4}\,dx$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^4,x)
```

output
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.26

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^4}\,dx = \frac{-4\sqrt{x}\,\sqrt{cx+b}\,ab-16\sqrt{x}\,\sqrt{cx+b}\,acx-12\sqrt{x}\,\sqrt{cx+b}\,b^2x+6\sqrt{x}\,\sqrt{cx -}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^4,x)
```

output
```
( - 4*sqrt(x)*sqrt(b + c*x)*a*b - 16*sqrt(x)*sqrt(b + c*x)*a*c*x - 12*sqrt
(x)*sqrt(b + c*x)*b**2*x + 6*sqrt(x)*sqrt(b + c*x)*b*c*x**2 + 12*sqrt(c)*l
og((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*c*x**2 + 18*sqrt(c)*log((s
qrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**2*x**2 + 5*sqrt(c)*b**2*x**2)/
(6*x**2)
```

3.121 $\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^5} dx$

Optimal result

Integrand size = 22, antiderivative size = 98

$$\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^5} dx = -\frac{2bB\sqrt{bx+cx^2}}{3x^2} - \frac{8Bc\sqrt{bx+cx^2}}{3x} - \frac{2A(bx+cx^2)^{5/2}}{5bx^5} + 2Bc^{3/2}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)$$

output
```
-2/3*b*B*(c*x^2+b*x)^(1/2)/x^2-8/3*B*c*(c*x^2+b*x)^(1/2)/x-2/5*A*(c*x^2+b*
x)^(5/2)/b/x^5+2*B*c^(3/2)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^5} dx = -\frac{2\sqrt{x(b+cx)}\left(\sqrt{b+cx}(3A(b+cx)^2+5bBx(b+4cx))+15bBc^{3/2}x^{5/2}\log\left(-\sqrt{c}\sqrt{x}+\sqrt{b+cx}\right)\right)}{15bx^3\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^5,x]
```

output
```
(-2*Sqrt[x*(b + c*x)]*(Sqrt[b + c*x]*(3*A*(b + c*x)^2 + 5*b*B*x*(b + 4*c*x
)) + 15*b*B*c^(3/2)*x^(5/2)*Log[-(Sqrt[c]*Sqrt[x]) + Sqrt[b + c*x]]))/(15*
b*x^3*Sqrt[b + c*x])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1220, 1130, 1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^5}dx$$

$$\Big\downarrow \text{1220}$$

$$B\int \frac{\left(cx^2+bx\right)^{3/2}}{x^4}dx-\frac{2A\left(bx+cx^2\right)^{5/2}}{5bx^5}$$

$$\Big\downarrow \text{1130}$$

$$B\left(c\int \frac{\sqrt{cx^2+bx}}{x^2}dx-\frac{2\left(bx+cx^2\right)^{3/2}}{3x^3}\right)-\frac{2A\left(bx+cx^2\right)^{5/2}}{5bx^5}$$

$$\Big\downarrow \text{1125}$$

$$B\left(c\left(-\int -\frac{c}{\sqrt{cx^2+bx}}dx-\frac{2\sqrt{bx+cx^2}}{x}\right)-\frac{2\left(bx+cx^2\right)^{3/2}}{3x^3}\right)-\frac{2A\left(bx+cx^2\right)^{5/2}}{5bx^5}$$

$$\Big\downarrow \text{25}$$

$$B\left(c\left(\int \frac{c}{\sqrt{cx^2+bx}}dx-\frac{2\sqrt{bx+cx^2}}{x}\right)-\frac{2\left(bx+cx^2\right)^{3/2}}{3x^3}\right)-\frac{2A\left(bx+cx^2\right)^{5/2}}{5bx^5}$$

$$\Big\downarrow \text{27}$$

$$B\left(c\left(c\int \frac{1}{\sqrt{cx^2+bx}}dx-\frac{2\sqrt{bx+cx^2}}{x}\right)-\frac{2\left(bx+cx^2\right)^{3/2}}{3x^3}\right)-\frac{2A\left(bx+cx^2\right)^{5/2}}{5bx^5}$$

$$\Big\downarrow \text{1091}$$

$$B\left(c\left(2c\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}-\frac{2\sqrt{bx+cx^2}}{x}\right)-\frac{2\left(bx+cx^2\right)^{3/2}}{3x^3}\right)-\frac{2A\left(bx+cx^2\right)^{5/2}}{5bx^5}$$

$$\Big\downarrow \text{219}$$

$$B\left(c\left(2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)-\frac{2\sqrt{bx+cx^2}}{x}\right)-\frac{2\left(bx+cx^2\right)^{3/2}}{3x^3}\right)-\frac{2A\left(bx+cx^2\right)^{5/2}}{5bx^5}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^5,x]
```

output
```
(-2*A*(b*x + c*x^2)^(5/2))/(5*b*x^5) + B*((-2*(b*x + c*x^2)^(3/2))/(3*x^3)
 + c*((-2*Sqrt[b*x + c*x^2])/x + 2*Sqrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x +
c*x^2]]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.80

method	result
pseudoelliptic	$\frac{2Bb\,c^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)x^3-\frac{2\sqrt{x(cx+b)}\left(\left(\frac{5Bx}{3}+A\right)b^2+2cx\left(\frac{10Bx}{3}+A\right)b+A\,c^2x^2\right)}{5}}{b\,x^3}$
risch	$-\frac{2(cx+b)(3A\,c^2x^2+20x^2Bbc+6Abcx+5xB\,b^2+3b^2A)}{15x^2\sqrt{x(cx+b)}\,b}+B\,c^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)$
default	$-\frac{2A(c\,x^2+bx)^{\frac{5}{2}}}{5b\,x^5}+B\left(-\frac{2(c\,x^2+bx)^{\frac{5}{2}}}{3b\,x^4}+\frac{2c\left(-\frac{2(c\,x^2+bx)^{\frac{5}{2}}}{b\,x^3}+\frac{4c\left(\frac{2(c\,x^2+bx)^{\frac{5}{2}}}{b\,x^2}-\frac{6c\left(\frac{(c\,x^2+bx)^{\frac{3}{2}}}{3}+\frac{b\left(\frac{(2cx+b)\sqrt{c\,x^2}}{4c}\right.}{}\right.}{}\right.}{b}\right.}{3b}\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
2/5*(5*B*b*c^(3/2)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))*x^3-(x*(c*x+b))^(1
/2)*((5/3*B*x+A)*b^2+2*c*x*(10/3*B*x+A)*b+A*c^2*x^2))/b/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.93

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^5}\,dx = \left[\frac{15\,Bbc^{\frac{3}{2}}x^3\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(3\,Ab^2+(20\,Bbc+3\,Ac^2)\right.$$

$$\left.-\frac{2\left(15\,Bb\sqrt{-c}cx^3\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)+(3\,Ab^2+(20\,Bbc+3\,Ac^2)x^2+(5\,Bb^2+6\,Abc)x)\sqrt{cx^2+bx}\right)}{15\,bx^3}\right]$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/15*(15*B*b*c^(3/2)*x^3*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2
*(3*A*b^2 + (20*B*b*c + 3*A*c^2)*x^2 + (5*B*b^2 + 6*A*b*c)*x)*sqrt(c*x^2 +
 b*x))/(b*x^3), -2/15*(15*B*b*sqrt(-c)*c*x^3*arctan(sqrt(c*x^2 + b*x)*sqrt
(-c)/(c*x + b)) + (3*A*b^2 + (20*B*b*c + 3*A*c^2)*x^2 + (5*B*b^2 + 6*A*b*c
)*x)*sqrt(c*x^2 + b*x))/(b*x^3)]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^5}\,dx = \int \frac{(x(b+cx))^{\frac{3}{2}}\,(A+Bx)}{x^5}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**5,x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.61

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^5}\,dx = Bc^{\frac{3}{2}}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right) - \frac{7\,\sqrt{cx^2+bx}Bc}{3\,x} - \frac{2\,\sqrt{cx^2+bx}Ac^2}{5\,bx} - \frac{\sqrt{cx^2+bx}Bb}{3\,x^2} + \frac{\sqrt{cx^2+bx}Ac}{5\,x^2} - \frac{\left(cx^2+bx\right)^{\frac{3}{2}}B}{3\,x^3} + \frac{3\,\sqrt{cx^2+bx}Ab}{5\,x^3} - \frac{\left(cx^2+bx\right)^{\frac{3}{2}}A}{x^4}$$

input

```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^5,x, algorithm="maxima")
```

output

```
B*c^(3/2)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 7/3*sqrt(c*x^2 +
b*x)*B*c/x - 2/5*sqrt(c*x^2 + b*x)*A*c^2/(b*x) - 1/3*sqrt(c*x^2 + b*x)*B*b
/x^2 + 1/5*sqrt(c*x^2 + b*x)*A*c/x^2 - 1/3*(c*x^2 + b*x)^(3/2)*B/x^3 + 3/5
*sqrt(c*x^2 + b*x)*A*b/x^3 - (c*x^2 + b*x)^(3/2)*A/x^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(80) = 160.

Time = 0.27 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.64

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^5}\,dx = -Bc^{\frac{3}{2}}\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right) + \frac{2\left(30\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4Bbc+15\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4Ac^2+15\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3Bb^2\sqrt{c}+30\left(\sqrt{c}\right.\right.}{1}$$

input

```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^5,x, algorithm="giac")
```

output
```
-B*c^(3/2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b)) + 2/15*
(30*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*B*b*c + 15*(sqrt(c)*x - sqrt(c*x^2 +
 b*x))^4*A*c^2 + 15*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*B*b^2*sqrt(c) + 30*(
sqrt(c)*x - sqrt(c*x^2 + b*x))^3*A*b*c^(3/2) + 5*(sqrt(c)*x - sqrt(c*x^2 +
 b*x))^2*B*b^3 + 30*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*A*b^2*c + 15*(sqrt(c
)*x - sqrt(c*x^2 + b*x))*A*b^3*sqrt(c) + 3*A*b^4)/(sqrt(c)*x - sqrt(c*x^2
+ b*x))^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^5}\,dx=\int \frac{{\left(c\,x^2+b\,x\right)}^{3/2}\,\left(A+B\,x\right)}{x^5}\,dx$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^5,x)
```

output
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.39

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^5}\,dx=\frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^2}{5}-\frac{4\sqrt{x}\,\sqrt{cx+b}\,abcx}{5}-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,c^2x^2}{5}-\frac{2\sqrt{x}\,\sqrt{cx+b}\,b^3x}{3}-\frac{8\sqrt{x}\,\sqrt{cx+b}}{3}}{b\,x^3}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^5,x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(b + c*x)*a*b**2 - 6*sqrt(x)*sqrt(b + c*x)*a*b*c*x -
3*sqrt(x)*sqrt(b + c*x)*a*c**2*x**2 - 5*sqrt(x)*sqrt(b + c*x)*b**3*x - 20*
sqrt(x)*sqrt(b + c*x)*b**2*c*x**2 + 15*sqrt(c)*log((sqrt(b + c*x) + sqrt(x
)*sqrt(c))/sqrt(b))*b**2*c*x**3 - 3*sqrt(c)*a*c**2*x**3 + 8*sqrt(c)*b**2*c
*x**3))/(15*b*x**3)
```

3.122 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^6} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^6} \, dx=-\frac{2A(bx+cx^2)^{5/2}}{7bx^6}-\frac{2(7bB-2Ac)\left(bx+cx^2\right)^{5/2}}{35b^2x^5}$$

output
```
-2/7*A*(c*x^2+b*x)^(5/2)/b/x^6-2/35*(-2*A*c+7*B*b)*(c*x^2+b*x)^(5/2)/b^2/x
^5
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^6} \, dx=-\frac{2(x(b+cx))^{5/2}(5Ab+7bBx-2Acx)}{35b^2x^6}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^6,x]
```

output
```
(-2*(x*(b + c*x))^(5/2)*(5*A*b + 7*b*B*x - 2*A*c*x))/(35*b^2*x^6)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1220, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^6}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(7bB-2Ac)\int \frac{\left(cx^2+bx\right)^{3/2}}{x^5}dx}{7b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{7bx^6}$$

$$\Big\downarrow \text{1123}$$

$$-\frac{2\left(bx+cx^2\right)^{5/2}(7bB-2Ac)}{35b^2x^5}-\frac{2A\left(bx+cx^2\right)^{5/2}}{7bx^6}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^6,x]
```

output
```
(-2*A*(b*x + c*x^2)^(5/2))/(7*b*x^6) - (2*(7*b*B - 2*A*c)*(b*x + c*x^2)^(5
/2))/(35*b^2*x^5)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$-\frac{2(cx+b)^{2}\sqrt{x(cx+b)}\left(\left(\frac{7Bx}{5}+A\right)b-\frac{2Acx}{5}\right)}{7x^{4}b^{2}}$	39
gosper	$-\frac{2(cx+b)(-2Acx+7Bbx+5Ab)\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{35b^{2}x^{5}}$	40
orering	$-\frac{2(cx+b)(-2Acx+7Bbx+5Ab)\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{35b^{2}x^{5}}$	40
default	$A\left(-\frac{2\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{7b\,x^{6}}+\frac{4c\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{35b^{2}x^{5}}\right)-\frac{2B\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{5b\,x^{5}}$	64
trager	$-\frac{2\left(-2A\,c^{3}x^{3}+7x^{3}Bb\,c^{2}+Ab\,c^{2}x^{2}+14x^{2}B\,b^{2}c+8A\,b^{2}cx+7xB\,b^{3}+5A\,b^{3}\right)\sqrt{c\,x^{2}+bx}}{35b^{2}x^{4}}$	80
risch	$-\frac{2(cx+b)\left(-2A\,c^{3}x^{3}+7x^{3}Bb\,c^{2}+Ab\,c^{2}x^{2}+14x^{2}B\,b^{2}c+8A\,b^{2}cx+7xB\,b^{3}+5A\,b^{3}\right)}{35x^{3}\sqrt{x(cx+b)}\,b^{2}}$	83

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/7*(c*x+b)^2*(x*(c*x+b))^(1/2)*((7/5*B*x+A)*b-2/5*A*c*x)/x^4/b^2
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.37

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^6}\,dx = -\frac{2\left(5\,Ab^3+(7\,Bbc^2-2\,Ac^3)x^3+(14\,Bb^2c+Abc^2)x^2+(7\,Bb^3+8\,Ab^2c)x\right)\sqrt{cx^2+bx}}{35\,b^2x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
-2/35*(5*A*b^3 + (7*B*b*c^2 - 2*A*c^3)*x^3 + (14*B*b^2*c + A*b*c^2)*x^2 +
(7*B*b^3 + 8*A*b^2*c)*x)*sqrt(c*x^2 + b*x)/(b^2*x^4)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^6}\,dx = \int \frac{(x(b+cx))^{\frac{3}{2}}\,(A+Bx)}{x^6}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**6,x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/x**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(49) = 98.

Time = 0.03 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.09

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^6}\,dx = -\frac{2\,\sqrt{cx^2+bx}Bc^2}{5\,bx}+\frac{4\,\sqrt{cx^2+bx}Ac^3}{35\,b^2x} + \frac{\sqrt{cx^2+bx}Bc}{5\,x^2}-\frac{2\,\sqrt{cx^2+bx}Ac^2}{35\,bx^2}+\frac{3\,\sqrt{cx^2+bx}Bb}{5\,x^3} + \frac{3\,\sqrt{cx^2+bx}Ac}{70\,x^3}-\frac{(cx^2+bx)^{\frac{3}{2}}B}{x^4}+\frac{3\,\sqrt{cx^2+bx}Ab}{14\,x^4}-\frac{(cx^2+bx)^{\frac{3}{2}}A}{2\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
-2/5*sqrt(c*x^2 + b*x)*B*c^2/(b*x) + 4/35*sqrt(c*x^2 + b*x)*A*c^3/(b^2*x)
+ 1/5*sqrt(c*x^2 + b*x)*B*c/x^2 - 2/35*sqrt(c*x^2 + b*x)*A*c^2/(b*x^2) + 3
/5*sqrt(c*x^2 + b*x)*B*b/x^3 + 3/70*sqrt(c*x^2 + b*x)*A*c/x^3 - (c*x^2 + b
*x)^(3/2)*B/x^4 + 3/14*sqrt(c*x^2 + b*x)*A*b/x^4 - 1/2*(c*x^2 + b*x)^(3/2)
*A/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(49) = 98$.

Time = 0.19 (sec) , antiderivative size = 311, normalized size of antiderivative = 5.46

$$\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^6}\,dx = \frac{2\left(35\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^6Bc^2+70\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5Bbc^{\frac{3}{2}}+35\left(\sqrt{c}x\right.\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^6,x, algorithm="giac")
```

output
```
2/35*(35*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*B*c^2 + 70*(sqrt(c)*x - sqrt(c*
x^2 + b*x))^5*B*b*c^(3/2) + 35*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*A*c^(5/2)
 + 70*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*B*b^2*c + 105*(sqrt(c)*x - sqrt(c*
x^2 + b*x))^4*A*b*c^2 + 35*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*B*b^3*sqrt(c)
 + 140*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*A*b^2*c^(3/2) + 7*(sqrt(c)*x - sq
rt(c*x^2 + b*x))^2*B*b^4 + 98*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*A*b^3*c +
35*(sqrt(c)*x - sqrt(c*x^2 + b*x))*A*b^4*sqrt(c) + 5*A*b^5)/(sqrt(c)*x - s
qrt(c*x^2 + b*x))^7
```

Mupad [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.49

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^6}\,dx = \frac{4\,A\,c^3\,\sqrt{c\,x^2+b\,x}}{35\,b^2\,x} - \frac{16\,A\,c\,\sqrt{c\,x^2+b\,x}}{35\,x^3} - \frac{2\,B\,b\,\sqrt{c\,x^2+b\,x}}{5\,x^3} - \frac{4\,B\,c\,\sqrt{c\,x^2+b\,x}}{5\,x^2} - \frac{2\,A\,c^2\,\sqrt{c\,x^2+b\,x}}{35\,b\,x^2} - \frac{2\,A\,b\,\sqrt{c\,x^2+b\,x}}{7\,x^4} - \frac{2\,B\,c^2\,\sqrt{c\,x^2+b\,x}}{5\,b\,x}$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^6,x)
```

output
```
(4*A*c^3*(b*x + c*x^2)^(1/2))/(35*b^2*x) - (16*A*c*(b*x + c*x^2)^(1/2))/(3
5*x^3) - (2*B*b*(b*x + c*x^2)^(1/2))/(5*x^3) - (4*B*c*(b*x + c*x^2)^(1/2))
/(5*x^2) - (2*A*c^2*(b*x + c*x^2)^(1/2))/(35*b*x^2) - (2*A*b*(b*x + c*x^2)
^(1/2))/(7*x^4) - (2*B*c^2*(b*x + c*x^2)^(1/2))/(5*b*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.60

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^6}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^3}{7} - \frac{16\sqrt{x}\,\sqrt{cx+b}\,a\,b^2cx}{35} - \frac{2\sqrt{x}\,\sqrt{cx+b}\,ab\,c^2x^2}{35} + \frac{4\sqrt{x}\,\sqrt{cx+b}\,a\,c^3x^3}{35} - \frac{2\sqrt{x}\cdots}{}}{b^2x^4}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^6,x)
```

output
```
(2*( - 5*sqrt(x)*sqrt(b + c*x)*a*b**3 - 8*sqrt(x)*sqrt(b + c*x)*a*b**2*c*x
 - sqrt(x)*sqrt(b + c*x)*a*b*c**2*x**2 + 2*sqrt(x)*sqrt(b + c*x)*a*c**3*x*
*3 - 7*sqrt(x)*sqrt(b + c*x)*b**4*x - 14*sqrt(x)*sqrt(b + c*x)*b**3*c*x**2
 - 7*sqrt(x)*sqrt(b + c*x)*b**2*c**2*x**3 - 2*sqrt(c)*a*c**3*x**4 - 3*sqrt
(c)*b**2*c**2*x**4))/(35*b**2*x**4)
```

3.123 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^7} dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^7} dx = -\frac{2A\left(bx+cx^2\right)^{5/2}}{9bx^7} - \frac{2(9bB-4Ac)\left(bx+cx^2\right)^{5/2}}{63b^2x^6} + \frac{4c(9bB-4Ac)\left(bx+cx^2\right)^{5/2}}{315b^3x^5}$$

output
```
-2/9*A*(c*x^2+b*x)^(5/2)/b/x^7-2/63*(-4*A*c+9*B*b)*(c*x^2+b*x)^(5/2)/b^2/x
^6+4/315*c*(-4*A*c+9*B*b)*(c*x^2+b*x)^(5/2)/b^3/x^5
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^7} dx = \frac{2(x(b+cx))^{5/2}\left(9bBx(-5b+2cx)+A\left(-35b^2+20bcx-8c^2x^2\right)\right)}{315b^3x^7}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^7,x]
```

output
```
(2*(x*(b + c*x))^(5/2)*(9*b*B*x*(-5*b + 2*c*x) + A*(-35*b^2 + 20*b*c*x - 8
*c^2*x^2)))/(315*b^3*x^7)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^7}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(9bB-4Ac)\int \frac{\left(cx^2+bx\right)^{3/2}}{x^6}dx}{9b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{9bx^7}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(9bB-4Ac)\left(-\frac{2c\int \frac{\left(cx^2+bx\right)^{3/2}}{x^5}dx}{7b}-\frac{2\left(bx+cx^2\right)^{5/2}}{7bx^6}\right)}{9b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{9bx^7}$$

$$\Big\downarrow \text{1123}$$

$$\frac{\left(\frac{4c\left(bx+cx^2\right)^{5/2}}{35b^2x^5}-\frac{2\left(bx+cx^2\right)^{5/2}}{7bx^6}\right)(9bB-4Ac)}{9b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{9bx^7}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^7,x]
```

output
```
(-2*A*(b*x + c*x^2)^(5/2))/(9*b*x^7) + ((9*b*B - 4*A*c)*((-2*(b*x + c*x^2)
^(5/2))/(7*b*x^6) + (4*c*(b*x + c*x^2)^(5/2))/(35*b^2*x^5)))/(9*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

method	result	size
pseudoelliptic	$-\frac{2(cx+b)^2\sqrt{x(cx+b)}\left(\left(\frac{9Bx}{7}+A\right)b^2-\frac{4c\left(\frac{9Bx}{10}+A\right)xb}{7}+\frac{8A\,c^2x^2}{35}\right)}{9x^5b^3}$	56
gosper	$-\frac{2(cx+b)\left(8A\,c^2x^2-18x^2Bbc-20Abcx+45xB\,b^2+35b^2A\right)\left(c\,x^2+bx\right)^{\frac{3}{2}}}{315b^3x^6}$	62
orering	$-\frac{2(cx+b)\left(8A\,c^2x^2-18x^2Bbc-20Abcx+45xB\,b^2+35b^2A\right)\left(c\,x^2+bx\right)^{\frac{3}{2}}}{315b^3x^6}$	62
trager	$-\frac{2\left(8A\,c^4x^4-18Bb\,c^3x^4-4Ab\,c^3x^3+9B\,b^2c^2x^3+3A\,b^2c^2x^2+72B\,b^3c\,x^2+50A\,b^3cx+45B\,b^4x+35A\,b^4\right)\sqrt{c\,x^2+bx}}{315b^3x^5}$	105
risch	$-\frac{2(cx+b)\left(8A\,c^4x^4-18Bb\,c^3x^4-4Ab\,c^3x^3+9B\,b^2c^2x^3+3A\,b^2c^2x^2+72B\,b^3c\,x^2+50A\,b^3cx+45B\,b^4x+35A\,b^4\right)}{315x^4\sqrt{x(cx+b)}\,b^3}$	108
default	$A\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{5}{2}}}{9b\,x^7}-\frac{4c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{5}{2}}}{7b\,x^6}+\frac{4c\left(c\,x^2+bx\right)^{\frac{5}{2}}}{35b^2x^5}\right)}{9b}\right)+B\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{5}{2}}}{7b\,x^6}+\frac{4c\left(c\,x^2+bx\right)^{\frac{5}{2}}}{35b^2x^5}\right)$	112

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-2/9*(c*x+b)^2*(x*(c*x+b))^(1/2)*((9/7*B*x+A)*b^2-4/7*c*(9/10*B*x+A)*x*b+8
/35*A*c^2*x^2)/x^5/b^3
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.16

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^7}\,dx =$$
$$-\frac{2\left(35\,Ab^4-2\left(9\,Bbc^3-4\,Ac^4\right)x^4+\left(9\,Bb^2c^2-4\,Abc^3\right)x^3+3\left(24\,Bb^3c+Ab^2c^2\right)x^2+5\left(9\,Bb^4+10\,Ab^3c\right)x}{315\,b^3x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
-2/315*(35*A*b^4 - 2*(9*B*b*c^3 - 4*A*c^4)*x^4 + (9*B*b^2*c^2 - 4*A*b*c^3)
*x^3 + 3*(24*B*b^3*c + A*b^2*c^2)*x^2 + 5*(9*B*b^4 + 10*A*b^3*c)*x)*sqrt(c
*x^2 + b*x)/(b^3*x^5)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^7}\,dx=\int \frac{(x(b+cx))^{\frac{3}{2}}\,(A+Bx)}{x^7}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**7,x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(78) = 156.

Time = 0.04 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.47

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^7}\,dx=\frac{4\,\sqrt{cx^2+bx}Bc^3}{35\,b^2x}-\frac{16\,\sqrt{cx^2+bx}Ac^4}{315\,b^3x}-\frac{2\,\sqrt{cx^2+bx}Bc^2}{35\,bx^2}+\frac{8\,\sqrt{cx^2+bx}Ac^3}{315\,b^2x^2}+\frac{3\,\sqrt{cx^2+bx}Bc}{70\,x^3}-\frac{2\,\sqrt{cx^2+bx}Ac^2}{105\,bx^3}+\frac{3\,\sqrt{cx^2+bx}Bb}{14\,x^4}+\frac{\sqrt{cx^2+bx}Ac}{63\,x^4}-\frac{(cx^2+bx)^{\frac{3}{2}}B}{2\,x^5}+\frac{\sqrt{cx^2+bx}Ab}{9\,x^5}-\frac{(cx^2+bx)^{\frac{3}{2}}A}{3\,x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
4/35*sqrt(c*x^2 + b*x)*B*c^3/(b^2*x) - 16/315*sqrt(c*x^2 + b*x)*A*c^4/(b^3
*x) - 2/35*sqrt(c*x^2 + b*x)*B*c^2/(b*x^2) + 8/315*sqrt(c*x^2 + b*x)*A*c^3
/(b^2*x^2) + 3/70*sqrt(c*x^2 + b*x)*B*c/x^3 - 2/105*sqrt(c*x^2 + b*x)*A*c^
2/(b*x^3) + 3/14*sqrt(c*x^2 + b*x)*B*b/x^4 + 1/63*sqrt(c*x^2 + b*x)*A*c/x^
4 - 1/2*(c*x^2 + b*x)^(3/2)*B/x^5 + 1/9*sqrt(c*x^2 + b*x)*A*b/x^5 - 1/3*(c
*x^2 + b*x)^(3/2)*A/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(78) = 156$.

Time = 0.13 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.12

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^7}\,dx = \frac{2\left(315\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^7Bc^{\frac{5}{2}}+945\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^6Bbc^2+420\left(\right.\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^7,x, algorithm="giac")
```

output
```
2/315*(315*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*B*c^(5/2) + 945*(sqrt(c)*x -
sqrt(c*x^2 + b*x))^6*B*b*c^2 + 420*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*A*c^3
 + 1260*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*B*b^2*c^(3/2) + 1575*(sqrt(c)*x
- sqrt(c*x^2 + b*x))^5*A*b*c^(5/2) + 882*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4
*B*b^3*c + 2583*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*A*b^2*c^2 + 315*(sqrt(c)
*x - sqrt(c*x^2 + b*x))^3*B*b^4*sqrt(c) + 2310*(sqrt(c)*x - sqrt(c*x^2 + b
*x))^3*A*b^3*c^(3/2) + 45*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*b^5 + 1170*(
sqrt(c)*x - sqrt(c*x^2 + b*x))^2*A*b^4*c + 315*(sqrt(c)*x - sqrt(c*x^2 + b
*x))*A*b^5*sqrt(c) + 35*A*b^6)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^9
```

Mupad [B] (verification not implemented)

Time = 5.96 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.09

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^7}\,dx = \frac{8\,A\,c^3\,\sqrt{c\,x^2+b\,x}}{315\,b^2\,x^2}-\frac{20\,A\,c\,\sqrt{c\,x^2+b\,x}}{63\,x^4} -\frac{2\,B\,b\,\sqrt{c\,x^2+b\,x}}{7\,x^4}-\frac{16\,B\,c\,\sqrt{c\,x^2+b\,x}}{35\,x^3}-\frac{2\,A\,c^2\,\sqrt{c\,x^2+b\,x}}{105\,b\,x^3}-\frac{2\,A\,b\,\sqrt{c\,x^2+b\,x}}{9\,x^5} -\frac{16\,A\,c^4\,\sqrt{c\,x^2+b\,x}}{315\,b^3\,x}-\frac{2\,B\,c^2\,\sqrt{c\,x^2+b\,x}}{35\,b\,x^2}+\frac{4\,B\,c^3\,\sqrt{c\,x^2+b\,x}}{35\,b^2\,x}$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^7,x)
```

output
```
(8*A*c^3*(b*x + c*x^2)^(1/2))/(315*b^2*x^2) - (20*A*c*(b*x + c*x^2)^(1/2))
/(63*x^4) - (2*B*b*(b*x + c*x^2)^(1/2))/(7*x^4) - (16*B*c*(b*x + c*x^2)^(1
/2))/(35*x^3) - (2*A*c^2*(b*x + c*x^2)^(1/2))/(105*b*x^3) - (2*A*b*(b*x +
c*x^2)^(1/2))/(9*x^5) - (16*A*c^4*(b*x + c*x^2)^(1/2))/(315*b^3*x) - (2*B*
c^2*(b*x + c*x^2)^(1/2))/(35*b*x^2) + (4*B*c^3*(b*x + c*x^2)^(1/2))/(35*b^
2*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.08

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^7}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^4}{9}-\frac{20\sqrt{x}\,\sqrt{cx+b}\,a\,b^3cx}{63}-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^2x^2}{105}+\frac{8\sqrt{x}\,\sqrt{cx+b}\,ab\,c^3x^3}{315}-16\ldots}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^7,x)
```

output
```
(2*( - 35*sqrt(x)*sqrt(b + c*x)*a*b**4 - 50*sqrt(x)*sqrt(b + c*x)*a*b**3*c
*x - 3*sqrt(x)*sqrt(b + c*x)*a*b**2*c**2*x**2 + 4*sqrt(x)*sqrt(b + c*x)*a*
b*c**3*x**3 - 8*sqrt(x)*sqrt(b + c*x)*a*c**4*x**4 - 45*sqrt(x)*sqrt(b + c*
x)*b**5*x - 72*sqrt(x)*sqrt(b + c*x)*b**4*c*x**2 - 9*sqrt(x)*sqrt(b + c*x)
*b**3*c**2*x**3 + 18*sqrt(x)*sqrt(b + c*x)*b**2*c**3*x**4 + 8*sqrt(c)*a*c*
*4*x**5 - 18*sqrt(c)*b**2*c**3*x**5))/(315*b**3*x**5)
```

3.124 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^8} dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^8} dx = -\frac{2A(bx+cx^2)^{5/2}}{11bx^8} - \frac{2(11bB-6Ac)\left(bx+cx^2\right)^{5/2}}{99b^2x^7} + \frac{8c(11bB-6Ac)\left(bx+cx^2\right)^{5/2}}{693b^3x^6} - \frac{16c^2(11bB-6Ac)\left(bx+cx^2\right)^{5/2}}{3465b^4x^5}$$

output
```
-2/11*A*(c*x^2+b*x)^(5/2)/b/x^8-2/99*(-6*A*c+11*B*b)*(c*x^2+b*x)^(5/2)/b^2
/x^7+8/693*c*(-6*A*c+11*B*b)*(c*x^2+b*x)^(5/2)/b^3/x^6-16/3465*c^2*(-6*A*c
+11*B*b)*(c*x^2+b*x)^(5/2)/b^4/x^5
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.63

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^8} dx = -\frac{2(x(b+cx))^{5/2}\left(11bBx(35b^2-20bcx+8c^2x^2)+3A(105b^3-70b^2cx+40bc^2x^2-16c^3x^3)\right)}{3465b^4x^8}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^8,x]
```

output
```
(-2*(x*(b + c*x))^(5/2)*(11*b*B*x*(35*b^2 - 20*b*c*x + 8*c^2*x^2) + 3*A*(1
05*b^3 - 70*b^2*c*x + 40*b*c^2*x^2 - 16*c^3*x^3)))/(3465*b^4*x^8)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^8}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(11bB-6Ac)\int \frac{\left(cx^2+bx\right)^{3/2}}{x^7}dx}{11b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{11bx^8}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(11bB-6Ac)\left(-\frac{4c\int \frac{\left(cx^2+bx\right)^{3/2}}{x^6}dx}{9b}-\frac{2\left(bx+cx^2\right)^{5/2}}{9bx^7}\right)}{11b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{11bx^8}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(11bB-6Ac)\left(-\frac{4c\left(-\frac{2c\int \frac{\left(cx^2+bx\right)^{3/2}}{x^5}dx}{7b}-\frac{2\left(bx+cx^2\right)^{5/2}}{7bx^6}\right)}{9b}-\frac{2\left(bx+cx^2\right)^{5/2}}{9bx^7}\right)}{11b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{11bx^8}$$

$$\Big\downarrow \text{ 1123}$$

$$\frac{\left(-\frac{4c\left(\frac{4c\left(bx+cx^2\right)^{5/2}}{35b^2x^5}-\frac{2\left(bx+cx^2\right)^{5/2}}{7bx^6}\right)}{9b}-\frac{2\left(bx+cx^2\right)^{5/2}}{9bx^7}\right)(11bB-6Ac)}{11b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{11bx^8}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^8,x]
```

output
```
(-2*A*(b*x + c*x^2)^(5/2))/(11*b*x^8) + ((11*b*B - 6*A*c)*((-2*(b*x + c*x^
2)^(5/2))/(9*b*x^7) - (4*c*((-2*(b*x + c*x^2)^(5/2))/(7*b*x^6) + (4*c*(b*x
 + c*x^2)^(5/2))/(35*b^2*x^5)))/(9*b)))/(11*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$-\frac{2(cx+b)^2\sqrt{x(cx+b)}\left(\left(\frac{11Bx}{9}+A\right)b^3-\frac{2cx\left(\frac{22Bx}{21}+A\right)b^2}{3}+\frac{8c^2\left(\frac{11Bx}{15}+A\right)x^2b}{21}-\frac{16Ac^3x^3}{105}\right)}{11x^6b^4}$
gosper	$-\frac{2(cx+b)(-48Ac^3x^3+88x^3Bbc^2+120Abc^2x^2-220x^2Bb^2c-210Ab^2cx+385xBb^3+315Ab^3)(cx^2+bx)^{\frac{3}{2}}}{3465x^7b^4}$
orering	$-\frac{2(cx+b)(-48Ac^3x^3+88x^3Bbc^2+120Abc^2x^2-220x^2Bb^2c-210Ab^2cx+385xBb^3+315Ab^3)(cx^2+bx)^{\frac{3}{2}}}{3465x^7b^4}$
trager	$-\frac{2(-48Ac^5x^5+88Bbc^4x^5+24Abc^4x^4-44Bb^2c^3x^4-18Ab^2c^3x^3+33Bb^3c^2x^3+15Ab^3c^2x^2+550Bb^4cx^2+420Ab^4cx+3}{3465b^4x^6}$
risch	$-\frac{2(cx+b)(-48Ac^5x^5+88Bbc^4x^5+24Abc^4x^4-44Bb^2c^3x^4-18Ab^2c^3x^3+33Bb^3c^2x^3+15Ab^3c^2x^2+550Bb^4cx^2+420A}{3465x^5\sqrt{x(cx+b)}\,b^4}$
default	$A\left(-\frac{2(cx^2+bx)^{\frac{5}{2}}}{11bx^8}-\frac{6c\left(-\frac{2(cx^2+bx)^{\frac{5}{2}}}{9bx^7}-\frac{4c\left(-\frac{2(cx^2+bx)^{\frac{5}{2}}}{7bx^6}+\frac{4c(cx^2+bx)^{\frac{5}{2}}}{35b^2x^5}\right)}{9b}\right)}{11b}\right)+B\left(-\frac{2(cx^2+bx)^{\frac{5}{2}}}{9bx^7}-\frac{4c}{}\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-2/11*(c*x+b)^2*(x*(c*x+b))^(1/2)*((11/9*B*x+A)*b^3-2/3*c*x*(22/21*B*x+A)*
b^2+8/21*c^2*(11/15*B*x+A)*x^2*b-16/105*A*c^3*x^3)/x^6/b^4
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.04

$$\int\frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^8}\,dx=$$

$$-\frac{2\left(315\,Ab^5+8\,(11\,Bbc^4-6\,Ac^5)x^5-4\,(11\,Bb^2c^3-6\,Abc^4)x^4+3\,(11\,Bb^3c^2-6\,Ab^2c^3)x^3+5\,(110\,Bb^4c\right.}{3465\,b^4x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
-2/3465*(315*A*b^5 + 8*(11*B*b*c^4 - 6*A*c^5)*x^5 - 4*(11*B*b^2*c^3 - 6*A*
b*c^4)*x^4 + 3*(11*B*b^3*c^2 - 6*A*b^2*c^3)*x^3 + 5*(110*B*b^4*c + 3*A*b^3
*c^2)*x^2 + 35*(11*B*b^5 + 12*A*b^4*c)*x)*sqrt(c*x^2 + b*x)/(b^4*x^6)
```

Sympy [F]

$$\int \frac{(A+Bx)\,(bx+cx^2)^{3/2}}{x^8}\,dx = \int \frac{(x(b+cx))^{\frac{3}{2}}\,(A+Bx)}{x^8}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**8,x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/x**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. 2(109) = 218.

Time = 0.04 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.14

$$\begin{aligned}\int \frac{(A+Bx)\,(bx+cx^2)^{3/2}}{x^8}\,dx = & -\frac{16\sqrt{cx^2+bx}Bc^4}{315\,b^3x} + \frac{32\sqrt{cx^2+bx}Ac^5}{1155\,b^4x} \\ & + \frac{8\sqrt{cx^2+bx}Bc^3}{315\,b^2x^2} - \frac{16\sqrt{cx^2+bx}Ac^4}{1155\,b^3x^2} - \frac{2\sqrt{cx^2+bx}Bc^2}{105\,bx^3} \\ & + \frac{4\sqrt{cx^2+bx}Ac^3}{385\,b^2x^3} + \frac{\sqrt{cx^2+bx}Bc}{63\,x^4} - \frac{2\sqrt{cx^2+bx}Ac^2}{231\,bx^4} + \frac{\sqrt{cx^2+bx}Bb}{9\,x^5} \\ & + \frac{\sqrt{cx^2+bx}Ac}{132\,x^5} - \frac{(cx^2+bx)^{\frac{3}{2}}B}{3\,x^6} + \frac{3\sqrt{cx^2+bx}Ab}{44\,x^6} - \frac{(cx^2+bx)^{\frac{3}{2}}A}{4\,x^7}\end{aligned}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
-16/315*sqrt(c*x^2 + b*x)*B*c^4/(b^3*x) + 32/1155*sqrt(c*x^2 + b*x)*A*c^5/
(b^4*x) + 8/315*sqrt(c*x^2 + b*x)*B*c^3/(b^2*x^2) - 16/1155*sqrt(c*x^2 + b
*x)*A*c^4/(b^3*x^2) - 2/105*sqrt(c*x^2 + b*x)*B*c^2/(b*x^3) + 4/385*sqrt(c
*x^2 + b*x)*A*c^3/(b^2*x^3) + 1/63*sqrt(c*x^2 + b*x)*B*c/x^4 - 2/231*sqrt(
c*x^2 + b*x)*A*c^2/(b*x^4) + 1/9*sqrt(c*x^2 + b*x)*B*b/x^5 + 1/132*sqrt(c*
x^2 + b*x)*A*c/x^5 - 1/3*(c*x^2 + b*x)^(3/2)*B/x^6 + 3/44*sqrt(c*x^2 + b*x
)*A*b/x^6 - 1/4*(c*x^2 + b*x)^(3/2)*A/x^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. $2(109) = 218$.

Time = 0.15 (sec) , antiderivative size = 431, normalized size of antiderivative = 3.45

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^8}\,dx = \frac{2\left(4620\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^8Bc^3+17325\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^7Bbc^{\frac{5}{2}}+69\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^8,x, algorithm="giac")
```

output
```
2/3465*(4620*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*B*c^3 + 17325*(sqrt(c)*x -
sqrt(c*x^2 + b*x))^7*B*b*c^(5/2) + 6930*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*
A*c^(7/2) + 28413*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*B*b^2*c^2 + 30492*(sqr
t(c)*x - sqrt(c*x^2 + b*x))^6*A*b*c^3 + 25410*(sqrt(c)*x - sqrt(c*x^2 + b*
x))^5*B*b^3*c^(3/2) + 58905*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*A*b^2*c^(5/2
) + 12870*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*B*b^4*c + 63855*(sqrt(c)*x - s
qrt(c*x^2 + b*x))^4*A*b^3*c^2 + 3465*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*B*b
^5*sqrt(c) + 41580*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*A*b^4*c^(3/2) + 385*(
sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*b^6 + 16170*(sqrt(c)*x - sqrt(c*x^2 + b
*x))^2*A*b^5*c + 3465*(sqrt(c)*x - sqrt(c*x^2 + b*x))*A*b^6*sqrt(c) + 315*
A*b^7)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^11
```

Mupad [B] (verification not implemented)

Time = 6.21 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.87

$$\int \frac{(A+Bx)\,(bx+cx^2)^{3/2}}{x^8}\,dx = \frac{4\,A\,c^3\,\sqrt{c\,x^2+b\,x}}{385\,b^2\,x^3} - \frac{8\,A\,c\,\sqrt{c\,x^2+b\,x}}{33\,x^5} - \frac{2\,B\,b\,\sqrt{c\,x^2+b\,x}}{9\,x^5} - \frac{20\,B\,c\,\sqrt{c\,x^2+b\,x}}{63\,x^4} - \frac{2\,A\,c^2\,\sqrt{c\,x^2+b\,x}}{231\,b\,x^4} - \frac{2\,A\,b\,\sqrt{c\,x^2+b\,x}}{11\,x^6} - \frac{16\,A\,c^4\,\sqrt{c\,x^2+b\,x}}{1155\,b^3\,x^2} + \frac{32\,A\,c^5\,\sqrt{c\,x^2+b\,x}}{1155\,b^4\,x} - \frac{2\,B\,c^2\,\sqrt{c\,x^2+b\,x}}{105\,b\,x^3} + \frac{8\,B\,c^3\,\sqrt{c\,x^2+b\,x}}{315\,b^2\,x^2} - \frac{16\,B\,c^4\,\sqrt{c\,x^2+b\,x}}{315\,b^3\,x}$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^8,x)
```

output
```
(4*A*c^3*(b*x + c*x^2)^(1/2))/(385*b^2*x^3) - (8*A*c*(b*x + c*x^2)^(1/2))/
(33*x^5) - (2*B*b*(b*x + c*x^2)^(1/2))/(9*x^5) - (20*B*c*(b*x + c*x^2)^(1/
2))/(63*x^4) - (2*A*c^2*(b*x + c*x^2)^(1/2))/(231*b*x^4) - (2*A*b*(b*x + c
*x^2)^(1/2))/(11*x^6) - (16*A*c^4*(b*x + c*x^2)^(1/2))/(1155*b^3*x^2) + (3
2*A*c^5*(b*x + c*x^2)^(1/2))/(1155*b^4*x) - (2*B*c^2*(b*x + c*x^2)^(1/2))/
(105*b*x^3) + (8*B*c^3*(b*x + c*x^2)^(1/2))/(315*b^2*x^2) - (16*B*c^4*(b*x
 + c*x^2)^(1/2))/(315*b^3*x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.81

$$\int \frac{(A+Bx)\,(bx+cx^2)^{3/2}}{x^8}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^5}{11} - \frac{8\sqrt{x}\,\sqrt{cx+b}\,a\,b^4cx}{33} - \frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^3c^2x^2}{231} + \frac{4\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^3x^3}{385} - 16\sqrt{}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^8,x)
```

output
```
(2*( - 315*sqrt(x)*sqrt(b + c*x)*a*b**5 - 420*sqrt(x)*sqrt(b + c*x)*a*b**4
*c*x - 15*sqrt(x)*sqrt(b + c*x)*a*b**3*c**2*x**2 + 18*sqrt(x)*sqrt(b + c*x
)*a*b**2*c**3*x**3 - 24*sqrt(x)*sqrt(b + c*x)*a*b*c**4*x**4 + 48*sqrt(x)*s
qrt(b + c*x)*a*c**5*x**5 - 385*sqrt(x)*sqrt(b + c*x)*b**6*x - 550*sqrt(x)*
sqrt(b + c*x)*b**5*c*x**2 - 33*sqrt(x)*sqrt(b + c*x)*b**4*c**2*x**3 + 44*s
qrt(x)*sqrt(b + c*x)*b**3*c**3*x**4 - 88*sqrt(x)*sqrt(b + c*x)*b**2*c**4*x
**5 - 48*sqrt(c)*a*c**5*x**6 + 88*sqrt(c)*b**2*c**4*x**6))/(3465*b**4*x**6
)
```

3.125 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^9}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^9}\,dx = -\frac{2A(bx+cx^2)^{5/2}}{13bx^9} - \frac{2(13bB-8Ac)\left(bx+cx^2\right)^{5/2}}{143b^2x^8} + \frac{4c(13bB-8Ac)\left(bx+cx^2\right)^{5/2}}{429b^3x^7} - \frac{16c^2(13bB-8Ac)\left(bx+cx^2\right)^{5/2}}{3003b^4x^6} + \frac{32c^3(13bB-8Ac)\left(bx+cx^2\right)^{5/2}}{15015b^5x^5}$$

output
```
-2/13*A*(c*x^2+b*x)^(5/2)/b/x^9-2/143*(-8*A*c+13*B*b)*(c*x^2+b*x)^(5/2)/b^
2/x^8+4/429*c*(-8*A*c+13*B*b)*(c*x^2+b*x)^(5/2)/b^3/x^7-16/3003*c^2*(-8*A*
c+13*B*b)*(c*x^2+b*x)^(5/2)/b^4/x^6+32/15015*c^3*(-8*A*c+13*B*b)*(c*x^2+b*
x)^(5/2)/b^5/x^5
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^9}\,dx = \frac{2(x(b+cx))^{5/2}\left(13bBx(-105b^3+70b^2cx-40bc^2x^2+16c^3x^3)+A(-1155\right.}{15015b^5x^9}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^9,x]
```

output
```
(2*(x*(b + c*x))^(5/2)*(13*b*B*x*(-105*b^3 + 70*b^2*c*x - 40*b*c^2*x^2 + 1
6*c^3*x^3) + A*(-1155*b^4 + 840*b^3*c*x - 560*b^2*c^2*x^2 + 320*b*c^3*x^3
- 128*c^4*x^4)))/(15015*b^5*x^9)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^9}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(13bB-8Ac)\int \frac{\left(cx^2+bx\right)^{3/2}}{x^8}dx}{13b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{13bx^9}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(13bB-8Ac)\left(-\frac{6c\int \frac{\left(cx^2+bx\right)^{3/2}}{x^7}dx}{11b}-\frac{2\left(bx+cx^2\right)^{5/2}}{11bx^8}\right)}{13b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{13bx^9}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(13bB-8Ac)\left(-\frac{6c\left(-\frac{4c\int \frac{\left(cx^2+bx\right)^{3/2}}{x^6}dx}{9b}-\frac{2\left(bx+cx^2\right)^{5/2}}{9bx^7}\right)}{11b}-\frac{2\left(bx+cx^2\right)^{5/2}}{11bx^8}\right)}{13b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{13bx^9}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(13bB-8Ac)\left(-\frac{6c\left(-\frac{4c\left(-\frac{2c\int \frac{\left(cx^2+bx\right)^{3/2}}{x^5}dx}{7b}-\frac{2\left(bx+cx^2\right)^{5/2}}{7bx^6}\right)}{9b}-\frac{2\left(bx+cx^2\right)^{5/2}}{9bx^7}\right)}{11b}-\frac{2\left(bx+cx^2\right)^{5/2}}{11bx^8}\right)}{13b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{13bx^9}$$

$$\Big\downarrow \text{ 1123}$$

$$\frac{\left(-\frac{6c\left(-\frac{4c\left(\frac{4c\left(bx+cx^2\right)^{5/2}}{35b^2x^5}-\frac{2\left(bx+cx^2\right)^{5/2}}{7bx^6}\right)}{9b}-\frac{2\left(bx+cx^2\right)^{5/2}}{9bx^7}\right)}{11b}-\frac{2\left(bx+cx^2\right)^{5/2}}{11bx^8}\right)(13bB-8Ac)}{13b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{13bx^9}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^9,x]
```

output
```
(-2*A*(b*x + c*x^2)^(5/2))/(13*b*x^9) + ((13*b*B - 8*A*c)*((-2*(b*x + c*x^
2)^(5/2))/(11*b*x^8) - (6*c*((-2*(b*x + c*x^2)^(5/2))/(9*b*x^7) - (4*c*((-
2*(b*x + c*x^2)^(5/2))/(7*b*x^6) + (4*c*(b*x + c*x^2)^(5/2))/(35*b^2*x^5))
)/(9*b)))/(11*b)))/(13*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.56

method	result
pseudoelliptic	$-\frac{2(cx+b)^2\sqrt{x(cx+b)}\left(\left(\frac{13Bx}{11}+A\right)b^4-\frac{8cx\left(\frac{13Bx}{12}+A\right)b^3}{11}+\frac{16c^2x^2\left(\frac{13Bx}{14}+A\right)b^2}{33}-\frac{64c^3x^3\left(\frac{13Bx}{20}+A\right)b}{231}+\frac{128A\,c^4x^4}{1155}\right)}{13x^7b^5}$
gosper	$-\frac{2(cx+b)(128A\,c^4x^4-208Bb\,c^3x^4-320Ab\,c^3x^3+520B\,b^2c^2x^3+560A\,b^2c^2x^2-910B\,b^3c\,x^2-840A\,b^3cx+1365B\,b^4x+1155}{15015x^8b^5}$
orering	$-\frac{2(cx+b)(128A\,c^4x^4-208Bb\,c^3x^4-320Ab\,c^3x^3+520B\,b^2c^2x^3+560A\,b^2c^2x^2-910B\,b^3c\,x^2-840A\,b^3cx+1365B\,b^4x+1155}{15015x^8b^5}$
trager	$-\frac{2(128A\,c^6x^6-208Bb\,c^5x^6-64Ab\,c^5x^5+104B\,b^2c^4x^5+48A\,b^2c^4x^4-78B\,b^3c^3x^4-40A\,b^3c^3x^3+65B\,b^4c^2x^3+35A\,b^4c^2x^2}{15015b^5x^7}$
risch	$-\frac{2(cx+b)(128A\,c^6x^6-208Bb\,c^5x^6-64Ab\,c^5x^5+104B\,b^2c^4x^5+48A\,b^2c^4x^4-78B\,b^3c^3x^4-40A\,b^3c^3x^3+65B\,b^4c^2x^3+35A}{15015x^6\sqrt{x(cx+b)}\,b^5}$
default	$A\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{5}{2}}}{13b\,x^9}-\frac{8c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{5}{2}}}{11b\,x^8}-\frac{6c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{5}{2}}}{9b\,x^7}-\frac{4c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{5}{2}}}{7b\,x^6}+\frac{4c\left(c\,x^2+bx\right)^{\frac{5}{2}}}{35b^2x^5}\right)}{9b}\right)}{11b}\right)}{13b}\right)+B\left(-\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-2/13*(c*x+b)^2*(x*(c*x+b))^(1/2)*((13/11*B*x+A)*b^4-8/11*c*x*(13/12*B*x+A
)*b^3+16/33*c^2*x^2*(13/14*B*x+A)*b^2-64/231*c^3*x^3*(13/20*B*x+A)*b+128/1
155*A*c^4*x^4)/x^7/b^5
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^9}\,dx =$$
$$-\frac{2\,(1155\,Ab^6-16\,(13\,Bbc^5-8\,Ac^6)x^6+8\,(13\,Bb^2c^4-8\,Abc^5)x^5-6\,(13\,Bb^3c^3-8\,Ab^2c^4)x^4+5\,(13\,Bb^4c}{15015\,b^5x^7}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
-2/15015*(1155*A*b^6 - 16*(13*B*b*c^5 - 8*A*c^6)*x^6 + 8*(13*B*b^2*c^4 - 8
*A*b*c^5)*x^5 - 6*(13*B*b^3*c^3 - 8*A*b^2*c^4)*x^4 + 5*(13*B*b^4*c^2 - 8*A
*b^3*c^3)*x^3 + 35*(52*B*b^5*c + A*b^4*c^2)*x^2 + 105*(13*B*b^6 + 14*A*b^5
*c)*x)*sqrt(c*x^2 + b*x)/(b^5*x^7)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^9}\,dx = \int \frac{\left(x(b+cx)\right)^{\frac{3}{2}}(A+Bx)}{x^9}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**9,x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/x**9, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 314 vs. $2(140) = 280$.

Time = 0.04 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.96

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^9}\,dx = \frac{32\sqrt{cx^2+bx}Bc^5}{1155\,b^4x} - \frac{256\sqrt{cx^2+bx}Ac^6}{15015\,b^5x} - \frac{16\sqrt{cx^2+bx}Bc^4}{1155\,b^3x^2} + \frac{128\sqrt{cx^2+bx}Ac^5}{15015\,b^4x^2} + \frac{4\sqrt{cx^2+bx}Bc^3}{385\,b^2x^3} - \frac{32\sqrt{cx^2+bx}Ac^4}{5005\,b^3x^3} - \frac{2\sqrt{cx^2+bx}Bc^2}{231\,bx^4} + \frac{16\sqrt{cx^2+bx}Ac^3}{3003\,b^2x^4} + \frac{\sqrt{cx^2+bx}Bc}{132\,x^5} - \frac{2\sqrt{cx^2+bx}Ac^2}{429\,bx^5} + \frac{3\sqrt{cx^2+bx}Bb}{44\,x^6} + \frac{3\sqrt{cx^2+bx}Ac}{715\,x^6} - \frac{(cx^2+bx)^{\frac{3}{2}}B}{4\,x^7} + \frac{3\sqrt{cx^2+bx}Ab}{65\,x^7} - \frac{(cx^2+bx)^{\frac{3}{2}}A}{5\,x^8}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
32/1155*sqrt(c*x^2 + b*x)*B*c^5/(b^4*x) - 256/15015*sqrt(c*x^2 + b*x)*A*c^
6/(b^5*x) - 16/1155*sqrt(c*x^2 + b*x)*B*c^4/(b^3*x^2) + 128/15015*sqrt(c*x
^2 + b*x)*A*c^5/(b^4*x^2) + 4/385*sqrt(c*x^2 + b*x)*B*c^3/(b^2*x^3) - 32/5
005*sqrt(c*x^2 + b*x)*A*c^4/(b^3*x^3) - 2/231*sqrt(c*x^2 + b*x)*B*c^2/(b*x
^4) + 16/3003*sqrt(c*x^2 + b*x)*A*c^3/(b^2*x^4) + 1/132*sqrt(c*x^2 + b*x)*
B*c/x^5 - 2/429*sqrt(c*x^2 + b*x)*A*c^2/(b*x^5) + 3/44*sqrt(c*x^2 + b*x)*B
*b/x^6 + 3/715*sqrt(c*x^2 + b*x)*A*c/x^6 - 1/4*(c*x^2 + b*x)^(3/2)*B/x^7 +
 3/65*sqrt(c*x^2 + b*x)*A*b/x^7 - 1/5*(c*x^2 + b*x)^(3/2)*A/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. 2(140) = 280.

Time = 0.16 (sec) , antiderivative size = 491, normalized size of antiderivative = 3.07

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^9}\,dx = \frac{2\left(30030\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^9Bc^{\frac{7}{2}}+132132\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^8Bbc^3+\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^9,x, algorithm="giac")
```

output
```
2/15015*(30030*(sqrt(c)*x - sqrt(c*x^2 + b*x))^9*B*c^(7/2) + 132132*(sqrt(
c)*x - sqrt(c*x^2 + b*x))^8*B*b*c^3 + 48048*(sqrt(c)*x - sqrt(c*x^2 + b*x)
)^8*A*c^4 + 255255*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*B*b^2*c^(5/2) + 24024
0*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*A*b*c^(7/2) + 276705*(sqrt(c)*x - sqrt
(c*x^2 + b*x))^6*B*b^3*c^2 + 531960*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*A*b^
2*c^3 + 180180*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*B*b^4*c^(3/2) + 675675*(s
qrt(c)*x - sqrt(c*x^2 + b*x))^5*A*b^3*c^(5/2) + 70070*(sqrt(c)*x - sqrt(c*
x^2 + b*x))^4*B*b^5*c + 535535*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*A*b^4*c^2
 + 15015*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*B*b^6*sqrt(c) + 270270*(sqrt(c)
*x - sqrt(c*x^2 + b*x))^3*A*b^5*c^(3/2) + 1365*(sqrt(c)*x - sqrt(c*x^2 + b
*x))^2*B*b^7 + 84630*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*A*b^6*c + 15015*(sq
rt(c)*x - sqrt(c*x^2 + b*x))*A*b^7*sqrt(c) + 1155*A*b^8)/(sqrt(c)*x - sqrt
(c*x^2 + b*x))^13
```

Mupad [B] (verification not implemented)

Time = 6.50 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.75

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^9}\,dx = \frac{16\,A\,c^3\,\sqrt{c\,x^2+b\,x}}{3003\,b^2\,x^4} - \frac{28\,A\,c\,\sqrt{c\,x^2+b\,x}}{143\,x^6} - \frac{2\,B\,b\,\sqrt{c\,x^2+b\,x}}{11\,x^6} - \frac{8\,B\,c\,\sqrt{c\,x^2+b\,x}}{33\,x^5} - \frac{2\,A\,c^2\,\sqrt{c\,x^2+b\,x}}{429\,b\,x^5} - \frac{2\,A\,b\,\sqrt{c\,x^2+b\,x}}{13\,x^7} - \frac{32\,A\,c^4\,\sqrt{c\,x^2+b\,x}}{5005\,b^3\,x^3} + \frac{128\,A\,c^5\,\sqrt{c\,x^2+b\,x}}{15015\,b^4\,x^2} - \frac{256\,A\,c^6\,\sqrt{c\,x^2+b\,x}}{15015\,b^5\,x} - \frac{2\,B\,c^2\,\sqrt{c\,x^2+b\,x}}{231\,b\,x^4} + \frac{4\,B\,c^3\,\sqrt{c\,x^2+b\,x}}{385\,b^2\,x^3} - \frac{16\,B\,c^4\,\sqrt{c\,x^2+b\,x}}{1155\,b^3\,x^2} + \frac{32\,B\,c^5\,\sqrt{c\,x^2+b\,x}}{1155\,b^4\,x}$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^9,x)
```

output
```
(16*A*c^3*(b*x + c*x^2)^(1/2))/(3003*b^2*x^4) - (28*A*c*(b*x + c*x^2)^(1/2
))/(143*x^6) - (2*B*b*(b*x + c*x^2)^(1/2))/(11*x^6) - (8*B*c*(b*x + c*x^2)
^(1/2))/(33*x^5) - (2*A*c^2*(b*x + c*x^2)^(1/2))/(429*b*x^5) - (2*A*b*(b*x
 + c*x^2)^(1/2))/(13*x^7) - (32*A*c^4*(b*x + c*x^2)^(1/2))/(5005*b^3*x^3)
+ (128*A*c^5*(b*x + c*x^2)^(1/2))/(15015*b^4*x^2) - (256*A*c^6*(b*x + c*x^
2)^(1/2))/(15015*b^5*x) - (2*B*c^2*(b*x + c*x^2)^(1/2))/(231*b*x^4) + (4*B
*c^3*(b*x + c*x^2)^(1/2))/(385*b^2*x^3) - (16*B*c^4*(b*x + c*x^2)^(1/2))/(
1155*b^3*x^2) + (32*B*c^5*(b*x + c*x^2)^(1/2))/(1155*b^4*x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.66

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^9}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^6}{13}-\frac{28\sqrt{x}\,\sqrt{cx+b}\,a\,b^5cx}{143}-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^4c^2x^2}{429}+\frac{16\sqrt{x}\,\sqrt{cx+b}\,a\,b^3c^3x^3}{3003}-3}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^9,x)
```

output
```
(2*( - 1155*sqrt(x)*sqrt(b + c*x)*a*b**6 - 1470*sqrt(x)*sqrt(b + c*x)*a*b*
*5*c*x - 35*sqrt(x)*sqrt(b + c*x)*a*b**4*c**2*x**2 + 40*sqrt(x)*sqrt(b + c
*x)*a*b**3*c**3*x**3 - 48*sqrt(x)*sqrt(b + c*x)*a*b**2*c**4*x**4 + 64*sqrt
(x)*sqrt(b + c*x)*a*b*c**5*x**5 - 128*sqrt(x)*sqrt(b + c*x)*a*c**6*x**6 -
1365*sqrt(x)*sqrt(b + c*x)*b**7*x - 1820*sqrt(x)*sqrt(b + c*x)*b**6*c*x**2
 - 65*sqrt(x)*sqrt(b + c*x)*b**5*c**2*x**3 + 78*sqrt(x)*sqrt(b + c*x)*b**4
*c**3*x**4 - 104*sqrt(x)*sqrt(b + c*x)*b**3*c**4*x**5 + 208*sqrt(x)*sqrt(b
 + c*x)*b**2*c**5*x**6 + 128*sqrt(c)*a*c**6*x**7 - 208*sqrt(c)*b**2*c**5*x
**7))/(15015*b**5*x**7)
```

3.126 $\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 195

$$\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^{10}}\,dx = -\frac{2A(bx+cx^2)^{5/2}}{15bx^{10}} - \frac{2(3bB-2Ac)(bx+cx^2)^{5/2}}{39b^2x^9} + \frac{16c(3bB-2Ac)(bx+cx^2)^{5/2}}{429b^3x^8} - \frac{32c^2(3bB-2Ac)(bx+cx^2)^{5/2}}{1287b^4x^7} + \frac{128c^3(3bB-2Ac)(bx+cx^2)^{5/2}}{9009b^5x^6} - \frac{256c^4(3bB-2Ac)(bx+cx^2)^{5/2}}{45045b^6x^5}$$

output
```
-2/15*A*(c*x^2+b*x)^(5/2)/b/x^10-2/39*(-2*A*c+3*B*b)*(c*x^2+b*x)^(5/2)/b^2
/x^9+16/429*c*(-2*A*c+3*B*b)*(c*x^2+b*x)^(5/2)/b^3/x^8-32/1287*c^2*(-2*A*c
+3*B*b)*(c*x^2+b*x)^(5/2)/b^4/x^7+128/9009*c^3*(-2*A*c+3*B*b)*(c*x^2+b*x)^
(5/2)/b^5/x^6-256/45045*c^4*(-2*A*c+3*B*b)*(c*x^2+b*x)^(5/2)/b^6/x^5
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.63

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{10}}\,dx =$$
$$-\frac{2(x(b+cx))^{5/2}\left(3bBx(1155b^4-840b^3cx+560b^2c^2x^2-320bc^3x^3+128c^4x^4)+A(3003b^5-2310b^4cx+\right.}{45045b^6x^{10}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^10,x]
```

output
```
(-2*(x*(b + c*x))^(5/2)*(3*b*B*x*(1155*b^4 - 840*b^3*c*x + 560*b^2*c^2*x^2
 - 320*b*c^3*x^3 + 128*c^4*x^4) + A*(3003*b^5 - 2310*b^4*c*x + 1680*b^3*c^
2*x^2 - 1120*b^2*c^3*x^3 + 640*b*c^4*x^4 - 256*c^5*x^5)))/(45045*b^6*x^10)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1220, 1129, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{10}}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(3bB-2Ac)\int \frac{\left(cx^2+bx\right)^{3/2}}{x^9}dx}{3b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{15bx^{10}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(3bB-2Ac)\left(-\frac{8c\int \frac{\left(cx^2+bx\right)^{3/2}}{x^8}dx}{13b}-\frac{2\left(bx+cx^2\right)^{5/2}}{13bx^9}\right)}{3b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{15bx^{10}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(3bB-2Ac)\left(-\frac{8c\left(-\frac{6c\int\frac{\left(cx^2+bx\right)^{3/2}}{x^7}dx}{11b}-\frac{2\left(bx+cx^2\right)^{5/2}}{11bx^8}\right)}{13b}-\frac{2\left(bx+cx^2\right)^{5/2}}{13bx^9}\right)}{3b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{15bx^{10}}$$

$\big\downarrow$ 1129

$$\frac{(3bB-2Ac)\left(-\frac{8c\left(-\frac{6c\left(-\frac{4c\int\frac{\left(cx^2+bx\right)^{3/2}}{x^6}dx}{9b}-\frac{2\left(bx+cx^2\right)^{5/2}}{9bx^7}\right)}{11b}-\frac{2\left(bx+cx^2\right)^{5/2}}{11bx^8}\right)}{13b}-\frac{2\left(bx+cx^2\right)^{5/2}}{13bx^9}\right)}{3b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{15bx^{10}}$$

$\big\downarrow$ 1129

$$\frac{(3bB-2Ac)\left(-\frac{8c\left(-\frac{6c\left(-\frac{4c\left(-\frac{2c\int\frac{\left(cx^2+bx\right)^{3/2}}{x^5}dx}{7b}-\frac{2\left(bx+cx^2\right)^{5/2}}{7bx^6}\right)}{9b}-\frac{2\left(bx+cx^2\right)^{5/2}}{9bx^7}\right)}{11b}-\frac{2\left(bx+cx^2\right)^{5/2}}{11bx^8}\right)}{13b}-\frac{2\left(bx+cx^2\right)^{5/2}}{13bx^9}\right)}{3b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{15bx^{10}}$$

$\downarrow$ 1123

$$\frac{\left(-\frac{8c\left(-\frac{6c\left(-\frac{4c\left(\frac{4c\left(bx+cx^2\right)^{5/2}}{35b^2x^5}-\frac{2\left(bx+cx^2\right)^{5/2}}{7bx^6}\right)}{9b}-\frac{2\left(bx+cx^2\right)^{5/2}}{9bx^7}\right)}{11b}-\frac{2\left(bx+cx^2\right)^{5/2}}{11bx^8}\right)}{13b}-\frac{2\left(bx+cx^2\right)^{5/2}}{13bx^9}\right)(3bB-2Ac)}{3b}-\frac{2A\left(bx+cx^2\right)^{5/2}}{15bx^{10}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^10,x]
```

output
```
(-2*A*(b*x + c*x^2)^(5/2))/(15*b*x^10) + ((3*b*B - 2*A*c)*((-2*(b*x + c*x^
2)^(5/2))/(13*b*x^9) - (8*c*((-2*(b*x + c*x^2)^(5/2))/(11*b*x^8) - (6*c*((
-2*(b*x + c*x^2)^(5/2))/(9*b*x^7) - (4*c*((-2*(b*x + c*x^2)^(5/2))/(7*b*x^
6) + (4*c*(b*x + c*x^2)^(5/2))/(35*b^2*x^5)))/(9*b)))/(11*b)))/(13*b)))/(3
*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.54

method	result
pseudoelliptic	$-\frac{2(cx+b)^2\sqrt{x(cx+b)}\left(\left(\left(\frac{15Bx}{13}+A\right)b^5-\frac{10cx\left(\frac{12Bx}{11}+A\right)b^4}{13}+\frac{80c^2x^2(Bx+A)b^3}{143}-\frac{160c^3x^3\left(\frac{6Bx}{7}+A\right)b^2}{429}+\frac{640c^4x^4\left(\frac{3Bx}{5}+A\right)b}{3003}-\right.\right.}{15x^8b^6}$
gosper	$-\frac{2(cx+b)\left(-256A\,c^5x^5+384Bb\,c^4x^5+640Ab\,c^4x^4-960B\,b^2c^3x^4-1120A\,b^2c^3x^3+1680B\,b^3c^2x^3+1680A\,b^3c^2x^2-2520B\,b\right.}{45045x^9b^6}$
orering	$-\frac{2(cx+b)\left(-256A\,c^5x^5+384Bb\,c^4x^5+640Ab\,c^4x^4-960B\,b^2c^3x^4-1120A\,b^2c^3x^3+1680B\,b^3c^2x^3+1680A\,b^3c^2x^2-2520B\,b\right.}{45045x^9b^6}$
trager	$-\frac{2\left(-256A\,c^7x^7+384Bb\,c^6x^7+128Ab\,c^6x^6-192B\,b^2c^5x^6-96A\,b^2c^5x^5+144B\,b^3c^4x^5+80A\,b^3c^4x^4-120B\,b^4c^3x^4-70A\,b^4\right.}{45045b^6x^8}$
risch	$-\frac{2(cx+b)\left(-256A\,c^7x^7+384Bb\,c^6x^7+128Ab\,c^6x^6-192B\,b^2c^5x^6-96A\,b^2c^5x^5+144B\,b^3c^4x^5+80A\,b^3c^4x^4-120B\,b^4c^3x^4-\right.}{45045x^7\sqrt{x(cx+b)}\,b^6}$
default	$A\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{5}{2}}}{15b\,x^{10}}-\frac{2c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{5}{2}}}{13b\,x^9}-\frac{8c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{5}{2}}}{11b\,x^8}-\frac{6c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{5}{2}}}{9b\,x^7}-\frac{4c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{5}{2}}}{7b\,x^6}+\frac{4c\left(c\,x^2+bx\right)^{\frac{5}{2}}}{35b^2x^5}\right.}{9b}\right.}{11b}\right.}{13b}\right.}{3b}\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(c*x+b)^2*(x*(c*x+b))^(1/2)*((15/13*B*x+A)*b^5-10/13*c*x*(12/11*B*x+
A)*b^4+80/143*c^2*x^2*(B*x+A)*b^3-160/429*c^3*x^3*(6/7*B*x+A)*b^2+640/3003
*c^4*x^4*(3/5*B*x+A)*b-256/3003*A*c^5*x^5)/x^8/b^6
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{10}}\,dx =$$
$$-\frac{2\,(3003\,Ab^7+128\,(3\,Bbc^6-2\,Ac^7)x^7-64\,(3\,Bb^2c^5-2\,Abc^6)x^6+48\,(3\,Bb^3c^4-2\,Ab^2c^5)x^5-40\,(3\,Bb^4c}{450}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^10,x, algorithm="fricas")
```

output
```
-2/45045*(3003*A*b^7 + 128*(3*B*b*c^6 - 2*A*c^7)*x^7 - 64*(3*B*b^2*c^5 - 2
*A*b*c^6)*x^6 + 48*(3*B*b^3*c^4 - 2*A*b^2*c^5)*x^5 - 40*(3*B*b^4*c^3 - 2*A
*b^3*c^4)*x^4 + 35*(3*B*b^5*c^2 - 2*A*b^4*c^3)*x^3 + 63*(70*B*b^6*c + A*b^
5*c^2)*x^2 + 231*(15*B*b^7 + 16*A*b^6*c)*x)*sqrt(c*x^2 + b*x)/(b^6*x^8)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{10}}\,dx = \int \frac{(x(b+cx))^{\frac{3}{2}}\,(A+Bx)}{x^{10}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**10,x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/x**10, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(171) = 342$.

Time = 0.05 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.85

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{10}}\,dx = -\frac{256\,\sqrt{cx^2+bx}Bc^6}{15015\,b^5x} + \frac{512\,\sqrt{cx^2+bx}Ac^7}{45045\,b^6x} + \frac{128\,\sqrt{cx^2+bx}Bc^5}{15015\,b^4x^2} - \frac{256\,\sqrt{cx^2+bx}Ac^6}{45045\,b^5x^2} - \frac{32\,\sqrt{cx^2+bx}Bc^4}{5005\,b^3x^3} + \frac{64\,\sqrt{cx^2+bx}Ac^5}{15015\,b^4x^3} + \frac{16\,\sqrt{cx^2+bx}Bc^3}{3003\,b^2x^4} - \frac{32\,\sqrt{cx^2+bx}Ac^4}{9009\,b^3x^4} - \frac{2\,\sqrt{cx^2+bx}Bc^2}{429\,bx^5} + \frac{4\,\sqrt{cx^2+bx}Ac^3}{1287\,b^2x^5} + \frac{3\,\sqrt{cx^2+bx}Bc}{715\,x^6} - \frac{2\,\sqrt{cx^2+bx}Ac^2}{715\,bx^6} + \frac{3\,\sqrt{cx^2+bx}Bb}{65\,x^7} + \frac{\sqrt{cx^2+bx}Ac}{390\,x^7} - \frac{(cx^2+bx)^{\frac{3}{2}}B}{5\,x^8} + \frac{\sqrt{cx^2+bx}Ab}{30\,x^8} - \frac{(cx^2+bx)^{\frac{3}{2}}A}{6\,x^9}$$

input

```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^10,x, algorithm="maxima")
```

output

```
-256/15015*sqrt(c*x^2 + b*x)*B*c^6/(b^5*x) + 512/45045*sqrt(c*x^2 + b*x)*A
*c^7/(b^6*x) + 128/15015*sqrt(c*x^2 + b*x)*B*c^5/(b^4*x^2) - 256/45045*sqr
t(c*x^2 + b*x)*A*c^6/(b^5*x^2) - 32/5005*sqrt(c*x^2 + b*x)*B*c^4/(b^3*x^3)
 + 64/15015*sqrt(c*x^2 + b*x)*A*c^5/(b^4*x^3) + 16/3003*sqrt(c*x^2 + b*x)*
B*c^3/(b^2*x^4) - 32/9009*sqrt(c*x^2 + b*x)*A*c^4/(b^3*x^4) - 2/429*sqrt(c
*x^2 + b*x)*B*c^2/(b*x^5) + 4/1287*sqrt(c*x^2 + b*x)*A*c^3/(b^2*x^5) + 3/7
15*sqrt(c*x^2 + b*x)*B*c/x^6 - 2/715*sqrt(c*x^2 + b*x)*A*c^2/(b*x^6) + 3/6
5*sqrt(c*x^2 + b*x)*B*b/x^7 + 1/390*sqrt(c*x^2 + b*x)*A*c/x^7 - 1/5*(c*x^2
 + b*x)^(3/2)*B/x^8 + 1/30*sqrt(c*x^2 + b*x)*A*b/x^8 - 1/6*(c*x^2 + b*x)^(
3/2)*A/x^9
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(171) = 342$.

Time = 0.22 (sec) , antiderivative size = 551, normalized size of antiderivative = 2.83

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{10}}\,dx = \frac{2\left(144144\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^{10}Bc^4 + 720720\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^{9}Bbc^{\frac{7}{2}}\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^10,x, algorithm="giac")
```

output
```
2/45045*(144144*(sqrt(c)*x - sqrt(c*x^2 + b*x))^10*B*c^4 + 720720*(sqrt(c)
*x - sqrt(c*x^2 + b*x))^9*B*b*c^(7/2) + 240240*(sqrt(c)*x - sqrt(c*x^2 + b
*x))^9*A*c^(9/2) + 1595880*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*B*b^2*c^3 + 1
338480*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*A*b*c^4 + 2027025*(sqrt(c)*x - sq
rt(c*x^2 + b*x))^7*B*b^3*c^(5/2) + 3333330*(sqrt(c)*x - sqrt(c*x^2 + b*x))
^7*A*b^2*c^(7/2) + 1606605*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*B*b^4*c^2 + 4
844840*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*A*b^3*c^3 + 810810*(sqrt(c)*x - s
qrt(c*x^2 + b*x))^5*B*b^5*c^(3/2) + 4513509*(sqrt(c)*x - sqrt(c*x^2 + b*x)
)^5*A*b^4*c^(5/2) + 253890*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*B*b^6*c + 278
8695*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*A*b^5*c^2 + 45045*(sqrt(c)*x - sqrt
(c*x^2 + b*x))^3*B*b^7*sqrt(c) + 1141140*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3
*A*b^6*c^(3/2) + 3465*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*b^8 + 297990*(sq
rt(c)*x - sqrt(c*x^2 + b*x))^2*A*b^7*c + 45045*(sqrt(c)*x - sqrt(c*x^2 + b
*x))*A*b^8*sqrt(c) + 3003*A*b^9)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^15
```

Mupad [B] (verification not implemented)

Time = 6.87 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.67

$$\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^{10}}\,dx = \frac{4\,A\,c^3\,\sqrt{c\,x^2+b\,x}}{1287\,b^2\,x^5} - \frac{32\,A\,c\,\sqrt{c\,x^2+b\,x}}{195\,x^7} - \frac{2\,B\,b\,\sqrt{c\,x^2+b\,x}}{13\,x^7} - \frac{28\,B\,c\,\sqrt{c\,x^2+b\,x}}{143\,x^6} - \frac{2\,A\,c^2\,\sqrt{c\,x^2+b\,x}}{715\,b\,x^6} - \frac{2\,A\,b\,\sqrt{c\,x^2+b\,x}}{15\,x^8} - \frac{32\,A\,c^4\,\sqrt{c\,x^2+b\,x}}{9009\,b^3\,x^4} + \frac{64\,A\,c^5\,\sqrt{c\,x^2+b\,x}}{15015\,b^4\,x^3} - \frac{256\,A\,c^6\,\sqrt{c\,x^2+b\,x}}{45045\,b^5\,x^2} + \frac{512\,A\,c^7\,\sqrt{c\,x^2+b\,x}}{45045\,b^6\,x} - \frac{2\,B\,c^2\,\sqrt{c\,x^2+b\,x}}{429\,b\,x^5} + \frac{16\,B\,c^3\,\sqrt{c\,x^2+b\,x}}{3003\,b^2\,x^4} - \frac{32\,B\,c^4\,\sqrt{c\,x^2+b\,x}}{5005\,b^3\,x^3} + \frac{128\,B\,c^5\,\sqrt{c\,x^2+b\,x}}{15015\,b^4\,x^2} - \frac{256\,B\,c^6\,\sqrt{c\,x^2+b\,x}}{15015\,b^5\,x}$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^10,x)
```

output
```
(4*A*c^3*(b*x + c*x^2)^(1/2))/(1287*b^2*x^5) - (32*A*c*(b*x + c*x^2)^(1/2)
)/(195*x^7) - (2*B*b*(b*x + c*x^2)^(1/2))/(13*x^7) - (28*B*c*(b*x + c*x^2)
^(1/2))/(143*x^6) - (2*A*c^2*(b*x + c*x^2)^(1/2))/(715*b*x^6) - (2*A*b*(b*
x + c*x^2)^(1/2))/(15*x^8) - (32*A*c^4*(b*x + c*x^2)^(1/2))/(9009*b^3*x^4)
 + (64*A*c^5*(b*x + c*x^2)^(1/2))/(15015*b^4*x^3) - (256*A*c^6*(b*x + c*x^
2)^(1/2))/(45045*b^5*x^2) + (512*A*c^7*(b*x + c*x^2)^(1/2))/(45045*b^6*x)
- (2*B*c^2*(b*x + c*x^2)^(1/2))/(429*b*x^5) + (16*B*c^3*(b*x + c*x^2)^(1/2
))/(3003*b^2*x^4) - (32*B*c^4*(b*x + c*x^2)^(1/2))/(5005*b^3*x^3) + (128*B
*c^5*(b*x + c*x^2)^(1/2))/(15015*b^4*x^2) - (256*B*c^6*(b*x + c*x^2)^(1/2)
)/(15015*b^5*x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.56

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{10}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^7}{15}-\frac{32\sqrt{x}\,\sqrt{cx+b}\,a\,b^6cx}{195}-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^5c^2x^2}{715}+\frac{4\sqrt{x}\,\sqrt{cx+b}\,a\,b^4c^3x^3}{1287}-32}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^10,x)
```

output
```
(2*( - 3003*sqrt(x)*sqrt(b + c*x)*a*b**7 - 3696*sqrt(x)*sqrt(b + c*x)*a*b*
*6*c*x - 63*sqrt(x)*sqrt(b + c*x)*a*b**5*c**2*x**2 + 70*sqrt(x)*sqrt(b + c
*x)*a*b**4*c**3*x**3 - 80*sqrt(x)*sqrt(b + c*x)*a*b**3*c**4*x**4 + 96*sqrt
(x)*sqrt(b + c*x)*a*b**2*c**5*x**5 - 128*sqrt(x)*sqrt(b + c*x)*a*b*c**6*x*
*6 + 256*sqrt(x)*sqrt(b + c*x)*a*c**7*x**7 - 3465*sqrt(x)*sqrt(b + c*x)*b*
*8*x - 4410*sqrt(x)*sqrt(b + c*x)*b**7*c*x**2 - 105*sqrt(x)*sqrt(b + c*x)*
b**6*c**2*x**3 + 120*sqrt(x)*sqrt(b + c*x)*b**5*c**3*x**4 - 144*sqrt(x)*sq
rt(b + c*x)*b**4*c**4*x**5 + 192*sqrt(x)*sqrt(b + c*x)*b**3*c**5*x**6 - 38
4*sqrt(x)*sqrt(b + c*x)*b**2*c**6*x**7 - 256*sqrt(c)*a*c**7*x**8 + 384*sqr
t(c)*b**2*c**6*x**8))/(45045*b**6*x**8)
```

3.127 $\int x^2(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 332

$$\begin{aligned}
\int x^2(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx = & -\frac{5b^7(11bB-18Ac)\sqrt{bx+cx^2}}{32768c^6}+\frac{5b^6(11bB-18Ac)x\sqrt{bx+cx^2}}{49152c^5} \\
& -\frac{b^5(11bB-18Ac)x^2\sqrt{bx+cx^2}}{12288c^4}+\frac{b^4(11bB-18Ac)x^3\sqrt{bx+cx^2}}{14336c^3} \\
& -\frac{b^3(11bB-18Ac)x^4\sqrt{bx+cx^2}}{16128c^2}-\frac{9b^2(11bB-18Ac)x^5\sqrt{bx+cx^2}}{896c} \\
& -\frac{11}{672}b(11bB-18Ac)x^6\sqrt{bx+cx^2}-\frac{1}{144}c(11bB-18Ac)x^7\sqrt{bx+cx^2} \\
& +\frac{Bx^2\left(bx+cx^2\right)^{7/2}}{9c}+\frac{5b^8(11bB-18Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{32768c^{13/2}}
\end{aligned}$$

output

```
-5/32768*b^7*(-18*A*c+11*B*b)*(c*x^2+b*x)^(1/2)/c^6+5/49152*b^6*(-18*A*c+1
1*B*b)*x*(c*x^2+b*x)^(1/2)/c^5-1/12288*b^5*(-18*A*c+11*B*b)*x^2*(c*x^2+b*x
)^(1/2)/c^4+1/14336*b^4*(-18*A*c+11*B*b)*x^3*(c*x^2+b*x)^(1/2)/c^3-1/16128
*b^3*(-18*A*c+11*B*b)*x^4*(c*x^2+b*x)^(1/2)/c^2-9/896*b^2*(-18*A*c+11*B*b)
*x^5*(c*x^2+b*x)^(1/2)/c-11/672*b*(-18*A*c+11*B*b)*x^6*(c*x^2+b*x)^(1/2)-1
/144*c*(-18*A*c+11*B*b)*x^7*(c*x^2+b*x)^(1/2)+1/9*B*x^2*(c*x^2+b*x)^(7/2)/
c+5/32768*b^8*(-18*A*c+11*B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(13/
2)
```

Mathematica [A] (verified)

Time = 1.98 (sec) , antiderivative size = 277, normalized size of antiderivative = 0.83

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx = \frac{\sqrt{x}\sqrt{b+cx}\left(\sqrt{c}\sqrt{x}\sqrt{b+cx}(-3465b^8B+256b^3c^5x^4(9A+5Bx)+28672c^8x^7(9A+8Bx)+\right.}{}$$

input
```
Integrate[x^2*(A + B*x)*(b*x + c*x^2)^(5/2),x]
```

output
```
(Sqrt[x]*Sqrt[b + c*x]*(Sqrt[c]*Sqrt[x]*Sqrt[b + c*x]*(-3465*b^8*B + 256*b
^3*c^5*x^4*(9*A + 5*B*x) + 28672*c^8*x^7*(9*A + 8*B*x) + 144*b^5*c^3*x^2*(
21*A + 11*B*x) + 210*b^7*c*(27*A + 11*B*x) - 84*b^6*c^2*x*(45*A + 22*B*x)
- 32*b^4*c^4*x^3*(81*A + 44*B*x) + 1536*b^2*c^6*x^5*(243*A + 206*B*x) + 20
48*b*c^7*x^6*(297*A + 259*B*x)) + 11340*A*b^8*c*ArcTanh[(Sqrt[c]*Sqrt[x])/
(Sqrt[b] - Sqrt[b + c*x])] + 6930*b^9*B*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b
] + Sqrt[b + c*x])]))/(2064384*c^(13/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.70, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1221, 1134, 1160, 1087, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{Bx^2\left(bx+cx^2\right)^{7/2}}{9c}-\frac{(11bB-18Ac)\int x^2\left(cx^2+bx\right)^{5/2}dx}{18c}$$

$$\Big\downarrow \text{1134}$$

$$\frac{Bx^2\left(bx+cx^2\right)^{7/2}}{9c}-\frac{(11bB-18Ac)\left(\frac{x\left(bx+cx^2\right)^{7/2}}{8c}-\frac{9b\int x\left(cx^2+bx\right)^{5/2}dx}{16c}\right)}{18c}$$

$$\Big\downarrow \text{1160}$$

$$\frac{Bx^2\left(bx+cx^2\right)^{7/2}}{9c}-\frac{(11bB-18Ac)\left(\frac{x\left(bx+cx^2\right)^{7/2}}{8c}-\frac{9b\left(\frac{\left(bx+cx^2\right)^{7/2}}{7c}-\frac{b\int\left(cx^2+bx\right)^{5/2}dx}{2c}\right)}{16c}\right)}{18c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{Bx^2\left(bx+cx^2\right)^{7/2}}{9c}-\frac{(11bB-18Ac)\left(\frac{x\left(bx+cx^2\right)^{7/2}}{8c}-\frac{9b\left(\frac{\left(bx+cx^2\right)^{7/2}}{7c}-\frac{b\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\int\left(cx^2+bx\right)^{3/2}dx}{24c}\right)}{2c}\right)}{16c}\right)}{18c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{Bx^2\left(bx+cx^2\right)^{7/2}}{9c}-\frac{(11bB-18Ac)\left(\frac{x\left(bx+cx^2\right)^{7/2}}{8c}-\frac{9b\left(\frac{\left(bx+cx^2\right)^{7/2}}{7c}-\frac{b\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int\sqrt{cx^2+bx}dx}{16c}\right)}{24c}\right)}{2c}\right)}{16c}\right)}{18c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{Bx^2\left(bx+cx^2\right)^{7/2}}{9c} - \frac{(11bB-18Ac)\left(\frac{x\left(bx+cx^2\right)^{7/2}}{8c} - \frac{9b\left(\frac{\left(bx+cx^2\right)^{7/2}}{7c} - \frac{b\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c} - \frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c} - \frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c} - \frac{b^2\int \frac{\cdots}{\sqrt{cx\cdots}}}{8\cdots}\cdots\right.}{16c}\cdots\right.}{24c}\cdots\right.}{2c}\cdots\right.}{16c}\cdots\right.}{18c}$$

↓ 1091

$$\frac{Bx^2\left(bx+cx^2\right)^{7/2}}{9c}-\frac{(11bB-18Ac)\left(\frac{x\left(bx+cx^2\right)^{7/2}}{8c}-\frac{9b\left(\frac{\left(bx+cx^2\right)^{7/2}}{7c}-\frac{b\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{\cdots}{1-\frac{\cdots}{c}}\cdots}{\cdots}\right.}{16c}\right.}{24c}\right.}{2c}\right.}{16c}\right.}{18c}$$

$$\Big\downarrow \; 219$$

$$\frac{Bx^2\left(bx+cx^2\right)^{7/2}}{9c} - \frac{(11bB-18Ac)\left(\frac{x\left(bx+cx^2\right)^{7/2}}{8c} - \frac{9b\left(\frac{\left(bx+cx^2\right)^{7/2}}{7c} - \frac{b\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c} - \frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c} - \frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c} - \frac{b^2\text{arcta}\cdots}{\cdots}\right.}{16c}\right.}{24c}\right.}{2c}\right.}{16c}\right.}{18c}$$

input
```
Int[x^2*(A + B*x)*(b*x + c*x^2)^(5/2),x]
```

output
```
(B*x^2*(b*x + c*x^2)^(7/2))/(9*c) - ((11*b*B - 18*A*c)*((x*(b*x + c*x^2)^(
7/2))/(8*c) - (9*b*((b*x + c*x^2)^(7/2)/(7*c) - (b*(((b + 2*c*x)*(b*x + c*
x^2)^(5/2))/(12*c) - (5*b^2*(((b + 2*c*x)*(b*x + c*x^2)^(3/2))/(8*c) - (3*
b^2*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt
[b*x + c*x^2]])/(4*c^(3/2))))/(16*c)))/(24*c)))/(2*c)))/(16*c)))/(18*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 241, normalized size of antiderivative = 0.73

method	result
risch	$\frac{(229376B\,c^{8}x^{8}+258048A\,c^{8}x^{7}+530432Bb\,c^{7}x^{7}+608256Ab\,c^{7}x^{6}+316416B\,b^{2}c^{6}x^{6}+373248A\,b^{2}c^{6}x^{5}+1280B\,b^{3}c^{5}x^{5}+2304A\,b^{3}c^{5}x}{20643}$
default	$A\left(\frac{x(c\,x^{2}+bx)^{\frac{7}{2}}}{8c}-\frac{9b\left(\frac{(c\,x^{2}+bx)^{\frac{7}{2}}}{7c}-\frac{b\left(\frac{(2cx+b)(c\,x^{2}+bx)^{\frac{5}{2}}}{12c}-\frac{5b^{2}\left(\frac{(2cx+b)(c\,x^{2}+bx)^{\frac{3}{2}}}{8c}-\frac{3b^{2}\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx}}{4c}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{}\right.}{8c^{\frac{3}{2}}}\right.}{16c}\right.}{24c}\right.}{2c}\right.}{16c}\right.$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2064384/c^6*(229376*B*c^8*x^8+258048*A*c^8*x^7+530432*B*b*c^7*x^7+608256
*A*b*c^7*x^6+316416*B*b^2*c^6*x^6+373248*A*b^2*c^6*x^5+1280*B*b^3*c^5*x^5+
2304*A*b^3*c^5*x^4-1408*B*b^4*c^4*x^4-2592*A*b^4*c^4*x^3+1584*B*b^5*c^3*x^
3+3024*A*b^5*c^3*x^2-1848*B*b^6*c^2*x^2-3780*A*b^6*c^2*x+2310*B*b^7*c*x+56
70*A*b^7*c-3465*B*b^8)*x*(c*x+b)/(x*(c*x+b))^(1/2)-5/65536*b^8*(18*A*c-11*
B*b)/c^(13/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 497, normalized size of antiderivative = 1.50

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\left[-\frac{315\,(11\,Bb^9-18\,Ab^8c)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(229376\,Bc^9x^8-3465\,Bb\right.$$

$$-\frac{315\,(11\,Bb^9-18\,Ab^8c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(229376\,Bc^9x^8-3465\,Bb^8c+5670\,Ab^7c^2+14336\,(3$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/4128768*(315*(11*B*b^9 - 18*A*b^8*c)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*
x^2 + b*x)*sqrt(c)) - 2*(229376*B*c^9*x^8 - 3465*B*b^8*c + 5670*A*b^7*c^2
+ 14336*(37*B*b*c^8 + 18*A*c^9)*x^7 + 3072*(103*B*b^2*c^7 + 198*A*b*c^8)*x
^6 + 256*(5*B*b^3*c^6 + 1458*A*b^2*c^7)*x^5 - 128*(11*B*b^4*c^5 - 18*A*b^3
*c^6)*x^4 + 144*(11*B*b^5*c^4 - 18*A*b^4*c^5)*x^3 - 168*(11*B*b^6*c^3 - 18
*A*b^5*c^4)*x^2 + 210*(11*B*b^7*c^2 - 18*A*b^6*c^3)*x)*sqrt(c*x^2 + b*x))/
c^7, -1/2064384*(315*(11*B*b^9 - 18*A*b^8*c)*sqrt(-c)*arctan(sqrt(c*x^2 +
b*x)*sqrt(-c)/(c*x + b)) - (229376*B*c^9*x^8 - 3465*B*b^8*c + 5670*A*b^7*c
^2 + 14336*(37*B*b*c^8 + 18*A*c^9)*x^7 + 3072*(103*B*b^2*c^7 + 198*A*b*c^8
)*x^6 + 256*(5*B*b^3*c^6 + 1458*A*b^2*c^7)*x^5 - 128*(11*B*b^4*c^5 - 18*A*
b^3*c^6)*x^4 + 144*(11*B*b^5*c^4 - 18*A*b^4*c^5)*x^3 - 168*(11*B*b^6*c^3 -
 18*A*b^5*c^4)*x^2 + 210*(11*B*b^7*c^2 - 18*A*b^6*c^3)*x)*sqrt(c*x^2 + b*x
))/c^7]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(323) = 646$.

Time = 0.58 (sec) , antiderivative size = 741, normalized size of antiderivative = 2.23

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(x**2*(B*x+A)*(c*x**2+b*x)**(5/2),x)
```

output
```
Piecewise((-63*b**5*(A*b**3 - 11*b*(3*A*b**2*c + B*b**3 - 13*b*(3*A*b*c**2
 + 3*B*b**2*c - 15*b*(A*c**3 + 37*B*b*c**2/18)/(16*c))/(14*c))/(12*c))*Pie
cewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c,
0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(256
*c**5) + sqrt(b*x + c*x**2)*(B*c**2*x**8/9 + 63*b**4*(A*b**3 - 11*b*(3*A*b
**2*c + B*b**3 - 13*b*(3*A*b*c**2 + 3*B*b**2*c - 15*b*(A*c**3 + 37*B*b*c**
2/18)/(16*c))/(14*c))/(12*c))/(128*c**5) - 21*b**3*x*(A*b**3 - 11*b*(3*A*b
**2*c + B*b**3 - 13*b*(3*A*b*c**2 + 3*B*b**2*c - 15*b*(A*c**3 + 37*B*b*c**
2/18)/(16*c))/(14*c))/(12*c))/(64*c**4) + 21*b**2*x**2*(A*b**3 - 11*b*(3*A
*b**2*c + B*b**3 - 13*b*(3*A*b*c**2 + 3*B*b**2*c - 15*b*(A*c**3 + 37*B*b*c
**2/18)/(16*c))/(14*c))/(12*c))/(80*c**3) - 9*b*x**3*(A*b**3 - 11*b*(3*A*b
**2*c + B*b**3 - 13*b*(3*A*b*c**2 + 3*B*b**2*c - 15*b*(A*c**3 + 37*B*b*c**
2/18)/(16*c))/(14*c))/(12*c))/(40*c**2) + x**7*(A*c**3 + 37*B*b*c**2/18)/(
8*c) + x**6*(3*A*b*c**2 + 3*B*b**2*c - 15*b*(A*c**3 + 37*B*b*c**2/18)/(16*
c))/(7*c) + x**5*(3*A*b**2*c + B*b**3 - 13*b*(3*A*b*c**2 + 3*B*b**2*c - 15
*b*(A*c**3 + 37*B*b*c**2/18)/(16*c))/(14*c))/(6*c) + x**4*(A*b**3 - 11*b*(
3*A*b**2*c + B*b**3 - 13*b*(3*A*b*c**2 + 3*B*b**2*c - 15*b*(A*c**3 + 37*B*
b*c**2/18)/(16*c))/(14*c))/(12*c))/(5*c)), Ne(c, 0)), (2*(A*(b*x)**(11/2)/
11 + B*(b*x)**(13/2)/(13*b))/b**3, Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.22

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\frac{(cx^2+bx)^{\frac{7}{2}}Bx^2}{9\,c}-\frac{55\,\sqrt{cx^2+bx}Bb^7x}{16384\,c^5}+\frac{55\,(cx^2+bx)^{\frac{3}{2}}Bb^5x}{6144\,c^4}+\frac{45\,\sqrt{cx^2+bx}Ab^6x}{8192\,c^4}-\frac{11\,(cx^2+bx)^{\frac{5}{2}}Bb^3x}{384\,c^3}-\frac{15\,(cx^2+bx)^{\frac{3}{2}}Ab^4x}{1024\,c^3}-\frac{11\,(cx^2+bx)^{\frac{7}{2}}Bbx}{144\,c^2}+\frac{3\,(cx^2+bx)^{\frac{5}{2}}Ab^2x}{64\,c^2}+\frac{(cx^2+bx)^{\frac{7}{2}}Ax}{8\,c}+\frac{55\,Bb^9\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{65536\,c^{\frac{13}{2}}}-\frac{45\,Ab^8\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{32768\,c^{\frac{11}{2}}}-\frac{55\,\sqrt{cx^2+bx}Bb^8}{32768\,c^6}+\frac{55\,(cx^2+bx)^{\frac{3}{2}}Bb^6}{12288\,c^5}+\frac{45\,\sqrt{cx^2+bx}Ab^7}{16384\,c^5}-\frac{11\,(cx^2+bx)^{\frac{5}{2}}Bb^4}{768\,c^4}-\frac{15\,(cx^2+bx)^{\frac{3}{2}}Ab^5}{2048\,c^4}+\frac{11\,(cx^2+bx)^{\frac{7}{2}}Bb^2}{224\,c^3}+\frac{3\,(cx^2+bx)^{\frac{5}{2}}Ab^3}{128\,c^3}-\frac{9\,(cx^2+bx)^{\frac{7}{2}}Ab}{112\,c^2}$$

input

```
integrate(x^2*(B*x+A)*(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output

```
1/9*(c*x^2 + b*x)^(7/2)*B*x^2/c - 55/16384*sqrt(c*x^2 + b*x)*B*b^7*x/c^5 +
 55/6144*(c*x^2 + b*x)^(3/2)*B*b^5*x/c^4 + 45/8192*sqrt(c*x^2 + b*x)*A*b^6
*x/c^4 - 11/384*(c*x^2 + b*x)^(5/2)*B*b^3*x/c^3 - 15/1024*(c*x^2 + b*x)^(3
/2)*A*b^4*x/c^3 - 11/144*(c*x^2 + b*x)^(7/2)*B*b*x/c^2 + 3/64*(c*x^2 + b*x
)^(5/2)*A*b^2*x/c^2 + 1/8*(c*x^2 + b*x)^(7/2)*A*x/c + 55/65536*B*b^9*log(2
*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(13/2) - 45/32768*A*b^8*log(2*c*
x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(11/2) - 55/32768*sqrt(c*x^2 + b*x)
*B*b^8/c^6 + 55/12288*(c*x^2 + b*x)^(3/2)*B*b^6/c^5 + 45/16384*sqrt(c*x^2
+ b*x)*A*b^7/c^5 - 11/768*(c*x^2 + b*x)^(5/2)*B*b^4/c^4 - 15/2048*(c*x^2 +
 b*x)^(3/2)*A*b^5/c^4 + 11/224*(c*x^2 + b*x)^(7/2)*B*b^2/c^3 + 3/128*(c*x^
2 + b*x)^(5/2)*A*b^3/c^3 - 9/112*(c*x^2 + b*x)^(7/2)*A*b/c^2
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.84

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\frac{1}{2064384}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\left(2\left(4\left(14\left(16\,Bc^2x+\frac{37\,Bbc^9+18\,Ac^{10}}{c^8}\right)x+\frac{3\,(103\,Bb^2\right.\right.\right.\right.\right.\right.\right.$$
$$-\frac{5\,(11\,Bb^9-18\,Ab^8c)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{65536\,c^{\frac{13}{2}}}$$

input
```
integrate(x^2*(B*x+A)*(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
1/2064384*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(2*(4*(14*(16*B*c^2*x + (37*B*b*c^
9 + 18*A*c^10)/c^8)*x + 3*(103*B*b^2*c^8 + 198*A*b*c^9)/c^8)*x + (5*B*b^3*
c^7 + 1458*A*b^2*c^8)/c^8)*x - (11*B*b^4*c^6 - 18*A*b^3*c^7)/c^8)*x + 9*(1
1*B*b^5*c^5 - 18*A*b^4*c^6)/c^8)*x - 21*(11*B*b^6*c^4 - 18*A*b^5*c^5)/c^8)
*x + 105*(11*B*b^7*c^3 - 18*A*b^6*c^4)/c^8)*x - 315*(11*B*b^8*c^2 - 18*A*b
^7*c^3)/c^8) - 5/65536*(11*B*b^9 - 18*A*b^8*c)*log(abs(2*(sqrt(c)*x - sqrt
(c*x^2 + b*x))*sqrt(c) + b))/c^(13/2)
```

Mupad [F(-1)]

Timed out.

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\int x^2\,\left(c\,x^2+b\,x\right)^{5/2}\,(A+B\,x)\,dx$$

input
```
int(x^2*(b*x + c*x^2)^(5/2)*(A + B*x),x)
```

output
```
int(x^2*(b*x + c*x^2)^(5/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.11

$$\int x^2(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx = \frac{5670\sqrt{x}\,\sqrt{cx+b}\,a\,b^7c^2-3780\sqrt{x}\,\sqrt{cx+b}\,a\,b^6c^3x+3024\sqrt{x}\,\sqrt{cx+b}\,a\,b^5c^4x^2-2592\sqrt{x}\,\sqrt{}}{}$$

input
```
int(x^2*(B*x+A)*(c*x^2+b*x)^(5/2),x)
```

output
```
(5670*sqrt(x)*sqrt(b + c*x)*a*b**7*c**2 - 3780*sqrt(x)*sqrt(b + c*x)*a*b**
6*c**3*x + 3024*sqrt(x)*sqrt(b + c*x)*a*b**5*c**4*x**2 - 2592*sqrt(x)*sqrt
(b + c*x)*a*b**4*c**5*x**3 + 2304*sqrt(x)*sqrt(b + c*x)*a*b**3*c**6*x**4 +
 373248*sqrt(x)*sqrt(b + c*x)*a*b**2*c**7*x**5 + 608256*sqrt(x)*sqrt(b + c
*x)*a*b*c**8*x**6 + 258048*sqrt(x)*sqrt(b + c*x)*a*c**9*x**7 - 3465*sqrt(x
)*sqrt(b + c*x)*b**9*c + 2310*sqrt(x)*sqrt(b + c*x)*b**8*c**2*x - 1848*sqr
t(x)*sqrt(b + c*x)*b**7*c**3*x**2 + 1584*sqrt(x)*sqrt(b + c*x)*b**6*c**4*x
**3 - 1408*sqrt(x)*sqrt(b + c*x)*b**5*c**5*x**4 + 1280*sqrt(x)*sqrt(b + c*
x)*b**4*c**6*x**5 + 316416*sqrt(x)*sqrt(b + c*x)*b**3*c**7*x**6 + 530432*s
qrt(x)*sqrt(b + c*x)*b**2*c**8*x**7 + 229376*sqrt(x)*sqrt(b + c*x)*b*c**9*
x**8 - 5670*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*b**8*
c + 3465*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**10)/(20
64384*c**7)
```

3.128 $\int x(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 295

$$\int x(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\frac{5b^6(9bB-16Ac)\sqrt{bx+cx^2}}{16384c^5}-\frac{5b^5(9bB-16Ac)x\sqrt{bx+cx^2}}{24576c^4}+\frac{b^4(9bB-16Ac)x^2\sqrt{bx+cx^2}}{6144c^3}-\frac{b^3(9bB-16Ac)x^3\sqrt{bx+cx^2}}{7168c^2}-\frac{37b^2(9bB-16Ac)x^4\sqrt{bx+cx^2}}{2688c}-\frac{29b(9bB-16Ac)x^5\sqrt{bx+cx^2}}{1344}-\frac{1}{112}c(9bB-16Ac)x^6\sqrt{bx+cx^2}+\frac{Bx\left(bx+cx^2\right)^{7/2}}{8c}-\frac{5b^7(9bB-16Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{16384c^{11/2}}$$

output
```
5/16384*b^6*(-16*A*c+9*B*b)*(c*x^2+b*x)^(1/2)/c^5-5/24576*b^5*(-16*A*c+9*B
*b)*x*(c*x^2+b*x)^(1/2)/c^4+1/6144*b^4*(-16*A*c+9*B*b)*x^2*(c*x^2+b*x)^(1/
2)/c^3-1/7168*b^3*(-16*A*c+9*B*b)*x^3*(c*x^2+b*x)^(1/2)/c^2-37/2688*b^2*(-
16*A*c+9*B*b)*x^4*(c*x^2+b*x)^(1/2)/c-29/1344*b*(-16*A*c+9*B*b)*x^5*(c*x^2
+b*x)^(1/2)-1/112*c*(-16*A*c+9*B*b)*x^6*(c*x^2+b*x)^(1/2)+1/8*B*x*(c*x^2+b
*x)^(7/2)/c-5/16384*b^7*(-16*A*c+9*B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2
))/c^(11/2)
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 257, normalized size of antiderivative = 0.87

$$\int x(A+Bx)\left(bx+cx^2\right)^{5/2}dx=\frac{\sqrt{x}\sqrt{b+cx}\left(\sqrt{c}\sqrt{x}\sqrt{b+cx}(945b^7B+384b^3c^4x^3(2A+Bx)-210b^6c(8A+3Bx)+6144c^7x$$

input
```
Integrate[x*(A + B*x)*(b*x + c*x^2)^(5/2),x]
```

output
```
(Sqrt[x]*Sqrt[b + c*x]*(Sqrt[c]*Sqrt[x]*Sqrt[b + c*x]*(945*b^7*B + 384*b^3
*c^4*x^3*(2*A + B*x) - 210*b^6*c*(8*A + 3*B*x) + 6144*c^7*x^6*(8*A + 7*B*x
) + 56*b^5*c^2*x*(20*A + 9*B*x) - 16*b^4*c^3*x^2*(56*A + 27*B*x) + 1024*b*
c^6*x^5*(116*A + 99*B*x) + 256*b^2*c^5*x^4*(296*A + 243*B*x)) + 1890*b^8*B
*ArcTanh[(Sqrt[c]*Sqrt[x])/(Sqrt[b] - Sqrt[b + c*x])] + 3360*A*b^7*c*ArcTa
nh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])]))/(344064*c^(11/2)*Sqrt[x
*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.63, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1225, 1087, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(A+Bx)\left(bx+cx^2\right)^{5/2}dx$$

$$\Big\downarrow \text{1225}$$

$$\frac{b(9bB-16Ac)\int \left(cx^2+bx\right)^{5/2}dx}{32c^2}-\frac{\left(bx+cx^2\right)^{7/2}(-16Ac+9bB-14Bcx)}{112c^2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{b(9bB-16Ac)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\int\left(cx^2+bx\right)^{3/2}dx}{24c}\right)}{32c^2}-\frac{\left(bx+cx^2\right)^{7/2}(-16Ac+9bB-14Bcx)}{112c^2}$$

$$\Big\downarrow \; 1087$$

$$\frac{b(9bB-16Ac)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int\sqrt{cx^2+bx}dx}{16c}\right)}{24c}\right)}{32c^2}-\frac{\left(bx+cx^2\right)^{7/2}(-16Ac+9bB-14Bcx)}{112c^2}$$

$$\Big\downarrow \; 1087$$

$$\frac{b(9bB-16Ac)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)}{24c}\right)}{32c^2}-\frac{\left(bx+cx^2\right)^{7/2}(-16Ac+9bB-14Bcx)}{112c^2}$$

$$\Big\downarrow \; 1091$$

$$\frac{b(9bB-16Ac)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)}{24c}\right)}{32c^2}-\frac{\left(bx+cx^2\right)^{7/2}(-16Ac+9bB-14Bcx)}{112c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{b(9bB-16Ac)\left(\frac{(b+2cx)(bx+cx^2)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)}{24c}\right)}{32c^2}-\frac{(bx+cx^2)^{7/2}(-16Ac+9bB-14Bcx)}{112c^2}$$

input
```
Int[x*(A + B*x)*(b*x + c*x^2)^(5/2),x]
```

output
```
-1/112*((9*b*B - 16*A*c - 14*B*c*x)*(b*x + c*x^2)^(7/2))/c^2 + (b*(9*b*B -
 16*A*c)*(((b + 2*c*x)*(b*x + c*x^2)^(5/2))/(12*c) - (5*b^2*(((b + 2*c*x)*
(b*x + c*x^2)^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c)
 - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))/(16*c)))/(24
*c)))/(32*c^2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(-43008 B\,c^7 x^7-49152 A\,c^7 x^6-101376 B b\,c^6 x^6-118784 A b\,c^6 x^5-62208 B\,b^2 c^5 x^5-75776 A\,b^2 c^5 x^4-384 B\,b^3 c^4 x^4-768 A\,b^3 c^4 x^3+}{344064 c^5 \sqrt{x(cx+b)}}$
default	$A\left(\frac{(c\,x^2+bx)^{\frac{7}{2}}}{7c}-\frac{b\left(\frac{(2cx+b)(c\,x^2+bx)^{\frac{5}{2}}}{12c}-\frac{5b^2\left(\frac{(2cx+b)(c\,x^2+bx)^{\frac{3}{2}}}{8c}-\frac{3b^2\left(\frac{(2cx+b)\sqrt{c\,x^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{24c}\right)}{2c}\right)$

input
```
int(x*(B*x+A)*(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/344064/c^5*(-43008*B*c^7*x^7-49152*A*c^7*x^6-101376*B*b*c^6*x^6-118784*
A*b*c^6*x^5-62208*B*b^2*c^5*x^5-75776*A*b^2*c^5*x^4-384*B*b^3*c^4*x^4-768*
A*b^3*c^4*x^3+432*B*b^4*c^3*x^3+896*A*b^4*c^3*x^2-504*B*b^5*c^2*x^2-1120*A
*b^5*c^2*x+630*B*b^6*c*x+1680*A*b^6*c-945*B*b^7)*x*(c*x+b)/(x*(c*x+b))^(1/
2)+5/32768*b^7*(16*A*c-9*B*b)/c^(11/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^
(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 448, normalized size of antiderivative = 1.52

$$\int x(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx = \left[-\frac{105\,(9\,Bb^8-16\,Ab^7c)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(43008\,Bc^8x^7+945\,Bb^7c-\right.$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/688128*(105*(9*B*b^8 - 16*A*b^7*c)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^
2 + b*x)*sqrt(c)) - 2*(43008*B*c^8*x^7 + 945*B*b^7*c - 1680*A*b^6*c^2 + 30
72*(33*B*b*c^7 + 16*A*c^8)*x^6 + 256*(243*B*b^2*c^6 + 464*A*b*c^7)*x^5 + 1
28*(3*B*b^3*c^5 + 592*A*b^2*c^6)*x^4 - 48*(9*B*b^4*c^4 - 16*A*b^3*c^5)*x^3
 + 56*(9*B*b^5*c^3 - 16*A*b^4*c^4)*x^2 - 70*(9*B*b^6*c^2 - 16*A*b^5*c^3)*x
)*sqrt(c*x^2 + b*x))/c^6, 1/344064*(105*(9*B*b^8 - 16*A*b^7*c)*sqrt(-c)*ar
ctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (43008*B*c^8*x^7 + 945*B*b^7*
c - 1680*A*b^6*c^2 + 3072*(33*B*b*c^7 + 16*A*c^8)*x^6 + 256*(243*B*b^2*c^6
 + 464*A*b*c^7)*x^5 + 128*(3*B*b^3*c^5 + 592*A*b^2*c^6)*x^4 - 48*(9*B*b^4*
c^4 - 16*A*b^3*c^5)*x^3 + 56*(9*B*b^5*c^3 - 16*A*b^4*c^4)*x^2 - 70*(9*B*b^
6*c^2 - 16*A*b^5*c^3)*x)*sqrt(c*x^2 + b*x))/c^6]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. 2(287) = 574.

Time = 0.64 (sec) , antiderivative size = 656, normalized size of antiderivative = 2.22

$$\int x(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\text{Too large to display}$$

input
```
integrate(x*(B*x+A)*(c*x**2+b*x)**(5/2),x)
```

output
```
Piecewise((35*b**4*(A*b**3 - 9*b*(3*A*b**2*c + B*b**3 - 11*b*(3*A*b*c**2 +
 3*B*b**2*c - 13*b*(A*c**3 + 33*B*b*c**2/16)/(14*c))/(12*c))/(10*c))*Piece
wise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)
), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(128*c
**4) + sqrt(b*x + c*x**2)*(B*c**2*x**7/8 - 35*b**3*(A*b**3 - 9*b*(3*A*b**2
*c + B*b**3 - 11*b*(3*A*b*c**2 + 3*B*b**2*c - 13*b*(A*c**3 + 33*B*b*c**2/1
6)/(14*c))/(12*c))/(10*c))/(64*c**4) + 35*b**2*x*(A*b**3 - 9*b*(3*A*b**2*c
 + B*b**3 - 11*b*(3*A*b*c**2 + 3*B*b**2*c - 13*b*(A*c**3 + 33*B*b*c**2/16)
/(14*c))/(12*c))/(10*c))/(96*c**3) - 7*b*x**2*(A*b**3 - 9*b*(3*A*b**2*c +
B*b**3 - 11*b*(3*A*b*c**2 + 3*B*b**2*c - 13*b*(A*c**3 + 33*B*b*c**2/16)/(1
4*c))/(12*c))/(10*c))/(24*c**2) + x**6*(A*c**3 + 33*B*b*c**2/16)/(7*c) + x
**5*(3*A*b*c**2 + 3*B*b**2*c - 13*b*(A*c**3 + 33*B*b*c**2/16)/(14*c))/(6*c
) + x**4*(3*A*b**2*c + B*b**3 - 11*b*(3*A*b*c**2 + 3*B*b**2*c - 13*b*(A*c*
*3 + 33*B*b*c**2/16)/(14*c))/(12*c))/(5*c) + x**3*(A*b**3 - 9*b*(3*A*b**2*
c + B*b**3 - 11*b*(3*A*b*c**2 + 3*B*b**2*c - 13*b*(A*c**3 + 33*B*b*c**2/16
)/(14*c))/(12*c))/(10*c))/(4*c)), Ne(c, 0)), (2*(A*(b*x)**(9/2)/9 + B*(b*x
)**(11/2)/(11*b))/b**2, Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.23

$$\int x(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\frac{45\,\sqrt{cx^2+bx}Bb^6x}{8192\,c^4}-\frac{15\,(cx^2+bx)^{\frac{3}{2}}Bb^4x}{1024\,c^3}-\frac{5\,\sqrt{cx^2+bx}Ab^5x}{512\,c^3}+\frac{3\,(cx^2+bx)^{\frac{5}{2}}Bb^2x}{64\,c^2}+\frac{5\,(cx^2+bx)^{\frac{3}{2}}Ab^3x}{192\,c^2}+\frac{(cx^2+bx)^{\frac{7}{2}}Bx}{8\,c}-\frac{(cx^2+bx)^{\frac{5}{2}}Abx}{12\,c}-\frac{45\,Bb^8\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{32768\,c^{\frac{11}{2}}}+\frac{5\,Ab^7\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{2048\,c^{\frac{9}{2}}}+\frac{45\,\sqrt{cx^2+bx}Bb^7}{16384\,c^5}-\frac{15\,(cx^2+bx)^{\frac{3}{2}}Bb^5}{2048\,c^4}-\frac{5\,\sqrt{cx^2+bx}Ab^6}{1024\,c^4}+\frac{3\,(cx^2+bx)^{\frac{5}{2}}Bb^3}{128\,c^3}+\frac{5\,(cx^2+bx)^{\frac{3}{2}}Ab^4}{384\,c^3}-\frac{9\,(cx^2+bx)^{\frac{7}{2}}Bb}{112\,c^2}-\frac{(cx^2+bx)^{\frac{5}{2}}Ab^2}{24\,c^2}+\frac{(cx^2+bx)^{\frac{7}{2}}A}{7\,c}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
45/8192*sqrt(c*x^2 + b*x)*B*b^6*x/c^4 - 15/1024*(c*x^2 + b*x)^(3/2)*B*b^4*
x/c^3 - 5/512*sqrt(c*x^2 + b*x)*A*b^5*x/c^3 + 3/64*(c*x^2 + b*x)^(5/2)*B*b
^2*x/c^2 + 5/192*(c*x^2 + b*x)^(3/2)*A*b^3*x/c^2 + 1/8*(c*x^2 + b*x)^(7/2)
*B*x/c - 1/12*(c*x^2 + b*x)^(5/2)*A*b*x/c - 45/32768*B*b^8*log(2*c*x + b +
 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(11/2) + 5/2048*A*b^7*log(2*c*x + b + 2*sq
rt(c*x^2 + b*x)*sqrt(c))/c^(9/2) + 45/16384*sqrt(c*x^2 + b*x)*B*b^7/c^5 -
15/2048*(c*x^2 + b*x)^(3/2)*B*b^5/c^4 - 5/1024*sqrt(c*x^2 + b*x)*A*b^6/c^4
 + 3/128*(c*x^2 + b*x)^(5/2)*B*b^3/c^3 + 5/384*(c*x^2 + b*x)^(3/2)*A*b^4/c
^3 - 9/112*(c*x^2 + b*x)^(7/2)*B*b/c^2 - 1/24*(c*x^2 + b*x)^(5/2)*A*b^2/c^
2 + 1/7*(c*x^2 + b*x)^(7/2)*A/c
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 251, normalized size of antiderivative = 0.85

$$\int x(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\frac{1}{344064}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\left(2\left(12\left(14\,Bc^2x+\frac{33\,Bbc^8+16\,Ac^9}{c^7}\right)x+\frac{243\,Bb^2c^7+46\cdots}{c^7}\right.\right.\right.\right.\right.\right. + \frac{5\left(9\,Bb^8-16\,Ab^7c\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{32768\,c^{\frac{11}{2}}}$$

input
```
integrate(x*(B*x+A)*(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
1/344064*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(2*(12*(14*B*c^2*x + (33*B*b*c^8 +
16*A*c^9)/c^7)*x + (243*B*b^2*c^7 + 464*A*b*c^8)/c^7)*x + (3*B*b^3*c^6 + 5
92*A*b^2*c^7)/c^7)*x - 3*(9*B*b^4*c^5 - 16*A*b^3*c^6)/c^7)*x + 7*(9*B*b^5*
c^4 - 16*A*b^4*c^5)/c^7)*x - 35*(9*B*b^6*c^3 - 16*A*b^5*c^4)/c^7)*x + 105*
(9*B*b^7*c^2 - 16*A*b^6*c^3)/c^7) + 5/32768*(9*B*b^8 - 16*A*b^7*c)*log(abs
(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int x(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\int x\left(c\,x^2+b\,x\right)^{5/2}\left(A+B\,x\right)\,dx$$

input
```
int(x*(b*x + c*x^2)^(5/2)*(A + B*x),x)
```

output
```
int(x*(b*x + c*x^2)^(5/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.12

$$\int x(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\frac{-1680\sqrt{x}\,\sqrt{cx+b}\,a\,b^6c^2+1120\sqrt{x}\,\sqrt{cx+b}\,a\,b^5c^3x-896\sqrt{x}\,\sqrt{cx+b}\,a\,b^4c^4x^2+768\sqrt{x}\,\sqrt{c}}{}$$

input
```
int(x*(B*x+A)*(c*x^2+b*x)^(5/2),x)
```

output
```
( - 1680*sqrt(x)*sqrt(b + c*x)*a*b**6*c**2 + 1120*sqrt(x)*sqrt(b + c*x)*a*
b**5*c**3*x - 896*sqrt(x)*sqrt(b + c*x)*a*b**4*c**4*x**2 + 768*sqrt(x)*sqr
t(b + c*x)*a*b**3*c**5*x**3 + 75776*sqrt(x)*sqrt(b + c*x)*a*b**2*c**6*x**4
 + 118784*sqrt(x)*sqrt(b + c*x)*a*b*c**7*x**5 + 49152*sqrt(x)*sqrt(b + c*x
)*a*c**8*x**6 + 945*sqrt(x)*sqrt(b + c*x)*b**8*c - 630*sqrt(x)*sqrt(b + c*
x)*b**7*c**2*x + 504*sqrt(x)*sqrt(b + c*x)*b**6*c**3*x**2 - 432*sqrt(x)*sq
rt(b + c*x)*b**5*c**4*x**3 + 384*sqrt(x)*sqrt(b + c*x)*b**4*c**5*x**4 + 62
208*sqrt(x)*sqrt(b + c*x)*b**3*c**6*x**5 + 101376*sqrt(x)*sqrt(b + c*x)*b*
*2*c**7*x**6 + 43008*sqrt(x)*sqrt(b + c*x)*b*c**8*x**7 + 1680*sqrt(c)*log(
(sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*b**7*c - 945*sqrt(c)*log((sqr
t(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**9)/(344064*c**6)
```

3.129 $\int (A+Bx)\left(bx+cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 252

$$\int (A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=-\frac{5b^5(bB-2Ac)\sqrt{bx+cx^2}}{1024c^4}+\frac{5b^4(bB-2Ac)x\sqrt{bx+cx^2}}{1536c^3}-\frac{b^3(bB-2Ac)x^2\sqrt{bx+cx^2}}{384c^2}-\frac{9b^2(bB-2Ac)x^3\sqrt{bx+cx^2}}{64c}-\frac{5}{24}b(bB-2Ac)x^4\sqrt{bx+cx^2}-\frac{1}{12}c(bB-2Ac)x^5\sqrt{bx+cx^2}+\frac{B\left(bx+cx^2\right)^{7/2}}{7c}+\frac{5b^6(bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{1024c^{9/2}}$$

output
```
-5/1024*b^5*(-2*A*c+B*b)*(c*x^2+b*x)^(1/2)/c^4+5/1536*b^4*(-2*A*c+B*b)*x*(
c*x^2+b*x)^(1/2)/c^3-1/384*b^3*(-2*A*c+B*b)*x^2*(c*x^2+b*x)^(1/2)/c^2-9/64
*b^2*(-2*A*c+B*b)*x^3*(c*x^2+b*x)^(1/2)/c-5/24*b*(-2*A*c+B*b)*x^4*(c*x^2+b
*x)^(1/2)-1/12*c*(-2*A*c+B*b)*x^5*(c*x^2+b*x)^(1/2)+1/7*B*(c*x^2+b*x)^(7/2
)/c+5/1024*b^6*(-2*A*c+B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.94

$$\int (A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\frac{\sqrt{x}\sqrt{b+cx}\left(\sqrt{c}\sqrt{x}\sqrt{b+cx}(-105b^6B+70b^5c(3A+Bx)-28b^4c^2x(5A+2Bx)+16b^3c^3x^2\right.}{}$$

input
```
Integrate[(A + B*x)*(b*x + c*x^2)^(5/2),x]
```

output
```
(Sqrt[x]*Sqrt[b + c*x]*(Sqrt[c]*Sqrt[x]*Sqrt[b + c*x]*(-105*b^6*B + 70*b^5
*c*(3*A + B*x) - 28*b^4*c^2*x*(5*A + 2*B*x) + 16*b^3*c^3*x^2*(7*A + 3*B*x)
 + 512*c^6*x^5*(7*A + 6*B*x) + 256*b*c^5*x^4*(35*A + 29*B*x) + 32*b^2*c^4*
x^3*(189*A + 148*B*x)) + 420*A*b^6*c*ArcTanh[(Sqrt[c]*Sqrt[x])/(Sqrt[b] -
Sqrt[b + c*x])] + 210*b^7*B*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b +
 c*x])]))/(21504*c^(9/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.68, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {1160, 1087, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (A+Bx)\left(bx+cx^2\right)^{5/2}\,dx$$

$$\Big\downarrow \ 1160$$

$$\frac{B\left(bx+cx^2\right)^{7/2}}{7c}-\frac{(bB-2Ac)\int \left(cx^2+bx\right)^{5/2}dx}{2c}$$

$$\Big\downarrow \ 1087$$

$$\frac{B\left(bx+cx^2\right)^{7/2}}{7c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\int \left(cx^2+bx\right)^{3/2}dx}{24c}\right)}{2c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{B\left(bx+cx^2\right)^{7/2}}{7c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int\sqrt{cx^2+bx}dx}{16c}\right)}{24c}\right)}{2c}$$

$$\Big\downarrow \text{1087}$$

$$\frac{B\left(bx+cx^2\right)^{7/2}}{7c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)}{24c}\right)}{2c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{B\left(bx+cx^2\right)^{7/2}}{7c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)\left(bx+cx^2\right)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)}{24c}\right)}{2c}$$

$$\Big\downarrow \text{219}$$

$$\frac{B(bx+cx^2)^{7/2}}{7c}-\frac{(bB-2Ac)\left(\frac{(b+2cx)(bx+cx^2)^{5/2}}{12c}-\frac{5b^2\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)}{24c}\right)}{2c}$$

input
```
Int[(A + B*x)*(b*x + c*x^2)^(5/2),x]
```

output
```
(B*(b*x + c*x^2)^(7/2))/(7*c) - ((b*B - 2*A*c)*(((b + 2*c*x)*(b*x + c*x^2)
^(5/2))/(12*c) - (5*b^2*(((b + 2*c*x)*(b*x + c*x^2)^(3/2))/(8*c) - (3*b^2*
(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x
 + c*x^2]])/(4*c^(3/2))))/(16*c)))/(24*c)))/(2*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.77

method	result
risch	$\frac{(3072B\,c^6x^6+3584A\,c^6x^5+7424Bb\,c^5x^5+8960Ab\,c^5x^4+4736B\,b^2c^4x^4+6048A\,b^2c^4x^3+48B\,b^3c^3x^3+112A\,b^3c^3x^2-56B\,b^4c^2x^2-1}{21504c^4\sqrt{x(cx+b)}}$
default	$A\left(\frac{(2cx+b)\left(c\,x^2+bx\right)^{\frac{5}{2}}}{12c}-\frac{5b^2\left(\frac{(2cx+b)\left(c\,x^2+bx\right)^{\frac{3}{2}}}{8c}-\frac{3b^2\left(\frac{(2cx+b)\sqrt{c\,x^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{24c}\right)+B\left(\frac{(c\,x^2+b}{7c}\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/21504/c^4*(3072*B*c^6*x^6+3584*A*c^6*x^5+7424*B*b*c^5*x^5+8960*A*b*c^5*x
^4+4736*B*b^2*c^4*x^4+6048*A*b^2*c^4*x^3+48*B*b^3*c^3*x^3+112*A*b^3*c^3*x^
2-56*B*b^4*c^2*x^2-140*A*b^4*c^2*x+70*B*b^5*c*x+210*A*b^5*c-105*B*b^6)*x*(
c*x+b)/(x*(c*x+b))^(1/2)-5/2048*b^6*(2*A*c-B*b)/c^(9/2)*ln((1/2*b+c*x)/c^(
1/2)+(c*x^2+b*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 393, normalized size of antiderivative = 1.56

$$\int (A+Bx)\left(bx+cx^2\right)^{5/2}\,dx = \left[-\frac{105\,(Bb^7-2\,Ab^6c)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(3072\,Bc^7x^6-105\,Bb^6c+21\cdots}{\cdots}\right.$$

$$-\frac{105\,(Bb^7-2\,Ab^6c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(3072\,Bc^7x^6-105\,Bb^6c+210\,Ab^5c^2+256\,(29\,Bbc^6+1\cdots}{\cdots}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/43008*(105*(B*b^7 - 2*A*b^6*c)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 +
b*x)*sqrt(c)) - 2*(3072*B*c^7*x^6 - 105*B*b^6*c + 210*A*b^5*c^2 + 256*(29*
B*b*c^6 + 14*A*c^7)*x^5 + 128*(37*B*b^2*c^5 + 70*A*b*c^6)*x^4 + 48*(B*b^3*
c^4 + 126*A*b^2*c^5)*x^3 - 56*(B*b^4*c^3 - 2*A*b^3*c^4)*x^2 + 70*(B*b^5*c^
2 - 2*A*b^4*c^3)*x)*sqrt(c*x^2 + b*x))/c^5, -1/21504*(105*(B*b^7 - 2*A*b^6
*c)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) - (3072*B*c^7*x^
6 - 105*B*b^6*c + 210*A*b^5*c^2 + 256*(29*B*b*c^6 + 14*A*c^7)*x^5 + 128*(3
7*B*b^2*c^5 + 70*A*b*c^6)*x^4 + 48*(B*b^3*c^4 + 126*A*b^2*c^5)*x^3 - 56*(B
*b^4*c^3 - 2*A*b^3*c^4)*x^2 + 70*(B*b^5*c^2 - 2*A*b^4*c^3)*x)*sqrt(c*x^2 +
 b*x))/c^5]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(240) = 480$.

Time = 0.52 (sec) , antiderivative size = 570, normalized size of antiderivative = 2.26

$$\int (A+Bx)\left(bx+cx^2\right)^{5/2}\,dx = \begin{cases} -\frac{5b^3\left(Ab^3-\frac{7b\left(3Ab^2c+Bb^3-\frac{9b\left(3Abc^2+3Bb^2c-\frac{11b\left(Ac^3+\frac{29Bbc^2}{14}\right)}{12c}\right)}{10c}\right)}{8c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right.}{16c^3} \\ \frac{2\left(\frac{A(bx)^{\frac{7}{2}}}{7}+\frac{B(bx)^{\frac{9}{2}}}{9b}\right)}{b} \\ 0 \end{cases}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2),x)
```

output
```
Piecewise((-5*b**3*(A*b**3 - 7*b*(3*A*b**2*c + B*b**3 - 9*b*(3*A*b*c**2 +
3*B*b**2*c - 11*b*(A*c**3 + 29*B*b*c**2/14)/(12*c))/(10*c))/(8*c))*Piecewi
se((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)),
 ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(16*c**3
) + sqrt(b*x + c*x**2)*(B*c**2*x**6/7 + 5*b**2*(A*b**3 - 7*b*(3*A*b**2*c +
 B*b**3 - 9*b*(3*A*b*c**2 + 3*B*b**2*c - 11*b*(A*c**3 + 29*B*b*c**2/14)/(1
2*c))/(10*c))/(8*c))/(8*c**3) - 5*b*x*(A*b**3 - 7*b*(3*A*b**2*c + B*b**3 -
 9*b*(3*A*b*c**2 + 3*B*b**2*c - 11*b*(A*c**3 + 29*B*b*c**2/14)/(12*c))/(10
*c))/(8*c))/(12*c**2) + x**5*(A*c**3 + 29*B*b*c**2/14)/(6*c) + x**4*(3*A*b
*c**2 + 3*B*b**2*c - 11*b*(A*c**3 + 29*B*b*c**2/14)/(12*c))/(5*c) + x**3*(
3*A*b**2*c + B*b**3 - 9*b*(3*A*b*c**2 + 3*B*b**2*c - 11*b*(A*c**3 + 29*B*b
*c**2/14)/(12*c))/(10*c))/(4*c) + x**2*(A*b**3 - 7*b*(3*A*b**2*c + B*b**3
- 9*b*(3*A*b*c**2 + 3*B*b**2*c - 11*b*(A*c**3 + 29*B*b*c**2/14)/(12*c))/(1
0*c))/(8*c))/(3*c)), Ne(c, 0)), (2*(A*(b*x)**(7/2)/7 + B*(b*x)**(9/2)/(9*b
))/b, Ne(b, 0)), (0, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.26

$$\begin{aligned}\int (A+Bx)\left(bx+cx^2\right)^{5/2}\,dx &= \frac{1}{6}\left(cx^2+bx\right)^{\frac{5}{2}}Ax - \frac{5\sqrt{cx^2+bx}Bb^5x}{512\,c^3} \\ &+ \frac{5\left(cx^2+bx\right)^{\frac{3}{2}}Bb^3x}{192\,c^2} + \frac{5\sqrt{cx^2+bx}Ab^4x}{256\,c^2} - \frac{\left(cx^2+bx\right)^{\frac{5}{2}}Bbx}{12\,c} - \frac{5\left(cx^2+bx\right)^{\frac{3}{2}}Ab^2x}{96\,c} \\ &+ \frac{5\,Bb^7\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{2048\,c^{\frac{9}{2}}} - \frac{5\,Ab^6\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{1024\,c^{\frac{7}{2}}} \\ &- \frac{5\sqrt{cx^2+bx}Bb^6}{1024\,c^4} + \frac{5\left(cx^2+bx\right)^{\frac{3}{2}}Bb^4}{384\,c^3} + \frac{5\sqrt{cx^2+bx}Ab^5}{512\,c^3} \\ &- \frac{\left(cx^2+bx\right)^{\frac{5}{2}}Bb^2}{24\,c^2} - \frac{5\left(cx^2+bx\right)^{\frac{3}{2}}Ab^3}{192\,c^2} + \frac{\left(cx^2+bx\right)^{\frac{7}{2}}B}{7\,c} + \frac{\left(cx^2+bx\right)^{\frac{5}{2}}Ab}{12\,c}\end{aligned}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
1/6*(c*x^2 + b*x)^(5/2)*A*x - 5/512*sqrt(c*x^2 + b*x)*B*b^5*x/c^3 + 5/192*
(c*x^2 + b*x)^(3/2)*B*b^3*x/c^2 + 5/256*sqrt(c*x^2 + b*x)*A*b^4*x/c^2 - 1/
12*(c*x^2 + b*x)^(5/2)*B*b*x/c - 5/96*(c*x^2 + b*x)^(3/2)*A*b^2*x/c + 5/20
48*B*b^7*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(9/2) - 5/1024*A*b
^6*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(7/2) - 5/1024*sqrt(c*x^
2 + b*x)*B*b^6/c^4 + 5/384*(c*x^2 + b*x)^(3/2)*B*b^4/c^3 + 5/512*sqrt(c*x^
2 + b*x)*A*b^5/c^3 - 1/24*(c*x^2 + b*x)^(5/2)*B*b^2/c^2 - 5/192*(c*x^2 + b
*x)^(3/2)*A*b^3/c^2 + 1/7*(c*x^2 + b*x)^(7/2)*B/c + 1/12*(c*x^2 + b*x)^(5/
2)*A*b/c
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.87

$$\int (A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\frac{1}{21504}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\left(2\left(12\,Bc^2x+\frac{29\,Bbc^7+14\,Ac^8}{c^6}\right)x+\frac{37\,Bb^2c^6+70\,Abc^7}{c^6}\right)\right.\right.\right.\right.\right. - \frac{5\left(Bb^7-2\,Ab^6c\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{2048\,c^{\frac{9}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
1/21504*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(2*(12*B*c^2*x + (29*B*b*c^7 + 14*A*
c^8)/c^6)*x + (37*B*b^2*c^6 + 70*A*b*c^7)/c^6)*x + 3*(B*b^3*c^5 + 126*A*b^
2*c^6)/c^6)*x - 7*(B*b^4*c^4 - 2*A*b^3*c^5)/c^6)*x + 35*(B*b^5*c^3 - 2*A*b
^4*c^4)/c^6)*x - 105*(B*b^6*c^2 - 2*A*b^5*c^3)/c^6) - 5/2048*(B*b^7 - 2*A*
b^6*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int (A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\int \left(c\,x^2+b\,x\right)^{5/2}\,\left(A+B\,x\right)\,dx$$

input
```
int((b*x + c*x^2)^(5/2)*(A + B*x),x)
```

output
```
int((b*x + c*x^2)^(5/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.16

$$\int (A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\frac{210\sqrt{x}\,\sqrt{cx+b}\,a\,b^5c^2-140\sqrt{x}\,\sqrt{cx+b}\,a\,b^4c^3x+112\sqrt{x}\,\sqrt{cx+b}\,a\,b^3c^4x^2+6048\sqrt{x}\,\sqrt{cx-}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2),x)
```

output
```
(210*sqrt(x)*sqrt(b + c*x)*a*b**5*c**2 - 140*sqrt(x)*sqrt(b + c*x)*a*b**4*
c**3*x + 112*sqrt(x)*sqrt(b + c*x)*a*b**3*c**4*x**2 + 6048*sqrt(x)*sqrt(b
+ c*x)*a*b**2*c**5*x**3 + 8960*sqrt(x)*sqrt(b + c*x)*a*b*c**6*x**4 + 3584*
sqrt(x)*sqrt(b + c*x)*a*c**7*x**5 - 105*sqrt(x)*sqrt(b + c*x)*b**7*c + 70*
sqrt(x)*sqrt(b + c*x)*b**6*c**2*x - 56*sqrt(x)*sqrt(b + c*x)*b**5*c**3*x**
2 + 48*sqrt(x)*sqrt(b + c*x)*b**4*c**4*x**3 + 4736*sqrt(x)*sqrt(b + c*x)*b
**3*c**5*x**4 + 7424*sqrt(x)*sqrt(b + c*x)*b**2*c**6*x**5 + 3072*sqrt(x)*s
qrt(b + c*x)*b*c**7*x**6 - 210*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c
))/sqrt(b))*a*b**6*c + 105*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/s
qrt(b))*b**8)/(21504*c**5)
```

3.130 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 227

$$\begin{aligned}\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x}\,dx &= \frac{b^4(5bB-12Ac)\sqrt{bx+cx^2}}{512c^3}\\ &-\frac{b^3(5bB-12Ac)x\sqrt{bx+cx^2}}{768c^2}-\frac{31b^2(5bB-12Ac)x^2\sqrt{bx+cx^2}}{960c}\\ &-\frac{7}{160}b(5bB-12Ac)x^3\sqrt{bx+cx^2}-\frac{1}{60}c(5bB-12Ac)x^4\sqrt{bx+cx^2}\\ &+\frac{B\left(bx+cx^2\right)^{7/2}}{6cx}-\frac{b^5(5bB-12Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{512c^{7/2}}\end{aligned}$$

output
```
1/512*b^4*(-12*A*c+5*B*b)*(c*x^2+b*x)^(1/2)/c^3-1/768*b^3*(-12*A*c+5*B*b)*
x*(c*x^2+b*x)^(1/2)/c^2-31/960*b^2*(-12*A*c+5*B*b)*x^2*(c*x^2+b*x)^(1/2)/c
-7/160*b*(-12*A*c+5*B*b)*x^3*(c*x^2+b*x)^(1/2)-1/60*c*(-12*A*c+5*B*b)*x^4*
(c*x^2+b*x)^(1/2)+1/6*B*(c*x^2+b*x)^(7/2)/c/x-1/512*b^5*(-12*A*c+5*B*b)*ar
ctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x}\,dx = \frac{(x(b+cx))^{5/2}\left(75b^5B-180Ab^4c-50b^4Bcx+120Ab^3c^2x+40b^3Bc^2x^2+\right.}{76}$$
$$-\frac{b^5(5bB-12Ac)(x(b+cx))^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{x}}{-\sqrt{b}+\sqrt{b+cx}}\right)}{256c^{7/2}x^{5/2}(b+cx)^{5/2}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x,x]
```

output
```
((x*(b + c*x))^(5/2)*(75*b^5*B - 180*A*b^4*c - 50*b^4*B*c*x + 120*A*b^3*c^
2*x + 40*b^3*B*c^2*x^2 + 2976*A*b^2*c^3*x^2 + 2160*b^2*B*c^3*x^3 + 4032*A*
b*c^4*x^3 + 3200*b*B*c^4*x^4 + 1536*A*c^5*x^4 + 1280*B*c^5*x^5))/(7680*c^3
*x^2*(b + c*x)^2) - (b^5*(5*b*B - 12*A*c)*(x*(b + c*x))^(5/2)*ArcTanh[(Sqr
t[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])])/(256*c^(7/2)*x^(5/2)*(b + c*x)^
(5/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.71, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1221, 1131, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x}dx$$

$$\Big\downarrow \; 1221$$

$$\frac{B\left(bx+cx^2\right)^{7/2}}{6cx}-\frac{(5bB-12Ac)\int \frac{\left(cx^2+bx\right)^{5/2}}{x}dx}{12c}$$

$$\Big\downarrow \; 1131$$

$$\frac{B\left(bx+cx^2\right)^{7/2}}{6cx}-\frac{(5bB-12Ac)\left(\frac{1}{2}b\int \left(cx^2+bx\right)^{3/2}dx+\frac{1}{5}\left(bx+cx^2\right)^{5/2}\right)}{12c}$$

$\Big\downarrow$ 1087

$$\frac{B\left(bx+cx^2\right)^{7/2}}{6cx}-\frac{(5bB-12Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\int \sqrt{cx^2+bx}dx}{16c}\right)+\frac{1}{5}\left(bx+cx^2\right)^{5/2}\right)}{12c}$$

$\Big\downarrow$ 1087

$$\frac{B\left(bx+cx^2\right)^{7/2}}{6cx}-\frac{(5bB-12Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)+\frac{1}{5}\left(bx+cx^2\right)^{5/2}\right)}{12c}$$

$\Big\downarrow$ 1091

$$\frac{B\left(bx+cx^2\right)^{7/2}}{6cx}-\frac{(5bB-12Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)+\frac{1}{5}\left(bx+cx^2\right)^{5/2}\right)}{12c}$$

$\Big\downarrow$ 219

$$\frac{B\left(bx+cx^2\right)^{7/2}}{6cx}-\frac{(5bB-12Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx)\left(bx+cx^2\right)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)+\frac{1}{5}\left(bx+cx^2\right)^{5/2}\right)}{12c}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x,x]
```

output
```
(B*(b*x + c*x^2)^(7/2))/(6*c*x) - ((5*b*B - 12*A*c)*((b*x + c*x^2)^(5/2)/5
 + (b*(((b + 2*c*x)*(b*x + c*x^2)^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x)*Sqrt
[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(
3/2))))/(16*c)))/2))/(12*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(-1280B\,c^5x^5-1536A\,c^5x^4-3200Bb\,c^4x^4-4032Ab\,c^4x^3-2160B\,b^2c^3x^3-2976A\,b^2c^3x^2-40B\,b^3c^2x^2-120A\,b^3c^2x+50B\,b^4cx+}{7680c^3\sqrt{x(cx+b)}}$
default	$B\left(\frac{(2cx+b)(c\,x^2+bx)^{\frac{5}{2}}}{12c}-\frac{5b^2\left(\frac{(2cx+b)(c\,x^2+bx)^{\frac{3}{2}}}{8c}-\frac{3b^2\left(\frac{(2cx+b)\sqrt{c\,x^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{24c}\right)+A\left(\frac{(c\,x^2+b}{5}\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/7680/c^3*(-1280*B*c^5*x^5-1536*A*c^5*x^4-3200*B*b*c^4*x^4-4032*A*b*c^4*
x^3-2160*B*b^2*c^3*x^3-2976*A*b^2*c^3*x^2-40*B*b^3*c^2*x^2-120*A*b^3*c^2*x
+50*B*b^4*c*x+180*A*b^4*c-75*B*b^5)*x*(c*x+b)/(x*(c*x+b))^(1/2)+1/1024*b^5
*(12*A*c-5*B*b)/c^(7/2)*ln((1/2*b+c*x)/c^(1/2)+(c*x^2+b*x)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.55

$$\int \frac{(A+Bx)\,(bx+cx^2)^{5/2}}{x}\,dx = \left[-\frac{15\,(5\,Bb^6-12\,Ab^5c)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(1280\,Bc^6}{}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x,x, algorithm="fricas")
```

output
```
[-1/15360*(15*(5*B*b^6 - 12*A*b^5*c)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2
+ b*x)*sqrt(c)) - 2*(1280*B*c^6*x^5 + 75*B*b^5*c - 180*A*b^4*c^2 + 128*(25
*B*b*c^5 + 12*A*c^6)*x^4 + 144*(15*B*b^2*c^4 + 28*A*b*c^5)*x^3 + 8*(5*B*b^
3*c^3 + 372*A*b^2*c^4)*x^2 - 10*(5*B*b^4*c^2 - 12*A*b^3*c^3)*x)*sqrt(c*x^2
 + b*x))/c^4, 1/7680*(15*(5*B*b^6 - 12*A*b^5*c)*sqrt(-c)*arctan(sqrt(c*x^2
 + b*x)*sqrt(-c)/(c*x + b)) + (1280*B*c^6*x^5 + 75*B*b^5*c - 180*A*b^4*c^2
 + 128*(25*B*b*c^5 + 12*A*c^6)*x^4 + 144*(15*B*b^2*c^4 + 28*A*b*c^5)*x^3 +
 8*(5*B*b^3*c^3 + 372*A*b^2*c^4)*x^2 - 10*(5*B*b^4*c^2 - 12*A*b^3*c^3)*x)*
sqrt(c*x^2 + b*x))/c^4]
```

Sympy [A] (verification not implemented)

Time = 2.98 (sec) , antiderivative size = 886, normalized size of antiderivative = 3.90

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x,x)
```

output
```
A*b**2*Piecewise((b**3*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2
*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(
2*c) + x)**2), True))/(16*c**2) + sqrt(b*x + c*x**2)*(-b**2/(8*c**2) + b*x
/(12*c) + x**2/3), Ne(c, 0)), (2*(b*x)**(5/2)/(5*b**2), Ne(b, 0)), (0, Tru
e)) + 2*A*b*c*Piecewise((-5*b**4*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c
*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/s
qrt(c*(b/(2*c) + x)**2), True))/(128*c**3) + sqrt(b*x + c*x**2)*(5*b**3/(6
4*c**3) - 5*b**2*x/(96*c**2) + b*x**2/(24*c) + x**3/4), Ne(c, 0)), (2*(b*x
)**(7/2)/(7*b**3), Ne(b, 0)), (0, True)) + A*c**2*Piecewise((7*b**5*Piecew
ise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0))
, ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(256*c*
*4) + sqrt(b*x + c*x**2)*(-7*b**4/(128*c**4) + 7*b**3*x/(192*c**3) - 7*b**
2*x**2/(240*c**2) + b*x**3/(40*c) + x**4/5), Ne(c, 0)), (2*(b*x)**(9/2)/(9
*b**4), Ne(b, 0)), (0, True)) + B*b**2*Piecewise((-5*b**4*Piecewise((log(b
 + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c
) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(128*c**3) + sqrt
(b*x + c*x**2)*(5*b**3/(64*c**3) - 5*b**2*x/(96*c**2) + b*x**2/(24*c) + x*
*3/4), Ne(c, 0)), (2*(b*x)**(7/2)/(7*b**3), Ne(b, 0)), (0, True)) + 2*B*b*
c*Piecewise((7*b**5*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*
x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.19

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x}\,dx = \frac{1}{6}\left(cx^2+bx\right)^{\frac{5}{2}}Bx + \frac{1}{8}\left(cx^2+bx\right)^{\frac{3}{2}}Abx + \frac{5\sqrt{cx^2+bx}Bb^4x}{256\,c^2} - \frac{5\left(cx^2+bx\right)^{\frac{3}{2}}Bb^2x}{96\,c} - \frac{3\sqrt{cx^2+bx}Ab^3x}{64\,c} - \frac{5\,Bb^6\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{1024\,c^{\frac{7}{2}}} + \frac{3\,Ab^5\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{256\,c^{\frac{5}{2}}} + \frac{1}{5}\left(cx^2+bx\right)^{\frac{5}{2}}A + \frac{5\sqrt{cx^2+bx}Bb^5}{512\,c^3} - \frac{5\left(cx^2+bx\right)^{\frac{3}{2}}Bb^3}{192\,c^2} - \frac{3\sqrt{cx^2+bx}Ab^4}{128\,c^2} + \frac{\left(cx^2+bx\right)^{\frac{5}{2}}Bb}{12\,c} + \frac{\left(cx^2+bx\right)^{\frac{3}{2}}Ab^2}{16\,c}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x,x, algorithm="maxima")
```

output
```
1/6*(c*x^2 + b*x)^(5/2)*B*x + 1/8*(c*x^2 + b*x)^(3/2)*A*b*x + 5/256*sqrt(c
*x^2 + b*x)*B*b^4*x/c^2 - 5/96*(c*x^2 + b*x)^(3/2)*B*b^2*x/c - 3/64*sqrt(c
*x^2 + b*x)*A*b^3*x/c - 5/1024*B*b^6*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*s
qrt(c))/c^(7/2) + 3/256*A*b^5*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))
/c^(5/2) + 1/5*(c*x^2 + b*x)^(5/2)*A + 5/512*sqrt(c*x^2 + b*x)*B*b^5/c^3 -
 5/192*(c*x^2 + b*x)^(3/2)*B*b^3/c^2 - 3/128*sqrt(c*x^2 + b*x)*A*b^4/c^2 +
 1/12*(c*x^2 + b*x)^(5/2)*B*b/c + 1/16*(c*x^2 + b*x)^(3/2)*A*b^2/c
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x}\,dx = \frac{1}{7680}\sqrt{cx^2+bx}\left(2\left(4\left(2\left(8\left(10\,Bc^2x+\frac{25\,Bbc^6+12\,Ac^7}{c^5}\right)x+\frac{9\,(15\,Bb\right.\right.\right.\right.\right.$$
$$+\frac{\left(5\,Bb^6-12\,Ab^5c\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{1024\,c^{\frac{7}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x,x, algorithm="giac")
```

output
```
1/7680*sqrt(c*x^2 + b*x)*(2*(4*(2*(8*(10*B*c^2*x + (25*B*b*c^6 + 12*A*c^7)
/c^5)*x + 9*(15*B*b^2*c^5 + 28*A*b*c^6)/c^5)*x + (5*B*b^3*c^4 + 372*A*b^2*
c^5)/c^5)*x - 5*(5*B*b^4*c^3 - 12*A*b^3*c^4)/c^5)*x + 15*(5*B*b^5*c^2 - 12
*A*b^4*c^3)/c^5) + 1/1024*(5*B*b^6 - 12*A*b^5*c)*log(abs(2*(sqrt(c)*x - sq
rt(c*x^2 + b*x))*sqrt(c) + b))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x}\,dx = \int \frac{\left(c\,x^2+b\,x\right)^{5/2}\,\left(A+B\,x\right)}{x}\,dx$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x,x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\,(bx+cx^2)^{5/2}}{x}\,dx = \frac{-180\sqrt{x}\,\sqrt{cx+b}\,a\,b^4c^2+120\sqrt{x}\,\sqrt{cx+b}\,a\,b^3c^3x+2976\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x,x)
```

output
```
( - 180*sqrt(x)*sqrt(b + c*x)*a*b**4*c**2 + 120*sqrt(x)*sqrt(b + c*x)*a*b*
*3*c**3*x + 2976*sqrt(x)*sqrt(b + c*x)*a*b**2*c**4*x**2 + 4032*sqrt(x)*sqr
t(b + c*x)*a*b*c**5*x**3 + 1536*sqrt(x)*sqrt(b + c*x)*a*c**6*x**4 + 75*sqr
t(x)*sqrt(b + c*x)*b**6*c - 50*sqrt(x)*sqrt(b + c*x)*b**5*c**2*x + 40*sqrt
(x)*sqrt(b + c*x)*b**4*c**3*x**2 + 2160*sqrt(x)*sqrt(b + c*x)*b**3*c**4*x*
*3 + 3200*sqrt(x)*sqrt(b + c*x)*b**2*c**5*x**4 + 1280*sqrt(x)*sqrt(b + c*x
)*b*c**6*x**5 + 180*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))
*a*b**5*c - 75*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**7
)/(7680*c**4)
```

3.131 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 192

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^2}\,dx = -\frac{b^3(3bB-10Ac)\sqrt{bx+cx^2}}{128c^2} - \frac{59b^2(3bB-10Ac)x\sqrt{bx+cx^2}}{960c} - \frac{17}{240}b(3bB-10Ac)x^2\sqrt{bx+cx^2} - \frac{1}{40}c(3bB-10Ac)x^3\sqrt{bx+cx^2} + \frac{B\left(bx+cx^2\right)^{7/2}}{5cx^2} + \frac{b^4(3bB-10Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{128c^{5/2}}$$

output
```
-1/128*b^3*(-10*A*c+3*B*b)*(c*x^2+b*x)^(1/2)/c^2-59/960*b^2*(-10*A*c+3*B*b
)*x*(c*x^2+b*x)^(1/2)/c-17/240*b*(-10*A*c+3*B*b)*x^2*(c*x^2+b*x)^(1/2)-1/4
0*c*(-10*A*c+3*B*b)*x^3*(c*x^2+b*x)^(1/2)+1/5*B*(c*x^2+b*x)^(7/2)/c/x^2+1/
128*b^4*(-10*A*c+3*B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^2}\,dx = \frac{\sqrt{x}\sqrt{b+cx}\left(\sqrt{c}\sqrt{x}\sqrt{b+cx}(-45b^4B+30b^3c(5A+Bx)+96c^4x^3(5A+4\right.}{}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^2,x]
```

output
```
(Sqrt[x]*Sqrt[b + c*x]*(Sqrt[c]*Sqrt[x]*Sqrt[b + c*x]*(-45*b^4*B + 30*b^3*
c*(5*A + B*x) + 96*c^4*x^3*(5*A + 4*B*x) + 16*b*c^3*x^2*(85*A + 63*B*x) +
4*b^2*c^2*x*(295*A + 186*B*x)) + 300*A*b^4*c*ArcTanh[(Sqrt[c]*Sqrt[x])/(Sq
rt[b] - Sqrt[b + c*x])] + 90*b^5*B*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + S
qrt[b + c*x])]))/(1920*c^(5/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.84, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1220, 1131, 1087, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^2}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(3bB-10Ac)\int \frac{\left(cx^2+bx\right)^{5/2}}{x}dx}{3b}+\frac{2A\left(bx+cx^2\right)^{7/2}}{3bx^2}$$

$$\Big\downarrow \text{1131}$$

$$\frac{(3bB-10Ac)\left(\frac{1}{2}b\int \left(cx^2+bx\right)^{3/2}dx+\frac{1}{5}\left(bx+cx^2\right)^{5/2}\right)}{3b}+\frac{2A\left(bx+cx^2\right)^{7/2}}{3bx^2}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(3bB-10Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\int\sqrt{cx^2+bx}dx}{16c}\right)+\frac{1}{5}(bx+cx^2)^{5/2}\right)}{3b}+\frac{2A(bx+cx^2)^{7/2}}{3bx^2}$$

$$\Big\downarrow \; 1087$$

$$\frac{(3bB-10Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)}{16c}\right)+\frac{1}{5}(bx+cx^2)^{5/2}\right)}{3b}+\frac{2A(bx+cx^2)^{7/2}}{3bx^2}$$

$$\Big\downarrow \; 1091$$

$$\frac{(3bB-10Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)}{16c}\right)+\frac{1}{5}(bx+cx^2)^{5/2}\right)}{3b}+\frac{2A(bx+cx^2)^{7/2}}{3bx^2}$$

$$\Big\downarrow \; 219$$

$$\frac{(3bB-10Ac)\left(\frac{1}{2}b\left(\frac{(b+2cx)(bx+cx^2)^{3/2}}{8c}-\frac{3b^2\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)}{16c}\right)+\frac{1}{5}(bx+cx^2)^{5/2}\right)}{3b}+\frac{2A(bx+cx^2)^{7/2}}{3bx^2}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^2,x]
```

output
```
(2*A*(b*x + c*x^2)^(7/2))/(3*b*x^2) + ((3*b*B - 10*A*c)*((b*x + c*x^2)^(5/
2)/5 + (b*(((b + 2*c*x)*(b*x + c*x^2)^(3/2))/(8*c) - (3*b^2*(((b + 2*c*x)*
Sqrt[b*x + c*x^2])/(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4
*c^(3/2))))/(16*c)))/2))/(3*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.60

method	result
pseudoelliptic	$\frac{\frac{59\left(-\frac{15}{118} A b^{4} c+\frac{9}{236} b^{5} B\right) \operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{96}+\frac{59\left(\frac{15\left(\frac{Bx}{5}+A\right) b^{3} c^{\frac{3}{2}}}{118}+b^{2} x\left(\frac{186Bx}{295}+A\right) c^{\frac{5}{2}}+\frac{68x^{2}\left(\frac{63Bx}{85}+A\right) b\, c^{\frac{7}{2}}}{59}+\frac{24x^{3}\left(\frac{4Bx}{5}+A\right.}{59}\right.}{96}}{c^{\frac{5}{2}}}$
risch	$\frac{\left(384B\,c^{4}x^{4}+480A\,c^{4}x^{3}+1008Bb\,c^{3}x^{3}+1360Ab\,c^{3}x^{2}+744x^{2}B\,b^{2}c^{2}+1180A\,b^{2}c^{2}x+30B\,b^{3}cx+150A\,b^{3}c-45B\,b^{4}\right)x(cx+b)}{1920c^{2}\sqrt{x(cx+b)}}$
default	$A\left(\frac{2\left(c\,x^{2}+bx\right)^{\frac{7}{2}}}{3b\,x^{2}}-\frac{10c\left(\frac{\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{5}+\frac{b\left(\frac{(2cx+b)\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{8c}-\frac{3b^{2}\left(\frac{(2cx+b)\sqrt{c\,x^{2}+bx}}{4c}-\frac{b^{2}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{8c^{\frac{3}{2}}}\right)}{16c}\right)}{2}\right)}{3b}\right.$

input

```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output

```
59/96/c^(5/2)*((-15/118*A*b^4*c+9/236*b^5*B)*arctanh((x*(c*x+b))^(1/2)/x/c
^(1/2))+(15/118*(1/5*B*x+A)*b^3*c^(3/2)+b^2*x*(186/295*B*x+A)*c^(5/2)+68/5
9*x^2*(63/85*B*x+A)*b*c^(7/2)+24/59*x^3*(4/5*B*x+A)*c^(9/2)-9/236*B*c^(1/2
)*b^4)*(x*(c*x+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.59

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^2}\,dx = \left[-\frac{15\,(3\,Bb^5-10\,Ab^4c)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(384\,Bc^5x\right.$$

$$-\frac{15\,(3\,Bb^5-10\,Ab^4c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(384\,Bc^5x^4-45\,Bb^4c+150\,Ab^3c^2+48\,(21\,Bbc^4+10\,}{1920\,c^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
[-1/3840*(15*(3*B*b^5 - 10*A*b^4*c)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 +
 b*x)*sqrt(c)) - 2*(384*B*c^5*x^4 - 45*B*b^4*c + 150*A*b^3*c^2 + 48*(21*B*
b*c^4 + 10*A*c^5)*x^3 + 8*(93*B*b^2*c^3 + 170*A*b*c^4)*x^2 + 10*(3*B*b^3*c
^2 + 118*A*b^2*c^3)*x)*sqrt(c*x^2 + b*x))/c^3, -1/1920*(15*(3*B*b^5 - 10*A
*b^4*c)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) - (384*B*c^5
*x^4 - 45*B*b^4*c + 150*A*b^3*c^2 + 48*(21*B*b*c^4 + 10*A*c^5)*x^3 + 8*(93
*B*b^2*c^3 + 170*A*b*c^4)*x^2 + 10*(3*B*b^3*c^2 + 118*A*b^2*c^3)*x)*sqrt(c
*x^2 + b*x))/c^3]
```

Sympy [A] (verification not implemented)

Time = 0.79 (sec) , antiderivative size = 796, normalized size of antiderivative = 4.15

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^2}\,dx = \text{Too large to display}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**2,x)
```

output
```
A*b**2*Piecewise((-b**2*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) +
2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/
(2*c) + x)**2), True))/(8*c) + (b/(4*c) + x/2)*sqrt(b*x + c*x**2), Ne(c, 0
)), (2*(b*x)**(3/2)/(3*b), Ne(b, 0)), (0, True)) + 2*A*b*c*Piecewise((b**3
*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2
/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/
(16*c**2) + sqrt(b*x + c*x**2)*(-b**2/(8*c**2) + b*x/(12*c) + x**2/3), Ne(
c, 0)), (2*(b*x)**(5/2)/(5*b**2), Ne(b, 0)), (0, True)) + A*c**2*Piecewise
((-5*b**4*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c)
, Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2)
, True))/(128*c**3) + sqrt(b*x + c*x**2)*(5*b**3/(64*c**3) - 5*b**2*x/(96*
c**2) + b*x**2/(24*c) + x**3/4), Ne(c, 0)), (2*(b*x)**(7/2)/(7*b**3), Ne(b
, 0)), (0, True)) + B*b**2*Piecewise((b**3*Piecewise((log(b + 2*sqrt(c)*sq
rt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2
*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(16*c**2) + sqrt(b*x + c*x**2)*(
-b**2/(8*c**2) + b*x/(12*c) + x**2/3), Ne(c, 0)), (2*(b*x)**(5/2)/(5*b**2)
, Ne(b, 0)), (0, True)) + 2*B*b*c*Piecewise((-5*b**4*Piecewise((log(b + 2*
sqrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x
)*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True))/(128*c**3) + sqrt(b*x
+ c*x**2)*(5*b**3/(64*c**3) - 5*b**2*x/(96*c**2) + b*x**2/(24*c) + x**3...
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^2}\,dx = \frac{1}{8}\left(cx^2+bx\right)^{\frac{3}{2}}Bbx + \frac{5}{32}\sqrt{cx^2+bx}Ab^2x - \frac{3\sqrt{cx^2+bx}Bb^3x}{64\,c} + \frac{3\,Bb^5\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{256\,c^{\frac{5}{2}}} - \frac{5\,Ab^4\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{128\,c^{\frac{3}{2}}} + \frac{1}{5}\left(cx^2+bx\right)^{\frac{5}{2}}B + \frac{5}{24}\left(cx^2+bx\right)^{\frac{3}{2}}Ab - \frac{3\sqrt{cx^2+bx}Bb^4}{128\,c^2} + \frac{\left(cx^2+bx\right)^{\frac{3}{2}}Bb^2}{16\,c} + \frac{5\sqrt{cx^2+bx}Ab^3}{64\,c} + \frac{\left(cx^2+bx\right)^{\frac{5}{2}}A}{4\,x}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
1/8*(c*x^2 + b*x)^(3/2)*B*b*x + 5/32*sqrt(c*x^2 + b*x)*A*b^2*x - 3/64*sqrt
(c*x^2 + b*x)*B*b^3*x/c + 3/256*B*b^5*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*
sqrt(c))/c^(5/2) - 5/128*A*b^4*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)
)/c^(3/2) + 1/5*(c*x^2 + b*x)^(5/2)*B + 5/24*(c*x^2 + b*x)^(3/2)*A*b - 3/1
28*sqrt(c*x^2 + b*x)*B*b^4/c^2 + 1/16*(c*x^2 + b*x)^(3/2)*B*b^2/c + 5/64*s
qrt(c*x^2 + b*x)*A*b^3/c + 1/4*(c*x^2 + b*x)^(5/2)*A/x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^2}\,dx = \frac{1}{1920}\sqrt{cx^2+bx}\left(2\left(4\left(6\left(8\,Bc^2x+\frac{21\,Bbc^5+10\,Ac^6}{c^4}\right)x+\frac{93\,Bb^2c^4+1}{c^4}\right.\right.\right.$$
$$-\frac{\left(3\,Bb^5-10\,Ab^4c\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{256\,c^{\frac{5}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^2,x, algorithm="giac")
```

output
```
1/1920*sqrt(c*x^2 + b*x)*(2*(4*(6*(8*B*c^2*x + (21*B*b*c^5 + 10*A*c^6)/c^4
)*x + (93*B*b^2*c^4 + 170*A*b*c^5)/c^4)*x + 5*(3*B*b^3*c^3 + 118*A*b^2*c^4
)/c^4)*x - 15*(3*B*b^4*c^2 - 10*A*b^3*c^3)/c^4) - 1/256*(3*B*b^5 - 10*A*b^
4*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^2}\,dx = \int \frac{{\left(c\,x^2+b\,x\right)}^{5/2}\,\left(A+B\,x\right)}{x^2} \,dx$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^2,x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\,(bx+cx^2)^{5/2}}{x^2}\,dx = \frac{150\sqrt{x}\,\sqrt{cx+b}\,a\,b^3c^2+1180\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^3x+1360\sqrt{x}\,\sqrt{cx+b}\,ab\,c^4x}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^2,x)
```

output
```
(150*sqrt(x)*sqrt(b + c*x)*a*b**3*c**2 + 1180*sqrt(x)*sqrt(b + c*x)*a*b**2
*c**3*x + 1360*sqrt(x)*sqrt(b + c*x)*a*b*c**4*x**2 + 480*sqrt(x)*sqrt(b +
c*x)*a*c**5*x**3 - 45*sqrt(x)*sqrt(b + c*x)*b**5*c + 30*sqrt(x)*sqrt(b + c
*x)*b**4*c**2*x + 744*sqrt(x)*sqrt(b + c*x)*b**3*c**3*x**2 + 1008*sqrt(x)*
sqrt(b + c*x)*b**2*c**4*x**3 + 384*sqrt(x)*sqrt(b + c*x)*b*c**5*x**4 - 150
*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*b**4*c + 45*sqrt
(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**6)/(1920*c**3)
```

3.132 $\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^3} dx$

Optimal result

Integrand size = 22, antiderivative size = 153

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^3} dx = -\frac{11b^2(bB-8Ac)\sqrt{bx+cx^2}}{64c} - \frac{13}{96}b(bB-8Ac)x\sqrt{bx+cx^2} - \frac{1}{24}c(bB-8Ac)x^2\sqrt{bx+cx^2} + \frac{B(bx+cx^2)^{7/2}}{4cx^3} - \frac{5b^3(bB-8Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{3/2}}$$

output
```
-11/64*b^2*(-8*A*c+B*b)*(c*x^2+b*x)^(1/2)/c-13/96*b*(-8*A*c+B*b)*x*(c*x^2+
b*x)^(1/2)-1/24*c*(-8*A*c+B*b)*x^2*(c*x^2+b*x)^(1/2)+1/4*B*(c*x^2+b*x)^(7/
2)/c/x^3-5/64*b^3*(-8*A*c+B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(3/2
)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^3}\,dx = \frac{\sqrt{x(b+cx)}\left(\sqrt{c}(15b^3B+16c^3x^2(4A+3Bx)+8bc^2x(26A+17Bx)+2b^2\right.}{192c^{3/2}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^3,x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[c]*(15*b^3*B + 16*c^3*x^2*(4*A + 3*B*x) + 8*b*c^2
*x*(26*A + 17*B*x) + 2*b^2*c*(132*A + 59*B*x)) + (15*b^3*(b*B - 8*A*c)*Log
[-(Sqrt[c]*Sqrt[x]) + Sqrt[b + c*x]])/(Sqrt[x]*Sqrt[b + c*x])))/(192*c^(3/
2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1220, 1131, 1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^3}dx$$

$$\Big\downarrow \; 1220$$

$$\frac{(bB-8Ac)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^2}dx}{b}+\frac{2A\left(bx+cx^2\right)^{7/2}}{bx^3}$$

$$\Big\downarrow \; 1131$$

$$\frac{(bB-8Ac)\left(\frac{5}{8}b\int \frac{\left(cx^2+bx\right)^{3/2}}{x}dx+\frac{\left(bx+cx^2\right)^{5/2}}{4x}\right)}{b}+\frac{2A\left(bx+cx^2\right)^{7/2}}{bx^3}$$

$$\Big\downarrow \; 1131$$

$$\frac{(bB-8Ac)\left(\frac{5}{8}b\left(\frac{1}{2}b\int\sqrt{cx^2+bx}dx+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)+\frac{\left(bx+cx^2\right)^{5/2}}{4x}\right)}{b}+\frac{2A\left(bx+cx^2\right)^{7/2}}{bx^3}$$

$$\Big\downarrow \text{1087}$$

$$\frac{(bB-8Ac)\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)+\frac{\left(bx+cx^2\right)^{5/2}}{4x}\right)}{b}+\frac{2A\left(bx+cx^2\right)^{7/2}}{bx^3}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(bB-8Ac)\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)+\frac{\left(bx+cx^2\right)^{5/2}}{4x}\right)}{b}+\frac{2A\left(bx+cx^2\right)^{7/2}}{bx^3}$$

$$\Big\downarrow \text{219}$$

$$\frac{(bB-8Ac)\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)+\frac{\left(bx+cx^2\right)^{5/2}}{4x}\right)}{b}+\frac{2A\left(bx+cx^2\right)^{7/2}}{bx^3}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^3,x]
```

output
```
(2*A*(b*x + c*x^2)^(7/2))/(b*x^3) + ((b*B - 8*A*c)*((b*x + c*x^2)^(5/2)/(4
*x) + (5*b*((b*x + c*x^2)^(3/2)/3 + (b*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/(4
*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))/2))/8))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.63

method	result
pseudoelliptic	$\frac{\frac{5\left(Ac-\frac{Bb}{8}\right)b^3\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{8}+\frac{13\sqrt{x(cx+b)}\left(\frac{33b^2\left(\frac{59Bx}{132}+A\right)c^{\frac{3}{2}}}{26}+bx\left(\frac{17Bx}{26}+A\right)c^{\frac{5}{2}}+\frac{4x^2\left(\frac{3Bx}{4}+A\right)c^{\frac{7}{2}}}{13}+\frac{15B\sqrt{c}\,b^3}{208}\right)}{12}}{c^{\frac{3}{2}}}$
risch	$\frac{(48Bc^3x^3+64Ac^3x^2+136Bbc^2x^2+208Abc^2x+118Bb^2cx+264Ab^2c+15Bb^3)x(cx+b)}{192c\sqrt{x(cx+b)}}+\frac{5b^3(8Ac-Bb)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\right.}}{128c^{\frac{3}{2}}}$
default	$A\left(\frac{2(cx^2+bx)^{\frac{7}{2}}}{bx^3}-\frac{8c\left(\frac{2(cx^2+bx)^{\frac{7}{2}}}{3bx^2}-\frac{10c\left(\frac{(cx^2+bx)^{\frac{5}{2}}}{5}+\frac{b\left(\frac{(2cx+b)(cx^2+bx)^{\frac{3}{2}}}{8c}-\frac{3b^2\left(\frac{(2cx+b)\sqrt{cx^2+bx}}{4c}-\frac{b^2\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\right.}{8c^{\frac{3}{2}}}\right.}{16c}\right.}{2}\right.}{3b}\right.}{b}\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
13/12/c^(3/2)*(15/26*(A*c-1/8*B*b)*b^3*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2)
)+(x*(c*x+b))^(1/2)*(33/26*b^2*(59/132*B*x+A)*c^(3/2)+b*x*(17/26*B*x+A)*c^
(5/2)+4/13*x^2*(3/4*B*x+A)*c^(7/2)+15/208*B*c^(1/2)*b^3))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.66

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^3}\,dx = \left[-\frac{15\left(Bb^4-8\,Ab^3c\right)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\left(48\,Bc^4x^3+\right.}{}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
[-1/384*(15*(B*b^4 - 8*A*b^3*c)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x
)*sqrt(c)) - 2*(48*B*c^4*x^3 + 15*B*b^3*c + 264*A*b^2*c^2 + 8*(17*B*b*c^3
+ 8*A*c^4)*x^2 + 2*(59*B*b^2*c^2 + 104*A*b*c^3)*x)*sqrt(c*x^2 + b*x))/c^2,
 1/192*(15*(B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/
(c*x + b)) + (48*B*c^4*x^3 + 15*B*b^3*c + 264*A*b^2*c^2 + 8*(17*B*b*c^3 +
8*A*c^4)*x^2 + 2*(59*B*b^2*c^2 + 104*A*b*c^3)*x)*sqrt(c*x^2 + b*x))/c^2]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^3}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{x^3}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**3,x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.22

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^3}\,dx = \frac{5}{32}\sqrt{cx^2+bx}Bb^2x - \frac{5\,Bb^4\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{128\,c^{\frac{3}{2}}} + \frac{5\,Ab^3\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{16\,\sqrt{c}} + \frac{5}{24}\left(cx^2+bx\right)^{\frac{3}{2}}Bb + \frac{5}{8}\sqrt{cx^2+bx}Ab^2 + \frac{5\,\sqrt{cx^2+bx}Bb^3}{64\,c} + \frac{\left(cx^2+bx\right)^{\frac{5}{2}}B}{4\,x} + \frac{5\left(cx^2+bx\right)^{\frac{3}{2}}Ab}{12\,x} + \frac{\left(cx^2+bx\right)^{\frac{5}{2}}A}{3\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
5/32*sqrt(c*x^2 + b*x)*B*b^2*x - 5/128*B*b^4*log(2*c*x + b + 2*sqrt(c*x^2
+ b*x)*sqrt(c))/c^(3/2) + 5/16*A*b^3*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*s
qrt(c))/sqrt(c) + 5/24*(c*x^2 + b*x)^(3/2)*B*b + 5/8*sqrt(c*x^2 + b*x)*A*b
^2 + 5/64*sqrt(c*x^2 + b*x)*B*b^3/c + 1/4*(c*x^2 + b*x)^(5/2)*B/x + 5/12*(
c*x^2 + b*x)^(3/2)*A*b/x + 1/3*(c*x^2 + b*x)^(5/2)*A/x^2
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.91

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^3}\,dx = \frac{1}{192}\sqrt{cx^2+bx}\left(2\left(4\left(6\,Bc^2x+\frac{17\,Bbc^4+8\,Ac^5}{c^3}\right)x+\frac{59\,Bb^2c^3+104\,Ab}{c^3}\right.\right.$$

$$+\frac{5\left(Bb^4-8\,Ab^3c\right)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^3,x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x)*(2*(4*(6*B*c^2*x + (17*B*b*c^4 + 8*A*c^5)/c^3)*x +
 (59*B*b^2*c^3 + 104*A*b*c^4)/c^3)*x + 3*(5*B*b^3*c^2 + 88*A*b^2*c^3)/c^3)
 + 5/128*(B*b^4 - 8*A*b^3*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqr
t(c) + b))/c^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^3}\,dx=\int \frac{\left(c\,x^2+b\,x\right)^{5/2}\,\left(A+B\,x\right)}{x^3}\,dx$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^3,x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^3}\,dx=\frac{264\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^2+208\sqrt{x}\,\sqrt{cx+b}\,ab\,c^3x+64\sqrt{x}\,\sqrt{cx+b}\,a\,c^4x^2+1$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^3,x)
```

output
```
(264*sqrt(x)*sqrt(b + c*x)*a*b**2*c**2 + 208*sqrt(x)*sqrt(b + c*x)*a*b*c**
3*x + 64*sqrt(x)*sqrt(b + c*x)*a*c**4*x**2 + 15*sqrt(x)*sqrt(b + c*x)*b**4
*c + 118*sqrt(x)*sqrt(b + c*x)*b**3*c**2*x + 136*sqrt(x)*sqrt(b + c*x)*b**
2*c**3*x**2 + 48*sqrt(x)*sqrt(b + c*x)*b*c**4*x**3 + 120*sqrt(c)*log((sqrt
(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*b**3*c - 15*sqrt(c)*log((sqrt(b +
c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**5)/(192*c**2)
```

3.133 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^4}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 135

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^4}\,dx = \frac{25}{24}b(bB+6Ac)\sqrt{bx+cx^2} + \frac{5}{12}c(bB+6Ac)x\sqrt{bx+cx^2} - \frac{2A\left(bx+cx^2\right)^{5/2}}{x^3} + \frac{B\left(bx+cx^2\right)^{5/2}}{3x^2} + \frac{5b^2(bB+6Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8\sqrt{c}}$$

output
```
25/24*b*(6*A*c+B*b)*(c*x^2+b*x)^(1/2)+5/12*c*(6*A*c+B*b)*x*(c*x^2+b*x)^(1/
2)-2*A*(c*x^2+b*x)^(5/2)/x^3+1/3*B*(c*x^2+b*x)^(5/2)/x^2+5/8*b^2*(6*A*c+B*
b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^4}\,dx = \frac{\sqrt{x(b+cx)}\left(-6A(8b^2-9bcx-2c^2x^2)+Bx(33b^2+26bcx+8c^2x^2)+\frac{30\cdots}{\cdots}\right.}{24x}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^4,x]
```

output
```
(Sqrt[x*(b + c*x)]*(-6*A*(8*b^2 - 9*b*c*x - 2*c^2*x^2) + B*x*(33*b^2 + 26*
b*c*x + 8*c^2*x^2) + (30*b^2*(b*B + 6*A*c)*Sqrt[x]*ArcTanh[(Sqrt[c]*Sqrt[x
])/(-Sqrt[b] + Sqrt[b + c*x])])/(Sqrt[c]*Sqrt[b + c*x])))/(24*x)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1220, 1130, 1131, 1087, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^4}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(6Ac+bB)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^3}dx}{b} - \frac{2A\left(bx+cx^2\right)^{7/2}}{bx^4}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(6Ac+bB)\left(\frac{2\left(bx+cx^2\right)^{5/2}}{x^2} - 5c\int \frac{\left(cx^2+bx\right)^{3/2}}{x}dx\right)}{b} - \frac{2A\left(bx+cx^2\right)^{7/2}}{bx^4}$$

$$\Big\downarrow \text{1131}$$

$$\frac{(6Ac+bB)\left(\frac{2\left(bx+cx^2\right)^{5/2}}{x^2}-5c\left(\frac{1}{2}b\int\sqrt{cx^2+bx}dx+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)\right)}{b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{bx^4}$$

$\Big\downarrow$ 1087

$$\frac{(6Ac+bB)\left(\frac{2\left(bx+cx^2\right)^{5/2}}{x^2}-5c\left(\frac{1}{2}b\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{\sqrt{cx^2+bx}}dx}{8c}\right)+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)\right)}{b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{bx^4}$$

$\Big\downarrow$ 1091

$$\frac{(6Ac+bB)\left(\frac{2\left(bx+cx^2\right)^{5/2}}{x^2}-5c\left(\frac{1}{2}b\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c}\right)+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)\right)}{b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{bx^4}$$

$\Big\downarrow$ 219

$$\frac{(6Ac+bB)\left(\frac{2\left(bx+cx^2\right)^{5/2}}{x^2}-5c\left(\frac{1}{2}b\left(\frac{(b+2cx)\sqrt{bx+cx^2}}{4c}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{3/2}}\right)+\frac{1}{3}\left(bx+cx^2\right)^{3/2}\right)\right)}{b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{bx^4}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^4,x]
```

output
```
(-2*A*(b*x + c*x^2)^(7/2))/(b*x^4) + ((b*B + 6*A*c)*((2*(b*x + c*x^2)^(5/2
))/x^2 - 5*c*((b*x + c*x^2)^(3/2)/3 + (b*(((b + 2*c*x)*Sqrt[b*x + c*x^2])/
(4*c) - (b^2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(3/2))))/2)))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1087
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Simp[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1)))   Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] &&
GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.68

method	result
pseudoelliptic	$-\frac{2\left(-\frac{15 x b^{2}\left(A c+\frac{B b}{6}\right) \operatorname{arctanh}\left(\frac{\sqrt{x(c x+b)}}{x \sqrt{c}}\right)}{8}+\left(-\frac{9\left(\frac{13 B x}{27}+A\right) x b c^{\frac{3}{2}}}{8}-\frac{x^{2}\left(\frac{2 B x}{3}+A\right) c^{\frac{5}{2}}}{4}+b^{2} \sqrt{c}\left(-\frac{11 B x}{16}+A\right)\right) \sqrt{x(c x+b)}\right)}{\sqrt{c}\, x}$
risch	$-\frac{(c x+b)\left(-8 B c^{2} x^{3}-12 A c^{2} x^{2}-26 x^{2} B b c-54 A b c x-33 x B b^{2}+48 b^{2} A\right)}{24 \sqrt{x(c x+b)}}+\frac{5(6 A c+B b) b^{2} \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c x^{2}+b x}\right)}{16 \sqrt{c}}$
default	$A\left(-\frac{2\left(c x^{2}+b x\right)^{\frac{7}{2}}}{b x^{4}}+\frac{6 c\left(\frac{2\left(c x^{2}+b x\right)^{\frac{7}{2}}}{b x^{3}}-\frac{8 c\left(\frac{2\left(c x^{2}+b x\right)^{\frac{7}{2}}}{3 b x^{2}}-\frac{10 c\left(\frac{\left(c x^{2}+b x\right)^{\frac{5}{2}}}{5}+\frac{b\left(\frac{(2 c x+b)\left(c x^{2}+b x\right)^{\frac{3}{2}}}{8 c}-\frac{3 b^{2}\left(\frac{(2 c x+b) \sqrt{c \cdots}}{4 c} \cdots\right)}{\cdots}\right)}{2 \cdots}\right)}{3 b \cdots}\right)}{b \cdots}\right)}{b \cdots}\cdots\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-2/c^(1/2)*(-15/8*x*b^2*(A*c+1/6*B*b)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))
+(-9/8*(13/27*B*x+A)*x*b*c^(3/2)-1/4*x^2*(2/3*B*x+A)*c^(5/2)+b^2*c^(1/2)*(
-11/16*B*x+A))*(x*(c*x+b))^(1/2))/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.79

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^4}\,dx = \left[\frac{15\,(Bb^3+6\,Ab^2c)\sqrt{c}x\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,(8\,Bc^3x^3-48}{48\,cx}\right.$$

$$-\frac{15\,(Bb^3+6\,Ab^2c)\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(8\,Bc^3x^3-48\,Ab^2c+2\,(13\,Bbc^2+6\,Ac^3)x^2+3\,(11\,Bb^2}{24\,cx}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
[1/48*(15*(B*b^3 + 6*A*b^2*c)*sqrt(c)*x*log(2*c*x + b + 2*sqrt(c*x^2 + b*x
)*sqrt(c)) + 2*(8*B*c^3*x^3 - 48*A*b^2*c + 2*(13*B*b*c^2 + 6*A*c^3)*x^2 +
3*(11*B*b^2*c + 18*A*b*c^2)*x)*sqrt(c*x^2 + b*x))/(c*x), -1/24*(15*(B*b^3
+ 6*A*b^2*c)*sqrt(-c)*x*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) - (8*
B*c^3*x^3 - 48*A*b^2*c + 2*(13*B*b*c^2 + 6*A*c^3)*x^2 + 3*(11*B*b^2*c + 18
*A*b*c^2)*x)*sqrt(c*x^2 + b*x))/(c*x)]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^4}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{x^4}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**4,x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^4}\,dx = \frac{5\,Bb^3\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{16\,\sqrt{c}} + \frac{15}{8}\,Ab^2\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right) + \frac{5}{8}\,\sqrt{cx^2+bx}Bb^2 + \frac{5\,(cx^2+bx)^{\frac{3}{2}}Bb}{12\,x} - \frac{15\,\sqrt{cx^2+bx}Ab^2}{4\,x} + \frac{(cx^2+bx)^{\frac{5}{2}}B}{3\,x^2} + \frac{5\,(cx^2+bx)^{\frac{3}{2}}Ab}{4\,x^2} + \frac{(cx^2+bx)^{\frac{5}{2}}A}{2\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
5/16*B*b^3*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/sqrt(c) + 15/8*A*b
^2*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 5/8*sqrt(c*x^2 +
 b*x)*B*b^2 + 5/12*(c*x^2 + b*x)^(3/2)*B*b/x - 15/4*sqrt(c*x^2 + b*x)*A*b^
2/x + 1/3*(c*x^2 + b*x)^(5/2)*B/x^2 + 5/4*(c*x^2 + b*x)^(3/2)*A*b/x^2 + 1/
2*(c*x^2 + b*x)^(5/2)*A/x^3
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.03

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^4}\,dx = \frac{2\,Ab^3}{\sqrt{c}x-\sqrt{cx^2+bx}} + \frac{1}{24}\,\sqrt{cx^2+bx}\left(2\,\left(4\,Bc^2x+\frac{13\,Bbc^3+6\,Ac^4}{c^2}\right)x+\frac{3\,(11\,Bb^2c^2+18\,Abc^3)}{c^2}\right) - \frac{5\,(Bb^3+6\,Ab^2c)\log\left(\left|2\,\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{16\,\sqrt{c}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^4,x, algorithm="giac")
```

output
```
2*A*b^3/(sqrt(c)*x - sqrt(c*x^2 + b*x)) + 1/24*sqrt(c*x^2 + b*x)*(2*(4*B*c
^2*x + (13*B*b*c^3 + 6*A*c^4)/c^2)*x + 3*(11*B*b^2*c^2 + 18*A*b*c^3)/c^2)
- 5/16*(B*b^3 + 6*A*b^2*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(
c) + b))/sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^4}\,dx=\int \frac{\left(c\,x^2+b\,x\right)^{5/2}\,\left(A+B\,x\right)}{x^4}\,dx$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^4,x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.33

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^4}\,dx=\frac{-384\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c+432\sqrt{x}\,\sqrt{cx+b}\,ab\,c^2x+96\sqrt{x}\,\sqrt{cx+b}\,a\,c^3x^2+}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^4,x)
```

output
```
( - 384*sqrt(x)*sqrt(b + c*x)*a*b**2*c + 432*sqrt(x)*sqrt(b + c*x)*a*b*c**
2*x + 96*sqrt(x)*sqrt(b + c*x)*a*c**3*x**2 + 264*sqrt(x)*sqrt(b + c*x)*b**
3*c*x + 208*sqrt(x)*sqrt(b + c*x)*b**2*c**2*x**2 + 64*sqrt(x)*sqrt(b + c*x
)*b*c**3*x**3 + 720*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))
*a*b**2*c*x + 120*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b
**4*x - 480*sqrt(c)*a*b**2*c*x - 45*sqrt(c)*b**4*x)/(192*c*x)
```

3.134 $\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^5} dx$

Optimal result

Integrand size = 22, antiderivative size = 140

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^5} dx = \frac{5}{4}c(3bB+4Ac)\sqrt{bx+cx^2} - \frac{2(3bB+5Ac)\left(bx+cx^2\right)^{3/2}}{3x^2} + \frac{Bc\left(bx+cx^2\right)^{3/2}}{2x} - \frac{2A\left(bx+cx^2\right)^{5/2}}{3x^4} + \frac{5}{4}b\sqrt{c}(3bB+4Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)$$

output
```
5/4*c*(4*A*c+3*B*b)*(c*x^2+b*x)^(1/2)-2/3*(5*A*c+3*B*b)*(c*x^2+b*x)^(3/2)/
x^2+1/2*B*c*(c*x^2+b*x)^(3/2)/x-2/3*A*(c*x^2+b*x)^(5/2)/x^4+5/4*b*c^(1/2)*
(4*A*c+3*B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.98

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^5} dx = \frac{\sqrt{x(b+cx)}\left(\sqrt{b+cx}(-4A(2b^2+14bcx-3c^2x^2)+3Bx(-8b^2+9bcx+2\right.}{12x^2\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^5,x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[b + c*x]*(-4*A*(2*b^2 + 14*b*c*x - 3*c^2*x^2) + 3
*B*x*(-8*b^2 + 9*b*c*x + 2*c^2*x^2)) + 30*b*Sqrt[c]*(3*b*B + 4*A*c)*x^(3/2
)*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])]))/(12*x^2*Sqrt[b +
 c*x])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1220, 1125, 25, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^5}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(4Ac+3bB)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^4}dx}{3b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{3bx^5}$$

$$\Big\downarrow \text{1125}$$

$$\frac{(4Ac+3bB)\left(-\int -\frac{x^2c^3+3bxc^2+3b^2c}{\sqrt{cx^2+bx}}dx-\frac{2b^2\sqrt{bx+cx^2}}{x}\right)}{3b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{3bx^5}$$

$$\Big\downarrow \text{25}$$

$$\frac{(4Ac+3bB)\left(\int \frac{x^2c^3+3bxc^2+3b^2c}{\sqrt{cx^2+bx}}dx-\frac{2b^2\sqrt{bx+cx^2}}{x}\right)}{3b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{3bx^5}$$

$$\Big\downarrow \text{2192}$$

$$\frac{(4Ac+3bB)\left(\frac{\int \frac{3bc^2(4b+3cx)}{2\sqrt{cx^2+bx}}dx}{2c}-\frac{2b^2\sqrt{bx+cx^2}}{x}+\frac{1}{2}c^2x\sqrt{bx+cx^2}\right)}{3b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{3bx^5}$$

$$\Big\downarrow \text{27}$$

$$\frac{(4Ac+3bB)\left(\frac{3}{4}bc\int \frac{4b+3cx}{\sqrt{cx^2+bx}}dx-\frac{2b^2\sqrt{bx+cx^2}}{x}+\frac{1}{2}c^2x\sqrt{bx+cx^2}\right)}{3b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{3bx^5}$$

$$\Big\downarrow \text{1160}$$

$$\frac{(4Ac+3bB)\left(\frac{3}{4}bc\left(\frac{5}{2}b\int\frac{1}{\sqrt{cx^2+bx}}dx+3\sqrt{bx+cx^2}\right)-\frac{2b^2\sqrt{bx+cx^2}}{x}+\frac{1}{2}c^2x\sqrt{bx+cx^2}\right)}{3b}-\frac{2A(bx+cx^2)^{7/2}}{3bx^5}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(4Ac+3bB)\left(\frac{3}{4}bc\left(5b\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}+3\sqrt{bx+cx^2}\right)-\frac{2b^2\sqrt{bx+cx^2}}{x}+\frac{1}{2}c^2x\sqrt{bx+cx^2}\right)}{3b}-\frac{2A(bx+cx^2)^{7/2}}{3bx^5}$$

$$\Big\downarrow \text{219}$$

$$\frac{(4Ac+3bB)\left(\frac{3}{4}bc\left(\frac{5b\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}+3\sqrt{bx+cx^2}\right)-\frac{2b^2\sqrt{bx+cx^2}}{x}+\frac{1}{2}c^2x\sqrt{bx+cx^2}\right)}{3b}-\frac{2A(bx+cx^2)^{7/2}}{3bx^5}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^5,x]
```

output
```
(-2*A*(b*x + c*x^2)^(7/2))/(3*b*x^5) + ((3*b*B + 4*A*c)*((-2*b^2*Sqrt[b*x
+ c*x^2])/x + (c^2*x*Sqrt[b*x + c*x^2])/2 + (3*b*c*(3*Sqrt[b*x + c*x^2] +
(5*b*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c]))/4))/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.66

method	result
pseudoelliptic	$-\frac{2\left(-\frac{15 c x^{2}\left(A c+\frac{3 B b}{4}\right) b \operatorname{arctanh}\left(\frac{\sqrt{x(c x+b)}}{x \sqrt{c}}\right)}{2}+\left(7\left(-\frac{27 B x}{56}+A\right) x b c^{\frac{3}{2}}-\frac{3\left(\frac{B x}{2}+A\right) x^{2} c^{\frac{5}{2}}}{2}+b^{2} \sqrt{c}\,(3 B x+A)\right) \sqrt{x(c x+b)}\right)}{3 \sqrt{c}\, x^{2}}$
risch	$-\frac{(c x+b)\left(-6 B c^{2} x^{3}-12 A c^{2} x^{2}-27 x^{2} B b c+56 A b c x+24 x B b^{2}+8 b^{2} A\right)}{12 x \sqrt{x(c x+b)}}+\frac{5(4 A c+3 B b) \sqrt{c}\, b \ln \left(\frac{\frac{b}{2}+c x}{\sqrt{c}}+\sqrt{c x^{2}+b x}\right)}{8}$
	$4c\left(-\frac{2\left(c x^{2}+b x\right)^{\frac{7}{2}}}{b x^{4}}+\frac{6c\left(\frac{2\left(c x^{2}+b x\right)^{\frac{7}{2}}}{b x^{3}}-\frac{8c\left(\frac{2\left(c x^{2}+b x\right)^{\frac{7}{2}}}{3 b x^{2}}-\frac{10c\left(\frac{\left(c x^{2}+b x\right)^{\frac{5}{2}}}{5}+\frac{b\left(\frac{(2 c x+b)\left(c x^{2}+b x\right.}{8 c}\right.}{}\right.}{}\right.}{}\right.}{b}\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-15/2*c*x^2*(A*c+3/4*B*b)*b*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))+(7*
(-27/56*B*x+A)*x*b*c^(3/2)-3/2*(1/2*B*x+A)*x^2*c^(5/2)+b^2*c^(1/2)*(3*B*x+
A))*(x*(c*x+b))^(1/2))/c^(1/2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.61

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^5}\,dx = \left[\frac{15\,(3\,Bb^2+4\,Abc)\sqrt{c}x^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,(6\,Bc^2x^3-8}{24\,x^2}\right.$$

$$-\frac{15\,(3\,Bb^2+4\,Abc)\sqrt{-c}x^2\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(6\,Bc^2x^3-8\,Ab^2+3\,(9\,Bbc+4\,Ac^2)x^2-8\,(3\,Bb^2+7}{12\,x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^5,x, algorithm="fricas")
```

output
```
[1/24*(15*(3*B*b^2 + 4*A*b*c)*sqrt(c)*x^2*log(2*c*x + b + 2*sqrt(c*x^2 + b
*x)*sqrt(c)) + 2*(6*B*c^2*x^3 - 8*A*b^2 + 3*(9*B*b*c + 4*A*c^2)*x^2 - 8*(3
*B*b^2 + 7*A*b*c)*x)*sqrt(c*x^2 + b*x))/x^2, -1/12*(15*(3*B*b^2 + 4*A*b*c)
*sqrt(-c)*x^2*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) - (6*B*c^2*x^3
- 8*A*b^2 + 3*(9*B*b*c + 4*A*c^2)*x^2 - 8*(3*B*b^2 + 7*A*b*c)*x)*sqrt(c*x^
2 + b*x))/x^2]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^5}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{x^5}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**5,x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.36

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^5}\,dx = \frac{15}{8}Bb^2\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right) + \frac{5}{2}Abc^{\frac{3}{2}}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right) - \frac{15\,\sqrt{cx^2+bx}Bb^2}{4\,x} - \frac{35\,\sqrt{cx^2+bx}Abc}{6\,x} + \frac{5\,(cx^2+bx)^{\frac{3}{2}}Bb}{4\,x^2} - \frac{5\,\sqrt{cx^2+bx}Ab^2}{6\,x^2} + \frac{(cx^2+bx)^{\frac{5}{2}}B}{2\,x^3} - \frac{5\,(cx^2+bx)^{\frac{3}{2}}Ab}{6\,x^3} + \frac{(cx^2+bx)^{\frac{5}{2}}A}{x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^5,x, algorithm="maxima")
```

output
```
15/8*B*b^2*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + 5/2*A*b*
c^(3/2)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 15/4*sqrt(c*x^2 + b
*x)*B*b^2/x - 35/6*sqrt(c*x^2 + b*x)*A*b*c/x + 5/4*(c*x^2 + b*x)^(3/2)*B*b
/x^2 - 5/6*sqrt(c*x^2 + b*x)*A*b^2/x^2 + 1/2*(c*x^2 + b*x)^(5/2)*B/x^3 - 5
/6*(c*x^2 + b*x)^(3/2)*A*b/x^3 + (c*x^2 + b*x)^(5/2)*A/x^4
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^5}\,dx = \frac{1}{4}\left(2\,Bc^2x+\frac{9\,Bbc^2+4\,Ac^3}{c}\right)\sqrt{cx^2+bx} - \frac{5\,(3\,Bb^2c+4\,Abc^2)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{8\,\sqrt{c}} + \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2Bb^3+9\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2Ab^2c+3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)Ab^3\sqrt{c}+Ab^4\right)}{3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^5,x, algorithm="giac")
```

output
```
1/4*(2*B*c^2*x + (9*B*b*c^2 + 4*A*c^3)/c)*sqrt(c*x^2 + b*x) - 5/8*(3*B*b^2
*c + 4*A*b*c^2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/sq
rt(c) + 2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*b^3 + 9*(sqrt(c)*x - sq
rt(c*x^2 + b*x))^2*A*b^2*c + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x))*A*b^3*sqrt(
c) + A*b^4)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^5}\,dx=\int \frac{\left(c\,x^2+b\,x\right)^{5/2}\,\left(A+B\,x\right)}{x^5}\,dx$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^5,x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^5}\,dx=\frac{-64\sqrt{x}\,\sqrt{cx+b}\,a\,b^2-448\sqrt{x}\,\sqrt{cx+b}\,abcx+96\sqrt{x}\,\sqrt{cx+b}\,a\,c^2x^2-192}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^5,x)
```

output
```
( - 64*sqrt(x)*sqrt(b + c*x)*a*b**2 - 448*sqrt(x)*sqrt(b + c*x)*a*b*c*x +
96*sqrt(x)*sqrt(b + c*x)*a*c**2*x**2 - 192*sqrt(x)*sqrt(b + c*x)*b**3*x +
216*sqrt(x)*sqrt(b + c*x)*b**2*c*x**2 + 48*sqrt(x)*sqrt(b + c*x)*b*c**2*x*
*3 + 480*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*b*c*x**2
 + 360*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**3*x**2 +
80*sqrt(c)*a*b*c*x**2 + 95*sqrt(c)*b**3*x**2)/(96*x**2)
```

3.135 $\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^6}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 130

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^6}\,dx = Bc^2\sqrt{bx+cx^2} - \frac{2c(2bB+Ac)\sqrt{bx+cx^2}}{x} - \frac{2(bB+Ac)\left(bx+cx^2\right)^{3/2}}{3x^3} - \frac{2A(bx+cx^2)^{5/2}}{5x^5} + c^{3/2}(5bB+2Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)$$

output
```
B*c^2*(c*x^2+b*x)^(1/2)-2*c*(A*c+2*B*b)*(c*x^2+b*x)^(1/2)/x-2/3*(A*c+B*b)*
(c*x^2+b*x)^(3/2)/x^3-2/5*A*(c*x^2+b*x)^(5/2)/x^5+c^(3/2)*(2*A*c+5*B*b)*ar
ctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.05

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^6}\,dx = \frac{\sqrt{x(b+cx)}\left(-\sqrt{b+cx}(5Bx(2b^2+14bcx-3c^2x^2)+A(6b^2+22bcx+46\right.}{15x^3\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^6,x]
```

output
```
(Sqrt[x*(b + c*x)]*(-(Sqrt[b + c*x]*(5*B*x*(2*b^2 + 14*b*c*x - 3*c^2*x^2)
+ A*(6*b^2 + 22*b*c*x + 46*c^2*x^2))) + 30*c^(3/2)*(5*b*B + 2*A*c)*x^(5/2)
*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])]))/(15*x^3*Sqrt[b +
c*x])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.02, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1220, 1130, 1125, 25, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^6}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(2Ac+5bB)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^5}dx}{5b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{5bx^6}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(2Ac+5bB)\left(\frac{5}{3}c\int \frac{\left(cx^2+bx\right)^{3/2}}{x^3}dx-\frac{2\left(bx+cx^2\right)^{5/2}}{3x^4}\right)}{5b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{5bx^6}$$

$$\Big\downarrow \text{1125}$$

$$\frac{(2Ac+5bB)\left(\frac{5}{3}c\left(-\int -\frac{c(2b+cx)}{\sqrt{cx^2+bx}}dx-\frac{2b\sqrt{bx+cx^2}}{x}\right)-\frac{2\left(bx+cx^2\right)^{5/2}}{3x^4}\right)}{5b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{5bx^6}$$

$$\Big\downarrow \text{25}$$

$$\frac{(2Ac+5bB)\left(\frac{5}{3}c\left(\int \frac{c(2b+cx)}{\sqrt{cx^2+bx}}dx-\frac{2b\sqrt{bx+cx^2}}{x}\right)-\frac{2\left(bx+cx^2\right)^{5/2}}{3x^4}\right)}{5b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{5bx^6}$$

$$\Big\downarrow \text{27}$$

$$\frac{(2Ac+5bB)\left(\frac{5}{3}c\left(c\int \frac{2b+cx}{\sqrt{cx^2+bx}}dx-\frac{2b\sqrt{bx+cx^2}}{x}\right)-\frac{2\left(bx+cx^2\right)^{5/2}}{3x^4}\right)}{5b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{5bx^6}$$

$$\Big\downarrow \text{1160}$$

$$\frac{(2Ac+5bB)\left(\frac{5}{3}c\left(c\left(\frac{3}{2}b\int\frac{1}{\sqrt{cx^2+bx}}dx+\sqrt{bx+cx^2}\right)-\frac{2b\sqrt{bx+cx^2}}{x}\right)-\frac{2(bx+cx^2)^{5/2}}{3x^4}\right)}{5b}-\frac{2A(bx+cx^2)^{7/2}}{5bx^6}$$

$$\Big\downarrow \text{1091}$$

$$\frac{(2Ac+5bB)\left(\frac{5}{3}c\left(c\left(3b\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}+\sqrt{bx+cx^2}\right)-\frac{2b\sqrt{bx+cx^2}}{x}\right)-\frac{2(bx+cx^2)^{5/2}}{3x^4}\right)}{5b}-\frac{2A(bx+cx^2)^{7/2}}{5bx^6}$$

$$\Big\downarrow \text{219}$$

$$\frac{(2Ac+5bB)\left(\frac{5}{3}c\left(c\left(\frac{3b\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}+\sqrt{bx+cx^2}\right)-\frac{2b\sqrt{bx+cx^2}}{x}\right)-\frac{2(bx+cx^2)^{5/2}}{3x^4}\right)}{5b}-\frac{2A(bx+cx^2)^{7/2}}{5bx^6}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^6,x]
```

output
```
(-2*A*(b*x + c*x^2)^(7/2))/(5*b*x^6) + ((5*b*B + 2*A*c)*((-2*(b*x + c*x^2)
^(5/2))/(3*x^4) + (5*c*((-2*b*Sqrt[b*x + c*x^2])/x + c*(Sqrt[b*x + c*x^2]
+ (3*b*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c])))/3))/(5*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.75

method	result
pseudoelliptic	$-\frac{2\left(-5c^{2}x^{3}\left(Ac+\frac{5Bb}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)+\left(\frac{11\left(\frac{35Bx}{11}+A\right)xb\,c^{\frac{3}{2}}}{3}+\left(-\frac{5}{2}B\,x^{3}+\frac{23}{3}A\,x^{2}\right)c^{\frac{5}{2}}+b^{2}\sqrt{c}\left(\frac{5Bx}{3}+A\right)\right)\sqrt{x(cx+}}{5\sqrt{c}\,x^{3}}$
risch	$-\frac{(cx+b)\left(-15B\,c^{2}x^{3}+46A\,c^{2}x^{2}+70x^{2}Bbc+22Abcx+10xB\,b^{2}+6b^{2}A\right)}{15x^{2}\sqrt{x(cx+b)}}+\frac{(2Ac+5Bb)c^{\frac{3}{2}}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{2}$
	$10c\left(\frac{\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{5}\right.$ $8c\left(\frac{2\left(c\,x^{2}+bx\right)^{\frac{7}{2}}}{3b\,x^{2}}-\right.$ $6c\left(\frac{2\left(c\,x^{2}+bx\right)^{\frac{7}{2}}}{b\,x^{3}}-\right.$ $4c\left(-\frac{2\left(c\,x^{2}+bx\right)^{\frac{7}{2}}}{b\,x^{4}}+\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
-2/5/c^(1/2)*(-5*c^2*x^3*(A*c+5/2*B*b)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2)
)+(11/3*(35/11*B*x+A)*x*b*c^(3/2)+(-5/2*B*x^3+23/3*A*x^2)*c^(5/2)+b^2*c^(1
/2)*(5/3*B*x+A))*(x*(c*x+b))^(1/2))/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.73

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^6}\,dx = \left[\frac{15\,(5\,Bbc+2\,Ac^2)\sqrt{c}x^3\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)+2\,(15\,Bc^2x^3-}{30\,x^3}\right.$$

$$-\frac{15\,(5\,Bbc+2\,Ac^2)\sqrt{-c}x^3\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(15\,Bc^2x^3-6\,Ab^2-2\,(35\,Bbc+23\,Ac^2)x^2-2\,(5\,Bb^2}{15\,x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^6,x, algorithm="fricas")
```

output
```
[1/30*(15*(5*B*b*c + 2*A*c^2)*sqrt(c)*x^3*log(2*c*x + b + 2*sqrt(c*x^2 + b
*x)*sqrt(c)) + 2*(15*B*c^2*x^3 - 6*A*b^2 - 2*(35*B*b*c + 23*A*c^2)*x^2 - 2
*(5*B*b^2 + 11*A*b*c)*x)*sqrt(c*x^2 + b*x))/x^3, -1/15*(15*(5*B*b*c + 2*A*
c^2)*sqrt(-c)*x^3*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) - (15*B*c^2
*x^3 - 6*A*b^2 - 2*(35*B*b*c + 23*A*c^2)*x^2 - 2*(5*B*b^2 + 11*A*b*c)*x)*s
qrt(c*x^2 + b*x))/x^3]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^6}\,dx = \int \frac{\left(x(b+cx)\right)^{\frac{5}{2}}(A+Bx)}{x^6}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**6,x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. 2(112) = 224.

Time = 0.04 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.88

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^6}\,dx = \frac{5}{2}\,Bbc^{\frac{3}{2}}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right) + Ac^{\frac{5}{2}}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right) - \frac{35\,\sqrt{cx^2+bx}Bbc}{6\,x} - \frac{38\,\sqrt{cx^2+bx}Ac^2}{15\,x} - \frac{5\,\sqrt{cx^2+bx}Bb^2}{6\,x^2} - \frac{7\,\sqrt{cx^2+bx}Abc}{30\,x^2} - \frac{5\,(cx^2+bx)^{\frac{3}{2}}Bb}{6\,x^3} + \frac{3\,\sqrt{cx^2+bx}Ab^2}{10\,x^3} - \frac{(cx^2+bx)^{\frac{3}{2}}Ac}{3\,x^3} + \frac{(cx^2+bx)^{\frac{5}{2}}B}{x^4} - \frac{(cx^2+bx)^{\frac{3}{2}}Ab}{2\,x^4} - \frac{(cx^2+bx)^{\frac{5}{2}}A}{5\,x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^6,x, algorithm="maxima")
```

output
```
5/2*B*b*c^(3/2)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) + A*c^(5/2)*l
og(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 35/6*sqrt(c*x^2 + b*x)*B*b*c
/x - 38/15*sqrt(c*x^2 + b*x)*A*c^2/x - 5/6*sqrt(c*x^2 + b*x)*B*b^2/x^2 - 7
/30*sqrt(c*x^2 + b*x)*A*b*c/x^2 - 5/6*(c*x^2 + b*x)^(3/2)*B*b/x^3 + 3/10*s
qrt(c*x^2 + b*x)*A*b^2/x^3 - 1/3*(c*x^2 + b*x)^(3/2)*A*c/x^3 + (c*x^2 + b*
x)^(5/2)*B/x^4 - 1/2*(c*x^2 + b*x)^(3/2)*A*b/x^4 - 1/5*(c*x^2 + b*x)^(5/2)
*A/x^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. 2(112) = 224.

Time = 0.20 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.25

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^6}\,dx = \sqrt{cx^2+bx}Bc^2 - \frac{(5\,Bbc^2+2\,Ac^3)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{2\sqrt{c}} + \frac{2\left(45\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4Bb^2c+45\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4Abc^2+15\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3Bb^3\sqrt{c}+45\left(\sqrt{}\right.\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^6,x, algorithm="giac")
```

output
```
sqrt(c*x^2 + b*x)*B*c^2 - 1/2*(5*B*b*c^2 + 2*A*c^3)*log(abs(2*(sqrt(c)*x -
 sqrt(c*x^2 + b*x))*sqrt(c) + b))/sqrt(c) + 2/15*(45*(sqrt(c)*x - sqrt(c*x
^2 + b*x))^4*B*b^2*c + 45*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*A*b*c^2 + 15*(
sqrt(c)*x - sqrt(c*x^2 + b*x))^3*B*b^3*sqrt(c) + 45*(sqrt(c)*x - sqrt(c*x^
2 + b*x))^3*A*b^2*c^(3/2) + 5*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*b^4 + 35
*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*A*b^3*c + 15*(sqrt(c)*x - sqrt(c*x^2 +
b*x))*A*b^4*sqrt(c) + 3*A*b^5)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^6}\,dx = \int \frac{(c\,x^2+b\,x)^{5/2}\,(A+B\,x)}{x^6}\,dx$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^6,x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.38

$$\int \frac{(A+Bx)\,(bx+cx^2)^{5/2}}{x^6}\,dx = \frac{-24\sqrt{x}\,\sqrt{cx+b}\,a\,b^2-88\sqrt{x}\,\sqrt{cx+b}\,abcx-184\sqrt{x}\,\sqrt{cx+b}\,a\,c^2x^2-40\sqrt{}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^6,x)
```

output
```
( - 24*sqrt(x)*sqrt(b + c*x)*a*b**2 - 88*sqrt(x)*sqrt(b + c*x)*a*b*c*x - 1
84*sqrt(x)*sqrt(b + c*x)*a*c**2*x**2 - 40*sqrt(x)*sqrt(b + c*x)*b**3*x - 2
80*sqrt(x)*sqrt(b + c*x)*b**2*c*x**2 + 60*sqrt(x)*sqrt(b + c*x)*b*c**2*x**
3 + 120*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*c**2*x**3
 + 300*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**2*c*x**3
+ 40*sqrt(c)*a*c**2*x**3 + 163*sqrt(c)*b**2*c*x**3)/(60*x**3)
```

3.136 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^7}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^7}\,dx = -\frac{2b^2B\sqrt{bx+cx^2}}{5x^3}-\frac{22bBc\sqrt{bx+cx^2}}{15x^2}-\frac{46Bc^2\sqrt{bx+cx^2}}{15x}-\frac{2A(bx+cx^2)^{7/2}}{7bx^7}+2Bc^{5/2}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)$$

output
```
-2/5*b^2*B*(c*x^2+b*x)^(1/2)/x^3-22/15*b*B*c*(c*x^2+b*x)^(1/2)/x^2-46/15*B
*c^2*(c*x^2+b*x)^(1/2)/x-2/7*A*(c*x^2+b*x)^(7/2)/b/x^7+2*B*c^(5/2)*arctanh
(c^(1/2)*x/(c*x^2+b*x)^(1/2))
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^7}\,dx = -\frac{2\sqrt{x(b+cx)}\left(\sqrt{b+cx}(15A(b+cx)^3+7bBx(3b^2+11bcx+23c^2x^2))+105bBc^{5/2}x^{7/2}\log\left(-\sqrt{c}\sqrt{x}+\sqrt{\right.}\right.}{105bx^4\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^7,x]
```

output
```
(-2*Sqrt[x*(b + c*x)]*(Sqrt[b + c*x]*(15*A*(b + c*x)^3 + 7*b*B*x*(3*b^2 +
11*b*c*x + 23*c^2*x^2)) + 105*b*B*c^(5/2)*x^(7/2)*Log[-(Sqrt[c]*Sqrt[x]) +
 Sqrt[b + c*x]]))/(105*b*x^4*Sqrt[b + c*x])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1220, 1130, 1130, 1125, 25, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^7}dx$$

$$\Big\downarrow \text{1220}$$

$$B\int \frac{\left(cx^2+bx\right)^{5/2}}{x^6}dx-\frac{2A\left(bx+cx^2\right)^{7/2}}{7bx^7}$$

$$\Big\downarrow \text{1130}$$

$$B\left(c\int \frac{\left(cx^2+bx\right)^{3/2}}{x^4}dx-\frac{2\left(bx+cx^2\right)^{5/2}}{5x^5}\right)-\frac{2A\left(bx+cx^2\right)^{7/2}}{7bx^7}$$

$$\Big\downarrow \text{1130}$$

$$B\left(c\left(c\int \frac{\sqrt{cx^2+bx}}{x^2}dx-\frac{2\left(bx+cx^2\right)^{3/2}}{3x^3}\right)-\frac{2\left(bx+cx^2\right)^{5/2}}{5x^5}\right)-\frac{2A\left(bx+cx^2\right)^{7/2}}{7bx^7}$$

$$\Big\downarrow \text{1125}$$

$$B\left(c\left(c\left(-\int -\frac{c}{\sqrt{cx^2+bx}}dx-\frac{2\sqrt{bx+cx^2}}{x}\right)-\frac{2\left(bx+cx^2\right)^{3/2}}{3x^3}\right)-\frac{2\left(bx+cx^2\right)^{5/2}}{5x^5}\right)-\frac{2A\left(bx+cx^2\right)^{7/2}}{7bx^7}$$

$$\Big\downarrow \text{25}$$

$$B\left(c\left(c\left(\int \frac{c}{\sqrt{cx^2+bx}}dx-\frac{2\sqrt{bx+cx^2}}{x}\right)-\frac{2\left(bx+cx^2\right)^{3/2}}{3x^3}\right)-\frac{2\left(bx+cx^2\right)^{5/2}}{5x^5}\right)-\frac{2A\left(bx+cx^2\right)^{7/2}}{7bx^7}$$

$\Big\downarrow$ 27

$$B\left(c\left(c\left(c\int \frac{1}{\sqrt{cx^2+bx}}dx-\frac{2\sqrt{bx+cx^2}}{x}\right)-\frac{2\left(bx+cx^2\right)^{3/2}}{3x^3}\right)-\frac{2\left(bx+cx^2\right)^{5/2}}{5x^5}\right)-\frac{2A\left(bx+cx^2\right)^{7/2}}{7bx^7}$$

$\Big\downarrow$ 1091

$$B\left(c\left(c\left(2c\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}-\frac{2\sqrt{bx+cx^2}}{x}\right)-\frac{2\left(bx+cx^2\right)^{3/2}}{3x^3}\right)-\frac{2\left(bx+cx^2\right)^{5/2}}{5x^5}\right)-\frac{2A\left(bx+cx^2\right)^{7/2}}{7bx^7}$$

$\Big\downarrow$ 219

$$B\left(c\left(c\left(2\sqrt{c}\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)-\frac{2\sqrt{bx+cx^2}}{x}\right)-\frac{2\left(bx+cx^2\right)^{3/2}}{3x^3}\right)-\frac{2\left(bx+cx^2\right)^{5/2}}{5x^5}\right)-\frac{2A\left(bx+cx^2\right)^{7/2}}{7bx^7}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^7,x]
```

output
```
(-2*A*(b*x + c*x^2)^(7/2))/(7*b*x^7) + B*((-2*(b*x + c*x^2)^(5/2))/(5*x^5)
 + c*((-2*(b*x + c*x^2)^(3/2))/(3*x^3) + c*((-2*Sqrt[b*x + c*x^2])/x + 2*S
qrt[c]*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1125
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_
Symbol] :> Simp[-2*e^(2*m + 3)*(Sqrt[a + b*x + c*x^2]/((-2*c*d + b*e)^(m +
2)*(d + e*x))), x] - Simp[e^(2*m + 2)   Int[(1/Sqrt[a + b*x + c*x^2])*Expan
dToSum[((-2*c*d + b*e)^(-m - 1) - ((-c)*d + b*e + c*e*x)^(-m - 1))/(d + e*x
), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && ILtQ[m, 0] && EqQ[m + p, -3/2]
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.76

method	result
pseudoelliptic	$\frac{2Bb\,c^{\frac{5}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)x^4-\frac{2\sqrt{x(cx+b)}\left(\left(\frac{7Bx}{5}+A\right)b^3+3cx\left(\frac{77Bx}{45}+A\right)b^2+3c^2\left(\frac{161Bx}{45}+A\right)x^2b+A\,c^3x^3\right)}{7}}{b\,x^4}$
risch	$-\frac{2(cx+b)(15A\,c^3x^3+161x^3Bb\,c^2+45Ab\,c^2x^2+77x^2B\,b^2c+45A\,b^2cx+21xB\,b^3+15A\,b^3)}{105x^3\sqrt{x(cx+b)}\,b}+B\,c^{\frac{5}{2}}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,\ldots}\right.$
	$\ldots\ 8c\left(\frac{2\left(c\,x^2+b\ldots\right.}{3b\,x^2}\ldots\right.$ $\ldots\ 6c\left(\frac{2\left(c\,x^2+bx\right)^{\frac{7}{2}}}{b\,x^3}-\ldots\right.$ $\ldots\ 4c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{7}{2}}}{b\,x^4}+\ldots\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
2/7*(7*B*b*c^(5/2)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))*x^4-(x*(c*x+b))^(1
/2)*((7/5*B*x+A)*b^3+3*c*x*(77/45*B*x+A)*b^2+3*c^2*(161/45*B*x+A)*x^2*b+A*
c^3*x^3))/b/x^4
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.91

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^7}\,dx = \left[\frac{105\,Bbc^{\frac{5}{2}}x^4\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(15\,Ab^3+(161\,Bbc^2+1}{105\,b}\right.$$

$$-\frac{2\left(105\,Bb\sqrt{-c}c^2x^4\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)+(15\,Ab^3+(161\,Bbc^2+15\,Ac^3)x^3+(77\,Bb^2c+45\,Abc^2)x^2+\right.}{105\,bx^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^7,x, algorithm="fricas")
```

output
```
[1/105*(105*B*b*c^(5/2)*x^4*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) -
 2*(15*A*b^3 + (161*B*b*c^2 + 15*A*c^3)*x^3 + (77*B*b^2*c + 45*A*b*c^2)*x^
2 + 3*(7*B*b^3 + 15*A*b^2*c)*x)*sqrt(c*x^2 + b*x))/(b*x^4), -2/105*(105*B*
b*sqrt(-c)*c^2*x^4*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (15*A*b^
3 + (161*B*b*c^2 + 15*A*c^3)*x^3 + (77*B*b^2*c + 45*A*b*c^2)*x^2 + 3*(7*B*
b^3 + 15*A*b^2*c)*x)*sqrt(c*x^2 + b*x))/(b*x^4)]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^7}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{x^7}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**7,x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**7, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(103) = 206$.

Time = 0.04 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.06

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^7}\,dx = Bc^{\frac{5}{2}}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right) - \frac{38\,\sqrt{cx^2+bx}Bc^2}{15\,x} - \frac{2\,\sqrt{cx^2+bx}Ac^3}{7\,bx} - \frac{7\,\sqrt{cx^2+bx}Bbc}{30\,x^2} + \frac{\sqrt{cx^2+bx}Ac^2}{7\,x^2} + \frac{3\,\sqrt{cx^2+bx}Bb^2}{10\,x^3} - \frac{(cx^2+bx)^{\frac{3}{2}}Bc}{3\,x^3} - \frac{3\,\sqrt{cx^2+bx}Abc}{28\,x^3} - \frac{(cx^2+bx)^{\frac{3}{2}}Bb}{2\,x^4} - \frac{15\,\sqrt{cx^2+bx}Ab^2}{28\,x^4} - \frac{(cx^2+bx)^{\frac{5}{2}}B}{5\,x^5} + \frac{5\,(cx^2+bx)^{\frac{3}{2}}Ab}{4\,x^5} - \frac{(cx^2+bx)^{\frac{5}{2}}A}{x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^7,x, algorithm="maxima")
```

output
```
B*c^(5/2)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 38/15*sqrt(c*x^2
+ b*x)*B*c^2/x - 2/7*sqrt(c*x^2 + b*x)*A*c^3/(b*x) - 7/30*sqrt(c*x^2 + b*x
)*B*b*c/x^2 + 1/7*sqrt(c*x^2 + b*x)*A*c^2/x^2 + 3/10*sqrt(c*x^2 + b*x)*B*b
^2/x^3 - 1/3*(c*x^2 + b*x)^(3/2)*B*c/x^3 - 3/28*sqrt(c*x^2 + b*x)*A*b*c/x^
3 - 1/2*(c*x^2 + b*x)^(3/2)*B*b/x^4 - 15/28*sqrt(c*x^2 + b*x)*A*b^2/x^4 -
1/5*(c*x^2 + b*x)^(5/2)*B/x^5 + 5/4*(c*x^2 + b*x)^(3/2)*A*b/x^5 - (c*x^2 +
 b*x)^(5/2)*A/x^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(103) = 206$.

Time = 0.15 (sec) , antiderivative size = 379, normalized size of antiderivative = 3.03

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^7}\,dx = -Bc^{\frac{5}{2}}\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right) + \frac{2\left(315\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^6Bbc^2+105\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^6Ac^3+315\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5Bb^2c^{\frac{3}{2}}+315\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^7,x, algorithm="giac")
```

output
```
-B*c^(5/2)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b)) + 2/105
*(315*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*B*b*c^2 + 105*(sqrt(c)*x - sqrt(c*
x^2 + b*x))^6*A*c^3 + 315*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*B*b^2*c^(3/2)
+ 315*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*A*b*c^(5/2) + 245*(sqrt(c)*x - sqr
t(c*x^2 + b*x))^4*B*b^3*c + 525*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*A*b^2*c^
2 + 105*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*B*b^4*sqrt(c) + 525*(sqrt(c)*x -
 sqrt(c*x^2 + b*x))^3*A*b^3*c^(3/2) + 21*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2
*B*b^5 + 315*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*A*b^4*c + 105*(sqrt(c)*x -
sqrt(c*x^2 + b*x))*A*b^5*sqrt(c) + 15*A*b^6)/(sqrt(c)*x - sqrt(c*x^2 + b*x
))^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^7}\,dx=\int \frac{{\left(c\,x^2+b\,x\right)}^{5/2}\,\left(A+B\,x\right)}{x^7}\,dx$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^7,x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.43

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^7}\,dx=\frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^3}{7}-\frac{6\sqrt{x}\,\sqrt{cx+b}\,a\,b^2cx}{7}-\frac{6\sqrt{x}\,\sqrt{cx+b}\,ab\,c^2x^2}{7}-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,c^3x^3}{7}-2\sqrt{x}\,\sqrt{}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^7,x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(b + c*x)*a*b**3 - 45*sqrt(x)*sqrt(b + c*x)*a*b**2*c
*x - 45*sqrt(x)*sqrt(b + c*x)*a*b*c**2*x**2 - 15*sqrt(x)*sqrt(b + c*x)*a*c
**3*x**3 - 21*sqrt(x)*sqrt(b + c*x)*b**4*x - 77*sqrt(x)*sqrt(b + c*x)*b**3
*c*x**2 - 161*sqrt(x)*sqrt(b + c*x)*b**2*c**2*x**3 + 105*sqrt(c)*log((sqrt
(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**2*c**2*x**4 - 15*sqrt(c)*a*c**3*x
**4 + 71*sqrt(c)*b**2*c**2*x**4))/(105*b*x**4)
```

3.137 $\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^8} dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^8} dx = -\frac{2A(bx+cx^2)^{7/2}}{9bx^8} - \frac{2(9bB-2Ac)\left(bx+cx^2\right)^{7/2}}{63b^2x^7}$$

output
```
-2/9*A*(c*x^2+b*x)^(7/2)/b/x^8-2/63*(-2*A*c+9*B*b)*(c*x^2+b*x)^(7/2)/b^2/x
^7
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^8} dx = -\frac{2(x(b+cx))^{7/2}(7Ab+9bBx-2Acx)}{63b^2x^8}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^8,x]
```

output
```
(-2*(x*(b + c*x))^(7/2)*(7*A*b + 9*b*B*x - 2*A*c*x))/(63*b^2*x^8)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1220, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^8}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(9bB-2Ac)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^7}dx}{9b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{9bx^8}$$

$$\Big\downarrow \text{ 1123}$$

$$-\frac{2\left(bx+cx^2\right)^{7/2}(9bB-2Ac)}{63b^2x^7}-\frac{2A\left(bx+cx^2\right)^{7/2}}{9bx^8}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^8,x]
```

output
```
(-2*A*(b*x + c*x^2)^(7/2))/(9*b*x^8) - (2*(9*b*B - 2*A*c)*(b*x + c*x^2)^(7
/2))/(63*b^2*x^7)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.68

method	result	size
pseudoelliptic	$-\frac{2(cx+b)^{3}\sqrt{x(cx+b)}\left(\left(\frac{9Bx}{7}+A\right)b-\frac{2Acx}{7}\right)}{9x^{5}b^{2}}$	39
gosper	$-\frac{2(cx+b)(-2Acx+9Bbx+7Ab)\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{63b^{2}x^{7}}$	40
orering	$-\frac{2(cx+b)(-2Acx+9Bbx+7Ab)\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{63b^{2}x^{7}}$	40
default	$A\left(-\frac{2\left(c\,x^{2}+bx\right)^{\frac{7}{2}}}{9b\,x^{8}}+\frac{4c\left(c\,x^{2}+bx\right)^{\frac{7}{2}}}{63b^{2}x^{7}}\right)-\frac{2B\left(c\,x^{2}+bx\right)^{\frac{7}{2}}}{7b\,x^{7}}$	64
trager	$-\frac{2\left(-2A\,c^{4}x^{4}+9Bb\,c^{3}x^{4}+Ab\,c^{3}x^{3}+27B\,b^{2}c^{2}x^{3}+15A\,b^{2}c^{2}x^{2}+27B\,b^{3}c\,x^{2}+19A\,b^{3}cx+9B\,b^{4}x+7A\,b^{4}\right)\sqrt{c\,x^{2}+bx}}{63b^{2}x^{5}}$	104
risch	$-\frac{2(cx+b)\left(-2A\,c^{4}x^{4}+9Bb\,c^{3}x^{4}+Ab\,c^{3}x^{3}+27B\,b^{2}c^{2}x^{3}+15A\,b^{2}c^{2}x^{2}+27B\,b^{3}c\,x^{2}+19A\,b^{3}cx+9B\,b^{4}x+7A\,b^{4}\right)}{63x^{4}\sqrt{x(cx+b)}\,b^{2}}$	107

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-2/9*(c*x+b)^3*(x*(c*x+b))^(1/2)*((9/7*B*x+A)*b-2/7*A*c*x)/x^5/b^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(49) = 98$.

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.79

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^8}\,dx =$$
$$-\frac{2\,(7\,Ab^4+(9\,Bbc^3-2\,Ac^4)x^4+(27\,Bb^2c^2+Abc^3)x^3+3\,(9\,Bb^3c+5\,Ab^2c^2)x^2+(9\,Bb^4+19\,Ab^3c)x)\sqrt{c}}{63\,b^2x^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^8,x, algorithm="fricas")
```

output
```
-2/63*(7*A*b^4 + (9*B*b*c^3 - 2*A*c^4)*x^4 + (27*B*b^2*c^2 + A*b*c^3)*x^3
+ 3*(9*B*b^3*c + 5*A*b^2*c^2)*x^2 + (9*B*b^4 + 19*A*b^3*c)*x)*sqrt(c*x^2 +
 b*x)/(b^2*x^5)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^8}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{x^8}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**8,x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**8, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(49) = 98$.

Time = 0.04 (sec) , antiderivative size = 258, normalized size of antiderivative = 4.53

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^8}\,dx = -\frac{2\sqrt{cx^2+bx}\,Bc^3}{7\,bx}+\frac{4\sqrt{cx^2+bx}\,Ac^4}{63\,b^2x}+\frac{\sqrt{cx^2+bx}\,Bc^2}{7\,x^2}-\frac{2\sqrt{cx^2+bx}\,Ac^3}{63\,bx^2}-\frac{3\sqrt{cx^2+bx}\,Bbc}{28\,x^3}+\frac{\sqrt{cx^2+bx}\,Ac^2}{42\,x^3}-\frac{15\sqrt{cx^2+bx}\,Bb^2}{28\,x^4}-\frac{5\sqrt{cx^2+bx}\,Abc}{252\,x^4}+\frac{5\left(cx^2+bx\right)^{\frac{3}{2}}Bb}{4\,x^5}-\frac{5\sqrt{cx^2+bx}\,Ab^2}{36\,x^5}-\frac{\left(cx^2+bx\right)^{\frac{5}{2}}B}{x^6}+\frac{5\left(cx^2+bx\right)^{\frac{3}{2}}Ab}{12\,x^6}-\frac{\left(cx^2+bx\right)^{\frac{5}{2}}A}{2\,x^7}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^8,x, algorithm="maxima")
```

output
```
-2/7*sqrt(c*x^2 + b*x)*B*c^3/(b*x) + 4/63*sqrt(c*x^2 + b*x)*A*c^4/(b^2*x)
+ 1/7*sqrt(c*x^2 + b*x)*B*c^2/x^2 - 2/63*sqrt(c*x^2 + b*x)*A*c^3/(b*x^2) -
 3/28*sqrt(c*x^2 + b*x)*B*b*c/x^3 + 1/42*sqrt(c*x^2 + b*x)*A*c^2/x^3 - 15/
28*sqrt(c*x^2 + b*x)*B*b^2/x^4 - 5/252*sqrt(c*x^2 + b*x)*A*b*c/x^4 + 5/4*(
c*x^2 + b*x)^(3/2)*B*b/x^5 - 5/36*sqrt(c*x^2 + b*x)*A*b^2/x^5 - (c*x^2 + b
*x)^(5/2)*B/x^6 + 5/12*(c*x^2 + b*x)^(3/2)*A*b/x^6 - 1/2*(c*x^2 + b*x)^(5/
2)*A/x^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 431 vs. 2(49) = 98.

Time = 0.13 (sec) , antiderivative size = 431, normalized size of antiderivative = 7.56

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^8}\,dx = \frac{2\left(63\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^8Bc^3+189\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^7Bbc^{\frac{5}{2}}+63\left(\sqrt{c}\right.\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^8,x, algorithm="giac")
```

output
```
2/63*(63*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*B*c^3 + 189*(sqrt(c)*x - sqrt(c
*x^2 + b*x))^7*B*b*c^(5/2) + 63*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*A*c^(7/2
) + 315*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*B*b^2*c^2 + 273*(sqrt(c)*x - sqr
t(c*x^2 + b*x))^6*A*b*c^3 + 315*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*B*b^3*c^
(3/2) + 567*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*A*b^2*c^(5/2) + 189*(sqrt(c)
*x - sqrt(c*x^2 + b*x))^4*B*b^4*c + 693*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*
A*b^3*c^2 + 63*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*B*b^5*sqrt(c) + 525*(sqrt
(c)*x - sqrt(c*x^2 + b*x))^3*A*b^4*c^(3/2) + 9*(sqrt(c)*x - sqrt(c*x^2 + b
*x))^2*B*b^6 + 243*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*A*b^5*c + 63*(sqrt(c)
*x - sqrt(c*x^2 + b*x))*A*b^6*sqrt(c) + 7*A*b^7)/(sqrt(c)*x - sqrt(c*x^2 +
 b*x))^9
```

Mupad [B] (verification not implemented)

Time = 6.86 (sec) , antiderivative size = 188, normalized size of antiderivative = 3.30

$$\int \frac{(A+Bx)\,(bx+cx^2)^{5/2}}{x^8}\,dx = \frac{4\,A\,c^4\,\sqrt{c\,x^2+b\,x}}{63\,b^2\,x} - \frac{10\,A\,c^2\,\sqrt{c\,x^2+b\,x}}{21\,x^3} - \frac{2\,B\,b^2\,\sqrt{c\,x^2+b\,x}}{7\,x^4} - \frac{6\,B\,c^2\,\sqrt{c\,x^2+b\,x}}{7\,x^2} - \frac{2\,A\,c^3\,\sqrt{c\,x^2+b\,x}}{63\,b\,x^2} - \frac{2\,A\,b^2\,\sqrt{c\,x^2+b\,x}}{9\,x^5} - \frac{2\,B\,c^3\,\sqrt{c\,x^2+b\,x}}{7\,b\,x} - \frac{38\,A\,b\,c\,\sqrt{c\,x^2+b\,x}}{63\,x^4} - \frac{6\,B\,b\,c\,\sqrt{c\,x^2+b\,x}}{7\,x^3}$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^8,x)
```

output
```
(4*A*c^4*(b*x + c*x^2)^(1/2))/(63*b^2*x) - (10*A*c^2*(b*x + c*x^2)^(1/2))/
(21*x^3) - (2*B*b^2*(b*x + c*x^2)^(1/2))/(7*x^4) - (6*B*c^2*(b*x + c*x^2)^
(1/2))/(7*x^2) - (2*A*c^3*(b*x + c*x^2)^(1/2))/(63*b*x^2) - (2*A*b^2*(b*x
+ c*x^2)^(1/2))/(9*x^5) - (2*B*c^3*(b*x + c*x^2)^(1/2))/(7*b*x) - (38*A*b*
c*(b*x + c*x^2)^(1/2))/(63*x^4) - (6*B*b*c*(b*x + c*x^2)^(1/2))/(7*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 3.28

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^8}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^4}{9}-\frac{38\sqrt{x}\,\sqrt{cx+b}\,a\,b^3cx}{63}-\frac{10\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^2x^2}{21}-\frac{2\sqrt{x}\,\sqrt{cx+b}\,ab\,c^3x^3}{63}+\frac{4\sqrt{}}{}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^8,x)
```

output
```
(2*( - 7*sqrt(x)*sqrt(b + c*x)*a*b**4 - 19*sqrt(x)*sqrt(b + c*x)*a*b**3*c*
x - 15*sqrt(x)*sqrt(b + c*x)*a*b**2*c**2*x**2 - sqrt(x)*sqrt(b + c*x)*a*b*
c**3*x**3 + 2*sqrt(x)*sqrt(b + c*x)*a*c**4*x**4 - 9*sqrt(x)*sqrt(b + c*x)*
b**5*x - 27*sqrt(x)*sqrt(b + c*x)*b**4*c*x**2 - 27*sqrt(x)*sqrt(b + c*x)*b
**3*c**2*x**3 - 9*sqrt(x)*sqrt(b + c*x)*b**2*c**3*x**4 - 2*sqrt(c)*a*c**4*
x**5 - 5*sqrt(c)*b**2*c**3*x**5))/(63*b**2*x**5)
```

3.138 $\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^9} dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^9} dx = -\frac{2A(bx+cx^2)^{7/2}}{11bx^9} - \frac{2(11bB-4Ac)(bx+cx^2)^{7/2}}{99b^2x^8} + \frac{4c(11bB-4Ac)(bx+cx^2)^{7/2}}{693b^3x^7}$$

output
```
-2/11*A*(c*x^2+b*x)^(7/2)/b/x^9-2/99*(-4*A*c+11*B*b)*(c*x^2+b*x)^(7/2)/b^2
/x^8+4/693*c*(-4*A*c+11*B*b)*(c*x^2+b*x)^(7/2)/b^3/x^7
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.70

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^9} dx = -\frac{2(b+cx)(x(b+cx))^{5/2}(63Ab^2+77b^2Bx-28Abcx-22bBcx^2+8Ac^2x^2)}{693b^3x^8}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^9,x]
```

output
```
(-2*(b + c*x)*(x*(b + c*x))^(5/2)*(63*A*b^2 + 77*b^2*B*x - 28*A*b*c*x - 22
*b*B*c*x^2 + 8*A*c^2*x^2))/(693*b^3*x^8)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^9}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(11bB-4Ac)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^8}dx}{11b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{11bx^9}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(11bB-4Ac)\left(-\frac{2c\int \frac{\left(cx^2+bx\right)^{5/2}}{x^7}dx}{9b}-\frac{2\left(bx+cx^2\right)^{7/2}}{9bx^8}\right)}{11b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{11bx^9}$$

$$\Big\downarrow \text{ 1123}$$

$$\frac{\left(\frac{4c\left(bx+cx^2\right)^{7/2}}{63b^2x^7}-\frac{2\left(bx+cx^2\right)^{7/2}}{9bx^8}\right)(11bB-4Ac)}{11b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{11bx^9}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^9,x]
```

output
```
(-2*A*(b*x + c*x^2)^(7/2))/(11*b*x^9) + ((11*b*B - 4*A*c)*((-2*(b*x + c*x^
2)^(7/2))/(9*b*x^8) + (4*c*(b*x + c*x^2)^(7/2))/(63*b^2*x^7)))/(11*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

method	result
pseudoelliptic	$-\frac{2(cx+b)^3\sqrt{x(cx+b)}\left(\left(\frac{11Bx}{9}+A\right)b^2-\frac{4cx\left(\frac{11Bx}{14}+A\right)b}{9}+\frac{8Ac^2x^2}{63}\right)}{11x^6b^3}$
gosper	$-\frac{2(cx+b)(8Ac^2x^2-22x^2Bbc-28Abcx+77xBb^2+63b^2A)(cx^2+bx)^{\frac{5}{2}}}{693x^8b^3}$
orering	$-\frac{2(cx+b)(8Ac^2x^2-22x^2Bbc-28Abcx+77xBb^2+63b^2A)(cx^2+bx)^{\frac{5}{2}}}{693x^8b^3}$
default	$A\left(-\frac{2(cx^2+bx)^{\frac{7}{2}}}{11bx^9}-\frac{4c\left(-\frac{2(cx^2+bx)^{\frac{7}{2}}}{9bx^8}+\frac{4c(cx^2+bx)^{\frac{7}{2}}}{63b^2x^7}\right)}{11b}\right)+B\left(-\frac{2(cx^2+bx)^{\frac{7}{2}}}{9bx^8}+\frac{4c(cx^2+bx)^{\frac{7}{2}}}{63b^2x^7}\right)$
trager	$-\frac{2(8Ac^5x^5-22Bbc^4x^5-4Abc^4x^4+11Bb^2c^3x^4+3Ab^2c^3x^3+165Bb^3c^2x^3+113Ab^3c^2x^2+209Bb^4cx^2+161Ab^4cx+77B}{693b^3x^6}$
risch	$-\frac{2(cx+b)(8Ac^5x^5-22Bbc^4x^5-4Abc^4x^4+11Bb^2c^3x^4+3Ab^2c^3x^3+165Bb^3c^2x^3+113Ab^3c^2x^2+209Bb^4cx^2+161Ab^4c}{693x^5\sqrt{x(cx+b)}\,b^3}$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-2/11*(c*x+b)^3*(x*(c*x+b))^(1/2)*((11/9*B*x+A)*b^2-4/9*c*x*(11/14*B*x+A)*
b+8/63*A*c^2*x^2)/x^6/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.41

$$\int\frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^9}\,dx=$$

$$-\frac{2\,(63\,Ab^5-2\,(11\,Bbc^4-4\,Ac^5)x^5+(11\,Bb^2c^3-4\,Abc^4)x^4+3\,(55\,Bb^3c^2+Ab^2c^3)x^3+(209\,Bb^4c+113}{693\,b^3x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^9,x, algorithm="fricas")
```

output
```
-2/693*(63*A*b^5 - 2*(11*B*b*c^4 - 4*A*c^5)*x^5 + (11*B*b^2*c^3 - 4*A*b*c^
4)*x^4 + 3*(55*B*b^3*c^2 + A*b^2*c^3)*x^3 + (209*B*b^4*c + 113*A*b^3*c^2)*
x^2 + 7*(11*B*b^5 + 23*A*b^4*c)*x)*sqrt(c*x^2 + b*x)/(b^3*x^6)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^9}\,dx=\int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{x^9}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**9,x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**9, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(78) = 156.

Time = 0.04 (sec) , antiderivative size = 304, normalized size of antiderivative = 3.38

$$\begin{aligned}\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^9}\,dx=&\frac{4\,\sqrt{cx^2+bx}Bc^4}{63\,b^2x}-\frac{16\,\sqrt{cx^2+bx}Ac^5}{693\,b^3x}\\&-\frac{2\,\sqrt{cx^2+bx}Bc^3}{63\,bx^2}+\frac{8\,\sqrt{cx^2+bx}Ac^4}{693\,b^2x^2}+\frac{\sqrt{cx^2+bx}Bc^2}{42\,x^3}\\&-\frac{2\,\sqrt{cx^2+bx}Ac^3}{231\,bx^3}-\frac{5\,\sqrt{cx^2+bx}Bbc}{252\,x^4}+\frac{5\,\sqrt{cx^2+bx}Ac^2}{693\,x^4}\\&-\frac{5\,\sqrt{cx^2+bx}Bb^2}{36\,x^5}-\frac{5\,\sqrt{cx^2+bx}Abc}{792\,x^5}+\frac{5\,(cx^2+bx)^{\frac{3}{2}}Bb}{12\,x^6}\\&-\frac{5\,\sqrt{cx^2+bx}Ab^2}{88\,x^6}-\frac{(cx^2+bx)^{\frac{5}{2}}B}{2\,x^7}+\frac{5\,(cx^2+bx)^{\frac{3}{2}}Ab}{24\,x^7}-\frac{(cx^2+bx)^{\frac{5}{2}}A}{3\,x^8}\end{aligned}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^9,x, algorithm="maxima")
```

output
```
4/63*sqrt(c*x^2 + b*x)*B*c^4/(b^2*x) - 16/693*sqrt(c*x^2 + b*x)*A*c^5/(b^3
*x) - 2/63*sqrt(c*x^2 + b*x)*B*c^3/(b*x^2) + 8/693*sqrt(c*x^2 + b*x)*A*c^4
/(b^2*x^2) + 1/42*sqrt(c*x^2 + b*x)*B*c^2/x^3 - 2/231*sqrt(c*x^2 + b*x)*A*
c^3/(b*x^3) - 5/252*sqrt(c*x^2 + b*x)*B*b*c/x^4 + 5/693*sqrt(c*x^2 + b*x)*
A*c^2/x^4 - 5/36*sqrt(c*x^2 + b*x)*B*b^2/x^5 - 5/792*sqrt(c*x^2 + b*x)*A*b
*c/x^5 + 5/12*(c*x^2 + b*x)^(3/2)*B*b/x^6 - 5/88*sqrt(c*x^2 + b*x)*A*b^2/x
^6 - 1/2*(c*x^2 + b*x)^(5/2)*B/x^7 + 5/24*(c*x^2 + b*x)^(3/2)*A*b/x^7 - 1/
3*(c*x^2 + b*x)^(5/2)*A/x^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(78) = 156$.

Time = 0.14 (sec) , antiderivative size = 491, normalized size of antiderivative = 5.46

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^9}\,dx = \frac{2\left(693\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^9Bc^{\frac{7}{2}}+3003\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^8Bbc^3+924\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^9,x, algorithm="giac")
```

output
```
2/693*(693*(sqrt(c)*x - sqrt(c*x^2 + b*x))^9*B*c^(7/2) + 3003*(sqrt(c)*x -
 sqrt(c*x^2 + b*x))^8*B*b*c^3 + 924*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*A*c^
4 + 6237*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*B*b^2*c^(5/2) + 4851*(sqrt(c)*x
 - sqrt(c*x^2 + b*x))^7*A*b*c^(7/2) + 7623*(sqrt(c)*x - sqrt(c*x^2 + b*x))
^6*B*b^3*c^2 + 11781*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*A*b^2*c^3 + 5775*(s
qrt(c)*x - sqrt(c*x^2 + b*x))^5*B*b^4*c^(3/2) + 16863*(sqrt(c)*x - sqrt(c*
x^2 + b*x))^5*A*b^3*c^(5/2) + 2673*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*B*b^5
*c + 15345*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*A*b^4*c^2 + 693*(sqrt(c)*x -
sqrt(c*x^2 + b*x))^3*B*b^6*sqrt(c) + 9009*(sqrt(c)*x - sqrt(c*x^2 + b*x))^
3*A*b^5*c^(3/2) + 77*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*b^7 + 3311*(sqrt(
c)*x - sqrt(c*x^2 + b*x))^2*A*b^6*c + 693*(sqrt(c)*x - sqrt(c*x^2 + b*x))*
A*b^7*sqrt(c) + 63*A*b^8)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^11
```

Mupad [B] (verification not implemented)

Time = 7.20 (sec) , antiderivative size = 234, normalized size of antiderivative = 2.60

$$\begin{aligned}\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^9}\,dx &= \frac{8\,A\,c^4\,\sqrt{c\,x^2+b\,x}}{693\,b^2\,x^2}-\frac{226\,A\,c^2\,\sqrt{c\,x^2+b\,x}}{693\,x^4}\\&-\frac{2\,B\,b^2\,\sqrt{c\,x^2+b\,x}}{9\,x^5}-\frac{10\,B\,c^2\,\sqrt{c\,x^2+b\,x}}{21\,x^3}-\frac{2\,A\,c^3\,\sqrt{c\,x^2+b\,x}}{231\,b\,x^3}\\&-\frac{2\,A\,b^2\,\sqrt{c\,x^2+b\,x}}{11\,x^6}-\frac{16\,A\,c^5\,\sqrt{c\,x^2+b\,x}}{693\,b^3\,x}-\frac{2\,B\,c^3\,\sqrt{c\,x^2+b\,x}}{63\,b\,x^2}\\&+\frac{4\,B\,c^4\,\sqrt{c\,x^2+b\,x}}{63\,b^2\,x}-\frac{46\,A\,b\,c\,\sqrt{c\,x^2+b\,x}}{99\,x^5}-\frac{38\,B\,b\,c\,\sqrt{c\,x^2+b\,x}}{63\,x^4}\end{aligned}$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^9,x)
```

output
```
(8*A*c^4*(b*x + c*x^2)^(1/2))/(693*b^2*x^2) - (226*A*c^2*(b*x + c*x^2)^(1/
2))/(693*x^4) - (2*B*b^2*(b*x + c*x^2)^(1/2))/(9*x^5) - (10*B*c^2*(b*x + c
*x^2)^(1/2))/(21*x^3) - (2*A*c^3*(b*x + c*x^2)^(1/2))/(231*b*x^3) - (2*A*b
^2*(b*x + c*x^2)^(1/2))/(11*x^6) - (16*A*c^5*(b*x + c*x^2)^(1/2))/(693*b^3
*x) - (2*B*c^3*(b*x + c*x^2)^(1/2))/(63*b*x^2) + (4*B*c^4*(b*x + c*x^2)^(1
/2))/(63*b^2*x) - (46*A*b*c*(b*x + c*x^2)^(1/2))/(99*x^5) - (38*B*b*c*(b*x
 + c*x^2)^(1/2))/(63*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.51

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^9}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^5}{11} - \frac{46\sqrt{x}\,\sqrt{cx+b}\,a\,b^4cx}{99} - \frac{226\sqrt{x}\,\sqrt{cx+b}\,a\,b^3c^2x^2}{693} - \frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^3x^3}{231} +$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^9,x)
```

output
```
(2*( - 63*sqrt(x)*sqrt(b + c*x)*a*b**5 - 161*sqrt(x)*sqrt(b + c*x)*a*b**4*
c*x - 113*sqrt(x)*sqrt(b + c*x)*a*b**3*c**2*x**2 - 3*sqrt(x)*sqrt(b + c*x)
*a*b**2*c**3*x**3 + 4*sqrt(x)*sqrt(b + c*x)*a*b*c**4*x**4 - 8*sqrt(x)*sqrt
(b + c*x)*a*c**5*x**5 - 77*sqrt(x)*sqrt(b + c*x)*b**6*x - 209*sqrt(x)*sqrt
(b + c*x)*b**5*c*x**2 - 165*sqrt(x)*sqrt(b + c*x)*b**4*c**2*x**3 - 11*sqrt
(x)*sqrt(b + c*x)*b**3*c**3*x**4 + 22*sqrt(x)*sqrt(b + c*x)*b**2*c**4*x**5
 + 8*sqrt(c)*a*c**5*x**6 - 22*sqrt(c)*b**2*c**4*x**6))/(693*b**3*x**6)
```

3.139 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{10}}\,dx=-\frac{2A\left(bx+cx^2\right)^{7/2}}{13bx^{10}}-\frac{2(13bB-6Ac)\left(bx+cx^2\right)^{7/2}}{143b^2x^9}+\frac{8c(13bB-6Ac)\left(bx+cx^2\right)^{7/2}}{1287b^3x^8}-\frac{16c^2(13bB-6Ac)\left(bx+cx^2\right)^{7/2}}{9009b^4x^7}$$

output
```
-2/13*A*(c*x^2+b*x)^(7/2)/b/x^10-2/143*(-6*A*c+13*B*b)*(c*x^2+b*x)^(7/2)/b
^2/x^9+8/1287*c*(-6*A*c+13*B*b)*(c*x^2+b*x)^(7/2)/b^3/x^8-16/9009*c^2*(-6*
A*c+13*B*b)*(c*x^2+b*x)^(7/2)/b^4/x^7
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{10}}\,dx=-\frac{2(b+cx)(x(b+cx))^{5/2}\left(693Ab^3+819b^3Bx-378Ab^2cx-364b^2Bcx^2+168Abc^2x^2+104bBc^2x^3-48A\right.}{9009b^4x^9}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^10,x]
```

output
```
(-2*(b + c*x)*(x*(b + c*x))^(5/2)*(693*A*b^3 + 819*b^3*B*x - 378*A*b^2*c*x
 - 364*b^2*B*c*x^2 + 168*A*b*c^2*x^2 + 104*b*B*c^2*x^3 - 48*A*c^3*x^3))/(9
009*b^4*x^9)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{10}}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(13bB-6Ac)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^9}dx}{13b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{13bx^{10}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(13bB-6Ac)\left(-\frac{4c\int \frac{\left(cx^2+bx\right)^{5/2}}{x^8}dx}{11b}-\frac{2\left(bx+cx^2\right)^{7/2}}{11bx^9}\right)}{13b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{13bx^{10}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(13bB-6Ac)\left(-\frac{4c\left(-\frac{2c\int \frac{\left(cx^2+bx\right)^{5/2}}{x^7}dx}{9b}-\frac{2\left(bx+cx^2\right)^{7/2}}{9bx^8}\right)}{11b}-\frac{2\left(bx+cx^2\right)^{7/2}}{11bx^9}\right)}{13b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{13bx^{10}}$$

$$\Big\downarrow \text{1123}$$

$$\frac{\left(-\frac{4c\left(\frac{4c\left(bx+cx^2\right)^{7/2}}{63b^2x^7}-\frac{2\left(bx+cx^2\right)^{7/2}}{9bx^8}\right)}{11b}-\frac{2\left(bx+cx^2\right)^{7/2}}{11bx^9}\right)(13bB-6Ac)}{13b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{13bx^{10}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^10,x]
```

output
```
(-2*A*(b*x + c*x^2)^(7/2))/(13*b*x^10) + ((13*b*B - 6*A*c)*((-2*(b*x + c*x
^2)^(7/2))/(11*b*x^9) - (4*c*((-2*(b*x + c*x^2)^(7/2))/(9*b*x^8) + (4*c*(b
*x + c*x^2)^(7/2))/(63*b^2*x^7)))/(11*b)))/(13*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$-\frac{2(cx+b)^3\left(\left(\frac{13Bx}{11}+A\right)b^3-\frac{6cx\left(\frac{26Bx}{27}+A\right)b^2}{11}+\frac{8c^2x^2\left(\frac{13Bx}{21}+A\right)b}{33}-\frac{16A\,c^3x^3}{231}\right)\sqrt{x(cx+b)}}{13x^7b^4}$
gosper	$-\frac{2(cx+b)\left(-48A\,c^3x^3+104x^3Bb\,c^2+168Ab\,c^2x^2-364x^2B\,b^2c-378A\,b^2cx+819xB\,b^3+693A\,b^3\right)\left(c\,x^2+bx\right)^{\frac{5}{2}}}{9009x^9b^4}$
orering	$-\frac{2(cx+b)\left(-48A\,c^3x^3+104x^3Bb\,c^2+168Ab\,c^2x^2-364x^2B\,b^2c-378A\,b^2cx+819xB\,b^3+693A\,b^3\right)\left(c\,x^2+bx\right)^{\frac{5}{2}}}{9009x^9b^4}$
trager	$-\frac{2\left(-48A\,c^6x^6+104Bb\,c^5x^6+24Ab\,c^5x^5-52B\,b^2c^4x^5-18A\,b^2c^4x^4+39B\,b^3c^3x^4+15A\,b^3c^3x^3+1469B\,b^4c^2x^3+1113A\,b^4c\cdots\right.}{9009b^4x^7}$
risch	$-\frac{2(cx+b)\left(-48A\,c^6x^6+104Bb\,c^5x^6+24Ab\,c^5x^5-52B\,b^2c^4x^5-18A\,b^2c^4x^4+39B\,b^3c^3x^4+15A\,b^3c^3x^3+1469B\,b^4c^2x^3+111\cdots\right.}{9009x^6\sqrt{x(cx+b)}\,b^4}$
default	$A\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{7}{2}}}{13b\,x^{10}}-\frac{6c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{7}{2}}}{11b\,x^9}-\frac{4c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{7}{2}}}{9b\,x^8}+\frac{4c\left(c\,x^2+bx\right)^{\frac{7}{2}}}{63b^2x^7}\right)}{11b}\right)}{13b}\right)+B\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{7}{2}}}{11b\,x^9}-\frac{4c\cdots}{\cdots}\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-2/13*(c*x+b)^3*((13/11*B*x+A)*b^3-6/11*c*x*(26/27*B*x+A)*b^2+8/33*c^2*x^2
*(13/21*B*x+A)*b-16/231*A*c^3*x^3)*(x*(c*x+b))^(1/2)/x^7/b^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.22

$$\int\frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{10}}\,dx=$$
$$-\frac{2\left(693\,Ab^6+8\left(13\,Bbc^5-6\,Ac^6\right)x^6-4\left(13\,Bb^2c^4-6\,Abc^5\right)x^5+3\left(13\,Bb^3c^3-6\,Ab^2c^4\right)x^4+\left(1469\,Bb^4c^2\cdots\right.\right.}{9009\,b^4x^7}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^10,x, algorithm="fricas")
```

output
```
-2/9009*(693*A*b^6 + 8*(13*B*b*c^5 - 6*A*c^6)*x^6 - 4*(13*B*b^2*c^4 - 6*A*
b*c^5)*x^5 + 3*(13*B*b^3*c^3 - 6*A*b^2*c^4)*x^4 + (1469*B*b^4*c^2 + 15*A*b
^3*c^3)*x^3 + 7*(299*B*b^5*c + 159*A*b^4*c^2)*x^2 + 63*(13*B*b^6 + 27*A*b^
5*c)*x)*sqrt(c*x^2 + b*x)/(b^4*x^7)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{10}}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{x^{10}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**10,x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**10, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(109) = 218$.

Time = 0.04 (sec) , antiderivative size = 350, normalized size of antiderivative = 2.80

$$\begin{aligned}\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{10}}\,dx = &-\frac{16\sqrt{cx^2+bx}Bc^5}{693\,b^3x}+\frac{32\sqrt{cx^2+bx}Ac^6}{3003\,b^4x}\\&+\frac{8\sqrt{cx^2+bx}Bc^4}{693\,b^2x^2}-\frac{16\sqrt{cx^2+bx}Ac^5}{3003\,b^3x^2}-\frac{2\sqrt{cx^2+bx}Bc^3}{231\,bx^3}\\&+\frac{4\sqrt{cx^2+bx}Ac^4}{1001\,b^2x^3}+\frac{5\sqrt{cx^2+bx}Bc^2}{693\,x^4}-\frac{10\sqrt{cx^2+bx}Ac^3}{3003\,bx^4}-\frac{5\sqrt{cx^2+bx}Bbc}{792\,x^5}\\&+\frac{5\sqrt{cx^2+bx}Ac^2}{1716\,x^5}-\frac{5\sqrt{cx^2+bx}Bb^2}{88\,x^6}-\frac{3\sqrt{cx^2+bx}Abc}{1144\,x^6}+\frac{5\left(cx^2+bx\right)^{\frac{3}{2}}Bb}{24\,x^7}\\&-\frac{3\sqrt{cx^2+bx}Ab^2}{104\,x^7}-\frac{\left(cx^2+bx\right)^{\frac{5}{2}}B}{3\,x^8}+\frac{\left(cx^2+bx\right)^{\frac{3}{2}}Ab}{8\,x^8}-\frac{\left(cx^2+bx\right)^{\frac{5}{2}}A}{4\,x^9}\end{aligned}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^10,x, algorithm="maxima")
```

output
```
-16/693*sqrt(c*x^2 + b*x)*B*c^5/(b^3*x) + 32/3003*sqrt(c*x^2 + b*x)*A*c^6/
(b^4*x) + 8/693*sqrt(c*x^2 + b*x)*B*c^4/(b^2*x^2) - 16/3003*sqrt(c*x^2 + b
*x)*A*c^5/(b^3*x^2) - 2/231*sqrt(c*x^2 + b*x)*B*c^3/(b*x^3) + 4/1001*sqrt(
c*x^2 + b*x)*A*c^4/(b^2*x^3) + 5/693*sqrt(c*x^2 + b*x)*B*c^2/x^4 - 10/3003
*sqrt(c*x^2 + b*x)*A*c^3/(b*x^4) - 5/792*sqrt(c*x^2 + b*x)*B*b*c/x^5 + 5/1
716*sqrt(c*x^2 + b*x)*A*c^2/x^5 - 5/88*sqrt(c*x^2 + b*x)*B*b^2/x^6 - 3/114
4*sqrt(c*x^2 + b*x)*A*b*c/x^6 + 5/24*(c*x^2 + b*x)^(3/2)*B*b/x^7 - 3/104*s
qrt(c*x^2 + b*x)*A*b^2/x^7 - 1/3*(c*x^2 + b*x)^(5/2)*B/x^8 + 1/8*(c*x^2 +
b*x)^(3/2)*A*b/x^8 - 1/4*(c*x^2 + b*x)^(5/2)*A/x^9
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 551 vs. $2(109) = 218$.

Time = 0.13 (sec) , antiderivative size = 551, normalized size of antiderivative = 4.41

$$\int \frac{(A+Bx)\,(bx+cx^2)^{5/2}}{x^{10}}\,dx = \frac{2\left(12012\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^{10}Bc^4+63063\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^{9}Bbc^{\frac{7}{2}}+\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^10,x, algorithm="giac")
```

output
```
2/9009*(12012*(sqrt(c)*x - sqrt(c*x^2 + b*x))^10*B*c^4 + 63063*(sqrt(c)*x
- sqrt(c*x^2 + b*x))^9*B*b*c^(7/2) + 18018*(sqrt(c)*x - sqrt(c*x^2 + b*x))
^9*A*c^(9/2) + 153153*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*B*b^2*c^3 + 108108
*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*A*b*c^4 + 219219*(sqrt(c)*x - sqrt(c*x^
2 + b*x))^7*B*b^3*c^(5/2) + 297297*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*A*b^2
*c^(7/2) + 199485*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*B*b^4*c^2 + 485199*(sq
rt(c)*x - sqrt(c*x^2 + b*x))^6*A*b^3*c^3 + 117117*(sqrt(c)*x - sqrt(c*x^2
+ b*x))^5*B*b^5*c^(3/2) + 513513*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*A*b^4*c
^(5/2) + 43043*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*B*b^6*c + 363363*(sqrt(c)
*x - sqrt(c*x^2 + b*x))^4*A*b^5*c^2 + 9009*(sqrt(c)*x - sqrt(c*x^2 + b*x))
^3*B*b^7*sqrt(c) + 171171*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*A*b^6*c^(3/2)
+ 819*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*b^8 + 51597*(sqrt(c)*x - sqrt(c*
x^2 + b*x))^2*A*b^7*c + 9009*(sqrt(c)*x - sqrt(c*x^2 + b*x))*A*b^8*sqrt(c)
 + 693*A*b^9)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^13
```

Mupad [B] (verification not implemented)

Time = 7.45 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.24

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{10}}\,dx = \frac{4\,A\,c^4\,\sqrt{c\,x^2+b\,x}}{1001\,b^2\,x^3} - \frac{106\,A\,c^2\,\sqrt{c\,x^2+b\,x}}{429\,x^5} - \frac{2\,B\,b^2\,\sqrt{c\,x^2+b\,x}}{11\,x^6} - \frac{226\,B\,c^2\,\sqrt{c\,x^2+b\,x}}{693\,x^4} - \frac{10\,A\,c^3\,\sqrt{c\,x^2+b\,x}}{3003\,b\,x^4} - \frac{2\,A\,b^2\,\sqrt{c\,x^2+b\,x}}{13\,x^7} - \frac{16\,A\,c^5\,\sqrt{c\,x^2+b\,x}}{3003\,b^3\,x^2} + \frac{32\,A\,c^6\,\sqrt{c\,x^2+b\,x}}{3003\,b^4\,x} - \frac{2\,B\,c^3\,\sqrt{c\,x^2+b\,x}}{231\,b\,x^3} + \frac{8\,B\,c^4\,\sqrt{c\,x^2+b\,x}}{693\,b^2\,x^2} - \frac{16\,B\,c^5\,\sqrt{c\,x^2+b\,x}}{693\,b^3\,x} - \frac{54\,A\,b\,c\,\sqrt{c\,x^2+b\,x}}{143\,x^6} - \frac{46\,B\,b\,c\,\sqrt{c\,x^2+b\,x}}{99\,x^5}$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^10,x)
```

output
```
(4*A*c^4*(b*x + c*x^2)^(1/2))/(1001*b^2*x^3) - (106*A*c^2*(b*x + c*x^2)^(1
/2))/(429*x^5) - (2*B*b^2*(b*x + c*x^2)^(1/2))/(11*x^6) - (226*B*c^2*(b*x
+ c*x^2)^(1/2))/(693*x^4) - (10*A*c^3*(b*x + c*x^2)^(1/2))/(3003*b*x^4) -
(2*A*b^2*(b*x + c*x^2)^(1/2))/(13*x^7) - (16*A*c^5*(b*x + c*x^2)^(1/2))/(3
003*b^3*x^2) + (32*A*c^6*(b*x + c*x^2)^(1/2))/(3003*b^4*x) - (2*B*c^3*(b*x
 + c*x^2)^(1/2))/(231*b*x^3) + (8*B*c^4*(b*x + c*x^2)^(1/2))/(693*b^2*x^2)
 - (16*B*c^5*(b*x + c*x^2)^(1/2))/(693*b^3*x) - (54*A*b*c*(b*x + c*x^2)^(1
/2))/(143*x^6) - (46*B*b*c*(b*x + c*x^2)^(1/2))/(99*x^5)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.12

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{10}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^6}{13} - \frac{54\sqrt{x}\,\sqrt{cx+b}\,a\,b^5cx}{143} - \frac{106\sqrt{x}\,\sqrt{cx+b}\,a\,b^4c^2x^2}{429} - \frac{10\sqrt{x}\,\sqrt{cx+b}\,a\,b^3c^3x^3}{3003} + \cdots}{\cdots}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^10,x)
```

output
```
(2*( - 693*sqrt(x)*sqrt(b + c*x)*a*b**6 - 1701*sqrt(x)*sqrt(b + c*x)*a*b**
5*c*x - 1113*sqrt(x)*sqrt(b + c*x)*a*b**4*c**2*x**2 - 15*sqrt(x)*sqrt(b +
c*x)*a*b**3*c**3*x**3 + 18*sqrt(x)*sqrt(b + c*x)*a*b**2*c**4*x**4 - 24*sqr
t(x)*sqrt(b + c*x)*a*b*c**5*x**5 + 48*sqrt(x)*sqrt(b + c*x)*a*c**6*x**6 -
819*sqrt(x)*sqrt(b + c*x)*b**7*x - 2093*sqrt(x)*sqrt(b + c*x)*b**6*c*x**2
- 1469*sqrt(x)*sqrt(b + c*x)*b**5*c**2*x**3 - 39*sqrt(x)*sqrt(b + c*x)*b**
4*c**3*x**4 + 52*sqrt(x)*sqrt(b + c*x)*b**3*c**4*x**5 - 104*sqrt(x)*sqrt(b
 + c*x)*b**2*c**5*x**6 - 48*sqrt(c)*a*c**6*x**7 + 104*sqrt(c)*b**2*c**5*x*
*7))/(9009*b**4*x**7)
```

3.140 $\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^{11}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11}}\,dx = -\frac{2A\left(bx+cx^2\right)^{7/2}}{15bx^{11}} - \frac{2(15bB-8Ac)\left(bx+cx^2\right)^{7/2}}{195b^2x^{10}} + \frac{4c(15bB-8Ac)\left(bx+cx^2\right)^{7/2}}{715b^3x^9} - \frac{16c^2(15bB-8Ac)\left(bx+cx^2\right)^{7/2}}{6435b^4x^8} + \frac{32c^3(15bB-8Ac)\left(bx+cx^2\right)^{7/2}}{45045b^5x^7}$$

output
```
-2/15*A*(c*x^2+b*x)^(7/2)/b/x^11-2/195*(-8*A*c+15*B*b)*(c*x^2+b*x)^(7/2)/b
^2/x^10+4/715*c*(-8*A*c+15*B*b)*(c*x^2+b*x)^(7/2)/b^3/x^9-16/6435*c^2*(-8*
A*c+15*B*b)*(c*x^2+b*x)^(7/2)/b^4/x^8+32/45045*c^3*(-8*A*c+15*B*b)*(c*x^2+
b*x)^(7/2)/b^5/x^7
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.67

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11}}\,dx = \frac{2(b+cx)^3\sqrt{x(b+cx)}(15bBx(-231b^3+126b^2cx-56bc^2x^2+16c^3x^3)+A}{45045b^5x^8}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^11,x]
```

output
```
(2*(b + c*x)^3*Sqrt[x*(b + c*x)]*(15*b*B*x*(-231*b^3 + 126*b^2*c*x - 56*b*
c^2*x^2 + 16*c^3*x^3) + A*(-3003*b^4 + 1848*b^3*c*x - 1008*b^2*c^2*x^2 + 4
48*b*c^3*x^3 - 128*c^4*x^4)))/(45045*b^5*x^8)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(15bB-8Ac)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{10}}dx}{15b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{15bx^{11}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(15bB-8Ac)\left(-\frac{6c\int \frac{\left(cx^2+bx\right)^{5/2}}{x^9}dx}{13b}-\frac{2\left(bx+cx^2\right)^{7/2}}{13bx^{10}}\right)}{15b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{15bx^{11}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(15bB-8Ac)\left(-\frac{6c\left(-\frac{4c\int \frac{\left(cx^2+bx\right)^{5/2}}{x^8}dx}{11b}-\frac{2\left(bx+cx^2\right)^{7/2}}{11bx^9}\right)}{13b}-\frac{2\left(bx+cx^2\right)^{7/2}}{13bx^{10}}\right)}{15b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{15bx^{11}}$$

$\Big\downarrow$ 1129

$$\frac{(15bB-8Ac)\left(-\frac{6c\left(-\frac{4c\left(-\frac{2c\int \frac{\left(cx^2+bx\right)^{5/2}}{x^7}dx}{9b}-\frac{2\left(bx+cx^2\right)^{7/2}}{9bx^8}\right)}{11b}-\frac{2\left(bx+cx^2\right)^{7/2}}{11bx^9}\right)}{13b}-\frac{2\left(bx+cx^2\right)^{7/2}}{13bx^{10}}\right)}{15b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{15bx^{11}}$$

$\Big\downarrow$ 1123

$$\frac{\left(-\frac{6c\left(-\frac{4c\left(\frac{4c\left(bx+cx^2\right)^{7/2}}{63b^2x^7}-\frac{2\left(bx+cx^2\right)^{7/2}}{9bx^8}\right)}{11b}-\frac{2\left(bx+cx^2\right)^{7/2}}{11bx^9}\right)}{13b}-\frac{2\left(bx+cx^2\right)^{7/2}}{13bx^{10}}\right)(15bB-8Ac)}{15b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{15bx^{11}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^11,x]
```

output
```
(-2*A*(b*x + c*x^2)^(7/2))/(15*b*x^11) + ((15*b*B - 8*A*c)*((-2*(b*x + c*x
^2)^(7/2))/(13*b*x^10) - (6*c*((-2*(b*x + c*x^2)^(7/2))/(11*b*x^9) - (4*c*
((-2*(b*x + c*x^2)^(7/2))/(9*b*x^8) + (4*c*(b*x + c*x^2)^(7/2))/(63*b^2*x^
7)))/(11*b)))/(13*b)))/(15*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.56

method	result
pseudoelliptic	$-\frac{2(cx+b)^3\sqrt{x(cx+b)}\left(\left(\frac{15Bx}{13}+A\right)b^4-\frac{8cx\left(\frac{45Bx}{44}+A\right)b^3}{13}+\frac{48c^2x^2\left(\frac{5Bx}{6}+A\right)b^2}{143}-\frac{64c^3x^3\left(\frac{15Bx}{28}+A\right)b}{429}+\frac{128Ac^4x^4}{3003}\right)}{15x^8b^5}$
gosper	$-\frac{2(cx+b)(128Ac^4x^4-240Bbc^3x^4-448Abc^3x^3+840Bb^2c^2x^3+1008Ab^2c^2x^2-1890Bb^3cx^2-1848Ab^3cx+3465Bb^4x+}{45045x^{10}b^5}$
orering	$-\frac{2(cx+b)(128Ac^4x^4-240Bbc^3x^4-448Abc^3x^3+840Bb^2c^2x^3+1008Ab^2c^2x^2-1890Bb^3cx^2-1848Ab^3cx+3465Bb^4x+}{45045x^{10}b^5}$
trager	$-\frac{2(128Ac^7x^7-240Bbc^6x^7-64Abc^6x^6+120Bb^2c^5x^6+48Ab^2c^5x^5-90Bb^3c^4x^5-40Ab^3c^4x^4+75Bb^4c^3x^4+35Ab^4c^3x^3}{45045b^5x^8}$
risch	$-\frac{2(cx+b)(128Ac^7x^7-240Bbc^6x^7-64Abc^6x^6+120Bb^2c^5x^6+48Ab^2c^5x^5-90Bb^3c^4x^5-40Ab^3c^4x^4+75Bb^4c^3x^4+35A}{45045x^7\sqrt{x(cx+b)}\,b^5}$
default	$A\left(-\frac{2(cx^2+bx)^{\frac{7}{2}}}{15bx^{11}}-\frac{8c\left(-\frac{2(cx^2+bx)^{\frac{7}{2}}}{13bx^{10}}-\frac{6c\left(-\frac{2(cx^2+bx)^{\frac{7}{2}}}{11bx^9}-\frac{4c\left(-\frac{2(cx^2+bx)^{\frac{7}{2}}}{9bx^8}+\frac{4c(cx^2+bx)^{\frac{7}{2}}}{63b^2x^7}\right)}{11b}\right)}{13b}\right)}{15b}\right)+B\Bigg(-$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(c*x+b)^3*(x*(c*x+b))^(1/2)*((15/13*B*x+A)*b^4-8/13*c*x*(45/44*B*x+A
)*b^3+48/143*c^2*x^2*(5/6*B*x+A)*b^2-64/429*c^3*x^3*(15/28*B*x+A)*b+128/30
03*A*c^4*x^4)/x^8/b^5
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11}}\,dx =$$
$$-\frac{2\,(3003\,Ab^7-16\,(15\,Bbc^6-8\,Ac^7)x^7+8\,(15\,Bb^2c^5-8\,Abc^6)x^6-6\,(15\,Bb^3c^4-8\,Ab^2c^5)x^5+5\,(15\,Bb^4c}{45}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^11,x, algorithm="fricas")
```

output
```
-2/45045*(3003*A*b^7 - 16*(15*B*b*c^6 - 8*A*c^7)*x^7 + 8*(15*B*b^2*c^5 - 8
*A*b*c^6)*x^6 - 6*(15*B*b^3*c^4 - 8*A*b^2*c^5)*x^5 + 5*(15*B*b^4*c^3 - 8*A
*b^3*c^4)*x^4 + 35*(159*B*b^5*c^2 + A*b^4*c^3)*x^3 + 63*(135*B*b^6*c + 71*
A*b^5*c^2)*x^2 + 231*(15*B*b^7 + 31*A*b^6*c)*x)*sqrt(c*x^2 + b*x)/(b^5*x^8
)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11}}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{x^{11}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**11,x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**11, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. 2(140) = 280.

Time = 0.04 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.48

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^{11}}\,dx = \frac{32\sqrt{cx^2+bx}Bc^6}{3003\,b^4x} - \frac{256\sqrt{cx^2+bx}Ac^7}{45045\,b^5x} - \frac{16\sqrt{cx^2+bx}Bc^5}{3003\,b^3x^2} + \frac{128\sqrt{cx^2+bx}Ac^6}{45045\,b^4x^2} + \frac{4\sqrt{cx^2+bx}Bc^4}{1001\,b^2x^3} - \frac{32\sqrt{cx^2+bx}Ac^5}{15015\,b^3x^3} - \frac{10\sqrt{cx^2+bx}Bc^3}{3003\,bx^4} + \frac{16\sqrt{cx^2+bx}Ac^4}{9009\,b^2x^4} + \frac{5\sqrt{cx^2+bx}Bc^2}{1716\,x^5} - \frac{2\sqrt{cx^2+bx}Ac^3}{1287\,bx^5} - \frac{3\sqrt{cx^2+bx}Bbc}{1144\,x^6} + \frac{\sqrt{cx^2+bx}Ac^2}{715\,x^6} - \frac{3\sqrt{cx^2+bx}Bb^2}{104\,x^7} - \frac{\sqrt{cx^2+bx}Abc}{780\,x^7} + \frac{(cx^2+bx)^{\frac{3}{2}}Bb}{8\,x^8} - \frac{\sqrt{cx^2+bx}Ab^2}{60\,x^8} - \frac{(cx^2+bx)^{\frac{5}{2}}B}{4\,x^9} + \frac{(cx^2+bx)^{\frac{3}{2}}Ab}{12\,x^9} - \frac{(cx^2+bx)^{\frac{5}{2}}A}{5\,x^{10}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^11,x, algorithm="maxima")
```

output
```
32/3003*sqrt(c*x^2 + b*x)*B*c^6/(b^4*x) - 256/45045*sqrt(c*x^2 + b*x)*A*c^
7/(b^5*x) - 16/3003*sqrt(c*x^2 + b*x)*B*c^5/(b^3*x^2) + 128/45045*sqrt(c*x
^2 + b*x)*A*c^6/(b^4*x^2) + 4/1001*sqrt(c*x^2 + b*x)*B*c^4/(b^2*x^3) - 32/
15015*sqrt(c*x^2 + b*x)*A*c^5/(b^3*x^3) - 10/3003*sqrt(c*x^2 + b*x)*B*c^3/
(b*x^4) + 16/9009*sqrt(c*x^2 + b*x)*A*c^4/(b^2*x^4) + 5/1716*sqrt(c*x^2 +
b*x)*B*c^2/x^5 - 2/1287*sqrt(c*x^2 + b*x)*A*c^3/(b*x^5) - 3/1144*sqrt(c*x^
2 + b*x)*B*b*c/x^6 + 1/715*sqrt(c*x^2 + b*x)*A*c^2/x^6 - 3/104*sqrt(c*x^2
+ b*x)*B*b^2/x^7 - 1/780*sqrt(c*x^2 + b*x)*A*b*c/x^7 + 1/8*(c*x^2 + b*x)^(
3/2)*B*b/x^8 - 1/60*sqrt(c*x^2 + b*x)*A*b^2/x^8 - 1/4*(c*x^2 + b*x)^(5/2)*
B/x^9 + 1/12*(c*x^2 + b*x)^(3/2)*A*b/x^9 - 1/5*(c*x^2 + b*x)^(5/2)*A/x^10
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 611 vs. $2(140) = 280$.

Time = 0.12 (sec) , antiderivative size = 611, normalized size of antiderivative = 3.82

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^{11}}\,dx = \frac{2\left(90090\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^{11}Bc^{\frac{9}{2}} + 540540\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^{10}Bbc^4 \cdots\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^11,x, algorithm="giac")
```

output
```
2/45045*(90090*(sqrt(c)*x - sqrt(c*x^2 + b*x))^11*B*c^(9/2) + 540540*(sqrt
(c)*x - sqrt(c*x^2 + b*x))^10*B*b*c^4 + 144144*(sqrt(c)*x - sqrt(c*x^2 + b
*x))^10*A*c^5 + 1486485*(sqrt(c)*x - sqrt(c*x^2 + b*x))^9*B*b^2*c^(7/2) +
960960*(sqrt(c)*x - sqrt(c*x^2 + b*x))^9*A*b*c^(9/2) + 2425995*(sqrt(c)*x
- sqrt(c*x^2 + b*x))^8*B*b^3*c^3 + 2934360*(sqrt(c)*x - sqrt(c*x^2 + b*x))
^8*A*b^2*c^4 + 2567565*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*B*b^4*c^(5/2) + 5
360355*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*A*b^3*c^(7/2) + 1816815*(sqrt(c)*
x - sqrt(c*x^2 + b*x))^6*B*b^5*c^2 + 6451445*(sqrt(c)*x - sqrt(c*x^2 + b*x
))^6*A*b^4*c^3 + 855855*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*B*b^6*c^(3/2) +
5324319*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*A*b^5*c^(5/2) + 257985*(sqrt(c)*
x - sqrt(c*x^2 + b*x))^4*B*b^7*c + 3042585*(sqrt(c)*x - sqrt(c*x^2 + b*x))
^4*A*b^6*c^2 + 45045*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*B*b^8*sqrt(c) + 118
6185*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*A*b^7*c^(3/2) + 3465*(sqrt(c)*x - s
qrt(c*x^2 + b*x))^2*B*b^9 + 301455*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*A*b^8
*c + 45045*(sqrt(c)*x - sqrt(c*x^2 + b*x))*A*b^9*sqrt(c) + 3003*A*b^10)/(s
qrt(c)*x - sqrt(c*x^2 + b*x))^15
```

Mupad [B] (verification not implemented)

Time = 7.91 (sec) , antiderivative size = 326, normalized size of antiderivative = 2.04

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11}}\,dx = \frac{16\,A\,c^4\,\sqrt{c\,x^2+b\,x}}{9009\,b^2\,x^4} - \frac{142\,A\,c^2\,\sqrt{c\,x^2+b\,x}}{715\,x^6} - \frac{2\,B\,b^2\,\sqrt{c\,x^2+b\,x}}{13\,x^7} - \frac{106\,B\,c^2\,\sqrt{c\,x^2+b\,x}}{429\,x^5} - \frac{2\,A\,c^3\,\sqrt{c\,x^2+b\,x}}{1287\,b\,x^5} - \frac{2\,A\,b^2\,\sqrt{c\,x^2+b\,x}}{15\,x^8} - \frac{32\,A\,c^5\,\sqrt{c\,x^2+b\,x}}{15015\,b^3\,x^3} + \frac{128\,A\,c^6\,\sqrt{c\,x^2+b\,x}}{45045\,b^4\,x^2} - \frac{256\,A\,c^7\,\sqrt{c\,x^2+b\,x}}{45045\,b^5\,x} - \frac{10\,B\,c^3\,\sqrt{c\,x^2+b\,x}}{3003\,b\,x^4} + \frac{4\,B\,c^4\,\sqrt{c\,x^2+b\,x}}{1001\,b^2\,x^3} - \frac{16\,B\,c^5\,\sqrt{c\,x^2+b\,x}}{3003\,b^3\,x^2} + \frac{32\,B\,c^6\,\sqrt{c\,x^2+b\,x}}{3003\,b^4\,x} - \frac{62\,A\,b\,c\,\sqrt{c\,x^2+b\,x}}{195\,x^7} - \frac{54\,B\,b\,c\,\sqrt{c\,x^2+b\,x}}{143\,x^6}$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^11,x)
```

output
```
(16*A*c^4*(b*x + c*x^2)^(1/2))/(9009*b^2*x^4) - (142*A*c^2*(b*x + c*x^2)^(
1/2))/(715*x^6) - (2*B*b^2*(b*x + c*x^2)^(1/2))/(13*x^7) - (106*B*c^2*(b*x
 + c*x^2)^(1/2))/(429*x^5) - (2*A*c^3*(b*x + c*x^2)^(1/2))/(1287*b*x^5) -
(2*A*b^2*(b*x + c*x^2)^(1/2))/(15*x^8) - (32*A*c^5*(b*x + c*x^2)^(1/2))/(1
5015*b^3*x^3) + (128*A*c^6*(b*x + c*x^2)^(1/2))/(45045*b^4*x^2) - (256*A*c
^7*(b*x + c*x^2)^(1/2))/(45045*b^5*x) - (10*B*c^3*(b*x + c*x^2)^(1/2))/(30
03*b*x^4) + (4*B*c^4*(b*x + c*x^2)^(1/2))/(1001*b^2*x^3) - (16*B*c^5*(b*x
+ c*x^2)^(1/2))/(3003*b^3*x^2) + (32*B*c^6*(b*x + c*x^2)^(1/2))/(3003*b^4*
x) - (62*A*b*c*(b*x + c*x^2)^(1/2))/(195*x^7) - (54*B*b*c*(b*x + c*x^2)^(1
/2))/(143*x^6)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.90

$$\int \frac{(A+Bx)\,(bx+cx^2)^{5/2}}{x^{11}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^7}{15} - \frac{62\sqrt{x}\,\sqrt{cx+b}\,a\,b^6cx}{195} - \frac{142\sqrt{x}\,\sqrt{cx+b}\,a\,b^5c^2x^2}{715} - \frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^4c^3x^3}{1287} + \cdots}{\cdots}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^11,x)
```

output
```
(2*( - 3003*sqrt(x)*sqrt(b + c*x)*a*b**7 - 7161*sqrt(x)*sqrt(b + c*x)*a*b*
*6*c*x - 4473*sqrt(x)*sqrt(b + c*x)*a*b**5*c**2*x**2 - 35*sqrt(x)*sqrt(b +
 c*x)*a*b**4*c**3*x**3 + 40*sqrt(x)*sqrt(b + c*x)*a*b**3*c**4*x**4 - 48*sq
rt(x)*sqrt(b + c*x)*a*b**2*c**5*x**5 + 64*sqrt(x)*sqrt(b + c*x)*a*b*c**6*x
**6 - 128*sqrt(x)*sqrt(b + c*x)*a*c**7*x**7 - 3465*sqrt(x)*sqrt(b + c*x)*b
**8*x - 8505*sqrt(x)*sqrt(b + c*x)*b**7*c*x**2 - 5565*sqrt(x)*sqrt(b + c*x
)*b**6*c**2*x**3 - 75*sqrt(x)*sqrt(b + c*x)*b**5*c**3*x**4 + 90*sqrt(x)*sq
rt(b + c*x)*b**4*c**4*x**5 - 120*sqrt(x)*sqrt(b + c*x)*b**3*c**5*x**6 + 24
0*sqrt(x)*sqrt(b + c*x)*b**2*c**6*x**7 + 128*sqrt(c)*a*c**7*x**8 - 240*sqr
t(c)*b**2*c**6*x**8))/(45045*b**5*x**8)
```

3.141 $\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^{12}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 195

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^{12}}\,dx = -\frac{2A(bx+cx^2)^{7/2}}{17bx^{12}} - \frac{2(17bB-10Ac)(bx+cx^2)^{7/2}}{255b^2x^{11}} + \frac{16c(17bB-10Ac)(bx+cx^2)^{7/2}}{3315b^3x^{10}} - \frac{32c^2(17bB-10Ac)(bx+cx^2)^{7/2}}{12155b^4x^9} + \frac{128c^3(17bB-10Ac)(bx+cx^2)^{7/2}}{109395b^5x^8} - \frac{256c^4(17bB-10Ac)(bx+cx^2)^{7/2}}{765765b^6x^7}$$

output
```
-2/17*A*(c*x^2+b*x)^(7/2)/b/x^12-2/255*(-10*A*c+17*B*b)*(c*x^2+b*x)^(7/2)/
b^2/x^11+16/3315*c*(-10*A*c+17*B*b)*(c*x^2+b*x)^(7/2)/b^3/x^10-32/12155*c^
2*(-10*A*c+17*B*b)*(c*x^2+b*x)^(7/2)/b^4/x^9+128/109395*c^3*(-10*A*c+17*B*
b)*(c*x^2+b*x)^(7/2)/b^5/x^8-256/765765*c^4*(-10*A*c+17*B*b)*(c*x^2+b*x)^(
7/2)/b^6/x^7
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.67

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{12}}\,dx = -\frac{2(b+cx)^3\sqrt{x(b+cx)}(17bBx(3003b^4-1848b^3cx+1008b^2c^2x^2-448bc^3x^3+128c^4x^4)+5A(9009b^5-6\cdots}{765765b^6x^9}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^12,x]
```

output
```
(-2*(b + c*x)^3*Sqrt[x*(b + c*x)]*(17*b*B*x*(3003*b^4 - 1848*b^3*c*x + 100
8*b^2*c^2*x^2 - 448*b*c^3*x^3 + 128*c^4*x^4) + 5*A*(9009*b^5 - 6006*b^4*c*
x + 3696*b^3*c^2*x^2 - 2016*b^2*c^3*x^3 + 896*b*c^4*x^4 - 256*c^5*x^5)))/(
765765*b^6*x^9)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1220, 1129, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{12}}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(17bB-10Ac)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{11}}dx}{17b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{17bx^{12}}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(17bB-10Ac)\left(-\frac{8c\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{10}}dx}{15b}-\frac{2\left(bx+cx^2\right)^{7/2}}{15bx^{11}}\right)}{17b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{17bx^{12}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(17bB-10Ac)\left(-\frac{8c\left(-\frac{6c\int\frac{(cx^2+bx)^{5/2}}{x^9}dx}{13b}-\frac{2(bx+cx^2)^{7/2}}{13bx^{10}}\right)}{15b}-\frac{2(bx+cx^2)^{7/2}}{15bx^{11}}\right)}{17b}-\frac{2A(bx+cx^2)^{7/2}}{17bx^{12}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(17bB-10Ac)\left(-\frac{8c\left(-\frac{6c\left(-\frac{4c\int\frac{(cx^2+bx)^{5/2}}{x^8}dx}{11b}-\frac{2(bx+cx^2)^{7/2}}{11bx^9}\right)}{13b}-\frac{2(bx+cx^2)^{7/2}}{13bx^{10}}\right)}{15b}-\frac{2(bx+cx^2)^{7/2}}{15bx^{11}}\right)}{17b}-\frac{2A(bx+cx^2)^{7/2}}{17bx^{12}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(17bB-10Ac)\left(-\frac{8c\left(-\frac{6c\left(-\frac{4c\left(-\frac{2c\int \frac{\left(cx^2+bx\right)^{5/2}}{x^7}dx}{9b}-\frac{2\left(bx+cx^2\right)^{7/2}}{9bx^8}\right)}{11b}-\frac{2\left(bx+cx^2\right)^{7/2}}{11bx^9}\right)}{13b}-\frac{2\left(bx+cx^2\right)^{7/2}}{13bx^{10}}\right)}{15b}-\frac{2\left(bx+cx^2\right)^{7/2}}{15bx^{11}}\right)}{17b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{17bx^{12}}$$

$$\Big\downarrow 1123$$

$$\frac{\left(-\frac{8c\left(-\frac{6c\left(-\frac{4c\left(\frac{4c\left(bx+cx^2\right)^{7/2}}{63b^2x^7}-\frac{2\left(bx+cx^2\right)^{7/2}}{9bx^8}\right)}{11b}-\frac{2\left(bx+cx^2\right)^{7/2}}{11bx^9}\right)}{13b}-\frac{2\left(bx+cx^2\right)^{7/2}}{13bx^{10}}\right)}{15b}-\frac{2\left(bx+cx^2\right)^{7/2}}{15bx^{11}}\right)(17bB-10Ac)}{17b}-\frac{2A\left(bx+cx^2\right)^{7/2}}{17bx^{12}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^12,x]
```

output
```
(-2*A*(b*x + c*x^2)^(7/2))/(17*b*x^12) + ((17*b*B - 10*A*c)*((-2*(b*x + c*
x^2)^(7/2))/(15*b*x^11) - (8*c*((-2*(b*x + c*x^2)^(7/2))/(13*b*x^10) - (6*
c*((-2*(b*x + c*x^2)^(7/2))/(11*b*x^9) - (4*c*((-2*(b*x + c*x^2)^(7/2))/(9
*b*x^8) + (4*c*(b*x + c*x^2)^(7/2))/(63*b^2*x^7)))/(11*b)))/(13*b)))/(15*b
)))/(17*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.55

method	result
pseudoelliptic	$-\frac{2(cx+b)^3\sqrt{x(cx+b)}\left(\left(\left(\frac{17Bx}{15}+A\right)b^5-\frac{2cx\left(\frac{68Bx}{65}+A\right)b^4}{3}+\frac{16c^2x^2\left(\frac{51Bx}{55}+A\right)b^3}{39}-\frac{32c^3x^3\left(\frac{34Bx}{45}+A\right)b^2}{143}+\frac{128c^4x^4\left(\frac{17Bx}{35}+A\right)}{1287}\right.\right.}{17x^9b^6}$
gosper	$-\frac{2(cx+b)\left(-1280A\,c^5x^5+2176Bb\,c^4x^5+4480Ab\,c^4x^4-7616B\,b^2c^3x^4-10080A\,b^2c^3x^3+17136B\,b^3c^2x^3+18480A\,b^3c^2x^2-\right.}{765765x^{11}b^6}$
orering	$-\frac{2(cx+b)\left(-1280A\,c^5x^5+2176Bb\,c^4x^5+4480Ab\,c^4x^4-7616B\,b^2c^3x^4-10080A\,b^2c^3x^3+17136B\,b^3c^2x^3+18480A\,b^3c^2x^2-\right.}{765765x^{11}b^6}$
trager	$-\frac{2\left(-1280A\,c^8x^8+2176Bb\,c^7x^8+640Ab\,c^7x^7-1088B\,b^2c^6x^7-480A\,b^2c^6x^6+816B\,b^3c^5x^6+400A\,b^3c^5x^5-680B\,b^4c^4x^5-\right.}{}$
risch	$-\frac{2(cx+b)\left(-1280A\,c^8x^8+2176Bb\,c^7x^8+640Ab\,c^7x^7-1088B\,b^2c^6x^7-480A\,b^2c^6x^6+816B\,b^3c^5x^6+400A\,b^3c^5x^5-680B\,b^4\right.}{}$
default	$A\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{7}{2}}}{17b\,x^{12}}-\frac{10c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{7}{2}}}{15b\,x^{11}}-\frac{8c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{7}{2}}}{13b\,x^{10}}-\frac{6c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{7}{2}}}{11b\,x^9}-\frac{4c\left(-\frac{2\left(c\,x^2+bx\right)^{\frac{7}{2}}}{9b\,x^8}+\frac{4c\left(c\,x^2+bx\right)^{\frac{7}{2}}}{63b^2x^7}\right.}{11b}\right.}{13b}\right.}{15b}\right.}{17b}\right.$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-2/17*(c*x+b)^3*(x*(c*x+b))^(1/2)*((17/15*B*x+A)*b^5-2/3*c*x*(68/65*B*x+A)
*b^4+16/39*c^2*x^2*(51/55*B*x+A)*b^3-32/143*c^3*x^3*(34/45*B*x+A)*b^2+128/
1287*c^4*x^4*(17/35*B*x+A)*b-256/9009*A*c^5*x^5)/x^9/b^6
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{12}}\,dx =$$
$$-\frac{2\,(45045\,Ab^8+128\,(17\,Bbc^7-10\,Ac^8)x^8-64\,(17\,Bb^2c^6-10\,Abc^7)x^7+48\,(17\,Bb^3c^5-10\,Ab^2c^6)x^6-4\cdots}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^12,x, algorithm="fricas")
```

output
```
-2/765765*(45045*A*b^8 + 128*(17*B*b*c^7 - 10*A*c^8)*x^8 - 64*(17*B*b^2*c^
6 - 10*A*b*c^7)*x^7 + 48*(17*B*b^3*c^5 - 10*A*b^2*c^6)*x^6 - 40*(17*B*b^4*
c^4 - 10*A*b^3*c^5)*x^5 + 35*(17*B*b^5*c^3 - 10*A*b^4*c^4)*x^4 + 63*(1207*
B*b^6*c^2 + 5*A*b^5*c^3)*x^3 + 231*(527*B*b^7*c + 275*A*b^6*c^2)*x^2 + 300
3*(17*B*b^8 + 35*A*b^7*c)*x)*sqrt(c*x^2 + b*x)/(b^6*x^9)
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{12}}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{x^{12}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**12,x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**12, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 442 vs. $2(171) = 342$.

Time = 0.04 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.27

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{12}}\,dx = -\frac{256\sqrt{cx^2+bx}Bc^7}{45045\,b^5x} + \frac{512\sqrt{cx^2+bx}Ac^8}{153153\,b^6x} + \frac{128\sqrt{cx^2+bx}Bc^6}{45045\,b^4x^2} - \frac{256\sqrt{cx^2+bx}Ac^7}{153153\,b^5x^2} - \frac{32\sqrt{cx^2+bx}Bc^5}{15015\,b^3x^3} + \frac{64\sqrt{cx^2+bx}Ac^6}{51051\,b^4x^3} + \frac{16\sqrt{cx^2+bx}Bc^4}{9009\,b^2x^4} - \frac{160\sqrt{cx^2+bx}Ac^5}{153153\,b^3x^4} - \frac{2\sqrt{cx^2+bx}Bc^3}{1287\,bx^5} + \frac{20\sqrt{cx^2+bx}Ac^4}{21879\,b^2x^5} + \frac{\sqrt{cx^2+bx}Bc^2}{715\,x^6} - \frac{2\sqrt{cx^2+bx}Ac^3}{2431\,bx^6} - \frac{\sqrt{cx^2+bx}Bbc}{780\,x^7} + \frac{\sqrt{cx^2+bx}Ac^2}{1326\,x^7} - \frac{\sqrt{cx^2+bx}Bb^2}{60\,x^8} - \frac{\sqrt{cx^2+bx}Abc}{1428\,x^8} + \frac{(cx^2+bx)^{\frac{3}{2}}Bb}{12\,x^9} - \frac{5\sqrt{cx^2+bx}Ab^2}{476\,x^9} - \frac{(cx^2+bx)^{\frac{5}{2}}B}{5\,x^{10}} + \frac{5\,(cx^2+bx)^{\frac{3}{2}}Ab}{84\,x^{10}} - \frac{(cx^2+bx)^{\frac{5}{2}}A}{6\,x^{11}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^12,x, algorithm="maxima")
```

output
```
-256/45045*sqrt(c*x^2 + b*x)*B*c^7/(b^5*x) + 512/153153*sqrt(c*x^2 + b*x)*
A*c^8/(b^6*x) + 128/45045*sqrt(c*x^2 + b*x)*B*c^6/(b^4*x^2) - 256/153153*s
qrt(c*x^2 + b*x)*A*c^7/(b^5*x^2) - 32/15015*sqrt(c*x^2 + b*x)*B*c^5/(b^3*x
^3) + 64/51051*sqrt(c*x^2 + b*x)*A*c^6/(b^4*x^3) + 16/9009*sqrt(c*x^2 + b*
x)*B*c^4/(b^2*x^4) - 160/153153*sqrt(c*x^2 + b*x)*A*c^5/(b^3*x^4) - 2/1287
*sqrt(c*x^2 + b*x)*B*c^3/(b*x^5) + 20/21879*sqrt(c*x^2 + b*x)*A*c^4/(b^2*x
^5) + 1/715*sqrt(c*x^2 + b*x)*B*c^2/x^6 - 2/2431*sqrt(c*x^2 + b*x)*A*c^3/(
b*x^6) - 1/780*sqrt(c*x^2 + b*x)*B*b*c/x^7 + 1/1326*sqrt(c*x^2 + b*x)*A*c^
2/x^7 - 1/60*sqrt(c*x^2 + b*x)*B*b^2/x^8 - 1/1428*sqrt(c*x^2 + b*x)*A*b*c/
x^8 + 1/12*(c*x^2 + b*x)^(3/2)*B*b/x^9 - 5/476*sqrt(c*x^2 + b*x)*A*b^2/x^9
 - 1/5*(c*x^2 + b*x)^(5/2)*B/x^10 + 5/84*(c*x^2 + b*x)^(3/2)*A*b/x^10 - 1/
6*(c*x^2 + b*x)^(5/2)*A/x^11
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(171) = 342$.

Time = 0.13 (sec) , antiderivative size = 671, normalized size of antiderivative = 3.44

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{12}}\,dx = \frac{2\left(2450448\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^{12}Bc^5+16336320\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^{11}B\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^12,x, algorithm="giac")
```

output
```
2/765765*(2450448*(sqrt(c)*x - sqrt(c*x^2 + b*x))^12*B*c^5 + 16336320*(sqr
t(c)*x - sqrt(c*x^2 + b*x))^11*B*b*c^(9/2) + 4084080*(sqrt(c)*x - sqrt(c*x
^2 + b*x))^11*A*c^(11/2) + 49884120*(sqrt(c)*x - sqrt(c*x^2 + b*x))^10*B*b
^2*c^4 + 29755440*(sqrt(c)*x - sqrt(c*x^2 + b*x))^10*A*b*c^5 + 91126035*(s
qrt(c)*x - sqrt(c*x^2 + b*x))^9*B*b^3*c^(7/2) + 99549450*(sqrt(c)*x - sqrt
(c*x^2 + b*x))^9*A*b^2*c^(9/2) + 109674565*(sqrt(c)*x - sqrt(c*x^2 + b*x))
^8*B*b^4*c^3 + 200800600*(sqrt(c)*x - sqrt(c*x^2 + b*x))^8*A*b^3*c^4 + 905
13423*(sqrt(c)*x - sqrt(c*x^2 + b*x))^7*B*b^5*c^(5/2) + 270315045*(sqrt(c)
*x - sqrt(c*x^2 + b*x))^7*A*b^4*c^(7/2) + 51723945*(sqrt(c)*x - sqrt(c*x^2
 + b*x))^6*B*b^6*c^2 + 254303595*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*A*b^5*c
^3 + 20165145*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*B*b^7*c^(3/2) + 170255085*
(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*A*b^6*c^(5/2) + 5124735*(sqrt(c)*x - sqr
t(c*x^2 + b*x))^4*B*b^8*c + 80994375*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*A*b
^7*c^2 + 765765*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*B*b^9*sqrt(c) + 26801775
*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*A*b^8*c^(3/2) + 51051*(sqrt(c)*x - sqrt
(c*x^2 + b*x))^2*B*b^10 + 5870865*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*A*b^9*
c + 765765*(sqrt(c)*x - sqrt(c*x^2 + b*x))*A*b^10*sqrt(c) + 45045*A*b^11)/
(sqrt(c)*x - sqrt(c*x^2 + b*x))^17
```

Mupad [B] (verification not implemented)

Time = 8.32 (sec) , antiderivative size = 372, normalized size of antiderivative = 1.91

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{12}}\,dx = \frac{20\,A\,c^4\,\sqrt{c\,x^2+b\,x}}{21879\,b^2\,x^5}-\frac{110\,A\,c^2\,\sqrt{c\,x^2+b\,x}}{663\,x^7}-\frac{2\,B\,b^2\,\sqrt{c\,x^2+b\,x}}{15\,x^8}-\frac{142\,B\,c^2\,\sqrt{c\,x^2+b\,x}}{715\,x^6}-\frac{2\,A\,c^3\,\sqrt{c\,x^2+b\,x}}{2431\,b\,x^6}-\frac{2\,A\,b^2\,\sqrt{c\,x^2+b\,x}}{17\,x^9}-\frac{160\,A\,c^5\,\sqrt{c\,x^2+b\,x}}{153153\,b^3\,x^4}+\frac{64\,A\,c^6\,\sqrt{c\,x^2+b\,x}}{51051\,b^4\,x^3}-\frac{256\,A\,c^7\,\sqrt{c\,x^2+b\,x}}{153153\,b^5\,x^2}+\frac{512\,A\,c^8\,\sqrt{c\,x^2+b\,x}}{153153\,b^6\,x}-\frac{2\,B\,c^3\,\sqrt{c\,x^2+b\,x}}{1287\,b\,x^5}+\frac{16\,B\,c^4\,\sqrt{c\,x^2+b\,x}}{9009\,b^2\,x^4}-\frac{32\,B\,c^5\,\sqrt{c\,x^2+b\,x}}{15015\,b^3\,x^3}+\frac{128\,B\,c^6\,\sqrt{c\,x^2+b\,x}}{45045\,b^4\,x^2}-\frac{256\,B\,c^7\,\sqrt{c\,x^2+b\,x}}{45045\,b^5\,x}-\frac{14\,A\,b\,c\,\sqrt{c\,x^2+b\,x}}{51\,x^8}-\frac{62\,B\,b\,c\,\sqrt{c\,x^2+b\,x}}{195\,x^7}$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^12,x)
```

output
```
(20*A*c^4*(b*x + c*x^2)^(1/2))/(21879*b^2*x^5) - (110*A*c^2*(b*x + c*x^2)^
(1/2))/(663*x^7) - (2*B*b^2*(b*x + c*x^2)^(1/2))/(15*x^8) - (142*B*c^2*(b*
x + c*x^2)^(1/2))/(715*x^6) - (2*A*c^3*(b*x + c*x^2)^(1/2))/(2431*b*x^6) -
 (2*A*b^2*(b*x + c*x^2)^(1/2))/(17*x^9) - (160*A*c^5*(b*x + c*x^2)^(1/2))/
(153153*b^3*x^4) + (64*A*c^6*(b*x + c*x^2)^(1/2))/(51051*b^4*x^3) - (256*A
*c^7*(b*x + c*x^2)^(1/2))/(153153*b^5*x^2) + (512*A*c^8*(b*x + c*x^2)^(1/2
))/(153153*b^6*x) - (2*B*c^3*(b*x + c*x^2)^(1/2))/(1287*b*x^5) + (16*B*c^4
*(b*x + c*x^2)^(1/2))/(9009*b^2*x^4) - (32*B*c^5*(b*x + c*x^2)^(1/2))/(150
15*b^3*x^3) + (128*B*c^6*(b*x + c*x^2)^(1/2))/(45045*b^4*x^2) - (256*B*c^7
*(b*x + c*x^2)^(1/2))/(45045*b^5*x) - (14*A*b*c*(b*x + c*x^2)^(1/2))/(51*x
^8) - (62*B*b*c*(b*x + c*x^2)^(1/2))/(195*x^7)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.76

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{12}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^8}{17}-\frac{14\sqrt{x}\,\sqrt{cx+b}\,a\,b^7cx}{51}-\frac{110\sqrt{x}\,\sqrt{cx+b}\,a\,b^6c^2x^2}{663}-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^5c^3x^3}{2431}+}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^12,x)
```

output
```
(2*( - 45045*sqrt(x)*sqrt(b + c*x)*a*b**8 - 105105*sqrt(x)*sqrt(b + c*x)*a
*b**7*c*x - 63525*sqrt(x)*sqrt(b + c*x)*a*b**6*c**2*x**2 - 315*sqrt(x)*sqr
t(b + c*x)*a*b**5*c**3*x**3 + 350*sqrt(x)*sqrt(b + c*x)*a*b**4*c**4*x**4 -
 400*sqrt(x)*sqrt(b + c*x)*a*b**3*c**5*x**5 + 480*sqrt(x)*sqrt(b + c*x)*a*
b**2*c**6*x**6 - 640*sqrt(x)*sqrt(b + c*x)*a*b*c**7*x**7 + 1280*sqrt(x)*sq
rt(b + c*x)*a*c**8*x**8 - 51051*sqrt(x)*sqrt(b + c*x)*b**9*x - 121737*sqrt
(x)*sqrt(b + c*x)*b**8*c*x**2 - 76041*sqrt(x)*sqrt(b + c*x)*b**7*c**2*x**3
 - 595*sqrt(x)*sqrt(b + c*x)*b**6*c**3*x**4 + 680*sqrt(x)*sqrt(b + c*x)*b*
*5*c**4*x**5 - 816*sqrt(x)*sqrt(b + c*x)*b**4*c**5*x**6 + 1088*sqrt(x)*sqr
t(b + c*x)*b**3*c**6*x**7 - 2176*sqrt(x)*sqrt(b + c*x)*b**2*c**7*x**8 - 12
80*sqrt(c)*a*c**8*x**9 + 2176*sqrt(c)*b**2*c**7*x**9))/(765765*b**6*x**9)
```

3.142 $\int \frac{x^4(A+Bx)}{\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 197

$$\int \frac{x^4(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{7b^3(9bB-10Ac)\sqrt{bx+cx^2}}{128c^5} - \frac{7b^2(9bB-10Ac)x\sqrt{bx+cx^2}}{192c^4} + \frac{7b(9bB-10Ac)x^2\sqrt{bx+cx^2}}{240c^3} - \frac{(9bB-10Ac)x^3\sqrt{bx+cx^2}}{40c^2} + \frac{Bx^4\sqrt{bx+cx^2}}{5c} - \frac{7b^4(9bB-10Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{128c^{11/2}}$$

output
```
7/128*b^3*(-10*A*c+9*B*b)*(c*x^2+b*x)^(1/2)/c^5-7/192*b^2*(-10*A*c+9*B*b)*
x*(c*x^2+b*x)^(1/2)/c^4+7/240*b*(-10*A*c+9*B*b)*x^2*(c*x^2+b*x)^(1/2)/c^3-
1/40*(-10*A*c+9*B*b)*x^3*(c*x^2+b*x)^(1/2)/c^2+1/5*B*x^4*(c*x^2+b*x)^(1/2)
/c-7/128*b^4*(-10*A*c+9*B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(11/2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.83

$$\int \frac{x^4(A+Bx)}{\sqrt{bx+cx^2}}\,dx$$
$$= \frac{\sqrt{c}x(b+cx)\left(945b^4B-210b^3c(5A+3Bx)+96c^4x^3(5A+4Bx)+28b^2c^2x(25A+18Bx)-16bc^3x^2(35\right.}{1920c^{11/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[(x^4*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[c]*x*(b + c*x)*(945*b^4*B - 210*b^3*c*(5*A + 3*B*x) + 96*c^4*x^3*(5*
A + 4*B*x) + 28*b^2*c^2*x*(25*A + 18*B*x) - 16*b*c^3*x^2*(35*A + 27*B*x))
+ 210*b^4*(9*b*B - 10*A*c)*Sqrt[x]*Sqrt[b + c*x]*ArcTanh[(Sqrt[c]*Sqrt[x])
/(Sqrt[b] - Sqrt[b + c*x])])/(1920*c^(11/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {1221, 1134, 1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{\sqrt{bx+cx^2}}\,dx$$
$$\Big\downarrow \; 1221$$
$$\frac{Bx^4\sqrt{bx+cx^2}}{5c}-\frac{(9bB-10Ac)\int \frac{x^4}{\sqrt{cx^2+bx}}dx}{10c}$$
$$\Big\downarrow \; 1134$$
$$\frac{Bx^4\sqrt{bx+cx^2}}{5c}-\frac{(9bB-10Ac)\left(\frac{x^3\sqrt{bx+cx^2}}{4c}-\frac{7b\int \frac{x^3}{\sqrt{cx^2+bx}}dx}{8c}\right)}{10c}$$

$\Big\downarrow$ 1134

$$\frac{Bx^4\sqrt{bx+cx^2}}{5c}-\frac{(9bB-10Ac)\left(\frac{x^3\sqrt{bx+cx^2}}{4c}-\frac{7b\left(\frac{x^2\sqrt{bx+cx^2}}{3c}-\frac{5b\int\frac{x^2}{\sqrt{cx^2+bx}}dx}{6c}\right)}{8c}\right)}{10c}$$

$\Big\downarrow$ 1134

$$\frac{Bx^4\sqrt{bx+cx^2}}{5c}-\frac{(9bB-10Ac)\left(\frac{x^3\sqrt{bx+cx^2}}{4c}-\frac{7b\left(\frac{x^2\sqrt{bx+cx^2}}{3c}-\frac{5b\left(\frac{x\sqrt{bx+cx^2}}{2c}-\frac{3b\int\frac{x}{\sqrt{cx^2+bx}}dx}{4c}\right)}{6c}\right)}{8c}\right)}{10c}$$

$\Big\downarrow$ 1160

$$\frac{Bx^4\sqrt{bx+cx^2}}{5c}-\frac{(9bB-10Ac)\left(\frac{x^3\sqrt{bx+cx^2}}{4c}-\frac{7b\left(\frac{x^2\sqrt{bx+cx^2}}{3c}-\frac{5b\left(\frac{x\sqrt{bx+cx^2}}{2c}-\frac{3b\left(\frac{\sqrt{bx+cx^2}}{c}-\frac{b\int\frac{1}{\sqrt{cx^2+bx}}dx}{2c}\right)}{4c}\right)}{6c}\right)}{8c}\right)}{10c}$$

$\Big\downarrow$ 1091

$$\frac{\frac{Bx^4\sqrt{bx+cx^2}}{5c}-(9bB-10Ac)\left(\frac{x^3\sqrt{bx+cx^2}}{4c}-\frac{7b\left(\frac{x^2\sqrt{bx+cx^2}}{3c}-\frac{5b\left(\frac{x\sqrt{bx+cx^2}}{2c}-\frac{3b\left(\frac{\sqrt{bx+cx^2}}{c}-\frac{b\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{c}\right)}{4c}\right)}{6c}\right)}{8c}\right)}{10c}$$

$$\Big\downarrow \; 219$$

$$\frac{Bx^4\sqrt{bx+cx^2}}{5c} - \frac{(9bB-10Ac)\left(\frac{x^3\sqrt{bx+cx^2}}{4c} - \frac{7b\left(\frac{x^2\sqrt{bx+cx^2}}{3c} - \frac{5b\left(\frac{x\sqrt{bx+cx^2}}{2c} - \frac{3b\left(\frac{\sqrt{bx+cx^2}}{c} - \frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}\right)}{4c}\right)}{6c}\right)}{8c}\right)}{10c}$$

input
```
Int[(x^4*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
(B*x^4*Sqrt[b*x + c*x^2])/(5*c) - ((9*b*B - 10*A*c)*((x^3*Sqrt[b*x + c*x^2
])/(4*c) - (7*b*((x^2*Sqrt[b*x + c*x^2])/(3*c) - (5*b*((x*Sqrt[b*x + c*x^2
])/(2*c) - (3*b*(Sqrt[b*x + c*x^2]/c - (b*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c
*x^2]])/c^(3/2)))/(4*c)))/(6*c)))/(8*c)))/(10*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.59

method	result
pseudoelliptic	$-\frac{7\left(\left(-\frac{15}{8}A\,b^{4}c+\frac{27}{16}b^{5}B\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)+\left(\frac{15\left(\frac{3Bx}{5}+A\right)b^{3}c^{\frac{3}{2}}}{8}-\frac{5\left(\frac{18Bx}{25}+A\right)x\,b^{2}c^{\frac{5}{2}}}{4}+b\,x^{2}\left(\frac{27Bx}{35}+A\right)c^{\frac{7}{2}}-\frac{6x^{3}\left(\frac{4Bx}{5}\right.}{7}\right.}{24c^{\frac{11}{2}}}$
risch	$-\frac{\left(-384B\,c^{4}x^{4}-480A\,c^{4}x^{3}+432B\,c^{3}x^{3}b+560Ab\,c^{3}x^{2}-504c^{2}x^{2}B\,b^{2}-700A\,b^{2}c^{2}x+630B\,b^{3}cx+1050A\,b^{3}c-945B\,b^{4}\right)x(cx}{1920c^{5}\sqrt{x(cx+b)}}$
default	$A\left(\frac{x^{3}\sqrt{c\,x^{2}+bx}}{4c}-\frac{7b\left(\frac{x^{2}\sqrt{c\,x^{2}+bx}}{3c}-\frac{5b\left(\frac{x\sqrt{c\,x^{2}+bx}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^{2}+bx}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{2c^{\frac{3}{2}}}\right)}{4c}\right)}{6c}\right)}{8c}\right)+B\left(\frac{x^{4}\sqrt{c\,x}}{5}\right.$

input
```
int(x^4*(B*x+A)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-7/24/c^(11/2)*((-15/8*A*b^4*c+27/16*b^5*B)*arctanh((x*(c*x+b))^(1/2)/x/c^
(1/2))+(15/8*(3/5*B*x+A)*b^3*c^(3/2)-5/4*(18/25*B*x+A)*x*b^2*c^(5/2)+b*x^2
*(27/35*B*x+A)*c^(7/2)-6/7*x^3*(4/5*B*x+A)*c^(9/2)-27/16*B*c^(1/2)*b^4)*(x
*(c*x+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.54

$$\int \frac{x^4(A+Bx)}{\sqrt{bx+cx^2}}\,dx$$

$$= \left[-\frac{105\,(9\,Bb^5-10\,Ab^4c)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(384\,Bc^5x^4+945\,Bb^4c-1050\,Ab^3c^2-}{3840\,c^6}\right.$$

input
```
integrate(x^4*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/3840*(105*(9*B*b^5 - 10*A*b^4*c)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2
+ b*x)*sqrt(c)) - 2*(384*B*c^5*x^4 + 945*B*b^4*c - 1050*A*b^3*c^2 - 48*(9*
B*b*c^4 - 10*A*c^5)*x^3 + 56*(9*B*b^2*c^3 - 10*A*b*c^4)*x^2 - 70*(9*B*b^3*
c^2 - 10*A*b^2*c^3)*x)*sqrt(c*x^2 + b*x))/c^6, 1/1920*(105*(9*B*b^5 - 10*A
*b^4*c)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (384*B*c^5
*x^4 + 945*B*b^4*c - 1050*A*b^3*c^2 - 48*(9*B*b*c^4 - 10*A*c^5)*x^3 + 56*(
9*B*b^2*c^3 - 10*A*b*c^4)*x^2 - 70*(9*B*b^3*c^2 - 10*A*b^2*c^3)*x)*sqrt(c*
x^2 + b*x))/c^6]
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.16

$$\int \frac{x^4(A+Bx)}{\sqrt{bx+cx^2}}\,dx$$
$$= \begin{cases} \frac{35b^4\left(A-\frac{9Bb}{10c}\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{128c^4} + \sqrt{bx+cx^2}\left(\frac{Bx^4}{5c} - \frac{35b^3\left(A-\frac{9Bb}{10c}\right)}{64c^4} + \frac{35b^2x\left(A-\frac{9Bb}{10c}\right)}{96c^3} - \right. \\ \frac{2\left(\frac{A(bx)^{\frac{9}{2}}}{9}+\frac{B(bx)^{\frac{11}{2}}}{11b}\right)}{b^5} \\ \tilde{\infty}\left(\frac{Ax^5}{5}+\frac{Bx^6}{6}\right) \end{cases}$$

input
```
integrate(x**4*(B*x+A)/(c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((35*b**4*(A - 9*B*b/(10*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*
x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) +
 x)/sqrt(c*(b/(2*c) + x)**2), True))/(128*c**4) + sqrt(b*x + c*x**2)*(B*x*
*4/(5*c) - 35*b**3*(A - 9*B*b/(10*c))/(64*c**4) + 35*b**2*x*(A - 9*B*b/(10
*c))/(96*c**3) - 7*b*x**2*(A - 9*B*b/(10*c))/(24*c**2) + x**3*(A - 9*B*b/(
10*c))/(4*c)), Ne(c, 0)), (2*(A*(b*x)**(9/2)/9 + B*(b*x)**(11/2)/(11*b))/b
**5, Ne(b, 0)), (zoo*(A*x**5/5 + B*x**6/6), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.28

$$\int \frac{x^4(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{\sqrt{cx^2+bx}Bx^4}{5\,c} - \frac{9\,\sqrt{cx^2+bx}Bbx^3}{40\,c^2} + \frac{\sqrt{cx^2+bx}Ax^3}{4\,c} + \frac{21\,\sqrt{cx^2+bx}Bb^2x^2}{80\,c^3} - \frac{7\,\sqrt{cx^2+bx}Abx^2}{24\,c^2} - \frac{21\,\sqrt{cx^2+bx}Bb^3x}{64\,c^4} + \frac{35\,\sqrt{cx^2+bx}Ab^2x}{96\,c^3} - \frac{63\,Bb^5\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{256\,c^{\frac{11}{2}}} + \frac{35\,Ab^4\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{128\,c^{\frac{9}{2}}} + \frac{63\,\sqrt{cx^2+bx}Bb^4}{128\,c^5} - \frac{35\,\sqrt{cx^2+bx}Ab^3}{64\,c^4}$$

input
```
integrate(x^4*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
1/5*sqrt(c*x^2 + b*x)*B*x^4/c - 9/40*sqrt(c*x^2 + b*x)*B*b*x^3/c^2 + 1/4*s
qrt(c*x^2 + b*x)*A*x^3/c + 21/80*sqrt(c*x^2 + b*x)*B*b^2*x^2/c^3 - 7/24*sq
rt(c*x^2 + b*x)*A*b*x^2/c^2 - 21/64*sqrt(c*x^2 + b*x)*B*b^3*x/c^4 + 35/96*
sqrt(c*x^2 + b*x)*A*b^2*x/c^3 - 63/256*B*b^5*log(2*c*x + b + 2*sqrt(c*x^2
+ b*x)*sqrt(c))/c^(11/2) + 35/128*A*b^4*log(2*c*x + b + 2*sqrt(c*x^2 + b*x
)*sqrt(c))/c^(9/2) + 63/128*sqrt(c*x^2 + b*x)*B*b^4/c^5 - 35/64*sqrt(c*x^2
 + b*x)*A*b^3/c^4
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.83

$$\int \frac{x^4(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{1}{1920}\,\sqrt{cx^2+bx}\left(2\left(4\left(6\left(\frac{8\,Bx}{c} - \frac{9\,Bbc^3-10\,Ac^4}{c^5}\right)x + \frac{7\,(9\,Bb^2c^2-10\,Abc^3)}{c^5}\right)x - \frac{35\,(9\,Bb^3c-10\,A}{c^5}\right.\right. + \frac{7\,(9\,Bb^5-10\,Ab^4c)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{256\,c^{\frac{11}{2}}}$$

input
```
integrate(x^4*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/1920*sqrt(c*x^2 + b*x)*(2*(4*(6*(8*B*x/c - (9*B*b*c^3 - 10*A*c^4)/c^5)*x
 + 7*(9*B*b^2*c^2 - 10*A*b*c^3)/c^5)*x - 35*(9*B*b^3*c - 10*A*b^2*c^2)/c^5
)*x + 105*(9*B*b^4 - 10*A*b^3*c)/c^5) + 7/256*(9*B*b^5 - 10*A*b^4*c)*log(a
bs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(11/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \int \frac{x^4\,(A+B\,x)}{\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((x^4*(A + B*x))/(b*x + c*x^2)^(1/2),x)
```

output
```
int((x^4*(A + B*x))/(b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.09

$$\int \frac{x^4(A+Bx)}{\sqrt{bx+cx^2}}\,dx$$
$$= \frac{-1050\sqrt{x}\,\sqrt{cx+b}\,a\,b^3c^2 + 700\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^3x - 560\sqrt{x}\,\sqrt{cx+b}\,ab\,c^4x^2 + 480\sqrt{x}\,\sqrt{cx+b}\,a\,c^5x^3 +}{}$$

input
```
int(x^4*(B*x+A)/(c*x^2+b*x)^(1/2),x)
```

output
```
( - 1050*sqrt(x)*sqrt(b + c*x)*a*b**3*c**2 + 700*sqrt(x)*sqrt(b + c*x)*a*b
**2*c**3*x - 560*sqrt(x)*sqrt(b + c*x)*a*b*c**4*x**2 + 480*sqrt(x)*sqrt(b
+ c*x)*a*c**5*x**3 + 945*sqrt(x)*sqrt(b + c*x)*b**5*c - 630*sqrt(x)*sqrt(b
 + c*x)*b**4*c**2*x + 504*sqrt(x)*sqrt(b + c*x)*b**3*c**3*x**2 - 432*sqrt(
x)*sqrt(b + c*x)*b**2*c**4*x**3 + 384*sqrt(x)*sqrt(b + c*x)*b*c**5*x**4 +
1050*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*b**4*c - 945
*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**6)/(1920*c**6)
```

3.143 $\int \frac{x^3(A+Bx)}{\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 162

$$\int \frac{x^3(A+Bx)}{\sqrt{bx+cx^2}}\,dx = -\frac{5b^2(7bB-8Ac)\sqrt{bx+cx^2}}{64c^4}+\frac{5b(7bB-8Ac)x\sqrt{bx+cx^2}}{96c^3} - \frac{(7bB-8Ac)x^2\sqrt{bx+cx^2}}{24c^2}+\frac{Bx^3\sqrt{bx+cx^2}}{4c} + \frac{5b^3(7bB-8Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{64c^{9/2}}$$

output

```
-5/64*b^2*(-8*A*c+7*B*b)*(c*x^2+b*x)^(1/2)/c^4+5/96*b*(-8*A*c+7*B*b)*x*(c*
x^2+b*x)^(1/2)/c^3-1/24*(-8*A*c+7*B*b)*x^2*(c*x^2+b*x)^(1/2)/c^2+1/4*B*x^3
*(c*x^2+b*x)^(1/2)/c+5/64*b^3*(-8*A*c+7*B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)
^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.16

$$\int \frac{x^3(A+Bx)}{\sqrt{bx+cx^2}}\,dx$$
$$= \frac{\sqrt{x}(b+cx)\left(-105b^3B\sqrt{x}+120Ab^2c\sqrt{x}+70b^2Bcx^{3/2}-80Abc^2x^{3/2}-56bBc^2x^{5/2}+64Ac^3x^{5/2}+48Bc\right.}{192c^4\sqrt{x(b+cx)}}$$
$$+\frac{5b^3(7bB-8Ac)\sqrt{x}\sqrt{b+cx}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{x}}{-\sqrt{b}+\sqrt{b+cx}}\right)}{32c^{9/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[(x^3*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[x]*(b + c*x)*(-105*b^3*B*Sqrt[x] + 120*A*b^2*c*Sqrt[x] + 70*b^2*B*c*
x^(3/2) - 80*A*b*c^2*x^(3/2) - 56*b*B*c^2*x^(5/2) + 64*A*c^3*x^(5/2) + 48*
B*c^3*x^(7/2)))/(192*c^4*Sqrt[x*(b + c*x)]) + (5*b^3*(7*b*B - 8*A*c)*Sqrt[
x]*Sqrt[b + c*x]*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])])/(3
2*c^(9/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1221, 1134, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{\sqrt{bx+cx^2}}\,dx$$
$$\Big\downarrow \ 1221$$
$$\frac{Bx^3\sqrt{bx+cx^2}}{4c}-\frac{(7bB-8Ac)\int\frac{x^3}{\sqrt{cx^2+bx}}dx}{8c}$$
$$\Big\downarrow \ 1134$$

$$\frac{Bx^3\sqrt{bx+cx^2}}{4c}-\frac{(7bB-8Ac)\left(\frac{x^2\sqrt{bx+cx^2}}{3c}-\frac{5b\int\frac{x^2}{\sqrt{cx^2+bx}}dx}{6c}\right)}{8c}$$

$$\Big\downarrow \text{1134}$$

$$\frac{Bx^3\sqrt{bx+cx^2}}{4c}-\frac{(7bB-8Ac)\left(\frac{x^2\sqrt{bx+cx^2}}{3c}-\frac{5b\left(\frac{x\sqrt{bx+cx^2}}{2c}-\frac{3b\int\frac{x}{\sqrt{cx^2+bx}}dx}{4c}\right)}{6c}\right)}{8c}$$

$$\Big\downarrow \text{1160}$$

$$\frac{Bx^3\sqrt{bx+cx^2}}{4c}-\frac{(7bB-8Ac)\left(\frac{x^2\sqrt{bx+cx^2}}{3c}-\frac{5b\left(\frac{x\sqrt{bx+cx^2}}{2c}-\frac{3b\left(\frac{\sqrt{bx+cx^2}}{c}-\frac{b\int\frac{1}{\sqrt{cx^2+bx}}dx}{2c}\right)}{4c}\right)}{6c}\right)}{8c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{Bx^3\sqrt{bx+cx^2}}{4c}-\frac{(7bB-8Ac)\left(\frac{x^2\sqrt{bx+cx^2}}{3c}-\frac{5b\left(\frac{x\sqrt{bx+cx^2}}{2c}-\frac{3b\left(\frac{\sqrt{bx+cx^2}}{c}-\frac{b\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{c}\right)}{4c}\right)}{6c}\right)}{8c}$$

$$\Big\downarrow \text{219}$$

$$\frac{Bx^3\sqrt{bx+cx^2}}{4c}-\frac{(7bB-8Ac)\left(\frac{x^2\sqrt{bx+cx^2}}{3c}-\frac{5b\left(\frac{x\sqrt{bx+cx^2}}{2c}-\frac{3b\left(\frac{\sqrt{bx+cx^2}}{c}-\frac{b\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}\right)}{4c}\right)}{6c}\right)}{8c}$$

input
```
Int[(x^3*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
(B*x^3*Sqrt[b*x + c*x^2])/(4*c) - ((7*b*B - 8*A*c)*((x^2*Sqrt[b*x + c*x^2]
)/(3*c) - (5*b*((x*Sqrt[b*x + c*x^2])/(2*c) - (3*b*(Sqrt[b*x + c*x^2]/c -
(b*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/c^(3/2)))/(4*c)))/(6*c)))/(8*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$-\frac{5\left(\frac{3\left(A\,b^{3}c-\frac{7}{8}B\,b^{4}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{2}+\sqrt{x(cx+b)}\,\left(-\frac{3\left(\frac{7Bx}{12}+A\right)b^{2}c^{\frac{3}{2}}}{2}+bx\left(\frac{7Bx}{10}+A\right)c^{\frac{5}{2}}-\frac{4x^{2}\left(\frac{3Bx}{4}+A\right)c^{\frac{7}{2}}}{5}+\frac{21B\sqrt{c}\,b^{3}}{16}\right.\right.}{12c^{\frac{9}{2}}}$
risch	$\frac{(48B\,c^{3}x^{3}+64A\,c^{3}x^{2}-56Bb\,c^{2}x^{2}-80Ab\,c^{2}x+70B\,b^{2}cx+120A\,b^{2}c-105B\,b^{3})x(cx+b)}{192c^{4}\sqrt{x(cx+b)}}-\frac{5b^{3}(8Ac-7Bb)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x\right.}{128c^{\frac{9}{2}}}$
default	$A\left(\frac{x^{2}\sqrt{c\,x^{2}+bx}}{3c}-\frac{5b\left(\frac{x\sqrt{c\,x^{2}+bx}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^{2}+bx}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{2c^{\frac{3}{2}}}\right)}{4c}\right)}{6c}\right)+B\left(\frac{x^{3}\sqrt{c\,x^{2}+bx}}{4c}-\frac{7b\left(\frac{x^{2}\sqrt{}\right.}{}\right.$

input
```
int(x^3*(B*x+A)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-5/12/c^(9/2)*(3/2*(A*b^3*c-7/8*B*b^4)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2)
)+(x*(c*x+b))^(1/2)*(-3/2*(7/12*B*x+A)*b^2*c^(3/2)+b*x*(7/10*B*x+A)*c^(5/2
)-4/5*x^2*(3/4*B*x+A)*c^(7/2)+21/16*B*c^(1/2)*b^3))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.59

$$\int \frac{x^3(A+Bx)}{\sqrt{bx+cx^2}}\,dx$$
$$= \left[-\frac{15\,(7\,Bb^4-8\,Ab^3c)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(48\,Bc^4x^3-105\,Bb^3c+120\,Ab^2c^2-8\,(7}{384\,c^5}\right.$$
$$-\frac{15\,(7\,Bb^4-8\,Ab^3c)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(48\,Bc^4x^3-105\,Bb^3c+120\,Ab^2c^2-8\,(7\,Bbc^3-8\,A}{192\,c^5}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*(7*B*b^4 - 8*A*b^3*c)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b
*x)*sqrt(c)) - 2*(48*B*c^4*x^3 - 105*B*b^3*c + 120*A*b^2*c^2 - 8*(7*B*b*c^
3 - 8*A*c^4)*x^2 + 10*(7*B*b^2*c^2 - 8*A*b*c^3)*x)*sqrt(c*x^2 + b*x))/c^5,
 -1/192*(15*(7*B*b^4 - 8*A*b^3*c)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-
c)/(c*x + b)) - (48*B*c^4*x^3 - 105*B*b^3*c + 120*A*b^2*c^2 - 8*(7*B*b*c^3
 - 8*A*c^4)*x^2 + 10*(7*B*b^2*c^2 - 8*A*b*c^3)*x)*sqrt(c*x^2 + b*x))/c^5]
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.26

$$\int \frac{x^3(A+Bx)}{\sqrt{bx+cx^2}}\,dx$$
$$= \begin{cases} -\frac{5b^3\left(A-\frac{7Bb}{8c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0\\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{16c^3}+\sqrt{bx+cx^2}\left(\frac{Bx^3}{4c}+\frac{5b^2\left(A-\frac{7Bb}{8c}\right)}{8c^3}-\frac{5bx\left(A-\frac{7Bb}{8c}\right)}{12c^2}+\frac{x^2}{}\right. \\ \frac{2\left(\frac{A(bx)^{\frac{7}{2}}}{7}+\frac{B(bx)^{\frac{9}{2}}}{9b}\right)}{b^4} \\ \tilde{\infty}\left(\frac{Ax^4}{4}+\frac{Bx^5}{5}\right) \end{cases}$$

input
```
integrate(x**3*(B*x+A)/(c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((-5*b**3*(A - 7*B*b/(8*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x
 + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) +
x)/sqrt(c*(b/(2*c) + x)**2), True))/(16*c**3) + sqrt(b*x + c*x**2)*(B*x**3
/(4*c) + 5*b**2*(A - 7*B*b/(8*c))/(8*c**3) - 5*b*x*(A - 7*B*b/(8*c))/(12*c
**2) + x**2*(A - 7*B*b/(8*c))/(3*c)), Ne(c, 0)), (2*(A*(b*x)**(7/2)/7 + B*
(b*x)**(9/2)/(9*b))/b**4, Ne(b, 0)), (zoo*(A*x**4/4 + B*x**5/5), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.27

$$\int \frac{x^3(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{\sqrt{cx^2+bx}Bx^3}{4\,c} - \frac{7\,\sqrt{cx^2+bx}Bbx^2}{24\,c^2} + \frac{\sqrt{cx^2+bx}Ax^2}{3\,c} + \frac{35\,\sqrt{cx^2+bx}Bb^2x}{96\,c^3} - \frac{5\,\sqrt{cx^2+bx}Abx}{12\,c^2} + \frac{35\,Bb^4\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{128\,c^{\frac{9}{2}}} - \frac{5\,Ab^3\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{16\,c^{\frac{7}{2}}} - \frac{35\,\sqrt{cx^2+bx}Bb^3}{64\,c^4} + \frac{5\,\sqrt{cx^2+bx}Ab^2}{8\,c^3}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
1/4*sqrt(c*x^2 + b*x)*B*x^3/c - 7/24*sqrt(c*x^2 + b*x)*B*b*x^2/c^2 + 1/3*s
qrt(c*x^2 + b*x)*A*x^2/c + 35/96*sqrt(c*x^2 + b*x)*B*b^2*x/c^3 - 5/12*sqrt
(c*x^2 + b*x)*A*b*x/c^2 + 35/128*B*b^4*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)
*sqrt(c))/c^(9/2) - 5/16*A*b^3*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)
)/c^(7/2) - 35/64*sqrt(c*x^2 + b*x)*B*b^3/c^4 + 5/8*sqrt(c*x^2 + b*x)*A*b^
2/c^3
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.83

$$\int \frac{x^3(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{1}{192}\sqrt{cx^2+bx}\left(2\left(4\left(\frac{6\,Bx}{c}-\frac{7\,Bbc^2-8\,Ac^3}{c^4}\right)x+\frac{5\,(7\,Bb^2c-8\,Abc^2)}{c^4}\right)x-\frac{15\,(7\,Bb^3-8\,Ab^2c)}{c^4}\right) - \frac{5\,(7\,Bb^4-8\,Ab^3c)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{128\,c^{\frac{9}{2}}}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/192*sqrt(c*x^2 + b*x)*(2*(4*(6*B*x/c - (7*B*b*c^2 - 8*A*c^3)/c^4)*x + 5*
(7*B*b^2*c - 8*A*b*c^2)/c^4)*x - 15*(7*B*b^3 - 8*A*b^2*c)/c^4) - 5/128*(7*
B*b^4 - 8*A*b^3*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))
/c^(9/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \int \frac{x^3\,(A+B\,x)}{\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((x^3*(A + B*x))/(b*x + c*x^2)^(1/2),x)
```

output
```
int((x^3*(A + B*x))/(b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.08

$$\int \frac{x^3(A+Bx)}{\sqrt{bx+cx^2}}\,dx$$
$$= \frac{120\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^2 - 80\sqrt{x}\,\sqrt{cx+b}\,ab\,c^3x + 64\sqrt{x}\,\sqrt{cx+b}\,a\,c^4x^2 - 105\sqrt{x}\,\sqrt{cx+b}\,b^4c + 70\sqrt{x}\,\sqrt{cx}}{}$$

input
```
int(x^3*(B*x+A)/(c*x^2+b*x)^(1/2),x)
```

output
```
(120*sqrt(x)*sqrt(b + c*x)*a*b**2*c**2 - 80*sqrt(x)*sqrt(b + c*x)*a*b*c**3
*x + 64*sqrt(x)*sqrt(b + c*x)*a*c**4*x**2 - 105*sqrt(x)*sqrt(b + c*x)*b**4
*c + 70*sqrt(x)*sqrt(b + c*x)*b**3*c**2*x - 56*sqrt(x)*sqrt(b + c*x)*b**2*
c**3*x**2 + 48*sqrt(x)*sqrt(b + c*x)*b*c**4*x**3 - 120*sqrt(c)*log((sqrt(b
 + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*b**3*c + 105*sqrt(c)*log((sqrt(b + c
*x) + sqrt(x)*sqrt(c))/sqrt(b))*b**5)/(192*c**5)
```

3.144 $\int \frac{x^2(A+Bx)}{\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 127

$$\int \frac{x^2(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{b(5bB-6Ac)\sqrt{bx+cx^2}}{8c^3} - \frac{(5bB-6Ac)x\sqrt{bx+cx^2}}{12c^2} + \frac{Bx^2\sqrt{bx+cx^2}}{3c} - \frac{b^2(5bB-6Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{7/2}}$$

output
```
1/8*b*(-6*A*c+5*B*b)*(c*x^2+b*x)^(1/2)/c^3-1/12*(-6*A*c+5*B*b)*x*(c*x^2+b*
x)^(1/2)/c^2+1/3*B*x^2*(c*x^2+b*x)^(1/2)/c-1/8*b^2*(-6*A*c+5*B*b)*arctanh(
c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.98

$$\int \frac{x^2(A+Bx)}{\sqrt{bx+cx^2}}\,dx$$
$$= \frac{\sqrt{c}x(b+cx)\left(15b^2B+4c^2x(3A+2Bx)-2bc(9A+5Bx)\right)+6b^2(5bB-6Ac)\sqrt{x}\sqrt{b+cx}\,\text{arctanh}\left(\frac{\sqrt{c}\ldots}{\sqrt{b}-\sqrt{\ldots}}\right.}{24c^{7/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[(x^2*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[c]*x*(b + c*x)*(15*b^2*B + 4*c^2*x*(3*A + 2*B*x) - 2*b*c*(9*A + 5*B*
x)) + 6*b^2*(5*b*B - 6*A*c)*Sqrt[x]*Sqrt[b + c*x]*ArcTanh[(Sqrt[c]*Sqrt[x]
)/(Sqrt[b] - Sqrt[b + c*x])])/(24*c^(7/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1221, 1134, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{Bx^2\sqrt{bx+cx^2}}{3c}-\frac{(5bB-6Ac)\int \frac{x^2}{\sqrt{cx^2+bx}}dx}{6c}$$

$$\Big\downarrow \text{1134}$$

$$\frac{Bx^2\sqrt{bx+cx^2}}{3c}-\frac{(5bB-6Ac)\left(\frac{x\sqrt{bx+cx^2}}{2c}-\frac{3b\int \frac{x}{\sqrt{cx^2+bx}}dx}{4c}\right)}{6c}$$

$$\Big\downarrow \text{1160}$$

$$\frac{Bx^2\sqrt{bx+cx^2}}{3c}-\frac{(5bB-6Ac)\left(\frac{x\sqrt{bx+cx^2}}{2c}-\frac{3b\left(\frac{\sqrt{bx+cx^2}}{c}-\frac{b\int \frac{1}{\sqrt{cx^2+bx}}dx}{2c}\right)}{4c}\right)}{6c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{Bx^2\sqrt{bx+cx^2}}{3c}-\frac{(5bB-6Ac)\left(\frac{x\sqrt{bx+cx^2}}{2c}-\frac{3b\left(\frac{\sqrt{bx+cx^2}}{c}-\frac{b\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{c}\right)}{4c}\right)}{6c}$$

$\downarrow$ 219

$$\frac{Bx^2\sqrt{bx+cx^2}}{3c}-\frac{(5bB-6Ac)\left(\frac{x\sqrt{bx+cx^2}}{2c}-\frac{3b\left(\frac{\sqrt{bx+cx^2}}{c}-\frac{b\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}\right)}{4c}\right)}{6c}$$

input
```
Int[(x^2*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
(B*x^2*Sqrt[b*x + c*x^2])/(3*c) - ((5*b*B - 6*A*c)*((x*Sqrt[b*x + c*x^2])/
(2*c) - (3*b*(Sqrt[b*x + c*x^2]/c - (b*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^
2]])/c^(3/2)))/(4*c)))/(6*c)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1134
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[(m + p)*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d + e*x)^
(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && EqQ[
c*d^2 - b*d*e + a*e^2, 0] && GtQ[m, 1] && NeQ[m + 2*p + 1, 0] && IntegerQ[2
*p]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$\frac{\left(\frac{3}{2}A\,b^{2}c-\frac{5}{4}B\,b^{3}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)+\left(-\frac{3\left(\frac{5Bx}{9}+A\right)b\,c^{\frac{3}{2}}}{2}+x\left(\frac{2Bx}{3}+A\right)c^{\frac{5}{2}}+\frac{5B\sqrt{c}\,b^{2}}{4}\right)\sqrt{x(cx+b)}}{2c^{\frac{7}{2}}}$
risch	$-\frac{\left(-8B\,c^{2}x^{2}-12A\,c^{2}x+10Bbcx+18Abc-15B\,b^{2}\right)x(cx+b)}{24c^{3}\sqrt{x(cx+b)}}+\frac{b^{2}(6Ac-5Bb)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{16c^{\frac{7}{2}}}$
default	$A\left(\frac{x\sqrt{c\,x^{2}+bx}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^{2}+bx}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{2c^{\frac{3}{2}}}\right)}{4c}\right)+B\left(\frac{x^{2}\sqrt{c\,x^{2}+bx}}{3c}-\frac{5b\left(\frac{x\sqrt{c\,x^{2}+bx}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^{2}+}}{c}\right.}{}\right.}{}\right.$

input
```
int(x^2*(B*x+A)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*((3/2*A*b^2*c-5/4*B*b^3)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))+(-3/2*(5
/9*B*x+A)*b*c^(3/2)+x*(2/3*B*x+A)*c^(5/2)+5/4*B*c^(1/2)*b^2)*(x*(c*x+b))^(
1/2))/c^(7/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.64

$$\int \frac{x^2(A+Bx)}{\sqrt{bx+cx^2}}\,dx$$
$$= \left[-\frac{3\,(5\,Bb^3-6\,Ab^2c)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(8\,Bc^3x^2+15\,Bb^2c-18\,Abc^2-2\,(5\,Bbc^2}{48\,c^4}\right.$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(5*B*b^3 - 6*A*b^2*c)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x
)*sqrt(c)) - 2*(8*B*c^3*x^2 + 15*B*b^2*c - 18*A*b*c^2 - 2*(5*B*b*c^2 - 6*A
*c^3)*x)*sqrt(c*x^2 + b*x))/c^4, 1/24*(3*(5*B*b^3 - 6*A*b^2*c)*sqrt(-c)*ar
ctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (8*B*c^3*x^2 + 15*B*b^2*c - 1
8*A*b*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x)*sqrt(c*x^2 + b*x))/c^4]
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.42

$$\int \frac{x^2(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \begin{cases} \frac{3b^2\left(A-\frac{5Bb}{6c}\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise} \end{cases}\right)}{8c^2} + \sqrt{bx+cx^2}\left(\frac{Bx^2}{3c} - \frac{3b\left(A-\frac{5Bb}{6c}\right)}{4c^2} + \frac{x\left(A-\frac{5Bb}{6c}\right)}{2c}\right) & \text{for } c \neq \\ \frac{2\left(\frac{A(bx)^{\frac{5}{2}}}{5}+\frac{B(bx)^{\frac{7}{2}}}{7b}\right)}{b^3} & \text{for } b \neq \\ \tilde{\infty}\left(\frac{Ax^3}{3}+\frac{Bx^4}{4}\right) & \text{other} \end{cases}$$

input

```
integrate(x**2*(B*x+A)/(c*x**2+b*x)**(1/2),x)
```

output

```
Piecewise((3*b**2*(A - 5*B*b/(6*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x
+ c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x
)/sqrt(c*(b/(2*c) + x)**2), True))/(8*c**2) + sqrt(b*x + c*x**2)*(B*x**2/(
3*c) - 3*b*(A - 5*B*b/(6*c))/(4*c**2) + x*(A - 5*B*b/(6*c))/(2*c)), Ne(c,
0)), (2*(A*(b*x)**(5/2)/5 + B*(b*x)**(7/2)/(7*b))/b**3, Ne(b, 0)), (zoo*(A
*x**3/3 + B*x**4/4), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.26

$$\begin{aligned}\int \frac{x^2(A+Bx)}{\sqrt{bx+cx^2}}\,dx &= \frac{\sqrt{cx^2+bx}Bx^2}{3\,c} - \frac{5\,\sqrt{cx^2+bx}Bbx}{12\,c^2} + \frac{\sqrt{cx^2+bx}Ax}{2\,c} \\ &\quad - \frac{5\,Bb^3\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{16\,c^{\frac{7}{2}}} \\ &\quad + \frac{3\,Ab^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,c^{\frac{5}{2}}} \\ &\quad + \frac{5\,\sqrt{cx^2+bx}Bb^2}{8\,c^3} - \frac{3\,\sqrt{cx^2+bx}Ab}{4\,c^2}\end{aligned}$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
1/3*sqrt(c*x^2 + b*x)*B*x^2/c - 5/12*sqrt(c*x^2 + b*x)*B*b*x/c^2 + 1/2*sqr
t(c*x^2 + b*x)*A*x/c - 5/16*B*b^3*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt
(c))/c^(7/2) + 3/8*A*b^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(5
/2) + 5/8*sqrt(c*x^2 + b*x)*B*b^2/c^3 - 3/4*sqrt(c*x^2 + b*x)*A*b/c^2
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int \frac{x^2(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{1}{24}\sqrt{cx^2+bx}\left(2\left(\frac{4\,Bx}{c}-\frac{5\,Bbc-6\,Ac^2}{c^3}\right)x+\frac{3\,(5\,Bb^2-6\,Abc)}{c^3}\right) + \frac{(5\,Bb^3-6\,Ab^2c)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{7}{2}}}$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x)*(2*(4*B*x/c - (5*B*b*c - 6*A*c^2)/c^3)*x + 3*(5*B*b
^2 - 6*A*b*c)/c^3) + 1/16*(5*B*b^3 - 6*A*b^2*c)*log(abs(2*(sqrt(c)*x - sqr
t(c*x^2 + b*x))*sqrt(c) + b))/c^(7/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \int \frac{x^2\,(A+B\,x)}{\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((x^2*(A + B*x))/(b*x + c*x^2)^(1/2),x)
```

output
```
int((x^2*(A + B*x))/(b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.07

$$\int \frac{x^2(A+Bx)}{\sqrt{bx+cx^2}}\,dx$$
$$= \frac{-18\sqrt{x}\,\sqrt{cx+b}\,ab\,c^2+12\sqrt{x}\,\sqrt{cx+b}\,a\,c^3x+15\sqrt{x}\,\sqrt{cx+b}\,b^3c-10\sqrt{x}\,\sqrt{cx+b}\,b^2c^2x+8\sqrt{x}\,\sqrt{cx+b}}{24c^4}$$

input
```
int(x^2*(B*x+A)/(c*x^2+b*x)^(1/2),x)
```

output
```
( - 18*sqrt(x)*sqrt(b + c*x)*a*b*c**2 + 12*sqrt(x)*sqrt(b + c*x)*a*c**3*x
+ 15*sqrt(x)*sqrt(b + c*x)*b**3*c - 10*sqrt(x)*sqrt(b + c*x)*b**2*c**2*x +
 8*sqrt(x)*sqrt(b + c*x)*b*c**3*x**2 + 18*sqrt(c)*log((sqrt(b + c*x) + sqr
t(x)*sqrt(c))/sqrt(b))*a*b**2*c - 15*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*
sqrt(c))/sqrt(b))*b**4)/(24*c**4)
```

3.145 $\int \frac{x(A+Bx)}{\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 92

$$\int \frac{x(A+Bx)}{\sqrt{bx+cx^2}}\,dx = -\frac{(3bB-4Ac)\sqrt{bx+cx^2}}{4c^2}+\frac{Bx\sqrt{bx+cx^2}}{2c}+\frac{b(3bB-4Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{5/2}}$$

output
```
-1/4*(-4*A*c+3*B*b)*(c*x^2+b*x)^(1/2)/c^2+1/2*B*x*(c*x^2+b*x)^(1/2)/c+1/4*
b*(-4*A*c+3*B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17

$$\int \frac{x(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{\sqrt{x}\left(\sqrt{c}\sqrt{x}(b+cx)(-3bB+4Ac+2Bcx)+2b(3bB-4Ac)\sqrt{b+cx}\operatorname{arctanh}\left(\frac{\sqrt{c}\sqrt{x}}{-\sqrt{b}+\sqrt{b+cx}}\right)\right)}{4c^{5/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[(x*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
(Sqrt[x]*(Sqrt[c]*Sqrt[x]*(b + c*x)*(-3*b*B + 4*A*c + 2*B*c*x) + 2*b*(3*b*
B - 4*A*c)*Sqrt[b + c*x]*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*
x])]))/(4*c^(5/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1225, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{1225}$$

$$\frac{b(3bB-4Ac)\int \frac{1}{\sqrt{cx^2+bx}}dx}{8c^2}-\frac{\sqrt{bx+cx^2}(-4Ac+3bB-2Bcx)}{4c^2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{b(3bB-4Ac)\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{4c^2}-\frac{\sqrt{bx+cx^2}(-4Ac+3bB-2Bcx)}{4c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{b(3bB-4Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{5/2}}-\frac{\sqrt{bx+cx^2}(-4Ac+3bB-2Bcx)}{4c^2}$$

input
```
Int[(x*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
-1/4*((3*b*B - 4*A*c - 2*B*c*x)*Sqrt[b*x + c*x^2])/c^2 + (b*(3*b*B - 4*A*c
)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/(4*c^(5/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1225
```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Simp[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3))   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c
, d, e, f, g, p}, x] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.70

method	result
pseudoelliptic	$\frac{\left(-Abc+\frac{3}{4}B\,b^{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)+\left(\left(\frac{Bx}{2}+A\right)c^{\frac{3}{2}}-\frac{3B\sqrt{c}\,b}{4}\right)\sqrt{x(cx+b)}}{c^{\frac{5}{2}}}$
risch	$\frac{(2Bcx+4Ac-3Bb)x(cx+b)}{4c^{2}\sqrt{x(cx+b)}}-\frac{b(4Ac-3Bb)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{8c^{\frac{5}{2}}}$
default	$A\left(\frac{\sqrt{c\,x^{2}+bx}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{2c^{\frac{3}{2}}}\right)+B\left(\frac{x\sqrt{c\,x^{2}+bx}}{2c}-\frac{3b\left(\frac{\sqrt{c\,x^{2}+bx}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{2c^{\frac{3}{2}}}\right)}{4c}\right)$

input
```
int(x*(B*x+A)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^(5/2)*((-A*b*c+3/4*B*b^2)*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))+((1/2*B
*x+A)*c^(3/2)-3/4*B*c^(1/2)*b)*(x*(c*x+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.73

$$\int \frac{x(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \left[-\frac{(3\,Bb^2-4\,Abc)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(2\,Bc^2x-3\,Bbc+4\,Ac^2)\sqrt{cx^2+bx}}{8\,c^3},\right.$$
$$\left.-\frac{(3\,Bb^2-4\,Abc)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(2\,Bc^2x-3\,Bbc+4\,Ac^2)\sqrt{cx^2+bx}}{4\,c^3}\right]$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((3*B*b^2 - 4*A*b*c)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqr
t(c)) - 2*(2*B*c^2*x - 3*B*b*c + 4*A*c^2)*sqrt(c*x^2 + b*x))/c^3, -1/4*((3
*B*b^2 - 4*A*b*c)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) -
(2*B*c^2*x - 3*B*b*c + 4*A*c^2)*sqrt(c*x^2 + b*x))/c^3]
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.64

$$\int \frac{x(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \begin{cases} -\frac{b\left(A-\frac{3Bb}{4c}\right)\left(\begin{cases}\frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for } \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise}\end{cases}\right)}{2c} + \sqrt{bx+cx^2}\left(\frac{Bx}{2c}+\frac{A-\frac{3Bb}{4c}}{c}\right) & \text{for } c\neq 0 \\ \frac{2\left(\frac{A(bx)^{\frac{3}{2}}}{3}+\frac{B(bx)^{\frac{5}{2}}}{5b}\right)}{b^2} & \text{for } b\neq 0 \\ \tilde{\infty}\left(\frac{Ax^2}{2}+\frac{Bx^3}{3}\right) & \text{otherwise}\end{cases}$$

input
```
integrate(x*(B*x+A)/(c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((-b*(A - 3*B*b/(4*c))*Piecewise((log(b + 2*sqrt(c)*sqrt(b*x + c*
x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)*log(b/(2*c) + x)/sq
rt(c*(b/(2*c) + x)**2), True))/(2*c) + sqrt(b*x + c*x**2)*(B*x/(2*c) + (A
- 3*B*b/(4*c))/c), Ne(c, 0)), (2*(A*(b*x)**(3/2)/3 + B*(b*x)**(5/2)/(5*b))
/b**2, Ne(b, 0)), (zoo*(A*x**2/2 + B*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.25

$$\int \frac{x(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{\sqrt{cx^2+bx}Bx}{2\,c} + \frac{3\,Bb^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,c^{\frac{5}{2}}} - \frac{Ab\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{2\,c^{\frac{3}{2}}} - \frac{3\,\sqrt{cx^2+bx}Bb}{4\,c^2} + \frac{\sqrt{cx^2+bx}A}{c}$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*sqrt(c*x^2 + b*x)*B*x/c + 3/8*B*b^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x
)*sqrt(c))/c^(5/2) - 1/2*A*b*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/
c^(3/2) - 3/4*sqrt(c*x^2 + b*x)*B*b/c^2 + sqrt(c*x^2 + b*x)*A/c
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.88

$$\int \frac{x(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{1}{4}\sqrt{cx^2+bx}\left(\frac{2\,Bx}{c}-\frac{3\,Bb-4\,Ac}{c^2}\right) - \frac{(3\,Bb^2-4\,Abc)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{5}{2}}}$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x)*(2*B*x/c - (3*B*b - 4*A*c)/c^2) - 1/8*(3*B*b^2 - 4*A
*b*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \int \frac{x\,(A+B\,x)}{\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((x*(A + B*x))/(b*x + c*x^2)^(1/2),x)
```

output
```
int((x*(A + B*x))/(b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.08

$$\int \frac{x(A+Bx)}{\sqrt{bx+cx^2}}\,dx$$
$$= \frac{4\sqrt{x}\,\sqrt{cx+b}\,a\,c^2 - 3\sqrt{x}\,\sqrt{cx+b}\,b^2c + 2\sqrt{x}\,\sqrt{cx+b}\,b\,c^2x - 4\sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)abc + 3\sqrt{c}\log\left(\frac{\sqrt{cx+}}{}\right.}{4c^3}$$

input
```
int(x*(B*x+A)/(c*x^2+b*x)^(1/2),x)
```

output
```
(4*sqrt(x)*sqrt(b + c*x)*a*c**2 - 3*sqrt(x)*sqrt(b + c*x)*b**2*c + 2*sqrt(
x)*sqrt(b + c*x)*b*c**2*x - 4*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c)
)/sqrt(b))*a*b*c + 3*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b)
)*b**3)/(4*c**3)
```

3.146 $\int \frac{A+Bx}{\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 55

$$\int \frac{A+Bx}{\sqrt{bx+cx^2}}\,dx=\frac{B\sqrt{bx+cx^2}}{c}-\frac{(bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}$$

output
```
B*(c*x^2+b*x)^(1/2)/c-(-2*A*c+B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^
(3/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.56

$$\int \frac{A+Bx}{\sqrt{bx+cx^2}}\,dx=\frac{B\sqrt{c}x(b+cx)+2(bB-2Ac)\sqrt{x}\sqrt{b+cx}\text{arctanh}\left(\frac{\sqrt{c}\sqrt{x}}{\sqrt{b}-\sqrt{b+cx}}\right)}{c^{3/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[(A + B*x)/Sqrt[b*x + c*x^2],x]
```

output
```
(B*Sqrt[c]*x*(b + c*x) + 2*(b*B - 2*A*c)*Sqrt[x]*Sqrt[b + c*x]*ArcTanh[(Sq
rt[c]*Sqrt[x])/(Sqrt[b] - Sqrt[b + c*x])])/(c^(3/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{1160}$$

$$\frac{B\sqrt{bx+cx^2}}{c}-\frac{(bB-2Ac)\int \frac{1}{\sqrt{cx^2+bx}}dx}{2c}$$

$$\Big\downarrow \text{1091}$$

$$\frac{B\sqrt{bx+cx^2}}{c}-\frac{(bB-2Ac)\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{c}$$

$$\Big\downarrow \text{219}$$

$$\frac{B\sqrt{bx+cx^2}}{c}-\frac{(bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}$$

input
```
Int[(A + B*x)/Sqrt[b*x + c*x^2],x]
```

output
```
(B*Sqrt[b*x + c*x^2])/c - ((b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*
x^2]])/c^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.84

method	result	size
pseudoelliptic	$\frac{B\sqrt{x(cx+b)}}{c}+\frac{(2Ac-Bb)\,\mathrm{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{c^{\frac{3}{2}}}$	46
risch	$\frac{Bx(cx+b)}{c\sqrt{x(cx+b)}}+\frac{(2Ac-Bb)\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{2c^{\frac{3}{2}}}$	60
default	$\frac{A\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{\sqrt{c}}+B\left(\frac{\sqrt{c\,x^2+bx}}{c}-\frac{b\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{2c^{\frac{3}{2}}}\right)$	79

input
```
int((B*x+A)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
B*(x*(c*x+b))^(1/2)/c+(2*A*c-B*b)/c^(3/2)*arctanh((x*(c*x+b))^(1/2)/x/c^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.11

$$\int\frac{A+Bx}{\sqrt{bx+cx^2}}\,dx$$
$$=\left[\frac{2\sqrt{cx^2+bx}Bc-(Bb-2\,Ac)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{2\,c^2},\frac{\sqrt{cx^2+bx}Bc+(Bb-2\,Ac)\sqrt{-c}\,\mathrm{ar}}{c^2}\right.$$

input
```
integrate((B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[1/2*(2*sqrt(c*x^2 + b*x)*B*c - (B*b - 2*A*c)*sqrt(c)*log(2*c*x + b + 2*sq
rt(c*x^2 + b*x)*sqrt(c)))/c^2, (sqrt(c*x^2 + b*x)*B*c + (B*b - 2*A*c)*sqrt
(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)))/c^2]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(48) = 96$.

Time = 0.37 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.22

$$\int \frac{A+Bx}{\sqrt{bx+cx^2}}\,dx = \begin{cases} \frac{B\sqrt{bx+cx^2}}{c} + \left(A-\frac{Bb}{2c}\right)\left(\begin{cases} \frac{\log\left(b+2\sqrt{c}\sqrt{bx+cx^2}+2cx\right)}{\sqrt{c}} & \text{for}\ \frac{b^2}{c}\neq 0 \\ \frac{\left(\frac{b}{2c}+x\right)\log\left(\frac{b}{2c}+x\right)}{\sqrt{c\left(\frac{b}{2c}+x\right)^2}} & \text{otherwise} \end{cases}\right) & \text{for}\ c\neq 0 \\ \frac{2A\sqrt{bx}+\frac{2B(bx)^{\frac{3}{2}}}{3b}}{b} & \text{for}\ b\neq 0 \\ \tilde{\infty}\left(Ax+\frac{Bx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((B*x+A)/(c*x**2+b*x)**(1/2),x)
```

output
```
Piecewise((B*sqrt(b*x + c*x**2)/c + (A - B*b/(2*c))*Piecewise((log(b + 2*s
qrt(c)*sqrt(b*x + c*x**2) + 2*c*x)/sqrt(c), Ne(b**2/c, 0)), ((b/(2*c) + x)
*log(b/(2*c) + x)/sqrt(c*(b/(2*c) + x)**2), True)), Ne(c, 0)), ((2*A*sqrt(
b*x) + 2*B*(b*x)**(3/2)/(3*b))/b, Ne(b, 0)), (zoo*(A*x + B*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.36

$$\int \frac{A+Bx}{\sqrt{bx+cx^2}}\,dx = -\frac{Bb\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{2\,c^{\frac{3}{2}}} + \frac{A\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{\sqrt{c}} + \frac{\sqrt{cx^2+bx}B}{c}$$

input
```
integrate((B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*B*b*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(3/2) + A*log(2*c*
x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/sqrt(c) + sqrt(c*x^2 + b*x)*B/c
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int \frac{A+Bx}{\sqrt{bx+cx^2}}\,dx = \frac{\sqrt{cx^2+bx}B}{c} + \frac{(Bb-2\,Ac)\log\left(\left|2\,\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
sqrt(c*x^2 + b*x)*B/c + 1/2*(B*b - 2*A*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^
2 + b*x))*sqrt(c) + b))/c^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.79 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.40

$$\int \frac{A+Bx}{\sqrt{bx+cx^2}}\,dx = \frac{A\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{\sqrt{c}} + \frac{B\,\sqrt{c\,x^2+b\,x}}{c} - \frac{B\,b\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{2\,c^{3/2}}$$

input
```
int((A + B*x)/(b*x + c*x^2)^(1/2),x)
```

output
```
(A*log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/c^(1/2) + (B*(b*x + c*x
^2)^(1/2))/c - (B*b*log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/(2*c^(
3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx}{\sqrt{bx+cx^2}}\,dx=\frac{\sqrt{x}\,\sqrt{cx+b}\,bc+2\sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)ac-\sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^2}{c^2}$$

input
```
int((B*x+A)/(c*x^2+b*x)^(1/2),x)
```

output
```
(sqrt(x)*sqrt(b + c*x)*b*c + 2*sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c
))/sqrt(b))*a*c - sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b
**2)/c**2
```

3.147 $\int \frac{A+Bx}{x\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 52

$$\int \frac{A+Bx}{x\sqrt{bx+cx^2}}\,dx = -\frac{2A\sqrt{bx+cx^2}}{bx}+\frac{2B\,\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}$$

output
```
-2*A*(c*x^2+b*x)^(1/2)/b/x+2*B*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(1/2
)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.42

$$\int \frac{A+Bx}{x\sqrt{bx+cx^2}}\,dx = -\frac{2\left(A\sqrt{c}(b+cx)+bB\sqrt{x}\sqrt{b+cx}\log\left(-\sqrt{c}\sqrt{x}+\sqrt{b+cx}\right)\right)}{b\sqrt{c}\sqrt{x(b+cx)}}$$

input
```
Integrate[(A + B*x)/(x*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*(A*Sqrt[c]*(b + c*x) + b*B*Sqrt[x]*Sqrt[b + c*x]*Log[-(Sqrt[c]*Sqrt[x]
) + Sqrt[b + c*x]]))/(b*Sqrt[c]*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {1220, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{ 1220}$$

$$B\int \frac{1}{\sqrt{cx^2+bx}}dx-\frac{2A\sqrt{bx+cx^2}}{bx}$$

$$\Big\downarrow \text{ 1091}$$

$$2B\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}-\frac{2A\sqrt{bx+cx^2}}{bx}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2B\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}-\frac{2A\sqrt{bx+cx^2}}{bx}$$

input
```
Int[(A + B*x)/(x*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*A*Sqrt[b*x + c*x^2])/(b*x) + (2*B*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2
]])/Sqrt[c]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

method	result	size
pseudoelliptic	$-\frac{2\left(-B\,\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)bx+A\sqrt{x(cx+b)}\,\sqrt{c}\right)}{x\sqrt{c}\,b}$	49
default	$\frac{B\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{\sqrt{c}}-\frac{2A\sqrt{c\,x^{2}+bx}}{bx}$	51
risch	$-\frac{2A(cx+b)}{b\sqrt{x(cx+b)}}+\frac{B\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{\sqrt{c}}$	51

input
```
int((B*x+A)/x/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(-B*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))*b*x+A*(x*(c*x+b))^(1/2)*c^(1/2
))/x/c^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.25

$$\int \frac{A+Bx}{x\sqrt{bx+cx^2}}\,dx = \left[\frac{Bb\sqrt{c}x\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,\sqrt{cx^2+bx}Ac}{bcx},\right.$$
$$\left.-\frac{2\left(Bb\sqrt{-c}x\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)+\sqrt{cx^2+bx}Ac\right)}{bcx}\right]$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[(B*b*sqrt(c)*x*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*sqrt(c*x^
2 + b*x)*A*c)/(b*c*x), -2*(B*b*sqrt(-c)*x*arctan(sqrt(c*x^2 + b*x)*sqrt(-c
)/(c*x + b)) + sqrt(c*x^2 + b*x)*A*c)/(b*c*x)]
```

Sympy [F]

$$\int \frac{A+Bx}{x\sqrt{bx+cx^2}}\,dx = \int \frac{A+Bx}{x\sqrt{x\left(b+cx\right)}}\,dx$$

input
```
integrate((B*x+A)/x/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral((A + B*x)/(x*sqrt(x*(b + c*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{x\sqrt{bx+cx^2}}\,dx = \frac{B\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{\sqrt{c}} - \frac{2\,\sqrt{cx^2+bx}A}{bx}$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
B*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/sqrt(c) - 2*sqrt(c*x^2 + b*
x)*A/(b*x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.13

$$\int \frac{A+Bx}{x\sqrt{bx+cx^2}}\,dx = -\frac{B\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{\sqrt{c}}+\frac{2\,A}{\sqrt{c}x-\sqrt{cx^2+bx}}$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
-B*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/sqrt(c) + 2*A/(
sqrt(c)*x - sqrt(c*x^2 + b*x))
```

Mupad [B] (verification not implemented)

Time = 5.77 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{A+Bx}{x\sqrt{bx+cx^2}}\,dx = \frac{B\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{\sqrt{c}}-\frac{2\,A\,\sqrt{c\,x^2+b\,x}}{b\,x}$$

input
```
int((A + B*x)/(x*(b*x + c*x^2)^(1/2)),x)
```

output
```
(B*log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/c^(1/2) - (2*A*(b*x + c
*x^2)^(1/2))/(b*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.08

$$\int \frac{A+Bx}{x\sqrt{bx+cx^2}}\,dx = \frac{-2\sqrt{x}\,\sqrt{cx+b}\,ac+2\sqrt{c}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^2x-2\sqrt{c}\,acx}{bcx}$$

input
```
int((B*x+A)/x/(c*x^2+b*x)^(1/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(b + c*x)*a*c + sqrt(c)*log((sqrt(b + c*x) + sqrt(x)*sq
rt(c))/sqrt(b))*b**2*x - sqrt(c)*a*c*x))/(b*c*x)
```

3.148 $\int \frac{A+Bx}{x^2\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 57

$$\int \frac{A+Bx}{x^2\sqrt{bx+cx^2}}\,dx=-\frac{2A\sqrt{bx+cx^2}}{3bx^2}-\frac{2(3bB-2Ac)\sqrt{bx+cx^2}}{3b^2x}$$

output
```
-2/3*A*(c*x^2+b*x)^(1/2)/b/x^2-2/3*(-2*A*c+3*B*b)*(c*x^2+b*x)^(1/2)/b^2/x
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx}{x^2\sqrt{bx+cx^2}}\,dx=-\frac{2\sqrt{x(b+cx)}(3bBx+A(b-2cx))}{3b^2x^2}$$

input
```
Integrate[(A + B*x)/(x^2*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*Sqrt[x*(b + c*x)]*(3*b*B*x + A*(b - 2*c*x)))/(3*b^2*x^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1220, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(3bB-2Ac)\int \frac{1}{x\sqrt{cx^2+bx}}dx}{3b}-\frac{2A\sqrt{bx+cx^2}}{3bx^2}$$

$$\Big\downarrow \text{ 1123}$$

$$-\frac{2\sqrt{bx+cx^2}(3bB-2Ac)}{3b^2x}-\frac{2A\sqrt{bx+cx^2}}{3bx^2}$$

input
```
Int[(A + B*x)/(x^2*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*A*Sqrt[b*x + c*x^2])/(3*b*x^2) - (2*(3*b*B - 2*A*c)*Sqrt[b*x + c*x^2])
/(3*b^2*x)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.88 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.56

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x(cx+b)}\,((3Bx+A)b-2Acx)}{3b^2x^2}$	32
trager	$-\frac{2(-2Acx+3Bbx+Ab)\sqrt{c\,x^2+bx}}{3b^2x^2}$	34
risch	$-\frac{2(cx+b)(-2Acx+3Bbx+Ab)}{3b^2x\sqrt{x(cx+b)}}$	37
gosper	$-\frac{2(cx+b)(-2Acx+3Bbx+Ab)}{3x\,b^2\sqrt{c\,x^2+bx}}$	39
orering	$-\frac{2(cx+b)(-2Acx+3Bbx+Ab)}{3x\,b^2\sqrt{c\,x^2+bx}}$	39
default	$A\left(-\frac{2\sqrt{c\,x^2+bx}}{3b\,x^2}+\frac{4c\sqrt{c\,x^2+bx}}{3b^2x}\right)-\frac{2B\sqrt{c\,x^2+bx}}{bx}$	64

input
```
int((B*x+A)/x^2/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(x*(c*x+b))^(1/2)*((3*B*x+A)*b-2*A*c*x)/b^2/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx}{x^2\sqrt{bx+cx^2}}\,dx = -\frac{2\sqrt{cx^2+bx}(Ab+(3\,Bb-2\,Ac)x)}{3\,b^2x^2}$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(c*x^2 + b*x)*(A*b + (3*B*b - 2*A*c)*x)/(b^2*x^2)
```

Sympy [F]

$$\int \frac{A+Bx}{x^2\sqrt{bx+cx^2}}\,dx = \int \frac{A+Bx}{x^2\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate((B*x+A)/x**2/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**2*sqrt(x*(b + c*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx}{x^2\sqrt{bx+cx^2}}\,dx = -\frac{2\sqrt{cx^2+bx}B}{bx}+\frac{4\sqrt{cx^2+bx}Ac}{3\,b^2x}-\frac{2\sqrt{cx^2+bx}A}{3\,bx^2}$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
-2*sqrt(c*x^2 + b*x)*B/(b*x) + 4/3*sqrt(c*x^2 + b*x)*A*c/(b^2*x) - 2/3*sqr
t(c*x^2 + b*x)*A/(b*x^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \frac{A+Bx}{x^2\sqrt{bx+cx^2}}\,dx = \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2B+3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)A\sqrt{c}+Ab\right)}{3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3}$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B + 3*(sqrt(c)*x - sqrt(c*x^2 + b
*x))*A*sqrt(c) + A*b)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^3
```

Mupad [B] (verification not implemented)

Time = 5.44 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.58

$$\int \frac{A+Bx}{x^2\sqrt{bx+cx^2}}\,dx = -\frac{2\sqrt{c\,x^2+b\,x}\,(A\,b-2\,A\,c\,x+3\,B\,b\,x)}{3\,b^2\,x^2}$$

input
```
int((A + B*x)/(x^2*(b*x + c*x^2)^(1/2)),x)
```

output
```
-(2*(b*x + c*x^2)^(1/2)*(A*b - 2*A*c*x + 3*B*b*x))/(3*b^2*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{x^2\sqrt{bx+cx^2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,ab}{3}+\frac{4\sqrt{x}\,\sqrt{cx+b}\,acx}{3}-2\sqrt{x}\,\sqrt{cx+b}\,b^2x-\frac{4\sqrt{c}\,ac\,x^2}{3}+\frac{2\sqrt{c}\,b^2x^2}{3}}{b^2x^2}$$

input
```
int((B*x+A)/x^2/(c*x^2+b*x)^(1/2),x)
```

output
```
(2*( - sqrt(x)*sqrt(b + c*x)*a*b + 2*sqrt(x)*sqrt(b + c*x)*a*c*x - 3*sqrt(
x)*sqrt(b + c*x)*b**2*x - 2*sqrt(c)*a*c*x**2 + sqrt(c)*b**2*x**2))/(3*b**2
*x**2)
```

3.149 $\int \frac{A+Bx}{x^3\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 90

$$\int \frac{A+Bx}{x^3\sqrt{bx+cx^2}}\,dx = -\frac{2A\sqrt{bx+cx^2}}{5bx^3} - \frac{2(5bB-4Ac)\sqrt{bx+cx^2}}{15b^2x^2} + \frac{4c(5bB-4Ac)\sqrt{bx+cx^2}}{15b^3x}$$

output
```
-2/5*A*(c*x^2+b*x)^(1/2)/b/x^3-2/15*(-4*A*c+5*B*b)*(c*x^2+b*x)^(1/2)/b^2/x
^2+4/15*c*(-4*A*c+5*B*b)*(c*x^2+b*x)^(1/2)/b^3/x
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx}{x^3\sqrt{bx+cx^2}}\,dx = -\frac{2\sqrt{x(b+cx)}(5bBx(b-2cx)+A(3b^2-4bcx+8c^2x^2))}{15b^3x^3}$$

input
```
Integrate[(A + B*x)/(x^3*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*Sqrt[x*(b + c*x)]*(5*b*B*x*(b - 2*c*x) + A*(3*b^2 - 4*b*c*x + 8*c^2*x^
2)))/(15*b^3*x^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1220, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3\sqrt{bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(5bB-4Ac)\int \frac{1}{x^2\sqrt{cx^2+bx}}dx}{5b}-\frac{2A\sqrt{bx+cx^2}}{5bx^3}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(5bB-4Ac)\left(-\frac{2c\int \frac{1}{x\sqrt{cx^2+bx}}dx}{3b}-\frac{2\sqrt{bx+cx^2}}{3bx^2}\right)}{5b}-\frac{2A\sqrt{bx+cx^2}}{5bx^3}$$

$$\Big\downarrow \text{1123}$$

$$\frac{\left(\frac{4c\sqrt{bx+cx^2}}{3b^2x}-\frac{2\sqrt{bx+cx^2}}{3bx^2}\right)(5bB-4Ac)}{5b}-\frac{2A\sqrt{bx+cx^2}}{5bx^3}$$

input
```
Int[(A + B*x)/(x^3*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*A*Sqrt[b*x + c*x^2])/(5*b*x^3) + ((5*b*B - 4*A*c)*((-2*Sqrt[b*x + c*x^
2])/(3*b*x^2) + (4*c*Sqrt[b*x + c*x^2])/(3*b^2*x)))/(5*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.54

method	result	size
pseudoelliptic	$-\frac{2\sqrt{x(cx+b)}\left(\left(\frac{5Bx}{3}+A\right)b^2-\frac{4cx\left(\frac{5Bx}{2}+A\right)b}{3}+\frac{8Ac^2x^2}{3}\right)}{5b^3x^3}$	49
trager	$-\frac{2(8Ac^2x^2-10x^2Bbc-4Abcx+5xBb^2+3b^2A)\sqrt{cx^2+bx}}{15b^3x^3}$	57
risch	$-\frac{2(cx+b)(8Ac^2x^2-10x^2Bbc-4Abcx+5xBb^2+3b^2A)}{15b^3x^2\sqrt{x(cx+b)}}$	60
gosper	$-\frac{2(cx+b)(8Ac^2x^2-10x^2Bbc-4Abcx+5xBb^2+3b^2A)}{15x^2b^3\sqrt{cx^2+bx}}$	62
orering	$-\frac{2(cx+b)(8Ac^2x^2-10x^2Bbc-4Abcx+5xBb^2+3b^2A)}{15x^2b^3\sqrt{cx^2+bx}}$	62
default	$A\left(-\frac{2\sqrt{cx^2+bx}}{5b\,x^3}-\frac{4c\left(-\frac{2\sqrt{cx^2+bx}}{3b\,x^2}+\frac{4c\sqrt{cx^2+bx}}{3b^2x}\right)}{5b}\right)+B\left(-\frac{2\sqrt{cx^2+bx}}{3b\,x^2}+\frac{4c\sqrt{cx^2+bx}}{3b^2x}\right)$	112

input
```
int((B*x+A)/x^3/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5*(x*(c*x+b))^(1/2)*((5/3*B*x+A)*b^2-4/3*c*x*(5/2*B*x+A)*b+8/3*A*c^2*x^
2)/b^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx}{x^3\sqrt{bx+cx^2}}\,dx = -\frac{2\,(3\,Ab^2-2\,(5\,Bbc-4\,Ac^2)x^2+(5\,Bb^2-4\,Abc)x)\sqrt{cx^2+bx}}{15\,b^3x^3}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*(3*A*b^2 - 2*(5*B*b*c - 4*A*c^2)*x^2 + (5*B*b^2 - 4*A*b*c)*x)*sqrt(c
*x^2 + b*x)/(b^3*x^3)
```

Sympy [F]

$$\int \frac{A+Bx}{x^3\sqrt{bx+cx^2}}\,dx=\int \frac{A+Bx}{x^3\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate((B*x+A)/x**3/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**3*sqrt(x*(b + c*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx}{x^3\sqrt{bx+cx^2}}\,dx=\frac{4\,\sqrt{cx^2+bx}Bc}{3\,b^2x}-\frac{16\,\sqrt{cx^2+bx}Ac^2}{15\,b^3x}-\frac{2\,\sqrt{cx^2+bx}B}{3\,bx^2}+\frac{8\,\sqrt{cx^2+bx}Ac}{15\,b^2x^2}-\frac{2\,\sqrt{cx^2+bx}A}{5\,bx^3}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
4/3*sqrt(c*x^2 + b*x)*B*c/(b^2*x) - 16/15*sqrt(c*x^2 + b*x)*A*c^2/(b^3*x)
- 2/3*sqrt(c*x^2 + b*x)*B/(b*x^2) + 8/15*sqrt(c*x^2 + b*x)*A*c/(b^2*x^2) -
 2/5*sqrt(c*x^2 + b*x)*A/(b*x^3)
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx}{x^3\sqrt{bx+cx^2}}\,dx$$

$$=\frac{2\left(15\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3B\sqrt{c}+5\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2Bb+20\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2Ac+15\left(\sqrt{c}x-\right.\right.}{15\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
2/15*(15*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*B*sqrt(c) + 5*(sqrt(c)*x - sqrt
(c*x^2 + b*x))^2*B*b + 20*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*A*c + 15*(sqrt
(c)*x - sqrt(c*x^2 + b*x))*A*b*sqrt(c) + 3*A*b^2)/(sqrt(c)*x - sqrt(c*x^2
+ b*x))^5
```

Mupad [B] (verification not implemented)

Time = 5.29 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx}{x^3\sqrt{bx+cx^2}}\,dx = -\frac{2\sqrt{c\,x^2+b\,x}\,\left(5\,B\,b^2\,x+3\,A\,b^2-10\,B\,b\,c\,x^2-4\,A\,b\,c\,x+8\,A\,c^2\,x^2\right)}{15\,b^3\,x^3}$$

input
```
int((A + B*x)/(x^3*(b*x + c*x^2)^(1/2)),x)
```

output
```
-(2*(b*x + c*x^2)^(1/2)*(3*A*b^2 + 8*A*c^2*x^2 + 5*B*b^2*x - 10*B*b*c*x^2
- 4*A*b*c*x))/(15*b^3*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{x^3\sqrt{bx+cx^2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^2}{5}+\frac{8\sqrt{x}\,\sqrt{cx+b}\,abcx}{15}-\frac{16\sqrt{x}\,\sqrt{cx+b}\,a\,c^2x^2}{15}-\frac{2\sqrt{x}\,\sqrt{cx+b}\,b^3x}{3}+\frac{4\sqrt{x}\,\sqrt{cx+b}\,b^2c\,x^2}{3}+\frac{16\sqrt{c}\,a\,c^2x^3}{15}-\frac{4\sqrt{c}\,b^2c\,x^3}{3}}{b^3x^3}$$

input
```
int((B*x+A)/x^3/(c*x^2+b*x)^(1/2),x)
```

output
```
(2*( - 3*sqrt(x)*sqrt(b + c*x)*a*b**2 + 4*sqrt(x)*sqrt(b + c*x)*a*b*c*x -
8*sqrt(x)*sqrt(b + c*x)*a*c**2*x**2 - 5*sqrt(x)*sqrt(b + c*x)*b**3*x + 10*
sqrt(x)*sqrt(b + c*x)*b**2*c*x**2 + 8*sqrt(c)*a*c**2*x**3 - 10*sqrt(c)*b**
2*c*x**3))/(15*b**3*x**3)
```

3.150 $\int \frac{A+Bx}{x^4\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 125

$$\int \frac{A+Bx}{x^4\sqrt{bx+cx^2}}\,dx=-\frac{2A\sqrt{bx+cx^2}}{7bx^4}-\frac{2(7bB-6Ac)\sqrt{bx+cx^2}}{35b^2x^3}+\frac{8c(7bB-6Ac)\sqrt{bx+cx^2}}{105b^3x^2}-\frac{16c^2(7bB-6Ac)\sqrt{bx+cx^2}}{105b^4x}$$

output
```
-2/7*A*(c*x^2+b*x)^(1/2)/b/x^4-2/35*(-6*A*c+7*B*b)*(c*x^2+b*x)^(1/2)/b^2/x
^3+8/105*c*(-6*A*c+7*B*b)*(c*x^2+b*x)^(1/2)/b^3/x^2-16/105*c^2*(-6*A*c+7*B
*b)*(c*x^2+b*x)^(1/2)/b^4/x
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx}{x^4\sqrt{bx+cx^2}}\,dx=-\frac{2\sqrt{x(b+cx)}(7bBx(3b^2-4bcx+8c^2x^2)+3A(5b^3-6b^2cx+8bc^2x^2-16c^3x^3))}{105b^4x^4}$$

input
```
Integrate[(A + B*x)/(x^4*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*Sqrt[x*(b + c*x)]*(7*b*B*x*(3*b^2 - 4*b*c*x + 8*c^2*x^2) + 3*A*(5*b^3
- 6*b^2*c*x + 8*b*c^2*x^2 - 16*c^3*x^3)))/(105*b^4*x^4)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1220, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^4\sqrt{bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(7bB-6Ac)\int \frac{1}{x^3\sqrt{cx^2+bx}}dx}{7b}-\frac{2A\sqrt{bx+cx^2}}{7bx^4}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(7bB-6Ac)\left(-\frac{4c\int \frac{1}{x^2\sqrt{cx^2+bx}}dx}{5b}-\frac{2\sqrt{bx+cx^2}}{5bx^3}\right)}{7b}-\frac{2A\sqrt{bx+cx^2}}{7bx^4}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(7bB-6Ac)\left(-\frac{4c\left(-\frac{2c\int \frac{1}{x\sqrt{cx^2+bx}}dx}{3b}-\frac{2\sqrt{bx+cx^2}}{3bx^2}\right)}{5b}-\frac{2\sqrt{bx+cx^2}}{5bx^3}\right)}{7b}-\frac{2A\sqrt{bx+cx^2}}{7bx^4}$$

$$\Big\downarrow \text{1123}$$

$$\frac{\left(-\frac{4c\left(\frac{4c\sqrt{bx+cx^2}}{3b^2x}-\frac{2\sqrt{bx+cx^2}}{3bx^2}\right)}{5b}-\frac{2\sqrt{bx+cx^2}}{5bx^3}\right)(7bB-6Ac)}{7b}-\frac{2A\sqrt{bx+cx^2}}{7bx^4}$$

input
```
Int[(A + B*x)/(x^4*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*A*Sqrt[b*x + c*x^2])/(7*b*x^4) + ((7*b*B - 6*A*c)*((-2*Sqrt[b*x + c*x^
2])/(5*b*x^3) - (4*c*((-2*Sqrt[b*x + c*x^2])/(3*b*x^2) + (4*c*Sqrt[b*x + c
*x^2])/(3*b^2*x)))/(5*b)))/(7*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.53

method	result
pseudoelliptic	$-\frac{2\sqrt{x(cx+b)}\left(\left(\frac{7Bx}{5}+A\right)b^3-\frac{6cx\left(\frac{14Bx}{9}+A\right)b^2}{5}+\frac{8c^2\left(\frac{7Bx}{3}+A\right)x^2b}{5}-\frac{16Ac^3x^3}{5}\right)}{7b^4x^4}$
trager	$-\frac{2\left(-48Ac^3x^3+56x^3Bbc^2+24Abc^2x^2-28x^2Bb^2c-18Ab^2cx+21xBb^3+15Ab^3\right)\sqrt{cx^2+bx}}{105b^4x^4}$
risch	$-\frac{2(cx+b)\left(-48Ac^3x^3+56x^3Bbc^2+24Abc^2x^2-28x^2Bb^2c-18Ab^2cx+21xBb^3+15Ab^3\right)}{105b^4x^3\sqrt{x(cx+b)}}$
gosper	$-\frac{2(cx+b)\left(-48Ac^3x^3+56x^3Bbc^2+24Abc^2x^2-28x^2Bb^2c-18Ab^2cx+21xBb^3+15Ab^3\right)}{105x^3b^4\sqrt{cx^2+bx}}$
orering	$-\frac{2(cx+b)\left(-48Ac^3x^3+56x^3Bbc^2+24Abc^2x^2-28x^2Bb^2c-18Ab^2cx+21xBb^3+15Ab^3\right)}{105x^3b^4\sqrt{cx^2+bx}}$
default	$A\left(-\frac{2\sqrt{cx^2+bx}}{7bx^4}-\frac{6c\left(-\frac{2\sqrt{cx^2+bx}}{5bx^3}-\frac{4c\left(-\frac{2\sqrt{cx^2+bx}}{3bx^2}+\frac{4c\sqrt{cx^2+bx}}{3b^2x}\right)}{5b}\right)}{7b}\right)+B\left(-\frac{2\sqrt{cx^2+bx}}{5bx^3}-\frac{4c\left(-\frac{2\sqrt{cx^2+}}{3bx^2}\right.}{}\right.$

input
```
int((B*x+A)/x^4/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*(x*(c*x+b))^(1/2)*((7/5*B*x+A)*b^3-6/5*c*x*(14/9*B*x+A)*b^2+8/5*c^2*(
7/3*B*x+A)*x^2*b-16/5*A*c^3*x^3)/b^4/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{x^4\sqrt{bx+cx^2}}\,dx = -\frac{2\left(15\,Ab^3+8\left(7\,Bbc^2-6\,Ac^3\right)x^3-4\left(7\,Bb^2c-6\,Abc^2\right)x^2+3\left(7\,Bb^3-6\,Ab^2c\right)x\right)\sqrt{cx^2+bx}}{105\,b^4x^4}$$

input
```
integrate((B*x+A)/x^4/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(15*A*b^3 + 8*(7*B*b*c^2 - 6*A*c^3)*x^3 - 4*(7*B*b^2*c - 6*A*b*c^2)
*x^2 + 3*(7*B*b^3 - 6*A*b^2*c)*x)*sqrt(c*x^2 + b*x)/(b^4*x^4)
```

Sympy [F]

$$\int \frac{A+Bx}{x^4\sqrt{bx+cx^2}}\,dx = \int \frac{A+Bx}{x^4\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate((B*x+A)/x**4/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**4*sqrt(x*(b + c*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx}{x^4\sqrt{bx+cx^2}}\,dx = -\frac{16\,\sqrt{cx^2+bx}Bc^2}{15\,b^3x} + \frac{32\,\sqrt{cx^2+bx}Ac^3}{35\,b^4x} + \frac{8\,\sqrt{cx^2+bx}Bc}{15\,b^2x^2} - \frac{16\,\sqrt{cx^2+bx}Ac^2}{35\,b^3x^2} - \frac{2\,\sqrt{cx^2+bx}B}{5\,bx^3} + \frac{12\,\sqrt{cx^2+bx}Ac}{35\,b^2x^3} - \frac{2\,\sqrt{cx^2+bx}A}{7\,bx^4}$$

input
```
integrate((B*x+A)/x^4/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
-16/15*sqrt(c*x^2 + b*x)*B*c^2/(b^3*x) + 32/35*sqrt(c*x^2 + b*x)*A*c^3/(b^
4*x) + 8/15*sqrt(c*x^2 + b*x)*B*c/(b^2*x^2) - 16/35*sqrt(c*x^2 + b*x)*A*c^
2/(b^3*x^2) - 2/5*sqrt(c*x^2 + b*x)*B/(b*x^3) + 12/35*sqrt(c*x^2 + b*x)*A*
c/(b^2*x^3) - 2/7*sqrt(c*x^2 + b*x)*A/(b*x^4)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.53

$$\int \frac{A+Bx}{x^4\sqrt{bx+cx^2}}\,dx = \frac{2\left(140\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4Bc+105\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3Bb\sqrt{c}+210\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^3Ac^{\frac{3}{2}}+21\left(\cdots\right.\right.}{105\left(\sqrt{c}x-\sqrt{\cdots}\right.}$$

input
```
integrate((B*x+A)/x^4/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
2/105*(140*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*B*c + 105*(sqrt(c)*x - sqrt(c
*x^2 + b*x))^3*B*b*sqrt(c) + 210*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*A*c^(3/
2) + 21*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*b^2 + 252*(sqrt(c)*x - sqrt(c*
x^2 + b*x))^2*A*b*c + 105*(sqrt(c)*x - sqrt(c*x^2 + b*x))*A*b^2*sqrt(c) +
15*A*b^3)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^7
```

Mupad [B] (verification not implemented)

Time = 5.27 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{x^4\sqrt{bx+cx^2}}\,dx = \frac{\sqrt{c\,x^2+b\,x}\,\left(96\,A\,c^3-112\,B\,b\,c^2\right)}{105\,b^4\,x} - \frac{\left(48\,A\,c^2-56\,B\,b\,c\right)\,\sqrt{c\,x^2+b\,x}}{105\,b^3\,x^2} - \frac{2\,A\,\sqrt{c\,x^2+b\,x}}{7\,b\,x^4} + \frac{\sqrt{c\,x^2+b\,x}\,\left(12\,A\,c-14\,B\,b\right)}{35\,b^2\,x^3}$$

input
```
int((A + B*x)/(x^4*(b*x + c*x^2)^(1/2)),x)
```

output
```
((b*x + c*x^2)^(1/2)*(96*A*c^3 - 112*B*b*c^2))/(105*b^4*x) - ((48*A*c^2 -
56*B*b*c)*(b*x + c*x^2)^(1/2))/(105*b^3*x^2) - (2*A*(b*x + c*x^2)^(1/2))/(
7*b*x^4) + ((b*x + c*x^2)^(1/2)*(12*A*c - 14*B*b))/(35*b^2*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx}{x^4\sqrt{bx+cx^2}}\,dx$$
$$= \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^3}{7}+\frac{12\sqrt{x}\,\sqrt{cx+b}\,a\,b^2cx}{35}-\frac{16\sqrt{x}\,\sqrt{cx+b}\,ab\,c^2x^2}{35}+\frac{32\sqrt{x}\,\sqrt{cx+b}\,a\,c^3x^3}{35}-\frac{2\sqrt{x}\,\sqrt{cx+b}\,b^4x}{5}+\frac{8\sqrt{x}\,\sqrt{cx+b}\,b^3c\,x^2}{15}-1}{b^4x^4}$$

input
```
int((B*x+A)/x^4/(c*x^2+b*x)^(1/2),x)
```

output
```
(2*( - 15*sqrt(x)*sqrt(b + c*x)*a*b**3 + 18*sqrt(x)*sqrt(b + c*x)*a*b**2*c
*x - 24*sqrt(x)*sqrt(b + c*x)*a*b*c**2*x**2 + 48*sqrt(x)*sqrt(b + c*x)*a*c
**3*x**3 - 21*sqrt(x)*sqrt(b + c*x)*b**4*x + 28*sqrt(x)*sqrt(b + c*x)*b**3
*c*x**2 - 56*sqrt(x)*sqrt(b + c*x)*b**2*c**2*x**3 - 48*sqrt(c)*a*c**3*x**4
 + 56*sqrt(c)*b**2*c**2*x**4))/(105*b**4*x**4)
```

3.151 $\int \frac{A+Bx}{x^5\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 160

$$\int \frac{A+Bx}{x^5\sqrt{bx+cx^2}}\,dx = -\frac{2A\sqrt{bx+cx^2}}{9bx^5} - \frac{2(9bB-8Ac)\sqrt{bx+cx^2}}{63b^2x^4} + \frac{4c(9bB-8Ac)\sqrt{bx+cx^2}}{105b^3x^3} - \frac{16c^2(9bB-8Ac)\sqrt{bx+cx^2}}{315b^4x^2} + \frac{32c^3(9bB-8Ac)\sqrt{bx+cx^2}}{315b^5x}$$

output
```
-2/9*A*(c*x^2+b*x)^(1/2)/b/x^5-2/63*(-8*A*c+9*B*b)*(c*x^2+b*x)^(1/2)/b^2/x
^4+4/105*c*(-8*A*c+9*B*b)*(c*x^2+b*x)^(1/2)/b^3/x^3-16/315*c^2*(-8*A*c+9*B
*b)*(c*x^2+b*x)^(1/2)/b^4/x^2+32/315*c^3*(-8*A*c+9*B*b)*(c*x^2+b*x)^(1/2)/
b^5/x
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx}{x^5\sqrt{bx+cx^2}}\,dx = -\frac{2\sqrt{x(b+cx)}(9bBx(5b^3-6b^2cx+8bc^2x^2-16c^3x^3)+A(35b^4-40b^3cx+48b^2c^2x^2-64bc^3x^3+128c}{315b^5x^5}$$

input
```
Integrate[(A + B*x)/(x^5*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*Sqrt[x*(b + c*x)]*(9*b*B*x*(5*b^3 - 6*b^2*c*x + 8*b*c^2*x^2 - 16*c^3*x
^3) + A*(35*b^4 - 40*b^3*c*x + 48*b^2*c^2*x^2 - 64*b*c^3*x^3 + 128*c^4*x^4
)))/(315*b^5*x^5)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1220, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^5\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(9bB-8Ac)\int \frac{1}{x^4\sqrt{cx^2+bx}}dx}{9b}-\frac{2A\sqrt{bx+cx^2}}{9bx^5}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(9bB-8Ac)\left(-\frac{6c\int \frac{1}{x^3\sqrt{cx^2+bx}}dx}{7b}-\frac{2\sqrt{bx+cx^2}}{7bx^4}\right)}{9b}-\frac{2A\sqrt{bx+cx^2}}{9bx^5}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(9bB-8Ac)\left(-\frac{6c\left(-\frac{4c\int \frac{1}{x^2\sqrt{cx^2+bx}}dx}{5b}-\frac{2\sqrt{bx+cx^2}}{5bx^3}\right)}{7b}-\frac{2\sqrt{bx+cx^2}}{7bx^4}\right)}{9b}-\frac{2A\sqrt{bx+cx^2}}{9bx^5}$$

$\Big\downarrow$ 1129

$$\frac{(9bB-8Ac)\left(-\frac{6c\left(-\frac{4c\left(-\frac{2c\int \frac{1}{x\sqrt{cx^2+bx}}dx}{3b}-\frac{2\sqrt{bx+cx^2}}{3bx^2}\right)}{5b}-\frac{2\sqrt{bx+cx^2}}{5bx^3}\right)}{7b}-\frac{2\sqrt{bx+cx^2}}{7bx^4}\right)}{9b}-\frac{2A\sqrt{bx+cx^2}}{9bx^5}$$

$\Big\downarrow$ 1123

$$\frac{\left(-\frac{6c\left(-\frac{4c\left(\frac{4c\sqrt{bx+cx^2}}{3b^2x}-\frac{2\sqrt{bx+cx^2}}{3bx^2}\right)}{5b}-\frac{2\sqrt{bx+cx^2}}{5bx^3}\right)}{7b}-\frac{2\sqrt{bx+cx^2}}{7bx^4}\right)(9bB-8Ac)}{9b}-\frac{2A\sqrt{bx+cx^2}}{9bx^5}$$

input
```
Int[(A + B*x)/(x^5*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*A*Sqrt[b*x + c*x^2])/(9*b*x^5) + ((9*b*B - 8*A*c)*((-2*Sqrt[b*x + c*x^
2])/(7*b*x^4) - (6*c*((-2*Sqrt[b*x + c*x^2])/(5*b*x^3) - (4*c*((-2*Sqrt[b*
x + c*x^2])/(3*b*x^2) + (4*c*Sqrt[b*x + c*x^2])/(3*b^2*x)))/(5*b)))/(7*b))
)/(9*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.52

method	result
pseudoelliptic	$-\frac{2\sqrt{x(cx+b)}\left(\left(\frac{9Bx}{7}+A\right)b^4-\frac{8cx\left(\frac{27Bx}{20}+A\right)b^3}{7}+\frac{48c^2x^2\left(\frac{3Bx}{2}+A\right)b^2}{35}-\frac{64c^3x^3\left(\frac{9Bx}{4}+A\right)b}{35}+\frac{128Ac^4x^4}{35}\right)}{9b^5x^5}$
trager	$-\frac{2\left(128Ac^4x^4-144Bbc^3x^4-64Abc^3x^3+72Bb^2c^2x^3+48Ab^2c^2x^2-54Bb^3cx^2-40Ab^3cx+45Bb^4x+35Ab^4\right)\sqrt{cx^2+bx}}{315b^5x^5}$
risch	$-\frac{2(cx+b)\left(128Ac^4x^4-144Bbc^3x^4-64Abc^3x^3+72Bb^2c^2x^3+48Ab^2c^2x^2-54Bb^3cx^2-40Ab^3cx+45Bb^4x+35Ab^4\right)}{315b^5x^4\sqrt{x(cx+b)}}$
gosper	$-\frac{2(cx+b)\left(128Ac^4x^4-144Bbc^3x^4-64Abc^3x^3+72Bb^2c^2x^3+48Ab^2c^2x^2-54Bb^3cx^2-40Ab^3cx+45Bb^4x+35Ab^4\right)}{315x^4b^5\sqrt{cx^2+bx}}$
orering	$-\frac{2(cx+b)\left(128Ac^4x^4-144Bbc^3x^4-64Abc^3x^3+72Bb^2c^2x^3+48Ab^2c^2x^2-54Bb^3cx^2-40Ab^3cx+45Bb^4x+35Ab^4\right)}{315x^4b^5\sqrt{cx^2+bx}}$
default	$A\left(-\frac{2\sqrt{cx^2+bx}}{9bx^5}-\frac{8c\left(-\frac{2\sqrt{cx^2+bx}}{7bx^4}-\frac{6c\left(-\frac{2\sqrt{cx^2+bx}}{5bx^3}-\frac{4c\left(-\frac{2\sqrt{cx^2+bx}}{3bx^2}+\frac{4c\sqrt{cx^2+bx}}{3b^2x}\right)}{5b}\right)}{7b}\right)}{9b}\right)+B\left(-\frac{2\sqrt{cx^2+b}}{7bx^4}\right.$

input
```
int((B*x+A)/x^5/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9*(x*(c*x+b))^(1/2)*((9/7*B*x+A)*b^4-8/7*c*x*(27/20*B*x+A)*b^3+48/35*c^
2*x^2*(3/2*B*x+A)*b^2-64/35*c^3*x^3*(9/4*B*x+A)*b+128/35*A*c^4*x^4)/b^5/x^
5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{x^5\sqrt{bx+cx^2}}\,dx = -\frac{2\left(35\,Ab^4-16\left(9\,Bbc^3-8\,Ac^4\right)x^4+8\left(9\,Bb^2c^2-8\,Abc^3\right)x^3-6\left(9\,Bb^3c-8\,Ab^2c^2\right)x^2+5\left(9\,Bb^4-8\,A\right.}{315\,b^5x^5}$$

input
```
integrate((B*x+A)/x^5/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/315*(35*A*b^4 - 16*(9*B*b*c^3 - 8*A*c^4)*x^4 + 8*(9*B*b^2*c^2 - 8*A*b*c
^3)*x^3 - 6*(9*B*b^3*c - 8*A*b^2*c^2)*x^2 + 5*(9*B*b^4 - 8*A*b^3*c)*x)*sqr
t(c*x^2 + b*x)/(b^5*x^5)
```

Sympy [F]

$$\int \frac{A+Bx}{x^5\sqrt{bx+cx^2}}\,dx = \int \frac{A+Bx}{x^5\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate((B*x+A)/x**5/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**5*sqrt(x*(b + c*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx}{x^5\sqrt{bx+cx^2}}\,dx = \frac{32\sqrt{cx^2+bx}Bc^3}{35\,b^4x} - \frac{256\sqrt{cx^2+bx}Ac^4}{315\,b^5x} - \frac{16\sqrt{cx^2+bx}Bc^2}{35\,b^3x^2} + \frac{128\sqrt{cx^2+bx}Ac^3}{315\,b^4x^2} + \frac{12\sqrt{cx^2+bx}Bc}{35\,b^2x^3} - \frac{32\sqrt{cx^2+bx}Ac^2}{105\,b^3x^3} - \frac{2\sqrt{cx^2+bx}B}{7\,bx^4} + \frac{16\sqrt{cx^2+bx}Ac}{63\,b^2x^4} - \frac{2\sqrt{cx^2+bx}A}{9\,bx^5}$$

input
```
integrate((B*x+A)/x^5/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
32/35*sqrt(c*x^2 + b*x)*B*c^3/(b^4*x) - 256/315*sqrt(c*x^2 + b*x)*A*c^4/(b
^5*x) - 16/35*sqrt(c*x^2 + b*x)*B*c^2/(b^3*x^2) + 128/315*sqrt(c*x^2 + b*x
)*A*c^3/(b^4*x^2) + 12/35*sqrt(c*x^2 + b*x)*B*c/(b^2*x^3) - 32/105*sqrt(c*
x^2 + b*x)*A*c^2/(b^3*x^3) - 2/7*sqrt(c*x^2 + b*x)*B/(b*x^4) + 16/63*sqrt(
c*x^2 + b*x)*A*c/(b^2*x^4) - 2/9*sqrt(c*x^2 + b*x)*A/(b*x^5)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.57

$$\int \frac{A+Bx}{x^5\sqrt{bx+cx^2}}\,dx = \frac{2\left(630\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5Bc^{\frac{3}{2}} + 756\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4Bbc + 1008\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^4Ac^2 + 315\right.}{}$$

input
```
integrate((B*x+A)/x^5/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
2/315*(630*(sqrt(c)*x - sqrt(c*x^2 + b*x))^5*B*c^(3/2) + 756*(sqrt(c)*x -
sqrt(c*x^2 + b*x))^4*B*b*c + 1008*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*A*c^2
+ 315*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*B*b^2*sqrt(c) + 1680*(sqrt(c)*x -
sqrt(c*x^2 + b*x))^3*A*b*c^(3/2) + 45*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*
b^3 + 1080*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*A*b^2*c + 315*(sqrt(c)*x - sq
rt(c*x^2 + b*x))*A*b^3*sqrt(c) + 35*A*b^4)/(sqrt(c)*x - sqrt(c*x^2 + b*x))
^9
```

Mupad [B] (verification not implemented)

Time = 5.34 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{x^5\sqrt{bx+cx^2}}\,dx = \frac{\sqrt{c\,x^2+b\,x}\,\left(128\,A\,c^3-144\,B\,b\,c^2\right)}{315\,b^4\,x^2} - \frac{\sqrt{c\,x^2+b\,x}\,\left(256\,A\,c^4-288\,B\,b\,c^3\right)}{315\,b^5\,x} - \frac{\left(32\,A\,c^2-36\,B\,b\,c\right)\,\sqrt{c\,x^2+b\,x}}{105\,b^3\,x^3} - \frac{2\,A\,\sqrt{c\,x^2+b\,x}}{9\,b\,x^5} + \frac{\sqrt{c\,x^2+b\,x}\,\left(16\,A\,c-18\,B\,b\right)}{63\,b^2\,x^4}$$

input
```
int((A + B*x)/(x^5*(b*x + c*x^2)^(1/2)),x)
```

output
```
((b*x + c*x^2)^(1/2)*(128*A*c^3 - 144*B*b*c^2))/(315*b^4*x^2) - ((b*x + c*
x^2)^(1/2)*(256*A*c^4 - 288*B*b*c^3))/(315*b^5*x) - ((32*A*c^2 - 36*B*b*c)
*(b*x + c*x^2)^(1/2))/(105*b^3*x^3) - (2*A*(b*x + c*x^2)^(1/2))/(9*b*x^5)
+ ((b*x + c*x^2)^(1/2)*(16*A*c - 18*B*b))/(63*b^2*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx}{x^5\sqrt{bx+cx^2}}\,dx = \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^4}{9}+\frac{16\sqrt{x}\,\sqrt{cx+b}\,a\,b^3cx}{63}-\frac{32\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^2x^2}{105}+\frac{128\sqrt{x}\,\sqrt{cx+b}\,ab\,c^3x^3}{315}-\frac{256\sqrt{x}\,\sqrt{cx+b}\,a\,c^4x^4}{315}-\frac{2\sqrt{x}\,\sqrt{cx+b}\,b^5x}{7}}{b^5x^5}$$

input
```
int((B*x+A)/x^5/(c*x^2+b*x)^(1/2),x)
```

output
```
(2*( - 35*sqrt(x)*sqrt(b + c*x)*a*b**4 + 40*sqrt(x)*sqrt(b + c*x)*a*b**3*c
*x - 48*sqrt(x)*sqrt(b + c*x)*a*b**2*c**2*x**2 + 64*sqrt(x)*sqrt(b + c*x)*
a*b*c**3*x**3 - 128*sqrt(x)*sqrt(b + c*x)*a*c**4*x**4 - 45*sqrt(x)*sqrt(b
+ c*x)*b**5*x + 54*sqrt(x)*sqrt(b + c*x)*b**4*c*x**2 - 72*sqrt(x)*sqrt(b +
 c*x)*b**3*c**2*x**3 + 144*sqrt(x)*sqrt(b + c*x)*b**2*c**3*x**4 + 128*sqrt
(c)*a*c**4*x**5 - 144*sqrt(c)*b**2*c**3*x**5))/(315*b**5*x**5)
```

3.152 $\int \frac{A+Bx}{x^6\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 195

$$\int \frac{A+Bx}{x^6\sqrt{bx+cx^2}}\,dx = -\frac{2A\sqrt{bx+cx^2}}{11bx^6} - \frac{2(11bB-10Ac)\sqrt{bx+cx^2}}{99b^2x^5} + \frac{16c(11bB-10Ac)\sqrt{bx+cx^2}}{693b^3x^4} - \frac{32c^2(11bB-10Ac)\sqrt{bx+cx^2}}{1155b^4x^3} + \frac{128c^3(11bB-10Ac)\sqrt{bx+cx^2}}{3465b^5x^2} - \frac{256c^4(11bB-10Ac)\sqrt{bx+cx^2}}{3465b^6x}$$

output
```
-2/11*A*(c*x^2+b*x)^(1/2)/b/x^6-2/99*(-10*A*c+11*B*b)*(c*x^2+b*x)^(1/2)/b^
2/x^5+16/693*c*(-10*A*c+11*B*b)*(c*x^2+b*x)^(1/2)/b^3/x^4-32/1155*c^2*(-10
*A*c+11*B*b)*(c*x^2+b*x)^(1/2)/b^4/x^3+128/3465*c^3*(-10*A*c+11*B*b)*(c*x^
2+b*x)^(1/2)/b^5/x^2-256/3465*c^4*(-10*A*c+11*B*b)*(c*x^2+b*x)^(1/2)/b^6/x
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.63

$$\int \frac{A+Bx}{x^6\sqrt{bx+cx^2}}\,dx = -\frac{2\sqrt{x(b+cx)}(11bBx(35b^4-40b^3cx+48b^2c^2x^2-64bc^3x^3+128c^4x^4)+5A(63b^5-70b^4cx+80b^3c^2x^2}{3465b^6x^6}$$

input
```
Integrate[(A + B*x)/(x^6*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*Sqrt[x*(b + c*x)]*(11*b*B*x*(35*b^4 - 40*b^3*c*x + 48*b^2*c^2*x^2 - 64
*b*c^3*x^3 + 128*c^4*x^4) + 5*A*(63*b^5 - 70*b^4*c*x + 80*b^3*c^2*x^2 - 96
*b^2*c^3*x^3 + 128*b*c^4*x^4 - 256*c^5*x^5)))/(3465*b^6*x^6)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1220, 1129, 1129, 1129, 1129, 1123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^6\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(11bB-10Ac)\int \frac{1}{x^5\sqrt{cx^2+bx}}dx}{11b}-\frac{2A\sqrt{bx+cx^2}}{11bx^6}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(11bB-10Ac)\left(-\frac{8c\int \frac{1}{x^4\sqrt{cx^2+bx}}dx}{9b}-\frac{2\sqrt{bx+cx^2}}{9bx^5}\right)}{11b}-\frac{2A\sqrt{bx+cx^2}}{11bx^6}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(11bB-10Ac)\left(-\frac{8c\left(-\frac{6c\int\frac{1}{x^3\sqrt{cx^2+bx}}dx}{7b}-\frac{2\sqrt{bx+cx^2}}{7bx^4}\right)}{9b}-\frac{2\sqrt{bx+cx^2}}{9bx^5}\right)}{11b}-\frac{2A\sqrt{bx+cx^2}}{11bx^6}$$

$\downarrow$ 1129

$$\frac{(11bB-10Ac)\left(-\frac{8c\left(-\frac{6c\left(-\frac{4c\int\frac{1}{x^2\sqrt{cx^2+bx}}dx}{5b}-\frac{2\sqrt{bx+cx^2}}{5bx^3}\right)}{7b}-\frac{2\sqrt{bx+cx^2}}{7bx^4}\right)}{9b}-\frac{2\sqrt{bx+cx^2}}{9bx^5}\right)}{11b}-\frac{2A\sqrt{bx+cx^2}}{11bx^6}$$

$\downarrow$ 1129

$$\frac{(11bB-10Ac)\left(-\frac{8c\left(-\frac{6c\left(-\frac{4c\left(-\frac{2c\int\frac{1}{x\sqrt{cx^2+bx}}dx}{3b}-\frac{2\sqrt{bx+cx^2}}{3bx^2}\right)}{5b}-\frac{2\sqrt{bx+cx^2}}{5bx^3}\right)}{7b}-\frac{2\sqrt{bx+cx^2}}{7bx^4}\right)}{9b}-\frac{2\sqrt{bx+cx^2}}{9bx^5}\right)}{11b}-\frac{2A\sqrt{bx+cx^2}}{11bx^6}$$

$\downarrow$ 1123

$$\frac{\left(-\frac{8c\left(-\frac{6c\left(-\frac{4c\left(\frac{4c\sqrt{bx+cx^2}}{3b^2x}-\frac{2\sqrt{bx+cx^2}}{3bx^2}\right)}{5b}-\frac{2\sqrt{bx+cx^2}}{5bx^3}\right)}{7b}-\frac{2\sqrt{bx+cx^2}}{7bx^4}\right)}{9b}-\frac{2\sqrt{bx+cx^2}}{9bx^5}\right)(11bB-10Ac)}{11b}-\frac{2A\sqrt{bx+cx^2}}{11bx^6}$$

input
```
Int[(A + B*x)/(x^6*Sqrt[b*x + c*x^2]),x]
```

output
```
(-2*A*Sqrt[b*x + c*x^2])/(11*b*x^6) + ((11*b*B - 10*A*c)*((-2*Sqrt[b*x + c
*x^2])/(9*b*x^5) - (8*c*((-2*Sqrt[b*x + c*x^2])/(7*b*x^4) - (6*c*((-2*Sqrt
[b*x + c*x^2])/(5*b*x^3) - (4*c*((-2*Sqrt[b*x + c*x^2])/(3*b*x^2) + (4*c*S
qrt[b*x + c*x^2])/(3*b^2*x)))/(5*b)))/(7*b)))/(9*b)))/(11*b)
```

Defintions of rubi rules used

rule 1123
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(2*c*d - b
*e))), x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2,
0] && EqQ[m + 2*p + 2, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.51

method	result
pseudoelliptic	$-\frac{2\sqrt{x(cx+b)}\left(\left(\frac{11Bx}{9}+A\right)b^5-\frac{10\left(\frac{44Bx}{35}+A\right)cx\,b^4}{9}+\frac{80c^2\left(\frac{33Bx}{25}+A\right)x^2b^3}{63}-\frac{32\left(\frac{22Bx}{15}+A\right)c^3x^3b^2}{21}+\frac{128c^4x^4\left(\frac{11Bx}{5}+A\right)b}{63}-\frac{256}{}\cdots\right)}{11b^6x^6}$
trager	$-\frac{2(-1280A\,c^5x^5+1408Bb\,c^4x^5+640Ab\,c^4x^4-704B\,b^2c^3x^4-480A\,b^2c^3x^3+528B\,b^3c^2x^3+400A\,b^3c^2x^2-440B\,b^4c\,x^2-35\cdots)}{3465b^6x^6}$
risch	$-\frac{2(cx+b)(-1280A\,c^5x^5+1408Bb\,c^4x^5+640Ab\,c^4x^4-704B\,b^2c^3x^4-480A\,b^2c^3x^3+528B\,b^3c^2x^3+400A\,b^3c^2x^2-440B\,b^4c\cdots)}{3465b^6x^5\sqrt{x(cx+b)}}$
gosper	$-\frac{2(cx+b)(-1280A\,c^5x^5+1408Bb\,c^4x^5+640Ab\,c^4x^4-704B\,b^2c^3x^4-480A\,b^2c^3x^3+528B\,b^3c^2x^3+400A\,b^3c^2x^2-440B\,b^4c\cdots)}{3465x^5b^6\sqrt{c\,x^2+bx}}$
orering	$-\frac{2(cx+b)(-1280A\,c^5x^5+1408Bb\,c^4x^5+640Ab\,c^4x^4-704B\,b^2c^3x^4-480A\,b^2c^3x^3+528B\,b^3c^2x^3+400A\,b^3c^2x^2-440B\,b^4c\cdots)}{3465x^5b^6\sqrt{c\,x^2+bx}}$
default	$A\left(-\frac{2\sqrt{c\,x^2+bx}}{11b\,x^6}-\frac{10c\left(-\frac{2\sqrt{c\,x^2+bx}}{9b\,x^5}-\frac{8c\left(-\frac{2\sqrt{c\,x^2+bx}}{7b\,x^4}-\frac{6c\left(-\frac{2\sqrt{c\,x^2+bx}}{5b\,x^3}-\frac{4c\left(-\frac{2\sqrt{c\,x^2+bx}}{3b\,x^2}+\frac{4c\sqrt{c\,x^2+bx}}{3b^2x}\right)}{5b}\right)}{7b}\right)}{9b}\right)}{11b}\right)\cdots$

input
```
int((B*x+A)/x^6/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/11*(x*(c*x+b))^(1/2)*((11/9*B*x+A)*b^5-10/9*(44/35*B*x+A)*c*x*b^4+80/63
*c^2*(33/25*B*x+A)*x^2*b^3-32/21*(22/15*B*x+A)*c^3*x^3*b^2+128/63*c^4*x^4*
(11/5*B*x+A)*b-256/63*A*c^5*x^5)/b^6/x^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.67

$$\int \frac{A+Bx}{x^6\sqrt{bx+cx^2}}\,dx = -\frac{2\,(315\,Ab^5+128\,(11\,Bbc^4-10\,Ac^5)x^5-64\,(11\,Bb^2c^3-10\,Abc^4)x^4+48\,(11\,Bb^3c^2-10\,Ab^2c^3)x^3-4}{3465\,b^6x^6}$$

input
```
integrate((B*x+A)/x^6/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
-2/3465*(315*A*b^5 + 128*(11*B*b*c^4 - 10*A*c^5)*x^5 - 64*(11*B*b^2*c^3 -
10*A*b*c^4)*x^4 + 48*(11*B*b^3*c^2 - 10*A*b^2*c^3)*x^3 - 40*(11*B*b^4*c -
10*A*b^3*c^2)*x^2 + 35*(11*B*b^5 - 10*A*b^4*c)*x)*sqrt(c*x^2 + b*x)/(b^6*x
^6)
```

Sympy [F]

$$\int \frac{A+Bx}{x^6\sqrt{bx+cx^2}}\,dx = \int \frac{A+Bx}{x^6\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate((B*x+A)/x**6/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**6*sqrt(x*(b + c*x))), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.25

$$\int \frac{A+Bx}{x^6\sqrt{bx+cx^2}}\,dx = -\frac{256\sqrt{cx^2+bx}Bc^4}{315\,b^5x} + \frac{512\sqrt{cx^2+bx}Ac^5}{693\,b^6x} + \frac{128\sqrt{cx^2+bx}Bc^3}{315\,b^4x^2} - \frac{256\sqrt{cx^2+bx}Ac^4}{693\,b^5x^2} - \frac{32\sqrt{cx^2+bx}Bc^2}{105\,b^3x^3} + \frac{64\sqrt{cx^2+bx}Ac^3}{231\,b^4x^3} + \frac{16\sqrt{cx^2+bx}Bc}{63\,b^2x^4} - \frac{160\sqrt{cx^2+bx}Ac^2}{693\,b^3x^4} - \frac{2\sqrt{cx^2+bx}B}{9\,bx^5} + \frac{20\sqrt{cx^2+bx}Ac}{99\,b^2x^5} - \frac{2\sqrt{cx^2+bx}A}{11\,bx^6}$$

input
```
integrate((B*x+A)/x^6/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
-256/315*sqrt(c*x^2 + b*x)*B*c^4/(b^5*x) + 512/693*sqrt(c*x^2 + b*x)*A*c^5
/(b^6*x) + 128/315*sqrt(c*x^2 + b*x)*B*c^3/(b^4*x^2) - 256/693*sqrt(c*x^2
+ b*x)*A*c^4/(b^5*x^2) - 32/105*sqrt(c*x^2 + b*x)*B*c^2/(b^3*x^3) + 64/231
*sqrt(c*x^2 + b*x)*A*c^3/(b^4*x^3) + 16/63*sqrt(c*x^2 + b*x)*B*c/(b^2*x^4)
 - 160/693*sqrt(c*x^2 + b*x)*A*c^2/(b^3*x^4) - 2/9*sqrt(c*x^2 + b*x)*B/(b*
x^5) + 20/99*sqrt(c*x^2 + b*x)*A*c/(b^2*x^5) - 2/11*sqrt(c*x^2 + b*x)*A/(b
*x^6)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.59

$$\int \frac{A+Bx}{x^6\sqrt{bx+cx^2}}\,dx = \frac{2\left(11088\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^6Bc^2 + 18480\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5Bbc^{\frac{3}{2}} + 18480\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^5Ac^{\frac{5}{2}}\right.}{}$$

input
```
integrate((B*x+A)/x^6/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
2/3465*(11088*(sqrt(c)*x - sqrt(c*x^2 + b*x))^6*B*c^2 + 18480*(sqrt(c)*x -
 sqrt(c*x^2 + b*x))^5*B*b*c^(3/2) + 18480*(sqrt(c)*x - sqrt(c*x^2 + b*x))^
5*A*c^(5/2) + 11880*(sqrt(c)*x - sqrt(c*x^2 + b*x))^4*B*b^2*c + 39600*(sqr
t(c)*x - sqrt(c*x^2 + b*x))^4*A*b*c^2 + 3465*(sqrt(c)*x - sqrt(c*x^2 + b*x
))^3*B*b^3*sqrt(c) + 34650*(sqrt(c)*x - sqrt(c*x^2 + b*x))^3*A*b^2*c^(3/2)
 + 385*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*b^4 + 15400*(sqrt(c)*x - sqrt(c
*x^2 + b*x))^2*A*b^3*c + 3465*(sqrt(c)*x - sqrt(c*x^2 + b*x))*A*b^4*sqrt(c
) + 315*A*b^5)/(sqrt(c)*x - sqrt(c*x^2 + b*x))^11
```

Mupad [B] (verification not implemented)

Time = 5.26 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{x^6\sqrt{bx+cx^2}}\,dx = \frac{\sqrt{c\,x^2+b\,x}\,(320\,A\,c^3-352\,B\,b\,c^2)}{1155\,b^4\,x^3} - \frac{\sqrt{c\,x^2+b\,x}\,(1280\,A\,c^4-1408\,B\,b\,c^3)}{3465\,b^5\,x^2} - \frac{(160\,A\,c^2-176\,B\,b\,c)\,\sqrt{c\,x^2+b\,x}}{693\,b^3\,x^4} - \frac{2\,A\,\sqrt{c\,x^2+b\,x}}{11\,b\,x^6} + \frac{\sqrt{c\,x^2+b\,x}\,(20\,A\,c-22\,B\,b)}{99\,b^2\,x^5} + \frac{256\,c^4\,\sqrt{c\,x^2+b\,x}\,(10\,A\,c-11\,B\,b)}{3465\,b^6\,x}$$

input
```
int((A + B*x)/(x^6*(b*x + c*x^2)^(1/2)),x)
```

output
```
((b*x + c*x^2)^(1/2)*(320*A*c^3 - 352*B*b*c^2))/(1155*b^4*x^3) - ((b*x + c
*x^2)^(1/2)*(1280*A*c^4 - 1408*B*b*c^3))/(3465*b^5*x^2) - ((160*A*c^2 - 17
6*B*b*c)*(b*x + c*x^2)^(1/2))/(693*b^3*x^4) - (2*A*(b*x + c*x^2)^(1/2))/(1
1*b*x^6) + ((b*x + c*x^2)^(1/2)*(20*A*c - 22*B*b))/(99*b^2*x^5) + (256*c^4
*(b*x + c*x^2)^(1/2)*(10*A*c - 11*B*b))/(3465*b^6*x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{x^6\sqrt{bx+cx^2}}\,dx$$
$$= \frac{-\frac{2\sqrt{x}\,\sqrt{cx+b}\,a\,b^5}{11}+\frac{20\sqrt{x}\,\sqrt{cx+b}\,a\,b^4cx}{99}-\frac{160\sqrt{x}\,\sqrt{cx+b}\,a\,b^3c^2x^2}{693}+\frac{64\sqrt{x}\,\sqrt{cx+b}\,a\,b^2c^3x^3}{231}-\frac{256\sqrt{x}\,\sqrt{cx+b}\,ab\,c^4x^4}{693}+\frac{512\sqrt{x}\,\sqrt{cx+}}{693}}{}$$

input
```
int((B*x+A)/x^6/(c*x^2+b*x)^(1/2),x)
```

output
```
(2*( - 315*sqrt(x)*sqrt(b + c*x)*a*b**5 + 350*sqrt(x)*sqrt(b + c*x)*a*b**4
*c*x - 400*sqrt(x)*sqrt(b + c*x)*a*b**3*c**2*x**2 + 480*sqrt(x)*sqrt(b + c
*x)*a*b**2*c**3*x**3 - 640*sqrt(x)*sqrt(b + c*x)*a*b*c**4*x**4 + 1280*sqrt
(x)*sqrt(b + c*x)*a*c**5*x**5 - 385*sqrt(x)*sqrt(b + c*x)*b**6*x + 440*sqr
t(x)*sqrt(b + c*x)*b**5*c*x**2 - 528*sqrt(x)*sqrt(b + c*x)*b**4*c**2*x**3
+ 704*sqrt(x)*sqrt(b + c*x)*b**3*c**3*x**4 - 1408*sqrt(x)*sqrt(b + c*x)*b*
*2*c**4*x**5 - 1280*sqrt(c)*a*c**5*x**6 + 1408*sqrt(c)*b**2*c**4*x**6))/(3
465*b**6*x**6)
```

3.153 $\int \frac{x^4(A+Bx)}{(bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 156

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{3/2}}\,dx=\frac{2(bB-Ac)x^3}{c^2\sqrt{bx+cx^2}}+\frac{5b(7bB-6Ac)\sqrt{bx+cx^2}}{8c^4}-\frac{5(7bB-6Ac)x\sqrt{bx+cx^2}}{12c^3}+\frac{Bx^2\sqrt{bx+cx^2}}{3c^2}-\frac{5b^2(7bB-6Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{8c^{9/2}}$$

output
```
2*(-A*c+B*b)*x^3/c^2/(c*x^2+b*x)^(1/2)+5/8*b*(-6*A*c+7*B*b)*(c*x^2+b*x)^(1
/2)/c^4-5/12*(-6*A*c+7*B*b)*x*(c*x^2+b*x)^(1/2)/c^3+1/3*B*x^2*(c*x^2+b*x)^
(1/2)/c^2-5/8*b^2*(-6*A*c+7*B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(9
/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{3/2}}\,dx=\frac{x^{3/2}\left(\sqrt{c}\sqrt{x}(b+cx)\left(105b^3B+4c^3x^2(3A+2Bx)-2bc^2x(15A+7Bx)+b^2(-90Ac\right.\right.}{24c^{9/2}(x(b+cx))^{3/2}}$$

input
```
Integrate[(x^4*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(x^(3/2)*(Sqrt[c]*Sqrt[x]*(b + c*x)*(105*b^3*B + 4*c^3*x^2*(3*A + 2*B*x) -
 2*b*c^2*x*(15*A + 7*B*x) + b^2*(-90*A*c + 35*B*c*x)) - 30*b^2*(7*b*B - 6*
A*c)*(b + c*x)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])]
))/(24*c^(9/2)*(x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1211, 25, 2192, 27, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int -\frac{-Bc^3x^3+c^2(bB-Ac)x^2-bc(bB-Ac)x+b^2(bB-Ac)}{\sqrt{cx^2+bx}}dx}{c^4}+\frac{2b^2x(bB-Ac)}{c^4\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2b^2x(bB-Ac)}{c^4\sqrt{bx+cx^2}}-\frac{\int \frac{-Bc^3x^3+c^2(bB-Ac)x^2-bc(bB-Ac)x+b^2(bB-Ac)}{\sqrt{cx^2+bx}}dx}{c^4}$$

$$\Big\downarrow \text{2192}$$

$$\frac{2b^2x(bB-Ac)}{c^4\sqrt{bx+cx^2}}-\frac{\frac{\int \frac{(11bB-6Ac)x^2c^3-6b(bB-Ac)xc^2+6b^2(bB-Ac)c}{2\sqrt{cx^2+bx}}dx}{3c}-\frac{1}{3}Bc^2x^2\sqrt{bx+cx^2}}{c^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b^2x(bB-Ac)}{c^4\sqrt{bx+cx^2}}-\frac{\frac{\int \frac{(11bB-6Ac)x^2c^3-6b(bB-Ac)xc^2+6b^2(bB-Ac)c}{\sqrt{cx^2+bx}}dx}{6c}-\frac{1}{3}Bc^2x^2\sqrt{bx+cx^2}}{c^4}$$

$$\Big\downarrow \text{2192}$$

$$\frac{2b^2x(bB-Ac)}{c^4\sqrt{bx+cx^2}}-\frac{\frac{\frac{\int \frac{3bc^2(8b(bB-Ac)-c(19bB-14Ac)x)}{2\sqrt{cx^2+bx}}dx}{2c}+\frac{1}{2}c^2x\sqrt{bx+cx^2}(11bB-6Ac)}{6c}-\frac{1}{3}Bc^2x^2\sqrt{bx+cx^2}}{c^4}$$

$$\Big\downarrow 27$$

$$\frac{2b^2x(bB-Ac)}{c^4\sqrt{bx+cx^2}}-\frac{\frac{\frac{3}{4}bc\int\frac{8b(bB-Ac)-c(19bB-14Ac)x}{\sqrt{cx^2+bx}}dx+\frac{1}{2}c^2x\sqrt{bx+cx^2}(11bB-6Ac)}{6c}-\frac{1}{3}Bc^2x^2\sqrt{bx+cx^2}}{c^4}$$

$$\Big\downarrow 1160$$

$$\frac{2b^2x(bB-Ac)}{c^4\sqrt{bx+cx^2}}-\frac{\frac{\frac{3}{4}bc\left(\frac{5}{2}b(7bB-6Ac)\int\frac{1}{\sqrt{cx^2+bx}}dx-\sqrt{bx+cx^2}(19bB-14Ac)\right)+\frac{1}{2}c^2x\sqrt{bx+cx^2}(11bB-6Ac)}{6c}-\frac{1}{3}Bc^2x^2\sqrt{bx+cx^2}}{c^4}$$

$$\Big\downarrow 1091$$

$$\frac{2b^2x(bB-Ac)}{c^4\sqrt{bx+cx^2}}-\frac{\frac{\frac{3}{4}bc\left(5b(7bB-6Ac)\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}-\sqrt{bx+cx^2}(19bB-14Ac)\right)+\frac{1}{2}c^2x\sqrt{bx+cx^2}(11bB-6Ac)}{6c}-\frac{1}{3}Bc^2x^2\sqrt{bx+cx^2}}{c^4}$$

$$\Big\downarrow 219$$

$$\frac{2b^2x(bB-Ac)}{c^4\sqrt{bx+cx^2}}-\frac{\frac{\frac{3}{4}bc\left(\frac{5b(7bB-6Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}-\sqrt{bx+cx^2}(19bB-14Ac)\right)+\frac{1}{2}c^2x\sqrt{bx+cx^2}(11bB-6Ac)}{6c}-\frac{1}{3}Bc^2x^2\sqrt{bx+cx^2}}{c^4}$$

input
```
Int[(x^4*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(2*b^2*(b*B - A*c)*x)/(c^4*Sqrt[b*x + c*x^2]) - (-1/3*(B*c^2*x^2*Sqrt[b*x
+ c*x^2]) + ((c^2*(11*b*B - 6*A*c)*x*Sqrt[b*x + c*x^2])/2 + (3*b*c*(-((19*
b*B - 14*A*c)*Sqrt[b*x + c*x^2]) + (5*b*(7*b*B - 6*A*c)*ArcTanh[(Sqrt[c]*x
)/Sqrt[b*x + c*x^2]])/Sqrt[c]))/4)/(6*c))/c^4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.69

method	result
pseudoelliptic	$-\frac{5\left(-3\left(Ac-\frac{7Bb}{6}\right)b^2\sqrt{x(cx+b)}\,\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)+x\left(3\left(-\frac{7Bx}{18}+A\right)b^2c^{\frac{3}{2}}+bx\left(\frac{7Bx}{15}+A\right)c^{\frac{5}{2}}-\frac{2x^2\left(\frac{2Bx}{3}+A\right)c^{\frac{7}{2}}}{5}-\frac{7B\sqrt{}}{}\right.}{4\sqrt{x(cx+b)}\,c^{\frac{9}{2}}}$
risch	$-\frac{(-8Bc^2x^2-12Ac^2x+22Bbcx+42Abc-57Bb^2)x(cx+b)}{24c^4\sqrt{x(cx+b)}}+\frac{b^2\left(30A\sqrt{c}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx}\right)-\frac{35Bb\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx}\right.}{\sqrt{c}}\right.}{16c^4}$
default	$A\left(\frac{x^3}{2c\sqrt{cx^2+bx}}-\frac{5b\left(\frac{x^2}{c\sqrt{cx^2+bx}}-\frac{3b\left(-\frac{x}{c\sqrt{cx^2+bx}}-\frac{b\left(-\frac{1}{c\sqrt{cx^2+bx}}+\frac{2cx+b}{bc\sqrt{cx^2+bx}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{cx^2+bx}\right)}{c^{\frac{3}{2}}}\right)}{2c}\right)}{4c}\right)+$

input
```
int(x^4*(B*x+A)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-5/4/(x*(c*x+b))^(1/2)/c^(9/2)*(-3*(A*c-7/6*B*b)*b^2*(x*(c*x+b))^(1/2)*arc
tanh((x*(c*x+b))^(1/2)/x/c^(1/2))+x*(3*(-7/18*B*x+A)*b^2*c^(3/2)+b*x*(7/15
*B*x+A)*c^(5/2)-2/5*x^2*(2/3*B*x+A)*c^(7/2)-7/2*B*c^(1/2)*b^3))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.01

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \left[-\frac{15\,(7\,Bb^4-6\,Ab^3c+(7\,Bb^3c-6\,Ab^2c^2)x)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\right.}{4}$$

input
```
integrate(x^4*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/48*(15*(7*B*b^4 - 6*A*b^3*c + (7*B*b^3*c - 6*A*b^2*c^2)*x)*sqrt(c)*log
(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(8*B*c^4*x^3 + 105*B*b^3*c -
 90*A*b^2*c^2 - 2*(7*B*b*c^3 - 6*A*c^4)*x^2 + 5*(7*B*b^2*c^2 - 6*A*b*c^3)*
x)*sqrt(c*x^2 + b*x))/(c^6*x + b*c^5), 1/24*(15*(7*B*b^4 - 6*A*b^3*c + (7*
B*b^3*c - 6*A*b^2*c^2)*x)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x
+ b)) + (8*B*c^4*x^3 + 105*B*b^3*c - 90*A*b^2*c^2 - 2*(7*B*b*c^3 - 6*A*c^4
)*x^2 + 5*(7*B*b^2*c^2 - 6*A*b*c^3)*x)*sqrt(c*x^2 + b*x))/(c^6*x + b*c^5)]
```

Sympy [F]

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{x^4(A+Bx)}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(B*x+A)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(x**4*(A + B*x)/(x*(b + c*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.36

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{Bx^4}{3\sqrt{cx^2+bx}c} - \frac{7\,Bbx^3}{12\sqrt{cx^2+bx}c^2} + \frac{Ax^3}{2\sqrt{cx^2+bx}c} + \frac{35\,Bb^2x^2}{24\sqrt{cx^2+bx}c^3} - \frac{5\,Abx^2}{4\sqrt{cx^2+bx}c^2} + \frac{35\,Bb^3x}{8\sqrt{cx^2+bx}c^4} - \frac{15\,Ab^2x}{4\sqrt{cx^2+bx}c^3} - \frac{35\,Bb^3\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{16\,c^{\frac{9}{2}}} + \frac{15\,Ab^2\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,c^{\frac{7}{2}}}$$

input
```
integrate(x^4*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
1/3*B*x^4/(sqrt(c*x^2 + b*x)*c) - 7/12*B*b*x^3/(sqrt(c*x^2 + b*x)*c^2) + 1
/2*A*x^3/(sqrt(c*x^2 + b*x)*c) + 35/24*B*b^2*x^2/(sqrt(c*x^2 + b*x)*c^3) -
 5/4*A*b*x^2/(sqrt(c*x^2 + b*x)*c^2) + 35/8*B*b^3*x/(sqrt(c*x^2 + b*x)*c^4
) - 15/4*A*b^2*x/(sqrt(c*x^2 + b*x)*c^3) - 35/16*B*b^3*log(2*c*x + b + 2*s
qrt(c*x^2 + b*x)*sqrt(c))/c^(9/2) + 15/8*A*b^2*log(2*c*x + b + 2*sqrt(c*x^
2 + b*x)*sqrt(c))/c^(7/2)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.05

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{1}{24}\sqrt{cx^2+bx}\left(2\,x\left(\frac{4\,Bx}{c^2} - \frac{11\,Bbc^{10}-6\,Ac^{11}}{c^{13}}\right) + \frac{3\,(19\,Bb^2c^9-14\,Abc^{10})}{c^{13}}\right) + \frac{5\,(7\,Bb^3-6\,Ab^2c)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{16\,c^{\frac{9}{2}}} + \frac{2\left(Bb^4\sqrt{c}-Ab^3c^{\frac{3}{2}}\right)}{\left(\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right)c^5}$$

input
```
integrate(x^4*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
1/24*sqrt(c*x^2 + b*x)*(2*x*(4*B*x/c^2 - (11*B*b*c^10 - 6*A*c^11)/c^13) +
3*(19*B*b^2*c^9 - 14*A*b*c^10)/c^13) + 5/16*(7*B*b^3 - 6*A*b^2*c)*log(abs(
2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(9/2) + 2*(B*b^4*sqrt(c)
 - A*b^3*c^(3/2))/(((sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b)*c^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{x^4\,(A+B\,x)}{(c\,x^2+b\,x)^{3/2}}\,dx$$

input
```
int((x^4*(A + B*x))/(b*x + c*x^2)^(3/2),x)
```

output
```
int((x^4*(A + B*x))/(b*x + c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.16

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{720\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)a\,b^2c-840\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^4-480}{}$$

input
```
int(x^4*(B*x+A)/(c*x^2+b*x)^(3/2),x)
```

output
```
(720*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*
a*b**2*c - 840*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))
/sqrt(b))*b**4 - 480*sqrt(c)*sqrt(b + c*x)*a*b**2*c + 525*sqrt(c)*sqrt(b +
 c*x)*b**4 - 720*sqrt(x)*a*b**2*c**2 - 240*sqrt(x)*a*b*c**3*x + 96*sqrt(x)
*a*c**4*x**2 + 840*sqrt(x)*b**4*c + 280*sqrt(x)*b**3*c**2*x - 112*sqrt(x)*
b**2*c**3*x**2 + 64*sqrt(x)*b*c**4*x**3)/(192*sqrt(b + c*x)*c**5)
```

3.154 $\int \frac{x^3(A+Bx)}{(bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{2(bB-Ac)x^2}{c^2\sqrt{bx+cx^2}} - \frac{3(5bB-4Ac)\sqrt{bx+cx^2}}{4c^3} + \frac{Bx\sqrt{bx+cx^2}}{2c^2} + \frac{3b(5bB-4Ac)\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{7/2}}$$

output
```
2*(-A*c+B*b)*x^2/c^2/(c*x^2+b*x)^(1/2)-3/4*(-4*A*c+5*B*b)*(c*x^2+b*x)^(1/2
)/c^3+1/2*B*x*(c*x^2+b*x)^(1/2)/c^2+3/4*b*(-4*A*c+5*B*b)*arctanh(c^(1/2)*x
/(c*x^2+b*x)^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{x^{3/2}\left(\sqrt{c}\sqrt{x}(b+cx)\left(-15b^2B+bc(12A-5Bx)+2c^2x(2A+Bx)\right)+6b(5bB-4Ac\right.}{4c^{7/2}(x(b+cx))^{3/2}}$$

input
```
Integrate[(x^3*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(x^(3/2)*(Sqrt[c]*Sqrt[x]*(b + c*x)*(-15*b^2*B + b*c*(12*A - 5*B*x)) + 2*c^
2*x*(2*A + B*x)) + 6*b*(5*b*B - 4*A*c)*(b + c*x)^(3/2)*ArcTanh[(Sqrt[c]*Sq
rt[x])/(-Sqrt[b] + Sqrt[b + c*x])]))/(4*c^(7/2)*(x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1211, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int \frac{Bc^2x^2-c(bB-Ac)x+b(bB-Ac)}{\sqrt{cx^2+bx}}dx}{c^3}-\frac{2bx(bB-Ac)}{c^3\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{2192}$$

$$\frac{\frac{\int \frac{c(4b(bB-Ac)-c(7bB-4Ac)x)}{2\sqrt{cx^2+bx}}dx}{2c}+\frac{1}{2}Bcx\sqrt{bx+cx^2}}{c^3}-\frac{2bx(bB-Ac)}{c^3\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}\int \frac{4b(bB-Ac)-c(7bB-4Ac)x}{\sqrt{cx^2+bx}}dx+\frac{1}{2}Bcx\sqrt{bx+cx^2}}{c^3}-\frac{2bx(bB-Ac)}{c^3\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1160}$$

$$\frac{\frac{1}{4}\left(\frac{3}{2}b(5bB-4Ac)\int \frac{1}{\sqrt{cx^2+bx}}dx-\sqrt{bx+cx^2}(7bB-4Ac)\right)+\frac{1}{2}Bcx\sqrt{bx+cx^2}}{c^3}-\frac{2bx(bB-Ac)}{c^3\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1091}$$

$$\frac{\frac{1}{4}\left(3b(5bB-4Ac)\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}-\sqrt{bx+cx^2}(7bB-4Ac)\right)+\frac{1}{2}Bcx\sqrt{bx+cx^2}}{c^3}-\frac{2bx(bB-Ac)}{c^3\sqrt{bx+cx^2}}$$

$$\Big\downarrow 219$$

$$\frac{\frac{1}{4}\left(\frac{3b(5bB-4Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}-\sqrt{bx+cx^2}(7bB-4Ac)\right)+\frac{1}{2}Bcx\sqrt{bx+cx^2}}{c^3}-\frac{2bx(bB-Ac)}{c^3\sqrt{bx+cx^2}}$$

input
```
Int[(x^3*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*b*(b*B - A*c)*x)/(c^3*Sqrt[b*x + c*x^2]) + ((B*c*x*Sqrt[b*x + c*x^2])/
2 + (-((7*b*B - 4*A*c)*Sqrt[b*x + c*x^2]) + (3*b*(5*b*B - 4*A*c)*ArcTanh[(
Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c])/4)/c^3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

method	result
pseudoelliptic	$\frac{-3\sqrt{x(cx+b)}\left(Ac-\frac{5Bb}{4}\right)b\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)+\left(3\left(-\frac{5Bx}{12}+A\right)b\,c^{\frac{3}{2}}+\left(\frac{Bx}{2}+A\right)x\,c^{\frac{5}{2}}-\frac{15B\sqrt{c}\,b^{2}}{4}\right)x}{c^{\frac{7}{2}}\sqrt{x(cx+b)}}$
risch	$\frac{(2Bcx+4Ac-7Bb)x(cx+b)}{4c^{3}\sqrt{x(cx+b)}}-\frac{b\left(12A\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)-\frac{15Bb\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{\sqrt{c}}-\frac{16(Ac-Bb)\sqrt{c\left(\frac{b}{c}+x\right)^{2}-(}}{c\left(\frac{b}{c}+x\right)}\right.}{8c^{3}}$
default	$A\left(\frac{x^{2}}{c\sqrt{c\,x^{2}+bx}}-\frac{3b\left(-\frac{x}{c\sqrt{c\,x^{2}+bx}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^{2}+bx}}+\frac{2cx+b}{bc\sqrt{c\,x^{2}+bx}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{c^{\frac{3}{2}}}\right)}{2c}\right)+B\left(\frac{x^{3}}{2c\sqrt{c\,x^{2}+bx}}\right.$

input
```
int(x^3*(B*x+A)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^(7/2)*(-3*(x*(c*x+b))^(1/2)*(A*c-5/4*B*b)*b*arctanh((x*(c*x+b))^(1/2)/
x/c^(1/2))+(3*(-5/12*B*x+A)*b*c^(3/2)+(1/2*B*x+A)*x*c^(5/2)-15/4*B*c^(1/2)
*b^2)*x)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.17

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \left[-\frac{3\,(5\,Bb^3-4\,Ab^2c+(5\,Bb^2c-4\,Abc^2)x)\sqrt{c}\log\left(2\,cx+b-2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(}{8\,(c^5x+bc^4)}\right.$$

$$-\frac{3\,(5\,Bb^3-4\,Ab^2c+(5\,Bb^2c-4\,Abc^2)x)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)-(2\,Bc^3x^2-15\,Bb^2c+12\,Abc^2-(5}{4\,(c^5x+bc^4)}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*(5*B*b^3 - 4*A*b^2*c + (5*B*b^2*c - 4*A*b*c^2)*x)*sqrt(c)*log(2*c
*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(2*B*c^3*x^2 - 15*B*b^2*c + 12*A
*b*c^2 - (5*B*b*c^2 - 4*A*c^3)*x)*sqrt(c*x^2 + b*x))/(c^5*x + b*c^4), -1/4
*(3*(5*B*b^3 - 4*A*b^2*c + (5*B*b^2*c - 4*A*b*c^2)*x)*sqrt(-c)*arctan(sqrt
(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) - (2*B*c^3*x^2 - 15*B*b^2*c + 12*A*b*c^2
 - (5*B*b*c^2 - 4*A*c^3)*x)*sqrt(c*x^2 + b*x))/(c^5*x + b*c^4)]
```

Sympy [F]

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{x^3(A+Bx)}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(B*x+A)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(x**3*(A + B*x)/(x*(b + c*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.35

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{Bx^3}{2\sqrt{cx^2+bx}c} - \frac{5\,Bbx^2}{4\sqrt{cx^2+bx}c^2} + \frac{Ax^2}{\sqrt{cx^2+bx}c} - \frac{15\,Bb^2x}{4\sqrt{cx^2+bx}c^3} + \frac{3\,Abx}{\sqrt{cx^2+bx}c^2} + \frac{15\,Bb^2\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,c^{\frac{7}{2}}} - \frac{3\,Ab\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{2\,c^{\frac{5}{2}}}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
1/2*B*x^3/(sqrt(c*x^2 + b*x)*c) - 5/4*B*b*x^2/(sqrt(c*x^2 + b*x)*c^2) + A*
x^2/(sqrt(c*x^2 + b*x)*c) - 15/4*B*b^2*x/(sqrt(c*x^2 + b*x)*c^3) + 3*A*b*x
/(sqrt(c*x^2 + b*x)*c^2) + 15/8*B*b^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*
sqrt(c))/c^(7/2) - 3/2*A*b*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^
(5/2)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.12

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{1}{4}\sqrt{cx^2+bx}\left(\frac{2\,Bx}{c^2} - \frac{7\,Bbc^5-4\,Ac^6}{c^8}\right) - \frac{3\,(5\,Bb^2-4\,Abc)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{7}{2}}} - \frac{2\left(Bb^3\sqrt{c}-Ab^2c^{\frac{3}{2}}\right)}{\left(\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right)c^4}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x)*(2*B*x/c^2 - (7*B*b*c^5 - 4*A*c^6)/c^8) - 3/8*(5*B*b
^2 - 4*A*b*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(7
/2) - 2*(B*b^3*sqrt(c) - A*b^2*c^(3/2))/(((sqrt(c)*x - sqrt(c*x^2 + b*x))*
sqrt(c) + b)*c^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{x^3\,(A+B\,x)}{(c\,x^2+b\,x)^{3/2}}\,dx$$

input
```
int((x^3*(A + B*x))/(b*x + c*x^2)^(3/2),x)
```

output
```
int((x^3*(A + B*x))/(b*x + c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.24

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{-12\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)abc+15\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^3+9\sqrt{c}\,\sqrt{}}{}$$

input
```
int(x^3*(B*x+A)/(c*x^2+b*x)^(3/2),x)
```

output
```
( - 12*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b)
)*a*b*c + 15*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/s
qrt(b))*b**3 + 9*sqrt(c)*sqrt(b + c*x)*a*b*c - 10*sqrt(c)*sqrt(b + c*x)*b*
*3 + 12*sqrt(x)*a*b*c**2 + 4*sqrt(x)*a*c**3*x - 15*sqrt(x)*b**3*c - 5*sqrt
(x)*b**2*c**2*x + 2*sqrt(x)*b*c**3*x**2)/(4*sqrt(b + c*x)*c**4)
```

3.155 $\int \frac{x^2(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 83

$$\int \frac{x^2(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx=\frac{2(bB-Ac)x}{c^2\sqrt{bx+cx^2}}+\frac{B\sqrt{bx+cx^2}}{c^2}-\frac{(3bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{5/2}}$$

output
```
2*(-A*c+B*b)*x/c^2/(c*x^2+b*x)^(1/2)+B*(c*x^2+b*x)^(1/2)/c^2-(-2*A*c+3*B*b
)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.24

$$\int \frac{x^2(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx=\frac{x^{3/2}\left(\sqrt{c}\sqrt{x}(b+cx)(3bB-2Ac+Bcx)+2(-3bB+2Ac)(b+cx)^{3/2}\text{arctanh}\left(\right.\right.}{c^{5/2}(x(b+cx))^{3/2}}$$

input
```
Integrate[(x^2*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(x^(3/2)*(Sqrt[c]*Sqrt[x]*(b + c*x)*(3*b*B - 2*A*c + B*c*x) + 2*(-3*b*B +
2*A*c)*(b + c*x)^(3/2)*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x]
)]))/(c^(5/2)*(x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1211, 25, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{(bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int -\frac{bB-cxB-Ac}{\sqrt{cx^2+bx}}dx}{c^2}+\frac{2x(bB-Ac)}{c^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{25}$$

$$\frac{2x(bB-Ac)}{c^2\sqrt{bx+cx^2}}-\frac{\int \frac{bB-cxB-Ac}{\sqrt{cx^2+bx}}dx}{c^2}$$

$$\Big\downarrow \text{1160}$$

$$\frac{2x(bB-Ac)}{c^2\sqrt{bx+cx^2}}-\frac{\frac{1}{2}(3bB-2Ac)\int \frac{1}{\sqrt{cx^2+bx}}dx-B\sqrt{bx+cx^2}}{c^2}$$

$$\Big\downarrow \text{1091}$$

$$\frac{2x(bB-Ac)}{c^2\sqrt{bx+cx^2}}-\frac{(3bB-2Ac)\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}-B\sqrt{bx+cx^2}}{c^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{2x(bB-Ac)}{c^2\sqrt{bx+cx^2}}-\frac{\frac{(3bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}-B\sqrt{bx+cx^2}}{c^2}$$

input
```
Int[(x^2*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(2*(b*B - A*c)*x)/(c^2*Sqrt[b*x + c*x^2]) - (-(B*Sqrt[b*x + c*x^2]) + ((3*
b*B - 2*A*c)*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c])/c^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

method	result
pseudoelliptic	$\frac{B\sqrt{x(cx+b)}+\frac{(2Ac-3Bb)\,\mathrm{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{\sqrt{c}}-\frac{2x(Ac-Bb)}{\sqrt{x(cx+b)}}}{c^{2}}$
risch	$\frac{Bx(cx+b)}{c^{2}\sqrt{x(cx+b)}}+\frac{2A\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)-\frac{3Bb\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{\sqrt{c}}-\frac{4(Ac-Bb)\sqrt{c\left(\frac{b}{c}+x\right)^{2}-\left(\frac{b}{c}+x\right)b}}{c\left(\frac{b}{c}+x\right)}}{2c^{2}}$
default	$A\left(-\frac{x}{c\sqrt{c\,x^{2}+bx}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^{2}+bx}}+\frac{2cx+b}{bc\sqrt{c\,x^{2}+bx}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{c^{\frac{3}{2}}}\right)+B\left(\frac{x^{2}}{c\sqrt{c\,x^{2}+bx}}-\frac{3b\left(-\frac{\ldots}{c\sqrt{c\ldots}}\right.}{\ldots}\right.$

input
```
int(x^2*(B*x+A)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/c^2*(B*(x*(c*x+b))^(1/2)+(2*A*c-3*B*b)/c^(1/2)*arctanh((x*(c*x+b))^(1/2)
/x/c^(1/2))-2*x*(A*c-B*b)/(x*(c*x+b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.45

$$\int \frac{x^2(A+Bx)}{(bx+cx^2)^{3/2}}\,dx=\left[-\frac{(3\,Bb^2-2\,Abc+(3\,Bbc-2\,Ac^2)x)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(Bc^2\ldots}{2\,(c^4x+bc^3)}\right.$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*((3*B*b^2 - 2*A*b*c + (3*B*b*c - 2*A*c^2)*x)*sqrt(c)*log(2*c*x + b +
 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(B*c^2*x + 3*B*b*c - 2*A*c^2)*sqrt(c*x^2
 + b*x))/(c^4*x + b*c^3), ((3*B*b^2 - 2*A*b*c + (3*B*b*c - 2*A*c^2)*x)*sqr
t(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (B*c^2*x + 3*B*b*c -
2*A*c^2)*sqrt(c*x^2 + b*x))/(c^4*x + b*c^3)]
```

Sympy [F]

$$\int \frac{x^2(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{x^2(A+Bx)}{(x(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(B*x+A)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(x**2*(A + B*x)/(x*(b + c*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.39

$$\int \frac{x^2(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{Bx^2}{\sqrt{cx^2+bx}c} + \frac{3\,Bbx}{\sqrt{cx^2+bx}c^2} - \frac{2\,Ax}{\sqrt{cx^2+bx}c} - \frac{3\,Bb\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{2\,c^{\frac{5}{2}}} + \frac{A\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{c^{\frac{3}{2}}}$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
B*x^2/(sqrt(c*x^2 + b*x)*c) + 3*B*b*x/(sqrt(c*x^2 + b*x)*c^2) - 2*A*x/(sqr
t(c*x^2 + b*x)*c) - 3/2*B*b*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c
^(5/2) + A*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.29

$$\int \frac{x^2(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{\sqrt{cx^2+bx}B}{c^2} + \frac{(3\,Bb-2\,Ac)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{5}{2}}} + \frac{2\left(Bb^2\sqrt{c}-Abc^{\frac{3}{2}}\right)}{\left(\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right)c^3}$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
sqrt(c*x^2 + b*x)*B/c^2 + 1/2*(3*B*b - 2*A*c)*log(abs(2*(sqrt(c)*x - sqrt(
c*x^2 + b*x))*sqrt(c) + b))/c^(5/2) + 2*(B*b^2*sqrt(c) - A*b*c^(3/2))/(((s
qrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b)*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{x^2\,(A+B\,x)}{(c\,x^2+b\,x)^{3/2}}\,dx$$

input
```
int((x^2*(A + B*x))/(b*x + c*x^2)^(3/2),x)
```

output
```
int((x^2*(A + B*x))/(b*x + c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.51

$$\int \frac{x^2(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{8\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)ac-12\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^2-8\sqrt{c}\,\sqrt{cx}}{4\sqrt{cx+b}\,c^3}$$

input
```
int(x^2*(B*x+A)/(c*x^2+b*x)^(3/2),x)
```

output
```
(8*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a*
c - 12*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b)
)*b**2 - 8*sqrt(c)*sqrt(b + c*x)*a*c + 9*sqrt(c)*sqrt(b + c*x)*b**2 - 8*sq
rt(x)*a*c**2 + 12*sqrt(x)*b**2*c + 4*sqrt(x)*b*c**2*x)/(4*sqrt(b + c*x)*c*
*3)
```

3.156 $\int \frac{x(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int \frac{x(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx=-\frac{2(bB-Ac)x}{bc\sqrt{bx+cx^2}}+\frac{2B\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}$$

output
```
-2*(-A*c+B*b)*x/b/c/(c*x^2+b*x)^(1/2)+2*B*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1
/2))/c^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.28

$$\int \frac{x(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx=-\frac{2\left(\sqrt{c}(bB-Ac)x+bB\sqrt{x}\sqrt{b+cx}\log\left(-\sqrt{c}\sqrt{x}+\sqrt{b+cx}\right)\right)}{bc^{3/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[(x*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(Sqrt[c]*(b*B - A*c)*x + b*B*Sqrt[x]*Sqrt[b + c*x]*Log[-(Sqrt[c]*Sqrt[
x]) + Sqrt[b + c*x]]))/(b*c^(3/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1211, 27, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{1211}$$

$$\frac{\int \frac{B}{\sqrt{cx^2+bx}}dx}{c}-\frac{2x(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{B\int \frac{1}{\sqrt{cx^2+bx}}dx}{c}-\frac{2x(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1091}$$

$$\frac{2B\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{c}-\frac{2x(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2B\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}-\frac{2x(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

input
```
Int[(x*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(b*B - A*c)*x)/(b*c*Sqrt[b*x + c*x^2]) + (2*B*ArcTanh[(Sqrt[c]*x)/Sqrt
[b*x + c*x^2]])/c^(3/2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1211
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_.) + (b_.)*
(x_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[-2*(2*c*d - b*e)^(m - 2)*(c*(
e*f + d*g) - b*e*g)^n*((d + e*x)/(c^(m + n - 1)*e^(n - 1)*Sqrt[a + b*x + c*
x^2])), x] + Simp[1/(c^(m + n - 1)*e^(n - 2))   Int[ExpandToSum[((2*c*d - b
*e)^(m - 1)*(c*(e*f + d*g) - b*e*g)^n - c^(m + n - 1)*e^n*(d + e*x)^(m - 1)
*(f + g*x)^n)/(c*d - b*e - c*e*x), x]/Sqrt[a + b*x + c*x^2], x], x] /; Free
Q[{a, b, c, d, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[m, 0]
&& IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{2A\,c^{\frac{3}{2}}x-2Bbx\sqrt{c}+2B\,\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)b\sqrt{x(cx+b)}}{c^{\frac{3}{2}}\sqrt{x(cx+b)}\,b}$
default	$A\left(-\frac{1}{c\sqrt{c\,x^{2}+bx}}+\frac{2cx+b}{bc\sqrt{c\,x^{2}+bx}}\right)+B\left(-\frac{x}{c\sqrt{c\,x^{2}+bx}}-\frac{b\left(-\frac{1}{c\sqrt{c\,x^{2}+bx}}+\frac{2cx+b}{bc\sqrt{c\,x^{2}+bx}}\right)}{2c}+\frac{\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{c^{\frac{3}{2}}}\right.$

input
```
int(x*(B*x+A)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
(2*A*c^(3/2)*x-2*B*b*x*c^(1/2)+2*B*arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))*b*
(x*(c*x+b))^(1/2))/c^(3/2)/(x*(c*x+b))^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.75

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \left[\frac{(Bbcx+Bb^2)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(Bbc-Ac^2)\sqrt{cx^2+bx}}{bc^3x+b^2c^2},\right.$$
$$\left.-\frac{2\left((Bbcx+Bb^2)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)+(Bbc-Ac^2)\sqrt{cx^2+bx}\right)}{bc^3x+b^2c^2}\right]$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[((B*b*c*x + B*b^2)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) -
 2*(B*b*c - A*c^2)*sqrt(c*x^2 + b*x))/(b*c^3*x + b^2*c^2), -2*((B*b*c*x +
B*b^2)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (B*b*c - A*
c^2)*sqrt(c*x^2 + b*x))/(b*c^3*x + b^2*c^2)]
```

Sympy [F]

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{x(A+Bx)}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(B*x+A)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(x*(A + B*x)/(x*(b + c*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{2\,Ax}{\sqrt{cx^2+bx}b} - \frac{2\,Bx}{\sqrt{cx^2+bx}c} + \frac{B\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{c^{\frac{3}{2}}}$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
2*A*x/(sqrt(c*x^2 + b*x)*b) - 2*B*x/(sqrt(c*x^2 + b*x)*c) + B*log(2*c*x +
b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.33

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = -\frac{B\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{c^{\frac{3}{2}}} - \frac{2\left(Bb\sqrt{c}-Ac^{\frac{3}{2}}\right)}{\left(\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right)c^2}$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
-B*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(3/2) - 2*(B*
b*sqrt(c) - A*c^(3/2))/(((sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b)*c^2)
```

Mupad [B] (verification not implemented)

Time = 5.42 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{B\,\ln\left(\frac{\frac{b}{2}+c\,x}{\sqrt{c}}+\sqrt{c\,x^2+b\,x}\right)}{c^{3/2}}+\frac{2\,A\,x}{b\,\sqrt{x\,\left(b+c\,x\right)}}-\frac{2\,B\,x}{c\,\sqrt{c\,x^2+b\,x}}$$

input
```
int((x*(A + B*x))/(b*x + c*x^2)^(3/2),x)
```

output
```
(B*log((b/2 + c*x)/c^(1/2) + (b*x + c*x^2)^(1/2)))/c^(3/2) + (2*A*x)/(b*(x
*(b + c*x))^(1/2)) - (2*B*x)/(c*(b*x + c*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.43

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^2+2\sqrt{c}\,\sqrt{cx+b}\,ac-2\sqrt{c}\,\sqrt{cx+b}\,b^2+2\sqrt{x}\,a\,c^2-}{\sqrt{cx+b}\,b\,c^2}$$

input
```
int(x*(B*x+A)/(c*x^2+b*x)^(3/2),x)
```

output
```
(2*(sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*b
**2 + sqrt(c)*sqrt(b + c*x)*a*c - sqrt(c)*sqrt(b + c*x)*b**2 + sqrt(x)*a*c
**2 - sqrt(x)*b**2*c))/(sqrt(b + c*x)*b*c**2)
```

3.157 $\int \frac{A+Bx}{(bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 47

$$\int \frac{A+Bx}{(bx+cx^2)^{3/2}}\,dx=-\frac{2A}{b\sqrt{bx+cx^2}}+\frac{2(bB-2Ac)x}{b^2\sqrt{bx+cx^2}}$$

output
```
-2*A/b/(c*x^2+b*x)^(1/2)+2*(-2*A*c+B*b)*x/b^2/(c*x^2+b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx}{(bx+cx^2)^{3/2}}\,dx=\frac{2bBx-2A(b+2cx)}{b^2\sqrt{x(b+cx)}}$$

input
```
Integrate[(A + B*x)/(b*x + c*x^2)^(3/2),x]
```

output
```
(2*b*B*x - 2*A*(b + 2*c*x))/(b^2*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {1158}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1158$$

$$-\frac{2(Ab-x(bB-2Ac))}{b^2\sqrt{bx+cx^2}}$$

input
```
Int[(A + B*x)/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(A*b - (b*B - 2*A*c)*x))/(b^2*Sqrt[b*x + c*x^2])
```

Defintions of rubi rules used

rule 1158
```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.64

method	result	size
pseudoelliptic	$\frac{(-4cx-2b)A+2Bbx}{\sqrt{x(cx+b)}\,b^2}$	30
gosper	$-\frac{2x(cx+b)(2Acx-Bbx+Ab)}{b^2(c\,x^2+bx)^{\frac{3}{2}}}$	37
orering	$-\frac{2x(cx+b)(2Acx-Bbx+Ab)}{b^2(c\,x^2+bx)^{\frac{3}{2}}}$	37
trager	$-\frac{2(2Acx-Bbx+Ab)\sqrt{c\,x^2+bx}}{(cx+b)b^2x}$	41
risch	$-\frac{2A(cx+b)}{b^2\sqrt{x(cx+b)}}-\frac{2x(Ac-Bb)}{\sqrt{x(cx+b)}\,b^2}$	45
default	$-\frac{2A(2cx+b)}{b^2\sqrt{c\,x^2+bx}}+B\left(-\frac{1}{c\sqrt{c\,x^2+bx}}+\frac{2cx+b}{bc\sqrt{c\,x^2+bx}}\right)$	68

input
```
int((B*x+A)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((-4*c*x-2*b)*A+2*B*b*x)/(x*(c*x+b))^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{(bx+cx^2)^{3/2}}\,dx = -\frac{2\sqrt{cx^2+bx}(Ab-(Bb-2\,Ac)x)}{b^2cx^2+b^3x}$$

input
```
integrate((B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(c*x^2 + b*x)*(A*b - (B*b - 2*A*c)*x)/(b^2*c*x^2 + b^3*x)
```

Sympy [F]

$$\int \frac{A + Bx}{(bx + cx^2)^{3/2}}\, dx = \int \frac{A + Bx}{(x(b + cx))^{\frac{3}{2}}}\, dx$$

input
```
integrate((B*x+A)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((A + B*x)/(x*(b + c*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.17

$$\int \frac{A + Bx}{(bx + cx^2)^{3/2}}\, dx = \frac{2\,Bx}{\sqrt{cx^2 + bx}b} - \frac{4\,Acx}{\sqrt{cx^2 + bx}b^2} - \frac{2\,A}{\sqrt{cx^2 + bx}b}$$

input
```
integrate((B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
2*B*x/(sqrt(c*x^2 + b*x)*b) - 4*A*c*x/(sqrt(c*x^2 + b*x)*b^2) - 2*A/(sqrt(
c*x^2 + b*x)*b)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.70

$$\int \frac{A + Bx}{(bx + cx^2)^{3/2}}\, dx = -\frac{2\left(\frac{A}{b} - \frac{(Bb - 2\,Ac)x}{b^2}\right)}{\sqrt{cx^2 + bx}}$$

input
```
integrate((B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
-2*(A/b - (B*b - 2*A*c)*x/b^2)/sqrt(c*x^2 + b*x)
```

Mupad [B] (verification not implemented)

Time = 5.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.66

$$\int \frac{A+Bx}{(bx+cx^2)^{3/2}}\,dx = -\frac{2\,A\,b+4\,A\,c\,x-2\,B\,b\,x}{b^2\,\sqrt{c\,x^2+b\,x}}$$

input
```
int((A + B*x)/(b*x + c*x^2)^(3/2),x)
```

output
```
-(2*A*b + 4*A*c*x - 2*B*b*x)/(b^2*(b*x + c*x^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.49

$$\int \frac{A+Bx}{(bx+cx^2)^{3/2}}\,dx = \frac{-4\sqrt{c}\,\sqrt{cx+b}\,acx+2\sqrt{c}\,\sqrt{cx+b}\,b^2x-2\sqrt{x}\,abc-4\sqrt{x}\,a\,c^2x+2\sqrt{x}\,b^2cx}{\sqrt{cx+b}\,b^2cx}$$

input
```
int((B*x+A)/(c*x^2+b*x)^(3/2),x)
```

output
```
(2*( - 2*sqrt(c)*sqrt(b + c*x)*a*c*x + sqrt(c)*sqrt(b + c*x)*b**2*x - sqrt
(x)*a*b*c - 2*sqrt(x)*a*c**2*x + sqrt(x)*b**2*c*x))/(sqrt(b + c*x)*b**2*c*
x)
```

3.158 $\int \frac{A+Bx}{x(bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 86

$$\int \frac{A+Bx}{x(bx+cx^2)^{3/2}}\,dx = \frac{2(3bB-4Ac)}{3b^2\sqrt{bx+cx^2}} - \frac{2A}{3bx\sqrt{bx+cx^2}} - \frac{4(3bB-4Ac)\sqrt{bx+cx^2}}{3b^3x}$$

output
```
2/3*(-4*A*c+3*B*b)/b^2/(c*x^2+b*x)^(1/2)-2/3*A/b/x/(c*x^2+b*x)^(1/2)-4/3*(
-4*A*c+3*B*b)*(c*x^2+b*x)^(1/2)/b^3/x
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.60

$$\int \frac{A+Bx}{x(bx+cx^2)^{3/2}}\,dx = -\frac{2(3bBx(b+2cx)+A(b^2-4bcx-8c^2x^2))}{3b^3x\sqrt{x(b+cx)}}$$

input
```
Integrate[(A + B*x)/(x*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(3*b*B*x*(b + 2*c*x) + A*(b^2 - 4*b*c*x - 8*c^2*x^2)))/(3*b^3*x*Sqrt[x
*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.70, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1220, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\left(bx+cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(3bB-4Ac)\int \frac{1}{(cx^2+bx)^{3/2}}dx}{3b}-\frac{2A}{3bx\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{2(b+2cx)(3bB-4Ac)}{3b^3\sqrt{bx+cx^2}}-\frac{2A}{3bx\sqrt{bx+cx^2}}$$

input
```
Int[(A + B*x)/(x*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*A)/(3*b*x*Sqrt[b*x + c*x^2]) - (2*(3*b*B - 4*A*c)*(b + 2*c*x))/(3*b^3*
Sqrt[b*x + c*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.57

method	result	size
pseudoelliptic	$-\frac{2\left((3Bx+A)b^2-4cx\left(-\frac{3Bx}{2}+A\right)b-8A\,c^2x^2\right)}{3\sqrt{x(cx+b)}\,x\,b^3}$	49
gosper	$-\frac{2(cx+b)\left(-8A\,c^2x^2+6x^2Bbc-4Abcx+3xB\,b^2+b^2A\right)}{3b^3\left(c\,x^2+bx\right)^{\frac{3}{2}}}$	58
orering	$-\frac{2(cx+b)\left(-8A\,c^2x^2+6x^2Bbc-4Abcx+3xB\,b^2+b^2A\right)}{3b^3\left(c\,x^2+bx\right)^{\frac{3}{2}}}$	58
risch	$-\frac{2(cx+b)(-5Acx+3Bbx+Ab)}{3b^3x\sqrt{x(cx+b)}}+\frac{2c(Ac-Bb)x}{\sqrt{x(cx+b)}\,b^3}$	62
trager	$-\frac{2\left(-8A\,c^2x^2+6x^2Bbc-4Abcx+3xB\,b^2+b^2A\right)\sqrt{c\,x^2+bx}}{3(cx+b)b^3x^2}$	63
default	$-\frac{2B(2cx+b)}{b^2\sqrt{c\,x^2+bx}}+A\left(-\frac{2}{3bx\sqrt{c\,x^2+bx}}+\frac{8c(2cx+b)}{3b^3\sqrt{c\,x^2+bx}}\right)$	70

input
```
int((B*x+A)/x/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(x*(c*x+b))^(1/2)*((3*B*x+A)*b^2-4*c*x*(-3/2*B*x+A)*b-8*A*c^2*x^2)/x/
b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx}{x\,(bx+cx^2)^{3/2}}\,dx = -\frac{2\,(Ab^2+2\,(3\,Bbc-4\,Ac^2)x^2+(3\,Bb^2-4\,Abc)x)\sqrt{cx^2+bx}}{3\,(b^3cx^3+b^4x^2)}$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*(A*b^2 + 2*(3*B*b*c - 4*A*c^2)*x^2 + (3*B*b^2 - 4*A*b*c)*x)*sqrt(c*x^
2 + b*x)/(b^3*c*x^3 + b^4*x^2)
```

Sympy [F]

$$\int \frac{A+Bx}{x\,(bx+cx^2)^{3/2}}\,dx = \int \frac{A+Bx}{x\,(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((A + B*x)/(x*(x*(b + c*x))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx}{x\,(bx+cx^2)^{3/2}}\,dx = -\frac{4\,Bcx}{\sqrt{cx^2+bx}b^2} + \frac{16\,Ac^2x}{3\,\sqrt{cx^2+bx}b^3} - \frac{2\,B}{\sqrt{cx^2+bx}b} + \frac{8\,Ac}{3\,\sqrt{cx^2+bx}b^2} - \frac{2\,A}{3\,\sqrt{cx^2+bx}bx}$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
-4*B*c*x/(sqrt(c*x^2 + b*x)*b^2) + 16/3*A*c^2*x/(sqrt(c*x^2 + b*x)*b^3) -
2*B/(sqrt(c*x^2 + b*x)*b) + 8/3*A*c/(sqrt(c*x^2 + b*x)*b^2) - 2/3*A/(sqrt(
c*x^2 + b*x)*b*x)
```

Giac [F]

$$\int \frac{A+Bx}{x\,(bx+cx^2)^{3/2}}\,dx=\int \frac{Bx+A}{(cx^2+bx)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/((c*x^2 + b*x)^(3/2)*x), x)
```

Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx}{x\,(bx+cx^2)^{3/2}}\,dx=-\frac{2\,\sqrt{c\,x^2+b\,x}\,\left(3\,B\,b^2\,x+A\,b^2+6\,B\,b\,c\,x^2-4\,A\,b\,c\,x-8\,A\,c^2\,x^2\right)}{3\,b^3\,x^2\,\left(b+c\,x\right)}$$

input
```
int((A + B*x)/(x*(b*x + c*x^2)^(3/2)),x)
```

output
```
-(2*(b*x + c*x^2)^(1/2)*(A*b^2 - 8*A*c^2*x^2 + 3*B*b^2*x + 6*B*b*c*x^2 - 4
*A*b*c*x))/(3*b^3*x^2*(b + c*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

$$\int \frac{A+Bx}{x\left(bx+cx^2\right)^{3/2}}\,dx = \frac{-\frac{16\sqrt{c}\,\sqrt{cx+b}\,ac\,x^2}{3}+4\sqrt{c}\,\sqrt{cx+b}\,b^2x^2-\frac{2\sqrt{x}\,a\,b^2}{3}+\frac{8\sqrt{x}\,abcx}{3}+\frac{16\sqrt{x}\,a\,c^2x^2}{3}-2\sqrt{x}\,b^3x-}{\sqrt{cx+b}\,b^3x^2}$$

input
```
int((B*x+A)/x/(c*x^2+b*x)^(3/2),x)
```

output
```
(2*( - 8*sqrt(c)*sqrt(b + c*x)*a*c*x**2 + 6*sqrt(c)*sqrt(b + c*x)*b**2*x**
2 - sqrt(x)*a*b**2 + 4*sqrt(x)*a*b*c*x + 8*sqrt(x)*a*c**2*x**2 - 3*sqrt(x)
*b**3*x - 6*sqrt(x)*b**2*c*x**2))/(3*sqrt(b + c*x)*b**3*x**2)
```

3.159 $\int \frac{A+Bx}{x^2(bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 122

$$\int \frac{A+Bx}{x^2\left(bx+cx^2\right)^{3/2}}\,dx=-\frac{2A}{5bx^2\sqrt{bx+cx^2}}+\frac{2(5bB-6Ac)}{5b^2x\sqrt{bx+cx^2}}-\frac{8(5bB-6Ac)\sqrt{bx+cx^2}}{15b^3x^2}+\frac{16c(5bB-6Ac)\sqrt{bx+cx^2}}{15b^4x}$$

output
```
-2/5*A/b/x^2/(c*x^2+b*x)^(1/2)+2/5*(-6*A*c+5*B*b)/b^2/x/(c*x^2+b*x)^(1/2)-
8/15*(-6*A*c+5*B*b)*(c*x^2+b*x)^(1/2)/b^3/x^2+16/15*c*(-6*A*c+5*B*b)*(c*x^
2+b*x)^(1/2)/b^4/x
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.61

$$\int \frac{A+Bx}{x^2\left(bx+cx^2\right)^{3/2}}\,dx=-\frac{2(5bBx(b^2-4bcx-8c^2x^2)+3A(b^3-2b^2cx+8bc^2x^2+16c^3x^3))}{15b^4x^2\sqrt{x(b+cx)}}$$

input
```
Integrate[(A + B*x)/(x^2*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(5*b*B*x*(b^2 - 4*b*c*x - 8*c^2*x^2) + 3*A*(b^3 - 2*b^2*c*x + 8*b*c^2*
x^2 + 16*c^3*x^3)))/(15*b^4*x^2*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1220, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2 (bx+cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(5bB-6Ac)\int \frac{1}{x(cx^2+bx)^{3/2}}dx}{5b} - \frac{2A}{5bx^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(5bB-6Ac)\left(-\frac{4c\int \frac{1}{(cx^2+bx)^{3/2}}dx}{3b} - \frac{2}{3bx\sqrt{bx+cx^2}}\right)}{5b} - \frac{2A}{5bx^2\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{\left(\frac{8c(b+2cx)}{3b^3\sqrt{bx+cx^2}} - \frac{2}{3bx\sqrt{bx+cx^2}}\right)(5bB-6Ac)}{5b} - \frac{2A}{5bx^2\sqrt{bx+cx^2}}$$

input
```
Int[(A + B*x)/(x^2*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*A)/(5*b*x^2*Sqrt[b*x + c*x^2]) + ((5*b*B - 6*A*c)*(-2/(3*b*x*Sqrt[b*x
+ c*x^2]) + (8*c*(b + 2*c*x))/(3*b^3*Sqrt[b*x + c*x^2])))/(5*b)
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.54

method	result	size
pseudoelliptic	$-\frac{2\left(\left(\frac{5Bx}{3}+A\right)b^3-2cx\left(\frac{10Bx}{3}+A\right)b^2+8c^2x^2\left(-\frac{5Bx}{3}+A\right)b+16A\,c^3x^3\right)}{5\sqrt{x(cx+b)}\,x^2b^4}$	66
gosper	$-\frac{2(cx+b)\left(48A\,c^3x^3-40x^3Bb\,c^2+24Ab\,c^2x^2-20x^2B\,b^2c-6A\,b^2cx+5xB\,b^3+3A\,b^3\right)}{15x\,b^4\left(c\,x^2+bx\right)^{\frac{3}{2}}}$	86
orering	$-\frac{2(cx+b)\left(48A\,c^3x^3-40x^3Bb\,c^2+24Ab\,c^2x^2-20x^2B\,b^2c-6A\,b^2cx+5xB\,b^3+3A\,b^3\right)}{15x\,b^4\left(c\,x^2+bx\right)^{\frac{3}{2}}}$	86
risch	$-\frac{2(cx+b)\left(33A\,c^2x^2-25x^2Bbc-9Abcx+5xB\,b^2+3b^2A\right)}{15b^4x^2\sqrt{x(cx+b)}}-\frac{2c^2(Ac-Bb)x}{\sqrt{x(cx+b)}\,b^4}$	87
trager	$-\frac{2\left(48A\,c^3x^3-40x^3Bb\,c^2+24Ab\,c^2x^2-20x^2B\,b^2c-6A\,b^2cx+5xB\,b^3+3A\,b^3\right)\sqrt{c\,x^2+bx}}{15(cx+b)b^4x^3}$	88
default	$A\left(-\frac{2}{5b\,x^2\sqrt{c\,x^2+bx}}-\frac{6c\left(-\frac{2}{3bx\sqrt{c\,x^2+bx}}+\frac{8c(2cx+b)}{3b^3\sqrt{c\,x^2+bx}}\right)}{5b}\right)+B\left(-\frac{2}{3bx\sqrt{c\,x^2+bx}}+\frac{8c(2cx+b)}{3b^3\sqrt{c\,x^2+bx}}\right)$	118

input
```
int((B*x+A)/x^2/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/5/(x*(c*x+b))^(1/2)*((5/3*B*x+A)*b^3-2*c*x*(10/3*B*x+A)*b^2+8*c^2*x^2*(
-5/3*B*x+A)*b+16*A*c^3*x^3)/x^2/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int \frac{A+Bx}{x^2\left(bx+cx^2\right)^{3/2}}\,dx =$$
$$-\frac{2\left(3\,Ab^3-8\left(5\,Bbc^2-6\,Ac^3\right)x^3-4\left(5\,Bb^2c-6\,Abc^2\right)x^2+\left(5\,Bb^3-6\,Ab^2c\right)x\right)\sqrt{cx^2+bx}}{15\left(b^4cx^4+b^5x^3\right)}$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(3*A*b^3 - 8*(5*B*b*c^2 - 6*A*c^3)*x^3 - 4*(5*B*b^2*c - 6*A*b*c^2)*x
^2 + (5*B*b^3 - 6*A*b^2*c)*x)*sqrt(c*x^2 + b*x)/(b^4*c*x^4 + b^5*x^3)
```

Sympy [F]

$$\int \frac{A+Bx}{x^2(bx+cx^2)^{3/2}}\,dx = \int \frac{A+Bx}{x^2(x(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**2/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((A + B*x)/(x**2*(x*(b + c*x))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.16

$$\int \frac{A+Bx}{x^2(bx+cx^2)^{3/2}}\,dx = \frac{16\,Bc^2x}{3\,\sqrt{cx^2+bx}b^3} - \frac{32\,Ac^3x}{5\,\sqrt{cx^2+bx}b^4} + \frac{8\,Bc}{3\,\sqrt{cx^2+bx}b^2} - \frac{16\,Ac^2}{5\,\sqrt{cx^2+bx}b^3} - \frac{2\,B}{3\,\sqrt{cx^2+bx}bx} + \frac{4\,Ac}{5\,\sqrt{cx^2+bx}b^2x} - \frac{2\,A}{5\,\sqrt{cx^2+bx}bx^2}$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
16/3*B*c^2*x/(sqrt(c*x^2 + b*x)*b^3) - 32/5*A*c^3*x/(sqrt(c*x^2 + b*x)*b^4
) + 8/3*B*c/(sqrt(c*x^2 + b*x)*b^2) - 16/5*A*c^2/(sqrt(c*x^2 + b*x)*b^3) -
 2/3*B/(sqrt(c*x^2 + b*x)*b*x) + 4/5*A*c/(sqrt(c*x^2 + b*x)*b^2*x) - 2/5*A
/(sqrt(c*x^2 + b*x)*b*x^2)
```

Giac [F]

$$\int \frac{A+Bx}{x^2(bx+cx^2)^{3/2}}\,dx = \int \frac{Bx+A}{(cx^2+bx)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/((c*x^2 + b*x)^(3/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 5.56 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.71

$$\int \frac{A+Bx}{x^2\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2\,\sqrt{c\,x^2+b\,x}\,\left(5\,B\,b^3\,x+3\,A\,b^3-20\,B\,b^2\,c\,x^2-6\,A\,b^2\,c\,x-40\,B\,b\,c^2\,x^3+24\,A\,b\,c^2\,x^2+48\,A\,c^3\,x^3\right)}{15\,b^4\,x^3\,\left(b+c\,x\right)}$$

input
```
int((A + B*x)/(x^2*(b*x + c*x^2)^(3/2)),x)
```

output
```
-(2*(b*x + c*x^2)^(1/2)*(3*A*b^3 + 48*A*c^3*x^3 + 5*B*b^3*x - 6*A*b^2*c*x
+ 24*A*b*c^2*x^2 - 20*B*b^2*c*x^2 - 40*B*b*c^2*x^3))/(15*b^4*x^3*(b + c*x)
)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx}{x^2\left(bx+cx^2\right)^{3/2}}\,dx = \frac{\frac{32\sqrt{c}\,\sqrt{cx+b}\,a\,c^2x^3}{5}-\frac{16\sqrt{c}\,\sqrt{cx+b}\,b^2c\,x^3}{3}-\frac{2\sqrt{x}\,a\,b^3}{5}+\frac{4\sqrt{x}\,a\,b^2cx}{5}-\frac{16\sqrt{x}\,ab\,c^2x^2}{5}-\frac{32\sqrt{x}\,a\,c^3x^3}{5}-}{\sqrt{cx+b}\,b^4x^3}$$

input
```
int((B*x+A)/x^2/(c*x^2+b*x)^(3/2),x)
```

output
```
(2*(48*sqrt(c)*sqrt(b + c*x)*a*c**2*x**3 - 40*sqrt(c)*sqrt(b + c*x)*b**2*c
*x**3 - 3*sqrt(x)*a*b**3 + 6*sqrt(x)*a*b**2*c*x - 24*sqrt(x)*a*b*c**2*x**2
 - 48*sqrt(x)*a*c**3*x**3 - 5*sqrt(x)*b**4*x + 20*sqrt(x)*b**3*c*x**2 + 40
*sqrt(x)*b**2*c**2*x**3))/(15*sqrt(b + c*x)*b**4*x**3)
```

3.160 $\int \frac{A+Bx}{x^3(bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 157

$$\int \frac{A+Bx}{x^3\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2A}{7bx^3\sqrt{bx+cx^2}} + \frac{2(7bB-8Ac)}{7b^2x^2\sqrt{bx+cx^2}} - \frac{12(7bB-8Ac)\sqrt{bx+cx^2}}{35b^3x^3} + \frac{16c(7bB-8Ac)\sqrt{bx+cx^2}}{35b^4x^2} - \frac{32c^2(7bB-8Ac)\sqrt{bx+cx^2}}{35b^5x}$$

output
```
-2/7*A/b/x^3/(c*x^2+b*x)^(1/2)+2/7*(-8*A*c+7*B*b)/b^2/x^2/(c*x^2+b*x)^(1/2
)-12/35*(-8*A*c+7*B*b)*(c*x^2+b*x)^(1/2)/b^3/x^3+16/35*c*(-8*A*c+7*B*b)*(c
*x^2+b*x)^(1/2)/b^4/x^2-32/35*c^2*(-8*A*c+7*B*b)*(c*x^2+b*x)^(1/2)/b^5/x
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx}{x^3\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2(7bBx(b^3-2b^2cx+8bc^2x^2+16c^3x^3)+A(5b^4-8b^3cx+16b^2c^2x^2-64bc^3x^3-128c^4x^4))}{35b^5x^3\sqrt{x(b+cx)}}$$

input
```
Integrate[(A + B*x)/(x^3*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(7*b*B*x*(b^3 - 2*b^2*c*x + 8*b*c^2*x^2 + 16*c^3*x^3) + A*(5*b^4 - 8*b
^3*c*x + 16*b^2*c^2*x^2 - 64*b*c^3*x^3 - 128*c^4*x^4)))/(35*b^5*x^3*Sqrt[x
*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1220, 1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3 (bx+cx^2)^{3/2}} \, dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(7bB-8Ac)\int \frac{1}{x^2(cx^2+bx)^{3/2}}dx}{7b}-\frac{2A}{7bx^3\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(7bB-8Ac)\left(-\frac{6c\int \frac{1}{x\left(cx^2+bx\right)^{3/2}}dx}{5b}-\frac{2}{5bx^2\sqrt{bx+cx^2}}\right)}{7b}-\frac{2A}{7bx^3\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{ 1129}$$

$$\frac{(7bB-8Ac)\left(-\frac{6c\left(-\frac{4c\int \frac{1}{\left(cx^2+bx\right)^{3/2}}dx}{3b}-\frac{2}{3bx\sqrt{bx+cx^2}}\right)}{5b}-\frac{2}{5bx^2\sqrt{bx+cx^2}}\right)}{7b}-\frac{2A}{7bx^3\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{ 1088}$$

$$\frac{\left(-\frac{6c\left(\frac{8c(b+2cx)}{3b^3\sqrt{bx+cx^2}}-\frac{2}{3bx\sqrt{bx+cx^2}}\right)}{5b}-\frac{2}{5bx^2\sqrt{bx+cx^2}}\right)(7bB-8Ac)}{7b}-\frac{2A}{7bx^3\sqrt{bx+cx^2}}$$

input
```
Int[(A + B*x)/(x^3*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*A)/(7*b*x^3*Sqrt[b*x + c*x^2]) + ((7*b*B - 8*A*c)*(-2/(5*b*x^2*Sqrt[b*
x + c*x^2]) - (6*c*(-2/(3*b*x*Sqrt[b*x + c*x^2]) + (8*c*(b + 2*c*x))/(3*b^
3*Sqrt[b*x + c*x^2])))/(5*b)))/(7*b)
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.53

method	result
pseudoelliptic	$-\frac{2\left(\left(\frac{7Bx}{5}+A\right)b^{4}-\frac{8\left(\frac{7Bx}{4}+A\right)cx\,b^{3}}{5}+\frac{16c^{2}x^{2}\left(\frac{7Bx}{2}+A\right)b^{2}}{5}-\frac{64c^{3}\left(-\frac{7Bx}{4}+A\right)x^{3}b}{5}-\frac{128A\,c^{4}x^{4}}{5}\right)}{7\sqrt{x(cx+b)}\,x^{3}b^{5}}$
gosper	$-\frac{2(cx+b)\left(-128A\,c^{4}x^{4}+112Bb\,c^{3}x^{4}-64Ab\,c^{3}x^{3}+56B\,b^{2}c^{2}x^{3}+16A\,b^{2}c^{2}x^{2}-14B\,b^{3}c\,x^{2}-8A\,b^{3}cx+7B\,b^{4}x+5A\,b^{4}\right)}{35x^{2}b^{5}\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}$
orering	$-\frac{2(cx+b)\left(-128A\,c^{4}x^{4}+112Bb\,c^{3}x^{4}-64Ab\,c^{3}x^{3}+56B\,b^{2}c^{2}x^{3}+16A\,b^{2}c^{2}x^{2}-14B\,b^{3}c\,x^{2}-8A\,b^{3}cx+7B\,b^{4}x+5A\,b^{4}\right)}{35x^{2}b^{5}\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}$
risch	$-\frac{2(cx+b)\left(-93A\,c^{3}x^{3}+77x^{3}Bb\,c^{2}+29Ab\,c^{2}x^{2}-21x^{2}B\,b^{2}c-13A\,b^{2}cx+7xB\,b^{3}+5A\,b^{3}\right)}{35b^{5}x^{3}\sqrt{x(cx+b)}}+\frac{2c^{3}(Ac-Bb)x}{\sqrt{x(cx+b)}\,b^{5}}$
trager	$-\frac{2\left(-128A\,c^{4}x^{4}+112Bb\,c^{3}x^{4}-64Ab\,c^{3}x^{3}+56B\,b^{2}c^{2}x^{3}+16A\,b^{2}c^{2}x^{2}-14B\,b^{3}c\,x^{2}-8A\,b^{3}cx+7B\,b^{4}x+5A\,b^{4}\right)\sqrt{c\,x^{2}+bx}}{35(cx+b)b^{5}x^{4}}$
default	$A\left(-\frac{2}{7b\,x^{3}\sqrt{c\,x^{2}+bx}}-\frac{8c\left(-\frac{2}{5b\,x^{2}\sqrt{c\,x^{2}+bx}}-\frac{6c\left(-\frac{2}{3bx\sqrt{c\,x^{2}+bx}}+\frac{8c(2cx+b)}{3b^{3}\sqrt{c\,x^{2}+bx}}\right)}{5b}\right)}{7b}\right)+B\left(-\frac{2}{5b\,x^{2}\sqrt{c\,x^{2}+bx}}-\right.$

input
```
int((B*x+A)/x^3/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/7*((7/5*B*x+A)*b^4-8/5*(7/4*B*x+A)*c*x*b^3+16/5*c^2*x^2*(7/2*B*x+A)*b^2
-64/5*c^3*(-7/4*B*x+A)*x^3*b-128/5*A*c^4*x^4)/(x*(c*x+b))^(1/2)/x^3/b^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx}{x^3\left(bx+cx^2\right)^{3/2}}\,dx =$$
$$-\frac{2\left(5\,Ab^4+16\left(7\,Bbc^3-8\,Ac^4\right)x^4+8\left(7\,Bb^2c^2-8\,Abc^3\right)x^3-2\left(7\,Bb^3c-8\,Ab^2c^2\right)x^2+\left(7\,Bb^4-8\,Ab^3c\right)x\right.}{35\left(b^5cx^5+b^6x^4\right)}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/35*(5*A*b^4 + 16*(7*B*b*c^3 - 8*A*c^4)*x^4 + 8*(7*B*b^2*c^2 - 8*A*b*c^3
)*x^3 - 2*(7*B*b^3*c - 8*A*b^2*c^2)*x^2 + (7*B*b^4 - 8*A*b^3*c)*x)*sqrt(c*
x^2 + b*x)/(b^5*c*x^5 + b^6*x^4)
```

Sympy [F]

$$\int \frac{A+Bx}{x^3(bx+cx^2)^{3/2}}\,dx = \int \frac{A+Bx}{x^3(x(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**3/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((A + B*x)/(x**3*(x*(b + c*x))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.20

$$\begin{aligned}\int \frac{A+Bx}{x^3(bx+cx^2)^{3/2}}\,dx = &-\frac{32\,Bc^3x}{5\,\sqrt{cx^2+bx}b^4}+\frac{256\,Ac^4x}{35\,\sqrt{cx^2+bx}b^5}\\&-\frac{16\,Bc^2}{5\,\sqrt{cx^2+bx}b^3}+\frac{128\,Ac^3}{35\,\sqrt{cx^2+bx}b^4}+\frac{4\,Bc}{5\,\sqrt{cx^2+bx}b^2x}-\frac{32\,Ac^2}{35\,\sqrt{cx^2+bx}b^3x}\\&-\frac{2\,B}{5\,\sqrt{cx^2+bx}bx^2}+\frac{16\,Ac}{35\,\sqrt{cx^2+bx}b^2x^2}-\frac{2\,A}{7\,\sqrt{cx^2+bx}bx^3}\end{aligned}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
-32/5*B*c^3*x/(sqrt(c*x^2 + b*x)*b^4) + 256/35*A*c^4*x/(sqrt(c*x^2 + b*x)*
b^5) - 16/5*B*c^2/(sqrt(c*x^2 + b*x)*b^3) + 128/35*A*c^3/(sqrt(c*x^2 + b*x
)*b^4) + 4/5*B*c/(sqrt(c*x^2 + b*x)*b^2*x) - 32/35*A*c^2/(sqrt(c*x^2 + b*x
)*b^3*x) - 2/5*B/(sqrt(c*x^2 + b*x)*b*x^2) + 16/35*A*c/(sqrt(c*x^2 + b*x)*
b^2*x^2) - 2/7*A/(sqrt(c*x^2 + b*x)*b*x^3)
```

Giac [F]

$$\int \frac{A+Bx}{x^3 (bx+cx^2)^{3/2}}\,dx = \int \frac{Bx+A}{(cx^2+bx)^{\frac{3}{2}}x^3}\,dx$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/((c*x^2 + b*x)^(3/2)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 5.72 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.03

$$\int \frac{A+Bx}{x^3 (bx+cx^2)^{3/2}}\,dx = -\frac{(14\,B\,b^2-26\,A\,b\,c)\,\sqrt{c\,x^2+b\,x}}{35\,b^4\,x^3} - \frac{2\,A\,\sqrt{c\,x^2+b\,x}}{7\,b^2\,x^4} - \frac{\sqrt{c\,x^2+b\,x}\,\left(x\,\left(\frac{116\,A\,c^4-84\,B\,b\,c^3}{35\,b^5}-\frac{4\,c^3\,(93\,A\,c-77\,B\,b)}{35\,b^5}\right)-\frac{2\,c^2\,(93\,A\,c-77\,B\,b)}{35\,b^4}\right)}{x\,(b+c\,x)} - \frac{2\,c\,\sqrt{c\,x^2+b\,x}\,(29\,A\,c-21\,B\,b)}{35\,b^4\,x^2}$$

input
```
int((A + B*x)/(x^3*(b*x + c*x^2)^(3/2)),x)
```

output
```
- ((14*B*b^2 - 26*A*b*c)*(b*x + c*x^2)^(1/2))/(35*b^4*x^3) - (2*A*(b*x + c
*x^2)^(1/2))/(7*b^2*x^4) - ((b*x + c*x^2)^(1/2)*(x*((116*A*c^4 - 84*B*b*c^
3)/(35*b^5) - (4*c^3*(93*A*c - 77*B*b))/(35*b^5)) - (2*c^2*(93*A*c - 77*B*
b))/(35*b^4)))/(x*(b + c*x)) - (2*c*(b*x + c*x^2)^(1/2)*(29*A*c - 21*B*b))
/(35*b^4*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.97

$$\int \frac{A+Bx}{x^3\left(bx+cx^2\right)^{3/2}}\,dx = \frac{-\frac{256\sqrt{c}\,\sqrt{cx+b}\,a\,c^3x^4}{35}+\frac{32\sqrt{c}\,\sqrt{cx+b}\,b^2c^2x^4}{5}-\frac{2\sqrt{x}\,a\,b^4}{7}+\frac{16\sqrt{x}\,a\,b^3cx}{35}-\frac{32\sqrt{x}\,a\,b^2c^2x^2}{35}+\frac{128\sqrt{x}\,ab}{35}}{\sqrt{cx+b}\,b^5x^4}$$

input
```
int((B*x+A)/x^3/(c*x^2+b*x)^(3/2),x)
```

output
```
(2*( - 128*sqrt(c)*sqrt(b + c*x)*a*c**3*x**4 + 112*sqrt(c)*sqrt(b + c*x)*b
**2*c**2*x**4 - 5*sqrt(x)*a*b**4 + 8*sqrt(x)*a*b**3*c*x - 16*sqrt(x)*a*b**
2*c**2*x**2 + 64*sqrt(x)*a*b*c**3*x**3 + 128*sqrt(x)*a*c**4*x**4 - 7*sqrt(
x)*b**5*x + 14*sqrt(x)*b**4*c*x**2 - 56*sqrt(x)*b**3*c**2*x**3 - 112*sqrt(
x)*b**2*c**3*x**4))/(35*sqrt(b + c*x)*b**5*x**4)
```

3.161 $\int \frac{A+Bx}{x^4(bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 192

$$\begin{aligned}\int \frac{A+Bx}{x^4(bx+cx^2)^{3/2}}\,dx = &-\frac{2A}{9bx^4\sqrt{bx+cx^2}}+\frac{2(9bB-10Ac)}{9b^2x^3\sqrt{bx+cx^2}}\\ &-\frac{16(9bB-10Ac)\sqrt{bx+cx^2}}{63b^3x^4}+\frac{32c(9bB-10Ac)\sqrt{bx+cx^2}}{105b^4x^3}\\ &-\frac{128c^2(9bB-10Ac)\sqrt{bx+cx^2}}{315b^5x^2}+\frac{256c^3(9bB-10Ac)\sqrt{bx+cx^2}}{315b^6x}\end{aligned}$$

output

```
-2/9*A/b/x^4/(c*x^2+b*x)^(1/2)+2/9*(-10*A*c+9*B*b)/b^2/x^3/(c*x^2+b*x)^(1/
2)-16/63*(-10*A*c+9*B*b)*(c*x^2+b*x)^(1/2)/b^3/x^4+32/105*c*(-10*A*c+9*B*b
)*(c*x^2+b*x)^(1/2)/b^4/x^3-128/315*c^2*(-10*A*c+9*B*b)*(c*x^2+b*x)^(1/2)/
b^5/x^2+256/315*c^3*(-10*A*c+9*B*b)*(c*x^2+b*x)^(1/2)/b^6/x
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.64

$$\int \frac{A+Bx}{x^4\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{2(9bBx(5b^4-8b^3cx+16b^2c^2x^2-64bc^3x^3-128c^4x^4)+5A(7b^5-10b^4cx+16b^3c^2x^2-32b^2c^3x^3+128b}{315b^6x^4\sqrt{x(b+cx)}}$$

input
```
Integrate[(A + B*x)/(x^4*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*(9*b*B*x*(5*b^4 - 8*b^3*c*x + 16*b^2*c^2*x^2 - 64*b*c^3*x^3 - 128*c^4*
x^4) + 5*A*(7*b^5 - 10*b^4*c*x + 16*b^3*c^2*x^2 - 32*b^2*c^3*x^3 + 128*b*c
^4*x^4 + 256*c^5*x^5)))/(315*b^6*x^4*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1220, 1129, 1129, 1129, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^4\left(bx+cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(9bB-10Ac)\int \frac{1}{x^3(cx^2+bx)^{3/2}}dx}{9b}-\frac{2A}{9bx^4\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(9bB-10Ac)\left(-\frac{8c\int \frac{1}{x^2\left(cx^2+bx\right)^{3/2}}dx}{7b}-\frac{2}{7bx^3\sqrt{bx+cx^2}}\right)}{9b}-\frac{2A}{9bx^4\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(9bB-10Ac)\left(-\frac{8c\left(-\frac{6c\int \frac{1}{x\left(cx^2+bx\right)^{3/2}}dx}{5b}-\frac{2}{5bx^2\sqrt{bx+cx^2}}\right)}{7b}-\frac{2}{7bx^3\sqrt{bx+cx^2}}\right)}{9b}-\frac{2A}{9bx^4\sqrt{bx+cx^2}}$$

$\Big\downarrow$ 1129

$$\frac{(9bB-10Ac)\left(-\frac{8c\left(-\frac{6c\left(-\frac{4c\int \frac{1}{\left(cx^2+bx\right)^{3/2}}dx}{3b}-\frac{2}{3bx\sqrt{bx+cx^2}}\right)}{5b}-\frac{2}{5bx^2\sqrt{bx+cx^2}}\right)}{7b}-\frac{2}{7bx^3\sqrt{bx+cx^2}}\right)}{9b}-\frac{2A}{9bx^4\sqrt{bx+cx^2}}$$

$\Big\downarrow$ 1088

$$\frac{\left(-\frac{8c\left(-\frac{6c\left(\frac{8c(b+2cx)}{3b^3\sqrt{bx+cx^2}}-\frac{2}{3bx\sqrt{bx+cx^2}}\right)}{5b}-\frac{2}{5bx^2\sqrt{bx+cx^2}}\right)}{7b}-\frac{2}{7bx^3\sqrt{bx+cx^2}}\right)(9bB-10Ac)}{9b}-\frac{2A}{9bx^4\sqrt{bx+cx^2}}$$

input
```
Int[(A + B*x)/(x^4*(b*x + c*x^2)^(3/2)),x]
```

output
```
(-2*A)/(9*b*x^4*Sqrt[b*x + c*x^2]) + ((9*b*B - 10*A*c)*(-2/(7*b*x^3*Sqrt[b
*x + c*x^2]) - (8*c*(-2/(5*b*x^2*Sqrt[b*x + c*x^2]) - (6*c*(-2/(3*b*x*Sqrt
[b*x + c*x^2]) + (8*c*(b + 2*c*x))/(3*b^3*Sqrt[b*x + c*x^2])))/(5*b)))/(7*
b)))/(9*b)
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] && !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.53

method	result
pseudoelliptic	$\frac{(-90Bx-70A)b^5+100c\left(\frac{36Bx}{25}+A\right)x\,b^4-160c^2x^2\left(\frac{9Bx}{5}+A\right)b^3+320c^3x^3\left(\frac{18Bx}{5}+A\right)b^2-1280c^4x^4\left(-\frac{9Bx}{5}+A\right)b-2560A\,c}{315\sqrt{x(cx+b)}\,x^4b^6}$
gosper	$-\frac{2(cx+b)\left(1280A\,c^5x^5-1152Bb\,c^4x^5+640Ab\,c^4x^4-576B\,b^2c^3x^4-160A\,b^2c^3x^3+144B\,b^3c^2x^3+80A\,b^3c^2x^2-72B\,b^4c\,x^2-\right.}{315x^3b^6(c\,x^2+bx)^{\frac{3}{2}}}$
orering	$-\frac{2(cx+b)\left(1280A\,c^5x^5-1152Bb\,c^4x^5+640Ab\,c^4x^4-576B\,b^2c^3x^4-160A\,b^2c^3x^3+144B\,b^3c^2x^3+80A\,b^3c^2x^2-72B\,b^4c\,x^2-\right.}{315x^3b^6(c\,x^2+bx)^{\frac{3}{2}}}$
risch	$-\frac{2(cx+b)\left(965A\,c^4x^4-837Bb\,c^3x^4-325Ab\,c^3x^3+261B\,b^2c^2x^3+165A\,b^2c^2x^2-117B\,b^3c\,x^2-85A\,b^3cx+45B\,b^4x+35A\,b^4\right)}{315b^6x^4\sqrt{x(cx+b)}}$
trager	$-\frac{2\left(1280A\,c^5x^5-1152Bb\,c^4x^5+640Ab\,c^4x^4-576B\,b^2c^3x^4-160A\,b^2c^3x^3+144B\,b^3c^2x^3+80A\,b^3c^2x^2-72B\,b^4c\,x^2-50A\,b^4\right.}{315(cx+b)b^6x^5}$
default	$A\left(-\frac{2}{9b\,x^4\sqrt{c\,x^2+bx}}-\frac{10c\left(-\frac{2}{7b\,x^3\sqrt{c\,x^2+bx}}-\frac{8c\left(-\frac{2}{5b\,x^2\sqrt{c\,x^2+bx}}-\frac{6c\left(-\frac{2}{3bx\sqrt{c\,x^2+bx}}+\frac{8c(2cx+b)}{3b^3\sqrt{c\,x^2+bx}}\right)}{5b}\right)}{7b}\right)}{9b}\right)+B$

input
```
int((B*x+A)/x^4/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/315*((-90*B*x-70*A)*b^5+100*c*(36/25*B*x+A)*x*b^4-160*c^2*x^2*(9/5*B*x+A
)*b^3+320*c^3*x^3*(18/5*B*x+A)*b^2-1280*c^4*x^4*(-9/5*B*x+A)*b-2560*A*c^5*
x^5)/(x*(c*x+b))^(1/2)/x^4/b^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx}{x^4\left(bx+cx^2\right)^{3/2}}\,dx =$$
$$-\frac{2\left(35\,Ab^5-128\,(9\,Bbc^4-10\,Ac^5)x^5-64\,(9\,Bb^2c^3-10\,Abc^4)x^4+16\,(9\,Bb^3c^2-10\,Ab^2c^3)x^3-8\,(9\,Bb^4c\right.}{315\,(b^6cx^6+b^7x^5)}$$

input
```
integrate((B*x+A)/x^4/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
-2/315*(35*A*b^5 - 128*(9*B*b*c^4 - 10*A*c^5)*x^5 - 64*(9*B*b^2*c^3 - 10*A
*b*c^4)*x^4 + 16*(9*B*b^3*c^2 - 10*A*b^2*c^3)*x^3 - 8*(9*B*b^4*c - 10*A*b^
3*c^2)*x^2 + 5*(9*B*b^5 - 10*A*b^4*c)*x)*sqrt(c*x^2 + b*x)/(b^6*c*x^6 + b^
7*x^5)
```

Sympy [F]

$$\int \frac{A+Bx}{x^4\left(bx+cx^2\right)^{3/2}}\,dx=\int \frac{A+Bx}{x^4\left(x\left(b+cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**4/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((A + B*x)/(x**4*(x*(b + c*x))**(3/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.22

$$\int \frac{A+Bx}{x^4\left(bx+cx^2\right)^{3/2}}\,dx=\frac{256\,Bc^4x}{35\,\sqrt{cx^2+bx}b^5}-\frac{512\,Ac^5x}{63\,\sqrt{cx^2+bx}b^6}+\frac{128\,Bc^3}{35\,\sqrt{cx^2+bx}b^4}-\frac{256\,Ac^4}{63\,\sqrt{cx^2+bx}b^5}-\frac{32\,Bc^2}{35\,\sqrt{cx^2+bx}b^3x}+\frac{64\,Ac^3}{63\,\sqrt{cx^2+bx}b^4x}+\frac{16\,Bc}{35\,\sqrt{cx^2+bx}b^2x^2}-\frac{32\,Ac^2}{63\,\sqrt{cx^2+bx}b^3x^2}-\frac{2\,B}{7\,\sqrt{cx^2+bx}bx^3}+\frac{20\,Ac}{63\,\sqrt{cx^2+bx}b^2x^3}-\frac{2\,A}{9\,\sqrt{cx^2+bx}bx^4}$$

input
```
integrate((B*x+A)/x^4/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
256/35*B*c^4*x/(sqrt(c*x^2 + b*x)*b^5) - 512/63*A*c^5*x/(sqrt(c*x^2 + b*x)
*b^6) + 128/35*B*c^3/(sqrt(c*x^2 + b*x)*b^4) - 256/63*A*c^4/(sqrt(c*x^2 +
b*x)*b^5) - 32/35*B*c^2/(sqrt(c*x^2 + b*x)*b^3*x) + 64/63*A*c^3/(sqrt(c*x^
2 + b*x)*b^4*x) + 16/35*B*c/(sqrt(c*x^2 + b*x)*b^2*x^2) - 32/63*A*c^2/(sqr
t(c*x^2 + b*x)*b^3*x^2) - 2/7*B/(sqrt(c*x^2 + b*x)*b*x^3) + 20/63*A*c/(sqr
t(c*x^2 + b*x)*b^2*x^3) - 2/9*A/(sqrt(c*x^2 + b*x)*b*x^4)
```

Giac [F]

$$\int \frac{A+Bx}{x^4 (bx+cx^2)^{3/2}}\,dx = \int \frac{Bx+A}{(cx^2+bx)^{\frac{3}{2}}x^4}\,dx$$

input
```
integrate((B*x+A)/x^4/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/((c*x^2 + b*x)^(3/2)*x^4), x)
```

Mupad [B] (verification not implemented)

Time = 5.75 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.99

$$\int \frac{A+Bx}{x^4 (bx+cx^2)^{3/2}}\,dx = \frac{\sqrt{c\,x^2+b\,x}\,\left(x\,\left(\frac{1300\,A\,c^5-1044\,B\,b\,c^4}{315\,b^6}-\frac{4\,c^4\,(965\,A\,c-837\,B\,b)}{315\,b^6}\right)-\frac{2\,c^3\,(965\,A\,c-837\,B\,b)}{315\,b^5}\right)}{x\,\left(b+c\,x\right)} - \frac{2\,A\,\sqrt{c\,x^2+b\,x}}{9\,b^2\,x^5} - \frac{(18\,B\,b^2-34\,A\,b\,c)\,\sqrt{c\,x^2+b\,x}}{63\,b^4\,x^4} - \frac{2\,c\,\sqrt{c\,x^2+b\,x}\,(55\,A\,c-39\,B\,b)}{105\,b^4\,x^3} + \frac{2\,c^2\,\sqrt{c\,x^2+b\,x}\,(325\,A\,c-261\,B\,b)}{315\,b^5\,x^2}$$

input
```
int((A + B*x)/(x^4*(b*x + c*x^2)^(3/2)),x)
```

output
```
((b*x + c*x^2)^(1/2)*(x*((1300*A*c^5 - 1044*B*b*c^4)/(315*b^6) - (4*c^4*(9
65*A*c - 837*B*b))/(315*b^6)) - (2*c^3*(965*A*c - 837*B*b))/(315*b^5)))/(x
*(b + c*x)) - (2*A*(b*x + c*x^2)^(1/2))/(9*b^2*x^5) - ((18*B*b^2 - 34*A*b*
c)*(b*x + c*x^2)^(1/2))/(63*b^4*x^4) - (2*c*(b*x + c*x^2)^(1/2)*(55*A*c -
39*B*b))/(105*b^4*x^3) + (2*c^2*(b*x + c*x^2)^(1/2)*(325*A*c - 261*B*b))/(
315*b^5*x^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{x^4 (bx+cx^2)^{3/2}}\, dx = \frac{\frac{512\sqrt{c}\,\sqrt{cx+b}\,a\,c^4x^5}{63} - \frac{256\sqrt{c}\,\sqrt{cx+b}\,b^2c^3x^5}{35} - \frac{2\sqrt{x}\,a\,b^5}{9} + \frac{20\sqrt{x}\,a\,b^4cx}{63} - \frac{32\sqrt{x}\,a\,b^3c^2x^2}{63} + \frac{64\sqrt{x}\,a\,b^2c}{63}}{\sqrt{}}$$

input
```
int((B*x+A)/x^4/(c*x^2+b*x)^(3/2),x)
```

output
```
(2*(1280*sqrt(c)*sqrt(b + c*x)*a*c**4*x**5 - 1152*sqrt(c)*sqrt(b + c*x)*b*
*2*c**3*x**5 - 35*sqrt(x)*a*b**5 + 50*sqrt(x)*a*b**4*c*x - 80*sqrt(x)*a*b*
*3*c**2*x**2 + 160*sqrt(x)*a*b**2*c**3*x**3 - 640*sqrt(x)*a*b*c**4*x**4 -
1280*sqrt(x)*a*c**5*x**5 - 45*sqrt(x)*b**6*x + 72*sqrt(x)*b**5*c*x**2 - 14
4*sqrt(x)*b**4*c**2*x**3 + 576*sqrt(x)*b**3*c**3*x**4 + 1152*sqrt(x)*b**2*
c**4*x**5))/(315*sqrt(b + c*x)*b**6*x**5)
```

3.162 $\int \frac{x^5(A+Bx)}{(bx+cx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 155

$$\int \frac{x^5(A+Bx)}{(bx+cx^2)^{5/2}} \, dx = \frac{2(bB-Ac)x^4}{3c^2(bx+cx^2)^{3/2}} + \frac{2(8bB-5Ac)x^2}{3c^3\sqrt{bx+cx^2}} - \frac{5(7bB-4Ac)\sqrt{bx+cx^2}}{4c^4} + \frac{Bx\sqrt{bx+cx^2}}{2c^3} + \frac{5b(7bB-4Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{4c^{9/2}}$$

output
```
2/3*(-A*c+B*b)*x^4/c^2/(c*x^2+b*x)^(3/2)+2/3*(-5*A*c+8*B*b)*x^2/c^3/(c*x^2
+b*x)^(1/2)-5/4*(-4*A*c+7*B*b)*(c*x^2+b*x)^(1/2)/c^4+1/2*B*x*(c*x^2+b*x)^(
1/2)/c^3+5/4*b*(-4*A*c+7*B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(9/2)
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.16

$$\int \frac{x^5(A+Bx)}{(bx+cx^2)^{5/2}} \, dx = \frac{x\left(\sqrt{c}x(-105b^3B+bc^2x(80A-21Bx)+20b^2c(3A-7Bx)+6c^3x^2(2A+Bx))+1\right.}{12c^{9/2}(x}$$

input
```
Integrate[(x^5*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(x*(Sqrt[c]*x*(-105*b^3*B + b*c^2*x*(80*A - 21*B*x) + 20*b^2*c*(3*A - 7*B*
x) + 6*c^3*x^2*(2*A + B*x)) + 120*A*b*c*Sqrt[x]*(b + c*x)^(3/2)*ArcTanh[(S
qrt[c]*Sqrt[x])/(Sqrt[b] - Sqrt[b + c*x])] + 210*b^2*B*Sqrt[x]*(b + c*x)^(
3/2)*ArcTanh[(Sqrt[c]*Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])]))/(12*c^(9/2)*(
x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1218, 1124, 2192, 27, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(A+Bx)}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1218}$$

$$-\frac{1}{3}\left(\frac{4A}{b}-\frac{7B}{c}\right)\int \frac{x^4}{(cx^2+bx)^{3/2}}dx-\frac{2x^5(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1124}$$

$$-\frac{1}{3}\left(\frac{4A}{b}-\frac{7B}{c}\right)\left(\frac{\int \frac{b^2-cxb+c^2x^2}{\sqrt{cx^2+bx}}dx}{c^3}-\frac{2b^2x}{c^3\sqrt{bx+cx^2}}\right)-\frac{2x^5(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{2192}$$

$$-\frac{1}{3}\left(\frac{4A}{b}-\frac{7B}{c}\right)\left(\frac{\frac{\int \frac{bc(4b-7cx)}{2\sqrt{cx^2+bx}}dx}{2c}+\frac{1}{2}cx\sqrt{bx+cx^2}}{c^3}-\frac{2b^2x}{c^3\sqrt{bx+cx^2}}\right)-\frac{2x^5(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$-\frac{1}{3}\left(\frac{4A}{b}-\frac{7B}{c}\right)\left(\frac{\frac{1}{4}b\int \frac{4b-7cx}{\sqrt{cx^2+bx}}dx+\frac{1}{2}cx\sqrt{bx+cx^2}}{c^3}-\frac{2b^2x}{c^3\sqrt{bx+cx^2}}\right)-\frac{2x^5(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1160}$$

$$-\frac{1}{3}\left(\frac{4A}{b}-\frac{7B}{c}\right)\left(\frac{\frac{1}{4}b\left(\frac{15}{2}b\int\frac{1}{\sqrt{cx^2+bx}}dx-7\sqrt{bx+cx^2}\right)+\frac{1}{2}cx\sqrt{bx+cx^2}}{c^3}-\frac{2b^2x}{c^3\sqrt{bx+cx^2}}\right)-\frac{2x^5(bB-Ac)}{3bc\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1091}$$

$$-\frac{1}{3}\left(\frac{4A}{b}-\frac{7B}{c}\right)\left(\frac{\frac{1}{4}b\left(15b\int\frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}-7\sqrt{bx+cx^2}\right)+\frac{1}{2}cx\sqrt{bx+cx^2}}{c^3}-\frac{2b^2x}{c^3\sqrt{bx+cx^2}}\right)-\frac{2x^5(bB-Ac)}{3bc\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{3}\left(\frac{4A}{b}-\frac{7B}{c}\right)\left(\frac{\frac{1}{4}b\left(\frac{15b\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}-7\sqrt{bx+cx^2}\right)+\frac{1}{2}cx\sqrt{bx+cx^2}}{c^3}-\frac{2b^2x}{c^3\sqrt{bx+cx^2}}\right)-\frac{2x^5(bB-Ac)}{3bc\left(bx+cx^2\right)^{3/2}}$$

input
```
Int[(x^5*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(b*B - A*c)*x^5)/(3*b*c*(b*x + c*x^2)^(3/2)) - (((4*A)/b - (7*B)/c)*((
-2*b^2*x)/(c^3*Sqrt[b*x + c*x^2]) + ((c*x*Sqrt[b*x + c*x^2])/2 + (b*(-7*Sq
rt[b*x + c*x^2] + (15*b*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c]))/
4)/c^3))/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

rule 2192
```
Int[(Pq_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> With[{q =
Expon[Pq, x], e = Coeff[Pq, x, Expon[Pq, x]]}, Simp[e*x^(q - 1)*((a + b*x +
 c*x^2)^(p + 1)/(c*(q + 2*p + 1))), x] + Simp[1/(c*(q + 2*p + 1))   Int[(a
+ b*x + c*x^2)^p*ExpandToSum[c*(q + 2*p + 1)*Pq - a*e*(q - 1)*x^(q - 2) - b
*e*(q + p)*x^(q - 1) - c*e*(q + 2*p + 1)*x^q, x], x], x]] /; FreeQ[{a, b, c
, p}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.77

method	result
pseudoelliptic	$-\frac{5\left(-x\left(-\frac{7Bx}{3}+A\right)b^{2}c^{\frac{3}{2}}-\frac{4x^{2}b\left(-\frac{21Bx}{80}+A\right)c^{\frac{5}{2}}}{3}-\frac{\left(\frac{Bx}{2}+A\right)x^{3}c^{\frac{7}{2}}}{5}+\left(\frac{7B\sqrt{c}\,b^{2}x}{4}+(cx+b)\,\text{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)\sqrt{x(cx+b)}\,\left(\right.\right.}{\sqrt{x(cx+b)}\,c^{\frac{9}{2}}\,(cx+b)}$
risch	$\frac{(2Bcx+4Ac-11Bb)x(cx+b)}{4c^{4}\sqrt{x(cx+b)}}-\frac{b\left(20A\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)-\frac{35Bb\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^{2}+bx}\right)}{\sqrt{c}}-\frac{16(3Ac-4Bb)\sqrt{c\left(\frac{b}{c}+x\right)^{2}}}{c\left(\frac{b}{c}+x\right)}\right.}{8c^{4}}$
	$b\left(-\frac{x}{2c\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}-\frac{b\left(-\frac{1}{3c\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}-\frac{b\left(-\frac{2(2cx+b)}{3b^{2}\left(c\,x^{2}+bx\right)}\right.}{}\right)}{4c}\right.$

input
```
int(x^5*(B*x+A)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-5*(-x*(-7/3*B*x+A)*b^2*c^(3/2)-4/3*x^2*b*(-21/80*B*x+A)*c^(5/2)-1/5*(1/2*
B*x+A)*x^3*c^(7/2)+(7/4*B*c^(1/2)*b^2*x+(c*x+b)*arctanh((x*(c*x+b))^(1/2)/
x/c^(1/2))*(x*(c*x+b))^(1/2)*(A*c-7/4*B*b))*b)/(x*(c*x+b))^(1/2)/c^(9/2)/(
c*x+b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.46

$$\int \frac{x^5(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \left[-\frac{15\,(7\,Bb^4 - 4\,Ab^3c + (7\,Bb^2c^2 - 4\,Abc^3)x^2 + 2\,(7\,Bb^3c - 4\,Ab^2c^2)x)\sqrt{c}\log\left(2\,cx + \right.}{} \right.$$

$$-\frac{15\,(7\,Bb^4 - 4\,Ab^3c + (7\,Bb^2c^2 - 4\,Abc^3)x^2 + 2\,(7\,Bb^3c - 4\,Ab^2c^2)x)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right) - (6\,Bc^4x}{12\,(c^7x^2 + 2\,bc^6x + b^2}$$

input
```
integrate(x^5*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(15*(7*B*b^4 - 4*A*b^3*c + (7*B*b^2*c^2 - 4*A*b*c^3)*x^2 + 2*(7*B*b
^3*c - 4*A*b^2*c^2)*x)*sqrt(c)*log(2*c*x + b - 2*sqrt(c*x^2 + b*x)*sqrt(c)
) - 2*(6*B*c^4*x^3 - 105*B*b^3*c + 60*A*b^2*c^2 - 3*(7*B*b*c^3 - 4*A*c^4)*
x^2 - 20*(7*B*b^2*c^2 - 4*A*b*c^3)*x)*sqrt(c*x^2 + b*x))/(c^7*x^2 + 2*b*c^
6*x + b^2*c^5), -1/12*(15*(7*B*b^4 - 4*A*b^3*c + (7*B*b^2*c^2 - 4*A*b*c^3)
*x^2 + 2*(7*B*b^3*c - 4*A*b^2*c^2)*x)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sq
rt(-c)/(c*x + b)) - (6*B*c^4*x^3 - 105*B*b^3*c + 60*A*b^2*c^2 - 3*(7*B*b*c
^3 - 4*A*c^4)*x^2 - 20*(7*B*b^2*c^2 - 4*A*b*c^3)*x)*sqrt(c*x^2 + b*x))/(c^
7*x^2 + 2*b*c^6*x + b^2*c^5)]
```

Sympy [F]

$$\int \frac{x^5(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x^5(A+Bx)}{(x(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5*(B*x+A)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(x**5*(A + B*x)/(x*(b + c*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. 2(131) = 262.

Time = 0.04 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.34

$$\begin{aligned}\int \frac{x^5(A+Bx)}{(bx+cx^2)^{5/2}}\,dx &= \frac{Bx^5}{2\,(cx^2+bx)^{\frac{3}{2}}c} \\ &- \frac{35\,Bb^2x\left(\frac{3\,x^2}{(cx^2+bx)^{\frac{3}{2}}c} + \frac{bx}{(cx^2+bx)^{\frac{3}{2}}c^2} - \frac{2\,x}{\sqrt{cx^2+bx}bc} - \frac{1}{\sqrt{cx^2+bx}c^2}\right)}{24\,c^2} \\ &+ \frac{5\,Abx\left(\frac{3\,x^2}{(cx^2+bx)^{\frac{3}{2}}c} + \frac{bx}{(cx^2+bx)^{\frac{3}{2}}c^2} - \frac{2\,x}{\sqrt{cx^2+bx}bc} - \frac{1}{\sqrt{cx^2+bx}c^2}\right)}{6\,c} \\ &- \frac{7\,Bbx^4}{4\,(cx^2+bx)^{\frac{3}{2}}c^2} + \frac{Ax^4}{(cx^2+bx)^{\frac{3}{2}}c} - \frac{35\,Bb^2x}{6\,\sqrt{cx^2+bx}c^4} \\ &+ \frac{10\,Abx}{3\,\sqrt{cx^2+bx}c^3} + \frac{35\,Bb^2\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{8\,c^{\frac{9}{2}}} \\ &- \frac{5\,Ab\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{2\,c^{\frac{7}{2}}} - \frac{35\,\sqrt{cx^2+bx}Bb}{12\,c^4} + \frac{5\,\sqrt{cx^2+bx}A}{3\,c^3}\end{aligned}$$

input
```
integrate(x^5*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
1/2*B*x^5/((c*x^2 + b*x)^(3/2)*c) - 35/24*B*b^2*x*(3*x^2/((c*x^2 + b*x)^(3
/2)*c) + b*x/((c*x^2 + b*x)^(3/2)*c^2) - 2*x/(sqrt(c*x^2 + b*x)*b*c) - 1/(
sqrt(c*x^2 + b*x)*c^2))/c^2 + 5/6*A*b*x*(3*x^2/((c*x^2 + b*x)^(3/2)*c) + b
*x/((c*x^2 + b*x)^(3/2)*c^2) - 2*x/(sqrt(c*x^2 + b*x)*b*c) - 1/(sqrt(c*x^2
 + b*x)*c^2))/c - 7/4*B*b*x^4/((c*x^2 + b*x)^(3/2)*c^2) + A*x^4/((c*x^2 +
b*x)^(3/2)*c) - 35/6*B*b^2*x/(sqrt(c*x^2 + b*x)*c^4) + 10/3*A*b*x/(sqrt(c*
x^2 + b*x)*c^3) + 35/8*B*b^2*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/
c^(9/2) - 5/2*A*b*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(7/2) - 3
5/12*sqrt(c*x^2 + b*x)*B*b/c^4 + 5/3*sqrt(c*x^2 + b*x)*A/c^3
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.59

$$\int \frac{x^5(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{1}{4}\sqrt{cx^2+bx}\left(\frac{2\,Bx}{c^3}-\frac{11\,Bbc^7-4\,Ac^8}{c^{11}}\right) - \frac{5\,(7\,Bb^2-4\,Abc)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{8\,c^{\frac{9}{2}}} - \frac{2\left(12\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2Bb^3c-9\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2Ab^2c^2+21\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)Bb^4\sqrt{c}-15\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)Ab^3c^{\frac{3}{2}}+10\,Bb^5-7\,Ab^4c\right)}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right)^3c^{\frac{9}{2}}}$$

input
```
integrate(x^5*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
1/4*sqrt(c*x^2 + b*x)*(2*B*x/c^3 - (11*B*b*c^7 - 4*A*c^8)/c^11) - 5/8*(7*B
*b^2 - 4*A*b*c)*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^
(9/2) - 2/3*(12*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*b^3*c - 9*(sqrt(c)*x -
 sqrt(c*x^2 + b*x))^2*A*b^2*c^2 + 21*(sqrt(c)*x - sqrt(c*x^2 + b*x))*B*b^4
*sqrt(c) - 15*(sqrt(c)*x - sqrt(c*x^2 + b*x))*A*b^3*c^(3/2) + 10*B*b^5 - 7
*A*b^4*c)/(((sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b)^3*c^(9/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x^5\,(A+B\,x)}{(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((x^5*(A + B*x))/(b*x + c*x^2)^(5/2),x)
```

output
```
int((x^5*(A + B*x))/(b*x + c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.85

$$\int \frac{x^5(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{-480\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)a\,b^2c-480\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)ab\,c^2x}{}$$

input
```
int(x^5*(B*x+A)/(c*x^2+b*x)^(5/2),x)
```

output
```
( - 480*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b
))*a*b**2*c - 480*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(
c))/sqrt(b))*a*b*c**2*x + 840*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + s
qrt(x)*sqrt(c))/sqrt(b))*b**4 + 840*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*
x) + sqrt(x)*sqrt(c))/sqrt(b))*b**3*c*x - 80*sqrt(c)*sqrt(b + c*x)*a*b**2*
c - 80*sqrt(c)*sqrt(b + c*x)*a*b*c**2*x + 175*sqrt(c)*sqrt(b + c*x)*b**4 +
 175*sqrt(c)*sqrt(b + c*x)*b**3*c*x + 480*sqrt(x)*a*b**2*c**2 + 640*sqrt(x
)*a*b*c**3*x + 96*sqrt(x)*a*c**4*x**2 - 840*sqrt(x)*b**4*c - 1120*sqrt(x)*
b**3*c**2*x - 168*sqrt(x)*b**2*c**3*x**2 + 48*sqrt(x)*b*c**4*x**3)/(96*sqr
t(b + c*x)*c**5*(b + c*x))
```

3.163 $\int \frac{x^4(A+Bx)}{(bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 115

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2(bB-Ac)x^3}{3c^2\left(bx+cx^2\right)^{3/2}} + \frac{2(2bB-Ac)x}{c^3\sqrt{bx+cx^2}} + \frac{B\sqrt{bx+cx^2}}{c^3} - \frac{(5bB-2Ac)\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{7/2}}$$

output
```
2/3*(-A*c+B*b)*x^3/c^2/(c*x^2+b*x)^(3/2)+2*(-A*c+2*B*b)*x/c^3/(c*x^2+b*x)^
(1/2)+B*(c*x^2+b*x)^(1/2)/c^3-(-2*A*c+5*B*b)*arctanh(c^(1/2)*x/(c*x^2+b*x)
^(1/2))/c^(7/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.09

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{x^{5/2}\left(\sqrt{c}\sqrt{x}(b+cx)\left(15b^2B+c^2x(-8A+3Bx)+b(-6Ac+20Bcx)\right)+6(-5bB+\right.}{3c^{7/2}(x(b+cx))^{5/2}}$$

input
```
Integrate[(x^4*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(x^(5/2)*(Sqrt[c]*Sqrt[x]*(b + c*x)*(15*b^2*B + c^2*x*(-8*A + 3*B*x) + b*(
-6*A*c + 20*B*c*x)) + 6*(-5*b*B + 2*A*c)*(b + c*x)^(5/2)*ArcTanh[(Sqrt[c]*
Sqrt[x])/(-Sqrt[b] + Sqrt[b + c*x])]))/(3*c^(7/2)*(x*(b + c*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1218, 1124, 25, 1160, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$-\frac{1}{3}\left(\frac{2A}{b}-\frac{5B}{c}\right)\int \frac{x^3}{(cx^2+bx)^{3/2}}dx-\frac{2x^4(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1124}$$

$$-\frac{1}{3}\left(\frac{2A}{b}-\frac{5B}{c}\right)\left(\frac{\int -\frac{b-cx}{\sqrt{cx^2+bx}}dx}{c^2}+\frac{2bx}{c^2\sqrt{bx+cx^2}}\right)-\frac{2x^4(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{25}$$

$$-\frac{1}{3}\left(\frac{2A}{b}-\frac{5B}{c}\right)\left(\frac{2bx}{c^2\sqrt{bx+cx^2}}-\frac{\int \frac{b-cx}{\sqrt{cx^2+bx}}dx}{c^2}\right)-\frac{2x^4(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1160}$$

$$-\frac{1}{3}\left(\frac{2A}{b}-\frac{5B}{c}\right)\left(\frac{2bx}{c^2\sqrt{bx+cx^2}}-\frac{\frac{3}{2}b\int \frac{1}{\sqrt{cx^2+bx}}dx-\sqrt{bx+cx^2}}{c^2}\right)-\frac{2x^4(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1091}$$

$$-\frac{1}{3}\left(\frac{2A}{b}-\frac{5B}{c}\right)\left(\frac{2bx}{c^2\sqrt{bx+cx^2}}-\frac{3b\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}-\sqrt{bx+cx^2}}{c^2}\right)-\frac{2x^4(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{1}{3}\left(\frac{2A}{b}-\frac{5B}{c}\right)\left(\frac{2bx}{c^2\sqrt{bx+cx^2}}-\frac{\frac{3b\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{\sqrt{c}}-\sqrt{bx+cx^2}}{c^2}\right)-\frac{2x^4(bB-Ac)}{3bc\left(bx+cx^2\right)^{3/2}}$$

input
```
Int[(x^4*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(b*B - A*c)*x^4)/(3*b*c*(b*x + c*x^2)^(3/2)) - (((2*A)/b - (5*B)/c)*((
2*b*x)/(c^2*Sqrt[b*x + c*x^2]) - (-Sqrt[b*x + c*x^2] + (3*b*ArcTanh[(Sqrt[
c]*x)/Sqrt[b*x + c*x^2]])/Sqrt[c])/c^2))/3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1160
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Simp[(2*c*d - b
*e)/(2*c)   Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
 && NeQ[p, -1]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.90

method	result
pseudoelliptic	$\frac{-6xb\left(-\frac{10Bx}{3}+A\right)c^{\frac{3}{2}}-8\left(-\frac{3Bx}{8}+A\right)x^2c^{\frac{5}{2}}+15B\sqrt{c}\,b^2x+6(cx+b)\sqrt{x(cx+b)}\left(-\frac{5Bb}{2}+Ac\right)\operatorname{arctanh}\left(\frac{\sqrt{x(cx+b)}}{x\sqrt{c}}\right)}{c^{\frac{7}{2}}\sqrt{x(cx+b)}\,(3cx+3b)}$
risch	$\frac{Bx(cx+b)}{c^3\sqrt{x(cx+b)}}+\frac{2A\sqrt{c}\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)-\frac{5Bb\ln\left(\frac{\frac{b}{2}+cx}{\sqrt{c}}+\sqrt{c\,x^2+bx}\right)}{\sqrt{c}}-\frac{4(2Ac-3Bb)\sqrt{c\left(\frac{b}{c}+x\right)^2-\left(\frac{b}{c}+x\right)b}}{c\left(\frac{b}{c}+x\right)}+\frac{2b^2(Ac-Bb}{}}{2c^3}$
default	$A\left(-\frac{x^3}{3c(c\,x^2+bx)^{\frac{3}{2}}}-\frac{b\left(-\frac{x^2}{c\left(c\,x^2+bx\right)^{\frac{3}{2}}}+\frac{b\left(-\frac{x}{2c\left(c\,x^2+bx\right)^{\frac{3}{2}}}-\frac{b\left(-\frac{1}{3c\left(c\,x^2+bx\right)^{\frac{3}{2}}}-\frac{b\left(-\frac{2(2cx+b)}{3b^2\left(c\,x^2+bx\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{c\,x^2+bx}}\right)}{2c}\right)}{4c}\right)}{2c}\right)}{2c}\right.$

input
```
int(x^4*(B*x+A)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
6/c^(7/2)*(-x*b*(-10/3*B*x+A)*c^(3/2)-4/3*(-3/8*B*x+A)*x^2*c^(5/2)+5/2*B*c
^(1/2)*b^2*x+(c*x+b)*(x*(c*x+b))^(1/2)*(-5/2*B*b+A*c)*arctanh((x*(c*x+b))^
(1/2)/x/c^(1/2)))/(x*(c*x+b))^(1/2)/(3*c*x+3*b)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.80

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \left[-\frac{3\,(5\,Bb^3-2\,Ab^2c+(5\,Bbc^2-2\,Ac^3)x^2+2\,(5\,Bb^2c-2\,Abc^2)x)\sqrt{c}\log\left(2\,cx+b\right.}{6\,(c^6x^2+}\right.$$

input
```
integrate(x^4*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*(5*B*b^3 - 2*A*b^2*c + (5*B*b*c^2 - 2*A*c^3)*x^2 + 2*(5*B*b^2*c -
 2*A*b*c^2)*x)*sqrt(c)*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c)) - 2*(3
*B*c^3*x^2 + 15*B*b^2*c - 6*A*b*c^2 + 4*(5*B*b*c^2 - 2*A*c^3)*x)*sqrt(c*x^
2 + b*x))/(c^6*x^2 + 2*b*c^5*x + b^2*c^4), 1/3*(3*(5*B*b^3 - 2*A*b^2*c + (
5*B*b*c^2 - 2*A*c^3)*x^2 + 2*(5*B*b^2*c - 2*A*b*c^2)*x)*sqrt(-c)*arctan(sq
rt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (3*B*c^3*x^2 + 15*B*b^2*c - 6*A*b*c^
2 + 4*(5*B*b*c^2 - 2*A*c^3)*x)*sqrt(c*x^2 + b*x))/(c^6*x^2 + 2*b*c^5*x + b
^2*c^4)]
```

Sympy [F]

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x^4(A+Bx)}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(B*x+A)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(x**4*(A + B*x)/(x*(b + c*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(101) = 202$.

Time = 0.04 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.70

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = -\frac{1}{3}\,Ax\left(\frac{3\,x^2}{(cx^2+bx)^{\frac{3}{2}}c}+\frac{bx}{(cx^2+bx)^{\frac{3}{2}}c^2}-\frac{2\,x}{\sqrt{cx^2+bx}bc}-\frac{1}{\sqrt{cx^2+bx}c^2}\right) + \frac{5\,Bbx\left(\frac{3\,x^2}{(cx^2+bx)^{\frac{3}{2}}c}+\frac{bx}{(cx^2+bx)^{\frac{3}{2}}c^2}-\frac{2\,x}{\sqrt{cx^2+bx}bc}-\frac{1}{\sqrt{cx^2+bx}c^2}\right)}{6\,c} + \frac{Bx^4}{(cx^2+bx)^{\frac{3}{2}}c} + \frac{10\,Bbx}{3\sqrt{cx^2+bx}c^3} - \frac{4\,Ax}{3\sqrt{cx^2+bx}c^2} - \frac{5\,Bb\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{2\,c^{\frac{7}{2}}} + \frac{A\log\left(2\,cx+b+2\sqrt{cx^2+bx}\sqrt{c}\right)}{c^{\frac{5}{2}}} + \frac{5\sqrt{cx^2+bx}B}{3\,c^3} - \frac{2\sqrt{cx^2+bx}A}{3\,bc^2}$$

input
```
integrate(x^4*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*A*x*(3*x^2/((c*x^2 + b*x)^(3/2)*c) + b*x/((c*x^2 + b*x)^(3/2)*c^2) -
2*x/(sqrt(c*x^2 + b*x)*b*c) - 1/(sqrt(c*x^2 + b*x)*c^2)) + 5/6*B*b*x*(3*x^
2/((c*x^2 + b*x)^(3/2)*c) + b*x/((c*x^2 + b*x)^(3/2)*c^2) - 2*x/(sqrt(c*x^
2 + b*x)*b*c) - 1/(sqrt(c*x^2 + b*x)*c^2))/c + B*x^4/((c*x^2 + b*x)^(3/2)*
c) + 10/3*B*b*x/(sqrt(c*x^2 + b*x)*c^3) - 4/3*A*x/(sqrt(c*x^2 + b*x)*c^2)
- 5/2*B*b*log(2*c*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(7/2) + A*log(2*c
*x + b + 2*sqrt(c*x^2 + b*x)*sqrt(c))/c^(5/2) + 5/3*sqrt(c*x^2 + b*x)*B/c^
3 - 2/3*sqrt(c*x^2 + b*x)*A/(b*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(101) = 202$.

Time = 0.14 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.89

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{\sqrt{cx^2+bx}B}{c^3} + \frac{(5\,Bb-2\,Ac)\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{2\,c^{\frac{7}{2}}} + \frac{2\left(9\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2Bb^2c-6\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2Abc^2+15\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)Bb^3\sqrt{c}-9\left(\sqrt{c}x\right.\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right)^3c^{\frac{7}{2}}}$$

input
```
integrate(x^4*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
sqrt(c*x^2 + b*x)*B/c^3 + 1/2*(5*B*b - 2*A*c)*log(abs(2*(sqrt(c)*x - sqrt(
c*x^2 + b*x))*sqrt(c) + b))/c^(7/2) + 2/3*(9*(sqrt(c)*x - sqrt(c*x^2 + b*x
))^2*B*b^2*c - 6*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*A*b*c^2 + 15*(sqrt(c)*x
 - sqrt(c*x^2 + b*x))*B*b^3*sqrt(c) - 9*(sqrt(c)*x - sqrt(c*x^2 + b*x))*A*
b^2*c^(3/2) + 7*B*b^4 - 4*A*b^3*c)/(((sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(
c) + b)^3*c^(7/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x^4\,(A+B\,x)}{{(c\,x^2+b\,x)}^{5/2}}\,dx$$

input
```
int((x^4*(A + B*x))/(b*x + c*x^2)^(5/2),x)
```

output
```
int((x^4*(A + B*x))/(b*x + c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.96

$$\int \frac{x^4(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{12\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)abc+12\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)a\,c^2x-30\sqrt{c}}{}$$

input
```
int(x^4*(B*x+A)/(c*x^2+b*x)^(5/2),x)
```

output
```
(12*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))*a
*b*c + 12*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt
(b))*a*c**2*x - 30*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt
(c))/sqrt(b))*b**3 - 30*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)
*sqrt(c))/sqrt(b))*b**2*c*x - 5*sqrt(c)*sqrt(b + c*x)*b**3 - 5*sqrt(c)*sqr
t(b + c*x)*b**2*c*x - 12*sqrt(x)*a*b*c**2 - 16*sqrt(x)*a*c**3*x + 30*sqrt(
x)*b**3*c + 40*sqrt(x)*b**2*c**2*x + 6*sqrt(x)*b*c**3*x**2)/(6*sqrt(b + c*
x)*c**4*(b + c*x))
```

3.164 $\int \frac{x^3(A+Bx)}{(bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 84

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = -\frac{2(bB-Ac)x^3}{3bc\,(bx+cx^2)^{3/2}} - \frac{2Bx}{c^2\sqrt{bx+cx^2}} + \frac{2B\operatorname{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{5/2}}$$

output
```
-2/3*(-A*c+B*b)*x^3/b/c/(c*x^2+b*x)^(3/2)-2*B*x/c^2/(c*x^2+b*x)^(1/2)+2*B*
arctanh(c^(1/2)*x/(c*x^2+b*x)^(1/2))/c^(5/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = -\frac{2x\left(\sqrt{c}x(3b^2B+4bBcx-Ac^2x)+3bB\sqrt{x}(b+cx)^{3/2}\log\left(-\sqrt{c}\sqrt{x}+\sqrt{b+cx}\right)\right)}{3bc^{5/2}(x(b+cx))^{3/2}}$$

input
```
Integrate[(x^3*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*x*(Sqrt[c]*x*(3*b^2*B + 4*b*B*c*x - A*c^2*x) + 3*b*B*Sqrt[x]*(b + c*x)
^(3/2)*Log[-(Sqrt[c]*Sqrt[x]) + Sqrt[b + c*x]]))/(3*b*c^(5/2)*(x*(b + c*x)
)^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1218, 1124, 1091, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{B\int \frac{x^2}{(cx^2+bx)^{3/2}}dx}{c}-\frac{2x^3(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1124}$$

$$\frac{B\left(\frac{\int \frac{1}{\sqrt{cx^2+bx}}dx}{c}-\frac{2x}{c\sqrt{bx+cx^2}}\right)}{c}-\frac{2x^3(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1091}$$

$$\frac{B\left(\frac{2\int \frac{1}{1-\frac{cx^2}{cx^2+bx}}d\frac{x}{\sqrt{cx^2+bx}}}{c}-\frac{2x}{c\sqrt{bx+cx^2}}\right)}{c}-\frac{2x^3(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{B\left(\frac{2\text{arctanh}\left(\frac{\sqrt{c}x}{\sqrt{bx+cx^2}}\right)}{c^{3/2}}-\frac{2x}{c\sqrt{bx+cx^2}}\right)}{c}-\frac{2x^3(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

input
```
Int[(x^3*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(b*B - A*c)*x^3)/(3*b*c*(b*x + c*x^2)^(3/2)) + (B*((-2*x)/(c*Sqrt[b*x
+ c*x^2]) + (2*ArcTanh[(Sqrt[c]*x)/Sqrt[b*x + c*x^2]])/c^(3/2)))/c
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1091
```
Int[1/Sqrt[(b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Simp[2   Subst[Int[1/(1
 - c*x^2), x], x, x/Sqrt[b*x + c*x^2]], x] /; FreeQ[{b, c}, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.05 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.05

method	result
pseudoelliptic	$\frac{-\frac{8 B c^{\frac{3}{2}} x^{2} b}{3}+\frac{2 A c^{\frac{5}{2}} x^{2}}{3}+2\left(-b x \sqrt{c}+\sqrt{x(c x+b)}\, \operatorname{arctanh}\left(\frac{\sqrt{x(c x+b)}}{x \sqrt{c}}\right)(c x+b)\right) B b}{c^{\frac{5}{2}}(c x+b) \sqrt{x(c x+b)}\, b}$
default	$A\left(-\frac{x^{2}}{c(c x^{2}+b x)^{\frac{3}{2}}}+\frac{b\left(-\frac{x}{2 c\left(c x^{2}+b x\right)^{\frac{3}{2}}}-\frac{b\left(-\frac{1}{3 c\left(c x^{2}+b x\right)^{\frac{3}{2}}}-\frac{b\left(-\frac{2(2 c x+b)}{3 b^{2}\left(c x^{2}+b x\right)^{\frac{3}{2}}}+\frac{16 c(2 c x+b)}{3 b^{4} \sqrt{c x^{2}+b x}}\right)}{2 c}\right)}{4 c}\right)}{2 c}\right)+B\Bigg(-$

input
```
int(x^3*(B*x+A)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2*(-4/3*B*c^(3/2)*x^2*b+1/3*A*c^(5/2)*x^2+(-b*x*c^(1/2)+(x*(c*x+b))^(1/2)*
arctanh((x*(c*x+b))^(1/2)/x/c^(1/2))*(c*x+b))*B*b)/c^(5/2)/(x*(c*x+b))^(1/
2)/(c*x+b)/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 240, normalized size of antiderivative = 2.86

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \left[\frac{3\,(Bbc^2x^2+2\,Bb^2cx+Bb^3)\sqrt{c}\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)-2\,(3\,Bb^2c+(4\,B}{3\,(bc^5x^2+2\,b^2c^4x+b^3c^3)}\right.$$

$$\left.-\frac{2\left(3\,(Bbc^2x^2+2\,Bb^2cx+Bb^3)\sqrt{-c}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-c}}{cx+b}\right)+(3\,Bb^2c+(4\,Bbc^2-Ac^3)x)\sqrt{cx^2+bx}\right)}{3\,(bc^5x^2+2\,b^2c^4x+b^3c^3)}\right]$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(B*b*c^2*x^2 + 2*B*b^2*c*x + B*b^3)*sqrt(c)*log(2*c*x + b + 2*sqrt
(c*x^2 + b*x)*sqrt(c)) - 2*(3*B*b^2*c + (4*B*b*c^2 - A*c^3)*x)*sqrt(c*x^2
+ b*x))/(b*c^5*x^2 + 2*b^2*c^4*x + b^3*c^3), -2/3*(3*(B*b*c^2*x^2 + 2*B*b^
2*c*x + B*b^3)*sqrt(-c)*arctan(sqrt(c*x^2 + b*x)*sqrt(-c)/(c*x + b)) + (3*
B*b^2*c + (4*B*b*c^2 - A*c^3)*x)*sqrt(c*x^2 + b*x))/(b*c^5*x^2 + 2*b^2*c^4
*x + b^3*c^3)]
```

Sympy [F]

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x^3(A+Bx)}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(B*x+A)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(x**3*(A + B*x)/(x*(b + c*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. $2(72) = 144$.

Time = 0.04 (sec) , antiderivative size = 221, normalized size of antiderivative = 2.63

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = -\frac{1}{3}\,Bx\left(\frac{3\,x^2}{(cx^2+bx)^{\frac{3}{2}}c}+\frac{bx}{(cx^2+bx)^{\frac{3}{2}}c^2}-\frac{2\,x}{\sqrt{cx^2+bx}bc}-\frac{1}{\sqrt{cx^2+bx}c^2}\right) - \frac{Ax^2}{(cx^2+bx)^{\frac{3}{2}}c}-\frac{4\,Bx}{3\,\sqrt{cx^2+bx}c^2}-\frac{Abx}{3\,(cx^2+bx)^{\frac{3}{2}}c^2}+\frac{2\,Ax}{3\,\sqrt{cx^2+bx}bc} + \frac{B\log\left(2\,cx+b+2\,\sqrt{cx^2+bx}\sqrt{c}\right)}{c^{\frac{5}{2}}}+\frac{A}{3\,\sqrt{cx^2+bx}c^2}-\frac{2\,\sqrt{cx^2+bx}B}{3\,bc^2}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
-1/3*B*x*(3*x^2/((c*x^2 + b*x)^(3/2)*c) + b*x/((c*x^2 + b*x)^(3/2)*c^2) -
2*x/(sqrt(c*x^2 + b*x)*b*c) - 1/(sqrt(c*x^2 + b*x)*c^2)) - A*x^2/((c*x^2 +
 b*x)^(3/2)*c) - 4/3*B*x/(sqrt(c*x^2 + b*x)*c^2) - 1/3*A*b*x/((c*x^2 + b*x
)^(3/2)*c^2) + 2/3*A*x/(sqrt(c*x^2 + b*x)*b*c) + B*log(2*c*x + b + 2*sqrt(
c*x^2 + b*x)*sqrt(c))/c^(5/2) + 1/3*A/(sqrt(c*x^2 + b*x)*c^2) - 2/3*sqrt(c
*x^2 + b*x)*B/(b*c^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(72) = 144$.

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.24

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = -\frac{B\log\left(\left|2\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right|\right)}{c^{\frac{5}{2}}} - \frac{2\left(6\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2Bbc-3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2Ac^2+9\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)Bb^2\sqrt{c}-3\left(\sqrt{c}x-\right.\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right)^3c^{\frac{5}{2}}}$$

input
```
integrate(x^3*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
-B*log(abs(2*(sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b))/c^(5/2) - 2/3*(
6*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*b*c - 3*(sqrt(c)*x - sqrt(c*x^2 + b*
x))^2*A*c^2 + 9*(sqrt(c)*x - sqrt(c*x^2 + b*x))*B*b^2*sqrt(c) - 3*(sqrt(c)
*x - sqrt(c*x^2 + b*x))*A*b*c^(3/2) + 4*B*b^3 - A*b^2*c)/(((sqrt(c)*x - sq
rt(c*x^2 + b*x))*sqrt(c) + b)^3*c^(5/2))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x^3\,(A+B\,x)}{{(c\,x^2+b\,x)}^{5/2}}\,dx$$

input
```
int((x^3*(A + B*x))/(b*x + c*x^2)^(5/2),x)
```

output
```
int((x^3*(A + B*x))/(b*x + c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.68

$$\int \frac{x^3(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^3 + 2\sqrt{c}\,\sqrt{cx+b}\log\left(\frac{\sqrt{cx+b}+\sqrt{x}\,\sqrt{c}}{\sqrt{b}}\right)b^2cx + \frac{2\sqrt{c}\,\sqrt{cx+b}}{3}}{\sqrt{cx+b}\,b\,c^3\,(cx+b)}$$

input
```
int(x^3*(B*x+A)/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*(3*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt(b))
*b**3 + 3*sqrt(c)*sqrt(b + c*x)*log((sqrt(b + c*x) + sqrt(x)*sqrt(c))/sqrt
(b))*b**2*c*x + sqrt(c)*sqrt(b + c*x)*a*b*c + sqrt(c)*sqrt(b + c*x)*a*c**2
*x + sqrt(x)*a*c**3*x - 3*sqrt(x)*b**3*c - 4*sqrt(x)*b**2*c**2*x))/(3*sqrt
(b + c*x)*b*c**3*(b + c*x))
```

3.165 $\int \frac{x^2(A+Bx)}{\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 67

$$\int \frac{x^2(A+Bx)}{\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2(bB-Ac)x^2}{3bc\left(bx+cx^2\right)^{3/2}} + \frac{2(bB+2Ac)x}{3b^2c\sqrt{bx+cx^2}}$$

output
```
-2/3*(-A*c+B*b)*x^2/b/c/(c*x^2+b*x)^(3/2)+2/3*(2*A*c+B*b)*x/b^2/c/(c*x^2+b
*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.52

$$\int \frac{x^2(A+Bx)}{\left(bx+cx^2\right)^{5/2}}\,dx = \frac{2x^2(3Ab+bBx+2Acx)}{3b^2(x(b+cx))^{3/2}}$$

input
```
Integrate[(x^2*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(2*x^2*(3*A*b + b*B*x + 2*A*c*x))/(3*b^2*(x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1218, 1124, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(A+Bx)}{(bx+cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{ 1218}$$

$$\frac{(2Ac+bB)\int \frac{x}{(cx^2+bx)^{3/2}}dx}{3bc}-\frac{2x^2(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 1124}$$

$$\frac{(2Ac+bB)\left(\int 0dx+\frac{2x}{b\sqrt{bx+cx^2}}\right)}{3bc}-\frac{2x^2(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{2x(2Ac+bB)}{3b^2c\sqrt{bx+cx^2}}-\frac{2x^2(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

input
```
Int[(x^2*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(b*B - A*c)*x^2)/(3*b*c*(b*x + c*x^2)^(3/2)) + (2*(b*B + 2*A*c)*x)/(3*
b^2*c*Sqrt[b*x + c*x^2])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 1124
```
Int[((d_.) + (e_.)*(x_))^(m_.)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x
_Symbol] :> Simp[-2*e*(2*c*d - b*e)^(m - 2)*((d + e*x)/(c^(m - 1)*Sqrt[a +
b*x + c*x^2])), x] + Simp[e^2/c^(m - 1)   Int[(1/Sqrt[a + b*x + c*x^2])*Exp
andToSum[((2*c*d - b*e)^(m - 1) - c^(m - 1)*(d + e*x)^(m - 1))/(c*d - b*e -
 c*e*x), x], x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e
^2, 0] && IGtQ[m, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.55

method	result
pseudoelliptic	$\frac{2x\left(\left(\frac{Bx}{3}+A\right)b+\frac{2Acx}{3}\right)}{\sqrt{x(cx+b)}\,(cx+b)b^2}$
trager	$\frac{2(2Acx+Bbx+3Ab)\sqrt{c\,x^2+bx}}{3b^2(cx+b)^2}$
gosper	$\frac{2x^3(cx+b)(2Acx+Bbx+3Ab)}{3b^2(c\,x^2+bx)^{\frac{5}{2}}}$
orering	$\frac{2x^3(cx+b)(2Acx+Bbx+3Ab)}{3b^2(c\,x^2+bx)^{\frac{5}{2}}}$
default	$A\left(-\frac{x}{2c(c\,x^2+bx)^{\frac{3}{2}}}-\frac{b\left(-\frac{1}{3c\left(c\,x^2+bx\right)^{\frac{3}{2}}}-\frac{b\left(-\frac{2(2cx+b)}{3b^2\left(c\,x^2+bx\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{c\,x^2+bx}}\right)}{2c}\right)}{4c}\right)+B\left(-\frac{x^2}{c(c\,x^2+bx)^{\frac{3}{2}}}+\frac{\ldots b}{}\right.$

input
```
int(x^2*(B*x+A)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/(x*(c*x+b))^(1/2)*x*((1/3*B*x+A)*b+2/3*A*c*x)/(c*x+b)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.76

$$\int \frac{x^2(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2\sqrt{cx^2+bx}(3\,Ab+(Bb+2\,Ac)x)}{3\,(b^2c^2x^2+2\,b^3cx+b^4)}$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(c*x^2 + b*x)*(3*A*b + (B*b + 2*A*c)*x)/(b^2*c^2*x^2 + 2*b^3*c*x +
 b^4)
```

Sympy [F]

$$\int \frac{x^2(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x^2(A+Bx)}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(B*x+A)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(x**2*(A + B*x)/(x*(b + c*x))**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(59) = 118.

Time = 0.03 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.00

$$\int \frac{x^2(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = -\frac{Bx^2}{(cx^2+bx)^{\frac{3}{2}}c} + \frac{4\,Ax}{3\,\sqrt{cx^2+bx}b^2} - \frac{Bbx}{3\,(cx^2+bx)^{\frac{3}{2}}c^2} - \frac{2\,Ax}{3\,(cx^2+bx)^{\frac{3}{2}}c} + \frac{2\,Bx}{3\,\sqrt{cx^2+bx}bc} + \frac{B}{3\,\sqrt{cx^2+bx}c^2} + \frac{2\,A}{3\,\sqrt{cx^2+bx}bc}$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
-B*x^2/((c*x^2 + b*x)^(3/2)*c) + 4/3*A*x/(sqrt(c*x^2 + b*x)*b^2) - 1/3*B*b
*x/((c*x^2 + b*x)^(3/2)*c^2) - 2/3*A*x/((c*x^2 + b*x)^(3/2)*c) + 2/3*B*x/(
sqrt(c*x^2 + b*x)*b*c) + 1/3*B/(sqrt(c*x^2 + b*x)*c^2) + 2/3*A/(sqrt(c*x^2
 + b*x)*b*c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 119 vs. 2(59) = 118.

Time = 0.19 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.78

$$\int \frac{x^2(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2\left(3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)^2Bc+3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)Bb\sqrt{c}+3\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\right.}{3\left(\left(\sqrt{c}x-\sqrt{cx^2+bx}\right)\sqrt{c}+b\right)^3c^{\frac{3}{2}}}$$

input
```
integrate(x^2*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
2/3*(3*(sqrt(c)*x - sqrt(c*x^2 + b*x))^2*B*c + 3*(sqrt(c)*x - sqrt(c*x^2 +
 b*x))*B*b*sqrt(c) + 3*(sqrt(c)*x - sqrt(c*x^2 + b*x))*A*c^(3/2) + B*b^2 +
 2*A*b*c)/(((sqrt(c)*x - sqrt(c*x^2 + b*x))*sqrt(c) + b)^3*c^(3/2))
```

Mupad [B] (verification not implemented)

Time = 5.53 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.55

$$\int \frac{x^2(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2\sqrt{c\,x^2+b\,x}\,(3\,A\,b+2\,A\,c\,x+B\,b\,x)}{3\,b^2\,(b+c\,x)^2}$$

input
```
int((x^2*(A + B*x))/(b*x + c*x^2)^(5/2),x)
```

output
```
(2*(b*x + c*x^2)^(1/2)*(3*A*b + 2*A*c*x + B*b*x))/(3*b^2*(b + c*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.58

$$\int \frac{x^2(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{-\frac{4\sqrt{c}\,\sqrt{cx+b}\,abc}{3}-\frac{4\sqrt{c}\,\sqrt{cx+b}\,a\,c^2x}{3}+\frac{2\sqrt{c}\,\sqrt{cx+b}\,b^3}{3}+\frac{2\sqrt{c}\,\sqrt{cx+b}\,b^2cx}{3}+2\sqrt{x}\,ab\,c^2+\frac{4\sqrt{x}\,a\,c^3x}{3}+}{\sqrt{cx+b}\,b^2c^2\,(cx+b)}$$

input
```
int(x^2*(B*x+A)/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*( - 2*sqrt(c)*sqrt(b + c*x)*a*b*c - 2*sqrt(c)*sqrt(b + c*x)*a*c**2*x +
sqrt(c)*sqrt(b + c*x)*b**3 + sqrt(c)*sqrt(b + c*x)*b**2*c*x + 3*sqrt(x)*a*
b*c**2 + 2*sqrt(x)*a*c**3*x + sqrt(x)*b**2*c**2*x))/(3*sqrt(b + c*x)*b**2*
c**2*(b + c*x))
```

3.166 $\int \frac{x(A+Bx)}{(bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 81

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = -\frac{2Ax}{b\,(bx+cx^2)^{3/2}}+\frac{2(bB-4Ac)x^2}{3b^2\,(bx+cx^2)^{3/2}}+\frac{4(bB-4Ac)x}{3b^3\sqrt{bx+cx^2}}$$

output
```
-2*A*x/b/(c*x^2+b*x)^(3/2)+2/3*(-4*A*c+B*b)*x^2/b^2/(c*x^2+b*x)^(3/2)+4/3*
(-4*A*c+B*b)*x/b^3/(c*x^2+b*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{x(2bBx(3b+2cx)-2A(3b^2+12bcx+8c^2x^2))}{3b^3(x(b+cx))^{3/2}}$$

input
```
Integrate[(x*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(x*(2*b*B*x*(3*b + 2*c*x) - 2*A*(3*b^2 + 12*b*c*x + 8*c^2*x^2)))/(3*b^3*(x
*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {1218, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$-\frac{(bB-4Ac)\int \frac{1}{(cx^2+bx)^{3/2}}dx}{3bc}-\frac{2x(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{2(b+2cx)(bB-4Ac)}{3b^3c\sqrt{bx+cx^2}}-\frac{2x(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

input
```
Int[(x*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(b*B - A*c)*x)/(3*b*c*(b*x + c*x^2)^(3/2)) + (2*(b*B - 4*A*c)*(b + 2*c
*x))/(3*b^3*c*Sqrt[b*x + c*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.65

method	result
pseudoelliptic	$-\frac{2\left(\left(-Bx+A\right)b^{2}+4cx\left(-\frac{Bx}{6}+A\right)b+\frac{8A\,c^{2}x^{2}}{3}\right)}{\sqrt{x\left(cx+b\right)}\,\left(cx+b\right)b^{3}}$
gosper	$-\frac{2x^{2}\left(cx+b\right)\left(8A\,c^{2}x^{2}-2x^{2}Bbc+12Abcx-3xB\,b^{2}+3b^{2}A\right)}{3b^{3}\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}$
orering	$-\frac{2x^{2}\left(cx+b\right)\left(8A\,c^{2}x^{2}-2x^{2}Bbc+12Abcx-3xB\,b^{2}+3b^{2}A\right)}{3b^{3}\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}$
trager	$-\frac{2\left(8A\,c^{2}x^{2}-2x^{2}Bbc+12Abcx-3xB\,b^{2}+3b^{2}A\right)\sqrt{c\,x^{2}+bx}}{3b^{3}\left(cx+b\right)^{2}x}$
risch	$-\frac{2A\left(cx+b\right)}{b^{3}\sqrt{x\left(cx+b\right)}}-\frac{2\left(5A\,c^{2}x-2Bbcx+6Abc-3B\,b^{2}\right)x}{3\sqrt{x\left(cx+b\right)}\,\left(cx+b\right)b^{3}}$
default	$A\left(-\frac{1}{3c\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}-\frac{b\left(-\frac{2\left(2cx+b\right)}{3b^{2}\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}+\frac{16c\left(2cx+b\right)}{3b^{4}\sqrt{c\,x^{2}+bx}}\right)}{2c}\right)+B\left(-\frac{x}{2c\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}-\frac{b\left(-\frac{1}{3c\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}-\frac{b\left(\ldots\right.}{\ldots}\right.}{\ldots}\right.$

input
```
int(x*(B*x+A)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((-B*x+A)*b^2+4*c*x*(-1/6*B*x+A)*b+8/3*A*c^2*x^2)/(x*(c*x+b))^(1/2)/(c*
x+b)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = -\frac{2\,(3\,Ab^2-2\,(Bbc-4\,Ac^2)x^2-3\,(Bb^2-4\,Abc)x)\sqrt{cx^2+bx}}{3\,(b^3c^2x^3+2\,b^4cx^2+b^5x)}$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(3*A*b^2 - 2*(B*b*c - 4*A*c^2)*x^2 - 3*(B*b^2 - 4*A*b*c)*x)*sqrt(c*x^
2 + b*x)/(b^3*c^2*x^3 + 2*b^4*c*x^2 + b^5*x)
```

Sympy [F]

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x(A+Bx)}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(B*x+A)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(x*(A + B*x)/(x*(b + c*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{4\,Bx}{3\,\sqrt{cx^2+bx}b^2}+\frac{2\,Ax}{3\,(cx^2+bx)^{\frac{3}{2}}b} - \frac{2\,Bx}{3\,(cx^2+bx)^{\frac{3}{2}}c}-\frac{16\,Acx}{3\,\sqrt{cx^2+bx}b^3}-\frac{8\,A}{3\,\sqrt{cx^2+bx}b^2}+\frac{2\,B}{3\,\sqrt{cx^2+bx}bc}$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
4/3*B*x/(sqrt(c*x^2 + b*x)*b^2) + 2/3*A*x/((c*x^2 + b*x)^(3/2)*b) - 2/3*B*
x/((c*x^2 + b*x)^(3/2)*c) - 16/3*A*c*x/(sqrt(c*x^2 + b*x)*b^3) - 8/3*A/(sq
rt(c*x^2 + b*x)*b^2) + 2/3*B/(sqrt(c*x^2 + b*x)*b*c)
```

Giac [F]

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)x}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)*x/(c*x^2 + b*x)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 5.51 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.78

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = -\frac{2\sqrt{c\,x^2+b\,x}\,\left(-3\,B\,b^2\,x+3\,A\,b^2-2\,B\,b\,c\,x^2+12\,A\,b\,c\,x+8\,A\,c^2\,x^2\right)}{3\,b^3\,x\,{\left(b+c\,x\right)}^2}$$

input
```
int((x*(A + B*x))/(b*x + c*x^2)^(5/2),x)
```

output
```
-(2*(b*x + c*x^2)^(1/2)*(3*A*b^2 + 8*A*c^2*x^2 - 3*B*b^2*x - 2*B*b*c*x^2 +
 12*A*b*c*x))/(3*b^3*x*(b + c*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.74

$$\int \frac{x(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{\frac{16\sqrt{c}\,\sqrt{cx+b}\,abcx}{3}+\frac{16\sqrt{c}\,\sqrt{cx+b}\,a\,c^2x^2}{3}-\frac{4\sqrt{c}\,\sqrt{cx+b}\,b^3x}{3}-\frac{4\sqrt{c}\,\sqrt{cx+b}\,b^2c\,x^2}{3}-2\sqrt{x}\,a\,b^2c-8\sqrt{x}\,a}{\sqrt{cx+b}\,b^3cx\,(cx+b)}$$

input
```
int(x*(B*x+A)/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*(8*sqrt(c)*sqrt(b + c*x)*a*b*c*x + 8*sqrt(c)*sqrt(b + c*x)*a*c**2*x**2
- 2*sqrt(c)*sqrt(b + c*x)*b**3*x - 2*sqrt(c)*sqrt(b + c*x)*b**2*c*x**2 - 3
*sqrt(x)*a*b**2*c - 12*sqrt(x)*a*b*c**2*x - 8*sqrt(x)*a*c**3*x**2 + 3*sqrt
(x)*b**3*c*x + 2*sqrt(x)*b**2*c**2*x**2))/(3*sqrt(b + c*x)*b**3*c*x*(b + c
*x))
```

3.167 $\int \frac{A+Bx}{\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 110

$$\int \frac{A+Bx}{\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2A}{3b\left(bx+cx^2\right)^{3/2}}+\frac{2(bB-2Ac)x}{3b^2\left(bx+cx^2\right)^{3/2}}+\frac{8(bB-2Ac)}{3b^3\sqrt{bx+cx^2}}-\frac{16(bB-2Ac)\sqrt{bx+cx^2}}{3b^4x}$$

output
```
-2/3*A/b/(c*x^2+b*x)^(3/2)+2/3*(-2*A*c+B*b)*x/b^2/(c*x^2+b*x)^(3/2)+8/3*(-
2*A*c+B*b)/b^3/(c*x^2+b*x)^(1/2)-16/3*(-2*A*c+B*b)*(c*x^2+b*x)^(1/2)/b^4/x
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx}{\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2(bBx(3b^2+12bcx+8c^2x^2)+A(b^3-6b^2cx-24bc^2x^2-16c^3x^3))}{3b^4(x(b+cx))^{3/2}}$$

input
```
Integrate[(A + B*x)/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(b*B*x*(3*b^2 + 12*b*c*x + 8*c^2*x^2) + A*(b^3 - 6*b^2*c*x - 24*b*c^2*
x^2 - 16*c^3*x^3)))/(3*b^4*(x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {1159, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1159}$$

$$\frac{4(bB-2Ac)\int \frac{1}{(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(Ab-x(bB-2Ac))}{3b^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$-\frac{8(b+2cx)(bB-2Ac)}{3b^4\sqrt{bx+cx^2}}-\frac{2(Ab-x(bB-2Ac))}{3b^2\left(bx+cx^2\right)^{3/2}}$$

input
```
Int[(A + B*x)/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(A*b - (b*B - 2*A*c)*x))/(3*b^2*(b*x + c*x^2)^(3/2)) - (8*(b*B - 2*A*c
)*(b + 2*c*x))/(3*b^4*Sqrt[b*x + c*x^2])
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1159
```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Simp[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &
& LtQ[p, -1] && NeQ[p, -3/2]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.66

method	result	siz
pseudoelliptic	$-\frac{2\left((3Bx+A)b^3-6cx(-2Bx+A)b^2-24c^2x^2\left(-\frac{Bx}{3}+A\right)b-16A\,c^3x^3\right)}{3\sqrt{x(cx+b)}\,x(cx+b)b^4}$	73
gosper	$-\frac{2x(cx+b)\left(-16A\,c^3x^3+8x^3Bb\,c^2-24Ab\,c^2x^2+12x^2B\,b^2c-6A\,b^2cx+3xB\,b^3+A\,b^3\right)}{3b^4(c\,x^2+bx)^{\frac{5}{2}}}$	83
orering	$-\frac{2x(cx+b)\left(-16A\,c^3x^3+8x^3Bb\,c^2-24Ab\,c^2x^2+12x^2B\,b^2c-6A\,b^2cx+3xB\,b^3+A\,b^3\right)}{3b^4(c\,x^2+bx)^{\frac{5}{2}}}$	83
risch	$-\frac{2(cx+b)(-8Acx+3Bbx+Ab)}{3b^4x\sqrt{x(cx+b)}}+\frac{2c\left(8A\,c^2x-5Bbcx+9Abc-6B\,b^2\right)x}{3\sqrt{x(cx+b)}\,(cx+b)b^4}$	86
trager	$-\frac{2\left(-16A\,c^3x^3+8x^3Bb\,c^2-24Ab\,c^2x^2+12x^2B\,b^2c-6A\,b^2cx+3xB\,b^3+A\,b^3\right)\sqrt{c\,x^2+bx}}{3b^4x^2(cx+b)^2}$	87
default	$A\left(-\frac{2(2cx+b)}{3b^2(c\,x^2+bx)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{c\,x^2+bx}}\right)+B\left(-\frac{1}{3c(c\,x^2+bx)^{\frac{3}{2}}}-\frac{b\left(-\frac{2(2cx+b)}{3b^2\left(c\,x^2+bx\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{c\,x^2+bx}}\right)}{2c}\right)$	12

input
```
int((B*x+A)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(x*(c*x+b))^(1/2)*((3*B*x+A)*b^3-6*c*x*(-2*B*x+A)*b^2-24*c^2*x^2*(-1/
3*B*x+A)*b-16*A*c^3*x^3)/x/(c*x+b)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{(bx+cx^2)^{5/2}}\,dx = -\frac{2\,(Ab^3+8\,(Bbc^2-2\,Ac^3)x^3+12\,(Bb^2c-2\,Abc^2)x^2+3\,(Bb^3-2\,Ab^2c)x)\sqrt{cx^2+bx}}{3\,(b^4c^2x^4+2\,b^5cx^3+b^6x^2)}$$

input
```
integrate((B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(A*b^3 + 8*(B*b*c^2 - 2*A*c^3)*x^3 + 12*(B*b^2*c - 2*A*b*c^2)*x^2 + 3
*(B*b^3 - 2*A*b^2*c)*x)*sqrt(c*x^2 + b*x)/(b^4*c^2*x^4 + 2*b^5*c*x^3 + b^6
*x^2)
```

Sympy [F]

$$\int \frac{A+Bx}{(bx+cx^2)^{5/2}}\,dx = \int \frac{A+Bx}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral((A + B*x)/(x*(b + c*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.18

$$\int \frac{A+Bx}{(bx+cx^2)^{5/2}}\,dx = \frac{2\,Bx}{3\,(cx^2+bx)^{\frac{3}{2}}b} - \frac{16\,Bcx}{3\,\sqrt{cx^2+bx}b^3} - \frac{4\,Acx}{3\,(cx^2+bx)^{\frac{3}{2}}b^2} + \frac{32\,Ac^2x}{3\,\sqrt{cx^2+bx}b^4} - \frac{8\,B}{3\,\sqrt{cx^2+bx}b^2} - \frac{2\,A}{3\,(cx^2+bx)^{\frac{3}{2}}b} + \frac{16\,Ac}{3\,\sqrt{cx^2+bx}b^3}$$

input
```
integrate((B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
2/3*B*x/((c*x^2 + b*x)^(3/2)*b) - 16/3*B*c*x/(sqrt(c*x^2 + b*x)*b^3) - 4/3
*A*c*x/((c*x^2 + b*x)^(3/2)*b^2) + 32/3*A*c^2*x/(sqrt(c*x^2 + b*x)*b^4) -
8/3*B/(sqrt(c*x^2 + b*x)*b^2) - 2/3*A/((c*x^2 + b*x)^(3/2)*b) + 16/3*A*c/(
sqrt(c*x^2 + b*x)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx}{\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2\left(\left(4\,x\left(\frac{2\,(Bbc^2-2\,Ac^3)x}{b^4}+\frac{3\,(Bb^2c-2\,Abc^2)}{b^4}\right)+\frac{3\,(Bb^3-2\,Ab^2c)}{b^4}\right)x+\frac{A}{b}\right)}{3\,(cx^2+bx)^{\frac{3}{2}}}$$

input
```
integrate((B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
-2/3*((4*x*(2*(B*b*c^2 - 2*A*c^3)*x/b^4 + 3*(B*b^2*c - 2*A*b*c^2)/b^4) + 3
*(B*b^3 - 2*A*b^2*c)/b^4)*x + A/b)/(c*x^2 + b*x)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 5.49 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.69

$$\int \frac{A+Bx}{\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2\,\left(3\,B\,b^3\,x+A\,b^3+12\,B\,b^2\,c\,x^2-6\,A\,b^2\,c\,x+8\,B\,b\,c^2\,x^3-24\,A\,b\,c^2\,x^2-16\,A\,c^3\,x^3\right)}{3\,b^4\,{\left(c\,x^2+b\,x\right)}^{3/2}}$$

input
```
int((A + B*x)/(b*x + c*x^2)^(5/2),x)
```

output
```
-(2*(A*b^3 - 16*A*c^3*x^3 + 3*B*b^3*x - 6*A*b^2*c*x - 24*A*b*c^2*x^2 + 12*
B*b^2*c*x^2 + 8*B*b*c^2*x^3))/(3*b^4*(b*x + c*x^2)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.48

$$\int \frac{A+Bx}{(bx+cx^2)^{5/2}}\,dx = \frac{-\frac{32\sqrt{c}\,\sqrt{cx+b}\,abc\,x^2}{3} - \frac{32\sqrt{c}\,\sqrt{cx+b}\,a\,c^2x^3}{3} + \frac{16\sqrt{c}\,\sqrt{cx+b}\,b^3x^2}{3} + \frac{16\sqrt{c}\,\sqrt{cx+b}\,b^2c\,x^3}{3} - \frac{2\sqrt{x}\,a\,b^3}{3} + 4\sqrt{}}{\sqrt{cx+b}\,b^4x^2\,(cx}$$

input
```
int((B*x+A)/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*( - 16*sqrt(c)*sqrt(b + c*x)*a*b*c*x**2 - 16*sqrt(c)*sqrt(b + c*x)*a*c*
*2*x**3 + 8*sqrt(c)*sqrt(b + c*x)*b**3*x**2 + 8*sqrt(c)*sqrt(b + c*x)*b**2
*c*x**3 - sqrt(x)*a*b**3 + 6*sqrt(x)*a*b**2*c*x + 24*sqrt(x)*a*b*c**2*x**2
 + 16*sqrt(x)*a*c**3*x**3 - 3*sqrt(x)*b**4*x - 12*sqrt(x)*b**3*c*x**2 - 8*
sqrt(x)*b**2*c**2*x**3))/(3*sqrt(b + c*x)*b**4*x**2*(b + c*x))
```

3.168 $\int \frac{A+Bx}{x\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 151

$$\int \frac{A+Bx}{x\left(bx+cx^2\right)^{5/2}}\,dx=\frac{2(5bB-8Ac)}{15b^2\left(bx+cx^2\right)^{3/2}}-\frac{2A}{5bx\left(bx+cx^2\right)^{3/2}}+\frac{4(5bB-8Ac)}{5b^3x\sqrt{bx+cx^2}}-\frac{16(5bB-8Ac)\sqrt{bx+cx^2}}{15b^4x^2}+\frac{32c(5bB-8Ac)\sqrt{bx+cx^2}}{15b^5x}$$

output
```
2/15*(-8*A*c+5*B*b)/b^2/(c*x^2+b*x)^(3/2)-2/5*A/b/x/(c*x^2+b*x)^(3/2)+4/5*
(-8*A*c+5*B*b)/b^3/x/(c*x^2+b*x)^(1/2)-16/15*(-8*A*c+5*B*b)*(c*x^2+b*x)^(1
/2)/b^4/x^2+32/15*c*(-8*A*c+5*B*b)*(c*x^2+b*x)^(1/2)/b^5/x
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx}{x\left(bx+cx^2\right)^{5/2}}\,dx=-\frac{2(5bBx(b^3-6b^2cx-24bc^2x^2-16c^3x^3)+A(3b^4-8b^3cx+48b^2c^2x^2+192bc^3x^3+128c^4x^4))}{15b^5x(x(b+cx))^{3/2}}$$

input
```
Integrate[(A + B*x)/(x*(b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*(5*b*B*x*(b^3 - 6*b^2*c*x - 24*b*c^2*x^2 - 16*c^3*x^3) + A*(3*b^4 - 8*
b^3*c*x + 48*b^2*c^2*x^2 + 192*b*c^3*x^3 + 128*c^4*x^4)))/(15*b^5*x*(x*(b
+ c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.63, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1220, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x\left(bx+cx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(5bB-8Ac)\int \frac{1}{\left(cx^2+bx\right)^{5/2}}dx}{5b}-\frac{2A}{5bx\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{(5bB-8Ac)\left(-\frac{8c\int \frac{1}{\left(cx^2+bx\right)^{3/2}}dx}{3b^2}-\frac{2(b+2cx)}{3b^2\left(bx+cx^2\right)^{3/2}}\right)}{5b}-\frac{2A}{5bx\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1088}$$

$$\frac{\left(\frac{16c(b+2cx)}{3b^4\sqrt{bx+cx^2}}-\frac{2(b+2cx)}{3b^2\left(bx+cx^2\right)^{3/2}}\right)(5bB-8Ac)}{5b}-\frac{2A}{5bx\left(bx+cx^2\right)^{3/2}}$$

input
```
Int[(A + B*x)/(x*(b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*A)/(5*b*x*(b*x + c*x^2)^(3/2)) + ((5*b*B - 8*A*c)*((-2*(b + 2*c*x))/(3
*b^2*(b*x + c*x^2)^(3/2)) + (16*c*(b + 2*c*x))/(3*b^4*Sqrt[b*x + c*x^2])))
/(5*b)
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.61

method	result
pseudoelliptic	$\frac{(-10Bx-6A)b^4+16c\left(\frac{15Bx}{4}+A\right)x\,b^3-96c^2x^2\left(-\frac{5Bx}{2}+A\right)b^2-384\left(-\frac{5Bx}{12}+A\right)c^3x^3b-256A\,c^4x^4}{15\sqrt{x(cx+b)}\,x^2(cx+b)b^5}$
gosper	$-\frac{2(cx+b)\left(128A\,c^4x^4-80Bb\,c^3x^4+192Ab\,c^3x^3-120B\,b^2c^2x^3+48A\,b^2c^2x^2-30B\,b^3c\,x^2-8A\,b^3cx+5B\,b^4x+3A\,b^4\right)}{15b^5\left(c\,x^2+bx\right)^{\frac{5}{2}}}$
orering	$-\frac{2(cx+b)\left(128A\,c^4x^4-80Bb\,c^3x^4+192Ab\,c^3x^3-120B\,b^2c^2x^3+48A\,b^2c^2x^2-30B\,b^3c\,x^2-8A\,b^3cx+5B\,b^4x+3A\,b^4\right)}{15b^5\left(c\,x^2+bx\right)^{\frac{5}{2}}}$
risch	$-\frac{2(cx+b)\left(73A\,c^2x^2-40x^2Bbc-14Abcx+5xB\,b^2+3b^2A\right)}{15b^5x^2\sqrt{x(cx+b)}}-\frac{2c^2\left(11A\,c^2x-8Bbcx+12Abc-9B\,b^2\right)x}{3\sqrt{x(cx+b)}\,(cx+b)b^5}$
trager	$-\frac{2\left(128A\,c^4x^4-80Bb\,c^3x^4+192Ab\,c^3x^3-120B\,b^2c^2x^3+48A\,b^2c^2x^2-30B\,b^3c\,x^2-8A\,b^3cx+5B\,b^4x+3A\,b^4\right)\sqrt{c\,x^2+bx}}{15b^5(cx+b)^2x^3}$
default	$B\left(-\frac{2(2cx+b)}{3b^2(c\,x^2+bx)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{c\,x^2+bx}}\right)+A\left(-\frac{2}{5bx(c\,x^2+bx)^{\frac{3}{2}}}-\frac{8c\left(-\frac{2(2cx+b)}{3b^2\left(c\,x^2+bx\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{c\,x^2+bx}}\right)}{5b}\right)$

input
```
int((B*x+A)/x/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*((-10*B*x-6*A)*b^4+16*c*(15/4*B*x+A)*x*b^3-96*c^2*x^2*(-5/2*B*x+A)*b^
2-384*(-5/12*B*x+A)*c^3*x^3*b-256*A*c^4*x^4)/(x*(c*x+b))^(1/2)/x^2/(c*x+b)
/b^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.85

$$\int \frac{A+Bx}{x\,(bx+cx^2)^{5/2}}\,dx =$$

$$-\frac{2\,(3\,Ab^4-16\,(5\,Bbc^3-8\,Ac^4)x^4-24\,(5\,Bb^2c^2-8\,Abc^3)x^3-6\,(5\,Bb^3c-8\,Ab^2c^2)x^2+(5\,Bb^4-8\,Ab^3c)}{15\,(b^5c^2x^5+2\,b^6cx^4+b^7x^3)}$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(3*A*b^4 - 16*(5*B*b*c^3 - 8*A*c^4)*x^4 - 24*(5*B*b^2*c^2 - 8*A*b*c^
3)*x^3 - 6*(5*B*b^3*c - 8*A*b^2*c^2)*x^2 + (5*B*b^4 - 8*A*b^3*c)*x)*sqrt(c
*x^2 + b*x)/(b^5*c^2*x^5 + 2*b^6*c*x^4 + b^7*x^3)
```

Sympy [F]

$$\int \frac{A+Bx}{x\left(bx+cx^2\right)^{5/2}}\,dx = \int \frac{A+Bx}{x\left(x\left(b+cx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/x/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral((A + B*x)/(x*(x*(b + c*x))**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx}{x\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{4\,Bcx}{3\left(cx^2+bx\right)^{\frac{3}{2}}b^2} + \frac{32\,Bc^2x}{3\sqrt{cx^2+bx}b^4} + \frac{32\,Ac^2x}{15\left(cx^2+bx\right)^{\frac{3}{2}}b^3} - \frac{256\,Ac^3x}{15\sqrt{cx^2+bx}b^5} - \frac{2\,B}{3\left(cx^2+bx\right)^{\frac{3}{2}}b} + \frac{16\,Bc}{3\sqrt{cx^2+bx}b^3} + \frac{16\,Ac}{15\left(cx^2+bx\right)^{\frac{3}{2}}b^2} - \frac{128\,Ac^2}{15\sqrt{cx^2+bx}b^4} - \frac{2\,A}{5\left(cx^2+bx\right)^{\frac{3}{2}}bx}$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
-4/3*B*c*x/((c*x^2 + b*x)^(3/2)*b^2) + 32/3*B*c^2*x/(sqrt(c*x^2 + b*x)*b^4
) + 32/15*A*c^2*x/((c*x^2 + b*x)^(3/2)*b^3) - 256/15*A*c^3*x/(sqrt(c*x^2 +
 b*x)*b^5) - 2/3*B/((c*x^2 + b*x)^(3/2)*b) + 16/3*B*c/(sqrt(c*x^2 + b*x)*b
^3) + 16/15*A*c/((c*x^2 + b*x)^(3/2)*b^2) - 128/15*A*c^2/(sqrt(c*x^2 + b*x
)*b^4) - 2/5*A/((c*x^2 + b*x)^(3/2)*b*x)
```

Giac [F]

$$\int \frac{A+Bx}{x\,(bx+cx^2)^{5/2}}\,dx=\int \frac{Bx+A}{(cx^2+bx)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((B*x+A)/x/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/((c*x^2 + b*x)^(5/2)*x), x)
```

Mupad [B] (verification not implemented)

Time = 5.74 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.74

$$\int \frac{A+Bx}{x\,(bx+cx^2)^{5/2}}\,dx = -\frac{2\sqrt{c\,x^2+b\,x}\,(5\,B\,b^4\,x+3\,A\,b^4-30\,B\,b^3\,c\,x^2-8\,A\,b^3\,c\,x-120\,B\,b^2\,c^2\,x^3+48\,A\,b^2\,c^2\,x^2-80\,B\,b\,c^3\,x^4+}{15\,b^5\,x^3\,(b+c\,x)^2}$$

input
```
int((A + B*x)/(x*(b*x + c*x^2)^(5/2)),x)
```

output
```
-(2*(b*x + c*x^2)^(1/2)*(3*A*b^4 + 128*A*c^4*x^4 + 5*B*b^4*x - 8*A*b^3*c*x
 + 192*A*b*c^3*x^3 - 30*B*b^3*c*x^2 - 80*B*b*c^3*x^4 + 48*A*b^2*c^2*x^2 -
120*B*b^2*c^2*x^3))/(15*b^5*x^3*(b + c*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.29

$$\int \frac{A+Bx}{x\,(bx+cx^2)^{5/2}}\,dx=\frac{\frac{256\sqrt{c}\,\sqrt{cx+b}\,ab\,c^2x^3}{15}+\frac{256\sqrt{c}\,\sqrt{cx+b}\,a\,c^3x^4}{15}-\frac{32\sqrt{c}\,\sqrt{cx+b}\,b^3c\,x^3}{3}-\frac{32\sqrt{c}\,\sqrt{cx+b}\,b^2c^2x^4}{3}-\frac{2\sqrt{x}\,a\,b^4}{5}+}{}$$

input
```
int((B*x+A)/x/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*(128*sqrt(c)*sqrt(b + c*x)*a*b*c**2*x**3 + 128*sqrt(c)*sqrt(b + c*x)*a*
c**3*x**4 - 80*sqrt(c)*sqrt(b + c*x)*b**3*c*x**3 - 80*sqrt(c)*sqrt(b + c*x
)*b**2*c**2*x**4 - 3*sqrt(x)*a*b**4 + 8*sqrt(x)*a*b**3*c*x - 48*sqrt(x)*a*
b**2*c**2*x**2 - 192*sqrt(x)*a*b*c**3*x**3 - 128*sqrt(x)*a*c**4*x**4 - 5*s
qrt(x)*b**5*x + 30*sqrt(x)*b**4*c*x**2 + 120*sqrt(x)*b**3*c**2*x**3 + 80*s
qrt(x)*b**2*c**3*x**4))/(15*sqrt(b + c*x)*b**5*x**3*(b + c*x))
```

3.169 $\int \frac{A+Bx}{x^2(bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 189

$$\int \frac{A+Bx}{x^2(bx+cx^2)^{5/2}}\,dx = -\frac{2A}{7bx^2(bx+cx^2)^{3/2}}+\frac{2(7bB-10Ac)}{21b^2x(bx+cx^2)^{3/2}}+\frac{16(7bB-10Ac)}{21b^3x^2\sqrt{bx+cx^2}}-\frac{32(7bB-10Ac)\sqrt{bx+cx^2}}{35b^4x^3}+\frac{128c(7bB-10Ac)\sqrt{bx+cx^2}}{105b^5x^2}-\frac{256c^2(7bB-10Ac)\sqrt{bx+cx^2}}{105b^6x}$$

output
```
-2/7*A/b/x^2/(c*x^2+b*x)^(3/2)+2/21*(-10*A*c+7*B*b)/b^2/x/(c*x^2+b*x)^(3/2
)+16/21*(-10*A*c+7*B*b)/b^3/x^2/(c*x^2+b*x)^(1/2)-32/35*(-10*A*c+7*B*b)*(c
*x^2+b*x)^(1/2)/b^4/x^3+128/105*c*(-10*A*c+7*B*b)*(c*x^2+b*x)^(1/2)/b^5/x^
2-256/105*c^2*(-10*A*c+7*B*b)*(c*x^2+b*x)^(1/2)/b^6/x
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx}{x^2\left(bx+cx^2\right)^{5/2}}\,dx = -\frac{2(7bBx(3b^4-8b^3cx+48b^2c^2x^2+192bc^3x^3+128c^4x^4)+5A(3b^5-6b^4cx+16b^3c^2x^2-96b^2c^3x^3-384b}{105b^6x^2(x(b+cx))^{3/2}}$$

input
```
Integrate[(A + B*x)/(x^2*(b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*(7*b*B*x*(3*b^4 - 8*b^3*c*x + 48*b^2*c^2*x^2 + 192*b*c^3*x^3 + 128*c^4
*x^4) + 5*A*(3*b^5 - 6*b^4*c*x + 16*b^3*c^2*x^2 - 96*b^2*c^3*x^3 - 384*b*c
^4*x^4 - 256*c^5*x^5)))/(105*b^6*x^2*(x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.67, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1220, 1129, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^2\left(bx+cx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(7bB-10Ac)\int \frac{1}{x(cx^2+bx)^{5/2}}dx}{7b}-\frac{2A}{7bx^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(7bB-10Ac)\left(-\frac{8c\int \frac{1}{\left(cx^2+bx\right)^{5/2}}dx}{5b}-\frac{2}{5bx(bx+cx^2)^{3/2}}\right)}{7b}-\frac{2A}{7bx^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1089}$$

$$\frac{(7bB-10Ac)\left(-\frac{8c\left(-\frac{8c\int \frac{1}{\left(cx^2+bx\right)^{3/2}}dx}{3b^2}-\frac{2(b+2cx)}{3b^2\left(bx+cx^2\right)^{3/2}}\right)}{5b}-\frac{2}{5bx(bx+cx^2)^{3/2}}\right)}{7b}-\frac{2A}{7bx^2\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 1088$$

$$\frac{\left(-\frac{8c\left(\frac{16c(b+2cx)}{3b^4\sqrt{bx+cx^2}}-\frac{2(b+2cx)}{3b^2\left(bx+cx^2\right)^{3/2}}\right)}{5b}-\frac{2}{5bx(bx+cx^2)^{3/2}}\right)(7bB-10Ac)}{7b}-\frac{2A}{7bx^2\left(bx+cx^2\right)^{3/2}}$$

input
```
Int[(A + B*x)/(x^2*(b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*A)/(7*b*x^2*(b*x + c*x^2)^(3/2)) + ((7*b*B - 10*A*c)*(-2/(5*b*x*(b*x +
 c*x^2)^(3/2)) - (8*c*((-2*(b + 2*c*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (16*
c*(b + 2*c*x))/(3*b^4*Sqrt[b*x + c*x^2])))/(5*b)))/(7*b)
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1129

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.58

method	result
pseudoelliptic	$\frac{(-42Bx-30A)b^5+60cx\left(\frac{28Bx}{15}+A\right)b^4-160c^2x^2\left(\frac{21Bx}{5}+A\right)b^3+960c^3\left(-\frac{14Bx}{5}+A\right)x^3b^2+3840c^4\left(-\frac{7Bx}{15}+A\right)x^4b+2560A}{105\sqrt{x(cx+b)}\,x^3(cx+b)b^6}$
gosper	$-\frac{2(cx+b)\left(-1280A\,c^5x^5+896Bb\,c^4x^5-1920Ab\,c^4x^4+1344B\,b^2c^3x^4-480A\,b^2c^3x^3+336B\,b^3c^2x^3+80A\,b^3c^2x^2-56B\,b^4c\,x\right.}{105x\,b^6(c\,x^2+bx)^{\frac{5}{2}}}$
orering	$-\frac{2(cx+b)\left(-1280A\,c^5x^5+896Bb\,c^4x^5-1920Ab\,c^4x^4+1344B\,b^2c^3x^4-480A\,b^2c^3x^3+336B\,b^3c^2x^3+80A\,b^3c^2x^2-56B\,b^4c\,x\right.}{105x\,b^6(c\,x^2+bx)^{\frac{5}{2}}}$
risch	$-\frac{2(cx+b)\left(-790A\,c^3x^3+511x^3Bb\,c^2+185Ab\,c^2x^2-98x^2B\,b^2c-60A\,b^2cx+21xB\,b^3+15A\,b^3\right)}{105b^6x^3\sqrt{x(cx+b)}}+\frac{2c^3\left(14A\,c^2x-11Bbcx+\right.}{3\sqrt{x(cx+b)}\,(c}$
trager	$-\frac{2\left(-1280A\,c^5x^5+896Bb\,c^4x^5-1920Ab\,c^4x^4+1344B\,b^2c^3x^4-480A\,b^2c^3x^3+336B\,b^3c^2x^3+80A\,b^3c^2x^2-56B\,b^4c\,x^2-30A\right.}{105b^6(cx+b)^2x^4}$
default	$A\left(-\frac{2}{7b\,x^2(c\,x^2+bx)^{\frac{3}{2}}}-\frac{10c\left(-\frac{2}{5bx\left(c\,x^2+bx\right)^{\frac{3}{2}}}-\frac{8c\left(-\frac{2(2cx+b)}{3b^2\left(c\,x^2+bx\right)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{c\,x^2+bx}}\right)}{5b}\right)}{7b}\right)+B\left(-\frac{2}{5bx(c\,x^2+bx)}\right.$

input
```
int((B*x+A)/x^2/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/105*((-42*B*x-30*A)*b^5+60*c*x*(28/15*B*x+A)*b^4-160*c^2*x^2*(21/5*B*x+A
)*b^3+960*c^3*(-14/5*B*x+A)*x^3*b^2+3840*c^4*(-7/15*B*x+A)*x^4*b+2560*A*c^
5*x^5)/(x*(c*x+b))^(1/2)/x^3/(c*x+b)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{x^2\,(bx+cx^2)^{5/2}}\,dx = -\frac{2\,(15\,Ab^5 + 128\,(7\,Bbc^4 - 10\,Ac^5)x^5 + 192\,(7\,Bb^2c^3 - 10\,Abc^4)x^4 + 48\,(7\,Bb^3c^2 - 10\,Ab^2c^3)x^3 - 8\,(7\,Bb^4}{105\,(b^6c^2x^6 + 2\,b^7cx^5 + b^8x^4)}$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/105*(15*A*b^5 + 128*(7*B*b*c^4 - 10*A*c^5)*x^5 + 192*(7*B*b^2*c^3 - 10*
A*b*c^4)*x^4 + 48*(7*B*b^3*c^2 - 10*A*b^2*c^3)*x^3 - 8*(7*B*b^4*c - 10*A*b
^3*c^2)*x^2 + 3*(7*B*b^5 - 10*A*b^4*c)*x)*sqrt(c*x^2 + b*x)/(b^6*c^2*x^6 +
 2*b^7*c*x^5 + b^8*x^4)
```

Sympy [F]

$$\int \frac{A+Bx}{x^2\,(bx+cx^2)^{5/2}}\,dx = \int \frac{A+Bx}{x^2\,(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**2/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral((A + B*x)/(x**2*(x*(b + c*x))**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{x^2\left(bx+cx^2\right)^{5/2}}\,dx = \frac{32\,Bc^2x}{15\,(cx^2+bx)^{\frac{3}{2}}b^3} - \frac{256\,Bc^3x}{15\,\sqrt{cx^2+bx}b^5} - \frac{64\,Ac^3x}{21\,(cx^2+bx)^{\frac{3}{2}}b^4} + \frac{512\,Ac^4x}{21\,\sqrt{cx^2+bx}b^6} + \frac{16\,Bc}{15\,(cx^2+bx)^{\frac{3}{2}}b^2} - \frac{128\,Bc^2}{15\,\sqrt{cx^2+bx}b^4} - \frac{32\,Ac^2}{21\,(cx^2+bx)^{\frac{3}{2}}b^3} + \frac{256\,Ac^3}{21\,\sqrt{cx^2+bx}b^5} - \frac{2\,B}{5\,(cx^2+bx)^{\frac{3}{2}}bx} + \frac{4\,Ac}{7\,(cx^2+bx)^{\frac{3}{2}}b^2x} - \frac{2\,A}{7\,(cx^2+bx)^{\frac{3}{2}}bx^2}$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
32/15*B*c^2*x/((c*x^2 + b*x)^(3/2)*b^3) - 256/15*B*c^3*x/(sqrt(c*x^2 + b*x
)*b^5) - 64/21*A*c^3*x/((c*x^2 + b*x)^(3/2)*b^4) + 512/21*A*c^4*x/(sqrt(c*
x^2 + b*x)*b^6) + 16/15*B*c/((c*x^2 + b*x)^(3/2)*b^2) - 128/15*B*c^2/(sqrt
(c*x^2 + b*x)*b^4) - 32/21*A*c^2/((c*x^2 + b*x)^(3/2)*b^3) + 256/21*A*c^3/
(sqrt(c*x^2 + b*x)*b^5) - 2/5*B/((c*x^2 + b*x)^(3/2)*b*x) + 4/7*A*c/((c*x^
2 + b*x)^(3/2)*b^2*x) - 2/7*A/((c*x^2 + b*x)^(3/2)*b*x^2)
```

Giac [F]

$$\int \frac{A+Bx}{x^2\left(bx+cx^2\right)^{5/2}}\,dx = \int \frac{Bx+A}{(cx^2+bx)^{\frac{5}{2}}x^2}\,dx$$

input
```
integrate((B*x+A)/x^2/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/((c*x^2 + b*x)^(5/2)*x^2), x)
```

Mupad [B] (verification not implemented)

Time = 5.83 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.24

$$\int \frac{A+Bx}{x^2\,(bx+cx^2)^{5/2}}\,dx=\frac{\sqrt{c\,x^2+b\,x}\,\left(\frac{1280\,A\,c^3-896\,B\,b\,c^2}{105\,b^5}+\frac{2\,c\,x\,\left(1280\,A\,c^3-896\,B\,b\,c^2\right)}{105\,b^6}\right)}{x\,\left(b+c\,x\right)}$$
$$-\frac{\sqrt{c\,x^2+b\,x}\,\left(14\,B\,b^3-40\,A\,b^2\,c\right)}{35\,b^6\,x^3}$$
$$-\frac{\sqrt{c\,x^2+b\,x}\,\left(x\,\left(\frac{4\,c^2\,\left(185\,A\,c-98\,B\,b\right)}{105\,b^4}+\frac{2\,c^2\,\left(230\,A\,c-91\,B\,b\right)}{105\,b^4}+\frac{b\,\left(\frac{160\,A\,c^4-56\,B\,b\,c^3}{105\,b^5}-\frac{4\,c^3\,\left(230\,A\,c-91\,B\,b\right)}{105\,b^5}\right)}{c}\right)+\frac{2\,c\,(185\,A\,c-98\,B}{105\,b^3}}{x^2\,{\left(b+c\,x\right)}^2}$$
$$-\frac{2\,A\,\sqrt{c\,x^2+b\,x}}{7\,b^3\,x^4}$$

input
```
int((A + B*x)/(x^2*(b*x + c*x^2)^(5/2)),x)
```

output
```
((b*x + c*x^2)^(1/2)*((1280*A*c^3 - 896*B*b*c^2)/(105*b^5) + (2*c*x*(1280*
A*c^3 - 896*B*b*c^2))/(105*b^6)))/(x*(b + c*x)) - ((b*x + c*x^2)^(1/2)*(14
*B*b^3 - 40*A*b^2*c))/(35*b^6*x^3) - ((b*x + c*x^2)^(1/2)*(x*((4*c^2*(185*
A*c - 98*B*b))/(105*b^4) + (2*c^2*(230*A*c - 91*B*b))/(105*b^4) + (b*((160
*A*c^4 - 56*B*b*c^3)/(105*b^5) - (4*c^3*(230*A*c - 91*B*b))/(105*b^5)))/c)
 + (2*c*(185*A*c - 98*B*b))/(105*b^3)))/(x^2*(b + c*x)^2) - (2*A*(b*x + c*
x^2)^(1/2))/(7*b^3*x^4)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{x^2\,(bx+cx^2)^{5/2}}\,dx=\frac{-\frac{512\sqrt{c}\,\sqrt{cx+b}\,ab\,c^3x^4}{21}-\frac{512\sqrt{c}\,\sqrt{cx+b}\,a\,c^4x^5}{21}+\frac{256\sqrt{c}\,\sqrt{cx+b}\,b^3c^2x^4}{15}+\frac{256\sqrt{c}\,\sqrt{cx+b}\,b^2c^3x^5}{15}-\frac{2\sqrt{x}}{7}}{}$$

input
```
int((B*x+A)/x^2/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*( - 1280*sqrt(c)*sqrt(b + c*x)*a*b*c**3*x**4 - 1280*sqrt(c)*sqrt(b + c*
x)*a*c**4*x**5 + 896*sqrt(c)*sqrt(b + c*x)*b**3*c**2*x**4 + 896*sqrt(c)*sq
rt(b + c*x)*b**2*c**3*x**5 - 15*sqrt(x)*a*b**5 + 30*sqrt(x)*a*b**4*c*x - 8
0*sqrt(x)*a*b**3*c**2*x**2 + 480*sqrt(x)*a*b**2*c**3*x**3 + 1920*sqrt(x)*a
*b*c**4*x**4 + 1280*sqrt(x)*a*c**5*x**5 - 21*sqrt(x)*b**6*x + 56*sqrt(x)*b
**5*c*x**2 - 336*sqrt(x)*b**4*c**2*x**3 - 1344*sqrt(x)*b**3*c**3*x**4 - 89
6*sqrt(x)*b**2*c**4*x**5))/(105*sqrt(b + c*x)*b**6*x**4*(b + c*x))
```

3.170 $\int \frac{A+Bx}{x^3(bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 224

$$\int \frac{A+Bx}{x^3(bx+cx^2)^{5/2}}\,dx = -\frac{2A}{9bx^3(bx+cx^2)^{3/2}}+\frac{2(3bB-4Ac)}{9b^2x^2(bx+cx^2)^{3/2}} + \frac{20(3bB-4Ac)}{9b^3x^3\sqrt{bx+cx^2}}-\frac{160(3bB-4Ac)\sqrt{bx+cx^2}}{63b^4x^4}+\frac{64c(3bB-4Ac)\sqrt{bx+cx^2}}{21b^5x^3} - \frac{256c^2(3bB-4Ac)\sqrt{bx+cx^2}}{63b^6x^2}+\frac{512c^3(3bB-4Ac)\sqrt{bx+cx^2}}{63b^7x}$$

output

```
-2/9*A/b/x^3/(c*x^2+b*x)^(3/2)+2/9*(-4*A*c+3*B*b)/b^2/x^2/(c*x^2+b*x)^(3/2
)+20/9*(-4*A*c+3*B*b)/b^3/x^3/(c*x^2+b*x)^(1/2)-160/63*(-4*A*c+3*B*b)*(c*x
^2+b*x)^(1/2)/b^4/x^4+64/21*c*(-4*A*c+3*B*b)*(c*x^2+b*x)^(1/2)/b^5/x^3-256
/63*c^2*(-4*A*c+3*B*b)*(c*x^2+b*x)^(1/2)/b^6/x^2+512/63*c^3*(-4*A*c+3*B*b)
*(c*x^2+b*x)^(1/2)/b^7/x
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.65

$$\int \frac{A+Bx}{x^3\left(bx+cx^2\right)^{5/2}}\,dx = \frac{6bBx(-3b^5+6b^4cx-16b^3c^2x^2+96b^2c^3x^3+384bc^4x^4+256c^5x^5)-2A(7b^6-12}{63b^7x^3(x(b+cx))}$$

input
```
Integrate[(A + B*x)/(x^3*(b*x + c*x^2)^(5/2)),x]
```

output
```
(6*b*B*x*(-3*b^5 + 6*b^4*c*x - 16*b^3*c^2*x^2 + 96*b^2*c^3*x^3 + 384*b*c^4
*x^4 + 256*c^5*x^5) - 2*A*(7*b^6 - 12*b^5*c*x + 24*b^4*c^2*x^2 - 64*b^3*c^
3*x^3 + 384*b^2*c^4*x^4 + 1536*b*c^5*x^5 + 1024*c^6*x^6))/(63*b^7*x^3*(x*(
b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.71, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1220, 1129, 1129, 1089, 1088}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^3\left(bx+cx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(3bB-4Ac)\int \frac{1}{x^2(cx^2+bx)^{5/2}}dx}{3b}-\frac{2A}{9bx^3\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(3bB-4Ac)\left(-\frac{10c\int \frac{1}{x\left(cx^2+bx\right)^{5/2}}dx}{7b}-\frac{2}{7bx^2(bx+cx^2)^{3/2}}\right)}{3b}-\frac{2A}{9bx^3\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1129}$$

$$\frac{(3bB-4Ac)\left(-\frac{10c\left(-\frac{8c\int \frac{1}{(cx^2+bx)^{5/2}}dx}{5b}-\frac{2}{5bx(bx+cx^2)^{3/2}}\right)}{7b}-\frac{2}{7bx^2(bx+cx^2)^{3/2}}\right)}{3b}-\frac{2A}{9bx^3(bx+cx^2)^{3/2}}$$

$\Big\downarrow$ 1089

$$\frac{(3bB-4Ac)\left(-\frac{10c\left(-\frac{8c\left(-\frac{8c\int \frac{1}{(cx^2+bx)^{3/2}}dx}{3b^2}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}\right)}{5b}-\frac{2}{5bx(bx+cx^2)^{3/2}}\right)}{7b}-\frac{2}{7bx^2(bx+cx^2)^{3/2}}\right)}{3b}-\frac{2A}{9bx^3(bx+cx^2)^{3/2}}$$

$\Big\downarrow$ 1088

$$\frac{\left(-\frac{10c\left(-\frac{8c\left(\frac{16c(b+2cx)}{3b^4\sqrt{bx+cx^2}}-\frac{2(b+2cx)}{3b^2(bx+cx^2)^{3/2}}\right)}{5b}-\frac{2}{5bx(bx+cx^2)^{3/2}}\right)}{7b}-\frac{2}{7bx^2(bx+cx^2)^{3/2}}\right)(3bB-4Ac)}{3b}-\frac{2A}{9bx^3(bx+cx^2)^{3/2}}$$

input
```
Int[(A + B*x)/(x^3*(b*x + c*x^2)^(5/2)),x]
```

output
```
(-2*A)/(9*b*x^3*(b*x + c*x^2)^(3/2)) + ((3*b*B - 4*A*c)*(-2/(7*b*x^2*(b*x
+ c*x^2)^(3/2)) - (10*c*(-2/(5*b*x*(b*x + c*x^2)^(3/2)) - (8*c*((-2*(b + 2
*c*x))/(3*b^2*(b*x + c*x^2)^(3/2)) + (16*c*(b + 2*c*x))/(3*b^4*Sqrt[b*x +
c*x^2])))/(5*b)))/(7*b)))/(3*b)
```

Defintions of rubi rules used

rule 1088
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[-2*((b +
2*c*x)/((b^2 - 4*a*c)*Sqrt[a + b*x + c*x^2])), x] /; FreeQ[{a, b, c}, x] &&
 NeQ[b^2 - 4*a*c, 0]
```

rule 1089
```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Simp[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c)))   Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Fre
eQ[{a, b, c}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[3*p])
```

rule 1129
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*(Simplify[m + 2*p + 2]/((m + p + 1)*(2*c*d - b*e)
))   Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[Simplify[m + 2*p +
2], 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.56

method	result
pseudoelliptic	$\frac{(-18Bx-14A)b^6+24cx\left(\frac{3Bx}{2}+A\right)b^5-48c^2x^2(2Bx+A)b^4+128c^3x^3\left(\frac{9Bx}{2}+A\right)b^3-768c^4x^4(-3Bx+A)b^2-3072c^5\left(-\frac{Bx}{2}+\right.}{63\sqrt{x(cx+b)}\,x^4(cx+b)b^7}$
gosper	$-\frac{2(cx+b)\left(1024Ac^6x^6-768Bbc^5x^6+1536Abc^5x^5-1152Bb^2c^4x^5+384Ab^2c^4x^4-288Bb^3c^3x^4-64Ab^3c^3x^3+48Bb^4c^2x^3\right.}{63x^2b^7(cx^2+bx)^{\frac{5}{2}}}$
orering	$-\frac{2(cx+b)\left(1024Ac^6x^6-768Bbc^5x^6+1536Abc^5x^5-1152Bb^2c^4x^5+384Ab^2c^4x^4-288Bb^3c^3x^4-64Ab^3c^3x^3+48Bb^4c^2x^3\right.}{63x^2b^7(cx^2+bx)^{\frac{5}{2}}}$
risch	$-\frac{2(cx+b)\left(667Ac^4x^4-474Bbc^3x^4-176Abc^3x^3+111Bb^2c^2x^3+69Ab^2c^2x^2-36Bb^3cx^2-26Ab^3cx+9Bb^4x+7Ab^4\right)}{63b^7x^4\sqrt{x(cx+b)}}-$
trager	$-\frac{2\left(1024Ac^6x^6-768Bbc^5x^6+1536Abc^5x^5-1152Bb^2c^4x^5+384Ab^2c^4x^4-288Bb^3c^3x^4-64Ab^3c^3x^3+48Bb^4c^2x^3+24A\right.}{63b^7x^5(cx+b)^2}$
default	$A\left(-\frac{2}{9bx^3(cx^2+bx)^{\frac{3}{2}}}-\frac{4c\left(-\frac{2}{7bx^2(cx^2+bx)^{\frac{3}{2}}}-\frac{10c\left(-\frac{2}{5bx(cx^2+bx)^{\frac{3}{2}}}-\frac{8c\left(-\frac{2(2cx+b)}{3b^2(cx^2+bx)^{\frac{3}{2}}}+\frac{16c(2cx+b)}{3b^4\sqrt{cx^2+bx}}\right)}{5b}\right)}{7b}\right)}{3b}\right)$

input

```
int((B*x+A)/x^3/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output

```
1/63*((-18*B*x-14*A)*b^6+24*c*x*(3/2*B*x+A)*b^5-48*c^2*x^2*(2*B*x+A)*b^4+1
28*c^3*x^3*(9/2*B*x+A)*b^3-768*c^4*x^4*(-3*B*x+A)*b^2-3072*c^5*(-1/2*B*x+A
)*x^5*b-2048*A*c^6*x^6)/(x*(c*x+b))^(1/2)/x^4/(c*x+b)/b^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.79

$$\int \frac{A+Bx}{x^3\left(bx+cx^2\right)^{5/2}}\,dx =$$
$$-\frac{2\,(7\,Ab^6-256\,(3\,Bbc^5-4\,Ac^6)x^6-384\,(3\,Bb^2c^4-4\,Abc^5)x^5-96\,(3\,Bb^3c^3-4\,Ab^2c^4)x^4+16\,(3\,Bb^4c^2-}{63\,(b^7c^2x^7+2\,b^8cx^6+b^9x^5)}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/63*(7*A*b^6 - 256*(3*B*b*c^5 - 4*A*c^6)*x^6 - 384*(3*B*b^2*c^4 - 4*A*b*
c^5)*x^5 - 96*(3*B*b^3*c^3 - 4*A*b^2*c^4)*x^4 + 16*(3*B*b^4*c^2 - 4*A*b^3*
c^3)*x^3 - 6*(3*B*b^5*c - 4*A*b^4*c^2)*x^2 + 3*(3*B*b^6 - 4*A*b^5*c)*x)*sq
rt(c*x^2 + b*x)/(b^7*c^2*x^7 + 2*b^8*c*x^6 + b^9*x^5)
```

Sympy [F]

$$\int \frac{A+Bx}{x^3\left(bx+cx^2\right)^{5/2}}\,dx = \int \frac{A+Bx}{x^3\left(x\left(b+cx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**3/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral((A + B*x)/(x**3*(x*(b + c*x))**(5/2)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.21

$$\int \frac{A+Bx}{x^3(bx+cx^2)^{5/2}}\,dx = -\frac{64\,Bc^3x}{21\,(cx^2+bx)^{\frac{3}{2}}b^4} + \frac{512\,Bc^4x}{21\,\sqrt{cx^2+bx}b^6} + \frac{256\,Ac^4x}{63\,(cx^2+bx)^{\frac{3}{2}}b^5} - \frac{2048\,Ac^5x}{63\,\sqrt{cx^2+bx}b^7} - \frac{32\,Bc^2}{21\,(cx^2+bx)^{\frac{3}{2}}b^3} + \frac{256\,Bc^3}{21\,\sqrt{cx^2+bx}b^5} + \frac{128\,Ac^3}{63\,(cx^2+bx)^{\frac{3}{2}}b^4} - \frac{1024\,Ac^4}{63\,\sqrt{cx^2+bx}b^6} + \frac{4\,Bc}{7\,(cx^2+bx)^{\frac{3}{2}}b^2x} - \frac{16\,Ac^2}{21\,(cx^2+bx)^{\frac{3}{2}}b^3x} - \frac{2\,B}{7\,(cx^2+bx)^{\frac{3}{2}}bx^2} + \frac{8\,Ac}{21\,(cx^2+bx)^{\frac{3}{2}}b^2x^2} - \frac{2\,A}{9\,(cx^2+bx)^{\frac{3}{2}}bx^3}$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
-64/21*B*c^3*x/((c*x^2 + b*x)^(3/2)*b^4) + 512/21*B*c^4*x/(sqrt(c*x^2 + b*
x)*b^6) + 256/63*A*c^4*x/((c*x^2 + b*x)^(3/2)*b^5) - 2048/63*A*c^5*x/(sqrt
(c*x^2 + b*x)*b^7) - 32/21*B*c^2/((c*x^2 + b*x)^(3/2)*b^3) + 256/21*B*c^3/
(sqrt(c*x^2 + b*x)*b^5) + 128/63*A*c^3/((c*x^2 + b*x)^(3/2)*b^4) - 1024/63
*A*c^4/(sqrt(c*x^2 + b*x)*b^6) + 4/7*B*c/((c*x^2 + b*x)^(3/2)*b^2*x) - 16/
21*A*c^2/((c*x^2 + b*x)^(3/2)*b^3*x) - 2/7*B/((c*x^2 + b*x)^(3/2)*b*x^2) +
 8/21*A*c/((c*x^2 + b*x)^(3/2)*b^2*x^2) - 2/9*A/((c*x^2 + b*x)^(3/2)*b*x^3
)
```

Giac [F]

$$\int \frac{A+Bx}{x^3(bx+cx^2)^{5/2}}\,dx = \int \frac{Bx+A}{(cx^2+bx)^{\frac{5}{2}}x^3}\,dx$$

input
```
integrate((B*x+A)/x^3/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
integrate((B*x + A)/((c*x^2 + b*x)^(5/2)*x^3), x)
```

Mupad [B] (verification not implemented)

Time = 5.63 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.19

$$\int \frac{A+Bx}{x^3\left(bx+cx^2\right)^{5/2}}\,dx = \frac{\sqrt{c\,x^2+b\,x}\left(x\left(\frac{4\,c^3\,(176\,A\,c-111\,B\,b)}{63\,b^5}+\frac{2\,c^3\,(247\,A\,c-138\,B\,b)}{63\,b^5}+\frac{b\left(\frac{184\,A\,c^5-96\,B\,b\,c^4}{63\,b^6}-\frac{4\,c^4\,(247\,A\,c-}{63\,b^6}\right.}{c}\right.\right.}{x^2\,(b+c\,x)^2}$$
$$-\frac{\sqrt{c\,x^2+b\,x}\,\left(18\,B\,b^3-52\,A\,b^2\,c\right)}{63\,b^6\,x^4}$$
$$-\frac{\sqrt{c\,x^2+b\,x}\,\left(\frac{1024\,A\,c^4-768\,B\,b\,c^3}{63\,b^6}+\frac{2\,c\,x\,\left(1024\,A\,c^4-768\,B\,b\,c^3\right)}{63\,b^7}\right)}{x\,(b+c\,x)}$$
$$-\frac{2\,A\,\sqrt{c\,x^2+b\,x}}{9\,b^3\,x^5}-\frac{2\,c\,\sqrt{c\,x^2+b\,x}\,(23\,A\,c-12\,B\,b)}{21\,b^5\,x^3}$$

input
```
int((A + B*x)/(x^3*(b*x + c*x^2)^(5/2)),x)
```

output
```
((b*x + c*x^2)^(1/2)*(x*((4*c^3*(176*A*c - 111*B*b))/(63*b^5) + (2*c^3*(24
7*A*c - 138*B*b))/(63*b^5) + (b*((184*A*c^5 - 96*B*b*c^4)/(63*b^6) - (4*c^
4*(247*A*c - 138*B*b))/(63*b^6)))/c) + (2*c^2*(176*A*c - 111*B*b))/(63*b^4
)))/(x^2*(b + c*x)^2) - ((b*x + c*x^2)^(1/2)*(18*B*b^3 - 52*A*b^2*c))/(63*
b^6*x^4) - ((b*x + c*x^2)^(1/2)*((1024*A*c^4 - 768*B*b*c^3)/(63*b^6) + (2*
c*x*(1024*A*c^4 - 768*B*b*c^3))/(63*b^7)))/(x*(b + c*x)) - (2*A*(b*x + c*x
^2)^(1/2))/(9*b^3*x^5) - (2*c*(b*x + c*x^2)^(1/2)*(23*A*c - 12*B*b))/(21*b
^5*x^3)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.12

$$\int \frac{A+Bx}{x^3\left(bx+cx^2\right)^{5/2}}\,dx = \frac{\frac{2048\sqrt{c}\,\sqrt{cx+b}\,ab\,c^4x^5}{63}+\frac{2048\sqrt{c}\,\sqrt{cx+b}\,a\,c^5x^6}{63}-\frac{512\sqrt{c}\,\sqrt{cx+b}\,b^3c^3x^5}{21}-\frac{512\sqrt{c}\,\sqrt{cx+b}\,b^2c^4x^6}{21}-\frac{2\sqrt{x}\,c}{9}}{}$$

input
```
int((B*x+A)/x^3/(c*x^2+b*x)^(5/2),x)
```

output

```
(2*(1024*sqrt(c)*sqrt(b + c*x)*a*b*c**4*x**5 + 1024*sqrt(c)*sqrt(b + c*x)*
a*c**5*x**6 - 768*sqrt(c)*sqrt(b + c*x)*b**3*c**3*x**5 - 768*sqrt(c)*sqrt(
b + c*x)*b**2*c**4*x**6 - 7*sqrt(x)*a*b**6 + 12*sqrt(x)*a*b**5*c*x - 24*sq
rt(x)*a*b**4*c**2*x**2 + 64*sqrt(x)*a*b**3*c**3*x**3 - 384*sqrt(x)*a*b**2*
c**4*x**4 - 1536*sqrt(x)*a*b*c**5*x**5 - 1024*sqrt(x)*a*c**6*x**6 - 9*sqrt
(x)*b**7*x + 18*sqrt(x)*b**6*c*x**2 - 48*sqrt(x)*b**5*c**2*x**3 + 288*sqrt
(x)*b**4*c**3*x**4 + 1152*sqrt(x)*b**3*c**4*x**5 + 768*sqrt(x)*b**2*c**5*x
**6))/(63*sqrt(b + c*x)*b**7*x**5*(b + c*x))
```

3.171 $\int x^{7/2}(A+Bx)\sqrt{bx+cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\int x^{7/2}(A+Bx)\sqrt{bx+cx^2}\,dx=-\frac{2b^4(bB-Ac)\left(bx+cx^2\right)^{3/2}}{3c^6x^{3/2}}+\frac{2b^3(5bB-4Ac)\left(bx+cx^2\right)^{5/2}}{5c^6x^{5/2}}-\frac{4b^2(5bB-3Ac)\left(bx+cx^2\right)^{7/2}}{7c^6x^{7/2}}+\frac{4b(5bB-2Ac)\left(bx+cx^2\right)^{9/2}}{9c^6x^{9/2}}-\frac{2(5bB-Ac)\left(bx+cx^2\right)^{11/2}}{11c^6x^{11/2}}+\frac{2B\left(bx+cx^2\right)^{13/2}}{13c^6x^{13/2}}$$

output
```
-2/3*b^4*(-A*c+B*b)*(c*x^2+b*x)^(3/2)/c^6/x^(3/2)+2/5*b^3*(-4*A*c+5*B*b)*(
c*x^2+b*x)^(5/2)/c^6/x^(5/2)-4/7*b^2*(-3*A*c+5*B*b)*(c*x^2+b*x)^(7/2)/c^6/
x^(7/2)+4/9*b*(-2*A*c+5*B*b)*(c*x^2+b*x)^(9/2)/c^6/x^(9/2)-2/11*(-A*c+5*B*
b)*(c*x^2+b*x)^(11/2)/c^6/x^(11/2)+2/13*B*(c*x^2+b*x)^(13/2)/c^6/x^(13/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.55

$$\int x^{7/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2(x(b+cx))^{3/2}\left(-1280b^5B+315c^5x^4(13A+11Bx)+128b^4c(13A+15Bx)-96b^3c\right.}{45045c^6x^{3/2}}$$

input
```
Integrate[x^(7/2)*(A + B*x)*Sqrt[b*x + c*x^2],x]
```

output
```
(2*(x*(b + c*x))^(3/2)*(-1280*b^5*B + 315*c^5*x^4*(13*A + 11*B*x) + 128*b^
4*c*(13*A + 15*B*x) - 96*b^3*c^2*x*(26*A + 25*B*x) + 80*b^2*c^3*x^2*(39*A
+ 35*B*x) - 70*b*c^4*x^3*(52*A + 45*B*x)))/(45045*c^6*x^(3/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1221, 1128, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{7/2}(A+Bx)\sqrt{bx+cx^2}\,dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{2Bx^{7/2}\left(bx+cx^2\right)^{3/2}}{13c}-\frac{(10bB-13Ac)\int x^{7/2}\sqrt{cx^2+bx}dx}{13c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2Bx^{7/2}\left(bx+cx^2\right)^{3/2}}{13c}-\frac{(10bB-13Ac)\left(\frac{2x^{5/2}\left(bx+cx^2\right)^{3/2}}{11c}-\frac{8b\int x^{5/2}\sqrt{cx^2+bx}dx}{11c}\right)}{13c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{\frac{2Bx^{7/2}\left(bx+cx^2\right)^{3/2}}{13c}-(10bB-13Ac)\left(\frac{2x^{5/2}\left(bx+cx^2\right)^{3/2}}{11c}-\frac{8b\left(\frac{2x^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}-\frac{2b\int x^{3/2}\sqrt{cx^2+bx}dx}{3c}\right)}{11c}\right)}{13c}$$

$\downarrow$ 1128

$$\frac{\frac{2Bx^{7/2}\left(bx+cx^2\right)^{3/2}}{13c}-(10bB-13Ac)\left(\frac{2x^{5/2}\left(bx+cx^2\right)^{3/2}}{11c}-\frac{8b\left(\frac{2x^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}-\frac{2b\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}-\frac{4b\int\sqrt{x}\sqrt{cx^2+bx}dx}{7c}\right)}{3c}\right)}{11c}\right)}{13c}$$

$\downarrow$ 1128

$$\frac{\frac{2Bx^{7/2}\left(bx+cx^2\right)^{3/2}}{13c}-(10bB-13Ac)\left(\frac{2x^{5/2}\left(bx+cx^2\right)^{3/2}}{11c}-\frac{8b\left(\frac{2x^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}-\frac{2b\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{3/2}}{5c\sqrt{x}}-\frac{2b\int\frac{\sqrt{cx^2+bx}}{\sqrt{x}}dx}{5c}\right)}{7c}\right)}{3c}\right)}{11c}\right)}{13c}$$

$\downarrow$ 1122

$$\frac{2Bx^{7/2}(bx+cx^2)^{3/2}}{13c} - \frac{\left(\frac{2x^{5/2}(bx+cx^2)^{3/2}}{11c} - \frac{8b\left(\frac{2x^{3/2}(bx+cx^2)^{3/2}}{9c} - \frac{2b\left(\frac{2\sqrt{x}(bx+cx^2)^{3/2}}{7c} - \frac{4b\left(\frac{2(bx+cx^2)^{3/2}}{5c\sqrt{x}} - \frac{4b(bx+cx^2)^{3/2}}{15c^2x^{3/2}}\right)}{7c}\right)}{3c}\right)}{11c}\right)(10bB-13Ac)}{13c}$$

input
```
Int[x^(7/2)*(A + B*x)*Sqrt[b*x + c*x^2],x]
```

output
```
(2*B*x^(7/2)*(b*x + c*x^2)^(3/2))/(13*c) - ((10*b*B - 13*A*c)*((2*x^(5/2)*
(b*x + c*x^2)^(3/2))/(11*c) - (8*b*((2*x^(3/2)*(b*x + c*x^2)^(3/2))/(9*c)
- (2*b*((2*Sqrt[x]*(b*x + c*x^2)^(3/2))/(7*c) - (4*b*((-4*b*(b*x + c*x^2)^
(3/2))/(15*c^2*x^(3/2)) + (2*(b*x + c*x^2)^(3/2))/(5*c*Sqrt[x])))/(7*c)))/
(3*c)))/(11*c)))/(13*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.63

method	result
default	$\frac{2(cx+b)(3465B\,c^5x^5+4095A\,c^5x^4-3150Bb\,c^4x^4-3640Ab\,c^4x^3+2800B\,b^2c^3x^3+3120A\,b^2c^3x^2-2400B\,b^3c^2x^2-2496A\,b^3c^2x+192}{45045c^6\sqrt{x}}$
gosper	$\frac{2(cx+b)(3465B\,c^5x^5+4095A\,c^5x^4-3150Bb\,c^4x^4-3640Ab\,c^4x^3+2800B\,b^2c^3x^3+3120A\,b^2c^3x^2-2400B\,b^3c^2x^2-2496A\,b^3c^2x+192}{45045c^6\sqrt{x}}$
orering	$\frac{2(cx+b)(3465B\,c^5x^5+4095A\,c^5x^4-3150Bb\,c^4x^4-3640Ab\,c^4x^3+2800B\,b^2c^3x^3+3120A\,b^2c^3x^2-2400B\,b^3c^2x^2-2496A\,b^3c^2x+192}{45045c^6\sqrt{x}}$
risch	$\frac{2(cx+b)\sqrt{x}\,(3465B\,c^6x^6+4095A\,c^6x^5+315Bb\,c^5x^5+455Ab\,c^5x^4-350B\,b^2c^4x^4-520A\,b^2c^4x^3+400B\,b^3c^3x^3+624A\,b^3c^3x^2-480B}{45045\sqrt{x(cx+b)}\,c^6}$

input

```
int(x^(7/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output

```
2/45045*(c*x+b)*(3465*B*c^5*x^5+4095*A*c^5*x^4-3150*B*b*c^4*x^4-3640*A*b*c
^4*x^3+2800*B*b^2*c^3*x^3+3120*A*b^2*c^3*x^2-2400*B*b^3*c^2*x^2-2496*A*b^3
*c^2*x+1920*B*b^4*c*x+1664*A*b^4*c-1280*B*b^5)*(x*(c*x+b))^(1/2)/c^6/x^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.73

$$\int x^{7/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2\,(3465\,Bc^6x^6 - 1280\,Bb^6 + 1664\,Ab^5c + 315\,(Bbc^5 + 13\,Ac^6)x^5 - 35\,(10\,Bb^2c^4 - 13$$

input
```
integrate(x^(7/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2/45045*(3465*B*c^6*x^6 - 1280*B*b^6 + 1664*A*b^5*c + 315*(B*b*c^5 + 13*A*
c^6)*x^5 - 35*(10*B*b^2*c^4 - 13*A*b*c^5)*x^4 + 40*(10*B*b^3*c^3 - 13*A*b^
2*c^4)*x^3 - 48*(10*B*b^4*c^2 - 13*A*b^3*c^3)*x^2 + 64*(10*B*b^5*c - 13*A*
b^4*c^2)*x)*sqrt(c*x^2 + b*x)/(c^6*sqrt(x))
```

Sympy [F]

$$\int x^{7/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \int x^{\frac{7}{2}}\sqrt{x\,(b+cx)}(A+Bx)\;dx$$

input
```
integrate(x**(7/2)*(B*x+A)*(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(x**(7/2)*sqrt(x*(b + c*x))*(A + B*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.69

$$\int x^{7/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2\,(315\,c^5x^5 + 35\,bc^4x^4 - 40\,b^2c^3x^3 + 48\,b^3c^2x^2 - 64\,b^4cx + 128\,b^5)\sqrt{cx+b}A}{3465\,c^5} + \frac{2\,(693\,c^6x^6 + 63\,bc^5x^5 - 70\,b^2c^4x^4 + 80\,b^3c^3x^3 - 96\,b^4c^2x^2 + 128\,b^5cx - 256\,b^6)\sqrt{cx+b}B}{9009\,c^6}$$

input
```
integrate(x^(7/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
2/3465*(315*c^5*x^5 + 35*b*c^4*x^4 - 40*b^2*c^3*x^3 + 48*b^3*c^2*x^2 - 64*
b^4*c*x + 128*b^5)*sqrt(c*x + b)*A/c^5 + 2/9009*(693*c^6*x^6 + 63*b*c^5*x^
5 - 70*b^2*c^4*x^4 + 80*b^3*c^3*x^3 - 96*b^4*c^2*x^2 + 128*b^5*c*x - 256*b
^6)*sqrt(c*x + b)*B/c^6
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.67

$$\int x^{7/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2\left(315\,(cx+b)^{\frac{11}{2}} - 1540\,(cx+b)^{\frac{9}{2}}b + 2970\,(cx+b)^{\frac{7}{2}}b^2 - 2772\,(cx+b)^{\frac{5}{2}}b^3 + 1155\,(c\right.}{3465\,c^5} + \frac{2\left(693\,(cx+b)^{\frac{13}{2}} - 4095\,(cx+b)^{\frac{11}{2}}b + 10010\,(cx+b)^{\frac{9}{2}}b^2 - 12870\,(cx+b)^{\frac{7}{2}}b^3 + 9009\,(cx+b)^{\frac{5}{2}}b^4 - 3003\right.}{9009\,c^6}$$

input
```
integrate(x^(7/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
2/3465*(315*(c*x + b)^(11/2) - 1540*(c*x + b)^(9/2)*b + 2970*(c*x + b)^(7/
2)*b^2 - 2772*(c*x + b)^(5/2)*b^3 + 1155*(c*x + b)^(3/2)*b^4)*A/c^5 + 2/90
09*(693*(c*x + b)^(13/2) - 4095*(c*x + b)^(11/2)*b + 10010*(c*x + b)^(9/2)
*b^2 - 12870*(c*x + b)^(7/2)*b^3 + 9009*(c*x + b)^(5/2)*b^4 - 3003*(c*x +
b)^(3/2)*b^5)*B/c^6
```

Mupad [F(-1)]

Timed out.

$$\int x^{7/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \int x^{7/2}\,\sqrt{c\,x^2+b\,x}\,(A+B\,x)\,dx$$

input
```
int(x^(7/2)*(b*x + c*x^2)^(1/2)*(A + B*x),x)
```

output
```
int(x^(7/2)*(b*x + c*x^2)^(1/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.67

$$\int x^{7/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2\sqrt{cx+b}\,(3465b\,c^6x^6+4095a\,c^6x^5+315b^2c^5x^5+455ab\,c^5x^4-350b^3c^4x^4-520a\,b^2c$$

input
```
int(x^(7/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x)
```

output
```
(2*sqrt(b + c*x)*(1664*a*b**5*c - 832*a*b**4*c**2*x + 624*a*b**3*c**3*x**2
 - 520*a*b**2*c**4*x**3 + 455*a*b*c**5*x**4 + 4095*a*c**6*x**5 - 1280*b**7
 + 640*b**6*c*x - 480*b**5*c**2*x**2 + 400*b**4*c**3*x**3 - 350*b**3*c**4*
x**4 + 315*b**2*c**5*x**5 + 3465*b*c**6*x**6))/(45045*c**6)
```

3.172 $\int x^{5/2}(A+Bx)\sqrt{bx+cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 169

$$\int x^{5/2}(A+Bx)\sqrt{bx+cx^2}\,dx=\frac{2b^3(bB-Ac)\left(bx+cx^2\right)^{3/2}}{3c^5x^{3/2}}-\frac{2b^2(4bB-3Ac)\left(bx+cx^2\right)^{5/2}}{5c^5x^{5/2}}+\frac{6b(2bB-Ac)\left(bx+cx^2\right)^{7/2}}{7c^5x^{7/2}}-\frac{2(4bB-Ac)\left(bx+cx^2\right)^{9/2}}{9c^5x^{9/2}}+\frac{2B\left(bx+cx^2\right)^{11/2}}{11c^5x^{11/2}}$$

output
```
2/3*b^3*(-A*c+B*b)*(c*x^2+b*x)^(3/2)/c^5/x^(3/2)-2/5*b^2*(-3*A*c+4*B*b)*(c
*x^2+b*x)^(5/2)/c^5/x^(5/2)+6/7*b*(-A*c+2*B*b)*(c*x^2+b*x)^(7/2)/c^5/x^(7/
2)-2/9*(-A*c+4*B*b)*(c*x^2+b*x)^(9/2)/c^5/x^(9/2)+2/11*B*(c*x^2+b*x)^(11/2
)/c^5/x^(11/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.56

$$\int x^{5/2}(A+Bx)\sqrt{bx+cx^2}\,dx=\frac{2(x(b+cx))^{3/2}\left(128b^4B+35c^4x^3(11A+9Bx)+24b^2c^2x(11A+10Bx)-16b^3c(11A\right.}{3465c^5x^{3/2}}$$

input
```
Integrate[x^(5/2)*(A + B*x)*Sqrt[b*x + c*x^2],x]
```

output
```
(2*(x*(b + c*x))^(3/2)*(128*b^4*B + 35*c^4*x^3*(11*A + 9*B*x) + 24*b^2*c^2
*x*(11*A + 10*B*x) - 16*b^3*c*(11*A + 12*B*x) - 10*b*c^3*x^2*(33*A + 28*B*
x)))/(3465*c^5*x^(3/2))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1221, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(A+Bx)\sqrt{bx+cx^2}\,dx$$

$$\Big\downarrow \ 1221$$

$$\frac{2Bx^{5/2}\left(bx+cx^2\right)^{3/2}}{11c}-\frac{(8bB-11Ac)\int x^{5/2}\sqrt{cx^2+bx}dx}{11c}$$

$$\Big\downarrow \ 1128$$

$$\frac{2Bx^{5/2}\left(bx+cx^2\right)^{3/2}}{11c}-\frac{(8bB-11Ac)\left(\frac{2x^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}-\frac{2b\int x^{3/2}\sqrt{cx^2+bx}dx}{3c}\right)}{11c}$$

$$\Big\downarrow \ 1128$$

$$\frac{2Bx^{5/2}\left(bx+cx^2\right)^{3/2}}{11c}-\frac{(8bB-11Ac)\left(\frac{2x^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}-\frac{2b\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}-\frac{4b\int\sqrt{x}\sqrt{cx^2+bx}dx}{7c}\right)}{3c}\right)}{11c}$$

$\big\downarrow$ 1128

$$\frac{2Bx^{5/2}\left(bx+cx^2\right)^{3/2}}{11c}-\frac{(8bB-11Ac)\left(\frac{2x^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}-\frac{2b\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{3/2}}{5c\sqrt{x}}-\frac{2b\int\frac{\sqrt{cx^2+bx}}{\sqrt{x}}dx}{5c}\right)}{7c}\right)}{3c}\right)}{11c}$$

$\big\downarrow$ 1122

$$\frac{2Bx^{5/2}\left(bx+cx^2\right)^{3/2}}{11c}-\frac{\left(\frac{2x^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}-\frac{2b\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{3/2}}{5c\sqrt{x}}-\frac{4b\left(bx+cx^2\right)^{3/2}}{15c^2x^{3/2}}\right)}{7c}\right)}{3c}\right)(8bB-11Ac)}{11c}$$

input
```
Int[x^(5/2)*(A + B*x)*Sqrt[b*x + c*x^2],x]
```

output
```
(2*B*x^(5/2)*(b*x + c*x^2)^(3/2))/(11*c) - ((8*b*B - 11*A*c)*((2*x^(3/2)*(
b*x + c*x^2)^(3/2))/(9*c) - (2*b*((2*Sqrt[x]*(b*x + c*x^2)^(3/2))/(7*c) -
(4*b*((-4*b*(b*x + c*x^2)^(3/2))/(15*c^2*x^(3/2)) + (2*(b*x + c*x^2)^(3/2)
)/(5*c*Sqrt[x])))/(7*c)))/(3*c)))/(11*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.62

method	result
default	$-\frac{2(cx+b)\left(-315B\,c^4x^4-385A\,c^4x^3+280B\,c^3x^3b+330Ab\,c^3x^2-240c^2x^2B\,b^2-264A\,b^2c^2x+192B\,b^3cx+176A\,b^3c-128B\,b^4\right)\sqrt{x(c}}{3465c^5\sqrt{x}}$
gosper	$-\frac{2(cx+b)\left(-315B\,c^4x^4-385A\,c^4x^3+280B\,c^3x^3b+330Ab\,c^3x^2-240c^2x^2B\,b^2-264A\,b^2c^2x+192B\,b^3cx+176A\,b^3c-128B\,b^4\right)\sqrt{c\,x^2}}{3465c^5\sqrt{x}}$
orering	$-\frac{2(cx+b)\left(-315B\,c^4x^4-385A\,c^4x^3+280B\,c^3x^3b+330Ab\,c^3x^2-240c^2x^2B\,b^2-264A\,b^2c^2x+192B\,b^3cx+176A\,b^3c-128B\,b^4\right)\sqrt{c\,x^2}}{3465c^5\sqrt{x}}$
risch	$-\frac{2(cx+b)\sqrt{x}\,(-315B\,c^5x^5-385A\,c^5x^4-35Bb\,c^4x^4-55Ab\,c^4x^3+40B\,b^2c^3x^3+66A\,b^2c^3x^2-48B\,b^3c^2x^2-88A\,b^3c^2x+64B\,b^4cx+1}{3465\sqrt{x(cx+b)}\,c^5}$

input
```
int(x^(5/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3465*(c*x+b)*(-315*B*c^4*x^4-385*A*c^4*x^3+280*B*b*c^3*x^3+330*A*b*c^3*
x^2-240*B*b^2*c^2*x^2-264*A*b^2*c^2*x+192*B*b^3*c*x+176*A*b^3*c-128*B*b^4)
*(x*(c*x+b))^(1/2)/c^5/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.75

$$\int x^{5/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2\,(315\,Bc^5x^5+128\,Bb^5-176\,Ab^4c+35\,(Bbc^4+11\,Ac^5)x^4-5\,(8\,Bb^2c^3-11\,Abc^4)x}{3465\,c^5\sqrt{x}}$$

input
```
integrate(x^(5/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*B*c^5*x^5 + 128*B*b^5 - 176*A*b^4*c + 35*(B*b*c^4 + 11*A*c^5)*
x^4 - 5*(8*B*b^2*c^3 - 11*A*b*c^4)*x^3 + 6*(8*B*b^3*c^2 - 11*A*b^2*c^3)*x^
2 - 8*(8*B*b^4*c - 11*A*b^3*c^2)*x)*sqrt(c*x^2 + b*x)/(c^5*sqrt(x))
```

Sympy [F]

$$\int x^{5/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \int x^{\frac{5}{2}}\sqrt{x\,(b+cx)}(A+Bx)\,dx$$

input
```
integrate(x**(5/2)*(B*x+A)*(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(x**(5/2)*sqrt(x*(b + c*x))*(A + B*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.71

$$\int x^{5/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2\,(35\,c^4x^4+5\,bc^3x^3-6\,b^2c^2x^2+8\,b^3cx-16\,b^4)\sqrt{cx+b}A}{315\,c^4} + \frac{2\,(315\,c^5x^5+35\,bc^4x^4-40\,b^2c^3x^3+48\,b^3c^2x^2-64\,b^4cx+128\,b^5)\sqrt{cx+b}B}{3465\,c^5}$$

input
```
integrate(x^(5/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*c^4*x^4 + 5*b*c^3*x^3 - 6*b^2*c^2*x^2 + 8*b^3*c*x - 16*b^4)*sqrt
(c*x + b)*A/c^4 + 2/3465*(315*c^5*x^5 + 35*b*c^4*x^4 - 40*b^2*c^3*x^3 + 48
*b^3*c^2*x^2 - 64*b^4*c*x + 128*b^5)*sqrt(c*x + b)*B/c^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.67

$$\int x^{5/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2\left(35\,(cx+b)^{\frac{9}{2}}-135\,(cx+b)^{\frac{7}{2}}b+189\,(cx+b)^{\frac{5}{2}}b^2-105\,(cx+b)^{\frac{3}{2}}b^3\right)A}{315\,c^4} + \frac{2\left(315\,(cx+b)^{\frac{11}{2}}-1540\,(cx+b)^{\frac{9}{2}}b+2970\,(cx+b)^{\frac{7}{2}}b^2-2772\,(cx+b)^{\frac{5}{2}}b^3+1155\,(cx+b)^{\frac{3}{2}}b^4\right)B}{3465\,c^5}$$

input
```
integrate(x^(5/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
2/315*(35*(c*x + b)^(9/2) - 135*(c*x + b)^(7/2)*b + 189*(c*x + b)^(5/2)*b^
2 - 105*(c*x + b)^(3/2)*b^3)*A/c^4 + 2/3465*(315*(c*x + b)^(11/2) - 1540*(
c*x + b)^(9/2)*b + 2970*(c*x + b)^(7/2)*b^2 - 2772*(c*x + b)^(5/2)*b^3 + 1
155*(c*x + b)^(3/2)*b^4)*B/c^5
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \int x^{5/2}\,\sqrt{c\,x^2+b\,x}\,(A+B\,x)\,dx$$

input
```
int(x^(5/2)*(b*x + c*x^2)^(1/2)*(A + B*x),x)
```

output
```
int(x^(5/2)*(b*x + c*x^2)^(1/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.67

$$\int x^{5/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2\sqrt{cx+b}\,(315b\,c^5x^5+385a\,c^5x^4+35b^2c^4x^4+55ab\,c^4x^3-40b^3c^3x^3-66a\,b^2c^3x^2+}{3465c^5}$$

input
```
int(x^(5/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x)
```

output
```
(2*sqrt(b + c*x)*( - 176*a*b**4*c + 88*a*b**3*c**2*x - 66*a*b**2*c**3*x**2
 + 55*a*b*c**4*x**3 + 385*a*c**5*x**4 + 128*b**6 - 64*b**5*c*x + 48*b**4*c
**2*x**2 - 40*b**3*c**3*x**3 + 35*b**2*c**4*x**4 + 315*b*c**5*x**5))/(3465
*c**5)
```

3.173 $\int x^{3/2}(A+Bx)\sqrt{bx+cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int x^{3/2}(A+Bx)\sqrt{bx+cx^2}\,dx=-\frac{2b^2(bB-Ac)\left(bx+cx^2\right)^{3/2}}{3c^4x^{3/2}}+\frac{2b(3bB-2Ac)\left(bx+cx^2\right)^{5/2}}{5c^4x^{5/2}}-\frac{2(3bB-Ac)\left(bx+cx^2\right)^{7/2}}{7c^4x^{7/2}}+\frac{2B\left(bx+cx^2\right)^{9/2}}{9c^4x^{9/2}}$$

output
```
-2/3*b^2*(-A*c+B*b)*(c*x^2+b*x)^(3/2)/c^4/x^(3/2)+2/5*b*(-2*A*c+3*B*b)*(c*
x^2+b*x)^(5/2)/c^4/x^(5/2)-2/7*(-A*c+3*B*b)*(c*x^2+b*x)^(7/2)/c^4/x^(7/2)+
2/9*B*(c*x^2+b*x)^(9/2)/c^4/x^(9/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.55

$$\int x^{3/2}(A+Bx)\sqrt{bx+cx^2}\,dx=\frac{2(x(b+cx))^{3/2}\left(-16b^3B+24b^2c(A+Bx)-6bc^2x(6A+5Bx)+5c^3x^2(9A+7Bx)\right)}{315c^4x^{3/2}}$$

input
```
Integrate[x^(3/2)*(A + B*x)*Sqrt[b*x + c*x^2],x]
```

output
```
(2*(x*(b + c*x))^(3/2)*(-16*b^3*B + 24*b^2*c*(A + B*x) - 6*b*c^2*x*(6*A +
5*B*x) + 5*c^3*x^2*(9*A + 7*B*x)))/(315*c^4*x^(3/2))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1221, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(A+Bx)\sqrt{bx+cx^2}\,dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{2Bx^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}-\frac{(2bB-3Ac)\int x^{3/2}\sqrt{cx^2+bx}dx}{3c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2Bx^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}-\frac{(2bB-3Ac)\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}-\frac{4b\int \sqrt{x}\sqrt{cx^2+bx}dx}{7c}\right)}{3c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2Bx^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}-\frac{(2bB-3Ac)\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{3/2}}{5c\sqrt{x}}-\frac{2b\int \frac{\sqrt{cx^2+bx}}{\sqrt{x}}dx}{5c}\right)}{7c}\right)}{3c}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2Bx^{3/2}\left(bx+cx^2\right)^{3/2}}{9c}-\frac{\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{3/2}}{5c\sqrt{x}}-\frac{4b\left(bx+cx^2\right)^{3/2}}{15c^2x^{3/2}}\right)}{7c}\right)(2bB-3Ac)}{3c}$$

input
```
Int[x^(3/2)*(A + B*x)*Sqrt[b*x + c*x^2],x]
```

output
```
(2*B*x^(3/2)*(b*x + c*x^2)^(3/2))/(9*c) - ((2*b*B - 3*A*c)*((2*Sqrt[x]*(b*
x + c*x^2)^(3/2))/(7*c) - (4*b*((-4*b*(b*x + c*x^2)^(3/2))/(15*c^2*x^(3/2)
) + (2*(b*x + c*x^2)^(3/2))/(5*c*Sqrt[x])))/(7*c)))/(3*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.91 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{2(cx+b)\left(35B\,c^{3}x^{3}+45A\,c^{3}x^{2}-30Bb\,c^{2}x^{2}-36Ab\,c^{2}x+24B\,b^{2}cx+24A\,b^{2}c-16B\,b^{3}\right)\sqrt{x(cx+b)}}{315c^{4}\sqrt{x}}$	81
gosper	$\frac{2(cx+b)\left(35B\,c^{3}x^{3}+45A\,c^{3}x^{2}-30Bb\,c^{2}x^{2}-36Ab\,c^{2}x+24B\,b^{2}cx+24A\,b^{2}c-16B\,b^{3}\right)\sqrt{c\,x^{2}+bx}}{315c^{4}\sqrt{x}}$	83
orering	$\frac{2(cx+b)\left(35B\,c^{3}x^{3}+45A\,c^{3}x^{2}-30Bb\,c^{2}x^{2}-36Ab\,c^{2}x+24B\,b^{2}cx+24A\,b^{2}c-16B\,b^{3}\right)\sqrt{c\,x^{2}+bx}}{315c^{4}\sqrt{x}}$	83
risch	$\frac{2(cx+b)\sqrt{x}\,\left(35B\,c^{4}x^{4}+45A\,c^{4}x^{3}+5B\,c^{3}x^{3}b+9Ab\,c^{3}x^{2}-6c^{2}x^{2}B\,b^{2}-12A\,b^{2}c^{2}x+8B\,b^{3}cx+24A\,b^{3}c-16B\,b^{4}\right)}{315\sqrt{x(cx+b)}\,c^{4}}$	105

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/315*(c*x+b)*(35*B*c^3*x^3+45*A*c^3*x^2-30*B*b*c^2*x^2-36*A*b*c^2*x+24*B*
b^2*c*x+24*A*b^2*c-16*B*b^3)*(x*(c*x+b))^(1/2)/c^4/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.77

$$\int x^{3/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2\,(35\,Bc^4x^4 - 16\,Bb^4 + 24\,Ab^3c + 5\,(Bbc^3 + 9\,Ac^4)x^3 - 3\,(2\,Bb^2c^2 - 3\,Abc^3)x^2 + 4\,(}{315\,c^4\sqrt{x}}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*B*c^4*x^4 - 16*B*b^4 + 24*A*b^3*c + 5*(B*b*c^3 + 9*A*c^4)*x^3 -
3*(2*B*b^2*c^2 - 3*A*b*c^3)*x^2 + 4*(2*B*b^3*c - 3*A*b^2*c^2)*x)*sqrt(c*x^
2 + b*x)/(c^4*sqrt(x))
```

Sympy [F]

$$\int x^{3/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \int x^{\frac{3}{2}}\sqrt{x\,(b+cx)}(A+Bx)\;dx$$

input
```
integrate(x**(3/2)*(B*x+A)*(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(x**(3/2)*sqrt(x*(b + c*x))*(A + B*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.74

$$\int x^{3/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2\,(15\,c^3x^3+3\,bc^2x^2-4\,b^2cx+8\,b^3)\sqrt{cx+b}A}{105\,c^3} + \frac{2\,(35\,c^4x^4+5\,bc^3x^3-6\,b^2c^2x^2+8\,b^3cx-16\,b^4)\sqrt{cx+b}B}{315\,c^4}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
2/105*(15*c^3*x^3 + 3*b*c^2*x^2 - 4*b^2*c*x + 8*b^3)*sqrt(c*x + b)*A/c^3 +
 2/315*(35*c^4*x^4 + 5*b*c^3*x^3 - 6*b^2*c^2*x^2 + 8*b^3*c*x - 16*b^4)*sqr
t(c*x + b)*B/c^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.67

$$\int x^{3/2}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2\left(15\,(cx+b)^{\frac{7}{2}}-42\,(cx+b)^{\frac{5}{2}}b+35\,(cx+b)^{\frac{3}{2}}b^2\right)A}{105\,c^3} + \frac{2\left(35\,(cx+b)^{\frac{9}{2}}-135\,(cx+b)^{\frac{7}{2}}b+189\,(cx+b)^{\frac{5}{2}}b^2-105\,(cx+b)^{\frac{3}{2}}b^3\right)B}{315\,c^4}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
2/105*(15*(c*x + b)^(7/2) - 42*(c*x + b)^(5/2)*b + 35*(c*x + b)^(3/2)*b^2)
*A/c^3 + 2/315*(35*(c*x + b)^(9/2) - 135*(c*x + b)^(7/2)*b + 189*(c*x + b)
^(5/2)*b^2 - 105*(c*x + b)^(3/2)*b^3)*B/c^4
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(A+Bx)\sqrt{bx+cx^2}\,dx=\int x^{3/2}\,\sqrt{c\,x^2+b\,x}\,\left(A+B\,x\right)\,dx$$

input
```
int(x^(3/2)*(b*x + c*x^2)^(1/2)*(A + B*x),x)
```

output
```
int(x^(3/2)*(b*x + c*x^2)^(1/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.69

$$\int x^{3/2}(A+Bx)\sqrt{bx+cx^2}\,dx=\frac{2\sqrt{cx+b}\,(35b\,c^4x^4+45a\,c^4x^3+5b^2c^3x^3+9ab\,c^3x^2-6b^3c^2x^2-12a\,b^2c^2x+8b^4cx+}{315c^4}$$

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x)
```

output
```
(2*sqrt(b + c*x)*(24*a*b**3*c - 12*a*b**2*c**2*x + 9*a*b*c**3*x**2 + 45*a*
c**4*x**3 - 16*b**5 + 8*b**4*c*x - 6*b**3*c**2*x**2 + 5*b**2*c**3*x**3 + 3
5*b*c**4*x**4))/(315*c**4)
```

3.174 $\int \sqrt{x}(A+Bx)\sqrt{bx+cx^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \sqrt{x}(A+Bx)\sqrt{bx+cx^2}\,dx=\frac{2b(bB-Ac)\left(bx+cx^2\right)^{3/2}}{3c^3x^{3/2}}-\frac{2(2bB-Ac)\left(bx+cx^2\right)^{5/2}}{5c^3x^{5/2}}+\frac{2B\left(bx+cx^2\right)^{7/2}}{7c^3x^{7/2}}$$

output
```
2/3*b*(-A*c+B*b)*(c*x^2+b*x)^(3/2)/c^3/x^(3/2)-2/5*(-A*c+2*B*b)*(c*x^2+b*x
)^(5/2)/c^3/x^(5/2)+2/7*B*(c*x^2+b*x)^(7/2)/c^3/x^(7/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.59

$$\int \sqrt{x}(A+Bx)\sqrt{bx+cx^2}\,dx=\frac{2(x(b+cx))^{3/2}\left(8b^2B+3c^2x(7A+5Bx)-2bc(7A+6Bx)\right)}{105c^3x^{3/2}}$$

input
```
Integrate[Sqrt[x]*(A + B*x)*Sqrt[b*x + c*x^2],x]
```

output
```
(2*(x*(b + c*x))^(3/2)*(8*b^2*B + 3*c^2*x*(7*A + 5*B*x) - 2*b*c*(7*A + 6*B
*x)))/(105*c^3*x^(3/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1221, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(A+Bx)\sqrt{bx+cx^2}\,dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{2B\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}-\frac{(4bB-7Ac)\int \sqrt{x}\sqrt{cx^2+bx}dx}{7c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2B\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}-\frac{(4bB-7Ac)\left(\frac{2\left(bx+cx^2\right)^{3/2}}{5c\sqrt{x}}-\frac{2b\int \frac{\sqrt{cx^2+bx}}{\sqrt{x}}dx}{5c}\right)}{7c}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2B\sqrt{x}\left(bx+cx^2\right)^{3/2}}{7c}-\frac{\left(\frac{2\left(bx+cx^2\right)^{3/2}}{5c\sqrt{x}}-\frac{4b\left(bx+cx^2\right)^{3/2}}{15c^2x^{3/2}}\right)(4bB-7Ac)}{7c}$$

input
```
Int[Sqrt[x]*(A + B*x)*Sqrt[b*x + c*x^2],x]
```

output
```
(2*B*Sqrt[x]*(b*x + c*x^2)^(3/2))/(7*c) - ((4*b*B - 7*A*c)*((-4*b*(b*x + c
*x^2)^(3/2))/(15*c^2*x^(3/2)) + (2*(b*x + c*x^2)^(3/2))/(5*c*Sqrt[x])))/(7
*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{2(cx+b)\left(-15B\,c^{2}x^{2}-21A\,c^{2}x+12Bbcx+14Abc-8B\,b^{2}\right)\sqrt{x(cx+b)}}{105c^{3}\sqrt{x}}$	57
gosper	$-\frac{2(cx+b)\left(-15B\,c^{2}x^{2}-21A\,c^{2}x+12Bbcx+14Abc-8B\,b^{2}\right)\sqrt{c\,x^{2}+bx}}{105c^{3}\sqrt{x}}$	59
orering	$-\frac{2(cx+b)\left(-15B\,c^{2}x^{2}-21A\,c^{2}x+12Bbcx+14Abc-8B\,b^{2}\right)\sqrt{c\,x^{2}+bx}}{105c^{3}\sqrt{x}}$	59
risch	$-\frac{2(cx+b)\sqrt{x}\,\left(-15B\,c^{3}x^{3}-21A\,c^{3}x^{2}-3Bb\,c^{2}x^{2}-7Ab\,c^{2}x+4B\,b^{2}cx+14A\,b^{2}c-8B\,b^{3}\right)}{105\sqrt{x(cx+b)}\,c^{3}}$	81

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(c*x+b)*(-15*B*c^2*x^2-21*A*c^2*x+12*B*b*c*x+14*A*b*c-8*B*b^2)*(x*(
c*x+b))^(1/2)/c^3/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.82

$$\int \sqrt{x}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2\,(15\,Bc^3x^3+8\,Bb^3-14\,Ab^2c+3\,(Bbc^2+7\,Ac^3)x^2-(4\,Bb^2c-7\,Abc^2)x)\sqrt{cx^2+bx}}{105\,c^3\sqrt{x}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*B*c^3*x^3 + 8*B*b^3 - 14*A*b^2*c + 3*(B*b*c^2 + 7*A*c^3)*x^2 - (
4*B*b^2*c - 7*A*b*c^2)*x)*sqrt(c*x^2 + b*x)/(c^3*sqrt(x))
```

Sympy [F]

$$\int \sqrt{x}(A+Bx)\sqrt{bx+cx^2}\,dx = \int \sqrt{x}\sqrt{x\,(b+cx)}(A+Bx)\,dx$$

input
```
integrate(x**(1/2)*(B*x+A)*(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(sqrt(x)*sqrt(x*(b + c*x))*(A + B*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

$$\int \sqrt{x}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2\,(3\,c^2x^2+bcx-2\,b^2)\sqrt{cx+b}A}{15\,c^2} + \frac{2\,(15\,c^3x^3+3\,bc^2x^2-4\,b^2cx+8\,b^3)\sqrt{cx+b}B}{105\,c^3}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*c^2*x^2 + b*c*x - 2*b^2)*sqrt(c*x + b)*A/c^2 + 2/105*(15*c^3*x^3 +
 3*b*c^2*x^2 - 4*b^2*c*x + 8*b^3)*sqrt(c*x + b)*B/c^3
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.68

$$\int \sqrt{x}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2\left(3\,(cx+b)^{\frac{5}{2}}-5\,(cx+b)^{\frac{3}{2}}b\right)A}{15\,c^2} + \frac{2\left(15\,(cx+b)^{\frac{7}{2}}-42\,(cx+b)^{\frac{5}{2}}b+35\,(cx+b)^{\frac{3}{2}}b^2\right)B}{105\,c^3}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
2/15*(3*(c*x + b)^(5/2) - 5*(c*x + b)^(3/2)*b)*A/c^2 + 2/105*(15*(c*x + b)
^(7/2) - 42*(c*x + b)^(5/2)*b + 35*(c*x + b)^(3/2)*b^2)*B/c^3
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(A+Bx)\sqrt{bx+cx^2}\,dx = \int \sqrt{x}\,\sqrt{c\,x^2+b\,x}\,(A+B\,x)\,dx$$

input
```
int(x^(1/2)*(b*x + c*x^2)^(1/2)*(A + B*x),x)
```

output
```
int(x^(1/2)*(b*x + c*x^2)^(1/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int \sqrt{x}(A+Bx)\sqrt{bx+cx^2}\,dx = \frac{2\sqrt{cx+b}\,(15b\,c^3x^3+21a\,c^3x^2+3b^2c^2x^2+7ab\,c^2x-4b^3cx-14a\,b^2c+8b^4)}{105c^3}$$

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x)^(1/2),x)
```

output
```
(2*sqrt(b + c*x)*( - 14*a*b**2*c + 7*a*b*c**2*x + 21*a*c**3*x**2 + 8*b**4
- 4*b**3*c*x + 3*b**2*c**2*x**2 + 15*b*c**3*x**3))/(105*c**3)
```

3.175 $\int \frac{(A+Bx)\sqrt{bx+cx^2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{\sqrt{x}}\,dx = -\frac{2(bB-Ac)\left(bx+cx^2\right)^{3/2}}{3c^2x^{3/2}}+\frac{2B(bx+cx^2)^{5/2}}{5c^2x^{5/2}}$$

output
```
-2/3*(-A*c+B*b)*(c*x^2+b*x)^(3/2)/c^2/x^(3/2)+2/5*B*(c*x^2+b*x)^(5/2)/c^2/
x^(5/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{\sqrt{x}}\,dx = \frac{2(x(b+cx))^{3/2}(-2bB+5Ac+3Bcx)}{15c^2x^{3/2}}$$

input
```
Integrate[((A + B*x)*Sqrt[b*x + c*x^2])/Sqrt[x],x]
```

output
```
(2*(x*(b + c*x))^(3/2)*(-2*b*B + 5*A*c + 3*B*c*x))/(15*c^2*x^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{2B(bx+cx^2)^{3/2}}{5c\sqrt{x}}-\frac{(2bB-5Ac)\int \frac{\sqrt{cx^2+bx}}{\sqrt{x}}dx}{5c}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2B(bx+cx^2)^{3/2}}{5c\sqrt{x}}-\frac{2(bx+cx^2)^{3/2}(2bB-5Ac)}{15c^2x^{3/2}}$$

input
```
Int[((A + B*x)*Sqrt[b*x + c*x^2])/Sqrt[x],x]
```

output
```
(-2*(2*b*B - 5*A*c)*(b*x + c*x^2)^(3/2))/(15*c^2*x^(3/2)) + (2*B*(b*x + c*
x^2)^(3/2))/(5*c*Sqrt[x])
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

method	result	size
default	$\frac{2(cx+b)(3Bcx+5Ac-2Bb)\sqrt{x(cx+b)}}{15c^{2}\sqrt{x}}$	37
gosper	$\frac{2(cx+b)(3Bcx+5Ac-2Bb)\sqrt{c\,x^{2}+bx}}{15c^{2}\sqrt{x}}$	39
orering	$\frac{2(cx+b)(3Bcx+5Ac-2Bb)\sqrt{c\,x^{2}+bx}}{15c^{2}\sqrt{x}}$	39
risch	$\frac{2(cx+b)\sqrt{x}\,(3B\,c^{2}x^{2}+5A\,c^{2}x+Bbcx+5Abc-2B\,b^{2})}{15\sqrt{x(cx+b)}\,c^{2}}$	56

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(c*x+b)*(3*B*c*x+5*A*c-2*B*b)*(x*(c*x+b))^(1/2)/c^2/x^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{\sqrt{x}}\,dx = \frac{2\,(3\,Bc^2x^2-2\,Bb^2+5\,Abc+(Bbc+5\,Ac^2)x)\sqrt{cx^2+bx}}{15\,c^2\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*c^2*x^2 - 2*B*b^2 + 5*A*b*c + (B*b*c + 5*A*c^2)*x)*sqrt(c*x^2 +
b*x)/(c^2*sqrt(x))
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{\sqrt{x}}\,dx = \int \frac{\sqrt{x\,(b+cx)}(A+Bx)}{\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2)/x**(1/2),x)
```

output
```
Integral(sqrt(x*(b + c*x))*(A + B*x)/sqrt(x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{\sqrt{x}}\,dx = \frac{2\,(cx+b)^{\frac{3}{2}}A}{3\,c} + \frac{2\,(3\,c^2x^2+bcx-2\,b^2)\sqrt{cx+b}B}{15\,c^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(1/2),x, algorithm="maxima")
```

output
```
2/3*(c*x + b)^(3/2)*A/c + 2/15*(3*c^2*x^2 + b*c*x - 2*b^2)*sqrt(c*x + b)*B
/c^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(48) = 96.

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{\sqrt{x}}\,dx$$

$$= \frac{2\left(15\sqrt{cx+b}Ab + 5\left((cx+b)^{\frac{3}{2}} - 3\sqrt{cx+b}b\right)A + \frac{5\left((cx+b)^{\frac{3}{2}}-3\sqrt{cx+b}b\right)Bb}{c} + \frac{\left(3\,(cx+b)^{\frac{5}{2}}-10\,(cx+b)^{\frac{3}{2}}b+15\sqrt{cx+b}\right.}{c}\right.}{15\,c}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(1/2),x, algorithm="giac")
```

output
```
2/15*(15*sqrt(c*x + b)*A*b + 5*((c*x + b)^(3/2) - 3*sqrt(c*x + b)*b)*A + 5
*((c*x + b)^(3/2) - 3*sqrt(c*x + b)*b)*B*b/c + (3*(c*x + b)^(5/2) - 10*(c*
x + b)^(3/2)*b + 15*sqrt(c*x + b)*b^2)*B/c)/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{\sqrt{x}}\,dx = \int \frac{\sqrt{c\,x^2+b\,x}\,(A+B\,x)}{\sqrt{x}}\,dx$$

input
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^(1/2),x)
```

output
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{\sqrt{x}}\,dx = \frac{2\sqrt{cx+b}\,(3b\,c^2x^2+5a\,c^2x+b^2cx+5abc-2b^3)}{15c^2}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^(1/2),x)
```

output
```
(2*sqrt(b + c*x)*(5*a*b*c + 5*a*c**2*x - 2*b**3 + b**2*c*x + 3*b*c**2*x**2
))/(15*c**2)
```

3.176 $\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{3/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 81

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{3/2}} \, dx = \frac{2A\sqrt{bx+cx^2}}{\sqrt{x}} + \frac{2B(bx+cx^2)^{3/2}}{3cx^{3/2}} - 2A\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)$$

output
```
2*A*(c*x^2+b*x)^(1/2)/x^(1/2)+2/3*B*(c*x^2+b*x)^(3/2)/c/x^(3/2)-2*A*b^(1/2
)*arctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1/2))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{3/2}} \, dx = \frac{2\sqrt{x}\sqrt{b+cx}\left(\sqrt{b+cx}(bB+3Ac+Bcx) - 3A\sqrt{b}c\,\text{arctanh}\left(\frac{\sqrt{b+cx}}{\sqrt{b}}\right)\right)}{3c\sqrt{x(b+cx)}}$$

input
```
Integrate[((A + B*x)*Sqrt[b*x + c*x^2])/x^(3/2),x]
```

output
```
(2*Sqrt[x]*Sqrt[b + c*x]*(Sqrt[b + c*x]*(b*B + 3*A*c + B*c*x) - 3*A*Sqrt[b
]*c*ArcTanh[Sqrt[b + c*x]/Sqrt[b]]))/(3*c*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1221, 1131, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{3/2}}\,dx$$

$$\Big\downarrow \text{1221}$$

$$A\int \frac{\sqrt{cx^2+bx}}{x^{3/2}}dx+\frac{2B\left(bx+cx^2\right)^{3/2}}{3cx^{3/2}}$$

$$\Big\downarrow \text{1131}$$

$$A\left(b\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)+\frac{2B\left(bx+cx^2\right)^{3/2}}{3cx^{3/2}}$$

$$\Big\downarrow \text{1136}$$

$$A\left(2b\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)+\frac{2B\left(bx+cx^2\right)^{3/2}}{3cx^{3/2}}$$

$$\Big\downarrow \text{220}$$

$$A\left(\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}-2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)\right)+\frac{2B\left(bx+cx^2\right)^{3/2}}{3cx^{3/2}}$$

input
```
Int[((A + B*x)*Sqrt[b*x + c*x^2])/x^(3/2),x]
```

output
```
(2*B*(b*x + c*x^2)^(3/2))/(3*c*x^(3/2)) + A*((2*Sqrt[b*x + c*x^2])/Sqrt[x]
 - 2*Sqrt[b]*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.18 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{2\sqrt{x(cx+b)}\left(3A\sqrt{b}\,c\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)-Bcx\sqrt{cx+b}-3Ac\sqrt{cx+b}-Bb\sqrt{cx+b}\right)}{3\sqrt{x}\,\sqrt{cx+b}\,c}$	79

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(x*(c*x+b))^(1/2)*(3*A*b^(1/2)*c*arctanh((c*x+b)^(1/2)/b^(1/2))-B*c*x
*(c*x+b)^(1/2)-3*A*c*(c*x+b)^(1/2)-B*b*(c*x+b)^(1/2))/x^(1/2)/(c*x+b)^(1/2
)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.86

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{3/2}}\,dx = \left[\frac{3\,A\sqrt{b}cx\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,(Bcx+Bb+3\,Ac)\sqrt{cx^2+bx}\sqrt{\cdot}}{3\,cx}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(3/2),x, algorithm="fricas")
```

output
```
[1/3*(3*A*sqrt(b)*c*x*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b)*sq
rt(x))/x^2) + 2*(B*c*x + B*b + 3*A*c)*sqrt(c*x^2 + b*x)*sqrt(x))/(c*x), 2/
3*(3*A*sqrt(-b)*c*x*arctan(sqrt(c*x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) + (B*c*
x + B*b + 3*A*c)*sqrt(c*x^2 + b*x)*sqrt(x))/(c*x)]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{3/2}}\,dx = \int \frac{\sqrt{x\,(b+cx)}(A+Bx)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2)/x**(3/2),x)
```

output
```
Integral(sqrt(x*(b + c*x))*(A + B*x)/x**(3/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{3/2}}\,dx = \int \frac{\sqrt{cx^2+bx}(Bx+A)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(3/2),x, algorithm="maxima")
```

output
```
A*integrate(sqrt(c*x + b)/x, x) + 2/3*(B*c*x + B*b)*sqrt(c*x + b)/c
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{3/2}}\,dx = \frac{2\,Ab\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + \frac{2\left((cx+b)^{\frac{3}{2}}Bc^2+3\sqrt{cx+b}Ac^3\right)}{3\,c^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(3/2),x, algorithm="giac")
```

output
```
2*A*b*arctan(sqrt(c*x + b)/sqrt(-b))/sqrt(-b) + 2/3*((c*x + b)^(3/2)*B*c^2
 + 3*sqrt(c*x + b)*A*c^3)/c^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{3/2}}\,dx = \int \frac{\sqrt{c\,x^2+b\,x}\,(A+B\,x)}{x^{3/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^(3/2),x)
```

output
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{3/2}}\,dx = \frac{6\sqrt{cx+b}\,ac+2\sqrt{cx+b}\,b^2+2\sqrt{cx+b}\,bcx+3\sqrt{b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)ac-3}{3c}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^(3/2),x)
```

output
```
(6*sqrt(b + c*x)*a*c + 2*sqrt(b + c*x)*b**2 + 2*sqrt(b + c*x)*b*c*x + 3*sq
rt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c - 3*sqrt(b)*log(sqrt(b + c*x) + sqr
t(b))*a*c)/(3*c)
```

3.177 $\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{5/2}}\,dx = -\frac{A\sqrt{bx+cx^2}}{x^{3/2}} + \frac{2B\sqrt{bx+cx^2}}{\sqrt{x}} - \frac{(2bB+Ac)\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}$$

output
```
-A*(c*x^2+b*x)^(1/2)/x^(3/2)+2*B*(c*x^2+b*x)^(1/2)/x^(1/2)-(A*c+2*B*b)*arc
tanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{5/2}}\,dx = -\frac{\sqrt{x(b+cx)}\left(\sqrt{b}(A-2Bx)\sqrt{b+cx}+(2bB+Ac)x\,\text{arctanh}\left(\frac{\sqrt{b+cx}}{\sqrt{b}}\right)\right)}{\sqrt{b}x^{3/2}\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*Sqrt[b*x + c*x^2])/x^(5/2),x]
```

output
```
-((Sqrt[x*(b + c*x)]*(Sqrt[b]*(A - 2*B*x)*Sqrt[b + c*x] + (2*b*B + A*c)*x*
ArcTanh[Sqrt[b + c*x]/Sqrt[b]]))/(Sqrt[b]*x^(3/2)*Sqrt[b + c*x]))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1220, 1131, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{5/2}}\,dx$$

$$\Big\downarrow \ 1220$$

$$\frac{(Ac+2bB)\int \frac{\sqrt{cx^2+bx}}{x^{3/2}}dx}{2b}-\frac{A\left(bx+cx^2\right)^{3/2}}{bx^{5/2}}$$

$$\Big\downarrow \ 1131$$

$$\frac{(Ac+2bB)\left(b\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)}{2b}-\frac{A\left(bx+cx^2\right)^{3/2}}{bx^{5/2}}$$

$$\Big\downarrow \ 1136$$

$$\frac{(Ac+2bB)\left(2b\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)}{2b}-\frac{A\left(bx+cx^2\right)^{3/2}}{bx^{5/2}}$$

$$\Big\downarrow \ 220$$

$$\frac{(Ac+2bB)\left(\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}-2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)\right)}{2b}-\frac{A\left(bx+cx^2\right)^{3/2}}{bx^{5/2}}$$

input
```
Int[((A + B*x)*Sqrt[b*x + c*x^2])/x^(5/2),x]
```

output
```
-((A*(b*x + c*x^2)^(3/2))/(b*x^(5/2))) + ((2*b*B + A*c)*((2*Sqrt[b*x + c*x
^2])/Sqrt[x] - 2*Sqrt[b]*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])]))/(2
*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

method	result	size
risch	$-\frac{A(cx+b)}{\sqrt{x}\,\sqrt{x(cx+b)}}+\frac{\left(2B\sqrt{cx+b}-\frac{(Ac+2Bb)\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)}{\sqrt{b}}\right)\sqrt{cx+b}\,\sqrt{x}}{\sqrt{x(cx+b)}}$	78
default	$\frac{\left(-A\ \mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)cx+2B\sqrt{cx+b}\,x\sqrt{b}-2B\ \mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)bx-A\sqrt{cx+b}\,\sqrt{b}\right)\sqrt{x(cx+b)}}{x^{\frac{3}{2}}\sqrt{cx+b}\,\sqrt{b}}$	86

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-A*(c*x+b)/x^(1/2)/(x*(c*x+b))^(1/2)+(2*B*(c*x+b)^(1/2)-(A*c+2*B*b)/b^(1/2
)*arctanh((c*x+b)^(1/2)/b^(1/2)))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.93

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{5/2}}\,dx = \left[\frac{(2\,Bb+Ac)\sqrt{b}x^2\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,(2\,Bbx-Ab)\sqrt{cx^2+bx}}{2\,bx^2}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/2*((2*B*b + A*c)*sqrt(b)*x^2*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*
sqrt(b)*sqrt(x))/x^2) + 2*(2*B*b*x - A*b)*sqrt(c*x^2 + b*x)*sqrt(x))/(b*x^
2), ((2*B*b + A*c)*sqrt(-b)*x^2*arctan(sqrt(c*x^2 + b*x)*sqrt(-b)/(b*sqrt(
x))) + (2*B*b*x - A*b)*sqrt(c*x^2 + b*x)*sqrt(x))/(b*x^2)]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{5/2}}\,dx = \int \frac{\sqrt{x\,(b+cx)}(A+Bx)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2)/x**(5/2),x)
```

output
```
Integral(sqrt(x*(b + c*x))*(A + B*x)/x**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{5/2}}\,dx = \int \frac{\sqrt{cx^2+bx}(Bx+A)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x)*(B*x + A)/x^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{5/2}}\,dx = c\left(\frac{2\sqrt{cx+b}B}{c} + \frac{(2\,Bb+Ac)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}c} - \frac{\sqrt{cx+b}A}{cx}\right)$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(5/2),x, algorithm="giac")
```

output
```
c*(2*sqrt(c*x + b)*B/c + (2*B*b + A*c)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqr
t(-b)*c) - sqrt(c*x + b)*A/(c*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{5/2}}\,dx=\int \frac{\sqrt{c\,x^2+b\,x}\,(A+B\,x)}{x^{5/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^(5/2),x)
```

output
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.25

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{5/2}}\,dx=\frac{-2\sqrt{cx+b}\,ab+4\sqrt{cx+b}\,b^2x+\sqrt{b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)acx+2\sqrt{b}\log\left(\sqrt{cx\right.}}{2\ldots}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^(5/2),x)
```

output
```
( - 2*sqrt(b + c*x)*a*b + 4*sqrt(b + c*x)*b**2*x + sqrt(b)*log(sqrt(b + c*
x) - sqrt(b))*a*c*x + 2*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*b**2*x - sqrt
(b)*log(sqrt(b + c*x) + sqrt(b))*a*c*x - 2*sqrt(b)*log(sqrt(b + c*x) + sqr
t(b))*b**2*x)/(2*b*x)
```

3.178 $\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 101

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{7/2}}\,dx = -\frac{A\sqrt{bx+cx^2}}{2x^{5/2}} - \frac{(4bB+Ac)\sqrt{bx+cx^2}}{4bx^{3/2}} - \frac{c(4bB-Ac)\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4b^{3/2}}$$

output
```
-1/2*A*(c*x^2+b*x)^(1/2)/x^(5/2)-1/4*(A*c+4*B*b)*(c*x^2+b*x)^(1/2)/b/x^(3/
2)-1/4*c*(-A*c+4*B*b)*arctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.93

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{7/2}}\,dx = -\frac{\sqrt{x(b+cx)}\left(\sqrt{b}\sqrt{b+cx}(2Ab+4bBx+Acx)+c(4bB-Ac)x^2\text{arctanh}\left(\frac{\sqrt{b+cx}}{\sqrt{b}}\right)\right)}{4b^{3/2}x^{5/2}\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*Sqrt[b*x + c*x^2])/x^(7/2),x]
```

output
```
-1/4*(Sqrt[x*(b + c*x)]*(Sqrt[b]*Sqrt[b + c*x]*(2*A*b + 4*b*B*x + A*c*x) +
 c*(4*b*B - A*c)*x^2*ArcTanh[Sqrt[b + c*x]/Sqrt[b]]))/(b^(3/2)*x^(5/2)*Sqr
t[b + c*x])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1220, 1130, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{7/2}}\,dx$$

$$\Big\downarrow \ 1220$$

$$\frac{(4bB-Ac)\int \frac{\sqrt{cx^2+bx}}{x^{5/2}}dx}{4b}-\frac{A\left(bx+cx^2\right)^{3/2}}{2bx^{7/2}}$$

$$\Big\downarrow \ 1130$$

$$\frac{(4bB-Ac)\left(\frac{1}{2}c\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx-\frac{\sqrt{bx+cx^2}}{x^{3/2}}\right)}{4b}-\frac{A\left(bx+cx^2\right)^{3/2}}{2bx^{7/2}}$$

$$\Big\downarrow \ 1136$$

$$\frac{(4bB-Ac)\left(c\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}-\frac{\sqrt{bx+cx^2}}{x^{3/2}}\right)}{4b}-\frac{A\left(bx+cx^2\right)^{3/2}}{2bx^{7/2}}$$

$$\Big\downarrow \ 220$$

$$\frac{(4bB-Ac)\left(-\frac{c\,\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}-\frac{\sqrt{bx+cx^2}}{x^{3/2}}\right)}{4b}-\frac{A\left(bx+cx^2\right)^{3/2}}{2bx^{7/2}}$$

input
```
Int[((A + B*x)*Sqrt[b*x + c*x^2])/x^(7/2),x]
```

output
```
-1/2*(A*(b*x + c*x^2)^(3/2))/(b*x^(7/2)) + ((4*b*B - A*c)*(-(Sqrt[b*x + c*
x^2]/x^(3/2)) - (c*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/Sqrt[b]))
/(4*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.82

method	result	size
risch	$-\frac{(cx+b)(Acx+4Bbx+2Ab)}{4x^{\frac{3}{2}} b\sqrt{x(cx+b)}}+\frac{c(Ac-4Bb)\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)\sqrt{cx+b}\,\sqrt{x}}{4b^{\frac{3}{2}}\sqrt{x(cx+b)}}$	83
default	$\frac{\sqrt{x(cx+b)}\left(A\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^2x^2-4B\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)bc\,x^2-Acx\sqrt{cx+b}\,\sqrt{b}-4B\,b^{\frac{3}{2}}x\sqrt{cx+b}-2A\,b^{\frac{3}{2}}\sqrt{cx+b}\right)}{4b^{\frac{3}{2}}x^{\frac{5}{2}}\sqrt{cx+b}}$	108

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x+b)*(A*c*x+4*B*b*x+2*A*b)/x^(3/2)/b/(x*(c*x+b))^(1/2)+1/4*c*(A*c-
4*B*b)/b^(3/2)*arctanh((c*x+b)^(1/2)/b^(1/2))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*
x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.88

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{7/2}}\,dx = \left[-\frac{(4\,Bbc-Ac^2)\sqrt{b}x^3\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,(2\,Ab^2+(4\,Bb^2+A}{8\,b^2x^3}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(7/2),x, algorithm="fricas")
```

output
```
[-1/8*((4*B*b*c - A*c^2)*sqrt(b)*x^3*log(-(c*x^2 + 2*b*x + 2*sqrt(c*x^2 +
b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(2*A*b^2 + (4*B*b^2 + A*b*c)*x)*sqrt(c*x^2
+ b*x)*sqrt(x))/(b^2*x^3), 1/4*((4*B*b*c - A*c^2)*sqrt(-b)*x^3*arctan(sqrt
(c*x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) - (2*A*b^2 + (4*B*b^2 + A*b*c)*x)*sqrt
(c*x^2 + b*x)*sqrt(x))/(b^2*x^3)]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{7/2}}\,dx = \int \frac{\sqrt{x\,(b+cx)}(A+Bx)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2)/x**(7/2),x)
```

output
```
Integral(sqrt(x*(b + c*x))*(A + B*x)/x**(7/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{7/2}}\,dx = \int \frac{\sqrt{cx^2+bx}(Bx+A)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x)*(B*x + A)/x^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.09

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{7/2}}\,dx = \frac{\frac{(4\,Bbc^2-Ac^3)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} - \frac{4\,(cx+b)^{\frac{3}{2}}Bbc^2-4\,\sqrt{cx+b}Bb^2c^2+(cx+b)^{\frac{3}{2}}Ac^3+\sqrt{cx+b}Abc^3}{bc^2x^2}}{4\,c}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(7/2),x, algorithm="giac")
```

output
```
1/4*((4*B*b*c^2 - A*c^3)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b) - (4*
(c*x + b)^(3/2)*B*b*c^2 - 4*sqrt(c*x + b)*B*b^2*c^2 + (c*x + b)^(3/2)*A*c^
3 + sqrt(c*x + b)*A*b*c^3)/(b*c^2*x^2))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{7/2}}\,dx=\int \frac{\sqrt{c\,x^2+b\,x}\,(A+B\,x)}{x^{7/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^(7/2),x)
```

output
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.31

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{7/2}}\,dx=\frac{-4\sqrt{cx+b}\,a\,b^2-2\sqrt{cx+b}\,abcx-8\sqrt{cx+b}\,b^3x-\sqrt{b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)a}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^(7/2),x)
```

output
```
( - 4*sqrt(b + c*x)*a*b**2 - 2*sqrt(b + c*x)*a*b*c*x - 8*sqrt(b + c*x)*b**
3*x - sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c**2*x**2 + 4*sqrt(b)*log(sqr
t(b + c*x) - sqrt(b))*b**2*c*x**2 + sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*a
*c**2*x**2 - 4*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*b**2*c*x**2)/(8*b**2*x
**2)
```

3.179 $\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 138

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{9/2}}\,dx = -\frac{A\sqrt{bx+cx^2}}{3x^{7/2}}-\frac{(6bB+Ac)\sqrt{bx+cx^2}}{12bx^{5/2}} -\frac{c(2bB-Ac)\sqrt{bx+cx^2}}{8b^2x^{3/2}}+\frac{c^2(2bB-Ac)\operatorname{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{5/2}}$$

output
```
-1/3*A*(c*x^2+b*x)^(1/2)/x^(7/2)-1/12*(A*c+6*B*b)*(c*x^2+b*x)^(1/2)/b/x^(5
/2)-1/8*c*(-A*c+2*B*b)*(c*x^2+b*x)^(1/2)/b^2/x^(3/2)+1/8*c^2*(-A*c+2*B*b)*
arctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{9/2}}\,dx = -\frac{\sqrt{x(b+cx)}\left(\sqrt{b}\sqrt{b+cx}(6bBx(2b+cx)+A(8b^2+2bcx-3c^2x^2))+3c^2(-2bB+Ac)x^3\operatorname{arctanh}\left(\frac{\sqrt{b+cx}}{\sqrt{b}}\right)\right)}{24b^{5/2}x^{7/2}\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*Sqrt[b*x + c*x^2])/x^(9/2),x]
```

output
```
-1/24*(Sqrt[x*(b + c*x)]*(Sqrt[b]*Sqrt[b + c*x]*(6*b*B*x*(2*b + c*x) + A*(
8*b^2 + 2*b*c*x - 3*c^2*x^2)) + 3*c^2*(-2*b*B + A*c)*x^3*ArcTanh[Sqrt[b +
c*x]/Sqrt[b]]))/(b^(5/2)*x^(7/2)*Sqrt[b + c*x])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1220, 1130, 1135, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{9/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(2bB-Ac)\int \frac{\sqrt{cx^2+bx}}{x^{7/2}}dx}{2b}-\frac{A\left(bx+cx^2\right)^{3/2}}{3bx^{9/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(2bB-Ac)\left(\frac{1}{4}c\int \frac{1}{x^{3/2}\sqrt{cx^2+bx}}dx-\frac{\sqrt{bx+cx^2}}{2x^{5/2}}\right)}{2b}-\frac{A\left(bx+cx^2\right)^{3/2}}{3bx^{9/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{(2bB-Ac)\left(\frac{1}{4}c\left(-\frac{c\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{2b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)-\frac{\sqrt{bx+cx^2}}{2x^{5/2}}\right)}{2b}-\frac{A\left(bx+cx^2\right)^{3/2}}{3bx^{9/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(2bB-Ac)\left(\frac{1}{4}c\left(-\frac{c\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)-\frac{\sqrt{bx+cx^2}}{2x^{5/2}}\right)}{2b}-\frac{A\left(bx+cx^2\right)^{3/2}}{3bx^{9/2}}$$

$$\Big\downarrow \text{220}$$

$$\frac{(2bB-Ac)\left(\frac{1}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)-\frac{\sqrt{bx+cx^2}}{2x^{5/2}}\right)}{2b}-\frac{A\left(bx+cx^2\right)^{3/2}}{3bx^{9/2}}$$

input
```
Int[((A + B*x)*Sqrt[b*x + c*x^2])/x^(9/2),x]
```

output
```
-1/3*(A*(b*x + c*x^2)^(3/2))/(b*x^(9/2)) + ((2*b*B - A*c)*(-1/2*Sqrt[b*x +
 c*x^2]/x^(5/2) + (c*(-(Sqrt[b*x + c*x^2]/(b*x^(3/2))) + (c*ArcTanh[Sqrt[b
*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/b^(3/2)))/4))/(2*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{(cx+b)\left(-3A\,c^2x^2+6x^2Bbc+2Abcx+12xB\,b^2+8b^2A\right)}{24x^{\frac{5}{2}}b^2\sqrt{x(cx+b)}}-\frac{c^2(Ac-2Bb)\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)\sqrt{cx+b}\,\sqrt{x}}{8b^{\frac{5}{2}}\sqrt{x(cx+b)}}$
default	$-\frac{\sqrt{x(cx+b)}\,\left(3A\ \mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^3x^3-6B\ \mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)b\,c^2x^3-3A\,c^2x^2\sqrt{cx+b}\,\sqrt{b}+6B\,b^{\frac{3}{2}}\,c\,x^2\sqrt{cx+b}+2A\,b^{\frac{3}{2}}\,cx\sqrt{cx+b}+1}{24b^{\frac{5}{2}}\,x^{\frac{7}{2}}\sqrt{cx+b}}$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x+b)*(-3*A*c^2*x^2+6*B*b*c*x^2+2*A*b*c*x+12*B*b^2*x+8*A*b^2)/x^(5
/2)/b^2/(x*(c*x+b))^(1/2)-1/8*c^2*(A*c-2*B*b)/b^(5/2)*arctanh((c*x+b)^(1/2
)/b^(1/2))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.75

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{9/2}}\,dx=\left[-\frac{3\,(2\,Bbc^2-Ac^3)\sqrt{b}x^4\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,(8\,Ab^3+3\,(2\,Bb^2}{48\,b^3x^4}\right.$$

$$-\frac{3\,(2\,Bbc^2-Ac^3)\sqrt{-b}x^4\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-b}}{b\sqrt{x}}\right)+(8\,Ab^3+3\,(2\,Bb^2c-Abc^2)x^2+2\,(6\,Bb^3+Ab^2c)x)\sqrt{cx^2}}{24\,b^3x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(9/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(2*B*b*c^2 - A*c^3)*sqrt(b)*x^4*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x
^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(8*A*b^3 + 3*(2*B*b^2*c - A*b*c^2)*x^2
 + 2*(6*B*b^3 + A*b^2*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^3*x^4), -1/24*(3
*(2*B*b*c^2 - A*c^3)*sqrt(-b)*x^4*arctan(sqrt(c*x^2 + b*x)*sqrt(-b)/(b*sqr
t(x))) + (8*A*b^3 + 3*(2*B*b^2*c - A*b*c^2)*x^2 + 2*(6*B*b^3 + A*b^2*c)*x)
*sqrt(c*x^2 + b*x)*sqrt(x))/(b^3*x^4)]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{9/2}}\,dx = \int \frac{\sqrt{x\,(b+cx)}(A+Bx)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2)/x**(9/2),x)
```

output
```
Integral(sqrt(x*(b + c*x))*(A + B*x)/x**(9/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{9/2}}\,dx = \int \frac{\sqrt{cx^2+bx}(Bx+A)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x)*(B*x + A)/x^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.81 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{9/2}}\,dx =$$
$$-\frac{1}{24}c^3\left(\frac{3\,(2\,Bb-Ac)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2c}+\frac{6\,(cx+b)^{\frac{5}{2}}Bb-6\,\sqrt{cx+b}Bb^3-3\,(cx+b)^{\frac{5}{2}}Ac+8\,(cx+b)^{\frac{3}{2}}Ab}{b^2c^4x^3}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(9/2),x, algorithm="giac")
```

output
```
-1/24*c^3*(3*(2*B*b - A*c)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^2*c)
 + (6*(c*x + b)^(5/2)*B*b - 6*sqrt(c*x + b)*B*b^3 - 3*(c*x + b)^(5/2)*A*c
+ 8*(c*x + b)^(3/2)*A*b*c + 3*sqrt(c*x + b)*A*b^2*c)/(b^2*c^4*x^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{9/2}}\,dx = \int \frac{\sqrt{c\,x^2+b\,x}\,(A+B\,x)}{x^{9/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^(9/2),x)
```

output
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{9/2}}\,dx = \frac{-16\sqrt{cx+b}\,a\,b^3-4\sqrt{cx+b}\,a\,b^2cx+6\sqrt{cx+b}\,ab\,c^2x^2-24\sqrt{cx+b}\,b^4x-1}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^(9/2),x)
```

output
```
( - 16*sqrt(b + c*x)*a*b**3 - 4*sqrt(b + c*x)*a*b**2*c*x + 6*sqrt(b + c*x)
*a*b*c**2*x**2 - 24*sqrt(b + c*x)*b**4*x - 12*sqrt(b + c*x)*b**3*c*x**2 +
3*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c**3*x**3 - 6*sqrt(b)*log(sqrt(b
+ c*x) - sqrt(b))*b**2*c**2*x**3 - 3*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*
a*c**3*x**3 + 6*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*b**2*c**2*x**3)/(48*b
**3*x**3)
```

3.180 $\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 175

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{11/2}}\,dx = -\frac{A\sqrt{bx+cx^2}}{4x^{9/2}} - \frac{(8bB+Ac)\sqrt{bx+cx^2}}{24bx^{7/2}} - \frac{c(8bB-5Ac)\sqrt{bx+cx^2}}{96b^2x^{5/2}} + \frac{c^2(8bB-5Ac)\sqrt{bx+cx^2}}{64b^3x^{3/2}} - \frac{c^3(8bB-5Ac)\operatorname{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{64b^{7/2}}$$

output
```
-1/4*A*(c*x^2+b*x)^(1/2)/x^(9/2)-1/24*(A*c+8*B*b)*(c*x^2+b*x)^(1/2)/b/x^(7
/2)-1/96*c*(-5*A*c+8*B*b)*(c*x^2+b*x)^(1/2)/b^2/x^(5/2)+1/64*c^2*(-5*A*c+8
*B*b)*(c*x^2+b*x)^(1/2)/b^3/x^(3/2)-1/64*c^3*(-5*A*c+8*B*b)*arctanh((c*x^2
+b*x)^(1/2)/b^(1/2)/x^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{11/2}}\,dx =$$
$$-\frac{\sqrt{x(b+cx)}\left(\sqrt{b}\sqrt{b+cx}(8bBx(8b^2+2bcx-3c^2x^2)+A(48b^3+8b^2cx-10bc^2x^2+15c^3x^3))+3c^3(8bB\right.}{192b^{7/2}x^{9/2}\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*Sqrt[b*x + c*x^2])/x^(11/2),x]
```

output
```
-1/192*(Sqrt[x*(b + c*x)]*(Sqrt[b]*Sqrt[b + c*x]*(8*b*B*x*(8*b^2 + 2*b*c*x
 - 3*c^2*x^2) + A*(48*b^3 + 8*b^2*c*x - 10*b*c^2*x^2 + 15*c^3*x^3)) + 3*c^
3*(8*b*B - 5*A*c)*x^4*ArcTanh[Sqrt[b + c*x]/Sqrt[b]]))/(b^(7/2)*x^(9/2)*Sq
rt[b + c*x])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1130, 1135, 1135, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{11/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(8bB-5Ac)\int \frac{\sqrt{cx^2+bx}}{x^{9/2}}dx}{8b}-\frac{A\left(bx+cx^2\right)^{3/2}}{4bx^{11/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(8bB-5Ac)\left(\frac{1}{6}c\int \frac{1}{x^{5/2}\sqrt{cx^2+bx}}dx-\frac{\sqrt{bx+cx^2}}{3x^{7/2}}\right)}{8b}-\frac{A\left(bx+cx^2\right)^{3/2}}{4bx^{11/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{(8bB-5Ac)\left(\frac{1}{6}c\left(-\frac{3c\int \frac{1}{x^{3/2}\sqrt{cx^2+bx}}dx}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)-\frac{\sqrt{bx+cx^2}}{3x^{7/2}}\right)}{8b}-\frac{A(bx+cx^2)^{3/2}}{4bx^{11/2}}$$

$\Big\downarrow$ 1135

$$\frac{(8bB-5Ac)\left(\frac{1}{6}c\left(-\frac{3c\left(-\frac{c\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{2b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)-\frac{\sqrt{bx+cx^2}}{3x^{7/2}}\right)}{8b}-\frac{A(bx+cx^2)^{3/2}}{4bx^{11/2}}$$

$\Big\downarrow$ 1136

$$\frac{(8bB-5Ac)\left(\frac{1}{6}c\left(-\frac{3c\left(-\frac{c\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)-\frac{\sqrt{bx+cx^2}}{3x^{7/2}}\right)}{8b}-\frac{A(bx+cx^2)^{3/2}}{4bx^{11/2}}$$

$\Big\downarrow$ 220

$$\frac{(8bB-5Ac)\left(\frac{1}{6}c\left(-\frac{3c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)-\frac{\sqrt{bx+cx^2}}{3x^{7/2}}\right)}{8b}-\frac{A(bx+cx^2)^{3/2}}{4bx^{11/2}}$$

input

```
Int[((A + B*x)*Sqrt[b*x + c*x^2])/x^(11/2),x]
```

output

```
-1/4*(A*(b*x + c*x^2)^(3/2))/(b*x^(11/2)) + ((8*b*B - 5*A*c)*(-1/3*Sqrt[b*
x + c*x^2]/x^(7/2) + (c*(-1/2*Sqrt[b*x + c*x^2]/(b*x^(5/2)) - (3*c*(-(Sqrt
[b*x + c*x^2]/(b*x^(3/2))) + (c*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x]
)])/b^(3/2)))/(4*b)))/6))/(8*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{(cx+b)(15A\,c^3x^3-24x^3Bb\,c^2-10Ab\,c^2x^2+16x^2B\,b^2c+8A\,b^2cx+64xB\,b^3+48A\,b^3)}{192x^{\frac{7}{2}}b^3\sqrt{x(cx+b)}}+\frac{c^3(5Ac-8Bb)\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)\sqrt{cx+b}}{64b^{\frac{7}{2}}\sqrt{x(cx+b)}}$
default	$\frac{\sqrt{x(cx+b)}\left(15A\ \mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^4x^4-24B\ \mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)b\,c^3x^4-15A\,c^3x^3\sqrt{cx+b}\,\sqrt{b}+24B\,b^{\frac{3}{2}}c^2x^3\sqrt{cx+b}+10A\,b^{\frac{3}{2}}c^2x^2\sqrt{c}\right.}{192b^{\frac{7}{2}}x^{\frac{9}{2}}\sqrt{cx+b}}$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(c*x+b)*(15*A*c^3*x^3-24*B*b*c^2*x^3-10*A*b*c^2*x^2+16*B*b^2*c*x^2+
8*A*b^2*c*x+64*B*b^3*x+48*A*b^3)/x^(7/2)/b^3/(x*(c*x+b))^(1/2)+1/64*c^3*(5
*A*c-8*B*b)/b^(7/2)*arctanh((c*x+b)^(1/2)/b^(1/2))*(c*x+b)^(1/2)*x^(1/2)/(
x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.66

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{11/2}}\,dx = \left[-\frac{3\,(8\,Bbc^3-5\,Ac^4)\sqrt{b}x^5\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,(48\,Ab^4-3\,(8\,B}{38}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(11/2),x, algorithm="fricas")
```

output
```
[-1/384*(3*(8*B*b*c^3 - 5*A*c^4)*sqrt(b)*x^5*log(-(c*x^2 + 2*b*x + 2*sqrt(
c*x^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(48*A*b^4 - 3*(8*B*b^2*c^2 - 5*A*b*
c^3)*x^3 + 2*(8*B*b^3*c - 5*A*b^2*c^2)*x^2 + 8*(8*B*b^4 + A*b^3*c)*x)*sqrt
(c*x^2 + b*x)*sqrt(x))/(b^4*x^5), 1/192*(3*(8*B*b*c^3 - 5*A*c^4)*sqrt(-b)*
x^5*arctan(sqrt(c*x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) - (48*A*b^4 - 3*(8*B*b^
2*c^2 - 5*A*b*c^3)*x^3 + 2*(8*B*b^3*c - 5*A*b^2*c^2)*x^2 + 8*(8*B*b^4 + A*
b^3*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^4*x^5)]
```

Sympy [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{11/2}}\,dx=\int \frac{\sqrt{x\,(b+cx)}(A+Bx)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2)/x**(11/2),x)
```

output
```
Integral(sqrt(x*(b + c*x))*(A + B*x)/x**(11/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{11/2}}\,dx=\int \frac{\sqrt{cx^2+bx}(Bx+A)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(11/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x)*(B*x + A)/x^(11/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.01

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{11/2}}\,dx=\frac{\frac{3\,(8\,Bbc^4-5\,Ac^5)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3}+\frac{24\,(cx+b)^{\frac{7}{2}}Bbc^4-88\,(cx+b)^{\frac{5}{2}}Bb^2c^4+40\,(cx+b)^{\frac{3}{2}}Bb^3c^4+24\,\sqrt{cx+}}{}}{192\,c}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(11/2),x, algorithm="giac")
```

output
```
1/192*(3*(8*B*b*c^4 - 5*A*c^5)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^
3) + (24*(c*x + b)^(7/2)*B*b*c^4 - 88*(c*x + b)^(5/2)*B*b^2*c^4 + 40*(c*x
+ b)^(3/2)*B*b^3*c^4 + 24*sqrt(c*x + b)*B*b^4*c^4 - 15*(c*x + b)^(7/2)*A*c
^5 + 55*(c*x + b)^(5/2)*A*b*c^5 - 73*(c*x + b)^(3/2)*A*b^2*c^5 - 15*sqrt(c
*x + b)*A*b^3*c^5)/(b^3*c^4*x^4))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{11/2}}\,dx = \int \frac{\sqrt{c\,x^2+b\,x}\,(A+B\,x)}{x^{11/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^(11/2),x)
```

output
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{11/2}}\,dx = \frac{-96\sqrt{cx+b}\,a\,b^4-16\sqrt{cx+b}\,a\,b^3cx+20\sqrt{cx+b}\,a\,b^2c^2x^2-30\sqrt{cx+b}\,ab\,c^3}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^(11/2),x)
```

output
```
( - 96*sqrt(b + c*x)*a*b**4 - 16*sqrt(b + c*x)*a*b**3*c*x + 20*sqrt(b + c*
x)*a*b**2*c**2*x**2 - 30*sqrt(b + c*x)*a*b*c**3*x**3 - 128*sqrt(b + c*x)*b
**5*x - 32*sqrt(b + c*x)*b**4*c*x**2 + 48*sqrt(b + c*x)*b**3*c**2*x**3 - 1
5*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c**4*x**4 + 24*sqrt(b)*log(sqrt(b
 + c*x) - sqrt(b))*b**2*c**3*x**4 + 15*sqrt(b)*log(sqrt(b + c*x) + sqrt(b)
)*a*c**4*x**4 - 24*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*b**2*c**3*x**4)/(3
84*b**4*x**4)
```

3.181 $\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{13/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 212

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{13/2}}\,dx = -\frac{A\sqrt{bx+cx^2}}{5x^{11/2}} - \frac{(10bB+Ac)\sqrt{bx+cx^2}}{40bx^{9/2}} - \frac{c(10bB-7Ac)\sqrt{bx+cx^2}}{240b^2x^{7/2}} + \frac{c^2(10bB-7Ac)\sqrt{bx+cx^2}}{192b^3x^{5/2}} - \frac{c^3(10bB-7Ac)\sqrt{bx+cx^2}}{128b^4x^{3/2}} + \frac{c^4(10bB-7Ac)\operatorname{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{128b^{9/2}}$$

output

```
-1/5*A*(c*x^2+b*x)^(1/2)/x^(11/2)-1/40*(A*c+10*B*b)*(c*x^2+b*x)^(1/2)/b/x^
(9/2)-1/240*c*(-7*A*c+10*B*b)*(c*x^2+b*x)^(1/2)/b^2/x^(7/2)+1/192*c^2*(-7*
A*c+10*B*b)*(c*x^2+b*x)^(1/2)/b^3/x^(5/2)-1/128*c^3*(-7*A*c+10*B*b)*(c*x^2
+b*x)^(1/2)/b^4/x^(3/2)+1/128*c^4*(-7*A*c+10*B*b)*arctanh((c*x^2+b*x)^(1/2
)/b^(1/2)/x^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.75

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{13/2}}\,dx = \frac{-\sqrt{b}(b+cx)\left(10bBx(48b^3+8b^2cx-10bc^2x^2+15c^3x^3)+A(384b^4+48b^3cx\right.}{1920b^{9/2}x}$$

input
```
Integrate[((A + B*x)*Sqrt[b*x + c*x^2])/x^(13/2),x]
```

output
```
(-(Sqrt[b]*(b + c*x)*(10*b*B*x*(48*b^3 + 8*b^2*c*x - 10*b*c^2*x^2 + 15*c^3
*x^3) + A*(384*b^4 + 48*b^3*c*x - 56*b^2*c^2*x^2 + 70*b*c^3*x^3 - 105*c^4*
x^4))) + 15*c^4*(10*b*B - 7*A*c)*x^5*Sqrt[b + c*x]*ArcTanh[Sqrt[b + c*x]/S
qrt[b]])/(1920*b^(9/2)*x^(9/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1220, 1130, 1135, 1135, 1135, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{13/2}}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(10bB-7Ac)\int \frac{\sqrt{cx^2+bx}}{x^{11/2}}dx}{10b}-\frac{A\left(bx+cx^2\right)^{3/2}}{5bx^{13/2}}$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{(10bB-7Ac)\left(\frac{1}{8}c\int \frac{1}{x^{7/2}\sqrt{cx^2+bx}}dx-\frac{\sqrt{bx+cx^2}}{4x^{9/2}}\right)}{10b}-\frac{A\left(bx+cx^2\right)^{3/2}}{5bx^{13/2}}$$

$$\Big\downarrow \text{ 1135}$$

$$\frac{(10bB-7Ac)\left(\frac{1}{8}c\left(-\frac{5c\int \frac{1}{x^{5/2}\sqrt{cx^2+bx}}dx}{6b}-\frac{\sqrt{bx+cx^2}}{3bx^{7/2}}\right)-\frac{\sqrt{bx+cx^2}}{4x^{9/2}}\right)}{10b}-\frac{A\left(bx+cx^2\right)^{3/2}}{5bx^{13/2}}$$

$\Big\downarrow$ 1135

$$\frac{(10bB-7Ac)\left(\frac{1}{8}c\left(-\frac{5c\left(-\frac{3c\int \frac{1}{x^{3/2}\sqrt{cx^2+bx}}dx}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)}{6b}-\frac{\sqrt{bx+cx^2}}{3bx^{7/2}}\right)-\frac{\sqrt{bx+cx^2}}{4x^{9/2}}\right)}{10b}-\frac{A\left(bx+cx^2\right)^{3/2}}{5bx^{13/2}}$$

$\Big\downarrow$ 1135

$$\frac{(10bB-7Ac)\left(\frac{1}{8}c\left(-\frac{5c\left(-\frac{3c\left(-\frac{c\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{2b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)}{6b}-\frac{\sqrt{bx+cx^2}}{3bx^{7/2}}\right)-\frac{\sqrt{bx+cx^2}}{4x^{9/2}}\right)}{10b}-\frac{A\left(bx+cx^2\right)^{3/2}}{5bx^{13/2}}$$

$\Big\downarrow$ 1136

$$\frac{(10bB-7Ac)\left(\frac{1}{8}c\left(-\frac{5c\left(-\frac{3c\left(-\frac{c\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)}{6b}-\frac{\sqrt{bx+cx^2}}{3bx^{7/2}}\right)-\frac{\sqrt{bx+cx^2}}{4x^{9/2}}\right)}{10b}-\frac{A\left(bx+cx^2\right)^{3/2}}{5bx^{13/2}}$$

$\Big\downarrow$ 220

$$\frac{(10bB-7Ac)\left(\frac{1}{8}c\left(-\frac{5c\left(-\frac{3c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)}{6b}-\frac{\sqrt{bx+cx^2}}{3bx^{7/2}}\right)-\frac{\sqrt{bx+cx^2}}{4x^{9/2}}\right)}{10b}-\frac{A(bx+cx^2)^{3/2}}{5bx^{13/2}}$$

input
```
Int[((A + B*x)*Sqrt[b*x + c*x^2])/x^(13/2),x]
```

output
```
-1/5*(A*(b*x + c*x^2)^(3/2))/(b*x^(13/2)) + ((10*b*B - 7*A*c)*(-1/4*Sqrt[b
*x + c*x^2]/x^(9/2) + (c*(-1/3*Sqrt[b*x + c*x^2]/(b*x^(7/2)) - (5*c*(-1/2*
Sqrt[b*x + c*x^2]/(b*x^(5/2)) - (3*c*(-(Sqrt[b*x + c*x^2]/(b*x^(3/2))) + (
c*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/b^(3/2)))/(4*b)))/(6*b)))/
8))/(10*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(cx+b)\left(-105A\,c^4x^4+150Bb\,c^3x^4+70Ab\,c^3x^3-100B\,b^2c^2x^3-56A\,b^2c^2x^2+80B\,b^3c\,x^2+48A\,b^3cx+480B\,b^4x+384A\,b^4\right)}{1920x^{\frac{9}{2}}b^4\sqrt{x(cx+b)}}-\frac{c^4(7$
default	$-\frac{\sqrt{x(cx+b)}\left(105A\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^5x^5-150B\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)b\,c^4x^5-105A\,c^4x^4\sqrt{cx+b}\,\sqrt{b}+150B\,b^{\frac{3}{2}}c^3x^4\sqrt{cx+b}+70A\,b^{\frac{3}{2}}c}{1920b^{\frac{9}{2}}}$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1920*(c*x+b)*(-105*A*c^4*x^4+150*B*b*c^3*x^4+70*A*b*c^3*x^3-100*B*b^2*c
^2*x^3-56*A*b^2*c^2*x^2+80*B*b^3*c*x^2+48*A*b^3*c*x+480*B*b^4*x+384*A*b^4)
/x^(9/2)/b^4/(x*(c*x+b))^(1/2)-1/128*c^4*(7*A*c-10*B*b)/b^(9/2)*arctanh((c
*x+b)^(1/2)/b^(1/2))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.59

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{13/2}}\,dx = \left[-\frac{15\,(10\,Bbc^4-7\,Ac^5)\sqrt{b}x^6\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,(384\,Ab^5+1}{}\right.$$

$$-\frac{15\,(10\,Bbc^4-7\,Ac^5)\sqrt{-b}x^6\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-b}}{b\sqrt{x}}\right)+(384\,Ab^5+15\,(10\,Bb^2c^3-7\,Abc^4)x^4-10\,(10\,Bb^3c^2}{1920\,b^5x^6}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(13/2),x, algorithm="fricas")
```

output
```
[-1/3840*(15*(10*B*b*c^4 - 7*A*c^5)*sqrt(b)*x^6*log(-(c*x^2 + 2*b*x - 2*sq
rt(c*x^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(384*A*b^5 + 15*(10*B*b^2*c^3 -
7*A*b*c^4)*x^4 - 10*(10*B*b^3*c^2 - 7*A*b^2*c^3)*x^3 + 8*(10*B*b^4*c - 7*A
*b^3*c^2)*x^2 + 48*(10*B*b^5 + A*b^4*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^5
*x^6), -1/1920*(15*(10*B*b*c^4 - 7*A*c^5)*sqrt(-b)*x^6*arctan(sqrt(c*x^2 +
 b*x)*sqrt(-b)/(b*sqrt(x))) + (384*A*b^5 + 15*(10*B*b^2*c^3 - 7*A*b*c^4)*x
^4 - 10*(10*B*b^3*c^2 - 7*A*b^2*c^3)*x^3 + 8*(10*B*b^4*c - 7*A*b^3*c^2)*x^
2 + 48*(10*B*b^5 + A*b^4*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^5*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(1/2)/x**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{13/2}}\,dx = \int \frac{\sqrt{cx^2+bx}(Bx+A)}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(13/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(c*x^2 + b*x)*(B*x + A)/x^(13/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.85

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{13/2}}\,dx =$$

$$-\frac{1}{1920}c^5\left(\frac{15\,(10\,Bb-7\,Ac)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^4c}+\frac{150\,(cx+b)^{\frac{9}{2}}Bb-700\,(cx+b)^{\frac{7}{2}}Bb^2+1280\,(cx+b)^{\frac{5}{2}}Bb^3}{}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(1/2)/x^(13/2),x, algorithm="giac")
```

output
```
-1/1920*c^5*(15*(10*B*b - 7*A*c)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*
b^4*c) + (150*(c*x + b)^(9/2)*B*b - 700*(c*x + b)^(7/2)*B*b^2 + 1280*(c*x
+ b)^(5/2)*B*b^3 - 580*(c*x + b)^(3/2)*B*b^4 - 150*sqrt(c*x + b)*B*b^5 - 1
05*(c*x + b)^(9/2)*A*c + 490*(c*x + b)^(7/2)*A*b*c - 896*(c*x + b)^(5/2)*A
*b^2*c + 790*(c*x + b)^(3/2)*A*b^3*c + 105*sqrt(c*x + b)*A*b^4*c)/(b^4*c^6
*x^5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{13/2}}\,dx = \int \frac{\sqrt{c\,x^2+b\,x}\,(A+B\,x)}{x^{13/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^(13/2),x)
```

output
```
int(((b*x + c*x^2)^(1/2)*(A + B*x))/x^(13/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx)\sqrt{bx+cx^2}}{x^{13/2}}\,dx = \frac{-768\sqrt{cx+b}\,a\,b^5-96\sqrt{cx+b}\,a\,b^4cx+112\sqrt{cx+b}\,a\,b^3c^2x^2-140\sqrt{cx+b}\,a}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(1/2)/x^(13/2),x)
```

output
```
( - 768*sqrt(b + c*x)*a*b**5 - 96*sqrt(b + c*x)*a*b**4*c*x + 112*sqrt(b +
c*x)*a*b**3*c**2*x**2 - 140*sqrt(b + c*x)*a*b**2*c**3*x**3 + 210*sqrt(b +
c*x)*a*b*c**4*x**4 - 960*sqrt(b + c*x)*b**6*x - 160*sqrt(b + c*x)*b**5*c*x
**2 + 200*sqrt(b + c*x)*b**4*c**2*x**3 - 300*sqrt(b + c*x)*b**3*c**3*x**4
+ 105*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c**5*x**5 - 150*sqrt(b)*log(s
qrt(b + c*x) - sqrt(b))*b**2*c**4*x**5 - 105*sqrt(b)*log(sqrt(b + c*x) + s
qrt(b))*a*c**5*x**5 + 150*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*b**2*c**4*x
**5)/(3840*b**5*x**5)
```

3.182 $\int x^{5/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\int x^{5/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=-\frac{2b^4(bB-Ac)\left(bx+cx^2\right)^{5/2}}{5c^6x^{5/2}}+\frac{2b^3(5bB-4Ac)\left(bx+cx^2\right)^{7/2}}{7c^6x^{7/2}}-\frac{4b^2(5bB-3Ac)\left(bx+cx^2\right)^{9/2}}{9c^6x^{9/2}}+\frac{4b(5bB-2Ac)\left(bx+cx^2\right)^{11/2}}{11c^6x^{11/2}}-\frac{2(5bB-Ac)\left(bx+cx^2\right)^{13/2}}{13c^6x^{13/2}}+\frac{2B\left(bx+cx^2\right)^{15/2}}{15c^6x^{15/2}}$$

output

```
-2/5*b^4*(-A*c+B*b)*(c*x^2+b*x)^(5/2)/c^6/x^(5/2)+2/7*b^3*(-4*A*c+5*B*b)*(
c*x^2+b*x)^(7/2)/c^6/x^(7/2)-4/9*b^2*(-3*A*c+5*B*b)*(c*x^2+b*x)^(9/2)/c^6/
x^(9/2)+4/11*b*(-2*A*c+5*B*b)*(c*x^2+b*x)^(11/2)/c^6/x^(11/2)-2/13*(-A*c+5
*B*b)*(c*x^2+b*x)^(13/2)/c^6/x^(13/2)+2/15*B*(c*x^2+b*x)^(15/2)/c^6/x^(15/
2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.53

$$\int x^{5/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{2(x(b+cx))^{5/2}\left(-256b^5B+1680b^2c^3x^2(A+Bx)+128b^4c(3A+5Bx)-160b^3c^2x(6A+7Bx)\right.}{45045c^6x^{5/2}}$$

input
```
Integrate[x^(5/2)*(A + B*x)*(b*x + c*x^2)^(3/2),x]
```

output
```
(2*(x*(b + c*x))^(5/2)*(-256*b^5*B + 1680*b^2*c^3*x^2*(A + B*x) + 128*b^4*
c*(3*A + 5*B*x) - 160*b^3*c^2*x*(6*A + 7*B*x) - 210*b*c^4*x^3*(12*A + 11*B
*x) + 231*c^5*x^4*(15*A + 13*B*x)))/(45045*c^6*x^(5/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.250$, Rules used = {1221, 1128, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{5/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{2Bx^{5/2}\left(bx+cx^2\right)^{5/2}}{15c}-\frac{(2bB-3Ac)\int x^{5/2}\left(cx^2+bx\right)^{3/2}dx}{3c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2Bx^{5/2}\left(bx+cx^2\right)^{5/2}}{15c}-\frac{(2bB-3Ac)\left(\frac{2x^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}-\frac{8b\int x^{3/2}\left(cx^2+bx\right)^{3/2}dx}{13c}\right)}{3c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2Bx^{5/2}\left(bx+cx^2\right)^{5/2}}{15c}-\frac{(2bB-3Ac)\left(\frac{2x^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}-\frac{8b\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}-\frac{6b\int \sqrt{x}\left(cx^2+bx\right)^{3/2}dx}{11c}\right)}{13c}\right)}{3c}$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{2Bx^{5/2}\left(bx+cx^2\right)^{5/2}}{15c}-\frac{(2bB-3Ac)\left(\frac{2x^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}-\frac{8b\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}-\frac{6b\left(\frac{2\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}-\frac{4b\int \frac{\left(cx^2+bx\right)^{3/2}}{\sqrt{x}}dx}{9c}\right)}{11c}\right)}{13c}\right)}{3c}$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{2Bx^{5/2}\left(bx+cx^2\right)^{5/2}}{15c}-\frac{(2bB-3Ac)\left(\frac{2x^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}-\frac{8b\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}-\frac{6b\left(\frac{2\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{5/2}}{7cx^{3/2}}-\frac{2b\int\frac{\left(cx^2+bx\right)^{3/2}}{x^{3/2}}dx}{7c}\right)}{9c}\right)}{11c}\right)}{13c}\right)}{3c}$$

$$\Big\downarrow \; 1122$$

$$\frac{2Bx^{5/2}\left(bx+cx^2\right)^{5/2}}{15c}-\frac{\left(\frac{2x^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}-\frac{8b\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}-\frac{6b\left(\frac{2\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{5/2}}{7cx^{3/2}}-\frac{4b\left(bx+cx^2\right)^{5/2}}{35c^2x^{5/2}}\right)}{9c}\right)}{11c}\right)}{13c}\right)(2bB-3Ac)}{3c}$$

input
```
Int[x^(5/2)*(A + B*x)*(b*x + c*x^2)^(3/2),x]
```

output
```
(2*B*x^(5/2)*(b*x + c*x^2)^(5/2))/(15*c) - ((2*b*B - 3*A*c)*((2*x^(3/2)*(b
*x + c*x^2)^(5/2))/(13*c) - (8*b*((2*Sqrt[x]*(b*x + c*x^2)^(5/2))/(11*c) -
 (6*b*((2*(b*x + c*x^2)^(5/2))/(9*c*Sqrt[x]) - (4*b*((-4*b*(b*x + c*x^2)^(
5/2))/(35*c^2*x^(5/2)) + (2*(b*x + c*x^2)^(5/2))/(7*c*x^(3/2))))/(9*c)))/(
11*c)))/(13*c)))/(3*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

method	result
gosper	$\frac{2(cx+b)\left(3003B\,c^5x^5+3465A\,c^5x^4-2310Bb\,c^4x^4-2520Ab\,c^4x^3+1680B\,b^2c^3x^3+1680A\,b^2c^3x^2-1120B\,b^3c^2x^2-960A\,b^3c^2x+640B \cdots\right.}{45045c^6x^{\frac{3}{2}}}$
default	$\frac{2\sqrt{x(cx+b)}\,(cx+b)^2\left(3003B\,c^5x^5+3465A\,c^5x^4-2310Bb\,c^4x^4-2520Ab\,c^4x^3+1680B\,b^2c^3x^3+1680A\,b^2c^3x^2-1120B\,b^3c^2x^2-960A \cdots\right.}{45045\sqrt{x}\,c^6}$
orering	$\frac{2(cx+b)\left(3003B\,c^5x^5+3465A\,c^5x^4-2310Bb\,c^4x^4-2520Ab\,c^4x^3+1680B\,b^2c^3x^3+1680A\,b^2c^3x^2-1120B\,b^3c^2x^2-960A\,b^3c^2x+640B \cdots\right.}{45045c^6x^{\frac{3}{2}}}$
risch	$\frac{2(cx+b)\sqrt{x}\,\left(3003B\,c^7x^7+3465A\,c^7x^6+3696Bb\,c^6x^6+4410Ab\,c^6x^5+63B\,b^2c^5x^5+105A\,b^2c^5x^4-70B\,b^3c^4x^4-120A\,b^3c^4x^3+80B\,b \cdots\right.}{45045\sqrt{x(cx+b)}\,c^6}$

input
```
int(x^(5/2)*(B*x+A)*(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/45045*(c*x+b)*(3003*B*c^5*x^5+3465*A*c^5*x^4-2310*B*b*c^4*x^4-2520*A*b*c
^4*x^3+1680*B*b^2*c^3*x^3+1680*A*b^2*c^3*x^2-1120*B*b^3*c^2*x^2-960*A*b^3*
c^2*x+640*B*b^4*c*x+384*A*b^4*c-256*B*b^5)*(c*x^2+b*x)^(3/2)/c^6/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.84

$$\int x^{5/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{2\left(3003\,Bc^7x^7-256\,Bb^7+384\,Ab^6c+231\,(16\,Bbc^6+15\,Ac^7)x^6+63\,(Bb^2c^5+70\,Abc^6)x^5\right.}{}$$

input
```
integrate(x^(5/2)*(B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
2/45045*(3003*B*c^7*x^7 - 256*B*b^7 + 384*A*b^6*c + 231*(16*B*b*c^6 + 15*A
*c^7)*x^6 + 63*(B*b^2*c^5 + 70*A*b*c^6)*x^5 - 35*(2*B*b^3*c^4 - 3*A*b^2*c^
5)*x^4 + 40*(2*B*b^4*c^3 - 3*A*b^3*c^4)*x^3 - 48*(2*B*b^5*c^2 - 3*A*b^4*c^
3)*x^2 + 64*(2*B*b^6*c - 3*A*b^5*c^2)*x)*sqrt(c*x^2 + b*x)/(c^6*sqrt(x))
```

Sympy [F]

$$\int x^{5/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\int x^{\frac{5}{2}}(x(b+cx))^{\frac{3}{2}}\,(A+Bx)\,dx$$

input
```
integrate(x**(5/2)*(B*x+A)*(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(x**(5/2)*(x*(b + c*x))**(3/2)*(A + B*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 318, normalized size of antiderivative = 1.54

$$\int x^{5/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{2\left(5\left(693\,c^6x^6+63\,bc^5x^5-70\,b^2c^4x^4+80\,b^3c^3x^3-96\,b^4c^2x^2+128\,b^5cx-256\,b^6\right)x^5+13\,(3\right.}{45045\,c^5x^5}+\frac{2\left(\left(3003\,c^7x^7+231\,bc^6x^6-252\,b^2c^5x^5+280\,b^3c^4x^4-320\,b^4c^3x^3+384\,b^5c^2x^2-512\,b^6cx+1024\,b^7\right)x^6+\right.}{45045\,c^6x}$$

input
```
integrate(x^(5/2)*(B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
2/45045*(5*(693*c^6*x^6 + 63*b*c^5*x^5 - 70*b^2*c^4*x^4 + 80*b^3*c^3*x^3 -
 96*b^4*c^2*x^2 + 128*b^5*c*x - 256*b^6)*x^5 + 13*(315*b*c^5*x^6 + 35*b^2*
c^4*x^5 - 40*b^3*c^3*x^4 + 48*b^4*c^2*x^3 - 64*b^5*c*x^2 + 128*b^6*x)*x^4)
*sqrt(c*x + b)*A/(c^5*x^5) + 2/45045*((3003*c^7*x^7 + 231*b*c^6*x^6 - 252*
b^2*c^5*x^5 + 280*b^3*c^4*x^4 - 320*b^4*c^3*x^3 + 384*b^5*c^2*x^2 - 512*b^
6*c*x + 1024*b^7)*x^6 + 5*(693*b*c^6*x^7 + 63*b^2*c^5*x^6 - 70*b^3*c^4*x^5
 + 80*b^4*c^3*x^4 - 96*b^5*c^2*x^3 + 128*b^6*c*x^2 - 256*b^7*x)*x^5)*sqrt(
c*x + b)*B/(c^6*x^6)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.45

$$\int x^{5/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \frac{2\left(315\,(cx+b)^{\frac{11}{2}}-1540\,(cx+b)^{\frac{9}{2}}b+2970\,(cx+b)^{\frac{7}{2}}b^2-2772\,(cx+b)^{\frac{5}{2}}b^3+1155\,(cx+b)^{\frac{3}{2}}\right.}{3465\,c^5}$$
$$+\frac{2\left(693\,(cx+b)^{\frac{13}{2}}-4095\,(cx+b)^{\frac{11}{2}}b+10010\,(cx+b)^{\frac{9}{2}}b^2-12870\,(cx+b)^{\frac{7}{2}}b^3+9009\,(cx+b)^{\frac{5}{2}}b^4-3003\right.}{9009\,c^6}$$
$$+\frac{2\left(693\,(cx+b)^{\frac{13}{2}}-4095\,(cx+b)^{\frac{11}{2}}b+10010\,(cx+b)^{\frac{9}{2}}b^2-12870\,(cx+b)^{\frac{7}{2}}b^3+9009\,(cx+b)^{\frac{5}{2}}b^4-3003\right.}{9009\,c^5}$$
$$+\frac{2\left(3003\,(cx+b)^{\frac{15}{2}}-20790\,(cx+b)^{\frac{13}{2}}b+61425\,(cx+b)^{\frac{11}{2}}b^2-100100\,(cx+b)^{\frac{9}{2}}b^3+96525\,(cx+b)^{\frac{7}{2}}b^4-\right.}{45045\,c^6}$$

input
```
integrate(x^(5/2)*(B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
2/3465*(315*(c*x + b)^(11/2) - 1540*(c*x + b)^(9/2)*b + 2970*(c*x + b)^(7/
2)*b^2 - 2772*(c*x + b)^(5/2)*b^3 + 1155*(c*x + b)^(3/2)*b^4)*A*b/c^5 + 2/
9009*(693*(c*x + b)^(13/2) - 4095*(c*x + b)^(11/2)*b + 10010*(c*x + b)^(9/
2)*b^2 - 12870*(c*x + b)^(7/2)*b^3 + 9009*(c*x + b)^(5/2)*b^4 - 3003*(c*x
+ b)^(3/2)*b^5)*B*b/c^6 + 2/9009*(693*(c*x + b)^(13/2) - 4095*(c*x + b)^(1
1/2)*b + 10010*(c*x + b)^(9/2)*b^2 - 12870*(c*x + b)^(7/2)*b^3 + 9009*(c*x
 + b)^(5/2)*b^4 - 3003*(c*x + b)^(3/2)*b^5)*A/c^5 + 2/45045*(3003*(c*x + b
)^(15/2) - 20790*(c*x + b)^(13/2)*b + 61425*(c*x + b)^(11/2)*b^2 - 100100*
(c*x + b)^(9/2)*b^3 + 96525*(c*x + b)^(7/2)*b^4 - 54054*(c*x + b)^(5/2)*b^
5 + 15015*(c*x + b)^(3/2)*b^6)*B/c^6
```

Mupad [F(-1)]

Timed out.

$$\int x^{5/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\int x^{5/2}\,\left(c\,x^2+b\,x\right)^{3/2}\,\left(A+B\,x\right)\,dx$$

input
```
int(x^(5/2)*(b*x + c*x^2)^(3/2)*(A + B*x),x)
```

output
```
int(x^(5/2)*(b*x + c*x^2)^(3/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.78

$$\int x^{5/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{2\sqrt{cx+b}\,(3003b\,c^7x^7+3465a\,c^7x^6+3696b^2c^6x^6+4410ab\,c^6x^5+63b^3c^5x^5+105a\,b^2c^5x^4-$$

input
```
int(x^(5/2)*(B*x+A)*(c*x^2+b*x)^(3/2),x)
```

output

```
(2*sqrt(b + c*x)*(384*a*b**6*c - 192*a*b**5*c**2*x + 144*a*b**4*c**3*x**2
- 120*a*b**3*c**4*x**3 + 105*a*b**2*c**5*x**4 + 4410*a*b*c**6*x**5 + 3465*
a*c**7*x**6 - 256*b**8 + 128*b**7*c*x - 96*b**6*c**2*x**2 + 80*b**5*c**3*x
**3 - 70*b**4*c**4*x**4 + 63*b**3*c**5*x**5 + 3696*b**2*c**6*x**6 + 3003*b
*c**7*x**7))/(45045*c**6)
```

3.183 $\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 169

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{2b^3(bB-Ac)\left(bx+cx^2\right)^{5/2}}{5c^5x^{5/2}}-\frac{2b^2(4bB-3Ac)\left(bx+cx^2\right)^{7/2}}{7c^5x^{7/2}}+\frac{2b(2bB-Ac)\left(bx+cx^2\right)^{9/2}}{3c^5x^{9/2}}-\frac{2(4bB-Ac)\left(bx+cx^2\right)^{11/2}}{11c^5x^{11/2}}+\frac{2B\left(bx+cx^2\right)^{13/2}}{13c^5x^{13/2}}$$

output

```
2/5*b^3*(-A*c+B*b)*(c*x^2+b*x)^(5/2)/c^5/x^(5/2)-2/7*b^2*(-3*A*c+4*B*b)*(c
*x^2+b*x)^(7/2)/c^5/x^(7/2)+2/3*b*(-A*c+2*B*b)*(c*x^2+b*x)^(9/2)/c^5/x^(9/
2)-2/11*(-A*c+4*B*b)*(c*x^2+b*x)^(11/2)/c^5/x^(11/2)+2/13*B*(c*x^2+b*x)^(1
3/2)/c^5/x^(13/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.56

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{2(x(b+cx))^{5/2}\left(128b^4B+105c^4x^3(13A+11Bx)-70bc^3x^2(13A+12Bx)+40b^2c^2x(13A+\right.}{15015c^5x^{5/2}}$$

input
```
Integrate[x^(3/2)*(A + B*x)*(b*x + c*x^2)^(3/2),x]
```

output
```
(2*(x*(b + c*x))^(5/2)*(128*b^4*B + 105*c^4*x^3*(13*A + 11*B*x) - 70*b*c^3
*x^2*(13*A + 12*B*x) + 40*b^2*c^2*x*(13*A + 14*B*x) - 16*b^3*c*(13*A + 20*
B*x)))/(15015*c^5*x^(5/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1221, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{2Bx^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}-\frac{(8bB-13Ac)\int x^{3/2}\left(cx^2+bx\right)^{3/2}dx}{13c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2Bx^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}-\frac{(8bB-13Ac)\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}-\frac{6b\int\sqrt{x}\left(cx^2+bx\right)^{3/2}dx}{11c}\right)}{13c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2Bx^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}-\frac{(8bB-13Ac)\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}-\frac{6b\left(\frac{2\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}-\frac{4b\int\frac{\left(cx^2+bx\right)^{3/2}}{\sqrt{x}}dx}{9c}\right)}{11c}\right)}{13c}$$

$\Big\downarrow$ 1128

$$\frac{2Bx^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}-\frac{(8bB-13Ac)\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}-\frac{6b\left(\frac{2\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{5/2}}{7cx^{3/2}}-\frac{2b\int\frac{\left(cx^2+bx\right)^{3/2}}{x^{3/2}}dx}{7c}\right)}{9c}\right)}{11c}\right)}{13c}$$

$\Big\downarrow$ 1122

$$\frac{2Bx^{3/2}\left(bx+cx^2\right)^{5/2}}{13c}-\frac{\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}-\frac{6b\left(\frac{2\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{5/2}}{7cx^{3/2}}-\frac{4b\left(bx+cx^2\right)^{5/2}}{35c^2x^{5/2}}\right)}{9c}\right)}{11c}\right)(8bB-13Ac)}{13c}$$

input
```
Int[x^(3/2)*(A + B*x)*(b*x + c*x^2)^(3/2),x]
```

output
```
(2*B*x^(3/2)*(b*x + c*x^2)^(5/2))/(13*c) - ((8*b*B - 13*A*c)*((2*Sqrt[x]*(
b*x + c*x^2)^(5/2))/(11*c) - (6*b*((2*(b*x + c*x^2)^(5/2))/(9*c*Sqrt[x]) -
 (4*b*((-4*b*(b*x + c*x^2)^(5/2))/(35*c^2*x^(5/2)) + (2*(b*x + c*x^2)^(5/2
))/(7*c*x^(3/2))))/(9*c)))/(11*c)))/(13*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.63

method	result
gosper	$-\frac{2(cx+b)\left(-1155B\,c^4x^4-1365A\,c^4x^3+840B\,c^3x^3b+910Ab\,c^3x^2-560c^2x^2B\,b^2-520A\,b^2c^2x+320B\,b^3cx+208A\,b^3c-128B\,b^4\right)(c\,x}{15015c^5x^{\frac{3}{2}}}$
default	$-\frac{2\sqrt{x(cx+b)}\,(cx+b)^2\left(-1155B\,c^4x^4-1365A\,c^4x^3+840B\,c^3x^3b+910Ab\,c^3x^2-560c^2x^2B\,b^2-520A\,b^2c^2x+320B\,b^3cx+208A\,b^3c-\right.}{15015\sqrt{x}\,c^5}$
orering	$-\frac{2(cx+b)\left(-1155B\,c^4x^4-1365A\,c^4x^3+840B\,c^3x^3b+910Ab\,c^3x^2-560c^2x^2B\,b^2-520A\,b^2c^2x+320B\,b^3cx+208A\,b^3c-128B\,b^4\right)(c\,x}{15015c^5x^{\frac{3}{2}}}$
risch	$-\frac{2(cx+b)\sqrt{x}\left(-1155B\,c^6x^6-1365A\,c^6x^5-1470Bb\,c^5x^5-1820Ab\,c^5x^4-35B\,b^2c^4x^4-65A\,b^2c^4x^3+40B\,b^3c^3x^3+78A\,b^3c^3x^2-48B\right.}{15015\sqrt{x(cx+b)}\,c^5}$

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15015*(c*x+b)*(-1155*B*c^4*x^4-1365*A*c^4*x^3+840*B*b*c^3*x^3+910*A*b*c
^3*x^2-560*B*b^2*c^2*x^2-520*A*b^2*c^2*x+320*B*b^3*c*x+208*A*b^3*c-128*B*b
^4)*(c*x^2+b*x)^(3/2)/c^5/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{2\left(1155\,Bc^6x^6+128\,Bb^6-208\,Ab^5c+105\,(14\,Bbc^5+13\,Ac^6)x^5+35\,(Bb^2c^4+52\,Abc^5)x^4\right.}{15015}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
2/15015*(1155*B*c^6*x^6 + 128*B*b^6 - 208*A*b^5*c + 105*(14*B*b*c^5 + 13*A
*c^6)*x^5 + 35*(B*b^2*c^4 + 52*A*b*c^5)*x^4 - 5*(8*B*b^3*c^3 - 13*A*b^2*c^
4)*x^3 + 6*(8*B*b^4*c^2 - 13*A*b^3*c^3)*x^2 - 8*(8*B*b^5*c - 13*A*b^4*c^2)
*x)*sqrt(c*x^2 + b*x)/(c^5*sqrt(x))
```

Sympy [F]

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\int x^{\frac{3}{2}}(x(b+cx))^{\frac{3}{2}}\,(A+Bx)\,dx$$

input
```
integrate(x**(3/2)*(B*x+A)*(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(x**(3/2)*(x*(b + c*x))**(3/2)*(A + B*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.62

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{2\left((315\,c^5x^5+35\,bc^4x^4-40\,b^2c^3x^3+48\,b^3c^2x^2-64\,b^4cx+128\,b^5)x^4+11\,(35\,bc^4x^5+5\,b^2c^3\cdots\right.}{3465\,c^4x^4}+\frac{2\left(5\,(693\,c^6x^6+63\,bc^5x^5-70\,b^2c^4x^4+80\,b^3c^3x^3-96\,b^4c^2x^2+128\,b^5cx-256\,b^6)x^5+13\,(315\,bc^5x^6+35\cdots\right.}{45045\,c^5x^5}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
2/3465*((315*c^5*x^5 + 35*b*c^4*x^4 - 40*b^2*c^3*x^3 + 48*b^3*c^2*x^2 - 64
*b^4*c*x + 128*b^5)*x^4 + 11*(35*b*c^4*x^5 + 5*b^2*c^3*x^4 - 6*b^3*c^2*x^3
 + 8*b^4*c*x^2 - 16*b^5*x)*x^3)*sqrt(c*x + b)*A/(c^4*x^4) + 2/45045*(5*(69
3*c^6*x^6 + 63*b*c^5*x^5 - 70*b^2*c^4*x^4 + 80*b^3*c^3*x^3 - 96*b^4*c^2*x^
2 + 128*b^5*c*x - 256*b^6)*x^5 + 13*(315*b*c^5*x^6 + 35*b^2*c^4*x^5 - 40*b
^3*c^3*x^4 + 48*b^4*c^2*x^3 - 64*b^5*c*x^2 + 128*b^6*x)*x^4)*sqrt(c*x + b)
*B/(c^5*x^5)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.49

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{2\left(35\,(cx+b)^{\frac{9}{2}}-135\,(cx+b)^{\frac{7}{2}}b+189\,(cx+b)^{\frac{5}{2}}b^2-105\,(cx+b)^{\frac{3}{2}}b^3\right)Ab}{315\,c^4}+\frac{2\left(315\,(cx+b)^{\frac{11}{2}}-1540\,(cx+b)^{\frac{9}{2}}b+2970\,(cx+b)^{\frac{7}{2}}b^2-2772\,(cx+b)^{\frac{5}{2}}b^3+1155\,(cx+b)^{\frac{3}{2}}b^4\right)Bb}{3465\,c^5}+\frac{2\left(315\,(cx+b)^{\frac{11}{2}}-1540\,(cx+b)^{\frac{9}{2}}b+2970\,(cx+b)^{\frac{7}{2}}b^2-2772\,(cx+b)^{\frac{5}{2}}b^3+1155\,(cx+b)^{\frac{3}{2}}b^4\right)A}{3465\,c^4}+\frac{2\left(693\,(cx+b)^{\frac{13}{2}}-4095\,(cx+b)^{\frac{11}{2}}b+10010\,(cx+b)^{\frac{9}{2}}b^2-12870\,(cx+b)^{\frac{7}{2}}b^3+9009\,(cx+b)^{\frac{5}{2}}b^4-3003\right.}{9009\,c^5}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
2/315*(35*(c*x + b)^(9/2) - 135*(c*x + b)^(7/2)*b + 189*(c*x + b)^(5/2)*b^
2 - 105*(c*x + b)^(3/2)*b^3)*A*b/c^4 + 2/3465*(315*(c*x + b)^(11/2) - 1540
*(c*x + b)^(9/2)*b + 2970*(c*x + b)^(7/2)*b^2 - 2772*(c*x + b)^(5/2)*b^3 +
 1155*(c*x + b)^(3/2)*b^4)*B*b/c^5 + 2/3465*(315*(c*x + b)^(11/2) - 1540*(
c*x + b)^(9/2)*b + 2970*(c*x + b)^(7/2)*b^2 - 2772*(c*x + b)^(5/2)*b^3 + 1
155*(c*x + b)^(3/2)*b^4)*A/c^4 + 2/9009*(693*(c*x + b)^(13/2) - 4095*(c*x
+ b)^(11/2)*b + 10010*(c*x + b)^(9/2)*b^2 - 12870*(c*x + b)^(7/2)*b^3 + 90
09*(c*x + b)^(5/2)*b^4 - 3003*(c*x + b)^(3/2)*b^5)*B/c^5
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\int x^{3/2}\,{\left(c\,x^2+b\,x\right)}^{3/2}\,\left(A+B\,x\right)\,dx$$

input
```
int(x^(3/2)*(b*x + c*x^2)^(3/2)*(A + B*x),x)
```

output
```
int(x^(3/2)*(b*x + c*x^2)^(3/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.81

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{2\sqrt{cx+b}\,(1155b\,c^6x^6+1365a\,c^6x^5+1470b^2c^5x^5+1820ab\,c^5x^4+35b^3c^4x^4+65a\,b^2c^4x^3-\cdots}{15015c^5}$$

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x)^(3/2),x)
```

output
```
(2*sqrt(b + c*x)*( - 208*a*b**5*c + 104*a*b**4*c**2*x - 78*a*b**3*c**3*x**
2 + 65*a*b**2*c**4*x**3 + 1820*a*b*c**5*x**4 + 1365*a*c**6*x**5 + 128*b**7
 - 64*b**6*c*x + 48*b**5*c**2*x**2 - 40*b**4*c**3*x**3 + 35*b**3*c**4*x**4
 + 1470*b**2*c**5*x**5 + 1155*b*c**6*x**6))/(15015*c**5)
```

3.184 $\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=-\frac{2b^2(bB-Ac)\left(bx+cx^2\right)^{5/2}}{5c^4x^{5/2}}+\frac{2b(3bB-2Ac)\left(bx+cx^2\right)^{7/2}}{7c^4x^{7/2}}-\frac{2(3bB-Ac)\left(bx+cx^2\right)^{9/2}}{9c^4x^{9/2}}+\frac{2B\left(bx+cx^2\right)^{11/2}}{11c^4x^{11/2}}$$

output
```
-2/5*b^2*(-A*c+B*b)*(c*x^2+b*x)^(5/2)/c^4/x^(5/2)+2/7*b*(-2*A*c+3*B*b)*(c*
x^2+b*x)^(7/2)/c^4/x^(7/2)-2/9*(-A*c+3*B*b)*(c*x^2+b*x)^(9/2)/c^4/x^(9/2)+
2/11*B*(c*x^2+b*x)^(11/2)/c^4/x^(11/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.57

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{2(x(b+cx))^{5/2}\left(-48b^3B+35c^3x^2(11A+9Bx)+8b^2c(11A+15Bx)-10bc^2x(22A+21Bx\right.}{3465c^4x^{5/2}}$$

input
```
Integrate[Sqrt[x]*(A + B*x)*(b*x + c*x^2)^(3/2),x]
```

output
```
(2*(x*(b + c*x))^(5/2)*(-48*b^3*B + 35*c^3*x^2*(11*A + 9*B*x) + 8*b^2*c*(1
1*A + 15*B*x) - 10*b*c^2*x*(22*A + 21*B*x)))/(3465*c^4*x^(5/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1221, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{3/2}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{2B\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}-\frac{(6bB-11Ac)\int \sqrt{x}\left(cx^2+bx\right)^{3/2}dx}{11c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2B\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}-\frac{(6bB-11Ac)\left(\frac{2\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}-\frac{4b\int \frac{\left(cx^2+bx\right)^{3/2}}{\sqrt{x}}dx}{9c}\right)}{11c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2B\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}-\frac{(6bB-11Ac)\left(\frac{2\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{5/2}}{7cx^{3/2}}-\frac{2b\int \frac{\left(cx^2+bx\right)^{3/2}}{x^{3/2}}dx}{7c}\right)}{9c}\right)}{11c}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2B\sqrt{x}\left(bx+cx^2\right)^{5/2}}{11c}-\frac{\left(\frac{2\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{5/2}}{7cx^{3/2}}-\frac{4b\left(bx+cx^2\right)^{5/2}}{35c^2x^{5/2}}\right)}{9c}\right)(6bB-11Ac)}{11c}$$

input
```
Int[Sqrt[x]*(A + B*x)*(b*x + c*x^2)^(3/2),x]
```

output
```
(2*B*Sqrt[x]*(b*x + c*x^2)^(5/2))/(11*c) - ((6*b*B - 11*A*c)*((2*(b*x + c*
x^2)^(5/2))/(9*c*Sqrt[x]) - (4*b*((-4*b*(b*x + c*x^2)^(5/2))/(35*c^2*x^(5/
2)) + (2*(b*x + c*x^2)^(5/2))/(7*c*x^(3/2))))/(9*c)))/(11*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.63

method	result
gosper	$\frac{2(cx+b)\left(315B\,c^{3}x^{3}+385A\,c^{3}x^{2}-210Bb\,c^{2}x^{2}-220Ab\,c^{2}x+120B\,b^{2}cx+88A\,b^{2}c-48B\,b^{3}\right)\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{3465c^{4}x^{\frac{3}{2}}}$
default	$\frac{2\sqrt{x(cx+b)}\,(cx+b)^{2}\left(315B\,c^{3}x^{3}+385A\,c^{3}x^{2}-210Bb\,c^{2}x^{2}-220Ab\,c^{2}x+120B\,b^{2}cx+88A\,b^{2}c-48B\,b^{3}\right)}{3465\sqrt{x}\,c^{4}}$
orering	$\frac{2(cx+b)\left(315B\,c^{3}x^{3}+385A\,c^{3}x^{2}-210Bb\,c^{2}x^{2}-220Ab\,c^{2}x+120B\,b^{2}cx+88A\,b^{2}c-48B\,b^{3}\right)\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{3465c^{4}x^{\frac{3}{2}}}$
risch	$\frac{2(cx+b)\sqrt{x}\,(315B\,c^{5}x^{5}+385A\,c^{5}x^{4}+420Bb\,c^{4}x^{4}+550Ab\,c^{4}x^{3}+15B\,b^{2}c^{3}x^{3}+33A\,b^{2}c^{3}x^{2}-18B\,b^{3}c^{2}x^{2}-44A\,b^{3}c^{2}x+24B\,b^{4}cx+88A}{3465\sqrt{x(cx+b)}\,c^{4}}$

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3465*(c*x+b)*(315*B*c^3*x^3+385*A*c^3*x^2-210*B*b*c^2*x^2-220*A*b*c^2*x+
120*B*b^2*c*x+88*A*b^2*c-48*B*b^3)*(c*x^2+b*x)^(3/2)/c^4/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.96

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \frac{2\left(315\,Bc^5x^5-48\,Bb^5+88\,Ab^4c+35\,(12\,Bbc^4+11\,Ac^5)x^4+5\,(3\,Bb^2c^3+110\,Abc^4)x^3-3\right.}{3465\,c^4\sqrt{x}}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
2/3465*(315*B*c^5*x^5 - 48*B*b^5 + 88*A*b^4*c + 35*(12*B*b*c^4 + 11*A*c^5)
*x^4 + 5*(3*B*b^2*c^3 + 110*A*b*c^4)*x^3 - 3*(6*B*b^3*c^2 - 11*A*b^2*c^3)*
x^2 + 4*(6*B*b^4*c - 11*A*b^3*c^2)*x)*sqrt(c*x^2 + b*x)/(c^4*sqrt(x))
```

Sympy [F]

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\int \sqrt{x}(x(b+cx))^{\frac{3}{2}}\,(A+Bx)\,dx$$

input
```
integrate(x**(1/2)*(B*x+A)*(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(sqrt(x)*(x*(b + c*x))**(3/2)*(A + B*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(108) = 216$.

Time = 0.07 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.73

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{2\left((35\,c^4x^4+5\,bc^3x^3-6\,b^2c^2x^2+8\,b^3cx-16\,b^4)x^3+3\,(15\,bc^3x^4+3\,b^2c^2x^3-4\,b^3cx^2+8\,b^4\right.}{315\,c^3x^3}+\frac{2\left((315\,c^5x^5+35\,bc^4x^4-40\,b^2c^3x^3+48\,b^3c^2x^2-64\,b^4cx+128\,b^5)x^4+11\,(35\,bc^4x^5+5\,b^2c^3x^4-6\,b^3c^2x^3\right.}{3465\,c^4x^4}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
2/315*((35*c^4*x^4 + 5*b*c^3*x^3 - 6*b^2*c^2*x^2 + 8*b^3*c*x - 16*b^4)*x^3
 + 3*(15*b*c^3*x^4 + 3*b^2*c^2*x^3 - 4*b^3*c*x^2 + 8*b^4*x)*x^2)*sqrt(c*x
+ b)*A/(c^3*x^3) + 2/3465*((315*c^5*x^5 + 35*b*c^4*x^4 - 40*b^2*c^3*x^3 +
48*b^3*c^2*x^2 - 64*b^4*c*x + 128*b^5)*x^4 + 11*(35*b*c^4*x^5 + 5*b^2*c^3*
x^4 - 6*b^3*c^2*x^3 + 8*b^4*c*x^2 - 16*b^5*x)*x^3)*sqrt(c*x + b)*B/(c^4*x^
4)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.54

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \frac{2\left(15\,(cx+b)^{\frac{7}{2}}-42\,(cx+b)^{\frac{5}{2}}b+35\,(cx+b)^{\frac{3}{2}}b^2\right)Ab}{105\,c^3}$$
$$+\frac{2\left(35\,(cx+b)^{\frac{9}{2}}-135\,(cx+b)^{\frac{7}{2}}b+189\,(cx+b)^{\frac{5}{2}}b^2-105\,(cx+b)^{\frac{3}{2}}b^3\right)Bb}{315\,c^4}$$
$$+\frac{2\left(35\,(cx+b)^{\frac{9}{2}}-135\,(cx+b)^{\frac{7}{2}}b+189\,(cx+b)^{\frac{5}{2}}b^2-105\,(cx+b)^{\frac{3}{2}}b^3\right)A}{315\,c^3}$$
$$+\frac{2\left(315\,(cx+b)^{\frac{11}{2}}-1540\,(cx+b)^{\frac{9}{2}}b+2970\,(cx+b)^{\frac{7}{2}}b^2-2772\,(cx+b)^{\frac{5}{2}}b^3+1155\,(cx+b)^{\frac{3}{2}}b^4\right)B}{3465\,c^4}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
2/105*(15*(c*x + b)^(7/2) - 42*(c*x + b)^(5/2)*b + 35*(c*x + b)^(3/2)*b^2)
*A*b/c^3 + 2/315*(35*(c*x + b)^(9/2) - 135*(c*x + b)^(7/2)*b + 189*(c*x +
b)^(5/2)*b^2 - 105*(c*x + b)^(3/2)*b^3)*B*b/c^4 + 2/315*(35*(c*x + b)^(9/2
) - 135*(c*x + b)^(7/2)*b + 189*(c*x + b)^(5/2)*b^2 - 105*(c*x + b)^(3/2)*
b^3)*A/c^3 + 2/3465*(315*(c*x + b)^(11/2) - 1540*(c*x + b)^(9/2)*b + 2970*
(c*x + b)^(7/2)*b^2 - 2772*(c*x + b)^(5/2)*b^3 + 1155*(c*x + b)^(3/2)*b^4)
*B/c^4
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx = \int \sqrt{x}\,\left(c\,x^2+b\,x\right)^{3/2}\,(A+B\,x)\,dx$$

input
```
int(x^(1/2)*(b*x + c*x^2)^(3/2)*(A + B*x),x)
```

output
```
int(x^(1/2)*(b*x + c*x^2)^(3/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{3/2}\,dx=\frac{2\sqrt{cx+b}\,(315b\,c^5x^5+385a\,c^5x^4+420b^2c^4x^4+550ab\,c^4x^3+15b^3c^3x^3+33a\,b^2c^3x^2-18b^4c}{3465c^4}$$

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x)^(3/2),x)
```

output
```
(2*sqrt(b + c*x)*(88*a*b**4*c - 44*a*b**3*c**2*x + 33*a*b**2*c**3*x**2 + 5
50*a*b*c**4*x**3 + 385*a*c**5*x**4 - 48*b**6 + 24*b**5*c*x - 18*b**4*c**2*
x**2 + 15*b**3*c**3*x**3 + 420*b**2*c**4*x**4 + 315*b*c**5*x**5))/(3465*c*
*4)
```

3.185 $\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{\sqrt{x}}\,dx = \frac{2b(bB-Ac)\left(bx+cx^2\right)^{5/2}}{5c^3x^{5/2}} - \frac{2(2bB-Ac)\left(bx+cx^2\right)^{7/2}}{7c^3x^{7/2}} + \frac{2B\left(bx+cx^2\right)^{9/2}}{9c^3x^{9/2}}$$

output
```
2/5*b*(-A*c+B*b)*(c*x^2+b*x)^(5/2)/c^3/x^(5/2)-2/7*(-A*c+2*B*b)*(c*x^2+b*x
)^(7/2)/c^3/x^(7/2)+2/9*B*(c*x^2+b*x)^(9/2)/c^3/x^(9/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.59

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{\sqrt{x}}\,dx = \frac{2(x(b+cx))^{5/2}\left(8b^2B+5c^2x(9A+7Bx)-2bc(9A+10Bx)\right)}{315c^3x^{5/2}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/Sqrt[x],x]
```

output
```
(2*(x*(b + c*x))^(5/2)*(8*b^2*B + 5*c^2*x*(9*A + 7*B*x) - 2*b*c*(9*A + 10*
B*x)))/(315*c^3*x^(5/2))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1221, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{2B\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}-\frac{(4bB-9Ac)\int \frac{\left(cx^2+bx\right)^{3/2}}{\sqrt{x}}dx}{9c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2B\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}-\frac{(4bB-9Ac)\left(\frac{2\left(bx+cx^2\right)^{5/2}}{7cx^{3/2}}-\frac{2b\int \frac{\left(cx^2+bx\right)^{3/2}}{x^{3/2}}dx}{7c}\right)}{9c}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2B\left(bx+cx^2\right)^{5/2}}{9c\sqrt{x}}-\frac{\left(\frac{2\left(bx+cx^2\right)^{5/2}}{7cx^{3/2}}-\frac{4b\left(bx+cx^2\right)^{5/2}}{35c^2x^{5/2}}\right)(4bB-9Ac)}{9c}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/Sqrt[x],x]
```

output
```
(2*B*(b*x + c*x^2)^(5/2))/(9*c*Sqrt[x]) - ((4*b*B - 9*A*c)*((-4*b*(b*x + c
*x^2)^(5/2))/(35*c^2*x^(5/2)) + (2*(b*x + c*x^2)^(5/2))/(7*c*x^(3/2))))/(9
*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.62

method	result	size
gosper	$-\frac{2(cx+b)\left(-35B\,c^{2}x^{2}-45A\,c^{2}x+20Bbcx+18Abc-8B\,b^{2}\right)\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{315c^{3}x^{\frac{3}{2}}}$	59
default	$-\frac{2\sqrt{x(cx+b)}\,(cx+b)^{2}\left(-35B\,c^{2}x^{2}-45A\,c^{2}x+20Bbcx+18Abc-8B\,b^{2}\right)}{315\sqrt{x}\,c^{3}}$	59
orering	$-\frac{2(cx+b)\left(-35B\,c^{2}x^{2}-45A\,c^{2}x+20Bbcx+18Abc-8B\,b^{2}\right)\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{315c^{3}x^{\frac{3}{2}}}$	59
risch	$-\frac{2(cx+b)\sqrt{x}\left(-35B\,c^{4}x^{4}-45A\,c^{4}x^{3}-50B\,c^{3}x^{3}b-72Ab\,c^{3}x^{2}-3c^{2}x^{2}B\,b^{2}-9A\,b^{2}c^{2}x+4B\,b^{3}cx+18A\,b^{3}c-8B\,b^{4}\right)}{315\sqrt{x(cx+b)}\,c^{3}}$	105

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*(c*x+b)*(-35*B*c^2*x^2-45*A*c^2*x+20*B*b*c*x+18*A*b*c-8*B*b^2)*(c*x
^2+b*x)^(3/2)/c^3/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.07

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{\sqrt{x}}\,dx = \frac{2\,(35\,Bc^4x^4+8\,Bb^4-18\,Ab^3c+5\,(10\,Bbc^3+9\,Ac^4)x^3+3\,(Bb^2c^2+24\,A}{315\,c^3\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*B*c^4*x^4 + 8*B*b^4 - 18*A*b^3*c + 5*(10*B*b*c^3 + 9*A*c^4)*x^3
+ 3*(B*b^2*c^2 + 24*A*b*c^3)*x^2 - (4*B*b^3*c - 9*A*b^2*c^2)*x)*sqrt(c*x^2
 + b*x)/(c^3*sqrt(x))
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{\sqrt{x}}\,dx = \int \frac{(x(b+cx))^{\frac{3}{2}}\,(A+Bx)}{\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**(1/2),x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/sqrt(x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(77) = 154$.

Time = 0.04 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.92

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{\sqrt{x}}\,dx = \frac{2\left((15\,c^3x^3+3\,bc^2x^2-4\,b^2cx+8\,b^3)x^2+7\,(3\,bc^2x^3+b^2cx^2-2\,b^3x)x\right)\sqrt{c}}{105\,c^2x^2} + \frac{2\left((35\,c^4x^4+5\,bc^3x^3-6\,b^2c^2x^2+8\,b^3cx-16\,b^4)x^3+3\,(15\,bc^3x^4+3\,b^2c^2x^3-4\,b^3cx^2+8\,b^4x)x^2\right)\sqrt{cx+b}}{315\,c^3x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(1/2),x, algorithm="maxima")
```

output
```
2/105*((15*c^3*x^3 + 3*b*c^2*x^2 - 4*b^2*c*x + 8*b^3)*x^2 + 7*(3*b*c^2*x^3
 + b^2*c*x^2 - 2*b^3*x)*x)*sqrt(c*x + b)*A/(c^2*x^2) + 2/315*((35*c^4*x^4
+ 5*b*c^3*x^3 - 6*b^2*c^2*x^2 + 8*b^3*c*x - 16*b^4)*x^3 + 3*(15*b*c^3*x^4
+ 3*b^2*c^2*x^3 - 4*b^3*c*x^2 + 8*b^4*x)*x^2)*sqrt(c*x + b)*B/(c^3*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(77) = 154$.

Time = 0.16 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.63

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{\sqrt{x}}\,dx = \frac{2\left(3\,(cx+b)^{\frac{5}{2}}-5\,(cx+b)^{\frac{3}{2}}b\right)Ab}{15\,c^2} + \frac{2\left(15\,(cx+b)^{\frac{7}{2}}-42\,(cx+b)^{\frac{5}{2}}b+35\,(cx+b)^{\frac{3}{2}}b^2\right)Bb}{105\,c^3} + \frac{2\left(15\,(cx+b)^{\frac{7}{2}}-42\,(cx+b)^{\frac{5}{2}}b+35\,(cx+b)^{\frac{3}{2}}b^2\right)A}{105\,c^2} + \frac{2\left(35\,(cx+b)^{\frac{9}{2}}-135\,(cx+b)^{\frac{7}{2}}b+189\,(cx+b)^{\frac{5}{2}}b^2-105\,(cx+b)^{\frac{3}{2}}b^3\right)B}{315\,c^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(1/2),x, algorithm="giac")
```

output
```
2/15*(3*(c*x + b)^(5/2) - 5*(c*x + b)^(3/2)*b)*A*b/c^2 + 2/105*(15*(c*x +
b)^(7/2) - 42*(c*x + b)^(5/2)*b + 35*(c*x + b)^(3/2)*b^2)*B*b/c^3 + 2/105*
(15*(c*x + b)^(7/2) - 42*(c*x + b)^(5/2)*b + 35*(c*x + b)^(3/2)*b^2)*A/c^2
 + 2/315*(35*(c*x + b)^(9/2) - 135*(c*x + b)^(7/2)*b + 189*(c*x + b)^(5/2)
*b^2 - 105*(c*x + b)^(3/2)*b^3)*B/c^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{\sqrt{x}}\,dx=\int \frac{\left(c\,x^2+b\,x\right)^{3/2}\,\left(A+B\,x\right)}{\sqrt{x}}\,dx$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(1/2),x)
```

output
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{\sqrt{x}}\,dx=\frac{2\sqrt{cx+b}\,(35b\,c^4x^4+45a\,c^4x^3+50b^2c^3x^3+72ab\,c^3x^2+3b^3c^2x^2+9a\,b^2c^2}{315c^3}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(1/2),x)
```

output
```
(2*sqrt(b + c*x)*( - 18*a*b**3*c + 9*a*b**2*c**2*x + 72*a*b*c**3*x**2 + 45
*a*c**4*x**3 + 8*b**5 - 4*b**4*c*x + 3*b**3*c**2*x**2 + 50*b**2*c**3*x**3
+ 35*b*c**4*x**4))/(315*c**3)
```

3.186 $\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{3/2}}\,dx=-\frac{2(bB-Ac)\left(bx+cx^2\right)^{5/2}}{5c^2x^{5/2}}+\frac{2B\left(bx+cx^2\right)^{7/2}}{7c^2x^{7/2}}$$

output
```
-2/5*(-A*c+B*b)*(c*x^2+b*x)^(5/2)/c^2/x^(5/2)+2/7*B*(c*x^2+b*x)^(7/2)/c^2/
x^(7/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{3/2}}\,dx=\frac{2(x(b+cx))^{5/2}(-2bB+7Ac+5Bcx)}{35c^2x^{5/2}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^(3/2),x]
```

output
```
(2*(x*(b + c*x))^(5/2)*(-2*b*B + 7*A*c + 5*B*c*x))/(35*c^2*x^(5/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{3/2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{2B\left(bx+cx^2\right)^{5/2}}{7cx^{3/2}}-\frac{(2bB-7Ac)\int \frac{\left(cx^2+bx\right)^{3/2}}{x^{3/2}}dx}{7c}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2B\left(bx+cx^2\right)^{5/2}}{7cx^{3/2}}-\frac{2\left(bx+cx^2\right)^{5/2}(2bB-7Ac)}{35c^2x^{5/2}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^(3/2),x]
```

output
```
(-2*(2*b*B - 7*A*c)*(b*x + c*x^2)^(5/2))/(35*c^2*x^(5/2)) + (2*B*(b*x + c*
x^2)^(5/2))/(7*c*x^(3/2))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{2(cx+b)(5Bcx+7Ac-2Bb)\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{35c^{2}x^{\frac{3}{2}}}$	39
default	$\frac{2\sqrt{x(cx+b)}\,(cx+b)^{2}(5Bcx+7Ac-2Bb)}{35\sqrt{x}\,c^{2}}$	39
orering	$\frac{2(cx+b)(5Bcx+7Ac-2Bb)\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}{35c^{2}x^{\frac{3}{2}}}$	39
risch	$\frac{2(cx+b)\sqrt{x}\,\left(5B\,c^{3}x^{3}+7A\,c^{3}x^{2}+8Bb\,c^{2}x^{2}+14Ab\,c^{2}x+B\,b^{2}cx+7A\,b^{2}c-2B\,b^{3}\right)}{35\sqrt{x(cx+b)}\,c^{2}}$	80

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(c*x+b)*(5*B*c*x+7*A*c-2*B*b)*(c*x^2+b*x)^(3/2)/c^2/x^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{3/2}}\,dx=\frac{2\,(5\,Bc^3x^3-2\,Bb^3+7\,Ab^2c+(8\,Bbc^2+7\,Ac^3)x^2+(Bb^2c+14\,Abc^2)x)\sqrt{}}{35\,c^2\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(3/2),x, algorithm="fricas")
```

output
```
2/35*(5*B*c^3*x^3 - 2*B*b^3 + 7*A*b^2*c + (8*B*b*c^2 + 7*A*c^3)*x^2 + (B*b
^2*c + 14*A*b*c^2)*x)*sqrt(c*x^2 + b*x)/(c^2*sqrt(x))
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{3/2}}\,dx = \int \frac{(x(b+cx))^{\frac{3}{2}}\,(A+Bx)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**(3/2),x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/x**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(48) = 96$.

Time = 0.05 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.15

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{3/2}}\,dx = \frac{2\,(5\,bcx^2+5\,b^2x+(3\,c^2x^2+bcx-2\,b^2)x)\sqrt{cx+b}A}{15\,cx} + \frac{2\,((15\,c^3x^3+3\,bc^2x^2-4\,b^2cx+8\,b^3)x^2+7\,(3\,bc^2x^3+b^2cx^2-2\,b^3x)x)\sqrt{cx+b}B}{105\,c^2x^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(3/2),x, algorithm="maxima")
```

output
```
2/15*(5*b*c*x^2 + 5*b^2*x + (3*c^2*x^2 + b*c*x - 2*b^2)*x)*sqrt(c*x + b)*A
/(c*x) + 2/105*((15*c^3*x^3 + 3*b*c^2*x^2 - 4*b^2*c*x + 8*b^3)*x^2 + 7*(3*
b*c^2*x^3 + b^2*c*x^2 - 2*b^3*x)*x)*sqrt(c*x + b)*B/(c^2*x^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(48) = 96$.

Time = 0.14 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.77

$$\int \frac{(A+Bx)\,(bx+cx^2)^{3/2}}{x^{3/2}}\,dx = \frac{2\,(cx+b)^{\frac{3}{2}}Ab}{3\,c} + \frac{2\left(3\,(cx+b)^{\frac{5}{2}}-5\,(cx+b)^{\frac{3}{2}}b\right)Bb}{15\,c^2} + \frac{2\left(3\,(cx+b)^{\frac{5}{2}}-5\,(cx+b)^{\frac{3}{2}}b\right)A}{15\,c} + \frac{2\left(15\,(cx+b)^{\frac{7}{2}}-42\,(cx+b)^{\frac{5}{2}}b+35\,(cx+b)^{\frac{3}{2}}b^2\right)B}{105\,c^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(3/2),x, algorithm="giac")
```

output
```
2/3*(c*x + b)^(3/2)*A*b/c + 2/15*(3*(c*x + b)^(5/2) - 5*(c*x + b)^(3/2)*b)
*B*b/c^2 + 2/15*(3*(c*x + b)^(5/2) - 5*(c*x + b)^(3/2)*b)*A/c + 2/105*(15*
(c*x + b)^(7/2) - 42*(c*x + b)^(5/2)*b + 35*(c*x + b)^(3/2)*b^2)*B/c^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\,(bx+cx^2)^{3/2}}{x^{3/2}}\,dx = \int \frac{(c\,x^2+b\,x)^{3/2}\,(A+B\,x)}{x^{3/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(3/2),x)
```

output
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)\,(bx+cx^2)^{3/2}}{x^{3/2}}\,dx = \frac{2\sqrt{cx+b}\,(5b\,c^3x^3+7a\,c^3x^2+8b^2c^2x^2+14ab\,c^2x+b^3cx+7a\,b^2c-2b^4)}{35c^2}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(3/2),x)
```

output
```
(2*sqrt(b + c*x)*(7*a*b**2*c + 14*a*b*c**2*x + 7*a*c**3*x**2 - 2*b**4 + b*
*3*c*x + 8*b**2*c**2*x**2 + 5*b*c**3*x**3))/(35*c**2)
```

3.187 $\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^{5/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{5/2}}\,dx = \frac{2Ab\sqrt{bx+cx^2}}{\sqrt{x}} + \frac{2A(bx+cx^2)^{3/2}}{3x^{3/2}} + \frac{2B(bx+cx^2)^{5/2}}{5cx^{5/2}} - 2Ab^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)$$

output
```
2*A*b*(c*x^2+b*x)^(1/2)/x^(1/2)+2/3*A*(c*x^2+b*x)^(3/2)/x^(3/2)+2/5*B*(c*x
^2+b*x)^(5/2)/c/x^(5/2)-2*A*b^(3/2)*arctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1
/2))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.95

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{5/2}}\,dx = \frac{2\sqrt{x}\sqrt{b+cx}\left(\sqrt{b+cx}(3b^2B+c^2x(5A+3Bx)+b(20Ac+6Bcx)) - 15A\right.}{15c\sqrt{x(b+cx)}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^(5/2),x]
```

output
```
(2*Sqrt[x]*Sqrt[b + c*x]*(Sqrt[b + c*x]*(3*b^2*B + c^2*x*(5*A + 3*B*x) + b
*(20*A*c + 6*B*c*x)) - 15*A*b^(3/2)*c*ArcTanh[Sqrt[b + c*x]/Sqrt[b]]))/(15
*c*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1221, 1131, 1131, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{5/2}}dx$$

$$\Big\downarrow \text{ 1221}$$

$$A\int \frac{\left(cx^2+bx\right)^{3/2}}{x^{5/2}}dx+\frac{2B\left(bx+cx^2\right)^{5/2}}{5cx^{5/2}}$$

$$\Big\downarrow \text{ 1131}$$

$$A\left(b\int \frac{\sqrt{cx^2+bx}}{x^{3/2}}dx+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}\right)+\frac{2B\left(bx+cx^2\right)^{5/2}}{5cx^{5/2}}$$

$$\Big\downarrow \text{ 1131}$$

$$A\left(b\left(b\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}\right)+\frac{2B\left(bx+cx^2\right)^{5/2}}{5cx^{5/2}}$$

$$\Big\downarrow \text{ 1136}$$

$$A\left(b\left(2b\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}\right)+\frac{2B\left(bx+cx^2\right)^{5/2}}{5cx^{5/2}}$$

$$\Big\downarrow \text{ 220}$$

$$A\left(b\left(\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}-2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)\right)+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}\right)+\frac{2B\left(bx+cx^2\right)^{5/2}}{5cx^{5/2}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^(5/2),x]
```

output
```
(2*B*(b*x + c*x^2)^(5/2))/(5*c*x^(5/2)) + A*((2*(b*x + c*x^2)^(3/2))/(3*x^
(3/2)) + b*((2*Sqrt[b*x + c*x^2])/Sqrt[x] - 2*Sqrt[b]*ArcTanh[Sqrt[b*x + c
*x^2]/(Sqrt[b]*Sqrt[x])]))
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.08

method	result	si
default	$-\frac{2\sqrt{x(cx+b)}\left(-3B\,c^{2}x^{2}\sqrt{cx+b}+15A\,b^{\frac{3}{2}}c\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)-5A\,c^{2}x\sqrt{cx+b}-6Bbcx\sqrt{cx+b}-20Abc\sqrt{cx+b}-3B\,b^{2}\sqrt{cx+b}\right)}{15\sqrt{x}\,\sqrt{cx+b}\,c}$	1

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(x*(c*x+b))^(1/2)*(-3*B*c^2*x^2*(c*x+b)^(1/2)+15*A*b^(3/2)*c*arctanh
((c*x+b)^(1/2)/b^(1/2))-5*A*c^2*x*(c*x+b)^(1/2)-6*B*b*c*x*(c*x+b)^(1/2)-20
*A*b*c*(c*x+b)^(1/2)-3*B*b^2*(c*x+b)^(1/2))/x^(1/2)/(c*x+b)^(1/2)/c
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.89

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{5/2}}\,dx = \left[\frac{15\,Ab^{\frac{3}{2}}cx\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,(3\,Bc^2x^2+3\,Bb^2+20\,Abc+}{15\,cx}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(5/2),x, algorithm="fricas")
```

output
```
[1/15*(15*A*b^(3/2)*c*x*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b)*
sqrt(x))/x^2) + 2*(3*B*c^2*x^2 + 3*B*b^2 + 20*A*b*c + (6*B*b*c + 5*A*c^2)*
x)*sqrt(c*x^2 + b*x)*sqrt(x))/(c*x), 2/15*(15*A*sqrt(-b)*b*c*x*arctan(sqrt
(c*x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) + (3*B*c^2*x^2 + 3*B*b^2 + 20*A*b*c +
(6*B*b*c + 5*A*c^2)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(c*x)]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{5/2}}\,dx = \int \frac{\left(x(b+cx)\right)^{\frac{3}{2}}(A+Bx)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**(5/2),x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/x**(5/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{5/2}}\,dx = \int \frac{\left(cx^2+bx\right)^{\frac{3}{2}}(Bx+A)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(5/2),x, algorithm="maxima")
```

output
```
A*b*integrate(sqrt(c*x + b)/x, x) + 2/15*(5*(B*b*c + A*c^2)*x^2 + (3*B*c^2
*x^2 + B*b*c*x - 2*B*b^2)*x + 5*(B*b^2 + A*b*c)*x)*sqrt(c*x + b)/(c*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.69

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{5/2}}\,dx = \frac{2\,Ab^2\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + \frac{2\left(3\,(cx+b)^{\frac{5}{2}}Bc^4+5\,(cx+b)^{\frac{3}{2}}Ac^5+15\,\sqrt{cx+b}Abc^5\right)}{15\,c^5}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(5/2),x, algorithm="giac")
```

output
```
2*A*b^2*arctan(sqrt(c*x + b)/sqrt(-b))/sqrt(-b) + 2/15*(3*(c*x + b)^(5/2)*
B*c^4 + 5*(c*x + b)^(3/2)*A*c^5 + 15*sqrt(c*x + b)*A*b*c^5)/c^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{5/2}}\,dx = \int \frac{\left(c\,x^2+b\,x\right)^{3/2}\,(A+B\,x)}{x^{5/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(5/2),x)
```

output
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{5/2}}\,dx = \frac{40\sqrt{cx+b}\,abc+10\sqrt{cx+b}\,a\,c^2x+6\sqrt{cx+b}\,b^3+12\sqrt{cx+b}\,b^2cx+6\sqrt{c}}{1}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(5/2),x)
```

output
```
(40*sqrt(b + c*x)*a*b*c + 10*sqrt(b + c*x)*a*c**2*x + 6*sqrt(b + c*x)*b**3
 + 12*sqrt(b + c*x)*b**2*c*x + 6*sqrt(b + c*x)*b*c**2*x**2 + 15*sqrt(b)*lo
g(sqrt(b + c*x) - sqrt(b))*a*b*c - 15*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))
*a*b*c)/(15*c)
```

3.188 $\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{7/2}}\,dx = \frac{(2bB+3Ac)\sqrt{bx+cx^2}}{\sqrt{x}} - \frac{A\left(bx+cx^2\right)^{3/2}}{x^{5/2}} + \frac{2B\left(bx+cx^2\right)^{3/2}}{3x^{3/2}} - \sqrt{b}(2bB+3Ac)\operatorname{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)$$

output
```
(3*A*c+2*B*b)*(c*x^2+b*x)^(1/2)/x^(1/2)-A*(c*x^2+b*x)^(3/2)/x^(5/2)+2/3*B*
(c*x^2+b*x)^(3/2)/x^(3/2)-b^(1/2)*(3*A*c+2*B*b)*arctanh((c*x^2+b*x)^(1/2)/
b^(1/2)/x^(1/2))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{7/2}}\,dx = \frac{\sqrt{x(b+cx)}\left(\sqrt{b+cx}(-3A(b-2cx)+2Bx(4b+cx)) - 3\sqrt{b}(2bB+3Ac)\right)}{3x^{3/2}\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^(7/2),x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[b + c*x]*(-3*A*(b - 2*c*x) + 2*B*x*(4*b + c*x)) -
 3*Sqrt[b]*(2*b*B + 3*A*c)*x*ArcTanh[Sqrt[b + c*x]/Sqrt[b]]))/(3*x^(3/2)*S
qrt[b + c*x])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1220, 1131, 1131, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(3Ac+2bB)\int \frac{\left(cx^2+bx\right)^{3/2}}{x^{5/2}}dx}{2b}-\frac{A\left(bx+cx^2\right)^{5/2}}{bx^{7/2}}$$

$$\Big\downarrow \text{1131}$$

$$\frac{(3Ac+2bB)\left(b\int \frac{\sqrt{cx^2+bx}}{x^{3/2}}dx+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}\right)}{2b}-\frac{A\left(bx+cx^2\right)^{5/2}}{bx^{7/2}}$$

$$\Big\downarrow \text{1131}$$

$$\frac{(3Ac+2bB)\left(b\left(b\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}\right)}{2b}-\frac{A\left(bx+cx^2\right)^{5/2}}{bx^{7/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(3Ac+2bB)\left(b\left(2b\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}\right)}{2b}-\frac{A\left(bx+cx^2\right)^{5/2}}{bx^{7/2}}$$

$$\Big\downarrow \text{220}$$

$$\frac{(3Ac+2bB)\left(b\left(\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}-2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)\right)+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}\right)}{2b}-\frac{A\left(bx+cx^2\right)^{5/2}}{bx^{7/2}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^(7/2),x]
```

output
```
-((A*(b*x + c*x^2)^(5/2))/(b*x^(7/2))) + ((2*b*B + 3*A*c)*((2*(b*x + c*x^2
)^(3/2))/(3*x^(3/2)) + b*((2*Sqrt[b*x + c*x^2])/Sqrt[x] - 2*Sqrt[b]*ArcTan
h[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])))/(2*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

method	result
risch	$-\frac{bA(cx+b)}{\sqrt{x}\,\sqrt{x(cx+b)}}+\frac{\left(\frac{2B(cx+b)^{\frac{3}{2}}}{3}+2Ac\sqrt{cx+b}+2Bb\sqrt{cx+b}-\sqrt{b}\,(3Ac+2Bb)\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)\right)\sqrt{cx+b}\,\sqrt{x}}{\sqrt{x(cx+b)}}$
default	$-\frac{\sqrt{x(cx+b)}\,\left(-2Bc\,x^{2}\sqrt{b}\,\sqrt{cx+b}+9A\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)bcx-6Acx\sqrt{cx+b}\,\sqrt{b}+6B\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)b^{2}x-8B\,b^{\frac{3}{2}}x\sqrt{cx+b}+3A\,b^{\frac{3}{2}}}{3x^{\frac{3}{2}}\sqrt{cx+b}\,\sqrt{b}}$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-b*A*(c*x+b)/x^(1/2)/(x*(c*x+b))^(1/2)+(2/3*B*(c*x+b)^(3/2)+2*A*c*(c*x+b)^
(1/2)+2*B*b*(c*x+b)^(1/2)-b^(1/2)*(3*A*c+2*B*b)*arctanh((c*x+b)^(1/2)/b^(1
/2)))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.64

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{7/2}}\,dx=\left[\frac{3\,(2\,Bb+3\,Ac)\sqrt{b}x^2\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,(2\,Bcx^2-3\,Ab+}{6\,x^2}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(7/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(2*B*b + 3*A*c)*sqrt(b)*x^2*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b
*x)*sqrt(b)*sqrt(x))/x^2) + 2*(2*B*c*x^2 - 3*A*b + 2*(4*B*b + 3*A*c)*x)*sq
rt(c*x^2 + b*x)*sqrt(x))/x^2, 1/3*(3*(2*B*b + 3*A*c)*sqrt(-b)*x^2*arctan(s
qrt(c*x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) + (2*B*c*x^2 - 3*A*b + 2*(4*B*b + 3
*A*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/x^2]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{7/2}}\,dx=\int \frac{(x(b+cx))^{\frac{3}{2}}\,(A+Bx)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**(7/2),x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/x**(7/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{7/2}}\,dx=\int \frac{\left(cx^2+bx\right)^{\frac{3}{2}}(Bx+A)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(7/2),x, algorithm="maxima")
```

output
```
2/3*(B*c*x + B*b)*sqrt(c*x + b) + integrate((A*b + (B*b + A*c)*x)*sqrt(c*x
 + b)/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{7/2}}\,dx=$$
$$-\frac{1}{3}\,c\left(\frac{3\sqrt{cx+b}Ab}{cx}-\frac{3\left(2\,Bb^2+3\,Abc\right)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}c}-\frac{2\left((cx+b)^{\frac{3}{2}}Bc^2+3\sqrt{cx+b}Bbc^2+3\sqrt{cx+\cdots}\right.}{c^3}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(7/2),x, algorithm="giac")
```

output
```
-1/3*c*(3*sqrt(c*x + b)*A*b/(c*x) - 3*(2*B*b^2 + 3*A*b*c)*arctan(sqrt(c*x
+ b)/sqrt(-b))/(sqrt(-b)*c) - 2*((c*x + b)^(3/2)*B*c^2 + 3*sqrt(c*x + b)*B
*b*c^2 + 3*sqrt(c*x + b)*A*c^3)/c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{7/2}}\,dx=\int \frac{{\left(c\,x^2+b\,x\right)}^{3/2}\,\left(A+B\,x\right)}{x^{7/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(7/2),x)
```

output
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.11

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{7/2}}\,dx=\frac{-6\sqrt{cx+b}\,ab+12\sqrt{cx+b}\,acx+16\sqrt{cx+b}\,b^2x+4\sqrt{cx+b}\,bc\,x^2+9\sqrt{b}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(7/2),x)
```

output
```
( - 6*sqrt(b + c*x)*a*b + 12*sqrt(b + c*x)*a*c*x + 16*sqrt(b + c*x)*b**2*x
 + 4*sqrt(b + c*x)*b*c*x**2 + 9*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c*x
 + 6*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*b**2*x - 9*sqrt(b)*log(sqrt(b +
c*x) + sqrt(b))*a*c*x - 6*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*b**2*x)/(6*
x)
```

3.189 $\int \frac{(A+Bx)(bx+cx^2)^{3/2}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{9/2}}\,dx = -\frac{(4bB+3Ac)\sqrt{bx+cx^2}}{4x^{3/2}} + \frac{2Bc\sqrt{bx+cx^2}}{\sqrt{x}} - \frac{A(bx+cx^2)^{3/2}}{2x^{7/2}} - \frac{3c(4bB+Ac)\operatorname{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4\sqrt{b}}$$

output
```
-1/4*(3*A*c+4*B*b)*(c*x^2+b*x)^(1/2)/x^(3/2)+2*B*c*(c*x^2+b*x)^(1/2)/x^(1/
2)-1/2*A*(c*x^2+b*x)^(3/2)/x^(7/2)-3/4*c*(A*c+4*B*b)*arctanh((c*x^2+b*x)^(
1/2)/b^(1/2)/x^(1/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.84

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{9/2}}\,dx = -\frac{\sqrt{x(b+cx)}\left(\sqrt{b}\sqrt{b+cx}(4Bx(b-2cx)+A(2b+5cx))+3c(4bB+Ac)x^2\operatorname{arctanh}\left(\frac{\sqrt{b+cx}}{\sqrt{b}}\right)\right)}{4\sqrt{b}x^{5/2}\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^(9/2),x]
```

output
```
-1/4*(Sqrt[x*(b + c*x)]*(Sqrt[b]*Sqrt[b + c*x]*(4*B*x*(b - 2*c*x) + A*(2*b
 + 5*c*x)) + 3*c*(4*b*B + A*c)*x^2*ArcTanh[Sqrt[b + c*x]/Sqrt[b]]))/(Sqrt[
b]*x^(5/2)*Sqrt[b + c*x])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1220, 1130, 1131, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{9/2}}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(Ac+4bB)\int \frac{\left(cx^2+bx\right)^{3/2}}{x^{7/2}}dx}{4b}-\frac{A\left(bx+cx^2\right)^{5/2}}{2bx^{9/2}}$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{(Ac+4bB)\left(\frac{3}{2}c\int \frac{\sqrt{cx^2+bx}}{x^{3/2}}dx-\frac{\left(bx+cx^2\right)^{3/2}}{x^{5/2}}\right)}{4b}-\frac{A\left(bx+cx^2\right)^{5/2}}{2bx^{9/2}}$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{(Ac+4bB)\left(\frac{3}{2}c\left(b\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)-\frac{\left(bx+cx^2\right)^{3/2}}{x^{5/2}}\right)}{4b}-\frac{A\left(bx+cx^2\right)^{5/2}}{2bx^{9/2}}$$

$$\Big\downarrow \text{ 1136}$$

$$\frac{(Ac+4bB)\left(\frac{3}{2}c\left(2b\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)-\frac{\left(bx+cx^2\right)^{3/2}}{x^{5/2}}\right)}{4b}-\frac{A\left(bx+cx^2\right)^{5/2}}{2bx^{9/2}}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{(Ac+4bB)\left(\frac{3}{2}c\left(\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}-2\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)\right)-\frac{(bx+cx^2)^{3/2}}{x^{5/2}}\right)}{4b}-\frac{A(bx+cx^2)^{5/2}}{2bx^{9/2}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^(9/2),x]
```

output
```
-1/2*(A*(b*x + c*x^2)^(5/2))/(b*x^(9/2)) + ((4*b*B + A*c)*(-((b*x + c*x^2)
^(3/2)/x^(5/2)) + (3*c*((2*Sqrt[b*x + c*x^2])/Sqrt[x] - 2*Sqrt[b]*ArcTanh[
Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])]))/2))/(4*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{(cx+b)(5Acx+4Bbx+2Ab)}{4x^{\frac{3}{2}}\sqrt{x(cx+b)}}+\frac{c\left(16B\sqrt{cx+b}-\frac{2(3Ac+12Bb)\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)}{\sqrt{b}}\right)\sqrt{cx+b}\,\sqrt{x}}{8\sqrt{x(cx+b)}}$
default	$-\frac{\sqrt{x(cx+b)}\left(3A\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^{2}x^{2}+12B\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)bc\,x^{2}-8Bc\,x^{2}\sqrt{b}\,\sqrt{cx+b}+5Acx\sqrt{cx+b}\,\sqrt{b}+4B\,b^{\frac{3}{2}}x\sqrt{cx+b}+2A\,b\right.}{4x^{\frac{5}{2}}\sqrt{cx+b}\,\sqrt{b}}$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x+b)*(5*A*c*x+4*B*b*x+2*A*b)/x^(3/2)/(x*(c*x+b))^(1/2)+1/8*c*(16*B
*(c*x+b)^(1/2)-2*(3*A*c+12*B*b)/b^(1/2)*arctanh((c*x+b)^(1/2)/b^(1/2)))*(c
*x+b)^(1/2)*x^(1/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.74

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{9/2}}\,dx=\left[\frac{3\left(4\,Bbc+Ac^2\right)\sqrt{b}x^3\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\left(8\,Bbcx^2-2\,Ab^2\right.}{8\,bx^3}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(9/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(4*B*b*c + A*c^2)*sqrt(b)*x^3*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 +
 b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(8*B*b*c*x^2 - 2*A*b^2 - (4*B*b^2 + 5*A*b*
c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b*x^3), 1/4*(3*(4*B*b*c + A*c^2)*sqrt(-b
)*x^3*arctan(sqrt(c*x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) + (8*B*b*c*x^2 - 2*A*
b^2 - (4*B*b^2 + 5*A*b*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b*x^3)]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{9/2}}\,dx=\int \frac{(x(b+cx))^{\frac{3}{2}}\,(A+Bx)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**(9/2),x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/x**(9/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{9/2}}\,dx=\int \frac{(cx^2+bx)^{\frac{3}{2}}(Bx+A)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(9/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(3/2)*(B*x + A)/x^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{9/2}}\,dx = \frac{8\sqrt{cx+b}Bc^2 + \frac{3\left(4Bbc^2+Ac^3\right)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{4(cx+b)^{\frac{3}{2}}Bbc^2-4\sqrt{cx+b}Bb^2c^2+5(cx+}{c^2x^2}}{4c}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(9/2),x, algorithm="giac")
```

output
```
1/4*(8*sqrt(c*x + b)*B*c^2 + 3*(4*B*b*c^2 + A*c^3)*arctan(sqrt(c*x + b)/sq
rt(-b))/sqrt(-b) - (4*(c*x + b)^(3/2)*B*b*c^2 - 4*sqrt(c*x + b)*B*b^2*c^2
+ 5*(c*x + b)^(3/2)*A*c^3 - 3*sqrt(c*x + b)*A*b*c^3)/(c^2*x^2))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{9/2}}\,dx = \int \frac{\left(c\,x^2+b\,x\right)^{3/2}\,\left(A+B\,x\right)}{x^{9/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(9/2),x)
```

output
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.23

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{9/2}}\,dx = \frac{-4\sqrt{cx+b}\,a\,b^2-10\sqrt{cx+b}\,abcx-8\sqrt{cx+b}\,b^3x+16\sqrt{cx+b}\,b^2c\,x^2+3}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(9/2),x)
```

output
```
( - 4*sqrt(b + c*x)*a*b**2 - 10*sqrt(b + c*x)*a*b*c*x - 8*sqrt(b + c*x)*b*
*3*x + 16*sqrt(b + c*x)*b**2*c*x**2 + 3*sqrt(b)*log(sqrt(b + c*x) - sqrt(b
))*a*c**2*x**2 + 12*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*b**2*c*x**2 - 3*s
qrt(b)*log(sqrt(b + c*x) + sqrt(b))*a*c**2*x**2 - 12*sqrt(b)*log(sqrt(b +
c*x) + sqrt(b))*b**2*c*x**2)/(8*b*x**2)
```

3.190 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{11/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 134

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{11/2}} \, dx = -\frac{(2bB+Ac)\sqrt{bx+cx^2}}{4x^{5/2}} - \frac{c(10bB+Ac)\sqrt{bx+cx^2}}{8bx^{3/2}} - \frac{A\left(bx+cx^2\right)^{3/2}}{3x^{9/2}} - \frac{c^2(6bB-Ac)\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{3/2}}$$

output
```
-1/4*(A*c+2*B*b)*(c*x^2+b*x)^(1/2)/x^(5/2)-1/8*c*(A*c+10*B*b)*(c*x^2+b*x)^
(1/2)/b/x^(3/2)-1/3*A*A*(c*x^2+b*x)^(3/2)/x^(9/2)-1/8*c^2*(-A*c+6*B*b)*arcta
nh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{11/2}} \, dx = \frac{\sqrt{x(b+cx)}\left(-\sqrt{b}\sqrt{b+cx}(6bBx(2b+5cx)+A(8b^2+14bcx+3c^2x^2))+\right.}{24b^{3/2}x^{7/2}\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^(11/2),x]
```

output
```
(Sqrt[x*(b + c*x)]*(-(Sqrt[b]*Sqrt[b + c*x]*(6*b*B*x*(2*b + 5*c*x) + A*(8*
b^2 + 14*b*c*x + 3*c^2*x^2))) + 3*c^2*(-6*b*B + A*c)*x^3*ArcTanh[Sqrt[b +
c*x]/Sqrt[b]]))/(24*b^(3/2)*x^(7/2)*Sqrt[b + c*x])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1220, 1130, 1130, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{11/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(6bB-Ac)\int \frac{\left(cx^2+bx\right)^{3/2}}{x^{9/2}}dx}{6b}-\frac{A\left(bx+cx^2\right)^{5/2}}{3bx^{11/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(6bB-Ac)\left(\frac{3}{4}c\int \frac{\sqrt{cx^2+bx}}{x^{5/2}}dx-\frac{\left(bx+cx^2\right)^{3/2}}{2x^{7/2}}\right)}{6b}-\frac{A\left(bx+cx^2\right)^{5/2}}{3bx^{11/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(6bB-Ac)\left(\frac{3}{4}c\left(\frac{1}{2}c\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx-\frac{\sqrt{bx+cx^2}}{x^{3/2}}\right)-\frac{\left(bx+cx^2\right)^{3/2}}{2x^{7/2}}\right)}{6b}-\frac{A\left(bx+cx^2\right)^{5/2}}{3bx^{11/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(6bB-Ac)\left(\frac{3}{4}c\left(c\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}-\frac{\sqrt{bx+cx^2}}{x^{3/2}}\right)-\frac{\left(bx+cx^2\right)^{3/2}}{2x^{7/2}}\right)}{6b}-\frac{A\left(bx+cx^2\right)^{5/2}}{3bx^{11/2}}$$

$$\Big\downarrow \text{220}$$

$$\frac{(6bB-Ac)\left(\frac{3}{4}c\left(-\frac{c\,\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}-\frac{\sqrt{bx+cx^2}}{x^{3/2}}\right)-\frac{\left(bx+cx^2\right)^{3/2}}{2x^{7/2}}\right)}{6b}-\frac{A\left(bx+cx^2\right)^{5/2}}{3bx^{11/2}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^(11/2),x]
```

output
```
-1/3*(A*(b*x + c*x^2)^(5/2))/(b*x^(11/2)) + ((6*b*B - A*c)*(-1/2*(b*x + c*
x^2)^(3/2)/x^(7/2) + (3*c*(-(Sqrt[b*x + c*x^2]/x^(3/2)) - (c*ArcTanh[Sqrt[
b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/Sqrt[b]))/4))/(6*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{(cx+b)(3A\,c^2x^2+30x^2Bbc+14Abcx+12xB\,b^2+8b^2A)}{24x^{\frac{5}{2}}b\sqrt{x(cx+b)}}+\frac{c^2(Ac-6Bb)\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)\sqrt{cx+b}\,\sqrt{x}}{8b^{\frac{3}{2}}\sqrt{x(cx+b)}}$
default	$\frac{\sqrt{x(cx+b)}\left(3A\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^3x^3-18B\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)b\,c^2x^3-3A\,c^2x^2\sqrt{cx+b}\,\sqrt{b}-30B\,b^{\frac{3}{2}}c\,x^2\sqrt{cx+b}-14A\,b^{\frac{3}{2}}cx\sqrt{cx+b}-\right.}{24b^{\frac{3}{2}}x^{\frac{7}{2}}\sqrt{cx+b}}$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x+b)*(3*A*c^2*x^2+30*B*b*c*x^2+14*A*b*c*x+12*B*b^2*x+8*A*b^2)/x^(
5/2)/b/(x*(c*x+b))^(1/2)+1/8*c^2*(A*c-6*B*b)/b^(3/2)*arctanh((c*x+b)^(1/2)
/b^(1/2))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.81

$$\int \frac{(A+Bx)\,(bx+cx^2)^{3/2}}{x^{11/2}}\,dx = \left[-\frac{3\,(6\,Bbc^2-Ac^3)\sqrt{b}x^4\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,(8\,Ab^3+3\,(10\right.}{48\,b^2x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(11/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(6*B*b*c^2 - A*c^3)*sqrt(b)*x^4*log(-(c*x^2 + 2*b*x + 2*sqrt(c*x
^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(8*A*b^3 + 3*(10*B*b^2*c + A*b*c^2)*x^
2 + 2*(6*B*b^3 + 7*A*b^2*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^2*x^4), 1/24*
(3*(6*B*b*c^2 - A*c^3)*sqrt(-b)*x^4*arctan(sqrt(c*x^2 + b*x)*sqrt(-b)/(b*s
qrt(x))) - (8*A*b^3 + 3*(10*B*b^2*c + A*b*c^2)*x^2 + 2*(6*B*b^3 + 7*A*b^2*
c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^2*x^4)]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{11/2}}\,dx=\int \frac{(x(b+cx))^{\frac{3}{2}}\,(A+Bx)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**(11/2),x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/x**(11/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{11/2}}\,dx=\int \frac{(cx^2+bx)^{\frac{3}{2}}\,(Bx+A)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(11/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(3/2)*(B*x + A)/x^(11/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.96

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{11/2}}\,dx=\frac{1}{24}\,c^3\left(\frac{3\,(6\,Bb-Ac)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}bc}-\frac{30\,(cx+b)^{\frac{5}{2}}Bb-48\,(cx+b)^{\frac{3}{2}}Bb$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(11/2),x, algorithm="giac")
```

output
```
1/24*c^3*(3*(6*B*b - A*c)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b*c) -
(30*(c*x + b)^(5/2)*B*b - 48*(c*x + b)^(3/2)*B*b^2 + 18*sqrt(c*x + b)*B*b^
3 + 3*(c*x + b)^(5/2)*A*c + 8*(c*x + b)^(3/2)*A*b*c - 3*sqrt(c*x + b)*A*b^
2*c)/(b*c^4*x^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{11/2}}\,dx=\int \frac{\left(c\,x^2+b\,x\right)^{3/2}\,\left(A+B\,x\right)}{x^{11/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(11/2),x)
```

output
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.27

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{11/2}}\,dx=\frac{-16\sqrt{cx+b}\,a\,b^3-28\sqrt{cx+b}\,a\,b^2cx-6\sqrt{cx+b}\,ab\,c^2x^2-24\sqrt{cx+b}\,b^4x}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(11/2),x)
```

output
```
( - 16*sqrt(b + c*x)*a*b**3 - 28*sqrt(b + c*x)*a*b**2*c*x - 6*sqrt(b + c*x
)*a*b*c**2*x**2 - 24*sqrt(b + c*x)*b**4*x - 60*sqrt(b + c*x)*b**3*c*x**2 -
 3*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c**3*x**3 + 18*sqrt(b)*log(sqrt(
b + c*x) - sqrt(b))*b**2*c**2*x**3 + 3*sqrt(b)*log(sqrt(b + c*x) + sqrt(b)
)*a*c**3*x**3 - 18*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*b**2*c**2*x**3)/(4
8*b**2*x**3)
```

3.191 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{13/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{13/2}}\,dx = -\frac{(8bB+3Ac)\sqrt{bx+cx^2}}{24x^{7/2}} - \frac{c(56bB+3Ac)\sqrt{bx+cx^2}}{96bx^{5/2}} - \frac{c^2(8bB-3Ac)\sqrt{bx+cx^2}}{64b^2x^{3/2}} - \frac{A(bx+cx^2)^{3/2}}{4x^{11/2}} + \frac{c^3(8bB-3Ac)\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{64b^{5/2}}$$

output
```
-1/24*(3*A*c+8*B*b)*(c*x^2+b*x)^(1/2)/x^(7/2)-1/96*c*(3*A*c+56*B*b)*(c*x^2
+b*x)^(1/2)/b/x^(5/2)-1/64*c^2*(-3*A*c+8*B*b)*(c*x^2+b*x)^(1/2)/b^2/x^(3/2
)-1/4*A*(c*x^2+b*x)^(3/2)/x^(11/2)+1/64*c^3*(-3*A*c+8*B*b)*arctanh((c*x^2+
b*x)^(1/2)/b^(1/2)/x^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{13/2}}\,dx =$$
$$-\frac{\sqrt{x(b+cx)}\left(\sqrt{b}\sqrt{b+cx}(8bBx(8b^2+14bcx+3c^2x^2)+A(48b^3+72b^2cx+6bc^2x^2-9c^3x^3))+3c^3(-8b\right.}{192b^{5/2}x^{9/2}\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^(13/2),x]
```

output
```
-1/192*(Sqrt[x*(b + c*x)]*(Sqrt[b]*Sqrt[b + c*x]*(8*b*B*x*(8*b^2 + 14*b*c*
x + 3*c^2*x^2) + A*(48*b^3 + 72*b^2*c*x + 6*b*c^2*x^2 - 9*c^3*x^3)) + 3*c^
3*(-8*b*B + 3*A*c)*x^4*ArcTanh[Sqrt[b + c*x]/Sqrt[b]]))/(b^(5/2)*x^(9/2)*S
qrt[b + c*x])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1130, 1130, 1135, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{13/2}}\,dx$$

$$\Big\downarrow \ 1220$$

$$\frac{(8bB-3Ac)\int \frac{\left(cx^2+bx\right)^{3/2}}{x^{11/2}}dx}{8b}-\frac{A\left(bx+cx^2\right)^{5/2}}{4bx^{13/2}}$$

$$\Big\downarrow \ 1130$$

$$\frac{(8bB-3Ac)\left(\frac{1}{2}c\int \frac{\sqrt{cx^2+bx}}{x^{7/2}}dx-\frac{\left(bx+cx^2\right)^{3/2}}{3x^{9/2}}\right)}{8b}-\frac{A\left(bx+cx^2\right)^{5/2}}{4bx^{13/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(8bB-3Ac)\left(\frac{1}{2}c\left(\frac{1}{4}c\int \frac{1}{x^{3/2}\sqrt{cx^2+bx}}dx-\frac{\sqrt{bx+cx^2}}{2x^{5/2}}\right)-\frac{\left(bx+cx^2\right)^{3/2}}{3x^{9/2}}\right)}{8b}-\frac{A\left(bx+cx^2\right)^{5/2}}{4bx^{13/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{(8bB-3Ac)\left(\frac{1}{2}c\left(\frac{1}{4}c\left(-\frac{c\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{2b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)-\frac{\sqrt{bx+cx^2}}{2x^{5/2}}\right)-\frac{\left(bx+cx^2\right)^{3/2}}{3x^{9/2}}\right)}{8b}-\frac{A\left(bx+cx^2\right)^{5/2}}{4bx^{13/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(8bB-3Ac)\left(\frac{1}{2}c\left(\frac{1}{4}c\left(-\frac{c\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)-\frac{\sqrt{bx+cx^2}}{2x^{5/2}}\right)-\frac{\left(bx+cx^2\right)^{3/2}}{3x^{9/2}}\right)}{8b}-\frac{A\left(bx+cx^2\right)^{5/2}}{4bx^{13/2}}$$

$$\Big\downarrow \text{220}$$

$$\frac{(8bB-3Ac)\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)-\frac{\sqrt{bx+cx^2}}{2x^{5/2}}\right)-\frac{\left(bx+cx^2\right)^{3/2}}{3x^{9/2}}\right)}{8b}-\frac{A\left(bx+cx^2\right)^{5/2}}{4bx^{13/2}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^(13/2),x]
```

output
```
-1/4*(A*(b*x + c*x^2)^(5/2))/(b*x^(13/2)) + ((8*b*B - 3*A*c)*(-1/3*(b*x +
c*x^2)^(3/2)/x^(9/2) + (c*(-1/2*Sqrt[b*x + c*x^2]/x^(5/2) + (c*(-(Sqrt[b*x
 + c*x^2]/(b*x^(3/2))) + (c*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/
b^(3/2)))/4))/2))/(8*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{(cx+b)(-9A\,c^3x^3+24x^3Bb\,c^2+6Ab\,c^2x^2+112x^2B\,b^2c+72A\,b^2cx+64xB\,b^3+48A\,b^3)}{192x^{\frac{7}{2}}b^2\sqrt{x(cx+b)}}-\frac{c^3(3Ac-8Bb)\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)\sqrt{cx+}}{64b^{\frac{5}{2}}\sqrt{x(cx+b)}}$
default	$-\frac{\sqrt{x(cx+b)}\left(9A\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^4x^4-24B\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)b\,c^3x^4-9A\,c^3x^3\sqrt{cx+b}\,\sqrt{b}+24B\,b^{\frac{3}{2}}c^2x^3\sqrt{cx+b}+6A\,b^{\frac{3}{2}}c^2x^2\sqrt{cx}\right.}{192b^{\frac{5}{2}}x^{\frac{9}{2}}\sqrt{cx+b}}$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(c*x+b)*(-9*A*c^3*x^3+24*B*b*c^2*x^3+6*A*b*c^2*x^2+112*B*b^2*c*x^2+
72*A*b^2*c*x+64*B*b^3*x+48*A*b^3)/x^(7/2)/b^2/(x*(c*x+b))^(1/2)-1/64*c^3*(
3*A*c-8*B*b)/b^(5/2)*arctanh((c*x+b)^(1/2)/b^(1/2))*(c*x+b)^(1/2)*x^(1/2)/
(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.68

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{13/2}}\,dx = \left[-\frac{3\left(8\,Bbc^3-3\,Ac^4\right)\sqrt{b}x^5\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\left(48\,Ab^4+3\,(\right.}{}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(13/2),x, algorithm="fricas")
```

output
```
[-1/384*(3*(8*B*b*c^3 - 3*A*c^4)*sqrt(b)*x^5*log(-(c*x^2 + 2*b*x - 2*sqrt(
c*x^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(48*A*b^4 + 3*(8*B*b^2*c^2 - 3*A*b*
c^3)*x^3 + 2*(56*B*b^3*c + 3*A*b^2*c^2)*x^2 + 8*(8*B*b^4 + 9*A*b^3*c)*x)*s
qrt(c*x^2 + b*x)*sqrt(x))/(b^3*x^5), -1/192*(3*(8*B*b*c^3 - 3*A*c^4)*sqrt(
-b)*x^5*arctan(sqrt(c*x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) + (48*A*b^4 + 3*(8*
B*b^2*c^2 - 3*A*b*c^3)*x^3 + 2*(56*B*b^3*c + 3*A*b^2*c^2)*x^2 + 8*(8*B*b^4
 + 9*A*b^3*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^3*x^5)]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{13/2}}\,dx=\int \frac{(x(b+cx))^{\frac{3}{2}}\,(A+Bx)}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**(13/2),x)
```

output
```
Integral((x*(b + c*x))**(3/2)*(A + B*x)/x**(13/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{13/2}}\,dx=\int \frac{\left(cx^2+bx\right)^{\frac{3}{2}}(Bx+A)}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(13/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(3/2)*(B*x + A)/x^(13/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.02

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{13/2}}\,dx=$$
$$-\frac{\frac{3\left(8\,Bbc^4-3\,Ac^5\right)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{24\,(cx+b)^{\frac{7}{2}}\,Bbc^4+40\,(cx+b)^{\frac{5}{2}}\,Bb^2c^4-88\,(cx+b)^{\frac{3}{2}}\,Bb^3c^4+24\,\sqrt{cx+b}Bb^4c^4-9\,(cx+b)^{\frac{7}{2}}\,Ac^5+33\,(cx+b)^{\frac{5}{2}}\,\ldots}{b^2c^4x^4}}{192\,c}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(13/2),x, algorithm="giac")
```

output
```
-1/192*(3*(8*B*b*c^4 - 3*A*c^5)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b
^2) + (24*(c*x + b)^(7/2)*B*b*c^4 + 40*(c*x + b)^(5/2)*B*b^2*c^4 - 88*(c*x
 + b)^(3/2)*B*b^3*c^4 + 24*sqrt(c*x + b)*B*b^4*c^4 - 9*(c*x + b)^(7/2)*A*c
^5 + 33*(c*x + b)^(5/2)*A*b*c^5 + 33*(c*x + b)^(3/2)*A*b^2*c^5 - 9*sqrt(c*
x + b)*A*b^3*c^5)/(b^2*c^4*x^4))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{13/2}}\,dx=\int \frac{\left(c\,x^2+b\,x\right)^{3/2}\,\left(A+B\,x\right)}{x^{13/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(13/2),x)
```

output
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(13/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{13/2}}\,dx=\frac{-96\sqrt{cx+b}\,a\,b^4-144\sqrt{cx+b}\,a\,b^3cx-12\sqrt{cx+b}\,a\,b^2c^2x^2+18\sqrt{cx+b}\,a}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(13/2),x)
```

output
```
( - 96*sqrt(b + c*x)*a*b**4 - 144*sqrt(b + c*x)*a*b**3*c*x - 12*sqrt(b + c
*x)*a*b**2*c**2*x**2 + 18*sqrt(b + c*x)*a*b*c**3*x**3 - 128*sqrt(b + c*x)*
b**5*x - 224*sqrt(b + c*x)*b**4*c*x**2 - 48*sqrt(b + c*x)*b**3*c**2*x**3 +
 9*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c**4*x**4 - 24*sqrt(b)*log(sqrt(
b + c*x) - sqrt(b))*b**2*c**3*x**4 - 9*sqrt(b)*log(sqrt(b + c*x) + sqrt(b)
)*a*c**4*x**4 + 24*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*b**2*c**3*x**4)/(3
84*b**3*x**4)
```

3.192 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{15/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 209

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{15/2}}\,dx = -\frac{(10bB+3Ac)\sqrt{bx+cx^2}}{40x^{9/2}} - \frac{c(30bB+Ac)\sqrt{bx+cx^2}}{80bx^{7/2}} - \frac{c^2(2bB-Ac)\sqrt{bx+cx^2}}{64b^2x^{5/2}} + \frac{3c^3(2bB-Ac)\sqrt{bx+cx^2}}{128b^3x^{3/2}} - \frac{A\left(bx+cx^2\right)^{3/2}}{5x^{13/2}} - \frac{3c^4(2bB-Ac)\operatorname{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{128b^{7/2}}$$

output
```
-1/40*(3*A*c+10*B*b)*(c*x^2+b*x)^(1/2)/x^(9/2)-1/80*c*(A*c+30*B*b)*(c*x^2+
b*x)^(1/2)/b/x^(7/2)-1/64*c^2*(-A*c+2*B*b)*(c*x^2+b*x)^(1/2)/b^2/x^(5/2)+3
/128*c^3*(-A*c+2*B*b)*(c*x^2+b*x)^(1/2)/b^3/x^(3/2)-1/5*A*(c*x^2+b*x)^(3/2
)/x^(13/2)-3/128*c^4*(-A*c+2*B*b)*arctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1/2
))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.76

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{15/2}}\,dx = \frac{-\sqrt{b}(b+cx)\left(10bBx(16b^3+24b^2cx+2bc^2x^2-3c^3x^3)+A(128b^4+176b^3\right.}{640b^{7/2}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(3/2))/x^(15/2),x]
```

output
```
(-(Sqrt[b]*(b + c*x)*(10*b*B*x*(16*b^3 + 24*b^2*c*x + 2*b*c^2*x^2 - 3*c^3*
x^3) + A*(128*b^4 + 176*b^3*c*x + 8*b^2*c^2*x^2 - 10*b*c^3*x^3 + 15*c^4*x^
4))) + 15*c^4*(-2*b*B + A*c)*x^5*Sqrt[b + c*x]*ArcTanh[Sqrt[b + c*x]/Sqrt[
b]])/(640*b^(7/2)*x^(9/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1220, 1130, 1130, 1135, 1135, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{15/2}}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(2bB-Ac)\int \frac{(cx^2+bx)^{3/2}}{x^{13/2}}dx}{2b}-\frac{A\left(bx+cx^2\right)^{5/2}}{5bx^{15/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(2bB-Ac)\left(\frac{3}{8}c\int \frac{\sqrt{cx^2+bx}}{x^{9/2}}dx-\frac{(bx+cx^2)^{3/2}}{4x^{11/2}}\right)}{2b}-\frac{A\left(bx+cx^2\right)^{5/2}}{5bx^{15/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(2bB-Ac)\left(\frac{3}{8}c\left(\frac{1}{6}c\int\frac{1}{x^{5/2}\sqrt{cx^2+bx}}dx-\frac{\sqrt{bx+cx^2}}{3x^{7/2}}\right)-\frac{(bx+cx^2)^{3/2}}{4x^{11/2}}\right)}{2b}-\frac{A(bx+cx^2)^{5/2}}{5bx^{15/2}}$$

$$\Big\downarrow \ 1135$$

$$\frac{(2bB-Ac)\left(\frac{3}{8}c\left(\frac{1}{6}c\left(-\frac{3c\int\frac{1}{x^{3/2}\sqrt{cx^2+bx}}dx}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)-\frac{\sqrt{bx+cx^2}}{3x^{7/2}}\right)-\frac{(bx+cx^2)^{3/2}}{4x^{11/2}}\right)}{2b}-\frac{A(bx+cx^2)^{5/2}}{5bx^{15/2}}$$

$$\Big\downarrow \ 1135$$

$$\frac{(2bB-Ac)\left(\frac{3}{8}c\left(\frac{1}{6}c\left(-\frac{3c\left(-\frac{c\int\frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{2b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)-\frac{\sqrt{bx+cx^2}}{3x^{7/2}}\right)-\frac{(bx+cx^2)^{3/2}}{4x^{11/2}}\right)}{2b}-\frac{A(bx+cx^2)^{5/2}}{5bx^{15/2}}$$

$$\Big\downarrow \ 1136$$

$$\frac{(2bB-Ac)\left(\frac{3}{8}c\left(\frac{1}{6}c\left(-\frac{3c\left(-\frac{c\int\frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)-\frac{\sqrt{bx+cx^2}}{3x^{7/2}}\right)-\frac{(bx+cx^2)^{3/2}}{4x^{11/2}}\right)}{2b}-\frac{A(bx+cx^2)^{5/2}}{5bx^{15/2}}$$

$$\Big\downarrow \ 220$$

$$\frac{(2bB-Ac)\left(\frac{3}{8}c\left(\frac{1}{6}c\left(-\frac{3c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)-\frac{\sqrt{bx+cx^2}}{3x^{7/2}}\right)-\frac{(bx+cx^2)^{3/2}}{4x^{11/2}}\right)}{2b}-\frac{A(bx+cx^2)^{5/2}}{5bx^{15/2}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(3/2))/x^(15/2),x]
```

output
```
-1/5*(A*(b*x + c*x^2)^(5/2))/(b*x^(15/2)) + ((2*b*B - A*c)*(-1/4*(b*x + c*
x^2)^(3/2)/x^(11/2) + (3*c*(-1/3*Sqrt[b*x + c*x^2]/x^(7/2) + (c*(-1/2*Sqrt
[b*x + c*x^2]/(b*x^(5/2)) - (3*c*(-(Sqrt[b*x + c*x^2]/(b*x^(3/2))) + (c*Ar
cTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/b^(3/2)))/(4*b)))/6))/8))/(2*b
)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{(cx+b)(15A\,c^4x^4-30Bb\,c^3x^4-10Ab\,c^3x^3+20B\,b^2c^2x^3+8A\,b^2c^2x^2+240B\,b^3c\,x^2+176A\,b^3cx+160B\,b^4x+128A\,b^4)}{640x^{\frac{9}{2}}b^3\sqrt{x(cx+b)}}+\frac{3c^4(Ac-$
default	$\frac{\sqrt{x(cx+b)}\left(15A\,\text{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^5x^5-30B\,\text{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)b\,c^4x^5-15A\,c^4x^4\sqrt{cx+b}\,\sqrt{b}+30B\,b^{\frac{3}{2}}c^3x^4\sqrt{cx+b}+10A\,b^{\frac{3}{2}}c^3x^3\sqrt{c}\right.}{640b^{\frac{7}{2}}x^{\frac{11}{2}}\sqrt{}}$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-1/640*(c*x+b)*(15*A*c^4*x^4-30*B*b*c^3*x^4-10*A*b*c^3*x^3+20*B*b^2*c^2*x^
3+8*A*b^2*c^2*x^2+240*B*b^3*c*x^2+176*A*b^3*c*x+160*B*b^4*x+128*A*b^4)/x^(
9/2)/b^3/(x*(c*x+b))^(1/2)+3/128*c^4*(A*c-2*B*b)/b^(7/2)*arctanh((c*x+b)^(
1/2)/b^(1/2))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 338, normalized size of antiderivative = 1.62

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{15/2}}\,dx=\left[-\frac{15\left(2\,Bbc^4-Ac^5\right)\sqrt{b}x^6\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\left(128\,Ab^5-1\right.}{}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(15/2),x, algorithm="fricas")
```

output
```
[-1/1280*(15*(2*B*b*c^4 - A*c^5)*sqrt(b)*x^6*log(-(c*x^2 + 2*b*x + 2*sqrt(
c*x^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(128*A*b^5 - 15*(2*B*b^2*c^3 - A*b*
c^4)*x^4 + 10*(2*B*b^3*c^2 - A*b^2*c^3)*x^3 + 8*(30*B*b^4*c + A*b^3*c^2)*x
^2 + 16*(10*B*b^5 + 11*A*b^4*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^4*x^6), 1
/640*(15*(2*B*b*c^4 - A*c^5)*sqrt(-b)*x^6*arctan(sqrt(c*x^2 + b*x)*sqrt(-b
)/(b*sqrt(x))) - (128*A*b^5 - 15*(2*B*b^2*c^3 - A*b*c^4)*x^4 + 10*(2*B*b^3
*c^2 - A*b^2*c^3)*x^3 + 8*(30*B*b^4*c + A*b^3*c^2)*x^2 + 16*(10*B*b^5 + 11
*A*b^4*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^4*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\,(bx+cx^2)^{3/2}}{x^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(3/2)/x**(15/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)\,(bx+cx^2)^{3/2}}{x^{15/2}}\,dx = \int \frac{(cx^2+bx)^{\frac{3}{2}}(Bx+A)}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(15/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(3/2)*(B*x + A)/x^(15/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.80

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{15/2}}\,dx = \frac{1}{640}\,c^5\left(\frac{15\,(2\,Bb-Ac)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3c}+\frac{30\,(cx+b)^{\frac{9}{2}}Bb-140\,(cx+b)^{\frac{7}{2}}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(3/2)/x^(15/2),x, algorithm="giac")
```

output
```
1/640*c^5*(15*(2*B*b - A*c)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^3*c
) + (30*(c*x + b)^(9/2)*B*b - 140*(c*x + b)^(7/2)*B*b^2 + 140*(c*x + b)^(3
/2)*B*b^4 - 30*sqrt(c*x + b)*B*b^5 - 15*(c*x + b)^(9/2)*A*c + 70*(c*x + b)
^(7/2)*A*b*c - 128*(c*x + b)^(5/2)*A*b^2*c - 70*(c*x + b)^(3/2)*A*b^3*c +
15*sqrt(c*x + b)*A*b^4*c)/(b^3*c^6*x^5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{15/2}}\,dx = \int \frac{\left(c\,x^2+b\,x\right)^{3/2}\,(A+B\,x)}{x^{15/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(15/2),x)
```

output
```
int(((b*x + c*x^2)^(3/2)*(A + B*x))/x^(15/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.15

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{3/2}}{x^{15/2}}\,dx = \frac{-256\sqrt{cx+b}\,a\,b^5-352\sqrt{cx+b}\,a\,b^4cx-16\sqrt{cx+b}\,a\,b^3c^2x^2+20\sqrt{cx+b}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(3/2)/x^(15/2),x)
```

output
```
( - 256*sqrt(b + c*x)*a*b**5 - 352*sqrt(b + c*x)*a*b**4*c*x - 16*sqrt(b +
c*x)*a*b**3*c**2*x**2 + 20*sqrt(b + c*x)*a*b**2*c**3*x**3 - 30*sqrt(b + c*
x)*a*b*c**4*x**4 - 320*sqrt(b + c*x)*b**6*x - 480*sqrt(b + c*x)*b**5*c*x**
2 - 40*sqrt(b + c*x)*b**4*c**2*x**3 + 60*sqrt(b + c*x)*b**3*c**3*x**4 - 15
*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c**5*x**5 + 30*sqrt(b)*log(sqrt(b
+ c*x) - sqrt(b))*b**2*c**4*x**5 + 15*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))
*a*c**5*x**5 - 30*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*b**2*c**4*x**5)/(12
80*b**4*x**5)
```

3.193 $\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=-\frac{2b^4(bB-Ac)\left(bx+cx^2\right)^{7/2}}{7c^6x^{7/2}}+\frac{2b^3(5bB-4Ac)\left(bx+cx^2\right)^{9/2}}{9c^6x^{9/2}}-\frac{4b^2(5bB-3Ac)\left(bx+cx^2\right)^{11/2}}{11c^6x^{11/2}}+\frac{4b(5bB-2Ac)\left(bx+cx^2\right)^{13/2}}{13c^6x^{13/2}}-\frac{2(5bB-Ac)\left(bx+cx^2\right)^{15/2}}{15c^6x^{15/2}}+\frac{2B\left(bx+cx^2\right)^{17/2}}{17c^6x^{17/2}}$$

output
```
-2/7*b^4*(-A*c+B*b)*(c*x^2+b*x)^(7/2)/c^6/x^(7/2)+2/9*b^3*(-4*A*c+5*B*b)*(
c*x^2+b*x)^(9/2)/c^6/x^(9/2)-4/11*b^2*(-3*A*c+5*B*b)*(c*x^2+b*x)^(11/2)/c^
6/x^(11/2)+4/13*b*(-2*A*c+5*B*b)*(c*x^2+b*x)^(13/2)/c^6/x^(13/2)-2/15*(-A*
c+5*B*b)*(c*x^2+b*x)^(15/2)/c^6/x^(15/2)+2/17*B*(c*x^2+b*x)^(17/2)/c^6/x^(
17/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.58

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\frac{2(b+cx)^3\sqrt{x(b+cx)}(-1280b^5B+3003c^5x^4(17A+15Bx)+128b^4c(17A+35Bx)-224b}{765765c^6\sqrt{x}}$$

input
```
Integrate[x^(3/2)*(A + B*x)*(b*x + c*x^2)^(5/2),x]
```

output
```
(2*(b + c*x)^3*Sqrt[x*(b + c*x)]*(-1280*b^5*B + 3003*c^5*x^4*(17*A + 15*B*
x) + 128*b^4*c*(17*A + 35*B*x) - 224*b^3*c^2*x*(34*A + 45*B*x) + 336*b^2*c
^3*x^2*(51*A + 55*B*x) - 462*b*c^4*x^3*(68*A + 65*B*x)))/(765765*c^6*Sqrt[
x])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1221, 1128, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{2Bx^{3/2}\left(bx+cx^2\right)^{7/2}}{17c}-\frac{(10bB-17Ac)\int x^{3/2}\left(cx^2+bx\right)^{5/2}dx}{17c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2Bx^{3/2}\left(bx+cx^2\right)^{7/2}}{17c}-\frac{(10bB-17Ac)\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{7/2}}{15c}-\frac{8b\int\sqrt{x}\left(cx^2+bx\right)^{5/2}dx}{15c}\right)}{17c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2Bx^{3/2}\left(bx+cx^2\right)^{7/2}}{17c}-\frac{(10bB-17Ac)\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{7/2}}{15c}-\frac{8b\left(\frac{2\left(bx+cx^2\right)^{7/2}}{13c\sqrt{x}}-\frac{6b\int\frac{\left(cx^2+bx\right)^{5/2}}{\sqrt{x}}dx}{13c}\right)}{15c}\right)}{17c}$$

$\Big\downarrow$ 1128

$$\frac{2Bx^{3/2}\left(bx+cx^2\right)^{7/2}}{17c}-\frac{(10bB-17Ac)\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{7/2}}{15c}-\frac{8b\left(\frac{2\left(bx+cx^2\right)^{7/2}}{13c\sqrt{x}}-\frac{6b\left(\frac{2\left(bx+cx^2\right)^{7/2}}{11cx^{3/2}}-\frac{4b\int\frac{\left(cx^2+bx\right)^{5/2}}{x^{3/2}}dx}{11c}\right)}{13c}\right)}{15c}\right)}{17c}$$

$\Big\downarrow$ 1128

$$\frac{2Bx^{3/2}\left(bx+cx^2\right)^{7/2}}{17c}-\frac{(10bB-17Ac)\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{7/2}}{15c}-\frac{8b\left(\frac{2\left(bx+cx^2\right)^{7/2}}{13c\sqrt{x}}-\frac{6b\left(\frac{2\left(bx+cx^2\right)^{7/2}}{11cx^{3/2}}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{7/2}}{9cx^{5/2}}-\frac{2b\int\frac{\left(cx^2+bx\right)^{5/2}}{x^{5/2}}dx}{9c}\right)}{11c}\right)}{13c}\right)}{15c}\right)}{17c}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{\frac{2Bx^{3/2}\left(bx+cx^2\right)^{7/2}}{17c}-\left(\frac{2\sqrt{x}\left(bx+cx^2\right)^{7/2}}{15c}-\frac{8b\left(\frac{2\left(bx+cx^2\right)^{7/2}}{13c\sqrt{x}}-\frac{6b\left(\frac{2\left(bx+cx^2\right)^{7/2}}{11cx^{3/2}}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{7/2}}{9cx^{5/2}}-\frac{4b\left(bx+cx^2\right)^{7/2}}{63c^2x^{7/2}}\right)}{11c}\right)}{13c}\right)}{15c}\right)(10bB-17Ac)}{17c}$$

input

```
Int[x^(3/2)*(A + B*x)*(b*x + c*x^2)^(5/2),x]
```

output

```
(2*B*x^(3/2)*(b*x + c*x^2)^(7/2))/(17*c) - ((10*b*B - 17*A*c)*((2*Sqrt[x]*
(b*x + c*x^2)^(7/2))/(15*c) - (8*b*((2*(b*x + c*x^2)^(7/2))/(13*c*Sqrt[x])
 - (6*b*((2*(b*x + c*x^2)^(7/2))/(11*c*x^(3/2)) - (4*b*((-4*b*(b*x + c*x^2
)^(7/2))/(63*c^2*x^(7/2)) + (2*(b*x + c*x^2)^(7/2))/(9*c*x^(5/2))))/(11*c)
))/(13*c)))/(15*c)))/(17*c)
```

Defintions of rubi rules used

rule 1122

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.64

method	result
gosper	$\frac{2(cx+b)(45045B\,c^5x^5+51051A\,c^5x^4-30030Bb\,c^4x^4-31416Ab\,c^4x^3+18480B\,b^2c^3x^3+17136A\,b^2c^3x^2-10080B\,b^3c^2x^2-7616A\,b^3c}{765765c^6x^{\frac{5}{2}}}$
default	$\frac{2\sqrt{x(cx+b)}\,(cx+b)^3(45045B\,c^5x^5+51051A\,c^5x^4-30030Bb\,c^4x^4-31416Ab\,c^4x^3+18480B\,b^2c^3x^3+17136A\,b^2c^3x^2-10080B\,b^3c^2x^2}{765765\sqrt{x}\,c^6}$
orering	$\frac{2(cx+b)(45045B\,c^5x^5+51051A\,c^5x^4-30030Bb\,c^4x^4-31416Ab\,c^4x^3+18480B\,b^2c^3x^3+17136A\,b^2c^3x^2-10080B\,b^3c^2x^2-7616A\,b^3c}{765765c^6x^{\frac{5}{2}}}$
risch	$\frac{2(cx+b)\sqrt{x}\,(45045B\,c^8x^8+51051A\,c^8x^7+105105Bb\,c^7x^7+121737Ab\,c^7x^6+63525B\,b^2c^6x^6+76041A\,b^2c^6x^5+315B\,b^3c^5x^5+595A\,b}{765765\sqrt{x}}$

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/765765*(c*x+b)*(45045*B*c^5*x^5+51051*A*c^5*x^4-30030*B*b*c^4*x^4-31416*
A*b*c^4*x^3+18480*B*b^2*c^3*x^3+17136*A*b^2*c^3*x^2-10080*B*b^3*c^2*x^2-76
16*A*b^3*c^2*x+4480*B*b^4*c*x+2176*A*b^4*c-1280*B*b^5)*(c*x^2+b*x)^(5/2)/c
^6/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.97

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx = \frac{2\left(45045\,Bc^8x^8-1280\,Bb^8+2176\,Ab^7c+3003\left(35\,Bbc^7+17\,Ac^8\right)x^7+231\left(275\,Bb^2c^6+52\right.\right.}{}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/765765*(45045*B*c^8*x^8 - 1280*B*b^8 + 2176*A*b^7*c + 3003*(35*B*b*c^7 +
 17*A*c^8)*x^7 + 231*(275*B*b^2*c^6 + 527*A*b*c^7)*x^6 + 63*(5*B*b^3*c^5 +
 1207*A*b^2*c^6)*x^5 - 35*(10*B*b^4*c^4 - 17*A*b^3*c^5)*x^4 + 40*(10*B*b^5
*c^3 - 17*A*b^4*c^4)*x^3 - 48*(10*B*b^6*c^2 - 17*A*b^5*c^3)*x^2 + 64*(10*B
*b^7*c - 17*A*b^6*c^2)*x)*sqrt(c*x^2 + b*x)/(c^6*sqrt(x))
```

Sympy [F(-1)]

Timed out.

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(x**(3/2)*(B*x+A)*(c*x**2+b*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. 2(170) = 340.

Time = 0.06 (sec) , antiderivative size = 507, normalized size of antiderivative = 2.46

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx = \frac{2\left(\left(3003\,c^7x^7+231\,bc^6x^6-252\,b^2c^5x^5+280\,b^3c^4x^4-320\,b^4c^3x^3+384\,b^5c^2x^2-512\,b^6cx+\right.\right.}{} + \frac{2\left(7\left(6435\,c^8x^8+429\,bc^7x^7-462\,b^2c^6x^6+504\,b^3c^5x^5-560\,b^4c^4x^4+640\,b^5c^3x^3-768\,b^6c^2x^2+1024\,b^7cx\right.\right.}{}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
2/45045*((3003*c^7*x^7 + 231*b*c^6*x^6 - 252*b^2*c^5*x^5 + 280*b^3*c^4*x^4
 - 320*b^4*c^3*x^3 + 384*b^5*c^2*x^2 - 512*b^6*c*x + 1024*b^7)*x^6 + 10*(6
93*b*c^6*x^7 + 63*b^2*c^5*x^6 - 70*b^3*c^4*x^5 + 80*b^4*c^3*x^4 - 96*b^5*c
^2*x^3 + 128*b^6*c*x^2 - 256*b^7*x)*x^5 + 13*(315*b^2*c^5*x^7 + 35*b^3*c^4
*x^6 - 40*b^4*c^3*x^5 + 48*b^5*c^2*x^4 - 64*b^6*c*x^3 + 128*b^7*x^2)*x^4)*
sqrt(c*x + b)*A/(c^5*x^6) + 2/765765*(7*(6435*c^8*x^8 + 429*b*c^7*x^7 - 46
2*b^2*c^6*x^6 + 504*b^3*c^5*x^5 - 560*b^4*c^4*x^4 + 640*b^5*c^3*x^3 - 768*
b^6*c^2*x^2 + 1024*b^7*c*x - 2048*b^8)*x^7 + 34*(3003*b*c^7*x^8 + 231*b^2*
c^6*x^7 - 252*b^3*c^5*x^6 + 280*b^4*c^4*x^5 - 320*b^5*c^3*x^4 + 384*b^6*c^
2*x^3 - 512*b^7*c*x^2 + 1024*b^8*x)*x^6 + 85*(693*b^2*c^6*x^8 + 63*b^3*c^5
*x^7 - 70*b^4*c^4*x^6 + 80*b^5*c^3*x^5 - 96*b^6*c^2*x^4 + 128*b^7*c*x^3 -
256*b^8*x^2)*x^5)*sqrt(c*x + b)*B/(c^6*x^7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(170) = 340$.

Time = 0.48 (sec) , antiderivative size = 489, normalized size of antiderivative = 2.37

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^(3/2)*(B*x+A)*(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
2/3465*(315*(c*x + b)^(11/2) - 1540*(c*x + b)^(9/2)*b + 2970*(c*x + b)^(7/
2)*b^2 - 2772*(c*x + b)^(5/2)*b^3 + 1155*(c*x + b)^(3/2)*b^4)*A*b^2/c^5 +
2/9009*(693*(c*x + b)^(13/2) - 4095*(c*x + b)^(11/2)*b + 10010*(c*x + b)^(
9/2)*b^2 - 12870*(c*x + b)^(7/2)*b^3 + 9009*(c*x + b)^(5/2)*b^4 - 3003*(c*
x + b)^(3/2)*b^5)*B*b^2/c^6 + 4/9009*(693*(c*x + b)^(13/2) - 4095*(c*x + b
)^(11/2)*b + 10010*(c*x + b)^(9/2)*b^2 - 12870*(c*x + b)^(7/2)*b^3 + 9009*
(c*x + b)^(5/2)*b^4 - 3003*(c*x + b)^(3/2)*b^5)*A*b/c^5 + 4/45045*(3003*(c
*x + b)^(15/2) - 20790*(c*x + b)^(13/2)*b + 61425*(c*x + b)^(11/2)*b^2 - 1
00100*(c*x + b)^(9/2)*b^3 + 96525*(c*x + b)^(7/2)*b^4 - 54054*(c*x + b)^(5
/2)*b^5 + 15015*(c*x + b)^(3/2)*b^6)*B*b/c^6 + 2/45045*(3003*(c*x + b)^(15
/2) - 20790*(c*x + b)^(13/2)*b + 61425*(c*x + b)^(11/2)*b^2 - 100100*(c*x
+ b)^(9/2)*b^3 + 96525*(c*x + b)^(7/2)*b^4 - 54054*(c*x + b)^(5/2)*b^5 + 1
5015*(c*x + b)^(3/2)*b^6)*A/c^5 + 2/109395*(6435*(c*x + b)^(17/2) - 51051*
(c*x + b)^(15/2)*b + 176715*(c*x + b)^(13/2)*b^2 - 348075*(c*x + b)^(11/2)
*b^3 + 425425*(c*x + b)^(9/2)*b^4 - 328185*(c*x + b)^(7/2)*b^5 + 153153*(c
*x + b)^(5/2)*b^6 - 36465*(c*x + b)^(3/2)*b^7)*B/c^6
```

Mupad [F(-1)]

Timed out.

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\int x^{3/2}\,\left(c\,x^2+b\,x\right)^{5/2}\,\left(A+B\,x\right)\,dx$$

input
```
int(x^(3/2)*(b*x + c*x^2)^(5/2)*(A + B*x),x)
```

output
```
int(x^(3/2)*(b*x + c*x^2)^(5/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.89

$$\int x^{3/2}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\frac{2\sqrt{cx+b}\left(45045b\,c^8x^8+51051a\,c^8x^7+105105b^2c^7x^7+121737ab\,c^7x^6+63525b^3c^6x^6+760\right.}{}$$

input
```
int(x^(3/2)*(B*x+A)*(c*x^2+b*x)^(5/2),x)
```

output
```
(2*sqrt(b + c*x)*(2176*a*b**7*c - 1088*a*b**6*c**2*x + 816*a*b**5*c**3*x**
2 - 680*a*b**4*c**4*x**3 + 595*a*b**3*c**5*x**4 + 76041*a*b**2*c**6*x**5 +
 121737*a*b*c**7*x**6 + 51051*a*c**8*x**7 - 1280*b**9 + 640*b**8*c*x - 480
*b**7*c**2*x**2 + 400*b**6*c**3*x**3 - 350*b**5*c**4*x**4 + 315*b**4*c**5*
x**5 + 63525*b**3*c**6*x**6 + 105105*b**2*c**7*x**7 + 45045*b*c**8*x**8))/
(765765*c**6)
```

3.194 $\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 169

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\frac{2b^3(bB-Ac)\left(bx+cx^2\right)^{7/2}}{7c^5x^{7/2}}-\frac{2b^2(4bB-3Ac)\left(bx+cx^2\right)^{9/2}}{9c^5x^{9/2}}+\frac{6b(2bB-Ac)\left(bx+cx^2\right)^{11/2}}{11c^5x^{11/2}}-\frac{2(4bB-Ac)\left(bx+cx^2\right)^{13/2}}{13c^5x^{13/2}}+\frac{2B\left(bx+cx^2\right)^{15/2}}{15c^5x^{15/2}}$$

output
```
2/7*b^3*(-A*c+B*b)*(c*x^2+b*x)^(7/2)/c^5/x^(7/2)-2/9*b^2*(-3*A*c+4*B*b)*(c
*x^2+b*x)^(9/2)/c^5/x^(9/2)+6/11*b*(-A*c+2*B*b)*(c*x^2+b*x)^(11/2)/c^5/x^(
11/2)-2/13*(-A*c+4*B*b)*(c*x^2+b*x)^(13/2)/c^5/x^(13/2)+2/15*B*(c*x^2+b*x)
^(15/2)/c^5/x^(15/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.60

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\frac{2(b+cx)^3\sqrt{x(b+cx)}(128b^4B+168b^2c^2x(5A+6Bx)+231c^4x^3(15A+13Bx)-16b^3c(15\ldots}{45045c^5\sqrt{x}}$$

input
```
Integrate[Sqrt[x]*(A + B*x)*(b*x + c*x^2)^(5/2),x]
```

output
```
(2*(b + c*x)^3*Sqrt[x*(b + c*x)]*(128*b^4*B + 168*b^2*c^2*x*(5*A + 6*B*x)
+ 231*c^4*x^3*(15*A + 13*B*x) - 16*b^3*c*(15*A + 28*B*x) - 42*b*c^3*x^2*(4
5*A + 44*B*x)))/(45045*c^5*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1221, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{2B\sqrt{x}\left(bx+cx^2\right)^{7/2}}{15c}-\frac{(8bB-15Ac)\int \sqrt{x}\left(cx^2+bx\right)^{5/2}dx}{15c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2B\sqrt{x}\left(bx+cx^2\right)^{7/2}}{15c}-\frac{(8bB-15Ac)\left(\frac{2\left(bx+cx^2\right)^{7/2}}{13c\sqrt{x}}-\frac{6b\int \frac{\left(cx^2+bx\right)^{5/2}}{\sqrt{x}}dx}{13c}\right)}{15c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2B\sqrt{x}\left(bx+cx^2\right)^{7/2}}{15c}-\frac{(8bB-15Ac)\left(\frac{2\left(bx+cx^2\right)^{7/2}}{13c\sqrt{x}}-\frac{6b\left(\frac{2\left(bx+cx^2\right)^{7/2}}{11cx^{3/2}}-\frac{4b\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{3/2}}dx}{11c}\right)}{13c}\right)}{15c}$$

$\big\downarrow$ 1128

$$\frac{2B\sqrt{x}\left(bx+cx^2\right)^{7/2}}{15c}-\frac{(8bB-15Ac)\left(\frac{2\left(bx+cx^2\right)^{7/2}}{13c\sqrt{x}}-\frac{6b\left(\frac{2\left(bx+cx^2\right)^{7/2}}{11cx^{3/2}}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{7/2}}{9cx^{5/2}}-\frac{2b\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{5/2}}dx}{9c}\right)}{11c}\right)}{13c}\right)}{15c}$$

$\big\downarrow$ 1122

$$\frac{2B\sqrt{x}\left(bx+cx^2\right)^{7/2}}{15c}-\frac{\left(\frac{2\left(bx+cx^2\right)^{7/2}}{13c\sqrt{x}}-\frac{6b\left(\frac{2\left(bx+cx^2\right)^{7/2}}{11cx^{3/2}}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{7/2}}{9cx^{5/2}}-\frac{4b\left(bx+cx^2\right)^{7/2}}{63c^2x^{7/2}}\right)}{11c}\right)}{13c}\right)(8bB-15Ac)}{15c}$$

input
```
Int[Sqrt[x]*(A + B*x)*(b*x + c*x^2)^(5/2),x]
```

output
```
(2*B*Sqrt[x]*(b*x + c*x^2)^(7/2))/(15*c) - ((8*b*B - 15*A*c)*((2*(b*x + c*
x^2)^(7/2))/(13*c*Sqrt[x]) - (6*b*((2*(b*x + c*x^2)^(7/2))/(11*c*x^(3/2))
- (4*b*((-4*b*(b*x + c*x^2)^(7/2))/(63*c^2*x^(7/2)) + (2*(b*x + c*x^2)^(7/
2))/(9*c*x^(5/2))))/(11*c)))/(13*c)))/(15*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.63

method	result
gosper	$-\frac{2(cx+b)\left(-3003B\,c^4x^4-3465A\,c^4x^3+1848B\,c^3x^3b+1890Ab\,c^3x^2-1008c^2x^2B\,b^2-840A\,b^2c^2x+448B\,b^3cx+240A\,b^3c-128B\,b^4\right)}{45045c^5x^{\frac{5}{2}}}$
default	$-\frac{2\sqrt{x(cx+b)}\,(cx+b)^3\left(-3003B\,c^4x^4-3465A\,c^4x^3+1848B\,c^3x^3b+1890Ab\,c^3x^2-1008c^2x^2B\,b^2-840A\,b^2c^2x+448B\,b^3cx+240A\,b^3\right.}{45045\sqrt{x}\,c^5}$
orering	$-\frac{2(cx+b)\left(-3003B\,c^4x^4-3465A\,c^4x^3+1848B\,c^3x^3b+1890Ab\,c^3x^2-1008c^2x^2B\,b^2-840A\,b^2c^2x+448B\,b^3cx+240A\,b^3c-128B\,b^4\right)}{45045c^5x^{\frac{5}{2}}}$
risch	$-\frac{2(cx+b)\sqrt{x}\left(-3003B\,c^7x^7-3465A\,c^7x^6-7161Bb\,c^6x^6-8505Ab\,c^6x^5-4473B\,b^2c^5x^5-5565A\,b^2c^5x^4-35B\,b^3c^4x^4-75A\,b^3c^4x^3+\right.}{45045\sqrt{x(cx+b)}\,c^5}$

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/45045*(c*x+b)*(-3003*B*c^4*x^4-3465*A*c^4*x^3+1848*B*b*c^3*x^3+1890*A*b
*c^3*x^2-1008*B*b^2*c^2*x^2-840*A*b^2*c^2*x+448*B*b^3*c*x+240*A*b^3*c-128*
B*b^4)*(c*x^2+b*x)^(5/2)/c^5/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.03

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx = \frac{2\left(3003\,Bc^7x^7+128\,Bb^7-240\,Ab^6c+231\,(31\,Bbc^6+15\,Ac^7)x^6+63\,(71\,Bb^2c^5+135\,Abc^6\right.}{}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/45045*(3003*B*c^7*x^7 + 128*B*b^7 - 240*A*b^6*c + 231*(31*B*b*c^6 + 15*A
*c^7)*x^6 + 63*(71*B*b^2*c^5 + 135*A*b*c^6)*x^5 + 35*(B*b^3*c^4 + 159*A*b^
2*c^5)*x^4 - 5*(8*B*b^4*c^3 - 15*A*b^3*c^4)*x^3 + 6*(8*B*b^5*c^2 - 15*A*b^
4*c^3)*x^2 - 8*(8*B*b^6*c - 15*A*b^5*c^2)*x)*sqrt(c*x^2 + b*x)/(c^5*sqrt(x
))
```

Sympy [F]

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\int \sqrt{x}(x(b+cx))^{\frac{5}{2}}\,(A+Bx)\,dx$$

input
```
integrate(x**(1/2)*(B*x+A)*(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(sqrt(x)*(x*(b + c*x))**(5/2)*(A + B*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(139) = 278.

Time = 0.06 (sec) , antiderivative size = 441, normalized size of antiderivative = 2.61

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\frac{2\left(5\left(693\,c^6x^6+63\,bc^5x^5-70\,b^2c^4x^4+80\,b^3c^3x^3-96\,b^4c^2x^2+128\,b^5cx-256\,b^6\right)x^5+26\,(3\cdots\right.}{\cdots}+\frac{2\left(\left(3003\,c^7x^7+231\,bc^6x^6-252\,b^2c^5x^5+280\,b^3c^4x^4-320\,b^4c^3x^3+384\,b^5c^2x^2-512\,b^6cx+1024\,b^7\right)x^6+\cdots\right.}{\cdots}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
2/45045*(5*(693*c^6*x^6 + 63*b*c^5*x^5 - 70*b^2*c^4*x^4 + 80*b^3*c^3*x^3 -
 96*b^4*c^2*x^2 + 128*b^5*c*x - 256*b^6)*x^5 + 26*(315*b*c^5*x^6 + 35*b^2*
c^4*x^5 - 40*b^3*c^3*x^4 + 48*b^4*c^2*x^3 - 64*b^5*c*x^2 + 128*b^6*x)*x^4
+ 143*(35*b^2*c^4*x^6 + 5*b^3*c^3*x^5 - 6*b^4*c^2*x^4 + 8*b^5*c*x^3 - 16*b
^6*x^2)*x^3)*sqrt(c*x + b)*A/(c^4*x^5) + 2/45045*((3003*c^7*x^7 + 231*b*c^
6*x^6 - 252*b^2*c^5*x^5 + 280*b^3*c^4*x^4 - 320*b^4*c^3*x^3 + 384*b^5*c^2*
x^2 - 512*b^6*c*x + 1024*b^7)*x^6 + 10*(693*b*c^6*x^7 + 63*b^2*c^5*x^6 - 7
0*b^3*c^4*x^5 + 80*b^4*c^3*x^4 - 96*b^5*c^2*x^3 + 128*b^6*c*x^2 - 256*b^7*
x)*x^5 + 13*(315*b^2*c^5*x^7 + 35*b^3*c^4*x^6 - 40*b^4*c^3*x^5 + 48*b^5*c^
2*x^4 - 64*b^6*c*x^3 + 128*b^7*x^2)*x^4)*sqrt(c*x + b)*B/(c^5*x^6)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(139) = 278$.

Time = 0.13 (sec) , antiderivative size = 417, normalized size of antiderivative = 2.47

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx = \frac{2\left(35\,(cx+b)^{\frac{9}{2}}-135\,(cx+b)^{\frac{7}{2}}b+189\,(cx+b)^{\frac{5}{2}}b^2-105\,(cx+b)^{\frac{3}{2}}b^3\right)Ab^2}{315\,c^4} + \frac{2\left(315\,(cx+b)^{\frac{11}{2}}-1540\,(cx+b)^{\frac{9}{2}}b+2970\,(cx+b)^{\frac{7}{2}}b^2-2772\,(cx+b)^{\frac{5}{2}}b^3+1155\,(cx+b)^{\frac{3}{2}}b^4\right)Bb^2}{3465\,c^5} + \frac{4\left(315\,(cx+b)^{\frac{11}{2}}-1540\,(cx+b)^{\frac{9}{2}}b+2970\,(cx+b)^{\frac{7}{2}}b^2-2772\,(cx+b)^{\frac{5}{2}}b^3+1155\,(cx+b)^{\frac{3}{2}}b^4\right)Ab}{3465\,c^4} + \frac{4\left(693\,(cx+b)^{\frac{13}{2}}-4095\,(cx+b)^{\frac{11}{2}}b+10010\,(cx+b)^{\frac{9}{2}}b^2-12870\,(cx+b)^{\frac{7}{2}}b^3+9009\,(cx+b)^{\frac{5}{2}}b^4-3003}{9009\,c^5} + \frac{2\left(693\,(cx+b)^{\frac{13}{2}}-4095\,(cx+b)^{\frac{11}{2}}b+10010\,(cx+b)^{\frac{9}{2}}b^2-12870\,(cx+b)^{\frac{7}{2}}b^3+9009\,(cx+b)^{\frac{5}{2}}b^4-3003}{9009\,c^4} + \frac{2\left(3003\,(cx+b)^{\frac{15}{2}}-20790\,(cx+b)^{\frac{13}{2}}b+61425\,(cx+b)^{\frac{11}{2}}b^2-100100\,(cx+b)^{\frac{9}{2}}b^3+96525\,(cx+b)^{\frac{7}{2}}b^4-}{45045\,c^5}$$

input
```
integrate(x^(1/2)*(B*x+A)*(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
2/315*(35*(c*x + b)^(9/2) - 135*(c*x + b)^(7/2)*b + 189*(c*x + b)^(5/2)*b^
2 - 105*(c*x + b)^(3/2)*b^3)*A*b^2/c^4 + 2/3465*(315*(c*x + b)^(11/2) - 15
40*(c*x + b)^(9/2)*b + 2970*(c*x + b)^(7/2)*b^2 - 2772*(c*x + b)^(5/2)*b^3
 + 1155*(c*x + b)^(3/2)*b^4)*B*b^2/c^5 + 4/3465*(315*(c*x + b)^(11/2) - 15
40*(c*x + b)^(9/2)*b + 2970*(c*x + b)^(7/2)*b^2 - 2772*(c*x + b)^(5/2)*b^3
 + 1155*(c*x + b)^(3/2)*b^4)*A*b/c^4 + 4/9009*(693*(c*x + b)^(13/2) - 4095
*(c*x + b)^(11/2)*b + 10010*(c*x + b)^(9/2)*b^2 - 12870*(c*x + b)^(7/2)*b^
3 + 9009*(c*x + b)^(5/2)*b^4 - 3003*(c*x + b)^(3/2)*b^5)*B*b/c^5 + 2/9009*
(693*(c*x + b)^(13/2) - 4095*(c*x + b)^(11/2)*b + 10010*(c*x + b)^(9/2)*b^
2 - 12870*(c*x + b)^(7/2)*b^3 + 9009*(c*x + b)^(5/2)*b^4 - 3003*(c*x + b)^
(3/2)*b^5)*A/c^4 + 2/45045*(3003*(c*x + b)^(15/2) - 20790*(c*x + b)^(13/2)
*b + 61425*(c*x + b)^(11/2)*b^2 - 100100*(c*x + b)^(9/2)*b^3 + 96525*(c*x
+ b)^(7/2)*b^4 - 54054*(c*x + b)^(5/2)*b^5 + 15015*(c*x + b)^(3/2)*b^6)*B/
c^5
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\int \sqrt{x}\,\left(c\,x^2+b\,x\right)^{5/2}\,\left(A+B\,x\right)\,dx$$

input
```
int(x^(1/2)*(b*x + c*x^2)^(5/2)*(A + B*x),x)
```

output
```
int(x^(1/2)*(b*x + c*x^2)^(5/2)*(A + B*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.95

$$\int \sqrt{x}(A+Bx)\left(bx+cx^2\right)^{5/2}\,dx=\frac{2\sqrt{cx+b}\,(3003b\,c^7x^7+3465a\,c^7x^6+7161b^2c^6x^6+8505ab\,c^6x^5+4473b^3c^5x^5+5565a\,b^2c^5x$$

input
```
int(x^(1/2)*(B*x+A)*(c*x^2+b*x)^(5/2),x)
```

output
```
(2*sqrt(b + c*x)*( - 240*a*b**6*c + 120*a*b**5*c**2*x - 90*a*b**4*c**3*x**
2 + 75*a*b**3*c**4*x**3 + 5565*a*b**2*c**5*x**4 + 8505*a*b*c**6*x**5 + 346
5*a*c**7*x**6 + 128*b**8 - 64*b**7*c*x + 48*b**6*c**2*x**2 - 40*b**5*c**3*
x**3 + 35*b**4*c**4*x**4 + 4473*b**3*c**5*x**5 + 7161*b**2*c**6*x**6 + 300
3*b*c**7*x**7))/(45045*c**5)
```

3.195 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{\sqrt{x}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 132

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{\sqrt{x}}\,dx=-\frac{2b^2(bB-Ac)\left(bx+cx^2\right)^{7/2}}{7c^4x^{7/2}}+\frac{2b(3bB-2Ac)\left(bx+cx^2\right)^{9/2}}{9c^4x^{9/2}}-\frac{2(3bB-Ac)\left(bx+cx^2\right)^{11/2}}{11c^4x^{11/2}}+\frac{2B\left(bx+cx^2\right)^{13/2}}{13c^4x^{13/2}}$$

output
```
-2/7*b^2*(-A*c+B*b)*(c*x^2+b*x)^(7/2)/c^4/x^(7/2)+2/9*b*(-2*A*c+3*B*b)*(c*
x^2+b*x)^(9/2)/c^4/x^(9/2)-2/11*(-A*c+3*B*b)*(c*x^2+b*x)^(11/2)/c^4/x^(11/
2)+2/13*B*(c*x^2+b*x)^(13/2)/c^4/x^(13/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.62

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{\sqrt{x}}\,dx=\frac{2(b+cx)^3\sqrt{x(b+cx)}\left(-48b^3B+63c^3x^2(13A+11Bx)+8b^2c(13A+21B\right.}{9009c^4\sqrt{x}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/Sqrt[x],x]
```

output
```
(2*(b + c*x)^3*Sqrt[x*(b + c*x)]*(-48*b^3*B + 63*c^3*x^2*(13*A + 11*B*x) +
 8*b^2*c*(13*A + 21*B*x) - 14*b*c^2*x*(26*A + 27*B*x)))/(9009*c^4*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1221, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{\sqrt{x}}dx$$

$$\Big\downarrow \text{ 1221}$$

$$\frac{2B\left(bx+cx^2\right)^{7/2}}{13c\sqrt{x}}-\frac{(6bB-13Ac)\int \frac{\left(cx^2+bx\right)^{5/2}}{\sqrt{x}}dx}{13c}$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{2B\left(bx+cx^2\right)^{7/2}}{13c\sqrt{x}}-\frac{(6bB-13Ac)\left(\frac{2\left(bx+cx^2\right)^{7/2}}{11cx^{3/2}}-\frac{4b\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{3/2}}dx}{11c}\right)}{13c}$$

$$\Big\downarrow \text{ 1128}$$

$$\frac{2B\left(bx+cx^2\right)^{7/2}}{13c\sqrt{x}}-\frac{(6bB-13Ac)\left(\frac{2\left(bx+cx^2\right)^{7/2}}{11cx^{3/2}}-\frac{4b\left(\frac{2\left(bx+cx^2\right)^{7/2}}{9cx^{5/2}}-\frac{2b\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{5/2}}dx}{9c}\right)}{11c}\right)}{13c}$$

$$\Big\downarrow \text{ 1122}$$

$$\frac{2B(bx+cx^2)^{7/2}}{13c\sqrt{x}}-\frac{\left(\frac{2(bx+cx^2)^{7/2}}{11cx^{3/2}}-\frac{4b\left(\frac{2(bx+cx^2)^{7/2}}{9cx^{5/2}}-\frac{4b(bx+cx^2)^{7/2}}{63c^2x^{7/2}}\right)}{11c}\right)(6bB-13Ac)}{13c}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/Sqrt[x],x]
```

output
```
(2*B*(b*x + c*x^2)^(7/2))/(13*c*Sqrt[x]) - ((6*b*B - 13*A*c)*((2*(b*x + c*
x^2)^(7/2))/(11*c*x^(3/2)) - (4*b*((-4*b*(b*x + c*x^2)^(7/2))/(63*c^2*x^(7
/2)) + (2*(b*x + c*x^2)^(7/2))/(9*c*x^(5/2))))/(11*c)))/(13*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.63

method	result
gosper	$\frac{2(cx+b)\left(693B\,c^{3}x^{3}+819A\,c^{3}x^{2}-378Bb\,c^{2}x^{2}-364Ab\,c^{2}x+168B\,b^{2}cx+104A\,b^{2}c-48B\,b^{3}\right)\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{9009c^{4}x^{\frac{5}{2}}}$
default	$\frac{2\sqrt{x(cx+b)}\,(cx+b)^{3}\left(693B\,c^{3}x^{3}+819A\,c^{3}x^{2}-378Bb\,c^{2}x^{2}-364Ab\,c^{2}x+168B\,b^{2}cx+104A\,b^{2}c-48B\,b^{3}\right)}{9009\sqrt{x}\,c^{4}}$
orering	$\frac{2(cx+b)\left(693B\,c^{3}x^{3}+819A\,c^{3}x^{2}-378Bb\,c^{2}x^{2}-364Ab\,c^{2}x+168B\,b^{2}cx+104A\,b^{2}c-48B\,b^{3}\right)\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{9009c^{4}x^{\frac{5}{2}}}$
risch	$\frac{2(cx+b)\sqrt{x}\,\left(693B\,c^{6}x^{6}+819A\,c^{6}x^{5}+1701Bb\,c^{5}x^{5}+2093Ab\,c^{5}x^{4}+1113B\,b^{2}c^{4}x^{4}+1469A\,b^{2}c^{4}x^{3}+15B\,b^{3}c^{3}x^{3}+39A\,b^{3}c^{3}x^{2}-18B\,b\right.}{9009\sqrt{x(cx+b)}\,c^{4}}$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/9009*(c*x+b)*(693*B*c^3*x^3+819*A*c^3*x^2-378*B*b*c^2*x^2-364*A*b*c^2*x+
168*B*b^2*c*x+104*A*b^2*c-48*B*b^3)*(c*x^2+b*x)^(5/2)/c^4/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.14

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{\sqrt{x}}\,dx = \frac{2\left(693\,Bc^6x^6 - 48\,Bb^6 + 104\,Ab^5c + 63\,(27\,Bbc^5 + 13\,Ac^6)x^5 + 7\,(159\,Bb^2\right.}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(1/2),x, algorithm="fricas")
```

output
```
2/9009*(693*B*c^6*x^6 - 48*B*b^6 + 104*A*b^5*c + 63*(27*B*b*c^5 + 13*A*c^6
)*x^5 + 7*(159*B*b^2*c^4 + 299*A*b*c^5)*x^4 + (15*B*b^3*c^3 + 1469*A*b^2*c
^4)*x^3 - 3*(6*B*b^4*c^2 - 13*A*b^3*c^3)*x^2 + 4*(6*B*b^5*c - 13*A*b^4*c^2
)*x)*sqrt(c*x^2 + b*x)/(c^4*sqrt(x))
```

Sympy [F]

$$\int \frac{(A+Bx)\,(bx+cx^2)^{5/2}}{\sqrt{x}}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**(1/2),x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/sqrt(x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(108) = 216.

Time = 0.05 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.84

$$\int \frac{(A+Bx)\,(bx+cx^2)^{5/2}}{\sqrt{x}}\,dx = \frac{2\,((315\,c^5x^5+35\,bc^4x^4-40\,b^2c^3x^3+48\,b^3c^2x^2-64\,b^4cx+128\,b^5)x^4+22}{} + \frac{2\,(5\,(693\,c^6x^6+63\,bc^5x^5-70\,b^2c^4x^4+80\,b^3c^3x^3-96\,b^4c^2x^2+128\,b^5cx-256\,b^6)x^5+26\,(315\,bc^5x^6+35}{}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(1/2),x, algorithm="maxima")
```

output
```
2/3465*((315*c^5*x^5 + 35*b*c^4*x^4 - 40*b^2*c^3*x^3 + 48*b^3*c^2*x^2 - 64
*b^4*c*x + 128*b^5)*x^4 + 22*(35*b*c^4*x^5 + 5*b^2*c^3*x^4 - 6*b^3*c^2*x^3
 + 8*b^4*c*x^2 - 16*b^5*x)*x^3 + 33*(15*b^2*c^3*x^5 + 3*b^3*c^2*x^4 - 4*b^
4*c*x^3 + 8*b^5*x^2)*x^2)*sqrt(c*x + b)*A/(c^3*x^4) + 2/45045*(5*(693*c^6*
x^6 + 63*b*c^5*x^5 - 70*b^2*c^4*x^4 + 80*b^3*c^3*x^3 - 96*b^4*c^2*x^2 + 12
8*b^5*c*x - 256*b^6)*x^5 + 26*(315*b*c^5*x^6 + 35*b^2*c^4*x^5 - 40*b^3*c^3
*x^4 + 48*b^4*c^2*x^3 - 64*b^5*c*x^2 + 128*b^6*x)*x^4 + 143*(35*b^2*c^4*x^
6 + 5*b^3*c^3*x^5 - 6*b^4*c^2*x^4 + 8*b^5*c*x^3 - 16*b^6*x^2)*x^3)*sqrt(c*
x + b)*B/(c^4*x^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(108) = 216.

Time = 0.12 (sec) , antiderivative size = 345, normalized size of antiderivative = 2.61

$$
\begin{aligned}
\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{\sqrt{x}}\,dx &= \frac{2\left(15\,(cx+b)^{\frac{7}{2}}-42\,(cx+b)^{\frac{5}{2}}b+35\,(cx+b)^{\frac{3}{2}}b^2\right)Ab^2}{105\,c^3} \\
&+ \frac{2\left(35\,(cx+b)^{\frac{9}{2}}-135\,(cx+b)^{\frac{7}{2}}b+189\,(cx+b)^{\frac{5}{2}}b^2-105\,(cx+b)^{\frac{3}{2}}b^3\right)Bb^2}{315\,c^4} \\
&+ \frac{4\left(35\,(cx+b)^{\frac{9}{2}}-135\,(cx+b)^{\frac{7}{2}}b+189\,(cx+b)^{\frac{5}{2}}b^2-105\,(cx+b)^{\frac{3}{2}}b^3\right)Ab}{315\,c^3} \\
&+ \frac{4\left(315\,(cx+b)^{\frac{11}{2}}-1540\,(cx+b)^{\frac{9}{2}}b+2970\,(cx+b)^{\frac{7}{2}}b^2-2772\,(cx+b)^{\frac{5}{2}}b^3+1155\,(cx+b)^{\frac{3}{2}}b^4\right)Bb}{3465\,c^4} \\
&+ \frac{2\left(315\,(cx+b)^{\frac{11}{2}}-1540\,(cx+b)^{\frac{9}{2}}b+2970\,(cx+b)^{\frac{7}{2}}b^2-2772\,(cx+b)^{\frac{5}{2}}b^3+1155\,(cx+b)^{\frac{3}{2}}b^4\right)A}{3465\,c^3} \\
&+ \frac{2\left(693\,(cx+b)^{\frac{13}{2}}-4095\,(cx+b)^{\frac{11}{2}}b+10010\,(cx+b)^{\frac{9}{2}}b^2-12870\,(cx+b)^{\frac{7}{2}}b^3+9009\,(cx+b)^{\frac{5}{2}}b^4-3003\right.}{9009\,c^4}
\end{aligned}
$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(1/2),x, algorithm="giac")
```

output
```
2/105*(15*(c*x + b)^(7/2) - 42*(c*x + b)^(5/2)*b + 35*(c*x + b)^(3/2)*b^2)
*A*b^2/c^3 + 2/315*(35*(c*x + b)^(9/2) - 135*(c*x + b)^(7/2)*b + 189*(c*x
+ b)^(5/2)*b^2 - 105*(c*x + b)^(3/2)*b^3)*B*b^2/c^4 + 4/315*(35*(c*x + b)^
(9/2) - 135*(c*x + b)^(7/2)*b + 189*(c*x + b)^(5/2)*b^2 - 105*(c*x + b)^(3
/2)*b^3)*A*b/c^3 + 4/3465*(315*(c*x + b)^(11/2) - 1540*(c*x + b)^(9/2)*b +
 2970*(c*x + b)^(7/2)*b^2 - 2772*(c*x + b)^(5/2)*b^3 + 1155*(c*x + b)^(3/2
)*b^4)*B*b/c^4 + 2/3465*(315*(c*x + b)^(11/2) - 1540*(c*x + b)^(9/2)*b + 2
970*(c*x + b)^(7/2)*b^2 - 2772*(c*x + b)^(5/2)*b^3 + 1155*(c*x + b)^(3/2)*
b^4)*A/c^3 + 2/9009*(693*(c*x + b)^(13/2) - 4095*(c*x + b)^(11/2)*b + 1001
0*(c*x + b)^(9/2)*b^2 - 12870*(c*x + b)^(7/2)*b^3 + 9009*(c*x + b)^(5/2)*b
^4 - 3003*(c*x + b)^(3/2)*b^5)*B/c^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{\sqrt{x}}\,dx = \int \frac{\left(c\,x^2+b\,x\right)^{5/2}\,(A+B\,x)}{\sqrt{x}}\,dx$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(1/2),x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{\sqrt{x}}\,dx = \frac{2\sqrt{cx+b}\,(693b\,c^6x^6+819a\,c^6x^5+1701b^2c^5x^5+2093ab\,c^5x^4+1113b^3c^4x$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(1/2),x)
```

output
```
(2*sqrt(b + c*x)*(104*a*b**5*c - 52*a*b**4*c**2*x + 39*a*b**3*c**3*x**2 +
1469*a*b**2*c**4*x**3 + 2093*a*b*c**5*x**4 + 819*a*c**6*x**5 - 48*b**7 + 2
4*b**6*c*x - 18*b**5*c**2*x**2 + 15*b**4*c**3*x**3 + 1113*b**3*c**4*x**4 +
 1701*b**2*c**5*x**5 + 693*b*c**6*x**6))/(9009*c**4)
```

3.196 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{3/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 95

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{3/2}} \, dx = \frac{2b(bB-Ac)\left(bx+cx^2\right)^{7/2}}{7c^3x^{7/2}} - \frac{2(2bB-Ac)\left(bx+cx^2\right)^{9/2}}{9c^3x^{9/2}} + \frac{2B\left(bx+cx^2\right)^{11/2}}{11c^3x^{11/2}}$$

output
```
2/7*b*(-A*c+B*b)*(c*x^2+b*x)^(7/2)/c^3/x^(7/2)-2/9*(-A*c+2*B*b)*(c*x^2+b*x
)^(9/2)/c^3/x^(9/2)+2/11*B*(c*x^2+b*x)^(11/2)/c^3/x^(11/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.66

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{3/2}} \, dx = \frac{2(b+cx)^3\sqrt{x(b+cx)}(8b^2B+7c^2x(11A+9Bx)-2bc(11A+14Bx))}{693c^3\sqrt{x}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(3/2),x]
```

output
```
(2*(b + c*x)^3*Sqrt[x*(b + c*x)]*(8*b^2*B + 7*c^2*x*(11*A + 9*B*x) - 2*b*c
*(11*A + 14*B*x)))/(693*c^3*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1221, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{3/2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{2B\left(bx+cx^2\right)^{7/2}}{11cx^{3/2}}-\frac{(4bB-11Ac)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{3/2}}dx}{11c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2B\left(bx+cx^2\right)^{7/2}}{11cx^{3/2}}-\frac{(4bB-11Ac)\left(\frac{2\left(bx+cx^2\right)^{7/2}}{9cx^{5/2}}-\frac{2b\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{5/2}}dx}{9c}\right)}{11c}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2B\left(bx+cx^2\right)^{7/2}}{11cx^{3/2}}-\frac{\left(\frac{2\left(bx+cx^2\right)^{7/2}}{9cx^{5/2}}-\frac{4b\left(bx+cx^2\right)^{7/2}}{63c^2x^{7/2}}\right)(4bB-11Ac)}{11c}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(3/2),x]
```

output
```
(2*B*(b*x + c*x^2)^(7/2))/(11*c*x^(3/2)) - ((4*b*B - 11*A*c)*((-4*b*(b*x +
 c*x^2)^(7/2))/(63*c^2*x^(7/2)) + (2*(b*x + c*x^2)^(7/2))/(9*c*x^(5/2))))/
(11*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.62

method	result
gosper	$-\frac{2(cx+b)\left(-63B\,c^{2}x^{2}-77A\,c^{2}x+28Bbcx+22Abc-8B\,b^{2}\right)\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{693c^{3}x^{\frac{5}{2}}}$
default	$-\frac{2\sqrt{x(cx+b)}\,(cx+b)^{3}\left(-63B\,c^{2}x^{2}-77A\,c^{2}x+28Bbcx+22Abc-8B\,b^{2}\right)}{693\sqrt{x}\,c^{3}}$
orering	$-\frac{2(cx+b)\left(-63B\,c^{2}x^{2}-77A\,c^{2}x+28Bbcx+22Abc-8B\,b^{2}\right)\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{693c^{3}x^{\frac{5}{2}}}$
risch	$-\frac{2(cx+b)\sqrt{x}\,\left(-63B\,c^{5}x^{5}-77A\,c^{5}x^{4}-161Bb\,c^{4}x^{4}-209Ab\,c^{4}x^{3}-113B\,b^{2}c^{3}x^{3}-165A\,b^{2}c^{3}x^{2}-3B\,b^{3}c^{2}x^{2}-11A\,b^{3}c^{2}x+4B\,b^{4}cx+2\right.}{693\sqrt{x(cx+b)}\,c^{3}}$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/693*(c*x+b)*(-63*B*c^2*x^2-77*A*c^2*x+28*B*b*c*x+22*A*b*c-8*B*b^2)*(c*x
^2+b*x)^(5/2)/c^3/x^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.32

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{3/2}}\,dx = \frac{2\,(63\,Bc^5x^5+8\,Bb^5-22\,Ab^4c+7\,(23\,Bbc^4+11\,Ac^5)x^4+(113\,Bb^2c^3+2}{693\,c^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(3/2),x, algorithm="fricas")
```

output
```
2/693*(63*B*c^5*x^5 + 8*B*b^5 - 22*A*b^4*c + 7*(23*B*b*c^4 + 11*A*c^5)*x^4
 + (113*B*b^2*c^3 + 209*A*b*c^4)*x^3 + 3*(B*b^3*c^2 + 55*A*b^2*c^3)*x^2 -
(4*B*b^4*c - 11*A*b^3*c^2)*x)*sqrt(c*x^2 + b*x)/(c^3*sqrt(x))
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{3/2}}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**(3/2),x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(77) = 154$.

Time = 0.05 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.21

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{3/2}}\,dx = \frac{2\left(\left(35\,c^4x^4+5\,bc^3x^3-6\,b^2c^2x^2+8\,b^3cx-16\,b^4\right)x^3+6\left(15\,bc^3x^4+3\,b^2c^2x\right.\right.}{315\,c^2x^3}$$
$$+\frac{2\left(\left(315\,c^5x^5+35\,bc^4x^4-40\,b^2c^3x^3+48\,b^3c^2x^2-64\,b^4cx+128\,b^5\right)x^4+22\left(35\,bc^4x^5+5\,b^2c^3x^4-6\,b^3c^2x^3\right.\right.}{3465\,c^3x^4}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(3/2),x, algorithm="maxima")
```

output
```
2/315*((35*c^4*x^4 + 5*b*c^3*x^3 - 6*b^2*c^2*x^2 + 8*b^3*c*x - 16*b^4)*x^3
 + 6*(15*b*c^3*x^4 + 3*b^2*c^2*x^3 - 4*b^3*c*x^2 + 8*b^4*x)*x^2 + 21*(3*b^
2*c^2*x^4 + b^3*c*x^3 - 2*b^4*x^2)*x)*sqrt(c*x + b)*A/(c^2*x^3) + 2/3465*(
(315*c^5*x^5 + 35*b*c^4*x^4 - 40*b^2*c^3*x^3 + 48*b^3*c^2*x^2 - 64*b^4*c*x
 + 128*b^5)*x^4 + 22*(35*b*c^4*x^5 + 5*b^2*c^3*x^4 - 6*b^3*c^2*x^3 + 8*b^4
*c*x^2 - 16*b^5*x)*x^3 + 33*(15*b^2*c^3*x^5 + 3*b^3*c^2*x^4 - 4*b^4*c*x^3
+ 8*b^5*x^2)*x^2)*sqrt(c*x + b)*B/(c^3*x^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(77) = 154$.

Time = 0.14 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.87

$$\int \frac{(A+Bx)\,(bx+cx^2)^{5/2}}{x^{3/2}}\,dx = \frac{2\left(3\,(cx+b)^{\frac{5}{2}}-5\,(cx+b)^{\frac{3}{2}}b\right)Ab^2}{15\,c^2}$$
$$+\frac{2\left(15\,(cx+b)^{\frac{7}{2}}-42\,(cx+b)^{\frac{5}{2}}b+35\,(cx+b)^{\frac{3}{2}}b^2\right)Bb^2}{105\,c^3}$$
$$+\frac{4\left(15\,(cx+b)^{\frac{7}{2}}-42\,(cx+b)^{\frac{5}{2}}b+35\,(cx+b)^{\frac{3}{2}}b^2\right)Ab}{105\,c^2}$$
$$+\frac{4\left(35\,(cx+b)^{\frac{9}{2}}-135\,(cx+b)^{\frac{7}{2}}b+189\,(cx+b)^{\frac{5}{2}}b^2-105\,(cx+b)^{\frac{3}{2}}b^3\right)Bb}{315\,c^3}$$
$$+\frac{2\left(35\,(cx+b)^{\frac{9}{2}}-135\,(cx+b)^{\frac{7}{2}}b+189\,(cx+b)^{\frac{5}{2}}b^2-105\,(cx+b)^{\frac{3}{2}}b^3\right)A}{315\,c^2}$$
$$+\frac{2\left(315\,(cx+b)^{\frac{11}{2}}-1540\,(cx+b)^{\frac{9}{2}}b+2970\,(cx+b)^{\frac{7}{2}}b^2-2772\,(cx+b)^{\frac{5}{2}}b^3+1155\,(cx+b)^{\frac{3}{2}}b^4\right)B}{3465\,c^3}$$

input

```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(3/2),x, algorithm="giac")
```

output

```
2/15*(3*(c*x + b)^(5/2) - 5*(c*x + b)^(3/2)*b)*A*b^2/c^2 + 2/105*(15*(c*x
+ b)^(7/2) - 42*(c*x + b)^(5/2)*b + 35*(c*x + b)^(3/2)*b^2)*B*b^2/c^3 + 4/
105*(15*(c*x + b)^(7/2) - 42*(c*x + b)^(5/2)*b + 35*(c*x + b)^(3/2)*b^2)*A
*b/c^2 + 4/315*(35*(c*x + b)^(9/2) - 135*(c*x + b)^(7/2)*b + 189*(c*x + b)
^(5/2)*b^2 - 105*(c*x + b)^(3/2)*b^3)*B*b/c^3 + 2/315*(35*(c*x + b)^(9/2)
- 135*(c*x + b)^(7/2)*b + 189*(c*x + b)^(5/2)*b^2 - 105*(c*x + b)^(3/2)*b^
3)*A/c^2 + 2/3465*(315*(c*x + b)^(11/2) - 1540*(c*x + b)^(9/2)*b + 2970*(c
*x + b)^(7/2)*b^2 - 2772*(c*x + b)^(5/2)*b^3 + 1155*(c*x + b)^(3/2)*b^4)*B
/c^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{3/2}}\,dx=\int \frac{{\left(c\,x^2+b\,x\right)}^{5/2}\,\left(A+B\,x\right)}{x^{3/2}} \,d x$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(3/2),x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.20

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{3/2}}\,dx=\frac{2\sqrt{cx+b}\,(63b\,c^5x^5+77a\,c^5x^4+161b^2c^4x^4+209ab\,c^4x^3+113b^3c^3x^3+16}{693c^3}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(3/2),x)
```

output
```
(2*sqrt(b + c*x)*( - 22*a*b**4*c + 11*a*b**3*c**2*x + 165*a*b**2*c**3*x**2
 + 209*a*b*c**4*x**3 + 77*a*c**5*x**4 + 8*b**6 - 4*b**5*c*x + 3*b**4*c**2*
x**2 + 113*b**3*c**3*x**3 + 161*b**2*c**4*x**4 + 63*b*c**5*x**5))/(693*c**
3)
```

3.197 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{5/2}}\,dx=-\frac{2(bB-Ac)\left(bx+cx^2\right)^{7/2}}{7c^2x^{7/2}}+\frac{2B(bx+cx^2)^{9/2}}{9c^2x^{9/2}}$$

output
```
-2/7*(-A*c+B*b)*(c*x^2+b*x)^(7/2)/c^2/x^(7/2)+2/9*B*(c*x^2+b*x)^(9/2)/c^2/
x^(9/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{5/2}}\,dx=\frac{2(b+cx)(x(b+cx))^{5/2}(-9bB+9Ac+7B(b+cx))}{63c^2x^{5/2}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(5/2),x]
```

output
```
(2*(b + c*x)*(x*(b + c*x))^(5/2)*(-9*b*B + 9*A*c + 7*B*(b + c*x)))/(63*c^2
*x^(5/2))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{5/2}}dx$$
$$\Big\downarrow \text{1221}$$
$$\frac{2B\left(bx+cx^2\right)^{7/2}}{9cx^{5/2}}-\frac{(2bB-9Ac)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{5/2}}dx}{9c}$$
$$\Big\downarrow \text{1122}$$
$$\frac{2B\left(bx+cx^2\right)^{7/2}}{9cx^{5/2}}-\frac{2\left(bx+cx^2\right)^{7/2}(2bB-9Ac)}{63c^2x^{7/2}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(5/2),x]
```

output
```
(-2*(2*b*B - 9*A*c)*(b*x + c*x^2)^(7/2))/(63*c^2*x^(7/2)) + (2*B*(b*x + c*
x^2)^(7/2))/(9*c*x^(5/2))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{2(cx+b)(7Bcx+9Ac-2Bb)\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{63c^{2}x^{\frac{5}{2}}}$	39
default	$\frac{2\sqrt{x(cx+b)}\,(cx+b)^{3}(7Bcx+9Ac-2Bb)}{63\sqrt{x}\,c^{2}}$	39
orering	$\frac{2(cx+b)(7Bcx+9Ac-2Bb)\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}{63c^{2}x^{\frac{5}{2}}}$	39
risch	$\frac{2(cx+b)\sqrt{x}\,\left(7B\,c^{4}x^{4}+9A\,c^{4}x^{3}+19B\,c^{3}x^{3}b+27Ab\,c^{3}x^{2}+15c^{2}x^{2}B\,b^{2}+27A\,b^{2}c^{2}x+B\,b^{3}cx+9A\,b^{3}c-2B\,b^{4}\right)}{63\sqrt{x(cx+b)}\,c^{2}}$	104

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/63*(c*x+b)*(7*B*c*x+9*A*c-2*B*b)*(c*x^2+b*x)^(5/2)/c^2/x^(5/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. 2(48) = 96.

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.67

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{5/2}}\,dx = \frac{2\,(7\,Bc^4x^4-2\,Bb^4+9\,Ab^3c+(19\,Bbc^3+9\,Ac^4)x^3+3\,(5\,Bb^2c^2+9\,Abc^3)}{63\,c^2\sqrt{x}}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(5/2),x, algorithm="fricas")
```

output
```
2/63*(7*B*c^4*x^4 - 2*B*b^4 + 9*A*b^3*c + (19*B*b*c^3 + 9*A*c^4)*x^3 + 3*(
5*B*b^2*c^2 + 9*A*b*c^3)*x^2 + (B*b^3*c + 27*A*b^2*c^2)*x)*sqrt(c*x^2 + b*
x)/(c^2*sqrt(x))
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{5/2}}\,dx=\int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**(5/2),x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**(5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(48) = 96$.

Time = 0.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 3.83

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{5/2}}\,dx=\frac{2\left(35\,b^2cx^3+35\,b^3x^2+\left(15\,c^3x^3+3\,bc^2x^2-4\,b^2cx+8\,b^3\right)x^2+14\left(3\,bc^2x^3\right.\right.}{105\,cx^2} + \frac{2\left(\left(35\,c^4x^4+5\,bc^3x^3-6\,b^2c^2x^2+8\,b^3cx-16\,b^4\right)x^3+6\left(15\,bc^3x^4+3\,b^2c^2x^3-4\,b^3cx^2+8\,b^4x\right)x^2+21\left(3\,b\right.\right.}{315\,c^2x^3}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(5/2),x, algorithm="maxima")
```

output
```
2/105*(35*b^2*c*x^3 + 35*b^3*x^2 + (15*c^3*x^3 + 3*b*c^2*x^2 - 4*b^2*c*x +
 8*b^3)*x^2 + 14*(3*b*c^2*x^3 + b^2*c*x^2 - 2*b^3*x)*x)*sqrt(c*x + b)*A/(c
*x^2) + 2/315*((35*c^4*x^4 + 5*b*c^3*x^3 - 6*b^2*c^2*x^2 + 8*b^3*c*x - 16*
b^4)*x^3 + 6*(15*b*c^3*x^4 + 3*b^2*c^2*x^3 - 4*b^3*c*x^2 + 8*b^4*x)*x^2 +
21*(3*b^2*c^2*x^4 + b^3*c*x^3 - 2*b^4*x^2)*x)*sqrt(c*x + b)*B/(c^2*x^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(48) = 96$.

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 3.33

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^{5/2}}\,dx = \frac{2(cx+b)^{\frac{3}{2}}Ab^2}{3c} + \frac{2\left(3(cx+b)^{\frac{5}{2}} - 5(cx+b)^{\frac{3}{2}}b\right)Bb^2}{15c^2} + \frac{4\left(3(cx+b)^{\frac{5}{2}} - 5(cx+b)^{\frac{3}{2}}b\right)Ab}{15c} + \frac{4\left(15(cx+b)^{\frac{7}{2}} - 42(cx+b)^{\frac{5}{2}}b + 35(cx+b)^{\frac{3}{2}}b^2\right)Bb}{105c^2} + \frac{2\left(15(cx+b)^{\frac{7}{2}} - 42(cx+b)^{\frac{5}{2}}b + 35(cx+b)^{\frac{3}{2}}b^2\right)A}{105c} + \frac{2\left(35(cx+b)^{\frac{9}{2}} - 135(cx+b)^{\frac{7}{2}}b + 189(cx+b)^{\frac{5}{2}}b^2 - 105(cx+b)^{\frac{3}{2}}b^3\right)B}{315c^2}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(5/2),x, algorithm="giac")
```

output
```
2/3*(c*x + b)^(3/2)*A*b^2/c + 2/15*(3*(c*x + b)^(5/2) - 5*(c*x + b)^(3/2)*
b)*B*b^2/c^2 + 4/15*(3*(c*x + b)^(5/2) - 5*(c*x + b)^(3/2)*b)*A*b/c + 4/10
5*(15*(c*x + b)^(7/2) - 42*(c*x + b)^(5/2)*b + 35*(c*x + b)^(3/2)*b^2)*B*b
/c^2 + 2/105*(15*(c*x + b)^(7/2) - 42*(c*x + b)^(5/2)*b + 35*(c*x + b)^(3/
2)*b^2)*A/c + 2/315*(35*(c*x + b)^(9/2) - 135*(c*x + b)^(7/2)*b + 189*(c*x
 + b)^(5/2)*b^2 - 105*(c*x + b)^(3/2)*b^3)*B/c^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^{5/2}}\,dx = \int \frac{(c\,x^2+b\,x)^{5/2}\,(A+B\,x)}{x^{5/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(5/2),x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.50

$$\int \frac{(A+Bx)\,(bx+cx^2)^{5/2}}{x^{5/2}}\,dx = \frac{2\sqrt{cx+b}\,(7b\,c^4x^4+9a\,c^4x^3+19b^2c^3x^3+27ab\,c^3x^2+15b^3c^2x^2+27a\,b^2c^2}{63c^2}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(5/2),x)
```

output
```
(2*sqrt(b + c*x)*(9*a*b**3*c + 27*a*b**2*c**2*x + 27*a*b*c**3*x**2 + 9*a*c
**4*x**3 - 2*b**5 + b**4*c*x + 15*b**3*c**2*x**2 + 19*b**2*c**3*x**3 + 7*b
*c**4*x**4))/(63*c**2)
```

3.198 $\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^{7/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{7/2}}\,dx = \frac{2Ab^2\sqrt{bx+cx^2}}{\sqrt{x}} + \frac{2Ab\left(bx+cx^2\right)^{3/2}}{3x^{3/2}} + \frac{2A\left(bx+cx^2\right)^{5/2}}{5x^{5/2}} + \frac{2B\left(bx+cx^2\right)^{7/2}}{7cx^{7/2}} - 2Ab^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)$$

output
```
2*A*b^2*(c*x^2+b*x)^(1/2)/x^(1/2)+2/3*A*b*(c*x^2+b*x)^(3/2)/x^(3/2)+2/5*A*
(c*x^2+b*x)^(5/2)/x^(5/2)+2/7*B*(c*x^2+b*x)^(7/2)/c/x^(7/2)-2*A*b^(5/2)*ar
ctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1/2))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.87

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{7/2}}\,dx = \frac{2\sqrt{x}\Big((b+cx)\left(15b^3B+3c^3x^2(7A+5Bx)+bc^2x(77A+45Bx)+b^2c(161}{105c\sqrt{x(b+cx)}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(7/2),x]
```

output
```
(2*Sqrt[x]*((b + c*x)*(15*b^3*B + 3*c^3*x^2*(7*A + 5*B*x) + b*c^2*x*(77*A
+ 45*B*x) + b^2*c*(161*A + 45*B*x)) - 105*A*b^(5/2)*c*Sqrt[b + c*x]*ArcTan
h[Sqrt[b + c*x]/Sqrt[b]]))/(105*c*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1221, 1131, 1131, 1131, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{7/2}}\,dx$$

$$\Big\downarrow \text{1221}$$

$$A\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{7/2}}dx+\frac{2B\left(bx+cx^2\right)^{7/2}}{7cx^{7/2}}$$

$$\Big\downarrow \text{1131}$$

$$A\left(b\int \frac{\left(cx^2+bx\right)^{3/2}}{x^{5/2}}dx+\frac{2\left(bx+cx^2\right)^{5/2}}{5x^{5/2}}\right)+\frac{2B\left(bx+cx^2\right)^{7/2}}{7cx^{7/2}}$$

$$\Big\downarrow \text{1131}$$

$$A\left(b\left(b\int \frac{\sqrt{cx^2+bx}}{x^{3/2}}dx+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}\right)+\frac{2\left(bx+cx^2\right)^{5/2}}{5x^{5/2}}\right)+\frac{2B\left(bx+cx^2\right)^{7/2}}{7cx^{7/2}}$$

$$\Big\downarrow \text{1131}$$

$$A\left(b\left(b\left(b\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}\right)+\frac{2\left(bx+cx^2\right)^{5/2}}{5x^{5/2}}\right)+\frac{2B\left(bx+cx^2\right)^{7/2}}{7cx^{7/2}}$$

$$\Big\downarrow \text{1136}$$

$$A\left(b\left(b\left(2b\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}\right)+\frac{2\left(bx+cx^2\right)^{5/2}}{5x^{5/2}}\right)+\frac{2B\left(bx+cx^2\right)^{7/2}}{7cx^{7/2}}$$

$\Big\downarrow$ 220

$$A\left(b\left(b\left(\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}-2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)\right)+\frac{2\left(bx+cx^2\right)^{3/2}}{3x^{3/2}}\right)+\frac{2\left(bx+cx^2\right)^{5/2}}{5x^{5/2}}\right)+\frac{2B\left(bx+cx^2\right)^{7/2}}{7cx^{7/2}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(7/2),x]
```

output
```
(2*B*(b*x + c*x^2)^(7/2))/(7*c*x^(7/2)) + A*((2*(b*x + c*x^2)^(5/2))/(5*x^
(5/2)) + b*((2*(b*x + c*x^2)^(3/2))/(3*x^(3/2)) + b*((2*Sqrt[b*x + c*x^2])
/Sqrt[x] - 2*Sqrt[b]*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])))
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.15

method	result
default	$-\frac{2\sqrt{x(cx+b)}\left(-15B\,c^3x^3\sqrt{cx+b}-21A\,c^3x^2\sqrt{cx+b}-45Bb\,c^2x^2\sqrt{cx+b}+105A\,b^{\frac{5}{2}}c\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)-77Ab\,c^2x\sqrt{cx+b}-45B\,b^2cx\ldots}{105\sqrt{x}\,\sqrt{cx+b}\,c}$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(x*(c*x+b))^(1/2)*(-15*B*c^3*x^3*(c*x+b)^(1/2)-21*A*c^3*x^2*(c*x+b)
^(1/2)-45*B*b*c^2*x^2*(c*x+b)^(1/2)+105*A*b^(5/2)*c*arctanh((c*x+b)^(1/2)/
b^(1/2))-77*A*b*c^2*x*(c*x+b)^(1/2)-45*B*b^2*c*x*(c*x+b)^(1/2)-161*A*b^2*c
*(c*x+b)^(1/2)-15*B*b^3*(c*x+b)^(1/2))/x^(1/2)/(c*x+b)^(1/2)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.89

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{7/2}}\,dx = \left[\frac{105\,Ab^{\frac{5}{2}}cx\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,(15\,Bc^3x^3+15\,Bb^3+161\,A\ldots}{105\,cx\ldots}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(7/2),x, algorithm="fricas")
```

output
```
[1/105*(105*A*b^(5/2)*c*x*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b
)*sqrt(x))/x^2) + 2*(15*B*c^3*x^3 + 15*B*b^3 + 161*A*b^2*c + 3*(15*B*b*c^2
 + 7*A*c^3)*x^2 + (45*B*b^2*c + 77*A*b*c^2)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/
(c*x), 2/105*(105*A*sqrt(-b)*b^2*c*x*arctan(sqrt(c*x^2 + b*x)*sqrt(-b)/(b*
sqrt(x))) + (15*B*c^3*x^3 + 15*B*b^3 + 161*A*b^2*c + 3*(15*B*b*c^2 + 7*A*c
^3)*x^2 + (45*B*b^2*c + 77*A*b*c^2)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(c*x)]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{7/2}}\,dx=\int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**(7/2),x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**(7/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{7/2}}\,dx=\int \frac{(cx^2+bx)^{\frac{5}{2}}(Bx+A)}{x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(7/2),x, algorithm="maxima")
```

output
```
A*b^2*integrate(sqrt(c*x + b)/x, x) + 2/105*(35*(B*b^2*c + 2*A*b*c^2)*x^3
+ (15*B*c^3*x^3 + 3*B*b*c^2*x^2 - 4*B*b^2*c*x + 8*B*b^3)*x^2 + 35*(B*b^3 +
 2*A*b^2*c)*x^2 + 7*(3*(2*B*b*c^2 + A*c^3)*x^3 + (2*B*b^2*c + A*b*c^2)*x^2
 - 2*(2*B*b^3 + A*b^2*c)*x)*x)*sqrt(c*x + b)/(c*x^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.67

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{7/2}}\,dx = \frac{2\,Ab^3\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + \frac{2\left(15\,(cx+b)^{\frac{7}{2}}Bc^6+21\,(cx+b)^{\frac{5}{2}}Ac^7+35\,(cx+b)^{\frac{3}{2}}Abc^7+105\,\sqrt{cx+b}Ab^2c^7\right)}{105\,c^7}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(7/2),x, algorithm="giac")
```

output
```
2*A*b^3*arctan(sqrt(c*x + b)/sqrt(-b))/sqrt(-b) + 2/105*(15*(c*x + b)^(7/2
)*B*c^6 + 21*(c*x + b)^(5/2)*A*c^7 + 35*(c*x + b)^(3/2)*A*b*c^7 + 105*sqrt
(c*x + b)*A*b^2*c^7)/c^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{7/2}}\,dx = \int \frac{\left(c\,x^2+b\,x\right)^{5/2}\,(A+B\,x)}{x^{7/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(7/2),x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.10

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{7/2}}\,dx = \frac{322\sqrt{cx+b}\,a\,b^2c+154\sqrt{cx+b}\,ab\,c^2x+42\sqrt{cx+b}\,a\,c^3x^2+30\sqrt{cx+b}\,b^4}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(7/2),x)
```

output
```
(322*sqrt(b + c*x)*a*b**2*c + 154*sqrt(b + c*x)*a*b*c**2*x + 42*sqrt(b + c
*x)*a*c**3*x**2 + 30*sqrt(b + c*x)*b**4 + 90*sqrt(b + c*x)*b**3*c*x + 90*s
qrt(b + c*x)*b**2*c**2*x**2 + 30*sqrt(b + c*x)*b*c**3*x**3 + 105*sqrt(b)*l
og(sqrt(b + c*x) - sqrt(b))*a*b**2*c - 105*sqrt(b)*log(sqrt(b + c*x) + sqr
t(b))*a*b**2*c)/(105*c)
```

3.199 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{9/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 146

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{9/2}}\,dx = \frac{b(2bB+5Ac)\sqrt{bx+cx^2}}{\sqrt{x}} + \frac{(2bB+5Ac)\left(bx+cx^2\right)^{3/2}}{3x^{3/2}} - \frac{A\left(bx+cx^2\right)^{5/2}}{x^{7/2}} + \frac{2B\left(bx+cx^2\right)^{5/2}}{5x^{5/2}} - b^{3/2}(2bB+5Ac)\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)$$

output

```
b*(5*A*c+2*B*b)*(c*x^2+b*x)^(1/2)/x^(1/2)+1/3*(5*A*c+2*B*b)*(c*x^2+b*x)^(3
/2)/x^(3/2)-A*(c*x^2+b*x)^(5/2)/x^(7/2)+2/5*B*(c*x^2+b*x)^(5/2)/x^(5/2)-b^
(3/2)*(5*A*c+2*B*b)*arctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1/2))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{9/2}}\,dx = \frac{\sqrt{x(b+cx)}\Big(\sqrt{b+cx}\left(2Bx\left(23b^2+11bcx+3c^2x^2\right)+A\left(-15b^2+70bcx+\right.\right.}{15x^{3/2}\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(9/2),x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[b + c*x]*(2*B*x*(23*b^2 + 11*b*c*x + 3*c^2*x^2) +
 A*(-15*b^2 + 70*b*c*x + 10*c^2*x^2)) - 15*b^(3/2)*(2*b*B + 5*A*c)*x*ArcTa
nh[Sqrt[b + c*x]/Sqrt[b]]))/(15*x^(3/2)*Sqrt[b + c*x])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1131, 1131, 1131, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{9/2}}dx$$

$$\Big\downarrow \; 1220$$

$$\frac{(5Ac+2bB)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{7/2}}dx}{2b}-\frac{A\left(bx+cx^2\right)^{7/2}}{bx^{9/2}}$$

$$\Big\downarrow \; 1131$$

$$\frac{(5Ac+2bB)\left(b\int \frac{\left(cx^2+bx\right)^{3/2}}{x^{5/2}}dx+\frac{2\left(bx+cx^2\right)^{5/2}}{5x^{5/2}}\right)}{2b}-\frac{A\left(bx+cx^2\right)^{7/2}}{bx^{9/2}}$$

$$\Big\downarrow \; 1131$$

$$\frac{(5Ac+2bB)\left(b\left(b\int \frac{\sqrt{cx^2+bx}}{x^{3/2}}dx+\frac{2(bx+cx^2)^{3/2}}{3x^{3/2}}\right)+\frac{2(bx+cx^2)^{5/2}}{5x^{5/2}}\right)}{2b}-\frac{A(bx+cx^2)^{7/2}}{bx^{9/2}}$$

$$\Big\downarrow \text{1131}$$

$$\frac{(5Ac+2bB)\left(b\left(b\left(b\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)+\frac{2(bx+cx^2)^{3/2}}{3x^{3/2}}\right)+\frac{2(bx+cx^2)^{5/2}}{5x^{5/2}}\right)}{2b}-\frac{A(bx+cx^2)^{7/2}}{bx^{9/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(5Ac+2bB)\left(b\left(b\left(2b\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)+\frac{2(bx+cx^2)^{3/2}}{3x^{3/2}}\right)+\frac{2(bx+cx^2)^{5/2}}{5x^{5/2}}\right)}{2b}-\frac{A(bx+cx^2)^{7/2}}{bx^{9/2}}$$

$$\Big\downarrow \text{220}$$

$$\frac{(5Ac+2bB)\left(b\left(b\left(\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}-2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)\right)+\frac{2(bx+cx^2)^{3/2}}{3x^{3/2}}\right)+\frac{2(bx+cx^2)^{5/2}}{5x^{5/2}}\right)}{2b}-\frac{A(bx+cx^2)^{7/2}}{bx^{9/2}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(9/2),x]
```

output
```
-((A*(b*x + c*x^2)^(7/2))/(b*x^(9/2))) + ((2*b*B + 5*A*c)*((2*(b*x + c*x^2
)^(5/2))/(5*x^(5/2)) + b*((2*(b*x + c*x^2)^(3/2))/(3*x^(3/2)) + b*((2*Sqrt
[b*x + c*x^2])/Sqrt[x] - 2*Sqrt[b]*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt
[x])]))))/(2*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{b^{2} A \left(c x+b\right)}{\sqrt{x}\, \sqrt{x \left(c x+b\right)}}+\frac{\left(\frac{2 B \left(c x+b\right)^{\frac{5}{2}}}{5}+\frac{2 A c \left(c x+b\right)^{\frac{3}{2}}}{3}+\frac{2 B b \left(c x+b\right)^{\frac{3}{2}}}{3}+4 A b c \sqrt{c x+b}+2 B\, b^{2} \sqrt{c x+b}-b^{\frac{3}{2}} \left(5 A c+2 B b\right) \operatorname{arctanh}\left(\frac{\sqrt{c x+b}}{\sqrt{b}}\right)\right) \sqrt{c}}{\sqrt{x \left(c x+b\right)}}$
default	$-\frac{\sqrt{x \left(c x+b\right)}\, \left(-6 B\, c^{2} x^{3} \sqrt{c x+b}\, \sqrt{b}-10 A\, c^{2} x^{2} \sqrt{c x+b}\, \sqrt{b}-22 B\, b^{\frac{3}{2}} c\, x^{2} \sqrt{c x+b}+75 A \operatorname{arctanh}\left(\frac{\sqrt{c x+b}}{\sqrt{b}}\right) b^{2} c x-70 A\, b^{\frac{3}{2}} c x \sqrt{c x+b}+30 B}{15 x^{\frac{3}{2}} \sqrt{c x+b}\, \sqrt{b}}$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-b^2*A*(c*x+b)/x^(1/2)/(x*(c*x+b))^(1/2)+(2/5*B*(c*x+b)^(5/2)+2/3*A*c*(c*x
+b)^(3/2)+2/3*B*b*(c*x+b)^(3/2)+4*A*b*c*(c*x+b)^(1/2)+2*B*b^2*(c*x+b)^(1/2
)-b^(3/2)*(5*A*c+2*B*b)*arctanh((c*x+b)^(1/2)/b^(1/2)))*(c*x+b)^(1/2)*x^(1
/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.65

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{9/2}}\,dx = \left[\frac{15\left(2\,Bb^2+5\,Abc\right)\sqrt{b}x^2\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\left(6\,Bc^2x^3-15\right.}{30\,x^2}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(9/2),x, algorithm="fricas")
```

output
```
[1/30*(15*(2*B*b^2 + 5*A*b*c)*sqrt(b)*x^2*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x
^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(6*B*c^2*x^3 - 15*A*b^2 + 2*(11*B*b*c
+ 5*A*c^2)*x^2 + 2*(23*B*b^2 + 35*A*b*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/x^2
, 1/15*(15*(2*B*b^2 + 5*A*b*c)*sqrt(-b)*x^2*arctan(sqrt(c*x^2 + b*x)*sqrt(
-b)/(b*sqrt(x))) + (6*B*c^2*x^3 - 15*A*b^2 + 2*(11*B*b*c + 5*A*c^2)*x^2 +
2*(23*B*b^2 + 35*A*b*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/x^2]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{9/2}}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**(9/2),x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**(9/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{9/2}}\,dx = \int \frac{\left(cx^2+bx\right)^{\frac{5}{2}}(Bx+A)}{x^{\frac{9}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(9/2),x, algorithm="maxima")
```

output
```
2/15*(5*(2*B*b*c + A*c^2)*x^2 + (3*B*c^2*x^2 + B*b*c*x - 2*B*b^2)*x + 5*(2
*B*b^2 + A*b*c)*x)*sqrt(c*x + b)/x + integrate((A*b^2 + (B*b^2 + 2*A*b*c)*
x)*sqrt(c*x + b)/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{9/2}}\,dx = -\frac{1}{15}\left(\frac{15\sqrt{cx+b}Ab^2}{cx} - \frac{15\left(2Bb^3+5Ab^2c\right)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}c} - \frac{2\left(3(cx+b)^{\frac{5}{2}}Bc^4+5(cx+b)^{\frac{3}{2}}Bbc^4+15\cdots$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(9/2),x, algorithm="giac")
```

output
```
-1/15*(15*sqrt(c*x + b)*A*b^2/(c*x) - 15*(2*B*b^3 + 5*A*b^2*c)*arctan(sqrt
(c*x + b)/sqrt(-b))/(sqrt(-b)*c) - 2*(3*(c*x + b)^(5/2)*B*c^4 + 5*(c*x + b
)^(3/2)*B*b*c^4 + 15*sqrt(c*x + b)*B*b^2*c^4 + 5*(c*x + b)^(3/2)*A*c^5 + 3
0*sqrt(c*x + b)*A*b*c^5)/c^5)*c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{9/2}}\,dx=\int \frac{{\left(c\,x^2+b\,x\right)}^{5/2}\,\left(A+B\,x\right)}{x^{9/2}} \,dx$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(9/2),x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.12

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{9/2}}\,dx=\frac{-30\sqrt{cx+b}\,a\,b^2+140\sqrt{cx+b}\,abcx+20\sqrt{cx+b}\,a\,c^2x^2+92\sqrt{cx+b}\,b^3x$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(9/2),x)
```

output
```
( - 30*sqrt(b + c*x)*a*b**2 + 140*sqrt(b + c*x)*a*b*c*x + 20*sqrt(b + c*x)
*a*c**2*x**2 + 92*sqrt(b + c*x)*b**3*x + 44*sqrt(b + c*x)*b**2*c*x**2 + 12
*sqrt(b + c*x)*b*c**2*x**3 + 75*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*b*c
*x + 30*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*b**3*x - 75*sqrt(b)*log(sqrt(
b + c*x) + sqrt(b))*a*b*c*x - 30*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*b**3
*x)/(30*x)
```

3.200 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 155

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11/2}}\,dx = \frac{5c(4bB+3Ac)\sqrt{bx+cx^2}}{4\sqrt{x}} - \frac{(4bB+5Ac)\left(bx+cx^2\right)^{3/2}}{4x^{5/2}} + \frac{2Bc(bx+cx^2)^{3/2}}{3x^{3/2}} - \frac{A(bx+cx^2)^{5/2}}{2x^{9/2}} - \frac{5}{4}\sqrt{b}c(4bB+3Ac)\operatorname{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)$$

output
```
5/4*c*(3*A*c+4*B*b)*(c*x^2+b*x)^(1/2)/x^(1/2)-1/4*(5*A*c+4*B*b)*(c*x^2+b*x
)^(3/2)/x^(5/2)+2/3*B*c*(c*x^2+b*x)^(3/2)/x^(3/2)-1/2*A*(c*x^2+b*x)^(5/2)/
x^(9/2)-5/4*b^(1/2)*c*(3*A*c+4*B*b)*arctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1
/2))
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.79

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11/2}}\,dx = \frac{\sqrt{x(b+cx)}\left(\sqrt{b+cx}(-3A(2b^2+9bcx-8c^2x^2)+4Bx(-3b^2+14bcx+2\cdots\right.}{12x^{5/2}\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(11/2),x]
```

output
```
(Sqrt[x*(b + c*x)]*(Sqrt[b + c*x]*(-3*A*(2*b^2 + 9*b*c*x - 8*c^2*x^2) + 4*
B*x*(-3*b^2 + 14*b*c*x + 2*c^2*x^2)) - 15*Sqrt[b]*c*(4*b*B + 3*A*c)*x^2*Ar
cTanh[Sqrt[b + c*x]/Sqrt[b]]))/(12*x^(5/2)*Sqrt[b + c*x])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1130, 1131, 1131, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11/2}}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(3Ac+4bB)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{9/2}}dx}{4b}-\frac{A\left(bx+cx^2\right)^{7/2}}{2bx^{11/2}}$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{(3Ac+4bB)\left(\frac{5}{2}c\int \frac{\left(cx^2+bx\right)^{3/2}}{x^{5/2}}dx-\frac{\left(bx+cx^2\right)^{5/2}}{x^{7/2}}\right)}{4b}-\frac{A\left(bx+cx^2\right)^{7/2}}{2bx^{11/2}}$$

$$\Big\downarrow \text{ 1131}$$

$$\frac{(3Ac+4bB)\left(\frac{5}{2}c\left(b\int\frac{\sqrt{cx^2+bx}}{x^{3/2}}dx+\frac{2(bx+cx^2)^{3/2}}{3x^{3/2}}\right)-\frac{(bx+cx^2)^{5/2}}{x^{7/2}}\right)}{4b}-\frac{A(bx+cx^2)^{7/2}}{2bx^{11/2}}$$

$$\Big\downarrow \ 1131$$

$$\frac{(3Ac+4bB)\left(\frac{5}{2}c\left(b\left(b\int\frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)+\frac{2(bx+cx^2)^{3/2}}{3x^{3/2}}\right)-\frac{(bx+cx^2)^{5/2}}{x^{7/2}}\right)}{4b}-\frac{A(bx+cx^2)^{7/2}}{2bx^{11/2}}$$

$$\Big\downarrow \ 1136$$

$$\frac{(3Ac+4bB)\left(\frac{5}{2}c\left(b\left(2b\int\frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)+\frac{2(bx+cx^2)^{3/2}}{3x^{3/2}}\right)-\frac{(bx+cx^2)^{5/2}}{x^{7/2}}\right)}{4b}-\frac{A(bx+cx^2)^{7/2}}{2bx^{11/2}}$$

$$\Big\downarrow \ 220$$

$$\frac{(3Ac+4bB)\left(\frac{5}{2}c\left(b\left(\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}-2\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)\right)+\frac{2(bx+cx^2)^{3/2}}{3x^{3/2}}\right)-\frac{(bx+cx^2)^{5/2}}{x^{7/2}}\right)}{4b}-\frac{A(bx+cx^2)^{7/2}}{2bx^{11/2}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(11/2),x]
```

output
```
-1/2*(A*(b*x + c*x^2)^(7/2))/(b*x^(11/2)) + ((4*b*B + 3*A*c)*(-((b*x + c*x
^2)^(5/2)/x^(7/2)) + (5*c*((2*(b*x + c*x^2)^(3/2))/(3*x^(3/2)) + b*((2*Sqr
t[b*x + c*x^2])/Sqrt[x] - 2*Sqrt[b]*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqr
t[x])])))/2))/(4*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{b(cx+b)(9Acx+4Bbx+2Ab)}{4x^{\frac{3}{2}}\sqrt{x(cx+b)}}+\frac{c\left(\frac{16B(cx+b)^{\frac{3}{2}}}{3}+16Ac\sqrt{cx+b}+32Bb\sqrt{cx+b}-10\sqrt{b}\,(3Ac+4Bb)\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)\right)\sqrt{cx+b}\,\sqrt{x}}{8\sqrt{x(cx+b)}}$
default	$-\frac{\sqrt{x(cx+b)}\left(-8Bc^2x^3\sqrt{cx+b}\,\sqrt{b}+45A\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)bc^2x^2-24Ac^2x^2\sqrt{cx+b}\,\sqrt{b}+60B\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)b^2cx^2-56Bb^{\frac{3}{2}}cx\right.}{12x^{\frac{5}{2}}\sqrt{cx+b}\,\sqrt{b}}$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*b*(c*x+b)*(9*A*c*x+4*B*b*x+2*A*b)/x^(3/2)/(x*(c*x+b))^(1/2)+1/8*c*(16
/3*B*(c*x+b)^(3/2)+16*A*c*(c*x+b)^(1/2)+32*B*b*(c*x+b)^(1/2)-10*b^(1/2)*(3
*A*c+4*B*b)*arctanh((c*x+b)^(1/2)/b^(1/2)))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*x+
b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.55

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11/2}}\,dx=\left[\frac{15\left(4\,Bbc+3\,Ac^2\right)\sqrt{b}x^3\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\left(8\,Bc^2x^3-6\,A\right.}{24\,x^3}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(11/2),x, algorithm="fricas")
```

output
```
[1/24*(15*(4*B*b*c + 3*A*c^2)*sqrt(b)*x^3*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x
^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(8*B*c^2*x^3 - 6*A*b^2 + 8*(7*B*b*c +
3*A*c^2)*x^2 - 3*(4*B*b^2 + 9*A*b*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/x^3, 1/
12*(15*(4*B*b*c + 3*A*c^2)*sqrt(-b)*x^3*arctan(sqrt(c*x^2 + b*x)*sqrt(-b)/
(b*sqrt(x))) + (8*B*c^2*x^3 - 6*A*b^2 + 8*(7*B*b*c + 3*A*c^2)*x^2 - 3*(4*B
*b^2 + 9*A*b*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/x^3]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11/2}}\,dx=\int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**(11/2),x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**(11/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11/2}}\,dx=\int \frac{(cx^2+bx)^{\frac{5}{2}}(Bx+A)}{x^{\frac{11}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(11/2),x, algorithm="maxima")
```

output
```
2/3*(B*c^2*x + B*b*c)*sqrt(c*x + b) + integrate((A*b^2 + (2*B*b*c + A*c^2)
*x^2 + (B*b^2 + 2*A*b*c)*x)*sqrt(c*x + b)/x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.00

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11/2}}\,dx=\frac{8\,(cx+b)^{\frac{3}{2}}Bc^2+48\,\sqrt{cx+b}Bbc^2+24\,\sqrt{cx+b}Ac^3+\frac{15\,(4\,Bb^2c^2+3\,Abc^3)\,\mathrm{arcta}}{\sqrt{-b}}}{12\,c}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(11/2),x, algorithm="giac")
```

output
```
1/12*(8*(c*x + b)^(3/2)*B*c^2 + 48*sqrt(c*x + b)*B*b*c^2 + 24*sqrt(c*x + b
)*A*c^3 + 15*(4*B*b^2*c^2 + 3*A*b*c^3)*arctan(sqrt(c*x + b)/sqrt(-b))/sqrt
(-b) - 3*(4*(c*x + b)^(3/2)*B*b^2*c^2 - 4*sqrt(c*x + b)*B*b^3*c^2 + 9*(c*x
 + b)^(3/2)*A*b*c^3 - 7*sqrt(c*x + b)*A*b^2*c^3)/(c^2*x^2))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11/2}}\,dx=\int \frac{\left(c\,x^2+b\,x\right)^{5/2}\,\left(A+B\,x\right)}{x^{11/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(11/2),x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.13

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{11/2}}\,dx=\frac{-12\sqrt{cx+b}\,a\,b^2-54\sqrt{cx+b}\,abcx+48\sqrt{cx+b}\,a\,c^2x^2-24\sqrt{cx+b}\,b^3x-}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(11/2),x)
```

output
```
( - 12*sqrt(b + c*x)*a*b**2 - 54*sqrt(b + c*x)*a*b*c*x + 48*sqrt(b + c*x)*
a*c**2*x**2 - 24*sqrt(b + c*x)*b**3*x + 112*sqrt(b + c*x)*b**2*c*x**2 + 16
*sqrt(b + c*x)*b*c**2*x**3 + 45*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c**
2*x**2 + 60*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*b**2*c*x**2 - 45*sqrt(b)*
log(sqrt(b + c*x) + sqrt(b))*a*c**2*x**2 - 60*sqrt(b)*log(sqrt(b + c*x) +
sqrt(b))*b**2*c*x**2)/(24*x**2)
```

3.201 $\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^{13/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^{13/2}}\,dx = -\frac{c(14bB+5Ac)\sqrt{bx+cx^2}}{8x^{3/2}}+\frac{2Bc^2\sqrt{bx+cx^2}}{\sqrt{x}} - \frac{(6bB+5Ac)(bx+cx^2)^{3/2}}{12x^{7/2}}-\frac{A(bx+cx^2)^{5/2}}{3x^{11/2}}-\frac{5c^2(6bB+Ac)\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8\sqrt{b}}$$

output
```
-1/8*c*(5*A*c+14*B*b)*(c*x^2+b*x)^(1/2)/x^(3/2)+2*B*c^2*(c*x^2+b*x)^(1/2)/
x^(1/2)-1/12*(5*A*c+6*B*b)*(c*x^2+b*x)^(3/2)/x^(7/2)-1/3*A*(c*x^2+b*x)^(5/
2)/x^(11/2)-5/8*c^2*(A*c+6*B*b)*arctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1/2))
/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.81

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^{13/2}}\,dx =$$
$$-\frac{\sqrt{x(b+cx)}\left(\sqrt{b}\sqrt{b+cx}(6Bx(2b^2+9bcx-8c^2x^2)+A(8b^2+26bcx+33c^2x^2))+15c^2(6bB+Ac)x^3\text{arc}\right.}{24\sqrt{b}x^{7/2}\sqrt{b+cx}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(13/2),x]
```

output
```
-1/24*(Sqrt[x*(b + c*x)]*(Sqrt[b]*Sqrt[b + c*x]*(6*B*x*(2*b^2 + 9*b*c*x -
8*c^2*x^2) + A*(8*b^2 + 26*b*c*x + 33*c^2*x^2)) + 15*c^2*(6*b*B + A*c)*x^3
*ArcTanh[Sqrt[b + c*x]/Sqrt[b]]))/(Sqrt[b]*x^(7/2)*Sqrt[b + c*x])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1130, 1130, 1131, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{13/2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(Ac+6bB)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{11/2}}dx}{6b}-\frac{A\left(bx+cx^2\right)^{7/2}}{3bx^{13/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(Ac+6bB)\left(\frac{5}{4}c\int \frac{\left(cx^2+bx\right)^{3/2}}{x^{7/2}}dx-\frac{\left(bx+cx^2\right)^{5/2}}{2x^{9/2}}\right)}{6b}-\frac{A\left(bx+cx^2\right)^{7/2}}{3bx^{13/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(Ac+6bB)\left(\frac{5}{4}c\left(\frac{3}{2}c\int \frac{\sqrt{cx^2+bx}}{x^{3/2}}dx-\frac{\left(bx+cx^2\right)^{3/2}}{x^{5/2}}\right)-\frac{\left(bx+cx^2\right)^{5/2}}{2x^{9/2}}\right)}{6b}-\frac{A\left(bx+cx^2\right)^{7/2}}{3bx^{13/2}}$$

$$\Big\downarrow \text{1131}$$

$$\frac{(Ac+6bB)\left(\frac{5}{4}c\left(\frac{3}{2}c\left(b\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)-\frac{\left(bx+cx^2\right)^{3/2}}{x^{5/2}}\right)-\frac{\left(bx+cx^2\right)^{5/2}}{2x^{9/2}}\right)}{6b}-\frac{A\left(bx+cx^2\right)^{7/2}}{3bx^{13/2}}$$

$$\Big\downarrow \text{ 1136}$$

$$\frac{(Ac+6bB)\left(\frac{5}{4}c\left(\frac{3}{2}c\left(2b\int\frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}+\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}\right)-\frac{(bx+cx^2)^{3/2}}{x^{5/2}}\right)-\frac{(bx+cx^2)^{5/2}}{2x^{9/2}}\right)}{6b}-\frac{A(bx+cx^2)^{7/2}}{3bx^{13/2}}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{(Ac+6bB)\left(\frac{5}{4}c\left(\frac{3}{2}c\left(\frac{2\sqrt{bx+cx^2}}{\sqrt{x}}-2\sqrt{b}\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)\right)-\frac{(bx+cx^2)^{3/2}}{x^{5/2}}\right)-\frac{(bx+cx^2)^{5/2}}{2x^{9/2}}\right)}{6b}-\frac{A(bx+cx^2)^{7/2}}{3bx^{13/2}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(13/2),x]
```

output
```
-1/3*(A*(b*x + c*x^2)^(7/2))/(b*x^(13/2)) + ((6*b*B + A*c)*(-1/2*(b*x + c*
x^2)^(5/2)/x^(9/2) + (5*c*(-((b*x + c*x^2)^(3/2)/x^(5/2)) + (3*c*((2*Sqrt[
b*x + c*x^2])/Sqrt[x] - 2*Sqrt[b]*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[
x])]))/2))/4))/(6*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1131
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Simp[p*((2*c*d - b*e)/(e^2*(m + 2*p + 1)))   Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b
*d*e + a*e^2, 0] && GtQ[p, 0] && (LeQ[-2, m, 0] || EqQ[m + p + 1, 0]) && Ne
Q[m + 2*p + 1, 0] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{(cx+b)\left(33A\,c^2x^2+54x^2Bbc+26Abcx+12xB\,b^2+8b^2A\right)}{24x^{\frac{5}{2}}\sqrt{x(cx+b)}}+\frac{c^2\left(32B\sqrt{cx+b}-\frac{2(5Ac+30Bb)\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)}{\sqrt{b}}\right)\sqrt{cx+b}\,\sqrt{x}}{16\sqrt{x(cx+b)}}$
default	$-\frac{\sqrt{x(cx+b)}\,\left(15A\ \operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^3x^3+90B\ \operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)b\,c^2x^3-48B\,c^2x^3\sqrt{cx+b}\,\sqrt{b}+33A\,c^2x^2\sqrt{cx+b}\,\sqrt{b}+54B\,b^{\frac{3}{2}}\,c\,x^2}{24x^{\frac{7}{2}}\sqrt{cx+b}\,\sqrt{b}}$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x+b)*(33*A*c^2*x^2+54*B*b*c*x^2+26*A*b*c*x+12*B*b^2*x+8*A*b^2)/x^
(5/2)/(x*(c*x+b))^(1/2)+1/16*c^2*(32*B*(c*x+b)^(1/2)-2*(5*A*c+30*B*b)/b^(1
/2)*arctanh((c*x+b)^(1/2)/b^(1/2)))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*x+b))^(1/2
)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.67

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{13/2}}\,dx = \left[\frac{15\left(6\,Bbc^2+Ac^3\right)\sqrt{b}x^4\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\left(48\,Bbc^2x^3-8\right.}{48\,bx}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(13/2),x, algorithm="fricas")
```

output
```
[1/48*(15*(6*B*b*c^2 + A*c^3)*sqrt(b)*x^4*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x
^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(48*B*b*c^2*x^3 - 8*A*b^3 - 3*(18*B*b^
2*c + 11*A*b*c^2)*x^2 - 2*(6*B*b^3 + 13*A*b^2*c)*x)*sqrt(c*x^2 + b*x)*sqrt
(x))/(b*x^4), 1/24*(15*(6*B*b*c^2 + A*c^3)*sqrt(-b)*x^4*arctan(sqrt(c*x^2
+ b*x)*sqrt(-b)/(b*sqrt(x))) + (48*B*b*c^2*x^3 - 8*A*b^3 - 3*(18*B*b^2*c +
 11*A*b*c^2)*x^2 - 2*(6*B*b^3 + 13*A*b^2*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/
(b*x^4)]
```

Sympy [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{13/2}}\,dx = \int \frac{(x(b+cx))^{\frac{5}{2}}\,(A+Bx)}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**(13/2),x)
```

output
```
Integral((x*(b + c*x))**(5/2)*(A + B*x)/x**(13/2), x)
```

Maxima [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{13/2}}\,dx=\int \frac{(cx^2+bx)^{\frac{5}{2}}(Bx+A)}{x^{\frac{13}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(13/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(5/2)*(B*x + A)/x^(13/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.86

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{13/2}}\,dx=\frac{1}{24}\,c^3\left(\frac{48\,\sqrt{cx+b}B}{c}+\frac{15\,(6\,Bb+Ac)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}c}-\frac{54\,(cx+b)^{\frac{5}{2}}Bb}{}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(13/2),x, algorithm="giac")
```

output
```
1/24*c^3*(48*sqrt(c*x + b)*B/c + 15*(6*B*b + A*c)*arctan(sqrt(c*x + b)/sqr
t(-b))/(sqrt(-b)*c) - (54*(c*x + b)^(5/2)*B*b - 96*(c*x + b)^(3/2)*B*b^2 +
 42*sqrt(c*x + b)*B*b^3 + 33*(c*x + b)^(5/2)*A*c - 40*(c*x + b)^(3/2)*A*b*
c + 15*sqrt(c*x + b)*A*b^2*c)/(c^4*x^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{13/2}}\,dx=\int \frac{\left(c\,x^2+b\,x\right)^{5/2}\,\left(A+B\,x\right)}{x^{13/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(13/2),x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(13/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.20

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{13/2}}\,dx = \frac{-16\sqrt{cx+b}\,a\,b^3-52\sqrt{cx+b}\,a\,b^2cx-66\sqrt{cx+b}\,ab\,c^2x^2-24\sqrt{cx+b}\,b^4x}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(13/2),x)
```

output
```
( - 16*sqrt(b + c*x)*a*b**3 - 52*sqrt(b + c*x)*a*b**2*c*x - 66*sqrt(b + c*
x)*a*b*c**2*x**2 - 24*sqrt(b + c*x)*b**4*x - 108*sqrt(b + c*x)*b**3*c*x**2
 + 96*sqrt(b + c*x)*b**2*c**2*x**3 + 15*sqrt(b)*log(sqrt(b + c*x) - sqrt(b
))*a*c**3*x**3 + 90*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*b**2*c**2*x**3 -
15*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*a*c**3*x**3 - 90*sqrt(b)*log(sqrt(
b + c*x) + sqrt(b))*b**2*c**2*x**3)/(48*b*x**3)
```

3.202 $\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{15/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{15/2}}\,dx = -\frac{c(24bB+5Ac)\sqrt{bx+cx^2}}{32x^{5/2}} - \frac{c^2(88bB+5Ac)\sqrt{bx+cx^2}}{64bx^{3/2}} - \frac{(8bB+5Ac)\left(bx+cx^2\right)^{3/2}}{24x^{9/2}} - \frac{A\left(bx+cx^2\right)^{5/2}}{4x^{13/2}} - \frac{5c^3(8bB-Ac)\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{64b^{3/2}}$$

output
```
-1/32*c*(5*A*c+24*B*b)*(c*x^2+b*x)^(1/2)/x^(5/2)-1/64*c^2*(5*A*c+88*B*b)*(
c*x^2+b*x)^(1/2)/b/x^(3/2)-1/24*(5*A*c+8*B*b)*(c*x^2+b*x)^(3/2)/x^(9/2)-1/
4*A*(c*x^2+b*x)^(5/2)/x^(13/2)-5/64*c^3*(-A*c+8*B*b)*arctanh((c*x^2+b*x)^(
1/2)/b^(1/2)/x^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.82

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{15/2}}\,dx = \frac{\sqrt{x(b+cx)}\left(-\sqrt{b}\sqrt{b+cx}(8bBx(8b^2+26bcx+33c^2x^2)+A(48b^3+136b\right.}{192b^{3/2}x^{9/2}\sqrt{}}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(15/2),x]
```

output
```
(Sqrt[x*(b + c*x)]*(-(Sqrt[b]*Sqrt[b + c*x]*(8*b*B*x*(8*b^2 + 26*b*c*x + 3
3*c^2*x^2) + A*(48*b^3 + 136*b^2*c*x + 118*b*c^2*x^2 + 15*c^3*x^3))) + 15*
c^3*(-8*b*B + A*c)*x^4*ArcTanh[Sqrt[b + c*x]/Sqrt[b]]))/(192*b^(3/2)*x^(9/
2)*Sqrt[b + c*x])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1130, 1130, 1130, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{15/2}}dx$$

$$\Big\downarrow \; 1220$$

$$\frac{(8bB-Ac)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{13/2}}dx}{8b}-\frac{A\left(bx+cx^2\right)^{7/2}}{4bx^{15/2}}$$

$$\Big\downarrow \; 1130$$

$$\frac{(8bB-Ac)\left(\frac{5}{6}c\int \frac{\left(cx^2+bx\right)^{3/2}}{x^{9/2}}dx-\frac{\left(bx+cx^2\right)^{5/2}}{3x^{11/2}}\right)}{8b}-\frac{A\left(bx+cx^2\right)^{7/2}}{4bx^{15/2}}$$

$$\Big\downarrow \; 1130$$

$$\frac{(8bB-Ac)\left(\frac{5}{6}c\left(\frac{3}{4}c\int\frac{\sqrt{cx^2+bx}}{x^{5/2}}dx-\frac{(bx+cx^2)^{3/2}}{2x^{7/2}}\right)-\frac{(bx+cx^2)^{5/2}}{3x^{11/2}}\right)}{8b}-\frac{A(bx+cx^2)^{7/2}}{4bx^{15/2}}$$

$$\Big\downarrow \text{1130}$$

$$\frac{(8bB-Ac)\left(\frac{5}{6}c\left(\frac{3}{4}c\left(\frac{1}{2}c\int\frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx-\frac{\sqrt{bx+cx^2}}{x^{3/2}}\right)-\frac{(bx+cx^2)^{3/2}}{2x^{7/2}}\right)-\frac{(bx+cx^2)^{5/2}}{3x^{11/2}}\right)}{8b}-\frac{A(bx+cx^2)^{7/2}}{4bx^{15/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(8bB-Ac)\left(\frac{5}{6}c\left(\frac{3}{4}c\left(c\int\frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}-\frac{\sqrt{bx+cx^2}}{x^{3/2}}\right)-\frac{(bx+cx^2)^{3/2}}{2x^{7/2}}\right)-\frac{(bx+cx^2)^{5/2}}{3x^{11/2}}\right)}{8b}-\frac{A(bx+cx^2)^{7/2}}{4bx^{15/2}}$$

$$\Big\downarrow \text{220}$$

$$\frac{(8bB-Ac)\left(\frac{5}{6}c\left(\frac{3}{4}c\left(-\frac{c\,\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}-\frac{\sqrt{bx+cx^2}}{x^{3/2}}\right)-\frac{(bx+cx^2)^{3/2}}{2x^{7/2}}\right)-\frac{(bx+cx^2)^{5/2}}{3x^{11/2}}\right)}{8b}-\frac{A(bx+cx^2)^{7/2}}{4bx^{15/2}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(15/2),x]
```

output
```
-1/4*(A*(b*x + c*x^2)^(7/2))/(b*x^(15/2)) + ((8*b*B - A*c)*(-1/3*(b*x + c*
x^2)^(5/2)/x^(11/2) + (5*c*(-1/2*(b*x + c*x^2)^(3/2)/x^(7/2) + (3*c*(-(Sqr
t[b*x + c*x^2]/x^(3/2)) - (c*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])
/Sqrt[b]))/4))/6))/(8*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{(cx+b)\left(15A\,c^{3}x^{3}+264x^{3}Bb\,c^{2}+118Ab\,c^{2}x^{2}+208x^{2}B\,b^{2}c+136A\,b^{2}cx+64xB\,b^{3}+48A\,b^{3}\right)}{192x^{\frac{7}{2}}b\sqrt{x(cx+b)}}+\frac{5c^{3}(Ac-8Bb)\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)\sqrt{\cdots}}{64b^{\frac{3}{2}}\sqrt{x(cx+b)}}$
default	$\frac{\sqrt{x(cx+b)}\left(15A\ \mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^{4}x^{4}-120B\ \mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)b\,c^{3}x^{4}-15A\,c^{3}x^{3}\sqrt{cx+b}\,\sqrt{b}-264B\,b^{\frac{3}{2}}c^{2}x^{3}\sqrt{cx+b}-118A\,b^{\frac{3}{2}}c^{2}x^{2}\cdots\right.}{192b^{\frac{3}{2}}x^{\frac{9}{2}}\sqrt{cx+b}}$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(c*x+b)*(15*A*c^3*x^3+264*B*b*c^2*x^3+118*A*b*c^2*x^2+208*B*b^2*c*x
^2+136*A*b^2*c*x+64*B*b^3*x+48*A*b^3)/x^(7/2)/b/(x*(c*x+b))^(1/2)+5/64*c^3
*(A*c-8*B*b)/b^(3/2)*arctanh((c*x+b)^(1/2)/b^(1/2))*(c*x+b)^(1/2)*x^(1/2)/
(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.72

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^{15/2}}\,dx = \left[-\frac{15\,(8\,Bbc^3-Ac^4)\sqrt{b}x^5\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,(48\,Ab^4+3\,(\cdots}{\cdots}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(15/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*(8*B*b*c^3 - A*c^4)*sqrt(b)*x^5*log(-(c*x^2 + 2*b*x + 2*sqrt(c
*x^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(48*A*b^4 + 3*(88*B*b^2*c^2 + 5*A*b*
c^3)*x^3 + 2*(104*B*b^3*c + 59*A*b^2*c^2)*x^2 + 8*(8*B*b^4 + 17*A*b^3*c)*x
)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^2*x^5), 1/192*(15*(8*B*b*c^3 - A*c^4)*sqrt
(-b)*x^5*arctan(sqrt(c*x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) - (48*A*b^4 + 3*(8
8*B*b^2*c^2 + 5*A*b*c^3)*x^3 + 2*(104*B*b^3*c + 59*A*b^2*c^2)*x^2 + 8*(8*B
*b^4 + 17*A*b^3*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^2*x^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**(15/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{15/2}}\,dx=\int \frac{\left(cx^2+bx\right)^{\frac{5}{2}}(Bx+A)}{x^{\frac{15}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(15/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(5/2)*(B*x + A)/x^(15/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{15/2}}\,dx=\frac{\frac{15\,(8\,Bbc^4-Ac^5)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b}-\frac{264\,(cx+b)^{\frac{7}{2}}Bbc^4-584\,(cx+b)^{\frac{5}{2}}Bb^2c^4+440\,(cx+b)^{\frac{3}{2}}Bb^3c^4-1}{}}{192\,c}$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(15/2),x, algorithm="giac")
```

output
```
1/192*(15*(8*B*b*c^4 - A*c^5)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b)
- (264*(c*x + b)^(7/2)*B*b*c^4 - 584*(c*x + b)^(5/2)*B*b^2*c^4 + 440*(c*x
+ b)^(3/2)*B*b^3*c^4 - 120*sqrt(c*x + b)*B*b^4*c^4 + 15*(c*x + b)^(7/2)*A*
c^5 + 73*(c*x + b)^(5/2)*A*b*c^5 - 55*(c*x + b)^(3/2)*A*b^2*c^5 + 15*sqrt(
c*x + b)*A*b^3*c^5)/(b*c^4*x^4))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{15/2}}\,dx=\int \frac{\left(c\,x^2+b\,x\right)^{5/2}\,\left(A+B\,x\right)}{x^{15/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(15/2),x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(15/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.21

$$\int \frac{(A+Bx)\,(bx+cx^2)^{5/2}}{x^{15/2}}\,dx = \frac{-96\sqrt{cx+b}\,a\,b^4-272\sqrt{cx+b}\,a\,b^3cx-236\sqrt{cx+b}\,a\,b^2c^2x^2-30\sqrt{cx+b}}{}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(15/2),x)
```

output
```
( - 96*sqrt(b + c*x)*a*b**4 - 272*sqrt(b + c*x)*a*b**3*c*x - 236*sqrt(b +
c*x)*a*b**2*c**2*x**2 - 30*sqrt(b + c*x)*a*b*c**3*x**3 - 128*sqrt(b + c*x)
*b**5*x - 416*sqrt(b + c*x)*b**4*c*x**2 - 528*sqrt(b + c*x)*b**3*c**2*x**3
 - 15*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c**4*x**4 + 120*sqrt(b)*log(s
qrt(b + c*x) - sqrt(b))*b**2*c**3*x**4 + 15*sqrt(b)*log(sqrt(b + c*x) + sq
rt(b))*a*c**4*x**4 - 120*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*b**2*c**3*x*
*4)/(384*b**2*x**4)
```

3.203 $\int \frac{(A+Bx)(bx+cx^2)^{5/2}}{x^{17/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{17/2}}\,dx = -\frac{c(22bB+3Ac)\sqrt{bx+cx^2}}{48x^{7/2}} - \frac{c^2(118bB+3Ac)\sqrt{bx+cx^2}}{192bx^{5/2}} - \frac{c^3(10bB-3Ac)\sqrt{bx+cx^2}}{128b^2x^{3/2}} - \frac{(2bB+Ac)\left(bx+cx^2\right)^{3/2}}{8x^{11/2}} - \frac{A\left(bx+cx^2\right)^{5/2}}{5x^{15/2}} + \frac{c^4(10bB-3Ac)\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{128b^{5/2}}$$

output

```
-1/48*c*(3*A*c+22*B*b)*(c*x^2+b*x)^(1/2)/x^(7/2)-1/192*c^2*(3*A*c+118*B*b)
*(c*x^2+b*x)^(1/2)/b/x^(5/2)-1/128*c^3*(-3*A*c+10*B*b)*(c*x^2+b*x)^(1/2)/b
^2/x^(3/2)-1/8*(A*c+2*B*b)*(c*x^2+b*x)^(3/2)/x^(11/2)-1/5*A*(c*x^2+b*x)^(5
/2)/x^(15/2)+1/128*c^4*(-3*A*c+10*B*b)*arctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x
^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.78

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{17/2}}\,dx=\frac{-\sqrt{b}(b+cx)\left(10bBx(48b^3+136b^2cx+118bc^2x^2+15c^3x^3)+3A(128b^4+\right.}{192}$$

input
```
Integrate[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(17/2),x]
```

output
```
(-(Sqrt[b]*(b + c*x)*(10*b*B*x*(48*b^3 + 136*b^2*c*x + 118*b*c^2*x^2 + 15*
c^3*x^3) + 3*A*(128*b^4 + 336*b^3*c*x + 248*b^2*c^2*x^2 + 10*b*c^3*x^3 - 1
5*c^4*x^4))) + 15*c^4*(10*b*B - 3*A*c)*x^5*Sqrt[b + c*x]*ArcTanh[Sqrt[b +
c*x]/Sqrt[b]])/(1920*b^(5/2)*x^(9/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1220, 1130, 1130, 1130, 1135, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{17/2}}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(10bB-3Ac)\int \frac{\left(cx^2+bx\right)^{5/2}}{x^{15/2}}dx}{10b}-\frac{A\left(bx+cx^2\right)^{7/2}}{5bx^{17/2}}$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{(10bB-3Ac)\left(\frac{5}{8}c\int \frac{\left(cx^2+bx\right)^{3/2}}{x^{11/2}}dx-\frac{\left(bx+cx^2\right)^{5/2}}{4x^{13/2}}\right)}{10b}-\frac{A\left(bx+cx^2\right)^{7/2}}{5bx^{17/2}}$$

$$\Big\downarrow \text{ 1130}$$

$$\frac{(10bB-3Ac)\left(\frac{5}{8}c\left(\frac{1}{2}c\int\frac{\sqrt{cx^2+bx}}{x^{7/2}}dx-\frac{(bx+cx^2)^{3/2}}{3x^{9/2}}\right)-\frac{(bx+cx^2)^{5/2}}{4x^{13/2}}\right)}{10b}-\frac{A(bx+cx^2)^{7/2}}{5bx^{17/2}}$$

$\downarrow$ 1130

$$\frac{(10bB-3Ac)\left(\frac{5}{8}c\left(\frac{1}{2}c\left(\frac{1}{4}c\int\frac{1}{x^{3/2}\sqrt{cx^2+bx}}dx-\frac{\sqrt{bx+cx^2}}{2x^{5/2}}\right)-\frac{(bx+cx^2)^{3/2}}{3x^{9/2}}\right)-\frac{(bx+cx^2)^{5/2}}{4x^{13/2}}\right)}{10b}-\frac{A(bx+cx^2)^{7/2}}{5bx^{17/2}}$$

$\downarrow$ 1135

$$\frac{(10bB-3Ac)\left(\frac{5}{8}c\left(\frac{1}{2}c\left(\frac{1}{4}c\left(-\frac{c\int\frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{2b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)-\frac{\sqrt{bx+cx^2}}{2x^{5/2}}\right)-\frac{(bx+cx^2)^{3/2}}{3x^{9/2}}\right)-\frac{(bx+cx^2)^{5/2}}{4x^{13/2}}\right)}{10b}-\frac{A(bx+cx^2)^{7/2}}{5bx^{17/2}}$$

$\downarrow$ 1136

$$\frac{(10bB-3Ac)\left(\frac{5}{8}c\left(\frac{1}{2}c\left(\frac{1}{4}c\left(-\frac{c\int\frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)-\frac{\sqrt{bx+cx^2}}{2x^{5/2}}\right)-\frac{(bx+cx^2)^{3/2}}{3x^{9/2}}\right)-\frac{(bx+cx^2)^{5/2}}{4x^{13/2}}\right)}{10b}-\frac{A(bx+cx^2)^{7/2}}{5bx^{17/2}}$$

$\downarrow$ 220

$$\frac{(10bB-3Ac)\left(\frac{5}{8}c\left(\frac{1}{2}c\left(\frac{1}{4}c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)-\frac{\sqrt{bx+cx^2}}{2x^{5/2}}\right)-\frac{(bx+cx^2)^{3/2}}{3x^{9/2}}\right)-\frac{(bx+cx^2)^{5/2}}{4x^{13/2}}\right)}{10b}-\frac{A(bx+cx^2)^{7/2}}{5bx^{17/2}}$$

input
```
Int[((A + B*x)*(b*x + c*x^2)^(5/2))/x^(17/2),x]
```

output
```
-1/5*(A*(b*x + c*x^2)^(7/2))/(b*x^(17/2)) + ((10*b*B - 3*A*c)*(-1/4*(b*x +
 c*x^2)^(5/2)/x^(13/2) + (5*c*(-1/3*(b*x + c*x^2)^(3/2)/x^(9/2) + (c*(-1/2
*Sqrt[b*x + c*x^2]/x^(5/2) + (c*(-(Sqrt[b*x + c*x^2]/(b*x^(3/2))) + (c*Arc
Tanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/b^(3/2)))/4))/2))/8))/(10*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1130
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + p + 1))), x]
- Simp[c*(p/(e^2*(m + p + 1)))   Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)^(p
 - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && GtQ[p, 0] && (LtQ[m, -2] || EqQ[m + 2*p + 1, 0]) && NeQ[m + p + 1, 0] &
& IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{(cx+b)\left(-45A\,c^4x^4+150Bb\,c^3x^4+30Ab\,c^3x^3+1180B\,b^2c^2x^3+744A\,b^2c^2x^2+1360B\,b^3c\,x^2+1008A\,b^3cx+480B\,b^4x+384A\,b^4\right)}{1920x^{\frac{9}{2}}b^2\sqrt{x(cx+b)}}-$
default	$-\frac{\sqrt{x(cx+b)}\left(45A\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^5x^5-150B\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)b\,c^4x^5-45A\,c^4x^4\sqrt{cx+b}\,\sqrt{b}+150B\,b^{\frac{3}{2}}c^3x^4\sqrt{cx+b}+30A\,b^{\frac{3}{2}}c^3x}{1920b}$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(17/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1920*(c*x+b)*(-45*A*c^4*x^4+150*B*b*c^3*x^4+30*A*b*c^3*x^3+1180*B*b^2*c
^2*x^3+744*A*b^2*c^2*x^2+1360*B*b^3*c*x^2+1008*A*b^3*c*x+480*B*b^4*x+384*A
*b^4)/x^(9/2)/b^2/(x*(c*x+b))^(1/2)-1/128*c^4*(3*A*c-10*B*b)/b^(5/2)*arcta
nh((c*x+b)^(1/2)/b^(1/2))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.65

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{17/2}}\,dx=\left[-\frac{15\left(10\,Bbc^4-3\,Ac^5\right)\sqrt{b}x^6\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\left(384\,Ab^5+\right.}{}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(17/2),x, algorithm="fricas")
```

output
```
[-1/3840*(15*(10*B*b*c^4 - 3*A*c^5)*sqrt(b)*x^6*log(-(c*x^2 + 2*b*x - 2*sq
rt(c*x^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(384*A*b^5 + 15*(10*B*b^2*c^3 -
3*A*b*c^4)*x^4 + 10*(118*B*b^3*c^2 + 3*A*b^2*c^3)*x^3 + 8*(170*B*b^4*c + 9
3*A*b^3*c^2)*x^2 + 48*(10*B*b^5 + 21*A*b^4*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x)
)/(b^3*x^6), -1/1920*(15*(10*B*b*c^4 - 3*A*c^5)*sqrt(-b)*x^6*arctan(sqrt(c
*x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) + (384*A*b^5 + 15*(10*B*b^2*c^3 - 3*A*b*
c^4)*x^4 + 10*(118*B*b^3*c^2 + 3*A*b^2*c^3)*x^3 + 8*(170*B*b^4*c + 93*A*b^
3*c^2)*x^2 + 48*(10*B*b^5 + 21*A*b^4*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^3
*x^6)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)*(c*x**2+b*x)**(5/2)/x**(17/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{17/2}}\,dx = \int \frac{\left(cx^2+bx\right)^{\frac{5}{2}}(Bx+A)}{x^{\frac{17}{2}}}\,dx$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(17/2),x, algorithm="maxima")
```

output
```
integrate((c*x^2 + b*x)^(5/2)*(B*x + A)/x^(17/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.88

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{17/2}}\,dx =$$

$$-\frac{1}{1920}c^5\left(\frac{15\,(10\,Bb-3\,Ac)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2c}+\frac{150\,(cx+b)^{\frac{9}{2}}Bb+580\,(cx+b)^{\frac{7}{2}}Bb^2-1280\,(cx+b)^{\frac{5}{2}}Bb^3}{}\right.$$

input
```
integrate((B*x+A)*(c*x^2+b*x)^(5/2)/x^(17/2),x, algorithm="giac")
```

output
```
-1/1920*c^5*(15*(10*B*b - 3*A*c)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*
b^2*c) + (150*(c*x + b)^(9/2)*B*b + 580*(c*x + b)^(7/2)*B*b^2 - 1280*(c*x
+ b)^(5/2)*B*b^3 + 700*(c*x + b)^(3/2)*B*b^4 - 150*sqrt(c*x + b)*B*b^5 - 4
5*(c*x + b)^(9/2)*A*c + 210*(c*x + b)^(7/2)*A*b*c + 384*(c*x + b)^(5/2)*A*
b^2*c - 210*(c*x + b)^(3/2)*A*b^3*c + 45*sqrt(c*x + b)*A*b^4*c)/(b^2*c^6*x
^5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{17/2}}\,dx = \int \frac{\left(c\,x^2+b\,x\right)^{5/2}\,(A+B\,x)}{x^{17/2}}\,dx$$

input
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(17/2),x)
```

output
```
int(((b*x + c*x^2)^(5/2)*(A + B*x))/x^(17/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.17

$$\int \frac{(A+Bx)\left(bx+cx^2\right)^{5/2}}{x^{17/2}}\,dx = \frac{-768\sqrt{cx+b}\,a\,b^5-2016\sqrt{cx+b}\,a\,b^4cx-1488\sqrt{cx+b}\,a\,b^3c^2x^2-60\sqrt{cx}$$

input
```
int((B*x+A)*(c*x^2+b*x)^(5/2)/x^(17/2),x)
```

output
```
( - 768*sqrt(b + c*x)*a*b**5 - 2016*sqrt(b + c*x)*a*b**4*c*x - 1488*sqrt(b
 + c*x)*a*b**3*c**2*x**2 - 60*sqrt(b + c*x)*a*b**2*c**3*x**3 + 90*sqrt(b +
 c*x)*a*b*c**4*x**4 - 960*sqrt(b + c*x)*b**6*x - 2720*sqrt(b + c*x)*b**5*c
*x**2 - 2360*sqrt(b + c*x)*b**4*c**2*x**3 - 300*sqrt(b + c*x)*b**3*c**3*x*
*4 + 45*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c**5*x**5 - 150*sqrt(b)*log
(sqrt(b + c*x) - sqrt(b))*b**2*c**4*x**5 - 45*sqrt(b)*log(sqrt(b + c*x) +
sqrt(b))*a*c**5*x**5 + 150*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*b**2*c**4*
x**5)/(3840*b**3*x**5)
```

3.204 $\int \frac{x^{7/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 167

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2b^3(bB-Ac)\sqrt{bx+cx^2}}{c^5\sqrt{x}} - \frac{2b^2(4bB-3Ac)\left(bx+cx^2\right)^{3/2}}{3c^5x^{3/2}} + \frac{6b(2bB-Ac)\left(bx+cx^2\right)^{5/2}}{5c^5x^{5/2}} - \frac{2(4bB-Ac)\left(bx+cx^2\right)^{7/2}}{7c^5x^{7/2}} + \frac{2B\left(bx+cx^2\right)^{9/2}}{9c^5x^{9/2}}$$

output
```
2*b^3*(-A*c+B*b)*(c*x^2+b*x)^(1/2)/c^5/x^(1/2)-2/3*b^2*(-3*A*c+4*B*b)*(c*x
^2+b*x)^(3/2)/c^5/x^(3/2)+6/5*b*(-A*c+2*B*b)*(c*x^2+b*x)^(5/2)/c^5/x^(5/2)
-2/7*(-A*c+4*B*b)*(c*x^2+b*x)^(7/2)/c^5/x^(7/2)+2/9*B*(c*x^2+b*x)^(9/2)/c^
5/x^(9/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.56

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2\sqrt{x(b+cx)}(128b^4B + 24b^2c^2x(3A+2Bx) - 16b^3c(9A+4Bx) + 5c^4x^3(9A+7Bx}{315c^5\sqrt{x}}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
(2*Sqrt[x*(b + c*x)]*(128*b^4*B + 24*b^2*c^2*x*(3*A + 2*B*x) - 16*b^3*c*(9
*A + 4*B*x) + 5*c^4*x^3*(9*A + 7*B*x) - 2*b*c^3*x^2*(27*A + 20*B*x)))/(315
*c^5*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1221, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{2Bx^{7/2}\sqrt{bx+cx^2}}{9c}-\frac{(8bB-9Ac)\int \frac{x^{7/2}}{\sqrt{cx^2+bx}}dx}{9c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2Bx^{7/2}\sqrt{bx+cx^2}}{9c}-\frac{(8bB-9Ac)\left(\frac{2x^{5/2}\sqrt{bx+cx^2}}{7c}-\frac{6b\int \frac{x^{5/2}}{\sqrt{cx^2+bx}}dx}{7c}\right)}{9c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2Bx^{7/2}\sqrt{bx+cx^2}}{9c}-\frac{(8bB-9Ac)\left(\frac{2x^{5/2}\sqrt{bx+cx^2}}{7c}-\frac{6b\left(\frac{2x^{3/2}\sqrt{bx+cx^2}}{5c}-\frac{4b\int \frac{x^{3/2}}{\sqrt{cx^2+bx}}dx}{5c}\right)}{7c}\right)}{9c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2Bx^{7/2}\sqrt{bx+cx^2}}{9c}-\frac{(8bB-9Ac)\left(\frac{2x^{5/2}\sqrt{bx+cx^2}}{7c}-\frac{6b\left(\frac{2x^{3/2}\sqrt{bx+cx^2}}{5c}-\frac{4b\left(\frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{2b\int\frac{\sqrt{x}}{\sqrt{cx^2+bx}}dx}{3c}\right)}{5c}\right)}{7c}\right)}{9c}$$

$\big\downarrow$ 1122

$$\frac{2Bx^{7/2}\sqrt{bx+cx^2}}{9c}-\frac{\left(\frac{2x^{5/2}\sqrt{bx+cx^2}}{7c}-\frac{6b\left(\frac{2x^{3/2}\sqrt{bx+cx^2}}{5c}-\frac{4b\left(\frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{4b\sqrt{bx+cx^2}}{3c^2\sqrt{x}}\right)}{5c}\right)}{7c}\right)(8bB-9Ac)}{9c}$$

input
```
Int[(x^(7/2)*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
(2*B*x^(7/2)*Sqrt[b*x + c*x^2])/(9*c) - ((8*b*B - 9*A*c)*((2*x^(5/2)*Sqrt[
b*x + c*x^2])/(7*c) - (6*b*((2*x^(3/2)*Sqrt[b*x + c*x^2])/(5*c) - (4*b*((-
4*b*Sqrt[b*x + c*x^2])/(3*c^2*Sqrt[x]) + (2*Sqrt[x]*Sqrt[b*x + c*x^2])/(3*
c)))/(5*c)))/(7*c)))/(9*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.60

method	result	size
default	$-\frac{2\sqrt{x(cx+b)}\,(-35B\,c^4x^4-45A\,c^4x^3+40B\,c^3x^3b+54Ab\,c^3x^2-48c^2x^2B\,b^2-72A\,b^2c^2x+64B\,b^3cx+144A\,b^3c-128B\,b^4)}{315\sqrt{x}\,c^5}$	100
risch	$-\frac{2(cx+b)\sqrt{x}\,(-35B\,c^4x^4-45A\,c^4x^3+40B\,c^3x^3b+54Ab\,c^3x^2-48c^2x^2B\,b^2-72A\,b^2c^2x+64B\,b^3cx+144A\,b^3c-128B\,b^4)}{315\sqrt{x(cx+b)}\,c^5}$	105
gosper	$-\frac{2(cx+b)(-35B\,c^4x^4-45A\,c^4x^3+40B\,c^3x^3b+54Ab\,c^3x^2-48c^2x^2B\,b^2-72A\,b^2c^2x+64B\,b^3cx+144A\,b^3c-128B\,b^4)\sqrt{x}}{315c^5\sqrt{c\,x^2+bx}}$	107
orering	$-\frac{2(cx+b)(-35B\,c^4x^4-45A\,c^4x^3+40B\,c^3x^3b+54Ab\,c^3x^2-48c^2x^2B\,b^2-72A\,b^2c^2x+64B\,b^3cx+144A\,b^3c-128B\,b^4)\sqrt{x}}{315c^5\sqrt{c\,x^2+bx}}$	107

input
```
int(x^(7/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315/x^(1/2)*(x*(c*x+b))^(1/2)*(-35*B*c^4*x^4-45*A*c^4*x^3+40*B*b*c^3*x^
3+54*A*b*c^3*x^2-48*B*b^2*c^2*x^2-72*A*b^2*c^2*x+64*B*b^3*c*x+144*A*b^3*c-
128*B*b^4)/c^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.62

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2\,(35\,Bc^4x^4 + 128\,Bb^4 - 144\,Ab^3c - 5\,(8\,Bbc^3 - 9\,Ac^4)x^3 + 6\,(8\,Bb^2c^2 - 9\,Abc^3)x^2}{315\,c^5\sqrt{x}}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*B*c^4*x^4 + 128*B*b^4 - 144*A*b^3*c - 5*(8*B*b*c^3 - 9*A*c^4)*x^
3 + 6*(8*B*b^2*c^2 - 9*A*b*c^3)*x^2 - 8*(8*B*b^3*c - 9*A*b^2*c^2)*x)*sqrt(
c*x^2 + b*x)/(c^5*sqrt(x))
```

Sympy [F]

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \int \frac{x^{\frac{7}{2}}(A+Bx)}{\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate(x**(7/2)*(B*x+A)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(x**(7/2)*(A + B*x)/sqrt(x*(b + c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.72

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2\,(5\,c^4x^4 - bc^3x^3 + 2\,b^2c^2x^2 - 8\,b^3cx - 16\,b^4)A}{35\,\sqrt{cx+b}c^4} + \frac{2\,(35\,c^5x^5 - 5\,bc^4x^4 + 8\,b^2c^3x^3 - 16\,b^3c^2x^2 + 64\,b^4cx + 128\,b^5)B}{315\,\sqrt{cx+b}c^5}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
2/35*(5*c^4*x^4 - b*c^3*x^3 + 2*b^2*c^2*x^2 - 8*b^3*c*x - 16*b^4)*A/(sqrt(
c*x + b)*c^4) + 2/315*(35*c^5*x^5 - 5*b*c^4*x^4 + 8*b^2*c^3*x^3 - 16*b^3*c
^2*x^2 + 64*b^4*c*x + 128*b^5)*B/(sqrt(c*x + b)*c^5)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.69

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2\,(Bb^4-Ab^3c)\sqrt{cx+b}}{c^5} + \frac{2\left(35\,(cx+b)^{\frac{9}{2}}B-180\,(cx+b)^{\frac{7}{2}}Bb+378\,(cx+b)^{\frac{5}{2}}Bb^2-420\,(cx+b)^{\frac{3}{2}}Bb^3+45\,(cx+b)^{\frac{7}{2}}Ac-189\,(cx\right.}{315\,c^5}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
2*(B*b^4 - A*b^3*c)*sqrt(c*x + b)/c^5 + 2/315*(35*(c*x + b)^(9/2)*B - 180*
(c*x + b)^(7/2)*B*b + 378*(c*x + b)^(5/2)*B*b^2 - 420*(c*x + b)^(3/2)*B*b^
3 + 45*(c*x + b)^(7/2)*A*c - 189*(c*x + b)^(5/2)*A*b*c + 315*(c*x + b)^(3/
2)*A*b^2*c)/c^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \int \frac{x^{7/2}\,(A+B\,x)}{\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((x^(7/2)*(A + B*x))/(b*x + c*x^2)^(1/2),x)
```

output
```
int((x^(7/2)*(A + B*x))/(b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.54

$$\int \frac{x^{7/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2\sqrt{cx+b}\,(35b\,c^4x^4+45a\,c^4x^3-40b^2c^3x^3-54ab\,c^3x^2+48b^3c^2x^2+72a\,b^2c^2x-64b}{315c^5}$$

input
```
int(x^(7/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x)
```

output
```
(2*sqrt(b + c*x)*( - 144*a*b**3*c + 72*a*b**2*c**2*x - 54*a*b*c**3*x**2 +
45*a*c**4*x**3 + 128*b**5 - 64*b**4*c*x + 48*b**3*c**2*x**2 - 40*b**2*c**3
*x**3 + 35*b*c**4*x**4))/(315*c**5)
```

3.205 $\int \frac{x^{5/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = -\frac{2b^2(bB-Ac)\sqrt{bx+cx^2}}{c^4\sqrt{x}} + \frac{2b(3bB-2Ac)\left(bx+cx^2\right)^{3/2}}{3c^4x^{3/2}} - \frac{2(3bB-Ac)\left(bx+cx^2\right)^{5/2}}{5c^4x^{5/2}} + \frac{2B\left(bx+cx^2\right)^{7/2}}{7c^4x^{7/2}}$$

output
```
-2*b^2*(-A*c+B*b)*(c*x^2+b*x)^(1/2)/c^4/x^(1/2)+2/3*b*(-2*A*c+3*B*b)*(c*x^
2+b*x)^(3/2)/c^4/x^(3/2)-2/5*(-A*c+3*B*b)*(c*x^2+b*x)^(5/2)/c^4/x^(5/2)+2/
7*B*(c*x^2+b*x)^(7/2)/c^4/x^(7/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2\sqrt{x(b+cx)}(-48b^3B+8b^2c(7A+3Bx)+3c^3x^2(7A+5Bx)-2bc^2x(14A+9Bx)}{105c^4\sqrt{x}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
(2*Sqrt[x*(b + c*x)]*(-48*b^3*B + 8*b^2*c*(7*A + 3*B*x) + 3*c^3*x^2*(7*A +
 5*B*x) - 2*b*c^2*x*(14*A + 9*B*x)))/(105*c^4*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1221, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{2Bx^{5/2}\sqrt{bx+cx^2}}{7c}-\frac{(6bB-7Ac)\int \frac{x^{5/2}}{\sqrt{cx^2+bx}}dx}{7c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2Bx^{5/2}\sqrt{bx+cx^2}}{7c}-\frac{(6bB-7Ac)\left(\frac{2x^{3/2}\sqrt{bx+cx^2}}{5c}-\frac{4b\int \frac{x^{3/2}}{\sqrt{cx^2+bx}}dx}{5c}\right)}{7c}$$

$$\Big\downarrow \text{1128}$$

$$\frac{2Bx^{5/2}\sqrt{bx+cx^2}}{7c}-\frac{(6bB-7Ac)\left(\frac{2x^{3/2}\sqrt{bx+cx^2}}{5c}-\frac{4b\left(\frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{2b\int \frac{\sqrt{x}}{\sqrt{cx^2+bx}}dx}{3c}\right)}{5c}\right)}{7c}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2Bx^{5/2}\sqrt{bx+cx^2}}{7c}-\frac{\left(\frac{2x^{3/2}\sqrt{bx+cx^2}}{5c}-\frac{4b\left(\frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{4b\sqrt{bx+cx^2}}{3c^2\sqrt{x}}\right)}{5c}\right)(6bB-7Ac)}{7c}$$

input
```
Int[(x^(5/2)*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
(2*B*x^(5/2)*Sqrt[b*x + c*x^2])/(7*c) - ((6*b*B - 7*A*c)*((2*x^(3/2)*Sqrt[
b*x + c*x^2])/(5*c) - (4*b*((-4*b*Sqrt[b*x + c*x^2])/(3*c^2*Sqrt[x]) + (2*
Sqrt[x]*Sqrt[b*x + c*x^2])/(3*c)))/(5*c)))/(7*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

method	result	size
default	$\frac{2\sqrt{x(cx+b)}\,(15B\,c^3x^3+21A\,c^3x^2-18Bb\,c^2x^2-28Ab\,c^2x+24B\,b^2cx+56A\,b^2c-48B\,b^3)}{105\sqrt{x}\,c^4}$	76
risch	$\frac{2(cx+b)\sqrt{x}\,(15B\,c^3x^3+21A\,c^3x^2-18Bb\,c^2x^2-28Ab\,c^2x+24B\,b^2cx+56A\,b^2c-48B\,b^3)}{105\sqrt{x(cx+b)}\,c^4}$	81
gosper	$\frac{2(cx+b)(15B\,c^3x^3+21A\,c^3x^2-18Bb\,c^2x^2-28Ab\,c^2x+24B\,b^2cx+56A\,b^2c-48B\,b^3)\sqrt{x}}{105c^4\sqrt{c\,x^2+bx}}$	83
orering	$\frac{2(cx+b)(15B\,c^3x^3+21A\,c^3x^2-18Bb\,c^2x^2-28Ab\,c^2x+24B\,b^2cx+56A\,b^2c-48B\,b^3)\sqrt{x}}{105c^4\sqrt{c\,x^2+bx}}$	83

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/105/x^(1/2)*(x*(c*x+b))^(1/2)*(15*B*c^3*x^3+21*A*c^3*x^2-18*B*b*c^2*x^2-
28*A*b*c^2*x+24*B*b^2*c*x+56*A*b^2*c-48*B*b^3)/c^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.61

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2\,(15\,Bc^3x^3-48\,Bb^3+56\,Ab^2c-3\,(6\,Bbc^2-7\,Ac^3)x^2+4\,(6\,Bb^2c-7\,Abc^2)x)\sqrt{cx}}{105\,c^4\sqrt{x}}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*B*c^3*x^3 - 48*B*b^3 + 56*A*b^2*c - 3*(6*B*b*c^2 - 7*A*c^3)*x^2
+ 4*(6*B*b^2*c - 7*A*b*c^2)*x)*sqrt(c*x^2 + b*x)/(c^4*sqrt(x))
```

Sympy [F]

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \int \frac{x^{\frac{5}{2}}(A+Bx)}{\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate(x**(5/2)*(B*x+A)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(x**(5/2)*(A + B*x)/sqrt(x*(b + c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2\,(3\,c^3x^3 - bc^2x^2 + 4\,b^2cx + 8\,b^3)A}{15\,\sqrt{cx+b}c^3} + \frac{2\,(5\,c^4x^4 - bc^3x^3 + 2\,b^2c^2x^2 - 8\,b^3cx - 16\,b^4)B}{35\,\sqrt{cx+b}c^4}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*c^3*x^3 - b*c^2*x^2 + 4*b^2*c*x + 8*b^3)*A/(sqrt(c*x + b)*c^3) + 2
/35*(5*c^4*x^4 - b*c^3*x^3 + 2*b^2*c^2*x^2 - 8*b^3*c*x - 16*b^4)*B/(sqrt(c
*x + b)*c^4)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.68

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = -\frac{2\,(Bb^3 - Ab^2c)\sqrt{cx+b}}{c^4} + \frac{2\left(15\,(cx+b)^{\frac{7}{2}}B - 63\,(cx+b)^{\frac{5}{2}}Bb + 105\,(cx+b)^{\frac{3}{2}}Bb^2 + 21\,(cx+b)^{\frac{5}{2}}Ac - 70\,(cx+b)^{\frac{3}{2}}Abc\right)}{105\,c^4}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
-2*(B*b^3 - A*b^2*c)*sqrt(c*x + b)/c^4 + 2/105*(15*(c*x + b)^(7/2)*B - 63*
(c*x + b)^(5/2)*B*b + 105*(c*x + b)^(3/2)*B*b^2 + 21*(c*x + b)^(5/2)*A*c -
 70*(c*x + b)^(3/2)*A*b*c)/c^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \int \frac{x^{5/2}\,(A+B\,x)}{\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((x^(5/2)*(A + B*x))/(b*x + c*x^2)^(1/2),x)
```

output
```
int((x^(5/2)*(A + B*x))/(b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.52

$$\int \frac{x^{5/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2\sqrt{cx+b}\,(15b\,c^3x^3+21a\,c^3x^2-18b^2c^2x^2-28ab\,c^2x+24b^3cx+56a\,b^2c-48b^4)}{105c^4}$$

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x)
```

output
```
(2*sqrt(b + c*x)*(56*a*b**2*c - 28*a*b*c**2*x + 21*a*c**3*x**2 - 48*b**4 +
 24*b**3*c*x - 18*b**2*c**2*x**2 + 15*b*c**3*x**3))/(105*c**4)
```

3.206 $\int \frac{x^{3/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 93

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2b(bB-Ac)\sqrt{bx+cx^2}}{c^3\sqrt{x}} - \frac{2(2bB-Ac)\left(bx+cx^2\right)^{3/2}}{3c^3x^{3/2}} + \frac{2B\left(bx+cx^2\right)^{5/2}}{5c^3x^{5/2}}$$

output
```
2*b*(-A*c+B*b)*(c*x^2+b*x)^(1/2)/c^3/x^(1/2)-2/3*(-A*c+2*B*b)*(c*x^2+b*x)^
(3/2)/c^3/x^(3/2)+2/5*B*(c*x^2+b*x)^(5/2)/c^3/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2\sqrt{x(b+cx)}\left(8b^2B-2bc(5A+2Bx)+c^2x(5A+3Bx)\right)}{15c^3\sqrt{x}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
(2*Sqrt[x*(b + c*x)]*(8*b^2*B - 2*b*c*(5*A + 2*B*x) + c^2*x*(5*A + 3*B*x))
)/(15*c^3*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1221, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \ 1221$$

$$\frac{2Bx^{3/2}\sqrt{bx+cx^2}}{5c}-\frac{(4bB-5Ac)\int \frac{x^{3/2}}{\sqrt{cx^2+bx}}dx}{5c}$$

$$\Big\downarrow \ 1128$$

$$\frac{2Bx^{3/2}\sqrt{bx+cx^2}}{5c}-\frac{(4bB-5Ac)\left(\frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{2b\int \frac{\sqrt{x}}{\sqrt{cx^2+bx}}dx}{3c}\right)}{5c}$$

$$\Big\downarrow \ 1122$$

$$\frac{2Bx^{3/2}\sqrt{bx+cx^2}}{5c}-\frac{\left(\frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{4b\sqrt{bx+cx^2}}{3c^2\sqrt{x}}\right)(4bB-5Ac)}{5c}$$

input
```
Int[(x^(3/2)*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
(2*B*x^(3/2)*Sqrt[b*x + c*x^2])/(5*c) - ((4*b*B - 5*A*c)*((-4*b*Sqrt[b*x +
 c*x^2])/(3*c^2*Sqrt[x]) + (2*Sqrt[x]*Sqrt[b*x + c*x^2])/(3*c)))/(5*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.56

method	result	size
default	$-\frac{2 \sqrt{x \left(c x +b \right)}\, \left(-3 B \,c^{2} x^{2}-5 A \,c^{2} x +4 B b c x +10 A b c -8 B \,b^{2}\right)}{15 \sqrt{x}\, c^{3}}$	52
risch	$-\frac{2 \left(c x +b \right) \sqrt{x}\, \left(-3 B \,c^{2} x^{2}-5 A \,c^{2} x +4 B b c x +10 A b c -8 B \,b^{2}\right)}{15 \sqrt{x \left(c x +b \right)}\, c^{3}}$	57
gosper	$-\frac{2 \left(c x +b \right) \left(-3 B \,c^{2} x^{2}-5 A \,c^{2} x +4 B b c x +10 A b c -8 B \,b^{2}\right) \sqrt{x}}{15 c^{3} \sqrt{c \,x^{2}+b x}}$	59
orering	$-\frac{2 \left(c x +b \right) \left(-3 B \,c^{2} x^{2}-5 A \,c^{2} x +4 B b c x +10 A b c -8 B \,b^{2}\right) \sqrt{x}}{15 c^{3} \sqrt{c \,x^{2}+b x}}$	59

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15/x^(1/2)*(x*(c*x+b))^(1/2)*(-3*B*c^2*x^2-5*A*c^2*x+4*B*b*c*x+10*A*b*c
-8*B*b^2)/c^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.59

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2\,(3\,Bc^2x^2+8\,Bb^2-10\,Abc-(4\,Bbc-5\,Ac^2)x)\sqrt{cx^2+bx}}{15\,c^3\sqrt{x}}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*c^2*x^2 + 8*B*b^2 - 10*A*b*c - (4*B*b*c - 5*A*c^2)*x)*sqrt(c*x^2
 + b*x)/(c^3*sqrt(x))
```

Sympy [F]

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \int \frac{x^{\frac{3}{2}}(A+Bx)}{\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate(x**(3/2)*(B*x+A)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(x**(3/2)*(A + B*x)/sqrt(x*(b + c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.81

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2\,(c^2x^2-bcx-2\,b^2)A}{3\,\sqrt{cx+b}c^2} + \frac{2\,(3\,c^3x^3-bc^2x^2+4\,b^2cx+8\,b^3)B}{15\,\sqrt{cx+b}c^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(c^2*x^2 - b*c*x - 2*b^2)*A/(sqrt(c*x + b)*c^2) + 2/15*(3*c^3*x^3 - b*
c^2*x^2 + 4*b^2*c*x + 8*b^3)*B/(sqrt(c*x + b)*c^3)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2\,(Bb^2-Abc)\sqrt{cx+b}}{c^3} + \frac{2\left(3\,(cx+b)^{\frac{5}{2}}B-10\,(cx+b)^{\frac{3}{2}}Bb+5\,(cx+b)^{\frac{3}{2}}Ac\right)}{15\,c^3}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
2*(B*b^2 - A*b*c)*sqrt(c*x + b)/c^3 + 2/15*(3*(c*x + b)^(5/2)*B - 10*(c*x
+ b)^(3/2)*B*b + 5*(c*x + b)^(3/2)*A*c)/c^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \int \frac{x^{3/2}\,(A+B\,x)}{\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((x^(3/2)*(A + B*x))/(b*x + c*x^2)^(1/2),x)
```

output
```
int((x^(3/2)*(A + B*x))/(b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.48

$$\int \frac{x^{3/2}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2\sqrt{cx+b}\,(3b\,c^2x^2+5a\,c^2x-4b^2cx-10abc+8b^3)}{15c^3}$$

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x)
```

output
```
(2*sqrt(b + c*x)*( - 10*a*b*c + 5*a*c**2*x + 8*b**3 - 4*b**2*c*x + 3*b*c**
2*x**2))/(15*c**3)
```

3.207 $\int \frac{\sqrt{x}(A+Bx)}{\sqrt{bx+cx^2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{bx+cx^2}} \, dx = -\frac{2(bB-Ac)\sqrt{bx+cx^2}}{c^2\sqrt{x}}+\frac{2B(bx+cx^2)^{3/2}}{3c^2x^{3/2}}$$

output
```
-2*(-A*c+B*b)*(c*x^2+b*x)^(1/2)/c^2/x^(1/2)+2/3*B*(c*x^2+b*x)^(3/2)/c^2/x^
(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{bx+cx^2}} \, dx = \frac{2\sqrt{x(b+cx)}(-2bB+3Ac+Bcx)}{3c^2\sqrt{x}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
(2*Sqrt[x*(b + c*x)]*(-2*b*B + 3*A*c + B*c*x))/(3*c^2*Sqrt[x])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.05, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1221, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{1221}$$

$$\frac{2B\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{(2bB-3Ac)\int \frac{\sqrt{x}}{\sqrt{cx^2+bx}}dx}{3c}$$

$$\Big\downarrow \text{1122}$$

$$\frac{2B\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{2\sqrt{bx+cx^2}(2bB-3Ac)}{3c^2\sqrt{x}}$$

input
```
Int[(Sqrt[x]*(A + B*x))/Sqrt[b*x + c*x^2],x]
```

output
```
(-2*(2*b*B - 3*A*c)*Sqrt[b*x + c*x^2])/(3*c^2*Sqrt[x]) + (2*B*Sqrt[x]*Sqrt
[b*x + c*x^2])/(3*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.53

method	result	size
default	$\frac{2\sqrt{x(cx+b)}\,(Bcx+3Ac-2Bb)}{3\sqrt{x}\,c^2}$	31
risch	$\frac{2(cx+b)\sqrt{x}\,(Bcx+3Ac-2Bb)}{3\sqrt{x(cx+b)}\,c^2}$	36
gosper	$\frac{2(cx+b)(Bcx+3Ac-2Bb)\sqrt{x}}{3c^2\sqrt{c\,x^2+bx}}$	38
orering	$\frac{2(cx+b)(Bcx+3Ac-2Bb)\sqrt{x}}{3c^2\sqrt{c\,x^2+bx}}$	38

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/3/x^(1/2)*(x*(c*x+b))^(1/2)*(B*c*x+3*A*c-2*B*b)/c^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{bx+cx^2}}\,dx = \frac{2\,(Bcx-2\,Bb+3\,Ac)\sqrt{cx^2+bx}}{3\,c^2\sqrt{x}}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
2/3*(B*c*x - 2*B*b + 3*A*c)*sqrt(c*x^2 + b*x)/(c^2*sqrt(x))
```

Sympy [F]

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{bx+cx^2}}\,dx=\int \frac{\sqrt{x}(A+Bx)}{\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate(x**(1/2)*(B*x+A)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral(sqrt(x)*(A + B*x)/sqrt(x*(b + c*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{bx+cx^2}}\,dx=\frac{2\sqrt{cx+b}A}{c}+\frac{2\,(c^2x^2-bcx-2\,b^2)B}{3\sqrt{cx+b}c^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(c*x + b)*A/c + 2/3*(c^2*x^2 - b*c*x - 2*b^2)*B/(sqrt(c*x + b)*c^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.59

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{bx+cx^2}}\,dx=\frac{2\,(cx+b)^{\frac{3}{2}}B}{3\,c^2}-\frac{2\,(Bb-Ac)\sqrt{cx+b}}{c^2}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
2/3*(c*x + b)^(3/2)*B/c^2 - 2*(B*b - A*c)*sqrt(c*x + b)/c^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{bx+cx^2}}\,dx=\int \frac{\sqrt{x}\,(A+B\,x)}{\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((x^(1/2)*(A + B*x))/(b*x + c*x^2)^(1/2),x)
```

output
```
int((x^(1/2)*(A + B*x))/(b*x + c*x^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.43

$$\int \frac{\sqrt{x}(A+Bx)}{\sqrt{bx+cx^2}}\,dx=\frac{2\sqrt{cx+b}\,(bcx+3ac-2b^2)}{3c^2}$$

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x)^(1/2),x)
```

output
```
(2*sqrt(b + c*x)*(3*a*c - 2*b**2 + b*c*x))/(3*c**2)
```

3.208 $\int \frac{A+Bx}{\sqrt{x}\sqrt{bx+cx^2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{bx+cx^2}} dx = \frac{2B\sqrt{bx+cx^2}}{c\sqrt{x}} - \frac{2A\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}$$

output
```
2*B*(c*x^2+b*x)^(1/2)/c/x^(1/2)-2*A*arctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1
/2))/b^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.17

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{bx+cx^2}} dx = \frac{2\sqrt{x}\left(\sqrt{b}B(b+cx) - Ac\sqrt{b+cx}\text{arctanh}\left(\frac{\sqrt{b+cx}}{\sqrt{b}}\right)\right)}{\sqrt{b}c\sqrt{x(b+cx)}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*Sqrt[b*x + c*x^2]),x]
```

output
```
(2*Sqrt[x]*(Sqrt[b]*B*(b + c*x) - A*c*Sqrt[b + c*x]*ArcTanh[Sqrt[b + c*x]/
Sqrt[b]]))/(Sqrt[b]*c*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1221, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{bx+cx^2}}\,dx$$

$$\Big\downarrow \text{ 1221}$$

$$A\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx+\frac{2B\sqrt{bx+cx^2}}{c\sqrt{x}}$$

$$\Big\downarrow \text{ 1136}$$

$$2A\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}+\frac{2B\sqrt{bx+cx^2}}{c\sqrt{x}}$$

$$\Big\downarrow \text{ 220}$$

$$\frac{2B\sqrt{bx+cx^2}}{c\sqrt{x}}-\frac{2A\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{\sqrt{b}}$$

input
```
Int[(A + B*x)/(Sqrt[x]*Sqrt[b*x + c*x^2]),x]
```

output
```
(2*B*Sqrt[b*x + c*x^2])/(c*Sqrt[x]) - (2*A*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt
[b]*Sqrt[x])])/Sqrt[b]
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{2\sqrt{x(cx+b)}\left(Ac\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)-B\sqrt{cx+b}\,\sqrt{b}\right)}{\sqrt{x}\,\sqrt{cx+b}\,c\sqrt{b}}$	57

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/x^(1/2)*(x*(c*x+b))^(1/2)*(A*c*arctanh((c*x+b)^(1/2)/b^(1/2))-B*(c*x+b)
^(1/2)*b^(1/2))/(c*x+b)^(1/2)/c/b^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.31

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{bx+cx^2}}\,dx$$
$$=\left[\frac{A\sqrt{b}cx\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,\sqrt{cx^2+bx}Bb\sqrt{x}}{bcx},\ \frac{2\left(A\sqrt{-b}cx\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-b}}{b\sqrt{x}}\right)+\sqrt{cx^2\right.}{bcx}\right.$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[(A*sqrt(b)*c*x*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b)*sqrt(x))
/x^2) + 2*sqrt(c*x^2 + b*x)*B*b*sqrt(x))/(b*c*x), 2*(A*sqrt(-b)*c*x*arctan
(sqrt(c*x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) + sqrt(c*x^2 + b*x)*B*b*sqrt(x))/
(b*c*x)]
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{bx+cx^2}}\,dx=\int \frac{A+Bx}{\sqrt{x}\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate((B*x+A)/x**(1/2)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral((A + B*x)/(sqrt(x)*sqrt(x*(b + c*x))), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{bx+cx^2}}\,dx=\int \frac{Bx+A}{\sqrt{cx^2+bx}\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(c*x^2 + b*x)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{bx+cx^2}}\,dx = \frac{2\,A\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}} + \frac{2\,\sqrt{cx+b}B}{c}$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
2*A*arctan(sqrt(c*x + b)/sqrt(-b))/sqrt(-b) + 2*sqrt(c*x + b)*B/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{bx+cx^2}}\,dx = \int \frac{A+B\,x}{\sqrt{x}\,\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((A + B*x)/(x^(1/2)*(b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^(1/2)*(b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{\sqrt{x}\sqrt{bx+cx^2}}\,dx = \frac{2\sqrt{cx+b}\,b^2+\sqrt{b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)ac-\sqrt{b}\log\left(\sqrt{cx+b}+\sqrt{b}\right)ac}{bc}$$

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x)^(1/2),x)
```

output
```
(2*sqrt(b + c*x)*b**2 + sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c - sqrt(b)
*log(sqrt(b + c*x) + sqrt(b))*a*c)/(b*c)
```

3.209 $\int \frac{A+Bx}{x^{3/2}\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \frac{A+Bx}{x^{3/2}\sqrt{bx+cx^2}}\,dx=-\frac{A\sqrt{bx+cx^2}}{bx^{3/2}}-\frac{(2bB-Ac)\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}$$

output
```
-A*(c*x^2+b*x)^(1/2)/b/x^(3/2)-(-A*c+2*B*b)*arctanh((c*x^2+b*x)^(1/2)/b^(1
/2)/x^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx}{x^{3/2}\sqrt{bx+cx^2}}\,dx=\frac{-A\sqrt{b}(b+cx)-(2bB-Ac)x\sqrt{b+cx}\text{arctanh}\left(\frac{\sqrt{b+cx}}{\sqrt{b}}\right)}{b^{3/2}\sqrt{x}\sqrt{x(b+cx)}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*Sqrt[b*x + c*x^2]),x]
```

output
```
(-(A*Sqrt[b]*(b + c*x)) - (2*b*B - A*c)*x*Sqrt[b + c*x]*ArcTanh[Sqrt[b + c
*x]/Sqrt[b]])/(b^(3/2)*Sqrt[x]*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1220, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}\sqrt{bx+cx^2}}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(2bB-Ac)\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{2b}-\frac{A\sqrt{bx+cx^2}}{bx^{3/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(2bB-Ac)\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}-\frac{A\sqrt{bx+cx^2}}{bx^{3/2}}$$

$$\Big\downarrow \text{220}$$

$$-\frac{(2bB-Ac)\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}-\frac{A\sqrt{bx+cx^2}}{bx^{3/2}}$$

input
```
Int[(A + B*x)/(x^(3/2)*Sqrt[b*x + c*x^2]),x]
```

output
```
-((A*Sqrt[b*x + c*x^2])/(b*x^(3/2))) - ((2*b*B - A*c)*ArcTanh[Sqrt[b*x + c
*x^2]/(Sqrt[b]*Sqrt[x])])/b^(3/2)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

method	result	size
risch	$-\frac{A(cx+b)}{b\sqrt{x}\,\sqrt{x(cx+b)}}+\frac{(Ac-2Bb)\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)\sqrt{cx+b}\,\sqrt{x}}{b^{\frac{3}{2}}\sqrt{x(cx+b)}}$	68
default	$\frac{\sqrt{x(cx+b)}\,\left(A\ \mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)cx-2B\ \mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)bx-A\sqrt{cx+b}\,\sqrt{b}\right)}{b^{\frac{3}{2}}x^{\frac{3}{2}}\sqrt{cx+b}}$	71

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-A/b*(c*x+b)/x^(1/2)/(x*(c*x+b))^(1/2)+(A*c-2*B*b)/b^(3/2)*arctanh((c*x+b)
^(1/2)/b^(1/2))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.23

$$\int \frac{A+Bx}{x^{3/2}\sqrt{bx+cx^2}}\,dx = \left[-\frac{(2\,Bb-Ac)\sqrt{b}x^2\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,\sqrt{cx^2+bx}Ab\sqrt{x}}{2\,b^2x^2}, \frac{(2\,Bb-}{}\right.$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((2*B*b - A*c)*sqrt(b)*x^2*log(-(c*x^2 + 2*b*x + 2*sqrt(c*x^2 + b*x)
*sqrt(b)*sqrt(x))/x^2) + 2*sqrt(c*x^2 + b*x)*A*b*sqrt(x))/(b^2*x^2), ((2*B
*b - A*c)*sqrt(-b)*x^2*arctan(sqrt(c*x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) - sq
rt(c*x^2 + b*x)*A*b*sqrt(x))/(b^2*x^2)]
```

Sympy [F]

$$\int \frac{A+Bx}{x^{3/2}\sqrt{bx+cx^2}}\,dx = \int \frac{A+Bx}{x^{\frac{3}{2}}\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate((B*x+A)/x**(3/2)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**(3/2)*sqrt(x*(b + c*x))), x)
```

Maxima [F]

$$\int \frac{A+Bx}{x^{3/2}\sqrt{bx+cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{cx^2+bx}x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(c*x^2 + b*x)*x^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.86

$$\int \frac{A+Bx}{x^{3/2}\sqrt{bx+cx^2}}\,dx = c\left(\frac{(2\,Bb-Ac)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}bc}-\frac{\sqrt{cx+b}A}{bcx}\right)$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
c*((2*B*b - A*c)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b*c) - sqrt(c*x
+ b)*A/(b*c*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{3/2}\sqrt{bx+cx^2}}\,dx = \int \frac{A+B\,x}{x^{3/2}\,\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((A + B*x)/(x^(3/2)*(b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^(3/2)*(b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.39

$$\int \frac{A+Bx}{x^{3/2}\sqrt{bx+cx^2}}\,dx = \frac{-2\sqrt{cx+b}\,ab-\sqrt{b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)acx+2\sqrt{b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)b^2x+\sqrt{b}}{2b^2x}$$

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x)^(1/2),x)
```

output
```
( - 2*sqrt(b + c*x)*a*b - sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c*x + 2*s
qrt(b)*log(sqrt(b + c*x) - sqrt(b))*b**2*x + sqrt(b)*log(sqrt(b + c*x) + s
qrt(b))*a*c*x - 2*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*b**2*x)/(2*b**2*x)
```

3.210 $\int \frac{A+Bx}{x^{5/2}\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \frac{A+Bx}{x^{5/2}\sqrt{bx+cx^2}}\,dx = -\frac{A\sqrt{bx+cx^2}}{2bx^{5/2}} - \frac{(4bB-3Ac)\sqrt{bx+cx^2}}{4b^2x^{3/2}} + \frac{c(4bB-3Ac)\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4b^{5/2}}$$

output
```
-1/2*A*(c*x^2+b*x)^(1/2)/b/x^(5/2)-1/4*(-3*A*c+4*B*b)*(c*x^2+b*x)^(1/2)/b^
2/x^(3/2)+1/4*c*(-3*A*c+4*B*b)*arctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1/2))/
b^(5/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.88

$$\int \frac{A+Bx}{x^{5/2}\sqrt{bx+cx^2}}\,dx = \frac{-\sqrt{b}(b+cx)(2Ab+4bBx-3Acx)+c(4bB-3Ac)x^2\sqrt{b+cx}\,\text{arctanh}\left(\frac{\sqrt{b+cx}}{\sqrt{b}}\right)}{4b^{5/2}x^{3/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*Sqrt[b*x + c*x^2]),x]
```

output
```
(-(Sqrt[b]*(b + c*x)*(2*A*b + 4*b*B*x - 3*A*c*x)) + c*(4*b*B - 3*A*c)*x^2*
Sqrt[b + c*x]*ArcTanh[Sqrt[b + c*x]/Sqrt[b]])/(4*b^(5/2)*x^(3/2)*Sqrt[x*(b
 + c*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1220, 1135, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(4bB-3Ac)\int \frac{1}{x^{3/2}\sqrt{cx^2+bx}}dx}{4b}-\frac{A\sqrt{bx+cx^2}}{2bx^{5/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{(4bB-3Ac)\left(-\frac{c\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{2b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)}{4b}-\frac{A\sqrt{bx+cx^2}}{2bx^{5/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(4bB-3Ac)\left(-\frac{c\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)}{4b}-\frac{A\sqrt{bx+cx^2}}{2bx^{5/2}}$$

$$\Big\downarrow \text{220}$$

$$\frac{(4bB-3Ac)\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)}{4b}-\frac{A\sqrt{bx+cx^2}}{2bx^{5/2}}$$

input
```
Int[(A + B*x)/(x^(5/2)*Sqrt[b*x + c*x^2]),x]
```

output
```
-1/2*(A*Sqrt[b*x + c*x^2])/(b*x^(5/2)) + ((4*b*B - 3*A*c)*(-(Sqrt[b*x + c*
x^2]/(b*x^(3/2))) + (c*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/b^(3/
2)))/(4*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.81

method	result	size
risch	$-\frac{(cx+b)(-3Acx+4Bbx+2Ab)}{4b^{2}x^{\frac{3}{2}}\sqrt{x(cx+b)}}-\frac{c(3Ac-4Bb)\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)\sqrt{cx+b}\,\sqrt{x}}{4b^{\frac{5}{2}}\sqrt{x(cx+b)}}$	85
default	$-\frac{\sqrt{x(cx+b)}\left(3A\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^{2}x^{2}-4B\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)bc\,x^{2}-3Acx\sqrt{cx+b}\,\sqrt{b}+4B\,b^{\frac{3}{2}}x\sqrt{cx+b}+2A\,b^{\frac{3}{2}}\sqrt{cx+b}\right)}{4b^{\frac{5}{2}}x^{\frac{5}{2}}\sqrt{cx+b}}$	109

input
```
int((B*x+A)/x^(5/2)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x+b)*(-3*A*c*x+4*B*b*x+2*A*b)/b^2/x^(3/2)/(x*(c*x+b))^(1/2)-1/4*c*
(3*A*c-4*B*b)/b^(5/2)*arctanh((c*x+b)^(1/2)/b^(1/2))*(c*x+b)^(1/2)*x^(1/2)
/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.82

$$\int \frac{A+Bx}{x^{5/2}\sqrt{bx+cx^2}}\,dx = \left[-\frac{(4\,Bbc-3\,Ac^2)\sqrt{b}x^3\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,(2\,Ab^2+(4\,Bb^2-3\,Abc}{8\,b^3x^3}\right.$$
$$\left.-\frac{(4\,Bbc-3\,Ac^2)\sqrt{-b}x^3\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-b}}{b\sqrt{x}}\right)+(2\,Ab^2+(4\,Bb^2-3\,Abc)x)\sqrt{cx^2+bx}\sqrt{x}}{4\,b^3x^3}\right]$$

input
```
integrate((B*x+A)/x^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((4*B*b*c - 3*A*c^2)*sqrt(b)*x^3*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2
+ b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(2*A*b^2 + (4*B*b^2 - 3*A*b*c)*x)*sqrt(c*
x^2 + b*x)*sqrt(x))/(b^3*x^3), -1/4*((4*B*b*c - 3*A*c^2)*sqrt(-b)*x^3*arct
an(sqrt(c*x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) + (2*A*b^2 + (4*B*b^2 - 3*A*b*c
)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^3*x^3)]
```

Sympy [F]

$$\int \frac{A+Bx}{x^{5/2}\sqrt{bx+cx^2}}\,dx = \int \frac{A+Bx}{x^{\frac{5}{2}}\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate((B*x+A)/x**(5/2)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**(5/2)*sqrt(x*(b + c*x))), x)
```

Maxima [F]

$$\int \frac{A+Bx}{x^{5/2}\sqrt{bx+cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{cx^2+bx}\,x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/x^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(c*x^2 + b*x)*x^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.06

$$\int \frac{A+Bx}{x^{5/2}\sqrt{bx+cx^2}}\,dx = -\frac{\frac{(4\,Bbc^2-3\,Ac^3)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{4\,(cx+b)^{\frac{3}{2}}\,Bbc^2-4\,\sqrt{cx+b}Bb^2c^2-3\,(cx+b)^{\frac{3}{2}}\,Ac^3+5\,\sqrt{cx+b}Abc^3}{b^2c^2x^2}}{4\,c}$$

input
```
integrate((B*x+A)/x^(5/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
-1/4*((4*B*b*c^2 - 3*A*c^3)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^2)
+ (4*(c*x + b)^(3/2)*B*b*c^2 - 4*sqrt(c*x + b)*B*b^2*c^2 - 3*(c*x + b)^(3/
2)*A*c^3 + 5*sqrt(c*x + b)*A*b*c^3)/(b^2*c^2*x^2))/c
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{5/2}\sqrt{bx+cx^2}}\,dx = \int \frac{A+B\,x}{x^{5/2}\,\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((A + B*x)/(x^(5/2)*(b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^(5/2)*(b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.27

$$\int \frac{A+Bx}{x^{5/2}\sqrt{bx+cx^2}}\,dx = \frac{-4\sqrt{cx+b}\,a\,b^2+6\sqrt{cx+b}\,abcx-8\sqrt{cx+b}\,b^3x+3\sqrt{b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)a\,c^2x^2 \cdots}{\cdots}$$

input
```
int((B*x+A)/x^(5/2)/(c*x^2+b*x)^(1/2),x)
```

output
```
( - 4*sqrt(b + c*x)*a*b**2 + 6*sqrt(b + c*x)*a*b*c*x - 8*sqrt(b + c*x)*b**
3*x + 3*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c**2*x**2 - 4*sqrt(b)*log(s
qrt(b + c*x) - sqrt(b))*b**2*c*x**2 - 3*sqrt(b)*log(sqrt(b + c*x) + sqrt(b
))*a*c**2*x**2 + 4*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*b**2*c*x**2)/(8*b*
*3*x**2)
```

3.211 $\int \frac{A+Bx}{x^{7/2}\sqrt{bx+cx^2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \frac{A+Bx}{x^{7/2}\sqrt{bx+cx^2}}\,dx = -\frac{A\sqrt{bx+cx^2}}{3bx^{7/2}} - \frac{(6bB-5Ac)\sqrt{bx+cx^2}}{12b^2x^{5/2}} + \frac{c(6bB-5Ac)\sqrt{bx+cx^2}}{8b^3x^{3/2}} - \frac{c^2(6bB-5Ac)\operatorname{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{7/2}}$$

output
```
-1/3*A*(c*x^2+b*x)^(1/2)/b/x^(7/2)-1/12*(-5*A*c+6*B*b)*(c*x^2+b*x)^(1/2)/b
^2/x^(5/2)+1/8*c*(-5*A*c+6*B*b)*(c*x^2+b*x)^(1/2)/b^3/x^(3/2)-1/8*c^2*(-5*
A*c+6*B*b)*arctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.81

$$\int \frac{A+Bx}{x^{7/2}\sqrt{bx+cx^2}}\,dx = \frac{-\sqrt{b}(b+cx)\left(6bBx(2b-3cx)+A(8b^2-10bcx+15c^2x^2)\right)+3c^2(-6bB+5Ac)x^3}{24b^{7/2}x^{5/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[(A + B*x)/(x^(7/2)*Sqrt[b*x + c*x^2]),x]
```

output
```
(-(Sqrt[b]*(b + c*x)*(6*b*B*x*(2*b - 3*c*x) + A*(8*b^2 - 10*b*c*x + 15*c^2
*x^2))) + 3*c^2*(-6*b*B + 5*A*c)*x^3*Sqrt[b + c*x]*ArcTanh[Sqrt[b + c*x]/S
qrt[b]])/(24*b^(7/2)*x^(5/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1220, 1135, 1135, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{7/2}\sqrt{bx+cx^2}}dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(6bB-5Ac)\int \frac{1}{x^{5/2}\sqrt{cx^2+bx}}dx}{6b}-\frac{A\sqrt{bx+cx^2}}{3bx^{7/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{(6bB-5Ac)\left(-\frac{3c\int \frac{1}{x^{3/2}\sqrt{cx^2+bx}}dx}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)}{6b}-\frac{A\sqrt{bx+cx^2}}{3bx^{7/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{(6bB-5Ac)\left(-\frac{3c\left(-\frac{c\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{2b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)}{6b}-\frac{A\sqrt{bx+cx^2}}{3bx^{7/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(6bB-5Ac)\left(-\frac{3c\left(-\frac{c\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)}{6b}-\frac{A\sqrt{bx+cx^2}}{3bx^{7/2}}$$

$$\Big\downarrow \; 220$$

$$\frac{(6bB-5Ac)\left(-\frac{3c\left(\frac{c\,\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}-\frac{\sqrt{bx+cx^2}}{bx^{3/2}}\right)}{4b}-\frac{\sqrt{bx+cx^2}}{2bx^{5/2}}\right)}{6b}-\frac{A\sqrt{bx+cx^2}}{3bx^{7/2}}$$

input
```
Int[(A + B*x)/(x^(7/2)*Sqrt[b*x + c*x^2]),x]
```

output
```
-1/3*(A*Sqrt[b*x + c*x^2])/(b*x^(7/2)) + ((6*b*B - 5*A*c)*(-1/2*Sqrt[b*x +
 c*x^2]/(b*x^(5/2)) - (3*c*(-(Sqrt[b*x + c*x^2]/(b*x^(3/2))) + (c*ArcTanh[
Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/b^(3/2)))/(4*b)))/(6*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{(cx+b)(15A\,c^2x^2-18x^2Bbc-10Abcx+12xB\,b^2+8b^2A)}{24b^3x^{\frac{5}{2}}\sqrt{x(cx+b)}}+\frac{c^2(5Ac-6Bb)\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)\sqrt{cx+b}\,\sqrt{x}}{8b^{\frac{7}{2}}\sqrt{x(cx+b)}}$
default	$\frac{\sqrt{x(cx+b)}\left(15A\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^3x^3-18B\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)b\,c^2x^3-15A\,c^2x^2\sqrt{cx+b}\,\sqrt{b}+18B\,b^{\frac{3}{2}}c\,x^2\sqrt{cx+b}+10A\,b^{\frac{3}{2}}cx\sqrt{cx+b}\right.}{24b^{\frac{7}{2}}x^{\frac{7}{2}}\sqrt{cx+b}}$

input
```
int((B*x+A)/x^(7/2)/(c*x^2+b*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x+b)*(15*A*c^2*x^2-18*B*b*c*x^2-10*A*b*c*x+12*B*b^2*x+8*A*b^2)/b^
3/x^(5/2)/(x*(c*x+b))^(1/2)+1/8*c^2*(5*A*c-6*B*b)/b^(7/2)*arctanh((c*x+b)^
(1/2)/b^(1/2))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.72

$$\int \frac{A+Bx}{x^{7/2}\sqrt{bx+cx^2}}\,dx=\left[-\frac{3\,(6\,Bbc^2-5\,Ac^3)\sqrt{b}x^4\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\,(8\,Ab^3-3\,(6\,Bb^2c-}{48\,b^4x^4}\right.$$

input
```
integrate((B*x+A)/x^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(6*B*b*c^2 - 5*A*c^3)*sqrt(b)*x^4*log(-(c*x^2 + 2*b*x + 2*sqrt(c
*x^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(8*A*b^3 - 3*(6*B*b^2*c - 5*A*b*c^2)
*x^2 + 2*(6*B*b^3 - 5*A*b^2*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^4*x^4), 1/
24*(3*(6*B*b*c^2 - 5*A*c^3)*sqrt(-b)*x^4*arctan(sqrt(c*x^2 + b*x)*sqrt(-b)
/(b*sqrt(x))) - (8*A*b^3 - 3*(6*B*b^2*c - 5*A*b*c^2)*x^2 + 2*(6*B*b^3 - 5*
A*b^2*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^4*x^4)]
```

Sympy [F]

$$\int \frac{A+Bx}{x^{7/2}\sqrt{bx+cx^2}}\,dx = \int \frac{A+Bx}{x^{\frac{7}{2}}\sqrt{x\,(b+cx)}}\,dx$$

input
```
integrate((B*x+A)/x**(7/2)/(c*x**2+b*x)**(1/2),x)
```

output
```
Integral((A + B*x)/(x**(7/2)*sqrt(x*(b + c*x))), x)
```

Maxima [F]

$$\int \frac{A+Bx}{x^{7/2}\sqrt{bx+cx^2}}\,dx = \int \frac{Bx+A}{\sqrt{cx^2+bx}\,x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)/x^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/(sqrt(c*x^2 + b*x)*x^(7/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.89

$$\int \frac{A+Bx}{x^{7/2}\sqrt{bx+cx^2}}\,dx = \frac{1}{24}\,c^3\left(\frac{3\,(6\,Bb-5\,Ac)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3c}+\frac{18\,(cx+b)^{\frac{5}{2}}Bb-48\,(cx+b)^{\frac{3}{2}}Bb^2+30\,\sqrt{}\right.$$

input
```
integrate((B*x+A)/x^(7/2)/(c*x^2+b*x)^(1/2),x, algorithm="giac")
```

output
```
1/24*c^3*(3*(6*B*b - 5*A*c)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^3*c
) + (18*(c*x + b)^(5/2)*B*b - 48*(c*x + b)^(3/2)*B*b^2 + 30*sqrt(c*x + b)*
B*b^3 - 15*(c*x + b)^(5/2)*A*c + 40*(c*x + b)^(3/2)*A*b*c - 33*sqrt(c*x +
b)*A*b^2*c)/(b^3*c^4*x^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{7/2}\sqrt{bx+cx^2}}\,dx = \int \frac{A+B\,x}{x^{7/2}\,\sqrt{c\,x^2+b\,x}}\,dx$$

input
```
int((A + B*x)/(x^(7/2)*(b*x + c*x^2)^(1/2)),x)
```

output
```
int((A + B*x)/(x^(7/2)*(b*x + c*x^2)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.20

$$\int \frac{A+Bx}{x^{7/2}\sqrt{bx+cx^2}}\,dx = \frac{-16\sqrt{cx+b}\,a\,b^3+20\sqrt{cx+b}\,a\,b^2cx-30\sqrt{cx+b}\,ab\,c^2x^2-24\sqrt{cx+b}\,b^4x+36\sqrt{}}{}$$

input
```
int((B*x+A)/x^(7/2)/(c*x^2+b*x)^(1/2),x)
```

output
```
( - 16*sqrt(b + c*x)*a*b**3 + 20*sqrt(b + c*x)*a*b**2*c*x - 30*sqrt(b + c*
x)*a*b*c**2*x**2 - 24*sqrt(b + c*x)*b**4*x + 36*sqrt(b + c*x)*b**3*c*x**2
- 15*sqrt(b)*log(sqrt(b + c*x) - sqrt(b))*a*c**3*x**3 + 18*sqrt(b)*log(sqr
t(b + c*x) - sqrt(b))*b**2*c**2*x**3 + 15*sqrt(b)*log(sqrt(b + c*x) + sqrt
(b))*a*c**3*x**3 - 18*sqrt(b)*log(sqrt(b + c*x) + sqrt(b))*b**2*c**2*x**3)
/(48*b**4*x**3)
```

3.212 $\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 163

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = -\frac{2b^3(bB-Ac)\sqrt{x}}{c^5\sqrt{bx+cx^2}} - \frac{2b^2(4bB-3Ac)\sqrt{bx+cx^2}}{c^5\sqrt{x}} + \frac{2b(2bB-Ac)\left(bx+cx^2\right)^{3/2}}{c^5x^{3/2}} - \frac{2(4bB-Ac)\left(bx+cx^2\right)^{5/2}}{5c^5x^{5/2}} + \frac{2B\left(bx+cx^2\right)^{7/2}}{7c^5x^{7/2}}$$

output

```
-2*b^3*(-A*c+B*b)*x^(1/2)/c^5/(c*x^2+b*x)^(1/2)-2*b^2*(-3*A*c+4*B*b)*(c*x^
2+b*x)^(1/2)/c^5/x^(1/2)+2*b*(-A*c+2*B*b)*(c*x^2+b*x)^(3/2)/c^5/x^(3/2)-2/
5*(-A*c+4*B*b)*(c*x^2+b*x)^(5/2)/c^5/x^(5/2)+2/7*B*(c*x^2+b*x)^(7/2)/c^5/x
^(7/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.57

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{2\sqrt{x}(-128b^4B+16b^3c(7A-4Bx)+8b^2c^2x(7A+2Bx)-2bc^3x^2(7A+4Bx)+c^4}{35c^5\sqrt{x(b+cx)}}$$

input

```
Integrate[(x^(9/2)*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[x]*(-128*b^4*B + 16*b^3*c*(7*A - 4*B*x) + 8*b^2*c^2*x*(7*A + 2*B*x
) - 2*b*c^3*x^2*(7*A + 4*B*x) + c^4*x^3*(7*A + 5*B*x)))/(35*c^5*Sqrt[x*(b
+ c*x)])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1218, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{3/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$-\left(\frac{7A}{b}-\frac{8B}{c}\right)\int \frac{x^{7/2}}{\sqrt{cx^2+bx}}dx-\frac{2x^{9/2}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1128}$$

$$-\left(\frac{7A}{b}-\frac{8B}{c}\right)\left(\frac{2x^{5/2}\sqrt{bx+cx^2}}{7c}-\frac{6b\int \frac{x^{5/2}}{\sqrt{cx^2+bx}}dx}{7c}\right)-\frac{2x^{9/2}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1128}$$

$$-\left(\frac{7A}{b}-\frac{8B}{c}\right)\left(\frac{2x^{5/2}\sqrt{bx+cx^2}}{7c}-\frac{6b\left(\frac{2x^{3/2}\sqrt{bx+cx^2}}{5c}-\frac{4b\int \frac{x^{3/2}}{\sqrt{cx^2+bx}}dx}{5c}\right)}{7c}\right)-\frac{2x^{9/2}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1128}$$

$$-\left(\frac{7A}{b}-\frac{8B}{c}\right)\left(\frac{2x^{5/2}\sqrt{bx+cx^2}}{7c}-\frac{6b\left(\frac{2x^{3/2}\sqrt{bx+cx^2}}{5c}-\frac{4b\left(\frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{2b\int\frac{\sqrt{x}}{\sqrt{cx^2+bx}}dx}{3c}\right)}{5c}\right)}{7c}\right)-\frac{2x^{9/2}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

$$\Big\downarrow \ 1122$$

$$-\left(\frac{2x^{5/2}\sqrt{bx+cx^2}}{7c}-\frac{6b\left(\frac{2x^{3/2}\sqrt{bx+cx^2}}{5c}-\frac{4b\left(\frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{4b\sqrt{bx+cx^2}}{3c^2\sqrt{x}}\right)}{5c}\right)}{7c}\right)\left(\frac{7A}{b}-\frac{8B}{c}\right)-\frac{2x^{9/2}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

input
```
Int[(x^(9/2)*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(b*B - A*c)*x^(9/2))/(b*c*Sqrt[b*x + c*x^2]) - ((7*A)/b - (8*B)/c)*((2
*x^(5/2)*Sqrt[b*x + c*x^2])/(7*c) - (6*b*((2*x^(3/2)*Sqrt[b*x + c*x^2])/(5
*c) - (4*b*((-4*b*Sqrt[b*x + c*x^2])/(3*c^2*Sqrt[x]) + (2*Sqrt[x]*Sqrt[b*x
 + c*x^2])/(3*c)))/(5*c)))/(7*c))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.66

method	result	size
gosper	$\frac{2(cx+b)\left(5B\,c^{4}x^{4}+7A\,c^{4}x^{3}-8B\,c^{3}x^{3}b-14Ab\,c^{3}x^{2}+16c^{2}x^{2}B\,b^{2}+56A\,b^{2}c^{2}x-64B\,b^{3}cx+112A\,b^{3}c-128B\,b^{4}\right)x^{\frac{3}{2}}}{35c^{5}\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}$	107
default	$\frac{2\sqrt{x(cx+b)}\,\left(5B\,c^{4}x^{4}+7A\,c^{4}x^{3}-8B\,c^{3}x^{3}b-14Ab\,c^{3}x^{2}+16c^{2}x^{2}B\,b^{2}+56A\,b^{2}c^{2}x-64B\,b^{3}cx+112A\,b^{3}c-128B\,b^{4}\right)}{35\sqrt{x}\,(cx+b)c^{5}}$	107
orering	$\frac{2(cx+b)\left(5B\,c^{4}x^{4}+7A\,c^{4}x^{3}-8B\,c^{3}x^{3}b-14Ab\,c^{3}x^{2}+16c^{2}x^{2}B\,b^{2}+56A\,b^{2}c^{2}x-64B\,b^{3}cx+112A\,b^{3}c-128B\,b^{4}\right)x^{\frac{3}{2}}}{35c^{5}\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}$	107
risch	$\frac{2\left(5B\,c^{3}x^{3}+7A\,c^{3}x^{2}-13Bb\,c^{2}x^{2}-21Ab\,c^{2}x+29B\,b^{2}cx+77A\,b^{2}c-93B\,b^{3}\right)(cx+b)\sqrt{x}}{35c^{5}\sqrt{x(cx+b)}}+\frac{2b^{3}(Ac-Bb)\sqrt{x}}{c^{5}\sqrt{x(cx+b)}}$	110

input
```
int(x^(9/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/35*(c*x+b)*(5*B*c^4*x^4+7*A*c^4*x^3-8*B*b*c^3*x^3-14*A*b*c^3*x^2+16*B*b^
2*c^2*x^2+56*A*b^2*c^2*x-64*B*b^3*c*x+112*A*b^3*c-128*B*b^4)*x^(3/2)/c^5/(
c*x^2+b*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.71

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{2\left(5\,Bc^4x^4 - 128\,Bb^4 + 112\,Ab^3c - (8\,Bbc^3 - 7\,Ac^4)x^3 + 2\,(8\,Bb^2c^2 - 7\,Abc^3)x^2 - \right.}{35\,(c^6x^2 + bc^5x)}$$

input
```
integrate(x^(9/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
2/35*(5*B*c^4*x^4 - 128*B*b^4 + 112*A*b^3*c - (8*B*b*c^3 - 7*A*c^4)*x^3 +
2*(8*B*b^2*c^2 - 7*A*b*c^3)*x^2 - 8*(8*B*b^3*c - 7*A*b^2*c^2)*x)*sqrt(c*x^
2 + b*x)*sqrt(x)/(c^6*x^2 + b*c^5*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)*(B*x+A)/(c*x**2+b*x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(Bx+A)x^{\frac{9}{2}}}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(9/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
2/105*((15*B*c^5*x^3 + 3*B*b*c^4*x^2 - 4*B*b^2*c^3*x + 8*B*b^3*c^2)*x^4 +
(16*B*b^4*c - 3*(4*B*b*c^4 - 7*A*c^5)*x^3 - (8*B*b^2*c^3 - 7*A*b*c^4)*x^2
+ 2*(10*B*b^3*c^2 - 7*A*b^2*c^3)*x)*x^3 + 4*(2*B*b^5 + (9*B*b^2*c^3 - 7*A*
b*c^4)*x^3 + 2*(10*B*b^3*c^2 - 7*A*b^2*c^3)*x^2 + (13*B*b^4*c - 7*A*b^3*c^
2)*x)*x^2)*sqrt(c*x + b)/(c^7*x^4 + 2*b*c^6*x^3 + b^2*c^5*x^2) + integrate
(-4/15*(4*B*b^5 - 2*A*b^4*c + (9*B*b^3*c^2 - 7*A*b^2*c^3)*x^2 + (13*B*b^4*
c - 9*A*b^3*c^2)*x)*sqrt(c*x + b)*x^2/(c^7*x^5 + 3*b*c^6*x^4 + 3*b^2*c^5*x
^3 + b^3*c^4*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.82

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = -\frac{2\,(Bb^4-Ab^3c)}{\sqrt{cx+b}c^5} + \frac{2\left(5\,(cx+b)^{\frac{7}{2}}Bc^{30} - 28\,(cx+b)^{\frac{5}{2}}Bbc^{30} + 70\,(cx+b)^{\frac{3}{2}}Bb^2c^{30} - 140\,\sqrt{cx+b}Bb^3c^{30} + 7\,(cx+b)^{\frac{5}{2}}Ac^{31} - \right.}{35\,c^{35}}$$

input
```
integrate(x^(9/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
-2*(B*b^4 - A*b^3*c)/(sqrt(c*x + b)*c^5) + 2/35*(5*(c*x + b)^(7/2)*B*c^30
- 28*(c*x + b)^(5/2)*B*b*c^30 + 70*(c*x + b)^(3/2)*B*b^2*c^30 - 140*sqrt(c
*x + b)*B*b^3*c^30 + 7*(c*x + b)^(5/2)*A*c^31 - 35*(c*x + b)^(3/2)*A*b*c^3
1 + 105*sqrt(c*x + b)*A*b^2*c^31)/c^35
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{x^{9/2}\,(A+B\,x)}{{(c\,x^2+b\,x)}^{3/2}}\,dx$$

input
```
int((x^(9/2)*(A + B*x))/(b*x + c*x^2)^(3/2),x)
```

output
```
int((x^(9/2)*(A + B*x))/(b*x + c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.57

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{\frac{2}{7}b\,c^4x^4+\frac{2}{5}a\,c^4x^3-\frac{16}{35}b^2c^3x^3-\frac{4}{5}ab\,c^3x^2+\frac{32}{35}b^3c^2x^2+\frac{16}{5}a\,b^2c^2x-\frac{128}{35}b^4cx+\frac{32}{5}a\,b^3c}{\sqrt{cx+b}\,c^5}$$

input
```
int(x^(9/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x)
```

output
```
(2*(112*a*b**3*c + 56*a*b**2*c**2*x - 14*a*b*c**3*x**2 + 7*a*c**4*x**3 - 1
28*b**5 - 64*b**4*c*x + 16*b**3*c**2*x**2 - 8*b**2*c**3*x**3 + 5*b*c**4*x*
*4))/(35*sqrt(b + c*x)*c**5)
```

3.213 $\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 128

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{2b^2(bB-Ac)\sqrt{x}}{c^4\sqrt{bx+cx^2}} + \frac{2b(3bB-2Ac)\sqrt{bx+cx^2}}{c^4\sqrt{x}} - \frac{2(3bB-Ac)\left(bx+cx^2\right)^{3/2}}{3c^4x^{3/2}} + \frac{2B\left(bx+cx^2\right)^{5/2}}{5c^4x^{5/2}}$$

output
```
2*b^2*(-A*c+B*b)*x^(1/2)/c^4/(c*x^2+b*x)^(1/2)+2*b*(-2*A*c+3*B*b)*(c*x^2+b
*x)^(1/2)/c^4/x^(1/2)-2/3*(-A*c+3*B*b)*(c*x^2+b*x)^(3/2)/c^4/x^(3/2)+2/5*B
*(c*x^2+b*x)^(5/2)/c^4/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.58

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{2\sqrt{x}(48b^3B-8b^2c(5A-3Bx)+c^3x^2(5A+3Bx)-2bc^2x(10A+3Bx))}{15c^4\sqrt{x(b+cx)}}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[x]*(48*b^3*B - 8*b^2*c*(5*A - 3*B*x) + c^3*x^2*(5*A + 3*B*x) - 2*b
*c^2*x*(10*A + 3*B*x)))/(15*c^4*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1218, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1218}$$

$$-\left(\frac{5A}{b}-\frac{6B}{c}\right)\int \frac{x^{5/2}}{\sqrt{cx^2+bx}}dx-\frac{2x^{7/2}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1128}$$

$$-\left(\frac{5A}{b}-\frac{6B}{c}\right)\left(\frac{2x^{3/2}\sqrt{bx+cx^2}}{5c}-\frac{4b\int \frac{x^{3/2}}{\sqrt{cx^2+bx}}dx}{5c}\right)-\frac{2x^{7/2}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1128}$$

$$-\left(\frac{5A}{b}-\frac{6B}{c}\right)\left(\frac{2x^{3/2}\sqrt{bx+cx^2}}{5c}-\frac{4b\left(\frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{2b\int \frac{\sqrt{x}}{\sqrt{cx^2+bx}}dx}{3c}\right)}{5c}\right)-\frac{2x^{7/2}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1122}$$

$$-\left(\frac{2x^{3/2}\sqrt{bx+cx^2}}{5c}-\frac{4b\left(\frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{4b\sqrt{bx+cx^2}}{3c^2\sqrt{x}}\right)}{5c}\right)\left(\frac{5A}{b}-\frac{6B}{c}\right)-\frac{2x^{7/2}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

input
```
Int[(x^(7/2)*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(b*B - A*c)*x^(7/2))/(b*c*Sqrt[b*x + c*x^2]) - ((5*A)/b - (6*B)/c)*((2
*x^(3/2)*Sqrt[b*x + c*x^2])/(5*c) - (4*b*((-4*b*Sqrt[b*x + c*x^2])/(3*c^2*
Sqrt[x]) + (2*Sqrt[x]*Sqrt[b*x + c*x^2])/(3*c)))/(5*c))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{2(cx+b)\left(-3B\,c^{3}x^{3}-5A\,c^{3}x^{2}+6Bb\,c^{2}x^{2}+20Ab\,c^{2}x-24B\,b^{2}cx+40A\,b^{2}c-48B\,b^{3}\right)x^{\frac{3}{2}}}{15c^{4}\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}$	83
default	$-\frac{2\sqrt{x(cx+b)}\,\left(-3B\,c^{3}x^{3}-5A\,c^{3}x^{2}+6Bb\,c^{2}x^{2}+20Ab\,c^{2}x-24B\,b^{2}cx+40A\,b^{2}c-48B\,b^{3}\right)}{15\sqrt{x}\,(cx+b)c^{4}}$	83
orering	$-\frac{2(cx+b)\left(-3B\,c^{3}x^{3}-5A\,c^{3}x^{2}+6Bb\,c^{2}x^{2}+20Ab\,c^{2}x-24B\,b^{2}cx+40A\,b^{2}c-48B\,b^{3}\right)x^{\frac{3}{2}}}{15c^{4}\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}$	83
risch	$-\frac{2\left(-3B\,c^{2}x^{2}-5A\,c^{2}x+9Bbcx+25Abc-33B\,b^{2}\right)(cx+b)\sqrt{x}}{15c^{4}\sqrt{x(cx+b)}}-\frac{2b^{2}(Ac-Bb)\sqrt{x}}{c^{4}\sqrt{x(cx+b)}}$	86

input
```
int(x^(7/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(c*x+b)*(-3*B*c^3*x^3-5*A*c^3*x^2+6*B*b*c^2*x^2+20*A*b*c^2*x-24*B*b^
2*c*x+40*A*b^2*c-48*B*b^3)*x^(3/2)/c^4/(c*x^2+b*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{2\left(3\,Bc^{3}x^{3}+48\,Bb^{3}-40\,Ab^{2}c-(6\,Bbc^{2}-5\,Ac^{3})x^{2}+4\,(6\,Bb^{2}c-5\,Abc^{2})x\right)\sqrt{cx^{2}-}}{15\,(c^{5}x^{2}+bc^{4}x)}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*c^3*x^3 + 48*B*b^3 - 40*A*b^2*c - (6*B*b*c^2 - 5*A*c^3)*x^2 + 4*
(6*B*b^2*c - 5*A*b*c^2)*x)*sqrt(c*x^2 + b*x)*sqrt(x)/(c^5*x^2 + b*c^4*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(7/2)*(B*x+A)/(c*x**2+b*x)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(Bx+A)x^{\frac{7}{2}}}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
2/15*((3*B*c^3*x^2 + B*b*c^2*x - 2*B*b^2*c)*x^3 - (4*B*b^3 + (4*B*b*c^2 -
5*A*c^3)*x^2 + (8*B*b^2*c - 5*A*b*c^2)*x)*x^2)*sqrt(c*x + b)/(c^5*x^3 + 2*
b*c^4*x^2 + b^2*c^3*x) - integrate(-4/15*(2*B*b^4 + (7*B*b^2*c^2 - 5*A*b*c
^3)*x^2 + (9*B*b^3*c - 5*A*b^2*c^2)*x)*sqrt(c*x + b)*x^2/(c^6*x^5 + 3*b*c^
5*x^4 + 3*b^2*c^4*x^3 + b^3*c^3*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{2\,(Bb^3 - Ab^2c)}{\sqrt{cx+b}c^4} + \frac{2\left(3\,(cx+b)^{\frac{5}{2}}Bc^{16} - 15\,(cx+b)^{\frac{3}{2}}Bbc^{16} + 45\,\sqrt{cx+b}Bb^2c^{16} + 5\,(cx+b)^{\frac{3}{2}}Ac^{17} - 30\,\sqrt{cx+b}Abc^{17}\right)}{15\,c^{20}}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
2*(B*b^3 - A*b^2*c)/(sqrt(c*x + b)*c^4) + 2/15*(3*(c*x + b)^(5/2)*B*c^16 -
 15*(c*x + b)^(3/2)*B*b*c^16 + 45*sqrt(c*x + b)*B*b^2*c^16 + 5*(c*x + b)^(
3/2)*A*c^17 - 30*sqrt(c*x + b)*A*b*c^17)/c^20
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx=\int \frac{x^{7/2}\,(A+B\,x)}{(c\,x^2+b\,x)^{3/2}}\,dx$$

input
```
int((x^(7/2)*(A + B*x))/(b*x + c*x^2)^(3/2),x)
```

output
```
int((x^(7/2)*(A + B*x))/(b*x + c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.55

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx=\frac{\frac{2}{5}b\,c^3x^3+\frac{2}{3}a\,c^3x^2-\frac{4}{5}b^2c^2x^2-\frac{8}{3}ab\,c^2x+\frac{16}{5}b^3cx-\frac{16}{3}a\,b^2c+\frac{32}{5}b^4}{\sqrt{cx+b}\,c^4}$$

input
```
int(x^(7/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x)
```

output
```
(2*( - 40*a*b**2*c - 20*a*b*c**2*x + 5*a*c**3*x**2 + 48*b**4 + 24*b**3*c*x
 - 6*b**2*c**2*x**2 + 3*b*c**3*x**3))/(15*sqrt(b + c*x)*c**4)
```

3.214 $\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx=-\frac{2b(bB-Ac)\sqrt{x}}{c^3\sqrt{bx+cx^2}}-\frac{2(2bB-Ac)\sqrt{bx+cx^2}}{c^3\sqrt{x}}+\frac{2B(bx+cx^2)^{3/2}}{3c^3x^{3/2}}$$

output
```
-2*b*(-A*c+B*b)*x^(1/2)/c^3/(c*x^2+b*x)^(1/2)-2*(-A*c+2*B*b)*(c*x^2+b*x)^(
1/2)/c^3/x^(1/2)+2/3*B*(c*x^2+b*x)^(3/2)/c^3/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.59

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx=\frac{2\sqrt{x}(-8b^2B+c^2x(3A+Bx)+b(6Ac-4Bcx))}{3c^3\sqrt{x(b+cx)}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[x]*(-8*b^2*B + c^2*x*(3*A + B*x) + b*(6*A*c - 4*B*c*x)))/(3*c^3*Sq
rt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.12, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1218, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1218}$$

$$-\left(\frac{3A}{b}-\frac{4B}{c}\right)\int \frac{x^{3/2}}{\sqrt{cx^2+bx}}dx-\frac{2x^{5/2}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{ 1128}$$

$$-\left(\frac{3A}{b}-\frac{4B}{c}\right)\left(\frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{2b\int \frac{\sqrt{x}}{\sqrt{cx^2+bx}}dx}{3c}\right)-\frac{2x^{5/2}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{ 1122}$$

$$-\left(\frac{2\sqrt{x}\sqrt{bx+cx^2}}{3c}-\frac{4b\sqrt{bx+cx^2}}{3c^2\sqrt{x}}\right)\left(\frac{3A}{b}-\frac{4B}{c}\right)-\frac{2x^{5/2}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

input
```
Int[(x^(5/2)*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(b*B - A*c)*x^(5/2))/(b*c*Sqrt[b*x + c*x^2]) - ((3*A)/b - (4*B)/c)*((-
4*b*Sqrt[b*x + c*x^2])/(3*c^2*Sqrt[x]) + (2*Sqrt[x]*Sqrt[b*x + c*x^2])/(3*
c))
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{2(cx+b)\left(B\,c^{2}x^{2}+3A\,c^{2}x-4Bbcx+6Abc-8B\,b^{2}\right)x^{\frac{3}{2}}}{3c^{3}\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}$	58
default	$\frac{2\sqrt{x(cx+b)}\,\left(B\,c^{2}x^{2}+3A\,c^{2}x-4Bbcx+6Abc-8B\,b^{2}\right)}{3\sqrt{x}\,(cx+b)c^{3}}$	58
orering	$\frac{2(cx+b)\left(B\,c^{2}x^{2}+3A\,c^{2}x-4Bbcx+6Abc-8B\,b^{2}\right)x^{\frac{3}{2}}}{3c^{3}\left(c\,x^{2}+bx\right)^{\frac{3}{2}}}$	58
risch	$\frac{2(Bcx+3Ac-5Bb)(cx+b)\sqrt{x}}{3c^{3}\sqrt{x(cx+b)}}+\frac{2b(Ac-Bb)\sqrt{x}}{c^{3}\sqrt{x(cx+b)}}$	63

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(c*x+b)*(B*c^2*x^2+3*A*c^2*x-4*B*b*c*x+6*A*b*c-8*B*b^2)*x^(3/2)/c^3/(c
*x^2+b*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{2\,(Bc^2x^2-8\,Bb^2+6\,Abc-(4\,Bbc-3\,Ac^2)x)\sqrt{cx^2+bx}\sqrt{x}}{3\,(c^4x^2+bc^3x)}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(B*c^2*x^2 - 8*B*b^2 + 6*A*b*c - (4*B*b*c - 3*A*c^2)*x)*sqrt(c*x^2 + b
*x)*sqrt(x)/(c^4*x^2 + b*c^3*x)
```

Sympy [F]

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{x^{\frac{5}{2}}(A+Bx)}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(5/2)*(B*x+A)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(x**(5/2)*(A + B*x)/(x*(b + c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(Bx+A)x^{\frac{5}{2}}}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
2/3*(B*c*x + B*b)*sqrt(c*x + b)*x^2/(c^3*x^2 + 2*b*c^2*x + b^2*c) + integr
ate(1/3*(3*A*b*c*x^2 - (4*B*b^2 + (4*B*b*c - 3*A*c^2)*x)*x^2)*sqrt(c*x + b
)/(c^4*x^4 + 3*b*c^3*x^3 + 3*b^2*c^2*x^2 + b^3*c*x), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.76

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = -\frac{2\,(Bb^2-Abc)}{\sqrt{cx+b}c^3} + \frac{2\left((cx+b)^{\frac{3}{2}}Bc^6 - 6\sqrt{cx+b}Bbc^6 + 3\sqrt{cx+b}Ac^7\right)}{3\,c^9}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
-2*(B*b^2 - A*b*c)/(sqrt(c*x + b)*c^3) + 2/3*((c*x + b)^(3/2)*B*c^6 - 6*sq
rt(c*x + b)*B*b*c^6 + 3*sqrt(c*x + b)*A*c^7)/c^9
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{x^{5/2}\,(A+B\,x)}{(c\,x^2+b\,x)^{3/2}}\,dx$$

input
```
int((x^(5/2)*(A + B*x))/(b*x + c*x^2)^(3/2),x)
```

output
```
int((x^(5/2)*(A + B*x))/(b*x + c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.51

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{\frac{2}{3}b\,c^2x^2+2a\,c^2x-\frac{8}{3}b^2cx+4abc-\frac{16}{3}b^3}{\sqrt{cx+b}\,c^3}$$

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x)
```

output
```
(2*(6*a*b*c + 3*a*c**2*x - 8*b**3 - 4*b**2*c*x + b*c**2*x**2))/(3*sqrt(b +
 c*x)*c**3)
```

3.215 $\int \frac{x^{3/2}(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \frac{x^{3/2}(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx=\frac{2(bB-Ac)\sqrt{x}}{c^2\sqrt{bx+cx^2}}+\frac{2B\sqrt{bx+cx^2}}{c^2\sqrt{x}}$$

output
```
2*(-A*c+B*b)*x^(1/2)/c^2/(c*x^2+b*x)^(1/2)+2*B*(c*x^2+b*x)^(1/2)/c^2/x^(1/
2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \frac{x^{3/2}(A+Bx)}{\left(bx+cx^2\right)^{3/2}}\,dx=\frac{2\sqrt{x}(2bB-Ac+Bcx)}{c^2\sqrt{x(b+cx)}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(2*Sqrt[x]*(2*b*B - A*c + B*c*x))/(c^2*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.25, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1218, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \ 1218$$

$$\frac{(2bB-Ac)\int \frac{\sqrt{x}}{\sqrt{cx^2+bx}}dx}{bc}-\frac{2x^{3/2}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

$$\Big\downarrow \ 1122$$

$$\frac{2\sqrt{bx+cx^2}(2bB-Ac)}{bc^2\sqrt{x}}-\frac{2x^{3/2}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

input
```
Int[(x^(3/2)*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(b*B - A*c)*x^(3/2))/(b*c*Sqrt[b*x + c*x^2]) + (2*(2*b*B - A*c)*Sqrt[b
*x + c*x^2])/(b*c^2*Sqrt[x])
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{2(cx+b)(-Bcx+Ac-2Bb)x^{\frac{3}{2}}}{c^{2}(c\,x^{2}+bx)^{\frac{3}{2}}}$	38
default	$-\frac{2\sqrt{x(cx+b)}\,(-Bcx+Ac-2Bb)}{\sqrt{x}\,(cx+b)c^{2}}$	38
orering	$-\frac{2(cx+b)(-Bcx+Ac-2Bb)x^{\frac{3}{2}}}{c^{2}(c\,x^{2}+bx)^{\frac{3}{2}}}$	38
risch	$\frac{2B(cx+b)\sqrt{x}}{c^{2}\sqrt{x(cx+b)}}-\frac{2(Ac-Bb)\sqrt{x}}{c^{2}\sqrt{x(cx+b)}}$	50

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(c*x+b)*(-B*c*x+A*c-2*B*b)*x^(3/2)/c^2/(c*x^2+b*x)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.80

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx=\frac{2\,(Bcx+2\,Bb-Ac)\sqrt{cx^2+bx}\sqrt{x}}{c^3x^2+bc^2x}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
2*(B*c*x + 2*B*b - A*c)*sqrt(c*x^2 + b*x)*sqrt(x)/(c^3*x^2 + b*c^2*x)
```

Sympy [F]

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{x^{\frac{3}{2}}(A+Bx)}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(3/2)*(B*x+A)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(x**(3/2)*(A + B*x)/(x*(b + c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(Bx+A)x^{\frac{3}{2}}}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*x^(3/2)/(c*x^2 + b*x)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{2\sqrt{cx+b}B}{c^2} + \frac{2\,(Bb-Ac)}{\sqrt{cx+b}c^2}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
2*sqrt(c*x + b)*B/c^2 + 2*(B*b - A*c)/(sqrt(c*x + b)*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{x^{3/2}\,(A+B\,x)}{(c\,x^2+b\,x)^{3/2}}\,dx$$

input
```
int((x^(3/2)*(A + B*x))/(b*x + c*x^2)^(3/2),x)
```

output
```
int((x^(3/2)*(A + B*x))/(b*x + c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.48

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{2bcx-2ac+4b^2}{\sqrt{cx+b}\,c^2}$$

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x)
```

output
```
(2*( - a*c + 2*b**2 + b*c*x))/(sqrt(b + c*x)*c**2)
```

3.216 $\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 68

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = -\frac{2(bB-Ac)\sqrt{x}}{bc\sqrt{bx+cx^2}} - \frac{2A\operatorname{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}$$

output
```
-2*(-A*c+B*b)*x^(1/2)/b/c/(c*x^2+b*x)^(1/2)-2*A*arctanh((c*x^2+b*x)^(1/2)/
b^(1/2)/x^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = -\frac{2\sqrt{x}\left(\sqrt{b}(bB-Ac)+Ac\sqrt{b+cx}\operatorname{arctanh}\left(\frac{\sqrt{b+cx}}{\sqrt{b}}\right)\right)}{b^{3/2}c\sqrt{x(b+cx)}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*Sqrt[x]*(Sqrt[b]*(b*B - A*c) + A*c*Sqrt[b + c*x]*ArcTanh[Sqrt[b + c*x]
/Sqrt[b]]))/(b^(3/2)*c*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1218, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{A\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{b}-\frac{2\sqrt{x}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{2A\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}-\frac{2\sqrt{x}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{220}$$

$$-\frac{2A\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}-\frac{2\sqrt{x}(bB-Ac)}{bc\sqrt{bx+cx^2}}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(b*x + c*x^2)^(3/2),x]
```

output
```
(-2*(b*B - A*c)*Sqrt[x])/(b*c*Sqrt[b*x + c*x^2]) - (2*A*ArcTanh[Sqrt[b*x +
 c*x^2]/(Sqrt[b]*Sqrt[x])])/b^(3/2)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

method	result	size
default	$-\frac{2\sqrt{x(cx+b)}\left(A\,\mathrm{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c\sqrt{cx+b}-Ac\sqrt{b}+B\,b^{\frac{3}{2}}\right)}{b^{\frac{3}{2}}\sqrt{x}\,(cx+b)c}$	63

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(x*(c*x+b))^(1/2)/b^(3/2)*(A*arctanh((c*x+b)^(1/2)/b^(1/2))*c*(c*x+b)^(
1/2)-A*c*b^(1/2)+B*b^(3/2))/x^(1/2)/(c*x+b)/c
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.87

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \left[\frac{(Ac^2x^2+Abcx)\sqrt{b}\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)-2\,(Bb^2-Abc)\sqrt{cx^2+bx}\sqrt{x}}{b^2c^2x^2+b^3cx}, \right.$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[((A*c^2*x^2 + A*b*c*x)*sqrt(b)*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*
sqrt(b)*sqrt(x))/x^2) - 2*(B*b^2 - A*b*c)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^2*
c^2*x^2 + b^3*c*x), 2*((A*c^2*x^2 + A*b*c*x)*sqrt(-b)*arctan(sqrt(c*x^2 +
b*x)*sqrt(-b)/(b*sqrt(x))) - (B*b^2 - A*b*c)*sqrt(c*x^2 + b*x)*sqrt(x))/(b
^2*c^2*x^2 + b^3*c*x)]
```

Sympy [F]

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{\sqrt{x}\,(A+Bx)}{(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**(1/2)*(B*x+A)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral(sqrt(x)*(A + B*x)/(x*(b + c*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{(Bx+A)\sqrt{x}}{(cx^2+bx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*sqrt(x)/(c*x^2 + b*x)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.72

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{2\,A\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b} - \frac{2\,(Bb-Ac)}{\sqrt{cx+b}bc}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
2*A*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b) - 2*(B*b - A*c)/(sqrt(c*x
+ b)*b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \int \frac{\sqrt{x}\,(A+B\,x)}{(c\,x^2+b\,x)^{3/2}}\,dx$$

input
```
int((x^(1/2)*(A + B*x))/(b*x + c*x^2)^(3/2),x)
```

output
```
int((x^(1/2)*(A + B*x))/(b*x + c*x^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.04

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{3/2}}\,dx = \frac{\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)ac-\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}+\sqrt{b}\right)ac+2abc-2}{\sqrt{cx+b}\,b^2c}$$

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x)^(3/2),x)
```

output
```
(sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*a*c - sqrt(b)*sqrt(b +
 c*x)*log(sqrt(b + c*x) + sqrt(b))*a*c + 2*a*b*c - 2*b**3)/(sqrt(b + c*x)*
b**2*c)
```

3.217 $\int \frac{A+Bx}{\sqrt{x}(bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 97

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^{3/2}}\,dx=\frac{2(bB-Ac)\sqrt{x}}{b^2\sqrt{bx+cx^2}}-\frac{A\sqrt{bx+cx^2}}{b^2x^{3/2}}-\frac{(2bB-3Ac)\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{5/2}}$$

output
```
2*(-A*c+B*b)*x^(1/2)/b^2/(c*x^2+b*x)^(1/2)-A*(c*x^2+b*x)^(1/2)/b^2/x^(3/2)
-(-3*A*c+2*B*b)*arctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.84

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^{3/2}}\,dx=\frac{\sqrt{b}(2bBx-A(b+3cx))-(2bB-3Ac)x\sqrt{b+cx}\,\text{arctanh}\left(\frac{\sqrt{b+cx}}{\sqrt{b}}\right)}{b^{5/2}\sqrt{x}\sqrt{x(b+cx)}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(b*x + c*x^2)^(3/2)),x]
```

output
```
(Sqrt[b]*(2*b*B*x - A*(b + 3*c*x)) - (2*b*B - 3*A*c)*x*Sqrt[b + c*x]*ArcTa
nh[Sqrt[b + c*x]/Sqrt[b]])/(b^(5/2)*Sqrt[x]*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1220, 1132, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^{3/2}}\,dx$$

$$\Big\downarrow \; 1220$$

$$\frac{(2bB-3Ac)\int \frac{\sqrt{x}}{(cx^2+bx)^{3/2}}dx}{2b}-\frac{A}{b\sqrt{x}\sqrt{bx+cx^2}}$$

$$\Big\downarrow \; 1132$$

$$\frac{(2bB-3Ac)\left(\frac{\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{2b}-\frac{A}{b\sqrt{x}\sqrt{bx+cx^2}}$$

$$\Big\downarrow \; 1136$$

$$\frac{(2bB-3Ac)\left(\frac{2\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{2b}-\frac{A}{b\sqrt{x}\sqrt{bx+cx^2}}$$

$$\Big\downarrow \; 220$$

$$\frac{(2bB-3Ac)\left(\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}\right)}{2b}-\frac{A}{b\sqrt{x}\sqrt{bx+cx^2}}$$

input
```
Int[(A + B*x)/(Sqrt[x]*(b*x + c*x^2)^(3/2)),x]
```

output
```
-(A/(b*Sqrt[x]*Sqrt[b*x + c*x^2])) + ((2*b*B - 3*A*c)*((2*Sqrt[x])/(b*Sqrt
[b*x + c*x^2]) - (2*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/b^(3/2))
)/(2*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\sqrt{x(cx+b)}\left(3A\sqrt{cx+b}\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)cx-2B\sqrt{cx+b}\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)bx+2B\,b^{\frac{3}{2}}x-3A\sqrt{b}\,cx-A\,b^{\frac{3}{2}}\right)}{x^{\frac{3}{2}}(cx+b)b^{\frac{5}{2}}}$	94
risch	$-\frac{A(cx+b)}{b^2\sqrt{x}\,\sqrt{x(cx+b)}}-\frac{\left(-\frac{2(-2Ac+2Bb)}{\sqrt{cx+b}}-\frac{2(3Ac-2Bb)\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)}{\sqrt{b}}\right)\sqrt{cx+b}\,\sqrt{x}}{2b^2\sqrt{x(cx+b)}}$	94

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/x^(3/2)*(x*(c*x+b))^(1/2)*(3*A*(c*x+b)^(1/2)*arctanh((c*x+b)^(1/2)/b^(1/
2))*c*x-2*B*(c*x+b)^(1/2)*arctanh((c*x+b)^(1/2)/b^(1/2))*b*x+2*B*b^(3/2)*x
-3*A*b^(1/2)*c*x-A*b^(3/2))/(c*x+b)/b^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.59

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^{3/2}}\,dx = \left[-\frac{((2\,Bbc-3\,Ac^2)x^3+(2\,Bb^2-3\,Abc)x^2)\sqrt{b}\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2}{2\,(b^3cx^3+b^4x^2)}\right.$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(((2*B*b*c - 3*A*c^2)*x^3 + (2*B*b^2 - 3*A*b*c)*x^2)*sqrt(b)*log(-(c
*x^2 + 2*b*x + 2*sqrt(c*x^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(A*b^2 - (2*B
*b^2 - 3*A*b*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^3*c*x^3 + b^4*x^2), (((2*
B*b*c - 3*A*c^2)*x^3 + (2*B*b^2 - 3*A*b*c)*x^2)*sqrt(-b)*arctan(sqrt(c*x^2
 + b*x)*sqrt(-b)/(b*sqrt(x))) - (A*b^2 - (2*B*b^2 - 3*A*b*c)*x)*sqrt(c*x^2
 + b*x)*sqrt(x))/(b^3*c*x^3 + b^4*x^2)]
```

Sympy [F]

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^{3/2}}\,dx=\int \frac{A+Bx}{\sqrt{x}\,(x\,(b+cx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**(1/2)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((A + B*x)/(sqrt(x)*(x*(b + c*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^{3/2}}\,dx=\int \frac{Bx+A}{(cx^2+bx)^{\frac{3}{2}}\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((c*x^2 + b*x)^(3/2)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.90

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^{3/2}}\,dx=\frac{(2\,Bb-3\,Ac)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2}+\frac{2\,(cx+b)Bb-2\,Bb^2-3\,(cx+b)Ac+2\,Abc}{\left((cx+b)^{\frac{3}{2}}-\sqrt{cx+b}b\right)b^2}$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
(2*B*b - 3*A*c)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^2) + (2*(c*x +
b)*B*b - 2*B*b^2 - 3*(c*x + b)*A*c + 2*A*b*c)/(((c*x + b)^(3/2) - sqrt(c*x
 + b)*b)*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{x}\left(bx+cx^2\right)^{3/2}}\,dx=\int \frac{A+B\,x}{\sqrt{x}\left(c\,x^2+b\,x\right)^{3/2}}\,dx$$

input
```
int((A + B*x)/(x^(1/2)*(b*x + c*x^2)^(3/2)),x)
```

output
```
int((A + B*x)/(x^(1/2)*(b*x + c*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.37

$$\int \frac{A+Bx}{\sqrt{x}\left(bx+cx^2\right)^{3/2}}\,dx=\frac{-3\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)acx+2\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)b^2x}{}$$

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x)^(3/2),x)
```

output
```
( - 3*sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*a*c*x + 2*sqrt(b)
*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*b**2*x + 3*sqrt(b)*sqrt(b + c*
x)*log(sqrt(b + c*x) + sqrt(b))*a*c*x - 2*sqrt(b)*sqrt(b + c*x)*log(sqrt(b
 + c*x) + sqrt(b))*b**2*x - 2*a*b**2 - 6*a*b*c*x + 4*b**3*x)/(2*sqrt(b + c
*x)*b**3*x)
```

3.218 $\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 137

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{3/2}}\,dx=-\frac{2c(bB-Ac)\sqrt{x}}{b^3\sqrt{bx+cx^2}}-\frac{A\sqrt{bx+cx^2}}{2b^2x^{5/2}}-\frac{(4bB-7Ac)\sqrt{bx+cx^2}}{4b^3x^{3/2}}+\frac{3c(4bB-5Ac)\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4b^{7/2}}$$

output
```
-2*c*(-A*c+B*b)*x^(1/2)/b^3/(c*x^2+b*x)^(1/2)-1/2*A*(c*x^2+b*x)^(1/2)/b^2/
x^(5/2)-1/4*(-7*A*c+4*B*b)*(c*x^2+b*x)^(1/2)/b^3/x^(3/2)+3/4*c*(-5*A*c+4*B
*b)*arctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.77

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{3/2}}\,dx=\frac{\sqrt{b}(-4bBx(b+3cx)+A(-2b^2+5bcx+15c^2x^2))+3c(4bB-5Ac)x^2\sqrt{b+cx}a}{4b^{7/2}x^{3/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(b*x + c*x^2)^(3/2)),x]
```

output
```
(Sqrt[b]*(-4*b*B*x*(b + 3*c*x) + A*(-2*b^2 + 5*b*c*x + 15*c^2*x^2)) + 3*c*
(4*b*B - 5*A*c)*x^2*Sqrt[b + c*x]*ArcTanh[Sqrt[b + c*x]/Sqrt[b]])/(4*b^(7/
2)*x^(3/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1220, 1135, 1132, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(4bB-5Ac)\int \frac{1}{\sqrt{x}(cx^2+bx)^{3/2}}dx}{4b}-\frac{A}{2bx^{3/2}\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{ 1135}$$

$$\frac{(4bB-5Ac)\left(-\frac{3c\int \frac{\sqrt{x}}{\left(cx^2+bx\right)^{3/2}}dx}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{4b}-\frac{A}{2bx^{3/2}\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{ 1132}$$

$$\frac{(4bB-5Ac)\left(-\frac{3c\left(\frac{\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{4b}-\frac{A}{2bx^{3/2}\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{ 1136}$$

$$\frac{(4bB-5Ac)\left(-\frac{3c\left(\frac{2\int\frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{4b}-\frac{A}{2bx^{3/2}\sqrt{bx+cx^2}}$$

$\big\downarrow$ 220

$$\frac{(4bB-5Ac)\left(-\frac{3c\left(\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{4b}-\frac{A}{2bx^{3/2}\sqrt{bx+cx^2}}$$

input
```
Int[(A + B*x)/(x^(3/2)*(b*x + c*x^2)^(3/2)),x]
```

output
```
-1/2*A/(b*x^(3/2)*Sqrt[b*x + c*x^2]) + ((4*b*B - 5*A*c)*(-(1/(b*Sqrt[x]*Sq
rt[b*x + c*x^2])) - (3*c*((2*Sqrt[x])/(b*Sqrt[b*x + c*x^2]) - (2*ArcTanh[S
qrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/b^(3/2)))/(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.80

method	result
risch	$-\frac{(cx+b)(-7Acx+4Bbx+2Ab)}{4b^3x^{\frac{3}{2}}\sqrt{x(cx+b)}}+\frac{c\left(-\frac{2(-8Ac+8Bb)}{\sqrt{cx+b}}-\frac{2(15Ac-12Bb)\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)}{\sqrt{b}}\right)\sqrt{cx+b}\,\sqrt{x}}{8b^3\sqrt{x(cx+b)}}$
default	$-\frac{\sqrt{x(cx+b)}\left(15A\sqrt{cx+b}\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^2x^2-12B\sqrt{cx+b}\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)bc\,x^2+4B\,b^{\frac{5}{2}}x+12B\,b^{\frac{3}{2}}c\,x^2+2A\,b^{\frac{5}{2}}-5A\,b^{\frac{3}{2}}cx-1\right.}{4x^{\frac{5}{2}}(cx+b)b^{\frac{7}{2}}}$

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x+b)*(-7*A*c*x+4*B*b*x+2*A*b)/b^3/x^(3/2)/(x*(c*x+b))^(1/2)+1/8/b^
3*c*(-2*(-8*A*c+8*B*b)/(c*x+b)^(1/2)-2*(15*A*c-12*B*b)/b^(1/2)*arctanh((c*
x+b)^(1/2)/b^(1/2)))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 307, normalized size of antiderivative = 2.24

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{3/2}}\,dx = \left[-\frac{3\left((4\,Bbc^2-5\,Ac^3)x^4+(4\,Bb^2c-5\,Abc^2)x^3\right)\sqrt{b}\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right.}{8\,(b^4cx^4+}\right.$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*((4*B*b*c^2 - 5*A*c^3)*x^4 + (4*B*b^2*c - 5*A*b*c^2)*x^3)*sqrt(b)
*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(2*A*
b^3 + 3*(4*B*b^2*c - 5*A*b*c^2)*x^2 + (4*B*b^3 - 5*A*b^2*c)*x)*sqrt(c*x^2
+ b*x)*sqrt(x))/(b^4*c*x^4 + b^5*x^3), -1/4*(3*((4*B*b*c^2 - 5*A*c^3)*x^4
+ (4*B*b^2*c - 5*A*b*c^2)*x^3)*sqrt(-b)*arctan(sqrt(c*x^2 + b*x)*sqrt(-b)/
(b*sqrt(x))) + (2*A*b^3 + 3*(4*B*b^2*c - 5*A*b*c^2)*x^2 + (4*B*b^3 - 5*A*b
^2*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^4*c*x^4 + b^5*x^3)]
```

Sympy [F]

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{3/2}}\,dx = \int \frac{A+Bx}{x^{\frac{3}{2}}\left(x\left(b+cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**(3/2)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((A + B*x)/(x**(3/2)*(x*(b + c*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{3/2}}\,dx=\int \frac{Bx+A}{\left(cx^2+bx\right)^{\frac{3}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((c*x^2 + b*x)^(3/2)*x^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.91

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{3/2}}\,dx=-\frac{3\left(4\,Bbc-5\,Ac^2\right)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{4\sqrt{-b}b^3}-\frac{2\left(Bbc-Ac^2\right)}{\sqrt{cx+b}b^3}-\frac{4\left(cx+b\right)^{\frac{3}{2}}Bbc-4\sqrt{cx+b}Bb^2c-7\left(cx+b\right)^{\frac{3}{2}}Ac^2+9\sqrt{cx+b}Abc^2}{4\,b^3c^2x^2}$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
-3/4*(4*B*b*c - 5*A*c^2)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^3) - 2
*(B*b*c - A*c^2)/(sqrt(c*x + b)*b^3) - 1/4*(4*(c*x + b)^(3/2)*B*b*c - 4*sq
rt(c*x + b)*B*b^2*c - 7*(c*x + b)^(3/2)*A*c^2 + 9*sqrt(c*x + b)*A*b*c^2)/(
b^3*c^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{3/2}}\,dx=\int \frac{A+B\,x}{x^{3/2}\,\left(c\,x^2+b\,x\right)^{3/2}}\,dx$$

input
```
int((A + B*x)/(x^(3/2)*(b*x + c*x^2)^(3/2)),x)
```

output
```
int((A + B*x)/(x^(3/2)*(b*x + c*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.23

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{3/2}}\,dx = \frac{15\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)a\,c^2x^2-12\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)b}{}$$

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x)^(3/2),x)
```

output
```
(15*sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*a*c**2*x**2 - 12*sq
rt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*b**2*c*x**2 - 15*sqrt(b)*
sqrt(b + c*x)*log(sqrt(b + c*x) + sqrt(b))*a*c**2*x**2 + 12*sqrt(b)*sqrt(b
 + c*x)*log(sqrt(b + c*x) + sqrt(b))*b**2*c*x**2 - 4*a*b**3 + 10*a*b**2*c*
x + 30*a*b*c**2*x**2 - 8*b**4*x - 24*b**3*c*x**2)/(8*sqrt(b + c*x)*b**4*x*
*2)
```

3.219 $\int \frac{A+Bx}{x^{5/2}(bx+cx^2)^{3/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 176

$$\int \frac{A+Bx}{x^{5/2}(bx+cx^2)^{3/2}} \, dx = \frac{2c^2(bB-Ac)\sqrt{x}}{b^4\sqrt{bx+cx^2}} - \frac{A\sqrt{bx+cx^2}}{3b^2x^{7/2}} - \frac{(6bB-11Ac)\sqrt{bx+cx^2}}{12b^3x^{5/2}} + \frac{c(14bB-19Ac)\sqrt{bx+cx^2}}{8b^4x^{3/2}} - \frac{5c^2(6bB-7Ac)\operatorname{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{9/2}}$$

output
```
2*c^2*(-A*c+B*b)*x^(1/2)/b^4/(c*x^2+b*x)^(1/2)-1/3*A*(c*x^2+b*x)^(1/2)/b^2
/x^(7/2)-1/12*(-11*A*c+6*B*b)*(c*x^2+b*x)^(1/2)/b^3/x^(5/2)+1/8*c*(-19*A*c
+14*B*b)*(c*x^2+b*x)^(1/2)/b^4/x^(3/2)-5/8*c^2*(-7*A*c+6*B*b)*arctanh((c*x
^2+b*x)^(1/2)/b^(1/2)/x^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx}{x^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx = \frac{\sqrt{b}(6bBx(-2b^2+5bcx+15c^2x^2)-A(8b^3-14b^2cx+35bc^2x^2+105c^3x^3))+15}{24b^{9/2}x^{5/2}\sqrt{x(b+cx)}}$$

input
```
Integrate[(A + B*x)/(x^(5/2)*(b*x + c*x^2)^(3/2)),x]
```

output
```
(Sqrt[b]*(6*b*B*x*(-2*b^2 + 5*b*c*x + 15*c^2*x^2) - A*(8*b^3 - 14*b^2*c*x
+ 35*b*c^2*x^2 + 105*c^3*x^3)) + 15*c^2*(-6*b*B + 7*A*c)*x^3*Sqrt[b + c*x]
*ArcTanh[Sqrt[b + c*x]/Sqrt[b]])/(24*b^(9/2)*x^(5/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1220, 1135, 1135, 1132, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(6bB-7Ac)\int \frac{1}{x^{3/2}\left(cx^2+bx\right)^{3/2}}dx}{6b}-\frac{A}{3bx^{5/2}\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{(6bB-7Ac)\left(-\frac{5c\int \frac{1}{\sqrt{x}\left(cx^2+bx\right)^{3/2}}dx}{4b}-\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}\right)}{6b}-\frac{A}{3bx^{5/2}\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{(6bB-7Ac)\left(-\frac{5c\left(-\frac{3c\int \frac{\sqrt{x}}{\left(cx^2+bx\right)^{3/2}}dx}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{4b}-\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}\right)}{6b}-\frac{A}{3bx^{5/2}\sqrt{bx+cx^2}}$$

$\Big\downarrow$ 1132

$$\frac{(6bB-7Ac)\left(-\frac{5c\left(-\frac{3c\left(\frac{\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{4b}-\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}\right)}{6b}-\frac{A}{3bx^{5/2}\sqrt{bx+cx^2}}$$

$\Big\downarrow$ 1136

$$\frac{(6bB-7Ac)\left(-\frac{5c\left(-\frac{3c\left(\frac{2\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{4b}-\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}\right)}{6b}-\frac{A}{3bx^{5/2}\sqrt{bx+cx^2}}$$

$\Big\downarrow$ 220

$$\frac{(6bB-7Ac)\left(-\frac{5c\left(-\frac{3c\left(\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{4b}-\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}\right)}{6b}-\frac{A}{3bx^{5/2}\sqrt{bx+cx^2}}$$

input
```
Int[(A + B*x)/(x^(5/2)*(b*x + c*x^2)^(3/2)),x]
```

output
```
-1/3*A/(b*x^(5/2)*Sqrt[b*x + c*x^2]) + ((6*b*B - 7*A*c)*(-1/2*1/(b*x^(3/2)
*Sqrt[b*x + c*x^2]) - (5*c*(-(1/(b*Sqrt[x]*Sqrt[b*x + c*x^2])) - (3*c*((2*
Sqrt[x])/(b*Sqrt[b*x + c*x^2]) - (2*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqr
t[x])])/b^(3/2)))/(2*b)))/(4*b)))/(6*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{(cx+b)\left(57Ac^{2}x^{2}-42x^{2}Bbc-22Abcx+12xB\,b^{2}+8b^{2}A\right)}{24b^{4}x^{\frac{5}{2}}\sqrt{x(cx+b)}}-\frac{c^{2}\left(-\frac{2(-16Ac+16Bb)}{\sqrt{cx+b}}-\frac{2(35Ac-30Bb)\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)}{\sqrt{b}}\right)\sqrt{cx+b}\,\sqrt{x}}{16b^{4}\sqrt{x(cx+b)}}$
default	$\frac{\sqrt{x(cx+b)}\,\left(105A\sqrt{cx+b}\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^{3}x^{3}-90B\sqrt{cx+b}\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)b\,c^{2}x^{3}-12B\,b^{\frac{7}{2}}x+30B\,b^{\frac{5}{2}}c\,x^{2}+90B\,b^{\frac{3}{2}}c^{2}x^{3}-8A\,b\right.}{24x^{\frac{7}{2}}(cx+b)b^{\frac{9}{2}}}$

input
```
int((B*x+A)/x^(5/2)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x+b)*(57*A*c^2*x^2-42*B*b*c*x^2-22*A*b*c*x+12*B*b^2*x+8*A*b^2)/b^
4/x^(5/2)/(x*(c*x+b))^(1/2)-1/16/b^4*c^2*(-2*(-16*A*c+16*B*b)/(c*x+b)^(1/2
)-2*(35*A*c-30*B*b)/b^(1/2)*arctanh((c*x+b)^(1/2)/b^(1/2)))*(c*x+b)^(1/2)*
x^(1/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.06

$$\int \frac{A+Bx}{x^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx = \left[-\frac{15\left((6\,Bbc^3-7\,Ac^4)x^5+(6\,Bb^2c^2-7\,Abc^3)x^4\right)\sqrt{b}\log\left(-\frac{cx^2+2\,bx+2\,\sqrt{cx^2+bx}\sqrt{b}}{x^2}\right.}{}\right.$$

input
```
integrate((B*x+A)/x^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/48*(15*((6*B*b*c^3 - 7*A*c^4)*x^5 + (6*B*b^2*c^2 - 7*A*b*c^3)*x^4)*sqr
t(b)*log(-(c*x^2 + 2*b*x + 2*sqrt(c*x^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(
8*A*b^4 - 15*(6*B*b^2*c^2 - 7*A*b*c^3)*x^3 - 5*(6*B*b^3*c - 7*A*b^2*c^2)*x
^2 + 2*(6*B*b^4 - 7*A*b^3*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^5*c*x^5 + b^
6*x^4), 1/24*(15*((6*B*b*c^3 - 7*A*c^4)*x^5 + (6*B*b^2*c^2 - 7*A*b*c^3)*x^
4)*sqrt(-b)*arctan(sqrt(c*x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) - (8*A*b^4 - 15
*(6*B*b^2*c^2 - 7*A*b*c^3)*x^3 - 5*(6*B*b^3*c - 7*A*b^2*c^2)*x^2 + 2*(6*B*
b^4 - 7*A*b^3*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^5*c*x^5 + b^6*x^4)]
```

Sympy [F]

$$\int \frac{A+Bx}{x^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx = \int \frac{A+Bx}{x^{\frac{5}{2}}\left(x\left(b+cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**(5/2)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((A + B*x)/(x**(5/2)*(x*(b + c*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{x^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx=\int \frac{Bx+A}{\left(cx^2+bx\right)^{\frac{3}{2}}x^{\frac{5}{2}}}\,dx$$

input
```
integrate((B*x+A)/x^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((c*x^2 + b*x)^(3/2)*x^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{x^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx=\frac{5\left(6\,Bbc^2-7\,Ac^3\right)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{8\,\sqrt{-b}b^4}+\frac{2\left(Bbc^2-Ac^3\right)}{\sqrt{cx+b}b^4}$$
$$+\frac{42\,(cx+b)^{\frac{5}{2}}Bbc^2-96\,(cx+b)^{\frac{3}{2}}Bb^2c^2+54\,\sqrt{cx+b}Bb^3c^2-57\,(cx+b)^{\frac{5}{2}}Ac^3+136\,(cx+b)^{\frac{3}{2}}Abc^3-87\,\sqrt{}}{24\,b^4c^3x^3}$$

input
```
integrate((B*x+A)/x^(5/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
5/8*(6*B*b*c^2 - 7*A*c^3)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^4) +
2*(B*b*c^2 - A*c^3)/(sqrt(c*x + b)*b^4) + 1/24*(42*(c*x + b)^(5/2)*B*b*c^2
 - 96*(c*x + b)^(3/2)*B*b^2*c^2 + 54*sqrt(c*x + b)*B*b^3*c^2 - 57*(c*x + b
)^(5/2)*A*c^3 + 136*(c*x + b)^(3/2)*A*b*c^3 - 87*sqrt(c*x + b)*A*b^2*c^3)/
(b^4*c^3*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx=\int \frac{A+B\,x}{x^{5/2}\,\left(c\,x^2+b\,x\right)^{3/2}}\,dx$$

input
```
int((A + B*x)/(x^(5/2)*(b*x + c*x^2)^(3/2)),x)
```

output
```
int((A + B*x)/(x^(5/2)*(b*x + c*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.11

$$\int \frac{A+Bx}{x^{5/2}\left(bx+cx^2\right)^{3/2}}\,dx=\frac{-105\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)a\,c^3x^3+90\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right.}{}$$

input
```
int((B*x+A)/x^(5/2)/(c*x^2+b*x)^(3/2),x)
```

output
```
( - 105*sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*a*c**3*x**3 + 9
0*sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*b**2*c**2*x**3 + 105*
sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) + sqrt(b))*a*c**3*x**3 - 90*sqrt(b
)*sqrt(b + c*x)*log(sqrt(b + c*x) + sqrt(b))*b**2*c**2*x**3 - 16*a*b**4 +
28*a*b**3*c*x - 70*a*b**2*c**2*x**2 - 210*a*b*c**3*x**3 - 24*b**5*x + 60*b
**4*c*x**2 + 180*b**3*c**2*x**3)/(48*sqrt(b + c*x)*b**5*x**3)
```

3.220 $\int \frac{A+Bx}{x^{7/2}(bx+cx^2)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 213

$$\int \frac{A+Bx}{x^{7/2}(bx+cx^2)^{3/2}}\,dx = -\frac{2c^3(bB-Ac)\sqrt{x}}{b^5\sqrt{bx+cx^2}}-\frac{A\sqrt{bx+cx^2}}{4b^2x^{9/2}} -\frac{(8bB-15Ac)\sqrt{bx+cx^2}}{24b^3x^{7/2}}+\frac{c(88bB-123Ac)\sqrt{bx+cx^2}}{96b^4x^{5/2}} -\frac{c^2(152bB-187Ac)\sqrt{bx+cx^2}}{64b^5x^{3/2}}+\frac{35c^3(8bB-9Ac)\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{64b^{11/2}}$$

output
```
-2*c^3*(-A*c+B*b)*x^(1/2)/b^5/(c*x^2+b*x)^(1/2)-1/4*A*(c*x^2+b*x)^(1/2)/b^
2/x^(9/2)-1/24*(-15*A*c+8*B*b)*(c*x^2+b*x)^(1/2)/b^3/x^(7/2)+1/96*c*(-123*
A*c+88*B*b)*(c*x^2+b*x)^(1/2)/b^4/x^(5/2)-1/64*c^2*(-187*A*c+152*B*b)*(c*x
^2+b*x)^(1/2)/b^5/x^(3/2)+35/64*c^3*(-9*A*c+8*B*b)*arctanh((c*x^2+b*x)^(1/
2)/b^(1/2)/x^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx}{x^{7/2}\left(bx+cx^2\right)^{3/2}}\,dx = \frac{\sqrt{b}(-8bBx(8b^3-14b^2cx+35bc^2x^2+105c^3x^3)+A(-48b^4+72b^3cx-126b^2c^2x}{192b^{11/2}x^{7/2}\sqrt{}}$$

input
```
Integrate[(A + B*x)/(x^(7/2)*(b*x + c*x^2)^(3/2)),x]
```

output
```
(Sqrt[b]*(-8*b*B*x*(8*b^3 - 14*b^2*c*x + 35*b*c^2*x^2 + 105*c^3*x^3) + A*(
-48*b^4 + 72*b^3*c*x - 126*b^2*c^2*x^2 + 315*b*c^3*x^3 + 945*c^4*x^4)) + 1
05*c^3*(8*b*B - 9*A*c)*x^4*Sqrt[b + c*x]*ArcTanh[Sqrt[b + c*x]/Sqrt[b]])/(
192*b^(11/2)*x^(7/2)*Sqrt[x*(b + c*x)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1220, 1135, 1135, 1135, 1132, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{7/2}\left(bx+cx^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(8bB-9Ac)\int \frac{1}{x^{5/2}(cx^2+bx)^{3/2}}dx}{8b}-\frac{A}{4bx^{7/2}\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{ 1135}$$

$$\frac{(8bB-9Ac)\left(-\frac{7c\int \frac{1}{x^{3/2}\left(cx^2+bx\right)^{3/2}}dx}{6b}-\frac{1}{3bx^{5/2}\sqrt{bx+cx^2}}\right)}{8b}-\frac{A}{4bx^{7/2}\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{ 1135}$$

$$\frac{(8bB-9Ac)\left(-\frac{7c\left(-\frac{5c\int \frac{1}{\sqrt{x}\left(cx^2+bx\right)^{3/2}}dx}{4b}-\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}\right)}{6b}-\frac{1}{3bx^{5/2}\sqrt{bx+cx^2}}\right)}{8b}-\frac{A}{4bx^{7/2}\sqrt{bx+cx^2}}$$

$\downarrow$ 1135

$$\frac{(8bB-9Ac)\left(-\frac{7c\left(-\frac{5c\left(-\frac{3c\int \frac{\sqrt{x}}{\left(cx^2+bx\right)^{3/2}}dx}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{4b}-\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}\right)}{6b}-\frac{1}{3bx^{5/2}\sqrt{bx+cx^2}}\right)}{8b}-\frac{A}{4bx^{7/2}\sqrt{bx+cx^2}}$$

$\downarrow$ 1132

$$\frac{(8bB-9Ac)\left(-\frac{7c\left(-\frac{5c\left(-\frac{3c\left(\frac{\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{4b}-\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}\right)}{6b}-\frac{1}{3bx^{5/2}\sqrt{bx+cx^2}}\right)}{8b}-\frac{A}{4bx^{7/2}\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{(8bB-9Ac)\left(-\frac{7c\left(-\frac{5c\left(-\frac{3c\left(\frac{2\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{4b}-\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}\right)}{6b}-\frac{1}{3bx^{5/2}\sqrt{bx+cx^2}}\right)}{8b}-\frac{A}{4bx^{7/2}\sqrt{bx+cx^2}}$$

$$\Big\downarrow \text{220}$$

$$\frac{(8bB-9Ac)\left(-\frac{7c\left(-\frac{5c\left(-\frac{3c\left(\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{4b}-\frac{1}{2bx^{3/2}\sqrt{bx+cx^2}}\right)}{6b}-\frac{1}{3bx^{5/2}\sqrt{bx+cx^2}}\right)}{8b}-\frac{A}{4bx^{7/2}\sqrt{bx+cx^2}}$$

input
```
Int[(A + B*x)/(x^(7/2)*(b*x + c*x^2)^(3/2)),x]
```

output
```
-1/4*A/(b*x^(7/2)*Sqrt[b*x + c*x^2]) + ((8*b*B - 9*A*c)*(-1/3*1/(b*x^(5/2)
*Sqrt[b*x + c*x^2]) - (7*c*(-1/2*1/(b*x^(3/2)*Sqrt[b*x + c*x^2]) - (5*c*(-
(1/(b*Sqrt[x]*Sqrt[b*x + c*x^2])) - (3*c*((2*Sqrt[x])/(b*Sqrt[b*x + c*x^2]
) - (2*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/b^(3/2)))/(2*b)))/(4*
b)))/(6*b)))/(8*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(cx+b)\left(-561A\,c^3x^3+456x^3Bb\,c^2+246Ab\,c^2x^2-176x^2B\,b^2c-120A\,b^2cx+64xB\,b^3+48A\,b^3\right)}{192b^5x^{\frac{7}{2}}\sqrt{x(cx+b)}}+\frac{c^3\left(-\frac{2(-128Ac+128Bb)}{\sqrt{cx+b}}-\frac{2(315\ldots}{}\right.}{128}$
default	$-\frac{\sqrt{x(cx+b)}\left(945A\sqrt{cx+b}\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^4x^4-840B\sqrt{cx+b}\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)b\,c^3x^4+64B\,b^{\frac{9}{2}}x-112B\,b^{\frac{7}{2}}c\,x^2+280B\,b^{\frac{5}{2}}c^2x^3\ldots\right.}{192x^{\frac{9}{2}}(cx+b)b^{\frac{11}{2}}}$

input
```
int((B*x+A)/x^(7/2)/(c*x^2+b*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(c*x+b)*(-561*A*c^3*x^3+456*B*b*c^2*x^3+246*A*b*c^2*x^2-176*B*b^2*c
*x^2-120*A*b^2*c*x+64*B*b^3*x+48*A*b^3)/b^5/x^(7/2)/(x*(c*x+b))^(1/2)+1/12
8*c^3/b^5*(-2*(-128*A*c+128*B*b)/(c*x+b)^(1/2)-2*(315*A*c-280*B*b)/b^(1/2)
*arctanh((c*x+b)^(1/2)/b^(1/2)))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.92

$$\int \frac{A+Bx}{x^{7/2}\left(bx+cx^2\right)^{3/2}}\,dx = \left[-\frac{105\left((8\,Bbc^4-9\,Ac^5)x^6+(8\,Bb^2c^3-9\,Abc^4)x^5\right)\sqrt{b}\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\ldots}{x^2}\right.}{}\right.$$

input
```
integrate((B*x+A)/x^(7/2)/(c*x^2+b*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/384*(105*((8*B*b*c^4 - 9*A*c^5)*x^6 + (8*B*b^2*c^3 - 9*A*b*c^4)*x^5)*s
qrt(b)*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2
*(48*A*b^5 + 105*(8*B*b^2*c^3 - 9*A*b*c^4)*x^4 + 35*(8*B*b^3*c^2 - 9*A*b^2
*c^3)*x^3 - 14*(8*B*b^4*c - 9*A*b^3*c^2)*x^2 + 8*(8*B*b^5 - 9*A*b^4*c)*x)*
sqrt(c*x^2 + b*x)*sqrt(x))/(b^6*c*x^6 + b^7*x^5), -1/192*(105*((8*B*b*c^4
- 9*A*c^5)*x^6 + (8*B*b^2*c^3 - 9*A*b*c^4)*x^5)*sqrt(-b)*arctan(sqrt(c*x^2
 + b*x)*sqrt(-b)/(b*sqrt(x))) + (48*A*b^5 + 105*(8*B*b^2*c^3 - 9*A*b*c^4)*
x^4 + 35*(8*B*b^3*c^2 - 9*A*b^2*c^3)*x^3 - 14*(8*B*b^4*c - 9*A*b^3*c^2)*x^
2 + 8*(8*B*b^5 - 9*A*b^4*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^6*c*x^6 + b^7
*x^5)]
```

Sympy [F]

$$\int \frac{A+Bx}{x^{7/2}\left(bx+cx^2\right)^{3/2}}\,dx=\int \frac{A+Bx}{x^{\frac{7}{2}}\left(x\left(b+cx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x**(7/2)/(c*x**2+b*x)**(3/2),x)
```

output
```
Integral((A + B*x)/(x**(7/2)*(x*(b + c*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{A+Bx}{x^{7/2}\left(bx+cx^2\right)^{3/2}}\,dx=\int \frac{Bx+A}{\left(cx^2+bx\right)^{\frac{3}{2}}x^{\frac{7}{2}}}\,dx$$

input
```
integrate((B*x+A)/x^(7/2)/(c*x^2+b*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((c*x^2 + b*x)^(3/2)*x^(7/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.92

$$\int \frac{A+Bx}{x^{7/2}\left(bx+cx^2\right)^{3/2}}\,dx = -\frac{35\,(8\,Bbc^3-9\,Ac^4)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{64\,\sqrt{-b}b^5} - \frac{2\,(Bbc^3-Ac^4)}{\sqrt{cx+b}b^5}$$
$$-\frac{456\,(cx+b)^{\frac{7}{2}}Bbc^3-1544\,(cx+b)^{\frac{5}{2}}Bb^2c^3+1784\,(cx+b)^{\frac{3}{2}}Bb^3c^3-696\,\sqrt{cx+b}Bb^4c^3-561\,(cx+b)^{\frac{7}{2}}A}{192\,b^5c^4x^4}$$

input
```
integrate((B*x+A)/x^(7/2)/(c*x^2+b*x)^(3/2),x, algorithm="giac")
```

output
```
-35/64*(8*B*b*c^3 - 9*A*c^4)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^5)
 - 2*(B*b*c^3 - A*c^4)/(sqrt(c*x + b)*b^5) - 1/192*(456*(c*x + b)^(7/2)*B*
b*c^3 - 1544*(c*x + b)^(5/2)*B*b^2*c^3 + 1784*(c*x + b)^(3/2)*B*b^3*c^3 -
696*sqrt(c*x + b)*B*b^4*c^3 - 561*(c*x + b)^(7/2)*A*c^4 + 1929*(c*x + b)^(
5/2)*A*b*c^4 - 2295*(c*x + b)^(3/2)*A*b^2*c^4 + 975*sqrt(c*x + b)*A*b^3*c^
4)/(b^5*c^4*x^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{7/2}\left(bx+cx^2\right)^{3/2}}\,dx = \int \frac{A+B\,x}{x^{7/2}\,\left(c\,x^2+b\,x\right)^{3/2}}\,dx$$

input
```
int((A + B*x)/(x^(7/2)*(b*x + c*x^2)^(3/2)),x)
```

output
```
int((A + B*x)/(x^(7/2)*(b*x + c*x^2)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.02

$$\int \frac{A+Bx}{x^{7/2}\left(bx+cx^2\right)^{3/2}}\,dx = \frac{945\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)a\,c^4x^4-840\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right.}{}$$

input
```
int((B*x+A)/x^(7/2)/(c*x^2+b*x)^(3/2),x)
```

output
```
(945*sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*a*c**4*x**4 - 840*
sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*b**2*c**3*x**4 - 945*sq
rt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) + sqrt(b))*a*c**4*x**4 + 840*sqrt(b)
*sqrt(b + c*x)*log(sqrt(b + c*x) + sqrt(b))*b**2*c**3*x**4 - 96*a*b**5 + 1
44*a*b**4*c*x - 252*a*b**3*c**2*x**2 + 630*a*b**2*c**3*x**3 + 1890*a*b*c**
4*x**4 - 128*b**6*x + 224*b**5*c*x**2 - 560*b**4*c**2*x**3 - 1680*b**3*c**
3*x**4)/(384*sqrt(b + c*x)*b**6*x**4)
```

3.221 $\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^{5/2}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^{5/2}} \, dx = -\frac{2b^3(bB-Ac)x^{3/2}}{3c^5(bx+cx^2)^{3/2}} + \frac{2b^2(4bB-3Ac)\sqrt{x}}{c^5\sqrt{bx+cx^2}} + \frac{6b(2bB-Ac)\sqrt{bx+cx^2}}{c^5\sqrt{x}} - \frac{2(4bB-Ac)(bx+cx^2)^{3/2}}{3c^5x^{3/2}} + \frac{2B(bx+cx^2)^{5/2}}{5c^5x^{5/2}}$$

output
```
-2/3*b^3*(-A*c+B*b)*x^(3/2)/c^5/(c*x^2+b*x)^(3/2)+2*b^2*(-3*A*c+4*B*b)*x^(
1/2)/c^5/(c*x^2+b*x)^(1/2)+6*b*(-A*c+2*B*b)*(c*x^2+b*x)^(1/2)/c^5/x^(1/2)-
2/3*(-A*c+4*B*b)*(c*x^2+b*x)^(3/2)/c^5/x^(3/2)+2/5*B*(c*x^2+b*x)^(5/2)/c^5
/x^(5/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.56

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^{5/2}} \, dx = \frac{2x^{3/2}(128b^4B + 24b^2c^2x(-5A+2Bx) + c^4x^3(5A+3Bx) - 2bc^3x^2(15A+4Bx) + }{15c^5(x(b+cx))^{3/2}}$$

input
```
Integrate[(x^(11/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(2*x^(3/2)*(128*b^4*B + 24*b^2*c^2*x*(-5*A + 2*B*x) + c^4*x^3*(5*A + 3*B*x
) - 2*b*c^3*x^2*(15*A + 4*B*x) + b^3*(-80*A*c + 192*B*c*x)))/(15*c^5*(x*(b
 + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1218, 1128, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1218}$$

$$-\frac{1}{3}\left(\frac{5A}{b}-\frac{8B}{c}\right)\int \frac{x^{9/2}}{(cx^2+bx)^{3/2}}dx-\frac{2x^{11/2}(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1128}$$

$$-\frac{1}{3}\left(\frac{5A}{b}-\frac{8B}{c}\right)\left(\frac{2x^{7/2}}{5c\sqrt{bx+cx^2}}-\frac{6b\int \frac{x^{7/2}}{(cx^2+bx)^{3/2}}dx}{5c}\right)-\frac{2x^{11/2}(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1128}$$

$$-\frac{1}{3}\left(\frac{5A}{b}-\frac{8B}{c}\right)\left(\frac{2x^{7/2}}{5c\sqrt{bx+cx^2}}-\frac{6b\left(\frac{2x^{5/2}}{3c\sqrt{bx+cx^2}}-\frac{4b\int \frac{x^{5/2}}{\left(cx^2+bx\right)^{3/2}}dx}{3c}\right)}{5c}\right)-\frac{2x^{11/2}(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1128}$$

$$-\frac{1}{3}\left(\frac{5A}{b}-\frac{8B}{c}\right)\left(\frac{2x^{7/2}}{5c\sqrt{bx+cx^2}}-\frac{6b\left(\frac{2x^{5/2}}{3c\sqrt{bx+cx^2}}-\frac{4b\left(\frac{2x^{3/2}}{c\sqrt{bx+cx^2}}-\frac{2b\int\frac{x^{3/2}}{(cx^2+bx)^{3/2}}dx}{c}\right)}{3c}\right)}{5c}\right)-\frac{2x^{11/2}(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \ 1122$$

$$-\frac{1}{3}\left(\frac{2x^{7/2}}{5c\sqrt{bx+cx^2}}-\frac{6b\left(\frac{2x^{5/2}}{3c\sqrt{bx+cx^2}}-\frac{4b\left(\frac{4b\sqrt{x}}{c^2\sqrt{bx+cx^2}}+\frac{2x^{3/2}}{c\sqrt{bx+cx^2}}\right)}{3c}\right)}{5c}\right)\left(\frac{5A}{b}-\frac{8B}{c}\right)-\frac{2x^{11/2}(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

input
```
Int[(x^(11/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(b*B - A*c)*x^(11/2))/(3*b*c*(b*x + c*x^2)^(3/2)) - (((5*A)/b - (8*B)/
c)*((2*x^(7/2))/(5*c*Sqrt[b*x + c*x^2]) - (6*b*((2*x^(5/2))/(3*c*Sqrt[b*x
+ c*x^2]) - (4*b*((4*b*Sqrt[x])/(c^2*Sqrt[b*x + c*x^2]) + (2*x^(3/2))/(c*S
qrt[b*x + c*x^2])))/(3*c)))/(5*c)))/3
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{2(cx+b)\left(-3B\,c^{4}x^{4}-5A\,c^{4}x^{3}+8B\,c^{3}x^{3}b+30Ab\,c^{3}x^{2}-48c^{2}x^{2}B\,b^{2}+120A\,b^{2}c^{2}x-192B\,b^{3}cx+80A\,b^{3}c-128B\,b^{4}\right)x^{\frac{5}{2}}}{15c^{5}\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}$	107
default	$-\frac{2\sqrt{x(cx+b)}\,\left(-3B\,c^{4}x^{4}-5A\,c^{4}x^{3}+8B\,c^{3}x^{3}b+30Ab\,c^{3}x^{2}-48c^{2}x^{2}B\,b^{2}+120A\,b^{2}c^{2}x-192B\,b^{3}cx+80A\,b^{3}c-128B\,b^{4}\right)}{15\sqrt{x}\,(cx+b)^{2}c^{5}}$	107
orering	$-\frac{2(cx+b)\left(-3B\,c^{4}x^{4}-5A\,c^{4}x^{3}+8B\,c^{3}x^{3}b+30Ab\,c^{3}x^{2}-48c^{2}x^{2}B\,b^{2}+120A\,b^{2}c^{2}x-192B\,b^{3}cx+80A\,b^{3}c-128B\,b^{4}\right)x^{\frac{5}{2}}}{15c^{5}\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}$	107
risch	$-\frac{2\left(-3B\,c^{2}x^{2}-5A\,c^{2}x+14Bbcx+40Abc-73B\,b^{2}\right)(cx+b)\sqrt{x}}{15c^{5}\sqrt{x(cx+b)}}-\frac{2b^{2}\left(9A\,c^{2}x-12Bbcx+8Abc-11B\,b^{2}\right)\sqrt{x}}{3c^{5}(cx+b)\sqrt{x(cx+b)}}$	110

input
```
int(x^(11/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(c*x+b)*(-3*B*c^4*x^4-5*A*c^4*x^3+8*B*b*c^3*x^3+30*A*b*c^3*x^2-48*B*
b^2*c^2*x^2+120*A*b^2*c^2*x-192*B*b^3*c*x+80*A*b^3*c-128*B*b^4)*x^(5/2)/c^
5/(c*x^2+b*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.77

$$\int \frac{x^{11/2}(A+Bx)}{\left(bx+cx^2\right)^{5/2}}\,dx = \frac{2\,(3\,Bc^4x^4 + 128\,Bb^4 - 80\,Ab^3c - (8\,Bbc^3 - 5\,Ac^4)x^3 + 6\,(8\,Bb^2c^2 - 5\,Abc^3)x^2 + 2}{15\,(c^7x^3 + 2\,bc^6x^2 + b^2c^5x)}$$

input
```
integrate(x^(11/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*B*c^4*x^4 + 128*B*b^4 - 80*A*b^3*c - (8*B*b*c^3 - 5*A*c^4)*x^3 + 6
*(8*B*b^2*c^2 - 5*A*b*c^3)*x^2 + 24*(8*B*b^3*c - 5*A*b^2*c^2)*x)*sqrt(c*x^
2 + b*x)*sqrt(x)/(c^7*x^3 + 2*b*c^6*x^2 + b^2*c^5*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx)}{\left(bx+cx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(11/2)*(B*x+A)/(c*x**2+b*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)x^{\frac{11}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(11/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
2/15*((3*B*c^3*x^2 + B*b*c^2*x - 2*B*b^2*c)*x^4 - (6*B*b^3 + (6*B*b*c^2 -
5*A*c^3)*x^2 + (12*B*b^2*c - 5*A*b*c^2)*x)*x^3)*sqrt(c*x + b)/(c^6*x^4 + 3
*b*c^5*x^3 + 3*b^2*c^4*x^2 + b^3*c^3*x) - integrate(-2/5*(4*B*b^4 + (9*B*b
^2*c^2 - 5*A*b*c^3)*x^2 + (13*B*b^3*c - 5*A*b^2*c^2)*x)*sqrt(c*x + b)*x^3/
(c^7*x^6 + 4*b*c^6*x^5 + 6*b^2*c^5*x^4 + 4*b^3*c^4*x^3 + b^4*c^3*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.76

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2\left(12\,(cx+b)Bb^3 - Bb^4 - 9\,(cx+b)Ab^2c + Ab^3c\right)}{3\,(cx+b)^{\frac{3}{2}}c^5} + \frac{2\left(3\,(cx+b)^{\frac{5}{2}}Bc^{20} - 20\,(cx+b)^{\frac{3}{2}}Bbc^{20} + 90\,\sqrt{cx+b}Bb^2c^{20} + 5\,(cx+b)^{\frac{3}{2}}Ac^{21} - 45\,\sqrt{cx+b}Abc^{21}\right)}{15\,c^{25}}$$

input
```
integrate(x^(11/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
2/3*(12*(c*x + b)*B*b^3 - B*b^4 - 9*(c*x + b)*A*b^2*c + A*b^3*c)/((c*x + b
)^(3/2)*c^5) + 2/15*(3*(c*x + b)^(5/2)*B*c^20 - 20*(c*x + b)^(3/2)*B*b*c^2
0 + 90*sqrt(c*x + b)*B*b^2*c^20 + 5*(c*x + b)^(3/2)*A*c^21 - 45*sqrt(c*x +
 b)*A*b*c^21)/c^25
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x^{11/2}\,(A+B\,x)}{(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((x^(11/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x)
```

output
```
int((x^(11/2)*(A + B*x))/(b*x + c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.61

$$\int \frac{x^{11/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{\frac{2}{5}b\,c^4x^4+\frac{2}{3}a\,c^4x^3-\frac{16}{15}b^2c^3x^3-4ab\,c^3x^2+\frac{32}{5}b^3c^2x^2-16a\,b^2c^2x+\frac{128}{5}b^4cx-\frac{32}{3}a\,b^3}{\sqrt{cx+b}\,c^5\,(cx+b)}$$

input
```
int(x^(11/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*( - 80*a*b**3*c - 120*a*b**2*c**2*x - 30*a*b*c**3*x**2 + 5*a*c**4*x**3
+ 128*b**5 + 192*b**4*c*x + 48*b**3*c**2*x**2 - 8*b**2*c**3*x**3 + 3*b*c**
4*x**4))/(15*sqrt(b + c*x)*c**5*(b + c*x))
```

3.222 $\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 128

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2b^2(bB-Ac)x^{3/2}}{3c^4(bx+cx^2)^{3/2}} - \frac{2b(3bB-2Ac)\sqrt{x}}{c^4\sqrt{bx+cx^2}} - \frac{2(3bB-Ac)\sqrt{bx+cx^2}}{c^4\sqrt{x}} + \frac{2B(bx+cx^2)^{3/2}}{3c^4x^{3/2}}$$

output
```
2/3*b^2*(-A*c+B*b)*x^(3/2)/c^4/(c*x^2+b*x)^(3/2)-2*b*(-2*A*c+3*B*b)*x^(1/2
)/c^4/(c*x^2+b*x)^(1/2)-2*(-A*c+3*B*b)*(c*x^2+b*x)^(1/2)/c^4/x^(1/2)+2/3*B
*(c*x^2+b*x)^(3/2)/c^4/x^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.55

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2x^{3/2}(-16b^3B+8b^2c(A-3Bx)-6bc^2x(-2A+Bx)+c^3x^2(3A+Bx))}{3c^4(x(b+cx))^{3/2}}$$

input
```
Integrate[(x^(9/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(2*x^(3/2)*(-16*b^3*B + 8*b^2*c*(A - 3*B*x) - 6*b*c^2*x*(-2*A + B*x) + c^3
*x^2*(3*A + B*x)))/(3*c^4*(x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1218, 1128, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{(2bB-Ac)\int \frac{x^{7/2}}{(cx^2+bx)^{3/2}}dx}{bc}-\frac{2x^{9/2}(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(2bB-Ac)\left(\frac{2x^{5/2}}{3c\sqrt{bx+cx^2}}-\frac{4b\int \frac{x^{5/2}}{\left(cx^2+bx\right)^{3/2}}dx}{3c}\right)}{bc}-\frac{2x^{9/2}(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(2bB-Ac)\left(\frac{2x^{5/2}}{3c\sqrt{bx+cx^2}}-\frac{4b\left(\frac{2x^{3/2}}{c\sqrt{bx+cx^2}}-\frac{2b\int \frac{x^{3/2}}{\left(cx^2+bx\right)^{3/2}}dx}{c}\right)}{3c}\right)}{bc}-\frac{2x^{9/2}(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{\left(\frac{2x^{5/2}}{3c\sqrt{bx+cx^2}} - \frac{4b\left(\frac{4b\sqrt{x}}{c^2\sqrt{bx+cx^2}} + \frac{2x^{3/2}}{c\sqrt{bx+cx^2}}\right)}{3c}\right)(2bB - Ac)}{bc} - \frac{2x^{9/2}(bB - Ac)}{3bc\,(bx + cx^2)^{3/2}}$$

input
```
Int[(x^(9/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(b*B - A*c)*x^(9/2))/(3*b*c*(b*x + c*x^2)^(3/2)) + ((2*b*B - A*c)*((2*
x^(5/2))/(3*c*Sqrt[b*x + c*x^2]) - (4*b*((4*b*Sqrt[x])/(c^2*Sqrt[b*x + c*x
^2]) + (2*x^(3/2))/(c*Sqrt[b*x + c*x^2])))/(3*c)))/(b*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.64

method	result	size
gosper	$\frac{2(cx+b)(B\,c^3x^3+3A\,c^3x^2-6Bb\,c^2x^2+12Ab\,c^2x-24B\,b^2cx+8A\,b^2c-16B\,b^3)x^{\frac{5}{2}}}{3c^4(c\,x^2+bx)^{\frac{5}{2}}}$	82
default	$\frac{2\sqrt{x(cx+b)}\,(B\,c^3x^3+3A\,c^3x^2-6Bb\,c^2x^2+12Ab\,c^2x-24B\,b^2cx+8A\,b^2c-16B\,b^3)}{3\sqrt{x}\,(cx+b)^2c^4}$	82
orering	$\frac{2(cx+b)(B\,c^3x^3+3A\,c^3x^2-6Bb\,c^2x^2+12Ab\,c^2x-24B\,b^2cx+8A\,b^2c-16B\,b^3)x^{\frac{5}{2}}}{3c^4(c\,x^2+bx)^{\frac{5}{2}}}$	82
risch	$\frac{2(Bcx+3Ac-8Bb)(cx+b)\sqrt{x}}{3c^4\sqrt{x(cx+b)}}+\frac{2b(6A\,c^2x-9Bbcx+5Abc-8B\,b^2)\sqrt{x}}{3c^4(cx+b)\sqrt{x(cx+b)}}$	87

input
```
int(x^(9/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(c*x+b)*(B*c^3*x^3+3*A*c^3*x^2-6*B*b*c^2*x^2+12*A*b*c^2*x-24*B*b^2*c*x
+8*A*b^2*c-16*B*b^3)*x^(5/2)/c^4/(c*x^2+b*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.80

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2\left(Bc^3x^3-16\,Bb^3+8\,Ab^2c-3\,(2\,Bbc^2-Ac^3)x^2-12\,(2\,Bb^2c-Abc^2)x\right)\sqrt{cx^2+bx}}{3\left(c^6x^3+2\,bc^5x^2+b^2c^4x\right)}$$

input
```
integrate(x^(9/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(B*c^3*x^3 - 16*B*b^3 + 8*A*b^2*c - 3*(2*B*b*c^2 - A*c^3)*x^2 - 12*(2*
B*b^2*c - A*b*c^2)*x)*sqrt(c*x^2 + b*x)*sqrt(x)/(c^6*x^3 + 2*b*c^5*x^2 + b
^2*c^4*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(x**(9/2)*(B*x+A)/(c*x**2+b*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)x^{\frac{9}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(9/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(B*c*x + B*b)*sqrt(c*x + b)*x^3/(c^4*x^3 + 3*b*c^3*x^2 + 3*b^2*c^2*x +
 b^3*c) + integrate((A*b*c*x^3 - (2*B*b^2 + (2*B*b*c - A*c^2)*x)*x^3)*sqrt
(c*x + b)/(c^5*x^5 + 4*b*c^4*x^4 + 6*b^2*c^3*x^3 + 4*b^3*c^2*x^2 + b^4*c*x
), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.72

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = -\frac{2\left(9\,(cx+b)Bb^2 - Bb^3 - 6\,(cx+b)Abc + Ab^2c\right)}{3\,(cx+b)^{\frac{3}{2}}c^4} + \frac{2\left((cx+b)^{\frac{3}{2}}Bc^8 - 9\,\sqrt{cx+b}Bbc^8 + 3\,\sqrt{cx+b}Ac^9\right)}{3\,c^{12}}$$

input
```
integrate(x^(9/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(9*(c*x + b)*B*b^2 - B*b^3 - 6*(c*x + b)*A*b*c + A*b^2*c)/((c*x + b)^
(3/2)*c^4) + 2/3*((c*x + b)^(3/2)*B*c^8 - 9*sqrt(c*x + b)*B*b*c^8 + 3*sqrt
(c*x + b)*A*c^9)/c^12
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x^{9/2}\,(A+B\,x)}{(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((x^(9/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x)
```

output
```
int((x^(9/2)*(A + B*x))/(b*x + c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.59

$$\int \frac{x^{9/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{\frac{2}{3}b\,c^3x^3+2a\,c^3x^2-4b^2c^2x^2+8ab\,c^2x-16b^3cx+\frac{16}{3}a\,b^2c-\frac{32}{3}b^4}{\sqrt{cx+b}\,c^4\,(cx+b)}$$

input
```
int(x^(9/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*(8*a*b**2*c + 12*a*b*c**2*x + 3*a*c**3*x**2 - 16*b**4 - 24*b**3*c*x - 6
*b**2*c**2*x**2 + b*c**3*x**3))/(3*sqrt(b + c*x)*c**4*(b + c*x))
```

3.223 $\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = -\frac{2b(bB-Ac)x^{3/2}}{3c^3(bx+cx^2)^{3/2}}+\frac{2(2bB-Ac)\sqrt{x}}{c^3\sqrt{bx+cx^2}}+\frac{2B\sqrt{bx+cx^2}}{c^3\sqrt{x}}$$

output
```
-2/3*b*(-A*c+B*b)*x^(3/2)/c^3/(c*x^2+b*x)^(3/2)+2*(-A*c+2*B*b)*x^(1/2)/c^3
/(c*x^2+b*x)^(1/2)+2*B*(c*x^2+b*x)^(1/2)/c^3/x^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.58

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2x^{3/2}(8b^2B-2bc(A-6Bx)+3c^2x(-A+Bx))}{3c^3(x(b+cx))^{3/2}}$$

input
```
Integrate[(x^(7/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(2*x^(3/2)*(8*b^2*B - 2*b*c*(A - 6*B*x) + 3*c^2*x*(-A + B*x)))/(3*c^3*(x*(
b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.14, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1218, 1128, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{(4bB-Ac)\int \frac{x^{5/2}}{(cx^2+bx)^{3/2}}dx}{3bc}-\frac{2x^{7/2}(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1128}$$

$$\frac{(4bB-Ac)\left(\frac{2x^{3/2}}{c\sqrt{bx+cx^2}}-\frac{2b\int \frac{x^{3/2}}{\left(cx^2+bx\right)^{3/2}}dx}{c}\right)}{3bc}-\frac{2x^{7/2}(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1122}$$

$$\frac{\left(\frac{4b\sqrt{x}}{c^2\sqrt{bx+cx^2}}+\frac{2x^{3/2}}{c\sqrt{bx+cx^2}}\right)(4bB-Ac)}{3bc}-\frac{2x^{7/2}(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

input
```
Int[(x^(7/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(b*B - A*c)*x^(7/2))/(3*b*c*(b*x + c*x^2)^(3/2)) + ((4*b*B - A*c)*((4*
b*Sqrt[x])/(c^2*Sqrt[b*x + c*x^2]) + (2*x^(3/2))/(c*Sqrt[b*x + c*x^2])))/(
3*b*c)
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1128
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(m + 2*p +
 1))), x] + Simp[Simplify[m + p]*((2*c*d - b*e)/(c*(m + 2*p + 1)))   Int[(d
 + e*x)^(m - 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[Simplify[m + p], 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.65

method	result	size
gosper	$-\frac{2(cx+b)\left(-3B\,c^{2}x^{2}+3A\,c^{2}x-12Bbcx+2Abc-8B\,b^{2}\right)x^{\frac{5}{2}}}{3c^{3}\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}$	59
default	$-\frac{2\sqrt{x(cx+b)}\,\left(-3B\,c^{2}x^{2}+3A\,c^{2}x-12Bbcx+2Abc-8B\,b^{2}\right)}{3\sqrt{x}\,(cx+b)^{2}c^{3}}$	59
orering	$-\frac{2(cx+b)\left(-3B\,c^{2}x^{2}+3A\,c^{2}x-12Bbcx+2Abc-8B\,b^{2}\right)x^{\frac{5}{2}}}{3c^{3}\left(c\,x^{2}+bx\right)^{\frac{5}{2}}}$	59
risch	$\frac{2B(cx+b)\sqrt{x}}{c^{3}\sqrt{x(cx+b)}}-\frac{2\left(3A\,c^{2}x-6Bbcx+2Abc-5B\,b^{2}\right)\sqrt{x}}{3c^{3}(cx+b)\sqrt{x(cx+b)}}$	74

input
```
int(x^(7/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(c*x+b)*(-3*B*c^2*x^2+3*A*c^2*x-12*B*b*c*x+2*A*b*c-8*B*b^2)*x^(5/2)/c
^3/(c*x^2+b*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.87

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2\,(3\,Bc^2x^2+8\,Bb^2-2\,Abc+3\,(4\,Bbc-Ac^2)x)\sqrt{cx^2+bx}\sqrt{x}}{3\,(c^5x^3+2\,bc^4x^2+b^2c^3x)}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*B*c^2*x^2 + 8*B*b^2 - 2*A*b*c + 3*(4*B*b*c - A*c^2)*x)*sqrt(c*x^2 +
 b*x)*sqrt(x)/(c^5*x^3 + 2*b*c^4*x^2 + b^2*c^3*x)
```

Sympy [F]

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x^{\frac{7}{2}}(A+Bx)}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**(7/2)*(B*x+A)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(x**(7/2)*(A + B*x)/(x*(b + c*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)x^{\frac{7}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*x^(7/2)/(c*x^2 + b*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2\sqrt{cx+b}B}{c^3} + \frac{2\,(6\,(cx+b)Bb - Bb^2 - 3\,(cx+b)Ac + Abc)}{3\,(cx+b)^{\frac{3}{2}}c^3}$$

input
```
integrate(x^(7/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
2*sqrt(c*x + b)*B/c^3 + 2/3*(6*(c*x + b)*B*b - B*b^2 - 3*(c*x + b)*A*c + A
*b*c)/((c*x + b)^(3/2)*c^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x^{7/2}\,(A+B\,x)}{(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((x^(7/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x)
```

output
```
int((x^(7/2)*(A + B*x))/(b*x + c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.59

$$\int \frac{x^{7/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2b\,c^2x^2-2a\,c^2x+8b^2cx-\frac{4}{3}abc+\frac{16}{3}b^3}{\sqrt{cx+b}\,c^3\,(cx+b)}$$

input
```
int(x^(7/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*( - 2*a*b*c - 3*a*c**2*x + 8*b**3 + 12*b**2*c*x + 3*b*c**2*x**2))/(3*sq
rt(b + c*x)*c**3*(b + c*x))
```

3.224 $\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 58

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2(bB-Ac)x^{3/2}}{3c^2(bx+cx^2)^{3/2}} - \frac{2B\sqrt{x}}{c^2\sqrt{bx+cx^2}}$$

output
```
2/3*(-A*c+B*b)*x^(3/2)/c^2/(c*x^2+b*x)^(3/2)-2*B*x^(1/2)/c^2/(c*x^2+b*x)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = -\frac{2x^{3/2}(2bB+c(A+3Bx))}{3c^2(x(b+cx))^{3/2}}$$

input
```
Integrate[(x^(5/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*x^(3/2)*(2*b*B + c*(A + 3*B*x)))/(3*c^2*(x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1218, 1122}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{5/2}} \, dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{(Ac+2bB)\int \frac{x^{3/2}}{(cx^2+bx)^{3/2}}dx}{3bc} - \frac{2x^{5/2}(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1122}$$

$$-\frac{2\sqrt{x}(Ac+2bB)}{3bc^2\sqrt{bx+cx^2}} - \frac{2x^{5/2}(bB-Ac)}{3bc\,(bx+cx^2)^{3/2}}$$

input
```
Int[(x^(5/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(b*B - A*c)*x^(5/2))/(3*b*c*(b*x + c*x^2)^(3/2)) - (2*(2*b*B + A*c)*Sq
rt[x])/(3*b*c^2*Sqrt[b*x + c*x^2])
```

Defintions of rubi rules used

rule 1122
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m - 1)*((a + b*x + c*x^2)^(p + 1)/(c*(p + 1))),
 x] /; FreeQ[{a, b, c, d, e, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] &&
EqQ[m + p, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.66

method	result	size
gosper	$-\frac{2(cx+b)(3Bcx+Ac+2Bb)x^{\frac{5}{2}}}{3c^{2}(c\,x^{2}+bx)^{\frac{5}{2}}}$	38
default	$-\frac{2\sqrt{x(cx+b)}\,(3Bcx+Ac+2Bb)}{3\sqrt{x}\,(cx+b)^{2}c^{2}}$	38
orering	$-\frac{2(cx+b)(3Bcx+Ac+2Bb)x^{\frac{5}{2}}}{3c^{2}(c\,x^{2}+bx)^{\frac{5}{2}}}$	38

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(c*x+b)*(3*B*c*x+A*c+2*B*b)*x^(5/2)/c^2/(c*x^2+b*x)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = -\frac{2\,(3\,Bcx+2\,Bb+Ac)\sqrt{cx^2+bx}\sqrt{x}}{3\,(c^4x^3+2\,bc^3x^2+b^2c^2x)}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(3*B*c*x + 2*B*b + A*c)*sqrt(c*x^2 + b*x)*sqrt(x)/(c^4*x^3 + 2*b*c^3*
x^2 + b^2*c^2*x)
```

Sympy [F]

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x^{\frac{5}{2}}(A+Bx)}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**(5/2)*(B*x+A)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(x**(5/2)*(A + B*x)/(x*(b + c*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)x^{\frac{5}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*x^(5/2)/(c*x^2 + b*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.48

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = -\frac{2\,(3\,(cx+b)B-Bb+Ac)}{3\,(cx+b)^{\frac{3}{2}}c^2}$$

input
```
integrate(x^(5/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
-2/3*(3*(c*x + b)*B - B*b + A*c)/((c*x + b)^(3/2)*c^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x^{5/2}\,(A+B\,x)}{(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((x^(5/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x)
```

output
```
int((x^(5/2)*(A + B*x))/(b*x + c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.60

$$\int \frac{x^{5/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{-2bcx-\frac{2}{3}ac-\frac{4}{3}b^2}{\sqrt{cx+b}\,c^2\,(cx+b)}$$

input
```
int(x^(5/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x)
```

output
```
(2*( - a*c - 2*b**2 - 3*b*c*x))/(3*sqrt(b + c*x)*c**2*(b + c*x))
```

3.225 $\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = -\frac{2(bB-Ac)x^{3/2}}{3bc\,(bx+cx^2)^{3/2}}+\frac{2A\sqrt{x}}{b^2\sqrt{bx+cx^2}}-\frac{2A\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{5/2}}$$

output
```
-2/3*(-A*c+B*b)*x^(3/2)/b/c/(c*x^2+b*x)^(3/2)+2*A*x^(1/2)/b^2/(c*x^2+b*x)^
(1/2)-2*A*arctanh((c*x^2+b*x)^(1/2)/b^(1/2)/x^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.88

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2x^{3/2}\left(\sqrt{b}(-b^2B+4Abc+3Ac^2x)-3Ac(b+cx)^{3/2}\text{arctanh}\left(\frac{\sqrt{b+cx}}{\sqrt{b}}\right)\right)}{3b^{5/2}c(x(b+cx))^{3/2}}$$

input
```
Integrate[(x^(3/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(2*x^(3/2)*(Sqrt[b]*(-(b^2*B) + 4*A*b*c + 3*A*c^2*x) - 3*A*c*(b + c*x)^(3/
2)*ArcTanh[Sqrt[b + c*x]/Sqrt[b]]))/(3*b^(5/2)*c*(x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1218, 1132, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$\frac{A\int \frac{\sqrt{x}}{(cx^2+bx)^{3/2}}dx}{b}-\frac{2x^{3/2}(bB-Ac)}{3bc(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1132}$$

$$\frac{A\left(\frac{\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{b}-\frac{2x^{3/2}(bB-Ac)}{3bc(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1136}$$

$$\frac{A\left(\frac{2\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{b}-\frac{2x^{3/2}(bB-Ac)}{3bc(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{220}$$

$$\frac{A\left(\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}\right)}{b}-\frac{2x^{3/2}(bB-Ac)}{3bc(bx+cx^2)^{3/2}}$$

input

```
Int[(x^(3/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output

```
(-2*(b*B - A*c)*x^(3/2))/(3*b*c*(b*x + c*x^2)^(3/2)) + (A*((2*Sqrt[x])/(b*
Sqrt[b*x + c*x^2]) - (2*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/b^(3
/2)))/b
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07

method	result	size
default	$-\frac{2 \sqrt{x (c x +b)}\, \left(3 A \,\operatorname{arctanh}\left(\frac{\sqrt{c x +b}}{\sqrt{b}}\right) c^{2} x \sqrt{c x +b}+3 A c \,\operatorname{arctanh}\left(\frac{\sqrt{c x +b}}{\sqrt{b}}\right) b \sqrt{c x +b}-3 A \sqrt{b}\, c^{2} x -4 A \,b^{\frac{3}{2}} c +B \,b^{\frac{5}{2}}\right)}{3 b^{\frac{5}{2}} \sqrt{x}\, \left(c x +b \right)^{2} c}$	101

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(x*(c*x+b))^(1/2)/b^(5/2)*(3*A*arctanh((c*x+b)^(1/2)/b^(1/2))*c^2*x*(
c*x+b)^(1/2)+3*A*c*arctanh((c*x+b)^(1/2)/b^(1/2))*b*(c*x+b)^(1/2)-3*A*b^(1
/2)*c^2*x-4*A*b^(3/2)*c+B*b^(5/2))/x^(1/2)/(c*x+b)^2/c
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 265, normalized size of antiderivative = 2.82

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \left[\frac{3\left(Ac^3x^3+2\,Abc^2x^2+Ab^2cx\right)\sqrt{b}\log\left(-\frac{cx^2+2\,bx-2\,\sqrt{cx^2+bx}\sqrt{b}\sqrt{x}}{x^2}\right)+2\left(3\,Abc^2x-Bb\right.}{3\left(b^3c^3x^3+2\,b^4c^2x^2+b^5cx\right)}\right.$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(A*c^3*x^3 + 2*A*b*c^2*x^2 + A*b^2*c*x)*sqrt(b)*log(-(c*x^2 + 2*b*
x - 2*sqrt(c*x^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(3*A*b*c^2*x - B*b^3 + 4
*A*b^2*c)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^3*c^3*x^3 + 2*b^4*c^2*x^2 + b^5*c*
x), 2/3*(3*(A*c^3*x^3 + 2*A*b*c^2*x^2 + A*b^2*c*x)*sqrt(-b)*arctan(sqrt(c*
x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) + (3*A*b*c^2*x - B*b^3 + 4*A*b^2*c)*sqrt(
c*x^2 + b*x)*sqrt(x))/(b^3*c^3*x^3 + 2*b^4*c^2*x^2 + b^5*c*x)]
```

Sympy [F]

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x^{\frac{3}{2}}(A+Bx)}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**(3/2)*(B*x+A)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(x**(3/2)*(A + B*x)/(x*(b + c*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)x^{\frac{3}{2}}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*x^(3/2)/(c*x^2 + b*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2\,A\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^2} - \frac{2\,(Bb^2-3\,(cx+b)Ac-Abc)}{3\,(cx+b)^{\frac{3}{2}}b^2c}$$

input
```
integrate(x^(3/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
2*A*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^2) - 2/3*(B*b^2 - 3*(c*x +
b)*A*c - A*b*c)/((c*x + b)^(3/2)*b^2*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{x^{3/2}\,(A+B\,x)}{(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((x^(3/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x)
```

output
```
int((x^(3/2)*(A + B*x))/(b*x + c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.53

$$\int \frac{x^{3/2}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{3\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)abc+3\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)a\,c^2x-3}{3}$$

input
```
int(x^(3/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x)
```

output
```
(3*sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*a*b*c + 3*sqrt(b)*sq
rt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*a*c**2*x - 3*sqrt(b)*sqrt(b + c*x
)*log(sqrt(b + c*x) + sqrt(b))*a*b*c - 3*sqrt(b)*sqrt(b + c*x)*log(sqrt(b
+ c*x) + sqrt(b))*a*c**2*x + 8*a*b**2*c + 6*a*b*c**2*x - 2*b**4)/(3*sqrt(b
 + c*x)*b**3*c*(b + c*x))
```

3.226 $\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 130

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{2(bB-Ac)x^{3/2}}{3b^2(bx+cx^2)^{3/2}} + \frac{2(bB-2Ac)\sqrt{x}}{b^3\sqrt{bx+cx^2}} - \frac{A\sqrt{bx+cx^2}}{b^3x^{3/2}} - \frac{(2bB-5Ac)\operatorname{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{7/2}}$$

output
```
2/3*(-A*c+B*b)*x^(3/2)/b^2/(c*x^2+b*x)^(3/2)+2*(-2*A*c+B*b)*x^(1/2)/b^3/(c
*x^2+b*x)^(1/2)-A*(c*x^2+b*x)^(1/2)/b^3/x^(3/2)-(-5*A*c+2*B*b)*arctanh((c*
x^2+b*x)^(1/2)/b^(1/2)/x^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.81

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{\sqrt{x}\left(\sqrt{b}(2bBx(4b+3cx)-A(3b^2+20bcx+15c^2x^2))-3(2bB-5Ac)x(b+cx)^{3/2}\text{a}\right.}{3b^{7/2}(x(b+cx))^{3/2}}$$

input
```
Integrate[(Sqrt[x]*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(Sqrt[x]*(Sqrt[b]*(2*b*B*x*(4*b + 3*c*x) - A*(3*b^2 + 20*b*c*x + 15*c^2*x^
2)) - 3*(2*b*B - 5*A*c)*x*(b + c*x)^(3/2)*ArcTanh[Sqrt[b + c*x]/Sqrt[b]]))
/(3*b^(7/2)*(x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1218, 1135, 1132, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{5/2}}dx$$

$$\Big\downarrow \text{1218}$$

$$-\frac{(2bB-5Ac)\int \frac{1}{\sqrt{x}(cx^2+bx)^{3/2}}dx}{3bc}-\frac{2\sqrt{x}(bB-Ac)}{3bc(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1135}$$

$$-\frac{(2bB-5Ac)\left(-\frac{3c\int \frac{\sqrt{x}}{\left(cx^2+bx\right)^{3/2}}dx}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{3bc}-\frac{2\sqrt{x}(bB-Ac)}{3bc(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1132}$$

$$-\frac{(2bB-5Ac)\left(-\frac{3c\left(\frac{\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{3bc}-\frac{2\sqrt{x}(bB-Ac)}{3bc(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \text{1136}$$

$$-\frac{(2bB-5Ac)\left(-\frac{3c\left(\frac{2\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{3bc}-\frac{2\sqrt{x}(bB-Ac)}{3bc\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 220$$

$$-\frac{(2bB-5Ac)\left(-\frac{3c\left(\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{3bc}-\frac{2\sqrt{x}(bB-Ac)}{3bc\left(bx+cx^2\right)^{3/2}}$$

input
```
Int[(Sqrt[x]*(A + B*x))/(b*x + c*x^2)^(5/2),x]
```

output
```
(-2*(b*B - A*c)*Sqrt[x])/(3*b*c*(b*x + c*x^2)^(3/2)) - ((2*b*B - 5*A*c)*(-
(1/(b*Sqrt[x]*Sqrt[b*x + c*x^2])) - (3*c*((2*Sqrt[x])/(b*Sqrt[b*x + c*x^2]
) - (2*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/b^(3/2)))/(2*b)))/(3*
b*c)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1218
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(g*(c*d - b*e) + c*e*f)*(d + e*x)^m*((
a + b*x + c*x^2)^(p + 1)/(c*(p + 1)*(2*c*d - b*e))), x] - Simp[e*((m*(g*(c*
d - b*e) + c*e*f) + e*(p + 1)*(2*c*f - b*g))/(c*(p + 1)*(2*c*d - b*e)))   I
nt[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d
, e, f, g}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 1.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{A(cx+b)}{b^{3}\sqrt{x}\,\sqrt{x(cx+b)}}-\frac{\left(-\frac{2(-4Ac+2Bb)}{\sqrt{cx+b}}+\frac{4b(Ac-Bb)}{3(cx+b)^{\frac{3}{2}}}-\frac{2(5Ac-2Bb)\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)}{\sqrt{b}}\right)\sqrt{cx+b}\,\sqrt{x}}{2b^{3}\sqrt{x(cx+b)}}$
default	$\frac{\sqrt{x(cx+b)}\,\left(15A\sqrt{cx+b}\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^{2}x^{2}-6B\sqrt{cx+b}\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)bc\,x^{2}+15A\,\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)bcx\sqrt{cx+b}-15A\sqrt{b}\,c^{2}x^{2}\right.}{3x^{\frac{3}{2}}(cx+b)^{2}b^{\frac{7}{2}}}$

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/b^3*A*(c*x+b)/x^(1/2)/(x*(c*x+b))^(1/2)-1/2/b^3*(-2*(-4*A*c+2*B*b)/(c*x
+b)^(1/2)+4/3*b*(A*c-B*b)/(c*x+b)^(3/2)-2*(5*A*c-2*B*b)/b^(1/2)*arctanh((c
*x+b)^(1/2)/b^(1/2)))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.85

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \left[-\frac{3\left((2\,Bbc^2-5\,Ac^3)x^4+2\,(2\,Bb^2c-5\,Abc^2)x^3+(2\,Bb^3-5\,Ab^2c)x^2\right)\sqrt{b}\log\left(-\frac{cx}{}\right.}{6\,(b^4c^2}\right.$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*((2*B*b*c^2 - 5*A*c^3)*x^4 + 2*(2*B*b^2*c - 5*A*b*c^2)*x^3 + (2*B
*b^3 - 5*A*b^2*c)*x^2)*sqrt(b)*log(-(c*x^2 + 2*b*x + 2*sqrt(c*x^2 + b*x)*s
qrt(b)*sqrt(x))/x^2) + 2*(3*A*b^3 - 3*(2*B*b^2*c - 5*A*b*c^2)*x^2 - 4*(2*B
*b^3 - 5*A*b^2*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^4*c^2*x^4 + 2*b^5*c*x^3
 + b^6*x^2), 1/3*(3*((2*B*b*c^2 - 5*A*c^3)*x^4 + 2*(2*B*b^2*c - 5*A*b*c^2)
*x^3 + (2*B*b^3 - 5*A*b^2*c)*x^2)*sqrt(-b)*arctan(sqrt(c*x^2 + b*x)*sqrt(-
b)/(b*sqrt(x))) - (3*A*b^3 - 3*(2*B*b^2*c - 5*A*b*c^2)*x^2 - 4*(2*B*b^3 -
5*A*b^2*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^4*c^2*x^4 + 2*b^5*c*x^3 + b^6*
x^2)]
```

Sympy [F]

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{\sqrt{x}\,(A+Bx)}{(x\,(b+cx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**(1/2)*(B*x+A)/(c*x**2+b*x)**(5/2),x)
```

output
```
Integral(sqrt(x)*(A + B*x)/(x*(b + c*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{(Bx+A)\sqrt{x}}{(cx^2+bx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)*sqrt(x)/(c*x^2 + b*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.69

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{(2\,Bb-5\,Ac)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{\sqrt{-b}b^3} - \frac{\sqrt{cx+b}A}{b^3x} + \frac{2\,(3\,(cx+b)Bb+Bb^2-6\,(cx+b)Ac-Abc)}{3\,(cx+b)^{\frac{3}{2}}b^3}$$

input
```
integrate(x^(1/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
(2*B*b - 5*A*c)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^3) - sqrt(c*x +
 b)*A/(b^3*x) + 2/3*(3*(c*x + b)*B*b + B*b^2 - 6*(c*x + b)*A*c - A*b*c)/((
c*x + b)^(3/2)*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \int \frac{\sqrt{x}\,(A+B\,x)}{(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((x^(1/2)*(A + B*x))/(b*x + c*x^2)^(5/2),x)
```

output
```
int((x^(1/2)*(A + B*x))/(b*x + c*x^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.12

$$\int \frac{\sqrt{x}(A+Bx)}{(bx+cx^2)^{5/2}}\,dx = \frac{-15\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)abcx-15\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)a\,c^2x}{}$$

input
```
int(x^(1/2)*(B*x+A)/(c*x^2+b*x)^(5/2),x)
```

output
```
( - 15*sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*a*b*c*x - 15*sqr
t(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*a*c**2*x**2 + 6*sqrt(b)*sq
rt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*b**3*x + 6*sqrt(b)*sqrt(b + c*x)*
log(sqrt(b + c*x) - sqrt(b))*b**2*c*x**2 + 15*sqrt(b)*sqrt(b + c*x)*log(sq
rt(b + c*x) + sqrt(b))*a*b*c*x + 15*sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x
) + sqrt(b))*a*c**2*x**2 - 6*sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) + sqr
t(b))*b**3*x - 6*sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) + sqrt(b))*b**2*c
*x**2 - 6*a*b**3 - 40*a*b**2*c*x - 30*a*b*c**2*x**2 + 16*b**4*x + 12*b**3*
c*x**2)/(6*sqrt(b + c*x)*b**4*x*(b + c*x))
```

3.227 $\int \frac{A+Bx}{\sqrt{x}\left(bx+cx^2\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 172

$$\int \frac{A+Bx}{\sqrt{x}\left(bx+cx^2\right)^{5/2}}\,dx=-\frac{2c(bB-Ac)x^{3/2}}{3b^3\left(bx+cx^2\right)^{3/2}}-\frac{2c(2bB-3Ac)\sqrt{x}}{b^4\sqrt{bx+cx^2}}-\frac{A\sqrt{bx+cx^2}}{2b^3x^{5/2}}-\frac{(4bB-11Ac)\sqrt{bx+cx^2}}{4b^4x^{3/2}}+\frac{5c(4bB-7Ac)\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{4b^{9/2}}$$

output
```
-2/3*c*(-A*c+B*b)*x^(3/2)/b^3/(c*x^2+b*x)^(3/2)-2*c*(-3*A*c+2*B*b)*x^(1/2)
/b^4/(c*x^2+b*x)^(1/2)-1/2*A*(c*x^2+b*x)^(1/2)/b^3/x^(5/2)-1/4*(-11*A*c+4*
B*b)*(c*x^2+b*x)^(1/2)/b^4/x^(3/2)+5/4*c*(-7*A*c+4*B*b)*arctanh((c*x^2+b*x
)^(1/2)/b^(1/2)/x^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.75

$$\int \frac{A+Bx}{\sqrt{x}\left(bx+cx^2\right)^{5/2}}\,dx=\frac{\sqrt{b}(-4bBx(3b^2+20bcx+15c^2x^2)+A(-6b^3+21b^2cx+140bc^2x^2+105c^3x^3)) -}{12b^{9/2}\sqrt{x}(x(b+cx))^{3/2}}$$

input
```
Integrate[(A + B*x)/(Sqrt[x]*(b*x + c*x^2)^(5/2)),x]
```

output
```
(Sqrt[b]*(-4*b*B*x*(3*b^2 + 20*b*c*x + 15*c^2*x^2) + A*(-6*b^3 + 21*b^2*c*
x + 140*b*c^2*x^2 + 105*c^3*x^3)) + 15*c*(4*b*B - 7*A*c)*x^2*(b + c*x)^(3/
2)*ArcTanh[Sqrt[b + c*x]/Sqrt[b]])/(12*b^(9/2)*Sqrt[x]*(x*(b + c*x))^(3/2)
)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1220, 1132, 1135, 1132, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{\sqrt{x}\left(bx+cx^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 1220}$$

$$\frac{(4bB-7Ac)\int \frac{\sqrt{x}}{\left(cx^2+bx\right)^{5/2}}dx}{4b}-\frac{A}{2b\sqrt{x}\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1132}$$

$$\frac{(4bB-7Ac)\left(\frac{5\int \frac{1}{\sqrt{x}\left(cx^2+bx\right)^{3/2}}dx}{3b}+\frac{2\sqrt{x}}{3b\left(bx+cx^2\right)^{3/2}}\right)}{4b}-\frac{A}{2b\sqrt{x}\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1135}$$

$$\frac{(4bB-7Ac)\left(\frac{5\left(-\frac{3c\int \frac{\sqrt{x}}{\left(cx^2+bx\right)^{3/2}}dx}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{3b}+\frac{2\sqrt{x}}{3b\left(bx+cx^2\right)^{3/2}}\right)}{4b}-\frac{A}{2b\sqrt{x}\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{ 1132}$$

$$\frac{(4bB-7Ac)\left(\frac{5\left(-\frac{3c\left(\frac{\int \frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{3b}+\frac{2\sqrt{x}}{3b(bx+cx^2)^{3/2}}\right)}{4b}-\frac{A}{2b\sqrt{x}\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \; 1136$$

$$\frac{(4bB-7Ac)\left(\frac{5\left(-\frac{3c\left(\frac{2\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{3b}+\frac{2\sqrt{x}}{3b(bx+cx^2)^{3/2}}\right)}{4b}-\frac{A}{2b\sqrt{x}\,(bx+cx^2)^{3/2}}$$

$$\Big\downarrow \; 220$$

$$\frac{(4bB-7Ac)\left(\frac{5\left(-\frac{3c\left(\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{3b}+\frac{2\sqrt{x}}{3b(bx+cx^2)^{3/2}}\right)}{4b}-\frac{A}{2b\sqrt{x}\,(bx+cx^2)^{3/2}}$$

input
```
Int[(A + B*x)/(Sqrt[x]*(b*x + c*x^2)^(5/2)),x]
```

output
```
-1/2*A/(b*Sqrt[x]*(b*x + c*x^2)^(3/2)) + ((4*b*B - 7*A*c)*((2*Sqrt[x])/(3*
b*(b*x + c*x^2)^(3/2)) + (5*(-(1/(b*Sqrt[x]*Sqrt[b*x + c*x^2])) - (3*c*((2
*Sqrt[x])/(b*Sqrt[b*x + c*x^2]) - (2*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sq
rt[x])])/b^(3/2)))/(2*b)))/(3*b)))/(4*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1132
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135
```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136
```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{(cx+b)(-11Acx+4Bbx+2Ab)}{4b^4x^{\frac{3}{2}}\sqrt{x(cx+b)}}+\frac{c\left(-\frac{2(-24Ac+16Bb)}{\sqrt{cx+b}}+\frac{16b(Ac-Bb)}{3(cx+b)^{\frac{3}{2}}}-\frac{2(35Ac-20Bb)\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)}{\sqrt{b}}\right)\sqrt{cx+b}\,\sqrt{x}}{8b^4\sqrt{x(cx+b)}}$
default	$-\frac{\sqrt{x(cx+b)}\left(105A\sqrt{cx+b}\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^3x^3-60B\sqrt{cx+b}\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)bc^2x^3+105A\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)bc^2x^2\sqrt{cx+b}-1\right.}{12x^{\frac{5}{2}}(cx+}$

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(c*x+b)*(-11*A*c*x+4*B*b*x+2*A*b)/b^4/x^(3/2)/(x*(c*x+b))^(1/2)+1/8/b
^4*c*(-2*(-24*A*c+16*B*b)/(c*x+b)^(1/2)+16/3*b*(A*c-B*b)/(c*x+b)^(3/2)-2*(
35*A*c-20*B*b)/b^(1/2)*arctanh((c*x+b)^(1/2)/b^(1/2)))*(c*x+b)^(1/2)*x^(1/
2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.48

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^{5/2}}\,dx = \left[-\frac{15\left((4\,Bbc^3-7\,Ac^4)x^5+2\,(4\,Bb^2c^2-7\,Abc^3)x^4+(4\,Bb^3c-7\,Ab^2c^2)x^3\right)\sqrt{b}\,\mathrm{l}}{}\right.$$

$$-\frac{15\left((4\,Bbc^3-7\,Ac^4)x^5+2\,(4\,Bb^2c^2-7\,Abc^3)x^4+(4\,Bb^3c-7\,Ab^2c^2)x^3\right)\sqrt{-b}\arctan\left(\frac{\sqrt{cx^2+bx}\sqrt{-b}}{b\sqrt{x}}\right)+(6}{12\,(b^5c^2x^5+2\,b^6cx}$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(15*((4*B*b*c^3 - 7*A*c^4)*x^5 + 2*(4*B*b^2*c^2 - 7*A*b*c^3)*x^4 +
(4*B*b^3*c - 7*A*b^2*c^2)*x^3)*sqrt(b)*log(-(c*x^2 + 2*b*x - 2*sqrt(c*x^2
+ b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(6*A*b^4 + 15*(4*B*b^2*c^2 - 7*A*b*c^3)*x
^3 + 20*(4*B*b^3*c - 7*A*b^2*c^2)*x^2 + 3*(4*B*b^4 - 7*A*b^3*c)*x)*sqrt(c*
x^2 + b*x)*sqrt(x))/(b^5*c^2*x^5 + 2*b^6*c*x^4 + b^7*x^3), -1/12*(15*((4*B
*b*c^3 - 7*A*c^4)*x^5 + 2*(4*B*b^2*c^2 - 7*A*b*c^3)*x^4 + (4*B*b^3*c - 7*A
*b^2*c^2)*x^3)*sqrt(-b)*arctan(sqrt(c*x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) + (
6*A*b^4 + 15*(4*B*b^2*c^2 - 7*A*b*c^3)*x^3 + 20*(4*B*b^3*c - 7*A*b^2*c^2)*
x^2 + 3*(4*B*b^4 - 7*A*b^3*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^5*c^2*x^5 +
 2*b^6*c*x^4 + b^7*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{x}\left(bx+cx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(1/2)/(c*x**2+b*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{\sqrt{x}\left(bx+cx^2\right)^{5/2}}\,dx = \int \frac{Bx+A}{\left(cx^2+bx\right)^{\frac{5}{2}}\sqrt{x}}\,dx$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((c*x^2 + b*x)^(5/2)*sqrt(x)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.87

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^{5/2}}\,dx = -\frac{5\,(4\,Bbc-7\,Ac^2)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{4\,\sqrt{-b}b^4} - \frac{2\,(6\,(cx+b)Bbc+Bb^2c-9\,(cx+b)Ac^2-Abc^2)}{3\,(cx+b)^{\frac{3}{2}}b^4} - \frac{4\,(cx+b)^{\frac{3}{2}}Bbc-4\,\sqrt{cx+b}Bb^2c-11\,(cx+b)^{\frac{3}{2}}Ac^2+13\,\sqrt{cx+b}Abc^2}{4\,b^4c^2x^2}$$

input
```
integrate((B*x+A)/x^(1/2)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
-5/4*(4*B*b*c - 7*A*c^2)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^4) - 2
/3*(6*(c*x + b)*B*b*c + B*b^2*c - 9*(c*x + b)*A*c^2 - A*b*c^2)/((c*x + b)^
(3/2)*b^4) - 1/4*(4*(c*x + b)^(3/2)*B*b*c - 4*sqrt(c*x + b)*B*b^2*c - 11*(
c*x + b)^(3/2)*A*c^2 + 13*sqrt(c*x + b)*A*b*c^2)/(b^4*c^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^{5/2}}\,dx = \int \frac{A+B\,x}{\sqrt{x}\,(c\,x^2+b\,x)^{5/2}}\,dx$$

input
```
int((A + B*x)/(x^(1/2)*(b*x + c*x^2)^(5/2)),x)
```

output
```
int((A + B*x)/(x^(1/2)*(b*x + c*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.84

$$\int \frac{A+Bx}{\sqrt{x}\,(bx+cx^2)^{5/2}}\,dx = \frac{105\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)ab\,c^2x^2+105\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right.}{}$$

input
```
int((B*x+A)/x^(1/2)/(c*x^2+b*x)^(5/2),x)
```

output
```
(105*sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*a*b*c**2*x**2 + 10
5*sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*a*c**3*x**3 - 60*sqrt
(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*b**3*c*x**2 - 60*sqrt(b)*sq
rt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*b**2*c**2*x**3 - 105*sqrt(b)*sqrt
(b + c*x)*log(sqrt(b + c*x) + sqrt(b))*a*b*c**2*x**2 - 105*sqrt(b)*sqrt(b
+ c*x)*log(sqrt(b + c*x) + sqrt(b))*a*c**3*x**3 + 60*sqrt(b)*sqrt(b + c*x)
*log(sqrt(b + c*x) + sqrt(b))*b**3*c*x**2 + 60*sqrt(b)*sqrt(b + c*x)*log(s
qrt(b + c*x) + sqrt(b))*b**2*c**2*x**3 - 12*a*b**4 + 42*a*b**3*c*x + 280*a
*b**2*c**2*x**2 + 210*a*b*c**3*x**3 - 24*b**5*x - 160*b**4*c*x**2 - 120*b*
*3*c**2*x**3)/(24*sqrt(b + c*x)*b**5*x**2*(b + c*x))
```

3.228 $\int \frac{A+Bx}{x^{3/2}(bx+cx^2)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 213

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx = \frac{2c^2(bB-Ac)x^{3/2}}{3b^4\left(bx+cx^2\right)^{3/2}} + \frac{2c^2(3bB-4Ac)\sqrt{x}}{b^5\sqrt{bx+cx^2}} - \frac{A\sqrt{bx+cx^2}}{3b^3x^{7/2}} - \frac{(6bB-17Ac)\sqrt{bx+cx^2}}{12b^4x^{5/2}} + \frac{c(22bB-41Ac)\sqrt{bx+cx^2}}{8b^5x^{3/2}} - \frac{35c^2(2bB-3Ac)\,\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{8b^{11/2}}$$

output
```
2/3*c^2*(-A*c+B*b)*x^(3/2)/b^4/(c*x^2+b*x)^(3/2)+2*c^2*(-4*A*c+3*B*b)*x^(1
/2)/b^5/(c*x^2+b*x)^(1/2)-1/3*A*(c*x^2+b*x)^(1/2)/b^3/x^(7/2)-1/12*(-17*A*
c+6*B*b)*(c*x^2+b*x)^(1/2)/b^4/x^(5/2)+1/8*c*(-41*A*c+22*B*b)*(c*x^2+b*x)^
(1/2)/b^5/x^(3/2)-35/8*c^2*(-3*A*c+2*B*b)*arctanh((c*x^2+b*x)^(1/2)/b^(1/2
)/x^(1/2))/b^(11/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.72

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx = \frac{\sqrt{b}(2bBx(-6b^3+21b^2cx+140bc^2x^2+105c^3x^3)-A(8b^4-18b^3cx+63b^2c^2x^2 -}{24b^{11/2}x^{3/2}(x(}$$

input
```
Integrate[(A + B*x)/(x^(3/2)*(b*x + c*x^2)^(5/2)),x]
```

output
```
(Sqrt[b]*(2*b*B*x*(-6*b^3 + 21*b^2*c*x + 140*b*c^2*x^2 + 105*c^3*x^3) - A*
(8*b^4 - 18*b^3*c*x + 63*b^2*c^2*x^2 + 420*b*c^3*x^3 + 315*c^4*x^4)) + 105
*c^2*(-2*b*B + 3*A*c)*x^3*(b + c*x)^(3/2)*ArcTanh[Sqrt[b + c*x]/Sqrt[b]])/
(24*b^(11/2)*x^(3/2)*(x*(b + c*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1220, 1135, 1132, 1135, 1132, 1136, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{1220}$$

$$\frac{(2bB-3Ac)\int \frac{1}{\sqrt{x}\left(cx^2+bx\right)^{5/2}}dx}{2b}-\frac{A}{3bx^{3/2}\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1135}$$

$$\frac{(2bB-3Ac)\left(-\frac{7c\int \frac{\sqrt{x}}{\left(cx^2+bx\right)^{5/2}}dx}{4b}-\frac{1}{2b\sqrt{x}\left(bx+cx^2\right)^{3/2}}\right)}{2b}-\frac{A}{3bx^{3/2}\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \text{1132}$$

$$\frac{(2bB-3Ac)\left(-\frac{7c\left(\frac{5\int \frac{1}{\sqrt{x}\left(cx^2+bx\right)^{3/2}}dx}{3b}+\frac{2\sqrt{x}}{3b\left(bx+cx^2\right)^{3/2}}\right)}{4b}-\frac{1}{2b\sqrt{x}(bx+cx^2)^{3/2}}\right)}{2b}-\frac{A}{3bx^{3/2}\left(bx+cx^2\right)^{3/2}}$$

$\big\downarrow$ 1135

$$\frac{(2bB-3Ac)\left(-\frac{7c\left(\frac{5\left(-\frac{3c\int \frac{\sqrt{x}}{\left(cx^2+bx\right)^{3/2}}dx}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{3b}+\frac{2\sqrt{x}}{3b\left(bx+cx^2\right)^{3/2}}\right)}{4b}-\frac{1}{2b\sqrt{x}(bx+cx^2)^{3/2}}\right)}{2b}-\frac{A}{3bx^{3/2}\left(bx+cx^2\right)^{3/2}}$$

$\big\downarrow$ 1132

$$\frac{(2bB-3Ac)\left(-\frac{7c\left(\frac{5\left(-\frac{3c\left(\frac{\int\frac{1}{\sqrt{x}\sqrt{cx^2+bx}}dx}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{3b}+\frac{2\sqrt{x}}{3b\left(bx+cx^2\right)^{3/2}}\right)}{4b}-\frac{1}{2b\sqrt{x}(bx+cx^2)^{3/2}}\right)}{2b}-\frac{A}{3bx^{3/2}\left(bx+cx^2\right)^{3/2}}$$

$\downarrow$ 1136

$$\frac{(2bB-3Ac)\left(-\frac{7c\left(\frac{5\left(-\frac{3c\left(\frac{2\int \frac{1}{\frac{cx^2+bx}{x}-b}d\frac{\sqrt{cx^2+bx}}{\sqrt{x}}}{b}+\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{3b}+\frac{2\sqrt{x}}{3b\left(bx+cx^2\right)^{3/2}}\right)}{4b}-\frac{1}{2b\sqrt{x}(bx+cx^2)^{3/2}}\right)}{2b}-\frac{A}{3bx^{3/2}\left(bx+cx^2\right)^{3/2}}$$

$$\Big\downarrow \ 220$$

$$\frac{(2bB-3Ac)\left(-\frac{7c\left(\frac{5\left(-\frac{3c\left(\frac{2\sqrt{x}}{b\sqrt{bx+cx^2}}-\frac{2\text{arctanh}\left(\frac{\sqrt{bx+cx^2}}{\sqrt{b}\sqrt{x}}\right)}{b^{3/2}}\right)}{2b}-\frac{1}{b\sqrt{x}\sqrt{bx+cx^2}}\right)}{3b}+\frac{2\sqrt{x}}{3b\left(bx+cx^2\right)^{3/2}}\right)}{4b}-\frac{1}{2b\sqrt{x}(bx+cx^2)^{3/2}}\right)}{2b}-\frac{A}{3bx^{3/2}\left(bx+cx^2\right)^{3/2}}$$

input
```
Int[(A + B*x)/(x^(3/2)*(b*x + c*x^2)^(5/2)),x]
```

output
```
-1/3*A/(b*x^(3/2)*(b*x + c*x^2)^(3/2)) + ((2*b*B - 3*A*c)*(-1/2*1/(b*Sqrt[
x]*(b*x + c*x^2)^(3/2)) - (7*c*((2*Sqrt[x])/(3*b*(b*x + c*x^2)^(3/2)) + (5
*(-(1/(b*Sqrt[x]*Sqrt[b*x + c*x^2])) - (3*c*((2*Sqrt[x])/(b*Sqrt[b*x + c*x
^2]) - (2*ArcTanh[Sqrt[b*x + c*x^2]/(Sqrt[b]*Sqrt[x])])/b^(3/2)))/(2*b)))/
(3*b)))/(4*b)))/(2*b)
```

Defintions of rubi rules used

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 1132

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(2*c*d - b*e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/(e*(p +
 1)*(b^2 - 4*a*c))), x] - Simp[(2*c*d - b*e)*((m + 2*p + 2)/((p + 1)*(b^2 -
 4*a*c)))   Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c, d, e}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && LtQ
[0, m, 1] && IntegerQ[2*p]
```

rule 1135

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-e)*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1)/((m + p + 1)*(2*
c*d - b*e))), x] + Simp[c*((m + 2*p + 2)/((m + p + 1)*(2*c*d - b*e)))   Int
[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, 0] && NeQ[m + p + 1, 0] && I
ntegerQ[2*p]
```

rule 1136

```
Int[1/(Sqrt[(d_.) + (e_.)*(x_)]*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x
_Symbol] :> Simp[2*e   Subst[Int[1/(2*c*d - b*e + e^2*x^2), x], x, Sqrt[a +
 b*x + c*x^2]/Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && EqQ[c*d^2
- b*d*e + a*e^2, 0]
```

rule 1220

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d*g - e*f)*(d + e*x)^m*((a + b*x + c*x
^2)^(p + 1)/((2*c*d - b*e)*(m + p + 1))), x] + Simp[(m*(g*(c*d - b*e) + c*e
*f) + e*(p + 1)*(2*c*f - b*g))/(e*(2*c*d - b*e)*(m + p + 1))   Int[(d + e*x
)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p},
 x] && EqQ[c*d^2 - b*d*e + a*e^2, 0] && ((LtQ[m, -1] &&  !IGtQ[m + p + 1, 0
]) || (LtQ[m, 0] && LtQ[p, -1]) || EqQ[m + 2*p + 2, 0]) && NeQ[m + p + 1, 0
]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{(cx+b)(123A c^2x^2-66x^2Bbc-34Abcx+12xB b^2+8b^2A)}{24b^5x^{\frac{5}{2}}\sqrt{x(cx+b)}}-\frac{c^2\left(-\frac{2(-64Ac+48Bb)}{\sqrt{cx+b}}+\frac{32b(Ac-Bb)}{3(cx+b)^{\frac{3}{2}}}-\frac{2(105Ac-70Bb)\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right.}{\sqrt{b}}\right.}{16b^5\sqrt{x(cx+b)}}$
default	$\frac{\sqrt{x(cx+b)}\left(315A\sqrt{cx+b}\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)c^4x^4-210B\sqrt{cx+b}\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)bc^3x^4+315A\operatorname{arctanh}\left(\frac{\sqrt{cx+b}}{\sqrt{b}}\right)bc^3x^3\sqrt{cx+b}-315\right.}{}$

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(c*x+b)*(123*A*c^2*x^2-66*B*b*c*x^2-34*A*b*c*x+12*B*b^2*x+8*A*b^2)/b
^5/x^(5/2)/(x*(c*x+b))^(1/2)-1/16*c^2/b^5*(-2*(-64*A*c+48*B*b)/(c*x+b)^(1/
2)+32/3*b*(A*c-B*b)/(c*x+b)^(3/2)-2*(105*A*c-70*B*b)/b^(1/2)*arctanh((c*x+
b)^(1/2)/b^(1/2)))*(c*x+b)^(1/2)*x^(1/2)/(x*(c*x+b))^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 480, normalized size of antiderivative = 2.25

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx=\left[-\frac{105\left((2\,Bbc^4-3\,Ac^5)x^6+2\,(2\,Bb^2c^3-3\,Abc^4)x^5+(2\,Bb^3c^2-3\,Ab^2c^3)x^4\right)\sqrt{}}{}\right.$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/48*(105*((2*B*b*c^4 - 3*A*c^5)*x^6 + 2*(2*B*b^2*c^3 - 3*A*b*c^4)*x^5 +
 (2*B*b^3*c^2 - 3*A*b^2*c^3)*x^4)*sqrt(b)*log(-(c*x^2 + 2*b*x + 2*sqrt(c*x
^2 + b*x)*sqrt(b)*sqrt(x))/x^2) + 2*(8*A*b^5 - 105*(2*B*b^2*c^3 - 3*A*b*c^
4)*x^4 - 140*(2*B*b^3*c^2 - 3*A*b^2*c^3)*x^3 - 21*(2*B*b^4*c - 3*A*b^3*c^2
)*x^2 + 6*(2*B*b^5 - 3*A*b^4*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^6*c^2*x^6
 + 2*b^7*c*x^5 + b^8*x^4), 1/24*(105*((2*B*b*c^4 - 3*A*c^5)*x^6 + 2*(2*B*b
^2*c^3 - 3*A*b*c^4)*x^5 + (2*B*b^3*c^2 - 3*A*b^2*c^3)*x^4)*sqrt(-b)*arctan
(sqrt(c*x^2 + b*x)*sqrt(-b)/(b*sqrt(x))) - (8*A*b^5 - 105*(2*B*b^2*c^3 - 3
*A*b*c^4)*x^4 - 140*(2*B*b^3*c^2 - 3*A*b^2*c^3)*x^3 - 21*(2*B*b^4*c - 3*A*
b^3*c^2)*x^2 + 6*(2*B*b^5 - 3*A*b^4*c)*x)*sqrt(c*x^2 + b*x)*sqrt(x))/(b^6*
c^2*x^6 + 2*b^7*c*x^5 + b^8*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((B*x+A)/x**(3/2)/(c*x**2+b*x)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx = \int \frac{Bx+A}{\left(cx^2+bx\right)^{\frac{5}{2}}x^{\frac{3}{2}}}\,dx$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*x + A)/((c*x^2 + b*x)^(5/2)*x^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.94

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx = \frac{35\left(2\,Bbc^2-3\,Ac^3\right)\arctan\left(\frac{\sqrt{cx+b}}{\sqrt{-b}}\right)}{8\sqrt{-b}b^5}$$
$$+\frac{210\,(cx+b)^4Bbc^2-560\,(cx+b)^3Bb^2c^2+462\,(cx+b)^2Bb^3c^2-96\,(cx+b)Bb^4c^2-16\,Bb^5c^2-315\,(cx+\cdots}{24\left((cx+b)^{\frac{3}{2}}-\sqrt{cx+b}b\cdots\right.}$$

input
```
integrate((B*x+A)/x^(3/2)/(c*x^2+b*x)^(5/2),x, algorithm="giac")
```

output
```
35/8*(2*B*b*c^2 - 3*A*c^3)*arctan(sqrt(c*x + b)/sqrt(-b))/(sqrt(-b)*b^5) +
 1/24*(210*(c*x + b)^4*B*b*c^2 - 560*(c*x + b)^3*B*b^2*c^2 + 462*(c*x + b)
^2*B*b^3*c^2 - 96*(c*x + b)*B*b^4*c^2 - 16*B*b^5*c^2 - 315*(c*x + b)^4*A*c
^3 + 840*(c*x + b)^3*A*b*c^3 - 693*(c*x + b)^2*A*b^2*c^3 + 144*(c*x + b)*A
*b^3*c^3 + 16*A*b^4*c^3)/((((c*x + b)^(3/2) - sqrt(c*x + b)*b)^3*b^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx = \int \frac{A+B\,x}{x^{3/2}\,\left(c\,x^2+b\,x\right)^{5/2}}\,dx$$

input
```
int((A + B*x)/(x^(3/2)*(b*x + c*x^2)^(5/2)),x)
```

output
```
int((A + B*x)/(x^(3/2)*(b*x + c*x^2)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.61

$$\int \frac{A+Bx}{x^{3/2}\left(bx+cx^2\right)^{5/2}}\,dx = \frac{-315\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\sqrt{b}\right)ab\,c^3x^3-315\sqrt{b}\,\sqrt{cx+b}\log\left(\sqrt{cx+b}-\right.}{}$$

input
```
int((B*x+A)/x^(3/2)/(c*x^2+b*x)^(5/2),x)
```

output
```
( - 315*sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*a*b*c**3*x**3 -
 315*sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*a*c**4*x**4 + 210*
sqrt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*b**3*c**2*x**3 + 210*sq
rt(b)*sqrt(b + c*x)*log(sqrt(b + c*x) - sqrt(b))*b**2*c**3*x**4 + 315*sqrt
(b)*sqrt(b + c*x)*log(sqrt(b + c*x) + sqrt(b))*a*b*c**3*x**3 + 315*sqrt(b)
*sqrt(b + c*x)*log(sqrt(b + c*x) + sqrt(b))*a*c**4*x**4 - 210*sqrt(b)*sqrt
(b + c*x)*log(sqrt(b + c*x) + sqrt(b))*b**3*c**2*x**3 - 210*sqrt(b)*sqrt(b
 + c*x)*log(sqrt(b + c*x) + sqrt(b))*b**2*c**3*x**4 - 16*a*b**5 + 36*a*b**
4*c*x - 126*a*b**3*c**2*x**2 - 840*a*b**2*c**3*x**3 - 630*a*b*c**4*x**4 -
24*b**6*x + 84*b**5*c*x**2 + 560*b**4*c**2*x**3 + 420*b**3*c**3*x**4)/(48*
sqrt(b + c*x)*b**6*x**3*(b + c*x))
```

3.229 $\int (ex)^m(c+dx)\left(ax+bx^2\right)^3\,dx$

Optimal result

Integrand size = 22, antiderivative size = 121

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^3\,dx=\frac{a^3c(ex)^{4+m}}{e^4(4+m)}+\frac{a^2(3bc+ad)(ex)^{5+m}}{e^5(5+m)}+\frac{3ab(bc+ad)(ex)^{6+m}}{e^6(6+m)}+\frac{b^2(bc+3ad)(ex)^{7+m}}{e^7(7+m)}+\frac{b^3d(ex)^{8+m}}{e^8(8+m)}$$

output
```
a^3*c*(e*x)^(4+m)/e^4/(4+m)+a^2*(a*d+3*b*c)*(e*x)^(5+m)/e^5/(5+m)+3*a*b*(a
*d+b*c)*(e*x)^(6+m)/e^6/(6+m)+b^2*(3*a*d+b*c)*(e*x)^(7+m)/e^7/(7+m)+b^3*d*
(e*x)^(8+m)/e^8/(8+m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^3\,dx=\frac{x^4(ex)^m\left(d(a+bx)^4+(-ad(4+m)+bc(8+m))\left(\frac{a^3}{4+m}+\frac{3a^2bx}{5+m}+\frac{3ab^2x^2}{6+m}+\frac{b^3x^3}{7+m}\right)\right)}{b(8+m)}$$

input
```
Integrate[(e*x)^m*(c + d*x)*(a*x + b*x^2)^3,x]
```

output
```
(x^4*(e*x)^m*(d*(a + b*x)^4 + (-(a*d*(4 + m)) + b*c*(8 + m))*(a^3/(4 + m)
+ (3*a^2*b*x)/(5 + m) + (3*a*b^2*x^2)/(6 + m) + (b^3*x^3)/(7 + m))))/(b*(8
 + m))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)^3 (c+dx)(ex)^m\,dx$$

$$\Big\downarrow \text{9}$$

$$\frac{\int (ex)^{m+3}(a+bx)^3(c+dx)dx}{e^3}$$

$$\Big\downarrow \text{85}$$

$$\frac{\int \left(a^3c(ex)^{m+3}+\frac{a^2(3bc+ad)(ex)^{m+4}}{e}+\frac{3ab(bc+ad)(ex)^{m+5}}{e^2}+\frac{b^2(bc+3ad)(ex)^{m+6}}{e^3}+\frac{b^3d(ex)^{m+7}}{e^4}\right)dx}{e^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^3c(ex)^{m+4}}{e(m+4)}+\frac{a^2(ex)^{m+5}(ad+3bc)}{e^2(m+5)}+\frac{b^2(ex)^{m+7}(3ad+bc)}{e^4(m+7)}+\frac{3ab(ex)^{m+6}(ad+bc)}{e^3(m+6)}+\frac{b^3d(ex)^{m+8}}{e^5(m+8)}}{e^3}$$

input
```
Int[(e*x)^m*(c + d*x)*(a*x + b*x^2)^3,x]
```

output
```
((a^3*c*(e*x)^(4 + m))/(e*(4 + m)) + (a^2*(3*b*c + a*d)*(e*x)^(5 + m))/(e^
2*(5 + m)) + (3*a*b*(b*c + a*d)*(e*x)^(6 + m))/(e^3*(6 + m)) + (b^2*(b*c +
 3*a*d)*(e*x)^(7 + m))/(e^4*(7 + m)) + (b^3*d*(e*x)^(8 + m))/(e^5*(8 + m))
)/e^3
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.01

method	result
norman	$\frac{a^{2}(ad+3bc)x^{5}e^{m\ln(ex)}}{5+m}+\frac{b^{2}(3ad+bc)x^{7}e^{m\ln(ex)}}{7+m}+\frac{b^{3}d\,x^{8}e^{m\ln(ex)}}{8+m}+\frac{c\,a^{3}x^{4}e^{m\ln(ex)}}{4+m}+\frac{3ab(ad+bc)x^{6}e^{m\ln(ex)}}{6+m}$
gosper	$(ex)^{m}(b^{3}d\,m^{4}x^{4}+3a\,b^{2}d\,m^{4}x^{3}+b^{3}c\,m^{4}x^{3}+22b^{3}d\,m^{3}x^{4}+3a^{2}bd\,m^{4}x^{2}+3a\,b^{2}c\,m^{4}x^{2}+69a\,b^{2}d\,m^{3}x^{3}+23b^{3}c\,m^{3}x^{3}+179b^{3}d\,m$
risch	$(ex)^{m}(b^{3}d\,m^{4}x^{4}+3a\,b^{2}d\,m^{4}x^{3}+b^{3}c\,m^{4}x^{3}+22b^{3}d\,m^{3}x^{4}+3a^{2}bd\,m^{4}x^{2}+3a\,b^{2}c\,m^{4}x^{2}+69a\,b^{2}d\,m^{3}x^{3}+23b^{3}c\,m^{3}x^{3}+179b^{3}d\,m$
orering	$(b^{3}d\,m^{4}x^{4}+3a\,b^{2}d\,m^{4}x^{3}+b^{3}c\,m^{4}x^{3}+22b^{3}d\,m^{3}x^{4}+3a^{2}bd\,m^{4}x^{2}+3a\,b^{2}c\,m^{4}x^{2}+69a\,b^{2}d\,m^{3}x^{3}+23b^{3}c\,m^{3}x^{3}+179b^{3}d\,m^{2}x^{4}+a$
parallelrisch	$960x^{7}(ex)^{m}b^{3}c+690x^{5}(ex)^{m}a^{2}bc\,m^{2}+2760x^{5}(ex)^{m}a^{2}bcm+23x^{7}(ex)^{m}b^{3}c\,m^{3}+638x^{8}(ex)^{m}b^{3}dm+194x^{7}(ex)^{m}b^{3}c\,m^{2}+x^{5}$

input
```
int((e*x)^m*(d*x+c)*(b*x^2+a*x)^3,x,method=_RETURNVERBOSE)
```

output
```
a^2*(a*d+3*b*c)/(5+m)*x^5*exp(m*ln(e*x))+b^2*(3*a*d+b*c)/(7+m)*x^7*exp(m*l
n(e*x))+b^3*d/(8+m)*x^8*exp(m*ln(e*x))+c*a^3/(4+m)*x^4*exp(m*ln(e*x))+3*a*
b*(a*d+b*c)/(6+m)*x^6*exp(m*ln(e*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(121) = 242$.

Time = 0.08 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.17

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^3\,dx$$
$$= \frac{((b^3dm^4 + 22\,b^3dm^3 + 179\,b^3dm^2 + 638\,b^3dm + 840\,b^3d)x^8 + ((b^3c + 3\,ab^2d)m^4 + 960\,b^3c + 2880\,ab^2d + }{}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^2+a*x)^3,x, algorithm="fricas")
```

output
```
((b^3*d*m^4 + 22*b^3*d*m^3 + 179*b^3*d*m^2 + 638*b^3*d*m + 840*b^3*d)*x^8
+ ((b^3*c + 3*a*b^2*d)*m^4 + 960*b^3*c + 2880*a*b^2*d + 23*(b^3*c + 3*a*b^
2*d)*m^3 + 194*(b^3*c + 3*a*b^2*d)*m^2 + 712*(b^3*c + 3*a*b^2*d)*m)*x^7 +
3*((a*b^2*c + a^2*b*d)*m^4 + 1120*a*b^2*c + 1120*a^2*b*d + 24*(a*b^2*c + a
^2*b*d)*m^3 + 211*(a*b^2*c + a^2*b*d)*m^2 + 804*(a*b^2*c + a^2*b*d)*m)*x^6
 + ((3*a^2*b*c + a^3*d)*m^4 + 4032*a^2*b*c + 1344*a^3*d + 25*(3*a^2*b*c +
a^3*d)*m^3 + 230*(3*a^2*b*c + a^3*d)*m^2 + 920*(3*a^2*b*c + a^3*d)*m)*x^5
+ (a^3*c*m^4 + 26*a^3*c*m^3 + 251*a^3*c*m^2 + 1066*a^3*c*m + 1680*a^3*c)*x
^4)*(e*x)^m/(m^5 + 30*m^4 + 355*m^3 + 2070*m^2 + 5944*m + 6720)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2111 vs. $2(112) = 224$.

Time = 0.58 (sec) , antiderivative size = 2111, normalized size of antiderivative = 17.45

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(d*x+c)*(b*x**2+a*x)**3,x)
```

output

```
Piecewise(((-a**3*c/(4*x**4) - a**3*d/(3*x**3) - a**2*b*c/x**3 - 3*a**2*b*
d/(2*x**2) - 3*a*b**2*c/(2*x**2) - 3*a*b**2*d/x - b**3*c/x + b**3*d*log(x)
)/e**8, Eq(m, -8)), ((-a**3*c/(3*x**3) - a**3*d/(2*x**2) - 3*a**2*b*c/(2*x
**2) - 3*a**2*b*d/x - 3*a*b**2*c/x + 3*a*b**2*d*log(x) + b**3*c*log(x) + b
**3*d*x)/e**7, Eq(m, -7)), ((-a**3*c/(2*x**2) - a**3*d/x - 3*a**2*b*c/x +
3*a**2*b*d*log(x) + 3*a*b**2*c*log(x) + 3*a*b**2*d*x + b**3*c*x + b**3*d*x
**2/2)/e**6, Eq(m, -6)), ((-a**3*c/x + a**3*d*log(x) + 3*a**2*b*c*log(x) +
 3*a**2*b*d*x + 3*a*b**2*c*x + 3*a*b**2*d*x**2/2 + b**3*c*x**2/2 + b**3*d*
x**3/3)/e**5, Eq(m, -5)), ((a**3*c*log(x) + a**3*d*x + 3*a**2*b*c*x + 3*a*
*2*b*d*x**2/2 + 3*a*b**2*c*x**2/2 + a*b**2*d*x**3 + b**3*c*x**3/3 + b**3*d
*x**4/4)/e**4, Eq(m, -4)), (a**3*c*m**4*x**4*(e*x)**m/(m**5 + 30*m**4 + 35
5*m**3 + 2070*m**2 + 5944*m + 6720) + 26*a**3*c*m**3*x**4*(e*x)**m/(m**5 +
 30*m**4 + 355*m**3 + 2070*m**2 + 5944*m + 6720) + 251*a**3*c*m**2*x**4*(e
*x)**m/(m**5 + 30*m**4 + 355*m**3 + 2070*m**2 + 5944*m + 6720) + 1066*a**3
*c*m*x**4*(e*x)**m/(m**5 + 30*m**4 + 355*m**3 + 2070*m**2 + 5944*m + 6720)
 + 1680*a**3*c*x**4*(e*x)**m/(m**5 + 30*m**4 + 355*m**3 + 2070*m**2 + 5944
*m + 6720) + a**3*d*m**4*x**5*(e*x)**m/(m**5 + 30*m**4 + 355*m**3 + 2070*m
**2 + 5944*m + 6720) + 25*a**3*d*m**3*x**5*(e*x)**m/(m**5 + 30*m**4 + 355*
m**3 + 2070*m**2 + 5944*m + 6720) + 230*a**3*d*m**2*x**5*(e*x)**m/(m**5 +
30*m**4 + 355*m**3 + 2070*m**2 + 5944*m + 6720) + 920*a**3*d*m*x**5*(e*...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.33

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^3\,dx = \frac{b^3de^mx^8x^m}{m+8}+\frac{b^3ce^mx^7x^m}{m+7}+\frac{3\,ab^2de^mx^7x^m}{m+7}+\frac{3\,ab^2ce^mx^6x^m}{m+6}+\frac{3\,a^2bde^mx^6x^m}{m+6}+\frac{3\,a^2bce^mx^5x^m}{m+5}+\frac{a^3de^mx^5x^m}{m+5}+\frac{a^3ce^mx^4x^m}{m+4}$$

input

```
integrate((e*x)^m*(d*x+c)*(b*x^2+a*x)^3,x, algorithm="maxima")
```

output

```
b^3*d*e^m*x^8*x^m/(m + 8) + b^3*c*e^m*x^7*x^m/(m + 7) + 3*a*b^2*d*e^m*x^7*
x^m/(m + 7) + 3*a*b^2*c*e^m*x^6*x^m/(m + 6) + 3*a^2*b*d*e^m*x^6*x^m/(m + 6
) + 3*a^2*b*c*e^m*x^5*x^m/(m + 5) + a^3*d*e^m*x^5*x^m/(m + 5) + a^3*c*e^m*
x^4*x^m/(m + 4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. $2(121) = 242$.

Time = 0.14 (sec) , antiderivative size = 683, normalized size of antiderivative = 5.64

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^2+a*x)^3,x, algorithm="giac")
```

output
```
((e*x)^m*b^3*d*m^4*x^8 + (e*x)^m*b^3*c*m^4*x^7 + 3*(e*x)^m*a*b^2*d*m^4*x^7
 + 22*(e*x)^m*b^3*d*m^3*x^8 + 3*(e*x)^m*a*b^2*c*m^4*x^6 + 3*(e*x)^m*a^2*b*
d*m^4*x^6 + 23*(e*x)^m*b^3*c*m^3*x^7 + 69*(e*x)^m*a*b^2*d*m^3*x^7 + 179*(e
*x)^m*b^3*d*m^2*x^8 + 3*(e*x)^m*a^2*b*c*m^4*x^5 + (e*x)^m*a^3*d*m^4*x^5 +
72*(e*x)^m*a*b^2*c*m^3*x^6 + 72*(e*x)^m*a^2*b*d*m^3*x^6 + 194*(e*x)^m*b^3*
c*m^2*x^7 + 582*(e*x)^m*a*b^2*d*m^2*x^7 + 638*(e*x)^m*b^3*d*m*x^8 + (e*x)^
m*a^3*c*m^4*x^4 + 75*(e*x)^m*a^2*b*c*m^3*x^5 + 25*(e*x)^m*a^3*d*m^3*x^5 +
633*(e*x)^m*a*b^2*c*m^2*x^6 + 633*(e*x)^m*a^2*b*d*m^2*x^6 + 712*(e*x)^m*b^
3*c*m*x^7 + 2136*(e*x)^m*a*b^2*d*m*x^7 + 840*(e*x)^m*b^3*d*x^8 + 26*(e*x)^
m*a^3*c*m^3*x^4 + 690*(e*x)^m*a^2*b*c*m^2*x^5 + 230*(e*x)^m*a^3*d*m^2*x^5
+ 2412*(e*x)^m*a*b^2*c*m*x^6 + 2412*(e*x)^m*a^2*b*d*m*x^6 + 960*(e*x)^m*b^
3*c*x^7 + 2880*(e*x)^m*a*b^2*d*x^7 + 251*(e*x)^m*a^3*c*m^2*x^4 + 2760*(e*x
)^m*a^2*b*c*m*x^5 + 920*(e*x)^m*a^3*d*m*x^5 + 3360*(e*x)^m*a*b^2*c*x^6 + 3
360*(e*x)^m*a^2*b*d*x^6 + 1066*(e*x)^m*a^3*c*m*x^4 + 4032*(e*x)^m*a^2*b*c*
x^5 + 1344*(e*x)^m*a^3*d*x^5 + 1680*(e*x)^m*a^3*c*x^4)/(m^5 + 30*m^4 + 355
*m^3 + 2070*m^2 + 5944*m + 6720)
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.33

$$\int (ex)^m(c+dx) \left(ax+bx^2\right)^3 \, dx = (e\,x)^m\,\left(\frac{a^3\,c\,x^4\,\left(m^4+26\,m^3+251\,m^2+1066\,m+1680\right)}{m^5+30\,m^4+355\,m^3+2070\,m^2+5944\,m+6720} + \frac{b^3\,d\,x^8\,\left(m^4+22\,m^3+179\,m^2+638\,m+840\right)}{m^5+30\,m^4+355\,m^3+2070\,m^2+5944\,m+6720} + \frac{a^2\,x^5\,\left(a\,d+3\,b\,c\right)\,\left(m^4+25\,m^3+230\,m^2+920\,m+1344\right)}{m^5+30\,m^4+355\,m^3+2070\,m^2+5944\,m+6720} + \frac{b^2\,x^7\,\left(3\,a\,d+b\,c\right)\,\left(m^4+23\,m^3+194\,m^2+712\,m+960\right)}{m^5+30\,m^4+355\,m^3+2070\,m^2+5944\,m+6720} + \frac{3\,a\,b\,x^6\,\left(a\,d+b\,c\right)\,\left(m^4+24\,m^3+211\,m^2+804\,m+1120\right)}{m^5+30\,m^4+355\,m^3+2070\,m^2+5944\,m+6720}\right)$$

input
```
int((a*x + b*x^2)^3*(e*x)^m*(c + d*x),x)
```

output
```
(e*x)^m*((a^3*c*x^4*(1066*m + 251*m^2 + 26*m^3 + m^4 + 1680))/(5944*m + 20
70*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720) + (b^3*d*x^8*(638*m + 179*m^2 + 22
*m^3 + m^4 + 840))/(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720) + (
a^2*x^5*(a*d + 3*b*c)*(920*m + 230*m^2 + 25*m^3 + m^4 + 1344))/(5944*m + 2
070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720) + (b^2*x^7*(3*a*d + b*c)*(712*m +
 194*m^2 + 23*m^3 + m^4 + 960))/(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^
5 + 6720) + (3*a*b*x^6*(a*d + b*c)*(804*m + 211*m^2 + 24*m^3 + m^4 + 1120)
)/(5944*m + 2070*m^2 + 355*m^3 + 30*m^4 + m^5 + 6720))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.78

$$\int (ex)^m(c+dx) \left(ax+bx^2\right)^3 \, dx = \frac{x^m e^m x^4 (b^3 d\, m^4 x^4 + 3a\, b^2 d\, m^4 x^3 + b^3 c\, m^4 x^3 + 22 b^3 d\, m^3 x^4 + 3a^2 b d\, m^4 x^2 + 3a\, b^2 c\, m^4 x^2 + 69 a\, b^2 d\, m^3 x^3 + \cdots}{}$$

input
```
int((e*x)^m*(d*x+c)*(b*x^2+a*x)^3,x)
```

output
```
(x**m*e**m*x**4*(a**3*c*m**4 + 26*a**3*c*m**3 + 251*a**3*c*m**2 + 1066*a**
3*c*m + 1680*a**3*c + a**3*d*m**4*x + 25*a**3*d*m**3*x + 230*a**3*d*m**2*x
 + 920*a**3*d*m*x + 1344*a**3*d*x + 3*a**2*b*c*m**4*x + 75*a**2*b*c*m**3*x
 + 690*a**2*b*c*m**2*x + 2760*a**2*b*c*m*x + 4032*a**2*b*c*x + 3*a**2*b*d*
m**4*x**2 + 72*a**2*b*d*m**3*x**2 + 633*a**2*b*d*m**2*x**2 + 2412*a**2*b*d
*m*x**2 + 3360*a**2*b*d*x**2 + 3*a*b**2*c*m**4*x**2 + 72*a*b**2*c*m**3*x**
2 + 633*a*b**2*c*m**2*x**2 + 2412*a*b**2*c*m*x**2 + 3360*a*b**2*c*x**2 + 3
*a*b**2*d*m**4*x**3 + 69*a*b**2*d*m**3*x**3 + 582*a*b**2*d*m**2*x**3 + 213
6*a*b**2*d*m*x**3 + 2880*a*b**2*d*x**3 + b**3*c*m**4*x**3 + 23*b**3*c*m**3
*x**3 + 194*b**3*c*m**2*x**3 + 712*b**3*c*m*x**3 + 960*b**3*c*x**3 + b**3*
d*m**4*x**4 + 22*b**3*d*m**3*x**4 + 179*b**3*d*m**2*x**4 + 638*b**3*d*m*x*
*4 + 840*b**3*d*x**4))/(m**5 + 30*m**4 + 355*m**3 + 2070*m**2 + 5944*m + 6
720)
```

3.230 $\int (ex)^m(c+dx)\left(ax+bx^2\right)^2\,dx$

Optimal result

Integrand size = 22, antiderivative size = 91

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^2\,dx=\frac{a^2c(ex)^{3+m}}{e^3(3+m)}+\frac{a(2bc+ad)(ex)^{4+m}}{e^4(4+m)}+\frac{b(bc+2ad)(ex)^{5+m}}{e^5(5+m)}+\frac{b^2d(ex)^{6+m}}{e^6(6+m)}$$

output
```
a^2*c*(e*x)^(3+m)/e^3/(3+m)+a*(a*d+2*b*c)*(e*x)^(4+m)/e^4/(4+m)+b*(2*a*d+b
*c)*(e*x)^(5+m)/e^5/(5+m)+b^2*d*(e*x)^(6+m)/e^6/(6+m)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.81

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^2\,dx=\frac{x^3(ex)^m\left(d(a+bx)^3+(-ad(3+m)+bc(6+m))\left(\frac{a^2}{3+m}+\frac{2abx}{4+m}+\frac{b^2x^2}{5+m}\right)\right)}{b(6+m)}$$

input
```
Integrate[(e*x)^m*(c + d*x)*(a*x + b*x^2)^2,x]
```

output
```
(x^3*(e*x)^m*(d*(a + b*x)^3 + (-(a*d*(3 + m)) + b*c*(6 + m))*(a^2/(3 + m)
+ (2*a*b*x)/(4 + m) + (b^2*x^2)/(5 + m))))/(b*(6 + m))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)^2 (c+dx)(ex)^m \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\frac{\int (ex)^{m+2}(a+bx)^2(c+dx)dx}{e^2}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\int \left(a^2c(ex)^{m+2}+\frac{a(2bc+ad)(ex)^{m+3}}{e}+\frac{b(bc+2ad)(ex)^{m+4}}{e^2}+\frac{b^2d(ex)^{m+5}}{e^3}\right)dx}{e^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^2c(ex)^{m+3}}{e(m+3)}+\frac{b(ex)^{m+5}(2ad+bc)}{e^3(m+5)}+\frac{a(ex)^{m+4}(ad+2bc)}{e^2(m+4)}+\frac{b^2d(ex)^{m+6}}{e^4(m+6)}}{e^2}$$

input
```
Int[(e*x)^m*(c + d*x)*(a*x + b*x^2)^2,x]
```

output
```
((a^2*c*(e*x)^(3 + m))/(e*(3 + m)) + (a*(2*b*c + a*d)*(e*x)^(4 + m))/(e^2*
(4 + m)) + (b*(b*c + 2*a*d)*(e*x)^(5 + m))/(e^3*(5 + m)) + (b^2*d*(e*x)^(6
 + m))/(e^4*(6 + m)))/e^2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.99 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

method	result
norman	$\frac{a(ad+2bc)x^4 \mathrm{e}^{m\ln(ex)}}{4+m}+\frac{a^2 c\,x^3 \mathrm{e}^{m\ln(ex)}}{3+m}+\frac{b(2ad+bc)x^5 \mathrm{e}^{m\ln(ex)}}{5+m}+\frac{b^2 d\,x^6 \mathrm{e}^{m\ln(ex)}}{6+m}$
gosper	$(ex)^m(b^2 d\,m^3 x^3+2abd\,m^3 x^2+b^2 c\,m^3 x^2+12b^2 d\,m^2 x^3+a^2 d\,m^3 x+2abc\,m^3 x+26abd\,m^2 x^2+13b^2 c\,m^2 x^2+47m\,x^3 b^2 d+a^2 c\,m$
risch	$(ex)^m(b^2 d\,m^3 x^3+2abd\,m^3 x^2+b^2 c\,m^3 x^2+12b^2 d\,m^2 x^3+a^2 d\,m^3 x+2abc\,m^3 x+26abd\,m^2 x^2+13b^2 c\,m^2 x^2+47m\,x^3 b^2 d+a^2 c\,m$
orering	$(b^2 d\,m^3 x^3+2abd\,m^3 x^2+b^2 c\,m^3 x^2+12b^2 d\,m^2 x^3+a^2 d\,m^3 x+2abc\,m^3 x+26abd\,m^2 x^2+13b^2 c\,m^2 x^2+47m\,x^3 b^2 d+a^2 c\,m^3+14a$
parallelrisch	$60x^6(ex)^m b^2 d+2x^5(ex)^m abd\,m^3+26x^5(ex)^m abd\,m^2+2x^4(ex)^m abc\,m^3+108x^5(ex)^m abdm+28x^4(ex)^m abc\,m^2+126x^4(ex)$

input
```
int((e*x)^m*(d*x+c)*(b*x^2+a*x)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(a*d+2*b*c)/(4+m)*x^4*exp(m*ln(e*x))+a^2*c/(3+m)*x^3*exp(m*ln(e*x))+b*(2
*a*d+b*c)/(5+m)*x^5*exp(m*ln(e*x))+b^2*d/(6+m)*x^6*exp(m*ln(e*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. $2(91) = 182$.

Time = 0.09 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.41

$$\int (ex)^m (c+dx) \left(ax+bx^2\right)^2 \, dx = \frac{((b^2dm^3 + 12\,b^2dm^2 + 47\,b^2dm + 60\,b^2d)x^6 + ((b^2c + 2\,abd)m^3 + 72\,b^2c + 144\,abd + 13\,(b^2c + 2\,abd)m^2}{}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^2+a*x)^2,x, algorithm="fricas")
```

output
```
((b^2*d*m^3 + 12*b^2*d*m^2 + 47*b^2*d*m + 60*b^2*d)*x^6 + ((b^2*c + 2*a*b*
d)*m^3 + 72*b^2*c + 144*a*b*d + 13*(b^2*c + 2*a*b*d)*m^2 + 54*(b^2*c + 2*a
*b*d)*m)*x^5 + ((2*a*b*c + a^2*d)*m^3 + 180*a*b*c + 90*a^2*d + 14*(2*a*b*c
 + a^2*d)*m^2 + 63*(2*a*b*c + a^2*d)*m)*x^4 + (a^2*c*m^3 + 15*a^2*c*m^2 +
74*a^2*c*m + 120*a^2*c)*x^3)*(e*x)^m/(m^4 + 18*m^3 + 119*m^2 + 342*m + 360
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1081 vs. $2(83) = 166$.

Time = 0.41 (sec) , antiderivative size = 1081, normalized size of antiderivative = 11.88

$$\int (ex)^m (c+dx) \left(ax+bx^2\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(d*x+c)*(b*x**2+a*x)**2,x)
```

output
```
Piecewise(((-a**2*c/(3*x**3) - a**2*d/(2*x**2) - a*b*c/x**2 - 2*a*b*d/x -
b**2*c/x + b**2*d*log(x))/e**6, Eq(m, -6)), ((-a**2*c/(2*x**2) - a**2*d/x
- 2*a*b*c/x + 2*a*b*d*log(x) + b**2*c*log(x) + b**2*d*x)/e**5, Eq(m, -5)),
 ((-a**2*c/x + a**2*d*log(x) + 2*a*b*c*log(x) + 2*a*b*d*x + b**2*c*x + b**
2*d*x**2/2)/e**4, Eq(m, -4)), ((a**2*c*log(x) + a**2*d*x + 2*a*b*c*x + a*b
*d*x**2 + b**2*c*x**2/2 + b**2*d*x**3/3)/e**3, Eq(m, -3)), (a**2*c*m**3*x*
*3*(e*x)**m/(m**4 + 18*m**3 + 119*m**2 + 342*m + 360) + 15*a**2*c*m**2*x**
3*(e*x)**m/(m**4 + 18*m**3 + 119*m**2 + 342*m + 360) + 74*a**2*c*m*x**3*(e
*x)**m/(m**4 + 18*m**3 + 119*m**2 + 342*m + 360) + 120*a**2*c*x**3*(e*x)**
m/(m**4 + 18*m**3 + 119*m**2 + 342*m + 360) + a**2*d*m**3*x**4*(e*x)**m/(m
**4 + 18*m**3 + 119*m**2 + 342*m + 360) + 14*a**2*d*m**2*x**4*(e*x)**m/(m*
*4 + 18*m**3 + 119*m**2 + 342*m + 360) + 63*a**2*d*m*x**4*(e*x)**m/(m**4 +
 18*m**3 + 119*m**2 + 342*m + 360) + 90*a**2*d*x**4*(e*x)**m/(m**4 + 18*m*
*3 + 119*m**2 + 342*m + 360) + 2*a*b*c*m**3*x**4*(e*x)**m/(m**4 + 18*m**3
+ 119*m**2 + 342*m + 360) + 28*a*b*c*m**2*x**4*(e*x)**m/(m**4 + 18*m**3 +
119*m**2 + 342*m + 360) + 126*a*b*c*m*x**4*(e*x)**m/(m**4 + 18*m**3 + 119*
m**2 + 342*m + 360) + 180*a*b*c*x**4*(e*x)**m/(m**4 + 18*m**3 + 119*m**2 +
 342*m + 360) + 2*a*b*d*m**3*x**5*(e*x)**m/(m**4 + 18*m**3 + 119*m**2 + 34
2*m + 360) + 26*a*b*d*m**2*x**5*(e*x)**m/(m**4 + 18*m**3 + 119*m**2 + 342*
m + 360) + 108*a*b*d*m*x**5*(e*x)**m/(m**4 + 18*m**3 + 119*m**2 + 342*m...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.26

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^2\,dx = \frac{b^2de^mx^6x^m}{m+6}+\frac{b^2ce^mx^5x^m}{m+5}+\frac{2\,abde^mx^5x^m}{m+5}+\frac{2\,abce^mx^4x^m}{m+4}+\frac{a^2de^mx^4x^m}{m+4}+\frac{a^2ce^mx^3x^m}{m+3}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^2+a*x)^2,x, algorithm="maxima")
```

output
```
b^2*d*e^m*x^6*x^m/(m + 6) + b^2*c*e^m*x^5*x^m/(m + 5) + 2*a*b*d*e^m*x^5*x^
m/(m + 5) + 2*a*b*c*e^m*x^4*x^m/(m + 4) + a^2*d*e^m*x^4*x^m/(m + 4) + a^2*
c*e^m*x^3*x^m/(m + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(91) = 182$.

Time = 0.18 (sec) , antiderivative size = 388, normalized size of antiderivative = 4.26

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^2\,dx = \frac{(ex)^m b^2dm^3x^6 + (ex)^m b^2cm^3x^5 + 2\,(ex)^m abdm^3x^5 + 12\,(ex)^m b^2dm^2x^6 + 2\,(ex)^m abcm^3x^4 + (ex)^m a^2c}{}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^2+a*x)^2,x, algorithm="giac")
```

output
```
((e*x)^m*b^2*d*m^3*x^6 + (e*x)^m*b^2*c*m^3*x^5 + 2*(e*x)^m*a*b*d*m^3*x^5 +
 12*(e*x)^m*b^2*d*m^2*x^6 + 2*(e*x)^m*a*b*c*m^3*x^4 + (e*x)^m*a^2*d*m^3*x^
4 + 13*(e*x)^m*b^2*c*m^2*x^5 + 26*(e*x)^m*a*b*d*m^2*x^5 + 47*(e*x)^m*b^2*d
*m*x^6 + (e*x)^m*a^2*c*m^3*x^3 + 28*(e*x)^m*a*b*c*m^2*x^4 + 14*(e*x)^m*a^2
*d*m^2*x^4 + 54*(e*x)^m*b^2*c*m*x^5 + 108*(e*x)^m*a*b*d*m*x^5 + 60*(e*x)^m
*b^2*d*x^6 + 15*(e*x)^m*a^2*c*m^2*x^3 + 126*(e*x)^m*a*b*c*m*x^4 + 63*(e*x)
^m*a^2*d*m*x^4 + 72*(e*x)^m*b^2*c*x^5 + 144*(e*x)^m*a*b*d*x^5 + 74*(e*x)^m
*a^2*c*m*x^3 + 180*(e*x)^m*a*b*c*x^4 + 90*(e*x)^m*a^2*d*x^4 + 120*(e*x)^m*
a^2*c*x^3)/(m^4 + 18*m^3 + 119*m^2 + 342*m + 360)
```

Mupad [B] (verification not implemented)

Time = 8.83 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.99

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^2\,dx = (e\,x)^m\left(\frac{a\,x^4\,(a\,d+2\,b\,c)\,\left(m^3+14\,m^2+63\,m+90\right)}{m^4+18\,m^3+119\,m^2+342\,m+360} + \frac{b\,x^5\,(2\,a\,d+b\,c)\,\left(m^3+13\,m^2+54\,m+72\right)}{m^4+18\,m^3+119\,m^2+342\,m+360} + \frac{a^2\,c\,x^3\,\left(m^3+15\,m^2+74\,m+120\right)}{m^4+18\,m^3+119\,m^2+342\,m+360} + \frac{b^2\,d\,x^6\,\left(m^3+12\,m^2+47\,m+60\right)}{m^4+18\,m^3+119\,m^2+342\,m+360}\right)$$

input
```
int((a*x + b*x^2)^2*(e*x)^m*(c + d*x),x)
```

output
```
(e*x)^m*((a*x^4*(a*d + 2*b*c)*(63*m + 14*m^2 + m^3 + 90))/(342*m + 119*m^2
 + 18*m^3 + m^4 + 360) + (b*x^5*(2*a*d + b*c)*(54*m + 13*m^2 + m^3 + 72))/
(342*m + 119*m^2 + 18*m^3 + m^4 + 360) + (a^2*c*x^3*(74*m + 15*m^2 + m^3 +
 120))/(342*m + 119*m^2 + 18*m^3 + m^4 + 360) + (b^2*d*x^6*(47*m + 12*m^2
+ m^3 + 60))/(342*m + 119*m^2 + 18*m^3 + m^4 + 360))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.74

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^2\,dx$$
$$= \frac{x^m e^m x^3 (b^2 d\, m^3 x^3 + 2abd\, m^3 x^2 + b^2 c\, m^3 x^2 + 12 b^2 d\, m^2 x^3 + a^2 d\, m^3 x + 2abc\, m^3 x + 26abd\, m^2 x^2 + 13b^2 c\, m}{}$$

input
```
int((e*x)^m*(d*x+c)*(b*x^2+a*x)^2,x)
```

output
```
(x**m*e**m*x**3*(a**2*c*m**3 + 15*a**2*c*m**2 + 74*a**2*c*m + 120*a**2*c +
 a**2*d*m**3*x + 14*a**2*d*m**2*x + 63*a**2*d*m*x + 90*a**2*d*x + 2*a*b*c*
m**3*x + 28*a*b*c*m**2*x + 126*a*b*c*m*x + 180*a*b*c*x + 2*a*b*d*m**3*x**2
 + 26*a*b*d*m**2*x**2 + 108*a*b*d*m*x**2 + 144*a*b*d*x**2 + b**2*c*m**3*x*
*2 + 13*b**2*c*m**2*x**2 + 54*b**2*c*m*x**2 + 72*b**2*c*x**2 + b**2*d*m**3
*x**3 + 12*b**2*d*m**2*x**3 + 47*b**2*d*m*x**3 + 60*b**2*d*x**3))/(m**4 +
18*m**3 + 119*m**2 + 342*m + 360)
```

3.231 $\int (ex)^m(c+dx)\left(ax+bx^2\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 60

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)\,dx=\frac{ac(ex)^{2+m}}{e^2(2+m)}+\frac{(bc+ad)(ex)^{3+m}}{e^3(3+m)}+\frac{bd(ex)^{4+m}}{e^4(4+m)}$$

output
```
a*c*(e*x)^(2+m)/e^2/(2+m)+(a*d+b*c)*(e*x)^(3+m)/e^3/(3+m)+b*d*(e*x)^(4+m)/
e^4/(4+m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)\,dx =\frac{x^2(ex)^m(a(4+m)(c(3+m)+d(2+m)x)+b(2+m)x(c(4+m)+d(3+m)x))}{(2+m)(3+m)(4+m)}$$

input
```
Integrate[(e*x)^m*(c + d*x)*(a*x + b*x^2),x]
```

output
```
(x^2*(e*x)^m*(a*(4 + m)*(c*(3 + m) + d*(2 + m)*x) + b*(2 + m)*x*(c*(4 + m)
 + d*(3 + m)*x)))/((2 + m)*(3 + m)*(4 + m))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax+bx^2\right)(c+dx)(ex)^m\,dx$$

$$\Big\downarrow \; 9$$

$$\frac{\int (ex)^{m+1}(a+bx)(c+dx)dx}{e}$$

$$\Big\downarrow \; 85$$

$$\frac{\int \left(ac(ex)^{m+1}+\frac{(bc+ad)(ex)^{m+2}}{e}+\frac{bd(ex)^{m+3}}{e^2}\right)dx}{e}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{(ex)^{m+3}(ad+bc)}{e^2(m+3)}+\frac{ac(ex)^{m+2}}{e(m+2)}+\frac{bd(ex)^{m+4}}{e^3(m+4)}}{e}$$

input
```
Int[(e*x)^m*(c + d*x)*(a*x + b*x^2),x]
```

output
```
((a*c*(e*x)^(2 + m))/(e*(2 + m)) + ((b*c + a*d)*(e*x)^(3 + m))/(e^2*(3 + m
)) + (b*d*(e*x)^(4 + m))/(e^3*(4 + m)))/e
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

method	result
norman	$\frac{(ad+bc)x^3 e^{m\ln(ex)}}{3+m}+\frac{ac\,x^2 e^{m\ln(ex)}}{2+m}+\frac{bd\,x^4 e^{m\ln(ex)}}{4+m}$
gosper	$\frac{(ex)^m\left(bd\,m^2x^2+ad\,m^2x+bc\,m^2x+5bdm\,x^2+ac\,m^2+6admx+6bcmx+6bd\,x^2+7acm+8adx+8cbx+12ac\right)x^2}{(4+m)(3+m)(2+m)}$
risch	$\frac{(ex)^m\left(bd\,m^2x^2+ad\,m^2x+bc\,m^2x+5bdm\,x^2+ac\,m^2+6admx+6bcmx+6bd\,x^2+7acm+8adx+8cbx+12ac\right)x^2}{(4+m)(3+m)(2+m)}$
orering	$\frac{\left(bd\,m^2x^2+ad\,m^2x+bc\,m^2x+5bdm\,x^2+ac\,m^2+6admx+6bcmx+6bd\,x^2+7acm+8adx+8cbx+12ac\right)x(ex)^m\left(b\,x^2+ax\right)}{(4+m)(3+m)(2+m)(bx+a)}$
parallelrisch	$\frac{x^4(ex)^m bd\,m^2+5x^4(ex)^m bdm+x^3(ex)^m ad\,m^2+x^3(ex)^m bc\,m^2+6x^4(ex)^m bd+6x^3(ex)^m adm+6x^3(ex)^m bcm+x^2(ex)^m ac\,m}{(4+m)(3+m)(2+m)}$

input
```
int((e*x)^m*(d*x+c)*(b*x^2+a*x),x,method=_RETURNVERBOSE)
```

output
```
(a*d+b*c)/(3+m)*x^3*exp(m*ln(e*x))+a*c/(2+m)*x^2*exp(m*ln(e*x))+b*d/(4+m)*
x^4*exp(m*ln(e*x))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.60

$$\int (ex)^m(c+dx)\left(ax+bx^2\right) dx = \frac{((bdm^2+5\,bdm+6\,bd)x^4+((bc+ad)m^2+8\,bc+8\,ad+6\,(bc+ad)m)x^3+(acm^2+7\,acm+12\,ac)x^2}{m^3+9\,m^2+26\,m+24}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^2+a*x),x, algorithm="fricas")
```

output
```
((b*d*m^2 + 5*b*d*m + 6*b*d)*x^4 + ((b*c + a*d)*m^2 + 8*b*c + 8*a*d + 6*(b
*c + a*d)*m)*x^3 + (a*c*m^2 + 7*a*c*m + 12*a*c)*x^2)*(e*x)^m/(m^3 + 9*m^2
+ 26*m + 24)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(53) = 106.

Time = 0.29 (sec) , antiderivative size = 425, normalized size of antiderivative = 7.08

$$\int (ex)^m(c+dx)\left(ax+bx^2\right) dx = \begin{cases} \frac{-\frac{ac}{2x^2}-\frac{ad}{x}-\frac{bc}{x}+bd\log(x)}{e^4} \\ \frac{-\frac{ac}{x}+ad\log(x)+bc\log(x)+bdx}{e^3} \\ \frac{ac\log(x)+adx+bcx+\frac{bdx^2}{2}}{e^2} \\ \frac{acm^2x^2(ex)^m}{m^3+9m^2+26m+24}+\frac{7acmx^2(ex)^m}{m^3+9m^2+26m+24}+\frac{12acx^2(ex)^m}{m^3+9m^2+26m+24}+\frac{adm^2x^3(ex)^m}{m^3+9m^2+26m+24}+\frac{6admx^3(ex)^m}{m^3+9m^2+26m+24}+\frac{8adx^3(ex)^m}{m^3+9m^2+26m+} \end{cases}$$

input
```
integrate((e*x)**m*(d*x+c)*(b*x**2+a*x),x)
```

output
```
Piecewise(((-a*c/(2*x**2) - a*d/x - b*c/x + b*d*log(x))/e**4, Eq(m, -4)),
((-a*c/x + a*d*log(x) + b*c*log(x) + b*d*x)/e**3, Eq(m, -3)), ((a*c*log(x)
 + a*d*x + b*c*x + b*d*x**2/2)/e**2, Eq(m, -2)), (a*c*m**2*x**2*(e*x)**m/(
m**3 + 9*m**2 + 26*m + 24) + 7*a*c*m*x**2*(e*x)**m/(m**3 + 9*m**2 + 26*m +
 24) + 12*a*c*x**2*(e*x)**m/(m**3 + 9*m**2 + 26*m + 24) + a*d*m**2*x**3*(e
*x)**m/(m**3 + 9*m**2 + 26*m + 24) + 6*a*d*m*x**3*(e*x)**m/(m**3 + 9*m**2
+ 26*m + 24) + 8*a*d*x**3*(e*x)**m/(m**3 + 9*m**2 + 26*m + 24) + b*c*m**2*
x**3*(e*x)**m/(m**3 + 9*m**2 + 26*m + 24) + 6*b*c*m*x**3*(e*x)**m/(m**3 +
9*m**2 + 26*m + 24) + 8*b*c*x**3*(e*x)**m/(m**3 + 9*m**2 + 26*m + 24) + b*
d*m**2*x**4*(e*x)**m/(m**3 + 9*m**2 + 26*m + 24) + 5*b*d*m*x**4*(e*x)**m/(
m**3 + 9*m**2 + 26*m + 24) + 6*b*d*x**4*(e*x)**m/(m**3 + 9*m**2 + 26*m + 2
4), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.15

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)\,dx=\frac{bde^mx^4x^m}{m+4}+\frac{bce^mx^3x^m}{m+3}+\frac{ade^mx^3x^m}{m+3}+\frac{ace^mx^2x^m}{m+2}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^2+a*x),x, algorithm="maxima")
```

output
```
b*d*e^m*x^4*x^m/(m + 4) + b*c*e^m*x^3*x^m/(m + 3) + a*d*e^m*x^3*x^m/(m + 3
) + a*c*e^m*x^2*x^m/(m + 2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(60) = 120.

Time = 0.13 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.88

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)\,dx$$
$$=\frac{(ex)^m\,bdm^2x^4+(ex)^m\,bcm^2x^3+(ex)^m\,adm^2x^3+5\,(ex)^m\,bdmx^4+(ex)^m\,acm^2x^2+6\,(ex)^m\,bcmx^3+6\,(}{m^3+9\,m^2+26\,n}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^2+a*x),x, algorithm="giac")
```

output
```
((e*x)^m*b*d*m^2*x^4 + (e*x)^m*b*c*m^2*x^3 + (e*x)^m*a*d*m^2*x^3 + 5*(e*x)
^m*b*d*m*x^4 + (e*x)^m*a*c*m^2*x^2 + 6*(e*x)^m*b*c*m*x^3 + 6*(e*x)^m*a*d*m
*x^3 + 6*(e*x)^m*b*d*x^4 + 7*(e*x)^m*a*c*m*x^2 + 8*(e*x)^m*b*c*x^3 + 8*(e*
x)^m*a*d*x^3 + 12*(e*x)^m*a*c*x^2)/(m^3 + 9*m^2 + 26*m + 24)
```

Mupad [B] (verification not implemented)

Time = 8.62 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.65

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)\,dx=(e\,x)^m\left(\frac{x^3\,(a\,d+b\,c)\,\left(m^2+6\,m+8\right)}{m^3+9\,m^2+26\,m+24}+\frac{a\,c\,x^2\,\left(m^2+7\,m+12\right)}{m^3+9\,m^2+26\,m+24}+\frac{b\,d\,x^4\,\left(m^2+5\,m+6\right)}{m^3+9\,m^2+26\,m+24}\right)$$

input
```
int((a*x + b*x^2)*(e*x)^m*(c + d*x),x)
```

output
```
(e*x)^m*((x^3*(a*d + b*c)*(6*m + m^2 + 8))/(26*m + 9*m^2 + m^3 + 24) + (a*
c*x^2*(7*m + m^2 + 12))/(26*m + 9*m^2 + m^3 + 24) + (b*d*x^4*(5*m + m^2 +
6))/(26*m + 9*m^2 + m^3 + 24))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.68

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)\,dx =\frac{x^m e^m x^2(bd\,m^2x^2+ad\,m^2x+bc\,m^2x+5bdm\,x^2+ac\,m^2+6admx+6bcmx+6bd\,x^2+7acm+8adx+}{m^3+9m^2+26m+24}$$

input
```
int((e*x)^m*(d*x+c)*(b*x^2+a*x),x)
```

output
```
(x**m*e**m*x**2*(a*c*m**2 + 7*a*c*m + 12*a*c + a*d*m**2*x + 6*a*d*m*x + 8*
a*d*x + b*c*m**2*x + 6*b*c*m*x + 8*b*c*x + b*d*m**2*x**2 + 5*b*d*m*x**2 +
6*b*d*x**2))/(m**3 + 9*m**2 + 26*m + 24)
```

3.232 $\int \frac{(ex)^m(c+dx)}{ax+bx^2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 50

$$\int \frac{(ex)^m(c+dx)}{ax+bx^2}\,dx = \frac{d(ex)^m}{bm} + \frac{(bc-ad)(ex)^m \operatorname{Hypergeometric2F1}\left(1,m,1+m,-\frac{bx}{a}\right)}{abm}$$

output
```
d*(e*x)^m/b/m+(-a*d+b*c)*(e*x)^m*hypergeom([1, m],[1+m],-b*x/a)/a/b/m
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{(ex)^m(c+dx)}{ax+bx^2}\,dx = \frac{(ex)^m\left(ac(1+m)+(-bc+ad)mx \operatorname{Hypergeometric2F1}\left(1,1+m,2+m,-\frac{bx}{a}\right)\right)}{a^2m(1+m)}$$

input
```
Integrate[((e*x)^m*(c + d*x))/(a*x + b*x^2),x]
```

output
```
((e*x)^m*(a*c*(1 + m) + (-(b*c) + a*d)*m*x*Hypergeometric2F1[1, 1 + m, 2 +
 m, -((b*x)/a)]))/(a^2*m*(1 + m))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.24, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {9, 88, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ex)^m}{ax+bx^2}\,dx$$

$$\Big\downarrow \text{9}$$

$$e\int \frac{(ex)^{m-1}(c+dx)}{a+bx}dx$$

$$\Big\downarrow \text{88}$$

$$e\left(\frac{c(ex)^m}{aem}-\frac{(bc-ad)\int \frac{(ex)^m}{a+bx}dx}{ae}\right)$$

$$\Big\downarrow \text{74}$$

$$e\left(\frac{c(ex)^m}{aem}-\frac{(ex)^{m+1}(bc-ad)\,\text{Hypergeometric2F1}\left(1,m+1,m+2,-\frac{bx}{a}\right)}{a^2e^2(m+1)}\right)$$

input
```
Int[((e*x)^m*(c + d*x))/(a*x + b*x^2),x]
```

output
```
e*((c*(e*x)^m)/(a*e*m) - ((b*c - a*d)*(e*x)^(1 + m)*Hypergeometric2F1[1, 1
 + m, 2 + m, -((b*x)/a)])/(a^2*e^2*(1 + m)))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

Maple [F]

$$\int \frac{(ex)^m (dx + c)}{b\,x^2 + ax} dx$$

input
```
int((e*x)^m*(d*x+c)/(b*x^2+a*x),x)
```

output
```
int((e*x)^m*(d*x+c)/(b*x^2+a*x),x)
```

Fricas [F]

$$\int \frac{(ex)^m(c + dx)}{ax + bx^2}\, dx = \int \frac{(dx + c)(ex)^m}{bx^2 + ax}\, dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^2+a*x),x, algorithm="fricas")
```

output
```
integral((d*x + c)*(e*x)^m/(b*x^2 + a*x), x)
```

Sympy [F]

$$\int \frac{(ex)^m(c+dx)}{ax+bx^2}\,dx = \int \frac{(ex)^m\,(c+dx)}{x\,(a+bx)}\,dx$$

input
```
integrate((e*x)**m*(d*x+c)/(b*x**2+a*x),x)
```

output
```
Integral((e*x)**m*(c + d*x)/(x*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(ex)^m(c+dx)}{ax+bx^2}\,dx = \int \frac{(dx+c)(ex)^m}{bx^2+ax}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^2+a*x),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^2 + a*x), x)
```

Giac [F]

$$\int \frac{(ex)^m(c+dx)}{ax+bx^2}\,dx = \int \frac{(dx+c)(ex)^m}{bx^2+ax}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^2+a*x),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^2 + a*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(c+dx)}{ax+bx^2}\,dx = \int \frac{(e\,x)^m\,(c+d\,x)}{b\,x^2+a\,x}\,dx$$

input
```
int(((e*x)^m*(c + d*x))/(a*x + b*x^2),x)
```

output
```
int(((e*x)^m*(c + d*x))/(a*x + b*x^2), x)
```

Reduce [F]

$$\int \frac{(ex)^m(c+dx)}{ax+bx^2}\,dx = \frac{e^m\left(x^m d-\left(\int \frac{x^m}{b\,x^2+ax}dx\right)adm+\left(\int \frac{x^m}{b\,x^2+ax}dx\right)bcm\right)}{bm}$$

input
```
int((e*x)^m*(d*x+c)/(b*x^2+a*x),x)
```

output
```
(e**m*(x**m*d - int(x**m/(a*x + b*x**2),x)*a*d*m + int(x**m/(a*x + b*x**2)
,x)*b*c*m))/(b*m)
```

3.233 $\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^2} dx$

Optimal result

Integrand size = 22, antiderivative size = 79

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^2} dx = -\frac{de(ex)^{-1+m}}{b(2-m)(a+bx)} - \frac{e\left(\frac{c}{1-m}-\frac{ad}{2b-bm}\right)(ex)^{-1+m}\text{Hypergeometric2F1}\left(2,-1+m,m,-\frac{bx}{a}\right)}{a^2}$$

output
```
-d*e*(e*x)^(-1+m)/b/(2-m)/(b*x+a)-e*(c/(1-m)-a*d/(-b*m+2*b))*(e*x)^(-1+m)*
hypergeom([2, -1+m],[m],-b*x/a)/a^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.85

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^2} dx = \frac{(ex)^m\left(\frac{a(bc-ad)}{a+bx}-\frac{(bc(-2+m)-ad(-1+m))\text{Hypergeometric2F1}\left(1,-1+m,m,-\frac{bx}{a}\right)}{-1+m}\right)}{a^2bx}$$

input
```
Integrate[((e*x)^m*(c + d*x))/(a*x + b*x^2)^2,x]
```

output
```
((e*x)^m*((a*(b*c - a*d))/(a + b*x) - ((b*c*(-2 + m) - a*d*(-1 + m))*Hyper
geometric2F1[1, -1 + m, m, -((b*x)/a)])/(-1 + m)))/(a^2*b*x)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 87, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ex)^m}{(ax+bx^2)^2}dx$$

$$\Big\downarrow \; 9$$

$$e^2\int \frac{(ex)^{m-2}(c+dx)}{(a+bx)^2}dx$$

$$\Big\downarrow \; 87$$

$$e^2\left(\frac{(ex)^{m-1}(bc-ad)}{abe(a+bx)}-\frac{(ad(1-m)-bc(2-m))\int \frac{(ex)^{m-2}}{a+bx}dx}{ab}\right)$$

$$\Big\downarrow \; 74$$

$$e^2\left(\frac{(ex)^{m-1}(ad(1-m)-bc(2-m))\,\text{Hypergeometric2F1}\left(1,m-1,m,-\frac{bx}{a}\right)}{a^2be(1-m)}+\frac{(ex)^{m-1}(bc-ad)}{abe(a+bx)}\right)$$

input
```
Int[((e*x)^m*(c + d*x))/(a*x + b*x^2)^2,x]
```

output
```
e^2*(((b*c - a*d)*(e*x)^(-1 + m))/(a*b*e*(a + b*x)) + ((a*d*(1 - m) - b*c*
(2 - m))*(e*x)^(-1 + m)*Hypergeometric2F1[1, -1 + m, m, -((b*x)/a)])/(a^2*
b*e*(1 - m)))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [F]

$$\int \frac{(ex)^m (dx+c)}{(b\,x^2+ax)^2}dx$$

input
```
int((e*x)^m*(d*x+c)/(b*x^2+a*x)^2,x)
```

output
```
int((e*x)^m*(d*x+c)/(b*x^2+a*x)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^2}\,dx=\int \frac{(dx+c)(ex)^m}{(bx^2+ax)^2}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^2+a*x)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(e*x)^m/(b^2*x^4 + 2*a*b*x^3 + a^2*x^2), x)
```

Sympy [F]

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^2}\,dx=\int \frac{(ex)^m\,(c+dx)}{x^2\,(a+bx)^2}\,dx$$

input
```
integrate((e*x)**m*(d*x+c)/(b*x**2+a*x)**2,x)
```

output
```
Integral((e*x)**m*(c + d*x)/(x**2*(a + b*x)**2), x)
```

Maxima [F]

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^2}\,dx=\int \frac{(dx+c)(ex)^m}{(bx^2+ax)^2}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^2+a*x)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^2 + a*x)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^2}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^2+ax)^2}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^2+a*x)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^2 + a*x)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^2}\,dx = \int \frac{(e\,x)^m\,(c+d\,x)}{(b\,x^2+a\,x)^2}\,dx$$

input
```
int(((e*x)^m*(c + d*x))/(a*x + b*x^2)^2,x)
```

output
```
int(((e*x)^m*(c + d*x))/(a*x + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^2}\,dx$$
$$= \frac{e^m\left(x^m d - \left(\int \frac{x^m}{b^2m\,x^4+2abm\,x^3-2b^2x^4+a^2m\,x^2-4ab\,x^3-2a^2x^2}dx\right)a^2d\,m^2x + 3\left(\int \frac{x^m}{b^2m\,x^4+2abm\,x^3-2b^2x^4+a^2m\,x^2-4ab\,x^3-2\right.$$

input
```
int((e*x)^m*(d*x+c)/(b*x^2+a*x)^2,x)
```

output

```
(e**m*(x**m*d - int(x**m/(a**2*m*x**2 - 2*a**2*x**2 + 2*a*b*m*x**3 - 4*a*b
*x**3 + b**2*m*x**4 - 2*b**2*x**4),x)*a**2*d*m**2*x + 3*int(x**m/(a**2*m*x
**2 - 2*a**2*x**2 + 2*a*b*m*x**3 - 4*a*b*x**3 + b**2*m*x**4 - 2*b**2*x**4)
,x)*a**2*d*m*x - 2*int(x**m/(a**2*m*x**2 - 2*a**2*x**2 + 2*a*b*m*x**3 - 4*
a*b*x**3 + b**2*m*x**4 - 2*b**2*x**4),x)*a**2*d*x + int(x**m/(a**2*m*x**2
- 2*a**2*x**2 + 2*a*b*m*x**3 - 4*a*b*x**3 + b**2*m*x**4 - 2*b**2*x**4),x)*
a*b*c*m**2*x - 4*int(x**m/(a**2*m*x**2 - 2*a**2*x**2 + 2*a*b*m*x**3 - 4*a*
b*x**3 + b**2*m*x**4 - 2*b**2*x**4),x)*a*b*c*m*x + 4*int(x**m/(a**2*m*x**2
 - 2*a**2*x**2 + 2*a*b*m*x**3 - 4*a*b*x**3 + b**2*m*x**4 - 2*b**2*x**4),x)
*a*b*c*x - int(x**m/(a**2*m*x**2 - 2*a**2*x**2 + 2*a*b*m*x**3 - 4*a*b*x**3
 + b**2*m*x**4 - 2*b**2*x**4),x)*a*b*d*m**2*x**2 + 3*int(x**m/(a**2*m*x**2
 - 2*a**2*x**2 + 2*a*b*m*x**3 - 4*a*b*x**3 + b**2*m*x**4 - 2*b**2*x**4),x)
*a*b*d*m*x**2 - 2*int(x**m/(a**2*m*x**2 - 2*a**2*x**2 + 2*a*b*m*x**3 - 4*a
*b*x**3 + b**2*m*x**4 - 2*b**2*x**4),x)*a*b*d*x**2 + int(x**m/(a**2*m*x**2
 - 2*a**2*x**2 + 2*a*b*m*x**3 - 4*a*b*x**3 + b**2*m*x**4 - 2*b**2*x**4),x)
*b**2*c*m**2*x**2 - 4*int(x**m/(a**2*m*x**2 - 2*a**2*x**2 + 2*a*b*m*x**3 -
 4*a*b*x**3 + b**2*m*x**4 - 2*b**2*x**4),x)*b**2*c*m*x**2 + 4*int(x**m/(a*
*2*m*x**2 - 2*a**2*x**2 + 2*a*b*m*x**3 - 4*a*b*x**3 + b**2*m*x**4 - 2*b**2
*x**4),x)*b**2*c*x**2))/(b*x*(a*m - 2*a + b*m*x - 2*b*x))
```

3.234 $\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 85

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^3}\,dx = -\frac{de^2(ex)^{-2+m}}{b(4-m)(a+bx)^2} - \frac{e^2\left(\frac{c}{2-m}-\frac{ad}{4b-bm}\right)(ex)^{-2+m}\,\text{Hypergeometric2F1}\left(3,-2+m,-1+m,-\frac{bx}{a}\right)}{a^3}$$

output
```
-d*e^2*(e*x)^(-2+m)/b/(4-m)/(b*x+a)^2-e^2*(c/(2-m)-a*d/(-b*m+4*b))*(e*x)^(
-2+m)*hypergeom([3, -2+m],[-1+m],-b*x/a)/a^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.87

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^3}\,dx = \frac{(ex)^m\left(\frac{a^2(bc-ad)}{(a+bx)^2}-\frac{(bc(-4+m)-ad(-2+m))\,\text{Hypergeometric2F1}\left(2,-2+m,-1+m,-\frac{bx}{a}\right)}{-2+m}\right)}{2a^3bx^2}$$

input
```
Integrate[((e*x)^m*(c + d*x))/(a*x + b*x^2)^3,x]
```

output
```
((e*x)^m*((a^2*(b*c - a*d))/(a + b*x)^2 - ((b*c*(-4 + m) - a*d*(-2 + m))*H
ypergeometric2F1[2, -2 + m, -1 + m, -((b*x)/a)])/(-2 + m)))/(2*a^3*b*x^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 87, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ex)^m}{(ax+bx^2)^3}dx$$

$$\Big\downarrow 9$$

$$e^3\int \frac{(ex)^{m-3}(c+dx)}{(a+bx)^3}dx$$

$$\Big\downarrow 87$$

$$e^3\left(\frac{(ex)^{m-2}(bc-ad)}{2abe(a+bx)^2}-\frac{(ad(2-m)-bc(4-m))\int \frac{(ex)^{m-3}}{(a+bx)^2}dx}{2ab}\right)$$

$$\Big\downarrow 74$$

$$e^3\left(\frac{(ex)^{m-2}(ad(2-m)-bc(4-m))\,\text{Hypergeometric2F1}\left(2,m-2,m-1,-\frac{bx}{a}\right)}{2a^3be(2-m)}+\frac{(ex)^{m-2}(bc-ad)}{2abe(a+bx)^2}\right)$$

input
```
Int[((e*x)^m*(c + d*x))/(a*x + b*x^2)^3,x]
```

output
```
e^3*(((b*c - a*d)*(e*x)^(-2 + m))/(2*a*b*e*(a + b*x)^2) + ((a*d*(2 - m) -
b*c*(4 - m))*(e*x)^(-2 + m)*Hypergeometric2F1[2, -2 + m, -1 + m, -((b*x)/a
)])/(2*a^3*b*e*(2 - m)))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [F]

$$\int \frac{(ex)^m (dx+c)}{(b\,x^2+ax)^3}dx$$

input
```
int((e*x)^m*(d*x+c)/(b*x^2+a*x)^3,x)
```

output
```
int((e*x)^m*(d*x+c)/(b*x^2+a*x)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^3}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^2+ax)^3}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^2+a*x)^3,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(e*x)^m/(b^3*x^6 + 3*a*b^2*x^5 + 3*a^2*b*x^4 + a^3*x^3)
, x)
```

Sympy [F]

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^3}\,dx = \int \frac{(ex)^m\,(c+dx)}{x^3\,(a+bx)^3}\,dx$$

input
```
integrate((e*x)**m*(d*x+c)/(b*x**2+a*x)**3,x)
```

output
```
Integral((e*x)**m*(c + d*x)/(x**3*(a + b*x)**3), x)
```

Maxima [F]

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^3}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^2+ax)^3}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^2+a*x)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^2 + a*x)^3, x)
```

Giac [F]

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^3}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^2+ax)^3}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^2+a*x)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^2 + a*x)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^3}\,dx = \int \frac{(e\,x)^m\,(c+d\,x)}{(b\,x^2+a\,x)^3}\,dx$$

input
```
int(((e*x)^m*(c + d*x))/(a*x + b*x^2)^3,x)
```

output
```
int(((e*x)^m*(c + d*x))/(a*x + b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m(c+dx)}{(ax+bx^2)^3}\,dx = \text{Too large to display}$$

input
```
int((e*x)^m*(d*x+c)/(b*x^2+a*x)^3,x)
```

output
```
(e**m*(x**m*d - int(x**m/(a**3*m*x**3 - 4*a**3*x**3 + 3*a**2*b*m*x**4 - 12
*a**2*b*x**4 + 3*a*b**2*m*x**5 - 12*a*b**2*x**5 + b**3*m*x**6 - 4*b**3*x**
6),x)*a**3*d*m**2*x**2 + 6*int(x**m/(a**3*m*x**3 - 4*a**3*x**3 + 3*a**2*b*
m*x**4 - 12*a**2*b*x**4 + 3*a*b**2*m*x**5 - 12*a*b**2*x**5 + b**3*m*x**6 -
 4*b**3*x**6),x)*a**3*d*m*x**2 - 8*int(x**m/(a**3*m*x**3 - 4*a**3*x**3 + 3
*a**2*b*m*x**4 - 12*a**2*b*x**4 + 3*a*b**2*m*x**5 - 12*a*b**2*x**5 + b**3*
m*x**6 - 4*b**3*x**6),x)*a**3*d*x**2 + int(x**m/(a**3*m*x**3 - 4*a**3*x**3
 + 3*a**2*b*m*x**4 - 12*a**2*b*x**4 + 3*a*b**2*m*x**5 - 12*a*b**2*x**5 + b
**3*m*x**6 - 4*b**3*x**6),x)*a**2*b*c*m**2*x**2 - 8*int(x**m/(a**3*m*x**3
- 4*a**3*x**3 + 3*a**2*b*m*x**4 - 12*a**2*b*x**4 + 3*a*b**2*m*x**5 - 12*a*
b**2*x**5 + b**3*m*x**6 - 4*b**3*x**6),x)*a**2*b*c*m*x**2 + 16*int(x**m/(a
**3*m*x**3 - 4*a**3*x**3 + 3*a**2*b*m*x**4 - 12*a**2*b*x**4 + 3*a*b**2*m*x
**5 - 12*a*b**2*x**5 + b**3*m*x**6 - 4*b**3*x**6),x)*a**2*b*c*x**2 - 2*int
(x**m/(a**3*m*x**3 - 4*a**3*x**3 + 3*a**2*b*m*x**4 - 12*a**2*b*x**4 + 3*a*
b**2*m*x**5 - 12*a*b**2*x**5 + b**3*m*x**6 - 4*b**3*x**6),x)*a**2*b*d*m**2
*x**3 + 12*int(x**m/(a**3*m*x**3 - 4*a**3*x**3 + 3*a**2*b*m*x**4 - 12*a**2
*b*x**4 + 3*a*b**2*m*x**5 - 12*a*b**2*x**5 + b**3*m*x**6 - 4*b**3*x**6),x)
*a**2*b*d*m*x**3 - 16*int(x**m/(a**3*m*x**3 - 4*a**3*x**3 + 3*a**2*b*m*x**
4 - 12*a**2*b*x**4 + 3*a*b**2*m*x**5 - 12*a*b**2*x**5 + b**3*m*x**6 - 4*b*
*3*x**6),x)*a**2*b*d*x**3 + 2*int(x**m/(a**3*m*x**3 - 4*a**3*x**3 + 3*a...
```

3.235 $\int (ex)^m(c+dx)\left(ax+bx^2\right)^p\,dx$

Optimal result

Integrand size = 22, antiderivative size = 96

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^p\,dx=\frac{d(ex)^m\left(ax+bx^2\right)^{1+p}}{b(2+m+2p)}+\left(\frac{c}{a(1+m+p)}-\frac{d}{b(2+m+2p)}\right)(ex)^m\left(ax+bx^2\right)^{1+p}\text{Hypergeometric2F1}\left(1,2+m+2p,2+m+p,-\frac{bx}{a}\right)$$

output
```
d*(e*x)^m*(b*x^2+a*x)^(p+1)/b/(2+2*p+m)+(c/a/(1+m+p)-d/b/(2+2*p+m))*(e*x)^
m*(b*x^2+a*x)^(p+1)*hypergeom([1, 2+2*p+m],[2+m+p],-b*x/a)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.95

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^p\,dx$$
$$=\frac{x(ex)^m(x(a+bx))^p\left(d(a+bx)+\frac{(-ad(1+m+p)+bc(2+m+2p))\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,1+m+p,2+m+p,-\frac{bx}{a}\right)}{1+m+p}\right)}{b(2+m+2p)}$$

input
```
Integrate[(e*x)^m*(c + d*x)*(a*x + b*x^2)^p,x]
```

output
```
(x*(e*x)^m*(x*(a + b*x))^p*(d*(a + b*x) + ((-(a*d*(1 + m + p)) + b*c*(2 +
m + 2*p))*Hypergeometric2F1[-p, 1 + m + p, 2 + m + p, -((b*x)/a)])/((1 + m
 + p)*(1 + (b*x)/a)^p)))/(b*(2 + m + 2*p))
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1221, 1137, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(ex)^m\left(ax+bx^2\right)^p\,dx$$
$$\Big\downarrow \ 1221$$
$$\left(c-\frac{ad(m+p+1)}{b(m+2p+2)}\right)\int (ex)^m\left(bx^2+ax\right)^p dx+\frac{d(ex)^m\left(ax+bx^2\right)^{p+1}}{b(m+2p+2)}$$
$$\Big\downarrow \ 1137$$
$$(ex)^mx^{-m-p}(a+bx)^{-p}\left(ax+bx^2\right)^p\left(c-\frac{ad(m+p+1)}{b(m+2p+2)}\right)\int x^{m+p}(a+bx)^pdx+\frac{d(ex)^m\left(ax+bx^2\right)^{p+1}}{b(m+2p+2)}$$

$$\Big\downarrow \text{76}$$

$$(ex)^m x^{-m-p}\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p\left(c-\frac{ad(m+p+1)}{b(m+2p+2)}\right)\int x^{m+p}\left(\frac{bx}{a}+1\right)^p dx+\frac{d(ex)^m\left(ax+bx^2\right)^{p+1}}{b(m+2p+2)}$$

$$\Big\downarrow \text{74}$$

$$\frac{x(ex)^m\left(\frac{bx}{a}+1\right)^{-p}\left(ax+bx^2\right)^p\left(c-\frac{ad(m+p+1)}{b(m+2p+2)}\right)\text{Hypergeometric2F1}\left(-p,m+p+1,m+p+2,-\frac{bx}{a}\right)}{m+p+1}+\frac{d(ex)^m\left(ax+bx^2\right)^{p+1}}{b(m+2p+2)}$$

input
```
Int[(e*x)^m*(c + d*x)*(a*x + b*x^2)^p,x]
```

output
```
(d*(e*x)^m*(a*x + b*x^2)^(1 + p))/(b*(2 + m + 2*p)) + ((c - (a*d*(1 + m +
p))/(b*(2 + m + 2*p)))*x*(e*x)^m*(a*x + b*x^2)^p*Hypergeometric2F1[-p, 1 +
 m + p, 2 + m + p, -((b*x)/a)])/((1 + m + p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 1137
```
Int[((e_.)*(x_))^(m_)*((b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(
e*x)^m*((b*x + c*x^2)^p/(x^(m + p)*(b + c*x)^p))   Int[x^(m + p)*(b + c*x)^
p, x], x] /; FreeQ[{b, c, e, m}, x]
```

rule 1221
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] + Simp[(m*(g*(c*d - b*e) + c*e*f) + e*(p + 1)*(2*c
*f - b*g))/(c*e*(m + 2*p + 2))   Int[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x
] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*d^2 - b*d*e + a*e^2, 0]
 && NeQ[m + 2*p + 2, 0]
```

Maple [F]

$$\int (ex)^m (dx+c) \left(b\,x^2+ax\right)^p dx$$

input
```
int((e*x)^m*(d*x+c)*(b*x^2+a*x)^p,x)
```

output
```
int((e*x)^m*(d*x+c)*(b*x^2+a*x)^p,x)
```

Fricas [F]

$$\int (ex)^m(c+dx) \left(ax+bx^2\right)^p \, dx = \int (dx+c)\left(bx^2+ax\right)^p(ex)^m \, dx$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^2+a*x)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(b*x^2 + a*x)^p*(e*x)^m, x)
```

Sympy [F]

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^p\,dx=\int (ex)^m\,(x(a+bx))^p\,(c+dx)\,dx$$

input
```
integrate((e*x)**m*(d*x+c)*(b*x**2+a*x)**p,x)
```

output
```
Integral((e*x)**m*(x*(a + b*x))**p*(c + d*x), x)
```

Maxima [F]

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^p\,dx=\int (dx+c)\left(bx^2+ax\right)^p(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^2+a*x)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(b*x^2 + a*x)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m(c+dx)\left(ax+bx^2\right)^p\,dx=\int (dx+c)\left(bx^2+ax\right)^p(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^2+a*x)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*x^2 + a*x)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m(c+dx) \left(ax+bx^2\right)^p \, dx = \int \left(b\,x^2+a\,x\right)^p\,{\left(e\,x\right)}^m\,\left(c+d\,x\right) \,dx$$

input
```
int((a*x + b*x^2)^p*(e*x)^m*(c + d*x),x)
```

output
```
int((a*x + b*x^2)^p*(e*x)^m*(c + d*x), x)
```

Reduce [F]

$$\int (ex)^m(c+dx) \left(ax+bx^2\right)^p \, dx = \text{too large to display}$$

input
```
int((e*x)^m*(d*x+c)*(b*x^2+a*x)^p,x)
```

output
```
(e**m*( - x**m*(a*x + b*x**2)**p*a**2*d*m*p - x**m*(a*x + b*x**2)**p*a**2*
d*p**2 - x**m*(a*x + b*x**2)**p*a**2*d*p + x**m*(a*x + b*x**2)**p*a*b*c*m*
p + 2*x**m*(a*x + b*x**2)**p*a*b*c*p**2 + 2*x**m*(a*x + b*x**2)**p*a*b*c*p
 + x**m*(a*x + b*x**2)**p*a*b*d*m*p*x + 2*x**m*(a*x + b*x**2)**p*a*b*d*p**
2*x + x**m*(a*x + b*x**2)**p*b**2*c*m**2*x + 4*x**m*(a*x + b*x**2)**p*b**2
*c*m*p*x + 2*x**m*(a*x + b*x**2)**p*b**2*c*m*x + 4*x**m*(a*x + b*x**2)**p*
b**2*c*p**2*x + 4*x**m*(a*x + b*x**2)**p*b**2*c*p*x + x**m*(a*x + b*x**2)*
*p*b**2*d*m**2*x**2 + 4*x**m*(a*x + b*x**2)**p*b**2*d*m*p*x**2 + x**m*(a*x
 + b*x**2)**p*b**2*d*m*x**2 + 4*x**m*(a*x + b*x**2)**p*b**2*d*p**2*x**2 +
2*x**m*(a*x + b*x**2)**p*b**2*d*p*x**2 + int((x**m*(a*x + b*x**2)**p)/(a*m
**3*x + 6*a*m**2*p*x + 3*a*m**2*x + 12*a*m*p**2*x + 12*a*m*p*x + 2*a*m*x +
 8*a*p**3*x + 12*a*p**2*x + 4*a*p*x + b*m**3*x**2 + 6*b*m**2*p*x**2 + 3*b*
m**2*x**2 + 12*b*m*p**2*x**2 + 12*b*m*p*x**2 + 2*b*m*x**2 + 8*b*p**3*x**2
+ 12*b*p**2*x**2 + 4*b*p*x**2),x)*a**3*d*m**5*p + 8*int((x**m*(a*x + b*x**
2)**p)/(a*m**3*x + 6*a*m**2*p*x + 3*a*m**2*x + 12*a*m*p**2*x + 12*a*m*p*x
+ 2*a*m*x + 8*a*p**3*x + 12*a*p**2*x + 4*a*p*x + b*m**3*x**2 + 6*b*m**2*p*
x**2 + 3*b*m**2*x**2 + 12*b*m*p**2*x**2 + 12*b*m*p*x**2 + 2*b*m*x**2 + 8*b
*p**3*x**2 + 12*b*p**2*x**2 + 4*b*p*x**2),x)*a**3*d*m**4*p**2 + 4*int((x**
m*(a*x + b*x**2)**p)/(a*m**3*x + 6*a*m**2*p*x + 3*a*m**2*x + 12*a*m*p**2*x
 + 12*a*m*p*x + 2*a*m*x + 8*a*p**3*x + 12*a*p**2*x + 4*a*p*x + b*m**3*x...
```

3.236 $\int (ex)^{1+p}(2b+3cx)\left(bx+cx^2\right)^p\,dx$

Optimal result

Integrand size = 27, antiderivative size = 26

$$\int (ex)^{1+p}(2b+3cx)\left(bx+cx^2\right)^p\,dx=\frac{(ex)^{1+p}\left(bx+cx^2\right)^{1+p}}{1+p}$$

output
```
(e*x)^(p+1)*(c*x^2+b*x)^(p+1)/(p+1)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92

$$\int (ex)^{1+p}(2b+3cx)\left(bx+cx^2\right)^p\,dx=\frac{ex(ex)^p(x(b+cx))^{1+p}}{1+p}$$

input
```
Integrate[(e*x)^(1 + p)*(2*b + 3*c*x)*(b*x + c*x^2)^p,x]
```

output
```
(e*x*(e*x)^p*(x*(b + c*x))^(1 + p))/(1 + p)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1217}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2b+3cx)(ex)^{p+1}\left(bx+cx^2\right)^p\,dx$$

$$\Big\downarrow \ 1217$$

$$\frac{(ex)^{p+1}\left(bx+cx^2\right)^{p+1}}{p+1}$$

input
```
Int[(e*x)^(1 + p)*(2*b + 3*c*x)*(b*x + c*x^2)^p,x]
```

output
```
((e*x)^(1 + p)*(b*x + c*x^2)^(1 + p))/(1 + p)
```

Defintions of rubi rules used

rule 1217
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p + 1
)/(c*(m + 2*p + 2))), x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && EqQ[c*
d^2 - b*d*e + a*e^2, 0] && EqQ[c*e*f*(m + 2*p + 2) + g*(c*d*m - b*e*(m + p
+ 1)), 0]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.19

method	result
gosper	$\frac{x(cx+b)(ex)^{p+1}(c\,x^2+bx)^p}{p+1}$
orering	$\frac{x(cx+b)(ex)^{p+1}(c\,x^2+bx)^p}{p+1}$
parallelrisch	$\frac{x^2(ex)^{p+1}(x(cx+b))^p bc+x(ex)^{p+1}(x(cx+b))^p b^2}{b(p+1)}$
risch	$x(cx+b)x^p(cx+b)^p \mathrm{e}^{-\frac{i\,\mathrm{csgn}(ix(cx+b))\,\mathrm{csgn}(i(cx+b))\pi\,\mathrm{csgn}(ix)p}{2}+\frac{i\,\mathrm{csgn}(ix)\pi\,\mathrm{csgn}(iex)^2p}{2}+\frac{i\,\mathrm{csgn}(ie)\pi\,\mathrm{csgn}(iex)^2}{2}-\frac{i\,\mathrm{csgn}(ie)\,\mathrm{csgn}(ix)\pi}{2}$

input
```
int((e*x)^(p+1)*(3*c*x+2*b)*(c*x^2+b*x)^p,x,method=_RETURNVERBOSE)
```

output
```
x*(c*x+b)/(p+1)*(e*x)^(p+1)*(c*x^2+b*x)^p
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.27

$$\int (ex)^{1+p}(2b+3cx)\left(bx+cx^2\right)^p\,dx=\frac{(cx^2+bx)(cx^2+bx)^p(ex)^{p+1}}{p+1}$$

input
```
integrate((e*x)^(p+1)*(3*c*x+2*b)*(c*x^2+b*x)^p,x, algorithm="fricas")
```

output
```
(c*x^2 + b*x)*(c*x^2 + b*x)^p*(e*x)^(p + 1)/(p + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(20) = 40.

Time = 0.78 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int (ex)^{1+p}(2b+3cx)\left(bx+cx^2\right)^p\,dx=\begin{cases}\frac{bx(ex)^{p+1}(bx+cx^2)^p}{p+1}+\frac{cx^2(ex)^{p+1}(bx+cx^2)^p}{p+1} & \text{for } p\neq -1\\ 2\log{(x)}+\log{\left(\frac{b}{c}+x\right)} & \text{otherwise}\end{cases}$$

input
```
integrate((e*x)**(p+1)*(3*c*x+2*b)*(c*x**2+b*x)**p,x)
```

output
```
Piecewise((b*x*(e*x)**(p + 1)*(b*x + c*x**2)**p/(p + 1) + c*x**2*(e*x)**(p
 + 1)*(b*x + c*x**2)**p/(p + 1), Ne(p, -1)), (2*log(x) + log(b/c + x), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int (ex)^{1+p}(2b+3cx)\left(bx+cx^2\right)^p\,dx=\frac{(ce^{p+1}x^3+be^{p+1}x^2)e^{(p\log(cx+b)+2\,p\log(x))}}{p+1}$$

input
```
integrate((e*x)^(p+1)*(3*c*x+2*b)*(c*x^2+b*x)^p,x, algorithm="maxima")
```

output
```
(c*e^(p + 1)*x^3 + b*e^(p + 1)*x^2)*e^(p*log(c*x + b) + 2*p*log(x))/(p + 1
)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(26) = 52.

Time = 0.21 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.35

$$\int (ex)^{1+p}(2b+3cx)\left(bx+cx^2\right)^p\,dx$$
$$=\frac{cx^2e^{(p\log(cx+b)+p\log(e)+2\,p\log(x)+\log(e)+\log(x))}+bxe^{(p\log(cx+b)+p\log(e)+2\,p\log(x)+\log(e)+\log(x))}}{p+1}$$

input
```
integrate((e*x)^(p+1)*(3*c*x+2*b)*(c*x^2+b*x)^p,x, algorithm="giac")
```

output
```
(c*x^2*e^(p*log(c*x + b) + p*log(e) + 2*p*log(x) + log(e) + log(x)) + b*x*
e^(p*log(c*x + b) + p*log(e) + 2*p*log(x) + log(e) + log(x)))/(p + 1)
```

Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.73

$$\int (ex)^{1+p}(2b+3cx)\left(bx+cx^2\right)^p\,dx=\left(c\,x^2+b\,x\right)^p\left(\frac{c\,x^2\,(e\,x)^{p+1}}{p+1}+\frac{b\,x\,(e\,x)^{p+1}}{p+1}\right)$$

input
```
int((b*x + c*x^2)^p*(e*x)^(p + 1)*(2*b + 3*c*x),x)
```

output
```
(b*x + c*x^2)^p*((c*x^2*(e*x)^(p + 1))/(p + 1) + (b*x*(e*x)^(p + 1))/(p +
1))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int (ex)^{1+p}(2b+3cx)\left(bx+cx^2\right)^p\,dx=\frac{x^p e^p(c\,x^2+bx)^p\,e\,x^2(cx+b)}{p+1}$$

input
```
int((e*x)^(p+1)*(3*c*x+2*b)*(c*x^2+b*x)^p,x)
```

output
```
(x**p*e**p*(b*x + c*x**2)**p*e*x**2*(b + c*x))/(p + 1)
```

3.237 $\int x^2(c+dx)\sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 167

$$\int x^2(c+dx)\sqrt{ax^2+bx^3}\,dx = -\frac{2a^3(bc-ad)\left(ax^2+bx^3\right)^{3/2}}{3b^5x^3} + \frac{2a^2(3bc-4ad)\left(ax^2+bx^3\right)^{5/2}}{5b^5x^5} - \frac{6a(bc-2ad)\left(ax^2+bx^3\right)^{7/2}}{7b^5x^7} + \frac{2(bc-4ad)\left(ax^2+bx^3\right)^{9/2}}{9b^5x^9} + \frac{2d\left(ax^2+bx^3\right)^{11/2}}{11b^5x^{11}}$$

output
```
-2/3*a^3*(-a*d+b*c)*(b*x^3+a*x^2)^(3/2)/b^5/x^3+2/5*a^2*(-4*a*d+3*b*c)*(b*
x^3+a*x^2)^(5/2)/b^5/x^5-6/7*a*(-2*a*d+b*c)*(b*x^3+a*x^2)^(7/2)/b^5/x^7+2/
9*(-4*a*d+b*c)*(b*x^3+a*x^2)^(9/2)/b^5/x^9+2/11*d*(b*x^3+a*x^2)^(11/2)/b^5
/x^11
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.56

$$\int x^2(c+dx)\sqrt{ax^2+bx^3}\,dx$$
$$=\frac{2(x^2(a+bx))^{3/2}\left(128a^4d+35b^4x^3(11c+9dx)+24a^2b^2x(11c+10dx)-16a^3b(11c+12dx)-10ab^3x^2(3\right.}{3465b^5x^3}$$

input
```
Integrate[x^2*(c + d*x)*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*(x^2*(a + b*x))^(3/2)*(128*a^4*d + 35*b^4*x^3*(11*c + 9*d*x) + 24*a^2*b
^2*x*(11*c + 10*d*x) - 16*a^3*b*(11*c + 12*d*x) - 10*a*b^3*x^2*(33*c + 28*
d*x)))/(3465*b^5*x^3)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1945, 1922, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{ax^2+bx^3}(c+dx)\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(11bc-8ad)\int x^2\sqrt{bx^3+ax^2}dx}{11b}+\frac{2dx\left(ax^2+bx^3\right)^{3/2}}{11b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(11bc-8ad)\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{9b}-\frac{2a\int x\sqrt{bx^3+ax^2}dx}{3b}\right)}{11b}+\frac{2dx\left(ax^2+bx^3\right)^{3/2}}{11b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(11bc-8ad)\left(\frac{2(ax^2+bx^3)^{3/2}}{9b}-\frac{2a\left(\frac{2(ax^2+bx^3)^{3/2}}{7bx}-\frac{4a\int\sqrt{bx^3+ax^2}dx}{7b}\right)}{3b}\right)}{11b}+\frac{2dx(ax^2+bx^3)^{3/2}}{11b}$$

$\Big\downarrow$ 1908

$$\frac{(11bc-8ad)\left(\frac{2(ax^2+bx^3)^{3/2}}{9b}-\frac{2a\left(\frac{2(ax^2+bx^3)^{3/2}}{7bx}-\frac{4a\left(\frac{2(ax^2+bx^3)^{3/2}}{5bx^2}-\frac{2a\int\frac{\sqrt{bx^3+ax^2}}{x}dx}{5b}\right)}{7b}\right)}{3b}\right)}{11b}+\frac{2dx(ax^2+bx^3)^{3/2}}{11b}$$

$\Big\downarrow$ 1920

$$\frac{\left(\frac{2(ax^2+bx^3)^{3/2}}{9b}-\frac{2a\left(\frac{2(ax^2+bx^3)^{3/2}}{7bx}-\frac{4a\left(\frac{2(ax^2+bx^3)^{3/2}}{5bx^2}-\frac{4a(ax^2+bx^3)^{3/2}}{15b^2x^3}\right)}{7b}\right)}{3b}\right)(11bc-8ad)}{11b}+\frac{2dx(ax^2+bx^3)^{3/2}}{11b}$$

input
```
Int[x^2*(c + d*x)*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*d*x*(a*x^2 + b*x^3)^(3/2))/(11*b) + ((11*b*c - 8*a*d)*((2*(a*x^2 + b*x^
3)^(3/2))/(9*b) - (2*a*((2*(a*x^2 + b*x^3)^(3/2))/(7*b*x) - (4*a*((-4*a*(a
*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*(a*x^2 + b*x^3)^(3/2))/(5*b*x^2)))/
(7*b)))/(3*b)))/(11*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 1.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.34

method	result
pseudoelliptic	$-\frac{32(bx+a)^{\frac{3}{2}}\left(-\frac{45x^2\left(c+\frac{7dx}{9}\right)b^3}{16}+\frac{9xa\left(\frac{5dx}{6}+c\right)b^2}{4}-\frac{3a^2(dx+c)b}{2}+a^3d\right)}{315b^4}$
gosper	$\frac{2(bx+a)\left(315d\,x^4b^4-280a\,b^3d\,x^3+385b^4c\,x^3+240a^2b^2d\,x^2-330a\,b^3c\,x^2-192a^3bdx+264a^2b^2cx+128a^4d-176a^3bc\right)\sqrt{b\,x^3-}}{3465b^5x}$
default	$\frac{2(bx+a)\left(315d\,x^4b^4-280a\,b^3d\,x^3+385b^4c\,x^3+240a^2b^2d\,x^2-330a\,b^3c\,x^2-192a^3bdx+264a^2b^2cx+128a^4d-176a^3bc\right)\sqrt{b\,x^3-}}{3465b^5x}$
orering	$\frac{2(bx+a)\left(315d\,x^4b^4-280a\,b^3d\,x^3+385b^4c\,x^3+240a^2b^2d\,x^2-330a\,b^3c\,x^2-192a^3bdx+264a^2b^2cx+128a^4d-176a^3bc\right)\sqrt{b\,x^3-}}{3465b^5x}$
risch	$\frac{2\sqrt{x^2(bx+a)}\,(315b^5d\,x^5+35a\,b^4d\,x^4+385b^5c\,x^4-40a^2b^3d\,x^3+55a\,b^4c\,x^3+48a^3b^2d\,x^2-66a^2b^3c\,x^2-64a^4bdx+88a^3b^2cx+}{3465x\,b^5}$
trager	$\frac{2(315b^5d\,x^5+35a\,b^4d\,x^4+385b^5c\,x^4-40a^2b^3d\,x^3+55a\,b^4c\,x^3+48a^3b^2d\,x^2-66a^2b^3c\,x^2-64a^4bdx+88a^3b^2cx+128a^5d-176}{3465b^5x}$

input
```
int(x^2*(d*x+c)*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-32/315*(b*x+a)^(3/2)*(-45/16*x^2*(c+7/9*d*x)*b^3+9/4*x*a*(5/6*d*x+c)*b^2-
3/2*a^2*(d*x+c)*b+a^3*d)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.77

$$\int x^2(c+dx)\sqrt{ax^2+bx^3}\,dx$$
$$=\frac{2\left(315\,b^5dx^5-176\,a^4bc+128\,a^5d+35\,(11\,b^5c+ab^4d)x^4+5\,(11\,ab^4c-8\,a^2b^3d)x^3-6\,(11\,a^2b^3c-8\,a^3b^2\right.}{3465\,b^5x}$$

input
```
integrate(x^2*(d*x+c)*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/3465*(315*b^5*d*x^5 - 176*a^4*b*c + 128*a^5*d + 35*(11*b^5*c + a*b^4*d)*
x^4 + 5*(11*a*b^4*c - 8*a^2*b^3*d)*x^3 - 6*(11*a^2*b^3*c - 8*a^3*b^2*d)*x^
2 + 8*(11*a^3*b^2*c - 8*a^4*b*d)*x)*sqrt(b*x^3 + a*x^2)/(b^5*x)
```

Sympy [F]

$$\int x^2(c+dx)\sqrt{ax^2+bx^3}\,dx=\int x^2\sqrt{x^2\,(a+bx)}(c+dx)\;dx$$

input
```
integrate(x**2*(d*x+c)*(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**2*sqrt(x**2*(a + b*x))*(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.72

$$\int x^2(c+dx)\sqrt{ax^2+bx^3}\,dx = \frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx+a}c}{315\,b^4} + \frac{2\,(315\,b^5x^5+35\,ab^4x^4-40\,a^2b^3x^3+48\,a^3b^2x^2-64\,a^4bx+128\,a^5)\sqrt{bx+a}d}{3465\,b^5}$$

input
```
integrate(x^2*(d*x+c)*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqrt
(b*x + a)*c/b^4 + 2/3465*(315*b^5*x^5 + 35*a*b^4*x^4 - 40*a^2*b^3*x^3 + 48
*a^3*b^2*x^2 - 64*a^4*b*x + 128*a^5)*sqrt(b*x + a)*d/b^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.71

$$\int x^2(c+dx)\sqrt{ax^2+bx^3}\,dx = \frac{2\left(\frac{99\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)ac\,\mathrm{sgn}(x)}{b^3}+\frac{11\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)\right.}{b^3}\right.}{} + \frac{32\left(11\,a^{\frac{9}{2}}bc-8\,a^{\frac{11}{2}}d\right)\mathrm{sgn}(x)}{3465\,b^5}$$

input
```
integrate(x^2*(d*x+c)*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/3465*(99*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*
a^2 - 35*sqrt(b*x + a)*a^3)*a*c*sgn(x)/b^3 + 11*(35*(b*x + a)^(9/2) - 180*
(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 31
5*sqrt(b*x + a)*a^4)*c*sgn(x)/b^3 + 11*(35*(b*x + a)^(9/2) - 180*(b*x + a)
^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*
x + a)*a^4)*a*d*sgn(x)/b^4 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*
a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3
/2)*a^4 - 693*sqrt(b*x + a)*a^5)*d*sgn(x)/b^4)/b + 32/3465*(11*a^(9/2)*b*c
 - 8*a^(11/2)*d)*sgn(x)/b^5
```

Mupad [B] (verification not implemented)

Time = 8.82 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.72

$$\int x^2(c+dx)\sqrt{ax^2+bx^3}\,dx$$
$$= \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,d\,x^5}{11}+\frac{256\,a^5\,d-352\,a^4\,b\,c}{3465\,b^5}+\frac{x^4\,\left(770\,c\,b^5+70\,a\,d\,b^4\right)}{3465\,b^5}-\frac{2\,a\,x^3\,\left(8\,a\,d-11\,b\,c\right)}{693\,b^2}-\frac{16\,a^3\,x\,\left(8\,a\,d-11\,b\,c\right)}{3465\,b^4}+\frac{4\,a^2\,x^2\,\left(8\,a\,d\right.}{1155\,b}\right.}{x}$$

input
```
int(x^2*(a*x^2 + b*x^3)^(1/2)*(c + d*x),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((2*d*x^5)/11 + (256*a^5*d - 352*a^4*b*c)/(3465*b^5
) + (x^4*(770*b^5*c + 70*a*b^4*d))/(3465*b^5) - (2*a*x^3*(8*a*d - 11*b*c))
/(693*b^2) - (16*a^3*x*(8*a*d - 11*b*c))/(3465*b^4) + (4*a^2*x^2*(8*a*d -
11*b*c))/(1155*b^3)))/x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.70

$$\int x^2(c+dx)\sqrt{ax^2+bx^3}\,dx$$
$$=\frac{2\sqrt{bx+a}\,(315b^5d\,x^5+35a\,b^4d\,x^4+385b^5c\,x^4-40a^2b^3d\,x^3+55a\,b^4c\,x^3+48a^3b^2d\,x^2-66a^2b^3c\,x^2-64a}{3465b^5}$$

input
```
int(x^2*(d*x+c)*(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(128*a**5*d - 176*a**4*b*c - 64*a**4*b*d*x + 88*a**3*b**2
*c*x + 48*a**3*b**2*d*x**2 - 66*a**2*b**3*c*x**2 - 40*a**2*b**3*d*x**3 + 5
5*a*b**4*c*x**3 + 35*a*b**4*d*x**4 + 385*b**5*c*x**4 + 315*b**5*d*x**5))/(
3465*b**5)
```

3.238 $\int x(c+dx)\sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 131

$$\int x(c+dx)\sqrt{ax^2+bx^3}\,dx = \frac{2a^2(bc-ad)\left(ax^2+bx^3\right)^{3/2}}{3b^4x^3} - \frac{2a(2bc-3ad)\left(ax^2+bx^3\right)^{5/2}}{5b^4x^5} + \frac{2(bc-3ad)\left(ax^2+bx^3\right)^{7/2}}{7b^4x^7} + \frac{2d\left(ax^2+bx^3\right)^{9/2}}{9b^4x^9}$$

output
```
2/3*a^2*(-a*d+b*c)*(b*x^3+a*x^2)^(3/2)/b^4/x^3-2/5*a*(-3*a*d+2*b*c)*(b*x^3
+a*x^2)^(5/2)/b^4/x^5+2/7*(-3*a*d+b*c)*(b*x^3+a*x^2)^(7/2)/b^4/x^7+2/9*d*(
b*x^3+a*x^2)^(9/2)/b^4/x^9
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.55

$$\int x(c+dx)\sqrt{ax^2+bx^3}\,dx = \frac{2\left(x^2(a+bx)\right)^{3/2}\left(-16a^3d+24a^2b(c+dx)-6ab^2x(6c+5dx)+5b^3x^2(9c+7dx)\right)}{315b^4x^3}$$

input
```
Integrate[x*(c + d*x)*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*(x^2*(a + b*x))^(3/2)*(-16*a^3*d + 24*a^2*b*(c + d*x) - 6*a*b^2*x*(6*c
+ 5*d*x) + 5*b^3*x^2*(9*c + 7*d*x)))/(315*b^4*x^3)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1945, 1922, 1908, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{ax^2+bx^3}(c+dx)\,dx$$

$$\Big\downarrow \; 1945$$

$$\frac{(3bc-2ad)\int x\sqrt{bx^3+ax^2}dx}{3b}+\frac{2d\left(ax^2+bx^3\right)^{3/2}}{9b}$$

$$\Big\downarrow \; 1922$$

$$\frac{(3bc-2ad)\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\int\sqrt{bx^3+ax^2}dx}{7b}\right)}{3b}+\frac{2d\left(ax^2+bx^3\right)^{3/2}}{9b}$$

$$\Big\downarrow \; 1908$$

$$\frac{(3bc-2ad)\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{2a\int\frac{\sqrt{bx^3+ax^2}}{x}dx}{5b}\right)}{7b}\right)}{3b}+\frac{2d\left(ax^2+bx^3\right)^{3/2}}{9b}$$

$$\Big\downarrow \; 1920$$

$$\frac{\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{7bx}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{3/2}}{5bx^2}-\frac{4a\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}\right)}{7b}\right)(3bc-2ad)}{3b}+\frac{2d\left(ax^2+bx^3\right)^{3/2}}{9b}$$

input
```
Int[x*(c + d*x)*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*d*(a*x^2 + b*x^3)^(3/2))/(9*b) + ((3*b*c - 2*a*d)*((2*(a*x^2 + b*x^3)^(
3/2))/(7*b*x) - (4*a*((-4*a*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*(a*x^
2 + b*x^3)^(3/2))/(5*b*x^2)))/(7*b)))/(3*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.31

method	result	size
pseudoelliptic	$\frac{16(bx+a)^{\frac{3}{2}}\left(\frac{21x\left(\frac{5dx}{7}+c\right)b^{2}}{8}-\frac{7\left(\frac{6dx}{7}+c\right)ab}{4}+a^{2}d\right)}{105b^{3}}$	41
gosper	$-\frac{2(bx+a)\left(-35b^{3}d\,x^{3}+30a\,b^{2}d\,x^{2}-45b^{3}c\,x^{2}-24a^{2}bdx+36a\,b^{2}cx+16a^{3}d-24c\,a^{2}b\right)\sqrt{b\,x^{3}+a\,x^{2}}}{315b^{4}x}$	85
default	$-\frac{2(bx+a)\left(-35b^{3}d\,x^{3}+30a\,b^{2}d\,x^{2}-45b^{3}c\,x^{2}-24a^{2}bdx+36a\,b^{2}cx+16a^{3}d-24c\,a^{2}b\right)\sqrt{b\,x^{3}+a\,x^{2}}}{315b^{4}x}$	85
orering	$-\frac{2(bx+a)\left(-35b^{3}d\,x^{3}+30a\,b^{2}d\,x^{2}-45b^{3}c\,x^{2}-24a^{2}bdx+36a\,b^{2}cx+16a^{3}d-24c\,a^{2}b\right)\sqrt{b\,x^{3}+a\,x^{2}}}{315b^{4}x}$	85
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-35d\,x^{4}b^{4}-5a\,b^{3}d\,x^{3}-45b^{4}c\,x^{3}+6a^{2}b^{2}d\,x^{2}-9a\,b^{3}c\,x^{2}-8a^{3}bdx+12a^{2}b^{2}cx+16a^{4}d-24a^{3}bc\right)}{315x\,b^{4}}$	102
trager	$-\frac{2\left(-35d\,x^{4}b^{4}-5a\,b^{3}d\,x^{3}-45b^{4}c\,x^{3}+6a^{2}b^{2}d\,x^{2}-9a\,b^{3}c\,x^{2}-8a^{3}bdx+12a^{2}b^{2}cx+16a^{4}d-24a^{3}bc\right)\sqrt{b\,x^{3}+a\,x^{2}}}{315b^{4}x}$	104

input
```
int(x*(d*x+c)*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
16/105*(b*x+a)^(3/2)*(21/8*x*(5/7*d*x+c)*b^2-7/4*(6/7*d*x+c)*a*b+a^2*d)/b^
3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.79

$$\int x(c+dx)\sqrt{ax^2+bx^3}\,dx$$
$$=\frac{2\left(35\,b^4dx^4+24\,a^3bc-16\,a^4d+5\,(9\,b^4c+ab^3d)x^3+3\,(3\,ab^3c-2\,a^2b^2d)x^2-4\,(3\,a^2b^2c-2\,a^3bd)x\right)\sqrt{bx^3+ax^2}}{315\,b^4x}$$

input
```
integrate(x*(d*x+c)*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^4*d*x^4 + 24*a^3*b*c - 16*a^4*d + 5*(9*b^4*c + a*b^3*d)*x^3 +
3*(3*a*b^3*c - 2*a^2*b^2*d)*x^2 - 4*(3*a^2*b^2*c - 2*a^3*b*d)*x)*sqrt(b*x^
3 + a*x^2)/(b^4*x)
```

Sympy [F]

$$\int x(c+dx)\sqrt{ax^2+bx^3}\,dx=\int x\sqrt{x^2(a+bx)}(c+dx)\,dx$$

input
```
integrate(x*(d*x+c)*(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x*sqrt(x**2*(a + b*x))*(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.75

$$\int x(c+dx)\sqrt{ax^2+bx^3}\,dx =\frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}c}{105\,b^3} +\frac{2\,(35\,b^4x^4+5\,ab^3x^3-6\,a^2b^2x^2+8\,a^3bx-16\,a^4)\sqrt{bx+a}d}{315\,b^4}$$

input
```
integrate(x*(d*x+c)*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/105*(15*b^3*x^3 + 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)*c/b^3 +
 2/315*(35*b^4*x^4 + 5*a*b^3*x^3 - 6*a^2*b^2*x^2 + 8*a^3*b*x - 16*a^4)*sqr
t(b*x + a)*d/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(115) = 230.

Time = 0.11 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.81

$$\int x(c+dx)\sqrt{ax^2+bx^3}\,dx$$
$$= \frac{2\left(\frac{21\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)ac\mathrm{sgn}(x)}{b^2}+\frac{9\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)c\mathrm{sgn}(x)}{b^2}+\frac{9\left(5\,(b\right.}{}}{}$$
$$-\frac{16\left(3\,a^{\frac{7}{2}}bc-2\,a^{\frac{9}{2}}d\right)\mathrm{sgn}(x)}{315\,b^4}$$

input
```
integrate(x*(d*x+c)*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/315*(21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2
)*a*c*sgn(x)/b^2 + 9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x +
 a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*c*sgn(x)/b^2 + 9*(5*(b*x + a)^(7/2)
- 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*
d*sgn(x)/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)
^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d*sgn(x)/b^3
)/b - 16/315*(3*a^(7/2)*b*c - 2*a^(9/2)*d)*sgn(x)/b^4
```

Mupad [B] (verification not implemented)

Time = 8.78 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.77

$$\int x(c+dx)\sqrt{ax^2+bx^3}\,dx$$
$$= \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,d\,x^4}{9}-\frac{32\,a^4\,d-48\,a^3\,b\,c}{315\,b^4}+\frac{x^3\,\left(90\,c\,b^4+10\,a\,d\,b^3\right)}{315\,b^4}-\frac{2\,a\,x^2\,\left(2\,a\,d-3\,b\,c\right)}{105\,b^2}+\frac{8\,a^2\,x\,\left(2\,a\,d-3\,b\,c\right)}{315\,b^3}\right)}{x}$$

input
```
int(x*(a*x^2 + b*x^3)^(1/2)*(c + d*x),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((2*d*x^4)/9 - (32*a^4*d - 48*a^3*b*c)/(315*b^4) +
(x^3*(90*b^4*c + 10*a*b^3*d))/(315*b^4) - (2*a*x^2*(2*a*d - 3*b*c))/(105*b
^2) + (8*a^2*x*(2*a*d - 3*b*c))/(315*b^3)))/x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.71

$$\int x(c+dx)\sqrt{ax^2+bx^3}\,dx$$
$$=\frac{2\sqrt{bx+a}\,(35b^4d\,x^4+5a\,b^3d\,x^3+45b^4c\,x^3-6a^2b^2d\,x^2+9a\,b^3c\,x^2+8a^3bdx-12a^2b^2cx-16a^4d+24a^3}{315b^4}$$

input
```
int(x*(d*x+c)*(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 16*a**4*d + 24*a**3*b*c + 8*a**3*b*d*x - 12*a**2*b**2
*c*x - 6*a**2*b**2*d*x**2 + 9*a*b**3*c*x**2 + 5*a*b**3*d*x**3 + 45*b**4*c*
x**3 + 35*b**4*d*x**4))/(315*b**4)
```

3.239 $\int (c+dx)\sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int (c+dx)\sqrt{ax^2+bx^3}\,dx = -\frac{2a(bc-ad)\left(ax^2+bx^3\right)^{3/2}}{3b^3x^3} + \frac{2(bc-2ad)\left(ax^2+bx^3\right)^{5/2}}{5b^3x^5} + \frac{2d\left(ax^2+bx^3\right)^{7/2}}{7b^3x^7}$$

output
```
-2/3*a*(-a*d+b*c)*(b*x^3+a*x^2)^(3/2)/b^3/x^3+2/5*(-2*a*d+b*c)*(b*x^3+a*x^
2)^(5/2)/b^3/x^5+2/7*d*(b*x^3+a*x^2)^(7/2)/b^3/x^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.60

$$\int (c+dx)\sqrt{ax^2+bx^3}\,dx = \frac{2\left(x^2(a+bx)\right)^{3/2}\left(8a^2d+3b^2x(7c+5dx)-2ab(7c+6dx)\right)}{105b^3x^3}$$

input
```
Integrate[(c + d*x)*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*(x^2*(a + b*x))^(3/2)*(8*a^2*d + 3*b^2*x*(7*c + 5*d*x) - 2*a*b*(7*c + 6
*d*x)))/(105*b^3*x^3)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.45, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2450, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ax^2+bx^3}(c+dx)\,dx$$

$$\Big\downarrow \text{2450}$$

$$\int \left(c\sqrt{ax^2+bx^3}+dx\sqrt{ax^2+bx^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{16a^2d\left(ax^2+bx^3\right)^{3/2}}{105b^3x^3}-\frac{4ac\left(ax^2+bx^3\right)^{3/2}}{15b^2x^3}-\frac{8ad\left(ax^2+bx^3\right)^{3/2}}{35b^2x^2}+\frac{2c\left(ax^2+bx^3\right)^{3/2}}{5bx^2}+\frac{2d\left(ax^2+bx^3\right)^{3/2}}{7bx}$$

input
```
Int[(c + d*x)*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(-4*a*c*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (16*a^2*d*(a*x^2 + b*x^3)^(3
/2))/(105*b^3*x^3) + (2*c*(a*x^2 + b*x^3)^(3/2))/(5*b*x^2) - (8*a*d*(a*x^2
 + b*x^3)^(3/2))/(35*b^2*x^2) + (2*d*(a*x^2 + b*x^3)^(3/2))/(7*b*x)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2450
```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) && !IntegerQ[p] && NeQ[n, j]
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.29

method	result	size
pseudoelliptic	$-\frac{2(bx+a)^{\frac{3}{2}}(-3bdx+2ad-5bc)}{15b^2}$	27
gosper	$\frac{2(bx+a)(15b^2dx^2-12abdx+21b^2cx+8a^2d-14abc)\sqrt{bx^3+ax^2}}{105b^3x}$	61
default	$\frac{2(bx+a)(15b^2dx^2-12abdx+21b^2cx+8a^2d-14abc)\sqrt{bx^3+ax^2}}{105b^3x}$	61
orering	$\frac{2(bx+a)(15b^2dx^2-12abdx+21b^2cx+8a^2d-14abc)\sqrt{bx^3+ax^2}}{105b^3x}$	61
risch	$\frac{2\sqrt{x^2(bx+a)}\,(15b^3dx^3+3ab^2dx^2+21b^3cx^2-4a^2bdx+7ab^2cx+8a^3d-14ca^2b)}{105xb^3}$	78
trager	$\frac{2(15b^3dx^3+3ab^2dx^2+21b^3cx^2-4a^2bdx+7ab^2cx+8a^3d-14ca^2b)\sqrt{bx^3+ax^2}}{105b^3x}$	80

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(b*x+a)^(3/2)*(-3*b*d*x+2*a*d-5*b*c)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.84

$$\int (c+dx)\sqrt{ax^2+bx^3}\,dx = \frac{2\,(15\,b^3dx^3-14\,a^2bc+8\,a^3d+3\,(7\,b^3c+ab^2d)x^2+(7\,ab^2c-4\,a^2bd)x)\sqrt{bx^3+ax^2}}{105\,b^3x}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*b^3*d*x^3 - 14*a^2*b*c + 8*a^3*d + 3*(7*b^3*c + a*b^2*d)*x^2 + (
7*a*b^2*c - 4*a^2*b*d)*x)*sqrt(b*x^3 + a*x^2)/(b^3*x)
```

Sympy [F]

$$\int (c+dx)\sqrt{ax^2+bx^3}\,dx = \int \sqrt{x^2\,(a+bx)}(c+dx)\;dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(sqrt(x**2*(a + b*x))*(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.80

$$\int (c+dx)\sqrt{ax^2+bx^3}\,dx = \frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}c}{15\,b^2} + \frac{2\,(15\,b^3x^3+3\,ab^2x^2-4\,a^2bx+8\,a^3)\sqrt{bx+a}d}{105\,b^3}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)*c/b^2 + 2/105*(15*b^3*x^3 +
 3*a*b^2*x^2 - 4*a^2*b*x + 8*a^3)*sqrt(b*x + a)*d/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(82) = 164$.

Time = 0.11 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.00

$$\int (c+dx)\sqrt{ax^2+bx^3}\,dx = \frac{2\left(\frac{35\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)ac\,\mathrm{sgn}(x)}{b} + \frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)c\,\mathrm{sgn}(x)}{b} + \frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a \cdots\right.}{b^2} \cdots\right.}{105\,b} + \frac{4\left(7\,a^{\frac{5}{2}}bc-4\,a^{\frac{7}{2}}d\right)\mathrm{sgn}(x)}{105\,b^3}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/105*(35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a*c*sgn(x)/b + 7*(3*(b*x +
 a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*c*sgn(x)/b + 7*(3
*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a*d*sgn(x)
/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^
2 - 35*sqrt(b*x + a)*a^3)*d*sgn(x)/b^2)/b + 4/105*(7*a^(5/2)*b*c - 4*a^(7/
2)*d)*sgn(x)/b^3
```

Mupad [B] (verification not implemented)

Time = 8.74 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.86

$$\int (c+dx)\sqrt{ax^2+bx^3}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,d\,x^3}{7}+\frac{16\,a^3\,d-28\,a^2\,b\,c}{105\,b^3}+\frac{x^2\,\left(42\,c\,b^3+6\,a\,d\,b^2\right)}{105\,b^3}-\frac{2\,a\,x\,\left(4\,a\,d-7\,b\,c\right)}{105\,b^2}\right)}{x}$$

input
```
int((a*x^2 + b*x^3)^(1/2)*(c + d*x),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((2*d*x^3)/7 + (16*a^3*d - 28*a^2*b*c)/(105*b^3) +
(x^2*(42*b^3*c + 6*a*b^2*d))/(105*b^3) - (2*a*x*(4*a*d - 7*b*c))/(105*b^2)
))/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.73

$$\int (c+dx)\sqrt{ax^2+bx^3}\,dx = \frac{2\sqrt{bx+a}\,(15b^3d\,x^3+3a\,b^2d\,x^2+21b^3c\,x^2-4a^2bdx+7a\,b^2cx+8a^3d-14a^2bc)}{105b^3}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**3*d - 14*a**2*b*c - 4*a**2*b*d*x + 7*a*b**2*c*x + 3
*a*b**2*d*x**2 + 21*b**3*c*x**2 + 15*b**3*d*x**3))/(105*b**3)
```

3.240 $\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x} \, dx = \frac{2(bc-ad)\left(ax^2+bx^3\right)^{3/2}}{3b^2x^3} + \frac{2d\left(ax^2+bx^3\right)^{5/2}}{5b^2x^5}$$

output
```
2/3*(-a*d+b*c)*(b*x^3+a*x^2)^(3/2)/b^2/x^3+2/5*d*(b*x^3+a*x^2)^(5/2)/b^2/x
^5
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.62

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x} \, dx = \frac{2\left(x^2(a+bx)\right)^{3/2}(5bc-2ad+3bdx)}{15b^2x^3}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x^2 + b*x^3])/x,x]
```

output
```
(2*(x^2*(a + b*x))^(3/2)*(5*b*c - 2*a*d + 3*b*d*x))/(15*b^2*x^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1945, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}(c+dx)}{x} \, dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(5bc-2ad)\int \frac{\sqrt{bx^3+ax^2}}{x}dx}{5b}+\frac{2d\left(ax^2+bx^3\right)^{3/2}}{5bx^2}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\left(ax^2+bx^3\right)^{3/2}(5bc-2ad)}{15b^2x^3}+\frac{2d\left(ax^2+bx^3\right)^{3/2}}{5bx^2}$$

input
```
Int[((c + d*x)*Sqrt[a*x^2 + b*x^3])/x,x]
```

output
```
(2*(5*b*c - 2*a*d)*(a*x^2 + b*x^3)^(3/2))/(15*b^2*x^3) + (2*d*(a*x^2 + b*x
^3)^(3/2))/(5*b*x^2)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{2(bx+a)(-3bdx+2ad-5bc)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{2}x}$	41
default	$-\frac{2(bx+a)(-3bdx+2ad-5bc)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{2}x}$	41
orering	$-\frac{2(bx+a)(-3bdx+2ad-5bc)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{2}x}$	41
pseudoelliptic	$\frac{-6\sqrt{a}\,bc\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+2((dx+3c)b+ad)\sqrt{bx+a}}{3b}$	48
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-3b^{2}d\,x^{2}-abdx-5b^{2}cx+2a^{2}d-5abc\right)}{15x\,b^{2}}$	54
trager	$-\frac{2\left(-3b^{2}d\,x^{2}-abdx-5b^{2}cx+2a^{2}d-5abc\right)\sqrt{b\,x^{3}+a\,x^{2}}}{15b^{2}x}$	56

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/15*(b*x+a)*(-3*b*d*x+2*a*d-5*b*c)*(b*x^3+a*x^2)^(1/2)/b^2/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x}\,dx = \frac{2\,(3\,b^2dx^2+5\,abc-2\,a^2d+(5\,b^2c+abd)x)\sqrt{bx^3+ax^2}}{15\,b^2x}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x,x, algorithm="fricas")
```

output
```
2/15*(3*b^2*d*x^2 + 5*a*b*c - 2*a^2*d + (5*b^2*c + a*b*d)*x)*sqrt(b*x^3 +
a*x^2)/(b^2*x)
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}(c+dx)}{x}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(1/2)/x,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))*(c + d*x)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.75

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}c}{3\,b} + \frac{2\,(3\,b^2x^2+abx-2\,a^2)\sqrt{bx+a}d}{15\,b^2}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x,x, algorithm="maxima")
```

output
```
2/3*(b*x + a)^(3/2)*c/b + 2/15*(3*b^2*x^2 + a*b*x - 2*a^2)*sqrt(b*x + a)*d
/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(52) = 104$.

Time = 0.12 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.17

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x}\,dx$$
$$= \frac{2\left(15\sqrt{bx+a}\,ac\,\mathrm{sgn}(x)+5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)c\,\mathrm{sgn}(x)+\frac{5\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)ad\,\mathrm{sgn}(x)}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-\right.}{}\right.}{15\,b}$$
$$-\frac{2\left(5\,a^{\frac{3}{2}}bc-2\,a^{\frac{5}{2}}d\right)\mathrm{sgn}(x)}{15\,b^2}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x,x, algorithm="giac")
```

output
```
2/15*(15*sqrt(b*x + a)*a*c*sgn(x) + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a
)*c*sgn(x) + 5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a*d*sgn(x)/b + (3*(b*
x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d*sgn(x)/b)/b
- 2/15*(5*a^(3/2)*b*c - 2*a^(5/2)*d)*sgn(x)/b^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x}\,dx=\int \frac{\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)}{x}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/x,x)
```

output
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/x, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x}\,dx = \frac{2\sqrt{bx+a}\,(3b^2d\,x^2+abdx+5b^2cx-2a^2d+5abc)}{15b^2}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/x,x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a**2*d + 5*a*b*c + a*b*d*x + 5*b**2*c*x + 3*b**2*d*
x**2))/(15*b**2)
```

3.241 $\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 81

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^2}\,dx = \frac{2c\sqrt{ax^2+bx^3}}{x}+\frac{2d(ax^2+bx^3)^{3/2}}{3bx^3} - 2\sqrt{a}c\,\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)$$

output
```
2*c*(b*x^3+a*x^2)^(1/2)/x+2/3*d*(b*x^3+a*x^2)^(3/2)/b/x^3-2*a^(1/2)*c*arct
anh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^2}\,dx = \frac{2x\left((a+bx)(3bc+ad+bdx)-3\sqrt{a}bc\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3b\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x^2 + b*x^3])/x^2,x]
```

output
```
(2*x*((a + b*x)*(3*b*c + a*d + b*d*x) - 3*Sqrt[a]*b*c*Sqrt[a + b*x]*ArcTan
h[Sqrt[a + b*x]/Sqrt[a]]))/(3*b*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1945, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}(c+dx)}{x^2} \, dx$$

$$\Big\downarrow \text{1945}$$

$$c\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx+\frac{2d\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

$$\Big\downarrow \text{1927}$$

$$c\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)+\frac{2d\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

$$\Big\downarrow \text{1914}$$

$$c\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)+\frac{2d\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

$$\Big\downarrow \text{219}$$

$$c\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)+\frac{2d\left(ax^2+bx^3\right)^{3/2}}{3bx^3}$$

input
```
Int[((c + d*x)*Sqrt[a*x^2 + b*x^3])/x^2,x]
```

output
```
(2*d*(a*x^2 + b*x^3)^(3/2))/(3*b*x^3) + c*((2*Sqrt[a*x^2 + b*x^3])/x - 2*S
qrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.60

method	result	size
pseudoelliptic	$-\frac{(2ad+bc)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)x+\sqrt{bx+a}\,\sqrt{a}\,(-2dx+c)}{\sqrt{a}\,x}$	49
default	$\frac{2\sqrt{b\,x^{3}+a\,x^{2}}\left((bx+a)^{\frac{3}{2}}d-3\sqrt{a}\,bc\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+3\sqrt{bx+a}\,bc\right)}{3x\sqrt{bx+a}\,b}$	69

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/a^(1/2)*((2*a*d+b*c)*arctanh((b*x+a)^(1/2)/a^(1/2))*x+(b*x+a)^(1/2)*a^(
1/2)*(-2*d*x+c))/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.90

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^2}\,dx$$
$$=\left[\frac{3\sqrt{a}bcx\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(bdx+3\,bc+ad)}{3\,bx},\frac{2\left(3\,\sqrt{-a}bcx\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{\cdot}}{bx^2+ax}\right.\right.}{}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="fricas")
```

output
```
[1/3*(3*sqrt(a)*b*c*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/
x^2) + 2*sqrt(b*x^3 + a*x^2)*(b*d*x + 3*b*c + a*d))/(b*x), 2/3*(3*sqrt(-a)
*b*c*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a
*x^2)*(b*d*x + 3*b*c + a*d))/(b*x)]
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^2}\,dx=\int \frac{\sqrt{x^2\,(a+bx)}(c+dx)}{x^2}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(1/2)/x**2,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))*(c + d*x)/x**2, x)
```

Maxima [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^2}\,dx = \int \frac{\sqrt{bx^3+ax^2}(dx+c)}{x^2}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)*(d*x + c)/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^2}\,dx = \frac{2\,ac\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} - \frac{2\left(3\,abc\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+3\sqrt{-a}\sqrt{a}bc+\sqrt{-a}a^{\frac{3}{2}}d\right)\mathrm{sgn}(x)}{3\sqrt{-a}b} + \frac{2\left(3\sqrt{bx+a}b^3c\mathrm{sgn}(x)+(bx+a)^{\frac{3}{2}}b^2d\mathrm{sgn}(x)\right)}{3\,b^3}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^2,x, algorithm="giac")
```

output
```
2*a*c*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - 2/3*(3*a*b*c*arctan
(sqrt(a)/sqrt(-a)) + 3*sqrt(-a)*sqrt(a)*b*c + sqrt(-a)*a^(3/2)*d)*sgn(x)/(
sqrt(-a)*b) + 2/3*(3*sqrt(b*x + a)*b^3*c*sgn(x) + (b*x + a)^(3/2)*b^2*d*sg
n(x))/b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^2}\,dx = \int \frac{\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)}{x^2}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/x^2,x)
```

output
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^2}\,dx = \frac{2\sqrt{bx+a}\,ad+6\sqrt{bx+a}\,bc+2\sqrt{bx+a}\,bdx+3\sqrt{a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)bc-3\sqrt{a}\,\log\left(\sqrt{bx+a}+\sqrt{a}\right)bc}{3b}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^2,x)
```

output
```
(2*sqrt(a + b*x)*a*d + 6*sqrt(a + b*x)*b*c + 2*sqrt(a + b*x)*b*d*x + 3*sqr
t(a)*log(sqrt(a + b*x) - sqrt(a))*b*c - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt
(a))*b*c)/(3*b)
```

3.242 $\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 83

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^3}\,dx = -\frac{c\sqrt{ax^2+bx^3}}{x^2}+\frac{2d\sqrt{ax^2+bx^3}}{x} - \frac{(bc+2ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{\sqrt{a}}$$

output
```
-c*(b*x^3+a*x^2)^(1/2)/x^2+2*d*(b*x^3+a*x^2)^(1/2)/x-(2*a*d+b*c)*arctanh((
b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^3}\,dx = \frac{\sqrt{a}(a+bx)(-c+2dx)-(bc+2ad)x\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x^2 + b*x^3])/x^3,x]
```

output
```
(Sqrt[a]*(a + b*x)*(-c + 2*d*x) - (b*c + 2*a*d)*x*Sqrt[a + b*x]*ArcTanh[Sq
rt[a + b*x]/Sqrt[a]])/(Sqrt[a]*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1944, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}(c+dx)}{x^3}\,dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(2ad+bc)\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx}{2a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{ax^4}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(2ad+bc)\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)}{2a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{ax^4}$$

$$\Big\downarrow \text{1914}$$

$$\frac{(2ad+bc)\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)}{2a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{ax^4}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)(2ad+bc)}{2a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{ax^4}$$

input
```
Int[((c + d*x)*Sqrt[a*x^2 + b*x^3])/x^3,x]
```

output
```
-((c*(a*x^2 + b*x^3)^(3/2))/(a*x^4)) + ((b*c + 2*a*d)*((2*Sqrt[a*x^2 + b*x
^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]))/(2*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.77

method	result	size
pseudoelliptic	$-\frac{b\,x^{2}\left(a d-\frac{b c}{4}\right) \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)+\frac{\left(\left(4 d x+2 c\right) a^{\frac{3}{2}}+\sqrt{a}\,b c x\right) \sqrt{b x+a}}{4}}{a^{\frac{3}{2}} x^{2}}$	64
risch	$-\frac{c\sqrt{x^{2}\left(b x+a\right)}}{x^{2}}+\frac{\left(2 \sqrt{b x+a}\,d-\frac{\left(2 a d+b c\right) \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right) \sqrt{x^{2}\left(b x+a\right)}}{x\sqrt{b x+a}}$	77
default	$-\frac{\sqrt{b\,x^{3}+a\,x^{2}}\,\left(-2 \sqrt{b x+a}\,d x \sqrt{a}+2 \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right) a d x+\operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right) b c x+c \sqrt{b x+a}\,\sqrt{a}\right)}{x^{2} \sqrt{b x+a}\,\sqrt{a}}$	89

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/a^(3/2)*(b*x^2*(a*d-1/4*b*c)*arctanh((b*x+a)^(1/2)/a^(1/2))+1/4*((4*d*x
+2*c)*a^(3/2)+a^(1/2)*b*c*x)*(b*x+a)^(1/2))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.96

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^3}\,dx$$

$$= \left[\frac{(bc+2\,ad)\sqrt{a}x^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(2\,adx-ac)}{2\,ax^2}, \frac{(bc+2\,ad)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{}}{}\right.}{}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="fricas")
```

output
```
[1/2*((b*c + 2*a*d)*sqrt(a)*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)
*sqrt(a))/x^2) + 2*sqrt(b*x^3 + a*x^2)*(2*a*d*x - a*c))/(a*x^2), ((b*c + 2
*a*d)*sqrt(-a)*x^2*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sq
rt(b*x^3 + a*x^2)*(2*a*d*x - a*c))/(a*x^2)]
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^3}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}(c+dx)}{x^3}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(1/2)/x**3,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))*(c + d*x)/x**3, x)
```

Maxima [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^3}\,dx = \int \frac{\sqrt{bx^3+ax^2}(dx+c)}{x^3}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)*(d*x + c)/x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^3}\,dx = \left(\frac{2\,\sqrt{bx+a}d\mathrm{sgn}(x)}{b} + \frac{(bc\mathrm{sgn}(x)+2\,ad\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}b} - \frac{\sqrt{bx+a}c\mathrm{sgn}(x)}{bx}\right)b$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^3,x, algorithm="giac")
```

output
```
(2*sqrt(b*x + a)*d*sgn(x)/b + (b*c*sgn(x) + 2*a*d*sgn(x))*arctan(sqrt(b*x
+ a)/sqrt(-a))/(sqrt(-a)*b) - sqrt(b*x + a)*c*sgn(x)/(b*x))*b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^3}\,dx = \int \frac{\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)}{x^3}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/x^3,x)
```

output
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^3}\,dx = \frac{-2\sqrt{bx+a}\,ac+4\sqrt{bx+a}\,adx+2\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)adx+\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bcx-2\sqrt{a}\log}{2ax}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^3,x)
```

output
```
( - 2*sqrt(a + b*x)*a*c + 4*sqrt(a + b*x)*a*d*x + 2*sqrt(a)*log(sqrt(a + b
*x) - sqrt(a))*a*d*x + sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*c*x - 2*sqrt
(a)*log(sqrt(a + b*x) + sqrt(a))*a*d*x - sqrt(a)*log(sqrt(a + b*x) + sqrt(
a))*b*c*x)/(2*a*x)
```

3.243 $\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 100

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^4}\,dx = -\frac{c\sqrt{ax^2+bx^3}}{2x^3} - \frac{(bc+4ad)\sqrt{ax^2+bx^3}}{4ax^2} + \frac{b(bc-4ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{4a^{3/2}}$$

output
```
-1/2*c*(b*x^3+a*x^2)^(1/2)/x^3-1/4*(4*a*d+b*c)*(b*x^3+a*x^2)^(1/2)/a/x^2+1
/4*b*(-4*a*d+b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^4}\,dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(bcx+2a(c+2dx))+b(bc-4ad)x^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4a^{3/2}x^3\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x^2 + b*x^3])/x^4,x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(b*c*x + 2*a*(c + 2*d*x))) +
 b*(b*c - 4*a*d)*x^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(4*a^(3/2)*x^3*Sqrt[
a + b*x])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1944, 1926, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}(c+dx)}{x^4}\,dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(bc-4ad)\int \frac{\sqrt{bx^3+ax^2}}{x^3}dx}{4a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{2ax^5}$$

$$\Big\downarrow \text{1926}$$

$$-\frac{(bc-4ad)\left(\frac{1}{2}b\int \frac{1}{\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)}{4a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{2ax^5}$$

$$\Big\downarrow \text{1914}$$

$$-\frac{(bc-4ad)\left(-b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)}{4a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{2ax^5}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)(bc-4ad)}{4a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{2ax^5}$$

input
```
Int[((c + d*x)*Sqrt[a*x^2 + b*x^3])/x^4,x]
```

output
```
-1/2*(c*(a*x^2 + b*x^3)^(3/2))/(a*x^5) - ((b*c - 4*a*d)*(-(Sqrt[a*x^2 + b*
x^3]/x^2) - (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]))/(4*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.82

method	result
pseudoelliptic	$\frac{b^2x^3\left(ad-\frac{bc}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\frac{4\left(\frac{bx(3dx+c)a^{\frac{3}{2}}}{4}+\left(\frac{3dx}{2}+c\right)a^{\frac{5}{2}}-\frac{3\sqrt{a}\,b^2c\,x^2}{8}\right)\sqrt{bx+a}}{3}}{4a^{\frac{5}{2}}x^3}$
risch	$-\frac{(4adx+cbx+2ac)\sqrt{x^2(bx+a)}}{4x^3a}-\frac{(4ad-bc)b\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{4a^{\frac{3}{2}}x\sqrt{bx+a}}$
default	$-\frac{\sqrt{b\,x^3+a\,x^2}\left(4(bx+a)^{\frac{3}{2}}a^{\frac{5}{2}}d+(bx+a)^{\frac{3}{2}}a^{\frac{3}{2}}bc+4\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^2b^2d\,x^2-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^3c\,x^2-4\sqrt{bx+a}\,a^{\frac{7}{2}}d+\right.}{4b\,x^3\sqrt{bx+a}\,a^{\frac{5}{2}}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b^2*x^3*(a*d-1/2*b*c)*arctanh((b*x+a)^(1/2)/a^(1/2))-4/3*(1/4*b*x*(3*
d*x+c)*a^(3/2)+(3/2*d*x+c)*a^(5/2)-3/8*a^(1/2)*b^2*c*x^2)*(b*x+a)^(1/2))/a
^(5/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.90

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^4}\,dx$$
$$=\left[-\frac{(b^2c-4\,abd)\sqrt{a}x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(2\,a^2c+(abc+4\,a^2d)x)}{8\,a^2x^3},\right.$$
$$\left.-\frac{(b^2c-4\,abd)\sqrt{-a}x^3\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}(2\,a^2c+(abc+4\,a^2d)x)}{4\,a^2x^3}\right]$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="fricas")
```

output
```
[-1/8*((b^2*c - 4*a*b*d)*sqrt(a)*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a
*x^2)*sqrt(a))/x^2) + 2*sqrt(b*x^3 + a*x^2)*(2*a^2*c + (a*b*c + 4*a^2*d)*x
))/(a^2*x^3), -1/4*((b^2*c - 4*a*b*d)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a*x
^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*(2*a^2*c + (a*b*c + 4*a^
2*d)*x))/(a^2*x^3)]
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^4}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}(c+dx)}{x^4}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(1/2)/x**4,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))*(c + d*x)/x**4, x)
```

Maxima [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^4}\,dx = \int \frac{\sqrt{bx^3+ax^2}(dx+c)}{x^4}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)*(d*x + c)/x^4, x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^4}\,dx = -\frac{\frac{\left(b^3c\,\mathrm{sgn}(x)-4\,ab^2d\,\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{(bx+a)^{\frac{3}{2}}b^3c\,\mathrm{sgn}(x)+\sqrt{bx+a}ab^3c\,\mathrm{sgn}(x)+4\,(bx+a)^{\frac{3}{2}}ab^2d\,\mathrm{sgn}(x)-4\,\sqrt{bx+a}a^2b^2d\,\mathrm{sgn}(x)}{ab^2x^2}}{4\,b}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^4,x, algorithm="giac")
```

output
```
-1/4*((b^3*c*sgn(x) - 4*a*b^2*d*sgn(x))*arctan(sqrt(b*x + a)/sqrt(-a))/(sq
rt(-a)*a) + ((b*x + a)^(3/2)*b^3*c*sgn(x) + sqrt(b*x + a)*a*b^3*c*sgn(x) +
 4*(b*x + a)^(3/2)*a*b^2*d*sgn(x) - 4*sqrt(b*x + a)*a^2*b^2*d*sgn(x))/(a*b
^2*x^2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^4}\,dx = \int \frac{\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)}{x^4}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/x^4,x)
```

output
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^4}\,dx = \frac{-4\sqrt{bx+a}\,a^2c-8\sqrt{bx+a}\,a^2dx-2\sqrt{bx+a}\,abcx+4\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)abd\,x^2-\sqrt{a}\log\left(\sqrt{bx+a}\right.}{8a^2x^2}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^4,x)
```

output
```
( - 4*sqrt(a + b*x)*a**2*c - 8*sqrt(a + b*x)*a**2*d*x - 2*sqrt(a + b*x)*a*
b*c*x + 4*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b*d*x**2 - sqrt(a)*log(sq
rt(a + b*x) - sqrt(a))*b**2*c*x**2 - 4*sqrt(a)*log(sqrt(a + b*x) + sqrt(a)
)*a*b*d*x**2 + sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*c*x**2)/(8*a**2*x
**2)
```

3.244 $\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 136

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^5}\,dx = -\frac{c\sqrt{ax^2+bx^3}}{3x^4} - \frac{(bc+6ad)\sqrt{ax^2+bx^3}}{12ax^3} + \frac{b(bc-2ad)\sqrt{ax^2+bx^3}}{8a^2x^2} - \frac{b^2(bc-2ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{8a^{5/2}}$$

output
```
-1/3*c*(b*x^3+a*x^2)^(1/2)/x^4-1/12*(6*a*d+b*c)*(b*x^3+a*x^2)^(1/2)/a/x^3+
1/8*b*(-2*a*d+b*c)*(b*x^3+a*x^2)^(1/2)/a^2/x^2-1/8*b^2*(-2*a*d+b*c)*arctan
h((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^5}\,dx = -\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(-3b^2cx^2+2abx(c+3dx)+4a^2(2c+3dx))+3b^2(bc-2ad)x^3\text{arctanh}\left(\frac{\sqrt{a}}{\sqrt{}}\right.\right.}{24a^{5/2}x^4\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x^2 + b*x^3])/x^5,x]
```

output
```
-1/24*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(-3*b^2*c*x^2 + 2*a*b*x*
(c + 3*d*x) + 4*a^2*(2*c + 3*d*x)) + 3*b^2*(b*c - 2*a*d)*x^3*ArcTanh[Sqrt[
a + b*x]/Sqrt[a]]))/(a^(5/2)*x^4*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.91, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1944, 1926, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}(c+dx)}{x^5}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(bc-2ad)\int \frac{\sqrt{bx^3+ax^2}}{x^4}dx}{2a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{3ax^6}$$

$$\Big\downarrow \text{1926}$$

$$-\frac{(bc-2ad)\left(\frac{1}{4}b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)}{2a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{3ax^6}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{(bc-2ad)\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)}{2a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{3ax^6}$$

$$\Big\downarrow \text{1914}$$

$$-\frac{(bc-2ad)\left(\frac{1}{4}b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)}{2a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{3ax^6}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(\frac{1}{4}b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)(bc-2ad)}{2a}-\frac{c(ax^2+bx^3)^{3/2}}{3ax^6}$$

input
```
Int[((c + d*x)*Sqrt[a*x^2 + b*x^3])/x^5,x]
```

output
```
-1/3*(c*(a*x^2 + b*x^3)^(3/2))/(a*x^6) - ((b*c - 2*a*d)*(-1/2*Sqrt[a*x^2 +
 b*x^3]/x^3 + (b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/
Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/4))/(2*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.74

method	result
pseudoelliptic	$-\frac{b^3 x^4\left(ad-\frac{5bc}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{\sqrt{bx+a}\left(-\frac{5x^2b^2\left(\frac{12dx}{5}+c\right)a^{\frac{3}{2}}}{4}+bx(2dx+c)a^{\frac{5}{2}}+(8dx+6c)a^{\frac{7}{2}}+\frac{15\sqrt{a}\,b^3cx^3}{8}\right)}{3}}{8a^{\frac{7}{2}}x^4}$
risch	$-\frac{(6abd\,x^2-3b^2c\,x^2+12a^2dx+2abcx+8a^2c)\sqrt{x^2(bx+a)}}{24x^4a^2}+\frac{(2ad-bc)b^2\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{8a^{\frac{5}{2}}x\sqrt{bx+a}}$
default	$-\frac{\sqrt{b\,x^3+a\,x^2}\left(6(bx+a)^{\frac{5}{2}}a^{\frac{7}{2}}d-3(bx+a)^{\frac{5}{2}}a^{\frac{5}{2}}bc+8(bx+a)^{\frac{3}{2}}a^{\frac{7}{2}}bc-6\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^3b^3d\,x^3+3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^2b^4\right.}{24b\,x^4\sqrt{bx+a}\,a^{\frac{9}{2}}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^(7/2)*(b^3*x^4*(a*d-5/8*b*c)*arctanh((b*x+a)^(1/2)/a^(1/2))+1/3*(b*
x+a)^(1/2)*(-5/4*x^2*b^2*(12/5*d*x+c)*a^(3/2)+b*x*(2*d*x+c)*a^(5/2)+(8*d*x
+6*c)*a^(7/2)+15/8*a^(1/2)*b^3*c*x^3))/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.77

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^5}\,dx$$
$$= \left[-\frac{3\,(b^3c-2\,ab^2d)\sqrt{a}x^4\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(8\,a^3c-3\,(ab^2c-2\,a^2bd)x^2+2\,(a^2bc+6\,a^3d)x)}{48\,a^3x^4}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="fricas")
```

output
```
[-1/48*(3*(b^3*c - 2*a*b^2*d)*sqrt(a)*x^4*log((b*x^2 + 2*a*x + 2*sqrt(b*x^
3 + a*x^2)*sqrt(a))/x^2) + 2*(8*a^3*c - 3*(a*b^2*c - 2*a^2*b*d)*x^2 + 2*(a
^2*b*c + 6*a^3*d)*x)*sqrt(b*x^3 + a*x^2))/(a^3*x^4), 1/24*(3*(b^3*c - 2*a*
b^2*d)*sqrt(-a)*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - (
8*a^3*c - 3*(a*b^2*c - 2*a^2*b*d)*x^2 + 2*(a^2*b*c + 6*a^3*d)*x)*sqrt(b*x^
3 + a*x^2))/(a^3*x^4)]
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^5}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}(c+dx)}{x^5}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(1/2)/x**5,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))*(c + d*x)/x**5, x)
```

Maxima [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^5}\,dx = \int \frac{\sqrt{bx^3+ax^2}(dx+c)}{x^5}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)*(d*x + c)/x^5, x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^5}\,dx$$
$$= \frac{1}{24}b^3\left(\frac{3\,(bc\text{sgn}(x)-2\,ad\text{sgn}(x))\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2b} + \frac{3\,(bx+a)^{\frac{5}{2}}bc\text{sgn}(x)-8\,(bx+a)^{\frac{3}{2}}abc\text{sgn}(x)-3\,\sqrt{b}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^5,x, algorithm="giac")
```

output
```
1/24*b^3*(3*(b*c*sgn(x) - 2*a*d*sgn(x))*arctan(sqrt(b*x + a)/sqrt(-a))/(sq
rt(-a)*a^2*b) + (3*(b*x + a)^(5/2)*b*c*sgn(x) - 8*(b*x + a)^(3/2)*a*b*c*sg
n(x) - 3*sqrt(b*x + a)*a^2*b*c*sgn(x) - 6*(b*x + a)^(5/2)*a*d*sgn(x) + 6*s
qrt(b*x + a)*a^3*d*sgn(x))/(a^2*b^4*x^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^5}\,dx = \int \frac{\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)}{x^5}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/x^5,x)
```

output
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.25

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^5}\,dx$$
$$= \frac{-16\sqrt{bx+a}\,a^3c-24\sqrt{bx+a}\,a^3dx-4\sqrt{bx+a}\,a^2bcx-12\sqrt{bx+a}\,a^2bd\,x^2+6\sqrt{bx+a}\,a\,b^2c\,x^2-6\sqrt{a}\,}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^5,x)
```

output
```
( - 16*sqrt(a + b*x)*a**3*c - 24*sqrt(a + b*x)*a**3*d*x - 4*sqrt(a + b*x)*
a**2*b*c*x - 12*sqrt(a + b*x)*a**2*b*d*x**2 + 6*sqrt(a + b*x)*a*b**2*c*x**
2 - 6*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**2*d*x**3 + 3*sqrt(a)*log(s
qrt(a + b*x) - sqrt(a))*b**3*c*x**3 + 6*sqrt(a)*log(sqrt(a + b*x) + sqrt(a
))*a*b**2*d*x**3 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**3*c*x**3)/(48
*a**3*x**3)
```

3.245 $\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 175

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^6}\,dx = -\frac{c\sqrt{ax^2+bx^3}}{4x^5}-\frac{(bc+8ad)\sqrt{ax^2+bx^3}}{24ax^4}+\frac{b(5bc-8ad)\sqrt{ax^2+bx^3}}{96a^2x^3}-\frac{b^2(5bc-8ad)\sqrt{ax^2+bx^3}}{64a^3x^2}+\frac{b^3(5bc-8ad)\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{64a^{7/2}}$$

output
```
-1/4*c*(b*x^3+a*x^2)^(1/2)/x^5-1/24*(8*a*d+b*c)*(b*x^3+a*x^2)^(1/2)/a/x^4+
1/96*b*(-8*a*d+5*b*c)*(b*x^3+a*x^2)^(1/2)/a^2/x^3-1/64*b^2*(-8*a*d+5*b*c)*
(b*x^3+a*x^2)^(1/2)/a^3/x^2+1/64*b^3*(-8*a*d+5*b*c)*arctanh((b*x^3+a*x^2)^
(1/2)/a^(1/2)/x)/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.78

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^6}\,dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(15b^3cx^3+8a^2bx(c+2dx)+16a^3(3c+4dx)-2ab^2x^2(5c+12dx))+3b^3(5\right.}{192a^{7/2}x^5\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x^2 + b*x^3])/x^6,x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(15*b^3*c*x^3 + 8*a^2*b*x*(c
 + 2*d*x) + 16*a^3*(3*c + 4*d*x) - 2*a*b^2*x^2*(5*c + 12*d*x))) + 3*b^3*(5
*b*c - 8*a*d)*x^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(192*a^(7/2)*x^5*Sqrt[a
 + b*x])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.91, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1944, 1926, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}(c+dx)}{x^6}\,dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(5bc-8ad)\int \frac{\sqrt{bx^3+ax^2}}{x^5}dx}{8a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{4ax^7}$$

$$\Big\downarrow \text{1926}$$

$$-\frac{(5bc-8ad)\left(\frac{1}{6}b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)}{8a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{4ax^7}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{(5bc-8ad)\left(\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)}{8a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{4ax^7}$$

$$\Big\downarrow \; 1931$$

$$-\frac{(5bc-8ad)\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)}{8a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{4ax^7}$$

$$\Big\downarrow \; 1914$$

$$-\frac{(5bc-8ad)\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)}{8a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{4ax^7}$$

$$\Big\downarrow \; 219$$

$$-\frac{\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)(5bc-8ad)}{8a}-\frac{c\left(ax^2+bx^3\right)^{3/2}}{4ax^7}$$

input
```
Int[((c + d*x)*Sqrt[a*x^2 + b*x^3])/x^6,x]
```

output
```
-1/4*(c*(a*x^2 + b*x^3)^(3/2))/(a*x^7) - ((5*b*c - 8*a*d)*(-1/3*Sqrt[a*x^2
 + b*x^3]/x^4 + (b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2
+ b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))
/(4*a)))/6))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.67

method	result
pseudoelliptic	$\frac{\frac{5b^4x^5\left(ad-\frac{7bc}{10}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{64}+\frac{7\left(-\frac{5x^3\left(\frac{15dx}{7}+c\right)b^3a^{\frac{3}{2}}}{4}+b^2x^2\left(\frac{25dx}{14}+c\right)a^{\frac{5}{2}}-\frac{6xb\left(\frac{5dx}{3}+c\right)a^{\frac{7}{2}}}{7}+\frac{12(-5dx-4c)a^{\frac{9}{2}}}{7}+\frac{15\sqrt{a}\,b^4c}{8}\right.}{240}}{a^{\frac{9}{2}}x^5}$
risch	$-\frac{\left(-24a\,b^2d\,x^3+15b^3c\,x^3+16a^2bd\,x^2-10a\,b^2c\,x^2+64a^3dx+8a^2bcx+48c\,a^3\right)\sqrt{x^2(bx+a)}}{192x^5a^3}-\frac{(8ad-5bc)b^3\operatorname{arctanh}\left(\frac{\sqrt{bx+}}{\sqrt{a}}\right.}{64a^{\frac{7}{2}}x\sqrt{bx+c}}$
default	$\frac{\sqrt{b\,x^3+a\,x^2}\left(24(bx+a)^{\frac{7}{2}}a^{\frac{9}{2}}d-15(bx+a)^{\frac{7}{2}}a^{\frac{7}{2}}bc-88(bx+a)^{\frac{5}{2}}a^{\frac{11}{2}}d+55(bx+a)^{\frac{5}{2}}a^{\frac{9}{2}}bc+40(bx+a)^{\frac{3}{2}}a^{\frac{13}{2}}d-73(bx+a)^{\frac{3}{2}}a^{\frac{11}{2}}b\right.}{192b\,x^5\sqrt{bx+a}\,a^{\frac{13}{2}}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
5/64*(b^4*x^5*(a*d-7/10*b*c)*arctanh((b*x+a)^(1/2)/a^(1/2))+28/75*(-5/4*x^
3*(15/7*d*x+c)*b^3*a^(3/2)+b^2*x^2*(25/14*d*x+c)*a^(5/2)-6/7*x*b*(5/3*d*x+
c)*a^(7/2)+12/7*(-5*d*x-4*c)*a^(9/2)+15/8*a^(1/2)*b^4*c*x^4)*(b*x+a)^(1/2)
)/a^(9/2)/x^5
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^6}\,dx$$
$$=\left[-\frac{3\,(5\,b^4c-8\,ab^3d)\sqrt{a}x^5\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(48\,a^4c+3\,(5\,ab^3c-8\,a^2b^2d)x^3-2\,(5\,a^2b^2c-}{384\,a^4x^5}\right.$$
$$-\frac{3\,(5\,b^4c-8\,ab^3d)\sqrt{-a}x^5\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(48\,a^4c+3\,(5\,ab^3c-8\,a^2b^2d)x^3-2\,(5\,a^2b^2c-8\,a^3b}{192\,a^4x^5}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="fricas")
```

output
```
[-1/384*(3*(5*b^4*c - 8*a*b^3*d)*sqrt(a)*x^5*log((b*x^2 + 2*a*x - 2*sqrt(b
*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(48*a^4*c + 3*(5*a*b^3*c - 8*a^2*b^2*d)*x^
3 - 2*(5*a^2*b^2*c - 8*a^3*b*d)*x^2 + 8*(a^3*b*c + 8*a^4*d)*x)*sqrt(b*x^3
+ a*x^2))/(a^4*x^5), -1/192*(3*(5*b^4*c - 8*a*b^3*d)*sqrt(-a)*x^5*arctan(s
qrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (48*a^4*c + 3*(5*a*b^3*c - 8*
a^2*b^2*d)*x^3 - 2*(5*a^2*b^2*c - 8*a^3*b*d)*x^2 + 8*(a^3*b*c + 8*a^4*d)*x
)*sqrt(b*x^3 + a*x^2))/(a^4*x^5)]
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^6}\,dx=\int \frac{\sqrt{x^2\,(a+bx)}(c+dx)}{x^6}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(1/2)/x**6,x)
```

output
```
Integral(sqrt(x**2*(a + b*x))*(c + d*x)/x**6, x)
```

Maxima [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^6}\,dx=\int \frac{\sqrt{bx^3+ax^2}(dx+c)}{x^6}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)*(d*x + c)/x^6, x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^6}\,dx = -\frac{\frac{3\left(5\,b^5c\mathrm{sgn}(x)-8\,ab^4d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3}+\frac{15\,(bx+a)^{\frac{7}{2}}b^5c\mathrm{sgn}(x)-55\,(bx+a)^{\frac{5}{2}}ab^5c\mathrm{sgn}(x)+73\,(bx+a)^{\frac{3}{2}}a^2b^5c\mathrm{sgn}(x)+15\,\sqrt{bx+a}a^3b^5}{}}{192\,b}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^6,x, algorithm="giac")
```

output
```
-1/192*(3*(5*b^5*c*sgn(x) - 8*a*b^4*d*sgn(x))*arctan(sqrt(b*x + a)/sqrt(-a
))/(sqrt(-a)*a^3) + (15*(b*x + a)^(7/2)*b^5*c*sgn(x) - 55*(b*x + a)^(5/2)*
a*b^5*c*sgn(x) + 73*(b*x + a)^(3/2)*a^2*b^5*c*sgn(x) + 15*sqrt(b*x + a)*a^
3*b^5*c*sgn(x) - 24*(b*x + a)^(7/2)*a*b^4*d*sgn(x) + 88*(b*x + a)^(5/2)*a^
2*b^4*d*sgn(x) - 40*(b*x + a)^(3/2)*a^3*b^4*d*sgn(x) - 24*sqrt(b*x + a)*a^
4*b^4*d*sgn(x))/(a^3*b^4*x^4))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^6}\,dx = \int \frac{\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)}{x^6}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/x^6,x)
```

output
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.18

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{x^6}\,dx$$
$$= \frac{-96\sqrt{bx+a}\,a^4c-128\sqrt{bx+a}\,a^4dx-16\sqrt{bx+a}\,a^3bcx-32\sqrt{bx+a}\,a^3bd\,x^2+20\sqrt{bx+a}\,a^2b^2c\,x^2+4}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/x^6,x)
```

output
```
( - 96*sqrt(a + b*x)*a**4*c - 128*sqrt(a + b*x)*a**4*d*x - 16*sqrt(a + b*x
)*a**3*b*c*x - 32*sqrt(a + b*x)*a**3*b*d*x**2 + 20*sqrt(a + b*x)*a**2*b**2
*c*x**2 + 48*sqrt(a + b*x)*a**2*b**2*d*x**3 - 30*sqrt(a + b*x)*a*b**3*c*x*
*3 + 24*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**3*d*x**4 - 15*sqrt(a)*lo
g(sqrt(a + b*x) - sqrt(a))*b**4*c*x**4 - 24*sqrt(a)*log(sqrt(a + b*x) + sq
rt(a))*a*b**3*d*x**4 + 15*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**4*c*x**4
)/(384*a**4*x**4)
```

3.246 $\int x^2(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 240

$$\int x^2(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=-\frac{2a^5(bc-ad)\left(ax^2+bx^3\right)^{5/2}}{5b^7x^5}+\frac{2a^4(5bc-6ad)\left(ax^2+bx^3\right)^{7/2}}{7b^7x^7}-\frac{10a^3(2bc-3ad)\left(ax^2+bx^3\right)^{9/2}}{9b^7x^9}+\frac{20a^2(bc-2ad)\left(ax^2+bx^3\right)^{11/2}}{11b^7x^{11}}-\frac{10a(bc-3ad)\left(ax^2+bx^3\right)^{13/2}}{13b^7x^{13}}+\frac{2(bc-6ad)\left(ax^2+bx^3\right)^{15/2}}{15b^7x^{15}}+\frac{2d\left(ax^2+bx^3\right)^{17/2}}{17b^7x^{17}}$$

output
```
-2/5*a^5*(-a*d+b*c)*(b*x^3+a*x^2)^(5/2)/b^7/x^5+2/7*a^4*(-6*a*d+5*b*c)*(b*
x^3+a*x^2)^(7/2)/b^7/x^7-10/9*a^3*(-3*a*d+2*b*c)*(b*x^3+a*x^2)^(9/2)/b^7/x
^9+20/11*a^2*(-2*a*d+b*c)*(b*x^3+a*x^2)^(11/2)/b^7/x^11-10/13*a*(-3*a*d+b*
c)*(b*x^3+a*x^2)^(13/2)/b^7/x^13+2/15*(-6*a*d+b*c)*(b*x^3+a*x^2)^(15/2)/b^
7/x^15+2/17*d*(b*x^3+a*x^2)^(17/2)/b^7/x^17
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.57

$$\int x^2(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx = \frac{2x(a+bx)^3\left(3072a^6d+3003b^6x^5(17c+15dx)-1120a^3b^3x^2(17c+18dx)+640a^4b^2x(17c+\right.}{765765b^7\sqrt{x^2(a}}$$

input
```
Integrate[x^2*(c + d*x)*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(a + b*x)^3*(3072*a^6*d + 3003*b^6*x^5*(17*c + 15*d*x) - 1120*a^3*b^3
*x^2*(17*c + 18*d*x) + 640*a^4*b^2*x*(17*c + 21*d*x) - 256*a^5*b*(17*c + 3
0*d*x) + 840*a^2*b^4*x^3*(34*c + 33*d*x) - 462*a*b^5*x^4*(85*c + 78*d*x)))
/(765765*b^7*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1945, 1922, 1922, 1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(ax^2+bx^3\right)^{3/2}(c+dx)\,dx$$

$$\Big\downarrow \text{ 1945}$$

$$\frac{(17bc-12ad)\int x^2\left(bx^3+ax^2\right)^{3/2}dx}{17b}+\frac{2dx\left(ax^2+bx^3\right)^{5/2}}{17b}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{(17bc-12ad)\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{2a\int x\left(bx^3+ax^2\right)^{3/2}dx}{3b}\right)}{17b}+\frac{2dx\left(ax^2+bx^3\right)^{5/2}}{17b}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{(17bc-12ad)\left(\frac{2(ax^2+bx^3)^{5/2}}{15b}-\frac{2a\left(\frac{2(ax^2+bx^3)^{5/2}}{13bx}-\frac{8a\int (bx^3+ax^2)^{3/2}dx}{13b}\right)}{3b}\right)}{17b}+\frac{2dx(ax^2+bx^3)^{5/2}}{17b}$$

$\Big\downarrow$ 1908

$$\frac{(17bc-12ad)\left(\frac{2(ax^2+bx^3)^{5/2}}{15b}-\frac{2a\left(\frac{2(ax^2+bx^3)^{5/2}}{13bx}-\frac{8a\left(\frac{2(ax^2+bx^3)^{5/2}}{11bx^2}-\frac{6a\int \frac{(bx^3+ax^2)^{3/2}}{x}dx}{11b}\right)}{13b}\right)}{3b}\right)}{17b}+\frac{2dx(ax^2+bx^3)^{5/2}}{17b}$$

$\Big\downarrow$ 1922

$$\frac{(17bc-12ad)\left(\frac{2(ax^2+bx^3)^{5/2}}{15b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\int\frac{\left(bx^3+ax^2\right)^{3/2}}{x^2}dx}{9b}\right)}{11b}\right)}{13b}\right)}{3b}\right)}{17b}+\frac{2dx\left(ax^2+bx^3\right)^{5/2}}{17b}$$

$$\Big\downarrow \; 1922$$

$$\frac{(17bc-12ad)\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{15b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{2a\int\frac{\left(bx^3+ax^2\right)^{3/\cdots}}{x^3}\cdots}{7b}\cdots\right.}{9b}\cdots\right.}{11b}\cdots\right.}{13b}\cdots\right.}{3b}\cdots\right.}{17b}$$

$$\frac{2dx\left(ax^2+bx^3\right)^{5/2}}{17b}$$

$\downarrow$ 1920

$$\frac{\left(\frac{2(ax^2+bx^3)^{5/2}}{15b}-\frac{2a\left(\frac{2(ax^2+bx^3)^{5/2}}{13bx}-\frac{8a\left(\frac{2(ax^2+bx^3)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2(ax^2+bx^3)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2(ax^2+bx^3)^{5/2}}{7bx^4}-\frac{4a(ax^2+bx^3)^{5/2}}{35b^2x^5}\right)}{9b}\right)}{11b}\right)}{13b}\right)}{3b}\right)(17bc\ \cdots}{17b}$$

$$\frac{2dx(ax^2+bx^3)^{5/2}}{17b}$$

input
```
Int[x^2*(c + d*x)*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*d*x*(a*x^2 + b*x^3)^(5/2))/(17*b) + ((17*b*c - 12*a*d)*((2*(a*x^2 + b*x
^3)^(5/2))/(15*b) - (2*a*((2*(a*x^2 + b*x^3)^(5/2))/(13*b*x) - (8*a*((2*(a
*x^2 + b*x^3)^(5/2))/(11*b*x^2) - (6*a*((2*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3
) - (4*a*((-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(
5/2))/(7*b*x^4)))/(9*b)))/(11*b)))/(13*b)))/(3*b)))/(17*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.24

method	result
pseudoelliptic	$-\frac{32\left(-\frac{385\left(\frac{9dx}{11}+c\right)x^{2}b^{3}}{48}+\frac{55x\left(\frac{21dx}{22}+c\right)a\,b^{2}}{12}-\frac{11\left(\frac{15dx}{11}+c\right)a^{2}b}{6}+a^{3}d\right)(bx+a)^{\frac{5}{2}}}{1155b^{4}}$
gosper	$\frac{2(bx+a)\left(45045d\,x^{6}b^{6}-36036a\,b^{5}d\,x^{5}+51051b^{6}c\,x^{5}+27720a^{2}b^{4}d\,x^{4}-39270a\,b^{5}c\,x^{4}-20160a^{3}b^{3}d\,x^{3}+28560a^{2}b^{4}c\,x^{3}+1344}{765765b^{7}x^{3}}$
default	$\frac{2(bx+a)\left(45045d\,x^{6}b^{6}-36036a\,b^{5}d\,x^{5}+51051b^{6}c\,x^{5}+27720a^{2}b^{4}d\,x^{4}-39270a\,b^{5}c\,x^{4}-20160a^{3}b^{3}d\,x^{3}+28560a^{2}b^{4}c\,x^{3}+1344}{765765b^{7}x^{3}}$
orering	$\frac{2(bx+a)\left(45045d\,x^{6}b^{6}-36036a\,b^{5}d\,x^{5}+51051b^{6}c\,x^{5}+27720a^{2}b^{4}d\,x^{4}-39270a\,b^{5}c\,x^{4}-20160a^{3}b^{3}d\,x^{3}+28560a^{2}b^{4}c\,x^{3}+1344}{765765b^{7}x^{3}}$
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,\left(45045b^{8}d\,x^{8}+54054a\,b^{7}d\,x^{7}+51051b^{8}c\,x^{7}+693a^{2}b^{6}d\,x^{6}+62832a\,b^{7}c\,x^{6}-756a^{3}b^{5}d\,x^{5}+1071a^{2}b^{6}c\,x^{5}+840a}{7}$
trager	$\frac{2\left(45045b^{8}d\,x^{8}+54054a\,b^{7}d\,x^{7}+51051b^{8}c\,x^{7}+693a^{2}b^{6}d\,x^{6}+62832a\,b^{7}c\,x^{6}-756a^{3}b^{5}d\,x^{5}+1071a^{2}b^{6}c\,x^{5}+840a^{4}b^{4}d\,x^{4}-119}{7}$

input
```
int(x^2*(d*x+c)*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-32/1155*(-385/48*(9/11*d*x+c)*x^2*b^3+55/12*x*(21/22*d*x+c)*a*b^2-11/6*(1
5/11*d*x+c)*a^2*b+a^3*d)*(b*x+a)^(5/2)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.84

$$\int x^2(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(45045\,b^8dx^8-4352\,a^7bc+3072\,a^8d+3003\,(17\,b^8c+18\,ab^7d)x^7+231\,(272\,ab^7c+3\,a^2b^6d\right.}{}$$

input
```
integrate(x^2*(d*x+c)*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/765765*(45045*b^8*d*x^8 - 4352*a^7*b*c + 3072*a^8*d + 3003*(17*b^8*c + 1
8*a*b^7*d)*x^7 + 231*(272*a*b^7*c + 3*a^2*b^6*d)*x^6 + 63*(17*a^2*b^6*c -
12*a^3*b^5*d)*x^5 - 70*(17*a^3*b^5*c - 12*a^4*b^4*d)*x^4 + 80*(17*a^4*b^4*
c - 12*a^5*b^3*d)*x^3 - 96*(17*a^5*b^3*c - 12*a^6*b^2*d)*x^2 + 128*(17*a^6
*b^2*c - 12*a^7*b*d)*x)*sqrt(b*x^3 + a*x^2)/(b^7*x)
```

Sympy [F]

$$\int x^2(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\int x^2\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)\,dx$$

input
```
integrate(x**2*(d*x+c)*(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**2*(x**2*(a + b*x))**(3/2)*(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.78

$$\int x^2(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(3003\,b^7x^7+3696\,ab^6x^6+63\,a^2b^5x^5-70\,a^3b^4x^4+80\,a^4b^3x^3-96\,a^5b^2x^2+128\,a^6bx-256\right.}{45045\,b^6}+\frac{2\left(15015\,b^8x^8+18018\,ab^7x^7+231\,a^2b^6x^6-252\,a^3b^5x^5+280\,a^4b^4x^4-320\,a^5b^3x^3+384\,a^6b^2x^2-512\,a^7b\right.}{255255\,b^7}$$

input
```
integrate(x^2*(d*x+c)*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/45045*(3003*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 +
 80*a^4*b^3*x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x + a)*c/
b^6 + 2/255255*(15015*b^8*x^8 + 18018*a*b^7*x^7 + 231*a^2*b^6*x^6 - 252*a^
3*b^5*x^5 + 280*a^4*b^4*x^4 - 320*a^5*b^3*x^3 + 384*a^6*b^2*x^2 - 512*a^7*
b*x + 1024*a^8)*sqrt(b*x + a)*d/b^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(212) = 424$.

Time = 0.19 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.50

$$\int x^2(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(d*x+c)*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
2/765765*(1105*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a
)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sq
rt(b*x + a)*a^5)*a^2*c*sgn(x)/b^5 + 510*(231*(b*x + a)^(13/2) - 1638*(b*x
+ a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009
*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*
a*c*sgn(x)/b^5 + 255*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 500
5*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^
4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^2*d*sgn(x)/b^6 +
119*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/
2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*
x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*c*s
gn(x)/b^5 + 238*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b
*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4
 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x +
 a)*a^7)*a*d*sgn(x)/b^6 + 7*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2
)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(
b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a
^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*d*sgn(x)/b^6)/
b + 512/765765*(17*a^(15/2)*b*c - 12*a^(17/2)*d)*sgn(x)/b^7
```

Mupad [B] (verification not implemented)

Time = 9.30 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.71

$$\int x^2(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(x^7\,\left(\frac{12\,a\,d}{85}+\frac{2\,b\,c}{15}\right)+\frac{512\,a^7\,\left(12\,a\,d-17\,b\,c\right)}{765765\,b^7}+\frac{2\,b\,d\,x^8}{17}-\frac{256\,a^6\,x\,\left(12\,a\,d-17\,b\,c\right)}{765765\,b^6}+\frac{2\,a\,x^6\,\left(3\,a\,d+2\right.}{3315\,b}\right.}{x}$$

input
```
int(x^2*(a*x^2 + b*x^3)^(3/2)*(c + d*x),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*(x^7*((12*a*d)/85 + (2*b*c)/15) + (512*a^7*(12*a*d
- 17*b*c))/(765765*b^7) + (2*b*d*x^8)/17 - (256*a^6*x*(12*a*d - 17*b*c))/(
765765*b^6) + (2*a*x^6*(3*a*d + 272*b*c))/(3315*b) - (2*a^2*x^5*(12*a*d -
17*b*c))/(12155*b^2) + (4*a^3*x^4*(12*a*d - 17*b*c))/(21879*b^3) - (32*a^4
*x^3*(12*a*d - 17*b*c))/(153153*b^4) + (64*a^5*x^2*(12*a*d - 17*b*c))/(255
255*b^5)))/x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.79

$$\int x^2(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\sqrt{bx+a}\,(45045b^8d\,x^8+54054a\,b^7d\,x^7+51051b^8c\,x^7+693a^2b^6d\,x^6+62832a\,b^7c\,x^6-756a$$

input
```
int(x^2*(d*x+c)*(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*sqrt(a + b*x)*(3072*a**8*d - 4352*a**7*b*c - 1536*a**7*b*d*x + 2176*a**
6*b**2*c*x + 1152*a**6*b**2*d*x**2 - 1632*a**5*b**3*c*x**2 - 960*a**5*b**3
*d*x**3 + 1360*a**4*b**4*c*x**3 + 840*a**4*b**4*d*x**4 - 1190*a**3*b**5*c*
x**4 - 756*a**3*b**5*d*x**5 + 1071*a**2*b**6*c*x**5 + 693*a**2*b**6*d*x**6
 + 62832*a*b**7*c*x**6 + 54054*a*b**7*d*x**7 + 51051*b**8*c*x**7 + 45045*b
**8*d*x**8))/(765765*b**7)
```

3.247 $\int x(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 205

$$\int x(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2a^4(bc-ad)\left(ax^2+bx^3\right)^{5/2}}{5b^6x^5}-\frac{2a^3(4bc-5ad)\left(ax^2+bx^3\right)^{7/2}}{7b^6x^7}+\frac{4a^2(3bc-5ad)\left(ax^2+bx^3\right)^{9/2}}{9b^6x^9}-\frac{4a(2bc-5ad)\left(ax^2+bx^3\right)^{11/2}}{11b^6x^{11}}+\frac{2(bc-5ad)\left(ax^2+bx^3\right)^{13/2}}{13b^6x^{13}}+\frac{2d\left(ax^2+bx^3\right)^{15/2}}{15b^6x^{15}}$$

output
```
2/5*a^4*(-a*d+b*c)*(b*x^3+a*x^2)^(5/2)/b^6/x^5-2/7*a^3*(-5*a*d+4*b*c)*(b*x
^3+a*x^2)^(7/2)/b^6/x^7+4/9*a^2*(-5*a*d+3*b*c)*(b*x^3+a*x^2)^(9/2)/b^6/x^9
-4/11*a*(-5*a*d+2*b*c)*(b*x^3+a*x^2)^(11/2)/b^6/x^11+2/13*(-5*a*d+b*c)*(b*
x^3+a*x^2)^(13/2)/b^6/x^13+2/15*d*(b*x^3+a*x^2)^(15/2)/b^6/x^15
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.56

$$\int x(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2x(a+bx)^3\left(-256a^5d+1680a^2b^3x^2(c+dx)+128a^4b(3c+5dx)-160a^3b^2x(6c+7dx)-2\right.}{45045b^6\sqrt{x^2(a+bx)}}$$

input
```
Integrate[x*(c + d*x)*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(a + b*x)^3*(-256*a^5*d + 1680*a^2*b^3*x^2*(c + d*x) + 128*a^4*b*(3*c
 + 5*d*x) - 160*a^3*b^2*x*(6*c + 7*d*x) - 210*a*b^4*x^3*(12*c + 11*d*x) +
231*b^5*x^4*(15*c + 13*d*x)))/(45045*b^6*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1945, 1922, 1908, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax^2+bx^3\right)^{3/2}(c+dx)\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(3bc-2ad)\int x\left(bx^3+ax^2\right)^{3/2}dx}{3b}+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{15b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(3bc-2ad)\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\int\left(bx^3+ax^2\right)^{3/2}dx}{13b}\right)}{3b}+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{15b}$$

$$\Big\downarrow \text{1908}$$

$$\frac{(3bc-2ad)\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\int\frac{\left(bx^3+ax^2\right)^{3/2}}{x}dx}{11b}\right)}{13b}\right)}{3b}+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{15b}$$

$\downarrow$ 1922

$$\frac{(3bc-2ad)\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\int\frac{\left(bx^3+ax^2\right)^{3/2}}{x^2}dx}{9b}\right)}{11b}\right)}{13b}\right)}{3b}+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{15b}$$

$\downarrow$ 1922

$$\frac{(3bc-2ad)\left(\frac{2(ax^2+bx^3)^{5/2}}{13bx}-\frac{8a\left(\frac{2(ax^2+bx^3)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2(ax^2+bx^3)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2(ax^2+bx^3)^{5/2}}{7bx^4}-\frac{2a\int\frac{(bx^3+ax^2)^{3/2}}{x^3}dx}{7b}\right)}{9b}\right)}{11b}\right)}{13b}\right)}{3b}+\frac{2d(ax^2+bx^3)^{5/2}}{15b}$$

$\downarrow$ 1920

$$\frac{\left(\frac{2(ax^2+bx^3)^{5/2}}{13bx}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{11bx^2}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}\right)}{9b}\right)}{11b}\right)}{13b}\right)(3bc-2ad)}{3b}+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{15b}$$

input
```
Int[x*(c + d*x)*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*d*(a*x^2 + b*x^3)^(5/2))/(15*b) + ((3*b*c - 2*a*d)*((2*(a*x^2 + b*x^3)^
(5/2))/(13*b*x) - (8*a*((2*(a*x^2 + b*x^3)^(5/2))/(11*b*x^2) - (6*a*((2*(a
*x^2 + b*x^3)^(5/2))/(9*b*x^3) - (4*a*((-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^
2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b*x^4)))/(9*b)))/(11*b)))/(13*b)))/(
3*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.20

method	result
pseudoelliptic	$\frac{16(bx+a)^{\frac{5}{2}}\left(\frac{45x\left(c+\frac{7dx}{9}\right)b^2}{8}-\frac{9\left(\frac{10dx}{9}+c\right)ab}{4}+a^2d\right)}{315b^3}$
gosper	$-\frac{2(bx+a)\left(-3003d\,x^5b^5+2310a\,b^4d\,x^4-3465b^5c\,x^4-1680a^2b^3d\,x^3+2520a\,b^4c\,x^3+1120a^3b^2d\,x^2-1680a^2b^3c\,x^2-640a^4ba\right.}{45045b^6x^3}$
default	$-\frac{2(bx+a)\left(-3003d\,x^5b^5+2310a\,b^4d\,x^4-3465b^5c\,x^4-1680a^2b^3d\,x^3+2520a\,b^4c\,x^3+1120a^3b^2d\,x^2-1680a^2b^3c\,x^2-640a^4ba\right.}{45045b^6x^3}$
orering	$-\frac{2(bx+a)\left(-3003d\,x^5b^5+2310a\,b^4d\,x^4-3465b^5c\,x^4-1680a^2b^3d\,x^3+2520a\,b^4c\,x^3+1120a^3b^2d\,x^2-1680a^2b^3c\,x^2-640a^4ba\right.}{45045b^6x^3}$
risch	$-\frac{2\sqrt{x^2(bx+a)}\left(-3003b^7d\,x^7-3696a\,b^6d\,x^6-3465b^7c\,x^6-63a^2b^5d\,x^5-4410a\,b^6c\,x^5+70a^3b^4d\,x^4-105a^2b^5c\,x^4-80a^4b^3d\right.}{45045x\,b^6}$
trager	$-\frac{2\left(-3003b^7d\,x^7-3696a\,b^6d\,x^6-3465b^7c\,x^6-63a^2b^5d\,x^5-4410a\,b^6c\,x^5+70a^3b^4d\,x^4-105a^2b^5c\,x^4-80a^4b^3d\,x^3+120a^3b^4\right.}{45045b^6x}$

input
```
int(x*(d*x+c)*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
16/315*(b*x+a)^(5/2)*(45/8*x*(c+7/9*d*x)*b^2-9/4*(10/9*d*x+c)*a*b+a^2*d)/b
^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.86

$$\int x(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(3003\,b^7dx^7+384\,a^6bc-256\,a^7d+231\,(15\,b^7c+16\,ab^6d)x^6+63\,(70\,ab^6c+a^2b^5d)x^5+35\right.}{}$$

input
```
integrate(x*(d*x+c)*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/45045*(3003*b^7*d*x^7 + 384*a^6*b*c - 256*a^7*d + 231*(15*b^7*c + 16*a*b
^6*d)*x^6 + 63*(70*a*b^6*c + a^2*b^5*d)*x^5 + 35*(3*a^2*b^5*c - 2*a^3*b^4*
d)*x^4 - 40*(3*a^3*b^4*c - 2*a^4*b^3*d)*x^3 + 48*(3*a^4*b^3*c - 2*a^5*b^2*
d)*x^2 - 64*(3*a^5*b^2*c - 2*a^6*b*d)*x)*sqrt(b*x^3 + a*x^2)/(b^6*x)
```

Sympy [F]

$$\int x(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\int x\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)\,dx$$

input
```
integrate(x*(d*x+c)*(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x*(x**2*(a + b*x))**(3/2)*(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.80

$$\int x(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6\right)\sqrt{bx+a}c}{15015\,b^5}+\frac{2\left(3003\,b^7x^7+3696\,ab^6x^6+63\,a^2b^5x^5-70\,a^3b^4x^4+80\,a^4b^3x^3-96\,a^5b^2x^2+128\,a^6bx-256\,a^7\right)\sqrt{bx+a}d}{45045\,b^6}$$

input
```
integrate(x*(d*x+c)*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/15015*(1155*b^6*x^6 + 1470*a*b^5*x^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 +
 48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a^6)*sqrt(b*x + a)*c/b^5 + 2/45045*(300
3*b^7*x^7 + 3696*a*b^6*x^6 + 63*a^2*b^5*x^5 - 70*a^3*b^4*x^4 + 80*a^4*b^3*
x^3 - 96*a^5*b^2*x^2 + 128*a^6*b*x - 256*a^7)*sqrt(b*x + a)*d/b^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. 2(181) = 362.

Time = 0.19 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.58

$$\int x(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate(x*(d*x+c)*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
2/45045*(143*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(
5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^2*c*sgn(x)/b
^4 + 130*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2
)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x
 + a)*a^5)*a*c*sgn(x)/b^4 + 65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*
a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3
/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^2*d*sgn(x)/b^5 + 15*(231*(b*x + a)^(13/
2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(
7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt
(b*x + a)*a^6)*c*sgn(x)/b^4 + 30*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(1
1/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x +
 a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a*d*sgn
(x)/b^5 + 7*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x +
 a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 2
7027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*
a^7)*d*sgn(x)/b^5)/b - 256/45045*(3*a^(13/2)*b*c - 2*a^(15/2)*d)*sgn(x)/b^
6
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.74

$$\int x(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{\sqrt{b\,x^3+a\,x^2}\,\left(x^6\,\left(\frac{32\,a\,d}{195}+\frac{2\,b\,c}{13}\right)-\frac{512\,a^7\,d-768\,a^6\,b\,c}{45045\,b^6}+\frac{2\,b\,d\,x^7}{15}+\frac{128\,a^5\,x\,(2\,a\,d-3\,b\,c)}{45045\,b^5}+\frac{2\,a\,x^5\,(a\,d+70\,b\,c)}{715\,b}\right.}{x}$$

input
```
int(x*(a*x^2 + b*x^3)^(3/2)*(c + d*x),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*(x^6*((32*a*d)/195 + (2*b*c)/13) - (512*a^7*d - 768
*a^6*b*c)/(45045*b^6) + (2*b*d*x^7)/15 + (128*a^5*x*(2*a*d - 3*b*c))/(4504
5*b^5) + (2*a*x^5*(a*d + 70*b*c))/(715*b) - (2*a^2*x^4*(2*a*d - 3*b*c))/(1
287*b^2) + (16*a^3*x^3*(2*a*d - 3*b*c))/(9009*b^3) - (32*a^4*x^2*(2*a*d -
3*b*c))/(15015*b^4)))/x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.80

$$\int x(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\sqrt{bx+a}\,(3003b^7d\,x^7+3696a\,b^6d\,x^6+3465b^7c\,x^6+63a^2b^5d\,x^5+4410a\,b^6c\,x^5-70a^3b^4d\,x^4}{}$$

input
```
int(x*(d*x+c)*(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 256*a**7*d + 384*a**6*b*c + 128*a**6*b*d*x - 192*a**5
*b**2*c*x - 96*a**5*b**2*d*x**2 + 144*a**4*b**3*c*x**2 + 80*a**4*b**3*d*x*
*3 - 120*a**3*b**4*c*x**3 - 70*a**3*b**4*d*x**4 + 105*a**2*b**5*c*x**4 + 6
3*a**2*b**5*d*x**5 + 4410*a*b**6*c*x**5 + 3696*a*b**6*d*x**6 + 3465*b**7*c
*x**6 + 3003*b**7*d*x**7))/(45045*b**6)
```

3.248 $\int (c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 167

$$\begin{aligned}\int (c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx &= -\frac{2a^3(bc-ad)\left(ax^2+bx^3\right)^{5/2}}{5b^5x^5}\\
&+\frac{2a^2(3bc-4ad)\left(ax^2+bx^3\right)^{7/2}}{7b^5x^7}-\frac{2a(bc-2ad)\left(ax^2+bx^3\right)^{9/2}}{3b^5x^9}\\
&+\frac{2(bc-4ad)\left(ax^2+bx^3\right)^{11/2}}{11b^5x^{11}}+\frac{2d\left(ax^2+bx^3\right)^{13/2}}{13b^5x^{13}}\end{aligned}$$

output
```
-2/5*a^3*(-a*d+b*c)*(b*x^3+a*x^2)^(5/2)/b^5/x^5+2/7*a^2*(-4*a*d+3*b*c)*(b*
x^3+a*x^2)^(7/2)/b^5/x^7-2/3*a*(-2*a*d+b*c)*(b*x^3+a*x^2)^(9/2)/b^5/x^9+2/
11*(-4*a*d+b*c)*(b*x^3+a*x^2)^(11/2)/b^5/x^11+2/13*d*(b*x^3+a*x^2)^(13/2)/
b^5/x^13
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.59

$$\int (c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2x(a+bx)^3\left(128a^4d+105b^4x^3(13c+11dx)-70ab^3x^2(13c+12dx)+40a^2b^2x(13c+14dx)\right.}{15015b^5\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(a + b*x)^3*(128*a^4*d + 105*b^4*x^3*(13*c + 11*d*x) - 70*a*b^3*x^2*(
13*c + 12*d*x) + 40*a^2*b^2*x*(13*c + 14*d*x) - 16*a^3*b*(13*c + 20*d*x)))
/(15015*b^5*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.51, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2450, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^{3/2}(c+dx)\,dx$$

$$\Big\downarrow \text{2450}$$

$$\int \left(c\left(ax^2+bx^3\right)^{3/2}+dx\left(ax^2+bx^3\right)^{3/2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{256a^4d\left(ax^2+bx^3\right)^{5/2}}{15015b^5x^5}-\frac{32a^3c\left(ax^2+bx^3\right)^{5/2}}{1155b^4x^5}-\frac{128a^3d\left(ax^2+bx^3\right)^{5/2}}{3003b^4x^4}+\frac{16a^2c\left(ax^2+bx^3\right)^{5/2}}{231b^3x^4}+\frac{32a^2d\left(ax^2+bx^3\right)^{5/2}}{429b^3x^3}-\frac{4ac\left(ax^2+bx^3\right)^{5/2}}{33b^2x^3}-\frac{16ad\left(ax^2+bx^3\right)^{5/2}}{143b^2x^2}+\frac{2c\left(ax^2+bx^3\right)^{5/2}}{11bx^2}+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{13bx}$$

input
```
Int[(c + d*x)*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-32*a^3*c*(a*x^2 + b*x^3)^(5/2))/(1155*b^4*x^5) + (256*a^4*d*(a*x^2 + b*x
^3)^(5/2))/(15015*b^5*x^5) + (16*a^2*c*(a*x^2 + b*x^3)^(5/2))/(231*b^3*x^4
) - (128*a^3*d*(a*x^2 + b*x^3)^(5/2))/(3003*b^4*x^4) - (4*a*c*(a*x^2 + b*x
^3)^(5/2))/(33*b^2*x^3) + (32*a^2*d*(a*x^2 + b*x^3)^(5/2))/(429*b^3*x^3) +
 (2*c*(a*x^2 + b*x^3)^(5/2))/(11*b*x^2) - (16*a*d*(a*x^2 + b*x^3)^(5/2))/(
143*b^2*x^2) + (2*d*(a*x^2 + b*x^3)^(5/2))/(13*b*x)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2450
```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IntegerQ[p] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.16

method	result
pseudoelliptic	$-\frac{2(bx+a)^{\frac{5}{2}}(-5bdx+2ad-7bc)}{35b^2}$
gosper	$\frac{2(bx+a)\left(1155d\,x^4b^4-840a\,b^3d\,x^3+1365b^4c\,x^3+560a^2b^2d\,x^2-910a\,b^3c\,x^2-320a^3bdx+520a^2b^2cx+128a^4d-208a^3bc\right)(b\,x^3}{15015b^5x^3}$
default	$\frac{2(bx+a)\left(1155d\,x^4b^4-840a\,b^3d\,x^3+1365b^4c\,x^3+560a^2b^2d\,x^2-910a\,b^3c\,x^2-320a^3bdx+520a^2b^2cx+128a^4d-208a^3bc\right)(b\,x^3}{15015b^5x^3}$
orering	$\frac{2(bx+a)\left(1155d\,x^4b^4-840a\,b^3d\,x^3+1365b^4c\,x^3+560a^2b^2d\,x^2-910a\,b^3c\,x^2-320a^3bdx+520a^2b^2cx+128a^4d-208a^3bc\right)(b\,x^3}{15015b^5x^3}$
risch	$\frac{2\sqrt{x^2(bx+a)}\,(1155d\,x^6b^6+1470a\,b^5d\,x^5+1365b^6c\,x^5+35a^2b^4d\,x^4+1820a\,b^5c\,x^4-40a^3b^3d\,x^3+65a^2b^4c\,x^3+48a^4b^2d\,x^2-7}{15015x\,b^5}$
trager	$\frac{2(1155d\,x^6b^6+1470a\,b^5d\,x^5+1365b^6c\,x^5+35a^2b^4d\,x^4+1820a\,b^5c\,x^4-40a^3b^3d\,x^3+65a^2b^4c\,x^3+48a^4b^2d\,x^2-78a^3b^3c\,x^2-6}{15015b^5x}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(b*x+a)^(5/2)*(-5*b*d*x+2*a*d-7*b*c)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.91

$$\int (c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(1155\,b^6dx^6-208\,a^5bc+128\,a^6d+105\left(13\,b^6c+14\,ab^5d\right)x^5+35\left(52\,ab^5c+a^2b^4d\right)x^4+5\,(\right.}{15015\,b^5}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/15015*(1155*b^6*d*x^6 - 208*a^5*b*c + 128*a^6*d + 105*(13*b^6*c + 14*a*b
^5*d)*x^5 + 35*(52*a*b^5*c + a^2*b^4*d)*x^4 + 5*(13*a^2*b^4*c - 8*a^3*b^3*
d)*x^3 - 6*(13*a^3*b^3*c - 8*a^4*b^2*d)*x^2 + 8*(13*a^4*b^2*c - 8*a^5*b*d)
*x)*sqrt(b*x^3 + a*x^2)/(b^5*x)
```

Sympy [F]

$$\int (c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\int \left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)*(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.85

$$\int (c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx+a}c}{1155\,b^4}+\frac{2\left(1155\,b^6x^6+1470\,ab^5x^5+35\,a^2b^4x^4-40\,a^3b^3x^3+48\,a^4b^2x^2-64\,a^5bx+128\,a^6\right)\sqrt{bx+a}d}{15015\,b^5}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*a^3*b^2*x^2 + 8*a^
4*b*x - 16*a^5)*sqrt(b*x + a)*c/b^4 + 2/15015*(1155*b^6*x^6 + 1470*a*b^5*x
^5 + 35*a^2*b^4*x^4 - 40*a^3*b^3*x^3 + 48*a^4*b^2*x^2 - 64*a^5*b*x + 128*a
^6)*sqrt(b*x + a)*d/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(147) = 294.

Time = 0.16 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.73

$$\int (c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
2/45045*(1287*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/
2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2*c*sgn(x)/b^3 + 286*(35*(b*x + a)^(9/2)
- 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^
3 + 315*sqrt(b*x + a)*a^4)*a*c*sgn(x)/b^3 + 143*(35*(b*x + a)^(9/2) - 180*
(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 31
5*sqrt(b*x + a)*a^4)*a^2*d*sgn(x)/b^4 + 65*(63*(b*x + a)^(11/2) - 385*(b*x
 + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*
(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*c*sgn(x)/b^3 + 130*(63*(b*x +
 a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x +
 a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a*d*sgn(
x)/b^4 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x +
a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*
(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*d*sgn(x)/b^4)/b + 32/15015*(
13*a^(11/2)*b*c - 8*a^(13/2)*d)*sgn(x)/b^5
```

Mupad [B] (verification not implemented)

Time = 9.28 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.79

$$\int (c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{\sqrt{b\,x^3+a\,x^2}\,\left(x^5\,\left(\frac{28\,a\,d}{143}+\frac{2\,b\,c}{11}\right)+\frac{256\,a^6\,d-416\,a^5\,b\,c}{15015\,b^5}+\frac{2\,b\,d\,x^6}{13}-\frac{16\,a^4\,x\,(8\,a\,d-13\,b\,c)}{15015\,b^4}+\frac{2\,a\,x^4\,(a\,d+52\,b\,c)}{429\,b}\right.}{x}$$

input
```
int((a*x^2 + b*x^3)^(3/2)*(c + d*x),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*(x^5*((28*a*d)/143 + (2*b*c)/11) + (256*a^6*d - 416
*a^5*b*c)/(15015*b^5) + (2*b*d*x^6)/13 - (16*a^4*x*(8*a*d - 13*b*c))/(1501
5*b^4) + (2*a*x^4*(a*d + 52*b*c))/(429*b) - (2*a^2*x^3*(8*a*d - 13*b*c))/(
3003*b^2) + (4*a^3*x^2*(8*a*d - 13*b*c))/(5005*b^3)))/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.84

$$\int (c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx = \frac{2\sqrt{bx+a}\,(1155b^6d\,x^6+1470a\,b^5d\,x^5+1365b^6c\,x^5+35a^2b^4d\,x^4+1820a\,b^5c\,x^4-40a^3b^3d\,x^3}{15015b$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*sqrt(a + b*x)*(128*a**6*d - 208*a**5*b*c - 64*a**5*b*d*x + 104*a**4*b**
2*c*x + 48*a**4*b**2*d*x**2 - 78*a**3*b**3*c*x**2 - 40*a**3*b**3*d*x**3 +
65*a**2*b**4*c*x**3 + 35*a**2*b**4*d*x**4 + 1820*a*b**5*c*x**4 + 1470*a*b*
*5*d*x**5 + 1365*b**6*c*x**5 + 1155*b**6*d*x**6))/(15015*b**5)
```

3.249 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x} \, dx = \frac{2a^2(bc-ad)\left(ax^2+bx^3\right)^{5/2}}{5b^4x^5} - \frac{2a(2bc-3ad)\left(ax^2+bx^3\right)^{7/2}}{7b^4x^7} + \frac{2(bc-3ad)\left(ax^2+bx^3\right)^{9/2}}{9b^4x^9} + \frac{2d\left(ax^2+bx^3\right)^{11/2}}{11b^4x^{11}}$$

output
```
2/5*a^2*(-a*d+b*c)*(b*x^3+a*x^2)^(5/2)/b^4/x^5-2/7*a*(-3*a*d+2*b*c)*(b*x^3
+a*x^2)^(7/2)/b^4/x^7+2/9*(-3*a*d+b*c)*(b*x^3+a*x^2)^(9/2)/b^4/x^9+2/11*d*
(b*x^3+a*x^2)^(11/2)/b^4/x^11
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x} \, dx = \frac{2x(a+bx)^3\left(-48a^3d+35b^3x^2(11c+9dx)+8a^2b(11c+15dx)-10ab^2x(2\right.}{3465b^4\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x,x]
```

output
```
(2*x*(a + b*x)^3*(-48*a^3*d + 35*b^3*x^2*(11*c + 9*d*x) + 8*a^2*b*(11*c +
15*d*x) - 10*a*b^2*x*(22*c + 21*d*x)))/(3465*b^4*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1945, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}(c+dx)}{x}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(11bc-6ad)\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x}dx}{11b}+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{11bx^2}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(11bc-6ad)\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^2}dx}{9b}\right)}{11b}+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{11bx^2}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(11bc-6ad)\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{2a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^3}dx}{7b}\right)}{9b}\right)}{11b}+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{11bx^2}$$

$$\Big\downarrow \text{1920}$$

$$\frac{\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{9bx^3}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}\right)}{9b}\right)(11bc-6ad)}{11b}+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{11bx^2}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x,x]
```

output
```
(2*d*(a*x^2 + b*x^3)^(5/2))/(11*b*x^2) + ((11*b*c - 6*a*d)*((2*(a*x^2 + b*
x^3)^(5/2))/(9*b*x^3) - (4*a*((-4*a*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) +
(2*(a*x^2 + b*x^3)^(5/2))/(7*b*x^4)))/(9*b)))/(11*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.47 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.49

method	result
pseudoelliptic	$\frac{-2c\,a^{\frac{3}{2}}b\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{2\sqrt{bx+a}\left(\left(d\,x^2+\frac{5}{3}cx\right)b^2+\frac{20\left(\frac{3dx}{10}+c\right)ab}{3}+a^2d\right)}{5}}{b}$
gosper	$-\frac{2(bx+a)\left(-315b^3d\,x^3+210a\,b^2d\,x^2-385b^3c\,x^2-120a^2bdx+220a\,b^2cx+48a^3d-88c\,a^2b\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{3465b^4x^3}$
default	$-\frac{2(bx+a)\left(-315b^3d\,x^3+210a\,b^2d\,x^2-385b^3c\,x^2-120a^2bdx+220a\,b^2cx+48a^3d-88c\,a^2b\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{3465b^4x^3}$
orering	$-\frac{2(bx+a)\left(-315b^3d\,x^3+210a\,b^2d\,x^2-385b^3c\,x^2-120a^2bdx+220a\,b^2cx+48a^3d-88c\,a^2b\right)\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}}{3465b^4x^3}$
risch	$-\frac{2\sqrt{x^2(bx+a)}\,(-315d\,x^5b^5-420a\,b^4d\,x^4-385b^5c\,x^4-15a^2b^3d\,x^3-550a\,b^4c\,x^3+18a^3b^2d\,x^2-33a^2b^3c\,x^2-24a^4bdx+44a^3 \cdots}{3465x\,b^4}$
trager	$-\frac{2(-315d\,x^5b^5-420a\,b^4d\,x^4-385b^5c\,x^4-15a^2b^3d\,x^3-550a\,b^4c\,x^3+18a^3b^2d\,x^2-33a^2b^3c\,x^2-24a^4bdx+44a^3b^2cx+48a^5 \cdots}{3465b^4x}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/5*(-5*c*a^(3/2)*b*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^(1/2)*((d*x^2+5
/3*c*x)*b^2+20/3*(3/10*d*x+c)*a*b+a^2*d))/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x}\,dx = \frac{2\left(315\,b^5dx^5+88\,a^4bc-48\,a^5d+35\,(11\,b^5c+12\,ab^4d)x^4+5\,(110\,ab^4c+3 \cdots\right.}{3465 \cdots}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x,x, algorithm="fricas")
```

output
```
2/3465*(315*b^5*d*x^5 + 88*a^4*b*c - 48*a^5*d + 35*(11*b^5*c + 12*a*b^4*d)
*x^4 + 5*(110*a*b^4*c + 3*a^2*b^3*d)*x^3 + 3*(11*a^2*b^3*c - 6*a^3*b^2*d)*
x^2 - 4*(11*a^3*b^2*c - 6*a^4*b*d)*x)*sqrt(b*x^3 + a*x^2)/(b^4*x)
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)}{x}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/x,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)*(c + d*x)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x}\,dx=\frac{2\left(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4\right)\sqrt{bx+a}c}{315\,b^3}+\frac{2\left(105\,b^5x^5+140\,ab^4x^4+5\,a^2b^3x^3-6\,a^3b^2x^2+8\,a^4bx-16\,a^5\right)\sqrt{bx+a}d}{1155\,b^4}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x,x, algorithm="maxima")
```

output
```
2/315*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt
(b*x + a)*c/b^3 + 2/1155*(105*b^5*x^5 + 140*a*b^4*x^4 + 5*a^2*b^3*x^3 - 6*
a^3*b^2*x^2 + 8*a^4*b*x - 16*a^5)*sqrt(b*x + a)*d/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(115) = 230.

Time = 0.12 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.93

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x}\,dx=\frac{2\left(\frac{231\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)a^2c\mathrm{sgn}(x)}{b^2}+\frac{198\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\right.}{b}\right.}{}-\frac{16\left(11\,a^{\frac{9}{2}}bc-6\,a^{\frac{11}{2}}d\right)\mathrm{sgn}(x)}{3465\,b^4}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x,x, algorithm="giac")
```

output
```
2/3465*(231*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a
^2)*a^2*c*sgn(x)/b^2 + 198*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*
(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*c*sgn(x)/b^2 + 99*(5*(b*x +
a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a
)*a^3)*a^2*d*sgn(x)/b^3 + 11*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a +
 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4
)*c*sgn(x)/b^2 + 22*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x
 + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a*d*sgn
(x)/b^3 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(
7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(
b*x + a)*a^5)*d*sgn(x)/b^3)/b - 16/3465*(11*a^(9/2)*b*c - 6*a^(11/2)*d)*sg
n(x)/b^4
```

Mupad [B] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x}\,dx=\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{x^4\,\left(770\,c\,b^5+840\,a\,d\,b^4\right)}{3465\,b^4}-\frac{96\,a^5\,d-176\,a^4\,b\,c}{3465\,b^4}+\frac{2\,b\,d\,x^5}{11}+\frac{8\,a^3\,x\,(6\,a\,d-11\,b\,c)}{3465\,b^3}\right.}{x}$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/x,x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((x^4*(770*b^5*c + 840*a*b^4*d))/(3465*b^4) - (96*a
^5*d - 176*a^4*b*c)/(3465*b^4) + (2*b*d*x^5)/11 + (8*a^3*x*(6*a*d - 11*b*c
))/(3465*b^3) + (2*a*x^3*(3*a*d + 110*b*c))/(693*b) - (2*a^2*x^2*(6*a*d -
11*b*c))/(1155*b^2)))/x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x}\,dx = \frac{2\sqrt{bx+a}\,(315b^5d\,x^5+420a\,b^4d\,x^4+385b^5c\,x^4+15a^2b^3d\,x^3+550a\,b^4c\,x^3}{3465b^4}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x,x)
```

output
```
(2*sqrt(a + b*x)*( - 48*a**5*d + 88*a**4*b*c + 24*a**4*b*d*x - 44*a**3*b**
2*c*x - 18*a**3*b**2*d*x**2 + 33*a**2*b**3*c*x**2 + 15*a**2*b**3*d*x**3 +
550*a*b**4*c*x**3 + 420*a*b**4*d*x**4 + 385*b**5*c*x**4 + 315*b**5*d*x**5)
)/(3465*b**4)
```

3.250 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx=-\frac{2a(bc-ad)\left(ax^2+bx^3\right)^{5/2}}{5b^3x^5}+\frac{2(bc-2ad)\left(ax^2+bx^3\right)^{7/2}}{7b^3x^7}+\frac{2d\left(ax^2+bx^3\right)^{9/2}}{9b^3x^9}$$

output
```
-2/5*a*(-a*d+b*c)*(b*x^3+a*x^2)^(5/2)/b^3/x^5+2/7*(-2*a*d+b*c)*(b*x^3+a*x^
2)^(7/2)/b^3/x^7+2/9*d*(b*x^3+a*x^2)^(9/2)/b^3/x^9
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx=\frac{2x(a+bx)^3\left(8a^2d+5b^2x(9c+7dx)-2ab(9c+10dx)\right)}{315b^3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^2,x]
```

output
```
(2*x*(a + b*x)^3*(8*a^2*d + 5*b^2*x*(9*c + 7*d*x) - 2*a*b*(9*c + 10*d*x)))
/(315*b^3*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1945, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}(c+dx)}{x^2}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(9bc-4ad)\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^2}dx}{9b}+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{9bx^3}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(9bc-4ad)\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{2a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^3}dx}{7b}\right)}{9b}+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{9bx^3}$$

$$\Big\downarrow \text{1920}$$

$$\frac{\left(\frac{2\left(ax^2+bx^3\right)^{5/2}}{7bx^4}-\frac{4a\left(ax^2+bx^3\right)^{5/2}}{35b^2x^5}\right)(9bc-4ad)}{9b}+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{9bx^3}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^2,x]
```

output
```
(2*d*(a*x^2 + b*x^3)^(5/2))/(9*b*x^3) + ((9*b*c - 4*a*d)*((-4*a*(a*x^2 + b
*x^3)^(5/2))/(35*b^2*x^5) + (2*(a*x^2 + b*x^3)^(5/2))/(7*b*x^4)))/(9*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

method	result	size
gosper	$\frac{2(bx+a)\left(35b^{2}d\,x^{2}-20abdx+45b^{2}cx+8a^{2}d-18abc\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{315b^{3}x^{3}}$	61
default	$\frac{2(bx+a)\left(35b^{2}d\,x^{2}-20abdx+45b^{2}cx+8a^{2}d-18abc\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{315b^{3}x^{3}}$	61
orering	$\frac{2(bx+a)\left(35b^{2}d\,x^{2}-20abdx+45b^{2}cx+8a^{2}d-18abc\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{315b^{3}x^{3}}$	61
pseudoelliptic	$-\frac{2\left(ax\left(ad+\frac{3bc}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\left(\left(\frac{4dx}{3}-\frac{c}{2}\right)a^{\frac{3}{2}}+bx\sqrt{a}\left(\frac{dx}{3}+c\right)\right)\sqrt{bx+a}\right)}{\sqrt{a}\,x}$	67
risch	$\frac{2\sqrt{x^{2}(bx+a)}\left(35d\,x^{4}b^{4}+50a\,b^{3}d\,x^{3}+45b^{4}c\,x^{3}+3a^{2}b^{2}d\,x^{2}+72a\,b^{3}c\,x^{2}-4a^{3}bdx+9a^{2}b^{2}cx+8a^{4}d-18a^{3}bc\right)}{315x\,b^{3}}$	102
trager	$\frac{2\left(35d\,x^{4}b^{4}+50a\,b^{3}d\,x^{3}+45b^{4}c\,x^{3}+3a^{2}b^{2}d\,x^{2}+72a\,b^{3}c\,x^{2}-4a^{3}bdx+9a^{2}b^{2}cx+8a^{4}d-18a^{3}bc\right)\sqrt{b\,x^{3}+a\,x^{2}}}{315b^{3}x}$	104

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
2/315*(b*x+a)*(35*b^2*d*x^2-20*a*b*d*x+45*b^2*c*x+8*a^2*d-18*a*b*c)*(b*x^3
+a*x^2)^(3/2)/b^3/x^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx = \frac{2\left(35\,b^4dx^4-18\,a^3bc+8\,a^4d+5\,(9\,b^4c+10\,ab^3d)x^3+3\,(24\,ab^3c+a^2b^2d)x\right.}{315\,b^3x}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="fricas")
```

output
```
2/315*(35*b^4*d*x^4 - 18*a^3*b*c + 8*a^4*d + 5*(9*b^4*c + 10*a*b^3*d)*x^3
+ 3*(24*a*b^3*c + a^2*b^2*d)*x^2 + (9*a^2*b^2*c - 4*a^3*b*d)*x)*sqrt(b*x^3
 + a*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)}{x^2}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/x**2,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)*(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx = \frac{2\,(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3)\sqrt{bx+a}c}{35\,b^2} + \frac{2\,(35\,b^4x^4+50\,ab^3x^3+3\,a^2b^2x^2-4\,a^3bx+8\,a^4)\sqrt{bx+a}d}{315\,b^3}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="maxima")
```

output
```
2/35*(5*b^3*x^3 + 8*a*b^2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)*c/b^2 + 2/3
15*(35*b^4*x^4 + 50*a*b^3*x^3 + 3*a^2*b^2*x^2 - 4*a^3*b*x + 8*a^4)*sqrt(b*
x + a)*d/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(82) = 164$.

Time = 0.14 (sec) , antiderivative size = 309, normalized size of antiderivative = 3.29

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx = \frac{2\left(\frac{105\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)a^2c\operatorname{sgn}(x)}{b}+\frac{42\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\sqrt{bx+a}a^2\right)ac\operatorname{sgn}(x)}{b}\right.}{} + \frac{4\left(9\,a^{\frac{7}{2}}bc-4\,a^{\frac{9}{2}}d\right)\operatorname{sgn}(x)}{315\,b^3}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^2,x, algorithm="giac")
```

output
```
2/315*(105*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^2*c*sgn(x)/b + 42*(3*(b
*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a*c*sgn(x)/b
+ 21*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a^2
*d*sgn(x)/b^2 + 9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)
^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*c*sgn(x)/b + 18*(5*(b*x + a)^(7/2) - 21
*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*d*sg
n(x)/b^2 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/
2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d*sgn(x)/b^2)/b
+ 4/315*(9*a^(7/2)*b*c - 4*a^(9/2)*d)*sgn(x)/b^3
```

Mupad [B] (verification not implemented)

Time = 9.01 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{16\,a^4\,d-36\,a^3\,b\,c}{315\,b^3}+\frac{x^3\,\left(90\,c\,b^4+100\,a\,d\,b^3\right)}{315\,b^3}+\frac{2\,b\,d\,x^4}{9}-\frac{2\,a^2\,x\,\left(4\,a\,d-9\,b\,c\right)}{315\,b^2}+\right.}{x}$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/x^2,x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((16*a^4*d - 36*a^3*b*c)/(315*b^3) + (x^3*(90*b^4*c
 + 100*a*b^3*d))/(315*b^3) + (2*b*d*x^4)/9 - (2*a^2*x*(4*a*d - 9*b*c))/(31
5*b^2) + (2*a*x^2*(a*d + 24*b*c))/(105*b)))/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^2}\,dx = \frac{2\sqrt{bx+a}\,(35b^4d\,x^4+50a\,b^3d\,x^3+45b^4c\,x^3+3a^2b^2d\,x^2+72a\,b^3c\,x^2-4a^3}{315b^3}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^2,x)
```

output
```
(2*sqrt(a + b*x)*(8*a**4*d - 18*a**3*b*c - 4*a**3*b*d*x + 9*a**2*b**2*c*x
+ 3*a**2*b**2*d*x**2 + 72*a*b**3*c*x**2 + 50*a*b**3*d*x**3 + 45*b**4*c*x**
3 + 35*b**4*d*x**4))/(315*b**3)
```

3.251 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^3} \, dx = \frac{2(bc-ad)\left(ax^2+bx^3\right)^{5/2}}{5b^2x^5}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{7b^2x^7}$$

output
```
2/5*(-a*d+b*c)*(b*x^3+a*x^2)^(5/2)/b^2/x^5+2/7*d*(b*x^3+a*x^2)^(7/2)/b^2/x
^7
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^3} \, dx = \frac{2x(a+bx)^3(7bc-2ad+5bdx)}{35b^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^3,x]
```

output
```
(2*x*(a + b*x)^3*(7*b*c - 2*a*d + 5*b*d*x))/(35*b^2*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1945, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}(c+dx)}{x^3}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(7bc-2ad)\int \frac{(bx^3+ax^2)^{3/2}}{x^3}dx}{7b}+\frac{2d(ax^2+bx^3)^{5/2}}{7bx^4}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2(ax^2+bx^3)^{5/2}(7bc-2ad)}{35b^2x^5}+\frac{2d(ax^2+bx^3)^{5/2}}{7bx^4}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^3,x]
```

output
```
(2*(7*b*c - 2*a*d)*(a*x^2 + b*x^3)^(5/2))/(35*b^2*x^5) + (2*d*(a*x^2 + b*x
^3)^(5/2))/(7*b*x^4)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

method	result	size
gosper	$-\frac{2(bx+a)(-5bdx+2ad-7bc)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{3}}$	41
default	$-\frac{2(bx+a)(-5bdx+2ad-7bc)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{3}}$	41
orering	$-\frac{2(bx+a)(-5bdx+2ad-7bc)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{35b^{2}x^{3}}$	41
pseudoelliptic	$-\frac{3\left(b\,x^{2}\left(ad+\frac{bc}{4}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{5\left(\frac{2(2dx+c)a^{\frac{3}{2}}}{5}+bx\sqrt{a}\left(-\frac{8dx}{5}+c\right)\right)\sqrt{bx+a}}{12}\right)}{\sqrt{a}\,x^{2}}$	68
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-5b^{3}d\,x^{3}-8a\,b^{2}d\,x^{2}-7b^{3}c\,x^{2}-a^{2}bdx-14a\,b^{2}cx+2a^{3}d-7c\,a^{2}b\right)}{35x\,b^{2}}$	78
trager	$-\frac{2\left(-5b^{3}d\,x^{3}-8a\,b^{2}d\,x^{2}-7b^{3}c\,x^{2}-a^{2}bdx-14a\,b^{2}cx+2a^{3}d-7c\,a^{2}b\right)\sqrt{b\,x^{3}+a\,x^{2}}}{35b^{2}x}$	80

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/35*(b*x+a)*(-5*b*d*x+2*a*d-7*b*c)*(b*x^3+a*x^2)^(3/2)/b^2/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx = \frac{2\,(5\,b^3dx^3+7\,a^2bc-2\,a^3d+(7\,b^3c+8\,ab^2d)x^2+(14\,ab^2c+a^2bd)x)\sqrt{bx^3}}{35\,b^2x}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
2/35*(5*b^3*d*x^3 + 7*a^2*b*c - 2*a^3*d + (7*b^3*c + 8*a*b^2*d)*x^2 + (14*
a*b^2*c + a^2*b*d)*x)*sqrt(b*x^3 + a*x^2)/(b^2*x)
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)}{x^3}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/x**3,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)*(c + d*x)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.20

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx = \frac{2\,(b^2x^2+2\,abx+a^2)\sqrt{bx+a}c}{5\,b} + \frac{2\,(5\,b^3x^3+8\,ab^2x^2+a^2bx-2\,a^3)\sqrt{bx+a}d}{35\,b^2}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
2/5*(b^2*x^2 + 2*a*b*x + a^2)*sqrt(b*x + a)*c/b + 2/35*(5*b^3*x^3 + 8*a*b^
2*x^2 + a^2*b*x - 2*a^3)*sqrt(b*x + a)*d/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(52) = 104$.

Time = 0.13 (sec) , antiderivative size = 226, normalized size of antiderivative = 3.77

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx = \frac{2\left(105\sqrt{bx+a}\,a^2c\,\mathrm{sgn}(x)+70\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}\,a\right)ac\,\mathrm{sgn}(x)+\frac{35\left((\cdots\right.}{\cdots}\cdots\right.}{\cdots}$$
$$-\frac{2\left(7\,a^{\frac{5}{2}}bc-2\,a^{\frac{7}{2}}d\right)\mathrm{sgn}(x)}{35\,b^2}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^3,x, algorithm="giac")
```

output
```
2/105*(105*sqrt(b*x + a)*a^2*c*sgn(x) + 70*((b*x + a)^(3/2) - 3*sqrt(b*x +
 a)*a)*a*c*sgn(x) + 35*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^2*d*sgn(x)/
b + 7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*c*
sgn(x) + 14*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a
^2)*a*d*sgn(x)/b + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x +
 a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*d*sgn(x)/b)/b - 2/35*(7*a^(5/2)*b*c
- 2*a^(7/2)*d)*sgn(x)/b^2
```

Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx = \frac{2\sqrt{b\,x^3+a\,x^2}\,\left(a+b\,x\right)^2\left(7\,b\,c-2\,a\,d+5\,b\,d\,x\right)}{35\,b^2\,x}$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/x^3,x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^2*(7*b*c - 2*a*d + 5*b*d*x))/(35*b^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^3}\,dx = \frac{2\sqrt{bx+a}\,(5b^3d\,x^3+8a\,b^2d\,x^2+7b^3c\,x^2+a^2bdx+14a\,b^2cx-2a^3d+7a^2b}{35b^2}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^3,x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a**3*d + 7*a**2*b*c + a**2*b*d*x + 14*a*b**2*c*x +
8*a*b**2*d*x**2 + 7*b**3*c*x**2 + 5*b**3*d*x**3))/(35*b**2)
```

3.252 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^4}\,dx = \frac{2ac\sqrt{ax^2+bx^3}}{x}+\frac{2c(ax^2+bx^3)^{3/2}}{3x^3}+\frac{2d(ax^2+bx^3)^{5/2}}{5bx^5}-2a^{3/2}c\,\mathrm{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)$$

output
```
2*a*c*(b*x^3+a*x^2)^(1/2)/x+2/3*c*(b*x^3+a*x^2)^(3/2)/x^3+2/5*d*(b*x^3+a*x
^2)^(5/2)/b/x^5-2*a^(3/2)*c*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^4}\,dx = \frac{2x\left((a+bx)\left(3a^2d+b^2x(5c+3dx)+ab(20c+6dx)\right)-15a^{3/2}bc\sqrt{a+bx}a\right.}{15b\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^4,x]
```

output
```
(2*x*((a + b*x)*(3*a^2*d + b^2*x*(5*c + 3*d*x) + a*b*(20*c + 6*d*x)) - 15*
a^(3/2)*b*c*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(15*b*Sqrt[x^2*
(a + b*x)])
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1945, 1927, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}(c+dx)}{x^4}dx$$

$$\Big\downarrow \text{1945}$$

$$c\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^4}dx+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{5bx^5}$$

$$\Big\downarrow \text{1927}$$

$$c\left(a\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{5bx^5}$$

$$\Big\downarrow \text{1927}$$

$$c\left(a\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{5bx^5}$$

$$\Big\downarrow \text{1914}$$

$$c\left(a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{5bx^5}$$

$$\Big\downarrow \text{219}$$

$$c\left(a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{5bx^5}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^4,x]
```

output
```
(2*d*(a*x^2 + b*x^3)^(5/2))/(5*b*x^5) + c*((2*(a*x^2 + b*x^3)^(3/2))/(3*x^
3) + a*((2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x
^2 + b*x^3]]))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{2\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}} \left(3 d \left(b x +a \right)^{\frac{5}{2}}-15 c\,a^{\frac{3}{2}} b \,\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right)+5 \left(b x +a \right)^{\frac{3}{2}} b c +15 \sqrt{b x +a}\,a b c \right)}{15 x^{3} \left(b x +a \right)^{\frac{3}{2}} b}$	82
pseudoelliptic	$-\frac{3 \left(b^{2} x^{3} \left(a d -\frac{b c}{6}\right) \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right)+\frac{4 \left(\frac{7 x \left(\frac{15 d x}{7}+c \right) b\,a^{\frac{3}{2}}}{4}+\left(\frac{3 d x}{2}+c \right) a^{\frac{5}{2}}+\frac{3 \sqrt{a}\,b^{2} c\,x^{2}}{8}\right) \sqrt{b x +a}}{9}\right)}{4 a^{\frac{3}{2}} x^{3}}$	82

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/15*(b*x^3+a*x^2)^(3/2)*(3*d*(b*x+a)^(5/2)-15*c*a^(3/2)*b*arctanh((b*x+a)
^(1/2)/a^(1/2))+5*(b*x+a)^(3/2)*b*c+15*(b*x+a)^(1/2)*a*b*c)/x^3/(b*x+a)^(3
/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.91

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^4}\,dx = \left[\frac{15\,a^{\frac{3}{2}}bcx\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(3\,b^2dx^2+20\,abc+3\,a^2d+(5\,b^2}{15\,bx}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="fricas")
```

output
```
[1/15*(15*a^(3/2)*b*c*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a)
)/x^2) + 2*(3*b^2*d*x^2 + 20*a*b*c + 3*a^2*d + (5*b^2*c + 6*a*b*d)*x)*sqrt
(b*x^3 + a*x^2))/(b*x), 2/15*(15*sqrt(-a)*a*b*c*x*arctan(sqrt(b*x^3 + a*x^
2)*sqrt(-a)/(b*x^2 + a*x)) + (3*b^2*d*x^2 + 20*a*b*c + 3*a^2*d + (5*b^2*c
+ 6*a*b*d)*x)*sqrt(b*x^3 + a*x^2))/(b*x)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^4}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)}{x^4}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/x**4,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)*(c + d*x)/x**4, x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^4}\,dx=\int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}(dx+c)}{x^4}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/x^4, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^4}\,dx=\frac{2\,a^2c\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\mathrm{sgn}(x)}{\sqrt{-a}} - \frac{2\left(15\,a^2bc\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+20\,\sqrt{-a}a^{\frac{3}{2}}bc+3\,\sqrt{-a}a^{\frac{5}{2}}d\right)\mathrm{sgn}(x)}{15\,\sqrt{-a}b} + \frac{2\left(5\,(bx+a)^{\frac{3}{2}}b^5c\mathrm{sgn}(x)+15\,\sqrt{bx+a}ab^5c\mathrm{sgn}(x)+3\,(bx+a)^{\frac{5}{2}}b^4d\mathrm{sgn}(x)\right)}{15\,b^5}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^4,x, algorithm="giac")
```

output
```
2*a^2*c*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - 2/15*(15*a^2*b*c*
arctan(sqrt(a)/sqrt(-a)) + 20*sqrt(-a)*a^(3/2)*b*c + 3*sqrt(-a)*a^(5/2)*d)
*sgn(x)/(sqrt(-a)*b) + 2/15*(5*(b*x + a)^(3/2)*b^5*c*sgn(x) + 15*sqrt(b*x
+ a)*a*b^5*c*sgn(x) + 3*(b*x + a)^(5/2)*b^4*d*sgn(x))/b^5
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^4}\,dx=\int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}\,\left(c+d\,x\right)}{x^4}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/x^4,x)
```

output
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^4}\,dx=\frac{6\sqrt{bx+a}\,a^2d+40\sqrt{bx+a}\,abc+12\sqrt{bx+a}\,abdx+10\sqrt{bx+a}\,b^2cx+6\sqrt{}}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^4,x)
```

output
```
(6*sqrt(a + b*x)*a**2*d + 40*sqrt(a + b*x)*a*b*c + 12*sqrt(a + b*x)*a*b*d*
x + 10*sqrt(a + b*x)*b**2*c*x + 6*sqrt(a + b*x)*b**2*d*x**2 + 15*sqrt(a)*l
og(sqrt(a + b*x) - sqrt(a))*a*b*c - 15*sqrt(a)*log(sqrt(a + b*x) + sqrt(a)
)*a*b*c)/(15*b)
```

3.253 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^5}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 114

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^5}\,dx = \frac{(3bc+2ad)\sqrt{ax^2+bx^3}}{x} - \frac{c(ax^2+bx^3)^{3/2}}{x^4} + \frac{2d(ax^2+bx^3)^{3/2}}{3x^3} - \sqrt{a}(3bc+2ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)$$

output
```
(2*a*d+3*b*c)*(b*x^3+a*x^2)^(1/2)/x-c*(b*x^3+a*x^2)^(3/2)/x^4+2/3*d*(b*x^3
+a*x^2)^(3/2)/x^3-a^(1/2)*(2*a*d+3*b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2
)/x)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.77

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^5}\,dx = \frac{(a+bx)(2bx(3c+dx)+a(-3c+8dx))-3\sqrt{a}(3bc+2ad)x\sqrt{a+bx}\text{arcta}}{3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^5,x]
```

output
```
((a + b*x)*(2*b*x*(3*c + d*x) + a*(-3*c + 8*d*x)) - 3*Sqrt[a]*(3*b*c + 2*a
*d)*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(3*Sqrt[x^2*(a + b*x)]
)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1944, 1927, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}(c+dx)}{x^5}\,dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(2ad+3bc)\int \frac{(bx^3+ax^2)^{3/2}}{x^4}dx}{2a}-\frac{c(ax^2+bx^3)^{5/2}}{ax^6}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(2ad+3bc)\left(a\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}\right)}{2a}-\frac{c(ax^2+bx^3)^{5/2}}{ax^6}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(2ad+3bc)\left(a\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}\right)}{2a}-\frac{c(ax^2+bx^3)^{5/2}}{ax^6}$$

$$\Big\downarrow \text{1914}$$

$$\frac{(2ad+3bc)\left(a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}\right)}{2a}-\frac{c(ax^2+bx^3)^{5/2}}{ax^6}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}\right)(2ad+3bc)}{2a}-\frac{c(ax^2+bx^3)^{5/2}}{ax^6}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^5,x]
```

output
```
-((c*(a*x^2 + b*x^3)^(5/2))/(a*x^6)) + ((3*b*c + 2*a*d)*((2*(a*x^2 + b*x^3
)^(3/2))/(3*x^3) + a*((2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[
a]*x)/Sqrt[a*x^2 + b*x^3]])))/(2*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

method	result
pseudoelliptic	$\frac{b^{3}x^{4}\left(ad-\frac{3bc}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-3\left(\frac{b^{2}x^{2}(4dx+c)a^{\frac{3}{2}}}{12}+bx\left(\frac{14dx}{9}+c\right)a^{\frac{5}{2}}+\left(\frac{8dx}{9}+\frac{2c}{3}\right)a^{\frac{7}{2}}-\frac{\sqrt{a}\,b^{3}c\,x^{3}}{8}\right)\sqrt{bx+a}}{8a^{\frac{5}{2}}x^{4}}$
risch	$-\frac{ac\sqrt{x^{2}(bx+a)}}{x^{2}}+\frac{\left(\frac{2(bx+a)^{\frac{3}{2}}d}{3}+2\sqrt{bx+a}\,ad+2\sqrt{bx+a}\,bc-(2ad+3bc)\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)\sqrt{x^{2}(bx+a)}}{x\sqrt{bx+a}}$
default	$\frac{(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}\left(2(bx+a)^{\frac{3}{2}}dx\sqrt{a}+6a^{\frac{3}{2}}dx\sqrt{bx+a}+6bcx\sqrt{a}\sqrt{bx+a}-6\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^{2}dx-9\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abcx-3a\right.}{3x^{4}(bx+a)^{\frac{3}{2}}\sqrt{a}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/8*(b^3*x^4*(a*d-3/8*b*c)*arctanh((b*x+a)^(1/2)/a^(1/2))-3*(1/12*b^2*x^2*
(4*d*x+c)*a^(3/2)+b*x*(14/9*d*x+c)*a^(5/2)+(8/9*d*x+2/3*c)*a^(7/2)-1/8*a^(
1/2)*b^3*c*x^3)*(b*x+a)^(1/2))/a^(5/2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{3/2}}{x^5}\,dx = \left[\frac{3\,(3\,bc+2\,ad)\sqrt{a}x^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(2\,bdx^2-3\,ac+2\,(3\,b}{6\,x^2}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="fricas")
```

output
```
[1/6*(3*(3*b*c + 2*a*d)*sqrt(a)*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*
x^2)*sqrt(a))/x^2) + 2*(2*b*d*x^2 - 3*a*c + 2*(3*b*c + 4*a*d)*x)*sqrt(b*x^
3 + a*x^2))/x^2, 1/3*(3*(3*b*c + 2*a*d)*sqrt(-a)*x^2*arctan(sqrt(b*x^3 + a
*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (2*b*d*x^2 - 3*a*c + 2*(3*b*c + 4*a*d)*x)*
sqrt(b*x^3 + a*x^2))/x^2]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^5}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)}{x^5}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/x**5,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)*(c + d*x)/x**5, x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^5}\,dx=\int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}(dx+c)}{x^5}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/x^5, x)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^5}\,dx=$$
$$-\frac{1}{3}\left(\frac{3\sqrt{bx+a}\,ac\,\mathrm{sgn}(x)}{bx}-\frac{3\left(3\,abc\,\mathrm{sgn}(x)+2\,a^2d\,\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,b}-\frac{2\left(3\sqrt{bx+a}\,ab^3c\,\mathrm{sgn}(x)+(ba\right.}{}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^5,x, algorithm="giac")
```

output
```
-1/3*(3*sqrt(b*x + a)*a*c*sgn(x)/(b*x) - 3*(3*a*b*c*sgn(x) + 2*a^2*d*sgn(x
))*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*b) - 2*(3*sqrt(b*x + a)*b^3*c*
sgn(x) + (b*x + a)^(3/2)*b^2*d*sgn(x) + 3*sqrt(b*x + a)*a*b^2*d*sgn(x))/b^
3)*b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^5}\,dx=\int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}\,\left(c+d\,x\right)}{x^5} \,d x$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/x^5,x)
```

output
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^5}\,dx=\frac{-6\sqrt{bx+a}\,ac+16\sqrt{bx+a}\,adx+12\sqrt{bx+a}\,bcx+4\sqrt{bx+a}\,bd\,x^2+6\sqrt{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^5,x)
```

output
```
( - 6*sqrt(a + b*x)*a*c + 16*sqrt(a + b*x)*a*d*x + 12*sqrt(a + b*x)*b*c*x
+ 4*sqrt(a + b*x)*b*d*x**2 + 6*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*d*x
+ 9*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*c*x - 6*sqrt(a)*log(sqrt(a + b*
x) + sqrt(a))*a*d*x - 9*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*c*x)/(6*x)
```

3.254 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 120

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^6}\,dx=-\frac{(3bc+4ad)\sqrt{ax^2+bx^3}}{4x^2}+\frac{2bd\sqrt{ax^2+bx^3}}{x}-\frac{c(ax^2+bx^3)^{3/2}}{2x^5}-\frac{3b(bc+4ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{4\sqrt{a}}$$

output
```
-1/4*(4*a*d+3*b*c)*(b*x^3+a*x^2)^(1/2)/x^2+2*b*d*(b*x^3+a*x^2)^(1/2)/x-1/2
*c*(b*x^3+a*x^2)^(3/2)/x^5-3/4*b*(4*a*d+b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a
^(1/2)/x)/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^6}\,dx=-\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(bx(5c-8dx)+2a(c+2dx))+3b(bc+4ad)x^2\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{4\sqrt{a}x^3\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^6,x]
```

output
```
-1/4*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(b*x*(5*c - 8*d*x) + 2*a*
(c + 2*d*x)) + 3*b*(b*c + 4*a*d)*x^2*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(Sqr
t[a]*x^3*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1944, 1926, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}(c+dx)}{x^6}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(4ad+bc)\int \frac{(bx^3+ax^2)^{3/2}}{x^5}dx}{4a}-\frac{c(ax^2+bx^3)^{5/2}}{2ax^7}$$

$$\Big\downarrow \text{1926}$$

$$\frac{(4ad+bc)\left(\frac{3}{2}b\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx-\frac{(ax^2+bx^3)^{3/2}}{x^4}\right)}{4a}-\frac{c(ax^2+bx^3)^{5/2}}{2ax^7}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(4ad+bc)\left(\frac{3}{2}b\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}\right)}{4a}-\frac{c(ax^2+bx^3)^{5/2}}{2ax^7}$$

$$\Big\downarrow \text{1914}$$

$$\frac{(4ad+bc)\left(\frac{3}{2}b\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}\right)}{4a}-\frac{c(ax^2+bx^3)^{5/2}}{2ax^7}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{3}{2}b\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}\right)(4ad+bc)}{4a}-\frac{c(ax^2+bx^3)^{5/2}}{2ax^7}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^6,x]
```

output
```
-1/2*(c*(a*x^2 + b*x^3)^(5/2))/(a*x^7) + ((b*c + 4*a*d)*(-((a*x^2 + b*x^3)
^(3/2)/x^4) + (3*b*((2*Sqrt[a*x^2 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]
*x)/Sqrt[a*x^2 + b*x^3]]))/2))/(4*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1944

```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{(4adx+5cbx+2ac)\sqrt{x^2(bx+a)}}{4x^3}+\frac{b\left(16\sqrt{bx+a}\,d-\frac{2(12ad+3bc)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}\right)\sqrt{x^2(bx+a)}}{8x\sqrt{bx+a}}$
pseudoelliptic	$-\frac{3\left(b^4x^5\left(ad-\frac{bc}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{4\left(-\frac{5b^3x^3(3dx+c)a^{\frac{3}{2}}}{4}+b^2x^2\left(\frac{5dx}{2}+c\right)a^{\frac{5}{2}}+22xb\left(\frac{15dx}{11}+c\right)a^{\frac{7}{2}}+(20dx+16c)a^{\frac{9}{2}}+\frac{15\sqrt{a}\,b^4}{8}\right.}{15}\right.}{64a^{\frac{7}{2}}x^5}$
default	$-\frac{(b\,x^3+a\,x^2)^{\frac{3}{2}}\left(-8\sqrt{bx+a}\,d\,b^2x^2\sqrt{a}+12\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^2d\,x^2+3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^3c\,x^2+4(bx+a)^{\frac{3}{2}}a^{\frac{3}{2}}d+5(bx+\right.}{4b\,x^5(bx+a)^{\frac{3}{2}}\sqrt{a}}$

input

```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

output

```
-1/4*(4*a*d*x+5*b*c*x+2*a*c)/x^3*(x^2*(b*x+a))^(1/2)+1/8*b*(16*(b*x+a)^(1/
2)*d-2*(12*a*d+3*b*c)/a^(1/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))*(x^2*(b*x+a)
)^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.77

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^6}\,dx = \left[\frac{3\left(b^2c+4\,abd\right)\sqrt{a}x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(8\,abdx^2-2\,a^2c-(5\right.}{8\,ax^3}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="fricas")
```

output
```
[1/8*(3*(b^2*c + 4*a*b*d)*sqrt(a)*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 +
a*x^2)*sqrt(a))/x^2) + 2*(8*a*b*d*x^2 - 2*a^2*c - (5*a*b*c + 4*a^2*d)*x)*s
qrt(b*x^3 + a*x^2))/(a*x^3), 1/4*(3*(b^2*c + 4*a*b*d)*sqrt(-a)*x^3*arctan(
sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (8*a*b*d*x^2 - 2*a^2*c - (5*
a*b*c + 4*a^2*d)*x)*sqrt(b*x^3 + a*x^2))/(a*x^3)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^6}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)}{x^6}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/x**6,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)*(c + d*x)/x**6, x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^6}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}(dx+c)}{x^6}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/x^6, x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^6}\,dx = \frac{8\sqrt{bx+a}b^2d\text{sgn}(x)+\frac{3\left(b^3c\text{sgn}(x)+4\,ab^2d\text{sgn}(x)\right)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}-\frac{5\,(bx+a)^{\frac{3}{2}}b^3c\text{sgn}(x)}{}}{4\,b}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^6,x, algorithm="giac")
```

output
```
1/4*(8*sqrt(b*x + a)*b^2*d*sgn(x) + 3*(b^3*c*sgn(x) + 4*a*b^2*d*sgn(x))*ar
ctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - (5*(b*x + a)^(3/2)*b^3*c*sgn(x) -
3*sqrt(b*x + a)*a*b^3*c*sgn(x) + 4*(b*x + a)^(3/2)*a*b^2*d*sgn(x) - 4*sqrt
(b*x + a)*a^2*b^2*d*sgn(x))/(b^2*x^2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^6}\,dx = \int \frac{\left(b\,x^3+a\,x^2\right)^{3/2}\,\left(c+d\,x\right)}{x^6}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/x^6,x)
```

output
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^6}\,dx = \frac{-4\sqrt{bx+a}\,a^2c-8\sqrt{bx+a}\,a^2dx-10\sqrt{bx+a}\,abcx+16\sqrt{bx+a}\,abd\,x^2+}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^6,x)
```

output
```
( - 4*sqrt(a + b*x)*a**2*c - 8*sqrt(a + b*x)*a**2*d*x - 10*sqrt(a + b*x)*a
*b*c*x + 16*sqrt(a + b*x)*a*b*d*x**2 + 12*sqrt(a)*log(sqrt(a + b*x) - sqrt
(a))*a*b*d*x**2 + 3*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**2*c*x**2 - 12*
sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a*b*d*x**2 - 3*sqrt(a)*log(sqrt(a + b
*x) + sqrt(a))*b**2*c*x**2)/(8*a*x**2)
```

3.255 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 133

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx = -\frac{(bc+2ad)\sqrt{ax^2+bx^3}}{4x^3} - \frac{b(bc+10ad)\sqrt{ax^2+bx^3}}{8ax^2} - \frac{c(ax^2+bx^3)^{3/2}}{3x^6} + \frac{b^2(bc-6ad)\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{8a^{3/2}}$$

output
```
-1/4*(2*a*d+b*c)*(b*x^3+a*x^2)^(1/2)/x^3-1/8*b*(10*a*d+b*c)*(b*x^3+a*x^2)^
(1/2)/a/x^2-1/3*c*(b*x^3+a*x^2)^(3/2)/x^6+1/8*b^2*(-6*a*d+b*c)*arctanh((b*
x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(3b^2cx^2+4a^2(2c+3dx)+2abx(7c+15dx))\right.}{24a^{3/2}x^4\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^7,x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(3*b^2*c*x^2 + 4*a^2*(2*c +
3*d*x) + 2*a*b*x*(7*c + 15*d*x))) + 3*b^2*(b*c - 6*a*d)*x^3*ArcTanh[Sqrt[a
 + b*x]/Sqrt[a]]))/(24*a^(3/2)*x^4*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1944, 1926, 1926, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}(c+dx)}{x^7}dx$$
$$\Big\downarrow \text{1944}$$
$$-\frac{(bc-6ad)\int \frac{(bx^3+ax^2)^{3/2}}{x^6}dx}{6a}-\frac{c(ax^2+bx^3)^{5/2}}{3ax^8}$$
$$\Big\downarrow \text{1926}$$
$$-\frac{(bc-6ad)\left(\frac{3}{4}b\int \frac{\sqrt{bx^3+ax^2}}{x^3}dx-\frac{(ax^2+bx^3)^{3/2}}{2x^5}\right)}{6a}-\frac{c(ax^2+bx^3)^{5/2}}{3ax^8}$$
$$\Big\downarrow \text{1926}$$
$$-\frac{(bc-6ad)\left(\frac{3}{4}b\left(\frac{1}{2}b\int \frac{1}{\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{(ax^2+bx^3)^{3/2}}{2x^5}\right)}{6a}-\frac{c(ax^2+bx^3)^{5/2}}{3ax^8}$$
$$\Big\downarrow \text{1914}$$
$$-\frac{(bc-6ad)\left(\frac{3}{4}b\left(-b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{(ax^2+bx^3)^{3/2}}{2x^5}\right)}{6a}-\frac{c(ax^2+bx^3)^{5/2}}{3ax^8}$$
$$\Big\downarrow \text{219}$$
$$-\frac{\left(\frac{3}{4}b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{(ax^2+bx^3)^{3/2}}{2x^5}\right)(bc-6ad)}{6a}-\frac{c(ax^2+bx^3)^{5/2}}{3ax^8}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^7,x]
```

output
```
-1/3*(c*(a*x^2 + b*x^3)^(5/2))/(a*x^8) - ((b*c - 6*a*d)*(-1/2*(a*x^2 + b*x
^3)^(3/2)/x^5 + (3*b*(-(Sqrt[a*x^2 + b*x^3]/x^2) - (b*ArcTanh[(Sqrt[a]*x)/
Sqrt[a*x^2 + b*x^3]])/Sqrt[a]))/4))/(6*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\left(30abd\,x^{2}+3b^{2}c\,x^{2}+12a^{2}dx+14abcx+8a^{2}c\right)\sqrt{x^{2}\left(bx+a\right)}}{24x^{4}a}-\frac{\left(6ad-bc\right)b^{2}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{x^{2}\left(bx+a\right)}}{8a^{\frac{3}{2}}x\sqrt{bx+a}}$
pseudoelliptic	$\frac{\frac{3b^{5}x^{6}\left(ad-\frac{7bc}{12}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{128}+\frac{7\left(\frac{32(-6dx-5c)a^{\frac{11}{2}}}{7}+xb\left(-\frac{5\left(\frac{18dx}{7}+c\right)x^{3}b^{3}a^{\frac{3}{2}}}{4}+b^{2}x^{2}\left(\frac{15dx}{7}+c\right)a^{\frac{5}{2}}-\frac{6bx(2dx+c)a^{\frac{7}{2}}}{7}+\frac{8(-33}{}\right.\right.}{960}}{a^{\frac{9}{2}}x^{6}}$
default	$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(30(bx+a)^{\frac{5}{2}}a^{\frac{5}{2}}d+3(bx+a)^{\frac{5}{2}}a^{\frac{3}{2}}bc+18\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^{2}b^{3}d\,x^{3}-3\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^{4}c\,x^{3}-48(bx+a\right.}{24b\,x^{6}(bx+a)^{\frac{3}{2}}a^{\frac{5}{2}}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(30*a*b*d*x^2+3*b^2*c*x^2+12*a^2*d*x+14*a*b*c*x+8*a^2*c)/x^4/a*(x^2*
(b*x+a))^(1/2)-1/8*(6*a*d-b*c)*b^2/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*
(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.82

$$\int \frac{(c+dx)\left(ax^{2}+bx^{3}\right)^{3/2}}{x^{7}}\,dx=\left[-\frac{3\left(b^{3}c-6\,ab^{2}d\right)\sqrt{a}x^{4}\log\left(\frac{bx^{2}+2\,ax-2\,\sqrt{bx^{3}+ax^{2}}\sqrt{a}}{x^{2}}\right)+2\left(8\,a^{3}c+3\left(ab^{2}c+\right.\right.}{48\,a^{2}x^{4}}\right.$$

$$-\frac{3\left(b^{3}c-6\,ab^{2}d\right)\sqrt{-a}x^{4}\arctan\left(\frac{\sqrt{bx^{3}+ax^{2}}\sqrt{-a}}{bx^{2}+ax}\right)+\left(8\,a^{3}c+3\left(ab^{2}c+10\,a^{2}bd\right)x^{2}+2\left(7\,a^{2}bc+6\,a^{3}d\right)x\right)\sqrt{bx^{3}}}{24\,a^{2}x^{4}}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="fricas")
```

output
```
[-1/48*(3*(b^3*c - 6*a*b^2*d)*sqrt(a)*x^4*log((b*x^2 + 2*a*x - 2*sqrt(b*x^
3 + a*x^2)*sqrt(a))/x^2) + 2*(8*a^3*c + 3*(a*b^2*c + 10*a^2*b*d)*x^2 + 2*(
7*a^2*b*c + 6*a^3*d)*x)*sqrt(b*x^3 + a*x^2))/(a^2*x^4), -1/24*(3*(b^3*c -
6*a*b^2*d)*sqrt(-a)*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x))
 + (8*a^3*c + 3*(a*b^2*c + 10*a^2*b*d)*x^2 + 2*(7*a^2*b*c + 6*a^3*d)*x)*sq
rt(b*x^3 + a*x^2))/(a^2*x^4)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)}{x^7}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/x**7,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)*(c + d*x)/x**7, x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx=\int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}(dx+c)}{x^7}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/x^7, x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx =$$

$$-\frac{1}{24}b^3\left(\frac{3\left(bc\mathrm{sgn}(x)-6\,ad\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ab}+\frac{3\left(bx+a\right)^{\frac{5}{2}}bc\mathrm{sgn}(x)+8\left(bx+a\right)^{\frac{3}{2}}abc\mathrm{sgn}(x)-3\sqrt{b}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^7,x, algorithm="giac")
```

output
```
-1/24*b^3*(3*(b*c*sgn(x) - 6*a*d*sgn(x))*arctan(sqrt(b*x + a)/sqrt(-a))/(s
qrt(-a)*a*b) + (3*(b*x + a)^(5/2)*b*c*sgn(x) + 8*(b*x + a)^(3/2)*a*b*c*sgn
(x) - 3*sqrt(b*x + a)*a^2*b*c*sgn(x) + 30*(b*x + a)^(5/2)*a*d*sgn(x) - 48*
(b*x + a)^(3/2)*a^2*d*sgn(x) + 18*sqrt(b*x + a)*a^3*d*sgn(x))/(a*b^4*x^3))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx = \int \frac{\left(b\,x^3+a\,x^2\right)^{3/2}\,\left(c+d\,x\right)}{x^7}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/x^7,x)
```

output
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^7}\,dx = \frac{-16\sqrt{bx+a}\,a^3c-24\sqrt{bx+a}\,a^3dx-28\sqrt{bx+a}\,a^2bcx-60\sqrt{bx+a}\,a^2bd}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^7,x)
```

output
```
( - 16*sqrt(a + b*x)*a**3*c - 24*sqrt(a + b*x)*a**3*d*x - 28*sqrt(a + b*x)
*a**2*b*c*x - 60*sqrt(a + b*x)*a**2*b*d*x**2 - 6*sqrt(a + b*x)*a*b**2*c*x*
*2 + 18*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**2*d*x**3 - 3*sqrt(a)*log
(sqrt(a + b*x) - sqrt(a))*b**3*c*x**3 - 18*sqrt(a)*log(sqrt(a + b*x) + sqr
t(a))*a*b**2*d*x**3 + 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**3*c*x**3)/
(48*a**2*x**3)
```

3.256 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^8}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 173

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^8}\,dx = -\frac{(3bc+8ad)\sqrt{ax^2+bx^3}}{24x^4} - \frac{b(3bc+56ad)\sqrt{ax^2+bx^3}}{96ax^3} + \frac{b^2(3bc-8ad)\sqrt{ax^2+bx^3}}{64a^2x^2} - \frac{c(ax^2+bx^3)^{3/2}}{4x^7} - \frac{b^3(3bc-8ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{64a^{5/2}}$$

output

```
-1/24*(8*a*d+3*b*c)*(b*x^3+a*x^2)^(1/2)/x^4-1/96*b*(56*a*d+3*b*c)*(b*x^3+a
*x^2)^(1/2)/a/x^3+1/64*b^2*(-8*a*d+3*b*c)*(b*x^3+a*x^2)^(1/2)/a^2/x^2-1/4*
c*(b*x^3+a*x^2)^(3/2)/x^7-1/64*b^3*(-8*a*d+3*b*c)*arctanh((b*x^3+a*x^2)^(1
/2)/a^(1/2)/x)/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^8}\,dx =$$

$$-\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(-9b^3cx^3+6ab^2x^2(c+4dx)+16a^3(3c+4dx)+8a^2bx(9c+14dx))+3b^3(3bc\right.}{192a^{5/2}x^5\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^8,x]
```

output
```
-1/192*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(-9*b^3*c*x^3 + 6*a*b^2
*x^2*(c + 4*d*x) + 16*a^3*(3*c + 4*d*x) + 8*a^2*b*x*(9*c + 14*d*x)) + 3*b^
3*(3*b*c - 8*a*d)*x^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(a^(5/2)*x^5*Sqrt[a
 + b*x])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.88, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1944, 1926, 1926, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}(c+dx)}{x^8}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(3bc-8ad)\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^7}dx}{8a}-\frac{c\left(ax^2+bx^3\right)^{5/2}}{4ax^9}$$

$$\Big\downarrow \text{1926}$$

$$-\frac{(3bc-8ad)\left(\frac{1}{2}b\int \frac{\sqrt{bx^3+ax^2}}{x^4}dx-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}\right)}{8a}-\frac{c\left(ax^2+bx^3\right)^{5/2}}{4ax^9}$$

$$\Big\downarrow \; 1926$$

$$-\frac{(3bc-8ad)\left(\frac{1}{2}b\left(\frac{1}{4}b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{(ax^2+bx^3)^{3/2}}{3x^6}\right)}{8a}-\frac{c(ax^2+bx^3)^{5/2}}{4ax^9}$$

$$\Big\downarrow \; 1931$$

$$-\frac{(3bc-8ad)\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{(ax^2+bx^3)^{3/2}}{3x^6}\right)}{8a}-\frac{c(ax^2+bx^3)^{5/2}}{4ax^9}$$

$$\Big\downarrow \; 1914$$

$$-\frac{(3bc-8ad)\left(\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{(ax^2+bx^3)^{3/2}}{3x^6}\right)}{8a}-\frac{c(ax^2+bx^3)^{5/2}}{4ax^9}$$

$$\Big\downarrow \; 219$$

$$-\frac{\left(\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{(ax^2+bx^3)^{3/2}}{3x^6}\right)(3bc-8ad)}{8a}-\frac{c(ax^2+bx^3)^{5/2}}{4ax^9}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^8,x]
```

output
```
-1/4*(c*(a*x^2 + b*x^3)^(5/2))/(a*x^9) - ((3*b*c - 8*a*d)*(-1/3*(a*x^2 + b
*x^3)^(3/2)/x^6 + (b*(-1/2*Sqrt[a*x^2 + b*x^3]/x^3 + (b*(-(Sqrt[a*x^2 + b*
x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/4))
/2))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\left(24a\,b^{2}d\,x^{3}-9b^{3}c\,x^{3}+112a^{2}bd\,x^{2}+6a\,b^{2}c\,x^{2}+64a^{3}dx+72a^{2}bcx+48c\,a^{3}\right)\sqrt{x^{2}(bx+a)}}{192x^{5}a^{2}}+\frac{(8ad-3bc)b^{3}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right.}{64a^{\frac{5}{2}}x\sqrt{bx+a}}$
pseudoelliptic	$-\frac{7\left(b^{6}x^{7}\left(ad-\frac{9bc}{14}\right)\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\frac{72\left(-\frac{400xb\left(\frac{91dx}{75}+c\right)a^{\frac{11}{2}}}{9}+\frac{160\left(-\frac{7dx}{3}-2c\right)a^{\frac{13}{2}}}{9}+x^{2}b^{2}\left(\frac{35x^{3}b^{3}\left(\frac{7dx}{3}+c\right)a^{\frac{3}{2}}}{24}-\frac{7x^{2}b^{2}(}{}\right.\right.}{245}\right.}{512a^{\frac{11}{2}}x^{7}}$
default	$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(24(bx+a)^{\frac{7}{2}}a^{\frac{7}{2}}d-9(bx+a)^{\frac{7}{2}}a^{\frac{5}{2}}bc+40(bx+a)^{\frac{5}{2}}a^{\frac{9}{2}}d+33(bx+a)^{\frac{5}{2}}a^{\frac{7}{2}}bc-24\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^{3}b^{4}d\,x^{4}+9\right.}{192b\,x^{7}(bx+a)^{\frac{3}{2}}a^{\frac{9}{2}}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/192*(24*a*b^2*d*x^3-9*b^3*c*x^3+112*a^2*b*d*x^2+6*a*b^2*c*x^2+64*a^3*d*
x+72*a^2*b*c*x+48*a^3*c)/x^5/a^2*(x^2*(b*x+a))^(1/2)+1/64*(8*a*d-3*b*c)*b^
3/a^(5/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/
2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.71

$$\int \frac{(c+dx)\left(ax^{2}+bx^{3}\right)^{3/2}}{x^{8}}\,dx=\left[-\frac{3\left(3\,b^{4}c-8\,ab^{3}d\right)\sqrt{a}x^{5}\log\left(\frac{bx^{2}+2\,ax+2\,\sqrt{bx^{3}+ax^{2}}\sqrt{a}}{x^{2}}\right)+2\left(48\,a^{4}c-3\left(3\,ab^{3}\right.\right.}{38}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="fricas")
```

output
```
[-1/384*(3*(3*b^4*c - 8*a*b^3*d)*sqrt(a)*x^5*log((b*x^2 + 2*a*x + 2*sqrt(b
*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(48*a^4*c - 3*(3*a*b^3*c - 8*a^2*b^2*d)*x^
3 + 2*(3*a^2*b^2*c + 56*a^3*b*d)*x^2 + 8*(9*a^3*b*c + 8*a^4*d)*x)*sqrt(b*x
^3 + a*x^2))/(a^3*x^5), 1/192*(3*(3*b^4*c - 8*a*b^3*d)*sqrt(-a)*x^5*arctan
(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - (48*a^4*c - 3*(3*a*b^3*c -
8*a^2*b^2*d)*x^3 + 2*(3*a^2*b^2*c + 56*a^3*b*d)*x^2 + 8*(9*a^3*b*c + 8*a^4
*d)*x)*sqrt(b*x^3 + a*x^2))/(a^3*x^5)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^8}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)}{x^8}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/x**8,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)*(c + d*x)/x**8, x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^8}\,dx=\int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}(dx+c)}{x^8}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/x^8, x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^8}\,dx = \frac{\frac{3\,\left(3\,b^5 c\mathrm{sgn}(x)-8\,ab^4 d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2} + \frac{9\,(bx+a)^{\frac{7}{2}}b^5 c\mathrm{sgn}(x)-33\,(bx+a)^{\frac{5}{2}}ab^5 c\mathrm{sgn}(x)-33\,(}{}}{}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^8,x, algorithm="giac")
```

output
```
1/192*(3*(3*b^5*c*sgn(x) - 8*a*b^4*d*sgn(x))*arctan(sqrt(b*x + a)/sqrt(-a)
)/(sqrt(-a)*a^2) + (9*(b*x + a)^(7/2)*b^5*c*sgn(x) - 33*(b*x + a)^(5/2)*a*
b^5*c*sgn(x) - 33*(b*x + a)^(3/2)*a^2*b^5*c*sgn(x) + 9*sqrt(b*x + a)*a^3*b
^5*c*sgn(x) - 24*(b*x + a)^(7/2)*a*b^4*d*sgn(x) - 40*(b*x + a)^(5/2)*a^2*b
^4*d*sgn(x) + 88*(b*x + a)^(3/2)*a^3*b^4*d*sgn(x) - 24*sqrt(b*x + a)*a^4*b
^4*d*sgn(x))/(a^2*b^4*x^4))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^8}\,dx = \int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}\,\left(c+d\,x\right)}{x^8} \,dx$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/x^8,x)
```

output
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^8}\,dx = \frac{-96\sqrt{bx+a}\,a^4c-128\sqrt{bx+a}\,a^4dx-144\sqrt{bx+a}\,a^3bcx-224\sqrt{bx+a}\,a$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^8,x)
```

output
```
( - 96*sqrt(a + b*x)*a**4*c - 128*sqrt(a + b*x)*a**4*d*x - 144*sqrt(a + b*
x)*a**3*b*c*x - 224*sqrt(a + b*x)*a**3*b*d*x**2 - 12*sqrt(a + b*x)*a**2*b*
*2*c*x**2 - 48*sqrt(a + b*x)*a**2*b**2*d*x**3 + 18*sqrt(a + b*x)*a*b**3*c*
x**3 - 24*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**3*d*x**4 + 9*sqrt(a)*l
og(sqrt(a + b*x) - sqrt(a))*b**4*c*x**4 + 24*sqrt(a)*log(sqrt(a + b*x) + s
qrt(a))*a*b**3*d*x**4 - 9*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**4*c*x**4
)/(384*a**3*x**4)
```

3.257 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx = -\frac{(3bc+10ad)\sqrt{ax^2+bx^3}}{40x^5} - \frac{b(bc+30ad)\sqrt{ax^2+bx^3}}{80ax^4} + \frac{b^2(bc-2ad)\sqrt{ax^2+bx^3}}{64a^2x^3} - \frac{3b^3(bc-2ad)\sqrt{ax^2+bx^3}}{128a^3x^2} - \frac{c\left(ax^2+bx^3\right)^{3/2}}{5x^8} + \frac{3b^4(bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{128a^{7/2}}$$

output

```
-1/40*(10*a*d+3*b*c)*(b*x^3+a*x^2)^(1/2)/x^5-1/80*b*(30*a*d+b*c)*(b*x^3+a*
x^2)^(1/2)/a/x^4+1/64*b^2*(-2*a*d+b*c)*(b*x^3+a*x^2)^(1/2)/a^2/x^3-3/128*b
^3*(-2*a*d+b*c)*(b*x^3+a*x^2)^(1/2)/a^3/x^2-1/5*c*(b*x^3+a*x^2)^(3/2)/x^8+
3/128*b^4*(-2*a*d+b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.75

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{3/2}}{x^9}\,dx = \frac{\sqrt{x^2(a+bx)}\Big(-\sqrt{a}\sqrt{a+bx}(15b^4cx^4-10ab^3x^3(c+3dx)+4a^2b^2x^2(2c+5}{640a}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^9,x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(15*b^4*c*x^4 - 10*a*b^3*x^3
*(c + 3*d*x) + 4*a^2*b^2*x^2*(2*c + 5*d*x) + 32*a^4*(4*c + 5*d*x) + 16*a^3
*b*x*(11*c + 15*d*x))) + 15*b^4*(b*c - 2*a*d)*x^5*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]]))/(640*a^(7/2)*x^6*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1944, 1926, 1926, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}(c+dx)}{x^9}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(bc-2ad)\int \frac{(bx^3+ax^2)^{3/2}}{x^8}dx}{2a}-\frac{c(ax^2+bx^3)^{5/2}}{5ax^{10}}$$

$$\Big\downarrow \text{1926}$$

$$-\frac{(bc-2ad)\left(\frac{3}{8}b\int \frac{\sqrt{bx^3+ax^2}}{x^5}dx-\frac{(ax^2+bx^3)^{3/2}}{4x^7}\right)}{2a}-\frac{c(ax^2+bx^3)^{5/2}}{5ax^{10}}$$

$$\Big\downarrow \text{1926}$$

$$-\frac{(bc-2ad)\left(\frac{3}{8}b\left(\frac{1}{6}b\int\frac{1}{x^2\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{(ax^2+bx^3)^{3/2}}{4x^7}\right)}{2a}-\frac{c(ax^2+bx^3)^{5/2}}{5ax^{10}}$$

$\Big\downarrow$ 1931

$$-\frac{(bc-2ad)\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\int\frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{(ax^2+bx^3)^{3/2}}{4x^7}\right)}{2a}-\frac{c(ax^2+bx^3)^{5/2}}{5ax^{10}}$$

$\Big\downarrow$ 1931

$$-\frac{(bc-2ad)\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int\frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{(ax^2+bx^3)^{3/2}}{4x^7}\right)}{2a}-\frac{c(ax^2+bx^3)^{5/2}}{5ax^{10}}$$

$\Big\downarrow$ 1914

$$-\frac{(bc-2ad)\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\int\frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{(ax^2+bx^3)^{3/2}}{4x^7}\right)}{2a}-\frac{c(ax^2+bx^3)^{5/2}}{5ax^{10}}$$

$\Big\downarrow$ 219

$$-\frac{\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{(ax^2+bx^3)^{3/2}}{4x^7}\right)(bc-2ad)}{2a}-\frac{c(ax^2+bx^3)^{5/2}}{5ax^{10}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/x^9,x]
```

output
```
-1/5*(c*(a*x^2 + b*x^3)^(5/2))/(a*x^10) - ((b*c - 2*a*d)*(-1/4*(a*x^2 + b*
x^3)^(3/2)/x^7 + (3*b*(-1/3*Sqrt[a*x^2 + b*x^3]/x^4 + (b*(-1/2*Sqrt[a*x^2
+ b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt
[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/6))/8))/(2*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\left(-30x^{4}a\,b^{3}d+15x^{4}b^{4}c+20a^{2}b^{2}d\,x^{3}-10a\,b^{3}c\,x^{3}+240a^{3}bd\,x^{2}+8a^{2}b^{2}c\,x^{2}+160a^{4}dx+176a^{3}bcx+128c\,a^{4}\right)\sqrt{x^{2}\left(bx+a\right)}}{640x^{6}a^{3}} -$
pseudoelliptic	$\frac{\frac{9b^{7}x^{8}\left(ad-\frac{11bc}{16}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{1024}-\frac{99\left(\frac{80\left(\frac{8dx}{5}+c\right)x^{2}b^{2}a^{\frac{11}{2}}}{99}+\frac{5440x\left(\frac{20dx}{17}+c\right)b\,a^{\frac{13}{2}}}{99}+\left(\frac{5120dx}{99}+\frac{4480c}{99}\right)a^{\frac{15}{2}}+x^{3}b^{3}\left(\frac{35\left(\frac{24dx}{11}+\cdots\right.}{2\cdots}\right.}{35\cdots}}{a^{\frac{13}{2}}x^{8}}$
default	$\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(30(bx+a)^{\frac{9}{2}}a^{\frac{9}{2}}d-15(bx+a)^{\frac{9}{2}}a^{\frac{7}{2}}bc-140(bx+a)^{\frac{7}{2}}a^{\frac{11}{2}}d+70(bx+a)^{\frac{7}{2}}a^{\frac{9}{2}}bc-128(bx+a)^{\frac{5}{2}}a^{\frac{11}{2}}bc-30\operatorname{arctanh}\left(\cdots\right.\right.}{640b\,x^{8}\left(\cdots\right.}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/640*(-30*a*b^3*d*x^4+15*b^4*c*x^4+20*a^2*b^2*d*x^3-10*a*b^3*c*x^3+240*a
^3*b*d*x^2+8*a^2*b^2*c*x^2+160*a^4*d*x+176*a^3*b*c*x+128*a^4*c)/x^6/a^3*(x
^2*(b*x+a))^(1/2)-3/128*(2*a*d-b*c)*b^4/a^(7/2)*arctanh((b*x+a)^(1/2)/a^(1
/2))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.62

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx = \left[-\frac{15\,(b^5c-2\,ab^4d)\sqrt{a}x^6\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(128\,a^5c+15\,(ab\right.$$

$$-\frac{15\,(b^5c-2\,ab^4d)\sqrt{-a}x^6\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(128\,a^5c+15\,(ab^4c-2\,a^2b^3d)x^4-10\,(a^2b^3c-2\,a^3b^2d)}{640\,a^4x^6}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="fricas")
```

output
```
[-1/1280*(15*(b^5*c - 2*a*b^4*d)*sqrt(a)*x^6*log((b*x^2 + 2*a*x - 2*sqrt(b
*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(128*a^5*c + 15*(a*b^4*c - 2*a^2*b^3*d)*x^
4 - 10*(a^2*b^3*c - 2*a^3*b^2*d)*x^3 + 8*(a^3*b^2*c + 30*a^4*b*d)*x^2 + 16
*(11*a^4*b*c + 10*a^5*d)*x)*sqrt(b*x^3 + a*x^2))/(a^4*x^6), -1/640*(15*(b^
5*c - 2*a*b^4*d)*sqrt(-a)*x^6*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 +
 a*x)) + (128*a^5*c + 15*(a*b^4*c - 2*a^2*b^3*d)*x^4 - 10*(a^2*b^3*c - 2*a
^3*b^2*d)*x^3 + 8*(a^3*b^2*c + 30*a^4*b*d)*x^2 + 16*(11*a^4*b*c + 10*a^5*d
)*x)*sqrt(b*x^3 + a*x^2))/(a^4*x^6)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)}{x^9}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/x**9,x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)*(c + d*x)/x**9, x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}(dx+c)}{x^9}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/x^9, x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx =$$

$$-\frac{1}{640}\,b^5\left(\frac{15\,(bc\mathrm{sgn}(x)-2\,ad\mathrm{sgn}(x))\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3b}+\frac{15\,(bx+a)^{\frac{9}{2}}bc\mathrm{sgn}(x)-70\,(bx+a)^{\frac{7}{2}}abc\mathrm{sgn}(x)+\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^9,x, algorithm="giac")
```

output
```
-1/640*b^5*(15*(b*c*sgn(x) - 2*a*d*sgn(x))*arctan(sqrt(b*x + a)/sqrt(-a))/
(sqrt(-a)*a^3*b) + (15*(b*x + a)^(9/2)*b*c*sgn(x) - 70*(b*x + a)^(7/2)*a*b
*c*sgn(x) + 128*(b*x + a)^(5/2)*a^2*b*c*sgn(x) + 70*(b*x + a)^(3/2)*a^3*b*
c*sgn(x) - 15*sqrt(b*x + a)*a^4*b*c*sgn(x) - 30*(b*x + a)^(9/2)*a*d*sgn(x)
 + 140*(b*x + a)^(7/2)*a^2*d*sgn(x) - 140*(b*x + a)^(3/2)*a^4*d*sgn(x) + 3
0*sqrt(b*x + a)*a^5*d*sgn(x))/(a^3*b^6*x^5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx=\int \frac{\left(b\,x^3+a\,x^2\right)^{3/2}\,\left(c+d\,x\right)}{x^9}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/x^9,x)
```

output
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/x^9, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{x^9}\,dx=\frac{-256\sqrt{bx+a}\,a^5c-320\sqrt{bx+a}\,a^5dx-352\sqrt{bx+a}\,a^4bcx-480\sqrt{bx+a}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/x^9,x)
```

output
```
( - 256*sqrt(a + b*x)*a**5*c - 320*sqrt(a + b*x)*a**5*d*x - 352*sqrt(a + b
*x)*a**4*b*c*x - 480*sqrt(a + b*x)*a**4*b*d*x**2 - 16*sqrt(a + b*x)*a**3*b
**2*c*x**2 - 40*sqrt(a + b*x)*a**3*b**2*d*x**3 + 20*sqrt(a + b*x)*a**2*b**
3*c*x**3 + 60*sqrt(a + b*x)*a**2*b**3*d*x**4 - 30*sqrt(a + b*x)*a*b**4*c*x
**4 + 30*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**4*d*x**5 - 15*sqrt(a)*l
og(sqrt(a + b*x) - sqrt(a))*b**5*c*x**5 - 30*sqrt(a)*log(sqrt(a + b*x) + s
qrt(a))*a*b**4*d*x**5 + 15*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**5*c*x**
5)/(1280*a**4*x**5)
```

3.258 $\int x^2(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 314

$$\begin{aligned}\int x^2(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx &= -\frac{2a^7(bc-ad)\left(ax^2+bx^3\right)^{7/2}}{7b^9x^7}\\&+\frac{2a^6(7bc-8ad)\left(ax^2+bx^3\right)^{9/2}}{9b^9x^9}-\frac{14a^5(3bc-4ad)\left(ax^2+bx^3\right)^{11/2}}{11b^9x^{11}}\\&+\frac{14a^4(5bc-8ad)\left(ax^2+bx^3\right)^{13/2}}{13b^9x^{13}}-\frac{14a^3(bc-2ad)\left(ax^2+bx^3\right)^{15/2}}{3b^9x^{15}}\\&+\frac{14a^2(3bc-8ad)\left(ax^2+bx^3\right)^{17/2}}{17b^9x^{17}}-\frac{14a(bc-4ad)\left(ax^2+bx^3\right)^{19/2}}{19b^9x^{19}}\\&+\frac{2(bc-8ad)\left(ax^2+bx^3\right)^{21/2}}{21b^9x^{21}}+\frac{2d\left(ax^2+bx^3\right)^{23/2}}{23b^9x^{23}}\end{aligned}$$

output

```
-2/7*a^7*(-a*d+b*c)*(b*x^3+a*x^2)^(7/2)/b^9/x^7+2/9*a^6*(-8*a*d+7*b*c)*(b*
x^3+a*x^2)^(9/2)/b^9/x^9-14/11*a^5*(-4*a*d+3*b*c)*(b*x^3+a*x^2)^(11/2)/b^9
/x^11+14/13*a^4*(-8*a*d+5*b*c)*(b*x^3+a*x^2)^(13/2)/b^9/x^13-14/3*a^3*(-2*
a*d+b*c)*(b*x^3+a*x^2)^(15/2)/b^9/x^15+14/17*a^2*(-8*a*d+3*b*c)*(b*x^3+a*x
^2)^(17/2)/b^9/x^17-14/19*a*(-4*a*d+b*c)*(b*x^3+a*x^2)^(19/2)/b^9/x^19+2/2
1*(-8*a*d+b*c)*(b*x^3+a*x^2)^(21/2)/b^9/x^21+2/23*d*(b*x^3+a*x^2)^(23/2)/b
^9/x^23
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.56

$$\int x^2(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx = \frac{2x(a+bx)^4\left(32768a^8d+138567b^8x^7(23c+21dx)-48048a^3b^5x^4(23c+24dx)+29568a^4b^4\cdots\right.}{}$$

input
```
Integrate[x^2*(c + d*x)*(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*x*(a + b*x)^4*(32768*a^8*d + 138567*b^8*x^7*(23*c + 21*d*x) - 48048*a^3
*b^5*x^4*(23*c + 24*d*x) + 29568*a^4*b^4*x^3*(23*c + 26*d*x) + 7168*a^6*b^
2*x*(23*c + 36*d*x) - 2048*a^7*b*(23*c + 56*d*x) + 24024*a^2*b^6*x^5*(69*c
 + 68*d*x) - 5376*a^5*b^3*x^2*(69*c + 88*d*x) - 14586*a*b^7*x^6*(161*c + 1
52*d*x)))/(66927861*b^9*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 1.06 (sec) , antiderivative size = 294, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1945, 1922, 1922, 1908, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(ax^2+bx^3\right)^{5/2}(c+dx)\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(23bc-16ad)\int x^2\left(bx^3+ax^2\right)^{5/2}dx}{23b}+\frac{2dx\left(ax^2+bx^3\right)^{7/2}}{23b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(23bc-16ad)\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{21b}-\frac{2a\int x\left(bx^3+ax^2\right)^{5/2}dx}{3b}\right)}{23b}+\frac{2dx\left(ax^2+bx^3\right)^{7/2}}{23b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(23bc-16ad)\left(\frac{2(ax^2+bx^3)^{7/2}}{21b}-\frac{2a\left(\frac{2(ax^2+bx^3)^{7/2}}{19bx}-\frac{12a\int (bx^3+ax^2)^{5/2}dx}{19b}\right)}{3b}\right)}{23b}+\frac{2dx(ax^2+bx^3)^{7/2}}{23b}$$

$$\Big\downarrow \ 1908$$

$$\frac{(23bc-16ad)\left(\frac{2(ax^2+bx^3)^{7/2}}{21b}-\frac{2a\left(\frac{2(ax^2+bx^3)^{7/2}}{19bx}-\frac{12a\left(\frac{2(ax^2+bx^3)^{7/2}}{17bx^2}-\frac{10a\int \frac{(bx^3+ax^2)^{5/2}}{x}dx}{17b}\right)}{19b}\right)}{3b}\right)}{23b}+\frac{2dx(ax^2+bx^3)^{7/2}}{23b}$$

$$\Big\downarrow \ 1922$$

$$\frac{(23bc-16ad)\left(\frac{2(ax^2+bx^3)^{7/2}}{21b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{19bx}-\frac{12a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x^2}dx}{15b}\right)}{17b}\right)}{19b}\right)}{3b}\right)}{23b}+\frac{2dx\left(ax^2+bx^3\right)^{7/2}}{23b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(23bc-16ad)\left(\frac{2(ax^2+bx^3)^{7/2}}{21b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{19bx}-\frac{12a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\int\frac{\left(bx^3+ax^2\right)^{\cdots}}{x^3}}{13b}\cdots\right.}{15b}\cdots\right.}{17b}\cdots\right.}{19b}\cdots\right.}{3b}\cdots\right.}{23b}$$

$$\frac{2dx\left(ax^2+bx^3\right)^{7/2}}{23b}$$

↓ 1922

$$(23bc-16ad)\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{21b}-\frac{2a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{19bx}-\frac{12a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)}{11bx^5}\right.}{\cdots}\right.}{15b}\right.}{17b}\right.}{19b}\right.}{3b}\right.$$

↓ 1922

$$6a\left(\frac{2\left(ax^2+bx^3\right)}{11bx^5}\right.$$

$$8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\right.$$

$$10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\right.$$

$$12a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{}{17b}\right.$$

$$2a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{19bx}-\frac{}{19b}\right.$$

↓ 1920

$$\left(2a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{19bx}-\frac{12a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2\left(ax^2+b\ldots\right.}{9bx\ldots}\right.}{\ldots}\right.}{13b}\right.}{15b}\right.}{17b}\right.}{19b}\right.\right.$$

input
```
Int[x^2*(c + d*x)*(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*d*x*(a*x^2 + b*x^3)^(7/2))/(23*b) + ((23*b*c - 16*a*d)*((2*(a*x^2 + b*x
^3)^(7/2))/(21*b) - (2*a*((2*(a*x^2 + b*x^3)^(7/2))/(19*b*x) - (12*a*((2*(
a*x^2 + b*x^3)^(7/2))/(17*b*x^2) - (10*a*((2*(a*x^2 + b*x^3)^(7/2))/(15*b*
x^3) - (8*a*((2*(a*x^2 + b*x^3)^(7/2))/(13*b*x^4) - (6*a*((2*(a*x^2 + b*x^
3)^(7/2))/(11*b*x^5) - (4*a*((-4*a*(a*x^2 + b*x^3)^(7/2))/(63*b^2*x^7) + (
2*(a*x^2 + b*x^3)^(7/2))/(9*b*x^6)))/(11*b)))/(13*b)))/(15*b)))/(17*b)))/(
19*b)))/(3*b)))/(23*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.18

method	result
pseudoelliptic	$-\frac{32(bx+a)^{\frac{7}{2}}\left(-\frac{273\left(\frac{11dx}{13}+c\right)x^{2}b^{3}}{16}+\frac{91x\left(\frac{27dx}{26}+c\right)a\,b^{2}}{12}-\frac{13\left(\frac{21dx}{13}+c\right)a^{2}b}{6}+a^{3}d\right)}{3003b^{4}}$
gosper	$\frac{2(bx+a)\left(2909907d\,x^{8}b^{8}-2217072a\,b^{7}d\,x^{7}+3187041b^{8}c\,x^{7}+1633632a^{2}b^{6}d\,x^{6}-2348346a\,b^{7}c\,x^{6}-1153152a^{3}b^{5}d\,x^{5}+16576\right.}{}$
default	$\frac{2(bx+a)\left(2909907d\,x^{8}b^{8}-2217072a\,b^{7}d\,x^{7}+3187041b^{8}c\,x^{7}+1633632a^{2}b^{6}d\,x^{6}-2348346a\,b^{7}c\,x^{6}-1153152a^{3}b^{5}d\,x^{5}+16576\right.}{}$
orering	$\frac{2(bx+a)\left(2909907d\,x^{8}b^{8}-2217072a\,b^{7}d\,x^{7}+3187041b^{8}c\,x^{7}+1633632a^{2}b^{6}d\,x^{6}-2348346a\,b^{7}c\,x^{6}-1153152a^{3}b^{5}d\,x^{5}+16576\right.}{}$
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,\left(2909907b^{11}d\,x^{11}+6512649a\,b^{10}d\,x^{10}+3187041b^{11}c\,x^{10}+3712137a^{2}b^{9}d\,x^{9}+7212777a\,b^{10}c\,x^{9}+6435a^{3}b^{8}d\,x\right.}{}$
trager	$\frac{2\left(2909907b^{11}d\,x^{11}+6512649a\,b^{10}d\,x^{10}+3187041b^{11}c\,x^{10}+3712137a^{2}b^{9}d\,x^{9}+7212777a\,b^{10}c\,x^{9}+6435a^{3}b^{8}d\,x^{8}+4173741a\right.}{}$

input
```
int(x^2*(d*x+c)*(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-32/3003*(b*x+a)^(7/2)*(-273/16*(11/13*d*x+c)*x^2*b^3+91/12*x*(27/26*d*x+c
)*a*b^2-13/6*(21/13*d*x+c)*a^2*b+a^3*d)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 273, normalized size of antiderivative = 0.87

$$\int x^2(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx = \frac{2\left(2909907\,b^{11}dx^{11}-47104\,a^{10}bc+32768\,a^{11}d+138567\,(23\,b^{11}c+47\,ab^{10}d)x^{10}+7293\,(989\right.}{}$$

input
```
integrate(x^2*(d*x+c)*(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
2/66927861*(2909907*b^11*d*x^11 - 47104*a^10*b*c + 32768*a^11*d + 138567*(
23*b^11*c + 47*a*b^10*d)*x^10 + 7293*(989*a*b^10*c + 509*a^2*b^9*d)*x^9 +
1287*(3243*a^2*b^9*c + 5*a^3*b^8*d)*x^8 + 429*(23*a^3*b^8*c - 16*a^4*b^7*d
)*x^7 - 462*(23*a^4*b^7*c - 16*a^5*b^6*d)*x^6 + 504*(23*a^5*b^6*c - 16*a^6
*b^5*d)*x^5 - 560*(23*a^6*b^5*c - 16*a^7*b^4*d)*x^4 + 640*(23*a^7*b^4*c -
16*a^8*b^3*d)*x^3 - 768*(23*a^8*b^3*c - 16*a^9*b^2*d)*x^2 + 1024*(23*a^9*b
^2*c - 16*a^10*b*d)*x)*sqrt(b*x^3 + a*x^2)/(b^9*x)
```

Sympy [F]

$$\int x^2(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx = \int x^2\left(x^2(a+bx)\right)^{\frac{5}{2}}(c+dx)\,dx$$

input
```
integrate(x**2*(d*x+c)*(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral(x**2*(x**2*(a + b*x))**(5/2)*(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.80

$$\int x^2(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx = \frac{2\left(138567\,b^{10}x^{10}+313599\,ab^9x^9+181467\,a^2b^8x^8+429\,a^3b^7x^7-462\,a^4b^6x^6+504\,a^5b^5x^5-\right.}{2909907\,b^8} + \frac{2\left(2909907\,b^{11}x^{11}+6512649\,ab^{10}x^{10}+3712137\,a^2b^9x^9+6435\,a^3b^8x^8-6864\,a^4b^7x^7+7392\,a^5b^6x^6-806\right.}{66927861\,b^9}$$

input
```
integrate(x^2*(d*x+c)*(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
2/2909907*(138567*b^10*x^10 + 313599*a*b^9*x^9 + 181467*a^2*b^8*x^8 + 429*
a^3*b^7*x^7 - 462*a^4*b^6*x^6 + 504*a^5*b^5*x^5 - 560*a^6*b^4*x^4 + 640*a^
7*b^3*x^3 - 768*a^8*b^2*x^2 + 1024*a^9*b*x - 2048*a^10)*sqrt(b*x + a)*c/b^
8 + 2/66927861*(2909907*b^11*x^11 + 6512649*a*b^10*x^10 + 3712137*a^2*b^9*
x^9 + 6435*a^3*b^8*x^8 - 6864*a^4*b^7*x^7 + 7392*a^5*b^6*x^6 - 8064*a^6*b^
5*x^5 + 8960*a^7*b^4*x^4 - 10240*a^8*b^3*x^3 + 12288*a^9*b^2*x^2 - 16384*a
^10*b*x + 32768*a^11)*sqrt(b*x + a)*d/b^9
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1034 vs. 2(278) = 556.

Time = 0.22 (sec) , antiderivative size = 1034, normalized size of antiderivative = 3.29

$$\int x^2(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*(d*x+c)*(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
2/334639305*(52003*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285
*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*
a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*
x + a)*a^7)*a^3*c*sgn(x)/b^7 + 9177*(6435*(b*x + a)^(17/2) - 58344*(b*x +
a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 +
850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)
^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a^2*c*
sgn(x)/b^7 + 3059*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 2356
20*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(
9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 29172
0*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a^3*d*sgn(x)/b^8 + 4347*
(12155*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15
/2)*a^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 32
33230*(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a
)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqrt(b*x + a)*a^9)*a*c*s
gn(x)/b^7 + 4347*(12155*(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*a + 554
268*(b*x + a)^(15/2)*a^2 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b*x + a
)^(11/2)*a^4 - 3233230*(b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*a^6 -
 1662804*(b*x + a)^(5/2)*a^7 + 692835*(b*x + a)^(3/2)*a^8 - 230945*sqrt(b*
x + a)*a^9)*a^2*d*sgn(x)/b^8 + 345*(46189*(b*x + a)^(21/2) - 510510*(b*...
```

Mupad [B] (verification not implemented)

Time = 9.69 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.74

$$\int x^2(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx=\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,a\,x^9\,(509\,a\,d+989\,b\,c)}{9177}+\frac{2\,b\,x^{10}\,(47\,a\,d+23\,b\,c)}{483}+\frac{2\,b^2\,d\,x^{11}}{23}+\frac{4096\,a^{10}\,(16\,a\,d-23\,b\,c)}{66927861\,b^9}-\frac{2048\,a^9\,x\,(}{6692}\right.}{}$$

input
```
int(x^2*(a*x^2 + b*x^3)^(5/2)*(c + d*x),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((2*a*x^9*(509*a*d + 989*b*c))/9177 + (2*b*x^10*(47
*a*d + 23*b*c))/483 + (2*b^2*d*x^11)/23 + (4096*a^10*(16*a*d - 23*b*c))/(6
6927861*b^9) - (2048*a^9*x*(16*a*d - 23*b*c))/(66927861*b^8) - (2*a^3*x^7*
(16*a*d - 23*b*c))/(156009*b^2) + (4*a^4*x^6*(16*a*d - 23*b*c))/(289731*b^
3) - (16*a^5*x^5*(16*a*d - 23*b*c))/(1062347*b^4) + (160*a^6*x^4*(16*a*d -
 23*b*c))/(9561123*b^5) - (1280*a^7*x^3*(16*a*d - 23*b*c))/(66927861*b^6)
+ (512*a^8*x^2*(16*a*d - 23*b*c))/(22309287*b^7) + (2*a^2*x^8*(5*a*d + 324
3*b*c))/(52003*b)))/x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.83

$$\int x^2(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx=\frac{2\sqrt{bx+a}\,(2909907b^{11}d\,x^{11}+6512649a\,b^{10}d\,x^{10}+3187041b^{11}c\,x^{10}+3712137a^2b^9d\,x^9+721$$

input
```
int(x^2*(d*x+c)*(b*x^3+a*x^2)^(5/2),x)
```

output
```
(2*sqrt(a + b*x)*(32768*a**11*d - 47104*a**10*b*c - 16384*a**10*b*d*x + 23
552*a**9*b**2*c*x + 12288*a**9*b**2*d*x**2 - 17664*a**8*b**3*c*x**2 - 1024
0*a**8*b**3*d*x**3 + 14720*a**7*b**4*c*x**3 + 8960*a**7*b**4*d*x**4 - 1288
0*a**6*b**5*c*x**4 - 8064*a**6*b**5*d*x**5 + 11592*a**5*b**6*c*x**5 + 7392
*a**5*b**6*d*x**6 - 10626*a**4*b**7*c*x**6 - 6864*a**4*b**7*d*x**7 + 9867*
a**3*b**8*c*x**7 + 6435*a**3*b**8*d*x**8 + 4173741*a**2*b**9*c*x**8 + 3712
137*a**2*b**9*d*x**9 + 7212777*a*b**10*c*x**9 + 6512649*a*b**10*d*x**10 +
3187041*b**11*c*x**10 + 2909907*b**11*d*x**11))/(66927861*b**9)
```

3.259 $\int x(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 279

$$\int x(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx = \frac{2a^6(bc-ad)\left(ax^2+bx^3\right)^{7/2}}{7b^8x^7} - \frac{2a^5(6bc-7ad)\left(ax^2+bx^3\right)^{9/2}}{9b^8x^9} + \frac{6a^4(5bc-7ad)\left(ax^2+bx^3\right)^{11/2}}{11b^8x^{11}} - \frac{10a^3(4bc-7ad)\left(ax^2+bx^3\right)^{13/2}}{13b^8x^{13}} + \frac{2a^2(3bc-7ad)\left(ax^2+bx^3\right)^{15/2}}{3b^8x^{15}} - \frac{6a(2bc-7ad)\left(ax^2+bx^3\right)^{17/2}}{17b^8x^{17}} + \frac{2(bc-7ad)\left(ax^2+bx^3\right)^{19/2}}{19b^8x^{19}} + \frac{2d\left(ax^2+bx^3\right)^{21/2}}{21b^8x^{21}}$$

output
```
2/7*a^6*(-a*d+b*c)*(b*x^3+a*x^2)^(7/2)/b^8/x^7-2/9*a^5*(-7*a*d+6*b*c)*(b*x
^3+a*x^2)^(9/2)/b^8/x^9+6/11*a^4*(-7*a*d+5*b*c)*(b*x^3+a*x^2)^(11/2)/b^8/x
^11-10/13*a^3*(-7*a*d+4*b*c)*(b*x^3+a*x^2)^(13/2)/b^8/x^13+2/3*a^2*(-7*a*d
+3*b*c)*(b*x^3+a*x^2)^(15/2)/b^8/x^15-6/17*a*(-7*a*d+2*b*c)*(b*x^3+a*x^2)^
(17/2)/b^8/x^17+2/19*(-7*a*d+b*c)*(b*x^3+a*x^2)^(19/2)/b^8/x^19+2/21*d*(b*
x^3+a*x^2)^(21/2)/b^8/x^21
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.55

$$\int x(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx=\frac{2x(a+bx)^4\left(-2048a^7d+72072a^2b^5x^4(c+dx)-5376a^5b^2x(2c+3dx)+1024a^6b(3c+7dx\right.}{2909}$$

input
```
Integrate[x*(c + d*x)*(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*x*(a + b*x)^4*(-2048*a^7*d + 72072*a^2*b^5*x^4*(c + d*x) - 5376*a^5*b^2
*x*(2*c + 3*d*x) + 1024*a^6*b*(3*c + 7*d*x) + 2688*a^4*b^3*x^2*(9*c + 11*d
*x) - 3696*a^3*b^4*x^3*(12*c + 13*d*x) - 6006*a*b^6*x^5*(18*c + 17*d*x) +
7293*b^7*x^6*(21*c + 19*d*x)))/(2909907*b^8*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {1945, 1922, 1908, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(ax^2+bx^3\right)^{5/2}(c+dx)\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(3bc-2ad)\int x\left(bx^3+ax^2\right)^{5/2}dx}{3b}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{21b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(3bc-2ad)\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{19bx}-\frac{12a\int\left(bx^3+ax^2\right)^{5/2}dx}{19b}\right)}{3b}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{21b}$$

$$\Big\downarrow \text{1908}$$

$$\frac{(3bc-2ad)\left(\frac{2(ax^2+bx^3)^{7/2}}{19bx}-\frac{12a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x}dx}{17b}\right)}{19b}\right)}{3b}+\frac{2d(ax^2+bx^3)^{7/2}}{21b}$$

$$\Big\downarrow \; 1922$$

$$\frac{(3bc-2ad)\left(\frac{2(ax^2+bx^3)^{7/2}}{19bx}-\frac{12a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x^2}dx}{15b}\right)}{17b}\right)}{19b}\right)}{3b}+\frac{2d(ax^2+bx^3)^{7/2}}{21b}$$

$$\Big\downarrow \; 1922$$

$$\frac{(3bc-2ad)\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{19bx}-\frac{12a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x^3}dx}{13b}\right)}{15b}\right)}{17b}\right)}{19b}\right)}{3b}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{21b}$$

$\downarrow$ 1922

$$\frac{(3bc-2ad)\left(\frac{2(ax^2+bx^3)^{7/2}}{19bx}-\frac{12a\left(\frac{2(ax^2+bx^3)^{7/2}}{17bx^2}-\frac{10a\left(\frac{2(ax^2+bx^3)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2(ax^2+bx^3)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2(ax^2+bx^3)^{7/2}}{11bx^5}-\frac{4a\int\frac{(bx^3+ax^2)^{5/2}}{x^4}}{11b}\right.}{13b}\right.}{15b}\right.}{17b}\right.}{19b}\right.}{3b}$$

$$\frac{2d(ax^2+bx^3)^{7/2}}{21b}$$

↓ 1922

$$4a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{9bx^6}\right.$$

$$6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\right.$$

$$8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{\quad}{13b}\right.$$

$$10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{\quad}{15b}\right.$$

$$12a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2}-\frac{\quad}{17b}\right.$$

$$(3bc-2ad)\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{19bx}-\frac{\quad}{19b}\right.$$

↓ 1920

$$\frac{2\left(ax^2+bx^3\right)^{7/2}}{19bx} - \frac{12a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{17bx^2} - \frac{10a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3} - \frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4} - \frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5} - \frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{9bx^6} - \frac{4a\left(ax^2+bx^3\right)}{63b^2x^7}\right.}{11b}\right.}{13b}\right.}{15b}\right.}{17b}\right.}{19b}$$

input
```
Int[x*(c + d*x)*(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*d*(a*x^2 + b*x^3)^(7/2))/(21*b) + ((3*b*c - 2*a*d)*((2*(a*x^2 + b*x^3)^
(7/2))/(19*b*x) - (12*a*((2*(a*x^2 + b*x^3)^(7/2))/(17*b*x^2) - (10*a*((2*
(a*x^2 + b*x^3)^(7/2))/(15*b*x^3) - (8*a*((2*(a*x^2 + b*x^3)^(7/2))/(13*b*
x^4) - (6*a*((2*(a*x^2 + b*x^3)^(7/2))/(11*b*x^5) - (4*a*((-4*a*(a*x^2 + b
*x^3)^(7/2))/(63*b^2*x^7) + (2*(a*x^2 + b*x^3)^(7/2))/(9*b*x^6)))/(11*b)))
/(13*b)))/(15*b)))/(17*b)))/(19*b)))/(3*b)
```

Defintions of rubi rules used

rule 1908
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[(a*x^j +
b*x^n)^(p + 1)/(a*(j*p + 1)*x^(j - 1)), x] - Simp[b*((n*p + n - j + 1)/(a*(
j*p + 1)))   Int[x^(n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(n*p + n - j + 1)/(n
- j)], 0] && NeQ[j*p + 1, 0]
```

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.15

method	result
pseudoelliptic	$\frac{16(bx+a)^{\frac{7}{2}}\left(\frac{77\left(\frac{9dx}{11}+c\right)x\,b^2}{8}-\frac{11\left(\frac{14dx}{11}+c\right)ab}{4}+a^2d\right)}{693b^3}$
gosper	$-\frac{2(bx+a)\left(-138567d\,x^7b^7+102102a\,b^6d\,x^6-153153b^7c\,x^6-72072a^2b^5d\,x^5+108108a\,b^6c\,x^5+48048a^3b^4d\,x^4-72072a^2b^5c\right.}{}$
default	$-\frac{2(bx+a)\left(-138567d\,x^7b^7+102102a\,b^6d\,x^6-153153b^7c\,x^6-72072a^2b^5d\,x^5+108108a\,b^6c\,x^5+48048a^3b^4d\,x^4-72072a^2b^5c\right.}{}$
orering	$-\frac{2(bx+a)\left(-138567d\,x^7b^7+102102a\,b^6d\,x^6-153153b^7c\,x^6-72072a^2b^5d\,x^5+108108a\,b^6c\,x^5+48048a^3b^4d\,x^4-72072a^2b^5c\right.}{}$
risch	$-\frac{2\sqrt{x^2(bx+a)}\left(-138567b^{10}d\,x^{10}-313599a\,b^9d\,x^9-153153b^{10}c\,x^9-181467a^2b^8d\,x^8-351351a\,b^9c\,x^8-429a^3b^7d\,x^7-2072\right.}{}$
trager	$-\frac{2\left(-138567b^{10}d\,x^{10}-313599a\,b^9d\,x^9-153153b^{10}c\,x^9-181467a^2b^8d\,x^8-351351a\,b^9c\,x^8-429a^3b^7d\,x^7-207207a^2b^8c\,x^7+\right.}{}$

input
```
int(x*(d*x+c)*(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
16/693*(b*x+a)^(7/2)*(77/8*(9/11*d*x+c)*x*b^2-11/4*(14/11*d*x+c)*a*b+a^2*d
)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.89

$$\int x(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx=\frac{2\left(138567\,b^{10}dx^{10}+3072\,a^9bc-2048\,a^{10}d+7293\left(21\,b^{10}c+43\,ab^9d\right)x^9+3861\left(91\,ab^9c+47\right.\right.}{}$$

input
```
integrate(x*(d*x+c)*(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
2/2909907*(138567*b^10*d*x^10 + 3072*a^9*b*c - 2048*a^10*d + 7293*(21*b^10
*c + 43*a*b^9*d)*x^9 + 3861*(91*a*b^9*c + 47*a^2*b^8*d)*x^8 + 429*(483*a^2
*b^8*c + a^3*b^7*d)*x^7 + 231*(3*a^3*b^7*c - 2*a^4*b^6*d)*x^6 - 252*(3*a^4
*b^6*c - 2*a^5*b^5*d)*x^5 + 280*(3*a^5*b^5*c - 2*a^6*b^4*d)*x^4 - 320*(3*a
^6*b^4*c - 2*a^7*b^3*d)*x^3 + 384*(3*a^7*b^3*c - 2*a^8*b^2*d)*x^2 - 512*(3
*a^8*b^2*c - 2*a^9*b*d)*x)*sqrt(b*x^3 + a*x^2)/(b^8*x)
```

Sympy [F]

$$\int x(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx=\int x\left(x^2(a+bx)\right)^{\frac{5}{2}}(c+dx)\,dx$$

input
```
integrate(x*(d*x+c)*(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral(x*(x**2*(a + b*x))**(5/2)*(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.82

$$\int x(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx = \frac{2\,(51051\,b^9x^9+117117\,ab^8x^8+69069\,a^2b^7x^7+231\,a^3b^6x^6-252\,a^4b^5x^5+280\,a^5b^4x^4-320}{969969\,b^7} + \frac{2\,(138567\,b^{10}x^{10}+313599\,ab^9x^9+181467\,a^2b^8x^8+429\,a^3b^7x^7-462\,a^4b^6x^6+504\,a^5b^5x^5-560\,a^6b^4x^4+}{2909907\,b^8}$$

input
```
integrate(x*(d*x+c)*(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
2/969969*(51051*b^9*x^9 + 117117*a*b^8*x^8 + 69069*a^2*b^7*x^7 + 231*a^3*b
^6*x^6 - 252*a^4*b^5*x^5 + 280*a^5*b^4*x^4 - 320*a^6*b^3*x^3 + 384*a^7*b^2
*x^2 - 512*a^8*b*x + 1024*a^9)*sqrt(b*x + a)*c/b^7 + 2/2909907*(138567*b^1
0*x^10 + 313599*a*b^9*x^9 + 181467*a^2*b^8*x^8 + 429*a^3*b^7*x^7 - 462*a^4
*b^6*x^6 + 504*a^5*b^5*x^5 - 560*a^6*b^4*x^4 + 640*a^7*b^3*x^3 - 768*a^8*b
^2*x^2 + 1024*a^9*b*x - 2048*a^10)*sqrt(b*x + a)*d/b^8
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. 2(247) = 494.

Time = 0.14 (sec) , antiderivative size = 938, normalized size of antiderivative = 3.36

$$\int x(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate(x*(d*x+c)*(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
2/14549535*(4845*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b
*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 -
6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^3*c*sgn(x)/b^6 + 6783
*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*
a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x +
 a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*a^2*c*
sgn(x)/b^6 + 2261*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*
(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a
^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x
 + a)*a^7)*a^3*d*sgn(x)/b^7 + 399*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)
^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 85
0850*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(
5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a*c*sgn(
x)/b^6 + 399*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 235620*(b
*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x + a)^(9/2)*
a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(b*x + a)^(5/2)*a^6 - 291720*(b*
x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8)*a^2*d*sgn(x)/b^7 + 63*(12155*
(b*x + a)^(19/2) - 122265*(b*x + a)^(17/2)*a + 554268*(b*x + a)^(15/2)*a^2
 - 1492260*(b*x + a)^(13/2)*a^3 + 2645370*(b*x + a)^(11/2)*a^4 - 3233230*(
b*x + a)^(9/2)*a^5 + 2771340*(b*x + a)^(7/2)*a^6 - 1662804*(b*x + a)^(5...
```

Mupad [B] (verification not implemented)

Time = 9.26 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.76

$$\int x(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx=\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{6\,a\,x^8\,(47\,a\,d+91\,b\,c)}{2261}+\frac{2\,b\,x^9\,(43\,a\,d+21\,b\,c)}{399}+\frac{2\,b^2\,d\,x^{10}}{21}-\frac{2048\,a^9\,(2\,a\,d-3\,b\,c)}{2909907\,b^8}+\frac{1024\,a^8\,x\,(2\,a\,d-}{2909907\,b^7}$$

input
```
int(x*(a*x^2 + b*x^3)^(5/2)*(c + d*x),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((6*a*x^8*(47*a*d + 91*b*c))/2261 + (2*b*x^9*(43*a*
d + 21*b*c))/399 + (2*b^2*d*x^10)/21 - (2048*a^9*(2*a*d - 3*b*c))/(2909907
*b^8) + (1024*a^8*x*(2*a*d - 3*b*c))/(2909907*b^7) - (2*a^3*x^6*(2*a*d - 3
*b*c))/(12597*b^2) + (8*a^4*x^5*(2*a*d - 3*b*c))/(46189*b^3) - (80*a^5*x^4
*(2*a*d - 3*b*c))/(415701*b^4) + (640*a^6*x^3*(2*a*d - 3*b*c))/(2909907*b^
5) - (256*a^7*x^2*(2*a*d - 3*b*c))/(969969*b^6) + (2*a^2*x^7*(a*d + 483*b*
c))/(6783*b)))/x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.85

$$\int x(c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx=\frac{2\sqrt{bx+a}\,(138567b^{10}d\,x^{10}+313599a\,b^9d\,x^9+153153b^{10}c\,x^9+181467a^2b^8d\,x^8+351351a\,b^9}{}$$

input
```
int(x*(d*x+c)*(b*x^3+a*x^2)^(5/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 2048*a**10*d + 3072*a**9*b*c + 1024*a**9*b*d*x - 1536
*a**8*b**2*c*x - 768*a**8*b**2*d*x**2 + 1152*a**7*b**3*c*x**2 + 640*a**7*b
**3*d*x**3 - 960*a**6*b**4*c*x**3 - 560*a**6*b**4*d*x**4 + 840*a**5*b**5*c
*x**4 + 504*a**5*b**5*d*x**5 - 756*a**4*b**6*c*x**5 - 462*a**4*b**6*d*x**6
 + 693*a**3*b**7*c*x**6 + 429*a**3*b**7*d*x**7 + 207207*a**2*b**8*c*x**7 +
 181467*a**2*b**8*d*x**8 + 351351*a*b**9*c*x**8 + 313599*a*b**9*d*x**9 + 1
53153*b**10*c*x**9 + 138567*b**10*d*x**10))/(2909907*b**8)
```

3.260 $\int (c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 240

$$\begin{aligned}\int (c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx = & -\frac{2a^5(bc-ad)\left(ax^2+bx^3\right)^{7/2}}{7b^7x^7} \\ & +\frac{2a^4(5bc-6ad)\left(ax^2+bx^3\right)^{9/2}}{9b^7x^9}-\frac{10a^3(2bc-3ad)\left(ax^2+bx^3\right)^{11/2}}{11b^7x^{11}} \\ & +\frac{20a^2(bc-2ad)\left(ax^2+bx^3\right)^{13/2}}{13b^7x^{13}}-\frac{2a(bc-3ad)\left(ax^2+bx^3\right)^{15/2}}{3b^7x^{15}} \\ & +\frac{2(bc-6ad)\left(ax^2+bx^3\right)^{17/2}}{17b^7x^{17}}+\frac{2d\left(ax^2+bx^3\right)^{19/2}}{19b^7x^{19}}\end{aligned}$$

output

```
-2/7*a^5*(-a*d+b*c)*(b*x^3+a*x^2)^(7/2)/b^7/x^7+2/9*a^4*(-6*a*d+5*b*c)*(b*
x^3+a*x^2)^(9/2)/b^7/x^9-10/11*a^3*(-3*a*d+2*b*c)*(b*x^3+a*x^2)^(11/2)/b^7
/x^11+20/13*a^2*(-2*a*d+b*c)*(b*x^3+a*x^2)^(13/2)/b^7/x^13-2/3*a*(-3*a*d+b
*c)*(b*x^3+a*x^2)^(15/2)/b^7/x^15+2/17*(-6*a*d+b*c)*(b*x^3+a*x^2)^(17/2)/b
^7/x^17+2/19*d*(b*x^3+a*x^2)^(19/2)/b^7/x^19
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.57

$$\int (c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx=\frac{2x(a+bx)^4\left(3072a^6d+9009b^6x^5(19c+17dx)-6006ab^5x^4(19c+18dx)-2016a^3b^3x^2(19c\right.}{2909907b^7\sqrt{x^2(}}$$

input
```
Integrate[(c + d*x)*(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*x*(a + b*x)^4*(3072*a^6*d + 9009*b^6*x^5*(19*c + 17*d*x) - 6006*a*b^5*x
^4*(19*c + 18*d*x) - 2016*a^3*b^3*x^2*(19*c + 22*d*x) + 896*a^4*b^2*x*(19*
c + 27*d*x) + 1848*a^2*b^4*x^3*(38*c + 39*d*x) - 256*a^5*b*(19*c + 42*d*x)
))/(2909907*b^7*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.53, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2450, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^{5/2}(c+dx)\,dx$$

$$\Big\downarrow \text{ 2450}$$

$$\int \left(c\left(ax^2+bx^3\right)^{5/2}+dx\left(ax^2+bx^3\right)^{5/2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2048a^6d(ax^2+bx^3)^{7/2}}{969969b^7x^7}-\frac{512a^5c(ax^2+bx^3)^{7/2}}{153153b^6x^7}-\frac{1024a^5d(ax^2+bx^3)^{7/2}}{138567b^6x^6}+\frac{256a^4c(ax^2+bx^3)^{7/2}}{21879b^5x^6}+\frac{768a^4d(ax^2+bx^3)^{7/2}}{46189b^5x^5}-\frac{64a^3c(ax^2+bx^3)^{7/2}}{2431b^4x^5}-\frac{128a^3d(ax^2+bx^3)^{7/2}}{4199b^4x^4}+\frac{32a^2c(ax^2+bx^3)^{7/2}}{663b^3x^4}+\frac{16a^2d(ax^2+bx^3)^{7/2}}{323b^3x^3}-\frac{4ac(ax^2+bx^3)^{7/2}}{51b^2x^3}-\frac{24ad(ax^2+bx^3)^{7/2}}{323b^2x^2}+\frac{2c(ax^2+bx^3)^{7/2}}{17bx^2}+\frac{2d(ax^2+bx^3)^{7/2}}{19bx}$$

input
```
Int[(c + d*x)*(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(-512*a^5*c*(a*x^2 + b*x^3)^(7/2))/(153153*b^6*x^7) + (2048*a^6*d*(a*x^2 +
 b*x^3)^(7/2))/(969969*b^7*x^7) + (256*a^4*c*(a*x^2 + b*x^3)^(7/2))/(21879
*b^5*x^6) - (1024*a^5*d*(a*x^2 + b*x^3)^(7/2))/(138567*b^6*x^6) - (64*a^3*
c*(a*x^2 + b*x^3)^(7/2))/(2431*b^4*x^5) + (768*a^4*d*(a*x^2 + b*x^3)^(7/2)
)/(46189*b^5*x^5) + (32*a^2*c*(a*x^2 + b*x^3)^(7/2))/(663*b^3*x^4) - (128*
a^3*d*(a*x^2 + b*x^3)^(7/2))/(4199*b^4*x^4) - (4*a*c*(a*x^2 + b*x^3)^(7/2)
)/(51*b^2*x^3) + (16*a^2*d*(a*x^2 + b*x^3)^(7/2))/(323*b^3*x^3) + (2*c*(a*
x^2 + b*x^3)^(7/2))/(17*b*x^2) - (24*a*d*(a*x^2 + b*x^3)^(7/2))/(323*b^2*x
^2) + (2*d*(a*x^2 + b*x^3)^(7/2))/(19*b*x)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2450
```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IntegerQ[p] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.11

method	result
pseudoelliptic	$-\frac{2(bx+a)^{\frac{7}{2}}(-7bdx+2ad-9bc)}{63b^2}$
gosper	$\frac{2(bx+a)(153153d\,x^6b^6-108108a\,b^5d\,x^5+171171b^6c\,x^5+72072a^2b^4d\,x^4-114114a\,b^5c\,x^4-44352a^3b^3d\,x^3+70224a^2b^4c\,x^3+}{2909907b^7x^5}$
default	$\frac{2(bx+a)(153153d\,x^6b^6-108108a\,b^5d\,x^5+171171b^6c\,x^5+72072a^2b^4d\,x^4-114114a\,b^5c\,x^4-44352a^3b^3d\,x^3+70224a^2b^4c\,x^3+}{2909907b^7x^5}$
orering	$\frac{2(bx+a)(153153d\,x^6b^6-108108a\,b^5d\,x^5+171171b^6c\,x^5+72072a^2b^4d\,x^4-114114a\,b^5c\,x^4-44352a^3b^3d\,x^3+70224a^2b^4c\,x^3+}{2909907b^7x^5}$
risch	$\frac{2\sqrt{x^2(bx+a)}\,(153153b^9d\,x^9+351351a\,b^8d\,x^8+171171b^9c\,x^8+207207a^2b^7d\,x^7+399399a\,b^8c\,x^7+693a^3b^6d\,x^6+241395a^2b^7}{}$
trager	$\frac{2(153153b^9d\,x^9+351351a\,b^8d\,x^8+171171b^9c\,x^8+207207a^2b^7d\,x^7+399399a\,b^8c\,x^7+693a^3b^6d\,x^6+241395a^2b^7c\,x^6-756a^4b}{}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/63*(b*x+a)^(7/2)*(-7*b*d*x+2*a*d-9*b*c)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 225, normalized size of antiderivative = 0.94

$$\int (c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx=\frac{2\left(153153\,b^9dx^9-4864\,a^8bc+3072\,a^9d+9009\,(19\,b^9c+39\,ab^8d)x^8+3003\,(133\,ab^8c+69\,a\right.}{}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
2/2909907*(153153*b^9*d*x^9 - 4864*a^8*b*c + 3072*a^9*d + 9009*(19*b^9*c +
 39*a*b^8*d)*x^8 + 3003*(133*a*b^8*c + 69*a^2*b^7*d)*x^7 + 231*(1045*a^2*b
^7*c + 3*a^3*b^6*d)*x^6 + 63*(19*a^3*b^6*c - 12*a^4*b^5*d)*x^5 - 70*(19*a^
4*b^5*c - 12*a^5*b^4*d)*x^4 + 80*(19*a^5*b^4*c - 12*a^6*b^3*d)*x^3 - 96*(1
9*a^6*b^3*c - 12*a^7*b^2*d)*x^2 + 128*(19*a^7*b^2*c - 12*a^8*b*d)*x)*sqrt(
b*x^3 + a*x^2)/(b^7*x)
```

Sympy [F]

$$\int (c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx=\int \left(x^2(a+bx)\right)^{\frac{5}{2}}(c+dx)\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.87

$$\int (c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx=\frac{2\left(9009\,b^8x^8+21021\,ab^7x^7+12705\,a^2b^6x^6+63\,a^3b^5x^5-70\,a^4b^4x^4+80\,a^5b^3x^3-96\,a^6b^2x^2\right.}{153153\,b^6}+\frac{2\left(51051\,b^9x^9+117117\,ab^8x^8+69069\,a^2b^7x^7+231\,a^3b^6x^6-252\,a^4b^5x^5+280\,a^5b^4x^4-320\,a^6b^3x^3+384\right.}{969969\,b^7}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
2/153153*(9009*b^8*x^8 + 21021*a*b^7*x^7 + 12705*a^2*b^6*x^6 + 63*a^3*b^5*
x^5 - 70*a^4*b^4*x^4 + 80*a^5*b^3*x^3 - 96*a^6*b^2*x^2 + 128*a^7*b*x - 256
*a^8)*sqrt(b*x + a)*c/b^6 + 2/969969*(51051*b^9*x^9 + 117117*a*b^8*x^8 + 6
9069*a^2*b^7*x^7 + 231*a^3*b^6*x^6 - 252*a^4*b^5*x^5 + 280*a^5*b^4*x^4 - 3
20*a^6*b^3*x^3 + 384*a^7*b^2*x^2 - 512*a^8*b*x + 1024*a^9)*sqrt(b*x + a)*d
/b^7
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(212) = 424$.

Time = 0.13 (sec) , antiderivative size = 842, normalized size of antiderivative = 3.51

$$\int (c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
2/14549535*(20995*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x
+ a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693
*sqrt(b*x + a)*a^5)*a^3*c*sgn(x)/b^5 + 14535*(231*(b*x + a)^(13/2) - 1638*
(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 +
 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*
a^6)*a^2*c*sgn(x)/b^5 + 4845*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)
*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^
(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*a^3*d*sgn(x
)/b^6 + 6783*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a + 12285*(b*x
+ a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 -
27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)
*a^7)*a*c*sgn(x)/b^5 + 6783*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*
a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x +
a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 643
5*sqrt(b*x + a)*a^7)*a^2*d*sgn(x)/b^6 + 133*(6435*(b*x + a)^(17/2) - 58344
*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2
)*a^3 + 850850*(b*x + a)^(9/2)*a^4 - 875160*(b*x + a)^(7/2)*a^5 + 612612*(
b*x + a)^(5/2)*a^6 - 291720*(b*x + a)^(3/2)*a^7 + 109395*sqrt(b*x + a)*a^8
)*c*sgn(x)/b^5 + 399*(6435*(b*x + a)^(17/2) - 58344*(b*x + a)^(15/2)*a + 2
35620*(b*x + a)^(13/2)*a^2 - 556920*(b*x + a)^(11/2)*a^3 + 850850*(b*x ...
```

Mupad [B] (verification not implemented)

Time = 9.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.80

$$\int (c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx=\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,a\,x^7\,(69\,a\,d+133\,b\,c)}{969}+\frac{2\,b\,x^8\,(39\,a\,d+19\,b\,c)}{323}+\frac{2\,b^2\,d\,x^9}{19}+\frac{512\,a^8\,(12\,a\,d-19\,b\,c)}{2909907\,b^7}-\frac{256\,a^7\,x\,(12\,a\,d-}{2909907\,b}$$

input
```
int((a*x^2 + b*x^3)^(5/2)*(c + d*x),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((2*a*x^7*(69*a*d + 133*b*c))/969 + (2*b*x^8*(39*a*
d + 19*b*c))/323 + (2*b^2*d*x^9)/19 + (512*a^8*(12*a*d - 19*b*c))/(2909907
*b^7) - (256*a^7*x*(12*a*d - 19*b*c))/(2909907*b^6) - (2*a^3*x^5*(12*a*d -
 19*b*c))/(46189*b^2) + (20*a^4*x^4*(12*a*d - 19*b*c))/(415701*b^3) - (160
*a^5*x^3*(12*a*d - 19*b*c))/(2909907*b^4) + (64*a^6*x^2*(12*a*d - 19*b*c))
/(969969*b^5) + (2*a^2*x^6*(3*a*d + 1045*b*c))/(12597*b)))/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.89

$$\int (c+dx)\left(ax^2+bx^3\right)^{5/2}\,dx=\frac{2\sqrt{bx+a}\,(153153b^9d\,x^9+351351a\,b^8d\,x^8+171171b^9c\,x^8+207207a^2b^7d\,x^7+399399a\,b^8c\,x}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2),x)
```

output
```
(2*sqrt(a + b*x)*(3072*a**9*d - 4864*a**8*b*c - 1536*a**8*b*d*x + 2432*a**
7*b**2*c*x + 1152*a**7*b**2*d*x**2 - 1824*a**6*b**3*c*x**2 - 960*a**6*b**3
*d*x**3 + 1520*a**5*b**4*c*x**3 + 840*a**5*b**4*d*x**4 - 1330*a**4*b**5*c*
x**4 - 756*a**4*b**5*d*x**5 + 1197*a**3*b**6*c*x**5 + 693*a**3*b**6*d*x**6
 + 241395*a**2*b**7*c*x**6 + 207207*a**2*b**7*d*x**7 + 399399*a*b**8*c*x**
7 + 351351*a*b**8*d*x**8 + 171171*b**9*c*x**8 + 153153*b**9*d*x**9))/(2909
907*b**7)
```

3.261 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x} dx$

Optimal result

Integrand size = 24, antiderivative size = 205

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x} dx = \frac{2a^4(bc-ad)\left(ax^2+bx^3\right)^{7/2}}{7b^6x^7} - \frac{2a^3(4bc-5ad)\left(ax^2+bx^3\right)^{9/2}}{9b^6x^9} + \frac{4a^2(3bc-5ad)\left(ax^2+bx^3\right)^{11/2}}{11b^6x^{11}} - \frac{4a(2bc-5ad)\left(ax^2+bx^3\right)^{13/2}}{13b^6x^{13}} + \frac{2(bc-5ad)\left(ax^2+bx^3\right)^{15/2}}{15b^6x^{15}} + \frac{2d\left(ax^2+bx^3\right)^{17/2}}{17b^6x^{17}}$$

output
```
2/7*a^4*(-a*d+b*c)*(b*x^3+a*x^2)^(7/2)/b^6/x^7-2/9*a^3*(-5*a*d+4*b*c)*(b*x
^3+a*x^2)^(9/2)/b^6/x^9+4/11*a^2*(-5*a*d+3*b*c)*(b*x^3+a*x^2)^(11/2)/b^6/x
^11-4/13*a*(-5*a*d+2*b*c)*(b*x^3+a*x^2)^(13/2)/b^6/x^13+2/15*(-5*a*d+b*c)*
(b*x^3+a*x^2)^(15/2)/b^6/x^15+2/17*d*(b*x^3+a*x^2)^(17/2)/b^6/x^17
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.58

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x}\,dx = \frac{2x(a+bx)^4\left(-1280a^5d+3003b^5x^4(17c+15dx)+128a^4b(17c+35dx)-\right.}{765765b^6\sqrt{}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x,x]
```

output
```
(2*x*(a + b*x)^4*(-1280*a^5*d + 3003*b^5*x^4*(17*c + 15*d*x) + 128*a^4*b*(
17*c + 35*d*x) - 224*a^3*b^2*x*(34*c + 45*d*x) + 336*a^2*b^3*x^2*(51*c + 5
5*d*x) - 462*a*b^4*x^3*(68*c + 65*d*x)))/(765765*b^6*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1945, 1922, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{x}dx$$

$$\Big\downarrow \text{ 1945}$$

$$\frac{(17bc-10ad)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x}dx}{17b}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{17bx^2}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{(17bc-10ad)\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^2}dx}{15b}\right)}{17b}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{17bx^2}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{(17bc-10ad)\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x^3}dx}{13b}\right)}{15b}\right)}{17b}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{17bx^2}$$

$$\Big\downarrow \; 1922$$

$$\frac{(17bc-10ad)\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x^4}dx}{11b}\right)}{13b}\right)}{15b}\right)}{17b}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{17bx^2}$$

$$\Big\downarrow \; 1922$$

$$\frac{(17bc-10ad)\left(\frac{2(ax^2+bx^3)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{9bx^6}-\frac{2a\int\frac{\left(bx^3+ax^2\right)^{5/2}}{x^5}dx}{9b}\right)}{11b}\right)}{13b}\right)}{15b}\right)}{17b}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{17bx^2}$$

$\downarrow$ 1920

$$\frac{(17bc-10ad)\left(\frac{2(ax^2+bx^3)^{7/2}}{15bx^3}-\frac{8a\left(\frac{2(ax^2+bx^3)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2(ax^2+bx^3)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2(ax^2+bx^3)^{7/2}}{9bx^6}-\frac{4a(ax^2+bx^3)^{7/2}}{63b^2x^7}\right)}{11b}\right)}{13b}\right)}{15b}\right)}{17b}+\frac{2d(ax^2+bx^3)^{7/2}}{17bx^2}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x,x]
```

output
```
(2*d*(a*x^2 + b*x^3)^(7/2))/(17*b*x^2) + ((17*b*c - 10*a*d)*((2*(a*x^2 + b
*x^3)^(7/2))/(15*b*x^3) - (8*a*((2*(a*x^2 + b*x^3)^(7/2))/(13*b*x^4) - (6*
a*((2*(a*x^2 + b*x^3)^(7/2))/(11*b*x^5) - (4*a*((-4*a*(a*x^2 + b*x^3)^(7/2
))/(63*b^2*x^7) + (2*(a*x^2 + b*x^3)^(7/2))/(9*b*x^6)))/(11*b)))/(13*b)))/
(15*b)))/(17*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.49 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.40

method	result
pseudoelliptic	$\frac{-2a^{\frac{5}{2}}bc\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{2\sqrt{bx+a}\left(\left(d\,x^{3}+\frac{7}{5}c\,x^{2}\right)b^{3}+\frac{77xa\left(\frac{45dx}{77}+c\right)b^{2}}{15}+a^{2}\left(3dx+\frac{161c}{15}\right)b+a^{3}d\right)}{7}}{b}$
gosper	$-\frac{2(bx+a)\left(-45045d\,x^{5}b^{5}+30030a\,b^{4}d\,x^{4}-51051b^{5}c\,x^{4}-18480a^{2}b^{3}d\,x^{3}+31416a\,b^{4}c\,x^{3}+10080a^{3}b^{2}d\,x^{2}-17136a^{2}b^{3}c\,x^{2}-\right.}{765765b^{6}x^{5}}$
default	$-\frac{2(bx+a)\left(-45045d\,x^{5}b^{5}+30030a\,b^{4}d\,x^{4}-51051b^{5}c\,x^{4}-18480a^{2}b^{3}d\,x^{3}+31416a\,b^{4}c\,x^{3}+10080a^{3}b^{2}d\,x^{2}-17136a^{2}b^{3}c\,x^{2}-\right.}{765765b^{6}x^{5}}$
orering	$-\frac{2(bx+a)\left(-45045d\,x^{5}b^{5}+30030a\,b^{4}d\,x^{4}-51051b^{5}c\,x^{4}-18480a^{2}b^{3}d\,x^{3}+31416a\,b^{4}c\,x^{3}+10080a^{3}b^{2}d\,x^{2}-17136a^{2}b^{3}c\,x^{2}-\right.}{765765b^{6}x^{5}}$
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-45045d\,x^{8}b^{8}-105105a\,b^{7}d\,x^{7}-51051b^{8}c\,x^{7}-63525a^{2}b^{6}d\,x^{6}-121737a\,b^{7}c\,x^{6}-315a^{3}b^{5}d\,x^{5}-76041a^{2}b^{6}\right.}{}$
trager	$-\frac{2\left(-45045d\,x^{8}b^{8}-105105a\,b^{7}d\,x^{7}-51051b^{8}c\,x^{7}-63525a^{2}b^{6}d\,x^{6}-121737a\,b^{7}c\,x^{6}-315a^{3}b^{5}d\,x^{5}-76041a^{2}b^{6}c\,x^{5}+350a^{4}b\right.}{}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/7*(-7*a^(5/2)*b*c*arctanh((b*x+a)^(1/2)/a^(1/2))+(b*x+a)^(1/2)*((d*x^3+7
/5*c*x^2)*b^3+77/15*x*a*(45/77*d*x+c)*b^2+a^2*(3*d*x+161/15*c)*b+a^3*d))/b
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x}\,dx = \frac{2\left(45045\,b^8dx^8+2176\,a^7bc-1280\,a^8d+3003\left(17\,b^8c+35\,ab^7d\right)x^7+231\left(\right.\right.}{}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x,x, algorithm="fricas")
```

output
```
2/765765*(45045*b^8*d*x^8 + 2176*a^7*b*c - 1280*a^8*d + 3003*(17*b^8*c + 3
5*a*b^7*d)*x^7 + 231*(527*a*b^7*c + 275*a^2*b^6*d)*x^6 + 63*(1207*a^2*b^6*
c + 5*a^3*b^5*d)*x^5 + 35*(17*a^3*b^5*c - 10*a^4*b^4*d)*x^4 - 40*(17*a^4*b
^4*c - 10*a^5*b^3*d)*x^3 + 48*(17*a^5*b^3*c - 10*a^6*b^2*d)*x^2 - 64*(17*a
^6*b^2*c - 10*a^7*b*d)*x)*sqrt(b*x^3 + a*x^2)/(b^6*x)
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{5}{2}}(c+dx)}{x}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/x,x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.91

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x}\,dx = \frac{2\left(3003\,b^7x^7+7161\,ab^6x^6+4473\,a^2b^5x^5+35\,a^3b^4x^4-40\,a^4b^3x^3+48\,a^5b^2\right.}{45045\,b^5} + \frac{2\left(9009\,b^8x^8+21021\,ab^7x^7+12705\,a^2b^6x^6+63\,a^3b^5x^5-70\,a^4b^4x^4+80\,a^5b^3x^3-96\,a^6b^2x^2+128\,a^7bx-\right.}{153153\,b^6}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x,x, algorithm="maxima")
```

output
```
2/45045*(3003*b^7*x^7 + 7161*a*b^6*x^6 + 4473*a^2*b^5*x^5 + 35*a^3*b^4*x^4
 - 40*a^4*b^3*x^3 + 48*a^5*b^2*x^2 - 64*a^6*b*x + 128*a^7)*sqrt(b*x + a)*c
/b^5 + 2/153153*(9009*b^8*x^8 + 21021*a*b^7*x^7 + 12705*a^2*b^6*x^6 + 63*a
^3*b^5*x^5 - 70*a^4*b^4*x^4 + 80*a^5*b^3*x^3 - 96*a^6*b^2*x^2 + 128*a^7*b*
x - 256*a^8)*sqrt(b*x + a)*d/b^6
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 746 vs. 2(181) = 362.

Time = 0.13 (sec) , antiderivative size = 746, normalized size of antiderivative = 3.64

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x,x, algorithm="giac")
```

output
```
2/765765*(2431*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a)
^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a^3*c*sgn(x)
/b^4 + 3315*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(
7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(
b*x + a)*a^5)*a^2*c*sgn(x)/b^4 + 1105*(63*(b*x + a)^(11/2) - 385*(b*x + a)
^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x
+ a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^3*d*sgn(x)/b^5 + 765*(231*(b*x +
 a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*
x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3
003*sqrt(b*x + a)*a^6)*a*c*sgn(x)/b^4 + 765*(231*(b*x + a)^(13/2) - 1638*(
b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 +
9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a
^6)*a^2*d*sgn(x)/b^5 + 119*(429*(b*x + a)^(15/2) - 3465*(b*x + a)^(13/2)*a
 + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32175*(b*x + a
)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2)*a^6 - 6435
*sqrt(b*x + a)*a^7)*c*sgn(x)/b^4 + 357*(429*(b*x + a)^(15/2) - 3465*(b*x +
 a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/2)*a^3 + 32
175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*x + a)^(3/2
)*a^6 - 6435*sqrt(b*x + a)*a^7)*a*d*sgn(x)/b^5 + 7*(6435*(b*x + a)^(17/2)
- 58344*(b*x + a)^(15/2)*a + 235620*(b*x + a)^(13/2)*a^2 - 556920*(b*x ...
```

Mupad [B] (verification not implemented)

Time = 9.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{x}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,a\,x^6\,(275\,a\,d+527\,b\,c)}{3315}+\frac{2\,b\,x^7\,(35\,a\,d+17\,b\,c)}{255}+\frac{2\,b^2\,d\,x^8}{17}-\frac{256\,a^7\,(10\,a\,d-1}{765765\,b^6}\right.}{}$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x,x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((2*a*x^6*(275*a*d + 527*b*c))/3315 + (2*b*x^7*(35*
a*d + 17*b*c))/255 + (2*b^2*d*x^8)/17 - (256*a^7*(10*a*d - 17*b*c))/(76576
5*b^6) + (128*a^6*x*(10*a*d - 17*b*c))/(765765*b^5) - (2*a^3*x^4*(10*a*d -
 17*b*c))/(21879*b^2) + (16*a^4*x^3*(10*a*d - 17*b*c))/(153153*b^3) - (32*
a^5*x^2*(10*a*d - 17*b*c))/(255255*b^4) + (2*a^2*x^5*(5*a*d + 1207*b*c))/(
12155*b))/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x}\,dx = \frac{2\sqrt{bx+a}\,(45045b^8d\,x^8+105105a\,b^7d\,x^7+51051b^8c\,x^7+63525a^2b^6d\,x^6+$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x,x)
```

output
```
(2*sqrt(a + b*x)*( - 1280*a**8*d + 2176*a**7*b*c + 640*a**7*b*d*x - 1088*a
**6*b**2*c*x - 480*a**6*b**2*d*x**2 + 816*a**5*b**3*c*x**2 + 400*a**5*b**3
*d*x**3 - 680*a**4*b**4*c*x**3 - 350*a**4*b**4*d*x**4 + 595*a**3*b**5*c*x*
*4 + 315*a**3*b**5*d*x**5 + 76041*a**2*b**6*c*x**5 + 63525*a**2*b**6*d*x**
6 + 121737*a*b**7*c*x**6 + 105105*a*b**7*d*x**7 + 51051*b**8*c*x**7 + 4504
5*b**8*d*x**8))/(765765*b**6)
```

3.262 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^2} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 167

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^2} \, dx = -\frac{2a^3(bc-ad)\left(ax^2+bx^3\right)^{7/2}}{7b^5x^7} + \frac{2a^2(3bc-4ad)\left(ax^2+bx^3\right)^{9/2}}{9b^5x^9} - \frac{6a(bc-2ad)\left(ax^2+bx^3\right)^{11/2}}{11b^5x^{11}} + \frac{2(bc-4ad)\left(ax^2+bx^3\right)^{13/2}}{13b^5x^{13}} + \frac{2d\left(ax^2+bx^3\right)^{15/2}}{15b^5x^{15}}$$

output

```
-2/7*a^3*(-a*d+b*c)*(b*x^3+a*x^2)^(7/2)/b^5/x^7+2/9*a^2*(-4*a*d+3*b*c)*(b*
x^3+a*x^2)^(9/2)/b^5/x^9-6/11*a*(-2*a*d+b*c)*(b*x^3+a*x^2)^(11/2)/b^5/x^11
+2/13*(-4*a*d+b*c)*(b*x^3+a*x^2)^(13/2)/b^5/x^13+2/15*d*(b*x^3+a*x^2)^(15/
2)/b^5/x^15
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.59

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^2}\,dx = \frac{2x(a+bx)^4\left(128a^4d+168a^2b^2x(5c+6dx)+231b^4x^3(15c+13dx)-16a^3\right.}{45045b^5\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^2,x]
```

output
```
(2*x*(a + b*x)^4*(128*a^4*d + 168*a^2*b^2*x*(5*c + 6*d*x) + 231*b^4*x^3*(1
5*c + 13*d*x) - 16*a^3*b*(15*c + 28*d*x) - 42*a*b^3*x^2*(45*c + 44*d*x)))/
(45045*b^5*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1945, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{x^2}dx$$

$$\Big\downarrow \; 1945$$

$$\frac{(15bc-8ad)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^2}dx}{15b}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{15bx^3}$$

$$\Big\downarrow \; 1922$$

$$\frac{(15bc-8ad)\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{13bx^4}-\frac{6a\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^3}dx}{13b}\right)}{15b}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{15bx^3}$$

$$\Big\downarrow \; 1922$$

$$\frac{(15bc-8ad)\left(\frac{2(ax^2+bx^3)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2(ax^2+bx^3)^{7/2}}{11bx^5}-\frac{4a\int\frac{(bx^3+ax^2)^{5/2}}{x^4}dx}{11b}\right)}{13b}\right)}{15b}+\frac{2d(ax^2+bx^3)^{7/2}}{15bx^3}$$

$\Big\downarrow$ 1922

$$\frac{(15bc-8ad)\left(\frac{2(ax^2+bx^3)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2(ax^2+bx^3)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2(ax^2+bx^3)^{7/2}}{9bx^6}-\frac{2a\int\frac{(bx^3+ax^2)^{5/2}}{x^5}dx}{9b}\right)}{11b}\right)}{13b}\right)}{15b}+\frac{2d(ax^2+bx^3)^{7/2}}{15bx^3}$$

$\Big\downarrow$ 1920

$$\frac{\left(\frac{2(ax^2+bx^3)^{7/2}}{13bx^4}-\frac{6a\left(\frac{2(ax^2+bx^3)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2(ax^2+bx^3)^{7/2}}{9bx^6}-\frac{4a(ax^2+bx^3)^{7/2}}{63b^2x^7}\right)}{11b}\right)}{13b}\right)(15bc-8ad)}{15b}+\frac{2d(ax^2+bx^3)^{7/2}}{15bx^3}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^2,x]
```

output
```
(2*d*(a*x^2 + b*x^3)^(7/2))/(15*b*x^3) + ((15*b*c - 8*a*d)*((2*(a*x^2 + b*
x^3)^(7/2))/(13*b*x^4) - (6*a*((2*(a*x^2 + b*x^3)^(7/2))/(11*b*x^5) - (4*a
*((-4*a*(a*x^2 + b*x^3)^(7/2))/(63*b^2*x^7) + (2*(a*x^2 + b*x^3)^(7/2))/(9
*b*x^6)))/(11*b)))/(13*b)))/(15*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 3 vs. order 2.

Time = 0.55 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.51

method	result
pseudoelliptic	$-\frac{2\left(a^{2} x \left(a d+\frac{5 b c}{2}\right) \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)-\frac{\left(7 x b \left(\frac{11 d x}{35}+c \right) a^{\frac{3}{2}}+\left(\frac{23 d x}{5}-\frac{3 c}{2}\right) a^{\frac{5}{2}}+b^{2} x^{2} \sqrt{a}\, \left(\frac{3 d x}{5}+c \right)\right) \sqrt{b x+a}}{3}\right)}{\sqrt{a}\, x}$
gosper	$\frac{2(bx+a)\left(3003d\,x^{4}b^{4}-1848a\,b^{3}d\,x^{3}+3465b^{4}c\,x^{3}+1008a^{2}b^{2}d\,x^{2}-1890a\,b^{3}c\,x^{2}-448a^{3}bdx+840a^{2}b^{2}cx+128a^{4}d-240a^{3}bc\right)(}{45045b^{5}x^{5}}$
default	$\frac{2(bx+a)\left(3003d\,x^{4}b^{4}-1848a\,b^{3}d\,x^{3}+3465b^{4}c\,x^{3}+1008a^{2}b^{2}d\,x^{2}-1890a\,b^{3}c\,x^{2}-448a^{3}bdx+840a^{2}b^{2}cx+128a^{4}d-240a^{3}bc\right)(}{45045b^{5}x^{5}}$
orering	$\frac{2(bx+a)\left(3003d\,x^{4}b^{4}-1848a\,b^{3}d\,x^{3}+3465b^{4}c\,x^{3}+1008a^{2}b^{2}d\,x^{2}-1890a\,b^{3}c\,x^{2}-448a^{3}bdx+840a^{2}b^{2}cx+128a^{4}d-240a^{3}bc\right)(}{45045b^{5}x^{5}}$
risch	$\frac{2\sqrt{x^{2}(bx+a)}\,\left(3003b^{7}d\,x^{7}+7161a\,b^{6}d\,x^{6}+3465b^{7}c\,x^{6}+4473a^{2}b^{5}d\,x^{5}+8505a\,b^{6}c\,x^{5}+35a^{3}b^{4}d\,x^{4}+5565a^{2}b^{5}c\,x^{4}-40a^{4}b^{3}d\,x\right.}{45045x\,b^{5}}$
trager	$\frac{2\left(3003b^{7}d\,x^{7}+7161a\,b^{6}d\,x^{6}+3465b^{7}c\,x^{6}+4473a^{2}b^{5}d\,x^{5}+8505a\,b^{6}c\,x^{5}+35a^{3}b^{4}d\,x^{4}+5565a^{2}b^{5}c\,x^{4}-40a^{4}b^{3}d\,x^{3}+75a^{3}b^{4}c\,x\right.}{45045b^{5}x}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(a^2*x*(a*d+5/2*b*c)*arctanh((b*x+a)^(1/2)/a^(1/2))-1/3*(7*x*b*(11/35*d
*x+c)*a^(3/2)+(23/5*d*x-3/2*c)*a^(5/2)+b^2*x^2*a^(1/2)*(3/5*d*x+c))*(b*x+a
)^(1/2))/a^(1/2)/x
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^2}\,dx = \frac{2\,(3003\,b^7dx^7-240\,a^6bc+128\,a^7d+231\,(15\,b^7c+31\,ab^6d)x^6+63\,(135\,ab$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^2,x, algorithm="fricas")
```

output
```
2/45045*(3003*b^7*d*x^7 - 240*a^6*b*c + 128*a^7*d + 231*(15*b^7*c + 31*a*b
^6*d)*x^6 + 63*(135*a*b^6*c + 71*a^2*b^5*d)*x^5 + 35*(159*a^2*b^5*c + a^3*
b^4*d)*x^4 + 5*(15*a^3*b^4*c - 8*a^4*b^3*d)*x^3 - 6*(15*a^4*b^3*c - 8*a^5*
b^2*d)*x^2 + 8*(15*a^5*b^2*c - 8*a^6*b*d)*x)*sqrt(b*x^3 + a*x^2)/(b^5*x)
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^2}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{5}{2}}(c+dx)}{x^2}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/x**2,x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^2}\,dx=\frac{2\left(231\,b^6x^6+567\,ab^5x^5+371\,a^2b^4x^4+5\,a^3b^3x^3-6\,a^4b^2x^2+8\,a^5bx-16\,a\right.}{3003\,b^4} + \frac{2\left(3003\,b^7x^7+7161\,ab^6x^6+4473\,a^2b^5x^5+35\,a^3b^4x^4-40\,a^4b^3x^3+48\,a^5b^2x^2-64\,a^6bx+128\,a^7\right)\sqrt{bx+a}}{45045\,b^5}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^2,x, algorithm="maxima")
```

output
```
2/3003*(231*b^6*x^6 + 567*a*b^5*x^5 + 371*a^2*b^4*x^4 + 5*a^3*b^3*x^3 - 6*
a^4*b^2*x^2 + 8*a^5*b*x - 16*a^6)*sqrt(b*x + a)*c/b^4 + 2/45045*(3003*b^7*
x^7 + 7161*a*b^6*x^6 + 4473*a^2*b^5*x^5 + 35*a^3*b^4*x^4 - 40*a^4*b^3*x^3
+ 48*a^5*b^2*x^2 - 64*a^6*b*x + 128*a^7)*sqrt(b*x + a)*d/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. 2(147) = 294.

Time = 0.14 (sec) , antiderivative size = 650, normalized size of antiderivative = 3.89

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^2}\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^2,x, algorithm="giac")
```

output
```
2/45045*(1287*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/
2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^3*c*sgn(x)/b^3 + 429*(35*(b*x + a)^(9/2)
- 180*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^
3 + 315*sqrt(b*x + a)*a^4)*a^2*c*sgn(x)/b^3 + 143*(35*(b*x + a)^(9/2) - 18
0*(b*x + a)^(7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 +
315*sqrt(b*x + a)*a^4)*a^3*d*sgn(x)/b^4 + 195*(63*(b*x + a)^(11/2) - 385*(
b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 11
55*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a*c*sgn(x)/b^3 + 195*(63*(
b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(
b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a^2
*d*sgn(x)/b^4 + 15*(231*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*
(b*x + a)^(9/2)*a^2 - 8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4
- 6006*(b*x + a)^(3/2)*a^5 + 3003*sqrt(b*x + a)*a^6)*c*sgn(x)/b^3 + 45*(23
1*(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 -
8580*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)
*a^5 + 3003*sqrt(b*x + a)*a^6)*a*d*sgn(x)/b^4 + 7*(429*(b*x + a)^(15/2) -
3465*(b*x + a)^(13/2)*a + 12285*(b*x + a)^(11/2)*a^2 - 25025*(b*x + a)^(9/
2)*a^3 + 32175*(b*x + a)^(7/2)*a^4 - 27027*(b*x + a)^(5/2)*a^5 + 15015*(b*
x + a)^(3/2)*a^6 - 6435*sqrt(b*x + a)*a^7)*d*sgn(x)/b^4)/b + 32/45045*(15*
a^(13/2)*b*c - 8*a^(15/2)*d)*sgn(x)/b^5
```

Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^2}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{256\,a^7\,d-480\,a^6\,b\,c}{45045\,b^5}+\frac{2\,a\,x^5\,\left(71\,a\,d+135\,b\,c\right)}{715}+\frac{2\,b\,x^6\,\left(31\,a\,d+15\,b\,c\right)}{195}+\frac{2\,b^2\,d\,x}{15}\right.}{}$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^2,x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((256*a^7*d - 480*a^6*b*c)/(45045*b^5) + (2*a*x^5*(
71*a*d + 135*b*c))/715 + (2*b*x^6*(31*a*d + 15*b*c))/195 + (2*b^2*d*x^7)/1
5 - (16*a^5*x*(8*a*d - 15*b*c))/(45045*b^4) - (2*a^3*x^3*(8*a*d - 15*b*c))
/(9009*b^2) + (4*a^4*x^2*(8*a*d - 15*b*c))/(15015*b^3) + (2*a^2*x^4*(a*d +
 159*b*c))/(1287*b)))/x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^2}\,dx = \frac{2\sqrt{bx+a}\,(3003b^7d\,x^7+7161a\,b^6d\,x^6+3465b^7c\,x^6+4473a^2b^5d\,x^5+8505a$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^2,x)
```

output
```
(2*sqrt(a + b*x)*(128*a**7*d - 240*a**6*b*c - 64*a**6*b*d*x + 120*a**5*b**
2*c*x + 48*a**5*b**2*d*x**2 - 90*a**4*b**3*c*x**2 - 40*a**4*b**3*d*x**3 +
75*a**3*b**4*c*x**3 + 35*a**3*b**4*d*x**4 + 5565*a**2*b**5*c*x**4 + 4473*a
**2*b**5*d*x**5 + 8505*a*b**6*c*x**5 + 7161*a*b**6*d*x**6 + 3465*b**7*c*x*
*6 + 3003*b**7*d*x**7))/(45045*b**5)
```

3.263 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^3}\,dx=\frac{2a^2(bc-ad)\left(ax^2+bx^3\right)^{7/2}}{7b^4x^7}-\frac{2a(2bc-3ad)\left(ax^2+bx^3\right)^{9/2}}{9b^4x^9}+\frac{2(bc-3ad)\left(ax^2+bx^3\right)^{11/2}}{11b^4x^{11}}+\frac{2d\left(ax^2+bx^3\right)^{13/2}}{13b^4x^{13}}$$

output
```
2/7*a^2*(-a*d+b*c)*(b*x^3+a*x^2)^(7/2)/b^4/x^7-2/9*a*(-3*a*d+2*b*c)*(b*x^3
+a*x^2)^(9/2)/b^4/x^9+2/11*(-3*a*d+b*c)*(b*x^3+a*x^2)^(11/2)/b^4/x^11+2/13
*d*(b*x^3+a*x^2)^(13/2)/b^4/x^13
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^3}\,dx=\frac{2x(a+bx)^4\left(-48a^3d+63b^3x^2(13c+11dx)+8a^2b(13c+21dx)-14ab^2x(\right.}{9009b^4\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^3,x]
```

output
```
(2*x*(a + b*x)^4*(-48*a^3*d + 63*b^3*x^2*(13*c + 11*d*x) + 8*a^2*b*(13*c +
 21*d*x) - 14*a*b^2*x*(26*c + 27*d*x)))/(9009*b^4*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1945, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{x^3}dx$$

$$\Big\downarrow \text{ 1945}$$

$$\frac{(13bc-6ad)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^3}dx}{13b}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{13bx^4}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{(13bc-6ad)\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^4}dx}{11b}\right)}{13b}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{13bx^4}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{(13bc-6ad)\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{9bx^6}-\frac{2a\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^5}dx}{9b}\right)}{11b}\right)}{13b}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{13bx^4}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{11bx^5}-\frac{4a\left(\frac{2\left(ax^2+bx^3\right)^{7/2}}{9bx^6}-\frac{4a\left(ax^2+bx^3\right)^{7/2}}{63b^2x^7}\right)}{11b}\right)(13bc-6ad)}{13b}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{13bx^4}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^3,x]
```

output
```
(2*d*(a*x^2 + b*x^3)^(7/2))/(13*b*x^4) + ((13*b*c - 6*a*d)*((2*(a*x^2 + b*
x^3)^(7/2))/(11*b*x^5) - (4*a*((-4*a*(a*x^2 + b*x^3)^(7/2))/(63*b^2*x^7) +
 (2*(a*x^2 + b*x^3)^(7/2))/(9*b*x^6)))/(11*b)))/(13*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.65

method	result
gosper	$-\frac{2(bx+a)\left(-693b^3d\,x^3+378a\,b^2d\,x^2-819b^3c\,x^2-168a^2bdx+364a\,b^2cx+48a^3d-104c\,a^2b\right)\left(b\,x^3+a\,x^2\right)^{\frac{5}{2}}}{9009b^4x^5}$
default	$-\frac{2(bx+a)\left(-693b^3d\,x^3+378a\,b^2d\,x^2-819b^3c\,x^2-168a^2bdx+364a\,b^2cx+48a^3d-104c\,a^2b\right)\left(b\,x^3+a\,x^2\right)^{\frac{5}{2}}}{9009b^4x^5}$
orering	$-\frac{2(bx+a)\left(-693b^3d\,x^3+378a\,b^2d\,x^2-819b^3c\,x^2-168a^2bdx+364a\,b^2cx+48a^3d-104c\,a^2b\right)\left(b\,x^3+a\,x^2\right)^{\frac{5}{2}}}{9009b^4x^5}$
pseudoelliptic	$-\frac{10x^2b\left(ad+\frac{3bc}{4}\right)a\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{bx+a}\left(-4x^2\left(\frac{dx}{3}+c\right)b^2\sqrt{a}+\left(-\frac{28bd\,x^2}{3}+\left(2ad+\frac{9bc}{2}\right)x+ac\right)a^{\frac{3}{2}}\right)}{2\sqrt{a}\,x^2}$
risch	$-\frac{2\sqrt{x^2(bx+a)}\,\left(-693d\,x^6b^6-1701a\,b^5d\,x^5-819b^6c\,x^5-1113a^2b^4d\,x^4-2093a\,b^5c\,x^4-15a^3b^3d\,x^3-1469a^2b^4c\,x^3+18a^4b^2d\right.}{9009x\,b^4}$
trager	$-\frac{2\left(-693d\,x^6b^6-1701a\,b^5d\,x^5-819b^6c\,x^5-1113a^2b^4d\,x^4-2093a\,b^5c\,x^4-15a^3b^3d\,x^3-1469a^2b^4c\,x^3+18a^4b^2d\,x^2-39a^3b^3\right.}{9009b^4x}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-2/9009*(b*x+a)*(-693*b^3*d*x^3+378*a*b^2*d*x^2-819*b^3*c*x^2-168*a^2*b*d*
x+364*a*b^2*c*x+48*a^3*d-104*a^2*b*c)*(b*x^3+a*x^2)^(5/2)/b^4/x^5
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^3}\,dx = \frac{2\left(693\,b^6dx^6+104\,a^5bc-48\,a^6d+63\,(13\,b^6c+27\,ab^5d)x^5+7\,(299\,ab^5c+\right.}{}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^3,x, algorithm="fricas")
```

output
```
2/9009*(693*b^6*d*x^6 + 104*a^5*b*c - 48*a^6*d + 63*(13*b^6*c + 27*a*b^5*d
)*x^5 + 7*(299*a*b^5*c + 159*a^2*b^4*d)*x^4 + (1469*a^2*b^4*c + 15*a^3*b^3
*d)*x^3 + 3*(13*a^3*b^3*c - 6*a^4*b^2*d)*x^2 - 4*(13*a^4*b^2*c - 6*a^5*b*d
)*x)*sqrt(b*x^3 + a*x^2)/(b^4*x)
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^3}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{5}{2}}(c+dx)}{x^3}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/x**3,x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^3}\,dx=\frac{2\left(63\,b^5x^5+161\,ab^4x^4+113\,a^2b^3x^3+3\,a^3b^2x^2-4\,a^4bx+8\,a^5\right)\sqrt{bx+a}c}{693\,b^3}+\frac{2\left(231\,b^6x^6+567\,ab^5x^5+371\,a^2b^4x^4+5\,a^3b^3x^3-6\,a^4b^2x^2+8\,a^5bx-16\,a^6\right)\sqrt{bx+a}d}{3003\,b^4}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^3,x, algorithm="maxima")
```

output
```
2/693*(63*b^5*x^5 + 161*a*b^4*x^4 + 113*a^2*b^3*x^3 + 3*a^3*b^2*x^2 - 4*a^
4*b*x + 8*a^5)*sqrt(b*x + a)*c/b^3 + 2/3003*(231*b^6*x^6 + 567*a*b^5*x^5 +
 371*a^2*b^4*x^4 + 5*a^3*b^3*x^3 - 6*a^4*b^2*x^2 + 8*a^5*b*x - 16*a^6)*sqr
t(b*x + a)*d/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. 2(115) = 230.

Time = 0.38 (sec) , antiderivative size = 554, normalized size of antiderivative = 4.23

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^3}\,dx=\text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^3,x, algorithm="giac")
```

output
```
2/45045*(3003*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)
*a^2)*a^3*c*sgn(x)/b^2 + 3861*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a +
35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a^2*c*sgn(x)/b^2 + 1287*(5*
(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(
b*x + a)*a^3)*a^3*d*sgn(x)/b^3 + 429*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(
7/2)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x
+ a)*a^4)*a*c*sgn(x)/b^2 + 429*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a
 + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a
^4)*a^2*d*sgn(x)/b^3 + 65*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 9
90*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a
^4 - 693*sqrt(b*x + a)*a^5)*c*sgn(x)/b^2 + 195*(63*(b*x + a)^(11/2) - 385*
(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1
155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x + a)*a^5)*a*d*sgn(x)/b^3 + 15*(231*
(b*x + a)^(13/2) - 1638*(b*x + a)^(11/2)*a + 5005*(b*x + a)^(9/2)*a^2 - 85
80*(b*x + a)^(7/2)*a^3 + 9009*(b*x + a)^(5/2)*a^4 - 6006*(b*x + a)^(3/2)*a
^5 + 3003*sqrt(b*x + a)*a^6)*d*sgn(x)/b^3)/b - 16/9009*(13*a^(11/2)*b*c -
6*a^(13/2)*d)*sgn(x)/b^4
```

Mupad [B] (verification not implemented)

Time = 8.90 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{x^3}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,a\,x^4\,(159\,a\,d+299\,b\,c)}{1287}-\frac{96\,a^6\,d-208\,a^5\,b\,c}{9009\,b^4}+\frac{2\,b^2\,d\,x^6}{13}+\frac{x^5\,(1638\,c\,b^6+3402}{9009\,b^4}\right.}{x}$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^3,x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((2*a*x^4*(159*a*d + 299*b*c))/1287 - (96*a^6*d - 2
08*a^5*b*c)/(9009*b^4) + (2*b^2*d*x^6)/13 + (x^5*(1638*b^6*c + 3402*a*b^5*
d))/(9009*b^4) + (8*a^4*x*(6*a*d - 13*b*c))/(9009*b^3) - (2*a^3*x^2*(6*a*d
 - 13*b*c))/(3003*b^2) + (2*a^2*x^3*(15*a*d + 1469*b*c))/(9009*b))/x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{x^3}\,dx = \frac{2\sqrt{bx+a}\,(693b^6d\,x^6+1701a\,b^5d\,x^5+819b^6c\,x^5+1113a^2b^4d\,x^4+2093a\,b^5$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^3,x)
```

output
```
(2*sqrt(a + b*x)*( - 48*a**6*d + 104*a**5*b*c + 24*a**5*b*d*x - 52*a**4*b*
*2*c*x - 18*a**4*b**2*d*x**2 + 39*a**3*b**3*c*x**2 + 15*a**3*b**3*d*x**3 +
 1469*a**2*b**4*c*x**3 + 1113*a**2*b**4*d*x**4 + 2093*a*b**5*c*x**4 + 1701
*a*b**5*d*x**5 + 819*b**6*c*x**5 + 693*b**6*d*x**6))/(9009*b**4)
```

3.264 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^4}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 94

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^4}\,dx = -\frac{2a(bc-ad)\left(ax^2+bx^3\right)^{7/2}}{7b^3x^7} + \frac{2(bc-2ad)\left(ax^2+bx^3\right)^{9/2}}{9b^3x^9} + \frac{2d\left(ax^2+bx^3\right)^{11/2}}{11b^3x^{11}}$$

output
```
-2/7*a*(-a*d+b*c)*(b*x^3+a*x^2)^(7/2)/b^3/x^7+2/9*(-2*a*d+b*c)*(b*x^3+a*x^
2)^(9/2)/b^3/x^9+2/11*d*(b*x^3+a*x^2)^(11/2)/b^3/x^11
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^4}\,dx = \frac{2x(a+bx)^4\left(8a^2d+7b^2x(11c+9dx)-2ab(11c+14dx)\right)}{693b^3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^4,x]
```

output
```
(2*x*(a + b*x)^4*(8*a^2*d + 7*b^2*x*(11*c + 9*d*x) - 2*a*b*(11*c + 14*d*x)
))/(693*b^3*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1945, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{5/2}(c+dx)}{x^4}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(11bc-4ad)\int \frac{(bx^3+ax^2)^{5/2}}{x^4}dx}{11b}+\frac{2d(ax^2+bx^3)^{7/2}}{11bx^5}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(11bc-4ad)\left(\frac{2(ax^2+bx^3)^{7/2}}{9bx^6}-\frac{2a\int \frac{(bx^3+ax^2)^{5/2}}{x^5}dx}{9b}\right)}{11b}+\frac{2d(ax^2+bx^3)^{7/2}}{11bx^5}$$

$$\Big\downarrow \text{1920}$$

$$\frac{\left(\frac{2(ax^2+bx^3)^{7/2}}{9bx^6}-\frac{4a(ax^2+bx^3)^{7/2}}{63b^2x^7}\right)(11bc-4ad)}{11b}+\frac{2d(ax^2+bx^3)^{7/2}}{11bx^5}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^4,x]
```

output
```
(2*d*(a*x^2 + b*x^3)^(7/2))/(11*b*x^5) + ((11*b*c - 4*a*d)*((-4*a*(a*x^2 +
 b*x^3)^(7/2))/(63*b^2*x^7) + (2*(a*x^2 + b*x^3)^(7/2))/(9*b*x^6)))/(11*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.65

method	result
gosper	$\frac{2(bx+a)\left(63b^2d\,x^2-28abdx+77b^2cx+8a^2d-22abc\right)\left(b\,x^3+a\,x^2\right)^{\frac{5}{2}}}{693b^3x^5}$
default	$\frac{2(bx+a)\left(63b^2d\,x^2-28abdx+77b^2cx+8a^2d-22abc\right)\left(b\,x^3+a\,x^2\right)^{\frac{5}{2}}}{693b^3x^5}$
orering	$\frac{2(bx+a)\left(63b^2d\,x^2-28abdx+77b^2cx+8a^2d-22abc\right)\left(b\,x^3+a\,x^2\right)^{\frac{5}{2}}}{693b^3x^5}$
pseudoelliptic	$-\frac{15\left(b^2x^3\left(ad+\frac{bc}{6}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{11\left(\frac{26xb\left(\frac{27dx}{13}+c\right)a^{\frac{3}{2}}}{33}+\frac{4\left(dx+\frac{2c}{3}\right)a^{\frac{5}{2}}}{11}+b^2x^2\sqrt{a}\left(-\frac{16dx}{11}+c\right)\right)\sqrt{bx+a}}{30}\right)}{4\sqrt{a}\,x^3}$
risch	$\frac{2\sqrt{x^2(bx+a)}\,\left(63d\,x^5b^5+161a\,b^4d\,x^4+77b^5c\,x^4+113a^2b^3d\,x^3+209a\,b^4c\,x^3+3a^3b^2d\,x^2+165a^2b^3c\,x^2-4a^4bdx+11a^3b^2cx+\right.}{693x\,b^3}$
trager	$\frac{2\left(63d\,x^5b^5+161a\,b^4d\,x^4+77b^5c\,x^4+113a^2b^3d\,x^3+209a\,b^4c\,x^3+3a^3b^2d\,x^2+165a^2b^3c\,x^2-4a^4bdx+11a^3b^2cx+8a^5d-22a^4b\right.}{693b^3x}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^4,x,method=_RETURNVERBOSE)
```

output
```
2/693*(b*x+a)*(63*b^2*d*x^2-28*a*b*d*x+77*b^2*c*x+8*a^2*d-22*a*b*c)*(b*x^3
+a*x^2)^(5/2)/b^3/x^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.34

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^4}\,dx = \frac{2\left(63\,b^5dx^5-22\,a^4bc+8\,a^5d+7\,(11\,b^5c+23\,ab^4d)x^4+(209\,ab^4c+113\,a^2\right.}{693\,b^3x}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^4,x, algorithm="fricas")
```

output
```
2/693*(63*b^5*d*x^5 - 22*a^4*b*c + 8*a^5*d + 7*(11*b^5*c + 23*a*b^4*d)*x^4
 + (209*a*b^4*c + 113*a^2*b^3*d)*x^3 + 3*(55*a^2*b^3*c + a^3*b^2*d)*x^2 +
(11*a^3*b^2*c - 4*a^4*b*d)*x)*sqrt(b*x^3 + a*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{x^4}\,dx = \int \frac{(x^2(a+bx))^{\frac{5}{2}}\,(c+dx)}{x^4}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/x**4,x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{x^4}\,dx = \frac{2\,(7\,b^4x^4+19\,ab^3x^3+15\,a^2b^2x^2+a^3bx-2\,a^4)\sqrt{bx+a}c}{63\,b^2} + \frac{2\,(63\,b^5x^5+161\,ab^4x^4+113\,a^2b^3x^3+3\,a^3b^2x^2-4\,a^4bx+8\,a^5)\sqrt{bx+a}d}{693\,b^3}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^4,x, algorithm="maxima")
```

output
```
2/63*(7*b^4*x^4 + 19*a*b^3*x^3 + 15*a^2*b^2*x^2 + a^3*b*x - 2*a^4)*sqrt(b*
x + a)*c/b^2 + 2/693*(63*b^5*x^5 + 161*a*b^4*x^4 + 113*a^2*b^3*x^3 + 3*a^3
*b^2*x^2 - 4*a^4*b*x + 8*a^5)*sqrt(b*x + a)*d/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(82) = 164.

Time = 0.17 (sec) , antiderivative size = 456, normalized size of antiderivative = 4.85

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{x^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^4,x, algorithm="giac")
```

output
```
2/3465*(1155*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^3*c*sgn(x)/b + 693*(3
*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*a^2*c*sgn(
x)/b + 231*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^
2)*a^3*d*sgn(x)/b^2 + 297*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(
b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*a*c*sgn(x)/b + 297*(5*(b*x + a)
^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*
a^3)*a^2*d*sgn(x)/b^2 + 11*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 3
78*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*
c*sgn(x)/b + 33*(35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x + a
)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*a*d*sgn(x)/
b^2 + 5*(63*(b*x + a)^(11/2) - 385*(b*x + a)^(9/2)*a + 990*(b*x + a)^(7/2)
*a^2 - 1386*(b*x + a)^(5/2)*a^3 + 1155*(b*x + a)^(3/2)*a^4 - 693*sqrt(b*x
+ a)*a^5)*d*sgn(x)/b^2)/b + 4/693*(11*a^(9/2)*b*c - 4*a^(11/2)*d)*sgn(x)/b
^3
```

Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^4}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{16\,a^5\,d-44\,a^4\,b\,c}{693\,b^3}+\frac{2\,a\,x^3\,(113\,a\,d+209\,b\,c)}{693}+\frac{2\,b^2\,d\,x^5}{11}+\frac{x^4\,(154\,c\,b^5+322\,a\,d}{693\,b^3}\right.}{x}$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^4,x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((16*a^5*d - 44*a^4*b*c)/(693*b^3) + (2*a*x^3*(113*
a*d + 209*b*c))/693 + (2*b^2*d*x^5)/11 + (x^4*(154*b^5*c + 322*a*b^4*d))/(
693*b^3) - (2*a^3*x*(4*a*d - 11*b*c))/(693*b^2) + (2*a^2*x^2*(a*d + 55*b*c
))/(231*b)))/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^4}\,dx = \frac{2\sqrt{bx+a}\,(63b^5d\,x^5+161a\,b^4d\,x^4+77b^5c\,x^4+113a^2b^3d\,x^3+209a\,b^4c\,x^3+}{693b^3}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^4,x)
```

output
```
(2*sqrt(a + b*x)*(8*a**5*d - 22*a**4*b*c - 4*a**4*b*d*x + 11*a**3*b**2*c*x
 + 3*a**3*b**2*d*x**2 + 165*a**2*b**3*c*x**2 + 113*a**2*b**3*d*x**3 + 209*
a*b**4*c*x**3 + 161*a*b**4*d*x**4 + 77*b**5*c*x**4 + 63*b**5*d*x**5))/(693
*b**3)
```

3.265 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^5} dx$

Optimal result

Integrand size = 24, antiderivative size = 60

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^5} dx = \frac{2(bc-ad)\left(ax^2+bx^3\right)^{7/2}}{7b^2x^7} + \frac{2d\left(ax^2+bx^3\right)^{9/2}}{9b^2x^9}$$

output
```
2/7*(-a*d+b*c)*(b*x^3+a*x^2)^(7/2)/b^2/x^7+2/9*d*(b*x^3+a*x^2)^(9/2)/b^2/x
^9
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^5} dx = \frac{2x(a+bx)^4(9bc-2ad+7bdx)}{63b^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^5,x]
```

output
```
(2*x*(a + b*x)^4*(9*b*c - 2*a*d + 7*b*d*x))/(63*b^2*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {1945, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{5/2}(c+dx)}{x^5}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(9bc-2ad)\int \frac{(bx^3+ax^2)^{5/2}}{x^5}dx}{9b}+\frac{2d(ax^2+bx^3)^{7/2}}{9bx^6}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2(ax^2+bx^3)^{7/2}(9bc-2ad)}{63b^2x^7}+\frac{2d(ax^2+bx^3)^{7/2}}{9bx^6}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^5,x]
```

output
```
(2*(9*b*c - 2*a*d)*(a*x^2 + b*x^3)^(7/2))/(63*b^2*x^7) + (2*d*(a*x^2 + b*x
^3)^(7/2))/(9*b*x^6)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

method	result
gosper	$-\frac{2(bx+a)(-7bdx+2ad-9bc)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{63b^{2}x^{5}}$
default	$-\frac{2(bx+a)(-7bdx+2ad-9bc)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{63b^{2}x^{5}}$
orering	$-\frac{2(bx+a)(-7bdx+2ad-9bc)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{63b^{2}x^{5}}$
pseudoelliptic	$-\frac{5\left(b^{3}x^{4}\left(ad-\frac{bc}{8}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{17\sqrt{bx+a}\left(\frac{59x^{2}\left(\frac{132dx}{59}+c\right)b^{2}a^{\frac{3}{2}}}{68}+bx\left(\frac{26dx}{17}+c\right)a^{\frac{5}{2}}+\frac{2(4dx+3c)a^{\frac{7}{2}}}{17}+\frac{15\sqrt{a}\,b^{3}c\,x^{3}}{136}\right)}{15}\right)}{8a^{\frac{3}{2}}x^{4}}$
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-7d\,x^{4}b^{4}-19a\,b^{3}d\,x^{3}-9b^{4}c\,x^{3}-15a^{2}b^{2}d\,x^{2}-27a\,b^{3}c\,x^{2}-a^{3}bdx-27a^{2}b^{2}cx+2a^{4}d-9a^{3}bc\right)}{63x\,b^{2}}$
trager	$-\frac{2\left(-7d\,x^{4}b^{4}-19a\,b^{3}d\,x^{3}-9b^{4}c\,x^{3}-15a^{2}b^{2}d\,x^{2}-27a\,b^{3}c\,x^{2}-a^{3}bdx-27a^{2}b^{2}cx+2a^{4}d-9a^{3}bc\right)\sqrt{b\,x^{3}+a\,x^{2}}}{63b^{2}x}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-2/63*(b*x+a)*(-7*b*d*x+2*a*d-9*b*c)*(b*x^3+a*x^2)^(5/2)/b^2/x^5
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.70

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{x^5}\,dx = \frac{2\,(7\,b^4dx^4+9\,a^3bc-2\,a^4d+(9\,b^4c+19\,ab^3d)x^3+3\,(9\,ab^3c+5\,a^2b^2d)x^2+}{63\,b^2x}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^5,x, algorithm="fricas")
```

output
```
2/63*(7*b^4*d*x^4 + 9*a^3*b*c - 2*a^4*d + (9*b^4*c + 19*a*b^3*d)*x^3 + 3*(
9*a*b^3*c + 5*a^2*b^2*d)*x^2 + (27*a^2*b^2*c + a^3*b*d)*x)*sqrt(b*x^3 + a*
x^2)/(b^2*x)
```

Sympy [F]

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{x^5}\,dx = \int \frac{(x^2(a+bx))^{\frac{5}{2}}\,(c+dx)}{x^5}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/x**5,x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.57

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{x^5}\,dx = \frac{2\,(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)\sqrt{bx+a}c}{7\,b} + \frac{2\,(7\,b^4x^4+19\,ab^3x^3+15\,a^2b^2x^2+a^3bx-2\,a^4)\sqrt{bx+a}d}{63\,b^2}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^5,x, algorithm="maxima")
```

output
```
2/7*(b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*sqrt(b*x + a)*c/b + 2/63*(7*
b^4*x^4 + 19*a*b^3*x^3 + 15*a^2*b^2*x^2 + a^3*b*x - 2*a^4)*sqrt(b*x + a)*d
/b^2
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(52) = 104$.

Time = 0.12 (sec) , antiderivative size = 344, normalized size of antiderivative = 5.73

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^5}\,dx = \frac{2\left(315\sqrt{bx+a}a^3c\mathrm{sgn}(x)+315\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)a^2c\mathrm{sgn}(x)+\frac{10}{\ }\right.}{}$$

$$-\frac{2\left(9\,a^{\frac{7}{2}}bc-2\,a^{\frac{9}{2}}d\right)\mathrm{sgn}(x)}{63\,b^2}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^5,x, algorithm="giac")
```

output
```
2/315*(315*sqrt(b*x + a)*a^3*c*sgn(x) + 315*((b*x + a)^(3/2) - 3*sqrt(b*x
+ a)*a)*a^2*c*sgn(x) + 105*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*a^3*d*sgn
(x)/b + 63*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^
2)*a*c*sgn(x) + 63*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x
 + a)*a^2)*a^2*d*sgn(x)/b + 9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a +
35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a^3)*c*sgn(x) + 27*(5*(b*x + a)^
(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^2 - 35*sqrt(b*x + a)*a
^3)*a*d*sgn(x)/b + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2)*a + 378*(b*x
+ a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a)*a^4)*d*sgn(x)
/b)/b - 2/63*(9*a^(7/2)*b*c - 2*a^(9/2)*d)*sgn(x)/b^2
```

Mupad [B] (verification not implemented)

Time = 9.56 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.05

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^5}\,dx = \frac{6\,a^2\,c\,\sqrt{b\,x^3+a\,x^2}}{7} + \frac{2\,b^2\,c\,x^2\,\sqrt{b\,x^3+a\,x^2}}{7} + \frac{2\,a^3\,c\,\sqrt{b\,x^3+a\,x^2}}{7\,b\,x} + \frac{6\,a\,b\,c\,x\,\sqrt{b\,x^3+a\,x^2}}{7} - \frac{2\,d\,(2\,a-7\,b\,x)\,\sqrt{b\,x^3+a\,x^2}\,(a+b\,x)^3}{63\,b^2\,x}$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^5,x)
```

output
```
(6*a^2*c*(a*x^2 + b*x^3)^(1/2))/7 + (2*b^2*c*x^2*(a*x^2 + b*x^3)^(1/2))/7
+ (2*a^3*c*(a*x^2 + b*x^3)^(1/2))/(7*b*x) + (6*a*b*c*x*(a*x^2 + b*x^3)^(1/
2))/7 - (2*d*(2*a - 7*b*x)*(a*x^2 + b*x^3)^(1/2)*(a + b*x)^3)/(63*b^2*x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.53

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^5}\,dx = \frac{2\sqrt{bx+a}\,(7b^4d\,x^4+19a\,b^3d\,x^3+9b^4c\,x^3+15a^2b^2d\,x^2+27a\,b^3c\,x^2+a^3bd}{63b^2}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^5,x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a**4*d + 9*a**3*b*c + a**3*b*d*x + 27*a**2*b**2*c*x
 + 15*a**2*b**2*d*x**2 + 27*a*b**3*c*x**2 + 19*a*b**3*d*x**3 + 9*b**4*c*x*
*3 + 7*b**4*d*x**4))/(63*b**2)
```

3.266 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^6}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 131

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^6}\,dx = \frac{2a^2c\sqrt{ax^2+bx^3}}{x}+\frac{2ac(ax^2+bx^3)^{3/2}}{3x^3}+\frac{2c(ax^2+bx^3)^{5/2}}{5x^5}+\frac{2d(ax^2+bx^3)^{7/2}}{7bx^7}-2a^{5/2}c\,\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)$$

output
```
2*a^2*c*(b*x^3+a*x^2)^(1/2)/x+2/3*a*c*(b*x^3+a*x^2)^(3/2)/x^3+2/5*c*(b*x^3
+a*x^2)^(5/2)/x^5+2/7*d*(b*x^3+a*x^2)^(7/2)/b/x^7-2*a^(5/2)*c*arctanh((b*x
^3+a*x^2)^(1/2)/a^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^6}\,dx = \frac{2x\Big((a+bx)\left(15a^3d+3b^3x^2(7c+5dx)+ab^2x(77c+45dx)+a^2b(161c+4\right.}{105b\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^6,x]
```

output
```
(2*x*((a + b*x)*(15*a^3*d + 3*b^3*x^2*(7*c + 5*d*x) + a*b^2*x*(77*c + 45*d
*x) + a^2*b*(161*c + 45*d*x)) - 105*a^(5/2)*b*c*Sqrt[a + b*x]*ArcTanh[Sqrt
[a + b*x]/Sqrt[a]]))/(105*b*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1945, 1927, 1927, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{x^6}dx$$

$$\Big\downarrow \text{1945}$$

$$c\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^6}dx+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{7bx^7}$$

$$\Big\downarrow \text{1927}$$

$$c\left(a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^4}dx+\frac{2\left(ax^2+bx^3\right)^{5/2}}{5x^5}\right)+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{7bx^7}$$

$$\Big\downarrow \text{1927}$$

$$c\left(a\left(a\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)+\frac{2\left(ax^2+bx^3\right)^{5/2}}{5x^5}\right)+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{7bx^7}$$

$$\Big\downarrow \text{1927}$$

$$c\left(a\left(a\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)+\frac{2\left(ax^2+bx^3\right)^{5/2}}{5x^5}\right)+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{7bx^7}$$

$$\Big\downarrow \text{1914}$$

$$c\left(a\left(a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)+\frac{2\left(ax^2+bx^3\right)^{5/2}}{5x^5}\right)+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{7bx^7}$$

$\downarrow$ 219

$$c\left(a\left(a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)+\frac{2\left(ax^2+bx^3\right)^{5/2}}{5x^5}\right)+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{7bx^7}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^6,x]
```

output
```
(2*d*(a*x^2 + b*x^3)^(7/2))/(7*b*x^7) + c*((2*(a*x^2 + b*x^3)^(5/2))/(5*x^
5) + a*((2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) + a*((2*Sqrt[a*x^2 + b*x^3])/x -
 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.73

method	result
default	$\frac{2\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}\left(15d(bx+a)^{\frac{7}{2}}-105a^{\frac{5}{2}}bc\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+21(bx+a)^{\frac{5}{2}}bc+35(bx+a)^{\frac{3}{2}}abc+105\sqrt{bx+a}\,a^{2}bc\right)}{105x^{5}(bx+a)^{\frac{5}{2}}b}$
pseudoelliptic	$\frac{\frac{5b^{4}x^{5}\left(ad-\frac{3bc}{10}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{64}-\frac{31\left(\frac{5b^{3}x^{3}(5dx+c)a^{\frac{3}{2}}}{124}+b^{2}x^{2}\left(\frac{295dx}{186}+c\right)a^{\frac{5}{2}}+\frac{42x\left(\frac{85dx}{63}+c\right)b\,a^{\frac{7}{2}}}{31}+\frac{4(5dx+4c)a^{\frac{9}{2}}}{31}-\frac{15\sqrt{a}\,b^{4}c\,x^{4}}{248}\right.}{80}}{a^{\frac{5}{2}}x^{5}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^6,x,method=_RETURNVERBOSE)
```

output
```
2/105*(b*x^3+a*x^2)^(5/2)*(15*d*(b*x+a)^(7/2)-105*a^(5/2)*b*c*arctanh((b*x
+a)^(1/2)/a^(1/2))+21*(b*x+a)^(5/2)*b*c+35*(b*x+a)^(3/2)*a*b*c+105*(b*x+a)
^(1/2)*a^2*b*c)/x^5/(b*x+a)^(5/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.92

$$\int \frac{(c+dx)\left(ax^{2}+bx^{3}\right)^{5/2}}{x^{6}}\,dx = \left[\frac{105\,a^{\frac{5}{2}}bcx\log\left(\frac{bx^{2}+2\,ax-2\,\sqrt{bx^{3}+ax^{2}}\sqrt{a}}{x^{2}}\right)+2\left(15\,b^{3}dx^{3}+161\,a^{2}bc+15\,a^{3}d+\right.}{105\,bx}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^6,x, algorithm="fricas")
```

output
```
[1/105*(105*a^(5/2)*b*c*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(
a))/x^2) + 2*(15*b^3*d*x^3 + 161*a^2*b*c + 15*a^3*d + 3*(7*b^3*c + 15*a*b^
2*d)*x^2 + (77*a*b^2*c + 45*a^2*b*d)*x)*sqrt(b*x^3 + a*x^2))/(b*x), 2/105*
(105*sqrt(-a)*a^2*b*c*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x))
 + (15*b^3*d*x^3 + 161*a^2*b*c + 15*a^3*d + 3*(7*b^3*c + 15*a*b^2*d)*x^2 +
 (77*a*b^2*c + 45*a^2*b*d)*x)*sqrt(b*x^3 + a*x^2))/(b*x)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^6}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{5}{2}}(c+dx)}{x^6}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/x**6,x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x)/x**6, x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^6}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{x^6}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^6,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/x^6, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^6}\,dx = \frac{2\,a^3c\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)\operatorname{sgn}(x)}{\sqrt{-a}} - \frac{2\left(105\,a^3bc\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+161\,\sqrt{-a}a^{\frac{5}{2}}bc+15\,\sqrt{-a}a^{\frac{7}{2}}d\right)\operatorname{sgn}(x)}{105\,\sqrt{-a}b} + \frac{2\left(21\,(bx+a)^{\frac{5}{2}}b^7c\operatorname{sgn}(x)+35\,(bx+a)^{\frac{3}{2}}ab^7c\operatorname{sgn}(x)+105\,\sqrt{bx+a}a^2b^7c\operatorname{sgn}(x)+15\,(bx+a)^{\frac{7}{2}}b^6d\operatorname{sgn}(x)\right)}{105\,b^7}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^6,x, algorithm="giac")
```

output
```
2*a^3*c*arctan(sqrt(b*x + a)/sqrt(-a))*sgn(x)/sqrt(-a) - 2/105*(105*a^3*b*
c*arctan(sqrt(a)/sqrt(-a)) + 161*sqrt(-a)*a^(5/2)*b*c + 15*sqrt(-a)*a^(7/2
)*d)*sgn(x)/(sqrt(-a)*b) + 2/105*(21*(b*x + a)^(5/2)*b^7*c*sgn(x) + 35*(b*
x + a)^(3/2)*a*b^7*c*sgn(x) + 105*sqrt(b*x + a)*a^2*b^7*c*sgn(x) + 15*(b*x
 + a)^(7/2)*b^6*d*sgn(x))/b^7
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^6}\,dx = \int \frac{{\left(b\,x^3+a\,x^2\right)}^{5/2}\,\left(c+d\,x\right)}{x^6}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^6,x)
```

output
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^6, x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.11

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^6}\,dx = \frac{30\sqrt{bx+a}\,a^3d+322\sqrt{bx+a}\,a^2bc+90\sqrt{bx+a}\,a^2bdx+154\sqrt{bx+a}\,a\,b^2c$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^6,x)
```

output
```
(30*sqrt(a + b*x)*a**3*d + 322*sqrt(a + b*x)*a**2*b*c + 90*sqrt(a + b*x)*a
**2*b*d*x + 154*sqrt(a + b*x)*a*b**2*c*x + 90*sqrt(a + b*x)*a*b**2*d*x**2
+ 42*sqrt(a + b*x)*b**3*c*x**2 + 30*sqrt(a + b*x)*b**3*d*x**3 + 105*sqrt(a
)*log(sqrt(a + b*x) - sqrt(a))*a**2*b*c - 105*sqrt(a)*log(sqrt(a + b*x) +
sqrt(a))*a**2*b*c)/(105*b)
```

3.267 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^7}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 146

$$\begin{aligned}\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^7}\,dx &= \frac{a(5bc+2ad)\sqrt{ax^2+bx^3}}{x}\\ &+ \frac{(5bc+2ad)\left(ax^2+bx^3\right)^{3/2}}{3x^3} - \frac{c\left(ax^2+bx^3\right)^{5/2}}{x^6}\\ &+ \frac{2d\left(ax^2+bx^3\right)^{5/2}}{5x^5} - a^{3/2}(5bc+2ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)\end{aligned}$$

output
```
a*(2*a*d+5*b*c)*(b*x^3+a*x^2)^(1/2)/x+1/3*(2*a*d+5*b*c)*(b*x^3+a*x^2)^(3/2
)/x^3-c*(b*x^3+a*x^2)^(5/2)/x^6+2/5*d*(b*x^3+a*x^2)^(5/2)/x^5-a^(3/2)*(2*a
*d+5*b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.74

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^7}\,dx = \frac{(a+bx)\left(2b^2x^2(5c+3dx)+2abx(35c+11dx)+a^2(-15c+46dx)\right)-15a}{15\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^7,x]
```

output
```
((a + b*x)*(2*b^2*x^2*(5*c + 3*d*x) + 2*a*b*x*(35*c + 11*d*x) + a^2*(-15*c
 + 46*d*x)) - 15*a^(3/2)*(5*b*c + 2*a*d)*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a +
b*x]/Sqrt[a]])/(15*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1944, 1927, 1927, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{x^7}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(2ad+5bc)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^6}dx}{2a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{ax^8}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(2ad+5bc)\left(a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^4}dx+\frac{2\left(ax^2+bx^3\right)^{5/2}}{5x^5}\right)}{2a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{ax^8}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(2ad+5bc)\left(a\left(a\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}\right)+\frac{2(ax^2+bx^3)^{5/2}}{5x^5}\right)}{2a}-\frac{c(ax^2+bx^3)^{7/2}}{ax^8}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(2ad+5bc)\left(a\left(a\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}\right)+\frac{2(ax^2+bx^3)^{5/2}}{5x^5}\right)}{2a}-\frac{c(ax^2+bx^3)^{7/2}}{ax^8}$$

$$\Big\downarrow \text{1914}$$

$$\frac{(2ad+5bc)\left(a\left(a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}\right)+\frac{2(ax^2+bx^3)^{5/2}}{5x^5}\right)}{2a}-\frac{c(ax^2+bx^3)^{7/2}}{ax^8}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(a\left(a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}\right)+\frac{2(ax^2+bx^3)^{5/2}}{5x^5}\right)(2ad+5bc)}{2a}-\frac{c(ax^2+bx^3)^{7/2}}{ax^8}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^7,x]
```

output
```
-((c*(a*x^2 + b*x^3)^(7/2))/(a*x^8)) + ((5*b*c + 2*a*d)*((2*(a*x^2 + b*x^3
)^(5/2))/(5*x^5) + a*((2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) + a*((2*Sqrt[a*x^2
 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]))))/(2*a
)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.88

method	result
risch	$-\frac{a^{2}c\sqrt{x^{2}(bx+a)}}{x^{2}}+\frac{\left(\frac{2d(bx+a)^{\frac{5}{2}}}{5}+\frac{2(bx+a)^{\frac{3}{2}}ad}{3}+\frac{2(bx+a)^{\frac{3}{2}}bc}{3}+2\sqrt{bx+a}\,a^{2}d+4\sqrt{bx+a}\,abc-(2ad+5bc)a^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right.\right.}{x\sqrt{bx+a}}$
pseudoelliptic	$-\frac{3\left(b^{5}x^{6}\left(ad-\frac{5bc}{12}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{2\left(\left(\frac{192dx}{5}+32c\right)a^{\frac{11}{2}}+xb\left(-\frac{5\left(\frac{18dx}{5}+c\right)x^{3}b^{3}a^{\frac{3}{2}}}{4}+b^{2}x^{2}(3dx+c)a^{\frac{5}{2}}+54xb\left(\frac{62dx}{45}+c\right)a^{\frac{7}{2}}\right.\right.}{9}\right.}{128a^{\frac{7}{2}}x^{6}}$
default	$\frac{(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}\left(6d(bx+a)^{\frac{5}{2}}x\sqrt{a}+10(bx+a)^{\frac{3}{2}}a^{\frac{3}{2}}dx+10(bx+a)^{\frac{3}{2}}bcx\sqrt{a}+30a^{\frac{5}{2}}dx\sqrt{bx+a}+60a^{\frac{3}{2}}bcx\sqrt{bx+a}-30\operatorname{arctanh}\left(\right.\right.}{15x^{6}(bx+a)^{\frac{5}{2}}\sqrt{a}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^7,x,method=_RETURNVERBOSE)
```

output
```
-a^2*c/x^2*(x^2*(b*x+a))^(1/2)+(2/5*d*(b*x+a)^(5/2)+2/3*(b*x+a)^(3/2)*a*d+
2/3*(b*x+a)^(3/2)*b*c+2*(b*x+a)^(1/2)*a^2*d+4*(b*x+a)^(1/2)*a*b*c-(2*a*d+5
*b*c)*a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2)))*(x^2*(b*x+a))^(1/2)/x/(b*x+a
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx)\left(ax^{2}+bx^{3}\right)^{5/2}}{x^{7}}\,dx=\left[\frac{15\left(5\,abc+2\,a^{2}d\right)\sqrt{a}x^{2}\log\left(\frac{bx^{2}+2\,ax-2\,\sqrt{bx^{3}+ax^{2}}\sqrt{a}}{x^{2}}\right)+2\left(6\,b^{2}dx^{3}-15\,a^{2}c\right.}{30\,x^{2}}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^7,x, algorithm="fricas")
```

output
```
[1/30*(15*(5*a*b*c + 2*a^2*d)*sqrt(a)*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^
3 + a*x^2)*sqrt(a))/x^2) + 2*(6*b^2*d*x^3 - 15*a^2*c + 2*(5*b^2*c + 11*a*b
*d)*x^2 + 2*(35*a*b*c + 23*a^2*d)*x)*sqrt(b*x^3 + a*x^2))/x^2, 1/15*(15*(5
*a*b*c + 2*a^2*d)*sqrt(-a)*x^2*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2
+ a*x)) + (6*b^2*d*x^3 - 15*a^2*c + 2*(5*b^2*c + 11*a*b*d)*x^2 + 2*(35*a*b
*c + 23*a^2*d)*x)*sqrt(b*x^3 + a*x^2))/x^2]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^7}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{5}{2}}(c+dx)}{x^7}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/x**7,x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x)/x**7, x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^7}\,dx=\int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{x^7}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^7,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/x^7, x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.05

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^7}\,dx =$$
$$-\frac{1}{15}\left(\frac{15\sqrt{bx+a}a^2c\mathrm{sgn}(x)}{bx} - \frac{15\left(5a^2bc\mathrm{sgn}(x)+2a^3d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}b} - \frac{2\left(5(bx+a)^{\frac{3}{2}}b^5c\mathrm{sgn}(x\right.}{}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^7,x, algorithm="giac")
```

output
```
-1/15*(15*sqrt(b*x + a)*a^2*c*sgn(x)/(b*x) - 15*(5*a^2*b*c*sgn(x) + 2*a^3*
d*sgn(x))*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*b) - 2*(5*(b*x + a)^(3/
2)*b^5*c*sgn(x) + 30*sqrt(b*x + a)*a*b^5*c*sgn(x) + 3*(b*x + a)^(5/2)*b^4*
d*sgn(x) + 5*(b*x + a)^(3/2)*a*b^4*d*sgn(x) + 15*sqrt(b*x + a)*a^2*b^4*d*s
gn(x))/b^5)*b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^7}\,dx = \int \frac{\left(b\,x^3+a\,x^2\right)^{5/2}\,\left(c+d\,x\right)}{x^7}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^7,x)
```

output
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^7, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.13

$$\int \frac{(c+dx)(ax^2+bx^3)^{5/2}}{x^7}\,dx = \frac{-30\sqrt{bx+a}\,a^2c+92\sqrt{bx+a}\,a^2dx+140\sqrt{bx+a}\,abcx+44\sqrt{bx+a}\,abd\,x}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^7,x)
```

output
```
( - 30*sqrt(a + b*x)*a**2*c + 92*sqrt(a + b*x)*a**2*d*x + 140*sqrt(a + b*x
)*a*b*c*x + 44*sqrt(a + b*x)*a*b*d*x**2 + 20*sqrt(a + b*x)*b**2*c*x**2 + 1
2*sqrt(a + b*x)*b**2*d*x**3 + 30*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a**2
*d*x + 75*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b*c*x - 30*sqrt(a)*log(sq
rt(a + b*x) + sqrt(a))*a**2*d*x - 75*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*
a*b*c*x)/(30*x)
```

3.268 $\int \frac{(c+dx)(ax^2+bx^3)^{5/2}}{x^8} dx$

Optimal result

Integrand size = 24, antiderivative size = 155

$$\int \frac{(c+dx)(ax^2+bx^3)^{5/2}}{x^8} dx = \frac{5b(3bc+4ad)\sqrt{ax^2+bx^3}}{4x} - \frac{(5bc+4ad)(ax^2+bx^3)^{3/2}}{4x^4} + \frac{2bd(ax^2+bx^3)^{3/2}}{3x^3} - \frac{c(ax^2+bx^3)^{5/2}}{2x^7} - \frac{5}{4}\sqrt{a}b(3bc+4ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)$$

output
```
5/4*b*(4*a*d+3*b*c)*(b*x^3+a*x^2)^(1/2)/x-1/4*(4*a*d+5*b*c)*(b*x^3+a*x^2)^
(3/2)/x^4+2/3*b*d*(b*x^3+a*x^2)^(3/2)/x^3-1/2*c*(b*x^3+a*x^2)^(5/2)/x^7-5/
4*a^(1/2)*b*(4*a*d+3*b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.74

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^8}\,dx=\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a+bx}(8b^2x^2(3c+dx)-6a^2(c+2dx)+abx(-27c+56d\right.}{12x^3\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^8,x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(Sqrt[a + b*x]*(8*b^2*x^2*(3*c + d*x) - 6*a^2*(c + 2*
d*x) + a*b*x*(-27*c + 56*d*x)) - 15*Sqrt[a]*b*(3*b*c + 4*a*d)*x^2*ArcTanh[
Sqrt[a + b*x]/Sqrt[a]]))/(12*x^3*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1944, 1926, 1927, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{x^8}\,dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(4ad+3bc)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^7}dx}{4a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{2ax^9}$$

$$\Big\downarrow \text{1926}$$

$$\frac{(4ad+3bc)\left(\frac{5}{2}b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^4}dx-\frac{\left(ax^2+bx^3\right)^{5/2}}{x^6}\right)}{4a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{2ax^9}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(4ad+3bc)\left(\frac{5}{2}b\left(a\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx+\frac{2\left(ax^2+bx^3\right)^{3/2}}{3x^3}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{x^6}\right)}{4a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{2ax^9}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(4ad+3bc)\left(\frac{5}{2}b\left(a\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}\right)-\frac{(ax^2+bx^3)^{5/2}}{x^6}\right)}{4a}-\frac{c(ax^2+bx^3)^{7/2}}{2ax^9}$$

$$\Big\downarrow \text{1914}$$

$$\frac{(4ad+3bc)\left(\frac{5}{2}b\left(a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}\right)-\frac{(ax^2+bx^3)^{5/2}}{x^6}\right)}{4a}-\frac{c(ax^2+bx^3)^{7/2}}{2ax^9}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{5}{2}b\left(a\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)+\frac{2(ax^2+bx^3)^{3/2}}{3x^3}\right)-\frac{(ax^2+bx^3)^{5/2}}{x^6}\right)(4ad+3bc)}{4a}-\frac{c(ax^2+bx^3)^{7/2}}{2ax^9}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^8,x]
```

output
```
-1/2*(c*(a*x^2 + b*x^3)^(7/2))/(a*x^9) + ((3*b*c + 4*a*d)*(-((a*x^2 + b*x^
3)^(5/2)/x^6) + (5*b*((2*(a*x^2 + b*x^3)^(3/2))/(3*x^3) + a*((2*Sqrt[a*x^2
 + b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])))/2))/(
4*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.75

method	result
risch	$-\frac{a(4adx+9cbx+2ac)\sqrt{x^2(bx+a)}}{4x^3}+\frac{b\Big(\frac{16(bx+a)^{\frac{3}{2}}d}{3}+32\sqrt{bx+a}\,ad+16\sqrt{bx+a}\,bc-10\sqrt{a}\,(4ad+3bc)\operatorname{arctanh}\Big(\frac{\sqrt{bx+a}}{\sqrt{a}}\Big)\Big)}{8x\sqrt{bx+a}}$
pseudoelliptic	$\frac{\frac{5b^6x^7\left(ad-\frac{bc}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{512}+\frac{\Big(-\frac{928\left(\frac{35dx}{29}+c\right)xb\,a^{\frac{11}{2}}}{7}+\left(-64dx-\frac{384c}{7}\right)a^{\frac{13}{2}}+x^2b^2\Big(-\frac{5b^3x^3(3dx+c)a^{\frac{3}{2}}}{4}+b^2x^2\left(\frac{5dx}{2}+c\right)a^{\frac{5}{2}}-}{384}}{a^{\frac{9}{2}}x^7}$
default	$-\frac{(b\,x^3+a\,x^2)^{\frac{5}{2}}\Big(-8(bx+a)^{\frac{3}{2}}d\,b^2x^2\sqrt{a}-48\sqrt{bx+a}\,a^{\frac{3}{2}}d\,b^2x^2-24\sqrt{bx+a}\,b^3c\,x^2\sqrt{a}+60\operatorname{arctanh}\Big(\frac{\sqrt{bx+a}}{\sqrt{a}}\Big)a^2b^2d\,x^2+45\operatorname{arc}}{12b\,x^7(bx+a)^{\frac{5}{2}}\sqrt{a}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^8,x,method=_RETURNVERBOSE)
```

output
```
-1/4*a*(4*a*d*x+9*b*c*x+2*a*c)/x^3*(x^2*(b*x+a))^(1/2)+1/8*b*(16/3*(b*x+a)
^(3/2)*d+32*(b*x+a)^(1/2)*a*d+16*(b*x+a)^(1/2)*b*c-10*a^(1/2)*(4*a*d+3*b*c
)*arctanh((b*x+a)^(1/2)/a^(1/2)))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.57

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{x^8}\,dx = \Bigg[\frac{15\,(3\,b^2c+4\,abd)\sqrt{a}x^3\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(8\,b^2dx^3-6\,a^2c+}{24\,x^3}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^8,x, algorithm="fricas")
```

output
```
[1/24*(15*(3*b^2*c + 4*a*b*d)*sqrt(a)*x^3*log((b*x^2 + 2*a*x - 2*sqrt(b*x^
3 + a*x^2)*sqrt(a))/x^2) + 2*(8*b^2*d*x^3 - 6*a^2*c + 8*(3*b^2*c + 7*a*b*d
)*x^2 - 3*(9*a*b*c + 4*a^2*d)*x)*sqrt(b*x^3 + a*x^2))/x^3, 1/12*(15*(3*b^2
*c + 4*a*b*d)*sqrt(-a)*x^3*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*
x)) + (8*b^2*d*x^3 - 6*a^2*c + 8*(3*b^2*c + 7*a*b*d)*x^2 - 3*(9*a*b*c + 4*
a^2*d)*x)*sqrt(b*x^3 + a*x^2))/x^3]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^8}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{5}{2}}(c+dx)}{x^8}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/x**8,x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x)/x**8, x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^8}\,dx=\int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{x^8}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^8,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/x^8, x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^8}\,dx=\frac{24\sqrt{bx+a}\,b^3c\,\mathrm{sgn}(x)+8\,(bx+a)^{\frac{3}{2}}b^2d\,\mathrm{sgn}(x)+48\sqrt{bx+a}\,ab^2d\,\mathrm{sgn}(x)+\cdots}{\cdots}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^8,x, algorithm="giac")
```

output
```
1/12*(24*sqrt(b*x + a)*b^3*c*sgn(x) + 8*(b*x + a)^(3/2)*b^2*d*sgn(x) + 48*
sqrt(b*x + a)*a*b^2*d*sgn(x) + 15*(3*a*b^3*c*sgn(x) + 4*a^2*b^2*d*sgn(x))*
arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) - 3*(9*(b*x + a)^(3/2)*a*b^3*c*sgn
(x) - 7*sqrt(b*x + a)*a^2*b^3*c*sgn(x) + 4*(b*x + a)^(3/2)*a^2*b^2*d*sgn(x
) - 4*sqrt(b*x + a)*a^3*b^2*d*sgn(x))/(b^2*x^2))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^8}\,dx=\int \frac{\left(b\,x^3+a\,x^2\right)^{5/2}\,\left(c+d\,x\right)}{x^8}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^8,x)
```

output
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^8, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^8}\,dx=\frac{-12\sqrt{bx+a}\,a^2c-24\sqrt{bx+a}\,a^2dx-54\sqrt{bx+a}\,abcx+112\sqrt{bx+a}\,abd\,x$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^8,x)
```

output
```
( - 12*sqrt(a + b*x)*a**2*c - 24*sqrt(a + b*x)*a**2*d*x - 54*sqrt(a + b*x)
*a*b*c*x + 112*sqrt(a + b*x)*a*b*d*x**2 + 48*sqrt(a + b*x)*b**2*c*x**2 + 1
6*sqrt(a + b*x)*b**2*d*x**3 + 60*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b*
d*x**2 + 45*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**2*c*x**2 - 60*sqrt(a)*
log(sqrt(a + b*x) + sqrt(a))*a*b*d*x**2 - 45*sqrt(a)*log(sqrt(a + b*x) + s
qrt(a))*b**2*c*x**2)/(24*x**2)
```

3.269 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^9}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 156

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^9}\,dx=-\frac{b(5bc+14ad)\sqrt{ax^2+bx^3}}{8x^2}+\frac{2b^2d\sqrt{ax^2+bx^3}}{x}-\frac{(5bc+6ad)\left(ax^2+bx^3\right)^{3/2}}{12x^5}-\frac{c\left(ax^2+bx^3\right)^{5/2}}{3x^8}-\frac{5b^2(bc+6ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{8\sqrt{a}}$$

output
```
-1/8*b*(14*a*d+5*b*c)*(b*x^3+a*x^2)^(1/2)/x^2+2*b^2*d*(b*x^3+a*x^2)^(1/2)/
x-1/12*(6*a*d+5*b*c)*(b*x^3+a*x^2)^(3/2)/x^5-1/3*c*(b*x^3+a*x^2)^(5/2)/x^8
-5/8*b^2*(6*a*d+b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.80

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^9}\,dx=$$

$$-\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(3b^2x^2(11c-16dx)+4a^2(2c+3dx)+2abx(13c+27dx))+15b^2(bc+6ad)x^3\right.}{24\sqrt{a}x^4\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^9,x]
```

output
```
-1/24*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(3*b^2*x^2*(11*c - 16*d*
x) + 4*a^2*(2*c + 3*d*x) + 2*a*b*x*(13*c + 27*d*x)) + 15*b^2*(b*c + 6*a*d)
*x^3*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(Sqrt[a]*x^4*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 147, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1944, 1926, 1926, 1927, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{x^9}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(6ad+bc)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^8}dx}{6a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{3ax^{10}}$$

$$\Big\downarrow \text{1926}$$

$$\frac{(6ad+bc)\left(\frac{5}{4}b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^5}dx-\frac{\left(ax^2+bx^3\right)^{5/2}}{2x^7}\right)}{6a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{3ax^{10}}$$

$$\Big\downarrow \text{1926}$$

$$\frac{(6ad+bc)\left(\frac{5}{4}b\left(\frac{3}{2}b\int \frac{\sqrt{bx^3+ax^2}}{x^2}dx-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{2x^7}\right)}{6a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{3ax^{10}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(6ad+bc)\left(\frac{5}{4}b\left(\frac{3}{2}b\left(a\int \frac{1}{\sqrt{bx^3+ax^2}}dx+\frac{2\sqrt{ax^2+bx^3}}{x}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{x^4}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{2x^7}\right)}{6a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{3ax^{10}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{(6ad+bc)\left(\frac{5}{4}b\left(\frac{3}{2}b\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2a\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}\right)-\frac{(ax^2+bx^3)^{5/2}}{2x^7}\right)}{6a}-\frac{c(ax^2+bx^3)^{7/2}}{3ax^{10}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{5}{4}b\left(\frac{3}{2}b\left(\frac{2\sqrt{ax^2+bx^3}}{x}-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)\right)-\frac{(ax^2+bx^3)^{3/2}}{x^4}\right)-\frac{(ax^2+bx^3)^{5/2}}{2x^7}\right)(6ad+bc)}{6a}-\frac{c(ax^2+bx^3)^{7/2}}{3ax^{10}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^9,x]
```

output
```
-1/3*(c*(a*x^2 + b*x^3)^(7/2))/(a*x^10) + ((b*c + 6*a*d)*(-1/2*(a*x^2 + b*
x^3)^(5/2)/x^7 + (5*b*(-((a*x^2 + b*x^3)^(3/2)/x^4) + (3*b*((2*Sqrt[a*x^2
+ b*x^3])/x - 2*Sqrt[a]*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]]))/2))/4))
/(6*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\left(54 a b d \,x^{2}+33 b^{2} c \,x^{2}+12 a^{2} d x +26 a b c x +8 a^{2} c \right) \sqrt{x^{2} \left(b x +a \right)}}{24 x^{4}}+\frac{b^{2} \left(32 \sqrt{b x +a}\, d -\frac{2 \left(30 a d +5 b c \right) \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right)}{\sqrt{a}}\right) \sqrt{x^{2} (bx}{16 x \sqrt{b x +a}}$
pseudoelliptic	$-\frac{5 \left(b^{7} x^{8} \left(a d -\frac{9 b c}{16}\right) \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right)-\frac{9 \left(-144 \left(\frac{296 d x}{243}+c \right) x^{2} b^{2} a^{\frac{11}{2}}-\frac{704 x b \left(\frac{116 d x}{99}+c \right) a^{\frac{13}{2}}}{3}+\frac{128 \left(-8 d x -7 c \right) a^{\frac{15}{2}}}{9}+x^{3} b^{3} \left(\frac{35 x^{3}}{}\right.\right.}{}\right.}{1024 a^{\frac{11}{2}} x^{8}}$
default	$-\frac{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{5}{2}} \left(-48 \sqrt{b x +a}\, d \,b^{3} x^{3} \sqrt{a}+90 \,\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) a \,b^{3} d \,x^{3}+15 \,\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) b^{4} c \,x^{3}+54 \left(b x +a \right)^{\frac{5}{2}} a^{\frac{3}{2}} d +33(\right.}{24 b \,x^{8} \left(b x +a \right)^{\frac{5}{2}} \sqrt{a}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^9,x,method=_RETURNVERBOSE)
```

output
```
-1/24*(54*a*b*d*x^2+33*b^2*c*x^2+12*a^2*d*x+26*a*b*c*x+8*a^2*c)/x^4*(x^2*(
b*x+a))^(1/2)+1/16*b^2*(32*(b*x+a)^(1/2)*d-2*(30*a*d+5*b*c)/a^(1/2)*arctan
h((b*x+a)^(1/2)/a^(1/2)))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.69

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^9}\,dx = \left[\frac{15\left(b^3c+6\,ab^2d\right)\sqrt{a}x^4\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\left(48\,ab^2dx^3-8\,a^3c\right.}{48\,ax^4}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^9,x, algorithm="fricas")
```

output
```
[1/48*(15*(b^3*c + 6*a*b^2*d)*sqrt(a)*x^4*log((b*x^2 + 2*a*x - 2*sqrt(b*x^
3 + a*x^2)*sqrt(a))/x^2) + 2*(48*a*b^2*d*x^3 - 8*a^3*c - 3*(11*a*b^2*c + 1
8*a^2*b*d)*x^2 - 2*(13*a^2*b*c + 6*a^3*d)*x)*sqrt(b*x^3 + a*x^2))/(a*x^4),
 1/24*(15*(b^3*c + 6*a*b^2*d)*sqrt(-a)*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt
(-a)/(b*x^2 + a*x)) + (48*a*b^2*d*x^3 - 8*a^3*c - 3*(11*a*b^2*c + 18*a^2*b
*d)*x^2 - 2*(13*a^2*b*c + 6*a^3*d)*x)*sqrt(b*x^3 + a*x^2))/(a*x^4)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^9}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{5}{2}}(c+dx)}{x^9}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/x**9,x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x)/x**9, x)
```

Maxima [F]

$$\int \frac{(c+dx)(ax^2+bx^3)^{5/2}}{x^9}\,dx = \int \frac{(bx^3+ax^2)^{\frac{5}{2}}(dx+c)}{x^9}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^9,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/x^9, x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)(ax^2+bx^3)^{5/2}}{x^9}\,dx = \frac{1}{24}\left(\frac{48\sqrt{bx+a}d\text{sgn}(x)}{b} + \frac{15\,(bc\text{sgn}(x)+6\,ad\text{sgn}(x))\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}b} -\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^9,x, algorithm="giac")
```

output
```
1/24*(48*sqrt(b*x + a)*d*sgn(x)/b + 15*(b*c*sgn(x) + 6*a*d*sgn(x))*arctan(
sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*b) - (33*(b*x + a)^(5/2)*b*c*sgn(x) - 40
*(b*x + a)^(3/2)*a*b*c*sgn(x) + 15*sqrt(b*x + a)*a^2*b*c*sgn(x) + 54*(b*x
+ a)^(5/2)*a*d*sgn(x) - 96*(b*x + a)^(3/2)*a^2*d*sgn(x) + 42*sqrt(b*x + a)
*a^3*d*sgn(x))/(b^4*x^3))*b^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)(ax^2+bx^3)^{5/2}}{x^9}\,dx = \int \frac{(b\,x^3+a\,x^2)^{5/2}\,(c+d\,x)}{x^9}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^9,x)
```

output
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^9, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^9}\,dx = \frac{-16\sqrt{bx+a}\,a^3c-24\sqrt{bx+a}\,a^3dx-52\sqrt{bx+a}\,a^2bcx-108\sqrt{bx+a}\,a^2b\cdots}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^9,x)
```

output
```
( - 16*sqrt(a + b*x)*a**3*c - 24*sqrt(a + b*x)*a**3*d*x - 52*sqrt(a + b*x)
*a**2*b*c*x - 108*sqrt(a + b*x)*a**2*b*d*x**2 - 66*sqrt(a + b*x)*a*b**2*c*
x**2 + 96*sqrt(a + b*x)*a*b**2*d*x**3 + 90*sqrt(a)*log(sqrt(a + b*x) - sqr
t(a))*a*b**2*d*x**3 + 15*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**3*c*x**3
- 90*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*a*b**2*d*x**3 - 15*sqrt(a)*log(s
qrt(a + b*x) + sqrt(a))*b**3*c*x**3)/(48*a*x**3)
```

3.270 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{10}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 169

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{10}}\,dx = -\frac{b(5bc+24ad)\sqrt{ax^2+bx^3}}{32x^3} - \frac{b^2(5bc+88ad)\sqrt{ax^2+bx^3}}{64ax^2} - \frac{(5bc+8ad)\left(ax^2+bx^3\right)^{3/2}}{24x^6} - \frac{c(ax^2+bx^3)^{5/2}}{4x^9} + \frac{5b^3(bc-8ad)\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{64a^{3/2}}$$

output
```
-1/32*b*(24*a*d+5*b*c)*(b*x^3+a*x^2)^(1/2)/x^3-1/64*b^2*(88*a*d+5*b*c)*(b*
x^3+a*x^2)^(1/2)/a/x^2-1/24*(8*a*d+5*b*c)*(b*x^3+a*x^2)^(3/2)/x^6-1/4*c*(b
*x^3+a*x^2)^(5/2)/x^9+5/64*b^3*(-8*a*d+b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^
(1/2)/x)/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{10}}\,dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a}\sqrt{a+bx}(15b^3cx^3+16a^3(3c+4dx)+8a^2bx(17c+26d\right.}{192a^{3/2}x^5\sqrt{a}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^10,x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(15*b^3*c*x^3 + 16*a^3*(3*c
+ 4*d*x) + 8*a^2*b*x*(17*c + 26*d*x) + 2*a*b^2*x^2*(59*c + 132*d*x))) + 15
*b^3*(b*c - 8*a*d)*x^4*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(192*a^(3/2)*x^5*S
qrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1944, 1926, 1926, 1926, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{x^{10}}dx$$

$$\Big\downarrow \ 1944$$

$$-\frac{(bc-8ad)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^9}dx}{8a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{4ax^{11}}$$

$$\Big\downarrow \ 1926$$

$$-\frac{(bc-8ad)\left(\frac{5}{6}b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^6}dx-\frac{\left(ax^2+bx^3\right)^{5/2}}{3x^8}\right)}{8a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{4ax^{11}}$$

$$\Big\downarrow \ 1926$$

$$-\frac{(bc-8ad)\left(\frac{5}{6}b\left(\frac{3}{4}b\int\frac{\sqrt{bx^3+ax^2}}{x^3}dx-\frac{(ax^2+bx^3)^{3/2}}{2x^5}\right)-\frac{(ax^2+bx^3)^{5/2}}{3x^8}\right)}{8a}-\frac{c(ax^2+bx^3)^{7/2}}{4ax^{11}}$$

$\Big\downarrow$ 1926

$$-\frac{(bc-8ad)\left(\frac{5}{6}b\left(\frac{3}{4}b\left(\frac{1}{2}b\int\frac{1}{\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{(ax^2+bx^3)^{3/2}}{2x^5}\right)-\frac{(ax^2+bx^3)^{5/2}}{3x^8}\right)}{8a}-\frac{c(ax^2+bx^3)^{7/2}}{4ax^{11}}$$

$\Big\downarrow$ 1914

$$-\frac{(bc-8ad)\left(\frac{5}{6}b\left(\frac{3}{4}b\left(-b\int\frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{(ax^2+bx^3)^{3/2}}{2x^5}\right)-\frac{(ax^2+bx^3)^{5/2}}{3x^8}\right)}{8a}-\frac{c(ax^2+bx^3)^{7/2}}{4ax^{11}}$$

$\Big\downarrow$ 219

$$-\frac{\left(\frac{5}{6}b\left(\frac{3}{4}b\left(-\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}-\frac{\sqrt{ax^2+bx^3}}{x^2}\right)-\frac{(ax^2+bx^3)^{3/2}}{2x^5}\right)-\frac{(ax^2+bx^3)^{5/2}}{3x^8}\right)(bc-8ad)}{8a}-\frac{c(ax^2+bx^3)^{7/2}}{4ax^{11}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^10,x]
```

output
```
-1/4*(c*(a*x^2 + b*x^3)^(7/2))/(a*x^11) - ((b*c - 8*a*d)*(-1/3*(a*x^2 + b*
x^3)^(5/2)/x^8 + (5*b*(-1/2*(a*x^2 + b*x^3)^(3/2)/x^5 + (3*b*(-(Sqrt[a*x^2
 + b*x^3]/x^2) - (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/Sqrt[a]))/4)
)/6))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.78

method	result
risch	$-\frac{\left(264 a \,b^{2} d \,x^{3}+15 b^{3} c \,x^{3}+208 a^{2} b d \,x^{2}+118 a \,b^{2} c \,x^{2}+64 a^{3} d x +136 a^{2} b c x +48 c \,a^{3}\right) \sqrt{x^{2} \left(b x +a \right)}}{192 x^{5} a}-\frac{5 \left(8 a d -b c \right) b^{3} \operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.}{64 a^{\frac{3}{2}} x \sqrt{b}}$
pseudoelliptic	$\frac{\frac{45 b^{8} x^{9} \left(a d -\frac{11 b c}{18}\right) \operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right)}{16384}-\frac{11 \sqrt{b x +a}\, \left(\frac{80 x^{3} b^{3} \left(\frac{9 d x}{5}+c \right) a^{\frac{11}{2}}}{99}+\frac{6592 x^{2} \left(\frac{243 d x}{206}+c \right) b^{2} a^{\frac{13}{2}}}{33}+\frac{33152 x \left(\frac{297 d x}{259}+c \right) b \,a^{\frac{15}{2}}}{99}+\left(1\right.\right.}{}}{a^{\frac{13}{2}} x}$
default	$-\frac{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{5}{2}} \left(264 \left(b x +a \right)^{\frac{7}{2}} a^{\frac{5}{2}} d +15 \left(b x +a \right)^{\frac{7}{2}} a^{\frac{3}{2}} b c +120 \,\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) a^{2} b^{4} d \,x^{4}-15 \,\operatorname{arctanh}\left(\frac{\sqrt{b x +a}}{\sqrt{a}}\right) a \,b^{5} c \,x^{4}-584\right.}{192 b \,x^{9} \left(b x +a \right)^{\frac{5}{2}}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^10,x,method=_RETURNVERBOSE)
```

output
```
-1/192*(264*a*b^2*d*x^3+15*b^3*c*x^3+208*a^2*b*d*x^2+118*a*b^2*c*x^2+64*a^
3*d*x+136*a^2*b*c*x+48*a^3*c)/x^5/a*(x^2*(b*x+a))^(1/2)-5/64*(8*a*d-b*c)*b
^3/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1
/2)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.73

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{10}}\,dx = \left[-\frac{15\,(b^4c-8\,ab^3d)\sqrt{a}x^5\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(48\,a^4c+3\,(5\,ab^3}{3}\right.$$

$$-\frac{15\,(b^4c-8\,ab^3d)\sqrt{-a}x^5\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(48\,a^4c+3\,(5\,ab^3c+88\,a^2b^2d)x^3+2\,(59\,a^2b^2c+104\,a^3}{192\,a^2x^5}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^10,x, algorithm="fricas")
```

output
```
[-1/384*(15*(b^4*c - 8*a*b^3*d)*sqrt(a)*x^5*log((b*x^2 + 2*a*x - 2*sqrt(b*
x^3 + a*x^2)*sqrt(a))/x^2) + 2*(48*a^4*c + 3*(5*a*b^3*c + 88*a^2*b^2*d)*x^
3 + 2*(59*a^2*b^2*c + 104*a^3*b*d)*x^2 + 8*(17*a^3*b*c + 8*a^4*d)*x)*sqrt(
b*x^3 + a*x^2))/(a^2*x^5), -1/192*(15*(b^4*c - 8*a*b^3*d)*sqrt(-a)*x^5*arc
tan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (48*a^4*c + 3*(5*a*b^3*c
 + 88*a^2*b^2*d)*x^3 + 2*(59*a^2*b^2*c + 104*a^3*b*d)*x^2 + 8*(17*a^3*b*c
+ 8*a^4*d)*x)*sqrt(b*x^3 + a*x^2))/(a^2*x^5)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{10}}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{5}{2}}(c+dx)}{x^{10}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/x**10,x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x)/x**10, x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{10}}\,dx=\int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{x^{10}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^10,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/x^10, x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{10}}\,dx =$$

$$-\frac{\frac{15\left(b^5\mathrm{csgn}(x)-8\,ab^4d\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a}+\frac{15\,(bx+a)^{\frac{7}{2}}b^5\mathrm{csgn}(x)+73\,(bx+a)^{\frac{5}{2}}ab^5\mathrm{csgn}(x)-55\,(bx+a)^{\frac{3}{2}}a^2b^5\mathrm{csgn}(x)+15\sqrt{bx+a}a^3b^5\mathrm{csg}}{}}{192\,b}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^10,x, algorithm="giac")
```

output
```
-1/192*(15*(b^5*c*sgn(x) - 8*a*b^4*d*sgn(x))*arctan(sqrt(b*x + a)/sqrt(-a)
)/(sqrt(-a)*a) + (15*(b*x + a)^(7/2)*b^5*c*sgn(x) + 73*(b*x + a)^(5/2)*a*b
^5*c*sgn(x) - 55*(b*x + a)^(3/2)*a^2*b^5*c*sgn(x) + 15*sqrt(b*x + a)*a^3*b
^5*c*sgn(x) + 264*(b*x + a)^(7/2)*a*b^4*d*sgn(x) - 584*(b*x + a)^(5/2)*a^2
*b^4*d*sgn(x) + 440*(b*x + a)^(3/2)*a^3*b^4*d*sgn(x) - 120*sqrt(b*x + a)*a
^4*b^4*d*sgn(x))/(a*b^4*x^4))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{10}}\,dx = \int \frac{\left(b\,x^3+a\,x^2\right)^{5/2}\,(c+d\,x)}{x^{10}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^10,x)
```

output
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^10, x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{10}}\,dx = \frac{-96\sqrt{bx+a}\,a^4c-128\sqrt{bx+a}\,a^4dx-272\sqrt{bx+a}\,a^3bcx-416\sqrt{bx+a}\,a}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^10,x)
```

output
```
( - 96*sqrt(a + b*x)*a**4*c - 128*sqrt(a + b*x)*a**4*d*x - 272*sqrt(a + b*
x)*a**3*b*c*x - 416*sqrt(a + b*x)*a**3*b*d*x**2 - 236*sqrt(a + b*x)*a**2*b
**2*c*x**2 - 528*sqrt(a + b*x)*a**2*b**2*d*x**3 - 30*sqrt(a + b*x)*a*b**3*
c*x**3 + 120*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**3*d*x**4 - 15*sqrt(
a)*log(sqrt(a + b*x) - sqrt(a))*b**4*c*x**4 - 120*sqrt(a)*log(sqrt(a + b*x
) + sqrt(a))*a*b**3*d*x**4 + 15*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**4*
c*x**4)/(384*a**2*x**4)
```

3.271 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{11}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 206

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{11}}\,dx = -\frac{b(3bc+22ad)\sqrt{ax^2+bx^3}}{48x^4} - \frac{b^2(3bc+118ad)\sqrt{ax^2+bx^3}}{192ax^3} + \frac{b^3(3bc-10ad)\sqrt{ax^2+bx^3}}{128a^2x^2} - \frac{(bc+2ad)\left(ax^2+bx^3\right)^{3/2}}{8x^7} - \frac{c(ax^2+bx^3)^{5/2}}{5x^{10}} - \frac{b^4(3bc-10ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{128a^{5/2}}$$

output
```
-1/48*b*(22*a*d+3*b*c)*(b*x^3+a*x^2)^(1/2)/x^4-1/192*b^2*(118*a*d+3*b*c)*(
b*x^3+a*x^2)^(1/2)/a/x^3+1/128*b^3*(-10*a*d+3*b*c)*(b*x^3+a*x^2)^(1/2)/a^2
/x^2-1/8*(2*a*d+b*c)*(b*x^3+a*x^2)^(3/2)/x^7-1/5*c*(b*x^3+a*x^2)^(5/2)/x^1
0-1/128*b^4*(-10*a*d+3*b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.75

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{11}}\,dx =$$
$$-\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a}\sqrt{a+bx}(-45b^4cx^4+30ab^3x^3(c+5dx)+96a^4(4c+5dx)+16a^3bx(63c+85dx)+4a^2\right.}{1920a^{5/2}x^6\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^11,x]
```

output
```
-1/1920*(Sqrt[x^2*(a + b*x)]*(Sqrt[a]*Sqrt[a + b*x]*(-45*b^4*c*x^4 + 30*a*
b^3*x^3*(c + 5*d*x) + 96*a^4*(4*c + 5*d*x) + 16*a^3*b*x*(63*c + 85*d*x) +
4*a^2*b^2*x^2*(186*c + 295*d*x)) + 15*b^4*(3*b*c - 10*a*d)*x^5*ArcTanh[Sqr
t[a + b*x]/Sqrt[a]]))/(a^(5/2)*x^6*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1944, 1926, 1926, 1926, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{x^{11}}\,dx$$
$$\Big\downarrow \text{ 1944}$$
$$-\frac{(3bc-10ad)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^{10}}dx}{10a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{5ax^{12}}$$
$$\Big\downarrow \text{ 1926}$$
$$-\frac{(3bc-10ad)\left(\frac{5}{8}b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^7}dx-\frac{\left(ax^2+bx^3\right)^{5/2}}{4x^9}\right)}{10a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{5ax^{12}}$$

$$\Big\downarrow \text{1926}$$

$$-\frac{(3bc-10ad)\left(\frac{5}{8}b\left(\frac{1}{2}b\int \frac{\sqrt{bx^3+ax^2}}{x^4}dx-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{4x^9}\right)}{10a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{5ax^{12}}$$

$$\Big\downarrow \text{1926}$$

$$-\frac{(3bc-10ad)\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{4x^9}\right)}{10a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{5ax^{12}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{(3bc-10ad)\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{4x^9}\right)}{10a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{5ax^{12}}$$

$$\Big\downarrow \text{1914}$$

$$-\frac{(3bc-10ad)\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{4x^9}\right)}{10a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{5ax^{12}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(\frac{5}{8}b\left(\frac{1}{2}b\left(\frac{1}{4}b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)-\frac{\sqrt{ax^2+bx^3}}{2x^3}\right)-\frac{\left(ax^2+bx^3\right)^{3/2}}{3x^6}\right)-\frac{\left(ax^2+bx^3\right)^{5/2}}{4x^9}\right)(3bc-10ad)}{10a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{5ax^{12}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^11,x]
```

output
```
-1/5*(c*(a*x^2 + b*x^3)^(7/2))/(a*x^12) - ((3*b*c - 10*a*d)*(-1/4*(a*x^2 +
 b*x^3)^(5/2)/x^9 + (5*b*(-1/3*(a*x^2 + b*x^3)^(3/2)/x^6 + (b*(-1/2*Sqrt[a
*x^2 + b*x^3]/x^3 + (b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[
a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/4))/2))/8))/(10*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.76

method	result
risch	$-\frac{\left(150x^4ab^3d-45x^4b^4c+1180a^2b^2dx^3+30ab^3cx^3+1360a^3bdx^2+744a^2b^2cx^2+480a^4dx+1008a^3bcx+384ca^4\right)\sqrt{x^2(bx-}}{1920x^6a^2}$
pseudoelliptic	$-\frac{55\left(b^9x^{10}\left(ad-\frac{13bc}{20}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)-\frac{52\sqrt{bx+a}\left(\frac{80x^4b^4\left(\frac{22dx}{13}+c\right)a^{\frac{11}{2}}}{99}-\frac{320x^3b^3\left(\frac{5dx}{3}+c\right)a^{\frac{13}{2}}}{429}-\frac{343168x^2\left(\frac{3090dx}{2681}+c\right)b}{1287}\right.}{}\right.}{}$
default	$-\frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}\left(150(bx+a)^{\frac{9}{2}}a^{\frac{7}{2}}d-45(bx+a)^{\frac{9}{2}}a^{\frac{5}{2}}bc+580(bx+a)^{\frac{7}{2}}a^{\frac{9}{2}}d+210(bx+a)^{\frac{7}{2}}a^{\frac{7}{2}}bc-150\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^3b^5d\right.}{}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^11,x,method=_RETURNVERBOSE)
```

output
```
-1/1920*(150*a*b^3*d*x^4-45*b^4*c*x^4+1180*a^2*b^2*d*x^3+30*a*b^3*c*x^3+13
60*a^3*b*d*x^2+744*a^2*b^2*c*x^2+480*a^4*d*x+1008*a^3*b*c*x+384*a^4*c)/x^6
/a^2*(x^2*(b*x+a))^(1/2)+1/128*(10*a*d-3*b*c)*b^4/a^(5/2)*arctanh((b*x+a)^
(1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{x^{11}}\,dx = \left[-\frac{15\,(3\,b^5c-10\,ab^4d)\sqrt{a}x^6\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(384\,a^5c-15\,(\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^11,x, algorithm="fricas")
```

output
```
[-1/3840*(15*(3*b^5*c - 10*a*b^4*d)*sqrt(a)*x^6*log((b*x^2 + 2*a*x + 2*sqr
t(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(384*a^5*c - 15*(3*a*b^4*c - 10*a^2*b^3
*d)*x^4 + 10*(3*a^2*b^3*c + 118*a^3*b^2*d)*x^3 + 8*(93*a^3*b^2*c + 170*a^4
*b*d)*x^2 + 48*(21*a^4*b*c + 10*a^5*d)*x)*sqrt(b*x^3 + a*x^2))/(a^3*x^6),
1/1920*(15*(3*b^5*c - 10*a*b^4*d)*sqrt(-a)*x^6*arctan(sqrt(b*x^3 + a*x^2)*
sqrt(-a)/(b*x^2 + a*x)) - (384*a^5*c - 15*(3*a*b^4*c - 10*a^2*b^3*d)*x^4 +
 10*(3*a^2*b^3*c + 118*a^3*b^2*d)*x^3 + 8*(93*a^3*b^2*c + 170*a^4*b*d)*x^2
 + 48*(21*a^4*b*c + 10*a^5*d)*x)*sqrt(b*x^3 + a*x^2))/(a^3*x^6)]
```

Sympy [F]

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{x^{11}}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{5}{2}}(c+dx)}{x^{11}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/x**11,x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x)/x**11, x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{11}}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{x^{11}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^11,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/x^11, x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{11}}\,dx = \frac{1}{1920}\,b^5\left(\frac{15\,(3\,bc\text{sgn}(x)-10\,ad\text{sgn}(x))\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2b} + \frac{45\,(bx+a)^{\frac{9}{2}}bcs\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^11,x, algorithm="giac")
```

output
```
1/1920*b^5*(15*(3*b*c*sgn(x) - 10*a*d*sgn(x))*arctan(sqrt(b*x + a)/sqrt(-a
))/(sqrt(-a)*a^2*b) + (45*(b*x + a)^(9/2)*b*c*sgn(x) - 210*(b*x + a)^(7/2)
*a*b*c*sgn(x) - 384*(b*x + a)^(5/2)*a^2*b*c*sgn(x) + 210*(b*x + a)^(3/2)*a
^3*b*c*sgn(x) - 45*sqrt(b*x + a)*a^4*b*c*sgn(x) - 150*(b*x + a)^(9/2)*a*d*
sgn(x) - 580*(b*x + a)^(7/2)*a^2*d*sgn(x) + 1280*(b*x + a)^(5/2)*a^3*d*sgn
(x) - 700*(b*x + a)^(3/2)*a^4*d*sgn(x) + 150*sqrt(b*x + a)*a^5*d*sgn(x))/(
a^2*b^6*x^5))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{11}}\,dx=\int \frac{\left(b\,x^3+a\,x^2\right)^{5/2}\,(c+d\,x)}{x^{11}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^11,x)
```

output
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^11, x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{11}}\,dx=\frac{-768\sqrt{bx+a}\,a^5c-960\sqrt{bx+a}\,a^5dx-2016\sqrt{bx+a}\,a^4bcx-2720\sqrt{bx+}}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^11,x)
```

output
```
( - 768*sqrt(a + b*x)*a**5*c - 960*sqrt(a + b*x)*a**5*d*x - 2016*sqrt(a +
b*x)*a**4*b*c*x - 2720*sqrt(a + b*x)*a**4*b*d*x**2 - 1488*sqrt(a + b*x)*a*
*3*b**2*c*x**2 - 2360*sqrt(a + b*x)*a**3*b**2*d*x**3 - 60*sqrt(a + b*x)*a*
*2*b**3*c*x**3 - 300*sqrt(a + b*x)*a**2*b**3*d*x**4 + 90*sqrt(a + b*x)*a*b
**4*c*x**4 - 150*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**4*d*x**5 + 45*s
qrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**5*c*x**5 + 150*sqrt(a)*log(sqrt(a +
 b*x) + sqrt(a))*a*b**4*d*x**5 - 45*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b
**5*c*x**5)/(3840*a**3*x**5)
```

3.272 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{12}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 244

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{12}}\,dx = -\frac{b(5bc+52ad)\sqrt{ax^2+bx^3}}{160x^5} - \frac{b^2(5bc+372ad)\sqrt{ax^2+bx^3}}{960ax^4} + \frac{b^3(5bc-12ad)\sqrt{ax^2+bx^3}}{768a^2x^3} - \frac{b^4(5bc-12ad)\sqrt{ax^2+bx^3}}{512a^3x^2} - \frac{(5bc+12ad)\left(ax^2+bx^3\right)^{3/2}}{60x^8} - \frac{c\left(ax^2+bx^3\right)^{5/2}}{6x^{11}} + \frac{b^5(5bc-12ad)\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{512a^{7/2}}$$

output
```
-1/160*b*(52*a*d+5*b*c)*(b*x^3+a*x^2)^(1/2)/x^5-1/960*b^2*(372*a*d+5*b*c)*
(b*x^3+a*x^2)^(1/2)/a/x^4+1/768*b^3*(-12*a*d+5*b*c)*(b*x^3+a*x^2)^(1/2)/a^
2/x^3-1/512*b^4*(-12*a*d+5*b*c)*(b*x^3+a*x^2)^(1/2)/a^3/x^2-1/60*(12*a*d+5
*b*c)*(b*x^3+a*x^2)^(3/2)/x^8-1/6*c*(b*x^3+a*x^2)^(5/2)/x^11+1/512*b^5*(-1
2*a*d+5*b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.72

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{12}}\,dx = \frac{\sqrt{x^2(a+bx)}\Big(-\sqrt{a}\sqrt{a+bx}(75b^5cx^5+40a^2b^3x^3(c+3dx)+256a^5(5c+6$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^12,x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a]*Sqrt[a + b*x]*(75*b^5*c*x^5 + 40*a^2*b^3*x
^3*(c + 3*d*x) + 256*a^5*(5*c + 6*d*x) - 10*a*b^4*x^4*(5*c + 18*d*x) + 48*
a^3*b^2*x^2*(45*c + 62*d*x) + 64*a^4*b*x*(50*c + 63*d*x))) + 15*b^5*(5*b*c
 - 12*a*d)*x^6*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(7680*a^(7/2)*x^7*Sqrt[a +
 b*x])
```

Rubi [A] (verified)

Time = 0.80 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.88, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1944, 1926, 1926, 1926, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{x^{12}}\,dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(5bc-12ad)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{x^{11}}dx}{12a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{6ax^{13}}$$

$$\Big\downarrow \text{1926}$$

$$-\frac{(5bc-12ad)\left(\frac{1}{2}b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{x^8}dx-\frac{\left(ax^2+bx^3\right)^{5/2}}{5x^{10}}\right)}{12a}-\frac{c\left(ax^2+bx^3\right)^{7/2}}{6ax^{13}}$$

$$\Big\downarrow \text{1926}$$

$$-\frac{(5bc-12ad)\left(\frac{1}{2}b\left(\frac{3}{8}b\int \frac{\sqrt{bx^3+ax^2}}{x^5}dx-\frac{(ax^2+bx^3)^{3/2}}{4x^7}\right)-\frac{(ax^2+bx^3)^{5/2}}{5x^{10}}\right)}{12a}-\frac{c(ax^2+bx^3)^{7/2}}{6ax^{13}}$$

$$\Big\downarrow \text{ 1926}$$

$$-\frac{(5bc-12ad)\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{(ax^2+bx^3)^{3/2}}{4x^7}\right)-\frac{(ax^2+bx^3)^{5/2}}{5x^{10}}\right)}{12a}-\frac{c(ax^2+bx^3)^{7/2}}{6ax^{13}}$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{(5bc-12ad)\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{(ax^2+bx^3)^{3/2}}{4x^7}\right)-\frac{(ax^2+bx^3)^{5/2}}{5x^{10}}\right)}{12a}-\frac{c(ax^2+bx^3)^{7/2}}{6ax^{13}}$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{(5bc-12ad)\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{(ax^2+bx^3)^{3/2}}{4x^7}\right)-\frac{(ax \cdots}{\cdots}\right)}{12a} \cdots \frac{c(ax^2+bx^3)^{7/2}}{6ax^{13}}$$

$$\Big\downarrow \text{ 1914}$$

$$-\frac{(5bc-12ad)\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{(ax^2+bx^3)^{3/2}}{4x^7}\cdots\right)}{12a} \cdots \frac{c(ax^2+bx^3)^{7/2}}{6ax^{13}}$$

$$\Big\downarrow \text{ 219}$$

$$-\frac{\left(\frac{1}{2}b\left(\frac{3}{8}b\left(\frac{1}{6}b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)-\frac{\sqrt{ax^2+bx^3}}{3x^4}\right)-\frac{(ax^2+bx^3)^{3/2}}{4x^7}\right)-\frac{(ax^2+bx^3)^{5/}}{5x^{10}}\right.}{12a}$$

$$\frac{c(ax^2+bx^3)^{7/2}}{6ax^{13}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/x^12,x]
```

output
```
-1/6*(c*(a*x^2 + b*x^3)^(7/2))/(a*x^13) - ((5*b*c - 12*a*d)*(-1/5*(a*x^2 +
 b*x^3)^(5/2)/x^10 + (b*(-1/4*(a*x^2 + b*x^3)^(3/2)/x^7 + (3*b*(-1/3*Sqrt[
a*x^2 + b*x^3]/x^4 + (b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a
*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3
/2)))/(4*a)))/6))/8))/2))/(12*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.74

method	result
risch	$-\frac{\left(-180 a\,b^{4} d\,x^{5}+75 b^{5} c\,x^{5}+120 x^{4} a^{2} b^{3} d-50 x^{4} a\,b^{4} c+2976 a^{3} b^{2} d\,x^{3}+40 a^{2} b^{3} c\,x^{3}+4032 a^{4} b d\,x^{2}+2160 a^{3} b^{2} c\,x^{2}+1536 a^{5} d x \cdots\right.}{7680 x^{7} a^{3}}$
pseudoelliptic	$\frac{\frac{143 b^{10} x^{11}\left(a d-\frac{15 b c}{22}\right) \operatorname{arctanh}\left(\frac{\sqrt{b x+a}}{\sqrt{a}}\right)}{131072}-\frac{39 \sqrt{b x+a}\left(\frac{80 x^{5}\left(\frac{121 d x}{75}+c\right) b^{5} a^{\frac{11}{2}}}{99}-\frac{320 x^{4}\left(\frac{143 d x}{90}+c\right) b^{4} a^{\frac{13}{2}}}{429}+\frac{896 x^{3} b^{3}\left(\frac{11 d x}{7}+c\right) a^{\frac{15}{2}}}{1287}+\cdots\right.}{\cdots}}{\cdots}$
default	$\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}\left(180 (b x+a)^{\frac{11}{2}} a^{\frac{9}{2}} d-75 (b x+a)^{\frac{11}{2}} a^{\frac{7}{2}} b c-1020 (b x+a)^{\frac{9}{2}} a^{\frac{11}{2}} d+425 (b x+a)^{\frac{9}{2}} a^{\frac{9}{2}} b c-696 (b x+a)^{\frac{7}{2}} a^{\frac{13}{2}} d-990 (b x \cdots\right.}{\cdots}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^12,x,method=_RETURNVERBOSE)
```

output
```
-1/7680*(-180*a*b^4*d*x^5+75*b^5*c*x^5+120*a^2*b^3*d*x^4-50*a*b^4*c*x^4+29
76*a^3*b^2*d*x^3+40*a^2*b^3*c*x^3+4032*a^4*b*d*x^2+2160*a^3*b^2*c*x^2+1536
*a^5*d*x+3200*a^4*b*c*x+1280*a^5*c)/x^7/a^3*(x^2*(b*x+a))^(1/2)-1/512*(12*
a*d-5*b*c)*b^5/a^(7/2)*arctanh((b*x+a)^(1/2)/a^(1/2))*(x^2*(b*x+a))^(1/2)/
x/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.60

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{12}}\,dx = \left[-\frac{15\,(5\,b^6c-12\,ab^5d)\sqrt{a}x^7\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(1280\,a^6c+15\right.$$

$$-\frac{15\,(5\,b^6c-12\,ab^5d)\sqrt{-a}x^7\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(1280\,a^6c+15\,(5\,ab^5c-12\,a^2b^4d)x^5-10\,(5\,a^2b^4c-}{768}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^12,x, algorithm="fricas")
```

output
```
[-1/15360*(15*(5*b^6*c - 12*a*b^5*d)*sqrt(a)*x^7*log((b*x^2 + 2*a*x - 2*sq
rt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(1280*a^6*c + 15*(5*a*b^5*c - 12*a^2*b
^4*d)*x^5 - 10*(5*a^2*b^4*c - 12*a^3*b^3*d)*x^4 + 8*(5*a^3*b^3*c + 372*a^4
*b^2*d)*x^3 + 144*(15*a^4*b^2*c + 28*a^5*b*d)*x^2 + 128*(25*a^5*b*c + 12*a
^6*d)*x)*sqrt(b*x^3 + a*x^2))/(a^4*x^7), -1/7680*(15*(5*b^6*c - 12*a*b^5*d
)*sqrt(-a)*x^7*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (1280*
a^6*c + 15*(5*a*b^5*c - 12*a^2*b^4*d)*x^5 - 10*(5*a^2*b^4*c - 12*a^3*b^3*d
)*x^4 + 8*(5*a^3*b^3*c + 372*a^4*b^2*d)*x^3 + 144*(15*a^4*b^2*c + 28*a^5*b
*d)*x^2 + 128*(25*a^5*b*c + 12*a^6*d)*x)*sqrt(b*x^3 + a*x^2))/(a^4*x^7)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{12}}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{5}{2}}(c+dx)}{x^{12}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/x**12,x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x)/x**12, x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{12}}\,dx=\int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{x^{12}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^12,x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/x^12, x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{x^{12}}\,dx=$$
$$-\frac{\frac{15\left(5b^7c\,\mathrm{sgn}(x)-12ab^6d\,\mathrm{sgn}(x)\right)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^3}+\frac{75(bx+a)^{\frac{11}{2}}b^7c\,\mathrm{sgn}(x)-425(bx+a)^{\frac{9}{2}}ab^7c\,\mathrm{sgn}(x)+990(bx+a)^{\frac{7}{2}}a^2b^7c\,\mathrm{sgn}(x)+990(bx+a)^{\frac{5}{2}}}{}}{}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^12,x, algorithm="giac")
```

output
```
-1/7680*(15*(5*b^7*c*sgn(x) - 12*a*b^6*d*sgn(x))*arctan(sqrt(b*x + a)/sqrt
(-a))/(sqrt(-a)*a^3) + (75*(b*x + a)^(11/2)*b^7*c*sgn(x) - 425*(b*x + a)^(
9/2)*a*b^7*c*sgn(x) + 990*(b*x + a)^(7/2)*a^2*b^7*c*sgn(x) + 990*(b*x + a)
^(5/2)*a^3*b^7*c*sgn(x) - 425*(b*x + a)^(3/2)*a^4*b^7*c*sgn(x) + 75*sqrt(b
*x + a)*a^5*b^7*c*sgn(x) - 180*(b*x + a)^(11/2)*a*b^6*d*sgn(x) + 1020*(b*x
 + a)^(9/2)*a^2*b^6*d*sgn(x) + 696*(b*x + a)^(7/2)*a^3*b^6*d*sgn(x) - 2376
*(b*x + a)^(5/2)*a^4*b^6*d*sgn(x) + 1020*(b*x + a)^(3/2)*a^5*b^6*d*sgn(x)
- 180*sqrt(b*x + a)*a^6*b^6*d*sgn(x))/(a^3*b^6*x^6))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)(ax^2+bx^3)^{5/2}}{x^{12}}\,dx = \int \frac{(b\,x^3+a\,x^2)^{5/2}\,(c+d\,x)}{x^{12}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^12,x)
```

output
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/x^12, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)(ax^2+bx^3)^{5/2}}{x^{12}}\,dx = \frac{-2560\sqrt{bx+a}\,a^6c-3072\sqrt{bx+a}\,a^6dx-6400\sqrt{bx+a}\,a^5bcx-8064\sqrt{bx}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/x^12,x)
```

output
```
( - 2560*sqrt(a + b*x)*a**6*c - 3072*sqrt(a + b*x)*a**6*d*x - 6400*sqrt(a
+ b*x)*a**5*b*c*x - 8064*sqrt(a + b*x)*a**5*b*d*x**2 - 4320*sqrt(a + b*x)*
a**4*b**2*c*x**2 - 5952*sqrt(a + b*x)*a**4*b**2*d*x**3 - 80*sqrt(a + b*x)*
a**3*b**3*c*x**3 - 240*sqrt(a + b*x)*a**3*b**3*d*x**4 + 100*sqrt(a + b*x)*
a**2*b**4*c*x**4 + 360*sqrt(a + b*x)*a**2*b**4*d*x**5 - 150*sqrt(a + b*x)*
a*b**5*c*x**5 + 180*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**5*d*x**6 - 7
5*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b**6*c*x**6 - 180*sqrt(a)*log(sqrt(
a + b*x) + sqrt(a))*a*b**5*d*x**6 + 75*sqrt(a)*log(sqrt(a + b*x) + sqrt(a)
)*b**6*c*x**6)/(15360*a**4*x**6)
```

3.273 $\int \frac{x^4(c+dx)}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 165

$$\int \frac{x^4(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2a^3(bc-ad)\sqrt{ax^2+bx^3}}{b^5x}+\frac{2a^2(3bc-4ad)\left(ax^2+bx^3\right)^{3/2}}{3b^5x^3} -\frac{6a(bc-2ad)\left(ax^2+bx^3\right)^{5/2}}{5b^5x^5} +\frac{2(bc-4ad)\left(ax^2+bx^3\right)^{7/2}}{7b^5x^7}+\frac{2d\left(ax^2+bx^3\right)^{9/2}}{9b^5x^9}$$

output
```
-2*a^3*(-a*d+b*c)*(b*x^3+a*x^2)^(1/2)/b^5/x+2/3*a^2*(-4*a*d+3*b*c)*(b*x^3+
a*x^2)^(3/2)/b^5/x^3-6/5*a*(-2*a*d+b*c)*(b*x^3+a*x^2)^(5/2)/b^5/x^5+2/7*(-
4*a*d+b*c)*(b*x^3+a*x^2)^(7/2)/b^5/x^7+2/9*d*(b*x^3+a*x^2)^(9/2)/b^5/x^9
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.57

$$\int \frac{x^4(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{x^2(a+bx)}\left(128a^4d+24a^2b^2x(3c+2dx)-16a^3b(9c+4dx)+5b^4x^3(9c+7dx)-2ab^3x^2(27c+20dx\right.}{315b^5x}$$

input
```
Integrate[(x^4*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(128*a^4*d + 24*a^2*b^2*x*(3*c + 2*d*x) - 16*a^3*b*
(9*c + 4*d*x) + 5*b^4*x^3*(9*c + 7*d*x) - 2*a*b^3*x^2*(27*c + 20*d*x)))/(3
15*b^5*x)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1945, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx)}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{ 1945}$$

$$\frac{(9bc-8ad)\int \frac{x^4}{\sqrt{bx^3+ax^2}}dx}{9b}+\frac{2dx^3\sqrt{ax^2+bx^3}}{9b}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{(9bc-8ad)\left(\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\int \frac{x^3}{\sqrt{bx^3+ax^2}}dx}{7b}\right)}{9b}+\frac{2dx^3\sqrt{ax^2+bx^3}}{9b}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{(9bc-8ad)\left(\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}\right)}{7b}\right)}{9b}+\frac{2dx^3\sqrt{ax^2+bx^3}}{9b}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{(9bc-8ad)\left(\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int\frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}\right)}{7b}\right)}{9b}+\frac{2dx^3\sqrt{ax^2+bx^3}}{9b}$$

$\Big\downarrow$ 1920

$$\frac{\left(\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{5b}\right)}{7b}\right)(9bc-8ad)}{9b}+\frac{2dx^3\sqrt{ax^2+bx^3}}{9b}$$

input
```
Int[(x^4*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*d*x^3*Sqrt[a*x^2 + b*x^3])/(9*b) + ((9*b*c - 8*a*d)*((2*x^2*Sqrt[a*x^2
+ b*x^3])/(7*b) - (6*a*((2*x*Sqrt[a*x^2 + b*x^3])/(5*b) - (4*a*((2*Sqrt[a*
x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)))/(5*b)))/(7*b))
)/(9*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.56

method	result
pseudoelliptic	$-\frac{512\sqrt{bx+a}\,\left(-\frac{77x^{4}\left(\frac{9dx}{11}+c\right)b^{5}}{256}+\frac{11x^{3}\left(\frac{35dx}{44}+c\right)a\,b^{4}}{32}-\frac{33\left(\frac{25dx}{33}+c\right)x^{2}a^{2}b^{3}}{80}+\frac{11x\left(\frac{15dx}{22}+c\right)a^{3}b^{2}}{20}-\frac{11\left(\frac{5dx}{11}+c\right)a^{4}b}{10}+a^{5}d\right)}{693b^{6}}$
trager	$\frac{2\left(35d\,x^{4}b^{4}-40a\,b^{3}d\,x^{3}+45b^{4}c\,x^{3}+48a^{2}b^{2}d\,x^{2}-54a\,b^{3}c\,x^{2}-64a^{3}bdx+72a^{2}b^{2}cx+128a^{4}d-144a^{3}bc\right)\sqrt{b\,x^{3}+a\,x^{2}}}{315b^{5}x}$
risch	$\frac{2(bx+a)x\left(35d\,x^{4}b^{4}-40a\,b^{3}d\,x^{3}+45b^{4}c\,x^{3}+48a^{2}b^{2}d\,x^{2}-54a\,b^{3}c\,x^{2}-64a^{3}bdx+72a^{2}b^{2}cx+128a^{4}d-144a^{3}bc\right)}{315\sqrt{x^{2}(bx+a)}\,b^{5}}$
gosper	$\frac{2(bx+a)\left(35d\,x^{4}b^{4}-40a\,b^{3}d\,x^{3}+45b^{4}c\,x^{3}+48a^{2}b^{2}d\,x^{2}-54a\,b^{3}c\,x^{2}-64a^{3}bdx+72a^{2}b^{2}cx+128a^{4}d-144a^{3}bc\right)x}{315b^{5}\sqrt{b\,x^{3}+a\,x^{2}}}$
default	$\frac{2(bx+a)\left(35d\,x^{4}b^{4}-40a\,b^{3}d\,x^{3}+45b^{4}c\,x^{3}+48a^{2}b^{2}d\,x^{2}-54a\,b^{3}c\,x^{2}-64a^{3}bdx+72a^{2}b^{2}cx+128a^{4}d-144a^{3}bc\right)x}{315b^{5}\sqrt{b\,x^{3}+a\,x^{2}}}$
orering	$\frac{2(bx+a)\left(35d\,x^{4}b^{4}-40a\,b^{3}d\,x^{3}+45b^{4}c\,x^{3}+48a^{2}b^{2}d\,x^{2}-54a\,b^{3}c\,x^{2}-64a^{3}bdx+72a^{2}b^{2}cx+128a^{4}d-144a^{3}bc\right)x}{315b^{5}\sqrt{b\,x^{3}+a\,x^{2}}}$

input
```
int(x^4*(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-512/693*(b*x+a)^(1/2)*(-77/256*x^4*(9/11*d*x+c)*b^5+11/32*x^3*(35/44*d*x+
c)*a*b^4-33/80*(25/33*d*x+c)*x^2*a^2*b^3+11/20*x*(15/22*d*x+c)*a^3*b^2-11/
10*(5/11*d*x+c)*a^4*b+a^5*d)/b^6
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.64

$$\int \frac{x^4(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\left(35\,b^4dx^4-144\,a^3bc+128\,a^4d+5\,(9\,b^4c-8\,ab^3d)x^3-6\,(9\,ab^3c-8\,a^2b^2d)x^2+8\,(9\,a^2b^2c-8\,a^3bd)x\right)\sqrt{}}{315\,b^5x}$$

input
```
integrate(x^4*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/315*(35*b^4*d*x^4 - 144*a^3*b*c + 128*a^4*d + 5*(9*b^4*c - 8*a*b^3*d)*x^
3 - 6*(9*a*b^3*c - 8*a^2*b^2*d)*x^2 + 8*(9*a^2*b^2*c - 8*a^3*b*d)*x)*sqrt(
b*x^3 + a*x^2)/(b^5*x)
```

Sympy [F]

$$\int \frac{x^4(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^4(c+dx)}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**4*(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**4*(c + d*x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.73

$$\int \frac{x^4(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(5\,b^4x^4 - ab^3x^3 + 2\,a^2b^2x^2 - 8\,a^3bx - 16\,a^4)c}{35\,\sqrt{bx+a}b^4} + \frac{2\,(35\,b^5x^5 - 5\,ab^4x^4 + 8\,a^2b^3x^3 - 16\,a^3b^2x^2 + 64\,a^4bx + 128\,a^5)d}{315\,\sqrt{bx+a}b^5}$$

input
```
integrate(x^4*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)*c/(sqrt(
b*x + a)*b^4) + 2/315*(35*b^5*x^5 - 5*a*b^4*x^4 + 8*a^2*b^3*x^3 - 16*a^3*b
^2*x^2 + 64*a^4*b*x + 128*a^5)*d/(sqrt(b*x + a)*b^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.87

$$\int \frac{x^4(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\left(\frac{9\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)c}{b^3}+\frac{\left(35\,(bx+a)^{\frac{9}{2}}-180\,(bx+a)^{\frac{7}{2}}a+378\,(bx+a)^{\frac{5}{2}}a^2-420\,(bx+a)^{\frac{3}{2}}a^3+315\,\sqrt{}\right.}{b^4}}{315\,b\text{sgn}\,(x)} + \frac{32\left(9\,a^{\frac{7}{2}}bc-8\,a^{\frac{9}{2}}d\right)\text{sgn}(x)}{315\,b^5}$$

input
```
integrate(x^4*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/315*(9*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*a^
2 - 35*sqrt(b*x + a)*a^3)*c/b^3 + (35*(b*x + a)^(9/2) - 180*(b*x + a)^(7/2
)*a + 378*(b*x + a)^(5/2)*a^2 - 420*(b*x + a)^(3/2)*a^3 + 315*sqrt(b*x + a
)*a^4)*d/b^4)/(b*sgn(x)) + 32/315*(9*a^(7/2)*b*c - 8*a^(9/2)*d)*sgn(x)/b^5
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.63

$$\int \frac{x^4(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{256\,a^4\,d-288\,a^3\,b\,c}{315\,b^5}+\frac{2\,d\,x^4}{9\,b}+\frac{x^3\,\left(90\,b^4\,c-80\,a\,b^3\,d\right)}{315\,b^5}+\frac{4\,a\,x^2\,\left(8\,a\,d-9\,b\,c\right)}{105\,b^3}-\frac{16\,a^2\,x\,\left(8\,a\,d-9\,b\,c\right)}{315\,b^4}\right)}{x}$$

input
```
int((x^4*(c + d*x))/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((256*a^4*d - 288*a^3*b*c)/(315*b^5) + (2*d*x^4)/(9
*b) + (x^3*(90*b^4*c - 80*a*b^3*d))/(315*b^5) + (4*a*x^2*(8*a*d - 9*b*c))/
(105*b^3) - (16*a^2*x*(8*a*d - 9*b*c))/(315*b^4)))/x
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.56

$$\int \frac{x^4(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,(35b^4d\,x^4-40a\,b^3d\,x^3+45b^4c\,x^3+48a^2b^2d\,x^2-54a\,b^3c\,x^2-64a^3bdx+72a^2b^2cx+128a^4d-}{315b^5}$$

input
```
int(x^4*(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(128*a**4*d - 144*a**3*b*c - 64*a**3*b*d*x + 72*a**2*b**2
*c*x + 48*a**2*b**2*d*x**2 - 54*a*b**3*c*x**2 - 40*a*b**3*d*x**3 + 45*b**4
*c*x**3 + 35*b**4*d*x**4))/(315*b**5)
```

3.274 $\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 129

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}} \, dx = \frac{2a^2(bc-ad)\sqrt{ax^2+bx^3}}{b^4x} - \frac{2a(2bc-3ad)\left(ax^2+bx^3\right)^{3/2}}{3b^4x^3} + \frac{2(bc-3ad)\left(ax^2+bx^3\right)^{5/2}}{5b^4x^5} + \frac{2d\left(ax^2+bx^3\right)^{7/2}}{7b^4x^7}$$

output
```
2*a^2*(-a*d+b*c)*(b*x^3+a*x^2)^(1/2)/b^4/x-2/3*a*(-3*a*d+2*b*c)*(b*x^3+a*x
^2)^(3/2)/b^4/x^3+2/5*(-3*a*d+b*c)*(b*x^3+a*x^2)^(5/2)/b^4/x^5+2/7*d*(b*x^
3+a*x^2)^(7/2)/b^4/x^7
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}} \, dx = \frac{2\sqrt{x^2(a+bx)}\left(-48a^3d+8a^2b(7c+3dx)+3b^3x^2(7c+5dx)-2ab^2x(14c+9dx)\right)}{105b^4x}$$

input
```
Integrate[(x^3*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(-48*a^3*d + 8*a^2*b*(7*c + 3*d*x) + 3*b^3*x^2*(7*c
 + 5*d*x) - 2*a*b^2*x*(14*c + 9*d*x)))/(105*b^4*x)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1945, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(7bc-6ad)\int \frac{x^3}{\sqrt{bx^3+ax^2}}dx}{7b}+\frac{2dx^2\sqrt{ax^2+bx^3}}{7b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(7bc-6ad)\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}\right)}{7b}+\frac{2dx^2\sqrt{ax^2+bx^3}}{7b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(7bc-6ad)\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}\right)}{7b}+\frac{2dx^2\sqrt{ax^2+bx^3}}{7b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{5b}\right)(7bc-6ad)}{7b}+\frac{2dx^2\sqrt{ax^2+bx^3}}{7b}$$

input
```
Int[(x^3*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*d*x^2*Sqrt[a*x^2 + b*x^3])/(7*b) + ((7*b*c - 6*a*d)*((2*x*Sqrt[a*x^2 +
b*x^3])/(5*b) - (4*a*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*
x^3])/(3*b^2*x)))/(5*b)))/(7*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.58

method	result	size
pseudoelliptic	$\frac{256\sqrt{bx+a}\left(\frac{45x^{3}\left(c+\frac{7dx}{9}\right)b^{4}}{128}-\frac{27x^{2}a\left(\frac{20dx}{27}+c\right)b^{3}}{64}+\frac{9x\left(\frac{2dx}{3}+c\right)a^{2}b^{2}}{16}-\frac{9a^{3}\left(\frac{4dx}{9}+c\right)b}{8}+a^{4}d\right)}{315b^{5}}$	75
trager	$-\frac{2\left(-15b^{3}d\,x^{3}+18a\,b^{2}d\,x^{2}-21b^{3}c\,x^{2}-24a^{2}bdx+28a\,b^{2}cx+48a^{3}d-56c\,a^{2}b\right)\sqrt{b\,x^{3}+a\,x^{2}}}{105b^{4}x}$	80
risch	$-\frac{2(bx+a)x\left(-15b^{3}d\,x^{3}+18a\,b^{2}d\,x^{2}-21b^{3}c\,x^{2}-24a^{2}bdx+28a\,b^{2}cx+48a^{3}d-56c\,a^{2}b\right)}{105\sqrt{x^{2}(bx+a)}\,b^{4}}$	81
gosper	$-\frac{2(bx+a)\left(-15b^{3}d\,x^{3}+18a\,b^{2}d\,x^{2}-21b^{3}c\,x^{2}-24a^{2}bdx+28a\,b^{2}cx+48a^{3}d-56c\,a^{2}b\right)x}{105b^{4}\sqrt{b\,x^{3}+a\,x^{2}}}$	83
default	$-\frac{2(bx+a)\left(-15b^{3}d\,x^{3}+18a\,b^{2}d\,x^{2}-21b^{3}c\,x^{2}-24a^{2}bdx+28a\,b^{2}cx+48a^{3}d-56c\,a^{2}b\right)x}{105b^{4}\sqrt{b\,x^{3}+a\,x^{2}}}$	83
orering	$-\frac{2(bx+a)\left(-15b^{3}d\,x^{3}+18a\,b^{2}d\,x^{2}-21b^{3}c\,x^{2}-24a^{2}bdx+28a\,b^{2}cx+48a^{3}d-56c\,a^{2}b\right)x}{105b^{4}\sqrt{b\,x^{3}+a\,x^{2}}}$	83

input
```
int(x^3*(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
256/315*(b*x+a)^(1/2)*(45/128*x^3*(c+7/9*d*x)*b^4-27/64*x^2*a*(20/27*d*x+c
)*b^3+9/16*x*(2/3*d*x+c)*a^2*b^2-9/8*a^3*(4/9*d*x+c)*b+a^4*d)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.63

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\left(15\,b^3dx^3+56\,a^2bc-48\,a^3d+3\,(7\,b^3c-6\,ab^2d)x^2-4\,(7\,ab^2c-6\,a^2bd)x\right)\sqrt{bx^3+ax^2}}{105\,b^4x}$$

input
```
integrate(x^3*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/105*(15*b^3*d*x^3 + 56*a^2*b*c - 48*a^3*d + 3*(7*b^3*c - 6*a*b^2*d)*x^2
- 4*(7*a*b^2*c - 6*a^2*b*d)*x)*sqrt(b*x^3 + a*x^2)/(b^4*x)
```

Sympy [F]

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^3(c+dx)}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**3*(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**3*(c + d*x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.76

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(3\,b^3x^3-ab^2x^2+4\,a^2bx+8\,a^3)c}{15\,\sqrt{bx+a}b^3} + \frac{2\,(5\,b^4x^4-ab^3x^3+2\,a^2b^2x^2-8\,a^3bx-16\,a^4)d}{35\,\sqrt{bx+a}b^4}$$

input
```
integrate(x^3*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/15*(3*b^3*x^3 - a*b^2*x^2 + 4*a^2*b*x + 8*a^3)*c/(sqrt(b*x + a)*b^3) + 2
/35*(5*b^4*x^4 - a*b^3*x^3 + 2*a^2*b^2*x^2 - 8*a^3*b*x - 16*a^4)*d/(sqrt(b
*x + a)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.93

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\left(\frac{7\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)c}{b^2}+\frac{3\left(5\,(bx+a)^{\frac{7}{2}}-21\,(bx+a)^{\frac{5}{2}}a+35\,(bx+a)^{\frac{3}{2}}a^2-35\,\sqrt{bx+a}a^3\right)d}{b^3}\right)}{105\,b\mathrm{sgn}\,(x)} - \frac{16\left(7\,a^{\frac{5}{2}}bc-6\,a^{\frac{7}{2}}d\right)\mathrm{sgn}(x)}{105\,b^4}$$

input
```
integrate(x^3*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/105*(7*(3*(b*x + a)^(5/2) - 10*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)
*c/b^2 + 3*(5*(b*x + a)^(7/2) - 21*(b*x + a)^(5/2)*a + 35*(b*x + a)^(3/2)*
a^2 - 35*sqrt(b*x + a)*a^3)*d/b^3)/(b*sgn(x)) - 16/105*(7*a^(5/2)*b*c - 6*
a^(7/2)*d)*sgn(x)/b^4
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.65

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,d\,x^3}{7\,b}-\frac{96\,a^3\,d-112\,a^2\,b\,c}{105\,b^4}+\frac{x^2\,\left(42\,b^3\,c-36\,a\,b^2\,d\right)}{105\,b^4}+\frac{8\,a\,x\,\left(6\,a\,d-7\,b\,c\right)}{105\,b^3}\right)}{x}$$

input
```
int((x^3*(c + d*x))/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((2*d*x^3)/(7*b) - (96*a^3*d - 112*a^2*b*c)/(105*b^
4) + (x^2*(42*b^3*c - 36*a*b^2*d))/(105*b^4) + (8*a*x*(6*a*d - 7*b*c))/(10
5*b^3)))/x
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.53

$$\int \frac{x^3(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,(15b^3d\,x^3-18a\,b^2d\,x^2+21b^3c\,x^2+24a^2bdx-28a\,b^2cx-48a^3d+56a^2bc)}{105b^4}$$

input
```
int(x^3*(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 48*a**3*d + 56*a**2*b*c + 24*a**2*b*d*x - 28*a*b**2*c
*x - 18*a*b**2*d*x**2 + 21*b**3*c*x**2 + 15*b**3*d*x**3))/(105*b**4)
```

3.275 $\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}} \, dx = -\frac{2a(bc-ad)\sqrt{ax^2+bx^3}}{b^3x} + \frac{2(bc-2ad)\left(ax^2+bx^3\right)^{3/2}}{3b^3x^3} + \frac{2d\left(ax^2+bx^3\right)^{5/2}}{5b^3x^5}$$

output
```
-2*a*(-a*d+b*c)*(b*x^3+a*x^2)^(1/2)/b^3/x+2/3*(-2*a*d+b*c)*(b*x^3+a*x^2)^(
3/2)/b^3/x^3+2/5*d*(b*x^3+a*x^2)^(5/2)/b^3/x^5
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.60

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}} \, dx = \frac{2\sqrt{x^2(a+bx)}(8a^2d-2ab(5c+2dx)+b^2x(5c+3dx))}{15b^3x}$$

input
```
Integrate[(x^2*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(8*a^2*d - 2*a*b*(5*c + 2*d*x) + b^2*x*(5*c + 3*d*x
)))/(15*b^3*x)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1945, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(5bc-4ad)\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}+\frac{2dx\sqrt{ax^2+bx^3}}{5b}$$

$$\Big\downarrow \text{1922}$$

$$\frac{(5bc-4ad)\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}+\frac{2dx\sqrt{ax^2+bx^3}}{5b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)(5bc-4ad)}{5b}+\frac{2dx\sqrt{ax^2+bx^3}}{5b}$$

input
```
Int[(x^2*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*d*x*Sqrt[a*x^2 + b*x^3])/(5*b) + ((5*b*c - 4*a*d)*((2*Sqrt[a*x^2 + b*x^
3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)))/(5*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

method	result	size
trager	$\frac{2(3b^2d\,x^2-4abdx+5b^2cx+8a^2d-10abc)\sqrt{b\,x^3+a\,x^2}}{15b^3x}$	56
risch	$\frac{2(bx+a)x(3b^2d\,x^2-4abdx+5b^2cx+8a^2d-10abc)}{15\sqrt{x^2(bx+a)}\,b^3}$	57
pseudoelliptic	$-\frac{32\left(-\frac{7x^2\left(\frac{5dx}{7}+c\right)b^3}{16}+\frac{7\left(\frac{9dx}{14}+c\right)xa\,b^2}{12}-\frac{7\left(\frac{3dx}{7}+c\right)a^2b}{6}+a^3d\right)\sqrt{bx+a}}{35b^4}$	58
gosper	$\frac{2(bx+a)(3b^2d\,x^2-4abdx+5b^2cx+8a^2d-10abc)x}{15b^3\sqrt{b\,x^3+a\,x^2}}$	59
default	$\frac{2(bx+a)(3b^2d\,x^2-4abdx+5b^2cx+8a^2d-10abc)x}{15b^3\sqrt{b\,x^3+a\,x^2}}$	59
orering	$\frac{2(bx+a)(3b^2d\,x^2-4abdx+5b^2cx+8a^2d-10abc)x}{15b^3\sqrt{b\,x^3+a\,x^2}}$	59

input
```
int(x^2*(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(3*b^2*d*x^2-4*a*b*d*x+5*b^2*c*x+8*a^2*d-10*a*b*c)/b^3/x*(b*x^3+a*x^2
)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.61

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(3\,b^2dx^2-10\,abc+8\,a^2d+(5\,b^2c-4\,abd)x)\sqrt{bx^3+ax^2}}{15\,b^3x}$$

input
```
integrate(x^2*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^2*d*x^2 - 10*a*b*c + 8*a^2*d + (5*b^2*c - 4*a*b*d)*x)*sqrt(b*x^3
 + a*x^2)/(b^3*x)
```

Sympy [F]

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x^2(c+dx)}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x**2*(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x**2*(c + d*x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.82

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,(b^2x^2-abx-2\,a^2)c}{3\,\sqrt{bx+a}b^2}+\frac{2\,(3\,b^3x^3-ab^2x^2+4\,a^2bx+8\,a^3)d}{15\,\sqrt{bx+a}b^3}$$

input
```
integrate(x^2*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2/3*(b^2*x^2 - a*b*x - 2*a^2)*c/(sqrt(b*x + a)*b^2) + 2/15*(3*b^3*x^3 - a*
b^2*x^2 + 4*a^2*b*x + 8*a^3)*d/(sqrt(b*x + a)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.01

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\left(\frac{5\left((bx+a)^{\frac{3}{2}}-3\,\sqrt{bx+a}a\right)c}{b}+\frac{\left(3\,(bx+a)^{\frac{5}{2}}-10\,(bx+a)^{\frac{3}{2}}a+15\,\sqrt{bx+a}a^2\right)d}{b^2}\right)}{15\,b\mathrm{sgn}\left(x\right)} + \frac{4\left(5\,a^{\frac{3}{2}}bc-4\,a^{\frac{5}{2}}d\right)\mathrm{sgn}(x)}{15\,b^3}$$

input
```
integrate(x^2*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/15*(5*((b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*c/b + (3*(b*x + a)^(5/2) - 1
0*(b*x + a)^(3/2)*a + 15*sqrt(b*x + a)*a^2)*d/b^2)/(b*sgn(x)) + 4/15*(5*a^
(3/2)*b*c - 4*a^(5/2)*d)*sgn(x)/b^3
```

Mupad [B] (verification not implemented)

Time = 8.92 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.67

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{16\,a^2\,d-20\,a\,b\,c}{15\,b^3}+\frac{x\,\left(10\,b^2\,c-8\,a\,b\,d\right)}{15\,b^3}+\frac{2\,d\,x^2}{5\,b}\right)}{x}$$

input
```
int((x^2*(c + d*x))/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((16*a^2*d - 20*a*b*c)/(15*b^3) + (x*(10*b^2*c - 8*
a*b*d))/(15*b^3) + (2*d*x^2)/(5*b)))/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.49

$$\int \frac{x^2(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,(3b^2d\,x^2-4abdx+5b^2cx+8a^2d-10abc)}{15b^3}$$

input
```
int(x^2*(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*(8*a**2*d - 10*a*b*c - 4*a*b*d*x + 5*b**2*c*x + 3*b**2*d*
x**2))/(15*b**3)
```

3.276 $\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 58

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}} \, dx = \frac{2(bc-ad)\sqrt{ax^2+bx^3}}{b^2x} + \frac{2d(ax^2+bx^3)^{3/2}}{3b^2x^3}$$

output
```
2*(-a*d+b*c)*(b*x^3+a*x^2)^(1/2)/b^2/x+2/3*d*(b*x^3+a*x^2)^(3/2)/b^2/x^3
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}} \, dx = \frac{2\sqrt{x^2(a+bx)}(3bc-2ad+bdx)}{3b^2x}$$

input
```
Integrate[(x*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*Sqrt[x^2*(a + b*x)]*(3*b*c - 2*a*d + b*d*x))/(3*b^2*x)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1945, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(3bc-2ad)\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}+\frac{2d\sqrt{ax^2+bx^3}}{3b}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\sqrt{ax^2+bx^3}(3bc-2ad)}{3b^2x}+\frac{2d\sqrt{ax^2+bx^3}}{3b}$$

input
```
Int[(x*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*d*Sqrt[a*x^2 + b*x^3])/(3*b) + (2*(3*b*c - 2*a*d)*Sqrt[a*x^2 + b*x^3])/
(3*b^2*x)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.62

method	result	size
trager	$-\frac{2(-bdx+2ad-3bc)\sqrt{b\,x^{3}+a\,x^{2}}}{3b^{2}x}$	36
risch	$-\frac{2(bx+a)x(-bdx+2ad-3bc)}{3\sqrt{x^{2}(bx+a)}\,b^{2}}$	37
gosper	$-\frac{2(bx+a)(-bdx+2ad-3bc)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	39
default	$-\frac{2(bx+a)(-bdx+2ad-3bc)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	39
orering	$-\frac{2(bx+a)(-bdx+2ad-3bc)x}{3b^{2}\sqrt{b\,x^{3}+a\,x^{2}}}$	39
pseudoelliptic	$\frac{16\sqrt{bx+a}\left(\frac{5\left(\frac{3dx}{5}+c\right)x\,b^{2}}{8}-\frac{5\left(\frac{2dx}{5}+c\right)ab}{4}+a^{2}d\right)}{15b^{3}}$	41

input
```
int(x*(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-b*d*x+2*a*d-3*b*c)/b^2/x*(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.59

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,\sqrt{bx^3+ax^2}(bdx+3\,bc-2\,ad)}{3\,b^2x}$$

input
```
integrate(x*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
2/3*sqrt(b*x^3 + a*x^2)*(b*d*x + 3*b*c - 2*a*d)/(b^2*x)
```

Sympy [F]

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{x(c+dx)}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate(x*(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(x*(c + d*x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.78

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\,\sqrt{bx+a}c}{b} + \frac{2\,(b^2x^2-abx-2\,a^2)d}{3\,\sqrt{bx+a}b^2}$$

input
```
integrate(x*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(b*x + a)*c/b + 2/3*(b^2*x^2 - a*b*x - 2*a^2)*d/(sqrt(b*x + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.12

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = -\frac{2\left(3\sqrt{a}bc-2a^{\frac{3}{2}}d\right)\mathrm{sgn}(x)}{3b^2}+\frac{2\left(3\sqrt{bx+a}c+\frac{\left((bx+a)^{\frac{3}{2}}-3\sqrt{bx+a}a\right)d}{b}\right)}{3b\,\mathrm{sgn}(x)}$$

input
```
integrate(x*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/3*(3*sqrt(a)*b*c - 2*a^(3/2)*d)*sgn(x)/b^2 + 2/3*(3*sqrt(b*x + a)*c + (
(b*x + a)^(3/2) - 3*sqrt(b*x + a)*a)*d/b)/(b*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.69 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.69

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = -\frac{\left(\frac{4\,a\,d-6\,b\,c}{3\,b^2}-\frac{2\,d\,x}{3\,b}\right)\sqrt{b\,x^3+a\,x^2}}{x}$$

input
```
int((x*(c + d*x))/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
-(((4*a*d - 6*b*c)/(3*b^2) - (2*d*x)/(3*b))*(a*x^2 + b*x^3)^(1/2))/x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.41

$$\int \frac{x(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,(bdx-2ad+3bc)}{3b^2}$$

input
```
int(x*(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*( - 2*a*d + 3*b*c + b*d*x))/(3*b**2)
```

3.277 $\int \frac{c+dx}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 58

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}\,dx = \frac{2d\sqrt{ax^2+bx^3}}{bx} - \frac{2c\,\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{\sqrt{a}}$$

output
```
2*d*(b*x^3+a*x^2)^(1/2)/b/x-2*c*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(
1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.14

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}\,dx = \frac{2x\left(\sqrt{a}d(a+bx) - bc\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{\sqrt{a}b\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*x*(Sqrt[a]*d*(a + b*x) - b*c*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a
]]))/(Sqrt[a]*b*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.97, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2450, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \; 2450$$

$$\int \left(\frac{c}{\sqrt{ax^2+bx^3}}+\frac{dx}{\sqrt{ax^2+bx^3}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2d\sqrt{ax^2+bx^3}}{bx}-\frac{2c\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{a}}$$

input
```
Int[(c + d*x)/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(2*d*Sqrt[a*x^2 + b*x^3])/(b*x) - (2*c*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*
x^3]])/Sqrt[a]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2450
```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IntegerQ[p] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.47

method	result	size
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,(-bdx+2ad-3bc)}{3b^2}$	27
default	$\frac{2x\sqrt{bx+a}\left(\sqrt{bx+a}\,d\sqrt{a}-bc\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\right)}{\sqrt{b\,x^3+a\,x^2}\,b\sqrt{a}}$	59

input
```
int((d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)^(1/2)*(-b*d*x+2*a*d-3*b*c)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.36

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}\,dx$$
$$= \left[\frac{\sqrt{a}bcx\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}ad}{abx}, \frac{2\left(\sqrt{-a}bcx\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}\right.}{abx}\right.$$

input
```
integrate((d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(a)*b*c*x*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) +
 2*sqrt(b*x^3 + a*x^2)*a*d)/(a*b*x), 2*(sqrt(-a)*b*c*x*arctan(sqrt(b*x^3 +
 a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*a*d)/(a*b*x)]
```

Sympy [F]

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}\,dx=\int \frac{c+dx}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}\,dx=\int \frac{dx+c}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}\,dx=-\frac{2\left(bc\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}\sqrt{a}d\right)\mathrm{sgn}(x)}{\sqrt{-a}b}+\frac{2\left(\frac{c\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\sqrt{bx+a}d}{b}\right)}{\mathrm{sgn}\left(x\right)}$$

input
```
integrate((d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2*(b*c*arctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*sqrt(a)*d)*sgn(x)/(sqrt(-a)*b)
 + 2*(c*arctan(sqrt(b*x + a)/sqrt(-a))/sqrt(-a) + sqrt(b*x + a)*d/b)/sgn(x
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}\,dx=\int \frac{c+d\,x}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int((c + d*x)/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.88

$$\int \frac{c+dx}{\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}\,ad+\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bc-\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bc}{ab}$$

input
```
int((d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(a + b*x)*a*d + sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*c - sqrt(a)*
log(sqrt(a + b*x) + sqrt(a))*b*c)/(a*b)
```

3.278 $\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 64

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx=-\frac{c\sqrt{ax^2+bx^3}}{ax^2}+\frac{(bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{a^{3/2}}$$

output
```
-c*(b*x^3+a*x^2)^(1/2)/a/x^2+(-2*a*d+b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1
/2)/x)/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.06

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx=\frac{-\sqrt{a}c(a+bx)+(bc-2ad)x\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{3/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)/(x*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(Sqrt[a]*c*(a + b*x)) + (b*c - 2*a*d)*x*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b
*x]/Sqrt[a]])/(a^(3/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1944, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{ 1944}$$

$$-\frac{(bc-2ad)\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{c\sqrt{ax^2+bx^3}}{ax^2}$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{(bc-2ad)\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{c\sqrt{ax^2+bx^3}}{ax^2}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)(bc-2ad)}{a^{3/2}}-\frac{c\sqrt{ax^2+bx^3}}{ax^2}$$

input
```
Int[(c + d*x)/(x*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
-((c*Sqrt[a*x^2 + b*x^3])/(a*x^2)) + ((b*c - 2*a*d)*ArcTanh[(Sqrt[a]*x)/Sq
rt[a*x^2 + b*x^3]])/a^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.59

method	result	size
pseudoelliptic	$\frac{2\sqrt{bx+a}\, d\sqrt{a}-2bc \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{b\sqrt{a}}$	38
risch	$-\frac{c(bx+a)}{a\sqrt{x^2(bx+a)}}-\frac{(2ad-bc)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\, x}{a^{\frac{3}{2}}\sqrt{x^2(bx+a)}}$	69
default	$-\frac{\sqrt{bx+a}\left(a^{\frac{3}{2}}c\sqrt{bx+a}+2\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^2dx-\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abcx\right)}{\sqrt{b\,x^3+a\,x^2}\, a^{\frac{5}{2}}}$	76

input
```
int((d*x+c)/x/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((b*x+a)^(1/2)*d*a^(1/2)-b*c*arctanh((b*x+a)^(1/2)/a^(1/2)))/b/a^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.31

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx = \left[-\frac{(bc-2\,ad)\sqrt{a}x^2\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}ac}{2\,a^2x^2},\right.$$
$$\left.-\frac{(bc-2\,ad)\sqrt{-a}x^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}ac}{a^2x^2}\right]$$

input
```
integrate((d*x+c)/x/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((b*c - 2*a*d)*sqrt(a)*x^2*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2
)*sqrt(a))/x^2) + 2*sqrt(b*x^3 + a*x^2)*a*c)/(a^2*x^2), -((b*c - 2*a*d)*sq
rt(-a)*x^2*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3
 + a*x^2)*a*c)/(a^2*x^2)]
```

Sympy [F]

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+dx}{x\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/x/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)/(x*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+ax^2}x}\,dx$$

input
```
integrate((d*x+c)/x/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(sqrt(b*x^3 + a*x^2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.94

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx = -\frac{b\left(\frac{(bc-2\,ad)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}ab}+\frac{\sqrt{bx+a}c}{abx}\right)}{\operatorname{sgn}(x)}$$

input
```
integrate((d*x+c)/x/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-b*((b*c - 2*a*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a*b) + sqrt(b*x
 + a)*c/(a*b*x))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+d\,x}{x\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)/(x*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int((c + d*x)/(x*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.41

$$\int \frac{c+dx}{x\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{-2\sqrt{bx+a}\,ac+2\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)adx-\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bcx-2\sqrt{a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)}{2a^2x}$$

input
```
int((d*x+c)/x/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 2*sqrt(a + b*x)*a*c + 2*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*d*x - s
qrt(a)*log(sqrt(a + b*x) - sqrt(a))*b*c*x - 2*sqrt(a)*log(sqrt(a + b*x) +
sqrt(a))*a*d*x + sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*c*x)/(2*a**2*x)
```

3.279 $\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 105

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx=-\frac{c\sqrt{ax^2+bx^3}}{2ax^3}+\frac{(3bc-4ad)\sqrt{ax^2+bx^3}}{4a^2x^2}-\frac{b(3bc-4ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{4a^{5/2}}$$

output
```
-1/2*c*(b*x^3+a*x^2)^(1/2)/a/x^3+1/4*(-4*a*d+3*b*c)*(b*x^3+a*x^2)^(1/2)/a^
2/x^2-1/4*b*(-4*a*d+3*b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.89

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx=\frac{-\sqrt{a}(a+bx)(-3bcx+2a(c+2dx))-b(3bc-4ad)x^2\sqrt{a+bx}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{4a^{5/2}x\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(Sqrt[a]*(a + b*x)*(-3*b*c*x + 2*a*(c + 2*d*x))) - b*(3*b*c - 4*a*d)*x^2
*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(5/2)*x*Sqrt[x^2*(a +
b*x)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1944, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(3bc-4ad)\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{c\sqrt{ax^2+bx^3}}{2ax^3}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{(3bc-4ad)\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{c\sqrt{ax^2+bx^3}}{2ax^3}$$

$$\Big\downarrow \text{1914}$$

$$-\frac{(3bc-4ad)\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{c\sqrt{ax^2+bx^3}}{2ax^3}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)(3bc-4ad)}{4a}-\frac{c\sqrt{ax^2+bx^3}}{2ax^3}$$

input
```
Int[(c + d*x)/(x^2*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
-1/2*(c*Sqrt[a*x^2 + b*x^3])/(a*x^3) - ((3*b*c - 4*a*d)*(-(Sqrt[a*x^2 + b*
x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*
a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.41

method	result
pseudoelliptic	$-\frac{c\sqrt{bx+a}}{ax}-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\left(ad-\frac{bc}{2}\right)}{a^{\frac{3}{2}}}$
risch	$-\frac{(bx+a)(4adx-3cbx+2ac)}{4a^2x\sqrt{x^2(bx+a)}}+\frac{(4ad-3bc)b\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{4a^{\frac{5}{2}}\sqrt{x^2(bx+a)}}$
default	$-\frac{\sqrt{bx+a}\left(4(bx+a)^{\frac{3}{2}}a^{\frac{7}{2}}d-3(bx+a)^{\frac{3}{2}}a^{\frac{5}{2}}bc-4\sqrt{bx+a}\,a^{\frac{9}{2}}d+5\sqrt{bx+a}\,a^{\frac{7}{2}}bc-4\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^3b^2d\,x^2+3\,\mathrm{arctanh}\left(\frac{\sqrt{bx}}{\sqrt{}}\right.}{4xb\sqrt{b\,x^3+a\,x^2}\,a^{\frac{9}{2}}}$

input
```
int((d*x+c)/x^2/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-c/a*(b*x+a)^(1/2)/x-2*arctanh((b*x+a)^(1/2)/a^(1/2))*(a*d-1/2*b*c)/a^(3/2
)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.88

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx$$
$$=\left[-\frac{(3\,b^2c-4\,abd)\sqrt{a}x^3\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(2\,a^2c-(3\,abc-4\,a^2d)x)}{8\,a^3x^3},\frac{(3\,b^2c-4}{}\right.$$

input
```
integrate((d*x+c)/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((3*b^2*c - 4*a*b*d)*sqrt(a)*x^3*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 +
 a*x^2)*sqrt(a))/x^2) + 2*sqrt(b*x^3 + a*x^2)*(2*a^2*c - (3*a*b*c - 4*a^2*
d)*x))/(a^3*x^3), 1/4*((3*b^2*c - 4*a*b*d)*sqrt(-a)*x^3*arctan(sqrt(b*x^3
+ a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - sqrt(b*x^3 + a*x^2)*(2*a^2*c - (3*a*b*c
 - 4*a^2*d)*x))/(a^3*x^3)]
```

Sympy [F]

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx=\int \frac{c+dx}{x^2\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/x**2/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)/(x**2*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx=\int \frac{dx+c}{\sqrt{bx^3+ax^2}x^2}\,dx$$

input
```
integrate((d*x+c)/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(sqrt(b*x^3 + a*x^2)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.10

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx$$
$$=\frac{\frac{(3\,b^3c-4\,ab^2d)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2}+\frac{3\,(bx+a)^{\frac{3}{2}}b^3c-5\,\sqrt{bx+a}ab^3c-4\,(bx+a)^{\frac{3}{2}}ab^2d+4\,\sqrt{bx+a}a^2b^2d}{a^2b^2x^2}}{4\,b\mathrm{sgn}\left(x\right)}$$

input
```
integrate((d*x+c)/x^2/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*((3*b^3*c - 4*a*b^2*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2) +
 (3*(b*x + a)^(3/2)*b^3*c - 5*sqrt(b*x + a)*a*b^3*c - 4*(b*x + a)^(3/2)*a*
b^2*d + 4*sqrt(b*x + a)*a^2*b^2*d)/(a^2*b^2*x^2))/(b*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+d\,x}{x^2\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)/(x^2*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int((c + d*x)/(x^2*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.26

$$\int \frac{c+dx}{x^2\sqrt{ax^2+bx^3}}\,dx = \frac{-4\sqrt{bx+a}\,a^2c-8\sqrt{bx+a}\,a^2dx+6\sqrt{bx+a}\,abcx-4\sqrt{a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)abd\,x^2+3\sqrt{a}\log\left(\sqrt{bx+\right.}}{8a^3x^2}$$

input
```
int((d*x+c)/x^2/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 4*sqrt(a + b*x)*a**2*c - 8*sqrt(a + b*x)*a**2*d*x + 6*sqrt(a + b*x)*a*
b*c*x - 4*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b*d*x**2 + 3*sqrt(a)*log(
sqrt(a + b*x) - sqrt(a))*b**2*c*x**2 + 4*sqrt(a)*log(sqrt(a + b*x) + sqrt(
a))*a*b*d*x**2 - 3*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**2*c*x**2)/(8*a*
*3*x**2)
```

3.280 $\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 142

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx = -\frac{c\sqrt{ax^2+bx^3}}{3ax^4}+\frac{(5bc-6ad)\sqrt{ax^2+bx^3}}{12a^2x^3} - \frac{b(5bc-6ad)\sqrt{ax^2+bx^3}}{8a^3x^2}+\frac{b^2(5bc-6ad)\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{8a^{7/2}}$$

output
```
-1/3*c*(b*x^3+a*x^2)^(1/2)/a/x^4+1/12*(-6*a*d+5*b*c)*(b*x^3+a*x^2)^(1/2)/a
^2/x^3-1/8*b*(-6*a*d+5*b*c)*(b*x^3+a*x^2)^(1/2)/a^3/x^2+1/8*b^2*(-6*a*d+5*
b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx = \frac{-\sqrt{a}(a+bx)\left(15b^2cx^2+4a^2(2c+3dx)-2abx(5c+9dx)\right)+3b^2(5bc-6ad)x^3\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{24a^{7/2}x^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(Sqrt[a]*(a + b*x)*(15*b^2*c*x^2 + 4*a^2*(2*c + 3*d*x) - 2*a*b*x*(5*c +
9*d*x))) + 3*b^2*(5*b*c - 6*a*d)*x^3*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/S
qrt[a]])/(24*a^(7/2)*x^2*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1944, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(5bc-6ad)\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{c\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{(5bc-6ad)\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{c\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{(5bc-6ad)\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{c\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{1914}$$

$$-\frac{(5bc-6ad)\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{c\sqrt{ax^2+bx^3}}{3ax^4}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)(5bc-6ad)}{6a}-\frac{c\sqrt{ax^2+bx^3}}{3ax^4}$$

input
```
Int[(c + d*x)/(x^3*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
-1/3*(c*Sqrt[a*x^2 + b*x^3])/(a*x^4) - ((5*b*c - 6*a*d)*(-1/2*Sqrt[a*x^2 +
 b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[
a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.45

method	result
pseudoelliptic	$\frac{b\,x^{2}\left(ad-\frac{3bc}{4}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{3\sqrt{bx+a}\,\left(\frac{2(-2dx-c)a^{\frac{3}{2}}}{3}+\sqrt{a}\,bcx\right)}{4}}{a^{\frac{5}{2}}\,x^{2}}$
risch	$-\frac{(bx+a)\left(-18abd\,x^{2}+15b^{2}c\,x^{2}+12a^{2}dx-10abcx+8a^{2}c\right)}{24a^{3}x^{2}\sqrt{x^{2}(bx+a)}}-\frac{(6ad-5bc)b^{2}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)\sqrt{bx+a}\,x}{8a^{\frac{7}{2}}\sqrt{x^{2}(bx+a)}}$
default	$\frac{\sqrt{bx+a}\,\left(18(bx+a)^{\frac{5}{2}}a^{\frac{9}{2}}d-15(bx+a)^{\frac{5}{2}}a^{\frac{7}{2}}bc-48(bx+a)^{\frac{3}{2}}a^{\frac{11}{2}}d+40(bx+a)^{\frac{3}{2}}a^{\frac{9}{2}}bc+30\sqrt{bx+a}\,a^{\frac{13}{2}}d-33\sqrt{bx+a}\,a^{\frac{11}{2}}bc-18\ a\right.}{24x^{2}b\sqrt{b\,x^{3}+a\,x^{2}}\,a^{\frac{13}{2}}}$

input
```
int((d*x+c)/x^3/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/a^(5/2)*(b*x^2*(a*d-3/4*b*c)*arctanh((b*x+a)^(1/2)/a^(1/2))+3/4*(b*x+a)^
(1/2)*(2/3*(-2*d*x-c)*a^(3/2)+a^(1/2)*b*c*x))/x^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.73

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx$$
$$= \left[-\frac{3\,(5\,b^3c-6\,ab^2d)\sqrt{a}x^4\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(8\,a^3c+3\,(5\,ab^2c-6\,a^2bd)x^2-2\,(5\,a^2bc-6\,a}{48\,a^4x^4}\right.$$
$$-\frac{3\,(5\,b^3c-6\,ab^2d)\sqrt{-a}x^4\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(8\,a^3c+3\,(5\,ab^2c-6\,a^2bd)x^2-2\,(5\,a^2bc-6\,a^3d)x)}{24\,a^4x^4}$$

input
```
integrate((d*x+c)/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(5*b^3*c - 6*a*b^2*d)*sqrt(a)*x^4*log((b*x^2 + 2*a*x - 2*sqrt(b*
x^3 + a*x^2)*sqrt(a))/x^2) + 2*(8*a^3*c + 3*(5*a*b^2*c - 6*a^2*b*d)*x^2 -
2*(5*a^2*b*c - 6*a^3*d)*x)*sqrt(b*x^3 + a*x^2))/(a^4*x^4), -1/24*(3*(5*b^3
*c - 6*a*b^2*d)*sqrt(-a)*x^4*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 +
a*x)) + (8*a^3*c + 3*(5*a*b^2*c - 6*a^2*b*d)*x^2 - 2*(5*a^2*b*c - 6*a^3*d)
*x)*sqrt(b*x^3 + a*x^2))/(a^4*x^4)]
```

Sympy [F]

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+dx}{x^3\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/x**3/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)/(x**3*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+ax^2}x^3}\,dx$$

input
```
integrate((d*x+c)/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(sqrt(b*x^3 + a*x^2)*x^3), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.92

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx = -\frac{b^3\left(\frac{3\,(5\,bc-6\,ad)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3b}+\frac{15\,(bx+a)^{\frac{5}{2}}bc-40\,(bx+a)^{\frac{3}{2}}abc+33\,\sqrt{bx+a}a^2bc-18\,(bx+a)^{\frac{5}{2}}ad+48\,(bx+a)^{\frac{3}{2}}a^2d-30\,\sqrt{bx+a}a^3d}{a^3b^4x^3}\right)}{24\,\mathrm{sgn}\,(x)}$$

input
```
integrate((d*x+c)/x^3/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-1/24*b^3*(3*(5*b*c - 6*a*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3*
b) + (15*(b*x + a)^(5/2)*b*c - 40*(b*x + a)^(3/2)*a*b*c + 33*sqrt(b*x + a)
*a^2*b*c - 18*(b*x + a)^(5/2)*a*d + 48*(b*x + a)^(3/2)*a^2*d - 30*sqrt(b*x
 + a)*a^3*d)/(a^3*b^4*x^3))/sgn(x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+d\,x}{x^3\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)/(x^3*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int((c + d*x)/(x^3*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.20

$$\int \frac{c+dx}{x^3\sqrt{ax^2+bx^3}}\,dx = \frac{-16\sqrt{bx+a}\,a^3c-24\sqrt{bx+a}\,a^3dx+20\sqrt{bx+a}\,a^2bcx+36\sqrt{bx+a}\,a^2bd\,x^2-30\sqrt{bx+a}\,a\,b^2c\,x^2+18}{}$$

input
```
int((d*x+c)/x^3/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 16*sqrt(a + b*x)*a**3*c - 24*sqrt(a + b*x)*a**3*d*x + 20*sqrt(a + b*x)
*a**2*b*c*x + 36*sqrt(a + b*x)*a**2*b*d*x**2 - 30*sqrt(a + b*x)*a*b**2*c*x
**2 + 18*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**2*d*x**3 - 15*sqrt(a)*l
og(sqrt(a + b*x) - sqrt(a))*b**3*c*x**3 - 18*sqrt(a)*log(sqrt(a + b*x) + s
qrt(a))*a*b**2*d*x**3 + 15*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b**3*c*x**
3)/(48*a**4*x**3)
```

3.281 $\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 179

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx = -\frac{c\sqrt{ax^2+bx^3}}{4ax^5}+\frac{(7bc-8ad)\sqrt{ax^2+bx^3}}{24a^2x^4} - \frac{5b(7bc-8ad)\sqrt{ax^2+bx^3}}{96a^3x^3}+\frac{5b^2(7bc-8ad)\sqrt{ax^2+bx^3}}{64a^4x^2} - \frac{5b^3(7bc-8ad)\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{64a^{9/2}}$$

output
```
-1/4*c*(b*x^3+a*x^2)^(1/2)/a/x^5+1/24*(-8*a*d+7*b*c)*(b*x^3+a*x^2)^(1/2)/a
^2/x^4-5/96*b*(-8*a*d+7*b*c)*(b*x^3+a*x^2)^(1/2)/a^3/x^3+5/64*b^2*(-8*a*d+
7*b*c)*(b*x^3+a*x^2)^(1/2)/a^4/x^2-5/64*b^3*(-8*a*d+7*b*c)*arctanh((b*x^3+
a*x^2)^(1/2)/a^(1/2)/x)/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.75

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx$$
$$=\frac{-\sqrt{a}(a+bx)\left(-105b^3cx^3+16a^3(3c+4dx)-8a^2bx(7c+10dx)+10ab^2x^2(7c+12dx)\right)-15b^3(7bc-8a}{192a^{9/2}x^3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)/(x^4*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-(Sqrt[a]*(a + b*x)*(-105*b^3*c*x^3 + 16*a^3*(3*c + 4*d*x) - 8*a^2*b*x*(7
*c + 10*d*x) + 10*a*b^2*x^2*(7*c + 12*d*x))) - 15*b^3*(7*b*c - 8*a*d)*x^4*
Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(192*a^(9/2)*x^3*Sqrt[x^2*(a
 + b*x)])
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1944, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx$$
$$\Big\downarrow \text{1944}$$
$$-\frac{(7bc-8ad)\int \frac{1}{x^3\sqrt{bx^3+ax^2}}dx}{8a}-\frac{c\sqrt{ax^2+bx^3}}{4ax^5}$$
$$\Big\downarrow \text{1931}$$
$$-\frac{(7bc-8ad)\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{c\sqrt{ax^2+bx^3}}{4ax^5}$$
$$\Big\downarrow \text{1931}$$

$$-\frac{(7bc-8ad)\left(-\frac{5b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{c\sqrt{ax^2+bx^3}}{4ax^5}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{(7bc-8ad)\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{c\sqrt{ax^2+bx^3}}{4ax^5}$$

$$\Big\downarrow \text{1914}$$

$$-\frac{(7bc-8ad)\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{c\sqrt{ax^2+bx^3}}{4ax^5}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)(7bc-8ad)}{8a}-\frac{c\sqrt{ax^2+bx^3}}{4ax^5}$$

input
```
Int[(c + d*x)/(x^4*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
-1/4*(c*Sqrt[a*x^2 + b*x^3])/(a*x^5) - ((7*b*c - 8*a*d)*(-1/3*Sqrt[a*x^2 +
 b*x^3]/(a*x^4) - (5*b*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*
x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/
2)))/(4*a)))/(6*a)))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.46

method	result
pseudoelliptic	$-\frac{3\left(b^{2}x^{3}\left(ad-\frac{5bc}{6}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\frac{4\left(-\frac{5xb\left(\frac{9dx}{5}+c\right)a^{\frac{3}{2}}}{4}+\left(\frac{3dx}{2}+c\right)a^{\frac{5}{2}}+\frac{15\sqrt{a}\,b^{2}c\,x^{2}}{8}\right)\sqrt{bx+a}}{9}\right)}{4a^{\frac{7}{2}}x^{3}}$
risch	$-\frac{(bx+a)\left(120a\,b^{2}d\,x^{3}-105b^{3}c\,x^{3}-80a^{2}bd\,x^{2}+70a\,b^{2}c\,x^{2}+64a^{3}dx-56a^{2}bcx+48c\,a^{3}\right)}{192a^{4}x^{3}\sqrt{x^{2}(bx+a)}}+\frac{5(8ad-7bc)b^{3}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{64a^{\frac{9}{2}}\sqrt{x^{2}(bx+a)}}$
default	$-\frac{\sqrt{bx+a}\left(120(bx+a)^{\frac{7}{2}}a^{\frac{11}{2}}d-105(bx+a)^{\frac{7}{2}}a^{\frac{9}{2}}bc-440(bx+a)^{\frac{5}{2}}a^{\frac{13}{2}}d+385(bx+a)^{\frac{5}{2}}a^{\frac{11}{2}}bc+584(bx+a)^{\frac{3}{2}}a^{\frac{15}{2}}d-511(bx+a)\right.}{192x^{3}b\sqrt{b\,x^{3}+}}$

input
```
int((d*x+c)/x^4/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-3/4*(b^2*x^3*(a*d-5/6*b*c)*arctanh((b*x+a)^(1/2)/a^(1/2))+4/9*(-5/4*x*b*(
9/5*d*x+c)*a^(3/2)+(3/2*d*x+c)*a^(5/2)+15/8*a^(1/2)*b^2*c*x^2)*(b*x+a)^(1/
2))/a^(7/2)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.65

$$\int\frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx$$
$$=\left[-\frac{15\left(7\,b^{4}c-8\,ab^{3}d\right)\sqrt{a}x^{5}\log\left(\frac{bx^{2}+2\,ax+2\,\sqrt{bx^{3}+ax^{2}}\sqrt{a}}{x^{2}}\right)+2\left(48\,a^{4}c-15\left(7\,ab^{3}c-8\,a^{2}b^{2}d\right)x^{3}+10\left(7\,a^{2}b^{2}c\right.\right.}{384\,a^{5}x^{5}}\right.$$

input
```
integrate((d*x+c)/x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*(7*b^4*c - 8*a*b^3*d)*sqrt(a)*x^5*log((b*x^2 + 2*a*x + 2*sqrt(
b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(48*a^4*c - 15*(7*a*b^3*c - 8*a^2*b^2*d)*
x^3 + 10*(7*a^2*b^2*c - 8*a^3*b*d)*x^2 - 8*(7*a^3*b*c - 8*a^4*d)*x)*sqrt(b
*x^3 + a*x^2))/(a^5*x^5), 1/192*(15*(7*b^4*c - 8*a*b^3*d)*sqrt(-a)*x^5*arc
tan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - (48*a^4*c - 15*(7*a*b^3*
c - 8*a^2*b^2*d)*x^3 + 10*(7*a^2*b^2*c - 8*a^3*b*d)*x^2 - 8*(7*a^3*b*c - 8
*a^4*d)*x)*sqrt(b*x^3 + a*x^2))/(a^5*x^5)]
```

Sympy [F]

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+dx}{x^4\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/x**4/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)/(x**4*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+ax^2}x^4}\,dx$$

input
```
integrate((d*x+c)/x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(sqrt(b*x^3 + a*x^2)*x^4), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx = \frac{\frac{15\,(7\,b^5c-8\,ab^4d)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^4} + \frac{105\,(bx+a)^{\frac{7}{2}}b^5c-385\,(bx+a)^{\frac{5}{2}}ab^5c+511\,(bx+a)^{\frac{3}{2}}a^2b^5c-279\,\sqrt{bx+a}a^3b^5c-120\,(bx+a)^{\frac{7}{2}}ab^4d+440\,(bx+}{a^4b^4x^4}}{192\,b\,\mathrm{sgn}\left(x\right)}$$

input
```
integrate((d*x+c)/x^4/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/192*(15*(7*b^5*c - 8*a*b^4*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a
^4) + (105*(b*x + a)^(7/2)*b^5*c - 385*(b*x + a)^(5/2)*a*b^5*c + 511*(b*x
+ a)^(3/2)*a^2*b^5*c - 279*sqrt(b*x + a)*a^3*b^5*c - 120*(b*x + a)^(7/2)*a
*b^4*d + 440*(b*x + a)^(5/2)*a^2*b^4*d - 584*(b*x + a)^(3/2)*a^3*b^4*d + 2
64*sqrt(b*x + a)*a^4*b^4*d)/(a^4*b^4*x^4))/(b*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+d\,x}{x^4\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)/(x^4*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int((c + d*x)/(x^4*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.15

$$\int \frac{c+dx}{x^4\sqrt{ax^2+bx^3}}\,dx$$
$$= \frac{-96\sqrt{bx+a}\,a^4c-128\sqrt{bx+a}\,a^4dx+112\sqrt{bx+a}\,a^3bcx+160\sqrt{bx+a}\,a^3bd\,x^2-140\sqrt{bx+a}\,a^2b^2c\,x^2}{}$$

input
```
int((d*x+c)/x^4/(b*x^3+a*x^2)^(1/2),x)
```

output
```
( - 96*sqrt(a + b*x)*a**4*c - 128*sqrt(a + b*x)*a**4*d*x + 112*sqrt(a + b*
x)*a**3*b*c*x + 160*sqrt(a + b*x)*a**3*b*d*x**2 - 140*sqrt(a + b*x)*a**2*b
**2*c*x**2 - 240*sqrt(a + b*x)*a**2*b**2*d*x**3 + 210*sqrt(a + b*x)*a*b**3
*c*x**3 - 120*sqrt(a)*log(sqrt(a + b*x) - sqrt(a))*a*b**3*d*x**4 + 105*sqr
t(a)*log(sqrt(a + b*x) - sqrt(a))*b**4*c*x**4 + 120*sqrt(a)*log(sqrt(a + b
*x) + sqrt(a))*a*b**3*d*x**4 - 105*sqrt(a)*log(sqrt(a + b*x) + sqrt(a))*b*
*4*c*x**4)/(384*a**5*x**4)
```

3.282 $\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 159

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx=\frac{2a^3(bc-ad)x}{b^5\sqrt{ax^2+bx^3}}+\frac{2a^2(3bc-4ad)\sqrt{ax^2+bx^3}}{b^5x}-\frac{2a(bc-2ad)\left(ax^2+bx^3\right)^{3/2}}{b^5x^3}+\frac{2(bc-4ad)\left(ax^2+bx^3\right)^{5/2}}{5b^5x^5}+\frac{2d\left(ax^2+bx^3\right)^{7/2}}{7b^5x^7}$$

output
```
2*a^3*(-a*d+b*c)*x/b^5/(b*x^3+a*x^2)^(1/2)+2*a^2*(-4*a*d+3*b*c)*(b*x^3+a*x
^2)^(1/2)/b^5/x-2*a*(-2*a*d+b*c)*(b*x^3+a*x^2)^(3/2)/b^5/x^3+2/5*(-4*a*d+b
*c)*(b*x^3+a*x^2)^(5/2)/b^5/x^5+2/7*d*(b*x^3+a*x^2)^(7/2)/b^5/x^7
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.57

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx=\frac{2x(-128a^4d+16a^3b(7c-4dx)+8a^2b^2x(7c+2dx)-2ab^3x^2(7c+4dx)+b^4x^3(7c}{35b^5\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(x^6*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(-128*a^4*d + 16*a^3*b*(7*c - 4*d*x) + 8*a^2*b^2*x*(7*c + 2*d*x) - 2*
a*b^3*x^2*(7*c + 4*d*x) + b^4*x^3*(7*c + 5*d*x)))/(35*b^5*Sqrt[x^2*(a + b*
x)])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {1943, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{2x^5(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{(7bc-8ad)\int \frac{x^4}{\sqrt{bx^3+ax^2}}dx}{ab}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^5(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{(7bc-8ad)\left(\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\int \frac{x^3}{\sqrt{bx^3+ax^2}}dx}{7b}\right)}{ab}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^5(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{(7bc-8ad)\left(\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}\right)}{7b}\right)}{ab}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^5(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{(7bc-8ad)\left(\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int\frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}\right)}{7b}\right)}{ab}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{2x^5(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{\left(\frac{2x^2\sqrt{ax^2+bx^3}}{7b}-\frac{6a\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{5b}\right)}{7b}\right)(7bc-8ad)}{ab}$$

input
```
Int[(x^6*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*(b*c - a*d)*x^5)/(a*b*Sqrt[a*x^2 + b*x^3]) - ((7*b*c - 8*a*d)*((2*x^2*S
qrt[a*x^2 + b*x^3])/(7*b) - (6*a*((2*x*Sqrt[a*x^2 + b*x^3])/(5*b) - (4*a*(
(2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)))/(5*b
)))/(7*b)))/(a*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.69

method	result
gosper	$-\frac{2(bx+a)\left(-5d\,x^{4}b^{4}+8a\,b^{3}d\,x^{3}-7b^{4}c\,x^{3}-16a^{2}b^{2}d\,x^{2}+14a\,b^{3}c\,x^{2}+64a^{3}bdx-56a^{2}b^{2}cx+128a^{4}d-112a^{3}bc\right)x^{3}}{35b^{5}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$
default	$-\frac{2(bx+a)\left(-5d\,x^{4}b^{4}+8a\,b^{3}d\,x^{3}-7b^{4}c\,x^{3}-16a^{2}b^{2}d\,x^{2}+14a\,b^{3}c\,x^{2}+64a^{3}bdx-56a^{2}b^{2}cx+128a^{4}d-112a^{3}bc\right)x^{3}}{35b^{5}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$
orering	$-\frac{2(bx+a)\left(-5d\,x^{4}b^{4}+8a\,b^{3}d\,x^{3}-7b^{4}c\,x^{3}-16a^{2}b^{2}d\,x^{2}+14a\,b^{3}c\,x^{2}+64a^{3}bdx-56a^{2}b^{2}cx+128a^{4}d-112a^{3}bc\right)x^{3}}{35b^{5}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$
risch	$-\frac{2\left(-5b^{3}d\,x^{3}+13a\,b^{2}d\,x^{2}-7b^{3}c\,x^{2}-29a^{2}bdx+21a\,b^{2}cx+93a^{3}d-77c\,a^{2}b\right)(bx+a)x}{35b^{5}\sqrt{x^{2}(bx+a)}}-\frac{2a^{3}(ad-bc)x}{b^{5}\sqrt{x^{2}(bx+a)}}$
trager	$-\frac{2\left(-5d\,x^{4}b^{4}+8a\,b^{3}d\,x^{3}-7b^{4}c\,x^{3}-16a^{2}b^{2}d\,x^{2}+14a\,b^{3}c\,x^{2}+64a^{3}bdx-56a^{2}b^{2}cx+128a^{4}d-112a^{3}bc\right)\sqrt{b\,x^{3}+a\,x^{2}}}{35(bx+a)b^{5}x}$
pseudoelliptic	$\frac{\frac{2\left(\frac{11dx}{13}+c\right)x^{6}b^{7}}{11}-\frac{8x^{5}\left(\frac{21dx}{26}+c\right)a\,b^{6}}{33}+\frac{80x^{4}a^{2}\left(\frac{49dx}{65}+c\right)b^{5}}{231}-\frac{128x^{3}a^{3}\left(\frac{35dx}{52}+c\right)b^{4}}{231}+\frac{256x^{2}\left(\frac{7dx}{13}+c\right)a^{4}b^{3}}{231}-\frac{1024\left(\frac{7dx}{26}+c\right)x\,a^{5}b^{2}}{231}-}{b^{8}\sqrt{bx+a}}$

input
```
int(x^6*(d*x+c)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*(b*x+a)*(-5*b^4*d*x^4+8*a*b^3*d*x^3-7*b^4*c*x^3-16*a^2*b^2*d*x^2+14*
a*b^3*c*x^2+64*a^3*b*d*x-56*a^2*b^2*c*x+128*a^4*d-112*a^3*b*c)*x^3/b^5/(b*
x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.72

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(5\,b^4dx^4+112\,a^3bc-128\,a^4d+(7\,b^4c-8\,ab^3d)x^3-2\,(7\,ab^3c-8\,a^2b^2d)x^2+8\,(7}{35\,(b^6x^2+ab^5x)}$$

input
```
integrate(x^6*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/35*(5*b^4*d*x^4 + 112*a^3*b*c - 128*a^4*d + (7*b^4*c - 8*a*b^3*d)*x^3 -
2*(7*a*b^3*c - 8*a^2*b^2*d)*x^2 + 8*(7*a^2*b^2*c - 8*a^3*b*d)*x)*sqrt(b*x^
3 + a*x^2)/(b^6*x^2 + a*b^5*x)
```

Sympy [F]

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^6(c+dx)}{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**6*(d*x+c)/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**6*(c + d*x)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.61

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3)c}{5\,\sqrt{bx+a}b^4} + \frac{2\,(5\,b^4x^4-8\,ab^3x^3+16\,a^2b^2x^2-64\,a^3bx-128\,a^4)d}{35\,\sqrt{bx+a}b^5}$$

input
```
integrate(x^6*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/5*(b^3*x^3 - 2*a*b^2*x^2 + 8*a^2*b*x + 16*a^3)*c/(sqrt(b*x + a)*b^4) + 2
/35*(5*b^4*x^4 - 8*a*b^3*x^3 + 16*a^2*b^2*x^2 - 64*a^3*b*x - 128*a^4)*d/(s
qrt(b*x + a)*b^5)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{32\,(7\,a^3bc-8\,a^4d)\mathrm{sgn}(x)}{35\,\sqrt{a}b^5}+\frac{2\,(a^3bc-a^4d)}{\sqrt{bx+a}b^5\mathrm{sgn}\,(x)}+\frac{2\left(7\,(bx+a)^{\frac{5}{2}}b^{31}c-35\,(bx+a)^{\frac{3}{2}}ab^{31}c+105\,\sqrt{bx+a}a^2b^{31}c+5\,(bx+a)^{\frac{7}{2}}b^{30}d-28\,(bx+a)^{\frac{5}{2}}ab^{30}d+70\right.}{35\,b^{35}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^6*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-32/35*(7*a^3*b*c - 8*a^4*d)*sgn(x)/(sqrt(a)*b^5) + 2*(a^3*b*c - a^4*d)/(s
qrt(b*x + a)*b^5*sgn(x)) + 2/35*(7*(b*x + a)^(5/2)*b^31*c - 35*(b*x + a)^(
3/2)*a*b^31*c + 105*sqrt(b*x + a)*a^2*b^31*c + 5*(b*x + a)^(7/2)*b^30*d -
28*(b*x + a)^(5/2)*a*b^30*d + 70*(b*x + a)^(3/2)*a^2*b^30*d - 140*sqrt(b*x
 + a)*a^3*b^30*d)/(b^35*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.13 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.69

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,(-128\,d\,a^4-64\,d\,a^3\,b\,x+112\,c\,a^3\,b+16\,d\,a^2\,b^2\,x^2+56\,c\,a^2\,b^2\,x-8\,}{35\,b^5\,x\,(a+b\,x)}$$

input
```
int((x^6*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(7*b^4*c*x^3 - 128*a^4*d + 5*b^4*d*x^4 + 112*a^3*
b*c + 16*a^2*b^2*d*x^2 - 64*a^3*b*d*x + 56*a^2*b^2*c*x - 14*a*b^3*c*x^2 -
8*a*b^3*d*x^3))/(35*b^5*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.60

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\frac{2}{7}b^4d\,x^4 - \frac{16}{35}a\,b^3d\,x^3 + \frac{2}{5}b^4c\,x^3 + \frac{32}{35}a^2b^2d\,x^2 - \frac{4}{5}a\,b^3c\,x^2 - \frac{128}{35}a^3bdx + \frac{16}{5}a^2b^2cx - \frac{25}{35}}{\sqrt{bx+a}\,b^5}$$

input

```
int(x^6*(d*x+c)/(b*x^3+a*x^2)^(3/2),x)
```

output

```
(2*( - 128*a**4*d + 112*a**3*b*c - 64*a**3*b*d*x + 56*a**2*b**2*c*x + 16*a
**2*b**2*d*x**2 - 14*a*b**3*c*x**2 - 8*a*b**3*d*x**3 + 7*b**4*c*x**3 + 5*b
**4*d*x**4))/(35*sqrt(a + b*x)*b**5)
```

3.283 $\int \frac{x^5(c+dx)}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \frac{x^5(c+dx)}{\left(ax^2+bx^3\right)^{3/2}}\,dx=-\frac{2a^2(bc-ad)x}{b^4\sqrt{ax^2+bx^3}}-\frac{2a(2bc-3ad)\sqrt{ax^2+bx^3}}{b^4x}+\frac{2(bc-3ad)\left(ax^2+bx^3\right)^{3/2}}{3b^4x^3}+\frac{2d\left(ax^2+bx^3\right)^{5/2}}{5b^4x^5}$$

output
```
-2*a^2*(-a*d+b*c)*x/b^4/(b*x^3+a*x^2)^(1/2)-2*a*(-3*a*d+2*b*c)*(b*x^3+a*x^
2)^(1/2)/b^4/x+2/3*(-3*a*d+b*c)*(b*x^3+a*x^2)^(3/2)/b^4/x^3+2/5*d*(b*x^3+a
*x^2)^(5/2)/b^4/x^5
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.58

$$\int \frac{x^5(c+dx)}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2x\left(48a^3d-8a^2b(5c-3dx)+b^3x^2(5c+3dx)-2ab^2x(10c+3dx)\right)}{15b^4\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(x^5*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(48*a^3*d - 8*a^2*b*(5*c - 3*d*x) + b^3*x^2*(5*c + 3*d*x) - 2*a*b^2*x
*(10*c + 3*d*x)))/(15*b^4*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1943, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{2x^4(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{(5bc-6ad)\int \frac{x^3}{\sqrt{bx^3+ax^2}}dx}{ab}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^4(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{(5bc-6ad)\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}\right)}{ab}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^4(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{(5bc-6ad)\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}\right)}{ab}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2x^4(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{5b}\right)(5bc-6ad)}{ab}$$

input
```
Int[(x^5*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*(b*c - a*d)*x^4)/(a*b*Sqrt[a*x^2 + b*x^3]) - ((5*b*c - 6*a*d)*((2*x*Sqr
t[a*x^2 + b*x^3])/(5*b) - (4*a*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[
a*x^2 + b*x^3])/(3*b^2*x)))/(5*b)))/(a*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.68

method	result
gosper	$\frac{2(bx+a)\left(3b^{3}d\,x^{3}-6a\,b^{2}d\,x^{2}+5b^{3}c\,x^{2}+24a^{2}bdx-20a\,b^{2}cx+48a^{3}d-40c\,a^{2}b\right)x^{3}}{15b^{4}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$
default	$\frac{2(bx+a)\left(3b^{3}d\,x^{3}-6a\,b^{2}d\,x^{2}+5b^{3}c\,x^{2}+24a^{2}bdx-20a\,b^{2}cx+48a^{3}d-40c\,a^{2}b\right)x^{3}}{15b^{4}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$
orering	$\frac{2(bx+a)\left(3b^{3}d\,x^{3}-6a\,b^{2}d\,x^{2}+5b^{3}c\,x^{2}+24a^{2}bdx-20a\,b^{2}cx+48a^{3}d-40c\,a^{2}b\right)x^{3}}{15b^{4}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$
risch	$\frac{2\left(3b^{2}d\,x^{2}-9abdx+5b^{2}cx+33a^{2}d-25abc\right)(bx+a)x}{15b^{4}\sqrt{x^{2}(bx+a)}}+\frac{2a^{2}(ad-bc)x}{b^{4}\sqrt{x^{2}(bx+a)}}$
trager	$\frac{2\left(3b^{3}d\,x^{3}-6a\,b^{2}d\,x^{2}+5b^{3}c\,x^{2}+24a^{2}bdx-20a\,b^{2}cx+48a^{3}d-40c\,a^{2}b\right)\sqrt{b\,x^{3}+a\,x^{2}}}{15(bx+a)b^{4}x}$
pseudoelliptic	$-\frac{2048\left(-\frac{77x^{5}\left(\frac{9dx}{11}+c\right)b^{6}}{3072}+\frac{55\left(\frac{42dx}{55}+c\right)x^{4}a\,b^{5}}{1536}-\frac{11x^{3}a^{2}\left(\frac{15dx}{22}+c\right)b^{4}}{192}+\frac{11\left(\frac{6dx}{11}+c\right)x^{2}a^{3}b^{3}}{96}-\frac{11x\left(\frac{3dx}{11}+c\right)a^{4}b^{2}}{24}-\frac{11\left(-\frac{6dx}{11}+c\right)}{12}\right.}{231\sqrt{bx+a}\,b^{7}}$

input
```
int(x^5*(d*x+c)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(b*x+a)*(3*b^3*d*x^3-6*a*b^2*d*x^2+5*b^3*c*x^2+24*a^2*b*d*x-20*a*b^2*
c*x+48*a^3*d-40*a^2*b*c)*x^3/b^4/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.72

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\left(3\,b^3dx^3-40\,a^2bc+48\,a^3d+(5\,b^3c-6\,ab^2d)x^2-4\,(5\,ab^2c-6\,a^2bd)x\right)\sqrt{bx^3+ax^2}}{15\,(b^5x^2+ab^4x)}$$

input
```
integrate(x^5*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^3*d*x^3 - 40*a^2*b*c + 48*a^3*d + (5*b^3*c - 6*a*b^2*d)*x^2 - 4*
(5*a*b^2*c - 6*a^2*b*d)*x)*sqrt(b*x^3 + a*x^2)/(b^5*x^2 + a*b^4*x)
```

Sympy [F]

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^5(c+dx)}{(x^2(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**5*(d*x+c)/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**5*(c + d*x)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.59

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(b^2x^2-4\,abx-8\,a^2)c}{3\,\sqrt{bx+a}b^3} + \frac{2\,(b^3x^3-2\,ab^2x^2+8\,a^2bx+16\,a^3)d}{5\,\sqrt{bx+a}b^4}$$

input
```
integrate(x^5*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*c/(sqrt(b*x + a)*b^3) + 2/5*(b^3*x^3 - 2*a
*b^2*x^2 + 8*a^2*b*x + 16*a^3)*d/(sqrt(b*x + a)*b^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{16\,(5\,a^2bc-6\,a^3d)\mathrm{sgn}(x)}{15\,\sqrt{a}b^4} - \frac{2\,(a^2bc-a^3d)}{\sqrt{bx+a}b^4\mathrm{sgn}\,(x)} + \frac{2\left(5\,(bx+a)^{\frac{3}{2}}b^{17}c-30\,\sqrt{bx+a}ab^{17}c+3\,(bx+a)^{\frac{5}{2}}b^{16}d-15\,(bx+a)^{\frac{3}{2}}ab^{16}d+45\,\sqrt{bx+a}a^2b^{16}d\right)}{15\,b^{20}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^5*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
16/15*(5*a^2*b*c - 6*a^3*d)*sgn(x)/(sqrt(a)*b^4) - 2*(a^2*b*c - a^3*d)/(sq
rt(b*x + a)*b^4*sgn(x)) + 2/15*(5*(b*x + a)^(3/2)*b^17*c - 30*sqrt(b*x + a
)*a*b^17*c + 3*(b*x + a)^(5/2)*b^16*d - 15*(b*x + a)^(3/2)*a*b^16*d + 45*s
qrt(b*x + a)*a^2*b^16*d)/(b^20*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{b\,x^3+a\,x^2}\,\left(48\,d\,a^3+24\,d\,a^2\,b\,x-40\,c\,a^2\,b-6\,d\,a\,b^2\,x^2-20\,c\,a\,b^2\,x+3\,d\,b^3\,x^3+\right.}{15\,b^4\,x\,\left(a+b\,x\right)}$$

input
```
int((x^5*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(48*a^3*d + 5*b^3*c*x^2 + 3*b^3*d*x^3 - 40*a^2*b*
c - 20*a*b^2*c*x + 24*a^2*b*d*x - 6*a*b^2*d*x^2))/(15*b^4*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.57

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\frac{2}{5}b^3d\,x^3-\frac{4}{5}a\,b^2d\,x^2+\frac{2}{3}b^3c\,x^2+\frac{16}{5}a^2bdx-\frac{8}{3}a\,b^2cx+\frac{32}{5}a^3d-\frac{16}{3}a^2bc}{\sqrt{bx+a}\,b^4}$$

input
```
int(x^5*(d*x+c)/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*(48*a**3*d - 40*a**2*b*c + 24*a**2*b*d*x - 20*a*b**2*c*x - 6*a*b**2*d*x
**2 + 5*b**3*c*x**2 + 3*b**3*d*x**3))/(15*sqrt(a + b*x)*b**4)
```

3.284 $\int \frac{x^4(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{x^4(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2a(bc-ad)x}{b^3\sqrt{ax^2+bx^3}} + \frac{2(bc-2ad)\sqrt{ax^2+bx^3}}{b^3x} + \frac{2d(ax^2+bx^3)^{3/2}}{3b^3x^3}$$

output
```
2*a*(-a*d+b*c)*x/b^3/(b*x^3+a*x^2)^(1/2)+2*(-2*a*d+b*c)*(b*x^3+a*x^2)^(1/2
)/b^3/x+2/3*d*(b*x^3+a*x^2)^(3/2)/b^3/x^3
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.58

$$\int \frac{x^4(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2x(-8a^2d+ab(6c-4dx)+b^2x(3c+dx))}{3b^3\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(x^4*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(-8*a^2*d + a*b*(6*c - 4*d*x) + b^2*x*(3*c + d*x)))/(3*b^3*Sqrt[x^2*(
a + b*x)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.15, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1943, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx)}{(ax^2+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{2x^3(bc-ad)}{ab\sqrt{ax^2+bx^3}} - \frac{(3bc-4ad)\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{ab}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^3(bc-ad)}{ab\sqrt{ax^2+bx^3}} - \frac{(3bc-4ad)\left(\frac{2\sqrt{ax^2+bx^3}}{3b} - \frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{ab}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2x^3(bc-ad)}{ab\sqrt{ax^2+bx^3}} - \frac{\left(\frac{2\sqrt{ax^2+bx^3}}{3b} - \frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)(3bc-4ad)}{ab}$$

input
```
Int[(x^4*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*(b*c - a*d)*x^3)/(a*b*Sqrt[a*x^2 + b*x^3]) - ((3*b*c - 4*a*d)*((2*Sqrt[
a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)))/(a*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

method	result
gosper	$-\frac{2(bx+a)\left(-b^{2}d\,x^{2}+4abdx-3b^{2}cx+8a^{2}d-6abc\right)x^{3}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$
default	$-\frac{2(bx+a)\left(-b^{2}d\,x^{2}+4abdx-3b^{2}cx+8a^{2}d-6abc\right)x^{3}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$
orering	$-\frac{2(bx+a)\left(-b^{2}d\,x^{2}+4abdx-3b^{2}cx+8a^{2}d-6abc\right)x^{3}}{3b^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$
trager	$-\frac{2\left(-b^{2}d\,x^{2}+4abdx-3b^{2}cx+8a^{2}d-6abc\right)\sqrt{b\,x^{3}+a\,x^{2}}}{3(bx+a)b^{3}x}$
risch	$-\frac{2(-bdx+5ad-3bc)(bx+a)x}{3b^{3}\sqrt{x^{2}(bx+a)}}-\frac{2a(ad-bc)x}{b^{3}\sqrt{x^{2}(bx+a)}}$
pseudoelliptic	$\frac{\left(70d\,x^{5}+90c\,x^{4}\right)b^{5}-144\left(\frac{25dx}{36}+c\right)x^{3}a\,b^{4}+288\left(\frac{5dx}{9}+c\right)x^{2}a^{2}b^{3}-1152\left(\frac{5dx}{18}+c\right)x\,a^{3}b^{2}-2304\left(-\frac{5dx}{9}+c\right)a^{4}b+2560a^{5}d}{315\sqrt{bx+a}\,b^{6}}$

input
```
int(x^4*(d*x+c)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)*(-b^2*d*x^2+4*a*b*d*x-3*b^2*c*x+8*a^2*d-6*a*b*c)*x^3/b^3/(b*x
^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.74

$$\int \frac{x^4(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(b^2dx^2+6\,abc-8\,a^2d+(3\,b^2c-4\,abd)x)\sqrt{bx^3+ax^2}}{3\,(b^4x^2+ab^3x)}$$

input
```
integrate(x^4*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2/3*(b^2*d*x^2 + 6*a*b*c - 8*a^2*d + (3*b^2*c - 4*a*b*d)*x)*sqrt(b*x^3 + a
*x^2)/(b^4*x^2 + a*b^3*x)
```

Sympy [F]

$$\int \frac{x^4(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^4(c+dx)}{(x^2(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**4*(d*x+c)/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**4*(c + d*x)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.59

$$\int \frac{x^4(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(bx+2\,a)c}{\sqrt{bx+a}b^2} + \frac{2\,(b^2x^2-4\,abx-8\,a^2)d}{3\,\sqrt{bx+a}b^3}$$

input
```
integrate(x^4*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
2*(b*x + 2*a)*c/(sqrt(b*x + a)*b^2) + 2/3*(b^2*x^2 - 4*a*b*x - 8*a^2)*d/(s
qrt(b*x + a)*b^3)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.12

$$\int \frac{x^4(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{4\,(3\,abc-4\,a^2d)\mathrm{sgn}(x)}{3\,\sqrt{a}b^3} + \frac{2\,(abc-a^2d)}{\sqrt{bx+a}b^3\mathrm{sgn}\,(x)} + \frac{2\left(3\,\sqrt{bx+a}b^7c+(bx+a)^{\frac{3}{2}}b^6d-6\,\sqrt{bx+a}ab^6d\right)}{3\,b^9\mathrm{sgn}\,(x)}$$

input
```
integrate(x^4*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-4/3*(3*a*b*c - 4*a^2*d)*sgn(x)/(sqrt(a)*b^3) + 2*(a*b*c - a^2*d)/(sqrt(b*
x + a)*b^3*sgn(x)) + 2/3*(3*sqrt(b*x + a)*b^7*c + (b*x + a)^(3/2)*b^6*d -
6*sqrt(b*x + a)*a*b^6*d)/(b^9*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.95 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \frac{x^4(c+dx)}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2\,\sqrt{b\,x^3+a\,x^2}\,\left(-8\,d\,a^2-4\,d\,a\,b\,x+6\,c\,a\,b+d\,b^2\,x^2+3\,c\,b^2\,x\right)}{3\,b^3\,x\,\left(a+b\,x\right)}$$

input
```
int((x^4*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(b^2*d*x^2 - 8*a^2*d + 6*a*b*c + 3*b^2*c*x - 4*a*
b*d*x))/(3*b^3*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.52

$$\int \frac{x^4(c+dx)}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{\frac{2}{3}b^2d\,x^2-\frac{8}{3}abdx+2b^2cx-\frac{16}{3}a^2d+4abc}{\sqrt{bx+a}\,b^3}$$

input
```
int(x^4*(d*x+c)/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*( - 8*a**2*d + 6*a*b*c - 4*a*b*d*x + 3*b**2*c*x + b**2*d*x**2))/(3*sqrt
(a + b*x)*b**3)
```

3.285 $\int \frac{x^3(c+dx)}{(ax^2+bx^3)^{3/2}} dx$

Optimal result

Integrand size = 24, antiderivative size = 54

$$\int \frac{x^3(c+dx)}{(ax^2+bx^3)^{3/2}} dx = -\frac{2(bc-ad)x}{b^2\sqrt{ax^2+bx^3}} + \frac{2d\sqrt{ax^2+bx^3}}{b^2x}$$

output
```
-2*(-a*d+b*c)*x/b^2/(b*x^3+a*x^2)^(1/2)+2*d*(b*x^3+a*x^2)^(1/2)/b^2/x
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.59

$$\int \frac{x^3(c+dx)}{(ax^2+bx^3)^{3/2}} dx = \frac{2x(-bc+2ad+bdx)}{b^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(x^3*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*x*(-(b*c) + 2*a*d + b*d*x))/(b^2*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.28, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1943, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)}{(ax^2+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{2x^2(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{(bc-2ad)\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{ab}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2x^2(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{2\sqrt{ax^2+bx^3}(bc-2ad)}{ab^2x}$$

input
```
Int[(x^3*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*(b*c - a*d)*x^2)/(a*b*Sqrt[a*x^2 + b*x^3]) - (2*(b*c - 2*a*d)*Sqrt[a*x^
2 + b*x^3])/(a*b^2*x)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.74

method	result	size
gosper	$\frac{2(bx+a)(bdx+2ad-bc)x^{3}}{b^{2}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	40
default	$\frac{2(bx+a)(bdx+2ad-bc)x^{3}}{b^{2}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	40
orering	$\frac{2(bx+a)(bdx+2ad-bc)x^{3}}{b^{2}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	40
trager	$\frac{2(bdx+2ad-bc)\sqrt{b\,x^{3}+a\,x^{2}}}{(bx+a)b^{2}x}$	42
risch	$\frac{2d(bx+a)x}{b^{2}\sqrt{x^{2}(bx+a)}}+\frac{2(ad-bc)x}{b^{2}\sqrt{x^{2}(bx+a)}}$	50
pseudoelliptic	$-\frac{256\left(-\frac{7x^{3}\left(\frac{5dx}{7}+c\right)b^{4}}{128}+\frac{7x^{2}\left(\frac{4dx}{7}+c\right)a\,b^{3}}{64}-\frac{7\left(\frac{2dx}{7}+c\right)x\,a^{2}b^{2}}{16}-\frac{7a^{3}\left(-\frac{4dx}{7}+c\right)b}{8}+a^{4}d\right)}{35\sqrt{bx+a}\,b^{5}}$	75

input
```
int(x^3*(d*x+c)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(b*x+a)*(b*d*x+2*a*d-b*c)*x^3/b^2/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \frac{x^3(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{bx^3+ax^2}(bdx-bc+2\,ad)}{b^3x^2+ab^2x}$$

input
```
integrate(x^3*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
2*sqrt(b*x^3 + a*x^2)*(b*d*x - b*c + 2*a*d)/(b^3*x^2 + a*b^2*x)
```

Sympy [F]

$$\int \frac{x^3(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^3(c+dx)}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3*(d*x+c)/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**3*(c + d*x)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.63

$$\int \frac{x^3(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\,c}{\sqrt{bx+a}b}+\frac{2\,(bx+2\,a)d}{\sqrt{bx+a}b^2}$$

input
```
integrate(x^3*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
-2*c/(sqrt(b*x + a)*b) + 2*(b*x + 2*a)*d/(sqrt(b*x + a)*b^2)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{x^3(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,(bc-2\,ad)\mathrm{sgn}(x)}{\sqrt{a}b^2} + \frac{2\,\sqrt{bx+a}d}{b^2\mathrm{sgn}\,(x)} - \frac{2\,(bc-ad)}{\sqrt{bx+a}b^2\mathrm{sgn}\,(x)}$$

input
```
integrate(x^3*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
2*(b*c - 2*a*d)*sgn(x)/(sqrt(a)*b^2) + 2*sqrt(b*x + a)*d/(b^2*sgn(x)) - 2*
(b*c - a*d)/(sqrt(b*x + a)*b^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int \frac{x^3(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,\sqrt{b\,x^3+a\,x^2}\,(2\,a\,d-b\,c+b\,d\,x)}{b^2\,x\,(a+b\,x)}$$

input
```
int((x^3*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(2*a*d - b*c + b*d*x))/(b^2*x*(a + b*x))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.48

$$\int \frac{x^3(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2bdx+4ad-2bc}{\sqrt{bx+a}\,b^2}$$

input
```
int(x^3*(d*x+c)/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*(2*a*d - b*c + b*d*x))/(sqrt(a + b*x)*b**2)
```

3.286 $\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2(bc-ad)x}{ab\sqrt{ax^2+bx^3}} - \frac{2c\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{a^{3/2}}$$

output
```
2*(-a*d+b*c)*x/a/b/(b*x^3+a*x^2)^(1/2)-2*c*arctanh((b*x^3+a*x^2)^(1/2)/a^(
1/2)/x)/a^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.02

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2x\left(\sqrt{a}(-bc+ad)+bc\sqrt{a+bx}\operatorname{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{a^{3/2}b\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(x^2*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-2*x*(Sqrt[a]*(-(b*c) + a*d) + b*c*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sq
rt[a]]))/(a^(3/2)*b*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1943, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{ 1943}$$

$$\frac{c\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{a}+\frac{2x(bc-ad)}{ab\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1914}$$

$$\frac{2x(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{2c\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2x(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{2c\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}$$

input
```
Int[(x^2*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*(b*c - a*d)*x)/(a*b*Sqrt[a*x^2 + b*x^3]) - (2*c*ArcTanh[(Sqrt[a]*x)/Sqr
t[a*x^2 + b*x^3]])/a^(3/2)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.88

method	result	size
pseudoelliptic	$\frac{\frac{2x^2\left(\frac{3dx}{5}+c\right)b^3}{3}-\frac{8x\left(\frac{3dx}{10}+c\right)a\,b^2}{3}-\frac{16\left(-\frac{3dx}{5}+c\right)a^2b}{3}+\frac{32a^3d}{5}}{b^4\sqrt{bx+a}}$	58
default	$-\frac{2x^3(bx+a)\left(bc\ \operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\sqrt{bx+a}+a^{\frac{5}{2}}d-a^{\frac{3}{2}}bc\right)}{\left(b\,x^3+a\,x^2\right)^{\frac{3}{2}}b\,a^{\frac{5}{2}}}$	66

input
```
int(x^2*(d*x+c)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
32/5/(b*x+a)^(1/2)*(5/48*x^2*(3/5*d*x+c)*b^3-5/12*x*(3/10*d*x+c)*a*b^2-5/6
*(-3/5*d*x+c)*a^2*b+a^3*d)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.98

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \left[\frac{(b^2cx^2+abcx)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3+ax^2}(abc-a^2d)}{a^2b^2x^2+a^3bx}, \frac{2\left((b^2c\right.}{}\right.$$

input
```
integrate(x^2*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[((b^2*c*x^2 + a*b*c*x)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)
*sqrt(a))/x^2) + 2*sqrt(b*x^3 + a*x^2)*(a*b*c - a^2*d))/(a^2*b^2*x^2 + a^3
*b*x), 2*((b^2*c*x^2 + a*b*c*x)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-
a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*(a*b*c - a^2*d))/(a^2*b^2*x^2 + a^
3*b*x)]
```

Sympy [F]

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^2(c+dx)}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2*(d*x+c)/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x**2*(c + d*x)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(dx+c)x^2}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*x^2/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.61

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\,c\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a\text{sgn}\,(x)} - \frac{2\left(\sqrt{a}bc\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+\sqrt{-a}bc-\sqrt{-a}ad\right)\text{sgn}(x)}{\sqrt{-a}a^{\frac{3}{2}}b} + \frac{2\,(bc-ad)}{\sqrt{bx+a}ab\text{sgn}\,(x)}$$

input
```
integrate(x^2*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
2*c*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a*sgn(x)) - 2*(sqrt(a)*b*c*ar
ctan(sqrt(a)/sqrt(-a)) + sqrt(-a)*b*c - sqrt(-a)*a*d)*sgn(x)/(sqrt(-a)*a^(
3/2)*b) + 2*(b*c - a*d)/(sqrt(b*x + a)*a*b*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x^2\,(c+d\,x)}{(b\,x^3+a\,x^2)^{3/2}}\,dx$$

input
```
int((x^2*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
int((x^2*(c + d*x))/(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bc-\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}+\sqrt{a}\right)bc-2a^2d+}{\sqrt{bx+a}\,a^2b}$$

input
```
int(x^2*(d*x+c)/(b*x^3+a*x^2)^(3/2),x)
```

output

```
(sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b*c - sqrt(a)*sqrt(a +
 b*x)*log(sqrt(a + b*x) + sqrt(a))*b*c - 2*a**2*d + 2*a*b*c)/(sqrt(a + b*x
)*a**2*b)
```

3.287 $\int \frac{x(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 94

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2(bc-ad)x}{a^2\sqrt{ax^2+bx^3}}-\frac{c\sqrt{ax^2+bx^3}}{a^2x^2}+\frac{(3bc-2ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{a^{5/2}}$$

output
```
-2*(-a*d+b*c)*x/a^2/(b*x^3+a*x^2)^(1/2)-c*(b*x^3+a*x^2)^(1/2)/a^2/x^2+(-2*
a*d+3*b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{-\sqrt{a}(3bcx+a(c-2dx))+(3bc-2ad)x\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)}{a^{5/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(x*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-(Sqrt[a]*(3*b*c*x + a*(c - 2*d*x))) + (3*b*c - 2*a*d)*x*Sqrt[a + b*x]*Ar
cTanh[Sqrt[a + b*x]/Sqrt[a]])/(a^(5/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1944, 1929, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(3bc-2ad)\int \frac{x^2}{(bx^3+ax^2)^{3/2}}dx}{2a}-\frac{c}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1929}$$

$$-\frac{(3bc-2ad)\left(\frac{\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{a}+\frac{2x}{a\sqrt{ax^2+bx^3}}\right)}{2a}-\frac{c}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1914}$$

$$-\frac{(3bc-2ad)\left(\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}\right)}{2a}-\frac{c}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\left(\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}\right)(3bc-2ad)}{2a}-\frac{c}{a\sqrt{ax^2+bx^3}}$$

input
```
Int[(x*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
-(c/(a*Sqrt[a*x^2 + b*x^3])) - ((3*b*c - 2*a*d)*((2*x)/(a*Sqrt[a*x^2 + b*x
^3]) - (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(2*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.44

method	result	size
pseudoelliptic	$-\frac{16\left(-\frac{3x\left(\frac{dx}{3}+c\right)b^{2}}{8}-\frac{3a\left(-\frac{2dx}{3}+c\right)b}{4}+a^{2}d\right)}{3\sqrt{bx+a}\,b^{3}}$	41
risch	$-\frac{c(bx+a)}{a^{2}\sqrt{x^{2}(bx+a)}}+\frac{\left(\frac{2(-2ad+3bc)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{4ad-4bc}{\sqrt{bx+a}}\right)\sqrt{bx+a}\,x}{2a^{2}\sqrt{x^{2}(bx+a)}}$	93
default	$-\frac{x^{2}(bx+a)\left(2\sqrt{bx+a}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)adx-3\sqrt{bx+a}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bcx-2a^{\frac{3}{2}}dx+3\sqrt{a}\,bcx+a^{\frac{3}{2}}c\right)}{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}a^{\frac{5}{2}}}$	96

input
```
int(x*(d*x+c)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-16/3/(b*x+a)^(1/2)*(-3/8*x*(1/3*d*x+c)*b^2-3/4*a*(-2/3*d*x+c)*b+a^2*d)/b^
3
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.68

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \left[-\frac{((3\,b^2c-2\,abd)x^3+(3\,abc-2\,a^2d)x^2)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,\sqrt{bx^3}}{2\,(a^3bx^3+a^4x^2)}\right.$$

$$-\frac{((3\,b^2c-2\,abd)x^3+(3\,abc-2\,a^2d)x^2)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}(a^2c+(3\,abc-2\,a^2d)x)}{a^3bx^3+a^4x^2}$$

input
```
integrate(x*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(((3*b^2*c - 2*a*b*d)*x^3 + (3*a*b*c - 2*a^2*d)*x^2)*sqrt(a)*log((b*
x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*sqrt(b*x^3 + a*x^2)*
(a^2*c + (3*a*b*c - 2*a^2*d)*x))/(a^3*b*x^3 + a^4*x^2), -(((3*b^2*c - 2*a*
b*d)*x^3 + (3*a*b*c - 2*a^2*d)*x^2)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sq
rt(-a)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*(a^2*c + (3*a*b*c - 2*a^2*d)*x
))/(a^3*b*x^3 + a^4*x^2)]
```

Sympy [F]

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x(c+dx)}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(d*x+c)/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(x*(c + d*x)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(dx+c)x}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*x/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.03

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{(3\,bc-2\,ad)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}\,(x)} - \frac{3\,(bx+a)bc-2\,abc-2\,(bx+a)ad+2\,a^2d}{\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}a\right)a^2\mathrm{sgn}\,(x)}$$

input
```
integrate(x*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-(3*b*c - 2*a*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(x)) - (3
*(b*x + a)*b*c - 2*a*b*c - 2*(b*x + a)*a*d + 2*a^2*d)/(((b*x + a)^(3/2) -
sqrt(b*x + a)*a)*a^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{x\,(c+d\,x)}{{(b\,x^3+a\,x^2)}^{3/2}}\,dx$$

input
```
int((x*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
int((x*(c + d*x))/(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.40

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)adx-3\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)bcx-2}{}$$

input
```
int(x*(d*x+c)/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*d*x - 3*sqrt(a)*sq
rt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b*c*x - 2*sqrt(a)*sqrt(a + b*x)*l
og(sqrt(a + b*x) + sqrt(a))*a*d*x + 3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b
*x) + sqrt(a))*b*c*x - 2*a**2*c + 4*a**2*d*x - 6*a*b*c*x)/(2*sqrt(a + b*x)
*a**3*x)
```

3.288 $\int \frac{c+dx}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 135

$$\int \frac{c+dx}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2b(bc-ad)x}{a^3\sqrt{ax^2+bx^3}}-\frac{c\sqrt{ax^2+bx^3}}{2a^2x^3}+\frac{(7bc-4ad)\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{3b(5bc-4ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{4a^{7/2}}$$

output
```
2*b*(-a*d+b*c)*x/a^3/(b*x^3+a*x^2)^(1/2)-1/2*c*(b*x^3+a*x^2)^(1/2)/a^2/x^3
+1/4*(-4*a*d+7*b*c)*(b*x^3+a*x^2)^(1/2)/a^3/x^2-3/4*b*(-4*a*d+5*b*c)*arcta
nh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.78

$$\int \frac{c+dx}{\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{\sqrt{a}\left(15b^2cx^2+abx(5c-12dx)-2a^2(c+2dx)\right)-3b(5bc-4ad)x^2\sqrt{a+bx}\,\text{arctanh}}{4a^{7/2}x\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(Sqrt[a]*(15*b^2*c*x^2 + a*b*x*(5*c - 12*d*x) - 2*a^2*(c + 2*d*x)) - 3*b*(
5*b*c - 4*a*d)*x^2*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(4*a^(7/2
)*x*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.41, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2450, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(ax^2+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \text{2450}$$

$$\int \left(\frac{c}{(ax^2+bx^3)^{3/2}}+\frac{dx}{(ax^2+bx^3)^{3/2}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{15b^2c\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{4a^{7/2}}+\frac{3bd\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{5/2}}+\frac{15bc\sqrt{ax^2+bx^3}}{4a^3x^2}-\frac{5c\sqrt{ax^2+bx^3}}{2a^2x^3}-\frac{3d\sqrt{ax^2+bx^3}}{a^2x^2}+\frac{2c}{ax\sqrt{ax^2+bx^3}}+\frac{2d}{a\sqrt{ax^2+bx^3}}$$

input
```
Int[(c + d*x)/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*d)/(a*Sqrt[a*x^2 + b*x^3]) + (2*c)/(a*x*Sqrt[a*x^2 + b*x^3]) - (5*c*Sqr
t[a*x^2 + b*x^3])/(2*a^2*x^3) + (15*b*c*Sqrt[a*x^2 + b*x^3])/(4*a^3*x^2) -
 (3*d*Sqrt[a*x^2 + b*x^3])/(a^2*x^2) - (15*b^2*c*ArcTanh[(Sqrt[a]*x)/Sqrt[
a*x^2 + b*x^3]])/(4*a^(7/2)) + (3*b*d*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x
^3]])/a^(5/2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2450
```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IntegerQ[p] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.20

method	result
pseudoelliptic	$\frac{(2dx-2c)b+4ad}{b^{2}\sqrt{bx+a}}$
risch	$-\frac{(bx+a)(4adx-7cbx+2ac)}{4a^{3}x\sqrt{x^{2}(bx+a)}}-\frac{b\left(-\frac{2(12ad-15bc)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2(-8ad+8bc)}{\sqrt{bx+a}}\right)\sqrt{bx+a}\,x}{8a^{3}\sqrt{x^{2}(bx+a)}}$
default	$\frac{x(bx+a)\left(12\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abd\,x^{2}-15\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{2}c\,x^{2}-4a^{\frac{5}{2}}dx-12a^{\frac{3}{2}}bd\,x^{2}-2a^{\frac{5}{2}}c+5a^{\frac{3}{2}}bcx+1}{4(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}a^{\frac{7}{2}}}$

input
```
int((d*x+c)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
((2*d*x-2*c)*b+4*a*d)/b^2/(b*x+a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 313, normalized size of antiderivative = 2.32

$$\int \frac{c+dx}{(ax^2+bx^3)^{3/2}}\,dx = \left[-\frac{3\left((5\,b^3c-4\,ab^2d)x^4+(5\,ab^2c-4\,a^2bd)x^3\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(2}{8\,(a^4bx^4+a^5x^3)}\right.$$

input
```
integrate((d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*((5*b^3*c - 4*a*b^2*d)*x^4 + (5*a*b^2*c - 4*a^2*b*d)*x^3)*sqrt(a)
*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(2*a^3*c - 3
*(5*a*b^2*c - 4*a^2*b*d)*x^2 - (5*a^2*b*c - 4*a^3*d)*x)*sqrt(b*x^3 + a*x^2
))/(a^4*b*x^4 + a^5*x^3), 1/4*(3*((5*b^3*c - 4*a*b^2*d)*x^4 + (5*a*b^2*c -
 4*a^2*b*d)*x^3)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x
)) - (2*a^3*c - 3*(5*a*b^2*c - 4*a^2*b*d)*x^2 - (5*a^2*b*c - 4*a^3*d)*x)*s
qrt(b*x^3 + a*x^2))/(a^4*b*x^4 + a^5*x^3)]
```

Sympy [F]

$$\int \frac{c+dx}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{c+dx}{(x^2(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((c + d*x)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{c+dx}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{dx+c}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{3\,(5\,b^2c-4\,abd)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^3\mathrm{sgn}\left(x\right)}+\frac{2\,(b^2c-abd)}{\sqrt{bx+a}a^3\mathrm{sgn}\left(x\right)} + \frac{7\,(bx+a)^{\frac{3}{2}}b^2c-9\,\sqrt{bx+a}ab^2c-4\,(bx+a)^{\frac{3}{2}}abd+4\,\sqrt{bx+a}a^2bd}{4\,a^3b^2x^2\mathrm{sgn}\left(x\right)}$$

input

```
integrate((d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output

```
3/4*(5*b^2*c - 4*a*b*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3*sgn(x
)) + 2*(b^2*c - a*b*d)/(sqrt(b*x + a)*a^3*sgn(x)) + 1/4*(7*(b*x + a)^(3/2)
*b^2*c - 9*sqrt(b*x + a)*a*b^2*c - 4*(b*x + a)^(3/2)*a*b*d + 4*sqrt(b*x +
a)*a^2*b*d)/(a^3*b^2*x^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \int \frac{c+d\,x}{{\left(b\,x^3+a\,x^2\right)}^{3/2}} \,d x$$

input

```
int((c + d*x)/(a*x^2 + b*x^3)^(3/2),x)
```

output

```
int((c + d*x)/(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.24

$$\int \frac{c+dx}{(ax^2+bx^3)^{3/2}}\,dx = \frac{-12\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)abd\,x^2+15\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b}{}$$

input
```
int((d*x+c)/(b*x^3+a*x^2)^(3/2),x)
```

output
```
( - 12*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b*d*x**2 + 15*
sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**2*c*x**2 + 12*sqrt(a
)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b*d*x**2 - 15*sqrt(a)*sqrt(
a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**2*c*x**2 - 4*a**3*c - 8*a**3*d*x
+ 10*a**2*b*c*x - 24*a**2*b*d*x**2 + 30*a*b**2*c*x**2)/(8*sqrt(a + b*x)*a*
*4*x**2)
```

3.289 $\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 174

$$\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=-\frac{2b^2(bc-ad)x}{a^4\sqrt{ax^2+bx^3}}-\frac{c\sqrt{ax^2+bx^3}}{3a^2x^4}+\frac{(11bc-6ad)\sqrt{ax^2+bx^3}}{12a^3x^3}-\frac{b(19bc-14ad)\sqrt{ax^2+bx^3}}{8a^4x^2}+\frac{5b^2(7bc-6ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{8a^{9/2}}$$

output
```
-2*b^2*(-a*d+b*c)*x/a^4/(b*x^3+a*x^2)^(1/2)-1/3*c*(b*x^3+a*x^2)^(1/2)/a^2/
x^4+1/12*(-6*a*d+11*b*c)*(b*x^3+a*x^2)^(1/2)/a^3/x^3-1/8*b*(-14*a*d+19*b*c
)*(b*x^3+a*x^2)^(1/2)/a^4/x^2+5/8*b^2*(-6*a*d+7*b*c)*arctanh((b*x^3+a*x^2)
^(1/2)/a^(1/2)/x)/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.74

$$\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{\sqrt{a}(-105b^3cx^3-4a^3(2c+3dx)+2a^2bx(7c+15dx)+5ab^2x^2(-7c+18dx))+1}{24a^{9/2}x^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
(Sqrt[a]*(-105*b^3*c*x^3 - 4*a^3*(2*c + 3*d*x) + 2*a^2*b*x*(7*c + 15*d*x)
+ 5*a*b^2*x^2*(-7*c + 18*d*x)) + 15*b^2*(7*b*c - 6*a*d)*x^3*Sqrt[a + b*x]*
ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^(9/2)*x^2*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1944, 1912, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1944}$$

$$-\frac{(7bc-6ad)\int \frac{1}{\left(bx^3+ax^2\right)^{3/2}}dx}{6a}-\frac{c}{3ax^2\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1912}$$

$$-\frac{(7bc-6ad)\left(\frac{5\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{6a}-\frac{c}{3ax^2\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{(7bc-6ad)\left(\frac{5\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{6a}-\frac{c}{3ax^2\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{(7bc-6ad)\left(\frac{5\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{6a}-\frac{c}{3ax^2\sqrt{ax^2+bx^3}}$$

$\Big\downarrow$ 1914

$$-\frac{(7bc-6ad)\left(\frac{5\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{6a}-\frac{c}{3ax^2\sqrt{ax^2+bx^3}}$$

$\Big\downarrow$ 219

$$-\frac{\left(\frac{5\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)(7bc-6ad)}{6a}-\frac{c}{3ax^2\sqrt{ax^2+bx^3}}$$

input
```
Int[(c + d*x)/(x*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
-1/3*c/(a*x^2*Sqrt[a*x^2 + b*x^3]) - ((7*b*c - 6*a*d)*(2/(a*x*Sqrt[a*x^2 +
 b*x^3]) + (5*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x
^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a
)))/a))/(6*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.26

method	result
pseudoelliptic	$\frac{-\frac{2bc\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{2(ad-bc)}{a\sqrt{bx+a}}}{b}$
risch	$-\frac{(bx+a)\left(-42abd\,x^{2}+57b^{2}c\,x^{2}+12a^{2}dx-22abcx+8a^{2}c\right)}{24a^{4}x^{2}\sqrt{x^{2}(bx+a)}}+\frac{b^{2}\left(-\frac{2(30ad-35bc)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2(-16ad+16bc)}{\sqrt{bx+a}}\right)\sqrt{bx-}}{16a^{4}\sqrt{x^{2}(bx+a)}}$
default	$-\frac{(bx+a)\left(90\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^{2}d\,x^{3}-105\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{3}c\,x^{3}+12a^{\frac{7}{2}}dx-30a^{\frac{5}{2}}bd\,x^{2}-90a^{\frac{3}{2}}b^{2}d\,x^{3}-\right.}{24\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}a^{\frac{9}{2}}}$

input
```
int((d*x+c)/x/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/b*(-b*c/a^(3/2)*arctanh((b*x+a)^(1/2)/a^(1/2))-(a*d-b*c)/a/(b*x+a)^(1/2)
)
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.09

$$\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \left[-\frac{15\left((7\,b^4c-6\,ab^3d)x^5+(7\,ab^3c-6\,a^2b^2d)x^4\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+ \cdots}{48 \cdots}\right.$$

$$-\frac{15\left((7\,b^4c-6\,ab^3d)x^5+(7\,ab^3c-6\,a^2b^2d)x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\left(8\,a^4c+15\left(7\,ab^3c-6\,a^2b^2d\right) \cdots\right.}{24\left(a^5bx^5+a^6x^4\right)}$$

input
```
integrate((d*x+c)/x/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/48*(15*((7*b^4*c - 6*a*b^3*d)*x^5 + (7*a*b^3*c - 6*a^2*b^2*d)*x^4)*sqr
t(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(8*a^4*c
 + 15*(7*a*b^3*c - 6*a^2*b^2*d)*x^3 + 5*(7*a^2*b^2*c - 6*a^3*b*d)*x^2 - 2*
(7*a^3*b*c - 6*a^4*d)*x)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 + a^6*x^4), -1/24
*(15*((7*b^4*c - 6*a*b^3*d)*x^5 + (7*a*b^3*c - 6*a^2*b^2*d)*x^4)*sqrt(-a)*
arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (8*a^4*c + 15*(7*a*b^
3*c - 6*a^2*b^2*d)*x^3 + 5*(7*a^2*b^2*c - 6*a^3*b*d)*x^2 - 2*(7*a^3*b*c -
6*a^4*d)*x)*sqrt(b*x^3 + a*x^2))/(a^5*b*x^5 + a^6*x^4)]
```

Sympy [F]

$$\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{3/2}}\,dx = \int \frac{c+dx}{x\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/x/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((c + d*x)/(x*(x**2*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=\int \frac{dx+c}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x}\,dx$$

input
```
integrate((d*x+c)/x/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/((b*x^3 + a*x^2)^(3/2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.02

$$\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=-\frac{5\left(7\,b^3c-6\,ab^2d\right)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^4\mathrm{sgn}\left(x\right)}-\frac{2\left(b^3c-ab^2d\right)}{\sqrt{bx+a}a^4\mathrm{sgn}\left(x\right)}-\frac{57\left(bx+a\right)^{\frac{5}{2}}b^3c-136\left(bx+a\right)^{\frac{3}{2}}ab^3c+87\sqrt{bx+a}a^2b^3c-42\left(bx+a\right)^{\frac{5}{2}}ab^2d+96\left(bx+a\right)^{\frac{3}{2}}a^2b^2d-54\sqrt{}}{24\,a^4b^3x^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate((d*x+c)/x/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-5/8*(7*b^3*c - 6*a*b^2*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4*sg
n(x)) - 2*(b^3*c - a*b^2*d)/(sqrt(b*x + a)*a^4*sgn(x)) - 1/24*(57*(b*x + a
)^(5/2)*b^3*c - 136*(b*x + a)^(3/2)*a*b^3*c + 87*sqrt(b*x + a)*a^2*b^3*c -
 42*(b*x + a)^(5/2)*a*b^2*d + 96*(b*x + a)^(3/2)*a^2*b^2*d - 54*sqrt(b*x +
 a)*a^3*b^2*d)/(a^4*b^3*x^3*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=\int \frac{c+d\,x}{x\,\left(b\,x^3+a\,x^2\right)^{3/2}}\,dx$$

input
```
int((c + d*x)/(x*(a*x^2 + b*x^3)^(3/2)),x)
```

output
```
int((c + d*x)/(x*(a*x^2 + b*x^3)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.13

$$\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{90\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a\,b^2d\,x^3-105\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)}{}$$

input
```
int((d*x+c)/x/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(90*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**2*d*x**3 - 105
*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**3*c*x**3 - 90*sqrt(
a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**2*d*x**3 + 105*sqrt(a)*
sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**3*c*x**3 - 16*a**4*c - 24*a*
*4*d*x + 28*a**3*b*c*x + 60*a**3*b*d*x**2 - 70*a**2*b**2*c*x**2 + 180*a**2
*b**2*d*x**3 - 210*a*b**3*c*x**3)/(48*sqrt(a + b*x)*a**5*x**3)
```

3.290 $\int \frac{c+dx}{x^2(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 211

$$\int \frac{c+dx}{x^2(ax^2+bx^3)^{3/2}}\,dx = \frac{2b^3(bc-ad)x}{a^5\sqrt{ax^2+bx^3}} - \frac{c\sqrt{ax^2+bx^3}}{4a^2x^5} + \frac{(15bc-8ad)\sqrt{ax^2+bx^3}}{24a^3x^4} - \frac{b(123bc-88ad)\sqrt{ax^2+bx^3}}{96a^4x^3} + \frac{b^2(187bc-152ad)\sqrt{ax^2+bx^3}}{64a^5x^2} - \frac{35b^3(9bc-8ad)\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{64a^{11/2}}$$

output

```
2*b^3*(-a*d+b*c)*x/a^5/(b*x^3+a*x^2)^(1/2)-1/4*c*(b*x^3+a*x^2)^(1/2)/a^2/x
^5+1/24*(-8*a*d+15*b*c)*(b*x^3+a*x^2)^(1/2)/a^3/x^4-1/96*b*(-88*a*d+123*b*
c)*(b*x^3+a*x^2)^(1/2)/a^4/x^3+1/64*b^2*(-152*a*d+187*b*c)*(b*x^3+a*x^2)^(
1/2)/a^5/x^2-35/64*b^3*(-8*a*d+9*b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/
x)/a^(11/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.70

$$\int \frac{c+dx}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{\sqrt{a}(945b^4cx^4+105ab^3x^3(3c-8dx)-16a^4(3c+4dx)+8a^3bx(9c+14dx)-14\ldots}{192a^{11/2}x^3\sqrt{x^2(a\ldots}}$$

input
```
Integrate[(c + d*x)/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
(Sqrt[a]*(945*b^4*c*x^4 + 105*a*b^3*x^3*(3*c - 8*d*x) - 16*a^4*(3*c + 4*d*
x) + 8*a^3*b*x*(9*c + 14*d*x) - 14*a^2*b^2*x^2*(9*c + 20*d*x)) - 105*b^3*(
9*b*c - 8*a*d)*x^4*Sqrt[a + b*x]*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(192*a^(1
1/2)*x^3*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.92, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.292$, Rules used = {1944, 1929, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x^2\left(ax^2+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 1944}$$

$$-\frac{(9bc-8ad)\int \frac{1}{x\left(bx^3+ax^2\right)^{3/2}}dx}{8a}-\frac{c}{4ax^3\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1929}$$

$$-\frac{(9bc-8ad)\left(\frac{7\int \frac{1}{x^3\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}\right)}{8a}-\frac{c}{4ax^3\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1931}$$

$$-\frac{(9bc-8ad)\left(\frac{7\left(-\frac{5b\int\frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}\right)}{8a}-\frac{c}{4ax^3\sqrt{ax^2+bx^3}}$$

$\downarrow$ 1931

$$-\frac{(9bc-8ad)\left(\frac{7\left(-\frac{5b\left(-\frac{3b\int\frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}\right)}{8a}-\frac{c}{4ax^3\sqrt{ax^2+bx^3}}$$

$\downarrow$ 1931

$$-\frac{(9bc-8ad)\left(\frac{7\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int\frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}\right)}{8a}-\frac{c}{4ax^3\sqrt{ax^2+bx^3}}$$

$\downarrow$ 1914

$$-\frac{(9bc-8ad)\left(\frac{7\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}\right)}{8a}-\frac{c}{4ax^3\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \; 219$$

$$-\frac{\left(\frac{7\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{a}+\frac{2}{ax^2\sqrt{ax^2+bx^3}}\right)(9bc-8ad)}{8a}-\frac{c}{4ax^3\sqrt{ax^2+bx^3}}$$

input
```
Int[(c + d*x)/(x^2*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
-1/4*c/(a*x^3*Sqrt[a*x^2 + b*x^3]) - ((9*b*c - 8*a*d)*(2/(a*x^2*Sqrt[a*x^2
 + b*x^3]) + (7*(-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*b*(-1/2*Sqrt[a*x^2
+ b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt
[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)))/a))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.31

method	result
pseudoelliptic	$-\frac{2\sqrt{bx+a}\,x\left(ad-\frac{3bc}{2}\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)+\sqrt{a}\,((-2dx+c)a+3cbx)}{\sqrt{bx+a}\,x\,a^{\frac{5}{2}}}$
risch	$-\frac{(bx+a)\left(456a\,b^{2}d\,x^{3}-561b^{3}c\,x^{3}-176a^{2}bd\,x^{2}+246a\,b^{2}c\,x^{2}+64a^{3}dx-120a^{2}bcx+48c\,a^{3}\right)}{192a^{5}x^{3}\sqrt{x^{2}(bx+a)}}-\frac{b^{3}\left(-\frac{2(280ad-315bc)\operatorname{arctanh}}{\sqrt{a}}\right.}{128}$
default	$\frac{(bx+a)\left(840\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^{3}d\,x^{4}-945\sqrt{bx+a}\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{4}c\,x^{4}-64a^{\frac{9}{2}}dx+112a^{\frac{7}{2}}bd\,x^{2}-280a^{\frac{5}{2}}b^{2}d\,x^{3}\right.}{192x(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}a^{\frac{11}{2}}}$

input
```
int((d*x+c)/x^2/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/(b*x+a)^(1/2)*(2*(b*x+a)^(1/2)*x*(a*d-3/2*b*c)*arctanh((b*x+a)^(1/2)/a^
(1/2))+a^(1/2)*((-2*d*x+c)*a+3*c*b*x))/x/a^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 413, normalized size of antiderivative = 1.96

$$\int \frac{c+dx}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx = \left[-\frac{105\left((9\,b^5c-8\,ab^4d)x^6+(9\,ab^4c-8\,a^2b^3d)x^5\right)\sqrt{a}\log\left(\frac{bx^2+2\,ax+2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)}{}\right.$$

input
```
integrate((d*x+c)/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/384*(105*((9*b^5*c - 8*a*b^4*d)*x^6 + (9*a*b^4*c - 8*a^2*b^3*d)*x^5)*s
qrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(48*a^
5*c - 105*(9*a*b^4*c - 8*a^2*b^3*d)*x^4 - 35*(9*a^2*b^3*c - 8*a^3*b^2*d)*x
^3 + 14*(9*a^3*b^2*c - 8*a^4*b*d)*x^2 - 8*(9*a^4*b*c - 8*a^5*d)*x)*sqrt(b*
x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5), 1/192*(105*((9*b^5*c - 8*a*b^4*d)*x^6
 + (9*a*b^4*c - 8*a^2*b^3*d)*x^5)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt
(-a)/(b*x^2 + a*x)) - (48*a^5*c - 105*(9*a*b^4*c - 8*a^2*b^3*d)*x^4 - 35*(
9*a^2*b^3*c - 8*a^3*b^2*d)*x^3 + 14*(9*a^3*b^2*c - 8*a^4*b*d)*x^2 - 8*(9*a
^4*b*c - 8*a^5*d)*x)*sqrt(b*x^3 + a*x^2))/(a^6*b*x^6 + a^7*x^5)]
```

Sympy [F]

$$\int \frac{c+dx}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx = \int \frac{c+dx}{x^2\left(x^2\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/x**2/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((c + d*x)/(x**2*(x**2*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{c+dx}{x^2 \left(ax^2+bx^3\right)^{3/2}}\,dx = \int \frac{dx+c}{\left(bx^3+ax^2\right)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate((d*x+c)/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/((b*x^3 + a*x^2)^(3/2)*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.99

$$\int \frac{c+dx}{x^2 \left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{35\left(9\,b^4c-8\,ab^3d\right)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{64\,\sqrt{-a}a^5\mathrm{sgn}\left(x\right)} + \frac{2\left(b^4c-ab^3d\right)}{\sqrt{bx+a}a^5\mathrm{sgn}\left(x\right)} + \frac{561\left(bx+a\right)^{\frac{7}{2}}b^4c-1929\left(bx+a\right)^{\frac{5}{2}}ab^4c+2295\left(bx+a\right)^{\frac{3}{2}}a^2b^4c-975\,\sqrt{bx+a}a^3b^4c-456\left(bx+a\right)^{\frac{7}{2}}ab^3d+}{192\,a^5b^4x^4\mathrm{sgn}\left(x\right)}$$

input
```
integrate((d*x+c)/x^2/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
35/64*(9*b^4*c - 8*a*b^3*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^5*s
gn(x)) + 2*(b^4*c - a*b^3*d)/(sqrt(b*x + a)*a^5*sgn(x)) + 1/192*(561*(b*x
+ a)^(7/2)*b^4*c - 1929*(b*x + a)^(5/2)*a*b^4*c + 2295*(b*x + a)^(3/2)*a^2
*b^4*c - 975*sqrt(b*x + a)*a^3*b^4*c - 456*(b*x + a)^(7/2)*a*b^3*d + 1544*
(b*x + a)^(5/2)*a^2*b^3*d - 1784*(b*x + a)^(3/2)*a^3*b^3*d + 696*sqrt(b*x
+ a)*a^4*b^3*d)/(a^5*b^4*x^4*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx=\int \frac{c+d\,x}{x^2\,{\left(b\,x^3+a\,x^2\right)}^{3/2}} \,d x$$

input
```
int((c + d*x)/(x^2*(a*x^2 + b*x^3)^(3/2)),x)
```

output
```
int((c + d*x)/(x^2*(a*x^2 + b*x^3)^(3/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.04

$$\int \frac{c+dx}{x^2\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{-840\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)a\,b^3d\,x^4+945\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\right.}{}$$

input
```
int((d*x+c)/x^2/(b*x^3+a*x^2)^(3/2),x)
```

output
```
( - 840*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**3*d*x**4 +
 945*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**4*c*x**4 + 840*
sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**3*d*x**4 - 945*sqr
t(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**4*c*x**4 - 96*a**5*c -
128*a**5*d*x + 144*a**4*b*c*x + 224*a**4*b*d*x**2 - 252*a**3*b**2*c*x**2 -
 560*a**3*b**2*d*x**3 + 630*a**2*b**3*c*x**3 - 1680*a**2*b**3*d*x**4 + 189
0*a*b**4*c*x**4)/(384*sqrt(a + b*x)*a**6*x**4)
```

3.291 $\int \frac{x^8(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 161

$$\int \frac{x^8(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \frac{2a^3(bc-ad)x^3}{3b^5\left(ax^2+bx^3\right)^{3/2}} - \frac{2a^2(3bc-4ad)x}{b^5\sqrt{ax^2+bx^3}} - \frac{6a(bc-2ad)\sqrt{ax^2+bx^3}}{b^5x} + \frac{2(bc-4ad)\left(ax^2+bx^3\right)^{3/2}}{3b^5x^3} + \frac{2d\left(ax^2+bx^3\right)^{5/2}}{5b^5x^5}$$

output
```
2/3*a^3*(-a*d+b*c)*x^3/b^5/(b*x^3+a*x^2)^(3/2)-2*a^2*(-4*a*d+3*b*c)*x/b^5/
(b*x^3+a*x^2)^(1/2)-6*a*(-2*a*d+b*c)*(b*x^3+a*x^2)^(1/2)/b^5/x+2/3*(-4*a*d
+b*c)*(b*x^3+a*x^2)^(3/2)/b^5/x^3+2/5*d*(b*x^3+a*x^2)^(5/2)/b^5/x^5
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.57

$$\int \frac{x^8(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \frac{2x^3\left(128a^4d+24a^2b^2x(-5c+2dx)+b^4x^3(5c+3dx)-2ab^3x^2(15c+4dx)+a^3b(-\right.}{15b^5\left(x^2(a+bx)\right)^{3/2}}$$

input
```
Integrate[(x^8*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*x^3*(128*a^4*d + 24*a^2*b^2*x*(-5*c + 2*d*x) + b^4*x^3*(5*c + 3*d*x) -
2*a*b^3*x^2*(15*c + 4*d*x) + a^3*b*(-80*c + 192*d*x)))/(15*b^5*(x^2*(a + b
*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1943, 1921, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^8(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1943}$$

$$\frac{2x^7(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}-\frac{(5bc-8ad)\int \frac{x^6}{(bx^3+ax^2)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{2x^7(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}-\frac{(5bc-8ad)\left(\frac{6\int \frac{x^3}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}\right)}{3ab}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{2x^7(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}-\frac{(5bc-8ad)\left(\frac{6\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{5b}\right)}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}\right)}{3ab}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{\frac{2x^7(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}-}{\frac{(5bc-8ad)\left(\frac{6\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int\frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{5b}\right)}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}\right)}{3ab}}$$

$\downarrow$ 1920

$$\frac{2x^7(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}-\frac{\left(\frac{6\left(\frac{2x\sqrt{ax^2+bx^3}}{5b}-\frac{4a\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{5b}\right)}{b}-\frac{2x^4}{b\sqrt{ax^2+bx^3}}\right)(5bc-8ad)}{3ab}$$

input
```
Int[(x^8*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*(b*c - a*d)*x^7)/(3*a*b*(a*x^2 + b*x^3)^(3/2)) - ((5*b*c - 8*a*d)*((-2*
x^4)/(b*Sqrt[a*x^2 + b*x^3]) + (6*((2*x*Sqrt[a*x^2 + b*x^3])/(5*b) - (4*a*
((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt[a*x^2 + b*x^3])/(3*b^2*x)))/(5*
b)))/b))/(3*a*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.68

method	result
gosper	$\frac{2(bx+a)\left(3d\,x^{4} b^{4}-8a\,b^{3} d\,x^{3}+5b^{4} c\,x^{3}+48a^{2} b^{2} d\,x^{2}-30a\,b^{3} c\,x^{2}+192a^{3} bdx-120a^{2} b^{2} cx+128a^{4} d-80a^{3} bc\right) x^{5}}{15b^{5} \left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$
default	$\frac{2(bx+a)\left(3d\,x^{4} b^{4}-8a\,b^{3} d\,x^{3}+5b^{4} c\,x^{3}+48a^{2} b^{2} d\,x^{2}-30a\,b^{3} c\,x^{2}+192a^{3} bdx-120a^{2} b^{2} cx+128a^{4} d-80a^{3} bc\right) x^{5}}{15b^{5} \left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$
orering	$\frac{2(bx+a)\left(3d\,x^{4} b^{4}-8a\,b^{3} d\,x^{3}+5b^{4} c\,x^{3}+48a^{2} b^{2} d\,x^{2}-30a\,b^{3} c\,x^{2}+192a^{3} bdx-120a^{2} b^{2} cx+128a^{4} d-80a^{3} bc\right) x^{5}}{15b^{5} \left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$
risch	$\frac{2\left(3b^{2} d\,x^{2}-14abdx+5b^{2} cx+73a^{2} d-40abc\right)(bx+a)x}{15b^{5} \sqrt{x^{2}(bx+a)}}+\frac{2a^{2} \left(12abdx-9b^{2} cx+11a^{2} d-8abc\right)x}{3b^{5} (bx+a)\sqrt{x^{2}(bx+a)}}$
trager	$\frac{2\left(3d\,x^{4} b^{4}-8a\,b^{3} d\,x^{3}+5b^{4} c\,x^{3}+48a^{2} b^{2} d\,x^{2}-30a\,b^{3} c\,x^{2}+192a^{3} bdx-120a^{2} b^{2} cx+128a^{4} d-80a^{3} bc\right)\sqrt{b\,x^{3}+a\,x^{2}}}{15x\,b^{5} (bx+a)^{2}}$
pseudoelliptic	$\frac{\left(858d\,x^{9}+990c\,x^{8}\right)b^{9}-1440x^{7}\left(\frac{33dx}{40}+c\right)a\,b^{8}+2240\left(\frac{27dx}{35}+c\right)x^{6} a^{2} b^{7}-3840x^{5}\left(\frac{7dx}{10}+c\right)a^{3} b^{6}+7680x^{4}\left(\frac{3dx}{5}+c\right)a^{4} b^{5}-2048}{6435(bx+a)^{\frac{3}{2}} b^{10}}$

input
```
int(x^8*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(b*x+a)*(3*b^4*d*x^4-8*a*b^3*d*x^3+5*b^4*c*x^3+48*a^2*b^2*d*x^2-30*a*
b^3*c*x^2+192*a^3*b*d*x-120*a^2*b^2*c*x+128*a^4*d-80*a^3*b*c)*x^5/b^5/(b*x
^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.78

$$\int \frac{x^8(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\left(3\,b^4dx^4-80\,a^3bc+128\,a^4d+(5\,b^4c-8\,ab^3d)x^3-6\,(5\,ab^3c-8\,a^2b^2d)x^2-24\,(5}{15\,(b^7x^3+2\,ab^6x^2+a^2b^5x)}$$

input
```
integrate(x^8*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
2/15*(3*b^4*d*x^4 - 80*a^3*b*c + 128*a^4*d + (5*b^4*c - 8*a*b^3*d)*x^3 - 6
*(5*a*b^3*c - 8*a^2*b^2*d)*x^2 - 24*(5*a^2*b^2*c - 8*a^3*b*d)*x)*sqrt(b*x^
3 + a*x^2)/(b^7*x^3 + 2*a*b^6*x^2 + a^2*b^5*x)
```

Sympy [F]

$$\int \frac{x^8(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^8(c+dx)}{(x^2(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**8*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral(x**8*(c + d*x)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.73

$$\int \frac{x^8(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\,(b^3x^3-6\,ab^2x^2-24\,a^2bx-16\,a^3)c}{3\,(b^5x+ab^4)\sqrt{bx+a}} + \frac{2\,(3\,b^4x^4-8\,ab^3x^3+48\,a^2b^2x^2+192\,a^3bx+128\,a^4)d}{15\,(b^6x+ab^5)\sqrt{bx+a}}$$

input
```
integrate(x^8*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(b^3*x^3 - 6*a*b^2*x^2 - 24*a^2*b*x - 16*a^3)*c/((b^5*x + a*b^4)*sqrt(
b*x + a)) + 2/15*(3*b^4*x^4 - 8*a*b^3*x^3 + 48*a^2*b^2*x^2 + 192*a^3*b*x +
 128*a^4)*d/((b^6*x + a*b^5)*sqrt(b*x + a))
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98

$$\int \frac{x^8(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{32\,(5\,a^2bc-8\,a^3d)\mathrm{sgn}(x)}{15\,\sqrt{a}b^5} - \frac{2\,(9\,(bx+a)a^2bc-a^3bc-12\,(bx+a)a^3d+a^4d)}{3\,(bx+a)^{\frac{3}{2}}b^5\mathrm{sgn}\,(x)} + \frac{2\left(5\,(bx+a)^{\frac{3}{2}}b^{21}c-45\,\sqrt{bx+a}ab^{21}c+3\,(bx+a)^{\frac{5}{2}}b^{20}d-20\,(bx+a)^{\frac{3}{2}}ab^{20}d+90\,\sqrt{bx+a}a^2b^{20}d\right)}{15\,b^{25}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^8*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
32/15*(5*a^2*b*c - 8*a^3*d)*sgn(x)/(sqrt(a)*b^5) - 2/3*(9*(b*x + a)*a^2*b*
c - a^3*b*c - 12*(b*x + a)*a^3*d + a^4*d)/((b*x + a)^(3/2)*b^5*sgn(x)) + 2
/15*(5*(b*x + a)^(3/2)*b^21*c - 45*sqrt(b*x + a)*a*b^21*c + 3*(b*x + a)^(5
/2)*b^20*d - 20*(b*x + a)^(3/2)*a*b^20*d + 90*sqrt(b*x + a)*a^2*b^20*d)/(b
^25*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.68

$$\int \frac{x^8(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{b\,x^3+a\,x^2}\,(128\,d\,a^4+192\,d\,a^3\,b\,x-80\,c\,a^3\,b+48\,d\,a^2\,b^2\,x^2-120\,c\,a^2\,b^2\,x-8\,d}{15\,b^5\,x\,(a+b\,x)^2}$$

input
```
int((x^8*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(128*a^4*d + 5*b^4*c*x^3 + 3*b^4*d*x^4 - 80*a^3*b
*c + 48*a^2*b^2*d*x^2 + 192*a^3*b*d*x - 120*a^2*b^2*c*x - 30*a*b^3*c*x^2 -
 8*a*b^3*d*x^3))/(15*b^5*x*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.63

$$\int \frac{x^8(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{\frac{2}{5}b^4d\,x^4-\frac{16}{15}a\,b^3d\,x^3+\frac{2}{3}b^4c\,x^3+\frac{32}{5}a^2b^2d\,x^2-4a\,b^3c\,x^2+\frac{128}{5}a^3bdx-16a^2b^2cx+\frac{25}{15}}{\sqrt{bx+a}\,b^5\,(bx+a)}$$

input
```
int(x^8*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(2*(128*a**4*d - 80*a**3*b*c + 192*a**3*b*d*x - 120*a**2*b**2*c*x + 48*a**
2*b**2*d*x**2 - 30*a*b**3*c*x**2 - 8*a*b**3*d*x**3 + 5*b**4*c*x**3 + 3*b**
4*d*x**4))/(15*sqrt(a + b*x)*b**5*(a + b*x))
```

3.292 $\int \frac{x^7(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 125

$$\int \frac{x^7(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2a^2(bc-ad)x^3}{3b^4\left(ax^2+bx^3\right)^{3/2}}+\frac{2a(2bc-3ad)x}{b^4\sqrt{ax^2+bx^3}}+\frac{2(bc-3ad)\sqrt{ax^2+bx^3}}{b^4x}+\frac{2d\left(ax^2+bx^3\right)^{3/2}}{3b^4x^3}$$

output
```
-2/3*a^2*(-a*d+b*c)*x^3/b^4/(b*x^3+a*x^2)^(3/2)+2*a*(-3*a*d+2*b*c)*x/b^4/(
b*x^3+a*x^2)^(1/2)+2*(-3*a*d+b*c)*(b*x^3+a*x^2)^(1/2)/b^4/x+2/3*d*(b*x^3+a
*x^2)^(3/2)/b^4/x^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.56

$$\int \frac{x^7(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2x^3(-16a^3d+8a^2b(c-3dx)-6ab^2x(-2c+dx)+b^3x^2(3c+dx))}{3b^4\left(x^2(a+bx)\right)^{3/2}}$$

input
```
Integrate[(x^7*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*x^3*(-16*a^3*d + 8*a^2*b*(c - 3*d*x) - 6*a*b^2*x*(-2*c + d*x) + b^3*x^2
*(3*c + d*x)))/(3*b^4*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1943, 1921, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^7(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{2x^6(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}-\frac{(bc-2ad)\int \frac{x^5}{(bx^3+ax^2)^{3/2}}dx}{ab}$$

$$\Big\downarrow \text{1921}$$

$$\frac{2x^6(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}-\frac{(bc-2ad)\left(\frac{4\int \frac{x^2}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}\right)}{ab}$$

$$\Big\downarrow \text{1922}$$

$$\frac{2x^6(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}-\frac{(bc-2ad)\left(\frac{4\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{2a\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{3b}\right)}{b}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}\right)}{ab}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2x^6(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}-\frac{\left(\frac{4\left(\frac{2\sqrt{ax^2+bx^3}}{3b}-\frac{4a\sqrt{ax^2+bx^3}}{3b^2x}\right)}{b}-\frac{2x^3}{b\sqrt{ax^2+bx^3}}\right)(bc-2ad)}{ab}$$

input
```
Int[(x^7*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*(b*c - a*d)*x^6)/(3*a*b*(a*x^2 + b*x^3)^(3/2)) - ((b*c - 2*a*d)*((-2*x^
3)/(b*Sqrt[a*x^2 + b*x^3]) + (4*((2*Sqrt[a*x^2 + b*x^3])/(3*b) - (4*a*Sqrt
[a*x^2 + b*x^3])/(3*b^2*x)))/b))/(a*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 1.01 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.68

method	result
gosper	$-\frac{2(bx+a)\left(-b^{3} d\,x^{3}+6 a\,b^{2} d\,x^{2}-3 b^{3} c\,x^{2}+24 a^{2} b d x-12 a\,b^{2} c x+16 a^{3} d-8 c\,a^{2} b\right) x^{5}}{3 b^{4} \left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$
default	$-\frac{2(bx+a)\left(-b^{3} d\,x^{3}+6 a\,b^{2} d\,x^{2}-3 b^{3} c\,x^{2}+24 a^{2} b d x-12 a\,b^{2} c x+16 a^{3} d-8 c\,a^{2} b\right) x^{5}}{3 b^{4} \left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$
orering	$-\frac{2(bx+a)\left(-b^{3} d\,x^{3}+6 a\,b^{2} d\,x^{2}-3 b^{3} c\,x^{2}+24 a^{2} b d x-12 a\,b^{2} c x+16 a^{3} d-8 c\,a^{2} b\right) x^{5}}{3 b^{4} \left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$
trager	$-\frac{2\left(-b^{3} d\,x^{3}+6 a\,b^{2} d\,x^{2}-3 b^{3} c\,x^{2}+24 a^{2} b d x-12 a\,b^{2} c x+16 a^{3} d-8 c\,a^{2} b\right) \sqrt{b\,x^{3}+a\,x^{2}}}{3 x\,b^{4} (bx+a)^{2}}$
risch	$-\frac{2(-bdx+8ad-3bc)(bx+a)x}{3b^{4}\sqrt{x^{2}(bx+a)}}-\frac{2a\left(9abdx-6b^{2}cx+8a^{2}d-5abc\right)x}{3b^{4}(bx+a)\sqrt{x^{2}(bx+a)}}$
pseudoelliptic	$\frac{\frac{2\left(\frac{11dx}{13}+c\right)x^{7}b^{8}}{11}-\frac{28x^{6}a\left(\frac{72dx}{91}+c\right)b^{7}}{99}+\frac{16x^{5}\left(\frac{28dx}{39}+c\right)a^{2}b^{6}}{33}-\frac{32x^{4}\left(\frac{8dx}{13}+c\right)a^{3}b^{5}}{33}+\frac{256x^{3}\left(\frac{6dx}{13}+c\right)a^{4}b^{4}}{99}-\frac{512\left(\frac{8dx}{39}+c\right)x^{2}a^{5}b^{3}}{33}-2}{b^{9}(bx+a)^{\frac{3}{2}}}$

input
```
int(x^7*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)*(-b^3*d*x^3+6*a*b^2*d*x^2-3*b^3*c*x^2+24*a^2*b*d*x-12*a*b^2*c
*x+16*a^3*d-8*a^2*b*c)*x^5/b^4/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.79

$$\int \frac{x^7(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\left(b^3dx^3+8\,a^2bc-16\,a^3d+3\,(b^3c-2\,ab^2d)x^2+12\,(ab^2c-2\,a^2bd)x\right)\sqrt{bx^3+ax^2}}{3\left(b^6x^3+2\,ab^5x^2+a^2b^4x\right)}$$

input
```
integrate(x^7*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(b^3*d*x^3 + 8*a^2*b*c - 16*a^3*d + 3*(b^3*c - 2*a*b^2*d)*x^2 + 12*(a*
b^2*c - 2*a^2*b*d)*x)*sqrt(b*x^3 + a*x^2)/(b^6*x^3 + 2*a*b^5*x^2 + a^2*b^4
*x)
```

Sympy [F]

$$\int \frac{x^7(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^7(c+dx)}{(x^2(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**7*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral(x**7*(c + d*x)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.76

$$\int \frac{x^7(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\,(3\,b^2x^2+12\,abx+8\,a^2)c}{3\,(b^4x+ab^3)\sqrt{bx+a}} + \frac{2\,(b^3x^3-6\,ab^2x^2-24\,a^2bx-16\,a^3)d}{3\,(b^5x+ab^4)\sqrt{bx+a}}$$

input
```
integrate(x^7*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
2/3*(3*b^2*x^2 + 12*a*b*x + 8*a^2)*c/((b^4*x + a*b^3)*sqrt(b*x + a)) + 2/3
*(b^3*x^3 - 6*a*b^2*x^2 - 24*a^2*b*x - 16*a^3)*d/((b^5*x + a*b^4)*sqrt(b*x
 + a))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.97

$$\int \frac{x^7(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{16\,(abc-2\,a^2d)\mathrm{sgn}(x)}{3\,\sqrt{a}b^4} + \frac{2\,(6\,(bx+a)abc-a^2bc-9\,(bx+a)a^2d+a^3d)}{3\,(bx+a)^{\frac{3}{2}}b^4\mathrm{sgn}\,(x)} + \frac{2\left(3\,\sqrt{bx+a}ab^9c+(bx+a)^{\frac{3}{2}}b^8d-9\,\sqrt{bx+a}ab^8d\right)}{3\,b^{12}\mathrm{sgn}\,(x)}$$

input
```
integrate(x^7*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
-16/3*(a*b*c - 2*a^2*d)*sgn(x)/(sqrt(a)*b^4) + 2/3*(6*(b*x + a)*a*b*c - a^
2*b*c - 9*(b*x + a)*a^2*d + a^3*d)/((b*x + a)^(3/2)*b^4*sgn(x)) + 2/3*(3*s
qrt(b*x + a)*b^9*c + (b*x + a)^(3/2)*b^8*d - 9*sqrt(b*x + a)*a*b^8*d)/(b^1
2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.25 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.68

$$\int \frac{x^7(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{b\,x^3+a\,x^2}\,(-16\,d\,a^3-24\,d\,a^2\,b\,x+8\,c\,a^2\,b-6\,d\,a\,b^2\,x^2+12\,c\,a\,b^2\,x+d\,b^3\,x^3+}{3\,b^4\,x\,(a+b\,x)^2}$$

input
```
int((x^7*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(3*b^3*c*x^2 - 16*a^3*d + b^3*d*x^3 + 8*a^2*b*c +
 12*a*b^2*c*x - 24*a^2*b*d*x - 6*a*b^2*d*x^2))/(3*b^4*x*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.62

$$\int \frac{x^7(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{\frac{2}{3}b^3d\,x^3-4a\,b^2d\,x^2+2b^3c\,x^2-16a^2bdx+8a\,b^2cx-\frac{32}{3}a^3d+\frac{16}{3}a^2bc}{\sqrt{bx+a}\,b^4\,(bx+a)}$$

input
```
int(x^7*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(2*( - 16*a**3*d + 8*a**2*b*c - 24*a**2*b*d*x + 12*a*b**2*c*x - 6*a*b**2*d
*x**2 + 3*b**3*c*x**2 + b**3*d*x**3))/(3*sqrt(a + b*x)*b**4*(a + b*x))
```

3.293 $\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 88

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2a(bc-ad)x^3}{3b^3(ax^2+bx^3)^{3/2}} - \frac{2(bc-2ad)x}{b^3\sqrt{ax^2+bx^3}} + \frac{2d\sqrt{ax^2+bx^3}}{b^3x}$$

output
```
2/3*a*(-a*d+b*c)*x^3/b^3/(b*x^3+a*x^2)^(3/2)-2*(-2*a*d+b*c)*x/b^3/(b*x^3+a
*x^2)^(1/2)+2*d*(b*x^3+a*x^2)^(1/2)/b^3/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2x^3(8a^2d-2ab(c-6dx)+3b^2x(-c+dx))}{3b^3(x^2(a+bx))^{3/2}}$$

input
```
Integrate[(x^6*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*x^3*(8*a^2*d - 2*a*b*(c - 6*d*x) + 3*b^2*x*(-c + d*x)))/(3*b^3*(x^2*(a
+ b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.16, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {1943, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{5/2}} \, dx$$

$$\Big\downarrow \text{ 1943}$$

$$\frac{2x^5(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}-\frac{(bc-4ad)\int \frac{x^4}{(bx^3+ax^2)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{2x^5(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}-\frac{(bc-4ad)\left(\frac{2\int \frac{x}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}\right)}{3ab}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{2x^5(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}-\frac{\left(\frac{4\sqrt{ax^2+bx^3}}{b^2x}-\frac{2x^2}{b\sqrt{ax^2+bx^3}}\right)(bc-4ad)}{3ab}$$

input
```
Int[(x^6*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*(b*c - a*d)*x^5)/(3*a*b*(a*x^2 + b*x^3)^(3/2)) - ((b*c - 4*a*d)*((-2*x^
2)/(b*Sqrt[a*x^2 + b*x^3]) + (4*Sqrt[a*x^2 + b*x^3])/(b^2*x)))/(3*a*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

method	result
gosper	$\frac{2(bx+a)(3b^2d\,x^2+12abdx-3b^2cx+8a^2d-2abc)x^5}{3b^3(b\,x^3+a\,x^2)^{\frac{5}{2}}}$
default	$\frac{2(bx+a)(3b^2d\,x^2+12abdx-3b^2cx+8a^2d-2abc)x^5}{3b^3(b\,x^3+a\,x^2)^{\frac{5}{2}}}$
orering	$\frac{2(bx+a)(3b^2d\,x^2+12abdx-3b^2cx+8a^2d-2abc)x^5}{3b^3(b\,x^3+a\,x^2)^{\frac{5}{2}}}$
trager	$\frac{2(3b^2d\,x^2+12abdx-3b^2cx+8a^2d-2abc)\sqrt{b\,x^3+a\,x^2}}{3x\,b^3(bx+a)^2}$
risch	$\frac{2d(bx+a)x}{b^3\sqrt{x^2(bx+a)}}+\frac{2(6abdx-3b^2cx+5a^2d-2abc)x}{3b^3(bx+a)\sqrt{x^2(bx+a)}}$
pseudoelliptic	$-\frac{4096\left(-\frac{11x^6\left(\frac{9dx}{11}+c\right)b^7}{2048}+\frac{33x^5a\left(\frac{49dx}{66}+c\right)b^6}{3584}-\frac{33x^4\left(\frac{7dx}{11}+c\right)a^2b^5}{1792}+\frac{11x^3a^3\left(\frac{21dx}{44}+c\right)b^4}{224}-\frac{33\left(\frac{7dx}{33}+c\right)x^2a^4b^3}{112}-\frac{33x\,a^5\left(-\frac{7dx}{22}\right.}{28}\right.}{99(bx+a)^{\frac{3}{2}}b^8}$

input
```
int(x^6*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(b*x+a)*(3*b^2*d*x^2+12*a*b*d*x-3*b^2*c*x+8*a^2*d-2*a*b*c)*x^5/b^3/(b*
x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.88

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx=\frac{2\,(3\,b^2dx^2-2\,abc+8\,a^2d-3\,(b^2c-4\,abd)x)\sqrt{bx^3+ax^2}}{3\,(b^5x^3+2\,ab^4x^2+a^2b^3x)}$$

input
```
integrate(x^6*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*b^2*d*x^2 - 2*a*b*c + 8*a^2*d - 3*(b^2*c - 4*a*b*d)*x)*sqrt(b*x^3 +
 a*x^2)/(b^5*x^3 + 2*a*b^4*x^2 + a^2*b^3*x)
```

Sympy [F]

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^6(c+dx)}{(x^2(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**6*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral(x**6*(c + d*x)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.84

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\,(3\,bx+2\,a)c}{3\,(b^3x+ab^2)\sqrt{bx+a}} + \frac{2\,(3\,b^2x^2+12\,abx+8\,a^2)d}{3\,(b^4x+ab^3)\sqrt{bx+a}}$$

input
```
integrate(x^6*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(3*b*x + 2*a)*c/((b^3*x + a*b^2)*sqrt(b*x + a)) + 2/3*(3*b^2*x^2 + 12
*a*b*x + 8*a^2)*d/((b^4*x + a*b^3)*sqrt(b*x + a))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.92

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{4\,(bc-4\,ad)\mathrm{sgn}(x)}{3\,\sqrt{a}b^3} + \frac{2\,\sqrt{bx+a}d}{b^3\mathrm{sgn}\,(x)} - \frac{2\,(3\,(bx+a)bc-abc-6\,(bx+a)ad+a^2d)}{3\,(bx+a)^{\frac{3}{2}}b^3\mathrm{sgn}\,(x)}$$

input
```
integrate(x^6*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
4/3*(b*c - 4*a*d)*sgn(x)/(sqrt(a)*b^3) + 2*sqrt(b*x + a)*d/(b^3*sgn(x)) -
2/3*(3*(b*x + a)*b*c - a*b*c - 6*(b*x + a)*a*d + a^2*d)/((b*x + a)^(3/2)*b
^3*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.70

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{b\,x^3+a\,x^2}\,(8\,d\,a^2+12\,d\,a\,b\,x-2\,c\,a\,b+3\,d\,b^2\,x^2-3\,c\,b^2\,x)}{3\,b^3\,x\,(a+b\,x)^2}$$

input
```
int((x^6*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
(2*(a*x^2 + b*x^3)^(1/2)*(8*a^2*d + 3*b^2*d*x^2 - 2*a*b*c - 3*b^2*c*x + 12
*a*b*d*x))/(3*b^3*x*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.61

$$\int \frac{x^6(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2b^2d\,x^2+8abdx-2b^2cx+\frac{16}{3}a^2d-\frac{4}{3}abc}{\sqrt{bx+a}\,b^3\,(bx+a)}$$

input
```
int(x^6*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(2*(8*a**2*d - 2*a*b*c + 12*a*b*d*x - 3*b**2*c*x + 3*b**2*d*x**2))/(3*sqrt
(a + b*x)*b**3*(a + b*x))
```

3.294 $\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 56

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2(bc-ad)x^3}{3b^2(ax^2+bx^3)^{3/2}} - \frac{2dx}{b^2\sqrt{ax^2+bx^3}}$$

output
```
-2/3*(-a*d+b*c)*x^3/b^2/(b*x^3+a*x^2)^(3/2)-2*d*x/b^2/(b*x^3+a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.64

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2x^3(2ad+b(c+3dx))}{3b^2(x^2(a+bx))^{3/2}}$$

input
```
Integrate[(x^5*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(-2*x^3*(2*a*d + b*(c + 3*d*x)))/(3*b^2*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.27, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {1943, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{(2ad+bc)\int \frac{x^3}{(bx^3+ax^2)^{3/2}}dx}{3ab}+\frac{2x^4(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2x^4(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}-\frac{2x(2ad+bc)}{3ab^2\sqrt{ax^2+bx^3}}$$

input
```
Int[(x^5*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*(b*c - a*d)*x^4)/(3*a*b*(a*x^2 + b*x^3)^(3/2)) - (2*(b*c + 2*a*d)*x)/(3
*a*b^2*Sqrt[a*x^2 + b*x^3])
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.71

method	result
gosper	$-\frac{2(bx+a)(3bdx+2ad+bc)x^{5}}{3b^{2}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$
default	$-\frac{2(bx+a)(3bdx+2ad+bc)x^{5}}{3b^{2}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$
orering	$-\frac{2(bx+a)(3bdx+2ad+bc)x^{5}}{3b^{2}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$
trager	$-\frac{2(3bdx+2ad+bc)\sqrt{b\,x^{3}+a\,x^{2}}}{3x\,b^{2}(bx+a)^{2}}$
pseudoelliptic	$\frac{\left(14d\,x^{6}+18c\,x^{5}\right)b^{6}-36x^{4}\left(\frac{2dx}{3}+c\right)a\,b^{5}+96x^{3}\left(\frac{dx}{2}+c\right)a^{2}b^{4}-576\left(\frac{2dx}{9}+c\right)x^{2}a^{3}b^{3}-2304\left(-\frac{dx}{3}+c\right)x\,a^{4}b^{2}-1536a^{5}(-2dx+}{63(bx+a)^{\frac{3}{2}}b^{7}}$

input
```
int(x^5*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(b*x+a)*(3*b*d*x+2*a*d+b*c)*x^5/b^2/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\,\sqrt{bx^3+ax^2}(3\,bdx+bc+2\,ad)}{3\,(b^4x^3+2\,ab^3x^2+a^2b^2x)}$$

input
```
integrate(x^5*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(b*x^3 + a*x^2)*(3*b*d*x + b*c + 2*a*d)/(b^4*x^3 + 2*a*b^3*x^2 +
a^2*b^2*x)
```

Sympy [F]

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^5(c+dx)}{(x^2(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**5*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral(x**5*(c + d*x)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.95

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\,(3\,bx+2\,a)d}{3\,(b^3x+ab^2)\sqrt{bx+a}} - \frac{2\,c}{3\,(b^2x+ab)\sqrt{bx+a}}$$

input
```
integrate(x^5*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
-2/3*(3*b*x + 2*a)*d/((b^3*x + a*b^2)*sqrt(b*x + a)) - 2/3*c/((b^2*x + a*b
)*sqrt(b*x + a))
```

Giac [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\,(bc+2\,ad)\mathrm{sgn}(x)}{3\,a^{\frac{3}{2}}b^2} - \frac{2\,(bc+3\,(bx+a)d-ad)}{3\,(bx+a)^{\frac{3}{2}}b^2\mathrm{sgn}\,(x)}$$

input
```
integrate(x^5*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
2/3*(b*c + 2*a*d)*sgn(x)/(a^(3/2)*b^2) - 2/3*(b*c + 3*(b*x + a)*d - a*d)/(
(b*x + a)^(3/2)*b^2*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 8.86 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\,\sqrt{b\,x^3+a\,x^2}\,(2\,a\,d+b\,c+3\,b\,d\,x)}{3\,b^2\,x\,(a+b\,x)^2}$$

input
```
int((x^5*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
-(2*(a*x^2 + b*x^3)^(1/2)*(2*a*d + b*c + 3*b*d*x))/(3*b^2*x*(a + b*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.61

$$\int \frac{x^5(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{-2bdx-\frac{4}{3}ad-\frac{2}{3}bc}{\sqrt{bx+a}\,b^2\,(bx+a)}$$

input
```
int(x^5*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(2*( - 2*a*d - b*c - 3*b*d*x))/(3*sqrt(a + b*x)*b**2*(a + b*x))
```

3.295 $\int \frac{x^4(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 92

$$\int \frac{x^4(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx=\frac{2(bc-ad)x^3}{3ab\left(ax^2+bx^3\right)^{3/2}}+\frac{2cx}{a^2\sqrt{ax^2+bx^3}}-\frac{2c\,\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{a^{5/2}}$$

output
```
2/3*(-a*d+b*c)*x^3/a/b/(b*x^3+a*x^2)^(3/2)+2*c*x/a^2/(b*x^3+a*x^2)^(1/2)-2
*c*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(5/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.90

$$\int \frac{x^4(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx=\frac{2x^3\left(\sqrt{a}(4abc-a^2d+3b^2cx)-3bc(a+bx)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+bx}}{\sqrt{a}}\right)\right)}{3a^{5/2}b\left(x^2(a+bx)\right)^{3/2}}$$

input
```
Integrate[(x^4*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*x^3*(Sqrt[a]*(4*a*b*c - a^2*d + 3*b^2*c*x) - 3*b*c*(a + b*x)^(3/2)*ArcT
anh[Sqrt[a + b*x]/Sqrt[a]]))/(3*a^(5/2)*b*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1943, 1929, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \ 1943$$

$$\frac{c\int \frac{x^2}{(bx^3+ax^2)^{3/2}}dx}{a}+\frac{2x^3(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \ 1929$$

$$\frac{c\left(\frac{\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{a}+\frac{2x}{a\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2x^3(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \ 1914$$

$$\frac{c\left(\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}\right)}{a}+\frac{2x^3(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{c\left(\frac{2x}{a\sqrt{ax^2+bx^3}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}\right)}{a}+\frac{2x^3(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

input
```
Int[(x^4*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*(b*c - a*d)*x^3)/(3*a*b*(a*x^2 + b*x^3)^(3/2)) + (c*((2*x)/(a*Sqrt[a*x^
2 + b*x^3]) - (2*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/a
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.86

method	result	size
default	$-\frac{2x^{5}\left(bx+a\right)\left(d\,a^{\frac{9}{2}}-4a^{\frac{7}{2}}bc-3a^{\frac{5}{2}}b^{2}cx+3bc\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^{2}\left(bx+a\right)^{\frac{3}{2}}\right)}{3\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}b\,a^{\frac{9}{2}}}$	79
pseudoelliptic	$-\frac{512\left(-\frac{21x^{4}\left(\frac{5dx}{7}+c\right)b^{5}}{1280}+\frac{7x^{3}\left(\frac{15dx}{28}+c\right)a\,b^{4}}{160}-\frac{21x^{2}a^{2}\left(\frac{5dx}{21}+c\right)b^{3}}{80}-\frac{21\left(-\frac{5dx}{14}+c\right)x\,a^{3}b^{2}}{20}-\frac{7\left(-\frac{15dx}{7}+c\right)a^{4}b}{10}+a^{5}d\right)}{21\left(bx+a\right)^{\frac{3}{2}}b^{6}}$	92

input
```
int(x^4*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x^5*(b*x+a)*(d*a^(9/2)-4*a^(7/2)*b*c-3*a^(5/2)*b^2*c*x+3*b*c*arctanh(
(b*x+a)^(1/2)/a^(1/2))*a^2*(b*x+a)^(3/2))/(b*x^3+a*x^2)^(5/2)/b/a^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 268, normalized size of antiderivative = 2.91

$$\int \frac{x^4(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \left[\frac{3\,(b^3cx^3+2\,ab^2cx^2+a^2bcx)\sqrt{a}\log\left(\frac{bx^2+2\,ax-2\,\sqrt{bx^3+ax^2}\sqrt{a}}{x^2}\right)+2\,(3\,ab^2cx+4\,a^2bc-}{3\,(a^3b^3x^3+2\,a^4b^2x^2+a^5bx)}\right.$$

input
```
integrate(x^4*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(b^3*c*x^3 + 2*a*b^2*c*x^2 + a^2*b*c*x)*sqrt(a)*log((b*x^2 + 2*a*x
 - 2*sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(3*a*b^2*c*x + 4*a^2*b*c - a^3*
d)*sqrt(b*x^3 + a*x^2))/(a^3*b^3*x^3 + 2*a^4*b^2*x^2 + a^5*b*x), 2/3*(3*(b
^3*c*x^3 + 2*a*b^2*c*x^2 + a^2*b*c*x)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*
sqrt(-a)/(b*x^2 + a*x)) + (3*a*b^2*c*x + 4*a^2*b*c - a^3*d)*sqrt(b*x^3 + a
*x^2))/(a^3*b^3*x^3 + 2*a^4*b^2*x^2 + a^5*b*x)]
```

Sympy [F]

$$\int \frac{x^4(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^4(c+dx)}{(x^2\,(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**4*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral(x**4*(c + d*x)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^4(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(dx+c)x^4}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^4*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*x^4/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.30

$$\int \frac{x^4(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\,c\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^2\mathrm{sgn}\left(x\right)} - \frac{2\left(3\,\sqrt{a}bc\arctan\left(\frac{\sqrt{a}}{\sqrt{-a}}\right)+4\,\sqrt{-a}bc-\sqrt{-a}ad\right)\mathrm{sgn}(x)}{3\,\sqrt{-a}a^{\frac{5}{2}}b} + \frac{2\,(3\,(bx+a)bc+abc-a^2d)}{3\,(bx+a)^{\frac{3}{2}}a^2b\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^4*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
2*c*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^2*sgn(x)) - 2/3*(3*sqrt(a)*
b*c*arctan(sqrt(a)/sqrt(-a)) + 4*sqrt(-a)*b*c - sqrt(-a)*a*d)*sgn(x)/(sqrt
(-a)*a^(5/2)*b) + 2/3*(3*(b*x + a)*b*c + a*b*c - a^2*d)/((b*x + a)^(3/2)*a
^2*b*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^4\,(c+d\,x)}{(b\,x^3+a\,x^2)^{5/2}}\,dx$$

input
```
int((x^4*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
int((x^4*(c + d*x))/(a*x^2 + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.58

$$\int \frac{x^4(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{3\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)abc+3\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)b^2cx-3}{3}$$

input
```
int(x^4*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b*c + 3*sqrt(a)*sq
rt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**2*c*x - 3*sqrt(a)*sqrt(a + b*x
)*log(sqrt(a + b*x) + sqrt(a))*a*b*c - 3*sqrt(a)*sqrt(a + b*x)*log(sqrt(a
+ b*x) + sqrt(a))*b**2*c*x - 2*a**3*d + 8*a**2*b*c + 6*a*b**2*c*x)/(3*sqrt
(a + b*x)*a**3*b*(a + b*x))
```

3.296 $\int \frac{x^3(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 128

$$\int \frac{x^3(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx=-\frac{2(bc-ad)x^3}{3a^2\left(ax^2+bx^3\right)^{3/2}}-\frac{2(2bc-ad)x}{a^3\sqrt{ax^2+bx^3}}-\frac{c\sqrt{ax^2+bx^3}}{a^3x^2}+\frac{(5bc-2ad)\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{a^{7/2}}$$

output
```
-2/3*(-a*d+b*c)*x^3/a^2/(b*x^3+a*x^2)^(3/2)-2*(-a*d+2*b*c)*x/a^3/(b*x^3+a*
x^2)^(1/2)-c*(b*x^3+a*x^2)^(1/2)/a^3/x^2+(-2*a*d+5*b*c)*arctanh((b*x^3+a*x
^2)^(1/2)/a^(1/2)/x)/a^(7/2)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.81

$$\int \frac{x^3(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx=\frac{x^2\left(\sqrt{a}(-15b^2cx^2+2abx(-10c+3dx)+a^2(-3c+8dx))+3(5bc-2ad)x(a+bx)\right.}{3a^{7/2}\left(x^2(a+bx)\right)^{3/2}}$$

input
```
Integrate[(x^3*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(x^2*(Sqrt[a]*(-15*b^2*c*x^2 + 2*a*b*x*(-10*c + 3*d*x) + a^2*(-3*c + 8*d*x
)) + 3*(5*b*c - 2*a*d)*x*(a + b*x)^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]))/
(3*a^(7/2)*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.208$, Rules used = {1943, 1929, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{(5bc-2ad)\int \frac{x}{(bx^3+ax^2)^{3/2}}dx}{3ab}+\frac{2x^2(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1929}$$

$$\frac{(5bc-2ad)\left(\frac{3\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}\right)}{3ab}+\frac{2x^2(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{(5bc-2ad)\left(\frac{3\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}\right)}{3ab}+\frac{2x^2(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{(5bc-2ad)\left(\frac{3\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}\right)}{3ab}+\frac{2x^2(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{\left(\frac{3\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{a}+\frac{2}{a\sqrt{ax^2+bx^3}}\right)(5bc-2ad)}{3ab}+\frac{2x^2(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}$$

input
```
Int[(x^3*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*(b*c - a*d)*x^2)/(3*a*b*(a*x^2 + b*x^3)^(3/2)) + ((5*b*c - 2*a*d)*(2/(a
*Sqrt[a*x^2 + b*x^3]) + (3*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(S
qrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/a))/(3*a*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.59

method	result
pseudoelliptic	$\frac{\frac{2x^{3}\left(\frac{3dx}{5}+c\right)b^{4}}{3}-4x^{2}a\left(\frac{4dx}{15}+c\right)b^{3}-16x\left(-\frac{2dx}{5}+c\right)a^{2}b^{2}-\frac{32a^{3}\left(-\frac{12dx}{5}+c\right)b}{3}+\frac{256a^{4}d}{15}}{b^{5}\left(bx+a\right)^{\frac{3}{2}}}$
risch	$-\frac{c\left(bx+a\right)}{a^{3}\sqrt{x^{2}\left(bx+a\right)}}-\frac{\left(-\frac{2\left(-2ad+5bc\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\left(2ad-4bc\right)}{\sqrt{bx+a}}-\frac{4a\left(ad-bc\right)}{3\left(bx+a\right)^{\frac{3}{2}}}\right)\sqrt{bx+a}\,x}{2a^{3}\sqrt{x^{2}\left(bx+a\right)}}$
default	$-\frac{x^{4}\left(bx+a\right)\left(6\left(bx+a\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)adx-15\left(bx+a\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)bcx-8a^{\frac{5}{2}}dx-6a^{\frac{3}{2}}bd\,x^{2}+20a^{\frac{3}{2}}bcx+15\sqrt{a}\,b^{2}}{3\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}a^{\frac{7}{2}}}$

input
```
int(x^3*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
256/15/(b*x+a)^(3/2)*(5/128*x^3*(3/5*d*x+c)*b^4-15/64*x^2*a*(4/15*d*x+c)*b
^3-15/16*x*(-2/5*d*x+c)*a^2*b^2-5/8*a^3*(-12/5*d*x+c)*b+a^4*d)/b^5
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.91

$$\int \frac{x^3(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \left[-\frac{3\left((5\,b^3c-2\,ab^2d)x^4+2\,(5\,ab^2c-2\,a^2bd)x^3+(5\,a^2bc-2\,a^3d)x^2\right)\sqrt{a}\log\left(\frac{bx^2+2\,\cdots}{\cdots}\right)\cdots}{6\,(a^4b^2x^4+\cdots}\right.$$

$$-\frac{3\left((5\,b^3c-2\,ab^2d)x^4+2\,(5\,ab^2c-2\,a^2bd)x^3+(5\,a^2bc-2\,a^3d)x^2\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+(3\,a^3c+\cdots}{3\,(a^4b^2x^4+2\,a^5bx^3+a^6x^2)}$$

input
```
integrate(x^3*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*((5*b^3*c - 2*a*b^2*d)*x^4 + 2*(5*a*b^2*c - 2*a^2*b*d)*x^3 + (5*a
^2*b*c - 2*a^3*d)*x^2)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^3 + a*x^2)*
sqrt(a))/x^2) + 2*(3*a^3*c + 3*(5*a*b^2*c - 2*a^2*b*d)*x^2 + 4*(5*a^2*b*c
- 2*a^3*d)*x)*sqrt(b*x^3 + a*x^2))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2),
-1/3*(3*((5*b^3*c - 2*a*b^2*d)*x^4 + 2*(5*a*b^2*c - 2*a^2*b*d)*x^3 + (5*a^
2*b*c - 2*a^3*d)*x^2)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2
+ a*x)) + (3*a^3*c + 3*(5*a*b^2*c - 2*a^2*b*d)*x^2 + 4*(5*a^2*b*c - 2*a^3*
d)*x)*sqrt(b*x^3 + a*x^2))/(a^4*b^2*x^4 + 2*a^5*b*x^3 + a^6*x^2)]
```

Sympy [F]

$$\int \frac{x^3(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \int \frac{x^3(c+dx)}{\left(x^2\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**3*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral(x**3*(c + d*x)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^3(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(dx+c)x^3}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^3*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*x^3/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.80

$$\int \frac{x^3(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{(5\,bc-2\,ad)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{\sqrt{-a}a^3\mathrm{sgn}\left(x\right)} - \frac{\sqrt{bx+a}c}{a^3x\mathrm{sgn}\left(x\right)} - \frac{2\left(6\left(bx+a\right)bc+abc-3\left(bx+a\right)ad-a^2d\right)}{3\left(bx+a\right)^{\frac{3}{2}}a^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate(x^3*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
-(5*b*c - 2*a*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^3*sgn(x)) - sq
rt(b*x + a)*c/(a^3*x*sgn(x)) - 2/3*(6*(b*x + a)*b*c + a*b*c - 3*(b*x + a)*
a*d - a^2*d)/((b*x + a)^(3/2)*a^3*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^3\,(c+d\,x)}{(b\,x^3+a\,x^2)^{5/2}}\,dx$$

input
```
int((x^3*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
int((x^3*(c + d*x))/(a*x^2 + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.16

$$\int \frac{x^3(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{6\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)a^2dx-15\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)abcx}{}$$

input
```
int(x^3*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(6*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a**2*d*x - 15*sqrt(a
)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b*c*x + 6*sqrt(a)*sqrt(a +
b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b*d*x**2 - 15*sqrt(a)*sqrt(a + b*x)*lo
g(sqrt(a + b*x) - sqrt(a))*b**2*c*x**2 - 6*sqrt(a)*sqrt(a + b*x)*log(sqrt(
a + b*x) + sqrt(a))*a**2*d*x + 15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x)
+ sqrt(a))*a*b*c*x - 6*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*
a*b*d*x**2 + 15*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**2*c*
x**2 - 6*a**3*c + 16*a**3*d*x - 40*a**2*b*c*x + 12*a**2*b*d*x**2 - 30*a*b*
*2*c*x**2)/(6*sqrt(a + b*x)*a**4*x*(a + b*x))
```

3.297 $\int \frac{x^2(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 170

$$\int \frac{x^2(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \frac{2b(bc-ad)x^3}{3a^3\left(ax^2+bx^3\right)^{3/2}}+\frac{2b(3bc-2ad)x}{a^4\sqrt{ax^2+bx^3}}-\frac{c\sqrt{ax^2+bx^3}}{2a^3x^3}+\frac{(11bc-4ad)\sqrt{ax^2+bx^3}}{4a^4x^2}-\frac{5b(7bc-4ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{ax}}\right)}{4a^{9/2}}$$

output
```
2/3*b*(-a*d+b*c)*x^3/a^3/(b*x^3+a*x^2)^(3/2)+2*b*(-2*a*d+3*b*c)*x/a^4/(b*x
^3+a*x^2)^(1/2)-1/2*c*(b*x^3+a*x^2)^(1/2)/a^3/x^3+1/4*(-4*a*d+11*b*c)*(b*x
^3+a*x^2)^(1/2)/a^4/x^2-5/4*b*(-4*a*d+7*b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a
^(1/2)/x)/a^(9/2)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.72

$$\int \frac{x^2(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \frac{x\left(\sqrt{a}(105b^3cx^3+a^2bx(21c-80dx)+20ab^2x^2(7c-3dx)-6a^3(c+2dx))+15b(-\right.}{12a^{9/2}\left(x^2(a+bx)\right)^{3/2}}$$

input
```
Integrate[(x^2*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(x*(Sqrt[a]*(105*b^3*c*x^3 + a^2*b*x*(21*c - 80*d*x) + 20*a*b^2*x^2*(7*c -
 3*d*x) - 6*a^3*(c + 2*d*x)) + 15*b*(-7*b*c + 4*a*d)*x^2*(a + b*x)^(3/2)*A
rcTanh[Sqrt[a + b*x]/Sqrt[a]]))/(12*a^(9/2)*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.01, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1943, 1912, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{(7bc-4ad)\int \frac{1}{(bx^3+ax^2)^{3/2}}dx}{3ab}+\frac{2x(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1912}$$

$$\frac{(7bc-4ad)\left(\frac{5\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{3ab}+\frac{2x(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{(7bc-4ad)\left(\frac{5\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{3ab}+\frac{2x(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$\frac{(7bc-4ad)\left(\frac{5\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{3ab}+\frac{2x(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1914}$$

$$\frac{(7bc-4ad)\left(\frac{5\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{3ab}+\frac{2x(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\left(\frac{5\left(-\frac{3b\left(\frac{b\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)(7bc-4ad)}{3ab}+\frac{2x(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}$$

input
```
Int[(x^2*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*(b*c - a*d)*x)/(3*a*b*(a*x^2 + b*x^3)^(3/2)) + ((7*b*c - 4*a*d)*(2/(a*x
*Sqrt[a*x^2 + b*x^3]) + (5*(-1/2*Sqrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqr
t[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a
^(3/2)))/(4*a)))/a))/(3*a*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.34

method	result
pseudoelliptic	$-\frac{32\left(-\frac{3x^{2}\left(\frac{dx}{3}+c\right)b^{3}}{16}-\frac{3\left(-\frac{dx}{2}+c\right)xa\,b^{2}}{4}-\frac{a^{2}(-3dx+c)b}{2}+a^{3}d\right)}{3(bx+a)^{\frac{3}{2}}b^{4}}$
risch	$-\frac{(bx+a)(4adx-11cbx+2ac)}{4a^{4}x\sqrt{x^{2}(bx+a)}}-\frac{b\left(-\frac{2(20ad-35bc)\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2(-16ad+24bc)}{\sqrt{bx+a}}+\frac{16a(ad-bc)}{3(bx+a)^{\frac{3}{2}}}\right)\sqrt{bx+a}\,x}{8a^{4}\sqrt{x^{2}(bx+a)}}$
default	$\frac{x^{3}(bx+a)\left(60(bx+a)^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)abd\,x^{2}-105(bx+a)^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{2}c\,x^{2}-12a^{\frac{7}{2}}dx-80a^{\frac{5}{2}}bd\,x^{2}-60a^{\frac{3}{2}}b^{2}d\,x^{3}}{12(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}a^{\frac{9}{2}}}$

input
```
int(x^2*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-32/3/(b*x+a)^(3/2)*(-3/16*x^2*(1/3*d*x+c)*b^3-3/4*(-1/2*d*x+c)*x*a*b^2-1/
2*a^2*(-3*d*x+c)*b+a^3*d)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.54

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \left[-\frac{15\left((7\,b^4c-4\,ab^3d)x^5+2\,(7\,ab^3c-4\,a^2b^2d)x^4+(7\,a^2b^2c-4\,a^3bd)x^3\right)\sqrt{a}\log\left(\frac{bx}{}\right.}{}\right.$$

input
```
integrate(x^2*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(15*((7*b^4*c - 4*a*b^3*d)*x^5 + 2*(7*a*b^3*c - 4*a^2*b^2*d)*x^4 +
(7*a^2*b^2*c - 4*a^3*b*d)*x^3)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(b*x^3 +
 a*x^2)*sqrt(a))/x^2) + 2*(6*a^4*c - 15*(7*a*b^3*c - 4*a^2*b^2*d)*x^3 - 20
*(7*a^2*b^2*c - 4*a^3*b*d)*x^2 - 3*(7*a^3*b*c - 4*a^4*d)*x)*sqrt(b*x^3 + a
*x^2))/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a^7*x^3), 1/12*(15*((7*b^4*c - 4*a*b^3
*d)*x^5 + 2*(7*a*b^3*c - 4*a^2*b^2*d)*x^4 + (7*a^2*b^2*c - 4*a^3*b*d)*x^3)
*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) - (6*a^4*c -
15*(7*a*b^3*c - 4*a^2*b^2*d)*x^3 - 20*(7*a^2*b^2*c - 4*a^3*b*d)*x^2 - 3*(7
*a^3*b*c - 4*a^4*d)*x)*sqrt(b*x^3 + a*x^2))/(a^5*b^2*x^5 + 2*a^6*b*x^4 + a
^7*x^3)]
```

Sympy [F]

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^2(c+dx)}{(x^2\,(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate(x**2*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral(x**2*(c + d*x)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(dx+c)x^2}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x^2*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*x^2/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.95

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{5\,(7\,b^2c-4\,abd)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{4\,\sqrt{-a}a^4\mathrm{sgn}\,(x)} + \frac{2\,(9\,(bx+a)b^2c+ab^2c-6\,(bx+a)abd-a^2bd)}{3\,(bx+a)^{\frac{3}{2}}a^4\mathrm{sgn}\,(x)} + \frac{11\,(bx+a)^{\frac{3}{2}}b^2c-13\,\sqrt{bx+a}ab^2c-4\,(bx+a)^{\frac{3}{2}}abd+4\,\sqrt{bx+a}a^2bd}{4\,a^4b^2x^2\mathrm{sgn}\,(x)}$$

input

```
integrate(x^2*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output

```
5/4*(7*b^2*c - 4*a*b*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^4*sgn(x
)) + 2/3*(9*(b*x + a)*b^2*c + a*b^2*c - 6*(b*x + a)*a*b*d - a^2*b*d)/((b*x
 + a)^(3/2)*a^4*sgn(x)) + 1/4*(11*(b*x + a)^(3/2)*b^2*c - 13*sqrt(b*x + a)
*a*b^2*c - 4*(b*x + a)^(3/2)*a*b*d + 4*sqrt(b*x + a)*a^2*b*d)/(a^4*b^2*x^2
*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x^2\,(c+d\,x)}{{(b\,x^3+a\,x^2)}^{5/2}}\,dx$$

input

```
int((x^2*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output

```
int((x^2*(c + d*x))/(a*x^2 + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.88

$$\int \frac{x^2(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{-60\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a^2bd\,x^2+105\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)}{}$$

input
```
int(x^2*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
( - 60*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a**2*b*d*x**2 +
105*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**2*c*x**2 - 60*
sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**2*d*x**3 + 105*sqr
t(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**3*c*x**3 + 60*sqrt(a)*s
qrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a**2*b*d*x**2 - 105*sqrt(a)*sqrt
(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**2*c*x**2 + 60*sqrt(a)*sqrt(a +
 b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**2*d*x**3 - 105*sqrt(a)*sqrt(a + b*
x)*log(sqrt(a + b*x) + sqrt(a))*b**3*c*x**3 - 12*a**4*c - 24*a**4*d*x + 42
*a**3*b*c*x - 160*a**3*b*d*x**2 + 280*a**2*b**2*c*x**2 - 120*a**2*b**2*d*x
**3 + 210*a*b**3*c*x**3)/(24*sqrt(a + b*x)*a**5*x**2*(a + b*x))
```

3.298 $\int \frac{x(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 211

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2b^2(bc-ad)x^3}{3a^4(ax^2+bx^3)^{3/2}} - \frac{2b^2(4bc-3ad)x}{a^5\sqrt{ax^2+bx^3}} - \frac{c\sqrt{ax^2+bx^3}}{3a^3x^4} + \frac{(17bc-6ad)\sqrt{ax^2+bx^3}}{12a^4x^3} - \frac{b(41bc-22ad)\sqrt{ax^2+bx^3}}{8a^5x^2} + \frac{35b^2(3bc-2ad)\text{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{8a^{11/2}}$$

output

```
-2/3*b^2*(-a*d+b*c)*x^3/a^4/(b*x^3+a*x^2)^(3/2)-2*b^2*(-3*a*d+4*b*c)*x/a^5
/(b*x^3+a*x^2)^(1/2)-1/3*c*(b*x^3+a*x^2)^(1/2)/a^3/x^4+1/12*(-6*a*d+17*b*c
)*(b*x^3+a*x^2)^(1/2)/a^4/x^3-1/8*b*(-22*a*d+41*b*c)*(b*x^3+a*x^2)^(1/2)/a
^5/x^2+35/8*b^2*(-2*a*d+3*b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(1
1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.68

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{\sqrt{a}(-315b^4cx^4+210ab^3x^3(-2c+dx)-4a^4(2c+3dx)+6a^3bx(3c+7dx)+7a^2b^2}{24a^{11/2}\left(x^2(a+bx)\right.}$$

input
```
Integrate[(x*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(Sqrt[a]*(-315*b^4*c*x^4 + 210*a*b^3*x^3*(-2*c + d*x) - 4*a^4*(2*c + 3*d*x
) + 6*a^3*b*x*(3*c + 7*d*x) + 7*a^2*b^2*x^2*(-9*c + 40*d*x)) + 105*b^2*(3*
b*c - 2*a*d)*x^3*(a + b*x)^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(24*a^(11
/2)*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {1944, 1929, 1912, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \; 1944$$

$$-\frac{(3bc-2ad)\int \frac{x^2}{(bx^3+ax^2)^{5/2}}dx}{2a}-\frac{c}{3a\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \; 1929$$

$$-\frac{(3bc-2ad)\left(\frac{7\int \frac{1}{\left(bx^3+ax^2\right)^{3/2}}dx}{3a}+\frac{2x}{3a(ax^2+bx^3)^{3/2}}\right)}{2a}-\frac{c}{3a\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \; 1912$$

$$-\frac{(3bc-2ad)\left(\frac{7\left(\frac{5\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2x}{3a(ax^2+bx^3)^{3/2}}\right)}{2a}-\frac{c}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$\downarrow$ 1931

$$-\frac{(3bc-2ad)\left(\frac{7\left(\frac{5\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2x}{3a(ax^2+bx^3)^{3/2}}\right)}{2a}-\frac{c}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$\downarrow$ 1931

$$-\frac{(3bc-2ad)\left(\frac{7\left(\frac{5\left(-\frac{3b\left(-\frac{b\int \frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2x}{3a(ax^2+bx^3)^{3/2}}\right)}{2a}-\frac{c}{3a\left(ax^2+bx^3\right)^{3/2}}$$

$\downarrow$ 1914

$$-\frac{\frac{(3bc-2ad)\left(\frac{7\left(\frac{5\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2x}{3a(ax^2+bx^3)^{3/2}}\right)}{2a}-\frac{c}{3a\left(ax^2+bx^3\right)^{3/2}}}{}$$

$\Big\downarrow$ 219

$$-\frac{\left(\frac{7\left(\frac{5\left(-\frac{3b\left(\frac{b\,\text{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{a}+\frac{2}{ax\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2x}{3a(ax^2+bx^3)^{3/2}}\right)(3bc-2ad)}{2a}-\frac{c}{3a\left(ax^2+bx^3\right)^{3/2}}$$

input
```
Int[(x*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
-1/3*c/(a*(a*x^2 + b*x^3)^(3/2)) - ((3*b*c - 2*a*d)*((2*x)/(3*a*(a*x^2 + b
*x^3)^(3/2)) + (7*(2/(a*x*Sqrt[a*x^2 + b*x^3]) + (5*(-1/2*Sqrt[a*x^2 + b*x
^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*ArcTanh[(Sqrt[a]*x
)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/a))/(3*a)))/(2*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.21

method	result
pseudoelliptic	$\frac{2\left(d\,x^{2}-cx\right)b^{2}-\frac{4a\left(-6dx+c\right)b}{3}+\frac{16a^{2}d}{3}}{b^{3}\left(bx+a\right)^{\frac{3}{2}}}$
risch	$-\frac{\left(bx+a\right)\left(-66abd\,x^{2}+123b^{2}c\,x^{2}+12a^{2}dx-34abcx+8a^{2}c\right)}{24a^{5}x^{2}\sqrt{x^{2}\left(bx+a\right)}}+\frac{b^{2}\left(-\frac{2\left(70ad-105bc\right)\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2\left(-48ad+64bc\right)}{\sqrt{bx+a}}+\frac{32c}{3(\cdots}\right.}{16a^{5}\sqrt{x^{2}\left(bx+a\right)}}$
default	$-\frac{x^{2}\left(bx+a\right)\left(210\left(bx+a\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^{2}d\,x^{3}-315\left(bx+a\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{3}c\,x^{3}+12a^{\frac{9}{2}}dx-42a^{\frac{7}{2}}bd\,x^{2}-280a^{\frac{5}{2}}\cdots\right.}{24\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}a^{\frac{11}{2}}}$

input
```
int(x*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(3*(d*x^2-c*x)*b^2-2*a*(-6*d*x+c)*b+8*a^2*d)/(b*x+a)^(3/2)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 482, normalized size of antiderivative = 2.28

$$\int \frac{x(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx=\left[-\frac{105\left(\left(3\,b^5c-2\,ab^4d\right)x^6+2\left(3\,ab^4c-2\,a^2b^3d\right)x^5+\left(3\,a^2b^3c-2\,a^3b^2d\right)x^4\right)\sqrt{a}\log\left(\cdots\right.}{\cdots}\right.$$

$$-\frac{105\left(\left(3\,b^5c-2\,ab^4d\right)x^6+2\left(3\,ab^4c-2\,a^2b^3d\right)x^5+\left(3\,a^2b^3c-2\,a^3b^2d\right)x^4\right)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-a}}{bx^2+ax}\right)+\left(8\cdots\right.}{24\left(a^6b^{\cdots}\right.}$$

input
```
integrate(x*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/48*(105*((3*b^5*c - 2*a*b^4*d)*x^6 + 2*(3*a*b^4*c - 2*a^2*b^3*d)*x^5 +
 (3*a^2*b^3*c - 2*a^3*b^2*d)*x^4)*sqrt(a)*log((b*x^2 + 2*a*x - 2*sqrt(b*x^
3 + a*x^2)*sqrt(a))/x^2) + 2*(8*a^5*c + 105*(3*a*b^4*c - 2*a^2*b^3*d)*x^4
+ 140*(3*a^2*b^3*c - 2*a^3*b^2*d)*x^3 + 21*(3*a^3*b^2*c - 2*a^4*b*d)*x^2 -
 6*(3*a^4*b*c - 2*a^5*d)*x)*sqrt(b*x^3 + a*x^2))/(a^6*b^2*x^6 + 2*a^7*b*x^
5 + a^8*x^4), -1/24*(105*((3*b^5*c - 2*a*b^4*d)*x^6 + 2*(3*a*b^4*c - 2*a^2
*b^3*d)*x^5 + (3*a^2*b^3*c - 2*a^3*b^2*d)*x^4)*sqrt(-a)*arctan(sqrt(b*x^3
+ a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (8*a^5*c + 105*(3*a*b^4*c - 2*a^2*b^3*d
)*x^4 + 140*(3*a^2*b^3*c - 2*a^3*b^2*d)*x^3 + 21*(3*a^3*b^2*c - 2*a^4*b*d)
*x^2 - 6*(3*a^4*b*c - 2*a^5*d)*x)*sqrt(b*x^3 + a*x^2))/(a^6*b^2*x^6 + 2*a^
7*b*x^5 + a^8*x^4)]
```

Sympy [F]

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x(c+dx)}{\left(x^2\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral(x*(c + d*x)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(dx+c)x}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*x/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.99

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{35\,(3\,b^3c-2\,ab^2d)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}a^5\mathrm{sgn}\,(x)}$$
$$-\frac{315\,(bx+a)^4b^3c-840\,(bx+a)^3ab^3c+693\,(bx+a)^2a^2b^3c-144\,(bx+a)a^3b^3c-16\,a^4b^3c-210\,(bx+a}{24\left((bx+a)^{\frac{3}{2}}-\sqrt{bx+a}a\right)^3a^5}$$

input
```
integrate(x*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
-35/8*(3*b^3*c - 2*a*b^2*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^5*s
gn(x)) - 1/24*(315*(b*x + a)^4*b^3*c - 840*(b*x + a)^3*a*b^3*c + 693*(b*x
+ a)^2*a^2*b^3*c - 144*(b*x + a)*a^3*b^3*c - 16*a^4*b^3*c - 210*(b*x + a)^
4*a*b^2*d + 560*(b*x + a)^3*a^2*b^2*d - 462*(b*x + a)^2*a^3*b^2*d + 96*(b*
x + a)*a^4*b^2*d + 16*a^5*b^2*d)/(((b*x + a)^(3/2) - sqrt(b*x + a)*a)^3*a^
5*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{x\,(c+d\,x)}{{(b\,x^3+a\,x^2)}^{5/2}}\,dx$$

input
```
int((x*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
int((x*(c + d*x))/(a*x^2 + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 347, normalized size of antiderivative = 1.64

$$\int \frac{x(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{210\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a^2b^2d\,x^3 - 315\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)}{}$$

input
```
int(x*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(210*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a**2*b**2*d*x**3 -
 315*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**3*c*x**3 + 21
0*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**3*d*x**4 - 315*s
qrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**4*c*x**4 - 210*sqrt(a
)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a**2*b**2*d*x**3 + 315*sqrt(a
)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**3*c*x**3 - 210*sqrt(a)*s
qrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**3*d*x**4 + 315*sqrt(a)*sqrt
(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**4*c*x**4 - 16*a**5*c - 24*a**5*d
*x + 36*a**4*b*c*x + 84*a**4*b*d*x**2 - 126*a**3*b**2*c*x**2 + 560*a**3*b*
*2*d*x**3 - 840*a**2*b**3*c*x**3 + 420*a**2*b**3*d*x**4 - 630*a*b**4*c*x**
4)/(48*sqrt(a + b*x)*a**6*x**3*(a + b*x))
```

3.299 $\int \frac{c+dx}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 248

$$\int \frac{c+dx}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2b^3(bc-ad)x^3}{3a^5(ax^2+bx^3)^{3/2}} + \frac{2b^3(5bc-4ad)x}{a^6\sqrt{ax^2+bx^3}} - \frac{c\sqrt{ax^2+bx^3}}{4a^3x^5} + \frac{(23bc-8ad)\sqrt{ax^2+bx^3}}{24a^4x^4} - \frac{b(259bc-136ad)\sqrt{ax^2+bx^3}}{96a^5x^3} + \frac{b^2(515bc-328ad)\sqrt{ax^2+bx^3}}{64a^6x^2} - \frac{105b^3(11bc-8ad)\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{64a^{13/2}}$$

output
```
2/3*b^3*(-a*d+b*c)*x^3/a^5/(b*x^3+a*x^2)^(3/2)+2*b^3*(-4*a*d+5*b*c)*x/a^6/
(b*x^3+a*x^2)^(1/2)-1/4*c*(b*x^3+a*x^2)^(1/2)/a^3/x^5+1/24*(-8*a*d+23*b*c)
*(b*x^3+a*x^2)^(1/2)/a^4/x^4-1/96*b*(-136*a*d+259*b*c)*(b*x^3+a*x^2)^(1/2)
/a^5/x^3+1/64*b^2*(-328*a*d+515*b*c)*(b*x^3+a*x^2)^(1/2)/a^6/x^2-105/64*b^
3*(-8*a*d+11*b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(13/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.67

$$\int \frac{c+dx}{(ax^2+bx^3)^{5/2}}\,dx = \frac{\sqrt{a}(3465b^5cx^5+21a^2b^3x^3(33c-160dx)+420ab^4x^4(11c-6dx)-16a^5(3c+4dx)}{192a}$$

input
```
Integrate[(c + d*x)/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(Sqrt[a]*(3465*b^5*c*x^5 + 21*a^2*b^3*x^3*(33*c - 160*d*x) + 420*a*b^4*x^4
*(11*c - 6*d*x) - 16*a^5*(3*c + 4*d*x) + 8*a^4*b*x*(11*c + 18*d*x) - 18*a^
3*b^2*x^2*(11*c + 28*d*x)) + 315*b^3*(-11*b*c + 8*a*d)*x^4*(a + b*x)^(3/2)
*ArcTanh[Sqrt[a + b*x]/Sqrt[a]])/(192*a^(13/2)*x*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 363, normalized size of antiderivative = 1.46, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {2450, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 2450}$$

$$\int \left(\frac{c}{(ax^2+bx^3)^{5/2}}+\frac{dx}{(ax^2+bx^3)^{5/2}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{1155b^4c\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{64a^{13/2}}+\frac{105b^3d\operatorname{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{8a^{11/2}}+\frac{1155b^3c\sqrt{ax^2+bx^3}}{64a^6x^2}-\frac{385b^2c\sqrt{ax^2+bx^3}}{32a^5x^3}-\frac{105b^2d\sqrt{ax^2+bx^3}}{8a^5x^2}+\frac{77bc\sqrt{ax^2+bx^3}}{8a^4x^4}+\frac{35bd\sqrt{ax^2+bx^3}}{4a^4x^3}-\frac{33c\sqrt{ax^2+bx^3}}{4a^3x^5}-\frac{7d\sqrt{ax^2+bx^3}}{a^3x^4}+\frac{22c}{3a^2x^3\sqrt{ax^2+bx^3}}+\frac{6d}{a^2x^2\sqrt{ax^2+bx^3}}+\frac{2c}{3ax\left(ax^2+bx^3\right)^{3/2}}+\frac{2d}{3a\left(ax^2+bx^3\right)^{3/2}}$$

input
```
Int[(c + d*x)/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*d)/(3*a*(a*x^2 + b*x^3)^(3/2)) + (2*c)/(3*a*x*(a*x^2 + b*x^3)^(3/2)) +
(22*c)/(3*a^2*x^3*Sqrt[a*x^2 + b*x^3]) + (6*d)/(a^2*x^2*Sqrt[a*x^2 + b*x^3
]) - (33*c*Sqrt[a*x^2 + b*x^3])/(4*a^3*x^5) + (77*b*c*Sqrt[a*x^2 + b*x^3])
/(8*a^4*x^4) - (7*d*Sqrt[a*x^2 + b*x^3])/(a^3*x^4) - (385*b^2*c*Sqrt[a*x^2
 + b*x^3])/(32*a^5*x^3) + (35*b*d*Sqrt[a*x^2 + b*x^3])/(4*a^4*x^3) + (1155
*b^3*c*Sqrt[a*x^2 + b*x^3])/(64*a^6*x^2) - (105*b^2*d*Sqrt[a*x^2 + b*x^3])
/(8*a^5*x^2) - (1155*b^4*c*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(64*a
^(13/2)) + (105*b^3*d*ArcTanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/(8*a^(11/2
))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2450
```
Int[(Pq_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[Expan
dIntegrand[Pq*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, j, n, p}, x] && (Po
lyQ[Pq, x] || PolyQ[Pq, x^n]) &&  !IntegerQ[p] && NeQ[n, j]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.10

method	result
pseudoelliptic	$-\frac{4\left(\frac{(3dx+c)b}{2}+ad\right)}{3(bx+a)^{\frac{3}{2}}b^{2}}$
risch	$-\frac{(bx+a)\left(984a\,b^{2}d\,x^{3}-1545b^{3}c\,x^{3}-272a^{2}bd\,x^{2}+518a\,b^{2}c\,x^{2}+64a^{3}dx-184a^{2}bcx+48c\,a^{3}\right)}{192a^{6}x^{3}\sqrt{x^{2}(bx+a)}}-\frac{b^{3}\left(-\frac{2(840ad-1155bc)\,\mathrm{arcta}}{\sqrt{a}}\right.}{}$
default	$\frac{x(bx+a)\left(2520(bx+a)^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a\,b^{3}d\,x^{4}-3465(bx+a)^{\frac{3}{2}}\,\mathrm{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)b^{4}c\,x^{4}-64a^{\frac{11}{2}}dx+144a^{\frac{9}{2}}bd\,x^{2}-504a\right.}{192(b\,x^{3}+}$

input
```
int((d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3/(b*x+a)^(3/2)*(1/2*(3*d*x+c)*b+a*d)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 531, normalized size of antiderivative = 2.14

$$\int \frac{c+dx}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \left[-\frac{315\left((11\,b^6c-8\,ab^5d)x^7+2\,(11\,ab^5c-8\,a^2b^4d)x^6+(11\,a^2b^4c-8\,a^3b^3d)x^5\right)\sqrt{a}\,\mathrm{l}}{}\right.$$

input
```
integrate((d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/384*(315*((11*b^6*c - 8*a*b^5*d)*x^7 + 2*(11*a*b^5*c - 8*a^2*b^4*d)*x^
6 + (11*a^2*b^4*c - 8*a^3*b^3*d)*x^5)*sqrt(a)*log((b*x^2 + 2*a*x + 2*sqrt(
b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(48*a^6*c - 315*(11*a*b^5*c - 8*a^2*b^4*d
)*x^5 - 420*(11*a^2*b^4*c - 8*a^3*b^3*d)*x^4 - 63*(11*a^3*b^3*c - 8*a^4*b^
2*d)*x^3 + 18*(11*a^4*b^2*c - 8*a^5*b*d)*x^2 - 8*(11*a^5*b*c - 8*a^6*d)*x)
*sqrt(b*x^3 + a*x^2))/(a^7*b^2*x^7 + 2*a^8*b*x^6 + a^9*x^5), 1/192*(315*((
11*b^6*c - 8*a*b^5*d)*x^7 + 2*(11*a*b^5*c - 8*a^2*b^4*d)*x^6 + (11*a^2*b^4
*c - 8*a^3*b^3*d)*x^5)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2
 + a*x)) - (48*a^6*c - 315*(11*a*b^5*c - 8*a^2*b^4*d)*x^5 - 420*(11*a^2*b^
4*c - 8*a^3*b^3*d)*x^4 - 63*(11*a^3*b^3*c - 8*a^4*b^2*d)*x^3 + 18*(11*a^4*
b^2*c - 8*a^5*b*d)*x^2 - 8*(11*a^5*b*c - 8*a^6*d)*x)*sqrt(b*x^3 + a*x^2))/
(a^7*b^2*x^7 + 2*a^8*b*x^6 + a^9*x^5)]
```

Sympy [F]

$$\int \frac{c+dx}{(ax^2+bx^3)^{5/2}}\,dx=\int \frac{c+dx}{(x^2(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral((c + d*x)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{c+dx}{(ax^2+bx^3)^{5/2}}\,dx=\int \frac{dx+c}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.95

$$\int \frac{c+dx}{(ax^2+bx^3)^{5/2}}\,dx = \frac{105\,(11\,b^4c-8\,ab^3d)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{64\,\sqrt{-a}a^6\mathrm{sgn}\,(x)} + \frac{2\,(15\,(bx+a)b^4c+ab^4c-12\,(bx+a)ab^3d-a^2b^3d)}{3\,(bx+a)^{\frac{3}{2}}a^6\mathrm{sgn}\,(x)} + \frac{1545\,(bx+a)^{\frac{7}{2}}b^4c-5153\,(bx+a)^{\frac{5}{2}}ab^4c+5855\,(bx+a)^{\frac{3}{2}}a^2b^4c-2295\,\sqrt{bx+a}a^3b^4c-984\,(bx+a)^{\frac{7}{2}}ab^3d}{192\,a^6b^4x^4\mathrm{sgn}\,(x)}$$

input
```
integrate((d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
105/64*(11*b^4*c - 8*a*b^3*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*a^6
*sgn(x)) + 2/3*(15*(b*x + a)*b^4*c + a*b^4*c - 12*(b*x + a)*a*b^3*d - a^2*
b^3*d)/((b*x + a)^(3/2)*a^6*sgn(x)) + 1/192*(1545*(b*x + a)^(7/2)*b^4*c -
5153*(b*x + a)^(5/2)*a*b^4*c + 5855*(b*x + a)^(3/2)*a^2*b^4*c - 2295*sqrt(
b*x + a)*a^3*b^4*c - 984*(b*x + a)^(7/2)*a*b^3*d + 3224*(b*x + a)^(5/2)*a^
2*b^3*d - 3560*(b*x + a)^(3/2)*a^3*b^3*d + 1320*sqrt(b*x + a)*a^4*b^3*d)/(
a^6*b^4*x^4*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{c+d\,x}{{(b\,x^3+a\,x^2)}^{5/2}}\,dx$$

input
```
int((c + d*x)/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
int((c + d*x)/(a*x^2 + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.50

$$\int \frac{c+dx}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \frac{-2520\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\sqrt{a}\right)a^2b^3d\,x^4+3465\sqrt{a}\,\sqrt{bx+a}\log\left(\sqrt{bx+a}-\right.}{}$$

input
```
int((d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
( - 2520*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a**2*b**3*d*x*
*4 + 3465*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**4*c*x**4
 - 2520*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**4*d*x**5 +
 3465*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**5*c*x**5 + 252
0*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a**2*b**3*d*x**4 - 34
65*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**4*c*x**4 + 2520
*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**4*d*x**5 - 3465*s
qrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**5*c*x**5 - 96*a**6*c
- 128*a**6*d*x + 176*a**5*b*c*x + 288*a**5*b*d*x**2 - 396*a**4*b**2*c*x**2
 - 1008*a**4*b**2*d*x**3 + 1386*a**3*b**3*c*x**3 - 6720*a**3*b**3*d*x**4 +
 9240*a**2*b**4*c*x**4 - 5040*a**2*b**4*d*x**5 + 6930*a*b**5*c*x**5)/(384*
sqrt(a + b*x)*a**7*x**4*(a + b*x))
```

3.300 $\int \frac{c+dx}{x(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 285

$$\int \frac{c+dx}{x(ax^2+bx^3)^{5/2}}\,dx = -\frac{2b^4(bc-ad)x^3}{3a^6(ax^2+bx^3)^{3/2}} - \frac{2b^4(6bc-5ad)x}{a^7\sqrt{ax^2+bx^3}} - \frac{c\sqrt{ax^2+bx^3}}{5a^3x^6} + \frac{(29bc-10ad)\sqrt{ax^2+bx^3}}{40a^4x^5} - \frac{b(443bc-230ad)\sqrt{ax^2+bx^3}}{240a^5x^4} + \frac{b^2(827bc-518ad)\sqrt{ax^2+bx^3}}{192a^6x^3} - \frac{b^3(1467bc-1030ad)\sqrt{ax^2+bx^3}}{128a^7x^2} + \frac{231b^4(13bc-10ad)\operatorname{arctanh}\left(\frac{\sqrt{ax^2+bx^3}}{\sqrt{a}x}\right)}{128a^{15/2}}$$

output

```
-2/3*b^4*(-a*d+b*c)*x^3/a^6/(b*x^3+a*x^2)^(3/2)-2*b^4*(-5*a*d+6*b*c)*x/a^7
/(b*x^3+a*x^2)^(1/2)-1/5*c*(b*x^3+a*x^2)^(1/2)/a^3/x^6+1/40*(-10*a*d+29*b*
c)*(b*x^3+a*x^2)^(1/2)/a^4/x^5-1/240*b*(-230*a*d+443*b*c)*(b*x^3+a*x^2)^(1
/2)/a^5/x^4+1/192*b^2*(-518*a*d+827*b*c)*(b*x^3+a*x^2)^(1/2)/a^6/x^3-1/128
*b^3*(-1030*a*d+1467*b*c)*(b*x^3+a*x^2)^(1/2)/a^7/x^2+231/128*b^4*(-10*a*d
+13*b*c)*arctanh((b*x^3+a*x^2)^(1/2)/a^(1/2)/x)/a^(15/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.65

$$\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{5/2}}\,dx = \frac{\sqrt{a}(-45045b^6cx^6-96a^6(4c+5dx)+2310ab^5x^5(-26c+15dx)+198a^3b^3x^3(13c$$

input
```
Integrate[(c + d*x)/(x*(a*x^2 + b*x^3)^(5/2)),x]
```

output
```
(Sqrt[a]*(-45045*b^6*c*x^6 - 96*a^6*(4*c + 5*d*x) + 2310*a*b^5*x^5*(-26*c
+ 15*d*x) + 198*a^3*b^3*x^3*(13*c + 35*d*x) - 44*a^4*b^2*x^2*(26*c + 45*d*
x) + 16*a^5*b*x*(39*c + 55*d*x) + 231*a^2*b^4*x^4*(-39*c + 200*d*x)) + 346
5*b^4*(13*b*c - 10*a*d)*x^5*(a + b*x)^(3/2)*ArcTanh[Sqrt[a + b*x]/Sqrt[a]]
)/(1920*a^(15/2)*x^2*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.92, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {1944, 1912, 1929, 1931, 1931, 1931, 1931, 1914, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(13bc-10ad)\int \frac{1}{\left(bx^3+ax^2\right)^{5/2}}dx}{10a}-\frac{c}{5ax^2\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1912}$$

$$-\frac{(13bc-10ad)\left(\frac{11\int \frac{1}{x^2\left(bx^3+ax^2\right)^{3/2}}dx}{3a}+\frac{2}{3ax\left(ax^2+bx^3\right)^{3/2}}\right)}{10a}-\frac{c}{5ax^2\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1929}$$

$$-\frac{(13bc-10ad)\left(\frac{11\left(\frac{9\int \frac{1}{x^4\sqrt{bx^3+ax^2}}dx}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2}{3ax(ax^2+bx^3)^{3/2}}\right)}{10a}-\frac{c}{5ax^2\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{(13bc-10ad)\left(\frac{11\left(\frac{9\left(-\frac{7b\int \frac{1}{x^3\sqrt{bx^3+ax^2}}dx}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2}{3ax(ax^2+bx^3)^{3/2}}\right)}{10a}-\frac{c}{5ax^2\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{(13bc-10ad)\left(\frac{11\left(\frac{9\left(-\frac{7b\left(-\frac{5b\int \frac{1}{x^2\sqrt{bx^3+ax^2}}dx}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2}{3ax(ax^2+bx^3)^{3/2}}\right)}{10a}-\frac{c}{5ax^2\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1931}$$

$$-\frac{(13bc-10ad)\left(\frac{11\left(\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\int \frac{1}{x\sqrt{bx^3+ax^2}}dx}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2}{3ax(ax^2+bx^3)^{3/2}}\right)}{10a}$$

$$\frac{c}{5ax^2\left(ax^2+bx^3\right)^{3/2}}$$

$\downarrow$ 1931

$$(13bc-10ad)\left(\frac{11\left(\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(-\frac{b\int\frac{1}{\sqrt{bx^3+ax^2}}dx}{2a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}\right)}{3a}+\right.$$

↓ 1914

$$(13bc-10ad)\left(\frac{11\left(\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\int \frac{1}{1-\frac{ax^2}{bx^3+ax^2}}d\frac{x}{\sqrt{bx^3+ax^2}}}{a}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2\cdots}}}{3a}\right.$$

↓ 219

$$\left(\frac{11\left(\frac{9\left(-\frac{7b\left(-\frac{5b\left(-\frac{3b\left(\frac{b\,\mathrm{arctanh}\left(\frac{\sqrt{a}x}{\sqrt{ax^2+bx^3}}\right)}{a^{3/2}}-\frac{\sqrt{ax^2+bx^3}}{ax^2}\right)}{4a}-\frac{\sqrt{ax^2+bx^3}}{2ax^3}\right)}{6a}-\frac{\sqrt{ax^2+bx^3}}{3ax^4}\right)}{8a}-\frac{\sqrt{ax^2+bx^3}}{4ax^5}\right)}{a}+\frac{2}{ax^3\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2}{3ax(ax^2+}\right.$$

input
```
Int[(c + d*x)/(x*(a*x^2 + b*x^3)^(5/2)),x]
```

output
```
-1/5*c/(a*x^2*(a*x^2 + b*x^3)^(3/2)) - ((13*b*c - 10*a*d)*(2/(3*a*x*(a*x^2
 + b*x^3)^(3/2)) + (11*(2/(a*x^3*Sqrt[a*x^2 + b*x^3]) + (9*(-1/4*Sqrt[a*x^
2 + b*x^3]/(a*x^5) - (7*b*(-1/3*Sqrt[a*x^2 + b*x^3]/(a*x^4) - (5*b*(-1/2*S
qrt[a*x^2 + b*x^3]/(a*x^3) - (3*b*(-(Sqrt[a*x^2 + b*x^3]/(a*x^2)) + (b*Arc
Tanh[(Sqrt[a]*x)/Sqrt[a*x^2 + b*x^3]])/a^(3/2)))/(4*a)))/(6*a)))/(8*a)))/a
))/(3*a)))/(10*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1912
```
Int[((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> Simp[-(a*x^j +
 b*x^n)^(p + 1)/(a*(n - j)*(p + 1)*x^(j - 1)), x] + Simp[(n*p + n - j + 1)/
(a*(n - j)*(p + 1))   Int[(a*x^j + b*x^n)^(p + 1)/x^j, x], x] /; FreeQ[{a,
b}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && LtQ[p, -1]
```

rule 1914
```
Int[1/Sqrt[(a_.)*(x_)^2 + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp[2/(2 - n)
Subst[Int[1/(1 - a*x^2), x], x, x/Sqrt[a*x^2 + b*x^n]], x] /; FreeQ[{a, b,
n}, x] && NeQ[n, 2]
```

rule 1929
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, m}, x] &
&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p,
 -1]
```

rule 1931
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p}, x]
 &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[
m + j*p + 1, 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.23

method	result
pseudoelliptic	$-\frac{2\left(3bc\,\operatorname{arctanh}\left(\frac{\sqrt{bx+a}}{\sqrt{a}}\right)a^{2}(bx+a)^{\frac{3}{2}}+\left(-3b^{2}cx+a^{2}d-4abc\right)a^{\frac{5}{2}}\right)}{3(bx+a)^{\frac{3}{2}}\,b\,a^{\frac{9}{2}}}$
risch	$-\frac{(bx+a)\left(-15450x^{4}a\,b^{3}d+22005x^{4}b^{4}c+5180a^{2}b^{2}d\,x^{3}-8270a\,b^{3}c\,x^{3}-1840a^{3}bd\,x^{2}+3544a^{2}b^{2}c\,x^{2}+480a^{4}dx-1392a^{3}bcx\right.}{1920a^{7}x^{4}\sqrt{x^{2}(bx+a)}}$
default	$-\frac{(bx+a)\left(480a^{\frac{13}{2}}dx-880a^{\frac{11}{2}}bd\,x^{2}+1980a^{\frac{9}{2}}b^{2}d\,x^{3}-6930a^{\frac{7}{2}}b^{3}d\,x^{4}-46200a^{\frac{5}{2}}b^{4}d\,x^{5}-34650a^{\frac{3}{2}}b^{5}d\,x^{6}+34650\,\operatorname{arctanh}\left(\right.\right.}{}$

input
```
int((d*x+c)/x/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(b*x+a)^(3/2)*(3*b*c*arctanh((b*x+a)^(1/2)/a^(1/2))*a^2*(b*x+a)^(3/2)
+(-3*b^2*c*x+a^2*d-4*a*b*c)*a^(5/2))/b/a^(9/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 578, normalized size of antiderivative = 2.03

$$\int \frac{c+dx}{x\,(ax^2+bx^3)^{5/2}}\,dx = \left[-\frac{3465\,((13\,b^7c-10\,ab^6d)x^8+2\,(13\,ab^6c-10\,a^2b^5d)x^7+(13\,a^2b^5c-10\,a^3b^4d)x}{}\right.$$

$$-\frac{3465\,((13\,b^7c-10\,ab^6d)x^8+2\,(13\,ab^6c-10\,a^2b^5d)x^7+(13\,a^2b^5c-10\,a^3b^4d)x^6)\sqrt{-a}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-}}{bx^2+ax}\right.}{}$$

input
```
integrate((d*x+c)/x/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/3840*(3465*((13*b^7*c - 10*a*b^6*d)*x^8 + 2*(13*a*b^6*c - 10*a^2*b^5*d
)*x^7 + (13*a^2*b^5*c - 10*a^3*b^4*d)*x^6)*sqrt(a)*log((b*x^2 + 2*a*x - 2*
sqrt(b*x^3 + a*x^2)*sqrt(a))/x^2) + 2*(384*a^7*c + 3465*(13*a*b^6*c - 10*a
^2*b^5*d)*x^6 + 4620*(13*a^2*b^5*c - 10*a^3*b^4*d)*x^5 + 693*(13*a^3*b^4*c
 - 10*a^4*b^3*d)*x^4 - 198*(13*a^4*b^3*c - 10*a^5*b^2*d)*x^3 + 88*(13*a^5*
b^2*c - 10*a^6*b*d)*x^2 - 48*(13*a^6*b*c - 10*a^7*d)*x)*sqrt(b*x^3 + a*x^2
))/(a^8*b^2*x^8 + 2*a^9*b*x^7 + a^10*x^6), -1/1920*(3465*((13*b^7*c - 10*a
*b^6*d)*x^8 + 2*(13*a*b^6*c - 10*a^2*b^5*d)*x^7 + (13*a^2*b^5*c - 10*a^3*b
^4*d)*x^6)*sqrt(-a)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-a)/(b*x^2 + a*x)) + (
384*a^7*c + 3465*(13*a*b^6*c - 10*a^2*b^5*d)*x^6 + 4620*(13*a^2*b^5*c - 10
*a^3*b^4*d)*x^5 + 693*(13*a^3*b^4*c - 10*a^4*b^3*d)*x^4 - 198*(13*a^4*b^3*
c - 10*a^5*b^2*d)*x^3 + 88*(13*a^5*b^2*c - 10*a^6*b*d)*x^2 - 48*(13*a^6*b*
c - 10*a^7*d)*x)*sqrt(b*x^3 + a*x^2))/(a^8*b^2*x^8 + 2*a^9*b*x^7 + a^10*x^
6)]
```

Sympy [F]

$$\int \frac{c+dx}{x\,(ax^2+bx^3)^{5/2}}\,dx = \int \frac{c+dx}{x\,(x^2\,(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)/x/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral((c + d*x)/(x*(x**2*(a + b*x))**(5/2)), x)
```

Maxima [F]

$$\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{5/2}}\,dx=\int \frac{dx+c}{\left(bx^3+ax^2\right)^{\frac{5}{2}}x}\,dx$$

input
```
integrate((d*x+c)/x/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/((b*x^3 + a*x^2)^(5/2)*x), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.94

$$\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{5/2}}\,dx=-\frac{231\,(13\,b^5c-10\,ab^4d)\arctan\left(\frac{\sqrt{bx+a}}{\sqrt{-a}}\right)}{128\,\sqrt{-a}a^7\mathrm{sgn}\,(x)} - \frac{2\,(18\,(bx+a)b^5c+ab^5c-15\,(bx+a)ab^4d-a^2b^4d)}{3\,(bx+a)^{\frac{3}{2}}a^7\mathrm{sgn}\,(x)} - \frac{22005\,(bx+a)^{\frac{9}{2}}b^5c-96290\,(bx+a)^{\frac{7}{2}}ab^5c+160384\,(bx+a)^{\frac{5}{2}}a^2b^5c-121310\,(bx+a)^{\frac{3}{2}}a^3b^5c+35595\,\sqrt{}$$

input
```
integrate((d*x+c)/x/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
-231/128*(13*b^5*c - 10*a*b^4*d)*arctan(sqrt(b*x + a)/sqrt(-a))/(sqrt(-a)*
a^7*sgn(x)) - 2/3*(18*(b*x + a)*b^5*c + a*b^5*c - 15*(b*x + a)*a*b^4*d - a
^2*b^4*d)/((b*x + a)^(3/2)*a^7*sgn(x)) - 1/1920*(22005*(b*x + a)^(9/2)*b^5
*c - 96290*(b*x + a)^(7/2)*a*b^5*c + 160384*(b*x + a)^(5/2)*a^2*b^5*c - 12
1310*(b*x + a)^(3/2)*a^3*b^5*c + 35595*sqrt(b*x + a)*a^4*b^5*c - 15450*(b*
x + a)^(9/2)*a*b^4*d + 66980*(b*x + a)^(7/2)*a^2*b^4*d - 110080*(b*x + a)^
(5/2)*a^3*b^4*d + 81500*(b*x + a)^(3/2)*a^4*b^4*d - 22950*sqrt(b*x + a)*a^
5*b^4*d)/(a^7*b^5*x^5*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{5/2}}\,dx=\int \frac{c+d\,x}{x\,\left(b\,x^3+a\,x^2\right)^{5/2}}\,dx$$

input
```
int((c + d*x)/(x*(a*x^2 + b*x^3)^(5/2)),x)
```

output
```
int((c + d*x)/(x*(a*x^2 + b*x^3)^(5/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.39

$$\int \frac{c+dx}{x\left(ax^2+bx^3\right)^{5/2}}\,dx=\frac{34650\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}-\sqrt{a}\right)a^2b^4d\,x^5-45045\sqrt{a}\,\sqrt{bx+a}\,\log\left(\sqrt{bx+a}\right.}{}$$

input
```
int((d*x+c)/x/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(34650*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a**2*b**4*d*x**5
 - 45045*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**5*c*x**5
+ 34650*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*a*b**5*d*x**6 -
 45045*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) - sqrt(a))*b**6*c*x**6 - 34
650*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a**2*b**4*d*x**5 +
45045*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**5*c*x**5 - 3
4650*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*a*b**5*d*x**6 + 45
045*sqrt(a)*sqrt(a + b*x)*log(sqrt(a + b*x) + sqrt(a))*b**6*c*x**6 - 768*a
**7*c - 960*a**7*d*x + 1248*a**6*b*c*x + 1760*a**6*b*d*x**2 - 2288*a**5*b*
*2*c*x**2 - 3960*a**5*b**2*d*x**3 + 5148*a**4*b**3*c*x**3 + 13860*a**4*b**
3*d*x**4 - 18018*a**3*b**4*c*x**4 + 92400*a**3*b**4*d*x**5 - 120120*a**2*b
**5*c*x**5 + 69300*a**2*b**5*d*x**6 - 90090*a*b**6*c*x**6)/(3840*sqrt(a +
b*x)*a**8*x**5*(a + b*x))
```

3.301 $\int (ex)^{3/2}(c+dx)\sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 258

$$\begin{aligned}\int (ex)^{3/2}(c+dx)\sqrt{ax^2+bx^3}\,dx &= \frac{a^3(10bc-7ad)e^2\sqrt{ax^2+bx^3}}{128b^4\sqrt{ex}} \\ &- \frac{a^2(10bc-7ad)e\sqrt{ex}\sqrt{ax^2+bx^3}}{192b^3} + \frac{a(10bc-7ad)(ex)^{3/2}\sqrt{ax^2+bx^3}}{240b^2} \\ &+ \frac{(10bc-7ad)(ex)^{5/2}\sqrt{ax^2+bx^3}}{40be} + \frac{de\sqrt{ex}(ax^2+bx^3)^{3/2}}{5b} \\ &- \frac{a^4(10bc-7ad)e^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{128b^{9/2}}\end{aligned}$$

output
```
1/128*a^3*(-7*a*d+10*b*c)*e^2*(b*x^3+a*x^2)^(1/2)/b^4/(e*x)^(1/2)-1/192*a^
2*(-7*a*d+10*b*c)*e*(e*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/b^3+1/240*a*(-7*a*d+10
*b*c)*(e*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/b^2+1/40*(-7*a*d+10*b*c)*(e*x)^(5/2)
*(b*x^3+a*x^2)^(1/2)/b/e+1/5*d*e*(e*x)^(1/2)*(b*x^3+a*x^2)^(3/2)/b-1/128*a
^4*(-7*a*d+10*b*c)*e^(3/2)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^
2)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.81

$$\int (ex)^{3/2}(c+dx)\sqrt{ax^2+bx^3}\,dx = \frac{(ex)^{3/2}\sqrt{x^2(a+bx)}\Big(\sqrt{b}\sqrt{x}\sqrt{a+bx}(-105a^4d+16ab^3x^2(5c+3dx)+96b^4x^3(5c+4$$

input
```
Integrate[(e*x)^(3/2)*(c + d*x)*Sqrt[a*x^2 + b*x^3],x]
```

output
```
((e*x)^(3/2)*Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-105*a^4*
d + 16*a*b^3*x^2*(5*c + 3*d*x) + 96*b^4*x^3*(5*c + 4*d*x) + 10*a^3*b*(15*c
 + 7*d*x) - 4*a^2*b^2*x*(25*c + 14*d*x)) + 300*a^4*b*c*ArcTanh[(Sqrt[b]*Sq
rt[x])/(Sqrt[a] - Sqrt[a + b*x])] + 210*a^5*d*ArcTanh[(Sqrt[b]*Sqrt[x])/(-
Sqrt[a] + Sqrt[a + b*x])]))/(1920*b^(9/2)*x^(5/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.96, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1945, 1927, 1930, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ex)^{3/2}\sqrt{ax^2+bx^3}(c+dx)\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(10bc-7ad)\int (ex)^{3/2}\sqrt{bx^3+ax^2}dx}{10b}+\frac{de\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(10bc-7ad)\left(\frac{a\int \frac{(ex)^{7/2}}{\sqrt{bx^3+ax^2}}dx}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+bx^3}}{4e}\right)}{10b}+\frac{de\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{(10bc-7ad)\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\int\frac{(ex)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+bx^3}}{4e}\right)}{10b}+\frac{de\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{(10bc-7ad)\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\int\frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+bx^3}}{4e}\right)}{10b}+\frac{de\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{(10bc-7ad)\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int\frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+bx^3}}{4e}\right)}{10b}+\frac{de\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{1937}$$

$$\frac{(10bc-7ad)\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+bx^3}}{4e}\right)}{10b}+\frac{de\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{1935}$$

$$\frac{(10bc-7ad)\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2\cdots}}{4e}\cdots\right)}{10b}$$

$$\frac{de\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(10bc-7ad)\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+b}}{4e}\right.}{10b}$$

$$\frac{de\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5b}$$

input
```
Int[(e*x)^(3/2)*(c + d*x)*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(d*e*Sqrt[e*x]*(a*x^2 + b*x^3)^(3/2))/(5*b) + ((10*b*c - 7*a*d)*(((e*x)^(5
/2)*Sqrt[a*x^2 + b*x^3])/(4*e) + (a*((e^2*(e*x)^(3/2)*Sqrt[a*x^2 + b*x^3])
/(3*b) - (5*a*e*((e^2*Sqrt[e*x]*Sqrt[a*x^2 + b*x^3])/(2*b) - (3*a*e*((e^2*
Sqrt[a*x^2 + b*x^3])/(b*Sqrt[e*x]) - (a*e*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/
2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/(4*b)))/(6*b)))/(8*e^2)))/(1
0*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{\left(-384 d \,x^{4} b^{4}-48 a \,b^{3} d \,x^{3}-480 b^{4} c \,x^{3}+56 a^{2} b^{2} d \,x^{2}-80 a \,b^{3} c \,x^{2}-70 a^{3} b d x +100 a^{2} b^{2} c x +105 a^{4} d -150 a^{3} b c \right) e^{2} \sqrt{x^{2} \left(b x +a \right)}}{1920 b^{4} \sqrt{e x}}+\frac{a^{4} (7a$
default	$-\frac{\sqrt{b \,x^{3}+a \,x^{2}}\, \sqrt{e x}\, e \left(-768 b^{4} d \,x^{4} \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}-96 a \,b^{3} d \,x^{3} \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}-960 b^{4} c \,x^{3} \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}+112 a^{2} b^{2} d \,x^{2} \sqrt{e x (b x+}\right.}{}$

input
```
int((e*x)^(3/2)*(d*x+c)*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1920*(-384*b^4*d*x^4-48*a*b^3*d*x^3-480*b^4*c*x^3+56*a^2*b^2*d*x^2-80*a
*b^3*c*x^2-70*a^3*b*d*x+100*a^2*b^2*c*x+105*a^4*d-150*a^3*b*c)/b^4*e^2*(x^
2*(b*x+a))^(1/2)/(e*x)^(1/2)+1/256*a^4*(7*a*d-10*b*c)/b^4*ln((1/2*a*e+b*e*
x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)*e^2*(x^2*(b*x+a))^(1/2)/
x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 378, normalized size of antiderivative = 1.47

$$\int (ex)^{3/2}(c+dx)\sqrt{ax^2+bx^3}\,dx = \left[-\frac{15\,(10\,a^4bc-7\,a^5d)ex\sqrt{\frac{e}{b}}\log\left(\frac{2\,bex^2+aex+2\,\sqrt{bx^3+ax^2}\sqrt{ex}b\sqrt{\frac{e}{b}}}{x}\right)-2\,(384\,b^4dex^4+48}{}\right.$$

input
```
integrate((e*x)^(3/2)*(d*x+c)*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/3840*(15*(10*a^4*b*c - 7*a^5*d)*e*x*sqrt(e/b)*log((2*b*e*x^2 + a*e*x +
 2*sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(e/b))/x) - 2*(384*b^4*d*e*x^4 + 48
*(10*b^4*c + a*b^3*d)*e*x^3 + 8*(10*a*b^3*c - 7*a^2*b^2*d)*e*x^2 - 10*(10*
a^2*b^2*c - 7*a^3*b*d)*e*x + 15*(10*a^3*b*c - 7*a^4*d)*e)*sqrt(b*x^3 + a*x
^2)*sqrt(e*x))/(b^4*x), 1/1920*(15*(10*a^4*b*c - 7*a^5*d)*e*x*sqrt(-e/b)*a
rctan(sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(-e/b)/(b*e*x^2 + a*e*x)) + (384
*b^4*d*e*x^4 + 48*(10*b^4*c + a*b^3*d)*e*x^3 + 8*(10*a*b^3*c - 7*a^2*b^2*d
)*e*x^2 - 10*(10*a^2*b^2*c - 7*a^3*b*d)*e*x + 15*(10*a^3*b*c - 7*a^4*d)*e)
*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^4*x)]
```

Sympy [F]

$$\int (ex)^{3/2}(c+dx)\sqrt{ax^2+bx^3}\,dx = \int (ex)^{\frac{3}{2}}\sqrt{x^2(a+bx)}(c+dx)\,dx$$

input
```
integrate((e*x)**(3/2)*(d*x+c)*(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((e*x)**(3/2)*sqrt(x**2*(a + b*x))*(c + d*x), x)
```

Maxima [F]

$$\int (ex)^{3/2}(c+dx)\sqrt{ax^2+bx^3}\,dx = \int \sqrt{bx^3+ax^2}(dx+c)(ex)^{\frac{3}{2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x+c)*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)*(d*x + c)*(e*x)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.17

$$\int (ex)^{3/2}(c+dx)\sqrt{ax^2+bx^3}\,dx = \frac{1}{3840}\left(\frac{20\left(\frac{15\,a^4e^2\log\left(\left|-\sqrt{be}\sqrt{ex}+\sqrt{be^2x+ae^2}\right|\right)}{\sqrt{be}b^3}+\sqrt{be^2x+ae^2}\left(2\left(4\,ex\left(\frac{6\,x}{e^2}+\frac{a}{be^2}\right)-\frac{5\,a^2}{b^2e}\right)\right.\right.}{e^2}\right.$$

input
```
integrate((e*x)^(3/2)*(d*x+c)*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/3840*(20*(15*a^4*e^2*log(abs(-sqrt(b*e)*sqrt(e*x) + sqrt(b*e^2*x + a*e^2
)))/(sqrt(b*e)*b^3) + sqrt(b*e^2*x + a*e^2)*(2*(4*e*x*(6*x/e^2 + a/(b*e^2)
) - 5*a^2/(b^2*e))*e*x + 15*a^3/b^3)*sqrt(e*x))*c*abs(e)*sgn(x)/e^2 - 2*(1
05*a^5*e^6*log(abs(-sqrt(b*e)*sqrt(e*x) + sqrt(b*e^2*x + a*e^2)))/(sqrt(b*
e)*b^4) + (105*a^4*e^4/b^4 - 2*(35*a^3*e^3/b^3 + 4*(6*(8*e*x + a*e/b)*e*x
- 7*a^2*e^2/b^2)*e*x)*e*x)*sqrt(b*e^2*x + a*e^2)*sqrt(e*x))*d*abs(e)*sgn(x
)/e^6 - 15*(10*a^4*b*c*abs(e)*log(e^2*abs(a)) - 7*a^5*d*abs(e)*log(e^2*abs
(a)))*sgn(x)/(sqrt(b*e)*b^4))*e
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{3/2}(c+dx)\sqrt{ax^2+bx^3}\,dx = \int (e\,x)^{3/2}\,\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)\,dx$$

input
```
int((e*x)^(3/2)*(a*x^2 + b*x^3)^(1/2)*(c + d*x),x)
```

output
```
int((e*x)^(3/2)*(a*x^2 + b*x^3)^(1/2)*(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.85

$$\int (ex)^{3/2}(c+dx)\sqrt{ax^2+bx^3}\,dx = \frac{\sqrt{e}\,e\Big(-105\sqrt{x}\,\sqrt{bx+a}\,a^4bd+150\sqrt{x}\,\sqrt{bx+a}\,a^3b^2c+70\sqrt{x}\,\sqrt{bx+a}\,a^3b^2dx-100$$

input
```
int((e*x)^(3/2)*(d*x+c)*(b*x^3+a*x^2)^(1/2),x)
```

output
```
(sqrt(e)*e*( - 105*sqrt(x)*sqrt(a + b*x)*a**4*b*d + 150*sqrt(x)*sqrt(a + b
*x)*a**3*b**2*c + 70*sqrt(x)*sqrt(a + b*x)*a**3*b**2*d*x - 100*sqrt(x)*sqr
t(a + b*x)*a**2*b**3*c*x - 56*sqrt(x)*sqrt(a + b*x)*a**2*b**3*d*x**2 + 80*
sqrt(x)*sqrt(a + b*x)*a*b**4*c*x**2 + 48*sqrt(x)*sqrt(a + b*x)*a*b**4*d*x*
*3 + 480*sqrt(x)*sqrt(a + b*x)*b**5*c*x**3 + 384*sqrt(x)*sqrt(a + b*x)*b**
5*d*x**4 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5
*d - 150*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*b*c))
/(1920*b**5)
```

3.302 $\int \sqrt{ex}(c+dx)\sqrt{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 214

$$\begin{aligned}\int \sqrt{ex}(c+dx)\sqrt{ax^2+bx^3}\,dx = &-\frac{a^2(8bc-5ad)e\sqrt{ax^2+bx^3}}{64b^3\sqrt{ex}}\\ &+\frac{a(8bc-5ad)\sqrt{ex}\sqrt{ax^2+bx^3}}{96b^2}\\ &+\frac{(8bc-5ad)(ex)^{3/2}\sqrt{ax^2+bx^3}}{24be}+\frac{de(ax^2+bx^3)^{3/2}}{4b\sqrt{ex}}\\ &+\frac{a^3(8bc-5ad)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{64b^{7/2}}\end{aligned}$$

output
```
-1/64*a^2*(-5*a*d+8*b*c)*e*(b*x^3+a*x^2)^(1/2)/b^3/(e*x)^(1/2)+1/96*a*(-5*
a*d+8*b*c)*(e*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/b^2+1/24*(-5*a*d+8*b*c)*(e*x)^(
3/2)*(b*x^3+a*x^2)^(1/2)/b/e+1/4*d*e*(b*x^3+a*x^2)^(3/2)/b/(e*x)^(1/2)+1/6
4*a^3*(-5*a*d+8*b*c)*e^(1/2)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*
x^2)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.71

$$\int \sqrt{ex}(c+dx)\sqrt{ax^2+bx^3}\,dx$$
$$= \frac{\sqrt{ex}\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{x}(15a^3d+8ab^2x(2c+dx)+16b^3x^2(4c+3dx)-2a^2b(12c+5dx))+\frac{6a^3(-8bc+5a}{}\right.}{192b^{7/2}x^{3/2}}$$

input
```
Integrate[Sqrt[e*x]*(c + d*x)*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(Sqrt[e*x]*Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*(15*a^3*d + 8*a*b^2*x*(2*c
 + d*x) + 16*b^3*x^2*(4*c + 3*d*x) - 2*a^2*b*(12*c + 5*d*x)) + (6*a^3*(-8*
b*c + 5*a*d)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])])/Sqrt[a
+ b*x]))/(192*b^(7/2)*x^(3/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1945, 1927, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\sqrt{ax^2+bx^3}(c+dx)\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(8bc-5ad)\int \sqrt{ex}\sqrt{bx^3+ax^2}dx}{8b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{4b\sqrt{ex}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(8bc-5ad)\left(\frac{a\int \frac{(ex)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{4b\sqrt{ex}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{(8bc-5ad)\left(\frac{a\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\int \frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{4b\sqrt{ex}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{(8bc-5ad)\left(\frac{a\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{4b\sqrt{ex}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{(8bc-5ad)\left(\frac{a\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{4b\sqrt{ex}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{(8bc-5ad)\left(\frac{a\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{4b\sqrt{ex}}$$

$$\Big\downarrow \ 219$$

$$\frac{(8bc-5ad)\left(\frac{a\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{4b\sqrt{ex}}$$

input
```
Int[Sqrt[e*x]*(c + d*x)*Sqrt[a*x^2 + b*x^3],x]
```

output
```
(d*e*(a*x^2 + b*x^3)^(3/2))/(4*b*Sqrt[e*x]) + ((8*b*c - 5*a*d)*(((e*x)^(3/
2)*Sqrt[a*x^2 + b*x^3])/(3*e) + (a*((e^2*Sqrt[e*x]*Sqrt[a*x^2 + b*x^3])/(2
*b) - (3*a*e*((e^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[e*x]) - (a*e*Sqrt[e*x]*Arc
Tanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/(4*b)))/(
6*e^2)))/(8*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 171, normalized size of antiderivative = 0.80

method	result
risch	$\frac{(48b^3 d x^3+8a b^2 d x^2+64b^3 c x^2-10a^2 bdx+16a b^2 cx+15a^3 d-24c a^2 b) e\sqrt{x^2(bx+a)}}{192b^3\sqrt{ex}}-\frac{a^3(5ad-8bc)\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^2+aex}\right)e}{128b^3\sqrt{be}\,x(bx+a)\sqrt{}}$
default	$\frac{\sqrt{b x^3+a x^2}\,\sqrt{ex}\left(96b^3 d x^3\sqrt{ex(bx+a)}\,\sqrt{be}+16a b^2 d x^2\sqrt{ex(bx+a)}\,\sqrt{be}+128b^3 c x^2\sqrt{ex(bx+a)}\,\sqrt{be}-15\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}}{2\sqrt{be}}\right.\right.}{}$

input
```
int((e*x)^(1/2)*(d*x+c)*(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/192*(48*b^3*d*x^3+8*a*b^2*d*x^2+64*b^3*c*x^2-10*a^2*b*d*x+16*a*b^2*c*x+1
5*a^3*d-24*a^2*b*c)/b^3*e*(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)-1/128*a^3*(5*a*d
-8*b*c)/b^3*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1
/2)*e*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.46

$$\int \sqrt{ex}(c+dx)\sqrt{ax^2+bx^3}\,dx$$

$$= \left[-\frac{3\,(8\,a^3bc-5\,a^4d)x\sqrt{\frac{e}{b}}\log\left(\frac{2\,bex^2+aex-2\,\sqrt{bx^3+ax^2}\sqrt{ex}b\sqrt{\frac{e}{b}}}{x}\right)-2\,(48\,b^3dx^3-24\,a^2bc+15\,a^3d+8\,(8\,b^3c+}{384\,b^3x} \right.$$

$$-\frac{3\,(8\,a^3bc-5\,a^4d)x\sqrt{-\frac{e}{b}}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{ex}b\sqrt{-\frac{e}{b}}}{bex^2+aex}\right)-(48\,b^3dx^3-24\,a^2bc+15\,a^3d+8\,(8\,b^3c+ab^2d)}{192\,b^3x}$$

input
```
integrate((e*x)^(1/2)*(d*x+c)*(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(3*(8*a^3*b*c - 5*a^4*d)*x*sqrt(e/b)*log((2*b*e*x^2 + a*e*x - 2*sq
rt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(e/b))/x) - 2*(48*b^3*d*x^3 - 24*a^2*b*c
 + 15*a^3*d + 8*(8*b^3*c + a*b^2*d)*x^2 + 2*(8*a*b^2*c - 5*a^2*b*d)*x)*sqr
t(b*x^3 + a*x^2)*sqrt(e*x))/(b^3*x), -1/192*(3*(8*a^3*b*c - 5*a^4*d)*x*sqr
t(-e/b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(-e/b)/(b*e*x^2 + a*e*x
)) - (48*b^3*d*x^3 - 24*a^2*b*c + 15*a^3*d + 8*(8*b^3*c + a*b^2*d)*x^2 + 2
*(8*a*b^2*c - 5*a^2*b*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^3*x)]
```

Sympy [F]

$$\int \sqrt{ex}(c+dx)\sqrt{ax^2+bx^3}\,dx = \int \sqrt{ex}\sqrt{x^2\,(a+bx)}(c+dx)\,dx$$

input
```
integrate((e*x)**(1/2)*(d*x+c)*(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(sqrt(e*x)*sqrt(x**2*(a + b*x))*(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{ex}(c+dx)\sqrt{ax^2+bx^3}\,dx = \int \sqrt{bx^3+ax^2}(dx+c)\sqrt{ex}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x+c)*(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)*(d*x + c)*sqrt(e*x), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.26

$$\int \sqrt{ex}(c+dx)\sqrt{ax^2+bx^3}\,dx = \frac{\left(\frac{15\,a^4e^2\log\left(\left|-\sqrt{be}\sqrt{ex}+\sqrt{be^2x+ae^2}\right|\right)}{\sqrt{be}b^3}+\sqrt{be^2x+ae^2}\left(2\left(4\,ex\left(\frac{6\,x}{e^2}+\frac{a}{be^2}\right)-\frac{5\,a^2}{b^2e}\right)ex+\frac{15\,a^3}{b^3}\right)\sqrt{ex}\right)d|e|\mathrm{sgn}(x)}{192\,e^2} - \frac{\left(\frac{3\,a^3e^4\log\left(\left|-\sqrt{be}\sqrt{ex}+\sqrt{be^2x+ae^2}\right|\right)}{\sqrt{be}b^2}-\sqrt{be^2x+ae^2}\left(2\left(4\,ex+\frac{ae}{b}\right)ex-\frac{3\,a^2e^2}{b^2}\right)\sqrt{ex}\right)c|e|\mathrm{sgn}(x)}{24\,e^4} + \frac{\left(8\,a^3bc|e|\log\left(e^2|a|\right)-5\,a^4d|e|\log\left(e^2|a|\right)\right)\mathrm{sgn}(x)}{128\,\sqrt{be}b^3}$$

input
```
integrate((e*x)^(1/2)*(d*x+c)*(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/192*(15*a^4*e^2*log(abs(-sqrt(b*e)*sqrt(e*x) + sqrt(b*e^2*x + a*e^2)))/(
sqrt(b*e)*b^3) + sqrt(b*e^2*x + a*e^2)*(2*(4*e*x*(6*x/e^2 + a/(b*e^2)) - 5
*a^2/(b^2*e))*e*x + 15*a^3/b^3)*sqrt(e*x))*d*abs(e)*sgn(x)/e^2 - 1/24*(3*a
^3*e^4*log(abs(-sqrt(b*e)*sqrt(e*x) + sqrt(b*e^2*x + a*e^2)))/(sqrt(b*e)*b
^2) - sqrt(b*e^2*x + a*e^2)*(2*(4*e*x + a*e/b)*e*x - 3*a^2*e^2/b^2)*sqrt(e
*x))*c*abs(e)*sgn(x)/e^4 + 1/128*(8*a^3*b*c*abs(e)*log(e^2*abs(a)) - 5*a^4
*d*abs(e)*log(e^2*abs(a)))*sgn(x)/(sqrt(b*e)*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}(c+dx)\sqrt{ax^2+bx^3}\,dx=\int \sqrt{e\,x}\,\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)\,dx$$

input
```
int((e*x)^(1/2)*(a*x^2 + b*x^3)^(1/2)*(c + d*x),x)
```

output
```
int((e*x)^(1/2)*(a*x^2 + b*x^3)^(1/2)*(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 179, normalized size of antiderivative = 0.84

$$\int \sqrt{ex}(c+dx)\sqrt{ax^2+bx^3}\,dx$$
$$=\frac{\sqrt{e}\left(15\sqrt{x}\,\sqrt{bx+a}\,a^3bd-24\sqrt{x}\,\sqrt{bx+a}\,a^2b^2c-10\sqrt{x}\,\sqrt{bx+a}\,a^2b^2dx+16\sqrt{x}\,\sqrt{bx+a}\,a\,b^3cx+8\sqrt{x}\right.}{}$$

input
```
int((e*x)^(1/2)*(d*x+c)*(b*x^3+a*x^2)^(1/2),x)
```

output
```
(sqrt(e)*(15*sqrt(x)*sqrt(a + b*x)*a**3*b*d - 24*sqrt(x)*sqrt(a + b*x)*a**
2*b**2*c - 10*sqrt(x)*sqrt(a + b*x)*a**2*b**2*d*x + 16*sqrt(x)*sqrt(a + b*
x)*a*b**3*c*x + 8*sqrt(x)*sqrt(a + b*x)*a*b**3*d*x**2 + 64*sqrt(x)*sqrt(a
+ b*x)*b**4*c*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*d*x**3 - 15*sqrt(b)*log
((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*d + 24*sqrt(b)*log((sqrt(
a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*b*c))/(192*b**4)
```

3.303 $\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{\sqrt{ex}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 172

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{\sqrt{ex}}\,dx = \frac{a(2bc-ad)\sqrt{ax^2+bx^3}}{8b^2\sqrt{ex}} + \frac{(2bc-ad)\sqrt{ex}\sqrt{ax^2+bx^3}}{4be} + \frac{de(ax^2+bx^3)^{3/2}}{3b(ex)^{3/2}} - \frac{a^2(2bc-ad)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{8b^{5/2}\sqrt{e}}$$

output

```
1/8*a*(-a*d+2*b*c)*(b*x^3+a*x^2)^(1/2)/b^2/(e*x)^(1/2)+1/4*(-a*d+2*b*c)*(e
*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/b/e+1/3*d*e*(b*x^3+a*x^2)^(3/2)/b/(e*x)^(3/2
)-1/8*a^2*(-a*d+2*b*c)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(
1/2))/b^(5/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{\sqrt{ex}}\,dx = \frac{\sqrt{x^2(a+bx)}\left(6abc\sqrt{x}-3a^2d\sqrt{x}+12b^2cx^{3/2}+2abdx^{3/2}+8b^2dx^{5/2}\right)}{24b^2\sqrt{x}\sqrt{ex}} + \frac{a^2(-2bc+ad)\sqrt{x^2(a+bx)}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{4b^{5/2}\sqrt{x}\sqrt{ex}\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x^2 + b*x^3])/Sqrt[e*x],x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(6*a*b*c*Sqrt[x] - 3*a^2*d*Sqrt[x] + 12*b^2*c*x^(3/2)
 + 2*a*b*d*x^(3/2) + 8*b^2*d*x^(5/2)))/(24*b^2*Sqrt[x]*Sqrt[e*x]) + (a^2*(
-2*b*c + a*d)*Sqrt[x^2*(a + b*x)]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sq
rt[a + b*x])])/(4*b^(5/2)*Sqrt[x]*Sqrt[e*x]*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.95, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1945, 1927, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}(c+dx)}{\sqrt{ex}}\,dx$$

$$\Big\downarrow \text{ 1945}$$

$$\frac{(2bc-ad)\int \frac{\sqrt{bx^3+ax^2}}{\sqrt{ex}}dx}{2b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{3b(ex)^{3/2}}$$

$$\Big\downarrow \text{ 1927}$$

$$\frac{(2bc-ad)\left(\frac{a\int \frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4e^2}+\frac{\sqrt{ex}\sqrt{ax^2+bx^3}}{2e}\right)}{2b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{3b(ex)^{3/2}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{(2bc-ad)\left(\frac{a\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4e^2}+\frac{\sqrt{ex}\sqrt{ax^2+bx^3}}{2e}\right)}{2b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{3b(ex)^{3/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{(2bc-ad)\left(\frac{a\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4e^2}+\frac{\sqrt{ex}\sqrt{ax^2+bx^3}}{2e}\right)}{2b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{3b(ex)^{3/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{(2bc-ad)\left(\frac{a\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4e^2}+\frac{\sqrt{ex}\sqrt{ax^2+bx^3}}{2e}\right)}{2b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{3b(ex)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(2bc-ad)\left(\frac{a\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4e^2}+\frac{\sqrt{ex}\sqrt{ax^2+bx^3}}{2e}\right)}{2b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{3b(ex)^{3/2}}$$

input
```
Int[((c + d*x)*Sqrt[a*x^2 + b*x^3])/Sqrt[e*x],x]
```

output
```
(d*e*(a*x^2 + b*x^3)^(3/2))/(3*b*(e*x)^(3/2)) + ((2*b*c - a*d)*((Sqrt[e*x]
*Sqrt[a*x^2 + b*x^3])/(2*e) + (a*((e^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[e*x])
- (a*e*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*
Sqrt[x])))/(4*e^2)))/(2*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

method	result
risch	$-\frac{\left(-8b^2d\,x^2-2abdx-12b^2cx+3a^2d-6abc\right)\sqrt{x^2(bx+a)}}{24b^2\sqrt{ex}}+\frac{a^2(ad-2bc)\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^2+aex}\right)\sqrt{x^2(bx+a)}\,\sqrt{ex(bx+a)}}{16b^2\sqrt{be}\,x(bx+a)\sqrt{ex}}$
default	$-\frac{\sqrt{b\,x^3+a\,x^2}\left(-16b^2d\,x^2\sqrt{be}\,\sqrt{ex(bx+a)}-3\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)a^3de+6\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)a^2bce-4\sqrt{ex(}\right.}{48\sqrt{ex}\,\sqrt{ex(bx+a)}\,b^2\sqrt{be}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-8*b^2*d*x^2-2*a*b*d*x-12*b^2*c*x+3*a^2*d-6*a*b*c)/b^2*(x^2*(b*x+a)
)^(1/2)/(e*x)^(1/2)+1/16*a^2*(a*d-2*b*c)/b^2*ln((1/2*a*e+b*e*x)/(b*e)^(1/2
)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(e*x*(b
*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.57

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{\sqrt{ex}}\,dx$$

$$=\left[-\frac{3\left(2\,a^2bc-a^3d\right)\sqrt{be}x\log\left(\frac{2\,bex^2+aex+2\,\sqrt{bx^3+ax^2}\sqrt{be}\sqrt{ex}}{x}\right)-2\left(8\,b^3dx^2+6\,ab^2c-3\,a^2bd+2\left(6\,b^3c+ab^2c\right.\right.}{48\,b^3ex}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(2*a^2*b*c - a^3*d)*sqrt(b*e)*x*log((2*b*e*x^2 + a*e*x + 2*sqrt(
b*x^3 + a*x^2)*sqrt(b*e)*sqrt(e*x))/x) - 2*(8*b^3*d*x^2 + 6*a*b^2*c - 3*a^
2*b*d + 2*(6*b^3*c + a*b^2*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^3*e*x),
 1/24*(3*(2*a^2*b*c - a^3*d)*sqrt(-b*e)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(
-b*e)*sqrt(e*x)/(b*e*x^2 + a*e*x)) + (8*b^3*d*x^2 + 6*a*b^2*c - 3*a^2*b*d
+ 2*(6*b^3*c + a*b^2*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^3*e*x)]
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{\sqrt{ex}}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}(c+dx)}{\sqrt{ex}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(1/2)/(e*x)**(1/2),x)
```

output
```
Integral(sqrt(x**2*(a + b*x))*(c + d*x)/sqrt(e*x), x)
```

Maxima [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{\sqrt{ex}}\,dx = \int \frac{\sqrt{bx^3+ax^2}(dx+c)}{\sqrt{ex}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)*(d*x + c)/sqrt(e*x), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{\sqrt{ex}}\,dx$$
$$= \frac{\left(\sqrt{(bx+a)be-abe}\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)d\operatorname{sgn}(x)}{b^3e}+\frac{6\,b^7ce^2\operatorname{sgn}(x)-7\,ab^6de^2\operatorname{sgn}(x)}{b^9e^3}\right)-\frac{3\,(2\,ab^7ce^2\operatorname{sgn}(x)-a^2b^6de}{b^9e^3}\right.\right.}{24\,|b|}$$
$$-\frac{\left(2\,a^2bc\log\left(\sqrt{be}\sqrt{a}\right)-a^3d\log\left(\sqrt{be}\sqrt{a}\right)\right)\operatorname{sgn}(x)}{8\,\sqrt{be}b|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
1/24*(sqrt((b*x + a)*b*e - a*b*e)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)*
d*sgn(x)/(b^3*e) + (6*b^7*c*e^2*sgn(x) - 7*a*b^6*d*e^2*sgn(x))/(b^9*e^3))
- 3*(2*a*b^7*c*e^2*sgn(x) - a^2*b^6*d*e^2*sgn(x))/(b^9*e^3)) + 3*(2*a^2*b*
c*sgn(x) - a^3*d*sgn(x))*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt((b*x + a)
*b*e - a*b*e)))/(sqrt(b*e)*b^2))*b/abs(b) - 1/8*(2*a^2*b*c*log(sqrt(b*e)*s
qrt(a)) - a^3*d*log(sqrt(b*e)*sqrt(a)))*sgn(x)/(sqrt(b*e)*b*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{\sqrt{ex}}\,dx = \int \frac{\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)}{\sqrt{e\,x}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/(e*x)^(1/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/(e*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{\sqrt{ex}}\,dx$$
$$=\frac{\sqrt{e}\left(-3\sqrt{x}\,\sqrt{bx+a}\,a^2bd+6\sqrt{x}\,\sqrt{bx+a}\,a\,b^2c+2\sqrt{x}\,\sqrt{bx+a}\,a\,b^2dx+12\sqrt{x}\,\sqrt{bx+a}\,b^3cx+8\sqrt{x}\,\sqrt{bx}\right.}{24b^3e}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(1/2),x)
```

output
```
(sqrt(e)*( - 3*sqrt(x)*sqrt(a + b*x)*a**2*b*d + 6*sqrt(x)*sqrt(a + b*x)*a*
b**2*c + 2*sqrt(x)*sqrt(a + b*x)*a*b**2*d*x + 12*sqrt(x)*sqrt(a + b*x)*b**
3*c*x + 8*sqrt(x)*sqrt(a + b*x)*b**3*d*x**2 + 3*sqrt(b)*log((sqrt(a + b*x)
 + sqrt(x)*sqrt(b))/sqrt(a))*a**3*d - 6*sqrt(b)*log((sqrt(a + b*x) + sqrt(
x)*sqrt(b))/sqrt(a))*a**2*b*c))/(24*b**3*e)
```

3.304 $\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 131

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{3/2}}\,dx = \frac{(4bc-ad)\sqrt{ax^2+bx^3}}{4be\sqrt{ex}} + \frac{de(ax^2+bx^3)^{3/2}}{2b(ex)^{5/2}} + \frac{a(4bc-ad)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{4b^{3/2}e^{3/2}}$$

output
```
1/4*(-a*d+4*b*c)*(b*x^3+a*x^2)^(1/2)/b/e/(e*x)^(1/2)+1/2*d*e*(b*x^3+a*x^2)
^(3/2)/b/(e*x)^(5/2)+1/4*a*(-a*d+4*b*c)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2
)/(b*x^3+a*x^2)^(1/2))/b^(3/2)/e^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{3/2}}\,dx = \frac{\sqrt{x}\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(4bc+ad+2bdx)+a(-4bc+ad)\log\left(-\sqrt{b}\right.\right.}{4b^{3/2}(ex)^{3/2}\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x^2 + b*x^3])/(e*x)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(4*b*c + a*d +
 2*b*d*x) + a*(-4*b*c + a*d)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(4*
b^(3/2)*(e*x)^(3/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1945, 1927, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}(c+dx)}{(ex)^{3/2}}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(4bc-ad)\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{3/2}}dx}{4b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{2b(ex)^{5/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(4bc-ad)\left(\frac{a\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2e^2}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{2b(ex)^{5/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{(4bc-ad)\left(\frac{a\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{2b(ex)^{5/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{(4bc-ad)\left(\frac{a\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{2b(ex)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(4bc-ad)\left(\frac{a\sqrt{ex}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4b}+\frac{de\left(ax^2+bx^3\right)^{3/2}}{2b(ex)^{5/2}}$$

input
```
Int[((c + d*x)*Sqrt[a*x^2 + b*x^3])/(e*x)^(3/2),x]
```

output
```
(d*e*(a*x^2 + b*x^3)^(3/2))/(2*b*(e*x)^(5/2)) + ((4*b*c - a*d)*(Sqrt[a*x^2
 + b*x^3]/(e*Sqrt[e*x]) + (a*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^
2 + b*x^3]])/(Sqrt[b]*e^2*Sqrt[x])))/(4*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.97

method	result
risch	$\frac{(2bdx+ad+4bc)\sqrt{x^2(bx+a)}}{4be\sqrt{ex}}-\frac{a(ad-4bc)\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^2+aex}\right)\sqrt{x^2(bx+a)}\,\sqrt{ex(bx+a)}}{8b\sqrt{be}\,ex(bx+a)\sqrt{ex}}$
default	$\frac{\sqrt{b\,x^3+a\,x^2}\left(4\sqrt{ex(bx+a)}\,\sqrt{be}\,bdx-\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)a^2de+4\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)abce+2\sqrt{ex(bx+a)}\,\sqrt{b\right.}{8e\sqrt{ex}\,\sqrt{ex(bx+a)}\,b\sqrt{be}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*b*d*x+a*d+4*b*c)/b/e*(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)-1/8*a*(a*d-4*b
*c)/b*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)/e*
(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.69

$$\int\frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{3/2}}\,dx=\left[-\frac{(4\,abc-a^2d)\sqrt{be}x\log\left(\frac{2\,bex^2+aex-2\,\sqrt{bx^3+ax^2}\sqrt{be}\sqrt{ex}}{x}\right)-2\,(2\,b^2dx+4\,b^2c+a}{8\,b^2e^2x}\right.$$

$$\left.-\frac{(4\,abc-a^2d)\sqrt{-be}x\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-be}\sqrt{ex}}{bex^2+aex}\right)-(2\,b^2dx+4\,b^2c+abd)\sqrt{bx^3+ax^2}\sqrt{ex}}{4\,b^2e^2x}\right]$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*((4*a*b*c - a^2*d)*sqrt(b*e)*x*log((2*b*e*x^2 + a*e*x - 2*sqrt(b*x^3
 + a*x^2)*sqrt(b*e)*sqrt(e*x))/x) - 2*(2*b^2*d*x + 4*b^2*c + a*b*d)*sqrt(b
*x^3 + a*x^2)*sqrt(e*x))/(b^2*e^2*x), -1/4*((4*a*b*c - a^2*d)*sqrt(-b*e)*x
*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b*e)*sqrt(e*x)/(b*e*x^2 + a*e*x)) - (2*b
^2*d*x + 4*b^2*c + a*b*d)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^2*e^2*x)]
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{3/2}}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}(c+dx)}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(1/2)/(e*x)**(3/2),x)
```

output
```
Integral(sqrt(x**2*(a + b*x))*(c + d*x)/(e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{3/2}}\,dx = \int \frac{\sqrt{bx^3+ax^2}(dx+c)}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)*(d*x + c)/(e*x)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{3/2}}\,dx = \frac{\frac{\left(\sqrt{(bx+a)be-abe}\sqrt{bx+a}\left(\frac{2\,(bx+a)d\mathrm{sgn}(x)}{b^2e}+\frac{4\,b^3ce\mathrm{sgn}(x)-ab^2de\mathrm{sgn}(x)}{b^4e^2}\right)-\frac{\left(4\,abc\mathrm{sgn}(x)-a^2d\mathrm{sgn}(x)\right)\log\left(\left|-\sqrt{be}\right.\right.}{\sqrt{be}b}}{|b|}}{4\,e}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
1/4*((sqrt((b*x + a)*b*e - a*b*e)*sqrt(b*x + a)*(2*(b*x + a)*d*sgn(x)/(b^2
*e) + (4*b^3*c*e*sgn(x) - a*b^2*d*e*sgn(x))/(b^4*e^2)) - (4*a*b*c*sgn(x) -
 a^2*d*sgn(x))*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt((b*x + a)*b*e - a*b
*e)))/(sqrt(b*e)*b))*b/abs(b) + (4*a*b*c*log(sqrt(b*e)*sqrt(a)) - a^2*d*lo
g(sqrt(b*e)*sqrt(a)))*sgn(x)/(sqrt(b*e)*abs(b)))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{3/2}}\,dx = \int \frac{\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)}{{(e\,x)}^{3/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/(e*x)^(3/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/(e*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{3/2}}\,dx = \frac{\sqrt{e}\left(\sqrt{x}\,\sqrt{bx+a}\,abd+4\sqrt{x}\,\sqrt{bx+a}\,b^2c+2\sqrt{x}\,\sqrt{bx+a}\,b^2dx-\sqrt{b}\log\left(\frac{\sqrt{bx-}}{}\right.\right.}{4b^2e^2}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(3/2),x)
```

output
```
(sqrt(e)*(sqrt(x)*sqrt(a + b*x)*a*b*d + 4*sqrt(x)*sqrt(a + b*x)*b**2*c + 2
*sqrt(x)*sqrt(a + b*x)*b**2*d*x - sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqr
t(b))/sqrt(a))*a**2*d + 4*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sq
rt(a))*a*b*c))/(4*b**2*e**2)
```

3.305 $\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 109

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{5/2}}\,dx = -\frac{2c\sqrt{ax^2+bx^3}}{e(ex)^{3/2}} + \frac{d\sqrt{ax^2+bx^3}}{e^2\sqrt{ex}} + \frac{(2bc+ad)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}e^{5/2}}$$

output
```
-2*c*(b*x^3+a*x^2)^(1/2)/e/(e*x)^(3/2)+d*(b*x^3+a*x^2)^(1/2)/e^2/(e*x)^(1/
2)+(a*d+2*b*c)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/b^
(1/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{5/2}}\,dx = \frac{x\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{a+bx}(-2c+dx)+2(2bc+ad)\sqrt{x}\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)\right)}{\sqrt{b}(ex)^{5/2}\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x^2 + b*x^3])/(e*x)^(5/2),x]
```

output
```
(x*Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[a + b*x]*(-2*c + d*x) + 2*(2*b*c + a*
d)*Sqrt[x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(Sqrt[b
]*(e*x)^(5/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1944, 1927, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}(c+dx)}{(ex)^{5/2}}dx$$

$$\Big\downarrow \text{ 1944}$$

$$\frac{(ad+2bc)\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{3/2}}dx}{ae}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{a(ex)^{7/2}}$$

$$\Big\downarrow \text{ 1927}$$

$$\frac{(ad+2bc)\left(\frac{a\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2e^2}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{ae}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{a(ex)^{7/2}}$$

$$\Big\downarrow \text{ 1937}$$

$$\frac{(ad+2bc)\left(\frac{a\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{ae}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{a(ex)^{7/2}}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{(ad+2bc)\left(\frac{a\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{ae}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{a(ex)^{7/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(ad+2bc)\left(\frac{a\sqrt{ex}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{ae}-\frac{2ce(ax^2+bx^3)^{3/2}}{a(ex)^{7/2}}$$

input
```
Int[((c + d*x)*Sqrt[a*x^2 + b*x^3])/(e*x)^(5/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(3/2))/(a*(e*x)^(7/2)) + ((2*b*c + a*d)*(Sqrt[a*x^
2 + b*x^3]/(e*Sqrt[e*x]) + (a*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x
^2 + b*x^3]])/(Sqrt[b]*e^2*Sqrt[x])))/(a*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{(-dx+2c)\sqrt{x^2(bx+a)}}{e^2x\sqrt{ex}}+\frac{\left(\frac{ad}{2}+bc\right)\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^2+aex}\right)\sqrt{x^2(bx+a)}\,\sqrt{ex(bx+a)}}{\sqrt{be}\,e^2x(bx+a)\sqrt{ex}}$
default	$-\frac{\sqrt{b\,x^3+a\,x^2}\left(-\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)adex-2\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)bcex-2dx\sqrt{ex(bx+a)}\,\sqrt{be}+4\sqrt{ex(bx+a)}\right.}{2x\,e^2\sqrt{ex}\,\sqrt{ex(bx+a)}\,\sqrt{be}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-(-d*x+2*c)/e^2*(x^2*(b*x+a))^(1/2)/x/(e*x)^(1/2)+(1/2*a*d+b*c)*ln((1/2*a*
e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)/e^2*(x^2*(b*x+a))^
(1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.83

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{5/2}}\,dx = \left[\frac{(2\,bc+ad)\sqrt{be}x^2\log\left(\frac{2\,bex^2+aex+2\,\sqrt{bx^3+ax^2}\sqrt{be}\sqrt{ex}}{x}\right)+2\,\sqrt{bx^3+ax^2}(bdx-2\,b\right.}{2\,be^3x^2}$$

$$-\frac{(2\,bc+ad)\sqrt{-be}x^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-be}\sqrt{ex}}{bex^2+aex}\right)-\sqrt{bx^3+ax^2}(bdx-2\,bc)\sqrt{ex}}{be^3x^2}\right]$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
[1/2*((2*b*c + a*d)*sqrt(b*e)*x^2*log((2*b*e*x^2 + a*e*x + 2*sqrt(b*x^3 +
a*x^2)*sqrt(b*e)*sqrt(e*x))/x) + 2*sqrt(b*x^3 + a*x^2)*(b*d*x - 2*b*c)*sqr
t(e*x))/(b*e^3*x^2), -((2*b*c + a*d)*sqrt(-b*e)*x^2*arctan(sqrt(b*x^3 + a*
x^2)*sqrt(-b*e)*sqrt(e*x)/(b*e*x^2 + a*e*x)) - sqrt(b*x^3 + a*x^2)*(b*d*x
- 2*b*c)*sqrt(e*x))/(b*e^3*x^2)]
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{5/2}}\,dx=\int \frac{\sqrt{x^2\,(a+bx)}(c+dx)}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(1/2)/(e*x)**(5/2),x)
```

output
```
Integral(sqrt(x**2*(a + b*x))*(c + d*x)/(e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{5/2}}\,dx=\int \frac{\sqrt{bx^3+ax^2}(dx+c)}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)*(d*x + c)/(e*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{5/2}}\,dx = \frac{\left(\frac{\sqrt{bx+a}\left(\frac{(bx+a)d\mathrm{sgn}(x)}{b}-\frac{2\,b^2c\mathrm{sgn}(x)+abd\mathrm{sgn}(x)}{b^2}\right)}{\sqrt{(bx+a)be-abe}}-\frac{(2\,bc\mathrm{sgn}(x)+ad\mathrm{sgn}(x))\log\left(\left|-\sqrt{be}\sqrt{bx+a}+\sqrt{(bx+\dots}\right.\right.}{\sqrt{be}b}\right.}{e^2|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
(sqrt(b*x + a)*((b*x + a)*d*sgn(x)/b - (2*b^2*c*sgn(x) + a*b*d*sgn(x))/b^2
)/sqrt((b*x + a)*b*e - a*b*e) - (2*b*c*sgn(x) + a*d*sgn(x))*log(abs(-sqrt(
b*e)*sqrt(b*x + a) + sqrt((b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*b))*b^2/(e^2
*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{5/2}}\,dx = \int \frac{\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)}{(e\,x)^{5/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/(e*x)^(5/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/(e*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{5/2}}\,dx = \frac{\sqrt{e}\left(-8\sqrt{x}\,\sqrt{bx+a}\,bc+4\sqrt{x}\,\sqrt{bx+a}\,bdx+4\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)adx+8\dots\right.}{4b\,e^3x}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(5/2),x)
```

output
```
(sqrt(e)*( - 8*sqrt(x)*sqrt(a + b*x)*b*c + 4*sqrt(x)*sqrt(a + b*x)*b*d*x +
 4*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*d*x + 8*sqrt(b
)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b*c*x - sqrt(b)*a*d*x - 8
*sqrt(b)*b*c*x))/(4*b*e**3*x)
```

3.306 $\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 107

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{7/2}}\,dx = -\frac{2d\sqrt{ax^2+bx^3}}{e^2(ex)^{3/2}} - \frac{2ce(ax^2+bx^3)^{3/2}}{3a(ex)^{9/2}} + \frac{2\sqrt{b}d\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{e^{7/2}}$$

output
```
-2*d*(b*x^3+a*x^2)^(1/2)/e^2/(e*x)^(3/2)-2/3*c*e*(b*x^3+a*x^2)^(3/2)/a/(e*
x)^(9/2)+2*b^(1/2)*d*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/
2))/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{7/2}}\,dx = -\frac{2x\sqrt{x^2(a+bx)}\left(\sqrt{a+bx}(bcx+a(c+3dx))+3a\sqrt{b}dx^{3/2}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{3a(ex)^{7/2}\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x^2 + b*x^3])/(e*x)^(7/2),x]
```

output
```
(-2*x*Sqrt[x^2*(a + b*x)]*(Sqrt[a + b*x]*(b*c*x + a*(c + 3*d*x)) + 3*a*Sqr
t[b]*d*x^(3/2)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(3*a*(e*x)^(7/2)*
Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1944, 1926, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}(c+dx)}{(ex)^{7/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{d\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{5/2}}dx}{e}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{3a(ex)^{9/2}}$$

$$\Big\downarrow \text{1926}$$

$$\frac{d\left(\frac{b\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{e^3}-\frac{2\sqrt{ax^2+bx^3}}{e(ex)^{3/2}}\right)}{e}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{3a(ex)^{9/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{d\left(\frac{b\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{e^3\sqrt{x}}-\frac{2\sqrt{ax^2+bx^3}}{e(ex)^{3/2}}\right)}{e}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{3a(ex)^{9/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{d\left(\frac{2b\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{e^3\sqrt{x}}-\frac{2\sqrt{ax^2+bx^3}}{e(ex)^{3/2}}\right)}{e}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{3a(ex)^{9/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{2\sqrt{b}\sqrt{ex}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{e^3\sqrt{x}}-\frac{2\sqrt{ax^2+bx^3}}{e(ex)^{3/2}}\right)}{e}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{3a(ex)^{9/2}}$$

input
```
Int[((c + d*x)*Sqrt[a*x^2 + b*x^3])/(e*x)^(7/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(3/2))/(3*a*(e*x)^(9/2)) + (d*((-2*Sqrt[a*x^2 + b*
x^3])/(e*(e*x)^(3/2)) + (2*Sqrt[b]*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqr
t[a*x^2 + b*x^3]])/(e^3*Sqrt[x])))/e
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.11

method	result
risch	$-\frac{2(3adx+cbx+ac)\sqrt{x^{2}(bx+a)}}{3x^{2}a\,e^{3}\sqrt{ex}}+\frac{bd\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^{2}+aex}\right)\sqrt{x^{2}(bx+a)}\,\sqrt{ex(bx+a)}}{\sqrt{be}\,e^{3}x(bx+a)\sqrt{ex}}$
default	$-\frac{\sqrt{b\,x^{3}+a\,x^{2}}\left(-3\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)abde\,x^{2}+6adx\sqrt{ex(bx+a)}\,\sqrt{be}+2bcx\sqrt{ex(bx+a)}\,\sqrt{be}+2ac\sqrt{ex(bx+a)}\,\sqrt{be}\right)}{3x^{2}e^{3}\sqrt{ex}\,\sqrt{ex(bx+a)}\,a\sqrt{be}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*a*d*x+b*c*x+a*c)/x^2/a/e^3*(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)+b*d*ln(
(1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)/e^3*(x^2*(b
*x+a))^(1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.91

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{7/2}}\,dx = \left[\frac{3\,adex^3\sqrt{\frac{b}{e}}\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\sqrt{ex}\sqrt{\frac{b}{e}}}{x}\right)-2\,\sqrt{bx^3+ax^2}(ac+(bc+3\,ad)}{3\,ae^4x^3}\right.$$

$$\left.-\frac{2\left(3\,adex^3\sqrt{-\frac{b}{e}}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{ex}\sqrt{-\frac{b}{e}}}{bx^2+ax}\right)+\sqrt{bx^3+ax^2}(ac+(bc+3\,ad)x)\sqrt{ex}\right)}{3\,ae^4x^3}\right]$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(7/2),x, algorithm="fricas")
```

output
```
[1/3*(3*a*d*e*x^3*sqrt(b/e)*log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)*sqr
t(e*x)*sqrt(b/e))/x) - 2*sqrt(b*x^3 + a*x^2)*(a*c + (b*c + 3*a*d)*x)*sqrt(
e*x))/(a*e^4*x^3), -2/3*(3*a*d*e*x^3*sqrt(-b/e)*arctan(sqrt(b*x^3 + a*x^2)
*sqrt(e*x)*sqrt(-b/e)/(b*x^2 + a*x)) + sqrt(b*x^3 + a*x^2)*(a*c + (b*c + 3
*a*d)*x)*sqrt(e*x))/(a*e^4*x^3)]
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{7/2}}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}(c+dx)}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(1/2)/(e*x)**(7/2),x)
```

output
```
Integral(sqrt(x**2*(a + b*x))*(c + d*x)/(e*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{7/2}}\,dx = \int \frac{\sqrt{bx^3+ax^2}(dx+c)}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)*(d*x + c)/(e*x)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.14

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{7/2}}\,dx = -\frac{2\left(\frac{3\,d\log\left(\left|-\sqrt{be}\sqrt{bx+a}+\sqrt{(bx+a)be-abe}\right|\right)\mathrm{sgn}(x)}{\sqrt{be}b}-\frac{\left(3\,ade\mathrm{sgn}(x)-\frac{\left(b^2ce\mathrm{sgn}(x)+3\,abde\mathrm{sgn}(x)\right)(bx+a)}{ab}\right)\sqrt{bx+a}}{((bx+a)be-abe)^{\frac{3}{2}}}\right)b^3}{3\,e^3|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(7/2),x, algorithm="giac")
```

output
```
-2/3*(3*d*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt((b*x + a)*b*e - a*b*e)))
*sgn(x)/(sqrt(b*e)*b) - (3*a*d*e*sgn(x) - (b^2*c*e*sgn(x) + 3*a*b*d*e*sgn(
x))*(b*x + a)/(a*b))*sqrt(b*x + a)/((b*x + a)*b*e - a*b*e)^(3/2))*b^3/(e^3
*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{7/2}}\,dx = \int \frac{\sqrt{b\,x^3+a\,x^2}\,(c+d\,x)}{(e\,x)^{7/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/(e*x)^(7/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/(e*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{7/2}}\,dx = \frac{2\sqrt{e}\left(-\sqrt{x}\,\sqrt{bx+a}\,ac-3\sqrt{x}\,\sqrt{bx+a}\,adx-\sqrt{x}\,\sqrt{bx+a}\,bcx+3\sqrt{b}\log\left(\frac{\sqrt{b}}{}\right.\right.}{3a\,e^4x^2}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(7/2),x)
```

output
```
(2*sqrt(e)*( - sqrt(x)*sqrt(a + b*x)*a*c - 3*sqrt(x)*sqrt(a + b*x)*a*d*x -
 sqrt(x)*sqrt(a + b*x)*b*c*x + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt
(b))/sqrt(a))*a*d*x**2 + sqrt(b)*a*d*x**2 - sqrt(b)*b*c*x**2))/(3*a*e**4*x
**2)
```

3.307 $\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{9/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{9/2}}\,dx = -\frac{2ce(ax^2+bx^3)^{3/2}}{5a(ex)^{11/2}} + \frac{2(2bc-5ad)\,(ax^2+bx^3)^{3/2}}{15a^2(ex)^{9/2}}$$

output
```
-2/5*c*e*(b*x^3+a*x^2)^(3/2)/a/(e*x)^(11/2)+2/15*(-5*a*d+2*b*c)*(b*x^3+a*x
^2)^(3/2)/a^2/(e*x)^(9/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{9/2}}\,dx = -\frac{2e(x^2(a+bx))^{3/2}\,(3ac-2bcx+5adx)}{15a^2(ex)^{11/2}}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x^2 + b*x^3])/(e*x)^(9/2),x]
```

output
```
(-2*e*(x^2*(a + b*x))^(3/2)*(3*a*c - 2*b*c*x + 5*a*d*x))/(15*a^2*(e*x)^(11
/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1944, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}(c+dx)}{(ex)^{9/2}}dx$$

$$\Big\downarrow \text{ 1944}$$

$$-\frac{(2bc-5ad)\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{7/2}}dx}{5ae}-\frac{2ce(ax^2+bx^3)^{3/2}}{5a(ex)^{11/2}}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{2(ax^2+bx^3)^{3/2}(2bc-5ad)}{15a^2(ex)^{9/2}}-\frac{2ce(ax^2+bx^3)^{3/2}}{5a(ex)^{11/2}}$$

input
```
Int[((c + d*x)*Sqrt[a*x^2 + b*x^3])/(e*x)^(9/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(3/2))/(5*a*(e*x)^(11/2)) + (2*(2*b*c - 5*a*d)*(a*
x^2 + b*x^3)^(3/2))/(15*a^2*(e*x)^(9/2))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{2x(bx+a)(5adx-2cbx+3ac)\sqrt{b\,x^{3}+a\,x^{2}}}{15a^{2}(ex)^{\frac{9}{2}}}$	45
orering	$-\frac{2x(bx+a)(5adx-2cbx+3ac)\sqrt{b\,x^{3}+a\,x^{2}}}{15a^{2}(ex)^{\frac{9}{2}}}$	45
default	$-\frac{2\sqrt{b\,x^{3}+a\,x^{2}}\,(bx+a)(5adx-2cbx+3ac)}{15x^{3}e^{4}\sqrt{ex}\,a^{2}}$	50
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,(5abd\,x^{2}-2b^{2}c\,x^{2}+5a^{2}dx+abcx+3a^{2}c)}{15e^{4}x^{3}\sqrt{ex}\,a^{2}}$	64

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*x*(b*x+a)*(5*a*d*x-2*b*c*x+3*a*c)*(b*x^3+a*x^2)^(1/2)/a^2/(e*x)^(9/2
)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{9/2}}\,dx = -\frac{2\,\sqrt{bx^3+ax^2}(3\,a^2c-(2\,b^2c-5\,abd)x^2+(abc+5\,a^2d)x)\sqrt{ex}}{15\,a^2e^5x^4}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(9/2),x, algorithm="fricas")
```

output
```
-2/15*sqrt(b*x^3 + a*x^2)*(3*a^2*c - (2*b^2*c - 5*a*b*d)*x^2 + (a*b*c + 5*
a^2*d)*x)*sqrt(e*x)/(a^2*e^5*x^4)
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{9/2}}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}(c+dx)}{(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(1/2)/(e*x)**(9/2),x)
```

output
```
Integral(sqrt(x**2*(a + b*x))*(c + d*x)/(e*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{9/2}}\,dx = \int \frac{\sqrt{bx^3+ax^2}(dx+c)}{(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)*(d*x + c)/(e*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.40

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{9/2}}\,dx = \frac{2\,(bx+a)^{\frac{3}{2}}b\left(\frac{(2\,b^5ce^2\mathrm{sgn}(x)-5\,ab^4de^2\mathrm{sgn}(x))(bx+a)}{a^2}-\frac{5\,(ab^5ce^2\mathrm{sgn}(x)-a^2b^4de^2\mathrm{sgn}(x))}{a^2}\right)}{15\,((bx+a)be-abe)^{\frac{5}{2}}e^4|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(9/2),x, algorithm="giac")
```

output
```
2/15*(b*x + a)^(3/2)*b*((2*b^5*c*e^2*sgn(x) - 5*a*b^4*d*e^2*sgn(x))*(b*x +
 a)/a^2 - 5*(a*b^5*c*e^2*sgn(x) - a^2*b^4*d*e^2*sgn(x))/a^2)/(((b*x + a)*b
*e - a*b*e)^(5/2)*e^4*abs(b))
```

Mupad [B] (verification not implemented)

Time = 9.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.06

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{9/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,c}{5\,e^4}-\frac{x^2\,\left(4\,b^2\,c-10\,a\,b\,d\right)}{15\,a^2\,e^4}+\frac{x\,\left(10\,d\,a^2+2\,b\,c\,a\right)}{15\,a^2\,e^4}\right)}{x^3\,\sqrt{e\,x}}$$

input
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/(e*x)^(9/2),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*c)/(5*e^4) - (x^2*(4*b^2*c - 10*a*b*d))/(15*a^
2*e^4) + (x*(10*a^2*d + 2*a*b*c))/(15*a^2*e^4)))/(x^3*(e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.59

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{9/2}}\,dx = \frac{2\sqrt{e}\left(-3\sqrt{x}\,\sqrt{bx+a}\,a^2c-5\sqrt{x}\,\sqrt{bx+a}\,a^2dx-\sqrt{x}\,\sqrt{bx+a}\,abcx-5\sqrt{x}\,\sqrt{}\right.}{15a^2e^5x^3}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(9/2),x)
```

output
```
(2*sqrt(e)*( - 3*sqrt(x)*sqrt(a + b*x)*a**2*c - 5*sqrt(x)*sqrt(a + b*x)*a*
*2*d*x - sqrt(x)*sqrt(a + b*x)*a*b*c*x - 5*sqrt(x)*sqrt(a + b*x)*a*b*d*x**
2 + 2*sqrt(x)*sqrt(a + b*x)*b**2*c*x**2 - sqrt(b)*a*b*d*x**3 - 2*sqrt(b)*b
**2*c*x**3))/(15*a**2*e**5*x**3)
```

3.308 $\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 112

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{11/2}}\,dx = -\frac{2ce(ax^2+bx^3)^{3/2}}{7a(ex)^{13/2}} + \frac{2(4bc-7ad)\,(ax^2+bx^3)^{3/2}}{35a^2(ex)^{11/2}} - \frac{4b(4bc-7ad)\,(ax^2+bx^3)^{3/2}}{105a^3e(ex)^{9/2}}$$

output
```
-2/7*c*e*(b*x^3+a*x^2)^(3/2)/a/(e*x)^(13/2)+2/35*(-7*a*d+4*b*c)*(b*x^3+a*x
^2)^(3/2)/a^2/(e*x)^(11/2)-4/105*b*(-7*a*d+4*b*c)*(b*x^3+a*x^2)^(3/2)/a^3/
e/(e*x)^(9/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{11/2}}\,dx = -\frac{2x(a+bx)\sqrt{x^2(a+bx)}(15a^2c-12abcx+21a^2dx+8b^2cx^2-14abdx^2)}{105a^3(ex)^{11/2}}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x^2 + b*x^3])/(e*x)^(11/2),x]
```

output
```
(-2*x*(a + b*x)*Sqrt[x^2*(a + b*x)]*(15*a^2*c - 12*a*b*c*x + 21*a^2*d*x +
8*b^2*c*x^2 - 14*a*b*d*x^2))/(105*a^3*(e*x)^(11/2))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1944, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}(c+dx)}{(ex)^{11/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(4bc-7ad)\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{9/2}}dx}{7ae}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{7a(ex)^{13/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{(4bc-7ad)\left(-\frac{2b\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{7/2}}dx}{5ae}-\frac{2e\left(ax^2+bx^3\right)^{3/2}}{5a(ex)^{11/2}}\right)}{7ae}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{7a(ex)^{13/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{(4bc-7ad)\left(\frac{4b\left(ax^2+bx^3\right)^{3/2}}{15a^2(ex)^{9/2}}-\frac{2e\left(ax^2+bx^3\right)^{3/2}}{5a(ex)^{11/2}}\right)}{7ae}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{7a(ex)^{13/2}}$$

input
```
Int[((c + d*x)*Sqrt[a*x^2 + b*x^3])/(e*x)^(11/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(3/2))/(7*a*(e*x)^(13/2)) - ((4*b*c - 7*a*d)*((-2*
e*(a*x^2 + b*x^3)^(3/2))/(5*a*(e*x)^(11/2)) + (4*b*(a*x^2 + b*x^3)^(3/2))/
(15*a^2*(e*x)^(9/2))))/(7*a*e)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2x(bx+a)\left(-14abd\,x^{2}+8b^{2}c\,x^{2}+21a^{2}dx-12abcx+15a^{2}c\right)\sqrt{b\,x^{3}+a\,x^{2}}}{105a^{3}\left(ex\right)^{\frac{11}{2}}}$	67
orering	$-\frac{2x(bx+a)\left(-14abd\,x^{2}+8b^{2}c\,x^{2}+21a^{2}dx-12abcx+15a^{2}c\right)\sqrt{b\,x^{3}+a\,x^{2}}}{105a^{3}\left(ex\right)^{\frac{11}{2}}}$	67
default	$-\frac{2\sqrt{b\,x^{3}+a\,x^{2}}\,(bx+a)\left(-14abd\,x^{2}+8b^{2}c\,x^{2}+21a^{2}dx-12abcx+15a^{2}c\right)}{105x^{4}e^{5}\sqrt{ex}\,a^{3}}$	72
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-14a\,b^{2}d\,x^{3}+8b^{3}c\,x^{3}+7a^{2}bd\,x^{2}-4a\,b^{2}c\,x^{2}+21a^{3}dx+3a^{2}bcx+15c\,a^{3}\right)}{105e^{5}x^{4}\sqrt{ex}\,a^{3}}$	89

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*x*(b*x+a)*(-14*a*b*d*x^2+8*b^2*c*x^2+21*a^2*d*x-12*a*b*c*x+15*a^2*c
)*(b*x^3+a*x^2)^(1/2)/a^3/(e*x)^(11/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{11/2}}\,dx = -\frac{2\,(15\,a^3c+2\,(4\,b^3c-7\,ab^2d)x^3-(4\,ab^2c-7\,a^2bd)x^2+3\,(a^2bc+7\,a^3d)x)\sqrt{bx^3+ax^2}\sqrt{ex}}{105\,a^3e^6x^5}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(11/2),x, algorithm="fricas")
```

output
```
-2/105*(15*a^3*c + 2*(4*b^3*c - 7*a*b^2*d)*x^3 - (4*a*b^2*c - 7*a^2*b*d)*x
^2 + 3*(a^2*b*c + 7*a^3*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x)/(a^3*e^6*x^5)
```

Sympy [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{11/2}}\,dx = \int \frac{\sqrt{x^2\,(a+bx)}(c+dx)}{(ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(1/2)/(e*x)**(11/2),x)
```

output
```
Integral(sqrt(x**2*(a + b*x))*(c + d*x)/(e*x)**(11/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{11/2}}\,dx = \int \frac{\sqrt{bx^3+ax^2}(dx+c)}{(ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(11/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)*(d*x + c)/(e*x)^(11/2), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{11/2}}\,dx = -\frac{2\,(bx+a)^{\frac{3}{2}}\left((bx+a)\left(\frac{2\,(4\,b^3ce^3\mathrm{sgn}(x)-7\,ab^2de^3\mathrm{sgn}(x))(bx+a)}{a^3}-\frac{7\,(4\,ab^3ce^3\mathrm{sgn}(x)-7\,a^2b^2de^3\mathrm{sgn}(x))}{a^3}\right)+\frac{35\,(a^2b^3ce^3\mathrm{sgn}(x)-a^3}{a^3}\right.}{105\,((bx+a)be-abe)^{\frac{7}{2}}e^5|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(11/2),x, algorithm="giac")
```

output
```
-2/105*(b*x + a)^(3/2)*((b*x + a)*(2*(4*b^3*c*e^3*sgn(x) - 7*a*b^2*d*e^3*s
gn(x))*(b*x + a)/a^3 - 7*(4*a*b^3*c*e^3*sgn(x) - 7*a^2*b^2*d*e^3*sgn(x))/a
^3) + 35*(a^2*b^3*c*e^3*sgn(x) - a^3*b^2*d*e^3*sgn(x))/a^3)*b^5/(((b*x + a
)*b*e - a*b*e)^(7/2)*e^5*abs(b))
```

Mupad [B] (verification not implemented)

Time = 9.14 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{11/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,c}{7\,e^5}+\frac{x\,\left(42\,d\,a^3+6\,b\,c\,a^2\right)}{105\,a^3\,e^5}+\frac{x^3\,\left(16\,b^3\,c-28\,a\,b^2\,d\right)}{105\,a^3\,e^5}+\frac{2\,b\,x^2\,\left(7\,a\,d-4\,b\,c\right)}{105\,a^2\,e^5}\right)}{x^4\,\sqrt{e\,x}}$$

input
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/(e*x)^(11/2),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*c)/(7*e^5) + (x*(42*a^3*d + 6*a^2*b*c))/(105*a
^3*e^5) + (x^3*(16*b^3*c - 28*a*b^2*d))/(105*a^3*e^5) + (2*b*x^2*(7*a*d -
4*b*c))/(105*a^2*e^5)))/(x^4*(e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.37

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{11/2}}\,dx = \frac{2\sqrt{e}\left(-15\sqrt{x}\,\sqrt{bx+a}\,a^3c-21\sqrt{x}\,\sqrt{bx+a}\,a^3dx-3\sqrt{x}\,\sqrt{bx+a}\,a^2bcx-7\sqrt{}\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(11/2),x)
```

output
```
(2*sqrt(e)*( - 15*sqrt(x)*sqrt(a + b*x)*a**3*c - 21*sqrt(x)*sqrt(a + b*x)*
a**3*d*x - 3*sqrt(x)*sqrt(a + b*x)*a**2*b*c*x - 7*sqrt(x)*sqrt(a + b*x)*a*
*2*b*d*x**2 + 4*sqrt(x)*sqrt(a + b*x)*a*b**2*c*x**2 + 14*sqrt(x)*sqrt(a +
b*x)*a*b**2*d*x**3 - 8*sqrt(x)*sqrt(a + b*x)*b**3*c*x**3 - 14*sqrt(b)*a*b*
*2*d*x**4 + 8*sqrt(b)*b**3*c*x**4))/(105*a**3*e**6*x**4)
```

3.309 $\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{13/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{13/2}}\,dx=-\frac{2ce(ax^2+bx^3)^{3/2}}{9a(ex)^{15/2}}+\frac{2(2bc-3ad)\left(ax^2+bx^3\right)^{3/2}}{21a^2(ex)^{13/2}}-\frac{8b(2bc-3ad)\left(ax^2+bx^3\right)^{3/2}}{105a^3e(ex)^{11/2}}+\frac{16b^2(2bc-3ad)\left(ax^2+bx^3\right)^{3/2}}{315a^4e^2(ex)^{9/2}}$$

output
```
-2/9*c*e*(b*x^3+a*x^2)^(3/2)/a/(e*x)^(15/2)+2/21*(-3*a*d+2*b*c)*(b*x^3+a*x
^2)^(3/2)/a^2/(e*x)^(13/2)-8/105*b*(-3*a*d+2*b*c)*(b*x^3+a*x^2)^(3/2)/a^3/
e/(e*x)^(11/2)+16/315*b^2*(-3*a*d+2*b*c)*(b*x^3+a*x^2)^(3/2)/a^4/e^2/(e*x)
^(9/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.54

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{13/2}}\,dx=-\frac{2\sqrt{ex}\left(x^2(a+bx)\right)^{3/2}\left(-16b^3cx^3+24ab^2x^2(c+dx)-6a^2bx(5c+6dx)+5a^3(7c+9dx)\right)}{315a^4e^7x^8}$$

input
```
Integrate[((c + d*x)*Sqrt[a*x^2 + b*x^3])/(e*x)^(13/2),x]
```

output
```
(-2*Sqrt[e*x]*(x^2*(a + b*x))^(3/2)*(-16*b^3*c*x^3 + 24*a*b^2*x^2*(c + d*x
) - 6*a^2*b*x*(5*c + 6*d*x) + 5*a^3*(7*c + 9*d*x)))/(315*a^4*e^7*x^8)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1944, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ax^2+bx^3}(c+dx)}{(ex)^{13/2}}\,dx$$

$$\Big\downarrow \text{ 1944}$$

$$-\frac{(2bc-3ad)\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{11/2}}dx}{3ae}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{9a(ex)^{15/2}}$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{(2bc-3ad)\left(-\frac{4b\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{9/2}}dx}{7ae}-\frac{2e\left(ax^2+bx^3\right)^{3/2}}{7a(ex)^{13/2}}\right)}{3ae}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{9a(ex)^{15/2}}$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{(2bc-3ad)\left(-\frac{4b\left(-\frac{2b\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{7/2}}dx}{5ae}-\frac{2e\left(ax^2+bx^3\right)^{3/2}}{5a(ex)^{11/2}}\right)}{7ae}-\frac{2e\left(ax^2+bx^3\right)^{3/2}}{7a(ex)^{13/2}}\right)}{3ae}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{9a(ex)^{15/2}}$$

$$\Big\downarrow \text{ 1920}$$

$$-\frac{(2bc-3ad)\left(-\frac{4b\left(\frac{4b\left(ax^2+bx^3\right)^{3/2}}{15a^2(ex)^{9/2}}-\frac{2e\left(ax^2+bx^3\right)^{3/2}}{5a(ex)^{11/2}}\right)}{7ae}-\frac{2e\left(ax^2+bx^3\right)^{3/2}}{7a(ex)^{13/2}}\right)}{3ae}-\frac{2ce\left(ax^2+bx^3\right)^{3/2}}{9a(ex)^{15/2}}$$

input
```
Int[((c + d*x)*Sqrt[a*x^2 + b*x^3])/(e*x)^(13/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(3/2))/(9*a*(e*x)^(15/2)) - ((2*b*c - 3*a*d)*((-2*
e*(a*x^2 + b*x^3)^(3/2))/(7*a*(e*x)^(13/2)) - (4*b*((-2*e*(a*x^2 + b*x^3)^
(3/2))/(5*a*(e*x)^(11/2)) + (4*b*(a*x^2 + b*x^3)^(3/2))/(15*a^2*(e*x)^(9/2
))))/(7*a*e)))/(3*a*e)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
 + 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{2x(bx+a)\left(24a\,b^{2}d\,x^{3}-16b^{3}c\,x^{3}-36a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+45a^{3}dx-30a^{2}bcx+35c\,a^{3}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{315a^{4}(ex)^{\frac{13}{2}}}$	91
orering	$-\frac{2x(bx+a)\left(24a\,b^{2}d\,x^{3}-16b^{3}c\,x^{3}-36a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+45a^{3}dx-30a^{2}bcx+35c\,a^{3}\right)\sqrt{b\,x^{3}+a\,x^{2}}}{315a^{4}(ex)^{\frac{13}{2}}}$	91
default	$-\frac{2\sqrt{b\,x^{3}+a\,x^{2}}\,(bx+a)\left(24a\,b^{2}d\,x^{3}-16b^{3}c\,x^{3}-36a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+45a^{3}dx-30a^{2}bcx+35c\,a^{3}\right)}{315x^{5}e^{6}\sqrt{ex}\,a^{4}}$	96
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(24x^{4}a\,b^{3}d-16x^{4}b^{4}c-12a^{2}b^{2}d\,x^{3}+8a\,b^{3}c\,x^{3}+9a^{3}bd\,x^{2}-6a^{2}b^{2}c\,x^{2}+45a^{4}dx+5a^{3}bcx+35c\,a^{4}\right)}{315e^{6}x^{5}\sqrt{ex}\,a^{4}}$	113

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*x*(b*x+a)*(24*a*b^2*d*x^3-16*b^3*c*x^3-36*a^2*b*d*x^2+24*a*b^2*c*x^
2+45*a^3*d*x-30*a^2*b*c*x+35*a^3*c)*(b*x^3+a*x^2)^(1/2)/a^4/(e*x)^(13/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.74

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{13/2}}\,dx =$$
$$-\frac{2\,(35\,a^4c-8\,(2\,b^4c-3\,ab^3d)x^4+4\,(2\,ab^3c-3\,a^2b^2d)x^3-3\,(2\,a^2b^2c-3\,a^3bd)x^2+5\,(a^3bc+9\,a^4d)x)\sqrt{bx}}{315\,a^4e^7x^6}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(13/2),x, algorithm="fricas")
```

output
```
-2/315*(35*a^4*c - 8*(2*b^4*c - 3*a*b^3*d)*x^4 + 4*(2*a*b^3*c - 3*a^2*b^2*
d)*x^3 - 3*(2*a^2*b^2*c - 3*a^3*b*d)*x^2 + 5*(a^3*b*c + 9*a^4*d)*x)*sqrt(b
*x^3 + a*x^2)*sqrt(e*x)/(a^4*e^7*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(1/2)/(e*x)**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{13/2}}\,dx = \int \frac{\sqrt{bx^3+ax^2}(dx+c)}{(ex)^{\frac{13}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(13/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^3 + a*x^2)*(d*x + c)/(e*x)^(13/2), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{13/2}}\,dx = \frac{2\left((bx+a)\left(4\,(bx+a)\left(\frac{2\,(2\,b^9ce^4\mathrm{sgn}(x)-3\,ab^8de^4\mathrm{sgn}(x))\,(bx+a)}{a^4}-\frac{9\,(2\,ab^9ce^4\mathrm{sgn}(x)-3\,a^2b^8c}{a^4}\right.\right.\right.}{315\left((bx+\right.}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(13/2),x, algorithm="giac")
```

output
```
2/315*((b*x + a)*(4*(b*x + a)*(2*(2*b^9*c*e^4*sgn(x) - 3*a*b^8*d*e^4*sgn(x
))*(b*x + a)/a^4 - 9*(2*a*b^9*c*e^4*sgn(x) - 3*a^2*b^8*d*e^4*sgn(x))/a^4)
+ 63*(2*a^2*b^9*c*e^4*sgn(x) - 3*a^3*b^8*d*e^4*sgn(x))/a^4) - 105*(a^3*b^9
*c*e^4*sgn(x) - a^4*b^8*d*e^4*sgn(x))/a^4)*(b*x + a)^(3/2)*b/(((b*x + a)*b
*e - a*b*e)^(9/2)*e^6*abs(b))
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{13/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\left(\frac{2\,c}{9\,e^6}+\frac{x\,(90\,d\,a^4+10\,b\,c\,a^3)}{315\,a^4\,e^6}-\frac{x^4\,(32\,b^4\,c-48\,a\,b^3\,d)}{315\,a^4\,e^6}-\frac{8\,b^2\,x^3\,(3\,a\,d-2\,b\,c)}{315\,a^3\,e^6}+\frac{2\,b\,x^2\,(3\,a\,d-2\,b\,c)}{105\,a^2\,e^6}\right)}{x^5\,\sqrt{e\,x}}$$

input
```
int(((a*x^2 + b*x^3)^(1/2)*(c + d*x))/(e*x)^(13/2),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*c)/(9*e^6) + (x*(90*a^4*d + 10*a^3*b*c))/(315*
a^4*e^6) - (x^4*(32*b^4*c - 48*a*b^3*d))/(315*a^4*e^6) - (8*b^2*x^3*(3*a*d
 - 2*b*c))/(315*a^3*e^6) + (2*b*x^2*(3*a*d - 2*b*c))/(105*a^2*e^6)))/(x^5*
(e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx)\sqrt{ax^2+bx^3}}{(ex)^{13/2}}\,dx = \frac{2\sqrt{e}\left(-35\sqrt{x}\,\sqrt{bx+a}\,a^4c-45\sqrt{x}\,\sqrt{bx+a}\,a^4dx-5\sqrt{x}\,\sqrt{bx+a}\,a^3bcx-9\sqrt{\right.}}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(1/2)/(e*x)^(13/2),x)
```

output
```
(2*sqrt(e)*( - 35*sqrt(x)*sqrt(a + b*x)*a**4*c - 45*sqrt(x)*sqrt(a + b*x)*
a**4*d*x - 5*sqrt(x)*sqrt(a + b*x)*a**3*b*c*x - 9*sqrt(x)*sqrt(a + b*x)*a*
*3*b*d*x**2 + 6*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c*x**2 + 12*sqrt(x)*sqrt(a
 + b*x)*a**2*b**2*d*x**3 - 8*sqrt(x)*sqrt(a + b*x)*a*b**3*c*x**3 - 24*sqrt
(x)*sqrt(a + b*x)*a*b**3*d*x**4 + 16*sqrt(x)*sqrt(a + b*x)*b**4*c*x**4 + 2
4*sqrt(b)*a*b**3*d*x**5 - 16*sqrt(b)*b**4*c*x**5))/(315*a**4*e**7*x**5)
```

3.310 $\int \sqrt{ex}(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 343

$$\begin{aligned}\int \sqrt{ex}(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx &= -\frac{a^5(14bc-9ad)e\sqrt{ax^2+bx^3}}{1024b^5\sqrt{ex}} \\ &+\frac{a^4(14bc-9ad)\sqrt{ex}\sqrt{ax^2+bx^3}}{1536b^4}-\frac{a^3(14bc-9ad)(ex)^{3/2}\sqrt{ax^2+bx^3}}{1920b^3e} \\ &+\frac{a^2(14bc-9ad)(ex)^{5/2}\sqrt{ax^2+bx^3}}{2240b^2e^2}+\frac{13a(14bc-9ad)(ex)^{7/2}\sqrt{ax^2+bx^3}}{840be^3} \\ &+\frac{(14bc-9ad)(ex)^{9/2}\sqrt{ax^2+bx^3}}{84e^4}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{7b\sqrt{ex}} \\ &+\frac{a^6(14bc-9ad)\sqrt{e}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{1024b^{11/2}}\end{aligned}$$

output
```
-1/1024*a^5*(-9*a*d+14*b*c)*e*(b*x^3+a*x^2)^(1/2)/b^5/(e*x)^(1/2)+1/1536*a
^4*(-9*a*d+14*b*c)*(e*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/b^4-1/1920*a^3*(-9*a*d+
14*b*c)*(e*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/b^3/e+1/2240*a^2*(-9*a*d+14*b*c)*(
e*x)^(5/2)*(b*x^3+a*x^2)^(1/2)/b^2/e^2+13/840*a*(-9*a*d+14*b*c)*(e*x)^(7/2
)*(b*x^3+a*x^2)^(1/2)/b/e^3+1/84*(-9*a*d+14*b*c)*(e*x)^(9/2)*(b*x^3+a*x^2)
^(1/2)/e^4+1/7*d*e*(b*x^3+a*x^2)^(5/2)/b/(e*x)^(1/2)+1/1024*a^6*(-9*a*d+14
*b*c)*e^(1/2)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(
11/2)
```

Mathematica [A] (verified)

Time = 1.33 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.72

$$\int \sqrt{ex}(c+dx)\left(ax^2+bx^3\right)^{3/2} dx = \frac{\sqrt{x}\sqrt{ex}\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(945a^6d-210a^5b(7c+3dx)+96a^2b^4x^3(7c+4dx)+2560b\right.}{}$$

input
```
Integrate[Sqrt[e*x]*(c + d*x)*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(Sqrt[x]*Sqrt[e*x]*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(945*a^6*d
 - 210*a^5*b*(7*c + 3*d*x) + 96*a^2*b^4*x^3*(7*c + 4*d*x) + 2560*b^6*x^5*(
7*c + 6*d*x) + 28*a^4*b^2*x*(35*c + 18*d*x) - 16*a^3*b^3*x^2*(49*c + 27*d*
x) + 256*a*b^5*x^4*(91*c + 75*d*x)) + 1890*a^7*d*ArcTanh[(Sqrt[b]*Sqrt[x])
/(Sqrt[a] - Sqrt[a + b*x])] + 2940*a^6*b*c*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqr
t[a] + Sqrt[a + b*x])]))/(107520*b^(11/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 328, normalized size of antiderivative = 0.96, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1945, 1927, 1927, 1930, 1930, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{ex}\left(ax^2+bx^3\right)^{3/2}(c+dx)\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(14bc-9ad)\int \sqrt{ex}\left(bx^3+ax^2\right)^{3/2}dx}{14b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{7b\sqrt{ex}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(14bc-9ad)\left(\frac{a\int (ex)^{5/2}\sqrt{bx^3+ax^2}dx}{4e^2}+\frac{(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}{6e}\right)}{14b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{7b\sqrt{ex}}$$

$$\Big\downarrow \text{ 1927}$$

$$\frac{(14bc-9ad)\left(\frac{a\left(\frac{a\int \frac{(ex)^{9/2}}{\sqrt{bx^3+ax^2}}dx}{10e^2}+\frac{(ex)^{7/2}\sqrt{ax^2+bx^3}}{5e}\right)}{4e^2}+\frac{(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}{6e}\right)}{14b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{7b\sqrt{ex}}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{(14bc-9ad)\left(\frac{a\left(\frac{a\left(\frac{e^2(ex)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ae\int \frac{(ex)^{7/2}}{\sqrt{bx^3+ax^2}}dx}{8b}\right)}{10e^2}+\frac{(ex)^{7/2}\sqrt{ax^2+bx^3}}{5e}\right)}{4e^2}+\frac{(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}{6e}\right)}{14b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{7b\sqrt{ex}}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{(14bc-9ad)\left(\frac{a\left(\frac{a\left(\frac{e^2(ex)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ae\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\int\frac{(ex)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6b}\right)}{8b}\right)}{10e^2}+\frac{(ex)^{7/2}\sqrt{ax^2+bx^3}}{5e}\right)}{4e^2}+\frac{(ex)^{3/2}(ax^2+bx^3)^{3/2}}{6e}\right.}{14b}$$

$$\frac{de\left(ax^2+bx^3\right)^{5/2}}{7b\sqrt{ex}}$$

$\downarrow$ 1930

$$\frac{(14bc-9ad)\left(\frac{a\left(\frac{a\left(\frac{e^2(ex)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ae\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\int\frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6b}\right)}{8b}\right)}{10e^2}+\frac{(ex)^{7/2}\sqrt{ax^2+bx^3}}{5e}\right)}{4e^2}\right)}{14b}$$

$$\frac{de\left(ax^2+bx^3\right)^{5/2}}{7b\sqrt{ex}}$$

$$\downarrow 1930$$

$$(14bc-9ad)\left(\frac{a\left(\frac{a\left(\frac{e^2(ex)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ae\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int\frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6b}\right)}{8b}\right)}{10e^2}+}{4e^2}\right.$$

↓ 1937

$$(14bc-9ad)\left(\frac{a\left(\frac{a\left(\frac{e^2(ex)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ae\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8b}\right.}{10e^2}}{4e^2}\right.$$

↓ 1935

$$a\left(\frac{a\left(\frac{e^2(ex)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{7ae\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^3}{\sqrt{bx^3}}}{b\sqrt{x}}\right.}{4b}\right.}{6b}\right.}{8b}\right.}{10e^2}\right.$$

$(14b \quad 9a \quad d)$

↓ 219

$$3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\,\mathrm{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx}}\right.}{b^{3/2}\sqrt{x}}\right.$$

$$5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{\cdots}{4b}\right.$$

$$7ae\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{\cdots}{6b}\right.$$

$$a\left(\frac{e^2(ex)^{5/2}\sqrt{ax^2+bx^3}}{4b}-\frac{\cdots}{8b}\right.$$

$$a\,\frac{\cdots}{10e^2}$$

$$(14bc-9ad)\,\frac{\cdots}{4e^2}$$

input
```
Int[Sqrt[e*x]*(c + d*x)*(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(d*e*(a*x^2 + b*x^3)^(5/2))/(7*b*Sqrt[e*x]) + ((14*b*c - 9*a*d)*(((e*x)^(3
/2)*(a*x^2 + b*x^3)^(3/2))/(6*e) + (a*(((e*x)^(7/2)*Sqrt[a*x^2 + b*x^3])/(
5*e) + (a*((e^2*(e*x)^(5/2)*Sqrt[a*x^2 + b*x^3])/(4*b) - (7*a*e*((e^2*(e*x
)^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a*e*((e^2*Sqrt[e*x]*Sqrt[a*x^2 + b
*x^3])/(2*b) - (3*a*e*((e^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[e*x]) - (a*e*Sqrt
[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/
(4*b)))/(6*b)))/(8*b)))/(10*e^2)))/(4*e^2)))/(14*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.71

method	result
risch	$\frac{(15360b^6 d x^6+19200a b^5 d x^5+17920b^6 c x^5+384a^2 b^4 d x^4+23296a b^5 c x^4-432a^3 b^3 d x^3+672a^2 b^4 c x^3+504a^4 b^2 d x^2-784a^3 b^3 c x^2-}{107520b^5 \sqrt{ex}}$
default	$\frac{(b x^3+a x^2)^{\frac{3}{2}} \sqrt{ex} \left(30720b^6 d x^6 \sqrt{ex(bx+a)} \sqrt{be}+38400a b^5 d x^5 \sqrt{ex(bx+a)} \sqrt{be}+35840b^6 c x^5 \sqrt{ex(bx+a)} \sqrt{be}+768a^2 b^4 d x^4 \sqrt{ex}\right.}{}$

input
```
int((e*x)^(1/2)*(d*x+c)*(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/107520/b^5*(15360*b^6*d*x^6+19200*a*b^5*d*x^5+17920*b^6*c*x^5+384*a^2*b^
4*d*x^4+23296*a*b^5*c*x^4-432*a^3*b^3*d*x^3+672*a^2*b^4*c*x^3+504*a^4*b^2*
d*x^2-784*a^3*b^3*c*x^2-630*a^5*b*d*x+980*a^4*b^2*c*x+945*a^6*d-1470*a^5*b
*c)*e*(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)-1/2048*a^6/b^5*(9*a*d-14*b*c)*ln((1/
2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)*e*(x^2*(b*x+a)
)^(1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.34

$$\int \sqrt{ex}(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx = \left[-\frac{105\,(14\,a^6bc-9\,a^7d)x\sqrt{\frac{e}{b}}\log\left(\frac{2\,bex^2+aex-2\,\sqrt{bx^3+ax^2}\sqrt{ex}b\sqrt{\frac{e}{b}}}{x}\right)-2\,(15360\,b^6dx^6-1470\,a^5b\ldots}{\ldots}\right.$$

$$-\frac{105\,(14\,a^6bc-9\,a^7d)x\sqrt{-\frac{e}{b}}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{ex}b\sqrt{-\frac{e}{b}}}{bex^2+aex}\right)-(15360\,b^6dx^6-1470\,a^5bc+945\,a^6d+1280\,(14\ldots}{\ldots}$$

input
```
integrate((e*x)^(1/2)*(d*x+c)*(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/215040*(105*(14*a^6*b*c - 9*a^7*d)*x*sqrt(e/b)*log((2*b*e*x^2 + a*e*x
- 2*sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(e/b))/x) - 2*(15360*b^6*d*x^6 - 1
470*a^5*b*c + 945*a^6*d + 1280*(14*b^6*c + 15*a*b^5*d)*x^5 + 128*(182*a*b^
5*c + 3*a^2*b^4*d)*x^4 + 48*(14*a^2*b^4*c - 9*a^3*b^3*d)*x^3 - 56*(14*a^3*
b^3*c - 9*a^4*b^2*d)*x^2 + 70*(14*a^4*b^2*c - 9*a^5*b*d)*x)*sqrt(b*x^3 + a
*x^2)*sqrt(e*x))/(b^5*x), -1/107520*(105*(14*a^6*b*c - 9*a^7*d)*x*sqrt(-e/
b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(-e/b)/(b*e*x^2 + a*e*x)) -
(15360*b^6*d*x^6 - 1470*a^5*b*c + 945*a^6*d + 1280*(14*b^6*c + 15*a*b^5*d)
*x^5 + 128*(182*a*b^5*c + 3*a^2*b^4*d)*x^4 + 48*(14*a^2*b^4*c - 9*a^3*b^3*
d)*x^3 - 56*(14*a^3*b^3*c - 9*a^4*b^2*d)*x^2 + 70*(14*a^4*b^2*c - 9*a^5*b*
d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^5*x)]
```

Sympy [F]

$$\int \sqrt{ex}(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx = \int \sqrt{ex}\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)\,dx$$

input
```
integrate((e*x)**(1/2)*(d*x+c)*(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(sqrt(e*x)*(x**2*(a + b*x))**(3/2)*(c + d*x), x)
```

Maxima [F]

$$\int \sqrt{ex}(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\int \left(bx^3+ax^2\right)^{\frac{3}{2}}(dx+c)\sqrt{ex}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x+c)*(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)*sqrt(e*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(287) = 574$.

Time = 0.23 (sec) , antiderivative size = 655, normalized size of antiderivative = 1.91

$$\int \sqrt{ex}(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((e*x)^(1/2)*(d*x+c)*(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/7680*(315*a^6*e^2*log(abs(-sqrt(b*e)*sqrt(e*x) + sqrt(b*e^2*x + a*e^2)))
/(sqrt(b*e)*b^5) + sqrt(b*e^2*x + a*e^2)*(2*(4*(2*(8*e*x*(10*x/e^4 + a/(b*
e^4)) - 9*a^2/(b^2*e^3))*e*x + 21*a^3/(b^3*e^2))*e*x - 105*a^4/(b^4*e))*e*
x + 315*a^5/b^5)*sqrt(e*x))*b*c*abs(e)*sgn(x)/e^2 + 1/7680*(315*a^6*e^2*lo
g(abs(-sqrt(b*e)*sqrt(e*x) + sqrt(b*e^2*x + a*e^2)))/(sqrt(b*e)*b^5) + sqr
t(b*e^2*x + a*e^2)*(2*(4*(2*(8*e*x*(10*x/e^4 + a/(b*e^4)) - 9*a^2/(b^2*e^3
))*e*x + 21*a^3/(b^3*e^2))*e*x - 105*a^4/(b^4*e))*e*x + 315*a^5/b^5)*sqrt(
e*x))*a*d*abs(e)*sgn(x)/e^2 - 1/1920*(105*a^5*e^6*log(abs(-sqrt(b*e)*sqrt(
e*x) + sqrt(b*e^2*x + a*e^2)))/(sqrt(b*e)*b^4) + (105*a^4*e^4/b^4 - 2*(35*
a^3*e^3/b^3 + 4*(6*(8*e*x + a*e/b)*e*x - 7*a^2*e^2/b^2)*e*x)*e*x)*sqrt(b*e
^2*x + a*e^2)*sqrt(e*x))*a*c*abs(e)*sgn(x)/e^6 - 1/107520*(3465*a^7*e^8*lo
g(abs(-sqrt(b*e)*sqrt(e*x) + sqrt(b*e^2*x + a*e^2)))/(sqrt(b*e)*b^6) + (34
65*a^6*e^6/b^6 - 2*(1155*a^5*e^5/b^5 - 4*(231*a^4*e^4/b^4 - 2*(99*a^3*e^3/
b^3 + 8*(10*(12*e*x + a*e/b)*e*x - 11*a^2*e^2/b^2)*e*x)*e*x)*e*x)*e*x)*sqr
t(b*e^2*x + a*e^2)*sqrt(e*x))*b*d*abs(e)*sgn(x)/e^8 + 1/2048*(14*a^6*b*c*a
bs(e)*log(e^2*abs(a)) - 9*a^7*d*abs(e)*log(e^2*abs(a)))*sgn(x)/(sqrt(b*e)*
b^5)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{ex}(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\int \sqrt{e\,x}\,\left(b\,x^3+a\,x^2\right)^{3/2}\,\left(c+d\,x\right)\,dx$$

input
```
int((e*x)^(1/2)*(a*x^2 + b*x^3)^(3/2)*(c + d*x),x)
```

output
```
int((e*x)^(1/2)*(a*x^2 + b*x^3)^(3/2)*(c + d*x), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.87

$$\int \sqrt{ex}(c+dx)\left(ax^2+bx^3\right)^{3/2}\,dx=\frac{\sqrt{e}\left(945\sqrt{x}\,\sqrt{bx+a}\,a^6bd-1470\sqrt{x}\,\sqrt{bx+a}\,a^5b^2c-630\sqrt{x}\,\sqrt{bx+a}\,a^5b^2dx+980\sqrt{x}\,\sqrt{bx\right.}{}$$

input
```
int((e*x)^(1/2)*(d*x+c)*(b*x^3+a*x^2)^(3/2),x)
```

output
```
(sqrt(e)*(945*sqrt(x)*sqrt(a + b*x)*a**6*b*d - 1470*sqrt(x)*sqrt(a + b*x)*
a**5*b**2*c - 630*sqrt(x)*sqrt(a + b*x)*a**5*b**2*d*x + 980*sqrt(x)*sqrt(a
 + b*x)*a**4*b**3*c*x + 504*sqrt(x)*sqrt(a + b*x)*a**4*b**3*d*x**2 - 784*s
qrt(x)*sqrt(a + b*x)*a**3*b**4*c*x**2 - 432*sqrt(x)*sqrt(a + b*x)*a**3*b**
4*d*x**3 + 672*sqrt(x)*sqrt(a + b*x)*a**2*b**5*c*x**3 + 384*sqrt(x)*sqrt(a
 + b*x)*a**2*b**5*d*x**4 + 23296*sqrt(x)*sqrt(a + b*x)*a*b**6*c*x**4 + 192
00*sqrt(x)*sqrt(a + b*x)*a*b**6*d*x**5 + 17920*sqrt(x)*sqrt(a + b*x)*b**7*
c*x**5 + 15360*sqrt(x)*sqrt(a + b*x)*b**7*d*x**6 - 945*sqrt(b)*log((sqrt(a
 + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**7*d + 1470*sqrt(b)*log((sqrt(a + b*
x) + sqrt(x)*sqrt(b))/sqrt(a))*a**6*b*c))/(107520*b**6)
```

3.311 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{\sqrt{ex}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 301

$$\begin{aligned}\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{\sqrt{ex}}\,dx &= \frac{a^4(12bc-7ad)\sqrt{ax^2+bx^3}}{512b^4\sqrt{ex}} \\ &\quad - \frac{a^3(12bc-7ad)\sqrt{ex}\sqrt{ax^2+bx^3}}{768b^3e} + \frac{a^2(12bc-7ad)(ex)^{3/2}\sqrt{ax^2+bx^3}}{960b^2e^2} \\ &\quad + \frac{11a(12bc-7ad)(ex)^{5/2}\sqrt{ax^2+bx^3}}{480be^3} + \frac{(12bc-7ad)(ex)^{7/2}\sqrt{ax^2+bx^3}}{60e^4} \\ &\quad + \frac{de\left(ax^2+bx^3\right)^{5/2}}{6b(ex)^{3/2}} - \frac{a^5(12bc-7ad)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{512b^{9/2}\sqrt{e}}\end{aligned}$$

output
```
1/512*a^4*(-7*a*d+12*b*c)*(b*x^3+a*x^2)^(1/2)/b^4/(e*x)^(1/2)-1/768*a^3*(-
7*a*d+12*b*c)*(e*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/b^3/e+1/960*a^2*(-7*a*d+12*b
*c)*(e*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/b^2/e^2+11/480*a*(-7*a*d+12*b*c)*(e*x)
^(5/2)*(b*x^3+a*x^2)^(1/2)/b/e^3+1/60*(-7*a*d+12*b*c)*(e*x)^(7/2)*(b*x^3+a
*x^2)^(1/2)/e^4+1/6*d*e*(b*x^3+a*x^2)^(5/2)/b/(e*x)^(3/2)-1/512*a^5*(-7*a*
d+12*b*c)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(9/2)
/e^(1/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.76

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{\sqrt{ex}}\,dx = \frac{x^{3/2}\sqrt{a+bx}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(-105a^5d+48a^2b^3x^2(2c+dx)+256b^5x^4(6c\right.}{}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/Sqrt[e*x],x]
```

output
```
(x^(3/2)*Sqrt[a + b*x]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-105*a^5*d + 48*a^2
*b^3*x^2*(2*c + d*x) + 256*b^5*x^4*(6*c + 5*d*x) - 8*a^3*b^2*x*(15*c + 7*d
*x) + 10*a^4*b*(18*c + 7*d*x) + 64*a*b^4*x^3*(33*c + 26*d*x)) + 360*a^5*b*
c*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])] + 210*a^6*d*ArcTanh
[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(7680*b^(9/2)*Sqrt[e*x]*S
qrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.321, Rules used = {1945, 1927, 1927, 1930, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}(c+dx)}{\sqrt{ex}}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(12bc-7ad)\int \frac{\left(bx^3+ax^2\right)^{3/2}}{\sqrt{ex}}dx}{12b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{6b(ex)^{3/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(12bc-7ad)\left(\frac{3a\int (ex)^{3/2}\sqrt{bx^3+ax^2}dx}{10e^2}+\frac{\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5e}\right)}{12b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{6b(ex)^{3/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(12bc-7ad)\left(\frac{3a\left(\frac{a\int\frac{(ex)^{7/2}}{\sqrt{bx^3+ax^2}}dx}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+bx^3}}{4e}\right)}{10e^2}+\frac{\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5e}\right)}{12b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{6b(ex)^{3/2}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{(12bc-7ad)\left(\frac{3a\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\int\frac{(ex)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+bx^3}}{4e}\right)}{10e^2}+\frac{\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5e}\right)}{12b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{6b(ex)^{3/2}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{(12bc-7ad)\left(\frac{3a\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\int\frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+bx^3}}{4e}\right)}{10e^2}+\frac{\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5e}\right)}{12b}-$$

$$\frac{de\left(ax^2+bx^3\right)^{5/2}}{6b(ex)^{3/2}}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{(12bc-7ad)\left(\frac{3a\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int\frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+bx^3}}{4e}\right)}{10e^2}+\sqrt{e}\cdots\right)}{12b}$$

$$\frac{de\left(ax^2+bx^3\right)^{5/2}}{6b(ex)^{3/2}}$$

$\downarrow$ 1937

$$\frac{(12bc-7ad)\left(\frac{3a\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+bx^3}}{4e}\right)}{10e^2}+}{12b}$$

$$\frac{de\left(ax^2+bx^3\right)^{5/2}}{6b(ex)^{3/2}}$$

$\downarrow$ 1935

$$(12bc-7ad)\left(\frac{3a\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2}}{4e}}{10e^2}\right.$$

↓ 219

$$\frac{(12bc-7ad)\left(\frac{3a\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+\cdots}}{4e}\cdots}{10e^2}\cdots\right.}{12b}$$

$$\frac{de\left(ax^2+bx^3\right)^{5/2}}{6b(ex)^{3/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/Sqrt[e*x],x]
```

output
```
(d*e*(a*x^2 + b*x^3)^(5/2))/(6*b*(e*x)^(3/2)) + ((12*b*c - 7*a*d)*((Sqrt[e
*x]*(a*x^2 + b*x^3)^(3/2))/(5*e) + (3*a*(((e*x)^(5/2)*Sqrt[a*x^2 + b*x^3])
/(4*e) + (a*((e^2*(e*x)^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a*e*((e^2*Sq
rt[e*x]*Sqrt[a*x^2 + b*x^3])/(2*b) - (3*a*e*((e^2*Sqrt[a*x^2 + b*x^3])/(b*
Sqrt[e*x]) - (a*e*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]]
)/(b^(3/2)*Sqrt[x])))/(4*b)))/(6*b)))/(8*e^2)))/(10*e^2)))/(12*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.72

method	result
risch	$-\frac{\left(-1280 d\,x^{5} b^{5}-1664 a\,b^{4} d\,x^{4}-1536 b^{5} c\,x^{4}-48 a^{2} b^{3} d\,x^{3}-2112 a\,b^{4} c\,x^{3}+56 a^{3} b^{2} d\,x^{2}-96 a^{2} b^{3} c\,x^{2}-70 a^{4} b d x+120 a^{3} b^{2} c x+105 a^{5} d-1\right.}{7680 b^{4} \sqrt{e x}}$
default	$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}} \left(-2560 b^{5} d\,x^{5} \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}-3328 a\,b^{4} d\,x^{4} \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}-3072 b^{5} c\,x^{4} \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}-96 a^{2} b^{3} d\,x^{3} \sqrt{e x (b x+\right.}{}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/7680/b^4*(-1280*b^5*d*x^5-1664*a*b^4*d*x^4-1536*b^5*c*x^4-48*a^2*b^3*d*
x^3-2112*a*b^4*c*x^3+56*a^3*b^2*d*x^2-96*a^2*b^3*c*x^2-70*a^4*b*d*x+120*a^
3*b^2*c*x+105*a^5*d-180*a^4*b*c)*(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)+1/1024*a^
5/b^4*(7*a*d-12*b*c)*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))
/(b*e)^(1/2)*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.38

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{\sqrt{ex}}\,dx = \left[-\frac{15\left(12\,a^5bc-7\,a^6d\right)\sqrt{be}x\log\left(\frac{2\,bex^2+aex+2\,\sqrt{bx^3+ax^2}\sqrt{be}\sqrt{ex}}{x}\right)-2\left(1280\,b^6d\right.}{}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(1/2),x, algorithm="fricas")
```

output
```
[-1/15360*(15*(12*a^5*b*c - 7*a^6*d)*sqrt(b*e)*x*log((2*b*e*x^2 + a*e*x +
2*sqrt(b*x^3 + a*x^2)*sqrt(b*e)*sqrt(e*x))/x) - 2*(1280*b^6*d*x^5 + 180*a^
4*b^2*c - 105*a^5*b*d + 128*(12*b^6*c + 13*a*b^5*d)*x^4 + 48*(44*a*b^5*c +
 a^2*b^4*d)*x^3 + 8*(12*a^2*b^4*c - 7*a^3*b^3*d)*x^2 - 10*(12*a^3*b^3*c -
7*a^4*b^2*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^5*e*x), 1/7680*(15*(12*a
^5*b*c - 7*a^6*d)*sqrt(-b*e)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b*e)*sqrt(
e*x)/(b*e*x^2 + a*e*x)) + (1280*b^6*d*x^5 + 180*a^4*b^2*c - 105*a^5*b*d +
128*(12*b^6*c + 13*a*b^5*d)*x^4 + 48*(44*a*b^5*c + a^2*b^4*d)*x^3 + 8*(12*
a^2*b^4*c - 7*a^3*b^3*d)*x^2 - 10*(12*a^3*b^3*c - 7*a^4*b^2*d)*x)*sqrt(b*x
^3 + a*x^2)*sqrt(e*x))/(b^5*e*x)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{\sqrt{ex}}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)}{\sqrt{ex}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/(e*x)**(1/2),x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)*(c + d*x)/sqrt(e*x), x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{\sqrt{ex}}\,dx=\int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}(dx+c)}{\sqrt{ex}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(1/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/sqrt(e*x), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.23

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{\sqrt{ex}}\,dx=\frac{\left(\sqrt{(bx+a)be-abe}\left(2\left(4\left(2\,(bx+a)\left(8\,(bx+a)\left(\frac{10\,(bx+a)d\,\mathrm{sgn}(x)}{b^5e}+\frac{12\,b^{21}ce^{4}\cdots}{\cdots}\right.\right.\right.\right.\right.\right.}{\cdots}$$

$$-\frac{\left(12\,a^5bc\log\left(\sqrt{be}\sqrt{a}\right)-7\,a^6d\log\left(\sqrt{be}\sqrt{a}\right)\right)\mathrm{sgn}(x)}{512\,\sqrt{be}eb^3|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(1/2),x, algorithm="giac")
```

output
```
1/7680*(sqrt((b*x + a)*b*e - a*b*e)*(2*(4*(2*(b*x + a)*(8*(b*x + a)*(10*(b
*x + a)*d*sgn(x)/(b^5*e) + (12*b^21*c*e^4*sgn(x) - 37*a*b^20*d*e^4*sgn(x))
/(b^25*e^5)) - 9*(28*a*b^21*c*e^4*sgn(x) - 43*a^2*b^20*d*e^4*sgn(x))/(b^25
*e^5)) + (372*a^2*b^21*c*e^4*sgn(x) - 377*a^3*b^20*d*e^4*sgn(x))/(b^25*e^5
))*(b*x + a) - 5*(12*a^3*b^21*c*e^4*sgn(x) - 7*a^4*b^20*d*e^4*sgn(x))/(b^2
5*e^5))*(b*x + a) - 15*(12*a^4*b^21*c*e^4*sgn(x) - 7*a^5*b^20*d*e^4*sgn(x)
)/(b^25*e^5))*sqrt(b*x + a) + 15*(12*a^5*b*c*sgn(x) - 7*a^6*d*sgn(x))*log(
abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt((b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*b^
4))*b/abs(b) - 1/512*(12*a^5*b*c*log(sqrt(b*e)*sqrt(a)) - 7*a^6*d*log(sqrt
(b*e)*sqrt(a)))*sgn(x)/(sqrt(b*e)*b^3*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{\sqrt{ex}}\,dx=\int \frac{\left(b\,x^3+a\,x^2\right)^{3/2}\,\left(c+d\,x\right)}{\sqrt{e\,x}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(1/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{\sqrt{ex}}\,dx=\frac{\sqrt{e}\left(-105\sqrt{x}\,\sqrt{bx+a}\,a^5bd+180\sqrt{x}\,\sqrt{bx+a}\,a^4b^2c+70\sqrt{x}\,\sqrt{bx+a}\,a^4b^2$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(1/2),x)
```

output
```
(sqrt(e)*( - 105*sqrt(x)*sqrt(a + b*x)*a**5*b*d + 180*sqrt(x)*sqrt(a + b*x
)*a**4*b**2*c + 70*sqrt(x)*sqrt(a + b*x)*a**4*b**2*d*x - 120*sqrt(x)*sqrt(
a + b*x)*a**3*b**3*c*x - 56*sqrt(x)*sqrt(a + b*x)*a**3*b**3*d*x**2 + 96*sq
rt(x)*sqrt(a + b*x)*a**2*b**4*c*x**2 + 48*sqrt(x)*sqrt(a + b*x)*a**2*b**4*
d*x**3 + 2112*sqrt(x)*sqrt(a + b*x)*a*b**5*c*x**3 + 1664*sqrt(x)*sqrt(a +
b*x)*a*b**5*d*x**4 + 1536*sqrt(x)*sqrt(a + b*x)*b**6*c*x**4 + 1280*sqrt(x)
*sqrt(a + b*x)*b**6*d*x**5 + 105*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt
(b))/sqrt(a))*a**6*d - 180*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/s
qrt(a))*a**5*b*c))/(7680*b**5*e)
```

3.312 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 260

$$\begin{aligned}\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{3/2}}\,dx &= -\frac{3a^3(2bc-ad)\sqrt{ax^2+bx^3}}{128b^3e\sqrt{ex}} \\ &+ \frac{a^2(2bc-ad)\sqrt{ex}\sqrt{ax^2+bx^3}}{64b^2e^2} + \frac{3a(2bc-ad)(ex)^{3/2}\sqrt{ax^2+bx^3}}{16be^3} \\ &+ \frac{(2bc-ad)(ex)^{5/2}\sqrt{ax^2+bx^3}}{8e^4} + \frac{de\left(ax^2+bx^3\right)^{5/2}}{5b(ex)^{5/2}} \\ &+ \frac{3a^4(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{128b^{7/2}e^{3/2}}\end{aligned}$$

output
```
-3/128*a^3*(-a*d+2*b*c)*(b*x^3+a*x^2)^(1/2)/b^3/e/(e*x)^(1/2)+1/64*a^2*(-a
*d+2*b*c)*(e*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/b^2/e^2+3/16*a*(-a*d+2*b*c)*(e*x
)^(3/2)*(b*x^3+a*x^2)^(1/2)/b/e^3+1/8*(-a*d+2*b*c)*(e*x)^(5/2)*(b*x^3+a*x^
2)^(1/2)/e^4+1/5*d*e*(b*x^3+a*x^2)^(5/2)/b/(e*x)^(5/2)+3/128*a^4*(-a*d+2*b
*c)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(7/2)/e^(3/
2)
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{3/2}}\,dx = \frac{\left(x^2(a+bx)\right)^{3/2}\left(-30a^3bc+15a^4d+20a^2b^2cx-10a^3bdx+240ab^3cx^2+8a\right.}{640b^3x(ex)^{3/2}(a+bx)}$$
$$-\frac{3a^4(-2bc+ad)\left(x^2(a+bx)\right)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{64b^{7/2}x^{3/2}(ex)^{3/2}(a+bx)^{3/2}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(3/2),x]
```

output
```
((x^2*(a + b*x))^(3/2)*(-30*a^3*b*c + 15*a^4*d + 20*a^2*b^2*c*x - 10*a^3*b
*d*x + 240*a*b^3*c*x^2 + 8*a^2*b^2*d*x^2 + 160*b^4*c*x^3 + 176*a*b^3*d*x^3
 + 128*b^4*d*x^4))/(640*b^3*x*(e*x)^(3/2)*(a + b*x)) - (3*a^4*(-2*b*c + a*
d)*(x^2*(a + b*x))^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*
x])])/(64*b^(7/2)*x^(3/2)*(e*x)^(3/2)*(a + b*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 244, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1945, 1927, 1927, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}(c+dx)}{(ex)^{3/2}}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(2bc-ad)\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(ex)^{3/2}}dx}{2b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{5b(ex)^{5/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(2bc-ad)\left(\frac{3a\int \sqrt{ex}\sqrt{bx^3+ax^2}dx}{8e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4e\sqrt{ex}}\right)}{2b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{5b(ex)^{5/2}}$$

$\downarrow$ 1927

$$\frac{(2bc-ad)\left(\frac{3a\left(\frac{a\int \frac{(ex)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4e\sqrt{ex}}\right)}{2b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{5b(ex)^{5/2}}$$

$\downarrow$ 1930

$$\frac{(2bc-ad)\left(\frac{3a\left(\frac{a\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\int \frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4e\sqrt{ex}}\right)}{2b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{5b(ex)^{5/2}}$$

$\downarrow$ 1930

$$\frac{(2bc-ad)\left(\frac{3a\left(\frac{a\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int\frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4e\sqrt{ex}}\right)}{2b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{5b(ex)^{5/2}}$$

$$\Big\downarrow \; 1937$$

$$\frac{(2bc-ad)\left(\frac{3a\left(\frac{a\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4e\sqrt{ex}}\right)}{2b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{5b(ex)^{5/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{(2bc-ad)\left(\frac{3a\left(\frac{a\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4e\sqrt{ex}}\right)}{2b}+$$

$$\frac{de\left(ax^2+bx^3\right)^{5/2}}{5b(ex)^{5/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(2bc-ad)\left(\frac{3a\left(\frac{a\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8e^2}+\frac{(ax^2+bx^3)^{3/2}}{4e\sqrt{ex}}\right)}{2b}+\frac{de(ax^2+bx^3)^{5/2}}{5b(ex)^{5/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(3/2),x]
```

output
```
(d*e*(a*x^2 + b*x^3)^(5/2))/(5*b*(e*x)^(5/2)) + ((2*b*c - a*d)*((a*x^2 + b
*x^3)^(3/2)/(4*e*Sqrt[e*x]) + (3*a*(((e*x)^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*e
) + (a*((e^2*Sqrt[e*x]*Sqrt[a*x^2 + b*x^3])/(2*b) - (3*a*e*((e^2*Sqrt[a*x^
2 + b*x^3])/(b*Sqrt[e*x]) - (a*e*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[
a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/(4*b)))/(6*e^2)))/(8*e^2)))/(2*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.76

method	result
risch	$\frac{\left(128 d\,x^{4} b^{4}+176 a\,b^{3} d\,x^{3}+160 b^{4} c\,x^{3}+8 a^{2} b^{2} d\,x^{2}+240 a\,b^{3} c\,x^{2}-10 a^{3} b d x+20 a^{2} b^{2} c x+15 a^{4} d-30 a^{3} b c\right) \sqrt{x^{2} \left(b x+a\right)}}{640 b^{3} e \sqrt{e x}}-\frac{3 a^{4} \left(a d-2 b c\right) \mathrm{l}}{}$
default	$\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}} \left(256 b^{4} d\,x^{4} \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}+352 a\,b^{3} d\,x^{3} \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}+320 b^{4} c\,x^{3} \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}+16 a^{2} b^{2} d\,x^{2} \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}+\right.}{}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/640/b^3*(128*b^4*d*x^4+176*a*b^3*d*x^3+160*b^4*c*x^3+8*a^2*b^2*d*x^2+240
*a*b^3*c*x^2-10*a^3*b*d*x+20*a^2*b^2*c*x+15*a^4*d-30*a^3*b*c)/e*(x^2*(b*x+
a))^(1/2)/(e*x)^(1/2)-3/256*a^4/b^3*(a*d-2*b*c)*ln((1/2*a*e+b*e*x)/(b*e)^(
1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)/e*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(e
*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.41

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{3/2}}\,dx=\left[-\frac{15\left(2\,a^4bc-a^5d\right)\sqrt{be}x\log\left(\frac{2\,bex^2+aex-2\,\sqrt{bx^3+ax^2}\sqrt{be}\sqrt{ex}}{x}\right)-2\left(128\,b^5dx^4\right.}{}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(3/2),x, algorithm="fricas")
```

output
```
[-1/1280*(15*(2*a^4*b*c - a^5*d)*sqrt(b*e)*x*log((2*b*e*x^2 + a*e*x - 2*sq
rt(b*x^3 + a*x^2)*sqrt(b*e)*sqrt(e*x))/x) - 2*(128*b^5*d*x^4 - 30*a^3*b^2*
c + 15*a^4*b*d + 16*(10*b^5*c + 11*a*b^4*d)*x^3 + 8*(30*a*b^4*c + a^2*b^3*
d)*x^2 + 10*(2*a^2*b^3*c - a^3*b^2*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b
^4*e^2*x), -1/640*(15*(2*a^4*b*c - a^5*d)*sqrt(-b*e)*x*arctan(sqrt(b*x^3 +
 a*x^2)*sqrt(-b*e)*sqrt(e*x)/(b*e*x^2 + a*e*x)) - (128*b^5*d*x^4 - 30*a^3*
b^2*c + 15*a^4*b*d + 16*(10*b^5*c + 11*a*b^4*d)*x^3 + 8*(30*a*b^4*c + a^2*
b^3*d)*x^2 + 10*(2*a^2*b^3*c - a^3*b^2*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x)
)/(b^4*e^2*x)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{3/2}}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/(e*x)**(3/2),x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)*(c + d*x)/(e*x)**(3/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{3/2}}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}(dx+c)}{(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/(e*x)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{3/2}}\,dx = \frac{\left(\sqrt{(bx+a)be-abe}\left(2\left(4\,(bx+a)\left(2\,(bx+a)\left(\frac{8\,(bx+a)d\mathrm{sgn}(x)}{b^4e}+\frac{10\,b^{13}ce^3\mathrm{sgn}(x)-21\,ab^{12}de^3\mathrm{sgn}(x)}{b^{16}e^4}\right)-\frac{30\,a}{}\right.\right.\right.\right.}{}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(3/2),x, algorithm="giac")
```

output
```
1/640*((sqrt((b*x + a)*b*e - a*b*e)*(2*(4*(b*x + a)*(2*(b*x + a)*(8*(b*x +
 a)*d*sgn(x)/(b^4*e) + (10*b^13*c*e^3*sgn(x) - 21*a*b^12*d*e^3*sgn(x))/(b^
16*e^4)) - (30*a*b^13*c*e^3*sgn(x) - 31*a^2*b^12*d*e^3*sgn(x))/(b^16*e^4))
 + 5*(2*a^2*b^13*c*e^3*sgn(x) - a^3*b^12*d*e^3*sgn(x))/(b^16*e^4))*(b*x +
a) + 15*(2*a^3*b^13*c*e^3*sgn(x) - a^4*b^12*d*e^3*sgn(x))/(b^16*e^4))*sqrt
(b*x + a) - 15*(2*a^4*b*c*sgn(x) - a^5*d*sgn(x))*log(abs(-sqrt(b*e)*sqrt(b
*x + a) + sqrt((b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*b^3))*b/abs(b) + 15*(2*
a^4*b*c*log(sqrt(b*e)*sqrt(a)) - a^5*d*log(sqrt(b*e)*sqrt(a)))*sgn(x)/(sqr
t(b*e)*b^2*abs(b)))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{3/2}}\,dx = \int \frac{\left(b\,x^3+a\,x^2\right)^{3/2}\,(c+d\,x)}{(e\,x)^{3/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(3/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{3/2}}\,dx = \frac{\sqrt{e}\left(15\sqrt{x}\,\sqrt{bx+a}\,a^4bd-30\sqrt{x}\,\sqrt{bx+a}\,a^3b^2c-10\sqrt{x}\,\sqrt{bx+a}\,a^3b^2dx+\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(3/2),x)
```

output
```
(sqrt(e)*(15*sqrt(x)*sqrt(a + b*x)*a**4*b*d - 30*sqrt(x)*sqrt(a + b*x)*a**
3*b**2*c - 10*sqrt(x)*sqrt(a + b*x)*a**3*b**2*d*x + 20*sqrt(x)*sqrt(a + b*
x)*a**2*b**3*c*x + 8*sqrt(x)*sqrt(a + b*x)*a**2*b**3*d*x**2 + 240*sqrt(x)*
sqrt(a + b*x)*a*b**4*c*x**2 + 176*sqrt(x)*sqrt(a + b*x)*a*b**4*d*x**3 + 16
0*sqrt(x)*sqrt(a + b*x)*b**5*c*x**3 + 128*sqrt(x)*sqrt(a + b*x)*b**5*d*x**
4 - 15*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**5*d + 30*
sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*b*c))/(640*b**
4*e**2)
```

3.313 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 216

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{5/2}}\,dx = \frac{a^2(8bc-3ad)\sqrt{ax^2+bx^3}}{64b^2e^2\sqrt{ex}} + \frac{7a(8bc-3ad)\sqrt{ex}\sqrt{ax^2+bx^3}}{96be^3} + \frac{(8bc-3ad)(ex)^{3/2}\sqrt{ax^2+bx^3}}{24e^4} + \frac{de(ax^2+bx^3)^{5/2}}{4b(ex)^{7/2}} - \frac{a^3(8bc-3ad)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{64b^{5/2}e^{5/2}}$$

output

```
1/64*a^2*(-3*a*d+8*b*c)*(b*x^3+a*x^2)^(1/2)/b^2/e^2/(e*x)^(1/2)+7/96*a*(-3
*a*d+8*b*c)*(e*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/b/e^3+1/24*(-3*a*d+8*b*c)*(e*x
)^(3/2)*(b*x^3+a*x^2)^(1/2)/e^4+1/4*d*e*(b*x^3+a*x^2)^(5/2)/b/(e*x)^(7/2)-
1/64*a^3*(-3*a*d+8*b*c)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^
(1/2))/b^(5/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.88

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{5/2}}\,dx = \frac{x^{3/2}\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(-9a^3d+6a^2b(4c+dx)+16b^3x^2(4c+3\right.}{192b^5}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(5/2),x]
```

output
```
(x^(3/2)*Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-9*a^3*d + 6*
a^2*b*(4*c + d*x) + 16*b^3*x^2*(4*c + 3*d*x) + 8*a*b^2*x*(14*c + 9*d*x)) +
 48*a^3*b*c*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])] + 18*a^4*
d*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(192*b^(5/2)*(e*
x)^(5/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1945, 1927, 1927, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}(c+dx)}{(ex)^{5/2}}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(8bc-3ad)\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(ex)^{5/2}}dx}{8b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{4b(ex)^{7/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(8bc-3ad)\left(\frac{a\int \frac{\sqrt{bx^3+ax^2}}{\sqrt{ex}}dx}{2e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{3e(ex)^{3/2}}\right)}{8b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{4b(ex)^{7/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(8bc-3ad)\left(\frac{a\left(\frac{a\int \frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4e^2}+\frac{\sqrt{ex}\sqrt{ax^2+bx^3}}{2e}\right)}{2e^2}+\frac{(ax^2+bx^3)^{3/2}}{3e(ex)^{3/2}}\right)}{8b}+\frac{de(ax^2+bx^3)^{5/2}}{4b(ex)^{7/2}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{(8bc-3ad)\left(\frac{a\left(\frac{a\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4e^2}+\frac{\sqrt{ex}\sqrt{ax^2+bx^3}}{2e}\right)}{2e^2}+\frac{(ax^2+bx^3)^{3/2}}{3e(ex)^{3/2}}\right)}{8b}+\frac{de(ax^2+bx^3)^{5/2}}{4b(ex)^{7/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{(8bc-3ad)\left(\frac{a\left(\frac{a\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4e^2}+\frac{\sqrt{ex}\sqrt{ax^2+bx^3}}{2e}\right)}{2e^2}+\frac{(ax^2+bx^3)^{3/2}}{3e(ex)^{3/2}}\right)}{8b}+\frac{de(ax^2+bx^3)^{5/2}}{4b(ex)^{7/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{(8bc-3ad)\left(\frac{a\left(\frac{a\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4e^2}+\frac{\sqrt{ex}\sqrt{ax^2+bx^3}}{2e}\right)}{2e^2}+\frac{(ax^2+bx^3)^{3/2}}{3e(ex)^{3/2}}\right)}{8b}+\frac{de(ax^2+bx^3)^{5/2}}{4b(ex)^{7/2}}$$

$\downarrow$ 219

$$\frac{(8bc-3ad)\left(\frac{a\left(\frac{a\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4e^2}+\frac{\sqrt{ex}\sqrt{ax^2+bx^3}}{2e}\right)}{2e^2}+\frac{(ax^2+bx^3)^{3/2}}{3e(ex)^{3/2}}\right)}{8b}+\frac{de(ax^2+bx^3)^{5/2}}{4b(ex)^{7/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(5/2),x]
```

output
```
(d*e*(a*x^2 + b*x^3)^(5/2))/(4*b*(e*x)^(7/2)) + ((8*b*c - 3*a*d)*((a*x^2 +
 b*x^3)^(3/2)/(3*e*(e*x)^(3/2)) + (a*((Sqrt[e*x]*Sqrt[a*x^2 + b*x^3])/(2*e
) + (a*((e^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[e*x]) - (a*e*Sqrt[e*x]*ArcTanh[(
Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/(4*e^2)))/(2*e^
2))/(8*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\left(-48 b^{3} d \,x^{3}-72 a \,b^{2} d \,x^{2}-64 b^{3} c \,x^{2}-6 a^{2} b d x -112 a \,b^{2} c x +9 a^{3} d -24 c \,a^{2} b \right) \sqrt{x^{2} \left(b x +a \right)}}{192 b^{2} e^{2} \sqrt{e x}}+\frac{a^{3} \left(3 a d -8 b c \right) \ln \left(\frac{\frac{1}{2} a e +b e x}{\sqrt{b e}}+\sqrt{b e \,x^{2}+a e}\right.}{128 b^{2} \sqrt{b e}\, e^{2} x \left(b x +\right.}$
default	$-\frac{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \left(-96 b^{3} d \,x^{3} \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}-144 a \,b^{2} d \,x^{2} \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}-128 b^{3} c \,x^{2} \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}-9 \ln \left(\frac{2 b e x +2 \sqrt{e x \left(b x +a \right)}\, \sqrt{b}}{2 \sqrt{b e}}\right.\right.}{384}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192/b^2*(-48*b^3*d*x^3-72*a*b^2*d*x^2-64*b^3*c*x^2-6*a^2*b*d*x-112*a*b^
2*c*x+9*a^3*d-24*a^2*b*c)/e^2*(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)+1/128*a^3/b^
2*(3*a*d-8*b*c)*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e
)^(1/2)/e^2*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 320, normalized size of antiderivative = 1.48

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{5/2}}\,dx = \left[-\frac{3\left(8\,a^3bc-3\,a^4d\right)\sqrt{be}x\log\left(\frac{2\,bex^2+aex+2\,\sqrt{bx^3+ax^2}\sqrt{be}\sqrt{ex}}{x}\right)-2\left(48\,b^4dx^3+\right.}{}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/384*(3*(8*a^3*b*c - 3*a^4*d)*sqrt(b*e)*x*log((2*b*e*x^2 + a*e*x + 2*sq
rt(b*x^3 + a*x^2)*sqrt(b*e)*sqrt(e*x))/x) - 2*(48*b^4*d*x^3 + 24*a^2*b^2*c
 - 9*a^3*b*d + 8*(8*b^4*c + 9*a*b^3*d)*x^2 + 2*(56*a*b^3*c + 3*a^2*b^2*d)*
x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^3*e^3*x), 1/192*(3*(8*a^3*b*c - 3*a^4
*d)*sqrt(-b*e)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b*e)*sqrt(e*x)/(b*e*x^2
+ a*e*x)) + (48*b^4*d*x^3 + 24*a^2*b^2*c - 9*a^3*b*d + 8*(8*b^4*c + 9*a*b^
3*d)*x^2 + 2*(56*a*b^3*c + 3*a^2*b^2*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/
(b^3*e^3*x)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{5/2}}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/(e*x)**(5/2),x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)*(c + d*x)/(e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{5/2}}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}(dx+c)}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/(e*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.32

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{5/2}}\,dx = \frac{\left(\sqrt{(bx+a)be-abe}\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{6\,(bx+a)d\mathrm{sgn}(x)}{b^3e}+\frac{8\,b^7ce^2\mathrm{sgn}(x)-9\,a}{b^9e^3}\right.\right.\right.\right.}{} - \frac{\left(8\,a^3bc\log\left(\sqrt{be}\sqrt{a}\right)-3\,a^4d\log\left(\sqrt{be}\sqrt{a}\right)\right)\mathrm{sgn}(x)}{64\sqrt{be}be^2|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
1/192*(sqrt((b*x + a)*b*e - a*b*e)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)*
d*sgn(x)/(b^3*e) + (8*b^7*c*e^2*sgn(x) - 9*a*b^6*d*e^2*sgn(x))/(b^9*e^3))
- (8*a*b^7*c*e^2*sgn(x) - 3*a^2*b^6*d*e^2*sgn(x))/(b^9*e^3)) - 3*(8*a^2*b^
7*c*e^2*sgn(x) - 3*a^3*b^6*d*e^2*sgn(x))/(b^9*e^3))*sqrt(b*x + a) + 3*(8*a
^3*b*c*sgn(x) - 3*a^4*d*sgn(x))*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt((b
*x + a)*b*e - a*b*e)))/(sqrt(b*e)*b^2))*b/(e^2*abs(b)) - 1/64*(8*a^3*b*c*l
og(sqrt(b*e)*sqrt(a)) - 3*a^4*d*log(sqrt(b*e)*sqrt(a)))*sgn(x)/(sqrt(b*e)*
b*e^2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{5/2}}\,dx=\int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}\,\left(c+d\,x\right)}{{\left(e\,x\right)}^{5/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(5/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.84

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{5/2}}\,dx=\frac{\sqrt{e}\,\left(-9\sqrt{x}\,\sqrt{bx+a}\,a^3bd+24\sqrt{x}\,\sqrt{bx+a}\,a^2b^2c+6\sqrt{x}\,\sqrt{bx+a}\,a^2b^2dx+\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(5/2),x)
```

output
```
(sqrt(e)*( - 9*sqrt(x)*sqrt(a + b*x)*a**3*b*d + 24*sqrt(x)*sqrt(a + b*x)*a
**2*b**2*c + 6*sqrt(x)*sqrt(a + b*x)*a**2*b**2*d*x + 112*sqrt(x)*sqrt(a +
b*x)*a*b**3*c*x + 72*sqrt(x)*sqrt(a + b*x)*a*b**3*d*x**2 + 64*sqrt(x)*sqrt
(a + b*x)*b**4*c*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*d*x**3 + 9*sqrt(b)*l
og((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*d - 24*sqrt(b)*log((sqr
t(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*b*c))/(192*b**3*e**3)
```

3.314 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{7/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 172

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{7/2}}\,dx = \frac{5a(6bc-ad)\sqrt{ax^2+bx^3}}{24be^3\sqrt{ex}} + \frac{(6bc-ad)\sqrt{ex}\sqrt{ax^2+bx^3}}{12e^4} + \frac{de\left(ax^2+bx^3\right)^{5/2}}{3b(ex)^{9/2}} + \frac{a^2(6bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{8b^{3/2}e^{7/2}}$$

output
```
5/24*a*(-a*d+6*b*c)*(b*x^3+a*x^2)^(1/2)/b/e^3/(e*x)^(1/2)+1/12*(-a*d+6*b*c
)*(e*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/e^4+1/3*d*e*(b*x^3+a*x^2)^(5/2)/b/(e*x)^
(9/2)+1/8*a^2*(-a*d+6*b*c)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^
2)^(1/2))/b^(3/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.79

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{7/2}}\,dx = \frac{\sqrt{x}\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(3a^2d+4b^2x(3c+2dx)+2ab(15c+7dx\right.}{24b^{3/2}e^2(ex)^{3/2}\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(7/2),x]
```

output
```
(Sqrt[x]*Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(3*a^2*d + 4*b
^2*x*(3*c + 2*d*x) + 2*a*b*(15*c + 7*d*x)) + 3*a^2*(-6*b*c + a*d)*Log[-(Sq
rt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(24*b^(3/2)*e^2*(e*x)^(3/2)*Sqrt[a + b*x
])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 162, normalized size of antiderivative = 0.94, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1945, 1927, 1927, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}(c+dx)}{(ex)^{7/2}}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(6bc-ad)\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(ex)^{7/2}}dx}{6b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{3b(ex)^{9/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(6bc-ad)\left(\frac{3a\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{3/2}}dx}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{6b}+\frac{de\left(ax^2+bx^3\right)^{5/2}}{3b(ex)^{9/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(6bc-ad)\left(\frac{3a\left(\frac{a\int\frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2e^2}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{(ax^2+bx^3)^{3/2}}{2e(ex)^{5/2}}\right)}{6b}+\frac{de(ax^2+bx^3)^{5/2}}{3b(ex)^{9/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{(6bc-ad)\left(\frac{3a\left(\frac{a\sqrt{ex}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{(ax^2+bx^3)^{3/2}}{2e(ex)^{5/2}}\right)}{6b}+\frac{de(ax^2+bx^3)^{5/2}}{3b(ex)^{9/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{(6bc-ad)\left(\frac{3a\left(\frac{a\sqrt{ex}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{(ax^2+bx^3)^{3/2}}{2e(ex)^{5/2}}\right)}{6b}+\frac{de(ax^2+bx^3)^{5/2}}{3b(ex)^{9/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(6bc-ad)\left(\frac{3a\left(\frac{a\sqrt{ex}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{(ax^2+bx^3)^{3/2}}{2e(ex)^{5/2}}\right)}{6b}+\frac{de(ax^2+bx^3)^{5/2}}{3b(ex)^{9/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(7/2),x]
```

output
```
(d*e*(a*x^2 + b*x^3)^(5/2))/(3*b*(e*x)^(9/2)) + ((6*b*c - a*d)*((a*x^2 + b
*x^3)^(3/2)/(2*e*(e*x)^(5/2)) + (3*a*(Sqrt[a*x^2 + b*x^3]/(e*Sqrt[e*x]) +
(a*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(Sqrt[b]*e^2*
Sqrt[x])))/(4*e^2)))/(6*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.87

method	result
risch	$\frac{(8b^2d\,x^2+14abdx+12b^2cx+3a^2d+30abc)\sqrt{x^2(bx+a)}}{24b\,e^3\sqrt{ex}}-\frac{a^2(ad-6bc)\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^2+aex}\right)\sqrt{x^2(bx+a)}\,\sqrt{ex(bx+a)}}{16b\sqrt{be}\,e^3x(bx+a)\sqrt{ex}}$
default	$\frac{(b\,x^3+a\,x^2)^{\frac{3}{2}}\left(16b^2d\,x^2\sqrt{be}\,\sqrt{ex(bx+a)}-3\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)a^3de+18\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)a^2bce+28\sqrt{ex(\right.}{48x^2(bx+a)e^3b\sqrt{ex}\,\sqrt{ex(bx+a)}\,\sqrt{be}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/24/b*(8*b^2*d*x^2+14*a*b*d*x+12*b^2*c*x+3*a^2*d+30*a*b*c)/e^3*(x^2*(b*x+
a))^(1/2)/(e*x)^(1/2)-1/16*a^2/b*(a*d-6*b*c)*ln((1/2*a*e+b*e*x)/(b*e)^(1/2
)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)/e^3*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(e*
x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.59

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{3/2}}{(ex)^{7/2}}\,dx=\left[-\frac{3\,(6\,a^2bc-a^3d)\sqrt{be}x\log\left(\frac{2\,bex^2+aex-2\,\sqrt{bx^3+ax^2}\sqrt{be}\sqrt{ex}}{x}\right)-2\,(8\,b^3dx^2+30}{48\,b^2e^4x}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(7/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(6*a^2*b*c - a^3*d)*sqrt(b*e)*x*log((2*b*e*x^2 + a*e*x - 2*sqrt(
b*x^3 + a*x^2)*sqrt(b*e)*sqrt(e*x))/x) - 2*(8*b^3*d*x^2 + 30*a*b^2*c + 3*a
^2*b*d + 2*(6*b^3*c + 7*a*b^2*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^2*e^
4*x), -1/24*(3*(6*a^2*b*c - a^3*d)*sqrt(-b*e)*x*arctan(sqrt(b*x^3 + a*x^2)
*sqrt(-b*e)*sqrt(e*x)/(b*e*x^2 + a*e*x)) - (8*b^3*d*x^2 + 30*a*b^2*c + 3*a
^2*b*d + 2*(6*b^3*c + 7*a*b^2*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^2*e^
4*x)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{7/2}}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/(e*x)**(7/2),x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)*(c + d*x)/(e*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{7/2}}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}(dx+c)}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/(e*x)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.34

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{7/2}}\,dx = \frac{\left(\sqrt{(bx+a)be-abe}\sqrt{bx+a}\left(2\,(bx+a)\left(\frac{4\,(bx+a)d\mathrm{sgn}(x)}{b^2e}+\frac{6\,b^3ce\mathrm{sgn}(x)-ab^2de\mathrm{sg}}{b^4e^2}\right.\right.\right.}{}$$
$$+\frac{\left(6\,a^2bc\log\left(\sqrt{be}\sqrt{a}\right)-a^3d\log\left(\sqrt{be}\sqrt{a}\right)\right)\mathrm{sgn}(x)}{8\,\sqrt{be}e^3|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(7/2),x, algorithm="giac")
```

output
```
1/24*(sqrt((b*x + a)*b*e - a*b*e)*sqrt(b*x + a)*(2*(b*x + a)*(4*(b*x + a)*
d*sgn(x)/(b^2*e) + (6*b^3*c*e*sgn(x) - a*b^2*d*e*sgn(x))/(b^4*e^2)) + 3*(6
*a*b^3*c*e*sgn(x) - a^2*b^2*d*e*sgn(x))/(b^4*e^2)) - 3*(6*a^2*b*c*sgn(x) -
 a^3*d*sgn(x))*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt((b*x + a)*b*e - a*b
*e)))/(sqrt(b*e)*b))*b/(e^3*abs(b)) + 1/8*(6*a^2*b*c*log(sqrt(b*e)*sqrt(a)
) - a^3*d*log(sqrt(b*e)*sqrt(a)))*sgn(x)/(sqrt(b*e)*e^3*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{7/2}}\,dx=\int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}\,\left(c+d\,x\right)}{{\left(e\,x\right)}^{7/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(7/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.83

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{7/2}}\,dx=\frac{\sqrt{e}\,\Big(3\sqrt{x}\,\sqrt{bx+a}\,a^2bd+30\sqrt{x}\,\sqrt{bx+a}\,a\,b^2c+14\sqrt{x}\,\sqrt{bx+a}\,a\,b^2dx+1$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(7/2),x)
```

output
```
(sqrt(e)*(3*sqrt(x)*sqrt(a + b*x)*a**2*b*d + 30*sqrt(x)*sqrt(a + b*x)*a*b*
*2*c + 14*sqrt(x)*sqrt(a + b*x)*a*b**2*d*x + 12*sqrt(x)*sqrt(a + b*x)*b**3
*c*x + 8*sqrt(x)*sqrt(a + b*x)*b**3*d*x**2 - 3*sqrt(b)*log((sqrt(a + b*x)
+ sqrt(x)*sqrt(b))/sqrt(a))*a**3*d + 18*sqrt(b)*log((sqrt(a + b*x) + sqrt(
x)*sqrt(b))/sqrt(a))*a**2*b*c))/(24*b**2*e**4)
```

3.315 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{9/2}} dx$

Optimal result

Integrand size = 28, antiderivative size = 153

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{9/2}}\,dx=\frac{3(4bc+ad)\sqrt{ax^2+bx^3}}{4e^4\sqrt{ex}}-\frac{2c(ax^2+bx^3)^{3/2}}{e(ex)^{7/2}}+\frac{d(ax^2+bx^3)^{3/2}}{2e^2(ex)^{5/2}}+\frac{3a(4bc+ad)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{4\sqrt{b}e^{9/2}}$$

output
```
3/4*(a*d+4*b*c)*(b*x^3+a*x^2)^(1/2)/e^4/(e*x)^(1/2)-2*c*(b*x^3+a*x^2)^(3/2
)/e/(e*x)^(7/2)+1/2*d*(b*x^3+a*x^2)^(3/2)/e^2/(e*x)^(5/2)+3/4*a*(a*d+4*b*c
)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(1/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{9/2}}\,dx=\frac{\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{a+bx}(2bx(2c+dx)+a(-8c+5dx))+6a(4bc+ad)\cdots\right.}{4\sqrt{b}e^3(ex)^{3/2}\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(9/2),x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[a + b*x]*(2*b*x*(2*c + d*x) + a*(-8*c +
5*d*x)) + 6*a*(4*b*c + a*d)*Sqrt[x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] +
Sqrt[a + b*x])]))/(4*Sqrt[b]*e^3*(e*x)^(3/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1944, 1927, 1927, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}(c+dx)}{(ex)^{9/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(ad+4bc)\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(ex)^{7/2}}dx}{ae}-\frac{2ce\left(ax^2+bx^3\right)^{5/2}}{a(ex)^{11/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(ad+4bc)\left(\frac{3a\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{3/2}}dx}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{ae}-\frac{2ce\left(ax^2+bx^3\right)^{5/2}}{a(ex)^{11/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(ad+4bc)\left(\frac{3a\left(\frac{a\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2e^2}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{ae}-\frac{2ce\left(ax^2+bx^3\right)^{5/2}}{a(ex)^{11/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{(ad+4bc)\left(\frac{3a\left(\frac{a\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{ae}-\frac{2ce\left(ax^2+bx^3\right)^{5/2}}{a(ex)^{11/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{(ad+4bc)\left(\frac{3a\left(\frac{a\sqrt{ex}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{(ax^2+bx^3)^{3/2}}{2e(ex)^{5/2}}\right)}{ae}-\frac{2ce(ax^2+bx^3)^{5/2}}{a(ex)^{11/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{(ad+4bc)\left(\frac{3a\left(\frac{a\sqrt{ex}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{(ax^2+bx^3)^{3/2}}{2e(ex)^{5/2}}\right)}{ae}-\frac{2ce(ax^2+bx^3)^{5/2}}{a(ex)^{11/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(9/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(5/2))/(a*(e*x)^(11/2)) + ((4*b*c + a*d)*((a*x^2 +
 b*x^3)^(3/2)/(2*e*(e*x)^(5/2)) + (3*a*(Sqrt[a*x^2 + b*x^3]/(e*Sqrt[e*x])
+ (a*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(Sqrt[b]*e^
2*Sqrt[x])))/(4*e^2)))/(a*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\left(-2 b d \,x^{2}-5 a d x -4 c b x +8 a c \right) \sqrt{x^{2} \left(b x +a \right)}}{4 e^{4} x \sqrt{e x}}+\frac{3 a \left(a d +4 b c \right) \ln \left(\frac{\frac{1}{2} a e +b e x}{\sqrt{b e}}+\sqrt{b e \,x^{2}+a e x}\right) \sqrt{x^{2} \left(b x +a \right)}\, \sqrt{e x \left(b x +a \right)}}{8 \sqrt{b e}\, e^{4} x \left(b x +a \right) \sqrt{e x}}$
default	$-\frac{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \left(-3 \ln \left(\frac{2 b e x +2 \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}+a e}{2 \sqrt{b e}}\right) a^{2} d e x -12 \ln \left(\frac{2 b e x +2 \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}+a e}{2 \sqrt{b e}}\right) a b c e x -4 \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}\, b d \,x^{2}-10 a}{8 x^{3} \left(b x +a \right) e^{4} \sqrt{e x}\, \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*b*d*x^2-5*a*d*x-4*b*c*x+8*a*c)/e^4*(x^2*(b*x+a))^(1/2)/x/(e*x)^(1
/2)+3/8*a*(a*d+4*b*c)*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2)
)/(b*e)^(1/2)/e^4*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{9/2}}\,dx = \left[\frac{3\left(4\,abc+a^2d\right)\sqrt{be}x^2\log\left(\frac{2\,bex^2+aex+2\,\sqrt{bx^3+ax^2}\sqrt{be}\sqrt{ex}}{x}\right)+2\left(2\,b^2dx^2-8\,ab\right.}{8\,be^5x^2}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(9/2),x, algorithm="fricas")
```

output
```
[1/8*(3*(4*a*b*c + a^2*d)*sqrt(b*e)*x^2*log((2*b*e*x^2 + a*e*x + 2*sqrt(b*
x^3 + a*x^2)*sqrt(b*e)*sqrt(e*x))/x) + 2*(2*b^2*d*x^2 - 8*a*b*c + (4*b^2*c
 + 5*a*b*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b*e^5*x^2), -1/4*(3*(4*a*b*
c + a^2*d)*sqrt(-b*e)*x^2*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b*e)*sqrt(e*x)/
(b*e*x^2 + a*e*x)) - (2*b^2*d*x^2 - 8*a*b*c + (4*b^2*c + 5*a*b*d)*x)*sqrt(
b*x^3 + a*x^2)*sqrt(e*x))/(b*e^5*x^2)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{9/2}}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{3}{2}}(c+dx)}{(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/(e*x)**(9/2),x)
```

output
```
Integral((x**2*(a + b*x))**(3/2)*(c + d*x)/(e*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{9/2}}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}(dx+c)}{(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/(e*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{9/2}}\,dx = \frac{\left(\frac{\left((bx+a)\left(\frac{2\,(bx+a)d\mathrm{sgn}(x)}{b}+\frac{4\,b^2c\mathrm{sgn}(x)+abd\mathrm{sgn}(x)}{b^2}\right)-\frac{3\left(4\,ab^2c\mathrm{sgn}(x)+a^2bd\mathrm{sgn}(x)\right)}{b^2}\right)\sqrt{bx+a}}{\sqrt{(bx+a)be-abe}}-\frac{3\,(4\,a \cdots}{\cdots}\right.}{4\,e^4|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(9/2),x, algorithm="giac")
```

output
```
1/4*(((b*x + a)*(2*(b*x + a)*d*sgn(x)/b + (4*b^2*c*sgn(x) + a*b*d*sgn(x))/
b^2) - 3*(4*a*b^2*c*sgn(x) + a^2*b*d*sgn(x))/b^2)*sqrt(b*x + a)/sqrt((b*x
+ a)*b*e - a*b*e) - 3*(4*a*b*c*sgn(x) + a^2*d*sgn(x))*log(abs(-sqrt(b*e)*s
qrt(b*x + a) + sqrt((b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*b))*b^2/(e^4*abs(b
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{9/2}}\,dx=\int \frac{\left(b\,x^3+a\,x^2\right)^{3/2}\,(c+d\,x)}{(e\,x)^{9/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(9/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.93

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{9/2}}\,dx=\frac{\sqrt{e}\,\Big(-8\sqrt{x}\,\sqrt{bx+a}\,abc+5\sqrt{x}\,\sqrt{bx+a}\,abdx+4\sqrt{x}\,\sqrt{bx+a}\,b^2cx+2\sqrt{x}}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(9/2),x)
```

output
```
(sqrt(e)*( - 8*sqrt(x)*sqrt(a + b*x)*a*b*c + 5*sqrt(x)*sqrt(a + b*x)*a*b*d
*x + 4*sqrt(x)*sqrt(a + b*x)*b**2*c*x + 2*sqrt(x)*sqrt(a + b*x)*b**2*d*x**
2 + 3*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*d*x + 12
*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b*c*x - sqrt(b)*
a**2*d*x - 9*sqrt(b)*a*b*c*x))/(4*b*e**5*x)
```

3.316 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 147

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{11/2}}\,dx = -\frac{2(bc+ad)\sqrt{ax^2+bx^3}}{e^4(ex)^{3/2}}+\frac{bd\sqrt{ax^2+bx^3}}{e^5\sqrt{ex}} - \frac{2c(ax^2+bx^3)^{3/2}}{3e(ex)^{9/2}}+\frac{\sqrt{b}(2bc+3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{e^{11/2}}$$

output
```
-2*(a*d+b*c)*(b*x^3+a*x^2)^(1/2)/e^4/(e*x)^(3/2)+b*d*(b*x^3+a*x^2)^(1/2)/e
^5/(e*x)^(1/2)-2/3*c*(b*x^3+a*x^2)^(3/2)/e/(e*x)^(9/2)+b^(1/2)*(3*a*d+2*b*
c)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/e^(11/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{11/2}}\,dx = \frac{\sqrt{x^2(a+bx)}\left(-\sqrt{a+bx}(bx(8c-3dx)+2a(c+3dx))+6\sqrt{b}(2bc+3ad)x\right.}{3e^3(ex)^{5/2}\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(11/2),x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(-(Sqrt[a + b*x]*(b*x*(8*c - 3*d*x) + 2*a*(c + 3*d*x)
)) + 6*Sqrt[b]*(2*b*c + 3*a*d)*x^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a]
 + Sqrt[a + b*x])]))/(3*e^3*(e*x)^(5/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1944, 1926, 1927, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}(c+dx)}{(ex)^{11/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(3ad+2bc)\int \frac{(bx^3+ax^2)^{3/2}}{(ex)^{9/2}}dx}{3ae}-\frac{2ce(ax^2+bx^3)^{5/2}}{3a(ex)^{13/2}}$$

$$\Big\downarrow \text{1926}$$

$$\frac{(3ad+2bc)\left(\frac{3b\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{3/2}}dx}{e^3}-\frac{2(ax^2+bx^3)^{3/2}}{e(ex)^{7/2}}\right)}{3ae}-\frac{2ce(ax^2+bx^3)^{5/2}}{3a(ex)^{13/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(3ad+2bc)\left(\frac{3b\left(\frac{a\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2e^2}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{e^3}-\frac{2(ax^2+bx^3)^{3/2}}{e(ex)^{7/2}}\right)}{3ae}-\frac{2ce(ax^2+bx^3)^{5/2}}{3a(ex)^{13/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{(3ad+2bc)\left(\frac{3b\left(\frac{a\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{e^3}-\frac{2(ax^2+bx^3)^{3/2}}{e(ex)^{7/2}}\right)}{3ae}-\frac{2ce(ax^2+bx^3)^{5/2}}{3a(ex)^{13/2}}$$

$$\downarrow \text{1935}$$

$$\frac{(3ad+2bc)\left(\frac{3b\left(\frac{a\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{e^3}-\frac{2(ax^2+bx^3)^{3/2}}{e(ex)^{7/2}}\right)}{3ae}-\frac{2ce(ax^2+bx^3)^{5/2}}{3a(ex)^{13/2}}$$

$$\downarrow \text{219}$$

$$\frac{(3ad+2bc)\left(\frac{3b\left(\frac{a\sqrt{ex}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{e^3}-\frac{2(ax^2+bx^3)^{3/2}}{e(ex)^{7/2}}\right)}{3ae}-\frac{2ce(ax^2+bx^3)^{5/2}}{3a(ex)^{13/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(11/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(5/2))/(3*a*(e*x)^(13/2)) + ((2*b*c + 3*a*d)*((-2*
(a*x^2 + b*x^3)^(3/2))/(e*(e*x)^(7/2)) + (3*b*(Sqrt[a*x^2 + b*x^3]/(e*Sqrt
[e*x]) + (a*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(Sqr
t[b]*e^2*Sqrt[x])))/e^3))/(3*a*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.91

method	result
risch	$-\frac{\left(-3bd\,x^{2}+6adx+8cbx+2ac\right)\sqrt{x^{2}\left(bx+a\right)}}{3x^{2}e^{5}\sqrt{ex}}+\frac{b\left(3ad+2bc\right)\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^{2}+aex}\right)\sqrt{x^{2}\left(bx+a\right)}\,\sqrt{ex\left(bx+a\right)}}{2\sqrt{be}\,e^{5}x\left(bx+a\right)\sqrt{ex}}$
default	$\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(9\ln\left(\frac{2bex+2\sqrt{ex\left(bx+a\right)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)abde\,x^{2}+6\ln\left(\frac{2bex+2\sqrt{ex\left(bx+a\right)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)b^{2}ce\,x^{2}+6\sqrt{ex\left(bx+a\right)}\,\sqrt{be}\,bd\,x^{2}-12adx\cdots\right.}{6x^{4}\left(bx+a\right)e^{5}\sqrt{ex}\,\sqrt{ex\left(bx+a\right)}\,\sqrt{be}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(-3*b*d*x^2+6*a*d*x+8*b*c*x+2*a*c)/x^2/e^5*(x^2*(b*x+a))^(1/2)/(e*x)^
(1/2)+1/2*b*(3*a*d+2*b*c)*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(
1/2))/(b*e)^(1/2)/e^5*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e
*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.59

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{11/2}}\,dx = \left[\frac{3\,(2\,bc+3\,ad)ex^3\sqrt{\frac{b}{e}}\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\sqrt{ex}\sqrt{\frac{b}{e}}}{x}\right)+2\,(3\,bdx^2-2\,ac-}{6\,e^6x^3}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(11/2),x, algorithm="fricas")
```

output
```
[1/6*(3*(2*b*c + 3*a*d)*e*x^3*sqrt(b/e)*log((2*b*x^2 + a*x + 2*sqrt(b*x^3
+ a*x^2)*sqrt(e*x)*sqrt(b/e))/x) + 2*(3*b*d*x^2 - 2*a*c - 2*(4*b*c + 3*a*d
)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(e^6*x^3), -1/3*(3*(2*b*c + 3*a*d)*e*x
^3*sqrt(-b/e)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(e*x)*sqrt(-b/e)/(b*x^2 + a*x
)) - (3*b*d*x^2 - 2*a*c - 2*(4*b*c + 3*a*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*
x))/(e^6*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/(e*x)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{3/2}}{(ex)^{11/2}}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}(dx+c)}{(ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(11/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/(e*x)^(11/2), x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{3/2}}{(ex)^{11/2}}\,dx = -\frac{b^3\left(\frac{3\,(2\,bc\mathrm{sgn}(x)+3\,ad\mathrm{sgn}(x))\log\left(\left|-\sqrt{be}\sqrt{bx+a}+\sqrt{(bx+a)be-abe}\right|\right)}{\sqrt{be}b}-\frac{\left(\left(3\,(bx+a)de\mathrm{sgn}(x)-\frac{4\left(2\,ab^2ce^2\mathrm{sgn}(x)+3\,a^2bde^2\mathrm{sgn}(x)\right)}{abe}\right)(bx+a)+}{((bx+a)be-abe)^{\frac{3}{2}}}\right)}{3\,e^5|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(11/2),x, algorithm="giac")
```

output
```
-1/3*b^3*(3*(2*b*c*sgn(x) + 3*a*d*sgn(x))*log(abs(-sqrt(b*e)*sqrt(b*x + a)
 + sqrt((b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*b) - ((3*(b*x + a)*d*e*sgn(x)
- 4*(2*a*b^2*c*e^2*sgn(x) + 3*a^2*b*d*e^2*sgn(x))/(a*b*e))*(b*x + a) + 3*(
2*a^2*b^2*c*e^2*sgn(x) + 3*a^3*b*d*e^2*sgn(x))/(a*b*e))*sqrt(b*x + a)/((b*
x + a)*b*e - a*b*e)^(3/2))/(e^5*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{11/2}}\,dx = \int \frac{\left(b\,x^3+a\,x^2\right)^{3/2}\,(c+d\,x)}{(e\,x)^{11/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(11/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{11/2}}\,dx = \frac{\sqrt{e}\left(-4\sqrt{x}\,\sqrt{bx+a}\,ac-12\sqrt{x}\,\sqrt{bx+a}\,adx-16\sqrt{x}\,\sqrt{bx+a}\,bcx+6\sqrt{x}\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(11/2),x)
```

output
```
(sqrt(e)*( - 4*sqrt(x)*sqrt(a + b*x)*a*c - 12*sqrt(x)*sqrt(a + b*x)*a*d*x
- 16*sqrt(x)*sqrt(a + b*x)*b*c*x + 6*sqrt(x)*sqrt(a + b*x)*b*d*x**2 + 18*s
qrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*d*x**2 + 12*sqrt(b
)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b*c*x**2 + 5*sqrt(b)*a*d*
x**2))/(6*e**6*x**2)
```

3.317 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{13/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 141

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{13/2}}\,dx=-\frac{2ad\sqrt{ax^2+bx^3}}{3e^4(ex)^{5/2}}-\frac{8bd\sqrt{ax^2+bx^3}}{3e^5(ex)^{3/2}}-\frac{2ce(ax^2+bx^3)^{5/2}}{5a(ex)^{15/2}}+\frac{2b^{3/2}d\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{e^{13/2}}$$

output
```
-2/3*a*d*(b*x^3+a*x^2)^(1/2)/e^4/(e*x)^(5/2)-8/3*b*d*(b*x^3+a*x^2)^(1/2)/e
^5/(e*x)^(3/2)-2/5*c*e*(b*x^3+a*x^2)^(5/2)/a/(e*x)^(15/2)+2*b^(3/2)*d*arct
anh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/e^(13/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.89

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{13/2}}\,dx=-\frac{2\sqrt{ex}\sqrt{x^2(a+bx)}\left(\sqrt{a+bx}(3b^2cx^2+a^2(3c+5dx)+2abx(3c+10dx))+15ab^{3/2}dx^{5/2}\log\left(-\sqrt{b}\sqrt{x}+\right.\right.}{15ae^7x^4\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(13/2),x]
```

output
```
(-2*Sqrt[e*x]*Sqrt[x^2*(a + b*x)]*(Sqrt[a + b*x]*(3*b^2*c*x^2 + a^2*(3*c +
 5*d*x) + 2*a*b*x*(3*c + 10*d*x)) + 15*a*b^(3/2)*d*x^(5/2)*Log[-(Sqrt[b]*S
qrt[x]) + Sqrt[a + b*x]]))/(15*a*e^7*x^4*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.06, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1944, 1926, 1926, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}(c+dx)}{(ex)^{13/2}}dx$$

$$\Big\downarrow \ 1944$$

$$\frac{d\int \frac{(bx^3+ax^2)^{3/2}}{(ex)^{11/2}}dx}{e}-\frac{2ce(ax^2+bx^3)^{5/2}}{5a(ex)^{15/2}}$$

$$\Big\downarrow \ 1926$$

$$\frac{d\left(\frac{b\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{5/2}}dx}{e^3}-\frac{2(ax^2+bx^3)^{3/2}}{3e(ex)^{9/2}}\right)}{e}-\frac{2ce(ax^2+bx^3)^{5/2}}{5a(ex)^{15/2}}$$

$$\Big\downarrow \ 1926$$

$$\frac{d\left(\frac{b\left(\frac{b\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{e^3}-\frac{2\sqrt{ax^2+bx^3}}{e(ex)^{3/2}}\right)}{e^3}-\frac{2(ax^2+bx^3)^{3/2}}{3e(ex)^{9/2}}\right)}{e}-\frac{2ce(ax^2+bx^3)^{5/2}}{5a(ex)^{15/2}}$$

$$\Big\downarrow \ 1937$$

$$\frac{d\left(\frac{b\left(\frac{b\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{e^3\sqrt{x}}-\frac{2\sqrt{ax^2+bx^3}}{e(ex)^{3/2}}\right)}{e^3}-\frac{2(ax^2+bx^3)^{3/2}}{3e(ex)^{9/2}}\right)}{e}-\frac{2ce(ax^2+bx^3)^{5/2}}{5a(ex)^{15/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{d\left(\frac{b\left(\frac{2b\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{e^3\sqrt{x}}-\frac{2\sqrt{ax^2+bx^3}}{e(ex)^{3/2}}\right)}{e^3}-\frac{2(ax^2+bx^3)^{3/2}}{3e(ex)^{9/2}}\right)}{e}-\frac{2ce(ax^2+bx^3)^{5/2}}{5a(ex)^{15/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{b\left(\frac{2\sqrt{b}\sqrt{ex}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{e^3\sqrt{x}}-\frac{2\sqrt{ax^2+bx^3}}{e(ex)^{3/2}}\right)}{e^3}-\frac{2(ax^2+bx^3)^{3/2}}{3e(ex)^{9/2}}\right)}{e}-\frac{2ce(ax^2+bx^3)^{5/2}}{5a(ex)^{15/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(13/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(5/2))/(5*a*(e*x)^(15/2)) + (d*((-2*(a*x^2 + b*x^3
)^(3/2))/(3*e*(e*x)^(9/2)) + (b*((-2*Sqrt[a*x^2 + b*x^3])/(e*(e*x)^(3/2))
+ (2*Sqrt[b]*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(e^
3*Sqrt[x])))/e^3))/e
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{2\left(20 a b d \,x^{2}+3 b^{2} c \,x^{2}+5 a^{2} d x +6 a b c x +3 a^{2} c \right) \sqrt{x^{2} \left(b x +a \right)}}{15 x^{3} a \,e^{6} \sqrt{e x}}+\frac{b^{2} d \ln \left(\frac{\frac{1}{2} a e +b e x}{\sqrt{b e}}+\sqrt{b e \,x^{2}+a e x}\right) \sqrt{x^{2} \left(b x +a \right)}\, \sqrt{e x \left(b x +a \right)}}{\sqrt{b e}\, e^{6} x \left(b x +a \right) \sqrt{e x}}$
default	$\frac{\left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \left(15 \ln \left(\frac{2 b e x +2 \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}+a e}{2 \sqrt{b e}}\right) a \,b^{2} d e \,x^{3}-40 a b d \,x^{2} \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}-6 b^{2} c \,x^{2} \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}-10 a^{2} d x \sqrt{e x (b x +}$ $\overline{15 x^{5} \left(b x +a \right) e^{6} a \sqrt{e x}\, \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*(20*a*b*d*x^2+3*b^2*c*x^2+5*a^2*d*x+6*a*b*c*x+3*a^2*c)/x^3/a/e^6*(x^
2*(b*x+a))^(1/2)/(e*x)^(1/2)+b^2*d*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2
+a*e*x)^(1/2))/(b*e)^(1/2)/e^6*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(e*x*(b*x+a))
^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.79

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{13/2}}\,dx = \left[\frac{15\,abdex^4\sqrt{\frac{b}{e}}\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\sqrt{ex}\sqrt{\frac{b}{e}}}{x}\right) - 2\,\sqrt{bx^3+ax^2}(3\,a^2c+(3\,b}{15\,ae^7x^4}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(13/2),x, algorithm="fricas")
```

output
```
[1/15*(15*a*b*d*e*x^4*sqrt(b/e)*log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*x^2)
*sqrt(e*x)*sqrt(b/e))/x) - 2*sqrt(b*x^3 + a*x^2)*(3*a^2*c + (3*b^2*c + 20*
a*b*d)*x^2 + (6*a*b*c + 5*a^2*d)*x)*sqrt(e*x))/(a*e^7*x^4), -2/15*(15*a*b*
d*e*x^4*sqrt(-b/e)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(e*x)*sqrt(-b/e)/(b*x^2
+ a*x)) + sqrt(b*x^3 + a*x^2)*(3*a^2*c + (3*b^2*c + 20*a*b*d)*x^2 + (6*a*b
*c + 5*a^2*d)*x)*sqrt(e*x))/(a*e^7*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{13/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/(e*x)**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{13/2}}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}(dx+c)}{(ex)^{\frac{13}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(13/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/(e*x)^(13/2), x)
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{13/2}}\,dx = -\frac{2\Bigg(\frac{15\,b^2 d\log\left(\left|-\sqrt{be}\sqrt{bx+a}+\sqrt{(bx+a)be-abe}\right|\right)\operatorname{sgn}(x)}{\sqrt{be}} + \frac{\Big(15\,a^2b^4de^2\operatorname{sgn}(x) - \Big(35\,ab^4de^2\operatorname{sgn}(x) - \frac{\left(3\,ab^5ce^2\operatorname{sgn}(x)+20\,a^2b^4de^2\operatorname{sgn}(x)\right)(bx+a)}{a^2}}{((bx+a)be-abe)^{\frac{5}{2}}}}{15\,e^6|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(13/2),x, algorithm="giac")
```

output
```
-2/15*(15*b^2*d*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt((b*x + a)*b*e - a*
b*e)))*sgn(x)/sqrt(b*e) + (15*a^2*b^4*d*e^2*sgn(x) - (35*a*b^4*d*e^2*sgn(x
) - (3*a*b^5*c*e^2*sgn(x) + 20*a^2*b^4*d*e^2*sgn(x))*(b*x + a)/a^2)*(b*x +
 a))*sqrt(b*x + a)/((b*x + a)*b*e - a*b*e)^(5/2))*b/(e^6*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{13/2}}\,dx=\int \frac{{\left(b\,x^3+a\,x^2\right)}^{3/2}\,\left(c+d\,x\right)}{{\left(e\,x\right)}^{13/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(13/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(13/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.99

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{13/2}}\,dx=\frac{2\sqrt{e}\left(-3\sqrt{x}\,\sqrt{bx+a}\,a^2c-5\sqrt{x}\,\sqrt{bx+a}\,a^2dx-6\sqrt{x}\,\sqrt{bx+a}\,abcx-20\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(13/2),x)
```

output
```
(2*sqrt(e)*( - 3*sqrt(x)*sqrt(a + b*x)*a**2*c - 5*sqrt(x)*sqrt(a + b*x)*a*
*2*d*x - 6*sqrt(x)*sqrt(a + b*x)*a*b*c*x - 20*sqrt(x)*sqrt(a + b*x)*a*b*d*
x**2 - 3*sqrt(x)*sqrt(a + b*x)*b**2*c*x**2 + 15*sqrt(b)*log((sqrt(a + b*x)
 + sqrt(x)*sqrt(b))/sqrt(a))*a*b*d*x**3 + 8*sqrt(b)*a*b*d*x**3 - 3*sqrt(b)
*b**2*c*x**3))/(15*a*e**7*x**3)
```

3.318 $\int \frac{(c+dx)(ax^2+bx^3)^{3/2}}{(ex)^{15/2}} dx$

Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{15/2}} dx = -\frac{2ce(ax^2+bx^3)^{5/2}}{7a(ex)^{17/2}} + \frac{2(2bc-7ad)\left(ax^2+bx^3\right)^{5/2}}{35a^2(ex)^{15/2}}$$

output
```
-2/7*c*e*(b*x^3+a*x^2)^(5/2)/a/(e*x)^(17/2)+2/35*(-7*a*d+2*b*c)*(b*x^3+a*x
^2)^(5/2)/a^2/(e*x)^(15/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{15/2}} dx = -\frac{2e\left(x^2(a+bx)\right)^{5/2}(5ac-2bcx+7adx)}{35a^2(ex)^{17/2}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(15/2),x]
```

output
```
(-2*e*(x^2*(a + b*x))^(5/2)*(5*a*c - 2*b*c*x + 7*a*d*x))/(35*a^2*(e*x)^(17
/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {1944, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}(c+dx)}{(ex)^{15/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(2bc-7ad)\int \frac{(bx^3+ax^2)^{3/2}}{(ex)^{13/2}}dx}{7ae}-\frac{2ce(ax^2+bx^3)^{5/2}}{7a(ex)^{17/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2(ax^2+bx^3)^{5/2}(2bc-7ad)}{35a^2(ex)^{15/2}}-\frac{2ce(ax^2+bx^3)^{5/2}}{7a(ex)^{17/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(15/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(5/2))/(7*a*(e*x)^(17/2)) + (2*(2*b*c - 7*a*d)*(a*
x^2 + b*x^3)^(5/2))/(35*a^2*(e*x)^(15/2))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{2 x (b x+a) (7 a d x-2 c b x+5 a c) \left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}}}{35 a^{2} (e x)^{\frac{15}{2}}}$	45
orering	$-\frac{2 x (b x+a) (7 a d x-2 c b x+5 a c) \left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}}}{35 a^{2} (e x)^{\frac{15}{2}}}$	45
default	$-\frac{2 \left(b \,x^{3}+a \,x^{2}\right)^{\frac{3}{2}} \left(7 a b d \,x^{2}-2 b^{2} c \,x^{2}+7 a^{2} d x+3 a b c x+5 a^{2} c \right)}{35 x^{6} a^{2} e^{7} \sqrt{e x}}$	67
risch	$-\frac{2 \sqrt{x^{2} (b x+a)}\, \left(7 a \,b^{2} d \,x^{3}-2 b^{3} c \,x^{3}+14 a^{2} b d \,x^{2}+a \,b^{2} c \,x^{2}+7 a^{3} d x+8 a^{2} b c x+5 c \,a^{3}\right)}{35 e^{7} x^{4} \sqrt{e x}\, a^{2}}$	88

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*x*(b*x+a)*(7*a*d*x-2*b*c*x+5*a*c)*(b*x^3+a*x^2)^(3/2)/a^2/(e*x)^(15/
2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{15/2}}\,dx = -\frac{2\,(5\,a^3c-(2\,b^3c-7\,ab^2d)x^3+(ab^2c+14\,a^2bd)x^2+(8\,a^2bc+7\,a^3d)x)\sqrt{bx^3+ax^2}\sqrt{ex}}{35\,a^2e^8x^5}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(15/2),x, algorithm="fricas")
```

output
```
-2/35*(5*a^3*c - (2*b^3*c - 7*a*b^2*d)*x^3 + (a*b^2*c + 14*a^2*b*d)*x^2 +
(8*a^2*b*c + 7*a^3*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x)/(a^2*e^8*x^5)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/(e*x)**(15/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{15/2}}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}(dx+c)}{(ex)^{\frac{15}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(15/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/(e*x)^(15/2), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{15/2}}\,dx = \frac{2\,(bx+a)^{\frac{5}{2}}b^5\left(\frac{\left(2\,ab^3ce^3\mathrm{sgn}(x)-7\,a^2b^2de^3\mathrm{sgn}(x)\right)(bx+a)}{a^3}-\frac{7\,\left(a^2b^3ce^3\mathrm{sgn}(x)-a^3b^2de^3\mathrm{sgn}(x)\right)}{a^3}\right)}{35\,((bx+a)be-abe)^{\frac{7}{2}}e^7|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(15/2),x, algorithm="giac")
```

output
```
2/35*(b*x + a)^(5/2)*b^5*((2*a*b^3*c*e^3*sgn(x) - 7*a^2*b^2*d*e^3*sgn(x))*
(b*x + a)/a^3 - 7*(a^2*b^3*c*e^3*sgn(x) - a^3*b^2*d*e^3*sgn(x))/a^3)/(((b*
x + a)*b*e - a*b*e)^(7/2)*e^7*abs(b))
```

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.41

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{15/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,a\,c}{7\,e^7}+\frac{x\,\left(14\,d\,a^3+16\,b\,c\,a^2\right)}{35\,a^2\,e^7}-\frac{x^3\,\left(4\,b^3\,c-14\,a\,b^2\,d\right)}{35\,a^2\,e^7}+\frac{2\,b\,x^2\,\left(14\,a\,d+b\,c\right)}{35\,a\,e^7}\right)}{x^4\,\sqrt{e\,x}}$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(15/2),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*a*c)/(7*e^7) + (x*(14*a^3*d + 16*a^2*b*c))/(35
*a^2*e^7) - (x^3*(4*b^3*c - 14*a*b^2*d))/(35*a^2*e^7) + (2*b*x^2*(14*a*d +
 b*c))/(35*a*e^7)))/(x^4*(e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.19

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{15/2}}\,dx = \frac{2\sqrt{e}\left(-5\sqrt{x}\,\sqrt{bx+a}\,a^3c-7\sqrt{x}\,\sqrt{bx+a}\,a^3dx-8\sqrt{x}\,\sqrt{bx+a}\,a^2bcx-14\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(15/2),x)
```

output
```
(2*sqrt(e)*( - 5*sqrt(x)*sqrt(a + b*x)*a**3*c - 7*sqrt(x)*sqrt(a + b*x)*a*
*3*d*x - 8*sqrt(x)*sqrt(a + b*x)*a**2*b*c*x - 14*sqrt(x)*sqrt(a + b*x)*a**
2*b*d*x**2 - sqrt(x)*sqrt(a + b*x)*a*b**2*c*x**2 - 7*sqrt(x)*sqrt(a + b*x)
*a*b**2*d*x**3 + 2*sqrt(x)*sqrt(a + b*x)*b**3*c*x**3 - 3*sqrt(b)*a*b**2*d*
x**4 - 2*sqrt(b)*b**3*c*x**4))/(35*a**2*e**8*x**4)
```

3.319 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{17/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 112

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{17/2}}\,dx = -\frac{2ce\left(ax^2+bx^3\right)^{5/2}}{9a(ex)^{19/2}} + \frac{2(4bc-9ad)\left(ax^2+bx^3\right)^{5/2}}{63a^2(ex)^{17/2}} - \frac{4b(4bc-9ad)\left(ax^2+bx^3\right)^{5/2}}{315a^3e(ex)^{15/2}}$$

output
```
-2/9*c*e*(b*x^3+a*x^2)^(5/2)/a/(e*x)^(19/2)+2/63*(-9*a*d+4*b*c)*(b*x^3+a*x
^2)^(5/2)/a^2/(e*x)^(17/2)-4/315*b*(-9*a*d+4*b*c)*(b*x^3+a*x^2)^(5/2)/a^3/
e/(e*x)^(15/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.57

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{17/2}}\,dx = -\frac{2e\left(x^2(a+bx)\right)^{5/2}\left(8b^2cx^2+5a^2(7c+9dx)-2abx(10c+9dx)\right)}{315a^3(ex)^{19/2}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(17/2),x]
```

output
```
(-2*e*(x^2*(a + b*x))^(5/2)*(8*b^2*c*x^2 + 5*a^2*(7*c + 9*d*x) - 2*a*b*x*(
10*c + 9*d*x)))/(315*a^3*(e*x)^(19/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1944, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{3/2}(c+dx)}{(ex)^{17/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(4bc-9ad)\int \frac{(bx^3+ax^2)^{3/2}}{(ex)^{15/2}}dx}{9ae}-\frac{2ce(ax^2+bx^3)^{5/2}}{9a(ex)^{19/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{(4bc-9ad)\left(-\frac{2b\int \frac{(bx^3+ax^2)^{3/2}}{(ex)^{13/2}}dx}{7ae}-\frac{2e(ax^2+bx^3)^{5/2}}{7a(ex)^{17/2}}\right)}{9ae}-\frac{2ce(ax^2+bx^3)^{5/2}}{9a(ex)^{19/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{(4bc-9ad)\left(\frac{4b(ax^2+bx^3)^{5/2}}{35a^2(ex)^{15/2}}-\frac{2e(ax^2+bx^3)^{5/2}}{7a(ex)^{17/2}}\right)}{9ae}-\frac{2ce(ax^2+bx^3)^{5/2}}{9a(ex)^{19/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(17/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(5/2))/(9*a*(e*x)^(19/2)) - ((4*b*c - 9*a*d)*((-2*
e*(a*x^2 + b*x^3)^(5/2))/(7*a*(e*x)^(17/2)) + (4*b*(a*x^2 + b*x^3)^(5/2))/
(35*a^2*(e*x)^(15/2))))/(9*a*e)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2x(bx+a)\left(-18abd\,x^{2}+8b^{2}c\,x^{2}+45a^{2}dx-20abcx+35a^{2}c\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{315a^{3}\left(ex\right)^{\frac{17}{2}}}$	67
orering	$-\frac{2x(bx+a)\left(-18abd\,x^{2}+8b^{2}c\,x^{2}+45a^{2}dx-20abcx+35a^{2}c\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{315a^{3}\left(ex\right)^{\frac{17}{2}}}$	67
default	$-\frac{2\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(-18a\,b^{2}d\,x^{3}+8b^{3}c\,x^{3}+27a^{2}bd\,x^{2}-12a\,b^{2}c\,x^{2}+45a^{3}dx+15a^{2}bcx+35c\,a^{3}\right)}{315x^{7}a^{3}e^{8}\sqrt{ex}}$	91
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\left(-18x^{4}a\,b^{3}d+8x^{4}b^{4}c+9a^{2}b^{2}d\,x^{3}-4a\,b^{3}c\,x^{3}+72a^{3}bd\,x^{2}+3a^{2}b^{2}c\,x^{2}+45a^{4}dx+50a^{3}bcx+35c\,a^{4}\right)}{315e^{8}x^{5}\sqrt{ex}\,a^{3}}$	113

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(17/2),x,method=_RETURNVERBOSE)
```

output
```
-2/315*x*(b*x+a)*(-18*a*b*d*x^2+8*b^2*c*x^2+45*a^2*d*x-20*a*b*c*x+35*a^2*c
)*(b*x^3+a*x^2)^(3/2)/a^3/(e*x)^(17/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{17/2}}\,dx = -\frac{2\left(35\,a^4c+2\left(4\,b^4c-9\,ab^3d\right)x^4-\left(4\,ab^3c-9\,a^2b^2d\right)x^3+3\left(a^2b^2c+24\,a^3bd\right)x^2+5\left(10\,a^3bc+9\,a^4d\right)x\right)\sqrt{b}}{315\,a^3e^9x^6}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(17/2),x, algorithm="fricas")
```

output
```
-2/315*(35*a^4*c + 2*(4*b^4*c - 9*a*b^3*d)*x^4 - (4*a*b^3*c - 9*a^2*b^2*d)
*x^3 + 3*(a^2*b^2*c + 24*a^3*b*d)*x^2 + 5*(10*a^3*b*c + 9*a^4*d)*x)*sqrt(b
*x^3 + a*x^2)*sqrt(e*x)/(a^3*e^9*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/(e*x)**(17/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{17/2}}\,dx = \int \frac{(bx^3+ax^2)^{\frac{3}{2}}(dx+c)}{(ex)^{\frac{17}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(17/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/(e*x)^(17/2), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{17/2}}\,dx = -\frac{2\,(bx+a)^{\frac{5}{2}}\left((bx+a)\left(\frac{2\,(4\,ab^9ce^4\mathrm{sgn}(x)-9\,a^2b^8de^4\mathrm{sgn}(x))(bx+a)}{a^4}-\frac{9\,(4\,a^2b^9ce^4\mathrm{sgn}(x)-9\,a^3b^8de^4\mathrm{sgn}(x))}{a^4}\right)+\frac{63\,(a^3b^9ce^4\mathrm{sgn}(x)-}{a^4}\right.}{315\,((bx+a)be-abe)^{\frac{9}{2}}e^8|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(17/2),x, algorithm="giac")
```

output
```
-2/315*(b*x + a)^(5/2)*((b*x + a)*(2*(4*a*b^9*c*e^4*sgn(x) - 9*a^2*b^8*d*e
^4*sgn(x))*(b*x + a)/a^4 - 9*(4*a^2*b^9*c*e^4*sgn(x) - 9*a^3*b^8*d*e^4*sgn
(x))/a^4) + 63*(a^3*b^9*c*e^4*sgn(x) - a^4*b^8*d*e^4*sgn(x))/a^4)*b/(((b*x
 + a)*b*e - a*b*e)^(9/2)*e^8*abs(b))
```

Mupad [B] (verification not implemented)

Time = 9.38 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.09

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{17/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,a\,c}{9\,e^8}+\frac{x\,\left(90\,d\,a^4+100\,b\,c\,a^3\right)}{315\,a^3\,e^8}+\frac{x^4\,\left(16\,b^4\,c-36\,a\,b^3\,d\right)}{315\,a^3\,e^8}+\frac{2\,b^2\,x^3\,\left(9\,a\,d-4\,b\,c\right)}{315\,a^2\,e^8}+\frac{2\,b\,x^2\,\left(24\,a\,d+b\,c\right)}{105\,a\,e^8}\right)}{x^5\,\sqrt{e\,x}}$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(17/2),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*a*c)/(9*e^8) + (x*(90*a^4*d + 100*a^3*b*c))/(3
15*a^3*e^8) + (x^4*(16*b^4*c - 36*a*b^3*d))/(315*a^3*e^8) + (2*b^2*x^3*(9*
a*d - 4*b*c))/(315*a^2*e^8) + (2*b*x^2*(24*a*d + b*c))/(105*a*e^8)))/(x^5*
(e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.72

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{17/2}}\,dx = \frac{2\sqrt{e}\left(-35\sqrt{x}\,\sqrt{bx+a}\,a^4c-45\sqrt{x}\,\sqrt{bx+a}\,a^4dx-50\sqrt{x}\,\sqrt{bx+a}\,a^3bcx-\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(17/2),x)
```

output
```
(2*sqrt(e)*( - 35*sqrt(x)*sqrt(a + b*x)*a**4*c - 45*sqrt(x)*sqrt(a + b*x)*
a**4*d*x - 50*sqrt(x)*sqrt(a + b*x)*a**3*b*c*x - 72*sqrt(x)*sqrt(a + b*x)*
a**3*b*d*x**2 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c*x**2 - 9*sqrt(x)*sqrt(
a + b*x)*a**2*b**2*d*x**3 + 4*sqrt(x)*sqrt(a + b*x)*a*b**3*c*x**3 + 18*sqr
t(x)*sqrt(a + b*x)*a*b**3*d*x**4 - 8*sqrt(x)*sqrt(a + b*x)*b**4*c*x**4 - 1
8*sqrt(b)*a*b**3*d*x**5 + 8*sqrt(b)*b**4*c*x**5))/(315*a**3*e**9*x**5)
```

3.320 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{19/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{19/2}}\,dx=-\frac{2ce\left(ax^2+bx^3\right)^{5/2}}{11a(ex)^{21/2}}+\frac{2(6bc-11ad)\left(ax^2+bx^3\right)^{5/2}}{99a^2(ex)^{19/2}}-\frac{8b(6bc-11ad)\left(ax^2+bx^3\right)^{5/2}}{693a^3e(ex)^{17/2}}+\frac{16b^2(6bc-11ad)\left(ax^2+bx^3\right)^{5/2}}{3465a^4e^2(ex)^{15/2}}$$

output

```
-2/11*c*e*(b*x^3+a*x^2)^(5/2)/a/(e*x)^(21/2)+2/99*(-11*a*d+6*b*c)*(b*x^3+a
*x^2)^(5/2)/a^2/(e*x)^(19/2)-8/693*b*(-11*a*d+6*b*c)*(b*x^3+a*x^2)^(5/2)/a
^3/e/(e*x)^(17/2)+16/3465*b^2*(-11*a*d+6*b*c)*(b*x^3+a*x^2)^(5/2)/a^4/e^2/
(e*x)^(15/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.56

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{19/2}}\,dx = -\frac{2\left(x^2(a+bx)\right)^{5/2}\left(-48b^3cx^3+35a^3(9c+11dx)+8ab^2x^2(15c+11dx)-10a^2bx(21c+22dx)\right)}{3465a^4e^8x^9(ex)^{3/2}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(19/2),x]
```

output
```
(-2*(x^2*(a + b*x))^(5/2)*(-48*b^3*c*x^3 + 35*a^3*(9*c + 11*d*x) + 8*a*b^2
*x^2*(15*c + 11*d*x) - 10*a^2*b*x*(21*c + 22*d*x)))/(3465*a^4*e^8*x^9*(e*x
)^(3/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1944, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{3/2}(c+dx)}{(ex)^{19/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(6bc-11ad)\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(ex)^{17/2}}dx}{11ae}-\frac{2ce\left(ax^2+bx^3\right)^{5/2}}{11a(ex)^{21/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{(6bc-11ad)\left(-\frac{4b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(ex)^{15/2}}dx}{9ae}-\frac{2e\left(ax^2+bx^3\right)^{5/2}}{9a(ex)^{19/2}}\right)}{11ae}-\frac{2ce\left(ax^2+bx^3\right)^{5/2}}{11a(ex)^{21/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{(6bc-11ad)\left(-\frac{4b\left(-\frac{2b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(ex)^{13/2}}dx}{7ae}-\frac{2e\left(ax^2+bx^3\right)^{5/2}}{7a(ex)^{17/2}}\right)}{9ae}-\frac{2e\left(ax^2+bx^3\right)^{5/2}}{9a(ex)^{19/2}}\right)}{11ae}-\frac{2ce\left(ax^2+bx^3\right)^{5/2}}{11a(ex)^{21/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{(6bc-11ad)\left(-\frac{4b\left(\frac{4b\left(ax^2+bx^3\right)^{5/2}}{35a^2(ex)^{15/2}}-\frac{2e\left(ax^2+bx^3\right)^{5/2}}{7a(ex)^{17/2}}\right)}{9ae}-\frac{2e\left(ax^2+bx^3\right)^{5/2}}{9a(ex)^{19/2}}\right)}{11ae}-\frac{2ce\left(ax^2+bx^3\right)^{5/2}}{11a(ex)^{21/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(3/2))/(e*x)^(19/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(5/2))/(11*a*(e*x)^(21/2)) - ((6*b*c - 11*a*d)*((-
2*e*(a*x^2 + b*x^3)^(5/2))/(9*a*(e*x)^(19/2)) - (4*b*((-2*e*(a*x^2 + b*x^3
)^(5/2))/(7*a*(e*x)^(17/2)) + (4*b*(a*x^2 + b*x^3)^(5/2))/(35*a^2*(e*x)^(1
5/2))))/(9*a*e)))/(11*a*e)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.58

method	result
gosper	$-\frac{2x(bx+a)\left(88a\,b^{2}d\,x^{3}-48b^{3}c\,x^{3}-220a^{2}bd\,x^{2}+120a\,b^{2}c\,x^{2}+385a^{3}dx-210a^{2}bcx+315c\,a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{3465a^{4}(ex)^{\frac{19}{2}}}$
orering	$-\frac{2x(bx+a)\left(88a\,b^{2}d\,x^{3}-48b^{3}c\,x^{3}-220a^{2}bd\,x^{2}+120a\,b^{2}c\,x^{2}+385a^{3}dx-210a^{2}bcx+315c\,a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}{3465a^{4}(ex)^{\frac{19}{2}}}$
default	$-\frac{2\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\left(88x^{4}a\,b^{3}d-48x^{4}b^{4}c-132a^{2}b^{2}d\,x^{3}+72a\,b^{3}c\,x^{3}+165a^{3}bd\,x^{2}-90a^{2}b^{2}c\,x^{2}+385a^{4}dx+105a^{3}bcx+315c\,a^{4}\right)}{3465x^{8}a^{4}e^{9}\sqrt{ex}}$
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,(88a\,b^{4}d\,x^{5}-48b^{5}c\,x^{5}-44x^{4}a^{2}b^{3}d+24x^{4}a\,b^{4}c+33a^{3}b^{2}d\,x^{3}-18a^{2}b^{3}c\,x^{3}+550a^{4}bd\,x^{2}+15a^{3}b^{2}c\,x^{2}+385a^{5}dx+420a}{3465e^{9}x^{6}\sqrt{ex}\,a^{4}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(19/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3465*x*(b*x+a)*(88*a*b^2*d*x^3-48*b^3*c*x^3-220*a^2*b*d*x^2+120*a*b^2*c
*x^2+385*a^3*d*x-210*a^2*b*c*x+315*a^3*c)*(b*x^3+a*x^2)^(3/2)/a^4/(e*x)^(1
9/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.90

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{19/2}}\,dx = -\frac{2\left(315\,a^5c-8\left(6\,b^5c-11\,ab^4d\right)x^5+4\left(6\,ab^4c-11\,a^2b^3d\right)x^4-3\left(6\,a^2b^3c-11\,a^3b^2d\right)x^3+5\left(3\,a^3b^2c+110\right.\right.}{3465\,a^4e^{10}x^7}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(19/2),x, algorithm="fricas")
```

output
```
-2/3465*(315*a^5*c - 8*(6*b^5*c - 11*a*b^4*d)*x^5 + 4*(6*a*b^4*c - 11*a^2*
b^3*d)*x^4 - 3*(6*a^2*b^3*c - 11*a^3*b^2*d)*x^3 + 5*(3*a^3*b^2*c + 110*a^4
*b*d)*x^2 + 35*(12*a^4*b*c + 11*a^5*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x)/(a
^4*e^10*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{19/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(3/2)/(e*x)**(19/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{19/2}}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{3}{2}}(dx+c)}{(ex)^{\frac{19}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(19/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(3/2)*(d*x + c)/(e*x)^(19/2), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{19/2}}\,dx = \frac{2\left((bx+a)\left(4\,(bx+a)\left(\frac{2\,(6\,ab^5ce^5\mathrm{sgn}(x)-11\,a^2b^4de^5\mathrm{sgn}(x))(bx+a)}{a^5}-\frac{11\,(6\,a^2b^5ce^5\mathrm{sgn}(x)\right.\right.\right.}{3465}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(19/2),x, algorithm="giac")
```

output
```
2/3465*((b*x + a)*(4*(b*x + a)*(2*(6*a*b^5*c*e^5*sgn(x) - 11*a^2*b^4*d*e^5
*sgn(x))*(b*x + a)/a^5 - 11*(6*a^2*b^5*c*e^5*sgn(x) - 11*a^3*b^4*d*e^5*sgn
(x))/a^5) + 99*(6*a^3*b^5*c*e^5*sgn(x) - 11*a^4*b^4*d*e^5*sgn(x))/a^5) - 6
93*(a^4*b^5*c*e^5*sgn(x) - a^5*b^4*d*e^5*sgn(x))/a^5)*(b*x + a)^(5/2)*b^7/
(((b*x + a)*b*e - a*b*e)^(11/2)*e^9*abs(b))
```

Mupad [B] (verification not implemented)

Time = 9.29 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.94

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{19/2}}\,dx =$$

$$-\frac{\sqrt{b\,x^3+a\,x^2}\left(\frac{2\,a\,c}{11\,e^9}+\frac{x\,\left(770\,d\,a^5+840\,b\,c\,a^4\right)}{3465\,a^4\,e^9}-\frac{x^5\,\left(96\,b^5\,c-176\,a\,b^4\,d\right)}{3465\,a^4\,e^9}+\frac{2\,b^2\,x^3\,\left(11\,a\,d-6\,b\,c\right)}{1155\,a^2\,e^9}-\frac{8\,b^3\,x^4\,\left(11\,a\,d-6\,b\,c\right)}{3465\,a^3\,e^9}+\frac{2\,b\,x^2\,(11\right.}{x^6\,\sqrt{e\,x}}$$

input
```
int(((a*x^2 + b*x^3)^(3/2)*(c + d*x))/(e*x)^(19/2),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*a*c)/(11*e^9) + (x*(770*a^5*d + 840*a^4*b*c))/
(3465*a^4*e^9) - (x^5*(96*b^5*c - 176*a*b^4*d))/(3465*a^4*e^9) + (2*b^2*x^
3*(11*a*d - 6*b*c))/(1155*a^2*e^9) - (8*b^3*x^4*(11*a*d - 6*b*c))/(3465*a^
3*e^9) + (2*b*x^2*(110*a*d + 3*b*c))/(693*a*e^9)))/(x^6*(e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.49

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{3/2}}{(ex)^{19/2}}\,dx = \frac{2\sqrt{e}\left(-315\sqrt{x}\,\sqrt{bx+a}\,a^5c-385\sqrt{x}\,\sqrt{bx+a}\,a^5dx-420\sqrt{x}\,\sqrt{bx+a}\,a^4bc\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(3/2)/(e*x)^(19/2),x)
```

output
```
(2*sqrt(e)*( - 315*sqrt(x)*sqrt(a + b*x)*a**5*c - 385*sqrt(x)*sqrt(a + b*x
)*a**5*d*x - 420*sqrt(x)*sqrt(a + b*x)*a**4*b*c*x - 550*sqrt(x)*sqrt(a + b
*x)*a**4*b*d*x**2 - 15*sqrt(x)*sqrt(a + b*x)*a**3*b**2*c*x**2 - 33*sqrt(x)
*sqrt(a + b*x)*a**3*b**2*d*x**3 + 18*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c*x**
3 + 44*sqrt(x)*sqrt(a + b*x)*a**2*b**3*d*x**4 - 24*sqrt(x)*sqrt(a + b*x)*a
*b**4*c*x**4 - 88*sqrt(x)*sqrt(a + b*x)*a*b**4*d*x**5 + 48*sqrt(x)*sqrt(a
 + b*x)*b**5*c*x**5 + 88*sqrt(b)*a*b**4*d*x**6 - 48*sqrt(b)*b**5*c*x**6))/(
3465*a**4*e**10*x**6)
```

3.321 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 346

$$\begin{aligned}\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{5/2}}\,dx &= \frac{5a^5(2bc-ad)\sqrt{ax^2+bx^3}}{1024b^4e^2\sqrt{ex}} \\ &- \frac{5a^4(2bc-ad)\sqrt{ex}\sqrt{ax^2+bx^3}}{1536b^3e^3} + \frac{a^3(2bc-ad)(ex)^{3/2}\sqrt{ax^2+bx^3}}{384b^2e^4} \\ &+ \frac{9a^2(2bc-ad)(ex)^{5/2}\sqrt{ax^2+bx^3}}{64be^5} + \frac{5a(2bc-ad)(ex)^{7/2}\sqrt{ax^2+bx^3}}{24e^6} \\ &+ \frac{b(2bc-ad)(ex)^{9/2}\sqrt{ax^2+bx^3}}{12e^7} + \frac{de\left(ax^2+bx^3\right)^{7/2}}{7b(ex)^{7/2}} \\ &- \frac{5a^6(2bc-ad)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{1024b^{9/2}e^{5/2}}\end{aligned}$$

output

```
5/1024*a^5*(-a*d+2*b*c)*(b*x^3+a*x^2)^(1/2)/b^4/e^2/(e*x)^(1/2)-5/1536*a^4
*(-a*d+2*b*c)*(e*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/b^3/e^3+1/384*a^3*(-a*d+2*b*
c)*(e*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/b^2/e^4+9/64*a^2*(-a*d+2*b*c)*(e*x)^(5/
2)*(b*x^3+a*x^2)^(1/2)/b/e^5+5/24*a*(-a*d+2*b*c)*(e*x)^(7/2)*(b*x^3+a*x^2)
^(1/2)/e^6+1/12*b*(-a*d+2*b*c)*(e*x)^(9/2)*(b*x^3+a*x^2)^(1/2)/e^7+1/7*d*e
*(b*x^3+a*x^2)^(7/2)/b/(e*x)^(7/2)-5/1024*a^6*(-a*d+2*b*c)*arctanh(b^(1/2)
*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(9/2)/e^(5/2)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.71

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{5/2}}\,dx = \frac{x^{3/2}\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(-105a^6d+70a^5b(3c+dx)-28a^4b^2x(5\right.}{}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(5/2),x]
```

output
```
(x^(3/2)*Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-105*a^6*d + 
70*a^5*b*(3*c + d*x) - 28*a^4*b^2*x*(5*c + 2*d*x) + 16*a^3*b^3*x^2*(7*c + 
3*d*x) + 512*b^6*x^5*(7*c + 6*d*x) + 256*a*b^5*x^4*(35*c + 29*d*x) + 32*a^
2*b^4*x^3*(189*c + 148*d*x)) + 420*a^6*b*c*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt
[a] - Sqrt[a + b*x])] + 210*a^7*d*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sq
rt[a + b*x])]))/(21504*b^(9/2)*(e*x)^(5/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 324, normalized size of antiderivative = 0.94, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {1945, 1927, 1927, 1927, 1930, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{(ex)^{5/2}}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(2bc-ad)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{(ex)^{5/2}}dx}{2b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{7b(ex)^{7/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(2bc-ad)\left(\frac{5a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{\sqrt{ex}}dx}{12e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{6e(ex)^{3/2}}\right)}{2b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{7b(ex)^{7/2}}$$

$$\Big\downarrow \text{ 1927}$$

$$\frac{(2bc-ad)\left(\frac{5a\left(\frac{3a\int (ex)^{3/2}\sqrt{bx^3+ax^2}dx}{10e^2}+\frac{\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5e}\right)}{12e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{6e(ex)^{3/2}}\right)}{2b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{7b(ex)^{7/2}}$$

$$\Big\downarrow \text{ 1927}$$

$$\frac{(2bc-ad)\left(\frac{5a\left(\frac{3a\left(\frac{a\int \frac{(ex)^{7/2}}{\sqrt{bx^3+ax^2}}dx}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+bx^3}}{4e}\right)}{10e^2}+\frac{\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5e}\right)}{12e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{6e(ex)^{3/2}}\right)}{2b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{7b(ex)^{7/2}}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{(2bc-ad)\left(\frac{5a\left(\frac{3a\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\int\frac{(ex)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+bx^3}}{4e}\right)}{10e^2}+\frac{\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5e}\right)}{12e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{6e(ex)^{3/2}}\right)}{2b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{7b(ex)^{7/2}}$$

$\big\downarrow$ 1930

$$\frac{(2bc-ad)\left(\frac{5a\left(\frac{3a\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\int\frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+bx^3}}{4e}\right)}{10e^2}+\frac{\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}{5e}\right)}{12e^2}+\cdots\right)}{2b}$$

$$\frac{de\left(ax^2+bx^3\right)^{7/2}}{7b(ex)^{7/2}}$$

$\downarrow$ 1930

$$(2bc-ad)\left(\frac{5a\left(\frac{3a\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int\frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+bx^3}}{4e}\right)}{10e^2}+\frac{\sqrt{ex}}{}\right)}{12e^2}\right.$$

↓ 1937

$$(2bc-ad)\left(\frac{5a\left(\frac{3a\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2+bx^3}}{4e}\right)}{10e^2}+\cdots}{12e^2}\right.$$

↓ 1935

$$5a\left(\frac{3a\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax}}{4e}}{10e^2}\right.$$

$(2b \quad d)$

↓ 219

$$(2bc-ad)\left(\frac{5a\left(\frac{3a\left(\frac{a\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8e^2}+\frac{(ex)^{5/2}\sqrt{ax^2}}{4e}\right.}{10e^2}\right.}{12e^2}\right.$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(5/2),x]
```

output
```
(d*e*(a*x^2 + b*x^3)^(7/2))/(7*b*(e*x)^(7/2)) + ((2*b*c - a*d)*((a*x^2 + b
*x^3)^(5/2)/(6*e*(e*x)^(3/2)) + (5*a*((Sqrt[e*x]*(a*x^2 + b*x^3)^(3/2))/(5
*e) + (3*a*(((e*x)^(5/2)*Sqrt[a*x^2 + b*x^3])/(4*e) + (a*((e^2*(e*x)^(3/2)
*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a*e*((e^2*Sqrt[e*x]*Sqrt[a*x^2 + b*x^3])/
(2*b) - (3*a*e*((e^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[e*x]) - (a*e*Sqrt[e*x]*A
rcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/(4*b)))
/(6*b)))/(8*e^2)))/(10*e^2)))/(12*e^2)))/(2*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 246, normalized size of antiderivative = 0.71

method	result
risch	$-\frac{\left(-3072 b^{6} d\,x^{6}-7424 a\,b^{5} d\,x^{5}-3584 b^{6} c\,x^{5}-4736 a^{2} b^{4} d\,x^{4}-8960 a\,b^{5} c\,x^{4}-48 a^{3} b^{3} d\,x^{3}-6048 a^{2} b^{4} c\,x^{3}+56 a^{4} b^{2} d\,x^{2}-112 a^{3} b^{3} c\,x^{2}-7}{21504 b^{4} e^{2} \sqrt{e x}}$
default	$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}} \left(-6144 b^{6} d\,x^{6} \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}-14848 a\,b^{5} d\,x^{5} \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}-7168 b^{6} c\,x^{5} \sqrt{e x \left(b x +a \right)}\, \sqrt{b e}-9472 a^{2} b^{4} d\,x^{4} \sqrt{e x (b}\right.}{}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/21504/b^4*(-3072*b^6*d*x^6-7424*a*b^5*d*x^5-3584*b^6*c*x^5-4736*a^2*b^4
*d*x^4-8960*a*b^5*c*x^4-48*a^3*b^3*d*x^3-6048*a^2*b^4*c*x^3+56*a^4*b^2*d*x
^2-112*a^3*b^3*c*x^2-70*a^5*b*d*x+140*a^4*b^2*c*x+105*a^6*d-210*a^5*b*c)/e
^2*(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)+5/2048*a^6/b^4*(a*d-2*b*c)*ln((1/2*a*e+
b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)/e^2*(x^2*(b*x+a))^(1
/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.34

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{5/2}}\,dx = \left[-\frac{105\left(2a^6bc-a^7d\right)\sqrt{be}x\log\left(\frac{2bex^2+aex+2\sqrt{bx^3+ax^2}\sqrt{be}\sqrt{ex}}{x}\right)-2\left(3072b^7dx\right.}{}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(5/2),x, algorithm="fricas")
```

output
```
[-1/43008*(105*(2*a^6*b*c - a^7*d)*sqrt(b*e)*x*log((2*b*e*x^2 + a*e*x + 2*
sqrt(b*x^3 + a*x^2)*sqrt(b*e)*sqrt(e*x))/x) - 2*(3072*b^7*d*x^6 + 210*a^5*
b^2*c - 105*a^6*b*d + 256*(14*b^7*c + 29*a*b^6*d)*x^5 + 128*(70*a*b^6*c +
37*a^2*b^5*d)*x^4 + 48*(126*a^2*b^5*c + a^3*b^4*d)*x^3 + 56*(2*a^3*b^4*c -
 a^4*b^3*d)*x^2 - 70*(2*a^4*b^3*c - a^5*b^2*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt
(e*x))/(b^5*e^3*x), 1/21504*(105*(2*a^6*b*c - a^7*d)*sqrt(-b*e)*x*arctan(s
qrt(b*x^3 + a*x^2)*sqrt(-b*e)*sqrt(e*x)/(b*e*x^2 + a*e*x)) + (3072*b^7*d*x
^6 + 210*a^5*b^2*c - 105*a^6*b*d + 256*(14*b^7*c + 29*a*b^6*d)*x^5 + 128*(
70*a*b^6*c + 37*a^2*b^5*d)*x^4 + 48*(126*a^2*b^5*c + a^3*b^4*d)*x^3 + 56*(
2*a^3*b^4*c - a^4*b^3*d)*x^2 - 70*(2*a^4*b^3*c - a^5*b^2*d)*x)*sqrt(b*x^3
+ a*x^2)*sqrt(e*x))/(b^5*e^3*x)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{5/2}}\,dx = \int \frac{\left(x^2(a+bx)\right)^{\frac{5}{2}}(c+dx)}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/(e*x)**(5/2),x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x)/(e*x)**(5/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{5/2}}\,dx=\int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/(e*x)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 421, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{5/2}}\,dx=\frac{\left(\sqrt{(bx+a)be-abe}\left(2\left(4\left(2\left(8(bx+a)\left(2(bx+a)\left(\frac{12(bx+a)d\,\mathrm{sgn}(x)}{b^5e}+\frac{14b^2\cdots}{}\right.\right.\right.\right.\right.\right.\right.}{}$$

$$-\frac{5\left(2a^6bc\log\left(\sqrt{be}\sqrt{a}\right)-a^7d\log\left(\sqrt{be}\sqrt{a}\right)\right)\mathrm{sgn}(x)}{1024\sqrt{be}b^3e^2|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(5/2),x, algorithm="giac")
```

output
```
1/21504*(sqrt((b*x + a)*b*e - a*b*e)*(2*(4*(2*(8*(b*x + a)*(2*(b*x + a)*(1
2*(b*x + a)*d*sgn(x)/(b^5*e) + (14*b^21*c*e^4*sgn(x) - 43*a*b^20*d*e^4*sgn
(x))/(b^25*e^5)) - (70*a*b^21*c*e^4*sgn(x) - 107*a^2*b^20*d*e^4*sgn(x))/(b
^25*e^5)) + 3*(126*a^2*b^21*c*e^4*sgn(x) - 127*a^3*b^20*d*e^4*sgn(x))/(b^2
5*e^5))*(b*x + a) - 7*(2*a^3*b^21*c*e^4*sgn(x) - a^4*b^20*d*e^4*sgn(x))/(b
^25*e^5))*(b*x + a) - 35*(2*a^4*b^21*c*e^4*sgn(x) - a^5*b^20*d*e^4*sgn(x))
/(b^25*e^5))*(b*x + a) - 105*(2*a^5*b^21*c*e^4*sgn(x) - a^6*b^20*d*e^4*sgn
(x))/(b^25*e^5))*sqrt(b*x + a) + 105*(2*a^6*b*c*sgn(x) - a^7*d*sgn(x))*log
(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt((b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*b
^4))*b/(e^2*abs(b)) - 5/1024*(2*a^6*b*c*log(sqrt(b*e)*sqrt(a)) - a^7*d*log
(sqrt(b*e)*sqrt(a)))*sgn(x)/(sqrt(b*e)*b^3*e^2*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{5/2}}\,dx = \int \frac{\left(b\,x^3+a\,x^2\right)^{5/2}\,(c+d\,x)}{(e\,x)^{5/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(5/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 302, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{5/2}}\,dx = \frac{\sqrt{e}\,\Big(-105\sqrt{x}\,\sqrt{bx+a}\,a^6bd+210\sqrt{x}\,\sqrt{bx+a}\,a^5b^2c+70\sqrt{x}\,\sqrt{bx+a}\,a^5b^2}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(5/2),x)
```

output
```
(sqrt(e)*( - 105*sqrt(x)*sqrt(a + b*x)*a**6*b*d + 210*sqrt(x)*sqrt(a + b*x
)*a**5*b**2*c + 70*sqrt(x)*sqrt(a + b*x)*a**5*b**2*d*x - 140*sqrt(x)*sqrt(
a + b*x)*a**4*b**3*c*x - 56*sqrt(x)*sqrt(a + b*x)*a**4*b**3*d*x**2 + 112*s
qrt(x)*sqrt(a + b*x)*a**3*b**4*c*x**2 + 48*sqrt(x)*sqrt(a + b*x)*a**3*b**4
*d*x**3 + 6048*sqrt(x)*sqrt(a + b*x)*a**2*b**5*c*x**3 + 4736*sqrt(x)*sqrt(
a + b*x)*a**2*b**5*d*x**4 + 8960*sqrt(x)*sqrt(a + b*x)*a*b**6*c*x**4 + 742
4*sqrt(x)*sqrt(a + b*x)*a*b**6*d*x**5 + 3584*sqrt(x)*sqrt(a + b*x)*b**7*c*
x**5 + 3072*sqrt(x)*sqrt(a + b*x)*b**7*d*x**6 + 105*sqrt(b)*log((sqrt(a +
b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**7*d - 210*sqrt(b)*log((sqrt(a + b*x) +
 sqrt(x)*sqrt(b))/sqrt(a))*a**6*b*c))/(21504*b**5*e**3)
```

3.322 $\int \frac{(c+dx)(ax^2+bx^3)^{5/2}}{(ex)^{7/2}} dx$

Optimal result

Integrand size = 28, antiderivative size = 302

$$\begin{aligned}\int \frac{(c+dx)(ax^2+bx^3)^{5/2}}{(ex)^{7/2}} dx = &-\frac{a^4(12bc-5ad)\sqrt{ax^2+bx^3}}{512b^3e^3\sqrt{ex}} \\ &+\frac{a^3(12bc-5ad)\sqrt{ex}\sqrt{ax^2+bx^3}}{768b^2e^4}+\frac{31a^2(12bc-5ad)(ex)^{3/2}\sqrt{ax^2+bx^3}}{960be^5} \\ &+\frac{7a(12bc-5ad)(ex)^{5/2}\sqrt{ax^2+bx^3}}{160e^6}+\frac{b(12bc-5ad)(ex)^{7/2}\sqrt{ax^2+bx^3}}{60e^7} \\ &+\frac{de(ax^2+bx^3)^{7/2}}{6b(ex)^{9/2}}+\frac{a^5(12bc-5ad)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{512b^{7/2}e^{7/2}}\end{aligned}$$

output
```
-1/512*a^4*(-5*a*d+12*b*c)*(b*x^3+a*x^2)^(1/2)/b^3/e^3/(e*x)^(1/2)+1/768*a
^3*(-5*a*d+12*b*c)*(e*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/b^2/e^4+31/960*a^2*(-5*
a*d+12*b*c)*(e*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/b/e^5+7/160*a*(-5*a*d+12*b*c)*
(e*x)^(5/2)*(b*x^3+a*x^2)^(1/2)/e^6+1/60*b*(-5*a*d+12*b*c)*(e*x)^(7/2)*(b*
x^3+a*x^2)^(1/2)/e^7+1/6*d*e*(b*x^3+a*x^2)^(7/2)/b/(e*x)^(9/2)+1/512*a^5*(
-5*a*d+12*b*c)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/b^
(7/2)/e^(7/2)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 230, normalized size of antiderivative = 0.76

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{7/2}}\,dx = \frac{\left(x^2(a+bx)\right)^{5/2}\left(-180a^4bc+75a^5d+120a^3b^2cx-50a^4bdx+2976a^2b^3cx^2\right.}{7680b^3x}$$
$$-\frac{a^5(-12bc+5ad)\left(x^2(a+bx)\right)^{5/2}\operatorname{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{256b^{7/2}x^{3/2}(ex)^{7/2}(a+bx)^{5/2}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(7/2),x]
```

output
```
((x^2*(a + b*x))^(5/2)*(-180*a^4*b*c + 75*a^5*d + 120*a^3*b^2*c*x - 50*a^4
*b*d*x + 2976*a^2*b^3*c*x^2 + 40*a^3*b^2*d*x^2 + 4032*a*b^4*c*x^3 + 2160*a
^2*b^3*d*x^3 + 1536*b^5*c*x^4 + 3200*a*b^4*d*x^4 + 1280*b^5*d*x^5))/(7680*
b^3*x*(e*x)^(7/2)*(a + b*x)^2) - (a^5*(-12*b*c + 5*a*d)*(x^2*(a + b*x))^(5
/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])])/(256*b^(7/2)*x^
(3/2)*(e*x)^(7/2)*(a + b*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 282, normalized size of antiderivative = 0.93, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.321$, Rules used = {1945, 1927, 1927, 1927, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{(ex)^{7/2}}dx$$

$$\Big\downarrow \ 1945$$

$$\frac{(12bc-5ad)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{(ex)^{7/2}}dx}{12b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{6b(ex)^{9/2}}$$

$$\Big\downarrow \ 1927$$

$$\frac{(12bc-5ad)\left(\frac{a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(ex)^{3/2}}dx}{2e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{5e(ex)^{5/2}}\right)}{12b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{6b(ex)^{9/2}}$$

$\big\downarrow$ 1927

$$\frac{(12bc-5ad)\left(\frac{a\left(\frac{3a\int \sqrt{ex}\sqrt{bx^3+ax^2}dx}{8e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4e\sqrt{ex}}\right)}{2e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{5e(ex)^{5/2}}\right)}{12b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{6b(ex)^{9/2}}$$

$\big\downarrow$ 1927

$$\frac{(12bc-5ad)\left(\frac{a\left(\frac{3a\left(\frac{a\int \frac{(ex)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4e\sqrt{ex}}\right)}{2e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{5e(ex)^{5/2}}\right)}{12b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{6b(ex)^{9/2}}$$

$\big\downarrow$ 1930

$$\frac{(12bc-5ad)\left(\frac{a\left(\frac{3a\left(\frac{a\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\int\frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4e\sqrt{ex}}\right)}{2e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{5e(ex)^{5/2}}\right)}{12b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{6b(ex)^{9/2}}$$

$$\Big\downarrow \; 1930$$

$$\frac{(12bc-5ad)\left(\frac{a\left(\frac{3a\left(\frac{a\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int\frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4e\sqrt{ex}}\right)}{2e^2}+\frac{\left(ax^2+bx^3\right.}{5e(ex)^{5}}\right.}{12b}$$

$$\frac{de\left(ax^2+bx^3\right)^{7/2}}{6b(ex)^{9/2}}$$

$$\Big\downarrow \; 1937$$

$$\frac{(12bc-5ad)\left(\frac{a\left(\frac{3a\left(\frac{a\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4e\sqrt{ex}}\right)}{2e^2}+\frac{(ax^2+}{5e(e}\right.}{12b}$$

$$\frac{de\left(ax^2+bx^3\right)^{7/2}}{6b(ex)^{9/2}}$$

$\downarrow$ 1935

$$(12bc-5ad)\left(\frac{a\left(\frac{3a\left(\frac{a\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4e\sqrt{ex}}\right)}{2e^2}\right.$$

↓ 219

$$\frac{(12bc-5ad)\left(\frac{a\left(\frac{3a\left(\frac{a\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{6e^2}+\frac{(ex)^{3/2}\sqrt{ax^2+bx^3}}{3e}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{4e\sqrt{ex}}\right)}{2e^2} - \cdots\right)}{12b}$$

$$\frac{de\left(ax^2+bx^3\right)^{7/2}}{6b(ex)^{9/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(7/2),x]
```

output
```
(d*e*(a*x^2 + b*x^3)^(7/2))/(6*b*(e*x)^(9/2)) + ((12*b*c - 5*a*d)*((a*x^2
+ b*x^3)^(5/2)/(5*e*(e*x)^(5/2)) + (a*((a*x^2 + b*x^3)^(3/2)/(4*e*Sqrt[e*x
]) + (3*a*(((e*x)^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*e) + (a*((e^2*Sqrt[e*x]*Sq
rt[a*x^2 + b*x^3])/(2*b) - (3*a*e*((e^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[e*x])
 - (a*e*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)
*Sqrt[x])))/(4*b)))/(6*e^2)))/(8*e^2)))/(2*e^2)))/(12*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.74

method	result
risch	$\frac{(1280d\,x^5b^5+3200a\,b^4d\,x^4+1536b^5c\,x^4+2160a^2b^3d\,x^3+4032a\,b^4c\,x^3+40a^3b^2d\,x^2+2976a^2b^3c\,x^2-50a^4bdx+120a^3b^2cx+75a^5d-18}{7680b^3e^3\sqrt{ex}}$
default	$\frac{(b\,x^3+a\,x^2)^{\frac{5}{2}}\left(2560b^5d\,x^5\sqrt{ex(bx+a)}\,\sqrt{be}+6400a\,b^4d\,x^4\sqrt{ex(bx+a)}\,\sqrt{be}+3072b^5c\,x^4\sqrt{ex(bx+a)}\,\sqrt{be}+4320a^2b^3d\,x^3\sqrt{ex(bx+a)}\right.}{}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/7680/b^3*(1280*b^5*d*x^5+3200*a*b^4*d*x^4+1536*b^5*c*x^4+2160*a^2*b^3*d*
x^3+4032*a*b^4*c*x^3+40*a^3*b^2*d*x^2+2976*a^2*b^3*c*x^2-50*a^4*b*d*x+120*
a^3*b^2*c*x+75*a^5*d-180*a^4*b*c)/e^3*(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)-1/10
24*a^5/b^3*(5*a*d-12*b*c)*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(
1/2))/(b*e)^(1/2)/e^3*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e
*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 417, normalized size of antiderivative = 1.38

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{(ex)^{7/2}}\,dx = \left[-\frac{15\,(12\,a^5bc-5\,a^6d)\sqrt{be}x\log\left(\frac{2\,bex^2+aex-2\,\sqrt{bx^3+ax^2}\sqrt{be}\sqrt{ex}}{x}\right)-2\,(1280\,b^6d\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(7/2),x, algorithm="fricas")
```

output
```
[-1/15360*(15*(12*a^5*b*c - 5*a^6*d)*sqrt(b*e)*x*log((2*b*e*x^2 + a*e*x -
2*sqrt(b*x^3 + a*x^2)*sqrt(b*e)*sqrt(e*x))/x) - 2*(1280*b^6*d*x^5 - 180*a^
4*b^2*c + 75*a^5*b*d + 128*(12*b^6*c + 25*a*b^5*d)*x^4 + 144*(28*a*b^5*c +
 15*a^2*b^4*d)*x^3 + 8*(372*a^2*b^4*c + 5*a^3*b^3*d)*x^2 + 10*(12*a^3*b^3*
c - 5*a^4*b^2*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^4*e^4*x), -1/7680*(1
5*(12*a^5*b*c - 5*a^6*d)*sqrt(-b*e)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b*e
)*sqrt(e*x)/(b*e*x^2 + a*e*x)) - (1280*b^6*d*x^5 - 180*a^4*b^2*c + 75*a^5*
b*d + 128*(12*b^6*c + 25*a*b^5*d)*x^4 + 144*(28*a*b^5*c + 15*a^2*b^4*d)*x^
3 + 8*(372*a^2*b^4*c + 5*a^3*b^3*d)*x^2 + 10*(12*a^3*b^3*c - 5*a^4*b^2*d)*
x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^4*e^4*x)]
```

Sympy [F]

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{(ex)^{7/2}}\,dx = \int \frac{(x^2(a+bx))^{\frac{5}{2}}\,(c+dx)}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/(e*x)**(7/2),x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x)/(e*x)**(7/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{7/2}}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/(e*x)^(7/2), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.24

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{7/2}}\,dx = \frac{\left(\sqrt{(bx+a)be-abe}\left(2\left(4\left(2\,(bx+a)\left(8\,(bx+a)\left(\frac{10\,(bx+a)d\mathrm{sgn}(x)}{b^4e}+\frac{12\,b^{13}ce^{3}}{}\right.\right.\right.\right.\right.\right.}{} + \frac{\left(12\,a^5bc\log\left(\sqrt{be}\sqrt{a}\right)-5\,a^6d\log\left(\sqrt{be}\sqrt{a}\right)\right)\mathrm{sgn}(x)}{512\,\sqrt{be}b^2e^3|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(7/2),x, algorithm="giac")
```

output
```
1/7680*(sqrt((b*x + a)*b*e - a*b*e)*(2*(4*(2*(b*x + a)*(8*(b*x + a)*(10*(b
*x + a)*d*sgn(x)/(b^4*e) + (12*b^13*c*e^3*sgn(x) - 25*a*b^12*d*e^3*sgn(x))
/(b^16*e^4)) - 3*(44*a*b^13*c*e^3*sgn(x) - 45*a^2*b^12*d*e^3*sgn(x))/(b^16
*e^4)) + (12*a^2*b^13*c*e^3*sgn(x) - 5*a^3*b^12*d*e^3*sgn(x))/(b^16*e^4))*
(b*x + a) + 5*(12*a^3*b^13*c*e^3*sgn(x) - 5*a^4*b^12*d*e^3*sgn(x))/(b^16*e
^4))*(b*x + a) + 15*(12*a^4*b^13*c*e^3*sgn(x) - 5*a^5*b^12*d*e^3*sgn(x))/(
b^16*e^4))*sqrt(b*x + a) - 15*(12*a^5*b*c*sgn(x) - 5*a^6*d*sgn(x))*log(abs
(-sqrt(b*e)*sqrt(b*x + a) + sqrt((b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*b^3))
*b/(e^3*abs(b)) + 1/512*(12*a^5*b*c*log(sqrt(b*e)*sqrt(a)) - 5*a^6*d*log(s
qrt(b*e)*sqrt(a)))*sgn(x)/(sqrt(b*e)*b^2*e^3*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{7/2}}\,dx=\int \frac{\left(b\,x^3+a\,x^2\right)^{5/2}\,\left(c+d\,x\right)}{\left(e\,x\right)^{7/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(7/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.87

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{7/2}}\,dx=\frac{\sqrt{e}\left(75\sqrt{x}\,\sqrt{bx+a}\,a^5bd-180\sqrt{x}\,\sqrt{bx+a}\,a^4b^2c-50\sqrt{x}\,\sqrt{bx+a}\,a^4b^2dx\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(7/2),x)
```

output
```
(sqrt(e)*(75*sqrt(x)*sqrt(a + b*x)*a**5*b*d - 180*sqrt(x)*sqrt(a + b*x)*a*
*4*b**2*c - 50*sqrt(x)*sqrt(a + b*x)*a**4*b**2*d*x + 120*sqrt(x)*sqrt(a +
b*x)*a**3*b**3*c*x + 40*sqrt(x)*sqrt(a + b*x)*a**3*b**3*d*x**2 + 2976*sqrt
(x)*sqrt(a + b*x)*a**2*b**4*c*x**2 + 2160*sqrt(x)*sqrt(a + b*x)*a**2*b**4*
d*x**3 + 4032*sqrt(x)*sqrt(a + b*x)*a*b**5*c*x**3 + 3200*sqrt(x)*sqrt(a +
b*x)*a*b**5*d*x**4 + 1536*sqrt(x)*sqrt(a + b*x)*b**6*c*x**4 + 1280*sqrt(x)
*sqrt(a + b*x)*b**6*d*x**5 - 75*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(
b))/sqrt(a))*a**6*d + 180*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sq
rt(a))*a**5*b*c))/(7680*b**4*e**4)
```

3.323 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{9/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 258

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{9/2}}\,dx = \frac{a^3(10bc-3ad)\sqrt{ax^2+bx^3}}{128b^2e^4\sqrt{ex}} + \frac{59a^2(10bc-3ad)\sqrt{ex}\sqrt{ax^2+bx^3}}{960be^5} + \frac{17a(10bc-3ad)(ex)^{3/2}\sqrt{ax^2+bx^3}}{240e^6} + \frac{b(10bc-3ad)(ex)^{5/2}\sqrt{ax^2+bx^3}}{40e^7} + \frac{de\left(ax^2+bx^3\right)^{7/2}}{5b(ex)^{11/2}} - \frac{a^4(10bc-3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{128b^{5/2}e^{9/2}}$$

output

```
1/128*a^3*(-3*a*d+10*b*c)*(b*x^3+a*x^2)^(1/2)/b^2/e^4/(e*x)^(1/2)+59/960*a
^2*(-3*a*d+10*b*c)*(e*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/b/e^5+17/240*a*(-3*a*d+
10*b*c)*(e*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/e^6+1/40*b*(-3*a*d+10*b*c)*(e*x)^(
5/2)*(b*x^3+a*x^2)^(1/2)/e^7+1/5*d*e*(b*x^3+a*x^2)^(7/2)/b/(e*x)^(11/2)-1/
128*a^4*(-3*a*d+10*b*c)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^
(1/2))/b^(5/2)/e^(9/2)
```

Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 212, normalized size of antiderivative = 0.82

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{9/2}}\,dx = \frac{\sqrt{x}\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{x}\sqrt{a+bx}(-45a^4d+30a^3b(5c+dx)+96b^4x^3(5c+\right.}{}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(9/2),x]
```

output
```
(Sqrt[x]*Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(-45*a^4*d + 3
0*a^3*b*(5*c + d*x) + 96*b^4*x^3*(5*c + 4*d*x) + 16*a*b^3*x^2*(85*c + 63*d
*x) + 4*a^2*b^2*x*(295*c + 186*d*x)) + 300*a^4*b*c*ArcTanh[(Sqrt[b]*Sqrt[x
])/(Sqrt[a] - Sqrt[a + b*x])] + 90*a^5*d*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[
a] + Sqrt[a + b*x])]))/(1920*b^(5/2)*e^3*(e*x)^(3/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {1945, 1927, 1927, 1927, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{(ex)^{9/2}}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(10bc-3ad)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{(ex)^{9/2}}dx}{10b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{5b(ex)^{11/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(10bc-3ad)\left(\frac{5a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(ex)^{5/2}}dx}{8e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{4e(ex)^{7/2}}\right)}{10b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{5b(ex)^{11/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(10bc-3ad)\left(\frac{5a\left(\frac{a\int \frac{\sqrt{bx^3+ax^2}}{\sqrt{ex}}dx}{2e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{3e(ex)^{3/2}}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{4e(ex)^{7/2}}\right)}{10b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{5b(ex)^{11/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(10bc-3ad)\left(\frac{5a\left(\frac{a\left(\frac{a\int \frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4e^2}+\frac{\sqrt{ex}\sqrt{ax^2+bx^3}}{2e}\right)}{2e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{3e(ex)^{3/2}}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{4e(ex)^{7/2}}\right)}{10b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{5b(ex)^{11/2}}$$

$$\Big\downarrow \text{1930}$$

$$\frac{(10bc-3ad)\left(\frac{5a\left(\frac{a\left(\frac{a\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int\frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4e^2}+\frac{\sqrt{ex}\sqrt{ax^2+bx^3}}{2e}\right)}{2e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{3e(ex)^{3/2}}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{4e(ex)^{7/2}}\right)}{10b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{5b(ex)^{11/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{(10bc-3ad)\left(\frac{5a\left(\frac{a\left(\frac{a\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4e^2}+\frac{\sqrt{ex}\sqrt{ax^2+bx^3}}{2e}\right)}{2e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{3e(ex)^{3/2}}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{4e(ex)^{7/2}}\right)}{10b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{5b(ex)^{11/2}}$$

$$\Big\downarrow \textcolor{blue}{1935}$$

$$\frac{(10bc-3ad)\left(\frac{5a\left(\frac{a\left(\frac{a\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4e^2}+\frac{\sqrt{ex}\sqrt{ax^2+bx^3}}{2e}\right)}{2e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{3e(ex)^{3/2}}\right)}{8e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{4e(ex)^{7/2}}\right)}{10b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{5b(ex)^{11/2}}$$

$$\Big\downarrow \textcolor{blue}{219}$$

$$\frac{(10bc-3ad)\left(\frac{5a\left(\frac{a\left(\frac{a\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4e^2}+\frac{\sqrt{ex}\sqrt{ax^2+bx^3}}{2e}\right)}{2e^2}+\frac{(ax^2+bx^3)^{3/2}}{3e(ex)^{3/2}}\right)}{8e^2}+\frac{(ax^2+bx^3)^{5/2}}{4e(ex)^{7/2}}\right)}{10b}+\frac{de(ax^2+bx^3)^{7/2}}{5b(ex)^{11/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(9/2),x]
```

output
```
(d*e*(a*x^2 + b*x^3)^(7/2))/(5*b*(e*x)^(11/2)) + ((10*b*c - 3*a*d)*((a*x^2
 + b*x^3)^(5/2)/(4*e*(e*x)^(7/2)) + (5*a*((a*x^2 + b*x^3)^(3/2)/(3*e*(e*x)
^(3/2)) + (a*((Sqrt[e*x]*Sqrt[a*x^2 + b*x^3])/(2*e) + (a*((e^2*Sqrt[a*x^2
+ b*x^3])/(b*Sqrt[e*x]) - (a*e*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*
x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/(4*e^2)))/(2*e^2)))/(8*e^2)))/(10*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.77

method	result
risch	$-\frac{\left(-384 d\,x^{4} b^{4}-1008 a\,b^{3} d\,x^{3}-480 b^{4} c\,x^{3}-744 a^{2} b^{2} d\,x^{2}-1360 a\,b^{3} c\,x^{2}-30 a^{3} b d x-1180 a^{2} b^{2} c x+45 a^{4} d-150 a^{3} b c\right) \sqrt{x^{2} \left(b x+a\right)}}{1920 b^{2} e^{4} \sqrt{e x}}+$
default	$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}} \left(-768 b^{4} d\,x^{4} \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}-2016 a\,b^{3} d\,x^{3} \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}-960 b^{4} c\,x^{3} \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}-1488 a^{2} b^{2} d\,x^{2} \sqrt{e x (b x+}\right.}{}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(9/2),x,method=_RETURNVERBOSE)
```

output
```
-1/1920/b^2*(-384*b^4*d*x^4-1008*a*b^3*d*x^3-480*b^4*c*x^3-744*a^2*b^2*d*x
^2-1360*a*b^3*c*x^2-30*a^3*b*d*x-1180*a^2*b^2*c*x+45*a^4*d-150*a^3*b*c)/e^
4*(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)+1/256*a^4/b^2*(3*a*d-10*b*c)*ln((1/2*a*e
+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)/e^4*(x^2*(b*x+a))^(
1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.43

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{9/2}}\,dx = \left[-\frac{15\left(10\,a^4bc-3\,a^5d\right)\sqrt{be}x\log\left(\frac{2\,bex^2+aex+2\,\sqrt{bx^3+ax^2}\sqrt{be}\sqrt{ex}}{x}\right)-2\left(384\,b^5dx\right.}{}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(9/2),x, algorithm="fricas")
```

output
```
[-1/3840*(15*(10*a^4*b*c - 3*a^5*d)*sqrt(b*e)*x*log((2*b*e*x^2 + a*e*x + 2
*sqrt(b*x^3 + a*x^2)*sqrt(b*e)*sqrt(e*x))/x) - 2*(384*b^5*d*x^4 + 150*a^3*
b^2*c - 45*a^4*b*d + 48*(10*b^5*c + 21*a*b^4*d)*x^3 + 8*(170*a*b^4*c + 93*
a^2*b^3*d)*x^2 + 10*(118*a^2*b^3*c + 3*a^3*b^2*d)*x)*sqrt(b*x^3 + a*x^2)*s
qrt(e*x))/(b^3*e^5*x), 1/1920*(15*(10*a^4*b*c - 3*a^5*d)*sqrt(-b*e)*x*arct
an(sqrt(b*x^3 + a*x^2)*sqrt(-b*e)*sqrt(e*x)/(b*e*x^2 + a*e*x)) + (384*b^5*
d*x^4 + 150*a^3*b^2*c - 45*a^4*b*d + 48*(10*b^5*c + 21*a*b^4*d)*x^3 + 8*(1
70*a*b^4*c + 93*a^2*b^3*d)*x^2 + 10*(118*a^2*b^3*c + 3*a^3*b^2*d)*x)*sqrt(
b*x^3 + a*x^2)*sqrt(e*x))/(b^3*e^5*x)]
```

Sympy [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{9/2}}\,dx=\int \frac{\left(x^2(a+bx)\right)^{\frac{5}{2}}(c+dx)}{(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/(e*x)**(9/2),x)
```

output
```
Integral((x**2*(a + b*x))**(5/2)*(c + d*x)/(e*x)**(9/2), x)
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{9/2}}\,dx=\int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{(ex)^{\frac{9}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(9/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/(e*x)^(9/2), x)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 331, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{9/2}}\,dx = \frac{\left(\sqrt{(bx+a)be-abe}\left(2\left(4(bx+a)\left(6(bx+a)\left(\frac{8(bx+a)d\text{sgn}(x)}{b^3e}+\frac{10b^7ce^2\text{sgn}(x)}{\ldots}\right.\right.\right.\right.\right.}{\ldots}$$
$$-\frac{\left(10a^4bc\log\left(\sqrt{be}\sqrt{a}\right)-3a^5d\log\left(\sqrt{be}\sqrt{a}\right)\right)\text{sgn}(x)}{128\sqrt{be}be^4|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(9/2),x, algorithm="giac")
```

output
```
1/1920*(sqrt((b*x + a)*b*e - a*b*e)*(2*(4*(b*x + a)*(6*(b*x + a)*(8*(b*x +
 a)*d*sgn(x)/(b^3*e) + (10*b^7*c*e^2*sgn(x) - 11*a*b^6*d*e^2*sgn(x))/(b^9*
e^3)) - (10*a*b^7*c*e^2*sgn(x) - 3*a^2*b^6*d*e^2*sgn(x))/(b^9*e^3)) - 5*(1
0*a^2*b^7*c*e^2*sgn(x) - 3*a^3*b^6*d*e^2*sgn(x))/(b^9*e^3))*(b*x + a) - 15
*(10*a^3*b^7*c*e^2*sgn(x) - 3*a^4*b^6*d*e^2*sgn(x))/(b^9*e^3))*sqrt(b*x +
a) + 15*(10*a^4*b*c*sgn(x) - 3*a^5*d*sgn(x))*log(abs(-sqrt(b*e)*sqrt(b*x +
 a) + sqrt((b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*b^2))*b/(e^4*abs(b)) - 1/12
8*(10*a^4*b*c*log(sqrt(b*e)*sqrt(a)) - 3*a^5*d*log(sqrt(b*e)*sqrt(a)))*sgn
(x)/(sqrt(b*e)*b*e^4*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{9/2}}\,dx = \int \frac{\left(b\,x^3+a\,x^2\right)^{5/2}\,\left(c+d\,x\right)}{{\left(e\,x\right)}^{9/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(9/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(9/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.86

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{9/2}}\,dx = \frac{\sqrt{e}\left(-45\sqrt{x}\,\sqrt{bx+a}\,a^4bd+150\sqrt{x}\,\sqrt{bx+a}\,a^3b^2c+30\sqrt{x}\,\sqrt{bx+a}\,a^3b^2d\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(9/2),x)
```

output
```
(sqrt(e)*( - 45*sqrt(x)*sqrt(a + b*x)*a**4*b*d + 150*sqrt(x)*sqrt(a + b*x)
*a**3*b**2*c + 30*sqrt(x)*sqrt(a + b*x)*a**3*b**2*d*x + 1180*sqrt(x)*sqrt(
a + b*x)*a**2*b**3*c*x + 744*sqrt(x)*sqrt(a + b*x)*a**2*b**3*d*x**2 + 1360
*sqrt(x)*sqrt(a + b*x)*a*b**4*c*x**2 + 1008*sqrt(x)*sqrt(a + b*x)*a*b**4*d
*x**3 + 480*sqrt(x)*sqrt(a + b*x)*b**5*c*x**3 + 384*sqrt(x)*sqrt(a + b*x)*
b**5*d*x**4 + 45*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*
*5*d - 150*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*b*c
))/(1920*b**3*e**5)
```

3.324 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{11/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 214

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{11/2}}\,dx = \frac{11a^2(8bc-ad)\sqrt{ax^2+bx^3}}{64be^5\sqrt{ex}} + \frac{13a(8bc-ad)\sqrt{ex}\sqrt{ax^2+bx^3}}{96e^6} + \frac{b(8bc-ad)(ex)^{3/2}\sqrt{ax^2+bx^3}}{24e^7} + \frac{de\left(ax^2+bx^3\right)^{7/2}}{4b(ex)^{13/2}} + \frac{5a^3(8bc-ad)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{64b^{3/2}e^{11/2}}$$

output

```
11/64*a^2*(-a*d+8*b*c)*(b*x^3+a*x^2)^(1/2)/b/e^5/(e*x)^(1/2)+13/96*a*(-a*d
+8*b*c)*(e*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/e^6+1/24*b*(-a*d+8*b*c)*(e*x)^(3/2
)*(b*x^3+a*x^2)^(1/2)/e^7+1/4*d*e*(b*x^3+a*x^2)^(7/2)/b/(e*x)^(13/2)+5/64*
a^3*(-a*d+8*b*c)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/
b^(3/2)/e^(11/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.72

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{(ex)^{11/2}}\,dx = \frac{\sqrt{x}\sqrt{x^2(a+bx)}\Big(\sqrt{b}\sqrt{x}\sqrt{a+bx}(15a^3d+16b^3x^2(4c+3dx)+8ab^2x(26c-}{192b^{3/2}e^4(e}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(11/2),x]
```

output
```
(Sqrt[x]*Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[x]*Sqrt[a + b*x]*(15*a^3*d + 16
*b^3*x^2*(4*c + 3*d*x) + 8*a*b^2*x*(26*c + 17*d*x) + 2*a^2*b*(132*c + 59*d
*x)) + 15*a^3*(-8*b*c + a*d)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(19
2*b^(3/2)*e^4*(e*x)^(3/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1945, 1927, 1927, 1927, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{5/2}(c+dx)}{(ex)^{11/2}}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(8bc-ad)\int \frac{(bx^3+ax^2)^{5/2}}{(ex)^{11/2}}dx}{8b}+\frac{de(ax^2+bx^3)^{7/2}}{4b(ex)^{13/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(8bc-ad)\left(\frac{5a\int \frac{(bx^3+ax^2)^{3/2}}{(ex)^{7/2}}dx}{6e^2}+\frac{(ax^2+bx^3)^{5/2}}{3e(ex)^{9/2}}\right)}{8b}+\frac{de(ax^2+bx^3)^{7/2}}{4b(ex)^{13/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(8bc-ad)\left(\frac{5a\left(\frac{3a\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{3/2}}dx}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{6e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{9/2}}\right)}{8b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{4b(ex)^{13/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(8bc-ad)\left(\frac{5a\left(\frac{3a\left(\frac{a\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2e^2}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{6e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{9/2}}\right)}{8b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{4b(ex)^{13/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{(8bc-ad)\left(\frac{5a\left(\frac{3a\left(\frac{a\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{6e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{9/2}}\right)}{8b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{4b(ex)^{13/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{(8bc-ad)\left(\frac{5a\left(\frac{3a\left(\frac{a\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{6e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{9/2}}\right)}{8b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{4b(ex)^{13/2}}$$

$$\Big\downarrow \; 219$$

$$\frac{(8bc-ad)\left(\frac{5a\left(\frac{3a\left(\frac{a\sqrt{ex}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{6e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{9/2}}\right)}{8b}+\frac{de\left(ax^2+bx^3\right)^{7/2}}{4b(ex)^{13/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(11/2),x]
```

output
```
(d*e*(a*x^2 + b*x^3)^(7/2))/(4*b*(e*x)^(13/2)) + ((8*b*c - a*d)*((a*x^2 +
b*x^3)^(5/2)/(3*e*(e*x)^(9/2)) + (5*a*((a*x^2 + b*x^3)^(3/2)/(2*e*(e*x)^(5
/2)) + (3*a*(Sqrt[a*x^2 + b*x^3]/(e*Sqrt[e*x]) + (a*Sqrt[e*x]*ArcTanh[(Sqr
t[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(Sqrt[b]*e^2*Sqrt[x])))/(4*e^2)))/(6*e
^2)))/(8*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.81

method	result
risch	$\frac{(48b^3 d\, x^3+136a\, b^2 d\, x^2+64b^3 c\, x^2+118a^2 b d x+208a\, b^2 c x+15a^3 d+264c\, a^2 b)\sqrt{x^2(bx+a)}}{192b\, e^5\sqrt{ex}}-\frac{5a^3(ad-8bc)\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\, x^2+a}\right.}{128b\sqrt{be}\, e^5 x(bx+}$
default	$\frac{(b\, x^3+a\, x^2)^{\frac{5}{2}}\left(96b^3 d\, x^3\sqrt{ex(bx+a)}\,\sqrt{be}+272a\, b^2 d\, x^2\sqrt{ex(bx+a)}\,\sqrt{be}+128b^3 c\, x^2\sqrt{ex(bx+a)}\,\sqrt{be}-15\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+c}{2\sqrt{be}}\right.\right.}{384a}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(11/2),x,method=_RETURNVERBOSE)
```

output
```
1/192/b*(48*b^3*d*x^3+136*a*b^2*d*x^2+64*b^3*c*x^2+118*a^2*b*d*x+208*a*b^2
*c*x+15*a^3*d+264*a^2*b*c)/e^5*(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)-5/128*a^3/b
*(a*d-8*b*c)*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(
1/2)/e^5*(x^2*(b*x+a))^(1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 321, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{11/2}}\,dx = \left[-\frac{15\left(8\,a^3bc-a^4d\right)\sqrt{bex}\log\left(\frac{2\,bex^2+aex-2\,\sqrt{bx^3+ax^2}\sqrt{be}\sqrt{ex}}{x}\right)-2\left(48\,b^4dx^3+\right.}{}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(11/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*(8*a^3*b*c - a^4*d)*sqrt(b*e)*x*log((2*b*e*x^2 + a*e*x - 2*sqr
t(b*x^3 + a*x^2)*sqrt(b*e)*sqrt(e*x))/x) - 2*(48*b^4*d*x^3 + 264*a^2*b^2*c
 + 15*a^3*b*d + 8*(8*b^4*c + 17*a*b^3*d)*x^2 + 2*(104*a*b^3*c + 59*a^2*b^2
*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^2*e^6*x), -1/192*(15*(8*a^3*b*c -
 a^4*d)*sqrt(-b*e)*x*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b*e)*sqrt(e*x)/(b*e*
x^2 + a*e*x)) - (48*b^4*d*x^3 + 264*a^2*b^2*c + 15*a^3*b*d + 8*(8*b^4*c +
17*a*b^3*d)*x^2 + 2*(104*a*b^3*c + 59*a^2*b^2*d)*x)*sqrt(b*x^3 + a*x^2)*sq
rt(e*x))/(b^2*e^6*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{11/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/(e*x)**(11/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{11/2}}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{(ex)^{\frac{11}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(11/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/(e*x)^(11/2), x)
```

Giac [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{11/2}}\,dx = \frac{\left(\sqrt{(bx+a)be-abe}\left(2\,(bx+a)\left(4\,(bx+a)\left(\frac{6\,(bx+a)d\,\mathrm{sgn}(x)}{b^2e}+\frac{8\,b^3ce\,\mathrm{sgn}(x)-ab^2c}{b^4e^2}\right.\right.\right.\right.}{} + \frac{5\left(8\,a^3bc\log\left(\sqrt{be}\sqrt{a}\right)-a^4d\log\left(\sqrt{be}\sqrt{a}\right)\right)\mathrm{sgn}(x)}{64\sqrt{be}e^5|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(11/2),x, algorithm="giac")
```

output
```
1/192*(sqrt((b*x + a)*b*e - a*b*e)*(2*(b*x + a)*(4*(b*x + a)*(6*(b*x + a)*
d*sgn(x)/(b^2*e) + (8*b^3*c*e*sgn(x) - a*b^2*d*e*sgn(x))/(b^4*e^2)) + 5*(8
*a*b^3*c*e*sgn(x) - a^2*b^2*d*e*sgn(x))/(b^4*e^2)) + 15*(8*a^2*b^3*c*e*sgn
(x) - a^3*b^2*d*e*sgn(x))/(b^4*e^2))*sqrt(b*x + a) - 15*(8*a^3*b*c*sgn(x)
- a^4*d*sgn(x))*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt((b*x + a)*b*e - a*
b*e)))/(sqrt(b*e)*b))*b/(e^5*abs(b)) + 5/64*(8*a^3*b*c*log(sqrt(b*e)*sqrt(
a)) - a^4*d*log(sqrt(b*e)*sqrt(a)))*sgn(x)/(sqrt(b*e)*e^5*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{11/2}}\,dx=\int \frac{\left(b\,x^3+a\,x^2\right)^{5/2}\,(c+d\,x)}{(e\,x)^{11/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(11/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(11/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{11/2}}\,dx=\frac{\sqrt{e}\,\left(15\sqrt{x}\,\sqrt{bx+a}\,a^3bd+264\sqrt{x}\,\sqrt{bx+a}\,a^2b^2c+118\sqrt{x}\,\sqrt{bx+a}\,a^2b^2dx\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(11/2),x)
```

output
```
(sqrt(e)*(15*sqrt(x)*sqrt(a + b*x)*a**3*b*d + 264*sqrt(x)*sqrt(a + b*x)*a*
*2*b**2*c + 118*sqrt(x)*sqrt(a + b*x)*a**2*b**2*d*x + 208*sqrt(x)*sqrt(a +
 b*x)*a*b**3*c*x + 136*sqrt(x)*sqrt(a + b*x)*a*b**3*d*x**2 + 64*sqrt(x)*sq
rt(a + b*x)*b**4*c*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*d*x**3 - 15*sqrt(b
)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*d + 120*sqrt(b)*log(
(sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*b*c))/(192*b**2*e**6)
```

3.325 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{13/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 194

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{13/2}}\,dx = \frac{25a(6bc+ad)\sqrt{ax^2+bx^3}}{24e^6\sqrt{ex}} + \frac{5b(6bc+ad)\sqrt{ex}\sqrt{ax^2+bx^3}}{12e^7} - \frac{2c\left(ax^2+bx^3\right)^{5/2}}{e(ex)^{11/2}} + \frac{d\left(ax^2+bx^3\right)^{5/2}}{3e^2(ex)^{9/2}} + \frac{5a^2(6bc+ad)\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{8\sqrt{b}e^{13/2}}$$

output
```
25/24*a*(a*d+6*b*c)*(b*x^3+a*x^2)^(1/2)/e^6/(e*x)^(1/2)+5/12*b*(a*d+6*b*c)
*(e*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/e^7-2*c*(b*x^3+a*x^2)^(5/2)/e/(e*x)^(11/2
)+1/3*d*(b*x^3+a*x^2)^(5/2)/e^2/(e*x)^(9/2)+5/8*a^2*(a*d+6*b*c)*arctanh(b^
(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(1/2)/e^(13/2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.76

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{13/2}}\,dx = \frac{\sqrt{x^2(a+bx)}\left(\sqrt{b}\sqrt{a+bx}(4b^2x^2(3c+2dx)+2abx(27c+13dx)+a^2(-48\right.}{24\sqrt{b}e^5(ex)^{3/2}\sqrt{a+}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(13/2),x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(Sqrt[b]*Sqrt[a + b*x]*(4*b^2*x^2*(3*c + 2*d*x) + 2*a
*b*x*(27*c + 13*d*x) + a^2*(-48*c + 33*d*x)) + 30*a^2*(6*b*c + a*d)*Sqrt[x
]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(24*Sqrt[b]*e^5*
(e*x)^(3/2)*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1944, 1927, 1927, 1927, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{(ex)^{13/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(ad+6bc)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{(ex)^{11/2}}dx}{ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{a(ex)^{15/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(ad+6bc)\left(\frac{5a\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(ex)^{7/2}}dx}{6e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{9/2}}\right)}{ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{a(ex)^{15/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(ad+6bc)\left(\frac{5a\left(\frac{3a\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{3/2}}dx}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{6e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{9/2}}\right)}{ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{a(ex)^{15/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(ad+6bc)\left(\frac{5a\left(\frac{3a\left(\frac{a\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2e^2}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{6e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{9/2}}\right)}{ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{a(ex)^{15/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{(ad+6bc)\left(\frac{5a\left(\frac{3a\left(\frac{a\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{6e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{9/2}}\right)}{ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{a(ex)^{15/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{(ad+6bc)\left(\frac{5a\left(\frac{3a\left(\frac{a\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{6e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{9/2}}\right)}{ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{a(ex)^{15/2}}$$

$\downarrow$ 219

$$\frac{(ad+6bc)\left(\frac{5a\left(\frac{3a\left(\frac{a\sqrt{ex}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{6e^2}+\frac{\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{9/2}}\right)}{ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{a(ex)^{15/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(13/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(7/2))/(a*(e*x)^(15/2)) + ((6*b*c + a*d)*((a*x^2 +
 b*x^3)^(5/2)/(3*e*(e*x)^(9/2)) + (5*a*((a*x^2 + b*x^3)^(3/2)/(2*e*(e*x)^(
5/2)) + (3*a*(Sqrt[a*x^2 + b*x^3]/(e*Sqrt[e*x]) + (a*Sqrt[e*x]*ArcTanh[(Sq
rt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(Sqrt[b]*e^2*Sqrt[x])))/(4*e^2)))/(6*
e^2)))/(a*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{(-8b^2dx^3-26abdx^2-12b^2cx^2-33a^2dx-54abcx+48a^2c)\sqrt{x^2(bx+a)}}{24e^6x\sqrt{ex}}+\frac{5a^2(ad+6bc)\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^2+aex}\right)\sqrt{x^2(bx+a}}{16\sqrt{be}\,e^6x(bx+a)\sqrt{ex}}$
default	$-\frac{(b\,x^3+a\,x^2)^{\frac{5}{2}}\left(-16b^2d\,x^3\sqrt{ex(bx+a)}\sqrt{be}-15\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\sqrt{be}+ae}{2\sqrt{be}}\right)a^3dex-90\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\sqrt{be}+ae}{2\sqrt{be}}\right)a^2bcex-\right.}{48x^5(bx+a)^2e^6\sqrt{}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(13/2),x,method=_RETURNVERBOSE)
```

output
```
-1/24*(-8*b^2*d*x^3-26*a*b*d*x^2-12*b^2*c*x^2-33*a^2*d*x-54*a*b*c*x+48*a^2
*c)/e^6*(x^2*(b*x+a))^(1/2)/x/(e*x)^(1/2)+5/16*a^2*(a*d+6*b*c)*ln((1/2*a*e
+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)/e^6*(x^2*(b*x+a))^(
1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.55

$$\int\frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{(ex)^{13/2}}\,dx=\left[\frac{15\,(6\,a^2bc+a^3d)\sqrt{be}x^2\log\left(\frac{2\,bex^2+aex+2\,\sqrt{bx^3+ax^2}\sqrt{be}\sqrt{ex}}{x}\right)+2\,(8\,b^3dx^3-48}{48\,be}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(13/2),x, algorithm="fricas")
```

output
```
[1/48*(15*(6*a^2*b*c + a^3*d)*sqrt(b*e)*x^2*log((2*b*e*x^2 + a*e*x + 2*sqr
t(b*x^3 + a*x^2)*sqrt(b*e)*sqrt(e*x))/x) + 2*(8*b^3*d*x^3 - 48*a^2*b*c + 2
*(6*b^3*c + 13*a*b^2*d)*x^2 + 3*(18*a*b^2*c + 11*a^2*b*d)*x)*sqrt(b*x^3 +
a*x^2)*sqrt(e*x))/(b*e^7*x^2), -1/24*(15*(6*a^2*b*c + a^3*d)*sqrt(-b*e)*x^
2*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b*e)*sqrt(e*x)/(b*e*x^2 + a*e*x)) - (8*
b^3*d*x^3 - 48*a^2*b*c + 2*(6*b^3*c + 13*a*b^2*d)*x^2 + 3*(18*a*b^2*c + 11
*a^2*b*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b*e^7*x^2)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{13/2}}\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/(e*x)**(13/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{13/2}}\,dx=\int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{(ex)^{\frac{13}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(13/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/(e*x)^(13/2), x)
```

Giac [A] (verification not implemented)

Time $=0.26$ (sec) , antiderivative size $=193$, normalized size of antiderivative $=0.99$

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{13/2}}\,dx=\frac{\Bigg(\frac{\Bigg(\bigg(2\,(bx+a)\Big(\frac{4\,(bx+a)d\,\mathrm{sgn}(x)}{b}+\frac{6\,b^2c\,\mathrm{sgn}(x)+abd\,\mathrm{sgn}(x)}{b^2}\Big)+\frac{5\,\left(6\,ab^2c\,\mathrm{sgn}(x)+a^2bd\,\mathrm{sgn}(x)\right)}{b^2}\bigg)(bx+a)-\frac{15\,(6\cdots}{\cdots}\cdots}{\sqrt{(bx+a)be-abe}\cdots}\cdots}{\cdots}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(13/2),x, algorithm="giac")
```

output
```
1/24*(((2*(b*x + a)*(4*(b*x + a)*d*sgn(x)/b + (6*b^2*c*sgn(x) + a*b*d*sgn(
x))/b^2) + 5*(6*a*b^2*c*sgn(x) + a^2*b*d*sgn(x))/b^2)*(b*x + a) - 15*(6*a^
2*b^2*c*sgn(x) + a^3*b*d*sgn(x))/b^2)*sqrt(b*x + a)/sqrt((b*x + a)*b*e - a
*b*e) - 15*(6*a^2*b*c*sgn(x) + a^3*d*sgn(x))*log(abs(-sqrt(b*e)*sqrt(b*x +
 a) + sqrt((b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*b))*b^2/(e^6*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)(ax^2+bx^3)^{5/2}}{(ex)^{13/2}}\,dx = \int \frac{(b\,x^3+a\,x^2)^{5/2}\,(c+d\,x)}{(e\,x)^{13/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(13/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(13/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx)(ax^2+bx^3)^{5/2}}{(ex)^{13/2}}\,dx = \frac{\sqrt{e}\left(-384\sqrt{x}\,\sqrt{bx+a}\,a^2bc+264\sqrt{x}\,\sqrt{bx+a}\,a^2bdx+432\sqrt{x}\,\sqrt{bx+a}\,a\,b\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(13/2),x)
```

output
```
(sqrt(e)*( - 384*sqrt(x)*sqrt(a + b*x)*a**2*b*c + 264*sqrt(x)*sqrt(a + b*x
)*a**2*b*d*x + 432*sqrt(x)*sqrt(a + b*x)*a*b**2*c*x + 208*sqrt(x)*sqrt(a +
 b*x)*a*b**2*d*x**2 + 96*sqrt(x)*sqrt(a + b*x)*b**3*c*x**2 + 64*sqrt(x)*sq
rt(a + b*x)*b**3*d*x**3 + 120*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b)
)/sqrt(a))*a**3*d*x + 720*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sq
rt(a))*a**2*b*c*x - 45*sqrt(b)*a**3*d*x - 480*sqrt(b)*a**2*b*c*x))/(192*b*
e**7*x)
```

3.326 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{15/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 197

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{15/2}}\,dx = \frac{5b(4bc+3ad)\sqrt{ax^2+bx^3}}{4e^7\sqrt{ex}} - \frac{2(5bc+3ad)\left(ax^2+bx^3\right)^{3/2}}{3e^4(ex)^{7/2}} + \frac{bd(ax^2+bx^3)^{3/2}}{2e^5(ex)^{5/2}} - \frac{2c(ax^2+bx^3)^{5/2}}{3e(ex)^{13/2}} + \frac{5a\sqrt{b}(4bc+3ad)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{4e^{15/2}}$$

output
```
5/4*b*(3*a*d+4*b*c)*(b*x^3+a*x^2)^(1/2)/e^7/(e*x)^(1/2)-2/3*(3*a*d+5*b*c)*
(b*x^3+a*x^2)^(3/2)/e^4/(e*x)^(7/2)+1/2*b*d*(b*x^3+a*x^2)^(3/2)/e^5/(e*x)^
(5/2)-2/3*c*(b*x^3+a*x^2)^(5/2)/e/(e*x)^(13/2)+5/4*a*b^(1/2)*(3*a*d+4*b*c)
*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/e^(15/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.71

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{15/2}}\,dx=\frac{\sqrt{x^2(a+bx)}\left(\sqrt{a+bx}(6b^2x^2(2c+dx)-8a^2(c+3dx)+abx(-56c+27d\right.}{12e^5(ex)^{5/2}\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(15/2),x]
```

output
```
(Sqrt[x^2*(a + b*x)]*(Sqrt[a + b*x]*(6*b^2*x^2*(2*c + d*x) - 8*a^2*(c + 3*
d*x) + a*b*x*(-56*c + 27*d*x)) + 30*a*Sqrt[b]*(4*b*c + 3*a*d)*x^(3/2)*ArcT
anh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(12*e^5*(e*x)^(5/2)*Sq
rt[a + b*x])
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1944, 1926, 1927, 1927, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{(ex)^{15/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$\frac{(3ad+4bc)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{(ex)^{13/2}}dx}{3ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{3a(ex)^{17/2}}$$

$$\Big\downarrow \text{1926}$$

$$\frac{(3ad+4bc)\left(\frac{5b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(ex)^{7/2}}dx}{e^3}-\frac{2\left(ax^2+bx^3\right)^{5/2}}{e(ex)^{11/2}}\right)}{3ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{3a(ex)^{17/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(3ad+4bc)\left(\frac{5b\left(\frac{3a\int\frac{\sqrt{bx^3+ax^2}}{(ex)^{3/2}}dx}{4e^2}+\frac{(ax^2+bx^3)^{3/2}}{2e(ex)^{5/2}}\right)}{e^3}-\frac{2(ax^2+bx^3)^{5/2}}{e(ex)^{11/2}}\right)}{3ae}-\frac{2ce(ax^2+bx^3)^{7/2}}{3a(ex)^{17/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(3ad+4bc)\left(\frac{5b\left(\frac{3a\left(\frac{a\int\frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2e^2}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{(ax^2+bx^3)^{3/2}}{2e(ex)^{5/2}}\right)}{e^3}-\frac{2(ax^2+bx^3)^{5/2}}{e(ex)^{11/2}}\right)}{3ae}-\frac{2ce(ax^2+bx^3)^{7/2}}{3a(ex)^{17/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{(3ad+4bc)\left(\frac{5b\left(\frac{3a\left(\frac{a\sqrt{ex}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{(ax^2+bx^3)^{3/2}}{2e(ex)^{5/2}}\right)}{e^3}-\frac{2(ax^2+bx^3)^{5/2}}{e(ex)^{11/2}}\right)}{3ae}-\frac{2ce(ax^2+bx^3)^{7/2}}{3a(ex)^{17/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{(3ad+4bc)\left(\frac{5b\left(\frac{3a\left(\frac{a\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{e^3}-\frac{2\left(ax^2+bx^3\right)^{5/2}}{e(ex)^{11/2}}\right)}{3ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{3a(ex)^{17/2}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(3ad+4bc)\left(\frac{5b\left(\frac{3a\left(\frac{a\sqrt{ex}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{4e^2}+\frac{\left(ax^2+bx^3\right)^{3/2}}{2e(ex)^{5/2}}\right)}{e^3}-\frac{2\left(ax^2+bx^3\right)^{5/2}}{e(ex)^{11/2}}\right)}{3ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{3a(ex)^{17/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(15/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(7/2))/(3*a*(e*x)^(17/2)) + ((4*b*c + 3*a*d)*((-2*
(a*x^2 + b*x^3)^(5/2))/(e*(e*x)^(11/2)) + (5*b*((a*x^2 + b*x^3)^(3/2)/(2*e
*(e*x)^(5/2)) + (3*a*(Sqrt[a*x^2 + b*x^3]/(e*Sqrt[e*x]) + (a*Sqrt[e*x]*Arc
Tanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(Sqrt[b]*e^2*Sqrt[x])))/(4*e^
2)))/e^3))/(3*a*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.81

method	result
risch	$-\frac{\left(-6 b^{2} d\,x^{3}-27 a b d\,x^{2}-12 b^{2} c\,x^{2}+24 a^{2} d x+56 a b c x+8 a^{2} c\right) \sqrt{x^{2} \left(b x+a\right)}}{12 x^{2} e^{7} \sqrt{e x}}+\frac{5 a b \left(3 a d+4 b c\right) \ln\left(\frac{\frac{1}{2} a e+b e x}{\sqrt{b e}}+\sqrt{b e\,x^{2}+a e x}\right) \sqrt{x^{2} \left(b x+a\right)}}{8 \sqrt{b e}\, e^{7} x \left(b x+a\right) \sqrt{e x}}$
default	$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}} \left(-45 \ln\left(\frac{2 b e x+2 \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}+a e}{2 \sqrt{b e}}\right) a^{2} b d e\,x^{2}-60 \ln\left(\frac{2 b e x+2 \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}+a e}{2 \sqrt{b e}}\right) a\,b^{2} c e\,x^{2}-12 b^{2} d\,x^{3} \sqrt{e x \left(b x+a\right)}\,\sqrt{}}{24 x^{6} \left(b x+a\right)^{2} e^{7}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(15/2),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-6*b^2*d*x^3-27*a*b*d*x^2-12*b^2*c*x^2+24*a^2*d*x+56*a*b*c*x+8*a^2*
c)/x^2/e^7*(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)+5/8*a*b*(3*a*d+4*b*c)*ln((1/2*a
*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)/e^7*(x^2*(b*x+a))
^(1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.46

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{15/2}}\,dx = \left[\frac{15\,(4\,abc+3\,a^2d)ex^3\sqrt{\frac{b}{e}}\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\sqrt{ex}\sqrt{\frac{b}{e}}}{x}\right)+2\,(6\,b^2dx^3-8}{24\,e^8x^3}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(15/2),x, algorithm="fricas")
```

output
```
[1/24*(15*(4*a*b*c + 3*a^2*d)*e*x^3*sqrt(b/e)*log((2*b*x^2 + a*x + 2*sqrt(
b*x^3 + a*x^2)*sqrt(e*x)*sqrt(b/e))/x) + 2*(6*b^2*d*x^3 - 8*a^2*c + 3*(4*b
^2*c + 9*a*b*d)*x^2 - 8*(7*a*b*c + 3*a^2*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*
x))/(e^8*x^3), -1/12*(15*(4*a*b*c + 3*a^2*d)*e*x^3*sqrt(-b/e)*arctan(sqrt(
b*x^3 + a*x^2)*sqrt(e*x)*sqrt(-b/e)/(b*x^2 + a*x)) - (6*b^2*d*x^3 - 8*a^2*
c + 3*(4*b^2*c + 9*a*b*d)*x^2 - 8*(7*a*b*c + 3*a^2*d)*x)*sqrt(b*x^3 + a*x^
2)*sqrt(e*x))/(e^8*x^3)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{15/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/(e*x)**(15/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{15/2}}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{(ex)^{\frac{15}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(15/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/(e*x)^(15/2), x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.19

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{15/2}}\,dx =$$

$$-\frac{b^3\left(\frac{15\,(4\,abc\,\mathrm{sgn}(x)+3\,a^2d\,\mathrm{sgn}(x))\log\left(\left|-\sqrt{be}\sqrt{bx+a}+\sqrt{(bx+a)be-abe}\right|\right)}{\sqrt{be}b}-\frac{\left(\left(3\left(2\,(bx+a)de\,\mathrm{sgn}(x)+\frac{4\,ab^2ce^2\mathrm{sgn}(x)+3\,a^2bde^2\mathrm{sgn}(x)}{abe}\right)(bx+c\right.\right.}{}\right.}{12\,e^7|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(15/2),x, algorithm="giac")
```

output
```
-1/12*b^3*(15*(4*a*b*c*sgn(x) + 3*a^2*d*sgn(x))*log(abs(-sqrt(b*e)*sqrt(b*
x + a) + sqrt((b*x + a)*b*e - a*b*e)))/(sqrt(b*e)*b) - ((3*(2*(b*x + a)*d*
e*sgn(x) + (4*a*b^2*c*e^2*sgn(x) + 3*a^2*b*d*e^2*sgn(x))/(a*b*e))*(b*x + a
) - 20*(4*a^2*b^2*c*e^2*sgn(x) + 3*a^3*b*d*e^2*sgn(x))/(a*b*e))*(b*x + a)
+ 15*(4*a^3*b^2*c*e^2*sgn(x) + 3*a^4*b*d*e^2*sgn(x))/(a*b*e))*sqrt(b*x + a
)/((b*x + a)*b*e - a*b*e)^(3/2))/(e^7*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{15/2}}\,dx = \int \frac{\left(b\,x^3+a\,x^2\right)^{5/2}\,\left(c+d\,x\right)}{\left(e\,x\right)^{15/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(15/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(15/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.92

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{(ex)^{15/2}}\,dx = \frac{\sqrt{e}\left(-64\sqrt{x}\,\sqrt{bx+a}\,a^2c-192\sqrt{x}\,\sqrt{bx+a}\,a^2dx-448\sqrt{x}\,\sqrt{bx+a}\,abcx\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(15/2),x)
```

output
```
(sqrt(e)*( - 64*sqrt(x)*sqrt(a + b*x)*a**2*c - 192*sqrt(x)*sqrt(a + b*x)*a
**2*d*x - 448*sqrt(x)*sqrt(a + b*x)*a*b*c*x + 216*sqrt(x)*sqrt(a + b*x)*a*
b*d*x**2 + 96*sqrt(x)*sqrt(a + b*x)*b**2*c*x**2 + 48*sqrt(x)*sqrt(a + b*x)
*b**2*d*x**3 + 360*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*
a**2*d*x**2 + 480*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a
*b*c*x**2 + 95*sqrt(b)*a**2*d*x**2 + 80*sqrt(b)*a*b*c*x**2))/(96*e**8*x**2
)
```

3.327 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{17/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 187

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{17/2}}\,dx = -\frac{2b(bc+2ad)\sqrt{ax^2+bx^3}}{e^7(ex)^{3/2}} + \frac{b^2d\sqrt{ax^2+bx^3}}{e^8\sqrt{ex}} - \frac{2(bc+ad)\left(ax^2+bx^3\right)^{3/2}}{3e^4(ex)^{9/2}} - \frac{2c\left(ax^2+bx^3\right)^{5/2}}{5e(ex)^{15/2}} + \frac{b^{3/2}(2bc+5ad)\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{e^{17/2}}$$

output
```
-2*b*(2*a*d+b*c)*(b*x^3+a*x^2)^(1/2)/e^7/(e*x)^(3/2)+b^2*d*(b*x^3+a*x^2)^(
1/2)/e^8/(e*x)^(1/2)-2/3*(a*d+b*c)*(b*x^3+a*x^2)^(3/2)/e^4/(e*x)^(9/2)-2/5
*c*(b*x^3+a*x^2)^(5/2)/e/(e*x)^(15/2)+b^(3/2)*(5*a*d+2*b*c)*arctanh(b^(1/2
)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/e^(17/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.78

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{17/2}}\,dx = \frac{\sqrt{ex}\sqrt{x^2(a+bx)}\left(-\sqrt{a+bx}(b^2x^2(46c-15dx)+2a^2(3c+5dx)+2abx(1\right.}{15e^9x^4\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(17/2),x]
```

output
```
(Sqrt[e*x]*Sqrt[x^2*(a + b*x)]*(-(Sqrt[a + b*x]*(b^2*x^2*(46*c - 15*d*x) +
 2*a^2*(3*c + 5*d*x) + 2*a*b*x*(11*c + 35*d*x))) + 30*b^(3/2)*(2*b*c + 5*a
*d)*x^(5/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(15*e^
9*x^4*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1944, 1926, 1926, 1927, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{(ex)^{17/2}}dx$$

$$\Big\downarrow \text{ 1944}$$

$$\frac{(5ad+2bc)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{(ex)^{15/2}}dx}{5ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{5a(ex)^{19/2}}$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{(5ad+2bc)\left(\frac{5b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(ex)^{9/2}}dx}{3e^3}-\frac{2\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{13/2}}\right)}{5ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{5a(ex)^{19/2}}$$

$$\Big\downarrow \text{1926}$$

$$\frac{(5ad+2bc)\left(\frac{5b\left(\frac{3b\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{3/2}}dx}{e^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{e(ex)^{7/2}}\right)}{3e^3}-\frac{2\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{13/2}}\right)}{5ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{5a(ex)^{19/2}}$$

$$\Big\downarrow \text{1927}$$

$$\frac{(5ad+2bc)\left(\frac{5b\left(\frac{3b\left(\frac{a\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2e^2}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{e^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{e(ex)^{7/2}}\right)}{3e^3}-\frac{2\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{13/2}}\right)}{5ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{5a(ex)^{19/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{(5ad+2bc)\left(\frac{5b\left(\frac{3b\left(\frac{a\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{e^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{e(ex)^{7/2}}\right)}{3e^3}-\frac{2\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{13/2}}\right)}{5ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{5a(ex)^{19/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{(5ad+2bc)\left(\frac{5b\left(\frac{3b\left(\frac{a\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{e^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{e(ex)^{7/2}}\right)}{3e^3}-\frac{2\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{13/2}}\right)}{5ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{5a(ex)^{19/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{(5ad+2bc)\left(\frac{5b\left(\frac{3b\left(\frac{a\sqrt{ex}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}e^2\sqrt{x}}+\frac{\sqrt{ax^2+bx^3}}{e\sqrt{ex}}\right)}{e^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{e(ex)^{7/2}}\right)}{3e^3}-\frac{2\left(ax^2+bx^3\right)^{5/2}}{3e(ex)^{13/2}}\right)}{5ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{5a(ex)^{19/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(17/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(7/2))/(5*a*(e*x)^(19/2)) + ((2*b*c + 5*a*d)*((-2*
(a*x^2 + b*x^3)^(5/2))/(3*e*(e*x)^(13/2)) + (5*b*((-2*(a*x^2 + b*x^3)^(3/2
))/(e*(e*x)^(7/2)) + (3*b*(Sqrt[a*x^2 + b*x^3]/(e*Sqrt[e*x]) + (a*Sqrt[e*x
]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(Sqrt[b]*e^2*Sqrt[x])))/
e^3))/(3*e^3)))/(5*a*e)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1927
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*
(n - j)*(p/(c^j*(m + n*p + 1)))   Int[(c*x)^(m + j)*(a*x^j + b*x^n)^(p - 1)
, x], x] /; FreeQ[{a, b, c, m}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Int
egersQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.86

method	result
risch	$-\frac{\left(-15 b^{2} d\,x^{3}+70 a b d\,x^{2}+46 b^{2} c\,x^{2}+10 a^{2} d x+22 a b c x+6 a^{2} c\right) \sqrt{x^{2} \left(b x+a\right)}}{15 x^{3} e^{8} \sqrt{e x}}+\frac{b^{2} \left(5 a d+2 b c\right) \ln \left(\frac{\frac{1}{2} a e+b e x}{\sqrt{b e}}+\sqrt{b e\,x^{2}+a e x}\right) \sqrt{x^{2} \left(b x+a\right.}}{2 \sqrt{b e}\, e^{8} x \left(b x+a\right) \sqrt{e x}}$
default	$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}} \left(-75 \ln \left(\frac{2 b e x+2 \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}+a e}{2 \sqrt{b e}}\right) a\,b^{2} d e\,x^{3}-30 \ln \left(\frac{2 b e x+2 \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}+a e}{2 \sqrt{b e}}\right) b^{3} c e\,x^{3}-30 b^{2} d\,x^{3} \sqrt{e x \left(b x+a\right)}\, \sqrt{}\right.}{30 x^{7} \left(b x+a\right)^{2} e^{8}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(17/2),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(-15*b^2*d*x^3+70*a*b*d*x^2+46*b^2*c*x^2+10*a^2*d*x+22*a*b*c*x+6*a^2
*c)/x^3/e^8*(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)+1/2*b^2*(5*a*d+2*b*c)*ln((1/2*
a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)/e^8*(x^2*(b*x+a)
)^(1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.53

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{17/2}}\,dx=\left[\frac{15\,(2\,b^2c+5\,abd)ex^4\sqrt{\frac{b}{e}}\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\sqrt{ex}\sqrt{\frac{b}{e}}}{x}\right)+2\,(15\,b^2dx^3-}{30\,e^9x}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(17/2),x, algorithm="fricas")
```

output
```
[1/30*(15*(2*b^2*c + 5*a*b*d)*e*x^4*sqrt(b/e)*log((2*b*x^2 + a*x + 2*sqrt(
b*x^3 + a*x^2)*sqrt(e*x)*sqrt(b/e))/x) + 2*(15*b^2*d*x^3 - 6*a^2*c - 2*(23
*b^2*c + 35*a*b*d)*x^2 - 2*(11*a*b*c + 5*a^2*d)*x)*sqrt(b*x^3 + a*x^2)*sqr
t(e*x))/(e^9*x^4), -1/15*(15*(2*b^2*c + 5*a*b*d)*e*x^4*sqrt(-b/e)*arctan(s
qrt(b*x^3 + a*x^2)*sqrt(e*x)*sqrt(-b/e)/(b*x^2 + a*x)) - (15*b^2*d*x^3 - 6
*a^2*c - 2*(23*b^2*c + 35*a*b*d)*x^2 - 2*(11*a*b*c + 5*a^2*d)*x)*sqrt(b*x^
3 + a*x^2)*sqrt(e*x))/(e^9*x^4)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{(ex)^{17/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/(e*x)**(17/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{(ex)^{17/2}}\,dx = \int \frac{(bx^3+ax^2)^{\frac{5}{2}}(dx+c)}{(ex)^{\frac{17}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(17/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/(e*x)^(17/2), x)
```

Giac [A] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.31

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{17/2}}\,dx = -\frac{\left(\frac{15\left(2b^3c\,\mathrm{sgn}(x)+5ab^2d\,\mathrm{sgn}(x)\right)\log\left(\left|-\sqrt{be}\sqrt{bx+a}+\sqrt{(bx+a)be-abe}\right|\right)}{\sqrt{be}} - \frac{\left(\left(\left(15(bx+a)b^4de^2\,\mathrm{sgn}(x)-\frac{23\left(2a^2b^7ce^4\,\mathrm{sgn}(x)+5a^3b^6de^4\,\mathrm{sgn}(x)\right)}{a^2b^2e^2}\right.\right.\right.}{}\right.}{15e^8|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(17/2),x, algorithm="giac")
```

output
```
-1/15*(15*(2*b^3*c*sgn(x) + 5*a*b^2*d*sgn(x))*log(abs(-sqrt(b*e)*sqrt(b*x
+ a) + sqrt((b*x + a)*b*e - a*b*e)))/sqrt(b*e) - (((15*(b*x + a)*b^4*d*e^2
*sgn(x) - 23*(2*a^2*b^7*c*e^4*sgn(x) + 5*a^3*b^6*d*e^4*sgn(x))/(a^2*b^2*e^
2))*(b*x + a) + 35*(2*a^3*b^7*c*e^4*sgn(x) + 5*a^4*b^6*d*e^4*sgn(x))/(a^2*
b^2*e^2))*(b*x + a) - 15*(2*a^4*b^7*c*e^4*sgn(x) + 5*a^5*b^6*d*e^4*sgn(x))
/(a^2*b^2*e^2))*sqrt(b*x + a)/((b*x + a)*b*e - a*b*e)^(5/2))*b/(e^8*abs(b)
)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{17/2}}\,dx = \int \frac{\left(b\,x^3+a\,x^2\right)^{5/2}\,(c+d\,x)}{(e\,x)^{17/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(17/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(17/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{(ex)^{17/2}}\,dx = \frac{\sqrt{e}\left(-24\sqrt{x}\,\sqrt{bx+a}\,a^2c-40\sqrt{x}\,\sqrt{bx+a}\,a^2dx-88\sqrt{x}\,\sqrt{bx+a}\,abcx-2\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(17/2),x)
```

output
```
(sqrt(e)*( - 24*sqrt(x)*sqrt(a + b*x)*a**2*c - 40*sqrt(x)*sqrt(a + b*x)*a*
*2*d*x - 88*sqrt(x)*sqrt(a + b*x)*a*b*c*x - 280*sqrt(x)*sqrt(a + b*x)*a*b*
d*x**2 - 184*sqrt(x)*sqrt(a + b*x)*b**2*c*x**2 + 60*sqrt(x)*sqrt(a + b*x)*
b**2*d*x**3 + 300*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a
*b*d*x**3 + 120*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**
2*c*x**3 + 163*sqrt(b)*a*b*d*x**3 + 40*sqrt(b)*b**2*c*x**3))/(60*e**9*x**3
)
```

3.328 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{19/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 177

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{19/2}}\,dx = -\frac{2a^2d\sqrt{ax^2+bx^3}}{5e^6(ex)^{7/2}} - \frac{22abd\sqrt{ax^2+bx^3}}{15e^7(ex)^{5/2}} - \frac{46b^2d\sqrt{ax^2+bx^3}}{15e^8(ex)^{3/2}} - \frac{2ce(ax^2+bx^3)^{7/2}}{7a(ex)^{21/2}} + \frac{2b^{5/2}d\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{e^{19/2}}$$

output
```
-2/5*a^2*d*(b*x^3+a*x^2)^(1/2)/e^6/(e*x)^(7/2)-22/15*a*b*d*(b*x^3+a*x^2)^(
1/2)/e^7/(e*x)^(5/2)-46/15*b^2*d*(b*x^3+a*x^2)^(1/2)/e^8/(e*x)^(3/2)-2/7*c
*e*(b*x^3+a*x^2)^(7/2)/a/(e*x)^(21/2)+2*b^(5/2)*d*arctanh(b^(1/2)*(e*x)^(3
/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/e^(19/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.81

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{19/2}}\,dx =$$

$$-\frac{2\sqrt{x^2(a+bx)}\left(\sqrt{a+bx}(15b^3cx^3+3a^3(5c+7dx)+a^2bx(45c+77dx)+ab^2x^2(45c+161dx))+105ab^{5/}\right.}{105ae^9x^4\sqrt{ex}\sqrt{a+bx}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(19/2),x]
```

output
```
(-2*Sqrt[x^2*(a + b*x)]*(Sqrt[a + b*x]*(15*b^3*c*x^3 + 3*a^3*(5*c + 7*d*x)
 + a^2*b*x*(45*c + 77*d*x) + a*b^2*x^2*(45*c + 161*d*x)) + 105*a*b^(5/2)*d
*x^(7/2)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(105*a*e^9*x^4*Sqrt[e*x
]*Sqrt[a + b*x])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1944, 1926, 1926, 1926, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{(ex)^{19/2}}dx$$

$$\Big\downarrow \text{ 1944}$$

$$\frac{d\int \frac{\left(bx^3+ax^2\right)^{5/2}}{(ex)^{17/2}}dx}{e}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{7a(ex)^{21/2}}$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{d\left(\frac{b\int \frac{\left(bx^3+ax^2\right)^{3/2}}{(ex)^{11/2}}dx}{e^3}-\frac{2\left(ax^2+bx^3\right)^{5/2}}{5e(ex)^{15/2}}\right)}{e}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{7a(ex)^{21/2}}$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{d\left(\frac{b\left(\frac{b\int \frac{\sqrt{bx^3+ax^2}}{(ex)^{5/2}}dx}{e^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{3e(ex)^{9/2}}\right)}{e^3}-\frac{2\left(ax^2+bx^3\right)^{5/2}}{5e(ex)^{15/2}}\right)}{e}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{7a(ex)^{21/2}}$$

$$\Big\downarrow \text{ 1926}$$

$$\frac{d\left(\frac{b\left(\frac{b\left(\frac{b\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{e^3}-\frac{2\sqrt{ax^2+bx^3}}{e(ex)^{3/2}}\right)}{e^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{3e(ex)^{9/2}}\right)}{e^3}-\frac{2\left(ax^2+bx^3\right)^{5/2}}{5e(ex)^{15/2}}\right)}{e}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{7a(ex)^{21/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{d\left(\frac{b\left(\frac{b\left(\frac{b\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{e^3\sqrt{x}}-\frac{2\sqrt{ax^2+bx^3}}{e(ex)^{3/2}}\right)}{e^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{3e(ex)^{9/2}}\right)}{e^3}-\frac{2\left(ax^2+bx^3\right)^{5/2}}{5e(ex)^{15/2}}\right)}{e}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{7a(ex)^{21/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{d\left(\frac{b\left(\frac{b\left(\frac{2b\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{e^3\sqrt{x}}-\frac{2\sqrt{ax^2+bx^3}}{e(ex)^{3/2}}\right)}{e^3}-\frac{2\left(ax^2+bx^3\right)^{3/2}}{3e(ex)^{9/2}}\right)}{e^3}-\frac{2\left(ax^2+bx^3\right)^{5/2}}{5e(ex)^{15/2}}\right)}{e}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{7a(ex)^{21/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{d\left(\frac{b\left(\frac{b\left(\frac{2\sqrt{b}\sqrt{ex}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{e^3\sqrt{x}}-\frac{2\sqrt{ax^2+bx^3}}{e(ex)^{3/2}}\right)}{e^3}-\frac{2(ax^2+bx^3)^{3/2}}{3e(ex)^{9/2}}\right)}{e^3}-\frac{2(ax^2+bx^3)^{5/2}}{5e(ex)^{15/2}}\right)}{e}-\frac{2ce(ax^2+bx^3)^{7/2}}{7a(ex)^{21/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(19/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(7/2))/(7*a*(e*x)^(21/2)) + (d*((-2*(a*x^2 + b*x^3
)^(5/2))/(5*e*(e*x)^(15/2)) + (b*((-2*(a*x^2 + b*x^3)^(3/2))/(3*e*(e*x)^(9
/2)) + (b*((-2*Sqrt[a*x^2 + b*x^3])/(e*(e*x)^(3/2)) + (2*Sqrt[b]*Sqrt[e*x]
*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(e^3*Sqrt[x])))/e^3))/e^3
))/e
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1926
```
Int[((c_.)*(x_))^(m_)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol]
 :> Simp[(c*x)^(m + 1)*((a*x^j + b*x^n)^p/(c*(m + j*p + 1))), x] - Simp[b*p
*((n - j)/(c^n*(m + j*p + 1)))   Int[(c*x)^(m + n)*(a*x^j + b*x^n)^(p - 1),
 x], x] /; FreeQ[{a, b, c}, x] &&  !IntegerQ[p] && LtQ[0, j, n] && (Integer
sQ[j, n] || GtQ[c, 0]) && GtQ[p, 0] && LtQ[m + j*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 169, normalized size of antiderivative = 0.95

method	result
risch	$-\frac{2\left(161 a\,b^{2} d\,x^{3}+15 b^{3} c\,x^{3}+77 a^{2} b d\,x^{2}+45 a\,b^{2} c\,x^{2}+21 a^{3} d x+45 a^{2} b c x+15 c\,a^{3}\right) \sqrt{x^{2} \left(b x+a\right)}}{105 x^{4} a\,e^{9} \sqrt{e x}}+\frac{b^{3} d \ln\left(\frac{\frac{1}{2} a e+b e x}{\sqrt{b e}}+\sqrt{b e\,x^{2}+a e x}\right) \sqrt{}}{\sqrt{b e}\,e^{9} x \left(b x+a\right) \sqrt{}}$
default	$-\frac{\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}} \left(-105 \ln\left(\frac{2 b e x+2 \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}+a e}{2 \sqrt{b e}}\right) a\,b^{3} d e\,x^{4}+322 a\,b^{2} d\,x^{3} \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}+30 b^{3} c\,x^{3} \sqrt{e x \left(b x+a\right)}\, \sqrt{b e}+154 a^{2} b d\right.}{105 x^{8} \left(b x+a\right)^{2} e^{9} a \sqrt{e}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(19/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*(161*a*b^2*d*x^3+15*b^3*c*x^3+77*a^2*b*d*x^2+45*a*b^2*c*x^2+21*a^3*
d*x+45*a^2*b*c*x+15*a^3*c)/x^4/a/e^9*(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)+b^3*d
*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)/e^9*(x^
2*(b*x+a))^(1/2)/x/(b*x+a)*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.72

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{19/2}}\,dx = \left[\frac{105\,ab^2dex^5\sqrt{\frac{b}{e}}\log\left(\frac{2\,bx^2+ax+2\,\sqrt{bx^3+ax^2}\sqrt{ex}\sqrt{\frac{b}{e}}}{x}\right) - 2\,(15\,a^3c + (15\,b^3c + 16}{105\,a}\right.$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(19/2),x, algorithm="fricas")
```

output
```
[1/105*(105*a*b^2*d*e*x^5*sqrt(b/e)*log((2*b*x^2 + a*x + 2*sqrt(b*x^3 + a*
x^2)*sqrt(e*x)*sqrt(b/e))/x) - 2*(15*a^3*c + (15*b^3*c + 161*a*b^2*d)*x^3
+ (45*a*b^2*c + 77*a^2*b*d)*x^2 + 3*(15*a^2*b*c + 7*a^3*d)*x)*sqrt(b*x^3 +
 a*x^2)*sqrt(e*x))/(a*e^10*x^5), -2/105*(105*a*b^2*d*e*x^5*sqrt(-b/e)*arct
an(sqrt(b*x^3 + a*x^2)*sqrt(e*x)*sqrt(-b/e)/(b*x^2 + a*x)) + (15*a^3*c + (
15*b^3*c + 161*a*b^2*d)*x^3 + (45*a*b^2*c + 77*a^2*b*d)*x^2 + 3*(15*a^2*b*
c + 7*a^3*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(a*e^10*x^5)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{19/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/(e*x)**(19/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{19/2}}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{(ex)^{\frac{19}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(19/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/(e*x)^(19/2), x)
```

Giac [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{19/2}}\,dx = -\frac{2\left(\frac{105\,d\log\left(\left|-\sqrt{be}\sqrt{bx+a}+\sqrt{(bx+a)be-abe}\right|\right)\mathrm{sgn}(x)}{\sqrt{be}b} - \frac{\left(105\,a^3b^2de^3\mathrm{sgn}(x) - \left(350\,a^2b^2de^3\mathrm{sgn}(x) - \left(406\,ab^2de^3\mathrm{sgn}(x) - \frac{\left(15\,a^2b^4ce^3\mathrm{sgn}(x\right.}{}\right.\right.\right.}{((bx+a)be-abe)^{\frac{7}{2}}}\right.}{105\,e^9|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(19/2),x, algorithm="giac")
```

output
```
-2/105*(105*d*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt((b*x + a)*b*e - a*b*
e)))*sgn(x)/(sqrt(b*e)*b) - (105*a^3*b^2*d*e^3*sgn(x) - (350*a^2*b^2*d*e^3
*sgn(x) - (406*a*b^2*d*e^3*sgn(x) - (15*a^2*b^4*c*e^3*sgn(x) + 161*a^3*b^3
*d*e^3*sgn(x))*(b*x + a)/(a^3*b))*(b*x + a))*(b*x + a))*sqrt(b*x + a)/((b*
x + a)*b*e - a*b*e)^(7/2))*b^5/(e^9*abs(b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{(ex)^{19/2}}\,dx = \int \frac{(b\,x^3+a\,x^2)^{5/2}\,(c+d\,x)}{(e\,x)^{19/2}}\,dx$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(19/2),x)
```

output
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(19/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{(ex)^{19/2}}\,dx = \frac{2\sqrt{e}\left(-15\sqrt{x}\,\sqrt{bx+a}\,a^3c-21\sqrt{x}\,\sqrt{bx+a}\,a^3dx-45\sqrt{x}\,\sqrt{bx+a}\,a^2bcx-\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(19/2),x)
```

output
```
(2*sqrt(e)*( - 15*sqrt(x)*sqrt(a + b*x)*a**3*c - 21*sqrt(x)*sqrt(a + b*x)*
a**3*d*x - 45*sqrt(x)*sqrt(a + b*x)*a**2*b*c*x - 77*sqrt(x)*sqrt(a + b*x)*
a**2*b*d*x**2 - 45*sqrt(x)*sqrt(a + b*x)*a*b**2*c*x**2 - 161*sqrt(x)*sqrt(
a + b*x)*a*b**2*d*x**3 - 15*sqrt(x)*sqrt(a + b*x)*b**3*c*x**3 + 105*sqrt(b
)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b**2*d*x**4 + 71*sqrt(b
)*a*b**2*d*x**4 - 15*sqrt(b)*b**3*c*x**4))/(105*a*e**10*x**4)
```

3.329 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{21/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{21/2}}\,dx=-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{9a(ex)^{23/2}}+\frac{2(2bc-9ad)\left(ax^2+bx^3\right)^{7/2}}{63a^2(ex)^{21/2}}$$

output
```
-2/9*c*e*(b*x^3+a*x^2)^(7/2)/a/(e*x)^(23/2)+2/63*(-9*a*d+2*b*c)*(b*x^3+a*x
^2)^(7/2)/a^2/(e*x)^(21/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{21/2}}\,dx=-\frac{2e\left(x^2(a+bx)\right)^{7/2}(7ac-2bcx+9adx)}{63a^2(ex)^{23/2}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(21/2),x]
```

output
```
(-2*e*(x^2*(a + b*x))^(7/2)*(7*a*c - 2*b*c*x + 9*a*d*x))/(63*a^2*(e*x)^(23
/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1944, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{(ex)^{21/2}}dx$$

$$\Big\downarrow \text{ 1944}$$

$$-\frac{(2bc-9ad)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{(ex)^{19/2}}dx}{9ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{9a(ex)^{23/2}}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{2\left(ax^2+bx^3\right)^{7/2}(2bc-9ad)}{63a^2(ex)^{21/2}}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{9a(ex)^{23/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(21/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(7/2))/(9*a*(e*x)^(23/2)) + (2*(2*b*c - 9*a*d)*(a*
x^2 + b*x^3)^(7/2))/(63*a^2*(e*x)^(21/2))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

method	result	size
gosper	$-\frac{2x(bx+a)(9adx-2cbx+7ac)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{63a^{2}(ex)^{\frac{21}{2}}}$	45
orering	$-\frac{2x(bx+a)(9adx-2cbx+7ac)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{63a^{2}(ex)^{\frac{21}{2}}}$	45
default	$-\frac{2\left(9abd\,x^{2}-2b^{2}c\,x^{2}+9a^{2}dx+5abcx+7a^{2}c\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{63\sqrt{ex}\,e^{10}a^{2}x^{9}}$	67
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(9x^{4}a\,b^{3}d-2x^{4}b^{4}c+27a^{2}b^{2}d\,x^{3}+a\,b^{3}c\,x^{3}+27a^{3}bd\,x^{2}+15a^{2}b^{2}c\,x^{2}+9a^{4}dx+19a^{3}bcx+7c\,a^{4}\right)}{63e^{10}x^{5}\sqrt{ex}\,a^{2}}$	112

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(21/2),x,method=_RETURNVERBOSE)
```

output
```
-2/63*x*(b*x+a)*(9*a*d*x-2*b*c*x+7*a*c)*(b*x^3+a*x^2)^(5/2)/a^2/(e*x)^(21/
2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.61

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{21/2}}\,dx = -\frac{2\,(7\,a^4c-(2\,b^4c-9\,ab^3d)x^4+(ab^3c+27\,a^2b^2d)x^3+3\,(5\,a^2b^2c+9\,a^3bd)x^2+(19\,a^3bc+9\,a^4d)x)\sqrt{bx^3+}}{63\,a^2e^{11}x^6}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(21/2),x, algorithm="fricas")
```

output
```
-2/63*(7*a^4*c - (2*b^4*c - 9*a*b^3*d)*x^4 + (a*b^3*c + 27*a^2*b^2*d)*x^3
+ 3*(5*a^2*b^2*c + 9*a^3*b*d)*x^2 + (19*a^3*b*c + 9*a^4*d)*x)*sqrt(b*x^3 +
 a*x^2)*sqrt(e*x)/(a^2*e^11*x^6)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{21/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/(e*x)**(21/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{21/2}}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{(ex)^{\frac{21}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(21/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/(e*x)^(21/2), x)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.50

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{21/2}}\,dx=\frac{2\,(bx+a)^{\frac{7}{2}}b\Big(\frac{\left(2\,a^2b^9ce^4\mathrm{sgn}(x)-9\,a^3b^8de^4\mathrm{sgn}(x)\right)(bx+a)}{a^4}-\frac{9\,\left(a^3b^9ce^4\mathrm{sgn}(x)-a^4b^8de^4\mathrm{sgn}(x)\right)}{a^4}\Big)}{63\left((bx+a)be-abe\right)^{\frac{9}{2}}e^{10}|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(21/2),x, algorithm="giac")
```

output
```
2/63*(b*x + a)^(7/2)*b*((2*a^2*b^9*c*e^4*sgn(x) - 9*a^3*b^8*d*e^4*sgn(x))*
(b*x + a)/a^4 - 9*(a^3*b^9*c*e^4*sgn(x) - a^4*b^8*d*e^4*sgn(x))/a^4)/(((b*
x + a)*b*e - a*b*e)^(9/2)*e^10*abs(b))
```

Mupad [B] (verification not implemented)

Time = 9.44 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.73

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{21/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,a^2\,c}{9\,e^{10}}+\frac{2\,b\,x^2\,\left(9\,a\,d+5\,b\,c\right)}{21\,e^{10}}+\frac{x\,\left(18\,d\,a^4+38\,b\,c\,a^3\right)}{63\,a^2\,e^{10}}-\frac{x^4\,\left(4\,b^4\,c-18\,a\,b^3\,d\right)}{63\,a^2\,e^{10}}+\frac{2\,b^2\,x^3\,\left(27\,a\,d+b\,c\right)}{63\,a\,e^{10}}\right)}{x^5\,\sqrt{e\,x}}$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(21/2),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*a^2*c)/(9*e^10) + (2*b*x^2*(9*a*d + 5*b*c))/(2
1*e^10) + (x*(18*a^4*d + 38*a^3*b*c))/(63*a^2*e^10) - (x^4*(4*b^4*c - 18*a
*b^3*d))/(63*a^2*e^10) + (2*b^2*x^3*(27*a*d + b*c))/(63*a*e^10)))/(x^5*(e*
x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.76

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{(ex)^{21/2}}\,dx = \frac{2\sqrt{e}\,\Big(-7\sqrt{x}\,\sqrt{bx+a}\,a^4c-9\sqrt{x}\,\sqrt{bx+a}\,a^4dx-19\sqrt{x}\,\sqrt{bx+a}\,a^3bcx-2}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(21/2),x)
```

output
```
(2*sqrt(e)*( - 7*sqrt(x)*sqrt(a + b*x)*a**4*c - 9*sqrt(x)*sqrt(a + b*x)*a*
*4*d*x - 19*sqrt(x)*sqrt(a + b*x)*a**3*b*c*x - 27*sqrt(x)*sqrt(a + b*x)*a*
*3*b*d*x**2 - 15*sqrt(x)*sqrt(a + b*x)*a**2*b**2*c*x**2 - 27*sqrt(x)*sqrt(
a + b*x)*a**2*b**2*d*x**3 - sqrt(x)*sqrt(a + b*x)*a*b**3*c*x**3 - 9*sqrt(x
)*sqrt(a + b*x)*a*b**3*d*x**4 + 2*sqrt(x)*sqrt(a + b*x)*b**4*c*x**4 - 5*sq
rt(b)*a*b**3*d*x**5 - 2*sqrt(b)*b**4*c*x**5))/(63*a**2*e**11*x**5)
```

3.330 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{23/2}} dx$

Optimal result

Integrand size = 28, antiderivative size = 112

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{23/2}} dx = -\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{11a(ex)^{25/2}} + \frac{2(4bc-11ad)\left(ax^2+bx^3\right)^{7/2}}{99a^2(ex)^{23/2}} - \frac{4b(4bc-11ad)\left(ax^2+bx^3\right)^{7/2}}{693a^3e(ex)^{21/2}}$$

output
```
-2/11*c*e*(b*x^3+a*x^2)^(7/2)/a/(e*x)^(25/2)+2/99*(-11*a*d+4*b*c)*(b*x^3+a
*x^2)^(7/2)/a^2/(e*x)^(23/2)-4/693*b*(-11*a*d+4*b*c)*(b*x^3+a*x^2)^(7/2)/a
^3/e/(e*x)^(21/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.57

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{23/2}} dx = -\frac{2e\left(x^2(a+bx)\right)^{7/2}\left(8b^2cx^2+7a^2(9c+11dx)-2abx(14c+11dx)\right)}{693a^3(ex)^{25/2}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(23/2),x]
```

output
```
(-2*e*(x^2*(a + b*x))^(7/2)*(8*b^2*c*x^2 + 7*a^2*(9*c + 11*d*x) - 2*a*b*x*
(14*c + 11*d*x)))/(693*a^3*(e*x)^(25/2))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1944, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ax^2+bx^3)^{5/2}(c+dx)}{(ex)^{23/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(4bc-11ad)\int \frac{(bx^3+ax^2)^{5/2}}{(ex)^{21/2}}dx}{11ae}-\frac{2ce(ax^2+bx^3)^{7/2}}{11a(ex)^{25/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{(4bc-11ad)\left(-\frac{2b\int \frac{(bx^3+ax^2)^{5/2}}{(ex)^{19/2}}dx}{9ae}-\frac{2e(ax^2+bx^3)^{7/2}}{9a(ex)^{23/2}}\right)}{11ae}-\frac{2ce(ax^2+bx^3)^{7/2}}{11a(ex)^{25/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{(4bc-11ad)\left(\frac{4b(ax^2+bx^3)^{7/2}}{63a^2(ex)^{21/2}}-\frac{2e(ax^2+bx^3)^{7/2}}{9a(ex)^{23/2}}\right)}{11ae}-\frac{2ce(ax^2+bx^3)^{7/2}}{11a(ex)^{25/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(23/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(7/2))/(11*a*(e*x)^(25/2)) - ((4*b*c - 11*a*d)*((-
2*e*(a*x^2 + b*x^3)^(7/2))/(9*a*(e*x)^(23/2)) + (4*b*(a*x^2 + b*x^3)^(7/2)
)/(63*a^2*(e*x)^(21/2))))/(11*a*e)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.60

method	result
gosper	$-\frac{2x(bx+a)\left(-22abd\,x^{2}+8b^{2}c\,x^{2}+77a^{2}dx-28abcx+63a^{2}c\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{693a^{3}\left(ex\right)^{\frac{23}{2}}}$
orering	$-\frac{2x(bx+a)\left(-22abd\,x^{2}+8b^{2}c\,x^{2}+77a^{2}dx-28abcx+63a^{2}c\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{693a^{3}\left(ex\right)^{\frac{23}{2}}}$
default	$-\frac{2\left(-22a\,b^{2}d\,x^{3}+8b^{3}c\,x^{3}+55a^{2}bd\,x^{2}-20a\,b^{2}c\,x^{2}+77a^{3}dx+35a^{2}bcx+63c\,a^{3}\right)\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}{693\sqrt{ex}\,e^{11}a^{3}x^{10}}$
risch	$-\frac{2\sqrt{x^{2}(bx+a)}\,\left(-22a\,b^{4}d\,x^{5}+8b^{5}c\,x^{5}+11x^{4}a^{2}b^{3}d-4x^{4}a\,b^{4}c+165a^{3}b^{2}d\,x^{3}+3a^{2}b^{3}c\,x^{3}+209a^{4}bd\,x^{2}+113a^{3}b^{2}c\,x^{2}+77a^{5}dx+161a^{4}\right.}{693e^{11}x^{6}\sqrt{ex}\,a^{3}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(23/2),x,method=_RETURNVERBOSE)
```

output
```
-2/693*x*(b*x+a)*(-22*a*b*d*x^2+8*b^2*c*x^2+77*a^2*d*x-28*a*b*c*x+63*a^2*c
)*(b*x^3+a*x^2)^(5/2)/a^3/(e*x)^(23/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.23

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{23/2}}\,dx =$$
$$-\frac{2\left(63\,a^{5}c+2\,(4\,b^{5}c-11\,ab^{4}d)x^{5}-(4\,ab^{4}c-11\,a^{2}b^{3}d)x^{4}+3\,(a^{2}b^{3}c+55\,a^{3}b^{2}d)x^{3}+(113\,a^{3}b^{2}c+209\,a^{4}b\right.}{693\,a^{3}e^{12}x^{7}}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(23/2),x, algorithm="fricas")
```

output
```
-2/693*(63*a^5*c + 2*(4*b^5*c - 11*a*b^4*d)*x^5 - (4*a*b^4*c - 11*a^2*b^3*
d)*x^4 + 3*(a^2*b^3*c + 55*a^3*b^2*d)*x^3 + (113*a^3*b^2*c + 209*a^4*b*d)*
x^2 + 7*(23*a^4*b*c + 11*a^5*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x)/(a^3*e^12
*x^7)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{23/2}}\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/(e*x)**(23/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{23/2}}\,dx=\int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{(ex)^{\frac{23}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(23/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/(e*x)^(23/2), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.33

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{23/2}}\,dx=$$
$$-\frac{2\,(bx+a)^{\frac{7}{2}}\left((bx+a)\left(\frac{2\,\left(4\,a^2b^5ce^5\mathrm{sgn}(x)-11\,a^3b^4de^5\mathrm{sgn}(x)\right)(bx+a)}{a^5}-\frac{11\,\left(4\,a^3b^5ce^5\mathrm{sgn}(x)-11\,a^4b^4de^5\mathrm{sgn}(x)\right)}{a^5}\right)+\frac{99\,\left(a^4b^5ce^5\mathrm{sgr}\right.}{}\right.}{693\,\left((bx+a)be-abe\right)^{\frac{11}{2}}e^{11}|b|}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(23/2),x, algorithm="giac")
```

output
```
-2/693*(b*x + a)^(7/2)*((b*x + a)*(2*(4*a^2*b^5*c*e^5*sgn(x) - 11*a^3*b^4*
d*e^5*sgn(x))*(b*x + a)/a^5 - 11*(4*a^3*b^5*c*e^5*sgn(x) - 11*a^4*b^4*d*e^
5*sgn(x))/a^5) + 99*(a^4*b^5*c*e^5*sgn(x) - a^5*b^4*d*e^5*sgn(x))/a^5)*b^7
/(((b*x + a)*b*e - a*b*e)^(11/2)*e^11*abs(b))
```

Mupad [B] (verification not implemented)

Time = 9.59 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{23/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,a^2\,c}{11\,e^{11}}+\frac{2\,b\,x^2\,(209\,a\,d+113\,b\,c)}{693\,e^{11}}+\frac{x\,(154\,d\,a^5+322\,b\,c\,a^4)}{693\,a^3\,e^{11}}+\frac{x^5\,(16\,b^5\,c-44\,a\,b^4\,d)}{693\,a^3\,e^{11}}+\frac{2\,b^3\,x^4\,(11\,a\,d-4\,b\,c)}{693\,a^2\,e^{11}}+\frac{2\,b^2\,x^3}{23\dots}\right.}{x^6\,\sqrt{e\,x}}$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(23/2),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*a^2*c)/(11*e^11) + (2*b*x^2*(209*a*d + 113*b*c
))/(693*e^11) + (x*(154*a^5*d + 322*a^4*b*c))/(693*a^3*e^11) + (x^5*(16*b^
5*c - 44*a*b^4*d))/(693*a^3*e^11) + (2*b^3*x^4*(11*a*d - 4*b*c))/(693*a^2*
e^11) + (2*b^2*x^3*(55*a*d + b*c))/(231*a*e^11)))/(x^6*(e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.08

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{23/2}}\,dx = \frac{2\sqrt{e}\left(-63\sqrt{x}\,\sqrt{bx+a}\,a^5c-77\sqrt{x}\,\sqrt{bx+a}\,a^5dx-161\sqrt{x}\,\sqrt{bx+a}\,a^4bcx\dots\right.}{\dots}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(23/2),x)
```

output
```
(2*sqrt(e)*( - 63*sqrt(x)*sqrt(a + b*x)*a**5*c - 77*sqrt(x)*sqrt(a + b*x)*
a**5*d*x - 161*sqrt(x)*sqrt(a + b*x)*a**4*b*c*x - 209*sqrt(x)*sqrt(a + b*x
)*a**4*b*d*x**2 - 113*sqrt(x)*sqrt(a + b*x)*a**3*b**2*c*x**2 - 165*sqrt(x)
*sqrt(a + b*x)*a**3*b**2*d*x**3 - 3*sqrt(x)*sqrt(a + b*x)*a**2*b**3*c*x**3
 - 11*sqrt(x)*sqrt(a + b*x)*a**2*b**3*d*x**4 + 4*sqrt(x)*sqrt(a + b*x)*a*b
**4*c*x**4 + 22*sqrt(x)*sqrt(a + b*x)*a*b**4*d*x**5 - 8*sqrt(x)*sqrt(a + b
*x)*b**5*c*x**5 - 22*sqrt(b)*a*b**4*d*x**6 + 8*sqrt(b)*b**5*c*x**6))/(693*
a**3*e**12*x**6)
```

3.331 $\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{25/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{25/2}}\,dx=-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{13a(ex)^{27/2}}+\frac{2(6bc-13ad)\left(ax^2+bx^3\right)^{7/2}}{143a^2(ex)^{25/2}}-\frac{8b(6bc-13ad)\left(ax^2+bx^3\right)^{7/2}}{1287a^3e(ex)^{23/2}}+\frac{16b^2(6bc-13ad)\left(ax^2+bx^3\right)^{7/2}}{9009a^4e^2(ex)^{21/2}}$$

output
```
-2/13*c*e*(b*x^3+a*x^2)^(7/2)/a/(e*x)^(27/2)+2/143*(-13*a*d+6*b*c)*(b*x^3+
a*x^2)^(7/2)/a^2/(e*x)^(25/2)-8/1287*b*(-13*a*d+6*b*c)*(b*x^3+a*x^2)^(7/2)
/a^3/e/(e*x)^(23/2)+16/9009*b^2*(-13*a*d+6*b*c)*(b*x^3+a*x^2)^(7/2)/a^4/e^
2/(e*x)^(21/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.60

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{25/2}}\,dx=$$
$$-\frac{2x(a+bx)\left(x^2(a+bx)\right)^{5/2}\left(693a^3c-378a^2bcx+819a^3dx+168ab^2cx^2-364a^2bdx^2-48b^3cx^3+104ab^2\right.}{9009a^4(ex)^{25/2}}$$

input
```
Integrate[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(25/2),x]
```

output
```
(-2*x*(a + b*x)*(x^2*(a + b*x))^(5/2)*(693*a^3*c - 378*a^2*b*c*x + 819*a^3
*d*x + 168*a*b^2*c*x^2 - 364*a^2*b*d*x^2 - 48*b^3*c*x^3 + 104*a*b^2*d*x^3)
)/(9009*a^4*(e*x)^(25/2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1944, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(ax^2+bx^3\right)^{5/2}(c+dx)}{(ex)^{25/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(6bc-13ad)\int \frac{\left(bx^3+ax^2\right)^{5/2}}{(ex)^{23/2}}dx}{13ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{13a(ex)^{27/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{(6bc-13ad)\left(-\frac{4b\int \frac{\left(bx^3+ax^2\right)^{5/2}}{(ex)^{21/2}}dx}{11ae}-\frac{2e\left(ax^2+bx^3\right)^{7/2}}{11a(ex)^{25/2}}\right)}{13ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{13a(ex)^{27/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{(6bc-13ad)\left(-\frac{4b\left(-\frac{2b\int \frac{\left(bx^3+ax^2\right)^{5/2}}{(ex)^{19/2}}dx}{9ae}-\frac{2e\left(ax^2+bx^3\right)^{7/2}}{9a(ex)^{23/2}}\right)}{11ae}-\frac{2e\left(ax^2+bx^3\right)^{7/2}}{11a(ex)^{25/2}}\right)}{13ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{13a(ex)^{27/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{(6bc-13ad)\left(-\frac{4b\left(\frac{4b\left(ax^2+bx^3\right)^{7/2}}{63a^2(ex)^{21/2}}-\frac{2e\left(ax^2+bx^3\right)^{7/2}}{9a(ex)^{23/2}}\right)}{11ae}-\frac{2e\left(ax^2+bx^3\right)^{7/2}}{11a(ex)^{25/2}}\right)}{13ae}-\frac{2ce\left(ax^2+bx^3\right)^{7/2}}{13a(ex)^{27/2}}$$

input
```
Int[((c + d*x)*(a*x^2 + b*x^3)^(5/2))/(e*x)^(25/2),x]
```

output
```
(-2*c*e*(a*x^2 + b*x^3)^(7/2))/(13*a*(e*x)^(27/2)) - ((6*b*c - 13*a*d)*((-
2*e*(a*x^2 + b*x^3)^(7/2))/(11*a*(e*x)^(25/2)) - (4*b*((-2*e*(a*x^2 + b*x^
3)^(7/2))/(9*a*(e*x)^(23/2)) + (4*b*(a*x^2 + b*x^3)^(7/2))/(63*a^2*(e*x)^(
21/2))))/(11*a*e)))/(13*a*e)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.58

method	result
gosper	$-\frac{2 x \left(b x +a \right) \left(104 a \,b^{2} d \,x^{3}-48 b^{3} c \,x^{3}-364 a^{2} b d \,x^{2}+168 a \,b^{2} c \,x^{2}+819 a^{3} d x -378 a^{2} b c x +693 c \,a^{3}\right) \left(b \,x^{3}+a \,x^{2}\right)^{\frac{5}{2}}}{9009 a^{4} \left(e x \right)^{\frac{25}{2}}}$
orering	$-\frac{2 x \left(b x +a \right) \left(104 a \,b^{2} d \,x^{3}-48 b^{3} c \,x^{3}-364 a^{2} b d \,x^{2}+168 a \,b^{2} c \,x^{2}+819 a^{3} d x -378 a^{2} b c x +693 c \,a^{3}\right) \left(b \,x^{3}+a \,x^{2}\right)^{\frac{5}{2}}}{9009 a^{4} \left(e x \right)^{\frac{25}{2}}}$
default	$-\frac{2 \left(104 x^{4} a \,b^{3} d -48 x^{4} b^{4} c -260 a^{2} b^{2} d \,x^{3}+120 a \,b^{3} c \,x^{3}+455 a^{3} b d \,x^{2}-210 a^{2} b^{2} c \,x^{2}+819 a^{4} d x +315 a^{3} b c x +693 c \,a^{4}\right) \left(b \,x^{3}+a \,x^{2}\right)^{\frac{5}{2}}}{9009 \sqrt{e x}\, e^{12} a^{4} x^{11}}$
risch	$-\frac{2 \sqrt{x^{2} \left(b x +a \right)}\, \left(104 a \,b^{5} d \,x^{6}-48 b^{6} c \,x^{6}-52 a^{2} b^{4} d \,x^{5}+24 a \,b^{5} c \,x^{5}+39 a^{3} b^{3} d \,x^{4}-18 a^{2} b^{4} c \,x^{4}+1469 a^{4} b^{2} d \,x^{3}+15 a^{3} b^{3} c \,x^{3}+2093 a^{5} b d \,x}{9009 e^{12} x^{7} \sqrt{e x}\, a^{4}}$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(25/2),x,method=_RETURNVERBOSE)
```

output
```
-2/9009*x*(b*x+a)*(104*a*b^2*d*x^3-48*b^3*c*x^3-364*a^2*b*d*x^2+168*a*b^2*
c*x^2+819*a^3*d*x-378*a^2*b*c*x+693*a^3*c)*(b*x^3+a*x^2)^(5/2)/a^4/(e*x)^(
25/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{25/2}}\,dx =$$
$$-\frac{2\,(693\,a^6c-8\,(6\,b^6c-13\,ab^5d)x^6+4\,(6\,ab^5c-13\,a^2b^4d)x^5-3\,(6\,a^2b^4c-13\,a^3b^3d)x^4+(15\,a^3b^3c+1469}{9009\,a^4e^{13}x^8}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(25/2),x, algorithm="fricas")
```

output
```
-2/9009*(693*a^6*c - 8*(6*b^6*c - 13*a*b^5*d)*x^6 + 4*(6*a*b^5*c - 13*a^2*
b^4*d)*x^5 - 3*(6*a^2*b^4*c - 13*a^3*b^3*d)*x^4 + (15*a^3*b^3*c + 1469*a^4
*b^2*d)*x^3 + 7*(159*a^4*b^2*c + 299*a^5*b*d)*x^2 + 63*(27*a^5*b*c + 13*a^
6*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x)/(a^4*e^13*x^8)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{25/2}}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)*(b*x**3+a*x**2)**(5/2)/(e*x)**(25/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{25/2}}\,dx = \int \frac{\left(bx^3+ax^2\right)^{\frac{5}{2}}(dx+c)}{(ex)^{\frac{25}{2}}}\,dx$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(25/2),x, algorithm="maxima")
```

output
```
integrate((b*x^3 + a*x^2)^(5/2)*(d*x + c)/(e*x)^(25/2), x)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{(ex)^{25/2}}\,dx = \frac{2\left((bx+a)\left(4\,(bx+a)\left(\frac{2\,(6\,a^2b^{13}ce^6\text{sgn}(x)-13\,a^3b^{12}de^6\text{sgn}(x))\,(bx+a)}{a^6}-\frac{13\,(6\,a^3b^{13}ce^6\text{s}\ldots}{}\right.\right.\right.}{90\ldots}$$

input
```
integrate((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(25/2),x, algorithm="giac")
```

output
```
2/9009*((b*x + a)*(4*(b*x + a)*(2*(6*a^2*b^13*c*e^6*sgn(x) - 13*a^3*b^12*d
*e^6*sgn(x))*(b*x + a)/a^6 - 13*(6*a^3*b^13*c*e^6*sgn(x) - 13*a^4*b^12*d*e
^6*sgn(x))/a^6) + 143*(6*a^4*b^13*c*e^6*sgn(x) - 13*a^5*b^12*d*e^6*sgn(x))
/a^6) - 1287*(a^5*b^13*c*e^6*sgn(x) - a^6*b^12*d*e^6*sgn(x))/a^6)*(b*x + a
)^(7/2)*b/(((b*x + a)*b*e - a*b*e)^(13/2)*e^12*abs(b))
```

Mupad [B] (verification not implemented)

Time = 9.68 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)\,(ax^2+bx^3)^{5/2}}{(ex)^{25/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,a^2\,c}{13\,e^{12}}+\frac{2\,b\,x^2\,(299\,a\,d+159\,b\,c)}{1287\,e^{12}}+\frac{x\,(1638\,d\,a^6+3402\,b\,c\,a^5)}{9009\,a^4\,e^{12}}-\frac{x^6\,(96\,b^6\,c-208\,a\,b^5\,d)}{9009\,a^4\,e^{12}}+\frac{2\,b^3\,x^4\,(13\,a\,d-6\,b\,c)}{3003\,a^2\,e^{12}}-\frac{8\,b^{\ldots}}{}\right.}{x^7\,\sqrt{e\,x}}$$

input
```
int(((a*x^2 + b*x^3)^(5/2)*(c + d*x))/(e*x)^(25/2),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*a^2*c)/(13*e^12) + (2*b*x^2*(299*a*d + 159*b*c
))/(1287*e^12) + (x*(1638*a^6*d + 3402*a^5*b*c))/(9009*a^4*e^12) - (x^6*(9
6*b^6*c - 208*a*b^5*d))/(9009*a^4*e^12) + (2*b^3*x^4*(13*a*d - 6*b*c))/(30
03*a^2*e^12) - (8*b^4*x^5*(13*a*d - 6*b*c))/(9009*a^3*e^12) + (2*b^2*x^3*(
1469*a*d + 15*b*c))/(9009*a*e^12)))/(x^7*(e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.75

$$\int \frac{(c+dx)\left(ax^2+bx^3\right)^{5/2}}{(ex)^{25/2}}\,dx = \frac{2\sqrt{e}\left(-693\sqrt{x}\,\sqrt{bx+a}\,a^6c-819\sqrt{x}\,\sqrt{bx+a}\,a^6dx-1701\sqrt{x}\,\sqrt{bx+a}\,a^5\right.}{}$$

input
```
int((d*x+c)*(b*x^3+a*x^2)^(5/2)/(e*x)^(25/2),x)
```

output
```
(2*sqrt(e)*( - 693*sqrt(x)*sqrt(a + b*x)*a**6*c - 819*sqrt(x)*sqrt(a + b*x
)*a**6*d*x - 1701*sqrt(x)*sqrt(a + b*x)*a**5*b*c*x - 2093*sqrt(x)*sqrt(a +
 b*x)*a**5*b*d*x**2 - 1113*sqrt(x)*sqrt(a + b*x)*a**4*b**2*c*x**2 - 1469*s
qrt(x)*sqrt(a + b*x)*a**4*b**2*d*x**3 - 15*sqrt(x)*sqrt(a + b*x)*a**3*b**3
*c*x**3 - 39*sqrt(x)*sqrt(a + b*x)*a**3*b**3*d*x**4 + 18*sqrt(x)*sqrt(a +
b*x)*a**2*b**4*c*x**4 + 52*sqrt(x)*sqrt(a + b*x)*a**2*b**4*d*x**5 - 24*sqr
t(x)*sqrt(a + b*x)*a*b**5*c*x**5 - 104*sqrt(x)*sqrt(a + b*x)*a*b**5*d*x**6
 + 48*sqrt(x)*sqrt(a + b*x)*b**6*c*x**6 + 104*sqrt(b)*a*b**5*d*x**7 - 48*s
qrt(b)*b**6*c*x**7))/(9009*a**4*e**13*x**7)
```

3.332 $\int \frac{(ex)^{7/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 219

$$\int \frac{(ex)^{7/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{5a^2(8bc-7ad)e^4\sqrt{ax^2+bx^3}}{64b^4\sqrt{ex}} - \frac{5a(8bc-7ad)e^3\sqrt{ex}\sqrt{ax^2+bx^3}}{96b^3} + \frac{(8bc-7ad)e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{24b^2} + \frac{de(ex)^{5/2}\sqrt{ax^2+bx^3}}{4b} - \frac{5a^3(8bc-7ad)e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{64b^{9/2}}$$

output
```
5/64*a^2*(-7*a*d+8*b*c)*e^4*(b*x^3+a*x^2)^(1/2)/b^4/(e*x)^(1/2)-5/96*a*(-7
*a*d+8*b*c)*e^3*(e*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/b^3+1/24*(-7*a*d+8*b*c)*e^
2*(e*x)^(3/2)*(b*x^3+a*x^2)^(1/2)/b^2+1/4*d*e*(e*x)^(5/2)*(b*x^3+a*x^2)^(1
/2)/b-5/64*a^3*(-7*a*d+8*b*c)*e^(7/2)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/
(b*x^3+a*x^2)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.91

$$\int \frac{(ex)^{7/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{e^3\sqrt{x}\sqrt{ex}\left(\sqrt{b}\sqrt{x}(a+bx)\left(-105a^3d+16b^3x^2(4c+3dx)-8ab^2x(10c+7dx)+10\right.\right.}{19}$$

input
```
Integrate[((e*x)^(7/2)*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(e^3*Sqrt[x]*Sqrt[e*x]*(Sqrt[b]*Sqrt[x]*(a + b*x)*(-105*a^3*d + 16*b^3*x^2
*(4*c + 3*d*x) - 8*a*b^2*x*(10*c + 7*d*x) + 10*a^2*b*(12*c + 7*d*x)) + 240
*a^3*b*c*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])
] + 210*a^4*d*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a +
 b*x])]))/(192*b^(9/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1945, 1930, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(8bc-7ad)\int \frac{(ex)^{7/2}}{\sqrt{bx^3+ax^2}}dx}{8b}+\frac{de(ex)^{5/2}\sqrt{ax^2+bx^3}}{4b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{(8bc-7ad)\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\int \frac{(ex)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6b}\right)}{8b}+\frac{de(ex)^{5/2}\sqrt{ax^2+bx^3}}{4b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{(8bc-7ad)\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\int\frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6b}\right)}{8b}+\frac{de(ex)^{5/2}\sqrt{ax^2+bx^3}}{4b}$$

$\Big\downarrow$ 1930

$$\frac{(8bc-7ad)\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int\frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6b}\right)}{8b}+\frac{de(ex)^{5/2}\sqrt{ax^2+bx^3}}{4b}$$

$\Big\downarrow$ 1937

$$\frac{(8bc-7ad)\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8b}+\frac{de(ex)^{5/2}\sqrt{ax^2+bx^3}}{4b}$$

$\Big\downarrow$ 1935

$$\frac{(8bc-7ad)\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8b}+\frac{de(ex)^{5/2}\sqrt{ax^2+bx^3}}{4b}$$

$\Big\downarrow$ 219

$$\frac{(8bc-7ad)\left(\frac{e^2(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}-\frac{5ae\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{6b}\right)}{8b}+\frac{de(ex)^{5/2}\sqrt{ax^2+bx^3}}{4b}$$

input
```
Int[((e*x)^(7/2)*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(d*e*(e*x)^(5/2)*Sqrt[a*x^2 + b*x^3])/(4*b) + ((8*b*c - 7*a*d)*((e^2*(e*x)
^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) - (5*a*e*((e^2*Sqrt[e*x]*Sqrt[a*x^2 + b*
x^3])/(2*b) - (3*a*e*((e^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[e*x]) - (a*e*Sqrt[
e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/(
4*b)))/(6*b)))/(8*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 174, normalized size of antiderivative = 0.79

method	result
risch	$-\frac{(-48b^3 d\,x^3+56a\,b^2 d\,x^2-64b^3 c\,x^2-70a^2 bdx+80a\,b^2 cx+105a^3 d-120c\,a^2 b)x^2(bx+a)e^4}{192b^4\sqrt{x^2(bx+a)}\,\sqrt{ex}}+\frac{5a^3(7ad-8bc)\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^2}\right.}{128b^4\sqrt{be}\,\sqrt{x^2(bx+}}$
default	$-\frac{x(bx+a)e^3\sqrt{ex}\left(-96b^3 d\,x^3\sqrt{ex(bx+a)}\,\sqrt{be}+112a\,b^2 d\,x^2\sqrt{ex(bx+a)}\,\sqrt{be}-128b^3 c\,x^2\sqrt{ex(bx+a)}\,\sqrt{be}-105\ln\left(\frac{2bex+2\sqrt{ex(bx+a}}{2\sqrt{be}}\right.\right.}{}$

input
```
int((e*x)^(7/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/192*(-48*b^3*d*x^3+56*a*b^2*d*x^2-64*b^3*c*x^2-70*a^2*b*d*x+80*a*b^2*c*
x+105*a^3*d-120*a^2*b*c)*x^2*(b*x+a)/b^4*e^4/(x^2*(b*x+a))^(1/2)/(e*x)^(1/
2)+5/128*a^3*(7*a*d-8*b*c)/b^4*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e
*x)^(1/2))/(b*e)^(1/2)*e^4/(x^2*(b*x+a))^(1/2)*x*(e*x*(b*x+a))^(1/2)/(e*x)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 350, normalized size of antiderivative = 1.60

$$\int \frac{(ex)^{7/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \left[-\frac{15\,(8\,a^3bc-7\,a^4d)e^3x\sqrt{\frac{e}{b}}\log\left(\frac{2\,bex^2+aex+2\,\sqrt{bx^3+ax^2}\sqrt{ex}b\sqrt{\frac{e}{b}}}{x}\right)-2\,(48\,b^3de^3x^3+8}{}\right.$$

input
```
integrate((e*x)^(7/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/384*(15*(8*a^3*b*c - 7*a^4*d)*e^3*x*sqrt(e/b)*log((2*b*e*x^2 + a*e*x +
 2*sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(e/b))/x) - 2*(48*b^3*d*e^3*x^3 + 8
*(8*b^3*c - 7*a*b^2*d)*e^3*x^2 - 10*(8*a*b^2*c - 7*a^2*b*d)*e^3*x + 15*(8*
a^2*b*c - 7*a^3*d)*e^3)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^4*x), 1/192*(15*
(8*a^3*b*c - 7*a^4*d)*e^3*x*sqrt(-e/b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(e*x
)*b*sqrt(-e/b)/(b*e*x^2 + a*e*x)) + (48*b^3*d*e^3*x^3 + 8*(8*b^3*c - 7*a*b
^2*d)*e^3*x^2 - 10*(8*a*b^2*c - 7*a^2*b*d)*e^3*x + 15*(8*a^2*b*c - 7*a^3*d
)*e^3)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^4*x)]
```

Sympy [F]

$$\int \frac{(ex)^{7/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(ex)^{\frac{7}{2}}\,(c+dx)}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((e*x)**(7/2)*(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((e*x)**(7/2)*(c + d*x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(ex)^{7/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(dx+c)(ex)^{\frac{7}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate((e*x)^(7/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^(7/2)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.17

$$\int \frac{(ex)^{7/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{\left(\sqrt{be^2x+ae^2}\left(2\left(4\,ex\left(\frac{6\,dx}{be}+\frac{8\,b^6ce^2-7\,ab^5de^2}{b^7e^3}\right)-\frac{5\,(8\,ab^5ce^3-7\,a^2b^4de^3)}{b^7e^3}\right)ex+\frac{15\,(8\,a^2b^4ce^4}{b^7}\right.\right.}{192\,|e|\mathrm{sgn}\,(x)}$$
$$-\frac{5\,(8\,a^3bce^5\log(e^2|a|)-7\,a^4de^5\log(e^2|a|))\mathrm{sgn}(x)}{128\,\sqrt{be}b^4|e|}$$

input
```
integrate((e*x)^(7/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/192*(sqrt(b*e^2*x + a*e^2)*(2*(4*e*x*(6*d*x/(b*e) + (8*b^6*c*e^2 - 7*a*b
^5*d*e^2)/(b^7*e^3)) - 5*(8*a*b^5*c*e^3 - 7*a^2*b^4*d*e^3)/(b^7*e^3))*e*x
+ 15*(8*a^2*b^4*c*e^4 - 7*a^3*b^3*d*e^4)/(b^7*e^3))*sqrt(e*x) + 15*(8*a^3*
b*c*e^3 - 7*a^4*d*e^3)*log(abs(-sqrt(b*e)*sqrt(e*x) + sqrt(b*e^2*x + a*e^2
)))/(sqrt(b*e)*b^4))*e^2/(abs(e)*sgn(x)) - 5/128*(8*a^3*b*c*e^5*log(e^2*ab
s(a)) - 7*a^4*d*e^5*log(e^2*abs(a)))*sgn(x)/(sqrt(b*e)*b^4*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(e\,x)^{7/2}\,(c+d\,x)}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(((e*x)^(7/2)*(c + d*x))/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int(((e*x)^(7/2)*(c + d*x))/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.83

$$\int \frac{(ex)^{7/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{e}\,e^3\Big(-105\sqrt{x}\,\sqrt{bx+a}\,a^3bd+120\sqrt{x}\,\sqrt{bx+a}\,a^2b^2c+70\sqrt{x}\,\sqrt{bx+a}\,a^2b^2dx-8}{}$$

input
```
int((e*x)^(7/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(sqrt(e)*e**3*( - 105*sqrt(x)*sqrt(a + b*x)*a**3*b*d + 120*sqrt(x)*sqrt(a
+ b*x)*a**2*b**2*c + 70*sqrt(x)*sqrt(a + b*x)*a**2*b**2*d*x - 80*sqrt(x)*s
qrt(a + b*x)*a*b**3*c*x - 56*sqrt(x)*sqrt(a + b*x)*a*b**3*d*x**2 + 64*sqrt
(x)*sqrt(a + b*x)*b**4*c*x**2 + 48*sqrt(x)*sqrt(a + b*x)*b**4*d*x**3 + 105
*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**4*d - 120*sqrt(
b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*b*c))/(192*b**5)
```

3.333 $\int \frac{(ex)^{5/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 175

$$\int \frac{(ex)^{5/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = -\frac{a(6bc-5ad)e^3\sqrt{ax^2+bx^3}}{8b^3\sqrt{ex}} + \frac{(6bc-5ad)e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{12b^2} + \frac{de(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b} + \frac{a^2(6bc-5ad)e^{5/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{8b^{7/2}}$$

output

```
-1/8*a*(-5*a*d+6*b*c)*e^3*(b*x^3+a*x^2)^(1/2)/b^3/(e*x)^(1/2)+1/12*(-5*a*d
+6*b*c)*e^2*(e*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/b^2+1/3*d*e*(e*x)^(3/2)*(b*x^3
+a*x^2)^(1/2)/b+1/8*a^2*(-5*a*d+6*b*c)*e^(5/2)*arctanh(b^(1/2)*(e*x)^(3/2)
/e^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.79

$$\int \frac{(ex)^{5/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{(ex)^{5/2}\left(\sqrt{b}\sqrt{x}(a+bx)\left(15a^2d+4b^2x(3c+2dx)-2ab(9c+5dx)\right)+6a^2(-6bc+\right.}{24b^{7/2}x^{3/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((e*x)^(5/2)*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
((e*x)^(5/2)*(Sqrt[b]*Sqrt[x]*(a + b*x)*(15*a^2*d + 4*b^2*x*(3*c + 2*d*x)
- 2*a*b*(9*c + 5*d*x)) + 6*a^2*(-6*b*c + 5*a*d)*Sqrt[a + b*x]*ArcTanh[(Sqr
t[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])]))/(24*b^(7/2)*x^(3/2)*Sqrt[x^2*(a
 + b*x)])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1945, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(6bc-5ad)\int \frac{(ex)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{6b}+\frac{de(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{(6bc-5ad)\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\int \frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{6b}+\frac{de(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{(6bc-5ad)\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int\frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{6b}+\frac{de(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}$$

$$\Big\downarrow \text{1937}$$

$$\frac{(6bc-5ad)\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{6b}+\frac{de(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}$$

$$\Big\downarrow \text{1935}$$

$$\frac{(6bc-5ad)\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{6b}+\frac{de(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(6bc-5ad)\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{6b}+\frac{de(ex)^{3/2}\sqrt{ax^2+bx^3}}{3b}$$

input
```
Int[((e*x)^(5/2)*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(d*e*(e*x)^(3/2)*Sqrt[a*x^2 + b*x^3])/(3*b) + ((6*b*c - 5*a*d)*((e^2*Sqrt[
e*x]*Sqrt[a*x^2 + b*x^3])/(2*b) - (3*a*e*((e^2*Sqrt[a*x^2 + b*x^3])/(b*Sqr
t[e*x]) - (a*e*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(
b^(3/2)*Sqrt[x])))/(4*b)))/(6*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.86

method	result
risch	$\frac{(8b^2 d\,x^2-10abdx+12b^2cx+15a^2d-18abc)x^2(bx+a)e^3}{24b^3\sqrt{x^2(bx+a)}\,\sqrt{ex}}-\frac{a^2(5ad-6bc)\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^2+aex}\right)e^3x\sqrt{ex(bx+a)}}{16b^3\sqrt{be}\,\sqrt{x^2(bx+a)}\,\sqrt{ex}}$
default	$\frac{x(bx+a)e^2\sqrt{ex}\left(16b^2d\,x^2\sqrt{be}\,\sqrt{ex(bx+a)}-15\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)a^3de+18\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)a^2bce-20\sqrt{\cdots}\right.}{48\sqrt{b\,x^3+a\,x^2}\,b^3\sqrt{ex(bx+a)}\,\sqrt{be}\cdots}$

input
```
int((e*x)^(5/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/24*(8*b^2*d*x^2-10*a*b*d*x+12*b^2*c*x+15*a^2*d-18*a*b*c)*x^2*(b*x+a)/b^3
*e^3/(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)-1/16*a^2*(5*a*d-6*b*c)/b^3*ln((1/2*a*
e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)*e^3/(x^2*(b*x+a))^
(1/2)*x*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.70

$$\int \frac{(ex)^{5/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx=\left[-\frac{3\,(6\,a^2bc-5\,a^3d)e^2x\sqrt{\frac{e}{b}}\log\left(\frac{2\,bex^2+aex-2\,\sqrt{bx^3+ax^2}\sqrt{ex}b\sqrt{\frac{e}{b}}}{x}\right)-2\,(8\,b^2de^2x^2+2\,(\cdots}{48\,b^3x}\right.$$

$$-\frac{3\,(6\,a^2bc-5\,a^3d)e^2x\sqrt{-\frac{e}{b}}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{ex}b\sqrt{-\frac{e}{b}}}{bex^2+aex}\right)-(8\,b^2de^2x^2+2\,(6\,b^2c-5\,abd)e^2x-3\,(6\,abc-5\,\cdots}{24\,b^3x}$$

input
```
integrate((e*x)^(5/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/48*(3*(6*a^2*b*c - 5*a^3*d)*e^2*x*sqrt(e/b)*log((2*b*e*x^2 + a*e*x - 2
*sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(e/b))/x) - 2*(8*b^2*d*e^2*x^2 + 2*(6
*b^2*c - 5*a*b*d)*e^2*x - 3*(6*a*b*c - 5*a^2*d)*e^2)*sqrt(b*x^3 + a*x^2)*s
qrt(e*x))/(b^3*x), -1/24*(3*(6*a^2*b*c - 5*a^3*d)*e^2*x*sqrt(-e/b)*arctan(
sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(-e/b)/(b*e*x^2 + a*e*x)) - (8*b^2*d*e
^2*x^2 + 2*(6*b^2*c - 5*a*b*d)*e^2*x - 3*(6*a*b*c - 5*a^2*d)*e^2)*sqrt(b*x
^3 + a*x^2)*sqrt(e*x))/(b^3*x)]
```

Sympy [F]

$$\int \frac{(ex)^{5/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(ex)^{\frac{5}{2}}\,(c+dx)}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((e*x)**(5/2)*(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((e*x)**(5/2)*(c + d*x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(ex)^{5/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(dx+c)(ex)^{\frac{5}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate((e*x)^(5/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^(5/2)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.25

$$\int \frac{(ex)^{5/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{\left(\sqrt{be^2x+ae^2}\left(2\,ex\left(\frac{4\,dx}{be}+\frac{6\,b^4ce^2-5\,ab^3de^2}{b^5e^3}\right)-\frac{3\,(6\,ab^3ce^3-5\,a^2b^2de^3)}{b^5e^3}\right)\sqrt{ex}-\frac{3\,(6\,a^2bce^2-5}{}\right.}{24\,|e|\mathrm{sgn}\,(x)}$$
$$+\frac{(6\,a^2bce^4\log\left(e^2|a|\right)-5\,a^3de^4\log\left(e^2|a|\right))\mathrm{sgn}(x)}{16\,\sqrt{be}b^3|e|}$$

input
```
integrate((e*x)^(5/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/24*(sqrt(b*e^2*x + a*e^2)*(2*e*x*(4*d*x/(b*e) + (6*b^4*c*e^2 - 5*a*b^3*d
*e^2)/(b^5*e^3)) - 3*(6*a*b^3*c*e^3 - 5*a^2*b^2*d*e^3)/(b^5*e^3))*sqrt(e*x
) - 3*(6*a^2*b*c*e^2 - 5*a^3*d*e^2)*log(abs(-sqrt(b*e)*sqrt(e*x) + sqrt(b*
e^2*x + a*e^2)))/(sqrt(b*e)*b^3))*e^2/(abs(e)*sgn(x)) + 1/16*(6*a^2*b*c*e^
4*log(e^2*abs(a)) - 5*a^3*d*e^4*log(e^2*abs(a)))*sgn(x)/(sqrt(b*e)*b^3*abs
(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(e\,x)^{5/2}\,(c+d\,x)}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(((e*x)^(5/2)*(c + d*x))/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int(((e*x)^(5/2)*(c + d*x))/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.81

$$\int \frac{(ex)^{5/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{e}\,e^2\Big(15\sqrt{x}\,\sqrt{bx+a}\,a^2bd - 18\sqrt{x}\,\sqrt{bx+a}\,a\,b^2c - 10\sqrt{x}\,\sqrt{bx+a}\,a\,b^2dx + 12\sqrt{x}}{}$$

input
```
int((e*x)^(5/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(sqrt(e)*e**2*(15*sqrt(x)*sqrt(a + b*x)*a**2*b*d - 18*sqrt(x)*sqrt(a + b*x
)*a*b**2*c - 10*sqrt(x)*sqrt(a + b*x)*a*b**2*d*x + 12*sqrt(x)*sqrt(a + b*x
)*b**3*c*x + 8*sqrt(x)*sqrt(a + b*x)*b**3*d*x**2 - 15*sqrt(b)*log((sqrt(a
+ b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**3*d + 18*sqrt(b)*log((sqrt(a + b*x)
+ sqrt(x)*sqrt(b))/sqrt(a))*a**2*b*c))/(24*b**4)
```

3.334 $\int \frac{(ex)^{3/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 131

$$\int \frac{(ex)^{3/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx=\frac{(4bc-3ad)e^2\sqrt{ax^2+bx^3}}{4b^2\sqrt{ex}}+\frac{de\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{a(4bc-3ad)e^{3/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{4b^{5/2}}$$

output
```
1/4*(-3*a*d+4*b*c)*e^2*(b*x^3+a*x^2)^(1/2)/b^2/(e*x)^(1/2)+1/2*d*e*(e*x)^(
1/2)*(b*x^3+a*x^2)^(1/2)/b-1/4*a*(-3*a*d+4*b*c)*e^(3/2)*arctanh(b^(1/2)*(e
*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.85

$$\int \frac{(ex)^{3/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx=\frac{(ex)^{3/2}\left(\sqrt{b}(a+bx)(4bc-3ad+2bdx)+\frac{2a(-4bc+3ad)\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)}{\sqrt{x}}\right)}{4b^{5/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((e*x)^(3/2)*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
((e*x)^(3/2)*(Sqrt[b]*(a + b*x)*(4*b*c - 3*a*d + 2*b*d*x) + (2*a*(-4*b*c +
 3*a*d)*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])
])/Sqrt[x]))/(4*b^(5/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.179, Rules used = {1945, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}(c+dx)}{\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(4bc-3ad)\int \frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}+\frac{de\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}$$

$$\Big\downarrow \text{1930}$$

$$\frac{(4bc-3ad)\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}+\frac{de\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}$$

$$\Big\downarrow \text{1937}$$

$$\frac{(4bc-3ad)\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}+\frac{de\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}$$

$$\Big\downarrow \text{1935}$$

$$\frac{(4bc-3ad)\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}+\frac{de\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}$$

$$\Big\downarrow \text{219}$$

$$\frac{(4bc-3ad)\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}+\frac{de\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}$$

input
```
Int[((e*x)^(3/2)*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(d*e*Sqrt[e*x]*Sqrt[a*x^2 + b*x^3])/(2*b) + ((4*b*c - 3*a*d)*((e^2*Sqrt[a*
x^2 + b*x^3])/(b*Sqrt[e*x]) - (a*e*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqr
t[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/(4*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{(-2bdx+3ad-4bc)x^2(bx+a)e^2}{4b^2\sqrt{x^2(bx+a)}\sqrt{ex}}+\frac{a(3ad-4bc)\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^2+aex}\right)e^2x\sqrt{ex(bx+a)}}{8b^2\sqrt{be}\sqrt{x^2(bx+a)}\sqrt{ex}}$
default	$-\frac{x(bx+a)e\sqrt{ex}\left(-4\sqrt{ex(bx+a)}\sqrt{be}\,bdx-3\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\sqrt{be}+ae}{2\sqrt{be}}\right)a^2de+4\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\sqrt{be}+ae}{2\sqrt{be}}\right)abce+6\sqrt{ex(bx \cdots}{8\sqrt{b\,x^3+a\,x^2}\sqrt{ex(bx+a)}\,b^2\sqrt{be}}$

input
```
int((e*x)^(3/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*b*d*x+3*a*d-4*b*c)*x^2*(b*x+a)/b^2*e^2/(x^2*(b*x+a))^(1/2)/(e*x)^
(1/2)+1/8*a*(3*a*d-4*b*c)/b^2*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*
x)^(1/2))/(b*e)^(1/2)*e^2/(x^2*(b*x+a))^(1/2)*x*(e*x*(b*x+a))^(1/2)/(e*x)^
(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.73

$$\int \frac{(ex)^{3/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \left[-\frac{(4\,abc-3\,a^2d)ex\sqrt{\frac{e}{b}}\log\left(\frac{2\,bex^2+aex+2\,\sqrt{bx^3+ax^2}\sqrt{ex}b\sqrt{\frac{e}{b}}}{x}\right)-2\,(2\,bdex+(4\,bc-3\, \cdots}{8\,b^2x}\right.$$

input
```
integrate((e*x)^(3/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/8*((4*a*b*c - 3*a^2*d)*e*x*sqrt(e/b)*log((2*b*e*x^2 + a*e*x + 2*sqrt(b
*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(e/b))/x) - 2*(2*b*d*e*x + (4*b*c - 3*a*d)*e
)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^2*x), 1/4*((4*a*b*c - 3*a^2*d)*e*x*sqr
t(-e/b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(-e/b)/(b*e*x^2 + a*e*x
)) + (2*b*d*e*x + (4*b*c - 3*a*d)*e)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^2*x
)]
```

Sympy [F]

$$\int \frac{(ex)^{3/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(ex)^{\frac{3}{2}}\,(c+dx)}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((e*x)**(3/2)*(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((e*x)**(3/2)*(c + d*x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(ex)^{3/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(dx+c)(ex)^{\frac{3}{2}}}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^(3/2)/sqrt(b*x^3 + a*x^2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.31

$$\int \frac{(ex)^{3/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx=\frac{1}{8}e\left(\frac{2\left(\sqrt{be^2x+ae^2}\sqrt{ex}\left(\frac{2\,dx}{be^2}+\frac{4\,b^2ce^3-3\,abde^3}{b^3e^5}\right)+\frac{(4\,abc-3\,a^2d)\log\left(\left|-\sqrt{be}\sqrt{ex}+\sqrt{be^2x+ae^2}\right.\right.}{\sqrt{be}b^2}\right.}{|e|\text{sgn}\left(x\right)}\right.$$

input
```
integrate((e*x)^(3/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
1/8*e*(2*(sqrt(b*e^2*x + a*e^2)*sqrt(e*x)*(2*d*x/(b*e^2) + (4*b^2*c*e^3 -
3*a*b*d*e^3)/(b^3*e^5)) + (4*a*b*c - 3*a^2*d)*log(abs(-sqrt(b*e)*sqrt(e*x)
 + sqrt(b*e^2*x + a*e^2)))/(sqrt(b*e)*b^2))*e^2/(abs(e)*sgn(x)) - (4*a*b*c
*e^2*log(e^2*abs(a)) - 3*a^2*d*e^2*log(e^2*abs(a)))*sgn(x)/(sqrt(b*e)*b^2*
abs(e)))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx=\int \frac{{\left(e\,x\right)}^{3/2}\,\left(c+d\,x\right)}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(((e*x)^(3/2)*(c + d*x))/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int(((e*x)^(3/2)*(c + d*x))/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.78

$$\int \frac{(ex)^{3/2}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{e}\,e\left(-3\sqrt{x}\,\sqrt{bx+a}\,abd+4\sqrt{x}\,\sqrt{bx+a}\,b^2c+2\sqrt{x}\,\sqrt{bx+a}\,b^2dx+3\sqrt{b}\log\left(\frac{\sqrt{bx}}{}\right.\right.}{4b^3}$$

input
```
int((e*x)^(3/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(sqrt(e)*e*( - 3*sqrt(x)*sqrt(a + b*x)*a*b*d + 4*sqrt(x)*sqrt(a + b*x)*b**
2*c + 2*sqrt(x)*sqrt(a + b*x)*b**2*d*x + 3*sqrt(b)*log((sqrt(a + b*x) + sq
rt(x)*sqrt(b))/sqrt(a))*a**2*d - 4*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sq
rt(b))/sqrt(a))*a*b*c))/(4*b**3)
```

3.335 $\int \frac{\sqrt{ex}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 83

$$\int \frac{\sqrt{ex}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{de\sqrt{ax^2+bx^3}}{b\sqrt{ex}} + \frac{(2bc-ad)\sqrt{e}\,\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

output
```
d*e*(b*x^3+a*x^2)^(1/2)/b/(e*x)^(1/2)+(-a*d+2*b*c)*e^(1/2)*arctanh(b^(1/2)
*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.20

$$\int \frac{\sqrt{ex}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \frac{\sqrt{x}\sqrt{ex}\left(\sqrt{b}d\sqrt{x}(a+bx)+2(2bc-ad)\sqrt{a+bx}\,\text{arctanh}\left(\frac{\sqrt{b}\sqrt{x}}{-\sqrt{a}+\sqrt{a+bx}}\right)\right)}{b^{3/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(Sqrt[e*x]*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(Sqrt[x]*Sqrt[e*x]*(Sqrt[b]*d*Sqrt[x]*(a + b*x) + 2*(2*b*c - a*d)*Sqrt[a +
 b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(b^(3/2)*Sqr
t[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1945, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx$$

$$\Big\downarrow \text{1945}$$

$$\frac{(2bc-ad)\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}+\frac{de\sqrt{ax^2+bx^3}}{b\sqrt{ex}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{\sqrt{ex}(2bc-ad)\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}+\frac{de\sqrt{ax^2+bx^3}}{b\sqrt{ex}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{\sqrt{ex}(2bc-ad)\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}+\frac{de\sqrt{ax^2+bx^3}}{b\sqrt{ex}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{ex}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)(2bc-ad)}{b^{3/2}\sqrt{x}}+\frac{de\sqrt{ax^2+bx^3}}{b\sqrt{ex}}$$

input
```
Int[(Sqrt[e*x]*(c + d*x))/Sqrt[a*x^2 + b*x^3],x]
```

output
```
(d*e*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[e*x]) + ((2*b*c - a*d)*Sqrt[e*x]*ArcTanh
[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.30

method	result	size
risch	$\frac{d\,x^2(bx+a)e}{b\sqrt{x^2(bx+a)}\,\sqrt{ex}}-\frac{(ad-2bc)\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^2+aex}\right)ex\sqrt{ex(bx+a)}}{2b\sqrt{be}\,\sqrt{x^2(bx+a)}\,\sqrt{ex}}$	108
default	$\frac{x(bx+a)\sqrt{ex}\left(-\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)ade+2\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)bce+2\sqrt{be}\,\sqrt{ex(bx+a)}\,d\right)}{2\sqrt{b\,x^3+a\,x^2}\,\sqrt{ex(bx+a)}\,b\sqrt{be}}$	142

input
```
int((e*x)^(1/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
d/b*x^2*(b*x+a)*e/(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)-1/2*(a*d-2*b*c)/b*ln((1/
2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)*e/(x^2*(b*x+a)
)^(1/2)*x*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.23

$$\int \frac{\sqrt{ex}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \left[-\frac{(2\,bc-ad)x\sqrt{\frac{e}{b}}\log\left(\frac{2\,bex^2+aex-2\,\sqrt{bx^3+ax^2}\sqrt{ex}b\sqrt{\frac{e}{b}}}{x}\right)-2\,\sqrt{bx^3+ax^2}\sqrt{ex}d}{2\,bx},\ -\frac{(2\,bc-ad)x\sqrt{-\frac{e}{b}}\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{ex}b\sqrt{-\frac{e}{b}}}{bex^2+aex}\right)-\sqrt{bx^3+ax^2}\sqrt{ex}d}{bx}\right]$$

input
```
integrate((e*x)^(1/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[-1/2*((2*b*c - a*d)*x*sqrt(e/b)*log((2*b*e*x^2 + a*e*x - 2*sqrt(b*x^3 + a
*x^2)*sqrt(e*x)*b*sqrt(e/b))/x) - 2*sqrt(b*x^3 + a*x^2)*sqrt(e*x)*d)/(b*x)
, -((2*b*c - a*d)*x*sqrt(-e/b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt
(-e/b)/(b*e*x^2 + a*e*x)) - sqrt(b*x^3 + a*x^2)*sqrt(e*x)*d)/(b*x)]
```

Sympy [F]

$$\int \frac{\sqrt{ex}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{\sqrt{ex}(c+dx)}{\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((e*x)**(1/2)*(d*x+c)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral(sqrt(e*x)*(c + d*x)/sqrt(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{\sqrt{ex}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = \int \frac{(dx+c)\sqrt{ex}}{\sqrt{bx^3+ax^2}}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*sqrt(e*x)/sqrt(b*x^3 + a*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(67) = 134$.

Time = 0.14 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.63

$$\int \frac{\sqrt{ex}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx = -\frac{e^2\left(\frac{(2\,bc-ad)\log\left(\left|-\sqrt{be}\sqrt{ex}+\sqrt{be^2x+ae^2}\right|\right)}{\sqrt{be}b} - \frac{\sqrt{be^2x+ae^2}\sqrt{ex}d}{be^2}\right)}{|e|\mathrm{sgn}\,(x)} + \frac{(2\,bce^2\log\left(e^2|a|\right) - ade^2\log\left(e^2|a|\right))\mathrm{sgn}(x)}{2\,\sqrt{be}b|e|}$$

input
```
integrate((e*x)^(1/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-e^2*((2*b*c - a*d)*log(abs(-sqrt(b*e)*sqrt(e*x) + sqrt(b*e^2*x + a*e^2)))
/(sqrt(b*e)*b) - sqrt(b*e^2*x + a*e^2)*sqrt(e*x)*d/(b*e^2))/(abs(e)*sgn(x)
) + 1/2*(2*b*c*e^2*log(e^2*abs(a)) - a*d*e^2*log(e^2*abs(a)))*sgn(x)/(sqrt
(b*e)*b*abs(e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx=\int \frac{\sqrt{e\,x}\,(c+d\,x)}{\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int(((e*x)^(1/2)*(c + d*x))/(a*x^2 + b*x^3)^(1/2),x)
```

output
```
int(((e*x)^(1/2)*(c + d*x))/(a*x^2 + b*x^3)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.80

$$\int \frac{\sqrt{ex}(c+dx)}{\sqrt{ax^2+bx^3}}\,dx =\frac{\sqrt{e}\left(\sqrt{x}\,\sqrt{bx+a}\,bd-\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)ad+2\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)bc\right)}{b^2}$$

input
```
int((e*x)^(1/2)*(d*x+c)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(sqrt(e)*(sqrt(x)*sqrt(a + b*x)*b*d - sqrt(b)*log((sqrt(a + b*x) + sqrt(x)
*sqrt(b))/sqrt(a))*a*d + 2*sqrt(b)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b))/s
qrt(a))*b*c))/b**2
```

3.336 $\int \frac{c+dx}{\sqrt{ex}\sqrt{ax^2+bx^3}} dx$

Optimal result

Integrand size = 28, antiderivative size = 77

$$\int \frac{c+dx}{\sqrt{ex}\sqrt{ax^2+bx^3}} dx = -\frac{2ce\sqrt{ax^2+bx^3}}{a(ex)^{3/2}} + \frac{2d\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}\sqrt{e}}$$

output
```
-2*c*e*(b*x^3+a*x^2)^(1/2)/a/(e*x)^(3/2)+2*d*arctanh(b^(1/2)*(e*x)^(3/2)/e
^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(1/2)/e^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.09

$$\int \frac{c+dx}{\sqrt{ex}\sqrt{ax^2+bx^3}} dx = -\frac{2\left(\sqrt{b}cx(a+bx) + adx^{3/2}\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{a\sqrt{b}\sqrt{ex}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)/(Sqrt[e*x]*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*(Sqrt[b]*c*x*(a + b*x) + a*d*x^(3/2)*Sqrt[a + b*x]*Log[-(Sqrt[b]*Sqrt[
x]) + Sqrt[a + b*x]]))/(a*Sqrt[b]*Sqrt[e*x]*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1944, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{ex}\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \ 1944$$

$$\frac{d\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{e}-\frac{2ce\sqrt{ax^2+bx^3}}{a(ex)^{3/2}}$$

$$\Big\downarrow \ 1937$$

$$\frac{d\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{e\sqrt{x}}-\frac{2ce\sqrt{ax^2+bx^3}}{a(ex)^{3/2}}$$

$$\Big\downarrow \ 1935$$

$$\frac{2d\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{e\sqrt{x}}-\frac{2ce\sqrt{ax^2+bx^3}}{a(ex)^{3/2}}$$

$$\Big\downarrow \ 219$$

$$\frac{2d\sqrt{ex}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{\sqrt{b}e\sqrt{x}}-\frac{2ce\sqrt{ax^2+bx^3}}{a(ex)^{3/2}}$$

input
```
Int[(c + d*x)/(Sqrt[e*x]*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*c*e*Sqrt[a*x^2 + b*x^3])/(a*(e*x)^(3/2)) + (2*d*Sqrt[e*x]*ArcTanh[(Sqr
t[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(Sqrt[b]*e*Sqrt[x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.22

method	result	size
risch	$-\frac{2c(bx+a)x}{a\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}+\frac{d\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^{2}+aex}\right)x\sqrt{ex(bx+a)}}{\sqrt{be}\,\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}$	94
default	$-\frac{x(bx+a)\left(-\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)adex+2\sqrt{ex(bx+a)}\,\sqrt{be}\,c\right)}{\sqrt{b\,x^{3}+a\,x^{2}}\,a\sqrt{ex}\,\sqrt{ex(bx+a)}\,\sqrt{be}}$	104

input
```
int((d*x+c)/(e*x)^(1/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*c/a*(b*x+a)/(x^2*(b*x+a))^(1/2)*x/(e*x)^(1/2)+d*ln((1/2*a*e+b*e*x)/(b*e
)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)/(x^2*(b*x+a))^(1/2)*x*(e*x*(b*x
+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.30

$$\int \frac{c+dx}{\sqrt{ex}\sqrt{ax^2+bx^3}}\,dx = \left[\frac{\sqrt{be}adx^2\log\left(\frac{2\,bex^2+aex+2\,\sqrt{bx^3+ax^2}\sqrt{be}\sqrt{ex}}{x}\right)-2\,\sqrt{bx^3+ax^2}\sqrt{ex}bc}{abex^2}, -\frac{2\left(\sqrt{-be}adx^2\arctan\left(\frac{\sqrt{bx^3+ax^2}\sqrt{-be}\sqrt{ex}}{bex^2+aex}\right)+\sqrt{bx^3+ax^2}\sqrt{ex}bc\right)}{abex^2}\right]$$

input
```
integrate((d*x+c)/(e*x)^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
[(sqrt(b*e)*a*d*x^2*log((2*b*e*x^2 + a*e*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(b*
e)*sqrt(e*x))/x) - 2*sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*c)/(a*b*e*x^2), -2*(s
qrt(-b*e)*a*d*x^2*arctan(sqrt(b*x^3 + a*x^2)*sqrt(-b*e)*sqrt(e*x)/(b*e*x^2
 + a*e*x)) + sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*c)/(a*b*e*x^2)]
```

Sympy [F]

$$\int \frac{c+dx}{\sqrt{ex}\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+dx}{\sqrt{ex}\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/(e*x)**(1/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)/(sqrt(e*x)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{ex}\sqrt{ax^2+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+ax^2}\sqrt{ex}}\,dx$$

input
```
integrate((d*x+c)/(e*x)^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(sqrt(b*x^3 + a*x^2)*sqrt(e*x)), x)
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.10

$$\int \frac{c+dx}{\sqrt{ex}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,b^2\left(\frac{d\log\left(\left|-\sqrt{be}\sqrt{bx+a}+\sqrt{(bx+a)be-abe}\right|\right)}{\sqrt{be}b}+\frac{\sqrt{bx+a}c}{\sqrt{(bx+a)be-abe}a}\right)}{|b|\mathrm{sgn}\left(x\right)}$$

input
```
integrate((d*x+c)/(e*x)^(1/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2*b^2*(d*log(abs(-sqrt(b*e)*sqrt(b*x + a) + sqrt((b*x + a)*b*e - a*b*e)))
/(sqrt(b*e)*b) + sqrt(b*x + a)*c/(sqrt((b*x + a)*b*e - a*b*e)*a))/(abs(b)*
sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{ex}\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+d\,x}{\sqrt{e\,x}\,\sqrt{b\,x^3+a\,x^2}}\,dx$$

input
```
int((c + d*x)/((e*x)^(1/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
int((c + d*x)/((e*x)^(1/2)*(a*x^2 + b*x^3)^(1/2)), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int \frac{c+dx}{\sqrt{ex}\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{e}\left(-\sqrt{x}\,\sqrt{bx+a}\,bc+\sqrt{b}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)adx-\sqrt{b}\,bcx\right)}{abex}$$

input
```
int((d*x+c)/(e*x)^(1/2)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(e)*( - sqrt(x)*sqrt(a + b*x)*b*c + sqrt(b)*log((sqrt(a + b*x) + sq
rt(x)*sqrt(b))/sqrt(a))*a*d*x - sqrt(b)*b*c*x))/(a*b*e*x)
```

3.337 $\int \frac{c+dx}{(ex)^{3/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 70

$$\int \frac{c+dx}{(ex)^{3/2}\sqrt{ax^2+bx^3}}\,dx=-\frac{2ce\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}+\frac{2(2bc-3ad)\sqrt{ax^2+bx^3}}{3a^2(ex)^{3/2}}$$

output
```
-2/3*c*e*(b*x^3+a*x^2)^(1/2)/a/(e*x)^(5/2)+2/3*(-3*a*d+2*b*c)*(b*x^3+a*x^2
)^(1/2)/a^2/(e*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.60

$$\int \frac{c+dx}{(ex)^{3/2}\sqrt{ax^2+bx^3}}\,dx=-\frac{2e\sqrt{x^2(a+bx)}(-2bcx+a(c+3dx))}{3a^2(ex)^{5/2}}$$

input
```
Integrate[(c + d*x)/((e*x)^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*e*Sqrt[x^2*(a + b*x)]*(-2*b*c*x + a*(c + 3*d*x)))/(3*a^2*(e*x)^(5/2))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1944, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(ex)^{3/2}\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(2bc-3ad)\int \frac{1}{\sqrt{ex}\sqrt{bx^3+ax^2}}dx}{3ae}-\frac{2ce\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{2\sqrt{ax^2+bx^3}(2bc-3ad)}{3a^2(ex)^{3/2}}-\frac{2ce\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}$$

input
```
Int[(c + d*x)/((e*x)^(3/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*c*e*Sqrt[a*x^2 + b*x^3])/(3*a*(e*x)^(5/2)) + (2*(2*b*c - 3*a*d)*Sqrt[a
*x^2 + b*x^3])/(3*a^2*(e*x)^(3/2))
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

method	result	size
gosper	$-\frac{2x(bx+a)(3adx-2cbx+ac)}{3a^{2}(ex)^{\frac{3}{2}}\sqrt{b\,x^{3}+a\,x^{2}}}$	44
risch	$-\frac{2(bx+a)(3adx-2cbx+ac)}{3e\sqrt{x^{2}(bx+a)}\,\sqrt{ex}\,a^{2}}$	44
orering	$-\frac{2x(bx+a)(3adx-2cbx+ac)}{3a^{2}(ex)^{\frac{3}{2}}\sqrt{b\,x^{3}+a\,x^{2}}}$	44
default	$-\frac{2(bx+a)(3adx-2cbx+ac)}{3\sqrt{b\,x^{3}+a\,x^{2}}\,a^{2}e\sqrt{ex}}$	46

input
```
int((d*x+c)/(e*x)^(3/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(b*x+a)*(3*a*d*x-2*b*c*x+a*c)/a^2/(e*x)^(3/2)/(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.64

$$\int \frac{c+dx}{(ex)^{3/2}\sqrt{ax^2+bx^3}}\,dx=-\frac{2\,\sqrt{bx^3+ax^2}(ac-(2\,bc-3\,ad)x)\sqrt{ex}}{3\,a^2e^2x^3}$$

input
```
integrate((d*x+c)/(e*x)^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(b*x^3 + a*x^2)*(a*c - (2*b*c - 3*a*d)*x)*sqrt(e*x)/(a^2*e^2*x^3)
```

Sympy [F]

$$\int \frac{c+dx}{(ex)^{3/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+dx}{(ex)^{\frac{3}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/(e*x)**(3/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)/((e*x)**(3/2)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{c+dx}{(ex)^{3/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+ax^2}\,(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/(e*x)^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(sqrt(b*x^3 + a*x^2)*(e*x)^(3/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.11

$$\int \frac{c+dx}{(ex)^{3/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{bx+a}b^3\left(\frac{(2\,bce-3\,ade)(bx+a)}{a^2}-\frac{3\,(abce-a^2de)}{a^2}\right)}{3\,((bx+a)be-abe)^{\frac{3}{2}}e|b|\mathrm{sgn}\,(x)}$$

input
```
integrate((d*x+c)/(e*x)^(3/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/3*sqrt(b*x + a)*b^3*((2*b*c*e - 3*a*d*e)*(b*x + a)/a^2 - 3*(a*b*c*e - a^
2*d*e)/a^2)/(((b*x + a)*b*e - a*b*e)^(3/2)*e*abs(b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.66 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

$$\int \frac{c+dx}{(ex)^{3/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,c}{3\,a\,e}+\frac{x\,\left(6\,a\,d-4\,b\,c\right)}{3\,a^2\,e}\right)}{x^2\,\sqrt{e\,x}}$$

input
```
int((c + d*x)/((e*x)^(3/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*c)/(3*a*e) + (x*(6*a*d - 4*b*c))/(3*a^2*e)))/(
x^2*(e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.99

$$\int \frac{c+dx}{(ex)^{3/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{e}\left(-\sqrt{x}\,\sqrt{bx+a}\,ac-3\sqrt{x}\,\sqrt{bx+a}\,adx+2\sqrt{x}\,\sqrt{bx+a}\,bcx+\sqrt{b}\,ad\,x^2-\right.}{3a^2e^2x^2}$$

input
```
int((d*x+c)/(e*x)^(3/2)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(e)*( - sqrt(x)*sqrt(a + b*x)*a*c - 3*sqrt(x)*sqrt(a + b*x)*a*d*x +
 2*sqrt(x)*sqrt(a + b*x)*b*c*x + sqrt(b)*a*d*x**2 - 2*sqrt(b)*b*c*x**2))/(
3*a**2*e**2*x**2)
```

3.338 $\int \frac{c+dx}{(ex)^{5/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 112

$$\int \frac{c+dx}{(ex)^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2ce\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}} + \frac{2(4bc-5ad)\sqrt{ax^2+bx^3}}{15a^2(ex)^{5/2}} - \frac{4b(4bc-5ad)\sqrt{ax^2+bx^3}}{15a^3e(ex)^{3/2}}$$

output
```
-2/5*c*e*(b*x^3+a*x^2)^(1/2)/a/(e*x)^(7/2)+2/15*(-5*a*d+4*b*c)*(b*x^3+a*x^
2)^(1/2)/a^2/(e*x)^(5/2)-4/15*b*(-5*a*d+4*b*c)*(b*x^3+a*x^2)^(1/2)/a^3/e/(
e*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.56

$$\int \frac{c+dx}{(ex)^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2e\sqrt{x^2(a+bx)}(8b^2cx^2-2abx(2c+5dx)+a^2(3c+5dx))}{15a^3(ex)^{7/2}}$$

input
```
Integrate[(c + d*x)/((e*x)^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*e*Sqrt[x^2*(a + b*x)]*(8*b^2*c*x^2 - 2*a*b*x*(2*c + 5*d*x) + a^2*(3*c
+ 5*d*x)))/(15*a^3*(e*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1944, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(ex)^{5/2}\sqrt{ax^2+bx^3}}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(4bc-5ad)\int \frac{1}{(ex)^{3/2}\sqrt{bx^3+ax^2}}dx}{5ae}-\frac{2ce\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{(4bc-5ad)\left(-\frac{2b\int \frac{1}{\sqrt{ex}\sqrt{bx^3+ax^2}}dx}{3ae}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{5ae}-\frac{2ce\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{(4bc-5ad)\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(ex)^{3/2}}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{5ae}-\frac{2ce\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}}$$

input
```
Int[(c + d*x)/((e*x)^(5/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*c*e*Sqrt[a*x^2 + b*x^3])/(5*a*(e*x)^(7/2)) - ((4*b*c - 5*a*d)*((-2*e*S
qrt[a*x^2 + b*x^3])/(3*a*(e*x)^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*(
e*x)^(3/2))))/(5*a*e)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2x(bx+a)\left(-10abd\,x^{2}+8b^{2}c\,x^{2}+5a^{2}dx-4abcx+3a^{2}c\right)}{15a^{3}(ex)^{\frac{5}{2}}\sqrt{b\,x^{3}+a\,x^{2}}}$	67
orering	$-\frac{2x(bx+a)\left(-10abd\,x^{2}+8b^{2}c\,x^{2}+5a^{2}dx-4abcx+3a^{2}c\right)}{15a^{3}(ex)^{\frac{5}{2}}\sqrt{b\,x^{3}+a\,x^{2}}}$	67
risch	$-\frac{2(bx+a)\left(-10abd\,x^{2}+8b^{2}c\,x^{2}+5a^{2}dx-4abcx+3a^{2}c\right)}{15e^{2}\sqrt{x^{2}(bx+a)}\,x\sqrt{ex}\,a^{3}}$	70
default	$-\frac{2(bx+a)\left(-10abd\,x^{2}+8b^{2}c\,x^{2}+5a^{2}dx-4abcx+3a^{2}c\right)}{15\sqrt{b\,x^{3}+a\,x^{2}}\,x\,a^{3}e^{2}\sqrt{ex}}$	72

input
```
int((d*x+c)/(e*x)^(5/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*x*(b*x+a)*(-10*a*b*d*x^2+8*b^2*c*x^2+5*a^2*d*x-4*a*b*c*x+3*a^2*c)/a^
3/(e*x)^(5/2)/(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.61

$$\int \frac{c+dx}{(ex)^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{bx^3+ax^2}(3\,a^2c+2\,(4\,b^2c-5\,abd)x^2-(4\,abc-5\,a^2d)x)\sqrt{ex}}{15\,a^3e^3x^4}$$

input
```
integrate((d*x+c)/(e*x)^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/15*sqrt(b*x^3 + a*x^2)*(3*a^2*c + 2*(4*b^2*c - 5*a*b*d)*x^2 - (4*a*b*c
- 5*a^2*d)*x)*sqrt(e*x)/(a^3*e^3*x^4)
```

Sympy [F]

$$\int \frac{c+dx}{(ex)^{5/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{c+dx}{(ex)^{\frac{5}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/(e*x)**(5/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)/((e*x)**(5/2)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{c+dx}{(ex)^{5/2}\sqrt{ax^2+bx^3}}\,dx = \int \frac{dx+c}{\sqrt{bx^3+ax^2}\,(ex)^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)/(e*x)^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(sqrt(b*x^3 + a*x^2)*(e*x)^(5/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.18

$$\int \frac{c+dx}{(ex)^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\sqrt{bx+a}\left((bx+a)\left(\frac{2\,(4\,b^5ce^2-5\,ab^4de^2)(bx+a)}{a^3}-\frac{5\,(4\,ab^5ce^2-5\,a^2b^4de^2)}{a^3}\right)+\frac{15\,(a^2b^5ce^2-a^3b^4de^2)}{a^3}\right)b}{15\left((bx+a)be-abe\right)^{\frac{5}{2}}e^2|b|\mathrm{sgn}\left(x\right)}$$

input
```
integrate((d*x+c)/(e*x)^(5/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
-2/15*sqrt(b*x + a)*((b*x + a)*(2*(4*b^5*c*e^2 - 5*a*b^4*d*e^2)*(b*x + a)/
a^3 - 5*(4*a*b^5*c*e^2 - 5*a^2*b^4*d*e^2)/a^3) + 15*(a^2*b^5*c*e^2 - a^3*b
^4*d*e^2)/a^3)*b/(((b*x + a)*b*e - a*b*e)^(5/2)*e^2*abs(b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.74 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.69

$$\int \frac{c+dx}{(ex)^{5/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,c}{5\,a\,e^2}+\frac{x^2\,\left(16\,b^2\,c-20\,a\,b\,d\right)}{15\,a^3\,e^2}+\frac{x\,\left(10\,a^2\,d-8\,a\,b\,c\right)}{15\,a^3\,e^2}\right)}{x^3\,\sqrt{e\,x}}$$

input
```
int((c + d*x)/((e*x)^(5/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*c)/(5*a*e^2) + (x^2*(16*b^2*c - 20*a*b*d))/(15
*a^3*e^2) + (x*(10*a^2*d - 8*a*b*c))/(15*a^3*e^2)))/(x^3*(e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.99

$$\int \frac{c+dx}{(ex)^{5/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{e}\left(-3\sqrt{x}\,\sqrt{bx+a}\,a^2c-5\sqrt{x}\,\sqrt{bx+a}\,a^2dx+4\sqrt{x}\,\sqrt{bx+a}\,abcx+10\sqrt{x}\,\sqrt{}\right.}{15a^3e^3x^3}$$

input
```
int((d*x+c)/(e*x)^(5/2)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(e)*( - 3*sqrt(x)*sqrt(a + b*x)*a**2*c - 5*sqrt(x)*sqrt(a + b*x)*a*
*2*d*x + 4*sqrt(x)*sqrt(a + b*x)*a*b*c*x + 10*sqrt(x)*sqrt(a + b*x)*a*b*d*
x**2 - 8*sqrt(x)*sqrt(a + b*x)*b**2*c*x**2 - 10*sqrt(b)*a*b*d*x**3 + 8*sqr
t(b)*b**2*c*x**3))/(15*a**3*e**3*x**3)
```

3.339 $\int \frac{c+dx}{(ex)^{7/2}\sqrt{ax^2+bx^3}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 156

$$\int \frac{c+dx}{(ex)^{7/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2ce\sqrt{ax^2+bx^3}}{7a(ex)^{9/2}}+\frac{2(6bc-7ad)\sqrt{ax^2+bx^3}}{35a^2(ex)^{7/2}} - \frac{8b(6bc-7ad)\sqrt{ax^2+bx^3}}{105a^3e(ex)^{5/2}}+\frac{16b^2(6bc-7ad)\sqrt{ax^2+bx^3}}{105a^4e^2(ex)^{3/2}}$$

output
```
-2/7*c*e*(b*x^3+a*x^2)^(1/2)/a/(e*x)^(9/2)+2/35*(-7*a*d+6*b*c)*(b*x^3+a*x^
2)^(1/2)/a^2/(e*x)^(7/2)-8/105*b*(-7*a*d+6*b*c)*(b*x^3+a*x^2)^(1/2)/a^3/e/
(e*x)^(5/2)+16/105*b^2*(-7*a*d+6*b*c)*(b*x^3+a*x^2)^(1/2)/a^4/e^2/(e*x)^(3
/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.53

$$\int \frac{c+dx}{(ex)^{7/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2e\sqrt{x^2(a+bx)}(-48b^3cx^3+8ab^2x^2(3c+7dx)+3a^3(5c+7dx)-2a^2bx(9c+14dx))}{105a^4(ex)^{9/2}}$$

input
```
Integrate[(c + d*x)/((e*x)^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*e*Sqrt[x^2*(a + b*x)]*(-48*b^3*c*x^3 + 8*a*b^2*x^2*(3*c + 7*d*x) + 3*a
^3*(5*c + 7*d*x) - 2*a^2*b*x*(9*c + 14*d*x)))/(105*a^4*(e*x)^(9/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1944, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(ex)^{7/2}\sqrt{ax^2+bx^3}}dx$$
$$\Big\downarrow \text{1944}$$
$$-\frac{(6bc-7ad)\int \frac{1}{(ex)^{5/2}\sqrt{bx^3+ax^2}}dx}{7ae}-\frac{2ce\sqrt{ax^2+bx^3}}{7a(ex)^{9/2}}$$
$$\Big\downarrow \text{1922}$$
$$-\frac{(6bc-7ad)\left(-\frac{4b\int \frac{1}{(ex)^{3/2}\sqrt{bx^3+ax^2}}dx}{5ae}-\frac{2e\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}}\right)}{7ae}-\frac{2ce\sqrt{ax^2+bx^3}}{7a(ex)^{9/2}}$$
$$\Big\downarrow \text{1922}$$
$$-\frac{(6bc-7ad)\left(-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{ex}\sqrt{bx^3+ax^2}}dx}{3ae}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{5ae}-\frac{2e\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}}\right)}{7ae}-\frac{2ce\sqrt{ax^2+bx^3}}{7a(ex)^{9/2}}$$
$$\Big\downarrow \text{1920}$$
$$-\frac{(6bc-7ad)\left(-\frac{4b\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(ex)^{3/2}}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{5ae}-\frac{2e\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}}\right)}{7ae}-\frac{2ce\sqrt{ax^2+bx^3}}{7a(ex)^{9/2}}$$

input
```
Int[(c + d*x)/((e*x)^(7/2)*Sqrt[a*x^2 + b*x^3]),x]
```

output
```
(-2*c*e*Sqrt[a*x^2 + b*x^3])/(7*a*(e*x)^(9/2)) - ((6*b*c - 7*a*d)*((-2*e*S
qrt[a*x^2 + b*x^3])/(5*a*(e*x)^(7/2)) - (4*b*((-2*e*Sqrt[a*x^2 + b*x^3])/(
3*a*(e*x)^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*(e*x)^(3/2))))/(5*a*e)
))/(7*a*e)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.58

method	result	size
gosper	$-\frac{2x(bx+a)(56a\,b^{2}d\,x^{3}-48b^{3}c\,x^{3}-28a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+21a^{3}dx-18a^{2}bcx+15c\,a^{3})}{105a^{4}(ex)^{\frac{7}{2}}\sqrt{b\,x^{3}+a\,x^{2}}}$	91
orering	$-\frac{2x(bx+a)(56a\,b^{2}d\,x^{3}-48b^{3}c\,x^{3}-28a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+21a^{3}dx-18a^{2}bcx+15c\,a^{3})}{105a^{4}(ex)^{\frac{7}{2}}\sqrt{b\,x^{3}+a\,x^{2}}}$	91
risch	$-\frac{2(bx+a)(56a\,b^{2}d\,x^{3}-48b^{3}c\,x^{3}-28a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+21a^{3}dx-18a^{2}bcx+15c\,a^{3})}{105e^{3}\sqrt{x^{2}(bx+a)}\,x^{2}\sqrt{ex}\,a^{4}}$	94
default	$-\frac{2(bx+a)(56a\,b^{2}d\,x^{3}-48b^{3}c\,x^{3}-28a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+21a^{3}dx-18a^{2}bcx+15c\,a^{3})}{105\sqrt{b\,x^{3}+a\,x^{2}}\,x^{2}a^{4}e^{3}\sqrt{ex}}$	96

input
```
int((d*x+c)/(e*x)^(7/2)/(b*x^3+a*x^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*x*(b*x+a)*(56*a*b^2*d*x^3-48*b^3*c*x^3-28*a^2*b*d*x^2+24*a*b^2*c*x^
2+21*a^3*d*x-18*a^2*b*c*x+15*a^3*c)/a^4/(e*x)^(7/2)/(b*x^3+a*x^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.59

$$\int \frac{c+dx}{(ex)^{7/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{2\,(15\,a^3c-8\,(6\,b^3c-7\,ab^2d)x^3+4\,(6\,ab^2c-7\,a^2bd)x^2-3\,(6\,a^2bc-7\,a^3d)x)\sqrt{bx^3+ax^2}\sqrt{ex}}{105\,a^4e^4x^5}$$

input
```
integrate((d*x+c)/(e*x)^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="fricas")
```

output
```
-2/105*(15*a^3*c - 8*(6*b^3*c - 7*a*b^2*d)*x^3 + 4*(6*a*b^2*c - 7*a^2*b*d)
*x^2 - 3*(6*a^2*b*c - 7*a^3*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x)/(a^4*e^4*x
^5)
```

Sympy [F]

$$\int \frac{c+dx}{(ex)^{7/2}\sqrt{ax^2+bx^3}}\,dx=\int \frac{c+dx}{(ex)^{\frac{7}{2}}\sqrt{x^2\,(a+bx)}}\,dx$$

input
```
integrate((d*x+c)/(e*x)**(7/2)/(b*x**3+a*x**2)**(1/2),x)
```

output
```
Integral((c + d*x)/((e*x)**(7/2)*sqrt(x**2*(a + b*x))), x)
```

Maxima [F]

$$\int \frac{c+dx}{(ex)^{7/2}\sqrt{ax^2+bx^3}}\,dx=\int \frac{dx+c}{\sqrt{bx^3+ax^2}\,(ex)^{\frac{7}{2}}}\,dx$$

input
```
integrate((d*x+c)/(e*x)^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(sqrt(b*x^3 + a*x^2)*(e*x)^(7/2)), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.10

$$\int \frac{c+dx}{(ex)^{7/2}\sqrt{ax^2+bx^3}}\,dx=\frac{2\left((bx+a)\left(4\,(bx+a)\left(\frac{2\,(6\,b^3ce^3-7\,ab^2de^3)(bx+a)}{a^4}-\frac{7\,(6\,ab^3ce^3-7\,a^2b^2de^3)}{a^4}\right)+\frac{35\,(6\,a^2b^3c\cdots}{\cdots}\right.\right.}{105\,((bx+a)be-abe)^{\frac{7}{2}}e^3|b|\mathrm{sgn}\,(x\cdots}$$

input
```
integrate((d*x+c)/(e*x)^(7/2)/(b*x^3+a*x^2)^(1/2),x, algorithm="giac")
```

output
```
2/105*((b*x + a)*(4*(b*x + a)*(2*(6*b^3*c*e^3 - 7*a*b^2*d*e^3)*(b*x + a)/a
^4 - 7*(6*a*b^3*c*e^3 - 7*a^2*b^2*d*e^3)/a^4) + 35*(6*a^2*b^3*c*e^3 - 7*a^
3*b^2*d*e^3)/a^4) - 105*(a^3*b^3*c*e^3 - a^4*b^2*d*e^3)/a^4)*sqrt(b*x + a)
*b^5/(((b*x + a)*b*e - a*b*e)^(7/2)*e^3*abs(b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 10.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.65

$$\int \frac{c+dx}{(ex)^{7/2}\sqrt{ax^2+bx^3}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,c}{7\,a\,e^3}+\frac{x\,\left(42\,a^3\,d-36\,a^2\,b\,c\right)}{105\,a^4\,e^3}-\frac{x^3\,\left(96\,b^3\,c-112\,a\,b^2\,d\right)}{105\,a^4\,e^3}-\frac{8\,b\,x^2\,\left(7\,a\,d-6\,b\,c\right)}{105\,a^3\,e^3}\right)}{x^4\,\sqrt{e\,x}}$$

input
```
int((c + d*x)/((e*x)^(7/2)*(a*x^2 + b*x^3)^(1/2)),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*c)/(7*a*e^3) + (x*(42*a^3*d - 36*a^2*b*c))/(10
5*a^4*e^3) - (x^3*(96*b^3*c - 112*a*b^2*d))/(105*a^4*e^3) - (8*b*x^2*(7*a*
d - 6*b*c))/(105*a^3*e^3)))/(x^4*(e*x)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.98

$$\int \frac{c+dx}{(ex)^{7/2}\sqrt{ax^2+bx^3}}\,dx = \frac{2\sqrt{e}\left(-15\sqrt{x}\,\sqrt{bx+a}\,a^3c-21\sqrt{x}\,\sqrt{bx+a}\,a^3dx+18\sqrt{x}\,\sqrt{bx+a}\,a^2bcx+28\cdots\right.}{}$$

input
```
int((d*x+c)/(e*x)^(7/2)/(b*x^3+a*x^2)^(1/2),x)
```

output
```
(2*sqrt(e)*( - 15*sqrt(x)*sqrt(a + b*x)*a**3*c - 21*sqrt(x)*sqrt(a + b*x)*
a**3*d*x + 18*sqrt(x)*sqrt(a + b*x)*a**2*b*c*x + 28*sqrt(x)*sqrt(a + b*x)*
a**2*b*d*x**2 - 24*sqrt(x)*sqrt(a + b*x)*a*b**2*c*x**2 - 56*sqrt(x)*sqrt(a
 + b*x)*a*b**2*d*x**3 + 48*sqrt(x)*sqrt(a + b*x)*b**3*c*x**3 + 56*sqrt(b)*
a*b**2*d*x**4 - 48*sqrt(b)*b**3*c*x**4))/(105*a**4*e**4*x**4)
```

3.340 $\int \frac{(ex)^{9/2}(c+dx)}{\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 171

$$\int \frac{(ex)^{9/2}(c+dx)}{\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{2(bc-ad)e^2(ex)^{5/2}}{b^2\sqrt{ax^2+bx^3}}+\frac{3(4bc-5ad)e^5\sqrt{ax^2+bx^3}}{4b^3\sqrt{ex}} + \frac{de^4\sqrt{ex}\sqrt{ax^2+bx^3}}{2b^2}-\frac{3a(4bc-5ad)e^{9/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{4b^{7/2}}$$

output
```
-2*(-a*d+b*c)*e^2*(e*x)^(5/2)/b^2/(b*x^3+a*x^2)^(1/2)+3/4*(-5*a*d+4*b*c)*e
^5*(b*x^3+a*x^2)^(1/2)/b^3/(e*x)^(1/2)+1/2*d*e^4*(e*x)^(1/2)*(b*x^3+a*x^2)
^(1/2)/b^2-3/4*a*(-5*a*d+4*b*c)*e^(9/2)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2
)/(b*x^3+a*x^2)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00

$$\int \frac{(ex)^{9/2}(c+dx)}{\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{e^4\sqrt{x}\sqrt{ex}\left(\sqrt{b}\sqrt{x}(-15a^2d+ab(12c-5dx)+2b^2x(2c+dx))+24abc\sqrt{a+bx}\text{arct}\right.}{4b^{7/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((e*x)^(9/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(e^4*Sqrt[x]*Sqrt[e*x]*(Sqrt[b]*Sqrt[x]*(-15*a^2*d + a*b*(12*c - 5*d*x) +
2*b^2*x*(2*c + d*x)) + 24*a*b*c*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x])/(S
qrt[a] - Sqrt[a + b*x])] + 30*a^2*d*Sqrt[a + b*x]*ArcTanh[(Sqrt[b]*Sqrt[x]
)/(-Sqrt[a] + Sqrt[a + b*x])]))/(4*b^(7/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1943, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{9/2}(c+dx)}{(ax^2+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{2e(ex)^{7/2}(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{e^2(4bc-5ad)\int \frac{(ex)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{ab}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2e(ex)^{7/2}(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{e^2(4bc-5ad)\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\int \frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{ab}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2e(ex)^{7/2}(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{e^2(4bc-5ad)\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{ab}$$

$$\Big\downarrow \text{1937}$$

$$\frac{2e(ex)^{7/2}(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{e^2(4bc-5ad)\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{ab}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{2e(ex)^{7/2}(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{e^2(4bc-5ad)\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{ab}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2e(ex)^{7/2}(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{e^2(4bc-5ad)\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{ab}$$

input
```
Int[((e*x)^(9/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*(b*c - a*d)*e*(e*x)^(7/2))/(a*b*Sqrt[a*x^2 + b*x^3]) - ((4*b*c - 5*a*d)
*e^2*((e^2*Sqrt[e*x]*Sqrt[a*x^2 + b*x^3])/(2*b) - (3*a*e*((e^2*Sqrt[a*x^2
+ b*x^3])/(b*Sqrt[e*x]) - (a*e*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*
x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/(4*b)))/(a*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.25

method	result
risch	$-\frac{(-2bdx+7ad-4bc)x^2(bx+a)e^5}{4b^3\sqrt{x^2(bx+a)}\sqrt{ex}}+\frac{a\left(\frac{15ad\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^2+aex}\right)}{\sqrt{be}}-\frac{12bc\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^2+aex}\right)}{\sqrt{be}}-\frac{16(ad-bc)\sqrt{be\left(x+\frac{a}{b}\right)^2}}{be\left(x+\frac{a}{b}\right)}\right.}{8b^3\sqrt{x^2(bx+a)}\sqrt{ex}}$
default	$-\frac{x^3(bx+a)\left(-4b^2d\,x^2\sqrt{be}\,\sqrt{ex(bx+a)}-15\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)a^2bdex+12\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)a\,b^2cex+10\right.}{}$

input
```
int((e*x)^(9/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*b*d*x+7*a*d-4*b*c)*x^2*(b*x+a)/b^3*e^5/(x^2*(b*x+a))^(1/2)/(e*x)^
(1/2)+1/8*a/b^3*(15*a*d*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/
2))/(b*e)^(1/2)-12*b*c*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2
))/(b*e)^(1/2)-16*(a*d-b*c)/b/e/(x+a/b)*(b*e*(x+a/b)^2-a*e*(x+a/b))^(1/2))
*e^5/(x^2*(b*x+a))^(1/2)*x*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.12

$$\int \frac{(ex)^{9/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx=\left[-\frac{3\left((4\,ab^2c-5\,a^2bd)e^4x^2+(4\,a^2bc-5\,a^3d)e^4x\right)\sqrt{\frac{e}{b}}\log\left(\frac{2\,bex^2+aex+2\,\sqrt{bx^3+ax^2}\sqrt{ex}b}{x}\right.}{8\,(b^4x^2}\right.$$

input
```
integrate((e*x)^(9/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/8*(3*((4*a*b^2*c - 5*a^2*b*d)*e^4*x^2 + (4*a^2*b*c - 5*a^3*d)*e^4*x)*s
qrt(e/b)*log((2*b*e*x^2 + a*e*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(e
/b))/x) - 2*(2*b^2*d*e^4*x^2 + (4*b^2*c - 5*a*b*d)*e^4*x + 3*(4*a*b*c - 5*
a^2*d)*e^4)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^4*x^2 + a*b^3*x), 1/4*(3*((4
*a*b^2*c - 5*a^2*b*d)*e^4*x^2 + (4*a^2*b*c - 5*a^3*d)*e^4*x)*sqrt(-e/b)*ar
ctan(sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(-e/b)/(b*e*x^2 + a*e*x)) + (2*b^
2*d*e^4*x^2 + (4*b^2*c - 5*a*b*d)*e^4*x + 3*(4*a*b*c - 5*a^2*d)*e^4)*sqrt(
b*x^3 + a*x^2)*sqrt(e*x))/(b^4*x^2 + a*b^3*x)]
```

Sympy [F]

$$\int \frac{(ex)^{9/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(ex)^{\frac{9}{2}}\,(c+dx)}{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**(9/2)*(d*x+c)/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((e*x)**(9/2)*(c + d*x)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(ex)^{9/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(dx+c)(ex)^{\frac{9}{2}}}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(9/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^(9/2)/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.35

$$\int \frac{(ex)^{9/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\left(\left(\frac{2\,de^3x|e|}{b\mathrm{sgn}(x)}+\frac{4\,b^4ce^5|e|\mathrm{sgn}(x)-5\,ab^3de^5|e|\mathrm{sgn}(x)}{b^5e^2}\right)ex+\frac{3\,(4\,ab^3ce^6|e|\mathrm{sgn}(x)-5\,a^2b^2de^6|e|\mathrm{sgn}(x))}{b^5e^2}\right)\sqrt{ex}}{4\,\sqrt{be^2x+ae^2}} - \frac{3\,(4\,abce^4|e|\log\left(e^2|a|\right)-5\,a^2de^4|e|\log\left(e^2|a|\right))\mathrm{sgn}(x)}{8\,\sqrt{be}b^3} + \frac{3\,(4\,abce^4|e|-5\,a^2de^4|e|)\log\left(\left|-\sqrt{be}\sqrt{ex}+\sqrt{be^2x+ae^2}\right|\right)}{4\,\sqrt{be}b^3\mathrm{sgn}\left(x\right)}$$

input
```
integrate((e*x)^(9/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*((2*d*e^3*x*abs(e)/(b*sgn(x)) + (4*b^4*c*e^5*abs(e)*sgn(x) - 5*a*b^3*d
*e^5*abs(e)*sgn(x))/(b^5*e^2))*e*x + 3*(4*a*b^3*c*e^6*abs(e)*sgn(x) - 5*a^
2*b^2*d*e^6*abs(e)*sgn(x))/(b^5*e^2))*sqrt(e*x)/sqrt(b*e^2*x + a*e^2) - 3/
8*(4*a*b*c*e^4*abs(e)*log(e^2*abs(a)) - 5*a^2*d*e^4*abs(e)*log(e^2*abs(a))
)*sgn(x)/(sqrt(b*e)*b^3) + 3/4*(4*a*b*c*e^4*abs(e) - 5*a^2*d*e^4*abs(e))*l
og(abs(-sqrt(b*e)*sqrt(e*x) + sqrt(b*e^2*x + a*e^2)))/(sqrt(b*e)*b^3*sgn(x
))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{{\left(e\,x\right)}^{9/2}\,\left(c+d\,x\right)}{{\left(b\,x^3+a\,x^2\right)}^{3/2}} \,d x$$

input
```
int(((e*x)^(9/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
int(((e*x)^(9/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.92

$$\int \frac{(ex)^{9/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\sqrt{e}\,e^4\left(15\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2d-12\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)ab\right.}{}$$

input
```
int((e*x)^(9/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(sqrt(e)*e**4*(15*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(
b))/sqrt(a))*a**2*d - 12*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x
)*sqrt(b))/sqrt(a))*a*b*c - 10*sqrt(b)*sqrt(a + b*x)*a**2*d + 9*sqrt(b)*sq
rt(a + b*x)*a*b*c - 15*sqrt(x)*a**2*b*d + 12*sqrt(x)*a*b**2*c - 5*sqrt(x)*
a*b**2*d*x + 4*sqrt(x)*b**3*c*x + 2*sqrt(x)*b**3*d*x**2))/(4*sqrt(a + b*x)
*b**4)
```

3.341 $\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 123

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2(bc-ad)e^2(ex)^{3/2}}{b^2\sqrt{ax^2+bx^3}} + \frac{de^4\sqrt{ax^2+bx^3}}{b^2\sqrt{ex}} + \frac{(2bc-3ad)e^{7/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{b^{5/2}}$$

output
```
-2*(-a*d+b*c)*e^2*(e*x)^(3/2)/b^2/(b*x^3+a*x^2)^(1/2)+d*e^4*(b*x^3+a*x^2)^
(1/2)/b^2/(e*x)^(1/2)+(-3*a*d+2*b*c)*e^(7/2)*arctanh(b^(1/2)*(e*x)^(3/2)/e
^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.91

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{(ex)^{7/2}\left(\sqrt{b}\sqrt{x}(a+bx)(-2bc+3ad+bdx)+2(2bc-3ad)(a+bx)^{3/2}\text{arctanh}\left(\frac{\ }{-\sqrt{\ }}\right.\right.}{b^{5/2}\sqrt{x}\left(x^2(a+bx)\right)^{3/2}}$$

input
```
Integrate[((e*x)^(7/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
((e*x)^(7/2)*(Sqrt[b]*Sqrt[x]*(a + b*x)*(-2*b*c + 3*a*d + b*d*x) + 2*(2*b*
c - 3*a*d)*(a + b*x)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a +
b*x])]))/(b^(5/2)*Sqrt[x]*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1943, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{3/2}}dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{2e(ex)^{5/2}(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{e^2(2bc-3ad)\int \frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{ab}$$

$$\Big\downarrow \text{1930}$$

$$\frac{2e(ex)^{5/2}(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{e^2(2bc-3ad)\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{ab}$$

$$\Big\downarrow \text{1937}$$

$$\frac{2e(ex)^{5/2}(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{e^2(2bc-3ad)\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{ab}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2e(ex)^{5/2}(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{e^2(2bc-3ad)\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{ab}$$

$$\Big\downarrow \text{219}$$

$$\frac{2e(ex)^{5/2}(bc-ad)}{ab\sqrt{ax^2+bx^3}}-\frac{e^2(2bc-3ad)\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\operatorname{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{ab}$$

input
```
Int[((e*x)^(7/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*(b*c - a*d)*e*(e*x)^(5/2))/(a*b*Sqrt[a*x^2 + b*x^3]) - ((2*b*c - 3*a*d)
*e^2*((e^2*Sqrt[a*x^2 + b*x^3])/(b*Sqrt[e*x]) - (a*e*Sqrt[e*x]*ArcTanh[(Sq
rt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/(a*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.62

method	result
risch	$\frac{d\,x^{2}(bx+a)e^{4}}{b^{2}\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}-\frac{\left(\frac{3ad\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^{2}+aex}\right)}{\sqrt{be}}-\frac{2bc\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^{2}+aex}\right)}{\sqrt{be}}-\frac{4(ad-bc)\sqrt{be\left(x+\frac{a}{b}\right)^{2}-ae\left(x+\frac{a}{b}\right)}}{be\left(x+\frac{a}{b}\right)}\right)e^{4}x\sqrt{}}{2b^{2}\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}$
default	$\frac{x^{3}(bx+a)\left(-3\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)abdex+2\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)b^{2}cex+2\sqrt{ex(bx+a)}\,\sqrt{be}\,bdx-3\ln\left(\frac{2bex+2\sqrt{}}{}\right.\right.}{2(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}\sqrt{be}\,\sqrt{ex(}}$

input
```
int((e*x)^(7/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
d/b^2*x^2*(b*x+a)*e^4/(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)-1/2/b^2*(3*a*d*ln((1
/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)-2*b*c*ln((1/2
*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)-4*(a*d-b*c)/b/e
/(x+a/b)*(b*e*(x+a/b)^2-a*e*(x+a/b))^(1/2))*e^4/(x^2*(b*x+a))^(1/2)*x*(e*x
*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.45

$$\int\frac{(ex)^{7/2}(c+dx)}{(ax^{2}+bx^{3})^{3/2}}\,dx=\left[-\frac{((2\,b^{2}c-3\,abd)e^{3}x^{2}+(2\,abc-3\,a^{2}d)e^{3}x)\sqrt{\frac{e}{b}}\log\left(\frac{2\,bex^{2}+aex-2\,\sqrt{bx^{3}+ax^{2}}\sqrt{ex}b\sqrt{\frac{e}{b}}}{x}\right)}{2\,(b^{3}x^{2}+ab^{2}x)}\right.$$

input
```
integrate((e*x)^(7/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[-1/2*(((2*b^2*c - 3*a*b*d)*e^3*x^2 + (2*a*b*c - 3*a^2*d)*e^3*x)*sqrt(e/b)
*log((2*b*e*x^2 + a*e*x - 2*sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(e/b))/x)
- 2*(b*d*e^3*x - (2*b*c - 3*a*d)*e^3)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^3*
x^2 + a*b^2*x), -(((2*b^2*c - 3*a*b*d)*e^3*x^2 + (2*a*b*c - 3*a^2*d)*e^3*x
)*sqrt(-e/b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(-e/b)/(b*e*x^2 +
a*e*x)) - (b*d*e^3*x - (2*b*c - 3*a*d)*e^3)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))
/(b^3*x^2 + a*b^2*x)]
```

Sympy [F]

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(ex)^{\frac{7}{2}}\,(c+dx)}{\left(x^2\,(a+bx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**(7/2)*(d*x+c)/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((e*x)**(7/2)*(c + d*x)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(dx+c)(ex)^{\frac{7}{2}}}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^(7/2)/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.45

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\left(\frac{de^3x|e|}{b\text{sgn}(x)} - \frac{2\,b^2ce^5|e|\text{sgn}(x)-3\,abde^5|e|\text{sgn}(x)}{b^3e^2}\right)\sqrt{ex}}{\sqrt{be^2x+ae^2}} + \frac{(2\,bce^3|e|\log\left(e^2|a|\right) - 3\,ade^3|e|\log\left(e^2|a|\right))\text{sgn}(x)}{2\sqrt{be}b^2} - \frac{(2\,bce^3|e| - 3\,ade^3|e|)\log\left(\left|-\sqrt{be}\sqrt{ex} + \sqrt{be^2x+ae^2}\right|\right)}{\sqrt{be}b^2\text{sgn}\,(x)}$$

input

```
integrate((e*x)^(7/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output

```
(d*e^3*x*abs(e)/(b*sgn(x)) - (2*b^2*c*e^5*abs(e)*sgn(x) - 3*a*b*d*e^5*abs(
e)*sgn(x))/(b^3*e^2))*sqrt(e*x)/sqrt(b*e^2*x + a*e^2) + 1/2*(2*b*c*e^3*abs
(e)*log(e^2*abs(a)) - 3*a*d*e^3*abs(e)*log(e^2*abs(a)))*sgn(x)/(sqrt(b*e)*
b^2) - (2*b*c*e^3*abs(e) - 3*a*d*e^3*abs(e))*log(abs(-sqrt(b*e)*sqrt(e*x)
+ sqrt(b*e^2*x + a*e^2)))/(sqrt(b*e)*b^2*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(e\,x)^{7/2}\,(c+d\,x)}{(b\,x^3+a\,x^2)^{3/2}}\,dx$$

input

```
int(((e*x)^(7/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x)
```

output

```
int(((e*x)^(7/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.03

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{\sqrt{e}\,e^3\left(-12\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)ad+8\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)bc}{4\sqrt{bx+a}}$$

input
```
int((e*x)^(7/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(sqrt(e)*e**3*( - 12*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sq
rt(b))/sqrt(a))*a*d + 8*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)
*sqrt(b))/sqrt(a))*b*c + 9*sqrt(b)*sqrt(a + b*x)*a*d - 8*sqrt(b)*sqrt(a +
b*x)*b*c + 12*sqrt(x)*a*b*d - 8*sqrt(x)*b**2*c + 4*sqrt(x)*b**2*d*x))/(4*s
qrt(a + b*x)*b**3)
```

3.342 $\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 87

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx=\frac{2(bc-ad)e(ex)^{3/2}}{ab\sqrt{ax^2+bx^3}}+\frac{2de^{5/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{b^{3/2}}$$

output
```
2*(-a*d+b*c)*e*(e*x)^(3/2)/a/b/(b*x^3+a*x^2)^(1/2)+2*d*e^(5/2)*arctanh(b^(
1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*x^2)^(1/2))/b^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2(ex)^{5/2}\left(\sqrt{b}(-bc+ad)\sqrt{x}+ad\sqrt{a+bx}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{ab^{3/2}x^{3/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((e*x)^(5/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-2*(e*x)^(5/2)*(Sqrt[b]*(-(b*c) + a*d)*Sqrt[x] + a*d*Sqrt[a + b*x]*Log[-(
Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(a*b^(3/2)*x^(3/2)*Sqrt[x^2*(a + b*x)]
)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1943, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{3/2}} \, dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{de^2 \int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{b} + \frac{2e(ex)^{3/2}(bc-ad)}{ab\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{de^2\sqrt{ex} \int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{b\sqrt{x}} + \frac{2e(ex)^{3/2}(bc-ad)}{ab\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{2de^2\sqrt{ex} \int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}} + \frac{2e(ex)^{3/2}(bc-ad)}{ab\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2de^2\sqrt{ex}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}} + \frac{2e(ex)^{3/2}(bc-ad)}{ab\sqrt{ax^2+bx^3}}$$

input
```
Int[((e*x)^(5/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*(b*c - a*d)*e*(e*x)^(3/2))/(a*b*Sqrt[a*x^2 + b*x^3]) + (2*d*e^2*Sqrt[e*
x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(71) = 142$.

Time = 0.40 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.00

method	result
default	$-\frac{x^{3}(bx+a)\left(-\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)abdex-\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)a^{2}de+2\sqrt{ex(bx+a)}\,\sqrt{be}\,ad-2\sqrt{ex(bx+a)}\,\sqrt{}\right.}{(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}\,a\sqrt{be}\,\sqrt{ex(bx+a)}\,b}$

input
```
int((e*x)^(5/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-x^3*(b*x+a)*(-ln(1/2*(2*b*e*x+2*(e*x*(b*x+a))^(1/2)*(b*e)^(1/2)+a*e)/(b*e
)^(1/2))*a*b*d*e*x-ln(1/2*(2*b*e*x+2*(e*x*(b*x+a))^(1/2)*(b*e)^(1/2)+a*e)/
(b*e)^(1/2))*a^2*d*e+2*(e*x*(b*x+a))^(1/2)*(b*e)^(1/2)*a*d-2*(e*x*(b*x+a))
^(1/2)*(b*e)^(1/2)*b*c)*(e*x)^(1/2)*e^2/(b*x^3+a*x^2)^(3/2)/a/(b*e)^(1/2)/
(e*x*(b*x+a))^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 246, normalized size of antiderivative = 2.83

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \left[\frac{2\sqrt{bx^3+ax^2}(bc-ad)\sqrt{ex}e^2 + (abde^2x^2+a^2de^2x)\sqrt{\frac{e}{b}}\log\left(\frac{2\,bex^2+aex+2\,\sqrt{bx^3+ax^2}\right.}{x}}{ab^2x^2+a^2bx}\right.$$

input
```
integrate((e*x)^(5/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
[(2*sqrt(b*x^3 + a*x^2)*(b*c - a*d)*sqrt(e*x)*e^2 + (a*b*d*e^2*x^2 + a^2*d
*e^2*x)*sqrt(e/b)*log((2*b*e*x^2 + a*e*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(e*x)
*b*sqrt(e/b))/x))/(a*b^2*x^2 + a^2*b*x), 2*(sqrt(b*x^3 + a*x^2)*(b*c - a*d
)*sqrt(e*x)*e^2 - (a*b*d*e^2*x^2 + a^2*d*e^2*x)*sqrt(-e/b)*arctan(sqrt(b*x
^3 + a*x^2)*sqrt(e*x)*b*sqrt(-e/b)/(b*e*x^2 + a*e*x)))/(a*b^2*x^2 + a^2*b*
x)]
```

Sympy [F]

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(ex)^{\frac{5}{2}}\,(c+dx)}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**(5/2)*(d*x+c)/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((e*x)**(5/2)*(c + d*x)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(dx+c)(ex)^{\frac{5}{2}}}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^(5/2)/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{de^2|e|\log\left(e^2|a|\right)\operatorname{sgn}(x)}{\sqrt{be}b} - \frac{2\,de^2|e|\log\left(\left|-\sqrt{be}\sqrt{ex}+\sqrt{be^2x+ae^2}\right|\right)}{\sqrt{be}b\operatorname{sgn}(x)} + \frac{2\left(bce^4|e|\operatorname{sgn}(x)-ade^4|e|\operatorname{sgn}(x)\right)\sqrt{ex}}{\sqrt{be^2x+ae^2}abe^2}$$

input
```
integrate((e*x)^(5/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
d*e^2*abs(e)*log(e^2*abs(a))*sgn(x)/(sqrt(b*e)*b) - 2*d*e^2*abs(e)*log(abs
(-sqrt(b*e)*sqrt(e*x) + sqrt(b*e^2*x + a*e^2)))/(sqrt(b*e)*b*sgn(x)) + 2*(
b*c*e^4*abs(e)*sgn(x) - a*d*e^4*abs(e)*sgn(x))*sqrt(e*x)/(sqrt(b*e^2*x + a
*e^2)*a*b*e^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(e\,x)^{5/2}\,(c+d\,x)}{(b\,x^3+a\,x^2)^{3/2}}\,dx$$

input
```
int(((e*x)^(5/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
int(((e*x)^(5/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.01

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{e}\,e^2\left(\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)ad-\sqrt{b}\,\sqrt{bx+a}\,ad+\sqrt{b}\,\sqrt{bx+a}\,bc- \right.}{\sqrt{bx+a}\,a\,b^2}$$

input
```
int((e*x)^(5/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*sqrt(e)*e**2*(sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt(b
))/sqrt(a))*a*d - sqrt(b)*sqrt(a + b*x)*a*d + sqrt(b)*sqrt(a + b*x)*b*c -
sqrt(x)*a*b*d + sqrt(x)*b**2*c))/(sqrt(a + b*x)*a*b**2)
```

3.343 $\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 66

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2ce\sqrt{ex}}{a\sqrt{ax^2+bx^3}} - \frac{2(2bc-ad)(ex)^{3/2}}{a^2\sqrt{ax^2+bx^3}}$$

output
```
-2*c*e*(e*x)^(1/2)/a/(b*x^3+a*x^2)^(1/2)-2*(-a*d+2*b*c)*(e*x)^(3/2)/a^2/(b
*x^3+a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.61

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2e\sqrt{ex}(-ac-2bcx+adx)}{a^2\sqrt{x^2(a+bx)}}$$

input
```
Integrate[((e*x)^(3/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(2*e*Sqrt[e*x]*(-(a*c) - 2*b*c*x + a*d*x))/(a^2*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1944, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx$$

$$\Big\downarrow \ 1944$$

$$-\frac{(2bc-ad)\int \frac{(ex)^{5/2}}{(bx^3+ax^2)^{3/2}}dx}{ae}-\frac{2ce\sqrt{ex}}{a\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \ 1920$$

$$-\frac{2(ex)^{3/2}(2bc-ad)}{a^2\sqrt{ax^2+bx^3}}-\frac{2ce\sqrt{ex}}{a\sqrt{ax^2+bx^3}}$$

input
```
Int[((e*x)^(3/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-2*c*e*Sqrt[e*x])/(a*Sqrt[a*x^2 + b*x^3]) - (2*(2*b*c - a*d)*(e*x)^(3/2))
/(a^2*Sqrt[a*x^2 + b*x^3])
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.67

method	result	size
gosper	$-\frac{2x(bx+a)(-adx+2cbx+ac)(ex)^{\frac{3}{2}}}{a^{2}(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}}$	44
orering	$-\frac{2x(bx+a)(-adx+2cbx+ac)(ex)^{\frac{3}{2}}}{a^{2}(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}}$	44
default	$-\frac{2x^{2}(bx+a)(-adx+2cbx+ac)\sqrt{ex}\,e}{(b\,x^{3}+a\,x^{2})^{\frac{3}{2}}a^{2}}$	47
risch	$-\frac{2c(bx+a)e^{2}x}{a^{2}\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}+\frac{2(ad-bc)x^{2}e^{2}}{a^{2}\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}$	68

input
```
int((e*x)^(3/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*x*(b*x+a)*(-a*d*x+2*b*c*x+a*c)*(e*x)^(3/2)/a^2/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx=-\frac{2\sqrt{bx^3+ax^2}(ace+(2\,bc-ad)ex)\sqrt{ex}}{a^2bx^3+a^3x^2}$$

input
```
integrate((e*x)^(3/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(b*x^3 + a*x^2)*(a*c*e + (2*b*c - a*d)*e*x)*sqrt(e*x)/(a^2*b*x^3 +
a^3*x^2)
```

Sympy [F]

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx=\int \frac{(ex)^{\frac{3}{2}}\,(c+dx)}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**(3/2)*(d*x+c)/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((e*x)**(3/2)*(c + d*x)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx=\int \frac{(dx+c)(ex)^{\frac{3}{2}}}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^(3/2)/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.68

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx=-2\,e\left(\frac{2\,\sqrt{be}ce^3}{\left(ae^2-\left(\sqrt{be}\sqrt{ex}-\sqrt{be^2x+ae^2}\right)^2\right)a|e|\mathrm{sgn}\,(x)}+\frac{(bce^2-ade^2)\sqrt{ex}}{\sqrt{be^2x+ae^2}a^2|e|\mathrm{sgn}\,(x)}\right)$$

input
```
integrate((e*x)^(3/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-2*e*(2*sqrt(b*e)*c*e^3/((a*e^2 - (sqrt(b*e)*sqrt(e*x) - sqrt(b*e^2*x + a*
e^2))^2)*a*abs(e)*sgn(x)) + (b*c*e^2 - a*d*e^2)*sqrt(e*x)/(sqrt(b*e^2*x +
a*e^2)*a^2*abs(e)*sgn(x)))
```

Mupad [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.03

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,c\,e\,\sqrt{e\,x}}{a\,b}-\frac{2\,e\,x\,\sqrt{e\,x}\,(a\,d-2\,b\,c)}{a^2\,b}\right)}{x^3+\frac{a\,x^2}{b}}$$

input
```
int(((e*x)^(3/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*c*e*(e*x)^(1/2))/(a*b) - (2*e*x*(e*x)^(1/2)*(a
*d - 2*b*c))/(a^2*b)))/(x^3 + (a*x^2)/b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{e}\,e\left(\sqrt{b}\,\sqrt{bx+a}\,adx-2\sqrt{b}\,\sqrt{bx+a}\,bcx-\sqrt{x}\,abc+\sqrt{x}\,abdx-2\sqrt{x}\,b^2cx\right)}{\sqrt{bx+a}\,a^2bx}$$

input
```
int((e*x)^(3/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*sqrt(e)*e*(sqrt(b)*sqrt(a + b*x)*a*d*x - 2*sqrt(b)*sqrt(a + b*x)*b*c*x
- sqrt(x)*a*b*c + sqrt(x)*a*b*d*x - 2*sqrt(x)*b**2*c*x))/(sqrt(a + b*x)*a*
*2*b*x)
```

3.344 $\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 111

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2ce}{3a\sqrt{ex}\sqrt{ax^2+bx^3}} - \frac{2(4bc-3ad)\sqrt{ex}}{3a^2\sqrt{ax^2+bx^3}} + \frac{4(4bc-3ad)e^2\sqrt{ax^2+bx^3}}{3a^3(ex)^{3/2}}$$

output
```
-2/3*c*e/a/(e*x)^(1/2)/(b*x^3+a*x^2)^(1/2)-2/3*(-3*a*d+4*b*c)*(e*x)^(1/2)/
a^2/(b*x^3+a*x^2)^(1/2)+4/3*(-3*a*d+4*b*c)*e^2*(b*x^3+a*x^2)^(1/2)/a^3/(e*
x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.55

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2e(-8b^2cx^2+2abx(-2c+3dx)+a^2(c+3dx))}{3a^3\sqrt{ex}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(Sqrt[e*x]*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-2*e*(-8*b^2*c*x^2 + 2*a*b*x*(-2*c + 3*d*x) + a^2*(c + 3*d*x)))/(3*a^3*Sq
rt[e*x]*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.107$, Rules used = {1944, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{3/2}} dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(4bc-3ad)\int \frac{(ex)^{3/2}}{(bx^3+ax^2)^{3/2}}dx}{3ae} - \frac{2ce}{3a\sqrt{ex}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1921}$$

$$-\frac{(4bc-3ad)\left(\frac{2e^2\int \frac{1}{\sqrt{ex}\sqrt{bx^3+ax^2}}dx}{a} + \frac{2e\sqrt{ex}}{a\sqrt{ax^2+bx^3}}\right)}{3ae} - \frac{2ce}{3a\sqrt{ex}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{(4bc-3ad)\left(\frac{2e\sqrt{ex}}{a\sqrt{ax^2+bx^3}} - \frac{4e^3\sqrt{ax^2+bx^3}}{a^2(ex)^{3/2}}\right)}{3ae} - \frac{2ce}{3a\sqrt{ex}\sqrt{ax^2+bx^3}}$$

input
```
Int[(Sqrt[e*x]*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x]
```

output
```
(-2*c*e)/(3*a*Sqrt[e*x]*Sqrt[a*x^2 + b*x^3]) - ((4*b*c - 3*a*d)*((2*e*Sqrt
[e*x])/(a*Sqrt[a*x^2 + b*x^3]) - (4*e^3*Sqrt[a*x^2 + b*x^3])/(a^2*(e*x)^(3
/2))))/(3*a*e)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{2x(bx+a)\left(6abd\,x^{2}-8b^{2}c\,x^{2}+3a^{2}dx-4abcx+a^{2}c\right)\sqrt{ex}}{3a^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	66
default	$-\frac{2x(bx+a)\left(6abd\,x^{2}-8b^{2}c\,x^{2}+3a^{2}dx-4abcx+a^{2}c\right)\sqrt{ex}}{3a^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	66
orering	$-\frac{2x(bx+a)\left(6abd\,x^{2}-8b^{2}c\,x^{2}+3a^{2}dx-4abcx+a^{2}c\right)\sqrt{ex}}{3a^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	66
risch	$-\frac{2(bx+a)(3adx-5cbx+ac)e}{3a^{3}\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}-\frac{2b(ad-bc)x^{2}e}{a^{3}\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}$	77

input
```
int((e*x)^(1/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(b*x+a)*(6*a*b*d*x^2-8*b^2*c*x^2+3*a^2*d*x-4*a*b*c*x+a^2*c)*(e*x)^(
1/2)/a^3/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{bx^3+ax^2}(a^2c-2\,(4\,b^2c-3\,abd)x^2-(4\,abc-3\,a^2d)x)\sqrt{ex}}{3\,(a^3bx^4+a^4x^3)}$$

input
```
integrate((e*x)^(1/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-2/3*sqrt(b*x^3 + a*x^2)*(a^2*c - 2*(4*b^2*c - 3*a*b*d)*x^2 - (4*a*b*c - 3
*a^2*d)*x)*sqrt(e*x)/(a^3*b*x^4 + a^4*x^3)
```

Sympy [F]

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{\sqrt{ex}(c+dx)}{(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**(1/2)*(d*x+c)/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral(sqrt(e*x)*(c + d*x)/(x**2*(a + b*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \int \frac{(dx+c)\sqrt{ex}}{(bx^3+ax^2)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*sqrt(e*x)/(b*x^3 + a*x^2)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. $2(93) = 186$.

Time = 0.76 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.74

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\left(b^2ce^2-abde^2\right)\sqrt{ex}}{\sqrt{be^2x+ae^2}\,a^3|e|\mathrm{sgn}(x)} + \frac{4\left(5\sqrt{be}a^2bce^7-3\sqrt{be}a^3de^7-12\sqrt{be}\left(\sqrt{be}\sqrt{ex}-\sqrt{be^2x+ae^2}\right)^2abce^5+6\sqrt{be}\left(\sqrt{be}\sqrt{ex}-\sqrt{be^2x+a\cdots\right.\right.}{3\left(ae^2-\left(\sqrt{be}\sqrt{ex}-\sqrt{be^2x+\cdots\right.\right.}$$

input
```
integrate((e*x)^(1/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
2*(b^2*c*e^2 - a*b*d*e^2)*sqrt(e*x)/(sqrt(b*e^2*x + a*e^2)*a^3*abs(e)*sgn(
x)) + 4/3*(5*sqrt(b*e)*a^2*b*c*e^7 - 3*sqrt(b*e)*a^3*d*e^7 - 12*sqrt(b*e)*
(sqrt(b*e)*sqrt(e*x) - sqrt(b*e^2*x + a*e^2))^2*a*b*c*e^5 + 6*sqrt(b*e)*(s
qrt(b*e)*sqrt(e*x) - sqrt(b*e^2*x + a*e^2))^2*a^2*d*e^5 + 3*sqrt(b*e)*(sqr
t(b*e)*sqrt(e*x) - sqrt(b*e^2*x + a*e^2))^4*b*c*e^3 - 3*sqrt(b*e)*(sqrt(b*
e)*sqrt(e*x) - sqrt(b*e^2*x + a*e^2))^4*a*d*e^3)/((a*e^2 - (sqrt(b*e)*sqrt
(e*x) - sqrt(b*e^2*x + a*e^2))^2)^3*a^2*abs(e)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.88 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.88

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\left(\frac{2\,c\,\sqrt{e\,x}}{3\,a\,b}+\frac{x\,\sqrt{e\,x}\,\left(6\,a^2\,d-8\,a\,b\,c\right)}{3\,a^3\,b}-\frac{x^2\,\sqrt{e\,x}\,\left(16\,b^2\,c-12\,a\,b\,d\right)}{3\,a^3\,b}\right)}{x^4+\frac{a\,x^3}{b}}$$

input
```
int(((e*x)^(1/2)*(c + d*x))/(a*x^2 + b*x^3)^(3/2),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*c*(e*x)^(1/2))/(3*a*b) + (x*(e*x)^(1/2)*(6*a^2
*d - 8*a*b*c))/(3*a^3*b) - (x^2*(e*x)^(1/2)*(16*b^2*c - 12*a*b*d))/(3*a^3*
b)))/(x^4 + (a*x^3)/b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.86

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{e}\left(6\sqrt{b}\,\sqrt{bx+a}\,ad\,x^2-8\sqrt{b}\,\sqrt{bx+a}\,bc\,x^2-\sqrt{x}\,a^2c-3\sqrt{x}\,a^2dx+4\sqrt{x}\,abcx-\right.}{3\sqrt{bx+a}\,a^3x^2}$$

input
```
int((e*x)^(1/2)*(d*x+c)/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*sqrt(e)*(6*sqrt(b)*sqrt(a + b*x)*a*d*x**2 - 8*sqrt(b)*sqrt(a + b*x)*b*c
*x**2 - sqrt(x)*a**2*c - 3*sqrt(x)*a**2*d*x + 4*sqrt(x)*a*b*c*x - 6*sqrt(x
)*a*b*d*x**2 + 8*sqrt(x)*b**2*c*x**2))/(3*sqrt(a + b*x)*a**3*x**2)
```

3.345 $\int \frac{c+dx}{\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 151

$$\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{3/2}}\,dx = -\frac{2ce}{5a(ex)^{3/2}\sqrt{ax^2+bx^3}} - \frac{2(6bc-5ad)}{5a^2\sqrt{ex}\sqrt{ax^2+bx^3}} + \frac{8(6bc-5ad)e^2\sqrt{ax^2+bx^3}}{15a^3(ex)^{5/2}} - \frac{16b(6bc-5ad)e\sqrt{ax^2+bx^3}}{15a^4(ex)^{3/2}}$$

output
```
-2/5*c*e/a/(e*x)^(3/2)/(b*x^3+a*x^2)^(1/2)-2/5*(-5*a*d+6*b*c)/a^2/(e*x)^(1
/2)/(b*x^3+a*x^2)^(1/2)+8/15*(-5*a*d+6*b*c)*e^2*(b*x^3+a*x^2)^(1/2)/a^3/(e
*x)^(5/2)-16/15*b*(-5*a*d+6*b*c)*e*(b*x^3+a*x^2)^(1/2)/a^4/(e*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.54

$$\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{3/2}}\,dx = -\frac{2e(48b^3cx^3+8ab^2x^2(3c-5dx)+a^3(3c+5dx)-2a^2bx(3c+10dx))}{15a^4(ex)^{3/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)/(Sqrt[e*x]*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
(-2*e*(48*b^3*c*x^3 + 8*a*b^2*x^2*(3*c - 5*d*x) + a^3*(3*c + 5*d*x) - 2*a^
2*b*x*(3*c + 10*d*x)))/(15*a^4*(e*x)^(3/2)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1944, 1921, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(6bc-5ad)\int \frac{\sqrt{ex}}{(bx^3+ax^2)^{3/2}}dx}{5ae}-\frac{2ce}{5a(ex)^{3/2}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1921}$$

$$-\frac{(6bc-5ad)\left(\frac{4e^2\int \frac{1}{(ex)^{3/2}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2e}{a\sqrt{ex}\sqrt{ax^2+bx^3}}\right)}{5ae}-\frac{2ce}{5a(ex)^{3/2}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{(6bc-5ad)\left(\frac{4e^2\left(-\frac{2b\int \frac{1}{\sqrt{ex}\sqrt{bx^3+ax^2}}dx}{3ae}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{a}+\frac{2e}{a\sqrt{ex}\sqrt{ax^2+bx^3}}\right)}{5ae}-\frac{2ce}{5a(ex)^{3/2}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{(6bc-5ad)\left(\frac{4e^2\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(ex)^{3/2}}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{a}+\frac{2e}{a\sqrt{ex}\sqrt{ax^2+bx^3}}\right)}{5ae}-\frac{2ce}{5a(ex)^{3/2}\sqrt{ax^2+bx^3}}$$

input
```
Int[(c + d*x)/(Sqrt[e*x]*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
(-2*c*e)/(5*a*(e*x)^(3/2)*Sqrt[a*x^2 + b*x^3]) - ((6*b*c - 5*a*d)*((2*e)/(
a*Sqrt[e*x]*Sqrt[a*x^2 + b*x^3]) + (4*e^2*((-2*e*Sqrt[a*x^2 + b*x^3])/(3*a
*(e*x)^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*(e*x)^(3/2))))/a))/(5*a*e
)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2x(bx+a)\left(-40a\,b^{2}d\,x^{3}+48b^{3}c\,x^{3}-20a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+5a^{3}dx-6a^{2}bcx+3c\,a^{3}\right)}{15a^{4}\sqrt{ex}\,\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	91
default	$-\frac{2x(bx+a)\left(-40a\,b^{2}d\,x^{3}+48b^{3}c\,x^{3}-20a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+5a^{3}dx-6a^{2}bcx+3c\,a^{3}\right)}{15a^{4}\sqrt{ex}\,\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	91
orering	$-\frac{2x(bx+a)\left(-40a\,b^{2}d\,x^{3}+48b^{3}c\,x^{3}-20a^{2}bd\,x^{2}+24a\,b^{2}c\,x^{2}+5a^{3}dx-6a^{2}bcx+3c\,a^{3}\right)}{15a^{4}\sqrt{ex}\,\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	91
risch	$-\frac{2(bx+a)\left(-25abd\,x^{2}+33b^{2}c\,x^{2}+5a^{2}dx-9abcx+3a^{2}c\right)}{15a^{4}x\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}+\frac{2(ad-bc)b^{2}x^{2}}{a^{4}\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}$	103

input
```
int((d*x+c)/(e*x)^(1/2)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*x*(b*x+a)*(-40*a*b^2*d*x^3+48*b^3*c*x^3-20*a^2*b*d*x^2+24*a*b^2*c*x^
2+5*a^3*d*x-6*a^2*b*c*x+3*a^3*c)/a^4/(e*x)^(1/2)/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.68

$$\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\,(3\,a^3c+8\,(6\,b^3c-5\,ab^2d)x^3+4\,(6\,ab^2c-5\,a^2bd)x^2-(6\,a^2bc-5\,a^3d)x)\sqrt{bx^3+ax^2}\sqrt{ex}}{15\,(a^4bex^5+a^5ex^4)}$$

input
```
integrate((d*x+c)/(e*x)^(1/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-2/15*(3*a^3*c + 8*(6*b^3*c - 5*a*b^2*d)*x^3 + 4*(6*a*b^2*c - 5*a^2*b*d)*x
^2 - (6*a^2*b*c - 5*a^3*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x)/(a^4*b*e*x^5 +
 a^5*e*x^4)
```

Sympy [F]

$$\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{3/2}}\,dx = \int \frac{c+dx}{\sqrt{ex}\,(x^2\,(a+bx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/(e*x)**(1/2)/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((c + d*x)/(sqrt(e*x)*(x**2*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{3/2}}\,dx = \int \frac{dx+c}{(bx^3+ax^2)^{\frac{3}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((d*x+c)/(e*x)^(1/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/((b*x^3 + a*x^2)^(3/2)*sqrt(e*x)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(127) = 254$.

Time = 0.28 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.14

$$\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{3/2}}\,dx = -\frac{2\sqrt{bx+a}\left((bx+a)\left(\frac{(33\,a^5b^8ce^2-25\,a^6b^7de^2)(bx+a)}{a^9b^2|b|\mathrm{sgn}(x)}-\frac{5\,(15\,a^6b^8ce^2-11\,a^7b^7de^2)}{a^9b^2|b|\mathrm{sgn}(x)}\right)+\frac{15\,(3\,a^7b^8ce^2-2\,a^8b^7de^2)}{a^9b^2|b|\mathrm{sgn}(x)}\right)}{15\,((bx+a)be-abe)^{\frac{5}{2}}} - \frac{4\,(b^9c^2e-2\,ab^8cde+a^2b^7d^2e)}{\left(\sqrt{be}ab^5ce-\sqrt{be}a^2b^4de+\sqrt{be}\left(\sqrt{be}\sqrt{bx+a}-\sqrt{(bx+a)be-abe}\right)^2b^4c-\sqrt{be}\left(\sqrt{be}\sqrt{bx+a}-\sqrt{(bx+\right.\right.}$$

input
```
integrate((d*x+c)/(e*x)^(1/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
-2/15*sqrt(b*x + a)*((b*x + a)*((33*a^5*b^8*c*e^2 - 25*a^6*b^7*d*e^2)*(b*x
 + a)/(a^9*b^2*abs(b)*sgn(x)) - 5*(15*a^6*b^8*c*e^2 - 11*a^7*b^7*d*e^2)/(a
^9*b^2*abs(b)*sgn(x))) + 15*(3*a^7*b^8*c*e^2 - 2*a^8*b^7*d*e^2)/(a^9*b^2*a
bs(b)*sgn(x)))/((b*x + a)*b*e - a*b*e)^(5/2) - 4*(b^9*c^2*e - 2*a*b^8*c*d*
e + a^2*b^7*d^2*e)/((sqrt(b*e)*a*b^5*c*e - sqrt(b*e)*a^2*b^4*d*e + sqrt(b*
e)*(sqrt(b*e)*sqrt(b*x + a) - sqrt((b*x + a)*b*e - a*b*e))^2*b^4*c - sqrt(
b*e)*(sqrt(b*e)*sqrt(b*x + a) - sqrt((b*x + a)*b*e - a*b*e))^2*a*b^3*d)*a^
3*abs(b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.82 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{3/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,c}{5\,a\,b}-\frac{8\,x^2\,(5\,a\,d-6\,b\,c)}{15\,a^3}+\frac{x\,(10\,a^3\,d-12\,a^2\,b\,c)}{15\,a^4\,b}+\frac{x^3\,(96\,b^3\,c-80\,a\,b^2\,d)}{15\,a^4\,b}\right)}{x^4\,\sqrt{e\,x}+\frac{a\,x^3\,\sqrt{e\,x}}{b}}$$

input
```
int((c + d*x)/((e*x)^(1/2)*(a*x^2 + b*x^3)^(3/2)),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*c)/(5*a*b) - (8*x^2*(5*a*d - 6*b*c))/(15*a^3)
+ (x*(10*a^3*d - 12*a^2*b*c))/(15*a^4*b) + (x^3*(96*b^3*c - 80*a*b^2*d))/(
15*a^4*b)))/(x^4*(e*x)^(1/2) + (a*x^3*(e*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.85

$$\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{3/2}}\,dx = \frac{2\sqrt{e}\left(-40\sqrt{b}\,\sqrt{bx+a}\,abd\,x^3+48\sqrt{b}\,\sqrt{bx+a}\,b^2c\,x^3-3\sqrt{x}\,a^3c-5\sqrt{x}\,a^3dx+\right.}{15\sqrt{bx+}}$$

input
```
int((d*x+c)/(e*x)^(1/2)/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*sqrt(e)*( - 40*sqrt(b)*sqrt(a + b*x)*a*b*d*x**3 + 48*sqrt(b)*sqrt(a + b
*x)*b**2*c*x**3 - 3*sqrt(x)*a**3*c - 5*sqrt(x)*a**3*d*x + 6*sqrt(x)*a**2*b
*c*x + 20*sqrt(x)*a**2*b*d*x**2 - 24*sqrt(x)*a*b**2*c*x**2 + 40*sqrt(x)*a*
b**2*d*x**3 - 48*sqrt(x)*b**3*c*x**3))/(15*sqrt(a + b*x)*a**4*e*x**3)
```

3.346 $\int \frac{c+dx}{(ex)^{3/2}(ax^2+bx^3)^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 192

$$\int \frac{c+dx}{(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{2ce}{7a(ex)^{5/2}\sqrt{ax^2+bx^3}} - \frac{2(8bc-7ad)}{7a^2(ex)^{3/2}\sqrt{ax^2+bx^3}} + \frac{12(8bc-7ad)e^2\sqrt{ax^2+bx^3}}{35a^3(ex)^{7/2}} - \frac{16b(8bc-7ad)e\sqrt{ax^2+bx^3}}{35a^4(ex)^{5/2}} + \frac{32b^2(8bc-7ad)\sqrt{ax^2+bx^3}}{35a^5(ex)^{3/2}}$$

output
```
-2/7*c*e/a/(e*x)^(5/2)/(b*x^3+a*x^2)^(1/2)-2/7*(-7*a*d+8*b*c)/a^2/(e*x)^(3
/2)/(b*x^3+a*x^2)^(1/2)+12/35*(-7*a*d+8*b*c)*e^2*(b*x^3+a*x^2)^(1/2)/a^3/(
e*x)^(7/2)-16/35*b*(-7*a*d+8*b*c)*e*(b*x^3+a*x^2)^(1/2)/a^4/(e*x)^(5/2)+32
/35*b^2*(-7*a*d+8*b*c)*(b*x^3+a*x^2)^(1/2)/a^5/(e*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.53

$$\int \frac{c+dx}{(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{2e(-128b^4cx^4+16ab^3x^3(-4c+7dx)+8a^2b^2x^2(2c+7dx)-2a^3bx(4c+7dx)+a^4(5c+7dx))}{35a^5(ex)^{5/2}\sqrt{x^2(a+bx)}}$$

input
```
Integrate[(c + d*x)/((e*x)^(3/2)*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
(-2*e*(-128*b^4*c*x^4 + 16*a*b^3*x^3*(-4*c + 7*d*x) + 8*a^2*b^2*x^2*(2*c +
 7*d*x) - 2*a^3*b*x*(4*c + 7*d*x) + a^4*(5*c + 7*d*x)))/(35*a^5*(e*x)^(5/2
)*Sqrt[x^2*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 191, normalized size of antiderivative = 0.99, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1944, 1921, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(8bc-7ad)\int \frac{1}{\sqrt{ex}\left(bx^3+ax^2\right)^{3/2}}dx}{7ae}-\frac{2ce}{7a(ex)^{5/2}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1921}$$

$$-\frac{(8bc-7ad)\left(\frac{6e^2\int \frac{1}{(ex)^{5/2}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2e}{a(ex)^{3/2}\sqrt{ax^2+bx^3}}\right)}{7ae}-\frac{2ce}{7a(ex)^{5/2}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{(8bc-7ad)\left(\frac{6e^2\left(-\frac{4b\int \frac{1}{(ex)^{3/2}\sqrt{bx^3+ax^2}}dx}{5ae}-\frac{2e\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}}\right)}{a}+\frac{2e}{a(ex)^{3/2}\sqrt{ax^2+bx^3}}\right)}{7ae}-\frac{2ce}{7a(ex)^{5/2}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1922}$$

$$-\frac{(8bc-7ad)\left(\frac{6e^2\left(-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{ex}\sqrt{bx^3+ax^2}}dx}{3ae}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{5ae}-\frac{2e\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}}\right)}{a}+\frac{2e}{a(ex)^{3/2}\sqrt{ax^2+bx^3}}\right)}{7ae}-\frac{2ce}{7a(ex)^{5/2}\sqrt{ax^2+bx^3}}$$

$$\Big\downarrow \text{ 1920}$$

$$-\frac{(8bc-7ad)\left(\frac{6e^2\left(-\frac{4b\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(ex)^{3/2}}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{5ae}-\frac{2e\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}}\right)}{a}+\frac{2e}{a(ex)^{3/2}\sqrt{ax^2+bx^3}}\right)}{7ae}-\frac{2ce}{7a(ex)^{5/2}\sqrt{ax^2+bx^3}}$$

input
```
Int[(c + d*x)/((e*x)^(3/2)*(a*x^2 + b*x^3)^(3/2)),x]
```

output
```
(-2*c*e)/(7*a*(e*x)^(5/2)*Sqrt[a*x^2 + b*x^3]) - ((8*b*c - 7*a*d)*((2*e)/(
a*(e*x)^(3/2)*Sqrt[a*x^2 + b*x^3]) + (6*e^2*((-2*e*Sqrt[a*x^2 + b*x^3])/(5
*a*(e*x)^(7/2)) - (4*b*((-2*e*Sqrt[a*x^2 + b*x^3])/(3*a*(e*x)^(5/2)) + (4*
b*Sqrt[a*x^2 + b*x^3])/(3*a^2*(e*x)^(3/2))))/(5*a*e)))/a))/(7*a*e)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.60

method	result	size
gosper	$-\frac{2x(bx+a)\left(112x^{4}a\,b^{3}d-128x^{4}b^{4}c+56a^{2}b^{2}d\,x^{3}-64a\,b^{3}c\,x^{3}-14a^{3}bd\,x^{2}+16a^{2}b^{2}c\,x^{2}+7a^{4}dx-8a^{3}bcx+5c\,a^{4}\right)}{35a^{5}(ex)^{\frac{3}{2}}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	115
orering	$-\frac{2x(bx+a)\left(112x^{4}a\,b^{3}d-128x^{4}b^{4}c+56a^{2}b^{2}d\,x^{3}-64a\,b^{3}c\,x^{3}-14a^{3}bd\,x^{2}+16a^{2}b^{2}c\,x^{2}+7a^{4}dx-8a^{3}bcx+5c\,a^{4}\right)}{35a^{5}(ex)^{\frac{3}{2}}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}}$	115
default	$-\frac{2(bx+a)\left(112x^{4}a\,b^{3}d-128x^{4}b^{4}c+56a^{2}b^{2}d\,x^{3}-64a\,b^{3}c\,x^{3}-14a^{3}bd\,x^{2}+16a^{2}b^{2}c\,x^{2}+7a^{4}dx-8a^{3}bcx+5c\,a^{4}\right)}{35\left(b\,x^{3}+a\,x^{2}\right)^{\frac{3}{2}}\sqrt{ex}\,e\,a^{5}}$	117
risch	$-\frac{2(bx+a)\left(77a\,b^{2}d\,x^{3}-93b^{3}c\,x^{3}-21a^{2}bd\,x^{2}+29a\,b^{2}c\,x^{2}+7a^{3}dx-13a^{2}bcx+5c\,a^{3}\right)}{35a^{5}x^{2}e\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}-\frac{2b^{3}(ad-bc)x^{2}}{a^{5}e\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}$	133

input
```
int((d*x+c)/(e*x)^(3/2)/(b*x^3+a*x^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/35*x*(b*x+a)*(112*a*b^3*d*x^4-128*b^4*c*x^4+56*a^2*b^2*d*x^3-64*a*b^3*c
*x^3-14*a^3*b*d*x^2+16*a^2*b^2*c*x^2+7*a^4*d*x-8*a^3*b*c*x+5*a^4*c)/a^5/(e
*x)^(3/2)/(b*x^3+a*x^2)^(3/2)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.68

$$\int \frac{c+dx}{(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx =$$
$$-\frac{2\left(5\,a^4c-16\left(8\,b^4c-7\,ab^3d\right)x^4-8\left(8\,ab^3c-7\,a^2b^2d\right)x^3+2\left(8\,a^2b^2c-7\,a^3bd\right)x^2-\left(8\,a^3bc-7\,a^4d\right)x\right)\sqrt{bx}}{35\left(a^5be^2x^6+a^6e^2x^5\right)}$$

input
```
integrate((d*x+c)/(e*x)^(3/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="fricas")
```

output
```
-2/35*(5*a^4*c - 16*(8*b^4*c - 7*a*b^3*d)*x^4 - 8*(8*a*b^3*c - 7*a^2*b^2*d
)*x^3 + 2*(8*a^2*b^2*c - 7*a^3*b*d)*x^2 - (8*a^3*b*c - 7*a^4*d)*x)*sqrt(b*
x^3 + a*x^2)*sqrt(e*x)/(a^5*b*e^2*x^6 + a^6*e^2*x^5)
```

Sympy [F]

$$\int \frac{c+dx}{(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx=\int \frac{c+dx}{(ex)^{\frac{3}{2}}\left(x^2(a+bx)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/(e*x)**(3/2)/(b*x**3+a*x**2)**(3/2),x)
```

output
```
Integral((c + d*x)/((e*x)**(3/2)*(x**2*(a + b*x))**(3/2)), x)
```

Maxima [F]

$$\int \frac{c+dx}{(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx=\int \frac{dx+c}{\left(bx^3+ax^2\right)^{\frac{3}{2}}(ex)^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/(e*x)^(3/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/((b*x^3 + a*x^2)^(3/2)*(e*x)^(3/2)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(162) = 324.

Time = 0.41 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.95

$$\int \frac{c+dx}{(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx=\frac{2\left(\frac{\left((bx+a)\left((bx+a)\left(\frac{\left(93\,a^9b^{10}ce^3|b|\mathrm{sgn}(x)-77\,a^{10}b^9de^3|b|\mathrm{sgn}(x)\right)(bx+a)}{a^{14}b^4}-\frac{28\left(11\,a^{10}b^{10}ce^3|b|\mathrm{sgn}(x)-9\,a\cdots\right.}{a^{14}b^4}\cdots\right.\right.\right.}{\cdots}\cdots\right.}{\cdots}$$

input
```
integrate((d*x+c)/(e*x)^(3/2)/(b*x^3+a*x^2)^(3/2),x, algorithm="giac")
```

output
```
2/35*(((b*x + a)*((b*x + a)*((93*a^9*b^10*c*e^3*abs(b)*sgn(x) - 77*a^10*b^
9*d*e^3*abs(b)*sgn(x))*(b*x + a)/(a^14*b^4) - 28*(11*a^10*b^10*c*e^3*abs(b
)*sgn(x) - 9*a^11*b^9*d*e^3*abs(b)*sgn(x))/(a^14*b^4)) + 70*(5*a^11*b^10*c
*e^3*abs(b)*sgn(x) - 4*a^12*b^9*d*e^3*abs(b)*sgn(x))/(a^14*b^4)) - 35*(4*a
^12*b^10*c*e^3*abs(b)*sgn(x) - 3*a^13*b^9*d*e^3*abs(b)*sgn(x))/(a^14*b^4))
*sqrt(b*x + a)/((b*x + a)*b*e - a*b*e)^(7/2) + 70*(b^11*c^2*e - 2*a*b^10*c
*d*e + a^2*b^9*d^2*e)/((sqrt(b*e)*a*b^6*c*e - sqrt(b*e)*a^2*b^5*d*e + sqrt
(b*e)*(sqrt(b*e)*sqrt(b*x + a) - sqrt((b*x + a)*b*e - a*b*e))^2*b^5*c - sq
rt(b*e)*(sqrt(b*e)*sqrt(b*x + a) - sqrt((b*x + a)*b*e - a*b*e))^2*a*b^4*d)
*a^4*abs(b)*sgn(x)))/e
```

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.77

$$\int \frac{c+dx}{(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,c}{7\,a\,b\,e}-\frac{4\,x^2\,\left(7\,a\,d-8\,b\,c\right)}{35\,a^3\,e}-\frac{x^4\,\left(256\,b^4\,c-224\,a\,b^3\,d\right)}{35\,a^5\,b\,e}+\frac{16\,b\,x^3\,\left(7\,a\,d-8\,b\,c\right)}{35\,a^4\,e}+\frac{x\,\left(14\,a^4\,d-16\,a^3\,b\,c\right)}{35\,a^5\,b\,e}\right)}{x^5\,\sqrt{e\,x}+\frac{a\,x^4\,\sqrt{e\,x}}{b}}$$

input
```
int((c + d*x)/((e*x)^(3/2)*(a*x^2 + b*x^3)^(3/2)),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*c)/(7*a*b*e) - (4*x^2*(7*a*d - 8*b*c))/(35*a^3
*e) - (x^4*(256*b^4*c - 224*a*b^3*d))/(35*a^5*b*e) + (16*b*x^3*(7*a*d - 8*
b*c))/(35*a^4*e) + (x*(14*a^4*d - 16*a^3*b*c))/(35*a^5*b*e)))/(x^5*(e*x)^(
1/2) + (a*x^4*(e*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.83

$$\int \frac{c+dx}{(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}\,dx = \frac{2\sqrt{e}\,\Big(112\sqrt{b}\,\sqrt{bx+a}\,a\,b^2d\,x^4-128\sqrt{b}\,\sqrt{bx+a}\,b^3c\,x^4-5\sqrt{x}\,a^4c-7\sqrt{x}\,a^4}{}$$

input
```
int((d*x+c)/(e*x)^(3/2)/(b*x^3+a*x^2)^(3/2),x)
```

output
```
(2*sqrt(e)*(112*sqrt(b)*sqrt(a + b*x)*a*b**2*d*x**4 - 128*sqrt(b)*sqrt(a +
 b*x)*b**3*c*x**4 - 5*sqrt(x)*a**4*c - 7*sqrt(x)*a**4*d*x + 8*sqrt(x)*a**3
*b*c*x + 14*sqrt(x)*a**3*b*d*x**2 - 16*sqrt(x)*a**2*b**2*c*x**2 - 56*sqrt(
x)*a**2*b**2*d*x**3 + 64*sqrt(x)*a*b**3*c*x**3 - 112*sqrt(x)*a*b**3*d*x**4
 + 128*sqrt(x)*b**4*c*x**4))/(35*sqrt(a + b*x)*a**5*e**2*x**4)
```

3.347 $\int \frac{(ex)^{15/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 214

$$\int \frac{(ex)^{15/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2(bc-ad)e^2(ex)^{11/2}}{3b^2(ax^2+bx^3)^{3/2}} - \frac{2(5bc-8ad)e^5(ex)^{5/2}}{3b^3\sqrt{ax^2+bx^3}} + \frac{5(4bc-7ad)e^8\sqrt{ax^2+bx^3}}{4b^4\sqrt{ex}} + \frac{de^7\sqrt{ex}\sqrt{ax^2+bx^3}}{2b^3} - \frac{5a(4bc-7ad)e^{15/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{4b^{9/2}}$$

output
```
-2/3*(-a*d+b*c)*e^2*(e*x)^(11/2)/b^2/(b*x^3+a*x^2)^(3/2)-2/3*(-8*a*d+5*b*c
)*e^5*(e*x)^(5/2)/b^3/(b*x^3+a*x^2)^(1/2)+5/4*(-7*a*d+4*b*c)*e^8*(b*x^3+a*
x^2)^(1/2)/b^4/(e*x)^(1/2)+1/2*d*e^7*(e*x)^(1/2)*(b*x^3+a*x^2)^(1/2)/b^3-5
/4*a*(-7*a*d+4*b*c)*e^(15/2)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*
x^2)^(1/2))/b^(9/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.89

$$\int \frac{(ex)^{15/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{e^7x^{5/2}\sqrt{ex}\left(\sqrt{b}\sqrt{x}(-105a^3d+ab^2x(80c-21dx)+20a^2b(3c-7dx)+6b^3x^2(2c+\ldots\right.}{12b^{9/2}\ldots}$$

input
```
Integrate[((e*x)^(15/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(e^7*x^(5/2)*Sqrt[e*x]*(Sqrt[b]*Sqrt[x]*(-105*a^3*d + a*b^2*x*(80*c - 21*d
*x) + 20*a^2*b*(3*c - 7*d*x) + 6*b^3*x^2*(2*c + d*x)) + 120*a*b*c*(a + b*x
)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(Sqrt[a] - Sqrt[a + b*x])] + 210*a^2*d*(
a + b*x)^(3/2)*ArcTanh[(Sqrt[b]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(12
*b^(9/2)*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1943, 1928, 1930, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{15/2}(c+dx)}{(ax^2+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{2e(ex)^{13/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}-\frac{e^2(4bc-7ad)\int \frac{(ex)^{11/2}}{(bx^3+ax^2)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \text{1928}$$

$$\frac{2e(ex)^{13/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}-\frac{e^2(4bc-7ad)\left(\frac{5e^3\int \frac{(ex)^{5/2}}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2e^2(ex)^{7/2}}{b\sqrt{ax^2+bx^3}}\right)}{3ab}$$

$\Big\downarrow$ 1930

$$\frac{2e(ex)^{13/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}-\frac{e^2(4bc-7ad)\left(\frac{5e^3\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\int\frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{4b}\right)}{b}-\frac{2e^2(ex)^{7/2}}{b\sqrt{ax^2+bx^3}}\right)}{3ab}$$

$\Big\downarrow$ 1930

$$\frac{2e(ex)^{13/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}-\frac{e^2(4bc-7ad)\left(\frac{5e^3\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int\frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{4b}\right)}{b}-\frac{2e^2(ex)^{7/2}}{b\sqrt{ax^2+bx^3}}\right)}{3ab}$$

$\Big\downarrow$ 1937

$$\frac{2e(ex)^{13/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}-\frac{e^2(4bc-7ad)\left(\frac{5e^3\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{4b}\right)}{b}-\frac{2e^2(ex)^{7/2}}{b\sqrt{ax^2+bx^3}}\right)}{3ab}$$

$\Big\downarrow$ 1935

$$\frac{\frac{2e(ex)^{13/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}-\frac{e^2(4bc-7ad)\left(\frac{5e^3\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{4b}\right)}{b}-\frac{2e^2(ex)^{7/2}}{b\sqrt{ax^2+bx^3}}\right)}{3ab}}{}$$

$\downarrow$ 219

$$\frac{2e(ex)^{13/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}-\frac{e^2(4bc-7ad)\left(\frac{5e^3\left(\frac{e^2\sqrt{ex}\sqrt{ax^2+bx^3}}{2b}-\frac{3ae\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{4b}\right)}{b}-\frac{2e^2(ex)^{7/2}}{b\sqrt{ax^2+bx^3}}\right)}{3ab}$$

input
```
Int[((e*x)^(15/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*(b*c - a*d)*e*(e*x)^(13/2))/(3*a*b*(a*x^2 + b*x^3)^(3/2)) - ((4*b*c - 7
*a*d)*e^2*((-2*e^2*(e*x)^(7/2))/(b*Sqrt[a*x^2 + b*x^3]) + (5*e^3*((e^2*Sqr
t[e*x]*Sqrt[a*x^2 + b*x^3])/(2*b) - (3*a*e*((e^2*Sqrt[a*x^2 + b*x^3])/(b*S
qrt[e*x]) - (a*e*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])
/(b^(3/2)*Sqrt[x])))/(4*b)))/b))/(3*a*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.49

method	result
risch	$-\frac{(-2bdx+11ad-4bc)x^2(bx+a)e^8}{4b^4\sqrt{x^2(bx+a)}\,\sqrt{ex}}+\frac{a\left(\frac{35ad\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^2+aex}\right)}{\sqrt{be}}-\frac{20bc\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^2+aex}\right)}{\sqrt{be}}-\frac{16(4ad-3bc)\sqrt{be\left(x+\frac{a}{b}\right.}}{be\left(x+\frac{a}{b}\right.}\right.}{8b^4\sqrt{x}}$
default	$-\frac{x^5(bx+a)\left(-12b^3d\,x^3\sqrt{ex(bx+a)}\,\sqrt{be}-105\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)a^2b^2de\,x^2+60\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)a\,b^3ce\,x\right.}{}$

input
```
int((e*x)^(15/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(-2*b*d*x+11*a*d-4*b*c)*x^2*(b*x+a)/b^4*e^8/(x^2*(b*x+a))^(1/2)/(e*x)
^(1/2)+1/8*a/b^4*(35*a*d*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1
/2))/(b*e)^(1/2)-20*b*c*ln((1/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/
2))/(b*e)^(1/2)-16*(4*a*d-3*b*c)/b/e/(x+a/b)*(b*e*(x+a/b)^2-a*e*(x+a/b))^(
1/2)+8*a^2*(a*d-b*c)/b^2*(2/3/a/e/(x+a/b)^2*(b*e*(x+a/b)^2-a*e*(x+a/b))^(1
/2)+4/3*b/e/a^2/(x+a/b)*(b*e*(x+a/b)^2-a*e*(x+a/b))^(1/2)))*e^8/(x^2*(b*x+
a))^(1/2)*x*(e*x*(b*x+a))^(1/2)/(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 494, normalized size of antiderivative = 2.31

$$\int \frac{(ex)^{15/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \left[-\frac{15\left((4\,ab^3c-7\,a^2b^2d)e^7x^3+2\,(4\,a^2b^2c-7\,a^3bd)e^7x^2+(4\,a^3bc-7\,a^4d)e^7x\right)\sqrt{}}{}\right.$$

input
```
integrate((e*x)^(15/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/24*(15*((4*a*b^3*c - 7*a^2*b^2*d)*e^7*x^3 + 2*(4*a^2*b^2*c - 7*a^3*b*d
)*e^7*x^2 + (4*a^3*b*c - 7*a^4*d)*e^7*x)*sqrt(e/b)*log((2*b*e*x^2 + a*e*x
+ 2*sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(e/b))/x) - 2*(6*b^3*d*e^7*x^3 + 3
*(4*b^3*c - 7*a*b^2*d)*e^7*x^2 + 20*(4*a*b^2*c - 7*a^2*b*d)*e^7*x + 15*(4*
a^2*b*c - 7*a^3*d)*e^7)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^6*x^3 + 2*a*b^5*
x^2 + a^2*b^4*x), 1/12*(15*((4*a*b^3*c - 7*a^2*b^2*d)*e^7*x^3 + 2*(4*a^2*b
^2*c - 7*a^3*b*d)*e^7*x^2 + (4*a^3*b*c - 7*a^4*d)*e^7*x)*sqrt(-e/b)*arctan
(sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(-e/b)/(b*e*x^2 + a*e*x)) + (6*b^3*d*
e^7*x^3 + 3*(4*b^3*c - 7*a*b^2*d)*e^7*x^2 + 20*(4*a*b^2*c - 7*a^2*b*d)*e^7
*x + 15*(4*a^2*b*c - 7*a^3*d)*e^7)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^6*x^3
 + 2*a*b^5*x^2 + a^2*b^4*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{15/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(15/2)*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^{15/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx=\int \frac{(dx+c)(ex)^{\frac{15}{2}}}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(15/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^(15/2)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.36

$$\int \frac{(ex)^{15/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx=\frac{\left(\left(3\left(\frac{2\,de^7x|e|}{b\mathrm{sgn}(x)}+\frac{4\,ab^6ce^{10}|e|\mathrm{sgn}(x)-7\,a^2b^5de^{10}|e|\mathrm{sgn}(x)}{ab^7e^3}\right)ex+\frac{20\,(4\,a^2b^5ce^{11}|e|\mathrm{sgn}(x)-7\,a^3b^4de^{11}|e|\mathrm{sg}}{ab^7e^3}\right.\right.}{12\,(be^2x+ae^2)^{\frac{3}{2}}}-\frac{5\,(4\,abce^7|e|\log(e^2|a|)-7\,a^2de^7|e|\log(e^2|a|))\mathrm{sgn}(x)}{8\,\sqrt{be}b^4}+\frac{5\,(4\,abce^7|e|-7\,a^2de^7|e|)\log\left(\left|-\sqrt{be}\sqrt{ex}+\sqrt{be^2x+ae^2}\right|\right)}{4\,\sqrt{be}b^4\mathrm{sgn}\,(x)}$$

input
```
integrate((e*x)^(15/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
1/12*((3*(2*d*e^7*x*abs(e)/(b*sgn(x)) + (4*a*b^6*c*e^10*abs(e)*sgn(x) - 7*
a^2*b^5*d*e^10*abs(e)*sgn(x))/(a*b^7*e^3))*e*x + 20*(4*a^2*b^5*c*e^11*abs(
e)*sgn(x) - 7*a^3*b^4*d*e^11*abs(e)*sgn(x))/(a*b^7*e^3))*e*x + 15*(4*a^3*b
^4*c*e^12*abs(e)*sgn(x) - 7*a^4*b^3*d*e^12*abs(e)*sgn(x))/(a*b^7*e^3))*sqr
t(e*x)/(b*e^2*x + a*e^2)^(3/2) - 5/8*(4*a*b*c*e^7*abs(e)*log(e^2*abs(a)) -
 7*a^2*d*e^7*abs(e)*log(e^2*abs(a)))*sgn(x)/(sqrt(b*e)*b^4) + 5/4*(4*a*b*c
*e^7*abs(e) - 7*a^2*d*e^7*abs(e))*log(abs(-sqrt(b*e)*sqrt(e*x) + sqrt(b*e^
2*x + a*e^2)))/(sqrt(b*e)*b^4*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{15/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx=\int \frac{(e\,x)^{15/2}\,(c+d\,x)}{(b\,x^3+a\,x^2)^{5/2}}\,dx$$

input
```
int(((e*x)^(15/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
int(((e*x)^(15/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.38

$$\int \frac{(ex)^{15/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx=\frac{\sqrt{e}\,e^7\left(840\sqrt{b}\,\sqrt{bx+a}\,\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^3d-480\sqrt{b}\,\sqrt{bx+a}\,\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)\right.}{}$$

input
```
int((e*x)^(15/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(sqrt(e)*e**7*(840*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt
(b))/sqrt(a))*a**3*d - 480*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt
(x)*sqrt(b))/sqrt(a))*a**2*b*c + 840*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b
*x) + sqrt(x)*sqrt(b))/sqrt(a))*a**2*b*d*x - 480*sqrt(b)*sqrt(a + b*x)*log
((sqrt(a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*a*b**2*c*x + 175*sqrt(b)*sqrt(
a + b*x)*a**3*d - 80*sqrt(b)*sqrt(a + b*x)*a**2*b*c + 175*sqrt(b)*sqrt(a +
 b*x)*a**2*b*d*x - 80*sqrt(b)*sqrt(a + b*x)*a*b**2*c*x - 840*sqrt(x)*a**3*
b*d + 480*sqrt(x)*a**2*b**2*c - 1120*sqrt(x)*a**2*b**2*d*x + 640*sqrt(x)*a
*b**3*c*x - 168*sqrt(x)*a*b**3*d*x**2 + 96*sqrt(x)*b**4*c*x**2 + 48*sqrt(x
)*b**4*d*x**3))/(96*sqrt(a + b*x)*b**5*(a + b*x))
```

3.348 $\int \frac{(ex)^{13/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 163

$$\int \frac{(ex)^{13/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2(bc-ad)e^2(ex)^{9/2}}{3b^2\left(ax^2+bx^3\right)^{3/2}} - \frac{2(bc-2ad)e^5(ex)^{3/2}}{b^3\sqrt{ax^2+bx^3}} + \frac{de^7\sqrt{ax^2+bx^3}}{b^3\sqrt{ex}} + \frac{(2bc-5ad)e^{13/2}\operatorname{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{b^{7/2}}$$

output
```
-2/3*(-a*d+b*c)*e^2*(e*x)^(9/2)/b^2/(b*x^3+a*x^2)^(3/2)-2*(-2*a*d+b*c)*e^5
*(e*x)^(3/2)/b^3/(b*x^3+a*x^2)^(1/2)+d*e^7*(b*x^3+a*x^2)^(1/2)/b^3/(e*x)^(
1/2)+(-5*a*d+2*b*c)*e^(13/2)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b*x^3+a*
x^2)^(1/2))/b^(7/2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.82

$$\int \frac{(ex)^{13/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{(ex)^{13/2}\left(\sqrt{b}\sqrt{x}(a+bx)\left(15a^2d+b^2x(-8c+3dx)+ab(-6c+20dx)\right)+6(2bc-\right.}{3b^{7/2}x^{3/2}\left(x^2(a+bx)\right)^{5/2}}$$

input
```
Integrate[((e*x)^(13/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
((e*x)^(13/2)*(Sqrt[b]*Sqrt[x]*(a + b*x)*(15*a^2*d + b^2*x*(-8*c + 3*d*x)
+ a*b*(-6*c + 20*d*x)) + 6*(2*b*c - 5*a*d)*(a + b*x)^(5/2)*ArcTanh[(Sqrt[b
]*Sqrt[x])/(-Sqrt[a] + Sqrt[a + b*x])]))/(3*b^(7/2)*x^(3/2)*(x^2*(a + b*x)
)^(5/2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {1943, 1928, 1930, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{13/2}(c+dx)}{(ax^2+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{ 1943}$$

$$\frac{2e(ex)^{11/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}-\frac{e^2(2bc-5ad)\int \frac{(ex)^{9/2}}{(bx^3+ax^2)^{3/2}}dx}{3ab}$$

$$\Big\downarrow \text{ 1928}$$

$$\frac{2e(ex)^{11/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}-\frac{e^2(2bc-5ad)\left(\frac{3e^3\int \frac{(ex)^{3/2}}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2e^2(ex)^{5/2}}{b\sqrt{ax^2+bx^3}}\right)}{3ab}$$

$$\Big\downarrow \text{ 1930}$$

$$\frac{2e(ex)^{11/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}-\frac{e^2(2bc-5ad)\left(\frac{3e^3\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{2b}\right)}{b}-\frac{2e^2(ex)^{5/2}}{b\sqrt{ax^2+bx^3}}\right)}{3ab}$$

$$\Big\downarrow \text{ 1937}$$

$$\frac{2e(ex)^{11/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}-\frac{e^2(2bc-5ad)\left(\frac{3e^3\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{2b\sqrt{x}}\right)}{b}-\frac{2e^2(ex)^{5/2}}{b\sqrt{ax^2+bx^3}}\right)}{3ab}$$

$$\Big\downarrow \text{ 1935}$$

$$\frac{2e(ex)^{11/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}-\frac{e^2(2bc-5ad)\left(\frac{3e^3\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\int\frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}\right)}{b}-\frac{2e^2(ex)^{5/2}}{b\sqrt{ax^2+bx^3}}\right)}{3ab}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2e(ex)^{11/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}-\frac{e^2(2bc-5ad)\left(\frac{3e^3\left(\frac{e^2\sqrt{ax^2+bx^3}}{b\sqrt{ex}}-\frac{ae\sqrt{ex}\,\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}\right)}{b}-\frac{2e^2(ex)^{5/2}}{b\sqrt{ax^2+bx^3}}\right)}{3ab}$$

input
```
Int[((e*x)^(13/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*(b*c - a*d)*e*(e*x)^(11/2))/(3*a*b*(a*x^2 + b*x^3)^(3/2)) - ((2*b*c - 5
*a*d)*e^2*((-2*e^2*(e*x)^(5/2))/(b*Sqrt[a*x^2 + b*x^3]) + (3*e^3*((e^2*Sqr
t[a*x^2 + b*x^3])/(b*Sqrt[e*x]) - (a*e*Sqrt[e*x]*ArcTanh[(Sqrt[b]*x^(3/2))
/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/b))/(3*a*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1930
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(m + n*p
+ 1))), x] - Simp[a*c^(n - j)*((m + j*p - n + j + 1)/(b*(m + n*p + 1)))   I
nt[(c*x)^(m - (n - j))*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, p},
x] &&  !IntegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && Gt
Q[m + j*p - n + j + 1, 0] && NeQ[m + n*p + 1, 0]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(137) = 274$.

Time = 0.43 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.87

method	result
risch	$\frac{d\,x^{2}(bx+a)e^{7}}{b^{3}\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}-\frac{\left(\frac{5ad\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^{2}+aex}\right)}{\sqrt{be}}-\frac{2bc\ln\left(\frac{\frac{1}{2}ae+bex}{\sqrt{be}}+\sqrt{be\,x^{2}+aex}\right)}{\sqrt{be}}-\frac{4(3ad-2bc)\sqrt{be\left(x+\frac{a}{b}\right)^{2}-ae\left(x+\frac{a}{b}\right)}}{be\left(x+\frac{a}{b}\right)}+\frac{2a^{2}(\ldots}{\ldots}\right.}{2b^{3}\sqrt{x^{2}(bx+a)}\,\sqrt{ex}\ldots}$
default	$\frac{x^{5}(bx+a)\left(-15\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)a\,b^{2}de\,x^{2}+6\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)b^{3}ce\,x^{2}+6b^{2}d\,x^{2}\sqrt{be}\,\sqrt{ex(bx+a)}-30\ln\left(\ldots\right.\right.}{\ldots}$

input
```
int((e*x)^(13/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
d/b^3*x^2*(b*x+a)*e^7/(x^2*(b*x+a))^(1/2)/(e*x)^(1/2)-1/2/b^3*(5*a*d*ln((1
/2*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)-2*b*c*ln((1/2
*a*e+b*e*x)/(b*e)^(1/2)+(b*e*x^2+a*e*x)^(1/2))/(b*e)^(1/2)-4*(3*a*d-2*b*c)
/b/e/(x+a/b)*(b*e*(x+a/b)^2-a*e*(x+a/b))^(1/2)+2*a^2*(a*d-b*c)/b^2*(2/3/a/
e/(x+a/b)^2*(b*e*(x+a/b)^2-a*e*(x+a/b))^(1/2)+4/3*b/e/a^2/(x+a/b)*(b*e*(x+
a/b)^2-a*e*(x+a/b))^(1/2)))*e^7/(x^2*(b*x+a))^(1/2)*x*(e*x*(b*x+a))^(1/2)/
(e*x)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 431, normalized size of antiderivative = 2.64

$$\int \frac{(ex)^{13/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \left[-\frac{3\left((2\,b^3c-5\,ab^2d)e^6x^3+2\,(2\,ab^2c-5\,a^2bd)e^6x^2+(2\,a^2bc-5\,a^3d)e^6x\right)\sqrt{\frac{e}{b}}\log\right.$$

input
```
integrate((e*x)^(13/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
[-1/6*(3*((2*b^3*c - 5*a*b^2*d)*e^6*x^3 + 2*(2*a*b^2*c - 5*a^2*b*d)*e^6*x^
2 + (2*a^2*b*c - 5*a^3*d)*e^6*x)*sqrt(e/b)*log((2*b*e*x^2 + a*e*x - 2*sqrt
(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(e/b))/x) - 2*(3*b^2*d*e^6*x^2 - 4*(2*b^2*
c - 5*a*b*d)*e^6*x - 3*(2*a*b*c - 5*a^2*d)*e^6)*sqrt(b*x^3 + a*x^2)*sqrt(e
*x))/(b^5*x^3 + 2*a*b^4*x^2 + a^2*b^3*x), -1/3*(3*((2*b^3*c - 5*a*b^2*d)*e
^6*x^3 + 2*(2*a*b^2*c - 5*a^2*b*d)*e^6*x^2 + (2*a^2*b*c - 5*a^3*d)*e^6*x)*
sqrt(-e/b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(-e/b)/(b*e*x^2 + a*
e*x)) - (3*b^2*d*e^6*x^2 - 4*(2*b^2*c - 5*a*b*d)*e^6*x - 3*(2*a*b*c - 5*a^
2*d)*e^6)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(b^5*x^3 + 2*a*b^4*x^2 + a^2*b^3*
x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{13/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(13/2)*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^{13/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(dx+c)(ex)^{\frac{13}{2}}}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(13/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^(13/2)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.45

$$\int \frac{(ex)^{13/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{\left(\left(\frac{3\,de^7x|e|}{b\mathrm{sgn}(x)} - \frac{4\,(2\,ab^4ce^{10}|e|\mathrm{sgn}(x) - 5\,a^2b^3de^{10}|e|\mathrm{sgn}(x))}{ab^5e^3}\right)ex - \frac{3\,(2\,a^2b^3ce^{11}|e|\mathrm{sgn}(x) - 5\,a^3b^2de^{11}|e|\mathrm{sgn}}{ab^5e^3}}{3\,(be^2x+ae^2)^{\frac{3}{2}}}$$
$$+ \frac{(2\,bce^6|e|\log(e^2|a|) - 5\,ade^6|e|\log(e^2|a|))\mathrm{sgn}(x)}{2\,\sqrt{be}b^3}$$
$$- \frac{(2\,bce^6|e| - 5\,ade^6|e|)\log\left(\left|-\sqrt{be}\sqrt{ex} + \sqrt{be^2x+ae^2}\right|\right)}{\sqrt{be}b^3\mathrm{sgn}(x)}$$

input
```
integrate((e*x)^(13/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
1/3*((3*d*e^7*x*abs(e)/(b*sgn(x)) - 4*(2*a*b^4*c*e^10*abs(e)*sgn(x) - 5*a^
2*b^3*d*e^10*abs(e)*sgn(x))/(a*b^5*e^3))*e*x - 3*(2*a^2*b^3*c*e^11*abs(e)*
sgn(x) - 5*a^3*b^2*d*e^11*abs(e)*sgn(x))/(a*b^5*e^3))*sqrt(e*x)/(b*e^2*x +
 a*e^2)^(3/2) + 1/2*(2*b*c*e^6*abs(e)*log(e^2*abs(a)) - 5*a*d*e^6*abs(e)*l
og(e^2*abs(a)))*sgn(x)/(sqrt(b*e)*b^3) - (2*b*c*e^6*abs(e) - 5*a*d*e^6*abs
(e))*log(abs(-sqrt(b*e)*sqrt(e*x) + sqrt(b*e^2*x + a*e^2)))/(sqrt(b*e)*b^3
*sgn(x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{13/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(e\,x)^{13/2}\,(c+d\,x)}{(b\,x^3+a\,x^2)^{5/2}}\,dx$$

input
```
int(((e*x)^(13/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
int(((e*x)^(13/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.41

$$\int \frac{(ex)^{13/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{\sqrt{e}\,e^6\left(-30\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2d+12\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)\right.}{}$$

input
```
int((e*x)^(13/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(sqrt(e)*e**6*( - 30*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sq
rt(b))/sqrt(a))*a**2*d + 12*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqr
t(x)*sqrt(b))/sqrt(a))*a*b*c - 30*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x)
 + sqrt(x)*sqrt(b))/sqrt(a))*a*b*d*x + 12*sqrt(b)*sqrt(a + b*x)*log((sqrt(
a + b*x) + sqrt(x)*sqrt(b))/sqrt(a))*b**2*c*x - 5*sqrt(b)*sqrt(a + b*x)*a*
*2*d - 5*sqrt(b)*sqrt(a + b*x)*a*b*d*x + 30*sqrt(x)*a**2*b*d - 12*sqrt(x)*
a*b**2*c + 40*sqrt(x)*a*b**2*d*x - 16*sqrt(x)*b**3*c*x + 6*sqrt(x)*b**3*d*
x**2))/(6*sqrt(a + b*x)*b**4*(a + b*x))
```

3.349 $\int \frac{(ex)^{11/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 120

$$\int \frac{(ex)^{11/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2(bc-ad)e(ex)^{9/2}}{3ab\left(ax^2+bx^3\right)^{3/2}} - \frac{2de^4(ex)^{3/2}}{b^2\sqrt{ax^2+bx^3}} + \frac{2de^{11/2}\text{arctanh}\left(\frac{\sqrt{b}(ex)^{3/2}}{e^{3/2}\sqrt{ax^2+bx^3}}\right)}{b^{5/2}}$$

output
```
2/3*(-a*d+b*c)*e*(e*x)^(9/2)/a/b/(b*x^3+a*x^2)^(3/2)-2*d*e^4*(e*x)^(3/2)/b
^2/(b*x^3+a*x^2)^(1/2)+2*d*e^(11/2)*arctanh(b^(1/2)*(e*x)^(3/2)/e^(3/2)/(b
*x^3+a*x^2)^(1/2))/b^(5/2)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.90

$$\int \frac{(ex)^{11/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2e^5x^{5/2}\sqrt{ex}\left(\sqrt{b}\sqrt{x}(3a^2d-b^2cx+4abdx)+3ad(a+bx)^{3/2}\log\left(-\sqrt{b}\sqrt{x}+\sqrt{a+bx}\right)\right)}{3ab^{5/2}\left(x^2(a+bx)\right)^{3/2}}$$

input
```
Integrate[((e*x)^(11/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(-2*e^5*x^(5/2)*Sqrt[e*x]*(Sqrt[b]*Sqrt[x]*(3*a^2*d - b^2*c*x + 4*a*b*d*x)
 + 3*a*d*(a + b*x)^(3/2)*Log[-(Sqrt[b]*Sqrt[x]) + Sqrt[a + b*x]]))/(3*a*b^
(5/2)*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1943, 1928, 1937, 1935, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{11/2}(c+dx)}{(ax^2+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{de^2\int \frac{(ex)^{7/2}}{(bx^3+ax^2)^{3/2}}dx}{b}+\frac{2e(ex)^{9/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1928}$$

$$\frac{de^2\left(\frac{e^3\int \frac{\sqrt{ex}}{\sqrt{bx^3+ax^2}}dx}{b}-\frac{2e^2(ex)^{3/2}}{b\sqrt{ax^2+bx^3}}\right)}{b}+\frac{2e(ex)^{9/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1937}$$

$$\frac{de^2\left(\frac{e^3\sqrt{ex}\int \frac{\sqrt{x}}{\sqrt{bx^3+ax^2}}dx}{b\sqrt{x}}-\frac{2e^2(ex)^{3/2}}{b\sqrt{ax^2+bx^3}}\right)}{b}+\frac{2e(ex)^{9/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1935}$$

$$\frac{de^2\left(\frac{2e^3\sqrt{ex}\int \frac{1}{1-\frac{bx^3}{bx^3+ax^2}}d\frac{x^{3/2}}{\sqrt{bx^3+ax^2}}}{b\sqrt{x}}-\frac{2e^2(ex)^{3/2}}{b\sqrt{ax^2+bx^3}}\right)}{b}+\frac{2e(ex)^{9/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{de^2\left(\frac{2e^3\sqrt{ex}\text{arctanh}\left(\frac{\sqrt{b}x^{3/2}}{\sqrt{ax^2+bx^3}}\right)}{b^{3/2}\sqrt{x}}-\frac{2e^2(ex)^{3/2}}{b\sqrt{ax^2+bx^3}}\right)}{b}+\frac{2e(ex)^{9/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}$$

input
```
Int[((e*x)^(11/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*(b*c - a*d)*e*(e*x)^(9/2))/(3*a*b*(a*x^2 + b*x^3)^(3/2)) + (d*e^2*((-2*
e^2*(e*x)^(3/2))/(b*Sqrt[a*x^2 + b*x^3]) + (2*e^3*Sqrt[e*x]*ArcTanh[(Sqrt[
b]*x^(3/2))/Sqrt[a*x^2 + b*x^3]])/(b^(3/2)*Sqrt[x])))/b
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 1928
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(n - 1)*(c*x)^(m - n + 1)*((a*x^j + b*x^n)^(p + 1)/(b*(n - j)*(
p + 1))), x] - Simp[c^n*((m + j*p - n + j + 1)/(b*(n - j)*(p + 1)))   Int[(
c*x)^(m - n)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c}, x] &&  !In
tegerQ[p] && LtQ[0, j, n] && (IntegersQ[j, n] || GtQ[c, 0]) && LtQ[p, -1] &
& GtQ[m + j*p + 1, n - j]
```

rule 1935
```
Int[(x_)^(m_.)/Sqrt[(a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.)], x_Symbol] :> Simp
[-2/(n - j)   Subst[Int[1/(1 - a*x^2), x], x, x^(j/2)/Sqrt[a*x^j + b*x^n]],
 x] /; FreeQ[{a, b, j, n}, x] && EqQ[m, j/2 - 1] && NeQ[n, j]
```

rule 1937
```
Int[((c_)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[c^IntPart[m]*((c*x)^FracPart[m]/x^FracPart[m])   Int[x^m*(a*x^j + b
*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] && IntegerQ[p + 1/2] &&
NeQ[n, j] && EqQ[Simplify[m + j*p + 1], 0]
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(98) = 196$.

Time = 0.40 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.06

method	result
default	$-\frac{x^{5}(bx+a)\left(-3\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)a\,b^{2}de\,x^{2}-6\ln\left(\frac{2bex+2\sqrt{ex(bx+a)}\,\sqrt{be}+ae}{2\sqrt{be}}\right)a^{2}bdex+8\sqrt{ex(bx+a)}\,\sqrt{be}\,abdx-2\sqrt{ex}}{3(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}\sqrt{be}\,a\sqrt{ex(bx+a)}\,b^{2}}$

input
```
int((e*x)^(11/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*x^5*(b*x+a)*(-3*ln(1/2*(2*b*e*x+2*(e*x*(b*x+a))^(1/2)*(b*e)^(1/2)+a*e
)/(b*e)^(1/2))*a*b^2*d*e*x^2-6*ln(1/2*(2*b*e*x+2*(e*x*(b*x+a))^(1/2)*(b*e)
^(1/2)+a*e)/(b*e)^(1/2))*a^2*b*d*e*x+8*(e*x*(b*x+a))^(1/2)*(b*e)^(1/2)*a*b
*d*x-2*(e*x*(b*x+a))^(1/2)*(b*e)^(1/2)*b^2*c*x-3*ln(1/2*(2*b*e*x+2*(e*x*(b
*x+a))^(1/2)*(b*e)^(1/2)+a*e)/(b*e)^(1/2))*a^3*d*e+6*(e*x*(b*x+a))^(1/2)*(
b*e)^(1/2)*a^2*d)*(e*x)^(1/2)*e^5/(b*x^3+a*x^2)^(5/2)/(b*e)^(1/2)/a/(e*x*(
b*x+a))^(1/2)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.80

$$\int \frac{(ex)^{11/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \left[\frac{3\left(ab^2de^5x^3+2\,a^2bde^5x^2+a^3de^5x\right)\sqrt{\frac{e}{b}}\log\left(\frac{2\,bex^2+aex+2\,\sqrt{bx^3+ax^2}\sqrt{ex}b\sqrt{\frac{e}{b}}}{x}\right)-2\,(3}{3\left(ab^4x^3+2\,a^2b^3x^2+a^3b^2x\right)}\right.$$

input
```
integrate((e*x)^(11/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
[1/3*(3*(a*b^2*d*e^5*x^3 + 2*a^2*b*d*e^5*x^2 + a^3*d*e^5*x)*sqrt(e/b)*log(
(2*b*e*x^2 + a*e*x + 2*sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*sqrt(e/b))/x) - 2*(
3*a^2*d*e^5 - (b^2*c - 4*a*b*d)*e^5*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x))/(a*b
^4*x^3 + 2*a^2*b^3*x^2 + a^3*b^2*x), -2/3*(3*(a*b^2*d*e^5*x^3 + 2*a^2*b*d*
e^5*x^2 + a^3*d*e^5*x)*sqrt(-e/b)*arctan(sqrt(b*x^3 + a*x^2)*sqrt(e*x)*b*s
qrt(-e/b)/(b*e*x^2 + a*e*x)) + (3*a^2*d*e^5 - (b^2*c - 4*a*b*d)*e^5*x)*sqr
t(b*x^3 + a*x^2)*sqrt(e*x))/(a*b^4*x^3 + 2*a^2*b^3*x^2 + a^3*b^2*x)]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{11/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(11/2)*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^{11/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(dx+c)(ex)^{\frac{11}{2}}}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(11/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^(11/2)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.26

$$\int \frac{(ex)^{11/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{de^5|e|\log(e^2|a|)\,\mathrm{sgn}(x)}{\sqrt{be}b^2} - \frac{2\,de^5|e|\log\left(\left|-\sqrt{be}\sqrt{ex}+\sqrt{be^2x+ae^2}\right|\right)}{\sqrt{be}b^2\mathrm{sgn}\,(x)} - \frac{2\left(\frac{3\,ade^7|e|}{b^2\mathrm{sgn}(x)} - \frac{(b^3ce^9|e|\mathrm{sgn}(x)-4\,ab^2de^9|e|\mathrm{sgn}(x))x}{ab^3e^2}\right)\sqrt{ex}}{3\,(be^2x+ae^2)^{\frac{3}{2}}}$$

input
```
integrate((e*x)^(11/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
d*e^5*abs(e)*log(e^2*abs(a))*sgn(x)/(sqrt(b*e)*b^2) - 2*d*e^5*abs(e)*log(a
bs(-sqrt(b*e)*sqrt(e*x) + sqrt(b*e^2*x + a*e^2)))/(sqrt(b*e)*b^2*sgn(x)) -
 2/3*(3*a*d*e^7*abs(e)/(b^2*sgn(x)) - (b^3*c*e^9*abs(e)*sgn(x) - 4*a*b^2*d
*e^9*abs(e)*sgn(x))*x/(a*b^3*e^2))*sqrt(e*x)/(b*e^2*x + a*e^2)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^{11/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(e\,x)^{11/2}\,(c+d\,x)}{(b\,x^3+a\,x^2)^{5/2}}\,dx$$

input
```
int(((e*x)^(11/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
int(((e*x)^(11/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.22

$$\int \frac{(ex)^{11/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{e}\,e^5\left(3\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)a^2d+3\sqrt{b}\,\sqrt{bx+a}\log\left(\frac{\sqrt{bx+a}+\sqrt{x}\,\sqrt{b}}{\sqrt{a}}\right)abc}{3\sqrt{bx+a}\,a\,b}$$

input
```
int((e*x)^(11/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(2*sqrt(e)*e**5*(3*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x)*sqrt
(b))/sqrt(a))*a**2*d + 3*sqrt(b)*sqrt(a + b*x)*log((sqrt(a + b*x) + sqrt(x
)*sqrt(b))/sqrt(a))*a*b*d*x + sqrt(b)*sqrt(a + b*x)*a*b*c + sqrt(b)*sqrt(a
 + b*x)*b**2*c*x - 3*sqrt(x)*a**2*b*d - 4*sqrt(x)*a*b**2*d*x + sqrt(x)*b**
3*c*x))/(3*sqrt(a + b*x)*a*b**3*(a + b*x))
```

3.350 $\int \frac{(ex)^{9/2}(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 85

$$\int \frac{(ex)^{9/2}(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx=\frac{2(bc-ad)e(ex)^{7/2}}{3ab\left(ax^2+bx^3\right)^{3/2}}+\frac{2(2bc+ad)e^3(ex)^{3/2}}{3a^2b\sqrt{ax^2+bx^3}}$$

output
```
2/3*(-a*d+b*c)*e*(e*x)^(7/2)/a/b/(b*x^3+a*x^2)^(3/2)+2/3*(a*d+2*b*c)*e^3*(
e*x)^(3/2)/a^2/b/(b*x^3+a*x^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.49

$$\int \frac{(ex)^{9/2}(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx=\frac{2e(ex)^{7/2}(3ac+2bcx+adx)}{3a^2\left(x^2(a+bx)\right)^{3/2}}$$

input
```
Integrate[((e*x)^(9/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*e*(e*x)^(7/2)*(3*a*c + 2*b*c*x + a*d*x))/(3*a^2*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {1943, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{9/2}(c+dx)}{(ax^2+bx^3)^{5/2}} \, dx$$
$$\Big\downarrow \text{1943}$$
$$\frac{e^2(ad+2bc)\int \frac{(ex)^{5/2}}{(bx^3+ax^2)^{3/2}}dx}{3ab}+\frac{2e(ex)^{7/2}(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$
$$\Big\downarrow \text{1920}$$
$$\frac{2e^3(ex)^{3/2}(ad+2bc)}{3a^2b\sqrt{ax^2+bx^3}}+\frac{2e(ex)^{7/2}(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

input
```
Int[((e*x)^(9/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*(b*c - a*d)*e*(e*x)^(7/2))/(3*a*b*(a*x^2 + b*x^3)^(3/2)) + (2*(2*b*c +
a*d)*e^3*(e*x)^(3/2))/(3*a^2*b*Sqrt[a*x^2 + b*x^3])
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.52

method	result	size
gosper	$\frac{2x(bx+a)(adx+2cbx+3ac)(ex)^{\frac{9}{2}}}{3a^{2}(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}}$	44
orering	$\frac{2x(bx+a)(adx+2cbx+3ac)(ex)^{\frac{9}{2}}}{3a^{2}(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}}$	44
default	$\frac{2x^{5}(bx+a)(adx+2cbx+3ac)\sqrt{ex}\,e^{4}}{3(b\,x^{3}+a\,x^{2})^{\frac{5}{2}}\,a^{2}}$	49

input
```
int((e*x)^(9/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*x*(b*x+a)*(a*d*x+2*b*c*x+3*a*c)*(e*x)^(9/2)/a^2/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.80

$$\int \frac{(ex)^{9/2}(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx = \frac{2\left(3\,ace^4+(2\,bc+ad)e^4x\right)\sqrt{bx^3+ax^2}\sqrt{ex}}{3\left(a^2b^2x^3+2\,a^3bx^2+a^4x\right)}$$

input
```
integrate((e*x)^(9/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
2/3*(3*a*c*e^4 + (2*b*c + a*d)*e^4*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x)/(a^2*b
^2*x^3 + 2*a^3*b*x^2 + a^4*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^{9/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**(9/2)*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^{9/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(dx+c)(ex)^{\frac{9}{2}}}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(9/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^(9/2)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int \frac{(ex)^{9/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\left(\frac{3\,ce^6|e|}{a\mathrm{sgn}(x)}+\frac{(2\,b^2ce^8|e|\mathrm{sgn}(x)+abde^8|e|\mathrm{sgn}(x))x}{a^2be^2}\right)\sqrt{ex}}{3\,(be^2x+ae^2)^{\frac{3}{2}}}$$

input
```
integrate((e*x)^(9/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
2/3*(3*c*e^6*abs(e)/(a*sgn(x)) + (2*b^2*c*e^8*abs(e)*sgn(x) + a*b*d*e^8*ab
s(e)*sgn(x))*x/(a^2*b*e^2))*sqrt(e*x)/(b*e^2*x + a*e^2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 9.73 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.94

$$\int \frac{(ex)^{9/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{\left(\frac{2\,c\,e^4\,\sqrt{e\,x}}{a\,b^2}+\frac{2\,e^4\,x\,\sqrt{e\,x}\,(a\,d+2\,b\,c)}{3\,a^2\,b^2}\right)\,\sqrt{b\,x^3+a\,x^2}}{x^3+\frac{2\,a\,x^2}{b}+\frac{a^2\,x}{b^2}}$$

input
```
int(((e*x)^(9/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
(((2*c*e^4*(e*x)^(1/2))/(a*b^2) + (2*e^4*x*(e*x)^(1/2)*(a*d + 2*b*c))/(3*a
^2*b^2))*(a*x^2 + b*x^3)^(1/2))/(x^3 + (2*a*x^2)/b + (a^2*x)/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.29

$$\int \frac{(ex)^{9/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{e}\,e^4\left(\sqrt{b}\,\sqrt{bx+a}\,a^2d-2\sqrt{b}\,\sqrt{bx+a}\,abc+\sqrt{b}\,\sqrt{bx+a}\,abdx-2\sqrt{b}\,\sqrt{bx+a}\,b^2\right.}{3\sqrt{bx+a}\,a^2b^2\,(bx+a)}$$

input
```
int((e*x)^(9/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(2*sqrt(e)*e**4*(sqrt(b)*sqrt(a + b*x)*a**2*d - 2*sqrt(b)*sqrt(a + b*x)*a*
b*c + sqrt(b)*sqrt(a + b*x)*a*b*d*x - 2*sqrt(b)*sqrt(a + b*x)*b**2*c*x + 3
*sqrt(x)*a*b**2*c + sqrt(x)*a*b**2*d*x + 2*sqrt(x)*b**3*c*x))/(3*sqrt(a +
b*x)*a**2*b**2*(a + b*x))
```

3.351 $\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 109

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2ce(ex)^{5/2}}{a\left(ax^2+bx^3\right)^{3/2}} - \frac{2(4bc-ad)(ex)^{7/2}}{3a^2\left(ax^2+bx^3\right)^{3/2}} - \frac{4(4bc-ad)e^2(ex)^{3/2}}{3a^3\sqrt{ax^2+bx^3}}$$

output
```
-2*c*e*(e*x)^(5/2)/a/(b*x^3+a*x^2)^(3/2)-2/3*(-a*d+4*b*c)*(e*x)^(7/2)/a^2/
(b*x^3+a*x^2)^(3/2)-4/3*(-a*d+4*b*c)*e^2*(e*x)^(3/2)/a^3/(b*x^3+a*x^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.56

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2e(ex)^{5/2}\left(-8b^2cx^2-3a^2(c-dx)+2abx(-6c+dx)\right)}{3a^3\left(x^2(a+bx)\right)^{3/2}}$$

input
```
Integrate[((e*x)^(7/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*e*(e*x)^(5/2)*(-8*b^2*c*x^2 - 3*a^2*(c - d*x) + 2*a*b*x*(-6*c + d*x)))/
(3*a^3*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.13, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {1943, 1921, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{1943}$$

$$\frac{e^2(4bc-ad)\int \frac{(ex)^{3/2}}{(bx^3+ax^2)^{3/2}}dx}{3ab}+\frac{2e(ex)^{5/2}(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1921}$$

$$\frac{e^2(4bc-ad)\left(\frac{2e^2\int \frac{1}{\sqrt{ex}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2e\sqrt{ex}}{a\sqrt{ax^2+bx^3}}\right)}{3ab}+\frac{2e(ex)^{5/2}(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1920}$$

$$\frac{e^2(4bc-ad)\left(\frac{2e\sqrt{ex}}{a\sqrt{ax^2+bx^3}}-\frac{4e^3\sqrt{ax^2+bx^3}}{a^2(ex)^{3/2}}\right)}{3ab}+\frac{2e(ex)^{5/2}(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

input
```
Int[((e*x)^(7/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*(b*c - a*d)*e*(e*x)^(5/2))/(3*a*b*(a*x^2 + b*x^3)^(3/2)) + ((4*b*c - a*
d)*e^2*((2*e*Sqrt[e*x])/(a*Sqrt[a*x^2 + b*x^3]) - (4*e^3*Sqrt[a*x^2 + b*x^
3])/(a^2*(e*x)^(3/2))))/(3*a*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] && !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] && !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] && !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.61

method	result	size
gosper	$-\frac{2x(bx+a)\left(-2abd\,x^{2}+8b^{2}c\,x^{2}-3a^{2}dx+12abcx+3a^{2}c\right)(ex)^{\frac{7}{2}}}{3a^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	67
orering	$-\frac{2x(bx+a)\left(-2abd\,x^{2}+8b^{2}c\,x^{2}-3a^{2}dx+12abcx+3a^{2}c\right)(ex)^{\frac{7}{2}}}{3a^{3}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	67
default	$-\frac{2x^{4}(bx+a)\left(-2abd\,x^{2}+8b^{2}c\,x^{2}-3a^{2}dx+12abcx+3a^{2}c\right)\sqrt{ex}\,e^{3}}{3\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}a^{3}}$	72
risch	$-\frac{2c(bx+a)e^{4}x}{a^{3}\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}+\frac{2\left(2abdx-5b^{2}cx+3a^{2}d-6abc\right)x^{2}e^{4}}{3(bx+a)a^{3}\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}$	92

input
```
int((e*x)^(7/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(b*x+a)*(-2*a*b*d*x^2+8*b^2*c*x^2-3*a^2*d*x+12*a*b*c*x+3*a^2*c)*(e*
x)^(7/2)/a^3/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.89

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\,(3\,a^2ce^3+2\,(4\,b^2c-abd)e^3x^2+3\,(4\,abc-a^2d)e^3x)\sqrt{bx^3+ax^2}\sqrt{ex}}{3\,(a^3b^2x^4+2\,a^4bx^3+a^5x^2)}$$

input
```
integrate((e*x)^(7/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(3*a^2*c*e^3 + 2*(4*b^2*c - a*b*d)*e^3*x^2 + 3*(4*a*b*c - a^2*d)*e^3*
x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x)/(a^3*b^2*x^4 + 2*a^4*b*x^3 + a^5*x^2)
```

Sympy [F]

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(ex)^{\frac{7}{2}}\,(c+dx)}{(x^2\,(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)**(7/2)*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral((e*x)**(7/2)*(c + d*x)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(dx+c)(ex)^{\frac{7}{2}}}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(7/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^(7/2)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.57

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{4\sqrt{be}ce^6}{\left(ae^2-\left(\sqrt{be}\sqrt{ex}-\sqrt{be^2x+ae^2}\right)^2\right)a^2|e|\mathrm{sgn}\,(x)} - \frac{2\sqrt{ex}\left(\frac{(5\,a^2b^3ce^7|e|\mathrm{sgn}(x)-2\,a^3b^2de^7|e|\mathrm{sgn}(x))x}{a^5be^2}+\frac{3\,(2\,a^3b^2ce^8|e|\mathrm{sgn}(x)-a^4bde^8|e|\mathrm{sgn}(x))}{a^5be^3}\right)}{3\,(be^2x+ae^2)^{\frac{3}{2}}}$$

input
```
integrate((e*x)^(7/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
-4*sqrt(b*e)*c*e^6/((a*e^2 - (sqrt(b*e)*sqrt(e*x) - sqrt(b*e^2*x + a*e^2))
^2)*a^2*abs(e)*sgn(x)) - 2/3*sqrt(e*x)*((5*a^2*b^3*c*e^7*abs(e)*sgn(x) - 2
*a^3*b^2*d*e^7*abs(e)*sgn(x))*x/(a^5*b*e^2) + 3*(2*a^3*b^2*c*e^8*abs(e)*sg
n(x) - a^4*b*d*e^8*abs(e)*sgn(x))/(a^5*b*e^3))/(b*e^2*x + a*e^2)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 9.51 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{2\,e^3\,x\,\sqrt{e\,x}\,(a\,d-4\,b\,c)}{a^2\,b^2}-\frac{2\,c\,e^3\,\sqrt{e\,x}}{a\,b^2}+\frac{4\,e^3\,x^2\,\sqrt{e\,x}\,(a\,d-4\,b\,c)}{3\,a^3\,b}\right)}{x^4+\frac{2\,a\,x^3}{b}+\frac{a^2\,x^2}{b^2}}$$

input
```
int(((e*x)^(7/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((2*e^3*x*(e*x)^(1/2)*(a*d - 4*b*c))/(a^2*b^2) - (2
*c*e^3*(e*x)^(1/2))/(a*b^2) + (4*e^3*x^2*(e*x)^(1/2)*(a*d - 4*b*c))/(3*a^3
*b)))/(x^4 + (2*a*x^3)/b + (a^2*x^2)/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.34

$$\int \frac{(ex)^{7/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{e}\,e^3\left(-2\sqrt{b}\,\sqrt{bx+a}\,a^2dx+8\sqrt{b}\,\sqrt{bx+a}\,abcx-2\sqrt{b}\,\sqrt{bx+a}\,abd\,x^2+8\sqrt{b}\,\sqrt{\cdots}\right.}{3\sqrt{bx+a}\,a\cdots}$$

input
```
int((e*x)^(7/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(2*sqrt(e)*e**3*( - 2*sqrt(b)*sqrt(a + b*x)*a**2*d*x + 8*sqrt(b)*sqrt(a +
b*x)*a*b*c*x - 2*sqrt(b)*sqrt(a + b*x)*a*b*d*x**2 + 8*sqrt(b)*sqrt(a + b*x
)*b**2*c*x**2 - 3*sqrt(x)*a**2*b*c + 3*sqrt(x)*a**2*b*d*x - 12*sqrt(x)*a*b
**2*c*x + 2*sqrt(x)*a*b**2*d*x**2 - 8*sqrt(x)*b**3*c*x**2))/(3*sqrt(a + b*
x)*a**3*b*x*(a + b*x))
```

3.352 $\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 152

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2ce(ex)^{3/2}}{3a\left(ax^2+bx^3\right)^{3/2}} - \frac{2(2bc-ad)(ex)^{5/2}}{3a^2\left(ax^2+bx^3\right)^{3/2}} - \frac{8(2bc-ad)e^2\sqrt{ex}}{3a^3\sqrt{ax^2+bx^3}} + \frac{16(2bc-ad)e^4\sqrt{ax^2+bx^3}}{3a^4(ex)^{3/2}}$$

output
```
-2/3*c*e*(e*x)^(3/2)/a/(b*x^3+a*x^2)^(3/2)-2/3*(-a*d+2*b*c)*(e*x)^(5/2)/a^
2/(b*x^3+a*x^2)^(3/2)-8/3*(-a*d+2*b*c)*e^2*(e*x)^(1/2)/a^3/(b*x^3+a*x^2)^(
1/2)+16/3*(-a*d+2*b*c)*e^4*(b*x^3+a*x^2)^(1/2)/a^4/(e*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.51

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2e(ex)^{3/2}\left(-16b^3cx^3-6a^2bx(c-2dx)+8ab^2x^2(-3c+dx)+a^3(c+3dx)\right)}{3a^4\left(x^2(a+bx)\right)^{3/2}}$$

input
```
Integrate[((e*x)^(5/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(-2*e*(e*x)^(3/2)*(-16*b^3*c*x^3 - 6*a^2*b*x*(c - 2*d*x) + 8*a*b^2*x^2*(-3
*c + d*x) + a^3*(c + 3*d*x)))/(3*a^4*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.05, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1943, 1921, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{ 1943}$$

$$\frac{e^2(2bc-ad)\int \frac{\sqrt{ex}}{(bx^3+ax^2)^{3/2}}dx}{ab}+\frac{2e(ex)^{3/2}(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 1921}$$

$$\frac{e^2(2bc-ad)\left(\frac{4e^2\int \frac{1}{(ex)^{3/2}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2e}{a\sqrt{ex}\sqrt{ax^2+bx^3}}\right)}{ab}+\frac{2e(ex)^{3/2}(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 1922}$$

$$\frac{e^2(2bc-ad)\left(\frac{4e^2\left(-\frac{2b\int \frac{1}{\sqrt{ex}\sqrt{bx^3+ax^2}}dx}{3ae}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{a}+\frac{2e}{a\sqrt{ex}\sqrt{ax^2+bx^3}}\right)}{ab}+\frac{2e(ex)^{3/2}(bc-ad)}{3ab\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{ 1920}$$

$$\frac{e^2(2bc-ad)\left(\frac{4e^2\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(ex)^{3/2}}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{a}+\frac{2e}{a\sqrt{ex}\sqrt{ax^2+bx^3}}\right)}{ab}+\frac{2e(ex)^{3/2}(bc-ad)}{3ab\left(ax^2+bx^3\right)^{3/2}}$$

input
```
Int[((e*x)^(5/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*(b*c - a*d)*e*(e*x)^(3/2))/(3*a*b*(a*x^2 + b*x^3)^(3/2)) + ((2*b*c - a*
d)*e^2*((2*e)/(a*Sqrt[e*x]*Sqrt[a*x^2 + b*x^3]) + (4*e^2*((-2*e*Sqrt[a*x^2
 + b*x^3])/(3*a*(e*x)^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*(e*x)^(3/2
))))/a))/(a*b)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1943
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[(-e^(j - 1))*(b*c - a*d)*(e*x)^(m - j
+ 1)*((a*x^j + b*x^(j + n))^(p + 1)/(a*b*n*(p + 1))), x] - Simp[e^j*((a*d*(
m + j*p + 1) - b*c*(m + n + p*(j + n) + 1))/(a*b*n*(p + 1)))   Int[(e*x)^(m
 - j)*(a*x^j + b*x^(j + n))^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, j, m,
n}, x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && LtQ[p, -1
] && GtQ[j, 0] && LeQ[j, m] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{2x(bx+a)\left(8a\,b^{2}d\,x^{3}-16b^{3}c\,x^{3}+12a^{2}bd\,x^{2}-24a\,b^{2}c\,x^{2}+3a^{3}dx-6a^{2}bcx+c\,a^{3}\right)(ex)^{\frac{5}{2}}}{3a^{4}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	90
orering	$-\frac{2x(bx+a)\left(8a\,b^{2}d\,x^{3}-16b^{3}c\,x^{3}+12a^{2}bd\,x^{2}-24a\,b^{2}c\,x^{2}+3a^{3}dx-6a^{2}bcx+c\,a^{3}\right)(ex)^{\frac{5}{2}}}{3a^{4}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	90
default	$-\frac{2x^{3}(bx+a)\left(8a\,b^{2}d\,x^{3}-16b^{3}c\,x^{3}+12a^{2}bd\,x^{2}-24a\,b^{2}c\,x^{2}+3a^{3}dx-6a^{2}bcx+c\,a^{3}\right)\sqrt{ex}\,e^{2}}{3\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}a^{4}}$	95
risch	$-\frac{2(bx+a)(3adx-8cbx+ac)e^{3}}{3a^{4}\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}-\frac{2b\left(5abdx-8b^{2}cx+6a^{2}d-9abc\right)x^{2}e^{3}}{3(bx+a)a^{4}\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}$	105

input
```
int((e*x)^(5/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/3*x*(b*x+a)*(8*a*b^2*d*x^3-16*b^3*c*x^3+12*a^2*b*d*x^2-24*a*b^2*c*x^2+3
*a^3*d*x-6*a^2*b*c*x+a^3*c)*(e*x)^(5/2)/a^4/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81

$$\int \frac{(ex)^{5/2}(c+dx)}{\left(ax^2+bx^3\right)^{5/2}}\,dx =$$
$$-\frac{2\left(a^3ce^2-8\left(2\,b^3c-ab^2d\right)e^2x^3-12\left(2\,ab^2c-a^2bd\right)e^2x^2-3\left(2\,a^2bc-a^3d\right)e^2x\right)\sqrt{bx^3+ax^2}\sqrt{ex}}{3\left(a^4b^2x^5+2\,a^5bx^4+a^6x^3\right)}$$

input
```
integrate((e*x)^(5/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-2/3*(a^3*c*e^2 - 8*(2*b^3*c - a*b^2*d)*e^2*x^3 - 12*(2*a*b^2*c - a^2*b*d)
*e^2*x^2 - 3*(2*a^2*b*c - a^3*d)*e^2*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x)/(a^4
*b^2*x^5 + 2*a^5*b*x^4 + a^6*x^3)
```

Sympy [F]

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(ex)^{\frac{5}{2}}(c+dx)}{(x^2(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)**(5/2)*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral((e*x)**(5/2)*(c + d*x)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(dx+c)(ex)^{\frac{5}{2}}}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(5/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^(5/2)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 365 vs. $2(128) = 256$.

Time = 1.46 (sec) , antiderivative size = 365, normalized size of antiderivative = 2.40

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{ex}\left(\frac{(8a^3b^4ce^6|e|\mathrm{sgn}(x)-5a^4b^3de^6|e|\mathrm{sgn}(x))x}{a^7be^2}+\frac{3(3a^4b^3ce^7|e|\mathrm{sgn}(x)-2a^5b^2de^7|e|\mathrm{sgn}(x))}{a^7be^3}\right)}{3(be^2x+ae^2)^{\frac{3}{2}}} + \frac{4\left(8\sqrt{be}a^2bce^9-3\sqrt{be}a^3de^9-18\sqrt{be}\left(\sqrt{be}\sqrt{ex}-\sqrt{be^2x+ae^2}\right)^2abce^7+6\sqrt{be}\left(\sqrt{be}\sqrt{ex}-\sqrt{be^2x+a\right.\right.}{3\left(ae^2-\left(\sqrt{be}\sqrt{ex}-\sqrt{be^2x+\right.\right.}$$

input
```
integrate((e*x)^(5/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
2/3*sqrt(e*x)*((8*a^3*b^4*c*e^6*abs(e)*sgn(x) - 5*a^4*b^3*d*e^6*abs(e)*sgn
(x))*x/(a^7*b*e^2) + 3*(3*a^4*b^3*c*e^7*abs(e)*sgn(x) - 2*a^5*b^2*d*e^7*ab
s(e)*sgn(x))/(a^7*b*e^3))/(b*e^2*x + a*e^2)^(3/2) + 4/3*(8*sqrt(b*e)*a^2*b
*c*e^9 - 3*sqrt(b*e)*a^3*d*e^9 - 18*sqrt(b*e)*(sqrt(b*e)*sqrt(e*x) - sqrt(
b*e^2*x + a*e^2))^2*a*b*c*e^7 + 6*sqrt(b*e)*(sqrt(b*e)*sqrt(e*x) - sqrt(b*
e^2*x + a*e^2))^2*a^2*d*e^7 + 6*sqrt(b*e)*(sqrt(b*e)*sqrt(e*x) - sqrt(b*e^
2*x + a*e^2))^4*b*c*e^5 - 3*sqrt(b*e)*(sqrt(b*e)*sqrt(e*x) - sqrt(b*e^2*x
+ a*e^2))^4*a*d*e^5)/((a*e^2 - (sqrt(b*e)*sqrt(e*x) - sqrt(b*e^2*x + a*e^2
))^2)^3*a^3*abs(e)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.39 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.88

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\left(\frac{2\,c\,e^2\,\sqrt{e\,x}}{3\,a\,b^2}+\frac{16\,e^2\,x^3\,\sqrt{e\,x}\,(a\,d-2\,b\,c)}{3\,a^4}+\frac{2\,e^2\,x\,\sqrt{e\,x}\,(a\,d-2\,b\,c)}{a^2\,b^2}+\frac{8\,e^2\,x^2\,\sqrt{e\,x}\,(a\,d-2\,b\,c)}{a^3\,b}\right)}{x^5+\frac{2\,a\,x^4}{b}+\frac{a^2\,x^3}{b^2}}$$

input
```
int(((e*x)^(5/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((2*c*e^2*(e*x)^(1/2))/(3*a*b^2) + (16*e^2*x^3*(e*
x)^(1/2)*(a*d - 2*b*c))/(3*a^4) + (2*e^2*x*(e*x)^(1/2)*(a*d - 2*b*c))/(a^2
*b^2) + (8*e^2*x^2*(e*x)^(1/2)*(a*d - 2*b*c))/(a^3*b)))/(x^5 + (2*a*x^4)/b
 + (a^2*x^3)/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.11

$$\int \frac{(ex)^{5/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{e}\,e^2\left(8\sqrt{b}\,\sqrt{bx+a}\,a^2d\,x^2-16\sqrt{b}\,\sqrt{bx+a}\,abc\,x^2+8\sqrt{b}\,\sqrt{bx+a}\,abd\,x^3-16\sqrt{b}\right.}{}$$

input
```
int((e*x)^(5/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(2*sqrt(e)*e**2*(8*sqrt(b)*sqrt(a + b*x)*a**2*d*x**2 - 16*sqrt(b)*sqrt(a +
 b*x)*a*b*c*x**2 + 8*sqrt(b)*sqrt(a + b*x)*a*b*d*x**3 - 16*sqrt(b)*sqrt(a
+ b*x)*b**2*c*x**3 - sqrt(x)*a**3*c - 3*sqrt(x)*a**3*d*x + 6*sqrt(x)*a**2*
b*c*x - 12*sqrt(x)*a**2*b*d*x**2 + 24*sqrt(x)*a*b**2*c*x**2 - 8*sqrt(x)*a*
b**2*d*x**3 + 16*sqrt(x)*b**3*c*x**3))/(3*sqrt(a + b*x)*a**4*x**2*(a + b*x
))
```

3.353 $\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 194

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2ce\sqrt{ex}}{5a\,(ax^2+bx^3)^{3/2}} - \frac{2(8bc-5ad)(ex)^{3/2}}{15a^2\,(ax^2+bx^3)^{3/2}} - \frac{4(8bc-5ad)e^2}{5a^3\sqrt{ex}\sqrt{ax^2+bx^3}} + \frac{16(8bc-5ad)e^4\sqrt{ax^2+bx^3}}{15a^4(ex)^{5/2}} - \frac{32b(8bc-5ad)e^3\sqrt{ax^2+bx^3}}{15a^5(ex)^{3/2}}$$

output
```
-2/5*c*e*(e*x)^(1/2)/a/(b*x^3+a*x^2)^(3/2)-2/15*(-5*a*d+8*b*c)*(e*x)^(3/2)
/a^2/(b*x^3+a*x^2)^(3/2)-4/5*(-5*a*d+8*b*c)*e^2/a^3/(e*x)^(1/2)/(b*x^3+a*x
^2)^(1/2)+16/15*(-5*a*d+8*b*c)*e^4*(b*x^3+a*x^2)^(1/2)/a^4/(e*x)^(5/2)-32/
15*b*(-5*a*d+8*b*c)*e^3*(b*x^3+a*x^2)^(1/2)/a^5/(e*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.53

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2e\sqrt{ex}(-128b^4cx^4+16ab^3x^3(-12c+5dx)+24a^2b^2x^2(-2c+5dx)-a^4(3c+5dx}{15a^5\left(x^2(a+bx)\right)^{3/2}}$$

input
```
Integrate[((e*x)^(3/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(2*e*Sqrt[e*x]*(-128*b^4*c*x^4 + 16*a*b^3*x^3*(-12*c + 5*d*x) + 24*a^2*b^2
*x^2*(-2*c + 5*d*x) - a^4*(3*c + 5*d*x) + 2*a^3*b*x*(4*c + 15*d*x)))/(15*a
^5*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1944, 1921, 1921, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{5/2}}dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(8bc-5ad)\int \frac{(ex)^{5/2}}{(bx^3+ax^2)^{5/2}}dx}{5ae}-\frac{2ce\sqrt{ex}}{5a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1921}$$

$$-\frac{(8bc-5ad)\left(\frac{2e^2\int \frac{\sqrt{ex}}{\left(bx^3+ax^2\right)^{3/2}}dx}{a}+\frac{2e(ex)^{3/2}}{3a(ax^2+bx^3)^{3/2}}\right)}{5ae}-\frac{2ce\sqrt{ex}}{5a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1921}$$

$$-\frac{(8bc-5ad)\left(\frac{2e^2\left(\frac{4e^2\int \frac{1}{(ex)^{3/2}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2e}{a\sqrt{ex}\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2e(ex)^{3/2}}{3a(ax^2+bx^3)^{3/2}}\right)}{5ae}-\frac{2ce\sqrt{ex}}{5a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 1922$$

$$-\frac{(8bc-5ad)\left(\frac{2e^2\left(\frac{4e^2\left(-\frac{2b\int \frac{1}{\sqrt{ex}\sqrt{bx^3+ax^2}}dx}{3ae}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{a}+\frac{2e}{a\sqrt{ex}\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2e(ex)^{3/2}}{3a(ax^2+bx^3)^{3/2}}\right)}{5ae}-\frac{2ce\sqrt{ex}}{5a\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \; 1920$$

$$-\frac{(8bc-5ad)\left(\frac{2e^2\left(\frac{4e^2\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(ex)^{3/2}}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{a}+\frac{2e}{a\sqrt{ex}\sqrt{ax^2+bx^3}}\right)}{a}+\frac{2e(ex)^{3/2}}{3a(ax^2+bx^3)^{3/2}}\right)}{5ae}-\frac{2ce\sqrt{ex}}{5a\left(ax^2+bx^3\right)^{3/2}}$$

input
```
Int[((e*x)^(3/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(-2*c*e*Sqrt[e*x])/(5*a*(a*x^2 + b*x^3)^(3/2)) - ((8*b*c - 5*a*d)*((2*e*(e
*x)^(3/2))/(3*a*(a*x^2 + b*x^3)^(3/2)) + (2*e^2*((2*e)/(a*Sqrt[e*x]*Sqrt[a
*x^2 + b*x^3]) + (4*e^2*((-2*e*Sqrt[a*x^2 + b*x^3])/(3*a*(e*x)^(5/2)) + (4
*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*(e*x)^(3/2))))/a))/a))/(5*a*e)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.59

method	result	size
gosper	$-\frac{2x(bx+a)\left(-80x^{4}a\,b^{3}d+128x^{4}b^{4}c-120a^{2}b^{2}d\,x^{3}+192a\,b^{3}c\,x^{3}-30a^{3}bd\,x^{2}+48a^{2}b^{2}c\,x^{2}+5a^{4}dx-8a^{3}bcx+3c\,a^{4}\right)(ex)^{\frac{3}{2}}}{15a^{5}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	115
orering	$-\frac{2x(bx+a)\left(-80x^{4}a\,b^{3}d+128x^{4}b^{4}c-120a^{2}b^{2}d\,x^{3}+192a\,b^{3}c\,x^{3}-30a^{3}bd\,x^{2}+48a^{2}b^{2}c\,x^{2}+5a^{4}dx-8a^{3}bcx+3c\,a^{4}\right)(ex)^{\frac{3}{2}}}{15a^{5}\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}}$	115
default	$-\frac{2x^{2}(bx+a)\left(-80x^{4}a\,b^{3}d+128x^{4}b^{4}c-120a^{2}b^{2}d\,x^{3}+192a\,b^{3}c\,x^{3}-30a^{3}bd\,x^{2}+48a^{2}b^{2}c\,x^{2}+5a^{4}dx-8a^{3}bcx+3c\,a^{4}\right)\sqrt{ex}\,e}{15\left(b\,x^{3}+a\,x^{2}\right)^{\frac{5}{2}}a^{5}}$	118
risch	$-\frac{2(bx+a)\left(-40abd\,x^{2}+73b^{2}c\,x^{2}+5a^{2}dx-14abcx+3a^{2}c\right)e^{2}}{15a^{5}x\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}+\frac{2b^{2}\left(8abdx-11b^{2}cx+9a^{2}d-12abc\right)x^{2}e^{2}}{3(bx+a)a^{5}\sqrt{x^{2}(bx+a)}\,\sqrt{ex}}$	133

input
```
int((e*x)^(3/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/15*x*(b*x+a)*(-80*a*b^3*d*x^4+128*b^4*c*x^4-120*a^2*b^2*d*x^3+192*a*b^3
*c*x^3-30*a^3*b*d*x^2+48*a^2*b^2*c*x^2+5*a^4*d*x-8*a^3*b*c*x+3*a^4*c)*(e*x
)^(3/2)/a^5/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 0.73

$$\int \frac{(ex)^{3/2}(c+dx)}{\left(ax^{2}+bx^{3}\right)^{5/2}}\,dx = -\frac{2\left(3\,a^{4}ce+16\left(8\,b^{4}c-5\,ab^{3}d\right)ex^{4}+24\left(8\,ab^{3}c-5\,a^{2}b^{2}d\right)ex^{3}+6\left(8\,a^{2}b^{2}c-5\,a^{3}bd\right)ex^{2}-\left(8\,a^{3}bc-5\,a^{4}d\right)e}{15\left(a^{5}b^{2}x^{6}+2\,a^{6}bx^{5}+a^{7}x^{4}\right)}$$

input
```
integrate((e*x)^(3/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-2/15*(3*a^4*c*e + 16*(8*b^4*c - 5*a*b^3*d)*e*x^4 + 24*(8*a*b^3*c - 5*a^2*
b^2*d)*e*x^3 + 6*(8*a^2*b^2*c - 5*a^3*b*d)*e*x^2 - (8*a^3*b*c - 5*a^4*d)*e
*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x)/(a^5*b^2*x^6 + 2*a^6*b*x^5 + a^7*x^4)
```

Sympy [F]

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(ex)^{\frac{3}{2}}\,(c+dx)}{(x^2\,(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)**(3/2)*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral((e*x)**(3/2)*(c + d*x)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(dx+c)(ex)^{\frac{3}{2}}}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(3/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^(3/2)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(3/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 9.50 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.78

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{16\,e\,x^3\,\sqrt{e\,x}\,(5\,a\,d-8\,b\,c)}{5\,a^4}-\frac{2\,c\,e\,\sqrt{e\,x}}{5\,a\,b^2}-\frac{2\,e\,x\,\sqrt{e\,x}\,(5\,a\,d-8\,b\,c)}{15\,a^2\,b^2}+\frac{32\,b\,e\,x^4\,\sqrt{e\,x}\,(5\,a\,d-8}{15\,a^5}\right.}{x^6+\frac{2\,a\,x^5}{b}+\frac{a^2\,x^4}{b^2}}$$

input

```
int(((e*x)^(3/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output

```
((a*x^2 + b*x^3)^(1/2)*((16*e*x^3*(e*x)^(1/2)*(5*a*d - 8*b*c))/(5*a^4) - (
2*c*e*(e*x)^(1/2))/(5*a*b^2) - (2*e*x*(e*x)^(1/2)*(5*a*d - 8*b*c))/(15*a^2
*b^2) + (32*b*e*x^4*(e*x)^(1/2)*(5*a*d - 8*b*c))/(15*a^5) + (4*e*x^2*(e*x)
^(1/2)*(5*a*d - 8*b*c))/(5*a^3*b)))/(x^6 + (2*a*x^5)/b + (a^2*x^4)/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.03

$$\int \frac{(ex)^{3/2}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{e}\,e\Big(-80\sqrt{b}\,\sqrt{bx+a}\,a^2bd\,x^3+128\sqrt{b}\,\sqrt{bx+a}\,a\,b^2c\,x^3-80\sqrt{b}\,\sqrt{bx+a}\,a\,b^2d\,x^4}{}$$

input

```
int((e*x)^(3/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output

```
(2*sqrt(e)*e*( - 80*sqrt(b)*sqrt(a + b*x)*a**2*b*d*x**3 + 128*sqrt(b)*sqrt
(a + b*x)*a*b**2*c*x**3 - 80*sqrt(b)*sqrt(a + b*x)*a*b**2*d*x**4 + 128*sqr
t(b)*sqrt(a + b*x)*b**3*c*x**4 - 3*sqrt(x)*a**4*c - 5*sqrt(x)*a**4*d*x + 8
*sqrt(x)*a**3*b*c*x + 30*sqrt(x)*a**3*b*d*x**2 - 48*sqrt(x)*a**2*b**2*c*x*
*2 + 120*sqrt(x)*a**2*b**2*d*x**3 - 192*sqrt(x)*a*b**3*c*x**3 + 80*sqrt(x)
*a*b**3*d*x**4 - 128*sqrt(x)*b**4*c*x**4))/(15*sqrt(a + b*x)*a**5*x**3*(a
+ b*x))
```

3.354 $\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 238

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2ce}{7a\sqrt{ex}\,(ax^2+bx^3)^{3/2}} - \frac{2(10bc-7ad)\sqrt{ex}}{21a^2\,(ax^2+bx^3)^{3/2}} - \frac{16(10bc-7ad)e^2}{21a^3(ex)^{3/2}\sqrt{ax^2+bx^3}} + \frac{32(10bc-7ad)e^4\sqrt{ax^2+bx^3}}{35a^4(ex)^{7/2}} - \frac{128b(10bc-7ad)e^3\sqrt{ax^2+bx^3}}{105a^5(ex)^{5/2}} + \frac{256b^2(10bc-7ad)e^2\sqrt{ax^2+bx^3}}{105a^6(ex)^{3/2}}$$

output

```
-2/7*c*e/a/(e*x)^(1/2)/(b*x^3+a*x^2)^(3/2)-2/21*(-7*a*d+10*b*c)*(e*x)^(1/2
)/a^2/(b*x^3+a*x^2)^(3/2)-16/21*(-7*a*d+10*b*c)*e^2/a^3/(e*x)^(3/2)/(b*x^3
+a*x^2)^(1/2)+32/35*(-7*a*d+10*b*c)*e^4*(b*x^3+a*x^2)^(1/2)/a^4/(e*x)^(7/2
)-128/105*b*(-7*a*d+10*b*c)*e^3*(b*x^3+a*x^2)^(1/2)/a^5/(e*x)^(5/2)+256/10
5*b^2*(-7*a*d+10*b*c)*e^2*(b*x^3+a*x^2)^(1/2)/a^6/(e*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.51

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx =$$
$$-\frac{2e(-1280b^5cx^5+128ab^4x^4(-15c+7dx)+3a^5(5c+7dx)+96a^2b^3x^3(-5c+14dx)+16a^3b^2x^2(5c+21}{105a^6\sqrt{ex}\,(x^2(a+bx))^{3/2}}$$

input
```
Integrate[(Sqrt[e*x]*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(-2*e*(-1280*b^5*c*x^5 + 128*a*b^4*x^4*(-15*c + 7*d*x) + 3*a^5*(5*c + 7*d*
x) + 96*a^2*b^3*x^3*(-5*c + 14*d*x) + 16*a^3*b^2*x^2*(5*c + 21*d*x) - 2*a^
4*b*x*(15*c + 28*d*x)))/(105*a^6*Sqrt[e*x]*(x^2*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {1944, 1921, 1921, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx$$
$$\Big\downarrow \; 1944$$
$$-\frac{(10bc-7ad)\int \frac{(ex)^{3/2}}{(bx^3+ax^2)^{5/2}}dx}{7ae}-\frac{2ce}{7a\sqrt{ex}\,(ax^2+bx^3)^{3/2}}$$
$$\Big\downarrow \; 1921$$
$$-\frac{(10bc-7ad)\left(\frac{8e^2\int \frac{1}{\sqrt{ex}\left(bx^3+ax^2\right)^{3/2}}dx}{3a}+\frac{2e\sqrt{ex}}{3a(ax^2+bx^3)^{3/2}}\right)}{7ae}-\frac{2ce}{7a\sqrt{ex}\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1921}$$

$$-\frac{(10bc-7ad)\left(\frac{8e^2\left(\frac{6e^2\int \frac{1}{(ex)^{5/2}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2e}{a(ex)^{3/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2e\sqrt{ex}}{3a(ax^2+bx^3)^{3/2}}\right)}{7ae}-\frac{2ce}{7a\sqrt{ex}\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{(10bc-7ad)\left(\frac{8e^2\left(\frac{6e^2\left(-\frac{4b\int \frac{1}{(ex)^{3/2}\sqrt{bx^3+ax^2}}dx}{5ae}-\frac{2e\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}}\right)}{a}+\frac{2e}{a(ex)^{3/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2e\sqrt{ex}}{3a(ax^2+bx^3)^{3/2}}\right)}{7ae}-\frac{2ce}{7a\sqrt{ex}\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{(10bc-7ad)\left(\frac{8e^2\left(\frac{6e^2\left(-\frac{4b\left(-\frac{2b\int \frac{1}{\sqrt{ex}\sqrt{bx^3+ax^2}}dx}{3ae}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{5ae}-\frac{2e\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}}\right)}{a}+\frac{2e}{a(ex)^{3/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2e\sqrt{ex}}{3a(ax^2+bx^3)^{3/2}}\right)}{7ae}-\frac{2ce}{7a\sqrt{ex}\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1920}$$

$$-\frac{(10bc-7ad)\left(\frac{8e^2\left(\frac{6e^2\left(-\frac{4b\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(ex)^{3/2}}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{5ae}-\frac{2e\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}}\right)}{a}+\frac{2e}{a(ex)^{3/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2e\sqrt{ex}}{3a(ax^2+bx^3)^{3/2}}\right)}{7ae}-\frac{2ce}{7a\sqrt{ex}\,(ax^2+bx^3)^{3/2}}$$

input
```
Int[(Sqrt[e*x]*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x]
```

output
```
(-2*c*e)/(7*a*Sqrt[e*x]*(a*x^2 + b*x^3)^(3/2)) - ((10*b*c - 7*a*d)*((2*e*S
qrt[e*x])/(3*a*(a*x^2 + b*x^3)^(3/2)) + (8*e^2*((2*e)/(a*(e*x)^(3/2)*Sqrt[
a*x^2 + b*x^3]) + (6*e^2*((-2*e*Sqrt[a*x^2 + b*x^3])/(5*a*(e*x)^(7/2)) - (
4*b*((-2*e*Sqrt[a*x^2 + b*x^3])/(3*a*(e*x)^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^
3])/(3*a^2*(e*x)^(3/2))))/(5*a*e)))/a))/(3*a)))/(7*a*e)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.58

method	result
gosper	$-\frac{2x(bx+a)\left(896a\,b^4d\,x^5-1280b^5c\,x^5+1344x^4a^2b^3d-1920x^4a\,b^4c+336a^3b^2d\,x^3-480a^2b^3c\,x^3-56a^4bd\,x^2+80a^3b^2c\,x^2+21a^5dx-3\right.}{105a^6(b\,x^3+a\,x^2)^{\frac{5}{2}}}$
default	$-\frac{2x(bx+a)\left(896a\,b^4d\,x^5-1280b^5c\,x^5+1344x^4a^2b^3d-1920x^4a\,b^4c+336a^3b^2d\,x^3-480a^2b^3c\,x^3-56a^4bd\,x^2+80a^3b^2c\,x^2+21a^5dx-3\right.}{105a^6(b\,x^3+a\,x^2)^{\frac{5}{2}}}$
orering	$-\frac{2x(bx+a)\left(896a\,b^4d\,x^5-1280b^5c\,x^5+1344x^4a^2b^3d-1920x^4a\,b^4c+336a^3b^2d\,x^3-480a^2b^3c\,x^3-56a^4bd\,x^2+80a^3b^2c\,x^2+21a^5dx-3\right.}{105a^6(b\,x^3+a\,x^2)^{\frac{5}{2}}}$
risch	$-\frac{2(bx+a)\left(511a\,b^2d\,x^3-790b^3c\,x^3-98a^2bd\,x^2+185a\,b^2c\,x^2+21a^3dx-60a^2bcx+15c\,a^3\right)e}{105a^6x^2\sqrt{x^2(bx+a)}\,\sqrt{ex}}-\frac{2b^3\left(11abdx-14b^2cx+12a^2d-15abc\right)}{3(bx+a)a^6\sqrt{x^2(bx+a)}\,\sqrt{ex}}$

input
```
int((e*x)^(1/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/105*x*(b*x+a)*(896*a*b^4*d*x^5-1280*b^5*c*x^5+1344*a^2*b^3*d*x^4-1920*a
*b^4*c*x^4+336*a^3*b^2*d*x^3-480*a^2*b^3*c*x^3-56*a^4*b*d*x^2+80*a^3*b^2*c
*x^2+21*a^5*d*x-30*a^4*b*c*x+15*a^5*c)*(e*x)^(1/2)/a^6/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.67

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\,(15\,a^5c-128\,(10\,b^5c-7\,ab^4d)x^5-192\,(10\,ab^4c-7\,a^2b^3d)x^4-48\,(10\,a^2b^3c-7\,a^3b^2d)x^3+8\,(10\,a^3b^2c\cdots}{105\,(a^6b^2x^7+2\,a^7bx^6+a^8x^5)}$$

input
```
integrate((e*x)^(1/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-2/105*(15*a^5*c - 128*(10*b^5*c - 7*a*b^4*d)*x^5 - 192*(10*a*b^4*c - 7*a^
2*b^3*d)*x^4 - 48*(10*a^2*b^3*c - 7*a^3*b^2*d)*x^3 + 8*(10*a^3*b^2*c - 7*a
^4*b*d)*x^2 - 3*(10*a^4*b*c - 7*a^5*d)*x)*sqrt(b*x^3 + a*x^2)*sqrt(e*x)/(a
^6*b^2*x^7 + 2*a^7*b*x^6 + a^8*x^5)
```

Sympy [F]

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{\sqrt{ex}(c+dx)}{(x^2\,(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)**(1/2)*(d*x+c)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral(sqrt(e*x)*(c + d*x)/(x**2*(a + b*x))**(5/2), x)
```

Maxima [F]

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \int \frac{(dx+c)\sqrt{ex}}{(bx^3+ax^2)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^(1/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*sqrt(e*x)/(b*x^3 + a*x^2)^(5/2), x)
```

Giac [F(-1)]

Timed out.

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((e*x)^(1/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
Timed out
```

Mupad [B] (verification not implemented)

Time = 9.45 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.77

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = -\frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{32\,x^3\,\sqrt{e\,x}\,(7\,a\,d-10\,b\,c)}{35\,a^4}+\frac{2\,c\,\sqrt{e\,x}}{7\,a\,b^2}-\frac{x^5\,\sqrt{e\,x}\,\left(2560\,b^5\,c-1792\,a\,b^4\,d\right)}{105\,a^6\,b^2}+\frac{128\,b\,x^4\,\sqrt{e\,x}\,(7\,a\,d-10\,b\,c)}{35\,a^5}+\frac{x\,\sqrt{e\,x}\,(42\,a^5\,d}{105\,a^6}\right.}{x^7+\frac{2\,a\,x^6}{b}+\frac{a^2\,x^5}{b^2}}$$

input
```
int(((e*x)^(1/2)*(c + d*x))/(a*x^2 + b*x^3)^(5/2),x)
```

output
```
-((a*x^2 + b*x^3)^(1/2)*((32*x^3*(e*x)^(1/2)*(7*a*d - 10*b*c))/(35*a^4) +
(2*c*(e*x)^(1/2))/(7*a*b^2) - (x^5*(e*x)^(1/2)*(2560*b^5*c - 1792*a*b^4*d)
)/(105*a^6*b^2) + (128*b*x^4*(e*x)^(1/2)*(7*a*d - 10*b*c))/(35*a^5) + (x*(
e*x)^(1/2)*(42*a^5*d - 60*a^4*b*c))/(105*a^6*b^2) - (16*x^2*(e*x)^(1/2)*(7
*a*d - 10*b*c))/(105*a^3*b)))/(x^7 + (2*a*x^6)/b + (a^2*x^5)/b^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.96

$$\int \frac{\sqrt{ex}(c+dx)}{(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{e}\left(896\sqrt{b}\,\sqrt{bx+a}\,a^2b^2d\,x^4 - 1280\sqrt{b}\,\sqrt{bx+a}\,a\,b^3c\,x^4 + 896\sqrt{b}\,\sqrt{bx+a}\,a\,b^3d\,x\right.}{}$$

input
```
int((e*x)^(1/2)*(d*x+c)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(2*sqrt(e)*(896*sqrt(b)*sqrt(a + b*x)*a**2*b**2*d*x**4 - 1280*sqrt(b)*sqrt
(a + b*x)*a*b**3*c*x**4 + 896*sqrt(b)*sqrt(a + b*x)*a*b**3*d*x**5 - 1280*s
qrt(b)*sqrt(a + b*x)*b**4*c*x**5 - 15*sqrt(x)*a**5*c - 21*sqrt(x)*a**5*d*x
 + 30*sqrt(x)*a**4*b*c*x + 56*sqrt(x)*a**4*b*d*x**2 - 80*sqrt(x)*a**3*b**2
*c*x**2 - 336*sqrt(x)*a**3*b**2*d*x**3 + 480*sqrt(x)*a**2*b**3*c*x**3 - 13
44*sqrt(x)*a**2*b**3*d*x**4 + 1920*sqrt(x)*a*b**4*c*x**4 - 896*sqrt(x)*a*b
**4*d*x**5 + 1280*sqrt(x)*b**5*c*x**5))/(105*sqrt(a + b*x)*a**6*x**4*(a +
b*x))
```

3.355 $\int \frac{c+dx}{\sqrt{ex}\left(ax^2+bx^3\right)^{5/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 280

$$\begin{aligned}\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{5/2}}\,dx &= -\frac{2ce}{9a(ex)^{3/2}\,(ax^2+bx^3)^{3/2}} \\ &- \frac{2(4bc-3ad)}{9a^2\sqrt{ex}\,(ax^2+bx^3)^{3/2}} - \frac{20(4bc-3ad)e^2}{9a^3(ex)^{5/2}\sqrt{ax^2+bx^3}} \\ &+ \frac{160(4bc-3ad)e^4\sqrt{ax^2+bx^3}}{63a^4(ex)^{9/2}} - \frac{64b(4bc-3ad)e^3\sqrt{ax^2+bx^3}}{21a^5(ex)^{7/2}} \\ &+ \frac{256b^2(4bc-3ad)e^2\sqrt{ax^2+bx^3}}{63a^6(ex)^{5/2}} - \frac{512b^3(4bc-3ad)e\sqrt{ax^2+bx^3}}{63a^7(ex)^{3/2}}\end{aligned}$$

output
```
-2/9*c*e/a/(e*x)^(3/2)/(b*x^3+a*x^2)^(3/2)-2/9*(-3*a*d+4*b*c)/a^2/(e*x)^(1
/2)/(b*x^3+a*x^2)^(3/2)-20/9*(-3*a*d+4*b*c)*e^2/a^3/(e*x)^(5/2)/(b*x^3+a*x
^2)^(1/2)+160/63*(-3*a*d+4*b*c)*e^4*(b*x^3+a*x^2)^(1/2)/a^4/(e*x)^(9/2)-64
/21*b*(-3*a*d+4*b*c)*e^3*(b*x^3+a*x^2)^(1/2)/a^5/(e*x)^(7/2)+256/63*b^2*(-
3*a*d+4*b*c)*e^2*(b*x^3+a*x^2)^(1/2)/a^6/(e*x)^(5/2)-512/63*b^3*(-3*a*d+4*
b*c)*e*(b*x^3+a*x^2)^(1/2)/a^7/(e*x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.48

$$\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{5/2}}\,dx = -\frac{2e(1024b^6cx^6+384a^2b^4x^4(c-3dx)-768ab^5x^5(-2c+dx)+24a^4b^2x^2(c+2dx)-6a^5bx(2c+3dx)-3}{63a^7(ex)^{3/2}\,(x^2(a+bx))^{3/2}}$$

input
```
Integrate[(c + d*x)/(Sqrt[e*x]*(a*x^2 + b*x^3)^(5/2)),x]
```

output
```
(-2*e*(1024*b^6*c*x^6 + 384*a^2*b^4*x^4*(c - 3*d*x) - 768*a*b^5*x^5*(-2*c
+ d*x) + 24*a^4*b^2*x^2*(c + 2*d*x) - 6*a^5*b*x*(2*c + 3*d*x) - 32*a^3*b^3
*x^3*(2*c + 9*d*x) + a^6*(7*c + 9*d*x)))/(63*a^7*(e*x)^(3/2)*(x^2*(a + b*x
))^(3/2))
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1944, 1921, 1921, 1922, 1922, 1922, 1920}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{5/2}}\,dx$$

$$\Big\downarrow \text{1944}$$

$$-\frac{(4bc-3ad)\int \frac{\sqrt{ex}}{(bx^3+ax^2)^{5/2}}dx}{3ae}-\frac{2ce}{9a(ex)^{3/2}\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1921}$$

$$-\frac{(4bc-3ad)\left(\frac{10e^2\int \frac{1}{(ex)^{3/2}\left(bx^3+ax^2\right)^{3/2}}dx}{3a}+\frac{2e}{3a\sqrt{ex}(ax^2+bx^3)^{3/2}}\right)}{3ae}-\frac{2ce}{9a(ex)^{3/2}\,(ax^2+bx^3)^{3/2}}$$

$$\Big\downarrow \text{1921}$$

$$-\frac{(4bc-3ad)\left(\frac{10e^2\left(\frac{8e^2\int \frac{1}{(ex)^{7/2}\sqrt{bx^3+ax^2}}dx}{a}+\frac{2e}{a(ex)^{5/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2e}{3a\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}\right)}{3ae}-\frac{2ce}{9a(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{(4bc-3ad)\left(\frac{10e^2\left(\frac{8e^2\left(-\frac{6b\int \frac{1}{(ex)^{5/2}\sqrt{bx^3+ax^2}}dx}{7ae}-\frac{2e\sqrt{ax^2+bx^3}}{7a(ex)^{9/2}}\right)}{a}+\frac{2e}{a(ex)^{5/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2e}{3a\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}\right)}{3ae}-\frac{2ce}{9a(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}$$

$$\Big\downarrow \text{1922}$$

$$-\frac{(4bc-3ad)\left(\frac{10e^2\left(\frac{8e^2\left(-\frac{6b\left(-\frac{4b\int \frac{1}{(ex)^{3/2}\sqrt{bx^3+ax^2}}dx}{5ae}-\frac{2e\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}}\right)}{7ae}-\frac{2e\sqrt{ax^2+bx^3}}{7a(ex)^{9/2}}\right)}{a}+\frac{2e}{a(ex)^{5/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2e}{3a\sqrt{ex}\left(ax^2+bx^3\right)^{3/2}}\right)}{3ae}-\frac{2ce}{9a(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}$$

$\downarrow$ 1922

$$-\frac{(4bc-3ad)\left(\frac{10e^2\left(\frac{8e^2\left(-\frac{6b\left(-\frac{4b\left(-\frac{2b\int\frac{1}{\sqrt{ex}\sqrt{bx^3+ax^2}}dx}{3ae}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{5ae}-\frac{2e\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}}\right)}{7ae}-\frac{2e\sqrt{ax^2+bx^3}}{7a(ex)^{9/2}}\right)}{a}+\frac{2e}{a(ex)^{5/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{}{3a}\cdots\right)}{3ae}$$

$$\frac{2ce}{9a(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}$$

$\downarrow$ 1920

$$-\frac{(4bc-3ad)\left(\frac{10e^2\left(\frac{8e^2\left(-\frac{6b\left(-\frac{4b\left(\frac{4b\sqrt{ax^2+bx^3}}{3a^2(ex)^{3/2}}-\frac{2e\sqrt{ax^2+bx^3}}{3a(ex)^{5/2}}\right)}{5ae}-\frac{2e\sqrt{ax^2+bx^3}}{5a(ex)^{7/2}}\right)}{7ae}-\frac{2e\sqrt{ax^2+bx^3}}{7a(ex)^{9/2}}\right)}{a}+\frac{2e}{a(ex)^{5/2}\sqrt{ax^2+bx^3}}\right)}{3a}+\frac{2e}{3a\sqrt{ex}(ax^2\cdots}\right.}{3ae}$$
$$\frac{2ce}{9a(ex)^{3/2}\left(ax^2+bx^3\right)^{3/2}}$$

input
```
Int[(c + d*x)/(Sqrt[e*x]*(a*x^2 + b*x^3)^(5/2)),x]
```

output
```
(-2*c*e)/(9*a*(e*x)^(3/2)*(a*x^2 + b*x^3)^(3/2)) - ((4*b*c - 3*a*d)*((2*e)
/(3*a*Sqrt[e*x]*(a*x^2 + b*x^3)^(3/2)) + (10*e^2*((2*e)/(a*(e*x)^(5/2)*Sqr
t[a*x^2 + b*x^3]) + (8*e^2*((-2*e*Sqrt[a*x^2 + b*x^3])/(7*a*(e*x)^(9/2)) -
 (6*b*((-2*e*Sqrt[a*x^2 + b*x^3])/(5*a*(e*x)^(7/2)) - (4*b*((-2*e*Sqrt[a*x
^2 + b*x^3])/(3*a*(e*x)^(5/2)) + (4*b*Sqrt[a*x^2 + b*x^3])/(3*a^2*(e*x)^(3
/2))))/(5*a*e)))/(7*a*e)))/a))/(3*a)))/(3*a*e)
```

Defintions of rubi rules used

rule 1920
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !IntegerQ[p] && NeQ[
n, j] && EqQ[m + n*p + n - j + 1, 0] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1921
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[(-c^(j - 1))*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(n - j
)*(p + 1))), x] + Simp[c^j*((m + n*p + n - j + 1)/(a*(n - j)*(p + 1)))   In
t[(c*x)^(m - j)*(a*x^j + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, j, m, n}
, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)/(
n - j)], 0] && LtQ[p, -1] && (IntegerQ[j] || GtQ[c, 0])
```

rule 1922
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^(j - 1)*(c*x)^(m - j + 1)*((a*x^j + b*x^n)^(p + 1)/(a*(m + j*p
+ 1))), x] - Simp[b*((m + n*p + n - j + 1)/(a*c^(n - j)*(m + j*p + 1)))   I
nt[(c*x)^(m + n - j)*(a*x^j + b*x^n)^p, x], x] /; FreeQ[{a, b, c, j, m, n,
p}, x] &&  !IntegerQ[p] && NeQ[n, j] && ILtQ[Simplify[(m + n*p + n - j + 1)
/(n - j)], 0] && NeQ[m + j*p + 1, 0] && (IntegersQ[j, n] || GtQ[c, 0])
```

rule 1944
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[c*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(a*(m + j*p + 1))), x] + Simp[(a*d*(m + j*p + 1) - b
*c*(m + n + p*(j + n) + 1))/(a*e^n*(m + j*p + 1))   Int[(e*x)^(m + n)*(a*x^
j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, p}, x] && EqQ[jn, j
+ n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && (LtQ[m + j*p, -1
] || (IntegersQ[m - 1/2, p - 1/2] && LtQ[p, 0] && LtQ[m, (-n)*p - 1])) && (
GtQ[e, 0] || IntegersQ[j, n]) && NeQ[m + j*p + 1, 0] && NeQ[m - n + j*p + 1
, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.58

method	result
gosper	$-\frac{2x(bx+a)(-768a\,b^5d\,x^6+1024b^6c\,x^6-1152a^2b^4d\,x^5+1536a\,b^5c\,x^5-288a^3b^3d\,x^4+384a^2b^4c\,x^4+48a^4b^2d\,x^3-64a^3b^3c\,x^3-18a^5b}{63a^7\sqrt{ex}\,(b\,x^3+a\,x^2)^{\frac{5}{2}}}$
default	$-\frac{2x(bx+a)(-768a\,b^5d\,x^6+1024b^6c\,x^6-1152a^2b^4d\,x^5+1536a\,b^5c\,x^5-288a^3b^3d\,x^4+384a^2b^4c\,x^4+48a^4b^2d\,x^3-64a^3b^3c\,x^3-18a^5b}{63a^7\sqrt{ex}\,(b\,x^3+a\,x^2)^{\frac{5}{2}}}$
orering	$-\frac{2x(bx+a)(-768a\,b^5d\,x^6+1024b^6c\,x^6-1152a^2b^4d\,x^5+1536a\,b^5c\,x^5-288a^3b^3d\,x^4+384a^2b^4c\,x^4+48a^4b^2d\,x^3-64a^3b^3c\,x^3-18a^5b}{63a^7\sqrt{ex}\,(b\,x^3+a\,x^2)^{\frac{5}{2}}}$
risch	$-\frac{2(bx+a)(-474x^4a\,b^3d+667x^4b^4c+111a^2b^2d\,x^3-176a\,b^3c\,x^3-36a^3bd\,x^2+69a^2b^2c\,x^2+9a^4dx-26a^3bcx+7c\,a^4)}{63a^7x^3\sqrt{x^2(bx+a)}\,\sqrt{ex}}+\frac{2b^4(14abdx}{3(bx+}$

input
```
int((d*x+c)/(e*x)^(1/2)/(b*x^3+a*x^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-2/63*x*(b*x+a)*(-768*a*b^5*d*x^6+1024*b^6*c*x^6-1152*a^2*b^4*d*x^5+1536*a
*b^5*c*x^5-288*a^3*b^3*d*x^4+384*a^2*b^4*c*x^4+48*a^4*b^2*d*x^3-64*a^3*b^3
*c*x^3-18*a^5*b*d*x^2+24*a^4*b^2*c*x^2+9*a^6*d*x-12*a^5*b*c*x+7*a^6*c)/a^7
/(e*x)^(1/2)/(b*x^3+a*x^2)^(5/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.67

$$\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{5/2}}\,dx =$$

$$-\frac{2\,(7\,a^6c+256\,(4\,b^6c-3\,ab^5d)x^6+384\,(4\,ab^5c-3\,a^2b^4d)x^5+96\,(4\,a^2b^4c-3\,a^3b^3d)x^4-16\,(4\,a^3b^3c-3\,a}{63\,(a^7b^2ex^8+2\,a^8bex^7+a^9ex^6)}$$

input
```
integrate((d*x+c)/(e*x)^(1/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="fricas")
```

output
```
-2/63*(7*a^6*c + 256*(4*b^6*c - 3*a*b^5*d)*x^6 + 384*(4*a*b^5*c - 3*a^2*b^
4*d)*x^5 + 96*(4*a^2*b^4*c - 3*a^3*b^3*d)*x^4 - 16*(4*a^3*b^3*c - 3*a^4*b^
2*d)*x^3 + 6*(4*a^4*b^2*c - 3*a^5*b*d)*x^2 - 3*(4*a^5*b*c - 3*a^6*d)*x)*sq
rt(b*x^3 + a*x^2)*sqrt(e*x)/(a^7*b^2*e*x^8 + 2*a^8*b*e*x^7 + a^9*e*x^6)
```

Sympy [F]

$$\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{5/2}}\,dx = \int \frac{c+dx}{\sqrt{ex}\,(x^2\,(a+bx))^{\frac{5}{2}}}\,dx$$

input
```
integrate((d*x+c)/(e*x)**(1/2)/(b*x**3+a*x**2)**(5/2),x)
```

output
```
Integral((c + d*x)/(sqrt(e*x)*(x**2*(a + b*x))**(5/2)), x)
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{5/2}}\,dx = \int \frac{dx+c}{(bx^3+ax^2)^{\frac{5}{2}}\sqrt{ex}}\,dx$$

input
```
integrate((d*x+c)/(e*x)^(1/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/((b*x^3 + a*x^2)^(5/2)*sqrt(e*x)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 533 vs. 2(238) = 476.

Time = 2.07 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.90

$$\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{5/2}}\,dx = -\frac{2\left(\left(\left((bx+a)\left((bx+a)\left(\frac{(667\,a^{18}b^{14}ce^4-474\,a^{19}b^{13}de^4)(bx+a)}{a^{25}b^4|b|\mathrm{sgn}(x)}-\frac{9\,(316\,a^{19}b^{14}ce^4-223\,a^{20}b^{13}de^4)}{a^{25}b^4|b|\mathrm{sgn}(x)}\right)+\frac{63\,(73\,a^{20}b^{14}ce^4-51\,a^{21}b^{1}}{a^{25}b^4|b|\mathrm{sgn}(x)}\right.\right.\right.\right.}{63\,((bx+a)be-abe)^{\frac{9}{2}}} - \frac{4\left(17\sqrt{be}a^2b^8ce^2-14\sqrt{be}a^3b^7de^2+36\sqrt{be}\left(\sqrt{be}\sqrt{bx+a}-\sqrt{(bx+a)be-abe}\right)^2ab^7ce-30\sqrt{be}\left(\sqrt{be}\right.\right.}{3\left(abe+\left(\right.\right.}$$

input
```
integrate((d*x+c)/(e*x)^(1/2)/(b*x^3+a*x^2)^(5/2),x, algorithm="giac")
```

output
```
-2/63*(((b*x + a)*((b*x + a)*((667*a^18*b^14*c*e^4 - 474*a^19*b^13*d*e^4)*
(b*x + a)/(a^25*b^4*abs(b)*sgn(x)) - 9*(316*a^19*b^14*c*e^4 - 223*a^20*b^1
3*d*e^4)/(a^25*b^4*abs(b)*sgn(x))) + 63*(73*a^20*b^14*c*e^4 - 51*a^21*b^13
*d*e^4)/(a^25*b^4*abs(b)*sgn(x))) - 210*(16*a^21*b^14*c*e^4 - 11*a^22*b^13
*d*e^4)/(a^25*b^4*abs(b)*sgn(x)))*(b*x + a) + 315*(3*a^22*b^14*c*e^4 - 2*a
^23*b^13*d*e^4)/(a^25*b^4*abs(b)*sgn(x)))*sqrt(b*x + a)/((b*x + a)*b*e - a
*b*e)^(9/2) - 4/3*(17*sqrt(b*e)*a^2*b^8*c*e^2 - 14*sqrt(b*e)*a^3*b^7*d*e^2
 + 36*sqrt(b*e)*(sqrt(b*e)*sqrt(b*x + a) - sqrt((b*x + a)*b*e - a*b*e))^2*
a*b^7*c*e - 30*sqrt(b*e)*(sqrt(b*e)*sqrt(b*x + a) - sqrt((b*x + a)*b*e - a
*b*e))^2*a^2*b^6*d*e + 15*sqrt(b*e)*(sqrt(b*e)*sqrt(b*x + a) - sqrt((b*x +
 a)*b*e - a*b*e))^4*b^6*c - 12*sqrt(b*e)*(sqrt(b*e)*sqrt(b*x + a) - sqrt((
b*x + a)*b*e - a*b*e))^4*a*b^5*d)/((a*b*e + (sqrt(b*e)*sqrt(b*x + a) - sqr
t((b*x + a)*b*e - a*b*e))^2)^3*a^6*abs(b)*sgn(x))
```

Mupad [B] (verification not implemented)

Time = 9.37 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.65

$$\int \frac{c+dx}{\sqrt{ex}\left(ax^2+bx^3\right)^{5/2}}\,dx = \frac{\sqrt{b\,x^3+a\,x^2}\,\left(\frac{64\,b\,x^4\,(3\,a\,d-4\,b\,c)}{21\,a^5}-\frac{32\,x^3\,(3\,a\,d-4\,b\,c)}{63\,a^4}-\frac{2\,c}{9\,a\,b^2}-\frac{x\,(18\,a^6\,d-24\,a^5\,b\,c)}{63\,a^7\,b^2}+\frac{4\,x^2\,(\cdots}{2\cdots}\right.}{x^7\,\sqrt{e\,x}+\frac{a^2\,x^5\,\sqrt{e\,x}}{b^2}+\frac{2\,a\,x^6\,\sqrt{e\,x}}{b}\cdots}$$

input
```
int((c + d*x)/((e*x)^(1/2)*(a*x^2 + b*x^3)^(5/2)),x)
```

output
```
((a*x^2 + b*x^3)^(1/2)*((64*b*x^4*(3*a*d - 4*b*c))/(21*a^5) - (32*x^3*(3*a
*d - 4*b*c))/(63*a^4) - (2*c)/(9*a*b^2) - (x*(18*a^6*d - 24*a^5*b*c))/(63*
a^7*b^2) + (4*x^2*(3*a*d - 4*b*c))/(21*a^3*b) + (256*b^2*x^5*(3*a*d - 4*b*
c))/(21*a^6) + (512*b^3*x^6*(3*a*d - 4*b*c))/(63*a^7)))/(x^7*(e*x)^(1/2) +
 (a^2*x^5*(e*x)^(1/2))/b^2 + (2*a*x^6*(e*x)^(1/2))/b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.93

$$\int \frac{c+dx}{\sqrt{ex}\,(ax^2+bx^3)^{5/2}}\,dx = \frac{2\sqrt{e}\left(-768\sqrt{b}\,\sqrt{bx+a}\,a^2b^3d\,x^5+1024\sqrt{b}\,\sqrt{bx+a}\,a\,b^4c\,x^5-768\sqrt{b}\,\sqrt{bx+a}\right.}{}$$

input
```
int((d*x+c)/(e*x)^(1/2)/(b*x^3+a*x^2)^(5/2),x)
```

output
```
(2*sqrt(e)*( - 768*sqrt(b)*sqrt(a + b*x)*a**2*b**3*d*x**5 + 1024*sqrt(b)*s
qrt(a + b*x)*a*b**4*c*x**5 - 768*sqrt(b)*sqrt(a + b*x)*a*b**4*d*x**6 + 102
4*sqrt(b)*sqrt(a + b*x)*b**5*c*x**6 - 7*sqrt(x)*a**6*c - 9*sqrt(x)*a**6*d*
x + 12*sqrt(x)*a**5*b*c*x + 18*sqrt(x)*a**5*b*d*x**2 - 24*sqrt(x)*a**4*b**
2*c*x**2 - 48*sqrt(x)*a**4*b**2*d*x**3 + 64*sqrt(x)*a**3*b**3*c*x**3 + 288
*sqrt(x)*a**3*b**3*d*x**4 - 384*sqrt(x)*a**2*b**4*c*x**4 + 1152*sqrt(x)*a*
*2*b**4*d*x**5 - 1536*sqrt(x)*a*b**5*c*x**5 + 768*sqrt(x)*a*b**5*d*x**6 -
1024*sqrt(x)*b**6*c*x**6))/(63*sqrt(a + b*x)*a**7*e*x**5*(a + b*x))
```

3.356 $\int (ex)^m(c+dx) \left(ax^2+bx^3\right)^3 \, dx$

Optimal result

Integrand size = 24, antiderivative size = 121

$$\int (ex)^m(c+dx) \left(ax^2+bx^3\right)^3 \, dx = \frac{a^3c(ex)^{7+m}}{e^7(7+m)}+\frac{a^2(3bc+ad)(ex)^{8+m}}{e^8(8+m)} +\frac{3ab(bc+ad)(ex)^{9+m}}{e^9(9+m)} +\frac{b^2(bc+3ad)(ex)^{10+m}}{e^{10}(10+m)}+\frac{b^3d(ex)^{11+m}}{e^{11}(11+m)}$$

output
```
a^3*c*(e*x)^(7+m)/e^7/(7+m)+a^2*(a*d+3*b*c)*(e*x)^(8+m)/e^8/(8+m)+3*a*b*(a
*d+b*c)*(e*x)^(9+m)/e^9/(9+m)+b^2*(3*a*d+b*c)*(e*x)^(10+m)/e^10/(10+m)+b^3
*d*(e*x)^(11+m)/e^11/(11+m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.74

$$\int (ex)^m(c+dx) \left(ax^2+bx^3\right)^3 \, dx = \frac{x^7(ex)^m \left(d(a+bx)^4+(-ad(7+m)+bc(11+m)) \left(\frac{a^3}{7+m}+\frac{3a^2bx}{8+m}+\frac{3ab^2x^2}{9+m}+\frac{b^3x^3}{10+m}\right)\right)}{b(11+m)}$$

input
```
Integrate[(e*x)^m*(c + d*x)*(a*x^2 + b*x^3)^3,x]
```

output
```
(x^7*(e*x)^m*(d*(a + b*x)^4 + (-(a*d*(7 + m)) + b*c*(11 + m))*(a^3/(7 + m)
 + (3*a^2*b*x)/(8 + m) + (3*a*b^2*x^2)/(9 + m) + (b^3*x^3)/(10 + m))))/(b*
(11 + m))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^3 (c+dx)(ex)^m \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\frac{\int (ex)^{m+6}(a+bx)^3(c+dx)dx}{e^6}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\int \left(a^3c(ex)^{m+6}+\frac{a^2(3bc+ad)(ex)^{m+7}}{e}+\frac{3ab(bc+ad)(ex)^{m+8}}{e^2}+\frac{b^2(bc+3ad)(ex)^{m+9}}{e^3}+\frac{b^3d(ex)^{m+10}}{e^4}\right)dx}{e^6}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^3c(ex)^{m+7}}{e(m+7)}+\frac{a^2(ex)^{m+8}(ad+3bc)}{e^2(m+8)}+\frac{b^2(ex)^{m+10}(3ad+bc)}{e^4(m+10)}+\frac{3ab(ex)^{m+9}(ad+bc)}{e^3(m+9)}+\frac{b^3d(ex)^{m+11}}{e^5(m+11)}}{e^6}$$

input
```
Int[(e*x)^m*(c + d*x)*(a*x^2 + b*x^3)^3,x]
```

output
```
((a^3*c*(e*x)^(7 + m))/(e*(7 + m)) + (a^2*(3*b*c + a*d)*(e*x)^(8 + m))/(e^
2*(8 + m)) + (3*a*b*(b*c + a*d)*(e*x)^(9 + m))/(e^3*(9 + m)) + (b^2*(b*c +
 3*a*d)*(e*x)^(10 + m))/(e^4*(10 + m)) + (b^3*d*(e*x)^(11 + m))/(e^5*(11 +
 m)))/e^6
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(121) = 242$.

Time = 0.40 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.78

method	result
gosper	$\frac{(ex)^m (b^3 d m^4 x^4+3a b^2 d m^4 x^3+b^3 c m^4 x^3+34 b^3 d m^3 x^4+3a^2 b d m^4 x^2+3a b^2 c m^4 x^2+105a b^2 d m^3 x^3+35 b^3 c m^3 x^3+431 b^3 d m}{}$
risch	$\frac{(ex)^m (b^3 d m^4 x^4+3a b^2 d m^4 x^3+b^3 c m^4 x^3+34 b^3 d m^3 x^4+3a^2 b d m^4 x^2+3a b^2 c m^4 x^2+105a b^2 d m^3 x^3+35 b^3 c m^3 x^3+431 b^3 d m}{}$
orering	$\frac{(b^3 d m^4 x^4+3a b^2 d m^4 x^3+b^3 c m^4 x^3+34 b^3 d m^3 x^4+3a^2 b d m^4 x^2+3a b^2 c m^4 x^2+105a b^2 d m^3 x^3+35 b^3 c m^3 x^3+431 b^3 d m^2 x^4+}{}$
parallelrisch	$\frac{108x^9 (ex)^m a^2 b d m^3+108x^9 (ex)^m a b^2 c m^3+3x^8 (ex)^m a^2 b c m^4+7815x^{10} (ex)^m a b^2 d m+1443x^9 (ex)^m a^2 b d m^2+1443x^9 (ex)^}{}$

input
```
int((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^3,x,method=_RETURNVERBOSE)
```

output
```
(e*x)^m*(b^3*d*m^4*x^4+3*a*b^2*d*m^4*x^3+b^3*c*m^4*x^3+34*b^3*d*m^3*x^4+3*
a^2*b*d*m^4*x^2+3*a*b^2*c*m^4*x^2+105*a*b^2*d*m^3*x^3+35*b^3*c*m^3*x^3+431
*b^3*d*m^2*x^4+a^3*d*m^4*x+3*a^2*b*c*m^4*x+108*a^2*b*d*m^3*x^2+108*a*b^2*c
*m^3*x^2+1365*a*b^2*d*m^2*x^3+455*b^3*c*m^2*x^3+2414*b^3*d*m*x^4+a^3*c*m^4
+37*a^3*d*m^3*x+111*a^2*b*c*m^3*x+1443*a^2*b*d*m^2*x^2+1443*a*b^2*c*m^2*x^
2+7815*a*b^2*d*m*x^3+2605*b^3*c*m*x^3+5040*b^3*d*x^4+38*a^3*c*m^3+509*a^3*
d*m^2*x+1527*a^2*b*c*m^2*x+8478*a^2*b*d*m*x^2+8478*a*b^2*c*m*x^2+16632*a*b
^2*d*x^3+5544*b^3*c*x^3+539*a^3*c*m^2+3083*a^3*d*m*x+9249*a^2*b*c*m*x+1848
0*a^2*b*d*x^2+18480*a*b^2*c*x^2+3382*a^3*c*m+6930*a^3*d*x+20790*a^2*b*c*x+
7920*a^3*c)*x^7/(11+m)/(10+m)/(9+m)/(8+m)/(7+m)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(121) = 242$.

Time = 0.10 (sec) , antiderivative size = 383, normalized size of antiderivative = 3.17

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^3\,dx = \frac{((b^3dm^4+34\,b^3dm^3+431\,b^3dm^2+2414\,b^3dm+5040\,b^3d)x^{11}+((b^3c+3\,ab^2d)m^4+5544\,b^3c+16632\,a}{}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^3,x, algorithm="fricas")
```

output
```
((b^3*d*m^4 + 34*b^3*d*m^3 + 431*b^3*d*m^2 + 2414*b^3*d*m + 5040*b^3*d)*x^
11 + ((b^3*c + 3*a*b^2*d)*m^4 + 5544*b^3*c + 16632*a*b^2*d + 35*(b^3*c + 3
*a*b^2*d)*m^3 + 455*(b^3*c + 3*a*b^2*d)*m^2 + 2605*(b^3*c + 3*a*b^2*d)*m)*
x^10 + 3*((a*b^2*c + a^2*b*d)*m^4 + 6160*a*b^2*c + 6160*a^2*b*d + 36*(a*b^
2*c + a^2*b*d)*m^3 + 481*(a*b^2*c + a^2*b*d)*m^2 + 2826*(a*b^2*c + a^2*b*d
)*m)*x^9 + ((3*a^2*b*c + a^3*d)*m^4 + 20790*a^2*b*c + 6930*a^3*d + 37*(3*a
^2*b*c + a^3*d)*m^3 + 509*(3*a^2*b*c + a^3*d)*m^2 + 3083*(3*a^2*b*c + a^3*
d)*m)*x^8 + (a^3*c*m^4 + 38*a^3*c*m^3 + 539*a^3*c*m^2 + 3382*a^3*c*m + 792
0*a^3*c)*x^7)*(e*x)^m/(m^5 + 45*m^4 + 805*m^3 + 7155*m^2 + 31594*m + 55440
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2111 vs. $2(112) = 224$.

Time = 0.70 (sec) , antiderivative size = 2111, normalized size of antiderivative = 17.45

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^3\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(d*x+c)*(b*x**3+a*x**2)**3,x)
```

output
```
Piecewise(((-a**3*c/(4*x**4) - a**3*d/(3*x**3) - a**2*b*c/x**3 - 3*a**2*b*
d/(2*x**2) - 3*a*b**2*c/(2*x**2) - 3*a*b**2*d/x - b**3*c/x + b**3*d*log(x)
)/e**11, Eq(m, -11)), ((-a**3*c/(3*x**3) - a**3*d/(2*x**2) - 3*a**2*b*c/(2
*x**2) - 3*a**2*b*d/x - 3*a*b**2*c/x + 3*a*b**2*d*log(x) + b**3*c*log(x) +
 b**3*d*x)/e**10, Eq(m, -10)), ((-a**3*c/(2*x**2) - a**3*d/x - 3*a**2*b*c/
x + 3*a**2*b*d*log(x) + 3*a*b**2*c*log(x) + 3*a*b**2*d*x + b**3*c*x + b**3
*d*x**2/2)/e**9, Eq(m, -9)), ((-a**3*c/x + a**3*d*log(x) + 3*a**2*b*c*log(
x) + 3*a**2*b*d*x + 3*a*b**2*c*x + 3*a*b**2*d*x**2/2 + b**3*c*x**2/2 + b**
3*d*x**3/3)/e**8, Eq(m, -8)), ((a**3*c*log(x) + a**3*d*x + 3*a**2*b*c*x +
3*a**2*b*d*x**2/2 + 3*a*b**2*c*x**2/2 + a*b**2*d*x**3 + b**3*c*x**3/3 + b*
*3*d*x**4/4)/e**7, Eq(m, -7)), (a**3*c*m**4*x**7*(e*x)**m/(m**5 + 45*m**4
+ 805*m**3 + 7155*m**2 + 31594*m + 55440) + 38*a**3*c*m**3*x**7*(e*x)**m/(
m**5 + 45*m**4 + 805*m**3 + 7155*m**2 + 31594*m + 55440) + 539*a**3*c*m**2
*x**7*(e*x)**m/(m**5 + 45*m**4 + 805*m**3 + 7155*m**2 + 31594*m + 55440) +
 3382*a**3*c*m*x**7*(e*x)**m/(m**5 + 45*m**4 + 805*m**3 + 7155*m**2 + 3159
4*m + 55440) + 7920*a**3*c*x**7*(e*x)**m/(m**5 + 45*m**4 + 805*m**3 + 7155
*m**2 + 31594*m + 55440) + a**3*d*m**4*x**8*(e*x)**m/(m**5 + 45*m**4 + 805
*m**3 + 7155*m**2 + 31594*m + 55440) + 37*a**3*d*m**3*x**8*(e*x)**m/(m**5
+ 45*m**4 + 805*m**3 + 7155*m**2 + 31594*m + 55440) + 509*a**3*d*m**2*x**8
*(e*x)**m/(m**5 + 45*m**4 + 805*m**3 + 7155*m**2 + 31594*m + 55440) + 3...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.33

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^3\,dx=\frac{b^3de^mx^{11}x^m}{m+11}+\frac{b^3ce^mx^{10}x^m}{m+10}+\frac{3\,ab^2de^mx^{10}x^m}{m+10}+\frac{3\,ab^2ce^mx^9x^m}{m+9}+\frac{3\,a^2bde^mx^9x^m}{m+9}+\frac{3\,a^2bce^mx^8x^m}{m+8}+\frac{a^3de^mx^8x^m}{m+8}+\frac{a^3ce^mx^7x^m}{m+7}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^3,x, algorithm="maxima")
```

output
```
b^3*d*e^m*x^11*x^m/(m + 11) + b^3*c*e^m*x^10*x^m/(m + 10) + 3*a*b^2*d*e^m*
x^10*x^m/(m + 10) + 3*a*b^2*c*e^m*x^9*x^m/(m + 9) + 3*a^2*b*d*e^m*x^9*x^m/
(m + 9) + 3*a^2*b*c*e^m*x^8*x^m/(m + 8) + a^3*d*e^m*x^8*x^m/(m + 8) + a^3*
c*e^m*x^7*x^m/(m + 7)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(121) = 242.

Time = 0.39 (sec) , antiderivative size = 683, normalized size of antiderivative = 5.64

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^3\,dx=\text{Too large to display}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^3,x, algorithm="giac")
```

output
```
((e*x)^m*b^3*d*m^4*x^11 + (e*x)^m*b^3*c*m^4*x^10 + 3*(e*x)^m*a*b^2*d*m^4*x
^10 + 34*(e*x)^m*b^3*d*m^3*x^11 + 3*(e*x)^m*a*b^2*c*m^4*x^9 + 3*(e*x)^m*a^
2*b*d*m^4*x^9 + 35*(e*x)^m*b^3*c*m^3*x^10 + 105*(e*x)^m*a*b^2*d*m^3*x^10 +
 431*(e*x)^m*b^3*d*m^2*x^11 + 3*(e*x)^m*a^2*b*c*m^4*x^8 + (e*x)^m*a^3*d*m^
4*x^8 + 108*(e*x)^m*a*b^2*c*m^3*x^9 + 108*(e*x)^m*a^2*b*d*m^3*x^9 + 455*(e
*x)^m*b^3*c*m^2*x^10 + 1365*(e*x)^m*a*b^2*d*m^2*x^10 + 2414*(e*x)^m*b^3*d*
m*x^11 + (e*x)^m*a^3*c*m^4*x^7 + 111*(e*x)^m*a^2*b*c*m^3*x^8 + 37*(e*x)^m*
a^3*d*m^3*x^8 + 1443*(e*x)^m*a*b^2*c*m^2*x^9 + 1443*(e*x)^m*a^2*b*d*m^2*x^
9 + 2605*(e*x)^m*b^3*c*m*x^10 + 7815*(e*x)^m*a*b^2*d*m*x^10 + 5040*(e*x)^m
*b^3*d*x^11 + 38*(e*x)^m*a^3*c*m^3*x^7 + 1527*(e*x)^m*a^2*b*c*m^2*x^8 + 50
9*(e*x)^m*a^3*d*m^2*x^8 + 8478*(e*x)^m*a*b^2*c*m*x^9 + 8478*(e*x)^m*a^2*b*
d*m*x^9 + 5544*(e*x)^m*b^3*c*x^10 + 16632*(e*x)^m*a*b^2*d*x^10 + 539*(e*x)
^m*a^3*c*m^2*x^7 + 9249*(e*x)^m*a^2*b*c*m*x^8 + 3083*(e*x)^m*a^3*d*m*x^8 +
 18480*(e*x)^m*a*b^2*c*x^9 + 18480*(e*x)^m*a^2*b*d*x^9 + 3382*(e*x)^m*a^3*
c*m*x^7 + 20790*(e*x)^m*a^2*b*c*x^8 + 6930*(e*x)^m*a^3*d*x^8 + 7920*(e*x)^
m*a^3*c*x^7)/(m^5 + 45*m^4 + 805*m^3 + 7155*m^2 + 31594*m + 55440)
```

Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.33

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^3\,dx = (e\,x)^m\left(\frac{a^3\,c\,x^7\,\left(m^4+38\,m^3+539\,m^2+3382\,m+7920\right)}{m^5+45\,m^4+805\,m^3+7155\,m^2+31594\,m+55440}+\frac{b^3\,d\,x^{11}\,\left(m^4+34\,m^3+431\,m^2+2414\,m+5040\right)}{m^5+45\,m^4+805\,m^3+7155\,m^2+31594\,m+55440}+\frac{a^2\,x^8\,\left(a\,d+3\,b\,c\right)\,\left(m^4+37\,m^3+509\,m^2+3083\,m+6930\right)}{m^5+45\,m^4+805\,m^3+7155\,m^2+31594\,m+55440}+\frac{b^2\,x^{10}\,\left(3\,a\,d+b\,c\right)\,\left(m^4+35\,m^3+455\,m^2+2605\,m+5544\right)}{m^5+45\,m^4+805\,m^3+7155\,m^2+31594\,m+55440}+\frac{3\,a\,b\,x^9\,\left(a\,d+b\,c\right)\,\left(m^4+36\,m^3+481\,m^2+2826\,m+6160\right)}{m^5+45\,m^4+805\,m^3+7155\,m^2+31594\,m+55440}\right)$$

input
```
int((e*x)^m*(a*x^2 + b*x^3)^3*(c + d*x),x)
```

output
```
(e*x)^m*((a^3*c*x^7*(3382*m + 539*m^2 + 38*m^3 + m^4 + 7920))/(31594*m + 7
155*m^2 + 805*m^3 + 45*m^4 + m^5 + 55440) + (b^3*d*x^11*(2414*m + 431*m^2
+ 34*m^3 + m^4 + 5040))/(31594*m + 7155*m^2 + 805*m^3 + 45*m^4 + m^5 + 554
40) + (a^2*x^8*(a*d + 3*b*c)*(3083*m + 509*m^2 + 37*m^3 + m^4 + 6930))/(31
594*m + 7155*m^2 + 805*m^3 + 45*m^4 + m^5 + 55440) + (b^2*x^10*(3*a*d + b*
c)*(2605*m + 455*m^2 + 35*m^3 + m^4 + 5544))/(31594*m + 7155*m^2 + 805*m^3
 + 45*m^4 + m^5 + 55440) + (3*a*b*x^9*(a*d + b*c)*(2826*m + 481*m^2 + 36*m
^3 + m^4 + 6160))/(31594*m + 7155*m^2 + 805*m^3 + 45*m^4 + m^5 + 55440))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.78

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^3\,dx$$
$$=\frac{x^m e^m x^7(b^3d\,m^4x^4+3a\,b^2d\,m^4x^3+b^3c\,m^4x^3+34b^3d\,m^3x^4+3a^2bd\,m^4x^2+3a\,b^2c\,m^4x^2+105a\,b^2d\,m^3x^3}{}$$

input
```
int((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^3,x)
```

output
```
(x**m*e**m*x**7*(a**3*c*m**4 + 38*a**3*c*m**3 + 539*a**3*c*m**2 + 3382*a**
3*c*m + 7920*a**3*c + a**3*d*m**4*x + 37*a**3*d*m**3*x + 509*a**3*d*m**2*x
 + 3083*a**3*d*m*x + 6930*a**3*d*x + 3*a**2*b*c*m**4*x + 111*a**2*b*c*m**3
*x + 1527*a**2*b*c*m**2*x + 9249*a**2*b*c*m*x + 20790*a**2*b*c*x + 3*a**2*
b*d*m**4*x**2 + 108*a**2*b*d*m**3*x**2 + 1443*a**2*b*d*m**2*x**2 + 8478*a*
*2*b*d*m*x**2 + 18480*a**2*b*d*x**2 + 3*a*b**2*c*m**4*x**2 + 108*a*b**2*c*
m**3*x**2 + 1443*a*b**2*c*m**2*x**2 + 8478*a*b**2*c*m*x**2 + 18480*a*b**2*
c*x**2 + 3*a*b**2*d*m**4*x**3 + 105*a*b**2*d*m**3*x**3 + 1365*a*b**2*d*m**
2*x**3 + 7815*a*b**2*d*m*x**3 + 16632*a*b**2*d*x**3 + b**3*c*m**4*x**3 + 3
5*b**3*c*m**3*x**3 + 455*b**3*c*m**2*x**3 + 2605*b**3*c*m*x**3 + 5544*b**3
*c*x**3 + b**3*d*m**4*x**4 + 34*b**3*d*m**3*x**4 + 431*b**3*d*m**2*x**4 +
2414*b**3*d*m*x**4 + 5040*b**3*d*x**4))/(m**5 + 45*m**4 + 805*m**3 + 7155*
m**2 + 31594*m + 55440)
```

3.357 $\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^2\,dx$

Optimal result

Integrand size = 24, antiderivative size = 91

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^2\,dx=\frac{a^2c(ex)^{5+m}}{e^5(5+m)}+\frac{a(2bc+ad)(ex)^{6+m}}{e^6(6+m)}+\frac{b(bc+2ad)(ex)^{7+m}}{e^7(7+m)}+\frac{b^2d(ex)^{8+m}}{e^8(8+m)}$$

output
```
a^2*c*(e*x)^(5+m)/e^5/(5+m)+a*(a*d+2*b*c)*(e*x)^(6+m)/e^6/(6+m)+b*(2*a*d+b
*c)*(e*x)^(7+m)/e^7/(7+m)+b^2*d*(e*x)^(8+m)/e^8/(8+m)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.81

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^2\,dx=\frac{x^5(ex)^m\left(d(a+bx)^3+(-ad(5+m)+bc(8+m))\left(\frac{a^2}{5+m}+\frac{2abx}{6+m}+\frac{b^2x^2}{7+m}\right)\right)}{b(8+m)}$$

input
```
Integrate[(e*x)^m*(c + d*x)*(a*x^2 + b*x^3)^2,x]
```

output
```
(x^5*(e*x)^m*(d*(a + b*x)^3 + (-(a*d*(5 + m)) + b*c*(8 + m))*(a^2/(5 + m)
+ (2*a*b*x)/(6 + m) + (b^2*x^2)/(7 + m))))/(b*(8 + m))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)^2 (c+dx)(ex)^m \, dx$$

$$\Big\downarrow \text{ 9}$$

$$\frac{\int (ex)^{m+4}(a+bx)^2(c+dx)dx}{e^4}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\int \left(a^2c(ex)^{m+4}+\frac{a(2bc+ad)(ex)^{m+5}}{e}+\frac{b(bc+2ad)(ex)^{m+6}}{e^2}+\frac{b^2d(ex)^{m+7}}{e^3}\right)dx}{e^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a^2c(ex)^{m+5}}{e(m+5)}+\frac{b(ex)^{m+7}(2ad+bc)}{e^3(m+7)}+\frac{a(ex)^{m+6}(ad+2bc)}{e^2(m+6)}+\frac{b^2d(ex)^{m+8}}{e^4(m+8)}}{e^4}$$

input
```
Int[(e*x)^m*(c + d*x)*(a*x^2 + b*x^3)^2,x]
```

output
```
((a^2*c*(e*x)^(5 + m))/(e*(5 + m)) + (a*(2*b*c + a*d)*(e*x)^(6 + m))/(e^2*
(6 + m)) + (b*(b*c + 2*a*d)*(e*x)^(7 + m))/(e^3*(7 + m)) + (b^2*d*(e*x)^(8
 + m))/(e^4*(8 + m)))/e^4
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

method	result
norman	$\frac{a(ad+2bc)x^6 e^{m\ln(ex)}}{6+m}+\frac{a^2 c x^5 e^{m\ln(ex)}}{5+m}+\frac{b(2ad+bc)x^7 e^{m\ln(ex)}}{7+m}+\frac{b^2 d x^8 e^{m\ln(ex)}}{8+m}$
gosper	$(ex)^m (b^2 d m^3 x^3+2abd m^3 x^2+b^2 c m^3 x^2+18b^2 d m^2 x^3+a^2 d m^3 x+2abc m^3 x+38abd m^2 x^2+19b^2 c m^2 x^2+107m x^3 b^2 d+a^2 c m$
risch	$(ex)^m (b^2 d m^3 x^3+2abd m^3 x^2+b^2 c m^3 x^2+18b^2 d m^2 x^3+a^2 d m^3 x+2abc m^3 x+38abd m^2 x^2+19b^2 c m^2 x^2+107m x^3 b^2 d+a^2 c m$
orering	$(b^2 d m^3 x^3+2abd m^3 x^2+b^2 c m^3 x^2+18b^2 d m^2 x^3+a^2 d m^3 x+2abc m^3 x+38abd m^2 x^2+19b^2 c m^2 x^2+107m x^3 b^2 d+a^2 c m^3+20$
parallelrisch	$560x^6 (ex)^m abc+146x^5 (ex)^m a^2 cm+131x^6 (ex)^m a^2 dm+21x^5 (ex)^m a^2 c m^2+2x^7 (ex)^m abd m^3+38x^7 (ex)^m abd m^2+2x^6 (ex)$

input
```
int((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^2,x,method=_RETURNVERBOSE)
```

output
```
a*(a*d+2*b*c)/(6+m)*x^6*exp(m*ln(e*x))+a^2*c/(5+m)*x^5*exp(m*ln(e*x))+b*(2
*a*d+b*c)/(7+m)*x^7*exp(m*ln(e*x))+b^2*d/(8+m)*x^8*exp(m*ln(e*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(91) = 182.

Time = 0.10 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.41

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^2\,dx$$
$$=\frac{((b^2dm^3+18\,b^2dm^2+107\,b^2dm+210\,b^2d)x^8+((b^2c+2\,abd)m^3+240\,b^2c+480\,abd+19\,(b^2c+2\,abd)}{}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
((b^2*d*m^3 + 18*b^2*d*m^2 + 107*b^2*d*m + 210*b^2*d)*x^8 + ((b^2*c + 2*a*
b*d)*m^3 + 240*b^2*c + 480*a*b*d + 19*(b^2*c + 2*a*b*d)*m^2 + 118*(b^2*c +
 2*a*b*d)*m)*x^7 + ((2*a*b*c + a^2*d)*m^3 + 560*a*b*c + 280*a^2*d + 20*(2*
a*b*c + a^2*d)*m^2 + 131*(2*a*b*c + a^2*d)*m)*x^6 + (a^2*c*m^3 + 21*a^2*c*
m^2 + 146*a^2*c*m + 336*a^2*c)*x^5)*(e*x)^m/(m^4 + 26*m^3 + 251*m^2 + 1066
*m + 1680)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1081 vs. 2(83) = 166.

Time = 0.47 (sec) , antiderivative size = 1081, normalized size of antiderivative = 11.88

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((e*x)**m*(d*x+c)*(b*x**3+a*x**2)**2,x)
```

output
```
Piecewise(((-a**2*c/(3*x**3) - a**2*d/(2*x**2) - a*b*c/x**2 - 2*a*b*d/x -
b**2*c/x + b**2*d*log(x))/e**8, Eq(m, -8)), ((-a**2*c/(2*x**2) - a**2*d/x
- 2*a*b*c/x + 2*a*b*d*log(x) + b**2*c*log(x) + b**2*d*x)/e**7, Eq(m, -7)),
 ((-a**2*c/x + a**2*d*log(x) + 2*a*b*c*log(x) + 2*a*b*d*x + b**2*c*x + b**
2*d*x**2/2)/e**6, Eq(m, -6)), ((a**2*c*log(x) + a**2*d*x + 2*a*b*c*x + a*b
*d*x**2 + b**2*c*x**2/2 + b**2*d*x**3/3)/e**5, Eq(m, -5)), (a**2*c*m**3*x*
*5*(e*x)**m/(m**4 + 26*m**3 + 251*m**2 + 1066*m + 1680) + 21*a**2*c*m**2*x
**5*(e*x)**m/(m**4 + 26*m**3 + 251*m**2 + 1066*m + 1680) + 146*a**2*c*m*x*
*5*(e*x)**m/(m**4 + 26*m**3 + 251*m**2 + 1066*m + 1680) + 336*a**2*c*x**5*
(e*x)**m/(m**4 + 26*m**3 + 251*m**2 + 1066*m + 1680) + a**2*d*m**3*x**6*(e
*x)**m/(m**4 + 26*m**3 + 251*m**2 + 1066*m + 1680) + 20*a**2*d*m**2*x**6*(
e*x)**m/(m**4 + 26*m**3 + 251*m**2 + 1066*m + 1680) + 131*a**2*d*m*x**6*(e
*x)**m/(m**4 + 26*m**3 + 251*m**2 + 1066*m + 1680) + 280*a**2*d*x**6*(e*x)
**m/(m**4 + 26*m**3 + 251*m**2 + 1066*m + 1680) + 2*a*b*c*m**3*x**6*(e*x)*
*m/(m**4 + 26*m**3 + 251*m**2 + 1066*m + 1680) + 40*a*b*c*m**2*x**6*(e*x)*
*m/(m**4 + 26*m**3 + 251*m**2 + 1066*m + 1680) + 262*a*b*c*m*x**6*(e*x)**m
/(m**4 + 26*m**3 + 251*m**2 + 1066*m + 1680) + 560*a*b*c*x**6*(e*x)**m/(m*
*4 + 26*m**3 + 251*m**2 + 1066*m + 1680) + 2*a*b*d*m**3*x**7*(e*x)**m/(m**
4 + 26*m**3 + 251*m**2 + 1066*m + 1680) + 38*a*b*d*m**2*x**7*(e*x)**m/(m**
4 + 26*m**3 + 251*m**2 + 1066*m + 1680) + 236*a*b*d*m*x**7*(e*x)**m/(m*...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.26

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^2\,dx = \frac{b^2de^mx^8x^m}{m+8}+\frac{b^2ce^mx^7x^m}{m+7}+\frac{2\,abde^mx^7x^m}{m+7}+\frac{2\,abce^mx^6x^m}{m+6}+\frac{a^2de^mx^6x^m}{m+6}+\frac{a^2ce^mx^5x^m}{m+5}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
b^2*d*e^m*x^8*x^m/(m + 8) + b^2*c*e^m*x^7*x^m/(m + 7) + 2*a*b*d*e^m*x^7*x^
m/(m + 7) + 2*a*b*c*e^m*x^6*x^m/(m + 6) + a^2*d*e^m*x^6*x^m/(m + 6) + a^2*
c*e^m*x^5*x^m/(m + 5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(91) = 182$.

Time = 0.29 (sec) , antiderivative size = 388, normalized size of antiderivative = 4.26

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^2\,dx = \frac{(ex)^m b^2dm^3x^8 + (ex)^m b^2cm^3x^7 + 2(ex)^m abdm^3x^7 + 18(ex)^m b^2dm^2x^8 + 2(ex)^m abcm^3x^6 + (ex)^m a^2c}{}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
((e*x)^m*b^2*d*m^3*x^8 + (e*x)^m*b^2*c*m^3*x^7 + 2*(e*x)^m*a*b*d*m^3*x^7 +
 18*(e*x)^m*b^2*d*m^2*x^8 + 2*(e*x)^m*a*b*c*m^3*x^6 + (e*x)^m*a^2*d*m^3*x^
6 + 19*(e*x)^m*b^2*c*m^2*x^7 + 38*(e*x)^m*a*b*d*m^2*x^7 + 107*(e*x)^m*b^2*
d*m*x^8 + (e*x)^m*a^2*c*m^3*x^5 + 40*(e*x)^m*a*b*c*m^2*x^6 + 20*(e*x)^m*a^
2*d*m^2*x^6 + 118*(e*x)^m*b^2*c*m*x^7 + 236*(e*x)^m*a*b*d*m*x^7 + 210*(e*x
)^m*b^2*d*x^8 + 21*(e*x)^m*a^2*c*m^2*x^5 + 262*(e*x)^m*a*b*c*m*x^6 + 131*(
e*x)^m*a^2*d*m*x^6 + 240*(e*x)^m*b^2*c*x^7 + 480*(e*x)^m*a*b*d*x^7 + 146*(
e*x)^m*a^2*c*m*x^5 + 560*(e*x)^m*a*b*c*x^6 + 280*(e*x)^m*a^2*d*x^6 + 336*(
e*x)^m*a^2*c*x^5)/(m^4 + 26*m^3 + 251*m^2 + 1066*m + 1680)
```

Mupad [B] (verification not implemented)

Time = 8.94 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.99

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^2\,dx = (e\,x)^m\left(\frac{a\,x^6\,(a\,d+2\,b\,c)\,\left(m^3+20\,m^2+131\,m+280\right)}{m^4+26\,m^3+251\,m^2+1066\,m+1680} + \frac{b\,x^7\,(2\,a\,d+b\,c)\,\left(m^3+19\,m^2+118\,m+240\right)}{m^4+26\,m^3+251\,m^2+1066\,m+1680} + \frac{a^2\,c\,x^5\,\left(m^3+21\,m^2+146\,m+336\right)}{m^4+26\,m^3+251\,m^2+1066\,m+1680} + \frac{b^2\,d\,x^8\,\left(m^3+18\,m^2+107\,m+210\right)}{m^4+26\,m^3+251\,m^2+1066\,m+1680}\right)$$

input
```
int((e*x)^m*(a*x^2 + b*x^3)^2*(c + d*x),x)
```

output
```
(e*x)^m*((a*x^6*(a*d + 2*b*c)*(131*m + 20*m^2 + m^3 + 280))/(1066*m + 251*
m^2 + 26*m^3 + m^4 + 1680) + (b*x^7*(2*a*d + b*c)*(118*m + 19*m^2 + m^3 +
240))/(1066*m + 251*m^2 + 26*m^3 + m^4 + 1680) + (a^2*c*x^5*(146*m + 21*m^
2 + m^3 + 336))/(1066*m + 251*m^2 + 26*m^3 + m^4 + 1680) + (b^2*d*x^8*(107
*m + 18*m^2 + m^3 + 210))/(1066*m + 251*m^2 + 26*m^3 + m^4 + 1680))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.74

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^2\,dx$$
$$= \frac{x^m e^m x^5 (b^2 d\, m^3 x^3 + 2abd\, m^3 x^2 + b^2 c\, m^3 x^2 + 18b^2 d\, m^2 x^3 + a^2 d\, m^3 x + 2abc\, m^3 x + 38abd\, m^2 x^2 + 19b^2 c\,}{}$$

input
```
int((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^2,x)
```

output
```
(x**m*e**m*x**5*(a**2*c*m**3 + 21*a**2*c*m**2 + 146*a**2*c*m + 336*a**2*c
+ a**2*d*m**3*x + 20*a**2*d*m**2*x + 131*a**2*d*m*x + 280*a**2*d*x + 2*a*b
*c*m**3*x + 40*a*b*c*m**2*x + 262*a*b*c*m*x + 560*a*b*c*x + 2*a*b*d*m**3*x
**2 + 38*a*b*d*m**2*x**2 + 236*a*b*d*m*x**2 + 480*a*b*d*x**2 + b**2*c*m**3
*x**2 + 19*b**2*c*m**2*x**2 + 118*b**2*c*m*x**2 + 240*b**2*c*x**2 + b**2*d
*m**3*x**3 + 18*b**2*d*m**2*x**3 + 107*b**2*d*m*x**3 + 210*b**2*d*x**3))/(
m**4 + 26*m**3 + 251*m**2 + 1066*m + 1680)
```

3.358 $\int (ex)^m(c+dx)\left(ax^2+bx^3\right)\,dx$

Optimal result

Integrand size = 22, antiderivative size = 60

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)\,dx=\frac{ac(ex)^{3+m}}{e^3(3+m)}+\frac{(bc+ad)(ex)^{4+m}}{e^4(4+m)}+\frac{bd(ex)^{5+m}}{e^5(5+m)}$$

output
```
a*c*(e*x)^(3+m)/e^3/(3+m)+(a*d+b*c)*(e*x)^(4+m)/e^4/(4+m)+b*d*(e*x)^(5+m)/
e^5/(5+m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)\,dx =\frac{x^3(ex)^m(a(5+m)(c(4+m)+d(3+m)x)+b(3+m)x(c(5+m)+d(4+m)x))}{(3+m)(4+m)(5+m)}$$

input
```
Integrate[(e*x)^m*(c + d*x)*(a*x^2 + b*x^3),x]
```

output
```
(x^3*(e*x)^m*(a*(5 + m)*(c*(4 + m) + d*(3 + m)*x) + b*(3 + m)*x*(c*(5 + m)
 + d*(4 + m)*x)))/((3 + m)*(4 + m)*(5 + m))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.07, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {9, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(ax^2+bx^3\right)(c+dx)(ex)^m\,dx$$

$$\Big\downarrow \text{ 9}$$

$$\frac{\int (ex)^{m+2}(a+bx)(c+dx)dx}{e^2}$$

$$\Big\downarrow \text{ 85}$$

$$\frac{\int \left(ac(ex)^{m+2}+\frac{(bc+ad)(ex)^{m+3}}{e}+\frac{bd(ex)^{m+4}}{e^2}\right)dx}{e^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{(ex)^{m+4}(ad+bc)}{e^2(m+4)}+\frac{ac(ex)^{m+3}}{e(m+3)}+\frac{bd(ex)^{m+5}}{e^3(m+5)}}{e^2}$$

input
```
Int[(e*x)^m*(c + d*x)*(a*x^2 + b*x^3),x]
```

output
```
((a*c*(e*x)^(3 + m))/(e*(3 + m)) + ((b*c + a*d)*(e*x)^(4 + m))/(e^2*(4 + m
)) + (b*d*(e*x)^(5 + m))/(e^3*(5 + m)))/e^2
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02

method	result
norman	$\frac{(ad+bc)x^4 e^{m\ln(ex)}}{4+m}+\frac{ac\,x^3 e^{m\ln(ex)}}{3+m}+\frac{bd\,x^5 e^{m\ln(ex)}}{5+m}$
gosper	$\frac{(ex)^m\left(bd\,m^2x^2+ad\,m^2x+bc\,m^2x+7bdm\,x^2+ac\,m^2+8admx+8bcmx+12bd\,x^2+9acm+15adx+15cbx+20ac\right)x^3}{(5+m)(4+m)(3+m)}$
risch	$\frac{(ex)^m\left(bd\,m^2x^2+ad\,m^2x+bc\,m^2x+7bdm\,x^2+ac\,m^2+8admx+8bcmx+12bd\,x^2+9acm+15adx+15cbx+20ac\right)x^3}{(5+m)(4+m)(3+m)}$
orering	$\frac{\left(bd\,m^2x^2+ad\,m^2x+bc\,m^2x+7bdm\,x^2+ac\,m^2+8admx+8bcmx+12bd\,x^2+9acm+15adx+15cbx+20ac\right)x(ex)^m\left(b\,x^3+a\,x^2\right)}{(5+m)(4+m)(3+m)(bx+a)}$
parallelrisch	$\frac{x^5(ex)^m bd\,m^2+7x^5(ex)^m bdm+x^4(ex)^m ad\,m^2+x^4(ex)^m bc\,m^2+12x^5(ex)^m bd+8x^4(ex)^m adm+8x^4(ex)^m bcm+x^3(ex)^m ac}{(5+m)(4+m)(3+m)}$

input
```
int((e*x)^m*(d*x+c)*(b*x^3+a*x^2),x,method=_RETURNVERBOSE)
```

output
```
(a*d+b*c)/(4+m)*x^4*exp(m*ln(e*x))+a*c/(3+m)*x^3*exp(m*ln(e*x))+b*d/(5+m)*
x^5*exp(m*ln(e*x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.60

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)\,dx$$
$$=\frac{((bdm^2+7\,bdm+12\,bd)x^5+((bc+ad)m^2+15\,bc+15\,ad+8\,(bc+ad)m)x^4+(acm^2+9\,acm+20\,ac}{m^3+12\,m^2+47\,m+60}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
((b*d*m^2 + 7*b*d*m + 12*b*d)*x^5 + ((b*c + a*d)*m^2 + 15*b*c + 15*a*d + 8
*(b*c + a*d)*m)*x^4 + (a*c*m^2 + 9*a*c*m + 20*a*c)*x^3)*(e*x)^m/(m^3 + 12*
m^2 + 47*m + 60)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 425 vs. 2(53) = 106.

Time = 0.33 (sec) , antiderivative size = 425, normalized size of antiderivative = 7.08

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)\,dx$$
$$=\begin{cases}\frac{-\frac{ac}{2x^2}-\frac{ad}{x}-\frac{bc}{x}+bd\log(x)}{e^5}\\ \frac{-\frac{ac}{x}+ad\log(x)+bc\log(x)+bdx}{e^4}\\ \frac{ac\log(x)+adx+bcx+\frac{bdx^2}{2}}{e^3}\\ \frac{acm^2x^3(ex)^m}{m^3+12m^2+47m+60}+\frac{9acmx^3(ex)^m}{m^3+12m^2+47m+60}+\frac{20acx^3(ex)^m}{m^3+12m^2+47m+60}+\frac{adm^2x^4(ex)^m}{m^3+12m^2+47m+60}+\frac{8admx^4(ex)^m}{m^3+12m^2+47m+60}+\frac{15adx^4}{m^3+12m^2}\end{cases}$$

input
```
integrate((e*x)**m*(d*x+c)*(b*x**3+a*x**2),x)
```

output
```
Piecewise(((-a*c/(2*x**2) - a*d/x - b*c/x + b*d*log(x))/e**5, Eq(m, -5)),
((-a*c/x + a*d*log(x) + b*c*log(x) + b*d*x)/e**4, Eq(m, -4)), ((a*c*log(x)
 + a*d*x + b*c*x + b*d*x**2/2)/e**3, Eq(m, -3)), (a*c*m**2*x**3*(e*x)**m/(
m**3 + 12*m**2 + 47*m + 60) + 9*a*c*m*x**3*(e*x)**m/(m**3 + 12*m**2 + 47*m
 + 60) + 20*a*c*x**3*(e*x)**m/(m**3 + 12*m**2 + 47*m + 60) + a*d*m**2*x**4
*(e*x)**m/(m**3 + 12*m**2 + 47*m + 60) + 8*a*d*m*x**4*(e*x)**m/(m**3 + 12*
m**2 + 47*m + 60) + 15*a*d*x**4*(e*x)**m/(m**3 + 12*m**2 + 47*m + 60) + b*
c*m**2*x**4*(e*x)**m/(m**3 + 12*m**2 + 47*m + 60) + 8*b*c*m*x**4*(e*x)**m/
(m**3 + 12*m**2 + 47*m + 60) + 15*b*c*x**4*(e*x)**m/(m**3 + 12*m**2 + 47*m
 + 60) + b*d*m**2*x**5*(e*x)**m/(m**3 + 12*m**2 + 47*m + 60) + 7*b*d*m*x**
5*(e*x)**m/(m**3 + 12*m**2 + 47*m + 60) + 12*b*d*x**5*(e*x)**m/(m**3 + 12*
m**2 + 47*m + 60), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.15

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)\,dx=\frac{bde^mx^5x^m}{m+5}+\frac{bce^mx^4x^m}{m+4}+\frac{ade^mx^4x^m}{m+4}+\frac{ace^mx^3x^m}{m+3}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
b*d*e^m*x^5*x^m/(m + 5) + b*c*e^m*x^4*x^m/(m + 4) + a*d*e^m*x^4*x^m/(m + 4
) + a*c*e^m*x^3*x^m/(m + 3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(60) = 120$.

Time = 0.20 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.88

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)\,dx$$
$$=\frac{(ex)^m\,bdm^2x^5+(ex)^m\,bcm^2x^4+(ex)^m\,adm^2x^4+7\,(ex)^m\,bdmx^5+(ex)^m\,acm^2x^3+8\,(ex)^m\,bcmx^4+8\,(}{m^3+12\,m^2+4}$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
((e*x)^m*b*d*m^2*x^5 + (e*x)^m*b*c*m^2*x^4 + (e*x)^m*a*d*m^2*x^4 + 7*(e*x)
^m*b*d*m*x^5 + (e*x)^m*a*c*m^2*x^3 + 8*(e*x)^m*b*c*m*x^4 + 8*(e*x)^m*a*d*m
*x^4 + 12*(e*x)^m*b*d*x^5 + 9*(e*x)^m*a*c*m*x^3 + 15*(e*x)^m*b*c*x^4 + 15*
(e*x)^m*a*d*x^4 + 20*(e*x)^m*a*c*x^3)/(m^3 + 12*m^2 + 47*m + 60)
```

Mupad [B] (verification not implemented)

Time = 8.84 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.65

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)\,dx = (e\,x)^m\left(\frac{x^4\,(a\,d+b\,c)\,(m^2+8\,m+15)}{m^3+12\,m^2+47\,m+60} + \frac{a\,c\,x^3\,(m^2+9\,m+20)}{m^3+12\,m^2+47\,m+60} + \frac{b\,d\,x^5\,(m^2+7\,m+12)}{m^3+12\,m^2+47\,m+60}\right)$$

input
```
int((e*x)^m*(a*x^2 + b*x^3)*(c + d*x),x)
```

output
```
(e*x)^m*((x^4*(a*d + b*c)*(8*m + m^2 + 15))/(47*m + 12*m^2 + m^3 + 60) + (
a*c*x^3*(9*m + m^2 + 20))/(47*m + 12*m^2 + m^3 + 60) + (b*d*x^5*(7*m + m^2
 + 12))/(47*m + 12*m^2 + m^3 + 60))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.68

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)\,dx = \frac{x^m e^m x^3(bd\,m^2x^2+ad\,m^2x+bc\,m^2x+7bdm\,x^2+ac\,m^2+8admx+8bcmx+12bd\,x^2+9acm+15adx}{m^3+12m^2+47m+60}$$

input
```
int((e*x)^m*(d*x+c)*(b*x^3+a*x^2),x)
```

output
```
(x**m*e**m*x**3*(a*c*m**2 + 9*a*c*m + 20*a*c + a*d*m**2*x + 8*a*d*m*x + 15
*a*d*x + b*c*m**2*x + 8*b*c*m*x + 15*b*c*x + b*d*m**2*x**2 + 7*b*d*m*x**2
+ 12*b*d*x**2))/(m**3 + 12*m**2 + 47*m + 60)
```

3.359 $\int \frac{(ex)^m(c+dx)}{ax^2+bx^3}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \frac{(ex)^m(c+dx)}{ax^2+bx^3}\,dx = -\frac{de(ex)^{-1+m}}{b(1-m)} - \frac{(bc-ad)e(ex)^{-1+m}\,\text{Hypergeometric2F1}\left(1,-1+m,m,-\frac{bx}{a}\right)}{ab(1-m)}$$

output
```
-d*e*(e*x)^(-1+m)/b/(1-m)-(-a*d+b*c)*e*(e*x)^(-1+m)*hypergeom([1, -1+m],[m
],-b*x/a)/a/b/(1-m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.79

$$\int \frac{(ex)^m(c+dx)}{ax^2+bx^3}\,dx = \frac{(ex)^m\left(acm-(bc-ad)(-1+m)x\,\text{Hypergeometric2F1}\left(1,m,1+m,-\frac{bx}{a}\right)\right)}{a^2(-1+m)mx}$$

input
```
Integrate[((e*x)^m*(c + d*x))/(a*x^2 + b*x^3),x]
```

output
```
((e*x)^m*(a*c*m - (b*c - a*d)*(-1 + m)*x*Hypergeometric2F1[1, m, 1 + m, -(
(b*x)/a)]))/(a^2*(-1 + m)*m*x)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 88, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ex)^m}{ax^2+bx^3}\,dx$$

$$\Big\downarrow \text{9}$$

$$e^2\int \frac{(ex)^{m-2}(c+dx)}{a+bx}dx$$

$$\Big\downarrow \text{88}$$

$$e^2\left(-\frac{(bc-ad)\int \frac{(ex)^{m-1}}{a+bx}dx}{ae}-\frac{c(ex)^{m-1}}{ae(1-m)}\right)$$

$$\Big\downarrow \text{74}$$

$$e^2\left(-\frac{(ex)^m(bc-ad)\,\text{Hypergeometric2F1}\left(1,m,m+1,-\frac{bx}{a}\right)}{a^2e^2m}-\frac{c(ex)^{m-1}}{ae(1-m)}\right)$$

input
```
Int[((e*x)^m*(c + d*x))/(a*x^2 + b*x^3),x]
```

output
```
e^2*(-((c*(e*x)^(-1 + m))/(a*e*(1 - m))) - ((b*c - a*d)*(e*x)^m*Hypergeome
tric2F1[1, m, 1 + m, -((b*x)/a)])/(a^2*e^2*m))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 88
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^Simplify[p + 1],
 x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSimpl
erQ[p, 1]
```

Maple [F]

$$\int \frac{(ex)^m (dx+c)}{b\,x^3+a\,x^2}dx$$

input
```
int((e*x)^m*(d*x+c)/(b*x^3+a*x^2),x)
```

output
```
int((e*x)^m*(d*x+c)/(b*x^3+a*x^2),x)
```

Fricas [F]

$$\int \frac{(ex)^m(c+dx)}{ax^2+bx^3}\,dx = \int \frac{(dx+c)(ex)^m}{bx^3+ax^2}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^3+a*x^2),x, algorithm="fricas")
```

output
```
integral((d*x + c)*(e*x)^m/(b*x^3 + a*x^2), x)
```

Sympy [F]

$$\int \frac{(ex)^m(c+dx)}{ax^2+bx^3}\,dx = \int \frac{(ex)^m\,(c+dx)}{x^2\,(a+bx)}\,dx$$

input
```
integrate((e*x)**m*(d*x+c)/(b*x**3+a*x**2),x)
```

output
```
Integral((e*x)**m*(c + d*x)/(x**2*(a + b*x)), x)
```

Maxima [F]

$$\int \frac{(ex)^m(c+dx)}{ax^2+bx^3}\,dx = \int \frac{(dx+c)(ex)^m}{bx^3+ax^2}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^3+a*x^2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^3 + a*x^2), x)
```

Giac [F]

$$\int \frac{(ex)^m(c+dx)}{ax^2+bx^3}\,dx=\int \frac{(dx+c)(ex)^m}{bx^3+ax^2}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^3+a*x^2),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^3 + a*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(c+dx)}{ax^2+bx^3}\,dx=\int \frac{(e\,x)^m\,(c+d\,x)}{b\,x^3+a\,x^2}\,dx$$

input
```
int(((e*x)^m*(c + d*x))/(a*x^2 + b*x^3),x)
```

output
```
int(((e*x)^m*(c + d*x))/(a*x^2 + b*x^3), x)
```

Reduce [F]

$$\int \frac{(ex)^m(c+dx)}{ax^2+bx^3}\,dx=\frac{e^m\left(x^m d-\left(\int \frac{x^m}{b\,x^3+a\,x^2}dx\right)admx+\left(\int \frac{x^m}{b\,x^3+a\,x^2}dx\right)adx+\left(\int \frac{x^m}{b\,x^3+a\,x^2}dx\right)bcmx-\left(\int \frac{x^m}{b\,x^3+a\,x^2}dx\right)bcx\right)}{bx\,(m-1)}$$

input
```
int((e*x)^m*(d*x+c)/(b*x^3+a*x^2),x)
```

output
```
(e**m*(x**m*d - int(x**m/(a*x**2 + b*x**3),x)*a*d*m*x + int(x**m/(a*x**2 +
 b*x**3),x)*a*d*x + int(x**m/(a*x**2 + b*x**3),x)*b*c*m*x - int(x**m/(a*x*
*2 + b*x**3),x)*b*c*x))/(b*x*(m - 1))
```

3.360 $\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^2}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^2}\,dx = -\frac{de^3(ex)^{-3+m}}{b(4-m)(a+bx)} - \frac{e^3\left(\frac{c}{3-m}-\frac{ad}{4b-bm}\right)(ex)^{-3+m}\,\text{Hypergeometric2F1}\left(2,-3+m,-2+m,-\frac{bx}{a}\right)}{a^2}$$

output

```
-d*e^3*(e*x)^(-3+m)/b/(4-m)/(b*x+a)-e^3*(c/(3-m)-a*d/(-b*m+4*b))*(e*x)^(-3
+m)*hypergeom([2, -3+m],[-2+m],-b*x/a)/a^2
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.81

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^2}\,dx = \frac{(ex)^m\left(\frac{a(bc-ad)}{a+bx}-\frac{(bc(-4+m)-ad(-3+m))\,\text{Hypergeometric2F1}\left(1,-3+m,-2+m,-\frac{bx}{a}\right)}{-3+m}\right)}{a^2bx^3}$$

input
```
Integrate[((e*x)^m*(c + d*x))/(a*x^2 + b*x^3)^2,x]
```

output
```
((e*x)^m*((a*(b*c - a*d))/(a + b*x) - ((b*c*(-4 + m) - a*d*(-3 + m))*Hyper
geometric2F1[1, -3 + m, -2 + m, -((b*x)/a)])/(-3 + m)))/(a^2*b*x^3)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {9, 87, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ex)^m}{(ax^2+bx^3)^2}dx$$

$$\Big\downarrow 9$$

$$e^4\int \frac{(ex)^{m-4}(c+dx)}{(a+bx)^2}dx$$

$$\Big\downarrow 87$$

$$e^4\left(\frac{(ex)^{m-3}(bc-ad)}{abe(a+bx)}-\frac{(ad(3-m)-bc(4-m))\int \frac{(ex)^{m-4}}{a+bx}dx}{ab}\right)$$

$$\Big\downarrow 74$$

$$e^4\left(\frac{(ex)^{m-3}(ad(3-m)-bc(4-m))\,\text{Hypergeometric2F1}\left(1,m-3,m-2,-\frac{bx}{a}\right)}{a^2be(3-m)}+\frac{(ex)^{m-3}(bc-ad)}{abe(a+bx)}\right)$$

input
```
Int[((e*x)^m*(c + d*x))/(a*x^2 + b*x^3)^2,x]
```

output
```
e^4*(((b*c - a*d)*(e*x)^(-3 + m))/(a*b*e*(a + b*x)) + ((a*d*(3 - m) - b*c*
(4 - m))*(e*x)^(-3 + m)*Hypergeometric2F1[1, -3 + m, -2 + m, -((b*x)/a)])/
(a^2*b*e*(3 - m)))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [F]

$$\int \frac{(ex)^m (dx+c)}{(b\,x^3+a\,x^2)^2}dx$$

input
```
int((e*x)^m*(d*x+c)/(b*x^3+a*x^2)^2,x)
```

output
```
int((e*x)^m*(d*x+c)/(b*x^3+a*x^2)^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^2}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^3+ax^2)^2}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^3+a*x^2)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(e*x)^m/(b^2*x^6 + 2*a*b*x^5 + a^2*x^4), x)
```

Sympy [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^2}\,dx = \int \frac{(ex)^m\,(c+dx)}{x^4\,(a+bx)^2}\,dx$$

input
```
integrate((e*x)**m*(d*x+c)/(b*x**3+a*x**2)**2,x)
```

output
```
Integral((e*x)**m*(c + d*x)/(x**4*(a + b*x)**2), x)
```

Maxima [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^2}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^3+ax^2)^2}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^3+a*x^2)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^3 + a*x^2)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^2}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^3+ax^2)^2}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^3+a*x^2)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^3 + a*x^2)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^2}\,dx = \int \frac{(e\,x)^m\,(c+d\,x)}{(b\,x^3+a\,x^2)^2}\,dx$$

input
```
int(((e*x)^m*(c + d*x))/(a*x^2 + b*x^3)^2,x)
```

output
```
int(((e*x)^m*(c + d*x))/(a*x^2 + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^2}\,dx = \frac{e^m\left(x^m d-\left(\int \frac{x^m}{b^2m\,x^6+2abm\,x^5-4b^2x^6+a^2m\,x^4-8ab\,x^5-4a^2x^4}dx\right)a^2d\,m^2x^3+7\left(\int \frac{x^m}{b^2m\,x^6+2abm\,x^5-4b^2x^6+a^2m\,x^4-8ab\,x^5-\right.\right.}{}$$

input
```
int((e*x)^m*(d*x+c)/(b*x^3+a*x^2)^2,x)
```

output

```
(e**m*(x**m*d - int(x**m/(a**2*m*x**4 - 4*a**2*x**4 + 2*a*b*m*x**5 - 8*a*b
*x**5 + b**2*m*x**6 - 4*b**2*x**6),x)*a**2*d*m**2*x**3 + 7*int(x**m/(a**2*
m*x**4 - 4*a**2*x**4 + 2*a*b*m*x**5 - 8*a*b*x**5 + b**2*m*x**6 - 4*b**2*x*
*6),x)*a**2*d*m*x**3 - 12*int(x**m/(a**2*m*x**4 - 4*a**2*x**4 + 2*a*b*m*x*
*5 - 8*a*b*x**5 + b**2*m*x**6 - 4*b**2*x**6),x)*a**2*d*x**3 + int(x**m/(a*
*2*m*x**4 - 4*a**2*x**4 + 2*a*b*m*x**5 - 8*a*b*x**5 + b**2*m*x**6 - 4*b**2
*x**6),x)*a*b*c*m**2*x**3 - 8*int(x**m/(a**2*m*x**4 - 4*a**2*x**4 + 2*a*b*
m*x**5 - 8*a*b*x**5 + b**2*m*x**6 - 4*b**2*x**6),x)*a*b*c*m*x**3 + 16*int(
x**m/(a**2*m*x**4 - 4*a**2*x**4 + 2*a*b*m*x**5 - 8*a*b*x**5 + b**2*m*x**6
- 4*b**2*x**6),x)*a*b*c*x**3 - int(x**m/(a**2*m*x**4 - 4*a**2*x**4 + 2*a*b
*m*x**5 - 8*a*b*x**5 + b**2*m*x**6 - 4*b**2*x**6),x)*a*b*d*m**2*x**4 + 7*i
nt(x**m/(a**2*m*x**4 - 4*a**2*x**4 + 2*a*b*m*x**5 - 8*a*b*x**5 + b**2*m*x*
*6 - 4*b**2*x**6),x)*a*b*d*m*x**4 - 12*int(x**m/(a**2*m*x**4 - 4*a**2*x**4
 + 2*a*b*m*x**5 - 8*a*b*x**5 + b**2*m*x**6 - 4*b**2*x**6),x)*a*b*d*x**4 +
int(x**m/(a**2*m*x**4 - 4*a**2*x**4 + 2*a*b*m*x**5 - 8*a*b*x**5 + b**2*m*x
**6 - 4*b**2*x**6),x)*b**2*c*m**2*x**4 - 8*int(x**m/(a**2*m*x**4 - 4*a**2*
x**4 + 2*a*b*m*x**5 - 8*a*b*x**5 + b**2*m*x**6 - 4*b**2*x**6),x)*b**2*c*m*
x**4 + 16*int(x**m/(a**2*m*x**4 - 4*a**2*x**4 + 2*a*b*m*x**5 - 8*a*b*x**5
+ b**2*m*x**6 - 4*b**2*x**6),x)*b**2*c*x**4))/(b*x**3*(a*m - 4*a + b*m*x -
 4*b*x))
```

3.361 $\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^3} \, dx$

Optimal result

Integrand size = 24, antiderivative size = 85

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^3} \, dx = -\frac{de^5(ex)^{-5+m}}{b(7-m)(a+bx)^2} - \frac{e^5\left(\frac{c}{5-m}-\frac{ad}{7b-bm}\right)(ex)^{-5+m}\,\text{Hypergeometric2F1}\left(3,-5+m,-4+m,-\frac{bx}{a}\right)}{a^3}$$

output
```
-d*e^5*(e*x)^(-5+m)/b/(7-m)/(b*x+a)^2-e^5*(c/(5-m)-a*d/(-b*m+7*b))*(e*x)^(
-5+m)*hypergeom([3, -5+m],[-4+m],-b*x/a)/a^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.87

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^3} \, dx = \frac{(ex)^m\left(\frac{a^2(bc-ad)}{(a+bx)^2}-\frac{(bc(-7+m)-ad(-5+m))\,\text{Hypergeometric2F1}\left(2,-5+m,-4+m,-\frac{bx}{a}\right)}{-5+m}\right)}{2a^3bx^5}$$

input
```
Integrate[((e*x)^m*(c + d*x))/(a*x^2 + b*x^3)^3,x]
```

output
```
((e*x)^m*((a^2*(b*c - a*d))/(a + b*x)^2 - ((b*c*(-7 + m) - a*d*(-5 + m))*H
ypergeometric2F1[2, -5 + m, -4 + m, -((b*x)/a)])/(-5 + m)))/(2*a^3*b*x^5)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {9, 87, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ex)^m}{(ax^2+bx^3)^3}dx$$

$$\Big\downarrow \; 9$$

$$e^6\int \frac{(ex)^{m-6}(c+dx)}{(a+bx)^3}dx$$

$$\Big\downarrow \; 87$$

$$e^6\left(\frac{(ex)^{m-5}(bc-ad)}{2abe(a+bx)^2}-\frac{(ad(5-m)-bc(7-m))\int \frac{(ex)^{m-6}}{(a+bx)^2}dx}{2ab}\right)$$

$$\Big\downarrow \; 74$$

$$e^6\left(\frac{(ex)^{m-5}(ad(5-m)-bc(7-m))\,\text{Hypergeometric2F1}\left(2,m-5,m-4,-\frac{bx}{a}\right)}{2a^3be(5-m)}+\frac{(ex)^{m-5}(bc-ad)}{2abe(a+bx)^2}\right)$$

input
```
Int[((e*x)^m*(c + d*x))/(a*x^2 + b*x^3)^3,x]
```

output
```
e^6*(((b*c - a*d)*(e*x)^(-5 + m))/(2*a*b*e*(a + b*x)^2) + ((a*d*(5 - m) -
b*c*(7 - m))*(e*x)^(-5 + m)*Hypergeometric2F1[2, -5 + m, -4 + m, -((b*x)/a
)])/(2*a^3*b*e*(5 - m)))
```

Defintions of rubi rules used

rule 9
```
Int[(u_.)*(Px_)^(p_.)*((e_.)*(x_))^(m_.), x_Symbol] :> With[{r = Expon[Px,
x, Min]}, Simp[1/e^(p*r)   Int[u*(e*x)^(m + p*r)*ExpandToSum[Px/x^r, x]^p,
x], x] /; IGtQ[r, 0]] /; FreeQ[{e, m}, x] && PolyQ[Px, x] && IntegerQ[p] &&
  !MonomialQ[Px, x]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Maple [F]

$$\int \frac{(ex)^m (dx+c)}{(b\,x^3+a\,x^2)^3}dx$$

input
```
int((e*x)^m*(d*x+c)/(b*x^3+a*x^2)^3,x)
```

output
```
int((e*x)^m*(d*x+c)/(b*x^3+a*x^2)^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^3} \, dx = \int \frac{(dx+c)(ex)^m}{(bx^3+ax^2)^3} \, dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^3+a*x^2)^3,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(e*x)^m/(b^3*x^9 + 3*a*b^2*x^8 + 3*a^2*b*x^7 + a^3*x^6)
, x)
```

Sympy [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^3} \, dx = \int \frac{(ex)^m (c+dx)}{x^6 (a+bx)^3} \, dx$$

input
```
integrate((e*x)**m*(d*x+c)/(b*x**3+a*x**2)**3,x)
```

output
```
Integral((e*x)**m*(c + d*x)/(x**6*(a + b*x)**3), x)
```

Maxima [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^3} \, dx = \int \frac{(dx+c)(ex)^m}{(bx^3+ax^2)^3} \, dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^3+a*x^2)^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^3 + a*x^2)^3, x)
```

Giac [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^3}\,dx=\int \frac{(dx+c)(ex)^m}{(bx^3+ax^2)^3}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(b*x^3+a*x^2)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^3 + a*x^2)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^3}\,dx=\int \frac{(e\,x)^m\,(c+d\,x)}{(b\,x^3+a\,x^2)^3}\,dx$$

input
```
int(((e*x)^m*(c + d*x))/(a*x^2 + b*x^3)^3,x)
```

output
```
int(((e*x)^m*(c + d*x))/(a*x^2 + b*x^3)^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^2+bx^3)^3}\,dx=\text{Too large to display}$$

input
```
int((e*x)^m*(d*x+c)/(b*x^3+a*x^2)^3,x)
```

output

```
(e**m*(x**m*d - int(x**m/(a**3*m*x**6 - 7*a**3*x**6 + 3*a**2*b*m*x**7 - 21
*a**2*b*x**7 + 3*a*b**2*m*x**8 - 21*a*b**2*x**8 + b**3*m*x**9 - 7*b**3*x**
9),x)*a**3*d*m**2*x**5 + 12*int(x**m/(a**3*m*x**6 - 7*a**3*x**6 + 3*a**2*b
*m*x**7 - 21*a**2*b*x**7 + 3*a*b**2*m*x**8 - 21*a*b**2*x**8 + b**3*m*x**9
- 7*b**3*x**9),x)*a**3*d*m*x**5 - 35*int(x**m/(a**3*m*x**6 - 7*a**3*x**6 +
 3*a**2*b*m*x**7 - 21*a**2*b*x**7 + 3*a*b**2*m*x**8 - 21*a*b**2*x**8 + b**
3*m*x**9 - 7*b**3*x**9),x)*a**3*d*x**5 + int(x**m/(a**3*m*x**6 - 7*a**3*x*
*6 + 3*a**2*b*m*x**7 - 21*a**2*b*x**7 + 3*a*b**2*m*x**8 - 21*a*b**2*x**8 +
 b**3*m*x**9 - 7*b**3*x**9),x)*a**2*b*c*m**2*x**5 - 14*int(x**m/(a**3*m*x*
*6 - 7*a**3*x**6 + 3*a**2*b*m*x**7 - 21*a**2*b*x**7 + 3*a*b**2*m*x**8 - 21
*a*b**2*x**8 + b**3*m*x**9 - 7*b**3*x**9),x)*a**2*b*c*m*x**5 + 49*int(x**m
/(a**3*m*x**6 - 7*a**3*x**6 + 3*a**2*b*m*x**7 - 21*a**2*b*x**7 + 3*a*b**2*
m*x**8 - 21*a*b**2*x**8 + b**3*m*x**9 - 7*b**3*x**9),x)*a**2*b*c*x**5 - 2*
int(x**m/(a**3*m*x**6 - 7*a**3*x**6 + 3*a**2*b*m*x**7 - 21*a**2*b*x**7 + 3
*a*b**2*m*x**8 - 21*a*b**2*x**8 + b**3*m*x**9 - 7*b**3*x**9),x)*a**2*b*d*m
**2*x**6 + 24*int(x**m/(a**3*m*x**6 - 7*a**3*x**6 + 3*a**2*b*m*x**7 - 21*a
**2*b*x**7 + 3*a*b**2*m*x**8 - 21*a*b**2*x**8 + b**3*m*x**9 - 7*b**3*x**9)
,x)*a**2*b*d*m*x**6 - 70*int(x**m/(a**3*m*x**6 - 7*a**3*x**6 + 3*a**2*b*m*
x**7 - 21*a**2*b*x**7 + 3*a*b**2*m*x**8 - 21*a*b**2*x**8 + b**3*m*x**9 - 7
*b**3*x**9),x)*a**2*b*d*x**6 + 2*int(x**m/(a**3*m*x**6 - 7*a**3*x**6 + ...
```

3.362 $\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^p\,dx$

Optimal result

Integrand size = 24, antiderivative size = 111

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^p\,dx=\frac{de(ex)^{-1+m}\left(ax^2+bx^3\right)^{1+p}}{b(2+m+3p)}-e\left(\frac{d}{b(2+m+3p)}-\frac{c}{a+am+2ap}\right)(ex)^{-1+m}\left(ax^2+bx^3\right)^{1+p}\text{Hypergeometric2F1}\left(1,2+m+3p,2+m+2p,-\frac{bx}{a}\right)$$

output
```
d*e*(e*x)^(-1+m)*(b*x^3+a*x^2)^(p+1)/b/(2+m+3*p)-e*(d/b/(2+m+3*p)-c/(a*m+2
*a*p+a))*(e*x)^(-1+m)*(b*x^3+a*x^2)^(p+1)*hypergeom([1, 2+m+3*p],[2+2*p+m]
,-b*x/a)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.91

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^p\,dx$$
$$=\frac{x(ex)^m\left(x^2(a+bx)\right)^p\left(d(a+bx)+\frac{(-ad(1+m+2p)+bc(2+m+3p))\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,1+m+2p,2+m+2p,-\frac{bx}{a}\right.}{1+m+2p}\right.}{b(2+m+3p)}$$

input
```
Integrate[(e*x)^m*(c + d*x)*(a*x^2 + b*x^3)^p,x]
```

output
```
(x*(e*x)^m*(x^2*(a + b*x))^p*(d*(a + b*x) + ((-(a*d*(1 + m + 2*p)) + b*c*(
2 + m + 3*p))*Hypergeometric2F1[-p, 1 + m + 2*p, 2 + m + 2*p, -((b*x)/a)])
/((1 + m + 2*p)*(1 + (b*x)/a)^p)))/(b*(2 + m + 3*p))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1945, 1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(ex)^m\left(ax^2+bx^3\right)^p\,dx$$

$$\Big\downarrow \text{1945}$$

$$\left(c-\frac{ad(m+2p+1)}{b(m+3p+2)}\right)\int (ex)^m\left(bx^3+ax^2\right)^p dx+\frac{de(ex)^{m-1}\left(ax^2+bx^3\right)^{p+1}}{b(m+3p+2)}$$

$$\Big\downarrow \text{1938}$$

$$(ex)^m x^{-m-2p}(a+bx)^{-p}\left(ax^2+bx^3\right)^p\left(c-\frac{ad(m+2p+1)}{b(m+3p+2)}\right)\int x^{m+2p}(a+bx)^p dx+\frac{de(ex)^{m-1}\left(ax^2+bx^3\right)^{p+1}}{b(m+3p+2)}$$

$$\Big\downarrow \textbf{76}$$

$$(ex)^m x^{-m-2p}\left(\frac{bx}{a}+1\right)^{-p}(ax^2+bx^3)^p\left(c-\frac{ad(m+2p+1)}{b(m+3p+2)}\right)\int x^{m+2p}\left(\frac{bx}{a}+1\right)^p dx+\frac{de(ex)^{m-1}(ax^2+bx^3)^{p+1}}{b(m+3p+2)}$$

$$\Big\downarrow \textbf{74}$$

$$\frac{x(ex)^m\left(\frac{bx}{a}+1\right)^{-p}(ax^2+bx^3)^p\left(c-\frac{ad(m+2p+1)}{b(m+3p+2)}\right)\text{Hypergeometric2F1}\left(-p,m+2p+1,m+2p+2,-\frac{bx}{a}\right)}{m+2p+1}+\frac{de(ex)^{m-1}(ax^2+bx^3)^{p+1}}{b(m+3p+2)}$$

input
```
Int[(e*x)^m*(c + d*x)*(a*x^2 + b*x^3)^p,x]
```

output
```
(d*e*(e*x)^(-1 + m)*(a*x^2 + b*x^3)^(1 + p))/(b*(2 + m + 3*p)) + ((c - (a*
d*(1 + m + 2*p))/(b*(2 + m + 3*p)))*x*(e*x)^m*(a*x^2 + b*x^3)^p*Hypergeome
tric2F1[-p, 1 + m + 2*p, 2 + m + 2*p, -((b*x)/a)])/((1 + m + 2*p)*(1 + (b*
x)/a)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 && !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] && !Integer
Q[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0] && ((RationalQ[m] && !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) || !RationalQ[n])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [F]

$$\int (ex)^m (dx+c) \left(b\,x^3+a\,x^2\right)^p dx$$

input
```
int((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^p,x)
```

output
```
int((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^p,x)
```

Fricas [F]

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^p\,dx=\int (dx+c)\left(bx^3+ax^2\right)^p(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(b*x^3 + a*x^2)^p*(e*x)^m, x)
```

Sympy [F]

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^p\,dx=\int (ex)^m\left(x^2(a+bx)\right)^p(c+dx)\,dx$$

input
```
integrate((e*x)**m*(d*x+c)*(b*x**3+a*x**2)**p,x)
```

output
```
Integral((e*x)**m*(x**2*(a + b*x))**p*(c + d*x), x)
```

Maxima [F]

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^p\,dx=\int (dx+c)\left(bx^3+ax^2\right)^p(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(b*x^3 + a*x^2)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^p\,dx=\int (dx+c)\left(bx^3+ax^2\right)^p(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*x^3 + a*x^2)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^p\,dx=\int (e\,x)^m\,\left(b\,x^3+a\,x^2\right)^p\,(c+d\,x)\,dx$$

input
```
int((e*x)^m*(a*x^2 + b*x^3)^p*(c + d*x),x)
```

output
```
int((e*x)^m*(a*x^2 + b*x^3)^p*(c + d*x), x)
```

Reduce [F]

$$\int (ex)^m(c+dx)\left(ax^2+bx^3\right)^p\,dx=\text{too large to display}$$

input
```
int((e*x)^m*(d*x+c)*(b*x^3+a*x^2)^p,x)
```

output
```
(e**m*( - x**m*(a*x**2 + b*x**3)**p*a**2*d*m*p - 2*x**m*(a*x**2 + b*x**3)*
*p*a**2*d*p**2 - x**m*(a*x**2 + b*x**3)**p*a**2*d*p + x**m*(a*x**2 + b*x**
3)**p*a*b*c*m*p + 3*x**m*(a*x**2 + b*x**3)**p*a*b*c*p**2 + 2*x**m*(a*x**2
+ b*x**3)**p*a*b*c*p + x**m*(a*x**2 + b*x**3)**p*a*b*d*m*p*x + 3*x**m*(a*x
**2 + b*x**3)**p*a*b*d*p**2*x + x**m*(a*x**2 + b*x**3)**p*b**2*c*m**2*x +
6*x**m*(a*x**2 + b*x**3)**p*b**2*c*m*p*x + 2*x**m*(a*x**2 + b*x**3)**p*b**
2*c*m*x + 9*x**m*(a*x**2 + b*x**3)**p*b**2*c*p**2*x + 6*x**m*(a*x**2 + b*x
**3)**p*b**2*c*p*x + x**m*(a*x**2 + b*x**3)**p*b**2*d*m**2*x**2 + 6*x**m*(
a*x**2 + b*x**3)**p*b**2*d*m*p*x**2 + x**m*(a*x**2 + b*x**3)**p*b**2*d*m*x
**2 + 9*x**m*(a*x**2 + b*x**3)**p*b**2*d*p**2*x**2 + 3*x**m*(a*x**2 + b*x*
*3)**p*b**2*d*p*x**2 + int((x**m*(a*x**2 + b*x**3)**p)/(a*m**3*x + 9*a*m**
2*p*x + 3*a*m**2*x + 27*a*m*p**2*x + 18*a*m*p*x + 2*a*m*x + 27*a*p**3*x +
27*a*p**2*x + 6*a*p*x + b*m**3*x**2 + 9*b*m**2*p*x**2 + 3*b*m**2*x**2 + 27
*b*m*p**2*x**2 + 18*b*m*p*x**2 + 2*b*m*x**2 + 27*b*p**3*x**2 + 27*b*p**2*x
**2 + 6*b*p*x**2),x)*a**3*d*m**5*p + 13*int((x**m*(a*x**2 + b*x**3)**p)/(a
*m**3*x + 9*a*m**2*p*x + 3*a*m**2*x + 27*a*m*p**2*x + 18*a*m*p*x + 2*a*m*x
 + 27*a*p**3*x + 27*a*p**2*x + 6*a*p*x + b*m**3*x**2 + 9*b*m**2*p*x**2 + 3
*b*m**2*x**2 + 27*b*m*p**2*x**2 + 18*b*m*p*x**2 + 2*b*m*x**2 + 27*b*p**3*x
**2 + 27*b*p**2*x**2 + 6*b*p*x**2),x)*a**3*d*m**4*p**2 + 4*int((x**m*(a*x*
*2 + b*x**3)**p)/(a*m**3*x + 9*a*m**2*p*x + 3*a*m**2*x + 27*a*m*p**2*x ...
```

3.363 $\int (ex)^{1+p}(2b+3cx)\left(ax^2+bx^3\right)^p\,dx$

Optimal result

Integrand size = 29, antiderivative size = 106

$$\int (ex)^{1+p}(2b+3cx)\left(ax^2+bx^3\right)^p\,dx=\frac{3ce(ex)^p\left(ax^2+bx^3\right)^{1+p}}{b(3+4p)}-\frac{e\left(\frac{3c}{3+4p}-\frac{2b^2}{2a+3ap}\right)(ex)^p\left(ax^2+bx^3\right)^{1+p}\text{Hypergeometric2F1}\left(1,3+4p,3(1+p),-\frac{bx}{a}\right)}{b}$$

output
```
3*c*e*(e*x)^p*(b*x^3+a*x^2)^(p+1)/b/(3+4*p)-e*(3*c/(3+4*p)-2*b^2/(3*a*p+2*
a))*(e*x)^p*(b*x^3+a*x^2)^(p+1)*hypergeom([1, 3+4*p],[3*p+3],-b*x/a)/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.08

$$\int (ex)^{1+p}(2b+3cx)\left(ax^2+bx^3\right)^p\,dx=\frac{ex^2(ex)^p\left(x^2(a+bx)\right)^p\left(1+\frac{bx}{a}\right)^{-p}\left(3c(2+3p)(a+bx)\left(1+\frac{bx}{a}\right)^p+\left(-3ac(2+3p)+b^2(6+8p)\right)\text{Hyperge}}{b(2+3p)(3+4p)}$$

input
```
Integrate[(e*x)^(1 + p)*(2*b + 3*c*x)*(a*x^2 + b*x^3)^p,x]
```

output
```
(e*x^2*(e*x)^p*(x^2*(a + b*x))^p*(3*c*(2 + 3*p)*(a + b*x)*(1 + (b*x)/a)^p
+ (-3*a*c*(2 + 3*p) + b^2*(6 + 8*p))*Hypergeometric2F1[-p, 2 + 3*p, 3 + 3*
p, -((b*x)/a)]))/(b*(2 + 3*p)*(3 + 4*p)*(1 + (b*x)/a)^p)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1945, 1938, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (2b+3cx)(ex)^{p+1}\left(ax^2+bx^3\right)^p\,dx$$

$$\Big\downarrow \text{1945}$$

$$\left(2b-\frac{3ac(3p+2)}{b(4p+3)}\right)\int (ex)^{p+1}\left(bx^3+ax^2\right)^p dx+\frac{3ce(ex)^p\left(ax^2+bx^3\right)^{p+1}}{b(4p+3)}$$

$$\Big\downarrow \text{1938}$$

$$ex^{-3p}(ex)^p\left(2b-\frac{3ac(3p+2)}{b(4p+3)}\right)(a+bx)^{-p}\left(ax^2+bx^3\right)^p\int x^{3p+1}(a+bx)^p dx+\frac{3ce(ex)^p\left(ax^2+bx^3\right)^{p+1}}{b(4p+3)}$$

$$\Big\downarrow \text{76}$$

$$ex^{-3p}(ex)^p\left(2b-\frac{3ac(3p+2)}{b(4p+3)}\right)\left(\frac{bx}{a}+1\right)^{-p}\left(ax^2+bx^3\right)^p\int x^{3p+1}\left(\frac{bx}{a}+1\right)^p dx+\frac{3ce(ex)^p\left(ax^2+bx^3\right)^{p+1}}{b(4p+3)}$$

$$\Big\downarrow \text{74}$$

$$\frac{ex^2(ex)^p\left(2b-\frac{3ac(3p+2)}{b(4p+3)}\right)\left(ax^2+bx^3\right)^p\left(\frac{bx}{a}+1\right)^{-p}\text{Hypergeometric2F1}\left(-p,3p+2,3(p+1),-\frac{bx}{a}\right)}{3p+2}+\frac{3ce(ex)^p\left(ax^2+bx^3\right)^{p+1}}{b(4p+3)}$$

input
```
Int[(e*x)^(1 + p)*(2*b + 3*c*x)*(a*x^2 + b*x^3)^p,x]
```

output
```
(3*c*e*(e*x)^p*(a*x^2 + b*x^3)^(1 + p))/(b*(3 + 4*p)) + (e*(2*b - (3*a*c*(
2 + 3*p))/(b*(3 + 4*p)))*x^2*(e*x)^p*(a*x^2 + b*x^3)^p*Hypergeometric2F1[-
p, 2 + 3*p, 3*(1 + p), -((b*x)/a)])/((2 + 3*p)*(1 + (b*x)/a)^p)
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 1938
```
Int[((c_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(n_.))^(p_), x_Symbol
] :> Simp[c^IntPart[m]*(c*x)^FracPart[m]*((a*x^j + b*x^n)^FracPart[p]/(x^(F
racPart[m] + j*FracPart[p])*(a + b*x^(n - j))^FracPart[p]))   Int[x^(m + j*
p)*(a + b*x^(n - j))^p, x], x] /; FreeQ[{a, b, c, j, m, n, p}, x] &&  !Inte
gerQ[p] && NeQ[n, j] && PosQ[n - j]
```

rule 1945
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.)), x_Symbol] :> Simp[d*e^(j - 1)*(e*x)^(m - j + 1)*((a*x^j
+ b*x^(j + n))^(p + 1)/(b*(m + n + p*(j + n) + 1))), x] - Simp[(a*d*(m + j*
p + 1) - b*c*(m + n + p*(j + n) + 1))/(b*(m + n + p*(j + n) + 1))   Int[(e*
x)^m*(a*x^j + b*x^(j + n))^p, x], x] /; FreeQ[{a, b, c, d, e, j, m, n, p},
x] && EqQ[jn, j + n] &&  !IntegerQ[p] && NeQ[b*c - a*d, 0] && NeQ[m + n + p
*(j + n) + 1, 0] && (GtQ[e, 0] || IntegerQ[j])
```

Maple [F]

$$\int (ex)^{p+1} (3cx+2b) \left(b\,x^{3}+a\,x^{2}\right)^{p} dx$$

input
```
int((e*x)^(p+1)*(3*c*x+2*b)*(b*x^3+a*x^2)^p,x)
```

output
```
int((e*x)^(p+1)*(3*c*x+2*b)*(b*x^3+a*x^2)^p,x)
```

Fricas [F]

$$\int (ex)^{1+p}(2b+3cx) \left(ax^2+bx^3\right)^p \, dx = \int (3\,cx+2\,b)\left(bx^3+ax^2\right)^p(ex)^{p+1} \, dx$$

input
```
integrate((e*x)^(p+1)*(3*c*x+2*b)*(b*x^3+a*x^2)^p,x, algorithm="fricas")
```

output
```
integral((3*c*x + 2*b)*(b*x^3 + a*x^2)^p*(e*x)^(p + 1), x)
```

Sympy [F]

$$\int (ex)^{1+p}(2b+3cx) \left(ax^2+bx^3\right)^p \, dx = \int (ex)^{p+1} \left(x^2(a+bx)\right)^p (2b+3cx) \, dx$$

input
```
integrate((e*x)**(p+1)*(3*c*x+2*b)*(b*x**3+a*x**2)**p,x)
```

output
```
Integral((e*x)**(p + 1)*(x**2*(a + b*x))**p*(2*b + 3*c*x), x)
```

Maxima [F]

$$\int (ex)^{1+p}(2b+3cx)\left(ax^2+bx^3\right)^p\,dx=\int (3\,cx+2\,b)\left(bx^3+ax^2\right)^p(ex)^{p+1}\,dx$$

input
```
integrate((e*x)^(p+1)*(3*c*x+2*b)*(b*x^3+a*x^2)^p,x, algorithm="maxima")
```

output
```
integrate((3*c*x + 2*b)*(b*x^3 + a*x^2)^p*(e*x)^(p + 1), x)
```

Giac [F]

$$\int (ex)^{1+p}(2b+3cx)\left(ax^2+bx^3\right)^p\,dx=\int (3\,cx+2\,b)\left(bx^3+ax^2\right)^p(ex)^{p+1}\,dx$$

input
```
integrate((e*x)^(p+1)*(3*c*x+2*b)*(b*x^3+a*x^2)^p,x, algorithm="giac")
```

output
```
integrate((3*c*x + 2*b)*(b*x^3 + a*x^2)^p*(e*x)^(p + 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^{1+p}(2b+3cx)\left(ax^2+bx^3\right)^p\,dx=\int (e\,x)^{p+1}\,(2\,b+3\,c\,x)\,\left(b\,x^3+a\,x^2\right)^p\,dx$$

input
```
int((e*x)^(p + 1)*(2*b + 3*c*x)*(a*x^2 + b*x^3)^p,x)
```

output
```
int((e*x)^(p + 1)*(2*b + 3*c*x)*(a*x^2 + b*x^3)^p, x)
```

Reduce [F]

$$\int (ex)^{1+p}(2b+3cx)\left(ax^2+bx^3\right)^p\,dx = \text{Too large to display}$$

input
```
int((e*x)^(p+1)*(3*c*x+2*b)*(b*x^3+a*x^2)^p,x)
```

output
```
(e**p*e*(27*x**p*(a*x**2 + b*x**3)**p*a**3*c*p**2 + 27*x**p*(a*x**2 + b*x*
*3)**p*a**3*c*p + 6*x**p*(a*x**2 + b*x**3)**p*a**3*c - 24*x**p*(a*x**2 + b
*x**3)**p*a**2*b**2*p**2 - 26*x**p*(a*x**2 + b*x**3)**p*a**2*b**2*p - 6*x*
*p*(a*x**2 + b*x**3)**p*a**2*b**2 - 36*x**p*(a*x**2 + b*x**3)**p*a**2*b*c*
p**2*x - 24*x**p*(a*x**2 + b*x**3)**p*a**2*b*c*p*x + 32*x**p*(a*x**2 + b*x
**3)**p*a*b**3*p**2*x + 24*x**p*(a*x**2 + b*x**3)**p*a*b**3*p*x + 48*x**p*
(a*x**2 + b*x**3)**p*a*b**2*c*p**2*x**2 + 12*x**p*(a*x**2 + b*x**3)**p*a*b
**2*c*p*x**2 + 128*x**p*(a*x**2 + b*x**3)**p*b**4*p**2*x**2 + 128*x**p*(a*
x**2 + b*x**3)**p*b**4*p*x**2 + 24*x**p*(a*x**2 + b*x**3)**p*b**4*x**2 + 1
92*x**p*(a*x**2 + b*x**3)**p*b**3*c*p**2*x**3 + 144*x**p*(a*x**2 + b*x**3)
**p*b**3*c*p*x**3 + 24*x**p*(a*x**2 + b*x**3)**p*b**3*c*x**3 - 2592*int((x
**p*(a*x**2 + b*x**3)**p)/(32*a*p**3*x + 48*a*p**2*x + 22*a*p*x + 3*a*x +
32*b*p**3*x**2 + 48*b*p**2*x**2 + 22*b*p*x**2 + 3*b*x**2),x)*a**4*c*p**6 -
 6480*int((x**p*(a*x**2 + b*x**3)**p)/(32*a*p**3*x + 48*a*p**2*x + 22*a*p*
x + 3*a*x + 32*b*p**3*x**2 + 48*b*p**2*x**2 + 22*b*p*x**2 + 3*b*x**2),x)*a
**4*c*p**5 - 6246*int((x**p*(a*x**2 + b*x**3)**p)/(32*a*p**3*x + 48*a*p**2
*x + 22*a*p*x + 3*a*x + 32*b*p**3*x**2 + 48*b*p**2*x**2 + 22*b*p*x**2 + 3*
b*x**2),x)*a**4*c*p**4 - 2889*int((x**p*(a*x**2 + b*x**3)**p)/(32*a*p**3*x
 + 48*a*p**2*x + 22*a*p*x + 3*a*x + 32*b*p**3*x**2 + 48*b*p**2*x**2 + 22*b
*p*x**2 + 3*b*x**2),x)*a**4*c*p**3 - 639*int((x**p*(a*x**2 + b*x**3)**p...
```

3.364 $\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^3\,dx$

Optimal result

Integrand size = 26, antiderivative size = 146

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^3\,dx = \frac{3ab(bc+ad)x^{3(1+n)}(ex)^m}{3+m+3n}+\frac{a^3cx^{1+3n}(ex)^m}{1+m+3n} + \frac{a^2(3bc+ad)x^{2+3n}(ex)^m}{2+m+3n} + \frac{b^2(bc+3ad)x^{4+3n}(ex)^m}{4+m+3n}+\frac{b^3dx^{5+3n}(ex)^m}{5+m+3n}$$

output
```
3*a*b*(a*d+b*c)*x^(3+3*n)*(e*x)^m/(3+m+3*n)+a^3*c*x^(1+3*n)*(e*x)^m/(1+m+3
*n)+a^2*(a*d+3*b*c)*x^(2+3*n)*(e*x)^m/(2+m+3*n)+b^2*(3*a*d+b*c)*x^(4+3*n)*
(e*x)^m/(4+m+3*n)+b^3*d*x^(5+3*n)*(e*x)^m/(5+m+3*n)
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.79

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^3\,dx = \frac{x^{1+3n}(ex)^m\left(d(a+bx)^4+(-ad(1+m+3n)+bc(5+m+3n))\left(\frac{a^3}{1+m+3n}+\frac{3a^2bx}{2+m+3n}+\frac{3ab^2x^2}{3+m+3n}+\frac{b^3x^3}{4+m+3}\right.\right.}{b(5+m+3n)}$$

input
```
Integrate[(e*x)^m*(c + d*x)*(a*x^n + b*x^(1 + n))^3,x]
```

output
```
(x^(1 + 3*n)*(e*x)^m*(d*(a + b*x)^4 + (-(a*d*(1 + m + 3*n)) + b*c*(5 + m +
 3*n))*(a^3/(1 + m + 3*n) + (3*a^2*b*x)/(2 + m + 3*n) + (3*a*b^2*x^2)/(3 +
 m + 3*n) + (b^3*x^3)/(4 + m + 3*n))))/(b*(5 + m + 3*n))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2027, 30, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(ex)^m \left(ax^n+bx^{n+1}\right)^3 dx$$

$$\Big\downarrow \text{2027}$$

$$\int x^{3n}(a+bx)^3(c+dx)(ex)^m dx$$

$$\Big\downarrow \text{30}$$

$$x^{-m}(ex)^m \int x^{m+3n}(a+bx)^3(c+dx)dx$$

$$\Big\downarrow \text{85}$$

$$x^{-m}(ex)^m \int \left(a^3cx^{m+3n}+a^2(3bc+ad)x^{m+3n+1}+3ab(bc+ad)x^{m+3n+2}+b^2(bc+3ad)x^{m+3n+3}+b^3dx^{m+3n+4}\right.$$

$$\Big\downarrow \text{2009}$$

$$x^{-m}(ex)^m \left(\frac{a^3cx^{m+3n+1}}{m+3n+1}+\frac{a^2x^{m+3n+2}(ad+3bc)}{m+3n+2}+\frac{b^2x^{m+3n+4}(3ad+bc)}{m+3n+4}+\frac{3abx^{m+3n+3}(ad+bc)}{m+3n+3}+\frac{b^3dx^{m+3n}}{m+3n+}\right.$$

input
```
Int[(e*x)^m*(c + d*x)*(a*x^n + b*x^(1 + n))^3,x]
```

output
```
((e*x)^m*((a^3*c*x^(1 + m + 3*n))/(1 + m + 3*n) + (a^2*(3*b*c + a*d)*x^(2
+ m + 3*n))/(2 + m + 3*n) + (3*a*b*(b*c + a*d)*x^(3 + m + 3*n))/(3 + m + 3
*n) + (b^2*(b*c + 3*a*d)*x^(4 + m + 3*n))/(4 + m + 3*n) + (b^3*d*x^(5 + m
+ 3*n))/(5 + m + 3*n)))/x^m
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1453 vs. $2(146) = 292$.

Time = 7.01 (sec) , antiderivative size = 1454, normalized size of antiderivative = 9.96

method	result	size
orering	Expression too large to display	1454
risch	Expression too large to display	1478
parallelrisch	Expression too large to display	2971

input
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^3,x,method=_RETURNVERBOSE)
```

output
```
(b^3*d*m^4*x^4+12*b^3*d*m^3*n*x^4+54*b^3*d*m^2*n^2*x^4+108*b^3*d*m*n^3*x^4
+81*b^3*d*n^4*x^4+3*a*b^2*d*m^4*x^3+36*a*b^2*d*m^3*n*x^3+162*a*b^2*d*m^2*n
^2*x^3+324*a*b^2*d*m*n^3*x^3+243*a*b^2*d*n^4*x^3+b^3*c*m^4*x^3+12*b^3*c*m^
3*n*x^3+54*b^3*c*m^2*n^2*x^3+108*b^3*c*m*n^3*x^3+81*b^3*c*n^4*x^3+10*b^3*d
*m^3*x^4+90*b^3*d*m^2*n*x^4+270*b^3*d*m*n^2*x^4+270*b^3*d*n^3*x^4+3*a^2*b*
d*m^4*x^2+36*a^2*b*d*m^3*n*x^2+162*a^2*b*d*m^2*n^2*x^2+324*a^2*b*d*m*n^3*x
^2+243*a^2*b*d*n^4*x^2+3*a*b^2*c*m^4*x^2+36*a*b^2*c*m^3*n*x^2+162*a*b^2*c*
m^2*n^2*x^2+324*a*b^2*c*m*n^3*x^2+243*a*b^2*c*n^4*x^2+33*a*b^2*d*m^3*x^3+2
97*a*b^2*d*m^2*n*x^3+891*a*b^2*d*m*n^2*x^3+891*a*b^2*d*n^3*x^3+11*b^3*c*m^
3*x^3+99*b^3*c*m^2*n*x^3+297*b^3*c*m*n^2*x^3+297*b^3*c*n^3*x^3+35*b^3*d*m^
2*x^4+210*b^3*d*m*n*x^4+315*b^3*d*n^2*x^4+a^3*d*m^4*x+12*a^3*d*m^3*n*x+54*
a^3*d*m^2*n^2*x+108*a^3*d*m*n^3*x+81*a^3*d*n^4*x+3*a^2*b*c*m^4*x+36*a^2*b*
c*m^3*n*x+162*a^2*b*c*m^2*n^2*x+324*a^2*b*c*m*n^3*x+243*a^2*b*c*n^4*x+36*a
^2*b*d*m^3*x^2+324*a^2*b*d*m^2*n*x^2+972*a^2*b*d*m*n^2*x^2+972*a^2*b*d*n^3
*x^2+36*a*b^2*c*m^3*x^2+324*a*b^2*c*m^2*n*x^2+972*a*b^2*c*m*n^2*x^2+972*a*
b^2*c*n^3*x^2+123*a*b^2*d*m^2*x^3+738*a*b^2*d*m*n*x^3+1107*a*b^2*d*n^2*x^3
+41*b^3*c*m^2*x^3+246*b^3*c*m*n*x^3+369*b^3*c*n^2*x^3+50*b^3*d*m*x^4+150*b
^3*d*n*x^4+a^3*c*m^4+12*a^3*c*m^3*n+54*a^3*c*m^2*n^2+108*a^3*c*m*n^3+81*a^
3*c*n^4+13*a^3*d*m^3*x+117*a^3*d*m^2*n*x+351*a^3*d*m*n^2*x+351*a^3*d*n^3*x
+39*a^2*b*c*m^3*x+351*a^2*b*c*m^2*n*x+1053*a^2*b*c*m*n^2*x+1053*a^2*b*c...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1171 vs. $2(146) = 292$.

Time = 0.14 (sec) , antiderivative size = 1171, normalized size of antiderivative = 8.02

$$\int (ex)^m(c+dx) \left(ax^n+bx^{1+n}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^3,x, algorithm="fricas")
```

output
```
(a^3*c*m^4 + 81*a^3*c*n^4 + 14*a^3*c*m^3 + 71*a^3*c*m^2 + 154*a^3*c*m + (b
^3*d*m^4 + 81*b^3*d*n^4 + 10*b^3*d*m^3 + 35*b^3*d*m^2 + 50*b^3*d*m + 24*b^
3*d + 54*(2*b^3*d*m + 5*b^3*d)*n^3 + 9*(6*b^3*d*m^2 + 30*b^3*d*m + 35*b^3*
d)*n^2 + 6*(2*b^3*d*m^3 + 15*b^3*d*m^2 + 35*b^3*d*m + 25*b^3*d)*n)*x^4 + 1
20*a^3*c + 54*(2*a^3*c*m + 7*a^3*c)*n^3 + ((b^3*c + 3*a*b^2*d)*m^4 + 81*(b
^3*c + 3*a*b^2*d)*n^4 + 30*b^3*c + 90*a*b^2*d + 11*(b^3*c + 3*a*b^2*d)*m^3
 + 27*(11*b^3*c + 33*a*b^2*d + 4*(b^3*c + 3*a*b^2*d)*m)*n^3 + 41*(b^3*c +
3*a*b^2*d)*m^2 + 9*(41*b^3*c + 123*a*b^2*d + 6*(b^3*c + 3*a*b^2*d)*m^2 + 3
3*(b^3*c + 3*a*b^2*d)*m)*n^2 + 61*(b^3*c + 3*a*b^2*d)*m + 3*(61*b^3*c + 18
3*a*b^2*d + 4*(b^3*c + 3*a*b^2*d)*m^3 + 33*(b^3*c + 3*a*b^2*d)*m^2 + 82*(b
^3*c + 3*a*b^2*d)*m)*n)*x^3 + 9*(6*a^3*c*m^2 + 42*a^3*c*m + 71*a^3*c)*n^2
+ 3*((a*b^2*c + a^2*b*d)*m^4 + 81*(a*b^2*c + a^2*b*d)*n^4 + 40*a*b^2*c + 4
0*a^2*b*d + 12*(a*b^2*c + a^2*b*d)*m^3 + 108*(3*a*b^2*c + 3*a^2*b*d + (a*b
^2*c + a^2*b*d)*m)*n^3 + 49*(a*b^2*c + a^2*b*d)*m^2 + 9*(49*a*b^2*c + 49*a
^2*b*d + 6*(a*b^2*c + a^2*b*d)*m^2 + 36*(a*b^2*c + a^2*b*d)*m)*n^2 + 78*(a
*b^2*c + a^2*b*d)*m + 6*(39*a*b^2*c + 39*a^2*b*d + 2*(a*b^2*c + a^2*b*d)*m
^3 + 18*(a*b^2*c + a^2*b*d)*m^2 + 49*(a*b^2*c + a^2*b*d)*m)*n)*x^2 + 6*(2*
a^3*c*m^3 + 21*a^3*c*m^2 + 71*a^3*c*m + 77*a^3*c)*n + ((3*a^2*b*c + a^3*d)
*m^4 + 81*(3*a^2*b*c + a^3*d)*n^4 + 180*a^2*b*c + 60*a^3*d + 13*(3*a^2*b*c
 + a^3*d)*m^3 + 27*(39*a^2*b*c + 13*a^3*d + 4*(3*a^2*b*c + a^3*d)*m)*n^...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 18686 vs. $2(138) = 276$.

Time = 94.52 (sec) , antiderivative size = 18686, normalized size of antiderivative = 127.99

$$\int (ex)^m (c + dx) \left(ax^n + bx^{1+n}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(d*x+c)*(a*x**n+b*x**(1+n))**3,x)
```

output
```
Piecewise((-a**3*c*x*x**(3*n)*(e*x)**(-3*n - 5)/4 - a**3*d*x**2*x**(3*n)*(
e*x)**(-3*n - 5)/3 - a**2*b*c*x*x**(2*n)*x**(n + 1)*(e*x)**(-3*n - 5) - 3*
a**2*b*d*x**2*x**(2*n)*x**(n + 1)*(e*x)**(-3*n - 5)/2 - 3*a*b**2*c*x*x**n*
x**(2*n + 2)*(e*x)**(-3*n - 5)/2 - 3*a*b**2*d*x**2*x**n*x**(2*n + 2)*(e*x)
**(-3*n - 5) - b**3*c*x*x**(3*n + 3)*(e*x)**(-3*n - 5) + b**3*d*x**2*x**(3
*n + 3)*(e*x)**(-3*n - 5)*log(x), Eq(m, -3*n - 5)), (-a**3*c*x*x**(3*n)*(e
*x)**(-3*n - 4)/3 - a**3*d*x**2*x**(3*n)*(e*x)**(-3*n - 4)/2 - 3*a**2*b*c*
x*x**(2*n)*x**(n + 1)*(e*x)**(-3*n - 4)/2 - 3*a**2*b*d*x**2*x**(2*n)*x**(n
 + 1)*(e*x)**(-3*n - 4) - 3*a*b**2*c*x*x**n*x**(2*n + 2)*(e*x)**(-3*n - 4)
 + 3*a*b**2*d*x**2*x**n*x**(2*n + 2)*(e*x)**(-3*n - 4)*log(x) + b**3*c*x*x
**(3*n + 3)*(e*x)**(-3*n - 4)*log(x) + b**3*d*x**2*x**(3*n + 3)*(e*x)**(-3
*n - 4), Eq(m, -3*n - 4)), (-a**3*c*x*x**(3*n)*(e*x)**(-3*n - 3)/2 - a**3*
d*x**2*x**(3*n)*(e*x)**(-3*n - 3) - 3*a**2*b*c*x*x**(2*n)*x**(n + 1)*(e*x)
**(-3*n - 3) + 3*a**2*b*d*x**2*x**(2*n)*x**(n + 1)*(e*x)**(-3*n - 3)*log(x
) + 3*a*b**2*c*x*x**n*x**(2*n + 2)*(e*x)**(-3*n - 3)*log(x) + 3*a*b**2*d*x
**2*x**n*x**(2*n + 2)*(e*x)**(-3*n - 3) + b**3*c*x*x**(3*n + 3)*(e*x)**(-3
*n - 3) + b**3*d*x**2*x**(3*n + 3)*(e*x)**(-3*n - 3)/2, Eq(m, -3*n - 3)),
(-a**3*c*x*x**(3*n)*(e*x)**(-3*n - 2) + a**3*d*x**2*x**(3*n)*(e*x)**(-3*n
- 2)*log(x) + 3*a**2*b*c*x*x**(2*n)*x**(n + 1)*(e*x)**(-3*n - 2)*log(x) +
3*a**2*b*d*x**2*x**(2*n)*x**(n + 1)*(e*x)**(-3*n - 2) + 3*a*b**2*c*x*x*...
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.69

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^3\,dx = \frac{b^3de^mx^5e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+5} + \frac{b^3ce^mx^4e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+4} + \frac{3\,ab^2de^mx^4e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+4} + \frac{3\,ab^2ce^mx^3e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+3} + \frac{3\,a^2bde^mx^3e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+3} + \frac{3\,a^2bce^mx^2e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+2} + \frac{a^3de^mx^2e^{(m\log(x)+3\,n\log(x))}}{m+3\,n+2} + \frac{a^3ce^mxe^{(m\log(x)+3\,n\log(x))}}{m+3\,n+1}$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^3,x, algorithm="maxima")
```

output
```
b^3*d*e^m*x^5*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 5) + b^3*c*e^m*x^4*e^(m
*log(x) + 3*n*log(x))/(m + 3*n + 4) + 3*a*b^2*d*e^m*x^4*e^(m*log(x) + 3*n*
log(x))/(m + 3*n + 4) + 3*a*b^2*c*e^m*x^3*e^(m*log(x) + 3*n*log(x))/(m + 3
*n + 3) + 3*a^2*b*d*e^m*x^3*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 3) + 3*a^
2*b*c*e^m*x^2*e^(m*log(x) + 3*n*log(x))/(m + 3*n + 2) + a^3*d*e^m*x^2*e^(m
*log(x) + 3*n*log(x))/(m + 3*n + 2) + a^3*c*e^m*x*e^(m*log(x) + 3*n*log(x)
)/(m + 3*n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3372 vs. $2(146) = 292$.

Time = 0.36 (sec) , antiderivative size = 3372, normalized size of antiderivative = 23.10

$$\int (ex)^m (c + dx) \left(ax^n + bx^{1+n}\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^3,x, algorithm="giac")
```

output
```
(b^3*d*m^4*x^5*x^(3*n)*e^(m*log(e) + m*log(x)) + 12*b^3*d*m^3*n*x^5*x^(3*n
)*e^(m*log(e) + m*log(x)) + 54*b^3*d*m^2*n^2*x^5*x^(3*n)*e^(m*log(e) + m*l
og(x)) + 108*b^3*d*m*n^3*x^5*x^(3*n)*e^(m*log(e) + m*log(x)) + 81*b^3*d*n^
4*x^5*x^(3*n)*e^(m*log(e) + m*log(x)) + b^3*c*m^4*x^4*x^(3*n)*e^(m*log(e)
+ m*log(x)) + 3*a*b^2*d*m^4*x^4*x^(3*n)*e^(m*log(e) + m*log(x)) + 12*b^3*c
*m^3*n*x^4*x^(3*n)*e^(m*log(e) + m*log(x)) + 36*a*b^2*d*m^3*n*x^4*x^(3*n)*
e^(m*log(e) + m*log(x)) + 54*b^3*c*m^2*n^2*x^4*x^(3*n)*e^(m*log(e) + m*log
(x)) + 162*a*b^2*d*m^2*n^2*x^4*x^(3*n)*e^(m*log(e) + m*log(x)) + 108*b^3*c
*m*n^3*x^4*x^(3*n)*e^(m*log(e) + m*log(x)) + 324*a*b^2*d*m*n^3*x^4*x^(3*n)
*e^(m*log(e) + m*log(x)) + 81*b^3*c*n^4*x^4*x^(3*n)*e^(m*log(e) + m*log(x)
) + 243*a*b^2*d*n^4*x^4*x^(3*n)*e^(m*log(e) + m*log(x)) + 10*b^3*d*m^3*x^5
*x^(3*n)*e^(m*log(e) + m*log(x)) + 90*b^3*d*m^2*n*x^5*x^(3*n)*e^(m*log(e)
+ m*log(x)) + 270*b^3*d*m*n^2*x^5*x^(3*n)*e^(m*log(e) + m*log(x)) + 270*b^
3*d*n^3*x^5*x^(3*n)*e^(m*log(e) + m*log(x)) + 3*a*b^2*c*m^4*x^3*x^(3*n)*e^
(m*log(e) + m*log(x)) + 3*a^2*b*d*m^4*x^3*x^(3*n)*e^(m*log(e) + m*log(x))
+ 36*a*b^2*c*m^3*n*x^3*x^(3*n)*e^(m*log(e) + m*log(x)) + 36*a^2*b*d*m^3*n*
x^3*x^(3*n)*e^(m*log(e) + m*log(x)) + 162*a*b^2*c*m^2*n^2*x^3*x^(3*n)*e^(m
*log(e) + m*log(x)) + 162*a^2*b*d*m^2*n^2*x^3*x^(3*n)*e^(m*log(e) + m*log(
x)) + 324*a*b^2*c*m*n^3*x^3*x^(3*n)*e^(m*log(e) + m*log(x)) + 324*a^2*b*d*
m*n^3*x^3*x^(3*n)*e^(m*log(e) + m*log(x)) + 243*a*b^2*c*n^4*x^3*x^(3*n)...
```

Mupad [B] (verification not implemented)

Time = 9.75 (sec) , antiderivative size = 1641, normalized size of antiderivative = 11.24

$$\int (ex)^m(c+dx) \left(ax^n+bx^{1+n}\right)^3 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(a*x^n + b*x^(n + 1))^3*(c + d*x),x)
```

output
```
(a^3*c*x*x^(3*n)*(e*x)^m*(154*m + 462*n + 426*m*n + 378*m*n^2 + 126*m^2*n
+ 108*m*n^3 + 12*m^3*n + 71*m^2 + 14*m^3 + m^4 + 639*n^2 + 378*n^3 + 81*n^
4 + 54*m^2*n^2 + 120))/(274*m + 822*n + 1350*m*n + 2295*m*n^2 + 765*m^2*n
+ 1620*m*n^3 + 180*m^3*n + 405*m*n^4 + 15*m^4*n + 225*m^2 + 85*m^3 + 15*m^
4 + m^5 + 2025*n^2 + 2295*n^3 + 1215*n^4 + 243*n^5 + 810*m^2*n^2 + 270*m^2
*n^3 + 90*m^3*n^2 + 120) + (b^3*c*x*x^(3*n + 3)*(e*x)^m*(61*m + 183*n + 24
6*m*n + 297*m*n^2 + 99*m^2*n + 108*m*n^3 + 12*m^3*n + 41*m^2 + 11*m^3 + m^
4 + 369*n^2 + 297*n^3 + 81*n^4 + 54*m^2*n^2 + 30))/(274*m + 822*n + 1350*m
*n + 2295*m*n^2 + 765*m^2*n + 1620*m*n^3 + 180*m^3*n + 405*m*n^4 + 15*m^4*
n + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 2025*n^2 + 2295*n^3 + 1215*n^4 + 243
*n^5 + 810*m^2*n^2 + 270*m^2*n^3 + 90*m^3*n^2 + 120) + (a^3*d*x^(3*n)*x^2*
(e*x)^m*(107*m + 321*n + 354*m*n + 351*m*n^2 + 117*m^2*n + 108*m*n^3 + 12*
m^3*n + 59*m^2 + 13*m^3 + m^4 + 531*n^2 + 351*n^3 + 81*n^4 + 54*m^2*n^2 +
60))/(274*m + 822*n + 1350*m*n + 2295*m*n^2 + 765*m^2*n + 1620*m*n^3 + 180
*m^3*n + 405*m*n^4 + 15*m^4*n + 225*m^2 + 85*m^3 + 15*m^4 + m^5 + 2025*n^2
 + 2295*n^3 + 1215*n^4 + 243*n^5 + 810*m^2*n^2 + 270*m^2*n^3 + 90*m^3*n^2
+ 120) + (b^3*d*x^(3*n + 3)*x^2*(e*x)^m*(50*m + 150*n + 210*m*n + 270*m*n^
2 + 90*m^2*n + 108*m*n^3 + 12*m^3*n + 35*m^2 + 10*m^3 + m^4 + 315*n^2 + 27
0*n^3 + 81*n^4 + 54*m^2*n^2 + 24))/(274*m + 822*n + 1350*m*n + 2295*m*n^2
+ 765*m^2*n + 1620*m*n^3 + 180*m^3*n + 405*m*n^4 + 15*m^4*n + 225*m^2 +...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 1508, normalized size of antiderivative = 10.33

$$\int (ex)^m(c+dx) \left(ax^n+bx^{1+n}\right)^3 \, dx = \text{Too large to display}$$

input
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^3,x)
```

output

```
(x**(m + 3*n)*e**m*x*(a**3*c*m**4 + 12*a**3*c*m**3*n + 14*a**3*c*m**3 + 54
*a**3*c*m**2*n**2 + 126*a**3*c*m**2*n + 71*a**3*c*m**2 + 108*a**3*c*m*n**3
 + 378*a**3*c*m*n**2 + 426*a**3*c*m*n + 154*a**3*c*m + 81*a**3*c*n**4 + 37
8*a**3*c*n**3 + 639*a**3*c*n**2 + 462*a**3*c*n + 120*a**3*c + a**3*d*m**4*
x + 12*a**3*d*m**3*n*x + 13*a**3*d*m**3*x + 54*a**3*d*m**2*n**2*x + 117*a*
*3*d*m**2*n*x + 59*a**3*d*m**2*x + 108*a**3*d*m*n**3*x + 351*a**3*d*m*n**2
*x + 354*a**3*d*m*n*x + 107*a**3*d*m*x + 81*a**3*d*n**4*x + 351*a**3*d*n**
3*x + 531*a**3*d*n**2*x + 321*a**3*d*n*x + 60*a**3*d*x + 3*a**2*b*c*m**4*x
 + 36*a**2*b*c*m**3*n*x + 39*a**2*b*c*m**3*x + 162*a**2*b*c*m**2*n**2*x +
351*a**2*b*c*m**2*n*x + 177*a**2*b*c*m**2*x + 324*a**2*b*c*m*n**3*x + 1053
*a**2*b*c*m*n**2*x + 1062*a**2*b*c*m*n*x + 321*a**2*b*c*m*x + 243*a**2*b*c
*n**4*x + 1053*a**2*b*c*n**3*x + 1593*a**2*b*c*n**2*x + 963*a**2*b*c*n*x +
 180*a**2*b*c*x + 3*a**2*b*d*m**4*x**2 + 36*a**2*b*d*m**3*n*x**2 + 36*a**2
*b*d*m**3*x**2 + 162*a**2*b*d*m**2*n**2*x**2 + 324*a**2*b*d*m**2*n*x**2 +
147*a**2*b*d*m**2*x**2 + 324*a**2*b*d*m*n**3*x**2 + 972*a**2*b*d*m*n**2*x*
*2 + 882*a**2*b*d*m*n*x**2 + 234*a**2*b*d*m*x**2 + 243*a**2*b*d*n**4*x**2
+ 972*a**2*b*d*n**3*x**2 + 1323*a**2*b*d*n**2*x**2 + 702*a**2*b*d*n*x**2 +
 120*a**2*b*d*x**2 + 3*a*b**2*c*m**4*x**2 + 36*a*b**2*c*m**3*n*x**2 + 36*a
*b**2*c*m**3*x**2 + 162*a*b**2*c*m**2*n**2*x**2 + 324*a*b**2*c*m**2*n*x**2
 + 147*a*b**2*c*m**2*x**2 + 324*a*b**2*c*m*n**3*x**2 + 972*a*b**2*c*m*n...
```

3.365 $\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^2\,dx$

Optimal result

Integrand size = 26, antiderivative size = 111

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^2\,dx=\frac{a(2bc+ad)x^{2(1+n)}(ex)^m}{2+m+2n}+\frac{b^2dx^{2(2+n)}(ex)^m}{4+m+2n}+\frac{a^2cx^{1+2n}(ex)^m}{1+m+2n}+\frac{b(bc+2ad)x^{3+2n}(ex)^m}{3+m+2n}$$

output
```
a*(a*d+2*b*c)*x^(2+2*n)*(e*x)^m/(2+m+2*n)+b^2*d*x^(4+2*n)*(e*x)^m/(4+m+2*n
)+a^2*c*x^(1+2*n)*(e*x)^m/(1+m+2*n)+b*(2*a*d+b*c)*x^(3+2*n)*(e*x)^m/(3+m+2
*n)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.86

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^2\,dx=\frac{x^{1+2n}(ex)^m\left(d(a+bx)^3+(-ad(1+m+2n)+bc(4+m+2n))\left(\frac{a^2}{1+m+2n}+\frac{2abx}{2+m+2n}+\frac{b^2x^2}{3+m+2n}\right)\right)}{b(4+m+2n)}$$

input
```
Integrate[(e*x)^m*(c + d*x)*(a*x^n + b*x^(1 + n))^2,x]
```

output
```
(x^(1 + 2*n)*(e*x)^m*(d*(a + b*x)^3 + (-(a*d*(1 + m + 2*n)) + b*c*(4 + m +
 2*n))*(a^2/(1 + m + 2*n) + (2*a*b*x)/(2 + m + 2*n) + (b^2*x^2)/(3 + m + 2
*n))))/(b*(4 + m + 2*n))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2027, 30, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(ex)^m \left(ax^n+bx^{n+1}\right)^2 dx$$

$$\Big\downarrow \text{ 2027}$$

$$\int x^{2n}(a+bx)^2(c+dx)(ex)^m dx$$

$$\Big\downarrow \text{ 30}$$

$$x^{-m}(ex)^m \int x^{m+2n}(a+bx)^2(c+dx)dx$$

$$\Big\downarrow \text{ 85}$$

$$x^{-m}(ex)^m \int \left(a^2cx^{m+2n}+a(2bc+ad)x^{m+2n+1}+b(bc+2ad)x^{m+2n+2}+b^2dx^{m+2n+3}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$x^{-m}(ex)^m \left(\frac{a^2cx^{m+2n+1}}{m+2n+1}+\frac{ax^{m+2n+2}(ad+2bc)}{m+2n+2}+\frac{bx^{m+2n+3}(2ad+bc)}{m+2n+3}+\frac{b^2dx^{m+2n+4}}{m+2n+4}\right)$$

input
```
Int[(e*x)^m*(c + d*x)*(a*x^n + b*x^(1 + n))^2,x]
```

output
```
((e*x)^m*((a^2*c*x^(1 + m + 2*n))/(1 + m + 2*n) + (a*(2*b*c + a*d)*x^(2 +
m + 2*n))/(2 + m + 2*n) + (b*(b*c + 2*a*d)*x^(3 + m + 2*n))/(3 + m + 2*n)
+ (b^2*d*x^(4 + m + 2*n))/(4 + m + 2*n)))/x^m
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 652 vs. $2(111) = 222$.

Time = 1.75 (sec) , antiderivative size = 653, normalized size of antiderivative = 5.88

method	result
orering	$\frac{(b^2 d m^3 x^3+6b^2 d m^2 n x^3+12b^2 dm n^2 x^3+8b^2 d n^3 x^3+2abd m^3 x^2+12abd m^2 n x^2+24abdm n^2 x^2+16abd n^3 x^2+b^2 c m^3 x^2+6b}{}$
risch	$\frac{x(b^2 d m^3 x^3+6b^2 d m^2 n x^3+12b^2 dm n^2 x^3+8b^2 d n^3 x^3+2abd m^3 x^2+12abd m^2 n x^2+24abdm n^2 x^2+16abd n^3 x^2+b^2 c m^3 x^2+6}{}$
parallelrisch	Expression too large to display

input
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^2,x,method=_RETURNVERBOSE)
```

output
```
(b^2*d*m^3*x^3+6*b^2*d*m^2*n*x^3+12*b^2*d*m*n^2*x^3+8*b^2*d*n^3*x^3+2*a*b*
d*m^3*x^2+12*a*b*d*m^2*n*x^2+24*a*b*d*m*n^2*x^2+16*a*b*d*n^3*x^2+b^2*c*m^3
*x^2+6*b^2*c*m^2*n*x^2+12*b^2*c*m*n^2*x^2+8*b^2*c*n^3*x^2+6*b^2*d*m^2*x^3+
24*b^2*d*m*n*x^3+24*b^2*d*n^2*x^3+a^2*d*m^3*x+6*a^2*d*m^2*n*x+12*a^2*d*m*n
^2*x+8*a^2*d*n^3*x+2*a*b*c*m^3*x+12*a*b*c*m^2*n*x+24*a*b*c*m*n^2*x+16*a*b*
c*n^3*x+14*a*b*d*m^2*x^2+56*a*b*d*m*n*x^2+56*a*b*d*n^2*x^2+7*b^2*c*m^2*x^2
+28*b^2*c*m*n*x^2+28*b^2*c*n^2*x^2+11*b^2*d*m*x^3+22*b^2*d*n*x^3+a^2*c*m^3
+6*a^2*c*m^2*n+12*a^2*c*m*n^2+8*a^2*c*n^3+8*a^2*d*m^2*x+32*a^2*d*m*n*x+32*
a^2*d*n^2*x+16*a*b*c*m^2*x+64*a*b*c*m*n*x+64*a*b*c*n^2*x+28*a*b*d*m*x^2+56
*a*b*d*n*x^2+14*b^2*c*m*x^2+28*b^2*c*n*x^2+6*b^2*d*x^3+9*a^2*c*m^2+36*a^2*
c*m*n+36*a^2*c*n^2+19*a^2*d*m*x+38*a^2*d*n*x+38*a*b*c*m*x+76*a*b*c*n*x+16*
a*b*d*x^2+8*b^2*c*x^2+26*a^2*c*m+52*a^2*c*n+12*a^2*d*x+24*a*b*c*x+24*a^2*c
)/(1+m+2*n)/(2+m+2*n)/(3+m+2*n)/(4+m+2*n)/(b*x+a)^2*x*(e*x)^m*(a*x^n+b*x^(
1+n))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(111) = 222$.

Time = 0.12 (sec) , antiderivative size = 565, normalized size of antiderivative = 5.09

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^2\,dx$$
$$= \frac{(a^2cm^3+8\,a^2cn^3+9\,a^2cm^2+26\,a^2cm+(b^2dm^3+8\,b^2dn^3+6\,b^2dm^2+11\,b^2dm+6\,b^2d+12\,(b^2dm+2}{}$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^2,x, algorithm="fricas")
```

output
```
(a^2*c*m^3 + 8*a^2*c*n^3 + 9*a^2*c*m^2 + 26*a^2*c*m + (b^2*d*m^3 + 8*b^2*d
*n^3 + 6*b^2*d*m^2 + 11*b^2*d*m + 6*b^2*d + 12*(b^2*d*m + 2*b^2*d)*n^2 + 2
*(3*b^2*d*m^2 + 12*b^2*d*m + 11*b^2*d)*n)*x^3 + 24*a^2*c + 12*(a^2*c*m + 3
*a^2*c)*n^2 + ((b^2*c + 2*a*b*d)*m^3 + 8*(b^2*c + 2*a*b*d)*n^3 + 8*b^2*c +
 16*a*b*d + 7*(b^2*c + 2*a*b*d)*m^2 + 4*(7*b^2*c + 14*a*b*d + 3*(b^2*c + 2
*a*b*d)*m)*n^2 + 14*(b^2*c + 2*a*b*d)*m + 2*(14*b^2*c + 28*a*b*d + 3*(b^2*
c + 2*a*b*d)*m^2 + 14*(b^2*c + 2*a*b*d)*m)*n)*x^2 + 2*(3*a^2*c*m^2 + 18*a^
2*c*m + 26*a^2*c)*n + ((2*a*b*c + a^2*d)*m^3 + 8*(2*a*b*c + a^2*d)*n^3 + 2
4*a*b*c + 12*a^2*d + 8*(2*a*b*c + a^2*d)*m^2 + 4*(16*a*b*c + 8*a^2*d + 3*(
2*a*b*c + a^2*d)*m)*n^2 + 19*(2*a*b*c + a^2*d)*m + 2*(38*a*b*c + 19*a^2*d
+ 3*(2*a*b*c + a^2*d)*m^2 + 16*(2*a*b*c + a^2*d)*m)*n)*x)*x^(2*n + 2)*e^(m
*log(e) + m*log(x))/((m^4 + 16*(2*m + 5)*n^3 + 16*n^4 + 10*m^3 + 4*(6*m^2
+ 30*m + 35)*n^2 + 35*m^2 + 4*(2*m^3 + 15*m^2 + 35*m + 25)*n + 50*m + 24)*
x)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6790 vs. $2(104) = 208$.

Time = 29.24 (sec) , antiderivative size = 6790, normalized size of antiderivative = 61.17

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(d*x+c)*(a*x**n+b*x**(1+n))**2,x)
```

output
```
Piecewise((-a**2*c*x*x**(2*n)*(e*x)**(-2*n - 4)/3 - a**2*d*x**2*x**(2*n)*(
e*x)**(-2*n - 4)/2 - a*b*c*x*x**n*x**(n + 1)*(e*x)**(-2*n - 4) - 2*a*b*d*x
**2*x**n*x**(n + 1)*(e*x)**(-2*n - 4) - b**2*c*x*x**(2*n + 2)*(e*x)**(-2*n
 - 4) + b**2*d*x**2*x**(2*n + 2)*(e*x)**(-2*n - 4)*log(x), Eq(m, -2*n - 4)
), (-a**2*c*x*x**(2*n)*(e*x)**(-2*n - 3)/2 - a**2*d*x**2*x**(2*n)*(e*x)**(
-2*n - 3) - 2*a*b*c*x*x**n*x**(n + 1)*(e*x)**(-2*n - 3) + 2*a*b*d*x**2*x**
n*x**(n + 1)*(e*x)**(-2*n - 3)*log(x) + b**2*c*x*x**(2*n + 2)*(e*x)**(-2*n
 - 3)*log(x) + b**2*d*x**2*x**(2*n + 2)*(e*x)**(-2*n - 3), Eq(m, -2*n - 3)
), (-a**2*c*x*x**(2*n)*(e*x)**(-2*n - 2) + a**2*d*x**2*x**(2*n)*(e*x)**(-2
*n - 2)*log(x) + 2*a*b*c*x*x**n*x**(n + 1)*(e*x)**(-2*n - 2)*log(x) + 2*a*
b*d*x**2*x**n*x**(n + 1)*(e*x)**(-2*n - 2) + b**2*c*x*x**(2*n + 2)*(e*x)**
(-2*n - 2) + b**2*d*x**2*x**(2*n + 2)*(e*x)**(-2*n - 2)/2, Eq(m, -2*n - 2)
), (a**2*c*x*x**(2*n)*(e*x)**(-2*n - 1)*log(x) + a**2*d*x**2*x**(2*n)*(e*x
)**(-2*n - 1) + 2*a*b*c*x*x**n*x**(n + 1)*(e*x)**(-2*n - 1) + a*b*d*x**2*x
**n*x**(n + 1)*(e*x)**(-2*n - 1) + b**2*c*x*x**(2*n + 2)*(e*x)**(-2*n - 1)
/2 + b**2*d*x**2*x**(2*n + 2)*(e*x)**(-2*n - 1)/3, Eq(m, -2*n - 1)), (a**2
*c*m**3*x*x**(2*n)*(e*x)**m/(m**4 + 8*m**3*n + 10*m**3 + 24*m**2*n**2 + 60
*m**2*n + 35*m**2 + 32*m*n**3 + 120*m*n**2 + 140*m*n + 50*m + 16*n**4 + 80
*n**3 + 140*n**2 + 100*n + 24) + 6*a**2*c*m**2*n*x*x**(2*n)*(e*x)**m/(m**4
 + 8*m**3*n + 10*m**3 + 24*m**2*n**2 + 60*m**2*n + 35*m**2 + 32*m*n**3 ...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.61

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^2\,dx=\frac{b^2de^mx^4e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+4}+\frac{b^2ce^mx^3e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+3}+\frac{2\,abde^mx^3e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+3}+\frac{2\,abce^mx^2e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+2}+\frac{a^2de^mx^2e^{(m\log(x)+2\,n\log(x))}}{m+2\,n+2}+\frac{a^2ce^mxe^{(m\log(x)+2\,n\log(x))}}{m+2\,n+1}$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^2,x, algorithm="maxima")
```

output
```
b^2*d*e^m*x^4*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 4) + b^2*c*e^m*x^3*e^(m
*log(x) + 2*n*log(x))/(m + 2*n + 3) + 2*a*b*d*e^m*x^3*e^(m*log(x) + 2*n*lo
g(x))/(m + 2*n + 3) + 2*a*b*c*e^m*x^2*e^(m*log(x) + 2*n*log(x))/(m + 2*n +
 2) + a^2*d*e^m*x^2*e^(m*log(x) + 2*n*log(x))/(m + 2*n + 2) + a^2*c*e^m*x*
e^(m*log(x) + 2*n*log(x))/(m + 2*n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1616 vs. $2(111) = 222$.

Time = 0.25 (sec) , antiderivative size = 1616, normalized size of antiderivative = 14.56

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^2,x, algorithm="giac")
```

output
```
(b^2*d*m^3*x^4*x^(2*n)*e^(m*log(e) + m*log(x)) + 6*b^2*d*m^2*n*x^4*x^(2*n)
*e^(m*log(e) + m*log(x)) + 12*b^2*d*m*n^2*x^4*x^(2*n)*e^(m*log(e) + m*log(
x)) + 8*b^2*d*n^3*x^4*x^(2*n)*e^(m*log(e) + m*log(x)) + b^2*c*m^3*x^3*x^(2
*n)*e^(m*log(e) + m*log(x)) + 2*a*b*d*m^3*x^3*x^(2*n)*e^(m*log(e) + m*log(
x)) + 6*b^2*c*m^2*n*x^3*x^(2*n)*e^(m*log(e) + m*log(x)) + 12*a*b*d*m^2*n*x
^3*x^(2*n)*e^(m*log(e) + m*log(x)) + 12*b^2*c*m*n^2*x^3*x^(2*n)*e^(m*log(e
) + m*log(x)) + 24*a*b*d*m*n^2*x^3*x^(2*n)*e^(m*log(e) + m*log(x)) + 8*b^2
*c*n^3*x^3*x^(2*n)*e^(m*log(e) + m*log(x)) + 16*a*b*d*n^3*x^3*x^(2*n)*e^(m
*log(e) + m*log(x)) + 6*b^2*d*m^2*x^4*x^(2*n)*e^(m*log(e) + m*log(x)) + 24
*b^2*d*m*n*x^4*x^(2*n)*e^(m*log(e) + m*log(x)) + 24*b^2*d*n^2*x^4*x^(2*n)*
e^(m*log(e) + m*log(x)) + 2*a*b*c*m^3*x^2*x^(2*n)*e^(m*log(e) + m*log(x))
 + a^2*d*m^3*x^2*x^(2*n)*e^(m*log(e) + m*log(x)) + 12*a*b*c*m^2*n*x^2*x^(2*
n)*e^(m*log(e) + m*log(x)) + 6*a^2*d*m^2*n*x^2*x^(2*n)*e^(m*log(e) + m*log
(x)) + 24*a*b*c*m*n^2*x^2*x^(2*n)*e^(m*log(e) + m*log(x)) + 12*a^2*d*m*n^2
*x^2*x^(2*n)*e^(m*log(e) + m*log(x)) + 16*a*b*c*n^3*x^2*x^(2*n)*e^(m*log(e
) + m*log(x)) + 8*a^2*d*n^3*x^2*x^(2*n)*e^(m*log(e) + m*log(x)) + 7*b^2*c*
m^2*x^3*x^(2*n)*e^(m*log(e) + m*log(x)) + 14*a*b*d*m^2*x^3*x^(2*n)*e^(m*lo
g(e) + m*log(x)) + 28*b^2*c*m*n*x^3*x^(2*n)*e^(m*log(e) + m*log(x)) + 56*a
*b*d*m*n*x^3*x^(2*n)*e^(m*log(e) + m*log(x)) + 28*b^2*c*n^2*x^3*x^(2*n)*e^
(m*log(e) + m*log(x)) + 56*a*b*d*n^2*x^3*x^(2*n)*e^(m*log(e) + m*log(x)...
```

Mupad [B] (verification not implemented)

Time = 8.68 (sec) , antiderivative size = 809, normalized size of antiderivative = 7.29

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^2\,dx$$
$$=\frac{a^2\,c\,x\,x^{2\,n}\,{\left(e\,x\right)}^m\,\left(m^3+6\,m^2\,n+9\,m^2+12\,m\,n^2+36\,m\,n+26\,m+8\,n^3+36\,n^2+52\right.}{m^4+8\,m^3\,n+10\,m^3+24\,m^2\,n^2+60\,m^2\,n+35\,m^2+32\,m\,n^3+120\,m\,n^2+140\,m\,n+50\,m+16\,n^4+8}$$
$$+\frac{b^2\,c\,x\,x^{2\,n+2}\,{\left(e\,x\right)}^m\,\left(m^3+6\,m^2\,n+7\,m^2+12\,m\,n^2+28\,m\,n+14\,m+8\,n^3+28\,n^2\right.}{m^4+8\,m^3\,n+10\,m^3+24\,m^2\,n^2+60\,m^2\,n+35\,m^2+32\,m\,n^3+120\,m\,n^2+140\,m\,n+50\,m+16\,n^4}$$
$$+\frac{a^2\,d\,x^{2\,n}\,x^2\,{\left(e\,x\right)}^m\,\left(m^3+6\,m^2\,n+8\,m^2+12\,m\,n^2+32\,m\,n+19\,m+8\,n^3+32\,n^2+\right.}{m^4+8\,m^3\,n+10\,m^3+24\,m^2\,n^2+60\,m^2\,n+35\,m^2+32\,m\,n^3+120\,m\,n^2+140\,m\,n+50\,m+16\,n^4}$$
$$+\frac{b^2\,d\,x^{2\,n+2}\,x^2\,{\left(e\,x\right)}^m\,\left(m^3+6\,m^2\,n+6\,m^2+12\,m\,n^2+24\,m\,n+11\,m+8\,n^3+24\,n^2\right.}{m^4+8\,m^3\,n+10\,m^3+24\,m^2\,n^2+60\,m^2\,n+35\,m^2+32\,m\,n^3+120\,m\,n^2+140\,m\,n+50\,m+16\,n^4}$$
$$+\frac{2\,a\,b\,c\,x\,x^n\,x^{n+1}\,{\left(e\,x\right)}^m\,\left(m^3+6\,m^2\,n+8\,m^2+12\,m\,n^2+32\,m\,n+19\,m+8\,n^3+32\,n^2\right.}{m^4+8\,m^3\,n+10\,m^3+24\,m^2\,n^2+60\,m^2\,n+35\,m^2+32\,m\,n^3+120\,m\,n^2+140\,m\,n+50\,m+16\,n^4}$$
$$+\frac{2\,a\,b\,d\,x^n\,x^{n+1}\,x^2\,{\left(e\,x\right)}^m\,\left(m^3+6\,m^2\,n+7\,m^2+12\,m\,n^2+28\,m\,n+14\,m+8\,n^3+28\,n\right.}{m^4+8\,m^3\,n+10\,m^3+24\,m^2\,n^2+60\,m^2\,n+35\,m^2+32\,m\,n^3+120\,m\,n^2+140\,m\,n+50\,m+16\,n^4}$$

input
```
int((e*x)^m*(a*x^n + b*x^(n + 1))^2*(c + d*x),x)
```

output
```
(a^2*c*x*x^(2*n)*(e*x)^m*(26*m + 52*n + 36*m*n + 12*m*n^2 + 6*m^2*n + 9*m^
2 + m^3 + 36*n^2 + 8*n^3 + 24))/(50*m + 100*n + 140*m*n + 120*m*n^2 + 60*m
^2*n + 32*m*n^3 + 8*m^3*n + 35*m^2 + 10*m^3 + m^4 + 140*n^2 + 80*n^3 + 16*
n^4 + 24*m^2*n^2 + 24) + (b^2*c*x*x^(2*n + 2)*(e*x)^m*(14*m + 28*n + 28*m*
n + 12*m*n^2 + 6*m^2*n + 7*m^2 + m^3 + 28*n^2 + 8*n^3 + 8))/(50*m + 100*n
+ 140*m*n + 120*m*n^2 + 60*m^2*n + 32*m*n^3 + 8*m^3*n + 35*m^2 + 10*m^3 +
m^4 + 140*n^2 + 80*n^3 + 16*n^4 + 24*m^2*n^2 + 24) + (a^2*d*x^(2*n)*x^2*(e
*x)^m*(19*m + 38*n + 32*m*n + 12*m*n^2 + 6*m^2*n + 8*m^2 + m^3 + 32*n^2 +
8*n^3 + 12))/(50*m + 100*n + 140*m*n + 120*m*n^2 + 60*m^2*n + 32*m*n^3 + 8
*m^3*n + 35*m^2 + 10*m^3 + m^4 + 140*n^2 + 80*n^3 + 16*n^4 + 24*m^2*n^2 +
24) + (b^2*d*x^(2*n + 2)*x^2*(e*x)^m*(11*m + 22*n + 24*m*n + 12*m*n^2 + 6*
m^2*n + 6*m^2 + m^3 + 24*n^2 + 8*n^3 + 6))/(50*m + 100*n + 140*m*n + 120*m
*n^2 + 60*m^2*n + 32*m*n^3 + 8*m^3*n + 35*m^2 + 10*m^3 + m^4 + 140*n^2 + 8
0*n^3 + 16*n^4 + 24*m^2*n^2 + 24) + (2*a*b*c*x*x^n*x^(n + 1)*(e*x)^m*(19*m
 + 38*n + 32*m*n + 12*m*n^2 + 6*m^2*n + 8*m^2 + m^3 + 32*n^2 + 8*n^3 + 12)
)/(50*m + 100*n + 140*m*n + 120*m*n^2 + 60*m^2*n + 32*m*n^3 + 8*m^3*n + 35
*m^2 + 10*m^3 + m^4 + 140*n^2 + 80*n^3 + 16*n^4 + 24*m^2*n^2 + 24) + (2*a*
b*d*x^n*x^(n + 1)*x^2*(e*x)^m*(14*m + 28*n + 28*m*n + 12*m*n^2 + 6*m^2*n +
 7*m^2 + m^3 + 28*n^2 + 8*n^3 + 8))/(50*m + 100*n + 140*m*n + 120*m*n^2 +
60*m^2*n + 32*m*n^3 + 8*m^3*n + 35*m^2 + 10*m^3 + m^4 + 140*n^2 + 80*n^...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 677, normalized size of antiderivative = 6.10

$$\int (ex)^m (c+dx) \left(ax^n+bx^{1+n}\right)^2 \, dx$$
$$= \frac{x^{m+2n} e^m x (b^2 d\, m^3 x^3 + 6b^2 d\, m^2 n\, x^3 + 12b^2 dm\, n^2 x^3 + 8b^2 d\, n^3 x^3 + 2abd\, m^3 x^2 + 12abd\, m^2 n\, x^2 + 24abdm}{}$$

input
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^2,x)
```

output

```
(x**(m + 2*n)*e***m*x*(a**2*c*m**3 + 6*a**2*c*m**2*n + 9*a**2*c*m**2 + 12*a
**2*c*m*n**2 + 36*a**2*c*m*n + 26*a**2*c*m + 8*a**2*c*n**3 + 36*a**2*c*n**
2 + 52*a**2*c*n + 24*a**2*c + a**2*d*m**3*x + 6*a**2*d*m**2*n*x + 8*a**2*d
*m**2*x + 12*a**2*d*m*n**2*x + 32*a**2*d*m*n*x + 19*a**2*d*m*x + 8*a**2*d*
n**3*x + 32*a**2*d*n**2*x + 38*a**2*d*n*x + 12*a**2*d*x + 2*a*b*c*m**3*x +
 12*a*b*c*m**2*n*x + 16*a*b*c*m**2*x + 24*a*b*c*m*n**2*x + 64*a*b*c*m*n*x
+ 38*a*b*c*m*x + 16*a*b*c*n**3*x + 64*a*b*c*n**2*x + 76*a*b*c*n*x + 24*a*b
*c*x + 2*a*b*d*m**3*x**2 + 12*a*b*d*m**2*n*x**2 + 14*a*b*d*m**2*x**2 + 24*
a*b*d*m*n**2*x**2 + 56*a*b*d*m*n*x**2 + 28*a*b*d*m*x**2 + 16*a*b*d*n**3*x*
*2 + 56*a*b*d*n**2*x**2 + 56*a*b*d*n*x**2 + 16*a*b*d*x**2 + b**2*c*m**3*x*
*2 + 6*b**2*c*m**2*n*x**2 + 7*b**2*c*m**2*x**2 + 12*b**2*c*m*n**2*x**2 + 2
8*b**2*c*m*n*x**2 + 14*b**2*c*m*x**2 + 8*b**2*c*n**3*x**2 + 28*b**2*c*n**2
*x**2 + 28*b**2*c*n*x**2 + 8*b**2*c*x**2 + b**2*d*m**3*x**3 + 6*b**2*d*m**
2*n*x**3 + 6*b**2*d*m**2*x**3 + 12*b**2*d*m*n**2*x**3 + 24*b**2*d*m*n*x**3
 + 11*b**2*d*m*x**3 + 8*b**2*d*n**3*x**3 + 24*b**2*d*n**2*x**3 + 22*b**2*d
*n*x**3 + 6*b**2*d*x**3))/(m**4 + 8*m**3*n + 10*m**3 + 24*m**2*n**2 + 60*m
**2*n + 35*m**2 + 32*m*n**3 + 120*m*n**2 + 140*m*n + 50*m + 16*n**4 + 80*n
**3 + 140*n**2 + 100*n + 24)
```

3.366 $\int (ex)^m (c+dx) \left(ax^n+bx^{1+n}\right) \, dx$

Optimal result

Integrand size = 24, antiderivative size = 63

$$\int (ex)^m (c+dx) \left(ax^n+bx^{1+n}\right) \, dx = \frac{acx^{1+n}(ex)^m}{1+m+n}+\frac{(bc+ad)x^{2+n}(ex)^m}{2+m+n}+\frac{bdx^{3+n}(ex)^m}{3+m+n}$$

output
```
a*c*x^(1+n)*(e*x)^m/(1+m+n)+(a*d+b*c)*x^(2+n)*(e*x)^m/(2+m+n)+b*d*x^(3+n)*
(e*x)^m/(3+m+n)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17

$$\int (ex)^m (c+dx) \left(ax^n+bx^{1+n}\right) \, dx = \frac{x^{1+n}(ex)^m \left(b(c+dx)^2-\frac{(bc(1+m+n)-ad(3+m+n))(c(2+m+n)+d(1+m+n)x)}{(1+m+n)(2+m+n)}\right)}{d(3+m+n)}$$

input
```
Integrate[(e*x)^m*(c + d*x)*(a*x^n + b*x^(1 + n)),x]
```

output
```
(x^(1 + n)*(e*x)^m*(b*(c + d*x)^2 - ((b*c*(1 + m + n) - a*d*(3 + m + n))*(
c*(2 + m + n) + d*(1 + m + n)*x))/((1 + m + n)*(2 + m + n))))/(d*(3 + m +
n))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {2027, 30, 85, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(ex)^m \left(ax^n+bx^{n+1}\right) \, dx$$

$$\Big\downarrow \; 2027$$

$$\int x^n(a+bx)(c+dx)(ex)^m dx$$

$$\Big\downarrow \; 30$$

$$x^{-m}(ex)^m \int x^{m+n}(a+bx)(c+dx)dx$$

$$\Big\downarrow \; 85$$

$$x^{-m}(ex)^m \int \left(acx^{m+n}+(bc+ad)x^{m+n+1}+bdx^{m+n+2}\right) dx$$

$$\Big\downarrow \; 2009$$

$$x^{-m}(ex)^m \left(\frac{x^{m+n+2}(ad+bc)}{m+n+2}+\frac{acx^{m+n+1}}{m+n+1}+\frac{bdx^{m+n+3}}{m+n+3}\right)$$

input
```
Int[(e*x)^m*(c + d*x)*(a*x^n + b*x^(1 + n)),x]
```

output
```
((e*x)^m*((a*c*x^(1 + m + n))/(1 + m + n) + ((b*c + a*d)*x^(2 + m + n))/(2
 + m + n) + (b*d*x^(3 + m + n))/(3 + m + n)))/x^m
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 85
```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_] :
> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b,
 d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*e + a*
f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] || (GeQ[n
 + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || EqQ[p,
1])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(63) = 126$.

Time = 0.41 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.25

method	result
orering	$\frac{(bd\,m^2x^2+2bdmn\,x^2+bd\,n^2x^2+ad\,m^2x+2admnx+ad\,n^2x+bc\,m^2x+2bcmnx+bc\,n^2x+3bdm\,x^2+3bdn\,x^2+ac\,m^2+2acmn+}{(1+m+n)(2+m+n)(3+m+n}$
risch	$\frac{x(bd\,m^2x^2+2bdmn\,x^2+bd\,n^2x^2+ad\,m^2x+2admnx+ad\,n^2x+bc\,m^2x+2bcmnx+bc\,n^2x+3bdm\,x^2+3bdn\,x^2+ac\,m^2+2acmn}{(1+}$
parallelrisch	$\frac{2x\,x^{1+n}(ex)^m bcmn+2x^2x^{1+n}(ex)^m bdmn+2x\,x^n(ex)^m acmn+2x^2x^n(ex)^m admn+x^2x^n(ex)^m ad\,m^2+x^2x^n(ex)^m ad\,n^2+x}{}$

input
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n)),x,method=_RETURNVERBOSE)
```

output
```
(b*d*m^2*x^2+2*b*d*m*n*x^2+b*d*n^2*x^2+a*d*m^2*x+2*a*d*m*n*x+a*d*n^2*x+b*c
*m^2*x+2*b*c*m*n*x+b*c*n^2*x+3*b*d*m*x^2+3*b*d*n*x^2+a*c*m^2+2*a*c*m*n+a*c
*n^2+4*a*d*m*x+4*a*d*n*x+4*b*c*m*x+4*b*c*n*x+2*b*d*x^2+5*a*c*m+5*a*c*n+3*a
*d*x+3*b*c*x+6*a*c)/(1+m+n)/(2+m+n)/(3+m+n)/(b*x+a)*x*(e*x)^m*(a*x^n+b*x^(
1+n))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(63) = 126$.

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.02

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)\,dx = \frac{(acm^2+acn^2+5\,acm+(bdm^2+bdn^2+3\,bdm+2\,bd+(2\,bdm+3\,bd)n)x^2+6\,ac+(2\,acm+5\,ac)n+}{m^3+3\,(m+2)n^2+n^3+6}$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n)),x, algorithm="fricas")
```

output
```
(a*c*m^2 + a*c*n^2 + 5*a*c*m + (b*d*m^2 + b*d*n^2 + 3*b*d*m + 2*b*d + (2*b
*d*m + 3*b*d)*n)*x^2 + 6*a*c + (2*a*c*m + 5*a*c)*n + ((b*c + a*d)*m^2 + (b
*c + a*d)*n^2 + 3*b*c + 3*a*d + 4*(b*c + a*d)*m + 2*(2*b*c + 2*a*d + (b*c
+ a*d)*m)*n)*x)*x^(n + 1)*e^(m*log(e) + m*log(x))/(m^3 + 3*(m + 2)*n^2 + n
^3 + 6*m^2 + (3*m^2 + 12*m + 11)*n + 11*m + 6)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1756 vs. $2(58) = 116$.

Time = 11.99 (sec) , antiderivative size = 1756, normalized size of antiderivative = 27.87

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x)**m*(d*x+c)*(a*x**n+b*x**(1+n)),x)
```

output
```
Piecewise((-a*c*x*x**n*(e*x)**(-n - 3)/2 - a*d*x**2*x**n*(e*x)**(-n - 3) -
 b*c*x*x**(n + 1)*(e*x)**(-n - 3) + b*d*x**2*x**(n + 1)*(e*x)**(-n - 3)*lo
g(x), Eq(m, -n - 3)), (-a*c*x*x**n*(e*x)**(-n - 2) + a*d*x**2*x**n*(e*x)**
(-n - 2)*log(x) + b*c*x*x**(n + 1)*(e*x)**(-n - 2)*log(x) + b*d*x**2*x**(n
 + 1)*(e*x)**(-n - 2), Eq(m, -n - 2)), (a*c*x*x**n*(e*x)**(-n - 1)*log(x)
+ a*d*x**2*x**n*(e*x)**(-n - 1) + b*c*x*x**(n + 1)*(e*x)**(-n - 1) + b*d*x
**2*x**(n + 1)*(e*x)**(-n - 1)/2, Eq(m, -n - 1)), (a*c*m**2*x*x**n*(e*x)**
m/(m**3 + 3*m**2*n + 6*m**2 + 3*m*n**2 + 12*m*n + 11*m + n**3 + 6*n**2 + 1
1*n + 6) + 2*a*c*m*n*x*x**n*(e*x)**m/(m**3 + 3*m**2*n + 6*m**2 + 3*m*n**2
+ 12*m*n + 11*m + n**3 + 6*n**2 + 11*n + 6) + 5*a*c*m*x*x**n*(e*x)**m/(m**
3 + 3*m**2*n + 6*m**2 + 3*m*n**2 + 12*m*n + 11*m + n**3 + 6*n**2 + 11*n +
6) + a*c*n**2*x*x**n*(e*x)**m/(m**3 + 3*m**2*n + 6*m**2 + 3*m*n**2 + 12*m*
n + 11*m + n**3 + 6*n**2 + 11*n + 6) + 5*a*c*n*x*x**n*(e*x)**m/(m**3 + 3*m
**2*n + 6*m**2 + 3*m*n**2 + 12*m*n + 11*m + n**3 + 6*n**2 + 11*n + 6) + 6*
a*c*x*x**n*(e*x)**m/(m**3 + 3*m**2*n + 6*m**2 + 3*m*n**2 + 12*m*n + 11*m +
 n**3 + 6*n**2 + 11*n + 6) + a*d*m**2*x**2*x**n*(e*x)**m/(m**3 + 3*m**2*n
+ 6*m**2 + 3*m*n**2 + 12*m*n + 11*m + n**3 + 6*n**2 + 11*n + 6) + 2*a*d*m*
n*x**2*x**n*(e*x)**m/(m**3 + 3*m**2*n + 6*m**2 + 3*m*n**2 + 12*m*n + 11*m
+ n**3 + 6*n**2 + 11*n + 6) + 4*a*d*m*x**2*x**n*(e*x)**m/(m**3 + 3*m**2*n
+ 6*m**2 + 3*m*n**2 + 12*m*n + 11*m + n**3 + 6*n**2 + 11*n + 6) + a*d*n...
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.57

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)\,dx=\frac{bde^mx^3e^{(m\log(x)+n\log(x))}}{m+n+3}+\frac{bce^mx^2e^{(m\log(x)+n\log(x))}}{m+n+2}+\frac{ade^mx^2e^{(m\log(x)+n\log(x))}}{m+n+2}+\frac{ace^mxe^{(m\log(x)+n\log(x))}}{m+n+1}$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n)),x, algorithm="maxima")
```

output
```
b*d*e^m*x^3*e^(m*log(x) + n*log(x))/(m + n + 3) + b*c*e^m*x^2*e^(m*log(x)
+ n*log(x))/(m + n + 2) + a*d*e^m*x^2*e^(m*log(x) + n*log(x))/(m + n + 2)
+ a*c*e^m*x*e^(m*log(x) + n*log(x))/(m + n + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(63) = 126$.

Time = 0.29 (sec) , antiderivative size = 544, normalized size of antiderivative = 8.63

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)\,dx = \text{Too large to display}$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n)),x, algorithm="giac")
```

output
```
(b*d*m^2*x^3*x^n*e^(m*log(e) + m*log(x)) + 2*b*d*m*n*x^3*x^n*e^(m*log(e) +
 m*log(x)) + b*d*n^2*x^3*x^n*e^(m*log(e) + m*log(x)) + b*c*m^2*x^2*x^n*e^(
m*log(e) + m*log(x)) + a*d*m^2*x^2*x^n*e^(m*log(e) + m*log(x)) + 2*b*c*m*n
*x^2*x^n*e^(m*log(e) + m*log(x)) + 2*a*d*m*n*x^2*x^n*e^(m*log(e) + m*log(x
)) + b*c*n^2*x^2*x^n*e^(m*log(e) + m*log(x)) + a*d*n^2*x^2*x^n*e^(m*log(e)
 + m*log(x)) + 3*b*d*m*x^3*x^n*e^(m*log(e) + m*log(x)) + 3*b*d*n*x^3*x^n*e
^(m*log(e) + m*log(x)) + a*c*m^2*x*x^n*e^(m*log(e) + m*log(x)) + 2*a*c*m*n
*x*x^n*e^(m*log(e) + m*log(x)) + a*c*n^2*x*x^n*e^(m*log(e) + m*log(x)) + 4
*b*c*m*x^2*x^n*e^(m*log(e) + m*log(x)) + 4*a*d*m*x^2*x^n*e^(m*log(e) + m*l
og(x)) + 4*b*c*n*x^2*x^n*e^(m*log(e) + m*log(x)) + 4*a*d*n*x^2*x^n*e^(m*lo
g(e) + m*log(x)) + 2*b*d*x^3*x^n*e^(m*log(e) + m*log(x)) + 5*a*c*m*x*x^n*e
^(m*log(e) + m*log(x)) + 5*a*c*n*x*x^n*e^(m*log(e) + m*log(x)) + 3*b*c*x^2
*x^n*e^(m*log(e) + m*log(x)) + 3*a*d*x^2*x^n*e^(m*log(e) + m*log(x)) + 6*a
*c*x*x^n*e^(m*log(e) + m*log(x)))/(m^3 + 3*m^2*n + 3*m*n^2 + n^3 + 6*m^2 +
 12*m*n + 6*n^2 + 11*m + 11*n + 6)
```

Mupad [B] (verification not implemented)

Time = 5.36 (sec) , antiderivative size = 297, normalized size of antiderivative = 4.71

$$\begin{aligned}\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)\,dx &= \frac{b\,d\,x^{n+1}\,x^2\,{(e\,x)}^m\,\left(m^2+2\,m\,n+3\,m+n^2+3\,n+2\right)}{m^3+3\,m^2\,n+6\,m^2+3\,m\,n^2+12\,m\,n+11\,m+n^3+6\,n^2+11\,n+6}\\ &+\frac{a\,c\,x\,x^n\,{(e\,x)}^m\,\left(m^2+2\,m\,n+5\,m+n^2+5\,n+6\right)}{m^3+3\,m^2\,n+6\,m^2+3\,m\,n^2+12\,m\,n+11\,m+n^3+6\,n^2+11\,n+6}\\ &+\frac{b\,c\,x\,x^{n+1}\,{(e\,x)}^m\,\left(m^2+2\,m\,n+4\,m+n^2+4\,n+3\right)}{m^3+3\,m^2\,n+6\,m^2+3\,m\,n^2+12\,m\,n+11\,m+n^3+6\,n^2+11\,n+6}\\ &+\frac{a\,d\,x^n\,x^2\,{(e\,x)}^m\,\left(m^2+2\,m\,n+4\,m+n^2+4\,n+3\right)}{m^3+3\,m^2\,n+6\,m^2+3\,m\,n^2+12\,m\,n+11\,m+n^3+6\,n^2+11\,n+6}\end{aligned}$$

input
```
int((e*x)^m*(a*x^n + b*x^(n + 1))*(c + d*x),x)
```

output
```
(b*d*x^(n + 1)*x^2*(e*x)^m*(3*m + 3*n + 2*m*n + m^2 + n^2 + 2))/(11*m + 11
*n + 12*m*n + 3*m*n^2 + 3*m^2*n + 6*m^2 + m^3 + 6*n^2 + n^3 + 6) + (a*c*x*
x^n*(e*x)^m*(5*m + 5*n + 2*m*n + m^2 + n^2 + 6))/(11*m + 11*n + 12*m*n + 3
*m*n^2 + 3*m^2*n + 6*m^2 + m^3 + 6*n^2 + n^3 + 6) + (b*c*x*x^(n + 1)*(e*x)
^m*(4*m + 4*n + 2*m*n + m^2 + n^2 + 3))/(11*m + 11*n + 12*m*n + 3*m*n^2 +
3*m^2*n + 6*m^2 + m^3 + 6*n^2 + n^3 + 6) + (a*d*x^n*x^2*(e*x)^m*(4*m + 4*n
 + 2*m*n + m^2 + n^2 + 3))/(11*m + 11*n + 12*m*n + 3*m*n^2 + 3*m^2*n + 6*m
^2 + m^3 + 6*n^2 + n^3 + 6)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.35

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)\,dx$$
$$= \frac{x^{m+n}e^m x(bd\,m^2x^2+2bdmn\,x^2+bd\,n^2x^2+ad\,m^2x+2admnx+ad\,n^2x+bc\,m^2x+2bcmnx+bc\,n^2x+}{m^3+3m^2n+3m\,n}$$

input
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n)),x)
```

output
```
(x**(m + n)*e**m*x*(a*c*m**2 + 2*a*c*m*n + 5*a*c*m + a*c*n**2 + 5*a*c*n +
6*a*c + a*d*m**2*x + 2*a*d*m*n*x + 4*a*d*m*x + a*d*n**2*x + 4*a*d*n*x + 3*
a*d*x + b*c*m**2*x + 2*b*c*m*n*x + 4*b*c*m*x + b*c*n**2*x + 4*b*c*n*x + 3*
b*c*x + b*d*m**2*x**2 + 2*b*d*m*n*x**2 + 3*b*d*m*x**2 + b*d*n**2*x**2 + 3*
b*d*n*x**2 + 2*b*d*x**2))/(m**3 + 3*m**2*n + 6*m**2 + 3*m*n**2 + 12*m*n +
11*m + n**3 + 6*n**2 + 11*n + 6)
```

3.367 $\int \frac{(ex)^m(c+dx)}{ax^n+bx^{1+n}}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 82

$$\int \frac{(ex)^m(c+dx)}{ax^n+bx^{1+n}}\,dx = \frac{dx^{1-n}(ex)^m}{b(1+m-n)} + \frac{(bc-ad)x^{1-n}(ex)^m \operatorname{Hypergeometric2F1}\left(1,1+m-n,2+m-n,-\frac{bx}{a}\right)}{ab(1+m-n)}$$

output
```
d*x^(1-n)*(e*x)^m/b/(1+m-n)+(-a*d+b*c)*x^(1-n)*(e*x)^m*hypergeom([1, 1+m-n
],[2+m-n],-b*x/a)/a/b/(1+m-n)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.74

$$\int \frac{(ex)^m(c+dx)}{ax^n+bx^{1+n}}\,dx = \frac{x^{1-n}(ex)^m\left(ad+(bc-ad)\operatorname{Hypergeometric2F1}\left(1,1+m-n,2+m-n,-\frac{bx}{a}\right)\right)}{ab(1+m-n)}$$

input
```
Integrate[((e*x)^m*(c + d*x))/(a*x^n + b*x^(1 + n)),x]
```

output
```
(x^(1 - n)*(e*x)^m*(a*d + (b*c - a*d)*Hypergeometric2F1[1, 1 + m - n, 2 +
m - n, -((b*x)/a)]))/(a*b*(1 + m - n))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.04, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2027, 30, 90, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ex)^m}{ax^n+bx^{n+1}}\,dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^{-n}(c+dx)(ex)^m}{a+bx}dx$$

$$\Big\downarrow \text{30}$$

$$x^{-m}(ex)^m\int \frac{x^{m-n}(c+dx)}{a+bx}dx$$

$$\Big\downarrow \text{90}$$

$$x^{-m}(ex)^m\left(\frac{(bc-ad)\int \frac{x^{m-n}}{a+bx}dx}{b}+\frac{dx^{m-n+1}}{b(m-n+1)}\right)$$

$$\Big\downarrow \text{74}$$

$$x^{-m}(ex)^m\left(\frac{x^{m-n+1}(bc-ad)\,\text{Hypergeometric2F1}\left(1,m-n+1,m-n+2,-\frac{bx}{a}\right)}{ab(m-n+1)}+\frac{dx^{m-n+1}}{b(m-n+1)}\right)$$

input
```
Int[((e*x)^m*(c + d*x))/(a*x^n + b*x^(1 + n)),x]
```

output
```
((e*x)^m*((d*x^(1 + m - n))/(b*(1 + m - n)) + ((b*c - a*d)*x^(1 + m - n)*H
ypergeometric2F1[1, 1 + m - n, 2 + m - n, -((b*x)/a)])/(a*b*(1 + m - n))))
/x^m
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [F]

$$\int \frac{(ex)^m (dx+c)}{a\,x^n+b\,x^{1+n}}dx$$

input
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n)),x)
```

output
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n)),x)
```

Fricas [F]

$$\int \frac{(ex)^m(c+dx)}{ax^n+bx^{1+n}}\,dx=\int \frac{(dx+c)(ex)^m}{bx^{n+1}+ax^n}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n)),x, algorithm="fricas")
```

output
```
integral((d*x + c)*(e*x)^m/(b*x^(n + 1) + a*x^n), x)
```

Sympy [F]

$$\int \frac{(ex)^m(c+dx)}{ax^n+bx^{1+n}}\,dx=\int \frac{(ex)^m\,(c+dx)}{ax^n+bx^{n+1}}\,dx$$

input
```
integrate((e*x)**m*(d*x+c)/(a*x**n+b*x**(1+n)),x)
```

output
```
Integral((e*x)**m*(c + d*x)/(a*x**n + b*x**(n + 1)), x)
```

Maxima [F]

$$\int \frac{(ex)^m(c+dx)}{ax^n+bx^{1+n}}\,dx=\int \frac{(dx+c)(ex)^m}{bx^{n+1}+ax^n}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n)),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^(n + 1) + a*x^n), x)
```

Giac [F]

$$\int \frac{(ex)^m(c+dx)}{ax^n+bx^{1+n}}\,dx=\int \frac{(dx+c)(ex)^m}{bx^{n+1}+ax^n}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n)),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^(n + 1) + a*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(c+dx)}{ax^n+bx^{1+n}}\,dx=\int \frac{(e\,x)^m\,(c+d\,x)}{a\,x^n+b\,x^{n+1}}\,dx$$

input
```
int(((e*x)^m*(c + d*x))/(a*x^n + b*x^(n + 1)),x)
```

output
```
int(((e*x)^m*(c + d*x))/(a*x^n + b*x^(n + 1)), x)
```

Reduce [F]

$$\int \frac{(ex)^m(c+dx)}{ax^n+bx^{1+n}}\,dx$$
$$=\frac{e^m\left(-x^m adm+x^m adn-x^m ad+x^m bcm-x^m bcn+x^m bc+x^m bdmx-x^m bdnx+x^n\left(\int \frac{x^m}{x^n ax+x^n b\,x^2}dx\right)\right.}{}$$

input
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n)),x)
```

output

```
(e**m*( - x**m*a*d*m + x**m*a*d*n - x**m*a*d + x**m*b*c*m - x**m*b*c*n + x
**m*b*c + x**m*b*d*m*x - x**m*b*d*n*x + x**n*int(x**m/(x**n*a*x + x**n*b*x
**2),x)*a**2*d*m**2 - 2*x**n*int(x**m/(x**n*a*x + x**n*b*x**2),x)*a**2*d*m
*n + x**n*int(x**m/(x**n*a*x + x**n*b*x**2),x)*a**2*d*m + x**n*int(x**m/(x
**n*a*x + x**n*b*x**2),x)*a**2*d*n**2 - x**n*int(x**m/(x**n*a*x + x**n*b*x
**2),x)*a**2*d*n - x**n*int(x**m/(x**n*a*x + x**n*b*x**2),x)*a*b*c*m**2 +
2*x**n*int(x**m/(x**n*a*x + x**n*b*x**2),x)*a*b*c*m*n - x**n*int(x**m/(x**
n*a*x + x**n*b*x**2),x)*a*b*c*m - x**n*int(x**m/(x**n*a*x + x**n*b*x**2),x
)*a*b*c*n**2 + x**n*int(x**m/(x**n*a*x + x**n*b*x**2),x)*a*b*c*n))/(x**n*b
**2*(m**2 - 2*m*n + m + n**2 - n))
```

3.368 $\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^2}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 94

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^2}\,dx = \frac{dx^{1-2n}(ex)^m}{b(m-2n)(a+bx)} + \frac{\left(\frac{c}{1+m-2n}-\frac{ad}{bm-2bn}\right)x^{1-2n}(ex)^m \operatorname{Hypergeometric2F1}\left(2,1+m-2n,2+m-2n,-\frac{bx}{a}\right)}{a^2}$$

output
```
d*x^(1-2*n)*(e*x)^m/b/(m-2*n)/(b*x+a)+(c/(1+m-2*n)-a*d/(b*m-2*b*n))*x^(1-2
*n)*(e*x)^m*hypergeom([2, 1+m-2*n],[2+m-2*n],-b*x/a)/a^2
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.91

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^2}\,dx = \frac{x^{1-2n}(ex)^m\left(\frac{a(bc-ad)}{a+bx}+\frac{(-bc(m-2n)+ad(1+m-2n))\operatorname{Hypergeometric2F1}\left(1,1+m-2n,2+m-2n,-\frac{bx}{a}\right)}{1+m-2n}\right)}{a^2b}$$

input
```
Integrate[((e*x)^m*(c + d*x))/(a*x^n + b*x^(1 + n))^2,x]
```

output
```
(x^(1 - 2*n)*(e*x)^m*((a*(b*c - a*d))/(a + b*x) + ((-(b*c*(m - 2*n)) + a*d
*(1 + m - 2*n))*Hypergeometric2F1[1, 1 + m - 2*n, 2 + m - 2*n, -((b*x)/a)]
)/(1 + m - 2*n)))/(a^2*b)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {2027, 30, 87, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ex)^m}{(ax^n+bx^{n+1})^2}dx$$

$$\Big\downarrow \text{2027}$$

$$\int \frac{x^{-2n}(c+dx)(ex)^m}{(a+bx)^2}dx$$

$$\Big\downarrow \text{30}$$

$$x^{-m}(ex)^m\int \frac{x^{m-2n}(c+dx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{87}$$

$$x^{-m}(ex)^m\left(\frac{x^{m-2n+1}(bc-ad)}{ab(a+bx)}-\frac{(bc(m-2n)-ad(m-2n+1))\int \frac{x^{m-2n}}{a+bx}dx}{ab}\right)$$

$$\Big\downarrow \text{74}$$

$$x^{-m}(ex)^m\left(\frac{x^{m-2n+1}(bc-ad)}{ab(a+bx)}-\frac{x^{m-2n+1}(bc(m-2n)-ad(m-2n+1))\,\text{Hypergeometric2F1}\left(1,m-2n+1,\right.}{a^2b(m-2n+1)}\right.$$

input
```
Int[((e*x)^m*(c + d*x))/(a*x^n + b*x^(1 + n))^2,x]
```

output
```
((e*x)^m*(((b*c - a*d)*x^(1 + m - 2*n))/(a*b*(a + b*x)) - ((b*c*(m - 2*n)
- a*d*(1 + m - 2*n))*x^(1 + m - 2*n)*Hypergeometric2F1[1, 1 + m - 2*n, 2 +
 m - 2*n, -((b*x)/a)])/(a^2*b*(1 + m - 2*n))))/x^m
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [F]

$$\int \frac{(ex)^m (dx + c)}{(a\,x^n + b\,x^{1+n})^2}dx$$

input
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^2,x)
```

output
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^2,x)
```

Fricas [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^2}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^{n+1}+ax^n)^2}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(e*x)^m/(2*a*b*x^(n + 1)*x^n + a^2*x^(2*n) + b^2*x^(2*n
 + 2)), x)
```

Sympy [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^2}\,dx = \int \frac{(ex)^m\,(c+dx)}{(ax^n+bx^{n+1})^2}\,dx$$

input
```
integrate((e*x)**m*(d*x+c)/(a*x**n+b*x**(1+n))**2,x)
```

output
```
Integral((e*x)**m*(c + d*x)/(a*x**n + b*x**(n + 1))**2, x)
```

Maxima [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^2}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^{n+1}+ax^n)^2}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^(n + 1) + a*x^n)^2, x)
```

Giac [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^2}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^{n+1}+ax^n)^2}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^(n + 1) + a*x^n)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^2}\,dx = \int \frac{(e\,x)^m\,(c+d\,x)}{(a\,x^n+b\,x^{n+1})^2}\,dx$$

input
```
int(((e*x)^m*(c + d*x))/(a*x^n + b*x^(n + 1))^2,x)
```

output
```
int(((e*x)^m*(c + d*x))/(a*x^n + b*x^(n + 1))^2, x)
```

Reduce [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^2}\,dx = \text{too large to display}$$

input
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^2,x)
```

output
```
(e**m*(x**m*c*x + x**(2*n)*int((x**m*x)/(x**(2*n)*a**2*m - 2*x**(2*n)*a**2
*n + x**(2*n)*a**2 + 2*x**(2*n)*a*b*m*x - 4*x**(2*n)*a*b*n*x + 2*x**(2*n)*
a*b*x + x**(2*n)*b**2*m*x**2 - 2*x**(2*n)*b**2*n*x**2 + x**(2*n)*b**2*x**2
),x)*a**2*d*m**2 - 4*x**(2*n)*int((x**m*x)/(x**(2*n)*a**2*m - 2*x**(2*n)*a
**2*n + x**(2*n)*a**2 + 2*x**(2*n)*a*b*m*x - 4*x**(2*n)*a*b*n*x + 2*x**(2*
n)*a*b*x + x**(2*n)*b**2*m*x**2 - 2*x**(2*n)*b**2*n*x**2 + x**(2*n)*b**2*x
**2),x)*a**2*d*m*n + 2*x**(2*n)*int((x**m*x)/(x**(2*n)*a**2*m - 2*x**(2*n)
*a**2*n + x**(2*n)*a**2 + 2*x**(2*n)*a*b*m*x - 4*x**(2*n)*a*b*n*x + 2*x**(
2*n)*a*b*x + x**(2*n)*b**2*m*x**2 - 2*x**(2*n)*b**2*n*x**2 + x**(2*n)*b**2
*x**2),x)*a**2*d*m + 4*x**(2*n)*int((x**m*x)/(x**(2*n)*a**2*m - 2*x**(2*n)
*a**2*n + x**(2*n)*a**2 + 2*x**(2*n)*a*b*m*x - 4*x**(2*n)*a*b*n*x + 2*x**(
2*n)*a*b*x + x**(2*n)*b**2*m*x**2 - 2*x**(2*n)*b**2*n*x**2 + x**(2*n)*b**2
*x**2),x)*a**2*d*n**2 - 4*x**(2*n)*int((x**m*x)/(x**(2*n)*a**2*m - 2*x**(2
*n)*a**2*n + x**(2*n)*a**2 + 2*x**(2*n)*a*b*m*x - 4*x**(2*n)*a*b*n*x + 2*x
**(2*n)*a*b*x + x**(2*n)*b**2*m*x**2 - 2*x**(2*n)*b**2*n*x**2 + x**(2*n)*b
**2*x**2),x)*a**2*d*n + x**(2*n)*int((x**m*x)/(x**(2*n)*a**2*m - 2*x**(2*n
)*a**2*n + x**(2*n)*a**2 + 2*x**(2*n)*a*b*m*x - 4*x**(2*n)*a*b*n*x + 2*x**
(2*n)*a*b*x + x**(2*n)*b**2*m*x**2 - 2*x**(2*n)*b**2*n*x**2 + x**(2*n)*b**
2*x**2),x)*a**2*d - x**(2*n)*int((x**m*x)/(x**(2*n)*a**2*m - 2*x**(2*n)*a*
*2*n + x**(2*n)*a**2 + 2*x**(2*n)*a*b*m*x - 4*x**(2*n)*a*b*n*x + 2*x**(...
```

3.369 $\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^3}\,dx$

Optimal result

Integrand size = 26, antiderivative size = 99

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^3}\,dx = -\frac{dx^{1-3n}(ex)^m}{b(1-m+3n)(a+bx)^2} + \frac{\left(\frac{c}{1+m-3n}+\frac{ad}{b-bm+3bn}\right)x^{1-3n}(ex)^m \text{Hypergeometric2F1}\left(3,1+m-3n,2+m-3n,-\frac{bx}{a}\right)}{a^3}$$

output
```
-d*x^(1-3*n)*(e*x)^m/b/(1-m+3*n)/(b*x+a)^2+(c/(1+m-3*n)+a*d/(-b*m+3*b*n+b)
)*x^(1-3*n)*(e*x)^m*hypergeom([3, 1+m-3*n],[2+m-3*n],-b*x/a)/a^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.94

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^3}\,dx = \frac{x^{1-3n}(ex)^m\left(\frac{a^2(bc-ad)}{(a+bx)^2}-\frac{(bc(-1+m-3n)-ad(1+m-3n))\,\text{Hypergeometric2F1}\left(2,1+m-3n,2+m-3n,-\frac{bx}{a}\right)}{1+m-3n}\right)}{2a^3b}$$

input
```
Integrate[((e*x)^m*(c + d*x))/(a*x^n + b*x^(1 + n))^3,x]
```

output
```
(x^(1 - 3*n)*(e*x)^m*((a^2*(b*c - a*d))/(a + b*x)^2 - ((b*c*(-1 + m - 3*n)
 - a*d*(1 + m - 3*n))*Hypergeometric2F1[2, 1 + m - 3*n, 2 + m - 3*n, -((b*
x)/a)])/(1 + m - 3*n)))/(2*a^3*b)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.14, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {2027, 30, 87, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ex)^m}{(ax^n+bx^{n+1})^3}dx$$

$$\Big\downarrow \ 2027$$

$$\int \frac{x^{-3n}(c+dx)(ex)^m}{(a+bx)^3}dx$$

$$\Big\downarrow \ 30$$

$$x^{-m}(ex)^m\int \frac{x^{m-3n}(c+dx)}{(a+bx)^3}dx$$

$$\Big\downarrow \ 87$$

$$x^{-m}(ex)^m\left(\frac{(ad(m-3n+1)+bc(-m+3n+1))\int \frac{x^{m-3n}}{(a+bx)^2}dx}{2ab}+\frac{x^{m-3n+1}(bc-ad)}{2ab(a+bx)^2}\right)$$

$$\Big\downarrow \ 74$$

$$x^{-m}(ex)^m\left(\frac{x^{m-3n+1}(ad(m-3n+1)+bc(-m+3n+1))\text{Hypergeometric2F1}\left(2,m-3n+1,m-3n+2,-\frac{b}{}\right.}{2a^3b(m-3n+1)}\right.$$

input
```
Int[((e*x)^m*(c + d*x))/(a*x^n + b*x^(1 + n))^3,x]
```

output
```
((e*x)^m*(((b*c - a*d)*x^(1 + m - 3*n))/(2*a*b*(a + b*x)^2) + ((a*d*(1 + m
 - 3*n) + b*c*(1 - m + 3*n))*x^(1 + m - 3*n)*Hypergeometric2F1[2, 1 + m -
3*n, 2 + m - 3*n, -((b*x)/a)])/(2*a^3*b*(1 + m - 3*n))))/x^m
```

Defintions of rubi rules used

rule 30
```
Int[(u_.)*((a_.)*(x_))^(m_.)*((b_.)*(x_)^(i_.))^(p_), x_Symbol] :> Simp[b^I
ntPart[p]*((b*x^i)^FracPart[p]/(a^(i*IntPart[p])*(a*x)^(i*FracPart[p])))
Int[u*(a*x)^(m + i*p), x], x] /; FreeQ[{a, b, i, m, p}, x] && IntegerQ[i] &
&  !IntegerQ[p]
```

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2027
```
Int[(Fx_.)*((a_.)*(x_)^(r_.) + (b_.)*(x_)^(s_.))^(p_.), x_Symbol] :> Int[x^
(p*r)*(a + b*x^(s - r))^p*Fx, x] /; FreeQ[{a, b, r, s}, x] && IntegerQ[p] &
& PosQ[s - r] &&  !(EqQ[p, 1] && EqQ[u, 1])
```

Maple [F]

$$\int \frac{(ex)^m (dx+c)}{(a\,x^n+b\,x^{1+n})^3}dx$$

input
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^3,x)
```

output
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^3,x)
```

Fricas [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^3}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^{n+1}+ax^n)^3}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^3,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(e*x)^m/(3*a^2*b*x^(2*n)*x^(n + 1) + a^3*x^(3*n) + (b^3
*x^(n + 1) + 3*a*b^2*x^n)*x^(2*n + 2)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^3}\,dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(d*x+c)/(a*x**n+b*x**(1+n))**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^3}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^{n+1}+ax^n)^3}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^(n + 1) + a*x^n)^3, x)
```

Giac [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^3}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^{n+1}+ax^n)^3}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^(n + 1) + a*x^n)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^3}\,dx = \int \frac{(e\,x)^m\,(c+d\,x)}{(a\,x^n+b\,x^{n+1})^3}\,dx$$

input
```
int(((e*x)^m*(c + d*x))/(a*x^n + b*x^(n + 1))^3,x)
```

output
```
int(((e*x)^m*(c + d*x))/(a*x^n + b*x^(n + 1))^3, x)
```

Reduce [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^3}\,dx = \text{too large to display}$$

input
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^3,x)
```

output
```
(e**m*(x**m*c*x + x**(3*n)*int((x**m*x)/(x**(3*n)*a**3*m - 3*x**(3*n)*a**3
*n + x**(3*n)*a**3 + 3*x**(3*n)*a**2*b*m*x - 9*x**(3*n)*a**2*b*n*x + 3*x**
(3*n)*a**2*b*x + 3*x**(3*n)*a*b**2*m*x**2 - 9*x**(3*n)*a*b**2*n*x**2 + 3*x
**(3*n)*a*b**2*x**2 + x**(3*n)*b**3*m*x**3 - 3*x**(3*n)*b**3*n*x**3 + x**(
3*n)*b**3*x**3),x)*a**3*d*m**2 - 6*x**(3*n)*int((x**m*x)/(x**(3*n)*a**3*m
 - 3*x**(3*n)*a**3*n + x**(3*n)*a**3 + 3*x**(3*n)*a**2*b*m*x - 9*x**(3*n)*a
**2*b*n*x + 3*x**(3*n)*a**2*b*x + 3*x**(3*n)*a*b**2*m*x**2 - 9*x**(3*n)*a*
b**2*n*x**2 + 3*x**(3*n)*a*b**2*x**2 + x**(3*n)*b**3*m*x**3 - 3*x**(3*n)*b
**3*n*x**3 + x**(3*n)*b**3*x**3),x)*a**3*d*m*n + 2*x**(3*n)*int((x**m*x)/(
x**(3*n)*a**3*m - 3*x**(3*n)*a**3*n + x**(3*n)*a**3 + 3*x**(3*n)*a**2*b*m*
x - 9*x**(3*n)*a**2*b*n*x + 3*x**(3*n)*a**2*b*x + 3*x**(3*n)*a*b**2*m*x**2
 - 9*x**(3*n)*a*b**2*n*x**2 + 3*x**(3*n)*a*b**2*x**2 + x**(3*n)*b**3*m*x**
3 - 3*x**(3*n)*b**3*n*x**3 + x**(3*n)*b**3*x**3),x)*a**3*d*m + 9*x**(3*n)*
int((x**m*x)/(x**(3*n)*a**3*m - 3*x**(3*n)*a**3*n + x**(3*n)*a**3 + 3*x**(
3*n)*a**2*b*m*x - 9*x**(3*n)*a**2*b*n*x + 3*x**(3*n)*a**2*b*x + 3*x**(3*n)
*a*b**2*m*x**2 - 9*x**(3*n)*a*b**2*n*x**2 + 3*x**(3*n)*a*b**2*x**2 + x**(3
*n)*b**3*m*x**3 - 3*x**(3*n)*b**3*n*x**3 + x**(3*n)*b**3*x**3),x)*a**3*d*n
**2 - 6*x**(3*n)*int((x**m*x)/(x**(3*n)*a**3*m - 3*x**(3*n)*a**3*n + x**(3
*n)*a**3 + 3*x**(3*n)*a**2*b*m*x - 9*x**(3*n)*a**2*b*n*x + 3*x**(3*n)*a**2
*b*x + 3*x**(3*n)*a*b**2*m*x**2 - 9*x**(3*n)*a*b**2*n*x**2 + 3*x**(3*n)...
```

3.370 $\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^{5/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 147

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^{5/2}\,dx=\frac{2dx^{1-n}(ex)^m\left(ax^n+bx^{1+n}\right)^{7/2}}{b(9+2m+5n)}$$
$$+\frac{2\left(bc-\frac{ad(2+2m+5n)}{9+2m+5n}\right)x^{-n}\left(-\frac{bx}{a}\right)^{-m-\frac{5n}{2}}(ex)^m\left(ax^n+bx^{1+n}\right)^{7/2}\text{Hypergeometric2F1}\left(\frac{7}{2},-m-\frac{5n}{2},\frac{9}{2},1+\frac{bx}{a}\right.}{7b^2}$$

output
```
2*d*x^(1-n)*(e*x)^m*(a*x^n+b*x^(1+n))^(7/2)/b/(9+2*m+5*n)+2/7*(b*c-a*d*(2+
2*m+5*n)/(9+2*m+5*n))*(-b*x/a)^(-m-5/2*n)*(e*x)^m*(a*x^n+b*x^(1+n))^(7/2)*
hypergeom([7/2, -m-5/2*n],[9/2],1+b*x/a)/b^2/(x^n)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.86

$$\int (ex)^m(c+dx)\left(ax^n\right.$$
$$\left.+bx^{1+n}\right)^{5/2}\,dx=\frac{2\left(-\frac{bx}{a}\right)^{-m-\frac{5n}{2}}(ex)^m(a+bx)\left(x^n(a+bx)\right)^{5/2}\left(7bdx\left(-\frac{bx}{a}\right)^{m+\frac{5n}{2}}+(-ad(2+2m+5n)+b\right.}{7b^2(9+2m+5n)}$$

input
```
Integrate[(e*x)^m*(c + d*x)*(a*x^n + b*x^(1 + n))^(5/2),x]
```

output
```
(2*(-((b*x)/a))^(-m - (5*n)/2)*(e*x)^m*(a + b*x)*(x^n*(a + b*x))^(5/2)*(7*
b*d*x*(-((b*x)/a))^(m + (5*n)/2) + (-(a*d*(2 + 2*m + 5*n)) + b*c*(9 + 2*m
+ 5*n))*Hypergeometric2F1[7/2, -m - (5*n)/2, 9/2, 1 + (b*x)/a]))/(7*b^2*(9
 + 2*m + 5*n))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1948, 90, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(ex)^m \left(ax^n+bx^{n+1}\right)^{5/2} dx$$

$$\Big\downarrow \text{ 1948}$$

$$\frac{(ex)^m x^{-m-\frac{n}{2}} \sqrt{ax^n+bx^{n+1}} \int x^{m+\frac{5n}{2}} (a+bx)^{5/2}(c+dx)dx}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{(ex)^m x^{-m-\frac{n}{2}} \sqrt{ax^n+bx^{n+1}} \left(\left(c-\frac{ad(2m+5n+2)}{b(2m+5n+9)}\right) \int x^{m+\frac{5n}{2}} (a+bx)^{5/2}dx+\frac{2d(a+bx)^{7/2}x^{m+\frac{5n}{2}+1}}{b(2m+5n+9)}\right)}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 77}$$

$$\frac{(ex)^m x^{-m-\frac{n}{2}} \sqrt{ax^n+bx^{n+1}} \left(x^{m+\frac{5n}{2}} \left(-\frac{bx}{a}\right)^{-m-\frac{5n}{2}} \left(c-\frac{ad(2m+5n+2)}{b(2m+5n+9)}\right) \int \left(-\frac{bx}{a}\right)^{m+\frac{5n}{2}} (a+bx)^{5/2}dx+\frac{2d(a+bx)^{7/2}x^{m}}{b(2m+5n}\right.}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 75}$$

$$\frac{(ex)^m x^{-m-\frac{n}{2}} \sqrt{ax^n+bx^{n+1}} \left(\frac{2(a+bx)^{7/2}x^{m+\frac{5n}{2}} \left(-\frac{bx}{a}\right)^{-m-\frac{5n}{2}} \left(c-\frac{ad(2m+5n+2)}{b(2m+5n+9)}\right) \text{Hypergeometric2F1}\left(\frac{7}{2},-m-\frac{5n}{2},\frac{9}{2},\frac{bx}{a}+1\right)}{7b}+\frac{2d(}{}\right.}{\sqrt{a+bx}}$$

input
```
Int[(e*x)^m*(c + d*x)*(a*x^n + b*x^(1 + n))^(5/2),x]
```

output
```
(x^(-m - n/2)*(e*x)^m*Sqrt[a*x^n + b*x^(1 + n)]*((2*d*x^(1 + m + (5*n)/2)*
(a + b*x)^(7/2))/(b*(9 + 2*m + 5*n)) + (2*(c - (a*d*(2 + 2*m + 5*n))/(b*(9
 + 2*m + 5*n)))*x^(m + (5*n)/2)*(-((b*x)/a))^(-m - (5*n)/2)*(a + b*x)^(7/2
)*Hypergeometric2F1[7/2, -m - (5*n)/2, 9/2, 1 + (b*x)/a])/(7*b)))/Sqrt[a +
 b*x]
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] && !IntegerQ[p]
 && NeQ[b*c - a*d, 0] && !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int (ex)^m (dx + c) \left(a\,x^n + b\,x^{1+n}\right)^{\frac{5}{2}} dx$$

input
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(5/2),x)
```

output
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^m(c + dx) \left(ax^n + bx^{1+n}\right)^{5/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m(c + dx) \left(ax^n + bx^{1+n}\right)^{5/2} dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(d*x+c)*(a*x**n+b*x**(1+n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^{5/2}\,dx=\int \left(bx^{n+1}+ax^n\right)^{\frac{5}{2}}(dx+c)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(5/2)*(d*x + c)*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^{5/2}\,dx=\int \left(bx^{n+1}+ax^n\right)^{\frac{5}{2}}(dx+c)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(5/2)*(d*x + c)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^{5/2}\,dx=\int (e\,x)^m\,\left(a\,x^n+b\,x^{n+1}\right)^{5/2}\,(c+d\,x)\,dx$$

input
```
int((e*x)^m*(a*x^n + b*x^(n + 1))^(5/2)*(c + d*x),x)
```

output
```
int((e*x)^m*(a*x^n + b*x^(n + 1))^(5/2)*(c + d*x), x)
```

Reduce [F]

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^{5/2}\,dx=\int (ex)^m\,(dx+c)\left(x^n a+b\,x^{1+n}\right)^{\frac{5}{2}}dx$$

input

```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(5/2),x)
```

output

```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(5/2),x)
```

3.371 $\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^{3/2}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 147

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^{3/2}\,dx=\frac{2dx^{1-n}(ex)^m\left(ax^n+bx^{1+n}\right)^{5/2}}{b(7+2m+3n)}+\frac{2\left(bc-\frac{ad(2+2m+3n)}{7+2m+3n}\right)x^{-n}\left(-\frac{bx}{a}\right)^{-m-\frac{3n}{2}}(ex)^m\left(ax^n+bx^{1+n}\right)^{5/2}\text{Hypergeometric2F1}\left(\frac{5}{2},-m-\frac{3n}{2},\frac{7}{2},1+\frac{bx}{a}\right)}{5b^2}$$

output
```
2*d*x^(1-n)*(e*x)^m*(a*x^n+b*x^(1+n))^(5/2)/b/(7+2*m+3*n)+2/5*(b*c-a*d*(2+
2*m+3*n)/(7+2*m+3*n))*(-b*x/a)^(-m-3/2*n)*(e*x)^m*(a*x^n+b*x^(1+n))^(5/2)*
hypergeom([5/2, -m-3/2*n],[7/2],1+b*x/a)/b^2/(x^n)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.86

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^{3/2}\,dx=\frac{2\left(-\frac{bx}{a}\right)^{-m-\frac{3n}{2}}(ex)^m(a+bx)\left(x^n(a+bx)\right)^{3/2}\left(5bdx\left(-\frac{bx}{a}\right)^{m+\frac{3n}{2}}+(-ad(2+2m+3n)+b\right.}{5b^2(7+2m+3n)}$$

input
```
Integrate[(e*x)^m*(c + d*x)*(a*x^n + b*x^(1 + n))^(3/2),x]
```

output
```
(2*(-((b*x)/a))^(-m - (3*n)/2)*(e*x)^m*(a + b*x)*(x^n*(a + b*x))^(3/2)*(5*
b*d*x*(-((b*x)/a))^(m + (3*n)/2) + (-(a*d*(2 + 2*m + 3*n)) + b*c*(7 + 2*m
+ 3*n))*Hypergeometric2F1[5/2, -m - (3*n)/2, 7/2, 1 + (b*x)/a]))/(5*b^2*(7
 + 2*m + 3*n))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1948, 90, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(ex)^m \left(ax^n+bx^{n+1}\right)^{3/2} dx$$

$$\Big\downarrow \text{ 1948}$$

$$\frac{(ex)^m x^{-m-\frac{n}{2}}\sqrt{ax^n+bx^{n+1}}\int x^{m+\frac{3n}{2}}(a+bx)^{3/2}(c+dx)dx}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 90}$$

$$\frac{(ex)^m x^{-m-\frac{n}{2}}\sqrt{ax^n+bx^{n+1}}\left(\left(c-\frac{ad(2m+3n+2)}{b(2m+3n+7)}\right)\int x^{m+\frac{3n}{2}}(a+bx)^{3/2}dx+\frac{2d(a+bx)^{5/2}x^{m+\frac{3n}{2}+1}}{b(2m+3n+7)}\right)}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 77}$$

$$\frac{(ex)^m x^{-m-\frac{n}{2}}\sqrt{ax^n+bx^{n+1}}\left(x^{m+\frac{3n}{2}}\left(-\frac{bx}{a}\right)^{-m-\frac{3n}{2}}\left(c-\frac{ad(2m+3n+2)}{b(2m+3n+7)}\right)\int \left(-\frac{bx}{a}\right)^{m+\frac{3n}{2}}(a+bx)^{3/2}dx+\frac{2d(a+bx)^{5/2}x^{m}}{b(2m+3n}\right.}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{ 75}$$

$$\frac{(ex)^m x^{-m-\frac{n}{2}}\sqrt{ax^n+bx^{n+1}}\left(\frac{2(a+bx)^{5/2}x^{m+\frac{3n}{2}}\left(-\frac{bx}{a}\right)^{-m-\frac{3n}{2}}\left(c-\frac{ad(2m+3n+2)}{b(2m+3n+7)}\right)\text{Hypergeometric2F1}\left(\frac{5}{2},-m-\frac{3n}{2},\frac{7}{2},\frac{bx}{a}+1\right)}{5b}+\frac{2d(}{}\right.}{\sqrt{a+bx}}$$

input
```
Int[(e*x)^m*(c + d*x)*(a*x^n + b*x^(1 + n))^(3/2),x]
```

output
```
(x^(-m - n/2)*(e*x)^m*Sqrt[a*x^n + b*x^(1 + n)]*((2*d*x^(1 + m + (3*n)/2)*
(a + b*x)^(5/2))/(b*(7 + 2*m + 3*n)) + (2*(c - (a*d*(2 + 2*m + 3*n))/(b*(7
 + 2*m + 3*n)))*x^(m + (3*n)/2)*(-((b*x)/a))^(-m - (3*n)/2)*(a + b*x)^(5/2
)*Hypergeometric2F1[5/2, -m - (3*n)/2, 7/2, 1 + (b*x)/a])/(5*b)))/Sqrt[a +
 b*x]
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] && !IntegerQ[p]
 && NeQ[b*c - a*d, 0] && !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int (ex)^m (dx+c) \left(a\,x^n+b\,x^{1+n}\right)^{\frac{3}{2}} dx$$

input
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^m(c+dx) \left(ax^n+bx^{1+n}\right)^{3/2} dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m(c+dx) \left(ax^n+bx^{1+n}\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((e*x)**m*(d*x+c)*(a*x**n+b*x**(1+n))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^{3/2}\,dx=\int \left(bx^{n+1}+ax^n\right)^{\frac{3}{2}}(dx+c)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(3/2)*(d*x + c)*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^{3/2}\,dx=\int \left(bx^{n+1}+ax^n\right)^{\frac{3}{2}}(dx+c)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*x^(n + 1) + a*x^n)^(3/2)*(d*x + c)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^{3/2}\,dx=\int {\left(e\,x\right)}^m\,{\left(a\,x^n+b\,x^{n+1}\right)}^{3/2}\,\left(c+d\,x\right)\,dx$$

input
```
int((e*x)^m*(a*x^n + b*x^(n + 1))^(3/2)*(c + d*x),x)
```

output
```
int((e*x)^m*(a*x^n + b*x^(n + 1))^(3/2)*(c + d*x), x)
```

Reduce [F]

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^{3/2}dx = e^m\left(\left(\int x^{m+\frac{3n}{2}}\sqrt{bx+a}\,x^2dx\right)bd+\left(\int x^{m+\frac{3n}{2}}\sqrt{bx+a}\,xdx\right)ad+\left(\int x^{m+\frac{3n}{2}}\sqrt{bx+a}\,xdx\right)bc+\left(\int x^{m+\frac{3n}{2}}\sqrt{bx+a}dx\right)ac\right)$$

input

```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(3/2),x)
```

output

```
e**m*(int(x**((2*m + 3*n)/2)*sqrt(a + b*x)*x**2,x)*b*d + int(x**((2*m + 3*
n)/2)*sqrt(a + b*x)*x,x)*a*d + int(x**((2*m + 3*n)/2)*sqrt(a + b*x)*x,x)*b
*c + int(x**((2*m + 3*n)/2)*sqrt(a + b*x),x)*a*c)
```

3.372 $\int (ex)^m(c+dx)\sqrt{ax^n+bx^{1+n}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 141

$$\int (ex)^m(c+dx)\sqrt{ax^n+bx^{1+n}}\,dx = \frac{2dx^{1-n}(ex)^m\left(ax^n+bx^{1+n}\right)^{3/2}}{b(5+2m+n)} + \frac{2\left(bc-\frac{ad(2+2m+n)}{5+2m+n}\right)x^{-n}\left(-\frac{bx}{a}\right)^{-m-\frac{n}{2}}(ex)^m\left(ax^n+bx^{1+n}\right)^{3/2}\text{Hypergeometric2F1}\left(\frac{3}{2},-m-\frac{n}{2},\frac{5}{2},1+\frac{bx}{a}\right.}{3b^2}$$

output
```
2*d*x^(1-n)*(e*x)^m*(a*x^n+b*x^(1+n))^(3/2)/b/(5+2*m+n)+2/3*(b*c-a*d*(2+2*
m+n)/(5+2*m+n))*(-b*x/a)^(-m-1/2*n)*(e*x)^m*(a*x^n+b*x^(1+n))^(3/2)*hyperg
eom([3/2, -m-1/2*n],[5/2],1+b*x/a)/b^2/(x^n)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.86

$$\int (ex)^m(c+dx)\sqrt{ax^n+bx^{1+n}}\,dx = \frac{2\left(-\frac{bx}{a}\right)^{-m-\frac{n}{2}}(ex)^m(a+bx)\sqrt{x^n(a+bx)}\left(3bdx\left(-\frac{bx}{a}\right)^{m+\frac{n}{2}}+(-ad(2+2m+n)+bc(5+2m+n))\,\text{Hyp}\right.}{3b^2(5+2m+n)}$$

input
```
Integrate[(e*x)^m*(c + d*x)*Sqrt[a*x^n + b*x^(1 + n)],x]
```

output
```
(2*(-((b*x)/a))^(-m - n/2)*(e*x)^m*(a + b*x)*Sqrt[x^n*(a + b*x)]*(3*b*d*x*
(-((b*x)/a))^(m + n/2) + (-(a*d*(2 + 2*m + n)) + b*c*(5 + 2*m + n))*Hyperg
eometric2F1[3/2, -m - n/2, 5/2, 1 + (b*x)/a]))/(3*b^2*(5 + 2*m + n))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1948, 90, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(ex)^m\sqrt{ax^n+bx^{n+1}}\,dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{(ex)^m x^{-m-\frac{n}{2}}\sqrt{ax^n+bx^{n+1}}\int x^{m+\frac{n}{2}}\sqrt{a+bx}(c+dx)dx}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{90}$$

$$\frac{(ex)^m x^{-m-\frac{n}{2}}\sqrt{ax^n+bx^{n+1}}\left(\left(c-\frac{ad(2m+n+2)}{b(2m+n+5)}\right)\int x^{m+\frac{n}{2}}\sqrt{a+bx}dx+\frac{2d(a+bx)^{3/2}x^{m+\frac{n}{2}+1}}{b(2m+n+5)}\right)}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{77}$$

$$\frac{(ex)^m x^{-m-\frac{n}{2}}\sqrt{ax^n+bx^{n+1}}\left(x^{m+\frac{n}{2}}\left(-\frac{bx}{a}\right)^{-m-\frac{n}{2}}\left(c-\frac{ad(2m+n+2)}{b(2m+n+5)}\right)\int \left(-\frac{bx}{a}\right)^{m+\frac{n}{2}}\sqrt{a+bx}dx+\frac{2d(a+bx)^{3/2}x^{m+\frac{n}{2}+1}}{b(2m+n+5)}\right.}{\sqrt{a+bx}}$$

$$\Big\downarrow \text{75}$$

$$\frac{(ex)^m x^{-m-\frac{n}{2}}\sqrt{ax^n+bx^{n+1}}\left(\frac{2(a+bx)^{3/2}x^{m+\frac{n}{2}}\left(-\frac{bx}{a}\right)^{-m-\frac{n}{2}}\left(c-\frac{ad(2m+n+2)}{b(2m+n+5)}\right)\text{Hypergeometric2F1}\left(\frac{3}{2},-m-\frac{n}{2},\frac{5}{2},\frac{bx}{a}+1\right)}{3b}+\frac{2d(a+b}{b(2}\right.}{\sqrt{a+bx}}$$

input
```
Int[(e*x)^m*(c + d*x)*Sqrt[a*x^n + b*x^(1 + n)],x]
```

output
```
(x^(-m - n/2)*(e*x)^m*Sqrt[a*x^n + b*x^(1 + n)]*((2*d*x^(1 + m + n/2)*(a +
 b*x)^(3/2))/(b*(5 + 2*m + n)) + (2*(c - (a*d*(2 + 2*m + n))/(b*(5 + 2*m +
 n)))*x^(m + n/2)*(-((b*x)/a))^(-m - n/2)*(a + b*x)^(3/2)*Hypergeometric2F
1[3/2, -m - n/2, 5/2, 1 + (b*x)/a])/(3*b)))/Sqrt[a + b*x]
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int (ex)^m (dx+c)\sqrt{a\,x^n+b\,x^{1+n}}dx$$

input
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(1/2),x)
```

output
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (ex)^m(c+dx)\sqrt{ax^n+bx^{1+n}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int (ex)^m(c+dx)\sqrt{ax^n+bx^{1+n}}\,dx = \int (ex)^m\,(c+dx)\,\sqrt{ax^n+bx^{n+1}}\,dx$$

input
```
integrate((e*x)**m*(d*x+c)*(a*x**n+b*x**(1+n))**(1/2),x)
```

output
```
Integral((e*x)**m*(c + d*x)*sqrt(a*x**n + b*x**(n + 1)), x)
```

Maxima [F]

$$\int (ex)^m(c+dx)\sqrt{ax^n+bx^{1+n}}\,dx = \int \sqrt{bx^{n+1}+ax^n}(dx+c)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*x^(n + 1) + a*x^n)*(d*x + c)*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m(c+dx)\sqrt{ax^n+bx^{1+n}}\,dx = \int \sqrt{bx^{n+1}+ax^n}(dx+c)(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*x^(n + 1) + a*x^n)*(d*x + c)*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m(c+dx)\sqrt{ax^n+bx^{1+n}}\,dx = \int (e\,x)^m\,\sqrt{a\,x^n+b\,x^{n+1}}\,(c+d\,x)\,dx$$

input
```
int((e*x)^m*(a*x^n + b*x^(n + 1))^(1/2)*(c + d*x),x)
```

output
```
int((e*x)^m*(a*x^n + b*x^(n + 1))^(1/2)*(c + d*x), x)
```

Reduce [F]

$$\int (ex)^m (c+dx)\sqrt{ax^n+bx^{1+n}}\,dx = e^m\left(\left(\int x^{m+\frac{n}{2}}\sqrt{bx+a}\,xdx\right)d + \left(\int x^{m+\frac{n}{2}}\sqrt{bx+a}dx\right)c\right)$$

input
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^(1/2),x)
```

output
```
e**m*(int(x**((2*m + n)/2)*sqrt(a + b*x)*x,x)*d + int(x**((2*m + n)/2)*sqr
t(a + b*x),x)*c)
```

3.373 $\int \frac{(ex)^m(c+dx)}{\sqrt{ax^n+bx^{1+n}}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 145

$$\int \frac{(ex)^m(c+dx)}{\sqrt{ax^n+bx^{1+n}}}\,dx = \frac{2dx^{1-n}(ex)^m\sqrt{ax^n+bx^{1+n}}}{b(3+2m-n)} + \frac{2\left(bc-\frac{ad(2+2m-n)}{3+2m-n}\right)x^{-n}\left(-\frac{bx}{a}\right)^{\frac{1}{2}(-2m+n)}(ex)^m\sqrt{ax^n+bx^{1+n}}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(-2m+n),\frac{3}{2},1\right.}{b^2}$$

output
```
2*d*x^(1-n)*(e*x)^m*(a*x^n+b*x^(1+n))^(1/2)/b/(3+2*m-n)+2*(b*c-a*d*(2+2*m-
n)/(3+2*m-n))*(-b*x/a)^(-m+1/2*n)*(e*x)^m*(a*x^n+b*x^(1+n))^(1/2)*hypergeo
m([1/2, -m+1/2*n],[3/2],1+b*x/a)/b^2/(x^n)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.73

$$\int \frac{(ex)^m(c+dx)}{\sqrt{ax^n+bx^{1+n}}}\,dx = \frac{2(ex)^m(a+bx)\left(bdx+(bc(3+2m-n)+ad(-2-2m+n))\left(-\frac{bx}{a}\right)^{\frac{1}{2}(-2m+n)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(\right.\right.}{b^2(3+2m-n)\sqrt{x^n(a+bx)}}$$

input
```
Integrate[((e*x)^m*(c + d*x))/Sqrt[a*x^n + b*x^(1 + n)],x]
```

output
```
(2*(e*x)^m*(a + b*x)*(b*d*x + (b*c*(3 + 2*m - n) + a*d*(-2 - 2*m + n))*(-(
(b*x)/a))^((-2*m + n)/2)*Hypergeometric2F1[1/2, (-2*m + n)/2, 3/2, 1 + (b*
x)/a]))/(b^2*(3 + 2*m - n)*Sqrt[x^n*(a + b*x)])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1948, 90, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ex)^m}{\sqrt{ax^n+bx^{n+1}}}dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{\sqrt{a+bx}(ex)^m x^{\frac{1}{2}(n-2m)}\int \frac{x^{m-\frac{n}{2}}(c+dx)}{\sqrt{a+bx}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{90}$$

$$\frac{\sqrt{a+bx}(ex)^m x^{\frac{1}{2}(n-2m)}\left(\left(c-\frac{ad(2m-n+2)}{b(2m-n+3)}\right)\int \frac{x^{m-\frac{n}{2}}}{\sqrt{a+bx}}dx+\frac{2d\sqrt{a+bx}x^{m-\frac{n}{2}+1}}{b(2m-n+3)}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{77}$$

$$\frac{\sqrt{a+bx}(ex)^m x^{\frac{1}{2}(n-2m)}\left(x^{m-\frac{n}{2}}\left(-\frac{bx}{a}\right)^{\frac{1}{2}(n-2m)}\left(c-\frac{ad(2m-n+2)}{b(2m-n+3)}\right)\int \frac{\left(-\frac{bx}{a}\right)^{m-\frac{n}{2}}}{\sqrt{a+bx}}dx+\frac{2d\sqrt{a+bx}x^{m-\frac{n}{2}+1}}{b(2m-n+3)}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{75}$$

$$\frac{\sqrt{a+bx}(ex)^m x^{\frac{1}{2}(n-2m)}\left(\frac{2\sqrt{a+bx}x^{m-\frac{n}{2}}\left(-\frac{bx}{a}\right)^{\frac{1}{2}(n-2m)}\left(c-\frac{ad(2m-n+2)}{b(2m-n+3)}\right)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1}{2}(n-2m),\frac{3}{2},\frac{bx}{a}+1\right)}{b}+\frac{2d\sqrt{a+bx}x^{m-\frac{n}{2}+1}}{b(2m-n+3)}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

input
```
Int[((e*x)^m*(c + d*x))/Sqrt[a*x^n + b*x^(1 + n)],x]
```

output
```
(x^((-2*m + n)/2)*(e*x)^m*Sqrt[a + b*x]*((2*d*x^(1 + m - n/2)*Sqrt[a + b*x
])/(b*(3 + 2*m - n)) + (2*(c - (a*d*(2 + 2*m - n))/(b*(3 + 2*m - n)))*x^(m
 - n/2)*(-((b*x)/a))^((-2*m + n)/2)*Sqrt[a + b*x]*Hypergeometric2F1[1/2, (
-2*m + n)/2, 3/2, 1 + (b*x)/a])/b))/Sqrt[a*x^n + b*x^(1 + n)]
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&
 !IntegerQ[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{(ex)^m (dx+c)}{\sqrt{a\,x^n+b\,x^{1+n}}}dx$$

input
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(1/2),x)
```

output
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ex)^m(c+dx)}{\sqrt{ax^n+bx^{1+n}}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(ex)^m(c+dx)}{\sqrt{ax^n+bx^{1+n}}}\,dx = \int \frac{(ex)^m\,(c+dx)}{\sqrt{ax^n+bx^{n+1}}}\,dx$$

input
```
integrate((e*x)**m*(d*x+c)/(a*x**n+b*x**(1+n))**(1/2),x)
```

output
```
Integral((e*x)**m*(c + d*x)/sqrt(a*x**n + b*x**(n + 1)), x)
```

Maxima [F]

$$\int \frac{(ex)^m(c+dx)}{\sqrt{ax^n+bx^{1+n}}}\,dx=\int \frac{(dx+c)(ex)^m}{\sqrt{bx^{n+1}+ax^n}}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^m/sqrt(b*x^(n + 1) + a*x^n), x)
```

Giac [F]

$$\int \frac{(ex)^m(c+dx)}{\sqrt{ax^n+bx^{1+n}}}\,dx=\int \frac{(dx+c)(ex)^m}{\sqrt{bx^{n+1}+ax^n}}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(1/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(e*x)^m/sqrt(b*x^(n + 1) + a*x^n), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(c+dx)}{\sqrt{ax^n+bx^{1+n}}}\,dx=\int \frac{{(e\,x)}^m\,(c+d\,x)}{\sqrt{a\,x^n+b\,x^{n+1}}}\,dx$$

input
```
int(((e*x)^m*(c + d*x))/(a*x^n + b*x^(n + 1))^(1/2),x)
```

output
```
int(((e*x)^m*(c + d*x))/(a*x^n + b*x^(n + 1))^(1/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m(c+dx)}{\sqrt{ax^n+bx^{1+n}}}\,dx = e^m\left(\left(\int \frac{x^m}{x^{\frac{n}{2}}\sqrt{bx+a}}dx\right)c+\left(\int \frac{x^m x}{x^{\frac{n}{2}}\sqrt{bx+a}}dx\right)d\right)$$

input
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(1/2),x)
```

output
```
e**m*(int(x**m/(x**(n/2)*sqrt(a + b*x)),x)*c + int((x**m*x)/(x**(n/2)*sqrt
(a + b*x)),x)*d)
```

3.374 $\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx$

Optimal result

Integrand size = 28, antiderivative size = 145

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx = \frac{2dx^{1-n}(ex)^m}{b(1+2m-3n)\sqrt{ax^n+bx^{1+n}}} - \frac{2\left(bc-\frac{ad(2+2m-3n)}{1+2m-3n}\right)x^{-n}\left(-\frac{bx}{a}\right)^{-m+\frac{3n}{2}}(ex)^m\,\text{Hypergeometric2F1}\left(-\frac{1}{2},-m+\frac{3n}{2},\frac{1}{2},1+\frac{bx}{a}\right)}{b^2\sqrt{ax^n+bx^{1+n}}}$$

output
```
2*d*x^(1-n)*(e*x)^m/b/(1+2*m-3*n)/(a*x^n+b*x^(1+n))^(1/2)-2*(b*c-a*d*(2+2*
m-3*n)/(1+2*m-3*n))*(-b*x/a)^(-m+3/2*n)*(e*x)^m*hypergeom([-1/2, -m+3/2*n]
,[1/2],1+b*x/a)/b^2/(x^n)/(a*x^n+b*x^(1+n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.79

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx = \frac{2(ex)^m(a+bx)\left(b(bc-ad)x-(bc(1+2m-3n)+ad(-2-2m+3n))\left(-\frac{bx}{a}\right)^{-}\right.}{ab^2\left(x^n(a+bx)\right)^{3/2}}$$

input
```
Integrate[((e*x)^m*(c + d*x))/(a*x^n + b*x^(1 + n))^(3/2),x]
```

output
```
(2*(e*x)^m*(a + b*x)*(b*(b*c - a*d)*x - (b*c*(1 + 2*m - 3*n) + a*d*(-2 - 2
*m + 3*n))*(-((b*x)/a))^(-m + (3*n)/2)*(a + b*x)*Hypergeometric2F1[1/2, -m
 + (3*n)/2, 3/2, 1 + (b*x)/a]))/(a*b^2*(x^n*(a + b*x))^(3/2))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {1948, 87, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ex)^m}{(ax^n+bx^{n+1})^{3/2}}\,dx$$

$$\Big\downarrow \text{1948}$$

$$\frac{\sqrt{a+bx}(ex)^m x^{\frac{1}{2}(n-2m)}\int \frac{x^{m-\frac{3n}{2}}(c+dx)}{(a+bx)^{3/2}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{87}$$

$$\frac{\sqrt{a+bx}(ex)^m x^{\frac{1}{2}(n-2m)}\left(\frac{2x^{m-\frac{3n}{2}+1}(bc-ad)}{ab\sqrt{a+bx}}-\frac{(bc(2m-3n+1)-ad(2m-3n+2))\int \frac{x^{m-\frac{3n}{2}}}{\sqrt{a+bx}}dx}{ab}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{77}$$

$$\frac{\sqrt{a+bx}(ex)^m x^{\frac{1}{2}(n-2m)}\left(\frac{2x^{m-\frac{3n}{2}+1}(bc-ad)}{ab\sqrt{a+bx}}-\frac{x^{m-\frac{3n}{2}}\left(-\frac{bx}{a}\right)^{\frac{3n}{2}-m}(bc(2m-3n+1)-ad(2m-3n+2))\int \frac{\left(-\frac{bx}{a}\right)^{m-\frac{3n}{2}}}{\sqrt{a+bx}}dx}{ab}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{75}$$

$$\frac{\sqrt{a+bx}(ex)^m x^{\frac{1}{2}(n-2m)}\left(\frac{2x^{m-\frac{3n}{2}+1}(bc-ad)}{ab\sqrt{a+bx}}-\frac{2\sqrt{a+bx}x^{m-\frac{3n}{2}}\left(-\frac{bx}{a}\right)^{\frac{3n}{2}-m}(bc(2m-3n+1)-ad(2m-3n+2))\,\text{Hypergeometric2F1}\left(\frac{1}{2}\right.}{ab^2}\right.}{\sqrt{ax^n+bx^{n+1}}}$$

input
```
Int[((e*x)^m*(c + d*x))/(a*x^n + b*x^(1 + n))^(3/2),x]
```

output
```
(x^((-2*m + n)/2)*(e*x)^m*Sqrt[a + b*x]*((2*(b*c - a*d)*x^(1 + m - (3*n)/2
))/(a*b*Sqrt[a + b*x]) - (2*(b*c*(1 + 2*m - 3*n) - a*d*(2 + 2*m - 3*n))*x^
(m - (3*n)/2)*(-((b*x)/a))^(-m + (3*n)/2)*Sqrt[a + b*x]*Hypergeometric2F1[
1/2, -m + (3*n)/2, 3/2, 1 + (b*x)/a])/(a*b^2)))/Sqrt[a*x^n + b*x^(1 + n)]
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] && !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{(ex)^m (dx + c)}{(a\,x^n + b\,x^{1+n})^{\frac{3}{2}}}dx$$

input
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(ex)^m\,(c+dx)}{(ax^n+bx^{n+1})^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)**m*(d*x+c)/(a*x**n+b*x**(1+n))**(3/2),x)
```

output
```
Integral((e*x)**m*(c + d*x)/(a*x**n + b*x**(n + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^{n+1}+ax^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^(n + 1) + a*x^n)^(3/2), x)
```

Giac [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^{n+1}+ax^n)^{\frac{3}{2}}}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^(n + 1) + a*x^n)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx = \int \frac{(e\,x)^m\,(c+d\,x)}{(a\,x^n+b\,x^{n+1})^{3/2}}\,dx$$

input
```
int(((e*x)^m*(c + d*x))/(a*x^n + b*x^(n + 1))^(3/2),x)
```

output
```
int(((e*x)^m*(c + d*x))/(a*x^n + b*x^(n + 1))^(3/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^{3/2}}\,dx = e^m\left(\left(\int \frac{x^m}{x^{\frac{3n}{2}}\sqrt{bx+a}\,a+x^{\frac{3n}{2}}\sqrt{bx+a}\,bx}dx\right)c + \left(\int \frac{x^m x}{x^{\frac{3n}{2}}\sqrt{bx+a}\,a+x^{\frac{3n}{2}}\sqrt{bx+a}\,bx}dx\right)d\right)$$

input
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(3/2),x)
```

output
```
e**m*(int(x**m/(x**((3*n)/2)*sqrt(a + b*x)*a + x**((3*n)/2)*sqrt(a + b*x)*
b*x),x)*c + int((x**m*x)/(x**((3*n)/2)*sqrt(a + b*x)*a + x**((3*n)/2)*sqrt
(a + b*x)*b*x),x)*d)
```

3.375 $\int \frac{(ex)^m(c+dx)}{\left(ax^n+bx^{1+n}\right)^{5/2}} \, dx$

Optimal result

Integrand size = 28, antiderivative size = 146

$$\int \frac{(ex)^m(c+dx)}{\left(ax^n+bx^{1+n}\right)^{5/2}} \, dx = -\frac{2dx^{1-n}(ex)^m}{b(1-2m+5n)\left(ax^n+bx^{1+n}\right)^{3/2}} - \frac{2\left(bc+\frac{ad(2+2m-5n)}{1-2m+5n}\right)x^{-n}\left(-\frac{bx}{a}\right)^{-m+\frac{5n}{2}}(ex)^m \text{Hypergeometric2F1}\left(-\frac{3}{2},-m+\frac{5n}{2},-\frac{1}{2},1+\frac{bx}{a}\right)}{3b^2\left(ax^n+bx^{1+n}\right)^{3/2}}$$

output
```
-2*d*x^(1-n)*(e*x)^m/b/(1-2*m+5*n)/(a*x^n+b*x^(1+n))^(3/2)-2/3*(b*c+a*d*(2
+2*m-5*n)/(1-2*m+5*n))*(-b*x/a)^(-m+5/2*n)*(e*x)^m*hypergeom([-3/2, -m+5/2
*n],[-1/2],1+b*x/a)/b^2/(x^n)/(a*x^n+b*x^(1+n))^(3/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.79

$$\int \frac{(ex)^m(c+dx)}{\left(ax^n+bx^{1+n}\right)^{5/2}} \, dx = \frac{2(ex)^m(a+bx)\left(b(bc-ad)x+(bc(-1+2m-5n)+ad(-2-2m+5n))\left(-\frac{bx}{a}\right)\right.}{3ab^2\left(x^n(a+bx)\right)^{5}}$$

input
```
Integrate[((e*x)^m*(c + d*x))/(a*x^n + b*x^(1 + n))^(5/2),x]
```

output
```
(2*(e*x)^m*(a + b*x)*(b*(b*c - a*d)*x + (b*c*(-1 + 2*m - 5*n) + a*d*(-2 -
2*m + 5*n))*(-((b*x)/a))^(-m + (5*n)/2)*(a + b*x)*Hypergeometric2F1[-1/2,
-m + (5*n)/2, 1/2, 1 + (b*x)/a]))/(3*a*b^2*(x^n*(a + b*x))^(5/2))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.18, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1948, 87, 77, 75}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)(ex)^m}{(ax^n+bx^{n+1})^{5/2}}dx$$

$$\Big\downarrow \text{ 1948}$$

$$\frac{\sqrt{a+bx}(ex)^m x^{\frac{1}{2}(n-2m)}\int \frac{x^{m-\frac{5n}{2}}(c+dx)}{(a+bx)^{5/2}}dx}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{ 87}$$

$$\frac{\sqrt{a+bx}(ex)^m x^{\frac{1}{2}(n-2m)}\left(\frac{(ad(2m-5n+2)+bc(-2m+5n+1))\int \frac{x^{m-\frac{5n}{2}}}{(a+bx)^{3/2}}dx}{3ab}+\frac{2x^{m-\frac{5n}{2}+1}(bc-ad)}{3ab(a+bx)^{3/2}}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{ 77}$$

$$\frac{\sqrt{a+bx}(ex)^m x^{\frac{1}{2}(n-2m)}\left(\frac{x^{m-\frac{5n}{2}}\left(-\frac{bx}{a}\right)^{\frac{5n}{2}-m}(ad(2m-5n+2)+bc(-2m+5n+1))\int \frac{\left(-\frac{bx}{a}\right)^{m-\frac{5n}{2}}}{(a+bx)^{3/2}}dx}{3ab}+\frac{2x^{m-\frac{5n}{2}+1}(bc-ad)}{3ab(a+bx)^{3/2}}\right)}{\sqrt{ax^n+bx^{n+1}}}$$

$$\Big\downarrow \text{ 75}$$

$$\frac{\sqrt{a+bx}(ex)^m x^{\frac{1}{2}(n-2m)}\left(\frac{2x^{m-\frac{5n}{2}+1}(bc-ad)}{3ab(a+bx)^{3/2}}-\frac{2x^{m-\frac{5n}{2}}\left(-\frac{bx}{a}\right)^{\frac{5n}{2}-m}(ad(2m-5n+2)+bc(-2m+5n+1))\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{5n}{2}\right.}{3ab^2\sqrt{a+bx}}\right.}{\sqrt{ax^n+bx^{n+1}}}$$

input
```
Int[((e*x)^m*(c + d*x))/(a*x^n + b*x^(1 + n))^(5/2),x]
```

output
```
(x^((-2*m + n)/2)*(e*x)^m*Sqrt[a + b*x]*((2*(b*c - a*d)*x^(1 + m - (5*n)/2
))/(3*a*b*(a + b*x)^(3/2)) - (2*(a*d*(2 + 2*m - 5*n) + b*c*(1 - 2*m + 5*n)
)*x^(m - (5*n)/2)*(-((b*x)/a))^(-m + (5*n)/2)*Hypergeometric2F1[-1/2, -m +
 (5*n)/2, 1/2, 1 + (b*x)/a])/(3*a*b^2*Sqrt[a + b*x])))/Sqrt[a*x^n + b*x^(1
 + n)]
```

Defintions of rubi rules used

rule 75
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

rule 77
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((-b)*(c/
d))^IntPart[m]*((b*x)^FracPart[m]/((-d)*(x/c))^FracPart[m])   Int[((-d)*(x/
c))^m*(c + d*x)^n, x], x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[m] &&
 !IntegerQ[n] && !GtQ[c, 0] && !GtQ[-d/(b*c), 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int \frac{(ex)^m (dx+c)}{(a\,x^n+b\,x^{1+n})^{\frac{5}{2}}}dx$$

input
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(5/2),x)
```

output
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(5/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^{5/2}}\,dx = \int \frac{(ex)^m\,(c+dx)}{(ax^n+bx^{n+1})^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)**m*(d*x+c)/(a*x**n+b*x**(1+n))**(5/2),x)
```

output
```
Integral((e*x)**m*(c + d*x)/(a*x**n + b*x**(n + 1))**(5/2), x)
```

Maxima [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^{5/2}}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^{n+1}+ax^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^(n + 1) + a*x^n)^(5/2), x)
```

Giac [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^{5/2}}\,dx = \int \frac{(dx+c)(ex)^m}{(bx^{n+1}+ax^n)^{\frac{5}{2}}}\,dx$$

input
```
integrate((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(e*x)^m/(b*x^(n + 1) + a*x^n)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^{5/2}}\,dx = \int \frac{(e\,x)^m\,(c+d\,x)}{(a\,x^n+b\,x^{n+1})^{5/2}}\,dx$$

input
```
int(((e*x)^m*(c + d*x))/(a*x^n + b*x^(n + 1))^(5/2),x)
```

output
```
int(((e*x)^m*(c + d*x))/(a*x^n + b*x^(n + 1))^(5/2), x)
```

Reduce [F]

$$\int \frac{(ex)^m(c+dx)}{(ax^n+bx^{1+n})^{5/2}}\,dx = e^m\left(\left(\int \frac{x^m}{x^{\frac{5n}{2}}\sqrt{bx+a}\,a^2+2x^{\frac{5n}{2}}\sqrt{bx+a}\,abx+x^{\frac{5n}{2}}\sqrt{bx+a}\,b^2x^2}dx\right)c + \left(\int \frac{x^m x}{x^{\frac{5n}{2}}\sqrt{bx+a}\,a^2+2x^{\frac{5n}{2}}\sqrt{bx+a}\,abx+x^{\frac{5n}{2}}\sqrt{bx+a}\,b^2x^2}dx\right)d\right)$$

input
```
int((e*x)^m*(d*x+c)/(a*x^n+b*x^(1+n))^(5/2),x)
```

output
```
e**m*(int(x**m/(x**((5*n)/2)*sqrt(a + b*x)*a**2 + 2*x**((5*n)/2)*sqrt(a +
b*x)*a*b*x + x**((5*n)/2)*sqrt(a + b*x)*b**2*x**2),x)*c + int((x**m*x)/(x*
*((5*n)/2)*sqrt(a + b*x)*a**2 + 2*x**((5*n)/2)*sqrt(a + b*x)*a*b*x + x**((
5*n)/2)*sqrt(a + b*x)*b**2*x**2),x)*d)
```

3.376 $\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^p\,dx$

Optimal result

Integrand size = 26, antiderivative size = 126

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^p\,dx=\frac{dx^{1-n}(ex)^m\left(ax^n+bx^{1+n}\right)^{1+p}}{b(2+m+p+np)}-\left(\frac{d}{b(2+m+p+np)}-\frac{c}{a+am+anp}\right)x^{1-n}(ex)^m\left(ax^n+bx^{1+n}\right)^{1+p}\text{Hypergeometric2F1}\left(1,2+m+p+np,2+m+np,-\frac{bx}{a}\right)$$

output
```
d*x^(1-n)*(e*x)^m*(a*x^n+b*x^(1+n))^(p+1)/b/(n*p+m+p+2)-(d/b/(n*p+m+p+2)-c
/(a*n*p+a*m+a))*x^(1-n)*(e*x)^m*(a*x^n+b*x^(1+n))^(p+1)*hypergeom([1, n*p+
m+p+2],[n*p+m+2],-b*x/a)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.82

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^p\,dx$$
$$=\frac{x(ex)^m\left(x^n(a+bx)\right)^p\left(d(a+bx)+\frac{(-ad(1+m+np)+bc(2+m+p+np))\left(1+\frac{bx}{a}\right)^{-p}\text{Hypergeometric2F1}\left(-p,1+m+np,2+m+np,\right.}{1+m+np}\right.}{b(2+m+p+np)}$$

input
```
Integrate[(e*x)^m*(c + d*x)*(a*x^n + b*x^(1 + n))^p,x]
```

output
```
(x*(e*x)^m*(x^n*(a + b*x))^p*(d*(a + b*x) + ((-(a*d*(1 + m + n*p)) + b*c*(
2 + m + p + n*p))*Hypergeometric2F1[-p, 1 + m + n*p, 2 + m + n*p, -((b*x)/
a)])/((1 + m + n*p)*(1 + (b*x)/a)^p)))/(b*(2 + m + p + n*p))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {1948, 90, 76, 74}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(ex)^m\left(ax^n+bx^{n+1}\right)^p\,dx$$

$$\Big\downarrow \text{1948}$$

$$(ex)^m(a+bx)^{-p}x^{-m-np}\left(ax^n+bx^{n+1}\right)^p\int x^{m+np}(a+bx)^p(c+dx)dx$$

$$\Big\downarrow \text{90}$$

$$(ex)^m(a+bx)^{-p}x^{-m-np}\left(ax^n+bx^{n+1}\right)^p\left(\left(c-\frac{ad(m+np+1)}{b(m+np+p+2)}\right)\int x^{m+np}(a+bx)^pdx+\frac{d(a+bx)^{p+1}x^{m+np+1}}{b(m+np+p+2)}\right)$$

$$\Big\downarrow \text{76}$$

$$(ex)^m(a+bx)^{-p}x^{-m-np}\left(ax^n+bx^{n+1}\right)^p\left((a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}\left(c-\frac{ad(m+np+1)}{b(m+np+p+2)}\right)\int x^{m+np}\left(\frac{bx}{a}+1\right)^p dx+\frac{d(a}{b(}\right.$$

$\big\downarrow$ 74

$$(ex)^m(a+bx)^{-p}x^{-m-np}\left(ax^n+bx^{n+1}\right)^p\left(\frac{(a+bx)^p\left(\frac{bx}{a}+1\right)^{-p}x^{m+np+1}\left(c-\frac{ad(m+np+1)}{b(m+np+p+2)}\right)\text{Hypergeometric2F1}\left(-p,m+\right.}{m+np+1}\right.$$

input
```
Int[(e*x)^m*(c + d*x)*(a*x^n + b*x^(1 + n))^p,x]
```

output
```
(x^(-m - n*p)*(e*x)^m*(a*x^n + b*x^(1 + n))^p*((d*x^(1 + m + n*p)*(a + b*x
)^(1 + p))/(b*(2 + m + p + n*p)) + ((c - (a*d*(1 + m + n*p))/(b*(2 + m + p
 + n*p)))*x^(1 + m + n*p)*(a + b*x)^p*Hypergeometric2F1[-p, 1 + m + n*p, 2
 + m + n*p, -((b*x)/a)])/((1 + m + n*p)*(1 + (b*x)/a)^p)))/(a + b*x)^p
```

Defintions of rubi rules used

rule 74
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^n*((b*x
)^(m + 1)/(b*(m + 1)))*Hypergeometric2F1[-n, m + 1, m + 2, (-d)*(x/c)], x]
/; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] && (IntegerQ[n] || (GtQ[c, 0]
 &&  !(EqQ[n, -2^(-1)] && EqQ[c^2 - d^2, 0] && GtQ[-d/(b*c), 0])))
```

rule 76
```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[c^IntPart
[n]*((c + d*x)^FracPart[n]/(1 + d*(x/c))^FracPart[n])   Int[(b*x)^m*(1 + d*
(x/c))^n, x], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[m] &&  !Integer
Q[n] &&  !GtQ[c, 0] &&  !GtQ[-d/(b*c), 0] && ((RationalQ[m] &&  !(EqQ[n, -2
^(-1)] && EqQ[c^2 - d^2, 0])) ||  !RationalQ[n])
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 1948
```
Int[((e_.)*(x_))^(m_.)*((a_.)*(x_)^(j_.) + (b_.)*(x_)^(jn_.))^(p_)*((c_) +
(d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Simp[e^IntPart[m]*(e*x)^FracPart[m]*(
(a*x^j + b*x^(j + n))^FracPart[p]/(x^(FracPart[m] + j*FracPart[p])*(a + b*x
^n)^FracPart[p]))   Int[x^(m + j*p)*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, j, m, n, p, q}, x] && EqQ[jn, j + n] &&  !IntegerQ[p]
 && NeQ[b*c - a*d, 0] &&  !(EqQ[n, 1] && EqQ[j, 1])
```

Maple [F]

$$\int (ex)^m (dx+c) \left(a\,x^n+b\,x^{1+n}\right)^p dx$$

input
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^p,x)
```

output
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^p,x)
```

Fricas [F]

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^p\,dx=\int (dx+c)\left(bx^{n+1}+ax^n\right)^p(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^p,x, algorithm="fricas")
```

output
```
integral((d*x + c)*(b*x^(n + 1) + a*x^n)^p*(e*x)^m, x)
```

Sympy [F(-1)]

Timed out.

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^p\,dx=\text{Timed out}$$

input
```
integrate((e*x)**m*(d*x+c)*(a*x**n+b*x**(1+n))**p,x)
```

output
```
Timed out
```

Maxima [F]

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^p\,dx=\int (dx+c)\left(bx^{n+1}+ax^n\right)^p(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^p,x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(b*x^(n + 1) + a*x^n)^p*(e*x)^m, x)
```

Giac [F]

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^p\,dx=\int (dx+c)\left(bx^{n+1}+ax^n\right)^p(ex)^m\,dx$$

input
```
integrate((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^p,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*x^(n + 1) + a*x^n)^p*(e*x)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^p\,dx=\int (e\,x)^m\,\left(a\,x^n+b\,x^{n+1}\right)^p\,(c+d\,x)\,dx$$

input
```
int((e*x)^m*(a*x^n + b*x^(n + 1))^p*(c + d*x),x)
```

output
```
int((e*x)^m*(a*x^n + b*x^(n + 1))^p*(c + d*x), x)
```

Reduce [F]

$$\int (ex)^m(c+dx)\left(ax^n+bx^{1+n}\right)^p\,dx = \text{too large to display}$$

input
```
int((e*x)^m*(d*x+c)*(a*x^n+b*x^(1+n))^p,x)
```

output
```
(e**m*( - x**m*(x**n*a + x**n*b*x)**p*a**2*d*m*p - x**m*(x**n*a + x**n*b*x
)**p*a**2*d*n*p**2 - x**m*(x**n*a + x**n*b*x)**p*a**2*d*p + x**m*(x**n*a +
 x**n*b*x)**p*a*b*c*m*p + x**m*(x**n*a + x**n*b*x)**p*a*b*c*n*p**2 + x**m*
(x**n*a + x**n*b*x)**p*a*b*c*p**2 + 2*x**m*(x**n*a + x**n*b*x)**p*a*b*c*p
+ x**m*(x**n*a + x**n*b*x)**p*a*b*d*m*p*x + x**m*(x**n*a + x**n*b*x)**p*a*
b*d*n*p**2*x + x**m*(x**n*a + x**n*b*x)**p*a*b*d*p**2*x + x**m*(x**n*a + x
**n*b*x)**p*b**2*c*m**2*x + 2*x**m*(x**n*a + x**n*b*x)**p*b**2*c*m*n*p*x +
 2*x**m*(x**n*a + x**n*b*x)**p*b**2*c*m*p*x + 2*x**m*(x**n*a + x**n*b*x)**
p*b**2*c*m*x + x**m*(x**n*a + x**n*b*x)**p*b**2*c*n**2*p**2*x + 2*x**m*(x*
*n*a + x**n*b*x)**p*b**2*c*n*p**2*x + 2*x**m*(x**n*a + x**n*b*x)**p*b**2*c
*n*p*x + x**m*(x**n*a + x**n*b*x)**p*b**2*c*p**2*x + 2*x**m*(x**n*a + x**n
*b*x)**p*b**2*c*p*x + x**m*(x**n*a + x**n*b*x)**p*b**2*d*m**2*x**2 + 2*x**
m*(x**n*a + x**n*b*x)**p*b**2*d*m*n*p*x**2 + 2*x**m*(x**n*a + x**n*b*x)**p
*b**2*d*m*p*x**2 + x**m*(x**n*a + x**n*b*x)**p*b**2*d*m*x**2 + x**m*(x**n*
a + x**n*b*x)**p*b**2*d*n**2*p**2*x**2 + 2*x**m*(x**n*a + x**n*b*x)**p*b**
2*d*n*p**2*x**2 + x**m*(x**n*a + x**n*b*x)**p*b**2*d*n*p*x**2 + x**m*(x**n
*a + x**n*b*x)**p*b**2*d*p**2*x**2 + x**m*(x**n*a + x**n*b*x)**p*b**2*d*p*
x**2 + int((x**m*(x**n*a + x**n*b*x)**p)/(a*m**3*x + 3*a*m**2*n*p*x + 3*a*
m**2*p*x + 3*a*m**2*x + 3*a*m*n**2*p**2*x + 6*a*m*n*p**2*x + 6*a*m*n*p*x +
 3*a*m*p**2*x + 6*a*m*p*x + 2*a*m*x + a*n**3*p**3*x + 3*a*n**2*p**3*x +...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```python
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
      return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Exp
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file